PATENT SPECIFICATION

Application Date: May 1, 1919. No. 10,869/19. **160,186**

Complete Left: Oct. 31, 1919.

Complete Accepted: Mar. 24, 1921.

PROVISIONAL SPECIFICATION.

Improvements in or relating to Logarithmic Computing Apparatus.

I, FRANCIS JAMES ANDERSON, C.B., Brigadier-General, on the active list of His Majesty's Army, of 4, Trebovir Road, Kensington, in the County of London, do hereby declare the nature of this invention to be as follows:—

This invention relates to logarithmic computing apparatus especially adapted for the use of statisticians and others whose work involves laborious arithmetical calculations.

According to this invention the apparatus comprises two distinct but related devices, one of which, serving for the actual calculating work, is of the general type described in the Specification of my Patent No. 22,762 of 1903, including a pair of discs with spirally arranged scales, with which is associated the second and smaller device, hereinafter referred to as a finder and consisting of a pair of concentric circular slide rules with contiguous scales and with sectors corresponding to the number of convolutions of the spirals in the main device, and marked to indicate the convolution to be observed in effecting the calculation on the main device. The small circular slide rule enables a rough calculation to be made very expeditiously so that on using the main device the part of the latter in which the full answer is to be found is indicated with certainty and without the delay which would usually be involved by the unassisted use of the main device. The finder therefore serves as a key to the results.

In a convenient form of finder the two concentric discs form an ordinary circular, decimally-divided, slide rule, with contiguous scales, but the outer disc (forming the standing part or limb of the rule) is divided into the aforesaid sectors, distinguished from each other by alternately thick and thin marginal circumferential bands, corresponding with alternately thick and thin convolutions of the spirals on the main device, and are further differentiated by carrying reference letters corresponding with lettering of the convolutions of the main device.

The inner disc, forming the slide of the finder may be removed and replaced by an alternative disc bearing on one face four concentric circles carrying square roots from 1 to 100, and on the other four similar circles carrying cube roots from 1 to 21.54. The use of this alternative disc, which can be clamped to certain fixed positions on the limb by a screw and slot connection, will be apparent from the description below. A transparent celluloid or other traveller is provided for use with this disc.

The main device, hereinafter referred to for convenience as "the abax", consists of two larger discs, the upper or slide concentrically pivoted on the lower or limb and capable of angular or rotary motion relatively to the latter. These two discs are marked with concentric plane spirals of a similar number of convolutions and graduated with similar decimally-divided logarithmic scales.

The positions of the various graduations are arrived at by multiplying the logarithms of the various numbers involved by some multiple of 360, and treating the results as degrees of angular measurement. In a typical instrument, a multiplier of 3,600 has been adopted, thus giving 10 complete convolutions of the spiral. The index (or radial line joining the graduations 1 and 10 respectively) on the slide (or inner disc) is continued outwards to command the spiral scale on the limb by means of an index-arm so attached to the slide that

its bevelled edge is in radial prolongation of the index of the scale on the latter.

This index-arm may carry a radial scale which serves to locate the position of any desired number on the limb and may also carry a series of parti-coloured bands indicating the quadrant of the limb in which any desired number is to be found. It preferably is provided with a self-clamping attachment by means of which it automatically clamps itself in any position on the limb to which it may be set. A touch of the finger releases this clamp, enabling the index-arm to be set afresh.

Pivoted at the centre of the instrument is a transparent traveller of celluloid, glass, or like material, carrying a radial hair-line, which commands the scales on both the slide and limb. It also bears (over both spirals) a series of radial divisions, differentiated by reference letters corresponding with the various convolutions of the spirals; thus, the inner radial division, bearing the letter A, covers the graduations from 1 to 1.2589 (as inscribed on the innermost convolution); the next radial division, marked B, covers the numbers 1.2589 to 1.585 (as inscribed on the second convolution); and so on up to K, which covers the graduations of the last, or outer, convolution from 7.9435 to 10. This traveller may also carry a self-clamping attachment similar to that on the index-arm.

The abax may, like the finder, be fitted with an alternative inner disc, bearing, on one face, square roots, and, on the other, cube roots, on similar lines to those described in the case of the finder.

The working of the combination is illustrated by the following examples, and it will be seen that the abax is governed by the same rules as the finder, the latter giving rough approximate answers, thereby establishing the exact spot at which the answers are to be read on the abax.

Multiplication.

Rule. Set the index of the slide to the first factor on the limb; opposite the second factor on the slide read the answer on the limb.

Example (1): Multiply 73.7 by 29.2;

On the finder:

I. Set index of slide to
II. 73.7 (as near as can be judged) on limb;
III. Opposite 29.2 on slide
IV. Read approximate answer 2160 on limb, noting that it is opposite a thin marginal line bearing the letter D.

On the abax:

I. Set index-arm of slide to
II. 73.7 on limb (being guided to its position by the radial scale on the index-arm), where it automatically clamps itself;
III. Opposite 29.2 on slide (guided by hair-line of traveller)
IV. Read answer 2152 on limb on the thin convolution D of the spiral (as ascertained by the finder), its position being identified by the scale of letters on the traveller.

Example (2): Multiply 11.7, 403.2, 217.4, and 861 by the constant 0.2913:

On the finder:

Setting the index to the constant (which we treat as the first factor) on the limb, where it remains clamped, we read the approximate answers in rotation as;
3.4 (opposite thin line F),
118 (opposite thick line A),
63.1 (opposite thin line H), and
250 (opposite thick line E).

On the abax:

Setting the index-arm to the constant, and using the hair-line of the traveller for alignment opposite the other factors in rotation, we read (guided by the approximate results obtained on the finder) the following answers on the limb;
3.408 on thin convolution F,
117.42 on thick convolution A,
63.33 on thin convolution H, and
250.8 on thick convolution E.

Division.

Rule. Set the divisor on slide to dividend on limb; opposite index of slide read answer on limb.

Example (1): Divide 197 by 831.29;

On the finder:

I. Set divisor 831 on slide
II. To dividend 197 on limb;
III. Opposite index of slide
IV. Read approximate answer .236 on limb, opposite thin marginal line D.

On the abax:

I. Using traveller for alignment, set 831.29 on slide
II. To 197 on limb;
III. Under index-arm
IV. Read answer .2369 on limb on thin convolution of spiral marked D.

Example (2): What is the birth-rate per mille if there be 728 births in a population of 29,835?

On the finder:

I. Set divisor 298 on slide
II. To dividend 728 on limb;
III. Opposite index of slide
IV. Read approximate answer 24.4 on limb, opposite thin marginal line D.

On the abax:

I. Using traveller for alignment, set 29835 on slide
II. To 728 on limb;
III. Under index-arm
IV. Read answer 24.40 on limb on thin convolution D.

Example (3): In a population of 40,245 there are 537 deaths in a year. What is the death-rate per mille?

On the finder:

I. Set divisor 4025 on slide
II. To 537 on limb;
III. Opposite index of slide
IV. Read approximate answer 13.3 on limb, opposite thin marginal line B.

On the abax:

I. Using traveller for alignments, set 40,245 on slide
II. To 537 on limb;
III. Under index-arm.
IV. Read answer 13.342 on limb on thin convolution B.

Proportion.

Rule. Set first term on slide to second term on limb; opposite third term on slide read answer on limb.

Example: 1.413: .004 : : 7 : ?

On the finder:

I. Set 141 on slide
II. To 4 on limb;
III. Opposite 7 on slide
IV. Read answer .019 approximately on limb opposite thick marginal line C.

On the abax:

I. Set hair-line of traveller to .004 on limb, and holding there,
II. Revolve the slide, by means of its index-arm, until 1.413 on it is under the hair-line;
III. Release the traveller and set it to 7 on slide.
IV. Read answer .01982 on limb, under hair-line of traveller, on thick convolution C.

Squares and Square Roots (on finder only).

Remove the slide of the finder and substitute for it the alternative disc provided (keeping the face marked square roots uppermost); clamp the index of the disc to that of the limb by means of a small screw which engages a slot in the limb.

The disc bears four circles, and it will be found that the numbers on the innermost circle are the square roots of those radially opposite to them on the limb (*i.e.* from 1 to 10); those on the next circle are the square roots of those radially opposite multiplied by 10 (*i.e.* from 10 to 100); those on the third circle are the square roots of those opposite multiplied by 100 (*i.e.* from 100 to 1,000); while those on the outer circle are the square roots of those opposite multiplied by 1,000 (*i.e.* 1,000 to 10,000). The converse of course holds good.

Examples: Using the small traveller provided for alignment, we read:

Square roots on disc:	Numbers on limb:
2.6 (on inner circle)	5.29
4.93 (on second circle)	24.3
9.15 (on third circle)	83.7
88 (on outer circle)	7744

and conversely

Squares on limb:	Numbers on disc:
4.25	2.06 (on inner circle)
92.5	9.62 (on second circle)
120	10.92 (on third circle)
3400	58.4 (on fourth or outer circle)

Radii & Areas of Circles (on finder only).

Unclamp the small screw of the disc, and clamp it in a second slot in the limb (to be found near the graduation 3.15 on the latter). Radii of circles on the disc will then have opposite to them on the limb the corresponding areas; thus

Radius on disc:	Area on limb:
2.7	22.90
5.15	81.7
30.3	2880

and conversely

Area on limb:	Radius on disc:
200	8
665	14.6
5.3	1.3

CUBES AND CUBE ROOTS (on finder only).

Reversing the disc so that the face marked "cube roots" is uppermost, we can read cubes and cube roots exactly on the same lines as squares and square roots; thus

Cube roots on disc:	Numbers on limb:
1.93 (on inner circle)	7.2
4.35 (on second circle)	82.3
9 (on third circle)	729
19 (on outer circle)	6860

and conversely

Numbers on disc:	Cubes on limb:
1.5 (on inner circle)	3.375
3.7 (on second circle)	50.6
8.4 (on third circle)	592
19.5 (on outer circle)	7415

Gauge points, or constants, may be inscribed in suitable positions on the scales of both the finder and the abax.

Dated this 1st day of May, 1919.

HASELTINE, LAKE & Co.,
28, Southampton Buildings, London, England, and
55, Liberty Street, New York City, U.S.A.,
Agents for the Applicant.

COMPLETE SPECIFICATION.

Improvements in or relating to Logarithmic Computing Apparatus.

I, Sir FRANCIS JAMES ANDERSON, K.B.E., C.B., Brigadier-General, on the active list of His Majesty's Army, of 4, Trebovir Road, Kensington, in the County of London, do hereby declare the nature of this invention and in what manner the same is to be performed, to be particularly described and ascertained in and by the following statement:—

This invention relates to logarithmic computing apparatus especially adapted for the use of statisticians and others whose work involves laborious arithmetical calculations.

According to this invention the apparatus comprises two distinct but related devices, one of which, serving for the actual calculating work, is of the general type described in the Specification of my Patent No. 22,762 of 1903, including a pair of discs with spirally arranged scales, with which is associated the second and smaller device hereinafter referred to as a finder and consisting of a pair of concentric members forming a circular slide rule with contiguous scales and with sectors corresponding to the number of convolutions of the spirals in the main device, and marked to indicate the convolution to be observed in effecting the calculation on the main device. The small circular slide rule enables a rough calculation to be made very expeditiously so that on using the main device (hereinafter referred to as the abax) the part of the latter in which the full answer is to be found is indicated with certainty and without the delay which would usually be involved by the unassisted use of the abax. The finder therefore serves as a key to the results.

In order that the said invention may be clearly understood and readily carried into effect the same will now be described more fully with reference to the accompanying drawings in which:—

Figure 1 is a plan of the main computing device or abax.

Figures 1^a and 1^b are detail views of parts to a larger scale.

Figure 2 is a plan of the finder or auxiliary computing device, and

Figures 3 and 4 are views of the two sides of a disc to be used with the auxiliary device for the purpose of obtaining square and cube roots, areas of circles and other results of computation.

A is the base or stationary disc of the main device. B is the movable disc or slide concentrically pivoted at *b* on the base or limb. C is the index arm rigidly mounted on the disc B and having its edge *c* continuous with the radial line b^1 on the inner disc. D is a transparent traveller also pivoted at *b*, marked with a fine hair line *d* which extends over both the slide and the limb. E, E are the spiral convolutions on the limb A. F, F^1 are the spiral convolutions on the slide B, the convolutions F^1 being marked in thicker lines so that the convolutions are alternately thick and thin for the pur-

pose hereinafter described, each convolution separating and terminating on the radial line b^1. G is the base or limb and H the slide or rotatable disc of the finder and J is the additional disc which may be used on the finder for obtaining squares, square roots, cubes, cube roots, and other results.

The finder in the form shown comprises the two concentric discs G and H, the disc H being mounted to turn round the central pin g on the limb G, a butterfly nut h or equivalent device retaining the disc H in position, while allowing it to turn as hereinafter described. The two concentric discs form an ordinary circular decimal divided slide rule with contiguous circular scales g^1 and h^1 divided to any practicable degree of fineness. Rough divisions only are shown in Figure 2 for the purpose of illustration. The outer disc or limb G is divided into ten sectors g^2 distinguished by the large letters A to K. The alternate letters are also distinguished from each other by the thick and thin circumferential bands g^3 and g^4. The sectors correspond with the alternately thick and thin convolutions of the abax and also with the lettering d^1 and d^2 on the transparent traveller D, the lettering d^1 having alternate thick bands d^3 to correspond to the thick bands g^3 of Figure 2, thus assisting the operator in finding the correct convolution of the abax according to the rough solution on the finder as hereinafter described.

Referring to the abax shown in Figure 1, the positions of the various graduations of the two sets of spiral convolutions E and F, F^1 are arrived at by multiplying the logarithms of the various numbers involved by some multiple of 360, and treating the results as degrees of angular measurement. In the typical instrument shown, a multiplier of 3,600 has been adopted,thus giving 10 complete convolutions of the spiral. The index b^1 (or radial line joining the graduations 1 and 10 respectively) on the slide B is continued outwards to command the spiral scale E on the limb A by means of the index-arm C so attached to the slide B that its bevelled edge is in radial prolongation of the index b^1 as already mentioned.

This index arm C (shown to a larger scale in Figure 1^b) may carry a radial scale c^1 which serves to locate the position of any desired number on the limb A and may also carry a series of parti-coloured bands c^2 indicating the quadrant of the limb in which any desired number is to be found. The bands c^2 are arranged in four colours, say, black, red, blue and yellow, and the quadrants of the limb are divided into corresponding colours by the small tabs or indicators a arranged at each quarter of the circle and each divided into two parts coloured to indicate the quadrant which lies between them.

The index-arm C which rotates with the inner disc or slide B is provided with an attachment in the form of a clip c^3 which engages the edge of the limb A and is released by the small knob c^4 under the control of a spring c^5 so that to turn the index-arm and slide the knob c^4 is pressed, releasing the clip and allowing the arm to turn. On releasing the knob the clip at once comes into operation and holds the index arm and slide in position. The transparent traveller D which moves over both the limb and the slide may be provided with a similar clip or a pair of similar clips which enable it to be secured to either the slide or the limb or both together.

The two sets of spirals on the limb and slide respectively are graduated as minutely as is found practicable, each having, as already described, ten convolutions. The main divisions are marked with large numerals as shown in Figure 1, the secondary divisions being marked with double numerals of smaller size. A third set of numerals is usually unnecessary but the divisions are sub-divided into as many parts as can be readily distinguished, a small portion of the divisions only being indicated in the drawing, which does not show the finer divisions as the actual device would usually be made to a larger scale than is shown in Figure 1.

The traveller D (shown to a larger scale in Figure 1^a) is of celluloid, glass or any other suitable transparent material and enables the graduations on the spirals below it to be readily seen, the hair line d being made accurately radial from the centre of the apparatus. The traveller bears over both spirals the lettered divisions d^1 and d^2 already referred to, these divisions corresponding with the various convolutions of the spirals, thus, the inner radial divisions, bearing the letter A, covers the graduations from 1 to 1.2589 (as inscribed on the innermost convolution); the next radial division, marked B, covers the numbers 1.2589 to 1.585 (as inscribed on the second convolution); and so on up to K, which covers the graduations of the last or outer convolution from 7.9435 to 10. The lettering d^1 of the traveller is provided with

alternate thick and thin bands to facilitate the finding of the proper convolution E. This is equivalent to the alternate thick and thin convolutions F F^1 on the slide. The convolutions E may instead be made alternately thick and thin.

The working of the combination is illustrated by the following examples, and it will be seen that the abax is governed by the same rules as the finder, the latter giving rough approximate answers, thereby establishing the exact spot at which the answers are to be read on the abax.

MULTIPLICATION.

RULE. Set the index of the slide to the first factor on the limb; opposite the second factor on the slide read the answer on the limb.

Example (1): Multiply 73.7 by 29.2;

On the finder:

I. Set index of slide H to
II. 73.7 (as near as can be judged) on limb G;
III. Opposite 29.2 on slide
IV. Read approximate answer 2160 on limb, noting that it is opposite a thin marginal line bearing the letter D.

On the abax:

I. Set index-arm C of slide B to
II. 73.7 on limb A (being guided to its position by the radial scale c^1 on the index-arm), where it automatically clamps itself by the clip c^3;
III. Opposite 29.2 on slide B (guided by hair-line *d* of the traveller D)
IV. Read answer 2152 on limb A on the thin convolution "D" of the spiral E (as ascertained by the finder), its position being identified by the scale of letters d^1 on the traveller D.

Example (2): Multiply 11.7, 403.2, 217.4, and 861 by the constant 0.2913:

On the finder:

Setting the index to the constant (which we treat as the first factor) on the limb G, where it remains clamped, we read the approximate answers in rotation as;
3.4 (opposite thin line F),
118 (opposite thick line A),
63.1 (opposite thin line H), and
250 (opposite thick line E).

On the abax:

Setting the index arm C to the constant, and using the hair-line *d* of the traveller D for alignment opposite the other factors in rotation, we read (guided by the approximate results obtained on the finder) the following answers on the limb;
3.408 on thin convolution F,
117.42 on thick convolution A,
63.33 on thin convolution H, and
250.8 on thick convolution E.

DIVISION.

Rule. Set the divisor on slide to dividend on limb; opposite index of slide read answer on limb.

Example (1): Divide 197 by 831.29;

On the finder:

I. Set divisor 831 on slide
II. To dividend 197 on limb,
III. Opposite index of slide
IV. Read approximate answer .236 on limb, opposite thin marginal line D.

On the abax:

I. Using traveller for alignment, set 831.29 on slide
II. To 197 on limb;
III. Under index-arm
IV. Read answer .2369 on limb on thin convolution of spiral marked "D".

Example (2): What is the birth-rate per mille if there be 728 births in a population of 29,835?

On the finder:

I. Set divisor 298 on slide
II. To dividend 728 on limb,
III. Opposite index of slide
IV. Read approximate answer 24.4 on limb, opposite thin marginal line D.

On the abax:

I. Using traveller for alignment, set 29835 on slide
II. To 728 on limb;
III. Under index-arm
IV. Read answer 24.40 on limb on thin convolution D.

Example (3): In a population of 40,245 there are 537 deaths in a year. What is the death-rate per mille?

On the finder:

I. Set divisor 4025 on slide

II. To 537 on limb;
III. Opposite index of slide
IV. Read approximate answer 13.3 on limb, opposite thin marginal line B.

On the abax:

I. Using traveller for alignment, set 40,245 on slide
II. To 537 on limb;
III. Under index-arm.
IV. Read answer 13.342 on limb on thin convolution B.

PROPORTION.

Rule. Set first term on slide to second term on limb; opposite third term on slide read answer on limb.

Example: 1.413: .004 : : 7 : ?

On the finder:

I. Set 141 on slide
II. To 4 on limb;
III. Opposite 7 on slide
IV. Read answer .019 approximately on limb opposite thick marginal line C.

On the abax:

I. Set hair-line of traveller to .004 on limb, and holding it there,
II. Revolve the slide, by means of its index-arm, until 1.413 on it is under the hair-line;
III. Release the traveller and set it to 7 on slide.
IV. Read answer .01982 on limb, under hair-line of traveller, on thick convolution C.

SQUARES AND SQUARE ROOTS (on finder only).

Remove the slide H of the finder and substitute for it the alternative disc J provided (keeping the face marked "square roots" (Figure 3) uppermost); clamp the index of the disc to that of the limb G by means of a small screw through the holes j and g^5 in the disc J and limb G.

The disc J bears four circles, and it will be found that the numbers on the innermost circle are the square roots of those radially opposite to them on the limb G (*i.e.* from 1 to 10); those on the next circle are the square roots of those radialy oppositely multiplied by 10 (i.e. from 10 to 100); those on the third circle are the square roots of those opposite multiplied by 100 (*i.e.* from 100 to 1,000); while those on the outer circle are the square roots of those opposite multiplied by 1,000 (*i.e.* 1,000 to 10,000). The converse of course holds good.

Examples: Using a small traveller or thread for alignment, we read:—

Square roots on disc J:	Numbers on limb G:
2.6 (on inner circle)	5.29
4.93 (on second circle)	24.3
9.15 (on third circle)	83.7
88 (on outer circle)	7744

and conversely

Squares on limb G:	Numbers on disc J:
4.25	2.06 (on inner circle)
92.5	9.62 (on second circle)
120	10.92 (on third circle)
3400	58.4 (on fourth or outer circle)

RADII & AREAS OF CIRCLES (on finder only).

Unclamp the disc J, and clamp it at the hole j in a second hole g^6 in the limb (to be found near the graduation 3.15 on the latter). Radii of circles on the disc will then have opposite to them on the limb the corresponding areas, thus

Radius on disc:	Area on limb:
2.7	22.90
5.15	81.7
30.3	2880

and conversely

Area on limb:	Radius on disc:
200	8
665	14.6
5.3	1.3

CUBES AND CUBE ROOTS (on finder only).

Reversing the disc so that the face marked "cube roots" (Figure 4) is uppermost, we can read cubes and cube roots exactly on the same lines as squares and square roots; thus

Cube roots on disc:	Numbers on limb:
1.93 (on inner circle)	7.2
4.35 (on second circle)	82.3
9 (on third circle)	729
19 (on outer circle)	6860

and conversely

Numbers on disc:	Cubes on limb:
1.5 (on inner circle)	3.375
3.7 (on second circle)	50.6
8.4 (on third circle)	592
19.5 (on outer circle)	7415

Gauge points, or constants, may be inscribed in suitable positions on the scales of both the finder and the abax.

Having now particularly described and ascertained the nature of my said inven-

tion and in what manner the same is to be performed, I declare that what I claim is:—

1. Logarithmic computing apparatus of the kind set forth, consisting of a main device comprising a fixed base and a rotatable disc both with spirally arranged scales forming a circular slide rule of high accuracy, and a second device or finder comprising a pair of concentric members, forming a circular slide rule with contiguous scales, and with sectors corresponding to the number of convolutions of the spirals in the main device and marked to indicate the convolution to be observed in effecting the calculation on the main device, a preliminary rough calculation being made upon the finder for this purpose.

2. In apparatus as claimed in Claim 1, an auxiliary disc on the finder, marked so as to indicate on one side the squares and square roots and on the other side cubes and cube roots, when associated with the base of the finder, the said disc being adapted to be secured in the required position on the base, for the purpose specified.

3. In apparatus as claimed in Claim 1, a transparent traveller mounted to turn over the two spiral scales of the main device and carrying radial divisions lettered to correspond to the convolutions of the two scales and a finder the base of which is divided into lettered segments corresponding to the lettering on the traveller, for the purpose specified.

4. Logarithmic computing apparatus arranged and adapted to operate substantially in the manner hereinbefore described with reference to the accompanying drawings, for the purpose specified.

Dated this 31st day of October, 1919.

HASELTINE, LAKE & Co.,
28, Southampton Buildings, London, England, and
55, Liberty Street, New York City, U.S.A.,
Agents for the Applicant.

Redhill: Printed for His Majesty's Stationery Office, by Love & Malcomson, Ltd.—1921.

[This Drawing is a reproduction of the Original on a reduced scale]

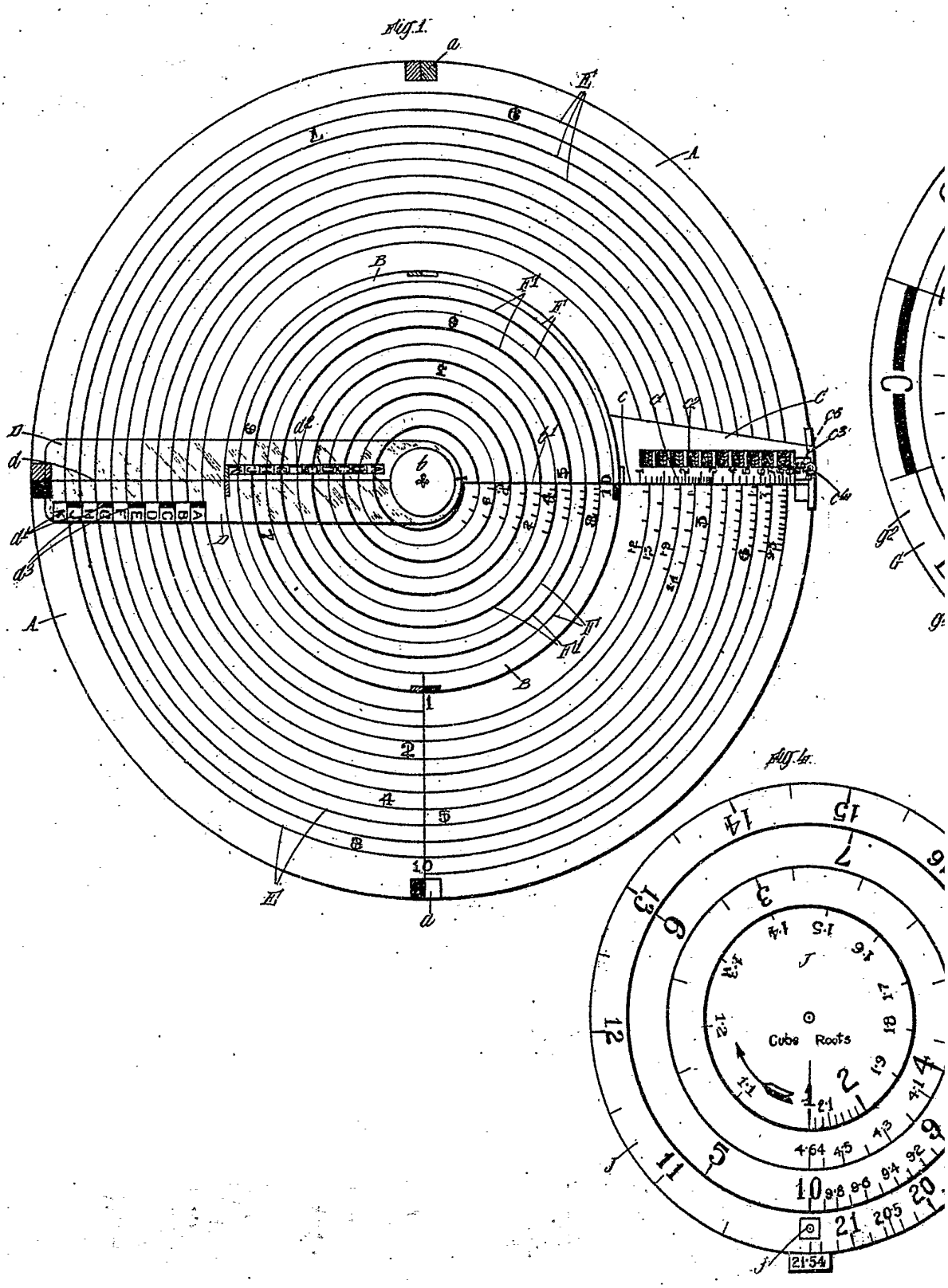


Fig. 2.

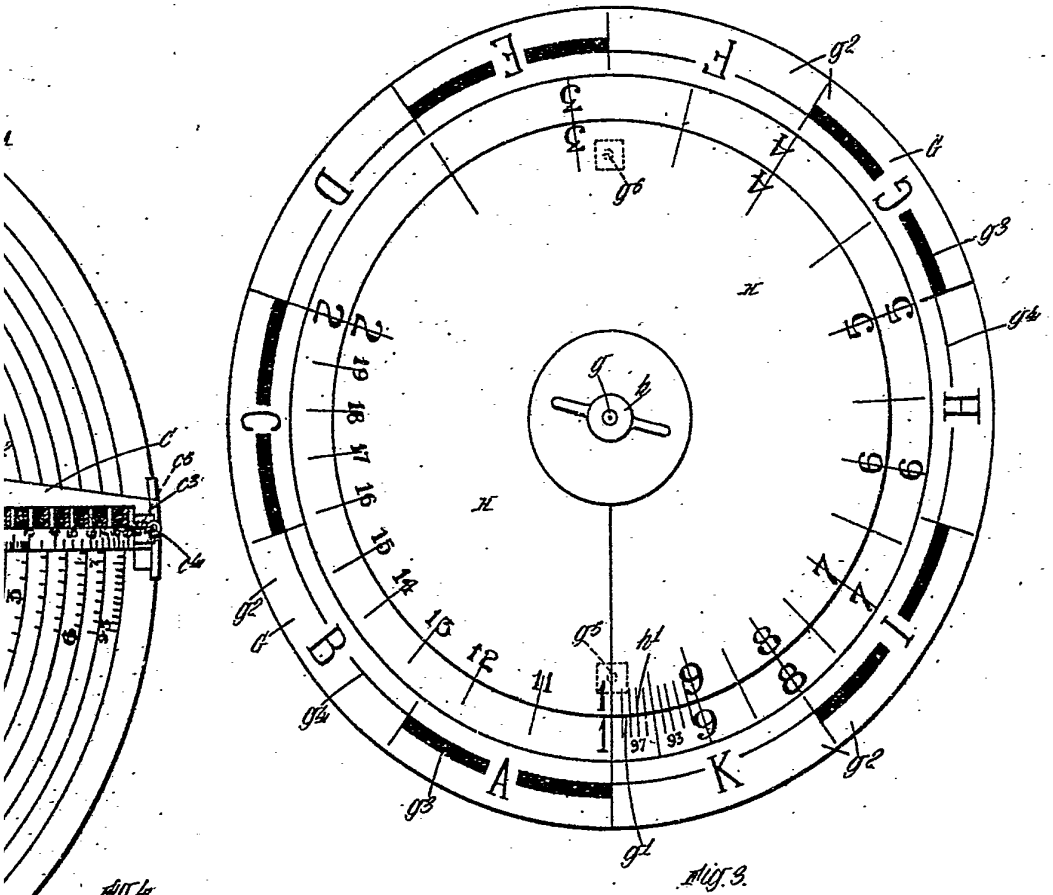
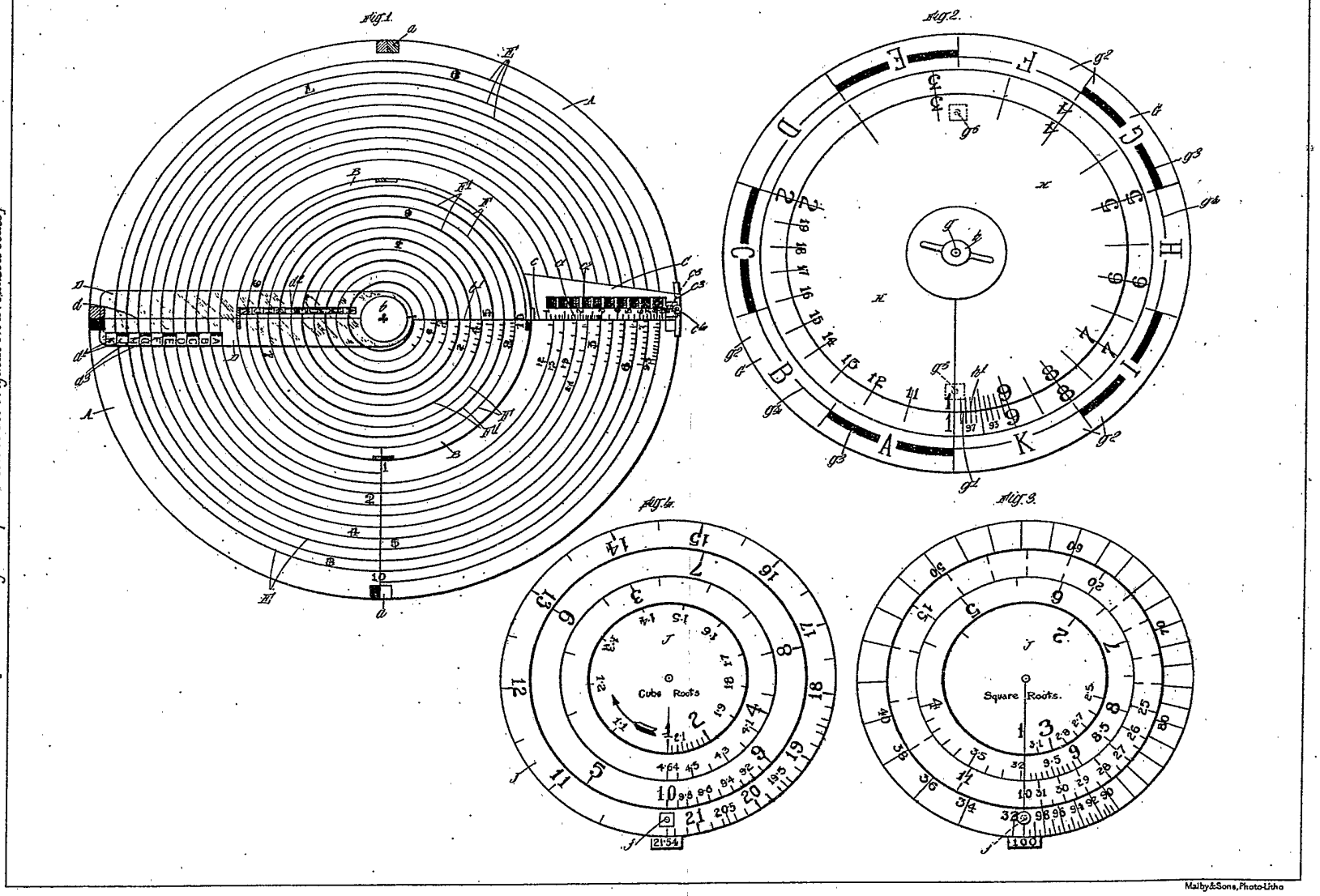


Fig. 4.

Fig. 3.

Malby&Sons, Photo-Litho

[This Drawing is a reproduction of the Original on a reduced scale.]



Malby&Sons, Photo-Litho

[This Drawing is a reproduction of the Original on a reduced scale.]

Fig. 1a

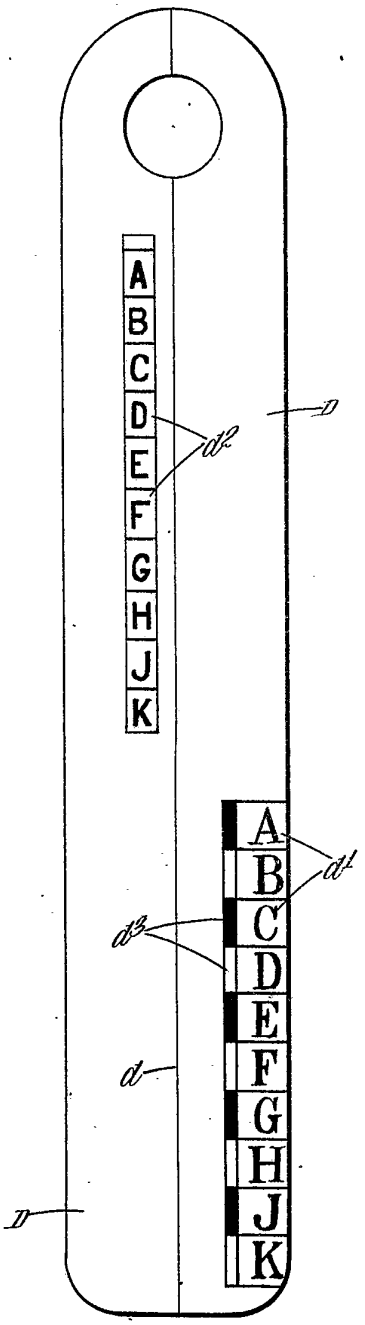
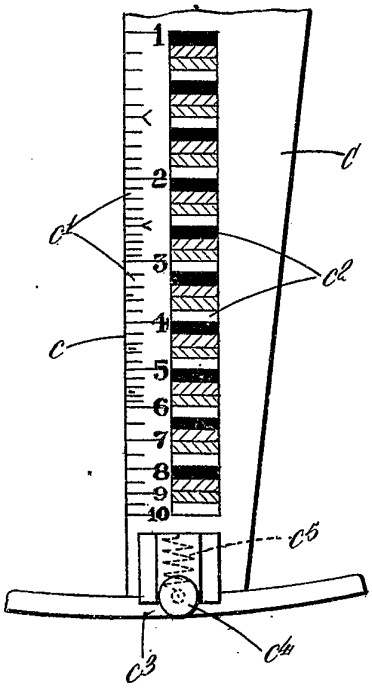


Fig. 1b



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