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II. RUN-OUT DISTANCES
III. CENTER-OF-MASS EARTHWORK COMPUTATIONS
CHAPTER III
EARTHWORK CALCULATIONS

On any highway or road construction project, the single largest cost item is almost always the earthwork. It is important, therefore, to make the best possible determination of the quantity of soil and rock materials that must be moved on the project. A number of different computational methods are used by various design organizations. The only method that will be discussed here will be the "Double End Area Method." This method is based on the determining the volume of a prism. The ends of this prism are represented by the cross sectional areas of cut and fill. The sides of the prism are the distances along the centerline.

I. The areas representing cut and fill respectively must be measured or calculated for each cross section.

A. Geometric Method. This method involves breaking up each area down into convenient geometric shapes. The areas of these shapes can then be determined. For forest road situations, the triangle is the most common geometric shape.

When finding the area of a triangle, there are usually two options for selecting the base and the height. Care must be exercised that the height of the triangle is perpendicular to the base.

If a ditch section is used in the design, the easiest method to account for it is to determine the area one time and treat it as a constant. This constant can then be added to each section where a ditch is utilized.
Area Triangle = \( \frac{1}{2} \) Base x Height = \( \frac{1}{2} \) B x H

B. Divider Method, Squares Method, "Avol Rule" Method, 50 Scale Method. These methods are all essentially the same. The squares method simply and painstakingly slowly counts the squares of cross section paper contained in each area of the cross section. The other three involve a mechanical technique for counting the squares.

Divide the area up with vertical lines drawn every five feet from centerline. The divider method measures the length of each vertical line, totaling their values and multiplying by the spacing.

Area Triangle (cut) = \( (O_1 + O_2) \times 5 \)
Area Triangle (fill) = \( (O_1 + O_2) \times 5 \)

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The "Avol Rule" is a commercially available plastic scale that automatically totals the vertical distance and multiplies by the spacing.

The 50 scale method involves measuring the vertical dimensions with the 50 scale of the engineer's scale. This system requires that the cross section be drawn at a scale of 1 inch equals 10 feet, and that the equal spacing of the vertical lines is five feet.

C. Digitizer Method. The most accurate and quickest method of determining the areas is the use of the electronic digitizer and the desktop calculator. Although few installations currently have this equipment available at the time of this writing, it will become much more common. It was not long ago that the hand held calculator was only for "university faculty and other nuts." The digitizer system electronically determines the coordinates of the various points on the cross section. These coordinates are then used to compute the areas. The areas are usually stored in the calculator to be used in later earthwork computations.

D. Matrix Method or Criss-Cross Method. This method treats the cross sections areas as traverses, and the various points along the cross section as coordinates. Standard Coordinate area computation procedures are used. A simplified procedure has been developed to facilitate the computation for the cross section cases.
- Gradeline @ \( \theta \) = \( \frac{0}{0} \)

1. Begin by computing the "coordinates where the groundline crosses the centerline. This step is too often not done by inexperienced users of the system, in which case the results are greatly in error.

2. Going in clockwise order, list the coordinates, left to right as shown below.

3. Carry out the mathematics as indicated in the examples.

\[
\begin{align*}
0 & \quad 8 & \quad 7.5 & \quad 5.2 & \quad 0 \\
0 & \quad 20 & \quad 36 & \quad 25.0 & \quad 5 \\
\end{align*}
\]

\[\begin{align*}
0 \times 0 & + 20 \times 8 & + 36 \times 7.5 & + 25 \times 5.2 & + 0 \times 5 = ? \\
0 & + 160 & + 270 & + 130 & + 26 & + 0 = 586 \\
\end{align*}\]

\[\begin{align*}
0 \times 20 & + 0 \times 36 & + 8 \times 25 & + 7.5 \times 0 & + 5 \times 0 = ? \\
0 & + 0 & + 200 & + 0 & + 0 & + 0 = \frac{200}{386} \\
\end{align*}\]

\[\frac{386}{2} = 193 \text{ ft}^2.\]

The fill area for this example is 193 ft\(^2\).

Discounting the digitizer because it's not universally available, the criss cross method is more accurate than the other methods. It is generally used to determine pay quantities. The planimeter method has been used to compute pay quantities when it is agreed to by both the contractor and the design agency.

E. Planimeter Method. This method involves a mechanical device to measure the area of the figure directly. Its accuracy is highly dependent on the operator and a number of other factors. The accuracy of a planimeter should be checked on each type of paper or drafting surface.
While determining the area one time with a planimeter is the fastest method, the accuracy is increased by repeating the measurement at least twice and preferably five times. The figures are then averaged to determine a more accurate figure.

Regardless of the method of measurement, the areas for the cut section and the fill section for each cross section must be measured and recorded individually.

Under certain circumstances, as when passing over a crest or when passing through a saddle, there may be two cut or fill areas separated by a fill or cut area, respectively. In these cases, each cut and fill area must be measured and recorded separately. The earthwork volume computations will include all three areas.

Refer to the EARTHWORK QUANTITY SHEET. For each cross section, record the station and the measured cut and fill areas. See items J,K,L.

II. There are a number of methods for calculating the volume of earthwork from the end areas. The method that is most often used for the forest road is the double end area method. What this method lacks in accuracy it makes up for in simplicity.

The double end area method determines the volume of a truncated prism. The computed volume may exceed the actual volume by as much as fifteen percent. The error is greatest when one end area is relatively large and the other is relatively small or zero.

Methods of correcting the volume computations exist, e.g., the prismoidal correction factor. For the low volume, low standard road, the corrections tend to balance out on the cut and fill areas for most cross

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sections. The additional effort required for the application of the correction factors is usually not justified except in computer calculated work.

In those cases where one end area value is zero, the formula for the volume of a prism is used.

\[
\text{Volume (yd}^3) = \frac{\text{End area (ft}^2)}{3} \times \text{distance (ft)} \times \frac{1}{27} \text{yd}^3
\]

Where the earthwork makes a transition from a full cut section or a full fill section, to either a cut-fill section or a full-fill or full cut section, a distance known as the run out distance needs to be determined. Refer to the appendix of this chapter for the discussion of the run out computations.

A. Determine the distance between cross sections by taking the difference between stations. Record this information in the appropriate column. See item M. Care must be exercised when the centerline of the roadway is on a horizontal curve. The centerline stationed distance does not represent the correct distance. Refer to the appendix discussion of this situation.

B. Sum the end areas of two adjacent stations. Record this information in the columns for the Double End Area. Keep the cut and fill areas separate. See columns N and O.

C. The Double End Area formula is as follows:

\[
\text{Volume} = \frac{\text{Area}_1 + \text{Area}_2}{2} \times \text{distance} \times \frac{1 \text{ cubic yard}}{27 \text{ cubic feet}}
\]

or

\[
\text{Volume} = \frac{(\text{Area}_1 + \text{Area}_2) \text{ distance}}{54}
\]
The volume of fill found by this equation can be entered on the EARTHWORK QUANTITY SHEET in the column EMBANKMENT and subtitled Actual. See item P. A running total of these volumes is carried in the next column, subtitled Total Accumulated. See item Q.

The volume of excavation computed in the above equation must be corrected for two special conditions.

The first is the Compaction or Shrinkage factor. Natural origins and placement of soils usually results in a relatively loose or uncompacted soil mass. This mass is not sufficiently strong to support the traffic loads of a roadway. During the construction process, the material is "remolded" by the equipment in such a way that the mass is more compact, more dense, and more able to carry the traffic loads. The price that must

![Diagram of soil layers and compaction]

ESTIMATED AVAILABLE MATERIAL: 500 ft³
ACTUAL AVAILABLE MATERIAL: 300 ft³

SHRINKAGE FACTOR = \frac{300}{500} = .6 \text{ or } 60\%
be paid for this is the reduction in the amount of common or soil material. A cubic yard may be compacted down to only two-thirds or three-quarters of a cubic yard. The percentage of this compaction is referred to as the compaction or shrinkage factor. The additional fact that material volume is lost because of root wads, duff, boulder holes, and other local problems may often result in further losses of material. The compaction factor should be determined by a competent soil mechanics specialist.

On the other side of the coin, when solid rock is excavated, or when larger boulders are excavated, broken or crushed and used in the fill, the resulting amount of fill material is greater than the cubic yardage of excavation.

ONE CUBIC FOOT OF ROCK
BEFORE EXCAVATION

AFTER EXCAVATION

EFFECTIVE VOLUME 1.5 ft³

\[
\text{SWELL FACTOR} = \frac{1.5}{1.0} = 150\%
\]

The amount of the increase in earth material as a percentage of the excavated material is often referred to as the "Swell Factor." The percentage

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increase is recorded on the EARTHWORK QUANTITY SHEET as a positive Compaction Factor. See item R.

The compaction factor considers a number of variables including losses due to root wads, duff and other clearing and grubbing items. Blasting, oversize, transportation losses and other construction practices also affect the compaction factor.

The second special condition to be accounted for is a consideration for solid rock. The percentage of material that is solid rock should be estimated by the locator, a geologist or soils specialist, or by the designer during a predesign walk of the "P" line. This percentage is recorded on the EARTHWORK QUANTITY SHEET in the appropriate column. The volume of solid rock is determined because of the construction cost differences between solid rock and common material.

D. Using the double end area formula from paragraph C above, compute the volume of excavation between two stations and record this volume in the Total EXCAVATION column. See item V.

E. Multiply the % Solid Rock times the Total EXCAVATION value to determine the volume of solid rock. Record the product in the Solid Rock Column. See item T.

The remainder of the excavation is considered to be common material. That volume is recorded in the column labeled U.

F. The Total EXCAVATION volume is then multiplied by the compaction factor to determine the amount of material that will be available for embankment construction. The resulting figure, the Adjusted Cut, is entered in the column labeled W.
G. The Total Accumulated Excavation column is simply a running total of the Adjusted Cut.

H. The final column on the EARTHWORK QUANTITY SHEET is the MASS ORDINATE. The mass ordinate is the sum of the Total Accumulated EXCAVATION and the Total Accumulated Embankment, where the embankment figure is negative. The mass ordinate at a station can be checked by taking the mass ordinate at the previous station, adding the Adjusted Cut between the two stations and subtracting the Actual EMBANKMENT.

Conceptually, the mass ordinate is the total excess or total shortage of earth material over the entire design up to the particular station. A positive mass ordinate indicates an excess of material available. A negative mass ordinate indicates a shortage of material.

Where the mass ordinate changes signs between two stations, a balance point exists. A balance point is the location where there is neither waste nor borrow over the project up to that point.

The exact station of a balance point can be determined by interpolation if necessary.

The MASS ORDINATE values will be used to develop the mass diagram which is the subject of the next chapter.
<table>
<thead>
<tr>
<th>STATION</th>
<th>Dist.</th>
<th>Cut End Area</th>
<th>Dbl. End Area</th>
<th>Fill End Area</th>
<th>Dbl. End Area</th>
<th>Compaction Factor</th>
<th>%SR</th>
<th>EXCAVATION</th>
<th>SOLID ROCK</th>
<th>COMMON</th>
<th>TOTAL</th>
<th>ADJUSTED CUT</th>
<th>TOTAL ACCUM.</th>
<th>EMBANKMENT</th>
<th>TOTAL ACCUM.</th>
<th>MASS ORDINATE</th>
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<tbody>
<tr>
<td>J</td>
<td>M</td>
<td>K</td>
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<td>L</td>
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COMPACTION FACTORS

The following is a guide for estimating compaction, shrinkage, and stripping losses for earthwork computations. Any combination can be used to determine the compaction factor.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>Below 25%</td>
<td>Through cuts</td>
</tr>
<tr>
<td></td>
<td>Previously cleared land</td>
</tr>
<tr>
<td></td>
<td>Extremely large cuts</td>
</tr>
<tr>
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<td>Rock excavation for deep fills</td>
</tr>
<tr>
<td></td>
<td>clay and clay-like shale soil</td>
</tr>
<tr>
<td>25% - 30%</td>
<td>Steep side slopes, greater than 50%</td>
</tr>
<tr>
<td></td>
<td>Light to medium size timber</td>
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<tr>
<td></td>
<td>Logged areas</td>
</tr>
<tr>
<td></td>
<td>Fractured rock or good shale fill material</td>
</tr>
<tr>
<td>30% - 35%</td>
<td>30% - 50% side slopes</td>
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<tr>
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<td>Medium to heavy clearing</td>
</tr>
<tr>
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<td>Medium size timber</td>
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<td></td>
<td>Sandy soil</td>
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<tr>
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<td>Fill-size boulders</td>
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<td>38% - 50%</td>
<td>Heavy clearing</td>
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<tr>
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<td>Large timber</td>
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<tr>
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<td>Low to medium percentage of large unusable boulders</td>
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<td></td>
<td>20% - 30% side slopes</td>
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<tr>
<td></td>
<td>Deep duff or organic soil</td>
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<tr>
<td>Over 50%</td>
<td>Flat ground</td>
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<td>High percentage of large unusable boulders</td>
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<td>Very small cuts or fills</td>
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<td>Old railroad on tram grades</td>
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