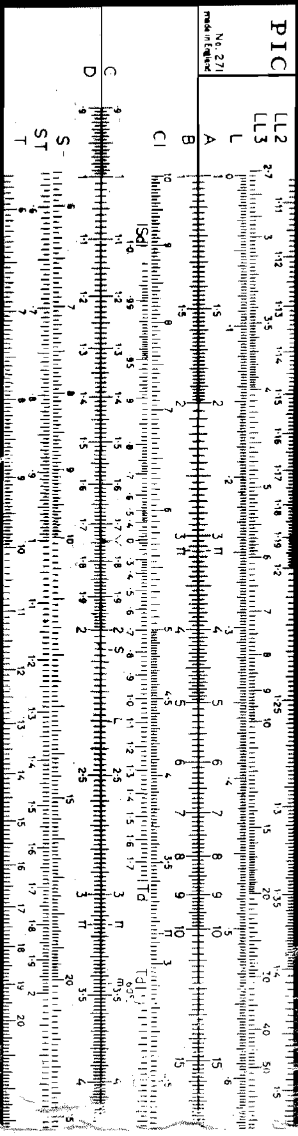
THORNTON

PIC Slide rules

Instructions for use



PIC slide rules Nos. 221, 271

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Instructions for use

ERRATA

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TO THE BEGINNER

It is easy to use a Slide Rule even though it may take practice to become really familiar with it. In using the various scales beginners will find it helpful to work out a simple problem which can be checked mentally before going on to more complicated calculations. In this way confidence and a proper understanding of the scales is quickly built up, together with an appreciation of the very great use which can be made of the slide rule.

Do not try to use the more advanced scales before you understand the basic scales and make a practice of rough checking your answer mentally — ask yourself 'Does it look right?' — and you will soon join the widening circle of slide rule initiates.

Introduction

Let us ignore all scales except the two identified by the letters C and D and which are located on the slide and stock respectively.

These two scales are easily the most frequently used on a slide rule and are the basic scales normally used for multiplication, division, ratios, etc.

Throughout these instructions references to scale subdivisions are related to the 25 cms (10 inch) models.

Closely inspect the C and D scales and note the numbering from left to right *viz:* 1, 1.1, 1.2, and so on 2, 3, 4 to 10. Possibly the beginner will find it advantageous to imagine the numbering as 100, 110, 120 . . . 200, 300, 400 . . . to 1,000, as this will assist in reading and setting the first three significant figures of numbers. With this imaginary numbering the range 100 to 200 is subdivided into 100 divisions, and adjacent line values in the third significant figure differ by *one.* The ranges 2 to 3, 3 to 4 and 4 to 5 are each subdivided into 50 divisions, and so adjacent lines differ in third significant figure value by *two.* Sections 5 to 6, 6 to 7, and so on to the end are each subdivided into 20, and so third significant place change is *five.* This *one, two, five* theme must be constantly observed and considered in assessing values of parts of a subdivision where a fourth significant figure is involved.

Now a word about the position of the decimal point. Usually you know the approximate value of the answer and therefore the position of the decimal point — if there is any doubt then do a rough calculation and decide the position of it by estimation.

The C and D scales are logarithmic scales numbered naturally and the following trial and observation lends emphasis to the important property of a logarithmic scale of numbers, namely, that it is the one and only scale that is of uniform proportional accuracy.

Move the slide so that 1 on C scale (*ie* C_1) aligns with 101 on D scale (*ie* D_{101}) and observe that:

C_2 aligns with D_{202}
C_3 aligns with D_{303}
C_5 aligns with D_{505}

ie a displacement of the C scale effects, along the entire length of the contact with the D scale, a fixed proportional relationship for all points in alignment.

Fig I

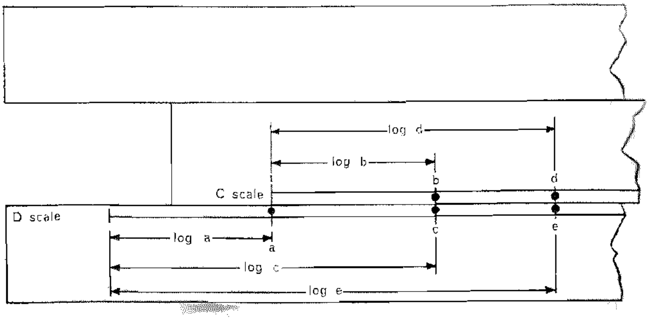


Fig 1 shows a C scale displaced in relation to the D scale and several transverse cursor lines are shown. From this we see that:

$\log a + \log b = \log. c$ *i.e.* $a \times b = c$

$\log a + \log d = \log. e$ $a \times d = e$

Alternatively

$\log c - \log b = \log a$ $\frac{c}{b} = a$

$\log e - \log d = \log a$ $\frac{e}{d} = a$

ie all values in alignment on the C and D scales respectively bear similar proportions to one another equal to '*a*'.

Multiplication and division

DIVISION

Example 1

Evaluate $\frac{84}{24}$

Set the cursor at 84 on the D scale and move the slide so that 24 on the C scale aligns with the cursor line. At C_1 read 35 on the D scale *ie* the significant figures of the value of $\frac{84}{24}$

Decimal point considered = 3.5

Example 2

Evaluate $\frac{30.6}{68}$

Set the cursor at 306 on the D scale and move the slide so that 68 on the C scale aligns with the cursor line. At C_{10} read 45 on the D scale, the significant figures of the value of $\frac{30.6}{68}$

Decimal point considered = .45

The two examples only differ in the respect that in (1) the answer aligns with C_1 on the D scale and in (2) the result is found at C_{10}.

MULTIPLICATION

Example 3 Evaluate 2.6×3.5

Move the slide bringing C_1 to D_{26}, set the cursor at C_{35} and read on the D scale 91, the significant figures of 2.6×3.5

Decimal point considered $2.6 \times 3.5 = 9.1$

Example 4 Evaluate 3.25×4.4

Cursor to D_{325}, C_{10} to cursor, cursor to C_{44} and read 143 on the D scale the significant figure value of 3.25×4.4, decimal point considered 14.3.

Again it will be noted that the only difference between examples 3 and 4 is in respect of applying either C_1 or C_{10}, the choice being such as to bring the second factor within the D scale range.

COMPOUND MULTIPLICATION and DIVISION

To evaluate a numerical expression of the type:

$$\frac{E \times G \times J}{F \times H}$$

(*i*) Set the cursor at 'E' on the D scale,

(*ii*) Move the slide so that 'F' on the C scale is at the cursor. (Observe for this position of the slide whether the 'G' reading on the C scale is in contact with the D scale).

If so

(*iii*) Move the cursor to 'G' on the C scale.

if not so

Carry out an intermediate sequence of operations known as 'End Switching' of the slide as follows:

(*iiia*) When the slide protrudes to the left of the stock move the cursor to C_{10} and then 'end switch' the slide by bringing C_1 to the cursor. Vice versa regarding C_1 and C_{10} when the slide protrudes to the right. Then 'G' on the C scale will be in contact with the D scale and so render operation (*iii*) possible.

(*iv*) Move the slide bringing 'H' on C scale to the cursor. (Observe the position of 'J' on the C scale, and if necessary, effect an 'end switch' operation).

(*v*) Move the cursor to 'J' on the C scale and read the significant figures of the compound value on the D scale at the cursor.

The sequence for example 5 (no 'end switch' involved), is tabulated so as to give significant figure results at each stage.

Example 5 Determine the value of:

$$\frac{161 \times 923 \times 152}{258 \times 172}$$

Instruction	Stage	Significant figures of result on D scale at: C_1	C_{10}	Cursor
(*i*) Set cursor at D_{161}	161			
(*ii*) Move slide bringing C_{258} to the cursor	$\frac{161}{258}$		624	
(*iii*) Cursor to C_{923}	$\frac{161}{258} \times 923$			576
(*iv*) Move slide bringing C_{172} to the cursor	$\frac{161 \times 923}{258 \times 172}$	335		
(*v*) Cursor to C_{152}	$\frac{161 \times 923 \times 152}{258 \times 172}$			509

decimal point considered 509.0

In the next example, if we take the factors as they occur, alternating from numerator to denominator etc, an 'end switch' is involved. Again instructions, stage etc, are tabulated to record intermediate values and an indication of where found.

Example 6 Determine the value of:

$$\frac{.0535 \times 741.0 \times 4.87}{.1925 \times .0524}$$

Instruction	Stage	Significant figures of result on D scale at:		
		C_1	C_{10}	Cursor
(*i*) Set cursor at D_{535}	535			
(*ii*) Move slide C_{1925} to cursor	$\frac{535}{1925}$	278		
'End switch' (*iii*) Cursor to C_1				
(*iv*) Move slide C_{10} to cursor				
(*v*) Cursor to C_{741}	$\frac{535 \times 741}{1925}$			206
(*vi*) Move slide C_{524} to cursor	$\frac{535 \times 741}{1925 \times 524}$		393	
(*vii*) Cursor to C_{487}	$\frac{535 \times 741 \times 487}{1925 \times 524}$			1914

decimal point considered 19140.0

Generalising for Compound Multiplication and Division we have:
First numerator value set on the D scale.
All other numerator and denominator values set on the slide.
Answer read on the D scale.

Movement of the *cursor* effects *multiplication*
Movement of the *slide* effects *division*
and these operations must take place alternately. If the cursor is moved last the 'result' is read at the cursor on the D scale. Where the slide is moved last the 'result' is the reading on the D scale against the C_1 or C_{10} line.

'*End switching*' is the equivalent of multiplying by 1 and dividing by 10 or vice versa. Thus these combined operations do not affect the significant figures of the evaluation.

From the preceding examples it will be seen that for continued multiplication or continued division — using C and D scales only — we must divide or multiply by unity or 10 as required.

Thus (using C and D scales only)

$$M \times N \times P$$

should be manipulated as:

$$M \div (1 \text{ or } 10) \times N \div (1 \text{ or } 10) \times P$$

Example 7 Evaluate $.0613 \times 19.25 \times .245 \times 56.4$

(*i*) Cursor to D_{613} (*ii*) C_{10} to cursor (*iii*) Cursor to C_{1925}

(*iv*) C_1 to cursor (*v*) Cursor to C_{245} (*vi*) C_{10} to cursor (*vii*) Cursor to C_{564} and on the D scale read 163 the significant figure value of the continuous multiplication; decimal point considered 16.3.

Similarly $\frac{1}{Q \times R \times S}$

should be worked as:

$$(1 \text{ or } 10) \div Q \times (1 \text{ or } 10) \div R \times (1 \text{ or } 10) \div S$$

Example 8 Determine the value of

$$\frac{1}{17.62 \times .846 \times 3.15}$$

(*i*) C_{1762} to D_1 (*ii*) Cursor to C_{10} (*iii*) C_{846} to cursor (*iv*) Cursor to C_{10} (*v*) C_{315} to cursor and read at C_1 on the D scale 213; decimal point considered .0213.

Examples 7 and 8 have been worked in order to contrast with examples 9 and 10 using the available additional Reciprocal of C scale (on the slide) in conjunction with the C and D scales.

CONTINUOUS MULTIPLICATION OR DIVISION

Using the Reciprocal of C Scale (CI) in conjunction with the normal C and D scales.

The form: $M \times N \times P$
must be treated as

$$M \div \frac{1}{N} \times P$$

the $\frac{1}{N}$ being the N value on the *Reciprocal Scale* (CI)

Example 9 Evaluate $.0613 \times 19.25 \times .245 \times 56.4$
treat as

$$.0613 \div \frac{1}{19.25} \times .245 \div \frac{1}{56.4}$$

(*i*) Cursor to D_{613} (*ii*) CI_{1925} to cursor (*iii*) Cursor to C_{245} (*iv*) CI_{564} to cursor and at C_1 on the D scale read 163; decimal point considered 16.3. Note only four settings required in place of seven in the corresponding example 7.

Similarly: $\frac{1}{Q \times R \times S}$
must be treated as

$$1 \div Q \times \frac{1}{R} \div S$$

Example 10 Determine the value of

$$\frac{1}{17.62 \times .846 \times 3.15}$$

(*i*) C_{1762} to D_{10} (*ii*) Cursor to CI_{846} (*iii*) C_{315} to Cursor and read at C_1 on the D scale 213; decimal point considered .0213.

Note the three settings in place of five required in example 8.

Apart from the additional obvious use of the Reciprocal Scale to obtain reciprocals by cursor projection from the C to the CI scale, many other uses will occur to the user in relation to his particular computations.

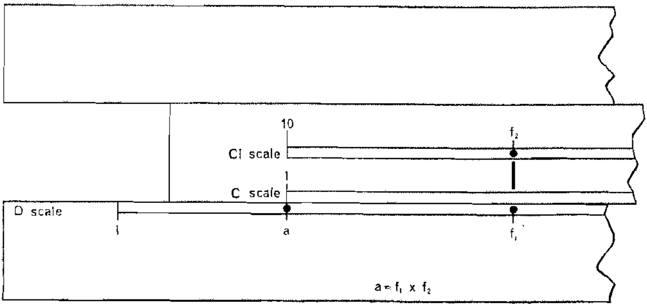


FIG 2

One relationship which merits special mention is:

Where $a = f_1 \times f_2$ and 'a' is fixed, to determine any of the infinite pairs of values of f_1 and f_2 that will satisfy (see fig.2).

Arrange slide so that C_1 (or C_{10}) is at value 'a' on the D scale. Using the Cursor for alignment from one factor on D scale, read other factor on the CI scale.

Particular care must be taken, when using the Reciprocal of C scale, to keep in mind the reverse direction of ascending significant figures.

USE OF CF AND DF SCALES

These scales are included on certain models of PIC slide rules and occupy the position normally used for the A and B scales with DF on the stock and CF on the slide. These two scales are simply C and D scales displaced by the factor π, and they have particular advantages in multiplication, division, proportions, etc, as they give complete factor range in conjunction with C and D scales.

This will be more easily appreciated by comparing the movements involved in a calculation using:

(*a*) C and D scales only

(*b*) CF and DF scales in conjunction with C and D

Complete Divisor Range

Example 11

Where a+b+c+d+e=T

express a, b, c, d, e as percentages of T given the following values:

a=41.3, b=25.8, c=89.4, d=128, e=84.5 and T=369

(*a*) *Using C and D scales only*

Move slide so that C_{369} aligns with D_1

Then for values 41.3, 84.5 and 89.4 move the cursor to

(*i*) C_{413}	(*ii*) C_{845}	(*iii*) C_{894}

and read the corresponding percentages on the D scale, *viz*:

11.2%	22.9%	24.2%

respectively.

To obtain the remaining percentages for 128 and 25.8 it is necessary to move the slide so that C_{369} aligns with D_{10}, when they can be read by moving the cursor to

(*iv*) C_{128}	(*v*) C_{258}
34.7%	7.0%

(*b*) *Using DF, CF, C and D scales*
Move the slide so that CF_{369} aligns wtih DF_{10}
With the slide in this position all values can be obtained by cursor projection, *viz*:
41.3, 84.5 and 89.4 from CF to DF scale
128 and 25.8 from C to D scale.

The next example shows how a given factor can be applied to a series of numbers.

Example 12

Multiply each of the following numbers:
12.7, 559, 173, 76.8, 24.6, 9.24 and 35.4 by .263
Using the combination DF and CF, C and D scales all the results can be obtained by a single displacement of the slide and relevant cursor projections, *viz*:
Move slide so that CF_{10} aligns with DF_{263}
With the slide in this position all values can be obtained by cursor projection as follows:
From CF to DF scale — the products of

(*i*) 559	(*ii*) 76.8	(*iii*) 9.24 are read as
147	20.2	2.43 respectively

and from C to D scale — the products of

(*iv*) 12.7	(*v*) 173	(*vi*) 24.6	(*vii*) 35.4 read as
3.34	45.5	6.47	9.31

Two positions of the slide would have been necessary in the above example if only C and D scales were used.

The next example deals with the familiar form

$$\frac{a \times b}{c}$$

Using C and D scales only, an 'end switch' is frequently necessary — as is the case in the example below — whereas with the combined displaced scales, the 'end switch' can always be avoided by proper selection of the scale to which the initial factor 'a' is applied (*ie* a choice between D or DF)

The correct scale to choose is that which results in more than half the slide engaging in the stock of the slide rule after the divisor has been set.

Example 13

Evaluate $\dfrac{3.1 \times 8.15}{1.64}$

(*a*) *Using C and D scales only — and following normal practice*

(*i*) Cursor to D_{31} (*ii*) C_{164} to cursor (*iii*) End switch bringing cursor to C_1 and then C_{10} to cursor

(*iv*) Cursor to C_{815} (*v*) Read at the cursor 154 on D scale.
After considering the decimal point

$$\frac{3.1 \times 8.15}{1.64} = 15.4$$

(*b*) *Using DF, CF, C and D scales*

(*i*) Cursor to D_{31} (*ii*) C_{164} to cursor (*iii*) Cursor to CF_{815}

(*iv*) Read at the cursor 154 on DF scale.
Decimal point considered $= 15.4$

Whilst the combination of the four scales minimises the need for 'end switching' it must not be concluded that it completely eliminates it in all cases of continuous compound calculations. With experience, by visualising the slide position before moving it, the user may often be able to select factors in an order, or choose scales to ensure that, after division, more than half of the slide is engaged with the stock. This will then mean that on either CF or C scales a complete significant figure range is in contact with either DF or D, thus enabling the cursor to be moved to the next factor without the need of an 'end switch'.

Note — It will be appreciated that cursor projection from C and D scales to CF and DF is the equivalent of multiplication by π of the C or D scale value. Thus circumference from diameter of circle (and vice versa) can be obtained at a single setting of the cursor.

Determination of Square Roots

It will be observed that Scale A of two $\frac{1}{2}$ unit sections is arranged in relation to the D scale (unit length) so that:

A_1 aligns with D_1
A_{100} aligns with D_{10}

Using the cursor for projection from D scale to A scale the following alignments can be observed.

(*a*) for involution

D	1	2	3	4	5	etc	10
A	1	4	9	16	25	etc	100

ie 'Squares' of values on D are in alignment on A

(*b*) for evolution, a reverse process provides 'square roots' of the values on A in alignment with D scale.

Since each section, *viz* 1 to 10 and 10 to 100, of the A scale provides a full cycle of significant figure range, in the case of 'square roots' the user has to decide which of the two sections is applicable to any particular evaluation.

Consider numbers whose significant figure value is
2788
Such values may occur in various forms
as
.0002788, .002788, etc
or
278.8, 278800, etc

By way of illustration let us consider determination of the square roots of three of these say
$\sqrt{.0002788}$, $\sqrt{278.8}$ and $\sqrt{278800}$

Starting from the decimal point, arrange bars over pairs of numbers as shown.

(1) $.\,\overline{00}\;\overline{02}\;\overline{78}\;\overline{80}$

(1a) $\overline{2}\;\overline{78}\,.\,\overline{80}$

(2) $\overline{27}\;\overline{88}\;\overline{00}\,.$

(1) and (1a) are alike in the respect that the first significant figure 2 is alone under a bar, whereas in (2) the first two significan figures, that is, 27 occur under the same bar.

In cases such as (1) or (1a) projection is from the first section of the A scale whilst case (2) calls for projection from the 2nd Section.

Evaluate $\sqrt{.\overline{00}\,\overline{02}\,\overline{78}\,\overline{8}}$

0 1 67

Cursor to 2788 on 1st Section of A scale and read on D scale 167.

Again consider the pairs, and with cipher or figure in the result for each pair, the significant figure 167, decimal point considered, becomes .0167

Evaluate $\sqrt{\overline{2}\ \overline{78}\ .\ \overline{80}}$

1 6 . 7

Cursor to 2788 on 1st Section of A and read on D scale 167. This significant figure value 167, decimal point considered, becomes 16.7

Evaluate $\sqrt{\overline{27}\ \overline{88}\ \overline{00}\ .}$

Cursor to 2788 on the 2nd Section of A scale and read on D scale 528

This significant figure group 528, decimal point considered, becomes 528.0

After a little practice, the bars for pairing figures can be imagined and so the first figure of a 'square root' together with its denomination, can readily be obtained by inspection.

Determination of Cube Roots

Models which incorporate a Cube Root scale denoted by K furnish a direct means of obtaining cube roots by cursor projection. The scale comprises three repeats of a $\frac{1}{3}$ unit but care must be exercised in selection of the section to be used.

Cube roots of numbers from 1 to 1000 are read off the D scale (C scale if the cube root scale is on the slide) by cursor projection from the K scale. For numbers above or below 1 to 1000 it is advisable to consider them in groups of three from the decimal point. Then the following basis can be applied:

One significant figure in excess of complete groups of three — use first section of K scale.

Two significant figures in excess of complete groups of three — use second section of K scale.

No significant figure in excess of complete groups of three — use third section of K scale.

eg Evaluate 56780 and .05678

In both cases the two significant figures in excess of complete triads indicate the use of the second section of K scale from which we obtain 38.44 and 0.3844 respectively as the cube roots.

Determination of Logarithms

The Logarithm scale denoted by L is a uniform scale related to the C and D scales and provides logarithms to base 10.

If we assign the definite values 1.0 to 10.0 to the significant figure scales C and D, then the length of L scale equals that of C or D from 1.0 to 10 and the extremes of the L scale are numbered 0.0 to 1.0 (Sub-divisions are in accord with decimal reading).

This combination functions as the equivalent of logarithm and antilog tables
To determine $\log_{10}5.2$

Use the cursor to project from D_{52} (C_{52} if L scale is on the slide) to the L scale and read $\text{Log}_{10}5.2 = .716$.

The reverse process provides logarithm to number conversions.

As with 'log' tables, only the 'mantissa' portion is obtainable from the rule and so in all cases, according to the position of the decimal point, the appropriate 'characteristic' must be applied.

Orthodox Trigonometrical Scales

These scales are:
Sine scale, denoted by S, for the angle range 5.7 to 90°
Tangent scale, denoted by T, for the angle range 5.7 to 45°
Sine and Tangent scale, denoted by ST, for the angle range 0.57 to 5.7°

All three scales are decimally sub-divided, are related to the C and D scales and values are read off directly by cursor projection.

eg Determine the value of Sine 20°

Set cursor to 20° on Sine scale and read on D scale at the cursor 342. Decimal point considered Sine 20° = .342

Note: The value of Cosec 20° $= \dfrac{1}{\text{Sin } 20°}$ can also be read off on the Reciprocal scale, *viz* 2.924 (provided, of course, that C_1 and D_1 are in alignment).

eg Determine Cos 16° and Secant 16°
Since Cos 16° = Sine 74° treat as Sin 74°
Similarly Sec 16° = Cosec 74°

Set cursor to 74° on Sine scale and read on D at the cursor 0.961 (the value of Sin 74° = Cos 16°) and read on CI scale at the cursor 1.040, the value of Cosec 74° = Sec 16°

eg Determine Tan 22° and Cotan 22°

Set cursor to 22° on Tan scale and read on D scale 0.404 = Tan 22° and on CI scale 2.475 = Cotan 22°

Note: For tangents of angles between 45° and 90° use the formula

$$\text{Tan} = \frac{1}{\text{Tan}\,(90 - \alpha)} = \text{Cot}\,(90 - \alpha)$$

It will be appreciated that determination of the angle when the function value is given involves the inverse of the above process.

eg To find the angle whose Sine is 0.41

Set cursor to D_{41} and read on S scale at the cursor 24.2°

The Sine and Tangent scale (ST) is used for both Sines and Tangents for the lower angle range below 5.7° and is the geometric mean of the two functions. In this respect it will be appreciated that Sin $\alpha \doteq$ Tan $\alpha \doteq \alpha$ in radians when α is small. Thus this scale may be used for converting degrees to radians and vice versa.

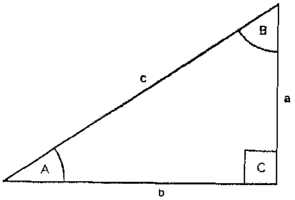
Note: For small angles care is required in positioning the decimal point when reading answers on D scale, and in the selection of the appropriate angle scale when the function value is given. As a guide the following rule is useful:
If the angle is on the ST scale then .0 will precede the significant figures read off the D scale.
If the function value on D scale is preceded by .0 then the angle is read on the ST scale.

SOLUTION OF TRIANGLES
It is important to remember the Sine rule

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

as these proportions can be usefully applied. It will be appreciated that if a given angle on the Sine scale is aligned with its respective given side on the C scale, then the other pairs of values are also in alignment. Thus if one of each of the other pairs is known the triangle can be solved by cursor projection between the Sine scale and the C scale.

RIGHT ANGLED TRIANGLES



The Sine rule becomes $\dfrac{a}{\text{Sin A}} = \dfrac{b}{\text{Sin B}} = \dfrac{c}{1}$

Case 1
Given Angle A = 35.3° and side c = 533
To find sides a and b

Then $\dfrac{a}{\text{Sin } 35.3} = \dfrac{b}{\text{Sin }(90 - 35.3)} = \dfrac{533}{1}$

(a) Set C_{533} to D_{10}
(b) Cursor to 35.3 on Sine scale and read on C scale 308 = side b
(c) Cursor to (90 — 35.3) = 54.7 on Sine scale and read on C scale 435 = side a

Case 2

Given a = 207 and c = 305
To find Angle A and side b

Then $\frac{207}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{305}{1}$

(a) Set C_{305} to D_{10}
(b) Cursor to C_{207} and read on the Sine scale 42.75 = Angle A
(c) Cursor to (90 — 42.75) = 47.25 on Sine scale and read on C scale 224 = side b.

Case 3

Given a = 133, b = 156
To find Angle A and side c

Then $\frac{133}{\text{Sin A}} = \frac{156}{\text{Sin B}} = \frac{c}{1}$

and Tan A = $\frac{133}{156}$

(a) Cursor to D_{133}
(b) C_{156} to cursor
(c) Cursor to C_{10} and read on Tangent scale 40.45 = Angle A
(d) Cursor to 40.45 on Sine scale
(e) C_{133} to cursor and at D_{10} read on C the value 205 = side c.

Alternatively
(a) C_1 to D_{133}
(b) Cursor to 156 on Reciprocal of C scale (giving 133 × $\frac{1}{156} = \frac{133}{156}$) and read on the tangent scale 40.45 = Angle A
(c) Cursor to 40.45 on Sine scale and read on Reciprocal of C scale 205 = side c.

PIC Differential Trigonometrical Scales

The group of scales consists of the following:

Sine Differential scale (denoted by Sd) of $\frac{\alpha}{\sin \alpha}$ for Sine range 0 to 90°

Tangent Differential scale (denoted by Td) of $\frac{\alpha}{\tan \alpha}$ for Tangent range 0 to 60°

Inverse Sine Differential scale (denoted by ISd) of $\frac{x}{\sin^{-1} x}$ for inverse of above Sine range

Inverse Tangent Differential scale (denoted by ITd) of $\frac{x}{\tan^{-1} x}$ for inverse of above Tangent range

The above four scales are positioned on the slide and together take up the equivalent of one 'scale length'. They are used in conjunction with C and D scales and are very simple to manipulate.

The principle is as follows:

$$\text{Since } Sd_{\propto} = \frac{\propto}{\text{Sin } \propto}$$

$$\text{Then } \frac{\propto}{Sd_{\propto}} = \frac{\propto}{\dfrac{\propto}{\text{Sin } \propto}} = \propto \times \frac{\text{Sin } \propto}{\propto} = \text{Sin } \propto$$

Thus by setting the $\propto$ value on D scale and dividing by $Sd_{\propto}$ (*ie* the $\propto$ value on the Sine Differential scale) the value of Sin $\propto$ is read on D scale at C_1 (or C_{10})

eg To find Sin 43° Treat as $\dfrac{43}{\dfrac{43}{\text{Sin } 43}}$ *ie* $\dfrac{43}{Sd_{43}}$

Cursor to D_{43}
Bring 43° on the Sine Differential scale (*ie* Sd_{43}) to the cursor
At C_{10} read on D scale 0.682 = Sin 43°

eg To find Tan 36.5°
Cursor to D_{365}
Bring 36.5° on the Tangent Differential scale (*ie* $Td_{36.5}$) to the cursor
At C_{10} read on D scale 0.740 = Tan 36.5°

eg To find the angle whose Sine is 0.66 (*ie* the value of Sin^{-1} 0.66)

Treat as $\dfrac{0.66}{\dfrac{0.66}{\text{Sin}^{-1}\, 0.66}}$ *ie* $\dfrac{0.66}{ISd_{0.66}}$

Cursor to D_{66}
Bring 0.66 on Inverse Sine Differential scale (*ie* $ISd_{0.66}$) to the cursor
At C_1 on D scale read 41.3 = Sin^{-1} 0.66

Similarly the angle whose Tangent is 0.9 is found to be 42° by using the Inverse Tangent Differential scale in conjunction with D scale.

eg To find the value of 73 Sin 52°
Cursor to D_{52}
Bring Sd_{52} to the cursor
Cursor to C_{73} and read on D scale at the cursor 57.5.

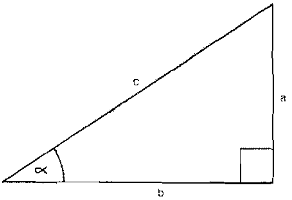
It will be appreciated that the Direct Sine and Tangent Differential scales provide the necessary *divisor correctives* which, when applied to angle readings on D scale, give the respective trigonometrical functions on D at C_1 or C_{10}. Similarly the Inverse Sine and Tangent Differential scales provide the *divisor correctives* which, when applied to function values on D scale, give the corresponding angles on D at C_1 or C_{10}.

These Differential Trigonometrical scales give consistent maximum accuracy over the complete angle range and experience with them soon reveals their superiority compared with the orthodox Trigonometrical scales.

The appearance of, for example, the Sine Differential Scale in the region of the 0° to 30° mark, in the respect that the distance is so comparatively short and the divisions so few, may suggest to the non-mathematical beginner that the accuracy is accordingly somewhat limited; but after a little practice with this scale, and thought regarding the nature of Sines, the user will correctly interpret the meaning of these small variations of the *divisor correctives* for the early angle range. Similar observations can be made with regard to the other Scales. After comparison with tabulated and calculated results, users will soon realise that the highest significant figure accuracy possible by the C and D scales is consistently maintained by the Differential Scales over the complete angle range.

On inspection of the Rule it will be observed that the Common Zero of the Direct Scales is at 'U' and coincides with the C scale reading 57.3, *ie* $\frac{180}{\pi}$. The Inverse Scales have their Common Zero at 'V' which corresponds to a C scale reading of 0.01746, *ie* $\frac{\pi}{180}$

RIGHT ANGLED TRIANGLES



eg Given a = 160 and b = 231
To find Angle α

$$\frac{a}{b} = \text{Tan}\,\alpha \qquad ie\ \alpha = \text{Tan}^{-1}\frac{a}{b} = \text{Tan}^{-1}\frac{160}{231}$$

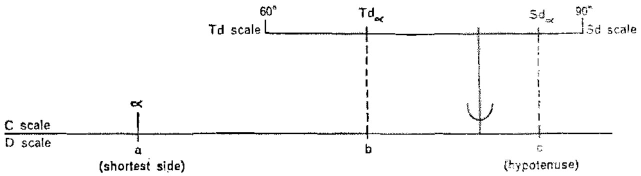
(a) Cursor to D_{160}
(b) Bring C_{231} to the cursor
(c) Cursor to C_{10} and read on D scale $0.6925 = \frac{a}{b}$

(d) ITd_{6925} to cursor and read at C_1 on D scale $34.7° = \alpha$

When given one side and an angle (other than the right angle) the following relationship is useful.

$$\frac{a}{\alpha} = \frac{b}{Td\alpha} = \frac{c}{Sd\alpha}$$

This is expressed diagrammatically by



Thus the remaining two sides are obtainable at a single setting of the slide (except where an 'end switch' is involved when a second movement of the slide is necessary).

eg Given c = 533, α = 35.3°

To find a and b

Cursor to D_{533} and bring $Sd_{35.3}$ in alignment.
Cursor to $Td_{35.3}$ and read on D in alignment 435 = b
Cursor to $C_{35.3}$ and read on D in alignment 308 = a

GAUGE MARKS V, U, m, s

These are conversion constants on the C scale for use as divisors as follows:

$$U = \frac{180}{\pi} = 57.2958 \text{ for Degrees to Radians}$$

$$V = \frac{\pi}{180} = 0.01746 \text{ for Radians to Degrees}$$

$$m = \frac{180 \times 60}{\pi} = 3437.75 \text{ for Minutes to Radians}$$

$$s = \frac{180 \times 60 \times 60}{\pi} = 206265.0 \text{ for Seconds to Radians}$$

Where $\alpha°$ is an angle expressed in Degrees
α' the angle expressed in Minutes
α'' the angle expressed in Seconds
ψ the angle expressed in Radians

Then $\frac{\alpha°}{U} = \frac{\alpha'}{m}$ or $\frac{\alpha''}{s} = \psi$

Note: For particular advantages of PIC Trigonometrical Differential scales in Tacheometric Surveying see Appendix.

Log Log Scales

Note: The following instructions cover models with three direct and three Reciprocal Log Log scales. Thus only certain portions of the instructions relate to models with two direct log log scales.

The Log Log scales and Reciprocal Log Log scales are arranged on the stock and are used for calculations involving the exponential form.

Any positive number N may be expressed as a particular power P of any positive base B thus $N = B^P$

Hence Log N = P Log B
and Log Log N = Log P Log Log B
or Log Log N — Log Log B = Log P

ie the values B and N on the Log Log scale are separated by a distance representing Log P.

eg Set cursor at 3 on LL3 scale and move slide bringing C_1 to the cursor.

Observe by cursor projection that
2 on C scale aligns with 9 on LL3
3 on C scale aligns with 27 on LL3
4 on C scale aligns with 81 on LL3
5 on C scale aligns with 243 on LL3
thus evaluating $3^2, 3^3, 3^4, 3^5$,

Similarly, on models with Reciprocal Log Log scales, by setting the cursor at .3 on the LL03 scale and bringing C_1 to the cursor
2 on C scale aligns with .09 on LL03
3 on C scale aligns with .027 on LL03
4 on C scale aligns with .0081 on LL03
thus evaluating $(.3)^2$, $(.3)^3$, $(.3)^4$

eg Evaluate $N = 3.5^{2.66}$
Cursor to 3.5 on LL3 scale
Bring C_1 to the cursor
Move cursor to C_{266} and read on LL3 scale at the cursor 28.0 = the value of N.

Cursor projection from LL1 to LL2 or LL2 to LL3 scales effects the process of raising to the 10th power (or vice versa extracting the 10th root).

Thus $3.5^{.266} = 1.395$ the figure in alignment on the LL2 scale.

When the base is less than unity the process is the same except that the Reciprocal Log Log scales are used.

Thus $0.35^{2.66} = 0.0612$
and $0.35^{.266} = 0.7563$

eg Evaluate $(.35)^{2.66}$

(a) Cursor to .35 on LL03, C_1 to cursor and then move cursor to $C_{2.66}$. In alignment at cursor read .0612 on the LL03 scale. $(.35)^{2.66} = 0.0612$

(b) Read on the LL02 scale in the same alignment the value .7563
$(.35)^{.266} = .7563$

Note: In order to decide which scale provides the value, mental approximation is necessary. It may be said that most students are more at ease with rough approximations of *powers* of quantities *greater than unity* than with those of quantities *less than unity.*

For example, it is easier to mentally appreciate that

$$3^{\frac{1}{2}} \text{ or } \sqrt{3} \simeq 1.7$$
$$3^3 = 27$$

than say

$$\sqrt{.4} \text{ or } .4^{\frac{1}{2}} \simeq .63$$
$$(.4)^3 = .064$$

Thus when dealing with the latter type as in the example it is useful to remember that at a particular setting on say the LL03 scale, the reciprocal of this value (greater than unity) appears in alignment on the related LL3 scale. The raising to the 'power' in question may then be observed on the greater than unity scales LL2 or LL3 and the final reading taken on the appropriate reciprocal log log scale.

The student should also observe in respect of reciprocals that those obtained by projections from LL2 to LL02 and vice versa are more accurate than those obtained by C, D or CI scales; but for the range covered by the LL3 and LL03 scales the advantage is with the primary scales C and D.

To solve for P when N and B are known, *ie* to determine the log of N to base B

Proceed as in the following example:
eg (Base > unity)
Solve $5.3^P = 92.0$

Set the cursor at 5.3 on LL3 scale and move the slide so that C_1 or C_{10} (in this case C_1) is at the cursor. Move the cursor to 92.0 on the LL3 scale and read the significant figures of P on the C scale at the cursor. *viz* 271.

$$\text{Then } 5.3^{2.71} = 92.0$$

or

$$\log_{5.3}92 = 2.71$$

eg (Base less than unity)

$$.452^P = .764$$

(Obviously the value of P is less than unity)

Set the cursor at .452 on the LL02 scale. Align C_{10} at the cursor and move the cursor to .764 on LL02. On C scale at the cursor read 339 the significant figures of P. . . decimal point considered .339.

To determine B when N and P are known
eg Determine B when $B^{2.14} = 40$

Set the cursor at 40 on LL3 and move the slide so that 2.14 on C is in alignment. Transfer the cursor to C_1 and note the readings on LL2 and LL3 namely 1.188 and 5.6 respectively.

Mental approximation will immediately select 5.6 as the required value.

$$\textit{ie } 5.6^{2.14} = 40$$

and note that

$$1.188^{21.4} = 40$$

Note:

(i) That N and B are positions on the log log scales and their denominations must be respected, inasmuch as values for 1.45, 14.5, 145 and 1450 are distinct positions at different parts of the scales.

(ii) That the significant figure value of P is employed on the C scale *ie* powers such as .25 and 2.5 are identical on that scale: denominations, supported by mental approximations, will dictate from which log log scale the N reading has to be taken.

(iii) That 1 or 10 and P value on the C scale respectively align with B and N on the log log scales.

(iv) That where roots occur they must be re-expressed as 'powers'

eg $\sqrt[3]{}$ as the .333 power

$\sqrt[4]{}$ as the .25 power

(v) Normally a negative base cannot be raised to a power *eg* $(-5.94)^{3.41}$ cannot be evaluated: exceptions are where the 'power' is either an integer or the reciprocal of an odd integer.

Where a 'power' and 'base' are such as to result in a value of N in excess of 10^5 as in the following
eg Evaluate $5.3^{7.8}$
Take the 'power' in parts as $7.8 = 4 + 3.8$
Fvaluate 5.3^4 and $5.3^{3.8}$ and obtain

$$5.3^4 = 790 \qquad 5.3^{3.8} = 565$$

$$\text{Then } 5.3^{7.8} = 790 \times 565 = 446000$$

When N is greater than 10^5, to determine P for a given B proceed as follows
eg Determine P when $5.3^P = 446000$
Factorise 446000 as 1000×446
Then consider $P = q + r$ where $5.3^q = 1000$ and $5.3^r = 446$
Evaluate q and r and obtain $q = 4.14$ and $r = 3.66$

$$\textit{ie } P = 4.14 + 3.66 = 7.8$$

Logarithms to the base e or evaluations of e^p
The log log scales are so positioned on the stock that projections therefrom on to the D scale furnish the significant figures of logarithms to the base e.

Provisionally assign to the D scale the values 1.0 to 10 and remember that numbers in alignment on the log log scales bear the relationship

$$(LL2)^{10} = LL3 \qquad (LL02)^{10} = LL03$$

Place the cursor at say 1.326 on the LL2 scale and note that the D scale reading in alignment is 2.822.

Also observe reading on LL3 16.8
observe reading on LL02 .754
observe reading on LL03 .0595

It will be seen that

$$\log_e 1.326 = 0.2822 \quad \text{or} \quad e^{.2822} = 1.326$$
$$\log_e 16.8 = 2.822 \quad \text{or} \quad e^{2.822} = 16.8$$
$$\log_e 0.754 = -0.2822 \quad \text{or} \quad e^{-.2822} = 0.754$$
$$\log_e 0.0595 = -2.822 \quad \text{or} \quad e^{-2.822} = 0.0595$$

The alignment of values is useful when evaluating hyperbolic functions since:

$$\text{Sin h } x = \frac{e^x - e^{-x}}{2}$$

$$\text{Cos h } x = \frac{e^x + e^{-x}}{2}$$

$$\text{Tan h } x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\text{Sin h } x}{\text{Cos h } x}$$

In the case of $x = 2.822$

$$\text{Then Sin h } x = \frac{16.8 - 0.0595}{2} = 8.37$$

$$\text{Cos h } x = \frac{16.8 + 0.0595}{2} = 8.43$$

$$\text{Tan h } x = \frac{8.37}{8.43} = 0.993$$

Use of the L Constant

The L constant at 2.3026 on C scale is useful for converting logs to base e to logs to base 10 since

$$\text{Log}_{10} N = \frac{\text{Loge } N}{2.3026} = \frac{\text{Loge } N}{L}$$

The conversion is effected by bringing the L mark on C scale in alignment by cursor projection with the N value on the log log scale and reading the value of $\text{Log}_{10}N$ on D scale at C_1 (or C_{10}).

It will be realised that logarithms to any base can be obtained by making a mark on C scale in the position which aligns with the particular base on the log log scales (with C_1 and D_1 in alignment of course) and by using the position marked on the C scale as a divisor for that base.

eg To find $\log_2 8$

(a) With C_1 and D_1 in alignment move cursor to 2.0 on log log scale and then make a mark on C scale at the cursor position, namely 693.

(b) Transfer cursor to 8 on log log scale

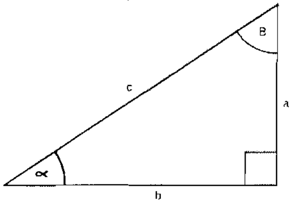
(c) Bring marked position at C_{693} to the cursor and read 3 on D scale at C_{10}

Then $\log_2 8 = 3$ (or $8 = 2^3$)

Vector Analysis Scales

Ps of $\sqrt{1-s^2}$ and Pt of $\sqrt{1+t^2}$

With the Ps scale of $\sqrt{1-s^2}$ it is possible, by cursor projection to the D scale, to obtain Cos α when Sin α is known or vice versa.



The difference between two squares is treated as follows

$$x = \sqrt{c^2 - a^2} = c\sqrt{1 - \left(\frac{a}{c}\right)^2}$$

Since $\frac{a}{c}$ = Sin α = s

Then $x = c\sqrt{1-s^2}$

eg Evaluate $\sqrt{5.3^2 - 2.8^2}$

$$= 5.3\sqrt{1 - \left(\frac{2.8}{5.3}\right)^2}$$

(i) Cursor to D_{28}, C_{53} to cursor and at C_{10} on D read

$$528 = \frac{2.8}{5.3}$$

(ii) Cursor to .528 on Ps scale, C_{10} to cursor, cursor to $C_{5\cdot}$ and read 45 on D scale

Then $\sqrt{5.3^2 - 2.8^2} = 4.5$

Note: Where s is greater than 0.995 the form $\sqrt{2(1-s)}$ may be used as a close approximation.

The Pt scale of $\sqrt{1+t^2}$ serves for determination of the square root of the sum of two squares as under:

$$h = \sqrt{a^2 + b^2} = b\sqrt{1 + \left(\frac{a}{b}\right)^2}$$

Since $\frac{a}{b}$ = Tan α = t

Then $h = b\sqrt{1+t^2}$

eg Evaluate $\sqrt{3^2 + 4^2}$

$$= 4\sqrt{1 + \left(\frac{3}{4}\right)^2} = 4\sqrt{1 + (0.75)^2}$$

(i) Cursor to 0.75 on Pt scale

(ii) Bring C_1 to cursor

(iii) Cursor to C_4 and read on D scale the value of h = 5

PIC Three Line Cursor

This cursor is an alternative to the single line cursor and has two lines added, one to the left and the other to the right of the main cursor line. To distinguish them from the main cursor line the additional lines are in red.

LEFT HAND LINE

This may be used for calculations involving areas of circles where diameter is given and vice versa and the distance between the left hand line and the main cursor line corresponds to the interval 0.7854 to 1 on the A scale.

By setting the main line to a diameter on D scale the corresponding area of the circle can be read on A scale at the left hand line since passing from D to A scale squares the diameter and reading at the left hand line is the equivalent of multiplying on A scale by $0.7854 = \frac{\pi}{4}$ The formula used is thus

$$\text{Area} = \frac{\pi}{4} d^2$$

It will be appreciated that area to diameter conversions may be made in the opposite way.

eg Given area of circle is 120 sq ins

Find the diameter

(a) Set left hand cursor line to 1.2 on A scale.

(b) From the main cursor line read on D scale 1236. Then diameter of circle equals 12.36 inches.

RIGHT HAND LINE

This may be used for horse power to kilowatt conversions, the distance between the main cursor line and the right hand line corresponding to the interval 0.746 to 1 on the A scale.

eg Find the number of kilowatts in 150 hp

(a) Set right hand cursor line to 1.5 on A scale

(b) At the main cursor line read 112. Then number of kilowatts equals 112.

Note: Removal of the single sided cursor may be effected as follows:

1 Move slide to one end of rule

2 Centralise the cursor

3 Compress the rule across its width in the region of the cursor which can now be removed.

Appendix
Tacheometric Surveying

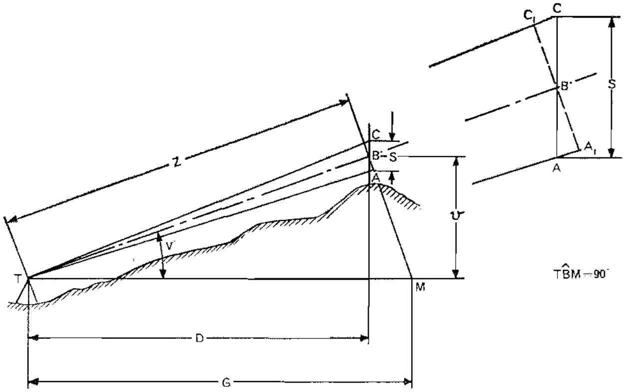
STADIA COMPUTATIONS

In the instructions for Differential Trigonometrical scales reference was made to the complete solution of the triangle given one side and one angle (other than the right angle).

Since the computations of heights and distances from stadia data call for the treatment of two overlapping right angle triangles, it will be seen from the following that a rule which incorporates the differential Sine and Tangent scales provides the means of effecting the evaluation of D, *v* and Z (see figure) when G and V° are known, by a *single setting of the slide* (occasional end switching excepted).

In the following it is assumed that the tacheometric instrument used is provided with an analatic lens, also that the measurement staff is held in a vertical position.

Where V° = the inclination of the telescope
S = the (vertical) staff reading = AC
r = the 'corrected' (staff) reading = A_1C_1
D = the horizontal distance
v = the vertical distance
Z = Distance on Collimation Line
G = Uncorrected distance Z = *k*.S
k = Constant for the wires (usually 100)



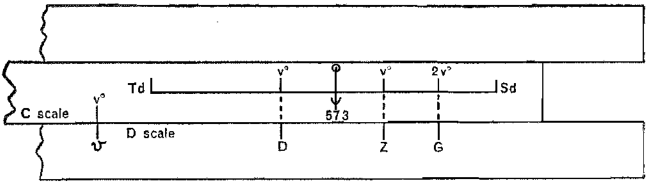
eg

Determine D, *v* and Z, when S = 4.5 ft and V° = 20° and the constant for the stadia lines = 100
G = 100 × 4.5 = 450, V° = 20°, 2V° = 40°

Set the slide so that the 40° mark on the Sd scale aligns with 450 on the D scale.

Then in alignment with:
20° on the Sd scale read the value of Z = 423 on the D scale
20° on the Td scale read the value of D = 397.4 on the D scale
20° on the C scale read the value of *v* = 144.6 on the D scale

That is, evaluate in accordance with the following diagram.



In the case of a value of G which occurs towards the left of the D scale such that $Sd_{2V°}$ when aligned with G results in $Sd_{V°}$, $Td_{V°}$ or $C_{V°}$ being to the left of D_1, then end switching must be effected by placing the cursor at C_{10} and then bringing C_1 to the cursor. This will enable the values of the remaining projections on the D scale to be made.

The mathematical proof which substantiates the correctness of the foregoing method follows.

From the figure and remembering that

$$\text{Sin V° Cos V°} = \frac{\text{Sin 2V°}}{2}$$

we get:

$$v = \text{G Sin V° Cos V°} = \text{Tan V° G.Cos}^2\text{ V°} = \text{D Tan V°} =$$

$$\text{Z Sin V°} = \text{G.}\frac{\text{Sin 2V°}}{2}$$

Then

$$\frac{\text{V°}}{v} = \frac{\text{V°}}{\text{G. Sin V° Cos V°}} = \frac{\text{V°}}{\text{Tan V°.G.Cos}^2\text{V°}}$$

$$= \frac{\text{V°}}{\text{D.Tan V°}}$$

$$= \frac{\text{V°}}{\text{Z Sin V°}} = \frac{\text{2V°}}{\text{G. Sin 2V°}}$$

$$\frac{\text{V°}}{v} = \frac{\text{V°}}{\frac{\text{Tan V°}}{\text{D}}} = \frac{\text{V°}}{\frac{\text{Sin V°}}{\text{Z}}} = \frac{\text{2V°}}{\frac{\text{Sin 2V°}}{\text{G}}}$$

ie

$$\frac{\text{V°}}{v} = \frac{\text{Td v°}}{\text{D}} = \frac{\text{Sd v°}}{\text{Z}} = \frac{\text{Sd 2v°}}{\text{G}} = \frac{\text{Sd 2v°}}{k.s.}$$

The slide rule interpretation of this form is given in the previous figure.

When the angle V° is small (say less than 3°), then within the accuracy limits of measuring 'S'

(i) $S \doteq r$ (ii) $Z \doteq 100r$

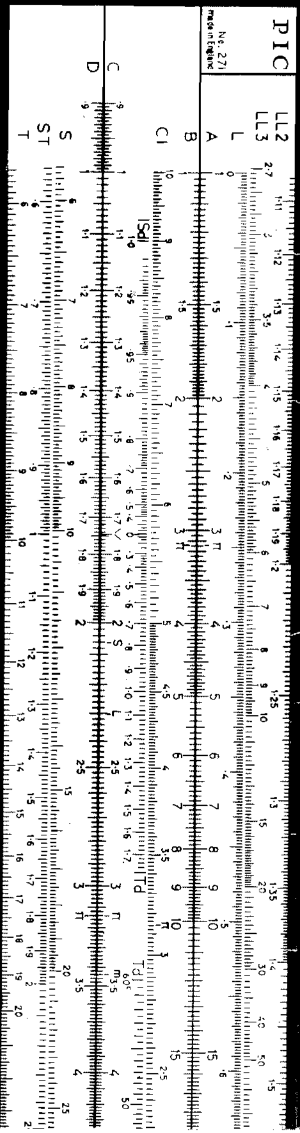
(iii) $D \doteq Z$ (iv) $v \doteq \dfrac{100r\ V°}{57.3}$

eg Given V° = 1° 14′ (*ie* less than 3°), and *r* = 6.17
To determine D and '*v*'
Express 1° 14′ as a degree and decimal = 1.233°

$$D \doteq 100 \times 6.17 = 617$$

To obtain '*v*' evaluate $\dfrac{\text{D.V°}}{57.3} = \dfrac{617 \times 1.233}{57.3}$

Set cursor at D_{617}, U (at $C_{57.3}$) to cursor, cursor to 1.233
Read '*v*' at cursor on D = 13.28 ft


PIC
No. 271
Made in England
LL2
LL3
L
A
B
CI
C
D
S
ST
T