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COMPLETE SPECIFICATION

Improvements in or relating to Slide Rules

We, HUBERT BOARDMAN, of 388 Bury Road, Rochdale, in the County of Lancaster, a British subject, and A. G. THORNTON LIMITED, of Harper Road, Wythenshawe, in the County of Lancaster, a British Company, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention concerns devices with relatively movable members bearing co-operating scales (hereinafter called "slide rules") and has for its object the provision of a slide rule which has scales enabling the real roots of quadratic or cubic equations more easily to be determined than has hitherto been possible. The term "slide rule" when used herein and in the claiming clauses hereof is intended to include not only the flat longitudinal slide type, but also types having a cylindrical, circular or other shape.

Quadratic equations may be considered to fall within one or other of two categories when "b" and "c" are positive, viz.,

Category I $x^2 \pm bx + c = 0$ (1)

Category II $x^2 \pm bx - c = 0$ (2)

In category I let $N_s = \dfrac{b}{\sqrt{c}}$

and

in category II let $N_D = \dfrac{b}{\sqrt{c}}$

Then category I has a related equation of the form:—

$$Q^2 \pm N_s Q + 1 = 0 \qquad (3)$$

and category II has a related equation of the form:—

$$Q^2 \pm N_D Q - 1 = 0 \qquad (4)$$

When the values of Q which satisfy equations (3) and (4) for specific values of N_s or N_D are REAL, one value of Q is of a magnitude less than unity. If this value be first determined a value of "x" that satisfies in appropriate equations (1) or (2) can be found since

$$x = Q\sqrt{c} \quad \text{or} \quad -Q\sqrt{c} \qquad (5)$$

In the case of category I the roots are real when N_s is equal to or greater than 2, and complex when N_s is less than 2. In category II all roots are REAL.

It is to be noted that inasmuch as the determination of complex roots can be effectively dealt with by using a combination of orthodox logarithmic scales A, B, C and D and a log $\sqrt{1-s^2}$ scale (where s is a given number), this application is not concerned with the determination of such roots.

The invention, insofar as it relates to the solution of quadratic equations, is based on the appreciation of the fact that quadratic equations of the two categories specified can be reduced to a related equation in one or other of the two forms also specified, and that then, the normal equations for the evaluation of the roots of quadratic equations, as applied to the related equation, contain only one unknown (N_s or N_D as the case may be). To illustrate this more specifically, the roots of the equations (1) and (2) would take one or other of the forms:—

$$\frac{-b \pm \sqrt{b^2-4c}}{2} \quad \text{or} \quad \frac{b \pm \sqrt{b^2-4c}}{2} \quad \text{or}$$

$$\frac{-b \pm \sqrt{b^2+4c}}{2} \quad \text{or} \quad \frac{b \pm \sqrt{b^2+4c}}{2}$$

Each of these expressions contains the separate variables b and c and as such does not permit of scale presentation.

The roots of the respective related equations are

$$-\frac{N_S}{2} \pm \sqrt{\left(\frac{N_S}{2}\right)^2-1} \quad \text{or} \quad \frac{N_S}{2} \pm \sqrt{\left(\frac{N_S}{2}\right)^2-1} \quad \text{or}$$

$$-\frac{N_D}{2} \pm \sqrt{\left(\frac{N_D}{2}\right)^2+1} \quad \text{or} \quad \frac{N_D}{2} \pm \sqrt{\left(\frac{N_D}{2}\right)^2+1}$$

Each of these expressions contain only one variable, N_s or N_D, and can be presented in scale form.

According to the present invention, a silde rule is characterised by the provision of a simple logarithmic scale and two further logarithmic scales, one covering values of N_s (as herein defined) from 2 to a chosen upper value and the other covering values of N_D (as herein defined) over a chosen range, the scales being in such fixed relationship one to the other that, for related equations of categories I and II (as herein defined) where the N_s or N_D value, as the case may be, is within the range of the respective N_s or N_D scale, the numerical value of Q (as herein defined) may be read off on the first-mentioned scale at a position corresponding with the N_s or N_D value on the respective N_s or N_D scale.

Turning now to cubic equations, a cubic equation of the form

$$x^3+ax^2+bx+c=0 \qquad (6)$$

and called herein an equation of category III, has a related equation of the form, applicable except when g (defined below) is zero,

$$Z^3-HZ+1=0 \qquad (7)$$

The value of H in terms of a, b and c may be derived as follows:—

Where

$$f=\frac{a^2}{3}-b \qquad (8)$$

and

$$g=\frac{2a^3}{27}-\frac{1ab}{3}+c \qquad (9)$$

Then

$$H=\frac{f}{\sqrt[3]{g^2}} \qquad (10)$$

Also

$$x=Z\sqrt[3]{g}-\frac{a}{3} \qquad (11)$$

Equation 7, with positive and negative values of Z considered, may be represented by a graph of the form shown in the single figure of the drawing accompanying the provisional specification. This graph can be considered in five sections:—

(1) o to n for negative values of Z range $-.1$ to -1.0

(2) n to m for negative values of Z range -1.0 to -10.0

(3) p to q for positive values of Z range .01 to .1

(4) q to r for positive values of Z range .1 to 1.0

(5) r to s for positive values of Z range 1.0 to 10.0

The invention, insofar as it relates to the solution of cubic equations, is based on the appreciation of the fact that a cubic equation of the form specified as category III can (except when *g* is zero) be reduced to a related equation of the form specified and that then, the roots of the related equation can be found by mathematical relationships involving only one variable.

If H scales corresponding to Z values of $-.1$ to -10.0 only are used then in nearly all cases it will be possible to evaluate a real root and to convert the equation eventually to a quadratic equation and to complete the solution by evaluating the roots of that quadratic equation in accordance with the invention, such scales as are necessary being, if desired, provided on the same rule. Alternatively H scales covering the full range of Z values hereinbefore specified may be used.

A slide rule for solving cubic equations, and designed in accordance with the principles hereinbefore set forth is separately described and claimed in our co-pending application No. 25406/60 [Serial No. 878,057] divided from the present application.

There will now be described, by way of example, a slide rule of the flat longitudinal slide type, especially designed for the evaluation of the real roots of quadratic equations and cubic equations.

Reference will be made to the accompanying drawings in which,

Fig. 1 is a front view of the rule,

Fig. 2 is a rear view of the rule, and

Fig. 3 shows the back of the slide.

For dealing with quadratic equations the rule incorporates the following scales. On the stock there will be:—

(1) A logarithmic scale called the D scale, numbered 1 to 10, the distance between the 1 and 10 being the length unit of reference.

(2) A logarithmic scale called the A scale, consisting of two sections of logarithmic scale each of $\frac{1}{2}$ unit and together numbered 1 to 100. It is positioned so that 1 on the D scale aligns with 1 on the A scale and 10 on the D scale aligns with 100 on the A scale.

(3) Also on the stock an N_s scale and an N_D scale, which will be described more fully hereinafter.

On the face of the slide there will be:

(1) A scale called the C scale similar to the D scale.

(2) A scale called the B scale similar to the A scale.

(3) A reverse of C scale called RC and

(4) A reverse of B scale called RB.

The 1 and 100 on the B scale
100 and 1 on the RB scale
10 and 1 on the RC scale
align respectively with
1 and 10 on the C scale

To facilitate determination of the real roots of an equation in category I, the N_s scale will be numbered according to values of N_s, the line dimensions from a datum being:—

$$\text{Log}\left[\frac{N_S}{2} - \sqrt{\left(\frac{N_S}{2}\right)^2 - 1}\right]$$

and when arranged on the stock this scale is so positioned that the line of the scale for $N_s=2$ aligns with 10 on the D scale. Projections from N_s values on this scale, by means of a cursor, will give on the D scale significant figures of an appropriate Q value less than unity that will satisfy equation (3).

To facilitate determination of the real roots of an equation in category II the N_D scale will be numbered according to values of N_D, the line dimensions from a datum being:—

$$\text{Log}\left[\sqrt{\left(\frac{N_D}{2}\right)^2 + 1} - \frac{N_D}{2}\right]$$

and when arranged on the stock this scale is positioned so that the line of the scale for $N_D=0$ aligns with the 10 on the D scale. Projections from N_D values on the N_D scale, by means of the cursor, will give on the D scale the significant figures of an appropriate Q value of magnitude less than unity that will satisfy equation (4).

One unit length of N_s scale will serve for a range of N_s values 2 to 10.1 and one unit length of N_D scale will serve for a range of N_D values 0 to 9.9. Additional lengths of N_s and N_D scales may be added by the usual fold back system employed in slide rule art, that is for the second unit lengths of N_s and N_D scale, 10.1 and 9.9 respectively would align with D_{10} and continue in the direction of D_1. For the purpose of projected values from the first sections of N_s or N_D to the D scale, the significant figure values of the D scale are for the range 0.1 to 1.0.

Using the rule described, i.e. with N_s and N_D scales on the stock, examples will illustrate the method of manipulation.

EXAMPLE 1:

Consider the determination of the roots of the equation

$$x^2 - 8.25x + 12 = 0$$

This quadratic equation falls in category I and will involve the use of the N_s scale. From textbook information regarding quadratic equations, conclusions would be drawn that both roots are of positive sign.

Proceed as follows:

(1) Set the cursor at 8.25 on the D scale

(2) Move the slide so that 12 on the B scale is at the cursor, and on the D scale at C_1 read 238 the significant figure value of $\frac{8.25}{\sqrt{12}}$ (i.e. $\frac{b}{\sqrt{c}}$). Decimal point considered 2.38.

(3) Move the cursor to 2.38 on the N_s scale.

(4) Bring 12 on the RB scale to the cursor and read on the D scale at C_1 the significant figures of the lesser numerical root $\sqrt{12} \times .545$ (i.e. $Q\sqrt{c}$) viz. 1.884.

(5) The magnitude of the other root
$= 8.25 - 1.884 = 6.366$
or may be obtained by transferring the cursor to C_1 and then moving the slide bringing C_{12} to the cursor. On the C scale at D_{10} read 6.366.

Hence the roots are

1.884 and 6.366.

EXAMPLE 2:

Consider the determination of the roots of the equation

$$x^2 + 7.35x - 24 = 0$$

which falls in category II and involves the use of the N_D scale. From text book information regarding quadratic equations, the conclusion would be reached that the roots are of opposite signs and that the root of lesser numerical magnitude is positive.

Proceed as follows:

(1) Set the cursor at 735 on the D scale.

(2) Move slide bringing 24 on the B scale to the cursor, and on the D scale at index C_1 read the significant figure value of $\frac{7.35}{\sqrt{24}}$ viz. 150, decimal point considered 1.50.

(3) Transfer the cursor to 1.5 on the N_D scale.

(4) Move the slide so that the 24 line on the RB scale is at the cursor and on the D scale at C_1 read the significant figures of $\sqrt{24} \times .5$ viz. 245, decimal point considered 2.45.

(5) The magnitude of other root is

$$= 7.35 + 2.45 = 9.8$$

or using the slide rule:

Transfer the cursor to C_1 and move slide bringing C_{24} to the cursor. On the C scale at D_{10} read 98, decimal point considered 9.8.

Hence the roots are:

$$+2.45 \text{ and } -9.80.$$

In order to illustrate the invention further the addition of scales enabling the roots of cubic equations to be evaluated to the slide rule previously described for the solution of quadratic equations will now be described.

Scales H_{on}, H_{nm}, H_{pq}, H_{qr} and H_{rs} numbered according to H values (positive or negative) will be prepared, corresponding to each section of the graph previously referred to, by the following method:—

A table of values of H for specific values of Z (positive or negative) is prepared by direct substitution in equation (7) and then, using selected Z values for purposes of inverse interpolation, Z values (positive and negative) for specific values of H are obtained.

Then, to the reference logarithmic unit of the D scale, line dimensions, from a datum, for H values on the scales to serve for portions o.n and n.m, i.e. for Scales H_{on} and H_{nm} will be according to

$$\text{Log}\left[-(Z)\right]$$

and for portions p.q, q.r and r.s, i.e. for scales H_{pq}, H_{qr}, and H_{rs} will be according to

$$\text{Log } Z$$

and assuming the scales are arranged on the stock of the slide rule, the datum of each of the scales will be in alignment with D_1 and run towards D_{10}.

Z values by projection from the new scales to the significant figure value D scale are as follows:—

For	H_{on}	$-.1$ to -1.0
	H_{nm}	-1.0 to -10.0
	H_{pq}	$+.01$ to $+.10$
	H_{qr}	$+.10$ to $+1.0$
	H_{rs}	$+1.0$ to $+10.0$

In an alternative arrangement, the scales are applied as reciprocals of H_{on}, H_{nm}, H_{pq}, H_{qr} or H_{rs} by longitudinal inversion.

In other embodiments either the direct or reciprocal forms of H_{on} and H_{mn} may be produced as a continuous scale and related to the A and B scales of the slide rule. The same applies to H_{pq}, and H_{qr} or H_{qr} and H_{rs}. Similarly scales H_{pq}, H_{qr} and H_{rs} may be continuous and related to a scale such as an F scale, which comprises three logarithmic sections, 1 to 10, 10 to 100 and 100 to 1000, each being one third the length of the unit D scale. 1 on the F scale aligns with 1 on the D scale and 1,000 on the F scale with 10 on the D scale.

For convenience the rule illustrated in the drawings has H_{nm} and H_{on} scales marked on the front of the rule stock for direct reading of Z values on the D scale by means of a cursor whilst H_{pq}, H_{qr} and H_{rs} scales are marked on the back of the slide, the H values being set against a datum line in the window illustrated and the Z values being read at C_1 on the D scale. The F scale is provided in the rule illustrated to facilitate the evaluation of cube roots, as will presently be apparent.

When H is positive and equal to or greater than $3\sqrt{6.75}$, viz. 1.88988 the related equation has three real roots. For values of a, b and c that result in $H = 3\sqrt{6.75}$, of the three real roots, two are equal.

In cases where H is less than $3\sqrt{6.75}$, of the three roots only one is real, namely, that which relates to the o, n, m portion of the graph.

The following examples will serve to illustrate the manipulation of the special scales just described.

EXAMPLE 3:

To determine the real root or real roots of the cubic equation

$$x^3 - 16x^2 + 73x - 90 = 0.$$

In this case with reference to equation (6)

$$a = -16 \quad b = 73 \quad c = -90$$

By normal arithmetical approach evaluate

f, from equation (8) $= 12.33$

and

g, from equation (9) $= -4.074$

The values of $3\sqrt{g}$ and $3\sqrt{g^2}$ may be found on the D and A scales respectively in alignment with the appropriate "g" value on the F scale, viz.

$$3\sqrt{-4.074} = -1.597, \quad 3\sqrt{g^2} = 2.550$$

Then evaluate

$$H = \frac{f}{3\sqrt{g^2}} = \frac{12.33}{2.550} = 4.836$$

Since this H value is positive and greater than $3\sqrt{6.75}$, there are three real roots.

H values of 4.836 will be found on the H_{nm}, H_{qr} and H_{rs} scales, and using the cursor and the window datum setting, as appropriate, for projection to the D scale, the respective Z values, viz. -2.296, .2086 and 2.088, may be obtained.

Then since

$$x = 3\sqrt{g}.Z - \frac{a}{3}$$

and

$$3\sqrt{g} = 1.597 \text{ and } a = -16$$

where x_1, x_2, and x_3 are the roots

$$x_1 = (-1.597)(-2.296) - \left(\frac{-16}{3}\right) = 3.67 + 5.33 = 9$$

$$x_2 = (-1.597)(.2086) - \left(\frac{-16}{3}\right) = -.333 + 5.333 = 5$$

$$x_3 = (-1.597)(2.088) - \left(\frac{-16}{3}\right) = -3.333 + 5.333 = 2$$

Thus the values of x which satisfy the equation are 9, 5 and 2.

EXAMPLE 4:

Solve:—

$$x^3 + 6x^2 + 16x - 38 = 0$$

with reference to equation (6)

$a = 6 \qquad b = 16 \qquad c = -38$

then

f from equation (8) $= -4$

g from equation (9) $= -54$

$3\sqrt{g}$ from the slide rule $= -3.78$

$3\sqrt{g^2}$ from the slide rule $= 14.29$

H from equation (10) $= -.28$

Here H is less than $3\sqrt{6.75}$ viz. 1.889 so there is only one real root. From the slide rule

when $H = -.28$

then $Z = -.907$

This is obtained by cursor projection from the $-.28$ line on the H_{on} scale to a D scale numbered $-.1$ to -1.0.

Then x from equation (11) since g, Z and a are known,

$$= (-3.78)(-.907) - \frac{6}{3} = 1.428$$

In order that the rule may be used for evaluating, according to known techniques, the complex roots of quadratic equations an f(s) scale is provided on the stock, aligned with the D scale, and enabling $\sqrt{1 - s^2}$ values to be read off on the f(s) scale from the s value on the D scale.

WHAT WE CLAIM IS:—

1. A slide rule characterised by the provision of a simple logarithmic scale and two further logarithmic scales, one covering values of N_s (as herein defined) from 2 to a chosen upper value and the other covering values of N_D (as herein defined) over a chosen range, the scales being in such fixed relationship one to the other that, for related equations of categories I and II (as herein defined) where the N_s or N_D value, as the case may be, is within the range of the respective N_s or N_D scale, the numerical value of Q (as herein defined) may be read off on the first-mentioned scale at a position corresponding with the N_s or N_D value on the respective N_s or N_D scale.

2. A slide rule as claimed in claim 1 further characterised by the provision of additional, conventional scales to enable the N_s or N_D value to be ascertained by simple manipulation of the rule according to the values of b and c in the quadratic equation to be solved.

3. A slide rule as claimed in claim 1 or 2 further characterised by the provision of additional conventional scales to enable the roots of the quadratic equation to be ascertained by direct manipulation according to the value of Q as read off in said first-mentioned scale.

4. A slide rule as claimed in any one of the preceding claims further characterised in that the N_s and N_D scales are in a plurality of separate lengths.

5. A slide rule as claimed in any one of the preceding claims further characterised by the provision of a further logarithmic scale covering values of H (as herein defined) for a chosen range or ranges of values of Z (as herein defined), the scale being in such fixed relationship to the simple log scale that, for related equations of category III where the H value is within the range of the H scale, the numerical value or values of Z may be read off on the first mentioned scale at a position or positions corresponding with the H value or values on the H scale.

6. A slide rule as claimed in claim 5 in which the H scale covers H values for a range of Z values from $-.1$ to -10.0.

7. A slide rule as claimed in claim 5 in which the H scale covers H values for ranges of Z values from $-.1$ to -10.0 and from .01 to 10.

8. A slide rule as claimed in any one of claims 5 to 7 further characterised by the provision of additional conventional scales for the determination of the squares of numbers and the cube roots of numbers by direct manipulation.

9. A slide rule as claimed in any one of the preceding claims further characterised by the provision of additional conventional scale or scales for the determination of values of $\sqrt{1 - s^2}$, where s is given, by simple manipulation.

10. A slide rule substantially as hereinbefore described with reference to and as illustrated in the accompanying drawings.

For the Applicants.

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PROVISIONAL SPECIFICATION

Improvements in or relating to Slide Rules

We, HUBERT BOARDMAN, of 388 Bury Road, Rochdale, in the County of Lancaster, a British subject, and A. G. THORNTON LIMITED, of Harper Road, Wythenshawe, in the County of Lancaster, a British Company, do hereby declare this invention to be described in and by the following statement:—

This invention concerns slide rules, and has for its object the provision of a slide rule (which may be of any type, e.g. flat or cylindrical) which has scales enabling the real roots of quadratic or cubic equations more easily to be determined than has hitherto been possible.

Quadratic equations may be considered to fall within one or other of two categories when "b" and "c" are positive, viz.,

Category I $x^2 \pm bx + c = 0$ (1)

Category II $x^2 \pm bx - c = 0$ (2)

In category (I) let $N_s = \dfrac{b}{\sqrt{c}}$

and

in category (II) let $N_D = \dfrac{b}{\sqrt{c}}$

Then category I has a related equation of the form:—

$$Q^2 \pm N_s Q + 1 = 0 \qquad (3)$$

and category II has a related equation of the form:—

$$Q^2 \pm N_D Q - 1 = 0 \qquad (4)$$

When the values of Q which satisfy equations (3) and (4) for specific values of N_s or N_D are REAL, one value of Q is of a magnitude less than unity. If this value be first determined a value of "x" that satisfies in appropriate equations (1) or (2) can be found since

$$x = Q\sqrt{c} \text{ or } -Q\sqrt{c} \qquad (5)$$

In the case of category I the roots are real when N_s is equal to or greater than 2, and complex when N_s is less than 2. In category II all roots are REAL.

It is to be noted that inasmuch as the determinations of complex roots can be effectively dealt with by using a combination of orthodox logarithmic scales A, B, C and D and a log $\sqrt{1-s^2}$ scale, this application is not concerned with the determination of such roots.

The invention, insofar as it relates to the solution of quadratic equations, is based on the appreciation of the fact that quadratic equations of the two categories specified can be reduced to a related equation in one or other of the two forms also specified, and that then the normal equations for the evaluation of the roots of quadratic equations, as applied to the related equation, contains only one unknown (N_s or N_D as the case may be). To illustrate this more specifically, the roots of the equations (1) and (2) would take one or other of the forms:—

$$\frac{-b \pm \sqrt{b^2 - 4c}}{2} \quad \text{or} \quad \frac{b \pm \sqrt{b^2 - 4c}}{2} \quad \text{or}$$

$$\frac{-b \pm \sqrt{b^2 + 4c}}{2} \quad \text{or} \quad \frac{b \pm \sqrt{b^2 + 4c}}{2}$$

Each of these expressions contains the separate variables b and c and as such does not permit of scale presentation.

The roots of the respective related equations are

$$\frac{N_s}{2} \pm \sqrt{\left(\frac{N_s}{2}\right)^2 - 1} \quad \text{or} \quad -\frac{N_s}{2} \pm \sqrt{\left(\frac{N_s}{2}\right)^2 - 1}$$

and

$$\frac{N_D}{2} \pm \sqrt{\left(\frac{N_D}{2}\right)^2 + 1} \quad \text{or} \quad -\frac{N_D}{2} \pm \sqrt{\left(\frac{N_D}{2}\right)^2 + 1}$$

Each of these expressions contain only one variable N_s or N_D and can be presented in scale form.

According to the present invention, therefore, a slide rule calculator includes N_s and N_D scales marked in relationship to other scales to enable the roots of the related equation of a given quadratic equation to be directly ascertained by positive movement. Preferably the rule will also have other, conventional, scales so arranged as to enable N_s and N_D values to be directly ascertained by positive movement, using the values of the co-efficients of the quadratic equation to be solved, and, after the roots of the related equation have been found, to enable the roots of the quadratic equation to be solved to be directly ascertained by positive movement.

There will now be described, by way of example, a slide rule of the flat longitudinal slide type, especially designed for the evaluation of the real roots of quadratic equations.

The rule incorporates the following scales.

On the stock there will be:—

(1) A logarithmic scale called the D scale, numbered 1 to 10, the distance between 1 and 10 being the length unit of reference.

(2) A logarithmic scale called the A scale, consisting of two sections of logarithmic scale each of $\frac{1}{2}$ unit and numbered 1 to 100. It is positioned so that 1 on the D scale aligns with 1 on the A scale and 10 on the D scale aligns with 100 on the A scale.

(3) Also on the stock an N_s scale and an N_D scale, which will be described more fully hereinafter.

On the face of the slide there will be:

(1) A scale called the C scale similar to the D scale.

(2) A scale called the B scale similar to the A scale.

(3) A reverse of C scale called RC and

(4) A reverse of B scale called RB.

The 1 and 100 on the B scale
100 and 1 on the RB scale
10 and 1 on the RC scale
align respectively with
1 and 10 on the C scale

To facilitate determination of the real roots of an equation in category I, the N_s scale will be numbered according to values of N_s, the line dimensions from a datum being:—

$$\text{Log}\left[\frac{N_s}{2} - \sqrt{\left(\frac{N_s}{2}\right)^2 - 1}\right]$$

and when arranged on the stock this scale is so positioned that the line of the scale for $N_s = 2$ aligns with 10 on the D scale. Projections from N_s values on this scale, by means of a cursor, will give the significant figures of an appropriate Q value less than unity that will satisfy equation (3).

To facilitate determination of the real roots of an equation in category II the N_D scale will be numbered according to value of N_D, the line dimensions from a datum being:—

$$\text{Log}\left[\sqrt{\left(\frac{N_D}{2}\right)^2 + 1} - \frac{N_D}{2}\right]$$

and when arranged on the stock this scale is positioned so that the line of the scale for $N_D = 0$ aligns with the 10 on the D scale. Projections from N_D values on the N_D scale, by means of a cursor, will give the significant figures of an appropriate Q value of magnitude less than unity that will satisfy equation (4).

One unit length of N_s scale will serve for a range of N_s values 2 to 10.1 and one unit length of N_D scale will serve for a range of N_D values 0 to 9.9. Additional lengths of N_s and N_D scales may be added by the usual fold back system employed in slide rule art, that is for the second unit lengths of N_s and N_D scale, 10.1 and 9.9 respectively would align with D_{10} and continue in the direction of D_1. For the purpose of projected values from the first section of N_s or N_D to the D scale, the significant figure values of the D scale are for the range 0.1 to 1.0.

Using the rule described, i.e. with N_s and N_D scales on the stock, examples will illustrate the method of manipulation.

EXAMPLE 1:

Consider the determination of the roots of the equation

$$x^2 - 8.25x + 12 = 0$$

This quadratic equation falls in category I and will involve the use of the N_s scale. From text-book information regarding quadratic equations, conclusions would be drawn that both roots are of positive sign.

Proceed as follows:

(1) Set the cursor at 8.25 on the D scale.

(2) Move the slide so that 12 on the B scale is at the cursor, and on the D scale at C_1 read 238 the significant figure value of $\frac{8.25}{\sqrt{12}}$ (i.e. $\frac{b}{\sqrt{c}}$). Decimal point considered 2.38.

(3) Move the cursor to 2.38 on the N_s scale (and made mental note of Q value in alignment on D, viz. .545).

(4) Bring 12 on the RB scale to the cursor and read on the D scale at C_1 the significant figures of the lesser numerical root $\sqrt{12} \times .545$ (i.e. $Q\sqrt{c}$) viz. 1.884.

(5) The magnitude of the other root $= 8.25 - 1.884 = 6.366$ or may be obtained by transferring the cursor to C_1 and then moving the slide bringing C_{12} to the cursor. On the C scale at D_{10} read 6.366.

Hence the roots are

1.884 and 6.366

EXAMPLE 2:

Consider the determination of the roots of the equation.

$$x^2 + 7.35x - 24 = 0$$

which falls in category II and involves the use of the N_D scale. From text book information regarding quadratic equations, the conclusion would be reached that the roots are of opposite signs and that the root of lesser numerical magnitude is positive.

Proceed as follows:

(1) Set the cursor at 735 on the D scale.

(2) Move slide bringing 24 on the B scale to the cursor, and on the D scale at index

C_1 read the significant figure value of $\frac{7.35}{\sqrt{24}}$ viz. 150, decimal point considered 1.50.

(3) Transfer the cursor to 1.5 on the N_D scale (mentally note the Q value on the D scale, viz. .5).

(4) Move the slide so that the 24 line on the RB scale is at the cursor and on the D scale at C_1 read the significant figures of $\sqrt{24} \times 1.5$ viz. 245, decimal point considered 2.45.

The magnitude of other root is

$$=7.35+2.45=9.8$$

or using the slide rule

(5) Transfer the cursor to C_1 and move slide bringing C_{24} to the cursor. On the C scale at D_{10} read 98, decimal point considered 9.8.

Hence the roots are:

$$+2.45 \text{ and } -9.80.$$

Turning now to the solution of cubic equations, a cubic equation of the form

$$x^3+ax^2+bx+c=0 \quad (6)$$

has a related equation of the form

$$z^3-HZ+1=0 \quad (7)$$

The value of H in terms of a, b and c may be derived as follows:—

Where

$$f=\frac{a^2}{3}-b \quad (8)$$

and

$$g=\frac{2a^3}{27}-\frac{1ab}{3}+c \quad (9)$$

Then

$$H=\frac{f}{3\sqrt{g^2}} \quad (10)$$

Also

$$x=3\sqrt{g}.Z-\frac{a}{3} \quad (11)$$

Equation 7, with positive and negative values of Z considered, may be represented by a graph of the form shown in the single figure of the accompanying drawing. This graph can be considered in five sections:—

(1) o to n for negative values of Z range −.1 to −1.0

(2) n to m for negative values of Z range −1.0 to −10.0

(3) p to q for positive values of Z range .01 to .1

(4) q to r for positive values of Z range .1 to 1.0

(5) r to s for positive values of Z range 1.0 to 10.0.

The invention, insofar as it relates to the solution of cubic equations, is based on the appreciation of the fact that a cubic equation of the form specified can be reduced to a related equation of the form also specified and that then, the roots of the related equation can be solved by mathematical relationships involving only one variable.

Also according to the invention, therefore, a slide rule calculator includes such H scales marked in relationship to other scales to enable at least one real root of the related equation of a given cubic equation to be directly ascertained by positive movement. If H scales corresponding to Z values of −.1 to −10.0 only are used then in nearly all cases it will be possible to evaluate a real root and the equation eventually converted to a quadratic equation and the solution completed by evaluating the roots of that quadratic equation in accordance with the invention, such scales as are necessary being, if desired, provided on the same rule. Alternatively H scales covering the full range of Z values hereinbefore specified may be used.

In order to illustrate the invention further the addition of scales enabling the roots of cubic equations to be evaluated to the slide rule previously described for the solution of quadratic equations will now be described.

Scales H_{on}, H_{nm}, H_{pq}, H_{qr} and H_{rs} numbered according to H values (positive or negative) will be prepared, corresponding to each section of the graph, by the following method:

A table of values of H for specific values of Z (positive or negative) is prepared by direct substitution in equation 7 and then, using selected Z values for purposes of inverse interpolation, Z values (positive or negative) for specific values of H are obtained.

Then, to the reference logarithmic unit of the D scale, line dimensions, from a datum, for H values on the scales to serve for portions o.n and n.m, i.e. for Scales H_{on} and H_{nm} will be according to

$$\text{Log}\left[-(Z)\right]$$

and for portions p.q, q.r and r.s, i.e. for scales H_{pq}, H_{qr}, and H_{rs} will be according to

$$\text{Log } Z$$

and assuming the scales are arranged on the stock of the slide rule, the datum of each of the scales will be in alignment with D_1 and run towards D_{10}.

Z values by projection from the new scales to the significant figure value D scale are as follows:

For	H_{on}	....	−.1 to −1.0
	H_{nm}	....	−1.0 to −10.0
	H_{pq}	....	+.01 to +.10
	H_{qr}	....	+.1 to 1.0
	H_{sr}	....	+1.0 to 10.0

In an alternative arrangement, the scales are applied as reciprocals of H_{on}, H_{nm}, H_{pq}, H_{qr} or H_{rs} by longitudinal inversion.

In other embodiments either the direct or reciprocal forms of H_{on} and H_{nm} may be produced as a continuous scale and related to the

A and B scales of the slide rule. The same applies to H_{pq}, and H_{qr} or H_{qr} and H_{rs}. Similarly scales H_{pq}, H_{qr} and H_{rs} may be continuous and related to a scale such as an F scale, which comprises three logarithmic sections, 1 to 10, 10 to 100 and 100 to 1000, each being one third the length of the unit D scale. 1 on the F scale aligns with 1 on the D scale and 1000 on the F scale with 10 on the D scale.

When H is a positive and equal to or greater than $\sqrt[3]{6.75}$, viz. 1.88988 the related equation has three real roots. For values of a, b and c that result in $H=\sqrt[3]{6.75}$, of the three real roots, two are equal.

In cases where H is less than $\sqrt[3]{6.75}$, of the three roots only one is real, namely, that which relates to the o, n, m portion of the graph.

The following examples will serve to illustrate the manipulation of the special cubic scales.

EXAMPLE 3:

To determine the real root or real roots of the cubic equation

$$x^3-16x^2+73x-90=0.$$

In this case with reference to equation (5)

$$a=-16 \qquad b=73 \qquad c=-90$$

By normal arithmetical approach evaluate

$$f, \text{ from equation (8)}=12.33$$

and

$$g, \text{ from equation (9)}=-4.074$$

The values of $\sqrt[3]{g}$ and $\sqrt[3]{g^2}$ may be found on the D and A scales respectively in alignment with the appropriate "g" value on the F scale, viz.

$$\sqrt[3]{-4.074}=-1.597, \quad \sqrt[3]{g^2}=2.550$$

Then evaluate

$$H=\frac{f}{\sqrt[3]{g^2}}=\frac{12.33}{2.550}=4.836$$

Since this H value is positive and greater than $\sqrt[3]{6.75}$, there are three real roots.

H values of +4.836 will be found on the H_{nm}, H_{qr} and H_{rs} scales, and using the cursor for projection to the D scale, the respective Z values, viz −2.296, .2086 and +2.088, may be obtained.

Then since

$$x=\sqrt[3]{g}\,.\,Z-\frac{a}{3}$$

and

$$\sqrt[3]{g}=1.597 \text{ and } a=-16$$

Where

x_1, x_2, and x_3 are the roots

$$x_1=(-1.597)(-2.296)-\left(\frac{-16}{3}\right)=3.67+5.33=9$$

$$x_2=(-1.597)(.2086)-\left(\frac{-16}{3}\right)=-.333+5.333=5$$

$$x_3=(-1.597)(2.088)-\left(\frac{-16}{3}\right)=-3.333+5.333=2$$

Thus the values of x which satisfy the equation are 9, 5 and 2.

When the H value lies within the range 10.01 and 100, the H_{nm}, H_{pq}, and H_{rs} scales would be involved.

EXAMPLE 4:

Solve:

$$x^3+6x^2+16x-38=0$$

with reference to equation (6)

$$a=6 \qquad b=16 \qquad c=-38$$

then

f from equation (8) = −4
g from equation (9) = −54
$\sqrt[3]{g}$ from the slide rule = −3.78
$\sqrt[3]{g^2}$ from the slide rule = 14.29
H from equation (10) = −.28.

Here H is less than $\sqrt[3]{6.75}$ viz. −1.889 so there is only one real root. From the slide rule

when H = −.28
then Z = −.907.

This is obtained by cursor projection from the −.28 line on the H_{on} scale to a D scale numbered −.1 to −1.0.

Then x from equation (11) since g, Z and a are known,

$$=(=3.78)(-.907)-\frac{6}{3}=1.428.$$

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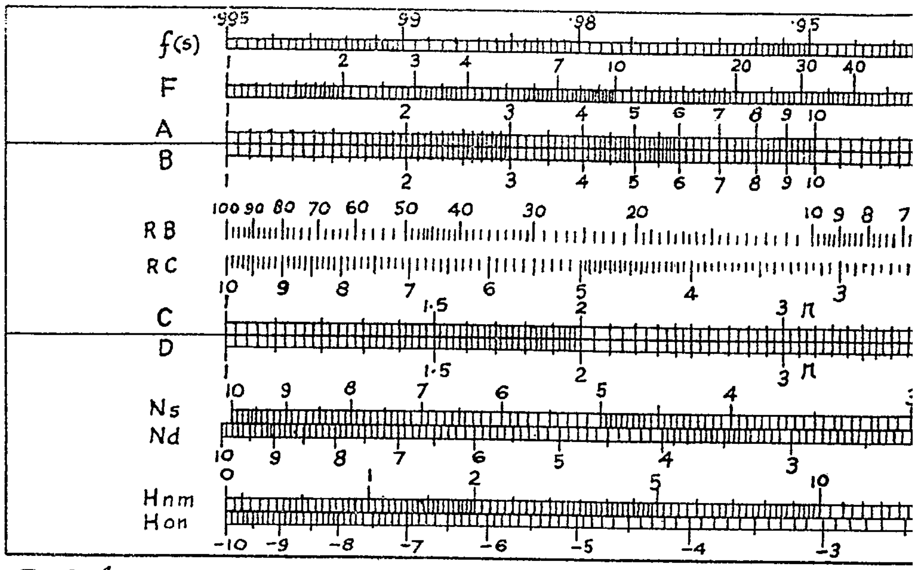

FIG. 1

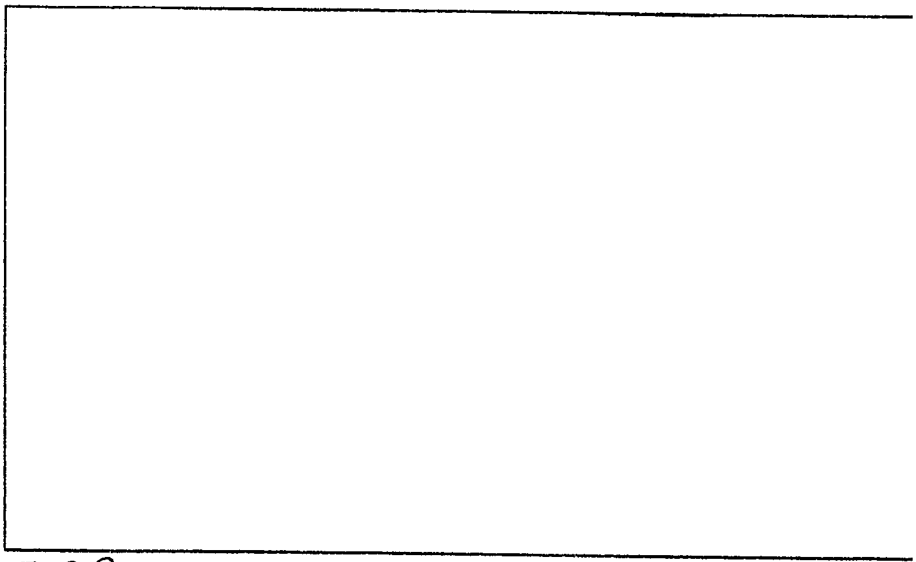
FIG. 2

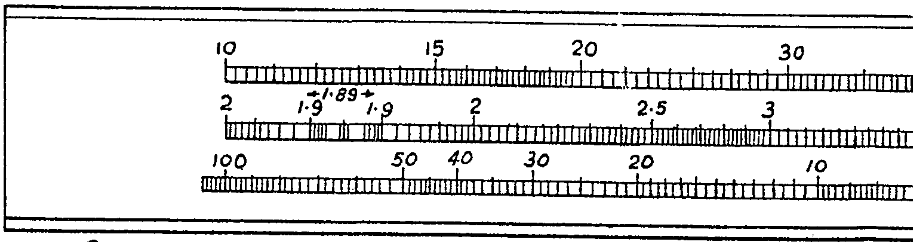

FIG. 3

878,056 COMPLETE SPECIFICATION

1 SHEET

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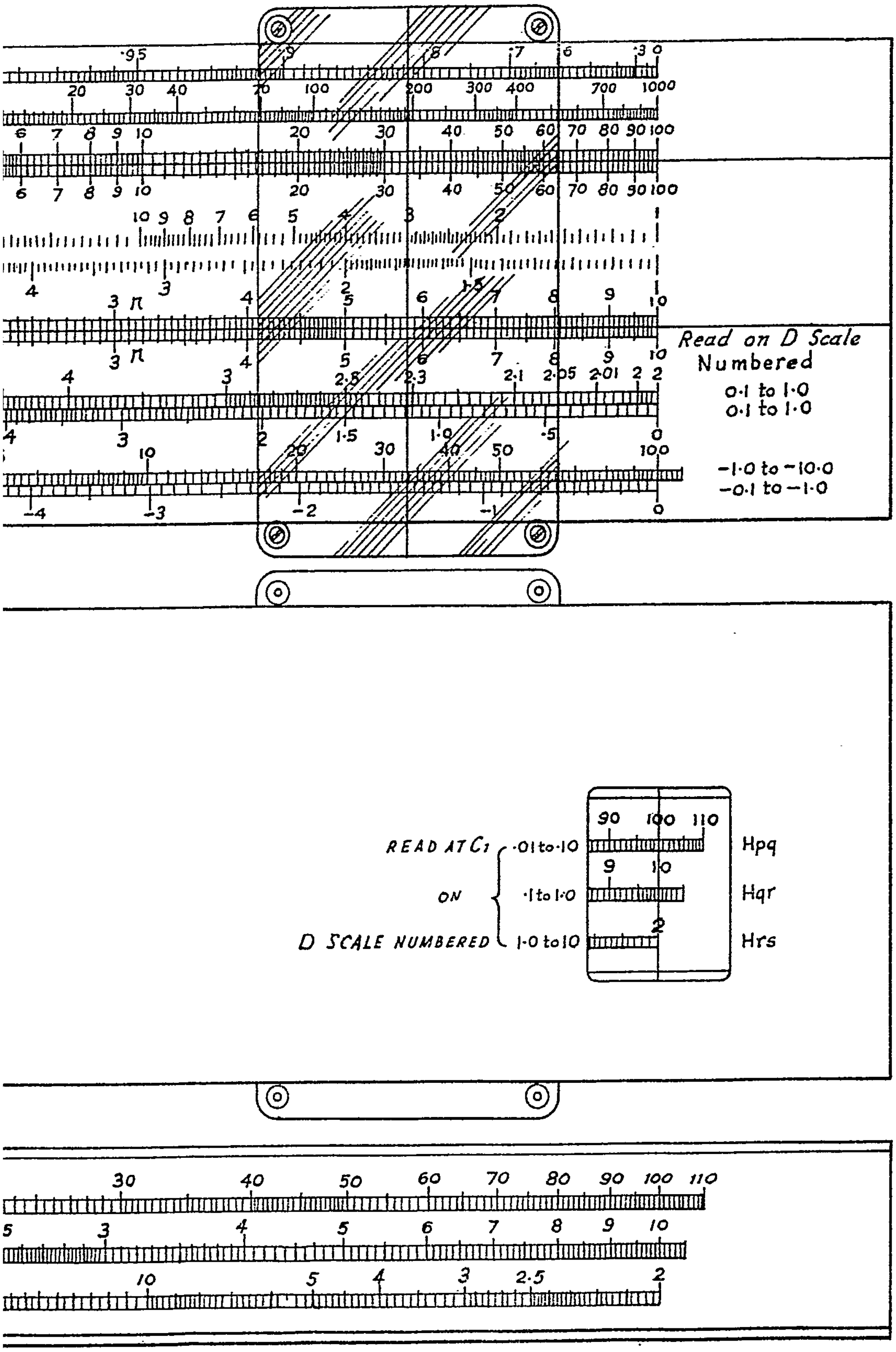

878,056 COMPLETE SPECIFICATION
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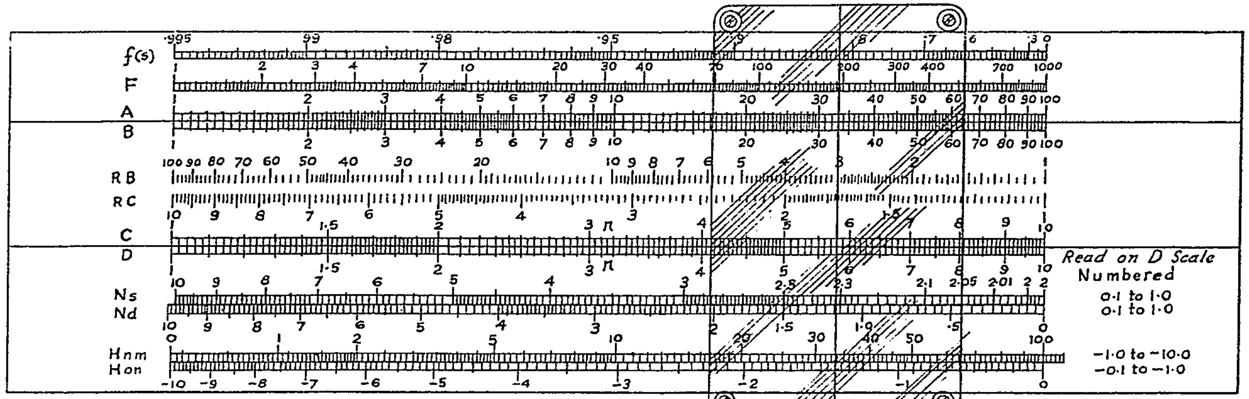

FIG. 1

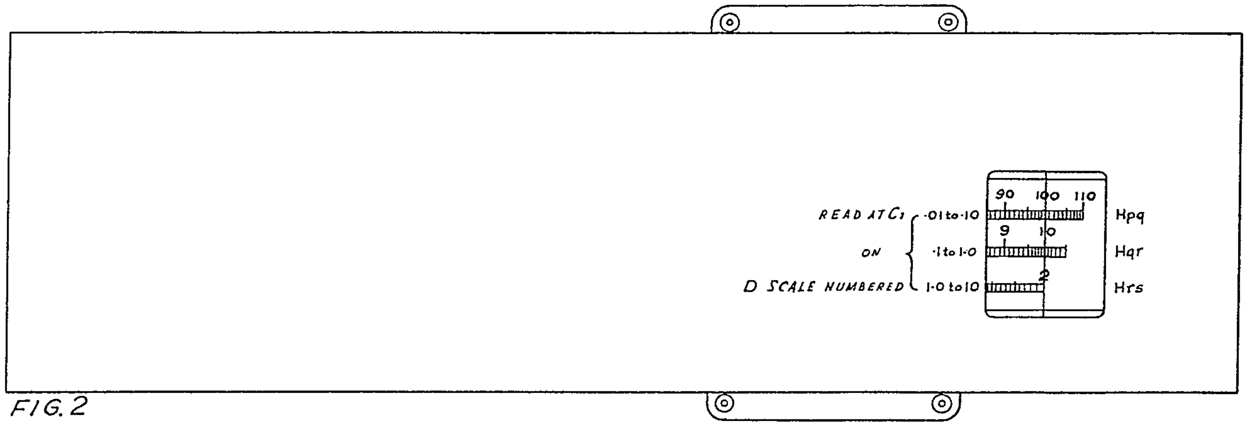

FIG. 2

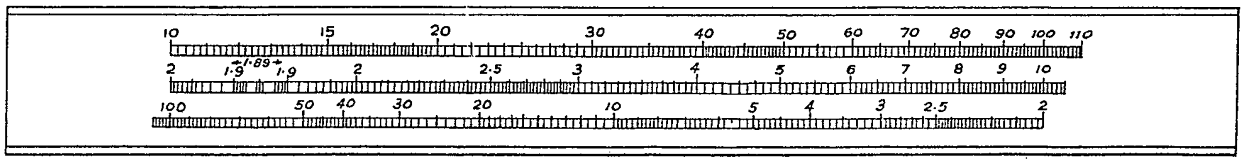

FIG. 3

878,056 PROVISIONAL SPECIFICATION
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