

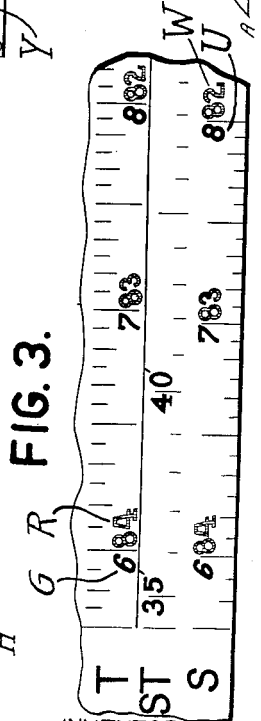
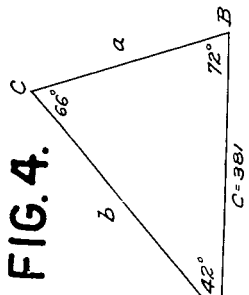
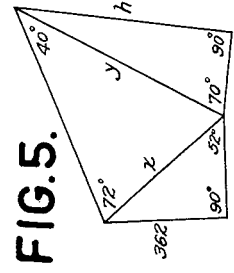
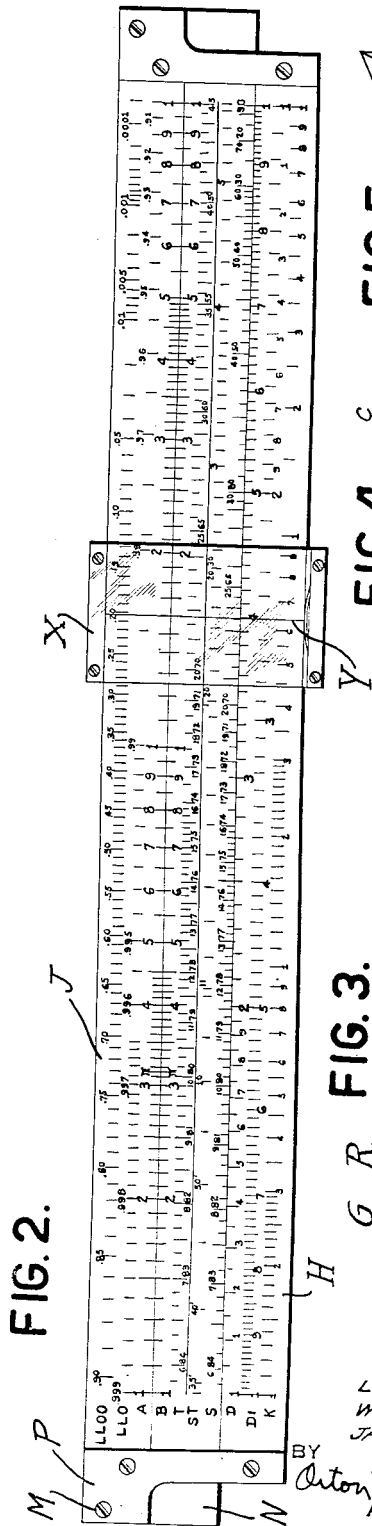
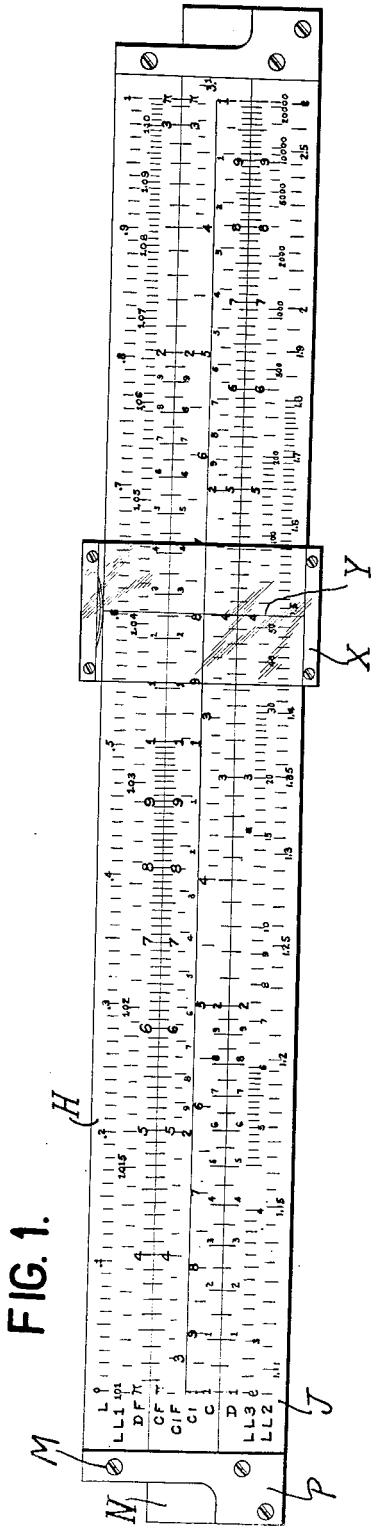
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SLIDE RULE

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# UNITED STATES PATENT OFFICE

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## SLIDE RULE

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This invention relates to slide rules of the kind in which indices in the form of scales adapted to coact with one another in accordance with well known laws are arranged on the respective relatively movable component elements.

Three types of slide rules are in general use. In one type, relatively movable elements reciprocate with respect to one another and on the respective elements are parallel coacting scales. Such slide rules comprise a body member composed of spaced parallel side bars between which reciprocates, as by means of a tongue and groove connection, a slide, the scales appearing on one or both surfaces of the rule. Another type of slide rule involves concentric rotatable discs of progressively varying diameter around the peripheries of which are arranged the coacting scales. So-called cylindrical slide rules consist, usually, in an axially rotatable cylinder on the surface of which are scales extending in the longitudinal direction of the rule while a rotary and usually axially movable slide concentric therewith carries parallel coacting scales. In all these rules the indices or scales are arranged in logarithmic proportions, the arrangement being based upon the principle that the sum of the logarithms of numbers is equal to the logarithm of the product of those numbers. To make computations in multiplication, for instance, it has been the practice to bring the index of one logarithmic scale to the scale divisions representing the logarithm of one of the numbers to be multiplied on a coacting scale and then read the product at the scale division representing the log of that product opposite the scale division on the moved scale representing that number by which it is multiplied. Division is performed as a reverse of this manipulation.

Simple computations have been thus readily effected by movement of the slide with respect to the body and, as the art has progressed, additional coacting scales have been added to slide rules enabling the solution of problems in square and cube root, in trigonometry and in the figuring of fractional powers and roots. A slide rule has been devised by which the right triangle has been solved by a single setting of the rule but in the solution of other problems a sequence of steps has been necessary, each step necessitating the proper relation of slide and body and the making of a notation of the results found after each step in order that the parts may be again manipulated in the performance of a further step using the result found in the previous computations. Each individual problem further required a dif-

ferent method of manipulating a rule so that great difficulty was experienced by all users in determining the proper manipulation to be used in solving problems with which they had not had recent and extensive practice. No slide rule heretofore proposed has ever overcome this difficulty.

The present invention seeks a slide rule, irrespective of type, by which, with the application of not more than two easily understood and readily memorized principles, the user is able to devise the best settings for any particular purpose and to recall settings which have been forgotten.

It is also an object of the invention to provide a slide rule in which problems involving numerical and/or trigonometric terms may be solved, irrespective of the number of steps therein, in a continuous manipulation, that is, without the necessity of having to set down a result in order to retain the same while a new setting is made.

A further object of the invention is a slide rule whereby problems involving exponential quantities may be similarly solved without the necessity of resetting the relatively movable parts to a result found in a previous manipulation.

The invention also resides in the selection and disposition of scales on the respective relatively moving parts by which the foregoing objects of the invention are realized.

These and other objects of the invention and the means for their attainment will be more apparent from the following detailed description, taken in connection with the accompanying drawing illustrating one embodiment by which the invention may be realized and in which:

Figure 1 is a plan view showing one face of the slide rule;

Figure 2 is a similar view showing the reverse face;

Figure 3 is a fragmentary view on an enlarged scale showing the face of the slide of Figure 2;

Figure 4 shows a triangle illustrative of the solution of various problems by the rule of this invention; and

Figure 5 shows a closed figure, problems in respect of which may be similarly solved.

In the drawing, side bars H and J are rigidly secured together by means of plates P which are secured thereto at M, so that a sliding bar N may be mounted between said bars H and J so as to be readily slidable longitudinally thereof. The slidable bar N has the usual tongue members on each edge adapted to slide in the usual groove members on the edges of the bars H and J contiguous to the sliding bar N so that the sliding bar N is

always held in engagement between the bars H and J in whatever position it occupies longitudinally thereof. A runner or indicator X, transparent on both faces and of any usual construction, is mounted over said bars H, J and N so as to be readily moved into any position desired between the plates P, and the runner X has a hairline Y on each side thereof.

On the front side of the rule, as shown in Figure 1, the upper scale on the bar H is designated as L and is a scale of equal parts from 0 to 1.0 and is used to obtain common logarithms when referred, as will be understood, to the D scale on bar J, referred to hereinafter. The scale next below the L scale on this face is designated as LL1 and has graduations representing logarithms of the logarithms of the numbers 1.01 to 1.11. The scale below the LL1 scale is designed as DF, and is a standard logarithmic scale of full unit length the same as the D scale which is yet to be described, except that it is folded and has its index at the center. This scale is proximate the inner edge of the upper side bar.

The front face, as viewed, of the slide is provided along its upper edge with a scale designated as CF and is identical with the DF scale, on the upper side bar H.

Also on the front face of the slide below the CF scale is a scale designated as CIF, which is a standard folded reciprocal logarithmic scale of full unit length graduated from 10 to 1 similar in every respect to the D scale soon to be described except that it is folded and inverted. Immediately below the CIF is the scale designated as CI which is a standard reciprocal logarithmic scale of full unit length graduated from 10 to 1. Below the CI scale and proximate the lower edge of the slide N is a fourth scale designated as C and has standard graduated logarithmic divisions of full unit length from 1 to 10. This scale is the same as the D scale on the body (bar J), next to be described.

On the upper edge of the side bar J proximate the slide is a scale designated as D, which is the same as the C scale and is a standard logarithmic scale of full unit length. Along the lower edge of the side bar J is a scale designated LL2 which has graduations representing the logarithms of logarithms of the numbers from 1.11 to 2.7. The middle scale on the front face of the lower side bar is a scale, designated as LL3, which has graduations representing the logarithms of logarithms of the numbers from 2.7 to 22,000. These three scales LL1, LL2 and LL3 are, conveniently, continuations of each other, as will be clear from the foregoing.

On the rear face of the slide rule, as shown in Figure 2, is a scale designated LL00 along the upper edge of the upper (as viewed) side bar J and immediately below that scale LL00 is a scale designated as LL0 both these scales together representing the logarithms of co-logarithms of the numbers between 0.00005 and 0.999. These scales LL0 and LL00 are conveniently continuations of each other. These scales are used to find powers and roots of numbers below unity and to find co-logarithms to any base of numbers below unity. The lower scale on the upper side bar J, on the rear face, is designated A and comprises a standard graduated logarithmic scale of two unit lengths from 1 to 10 and 10 to 100. The A scale represents the natural co-logarithms of the numbers on the LL0 and LL00 scales.

On the slide N, on this rear face of the rule, and along the upper edge thereof, is a scale designated

as B which has the same graduations as the scale A. Immediately below the B scale on the slide is a scale designated as T which is a tangent scale with divisions to represent angles from  $5^{\circ}45'$  to  $45^{\circ}$ . This range of angles will give tangents in connection with the C or D scale and cotangents in connection with the CI or DI scale. As shown in Figure 3, the numerals G representing the angles below  $45^{\circ}$  are indicated in one color, say, black, and slanted in the direction in which the scale is read, while the numerals R indicating the continuation of this scale ( $45^{\circ}$  to  $84^{\circ}15'$ ) in the opposite direction are in a different color, say, red, and slant in the opposite direction, i.e., the direction in which the angle increases and in which this scale is read. The angles noted in red will give tangents in connection with the CI or DI scales and cotangents in connection with the C or D scales.

Immediately below the tangent scale T is a scale designated as ST. This scale is used whenever an angle less than  $5^{\circ}44'$ , that is, an angle whose sine or tangent is less than 0.1, is involved in the solution of a trigonometrical problem. The lowermost scale on the slide N, on the rear face thereof, is the scale designated as S and is a scale graduated to the angles in degrees and minutes from  $5^{\circ}40'$  to  $90^{\circ}$  and is used with reference to the scales C or D from which scales are then read the sines of the angles indicated on the S scale. This scale obviously also enables a reading for the cosine to be obtained on the C or D scales when reading, however, in the opposite direction. The numerals U used in obtaining the sine are, therefore, in one color, say, black, and slant in the direction in which the angle increases, while the numerals W used in obtaining the cosine are in a different and contrasting color, say, red, and slant in an opposite direction, i.e., in the direction in which the angle increases and in which this scale is read.

On the lower side bar on the rear face (here the bar H), and along the edge proximate the slide N, is another scale designated as D which is exactly the same as the D scale on the front face hereinbefore described. Below the D scale is a scale designated as DI and is a standard reciprocal logarithmic scale of full unit length graduated from 10 to 1 the same as the D scale except that it is inverted. The lowermost scale on this face is a scale designated as K. This scale is graduated to show the cube of a number in the same transverse line on the D scale and comprises the standard graduated logarithmic scale of three unit lengths. Conversely, of course, the D scale shows the cube root of the corresponding graduation on the K scale. It will be noted that there are three parts to the K scale, each the same as the others except as to position. These parts will be referred to hereinafter as the left hand part, the middle part and the right hand part, or as K left, K middle and K right, respectively. The cube root of a given number is a second number whose cube is the given number.

In order to understand the simplicity of operation of this slide rule, as explained hereinbefore, whereby the operation thereof, irrespective of the number of steps, may be carried out continuously without resetting the parts to a result found in the previous step, the principle of operation for such simple problems as were possible with prior slide rules will first be explained, leading into more complicated problems which are only possible with the slide rule of this invention, in order to explain that the same simple procedure may

be applied to all problems whereby the user, having mastered the fundamental rules, is now able with this rule to perform all problems as they arise.

In the following illustration for the sake of brevity, the various scales are referred to merely by their letters such as C, D, etc., instead of "C" scale, "D" scale and the like:

*Example No. 1.—Evaluate*

$$\frac{73.6 \times 3.44}{.5}$$

Solution: The user reasons as follows: first divide 73.6 by .5 and then multiply the result by 3.44. This would suggest

Push hair line to 73.6 on D,  
Draw .5 on C under the hairline,  
Push hairline to 3.44 on C and  
Read 506 on D scale.

*Example No. 2.—Evaluate*

$$\frac{18 \times 45 \times 37}{23 \times 29}$$

Solution: Reason as follows: (a) divide 18 by 23, (b) multiply the result by 45, (c) divide this second result by 29, (d) multiply this third result by 37. This suggests to

Push hair line to 18 on D,  
Draw 23 on C to hairline,  
Push hairline to 45 on C,  
Draw 29 on C to hairline,  
Push hairline to 37 on C and  
Read 449 on D scale.

To determine the decimal point in the answer, a rough approximation is made. For the above example write

$$\frac{20 \times 40 \times 40}{20 \times 30} = \text{about } 50$$

Hence the answer is 44.9.

*Example 3.—Evaluate*

$$\frac{5 \times 31}{44 \times 18.4}$$

Solution: For solving a problem of combined multiplication and division on a slide rule it should be observed that the numerator always has to contain one more factor than the denominator. The above expression contains the same number of factors in the numerator as in the denominator. To fulfill the condition stated, there is added the factor 1 (one). This does not change the value of the expression which now reads:

$$\frac{5 \times 31 \times 1}{44 \times 18.4}$$

This suggests:

To 5 on D scale  
Draw 44 on C scale,  
Push hairline to 31 on C scale,  
Draw 18.4 on C to hairline,  
Push hairline to 1 on C scale and  
Read .1915 on D scale.

*Example 4.—Evaluate*

$$\frac{47.4}{3.88 \times 1.558}$$

Solution: This example has one more factor in the denominator than in the numerator. To ful-

fill the condition outlined in the solution of the previous example take one factor out of the denominator and replace it with its reciprocal in the numerator, thus:

$$\frac{47.4 \times \frac{1}{3.88}}{1.558}$$

To 47.4 on D scale  
Draw 1.558 on C scale,  
Push hairline to 3.88 on CI and  
Read 7.85 on D scale.

*Example 5.—Evaluate:*

$$1.843 \times 92 \times 2.45 \times 0.584 \times 365$$

Solution: This example has no denominator. In order to solve it on the slide rule in the shortest possible way, it will have to be rewritten with a denominator containing one less factor than the number of factors contained in the numerator. Thus:

$$\frac{1.843 \times 2.45 \times 365}{\frac{1}{92} \times \frac{1}{0.584}}$$

To 1.843 on D scale  
Draw 92 on CI scale,  
Push hairline to 2.45 on C scale,  
Draw .584 on CI scale to hairline,  
Push hairline to 365 on C scale and  
Read 88,600 on D scale.

A little reflection on the procedure of the preceding examples will enable the operator to evaluate by the shortest method expressions similar to those just considered. It should be observed that:

The D scale (scale located on the body) was used only twice, once at the beginning of the process and once at its end: The process for each number of the denominator consisted in drawing that number located on the C scale (or CI scale), under the hairline; the process for each number of the numerator consisted in pushing the hairline to that number located on the C scale (or CI scale).

The following group of examples are given to illustrate that the simplicity of the evaluation of expressions is not affected if some or all of the factors of an expression are squares or cubes, square roots or cube roots.

*Example 6.—Evaluate*

$$\frac{\sqrt{12} \times 1.14}{4.5}$$

Solution: To 12 on A scale  
Draw 4.5 on C scale,  
Push hairline to 1.14 on CF scale and  
Read .877 on DF scale.

*Example 7.—Evaluate*

$$\frac{170^3 \sqrt{.25}}{34.5}$$

Solution: To .25 on K scale (right)  
Draw 34.5 on C scale,  
Push hairline to 170 on C scale and  
Read 3.1 on D scale.

Example 8.—Evaluate

$$\frac{(.15)^2 .52}{12.4}$$

5 Solution: To .15 on D scale  
Draw 12.4 on B scale,  
Push hairline to .52 on B scale and  
Read .000943 on A scale.

10 Example 9.—Evaluate

$$5.05^3 \sqrt{\left(\frac{4.23}{2.82}\right)^3}$$

Solution: For solution this is rewritten

$$15 \quad \frac{(5.05 \sqrt[3]{4.23})^3}{\sqrt{2.82}}$$

To 4.23 on A scale  
Draw 2.82 on B scale  
20 Push hairline to 5.05 on C scale and  
Read 237 on K scale.

Example 10.—Evaluate

$$25 \quad \frac{69.3^3 \sqrt[3]{31.6} \sqrt{0.0317} \times 68.7}{\sqrt{87.6}}$$

$$= \frac{69.3^3 \sqrt[3]{31.6} \sqrt{0.0317}}{\sqrt{87.6} \frac{1}{68.7}}$$

30 Solution: To 31.6 on K scale  
Draw 87.6 on B scale,  
Push hairline to 69.3 on C scale,  
Draw 68.7 on CI scale under hairline,  
35 Push hairline to 0.0317 on B scale and  
Read 286 on D scale.

There are other slide rules by means of which the foregoing examples can be solved as readily as on the 'slide' rule of this invention. Their presentation here is only for the purpose of showing that the same principle and the same procedure used in solving these relatively simple problems are also applicable for solving problems involving trigonometric functions on this slide rule. This is possible due to the fact that all trigonometric scales are of a common full CD unit length and that they are rendered more flexible by being placed on the slide.

The following group of examples exemplify this:

Example 11.—Evaluate

$$\frac{73.6 \times 3.44}{\sin 30^\circ}$$

55 Solution: To 73.6 on D Scale,  
Draw 30° on S scale,  
Push hairline to 3.44 on C scale and  
Read 506 on D scale.

60 By comparing this solution with that of Example 1, it will be seen that the two are identical.

Example 12.—Evaluate

$$65 \quad \frac{53.5}{2.25 \sin 13^\circ 30'}$$

Solution: (Compare with Example 4)

$$70 \quad \frac{53.5}{2.25 \sin 13^\circ 30'} = \frac{53.5 \frac{1}{2.25}}{\sin 13^\circ 30'}$$

To 53.5 on D scale,  
Draw 13°30' on S scale,  
Push hairline to 2.25 on CIF scale and  
75 Read 101.9 on DF scale.

Example 13.—Evaluate

$$\frac{17 \cos 50^\circ 30'}{\tan 12^\circ \sin 59'}$$

Solution: (Compare with Example 2)

$$\frac{17 \cos 50^\circ 30'}{\tan 12^\circ \sin 59'} = \frac{17 \cos 50^\circ 30' \times 1}{\tan 12^\circ \sin 59'}$$

To 17 on D scale,  
Draw 12° on T scale,  
Push hairline to 50°30' on S (red),  
10 Draw 59' on ST scale to hairline,  
Push hairline to 1 (index) of C scale and  
Read 2960 on D scale.

Example 14.—Evaluate

$$\frac{\sqrt{2.9} \cos 16^\circ}{\tan 16^\circ}$$

Solution: (Compare with Example 6)

To 2.9 on A scale,  
20 Draw 16° on T scale,  
Push hairline to 16° on S (red) and  
Read 5.71 on D scale.

Example 15.—Evaluate

$$\frac{\tan 15^\circ}{\cos 20^\circ \sin 25^\circ}$$

Solution: This expression has one more factor in the denominator than in the numerator. To fulfill the statement given in Example 3, it would be necessary to transpose one factor from the denominator to the numerator in form of its reciprocal. But there are no inverted trigonometrical scales on the rule. However, by inserting unity twice in the numerator, we will have the desired condition. Thus:

$$\frac{\tan 15^\circ}{\cos 20^\circ \sin 25^\circ} = \frac{1 \times \tan 15^\circ \times 1}{\cos 20^\circ \sin 25^\circ}$$

To 1 on D scale,  
30 Draw 20° on S (red),  
Push hairline to 15° on T scale,  
Draw 25° on S scale to hairline,  
Push hairline to 1 on C scale and  
45 Read .675 on D scale.

Example 16.—Evaluate

$$\frac{22 \cos^2 33^\circ}{93}$$

Solution: To 22 on A scale,  
60 Draw 93 on B scale,  
Push hairline to 33° on S (red) and  
Read .166 on A scale.

Example 17.—Evaluate

$$\frac{(8.2)^2 \tan^2 21^\circ}{\sin^2 40^\circ}$$

Solution: To 8.2 on C scale,  
65 Draw 40° on S scale,  
Push hairline to 21° on T scale and  
Read 23.95 on A scale.

Example 18.—Evaluate

$$70 \quad \frac{3865 \sqrt{7.83} \times 75 \sin 38^\circ}{\tan 36^\circ \cos 25^\circ} = \frac{3865 \sqrt{7.83} \sin 38^\circ \times 1}{(1/75) \tan 36^\circ \cos 25^\circ}$$

Solution: To 3865 on D scale,  
70 Draw 75 on CI scale,  
Push hairline to 7.83 on B left,  
Draw 36° on T under hairline,  
Push hairline to 38 on S,  
Draw 25° on S (red) under hairline,  
At index of C scale,  
75 Read 758,800 on D scale.

**Example 19.—Evaluate**

$$\frac{\tan 25^\circ}{.939 \sin 15^\circ} = \frac{1}{.939} \frac{\tan 25^\circ}{\sin 15^\circ} = 1.919$$

To .939 on DI scale,  
 Draw 15° on S scale,  
 Push hairline to 25° on T scale and  
 Read 1.919 on D scale.

These examples show that the same settings for simple multiplication and division on the C and D scales are also used when trigonometric functions are introduced. To evaluate the expression of Example 18 on a rule with trigonometric scales of unequal unit lengths or a rule where trigonometric scales are placed on the body of the rule, one would probably first replace two of the trigonometric functions by their actual numerical values and then proceed as above. Much of the simplicity of operation of this slide rule consists in the fact that a single type of setting applies without exception in the case of combined multiplication and division problems whether they are simple or complex.

The arrangement of scales of this rule permit the use of the same law for solving triangles wherever possible without reverting to the opposite side of the rule. Thus to solve the triangle shown in Figure 4:

To 381 on D scale,  
 Draw 66 on S scale,  
 Push hairline to 42° on S scale  
 And read  $a=279$  (answer) on D scale;  
 Push hairline to 72° on S scale  
 And read  $b=396$  (answer) on D scale.

To obtain further knowledge of the simplicity of operation of this slide rule, attention is directed to the problem of finding the length  $h$  of the closed figure shown in Figure 5.

Solution: To 362 on D scale  
 Draw 52° on S scale,  
 Push hairline to 90° on S,  
 Draw 40° on S under hairline,  
 Push hairline to 72° on S,  
 Draw 90° on S under hairline,  
 Push hairline to 70° on S  
 And read  $h=637$  on D.

The process consists of a succession of movements of the slide and of the hairline with a final reading of the desired quantity. To solve the same problem on a rule with trigonometric scales located on the body, the three triangles would have to be solved separately by reading the sides marked  $x$  and  $y$  as intermittent results.

The following examples are solved with the aid of the scales LL<sup>1</sup>, LL<sup>2</sup> and LL<sup>3</sup>. The same principle as used with plain numbers, squares, square roots, cubes, cube roots and trigonometric functions is also applicable for these:

**Example 20.—Evaluate**

$$\sqrt[5]{(310)^{1.26}}$$

Solution:

$$\log \sqrt[5]{(310)^{1.26}} = \log (310)^{\frac{1.26}{5}} = \frac{1.26 \log 310}{5}$$

Compare with Example 1.

To 310 on LL3 scale  
 Draw 5 on C scale,  
 Push indicator to 1.26 on C scale and  
 Read 4.24 on LL3 scale.

**Example 21.—Evaluate**

$$\frac{3.2 \sqrt[8]{8}}{\sqrt{5^{2.6}}}$$

Solution:

$$\log \sqrt[8]{5^{2.6}} = \log 5^{\frac{3.2 \times 2.6}{8}} = \frac{8 \log 5}{3.2 \times 2.6} = \frac{8 \log 5 \times 1}{3.2 \times 2.6}$$

To 5 on LL3 scale  
 Draw 3.2 on C scale,  
 Push indicator to 8 on C scale,  
 Draw 2.6 on C scale under indicator,  
 Push indicator to 1 (index) to C scale and  
 Read 4.7 on LL3 scale.

**Example 22.—Evaluate**

$$\frac{17 \log_2 25}{\sqrt{31}}$$

Solution: To 25 on LL3 scale  
 Draw 31 on B scale,  
 Push indicator to 17 on CF scale and  
 Read 9.83 on DF scale.

**Example 23.—Evaluate**

$$\frac{\log_2 1.4 \sin 45^\circ}{\cos 70^\circ}$$

Solution: To 1.4 on LL2 scale  
 Draw 70° on S (red),  
 Push indicator to 45 on S scale and  
 Read .6955 on D scale.

**Example 24.—Evaluate**

$$\frac{\cos 79^\circ 05' \log_2 1.0498 \sec 70^\circ 10'}{\sin 55^\circ}$$

Solution: To 1.0498 on LL1  
 Draw 55° on S,  
 Push hairline to 79°05' on S (red);  
 Draw 70°10' on S (red) to hairline and  
 Read at the index .03319 on D.

**Example 25.—Evaluate**

$$\sqrt{16.2 \log_2 135 \csc 60^\circ}$$

Solution: To 135 on LL3 scale  
 Draw 60° on S scale,  
 Push hairline to 16.2 on B and  
 Read 22.8 on D scale.

An important use of scales LL00 and LL0 is to find the powers of numbers between 0.00005 and 0.999. These scales are so designed that when the hairline is set to a number on LL0 or LL00 the natural co-logarithm of the number is on scale A. Consequently scales LL0, LL00 are read against scales A and B.

The process of finding the powers of numbers between 0.00005 and 0.999 is the same as that for finding the powers of numbers greater than unity. As in the case of problems involving scales LL1, LL2, and LL3 a rough calculation may be made to determine the position of the decimal point in the answer. In this connection, it should be remembered that when a positive number less than unity is raised to a power greater than unity, the result is less than the number itself, and when it is raised to a decimal power, the result is greater than the number itself.

A number on scale LL00 is the 100th power of the number appearing directly opposite on scale LL0. The same relation may also be expressed by saying that any number on scale LL0 is the hundredth root of the number opposite on scale LL00.

*Example 26.*—Find  $0.64^{0.321}$ .

Solution: Since 0.64 is raised to a decimal power the answer is greater than 0.64. Hence to find  $0.64^{0.321}$

Push hairline to 0.64 on LL00,  
Draw index of B under hairline,  
Push hairline leftward to 321 on B,  
At the hairline read 0.8665 on LL00.

*Example 27.*—Find

$$0.64^{\frac{17.2}{6.34}}$$

Solution: Since 0.64 is raised to a power greater than unity we know that the required answer is less than 0.64. Now let

$$y = 0.64^{\frac{17.2}{6.34}}$$

and equate the logarithms of two members of this equation to obtain

$$\log y = \frac{17.2}{6.34} \log 0.64, \text{ or } \frac{\log y}{17.2} = \frac{\log 0.64}{6.34}$$

This proportion suggests the setting indicated in the following:

Push the hairline to 0.64 on LL00,  
Draw 634 on B under the hairline,  
Opposite 17.2 on the B scale  
Read 0.298 on LL00.

It is interesting to note that scales LL0 and LL00 can be used with scale A to find negative powers of  $e$ . Thus if the hairline is set to 5 on A left, we read  $e^{-0.005} = 0.99501$  on LL0 and  $e^{-0.5} = 0.6065$  on LL00 at the hairline; if the hairline is set to 5 on A right, we read  $e^{-0.05} = 0.9512$  on LL0 and  $e^{-5} = 0.0067$  on LL00 at the hairline.

*Example 28.*—Evaluate

$$\sqrt[64]{(.992)^{15}}$$

Solution:

$$\log \sqrt[64]{(.992)^{15}} = \log (.992)^{\frac{15}{64}} = \frac{15 \log .992}{.64}$$

To .992 on LL0 scale  
Draw .64 on B scale,  
Push indicator to 15 on B scale and  
Read .8285 on LL00 scale.

*Example 29.*—

$$\tan 28^\circ \sqrt{\frac{\log_e .998}{\sin^2 40^\circ}} = \sqrt{\frac{\log_e .998 \tan^2 28^\circ}{\sin^2 40^\circ}} = \sqrt{\frac{\log_e .998 \tan 28^\circ}{\sin 40^\circ}}$$

To .998 on LL0 scale  
Draw  $40^\circ$  on S scale,  
Push hairline to  $28^\circ$  on T scale,  
And read 0.0117 on D scale.

As said before, the first ten examples can be solved with some prior slide rules in the same manner as presented here. The advantage of this slide rule over all other rules is apparent in solving trigonometric problems. To demonstrate this advantage more clearly, the slide rule shown in United States Patent 1,488,686 is selected as the most advanced slide rule heretofore proposed, and the same examples as solved on the rule of the present invention are solved on the earlier rule in the following manner: By referring to the same examples noted, the difference between the two solutions will be clearly seen.

*Example 11.*—

$$\frac{73.6 \times 3.44}{\sin 30^\circ}$$

Solution: To 73.6 on scale A  
Set  $30^\circ$  on scale S  
At 3.44 on scale B,  
Read 506 on scale A.

This solution is the same as on the slide rule of this invention, with the exception that, on such slide rule, scales of 25 cm. unit length can be used while on the rule of the patent, scales of only 12.5 cm. unit length have to be used. This naturally tends to make the answer more accurate on the rule of this invention.

*Example 12.*—Evaluate

$$\frac{53.5}{2.25 \sin 13^\circ 30'}$$

Solution: To 53.5 on A scale  
Set  $13^\circ 30'$  on S scale,  
Hairline to left index of B scale,  
Set 2.25 on B scale under hairline,  
And opposite index on B scale,  
Read 101.9 on A scale.

Two movements of the slide are required on 12.5 cm. unit scales, while on the slide rule of this invention only one movement of the slide is necessary. The reason is that there 25 cm. unit scales are used, and, therefore, advantage of the inverted scales could be taken.

*Example 13.*—Evaluate

$$\frac{17 \cos 50^\circ 31'}{\tan 12^\circ \sin 59'}$$

Solution: The trigonometric scales of the rule of the patent are not numbered for complements; therefore

$$\cos 50^\circ 30' = \sin (90^\circ - 50^\circ 30') = \sin 39^\circ 30'$$

Since the T scale is of different unit length, it can not be used in conjunction with the S scale. Therefore, the natural tangent of  $12^\circ$  must first be found:

Opposite  $12^\circ$  on T scale,  
Read .2125 on C scale.

The problem now is

$$\frac{17 \sin 39^\circ 30'}{.2125 \sin 59'}$$

To 17 on A scale  
Set .2125 on B scale,  
Hairline to  $39^\circ 30'$  on S scale;  
 $59'$  on S scale to hairline;  
At index of B scale  
Read 2960 on A scale.

Usually a user will prefer to find the numerical values of all the trigonometric functions in the computation and then proceed as in regular multiplication and division, in order to use 25 cm. unit scales.

*Example 14.*—Evaluate

$$\sqrt{\frac{2.9 \cos 16^\circ}{\tan 16^\circ}}$$

Solution: Here the natural cosine of  $16^\circ$  must be inserted, since the square root is involved and this is found on the D scale.

$$\cos 16^\circ = \sin (90^\circ - 16^\circ) = \sin 74^\circ$$

At  $74^\circ$  on S scale read  
.96 on B scale.

The example now is

$$\sqrt{\frac{2.9 \times .96}{\tan 16^\circ}}$$

To 2.9 on A scale,  
Set 16° on T scale.  
Opposite .96 on C scale,  
Read 5.7 on D scale.

Example 15.—Evaluate

$$\frac{\tan 15^\circ}{\cos 20^\circ \sin 25^\circ}$$

Solution: At 15° on T scale  
Read .268 on C scale.  
At (90°-20°)=70° on S scale  
Read .94 on B scale.  
At 25° on S scale,  
Read .423 on B scale.

The example now is

$$\frac{.268}{.94 \times .423}$$

To .268 on D scale  
Set .423 on C scale.  
At .94 on CIF scale,  
Read .674 on DF scale.

It is easily seen that the solution on the rule of this invention is much simpler and quicker.

Example 16.—Evaluate

$$\frac{22 \cos^2 33^\circ}{93}$$

Solution:  
 $\cos^2 33^\circ = \sin^2 (90^\circ - 33^\circ) = \sin^2 57^\circ$   
At 57° on S scale  
Read .839 on B scale.

The problem now is

$$\frac{22(.839)^2}{93}$$

To 22 on A scale,  
Set 93 on B scale.  
At .839 on C scale,  
Read .166 on A scale.

Example 17.—Evaluate

$$\frac{(8.2)^2 \tan^2 21^\circ}{\sin^2 40^\circ}$$

Solution: At 40° on S scale  
Read  $\sin 40^\circ = .642$  on B scale.  
To 8.2 on D scale,  
Set .642 on C scale.  
At 21° on T scale,  
Read 24 on A scale.

Example 18.—Evaluate

$$\frac{3865 \sqrt{7.8375} \sin 38^\circ}{\tan 36^\circ \cos 25^\circ}$$

Solution: Again first find the numerical values of the trigonometric functions, and then proceed with multiplication and division.

At 38° on S scale  
Read .616 on B scale.  
At 90°-25°=65° on S scale,  
Read .906 on B scale.  
At 36° on T scale,  
Read .726 on D scale.

The problem now is:

$$\frac{3865 \sqrt{7.8375} \times .616}{.726 \times .906}$$

This is solved in the customary way.

It is seen that in almost every problem in which trigonometric functions occur either in the numerator or in the denominator, the solution on

the rule of this invention will be much simpler and the result in most cases more accurate than on the rule of the patent due to the fact that all trigonometric scales are of the same unit length and may be, for instance, 25 cm. long, which permits them to work in conjunction with all the other scales on the rule.

Example 23.—Evaluate

$$\frac{\log_e 1.4 \sin 45^\circ}{\cos 70^\circ}$$

Solution: The natural logarithms of numbers above unity are found on LL3, LL2, and LL1. These scales are of 25 cm. unit lengths while the S scale is of 12.5 cm. unit length. Therefore the natural functions of  $\sin 45^\circ$  and  $\cos 70^\circ$  have to be inserted in the above problem.

$$\cos 70^\circ = \sin (90^\circ - 70^\circ) = \sin 20^\circ$$

Opposite 20° on S scale  
Read .342 on B scale.  
Opposite 45° on S scale  
Read .707 on B scale.

The problem now reads:

$$\frac{\log_e 1.4 \times .707}{.342}$$

To 1.4 on LL2 scale  
Set .342 on C scale.  
Opposite .707 on C scale  
Read .695 on D scale.

Example 24.—Evaluate

$$\frac{\cos 79^\circ 05' \log_e 1.0498 \sec 70^\circ 10'}{\sin 55^\circ}$$

Solution: Here again all the trigonometric functions are found on a 12.5 cm. unit length scale, namely the S scale. Therefore their numerical value has to be found first.

$\cos 79^\circ 05' = \sin (90^\circ - 79^\circ 05') = \sin 10^\circ 55'$   
At 10°55' on S scale  
Read .1895 on B scale.

$$\sec 70^\circ 10' = \frac{1}{\cos 70^\circ 10'} = \frac{1}{\sin (90^\circ - 70^\circ 10')} = \frac{1}{\sin 19^\circ 50'}$$

At 19°50' on S scale,  
Read .34 on B scale.

$$\frac{1}{\sin 19^\circ 50'} = \frac{1}{.34}$$

At 55° on S scale,  
Read .82 on B scale.

The problem now is:

$$\frac{.1895 \log_e 1.0498}{.82 \times .34}$$

To 1.0498 on LLI scale  
Set .82 on C scale.  
Hairline to .1895 on C scale,  
.34 on C scale to hairline.  
At index of C read .033 on D scale.

Example 25.—Evaluate

$$\sqrt{16.2 \log_e 135 \csc 60^\circ}$$

Solution:

$$\csc 60^\circ = \frac{1}{\sin 60^\circ}$$

At 60° on S scale  
Read .866 on B scale.  
To 135 on LL3 scale  
Set .866 on C scale.  
At 16.2 on B scale,  
Read 22.75 on D scale.



It will thus be seen that a slide rule has been provided, all of the scales of which have been so disposed and so relatively related that substantially every problem arising can be solved as in the solution of simple problems in proportions.

Various modifications will occur to those skilled in the art in the type of slide rule employing this invention as well as in the selection and/or disposition of the scales and no limitation is intended by the phraseology of the foregoing specification or illustrations in the accompanying drawing except as indicated by the appended claims.

What is claimed is:

1. In a slide rule having two relatively fixed bars and a sliding bar, a parallel standard logarithmic scale along one edge of a bar on one face thereof, a sine and a tangent scale on the same face of the slide, identical parallel standard logarithmic scales along coacting edges of a bar and the slide on the other face thereof, all of said scales being to the same unit length whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

2. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member and identical logarithmic scales of the same unit length on the other face of the respective members whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

3. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members and identical logarithmic scales of a unit length one-half that of the trigonometric scales on the respective members on the same face as the first mentioned graduation whereby proportions involving trigonometric functions and squares or square roots are performed by combined operations in progressive manipulations.

4. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the trigonometric functions of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members, identical logarithmic scales of a unit length one-half that of the trigonometric scales on the respective members on the same face as the first mentioned graduations and a logarithmic scale of the same unit length on the same face of the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions and squares or square roots are performed by combined operations in progressive manipulations.

5. A slide rule comprising a body and a slide, said slide being graduated in trigonometric scales, a parallel logarithmic scale and a folded logarithmic scale of the same unit length as the first named scale on the body, a logarithmic scale identical to the first named scale on the slide, an

inverted logarithmic scale of the same unit length as the first named scale on the slide, a folded logarithmic scale of the same unit length on the slide and a folded inverted logarithmic scale of the same unit length on the slide and an inverted logarithmic scale of the same unit length on the body, said trigonometric and logarithmic scales being graduated to the same unit length whereby problems all as products of functions in both numerator and denominator may be performed.

6. A slide rule comprising a body and a slide, the slide being graduated in trigonometric scales, and folded logarithmic scales on the body and slide, all of said scales being to the same unit length whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

7. A slide rule comprising a body and a slide, the slide being graduated in trigonometric scales, folded logarithmic scales of the same unit length on both body and slide, and a folded inverted logarithmic scale on the slide, all of said scales being to the same unit length whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

8. A slide rule comprising a body and a slide, the slide being graduated in trigonometric scales, inverted logarithmic scales of the same unit length on both the body and the slide, and folded logarithmic scales on the body and slide, all of said scales being to the same unit length whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

9. A slide rule comprising two relatively movable members, one of said members bearing a scale graduated as a standard graduated logarithmic scale of two unit lengths graduated from 1 to 100 in a distance of twenty-five centimeters, and the other member being graduated as a standard graduated logarithmic scale identical with the first named scale and a trigonometric scale graduated to give angles in degrees and minutes from  $5^{\circ}40'$  to  $90^{\circ}$  in said same distance whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

10. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the trigonometric functions of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members and a logarithmic scale of the same unit length on the same face of the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

11. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member, a logarithmic scale of the same unit length on the other relatively movable member from that on which the first mentioned graduations appear, said slide rule being also graduated with a logarithmic scale of a unit length one-half that of the first mentioned logarithmic scale whereby proportions involving trigonometric functions

and the squares or square roots are performed by combined operations in progressive manipulations.

12. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member and identical logarithmic scales of the same unit length on the other face of the respective members and an inverted logarithmic scale of the same unit length on the same face of the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

13. A slide rule comprising two relatively movable members, one of said members being graduated in trigonometric scales of the same unit length and the other member being graduated in a logarithmic scale of the same unit length as the trigonometric scales and an inverted logarithmic scale of the same unit length on the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions are performed by combined operations in progressive manipulations.

14. A slide rule comprising two relatively movable members, one of said members being graduated in trigonometric scales of the same unit length and the other member being graduated in a logarithmic scale of the same unit length as the trigonometric scales and a scale representing logarithms of co-logarithms of numbers less than unity on the same face of the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions and logarithms of co-logarithms of numbers are performed by combined operations in progressive manipulations.

15. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members, identical logarithmic scales of a unit length one-half that of the trigonometric scales on the respective members on the same face as the first mentioned graduation and a scale representing logarithms of co-logarithms of numbers less than unity on the same face of the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions and squares or square roots and logarithms of co-logarithms of numbers are performed by combined operations in progressive manipulations.

16. A slide rule comprising two relatively movable members, one of said members being graduated in trigonometric scales of the same unit length and the other member being graduated in a logarithmic scale of the same unit length as the trigonometric scales and a scale representing logarithms of logarithms of numbers greater than unity on the other face of the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions and logarithms of logarithms of numbers are performed by combined operations in progressive manipulations.

17. A slide rule comprising two relatively movable

members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member and identical logarithmic scales of the same unit length on the other face of the respective members and a scale representing logarithms of logarithms of numbers greater than unity on the other face of the other member from that on which the trigonometric scales appear whereby proportions involving trigonometric functions and logarithms of logarithms of numbers are performed by combined operations in progressive manipulations.

18. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members and a scale graduated to give readings of the cube of a number on the first mentioned logarithmic scale whereby proportions involving trigonometric functions and cubes or cube roots are performed by combined operations in progressive manipulations.

19. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the sine and tangent of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members, identical logarithmic scales of a unit length one-half that of the trigonometric scales on the respective members on the same face as the first mentioned graduation and a scale graduated to give readings of the cube of a number on the first mentioned logarithmic scale whereby proportions involving trigonometric functions and squares or square roots and cubes or cube roots are performed by combined operations in progressive manipulations.

20. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the trigonometric functions of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members, a logarithmic scale of the same unit length on the same face of the other member from that on which the trigonometric scales appear and a scale graduated to give readings of the cube of a number on the first mentioned logarithmic scale whereby proportions involving trigonometric functions and cubes or cube roots are performed by combined operations in progressive manipulations.

21. A slide rule comprising two relatively movable members, one of said members being graduated in scale graduations of full unit length to give readings of angles the trigonometric functions of which are found by reference to a logarithmic scale on the other relatively movable member, identical logarithmic scales of the same unit length on the other face of the respective members, identical logarithmic scales of a unit length one-half that of the trigonometric scales on the respective members on the same face as

the first mentioned graduations, a logarithmic scale of the same unit length on the same face of the other member from that on which the trigonometric scales appear, and a scale graduated to give readings of the cube of a number on the first mentioned logarithmic scale whereby proportions involving trigonometric functions and

squares or square roots and cubes or cube roots are performed by combined operations in progressive manipulations.

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