THE SLIDE RULE

A Complete Manual

Alfred L. Slater

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PREFACE

The purpose of this manual is to provide the student with a large number of illustrative examples together with a plentiful supply of practice problems. Answers are given for all exercises. Separate exercise sets feature "formula-type" expressions, many of which require intermediate additions or subtractions. These are representative of computations the student will often encounter in practice.

The first twelve chapters cover the C, D, CI, CF, DF, CIF, A, B, and K scales. The double-length scale (R, Sq, $\sqrt{\ }$) is also discussed, and alternate procedures are presented utilizing this scale in place of the A-B scales. No knowledge of trigonometry or logarithms is assumed for this portion of the manual.

The remainder of the text covers the trigonometric scales (S, ST, T), the log scale (L), and the log-log scales (LL). The base-10 LL scales and the hyperbolic scales (Sh, Th) are briefly treated in the appendix.

The presentation is consistent throughout. Each new slide rule technique is introduced by examples which are carefully broken down step-by-step so that the student should have no difficulty following on his own slide rule. These are followed immediately by several similar drill problems with answers displayed. Thus, the student may test his understanding at once before getting to the main exercise set for the section. Normally, a good deal of classroom time is given to mastering the basic operational scales: C, D, CI, CF, DF, and CIF. Beyond this, time considerations will determine the selection of additional topics for class discussion. It is hoped that the text has sufficient clarity and detail to encourage a certain amount of independent learning by the student.

All of the material in this manual has been used in syllabus form for the past several years at Los Angeles City College. During this period, corrections and suggestions by the mathematics faculty at the college have been gratefully received by the author.

Los Angeles, California
February 1967

A.L.S.

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Chapter 1

INTRODUCTION: LOCATION OF DECIMAL POINT

1.1 Introduction

The slide rule is used principally to perform multiplication, division, and combinations of these operations. Also, it is very easy to find squares, cubes, square roots, and cube roots of numbers. Once you have learned to use the slide rule, you will be able to quickly find answers to the following:

$$463 \times 1.67 \times .0277 = ?$$

$$\frac{7.64 \times 5.22}{(3.62)^2} = ?$$

$$\sqrt{516} = ?$$

$$\sqrt[3]{.0744} = ?$$

$$\frac{175}{2.8}\sqrt{\frac{4.6 \times 17.2}{3.77}} = ?$$

All standard rules also have scales enabling you to find trigonometric functions and common logarithms; furthermore, if your rule has "log log" scales you can raise numbers to any power. Knowledge of these scales will enable you to compute the following:

$$14.2 \sin 46.5° = ?$$

$$\log 45.7 = ?$$

$$23^{1.4} = ?$$

$$\sqrt[5]{33.2} = ?$$

This first chapter is concerned only with the problem of placing the decimal point. With Chapter 2 you start using the slide rule and it is important that, beginning with this chapter, you read the manual with your slide rule always at hand. The illustrative examples are carefully broken down step-by-step, and you should reproduce each move on your slide rule as it is described. Be sure you understand thoroughly what each move accomplishes. There are many drill exercises (with answers) for you to practice on.

Mastery of the slide rule is a skill that can only be acquired through repetitive drill; only in this way will you reach the point where you automatically perform the operations quickly and accurately.

1.2 Slide rule does not ordinarily locate the decimal point

Aside from certain operations involving the L and LL scales, the slide rule does not indicate the position of the decimal point in the answer; it merely gives the principal digits, and you must place the decimal point yourself. Thus, if the slide rule is used to multiply 2.52 by 3.07, the answer will simply be read as a number whose digits are "seven-seven-four." Clearly, in this case, the answer must be 7.74. Again, suppose the problem is to divide 64.3 by 2.17. The slide rule will indicate the answer to be a number whose digits are "two-nine-six." Here, it is evident that the answer must be 29.6.

1.3 Locating the decimal point by inspection

In many slide rule operations, the decimal point may be placed simply by inspection. This is illustrated by the following examples:

Example 1: $3.96 \times 12.45 =$ "493"

By "493" we mean the slide rule reading with decimal point *unplaced;* this notation will be used throughout the text. In this example, we think of the product as approximately 4 times 12; hence, answer must be ***49.3.***

Example 2: $\dfrac{54.3}{2.78} =$ "1952"

Here, we are roughly dividing 54 by 3, so that quotient should be about 18. Thus, answer must be ***19.52.***

Example 3: $\dfrac{.00783}{2.16} =$ "363"

In this case, the answer is approximately .008 divided by 2, or about .004. Therefore, answer must be ***.00363.***

Example 4: $3.75 \times .0188 =$ "705"

We are roughly multiplying 4 by .02, so that product should be about .08. It follows that the result must be ***.0705.***

To give you some practice in doing this, the following exercise contains a number of simple products and quotients with the slide rule answers indicated; you are required to locate the decimal point properly.

Exercise 1-1

Locate the decimal point in the indicated slide rule answer:

1. $2.8 \times 4.2 =$ "1176"

2. $5.6 \times 6.2 =$ "347"

3. $26 \times 3.7 =$ "962"

4. $68 \times 0.26 =$ "1768"

5. $15.2 \times 0.47 =$ "714"

6. $165 \times 0.74 =$ "1221"

7. $\frac{14}{2.6} =$ "538"

8. $\frac{36}{5.7} =$ "632"

9. $\frac{75}{27} =$ "278"

10. $\frac{340}{7.2} =$ "472"

11. $1.75 \times 4.24 =$ "742"

12. $3.22 \times 15.7 =$ "505"

13. $26.2 \times 0.485 =$ "1270"

14. $25.8 \times 1.77 =$ "457"

15. $46.6 \times 2.07 =$ "966"

16. $3.79 \times 5.20 =$ "1970"

17. $\frac{56.2}{3.15} =$ "1785"

18. $\frac{.0776}{2.66} =$ "292"

19. $\frac{87.5}{2.39} =$ "366"

20. $\frac{114.5}{2.56} =$ "447"

21. $48.8 \times 0.283 =$ "1381"

22. $36.0 \times 3.24 =$ "1166"

23. $\frac{107.5}{45.2} =$ "238"

24. $547 \times 0.640 =$ "351"

25. $\frac{54}{77} =$ "702"

26. $2630 \times 0.533 =$ "1400"

27. $\frac{830}{5.25} =$ "1581"

28. $5050 \times 1.85 =$ "935"

29. $\frac{4.75}{52.7} =$ "902"

30. $25.9 \times 4.40 =$ "1140"

31. $42.6 \times 18.7 =$ "797"

32. .0366 × 7.23 = "265"

33. $\frac{12.66}{27.3}$ = "464"

34. $\frac{.00427}{8.15}$ = "524"

35. 12.75 × 5.62 = "716"

36. $\frac{243}{52.7}$ = "461"

37. $\frac{1}{3.22}$ = "311"

38. $\frac{148.5}{0.92}$ = "1615"

39. 43.6 × 21.7 = "946"

40. $\frac{.000862}{1.84}$ = "468"

1.4 Shifting decimal point in products and quotients

In a *product* of two numbers, if the decimal point of one of the numbers is moved a certain number of places in one direction, the decimal point of the other number must be moved the same number of places in the *opposite* direction. Ordinarily, the object is to make one of the factors a number between 1 and 10; it then becomes easier to place the decimal point in the answer.

Example 1: 1365 × .0000554 = "755"

Here, we may shift the decimal point in the first factor three places to the left, thus making it a number between 1 and 10. We must then shift the decimal point three places to the right in the second factor; this corresponds to dividing and multiplying by 1000. Thus, the product becomes:

1365. × .0000554 = 1.365 × .0554 = "755"

It is now clear that the answer must be ***.0755.***

Example 2: 304 × 1870 = "568"

In this case, we may shift the decimal point two places to the left in the first factor, and two places to the right in the second factor:

304. × 1870.00 = 3.04 × 187,000 = "568"

It is now evident that the answer must be ***568,000.***

In a *quotient* of two numbers, if the decimal point in the numerator is moved a certain number of places in one direction, the decimal point in the denominator must be moved the same number of places in the *same* direction. Here, the object is usually to make the denominator a number between 1 and 10, thus simplifying the placement of the decimal point.

Example 3: $\dfrac{.0713}{.000225} = \text{“317”}$

We shift the decimal point four places to the right in both numerator and denominator:

$$\frac{.0713}{.000225} = \frac{713}{2.25} = \text{“317”}$$

Note that this corresponds to multiplying numerator and denominator by 10,000. Clearly, the answer must be ***317.***

Example 4: $\dfrac{1}{4360} = \text{“229”}$

Shift decimal point three places to the left in both numerator and denominator:

$$\frac{001.}{4360.} = \frac{.001}{4.36} = \text{“229.”}$$ Answer must be ***.000229.***

Example 5: $\dfrac{32.1}{.00643} = \text{“500”}$

Shift decimal point three places to the right in both numerator and denominator:

$$\frac{32.100}{.00643} = \frac{32{,}100}{6.43} = \text{“500.”}$$ Answer must be ***5000.***

Exercise 1-2

Locate the decimal point in the indicated slide rule answers:

1. $2650 \times .000165 = \text{“437”}$

2. $324 \times 205 = \text{“664”}$

3. $.0642 \times .0322 = \text{“207”}$

4. $.000724 \times 217 = \text{“1570”}$

5. $.026 \times 1640 = \text{“426”}$

6. $\dfrac{361}{.0742} = \text{“486”}$

7. $\dfrac{.000724}{.026} = \text{“279”}$

8. $\dfrac{1.07}{4650} = \text{“230”}$

9. $\dfrac{27.6}{.0032} = \text{“862”}$

10. $\dfrac{12{,}750}{327} = \text{“390”}$

11. $.00724 \times .036 = $ "260"

12. $46{,}000 \times .0023 = $ "1059"

13. $\frac{2.75}{3750} = $ "733"

14. $\frac{0.622}{.00045} = $ "1382"

15. $0.43 \times .00026 = $ "1118"

16. $\frac{1}{.0073} = $ "1370"

17. $\frac{1}{3750} = $ "267"

18. $565 \times 407 = $ "230"

19. $.0064 \times .0024 = $ "1538"

20. $\frac{5640}{23.6} = $ "239"

21. $\frac{.0043}{.075} = $ "573"

22. $\frac{12.6}{.00165} = $ "763"

23. $43{,}000 \times .00611 = $ "262"

24. $265 \times 720 = $ "1908"

25. $.0045 \times .00133 = $ "598"

26. $\frac{1}{.000505} = $ "1980"

27. $\frac{1}{3700} = $ "270"

28. $\frac{14.6}{.000327} = $ "447"

29. $\frac{.0764}{.000223} = $ "343"

30. $420 \times .00065 = $ "273"

1.5 Expressions involving several numbers

In many cases involving more than two numbers, the decimal point may still be located by inspection. However, if the expression appears at all complicated, it is safer to rewrite it with the numbers rounded off to one significant digit.

Example 1: $\frac{37.6 \times 23.8}{71.6} = $ "1250"

Rounding off:

$$\frac{37.6 \times 23.8}{71.6} \approx \frac{40 \times 20}{70} = \frac{80}{7} \approx 11.$$ Answer must be ***12.50.***

Note: The "wavy" equal sign means "approximately equal to."

Example 2: $\frac{236 \times 61.4}{18.85 \times 4.45} = $ "1727"

Rounding off:

$$\frac{236 \times 61.4}{18.85 \times 4.45} \approx \frac{200 \times 60}{20 \times 4} = 150.$$ Answer must be ***172.7.***

Example 3: $$\frac{92.3}{2.37 \times 213} = \text{“1829”}$$

Rounding off:

$$\frac{92.3}{2.37 \times 213} \approx \frac{90}{2 \times 200} = \frac{90}{400} = \frac{9}{40} \approx 0.2.$$

Answer must be ***0.1829.***

Example 4: $$\frac{\pi \times 17.4 \times 2.75}{.0584} = \text{“257”}$$

Rounding off:

$$\frac{\pi \times 17.4 \times 2.75}{.0584} \approx \frac{3 \times 20 \times 3}{.06} = \frac{180}{.06} = \frac{18{,}000}{6} = 3000.$$

Clearly, answer must be ***2570.***

Example 5: $$\frac{.00332 \times 14.7}{.00072} = \text{“678”}$$

Shifting decimal points and rounding off:

$$\frac{.00332 \times 14.7}{.00072} = \frac{33.2 \times 14.7}{7.2} \approx \frac{30 \times 15}{7} = \frac{450}{7} \approx 60.$$

Answer must be ***67.8.***

Exercise 1-3

Locate the decimal point in the indicated slide rule answers:

1. $\frac{65.2 \times 71.6}{107} = \text{“436”}$

2. $\frac{11.45 \times 243}{25.6} = \text{“1087”}$

3. $\frac{26 \times 390}{4.2 \times 5} = \text{“483”}$

4. $\frac{2.61 \times 14.1}{6.05} = \text{“608”}$

5. $\frac{147}{3.45 \times 4.26} = $ "1000"

6. $\frac{362 \times .072}{645} = $ "404"

7. $\frac{32.4 \times 1.62}{11.4} = $ "460"

8. $\frac{37.4 \times 5.63}{9.22} = $ "228"

9. $\frac{148}{3.6 \times 4.12} = $ "997"

10. $\frac{\pi \times 21.2}{15.4} = $ "433"

11. $\frac{4600}{48.2 \times 2.06} = $ "463"

12. $\frac{5.72 \times 43.2}{2.40 \times 3.72} = $ "277"

13. $\frac{16.2 \times 27.2}{5.10 \times 5.90} = $ "1465"

14. $\frac{236 \times 5.60}{8.45 \times \pi} = $ "498"

15. $\frac{278 \times 562}{17.2 \times 8.41} = $ "1078"

16. $\frac{1750 \times 43.6}{36.3 \times 7.22} = $ "291"

17. $\frac{49.2 \times 576}{7250 \times \pi} = $ "1245"

18. $\frac{.0043 \times 56.2}{30.4} = $ "795"

19. $\frac{7850 \times .00034}{.00072} = $ "371"

20. $\frac{23 \times 356 \times 75}{12 \times 46} = $ "1113"

21. $\frac{582 \times 17 \times 62}{432 \times 108} = $ "1315"

22. $.0042 \times 563 \times 27 \times 7.6 = $ "485"

23. $0.46 \times 72.3 \times 5.22 = $ "1735"

24. $\frac{562{,}000 \times .00463}{2.77} = $ "940"

25. $\frac{.0475 \times 3460}{6.44} = $ "255"

26. $\frac{5.72 \times 4.65}{.00324} = $ "821"

27. $\frac{463 \times .00524}{3.22} = $ "753"

28. $\frac{245}{.000344 \times 6340} = $ "1124"

29. $\frac{415 \times 625}{765{,}000} = $ "339"

30. $\frac{3750 \times .00728}{.0263 \times 185} = $ "561"

31. $\frac{.00372 \times 15.6}{.00086} = $ "674"

32. $240 \times 36 \times .0057 \times 0.43 = $ "212"

33. $\frac{37.2 \times .072 \times 6.3}{256 \times .0041} = $ "1608"

34. $\frac{382 \times 624}{21 \times 43 \times 12} = $ "220"

35. $\frac{.036 \times .0073}{.0052} = $ "505"

36. $\frac{1}{.072 \times 6.4 \times 0.32} = $ "679"

37. $\frac{.00468 \times 6250}{3.81} = $ "767"

38. $\frac{2170}{25 \times 5.64 \times 3.44} = $ "447"

39. $\frac{.0032 \times .0843 \times 164}{21 \times 0.63} = $ "334"

40. $146 \times 21.6 \times .0072 = $ "227"

1.6 Scientific notation

A number is written in scientific notation when it is written in the form $M \times 10^n$, where M is a number between 1 and 10, and n is a positive or negative integer.

Example 1: $4630 = 4.63 \times 1000 = 4.63 \times 10^3$

Example 2: $.000706 = 7.06 \div 10{,}000 = 7.06 \times 10^{-4}$

It is clear that, in each case, the decimal point has been shifted to give a number between 1 and 10; furthermore, he number of places it has been shifted determines the exponent of the power of 10. If it has been moved to the left, the exponent is positive; if to the right, the exponent is negative.

Other examples of numbers converted to scientific notation follow:

number	*exponent*	*number in scientific notation*
5,740,000	6	5.74×10^6
.0000307	−5	3.07×10^{-5}
0.624	−1	6.24×10^{-1}
4.75	0	$4.75 \times 10^0 = 4.75$

1.7 Laws of exponents

In the next section we make use of the following laws of exponents:

1. When powers of the same base are *multiplied*, the exponents are *added*.
2. When powers of the same base are *divided*, the exponents are *subtracted*.

Examples:

a. $10^3 \times 10^{-2} \times 10^4 = 10^{3-2+4} = 10^5$

b. $\dfrac{10^5}{10^3} = 10^{5-3} = 10^2$

c. $\dfrac{10^5 \times 10^{-3}}{10^2} = 10^{5-3-2} = 10^0 = 1$

d. $\dfrac{10^2 \times 10^{-3} \times 10^4}{10^{-2} \times 10^8} = 10^{2-3+4+2-8} = 10^{-3}$

1.8 Approximation using scientific notation

When rounding off numbers to one significant digit, it is often helpful to put them in scientific notation at the same time. The following examples illustrate the procedure:

Example 1: $\dfrac{.00713 \times 42{,}300}{324 \times .0000516} = \text{“1802”}$

Round off to single digits using scientific notation:

$$\frac{.00713 \times 42{,}300}{324 \times .0000516} \approx \frac{7 \times 10^{-3} \times 4 \times 10^{4}}{3 \times 10^{2} \times 5 \times 10^{-5}}$$

Now, combining exponents:

$$\frac{7 \times 4}{3 \times 5} \times 10^{-3+4-2+5} = \frac{28}{15} \times 10^{4} \approx 2 \times 10^{4}$$

Therefore, answer must be ***1.802*** $\times$ ***10***^{4} or ***18,020.***

The above operation may be shortened by simply writing the exponent above each factor of the numerator, and below each factor of the denominator:

$$\frac{\overset{(-3)}{.00713} \times \overset{(4)}{42{,}300}}{\underset{(2)}{324} \times \underset{(-5)}{.0000516}} \approx \frac{7 \times 4}{3 \times 5} \times 10^{-3+4-2+5} \approx 2 \times 10^{4}$$

Example 2: $523 \times 46.2 \times 73{,}100 = \text{“1763”}$

Approximate as follows:

$$\overset{(2)}{523} \times \overset{(1)}{46.2} \times \overset{(4)}{73{,}100} \approx 5 \times 5 \times 7 \times 10^{2+1+4} = 175 \times 10^{7} = 1.75 \times 10^{9}$$

Answer must be ***1.763*** $\times$ ***10***^{9}.

Example 3: $\dfrac{.000273 \times 6.33 \times 6440}{0.821 \times 237{,}000} = \text{“572”}$

$$\frac{\overset{(-4)}{.000273} \times \overset{(0)}{6.33} \times \overset{(3)}{6440}}{\underset{(-1)}{0.821} \times \underset{(5)}{237{,}000}} \approx \frac{3 \times 6 \times 6}{8 \times 2} \times 10^{-5} = \frac{27}{4} \times 10^{-5} \approx 7 \times 10^{-5}$$

Answer must be ***5.72*** $\times$ ***10***$^{-5}$ or ***.0000572.***

Example 4: $\dfrac{4.12}{275 \times 36.2 \times .0074} = \text{“559”}$

$$\frac{\overset{(0)}{4.12}}{\underset{(2)}{275} \times \underset{(1)}{36.2} \times \underset{(-3)}{.0074}} \approx \frac{4}{3 \times 4 \times 7} \times 10^{0} = \frac{1}{21} \times 10^{0} \approx .05$$

Answer must be ***.0559.***

Example 5: $\dfrac{21.2 \times .00465}{0.733 \times 1740 \times 10^{-5}} = \text{“773”}$

$$\frac{\overset{(1)}{21.2} \times \overset{(-3)}{.00465}}{\underset{(-1)}{0.733} \times \underset{(3)}{1740} \times \underset{(-5)}{10^{-5}}} \approx \frac{2 \times 5}{7 \times 2 \times 1} \times 10^{1} = \frac{50}{7} \approx 7$$

Answer must be ***7.73.***

Exercise 1-4

Locate the decimal point using scientific notation. If the answer is very small or very large, it is better to leave it in the scientific notation form.

1. $.000482 \times .0000612 = \text{“295”}$
2. $3760 \times 28 \times 4810 = \text{“506”}$
3. $\dfrac{.0000216 \times 587}{317{,}000} = \text{“400”}$
4. $\dfrac{2430 \times 0.416}{.000136} = \text{“744”}$
5. $\dfrac{.000742}{124{,}500} = \text{“596”}$
6. $.00720 \times 2410 \times 35{,}000 = \text{“607”}$
7. $\dfrac{572 \times 43{,}000}{.00375} = \text{“656”}$
8. $\dfrac{6.73 \times 0.217 \times 5430}{.000245 \times 38.2} = \text{“846”}$
9. $\dfrac{4.35 \times .001675}{5840 \times 21.6} = \text{“578”}$
10. $\dfrac{1}{68.2 \times .000781 \times 0.207} = \text{“907”}$
11. $\dfrac{100}{1.63 \times 4830 \times 56.7} = \text{“224”}$
12. $.00713 \times 560 \times 10^{-3} = \text{“400”}$
13. $\dfrac{1645 \times .0724}{36.1 \times 4310} = \text{“767”}$
14. $\dfrac{10^{4}}{25.2 \times .0756 \times 644} = \text{“815”}$
15. $176 \times 93.5 \times .000455 = \text{“749”}$
16. $\dfrac{6530 \times 10^{5}}{1726 \times 23.7 \times 408} = \text{“391”}$
17. $\dfrac{.0756 \times 545 \times 10^{-3}}{.00534 \times 8670 \times 17.5} = \text{“509”}$
18. $.00577 \times 368 \times .0305 = \text{“648”}$
19. $\dfrac{1}{38.9 \times .00219 \times 6.44} = \text{“1824”}$
20. $24.9 \times 1740 \times 522 \times 10^{-4} = \text{“2260”}$
21. $\dfrac{.00683 \times .0468}{.000792 \times .0502} = \text{“803”}$
22. $\dfrac{56.3 \times 16.4 \times 10^{10}}{.000745 \times 136.5} = \text{“907”}$
23. $\dfrac{.000713 \times .0644}{43.9 \times 1.4 \times 10^{-6}} = \text{“747”}$
24. $\dfrac{340 \times 56.2 \times 755 \times 12}{.00439} = \text{“394”}$

25. $\dfrac{5.31 \times 66 \times 134 \times 10^{-4}}{.000294} = \text{"1598"}$

26. $\dfrac{1540 \times 36.2 \times .0426}{570} = \text{"416"}$

27. $\dfrac{.0774}{39.2 \times 143 \times 0.22} = \text{"627"}$

28. $436 \times 124 \times 15.6 \times 56.4 = \text{"475"}$

29. $\dfrac{19{,}000 \times 43.6}{285 \times .0622} = \text{"467"}$

30. $\dfrac{3.41 \times 563 \times .0071}{22.4 \times 172{,}000} = \text{"354"}$

Chapter 2

READING THE C AND D SCALES

2.1 Physical parts of the slide rule

That part of the slide rule which is fixed between the end plates is called the *body;* the long, movable center portion is called the *slide,* and the glass runner is called the *indicator.* The thin, vertical line on the indicator is referred to as the *hairline.* These parts are illustrated in Figure 2.1.

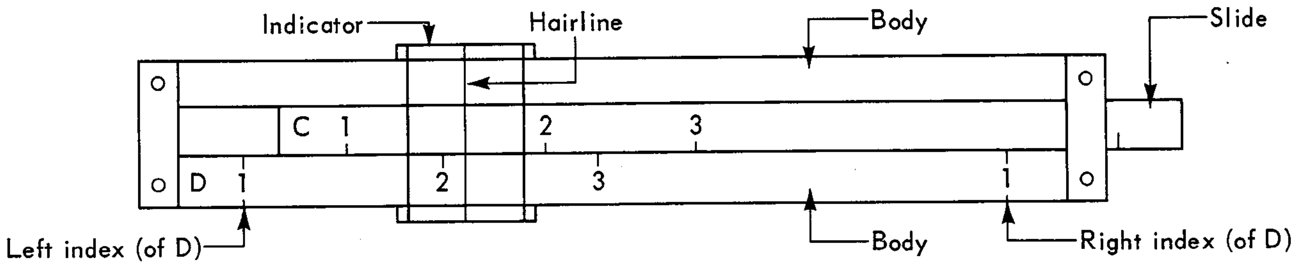


Figure 2.1

2.2 Uniform and nonuniform scales

An ordinary ruler or yardstick is an example of a uniform scale. Here, the distance, say, between the 2-inch mark and the 3-inch mark is the same as the distance between the 7-inch mark and the 8-inch mark; that is, the interval between consecutive inch marks is the same on all parts of the scale.

On a nonuniform scale, the distance between consecutive primary markings does not remain the same on all parts of the scale. Thus, the distance between 1 and 2 may be greater than the distance between 2 and 3, and so on. Now, if a conventional slide rule is examined, it will be seen that the L scale is the only uniform scale on the rule; all other scales are nonuniform.

2.3 Description of the C and D scales

The C and D scales are identical scales located on the slide and body respectively. You will note that these scales have ten primary marks which are numbered with the large numerals: 1, 2, 3, · · · 8, 9, 1. The primary mark corresponding to the large numeral 1 is called the index of the scale; hence, there are two indexes associated with each scale—a left index and a right index (see Figure 2.1).

Observe that the distance between the primary marks 1 and 2 is greater than the distance between 2 and 3, which, in turn, is greater than the distance between 3 and 4, etc. This nonuniform characteristic of the scale results from the fact that the scale distances are proportional to the logarithms of the corresponding numbers. Since the logarithm of 1 is zero, this explains why the scales start with the numeral 1.

Between the primary marks 1 and 2, there are ten secondary divisions which are numbered with the small numerals 1, 2, 3, · · · 8, 9. These secondary divisions are in turn divided into ten small intervals. Each of these smallest intervals may be taken to represent *one* unit.

Between the primary marks 2 and 3, there are also ten secondary divisions (not numbered), and each of these is in turn divided into five small intervals. Each of these smallest intervals then represents *two* units. The same is true for the portion of the rule between primary numbers 3 and 4.

Between 4 and 5, there are again ten secondary divisions, and each of these is divided into two small intervals; hence, each of these smallest intervals represents *five* units. The same holds true for the remaining primary divisions between 5 and the right index.

Portions of the D scale are shown in Figure 2.2, illustrating the markings.

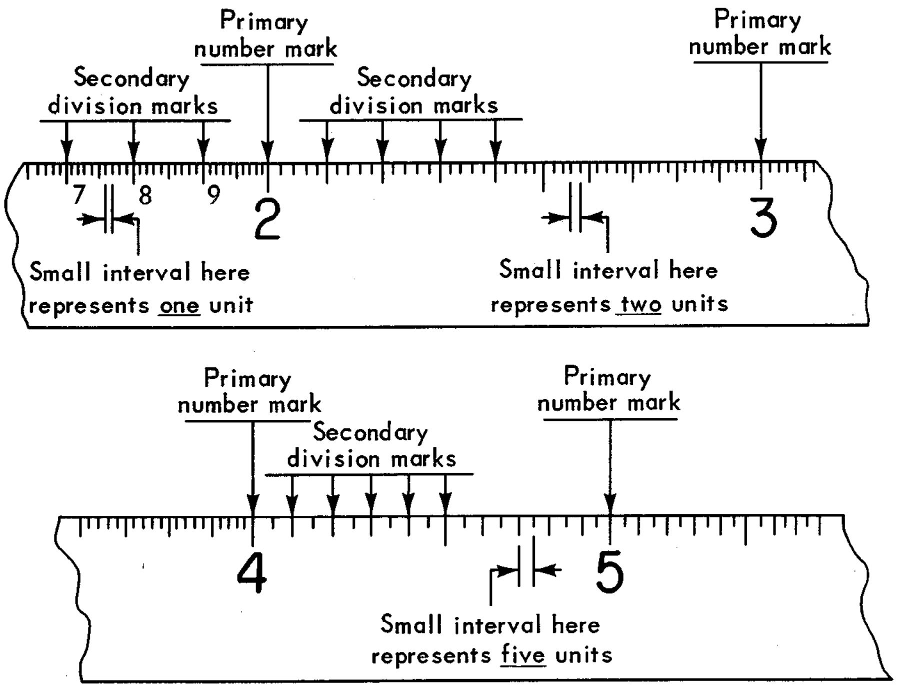


Figure 2.2

2.4 Scale setting independent of decimal point

As previously noted, the setting corresponding to a particular number is not affected by the position of the decimal point. Thus, the setting "237" may represent the numbers 23.7, 2.37, .00237, 2370; that is, the position on the scale is determined only by the digits "two-three-seven."

2.5 Reading the C and D scales

Consider that portion of the D (or C) scale between the primary numbers 3 and 4 (see Figure 2.3). Now if the hairline is positioned directly over the primary number 3, the setting corresponds to "300." This setting may represent the numbers 30, or .0300, or 30,000, and so on. If the hairline is moved over the first secondary division mark, the setting corresponds to "310"; that is, it may represent 3.10, or .00310, or 3100. If it is moved directly over the fourth secondary division mark, the setting corresponds to "340," and this may represent 34 or .034, and so forth.

Now consider the setting "346." It will be located between the secondary division marks corresponding to "340" and "350." Inasmuch as there are five small intervals, or spaces, between the secondary marks, each small space represents two units. Hence, to locate "346," the hairline is moved three small spaces to the right of "340." Again it is emphasized that the setting for "346" may represent the numbers 34.6, 3.46, 346, .00346, and so on.

As another example, consider "313." It will be located between the secondary division marks corresponding to "310" and "320." Remembering that each small interval represents two units, "313" is set by moving the hairline one and one-half small spaces to the right of "310."

These settings are illustrated in Figure 2.3, along with other typical settings in this range.

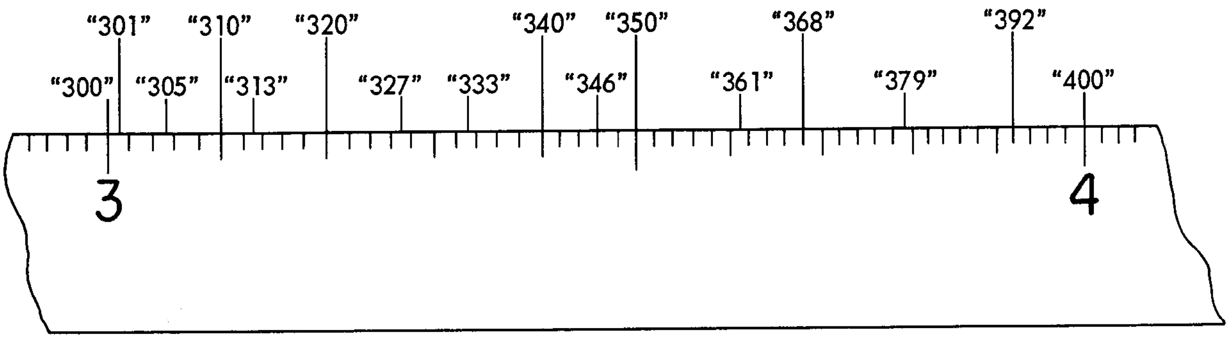


Figure 2.3

Now observe the portion of the scale between primary numbers 7 and 8, and consider the setting "733." It will be located between the secondary division marks corresponding to "730" and "740." Now if the hairline were to be moved over the small mark between "730" and "740" it would be at "735." Clearly, then, to locate "733," the hairline must be moved beyond "730" just three-fifths of the small interval between

"730" and "735." This position must be estimated, and it can be seen that it becomes difficult to account precisely for the third digit at this end of the scale.

This setting and others in this range are shown in Figure 2.4.

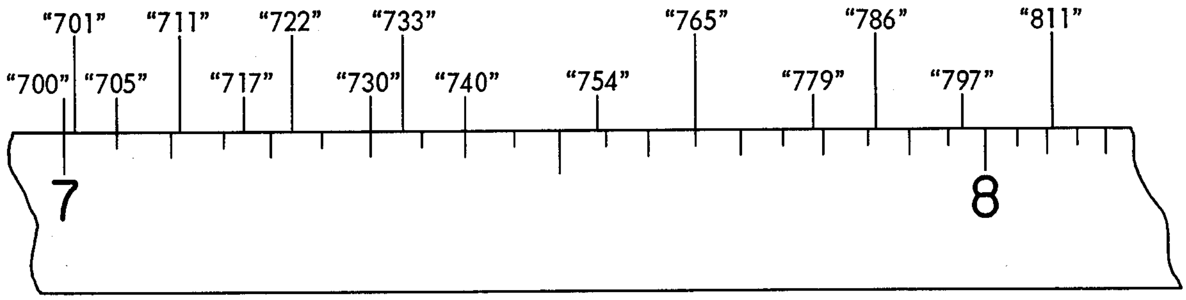


Figure 2.4

Finally, consider the left end of the scale between primary numbers 1 and 2. Because the distance between these two primary numbers is so large, the secondary division marks here are numbered with small numerals. You will note that when reading this part of the scale, it is possible to approximate a fourth digit. For example, consider the location of "1257." If the hairline is moved over the secondary division mark labeled with the small numeral 2, its position represents "1200." If it is now moved five small spaces further to the right, the reading is "1250." Finally, it must be carefully moved an additional distance estimated to be seven-tenths of the next small space. This locates "1257."

This setting and other typical settings are illustrated in Figure 2.5.

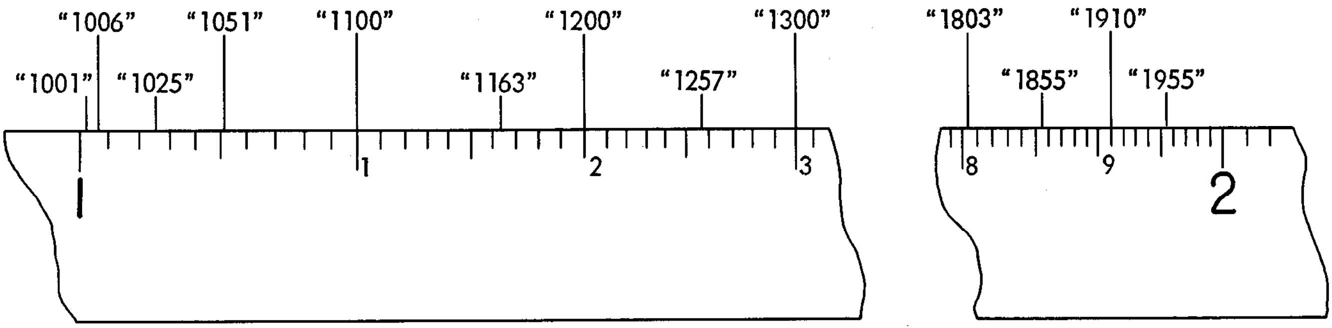


Figure 2.5

2.6 Accuracy of the slide rule

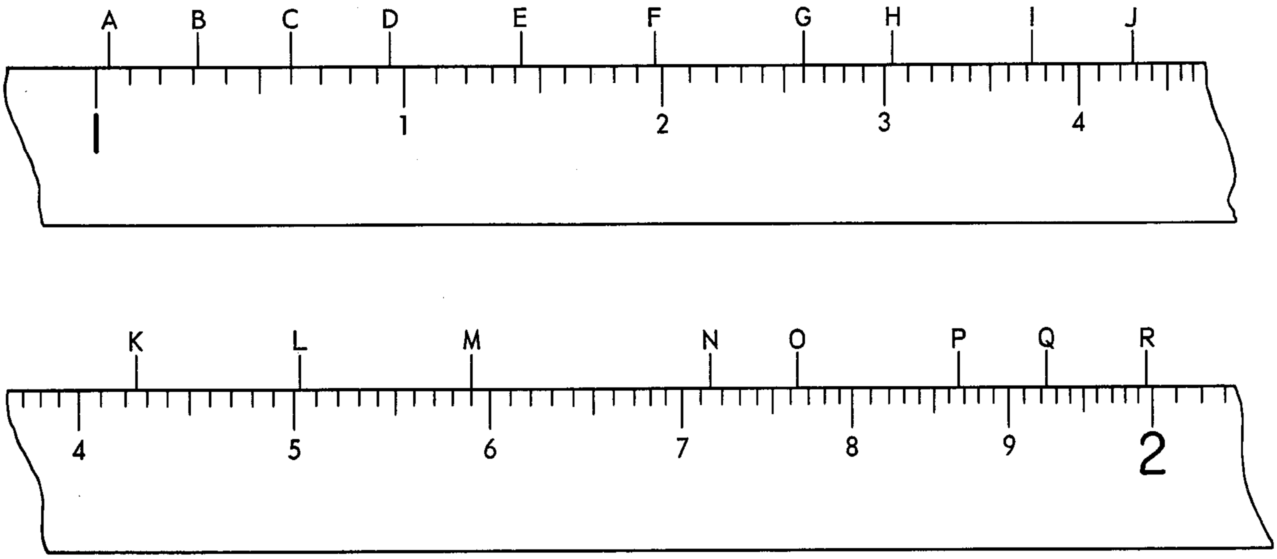
Suppose you possessed an ideal slide rule, precisely aligned and scribed, with no construction defects whatever. In reading such a rule, there would still be a possible error due to the limitation of your eye itself. On a 10-inch slide rule, the error in reading the C and D scales with the unaided eye is normally about 1 part in 1000. On a 20-inch scale, the observational error would be about 1 part in 2000, whereas on a 5-inch scale, the error would be about 1 part in 500.

Thus, on a 10-inch scale, if the observed position of the hairline is 1003, the possible error in the reading is about ±1, and the exact position of the hairline could be anywhere between 1002 and 1004. Again, if the observed hairline position is 313, the possible error is about ±0.3, and the exact position could be anywhere between 312.7 and 313.3. Finally, if the observed reading is 997, the possible error is again about ±1, and the exact position could be anywhere between 996 and 998.

It is apparent that for readings at the extreme left end of the scale, the fourth digit is good within ±1, whereas for readings at the extreme right end of the scale, the third digit is good within ±1. Ordinarily, the fourth digit is estimated for readings between primary marks 1 and 2; that is, numbers whose first digit is 1. All other settings are normally read to three digits only. Thus, typical slide rule readings (involving 10-inch C and D scales) would be: "1234," "602," "354," "1365," "437," "1187." To present a 10-inch scale reading as, say, "5837" would be unrealistic; here, the observational error is ±6, which makes the fourth digit meaningless.

Exercise 2-1

1. In Figure 2.6, read the indicated settings. Estimate the fourth digit.

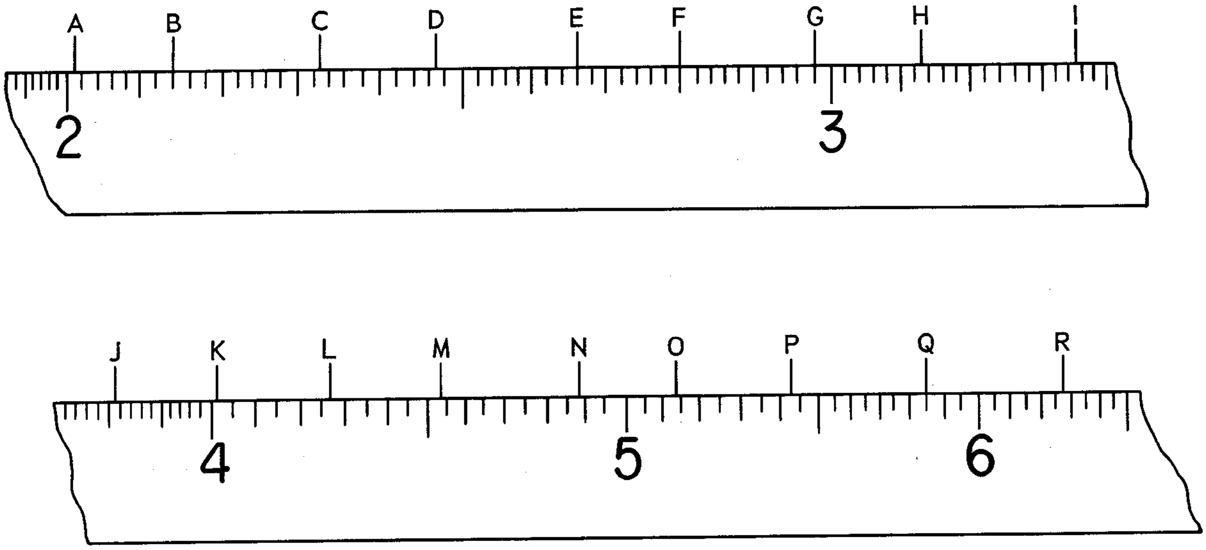


A________ B________ C________ D________ E________ F________ G________ H________ I________

J________ K________ L________ M________ N________ O________ P________ Q________ R________

Figure 2.6

2. In Figure 2.7, read the indicated settings to three significant digits.



A_______ B_______ C_______ D_______ E_______ F_______ G_______ H_______ I_______

J_______ K_______ L_______ M_______ N_______ O_______ P_______ Q_______ R_______

Figure 2.7

2.7 Reading the L scale

As mentioned before, the L scale is a uniform scale and, therefore, is straightforward to read. Although the principal use of this scale will be discussed in a later chapter, it may now be used to check your skill in reading the C and D scales on your own slide rule.

Examination of the L scale shows that it is divided into equal primary divisions marked 0, .1, .2, · · ·, .9, 1. (On some slide rules, the decimal point is omitted before the numeral; however, it is understood to be there, whether shown or not). Each primary division is divided into ten secondary divisions each representing .01, and each secondary division is, in turn, divided into five small intervals each representing .002. A portion of the left end of the L scale, together with some typical settings, is illustrated in Figure 2.8.

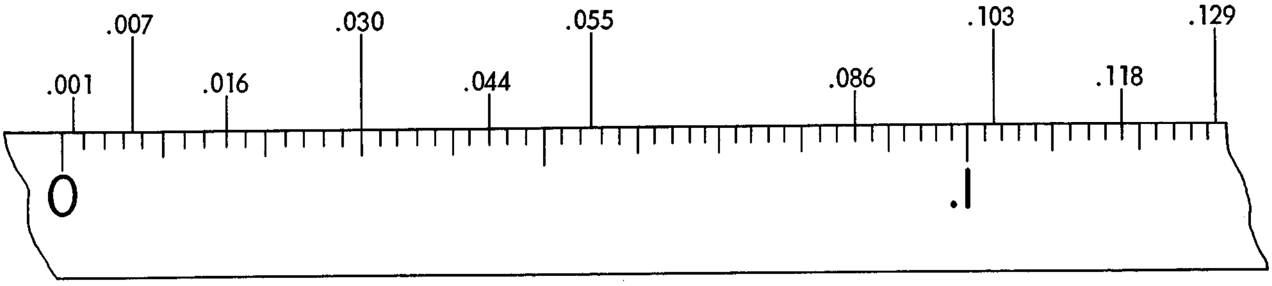


Figure 2.8

2.8 Using the L scale to check C and D readings

Let the slide rule be closed with C and D indexes exactly aligned. Now move the hairline over .312 on L, and observe the corresponding reading under the hairline on C and D. You should be reading "205" on C and D. Next, move the hairline over .617 on L; you should now be reading "414" on C and D. Finally, move the hairline over 15.45 on C and D (remember that the decimal point position does not affect the setting; the hairline is simply set at "1545"). The corresponding reading on L should now be .189.

The following exercise will further test your ability to properly read these scales.

Exercise 2-2

With rule closed (C and D indexes aligned), carefully note the corresponding readings and complete the following tables.

L	C or D
.100	"1259"
.200	
.300	
.400	
.500	
.600	
.700	
.800	
.900	
.480	
.132	
.950	
.312	
.020	
.002	
.405	

L	C or D
.989	
.266	
.810	
.006	
.850	
.030	
.517	
.998	
.389	245
	348
	107.4
	21.3
	0.398
	.0692
	15
	87,300

L	C or D
	74.4
	.00433
	100.7
	37
	10.65
	0.905
	5080
	.0101
	6.37
	4100
	.0022
	11
	96,300
	1095
	0.598
	1.765

Chapter 3

MULTIPLICATION AND DIVISION (C AND D SCALES)

3.1 Slide rule adds and subtracts lengths

The slide rule is an instrument which is designed to add or subtract lengths. If these lengths are proportional to the logarithms of numbers, it follows that adding such lengths corresponds to multiplying numbers, whereas subtracting the lengths corresponds to dividing numbers. The C and D scales are the basic scales used in these operations.

3.2 Multiplication of two numbers

The mechanics of multiplication may be illustrated by the following examples:

Example 1: $2 \times 4 = ?$ (Figure 3.1)

1. Set left index of C opposite 2 on D.
2. Move hairline over 4 on C.
3. Under hairline read "8" on D.

Essentially, we have performed the multiplication by adding logarithms:

$$\log 2 + \log 4 = \log(2 \cdot 4) = \log 8$$

More properly, we should state that the lengths which are added or subtracted are proportional to the *mantissas* of the logarithms. Thus, the slide rule combines the mantissas only and does not account for the characteristic. This is why the slide rule fails to locate the decimal point.

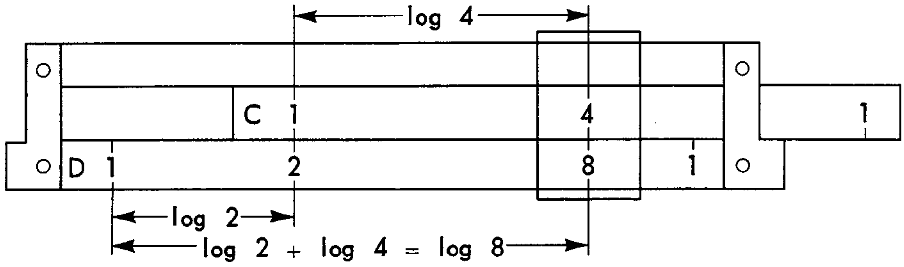


Figure 3.1

Example 2: 16 × 3 = ? (Figure 3.2)

1. Set left index of C opposite 16 on D.
2. Move HL (hairline) over 3 on C.
3. Under HL read "480" on D. Answer is ***48.***

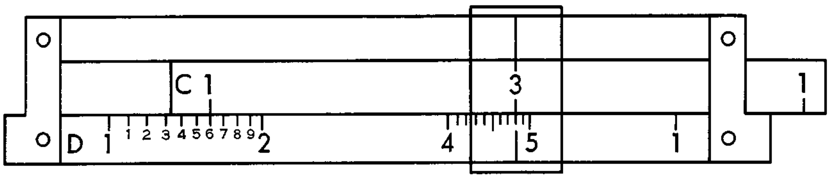


Figure 3.2

Example 3: 20.3 × 3.86 = ?

1. Set left index of C opposite 203 on D.
2. Move HL over 386 on C.
3. Under HL read "784" on D.

Rounding off: 20.3 × 3.86 ≈ 20 × 4 = 80. Answer must be ***78.4.***

Example 4: 1765 × .0000422 = ?

1. Set left index of C opposite 1765 on D.
2. Move HL over 422 on C.
3. Under HL read "745" on D.

Shifting decimal points and rounding off:

1765 × .0000422 = 1.765 × .0422 ≈ 2 × .04 = .08. Answer must be ***.0745.***

Example 5: 18.35% of 276 is ?

This corresponds to the product 0.1835 × 276.

1. Set left index of C opposite 1835 on D.

2. Move HL over 276 on C.
3. Under HL read "506" on D.

Rounding off: $0.1835 \times 276 \approx 0.2 \times 300 = 60$. Answer must be ***50.6.***

The foregoing examples illustrate the general procedure:

> *To multiply two numbers:*
> 1. Disregard the decimal point and set the index of C over one of the numbers on D.
> 2. Move HL over the other number on C.
> 3. Under HL read answer on D.
> 4. Locate decimal point in answer.

Exercise 3-1

1. $1.8 \times 3.6 =$

2. $2.4 \times 2.2 =$ "528"

3. $1.4 \times 5.8 =$

4. $15 \times 4.7 =$ "705"

5. $28 \times 3.7 =$

6. $2.2 \times 35 =$ "770"

7. $1.4 \times 5.2 =$

8. 11% of 175 = "1925"

9. $28 \times 3.3 =$

10. $14 \times 2.6 =$ "364"

11. 17.7% of 234 =

12. $2.07 \times 3.26 =$ "675"

13. $10.75 \times 6.42 =$

14. 23.4% of 26.3 = "615"

15. 29.6% of 308 =

16. $1.245 \times 14.7 =$ "1830"

17. $284 \times 1855 =$

18. $196 \times .0000207 =$ "406"

19. $1145 \times .000641 =$

20. $.000143 \times 463 =$ "662"

3.3 Either index may be used

Consider the product 5×0.4. If the left index of C is set opposite 5 on D, it is found that the number 4 is on the part of the C scale that projects beyond the right index of the D scale; thus it is impossible to read the answer on D. (Figure 3.3).

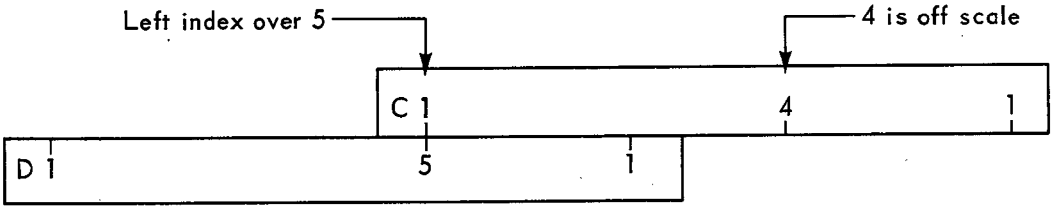


Figure 3.3

If we could somehow extend the D scale for another cycle, the answer would be located on this "extended" scale as shown in Figure 3.4.

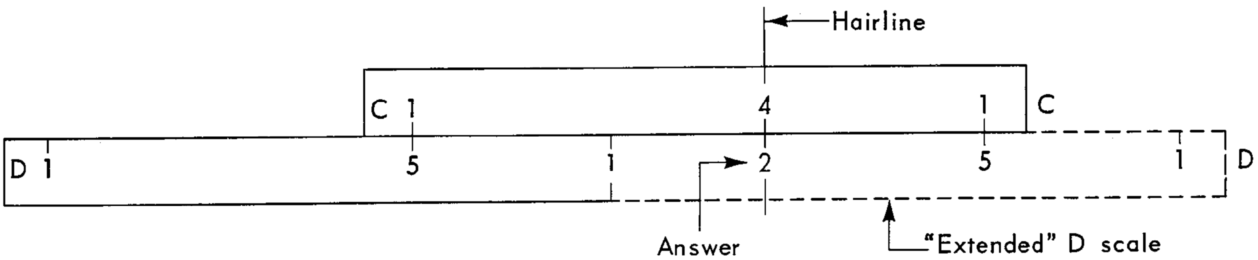


Figure 3.4

Now in Figure 3.4, note that the right index of C is opposite 5 on the "extended" scale. It is clear that the same multiplication may be accomplished by setting the *right index* of C opposite 5 on D, moving the hairline over 4 on C, and reading the result on D. This equivalent operation is illustrated in Figure 3.5.

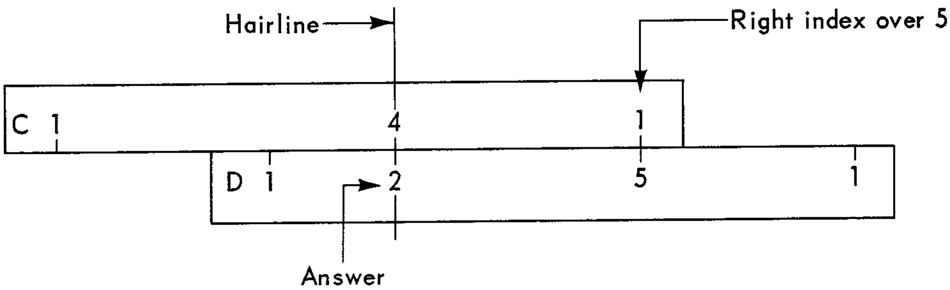


Figure 3.5

We summarize with the following statement:

> When multiplying two numbers, make the first setting with *either* the *left* or *right* index of C, whichever one will ensure that the answer can be read on the D scale.

Example 1: $9 \times .04 = ?$ (Figure 3.6)

1. Set right index of C opposite 9 on D.
2. Move HL over 4 on C.
3. Under HL read "360" on D. Answer is ***0.36.***

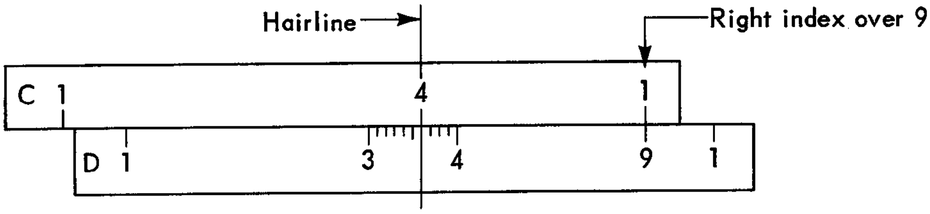


Figure 3.6

Example 2: $5.43 \times 3.26 = ?$

Again, the right index must be used.

1. Set right index of C opposite 543 on D.
2. Move HL over 326 on C.
3. Under HL read "1770" on D.

By inspection, answer must be ***17.70.***

Exercise 3-2

1. $6.5 \times 4.2 =$
2. $5.6 \times 5.7 =$ "319"
3. $8.4 \times 7.6 =$
4. $93 \times 1.7 =$ "1581"
5. $4.6 \times 3.2 =$
6. 37% of 475 = "1758"
7. 56% of 26.7 =
8. 43% of 3240 = "1393"
9. $6.25 \times 3.40 =$
10. $56 \times 40.7 =$ "228"
11. $0.74 \times 367 =$
12. $6.90 \times 22.4 =$ "1546"
13. $485 \times 2.53 =$
14. $65.4 \times 0.392 =$ "256"
15. $8360 \times .0206 =$
16. $7.14 \times 64.0 =$ "457"
17. $46.5 \times 31.2 =$
18. $.00945 \times 50.6 =$ "478"
19. $.000486 \times 366 =$
20. $408 \times 71.2 =$ "290"

3.4 Excessive extension of slide should be avoided

In multiplying two numbers, there is always a choice as to the order in which they are multiplied. Often, if the numbers are multiplied in one way the slide will extend far out of the body, whereas if the order is reversed this will not be the case. It is good practice in all slide rule operations to try to keep at least one half of the slide within the body.

Example 1: $38.4 \times .01245 = ?$

Note that if the left index of C is set opposite 384 on D, the slide extends far to the right; hence, it is better to begin by setting the index opposite 1245:

1. Set left index of C opposite 1245 on D.
2. Move HL over 384 on C.
3. Under HL read "478" on D. Answer is ***0.478.***

Example 2: $2.43 \times 8.16 = ?$

Here, the right index of C must be used, and the smaller extension of the slide occurs when the right index is set opposite 8.16:

1. Set right index of C opposite 816 on D.
2. Move HL over 243 on C.
3. Under HL read "1983" on D. Answer is ***19.83.***

Exercise 3-3

1. $1.6 \times 23 =$
2. $3.7 \times 12 =$ "444"
3. $2.1 \times 25 =$
4. $43 \times 10.2 =$ "439"
5. $5.6 \times 11 =$
6. $6.3 \times 13 =$ "819"
7. $230 \times 3.2 =$
8. $15 \times 19 =$ "285"
9. $27 \times 32 =$
10. $5.6 \times 3.24 =$ "1814"
11. $8.2 \times 4.06 =$
12. $71.2 \times 6.44 =$ "459"
13. $21 \times 63 =$
14. $185 \times 0.74 =$ "1369"
15. $4.66 \times 14.3 =$
16. $7.24 \times 10.65 =$ "771"
17. $27.5 \times 61 =$
18. $4.2 \times 83.6 =$ "351"
19. $35.1 \times 0.22 =$
20. $55 \times 3.7 =$ "204"

21. 7.6 × 10.45 =

22. 43 × 2.07 = "890"

23. 0.634 × 824 =

24. 12 × 26.1 = "313"

25. 36.2 × 0.209 =

26. 2.57 × 42.6 = "1095"

27. 3.75 × 92.5 =

28. 9.38 × 18.8 = "1763"

29. 32.6 × 3.15 =

30. 25.3 × 9.06 = "229"

31. 2.84 × 56.7 =

32. 3150 × 2.70 = "851"

33. 7.33 × 11.85 =

34. 30.4 × 7.56 = "230"

35. 0.466 × 50.2 =

36. 15.45 × 8.05 = "1244"

37. 623 × 0.124 =

38. 5.77 × 5.77 = "333"

39. 21.8 × 21.8 =

40. 2.56 × .00304 = "778"

41. .0245 × 432 =

42. 1015 × .00726 = "737"

43. 6.32 × .00124 =

44. 362 × 243 = "880"

45. 175 × 63.4 =

46. 5020 × .00318 = "1596"

47. 750 × .0222 =

48. 1085 × 0.986 = "1070"

49. .0784 × .001135 =

50. 436 × 2160 = "942"

51. 34,600 × .000375 =

52. .00643 × .00912 = "586"

53. 92.2 × 1250 =

54. .000254 × 412,000 = "1046"

55. 34.2 × .00664 =

56. 20.7 × 87.4 = "1809"

57. .00417 × .0505 =

58. 63,200 × .0001095 = "692"

59. .0759 × .0759 =

60. $(195)^2$ = "380"

61. .000612 × 56.7 =

62. 24.2% of 83.6 = "202"

63. 11.55% of 4310 =

64. 75.2% of .0905 = "681"

65. 43.6% of 7.62 =

66. 2.07% of 520 = "1076"

67. 33.3% of 1545 =

68. 123% of .0842 = "1036"

69. 3.05% of 23,400 =

70. 0.62% of 835 = "518"

3.5 Division

Division is the inverse of multiplication, and is illustrated by the following examples:

Example 1: $\frac{6}{3} = ?$ (Figure 3.7).

1. Move HL over 6 on D.
2. Slide 3 on C under HL.
3. Opposite left index of C read "200" on D. Answer is **2.**

Analysing this operation, it is clear that we have simply found the number which must multiply 3 to give 6. Also from Figure 3.7, it can be seen that the answer is located at (log 6 − log 3); hence, we have essentially performed the division by subtracting logarithms:

$$\log 6 - \log 3 = \log(6 \div 3) = \log 2$$

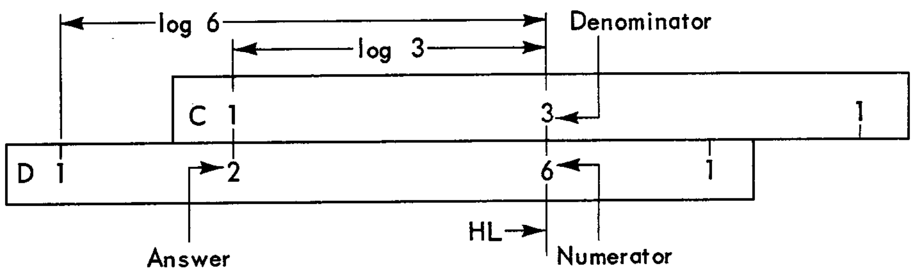


Figure 3.7

Note especially that the hairline is first moved over the *numerator* on *D*, the denominator is then pushed under the hairline on C, and the answer appears opposite the C index on D.

Example 2: $\frac{3}{5} = ?$ (Figure 3.8).

1. Move HL over 3 on D.
2. Slide 5 on C under HL.
3. Opposite right index of C read "600" on D. Answer is ***0.6.***

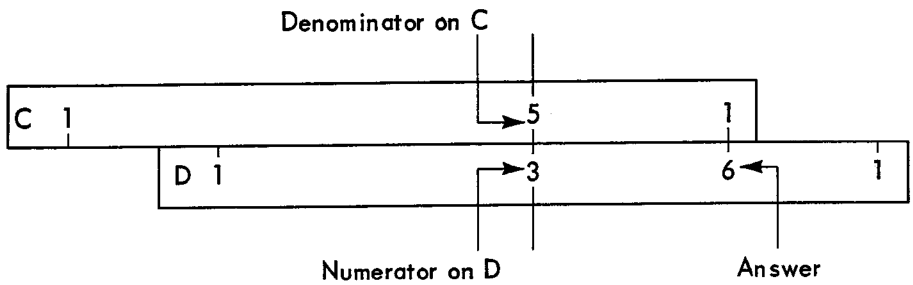


Figure 3.8

Example 3: $\frac{5.74}{3420} = ?$

1. Move HL over 574 on D.
2. Slide 342 on C under HL.
3. Opposite left index of C read "1678" on D.

Write: $\frac{5.74}{3420} = \frac{.00574}{3.42} \approx \frac{.006}{3} = .002.$ Answer must be ***.001678.***

Example 4: $312 \div 4.75 = ?$

1. Move HL over 312 on D.
2. Slide 475 on C under HL.
3. Opposite right index of C read "657" on D. Answer is ***65.7.***

The procedure may be summarized:

> *To divide two numbers:*
> 1. Disregard the decimal points, and move the hairline over the *numerator* on the *D scale.*
> 2. Move the slide so that the *denominator* on the *C scale* is under the hairline.
> 3. Read the answer on D opposite the right or left index of C, whichever is on scale.
> 4. Locate the decimal point in the answer.

Notice that there is only one order in which division may be performed using the C and D scales and, unlike multiplication, there is no possibility of the answer being off scale. However, there are certain combinations which will result in excessive slide extension. For example, if 11 is divided by 9, the slide must be moved almost entirely out of the body. As will be shown later, this may be avoided by using the inverted or folded scale.

Exercise 3-4

1. $\frac{62}{28} =$ **3.** $\frac{84}{25} =$ **5.** $\frac{13}{21} =$

2. $\frac{47}{19} =$ "247" **4.** $\frac{364}{142} =$ "256" **6.** $\frac{66}{145} =$ "455"

7. $\frac{1075}{240} =$

8. $\frac{520}{45} =$ "1155"

9. $\frac{17.6}{4.2} =$

10. $\frac{82.7}{31.6} =$ "262"

11. $\frac{143.5}{54} =$

12. $\frac{12.45}{3.26} =$ "382"

13. $\frac{282}{12} =$

14. $\frac{36}{4.1} =$ "878"

15. $\frac{756}{204} =$

16. $\frac{20.4}{5.66} =$ "360"

17. $\frac{38.4}{12.1} =$

18. $\frac{902}{23.1} =$ "390"

19. $\frac{82.4}{27.4} =$

20. $\frac{30.9}{5.24} =$ "590"

21. $\frac{1255}{63.1} =$

22. $\frac{420}{312} =$ "1346"

23. $\frac{5640}{2.03} =$

24. $\frac{67.7}{7.22} =$ "938"

25. $\frac{105.7}{82.4} =$

26. $\frac{62.7}{2.54} =$ "247"

27. $\frac{1945}{364} =$

28. $\frac{29.2}{11.43} =$ "255"

29. $\frac{71.9}{27.4} =$

30. $\frac{9.98}{5.04} =$ "1980"

31. $\frac{1}{175} =$

32. $\frac{.01}{76.6} =$ "1305"

33. $\frac{82.4}{524} =$

34. $\frac{9.46}{102} =$ "927"

35. $\frac{395}{48.2} =$

36. $\frac{26.2}{0.124} =$ "211"

37. $\frac{3.14}{16.4} =$

38. $\frac{295}{3600} =$ "819"

39. $\frac{32.4}{.0288} =$

40. $\frac{7.59}{248} =$ "306"

41. $\frac{920}{1436} =$

42. $\frac{3.24}{56{,}500} =$ "573"

43. $\frac{.00421}{2.17} =$

44. $\frac{754}{.00233} =$ "323"

45. $\frac{24.6}{.0824} =$

46. $\frac{368}{28.2} =$ "1305"

47. $\frac{1045}{54{,}200} =$

48. $\frac{.000411}{.0613} =$ "671"

49. $\frac{7240}{39.2} =$

50. $\frac{62{,}700}{382} =$ "1641"

51. $\frac{.00944}{543} =$

52. $\frac{237}{472{,}000} =$ "502"

53. $164.5 \div 3.14 =$

54. 100 ÷ 3750 = "267"

55. 28.2 ÷ .000466 =

56. 3.99 ÷ 46.2 = "863"

57. 783 ÷ 9020 =

58. $\frac{1}{49.2}$ = "203"

59. $\frac{.0504}{1675} =$

60. $\frac{72{,}100}{.0234}$ = "308"

61. $\frac{3.14}{3620} =$

62. $\frac{1000}{0.461}$ = "217"

63. $\frac{27.9}{0.123} =$

64. $\frac{49{,}200}{32.1}$ = "1533"

65. $\frac{.00723}{.0211} =$

66. $\frac{4300}{.0182}$ = "236"

67. $\frac{74{,}600}{25.2} =$

68. $\frac{.00207}{.000523}$ = "396"

69. $\frac{74.5}{4300} =$

70. $\frac{2.43}{.00071}$ = "342"

71. $\frac{5640}{0.920} =$

72. $\frac{293}{.0823}$ = "356"

73. $\frac{0.411}{.00624} =$

74. $\frac{9430}{.0912}$ = "1034"

75. $\frac{.00593}{2.66} =$

76. 4.67 is 35% of ______

77. 34.5 is 71.4% of ______

78. 16.25 is ______% of 23.4

79. .0744 is ______% of .0913

80. 234 is 123% of ______

81. 1.345 is ______% of 12.68

82. 57.4 is ______% of 41.6

83. 6430 is 74.2% of ______

84. 307,000 is 0.214% of ______

85. .0062 is ______% of 37.6

Chapter 4

COMBINED OPERATIONS WITH C AND D SCALES

4.1 Alternating division and multiplication most efficient

Example 1: $$\frac{234 \times 4.65}{312} = ?$$

Here, we may first divide 234 by 312 and then multiply by 4.65:

1. Move HL over 234 on D.
2. Slide 312 on C under HL. This divides 234 by 312; the result at this point is on D under right index of C. Now to multiply by 4.65, it is only necessary to:
3. Move HL over 465 on C.
4. Under HL read "349" on D. Answer is ***3.49.***

Clearly, we could have first multiplied 234 by 4.65, and then divided by 312. As will be seen, the important thing is to *alternate* the multiplication and division operations, regardless of which we choose as the initial operation.

Example 2: $$\frac{16 \times 3.40}{2.5 \times 2.9} = ?$$

In this case, we will first *divide* 16 by 2.5, then *multiply* this result by 3.4, and then *divide* by 2.9. The steps follow:

1. Move HL over 16 on D.
2. Slide 25 on C under HL. This divides 16 by 2.5; the result is now on D under right index of C. Next, we multiply this result by 3.4:

3. Move HL over 34 on C. The result at this point is under the hairline on D. Finally, in order to divide by 2.9:
4. Slide 29 on C under HL.
5. Opposite right index of C, read "750" on D. Answer is ***7.50.***

Note that we first move the hairline, then the slide, then the hairline, then the slide—*each move accomplishing an operation.* This is the most efficient way to proceed, and will always be the pattern if division and multiplication alternate with one another.

Verify the following:

1. $\dfrac{143 \times 3.7}{18} = 29.4$
2. $\dfrac{64 \times 28}{53} = 33.8$
3. $\dfrac{260 \times 42}{147} = 74.3$
4. $\dfrac{8.2 \times 25}{7.1 \times 1.8} = 16.04$
5. $\dfrac{163 \times 6.8}{23.2 \times 4.1} = 11.65$
6. $\dfrac{16 \times 6.3 \times 1.7}{2.5 \times 3.7} = 18.53$

Example 3: $\dfrac{12.1 \times 9.2}{8.5} = ?$

Here, if we first attempt to divide 12.1 by 8.5, the slide will extend far to the left; hence, it is better to first divide 9.2 by 8.5 and then multiply by 12.1.

Verify that the result is ***13.10.***

Exercise 4-1

1. $\dfrac{16 \times 4.6}{21} =$
2. $\dfrac{78 \times 12}{61} =$ "1535"
3. $\dfrac{14 \times 82}{73} =$
4. $\dfrac{210 \times 36}{125} =$ "605"
5. $\dfrac{11 \times 83}{7.3} =$
6. $\dfrac{38 \times 4.75}{5.2} =$ "347"
7. $\dfrac{74 \times 19}{16} =$
8. $\dfrac{35.2 \times 6.24}{25.6} =$ "858"
9. $\dfrac{2.40 \times 3.20}{1.94} =$
10. $\dfrac{15.6 \times 6.57}{9.21} =$ "1113"
11. $\dfrac{7.29 \times 500}{63.2} =$
12. $\dfrac{2.5 \times 5.2}{3.7 \times 4.6} =$ "764"

13. $\frac{6.4 \times 25}{4.8 \times 1.5} =$

14. $\frac{15 \times 3.7}{11 \times 8.6} =$ "587"

15. $\frac{6.3 \times 2.3}{1.3 \times 4.9} =$

16. $\frac{23 \times 56}{8.4 \times 19} =$ "807"

17. $\frac{246 \times 52.3}{252 \times 3.6} =$

18. $\frac{2.48 \times 51.6}{4.05 \times 1.25} =$ "253"

19. $\frac{2.8 \times 7.6 \times 11}{3.7 \times 5.1} =$

20. $\frac{5.6 \times 3.2 \times 9.5}{7.3 \times 4.1} =$ "569"

21. $\frac{18 \times 6.2 \times 15}{13 \times 8.4} =$

22. $\frac{64 \times 17 \times 73}{39 \times 41} =$ "496"

23. $\frac{8.4 \times 12 \times 6.3}{4.7 \times 1.4} =$

24. $\frac{36 \times 22 \times 4.3}{15 \times 9.5} =$ "239"

25. $\frac{12.65 \times 6.44 \times 41.7}{23.6 \times 18.45} =$

26. $\frac{7.66 \times 19.25 \times 31.2}{62.1 \times 10.75} =$ "689"

27. $\frac{3.6 \times 2.2 \times 5.8}{4.7 \times 1.3 \times 4.5} =$

28. $\frac{5.1 \times 9.6 \times 10.6}{7.8 \times 3.7 \times 2.3} =$ "782"

29. $\frac{13 \times 4.1 \times 83}{22 \times 54 \times 1.8} =$

30. $\frac{64.7 \times 1.85 \times 0.93}{41.2 \times 5.22 \times 0.306} =$ "1691"

31. $\frac{18.4 \times 7.66 \times 47.1}{25.3 \times 9.42 \times 1.76} =$

32. $\frac{.073 \times 4600}{625 \times .00375} =$ "1432"

33. $\frac{5600 \times 275}{43{,}600 \times .0122} =$

34. $\frac{.00468 \times 6450}{3.74} =$ "807"

35. $\frac{42 \times .064 \times 1.35}{310 \times .0073} =$

36. $\frac{564 \times 134 \times 0.413}{23.2 \times 174.5} =$ "771"

37. $\frac{7.43 \times 4.22 \times 27.4}{82 \times 6.84 \times 4.81} =$

38. $\frac{.0263 \times 314 \times 508}{1.235 \times 861} =$ "395"

39. $\frac{.0327 \times 82.1 \times 152.5}{.0662 \times 195 \times 63.1} =$

40. $\frac{.00234 \times 9640 \times 18.35}{.0582 \times 482 \times 1.036} =$ "1425"

41. $\frac{294 \times 4300}{14{,}300} =$

42. $\frac{.00468 \times 6450}{3.74} =$ "806"

43. $\frac{21.6 \times 8.75 \times 20.6}{43.5 \times 2.74 \times 5.66} =$

44. $\frac{10.75 \times 3060 \times 1250}{185 \times 216} =$ "1029"

45. $\frac{.0524 \times 5400 \times 4.73}{35.2 \times 8.22 \times .0293} =$

46. $\frac{426 \times 68.2 \times 6.42}{3.14 \times 92.3} =$ "644"

47. $\dfrac{47.6 \times .0543 \times 17.5}{3.96 \times 28.7} =$

48. $\dfrac{56.4 \times 2300 \times .0743}{37.6 \times 4.65 \times 63.1} =$ "874"

49. $\dfrac{.0346 \times 466 \times 22.8}{173.5 \times .0852} =$

50. $\dfrac{586 \times 2.67 \times 422}{72.4 \times 14.7 \times 706} =$ "879"

4.2 Alternating pattern not always possible with C and D

It often happens that you are unable to alternate the division and multiplication operations on the C and D scales. Later, you will see how the inverted and folded scales may be used to advantage in such cases; however, for the present, we will illustrate the procedure using just the C and D scales.

Example 1: $\dfrac{45}{2.6 \times 3.1} = ?$

1. Move HL over 45 on D.
2. Slide 26 on C under HL. We have now divided 45 by 2.6, and the result is opposite the C index on D. It now becomes necessary to move the hairline over this result on D so that we may divide it by 3.1:
3. Move HL over left index of C.
4. Slide 31 on C under HL. This divides by 3.1.
5. Opposite right index of C, read "558" on D. Answer is ***5.58.***

Notice that the hairline movement in step (3) was nonoperational in the sense that it served only to mark and hold a previous result on the D scale.

Example 2: $2.4 \times 3.1 \times 4.7 = ?$

1. Set left index of C opposite 24 on D.
2. Move HL over 31 on C. We have now multiplied 2.4 by 3.1 and the result is under the hairline on D. In order to multiply again, we must first reset the C index opposite this result:
3. Slide right index of C under HL. We are now in position to multiply by 4.7:
4. Move HL over 47 on C.
5. Under HL read "350" on D. Answer is ***35.0.***

Note that, in this case, the movement of the slide in step (3) was nonoperational in that it simply reset the index.

Verify the following:

1. $\dfrac{75}{4.2 \times 3.6} = 4.96$

2. $\dfrac{153}{2.7 \times 5.1} = 11.11$

3. $1.7 \times 2.8 \times 4.6 = 21.9$

4. $52 \times 0.43 \times 3.6 = 80.5$

5. $\dfrac{235}{11.5 \times 2.9 \times 3.22} = 2.19$

6. $\dfrac{3.14 \times (5.72)^2}{7.66} = 13.42$

Example 3: $\dfrac{27 \times 33 \times 6.2}{15 \times 19} = ?$

1. Move HL over 27 on D.
2. Slide 15 on C under HL. We have now divided 27 by 15.
3. Move HL over 33 on C. This multiplies by 33.
4. Slide 19 on C under HL. This divides by 19. We now observe that 62 on C is off-scale; hence, before multiplying by 6.2 it becomes necessary to reset the index.
5. Move HL over left index of C.
6. Slide right index of C under HL. Now multiply by 6.2:
7. Move HL over 62 on C.
8. Under HL read "1938" on D. Answer is ***19.38.***

In this example, both steps (5) and (6) were nonoperational in that they served only to interchange indexes. In the following chapters you will see how these nonoperational moves may usually be eliminated by proper use of the inverted and folded scales.

Verify the following:

1. $\dfrac{74 \times 21 \times 43}{33 \times 17} = 119.3$

2. $\dfrac{17 \times 65 \times 15}{41 \times 71} = 5.69$

3. $\dfrac{2.31 \times 2.94 \times 2.62}{5.41 \times 5.17 \times 4.83} = 0.1316$

4. $\dfrac{45.1 \times 38.2 \times 184.5}{71.6 \times 67.3 \times 83.2} = 0.793$

Exercise 4-2

1. $\dfrac{620}{35 \times 13} =$

2. $\dfrac{1}{(2.3)^2} =$ "1890"

3. $\dfrac{56}{3.4 \times 5.1} =$

4. $\dfrac{135}{21 \times 4.3} =$ "1495"

5. $\dfrac{36}{2.6 \times 7.1} =$

6. $\dfrac{95}{(4.6)^2} =$ "449"

7. $\dfrac{375}{(12.7)^2} =$

8. $15 \times 2.6 \times 1.9 =$ "742"

9. $2.8 \times 4.9 \times 3.1 =$

10. $72 \times 0.61 \times 1.73 =$ "760"

11. $3.14 \times (4.7)^2 =$

12. $3.14 \times (1.75)^2 =$ "962"

13. $1.6 \times 8.2 \times 0.43 \times 2.6 =$

14. $\frac{5.2 \times 3.1 \times 6.1}{1.9 \times 1.8} =$ "288"

15. $\frac{7.4 \times 26 \times 52}{3.9 \times 15} =$

16. $\frac{2.6 \times 81 \times 12}{5.3 \times 64} =$ "745"

17. $\frac{33 \times 27 \times 5.6}{14 \times 17} =$

18. $\frac{195 \times 74.4 \times 13.5}{51 \times 62 \times 22} =$ "281"

19. $\frac{324 \times 543 \times 224}{771 \times 635 \times 362} =$

20. $\frac{1}{2.64 \times 5.11 \times 0.344} =$ "215"

21. $\frac{62.5}{3.14 \times (1.64)^2} =$

22. $\frac{340}{3.14 \times (5.5)^2} =$ "358"

23. $\frac{57.2 \times 41.2}{3.14 \times (4.6)^2} =$

24. $\frac{405 \times 96}{821 \times 17.4 \times 3.83} =$ "710"

25. $\frac{37.5 \times 2.61 \times 3.04}{23.1} =$

26. $3.14 \times (2.2)^2 \times 1.6 =$ "243"

27. $3.14 \times (6.3)^2 \times 2.9 =$

28. $\frac{175 \times 46.3 \times 1.24 \times 0.7}{21.6} =$ "325"

29. $\frac{36.4 \times 51.9}{39 \times 1.74 \times 5.7} =$

30. $\frac{100}{2.7 \times 5.1 \times 3.4} =$ "214"

31. $\frac{19.2 \times 3.42 \times 2.77}{28.6} =$

32. $\frac{30.5 \times 27.6 \times 47.2}{13.15 \times 16.33 \times 1.52} =$ "1217"

Chapter 5

THE INVERTED SCALE (CI)

5.1 Description of the scale

You will note that the CI scale is identical with the C scale except that it reads from right to left instead of from left to right. The C and CI scales are related in the following manner:

> If the hairline is set over a number (N) on the **C** scale, the *reciprocal* of that number ($1/N$) is under the hairline on the **CI** scale.
>
> Conversely, if the hairline is over a number on the **CI** scale, its reciprocal is under the hairline on the **C** scale.

5.2 Using the CI scale to find reciprocals

Example 1: $\dfrac{1}{.00412} = ?$

1. Move HL over 412 on C.
2. Under HL read "243" on CI.

Shift decimal points: $\dfrac{1}{.00412} = \dfrac{1000}{4.12}$

It is now evident that the answer must be ***243.***

Example 2: Find the reciprocal of 6320.

1. Move HL over 632 on C.
2. Under HL read "1582" on CI.

   Shift decimal points: $\frac{1}{6320} = \frac{.001}{6.32}$

Answer is ***.0001582.***

5.3 The DI scale

Most modern slide rules have an inverted D scale located on the body. This scale is labeled "DI", and bears the same relation to the D scale as does the CI scale to the C scale.

Thus, if the hairline is moved over a number on the D scale, its reciprocal is under the hairline on the DI scale, and vice versa.

Exercise 5-1

Use the CI or DI scale to evaluate:

1. $\frac{1}{3.22} =$
2. $\frac{1}{4.75} =$
3. $\frac{1}{5.07} =$
4. $\frac{1}{2.63} =$
5. $\frac{1}{8.12} =$
6. $\frac{1}{9.35} =$
7. $\frac{1}{1.135} =$
8. $\frac{1}{32.6} =$
9. $\frac{1}{723} =$
10. $\frac{1}{0.285} =$
11. $\frac{1}{0.413} =$
12. $\frac{1}{635} =$
13. $\frac{1}{.01725} =$
14. $\frac{1}{.00346} =$
15. $\frac{1}{46.2} =$
16. $\frac{1}{.000248} =$
17. $\frac{1}{831} =$
18. $\frac{1}{.00604} =$
19. $\frac{1}{62.6} =$
20. $\frac{1}{283{,}000} =$
21. $\frac{1}{.0432} =$
22. $\frac{1}{840} =$
23. $\frac{1}{0.711} =$
24. $\frac{1}{2350} =$

5.4 Using the CI scale for division

Example 1: $234 \div 61 = ?$

This may be evaluated in the conventional manner, or it may be treated as the product: $234 \times (1/61)$. Clearly, instead of dividing by a number, we may multiply by the reciprocal of the number. Thus, the division may be accomplished as follows:

1. Set left index of C opposite 234 on D.
2. Move HL over 61 on CI. Note that HL is now over the reciprocal of 61 on C; hence, the rule is in the proper position for multiplying 234 on D by 1/61 on C.
3. Under HL read "384" on D. Answer is ***3.84.***

Example 2: $8.75 \div 1.275 = ?$

1. Set right index of C opposite 875 on D.
2. Move HL over 1275 on CI.
3. Under HL read "686" on D. Answer is ***6.86.***

Verify the following divisions (use the CI scale):

1. $1075 \div 240 = 4.48$
2. $143.5 \div 54 = 2.66$
3. $902 \div 23.1 = 39.1$
4. $5640 \div 2.03 = 2780$
5. $105.7 \div 82.4 = 1.282$
6. $920 \div 1436 = 0.641$
7. $368 \div 28.2 = 13.04$
8. $944 \div 543 = 1.738$
9. $27.9 \div 0.123 = 227$
10. $5640 \div 0.920 = 6130$

(*Exercise 3-4* may be used for more drill with this method.)

5.5 Using the CI scale for multiplication

Example 1: $24.5 \times 362 = ?$

This may be treated as the quotient: $24.5 \div (1/362)$. Obviously, instead of multiplying by a number, we may divide by its reciprocal. Thus, the product may be obtained as follows:

1. Move HL over 245 on D.
2. Slide 362 on CI under HL. Notice that the reciprocal of 362 is now under the

HL on C; hence, the rule is in the proper position for dividing 24.5 on D by (1/362) on C.

3. Opposite right index of C read "887" on D.

By inspection, answer is ***8870.***

Example 2: $0.327 \times 54.5 = ?$

Instead of multiplying by 54.5, we divide by the reciprocal of 54.5 as follows:

1. Move HL over 327 on D.
2. Slide 545 on CI under HL.
3. Opposite left index of C read "1782" on D.

By inspection, answer is ***17.82.***

Example 3: $.00435 \times 722 = ?$

Here, again, instead of multiplying by 722, we divide by its reciprocal:

1. Move HL over 435 on D.
2. Slide 722 on CI under HL.
3. Opposite left index of C read "314" on D.

Shifting decimal points, it is seen that answer must be ***3.14.***

These examples illustrate the procedure:

To multiply two numbers using CI scale:

1. Move HL over one of the numbers on **D.**
2. Move slide so that the other number on **CI** is under HL.
3. Read answer on **D** opposite **C index.**

The important feature of the CI scale is that it enables you to treat multiplication as division and vice versa. In the next chapter you will see how this scale may be used to advantage in combined operations.

Verify the following (use the CI scale):

1. $8.2 \times 4.06 = 33.3$
2. $27.5 \times 61 = 1678$

3. $43 \times 2.07 = 89.1$

4. $32.6 \times 3.15 = 102.7$

5. $30.4 \times 7.56 = 230$

6. $750 \times .0222 = 16.65$

7. $0.436 \times 7.62 = 3.32$

8. $36.2 \times 0.209 = 7.56$

9. $0.62 \times 835 = 517$

10. $.0245 \times 432 = 10.58$

(*Exercise 3-3* may be used for more drill with this method.)

Chapter 6

COMBINED OPERATIONS WITH C, D, AND CI SCALES

6.1 Application of the CI scale to continued division

Proper use of the CI scale often eliminates nonoperational moves when two or more divisions occur in succession. The following examples illustrate the technique.

Example 1: $\dfrac{410}{2.7 \times 36} = ?$

In Chapter 4, expressions of this type were evaluated using the C and D scales only. You will recall that it was necessary to move the hairline over the result of the first division before the second division could be performed.

However, we are now in a position to eliminate this nonoperational hairline movement. First, we divide 410 by 2.7. Then, instead of dividing again by 36, we multiply by the reciprocal of 36. Thus, the sequence of operations becomes $(410) \div (2.7) \times (1/36)$. Note that this exhibits the desired alternating pattern with no nonoperational moves.

The specific steps follow:

1. Move HL over 410 on D.
2. Slide 27 on C under HL. This divides by 2.7.
3. Move HL over 36 on CI. Note that hairline is now over (1/36) on C; hence, we have effectively multiplied by (1/36). This, of course, corresponds to dividing by 36.
4. Under HL read "422" on D. Answer is ***4.22.***

Verify the following:

1. $\dfrac{35}{19 \times 2.4} = 0.768$

2. $\dfrac{25}{5.4 \times 1.6} = 2.89$

3. $\dfrac{630}{84 \times 35} = 0.214$

4. $\dfrac{16.4}{2.75 \times 3.14} = 1.899$

5. $\dfrac{625}{37.2 \times 41.6} = 0.404$

6. $\dfrac{100}{8.6 \times 9.2} = 1.264$

Example 2: $\dfrac{152}{7.42 \times 1.85} = ?$

Here, the slide will be in better position if you first divide by 1.85, then multiply by the reciprocal of 7.42. Verify that the result is ***11.07.***

Example 3: $\dfrac{43}{2.6 \times 6.5 \times 5.7} = ?$

The sequence of operations is: (43) ÷ (2.6) × (1/6.5) ÷ (5.7)

1. Move HL over 43 on D.
2. Slide 26 on C under HL. This divides by 2.6.
3. Move HL over 65 on CI. This multiplies by (1/6.5).
4. Slide 57 on C under HL. This divides by 5.7.
5. Opposite right index of C, read "446" on D. Answer is ***0.446.***

Verify the following:

1. $\dfrac{76.2}{12.5 \times 8.6} = 0.708$

2. $\dfrac{164}{66 \times 2.05} = 1.211$

3. $\dfrac{45}{2.7 \times 4.1 \times 1.9} = 2.14$

4. $\dfrac{2.75 \times 63.1}{4.16 \times 3.04 \times 5.27} = 2.61$

Exercise 6-1

1. $\dfrac{340}{19 \times 22} =$

2. $\dfrac{27}{6.3 \times 2.1} =$ "204"

3. $\dfrac{54}{4.6 \times 7.5} =$

4. $\dfrac{19}{6.4 \times 15} =$ "1979"

5. $\dfrac{27}{17 \times 4.7 \times 1.3} =$

6. $\dfrac{53}{3.7 \times 6.2 \times 1.8} =$ "1284"

7. $\dfrac{100}{1.6 \times 31 \times 6.8} =$

8. $\dfrac{370}{4.7 \times 2.8 \times 8.2} =$ "343"

9. $\dfrac{64 \times 4.7}{3.8 \times 8.1 \times 1.2} =$

10. $\dfrac{51}{7.2 \times 1.8 \times 7.1} =$ "555"

11. $\dfrac{250 \times 23}{32 \times 1.6 \times 8.4} =$

12. $\dfrac{265}{4.7 \times 2.8 \times 8.2} =$ "246"

13. $\dfrac{21.3}{36.4 \times 2.19} =$

14. $\dfrac{243}{64.2 \times 2.21} =$ "1713"

15. $\dfrac{8.24}{2.17 \times 9.44} =$

16. $\dfrac{254}{18.2 \times 4.63 \times 1.35} =$ "223"

17. $\dfrac{8.17}{1.94 \times 9.26} =$

18. $\dfrac{1}{5.65 \times 1.27} =$ "1394"

19. $\dfrac{10}{7.82 \times 0.843} =$

20. $\dfrac{31.6 \times 68.4}{2.03 \times 9.2 \times 1.845} =$ "627"

21. $\dfrac{5.02}{7.23 \times 0.175 \times 7.12} =$

22. $\dfrac{115 \times 7.65}{21.8 \times 2.74 \times 9.12} =$ "1615"

23. $\dfrac{525}{7.16 \times 6.24} =$

24. $\dfrac{16.1}{270 \times .0154} =$ "387"

25. $\dfrac{6350}{41.5 \times 5.06 \times 1.475} =$

26. $\dfrac{1975}{.027 \times .00564 \times 32{,}100} =$ "404"

27. $\dfrac{1}{1.95 \times 21.6 \times 5.46} =$

28. $\dfrac{.025}{0.18 \times 2.6 \times .082 \times 3.4} =$ "1916"

29. $\dfrac{10^5}{84.2 \times .0365 \times 0.475 \times 19.6} =$

30. $\dfrac{435}{29.5 \times 1.76 \times 7.64 \times 1.42} =$ "772"

31. $\dfrac{803}{43 \times .0281 \times 1920} =$

32. $\dfrac{2350 \times 2.14}{.0316 \times 156.5 \times 8.4} =$ "1210"

33. $\dfrac{0.254}{18.2 \times 4.83 \times 1.35} =$

34. $\dfrac{3160 \times .00584}{2.03 \times 920 \times .01845} =$ "536"

35. $\dfrac{2500}{17.2 \times 26.2 \times .0462 \times 574} =$

6.2 Application of the CI scale to continued multiplication

Example 1: $4.2 \times 5.3 \times 3.2 \times 2.7 = ?$

Here, again, expressions of this type were evaluated in Chapter 4 using only the C and D scales. You will recall that it was necessary to reset the C index after each multiplication before the next multiplication could be performed.

However, using the CI scale, we may now eliminate these extra slide movements.

In the above example, the first multiplication may be handled as a division; that is, 4.2 may be divided by the reciprocal of 5.3. We then multiply by 3.2, and finally, instead of multiplying again, we divide by the reciprocal of 2.7. The sequence of operations thus becomes: (4.2) ÷ (1/5.3) × (3.2) ÷ (1/2.7). This has the desired alternating pattern.

The steps follow:

1. Move HL over 42 on D.
2. Slide 53 on CI under HL. This divides by (1/5.3) which corresponds to multiplying by 5.3.
3. Move HL over 32 on C. This multiplies by 3.2.
4. Slide 27 on CI under HL. This divides by (1/2.7) which corresponds to multiplying by 2.7.
5. Opposite left index of C read "1923" on D. Answer is ***192.3.***

Example 2:

$$\frac{462 \times .00334 \times 7.54 \times 125}{38.2} = ?$$

1. Move HL over 462 on D.
2. Slide 382 on C under HL. This divides by 38.2.
3. Move HL over 334 on C. This multiplies by .00334.
4. Slide 754 on CI under HL. This multiplies by 7.54.
5. Move HL over 125 on C. This multiplies by 125.
6. Under HL read "381" on D.

Rounding off using scientific notation:

$$\frac{\overset{(2)}{462} \times \overset{(-3)}{.00334} \times \overset{(0)}{7.54} \times \overset{(2)}{125}}{\underset{(1)}{38.2}} \approx \frac{5 \times 3 \times 8 \times 1}{4} \times 10^{2-3+0+2-1} = 30 \times 10^0. = 30.$$

Answer is ***38.1.***

Exercise 6-2

In each case, start with *division* and alternate the operations.

1. $3.2 \times 6.1 \times 2.6 =$
2. $4.5 \times 1.6 \times 1.8 =$ "1296"
3. $12 \times 0.44 \times 6.3 =$
4. $5.1 \times 4.3 \times 1.2 =$ "263"
5. $2.8 \times 5.4 \times 3.2 =$
6. $3.7 \times 1.9 \times 0.85 =$ "597"
7. $17 \times 3.2 \times 64 =$
8. $\frac{2.1 \times 3.4 \times 4.7}{1.3} =$ "258"
9. $\frac{4.3 \times 3.7 \times 7.2}{7.8} =$
10. $\frac{5.2 \times 4.1 \times 6.6}{4.9} =$ "287"
11. $6.4 \times 1.8 \times 3.5 \times 1.4 =$

12. $18 \times 0.8 \times 2.3 \times 2.4 =$ "795"

13. $3.7 \times 4.8 \times 1.3 \times 2.6 =$

14. $\frac{17 \times 4.4 \times 7.2 \times 2.5}{28} =$ "481"

15. $\frac{5.2 \times 6.4 \times 2.7 \times 33}{85} =$

16. $\frac{35 \times 1.5 \times 3.3 \times 0.52 \times 1.3}{24} =$ "488"

17. $\frac{6.1 \times 19 \times 6.5 \times 3.7}{43 \times 39} =$

18. $\frac{10.65 \times 82 \times 7.3 \times 0.24}{5.8 \times 19} =$ "1388"

19. $\frac{565 \times 16.45 \times 1.23}{21.7} =$

20. $\frac{74.2 \times 6.22 \times 3.47}{5.41} =$ "296"

21. $\frac{283 \times 34.6 \times .0405}{1570} =$

22. $5.42 \times 3.55 \times 1.72 \times 2.16 =$ "715"

23. $(2.31)^2 \times 0.62 \times 4.87 =$

24. $3.14 \times (4.4)^2 \times 2.63 =$ "1599"

25. $3.14 \times (2.13)^2 \times 37.8 =$

26. $3.14 \times (3.48)^2 \times 0.633 =$ "241"

27. $23.5 \times .066 \times 383 \times .001075 =$

28. $4.52 \times 131 \times .00766 \times 4.9 \times 330 =$ "733"

29. $3.22 \times 5.34 \times 12.65 \times 2.68 \times 0.845 =$

30. $210 \times 0.65 \times 314 \times .046 \times .0108 =$ "213"

31. $1.5 \times 4.4 \times 0.9 \times 2.3 \times 1.4 \times 0.85 =$

32. $1.9 \times 8.5 \times 2.42 \times 0.34 \times 5.6 \times 1.4 =$ "1042"

33. $\frac{0.563 \times 2.13 \times 6.15 \times 19.3}{31.5} =$

34. $\frac{175 \times 5.11 \times 0.335 \times 62.4}{234} =$ "799"

35. $\frac{376 \times .0394 \times .0574 \times 14.2}{.00523} =$

36. $\frac{6.1 \times 1.9 \times (2.4)^2 \times 5.6}{1.3 \times 4.3} =$ "669"

37. $\frac{54.1 \times .0322 \times 21 \times 164 \times 0.94}{890 \times 154.4} =$

38. $\frac{72.4 \times 32.5 \times 4.12 \times 0.533 \times 1.265}{51.6 \times 63.4} =$ "1998"

39. $\frac{3.17 \times 7.36 \times 14.2 \times 7.08 \times 1.335 \times 0.715}{5.55 \times 2.06 \times 43.2} =$

40. $\dfrac{.0284 \times 7140 \times 0.127 \times 69 \times .00121 \times 650}{.000504 \times 23.2 \times 4.06} = \text{“294”}$

6.3 Procedure when the next setting is off-scale

In the exercises presented thus far in this chapter, it has been possible to alternate the division and multiplication operations without running off the scale. It often happens that this is not the case, and in Chapter 7 you will see how the folded scales may be used when the next setting on C or CI is off-scale.

However, the following examples illustrate how this situation may be handled using just the C, D, and CI scales.

Example 1: $\dfrac{7.4}{3.4 \times 2.1 \times 1.4} = ?$

1. Move HL over 74 on D.
2. Slide 34 on C under HL. Now observe that both 2.1 and 1.4 on CI are beyond the D scale; hence, it is impossible to multiply by the reciprocal of either one. Therefore, divide by 2.1 as follows:
3. Move HL over left index of C. This is a nonoperational move.
4. Slide 21 on C under HL. This divides by 2.1. Now multiply by the reciprocal of 1.4:
5. Move HL over 14 on CI.
6. Under HL read “740” on D. Answer is ***0.740.***

Example 2: $2.1 \times 1.8 \times 26 \times 1.7 = ?$

1. Move HL over 21 on D.
2. Slide 18 on CI under HL. Note that both 26 and 1.7 on C are beyond the D scale; hence, it is impossible to multiply by either of them. Therefore, divide by the reciprocal of 26 as follows:
3. Move HL over right index of C. This is nonoperational.
4. Slide 26 on CI under HL. Now multiply by 1.7:
5. Move HL over 17 on C.
6. Under HL read “1670” on D. Answer is ***167.0.***

It is clear from the foregoing examples that if, during a combined or continuous operation on the C-D-CI scales, a multiplication is off-scale, you may handle it as a division and proceed.

Exercise 6-3

In each of the following, start with division:

1. $\dfrac{240}{39 \times 0.78} =$

2. $\dfrac{41}{3.1 \times 12} = \text{“1102”}$

3. $\frac{710}{29 \times 1.6} =$

4. $\frac{212}{42.6 \times 5.24} =$ "950"

5. $\frac{63}{3.4 \times 13 \times 1.7} =$

6. $\frac{83}{3.9 \times 20 \times 1.6} =$ "665"

7. $\frac{230}{4.1 \times 6.1 \times 12} =$

8. $\frac{14}{2.6 \times 6.1 \times 0.76} =$ "1162"

9. $\frac{706}{4.32 \times 15.25 \times 11.6} =$

10. $\frac{83.2}{6.47 \times 1.26 \times 11.5} =$ "888"

11. $\frac{196}{27 \times 7.95 \times 0.844} =$

12. $5.8 \times 3.6 \times 0.71 \times 6.3 =$ "934"

13. $7.6 \times 3.5 \times 7.1 \times 0.44 =$

14. $1.9 \times 2.2 \times 1.5 \times 1.4 =$ "878"

15. $7.15 \times 3.62 \times 0.734 \times 0.845 =$

16. $\frac{5.3 \times 5.5 \times 6.3}{2.6} =$ "706"

17. $\frac{2.8 \times 1.7 \times 1.9}{6.1} =$

18. $\frac{7.8 \times 6.4 \times 5.7}{4.2} =$ "677"

19. $\frac{2.48 \times 1.34 \times 1.165}{6.07} =$

20. $\frac{63.4 \times 7.21 \times 0.93}{3.54} =$ "1200"

21. $\frac{2.8 \times 5.6}{1.3 \times 1.5} =$

22. $\frac{3.2 \times 18}{6.5 \times 6.7} =$ "1323"

23. $\frac{39.4 \times 8.66}{1.73 \times 12.4} =$

24. $\frac{2.15 \times 21.7 \times 1.98}{5.06 \times 6.15} =$ "297"

25. $\frac{8.35 \times 81.3}{4.14 \times 1.62 \times 12.6} =$

Chapter 7

THE FOLDED SCALES (CF, DF, CIF)

7.1 Description of the scales

If you examine the scales on your rule marked CF and DF, you will see that they are identical to the regular C and D scales except that they begin and end at π, thus placing the index very nearly at the center of the scale. The CIF scale is the inverse of the CF scale; that is, if the hairline is set over a number on CIF, its reciprocal will be under the hairline on CF and vice versa.

7.2 Multiplication on the folded scales

An important relationship between the regular and folded scales is the following:

If either index of **C** is set opposite a number on **D**, the index of **CF** is opposite that *same* number on **DF.**

For example, set the right index of C opposite 6 on D, and note that the CF index is also opposite 6 on DF. Suppose you now go through the following moves with your slide rule:

1. Set left index of C opposite 2 on D. Observe that CF index is now opposite 2 on DF.
2. Move HL over 4 on C. Under HL read the product, $4 \times 2 = 8$, on D.
3. Move HL over 4 on CF. Under HL read the product, $4 \times 2 = 8$, on DF.

Inasmuch as our initial setting put the C index opposite 2 on D, and at the same time put the CF index opposite 2 on DF, we were in a position to perform the multiplication on *either* set of scales, regular or folded.

Let us repeat with another illustration:

1. Set right index of C opposite 7 on D. Note that CF index is now also opposite 7 on DF.
2. Move HL over 5 on C. Under HL read the product, $7 \times 5 = 35$, on D.
3. Move HL over 5 on CF. Under HL read 35 on DF.

7.3 Multiplying a number successively

Example 1: Given the equation $y = (2.3)x$. Find the corresponding values of y when x takes the values: 1.6, 3.8, 5.4, and 8.5.

1. Set left index of C opposite 23 on D. CF index is now also opposite 23 on DF.
2. Move HL over 16 on C. Under HL read "368" on D.
3. Move HL over 38 on C. Under HL read "874" on D.

   Observe that 54 is off-scale on C; hence, we now go to the folded scales:
4. Move HL over 54 on CF. Under HL read "1242" on DF.
5. Move HL over 85 on CF. Under HL read "1955" on DF.

Results are: ***3.68, 8.74, 12.42,*** and ***19.55.***

Example 2: Given the relation, $E = 2.47R$. Calculate E when R takes the successive values: 12.5, 27.6, 46.3, 78.2, and 145.

1. Set left index of C opposite 247 on D.
2. Move HL over 125 on C. Under HL read "309" on D.
3. Move HL over 276 on C. Under HL read "682" on D.
4. Note that 463 is off-scale on C; hence, move HL over 463 on CF. Under HL read "1144" on DF.
5. Move HL over 782 on CF. Under HL read "1931" on DF.
6. Move HL over 145 on C. Under HL read "358" on D.

The corresponding values of E are: ***30.9, 68.2, 114.4, 193.1,*** and ***358.***

Exercise 7-1

1. Given the formula, $R = 3.06T$. Find R corresponding to the following values of T: 1.84, 2.77, 5.75, and 8.24.
2. Given the formula, $P = .0534Q$. Find P corresponding to the following values of Q: 21.4, 56.2, 111, 175, and 321.
3. Given the equation, $y = (1.34)x$. Find the corresponding values of y when x takes the values: 1.25, 2.75, 3.50, 4.75, and 7.25.
4. Given the equation, $y = (0.635)x$. Find corresponding values of y when x takes the values: 1.25, 2.75, 5.45, 14.75, and 22.6.

5. A certain distance-time relationship is known to be $D = (3.06)T$, where D is distance in meters and T is time in seconds. Find D corresponding to the following values of T: 1.24, 2.35, 3.10, 4.66, 7.25, and 10.45.
6. Find 36.4% of the following numbers: 12.5, 39.0, 22.4, 185, 211, and 766.
7. Given the relationship, $R = (0.269)L$, where L represents the length of a wire in feet, and R represents the resistance in ohms. Find R corresponding to the following lengths: 150′, 320′, 535′, 820′, 1150′, 2040′, and 4050′.
8. A certain gasoline weighs 5.63 lbs per gallon. Find the weights of the following amounts: 12.5 gals, 28.4 gals, 56.7 gals, 82.4 gals, 130 gals, and 211 gals.
9. A firm is allowed a 45% discount off list on a certain item. Find the firm's net prices corresponding to the following list prices: $4.56, $8.20, $34.20, $11.50, $204.00, and $107.00. (Slide rule accuracy only.)
10. An alloy is 23.2% copper by weight. Find the pounds of copper in each of the following amounts of alloy: 140 lbs, 275 lbs, 560 lbs, 725 lbs, 1120 lbs, and 3060 lbs.
11. Given the relationship $e = (.000148)T$, where e is the stretch in inches of a steel wire, and T is the pull on the wire in lbs. Find e corresponding to the following values of T: 2500 lbs, 4250 lbs, 6100 lbs, 7800 lbs, 11,500 lbs, and 26,000 lbs.
12. To determine selling price, a merchant marks up his merchandise 37½% based on cost. Find the selling prices corresponding to the following cost prices: $0.36, $1.74, $0.82, $5.42, $.094, $0.26, and $3.24. Give selling price to nearest cent.
13. If a firm gets a trade discount of 20% less 15% less 15%, then the following formula applies: $N = (.80)(.85)(.85)L$, where L is the list price, and N is the net price. Find N corresponding to the following values of L: $10.50, $150.00, $43.50, $7.45, $16.50, and $340.00. (Slide rule accuracy only).
14. Given the formula: $V = \frac{\pi(2.83)N}{30}$, where V is rim speed in ft per sec., and N is angular speed in rpm. Find V corresponding to the following values of N: 750, 1830, 2400, 5500, and 12,500.

7.4 Dividing a number successively

Example: Given the relation, $P = (56.5)/V$. Calculate P when V takes the values: 17.2, 38.5, 61.4, 81.0, and 106.

When we wish to divide successively by several numbers, it is more convenient to multiply by the reciprocals.

1. Set right index of C opposite 565 on D.
2. Move HL over 172 on CI. Under HL read "328" on D.
3. Move HL over 385 on CI. Under HL read "1468" on D.
4. Observe that 614 is off-scale on CI; hence, move HL over 614 on CIF. Under HL read "920" on DF.
5. Move HL over 810 on CIF. Under HL read "698" on DF.
6. Move HL over 106 on CIF (or CI). Under HL read "533" on DF (or D).

Values of P are: ***3.28, 1.468, 0.920, 0.698,*** and ***0.533.***

Exercise 7-2

1. Given the equation, $y = 16/x$. Find y when x takes the values: 2.5, 4.5, 7.5, 10.5, and 14.5.
2. Given the equation, $y = 75/x$. Find y when x takes the values: 2.2, 3.3, 6.6, 7.7, and 8.8.
3. Given the equation, $y = (2450)/x$. Find y when x takes the values: 12.5, 16.7, 24.6, 43.2, and 95.0.
4. Given the equation, $y = (520)/x$. Find y when x takes the values: 15, 25, 34, 58.5, 74.5, 95, 145, and 225.
5. Given the relationship, $I = (12.5)/R$. Find I when R takes the values: 2.5, 35, 150, 435, and 1650.
6. Given the rate-time formula, $r = (47.5)/t$. Find r when t takes the values: 0.75, 2.65, 4.35, 13.6, 54.5.
7. The total investment necessary to return $2250 annually is given by the formula, $P = (2250)/r$, where r is the annual interest rate. Find P when r takes the values: .025, .045, .075, .125, and .150.
8. Given the formula, $N = (5400)/R$, where N is the rpm of a belt-driven pulley, and R is the radius of the pulley in inches. Find N corresponding to the following values of R: 3.25″, 4.25″, 6.50″, 8.75″, and 10.5″.
9. Given the temperature-time relationship, $T = \frac{360}{t} + 20$, where T is temperature in degrees Centigrade, and t is time in minutes. Find T corresponding to the following values of t: 25, 55, 125, 325, and 1500.
10. Given the formula, $B = \pi(2.7)^2/W$. Find B corresponding to the following values of W: 1.055, 1.645, 2.06, 4.85, and 7.22.

7.5 Other operations involving the folded scales

The preceding exercises have shown that if, during an operation on the regular scales, a hairline setting is off-scale, the setting may normally be made on the folded scales. Conversely, if the operation is proceeding on the folded scales and a hairline position is beyond the scale, the setting may usually be found on the regular scale. Therefore, proper use of the folded scales will eliminate the nonoperational moves which are often required when operating with just the C, D, and CI scales.

Example 1: $\frac{28 \times 13}{41} = ?$

1. Move HL over 28 on D.
2. Slide 41 on C under HL. The result of this division is now opposite the right index of C on D, and is also opposite the CF index on DF. Hence, the next multiplication may be carried out on either the regular or the folded scales.
3. Observe that 13 is off-scale on C; therefore, move HL over 13 on CF.
4. Under HL read "888" on DF. Answer is ***8.88***.

Example 2: $\frac{45 \times 5.70}{1.8 \times 1.2} = ?$

1. Move HL over 45 on D.
2. Slide 18 on C under HL. Now, as before, the next multiplication may be carried out on either the regular or folded scales. We note that 57 is off-scale on C; hence, we go to the folded scales:
3. Move HL over 57 on CF. Result at this point is under HL on DF; therefore, the next division must be made using the CF scale:
4. Slide 12 on CF under HL. Answer is now opposite CF index on DF, and is also opposite C index on D. This will always be the case; that is, whenever the result is opposite an index, it may be read on either DF or D. You will probably prefer reading on D.
5. Opposite left index of C read "1188" on D. Answer is ***118.8.***

Verify the following:

1. $\frac{29 \times 6.5}{14} = 13.47$
2. $\frac{23 \times 143}{42} = 78.3$
3. $\frac{48 \times 1.7}{8.3} = 9.83$
4. $\frac{35.2 \times 22.6}{8.41 \times 12.5} = 7.57$
5. $\frac{173 \times 154}{276 \times 77} = 1.254$
6. $\frac{26.4 \times 11.4}{4.18 \times 9.25} = 7.79$

Example 3: $\frac{48}{2.1 \times 1.1} = ?$

1. Move HL over 48 on D.
2. Slide 21 on C under HL. Now we must multiply by the reciprocal of 1.1. Observe that 11 is off-scale on CI; hence, go to the folded scales:
3. Move HL over 11 on CIF. Under HL read "208" on DF. Answer is ***20.8.***

Verify the following:

1. $\frac{22}{3.8 \times 6.4} = 0.905$
2. $\frac{175}{4.1 \times 5.6} = 7.62$
3. $\frac{68}{2.7 \times 1.55} = 16.25$
4. $\frac{280}{16.5 \times 12.3} = 1.380$
5. $\frac{56.3}{2.77 \times 1.085} = 18.74$
6. $\frac{183}{3.62 \times 5.74} = 8.81$

Example 4: $\dfrac{5200 \times 630}{2.9 \times .023 \times 0.27} = ?$

1. Move HL over 52 on D.
2. Slide 29 on C under HL.
3. Note that 63 is off-scale on C; hence, move HL over 63 on CF. The next division must be made on the folded scales:
4. Slide 23 on CF under HL. The result at this point is opposite CF index on DF, and is also opposite C index on D; therefore, the next multiplication may be carried out on either the folded or regular scales. In this case we must multiply by the reciprocal of 2.7.
5. Observe that 27 on CIF is off-scale; hence, move HL over 27 on CI.
6. Under HL read "1819" on D.

Rounding off using scientific notation:

$$\frac{\overset{(3)}{5200} \times \overset{(2)}{630}}{\underset{(0)}{2.9} \times \underset{(-2)}{.023} \times \underset{(-1)}{0.27}} \approx \frac{5 \times 6}{3 \times 2 \times 3} \times 10^{3+2-0+2+1} = \frac{5}{3} \times 10^{8} \approx 1.7 \times 10^{8}$$

Answer must be $\boldsymbol{1.819 \times 10^{8}}$.

It should now be clear that, after an operation which involves moving the slide, the hairline may then be positioned on either the regular or folded scales. If the regular scale is chosen, the operation must continue on the regular scales until the next hairline movement when, again, a choice may be made. If the folded scale is chosen, the operation must continue on the folded scales until the next hairline movement when, again, a choice exists.

It is important, then, to keep in mind the following:

> During a combined or continued operation, one may shift from the regular to the folded scales, or vice versa, *only when the hairline is moved.*

Verify the following:

1. $\dfrac{55 \times 13}{8.6 \times 6.8 \times 3.5} = 3.49$
2. $\dfrac{75 \times 53 \times 29}{36 \times 47} = 68.1$
3. $\dfrac{56}{31 \times 17 \times 1300} = 8.18 \times 10^{-5}$
4. $\dfrac{2.32 \times 6.25 \times 1.15}{4.71 \times 3.92} = 0.903$
5. $\dfrac{615}{3.14 \times 1.45 \times 1.45} = 93.1$
6. $\dfrac{136.5 \times 726}{0.273 \times .0514 \times 0.84} = 8.41 \times 10^{6}$

Example 5: $3.3 \times 4.1 \times 0.84 \times 0.76 \times 5.5. = ?$

1. Move HL over 33 on D.
2. Slide 41 on CI under HL.
3. Move HL over 84 on CF.
4. Slide 76 on CIF under HL. Note that 55 is now on-scale for both regular and folded scales. Suppose we choose the regular:
5. Move HL over 55 on C.
6. Under HL read "475" on D. Answer is ***47.5.***

Verify the following:

1. $1.8 \times 2.8 \times 1.4 \times 2.3 = 16.23$
2. $\dfrac{55.5 \times 15.4 \times 7.45 \times 5.2}{263} = 126.0$
3. $3.2 \times 2.1 \times 1.3 \times 1.2 \times 4.7 = 49.3$
4. $\dfrac{3.14 \times 2.06 \times (6.25)^2}{7.62} = 33.2$

Example 6: $\dfrac{11}{9.1 \times 0.85} = ?$

Here, the slide will be in better position if the operation starts on the folded scales. Verify that the result is ***1.422.***

Verify the following:

Start each operation on the folded scales:

1. $\dfrac{110}{8.2 \times 4.1} = 3.27$
2. $\dfrac{8.6}{1.3 \times 1.7} = 3.89$
3. $\dfrac{112 \times 14}{83} = 18.90$
4. $\dfrac{13.6 \times 12.4}{9.2 \times 5.6} = 3.27$
5. $\dfrac{94}{12.3 \times 3.72 \times 2.8} = 0.733$
6. $\dfrac{115 \times 1.08 \times 2.16}{9.15} = 29.3$

The techniques which have been illustrated thus far are basic, and they must be mastered if you are to use your slide rule with greatest efficiency. Many users of the slide rule have never developed the habit of properly utilizing the reciprocal and folded scales. This means that they make more moves and settings than necessary, which, in turn, increases the chance for error. It is important that you force yourself to use the reciprocal and folded scales until it becomes second nature; you should be just as comfortable and confident with these scales as with the C and D scales.

The following exercise set is designed to give you more practice with combined operations. See if you can go through them without making any nonoperational moves.

Exercise 7-3

1. $\frac{5.6 \times 7.8}{2.5} =$

2. $\frac{22}{4.7 \times 7.1} =$ "659"

3. $4.1 \times 4.9 \times 0.77 =$

4. $\frac{19 \times 21}{48 \times 13} =$ "640"

5. $\frac{12 \times 6.4 \times 1.7}{9.3} =$

6. $\frac{33}{(1.7)^2 \times 6.4} =$ "1785"

7. $1.9 \times 2.4 \times 2.1 \times 0.74 =$

8. $\frac{8.7 \times 21 \times 5.3}{11 \times 7.6} =$ "1158"

9. $\frac{37 \times 26}{81 \times 7.2} =$

10. $\frac{9.3 \times 8.4}{1.1 \times 1.3 \times 2.7} =$ "202"

11. $\frac{12 \times 10.5 \times 2.9}{8.8 \times 9.4} =$

12. $\frac{7.8 \times 2.1 \times 14}{3.5 \times 9.3} =$ "705"

13. $\frac{2.5 \times 4.9 \times 3.1}{1.6 \times 2.3} =$

14. $\frac{4.2 \times 27 \times 3.2}{6.3 \times 5.9} =$ "976"

15. $\frac{42 \times 4.5 \times 0.52}{17} =$

16. $\frac{76}{10.5 \times 1.2 \times 1.5} =$ "402"

17. $\frac{130}{8.6 \times (0.92)^2} =$

18. $4.1 \times (5.2)^2 =$ "1109"

19. $\frac{28 \times 19}{58 \times (2.1)^2} =$

20. $(5.2)^3 \times 1.25 =$ "1757"

21. $\frac{29 \times 32}{9.75 \times 9.3} =$

22. $\frac{74.1 \times 6.55 \times 9.25}{4.26} =$ "1054"

23. $\frac{19.6 \times 1.12 \times 2.43}{7.95 \times 0.844} =$

24. $\frac{83.2}{1.26 \times 6.47 \times 11.5} =$ "888"

25. $\frac{17.2 \times 120}{28.4 \times 34.2 \times 6.05} =$

26. $\frac{78.2}{(1.12)^3} =$ "557"

27. $\frac{46.5}{(2.1)^3} =$

28. $\frac{100}{4.52 \times 3.24 \times 1.85} =$ "369"

29. $\frac{62.5 \times 5.94 \times 6.47}{10.3 \times 1.13 \times 1.245} =$

30. $\frac{12.6 \times 15.7}{4.82 \times 7.11 \times 0.627} =$ "920"

31. $\frac{12.65 \times 6.21 \times 2.77}{8.34 \times 4.56 \times 3.69} =$

32. $\frac{24.6 \times 152}{3.85 \times 4.27 \times 6.23 \times 5.76} =$ "634"

33. $\frac{51.7 \times 64.7}{20.6 \times 2.35 \times 5.68 \times 17.2} =$

34. $7.12 \times 0.645 \times 4.73 \times 5.25 =$ "1140"

35. $5.84 \times 3.6 \times 0.211 \times (1.43)^2 =$

36. $\dfrac{47.8 \times 96.4 \times 12.4}{24.3 \times 5.62 \times 7.07 \times 19.2} =$ "308"

37. $\dfrac{.0364 \times 16.45}{723} =$

38. $\dfrac{485}{18.3 \times 2160} =$ "1227"

39. $.00264 \times .0663 \times 0.824 =$

40. $\dfrac{2650 \times .0624}{138 \times 149} =$ "804"

41. $\dfrac{926 \times 82.6}{.0113 \times 132 \times 264} =$

42. $\dfrac{.00473 \times 1150}{5.72 \times .0862} =$ "1103"

43. $\dfrac{.0224 \times 486 \times .00308}{0.154 \times .0196} =$

44. $\dfrac{.0226 \times 1940 \times 17.3}{845} =$ "898"

45. $\dfrac{184 \times 11.6 \times 23.6}{7840 \times 836} =$

46. $.00314 \times 2960 \times 52.6 \times .0345 =$ "1687"

47. $\dfrac{1}{13.6 \times (.0264)^2} =$

48. $\dfrac{718 \times .000617 \times 75.2}{.0246} =$ "1355"

49. $392 \times (.0616)^2 =$

50. $\dfrac{28.2}{.00416 \times 7.22 \times 32.4} =$ "290"

51. $\dfrac{.00743 \times (46.2)^2}{21.7} =$

52. $59.2 \times .00134 \times 410 \times 0.512 \times 8.3 =$ "1382"

53. $\dfrac{4.01 \times 232 \times 1.345}{96.8 \times 16.75} =$

54. $\dfrac{1}{.0764 \times 123 \times 2.75} = \text{“387”}$

55. $\dfrac{16{,}000}{46.2 \times 9.57 \times 100.4} =$

56. $\dfrac{.0743 \times 575}{.00524 \times 8540 \times 17.5 \times 10^{-3}} = \text{“546”}$

57. $\dfrac{.0816 \times 1075 \times 5.20 \times .0625}{426 \times .00347} =$

58. $\dfrac{6420 \times 10^4}{1735 \times 24.6 \times 402} = \text{“374”}$

59. $752 \times 14.65 \times 247 \times 60.3 =$

60. $\dfrac{4.62 \times 1575 \times .00288}{362 \times 5.71 \times 23.7 \times 10^{-3}} = \text{“428”}$

7.6 Multiplication and division by π

Inasmuch as the folded scales begin and end at π, the following relationship holds:

> If the hairline is moved over a number N on the **D** scale, then the product ($\pi \times N$) will be under the hairline on **DF.**
>
> Conversely, if the hairline is moved over a number N on **DF,** the quotient ($N \div \pi$) will be under the hairline on **D.** The C and CF scales are similarly related.

This means that, given the diameter, we can find the circumference of a circle with a single setting. Also, we may eliminate one operation when evaluating combined expressions involving π.

Example 1: Given circles with diameters 2.43 inches, 5.75 inches, and 12.6 inches. Find the respective circumferences.

1. Move HL over 243 on D. Under HL read “764” on DF.
2. Move HL over 575 on D. Under HL read “1806” on DF.
3. Move HL over 126 on D. Under HL read “396” on DF.

Answers are: ***7.64*** in., ***18.06*** in., and ***39.6*** in.

Example 2: $\dfrac{\pi \times 6.84}{8.31} = ?$

1. Move HL over 684 on D. Now $\pi \times 6.84$ is under HL on DF; hence, we need only divide by 8.31 on the folded scales:
2. Slide 831 on CF under HL.
3. Opposite left index of C, read "258" on D. Answer is ***2.58.***

Example 3: $\dfrac{13}{\pi \times 0.73} = ?$

1. Move HL over 13 on DF. Note that $13 \div \pi$ is now under HL on D; hence we continue on the regular scales:
2. Slide 73 on C under HL.
3. Opposite right index of C, read "567" on D. Answer is ***5.67.***

Exercise 7-4

1. Find circumferences of circles with following diameters: 2.68, 1.77, 7.05, 0.421, .0543, 17.6, 50.2, and 1.155.

2. Find diameters of circles with following circumferences: 20.4, 14.2, 83.7, 178.5, 0.417, 2430, .0206, and 1.09.

3. $\dfrac{36.1 \times \pi}{4.75} =$

4. $\dfrac{\pi \times 26}{7.2} =$ "1134"

5. $\dfrac{4.5 \times \pi}{23} =$

6. $\dfrac{\pi \times 1.73}{2.44} =$ "223"

7. $\pi \times 6.4 \times 1.3 =$

8. $\pi \times 27.3 \times 0.44 =$ "377"

9. $\pi \times (1.9)^2 =$

10. $\pi \times (4.1)^2 =$ "528"

11. $\dfrac{123}{\pi \times 6.34} =$

12. $\dfrac{17.2}{\pi \times 2.4} =$ "228"

13. $\dfrac{275}{\pi \times 8.46} =$

14. $\dfrac{56.2}{\pi \times 2.34} =$ "765"

15. $\dfrac{92.4}{\pi \times (4.75)^2} =$

7.7 Formula types

When substituting into formulas, we must often evaluate expressions similar to the following:

Example 1: $$\frac{26.5}{5.40 - (6.25 \times 0.123)} = ?$$

1. Verify that $6.25 \times 0.123 = 0.769$.
2. Expression now becomes:

$$\frac{26.5}{5.40 - 0.769} = \frac{26.5}{4.631} \approx \frac{26.5}{4.63}$$

Note that for slide rule computation, we round off the denominator to three significant figures.

3. Verify that answer is ***5.72.***

Example 2: $$\frac{244}{4.60\left[2.30 + \frac{6.70}{8.20}\right]} = ?$$

1. Verify that $\dfrac{6.70}{8.20} = 0.817$
2. Expression now becomes:

$$\frac{244}{4.60(2.30 + 0.817)} = \frac{244}{4.60 \times 3.117} \approx \frac{244}{4.60 \times 3.12}$$

3. Verify that answer is ***17.00.***

Example 3: $$\frac{22.2}{\frac{1}{2.6} + \frac{1}{3.2} + \frac{1}{4.8}} = ?$$

1. Use reciprocal scale to verify that:

$$\frac{1}{2.6} = 0.385; \frac{1}{3.2} = 0.312; \frac{1}{4.8} = 0.208$$

2. Expression becomes:

$$\frac{22.2}{0.385 + 0.312 + 0.208} = \frac{22.2}{0.905}$$

3. Verify that answer is ***24.5***.

Exercise 7-5

1. $\dfrac{41}{1 + (3.2 \times 1.7)} =$

2. $\dfrac{120}{1 - (45 \times .016)} =$

3. $7.60 \left[12.40 + \dfrac{36.3}{7.22}\right] =$

4. $\dfrac{520}{6.70 + \dfrac{3.25}{1.66}} =$

5. $\dfrac{28.6}{2.77 + \dfrac{1500}{3.62}} =$

6. $\dfrac{375}{2.88 \left[1.84 + \dfrac{32.6}{45.2}\right]} =$

7. $\dfrac{47{,}500}{165 \left[.027 + \dfrac{1}{230}\right]} =$

8. $\dfrac{1}{.00224 \left[2.63 - \dfrac{375}{162}\right]} =$

9. $\dfrac{1}{\dfrac{1}{18} + \dfrac{1}{35}} =$

10. $\dfrac{67.3}{\dfrac{1}{3.72} + \dfrac{1}{4.23} + \dfrac{1}{7.25}} =$

11. $\dfrac{.0865}{\dfrac{1}{450}+\dfrac{1}{175}+\dfrac{1}{620}} =$

12. $\dfrac{164}{15.2}\,[17.2 + (136 \times 0.245)] =$

13. $\dfrac{254}{6450}\,[368 - (26.6 \times 2.77)] =$

14. $(5.22 - 2.68)\left[37.2 + \dfrac{136}{(5.22)^2}\right] =$

15. $(2.75 - 1.37)\left[165 + \dfrac{237}{(2.75)^2}\right] =$

16. $26.5\left[\dfrac{3.72 \times 3.16}{3.72 - 3.16}\right] =$

17. $3.27 \times 10^{-6}\left[\dfrac{.01465 \times .01452}{.01465 - .01452}\right] =$

18. $\dfrac{16.3}{(1.72)^2}\left[\dfrac{3.75}{2.24} - \dfrac{3.25}{2.76}\right] =$

19. $\dfrac{2}{(.073)^2}\left[\dfrac{1.68}{75.2} - \dfrac{1.44}{86.7}\right] =$

20. Given the formula: $P = \dfrac{A}{1 + rt}$
Evaluate P if:
a. $A = 37{,}500$, $r = .0775$, $t = 14$
b. $A = 750$, $r = 0.125$, $t = 17$

21. Given the formula: $S = \dfrac{P}{A} + \dfrac{Mc}{I}$
Evaluate S if:
a. $P = 8400$, $A = 2.86$, $M = 6350$, $c = 3.20$, $I = 11.65$
b. $P = 12{,}500$, $A = 4.27$, $M = 7500$, $c = 4.62$, $I = 16.55$

22. Given the formula: $y_1 = \dfrac{I + Ay_0^2}{Ay_0}$
Evaluate y_1 if:
a. $I = 56.2$, $y_0 = 6.85$, $A = 19.4$
b. $I = 31.7$, $y_0 = 4.66$, $A = 12.7$

23. Given the formula: $\bar{x} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$
Evaluate $\bar{x}$ if:
a. $m_1 = 2.6$, $x_1 = 1.7$, $m_2 = 3.5$, $x_2 = 4.5$, $m_3 = 5.2$, $x_3 = 7.7$

b. $m_1 = .0375$, $x_1 = 10.8$, $m_2 = .0466$, $x_2 = -5.05$, $m_3 = .0226$, $x_3 = 21.2$

24. Given the formula: $V_0 = \dfrac{VH}{760(1 + at)}$

Evaluate V_0 if:
a. $V = 270$, $H = 524$, $a = .00367$, $t = 62.5$
b. $V = 1500$, $H = 1340$, $a = .00367$, $t = 140$

Chapter 8

RATIO AND PROPORTION

8.1 Definitions

The *ratio* of a number M to another number N is the *quotient* M/N. Thus, the ratio of 4 to 3 refers simply to the quotient 4/3.

A *proportion* is a statement of *equality* between *two ratios*. The following represent examples of proportions:

$$\frac{3}{8}=\frac{9}{24};\quad \frac{6}{4}=\frac{9}{6};\quad \frac{x}{3}=\frac{2}{5};\quad \frac{3.7}{6.5}=\frac{7.8}{y}$$

A statement of equality involving more than two ratios may be called a *continued proportion.* Following are examples of continued proportions:

$$\frac{3}{4}=\frac{6}{8}=\frac{18}{24};\quad \frac{2}{3}=\frac{6}{x}=\frac{5}{y};\quad \frac{6.1}{X}=\frac{Y}{2.8}=\frac{Z}{5}=\frac{7.3}{3.9}$$

It is often stated that M is *directly proportional* to N. This simply means that the *ratio* of M to N is *constant.*

8.2 Proportional settings on the C-D (CF-DF) scales

Suppose the slide is set so that the number M on the C scale is opposite the number N on the D scale. Then, for this position of the slide, the ratio of any other number on C to its opposite on D will be the same as the ratio of M to N. A similar relationship holds for the CF and DF scales.

Example 1: Find several ratios equivalent to 3/2.

1. Move HL over 2 on D.
2. Slide 3 on C under HL. The ratio 3/2 has now been set on the rule with the numerator on C opposite the denominator on D. Notice that 3 on CF is opposite 2 on DF; hence, opposite readings on the folded scales will also be in the ratio 3/2.
3. Move HL over 4 on C. Read "267" on D.
4. Move HL over 9 on C. Read "600" on D.
5. Move HL over 12 on CF. Read "800" on DF.

In this case, we have simply discovered the following continued proportion on the C-D (CF-DF) scales:

$$\frac{C(CF)}{D(DF)}: \frac{3}{2} = \frac{4}{2.67} = \frac{9}{6} = \frac{12}{8}$$

In the foregoing example, all the numerators appear on the C or CF scale, the denominators on the D or DF scale. Of course, the original ratio could have been set with the numerator on D and the denominator on C, in which case all the other numerators would be found on D (or DF) and denominators on C (or CF). In the following examples, the denominator will always be set on D (or DF).

Example 2: Solve the proportion:

$$\frac{2.7}{1.9} = \frac{x}{43}$$

This could be handled by first solving for x, and then performing the combined multiplication and division in the usual manner. However, in Figure 8.1 we illustrate how the proportion may be set up and solved directly on the slide rule without changing its form.

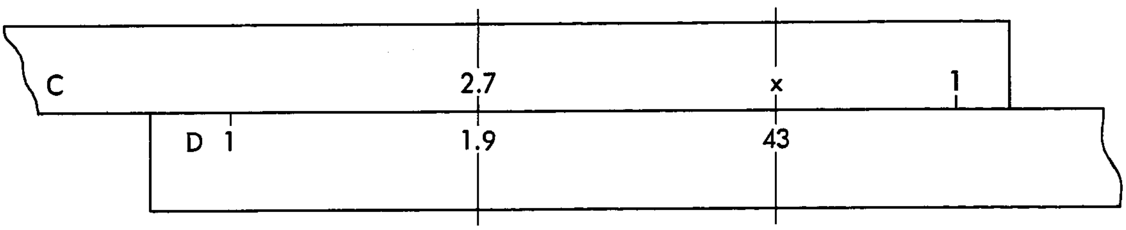


Figure 8.1

1. Move HL over 19 on D.
2. Slide 27 on C under HL. The known ratio is now set with numerator on C and denominator on D.
3. Move HL over 43 on D. Under HL read "611" on C.

Answer: $x = $ ***61.1.***

Example 3: Solve the following proportion on the C-D (CF-DF) scales.

$$\frac{C(CF)}{D(DF)}: \frac{45.1}{73} = \frac{2.2}{x} = \frac{y}{12.5}$$

1. Move HL over 73 on D.
2. Slide 451 on C under HL.
3. Move HL over 22 on C. Under HL read "356" on D.
4. Move HL over 125 on DF. Under HL read "772" on CF.

Answers: $x =$ ***3.56***, $y =$ ***7.72.***

Verify the following:

1. $\frac{215}{42} = \frac{y}{27}$; $y = 138.2$
2. $\frac{143}{R} = \frac{29.5}{6.75}$; $R = 32.7$
3. $\frac{71.5}{43.5} = \frac{124}{I}$; $I = 75.4$
4. $\frac{3.65}{5.20} = \frac{x}{22.4} = \frac{y}{0.63}$; $x = 15.72, y = 0.442$
5. $\frac{5.3}{V_1} = \frac{91}{44} = \frac{152}{V_2}$; $V_1 = 2.56, V_2 = 73.5$
6. $\frac{183}{34} = \frac{W}{16.4} = \frac{125}{L}$; $W = 88.2, L = 23.2$

Example 4: Solve the following proportion on the C-D (CF-DF) scales.

$$\frac{C\ (CF)}{D\ (DF)}: \frac{11.2}{8.63} = \frac{x}{47.5} = \frac{2.11}{y}$$

When the numerator and denominator of the given ratio are at opposite ends of the C and D scales, it is more convenient to set the ratio on the folded scales:

1. Move HL over 863 on DF.
2. Slide 112 on CF under HL.
3. Move HL over 475 on DF. Under HL read "616" on CF.
4. Move HL over 211 on CF. Under HL read "1627" on DF.

Answers: $x =$ ***61.6***, $y =$ ***1.627.***

Verify the following:

1. $\dfrac{125}{91} = \dfrac{x}{1.85};\ x = 2.54$

2. $\dfrac{205}{x} = \dfrac{8.34}{1.16};\ x = 28.5$

3. $\dfrac{545}{x} = \dfrac{y}{1.62} = \dfrac{13.2}{9.6};\ x = 396, y = 2.23$

4. $\dfrac{x}{61} = \dfrac{113}{88} = \dfrac{125}{y};\ x = 78.4, y = 97.3$

Example 5: $\dfrac{X}{.0275} = \dfrac{0.142}{56.2};\ X = ?$

We may shift the decimal points three places to the left in the right-hand ratio, thus making its denominator comparable with the denominator of the left-hand ratio:

$$\frac{X}{.0275} = \frac{.000142}{.0562}$$

Since .0275 is about one half of .0562, X must be about one half of .000142. Verify that $X = \boldsymbol{.0000695}$.

Example 6: $\dfrac{.0345}{162} = \dfrac{12.6}{X};\ X = ?$

Shift decimal points in the left-hand ratio:

$$\frac{3.45}{16{,}200} = \frac{12.6}{X}$$

Since 12.6 is about four times 3.45, X must be about four times 16,200. Verify that $X = \boldsymbol{59{,}100}$.

Exercise 8-1

Solve for the indicated letter or letters:

1. $\dfrac{X}{41.2} = \dfrac{3.84}{6.21}$

2. $\dfrac{X}{7.08} = \dfrac{21.2}{29.6}$

3. $\dfrac{X}{60.4} = \dfrac{179}{416}$

4. $\dfrac{4.16}{21.9} = \dfrac{X}{504}$

5. $\frac{3.14}{2.18} = \frac{X}{0.512}$

6. $\frac{X}{72.4} = \frac{3.18}{19.2}$

7. $\frac{T}{42.8} = \frac{12.2}{8.75}$

8. $\frac{254}{64.8} = \frac{T}{20.7}$

9. $\frac{9.60}{1.15} = \frac{39.6}{R}$

10. $\frac{5.74}{29.1} = \frac{8.68}{R}$

11. $\frac{41.6}{I} = \frac{105}{850}$

12. $\frac{14.25}{I} = \frac{.0253}{.00614}$

13. $\frac{546}{V} = \frac{182}{52.4}$

14. $\frac{.0362}{0.635} = \frac{7.54}{V}$

15. $\frac{194.5}{4.23} = \frac{39.6}{P}$

16. $\frac{24.3}{P} = \frac{.0053}{7.09}$

17. $\frac{X}{.0315} = \frac{0.219}{46.2}$

18. $\frac{1750}{X} = \frac{4.26}{.0615}$

19. $\frac{297}{71.4} = \frac{41.6}{L}$

20. $\frac{17.2}{283} = \frac{L}{147}$

21. $\frac{.00364}{W} = \frac{14.1}{22.6}$

22. $\frac{W}{5.63} = \frac{.0415}{.0022}$

23. $\frac{.0377}{12.75} = \frac{F}{6240}$

24. $\frac{.00452}{F} = \frac{0.724}{1255}$

25. $\frac{X}{3.54} = \frac{Y}{62.1} = \frac{370}{746}$

26. $\frac{X}{71.4} = \frac{Y}{2.07} = \frac{11.4}{9.10}$

27. $\frac{X}{0.824} = \frac{Y}{50.4} = \frac{12.55}{2.79}$

28. $\frac{856}{47.5} = \frac{X}{2.54} = \frac{Y}{5.21}$

29. $\frac{3.45}{23.7} = \frac{0.614}{V_1} = \frac{2.75}{V_2}$

30. $\frac{3.45}{x} = \frac{65.6}{5.12} = \frac{176}{y}$

31. $\frac{341}{76.1} = \frac{407}{V_1} = \frac{15.25}{V_2}$

32. $\frac{44.9}{V_1} = \frac{167}{V_2} = \frac{2.83}{5.49}$

33. $\frac{13.6}{74.5} = \frac{X}{6.24} = \frac{Y}{10.5} = \frac{4.20}{Z}$

34. $\frac{4.65}{X} = \frac{Y}{21.2} = \frac{14.25}{32.7} = \frac{Z}{7.60}$

35. $\frac{X}{90.4} = \frac{3.82}{Y} = \frac{16.38}{5.22} = \frac{640}{Z}$

36. $\frac{4.83}{2.11} = \frac{R_1}{.00482} = \frac{R_2}{0.204} = \frac{R_3}{0.651}$

37. $\frac{.0716}{0.477} = \frac{I_1}{38.2} = \frac{I_2}{275} = \frac{I_3}{1050}$

38. $\frac{1250}{83.5} = \frac{640}{T_1} = \frac{2670}{T_2} = \frac{135}{T_3}$

8.3 Conversion of units and other applications

Example 1: There are 16 ounces in a pound.
a. Convert to ounces: 4.3 lbs, 0.77 lbs.
b. Convert to pounds: 37.6 oz, 11.2 oz.

Here, the ratio of ounces to pounds is constant. Putting "ounces" on C opposite "pounds" on D, we may write:

$$\frac{\text{C: oz}}{\text{D: lbs}} = \text{constant}; \quad (\text{setting: } \textit{16 oz} \text{ opposite } \textit{1 lb})$$

The continued proportion may be indicated in tabular form:

oz	C (CF)	**16**	?	?	37.6	11.2
lbs	D (DF)	**1**	4.3	0.77	?	?

1. Slide 16 on C opposite left index of D. This sets up the desired ratio on C-D.
2. Move HL over 43 on D. Under HL read "688" on C.
3. Move HL over 77 on DF. Under HL read "1232" on CF.
4. Move HL over 376 on C. Under HL read "235" on D.
5. Move HL over 112 on CF. Under HL read "700" on DF.

Answers are:
a. ***68.8 oz, 12.32 oz;*** **b.** ***2.35 lbs, 0.700 lbs.***

Example 2: 33 knots is equivalent to 38 mph.
a. Convert to mph: 40 knots, 6 knots.
b. Convert to knots: 10 mph, 25 mph.

Putting "knots" on C opposite "mph" on D:

$$\frac{\text{C: knots}}{\text{D: mph}} = \text{constant}; \quad (\text{setting: } \textit{33 knots} \text{ opposite } \textit{38 mph})$$

knots	C (CF)	**33**	40	6	?	?
mph	D (DF)	**38**	?	?	10	25

1. Move HL over 38 on D. Slide 33 on C under HL. This sets up the ratio on C-D.
2. Verify the answers:
   a. ***46.1 mph, 6.91 mph;*** **b.** ***8.69 knots, 21.7 knots.***

Example 3: On a map 1 inch is equivalent to 75 miles.
a. Convert to miles: 5.2 inches, 0.34 inches.
b. Convert to inches: 950 miles, 225 miles.

Putting "inches" on C opposite "miles" on D:

$$\frac{\text{C: inches}}{\text{D: miles}} = \text{constant}; \quad (\text{setting: } \textit{1 inch} \text{ opposite } \textit{75 miles})$$

inches	C (CF)	**1**	5.2	0.34	?	?
miles	D (DF)	**75**	?	?	950	225

1. Set right index of C opposite 75 on D. This sets the constant ratio on C-D.
2. Verify the answers:
   a. ***390 miles, 25.5 miles***; b. ***12.67 inches, 3.00 inches.***

Example 4: A steel bar stretches .0073 inches under a load of 1200 lbs. If the stretch is directly proportional to the load, find the corresponding extensions of the bar for loads of 750 lbs, 2600 lbs, and 4250 lbs.

Inasmuch as this is a direct proportion, the ratio of stretch to load is constant. Putting "inches" on C opposite "lbs" on D:

$$\frac{\text{C: inches}}{\text{D: lbs}} = \text{constant}; \quad \text{(setting: } \textit{.0073 inches} \text{ opposite } \textit{1200 lbs)}$$

inches	C (CF)	**.0073**	?	?	?
lbs	D (DF)	**1200**	750	2600	4250

1. The slide will be in better position if the ratio is set on the *folded* scales. Move HL over 1200 on DF, slide 73 on CF under HL.
2. Verify the answers:
   .00456 inches, .0158 inches, and ***.0258 inches.***

Exercise 8-2

1. 53 liters is equivalent to 14 gallons.
   a. Convert to liters: 1 gal, 4.8 gal, 12.5 gal.
   b. Convert to gallons: 1 liter, 21 liters, 85 liters.

2. 30 mph is equivalent to 44 ft per sec.
   a. Convert to mph: 217 ft per sec, 68.2 ft per sec, 10 ft per sec.
   b. Convert to ft per sec: 48.2 mph, 21.7 mph, 85.6 mph.

3. 1 kilogram (kg) is equivalent to 2.2 lbs.
   a. Convert to lbs: 3.5 kg, 75.4 kg, 125 kg.
   b. Convert to kg: 34.6 lbs, 164 lbs, 4.75 lbs.

4. 14.7 lbs per sq in. is equivalent to 29.8 inches of mercury.
   a. Convert to inches of mercury: 21.2 lbs per sq in., 6.32 lbs per sq in., 135 lbs per sq in.
   b. Convert to lbs per sq in.: 0.723 inches mercury, 17.4 inches mercury.

5. On a map 1 inch is equivalent to 64 miles.
   a. Convert to miles: 2.4 in., 0.375 in., 4.75 in.
   b. Convert to inches: 150 miles, 275 miles.

6. 1 mile is equivalent to 1.61 kilometers (km).

a. Convert to km: 57.5 miles, 1245 miles, 845 miles.
b. Convert to miles: 452 km, 13.4 km.

7. 7.48 gals of water weighs 62.4 lbs.
a. Convert to lbs: 1 gal, 17.4 gals, 112 gals.
b. Convert to gals: 1940 lbs, 22.6 lbs.

8. 1 horsepower is equivalent to 746 watts.
a. Convert to hp: 124 watts, 36,500 watts, 1000 watts.
b. Convert to watts: 31.6 hp, 0.524 hp.

9. 31 square inches is approximately 200 square centimeters.
a. Convert to sq in: 350 sq cm, 62.5 sq cm, 756 sq cm.
b. Convert to sq cm: 148 sq in., 9.40 sq in.

10. In a circuit with constant resistance, the amperage (I) is proportional to the voltage (E). If $I = 8.74$ amps when $E = 120$ volts, find I corresponding to $E = 27.5$ volts, 110 volts, and 14.5 volts.

11. At constant velocity, distance is proportional to time. If it takes 10.5 hours to travel 645 miles, find the distance traveled in 1 hour, 6.25 hours, and 14.25 hours.

12. It requires 3.75 gallons of paint to cover 1450 square feet of surface. Assuming a direct proportion, find the amount of paint needed to cover 2100 sq ft, 6400 sq ft, 35,000 sq ft.

Chapter 9

SQUARES AND SQUARE ROOTS (A, B, R, SQ, AND $\sqrt{\ }$ SCALES)

9.1 Description of the A and B scales

The A and B scales are identical scales located on the body and slide respectively. Notice that each half of the A scale consists of a complete scale similar to D; thus, the A scale is made up of two half-length D scales, one next to the other. The two sections of the scale will be referred to as "A-left" and "A-right." Similar reference will be made to the B scale.

Since one complete A scale is half the length of the complete D scale, it follows that if the hairline is moved a certain distance along the D scale, it has effectively moved twice that distance relative to the A scale. For example, if the hairline is moved to the quarter point on D, it is at the half-way point on A. Inasmuch as these distances are proportional to the logarithms of the numbers, the reading on A must correspond to the *square* of the opposite reading on D. Conversely, the reading on D represents the *square root* of the opposite reading on A.

You can easily check this relationship by taking your slide rule and moving the hairline over "2" on D. Note that this puts the hairline over "4" on A-left. Now move the hairline over "3" on D; observe that hairline is over "9" on A-left. Finally, move the hairline over "5" on D and note that "25" is the opposite reading on A-right.

Summarizing:

> If the hairline is set over a number N on the **D** scale, then N^2 will be under the hairline on the **A** scale.
>
> Conversely, if the hairline is set over a number N on the *proper half* of the **A** scale, then $\sqrt{N}$ will be under the hairline on the **D** scale.

Because the A-B scales are half-length, you cannot read them as accurately as you read the C-D scales. Note especially that there are fewer subdivisions between primary numbers on the A-B scales.

9.2 The double-length scale (R, Sq, or √‾)

This scale appears with different labels on various commercial slide rules; depending on the make of the rule, you may find it identified as "R" (Frederick Post Co.), "Sq" (Keuffel & Esser Co.), or "√‾" (Pickett, Inc.) You will observe that the scale consists of two full-length sections, the first covers the same range as the left half of the D scale, whereas the second covers the same range as the right half of D. Thus, the two sections taken together represent one double-length D scale.

On the Post rule, these two sections are labeled R_1 and R_2 respectively; on the K & E rule they are denoted by Sq1 and Sq2. The Pickett model features the two sections of the √‾ scale "back-to-back"; that is, the two scales have a common axis, the upper markings corresponding to R_1 or Sq1, the lower markings corresponding to R_2 or Sq2. Hereafter, we shall simply use the R designation when referring to the double-length scale.

The R scale is related to the D scale in the same manner as the D scale is related to the A scale. Thus, the relationship may be described:

> If the hairline is set over a number N on the **R** scale, then N^2 will be under the hairline on the **D** scale.
>
> Conversely, if the hairline is set over a number N on the **D** scale, then $\sqrt{N}$ will be under the hairline on the *proper section* of the **R** scale.

The double-length scale may be read with greater accuracy than the A scale; hence, its advantage.

9.3 Squaring a number

Clearly, squaring a number may be handled as a multiplication in the conventional manner using the C (or CI) and D scales. However, with the A or R scales, squares may be obtained directly as illustrated in the following examples.

Example 1: $(2.48)^2 = ?$

Procedure with A scale:
1. Move HL over 248 on D.
2. Under HL read "615" on A.

Procedure with R scale:
1. Move HL over 248 on R_1.
2. Under HL read "615" on D.

Answer is ***6.15***.

Example 2: $(.00465)^2 = ?$

Write this: $(4.65 \times 10^{-3})^2 = (4.65)^2 \times 10^{-6}$.

Procedure with A scale:
1. Move HL over 465 on D.
2. Under HL read "216" on A.

Procedure with R scale:
1. Move HL over 465 on R_2.
2. Under HL read "216" on D.

Answer is ***21.6*** $\times$ ***10***$^{-6}$ or ***.0000216.***

Exercise 9-1

1. $(1.7)^2 =$

2. $(5.6)^2 =$

3. $(8.8)^2 =$

4. $(10.8)^2 =$

5. $(7.4)^2 =$

6. $(3.4)^2 =$

7. $(4.27)^2 =$

8. $(2.47)^2 =$

9. $(28.4)^2 =$

10. $(5.22)^2 =$

11. $(64.7)^2 =$

12. $(0.405)^2 =$

13. $(13.25)^2 =$

14. $(0.144)^2 =$

15. $(32.2)^2 =$

16. $(1.94 \times 10^2)^2 =$

17. $(2.77 \times 10^4)^2 =$

18. $(478)^2 =$

19. $(7840)^2 =$

20. $(56{,}000)^2 =$

21. $(162{,}500)^2 =$

22. $(4.05 \times 10^{-3})^2 =$

23. $(.0564)^2 =$

24. $(.00436)^2 =$

25. $(.000226)^2 =$

26. $(.0000362)^2 =$

27. $(18.65)^2 =$

28. $(56{,}400)^2 =$

29. $(207)^2 =$

30. $(456 \times 10^4)^2 =$

9.4 Finding square root of a number between 1 and 100

Example 1: $\sqrt{41.5} =$

Procedure with A scale:
Square roots are found by setting on A and reading on D. Note that the hairline may be

set over 415 on A-left or A-right. However, observe also that the desired square root is a number between 6 and 7; hence, it is clear that A-right is the proper choice.

1. Move HL over 415 on A-right.
2. Under HL read "644" on D. Answer is ***6.44.***

Procedure with R scale:
Square roots are found by setting on D and reading on R_1 or R_2. Here, we know the root is between 6 and 7; hence, answer must be on R_2.

1. Move HL over 415 on D.
2. Under HL read "644" on R_2. Answer is ***6.44.***

Example 2: $\sqrt{5.60} =$

By inspection, root is between 2 and 3.

Procedure with A scale:
1. Move HL over 560 on A-left.
2. Under HL read "237" on D.

Answer is ***2.37.***

Procedure with R scale:
1. Move HL over 560 on D.
2. Under HL read "2366" on R_1.

Answer is ***2.366.***

These examples illustrate the following rule:

1. When finding the square root of a number between *1 and 10*, use **A-left** or **R_1**.
2. For numbers between *10 and 100*, use **A-right** or **R_2**.

Again, you are reminded that square roots may also be obtained by reading from the proper half of the *B scale* to the *C scale*.

Verify the following:

1. $\sqrt{6.8} = 2.61$
2. $\sqrt{68} = 8.25$
3. $\sqrt{11.6} = 3.41$
4. $\sqrt{1.85} = 1.360$
5. $\sqrt{56.4} = 7.51$
6. $\sqrt{10} = 3.16$
7. $\sqrt{7.66} = 2.77$
8. $\sqrt{1.05} = 1.025$
9. $\sqrt{31.6} = 5.62$

9.5 A general procedure for square roots of numbers greater than 1

For numbers greater than 1, group the digits in *pairs* from right to left starting at the decimal point. Then:

> 1. If the leftmost group contains *one* digit, use **A-left** or **R_1**. If it contains *two* digits, use **A-right** or **R_2**.
>
> 2. The square root will contain the same number of *digits* to the *left* of the decimal point as there are *groups* to the *left* of the decimal point in the original number.

Example 1: $\sqrt{4,350,000} = ?$

1. Group the digits in pairs starting from the decimal point:

$$\sqrt{\underset{\smile}{4}\ \underset{\smile}{35}\ \underset{\smile}{00}\ \underset{\smile}{00}.}$$

↑ (leftmost group)

2. Leftmost group contains one digit; hence, use A-left or R_1.
3. Verify that slide rule reading is "209."
4. There are four groups to the left of the decimal point; therefore, answer will have four digits to left of decimal.

Answer is ***2090.***

Example 2: $\sqrt{167.5} = ?$

1. Group the digits in pairs starting from the decimal point:

$$\sqrt{\underset{\smile}{1}\ \underset{\smile}{67}.5}$$

↑ (leftmost group)

2. Leftmost group contains one digit; hence, use A-left or R_1.
3. Verify that slide rule reading is "1294."
4. There are two groups to the left of the decimal point; hence, answer will have two digits to left of decimal.

Answer is ***12.94.***

Example 3: $\sqrt{665,000} = ?$

1. Group the digits in pairs:

$$\sqrt{\underset{\smile}{66}\ \underset{\smile}{50}\ \underset{\smile}{00}.}$$

↑ (leftmost group)

2. Leftmost group contains two digits; hence, use A-right or R_2.
3. Verify that answer is ***815.***

Verify the following:

1. $\sqrt{352} = 18.76$

2. $\sqrt{2090} = 45.7$

3. $\sqrt{173.5} = 13.17$

4. $\sqrt{615{,}000} = 784$

5. $\sqrt{1{,}265{,}000} = 1125$

6. $\sqrt{100{,}000} = 316$

7. $\sqrt{1065} = 32.6$

8. $\sqrt{27{,}500} = 165.8$

9. $\sqrt{54{,}500{,}000} = 7380$

Example 4: $\sqrt[4]{3170} = ?$

We use the fact that the fourth root is the square root of the square root.

1. Verify that $\sqrt{3170} = 56.3$.
2. To find the fourth root, we extract the square root again:

$$\sqrt[4]{3170} = \sqrt{\sqrt{3170}} = \sqrt{56.3} = 7.50$$

On slide rules that have both the A scale and the double-length scale, the fourth root may be read directly. Thus, on such a rule, if the hairline is moved over a number N on the A scale, then $\sqrt[4]{N}$ will be under the hairline on the proper section of R (Sq or $\sqrt{\ }$).

Verify the following:

1. $\sqrt[4]{8.22} = 1.695$

2. $\sqrt[4]{15.7} = 1.990$

3. $\sqrt[4]{315} = 4.21$

4. $\sqrt[4]{16{,}500} = 11.35$

5. $\sqrt[4]{5620} = 8.66$

6. $\sqrt[4]{60{,}400{,}000} = 88.3$

9.6 General procedure for square roots of numbers less than 1

For numbers less than 1, group the digits in pairs from left to right starting at the decimal point. Then:

1. If the first nonzero group contains *one* significant digit, use **A-left** or $\mathbf{R_1}$. If it contains *two* significant digits, use **A-right** or $\mathbf{R_2}$.

2. For *each zero group* in the original number, the square root will have *one zero* immediately to the right of the decimal point.

Example 1: $\sqrt{.000617} = ?$

1. Group in pairs from left to right starting at decimal point:

$\sqrt{.00\ 06\ 17}$

(zero group) — (first nonzero group)

2. First nonzero group contains *one* significant digit; hence, use *A-left* or R_1. Verify that slide rule reading is "248."
3. There is *one* zero group; therefore, answer will have *one* zero immediately after the decimal point. Result is ***.0248.***

Example 2: $\sqrt{.0000309} = ?$

1. Group in pairs from left to right:

$\sqrt{.00\ 00\ 30\ 90}$

(zero groups) — (first nonzero group)

2. The first nonzero group contains *two* significant digits, hence use *A-right* or R_2. Verify that slide rule reading is "556."
3. There are *two* zero groups; therefore, answer will have *two* zeros immediately following the decimal point. Result is ***.00556.***

Example 3: $\sqrt{0.426} = ?$

1. Group in pairs from left to right:

$\sqrt{0.42\ 60}$

(first nonzero group)

2. First nonzero group contains *two* significant digits; hence, use *A-right* or R_2. Verify that slide rule reading is "653."
3. There are no zero groups; therefore, answer is ***0.653.***

Verify the following:

1. $\sqrt{.00475} = .0689$
2. $\sqrt{0.240} = 0.490$
3. $\sqrt{.0001425} = .01194$
4. $\sqrt{.0322} = 0.1794$
5. $\sqrt{.000061} = .00781$
6. $\sqrt{.00000845} = .00291$
7. $\sqrt[4]{.0050} = 0.266$
8. $\sqrt[4]{.0000008} = .0299$

9.7 Powers of 10 method

This method involves writing the original number in the form $M \times 10^n$, where M is a number between 1 and 100, and n is an *even* integer (positive or negative). The following examples illustrate:

Example 1: $\sqrt{430{,}000} = ?$

Rewrite as follows:

$$\sqrt{430{,}000} = \sqrt{43 \times 10^4} = \sqrt{43} \times 10^2 = \boldsymbol{6.56 \times 10^2}$$

Example 2: $\sqrt{.0000032} = ?$

Rewrite this:

$$\sqrt{.0000032} = \sqrt{3.2 \times 10^{-6}} = \sqrt{3.2} \times 10^{-3} = \boldsymbol{1.789 \times 10^{-3}}$$

Example 3: $\sqrt{2.25 \times 10^{-13}} = ?$

Rewrite:

$$\sqrt{2.25 \times 10^{-13}} = \sqrt{22.5 \times 10^{-14}} = \sqrt{22.5} \times 10^{-7} = \boldsymbol{4.74 \times 10^{-7}}$$

Example 4: $\sqrt{673 \times 10^7} = ?$

Rewrite this:

$$\sqrt{673 \times 10^7} = \sqrt{67.3 \times 10^8} = \sqrt{67.3} \times 10^4 = \boldsymbol{8.20 \times 10^4}$$

Again, you are reminded that the power of 10 must be *even;* that is, it must be *divisible by 2.*

Verify the following:

1. $\sqrt{6{,}100{,}000} = 2.47 \times 10^3$

2. $\sqrt{.00000055} = 7.42 \times 10^{-4}$

3. $\sqrt{2.75 \times 10^8} = 1.658 \times 10^4$

4. $\sqrt{375 \times 10^{-9}} = 6.12 \times 10^{-4}$

5. $\sqrt{14.2 \times 10^7} = 1.192 \times 10^4$

6. $\sqrt{10^{-5}} = 3.16 \times 10^{-3}$

Exercise 9-2

1. $\sqrt{48.6} =$

2. $\sqrt{5.43} =$

3. $\sqrt{12.65} =$

4. $\sqrt{1.06} =$

5. $\sqrt{59.8} =$

6. $\sqrt{10.8} =$

7. $\sqrt{4.78} =$

8. $\sqrt{364} =$

9. $\sqrt{3640} =$

10. $\sqrt{7460} =$

11. $\sqrt{992} =$

12. $\sqrt{19.65} =$

13. $\sqrt{\pi} =$

14. $\sqrt{27{,}400} =$

15. $\sqrt[4]{7200} =$

16. $\sqrt{31.4} =$

17. $\sqrt{0.764} =$

18. $\sqrt{0.415} =$

19. $\sqrt{.0423} =$

20. $\sqrt{.00177} =$

21. $\sqrt{.000717} =$

22. $\sqrt{656{,}000} =$

23. $\sqrt{.0000342} =$

24. $\sqrt[4]{.00643} =$

25. $\sqrt[4]{68{,}000} =$

26. $\sqrt{.000917} =$

27. $\sqrt{4.36 \times 10^6} =$

28. $\sqrt{2.43 \times 10^{-4}} =$

29. $\sqrt{5.64 \times 10^{-5}} =$

30. $\sqrt{27.4 \times 10^{-9}} =$

31. $\sqrt{2{,}060{,}000} =$

32. $\sqrt{.00060} =$

33. $\sqrt[4]{0.861} =$

34. $\sqrt{.0070} =$

35. $\sqrt{3.14 \times 10^{-3}} =$

36. $\sqrt{.0001925} =$

37. $\sqrt{37{,}000{,}000} =$

38. $\sqrt{2.17 \times 10^{-7}} =$

39. $\sqrt[4]{283} =$

40. $\sqrt{.000211} =$

41. $\sqrt{2.72 \times 10^6} =$

42. $\sqrt{2.18 \times 10^{-6}} =$

43. $\sqrt{8.45} =$

44. $\sqrt[4]{.00529} =$

45. $\sqrt{.000816} =$

46. $\sqrt{2.07 \times 10^{11}} =$

47. $\sqrt{5.64 \times 10^7} =$

48. $\sqrt{28{,}700} =$

49. $\sqrt[4]{0.385} =$

50. $\sqrt{5.82 \times 10^5} =$

9.8 Formula types

Example 1: $$\sqrt{3 + \frac{112}{16.2}} = ?$$

1. Verify that $\frac{112}{16.2} = 6.91$.

2. Expression may now be evaluated:

$$\sqrt{3 + 6.91} = \sqrt{9.91} = \boldsymbol{3.15}.$$

As illustrated in the next examples, it is sometimes better to change the form of an expression before evaluating.

Example 2: $\pi[(4.15)^2 - (4.12)^2] = ?$

To find the difference in squares which are of about the same magnitude, more accuracy is obtained by factoring first:

$$\pi[(4.15)^2 - (4.12)^2] = \pi(4.15 + 4.12)(4.15 - 4.12) = \pi(8.27)(.03)$$

Verify that the result is ***0.780.***

Example 3: $\dfrac{\sqrt{14} - \sqrt{13}}{2} = ?$

Here, we must find a small difference in square roots, and more accuracy is obtained if we first multiply and divide by $(\sqrt{14} + \sqrt{13})$:

$$\frac{\sqrt{14} - \sqrt{13}}{2} = \frac{(\sqrt{14} - \sqrt{13})(\sqrt{14} + \sqrt{13})}{2(\sqrt{14} + \sqrt{13})} = \frac{14 - 13}{2(\sqrt{14} + \sqrt{13})} = \frac{1}{2(\sqrt{14} + \sqrt{13})}$$

Verify that:

$$\frac{1}{2(\sqrt{14} + \sqrt{13})} = \frac{1}{2(3.74 + 3.61)} = \frac{1}{2(7.35)} = \boldsymbol{.0680.}$$

Exercise 9-3

1. $\sqrt{2 + \dfrac{16.3}{4.2}} =$
2. $\sqrt{28 - \dfrac{46}{3.5}} =$
3. $\pi[(3.22)^2 - (3.19)^2] =$
4. $\pi[(236)^2 - (234)^2] =$
5. $\sqrt{\dfrac{1}{22} + \dfrac{1}{44}} =$
6. $\sqrt{\dfrac{1}{2.5} + \dfrac{1}{3.5} + \dfrac{1}{5.5}} =$
7. $\dfrac{\sqrt{230} - \sqrt{228}}{2} =$

8. $\dfrac{\sqrt{31.2} - \sqrt{30.9}}{.034} =$

9. $\sqrt{\dfrac{36.7}{2.33} + \dfrac{165}{7.26}} =$

10. $\sqrt{\dfrac{53.7}{72.2} - (0.72)^2} =$

11. $\dfrac{2.65}{1 - (0.985)^2} =$

12. $\dfrac{1}{\sqrt{118} - \sqrt{116}} =$

13. $\dfrac{32.2}{\sqrt{12.8} - \sqrt{12.5}} =$

14. $\dfrac{31.6 + \sqrt{153}}{12.6} =$

15. $\dfrac{.0866 - \sqrt{.00324}}{2.77} =$

16. $\dfrac{13 + \sqrt{(13)^2 + (4 \times 12 \times 21)}}{2 \times 12} =$

17. $\dfrac{-6.82 + \sqrt{(6.82)^2 - (4 \times 2.33 \times 1.52)}}{2 \times 2.33} =$

18. $\sqrt{(2.6)^2 + (4.7)^2 + (5.2)^2} =$

19. $\dfrac{\sqrt{17.8} - \sqrt{6.22}}{0.27 \times 3.32} =$

20. $\dfrac{\sqrt{.0535} + \sqrt{0.136}}{.0064 \times 5.22} =$

In the following formulas, substitute as indicated and evaluate:

21. $T = \sqrt{a + \dfrac{1}{a}}$

a. $a = 1.65$
b. $a = 2.23$
c. $a = 0.644$

22. $t = \dfrac{KA}{A_0}(h_1^{1/2} - h_2^{1/2})$

a. $K = 12.6$, $A = 13.6$, $A_0 = 0.74$, $h_1 = 7$, $h_2 = 5$
b. $K = 22.8$, $A = 7.45$, $A_0 = 0.330$, $h_1 = 14$, $h_2 = 6$
c. $K = 7.25$, $A = 25.6$, $A_0 = 2.74$, $h_1 = 131$, $h_2 = 29$

23. $R = \dfrac{k + \sqrt[4]{1 + r^2}}{16.1}$

a. $k = 18.2$, $r = 1.77$
b. $k = 5.66$, $r = 0.785$
c. $k = 34.7$, $r = 6.55$

24. $h = \dfrac{\sqrt{p_0} + \sqrt{p_1}}{p_0 p_1}$

a. $p_0 = 7.65$, $p_1 = 1.66$
b. $p_0 = 5.20$, $p_1 = 28.6$
c. $p_0 = .0440$, $p_1 = 0.655$

25. $d = \sqrt{a^2 + b^2 + c^2}$

a. $a = 2.2$, $b = 3.6$, $c = 4.7$
b. $a = 13.7$, $b = 2.27$, $c = 8.66$

9.9 The quadratic formula

The two roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve for x: $2.84x^2 + 5.35x - 3.75 = 0$.

1. In this case, $a = 2.84$, $b = 5.35$, and $c = -3.75$. Substituting these values, the formula becomes:

$$x = \frac{-5.35 \pm \sqrt{(5.35)^2 - 4(2.84)(-3.75)}}{2(2.84)}$$

2. Verify that $x = \dfrac{-5.35 \pm \sqrt{28.6 + 42.5}}{5.68} = \dfrac{-5.35 \pm 8.44}{5.68}$

3. Finally verify that the two roots are ***0.544*** and ***−2.43.***

Exercise 9-4

Solve for x:

1. $2x^2 + 3x - 1 = 0$

2. $5x^2 - 17x + 6 = 0$

3. $7x^2 - 13x - 5 = 0$

4. $16x^2 + x - 0.30 = 0$

5. $x^2 - 11.7x + 4.55 = 0$

6. $2.65x^2 + 6.82x + 1.70 = 0$

7. $31.2x^2 + 23.7x + 2.18 = 0$

8. $0.810x^2 + 17.6x - 31.2 = 0$

Chapter 10

CUBES AND CUBE ROOTS (K AND $\sqrt[3]{\ }$ SCALES)

10.1 Description of the K scale*

The K scale consists of three complete scales, one next to the other, each similar to the D scale. Thus, each section represents a one-third-length D scale. We shall refer to these sections as "K-left," "K-middle," and "K-right." Because of the logarithmic nature of the scales, it follows that the one-third-length K scale leads to a cube-cube root relationship with the D scale. To verify this, suppose you now take your slide rule and move the hairline over "2" on the D scale. Note that this positions the hairline over "8" on K-left. Now move the hairline over "3" on D; note that this puts the hairline over "27" on K-middle. Finally, move the hairline over "5" on D and observe that the hairline is over "125" on K-right. In each case, the reading on K is the *cube* of the corresponding reading on D.

Summarizing:

> If the hairline is set over a number N on the **D** scale, then N^3 will be under the hairline on the **K** scale.
>
> Conversely, if the hairline is set over a number N on the *proper section* of the **K** scale, then $\sqrt[3]{N}$ will be under the hairline on the **D** scale.

In making settings and readings on the K scale, carefully note how the primary intervals are subdivided. Clearly, the use of this scale involves considerable loss in accuracy; over much of the scale you will find yourself straining to approximate the third digit.

*If your slide rule does not have a K scale, but does have a scale labeled $\sqrt[3]{\ }$, then refer to section 10.7 of this chapter.

10.2 Cubing a number using the K scale

Of course, cubing a number may be treated as successive multiplication using the C, CI, and D scales, thus retaining the accuracy of these scales. However, as indicated in the previous section, cubes may be read directly from D to K.

Example 1: $(3.45)^3 = ?$

1. Move HL over 345 on D.
2. Under HL read "410" on K-middle. Answer is ***41.0.***

Example 2: $(645)^3 = ?$

Write this: $(6.45 \times 10^2)^3 = (6.45)^3 \times 10^6$.
1. Move HL over 645 on D.
2. Under HL read "268" on K-right.

Answer is 268×10^6 or $\boldsymbol{2.68 \times 10^8}$.

Example 3: $(.00172)^3 = ?$

Write this: $(1.72 \times 10^{-3})^3 = (1.72)^3 \times 10^{-9}$.
1. Move HL over 172 on D.
2. Under HL read "509" on K-left. Answer is $\boldsymbol{5.09 \times 10^{-9}}$.

Exercise 10-1

1. $(1.6)^3 =$

2. $(2.4)^3 =$

3. $(6.5)^3 =$

4. $(7.2)^3 =$

5. $(1.23)^3 =$

6. $(5.82)^3 =$

7. $(3.24)^3 =$

8. $(2.76)^3 =$

9. $(8.04)^3 =$

10. $(0.764)^3 =$

11. $(0.512)^3 =$

12. $(10.65)^3 =$

13. $(12.7)^3 =$

14. $(8180)^3 =$

15. $(29{,}600)^3 =$

16. $(364)^3 =$

17. $(135)^3 =$

18. $(0.907)^3 =$

19. $(1265)^3 =$

20. $(4.65 \times 10^4)^3 =$

21. $(0.787)^3 =$

22. $(0.215)^3 =$

23. $(3.96 \times 10^{-3})^3 =$

24. $(0.404)^3 =$

25. $(.0643)^3 =$

26. $(.00284)^3 =$

27. $(.00847)^3 =$

28. $(17{,}550)^3 =$

29. $(68.2)^3 =$

30. $(.000545)^3 =$

10.3 Finding cube root of a number between 1 and 1000

As stated before, if the hairline is set over a number on the proper section of the K scale, the cube root of that number will be under the hairline on the D scale. You can easily verify the following rule for selecting the proper section of the K scale:

1. For cube roots of numbers between *1 and 10,* use ***K-left.***
2. For numbers between *10 and 100,* use ***K-middle.***
3. For numbers between *100 and 1000,* use ***K-right.***

Example 1: $\sqrt[3]{3.60} = ?$

1. The number is between 1 and 10; hence, move HL over 360 on K-left.
2. Under HL read "1532" on D. Answer is ***1.532.***

Example 2: $\sqrt[3]{45.4} = ?$

1. The number is between 10 and 100; hence, move HL over 454 on K-middle.
2. Under HL read "357" on D. Answer is ***3.57.***

Example 3: $\sqrt[3]{720} = ?$

1. The number is between 100 and 1000; hence, move HL over 720 on K-right.
2. Under HL read "896" on D. Answer is ***8.96.***

Verify the following:

1. $\sqrt[3]{7.4} = 1.950$ **2.** $\sqrt[3]{32} = 3.17$ **3.** $\sqrt[3]{615} = 8.50$

4. $\sqrt[3]{12.1} = 2.30$	**6.** $\sqrt[3]{10} = 2.15$	**8.** $\sqrt[3]{75} = 4.22$
5. $\sqrt[3]{4.25} = 1.620$	**7.** $\sqrt[3]{100} = 4.64$	**9.** $\sqrt[3]{134} = 5.12$

10.4 General procedure for cube roots

In order to determine which section of the K scale to use for any number, the digits may be grouped in a manner similar to that used for square roots. However, instead of marking the digits off in pairs, they are marked off in groups of *three*. The following rule then applies:

> *For numbers greater than 1:*
> 1. If the leftmost group contains *one* digit, use ***K-left;*** if it contains *two* digits, use ***K-middle;*** if it contains *three* digits, use ***K-right.***
> 2. The cube root will contain the same number of *digits to the left* of the decimal point as there are *groups to the left* of the decimal point in the original number.
>
> *For numbers less than 1:*
> 1. If the first nonzero group contains *one* significant digit, use ***K-left;*** if it contains *two* significant digits, use ***K-middle;*** if it contains *three* significant digits, use ***K-right.***
> 2. For *each zero group* in the original number, the cube root will contain *one zero* between the decimal point and the first significant figure.

Example 1: $\sqrt[3]{43{,}500} = ?$

1. Grouping the digits: $\sqrt[3]{\underbrace{43}\ \underbrace{500}}$
2. Leftmost group contains *two* digits; hence, use *K-middle.* Verify that the slide rule reading is "352."
3. There are *two* groups; hence, answer contains *two* digits to the left of the decimal point. Result is ***35.2.***

Example 2: $\sqrt[3]{2{,}140{,}000} = ?$

1. Grouping the digits: $\sqrt[3]{\underbrace{2}\ \underbrace{140}\ \underbrace{000}}$.
2. Leftmost group contains *one* digit; hence, use *K-left.* Verify that slide rule reading is "1288."
3. There are *three* groups; therefore, answer is ***128.8.***

Example 3: $\sqrt[3]{.000720} = ?$

1. Grouping the digits: $\sqrt[3]{.000\ 720}$
2. First nonzero group contains *three* significant digits; hence, use *K-right*. Slide rule reading is "896."
3. There is *one zero group;* therefore, there is *one zero* between the decimal point and the first significant digit. Answer is ***.0896.***

Example 4: $\sqrt[3]{.00564} = ?$

1. Grouping the digits: $\sqrt[3]{.005\ 640}$
2. First nonzero group contains *one* significant digit; hence, use *K-left*. Slide rule reading is "178."
3. There is *no* zero group; therefore, answer is ***0.178.***

Verify the following:

1. $\sqrt[3]{6400} = 18.57$
2. $\sqrt[3]{.0215} = 0.278$
3. $\sqrt[3]{365{,}000} = 71.5$
4. $\sqrt[3]{0.820} = 0.936$
5. $\sqrt[3]{.000100} = .0464$
6. $\sqrt[3]{.0000048} = .01687$
7. $\sqrt[3]{62{,}000{,}000} = 396$
8. $\sqrt[3]{.00000075} = .00909$
9. $\sqrt[3]{4{,}250{,}000} = 162$
10. $\sqrt[3]{.0325} = 0.319$

10.5 Powers of 10 method

Using this method, the original number is written in the form $M \times 10^n$, where M is a number between 1 and 1000, and n is a positive or negative integer which is *divisible by three.* The following examples illustrate:

Example 1: $\sqrt[3]{56{,}100{,}000} = ?$

Rewrite: $\sqrt[3]{56.1 \times 10^6} = \sqrt[3]{56.1} \times 10^2 = 3.83 \times 10^2 =$ ***383.***

Example 2: $\sqrt[3]{.000000750} = ?$

Rewrite: $\sqrt[3]{750 \times 10^{-9}} = \sqrt[3]{750} \times 10^{-3} = 9.09 \times 10^{-3} =$ ***.00909.***

Example 3: $\sqrt[3]{4.66 \times 10^{10}} = ?$

Rewrite: $\sqrt[3]{46.6 \times 10^9} = \sqrt[3]{46.6} \times 10^3 = 3.60 \times 10^3 =$ ***3600.***

Example 4: $\sqrt[3]{2.34 \times 10^{-10}} = ?$

Rewrite: $\sqrt[3]{234 \times 10^{-12}} = \sqrt[3]{234} \times 10^{-4} = 6.16 \times 10^{-4} =$ ***.000616.***

Again, you are reminded that the power of 10 must be *divisible by three.*

Verify the following:

1. $\sqrt[3]{1.84 \times 10^{7}} = 264$
2. $\sqrt[3]{6.47 \times 10^{-13}} = 8.65 \times 10^{-5}$
3. $\sqrt[3]{10^{8}} = 464$
4. $\sqrt[3]{10^{-11}} = .000216$
5. $\sqrt[3]{482 \times 10^{16}} = 1.69 \times 10^{6}$
6. $\sqrt[3]{42 \times 10^{-7}} = .01613$
7. $\sqrt[3]{36.5 \times 10^{8}} = 1540$
8. $\sqrt[3]{7.25 \times 10^{11}} = 8980$

Exercise 10-2

1. $\sqrt[3]{7.43} =$
2. $\sqrt[3]{8.22} =$
3. $\sqrt[3]{10.9} =$
4. $\sqrt[3]{96.2} =$
5. $\sqrt[3]{17.65} =$
6. $\sqrt[3]{124} =$
7. $\sqrt[3]{742} =$
8. $\sqrt[3]{2.94} =$
9. $\sqrt[3]{294} =$
10. $\sqrt[3]{57.4} =$
11. $\sqrt[3]{1.11} =$
12. $\sqrt[3]{11.1} =$
13. $\sqrt[3]{111} =$
14. $\sqrt[3]{.0624} =$
15. $\sqrt[3]{0.526} =$
16. $\sqrt[3]{0.912} =$
17. $\sqrt[3]{.0207} =$
18. $\sqrt[3]{.0611} =$
19. $\sqrt[3]{.0829} =$
20. $\sqrt[3]{.00299} =$
21. $\sqrt[3]{.00748} =$
22. $\sqrt[3]{3920} =$
23. $\sqrt[3]{4680} =$
24. $\sqrt[3]{9600} =$
25. $\sqrt[3]{2000} =$
26. $\sqrt[3]{7640} =$
27. $\sqrt[3]{10,700} =$
28. $\sqrt[3]{23,600} =$
29. $\sqrt[3]{64,300} =$
30. $\sqrt[3]{575,000} =$

31. $\sqrt[3]{207{,}000} =$

32. $\sqrt[3]{87{,}500} =$

33. $\sqrt[3]{.00726} =$

34. $\sqrt[3]{.000276} =$

35. $\sqrt[3]{.0000427} =$

36. $\sqrt[3]{23{,}600{,}000} =$

37. $\sqrt[3]{.00000717} =$

38. $\sqrt[3]{180{,}000{,}000} =$

39. $\sqrt[3]{.0000007} =$

40. $\sqrt[3]{76.1 \times 10^{6}} =$

41. $\sqrt[3]{2.76 \times 10^{-6}} =$

42. $\sqrt[3]{4.68 \times 10^{-5}} =$

43. $\sqrt[3]{5.64 \times 10^{-2}} =$

44. $\sqrt[3]{11.6 \times 10^{-4}} =$

45. $\sqrt[3]{1.95 \times 10^{5}} =$

46. $\sqrt[3]{12.65 \times 10^{8}} =$

47. $\sqrt[3]{0.416 \times 10^{10}} =$

48. $\sqrt[3]{4.26 \times 10^{-10}} =$

49. $\sqrt[3]{7.61 \times 10^{14}} =$

50. $\sqrt[3]{3.37 \times 10^{-11}} =$

10.6 Finding $(N)^{3/2}$ and $(N)^{2/3}$

The general problem of raising a number to any power is discussed later in connection with the log-log scales. However, certain fractional powers may be read directly using the A and K scales.

Example 1: $(56.2)^{3/2} = (\sqrt{56.2})^3 = ?$

1. Move HL over 562 on A-right. Note that $\sqrt{56.2}$ is now under HL on D, and $(\sqrt{56.2})^3$ is under HL on K.
2. Under HL read "421" on K. Answer is ***421***.

If you do not have the A scale, you may first evaluate $(56.2)^3$, then find the square root of this on R.

Example 2: $(7.2)^{2/3} = (\sqrt[3]{7.2})^2 = ?$

1. Move HL over 72 on K-left. Observe that $\sqrt[3]{7.2}$ is now under HL on D, and $(\sqrt[3]{7.2})^2$ is under HL on A.
2. Under HL read "373" on A. Answer is ***3.73***.

If no A scale is present, you may first find $\sqrt[3]{7.2}$, then multiply this by itself.

These examples illustrate the following relationship:

1. If HL is set over a number N on the **A** scale, then $(N)^{3/2}$ will be under HL on the **K** scale.
2. If HL is set over a number N on the **K** scale, then $(N)^{2/3}$ will be under HL on the **A** scale.

Example 3: $(2.75 \times 10^{13})^{2/3} = ?$

Write this: $(27.5 \times 10^{12})^{2/3} = (27.5)^{2/3} \times 10^{8}$.

1. Move HL over 275 on K-middle.
2. Under HL read "910" on A. Answer is $\boldsymbol{9.10 \times 10^{8}}$.

Exercise 10-3

1. $(4.5)^{3/2} =$	**11.** $(342)^{2/3} =$
2. $(37)^{3/2} =$	**12.** $(1.65)^{2/3} =$
3. $(16.3)^{3/2} =$	**13.** $(2340)^{2/3} =$
4. $(0.814)^{3/2} =$	**14.** $(.0621)^{2/3} =$
5. $(.0614)^{3/2} =$	**15.** $(.000175)^{2/3} =$
6. $(.00743)^{3/2} =$	**16.** $(0.445)^{3/2} =$
7. $(374)^{3/2} =$	**17.** $(3.22 \times 10^{-9})^{3/2} =$
8. $(.000514)^{3/2} =$	**18.** $(7.45 \times 10^{8})^{2/3} =$
9. $(2460)^{3/2} =$	**19.** $(63 \times 10^{-7})^{2/3} =$
10. $(46.2)^{2/3} =$	**20.** $(5.75 \times 10^{9})^{3/2} =$

10.7 The triple-length scale ($\sqrt[3]{\ }$)

Some Pickett models feature a $\sqrt[3]{\ }$ scale consisting of three full-length sections; the first covers the same range as the first third of the regular D scale (left index to "2154"), the second covers the range of the middle third of D ("2154 to "464"), and the third corresponds to the final third of the D scale ("464" to right index). We shall refer to these sections as $(\sqrt[3]{\ })_1$, $(\sqrt[3]{\ })_2$, and $(\sqrt[3]{\ })_3$ respectively. Clearly, the three sections taken together represent one triple-length D scale.

The $\sqrt[3]{\ }$ scale is related to the D scale as follows:

> If the hairline is set over a number N on the $\sqrt[3]{\ }$ scale, then N^3 will be under the hairline on the **D** scale.
>
> Conversely, if the hairline is set over a number N on the **D** scale, then $\sqrt[3]{N}$ will be under the hairline on the *proper section* of the $\sqrt[3]{\ }$ scale.

Example 1: $(3.47)^3 = ?$

1. Move HL over 347 on $(\sqrt[3]{\ })_2$.
2. Under HL read "418" on D. Answer is ***41.8.***

Example 2: $\sqrt[3]{6450} = ?$

To determine which section of the $\sqrt[3]{\ }$ scale applies, you should refer to sections 10.3 and 10.4 of this chapter—substituting $(\sqrt[3]{\ })_1$ for K-left, $(\sqrt[3]{\ })_2$ for K-middle, and $(\sqrt[3]{\ })_3$ for K-right.

1. Grouping the digits: $\sqrt[3]{\underset{\smile}{6}\ \underset{\smile}{450}}$.
2. Leftmost group contains *1 digit;* hence, answer is on $(\sqrt[3]{\ })_1$.
3. Move HL over 645 on D. Under HL read "1861" on $(\sqrt[3]{\ })_1$.
4. There are *two groups;* hence, there are *two digits* to the left of the decimal point in the answer. Result is ***18.61.***

Example 3: $\sqrt[3]{.000224} = ?$

1. Grouping the digits: $\sqrt[3]{.\underset{\smile}{000}\ \underset{\smile}{224}}$.
2. The first nonzero group contains *three* significant digits, hence answer is on $(\sqrt[3]{\ })_3$.
3. Move HL over 224 on D. Under HL read "607" on $(\sqrt[3]{\ })_3$.
4. There is *one zero group;* therefore, answer has *one zero* immediately following the decimal point. Result is ***.0607.***

Exercises 10-1 and 10-2 may be used for more drill with these scales.

Chapter 11

COMBINED OPERATIONS WITH SQUARES

11.1 The A and B scales as operational scales

We have seen how the A scale may be used with the D scale to find squares and square roots. However, inasmuch as the A and B scales are identical (one fixed and one movable), they may also be used as operational scales in the same manner as the C and D; that is, multiplication, division, and combinations of these operations may be carried out entirely on the A and B scales.

Example 1: $23.6 \times 1.85 = ?$ (Use A and B scales)

1. Set left index of B opposite 236 on A-left. (Note that middle index of B is also opposite 236 on A-right).
2. Move HL over 185 on B-left.
3. Under HL read "436" on A-left. Answer is ***43.6.***

Observe that hairline could have been moved over 185 on B-right, and the result read on A-right.

Example 2: $\dfrac{126 \times 0.83}{4.6} = ?$ (Use A and B scales)

1. Move HL over 126 on A-right. This is more centrally located on the rule than 126 on A-left.
2. Slide 46 on B-left under HL. (We could just as well choose B-right).
3. Move HL over 83 on B-left.
4. Under HL read "227" on A-right. Answer is ***22.7.***

Example 3: $\dfrac{56}{3.7 \times 2.9} = ?$ (Use A and B scales)

1. Move HL over 56 on A-left.
2. Slide 37 on B-left under HL. Before we can divide again, it is necessary to move HL over B index.
3. Move HL over middle B index. Slide 29 on B-right under HL.
4. Opposite right index of B, read "522" on A-right. Answer is ***5.22.***

Example 4: $\dfrac{1.2}{2.7} = \dfrac{x}{4.8} = \dfrac{7.6}{y}$; $x = ?, y = ?$

1. Move HL over 12 on A-left.
2. Slide 27 on B-left under HL.
3. Move HL over 48 on B-left (or right). Under HL read "213" on A-left (or right).
4. Move HL over 76 on A-left. Under HL read "171" on B-right.

Answers: $x =$ ***2.13***, $y =$ ***17.1.***

Clearly, the A-B scales are especially convenient for proportions; however, the disadvantage is that they cannot be read as accurately as the C and D scales. Also, most rules do not have a reciprocal B scale which reduces efficiency in combined operations. However, we shall see that, for combined operations involving the square or square root, the A-B scales can be used effectively in conjunction with the C and D scales.

Verify the following (use A-B scales):

1. $23 \times 4.6 = 106$
2. $\dfrac{56}{2.1} = 26.7$
3. $\dfrac{3.2 \times 7.1}{2.8} = 8.11$
4. $1.7 \times 3.8 \times 2.6 = 16.8$
5. $\dfrac{740}{26 \times 15} = 1.90$
6. $\dfrac{3.7}{7.1} = \dfrac{X}{2.6} = \dfrac{4.5}{Y}$ ($X = 1.355$; $Y = 8.63$)

11.2 Simple operations with squares

It is clear that any combined operation involving squared quantities may be evaluated in the conventional manner, the squares being treated simply as multiplication. In this section, however, we wish to show how the A-B or R scales may be used when the square is involved.

Example 1: $2.6 \times (5.4)^2 = ?$

Procedure with A-B scales:

1. Set right index of C opposite 54 on D. Note that $(5.4)^2$ is now on A opposite right index of B; hence, remaining multiplication by 2.6 may be carried out on the A-B scales.

2. Move HL over 26 on B.
3. Under HL read "758" on A. Answer is ***75.8.***

Alternatively, you may first set B index opposite 26 on A, then move hairline over 54 on C and read the result on A. Also, you could first move the hairline over 26 on A, then slide 54 on CI under hairline with the result appearing opposite B index on A.

Procedure with the R scale:

1. Move HL over 54 on R_2. HL is now over $(5.4)^2$ on D, and we may continue on the C-D scales.
2. Slide 26 on CI under HL. This multiplies by 2.6 on C-D.
3. Opposite right index of C read "758" on D. Answer is ***75.8.***

Example 2: $\dfrac{(6.75)^2}{3.5} = ?$

Procedure with A-B scales:

1. Move HL over 675 on D. HL is now over $(6.75)^2$ on A; hence, division by 3.5 may be carried out on the A-B scales.
2. Slide 35 on B-right under HL. (Observe that 35 on B-left could also be moved under HL; however, choosing B-right leaves slide in better position).
3. Opposite B index (either left or middle index) read "130" on A. Answer is ***13.0.***

Procedure with R scale:

1. Move HL over 675 on R_2. HL is now over $(6.75)^2$ on D; hence, division by 3.5 may be carried out on the C-D scales.
2. Slide 35 on C under HL.
3. Opposite left index of C, read "1302" on D. Answer is ***13.02.***

Example 3: $\dfrac{64.5}{(2.14)^2} = ?$

Procedure with A-B scales:

1. Move HL over 645 on A-left.
2. Slide 214 on C under HL. This positions $(2.14)^2$ on B under HL; thus, we have divided 64.5 by $(2.14)^2$ on the A-B scales.
3. Opposite left (or middle) index of B, read "141" on A.

Answer is ***14.1.***

Procedure with R scale:
The R scale is not convenient when the squared quantity appears in the denominator. We may simply treat the square as a product, and evaluate on C-D in the usual manner.

Verify that: $\dfrac{64.5}{(2.14)^2} = \dfrac{64.5}{2.14 \times 2.14} = \mathbf{14.08.}$

Observe in the foregoing examples that numbers to be squared are set on C-D, unsquared numbers are set on A-B, and the actual operation takes place on the A-B scales. If the R scale is used, a number to be squared is set on R (if it occurs in the numerator), unsquared numbers are set on C-D, and the actual operation takes place on the C-D scales.

Verify the following:

1. $3.4 \times (2.7)^2 = 24.8$
2. $\frac{(3.8)^2}{4.6} = 3.14$
3. $\frac{(27.2)^2}{64.5} = 11.5$
4. $1.65 \times (4.77)^2 = 37.5$
5. $\frac{5.7}{(1.8)^2} = 1.76$
6. $\frac{473}{(9.25)^2} = 5.52$

11.3 Area of a circle

The area of a circle is given by either of the two formulas:

$$A = \pi r^2 \quad \text{(where } r = \text{radius)}$$
$$A = \frac{\pi}{4} d^2 \quad \text{(where } d = \text{diameter)}$$

Both π and $\pi/4$ are usually marked for you on the A and B scales. The factor $\pi/4$ is approximately equal to 0.785 and, on most rules, you will find a scribed mark at this location on A-right and B-right.

Example 1: Find areas of circles with diameters equal to 2.45 in., 4.22 in., and 6.40 in., respectively.

1. Move HL over scribed mark at $\pi/4$ on A-right.
2. Slide right index of B under HL. You are now set up to multiply this factor by d^2. If the HL is moved over d on the C scale, d^2 will be under the HL on B and the desired area will be under the HL on A.
3. Move HL over 245 on C. Under HL read "471" on A.
4. Move HL over 422 on C. Under HL read "140" on A.
5. Move HL over 640 on C. Under HL read "321" on A.

Answers are: ***4.71*, *14.0*,** and ***32.1*** sq. in., respectively.

Example 2: Find areas of circles with radii equal to 1.45 cm., 2.24 cm., and 4.06 cm., respectively.

Procedure with A-B scales:

1. Move HL over scribed mark at π on A-left.
2. Slide left index of B under HL. You are now set up to multiply this factor by r^2. If the HL is moved over r on the C scale, r^2 will be under HL on B and the desired product will be under HL on A.

3. Move HL over 145 on C. Under HL read "660" on A.
4. Move HL over 224 on C. Under HL read "158" on A.
5. Move HL over 406 on C. Under HL read "518" on A.

Answers are: ***6.60, 15.8,*** and ***51.8*** sq. cm., respectively.

Procedure with R scale:
If the HL is set over r on the R scale, then r^2 will be under HL on D, and πr^2 will be under HL on DF. Therefore, if r is set on R, the area is read directly on DF.

1. Move HL over 145 on R_1. Under HL read "660" on DF.
2. Move HL over 224 on R_1. Under HL read "1576" on DF.
3. Move HL over 406 on R_2. Under HL read "518" on DF.

Answers are: ***6.60, 15.76,*** and ***51.8*** sq. cm. respectively.

This last example exhibits a convenient property of the R scale:

> *To find the area of a circle:*
> 1. Move HL over *radius* on R.
> 2. Under HL read *area* on DF.

Exercise 11-1

1. $3.24 \times (4.25)^2 =$

2. $1.68 \times (2.75)^2 =$ "127"

3. $21.4 \times (3.75)^2 =$

4. $12.6 \times (5.06)^2 =$ "323"

5. $4.63 \times (16.2)^2 =$

6. $64.3 \times (0.463)^2 =$ "138"

7. $.0723 \times (21.6)^2 =$

8. $\frac{(4.67)^2}{3.26} =$ "670"

9. $\frac{(26.7)^2}{19.2} =$

10. $\frac{(36.5)^2}{127} =$ "105"

11. $\frac{(11.45)^2}{16.2} =$

12. $\frac{4.65}{(1.67)^2} =$ "167"

13. $\frac{29.6}{(2.37)^2} =$

14. $\frac{347}{(14.2)^2} =$ "172"

15. $\frac{1275}{(21.6)^2} =$

16. $\frac{45.6}{(0.843)^2} =$ "642"

17. $\frac{(264)^2}{5.65} =$

18. $\dfrac{(515)^2}{28.5} =$ "930"

19. $230 \times (0.86)^2 =$

20. $0.76 \times (22.5)^2 =$ "385"

21. $\dfrac{7.44}{(.00362)^2} =$

22. $\dfrac{(1560)^2 \times 10^{-4}}{42.5} =$ "573"

23. $\dfrac{3.75 \times 10^{-7}}{(560)^2} =$

24. Find areas of circles with following radii:

a. 25.1 in. **b.** 5.82 in. **c.** 0.377 ft. **d.** 53.1 ft. **e.** .0104 cm.

25. Find areas of circles with following diameters:

a. 2.68 in. **b.** 1.77 in. **c.** 4.65 ft. **d.** 7.05 ft. **e.** 14.65 ft.

26. Evaluate $y = 2.84\, x^2$ for $x = 2, 3, 5, 7$, and 9.

27. Evaluate $s = 16.1\, t^2$ for $t = 1.5, 2.5, 3.5$, and 5.5.

28. Evaluate $P = .074\, I^2$ for $I = 2, 5, 12, 25$, and 60.

In the following exercises, use the CI scale in conjunction with A-B:

29. Evaluate $y = 14.7/x^2$ for $x = 3.4, 5.5, 7.5$, and 12.6.

30. Evaluate $y = 375/x^2$ for $x = 11, 15, 19$, and 25.

31. Evaluate $h = 180/r^2$ for $r = 2.2, 4.6, 7.5$, and 11.5.

11.4 Further operations with squares

Example 1: $\dfrac{(7.2)^2}{2.4 \times (3.1)^2} = ?$

Procedure with A-B scales:

1. Move HL over 72 on D. HL is now over $(7.2)^2$ on A.
2. Slide 24 on B-right under HL. This divides by 2.4 on A-B.
3. Move HL over 31 on CI. This positions HL over $1/(3.1)^2$ on B; therefore, we have multiplied by the reciprocal of $(3.1)^2$ on A-B.
4. Under HL read "225" on A. Answer is ***2.25.***

Procedure with R scale:

1. Move HL over 72 on R_2. HL is now over $(7.2)^2$ on D.
2. Slide 31 on C under HL. This divides by 3.1.
3. Move HL over 31 on CI. This divides again by 3.1.
4. Slide 24 on C under HL. This divides by 2.4.
5. Opposite left index of C, read "225" on D. Answer is ***2.25.***

Example 2: $\dfrac{(2.7)^2 \times (4.6)^2}{6.3} = ?$

Procedure with A-B scales:

1. Move HL over 27 on D.
2. Slide 63 on B-left under HL.
3. Move HL over 46 on C.
4. Under HL read "245" on A. Answer is ***24.5.***

Procedure with R scale:

1. Move HL over 27 on R_1.
2. Slide 63 on C under HL.
3. Move HL over 46 on C.
4. Slide 46 on CI under HL.
5. Opposite left index of C, read "245" on D. Answer is ***24.5.***

Example 3: $\dfrac{64}{(3.1 \times 2.2)^2} = \dfrac{64}{(3.1)^2 \times (2.2)^2} = ?$

Procedure with A-B scales:

1. Move HL over 64 on A-left.
2. Slide 31 on C under HL.
3. Move HL over 22 on CI.
4. Under HL read "138" on A. Answer is ***1.38.***

Procedure with R scale:
Since the squares occur in the denominator, the R scale is not convenient. This may be evaluated as a combined operation in the conventional manner, treating the squares as products. Verify that the result is ***1.377.***

Example 4: $\left[\dfrac{2.7 \times 4.1 \times 6.3}{7.2 \times 3.5}\right]^2 = ?$

Procedure with A-B scales:
In this case, all the settings are made on C-D and the result appears on the A scale. Verify that the answer is ***7.65.***

Procedure with R scale:
When several factors are to be squared, you may first evaluate the expression to be squared on the C-D scales. This result may then be multiplied by itself on C-D, or it may be transferred via hairline to the R scale with the answer appearing under the hairline on D. Verify that the result is ***7.65.***

Verify the following:

1. $\dfrac{(7.5)^2}{1.2 \times (3.7)^2} = 3.42$

2. $\dfrac{(15.2 \times 0.72)^2}{36.3} = 3.30$

3. $\dfrac{(26.4)^2}{\pi \times (19.3)^2} = 0.596$

4. $\dfrac{675}{(5.66)^2 \times 7.23} = 2.92$

5. $\dfrac{184}{(6.2 \times 5.6)^2} = 0.153$

6. $\left[\dfrac{6.34 \times 37.5}{1.23 \times 42.2}\right]^2 = 20.9$

Example 5: $\dfrac{(2.3)^3}{1.7} = ?$

Write: $\dfrac{(2.3)^3}{1.7} = \dfrac{(2.3)^2 \times 2.3}{1.7}$

Procedure with A-B scales:

1. Move HL over 23 on D. Operation now continues on A-B:
2. Slide 17 on B under HL.
3. Move HL over 23 on B.
4. Under HL read "715" on A. Answer is ***7.15.***

Procedure with R scale:

1. Move HL over 23 on R_1. Operation now continues on C-D:
2. Slide 17 on C under HL.
3. Move HL over 23 on C.
4. Under HL read "715" on D. Answer is ***7.15.***

If your slide rule has a $\sqrt[3]{\ }$ scale, you may obtain $(2.3)^3$ directly on D with a single setting; then simply divide by 1.7.

Example 6: $(2.85)^4 = ?$

Write: $(2.85)^4 = (2.85)^2 \times (2.85)^2$.

Procedure with A-B scales:

1. Set left index of C opposite 285 on D.
2. Move HL over 285 on C.
3. Under HL read "660" on A. Answer is ***66.0.***

Procedure with R scale:

1. Move HL over 285 on R_1. Observe that HL is over 812 on D.
2. Slide 812 on CI under HL.
3. Opposite left index of C read "660" on D.

Answer is ***66.0.***

Note that fourth powers may be read *directly* on slide rules that have both the A scale and the double-length scale. Thus, on such rules, if the hairline is set over a number N on R (Sq or $\sqrt{\ }$), N^4 will be under the hairline on A.

Example 7: $(0.82)^5 = ?$

Integral powers such as this arise frequently in probability calculations.

Verify that $(0.82)^5 = (0.82)^2 \times (0.82)^2 \times 0.82 =$ ***0.370.***

The techniques illustrated by the foregoing examples may be summarized as follows:

> *Combined Operations with Squares*
>
> *Procedure with A-B scales:*
> 1. Numbers to be squared are set on D, C, or CI, thus locating the squares on A or B.
> 2. Unsquared quantities are set directly on A or B, and the actual operation takes place on the A-B scales.
>
> *Procedure with R scale:*
> If a squared number is to be further multiplied or divided:
> 1. Number to be squared is set on R, thus locating its square on D.
> 2. Operation then continues on C-D; if more squares are involved, they are treated as products.

Exercise 11-2

1. $\dfrac{(2.6)^2 \times (5.3)^2}{45} =$

2. $\dfrac{(3.4)^2 \times (6.1)^2}{58} =$ "742"

3. $\dfrac{(3.4 \times 37.2)^2}{63.4} =$

4. $\dfrac{(2.7 \times 8.6)^2}{23} =$ "234"

5. $\dfrac{45}{(3.8)^2 \times (5.2)^2} =$

6. $\dfrac{28}{(2.6)^2 \times (1.8)^2} =$ "128"

7. $(3.17)^4 =$

8. $\dfrac{(1.56)^4}{3.40} =$ "174"

9. $\dfrac{73.2}{(2.15)^4} =$

10. $\dfrac{(5.2)^3}{4.6} =$ "305"

11. $\dfrac{(3.8)^3}{(2.6)^2} =$

12. $(2.32)^5 =$ "672"

13. $\dfrac{370}{(4.25 \times 0.660)^2} =$

14. $\dfrac{57.3}{(12.8 \times 0.455)^2} =$ "169"

15. $(2.44 \times 3.65 \times 5.22)^2 =$

16. $(2.65 \times 1.82 \times 3.22 \times 0.72)^2 =$ "125"

17. $(3.52 \times 4.62)^2 \times 0.711 =$

18. $(0.524)^2 \times (21.5)^2 \times 3.14 =$ "398"

19. $\dfrac{(3.78)^2 \times 14.2}{2.64} =$

20. $\dfrac{(5.27)^2 \times 34.6}{62.7} =$ "153"

21. $\pi \times (4.6)^2 \times 7.2 =$

22. $\pi \times (15.4)^2 \times 12.7 =$ "946"

23. $\left(\dfrac{94.3}{43.1}\right)^2 =$

24. $\left(\dfrac{52.4}{2.94}\right)^2 =$ "318"

25. $\left(\dfrac{28.2}{3.82 \times 2.64}\right)^2 =$

26. $\left(\dfrac{36.1}{4.20 \times 1.75}\right)^2 =$ "241"

27. $\dfrac{(1640 \times .0264)^2}{14.65} =$

28. $\dfrac{(64.3)^2 \times .0143}{(11.4)^2} =$ "455"

29. $\dfrac{(4.72)^4}{3.14 \times 10^6} =$

30. $\dfrac{(.0463)^2 \times 154}{(.0706)^2} =$ "661"

31. $\dfrac{(1.87)^3}{4.05} =$

32. $\dfrac{(21.4)^2}{(3.75)^3} =$ "869"

33. $\dfrac{(12.6)^3}{(5.42)^2} =$

34. $\dfrac{1}{36.3} \times \left(\dfrac{127}{4.21}\right)^2 =$ "251"

35. $\left(\dfrac{28.7 \times .0824}{.00365}\right)^2 =$

36. $\left(\dfrac{2.75 \times 3200}{1.64 \times .0346}\right)^2 =$ "240"

37. $(0.83)^4 =$

38. $(0.42)^4 =$ "311"

39. $(0.155)^5 =$

40. $(0.845)^5 =$ "430"

41. $(0.72)^6 =$

42. $(0.28)^6 =$ "481"

43. $(5.4)^5 =$

44. $(13)^7 =$ "628"

11.5 Formula types

Example 1: $\dfrac{19}{1.3} \times \left(\dfrac{1}{4.2} + \dfrac{1}{2.3}\right)^2 = ?$

1. Use CI or DI scale to verify that $\dfrac{1}{4.2} + \dfrac{1}{2.3} = 0.238 + 0.435 = 0.673$

2. Expression may now be evaluated as:

$$\frac{19 \times (0.673)^2}{1.3} = \boldsymbol{6.61}.$$

Example 2: Given the formula: $I = \dfrac{\pi}{32}(D_1^4 - D_2^4)$.

Find I when $D_1 = 4.25$ and $D_2 = 3.85$.

1. Substituting the given values:

$$I = \frac{\pi}{32}\left[(4.25)^4 - (3.85)^4\right].$$

2. Verify that $(4.25)^4 = 327$, and $(3.85)^4 = 220$.
3. Expression may now be evaluated:

$$I = \frac{\pi}{32}(327 - 220) = \frac{\pi}{32} \times 107 = \boldsymbol{10.5}.$$

Exercise 11-3

1. $32.4\,(1 + 16/3.7)^2 =$

2. $\dfrac{15000}{1 + \dfrac{(109)^2}{15000}} =$

3. $\dfrac{2.74 \times (3.66)^2}{(8.22)^2 - (3.66)^2} =$

4. $\dfrac{45.3 \times (13.5)^2}{(27.6)^2 - (13.5)^2} =$

5. $\dfrac{(216 - 212.4)^2}{212.4} + \dfrac{(115 - 107.6)^2}{107.6} + \dfrac{(64 - 61.2)^2}{61.2} =$

6. $\dfrac{(21.8 - 19.4)^2}{19.4} + \dfrac{(16.3 - 13.5)^2}{13.5} + \dfrac{(8.82 - 7.16)^2}{7.16} =$

7. $62.4\,[7.22 \times 8.13 - \pi\,(4.35)^2] =$

8. $\sqrt[3]{\dfrac{(2.6)^3(1.7) + (3.4)^3(2.2)}{1.7 + 2.2}} =$

9. $\sqrt[3]{\dfrac{(5.74)^3(2.66) + (7.22)^3(3.72)}{2.66 + 3.72}} =$

10. $\dfrac{64.5}{4.66}\left[\dfrac{1}{6.32} + \dfrac{1}{4.25}\right]^2 =$

11. $\dfrac{350}{52.4}\left[\dfrac{1}{3.66} + \dfrac{1}{7.25}\right]^2 =$

12. $\dfrac{(\sqrt{73} + \sqrt{6.4})^2}{\pi} =$

13. $\dfrac{(\sqrt{235} + \sqrt{61.2})^2}{\pi} =$

In the following formulas, substitute as indicated and evaluate:

14. $p = p_0 + \dfrac{1}{2} \rho V_0^2$

a. $p_0 = 275$, $\rho = .00238$, $V_0 = 124$
b. $p_0 = 1340$, $\rho = .00238$, $V_0 = 365$

15. $I = \dfrac{\pi D^4}{128}$

a. $D = 3.44$
b. $D = 6.32$

16. $\Theta = \dfrac{32\,TL}{\pi G\,(D_1^4 - D_2^4)}$

a. $T = 11{,}300$, $G = 12 \times 10^6$, $D_1 = 2.75$, $D_2 = 2.25$, $L = 5.50$
b. $T = 7500$, $G = 12 \times 10^6$, $D_1 = 1.85$, $D_2 = 1.65$, $L = 12.6$

17. $d = \dfrac{5wL^4}{384EI}$

a. $I = 168$, $w = 21.5$, $L = 115$, $E = 1.3 \times 10^6$
b. $I = 255$, $w = 36.2$, $L = 76$, $E = 1.3 \times 10^6$

18. $P = 15p^2q^4$

a. $p = 0.46$, $q = 0.54$
b. $p = 0.28$, $q = 0.72$

19. $P = 56\,p^5q^3$

a. $p = 0.42$, $q = 0.58$
b. $p = 0.86$, $q = 0.14$

20. $P = \dfrac{n!}{x!\,(n-x)!}\,p^x q^{n-x}$

a. $n = 7$, $x = 2$, $p = 0.12$, $q = 0.88$
b. $n = 7$, $x = 3$, $p = 0.33$, $q = 0.67$

Chapter 12

COMBINED OPERATIONS WITH ROOTS

12.1 Simple operations involving the square root

In this section we consider certain combined operations with the square root. Here again, the A-B scales may be used to advantage in conjunction with the C-D scales. The following examples illustrate the procedures with the A-B scales, and also with the R scale.

Example 1: $27 \times \sqrt{4.5} = ?$

Procedure with A-B scales:

1. Move HL over 45 on A-left. HL is now over $\sqrt{4.5}$ on D, and we are now in a position to multiply by 27 on the C-D scales.
2. Slide 27 on CI under HL. This multiplies by 27.
3. Opposite right index of C, read "573" on D. Answer is ***57.3.***

Note that in step (1) we are not free to choose either section of the A scale. We are locating the square root of 4.5 on D; therefore, we must select the proper section of A in accordance with the rules for square root.

Procedure with R scale:
You may find $\sqrt{4.5}$ on R, transfer the result to D, and then multiply by 27. An alternate procedure which avoids the transfer operation is the following:

1. Think of the given expression as $\sqrt{(27)^2 \times 4.5}$.
2. Move HL over 27 on R_1; this puts $(27)^2$ on D. Now multiply by 4.5:
3. Slide right index of C under HL; move HL over 45 on C. The result of this multiplication is now under HL on D, and the square root of this result is under the HL on R_1 or R_2. A rough estimate indicates that the answer must be on R_2.
4. Under HL read "573" on R_2. Answer is ***57.3.***

Example 2: $\dfrac{\sqrt{50.5}}{2.35} = ?$

Procedure with A-B scales:

1. Move HL over 505 on A-right. HL is now over $\sqrt{50.5}$ on D; therefore, the division is carried out on the C-D scales:
2. Slide 235 on C under HL.
3. Opposite left index of C, read "302" on D. Answer is ***3.02.***

Again, in step (1), we must choose the proper section of A.

Procedure with R scale:
Here, you may find $\sqrt{50.5}$ on R, transfer the result to D, and divide by 2.35.
Alternatively, you may think of the expression as $\sqrt{50.5/(2.35)^2}$. The steps would then be:

1. Move HL over 505 on D. Now divide by 2.35:
2. Slide 235 on C under HL. Now divide again by 2.35:
3. Move HL over 235 on CI. Square root is now under HL on R_1 or R_2. A rough estimate indicates that the result is on R_1.
4. Under HL read "3022" on R_1. Answer is ***3.022.***

Verify the following:

1. $\sqrt{21} \times 3.6 = 16.5$
2. $\sqrt{6.3} \times 12.7 = 31.9$
3. $0.45 \times \sqrt{140} = 5.33$
4. $\dfrac{\sqrt{3.2}}{1.4} = 1.278$
5. $\dfrac{\sqrt{43.0}}{5.70} = 1.150$
6. $14.2 \times \sqrt{39.1} = 88.8$
7. $243 \times \sqrt{.0241} = 37.7$
8. $37.5 \times \sqrt{.00425} = 2.44$
9. $\dfrac{\sqrt{12.4}}{4.73} = 0.745$
10. $\dfrac{\sqrt{5750}}{8.61} = 8.81$

Example 3: $\dfrac{18}{\sqrt{29}} = ?$

Procedure with A-B scales:

1. Move HL over 18 on D.
2. Slide 29 on B-right under HL. Note that $\sqrt{29}$ is now under HL on C, hence, the division has been performed on the C-D scales.
3. Opposite right index of C, read "334" on D. Answer is ***3.34.***

Procedure with R scale:
You may first find $\sqrt{29}$ on R, make a mental note of this (or write it down), move HL over 18 on D, and then complete the division. Alternatively, the expression may be evaluated as $\sqrt{(18)^2/29}$:

1. Move HL over 18 on R_1. HL is now over $(18)^2$ on D.
2. Slide 29 on C under HL. Move HL over left index of C.
3. Under HL read "3342" on R_1. Answer is ***3.342.***

Verify the following:

1. $\dfrac{26}{\sqrt{6.2}} = 10.44$
2. $\dfrac{68}{\sqrt{14}} = 18.2$
3. $134 \div \sqrt{210} = 9.25$
4. $\dfrac{43}{\sqrt{55}} = 5.80$
5. $\dfrac{10}{\sqrt{15.4}} = 2.55$
6. $\dfrac{12.5}{(7.25)^{1/2}} = 4.64$
7. $\dfrac{453}{(4150)^{1/2}} = 7.03$
8. $2.44 \div (.0382)^{1/2} = 12.5$

Exercise 12-1

1. $5.2 \times \sqrt{4.8} =$
2. $1.3 \times \sqrt{2.5} =$ "206"
3. $23 \times \sqrt{19} =$
4. $14 \times \sqrt{31} =$ "780"
5. $\dfrac{\sqrt{7.3}}{2.1} =$
6. $\dfrac{\sqrt{5.7}}{1.6} =$ "1492"
7. $\dfrac{\sqrt{71}}{4.8} =$
8. $\dfrac{\sqrt{43}}{3.2} =$ "205"
9. $\dfrac{14}{\sqrt{1.9}} =$
10. $\dfrac{34}{\sqrt{21}} =$ "742"
11. $\dfrac{120}{\sqrt{340}} =$
12. $\dfrac{7.4}{\sqrt{2.7}} =$ "450"
13. $12.5 \times \sqrt{423} =$
14. $8.22 \times \sqrt{0.725} =$ "700"
15. $\dfrac{\sqrt{5650}}{10.2} =$
16. $\dfrac{\sqrt{2450}}{15.6} =$ "317"
17. $\dfrac{230}{\sqrt{108}} =$
18. $\dfrac{9.25}{\sqrt{17.2}} =$ "223"
19. $41.2 \times (.061)^{1/2} =$
20. $.045 \times (735)^{1/2} =$ "1220"

21. $\dfrac{(.037)^{1/2}}{2.11} =$

22. $\dfrac{(.0071)^{1/2}}{1.85} =$ "456"

23. $\dfrac{1}{\sqrt{3.75}} =$

24. $\dfrac{1}{\sqrt{.077}} =$ "360"

25. $21.6 \times \sqrt{51.6} =$

26. $7.44 \times \sqrt{2.03} =$ "1060"

27. $\dfrac{\sqrt{344}}{6.48} =$

28. $\dfrac{\sqrt{41.2}}{2.75} =$ "233"

29. $0.821 \times \sqrt{18.4} =$

30. $\sqrt{78.2} \times 41.6 =$ "368"

31. $\dfrac{\sqrt{126}}{4.66} =$

32. $\dfrac{\sqrt{5670}}{2.41} =$ "312"

33. $\dfrac{25.8}{(0.73)^{1/2}} =$

34. $\dfrac{456}{\sqrt{234}} =$ "298"

35. $\dfrac{36.4}{\sqrt{9.55}} =$

36. $\dfrac{2.22}{\sqrt{0.431}} =$ "338"

37. $(138)^{1/2} \times 2.07 =$

38. $\pi \times \sqrt{0.811} =$ "283"

39. $\dfrac{\sqrt{10.45}}{0.823} =$

40. $\dfrac{\sqrt{65{,}400}}{27.2} =$ "940"

41. $.0124 \times \sqrt{1545} =$

42. $1.78 \times (473)^{1/2} =$ "387"

43. $\dfrac{\sqrt{51{,}200}}{43.6} =$

44. $\dfrac{52.1}{(17.2)^{1/2}} =$ "1256"

45. $\dfrac{(.0941)^{1/2}}{0.423} =$

46. $456 \times \sqrt{.000714} =$ "1218"

47. $\dfrac{\sqrt{.0555}}{1.755} =$

48. $126 \times \sqrt{.0545} =$ "294"

49. $.0452 \times \sqrt{84{,}300} =$

50. $\dfrac{\sqrt{165}}{.0144} =$ "892"

51. $\dfrac{\sqrt{495}}{36.7} =$

52. $\dfrac{\sqrt{0.204}}{821} =$ "550"

53. $\dfrac{\sqrt{9250} \times 10^4}{3400} =$

54. $\dfrac{\sqrt{175{,}000}}{6.55 \times 10^3} =$ "639"

55. $\dfrac{(75.6)^{1/2}}{0.334} =$

56. $\dfrac{204}{\sqrt{74.2}} =$ "237"

57. $\dfrac{1030}{\sqrt{4570}} =$

58. $(69.1)^{-1/2} =$ "1203"

59. $(.00504)^{-1/2} =$

12.2 Further operations with square roots

Example 1: $\sqrt{2.7 \times 6.3} = ?$

Procedure with A-B scales:
Think of the given expression as $\sqrt{2.7} \times \sqrt{6.3}$.

1. Set left index of B opposite 27 on A-left. Note that left C index is now opposite $\sqrt{2.7}$ on D.
2. Move HL over 63 on B-left. The hairline is now over $\sqrt{6.3}$ on C; hence, we have performed the desired multiplication on the C-D scales.
3. Under HL read "412" on D. Answer is ***4.12.***

Procedure with R scale:
The expression under the radical sign is evaluated on the C-D scales and the square root of this result is read on the appropriate R scale:

1. Set right index of C opposite 63 on D.
2. Move HL over 27 on C. The product under the radical sign is now under the HL on D; the square root of this is under HL on either R_1 or R_2. By approximation, it is evident that answer is on R_2.
3. Under HL read "4125" on R_2. Answer is ***4.125.***

Example 2: $\sqrt{\dfrac{21.7 \times 0.165}{6.24}} = ?$

Procedure with A-B scales:

Think of expression as: $\dfrac{\sqrt{21.7} \times \sqrt{0.165}}{\sqrt{6.24}}$

1. Move HL over 217 on A-right. HL is now over $\sqrt{21.7}$ on D.
2. Slide 624 on B-left under HL. This positions $\sqrt{6.24}$ on C under HL. Thus, we have divided $\sqrt{21.7}$ by $\sqrt{6.24}$ on the C-D scales. Now multiply by $\sqrt{0.165}$:
3. Move HL over 165 on B-right. HL is now over $\sqrt{0.165}$ on C; hence, result is under HL on D.
4. Under HL read "758" on D.

Approximating for decimal point:

$$\sqrt{\frac{21.7 \times 0.165}{6.24}} \approx \sqrt{\frac{20 \times 0.2}{6}} = \sqrt{\frac{4}{6}} \approx \sqrt{0.7} \approx 0.8$$

Answer must be ***0.758.***
(Again, you are reminded that the quantities under the radical sign must be set on the proper sections of the A and B scales.)

Procedure with R scale:
First approximate the answer to be about 0.8. Then evaluate the expression under the radical sign on the C-D scales, and read the square root on the appropriate R scale.

1. Set left index of C opposite 165 on D.
2. Move HL over 217 on C.
3. Slide 624 on C under HL. The expression under the radical sign has now been evaluated and the result is opposite C index on D. We wish to extract the square root of this:
4. Move HL over right index of C.
5. Under HL read "758" on R_2. Answer is ***0.758.***

Verify the following:

1. $\sqrt{6.45 \times 23.9} = 12.41$
2. $\sqrt{\dfrac{602}{46.5}} = 3.60$
3. $\sqrt{\dfrac{172}{4.95}} = 5.90$
4. $\sqrt{\dfrac{5.25 \times 31.2}{2.44}} = 8.20$
5. $\dfrac{\sqrt{55.5}}{\sqrt{.077}} = 26.8$
6. $\sqrt{.077 \times 266} = 4.53$
7. $\dfrac{\sqrt{127 \times .063}}{\sqrt{20.5}} = 0.625$
8. $\sqrt{\dfrac{31.2 \times 4.60}{2.55 \times 0.44}} = 11.31$
9. $\dfrac{\sqrt{23{,}500}}{\sqrt{0.912 \times 413}} = 7.90$
10. $\sqrt{\dfrac{4570}{23.1 \times 6.27}} = 5.62$

Example 3: $\dfrac{\sqrt{5210} \times 0.410}{\sqrt{.0755}} = ?$

Procedure with A-B scales:

1. Move HL over 521 on A-right.
2. Slide 755 on B-left under HL. We have now divided $\sqrt{5210}$ by $\sqrt{.0755}$. The operation has taken place on C-D, and the quotient is opposite left C index on D. To multiply by 0.410 on the C-D scales, we must now interchange indexes. However, this may be avoided by continuing on the folded scales:
3. Turn rule over and move HL over 410 on CF.
4. Under HL read "1077" on DF.

Approximating for the decimal point:

$$\frac{\sqrt{\underbrace{52}\ \underbrace{10}} \times 0.410}{\sqrt{.\underbrace{07}\ \underbrace{55}}} \approx \frac{70 \times 0.4}{0.3} = \frac{280}{3} \approx 90$$

Answer must be ***107.7.***

Procedure with R scale:

You may first divide 5210 by .0755 on C-D and find the square root of the result on R. Then transfer this back to D and multiply by 0.410.

Alternatively, the expression may be evaluated as $\sqrt{\dfrac{5210 \times (0.410)^2}{.0755}}$:

1. Move HL over 410 on R_2. This puts $(0.410)^2$ on D.
2. Slide 755 on C under HL.
3. Move HL over 521 on C. Answer is now under HL on R_1 or R_2; however, a rough approximation indicates that the answer must be on R_1.
4. Under HL read "1077" on R_1. Answer is ***107.7.***

Example 4: $\dfrac{286}{11.2 \times \sqrt{3.75}} = ?$

Procedure with A-B scales:

1. Move HL over 286 on D.
2. Slide 375 on B-left under HL. Now go to the folded scales:
3. Turn rule over and move HL over 112 on CIF.
4. Under HL read "1319" on DF. Answer is ***13.19.***

Procedure with R scale:

You may first find $\sqrt{3.75}$ on R, then evaluate as a combined operation on C-D.

Alternatively, the expression may be evaluated as $\sqrt{\dfrac{(286)^2}{(11.2)^2 \times 3.75}}$.

Verify that answer is ***13.19.***

Example 5: $\dfrac{\sqrt{28 \times 8.4}}{(2.2)^2} = ?$

When both square roots and squares are involved, the squares are simply treated as products.

Verify that $\dfrac{\sqrt{28 \times 8.4}}{(2.2)^2} = \dfrac{\sqrt{28 \times 8.4}}{2.2 \times 2.2} = \boldsymbol{3.17}$

Verify the following:

1. $\dfrac{\sqrt{5.4} \times 23}{\sqrt{11}} = 16.11$
2. $\dfrac{\sqrt{32.4} \times 1.46}{2.15} = 3.86$
3. $\dfrac{250}{\sqrt{2.6 \times 15.4}} = 39.5$
4. $\dfrac{\sqrt{5200} \times (3.4)^2}{\sqrt{.073}} = 3090$
5. $\dfrac{12.4}{0.362 \times \sqrt{.0017}} = 830$
6. $\dfrac{\sqrt{53.5 \times .026}}{375} = .00315$

Example 6: $\dfrac{(8.5)^{3/2}}{2.6} = ?$

Verify that $\dfrac{(8.5)^{3/2}}{2.6} = \dfrac{8.5 \times \sqrt{8.5}}{2.6} = \boldsymbol{9.53}$

Example 7: $(2.3)^{5/2} = ?$

Verify that $(2.3)^{5/2} = (2.3)^2 \times \sqrt{2.3} = \mathbf{8.03}$

Verify the following:

1. $37.5 \times (11.5)^{3/2} = 1462$

2. $2.7 \times (46)^{3/2} = 843$

3. $\dfrac{(52.5)^{3/2}}{6.44} = 59.1$

4. $\dfrac{450}{(6.45)^{3/2}} = 27.5$

5. $(12.5)^{5/2} = 553$

6. $(6.7)^{5/2} \div 3.7 = 31.4$

The foregoing techniques may be summarized as follows:

> *Combined operations with square roots*
>
> *Procedure with A-B scales:*
>
> 1. Numbers under radical signs are each set on the proper sections of the A-B scales, thus locating the square roots on C-D.
>
> 2. Numbers not under radical signs are set on C-D, and the actual operation takes place on the C-D scales.
>
> *Procedure with R scale:*
> 1. Square roots may first be evaluated on R, then transferred back to C-D if they are to be further multiplied or divided.
>
> 2. Alternatively, if the given expression is all contained under a single radical sign (or if it is put into this form), then the expression is evaluated on C-D with the square root appearing on the appropriate R scale.

Exercise 12-2

1. $\sqrt{\dfrac{1.8 \times 42}{2.9}} =$

2. $\sqrt{\dfrac{56 \times 3.4}{23}} =$ "288"

3. $\sqrt{\dfrac{120 \times 4.5}{37}} =$

4. $\sqrt{6.2 \times 3.4} =$ "459"

5. $\sqrt{15 \times 8.4} =$

6. $\sqrt{\dfrac{73}{2.1 \times 3.5}} =$ "315"

7. $\dfrac{\sqrt{220}}{\sqrt{6.2} \times \sqrt{10.5}} =$

8. $\dfrac{\sqrt{31}}{\sqrt{3.8}} =$ "286"

9. $\sqrt{\dfrac{260}{12.5}} =$

10. $\left(\dfrac{14}{37}\right)^{1/2} =$ "615"

11. $\sqrt{2.7 \times 5.6 \times 3.1} =$

12. $\sqrt{43 \times 0.35 \times 8.7} =$ "1145"

13. $\dfrac{14 \times \sqrt{23}}{\sqrt{3.5}} =$

14. $\dfrac{2.7 \times \sqrt{8.2}}{\sqrt{19}} =$ "1775"

15. $6.2 \times \sqrt{\dfrac{210}{5.7}} =$

16. $\dfrac{\sqrt{5.1} \times \sqrt{31}}{2.4} =$ "524"

17. $\dfrac{\sqrt{145} \times \sqrt{3.7}}{13} =$

18. $\dfrac{(61.2)^{3/2}}{32.5} =$ "1475"

19. $0.73 \times (11.5)^{3/2} =$

20. $1.82 \times (0.83)^{3/2} =$ "1376"

21. $\dfrac{12 \times 17}{\sqrt{5.1}} =$ "903"

22. $\dfrac{5.4 \times 75}{\sqrt{29}} =$

23. $\dfrac{\sqrt{340} \times 6.2}{23} =$ "497"

24. $\dfrac{170 \times \sqrt{0.42}}{53} =$

25. $\dfrac{(.00717)^{1/2} \times 0.811}{.00406} =$ "1691"

26. $\dfrac{7840}{\sqrt{21.2} \times 40.6} =$

27. $\dfrac{1}{\sqrt{0.814} \times .0243} =$ "456"

28. $\dfrac{37.4}{\sqrt{.0524} \times 6.44} =$

29. $\dfrac{2.76 \times 525}{\sqrt{40{,}800}} =$ "717"

30. $\dfrac{1.75 \times 4.77}{\sqrt{.0335}} =$

31. $\dfrac{23.4 \times 10^4}{(7430)^{1/2}} =$ "272"

32. $\dfrac{23.0 \times (8.45)^{3/2}}{6.32} =$

33. $\dfrac{246}{4.82 \times (8.25)^{3/2}} =$ "215"

34. $\sqrt{.00712} \times \sqrt{1.245} =$

35. $\sqrt{.0831 \times 3450} =$ "1694"

36. $\dfrac{\sqrt{4.73}}{\sqrt{19.2}} =$

37. $\dfrac{\sqrt{59.2}}{\sqrt{.00345}} =$ "1310"

38. $\dfrac{\sqrt{621}}{\sqrt{21.2} \times \sqrt{10.4}} =$

39. $\dfrac{\sqrt{7.25} \times \sqrt{31.6}}{\sqrt{85.2}} =$ "1640"

40. $(.0431)^{-1/2} =$

41. $\dfrac{\sqrt{902} \times \sqrt{0.412}}{\sqrt{.0521}} =$ "845"

42. $\sqrt{\dfrac{37.2}{4.75 \times .0714}} =$

43. $\sqrt{\dfrac{.00718 \times 5760}{.0436}} =$ "308"

44. $\sqrt{\dfrac{.0643 \times 384}{0.524}} =$

45. $\sqrt{\dfrac{342}{12.7 \times .0615}} =$ "209"

46. $\dfrac{2.46 \times \sqrt{41.6}}{\sqrt{714}} =$

47. $\dfrac{375 \times \sqrt{0.617}}{\sqrt{2070}} =$ "647"

48. $0.432 \times \sqrt{\dfrac{27{,}500}{.0643}} =$

49. $\dfrac{\sqrt{69.2} \times \sqrt{.00316}}{23.4} =$ "200"

50. $\dfrac{42.1 \times \sqrt{3.71}}{\sqrt{0.743}} =$

51. $6.28 \times \sqrt{\dfrac{143}{32.2}} =$ "1323"

52. $2.46 \times \sqrt{\dfrac{375}{19.2}} =$

53. $\dfrac{3.44 \times \sqrt{0.611}}{\sqrt{29.4}} =$ "496"

54. $\sqrt{\dfrac{9.36 \times 10^9}{5850}} =$ "1265"

55. $\dfrac{6.19 \times 10^5}{\sqrt{3.77 \times 10^7}} =$

56. $\dfrac{\sqrt{.075 \times 463}}{7.42} =$ "794"

57. $\dfrac{422}{\sqrt{7.06} \times \sqrt{213}} =$

58. $(2.4)^2 \times \sqrt{30.4} =$ "318"

59. $(3.7)^2 \times \sqrt{0.86} =$

60. $\dfrac{(11.5)^2}{\sqrt{235}} =$ "863"

61. $\dfrac{(0.76)^2}{\sqrt{14.25}} =$

62. $(31.2)^{5/2} =$ "5440"

63. $(.0072)^{5/2} =$

64. $\dfrac{2.3 \times 360}{(6.2)^{5/2}} =$ "865"

65. $\dfrac{(0.645)^{5/2}}{1.82} =$

66. $\dfrac{(1.63)^{5/2}}{26.5} =$ "1280"

67. $\dfrac{\sqrt{3450}}{(6.35)^2} =$ "1458"

68. $(5.2)^2 \times \sqrt{\dfrac{51.2}{6.31}} =$

69. $\dfrac{(.00206 \times 39.6)^{1/2}}{0.564} =$ "507"

70. $\dfrac{\sqrt{2160} \times \sqrt{7.25}}{4.89 \times 0.564} =$

71. $\dfrac{43.7 \times \sqrt{.0823}}{\sqrt{71.2} \times 0.821} =$ "1810"

72. $\dfrac{27.6 \times \sqrt{4750}}{\sqrt{157} \times 4.67} =$

73. $\left(\dfrac{37.2 \times .0168}{4.68 \times .00913}\right)^{1/2} =$ "383"

74. $\left(\dfrac{125 \times 5720}{.0823 \times 7.11}\right)^{1/2} =$

75. Evaluate $y = 3.6\sqrt{x}$ for $x = 1.5$, 3.6, 21.7, and 60.5.

76. Evaluate $y = 0.74\sqrt{x}$ for $x = 15$, 75, 250, and 1450.

77. Evaluate $y = \dfrac{\sqrt{x}}{2.6}$ for $x = 3$, 7, 11, and 45.

78. Evaluate $y = \dfrac{64}{\sqrt{x}}$ for $x = 0.5$, 8.5, 46.5, and 175.

79. Evaluate $y = (26.4)x^{-1/2}$ for $x = 0.70$, 1.85, 7.40, and 13.6.

80. Evaluate $y = \dfrac{150}{\pi\sqrt{x}}$ for $x = 5.75$, 12.6, 57.5, and 210.

12.3 Operations with cube roots and fourth roots

Example 1: $\sqrt[3]{12} \times 22 = ?$

1. Move HL over 12 on K-middle. This puts HL over $\sqrt[3]{12}$ on D. We may now multiply by 22 on C-D scales:
2. Slide 22 on CI under HL.
3. Opposite right index of C read "504" on D. Answer is ***50.4.***

Example 2: $\dfrac{\sqrt[3]{410}}{6.20} = ?$

1. Move HL over 410 on K-right.
2. Slide 62 on C under HL.
3. Opposite left index of C read "1198" on D. Answer is ***1.198.***

Example 3: $\dfrac{52}{\sqrt[3]{7.4}} = ?$

In this case, we divide $\sqrt[3]{7.4}$ by 52 and read the reciprocal.

1. Move HL over 74 on K-left.
2. Slide 52 on C under HL.
3. Move HL over right index of C. The quotient is now under HL on D, and its reciprocal is under HL on DI.
4. Under HL read "267" on DI. Answer is ***26.7.***

(Note that reciprocal may also be read opposite left D index on C.)

Verify the following:

1. $\sqrt[3]{4.5} \times 47 = 77.6$

2. $\dfrac{\sqrt[3]{5450}}{2.64} = 6.66$

3. $\dfrac{136}{\sqrt[3]{13.5}} = 57.2$

4. $\dfrac{\sqrt[3]{31.5} \times 16.2}{3.44} = 14.87$

5. $\dfrac{2.76 \times 67.5}{\sqrt[3]{0.37}} = 260$

6. $\dfrac{5210}{325 \times \sqrt[3]{.0460}} = 44.8$

Example 4: $1.7 \times \sqrt[4]{31.2} = ?$

Evaluate this as $1.7 \times \sqrt{\sqrt{31.2}}$.

Procedure with A-B scales:

1. Move HL over 312 on A-right.
2. Under HL read "559" on D; hence, $\sqrt{31.2} = 5.59$. Now take square root again.
3. Move HL over 559 on A-left. HL is now over $\sqrt[4]{31.2}$ on D, and we can multiply by 1.7.
4. Slide 17 on CI under HL.
5. Opposite right index of C read "401" on D. Answer is ***4.01***.

Procedure with R scale:

1. Use R scale twice to find $\sqrt[4]{31.2} = \sqrt{\sqrt{31.2}} = 2.36$.
2. Slide left index of C over 236 on D, and multiply by 1.7. Verify that answer is ***4.01***.

On rules that have both the A scale and the double-length scale (R, Sq, or $\sqrt{\ }$), you may read $\sqrt[4]{31.2}$ directly from A to R, transfer to D and multiply by 1.7.

Example 5: $(175)^{5/4} = ?$

Verify that $(175)^{5/4} = 175 \times (175)^{1/4} =$ ***636.***

Example 6: $(43)^{3/4} = ?$

Verify that $(43)^{3/4} = (43)^{1/4} \times (43)^{1/2} =$ ***16.8.***

Verify the following:

1. $4.6 \times \sqrt[4]{7.2} = 7.54$
2. $26.3 \times \sqrt[4]{2850} = 192.5$
3. $\sqrt[4]{.056} \div 1.8 = 0.270$
4. $(48)^{5/4} = 126.5$
5. $(0.82)^{5/4} = 0.78$
6. $(640)^{3/4} = 127.2$

Exercise 12-3

1. $\sqrt[3]{32} \times 4.6 =$
2. $\sqrt[3]{175} \times 8.5 =$ "476"

3. $\dfrac{\sqrt[3]{45}}{2.7} =$

4. $\dfrac{\sqrt[3]{560}}{8.2} =$ "1005"

5. $\dfrac{165}{\sqrt[3]{61}} =$

6. $\dfrac{34.6}{\sqrt[3]{2.75}} =$ "247"

7. $\dfrac{16.3 \times \sqrt[3]{760}}{57.4} =$

8. $\dfrac{\sqrt[3]{13.5} \times 2.46}{1.075} =$ "545"

9. $\dfrac{\sqrt[3]{96}}{4.75 \times 0.74} =$

10. $\dfrac{\sqrt[3]{1750}}{6.35 \times 2.48} =$ "765"

11. $\dfrac{1}{\sqrt[3]{4.6} \times .065} =$

12. $\dfrac{1}{\sqrt[3]{.065} \times 1.22} =$ "204"

13. $2.2 \times \sqrt[4]{185} =$

14. $10.6 \times \sqrt[4]{3650} =$ "824"

15. $0.72 \times \sqrt[4]{52} =$

16. $345 \times \sqrt[4]{.00665} =$ "985"

17. $\dfrac{\sqrt[4]{.0465}}{0.622} =$

18. $\dfrac{\sqrt[4]{74{,}000} \times 12.8}{5.22} =$ "404"

19. $(22.6)^{5/4} =$

20. $(0.63)^{5/4} =$ "561"

21. $(3750)^{5/4} =$

22. $\dfrac{(5.22)^{5/4}}{2.33} =$ "339"

23. $(9.2 \times 10^{-9})^{5/4} =$

24. $(230)^{3/4} =$ "591"

25. $(17.5)^{3/4} =$

26. $(0.445)^{3/4} =$ "545"

27. $(5.6 \times 10^{10})^{3/4} =$

28. $\dfrac{(1.9 \times 10^{-9})^{3/4}}{2.3} =$ "1252"

29. $\sqrt[3]{4.6 \times 3.5} =$

30. $\sqrt[3]{61.5 \times 15.3} =$ "980"

31. $\sqrt[3]{\dfrac{210}{4.8}} =$

32. $\sqrt[3]{\dfrac{76}{105}} =$ "898"

33. $\dfrac{16.7 \times 2.83}{\sqrt[3]{77}} =$

34. $\dfrac{0.425 \times 378}{\sqrt[3]{460}} =$ "208"

35. $(165)^{2/3} \times 3.77 =$

36. $(740)^{2/3} \div 11.6 =$ "705"

37. $\dfrac{(27.5)^{2/3} \times 6.35}{2.55} =$

38. $\sqrt[3]{145} \times \sqrt{6.3} =$ "1320"

39. $\dfrac{\sqrt[3]{1450}}{\sqrt{260}} =$

40. $\dfrac{\sqrt{510} \times 3.72}{\sqrt[3]{3160}} =$ "572"

12.4 Formula types

Example 1: $$\frac{\sqrt{1 + (1.46)^2}}{.0178 + (4.77)(0.133)} = ?$$

1. Verify that $(1.46)^2 = 2.13$.
2. Verify that $4.77 \times 0.133 = 0.634$.
3. Expression now becomes:

$$\frac{\sqrt{1 + 2.13}}{.0178 + 0.634} = \frac{\sqrt{3.13}}{0.6518} \approx \frac{\sqrt{3.13}}{0.652}$$

4. Verify that answer is ***2.72.***

Example 2: $$\frac{650 \times [(14.6)^2 - (8.25)^2]^{3/2}}{4320} = ?$$

1. Verify that $(14.6)^2 - (8.25)^2 = 213 - 68.0 = 145$.
2. Expression now becomes:

$$\frac{650 \times (145)^{3/2}}{4320} = \frac{650 \times 145 \times \sqrt{145}}{4320}$$

3. Verify that answer is ***263.***

Exercise 12-4

1. $\dfrac{\sqrt{17.8} - \sqrt{6.22}}{0.27 \times 3.32} =$

2. $\dfrac{\sqrt{1 + (2.7)^2}}{3.5 \times 1.7} =$

3. $\sqrt{\dfrac{17 \times 34}{2.6(1 + 34/19)}} =$

4. $\dfrac{360 \times [(21.3)^2 - (12.4)^2]^{3/2}}{1300} =$

5. $\dfrac{0.85}{\sqrt{1 - \left(\dfrac{1.7}{2.1}\right)^4}} =$

6. $32.2 \times \sqrt[4]{2.3 + (5.7)^2} =$

7. $\sqrt{\dfrac{32.2 \times [104 + (5.24)^2]}{26.7}} =$

8. $21 \times 6.2 \times \left[\dfrac{21 \times 6.2}{21 + 12.4}\right]^{2/3} =$

9. $\dfrac{(28.2)^2 - (11.7)^2}{6.35 + \pi(3.18)^{4/3}} =$

10. $\dfrac{\sqrt[3]{\dfrac{1}{.0032} + \dfrac{1}{.0075}}}{5.24 \times 10^7} =$

In the following formulas, substitute the given data and evaluate:

11. $d = \sqrt{pq(1/N_1 + 1/N_2)}$

a. $N_1 = 220$, $N_2 = 170$, $p = 0.65$, $q = 0.34$

b. $N_1 = 435$, $N_2 = 260$, $p = 0.83$, $q = 0.57$

12. $r = \sqrt{\dfrac{p_1 r_1^2 + p_2 r_2^2}{p_1 + p_2 - 2}}$

a. $p_1 = 23$, $p_2 = 17$, $r_1 = 0.36$, $r_2 = 0.17$

b. $p_1 = 74$, $p_2 = 55$, $r_1 = 0.67$, $r_2 = 0.29$

13. $R = \dfrac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$

a. $a = 11.5$, $b = 7.6$, $c = 8.3$, $s = 13.7$

b. $a = 21.6$, $b = 13.2$, $c = 12.4$, $s = 23.6$

14. $t = \dfrac{r\sqrt{N-2}}{\sqrt{1-r^2}}$

a. $r = 0.37$, $N = 20$

b. $r = 0.58$, $N = 16$

15. $C = 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$

a. $a = 26.4$, $b = 18.7$

b. $a = 475$, $b = 385$

16. $S = \pi r \sqrt{r^2 + h^2}$

a. $r = 16.7$, $h = 21.3$

b. $r = 5.84$, $h = 12.7$

17. $Q = c\pi r^2 \sqrt{2gh}$

a. $c = 0.594$, $r = .081$, $g = 32.2$, $h = 15$

b. $c = 0.655$, $r = 0.135$, $g = 32.2$, $h = 37$

18. $N = 50\pi d^2 \sqrt{\dfrac{ds}{4}}$

a. $d = 4.12$, $s = .00078$

b. $d = 12.75$, $s = .00342$

19. $Q = \frac{1.486}{n} R^{2/3} S^{1/2}$

a. $n = .015$, $R = 2.24$, $S = .00037$
b. $n = .034$, $R = 13.1$, $S = .0053$

20. $S = \pi (r_1 + r_2) \sqrt{h^2 + (r_1 - r_2)^2}$

a. $r_1 = 13.6$, $r_2 = 7.75$, $h = 8.18$
b. $r_1 = 63.2$, $r_2 = 28.6$, $h = 25.2$

21. $d = \frac{Pb\sqrt{3}}{27\,EIL} (L^2 - b^2)^{3/2}$

a. $I = 11.4$, $P = 4200$, $b = 24.5$, $L = 72$, $E = 30 \times 10^6$
b. $I = 7.35$, $P = 2650$, $b = 14.7$, $L = 48$, $E = 30 \times 10^6$

22. $T = \frac{2A_t}{CA_0 \sqrt{2g}} (h_1^{1/2} - h_2^{1/2})$

a. $A_t = 13.3$, $A_0 = 0.74$, $C = 0.85$, $g = 32.2$, $h_1 = 7$, $h_2 = 5$
b. $A_t = 21.2$, $A_0 = 0.66$, $C = 0.79$, $g = 32.2$, $h_1 = 11$, $h_2 = 7$

23. $W = \sqrt{\frac{3EI}{L^3 (M + 0.24m)}}$

a. $E = 30 \times 10^6$, $I = 13.2$, $L = 86$, $M = 10.7$, $m = 4.6$
b. $E = 30 \times 10^6$, $I = 22.7$, $L = 64$, $M = 14.6$, $m = 8.55$

24. $f = 0.4 \sqrt{\frac{EIg}{WL^3}}$

a. $E = 10^7$, $I = 2.34$, $g = 386$, $L = 76.5$, $W = 1000$
b. $E = 30 \times 10^6$, $I = 11.5$, $g = 386$, $L = 58.0$, $W = 12{,}500$

25. $r = \frac{CT_m - ph}{\sqrt{CT_1 - p^2}\,\sqrt{CT_2 - h^2}}$

a. $C = 0.594$, $T_1 = 450$, $T_2 = 480$, $T_m = 465$, $p = 12$, $h = 15$
b. $C = 0.655$, $T_1 = 510$, $T_2 = 550$, $T_m = 530$, $p = 14$, $h = 17$

Chapter 13

REVIEW EXERCISES

The following exercise sets are designed to give you additional drill with the techniques and scales covered in Chapters 1 through 12.

Exercise 13-1

1. $\dfrac{472}{17.7 \times 2150} =$

2. $\dfrac{x}{3.62} = \dfrac{365}{742};\ x =$

3. $\sqrt{.0615} =$

4. $\dfrac{\sqrt[3]{260}}{\pi} =$

5. $365 \times (.071)^{1/2} =$

6. $\dfrac{\sqrt{128}}{5.24} =$

Exercise 13-2

1. $11.7 \times (5.24)^2 =$

2. $\sqrt[3]{.0000416} =$

3. $(275)^{-1/2} =$

4. $\dfrac{.00428 \times 1160}{6.04 \times .0848} =$

5. $\sqrt{\dfrac{31.6}{4.66 \times .0685}} =$

6. Given three circles with diameters 2.44, 15.2, and 46.5 respectively.
   a. Find circumference of each; **b.** find area of each.

Exercise 13-3

1. $\dfrac{1}{.00266} =$

2. $\dfrac{22.7}{\sqrt{0.425}} =$

3. $\sqrt[4]{18.2} =$

4. $\sqrt{2.13 \times 10^{11}} =$

5. $\dfrac{.0744}{\dfrac{1}{350} + \dfrac{1}{240} + \dfrac{1}{560}} =$

6. $1.8 \times 2.3 \times 2.2 \times 0.78 =$

Exercise 13-4

1. $\dfrac{1230}{\pi \times 6.82} =$

2. $\dfrac{1640}{V} = \dfrac{4.15}{.0633}$; $V =$

3. $\sqrt[4]{39{,}000} =$

4. $(315)^{2/3} =$

5. $\dfrac{\sqrt{.00412 \times 722}}{3.78} =$

6. $\dfrac{58.3}{\sqrt[3]{13.6}} =$

Exercise 13-5

1. $\frac{5.86}{2.77} = \frac{R_1}{.00505} = \frac{R_2}{0.227}$; $R_1 =$, $R_2 =$

2. $\frac{\pi}{(.064)^2}\left[\frac{1.75}{72.3} - \frac{1.48}{84.6}\right] =$

3. Given: $S = \frac{P}{A} + \frac{Mc}{I}$

   Find S when $P = 6500$, $A = 4.77$, $M = 4650$, $c = 2.60$, and $I = 8.75$.

4. $\frac{915 \times 84.7}{.0117 \times 128 \times 246} =$

5. $\sqrt[3]{.00575} \times 4.66 =$

6. Given: $y = 3.6\sqrt{x}$. Find y when $x = 1.8$, 6.7, 41, and 116.

Exercise 13-6

1. Given the equation, $y = 16.3x$. Find y when x takes the values 1.45, 3.85, 7.45, and 9.25.

2. $\frac{16.8 \times 1300 \times 10^{-14}}{27.5 \times 33.6 \times 6.03} =$

3. $\frac{(3.70 \times 41.2)^2}{57.2} =$

4. $\sqrt{31.4 \times 10^{-9}} =$

5. The stretch of a spring is proportional to the applied force. If a force of 25 lbs stretches the spring 3.8 inches, find:
   a. stretch corresponding to 5.8 lbs, 18 lbs, and 46 lbs.
   b. force corresponding to 0.75 inches, 2.1 inches, and 5.4 inches.

6. $\sqrt{\frac{16 \times 450}{4.75(1 - 26/73)}} =$

Exercise 13-7

1. $\frac{\sqrt{4150} \times .0627}{\sqrt{.0633}} =$

2. $\frac{.00423}{F} = \frac{0.684}{1315}$; $F =$

3. $\sqrt[3]{2.46 \times 10^8} =$

4. $\dfrac{10^{13}}{.0754 \times 132 \times 2.65} =$

5. $56(0.82)^5(0.18)^3 =$

6. $\dfrac{.0815 \times 1065 \times 5.30 \times .0645}{436 \times .00358} =$

Exercise 13-8

1. $\dfrac{(\sqrt{52.3} + \sqrt{21.7})^2}{3.14} =$

2. $\dfrac{(46.2)^{3/2}}{2.44} =$

3. $\dfrac{(.00564)^2 \times 322 \times 10^{23}}{(.0714)^2} =$

4. $\dfrac{.0722 - \sqrt{.00357}}{3.42} =$

5. Given: $C = 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$
   Find C when $a = 13.7$ and $b = 8.75$.

6. $(2.17)^{5/2} =$

Chapter 14

THE TRIGONOMETRIC FUNCTIONS (S, ST, AND T SCALES)

14.1 Some important relations

In this chapter we shall make use of the following trigonometric relations:

The reciprocal relations:

$\cos x = 1/\sin x$
$\sec x = 1/\cos x$
$\cot x = 1/\tan x$

The complementary relations:

$\cos x = \sin(90° - x)$
$\cot x = \tan(90° - x)$
$\csc x = \sec(90° - x)$

14.2 The sine (S and ST scales)

The S and ST scales are normally located on the slide, and the numbers on these scales represent angles in degrees (K & E uses the designation "SRT" instead of "ST"). You will observe that the ST scale ranges from about 0.573° to 5.74°, and the S scale continues from 5.74° to 90°; thus, the two scales together form one continuous scale. You should carefully study the divisions on these scales so that you can quickly locate any angle with the hairline. On some slide rules, the subdivisions are such that fractions of a degree may be estimated in minutes; however, the trigonometric scales on most modern rules are subdivided in decimal fractions of a degree.

For angles in the range of the S scale (5.74° to 90°), the sine lies between 0.1 and 1.0. For angles in the ST range (0.573° to 5.74°), the sine is between .01 and 0.1. The S and ST scales are related to the C scale in the following way:

To find sin x:
1. Move HL over x on **S** or **ST.**
2. Under HL read *sin x* on **C** (or **D** if rule is closed).

To place the decimal point:
1. If x is located on **S,** sin x is between 0.1 and 1.0.
2. If x is located on **ST,** sin x is between .01 and 0.1.

When using the rule to simply read off trigonometric functions, it is a good idea to close the rule with the C and D indexes aligned. In this way, readings can be made on either C or D, and you eliminate the possibility of reading the wrong scale.

Example 1: sin 26° = ?

1. Close rule and move HL over 26° on S.
2. Under HL read "438" on C or D.

Angle is located on S; hence, result lies between 0.1 and 1.0. Answer is ***0.438.***

Example 2: sin 3.25° = ?

1. Close rule and move HL over 3.25° on ST.
2. Under HL read "567" on C or D.

Angle is located on ST; hence, result lies between .01 and 0.1. Answer is ***.0567.***

Example 3: sin 36° 37′ = ?

1. For rules with decimal subdivisions, we first divide 37′ by 60 to convert to a decimal fraction of a degree.
2. Verify that sin 36° 37′ ≈ sin 36.6° = ***0.596.***

Verify the following:

1. sin 36° = 0.588

2. sin 79° = 0.982

3. sin 14.2° = 0.245

4. sin 4.1° = .0715

5. sin 1.075° = .01877

6. sin 0.73° = .01274

7. sin 8° 15′ = 0.1435

8. sin 50° 12′ = 0.768

9. sin 2° 40′ = .0465

It is now clear that we can read the sine directly for all angles between 0.573° and 90°. Functions of angles smaller than 0.573° are discussed in Appendix A.

Exercise 14-1

1. sin 39° =	**16.** sin 1.86° =
2. sin 14° =	**17.** sin 43′ =
3. sin 68.5° =	**18.** sin 35° 12′ =
4. sin 10.2° =	**19.** sin 76° =
5. sin 45.6° =	**20.** sin 4.45° =
6. sin 2.3° =	**21.** sin 5° 22′ =
7. sin 1.14° =	**22.** sin 14.35° =
8. sin 17.6° =	**23.** sin 6.05° =
9. sin 57.3° =	**24.** sin 62.4° =
10. sin 82° =	**25.** sin 12° 17′ =
11. sin 30.5° =	**26.** sin 1° 11′ =
12. sin 0.745° =	**27.** sin 85° =
13. sin 7.66° =	**28.** sin 20.6° =
14. sin 26.3° =	**29.** sin 0.585° =
15. sin 15° 20″ =	**30.** sin 41° 40″ =

14.3 The complementary scale

You will recall that two angles are complementary if their sum is 90°. Thus, the complement of 24° is 66°, the complement of 31° is 59°, and so on. Now, on most slide rules, both the angle and its complement are indicated at the major division marks on the S scale (on some rules the complementary angles are indicated in red). At the 60° mark, for example, you will also see 30° indicated; at the 20° mark you will see 70°. In using these complementary markings, we will refer to the "complementary scale." Note, especially, that the complementary scale increases from *right* to *left*. The presence of the complementary scale enables you to locate the complement of an angle directly on S without actually subtracting from 90°.

Most slide rules do not have the complementary markings on the ST scale; to locate the complement on this scale you must first subtract from 90°.

14.4 The cosine

Here, we may use the complementary relation: $\cos x = \sin(90° - x)$. Thus, $\cos 25° = \sin 65°$, $\cos 4° = \sin 86°$, and so forth. Clearly, then, to find $\cos x$ we need only to locate the complement of x on S or ST; we may then read the cosine directly on C. Note also that the complement of x can be located on S simply by moving the hairline over x on the complementary scale.

Example 1: $\cos 62° = ?$

The complementary relation is: $\cos 62° = \sin(90° - 62°) = \sin 28°$.

1. Close rule and move HL over 62° on the complementary scale of S. Note that HL is now automatically over the complementary angle (28°) on the direct S scale, thus saving you the trouble of subtracting from 90°.
2. Under HL read "469" on C or D. Complement is on S; hence, result is between 0.1 and 1.0. Answer is ***0.469.***

This example illustrates the procedure:

> *To find cos x:*
> 1. Move HL over the *complement of x* on **S** or **ST.**
> 2. Under HL read *cos x* on **C** (or **D** if rule is closed).
>
> *To place the decimal point:*
> 1. If complement of x is on S, $\cos x$ is between 0.1 and 1.0.
> 2. If complement of x is on ST, $\cos x$ is between .01 and 0.1.

Example 2: $\cos 46.3° = ?$

1. Close rule and move HL over 46.3° on the complementary scale of S.
2. Under HL read "691" on C or D.

Complement is on S; hence, result is between 0.1 and 1.0. Answer is ***0.691.***

Example 3: $\cos 88.5° = ?$

If there is no complementary scale on ST, we must first obtain the complement: $90° - 88.5° = 1.5°$.

1. Close rule and move HL over 1.5° on ST.
2. Under HL read "262" on C or D.

Complement is on ST; hence, result is between .01 and 0.1. Answer is ***.0262.***

Verify the following:

1. cos 28° = 0.883
2. cos 64.5° = 0.430
3. cos 73° 30′ = 0.284
4. cos 87.25° = .0480
5. cos 46° 15′ = 0.692
6. cos 85.2° = .0837

Exercise 14-2

1. cos 43° =
2. cos 67° =
3. cos 22° =
4. cos 75.2° =
5. cos 58.4° =
6. cos 37.7° =
7. cos 83.15° =
8. cos 16° =
9. cos 89° =
10. cos 87.4° =
11. cos 89.24° =
12. cos 69.7° =
13. cos 81.45° =
14. cos 29.6° =
15. cos 5° =
16. cos 47° 30′ =
17. cos 87° 30′ =
18. cos 49° 20′ =
19. cos 73.6° =
20. cos 31.5° =
21. cos 55° 36′ =
22. cos 89° 13′ =
23. cos 28.4° =
24. cos 42.8° =
    sin 42.8° =
25. cos 72.7° =
    sin 72.7° =
26. cos 18.6° =
    sin 18.6° =
27. cos 52° 20′ =
    sin 52° 20′ =
28. cos 34.3° =
    sin 34.3° =

14.5 The tangent of angles less than 45° (ST and T scales)

For the range covered by the ST scale, the angles are small enough so that the sine and tangent are substantially equal. At least, they are close enough so that we cannot distinguish the difference on the slide rule. Therefore, if the hairline is moved over an angle on ST, the reading on C corresponds to either the sine or the tangent. This, of course, accounts for the scale designation "ST" (sine or tangent).

Example 1: tan 2.4° = ?

1. Close rule and move HL over 2.4° on ST.
2. Under HL read "419" on C or D. Answer is ***.0419.***

For angles beyond the ST range, we must refer to the T scale. If you examine this scale, you will see that it extends from 5.7° to 45° (tangents of angles in this range will lie between 0.1 and 1.0). Using either the ST or T scales, we may read tangents directly on C:

> *To find tan x (x less than 45°):*
> 1. Move HL over x on **T** or **ST.**
> 2. Under HL read *tan x* on **C** (or **D** if rule is closed).
>
> *To place the decimal point:*
> 1. If x is located on T, tan x is between 0.1 and 1.0.
> 2. If x is located on ST, tan x is between .01 and 0.1.

Example 2: tan 21° = ?

1. Close rule and move HL over 21° on T.
2. Under HL read "384" on C or D.

Angle is located on T; hence, result is between 0.1 and 1.0. Answer is ***0.384.***

Verify the following:

1. tan 17° = 0.306

2. tan 8.75° = 0.1539

3. tan 28.6° = 0.545

4. tan 21.2° = 0.388

5. tan 6.45° = 0.1130

6. tan 3.24° = .0565

7. tan 41° 20′ = 0.880

8. tan 4° 30′ = .0785

9. tan 12° 30′ = 0.222

10. tan 32.2° = 0.630

11. tan 0° 45′ = .01309

12. tan 1.65° = .0288

14.6 The tangent of angles greater than 45°

Again referring to the T scale, you will observe that, just as on the S scale, the complementary angles are indicated. Thus, although the regular T scale extends only to 45°, the complementary scale on T ranges from 45° to about 84.3°.

Recall now that $\cot x = \tan(90° - x)$; hence, if the hairline is set over an angle on the complementary scale of T, we will read the cotangent directly on C. But the tangent is just the reciprocal of the cotangent; therefore, the tangent will be under the hairline on CI (or DI if rule is closed).

Example 1: tan 63° = ?

1. Close rule and move HL over 63° on the complementary scale of T. The hairline is now over cot 63° on C, and over tan 63° on CI (or DI).
2. Under HL read "1963" on CI or DI.

The reading on C (or D) is between 0.1 and 1.0; hence, the reciprocal must be between 1 and 10. Answer is ***1.963.***

The general procedure may be stated:

To find tan x (x greater than 45°):
1. Move HL over *complement of x* on **T** or **ST.**
2. Under HL read *tan x* on **CI** (or **DI** if rule is closed).

To place the decimal point:
1. If complement of x is on T, tan x is between 1 and 10.
2. If complement of x is on ST, tan x is between 10 and 100.

Example 2: tan 78.5° = ?

1. Close rule and move HL over 78.5° on the complementary scale of T.
2. Under HL read "492" on CI or DI.

Complement is on T; hence, result is between 1 and 10. Answer is ***4.92.***

Example 3: tan 88.8° = ?

First obtain the complement: 90° − 88.8° = 1.2°.

1. Close rule and move HL over 1.2° on ST.
2. Under HL read "477" on CI or DI.

Complement is on ST; hence, result is between 10 and 100. Answer is ***47.7.***

Notice that by reading from the direct angle to C, or by reading from the complementary angle to CI, we are able to evaluate tangents of angles between 0.573° and 89.427°. The values of the tangents in this range lie between .01 and 100. If the rule is closed and you are reading on the C or D scales, the tangent is less than 1; when you are reading on the CI or DI scales, the tangent is greater than 1. For very small angles less than 0.573°, refer to Appendix A.

Verify the following:

1. tan 48.5° = 1.130
2. tan 62° = 1.881
3. tan 89.2° = 71.6
4. tan 46.2° = 1.043
5. tan 58.7° = 1.645
6. tan 84.8° = 11.02
7. tan 53° 18′ = 1.342
8. tan 80.6° = 6.04
9. tan 87° 30′ = 22.9

14.7 The extended T scale

Some slide rules have another direct T scale ranging from 45° to 84.3° which we shall refer to as the "extended" T scale (on Pickett rules, this scale is back-to-back with the regular T scale). If the hairline is set over an angle on this extended scale, the tangent may be read directly on the C scale. For angles on this scale, the tangent lies between 1 and 10. If the angle is greater than 84.3°, the tangent is found in the same manner described in the previous section.

Exercise 14-3

1. tan 33° =
2. tan 17° =
3. tan 55° =
4. tan 78° =
5. tan 2° =
6. tan 87° =
7. tan 13.4° =
8. tan 37.6° =
9. tan 51° =
10. tan 67.4° =
11. tan 3.7° =
12. tan 1.64° =
13. tan 14.3° =
14. tan 73.6° =
15. tan 49.4° =
16. tan 86.3° =
17. tan 89.15° =
18. tan 4° 15′ =

19. tan 0° 50′ =

20. tan 11.65° =

21. tan 80.4° =

22. tan 37° 20′ =

23. tan 21° 15′ =

24. tan 52.1° =

25. tan 5° 10′ =

26. tan 85.75° =

27. tan 2.66° =

28. tan 43° 40′ =

29. tan 84.05° =

30. tan 7° 50′ =

14.8 The cotangent, secant, and cosecant

To find the cotangent we either locate the angle on the complementary scale of T and read the cotangent on C, or locate the angle on the direct T scale and read the reciprocal on CI.

Example 1: cot 68° = ?

1. Close rule and move HL over 68° on complementary scale of T.
2. Under HL read "404" on C or D.

Complement is on T; hence, result is between 0.1 and 1.0. Answer is ***0.404.***

Example 2: cot 26° = ?

Here, the angle may be located directly on T, the tangent will be on C, and the desired cotangent is on CI.

1. Close rule and move HL over 26° on T.
2. Under HL read "205" on CI or DI.

The tangent on C is between 0.1 and 1.0; hence, the reciprocal on CI is between 1 and 10. Answer is ***2.05.***

Example 3: cot 86° = ?

First obtain the complement: 90° − 86° = 4°.

1. Close rule and move HL over 4° on ST.
2. Under HL read "699" on C or D.

Complement is on ST; hence, result on C is between .01 and 0.1. Answer is ***.0699.***

Verify the following:

1. cot 54° = 0.727	**4.** cot 33° 30′ = 1.511	**7.** cot 87.35° = .0463
2. cot 70.8° = 0.348	**5.** cot 4° 15′ = 13.47	**8.** cot 78.4° = 0.205
3. cot 12.4° = 4.55	**6.** cot 8.3° = 6.86	**9.** cot 2.44° = 23.5

The secant and cosecant can be evaluated as reciprocals of the cosine and sine respectively.

Example 4: sec 52° = ?

1. Close rule and move HL over 52° on the complementary scale of S. The cosine is now on C (or D), and the secant is on CI (or DI).
2. Under HL read "1624" on CI or DI.

The cosine is between 0.1 and 1.0; hence, reciprocal is between 1 and 10. Answer is ***1.624.***

Example 5: csc 3° = ?

We find sin 3° and read the reciprocal.

1. Close rule and move HL over 3° on ST. The sine is now on C and the cosecant is on CI (or DI).
2. Under HL read "1911" on CI or DI.

Angle is on ST; hence, sine is between .01 and 0.1 and the reciprocal must be between 10 and 100. Answer is ***19.11.***

Verify the following:

1. sec 28° = 1.133	**3.** csc 14.6° = 3.97	**5.** sec 85.7° = 13.33
2. sec 62.4° = 2.16	**4.** csc 54° 30′ = 1.228	**6.** csc 5.9° = 9.73

14.9 A summary of procedures with the trigonometric scales

The following summary may be useful; note especially the general remarks about decimal point placement.

To find sin x or csc x:
1. Move HL over x on **S** or **ST.**
2. Under HL read *sin x* on **C,** *csc x* on **CI.**

To find cos x or sec x:
1. Move HL over *complement of x* on **S** or **ST.**
2. Under HL read *cos x* on **C,** *sec x* on **CI.**

To find tan x or cot x:
Angles less than 45°:
1. Move HL over x on **T** or **ST.**
2. Under HL read *tan x* on **C,** *cot x* on **CI.**

Angles greater than 45°:
1. Move HL over *complement of x* on **T** or **ST.**
2. Under HL read *tan x* on **CI,** *cot x* on **C.**

To place the decimal point:
1. When reading from the S or T scale to:
   a. the C scale, answer is between 0.1 and 1.0 (.XXX).
   b. the CI scale, answer is between 1 and 10 (X.XX).
2. When reading from the ST scale to:
   a. The C scale, answer is between .01 and 0.1 (.0XXX).
   b. the CI scale, answer is between 10 and 100 (XX.X).

If rule is closed, readings may also be made on D and DI.

Exercise 14-4

All six trigonometric functions are represented in this exercise set.

1. cot 62° =

2. cot 24° =

3. cot 34° =

4. csc 56° =

5. csc 23° =

6. sec 47° =

7. cot 73.5° =

8. cot 56.2° =

9. cot 87.4° =

10. sec 85.7° =

11. sin 71° 20′ =

12. csc 17.3° =

13. tan 22.3° =

14. sin 28.3° =

15. cos 80.55° =

16. csc 35.6° =

17. tan 57.4° =

18. tan 72.6° =

19. cot 18.4° =

20. sin 9° 40′ =

21. csc 72.6° =

22. cos 86.66° =

23. sec 81° 30′ =

24. tan 1° 45′ =

25. cos 33° 15′ =

26. sin 46° 25′ =

27. cot 40.7° =

28. cos 15° =

29. tan 8.45° =

30. tan 88.22° =

31. sin 3° 40′ =

32. cos 58.4° =

33. cot 63.5° =

34. sin 6° 27′ =

35. cos 89.13° =

36. cot 2° 30′ =

37. csc 14.7° =

38. sin 4.05° =

39. sec 76.3° =

40. tan 41.2° =

41. sin 56° 50′ =

42. cot 27.7° =

43. sin 79° =

44. cot 85.8° =

45. csc 2° 20′ =

46. cos 89.15° =

47. tan 52° 45′ =

48. sec 72.5° =

Chapter 15

FURTHER OPERATIONS WITH THE TRIGONOMETRIC SCALES

15.1 Angles greater than 90°; negative angles

These examples assume familiarity with the algebraic signs of the functions in the various quadrants. The reference angle is the acute angle between the terminal side of the angle and the x axis. Negative angles are generated clockwise from the positive x axis.

Example 1: $\sin 217° = ?$

Here, the reference angle is $217° - 180° = 37°$. (See Figure 15.1.) The given angle is in the third quadrant; hence, the sine is negative. Thus, we may write:

$$\sin 217° = -\sin 37°$$

Verify that the result is ***−0.602.***

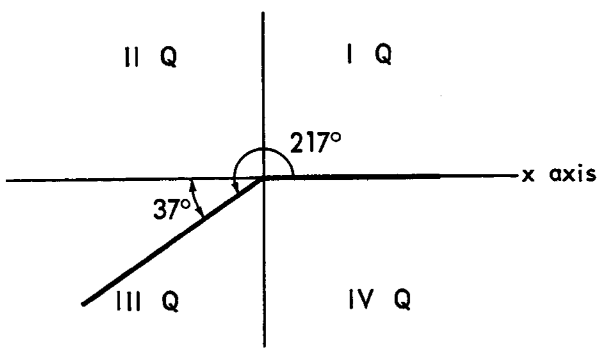


Figure 15.1

Example 2: $\cos 126° = ?$

The reference angle is $180° - 126° = 54°$. The given angle is in the second quadrant; hence, the cosine is negative. We may write:

$$\cos 126° = -\cos 54°$$

Verify that the result is ***−0.588.***

Example 3: $\tan(-114°) = ?$

The reference angle is $180° - 114° = 66°$. (See Figure 15.2.) The given angle is in the third quadrant; hence, the tangent is positive. Therefore, we may write:

$$\tan(-114°) = \tan 66°$$

Verify that the answer is ***2.25.***

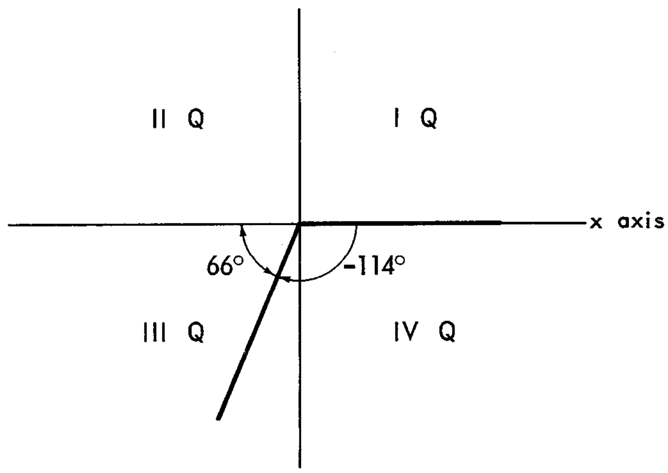


Figure 15.2

Exercise 15-1

1. $\sin 148° =$
2. $\sin 110° =$
3. $\sin 204° =$
4. $\cos 123° =$
5. $\cos 114° =$
6. $\tan 156° =$
7. $\tan 280° =$
8. $\tan 251° =$
9. $\sin(-136°) =$
10. $\sin 312° =$
11. $\cos 137.6° =$
12. $\cos(-32.4°) =$

13. tan 93.3° =

14. sin 176° 30′ =

15. cos 154° 30′ =

16. tan(−112°) =

17. sin 477° =

18. cos 283.7° =

15.2 The inverse trigonometric functions

Here, we have the reverse problem: given the value of a function, find the corresponding angle. You will recall that "arcsin x" or "$\sin^{-1} x$" are notations which simply mean "angle whose sine is x." Similarly, the notations "arctan x" or "$\tan^{-1} x$" mean "angle whose tangent is x," and so forth. We refer to these as "inverse trigonometric functions."

For example, consider the forms:

a. $\sin x = 0.245$; $x = ?$

b. $\sin^{-1} 0.245 = ?$

c. arcsin $0.245 = ?$

Although different notations are used, all three of these state exactly the same problem: find the angle whose sine is 0.245.

Clearly then, evaluating the inverse functions involves a reversal of the procedures outlined in the previous chapter. In the following examples and exercises, we shall assume that the desired angle is the *smallest positive angle* satisfying the condition.

Example 1: $\sin x = 0.676$; $x = ?$

1. Close rule and move HL over 676 on C or D. Note that the function is between 0.1 and 1.0; hence, angle is on S.
2. Under HL read ***42.5°*** on S.

Example 2: $\tan x = .0365$; $x = ?$

1. Close rule and move HL over 365 on C or D. Function is between .01 and 0.1; hence, angle is on ST.
2. Under HL read ***2.09°*** on ST.

Example 3: $\cos^{-1} 0.1875 = ?$

1. Close rule and move HL over 1875 on C or D. Function is between 0.1 and 1.0; hence, angle is on S. It is a *cosine* function, therefore, read the *complement.*
2. Under HL read ***79.2°*** on complementary scale of D.

Example 4: arctan $1.8 = ?$

Here, the tangent is between 1 and 10; hence, angle is greater than 45°. It follows, therefore, that the hairline is set on CI and the result is read on the complementary scale of T.

1. Close rule and move HL over 180 on CI or DI.
2. Under HL read ***60.95°*** on complementary scale of T.

Example 5: arcsin(−0.415) = ?

1. Close rule and move HL over 415 on C or D.
2. Under HL read "24.5°" on S.
3. The function is negative, and we know that the sine is negative in the third and fourth quadrants. Since it has been agreed to choose the smallest positive angle, we select the third quadrant. Hence, the desired angle is 180° + 24.5° = ***204.5°***.

Example 6: $\tan^{-1}(-0.161) = ?$

1. Close rule and move HL over 161 on C or D.
2. Under HL read "9.15°" on T.
3. The function is negative, and the tangent is negative in the second and fourth quadrants. The smallest positive angle will lie in the second quadrant; hence, result is 180° − 9.15° = ***170.85°***.

Exercise 15-2

Find the *smallest positive angle.*

1. $\sin x = 0.320;\ x =$
2. $\sin x = 0.575;\ x =$
3. $\sin x = 0.834;\ x =$
4. $\tan A = 0.322;\ A =$
5. $\tan A = 0.662;\ A =$
6. $\tan x = 0.815;\ x =$
7. $\cos A = 0.900;\ A =$
8. $\cos A = 0.600;\ A =$
9. $\cos B = 0.319;\ B =$
10. $\sin x = .0521;\ x =$
11. $\sin B = .0378;\ B =$
12. $\cos B = .0450;\ B =$
13. $\cos x = 0.891;\ x =$
14. $\tan x = .0366;\ x =$
15. $\tan x = .0610;\ x =$
16. $\sin C = 0.950;\ C =$
17. $\tan C = 1.303;\ C =$
18. arctan 4.00 =
19. arcsin .0901 =
20. arctan 0.1875 =
21. arctan 1.13 =
22. arccos 0.764 =
23. arctan 22.9 =
24. arctan 13.2 =
25. arccos 0.269 =
26. arcsin .0733 =
27. arcsin −0.381 =
28. arccos −0.205 =
29. arccos −0.474 =
30. arctan −0.924 =

31. arctan .01397 =

32. arcsin 0.1095 =

33. arctan −0.294 =

34. $\cos^{-1} -0.948$ =

35. $\cos^{-1}$ .0362 =

36. $\tan^{-1}$ 52.5 =

37. $\tan^{-1}$ 2.95 =

38. $\sin^{-1}$ .0194 =

39. $\sin^{-1}$ .0734 =

40. $\tan^{-1}$ 1.85 =

41. $\cos^{-1}$ −.0910 =

42. $\tan^{-1}$ 10.0 =

43. $\tan^{-1}$ 0.667 =

44. $\sin^{-1}$ .0276 =

45. $\cos^{-1}$ .0279 =

46. $\cos^{-1}$ .01535 =

47. $\sin^{-1}$ −0.245 =

48. $\tan^{-1}$ 1.085 =

49. $\cos^{-1}$ 0.332 =

50. $\tan^{-1}$ 15.45 =

51. $4\sin^2 A + 5\sin A - 6 = 0$; A =

52. $6\sin^2 x - 11\sin x + 3 = 0$; x =

15.3 Combined operations with trigonometric functions

When multiplying or dividing trigonometric functions by other numbers, simply bear in mind that the ST, S, and T scales are directly related to the C scale. In fact, you may think of these scales as "auxiliary C scales" which are merely graduated and labeled in different ways. In multiplication and division, therefore, the ST, S, and T scales operate in the same manner as does the C scale.

Example 1: 15.2 sin 22° = ?

1. Set left index of C opposite 152 on D.
2. Move HL over 22° on S. This puts HL over sin 22° on C; hence, we have performed the desired multiplication.
3. Under HL read "570" on D.

The angle occurs on the S scale; hence, sin 22° is between 0.1 and 1.0. The product, therefore, must be between 1.52 and 15.2.

Answer is ***5.70***.

Example 2: 435 sin 36.5° = ? 435 cos 36.5° = ?

1. Set right index of C opposite 435 on D.
2. Move HL over 36.5° on S. This multiplies by sin 36.5°.

3. Under HL read "259" on D.
4. Move HL over 36.5° on *complementary scale* of S. This multiplies by cos 36.5°.
5. Under HL read "350" on D.

Answers are ***259*** and ***350.***

Example 3: $\frac{26.4}{\cos 52.2°} = ?$

1. Move HL over 264 on D.
2. Slide 52.2° on *complementary scale* of S under HL.
3. Opposite right index of C read "431" on D.

Answer is ***43.1.***

Example 4: $1340 \tan 3.44° = ?$

1. Set left index of C opposite 1340 on D.
2. Move HL over 3.44° on ST.
3. Under HL read "805" on D.

The angle is on ST; hence, tan 3.44° is between .01 and 0.1, and the product must be between 13.4 and 134.

Answer is ***80.5.***

Verify the following:

1. $145 \cos 71.5° = 46.0$
2. $8.6 \sin 37° = 5.18$
3. $\frac{2.72}{\cos 46.2°} = 3.93$
4. $75 \tan 28.4° = 40.5$
5. $36.4 \sin 1.2° = 0.763$
6. $\frac{680}{\tan 18°30'} = 2030$

Example 5: $12.5 \tan 63° = ?$

Evaluate this as 12.5/cot 63°.

1. Move HL over 125 on D.
2. Slide 63° on complementary scale of T under HL.
3. Opposite right index of C, read "245" on D. Answer is ***24.5.***

Example 6: $6.25 \cot 37° = ?$

Evaluate this as 6.25/tan 37°. Verify that result is ***8.30.***

Example 7: $\dfrac{2350 \sin 2.3°}{4.66 \tan 17.5°} = ?$

1. Move HL over 235 on D.
2. Slide 466 on C under HL.
3. Move HL over 2.3° on ST.
4. Slide 17.5° on T under HL.
5. Opposite right index of C read "642" on D.

To place the decimal point, first observe that sin 2.3° is about .04, and tan 17.5° is about 0.3 (this can be done by simply glancing from the ST and T scales to the C scale). We may now write:

$$\frac{2350 \times \sin 2.3°}{4.66 \times \tan 17.5°} \approx \frac{2000 \times .04}{5 \times 0.3} = \frac{16}{0.3} \approx 50.$$ Answer must be ***64.2.***

Exercise 15-3

1. $12.8 \sin 46° =$
2. $64 \sin 33.4° =$
3. $7.6 \tan 26.2° =$
4. $125 \cos 54° =$
5. $1450 \sin 2.25° =$
6. $6.57 \tan 34.3° =$
7. $12.8 \sin 11° 30' =$
8. $630 \sin 62.3° =$
$630 \cos 62.3° =$
9. $12.65 \sin 22.4° =$
$12.65 \cos 22.4° =$
10. $7500 \sin 12.45° =$
$7500 \cos 12.45° =$
11. $344 \sin 37.2° =$
$344 \cos 37.2° =$
12. $115 \sin 127° =$
$115 \cos 127° =$
13. $9.60 \sin 206.2° =$
$9.60 \cos 206.2° =$
14. $478 \sin 18° 20' =$
$478 \cos 18° 20' =$
15. $\dfrac{36.2}{\sin 44°} =$
16. $\dfrac{267}{\cos 32°} =$
17. $1275 \tan 3° 30' =$
18. $\dfrac{2.85}{\sin 2.34°} =$
19. $\dfrac{345}{\tan 32.4°} =$
20. $\dfrac{72.5}{\sin 52°} =$
21. $\dfrac{2400}{\cos 58.2°} =$
22. $\dfrac{4.73}{\cos 15°} =$
23. $\dfrac{66.7}{\sin 18° 15'} =$
24. $6.8 \tan 71° =$
25. $2450 \tan 53.5° =$

26. $19.6 \cot 34^\circ =$

27. $480 \cot 18.4^\circ =$

28. $\dfrac{36.2 \sin 15^\circ}{2.75 \sin 38^\circ} =$

29. $\dfrac{\sin 46^\circ 30'}{.026 \sin 12^\circ 30'} =$

30. $\dfrac{175 \cos 48^\circ}{\sin 22^\circ} =$

31. $10.8 \cos 63^\circ \sin 23^\circ =$

32. $47.5 \sin 37^\circ \sin 2.4^\circ =$

33. $\dfrac{22.6 \sin^2 18.4^\circ}{1.66} =$

34. $\frac{1}{2}(21.3)(18.4) \sin 13^\circ 15' =$

35. $\frac{1}{2}(4.66)(7.43) \sin 128.5^\circ =$

36. $(16.45)(3.08) \cos 33.6^\circ =$

37. $(12.54)(4.88) \cos 129^\circ =$

38. $\dfrac{5600 \tan 3^\circ 30'}{24.4 \tan 65^\circ} =$

39. $\dfrac{246 \sin 32.4^\circ \cos 51.5^\circ}{\sin 11.25^\circ} =$

40. $\dfrac{6800 \sin 1.65^\circ}{23.5 \tan 31.2^\circ} =$

15.4 Radian measurement

Radians and degrees are related as follows:

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

Within slide rule accuracy, $\frac{180}{\pi} = 57.3$, and we use the approximate relation:

$$1 \text{ radian} = 57.3^\circ$$

The conversion formulas may be stated:

$$\text{angle(radians)} = \text{angle(degrees)} \div 57.3$$
$$\text{angle(degrees)} = \text{angle(radians)} \times 57.3$$

Many slide rules have a scribed mark at "573" on C or D (or both).

Example 1: Convert 27.6 ° to radians.

We must divide by 57.3:

1. Move HL over 276 on D.
2. Slide 573 on C under HL.
3. Opposite right index of C, read "482" on D.

Answer: 27.6° = ***0.482*** radians.

Example 2: sin 0.64 = ?

When the degree symbol is omitted, it is understood that the angle is in *radians*. Thus, sin 0.64 means "the sine of 0.64 radians." We must, therefore, first convert 0.64 radians to degrees, then find the sine using the S scale.

1. Verify that 0.64 radians = 36.7°.
2. Verify that sin 0.64 = sin 36.7° = ***0.598.***

Verify the following:

1. 28.2° = 0.492 radians	**5.** sin 0.44 = 0.426
2. 76.5° = 1.336 radians	**6.** tan 0.275 = 0.282
3. 1.44 radians = 82.5°	**7.** cos 1.1 = 0.454
4. 0.38 radians = 21.8°	**8.** sin 2.3 = 0.745

15.5 Using the ST scale for radian conversion

For small angles (in the range of ST or smaller), the sine or tangent is approximately equal to the angle itself expressed in radians. Thus, sin 2° is about numerically equal to the radian equivalent of 2°. Let us check this on the slide rule:

1. First move HL over 2° on ST, and verify that sin 2° = ***.0349.***
2. Now convert 2° to radians (divide by 57.3), and verify that 2° = ***.0349*** radians.

Therefore, to convert a small angle x (in the ST range) to radians, we simply find sin x using the ST scale, and this also represents the radian equivalent of the angle. For this reason, the ST scale on K & E slide rules is labeled "SRT"; that is, the scale may be used to find *sines, radian* equivalents, or *tangents* of small angles.

Example: Convert 2.4° to radians.

We proceed exactly as if we are finding sin 2.4°:

1. Move HL over 2.4° on ST.
2. Under HL read "419" on C.

Answer: 2.4° = ***.0419*** radians.

Exercise 15-4

1. Convert to radians:
   a. 32°, **b.** 68.4°, **c.** 11.4°, **d.** 145° 30′, **e.** 223°

2. Convert to radians (use ST scale):
a. 3.65°, **b.** 4.50°, **c.** 1.84°, **d.** 0° 45′, **e.** 2.76°

3. Convert to degrees:
a. 2.33 rad., **b.** 0.76 rad., **c.** 1.08 rad., **d.** 5.24 rad., **e.** 0.215 rad.

4. Convert to degrees (use ST scale):
a. .062 rad., **b.** .0345 rad., **c.** .0175 rad., **d.** .055 rad., **e.** .0765 rad.

5. sin 1.05 =

6. tan 0.64 =

7. cos 0.22 =

8. sin 0.445 =

9. sin 2.24 =

10. tan 0.36 =

11. cos 1.85 =

12. sin 1.26 =

13. tan 0.175 =

14. sin 4 =

15. sin(−0.72) =

16. cos(−2.9) =

15.6 Formula types

Example 1: $$\sqrt{\frac{36.4}{1 + 2.75 \sin^2 26.5^\circ}} = ?$$

1. Verify that $2.75 \sin^2 26.5^\circ = 0.547$.
2. Expression may now be evaluated:

$$\sqrt{\frac{36.4}{1 + 0.547}} = \sqrt{\frac{36.4}{1.547}} = \boldsymbol{4.85}$$

Example 2: Given the formula: $A = \frac{1}{2} r^2(\Theta - \sin \Theta)$.

Find A when $r = 12.6$ and $\Theta = 0.740$ radians.

1. Verify that $\sin 0.740 = \sin 42.4^\circ = 0.674$.
2. Formula becomes:

$$A = \frac{1}{2}(12.6)^2(0.740 - 0.674) = \frac{1}{2}(12.6)^2(.066) = \boldsymbol{5.24}.$$

Exercise 15-5

1. $53.5(1 - \cos 48^\circ) =$

2. $\frac{7.22}{3.15 + 2.75 \sin 32^\circ} =$

3. $\frac{3.65 \cos 36^\circ + 2.88}{\tan 36^\circ} =$

4. $\dfrac{82.2}{\cos 27^\circ \cot 15.2^\circ - \sin 27^\circ} =$

5. $\sqrt{14.7 \cos^2 71^\circ + 4.66} =$

6. $\sqrt{1 + \pi \sin^2 61^\circ} =$

7. $\dfrac{1 - 0.34 \tan 24^\circ}{1 + 0.34/\tan 24^\circ} =$

8. $\sqrt{\dfrac{253}{5.64 - 3.22 \sin^2 38.4^\circ}} =$

9. $\sqrt{(16.3)^2 + (7.44)^2 - 2(16.3)(7.44) \cos 56^\circ} =$

10. $\sqrt{(23.7)^2 + (38.2)^2 - 2(23.7)(38.2) \cos 118^\circ} =$

11. $\sin^{-1} \left(\dfrac{12.8 + 8.44}{12.8 \times 8.44}\right) =$

12. $\tan^{-1} \left[\dfrac{123 - 2.6(4.4)^2}{2.6 \times 4.4}\right] =$

13. $\cos^{-1} \left[\dfrac{(5.7)^2 + (6.2)^2 - (4.8)^2}{2 \times 5.7 \times 6.2}\right] =$

In the following formulas, substitute the given data and evaluate:

14. $S_n = \dfrac{1}{2} S(1 - \cos 2\Theta)$

a. $S = 2450$, $\Theta = 23.6^\circ$; **b.** $S = 16{,}700$, $\Theta = 35.2^\circ$

15. $R = \dfrac{V^2 \sin 2\Theta}{2g}$

a. $V = 1450$, $\Theta = 18^\circ\ 20'$, $g = 32.2$
b. $V = 825$, $\Theta = 28^\circ\ 30'$, $g = 32.2$

16. $\tan \Theta = \dfrac{a \cos \alpha}{a \sin \alpha + g}$ $(0^\circ < \Theta < 90^\circ)$

a. $\alpha = 28.2^\circ$, $a = 12.4$, $g = 32.2$. Find Θ.
b. $\alpha = 34.9^\circ$, $a = 21.6$, $g = 32.2$. Find Θ.

17. $n = \dfrac{\sin i}{\sin r}$

a. $i = 36.2^\circ$, $r = 25.4^\circ$; **b.** $i = 56.2^\circ$, $r = 32.8^\circ$

18. $A = \dfrac{1}{2} n\, r^2 \sin \left[\dfrac{360^\circ}{n}\right]$

a. $n = 14$, $r = 15.1$; **b.** $n = 17$, $r = 7.6$

19. $A = n\,r^2 \tan\left[\dfrac{180°}{n}\right]$

a. $n = 17$, $r = 8.2$; **b.** $n = 13$, $r = 21.5$

20. $c = \sqrt{a^2 + b^2 - 2ab\cos C}$

a. $a = 6.3$, $b = 4.7$, $C = 42°$

b. $a = 12.7$, $b = 16.2$, $C = 123°\ 30'$

21. $R = \dfrac{T\cos\Theta}{1 - \cos\phi\sin\Theta}$

a. $T = 350$, $\Theta = 31°$, $\phi = 58°$

b. $T = 2400$, $\Theta = 27.5°$, $\phi = 36.3°$

22. $\phi = \tan^{-1}\dfrac{2d/r}{(1/r)^2 - 1}$ $(0° < \phi < 90°)$

a. $r = 0.36$, $d = 3.47$; **b.** $r = 0.22$, $d = 4.65$

23. $T = \dfrac{\tan\alpha + f/\cos\Theta}{1 - f\tan\alpha/\cos\Theta}$

a. $f = 0.15$, $\alpha = 23°$, $\Theta = 32°$

b. $f = 0.24$, $\alpha = 18°$, $\Theta = 27°$

24. $\tan 2\Theta = \dfrac{S_{xy}}{\frac{1}{2}(S_y - S_x)}$ $(0° < \Theta < 45°)$

a. $S_{xy} = 7500$, $S_x = 12{,}400$, $S_y = 15{,}240$. Find Θ.

b. $S_{xy} = 13{,}500$, $S_x = 18{,}400$, $S_y = 22{,}600$. Find Θ.

25. $A = \dfrac{A_0\sin\Theta}{1 - (w/w_n)^2}$

a. $A_0 = 4.65$, $\Theta = 54°$, $w = 340$, $w_n = 825$

b. $A_0 = 21.2$, $\Theta = 48°$, $w = 460$, $w_n = 710$

26. $T = \dfrac{4R\sin\frac{1}{2}\Theta}{\Theta + \sin\Theta}$

a. $R = 8.64$, $\Theta = 0.42$ radians; **b.** $R = 21.7$, $\Theta = 1.22$ radians

27. $F_1 = F_2\left[\dfrac{\tan\Theta\cos\alpha}{\cos\beta} + \tan\beta\sin\alpha\right]$

a. $F_2 = 340$, $\Theta = 18°$, $\alpha = 22°$, $\beta = 26°$

b. $F_2 = 76$, $\Theta = 21.5°$, $\alpha = 25.6°$, $\beta = 29.5°$

28. $A = \dfrac{1}{2}\pi r^2 - \left[x\sqrt{r^2 - x^2} + r^2\sin^{-1}\left(\dfrac{x}{r}\right)\right]$ $\left(\sin^{-1}\left(\dfrac{x}{r}\right) \text{ in } \textit{radians}\right)$

a. $r = 11.7$, $x = 6.2$; **b.** $r = 6.25$, $x = 3.75$

29. $S = 2\pi b^2 + \dfrac{2\pi ab}{e}\sin^{-1}e$ ($\sin^{-1}e$ in *radians*)

a. $a = 8.5$, $b = 6.4$, $e = 0.66$

b. $a = 14.6$, $b = 13.5$, $e = 0.38$

30. $\cos \frac{1}{2}A = \sqrt{\frac{\sin s \sin(s - a)}{\sin b \sin c}}$ $(0° < A < 180°)$

a. $a = 60°$, $b = 135°$, $c = 110°$, $s = 152.5°$. Find A.

b. $a = 124°$, $b = 48°$, $c = 145°$, $s = 158.5°$. Find A.

31. $Y = e \sec\left[\frac{L}{2}\sqrt{\frac{P}{EI}}\right]$

a. $E = 29 \times 10^6$, $P = 22{,}000$, $I = 14.8$, $L = 87$, $e = 0.75$

b. $E = 29 \times 10^6$, $P = 18{,}500$, $I = 21.5$, $L = 120$, $e = 0.85$

Chapter 16

THE RIGHT TRIANGLE

16.1 Relations between the sides and angles

If A and B represent the acute angles of a right triangle, we let a and b denote the sides opposite A and B respectively, and let c denote the hypotenuse (Figure 16.1).

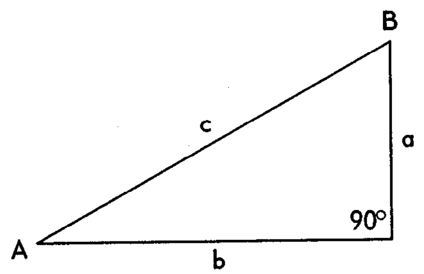


Figure 16.1

In the following section, we shall make use of the relation $A + B = 90°$, and we must also recall the definitions:

$$\sin A = \frac{a}{c} \qquad \sin B = \frac{b}{c}$$

$$\cos A = \frac{b}{c} \qquad \cos B = \frac{a}{c}$$

$$\tan A = \frac{a}{b} \qquad \tan B = \frac{b}{a}$$

16.2 Solution of the triangle using the function definitions

A right triangle is uniquely determined if an acute angle and a side are known, or if two sides are known. Given the known parts, the problem is to find the desired unknown parts. By choosing the proper function (sine, cosine, or tangent), we can always find the unknown parts in terms of the known. It is usually worthwhile to make a sketch roughly to scale as a check on the reasonableness of the solution.

Example 1: Given $A = 34.2°$ and $c = 346$. Find sides a and b (Figure 16.2).

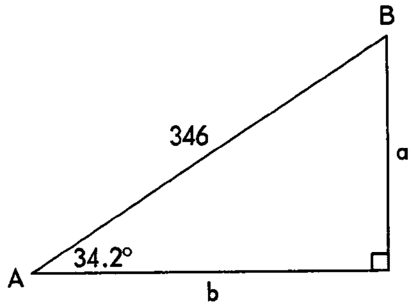


Figure 16.2

1. Write: $a = 346 \sin 34.2°$; $b = 346 \cos 34.2°$.
2. Verify that $a = \mathbf{194.5}$ and $b = \mathbf{286}$.

Example 2: Given $c = 46$ and $a = 21$. Find remaining parts (Figure 16.3).

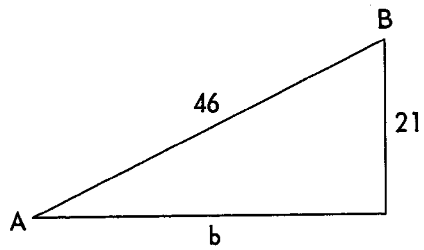


Figure 16.3

1. Write: $\sin A = \frac{21}{46}$. Verify that $A = \mathbf{27.2°}$.
2. Obtain $B = 90° - 27.2° = \mathbf{62.8°}$. This may also be read on the complementary scale in step (1).
3. Write: $b = 46 \cos 27.2°$. Verify that $b = \mathbf{40.9}$.

Example 3: Given $a = 4.2$ and $b = 3.4$. Find remaining parts (Figure 16.4).

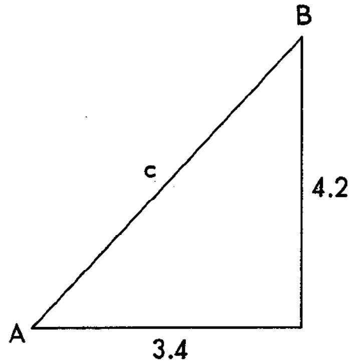


Figure 16.4

When the two legs are given, we first find the *smaller* acute angle using the tangent relation. The smaller angle will be less than 45°; hence, the answer will appear directly on the T (or ST) scale. In this example, B is the smaller angle (it is opposite the shorter side).

1. Write: $\tan B = \frac{3.4}{4.2}$. Verify that $B = \mathbf{39°}$.
2. Write: $A = 90° - 39° = \mathbf{51°}$. Again, this may be read on the complementary scale in step (1).
3. Write: $\sin 51° = \frac{4.2}{c}$, or $c = \frac{4.2}{\sin 51°}$. Verify that $c = \mathbf{5.4}$.

Exercise 16-1

Solve the right triangles (find remaining parts):

1. $A = 26°$, $c = 73$
2. $A = 56.2°$, $c = 315$
3. $a = 35$, $c = 52$
4. $b = 145$, $c = 220$
5. $a = 36$, $b = 59$
6. $a = 1500$, $b = 830$
7. $B = 48.4°$, $c = 425$
8. $a = 6.32$, $c = 9.40$

16.3 Solution using law of sines

Although any right triangle may be solved by the methods of the previous section, the law of sines provides a more efficient slide rule solution. For the right triangle, the law of sines takes the form:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 90°}{c}$$

This relationship will give a complete solution for all cases except where the known parts are the legs, a and b.

The proportion may be solved using the S and D scales. If the hairline is moved over an angle on S, it locates the sine of the angle on C; thus, although angles are set and read on S, the proportion is actually being solved on the C-D scales.

Example 1: Given $A = 38°$ and $a = 480$. Find remaining parts (Figure 16.5).

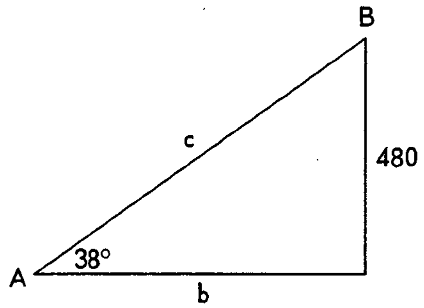


Figure 16.5

First, find B from the complementary relation: $B = 90° - 38° = $ ***52°***. Law of sines becomes:

$$\frac{\text{S}}{\text{D}}: \frac{\sin 38°}{480} = \frac{\sin 52°}{b} = \frac{\sin 90°}{c}$$

The slide rule settings for this proportion are illustrated in Figure 16.6.

1. Move HL over 480 on D.
2. Slide 38° on S under HL. This sets up the known ratio.
3. Move HL over 52° on S. Under HL read "615" on D.
4. Move HL over 90° on S. Under HL read "780" on D.

Answers are: $B =$ ***52°***, $b =$ ***615***, $c =$ ***780***.

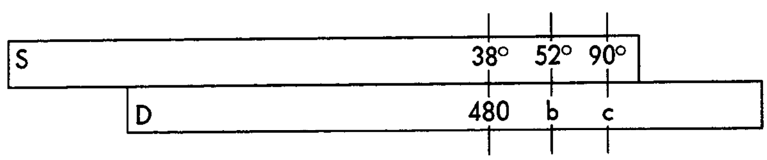


Figure 16.6

Example 2: Given $c = 4.3$ and $a = 1.8$. Find remaining parts (Figure 16.7).

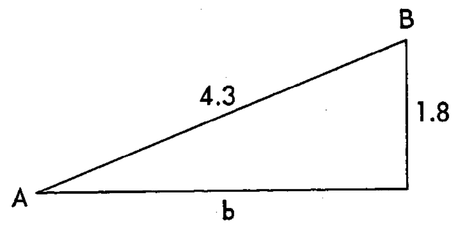


Figure 16.7

Writing the known ratio first, the law of sines becomes:

$$\frac{S}{D}: \frac{\sin 90°}{4.3} = \frac{\sin A}{1.8} = \frac{\sin B}{b}$$

1. Move HL over 43 on D.
2. Slide 90° on S under HL. This sets up the known ratio.
3. Move HL over 18 on D. Under HL read ***24.8°*** on S. This is angle A.
4. Under HL read ***65.2°*** on complementary scale of S. This is angle B.
5. Move HL over 65.2° on S. Under HL read "391" on D.

Answers are: A = ***24.8°***, B = ***65.2°***, b = ***3.91***

Example 3: Given $a = 5$, $c = 62$. Find remaining parts (Figure 16.8).

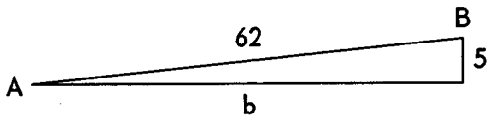


Figure 16.8

Law of sines becomes:

$$\frac{S}{D}: \frac{\sin 90°}{62} = \frac{\sin A}{5} = \frac{\sin B}{b}$$

1. Move HL over 62 on D.
2. Slide 90° on S under HL.
3. Move HL over 5 on D. Under HL read either 53.7° on S or 4.62° on ST. From the sketch it is evident that the smaller angle is the proper choice; hence, A = ***4.62°***. From the complementary relation, we have $B = 90° - 4.62°$ = ***85.38°***.
4. Move HL over 85° on S (this is all the accuracy we have at this end of the scale). Under HL read "618" on D.

Answers are: A = ***4.62°***, B = ***85.38°***, b = ***61.8***.

Example 4: Given $A = 50°$, $b = 7.2$. Find remaining parts (Figure 16.9).

From the complementary relation: $B = 90° - 50°$ = ***40°***. Law of sines becomes:

$$\frac{S}{D}: \frac{\sin 40°}{7.2} = \frac{\sin 50°}{A} = \frac{\sin 90°}{c}$$

1. Move HL over 72 on D.
2. Slide 40° on S under HL.
3. Move HL over 50° on S. Under HL read "858" on D.
4. Now 90° on S (right index of C) is off-scale; hence, we look to the left index of C. Opposite left C index read "1120" on D.

Answers are: B = ***40°***, a = ***8.58***, c = ***11.20***.

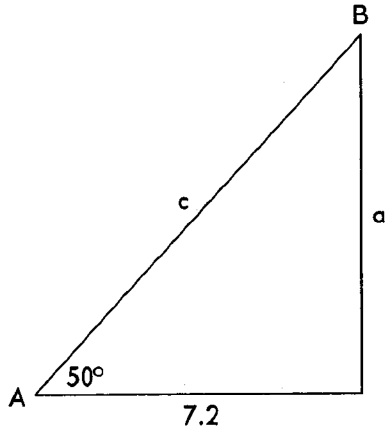


Figure 16.9

Example 5: Given $A = 63°$ and $a = 12$. Find remaining parts.

First, find angle B: $B = 90° - 63° = \mathbf{27°}$. Law of sines becomes:

$$\frac{S}{D}: \frac{\sin 63°}{12} = \frac{\sin 27°}{b} = \frac{\sin 90°}{c}$$

1. Move HL over 12 on D.
2. Slide 63° on S under HL.
3. Move HL over 90° on S. Under HL read "1348" on D.
4. Now, 27° on S is off-scale; hence, we interchange indexes (slide left index of C under HL).
5. Move HL over 27° on S. Under HL read "612" on D.

Answers are: $B = \mathbf{27°}$, $b = \mathbf{6.12}$, $c = \mathbf{13.48}$.

In example 5, the slide would have been in better position, if the ratio had been set up on S-DF rather than S-D. Note, however, that this requires using both sides of the rule. Also, keep in mind that if the ratio is originally set up on S-D, the proportion must be solved entirely on these scales. Similarly, if it is set up on S-DF, it must be solved entirely on S-DF. In either case, off-scale readings must be handled by interchanging indexes.

From the foregoing examples, you will observe that it is not really necessary to write down the law of sines. Since opposite parts of the triangle are located opposite each other on the slide rule (sides on D, angles on S), you can solve directly from the sketch itself.

Exercise 16-2

Solve the following right triangles. Use the law of sines.

1. $A = 25°$, $a = 32$
2. $A = 53°$, $c = 44$
3. $B = 35°$, $a = 6.3$
4. $B = 61°$, $b = 15.6$

5. $A = 47.3°, b = 320$

6. $a = 23, c = 37$

7. $a = 51, c = 83$

8. $b = 840, c = 2800$

9. $B = 17.5°, c = 4.24$

10. $a = 3.65, c = 58$

11. $A = 4.2°, a = 5.55$

12. $B = 13.6°, c = 4000$

13. $a = 1700, c = 3650$

14. $B = 86.2°, a = 3.45$

15. $B = 31.4°, b = 6.76$

16. $a = 12.2, c = 160$

17. $B = 5.1°, b = 1.85$

18. $b = 5600, c = 8250$

19. $A = 15.3°, c = 366$

20. $A = 71°, b = 21.6$

16.4 A special method for the case with 2 legs given

This case arises frequently in vector applications, and the following examples may be worth your special attention. The method involves finding the smaller acute angle using the tangent relation, then completing the solution with the law of sines.

Example 1: Given $a = 4.4$ and $b = 6.3$. Find remaining parts (Figure 16.10).

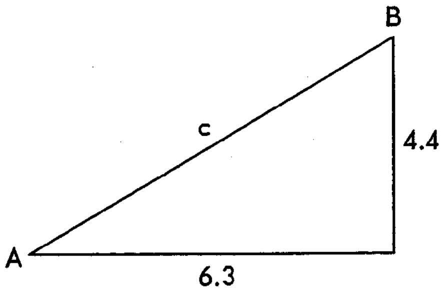


Figure 16.10

Angle A is the smaller angle, and we write the tangent relation:

$$\tan A = \frac{4.4}{6.3}, \text{ or, in proportion form: } \frac{\tan A}{4.4} = \frac{1}{6.3}$$

Solving the proportion:

1. Set right index of C opposite 63 on D.
2. Move HL over 44 on D.
3. Under HL read $A = \mathbf{34.9°}$ on T. (Read $B = \mathbf{55.1°}$ on complementary T scale.)

Applying law of sines: $\dfrac{\sin 34.9°}{4.4} = \dfrac{\sin 90°}{c}$

4. HL is already over 44 on D; hence, simply slide 34.9° on S under HL.
5. Move HL over 90° on S (C index). Under HL read c = ***7.7*** on D.

The foregoing procedure may be summarized as follows:

> *To solve a right triangle with 2 legs given:*
> 1. Set **C index** opposite *longer* leg on **D.**
> 2. Move HL over *shorter* leg on **D.**
> 3. Under HL read *smaller* angle on **T.** (Larger angle may be read on complementary T scale.)
> 4. Slide the *smaller* angle on **S** under HL.
> 5. Opposite **C index** read *hypotenuse* on **D.**

Example 2: Given $a = 345$ and $b = 270$. Find remaining parts (Figure 16.11).

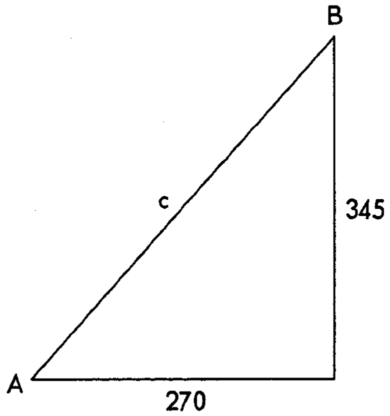


Figure 16.11

In this case, angle B is the smaller angle.

1. Set right index of C opposite 345 on D. (This is the longer leg.)
2. Move HL over 270 on D.
3. Under HL read B = ***38.1°*** on T. (Read A = ***51.9°*** on complementary scale).
4. Slide 38.1° on S under HL.
5. Opposite C index read c = ***438*** on D.

Example 3: Given $a = 7.25$ and $b = 14.60$. Find remaining parts.

Angle A is the smaller angle.

1. Set left C index opposite 146 on D.
2. Move HL over 725 on D.
3. Under HL read A = ***26.4°*** on T. (Read B = ***63.6°*** on complementary T scale.)

4. Slide 26.4° on S under HL.
5. Opposite C index read $c =$ ***16.30*** on D.

Verify the following:

1. Given $a = 65$, $b = 39$; verify that $A = 59°$, $B = 31°$, $c = 75.8$.
2. Given $a = 37.5$, $b = 72.4$; verify that $A = 27.4°$, $B = 62.6°$, $c = 81.5$.
3. Given $a = 1500$, $b = 840$; verify that $A = 60.8°$, $B = 29.2°$, $c = 1720$.

Example 4: Given $a = 24$, $b = 260$. Find remaining parts (Figure 16.12).

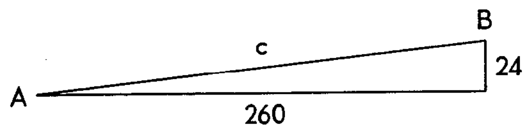


Figure 16.12

1. Set right index of C over 260 on D.
2. Move HL over 24 on D.
3. Under HL read 42.7° on T. However, from the sketch it is evident that angle A cannot be this large. A rough estimate shows us that $\tan A$ is about .09; hence, reading should be made on ST. Under HL read $A =$ ***5.29°*** on ST. (Angle $B = 90° - 5.29° =$ ***84.71°***.) Now if we try to slide 5.29° on S under HL, we are referred back to the same position on ST; therefore, when the smaller angle is in the ST range this procedure does not show any slide rule difference between the hypotenuse and the longer leg. This very slight difference does show up if we use the law of sines with the larger angle:

$$\frac{\sin 84.71°}{260} = \frac{\sin 90°}{c}$$

4. Move HL over 260 on D. Slide 84.71° (85°) on S under HL.
5. Opposite 90° on S (C index), read $c =$ ***261*** on D.

Exercise 16-3

Solve the following right triangles:

1. $a = 12$, $b = 17$
2. $a = 27$, $b = 16$
3. $a = 16$, $b = 41$
4. $a = 750$, $b = 1245$
5. $a = 9.5$, $b = 5.8$
6. $a = 35$, $b = 3$

7. $a = 6.5$, $b = 4.2$

8. $a = 12.5$, $b = 18.6$

9. $a = 230$, $b = 175$

10. $a = 3.6$, $b = 41$

11. $a = 10.75$, $b = 12.34$

12. $a = 0.29$, $b = 5.06$

13. $a = 635$, $b = 580$

14. $a = 1450$, $b = 3100$

15. $a = 25.2$, $b = 420$

16. $a = .075$, $b = .0266$

17. $a = 45.5$, $b = 112.5$

18. $a = 28{,}500$, $b = 26{,}500$

16.5 Complex numbers

The complex number $(a + bj)$, where $j = \sqrt{-1}$, may be represented in the complex plane by the point $P(a,b)$ as illustrated in Figure 16.13.

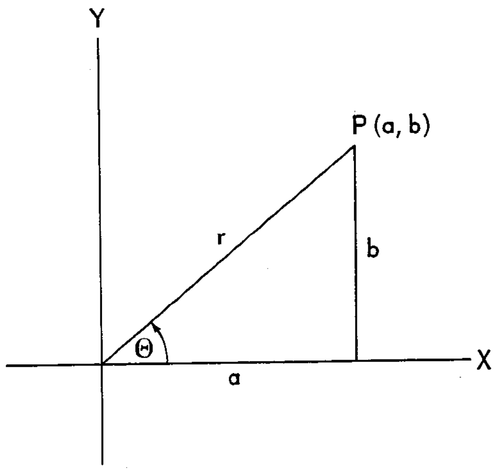


Figure 16.13

At times, it is desirable to represent the point P in terms of the polar coordinates (r,Θ), and from Figure 16.13 it is clear that the following relationships hold:

$$a = r \cos \Theta; \quad b = r \sin \Theta$$

Therefore, we may write:

$$a + bj = r(\cos \Theta + j \sin \Theta)$$

We refer to the left side as the *rectangular* form; the expression on the right is called the *polar* form. The polar form is often indicated by the compact notation: $r\underline{/\Theta}$. Clearly, complex numbers may be used to represent vector quantities, and this is one of their important applications.

It is evident that changing from polar to rectangular form, and vice versa, simply involves solving a right triangle with hypotenuse and side given, or with two legs given.

Example 1: Change $47\angle 38^\circ$ to rectangular form (Figure 16.14).

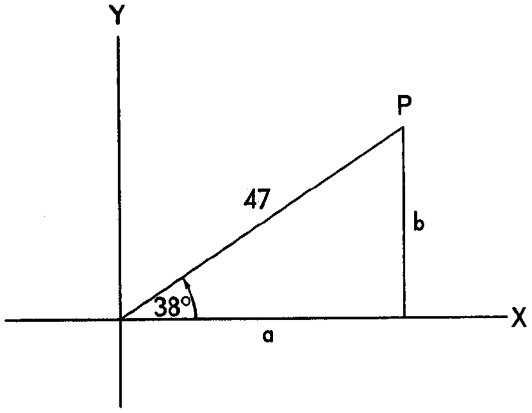


Figure 16.14

Verify that $a = 47 \cos 38^\circ = 37.0$; $b = 47 \sin 38^\circ = 28.9$
Therefore, $47\angle 38^\circ = \mathbf{37.0 + 28.9j}$.

Example 2: Change $(37 + 24j)$ to polar form (Figure 16.15).

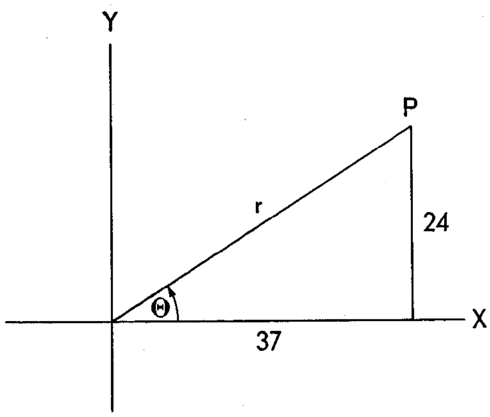


Figure 16.15

1. Set right index of C opposite 37 on D.
2. Move HL over 24 on D. Under HL read $\Theta = \mathbf{33^\circ}$ on T.
3. Slide 33° on S under HL. Opposite right index of C, read $r = \mathbf{44.1}$ on D.

Answer: $37 + 24j = \mathbf{44.1\angle 33^\circ}$.

Example 3: Change $(-6.3 + 8.2j)$ to polar form (Figure 16.16).

1. Set right index of C opposite 82 on D.
2. Move HL over 63 on D. Under HL read $\beta = 37.5^\circ$ on T. From the figure, it follows that $\Theta = 90^\circ + 37.5^\circ = \mathbf{127.5^\circ}$.

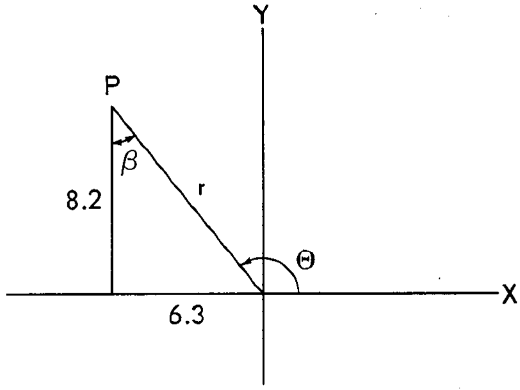


Figure 16.16

3. Slide 37.5° on S under HL. Opposite C index, read $r =$ ***10.35*** on D.

Answer $-6.3 + 8.2j =$ ***10.35/127.5°.***

Exercise 16-4

1. Change to rectangular form:
a. $53\angle 32°$, **b.** $34\angle 56°$, **c.** $180\angle 17°$, **d.** $475\angle 68°$, **e.** $10.6\angle 27.4°$,
f. $670\angle 3.6°$, **g.** $4.3\angle 134°$, **h.** $93.5\angle 152.5°$, **i.** $1060\angle 216°$,
j. $38.2\angle 312.5°$.

2. Change to polar form:
a. $(3 + 7j)$, **b.** $(21 + 52j)$, **c.** $(16 + 5j)$, **d.** $(5.6 + 2.3j)$,
e. $(210 + 750j)$, **f.** $(365 + 23j)$, **g.** $(-27 + 4j)$, **h.** $(-4.55 - 6.26j)$,
i. $(-0.82 - 0.22j)$, **j.** $(3400 - 720j)$.

3. To multiply two complex numbers, we may use the product formula:

$$r_1 \angle \Theta_1 \times r_2 \angle \Theta_2 = r_1 r_2 \angle \Theta_1 + \Theta_2.$$

Represent the following products in *rectangular* form:
a. $3.2\angle 23° \times 2.4\angle 31°$, **b.** $2.8\angle 14° \times 6.5\angle 48°$,
c. $48.2\angle 37.2° \times 0.585\angle 21.1°$, **d.** $5.33\angle 18.2° \times 7.05\angle 4.5°$,
e. $23.6\angle 52.7° \times 3.68\angle 73.3°$, **f.** $21\angle 110° \times 36\angle 140°$.

4. The division formula for complex numbers may be stated:

$$r_1 \angle \Theta_1 \div r_2 \angle \Theta_2 = r_1 / r_2 \angle \Theta_1 - \Theta_2.$$

Represent the following quotients in *rectangular* form:
a. $16\angle 57° \div 2.7\angle 17°$, **b.** $230\angle 96° \div 64\angle 32°$,
c. $125\angle 133° \div 344\angle 68°$, **d.** $5.35\angle 214° \div 7.40\angle 59°$,
e. $37.5\angle 317° \div 4.66\angle 85°$, **f.** $45.3\angle 27.4° \div 6.24\angle 68.2°$.

Chapter 17

THE OBLIQUE TRIANGLE

17.1 Relations between the sides and angles

If A, B, and C represent the angles of a triangle, we let a, b, and c denote the sides opposite A, B, and C, respectively (Figure 17.1).

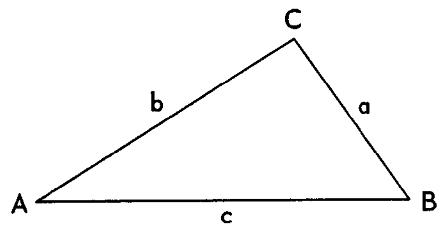


Figure 17.1

The following important relations hold for the general triangle:

Sum of the internal angles: $A + B + C = 180°$

Law of sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos A$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

We shall also make use of the identities:

$$\sin x = \sin(180^\circ - x)$$
$$\cos x = -\cos(180^\circ - x)$$

17.2 Three parts must be known

As with the right triangle, we are concerned here with finding the unknown parts of the triangle in terms of the known parts. For the oblique triangle, we must know three parts, at least one of which is a side. Thus, we may be given two angles and a side, two sides and an angle, or three sides.

In the following sections, slide rule procedures are described for handling these various cases.

17.3 Complete solution using the law of sines

If we are given two angles and a side, or two sides and an angle opposite one of the sides, the triangle may be solved completely using the law of sines.

Example 1: Given $a = 45$, $A = 52^\circ$, and $B = 62^\circ$.

Find the remaining parts (Figure 17.2).

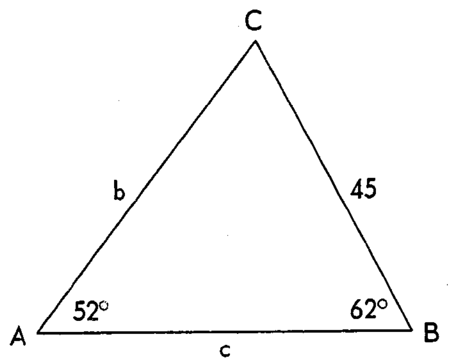


Figure 17.2

First, find angle C: $C = 180^\circ - (52^\circ + 62^\circ) = \boldsymbol{66^\circ}$.
Law of sines now becomes:

$$\frac{S}{D}: \frac{\sin 52^\circ}{45} = \frac{\sin 62^\circ}{b} = \frac{\sin 66^\circ}{c}$$

1. Move HL over 45 on D.
2. Slide 52° on S under HL. This sets up the known ratio.
3. Move HL over 62° on S. Under HL read $b =$ ***50.4*** on D.
4. Move HL over 66° on S. Under HL read $c =$ ***52.1*** on D.

Example 2: Given $C = 70°$, $a = 36$, and $c = 45$.

Find remaining parts (Figure 17.3).

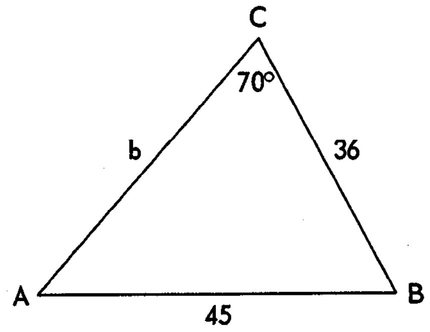


Figure 17.3

Applying the law of sines:

$$\frac{S}{D}: \frac{\sin A}{36} = \frac{\sin B}{b} = \frac{\sin 70°}{45}$$

1. Move HL over 45 on D.
2. Slide 70° on S under HL. This sets up the ratio.
3. Move HL over 36 on D. Under HL read A = ***48.7°*** on S. Solve for angle B: $B = 180° - (48.7° + 70°)$ = ***61.3°.***
4. Move HL over 61.3° on S. Under HL read b = ***42.0.***

Example 3: Given $A = 27.2°$, $C = 34.3°$, and $b = 137$.

Find remaining parts (Figure 17.4).

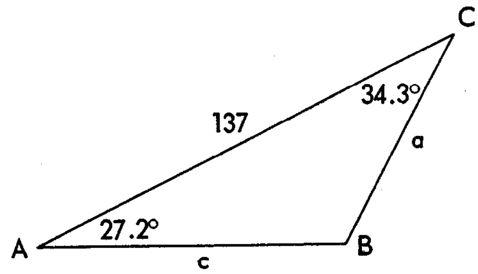


Figure 17.4

First, find angle B: $B = 180° - (27.2° + 34.3°)$ = ***118.5°.***
Law of sines becomes:

$$\frac{\sin 118.5°}{137} = \frac{\sin 27.2°}{a} = \frac{\sin 34.3°}{c}$$

Inasmuch as $\sin 118.5° = \sin(180° - 118.5°) = \sin 61.5°$, the sine law relation may be rewritten:

$$\frac{\sin 61.5°}{137} = \frac{\sin 27.2°}{a} = \frac{\sin 34.3°}{c}$$

Now observe that the slide will be in better position if the proportion is set up on the S-DF scales.

1. Move HL over 137 on DF.
2. Slide 61.5° on S under HL. This sets up the ratio.
3. Move HL over 27.2° on S. Under HL read $a = \mathbf{71.3}$ on DF.
4. Move HL over 34.3° on S. Under HL read $c = \mathbf{87.9}$ on DF.

Exercise 17-1

Solve the following triangles.

1. $A = 66°, B = 48°, a = 42$
2. $B = 37°, C = 63°, b = 26$
3. $a = 6.2, b = 4.6, A = 67°$
4. $a = 85, c = 57, A = 41°$
5. $B = 125°, b = 16, c = 3.5$
6. $A = 126°, C = 15°, b = 17$
7. $A = 75°, B = 62°, c = 10.4$
8. $B = 63.2°, C = 4.8°, a = 6.44$
9. $a = 17.2, c = 31.3, C = 72°$
10. $B = 46°, b = 7.65, c = 5.20$
11. $A = 80° 30', B = 56° 15', b = 7.05$
12. $A = 14° 20', C = 38° 30', a = 148$
13. $C = 154°, a = 21.2, c = 27.4$
14. $A = 3° 30', C = 58° 10', c = 115$
15. $A = 42°, a = 73, b = 28$
16. $b = 2.65, c = 13.7, C = 22.4°$
17. $A = 73°, B = 41.6°, a = 21.7$
18. $a = 12.7, c = 1.65, A = 100°$
19. $B = 159°, b = 355, c = 402$
20. $B = 43° 15', C = 119° 30', a = 236$

17.4 The case with two solutions

Example: Given $A = 45°$, $a = 16$, and $b = 20$. Find remaining parts.

Since the side opposite the given angle is shorter than the other given side, two triangles are possible (Figure 17.5).

Applying law of sines to triangle ABC:

$$\frac{\text{S}}{\text{D}}: \frac{\sin 45°}{16} = \frac{\sin B}{20} = \frac{\sin C}{c}$$

Verify that: $B = \mathbf{62°}$, $C = \mathbf{73°}$, and $c = \mathbf{21.6}$.

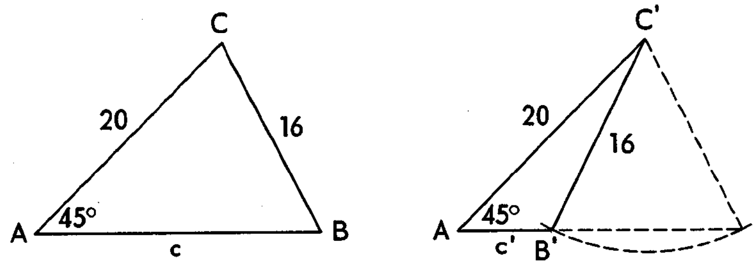


Figure 17.5

Now in triangle $AB'C'$, we first find angles B' and C':

$$B' = 180° - B = 180° - 62° = \boldsymbol{118°}$$
$$C' = 180° - (45° + 188°) = \boldsymbol{17°}.$$

Law of sines becomes:

$$\frac{\sin 45°}{16} = \frac{\sin 118°}{20} = \frac{\sin 17°}{c'}$$

The ratio is already set up on S-D; however, 17° is off-scale on S; hence, we must first interchange indexes. Verify that $c' = \boldsymbol{6.62}$. Summarizing the two solutions:

1. $B = \boldsymbol{62°}, C = \boldsymbol{73°}, c = \boldsymbol{21.6}$; **2.** $B' = \boldsymbol{118°}, C' = \boldsymbol{17°}, c' = \boldsymbol{6.62}$.

Note: If the original proportion had been set up on S-DF, it would not have been necessary to interchange indexes.

Exercise 17-2

Solve the triangles (two solutions possible).

1. $A = 26°$, $a = 4.5$, $b = 5.2$
2. $A = 37°$, $a = 13$, $b = 16$
3. $A = 41°$, $a = 22$, $b = 27$
4. $A = 8.4°$, $a = 2.66$, $c = 6.23$
5. $A = 43°$, $a = 48.2$, $c = 56$
6. $C = 5.3°$, $a = 12.3$, $c = 2.64$
7. $b = 8.65$, $c = 13.24$, $B = 14.6°$
8. $b = 0.65$, $c = 1.65$, $B = 22.6°$

17.5 Application of the law of cosines

The law of cosines is useful when we are given two sides and the included angle, or three sides.

Example 1: Given $A = 37°$, $b = 10$, $c = 15$. Find remaining parts (Figure 17.6).

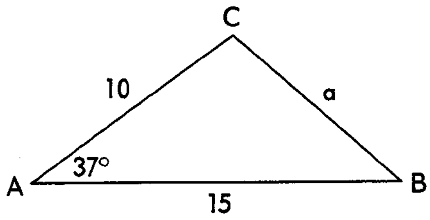


Figure 17.6

1. Use law of cosines to find side a:

$$a^2 = 10^2 + 15^2 - 2(10)(15)\cos 37°$$

$$a = \sqrt{100 + 225 - 300 \cos 37°}$$

Verify that $a = \boldsymbol{9.24}$.

2. Now apply law of sines:

$$\frac{\sin 37°}{9.24} = \frac{\sin B}{10} = \frac{\sin C}{15}$$

Solve for angle B first; it is the smaller angle, and must be acute. Verify that $B = \boldsymbol{40.7°}$.

3. Finally, $C = 180° - (40.7° + 37°) = \boldsymbol{102.3°}$.

Note that $\sin 102.3° = \sin(180° - 102.3°) = \sin 77.7°$, and verify that 77.7° satisfies the proportion; that is, 77.7° is opposite 15.

Example 2: Given $B = 126°$, $a = 13$, $c = 22$. Find remaining parts (Figure 17.7).

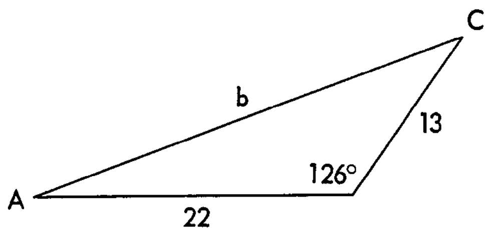


Figure 17.7

1. Use law of cosines to find side b:

$$b^2 = (13)^2 + (22)^2 - 2(13)(22)\cos 126°.$$

But $\cos 126° = -\cos(180° - 126°) = -\cos 54°$; hence, we write:

$$b = \sqrt{(13)^2 + (22)^2 + 2(13)(22)\cos 54°}$$

Verify that $b = \boldsymbol{31.5}$.

2. Apply law of sines:

$$\frac{\sin 126°}{31.5} = \frac{\sin A}{13} = \frac{\sin C}{22}$$

Both A and C must be acute. Verify that $A = \mathbf{19.5°}$, $C = \mathbf{34.5°}$.

Example 3: Given $a = 6.56$, $b = 4.30$, $c = 3.70$. Find remaining parts.

1. We use the law of cosines to find one of the acute angles; say, angle C:

$$\cos C = \frac{(6.56)^2 + (4.30)^2 - (3.70)^2}{2(6.56)(4.30)}$$

Verify that $\cos C = 0.848$, from which $C = \mathbf{32°}$.

2. Apply law of sines:

$$\frac{\sin A}{6.56} = \frac{\sin B}{4.30} = \frac{\sin 32°}{3.70}$$

Now solve for B first (which must also be acute), and then A. Verify that $B = \mathbf{38°}$ and $A = \mathbf{110°}$.

Exercise 17-3

Solve the triangles.

1. $a = 5$, $b = 7$, $C = 38°$

2. $b = 12$, $c = 17$, $A = 49°$

3. $a = 6$, $b = 8$, $c = 11$

4. $a = 12$, $b = 13$, $c = 13$

5. $a = 6$, $c = 11$, $B = 118°$

6. $C = 112.5°$, $a = 10$, $b = 15$

7. $a = 9$, $b = 4$, $c = 12$

8. $a = 11$, $b = 12$, $c = 17$

9. $a = 21.6$, $c = 36.4$, $B = 17.2°$

10. $a = 6.57$, $c = 14.15$, $B = 124.6°$

11. $a = 35$, $b = 29$, $c = 52$

12. $a = 4.75$, $b = 15.6$, $c = 17.25$

13. $a = 7.45$, $c = 3.62$, $B = 104.6°$

14. $A = 5°$, $b = 80$, $c = 60$

15. $a = 245$, $b = 280$, $c = 312$

16. $a = 56.4$, $b = 61.7$, $c = 111$

Chapter 18

LOGARITHMS TO BASE 10 (L SCALE)

18.1 Description of the L scale

The L scale is a uniform scale increasing from 0 to 1. The primary division marks are usually labeled 0, .1, .2, .3, and so forth; however, on some slide rules the decimal point is omitted. Whether shown or not, the decimal point is understood to be there. Depending upon the model, the L scale will be found either on the body of the rule, or on the slide.

18.2 Finding logarithms to base 10

In the following sections, if the base is omitted it is understood that the base 10 is intended; thus "log N" means "$\log_{10} N$."

We recall that, when using tables to find log N, the table gives us the mantissa only; the characteristic is determined by inspection. It will be seen that the L scale takes the place of the table; that is, it supplies the mantissa.

If the L scale is on the body of the rule, it is related to the D scale; if it is on the slide, it is related to the C scale. The relationship may be stated:

> *To find log N:*
> 1. Move HL over N on **D** scale (or **C** scale if L is on slide).
> 2. Under HL read *mantissa* of log N on the **L** scale.
> 3. Determine the characteristic by inspection, and add it to the mantissa for the complete log.

Example 1: log 36.4 = ?

1. Move HL over 364 on D (or C if L is on slide).
2. Under HL read 0.561 on L. This is the mantissa.

Characteristic is 1; hence, log 36.4 = ***1.561.***

Example 2: log 10,850 = ?

1. Move HL over 1085 on D (or C if L is on slide).
2. Under HL read 0.035 on L. This is the mantissa.

Characteristic is 4; hence, log 10,850 = ***4.035.***

Example 3: log .00784 = ?

1. Move HL over 784 on D (or C if L is on slide).
2. Under HL read 0.894 on L. This is the mantissa.

Characteristic is −3; hence, log .00784 = ***7.894 − 10.*** The result could also be presented as the negative number: ***−2.106.***

Example 4: $10^x = 52.7$; $x = ?$

Here, $x = \log 52.7$. Verify that $x =$ ***1.722.***

Verify the following:

1. log 17.5 = 1.243
2. log 524 = 2.719
3. log 1225 = 3.088
4. log .0738 = 8.868 − 10
5. log .00064 = −3.194
6. log 2.07 = 0.316
7. $10^x = 34{,}500$; $x = 4.538$
8. $10^x = .0124$; $x = -1.907$

18.3 Finding the antilogarithm

Here, we are concerned with the reverse problem: given log N, find N. This is called finding the antilogarithm of a number; thus "antilog x" means "the number whose log is x."

Example 1: antilog 3.497 = ?

The mantissa is 0.497; therefore, proceed as follows:

1. Move HL over 0.497 on L.
2. Under HL read "314" on D (or C if L is on slide).

The characteristic is 3; hence, result is ***3140.***

Example 2: $\log N = 9.022 - 10$; $N = ?$

1. Move HL over 0.022 on L.
2. Under HL read "1052" on D (or C if L is on slide).

Characteristic is −1; therefore, $N =$ ***0.1052.***

Example 3: antilog −1.375 = ?

In order to discover the mantissa and characteristic, we add and subtract 10:

$$-1.375 = (-1.375 + 10) - 10 = 8.625 - 10$$

We now see that the mantissa is 0.625 and the characteristic is −2.

1. Move HL over 0.625 on L.
2. Under HL read "422" on D (or C if L is on slide).

Result is ***.0422.***

Verify the following:

1. antilog 1.328 = 21.3
2. antilog 3.031 = 1074
3. antilog (8.444 − 10) = .0278
4. antilog −2.397 = .00401
5. antilog 8.064 = 1.159×10^8
6. antilog −18.943 = 1.14×10^{-19}

18.4 Finding 10^x

By definition, $10^x = \text{antilog } x$.

Example: $10^{1.45} = ?$

We merely find antilog 1.45:

1. Move HL over 0.45 on L.
2. Under HL read "282" on D (or C if L is on slide).

Characteristic is 1; hence, $10^{1.45} =$ ***28.2.***

Verify the following:

1. $10^{2.78} = 603$

2. $10^{.037} = 1.089$

3. $10^{4.853} = 71{,}300$

4. $10^{-2.974} = .001062$

Exercise 18-1

1. log 345 =

2. log 108 =

3. log 11.6 =

4. log .063 =

5. log .00545 =

6. log 425,000 =

7. log 2070 =

8. log 0.320 =

9. log .000185 =

10. $\log (4.23 \times 10^{7}) =$

11. $\log (2.03 \times 10^{12}) =$

12. $\log (1.77 \times 10^{-9}) =$

13. $\log (6.34 \times 10^{-21}) =$

14. $10^{x} = 27.5;\ x =$

15. $10^{x} = .00175;\ x =$

16. $10^{-x} = .0633;\ x =$

17. $10^{-2x} = 550;\ x =$

18. antilog 2.466 =

19. antilog (9.006 − 10) =

20. antilog (−1.264) =

21. antilog (−0.076) =

22. $10^{3.022} =$

23. $10^{0.315} =$

24. $10^{-1.267} =$

25. $10^{-2.705} =$

26. $10^{15.075} =$

27. $10^{-12.716} =$

28. $\log N = -1.436;\ N =$

29. $\log N = 9.825;\ N =$

30. $\log N = -11.307;\ N =$

18.5 Formula types

Example 1: In chemistry, the pH value of a solution is related to the hydrogen ion concentration (H^+) as follows:
$pH = -\log(H^+)$.

a. Find pH value if $(H^+) = .00450$.
1. Substituting in the formula: $pH = -\log .00450$.
2. Verify that $pH = -(7.653 - 10) = \mathbf{2.347}$.

b. Find (H^+) if pH = 6.150.

1. Substituting: 6.150 = $-\log(H^+)$.
2. Solve for $\log(H^+)$: $\log(H^+) = -6.150 = 3.850 - 10$.
3. Verify that (H^+) = ***7.08*** $\times$ ***10^{-7}***.

Verify the following:

1. (H^+) = .000325; pH = 3.488
2. (H^+) = .0644; pH = 1.191
3. (H^+) = 1.035×10^{-8}; pH = 7.985
4. pH = 3.123; (H^+) = .000753
5. pH = 1.624; (H^+) = .0238
6. pH = 9.045; (H^+) = 9.02×10^{-10}

Example 2: $\frac{1}{\pi}\log\left(\frac{156}{342}\right) = ?$

1. Divide 156 by 342 on C-D, and use indicator to read the corresponding log on L. Verify that the log thus obtained is $9.659 - 10 = -0.341$.
2. Move HL over 341 on DF. Under HL read "1085" on D.

Result is ***−0.1085.***

Example 3: Given the formula: $C = \dfrac{0.1208\,L}{\log\left(\frac{2D}{d}\right) - \frac{D^2}{8h^2}}$

Evaluate C when $L = 365$, $D = 4.25$, $d = 0.275$, $h = 6.3$.

1. Verify that $\log\left(\frac{2D}{d}\right) = \log\frac{2 \times 4.25}{0.275} = 1.49$.
2. Verify that $\frac{D^2}{8h^2} = \frac{(4.25)^2}{8(6.3)^2} = .0569$.
3. C may now be evaluated:

$$C = \frac{0.1208 \times 365}{1.49 - .0569} \approx \frac{0.1208 \times 365}{1.433} = \mathbf{30.7.}$$

Exercise 18-2

1. $3.12 \log 275 =$
2. $6.3 \log\left(\frac{1.84}{0.31}\right) =$
3. $\pi \log\left(\frac{26.3}{4.77}\right) =$
4. $(2.84)^2 \log\left(\frac{4.65}{175}\right) =$
5. $\dfrac{37.2 \log .00633}{2.05} =$
6. $\dfrac{2.72 \log 14.7}{\log 2800} =$

7. $\dfrac{\log .0664}{13.5 \log 7.23} =$

8. $(\log 655)^2 =$

9. $.045\sqrt{3.58 + \log 263} =$

10. $0.575 \times 10^{3.075} =$

11. $7.66 \times 10^{-9.622} =$

12. $(H^+) = .000109$; $pH =$

13. $(H^+) = 5.66 \times 10^{-10}$; $pH =$

14. $(H^+) = 1.074 \times 10^{-9}$; $pH =$

15. $pH = 7.024$; $(H^+) =$

16. $pH = 12.285$; $(H^+) =$

17. $pH = 8.628$; $(H^+) =$

In the following formulas, substitute the given data and evaluate.

18. $N = 10 \log \left(\dfrac{P_1}{P_2}\right)$
  a. $P_1 = 250$, $P_2 = 14.5$; b. $P_1 = 8500$, $P_2 = 175$

19. $U = 1.15 \log \left(\dfrac{1+r}{1-r}\right)$
  a. $r = 0.480$; b. $r = 0.235$

20. $R = 342 \log (W_f - D)$
  a. $W_f = 57000$, $D = 2600$; b. $W_f = 126{,}000$, $D = 5500$

21. $H = \dfrac{1.44 \times 10^{-6}}{d^2 - \log \left(\dfrac{h}{d}\right)}$
  a. $d = 2.88$, $h = 12.6$; b. $d = 5.75$, $h = 260$

22. $N = n \times 10^k$
  a. $n = 6.42$, $k = 5.44$; b. $n = 2.63$, $k = -8.125$

23. $I = \dfrac{k \log(L/g) - (L/r)}{r^2}$
  a. $k = 470$, $L = 124$, $g = 32.2$, $r = 0.725$
  b. $k = 650$, $L = 165$, $g = 32.2$, $r = 2.33$

24. $Q = \sqrt{1 - \dfrac{\log N_1}{\log N_2}}$
  a. $N_1 = 340$, $N_2 = 5630$; b. $N_1 = .0715$, $N_2 = .00306$

25. $s = \dfrac{k - \pi \log \sin \Theta}{t}$
  a. $k = 4.06$, $\Theta = 51°$, $t = 1.77$
  b. $k = 2.75$, $\Theta = 156°$, $t = 3.52$

26. $\tan \dfrac{1}{2}\Theta = k \log \left(\dfrac{m}{n}\right)$; $(0° < \Theta < 180°)$
  a. $k = 1.064$, $m = 1250$, $n = 325$. Find Θ.
  b. $k = 1.133$, $m = 46.7$, $n = 0.314$. Find Θ.

Chapter 19

THE LOG LOG SCALES (LL); POWERS OF E

19.1 The LL1, LL2, and LL3 scales

It will be observed that these scales form a continuous scale starting with 1.01 at the left end of LL1, and ending at about 22,000 at the right end of LL3. Note that, unlike the C and D scales, the decimal point is indicated for all numbers on the LL scales.

The range of each scale is associated with powers of e as follows:

LL1 extends from $e^{.01}$ to $e^{.10}$ (1.01 to 1.105)
LL2 extends from $e^{.10}$ to $e^{1.0}$ (1.105 to 2.718)
LL3 extends from $e^{1.0}$ to e^{10} (2.718 to 22,026)

On the K & E Deci-Lon slide rule, these scales are labeled Ln1, Ln2, and Ln3. Also, on certain Pickett models, the LL scales (or N scales) are associated with powers of 10 rather than with powers of e; these scales are discussed in Appendix D.

19.2 Finding e^x when x is positive

The LL scales are related to the D scale in the following manner:

> If the hairline is moved over a number x on the **D** scale, then e^x is located under the hairline on the appropriate **LL** scale:
>
> For x between .01 and 0.1, e^x is on LL1
> For x between 0.1 and 1.0, e^x is on LL2
> For x between 1.0 and 10, e^x is on LL3
>
> The range of x associated with each LL scale is usually indicated on the body of the rule.

Example 1: $e^{3.5} = ?$

1. Move HL over 35 on D. Exponent is between 1 and 10; hence, answer will be on LL3.
2. Under HL read ***33.1*** on LL3.

Observe that:

$e^{0.35} = 1.419$ is also under HL on LL2.
$e^{.035} = 1.0356$ is under HL on LL1.

Example 2: $e^{0.57} = ?$

1. Move HL over 57 on D. Exponent is between 0.1 and 1.0; hence, answer will be read on LL2.
2. Under HL read ***1.768*** on LL2.

Example 3: $e^{.042} = ?$

1. Move HL over 42 on D. Exponent is between .01 and 0.1; hence, answer is on LL1.
2. Under HL read ***1.0429*** on LL1.

Verify the following:

1. $e^{0.64} = 1.897$; $e^{.064} = 1.0661$
2. $e^{.0155} = 1.01561$; $e^{1.55} = 4.71$
3. $e^{3.55} = 34.8$; $e^{.0355} = 1.0361$
4. $e^{9.44} = 12{,}500$
5. $e^{0.835} = 2.305$
6. $e^{.0256} = 1.0259$

Example 4: $125\, e^{1.85} = ?$

1. First evaluate $e^{1.85} = 6.36$. Now transfer this to the D scale and multiply by 125:
2. Verify that $125\, e^{1.85} = 125 \times 6.36 =$ ***795.***

Exercise 19-1

1. $e^{1.55} =$
2. $e^{0.155} =$
3. $e^{.0155} =$
4. $e^{2.9} =$
5. $e^{.029} =$
6. $e^{5.6} =$
7. $e^{0.83} =$
8. $e^{0.28} =$
9. $e^{.0425} =$
10. $e^{.088} =$

11. $e^{1.075} =$

12. $e^{0.63} =$

13. $e^{2.05} =$

14. $e^{0.115} =$

15. $e^{.026} =$

16. $e^{7.22} =$

17. $e^{0.37} =$

18. $e^{9.4} =$

19. $e^{.0128} =$

20. $e^{1.245} =$

21. $e^{4.66} =$

22. $75\ e^{0.805} =$

23. $4300\ e^{.0177} =$

24. $.064\ e^{4.16} =$

25. $42.6\ e^{0.104} =$

26. $e^{.064} \sin 38° =$

27. $e^{0.52} \sin(0.24\pi) =$

28. $e^{3.06} \cos 73.5° =$

29. $e^{6.4} \cos 87° =$

30. $e^{.0227} \tan 1.25 =$

19.3 The "A-related" LL0 and LL00 scales

Some slide rules have the following described log log scales which are associated with the A scale:

1. A scale labeled "LL0" starting at 0.999 at the left and decreasing down to 0.905 on the right.
2. A scale labeled "LL00" which is a continuation of "LL0," and starts at 0.905 on the left, decreasing down to .000045 on the right.

The use of these scales is discussed in Appendix C.

19.4 The LLo1, LLo2, and LLo3 scales

These scales form a continuous scale starting at 0.990 at the left end of LLo1, and decreasing down to .000045 at the right end of LLo3. Thus, the scales are read from right to left in the same manner as we read the CI scale.

The range of each scale is associated with negative powers of e as follows:

LLo1 extends from $e^{-.01}$ to $e^{-.10}$ (.990 to .905)
LLo2 extends from $e^{-.10}$ to $e^{-1.0}$ (.905 to .368)
LLo3 extends from $e^{-1.0}$ to e^{-10} (.368 to .000045)

On the K & E Deci-Lon rule, these are labeled Ln-1, Ln-2, and Ln-3; on the Post Versalog model they are labeled LL/1, LL/2, and LL/3. The Pickett rules have these scales "back to back" with the LL1, LL2, and LL3 scales. Thus, the LL1 scale and the scale corresponding to LLo1 both share the same scale axis; the LL1 markings are above the axis (reading from left to right), and the LLo1 markings are below (reading from right to left).

19.5 Finding e^x when x is negative

The scales described in the preceding section are related to the D scale in the same manner as are the other LL scales. Hence, we may raise e to a negative power by reading directly from D to the appropriate LL scale, bearing in mind the range associated with each scale. Again, these ranges are usually indicated on the body of the rule.

Example 1: $e^{-.056} = ?$

1. Move HL over 56 on D. Exponent is between −.01 and −1.0; hence, answer is on LLo1.
2. Under HL read ***0.9455*** on LLo1.

Example 2: $e^{-4.51} = ?$

1. Move HL over 451 on D. Exponent is between −1.0 and −10; hence, answer is found on LLo3.
2. Under HL read ***.0110*** on LLo3.

Verify the following:

1. $e^{-2.65} = .0707$; $e^{-0.265} = 0.767$
2. $e^{-.064} = 0.938$; $e^{-6.4} = .00166$
3. $e^{-1.47} = 0.230$; $e^{-.0147} = 0.9854$
4. $e^{-.025} = 0.9753$
5. $170\ e^{-3.6} = 4.64$
6. $e^{-1.6} \sin 0.32 = .0635$

Exercise 19-2

1. $e^{-2.65} =$
2. $e^{-0.265} =$
3. $e^{-.064} =$
4. $e^{-6.4} =$
5. $e^{-3.7} =$
6. $e^{-1.03} =$
7. $e^{-.035} =$
8. $e^{-5.76} =$
9. $e^{-.073} =$
10. $e^{-0.1045} =$
11. $e^{-.0148} =$
12. $e^{-9.4} =$
13. $e^{-4.85} =$
14. $e^{-0.76} =$
15. $e^{-.066} =$
16. $e^{-0.10} =$
17. $e^{-3.22} =$
18. $e^{-0.28} =$

19. $e^{-0.1175} =$

20. $e^{-1.36} =$

21. $4.3\, e^{-0.14} =$

22. $12.6\, e^{-0.44} =$

23. $2700\, e^{-7.55} =$

24. $545\, e^{-.0147} =$

25. $68.2\, e^{-1.53} =$

26. $\sqrt{475}\, e^{-8.66} =$

27. $e^{-0.66} \sin 69° =$

28. $e^{-1.23} \sin(0.22\,\pi) =$

29. $e^{-.054} \cos 1.3 =$

30. $e^{-3.72} \tan 87.2° =$

19.6 An approximation for e^x (x near zero)

When raising e to very small powers, the following approximate relations are useful:

> If x is a positive number near zero:
> 1. $e^x \approx 1 + x$
> 2. $e^{-x} \approx 1 - x$

Example 1: $e^{.0042} = ?$

This is outside the range of LL1; hence, we use relation (1).

$$e^{.0042} \approx 1 + .0042 = \mathbf{1.0042} \text{ (approx.)}$$

Example 2: $e^{-.00074} = ?$

This is outside the range of LLo1; hence, we use relation (2).

$$e^{-.00074} \approx 1 - .00074 = \mathbf{0.99926} \text{ (approx.)}$$

19.7 Evaluating e^x for large positive or negative values of x

Example 1: $e^{11.6} = ?$

This is beyond the range of LL3; hence, we write:

$$e^{11.6} = (e^{5.8})^2$$

We may now find $e^{5.8}$ and square the result.
Verify that $e^{11.6} = (e^{5.8})^2 = (330)^2 = \mathbf{108{,}900}$.

Example 2: $e^{-22} = ?$

This is beyond the range of LLo3; hence, we write:

$$e^{-22} = (e^{-22/3})^3 = (e^{-7.33})^3$$

Verify that $e^{-22} = (.00065)^3 = \mathbf{2.74 \times 10^{-10}}$.

19.8 Slide rules with 8 LL scales

In the preceding sections we have described the six basic LL scales found on most conventional slide rules. However, some rules feature two additional scales in the neighborhood of 1. One of these extends from $e^{.001}$ to $e^{.01}$ (1.001 to 1.01), and we shall refer to this as the LL0 scale (on the Deci-Lon rule it is labeled Ln0). The other scale extends from $e^{-.001}$ to $e^{-.01}$ (0.999 to 0.990), and we shall refer to this as the LLo0 scale (on the Versalog rule this scale is labeled LL/0; on the Deci-Lon rule it is labeled Ln-0). If your slide rule has these scales, you may check the following examples:

Example 1: $e^{.0055} = ?$

1. Move HL over 55 on D.
2. Under HL read ***1.00551*** on LL0 (Ln0).

Example 2: $e^{-.0027} = ?$

1. Move HL over 27 on D.
2. Under HL read ***0.99731*** on LLo0 (Ln-0, LL/0).

Verify the following (use LL0 and LLo0 scales):

1. $e^{.009} = 1.00904$
2. $e^{.00235} = 1.00235$
3. $e^{.00421} = 1.00422$
4. $e^{-.006} = 0.99402$
5. $e^{-.00485} = 0.99517$
6. $e^{-.00163} = 0.99837$

If you check the foregoing examples using the approximate formulas of section 19.6, you will discover very close agreement.

Exercise 19-3

1. $e^{.0034} =$
2. $e^{-.00061} =$
3. $e^{.0072} =$
4. $e^{-.0014} =$

5. $e^{-.00037} =$

6. $e^{.000021} =$

7. $e^{13} =$

8. $e^{24} =$

9. $12{,}500\ e^{-15} =$

10. $475{,}000\ e^{-12} =$

19.9 Exponent in combined form

Example 1: $e^{(.032 \times 16.5)} = ?$

By approximation, the exponent is about 0.5; hence, result will be on LL2.

1. Set left index of C opposite 165 on D.
2. Move HL over 32 on C. HL is now over the product (.032 × 16.5) on D; therefore, *e* raised to this power is under HL on LL2.
3. Under HL read ***1.695*** on LL2.

Example 2: $e^{-35/16} = ?$

Exponent is about −2; hence, result will be on LLo3.

1. Move HL over 35 on D.
2. Slide 16 on C under HL.
3. Move HL over left index of C.
4. Under HL read ***0.112*** on LLo3.

Verify the following:

1. $e^{(2.3 \times 1.7)} = 50.0$
2. $e^{-2/7} = 0.7515$
3. $e^{26/\pi} = 3900$
4. $e^{1/15} = 1.0690$
5. $e^{(-.018 \times 4.7)} = 0.9190$
6. $e^{(-.0072 \times 16.2)} = 0.8900$

Example 3: Evaluate $e^{.016t}$ when (a) $t = 2.5$, (b) $t = 35$, (c) $t = 425$. By approximation, we see that the result of (a) will be on LL1, the result of (b) will be on LL2, and the result of (c) will be on LL3.

1. Set left index of C opposite 16 on D.
2. Move HL over 25 on C. Under HL read ***1.0408*** on LL1.
3. Move HL over 35 on C. Under HL read ***1.751*** on LL2.
4. Move HL over 425 on C. Under HL read ***900*** on LL3.

Exercise 19-4

1. $e^{3.2 \times 1.6} =$
2. $e^{1/9} =$
3. $e^{0.64 \times 5.2} =$
4. $e^{-.015 \times 2.7} =$
5. $e^{-3/7} =$
6. $e^{.06/2.4} =$
7. $e^{1/27} =$
8. $e^{.0045 \times 15} =$
9. $\sqrt[4]{e} =$
10. $e^{-1/6} =$
11. $e^{-11/240} =$
12. $e^{.06/4.2} =$
13. $e^{(1.3 \times 6.4)/15} =$
14. $e^{-(2.4 \times 6.1)/5} =$
15. $e^{(.0034 \times 2.6)/.015} =$
16. $e^{\pi/4} =$
17. $1600\, e^{-3\pi/2} =$
18. $e^{-(.035 \times 165)/77} =$
19. $e^{1/(2.6 \times 6.3)} =$
20. $e^{-5/(2.7 \times 0.43)} =$
21. $2.75\, e^{1/5.3} =$
22. $1250\, e^{-1.1\pi} =$
23. $e^{-7/60} \sin\left(2 - \frac{1}{2}\pi\right) =$
24. $e^{.024 \times 130} \sin(\pi/60) =$
25. $e^{-0.33 \times 11.4} \tan 72.3° =$
26. Evaluate $e^{-.0023t}$ for $t = 15, 30, 1250$.
27. Evaluate $e^{.073t}$ for $t = 15, 55, 110$.
28. Evaluate $e^{650/x}$ for $x = 125, 550, 12{,}000$.
29. Evaluate $450\, e^{-.063t}$ for $t = 25, 85, 125$.
30. Evaluate $1250\, e^{65/x}$ for $x = 12, 175, 2300$.

19.10 Hyperbolic functions

Certain combinations of exponential functions occur frequently in applied mathematics, and are known as "hyperbolic functions." They are defined as follows:

1. $\sinh x = \frac{1}{2}(e^x - e^{-x})$ (read "hyperbolic sine of x")
2. $\cosh x = \frac{1}{2}(e^x + e^{-x})$ (read "hyperbolic cosine of x")
3. $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x} - 1}{e^{2x} + 1}$ (read "hyperbolic tangent of x")

Definitions are also given for $\coth x$, $\operatorname{sech} x$, and $\operatorname{csch} x$; however, the listed functions are encountered most frequently.

Example 1: sinh 0.36 = ?

1. From the definition: $\sinh 0.36 = \frac{1}{2}(e^{0.36} - e^{-0.36})$.

2. Verify that $\sinh 0.36 = \frac{1}{2}(1.434 - 0.698) = \mathbf{0.368}$.

Example 2: $\cosh \pi/2 = ?$

1. From the definition: $\cosh \pi/2 = \frac{1}{2}(e^{\pi/2} + e^{-\pi/2})$.

2. Verify that $\cosh \pi/2 = \frac{1}{2}(4.81 + 0.208) = \mathbf{2.51}$.

Verify the following:

1. $\sinh 0.63 = 0.673$
2. $\cosh 1.35 = 2.06$
3. $\cosh(-0.84) = 1.374$
4. $\tanh 0.18 = 0.1781$
5. $\sinh(-\pi/7) = -0.464$
6. $\tanh(-1.10) = -0.800$

19.11 The Sh and Th scales

Certain slide rules have scales marked Sh and Th which enable you to read sinh x and tanh x directly. These scales are discussed in Appendix E.

Exercise 19-5

1. $\sinh 0.48 =$
2. $\cosh 1.25 =$
3. $\cosh -0.750 =$
4. $\tanh 1.43 =$
5. $\sinh -0.26 =$
6. $\cosh 0.09 =$
7. $\tanh -0.65 =$
8. $\sinh \pi/4 =$
9. $\cosh \pi/6 =$
10. $\tanh(.036 \times 5.7) =$
11. $\sinh \sqrt{1.75} =$
12. $\sinh(75/120) =$
13. A flexible cable hangs in a curve whose equation is: $y = 8.75 \cosh(x/8.75)$. Find y when x takes the values: 0, 5, 10, and 20.

14. If air resistance is taken into account, the velocity of a falling object is given by the formula: $V = P \tanh(2gt/P)$, where $P = \sqrt{W/k}$.

a. If $W = 2000$ lbs, $k = .0037$, $g = 32.2$ ft per sec^2, calculate V for $t = 0$ and $t = 15$ sec.

b. What value does V approach as t gets larger and larger?

19.12 Finding logarithms to base *e*

You will recall that if $N = e^x$, then $x = \log_e N$. We refer to $\log_e N$ as the *natural log* of N, and we shall write it *ln N.*

With reference to $N = e^x$, it is clear, then, that finding ln N amounts to determining the value of the exponent x corresponding to a known N. This is just the inverse of finding e^x. Accordingly, the relation between the LL scales and the D scale may be restated as follows:

> If the hairline is moved over a number N on the **LL** scale, then *ln N* will be under the hairline on the **D** scale.
>
> Conversely, if the hairline is over *ln N* on the **D** scale, then N will be under the hairline on the appropriate **LL** scale.

If your slide rule has a legend on the body indicating the range of the exponent associated with each LL scale, then these ranges, of course, also apply to ln N. In the following examples, you will find it helpful to refer to these range indications.

Example 1: ln 4.5 = ?

1. Move HL over 4.5 on LL3.
2. Under HL read "1504" on D. N is on LL3; hence, answer must be between 1 and 10. Therefore, ln 4.5 = ***1.504.***

Example 2: ln 1.0445 = ?

1. Move HL over 1.0445 on LL1.
2. Under HL read "435" on D. N is on LL1; hence, ln N is between .01 and 0.1. Answer is ***.0435.***

Example 3: ln 0.724 = ?

1. Move HL over 0.724 on LLo2.
2. Under HL read "323" on D. N is on LLo2; therefore, ln N is between −0.1 and −1.0. Answer is ***−0.323.***

Verify the following:

1. ln 68 = 4.22
2. ln 1.0223 = .0220
3. ln 0.814 = −0.206
4. ln 1.545 = 0.435
5. ln .0265 = −3.63
6. ln 0.9436 = −.0581

Example 4: ln N = 2.66; N = ?

We know ln N and we must find N; hence, we read from D to the proper LL scale ($N = e^{2.66}$).

1. Move HL over 266 on D. Exponent (or ln N) is between 1 and 10; hence, N will be located on LL3.
2. Under HL read ***14.3*** on LL3.

Example 5: ln N = −0.662; N = ?

1. Move HL over 662 on D. Exponent (or ln N) is between −0.1 and −1.0; hence, N will be located on LLo2.
2. Under HL read ***0.516*** on LLo2.

Verify the following:

1. Given ln N = 5.1; verify that N = 164.
2. Given ln N = −.0345; verify that N = 0.9661.
3. Given ln N = 0.720; verify that N = 2.055.
4. Given ln N = −1.84; verify that N = 0.158.

Exercise 19-6

1. ln 375 =
2. ln 1.23 =
3. ln 0.175 =
4. ln 2.85 =
5. ln 0.9075 =
6. ln 8.45 =
7. ln .00066 =
8. ln 164 =
9. ln 1.01645 =
10. ln 2500 =
11. ln .0034 =
12. ln 36.5 =

13. ln 1.01245 =

14. ln 16,500 =

15. ln 0.382 =

16. ln 2.77 =

17. ln .000175 =

18. ln 1.0103 =

19. ln 0.377 =

20. ln 8400 =

21. ln 15 =

22. ln .066 =

23. ln 210 =

24. ln N = 1.84; N =

25. ln N = 0.63; N =

26. ln N = −4.75; N =

27. ln N = −.082; N =

28. ln N = 7.2; N =

29. ln N = .0274; N =

30. ln N = −0.46; N =

19.13 Combined operations with ln *N*

Example 1: $4.5 \ln 2 = ?$

1. Move HL over 2 on LL2. This puts HL over ln 2 on D (note that its value is about 0.7). Now to multiply by 4.5:
2. Slide right index of C under HL. Move HL over 45 on C.
3. Under HL read "312" on D. Answer is ***3.12.***

Clearly, we could have divided by the reciprocal of 4.5 instead of multiplying as we did.

Example 2: $\dfrac{\ln 0.063}{7} = ?$

1. Move HL over .063 on LLo3. HL is now over ln 0.063 on D (note that its value is about −3). Now divide by 7:
2. Slide 7 on C under HL.
3. Opposite right index of C read "395" on D. Answer is ***−0.395.***

Observe that, because ln N appears on the D scale, it can easily be multiplied or divided by other numbers. This is an important feature of the LL scales, and in the next chapter we shall see how it enables us to evaluate powers of numbers in general.

Exercise 19-7

1. 1.4 ln 4.35 =

2. $\dfrac{\ln 7.44}{5.4} =$

3. $14.6 \ln 1.0164 =$

4. $\frac{\ln 0.814}{3.26} =$

5. $3.74 \ln 0.725 =$

6. $2.6 \ln 3.44 =$

7. $4.7 \ln 0.9645 =$

8. $0.38 \ln 650 =$

9. $\frac{\ln 1.84}{6.22} =$

10. $\frac{\ln 1650}{24.2} =$

11. $\frac{\ln .0145}{4.6} =$

12. $\frac{\ln .0044}{.0745} =$

13. $\frac{2.4 \ln 1.0246}{1.7} =$

14. $\frac{\pi \ln 115}{45} =$

15. $\frac{2.6 \ln 0.934}{3.55} =$

16. $\frac{6.8 \ln 0.564}{2.7} =$

Chapter 20

POWERS OF ANY POSITIVE NUMBER

20.1 Finding b^x (x between 1 and 10)

In this section we consider the evaluation of b^x where the base, b, may be any positive number, and the exponent, x, takes values between 1 and 10.

To illustrate the procedure, suppose you now go through the following moves on your slide rule:

1. Move HL over 3 on the LL3 scale.
2. Slide left index of C under HL.
3. Move HL over 2 on C. HL is now over 9 on LL3. Note that 9 is 3^2.
4. Move HL over 3 on C. HL is now over 27 on LL3. Note that 27 is 3^3.
5. Move HL over 4 on C. HL is now over 81 on LL3. Note that 81 is 3^4.

These observations suggest that if we set the C index opposite the base, b, on the LL scale, and then move the hairline over the exponent, x, on the C scale, we will find the power, b^x under the hairline on the LL scale. This is similar to multiplication, except that instead of using C and D, we use C and LL.

To see what actually is happening, consider the following examples:

Example 1: $4^3 = ?$

Suppose we let $N = 4^3$, and take natural log of both sides:

$$\ln N = \ln 4^3 = 3 \ln N$$

Now evaluate 3 ln 4:

1. Move HL over 4 on LL3. Note that HL is now over ln 4 on D; hence, it is easy to multiply by 3.

2. Slide left index of C under HL.
3. Move HL over 3 on C. HL is now over ln N on D, which means that it is over N on LL.
4. Under HL read 64 on LL3.

To evaluate $N = b^x$, then, we first move the hairline over the base b on the LL scale; this puts the hairline over ln b on the D scale. We then multiply ln b by the exponent x; this locates ln N on D, and N itself on the appropriate LL scale (see Figure 20.1).

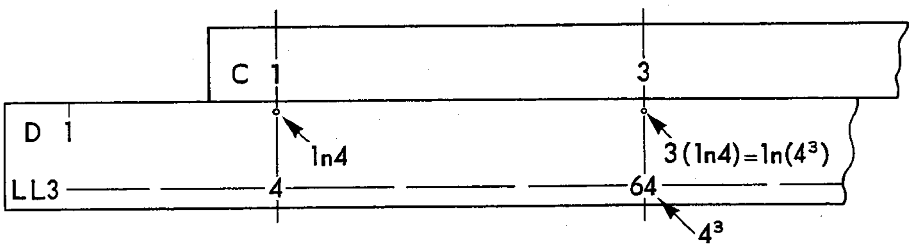


Figure 20.1

As stated in the previous chapter, the powerful feature of the LL scales is that ln b appears on the D scale, which is an operational scale, thus making it easy to further multiply or divide the logarithm by other numbers.

Example 2: Evaluate $(1.3)^x$ for $x = 2$, 3, 4.5, and 7.6.

1. Move HL over 1.3 on LL2. This puts HL over ln 1.3 on D. Now multiply the logarithm successively by the given values of x; the corresponding powers will be read on the LL scale.
2. Slide left index of C under HL.
3. Move HL over 2 on C. Under HL read ***1.69*** on LL2.
4. Move HL over 3 on C. Under HL read ***2.197*** on LL2.
5. Now we see that 4.5 on C is somewhat beyond the LL2 scale; hence answer must be on LL3 (which is just a continuation of LL2). Therefore, we interchange indexes and move HL over 45 on C. Under HL read ***3.26*** on LL3.
6. Move HL over 76 on C. Under HL read ***7.35*** on LL3.

In the foregoing example, you will notice that when the answer was to the *right* of the base, 1.3, it was located on the *same scale* (LL2). After interchanging indexes, the answer was to the *left* of 1.3, and it was located *one scale higher* on LL3.

Example 3: Evaluate $(0.70)^2$ and $(0.70)^{4.3}$.

1. Move HL over 0.70 on LLo2. This puts HL over ln 0.70 on D. Now multiply by 2, and then by 4.3.
2. Slide left index of C under HL.
3. Move HL over 2 on C. Under HL read ***0.49*** on LLo2.
4. Interchange indexes and move HL over 43 on C. Under HL read ***0.216*** on LLo3.

Here again, you will notice that when the answer was to the *right* of the base, 0.70, it was located on the *same scale* (LLo2). When the answer was to the *left* of 0.70, it was located *one scale higher* on LLo3. It should be understood that by a higher scale, we mean a scale with a higher number designation; for example, LL2 is "higher" than LL1, LLo3, is "higher" than LLo2, and so forth.

The procedure, then, may be summarized:

> *To evaluate b^x (x between 1 and 10):*
> 1. Move HL over b on LL scale. HL is now over ln b on D.
> 2. Multiply by x (slide C index under HL, move HL over x on C).
> 3. Under HL read b^x on the appropriate LL scale:
>   a. If the answer is to the *right* of b, it is on the *same* LL scale as b.
>   b. If the answer is to the *left* of b, it is located *one scale higher* than b.

Clearly, in step (2), we may multiply by x in the manner described, or we may divide by the reciprocal of x using the CI scale.

Example 4: $(14.2)^{3.3} = ?$

1. Move HL over 14.2 on LL3.
2. Slide left index of C under HL.
3. Move HL over 33 on C. Note that HL is to the *right* of 14.2; hence, answer is on the *same scale* (LL3).
4. Under HL read ***6400*** on LL3.

Example 5: $(1.055)^{6.25} = ?$

1. Move HL over 1.055 on LL1.
2. Slide right index of C under HL.
3. Move HL over 625 on C. HL is to the *left* of 1.055; hence, answer is *one scale higher* on LL2.
4. Under HL read ***1.398*** on LL2.

Example 6: $(.9766)^{3.4} = ?$

1. Move HL over .9766 on LLo1.
2. Slide left index of C under HL.
3. Move HL over 34 on C. HL is to the *right* of .9766; hence, answer is on the *same scale* (LLo1).
4. Under HL read ***0.9226*** on LLo1.

Example 7: $(0.864)^{9.2} = ?$

1. Move HL over 0.864 on LLo2. Note that slide will be in better position if we divide by the reciprocal of 9.2:
2. Slide 92 on CI under HL.
3. Move HL over left index of C. Observe that HL is to the *left* of 0.864; hence, answer is *one scale higher* on LLo3.
4. Under HL read ***0.261*** on LLo3.

Example 8: $\left(\frac{585}{126}\right)^{3.3} = ?$

1. Dividing on C-D, we find 585/126 = 4.64.
2. Verify that $(4.64)^{3.3} =$ ***158.***

Exercise 20-1

1. $(4)^{2.7} =$
2. $(8)^{1.35} =$
3. $(11)^{3.4} =$
4. $(1.25)^{1.8} =$
5. $(1.6)^{7} =$
6. $(1.044)^{5} =$
7. $(0.21)^{4} =$
8. $(0.56)^{6} =$
9. $(0.98)^{3.2} =$
10. $(0.94)^{4.5} =$
11. $(4.5)^{4} =$
12. $(7.3)^{3} =$
13. $(1.26)^{1.7} =$
14. $(0.75)^{3} =$
15. $(0.625)^{5} =$
16. $(1.1645)^{3.2} =$
17. Evaluate $(1.364)^{x}$ for $x = 2.2, 4.5, 7.3$.
18. Evaluate $(1.046)^{x}$ for $x = 1.8, 3.7, 10$.
19. Evaluate $(0.787)^{x}$ for $x = 4.5, 7.9, 10$.
20. Evaluate $(0.9515)^{x}$ for $x = 1.65, 2.80, 6.45$.
21. $(1.0534)^{7.5} =$
22. $(1.226)^{3.4} =$
23. $(5.75)^{4.6} =$
24. $(1.0945)^{1.1} =$
25. $(0.186)^{4.9} =$
26. $(1.534)^{5.75} =$
27. $(35.5)^{2.75} =$
28. $(1.0136)^{5.26} =$
29. $(0.8815)^{8.4} =$
30. $(.057)^{3.3} =$

31. $(0.954)^{3.6} =$

32. $(1.228)^{9.3} =$

33. $(0.354)^{6.72} =$

34. $(1.133)^{8.5} =$

35. $(1.765)^{3.45} =$

36. $(0.107)^{4.15} =$

37. $(0.444)^{1.15} =$

38. $(0.9445)^{7.64} =$

39. $(12.5)^{2.31} =$

40. $(0.9824)^{2.66} =$

41. $(0.634)^{5.75} =$

42. $(0.388)^{6.44} =$

43. $(2.84)^{6.9} =$

44. $(1.0366)^{8.4} =$

45. $(0.9863)^{7.6} =$

46. $(1.0545)^{1.93} =$

47. $\left(\frac{68}{85}\right)^{3.6} =$

48. $\left(\frac{244}{107}\right)^{2.8} =$

49. $\left(\frac{373}{950}\right)^{4.4} =$

50. $\left(\frac{0.37}{0.17}\right)^{6.8} =$

20.2 The "scale-shift" principle

The LL scales have an important property which we shall refer to as the "scale-shift" principle:

> If the HL is over b^x on one of the LL scales, then:
> 1. b^{10x} will be under HL on the next *higher* scale.
> 2. $b^{x/10}$ will be under HL on the next *lower* scale.

In other words, the decimal point in the exponent shifts *one place to the right* each time we move *one scale higher;* it shifts *one place to the left* each time we move *one scale lower.*

Example 1: **a.** $(1.2)^2 = ?$ **b.** $(1.2)^{20} = ?$ **c.** $(1.2)^{0.2} = ?$

a. Verify that $(1.2)^2$ is under HL on LL2 and is equal to ***1.44.***
b. Now, leaving the hairline in the same position, the "scale-shift" principle tells us that $(1.2)^{20}$ is located one scale higher on LL3 (decimal point in exponent has been shifted one place to the right). Under HL read ***38.3*** on LL3.
c. Again applying the "scale-shift" principle, we find $(1.2)^{0.2}$ on LL1 (one scale lower than LL2). Under HL read ***1.0372.***

These readings are illustrated in Figure 20.2.

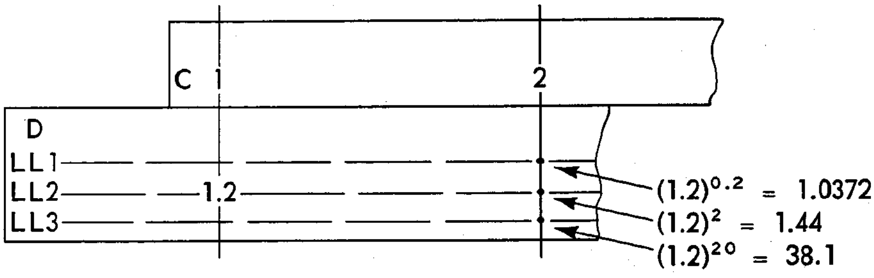


Figure 20.2

Example 2: **a.** $(.064)^2 = ?$ **b.** $(.064)^{.02} = ?$

a. Verify that $(.064)^2$ is under HL on LLo3, and is equal to ***.0041.***

b. Leaving hairline in the same position and applying the "scale-shift" principle, we locate $(.064)^{.02}$ two scales lower on LLo1 (decimal point in exponent has been shifted two places to the left). Under HL read ***0.9465*** on LLo1.

Example 3: $(2.1)^{.075} = ?$

a. First locate $(2.1)^{7.5}$. Verify that this is found under HL on LL3.

b. Applying "scale-shift" principle, $(2.1)^{.075}$ is located two scales lower on LL1. Verify that answer is ***1.0573.***

20.3 A general procedure for finding b^x (x positive)

The foregoing examples suggest how we may locate the answer when we are dealing with the more general case of b^x; that is, when the exponent is not restricted to a number between 1 and 10.

Example 1: $(1.015)^{300} = ?$

We first evaluate assuming that the exponent is between 1 and 10; we then apply the "scale-shift" principle to properly locate the answer.

1. Move HL over 1.015 on LL1. Slide left index of C under HL.
2. Move HL over 3 on C. Observe that HL is to the right of 1.015; hence, if exponent were 3.00 answer would be on the same scale (LL1). However, decimal point in exponent is actually *two* places to the *right* of this assumed position; therefore, answer is located *two* scales *higher* on LL3.
3. Under HL read ***87*** on LL3.

The procedure is illustrated in Figure 20.3.

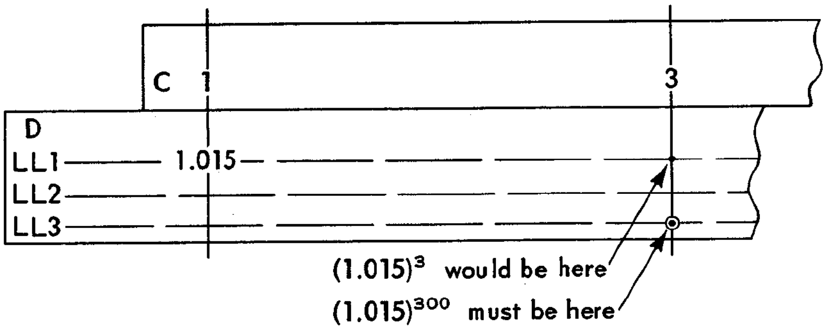


Figure 20.3

Example 2: $(1.0185)^{24} = ?$

1. Move HL over 1.0185 on LL1. Slide left index of C under HL.
2. Move HL over 24 on C. Observe that HL is to the right of 1.0185; hence, if exponent were 2.4, answer would be on the same scale (LL1). However, decimal point in exponent is actually *one* place to the *right* of this position; therefore, answer is located *one* scale *higher* on LL2.
3. Under HL read ***1.553*** on LL2.

Example 3: $(1.85)^{.07} = ?$

1. Move HL over 1.85 on LL2. Slide right index of C under HL.
2. Move HL over 7 on C. Note that HL is to the left of 1.85, hence if exponent were 7.00, answer would be on LL3. However, decimal point in exponent is actually *two* places to the *left* of this position; therefore, answer is located *two* scales *lower* on LL1.
3. Under HL read ***1.044*** on LL1.

We illustrate in Figure 20.4

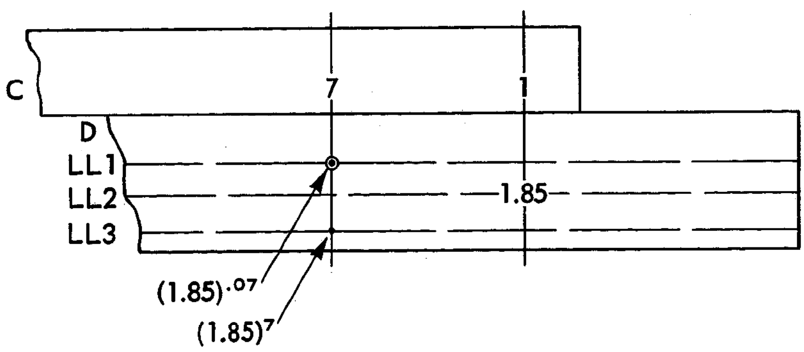


Figure 20.4

Example 4: $(62)^{.0073} = ?$

1. Move HL over 62 on LL3. Slide right index of C under HL.
2. Move HL over 73 on C. Note that HL is to the left of 62; hence, if exponent were 7.3, answer would be on "LL4" (if our slide rule had such a scale). However, decimal point in exponent is three places to the left of this assumed position; therefore, answer is located three scales lower on LL1.
3. Under HL read ***1.0306*** on LL1.

Example 5: $(0.82)^{0.175} = ?$

1. First locate $(0.82)^{1.75}$. Verify that this is on LLo2.
2. Applying "scale-shift" principle, $(0.82)^{0.175}$ is located one scale lower on LLo1. Verify that answer is ***0.9658.***

Example 6: $(0.964)^{25} = ?$

1. Verify that $(0.964)^{2.5}$ is on LLo1.
2. Applying "scale-shift" principle, $(0.964)^{25}$ will be located one scale higher on LLo2. Verify that answer is ***0.40.***

Example 7: $(.00325)^{.071} = ?$

1. Move HL over .00325 on LLo3. Slide right index of C under HL.
2. Move HL over 71 on C. HL is to the left of .00325; hence, if exponent were 7.1, answer would be on "LLo4" (if such a scale were present). Decimal point in exponent is actually two places to the left of this position; therefore, answer is two scales lower on LLo2.
3. Under HL read ***0.666*** on LLo2.

The procedure illustrated in the preceding examples may be formally stated:

To locate the result when evaluating b^x (x positive):
1. Determine the scale on which result would be found if decimal point in exponent were shifted to make it a number between 1 and 10.
2. Applying the "scale-shift" principle, move up or down as many scales as are necessary to adjust the decimal point to its true position.

Verify the following:

1. $(1.25)^{21} = 108$
2. $(28.5)^{.025} = 1.0874$

3. $(4700)^{.0032} = 1.0274$

4. $(0.9474)^{46} = .0835$

5. $(0.48)^{0.67} = 0.612$

6. $(1.01865)^{400} = 1610$

7. $(.0145)^{.0029} = 0.9878$

8. $(0.223)^{.083} = 0.883$

Exercise 20-2

1. $(1.186)^{34} =$

2. $(37)^{0.22} =$

3. $(145)^{.063} =$

4. $(1.164)^{3.7} =$

5. $(0.66)^{0.321} =$

6. $(0.59)^{.05} =$

7. $(1.244)^{25} =$

8. $(1.0665)^{7.3} =$

9. $(0.925)^{0.64} =$

10. $(0.50)^{0.265} =$

11. $(36)^{.08} =$

12. $(7500)^{.04} =$

13. $(1.174)^{0.55} =$

14. $(1.63)^{11.2} =$

15. $(1.107)^{45} =$

16. $(0.513)^{.035} =$

17. $(.00015)^{0.57} =$

18. $(1.024)^{100} =$

19. $(0.884)^{.092} =$

20. $(1.0625)^{0.74} =$

21. $(10)^{0.485} =$

22. $(100)^{.0064} =$

23. $(20{,}000)^{.0018} =$

24. $(0.9425)^{22.4} =$

25. $(1.425)^{.06} =$

26. $(0.568)^{9.5} =$

27. $(10)^{.041} =$

28. $(.9814)^{410} =$

29. $(0.117)^{0.255} =$

30. $(0.908)^{42} =$

31. $(1.264)^{31} =$

32. $(0.642)^{7.3} =$

33. $(650)^{.056} =$

34. $(.0245)^{.004} =$

35. $(1.375)^{18} =$

36. $(5200)^{.0021} =$

37. $(.00064)^{0.45} =$

38. $(1.1025)^{0.33} =$

39. $(1.214)^{47} =$

40. $(18{,}000)^{.027} =$

41. $(0.9684)^{290} =$

42. $(.00075)^{.062} =$

43. $(1.076)^{125} =$

44. $400(1 + .045)^{20} =$

45. $250(1 + .0375)^{45} =$

46. $24(1 + .03)^{300} =$

47. $\left(\frac{23}{72}\right)^{.036} =$

48. $\left(\frac{760}{635}\right)^{23} =$

49. $\left(\frac{5.34}{18.6}\right)^{0.55} =$

50. $\left(\frac{1.450}{.0026}\right)^{.075} =$

20.4 The reciprocal property of the LL scales

We have seen that the LL3 scale ranges from $e^{1.0}$ to e^{10}, whereas the LLo3 scale ranges from $e^{-1.0}$ to e^{-10}. This means that opposite readings on these two scales are reciprocals of each other. Similarly, the LL2 scale extends from $e^{0.1}$ to $e^{1.0}$, whereas the LLo2 scale extends from $e^{-0.1}$ to $e^{-1.0}$, and so forth. As a result, we may describe the following useful property of the LL scales:

> If the hairline is set over a number on LL3, its reciprocal will be under the hairline on LLo3, and vice versa.
>
> A similar reciprocal relation holds between LL2 and LLo2, and between LL1 and LLo1.

Example 1: Use LL scales to find reciprocal of 36.5.

1. Move HL over 36.5 on LL3.
2. Under HL read ***.0275*** on LLo3.

Example 2: $1/.0735 = ?$

1. Move HL over .0735 on LLo3.
2. Under HL read ***13.6*** on LL3.

Example 3: $(1.1635)^{-1} = ?$

1. Move HL over 1.1635 on LL2.
2. Under HL read ***0.8595*** on LLo2.

Verify the following:

1. $1/27 = .037$

2. $1/350 = .00286$

3. $1/0.645 = 1.55$

4. $1/.00085 = 1180$

5. $1/7.75 = 0.129$

6. $(.00235)^{-1} = 425$

7. $(14{,}500)^{-1} = .00007$

8. $(1.107)^{-1} = 0.9034$

9. $(0.383)^{-1} = 2.61$

10. $(1.0344)^{-1} = 0.9668$

20.5 Finding b^x (x negative)

Here, the reciprocal scales may be used to advantage as shown in the following examples:

Example 1: $5^{-4} = ?$

We simply proceed as if to evaluate 5^4, then read the reciprocal.

1. Move HL over 5 on LL3. Slide left index of C under HL.
2. Move HL over 4 on C. Now 5^4 is under HL on LL3; hence, 5^{-4} will be on LLo3.
3. Under HL read ***.00160*** on LLo3.

Example 2: $(0.15)^{-.25} = ?$

Verify that $(0.15)^{.25}$ is on LLo2; therefore, $(0.15)^{-.25}$ is found on LL2. Result is ***1.607.***

Example 3: $\left(\frac{223}{175}\right)^{-14.5} = ?$

1. First divide 223 by 175 and verify that expression becomes $(1.274)^{-14.5}$.
2. Now verify that $(1.274)^{14.5}$ is located on LL3 and, hence, $(1.274)^{-14.5}$ must be on LLo3. Result is ***.030.***

The foregoing illustrate the general procedure:

> *To evaluate b^x (x negative):*
> Proceed as if to evaluate b^x *(x positive)*, then read the result on the *reciprocal* LL scale.

Exercise 20-3

1. $6^{-4} =$

2. $3^{-5} =$

3. $(1.5)^{-7} =$

4. $(1.18)^{-2.5} =$

5. $(0.75)^{-3} =$

6. $(0.60)^{-5} =$

7. $(0.975)^{-30} =$

8. $(1.027)^{-18} =$

9. $(.006)^{-.07} =$

10. $(0.16)^{-.03} =$

11. $(264)^{-0.33} =$

12. $(.034)^{-0.26} =$

13. $(3700)^{-.025} =$

14. $(0.465)^{-.15} =$

15. $(56)^{-.252} =$

16. $(1.0276)^{-19.5} =$

17. $(.000075)^{-.064} =$

18. $(5600)^{-.007} =$

19. $(0.753)^{-8.2} =$

20. $(0.9783)^{-0.56} =$

21. $(.0575)^{-0.133} =$

22. $(.00013)^{-0.48} =$

23. $(0.9644)^{-28} =$

24. $(0.856)^{-52} =$

25. $3400(1 + 0.105)^{-14} =$

26. $25{,}000(1 + .035)^{-21} =$

27. $64{,}000(1 + .065)^{-34} =$

28. $\left(\frac{74}{15}\right)^{-.15} =$

29. $\left(\frac{465}{680}\right)^{-16} =$

30. $\left(\frac{6.9}{325}\right)^{-.072} =$

20.6 Evaluating b^x on slide rules with 8 LL scales

If your slide rule has 8 LL scales, you may check the following examples:

Example 1: $(1.00236)^{240} = ?$

1. Move HL over 1.00236 on LL0 (Ln0).
2. Slide left index of C under HL.
3. Move HL over 24 on C. Answer is located 2 scales higher on LL2.
4. Under HL read ***1.76*** on LL2.

Example 2: $(250)^{-.00065} = ?$

1. Move HL over 250 on LL3. Slide right index of C under HL.
2. Move HL over 65 on C. If exponent were positive, answer would be located 3 scales lower on LL0 (Ln0); exponent is negative, hence answer is on the reciprocal scale, LLo0 (Ln-0, LL/0).
3. Under HL read ***0.99642*** on LLo0 (Ln-0, LL/0).

Example 3: $(0.99805)^{1500} = ?$

1. Move HL over 0.99805 on LLo0 (Ln-0, LL/0). Slide left index of C under HL.
2. Move HL over 15 on C. Answer is located 3 scales higher on LLo3 (Ln-3, LL/3).
3. Under HL read ***.0536*** on LLo3 (Ln-3, LL/3).

Verify the following (use LL0 and LLo0 scales):

1. $(1.00164)^{30} = 1.0504$
2. $(1.00425)^{-820} = .0310$
3. $(1.223)^{.032} = 1.00646$
4. $(.0034)^{.00054} = 0.99694$
5. $(0.9763)^{-0.176} = 1.00423$
6. $(0.998745)^{4600} = .0031$
7. $(3200)^{.000185} = 1.001495$
8. $(0.99652)^{-23} = 1.0835$

20.7 Finding $b^{1/x}$

Example 1: $\sqrt[3]{72} = (72)^{1/3} = ?$

The logarithm may be written:

$$\ln(72)^{1/3} = (1/3)\ln(72)$$

In this case, then, we must *divide* ln(72) by 3. Note, also, that the exponent is *approximately 0.3.*

1. Move HL over 72 on LL3. This puts HL over ln(72) on D. Now divide by 3:
2. Slide 3 on C under HL.
3. Move HL over left index of C. HL is to the left of 72; hence, if exponent were about 3, answer would be on "LL4." Decimal point in exponent is actually one place to the left; therefore, answer is one scale lower on LL3.
4. Under HL read ***4.16*** on LL3.

Example 2: $\sqrt[12]{2.93} = (2.93)^{1/12} = ?$

Note that exponent is *about .08.*

1. Move HL over 2.93 on LL3. HL is now over ln(2.93) on D. Now *divide* by 12:
2. Slide 12 on C under HL.
3. Move HL over right index of C. HL is to the right of 2.93; hence, if exponent were about 8, answer would be on LL3. Decimal point in exponent is actually two places to the left of this position; therefore, answer is two scales lower on LL1.
4. Under HL read ***1.0938*** on LL1.

Example 3: $(.0115)^{1/60} = ?$

Note that exponent is *about .016*.

1. Move HL over .0115 on LLo3. Now *divide* by 60:
2. Slide 60 on C under HL.
3. Move HL over right index of C. HL is to right of .0115; hence, if exponent were about 1.6, answer would be on LLo3. Decimal point is actually two places to the left of this position; hence, answer is two scales lower on LLo1.
4. Under HL read ***0.9283*** on LLo1.

Example 4: $(0.9804)^{-1/.045} = ?$

Note that exponent is about −20.
Verify that result is on LL2, and is equal to ***1.553.***

Verify the following:

1. $\sqrt[4]{520} = 4.79$
2. $\sqrt[3]{2.64} = 1.382$
3. $\sqrt[12]{.0035} = 0.624$
4. $\sqrt[6]{0.550} = 0.9052$
5. $(1.345)^{1/4.4} = 1.0696$
6. $(0.243)^{-1/22} = 1.0665$
7. $(0.9605)^{1/.035} = 0.316$
8. $(15.5)^{-1/70} = 0.9616$

Example 5: Evaluate $10^{1/x}$ for $x = 3, 4, 5$, and 6.

Here, it is more convenient to perform the consecutive divisions using the CI scale.

1. Move HL over 10 on LL3.
2. Slide left index of C under HL.
3. Move HL successively over 3, 4, 5, and 6 on CI.

Verify that results are: ***2.155, 1.779, 1.585,*** and ***1.468.***

Example 6: $x^{2.3} = 36;\ x = ?$

1. Solving for x: $x = (36)^{1/2.3}$.
2. Verify that $x = \mathbf{4.75}$.

Example 7: $25(y^{.084}) = 47;\ y = ?$

1. Dividing 47 by 25, we obtain: $y^{.084} = 1.88$.
2. Solving for y: $y = (1.88)^{1/.084}$.
3. Verify that $y = \mathbf{1830}$.

Exercise 20-4

1. Evaluate $(32)^{1/x}$ for $x = 3, 5, 9, 15$.

2. $\sqrt[3]{275} =$

3. $\sqrt[3]{5.61} =$

4. $\sqrt[3]{.037} =$

5. $\sqrt[4]{2.55} =$

6. $\sqrt[4]{2200} =$

7. $\sqrt[5]{0.145} =$

8. $\sqrt[5]{55} =$

9. $\sqrt[7]{1.95} =$

10. $\sqrt[7]{0.474} =$

11. $\sqrt[6]{0.636} =$

12. $(12.5)^{1/55} =$

13. $(6.75)^{1/1.6} =$

14. $(0.515)^{-1/25} =$

15. $(.00014)^{1/2.2} =$

16. $(0.944)^{-1/.045} =$

17. $(145)^{1/15} =$

18. $(1.026)^{1/.024} =$

19. $(1.0245)^{-1/.007} =$

20. $\sqrt[10]{0.764} =$

21. $(2000)^{1/24} =$

22. $(0.9656)^{1/.032} =$

23. $(360)^{1/17} =$

24. $x^{2.2} = 1.7$; $x =$

25. $x^{0.3} = 5$; $x =$

26. $x^{15} = 0.78$; $x =$

27. $x^{1.75} = 0.575$; $x =$

28. $x^{-.053} = 0.9724$; $x =$

29. $2.76(x^{3.5}) = 400$; $x =$

30. $15(x^{-31}) = 3900$; $x =$

31. $340(x^{37.5}) = 0.289$; $x =$

32. $2400(x^{-3.56}) = 1740$; $x =$

33. $\left(\frac{56}{11}\right)^{1/7} =$

34. $\left(\frac{475}{620}\right)^{-1/.055} =$

35. $\left(\frac{2.4}{115}\right)^{1/14} =$

36. $\left(\frac{1.570}{.0042}\right)^{1/240} =$

37. $37.5\left(\frac{125}{340}\right)^{1/1.41} =$

38. $216\left(\frac{87.5}{31.0}\right)^{1/1.41} =$

20.8 More general powers of b

Example 1: $(1.055)^{25\times 2.8} = ?$

When the exponent is in combined form, we may first evaluate the exponent on C·D, then find the power. You may prefer this direct approach; however, in the following precedure you will note that it is not really necessary to evaluate the exponent.

1. Approximate the exponent to be about 70. Think of the logarithm: $\ln(1.055)^{25\times 2.8} = (25 \times 2.8)\ln(1.055)$.
2. Move HL over 1.055 on LL1. This puts HL over ln(1.055) on D. Now, instead of multiplying by 25, divide by its reciprocal:
3. Slide 25 on CI under HL. Now multiply by 2.8:
4. Move HL over 28 on C. Observing that HL is to the left of 1.055 with the exponent about 70, we determine that the answer must be on LL3.
5. Under HL read ***42.3*** on LL3.

Example 2: $(4)^{5/3} = ?$

1. Exponent is about 1.7. Think of the logarithm: $\ln(4)^{5/3} = (5/3)\ln(4)$.
2. Move HL over 4 on LL3. This puts HL over ln(4) on D. Now divide by 3:
3. Slide 3 on C under HL. Now multiply by 5:
4. Move HL over 5 on C. HL is to the right of 4 and exponent is about 1.7; hence, answer is on LL3.
5. Under HL read ***10.1*** on LL3.

Example 3: $(2.06)^{-(5.6\times 1.3)/155} = ?$

1. Approximate exponent to be about −.05.
2. Move HL over 2.06 on LL2. Slide right index of C under HL. Now multiply by 5.6:
3. Move HL over 56 on C. Now divide by 155:
4. Slide 155 on C under HL. Now multiply by 1.3:
5. Move HL over 13 on C. Observing that HL is to the left of 2.06, verify that, if exponent were positive, answer would be on LL1. It follows that answer must be on LLo1.
6. Under HL read ***0.9666*** on LLo1.

Example 4: $(.0145)^{1/(31\times 4.6)} = ?$

1. Approximate exponent to be about .007.
2. Verify that answer is located on LLo1, and is equal to ***0.97075.***

The techniques discussed in the preceding sections may finally be summarized as follows:

To raise b to any power:
1. If exponent is in reciprocal or combined form, estimate its value. This aids in locating the result on the proper LL scale.
2. Move HL over b on the LL scale. This puts HL over ln*(b)* on D.
3. Perform necessary operations on ln*(b)*—that is:
   multiplication (b^x, b^{xy}, etc.),
   division ($b^{1/x}$, $b^{1/xy}$, etc.),
   or both ($b^{x/y}$, $b^{xy/z}$, $b^{x/yz}$, etc.).
4. Read result on the appropriate LL scale.

Exercise 20-5

1. $(11)^{4/3} =$
2. $(8)^{5/7} =$
3. $(1.6)^{5/6} =$
4. $(36)^{-2/7} =$
5. $(0.7)^{4/9} =$
6. $(0.944)^{-13/7} =$
7. $(.034)^{-3/460} =$
8. $(1.0374)^{52 \times 0.34} =$
9. $(2.62)^{5.1 \times 1.8} =$
10. $(27.5)^{-(3.2 \times 4.7)/23} =$
11. $(.074)^{1/(4.5 \times 16.3)} =$
12. $(0.284)^{-100/(5.2 \times 3.7)} =$
13. $(4400)^{(3.7 \times 1.45)/34.6} =$
14. $(.00028)^{3/40} =$
15. $(0.9025)^{.064 \times 27} =$
16. $(1.0186)^{-100/(2.45 \times 3.14)} =$
17. $(650)^{(36 \times 2.7)/1250} =$
18. $(1.108)^{-(530 \times .077)} =$
19. $(.0135)^{2.24/(145 \times 1.95)} =$
20. $(0.9786)^{-68/\pi} =$
21. $(8.4)^{42/65} =$
22. $(1.0742)^{-(\pi \times 36)/2.3} =$
23. $x^{5/7} = 23;\ x =$
24. $x^{9/4} = 1.26;\ x =$
25. $36(x^{15/4}) = 250;\ x =$
26. $2.3(x^{1/6}) = 1.7;\ x =$
27. $x^{-5/16} = 0.955;\ x =$
28. $14(x^{-36/\pi}) = 175;\ x =$
29. $(1 + x)^{200/7} = 1.85;\ x =$
30. $(1 + x)^{-16/9} = 0.215;\ x =$
31. $360(1 - x)^{-25/11} = 570;\ x =$
32. $\left(1 + \frac{23}{670}\right)^{-4/.015} =$
33. $\left(1 - \frac{15}{463}\right)^{-240/13} =$
34. $624\left(\frac{6.4}{2.7}\right)^{0.415/1.415} =$
35. $545\left(\frac{34.2}{86.5}\right)^{0.415/1.415} =$
36. $\frac{575}{405} = \left(\frac{P}{50}\right)^{0.384/1.384};\ P =$
37. $\frac{425}{630} = \left(\frac{P}{7.4}\right)^{0.400/1.400};\ P =$
38. $\frac{560}{415} = \left(\frac{28.5}{P}\right)^{0.375/1.375};\ P =$
39. Evaluate $(0.98)^{-p/1.75}$ for: $p = 1, 12, 150,$ and 750.

20.9 Numbers outside the range of the LL scales.

The exercise sets of this chapter have been designed so that the initial settings and the results have, in all cases, been within the LL scale range. Methods for handling numbers outside the scale range are illustrated in Appendix B.

Chapter 21

FURTHER OPERATIONS WITH THE LL SCALES

21.1 Solving for the exponent in simple powers

In the previous chapter, we were concerned with evaluating powers directly. It often happens, however, that the power is known and we wish to find the exponent.

Example 1: $6^x = 1.55$; $x = ?$

Here, we know the result of raising 6 to a power, and we must determine the exponent. This is just the inverse of finding b^x.

1. Move HL over 6 on LL3. Slide left index of C under HL.
2. Move HL over 1.55 on LL2. Under HL read "245" on C. Observe that 1.55 is to the right of 6 and one scale lower. If it were on the same scale, exponent would be 2.45; however, it is one scale lower, hence exponent must be 0.245. It follows that $x =$ ***0.245.***

Example 2: $(0.955)^x = .064$; $x = ?$

1. Move HL over 0.955 on LLo1. Slide right index of C under HL.
2. Move HL over .064 on LLo3. Under HL read "596" on C. Note that .064 is to the left of 0.955 and is two scales higher. If it were one scale higher, exponent would be 5.96; however, it is two scales higher, hence $x =$ ***59.6.***

Example 3: $(1.22)^x = .028$; $x = ?$

1. Move HL over 1.22 on LL2. Slide left index of C under HL.

2. Move HL over .028 on LLo3. Under HL read "1800" on C. Note that .028 is to the right of 1.22 on LLo3. If it were on LL3, exponent would be 18.00; however, it is on the *reciprocal* LL scale, hence $x = -18.00$.

Example 4: $(.0165)^x = 1.022$; $x = ?$

1. Move HL over .0165 on LLo3. Slide right index of C under HL.
2. Move HL over 1.022 on LL1. Under HL read "530" on C. Observe that 1.022 is to the left of .0165 on LL1. If it were on LLo1, exponent would be .00530; however, it is on the reciprocal LL scale, hence $x = -.00530$.

Example 5: $e^x = 1.0466$; $x = ?$

When e is the base, we read directly from LL to D.

1. Move HL over 1.0466 on LL1.
2. Under HL read "455" on D. On LL1, exponent of e ranges from .01 to 0.1, hence $x = .0455$.

Example 6: $e^{-x} = .0033$; $x = ?$

1. Move HL over .0033 on LLo3.
2. Under HL read "571" on D. On LLo3, exponent of e ranges from −1 to −10. It follows that $-x = -5.71$, and $x = 5.71$.

Exercise 21-1

Determine x in the following:

1. $3^x = 20$
2. $10^x = 250$
3. $8^x = 54$
4. $4^x = 30$
5. $25^x = 7$
6. $(1.2)^x = 100$
7. $(1.03)^x = 200$
8. $e^x = 30$
9. $e^x = 550$
10. $e^x = 1.266$
11. $e^x = 0.9718$
12. $e^x = .00085$
13. $10^x = 1.34$
14. $(0.16)^x = .012$
15. $10^x = .02$
16. $5^x = 0.64$
17. $525^x = 7.5$
18. $(0.75)^x = 0.464$

19. $15^x = 0.65$

20. $(2.4)^x = 650$

21. $(1.165)^x = 1.0375$

22. $(1.84)^x = 0.766$

23. $(.037)^x = 0.49$

24. $(0.164)^x = 1.0525$

25. $(0.654)^x = .0475$

26. $e^x = 1.01675$

27. $e^{-x} = 0.562$

28. $e^{-x} = 0.106$

29. $10^x = 0.57$

30. $(2600)^{-x} = 0.124$

31. $(1.0164)^x = 64$

32. $e^{-x} = 0.9326$

33. $e^x = 1650$

34. $(0.586)^x = .00075$

35. $(3200)^{-x} = 0.794$

36. $(1.238)^{-x} = 0.513$

37. $(1.1645)^x = .0037$

38. $(0.386)^x = .084$

39. $(0.193)^x = 1.775$

40. $(1250)^x = 1.0138$

21.2 Finding logarithms to any base

Suppose we let $\log_b N = x$; it follows that $b^x = N$. Hence, finding $\log_b N$ leads to the same exponential form that was treated in the preceding section.

Example 1: $\log_2 5 = ?$

1. Let $\log_2 5 = x$; then $2^x = 5$.
2. Verify that $x = 2.32$; hence, $\log_2 5 =$ ***2.32.***

Example 2: $\log_3(0.62) = ?$

1. Let $\log_3(0.62) = x$; then $3^x = 0.62$.
2. Verify that $x = -0.435$; hence, $\log_3(0.62) =$ ***−0.435.***

Example 3: $\log_{10} 1.06 = ?$

1. Let $\log_{10} 1.06 = x$; then $10^x = 1.06$.
2. Verify that $x = .0253$; hence, $\log_{10} 1.06 =$ ***.0253.***

You will recall that we may find $\log_{10} N$ directly using the L scale. However, for values of N near 1, the LL scales yield more accuracy.

21.3 The DFM scale

The DFM scale is a D scale folded at $\log_{10}e = 0.434$. It is related to the D scale in the following way.

> If the hairline is over $\ln N$ on **D,** then $\log_{10}N$ is under the hairline on **DFM**

This scale makes it possible to directly convert logs from base e to base 10, and vice versa.

Example: **a.** $\ln 26 = ?$ **b.** $\log_{10}26 = ?$

1. Move HL over 26 on LL3.
2. Under HL read:
a. $\ln 26 =$ ***3.26*** on D.
b. $\log_{10}26 =$ ***1.415*** on DFM.

Exercise 21-2

1. $\log_2 7 =$

2. $\log_4 18 =$

3. $\log_8 35 =$

4. $\log_3 1.5 =$

5. $\log_{2.5} 100 =$

6. $\log_5 1.75 =$

7. $\log_{10} 1.155 =$

8. $\log_{10} 0.984 =$

9. $\log_{10} 0.9723 =$

10. $\log_{10} 1.01655 =$

11. $\log_{10} 1.0224 =$

12. $\log_3 0.635 =$

13. $\log_6 .0846 =$

14. $\log_{12} 350 =$

15. $\log_{7.5} 0.77 =$

16. $\log_{20} 1.27 =$

17. $\log_{10} 1.0444 =$

18. $\log_9 175 =$

19. $\log_{1.2} 1.88 =$

20. $\log_{2.6} 0.94 =$

21. $\log_8 4500 =$

22. $\log_{3.8} 0.665 =$

23. $\log_{10} 0.8645 =$

24. $\log_7 .0035 =$

21.4 Exponential equations

Any equation in which the unknown appears in an exponent may be called an exponential equation. In section 21.1, you were solving simple exponential equations; in this section we go further with equations of this type.

Example 1: $35e^{-.22t} = 0.106$; $t = ?$

1. Dividing 0.106 by 35 on the C-D scales, we obtain:

$$e^{-.22t} = .00303$$

2. Move HL over .00303 on LLo3.
3. Under HL read "580" on D. Therefore, $-.22t = -5.80$.
4. Dividing on C-D, we find $t =$ ***26.4.***

Example 2: $125(370)^x = 90$; $x = ?$

1. Dividing 90 by 125 on C-D, we obtain:

$$370^x = 0.720$$

2. Move HL over 370 on LL3. Slide right index of C under HL.
3. Move HL over 0.720 on LLo2. Under HL read "556" on C. Note that 0.720 is to the left of 370 on LLo2. If it were on LL2, exponent would have to be .0556; however, it is on the *reciprocal* LL scale, hence $x =$ ***−.0556.***

Example 3: $(0.73)^{.034x} = 0.865$; $x = ?$

1. Move HL over 0.73 on LLo2. Slide right index of C under HL.
2. Move HL over 0.865 on LLo2. Under HL read "461" on C. Note that 0.865 is to the left of 0.73 and on the same scale, hence exponent must be 0.461. It follows that $.034x = 0.461$.
3. Dividing on C-D, we obtain $x =$ ***13.56.***

Example 4: $75 = 1000(1.85)^{-14k}$; $k = ?$

1. Dividing 75 by 1000, we may write:

$$(1.85)^{-14k} = .075$$

2. Move HL over 1.85 on LL2. Slide right index of C under HL.
3. Move HL over .075 on LLo3. Under HL read "421" on C. Note that .075 is to the left of 1.85 on LLo3. If it were on LL3, exponent would be 4.21; however, it is on the reciprocal LL scale, hence exponent must be −4.21. Therefore, $-14k = -4.21$.
4. Dividing on C-D, we obtain $k =$ ***0.301.***

Alternate method:
Although more settings are required, you may prefer to first solve the given relation for k.

1. Returning to $(1.85)^{-14k} = .075$, we take natural log of both sides:

$$(-14k)\ln(1.85) = \ln(.075)$$

2. Solving for k:

$$k = \frac{\ln(.075)}{(-14)(\ln(1.85)}$$

3. Verify that $\ln(.075) = -2.59$, and $\ln(1.85) = 0.615$.
4. Substituting these values, k may be evaluated on C-D:

$$k = \frac{-2.59}{(-14)(0.615)} = \boldsymbol{0.301}.$$

Exercise 21-3

1. $e^{12x} = 1550;\ x =$
2. $e^{.0065x} = 1.0665;\ x =$
3. $e^{-10x} = 0.374;\ x =$
4. $e^{-.024x} = .0028;\ x =$
5. $3.5\ e^{x} = 460;\ x =$
6. $125\ e^{x} = 60;\ x =$
7. $550\ e^{-x} = 30;\ x =$
8. $e^{.0053x} = 1.267;\ x =$
9. $e^{k/12.5} = 1.0326;\ k =$
10. $4.63\ e^{3.6k} = 2150;\ k =$
11. $2000\ e^{-.064t} = 50;\ t =$
12. $1200\ e^{-.0075t} = 125;\ t =$
13. $10.7\ e^{75k} = 145{,}000;\ k =$
14. $100\ e^{-2\pi k} = 5.5;\ k =$
15. $26\ e^{t/16} = 94{,}000;\ t =$
16. $15{,}000\ e^{-t/25} = 220;\ t =$
17. $2.5(1.745)^{x} = 362;\ x =$
18. $6500(0.815)^{x} = 20.8;\ x =$
19. $24.6(0.586)^{-x} = 45.0;\ x =$
20. $1000(1.0625)^{n} = 3640;\ n =$
21. $2200(1.125)^{n} = 12{,}600;\ n =$
22. $1000(1.075)^{n} = 525;\ n =$
23. $8600(1.045)^{n} = 2700;\ n =$
24. $\left(\frac{12.6}{41.5}\right)^{x} = \frac{550}{640};\ x =$
25. $\left(\frac{34.5}{57.2}\right)^{x} = \frac{440}{510};\ x =$
26. $(1.074)^{-1.7k} = 0.948;\ k =$
27. $(840)^{.037x} = 1.056;\ x =$
28. $(250)^{-\pi x} = 0.744;\ x =$
29. $(0.162)^{.0045p} = 0.935;\ p =$

30. $(6000)^{.072t} = 64;\ t =$

31. $0.27(1.055)^{360t} = 50;\ t =$

32. $324(4.16)^{2\pi k} = 340;\ k =$

33. $3.6(1.15)^{.045t} = 31.0;\ t =$

34. $145(2.35)^{-.018t} = 26;\ t =$

35. $225(0.65)^{k/32} = 1.52;\ k =$

36. $1200(.00055)^{.0045p} = 1170;\ p =$

37. $0.410(26.5)^{-22x} = 0.336;\ x =$

38. $55(7500)^{-.0135t} = 24.2;\ t =$

39. $\frac{510}{620} = \left(\frac{8.25}{31.6}\right)^{(n-1)/n};\ n =$

40. $\frac{435}{520} = \left(\frac{126}{147}\right)^{(n-1)/n};\ n =$

41. $2e^{2x} - 12e^{x} + 5 = 0;\ x =$

42. $3e^{2x} - 8e^{x} + 3 = 0;\ x =$

43. $2e^{6x} - 31e^{3x} + 58 = 0;\ x =$

44. $12e^{-2x} - 11e^{-x} + 2 = 0;\ x =$

21.5 Formula types

Example 1: $\frac{(1 + .063\sqrt[3]{75})^{5.8}}{(0.65)^{3.2}} = ?$

1. Verify that $.063\sqrt[3]{75} = 0.266$
2. Expression now becomes: $\frac{(1.266)^{5.8}}{(0.65)^{3.2}}$.
3. Verify that $(1.266)^{5.8} = 3.93$; $(0.65)^{3.2} = 0.252$.
4. Expression finally becomes: $\frac{3.93}{0.252}$.
5. Dividing on C-D, verify that result is ***15.59.***

Example 2: Given the formula: $S = R\left[\frac{(1+i)^n - 1}{i}\right]$

If $S = 7500$, $R = 37$, $i = .035$, find n to the nearest integral value.

1. Substitute the given values:

$$7500 = \frac{37\,[(1.035)^n - 1]}{.035}$$

2. Verify that $(1.035)^n = \frac{.035 \times 7500}{37} + 1 = 7.1 + 1 = 8.1$.
3. Finally verify that $n = 60.8$; hence, to nearest integer, $n =$ ***61.***

Exercise 21-4

1. $\sqrt{29(1 + e^{-1.84})} =$

2. $15(0.32)^4(0.68)^2 =$

3. $120(0.72)^7(0.28)^3 =$

4. $\dfrac{7500}{1 + 12.5e^{-2.26}} =$

5. $.046e^{-2.7\pi} \sin 1.75 =$

6. $3.66 \times 10^7 \times \ln\left[\dfrac{6.44 - 1.27}{6.44 + 1.27}\right] =$

7. $\dfrac{1.32 \times 10^{-9} \times \sqrt{870}}{(.072)^{5/4}} =$

8. $\dfrac{(250)^{0.76}}{(25 + 12\sqrt{4.65})^{1.16}} =$

9. $\ln\left[\dfrac{1 + \sqrt{1 + (2.6)^2}}{2.6}\right] =$

10. $\dfrac{240\,[(1.0275)^{11} - 1]}{.0275} =$

11. $35\left[\dfrac{1 - (1.045)^{-15}}{.045}\right] =$

12. $\dfrac{10!}{8!2!}(0.84)^8(0.16)^2 =$

13. $\dfrac{10!}{6!4!}(0.43)^6(0.57)^4 =$

14. $(1260 \cos 53°)^{0.6}(0.164)^{2/3}(136)^{1/4} =$

15. $\dfrac{1670 \times 0.155}{(1 + 0.155)^{1/6} - 1} =$

16. $4.25 \times 10^6[50 + \pi/.065]^{1/23} =$

17. $67{,}500(0.86)^{5/2}\tan 62° =$

18. $[1 + .000275(27.2)^2]^{1.75/0.75} =$

19. $\dfrac{5.66 \times 10^{-17}}{\ln 7.24}[1 + \sqrt{7.25}] =$

20. $(0.72)^{-5}\,[e^{\pi/(0.72 \times 45)} - 1]^{-1} =$

In the following formulas, substitute the given data and evaluate:

21. $P = \dfrac{9.2\ D^{1.7}}{R + 5}$
a. $R = 3.75, D = 8$; **b.** $R = 8.60, D = 14.5$

22. $N = e^{2\pi d/\sqrt{1-d^2}}$
a. $d = 0.72$; **b.** $d = 0.46$

23. $x = \ln(u + \sqrt{u^2 + 1})$
a. $u = 2.20$; **b.** $u = -0.176$

24. $y = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$
a. $x = 0.25$; **b.** $x = 0.5$; **c.** $x = 1$; **d.** $x = 2.50$

25. $X = e^{-2.44t} \sin 0.46t$ (angle in radians)
a. $t = 1$; **b.** $t = 2.5$; **c.** $t = 5.7$

26. $V = 1.318\ C\ R^{0.63}\ S^{0.54}$
a. $C = 130, R = 0.25, S = .003$
b. $C = 750, R = .048, S = .0076$

27. $E = RT\ln(p_1/p_2)$
a. $R = 55.1, T = 500, p_1 = 54.7, p_2 = 36.0$
b. $R = 29.5, T = 350, p_1 = 24.2, p_2 = 18.6$

28. $A = \dfrac{250}{1 + 16e^{-.023t}}$
a. $t = 75$; **b.** $t = 125$

29. $p = \dfrac{b^x\ e^{-b}}{x!}$
a. $b = 0.2$, $x = 5$; **b.** $b = 2$, $x = 4$

30. $F_1 = F_2 e^{f\Theta}$ (Θ in radians).
a. $f = 0.25$, $\Theta = 210°$, $F_2 = 1800$
b. $f = 0.17$, $\Theta = 104°$, $F_2 = 2400$

31. $h = (p_0/w_0)\ \ln(p_0/p)$
a. $p_0 = 1830$, $w_0 = .0765$, $p = 1250$
b. $p_0 = 2620$, $w_0 = 1.140$, $p = 1780$

32. $Q = 2.38\ H^{5/2} \tan \dfrac{1}{2}\Theta$
a. $H = 3.6$, $\Theta = 70°$; **b.** $H = 6.2$, $\Theta = 55°$

33. $PV^{1.4} = 16000$
a. $V = 27.8$, find P; **b.** $V = 460$, find P; **c.** $P = 760$, find V;
d. $P = 1240$, find V.

34. $C = \dfrac{6}{\pi \ln(d_1/d_2)} \left[\dfrac{(d_1/d_2) - 1}{2}\right]$
a. $d_1 = 11.2$, $d_2 = 6.7$; **b.** $d_1 = 34.7$, $d_2 = 14.6$

35. $P = 650{,}000\ e^{kt}$
a. If $P = 925{,}000$ when $t = 10$, evaluate k.
b. Use this value of k to find P when $t = 18$.

36. $E = 14.2\, e^{-kt}$
a. If $E = 10.2$ when $t = 3$, evaluate k.
b. Use this value of k to find E when $t = 6.3$.

37. $P = \dfrac{N!}{x!(N - x)!}\, p^x\, q^{N-x}$
a. $N = 10$, $x = 4$, $p = 0.59$, $q = 0.41$
b. $N = 10$, $x = 7$, $p = 0.81$, $q = 0.19$
c. $N = 9$, $x = 3$, $p = 0.44$, $q = 0.56$
d. $N = 11$, $x = 5$, $p = 0.65$, $q = 0.35$

38. $A = P(1 + i)^n$
a. $P = 1000$, $A = 2250$, $n = 21$, find i;
b. $P = 1200$, $A = 2100$, $n = 12$, find i;
c. $P = 1200$, $A = 3000$, $i = .01625$, find n (nearest integral value);
d. $P = 650$, $A = 3500$, $i = .0275$, find n (nearest integral value).

39. $S = R\left[\frac{(1+i)^n - 1}{i}\right]$

a. $R = 55$, $i = .035$, $n = 15$; **b.** $R = 75$, $i = .0625$, $n = 22$;
c. $S = 2800$, $R = 75$, $i = .025$, find n (nearest integral value);
d. $S = 20{,}000$, $R = 825$, $i = .018$, find n (nearest integral value).

40. $A = R\left[\frac{1 - (1+i)^{-n}}{i}\right]$

a. $A = 1150$, $R = 63$, $i = .0325$, find n (nearest integral value);
b. $A = 7250$, $R = 215$, $i = .0125$, find n (nearest integral value).

41. $P = Fr/i + (V - Fr/i)(1+i)^{-n}$

a. $F = 2000$, $V = 2600$, $r = .045$, $i = .0625$, $n = 9$
b. $F = 15{,}000$, $V = 18{,}500$, $r = .035$, $i = .0825$, $n = 12$

42. $\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n}$

a. $P_1 = 14$, $P_2 = 73$, $T_2 = 550$, $n = 1.415$, find T_1;
b. $P_1 = 15$, $P_2 = 107$, $T_2 = 635$, $n = 1.346$, find T_1;
c. $T_1 = 520$, $T_2 = 625$, $P_1 = 15.5$, $P_2 = 39$, find n;
d. $T_1 = 575$, $T_2 = 740$, $P_1 = 19$, $P_2 = 42$, find n.

43. $V = V_f \tanh\left(\frac{gHt}{V_f L}\right)$

a. $H = 20$, $t = 10$, $V_f = 4.55$, $L = 2000$, $g = 32.2$
b. $H = 30$, $t = 12$, $V_f = 4.75$, $L = 1800$, $g = 32.2$

44. $S = 2\pi a^2 + \frac{\pi b^2}{e}\ln\left(\frac{1+e}{1-e}\right)$ (where $e = \sqrt{a^2 - b^2}/a$)

a. $a = 12$, $b = 7$; **b.** $a = 26.5$, $b = 18.3$

45. $E = \frac{p_1}{w_1}\left[\frac{k}{k-1}\right]\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$

a. $k = 1.32$, $p_1 = 2880$, $p_2 = 2120$, $w_1 = .0613$
b. $k = 1.55$, $p_1 = 3150$, $p_2 = 2120$, $w_1 = .0580$

46. $r = \left[1 + \left(\frac{k-1}{2}\right)\left(\frac{V}{c}\right)^2\right]^{k/(k-1)}$

a. $k = 1.4$, $V = 576$, $c = 1117$
b. $k = 1.65$, $V = 645$, $c = 1117$

Chapter 22

REVIEW EXERCISES

The following exercise sets require the use of all scales covered in Chapters 14 through 21.

Exercise 22-1

1. $\sin 37° =$

2. $\cos 86°30' =$

3. $\tan 47.5° =$

4. $\log_{10} 38.2 =$

5. $e^{0.134} =$

6. $e^{-.075} =$

7. $(1.075)^{27} =$

8. $(260)^{-0.32} =$

Exercise 22-2

1. Solve the right triangle: $A = 46.2°$, $b = 370$.

2. $\ln 26.2 =$

3. $(13)^{5/3} =$

4. $\cot 14.6° =$

5. $2600e^{.046} =$

6. $15(x^{.075}) = 27$; $x =$

7. $24 \sin 36° =$

8. $\sqrt[5]{0.138} =$

Exercise 22-3

1. $\dfrac{145 \cos 62°}{\sin 27°} =$

2. $\ln 1.0137 =$

3. $e^{-3.27} =$

4. $\tan 3.45° =$

5. $\left(\dfrac{6.22}{21.5}\right)^{0.57} =$

6. $\sin^{-1} 0.644 =$

7. $(520)^x = 4.25$; $x =$

8. Solve the oblique triangle: $a = 82$, $c = 55$, $A = 43°$.

Exercise 22-4

1. $(4400)^{.0026} =$

2. $\cos 17° 23' =$

3. $\tan^{-1} 1.25 =$

4. $e^{0.62} \sin 22° =$

5. $\log_{10} 56{,}500 =$

6. $\dfrac{\ln .0245}{\pi} =$

7. Change $24\angle 56°$ to rectangular form.

8. $\log_3 8 =$

Exercise 22-5

1. $(7500)^{.035} =$

2. $.0375 \log_{10}\left[\dfrac{16.25}{.00352}\right] =$

3. $e^{-0.455} \cos 0.62 =$

4. $\ln N = 1.46;\ N = ?$

5. $640 \left(\dfrac{24.2}{37.5}\right)^{0.43/1.43} =$

6. Convert to radians: **a.** 36°, **b.** 76.3°, **c.** 214°.

7. $\sin 0.76° =$

8. $2400\, e^{-.052t} = 75;\ t =$

Exercise 22-6

1. $\tan 257° =$

2. $e^{-3\pi/2} =$

3. Change $(16 + 7j)$ to polar form.

4. $\dfrac{\ln 0.742}{.076 \ln 12.7} =$

5. $(32)^{-(3.4\times 5.2)/310} =$

6. $175\, e^{-(3.6\times 1.22)} \sin(0.72) =$

7. $\sin^{-1} .0356 =$

8. $\ln N = -3.25;\ N =$

Exercise 22-7

1. $\cos 88.36° =$

2. $e^{(2.6 \times 4.3)/7.5} =$

3. Solve the oblique triangle: $A = 124°$, $C = 13°$, $b = 21$.

4. $(0.15)^x = 0.14$; $x =$

5. $\sqrt[7]{350} =$

6. $\frac{1}{2}(16.8)(23.2)\sin 24° 15' =$

7. $45(0.78)^8(0.22)^2 =$

8. $\frac{\ln 3.25}{2.3 \ln 0.422} =$

Exercise 22-8

1. Solve the right triangle: $a = 145$, $b = 67$.

2. $\cos 0.68 =$

3. $(0.9780)^{-0.48} =$

4. $\frac{132 \sin^2 19.3°}{\sin 2.46°} =$

5. Given: $V = \sqrt{\frac{2gR}{\sin 2\Theta}}$
   Find V if $R = 2400$, $g = 32.2$, $\Theta = 27.2°$.

6. $\sin^{-1}\left[\frac{12.8 + 8.44}{12.8 \times 8.44}\right] =$ (give answer in radians)

7. $\tan 86.5° =$

8. $165(2.75)^{-.022t} = 32$; $t =$

Appendix A

SMALL ANGLES (LESS THAN 0.573°)

A.1 Angles given in degrees

For angles in the range of the ST scale or smaller, the sine (or tangent) is proportional to the angle itself. Thus, if the angle is divided by 10, for example, its sine or tangent will also be divided by 10.

From the ST scale we may read sin 2.6° = .0454. Now, dividing both the sine and its angle by 10, we write: sin 0.26° = .00454. Clearly, we could have divided by 100 and obtained: sin .026° = .000454, and so forth.

It is emphasized that this procedure is only valid for very small angles; you should apply it only to angles that are too small to be read directly on the ST scale.

Example 1: sin .034° = ?

1. Move HL over 3.4° on ST.
2. Under HL read "593" on C.

Therefore, sin 3.4° = .0593, and sin .034° = ***.000593.***

Example 2: tan 89.22° = ?

1. Write: tan 89.22° = cot .08° = 1/tan .08°.
2. Verify that result is ***716.***

Verify the following:

1. sin 0.36° = .00628
2. sin .044° = .000767

3. tan 0.50° = .00872

4. tan .018° = .000314

5. tan 89.6° = 143.2

6. csc 0.15° = 382

A.2 Angles given in minutes or seconds

You may now verify that:

$$\sin 1' = \sin .01667° = .000291$$
$$\sin 1'' = \sin .000278° = .00000485$$

Since the sine (or tangent) of a small angle is proportional to the angle itself, the following relations may be used:

1. If angle is in *minutes*, then **sin x** or **(tan x)** = ***.000291 x.***
2. If angle is in *seconds*, then **sin x** or **(tan x)** = ***.00000485 x.***

The approximate values of these constants may be remembered as *"three zeros three"* and *"five zeros five."*

A.3 The "minutes" and "seconds" gauge marks

To facilitate finding the sine (or tangent) of small angles in minutes or seconds, some slide rules have two gauge marks related to the CI scale. These marks are scribed on the ST scale at positions opposite 291 and 485 on CI, and may be found at about 1.97° and 1.18°. They are usually labeled with the symbol for minutes (′) and the symbol for seconds (″) respectively.

On other rules, the marks are related to the C scale. They may be scribed directly on C at 291 and 485, or opposite these positions on ST at about 1.67° and 2.78°. Again, these are usually labeled with the minutes and seconds symbols.

The use of these gauge marks may be summarized as follows:

If gauge marks are related to CI:

1. Move HL over angle in minutes (or seconds) on D.
2. Slide "minutes" (or "seconds") scribe under HL.
3. Read sine (or tangent) of the angle opposite C index on D.

If gauge marks are related to C:

1. Set C index opposite angle in minutes (or seconds) on D.
2. Move HL over "minutes" (or "seconds") scribe.
3. Read sine (or tangent) of the angle under HL on D.

Example 1: $\sin 12.5' = ?$

A rough approximation is "three zeros three" times the angle, or .0003 × 12.5. Using the gauge marks, a more exact result is obtained.

Gauge marks related to CI:

1. Move HL over 125 on D.
2. Slide "minutes" scribe under HL.
3. Opposite right index of C read "364" on D.

From the rough approximation, we see that answer is ***.00364.***

Gauge marks related to C:

1. Set left index of C opposite 125 on D.
2. Move HL over "minutes" scribe.
3. Under HL read "364" on D. Answer is ***.00364.***

Example 2: tan 34″ = ?

A rough approximation is "five zeros five" times the angle, or .000005 × 34. Using the gauge mark, we proceed as follows:

Gauge marks related to CI:

1. Move HL over 34 on D.
2. Slide "seconds" scribe under HL.
3. Opposite left index of C read "1649" on D.

From the approximation, answer must be ***.0001649.***

Gauge marks related to C:

1. Set right index of C opposite 34 on D.
2. Move HL over "seconds" scribe.
3. Under HL read "1649" on D. Answer is ***.0001649.***

Verify the following:

1. sin 26′ = .00756
2. sin 7.5″ = .0000363
3. tan 14′30″ = .00422
4. sin 18.3′ = .00532
5. tan 20″ = .0000969
6. sin 3.7′ = .001077
7. tan 46″ = .000223
8. cot 25′18″ = 136
9. csc 12.4′ = 277

A.4 Conversion to radians

For the small angles we have been considering, the sine (or tangent) is about equal to the angle itself expressed in radians. Hence, to convert from degrees, minutes, or seconds, we simply find the sine of the angle and this also represents the radian equivalent of the angle.

Example 1: Convert 0.24° to radians.

1. Using the ST scale, verify that sin 0.24° = .00419.
2. It follows that 0.24° = ***.00419 radians.***

Example 2: Convert 25′ to radians.

1. Using the "minutes" gauge mark, verify that sin 25′ = .00727.
2. It follows that 25′ = ***.00727 radians.***

Verify the following:

1. 0.46° = .00802 radians
2. 37′ = .01076 radians
3. 28.5″ = .0001381 radians
4. .0375° = .000654 radians
5. 12.6′ = .00367 radians
6. 51.2″ = .000248 radians
7. 0.26° = .00454 radians
8. .071° = .00124 radians

Exercise A-1

1. sin 0.26° =
2. sin .077° =
3. tan .0145° =
4. sin .085° =
5. tan 89.7° =
6. cot 0.27° =
7. sin 0.46° =
8. tan 89.87° =
9. 1600 sin 0.4° =
10. 148 tan .065° =
11. Convert to radians:
    a. 0.35° b. .071°
    c. 0.185° d. .00225°
12. sin 5.8′ =
13. tan 12′15″ =
14. tan 35″ =
15. sin 2.44′ =
16. sin 13.5″ =
17. cot 22′30″ =
18. csc 11.25′ =
19. csc 33″ =
20. tan 89° 45.2′ =
21. tan 89° 59′26″ =
22. Convert to radians:
    a. 43′ b. 26″
    c. 8′40″ d. 11.5″
23. $\frac{1}{2}(170)(235)\sin 23' =$
24. $\frac{1}{2}(1350)(720)\sin 47'' =$

Appendix B

NUMBERS OUTSIDE THE RANGE OF THE LL SCALES

B.1 Numbers beyond the range of LL3 or LLo3

We have seen that the LL scales range from e^{-10} to e^{10}, or from .000045 to about 22,000. It is also evident that there is a discontinuity at the number 1; nowhere on the scale does this number appear. The scales approach 1 from above and from below, but never reach it. This is not surprising inasmuch as $1 = e^0$, and the number 0 does not appear on the D scale.

It follows that there are three ways in which we may fail to locate a number directly on the LL scales: the number may be *too large* (larger than 22,000), it may be *too small* (smaller than .000045), or it may be *too near 1*.

In this section we shall illustrate methods for dealing with numbers that are too large or too small (beyond the range of LL3 or LLo3). Numbers very near 1 are discussed in the next two sections.

Example 1: $(14)^5 = ?$

Rewrite this: $(1.4 \times 10)^5 = (1.4)^5 \times 10^5$.

Now, $(1.4)^5$ is within the range of the LL scale; hence, we may evaluate in the usual manner. Verify its value to be 5.38.

Therefore, result is $\boldsymbol{5.38 \times 10^5}$.

Example 2: $(.000032)^{-6} = ?$

Rewrite this: $(3.2 \times 10^{-5})^{-6} = (3.2)^{-6} \times 10^{30}$.

Evaluating in the usual manner: $(3.2)^{-6} = .00094$.

Hence, result is $.00094 \times 10^{30}$ or $\boldsymbol{9.4 \times 10^{26}}$.

Example 3: $(6350)^{5/2} = ?$

Rewrite this: $(0.635 \times 10^4)^{5/2} = (0.635)^{5/2} \times 10^{10}$.
Verify that $(0.635)^{5/2} = 0.322$.
Therefore, result is 0.322×10^{10} or $\boldsymbol{3.22 \times 10^9}$.

Example 4: $(5.6)^{-7.3} = ?$

Here, we divide the exponent by 2, then square the result.
Rewrite the expression: $[(5.6)^{-3.65}]^2$.
Verify that $(5.6)^{-3.65} = .00185$.
Result is $(.00185)^2 = (1.85 \times 10^{-3})^2 = \boldsymbol{3.42 \times 10^{-6}}$.

Example 5: $(155)^{14.3} = ?$

Rewrite this: $[1.55 \times 10^2]^{14.3} = (1.55)^{14.3} \times 10^{28.6}$.
This, in turn, may be written: $(1.55)^{14.3} \times 10^{0.6} \times 10^{28}$.
Verify that $(1.55)^{14.3} = 525$, and $10^{0.6} = 3.98$ (use L scale).
Result is $525 \times 3.98 \times 10^{28} = \boldsymbol{2.09 \times 10^{31}}$.

Example 6: $(.000425)^{7.22} = ?$

Rewrite this: $(0.425 \times 10^{-3})^{7.22} = (0.425)^{7.22} \times 10^{-21.66}$.
This may be written: $(0.425)^{7.22} \times 10^{0.34} \times 10^{-22}$.
Verify that $(0.425)^{7.22} = .0021$; $10^{0.34} = 2.19$ (use L scale).
Result is $.0021 \times 2.19 \times 10^{-22} = \boldsymbol{4.59 \times 10^{-25}}$.

Exercise B-1

1. $22^{4.6} =$

2. $e^{12} =$

3. $e^{-15} =$

4. $(1.032)^{500} =$

5. $12^{25} =$

6. $(.0072)^{12} =$

7. $(19)^{5.4} =$

8. $5^{7.3} =$

9. $(124)^{16.3} =$

10. $(76)^{12.7} =$

11. $(0.15)^{7.5} =$

12. $(145)^{20} =$

13. $(1230)^{-5.2} =$

14. $(1.25)^{51.6} =$

15. $(8.4)^{27} =$

16. $(9.6)^{-15} =$

17. $(.0064)^{8.75} =$

18. $(.062)^{-21.4} =$

19. $(.00053)^{-6.7} =$

20. $(27.5)^{14.6} =$

B.2 Numbers very near 1 (rules with 6 LL scales)

Numbers near 1 may be located on LL1 down to 1.01, and on LLo1 up to 0.99; therefore, any number between 0.99 and 1.01 is outside the scale range. In this section we illustrate procedures for working with such off-scale numbers, and we shall make use of the following approximate relations:

> If x is a positive number near zero:
>
> 1. $(1 + x)^n \approx 1 + nx$
> 2. $(1 - x)^n \approx 1 - nx$

Example 1: $(1.008)^{1.2} = ?$

Use relation (1):

$$(1.008)^{1.2} = (1 + .008)^{1.2} \approx 1 + (1.2)(.008) = \boldsymbol{1.0096} \text{ (approx.)}$$

Example 2: $(0.9996)^{3.5} = ?$

Use relation (2):

$$(0.9996)^{3.5} = (1 - .0004)^{3.5} \approx 1 - (3.5)(.0004) = \boldsymbol{0.9986} \text{ (approx.)}$$

Example 3: $(1.008)^{75} = ?$

Here, the exponent is quite large, and the result is in the range of the LL scales. We break up the exponent in the following way:

$$(1.008)^{75} = [(1.008)^k]^{75/k}$$

Now choose k just large enough to get $(1.008)^k$ on to the *left end* of the LL1 scale. Clearly, $k = 2$ will be satisfactory. Using relation (1) with $k = 2$:

$$(1.008)^2 = (1 + .008)^2 \approx 1 + 2(.008) = 1.016 \text{ (approx.)}$$

Hence, the original power may be written:

$$(1.008)^{75} \approx (1.016)^{75/2} = (1.016)^{37.5} \text{ (approx.)}$$

This may be evaluated on the LL scales in the usual manner.

Verify that the result is ***1.812.***

(This is, of course, an approximation. Using 5-place log tables, the answer is 1.818.)

Example 4: $(1.0045)^{146} = ?$

Write: $(1.0045)^{146} = [(1 + .0045)^k]^{146/k}$.

Now if we take $k = 3$, we will just get on to the left end of LL1. Using relation (1) with $k = 3$:

$$(1 + .0045)^3 \approx 1 + 3(.0045) = 1.0135 \text{ (approx.)}$$

Hence, we may write:

$$(1.0045)^{146} \approx (1.0135)^{146/3}$$

Evaluating as usual on the LL scales, verify that result is ***1.920*** (approx.) (Using 5-place logs, the answer is 1.9262.)

Example 5: $(1.00031)^{850} = ?$

Write: $(1.00031)^{850} = [(1 + .00031)^k]^{850/k}$.

In this case, $k = 40$ will bring us on to the left end of LL1.

Using relation (1): $(1 + .00031)^{40} \approx 1.0124$ (approx.)

Verify that $(1.00031)^{850} \approx (1.0124)^{850/40} =$ ***1.299*** (approx.)

Verify the following:

1. $(1.0005)^{4.4} \approx 1.0022$
2. $(0.9972)^{0.5} \approx 0.9986$
3. $(1.0026)^2 \approx 1.0052$
4. $(1.0063)^{60} \approx 1.455$
5. $(1.0023)^{115} \approx 1.301$
6. $(1.00037)^{700} \approx 1.294$

Example 6: $(0.99944)^{600} = ?$

In this case, we break up the exponent so that we can just get on to the *left end* of LLo1.

Write: $(0.99944)^{600} = [(1 - .00056)^k]^{600/k}$.

Here, $k = 20$ will just bring us on to the left end of LLo1.

Using relation (2) with $k = 20$:

$$(1 - .00056)^{20} \approx 1 - 20(.00056) = 0.9888 \text{ (approx.)}$$

The original power may now be written:

$$(0.99944)^{600} \approx (0.9888)^{600/20} = (0.9888)^{30} \text{ (approx.)}$$

Evaluating on the LL scales in the usual manner, verify that result is ***0.713*** (approx.)

Verify the following:

1. $(0.9938)^{100} \approx 0.536$

2. $(0.9965)^{80} \approx 0.755$

3. $(0.9986)^{240} \approx 0.714$

4. $(0.99935)^{10,000} \approx .00145$

Example 7: $(20)^{.00007} = ?$

We may write: $(20)^{.00007} = (20^k)^{.00007/k}$

Now set the index of C opposite 20 on LL3 and experiment with the hairline to discover the value of k that will just put 20^k within scale range (on to the left end of LL1).

Verify that $k = .0035$ will do this, and that $20^{.0035} = 1.01055$.

Thus, the original power may be written:

$$(20)^{.00007} = (1.01055)^{.00007/.0035} = (1.01055)^{.02}$$

Now use relation (1):

$$(20)^{.00007} = (1 + .01055)^{.02} \approx 1 + (.02)(.01055) = \boldsymbol{1.000211} \text{ (approx.)}$$

Exercise B-2

Approximate the following:

1. $(1.0006)^{3.5} =$

2. $(0.9996)^{5} =$

3. $(1.0037)^{40} =$

4. $(1.0011)^{80} =$

5. $(1.00062)^{1000} =$

6. $(1.0048)^{70} =$

7. $(1.00022)^{600} =$

8. $(1.00008)^{2000} =$

9. $(0.9962)^{40} =$

10. $(0.9925)^{120} =$

11. $(0.99940)^{500} =$

12. $(0.99926)^{1200} =$

13. $(0.9981)^{250} =$

14. $(1.7)^{.0008} =$

15. $(150)^{.00004} =$

16. $(0.25)^{.0003} =$

17. $(0.65)^{.0005} =$

18. $(4.4)^{-.00025} =$

19. $(1.075)^{-.0042} =$

20. $(0.85)^{-.0034} =$

B.3 Numbers very near 1 (rules with 8 LL scales)

As mentioned before, some slide rules have two additional lower scales in the neighborhood of 1. One of these ranges from 1.001 to 1.01, and we have referred to this as the LL0 scale (on the Deci-Lon rule it is labeled Ln0). The other scale ranges from 0.990 to 0.999 and we have referred to this as the LLo0 scale (on the Versalog rule it is labeled LL/0; on the Deci-Lon rule it is labeled Ln-0).

Example 1: $(1.00236)^{240} = ?$

1. Move HL over 1.00236 on LL0. Slide left index of C under HL.
2. Move HL over 24 on C. Answer is located 2 scales higher on LL2.
3. Under HL read ***1.761*** on LL2.

Example 2: $(250)^{.00065} = ?$

1. Move HL over 250 on LL3. Slide right index of C under HL.
2. Move HL over 65 on C. Answer is located 3 scales lower on LL0.
3. Under HL read ***1.00360*** on LL0.

Example 3: $(0.99805)^{1500} = ?$

1. Move HL over 0.99805 on LLo0. Slide left index of C under HL.
2. Move HL over 15 on C. Answer is located 3 scales higher on LLo3.
3. Under HL read ***.0536*** on LLo3.

We see that these lower scales enable us to make direct readings quite close to 1. However, they do more than that: actually, we can use them to make readings *as close to 1 as we wish.*

Suppose we imagine scales even lower than LL0, and let us use "$LL0_{-1}$" to describe *one* scale *lower* than LL0, "$LL0_{-2}$" to describe *two* scales *lower* than LL0, and so on. Now it happens that the LL0 scale range is so close to 1 that these lower scales may be mentally pictured very easily, even though they are not physically present. For example, the "$LL0_{-1}$" scale would look exactly the same as the LL0 scale except that the numbers would have an *extra zero* inserted to the right of the decimal point. The "$LL0_{-2}$" scale would look the same except that *two extra zeros* would be inserted. Thus, if you are reading 1.003 on LL0, this same location corresponds to 1.0003 on "$LL0_{-1}$," 1.00003 on "$LL0_{-2}$," and so forth. It follows that any reading on LL0 may be converted to a corresponding reading on a lower scale by simply inserting the proper number of zeros.

Example 4: $(1.03)^{.02} = ?$

1. Move HL over 1.03 on LL1. Slide left index of C under HL.
2. Move HL over 2 on C. Answer is located 2 scales lower on "$LL0_{-1}$."
3. Under HL read 1.00594 on LL0; therefore, the corresponding reading on "$LL0_{-1}$" is ***1.000594.***

Example 5: $(1.000054)^{4500} = ?$

1. Move HL over 1.0054 on LL0. If we now think of this as the "$LL0_{-2}$" scale, the hairline is over 1.000054.
2. Slide right index of C under HL.
3. Move HL over 45 on C. HL is to the left of 1.000054; hence, if exponent were 4.5, answer would be located one scale higher on "$LL0_{-1}$." Decimal point in exponent is actually 3 places to the right of this: hence, answer is on LL2 (3 scales higher than "$LL0_{-1}$").
4. Under HL read ***1.274*** on LL2.

Example 6: $(1.75)^{.0006} = ?$

1. Move HL over 1.75 on LL2. Slide right index of C under HL.
2. Move HL over 6 on C. Verify that result is 3 scales lower than LL2 on "$LL0_{-1}$."
3. Under HL read 1.00337 on LL0. To obtain corresponding reading on "$LL0_{-1}$," insert one zero. Answer is ***1.000337.***

In a similar way, we may refer to scales lower than LLo0 using the labels "$LLo0_{-1}$," "$LLo0_{-2}$," and so on. In this case, we simply insert *extra nines* instead of extra zeros. Thus, if we set at 0.9975 on LLo0, this same location corresponds to 0.99975 on "$LLo0_{-1}$," 0.999975 on "$LLo0_{-2}$," and so forth.

Example 7: $(0.978)^{.0023} = ?$

1. Move HL over 0.978 on LLo1. Slide left index of C under HL.
2. Move HL over 23 on C. Answer will be 3 scales lower on "$LLo0_{-2}$."
3. Under HL read 0.9949 on LLo0; therefore, inserting 2 nines, the corresponding reading on "$LLo0_{-2}$" is ***0.999949.***

Exercise B-3

1. $(1.00545)^{75} =$	**6.** $(0.99465)^{65} =$
2. $(6.5)^{.0031} =$	**7.** $(0.265)^{.0036} =$
3. $(1.00077)^{840} =$	**8.** $(0.947)^{.00075} =$
4. $10^{.000027} =$	**9.** $(0.99974)^{2600} =$
5. $(1.000024)^{-1200} =$	**10.** $(0.999905)^{-7500} =$

Exercise B-2 (at the end of the previous section) may be used for further drill.

Appendix C

THE "A-RELATED" LLO AND LLOO SCALES

C.1 Description of the scales

The LL0 and LL00 scales form a continuous scale starting at 0.999 at the left end of LL0, and decreasing down to about .000045 at the right end of LL00. It is convenient to break these scales up into four parts—the left and right halves of LL0, and the left and right halves of LL00. The range of each half-scale is associated with negative powers of e as follows:

LL0 (left half) extends from $e^{-.001}$ to $e^{-.01}$ (.999 to .990)
LL0 (right half) extends from $e^{-.01}$ to $e^{-.10}$ (.990 to .905)
LL00 (left half) extends from $e^{-.10}$ to $e^{-1.0}$ (.905 to .368)
LL00 (right half) extends from $e^{-1.0}$ to e^{-10} (.368 to .000045)

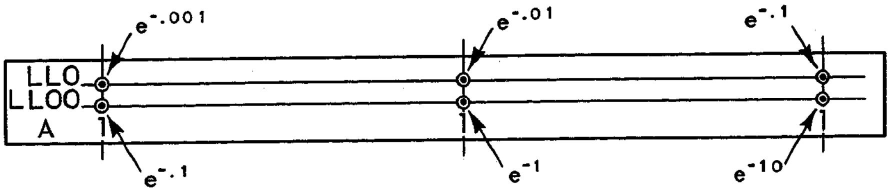


Figure C.1

It is easy to remember these ranges if you identify −10 with the right end of LL00 and then work back, shifting the decimal point one place to the left for each half-scale.

C.2 Finding e^x (x negative)

The left and right halves of LL0 and LL00 are related respectively to the left and right sections of the A scale in the same manner as the LL1, LL2, and LL3 scales are related to the D scale. Hence, we may raise e to negative powers by reading directly from A to the appropriate scale (LL0 or LL00), bearing in mind the range associated with each scale.

Example 1: $e^{-.056} = ?$

Inasmuch as −.056 is in the range of LL0 (right half), we set hairline on A-right.

1. Move HL over 56 on A-right.
2. Under HL read ***0.9455*** on LL0.

(Also note that $e^{-5.6}$ is under HL on LL00. Its value is .0037.)

Example 2: $e^{-0.35} = ?$

Note that −0.35 is in the range of LL00 (left half); hence, set hairline on A-left.

1. Move HL over 35 on A-left.
2. Under HL read ***0.705*** on LL00.

(Note that $e^{-.0035}$ is under HL on LL0. Its value is 0.9965.)

Example 3: $e^{-4.51} = ?$

Note that −4.51 is in the range of LL00 (right half); hence, set hairline on A-right.

1. Move HL over 451 on A-right.
2. Under HL read ***0.0110*** on LL00.

C.3 Finding ln *N* (*N* less than 1)

Here, we have the inverse problem: knowing e^x we must find x.

Example 1: ln 0.945 = ?

1. Move HL over 0.945 on LL0 (right half).
2. Under HL read "565" on A-right. Because of the range associated with LL0 (right half), we know that answer must be ***−.0565.***

Example 2: ln .065 = ?

1. Move HL over .065 on LL00 (right half).
2. Under HL read "274" on A-right. Noting the range associated with LL00 (right half), we see that answer must be ***−2.74.***

Verify the following:

1. $e^{-2.65} = .070$
2. $e^{-.064} = 0.938$
3. $e^{-.0147} = 0.9854$
4. $e^{-0.33} = 0.719$
5. $e^{-.0074} = 0.9926$
6. $e^{-100/22} = .0106$
7. $\ln 0.724 = -0.323$
8. $\ln .0225 = -3.79$

C.4 Finding b^x (x positive)

Inasmuch as the LL0 and LL00 scales are related to A-B, the hairline is set over the exponent on the B scale.

Example 1: Evaluate the following:
a. $(0.80)^2$; **b.** $(0.80)^{10}$; **c.** $(0.80)^{20}$.

1. Move HL over 0.80 on LL00. Slide left index of B under HL.
2. Move HL over 2 on B-left (we will refer to this as the "near half" of the B scale since it is the section nearest (0.80).
   Under HL read ***0.64***. This is $(0.80)^2$.
3. Move HL over middle index of B. Under HL read ***0.108***. This is $(0.80)^{10}$.
4. Move HL over 2 on the "far half" of B. Under HL read ***.0115***. This is $(0.80)^{20}$. The settings are illustrated in Figure C.2.

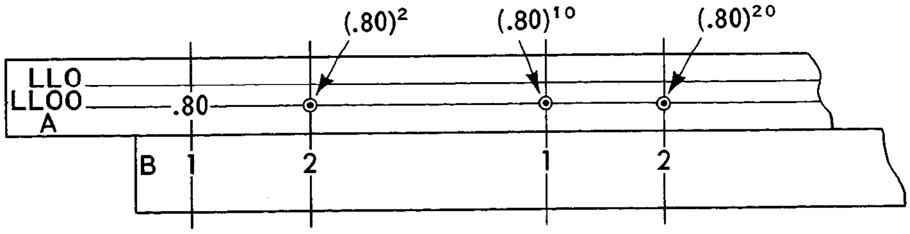


Figure C.2

Example 2: Evaluate: **a.** $(0.96)^{0.5}$; **b.** $(0.96)^{.05}$.

1. Move HL over 0.96 on LL0. Slide right index of B under HL.
2. Move HL over 5 on the near half of the B scale. Under HL read ***0.9798*** on LL0. This is $(0.96)^{0.5}$.
3. Move HL over 5 on the far half of B. Under HL read ***0.99796*** on LL0. This is $(0.96)^{.05}$. (See Figure C.3.)

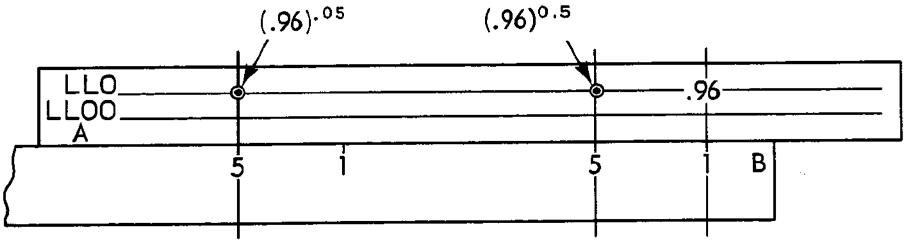


Figure C.3

From the foregoing examples we generalize as follows:

> 1. When evaluating b^x suppose the answer is to the *right* of b and on the *same scale*. Then:
>   a. If HL is positioned on the *near half* of B, the exponent must be *between 1 and 10* (X.XX).
>   b. If HL is on the *far half* of B, the exponent is *between 10 and 100* (XX.X).
>
> 2. Suppose the answer is the *left* of b and on the *same* scale. Then:
>   a. If HL is positioned on the *near half* of B, the exponent must be *between 0.1 and 1* (0.XXX).
>   b. If HL is positioned on the *far half* of B, the exponent is *between .01 and 0.1* (.0XXX).

Example 3: $(.05)^{.08} = ?$

1. Move HL over .05 on LL00. Slide right index of B under HL.
2. Move HL over 8 on the *far half* of B. Under HL read ***0.786*** on LL00.

Example 4: $(0.9954)^{4.5} = ?$

1. Move HL over 0.9954 on LL0. Slide left (or middle) index of B under HL.
2. Move HL over 45 on *near half* of B. Under HL read ***0.9795*** on LL0.

Verify the following:

1. $(0.75)^{15} = .0135$
2. $(0.97)^{0.35} = 0.9894$
3. $(.989)^{5.2} = 0.944$
4. $(.035)^{.075} = 0.778$

C.5 The "scale-shift" principle

The LL0 and LL00 scales are related to each other in the following way:

> 1. If the HL is over b^x on LL0, then b^{100x} is under the HL on LL00.
> 2. Conversely, if the HL is over b^x on LL00, then $b^{x/100}$ is under the HL on LL0.
>
> In other words, moving from LL0 to LL00 shifts the decimal point in the exponent *two* places to the *right*; moving from LL00 to LL0 shifts the decimal point *two* places to the *left.*

Example 1: $(0.564)^{.04} = ?$

1. Move HL over 0.564 on LL00. Slide left index of B under HL.
2. Move HL over 4 on near half of B. The HL is now over $(0.564)^4$ on LL00; therefore, $(0.564)^{.04}$ must be under HL on LL0.
3. Under HL read ***0.9773*** on LL0. See Figure C.4.

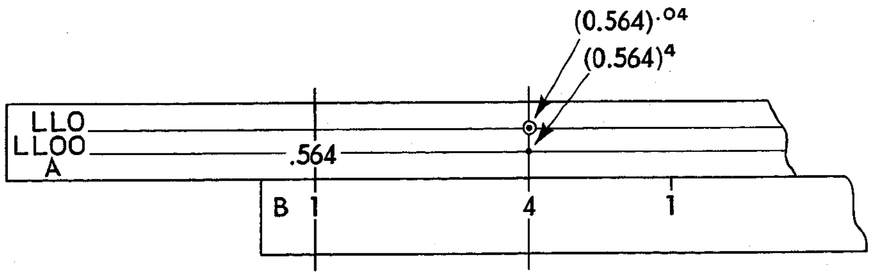


Figure C.4

Example 2: $(0.96)^5 = ?$

1. Move HL over 0.96 on LL0. Slide right index of B under HL.
2. Move HL over 5 on far half of B. The HL is now over $(0.96)^{.05}$ on LL0; therefore, $(0.96)^5$ is under HL on LL00.
3. Under HL read ***0.815*** on LL00.

Example 3: $(.064)^{.021} = ?$

1. Move HL over .064 on LL00. Slide middle index of B under HL.
2. Move HL to the right over 21 on the near half of B. HL is now over $(.064)^{2.1}$ on LL00; therefore, $(.064)^{.021}$ must be under HL on LL0.
3. Under HL read ***0.9439*** on LL0.

Example 4: $(.9971)^{250} = ?$

1. Move HL over .9971 on LL0. Slide left index of B under HL.
2. Move HL over 25 on near half of B. This puts HL over $(.9971)^{2.5}$ on LL0; therefore, HL must be over $(.9971)^{250}$ on LL00.
3. Under HL read ***0.484*** on LL00.

Verify the following:

1. $(0.48)^{0.67} = 0.612$
2. $(.0145)^{.0029} = 0.9878$
3. $(0.223)^{.083} = 0.883$
4. $(0.997)^{380} = 0.319$

C.6 Finding b^x (x negative)

Here, we may use either of the two relations:

1. $b^{-x} = \frac{1}{b^x}$
2. $b^{-x} = \left(\frac{1}{b}\right)^x$

Example 1: $(.985)^{-12} = ?$

Using relation (1), we may write: $(.985)^{-12} = \frac{1}{(.985)^{12}}$

1. Verify that $(.985)^{12} = 0.834$.
2. Find reciprocal of this in the conventional manner using C and CI scales (or D and DI). Verify that result is ***1.20.***

Example 2: $(0.55)^{-3.5} = ?$

Suppose we use relation (2): $(.055)^{-3.5} = \left(\frac{1}{0.55}\right)^{3.5}$

1. Verify that reciprocal of 0.55 is 1.82. Hence, we must evaluate $(1.82)^{3.5}$.
2. Verify that result is ***8.15.***

Example 3: $5^{-4.3} = ?$

Using relation (2): $5^{-4.3} = \left(\frac{1}{5}\right)^{4.3} = (0.2)^{4.3}$

Verify that result is ***.0010.***

Verify the following:

1. $8^{-3.5} = .00069$

2. $(0.94)^{-20} = 3.44$

3. $(0.75)^{-7.5} = 8.65$

4. $(65)^{-0.36} = 0.222$

Exercises in Chapters 19 and 20 may be used for further drill.

Appendix D

THE LL SCALES (BASE 10); THE FOLDED SCALES (CF/M, DF/M)

D.1 The LL scales (base 10)

Certain Pickett models have scales labeled N or LL which are associated with powers of 10 as follows:

LL1+ (or N_1) extends from $10^{.001}$ to $10^{.01}$
LL2+ (or N_2) extends from $10^{.01}$ to $10^{0.1}$
LL3+ (or N_3) extends from $10^{0.1}$ to $10^{1.0}$
LL4+ (or N_4) extends from $10^{1.0}$ to 10^{10}

Back-to-back with these are scales (increasing from right to left) which are associated with negative powers of 10 as follows:

LL1− (or $1/N_1$) extends from $10^{-.001}$ to $10^{-.01}$
LL2− (or $1/N_2$) extends from $10^{-.01}$ to $10^{-0.1}$
LL3− (or $1/N_3$) extends from $10^{-0.1}$ to $10^{-1.0}$
LL4− (or $1/N_4$) extends from $10^{-1.0}$ to 10^{-10}

In the following discussion, we shall use the "LL" rather than the "N" designation. The LL (base 10) scales are related to the **D** scale in the following manner:

> If the hairline is moved over a number x on **D**, then 10^x is located under the HL on the appropriate **LL** scale.
>
> Conversely, if the hairline is moved over a number N on **LL**, then $\log_{10}N$ is located under the HL on **D**.

Example 1: $10^{2.4} = ?$

Exponent is between 1 and 10; hence, answer will be on LL4+.

1. Move HL over 24 on D.
2. Under HL read ***250*** on LL4+.

Observe that $10^{-2.4}$ is located on LL4–, and is equal to .004.

Example 2: $10^{-.063} = ?$

Exponent is between –.01 and –0.10; hence, answer will be on LL2–.

1. Move HL over 63 on D.
2. Under HL read ***0.865*** on LL2–.

Observe that $10^{.063} = 1.156$ is on LL2+.

Example 3: $\log_{10} 1.1145 = ?$

1. Move HL over 1.1145 on LL2+.
2. Under HL read "471" on D. We are set on LL2+; hence, result must be between .01 and 0.10. Answer is ***.0471.***

D.2 The reciprocal property

Examples 1 and 2 of the preceding section illustrate the reciprocal property of the LL (base 10) scales:

> If the hairline is set over a number on LL4+, its reciprocal will be under the hairline on LL4–, and vice versa.
>
> A similar relation holds between LL3+ and LL3–, LL2+ and LL2–, and between LL1+ and LL1–.

Example: $1/1.73 = ?$

1. Move HL over 1.73 on LL3+.
2. Under HL read $1/1.73 =$ ***0.578*** on LL3–.

D.3 The "scale-shift" principle

Another useful property of the LL scales may be stated:

> If the hairline is over b^x on one of the LL scales, then:
> 1. b^{10x} is under HL on the *next higher* scale.
> 2. $b^{x/10}$ is under HL on the *next lower* scale.

In other words, the decimal point in the exponent shifts *one place* to the *right* each time we move *one scale higher;* it shifts *one place* to the *left* each time we move *one scale lower.* (By a higher scale we mean one with a higher number designation; LL3+ is higher than LL2+, LL4− is higher than LL3−, and so on.)

Example: Evaluate 10^x for $x = .022$, .0022, and 2.2.

1. Move HL over 22 on D.
2. Under HL read:
   a. $10^{.022} =$ ***1.01519*** on LL2+.
   b. $10^{.0022} =$ ***1.00507*** on LL1+ (one scale lower than LL2+).
   c. $10^{2.2} =$ ***159*** on LL4+ (two scales higher than LL2+).

Verify the following:

1. $1/6500 = .000154$
2. $1/0.99225 = 1.00781$
3. $\log_{10}1.00845 = 0.00366$
4. $\log_{10}0.9672 = -.01450$
5. $10^{5.81} = 650{,}000$; $10^{.0581} = 1.143$
6. $10^{-0.44} = 0.363$; $10^{-4.4} = .00004$

D.4 The CF/M and DF/M scales

These are C and D scales folded at $\log_e 10 = 2.30$. The DF/M scale is related to the base-10 LL scales as follows:

> If the hairline is moved over a number x on **DF/M**, then e^x is under HL on the appropriate **LL** scale.
>
> Conversely, if the hairline is over a number N on **LL**, then $\log_e N$ is under HL on **DF/M.**
>
> Also, if the hairline is over $\log_{10} N$ on **D**, then $\log_e N$ is under HL on **DF/M;** thus, conversion of logs from base e to base 10, and vice versa, may be accomplished directly.

Example 1: Evaluate e^x for $x = 1, 2, 3$, and 6.

1. Move HL over 1 on DF/M (we refer to this as the DF/M index). Under HL read $e =$ ***2.72*** on LL3+.
2. Move HL over 2 on DF/M. Under HL read $e^2 =$ ***7.40*** on LL3+.
3. Move HL over 3 on DF/M. Under HL read $e^3 =$ ***20.1*** on LL4+.
4. Move HL over 6 on DF/M. Under HL read $e^6 =$ ***405*** on LL4+.

The results of the foregoing example may be summarized:

> *When evaluating e^x (x between 1 and 10):*
> 1. Answer is on ***LL3***+ when HL is to the *right* of DF/M index.
> 2. Answer is on ***LL4***+ when HL is to the *left* of DF/M index.

Using the scale-shift principle and the reciprocal property, more general powers of e may be obtained.

Example 2: $e^{.043} = ?$

1. Move HL over 43 on DF/M. HL is to the left of the DF/M index; hence, if exponent were 4.3, answer would be on LL4+. Decimal point is two places to the left of this position; therefore, answer is two scales lower on LL2+.
2. Under HL read ***1.044*** on LL2+.

Example 3: $e^{-14.5} = ?$

1. Move HL over 145 on DF/M. HL is to the right of DF/M index; hence, if exponent were 1.45, answer would be on LL3+, and if exponent were 14.5, answer would be on LL4+. Exponent is −14.5; therefore, answer is on the reciprocal scale, LL4−.
2. Under HL read approximately $\boldsymbol{5 \times 10^{-7}}$ on LL4−.

Example 4: **a.** $\log_{10}1.76 = ?$ **b.** $\log_e 1.76 = ?$

It is helpful to remember that $\log_e N$ is a little more than twice $\log_{10} N$.

1. Move HL over 1.76 on LL3+.
2. Under HL read:
   a. $\log_{10}1.76 =$ ***0.246*** on D.
   b. $\log_e 1.76 =$ ***0.565*** on DF/M.

Verify the following:

1. $e^{1.46} = 4.31$	**6.** $e^{-2.12} = 0.120$
2. $e^{7.45} = 1720$	**7.** $e^{-.0075} = 0.99253$
3. $e^{0.65} = 1.916$	**8.** $e^{-16.6} = 6 \times 10^{-8}$
4. $e^{.015} = 1.0151$	**9.** $\log_{10}20 = 1.300$; $\log_e 20 = 3.00$
5. $e^{.0037} = 1.00371$	**10.** $\log_{10}0.466 = -0.332$; $\log_e 0.466 = -0.763$

Observe that CF/M and DF/M are *operational* scales, and may be used as another pair of folded scales in the same way that you use CF and DF. Also, you may use both scales in conjunction with the LL scales.

Example 5: $e^{(6.4 \times .076)/3.3} = ?$

Estimate the combined exponent to be about 0.15. We may now evaluate the exponent on the CF/M-DF/M scales and read the result on the appropriate LL scale.

1. Move HL over 64 on DF/M.
2. Slide 33 on CF/M under HL.
3. Move HL over 76 on CF/M. HL is to the right of the DF/M index; hence, if exponent were about 1.5, answer would be on LL3+. Exponent is about 0.15; therefore, result is one scale lower on LL2+.
4. Under HL read ***1.1587*** on LL2+.

Example 6: 5.6 ln 6.8 = ? (ln 6.8 means $\log_e 6.8$)

1. Move HL over 6.8 on LL3+. HL is now over ln 6.8 on DF/M. Now multiply by 5.6 using the CF/M-DF/M scales.
2. Slide CF/M index under HL. Move HL over 56 on CF/M.
3. Under HL read "1072" on DF/M. Answer is ***10.72.***

Example 7: $e^{-5/12} = ?$

Estimate exponent to be about −0.4.

1. Move HL over 5 on DF/M.
2. Slide 12 on CF/M under HL.
3. Move HL over CF/M index. Note that HL is to the left of DF/M index; hence, if exponent were about 0.4, answer would be on LL3+. Exponent is negative; therefore, answer is on the reciprocal scale, LL3−.
4. Under HL read ***0.659*** on LL3−.

Verify the following:

1. $e^{12/7} = 5.55$
2. $e^{7/60} = 1.1239$
3. $e^{-1/15} = 0.9355$
4. $e^{5\pi/4.6} = 30.2$
5. $e^{-30/(76 \times 3.75)} = 0.900$
6. $3.7 \ln 45 = 14.10$
7. $\dfrac{6.25 \ln .077}{2.44} = -6.57$

The exercises in Chapter 19 may be used for further practice.

D.5 Finding b^x

You should now refer to Chapter 20. The rules and techniques described in that chapter also apply to the base-10 scales. In following the illustrative examples in Chapter 20 with the base-10 scales, you will note certain differences in the scale designation. Thus, the number 36 is located on LL3 (base e), whereas on the base-10 scale it is found on LL4+. Also, when the left index is called for in Chapter 20, you may sometimes have to use the right index, and vice versa. You may also find it necessary to interchange indexes at a different point in the described procedure. However, the basic scale relationships and procedures outlined in Chapter 20 – the reciprocal property of the scales, the "scale-shift" principle, the rule for locating b^x on the proper LL scale – all these apply equally well to the base-10 scales.

The following examples assume familiarity with the material in Chapter 20.

Example 1: $(2.46)^{.052} = ?$

1. Move HL over 2.46 on LL3+. Slide right index of C under HL.
2. Move HL over 52 on C. Note that HL is to the left of 2.46; hence, if exponent were 5.2, answer would be one scale higher on LL4+. However, decimal point in exponent is actually 2 places to the left of this position; hence, result is two scales lower on LL2+.
3. Under HL read ***1.0479*** on LL2+.

Example 2: $(0.9265)^{-170} = ?$

1. Move HL over 0.9265 on LL2–. Slide left index of C under HL.
2. Move HL over 170 on C. HL is to the right of 0.9265; hence, if exponent were 1.70, answer would be on the same scale, LL2–. Decimal point in exponent is actually two places to the right of this position; hence, if exponent were 170, answer would be on LL4–. Exponent is negative; therefore, answer is on the reciprocal scale, LL4+.
3. Under HL read approximately ***440,000*** on LL4+.

Verify the following:

1. $(2.06)^{27.6} = 4.5 \times 10^8$

2. $(1.00442)^{260} = 3.15$

3. $(75{,}000)^{.00535} = 1.0618$

4. $(1.545)^{-22} = 0.00007$

5. $(0.99346)^{1800} = 7.4 \times 10^{-6}$

6. $(35{,}000)^{-.0044} = 0.9551$

The exercises in Chapter 20 will provide more drill. Methods for handling numbers outside the range of the scales are discussed in Appendix B.

Appendix E

THE HYPERBOLIC SCALES (SH, TH); THE PYTHAGOREAN SCALE (P)

E.1 The Sh and Th scales

For convenience, we repeat the definitions of the hyperbolic functions given in Chapter 19:

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

From these definitions, the following relations may also be obtained:

$$\begin{aligned} \sinh(-x) &= -\sinh x \\ \cosh(-x) &= \cosh x \\ \tanh(-x) &= -\tanh x \\ \cosh^2 x - \sinh^2 x &= 1 \end{aligned}$$

The Sh scale consists of two full-length scales labeled Sh1 and Sh2 (on the Pickett models, these are back-to-back; the upper scale corresponds to Sh1, the lower scale corresponds to Sh2). The two Sh scales taken together form one continuous scale ranging from about 0.10 to 3.0.

The Th scale is a single full-length scale which also ranges from about 0.10 to 3.0.

If the Sh and Th scales are on the slide, they are related to the C scale; if they are on the body of the rule, they are related to the D scale. Hyperbolic sines and tangents may be read directly as follows:

To find sinh x:
1. Move HL over x on **Sh1** or **Sh2.**
2. Under HL read *sinh x* on **C(D).**
3. To place the decimal point:
   a. When reading from **Sh1,** sinh x is *between 0.1 and 1.0.*
   b. When reading from **Sh2,** sinh x is *between 1.0 and 10.*

To find tanh x:
1. Move HL over x on **Th.**
2. Under HL read *tanh x* on **C(D).**
3. To place the decimal point:
   For all settings on **Th,** tanh x is *between 0.1 and 1.0.*

To find cosh x:

Use the relationship: $\cosh x = \dfrac{\sinh x}{\tanh x}$

Example 1: sinh 1.85 = ? tanh 0.545 = ?

1. Move HL over 1.85 on Sh2.
2. Under HL read sinh 1.85 = ***3.10*** on C(D).
3. Move HL over 0.545 on Th.
4. Under HL read tanh 0.545 = ***0.497*** on C(D).

Example 2: cosh 0.68 = ?

We use the relation: $\cosh 0.68 = \dfrac{\sinh 0.68}{\tanh 0.68}$

Verify that cosh 0.68 = ***1.24.***

Example 3: sinh x = 0.51; x = ?

1. Move HL over 51 on C(D).
2. Under HL read x = ***0.49*** on Sh1.

Result may also be written: $\sinh^{-1} 0.51$ = ***0.49.***

Example 4: $\cosh^{-1} 2.04$ = ?

This problem may also be stated: cosh x = 2.04; x = ?

1. Verify that $\sinh x = \sqrt{\cosh^2 x - 1} = \sqrt{(2.04)^2 - 1} = 1.778$.
2. Move HL over 1778 on C(D).

3. Under HL read $x = \mathbf{1.34}$ on Sh2.

Hence, $\cosh^{-1} 2.04 = \mathbf{1.34}$.

Verify the following:

1. $\sinh 0.55 = 0.578$

2. $\tanh(-1.26) = -0.851$

3. $\sinh 2.37 = 5.30$

4. $\cosh(-1.45) = 2.25$

5. $\sinh(-1.06) = -1.27$

6. $\cosh 0.89 = 1.423$

7. $\sinh^{-1} 4.19 = 2.14$

8. $\tanh^{-1} 0.493 = 0.54$

9. $\cosh^{-1} 1.32 = 0.78$

10. $\cosh^{-1} 2.70 = 1.65$

For values of x outside the range of the Sh and Th scales, the following approximations may be used:

1. For *large* values of x ($x > 3$):

$$\sinh x \approx \frac{1}{2}e^x \qquad \cosh x \approx \frac{1}{2}e^x \qquad \tanh x \approx 1$$

2. For *small* values of x ($0 < x < 0.1$):

$$\sinh x \approx x \qquad \cosh x \approx 1 \qquad \tanh x \approx x$$

E.2 The P scale

The P scale is often present on British and European slide rules. The scale is based on the Pythagorean relation, and ranges from 0 to 0.995 (increasing from right to left). It is usually on the body of the rule, and is related to the D scale as follows:

> If the hairline is over a number x on the **D** scale ($0.1 \leq x \leq 1$), then $\sqrt{1 - x^2}$ is under the hairline on the **P** scale.

Example 1: $\sqrt{1 - (0.46)^2} = ?$

1. Move HL over 46 on D.
2. Under HL read ***0.888*** on P.

Example 2: $\sin 40° = ?\quad \cos 40° = ?$

Recall that $\cos 40° = \sqrt{1 - \sin^2 40°}$

1. Close rule and move HL over 40° on S scale.
2. Under HL read:

$\sin 40° =$ ***0.643*** on D,
$\cos 40° =$ ***0.766*** on P.

Example 3: Given a right triangle with $c = 26$, $a = 17$. Find side b.

Write: $b = \sqrt{c^2 - a^2} = c\sqrt{1 - (a/c)^2}$.

Substituting: $b = 26\sqrt{1 - (17/26)^2}$.

1. Using P scale, verify that $\sqrt{1 - (17/26)^2} = 0.757$.
2. Multiplying 26 by 0.757 on C-D, verify that $b =$ ***19.65.***

Appendix F

SOME REPRESENTATIVE LOG LOG SLIDE RULES

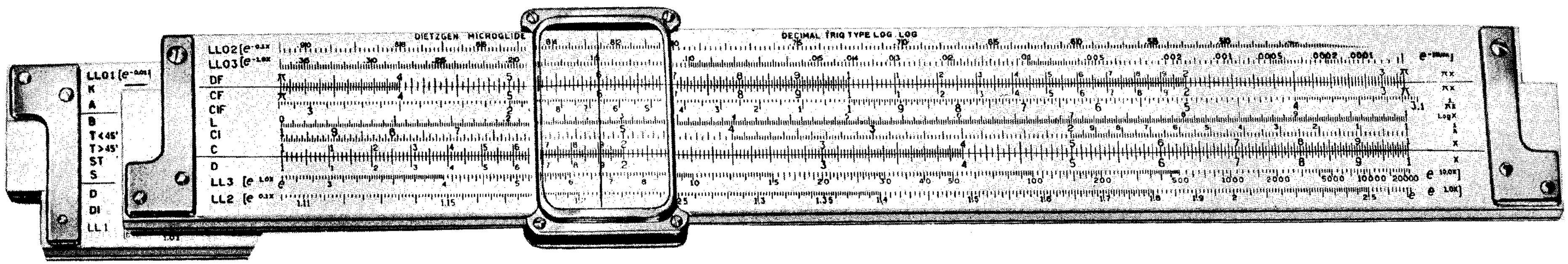


Figure F.1 DECIMAL TRIG LOG LOG TYPE (Courtesy Eugene Dietzgen Co.)

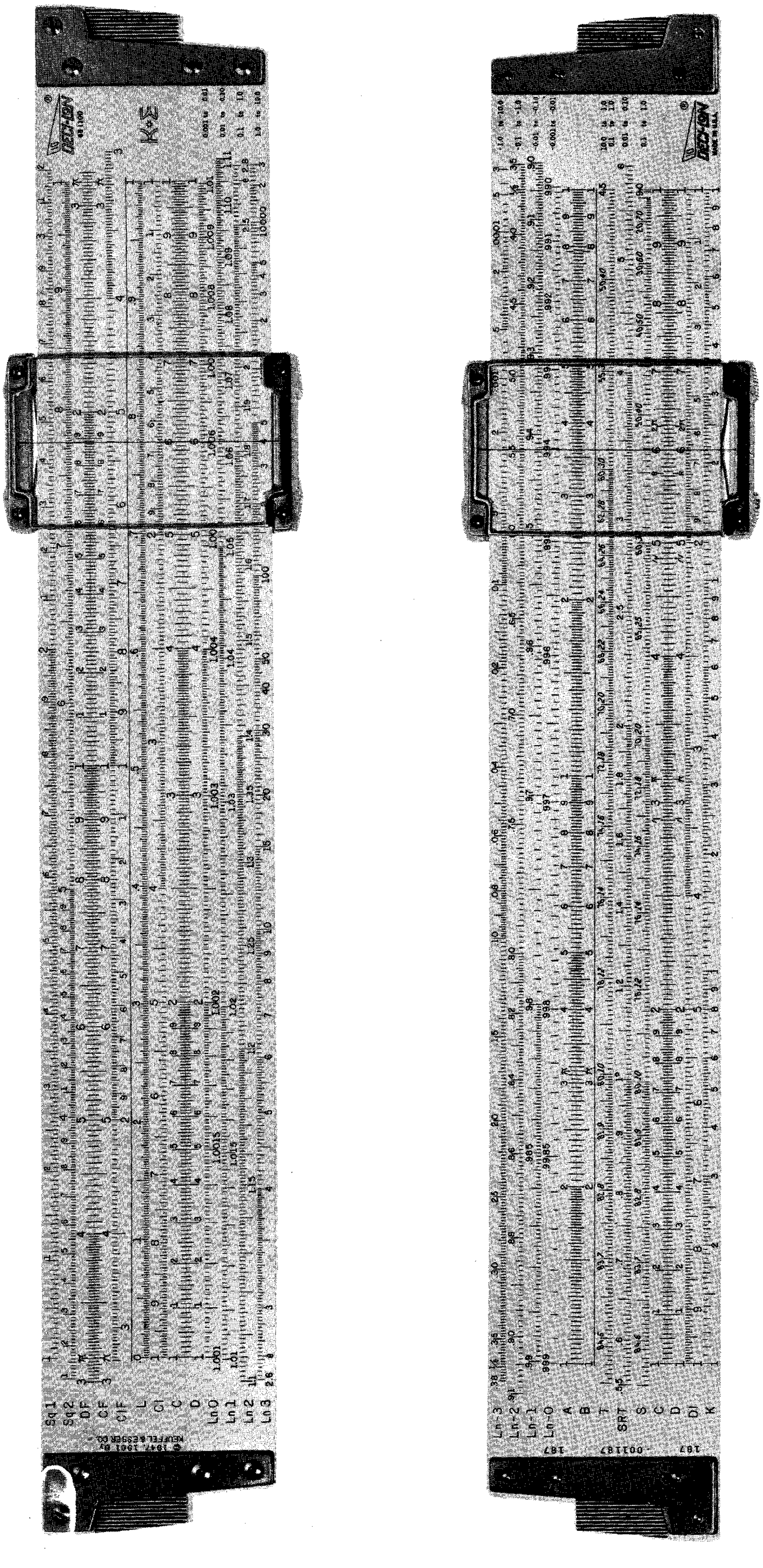


Figure F.2 DECI-LON (Courtesy Keuffel & Esser Co.)

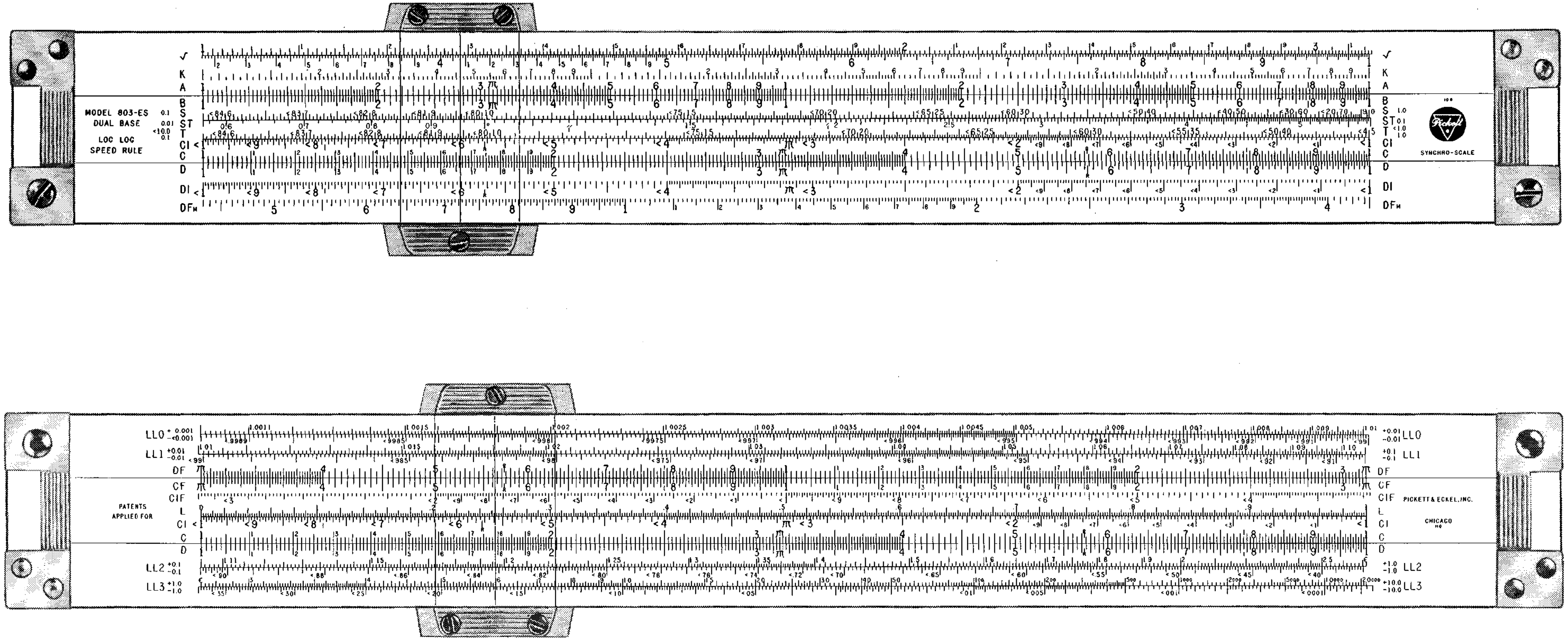


Figure F.3 DUAL BASE LOG LOG SPEED RULE, Model 803-ES (Courtesy Pickett, Inc.)

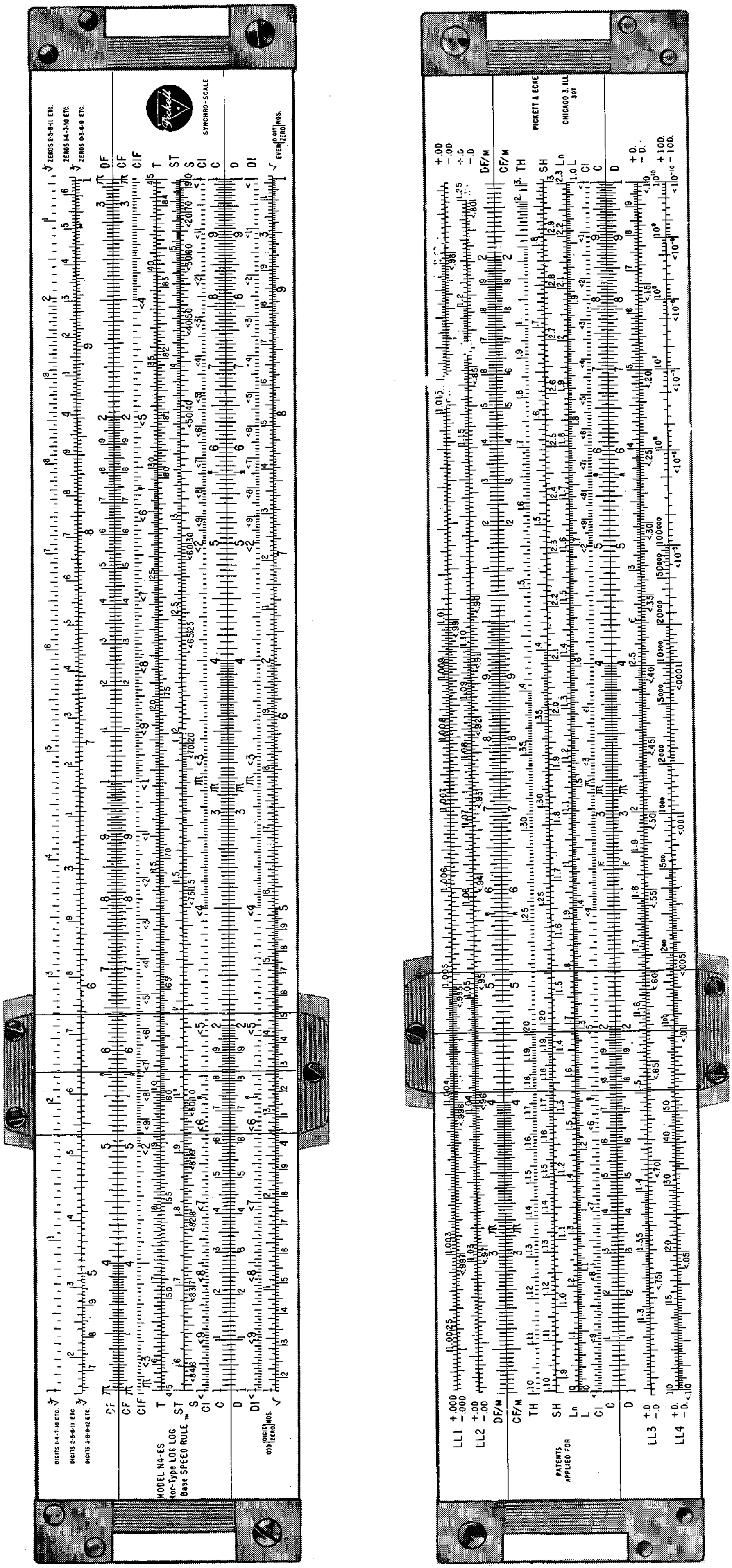


Figure F.4 VECTOR LOG LOG DUAL BASE SPEED RULE, Model N4-ES
(Courtesy Pickett, Inc.)

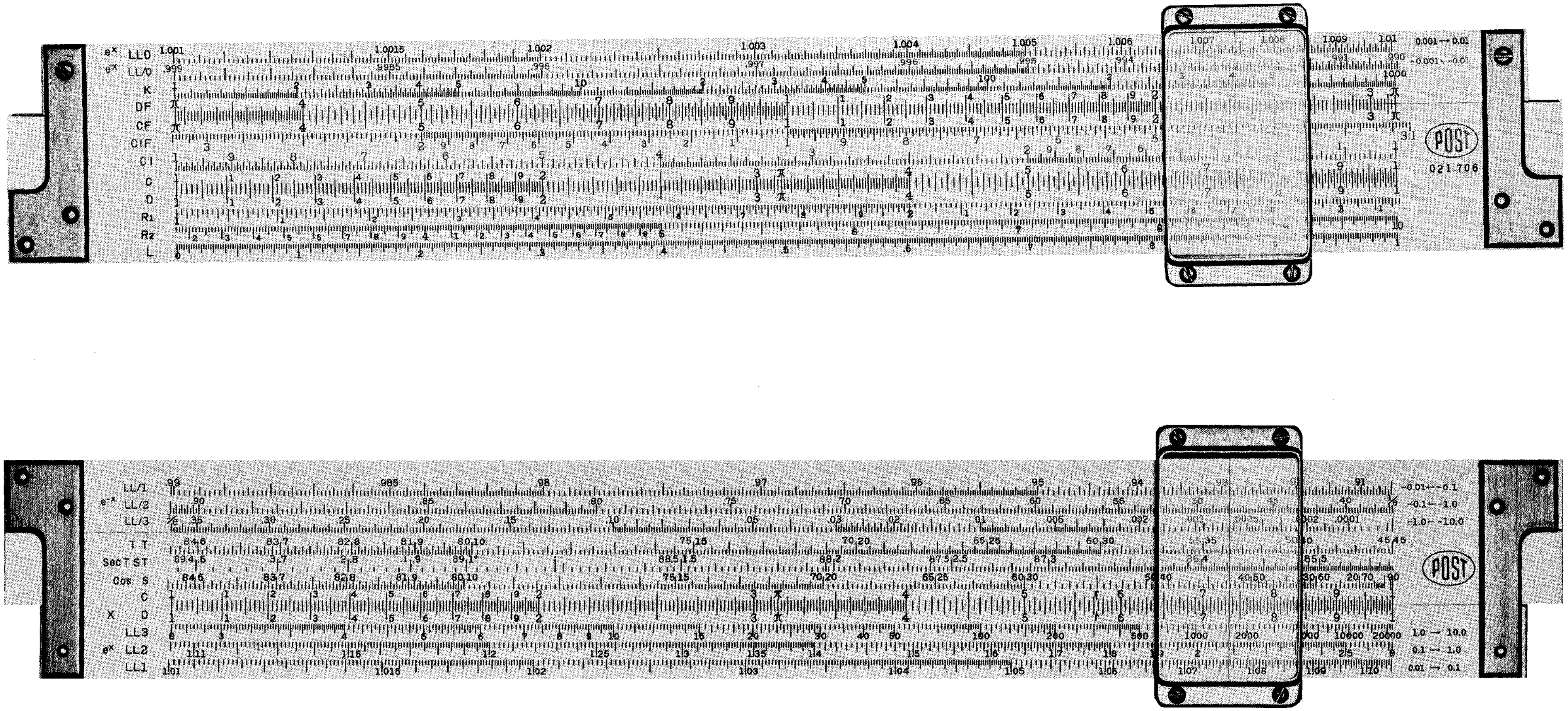


Figure F.5 VERSALOG (Courtesy Frederick Post Co.)

ANSWERS TO EXERCISES

Exercise 1-1

1. 11.76	**11.** 7.42	**21.** 13.81	**31.** 797
2. 34.7	**12.** 50.5	**22.** 116.6	**32.** 0.265
3. 96.2	**13.** 12.70	**23.** 2.38	**33.** 0.464
4. 17.68	**14.** 45.7	**24.** 351	**34.** .000524
5. 7.14	**15.** 96.6	**25.** 0.702	**35.** 71.6
6. 122.1	**16.** 19.70	**26.** 1400	**36.** 4.61
7. 5.38	**17.** 17.85	**27.** 158.1	**37.** 0.311
8. 6.32	**18.** .0292	**28.** 9350	**38.** 161.5
9. 2.78	**19.** 36.6	**29.** .0902	**39.** 946
10. 47.2	**20.** 44.7	**30.** 114.0	**40.** .000468

Exercise 1-2

1. 0.437	**8.** .000230	**15.** .0001118	**22.** 7630
2. 66,400	**9.** 8620	**16.** 137.0	**23.** 262
3. .00207	**10.** 39.0	**17.** .000267	**24.** 190,800
4. 0.1570	**11.** .000260	**18.** 230,000	**25.** .00000598
5. 42.6	**12.** 105.9	**19.** .00001538	**26.** 1980
6. 4860	**13.** .000733	**20.** 239	**27.** .000270
7. .0279	**14.** 1382	**21.** .0573	**28.** 44,700
	29. 343	**30.** 0.273	

Exercise 1-3

1. 43.6	**11.** 46.3	**21.** 13.15	**31.** 67.4
2. 108.7	**12.** 27.7	**22.** 485	**32.** 21.2
3. 483	**13.** 14.65	**23.** 173.5	**33.** 16.08
4. 6.08	**14.** 49.8	**24.** 940	**34.** 22.0
5. 10.00	**15.** 1078	**25.** 25.5	**35.** .0505
6. .0404	**16.** 291	**26.** 8210	**36.** 6.79
7. 4.60	**17.** 1.245	**27.** 0.753	**37.** 7.67
8. 22.8	**18.** .00795	**28.** 112.4	**38.** 4.47
9. 9.97	**19.** 3710	**29.** 0.339	**39.** .00334
10. 4.33	**20.** 1113	**30.** 5.61	**40.** 22.7

Exercises 1-4

1. 2.95×10^{-8}	**8.** 8.46×10^{5}	**15.** 7.49	**23.** 0.747
2. 5.06×10^{8}	**9.** 5.78×10^{-8}	**16.** 39.1	**24.** 3.94×10^{10}
3. 4.00×10^{-8}	**10.** 90.7	**17.** 5.09×10^{-5}	**25.** 15,980
4. 7.44×10^{6}	**11.** 2.24×10^{-4}	**18.** .0648	**26.** 4.16
5. 5.96×10^{-9}	**12.** 4.00×10^{-3}	**19.** 1.824	**27.** 6.27×10^{-5}
6. 6.07×10^{5}	**13.** 7.67×10^{-4}	**20.** 2260	**28.** 4.75×10^{7}
7. 6.56×10^{9}	**14.** 8.15	**21.** 8.03	**29.** 46,700
		22. 9.07×10^{13}	**30.** 3.54×10^{-6}

Exercise 2-1

1. A–"1003" B–"1031" C–"1060" D–"1095" E–"1143" F–"1197" G–"1260" H–"1303" I–"1372" J–"1428" K–"1425" L–"1502" M–"1590" N–"1714" O–"1763" P–"1867" Q–"1923" R–"1995"

2. A–"201" B–"214" C–"233" D–"247" E–"266" F–"280" G–"298" H–"313" I–"335" J–"381" K–"401" M–"453" N–"488" O–"514" P–"543" Q–"585" R–"627"

Exercise 2-2

(C or D)

"1259"	"501"	"205"	"646"
"1585"	"631"	"1047"	"1014"
"1995"	"794"	"1005"	"708"
"251"	"302"	"254"	"1072"
"316"	"1355"	"975"	"329"
"398"	"891"	"1845"	"995"

(L)

.389	.176	.027	.342
.542	.941	.957	.041
.031	.872	.706	.984
.328	.636	.004	.039
.600	.003	.804	.777
.840	.568	.613	.247

Exercise 3-1

1. 6.48	**6.** 77.0	**11.** 41.4	**16.** 18.30
2. 5.28	**7.** 7.28	**12.** 6.75	**17.** 527,000
3. 8.12	**8.** 19.25	**13.** 69.0	**18.** .00406
4. 70.5	**9.** 92.4	**14.** 6.15	**19.** 0.734
5. 103.6	**10.** 36.4	**15.** 91.2	**20.** 0.0662

Exercise 3-2

1. 27.3	**6.** 175.8	**11.** 272	**16.** 457
2. 31.9	**7.** 14.95	**12.** 154.6	**17.** 1450
3. 63.8	**8.** 1393	**13.** 1226	**18.** 0.478
4. 158.1	**9.** 21.2	**14.** 25.6	**19.** 0.1779
5. 14.72	**10.** 2280	**15.** 172.1	**20.** 29,000

Exercise 3-3

1. 36.8
2. 44.4
3. 52.5
4. 439
5. 61.6
6. 81.9
7. 736
8. 285
9. 864
10. 18.14
11. 33.3
12. 459
13. 1323
14. 136.9
15. 66.6
16. 77.1
17. 1678
18. 351
19. 7.72
20. 204
21. 79.4
22. 89.0
23. 522
24. 313
25. 7.57
26. 109.5
27. 347
28. 176.3
29. 102.7
30. 229
31. 161.0
32. 8510
33. 86.9
34. 230
35. 23.4
36. 124.4
37. 77.3
38. 33.3
39. 475
40. .00778
41. 10.58
42. 7.37
43. .00784
44. 88,000
45. 11,100
46. 15.96
47. 16.65
48. 1070
49. .0000890
50. 942,000
51. 12.98
52. .0000586
53. 115,200
54. 104.6
55. 0.227
56. 1809
57. .000211
58. 6.92
59. .00576
60. 38,000
61. .0347
62. 20.2
63. 498
64. .0681
65. 3.32
66. 10.76
67. 515
68. 0.1036
69. 714
70. 5.18

Exercise 3-4

1. 2.21
2. 2.47
3. 3.36
4. 2.56
5. 0.619
6. 0.455
7. 4.48
8. 11.55
9. 4.19
10. 2.62
11. 2.66
12. 3.82
13. 23.5
14. 8.78
15. 3.70
16. 3.60
17. 3.17
18. 39.0
19. 3.01
20. 5.90
21. 19.89
22. 1.346
23. 2780
24. 9.38

25. 1.283
26. 24.7
27. 5.34
28. 2.55
29. 2.62
30. 1.980
31. .00572
32. .0001305
33. 0.1573
34. .0927
35. 8.19
36. 211
37. 0.1915
38. .0819
39. 1125
40. .0306
41. 0.641
42. .0000573
43. .001940
44. 323,000
45. 299
46. 13.05
47. .01928
48. .00671
49. 184.7
50. 164.1
51. .00001738
52. .000502
53. 52.4
54. .0267
55. 60,500
56. .0863
57. .0868
58. .0203
59. .0000301
60. 3,080,000
61. .000867
62. 2170
63. 227
64. 1533
65. 0.343
66. 236,000
67. 2960
68. 3.96
69. .01733
70. 3420
71. 6130
72. 3560
73. 65.9
74. 103,400
75. .00223
76. 13.34
77. 48.3
78. 69.4
79. 81.5
80. 190.2
81. 10.61
82. 138.0
83. 8660
84. 143,500,000
85. .01649

Exercise 4-1

1. 3.50
2. 15.35
3. 15.73
4. 60.5
5. 125.1
6. 34.7
7. 87.9
8. 8.58
9. 3.96
10. 11.13
11. 57.6
12. 0.764
13. 22.2
14. 0.587
15. 2.27
16. 8.07
17. 14.18
18. 25.3
19. 12.40
20. 5.69
21. 15.33
22. 49.6
23. 96.5
24. 23.9
25. 7.80
26. 6.89
27. 1.671
28. 7.82
29. 2.07
30. 1.691
31. 15.83
32. 143.2

33. 2890
34. 8.07
35. 1.604
36. 7.71
37. 0.318
38. 3.95
39. 0.502
40. 14.25
41. 88.4
42. 8.06
43. 5.77
44. 1029
45. 157.9
46. 644
47. 0.398
48. 0.874
49. 24.9
50. 0.879

Exercise 4-2

1. 1.363
2. 0.1890
3. 3.23
4. 1.495
5. 1.950
6. 4.49
7. 2.32
8. 74.2
9. 42.5
10. 76.0
11. 69.4
12. 9.62
13. 14.67
14. 28.8
15. 171.0
16. 7.45
17. 21.0
18. 2.81
19. 0.222
20. 0.215
21. 7.40
22. 3.58
23. 35.5
24. 0.710
25. 12.88
26. 24.3
27. 362
28. 325
29. 4.88
30. 2.14
31. 6.36
32. 121.7

Exercise 5-1

1. 0.311
2. 0.211
3. 0.1972
4. 0.380
5. 0.1232
6. 0.1070
7. 0.881
8. .0307
9. .001383
10. 3.51
11. 2.42
12. .001575
13. 58.0
14. 289
15. .0216
16. 4030
17. .001203
18. 165.6
19. .01597
20. .00000353
21. 23.1
22. .001190
23. 1.406
24. .000426

Exercise 6-1

1. 0.813
2. 2.04
3. 1.565
4. 0.1979
5. 0.260
6. 1.284
7. 0.296
8. 3.43

9. 8.14
10. 0.555
11. 13.37
12. 2.46
13. 0.267
14. 1.713
15. 0.402
16. 2.23
17. 0.455
18. 0.1394
19. 1.517
20. 62.7
21. 0.557
22. 1.615
23. 11.75
24. 3.87
25. 20.5
26. 404
27. .00435
28. 0.1916
29. 3500
30. 0.772
31. 0.346
32. 121.0
33. .00214
34. 0.536
35. 0.209

Exercise 6-2

1. 50.8
2. 12.96
3. 33.3
4. 26.3
5. 48.4
6. 5.97
7. 3480
8. 25.8
9. 14.69
10. 28.7
11. 56.4
12. 79.5
13. 60.0
14. 48.1
15. 34.9
16. 4.88
17. 1.662
18. 13.88
19. 527
20. 296
21. 0.253
22. 71.5
23. 16.11
24. 159.9
25. 539
26. 24.1
27. 0.639
28. 7330
29. 493
30. 21.3
31. 16.26
32. 104.2
33. 4.52
34. 79.9
35. 2310
36. 66.9
37. .0410
38. 1.998
39. 4.53
40. 29,400

Exercise 6-3

1. 7.89
2. 1.102
3. 15.30
4. 0.950
5. 0.838
6. 0.665
7. 0.766
8. 1.162
9. 0.924
10. 0.888
11. 1.082
12. 93.4
13. 83.1
14. 8.78
15. 16.05
16. 70.6
17. 1.483
18. 67.7
19. 0.638
20. 120.0
21. 8.04
22. 1.323
23. 15.91
24. 2.97
25. 8.03

Exercise 7-1

1. 5.63, 8.48, 17.60, 25.2
2. 1.142, 3.00, 5.93, 9.34, 17.14
3. 1.675, 3.69, 4.69, 6.36, 9.72
4. 0.794, 1.746, 3.46, 9.36, 14.35
5. 3.79, 7.19, 9.49, 14.26, 22.2, 32.0 (meters)
6. 4.55, 14.20, 8.15, 67.3, 76.8, 279
7. 40.4, 86.1, 144.0, 221, 309, 549, 1089 (ohms)
8. 70.4, 159.9, 319, 464, 732, 1188 (lbs)
9. \$2.51, \$4.51, \$18.81, \$6.33, \$112.20, \$58.90
10. 32.5, 63.8, 129.9, 168.2, 260, 710 (lbs)
11. 0.370, 0.629, 0.903, 1.154, 1.702, 3.85 (inches)
12. \$0.50, \$2.39, \$1.13, \$7.45, \$0.13, \$0.36, \$4.46
13. \$6.07, \$86.70, \$25.10, \$4.31, \$9.54, \$196.50
14. 222, 542, 711, 1630, 3700 (ft per sec)

Exercise 7-2

1. 6.40, 3.56, 2.13, 1.523, 1.103
2. 34.1, 22.7, 11.36, 9.74, 8.52
3. 196.0, 146.7, 99.5, 56.7, 25.8
4. 34.7, 20.8, 15.29, 8.89, 6.98, 5.47, 3.59, 2.31
5. 5.00, 0.357, .0833, .0287, .00757
6. 63.3, 17.92, 10.92, 3.49, 0.872
7. \$90,000, \$50,000, \$30,000, \$18,000, \$15,000
8. 1662, 1271, 831, 617, 514 (rpm)
9. 34.4°, 26.54°, 22.88°, 21.108°, 20.24°
10. 21.7, 13.92, 11.12, 4.72, 3.17

Exercise 7-3

1. 17.47	**16.** 4.02	**31.** 1.551	**46.** 16.87
2. 0.659	**17.** 17.86	**32.** 6.34	**47.** 105.5
3. 15.47	**18.** 110.9	**33.** 0.707	**48.** 1355
4. 0.640	**19.** 2.08	**34.** 114.0	**49.** 1.487
5. 14.04	**20.** 175.7	**35.** 9.07	**50.** 29.0
6. 1.785	**21.** 10.23	**36.** 3.08	**51.** 0.731
7. 7.09	**22.** 1054	**37.** .000828	**52.** 138.2
8. 11.58	**23.** 7.95	**38.** .01227	**53.** 0.772
9. 1.650	**24.** 0.888	**39.** .0001442	**54.** .0387
10. 20.2	**25.** 0.351	**40.** .00804	**55.** 0.360
11. 4.42	**26.** 55.7	**41.** 194.2	**56.** 54.6
12. 7.05	**27.** 5.02	**42.** 11.03	**57.** 19.29
13. 10.32	**28.** 3.69	**43.** 11.11	**58.** 3.74
14. 9.76	**29.** 165.8	**44.** 0.898	**59.** 1.641×10^8
15. 5.78	**30.** 9.20	**45.** .00769	**60.** 0.428

Exercise 7-4

1. 8.42, 5.56, 22.1, 1.322, 0.1706, 55.3, 157.7, 3.63

2. 6.50, 4.52, 26.6, 56.9, 0.1329, 774, .00655, 0.347

3. 23.9	**7.** 26.1	**10.** 52.8	**13.** 10.35
4. 11.34	**8.** 37.7	**11.** 6.18	**14.** 7.65
5. 0.615	**9.** 11.34	**12.** 2.28	**15.** 1.304
6. 2.23			

Exercise 7-5

1. 6.37	**3.** 132.5	**5.** 0.0686	**7.** 9190
2. 429	**4.** 60.0	**6.** 50.8	**8.** 1420

9. 11.89
10. 104.6
11. 9.06
12. 545
13. 11.60
14. 107.1
15. 271
16. 556
17. 5.35×10^{-6}
18. 2.74
19. 2.15
20. **a.** 18,000 **b.** 240
21. **a.** 4685 **b.** 5020
22. **a.** 7.27 **b.** 5.20
23. **a.** 5.33 **b.** 6.08
24. **a.** 151.3 **b.** 1747

Exercise 8-1

1. $X = 25.5$
2. $X = 5.07$
3. $X = 26.0$
4. $X = 95.8$
5. $X = 0.738$
6. $X = 11.99$
7. $T = 59.6$
8. $T = 81.1$
9. $R = 4.74$
10. $R = 44.0$
11. $I = 337$
12. $I = 3.46$
13. $V = 157.2$
14. $V = 132.3$
15. $P = 0.861$
16. $P = 32{,}500$
17. $X = .0001493$
18. $X = 25.2$
19. $L = 10.00$
20. $L = 8.93$
21. $W = .00583$
22. $W = 106.2$
23. $F = 18.45$
24. $F = 7.84$
25. $X = 1.755$, $Y = 30.8$
26. $X = 89.4$, $Y = 2.59$
27. $X = 3.71$, $Y = 227$
28. $X = 45.8$, $Y = 94.0$
29. $V_1 = 4.22$, $V_2 = 18.89$
30. $x = 0.269$, $y = 13.74$
31. $V_1 = 90.9$, $V_2 = 3.40$
32. $V_1 = 87.0$, $V_2 = 324$
33. $X = 1.139$, $Y = 1.917$, $Z = 23.0$
34. $X = 10.67$, $Y = 9.24$, $Z = 3.31$
35. $X = 284$, $Y = 1.218$, $Z = 204$
36. $R_1 = .01103$, $R_2 = 0.467$, $R_3 = 1.490$
37. $I_1 = 5.74$, $I_2 = 41.3$, $I_3 = 157.7$
38. $T_1 = 42.7$, $T_2 = 178.2$, $T_3 = 9.01$

Exercise 8-2

1. **a.** 3.79, 18.17, 47.4 (liters)
b. 0.264, 5.55, 224 (gals)

2. **a.** 148, 46.5, 6.82 (mph)
b. 70.7, 31.8, 125.6 (ft/sec)

3. **a.** 7.70, 165.8, 275 (lbs)
b. 15.72, 74.5, 2.16 (kg)

4. **a.** 43.0, 12.81, 274 (in.merc.)
b. 0.356, 8.59 (lbs/sq in.)

5. **a.** 153.5, 24.0, 304 (miles)
b. 2.34, 4.30 (inches)

6. **a.** 92.6, 2000, 1360 (km)
b. 281, 8.32 (miles)

7. **a.** 8.34, 145.1, 934 (lbs)
b. 233, 2.71 (gals)

8. **a.** 0.1663, 48.9, 1.340 (hp)
b. 23,600, 390 (watts)

9. a. 54.2, 9.69, 117.1 (sq in.)
b. 955, 60.6 (sq cm)

10. 2.00, 8.01, 1.057 (amps)

11. 61.4, 384, 875 (miles)

12. 5.43, 16.55, 90.5 (gals)

Exercise 9-1

(D scale accuracy)

1. 2.89	**9.** 807	**17.** 7.67×10^8	**24.** .00001901
2. 31.4	**10.** 27.2	**18.** 2.28×10^5	**25.** 5.11×10^{-8}
3. 77.4	**11.** 4190	**19.** 6.15×10^7	**26.** 1.310×10^{-9}
4. 116.6	**12.** 0.1640	**20.** 3.14×10^9	**27.** 348
5. 54.8	**13.** 175.5	**21.** 2.64×10^{10}	**28.** 3.18×10^9
6. 11.56	**14.** .0207	**22.** 1.640×10^{-5}	**29.** 42,800
7. 18.23	**15.** 1037	**23.** .00318	**30.** 2.08×10^{13}
8. 6.10	**16.** 3.76×10^4		

Exercise 9-2

(R scale accuracy)

1. 6.97	**11.** 31.50	**21.** .02678	**31.** 1435
2. 2.330	**12.** 4.433	**22.** 810	**32.** .02450
3. 3.557	**13.** 1.772	**23.** .00585	**33.** 0.963
4. 1.030	**14.** 165.5	**24.** 0.2832	**34.** .0837
5. 7.73	**15.** 9.21	**25.** 16.15	**35.** .0560
6. 3.286	**16.** 5.60	**26.** .03028	**36.** .01387
7. 2.186	**17.** 0.874	**27.** 2088	**37.** 6080
8. 19.08	**18.** 0.644	**28.** .01559	**38.** .0004658
9. 60.3	**19.** 0.2057	**29.** .00751	**39.** 4.101
10. 86.4	**20.** .04207	**30.** 1.655×10^{-4}	**40.** .01453

41. 1649
42. .001476
43. 2.907
44. 0.2697
45. .02857
46. 4.550×10^5
47. 7510
48. 169.4
49. 0.788
50. 763

Exercise 9-3

1. 2.42
2. 3.86
3. 0.604
4. 2950
5. 0.261
6. 0.932
7. .0330
8. 0.791
9. 6.20
10. 0.474
11. 89.0
12. 10.84
13. 764
14. 3.49
15. 0.01071
16. 1.97
17. −0.243
18. 7.48
19. 1.93
20. 17.97
21. **a.** 1.502 **b.** 1.637 **c.** 1.483
22. **a.** 95.0 **b.** 664 **c.** 411
23. **a.** 1.220 **b.** 0.422 **c.** 2.32
24. **a.** 0.319 **b.** 0.0513 **c.** 35.4
25. **a.** 6.32 **b.** 16.37

Exercise 9-4

1. 0.280, −1.78
2. 3, 0.4
3. 2.18, −0.327
4. 0.1090, −0.1715
5. 11.3, − 0.40
6. −0.279, −2.29
7. −0.1069, −0.652
8. 1.65, −23.4

Exercise 10-1

1. 4.10
2. 13.8
3. 275
4. 373
5. 1.86
6. 197
7. 34.0
8. 21.0
9. 520
10. 0.446
11. 0.134
12. 1210
13. 2050
14. 5.47×10^{11}
15. 2.59×10^{13}
16. 4.82×10^7
17. 2.46×10^6
18. 0.745
19. 2.02×10^9
20. 1.01×10^{14}

21. 0.488
22. .00995
23. 6.20×10^{-8}
24. .0660
25. .000266
26. 2.29×10^{-8}
27. 6.08×10^{-7}
28. 5.40×10^{12}
29. 317,000
30. 1.62×10^{-10}

Exercise 10-2

1. 1.951
2. 2.02
3. 2.22
4. 4.58
5. 2.60
6. 4.99
7. 9.05
8. 1.433
9. 6.65
10. 3.86
11. 1.035
12. 2.23
13. 4.81
14. 0.397
15. 0.807
16. 0.970
17. 0.274
18. 0.394
19. 0.436
20. 0.1441
21. 0.1955
22. 15.77
23. 16.73
24. 21.3
25. 12.60
26. 19.70
27. 22.0
28. 28.7
29. 40.1
30. 83.1
31. 59.2
32. 44.4
33. 0.1936
34. .0651
35. .0350
36. 287
37. .01928
38. 565
39. .00888
40. 424
41. .01403
42. .0360
43. 0.384
44. 0.1051
45. 58.0
46. 1080
47. 1608
48. .000752
49. 91,300
50. .000323

Exercise 10-3

1. 9.60
2. 226
3. 66.0
4. 0.735
5. .0152
6. .000641
7. 7250
8. .0000117
9. 122,000
10. 12.9
11. 49.0
12. 1.40
13. 176
14. 0.157
15. .00313
16. 0.297
17. 1.83×10^{-13}
18. 820,000
19. .000341
20. 4.35×10^{14}

Exercise 11-1

(A scale accuracy)

1. 58.5
2. 12.7
3. 300
4. 323
5. 1220
6. 13.8
7. 33.7
8. 6.70
9. 37.1
10. 10.5
11. 8.09
12. 1.67
13. 5.27
14. 1.72
15. 2.73
16. 64.2
17. 12,300
18. 9300
19. 170
20. 385

21. 5.68×10^5

22. 5.73

23. 1.20×10^{-12}

24. **a.** 1980 sq in.
b. 106 sq in.
c. 0.446 sq ft
d. 8850 sq ft
e. .000340 sq cm

25. **a.** 5.64 sq in.
b. 2.46 sq in.
c. 17.0 sq ft
d. 39.0 sq ft
e. 169 sq ft

26. 11.4, 25.6, 71.0, 139, 230

27. 36.3, 101, 198, 487

28. 0.296, 1.85, 10.6, 46.3, 266

29. 1.27, 0.486, 0.261, .0925

30. 3.10, 1.67, 1.04, 0.60

31. 37.2, 8.51, 3.20, 1.36

Exercise 11-2

(A scale accuracy)

1. 4.22
2. 7.42
3. 252
4. 23.4
5. 0.115
6. 1.28
7. 101
8. 1.74
9. 3.43
10. 30.5
11. 8.12
12. 67.2
13. 47.0
14. 1.69
15. 2160
16. 125
17. 188
18. 398
19. 76.8
20. 15.3
21. 478
22. 9,460
23. 4.78
24. 318
25. 7.83
26. 24.1
27. 128
28. 0.455
29. 1.58×10^{-4}
30. 66.1
31. 1.61
32. 8.69

33. 68.0
34. 25.1
35. 4.20×10^5
36. 2.40×10^{10}
37. 0.475
38. .0311
39. 8.95×10^{-5}
40. 0.430
41. 0.139
42. 4.81×10^{-4}
43. 4590
44. 6.28×10^7

Exercise 11-3

1. 920
2. 8370
3. 0.677
4. 14.23
5. 0.698
6. 1.263
7. −43.7
8. 3.10
9. 6.68
10. 2.14
11. 1.13
12. 39.0
13. 171
14. a. 293.3
b. 1498.5
15. a. 3.44
b. 39.2
16. a. .001674
b. .01866
17. a. 0.224
b. 0.0475
18. a. 0.270
b. 0.315
19. a. 0.1427
b. 0.0723
20. a. 0.1595
b. 0.253

Exercise 12-1

1. 11.40
2. 2.06
3. 100.2
4. 78.0
5. 1.286
6. 1.492
7. 1.755
8. 2.05
9. 10.15
10. 7.42
11. 6.50
12. 4.50
13. 257
14. 7.00
15. 7.36
16. 3.17
17. 22.1
18. 2.23
19. 10.18
20. 1.220
21. .0912
22. .0456
23. 0.516
24. 3.60
25. 155.0
26. 10.60
27. 2.86
28. 2.33
29. 3.52
30. 368
31. 2.41
32. 31.2
33. 30.2
34. 29.8
35. 11.78
36. 3.38
37. 24.3
38. 2.83
39. 3.93
40. 9.40
41. 0.487
42. 38.7
43. 5.19
44. 12.56
45. 0.726
46. 12.18
47. 0.1342
48. 29.4
49. 13.12
50. 892
51. 0.606
52. .000550

53. 283
54. .0639
55. 26.0
56. 23.7
57. 15.24
58. 0.1203
59. 14.09

Exercise 12-2

1. 5.11
2. 2.88
3. 3.82
4. 4.59
5. 11.23
6. 3.15
7. 1.838
8. 2.86
9. 4.56
10. 0.615
11. 6.85
12. 11.45
13. 35.9
14. 1.775
15. 37.7
16. 5.24
17. 1.781
18. 14.75
19. 28.5
20. 1.376
21. 90.3
22. 75.2
23. 4.97
24. 2.08
25. 16.91
26. 41.9
27. 45.6
28. 25.4
29. 7.17
30. 45.6
31. 2720
32. 89.3
33. 2.15
34. .0942
35. 16.94
36. 0.496
37. 131.0
38. 1.680
39. 1.640
40. 4.82
41. 84.5
42. 10.48
43. 30.8
44. 6.86
45. 20.9
46. 0.594
47. 6.47
48. 282
49. .0200
50. 94.1
51. 13.23
52. 10.87
53. 0.496
54. 1265
55. 100.8
56. 0.794
57. 10.89
58. 31.8
59. 12.70
60. 8.63
61. 0.1530
62. 5440
63. 4.40×10^{-6}
64. 8.65
65. 0.1835
66. 0.1280
67. 1.458
68. 77.0
69. 0.507
70. 45.4
71. 1.810
72. 32.5
73. 3.83
74. 1105
75. 4.41, 6.84, 16.77, 28.0
76. 2.87, 6.40, 11.70, 28.2
77. 0.666, 1.019, 1.276, 2.58
78. 90.6, 22.0, 9.38, 4.84
79. 31.5, 19.41, 9.70, 7.16
80. 19.90, 13.45, 6.30, 3.29

Exercise 12-3

1. 14.60	**11.** 9.25	**21.** 29,400	**31.** 3.52
2. 47.6	**12.** 2.04	**22.** 3.39	**32.** 0.898
3. 1.318	**13.** 8.11	**23.** 9.00×10^{-11}	**33.** 11.11
4. 1.005	**14.** 82.4	**24.** 59.1	**34.** 20.8
5. 41.9	**15.** 1.933	**25.** 8.55	**35.** 113.5
6. 24.7	**16.** 98.5	**26.** 0.545	**36.** 7.05
7. 2.59	**17.** 0.748	**27.** 1.153×10^{8}	**37.** 22.7
8. 5.45	**18.** 40.4	**28.** 1.252×10^{-7}	**38.** 13.20
9. 1.302	**19.** 49.3	**29.** 2.53	**39.** 0.701
10. 0.765	**20.** 0.561	**30.** 9.80	**40.** 5.72

Exercise 12-4

1. 1.93

2. 0.484

3. 8.93

4. 1440

5. 1.126

6. 78.3

7. 12.59

8. 322

9. 31.3

10. 1.458×10^{-7}

11. **a.** .0480 **b.** 0.0539

12. **a.** 0.302 **b.** 0.546

13. **a.** 2.30 **b.** 3.14

14. **a.** 1.69 **b.** 2.66

15. **a.** 143.5 **b.** 2710

16. **a.** 1420 **b.** 256

17. **a.** 0.380 **b.** 1.830

18. **a.** 75.5 **b.** 2670

19. **a.** 3.26 **b.** 17.7

20. **a.** 675 **b.** 12,320

21. **a.** 0.0834 **b.** 0.0226

22. **a.** 2.16 **b.** 6.80

23. **a.** 12.59 **b.** 21.6

24. **a.** 1.798 **b.** 2.96

25. **a.** 1.118 **b.** 1.101

Exercise 13-1

1. .0124 **2.** 1.780 **3.** 0.248 **4.** 2.03 **5.** 97.3 **6.** 2.16

Exercise 13-2

1. 322 **2.** 0.0346 **3.** 0.0604 **4.** 9.70 **5.** 9.95 **6. a.** 7.67, 47.8, 146.0
b. 4.68, 181.5, 1698

Exercise 13-3

1. 376 **2.** 34.8 **3.** 2.07 **4.** 4.62×10^5 **5.** 8.44 **6.** 7.11

Exercise 13-4

1. 57.4 **2.** 25.0 **3.** 14.10 **4.** 46.3 **5.** 0.456 **6.** 24.4

Exercise 13-5

1. $R_1 = 0.01069$
$R_2 = 0.480$
2. 5.14 **3.** 2744 **4.** 210 **5.** 0.835 **6.** 4.83, 9.32, 23.0, 38.8

Exercise 13-6

1. 23.6, 62.8, 121.5, 150.8

2. 3.92×10^{-14}

3. 406

4. 1.772×10^{-4}

5. a. 0.881, 2.74, 6.99 (in.)
b. 4.93, 13.81, 35.5 (lbs)

6. 48.6

Exercise 13-7

1. 16.05 **2.** 8.12 **3.** 627 **4.** 3.79×10^{11} **5.** 0.121 **6.** 19.00

Exercise 13-8

1. 45.0 **2.** 128.8 **3.** 2.01×10^{23} **4.** .00363 **5.** 72.2 **6.** 6.95

Exercise 14-1

1. 0.630
2. 0.242
3. 0.930
4. 0.1770
5. 0.715
6. .0401
7. .01990
8. 0.302
9. 0.842
10. 0.990
11. 0.508
12. .0130
13. 0.1333
14. 0.443
15. 0.264
16. .0325
17. .0125
18. 0.576
19. 0.970
20. .0776
21. .0935
22. 0.248
23. 0.1054
24. 0.886
25. 0.213
26. .0206
27. 0.996
28. 0.352
29. .0102
30. 0.665

Exercise 14-2

1. 0.731
2. 0.391
3. 0.927
4. 0.255
5. 0.524
6. 0.791
7. 0.1193
8. 0.961
9. .01745
10. .0454
11. .01326
12. 0.347
13. 0.1485
14. 0.870
15. 0.996
16. 0.675
17. .0436
18. 0.652
19. 0.282
20. 0.853
21. 0.565
22. .01367
23. 0.880
24. 0.734
0.679
25. 0.297
0.955
26. 0.948
0.319
27. 0.611
0.791
28. 0.826
0.564

Exercise 14-3

1. 0.650
2. 0.306
3. 1.428
4. 4.70
5. .0349
6. 19.10
7. 0.238
8. 0.770
9. 1.235
10. 2.40
11. .0645
12. .0286
13. 0.255
14. 3.40
15. 1.167
16. 15.49

17. .01483 **18.** .0741 **19.** .01454 **20.** 0.206 **21.** 5.91 **22.** 0.762 **23.** 0.389 **24.** 1.285 **25.** .0900 **26.** 13.50 **27.** .0464 **28.** 0.955 **29.** 9.60 **30.** 0.1375

Exercise 14-4

1. 0.532 **2.** 2.25 **3.** 1.481 **4.** 1.206 **5.** 2.56 **6.** 1.466 **7.** 0.296 **8.** 0.670 **9.** .0454 **10.** 13.33 **11.** 0.947 **12.** 3.36 **13.** 0.410 **14.** 0.474 **15.** 0.1642 **16.** 1.717 **17.** 1.564 **18.** 3.19 **19.** 3.01 **20.** 0.1680 **21.** 1.048 **22.** .0583 **23.** 6.77 **24.** .0305 **25.** 0.836 **26.** 0.724 **27.** 1.163 **28.** 0.966 **29.** 0.1485 **30.** 32.2 **31.** .0640 **32.** 0.524 **33.** 0.499 **34.** 0.1123 **35.** .01520 **36.** 22.9 **37.** 3.94 **38.** .0706 **39.** 4.22 **40.** 0.875 **41.** 0.837 **42.** 1.905 **43.** 0.982 **44.** .0733 **45.** 24.6 **46.** .01483 **47.** 1.315 **48.** 3.33

Exercise 15-1

1. 0.530 **2.** 0.940 **3.** –0.407 **4.** –0.545 **5.** –0.407 **6.** –0.445 **7.** –5.67 **8.** 2.90 **9.** –0.695 **10.** –0.743 **11.** –0.738 **12.** 0.844 **13.** –17.34 **14.** 0.0611 **15.** –0.903 **16.** 2.48 **17.** 0.891 **18.** 0.237

Exercise 15-2

1. 18.66°
2. 35.1°
3. 56.5°
4. 17.85°
5. 33.5°
6. 39.2°
7. 25.9°
8. 53.1°
9. 71.4°
10. 2.99°
11. 2.17°
12. 87.42°
13. 27°
14. 2.10°
15. 3.5°
16. 72°
17. 52.5°
18. 75.96°
19. 5.17°
20. 10.6°
21. 48.5°
22. 40.2°
23. 87.5°
24. 85.66°
25. 74.4°
26. 4.20°
27. 202.4°
28. 101.82°
29. 118.3°
30. 137.3°
31. 0.80°
32. 6.29°
33. 163.6°
34. 161.5°
35. 87.93°
36. 88.91°
37. 71.3°
38. 1.11°
39. 4.21°
40. 61.6°
41. 174.78°
42. 84.29°
43. 33.7°
44. 1.58°
45. 88.4°
46. 89.12°
47. 194.2°
48. 47.4°
49. 70.6°
50. 86.29°
51. 48.6°
52. 19.45°

Exercise 15-3

1. 9.20
2. 35.2
3. 3.74
4. 73.5
5. 56.9
6. 4.48
7. 2.55
8. 558
293
9. 4.82
11.70
10. 1617
7320
11. 208
274
12. 91.9
−69.2
13. −4.24
−8.61
14. 150.1
454
15. 52.1
16. 315
17. 77.9
18. 69.8
19. 544
20. 92.0
21. 4560
22. 4.90
23. 213
24. 19.74
25. 3310
26. 29.1
27. 1442
28. 5.54
29. 129.0
30. 313
31. 1.918
32. 1.198
33. 1.356
34. 44.9
35. 13.54
36. 42.1

37. −38.5 **38.** 6.53 **39.** 421 **40.** 13.75

Exercise 15-4

1. a. 0.559 **b.** 1.193 **c.** 0.1190 **d.** 2.54 **e.** 3.89

2. a. 0.0636 **b.** 0.0785 **c.** 0.0321 **d.** .0131 **e.** 0.0482

3. a. 133.5° **b.** 43.5° **c.** 61.8° **d.** 300° **e.** 12.3°

4. a. 3.55° **b.** 1.97° **c.** 1.002° **d.** 3.15° **e.** 4.38°

5. 0.868

6. 0.745

7. 0.977

8. 0.430

9. 0.783

10. 0.377

11. −0.276

12. 0.952

13. 0.1770

14. −0.755

15. −0.660

16. −0.972

Exercise 15-5

1. 17.7

2. 1.565

3. 8.02

4. 29.1

5. 2.50

6. 1.844

7. 0.481

8. 7.59

9. 13.64

10. 43.3

11. 11.35°

12. 81.05°

13. 47.4°

14. a. 392 **b.** 5540

15. a. 19,500 **b.** 8860

16. a. 16° **b.** 21.7°

17. a. 1.375 **b.** 1.535

18. a. 693 **b.** 177.8

19. a. 213 **b.** 1480

20. a. 4.22 **b.** 25.5

21. a. 413 **b.** 3390

22. a. 70.8° **b.** 65.0°

23. a. 0.651 **b.** 0.650

24. a. 39.64° **b.** 40.58°

25. a. 4.53 **b.** 27.2

26. a. 8.68 **b.** 23.0

27. a. 176 **b.** 49.6

28. a. 76.9 **b.** 17.5

29. a. 630 **b.** 2610

30. a. 67.4° **b.** 91.4°

31. a. 0.788 **b.** 0.898

Exercise 16-1

1. $B = 64°$, $a = 32.0$, $b = 65.6$

2. $B = 33.8°$, $a = 262$, $b = 175$

3. $A = 42.3°$, $B = 47.7°$, $b = 38.5$

4. $A = 48.8°$, $B = 41.2°$, $a = 165.5$

5. $A = 31.4°$, $B = 58.6°$, $c = 69.1$

6. $A = 61.05°$, $B = 28.95°$, $c = 1715$

7. $A = 41.6°$, $a = 282$, $b = 318$

8. $A = 42.3°$, $B = 47.7°$, $b = 6.95$

Exercise 16-2

1. $B = 65°$, $b = 68.7$, $c = 75.8$

2. $B = 37°$, $a = 35.2$, $b = 26.5$

3. $A = 55°$, $b = 4.41$, $c = 7.69$

4. $A = 29°$, $a = 8.65$, $c = 17.83$

5. $B = 42.7°$, $a = 347$, $c = 472$

6. $A = 38.4°$, $B = 51.6°$, $b = 29.0$

7. $A = 37.9°$, $B = 52.1°$, $b = 65.5$

8. $A = 72.55°$, $B = 17.45°$, $a = 2670$

9. $A = 72.5°$, $a = 4.04$, $b = 1.275$

10. $A = 3.61°$, $B = 86.39°$, $b = 57.9$

11. $B = 85.8°$, $b = 75.5$, $c = 75.8$

12. $A = 76.4°$, $a = 3890$, $b = 940$

13. $A = 27.8°$, $B = 62.2°$, $b = 3230$

14. $A = 3.8°$, $b = 52.0$, $c = 52.1$

15. $A = 58.6°$, $a = 11.09$, $c = 12.99$

16. $A = 4.38°$, $B = 85.62°$, $b = 159.5$

17. $A = 84.9°$, $a = 20.7$, $c = 20.8$

18. $A = 47.3°$, $B = 42.7°$, $a = 6060$

19. $B = 74.7°$, $a = 96.6$, $b = 353$

20. $B = 19°$, $a = 62.7$, $c = 66.3$

Exercise 16-3

1. $A = 35.2°$, $B = 54.8°$, $c = 20.8$

2. $A = 59.3°$, $B = 30.7°$, $c = 31.4$

3. $A = 21.3°$, $B = 68.7°$, $c = 44.1$

4. $A = 31.0°$, $B = 59.0°$, $c = 1455$

5. $A = 58.6°$, $B = 31.4°$, $c = 11.13$

6. $A = 85.08°$, $B = 4.92°$, $c = 35.1$

7. $A = 57.1°$, $B = 32.9°$, $c = 7.74$

8. $A = 33.9°$, $B = 56.1°$, $c = 22.4$

9. $A = 52.7°$, $B = 37.3°$, $c = 289$

10. $A = 5.04°$, $B = 84.96°$, $c = 41.1$

11. $A = 41.0°$, $B = 49.0°$, $c = 16.37$

12. $A = 3.28°$, $B = 86.72°$, $c = 5.07$

13. $A = 47.6°$, $B = 42.4°$, $c = 861$

14. $A = 25.1°$, $B = 64.9°$, $c = 3420$

15. $A = 3.44°$, $B = 86.56°$, $c = 421$

16. $A = 70.5°$, $B = 19.5°$, $c = .0796$

17. $A = 22°$, $B = 68°$, $c = 121.5$

18. $A = 47.1°$, $B = 42.9°$, $c = 38{,}900$

Exercise 16-4

1. a. $45 + 28.1j$
   b. $19.01 + 28.2j$
   c. $172.1 + 52.6j$
   d. $178 + 440j$
   e. $9.41 + 4.88j$
   f. $669 + 42.1j$
   g. $-2.99 + 3.09j$
   h. $-82.9 + 43.1j$
   i. $-858 - 623j$
   j. $25.8 - 28.2j$

2. a. $7.61\angle 66.8°$
   b. $56\angle 68°$
   c. $16.75\angle 17.35°$
   d. $6.06\angle 22.3°$
   e. $779\angle 74.35°$
   f. $366\angle 3.61°$
   g. $27.3\angle 171.57°$
   h. $7.75\angle 234°$
   i. $0.85\angle 195°$
   j. $3480\angle 348.04°$

3. a. $4.51 + 6.21j$
   b. $8.55 + 16.09j$
   c. $14.81 + 24.0j$
   d. $34.7 + 14.5j$
   e. $-51.1 + 70.3j$
   f. $-259 - 711j$

4. a. $4.54 + 3.81j$
   b. $1.575 + 3.23j$
   c. $0.1535 + 0.329j$
   d. $-0.655 + 0.305j$
   e. $-4.95 - 6.34j$
   f. $5.49 - 4.74j$

Exercise 17-1

1. $C = 66°$, $b = 34.2$, $c = 42.0$
2. $A = 80°$, $a = 42.5$, $c = 38.5$
3. $B = 43.0°$, $C = 70.0°$, $c = 6.33$
4. $B = 112.9°$, $C = 26.1°$, $b = 119.4$
5. $A = 44.68°$, $C = 10.32°$, $a = 13.75$
6. $B = 39°$, $a = 21.8$, $c = 6.99$
7. $C = 43°$, $a = 14.71$, $b = 13.46$
8. $A = 112°$, $b = 6.20$, $c = 0.581$
9. $A = 31.5°$, $B = 76.5°$, $b = 32.0$
10. $A = 104.7°$, $c = 29.3°$, $a = 10.3$
11. $C = 43°15'$, $a = 8.36$, $c = 5.81$
12. $B = 127°10'$, $b = 476$, $c = 372$
13. $A = 19.8°$, $B = 6.2°$, $b = 6.75$
14. $B = 118°20'$, $a = 8.26$, $b = 119$
15. $B = 14.88°$, $C = 123.12°$, $c = 91.4$
16. $A = 153.37°$, $B = 4.23°$, $a = 16.10$
17. $C = 65.4°$, $b = 15.08$, $c = 20.6$
18. $B = 72.65°$, $C = 7.35°$, $b = 12.30$
19. No solution
20. $A = 17°15'$, $b = 545$, $c = 693$

Exercise 17-2

1. $B = 30.4°$, $C = 123.6°$, $c = 8.55$
   $B' = 149.6°$, $C' = 4.4°$, $c' = 0.788$
2. $B = 47.8°$, $C = 95.2°$, $c = 21.5$
   $B' = 132.2°$, $C' = 10.8°$, $c' = 4.05$
3. $B = 53.6°$, $C = 85.4°$, $c = 33.4$
   $B' = 126.4°$, $C' = 12.6°$, $c' = 7.31$
4. $A = 151.6°$, $C = 20°$, $a = 8.66$
   $A' = 11.6°$, $C' = 160°$, $a' = 3.66$

5. $B = 84.6°$, $C = 52.4°$, $b = 70.4$
$B' = 9.4°$, $C' = 127.6°$, $b' = 11.54$

6. $A = 25.5°$, $B = 149.2°$, $b = 14.62$
$A' = 154.5°$, $B' = 20.2°$, $b' = 9.87$

7. $A = 142.7°$, $C = 22.7°$, $a = 20.8$
$A' = 8.1°$, $C' = 157.3°$, $a' = 4.84$

8. $A = 80.4°$, $C = 77.0°$, $a = 1.670$
$A' = 54.4°$, $C' = 103°$, $a' = 1.376$

Exercise 17-3

1. $A = 45°$, $B = 97°$, $c = 4.35$

2. $B = 44.7°$, $C = 86.3°$, $a = 12.86$

3. $A = 32.1°$, $B = 45.1°$, $C = 102.8°$

4. $A = 55.0°$, $B = 62.5°$, $C = 62.5°$

5. $A = 21°$, $C = 41°$, $b = 14.8$

6. $A = 26.1°$, $B = 41.4°$, $c = 21.0$

7. $A = 34.8°$, $B = 14.7°$, $C = 130.5°$

8. $A = 40.1°$, $B = 44.6°$, $C = 95.3°$

9. $A = 22.1°$, $C = 140.7°$, $b = 17.0$

10. $A = 16.8°$, $C = 38.6°$, $b = 18.67$

11. $A = 39.8°$, $B = 32°$, $C = 108.2°$

12. $A = 15.6°$, $B = 62°$, $C = 102.4°$

13. $A = 52.6°$, $C = 22.8°$, $b = 9.07$

14. $B = 160.6°$, $C = 14.4°$, $a = 21.0$

15. $A = 48.5°$, $B = 58.9°$, $C = 72.6°$

16. $A = 19.2°$, $B = 21.1°$, $C = 139.7°$

Exercise 18-1

1. 2.538

2. 2.033

3. 1.064

4. 8.799 − 10

5. 7.736 − 10

6. 5.628

7. 3.316

8. 9.505 − 10

9. 6.267 − 10

10. 7.626

11. 12.307

12. 1.248 − 10

13. 9.802 − 30

14. 1.439

15. −2.757

16. 1.199

17. −1.370

18. 292

19. 0.1014

20. .0545

21. 0.840

22. 1052

23. 2.07

24. .0541

25. .00197

26. 1.19×10^{15}

27. 1.92×10^{-13}

28. .0366

29. 6.68×10^{9}

30. 4.93×10^{-12}

Exercise 18-2

1. 7.61

2. 4.87

3. 2.33

4. −12.7

5. −39.9

6. 0.920

7. −0.1017

8. 7.93

9. 0.1102

10. 684

11. 1.830×10^{-9}

12. 3.963

13. 9.247

14. 8.969

15. 9.46×10^{-8}

16. 5.19×10^{-13}

17. 2.36×10^{-9}

18. **a.** 12.37
b. 16.87

19. **a.** 0.523
b. 0.239

20. **a.** 1620
b. 1738

21. **a.** 1.88×10^{-7}
b. 4.59×10^{-8}

22. **a.** 1.77×10^{6}
b. 1.97×10^{-8}

23. **a.** 200
b. 71.9

24. **a.** 0.570
b. 0.737

25. **a.** 2.49
b. 1.13

26. **a.** 63.8°
b. 135.8°

Exercise 19-1

1. 4.71

2. 1.1678

3. 1.01562

4. 18.2

5. 1.0294

6. 270

7. 2.293

8. 1.323

9. 1.0434

10. 1.0920

11. 2.930

12. 1.878

13. 7.77

14. 1.122

15. 1.0264

16. 1370

17. 1.448

18. 12,000

19. 1.0129

20. 3.47

21. 106

22. 168

23. 4380

24. 4.10

25. 47.3

26. 0.656

27. 1.151

28. 6.05

29. 31.4

30. 3.08

Exercise 19-2

1. .0706

2. 0.767

3. 0.9380

4. .00166

5. .0247

6. 0.357

7. 0.9656

8. .00315

9. 0.9296

10. 0.9008

11. 0.9853

12. .000083

13. .0078

14. 0.468

15. 0.9361

16. 0.9048

17. .040

18. 0.756

19. 0.889

20. 0.257

21. 3.74

22. 8.11

23. 1.418

24. 536

25. 14.8

26. .00377

27. 0.483

28. 0.186

29. 0.254

30. 0.495

Exercise 19-3

1. 1.0034 (approx.)
2. 0.99939 (approx.)
3. 1.0072 (approx.)
4. 0.9986 (approx.)
5. 0.99963 (approx.)
6. 1.000021 (approx.)
7. 440,000 (approx.)
8. 2.65×10^{10} (approx.)
9. .0038 (approx.)
10. 2.9 (approx.)

Exercise 19-4

1. 168
2. 1.1175
3. 27.9
4. 0.9603
5. 0.651
6. 1.0253
7. 1.0378
8. 1.0699
9. 1.284
10. 0.8465
11. 0.9552
12. 1.0144
13. 1.741
14. .0535
15. 1.802
16. 2.195
17. .0090
18. 0.9277
19. 1.063
20. .0135
21. 3.32
22. 39.4
23. 0.370
24. 1.185
25. .0727
26. 0.9661, 0.9333, .0563
27. 2.99, 55, 3100
28. 181, 3.26, 1.0557
29. 93.1, 2.12, 0.171
30. 281,000, 1810, 1285

Exercise 19-5

1. 0.499
2. 1.89
3. 1.295
4. 0.892
5. −0.263
6. 1.0041
7. −0.572
8. 0.868
9. 1.140
10. 0.202
11. 1.75
12. 0.667
13. 8.75, 10.2, 15.1, 43.4
14. **a.** 0 ft/sec, 636 ft/sec
    b. 735 ft/sec

Exercise 19-6

1. 5.93
2. 0.207
3. −1.743
4. 1.047
5. −.0971
6. 2.13
7. −7.32
8. 5.10
9. .0163
10. 7.82
11. −5.68
12. 3.60
13. .01237
14. 9.71
15. −0.962
16. 1.019
17. −8.65
18. .01025
19. −0.975
20. 9.03
21. 2.71
22. −2.72
23. 5.35
24. 6.30
25. 1.878
26. .0087
27. 0.9213
28. 1340
29. 1.0278
30. 0.631

Exercise 19-7

1. 2.06
2. 0.372
3. 0.237
4. −.0631
5. −1.201
6. 3.21
7. −0.170
8. 2.46
9. .0980
10. 0.306
11. −0.920
12. −72.9
13. .0343
14. 0.331
15. −.0500
16. −1.441

Exercise 20-1

1. 42.3
2. 16.6
3. 3500
4. 1.494
5. 26.9
6. 1.240
7. .00195
8. .0309
9. 0.9374
10. 0.757
11. 410
12. 389
13. 1.482
14. 0.422
15. .0955
16. 1.628
17. 1.980
4.04
9.63
18. 1.0844
1.181
1.568
19. 0.34
0.151
.091
20. 0.9213
0.870
0.726
21. 1.477
22. 2.00
23. 3100
24. 1.1045
25. .000263
26. 11.7
27. 18,200
28. 1.0736
29. 0.347
30. .000079
31. 0.844
32. 6.75

33. .00094
34. 2.89
35. 7.10
36. .000094
37. 0.393
38. 0.647
39. 340
40. 0.9539
41. .0728
42. .00225
43. 1340
44. 1.352
45. 0.9005
46. 1.1078
47. 0.448
48. 10.03
49. .0164
50. 198

Exercise 20-2

1. 330
2. 2.213
3. 1.368
4. 1.754
5. 0.875
6. 0.97395
7. 235
8. 1.600
9. 0.9514
10. 0.832
11. 1.332
12. 1.429
13. 1.0923
14. 238
15. 97
16. 0.9769
17. .0066
18. 10.7
19. 0.98872
20. 1.0458
21. 3.06
22. 1.0299
23. 1.018
24. 0.266
25. 1.0215
26. .00465
27. 1.0991
28. .00045
29. 0.579
30. .0174
31. 1420
32. .0392
33. 1.437
34. 0.98528
35. 310
36. 1.01814
37. .0365
38. 1.0328
39. 9000
40. 1.303
41. .00009
42. 0.640
43. 9400
44. 964
45. 1309
46. 170,400
47. 0.9597
48. 63
49. 0.504
50. 1.608

Exercise 20-3

1. .00077
2. .0041
3. .0585
4. 0.661
5. 2.37
6. 12.85
7. 2.138
8. 0.619
9. 1.431
10. 1.0565
11. 0.159
12. 2.41
13. 0.8142
14. 1.1218
15. 0.363
16. 0.588

17. 1.837
18. 0.9414
19. 10.25
20. 1.01236
21. 1.462
22. 73
23. 2.76
24. 3250
25. 840
26. 12,150
27. 7550
28. 0.787
29. 435
30. 1.32

Exercise 20-4

1. 3.17
2.00
1.47
1.26
2. 6.50
3. 1.777
4. 0.333
5. 1.264
6. 6.85
7. 0.680
8. 2.23
9. 1.1001
10. 0.8989
11. 0.9273
12. 1.047
13. 3.30
14. 1.0269
15. .0177
16. 3.60
17. 1.394
18. 2.915
19. 0.0315
20. 0.9734
21. 1.373
22. 0.335
23. 1.414
24. 1.273
25. 214
26. 0.98358
27. 0.729
28. 1.696
29. 4.15
30. 0.836
31. 0.828
32. 1.0945
33. 1.262
34. 128
35. 0.7585
36. 1.025
37. 18.45
38. 451

Exercise 20-5

1. 24.5
2. 4.42
3. 1.480
4. 0.359
5. 0.8534
6. 1.113
7. 1.0223
8. 1.914
9. 6900
10. 0.1142
11. 0.9652
12. 700
13. 3.68
14. 0.5415
15. 0.8376
16. 0.787
17. 1.655
18. .0152
19. 0.9665
20. 1.597
21. 3.95
22. 0.0297
23. 81
24. 1.1081
25. 1.678
26. 0.163
27. 1.159
28. 0.8021
29. .02179
30. 1.375
31. 0.1835
32. .000125

33. 1.838 **34.** 804 **35.** 415 **36.** 177 **37.** 1.87 **38.** 9.51 **39.** 1.0116, 1.1485, 5.65, 5750

Exercise 21-1

1. 2.73 **2.** 2.40 **3.** 1.92 **4.** 2.45 **5.** 0.605 **6.** 25.3 **7.** 179.2 **8.** 3.40 **9.** 6.31 **10.** 0.236

11. –.0286 **12.** –7.07 **13.** 0.1270 **14.** 2.41 **15.** –1.70 **16.** –0.278 **17.** 0.322 **18.** 2.67 **19.** –0.159 **20.** 7.40

21. 0.241 **22.** –0.437 **23.** 0.216 **24.** –.0283 **25.** 7.18 **26.** .01660 **27.** 0.576 **28.** 2.24 **29.** –0.244 **30.** 0.265

31. 256 **32.** .0698 **33.** 7.41 **34.** 13.48 **35.** .0286 **36.** 3.13 **37.** –36.8 **38.** 2.60 **39.** –0.348 **40.** .00192

Exercise 21-2

1. 2.81 **2.** 2.09 **3.** 1.710 **4.** 0.368 **5.** 5.04 **6.** 0.348

7. .0625 **8.** –.00700 **9.** –.01220 **10.** .00712 **11.** .00961 **12.** –0.413

13. –1.379 **14.** 2.36 **15.** –0.1298 **16.** .0797 **17.** .01886 **18.** 2.35

19. 3.46 **20.** –.0648 **21.** 4.04 **22.** –0.306 **23.** –.0632 **24.** –2.91

Exercise 21-3

1. 0.612 **2.** 9.90 **3.** .0983 **4.** 245 **5.** 4.88 **6.** –0.735 **7.** 2.91 **8.** 44.6

9. 0.401
10. 1.705
11. 57.6
12. 302
13. 0.1268
14. 0.462
15. 131
16. 105.5
17. 8.94
18. 28.1
19. 1.130
20. 21.3
21. 14.81
22. −8.92
23. −26.4
24. 0.1277
25. 0.291
26. 0.440
27. 0.219
28. .01705
29. 8.20
30. 6.64
31. 0.271
32. .00545
33. 343
34. 112
35. 371
36. 0.749
37. .00275
38. 6.81
39. 1.17
40. −6.25
41. 1.714
−0.799
42. 0.795
−0.794
43. 0.863
0.259
44. 1.386
0.405

Exercise 21-4

Some deviation in these answers is to be expected—especially when intermediate steps involve readings on the less accurate portions of the LL scale.

1. 5.80
2. .0728
3. 0.264
4. 3250
5. 9.42×10^{-6}
6. -1.46×10^{7}
7. 1.047×10^{-6}
8. 0.692
9. 0.377
10. 3040
11. 375
12. 0.286
13. 0.140
14. 55.3
15. 10,660
16. 5.19×10^{6}
17. 87,100
18. 1.54
19. 1.054×10^{-16}
20. 50.7
21. **a.** 36
b. 63.6
22. **a.** 675
b. 26
23. **a.** 1.53
b. −0.1755
24. **a.** 0.387
b. 0.352
c. 0.242
d. .01755
25. **a.** .0387
b. .00205
c. 4.52×10^{-7}
26. **a.** 3.10
b. 10.50
27. **a.** 11,540
b. 2710
28. **a.** 65.0
b. 131.7
29. **a.** 2.18×10^{-6}
b. .090
30. **a.** 4500
b. 3270
31. **a.** 9,110
b. 886

32. **a.** 41
b. 119

33. **a.** 152
b. 3.0
c. 8.8
d. 6.2

34. **a.** 1.25
b. 1.52

35. **a.** .0352
b. 1,225,000

36. **a.** 0.11
b. 7.1

37. **a.** 0.121
b. 0.188
c. 0.221
d. .0985

38. **a.** .0394
b. .0478
c. 57
d. 62

39. **a.** 1060
b. 3350
c. 27
d. 20

40. **a.** 28
b. 44

41. **a.** 2113
b. 11,060

42. **a.** 339
b. 384
c. 1.25
d. 1.465

43. **a.** 2.77
b. 4.16

44. **a.** 1334
b. 7070

45. **a.** 13,890
b. 20,000

46. **a.** 1.199
b. 1.299

Exercise 22-1

1. 0.602
2. .0611
3. 1.091
4. 1.582
5. 1.1435
6. 0.9277
7. 7.05
8. 0.169

Exercise 22-2

1. $B = 43.8°$
$a = 386$
$c = 535$
2. 3.27
3. 72
4. 3.84
5. 2720
6. 2530
7. 14.10
8. 0.673

Exercise 22-3

1. 150
2. .0136
3. .038
4. .0602
5. 0.493
6. 40.1°
7. 0.231
8. $B = 109.8°$
$C = 27.$[illegible]°
$b = 113$

Exercise 22-4

1. 1.02205
2. 0.954
3. 51.3°
4. 0.696
5. 4.752
6. −1.181
7. $13.41 + 19.90j$
8. 1.890

Exercise 22-5

1. 1.367
2. 0.1373
3. 0.516
4. 4.31
5. 561
6. 0.628
1.331
3.73
7. .01327
8. 66.6

Exercise 22-6

1. 4.33
2. .0090
3. 17.5/23.6°
4. −1.545
5. 0.8207
6. 1.430
7. 2.04°
8. .0388

Exercise 22-7

1. .0286
2. 4.44
3. $B = 43°$
$a = 25.5$
$c = 6.93$
4. 1.037
5. 2.31
6. 80.0
7. 0.298
8. −0.594

Exercise 22-8

1. $A = 65.2°$
$B = 24.8°$
$c = 159.7$
2. 0.778
3. 1.01073
4. 336
5. 436
6. 0.198
7. 16.38
8. 73.6

Exercise A-1

1. .00454
2. .001345
3. .000253
4. .001485
5. 191
6. 212
7. .00802
8. 441
9. 11.17
10. 0.168
11. .00611
.00124
.00323
.0000393
12. .001687
13. .00356
14. .0001698
15. .000710
16. .0000655
17. 153
18. 306
19. 6250
20. 232
21. 6060
22. .0125
.0001260
.00252
.0000557
23. 133.7
24. 110.7

Exercise B-1

The following answers were obtained on a slide rule having 8 LL scales. They are more accurate than the approximate results obtained with 6 LL scales.

1. 1.497×10^{6}
2. 1.627×10^{5}
3. 3.060×10^{-7}
4. 6.918×10^{6}
5. 9.539×10^{26}
6. 1.941×10^{-26}
7. 8.040×10^{6}
8. 1.269×10^{5}
9. 1.327×10^{34}
10. 7.696×10^{23}
11. 6.617×10^{-7}
12. 1.688×10^{43}
13. 8.560×10^{-17}
14. 1.001×10^{5}
15. 9.027×10^{24}
16. 1.845×10^{-15}
17. 6.369×10^{-20}
18. 6.964×10^{25}
19. 8.858×10^{21}
20. 1.033×10^{21}

Exercise B-2

The following answers were obtained on a slide rule having 8 LL scales. They are more accurate than the approximate results obtained with 6 LL scales.

1. 1.0021
2. 0.9980
3. 1.1593
4. 1.0920
5. 1.855
6. 1.398
7. 1.1410
8. 1.1730
9. 0.8590
10. 0.406
11. 0.740
12. 0.411
13. 0.622
14. 1.000425
15. 1.000201
16. 0.999585
17. 0.999785
18. 0.999630
19. 0.999697
20. 1.000555

Exercise B-3

1. 1.504
2. 1.00582
3. 1.904
4. 1.0000624
5. 0.9717
6. 0.706
7. 0.99523
8. 0.9999592
9. 0.508
10. 2.045

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