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Versatrig

SLIDE RULE INSTRUCTIONS

FREDERICK POST COMPANY

PREFACE

The VERSATRIG—offering a lifetime of unquestioned accuracy and versatility to industrial technicians, scientific specialists, businessmen, students, tradesmen, anyone who uses mathematics—is truly a precision calculating instrument. The selection and logical arrangement of its 17 scales guarantees that the Versatrig will not become obsolete. The Versatrig is the only slide rule of its type to have R_1 and R_2 scales for greater accuracy in finding squares and square roots; the only slide rule of its type to have completely consistent color coding of the trigonometric scales in relationship to all the other scales.

Made of stable, self-lubricating laminated bamboo (a select variety available only in the Orient), the Versatrig resists dimensional change from temperature and humidity variations—will always slide smoothly, easily. Oils and powders are never needed to prevent the Versatrig from sticking, because the self-lubricating qualities of bamboo actually improve with use—a feature not found in wood, metal, plastic, or other materials. The scales are machine-divided with permanent graduations deep-cut into flat-white plastic faces. Settings are easily made and read, because of the contrast between the dense graduations and the white face, and because of the clean, uncrowded appearance.

This manual is intended for self-instruction. A mathematical background is not required for effective use of the rule, but a brief explanation of the theories involved is presented in Chapter 7. Examples of the operation of the Versatrig as applied to practical problems are illustrated in Chapter 6. A careful study of each section of this manual is recommended until the operation of the Versatrig is completely mastered.

For practicing engineers and others whose mathematical requirements extend beyond the Versatrig, the famed Versalog is recommended. With 23 scales, it presents the ultimate versatility in a log log type slide rule.

Versalog and Versatrig are registered trade names for slide rules designed by Frederick Post Company. They are unconditionally guaranteed against defects in materials or workmanship.

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W. Fred Blake
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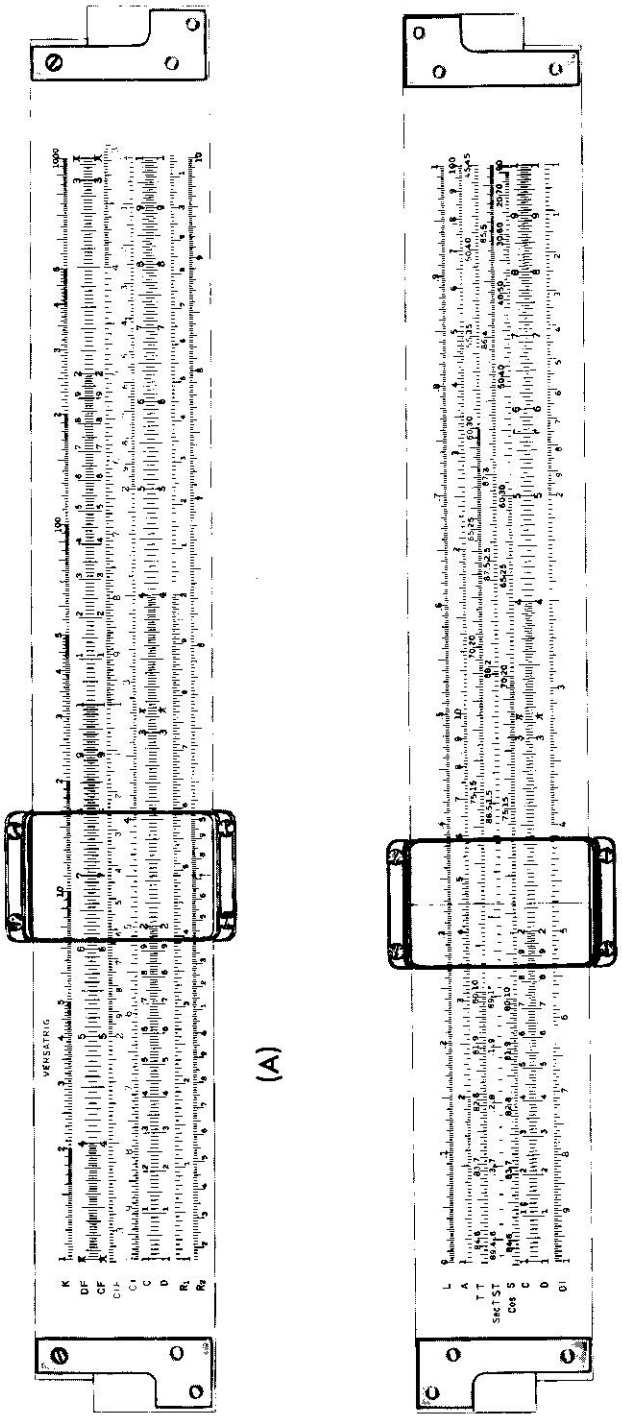
VERSATRIG
1450
SLIDE RULE



(A) (B)

FIGURE 1.1

CHAPTER 1

INTRODUCTION TO THE VERSATRIG SLIDE RULE

The slide rule is an instrument which enables the user to solve simple and involved arithmetical problems with speed and confidence impossible with written computations. Versatrig is a precision slide rule whose combination of features make it the outstanding trig rule.

This chapter covers the important topics of proper adjustment, care and manipulation of the slide rule, a brief description of the scales, and the method of locating numbers and reading the basic scales. Reading the scales should be mastered beforé proceeding to multiplication and division.

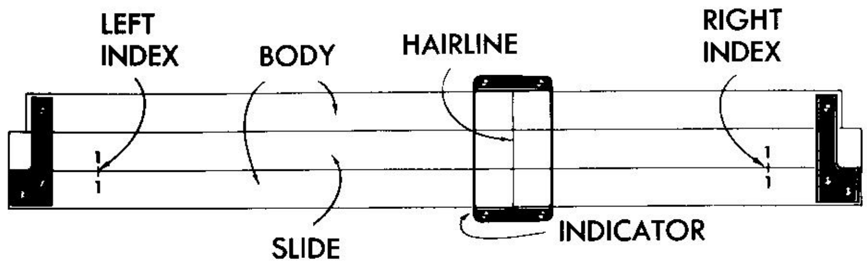


Figure 1.2

1.1 GENERAL DESCRIPTION

The construction of the slide rule is simple: essentially, there are three components; (1) the body, (2) the slide, and (3) the indicator. The body is comprised of the upper and lower fixed members. The slide is situated between these, and slides in grooves in the body. Scales are deep cut on both the body and the slide, on both front and back faces of the rule. The indicator (sometimes called the cursor) is composed of the frame which holds the two glass windows upon which the hairlines are etched. Calculations may be carried out on either side of the slide rule, or on both sides simultaneously.

The body and slide are made of laminated bamboo to which is permanently bonded the white plastic face. This type of construction offers the distinct advantages of insuring against warpage and providing exceptional dimensional stability, so that the rule will be accurate and operate smoothly over a wide range of weather conditions. Since bamboo is self-lubricating, the slide rule's action will improve with use. Each individual graduation is accurately cut into the white plastic face and will remain easy to read for a lifetime.

1.2 ADJUSTMENT

Your Versatrig slide rule should come to you in perfect adjustment. However, in case it is dropped or severely jarred, the precise adjustment may be lost. In any case, it is advisable to check the adjustment occasionally to be sure that the scale readings are as accurate as the instrument will allow.

There are three types of adjustment; scale alignment, hairline alignment, and slide tension.

Scale Alignment. Move the slide so that the C and D scales are in exact alignment. Check the CF and DF scales; they should also align perfectly. If they do not, loosen the screws in both the metal end bars. Check again for correct C and D alignment. Now move the upper body member to the right or left, being careful to hold the slide stationary, until DF and CF scales coincide. Carefully tighten the screws.

Hairline Alignment. Holding the rule with the shorter body member on top, position the hairline over the 1 mark on the extreme left of the D scale. (This is known as the "left index.") The hairline should simultaneously coincide with the π symbol on the extreme left of the DF scale. Imperfect alignment indicates that the hairline is not positioned perpendicular to the scales. Correction may be made as follows.

Loosen the four screws of the metal indicator frame. Slide the glass window as required for perfect alignment with the π of the DF scale and the left index of the D scale. Carefully retighten screws. Note that this adjustment requires perfect scale alignment.

Once the hairline on the front face of the slide rule is aligned, the hairline on the opposite side may be checked. Set the front hairline over the left index of the D scale and turn the slide rule over, being careful not to disturb the position of the indicator. The hairline on this reverse side should align exactly with the left indexes of the A and D scales. If it does not, the second hairline must be adjusted as described above. This adjustment of the hairlines is a delicate operation, and when aligning the second hairline, be careful not to move the previously aligned hairline.

When properly positioned, both hairlines will coincide perfectly, and operations using both faces of the rule may be performed with accuracy.

Slide Tension. If the operation of the slide seems too stiff, the body members may be gripping it too tightly. To correct this without losing the scale alignment, loosen the screw in either one of the metal end bars and pull the upper body member slightly away from the slide. Retighten the screw and repeat the operation at the other end bar. Since this is a trial and error procedure, it may have to be repeated several times to obtain the desired "action" of the slide.

1.3 CARE OF THE VERSATRIG

It is important to keep the slide rule as clean as possible. Having clean hands and keeping the rule in its case when not in use will help. To clean the body and slide, a slightly moist cloth may be used. Keep the running surface of the indicator clean, as dirt may accumulate here and cause an annoying "sticking" of the indicator. Particles of dirt may accumulate under the glass of the indicator. These may be removed by placing a strip of paper the width of the slide rule over the scales. Run the indicator over the paper and press down at the same time. The dirt particles will adhere to the paper.

The Versatrig needs no artificial lubrication; ordinary usage will utilize the bamboo's natural oils, and operation will improve with use.

1.4 MANIPULATION

The following suggestions are offered for manipulation of the slide rule. With experience, the reader may develop his own variations.

Grasp the slide rule between the thumb and forefinger of one hand at the end of the rule. Gripping the body in the center may cause a slight binding of the slide.

In setting the slide, move it to the general neighborhood of the required setting. This will generally cause one end of the slide to project beyond the body of the rule. In making the precise setting, grasp the extended end of the slide with the thumb and forefinger of the hand at that end, and bracing these against the end of the body, move the slide to the exact position required. When neither end of the slide projects very far beyond the body of the rule, the two forefingers can push against both ends of the slide—pushing harder on one end than the other until the exact setting is reached.

In setting the hairline, the indicator is pushed to the neighborhood of the desired location. For precise positioning, place the thumbs of both hands against each end of the indicator frame and push harder with one thumb than the other as required to set the hairline.

1.5 DESCRIPTION OF THE SCALES

There are 17 scales on the Versatrig slide rule; 9 on the front face and 8 on the reverse face. Each scale is designated by a letter or letters at the left end of the rule. The length of the scales is 25 cm. or 9.84 inches. The instrument is generally called a ten inch slide rule, even though the scales are somewhat shorter. Each scale and its function will be briefly described here. More detailed explanations will be found in the following chapters that deal with specific operations of the scales.

The **C** and **D** scales are the most basic and are most used. For convenience they appear on both sides of the slide rule. They are identical in markings and length; the C scale appearing on the slide, and the D scale on the lower body member. The C and D scales are used together for multiplication and division, and with each of the other scales for other computations.

The **CI** scale appears on the slide, and is identical to the C and D scales, except that its graduations and numbers run from the right to left. Those numbers appearing on the CI, or inverted scale, are reciprocals of those numbers directly opposite on the C scale. The CI scale is used with the D scale to provide efficient multiplication and division and with the other scales for various computations.

The **CF** and **DF** scales are folded scales. Their numbers and graduations are identical to those of the C and D scales, except that they begin and end with π. This arrangement results with the number 1 approximately at the center of the rule. The convenience of this arrangement for rapid work and for certain types of calculations is explained in Chapters 2, 3, and 4.

The **CIF** scale is an inverted CF scale (or a folded CI scale). It is identical to the CF scale, except that it is graduated and numbered from right to left, and numbers on the CIF scale are reciprocals of those directly opposite on the CF scale. It is used primarily for multiplication and division.

The **DI** scale is an inverted D scale, identical to the CI scale, but located on the body. It is used to perform some operations similar to the CI scale, and simplifies other trigonometric computations.

The **R_1** and **R_2** scales are primarily used to find squares and square roots of numbers, and are used with the D scale. Note that these two R scales actually constitute the halves of one long R, or root, scale. This length (nearly 20 inches) enables greater accuracy in reading squares and square roots.

The **A** scale is another scale for obtaining squares and square roots. It is a two section scale with a range from 1 to 100, and is used with the D scale. When the increased accuracy provided by the R_1 and R_2 scales is not required, the use of the A scale permits short cuts that speed computations.

The **K** scale is used for finding cubes and cube roots when used with the D scale. It is a three section scale with a range from 1 to 1,000.

The **L** scale, the reader will note, is the only scale that is uniformly divided. The reason for this may be found in the last chapter, which explains the principle upon which the slide rule works. The mantissa of the common logarithm (log to the base 10) is read on the L scale in conjunction with the D scale. The characteristic of the logarithm must be determined by the operator, as explained in Chapter 4.

The **Cos S** scale is used to find the values of the sine and cosine functions of angles. It is divided in degrees and decimals of degrees. For sines, the scale is graduated from left to right and ranges from 5.74 degrees to 90 degrees. For cosines, the scale is graduated from right to left and ranges from zero to 84.3 degrees. Sine and cosine functions of angles on the Cos and S scales (numbered in black) are found on the C scale (which is also numbered in black).

The **T** scale is used to obtain the values of the tangent function of angles from 5.71 to 84.3 degrees. Angles from 5.71 to 45 degrees are numbered from left to right and are numbered in black. Angles from 45 to 84.3 degrees are numbered from right to left on the T scale and are numbered in red. Functions of angles numbered in black are found on the C scale (which is also numbered in black); functions of angles numbered in red are found on the CI scale (which is also numbered in red).

The **Sec T ST** scale is provided for finding the tangent function of small angles varying from 0.57 to 5.73 degrees. It is graduated from left to right in this range, numbered in black, and is used with the C scale. It is also used for finding the sine function of small angles since the sine and tangent functions are nearly equal in this range. For large angles, the scale is graduated from right to left, numbered in red, and is used with the CI scale (numbered in red) to find tangent and secant functions. In this range, 84.27 to 89.43 degrees, the tangent and secant are nearly equal.

1.6 READING THE SCALES

The reading of the C and D scales is explained in detail. Applying these principles and noting the range and the direction of graduations, the reading of the other scales should be obvious.

The C and D scales are divided into 10 major divisions, numbered from 1 on the left (the left index) to the 1 mark (or 10) on the right (the right index). Since the slide rule is based on logarithms, the spaces between graduations are not uniform. All of the distances between the graduations are proportional to the differences of logarithms of the numbers represented. Figure 1.3 shows these major divisions.

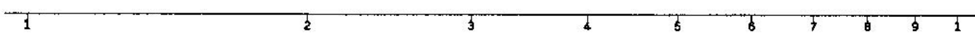


Figure 1.3 Major Divisions of the C and D Scales

The first digit (that is not zero) of a number is found in the major division. For example, 1745 is found in major division 1, which is between the major graduations numbered 1 and 2. Between the major graduations, there are ten secondary divisions, which indicate the location of the second digit of a number. In the first major division, these secondary divisions are also numbered.

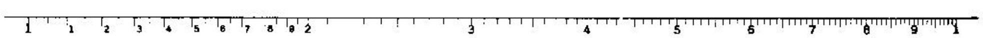


Figure 1.4 Major & Secondary Divisions of the C and D Scales

In the example 1745, the second digit, 7, is found in the secondary division 7. As space permits, the secondary divisions are divided into 10, 5, or 2 tertiary divisions, which indicate the position of the third digit of a number. Thus, the third digit, 4, of 1745 is found in the 4th tertiary division, between the secondary graduations 7 and 8. Between the major graduations 1 and 2, the tertiary divisions are large enough to allow accurate readings to the nearest tenth of this length. Settings to four significant digits can therefore be made in this portion of the scale. The fourth digit, 5, of 1745 is estimated between the fourth and fifth tertiary graduations. Figure 1.5 shows the steps in locating 1745 on the C or D scale.

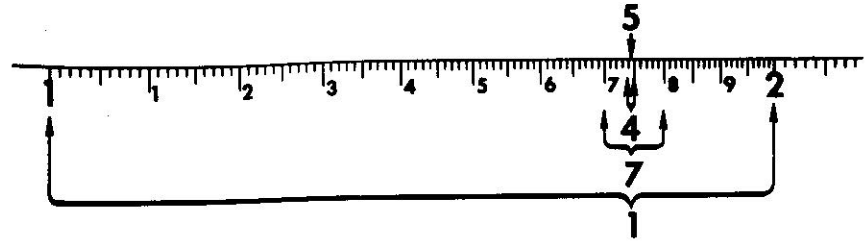


Figure 1.5 Steps in Locating 1745 on the C or D Scale

The location of the decimal point has no effect on the location of the number on the scale. All numbers that have the same digits (e.g. 274, 2.74, 27,400, and 0.00274) are located at the same point on the D scale regardless of the location of the decimal point. The examples in the following chapters assume that the decimal point is retained with the digits, even though it has no bearing on the location of the number on the scale. For example, when locating 2.74 on the scale, it is thought of as 2.74 and not as 274. This facilitates the placement of the decimal point in the answer. Several approaches to the decimal point placement are discussed in Chapter 2.

Figure 1.6 shows several examples of numbers on the C or D scales. After reviewing these, practice locating the same numbers on the CI scale, and perform the exercises that follow.

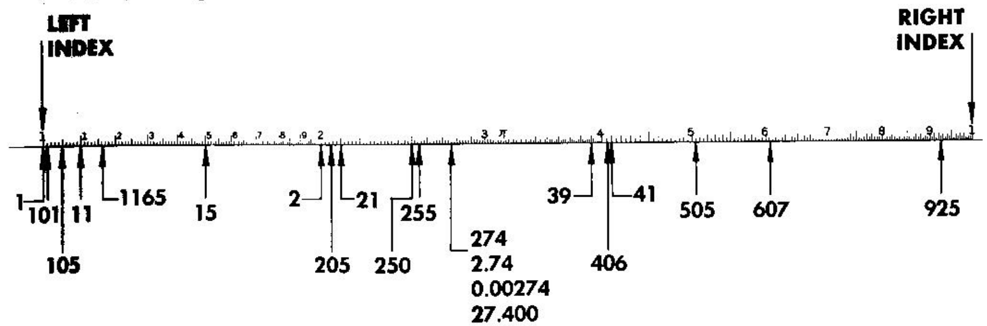
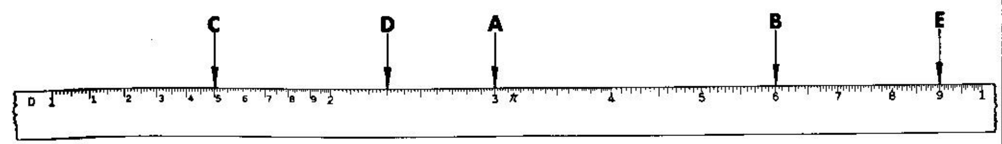
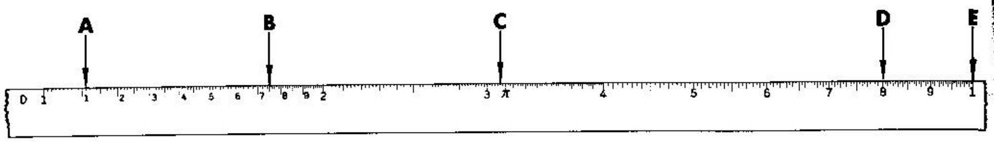


Figure 1.6 Examples in Reading the C and D Scales

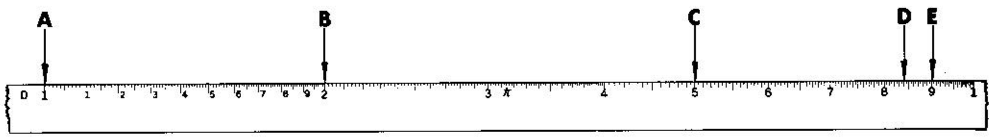
Exercises in Reading the Scale



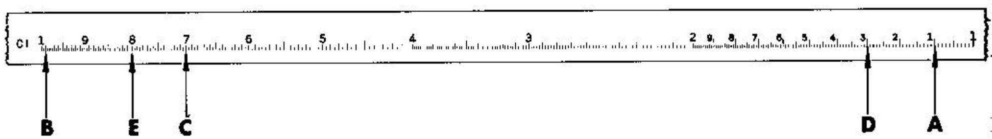
1. What are these values on the C or D scales? They range from 1 to 10. (Answers are located in the back of the book.)



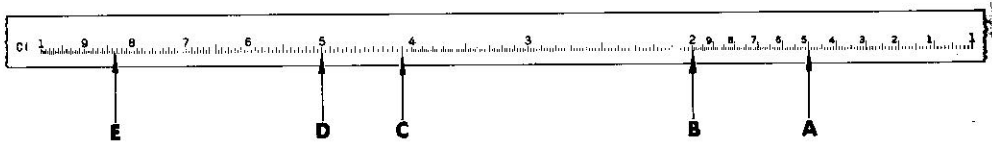
2. What are these values on the C or D scales? They range from 10 to 100.



3. What are these values on the C or D scales? They range from .01 to .1.



4. What are these values on the CI scale? They range from 1 to 10.



5. What are these values on the CI scale? They range from 100 to 1,000.

1.7 ACCURACY

The graduations on the Versatrig are highly accurate, but the accuracy of the slide rule is limited to the ability of the user to see, set, and read the desired numbers. Settings as accurate as four significant digits can be made for numbers having 1 as the first digit on the C or D scale. For other numbers, the scales can essentially only be read to an accuracy of three significant digits. Since the entire scale must be used, the accuracy as a whole, is limited to three digits, or 99.9%. Such accuracy is all that is required of ordinary design calculations, since it is unusual that all factors in a computation are so accurate that no factor contains an error larger than one part in one thousand.

CHAPTER 2

MULTIPLICATION AND DIVISION SIMPLE OPERATIONS

Since the slide rule is generally used far more for multiplication and division than for other computations, the Versatrig provides a selection of scales for optimum speed in any multiplication or division operation or series of computations. A familiarity of the alternates available can save time and steps in simple everyday computations, and a thorough understanding of the proper use of the scales is essential for efficient handling of sequences or series of computations.

This chapter and Chapter 3 present the scope of applications of the Versatrig to multiplication and division problems. The most efficient use of the rule is emphasized.

2.1 MULTIPLICATION: BASIC METHODS

The D and C Scales. The D and C scales are the most fundamental of the slide rule and are common to all types of rules, both simple and complex. Any multiplication or division operation may be performed solely through the use of the scales, though their exclusive use will not be the most efficient.

The following examples illustrate multiplication using the D and C scales.

Example 2.1

Problem: $2 \times 3 = 6$

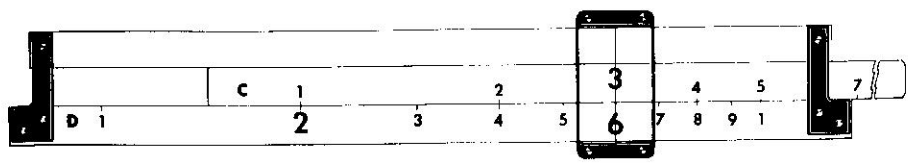


Figure 2.1

Operation: Set hairline to 2 on D.
Move left index of C to hairline.
(i.e., opposite 2 on D).
Move hairline to 3 on C.
Read answer, 6, on D at hairline.

The first step above is optional. Some people find it a convenience. The following examples omit this step.

Example 2.2

Problem: $9 \times 4 = 36$

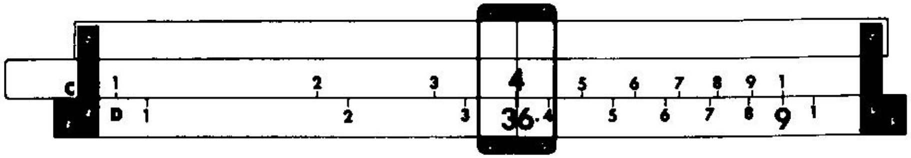


Figure 2.2

Operation: Set right index of C to 9 on D.
Move hairline to 4 on C.
Read product, 36, on D at hairline.

In this case, the right index must be used. Had the left index of the C scale been used, 4 on the C scale would project beyond the D scale and no answer could be obtained. Thus, the right index must obviously be used in this case. After a little practice, the selection of the proper index will become second nature.

Example 2.3

Problem: $1.5 \times 5.7 = 8.55$

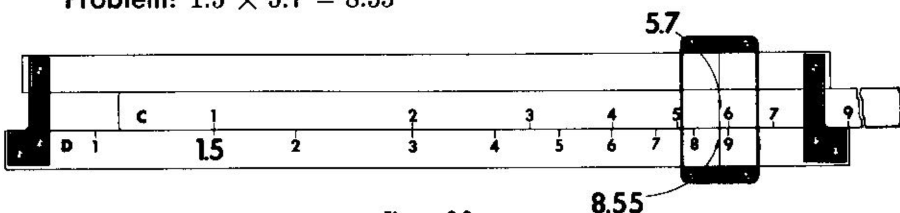


Figure 2.3

Operation: Set left index of C to 1.5 on D.
Move hairline to 5.7 on C.
Read 8.55 on D at hairline.

This computation requires a mental operation to locate the decimal point in the solution. This is true for any operation as the slide rule does not provide for the decimal point location. The topic of decimal point placement will be discussed later in this chapter.

From the examples above, a general procedure for multiplying using the D and C scales can be derived.

1. Set the hairline to one of the numbers to be multiplied on D.
2. Move the slide until an index of C coincides with the hairline.

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3. Move the hairline to the other number on C.
4. Read the product on D at the hairline.
5. If the slide extends beyond the body, to an extent that steps 3 and 4 cannot be performed, use the other index of C in step 2.

Since the D and C scales are adjacent, it is possible to multiply without the use of the indicator. Except in step 1, which may be omitted, the beginner is urged to make use of the indicator for it will save time and eliminate errors in reading the scale—particularly when using 3 and 4 digit numbers.

In multiplying numbers such as 4×9, the computation can be made by setting the right index of C at 4, moving the hairline to 9 on C, and reading 36 on D. However, considerable slide rule movement (and therefore time and effort) can be saved by mentally reversing the problem to read 9×4 and solving as in Example 2.2.

The D and CI Scales. Multiplication using the D and CI scales is often more efficient than using the D and C scales. It is often preferred because it is not necessary to determine which index of the C scale to use. Both this method and the preceding method should be used interchangeably, selecting the one most efficient for the particular problem. Remember that the values on the CI scale increase toward the left, while the values on the C scale increase toward the right. The procedure for multiplying using the D and CI scales follows:

1. Set the hairline to one of the numbers to be multiplied on D.
2. Move the slide until the other number on the CI scale coincides with the hairline.
3. Read the product of the two numbers on D at the index of C, (whichever index is adjacent to the D scale).

Example 2.4

Problem: $2 \times 3 = 6$

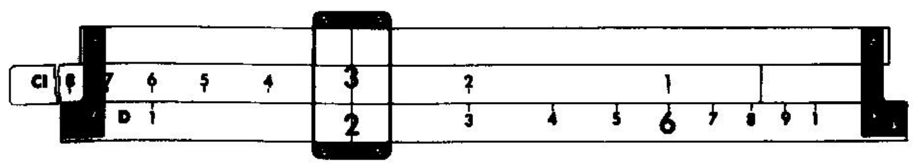


Figure 2.4

Operation: Set hairline at 2 on D.
Move 3 on CI to hairline.
Read 6 on D at right index of C.

Example 2.5

Problem: $52 \times 4 = 208$

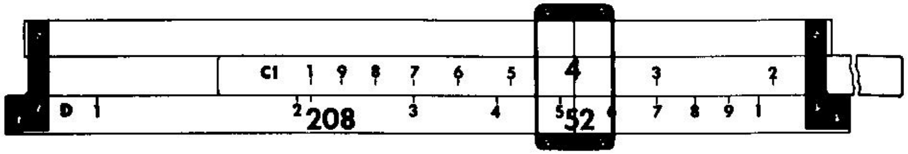


Figure 2.5

Operation: Set hairline to 52 on D.
Move 4 on CI to hairline.
Read 208 on D at left index of C.

Example 2.6

Problem: $64 \times .3 = 19.2$

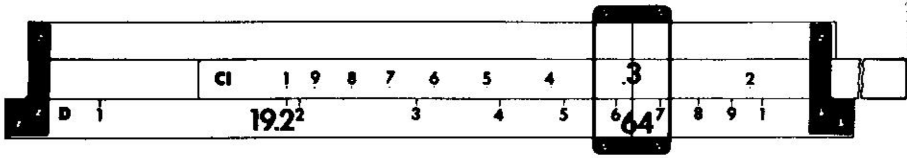


Figure 2.6

Operation: Set hairline to 64 on D.
Move .3 on CI to hairline.
Read 19.2 on D at index of C.

In this example, the solution must be less than 64 (since .3 is less than 1) and must be more than 6.4 (since .3 is more than .1). Therefore, the product of 64 and .3 must be 19.2.

Choice of the D and C or the D and CI Scale Combinations. In general, choose the scale combination that would require the least slide movement as this is the most efficient scale combination. For some computations, the choice of scale combinations makes little difference in efficiency, while for others, there is a decided difference. Whenever one scale combination requires moving the slide more than one-half its length, use another combination.

Exercise in Multiplication. In performing the following exercises, indicate the most advantageous scale combination. A rough mental approximation will serve to locate decimal points. No attempt should be made to read results more accurately than the instrument allows.

The accuracy is limited to four significant figures for numbers whose first non-zero digit is 1, but to only three significant digits for other numbers.

6. 8×3
7. 12×7
8. $.12 \times 70{,}000$
9. 1.37×27
10. 812×8.02
11. 7.92×6.4
12. 4×3
13. 29×19.4
14. 7.5×14.6
15. $.95 \times 1{,}074$
16. 572×1.452
17. 4.48×46.6

2.2 DIVISION: BASIC METHODS

The D and C Scales. The procedure for division using the D and C scales is as follows:

1. Set the hairline to the numerator on D.
2. Align the denominator on C with the hairline.
3. Read the quotient on D opposite the index of C.

Note that the procedure is the same as for multiplication using the D and CI scales, except when dividing, the D and C scales are used and the numerator is located on the D scale. Three examples follow.

Example 2.7

Problem: $\frac{6}{3} = 2$

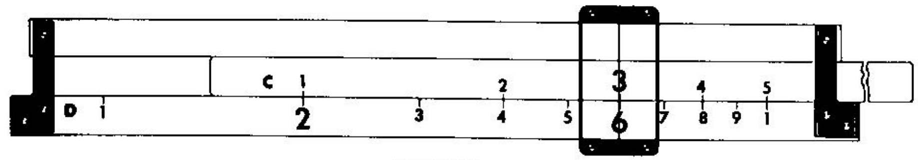


Figure 2.7

Operation: Set hairline to 6 on D.
Move 3 on C to hairline.
Read quotient, 2, on D at left index of C.

Example 2.8

Problem: $\frac{342}{6} = 57$

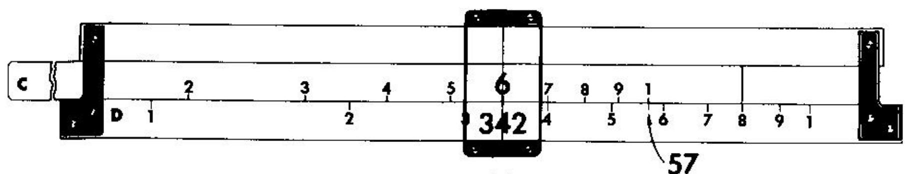


Figure 2.8

Operation: Set hairline to 342 on D.
Move 6 on C to hairline.
Read answer, 57, on D at right index of C.

Example 2.9

Problem: $\frac{9.56}{1.12} = 8.54$

Operation: Set hairline to 9.56 on D.
Move 1.12 on C to hairline.
Read quotient, 8.54, on D at left index of C.

The operation in Example 2.9 is grossly inefficient, because of the slide movement required. Using other scales, rather than the D and C scales for this and similar problems is preferable. Therefore, after mastering the use of the D and C scales for division, the use of the remaining scales should be studied.

The DF and CF Scales. These scales each have only one index (1 mark). Note that for any position of the slide, the reading on D, opposite the index of C, is identical to the reading on DF, opposite the index of CF. Except for the placement of the graduations, these scales are the same as the D and C scales. Instead of beginning and ending with 1, they begin and end with 3.14, π. Or, you might say that the D and C scales range from 1 to 10, while the DF and CF scales range from 3.14 to 31.4. In both multiplication and division, the DF scale is used like the D scale, and the CF scale is used like the C scale. The procedure for division using the DF and CF scales is the same as when using the D and C scales except the DF and CF scales are used in steps 1 and 2, as illustrated in examples 2.10, 2.11, and 2.12.

Example 2.10

Problem: $\frac{12}{6} = 2$

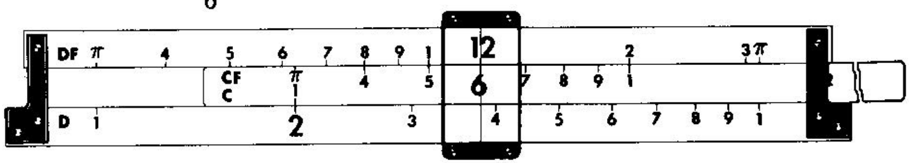


Figure 2.9

Operation: Set hairline to 12 on DF.
Move 6 on CF to hairline.
Read answer, 2, on D at index of C.

The answer can also be read on the DF scale opposite the CF index. However, developing the habit of looking for the answer in the same place (the D scale opposite the index of C) when using either the D and C or the DF and CF scale combinations simplifies the operation. Thus, even if the DF and CF combination is chosen when it is less efficient than the D and C combination and the CF index is not opposite the DF scale, the quotient can be read on the D scale without duplicating the operation on the D and C scales.

Example 2.11

Problem: $\frac{440}{5.5} = 80$

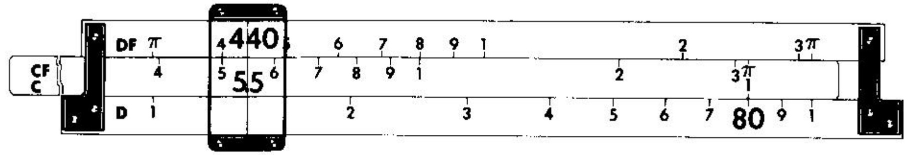


Figure 2.10

Operation: Set hairline to 440 on DF.
Move 5.5 on CF to hairline.
Read 80 on D at index of C.
(A quick mental calculation,
$\frac{500}{5} = 100$, locates the decimal point.)

Example 2.12

Problem: $\frac{9.56}{1.12} = 8.54$

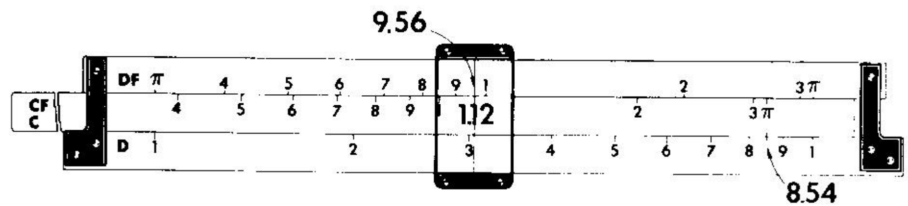


Figure 2.11

Operation: Set hairline to 9.56 on DF.
Move 1.12 on CF to hairline.
Read 8.54 on D at index of C.

This is the same problem as Example 2.9, but is performed far more efficiently using the DF and CF scales.

Choice of the D and C or DF and CF Scale Combinations. Again, choose the combination that requires the least slide movement. For some computations, there is no difference in efficiency, but for others, there is considerable difference. If the numerator and denominator are close to opposite ends of the D scale, choose the folded scale combination. It should never be necessary to move the slide more than half its length.

Exercise in Division. Perform the following operation and indicate the most advantageous scale combination.

18. 9.3 ÷ 3.08
19. 8.55 ÷ 2.96
20. 7.48 ÷ 2.63
21. 6.3 ÷ 0.27
22. 450 ÷ 19.2
23. 1950 ÷ 435
24. 9.3 ÷ 2.18
25. 8.55 ÷ 10.5
26. 7.48 ÷ 115
27. 6.3 ÷ 14.2
28. 450 ÷ 10.4
29. 1950 ÷ 94.5
30. 9.3 ÷ 6.5
31. 8.55 ÷ 5.12
32. 7.48 ÷ 3.54
33. 6.3 ÷ 7.5
34. 450 ÷ 57.2
35. 1950 ÷ 10.6

2.3 MULTIPLICATION: ALTERNATE METHODS

The DF and CF Scales. Multiplication can be performed strictly by using the DF and CF scales, but their more practical application is in conjunction with the D and C scales. Since every multiplication and division computation can be performed without moving the slide more than halfway out of the body of the rule, every value on a C scale (either C or CF) is opposite a value on a D scale (D or DF).

Multiplication using the DF and CF scales in conjunction with the D and C scales is illustrated in Examples 2.13 and 2.14.

Example 2.13
Problem: $21 \times 5 = 105$

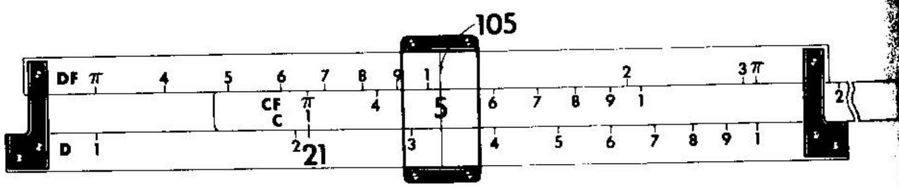


Figure 2.12

Operation: Set left index of C to 21 on D.
Move hairline to 5 on CF.
Read 105 on DF at hairline.

Example 2.14
Problem: $4.17 \times 2 = 8.34$

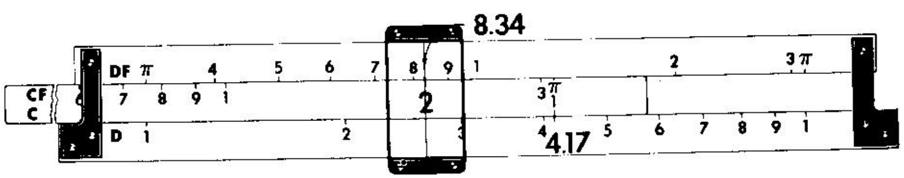


Figure 2.13

Operation: Set right index of C to 4.17 on D.
Move hairline to 2 on CF.
Read 8.34 on DF at hairline.

Multiplication using the DF and CF scales exclusively is demonstrated in the following example.

Example 2.15
Problem: $3 \times 6 = 18$

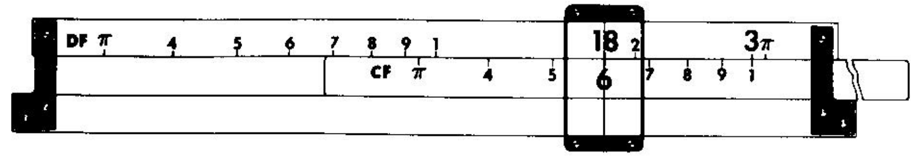


Figure 2.14

Operation: Set index of CF opposite 3 on DF.
Move hairline to 6 on CF.
Read 18 on DF at hairline.

The DF and CIF Scales. Multiplication using these folded scales is identical to using the D and CI scales, except the numbers to be multiplied together are located on the DF and CIF scales. The product can be read on the DF scale at the index of the CIF scale, but it is generally preferable to read the answer on the D scale, opposite an index of the C scale.

Example 2.16
Problem: $3 \times 6 = 18$

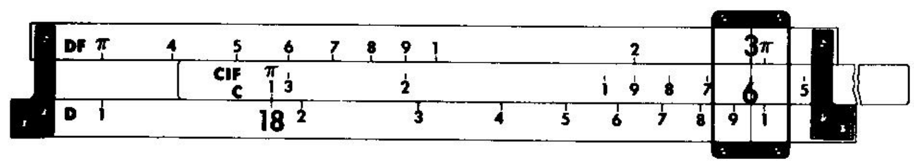


Figure 2.15

Operation: Set hairline at 3 on DF.
Move 6 on CIF to hairline.
Read 18 on D at left index of C.

The preceding multiplication exercises (6 to 17) can be worked again, using the DF and CF or the DF and CIF scale combinations, whichever is more efficient for each calculation.

2.4 DIVISION: ALTERNATE METHODS

The D and CI Scales. To divide, using the D and CI scales, the following procedure is used.

1. Set the hairline to the numerator on D (This step is optional since the scales are adjacent).
2. Move the slide until an index of CI coincides with the hairline.
3. Move the hairline to the denominator on the CI scale.
4. Read the quotient on D at the hairline.
5. If the slide extends beyond the body, to an extent that steps 3 and 4 cannot be performed, use the other index of CI in step 2.

Notice that the procedure is the same as used for multiplication using the D and C scales. Since the CI scale is a reciprocal scale, this method is, in effect, division by multiplication of the numerator by the reciprocal of the denominator. Two illustrations follow.

Example 2.17

Problem: $\frac{15}{3} = 5$ $\qquad (15 \times \frac{1}{3} = 5)$

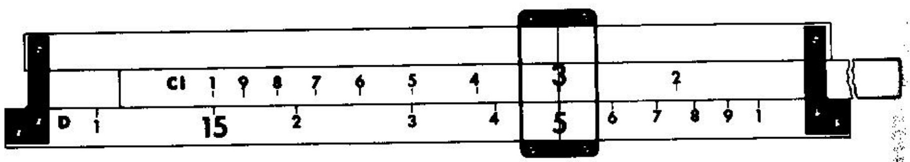


Figure 2.16

Operation: Set hairline to 15 on D.
Move left index of CI to hairline.
Move hairline to 3 on CI.
Read quotient, 5, on D at hairline.

Example 2.18

Problem: $\frac{3.94}{19.05} = .207$

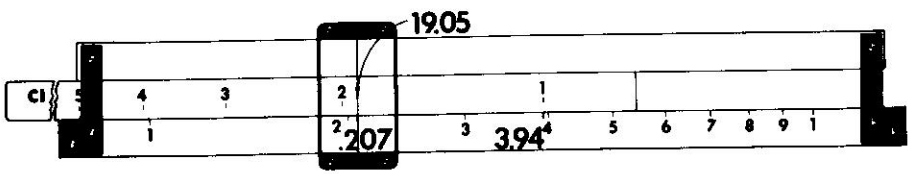


Figure 2.17

Operation: Set hairline to 3.94 on D.
Move right index of CI to hairline.
Move hairline to 19.05 on CI.
Read .207 on D at hairline.

The DF and CIF Scales. Some division computations can be performed with these scales alone, but their practical value comes from supporting the D and CI scales in division. The procedure is the same as described for the D and CI scales, except the CIF and DF scales are used in steps 3 and 4 respectively instead of the CI and D scales.

Example 2.19

Problem: $\frac{54}{7.5} = 7.2$

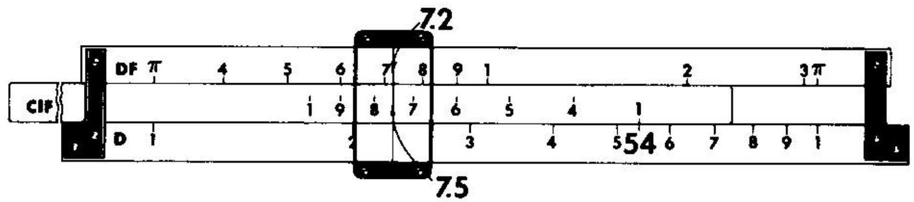


Figure 2.18

Operation: Set right index of CI (or C) to 54 on D.
Move hairline to 7.5 on CIF.
Read 7.2 on DF at hairline.

The preceding division exercises (18 to 35) can be worked again, using the D, CI, DF, and CIF scales.

2.5 DECIMAL POINT LOCATION

Thus far, the topic of the location of the decimal point has been just briefly mentioned. Since, as has been noted before, the slide rule has no provision for "carrying along" the decimal point in a given problem, some method must be adopted. Three methods are suggested. In all probability, all will be used at one time or another, depending on the complexity of the particular problem. Therefore, all three methods should be thoroughly understood.

The Common Sense Approach. For many problems, the combination of factors is simple enough that by inspection, the location of the decimal point is obvious. For instance, in the computation 9.2 ÷ 2, the slide rule reads 46, which certainly must be 4.6. Even for othe problems, which may be comprised of many factors and be more complicated mathematically, the result may be only reasonably interpreted in one way. For example, if the result of a calculation of the speed of an automobile in miles per hour was the digits 234, the correct speed would reasonably be 23.4 mph, not 234. or 2.34 mph.

The Approximation Approach. This method covers a greater range of problems. Essentially, it requires an estimate using rounded numbers. The product of 137.3 and 41.2 will read 566 on the slide rule. Since this is similar to 100×50, or 5,000, the answer is correctly interpreted as 5,660.

For more involved calculations, it may be more convenient to jot down the rounded numbers and cancel, as:

$$\frac{96 \times 55 \times 63 \times 57}{46 \times 2.7 \times 10{,}320 \times 688} \approx \frac{\cancel{100} \times \overset{2}{\cancel{60}} \times \cancel{60} \times \cancel{60}}{50 \times \cancel{3} \times 10{,}\cancel{000} \times 7\cancel{00}} = \frac{72}{3{,}500} = \frac{7.2}{350}$$

Since 7.2 is divided by a number greater than 100 but less than 1,000, the result is a number less than 0.072 but greater than 0.0072. In other words,

$$\frac{7.2}{350} > \frac{7.2}{1{,}000} \text{ and } \frac{7.2}{350} < \frac{7.2}{100,} \text{ then } \frac{7.2}{350} > .0072 \text{ and } \frac{7.2}{350} < .072$$

Using the original figures, the slide rule gives us three significant digits, 215, so the result must be .0215. See Chapter 3 for the solution of this type of problem.

The Standard Form Approach. This is the most exact method and is recommended for problems too complicated to be easily handled through either of the preceding methods. It is particularly helpful when dealing with numbers of very large or very small magnitude.

Placing a number in its standard form entails placing the decimal point after the first non-zero digit of the number and indicating the true location of the decimal point by multiplying it by the appropriate power-of-ten. The magnitude of the appropriate power-of-ten is determined by the number of digits that the decimal point was moved to place it after the first non-zero digit of the number. If the number is larger than 10, the decimal point is moved to the *left*, and the power-of-ten is *positive*. If the number is larger than 1, but less than 10, the decimal point is not relocated, and the power-of-ten is therefore zero, ($10^0 = 1$). If the number is smaller than 1, the decimal point is moved to the *right*, and the power-of-ten is *negative*.

The following examples are presented for clarification.

	NUMBER		STANDARD FORM
(a)	100	=	1×10^2
(b)	735	=	7.35×10^2
(c)	4,360,000	=	4.36×10^6
(d)	.0001354	=	1.354×10^{-4}
(e)	.0862	=	8.62×10^{-2}
(f)	26.9	=	2.69×10^1
(g)	.326	=	3.26×10^{-1}
(h)	7.1	=	7.1×10^0

When multiplying numbers in their standard form, the exponents are added; when dividing, they are subtracted. Thus, $7{,}000 \times .0006 = (7 \times 10^3)(6 \times 10^{-4}) = 42 \times 10^{-1} = 4.2$. Also note that $\frac{1}{10^{-3}} = 10^3$. Using this method with the approximation approach makes the slide rule solution of the following problem quick and error-free.

$$\frac{26 \times 79{,}800 \times .00633}{.0081 \times 7{,}800{,}000} = \frac{(2.6 \times 10^1)(7.98 \times 10^4)(6.33 \times 10^{-3})}{(8.1 \times 10^{-3})(7.8 \times 10^6)}$$

$$\approx \frac{(3)(\cancel{8})(6) \times 10^2}{(8)(\cancel{8}) \times 10^3}$$

$$\approx \frac{18}{8} \times 10^{-1} \text{ or } .225$$

While the simple and direct slide rule computation for this type of problem has not yet been explained, the solution would read 208, which is correctly interpreted as .208. Chapter 3 shows the solution to this computation with just two settings of the slide.

2.6 MULTIPLICATION AND DIVISION: SUMMARY

There are four pairs of scales generally used for multiplication and division; they are D and C, D and CI, DF and CF, and DF and CIF. There are two procedures of using these scales: 1) aligning the two factors and reading the answer opposite an index, and 2) aligning one factor and an index, and reading the answer opposite the other factor. Following the rule that numerators and answers are read on a D scale (D or DF), the four pairs of scales and two procedures of using them result in eight combinations; four yield multiplication and four yield division. These eight combinations are tabled below.

	Basic Methods		Alternate Methods	
	Scales	Procedure	Scales	Procedure
Multiplication	D & CI	1	DF & CIF	1
Multiplication	D & C	2	DF & CF	2
Division	D & C	1	D & CI	2
Division	DF & CF	1	DF & CIF	2

The object in perfecting more than one method for multiplying and one for dividing is to greatly increase the potential speed and efficiency. For both multiplication and division, two basic procedures should be mastered and used interchangeably to limit slide movement to half its length and so that no more than half of the slide projects beyond the body scales. Two alternate procedures should be learned for rapid handling of sequences and combinations of computations with the minimum number of settings.

The procedures recommended here are efficient, easily learned by beginners, and preferred by many teachers of the slide rule. However, some individuals prefer procedure 1 with the D and CI, and the DF and CIF scales as the basic methods for multiplication; while others prefer procedure 2 with the D and C and DF and CF scales. The important thing is that *all* of the procedures are understood and are used when required, and that the procedures selected as the basic ones are used efficiently and without error.

CHAPTER 3

MULTIPLICATION AND DIVISION COMPOUND OPERATIONS

A great advantage of the slide rule is that a number of calculations can be performed in one continuous operation. It is not necessary to record the answers to intermediate steps of a compound problem. A complete understanding of Chapter 2, however, is required.

3.1 COMBINED OPERATIONS

Multiplying or Dividing a Series of Numbers. Using two procedures for multiplication and two pairs of scales with their folded counterparts, a series of factors can be multiplied together with minimum time and effort. In the following examples, notice that the products of the intermediate calculations are available, but can be disregarded.

Example 3.1

Problem: $4.7 \times 5.24 \times 10.12 = 249$

Operation: Set hairline to 4.7 on D.
Move 5.24 on CI to hairline.
Move hairline to 10.12 on C.
Read 249 on D at hairline.

Example 3.2

Problem: $3.25 \times 4.28 \times 9.13 = 127$

Operation: Set hairline to 3.25 on D.
Move 4.28 on CI to hairline.
Move hairline to 9.13 on CF.
Read 127 on DF at hairline.

On th' slide rule, division is just as easy as multiplication. As previously mentioned, division of a number only requires multiplication by the reciprocal of the denominator, and thus the procedure of dividing a series of numbers is evident. Again, full use must be made of the various methods of division for efficient operation, as demonstrated in the following example.

Example 3.3

Problem: $\dfrac{1}{3.25 \times 4.28 \times 6.13} = .01173$

Operation: Set 3.25 on C to right index of D.
Move hairline to 4.28 on CI.
Move 6.13 on C to hairline.
Read .01173 on D at left index of C.

Multiplying and Dividing a Series of Numbers. The process of combining multiplication and division in a series of computations is as simple as combining the operations just illustrated. In the example that follows, two sequences of operations are described.

Example 3.4

Problem: $\dfrac{2.5 \times 5.85 \times 16.4}{4.35 \times 13.9 \times 3.36} = 1.18$

Operation (A): Set hairline to 2.5 on D.
Move 5.85 on CI to hairline.
Move hairline to 16.4 on C.
Move 4.35 on C to hairline.
Move hairline to 13.9 on CI.
Move 3.36 on C to hairline.
Read 1.18 on D at left index of C.

Operation (B): Set hairline to 2.5 on D.
Move 4.35 on C to hairline.
Move hairline to 5.85 on C.
Move 13.9 on C to hairline.
Move hairline to 16.4 on C.
Move 3.36 on C to hairline.
Read 1.18 on D at left index of C.

While there is no difference in the efficiency of the two sequences of operations, the first sequence, Operation A, is recommended. It is believed that fewer errors result by first using all factors in the numerator, then next using all factors in the denominator. In this way, one first concentrates on continuous multiplication, then on continuous division without alternating from one process to the other. Therefore, the first method shown for solving the last example is followed in the next examples.

Example 3.5

Problem: $\dfrac{120 \times 8.25 \times 19.1 \times 9.6}{40.5 \times 3.24 \times 50.4 \times 25} = 1.098$

Operation: Set hairline to 120 on D.
Move 8.25 on CI to hairline.
Move hairline to 19.1 on C.
Move 9.6 on CI to hairline.
Move hairline to 40.5 on CI.
Move 3.24 on C to hairline.
Move hairline to 50.4 on CI.
Move 25 on C to hairline.
Read 1.098 on D at left index of C.

Example 3.6

Problem: $\dfrac{26 \times 79{,}800 \times .00633}{.0081 \times 7{,}800{,}000} = .208$

Operation: Set hairline to 26 on D.
Move 79,800 on CI to hairline.
Move hairline to .00633 on CF.
Move .0081 on CF to hairline.
Move hairline to 7,800,000 on CI.
Read .208 on D at hairline.

Example 3.7

Problem: $\dfrac{100 \times (60.5)^3}{48 \times 3(10)^4 \times 655} = .0235$

Operation: Set hairline to 60.5 on DF.
Move 60.5 on CIF to hairline.
Move hairline to 60.5 on C.
Move 48 on C to hairline.
Move hairline to 300 on CI.
Move 655 on C to hairline.
Read .0235 on D at right index of C.

Example 3.8

Problem: $\dfrac{1.8 \times .625}{25.2 \times 97.3 \times .0076} = .0604$

Operation: Set hairline to 1.8 on D.
Move .625 on CI to hairline.
Move hairline to 25.2 on CI.
Move 97.3 on C to hairline.
Move hairline to .0076 on CIF.
Read .0604 on DF at hairline.

The DI Scale. The DI scale is identical to the CI scale, but located on the body rather than on the slide. It shows reciprocals of values directly opposite on the D scale. Adding the use of the DI scale to the other six scales used in combined multiplication and division calculations adds to the versatility of the Versatrig.

Example 3.9

Problem: $\dfrac{\frac{1}{18} \times 179}{7{,}250 \times 0.13} = .01055$

Operation: Set hairline to 18 on DI.
Move 179 on CI to hairline.
Move hairline to 7,250 on CI.
Move 0.13 on C to hairline.
Read .01055 on D at left index of C.

The problem shown in Example 3.3 can also be solved by multiplying the three terms in the denominator together and finding the reciprocal of this figure, on the DI scale, as shown in Example 3.10.

Example 3.10

Problem: $\dfrac{1}{3.25 \times 4.28 \times 6.13} = .01173$

Operation: Set hairline to 3.25 on D.
Move 4.28 on CI to hairline.
Move hairline to 6.13 on C.
Read .01173 on DI at hairline.

While the operations in Example 3.3 and 3.10 are equally efficient, the method followed in Example 3.3 is preferred. Some combinations of numbers require an additional step when following the method shown in Example 3.10. To verify this, change the last term in the denominator in the problem above from 6.13 to 9.13 and solve using both methods. However, the use of the DI scale in compound multiplication and division operations can occasionally save a step, or enable quick solution of a problem in the form in which it is encountered.

Multiplying and Dividing by π. Since the folded scales (DF, CF, and CIF) are folded at π, the value π on the three scales is opposite the indexes of the corresponding D, C, and CI scales. Multiplication and division, therefore, requires only a setting of the hairline.

To multiply by π, find the number on D and read π times that number on DF. To divide by π, find the numerator on DF and read the quotient on D. The C and CF, and the CI and CIF scales can be used in a similar manner to multiply and divide by π.

Example 3.11

Problem: $18\pi = 56.5$

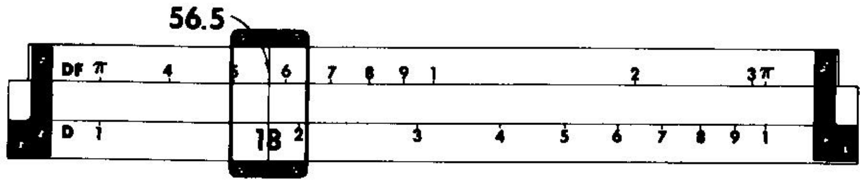


Figure 3.1

Operation: Set hairline to 18 on D.
Read 56.5 on DF at hairline.

Example 3.12

Problem: $\dfrac{13}{\pi} = 4.14$

Operation: Set hairline to 13 on DF.
Read 4.14 on D at hairline.

Example 3.13

Problem: $\dfrac{5.9 \times 2.2\,\pi}{25} = 1.63$

Operation: Set hairline to 5.9 on D.
Move 2.2 on CI to hairline.
Move hairline to 25 on CI.
Read 1.63 on DF at hairline.

Example 3.14

Problem: $\dfrac{21.2 \times 7.7}{8\,\pi} = 6.5$

Operation: Set hairline to 21.2 on D.
Move 7.7 on CI to hairline.
Move hairline to 8 on CIF.
Read 6.5 on D at hairline.

Exercises in Combined Operations.

36. $12.1 \times 2.36 \times 4.25$

37. $5.72 \times 6.25 \times 7.13$

38. $7.48 \times 802 \times 920$

39. $\dfrac{1}{1.04 \times 1.71 \times 9.25}$

40. $\dfrac{18.6}{4.1 \times 3.64 \times 2.04}$

41. $\dfrac{8.24 \times 9.13}{10.12 \times 14.7}$

42. $\dfrac{1080}{(29.4)^2 \times 7.6}$

43. $\dfrac{7.85 \times 204 \times 82.6}{6.55 \times 101.5 \times 71.9}$

44. $\dfrac{(21.2)^2 \times 895}{17.6 \times 61.7 \times 4.6}$

45. $\dfrac{\frac{1}{4.1} \times 18.6}{3.64 \times 2.04}$

46. $89.2\ \pi$

47. $\dfrac{6}{\pi}$

48. $\dfrac{37\ \pi}{93}$

49. $\dfrac{37}{9.3\ \pi}$

50. $\dfrac{16.9 \times 1.14 \times 7.05\ \pi}{50.2 \times 2.6 \times 2.17}$

3.2 MULTIPLICATION AND DIVISION OF A SINGLE FACTOR BY A SERIES OF NUMBERS, AND DIVISION OF A SERIES OF NUMBERS BY A SINGLE FACTOR.

It is frequently necessary to obtain the products or quotients of several different numbers, each multiplied or divided by a constant. With a single setting of the slide, it is only necessary to move the hairline to perform the successive computations. Remembering that the slide need not be moved more than one-half of its length, and since 3.16 is located approximately at its mid-point of the scale, neither index of the C scale should cross 3.16 on the D scale.

Multiplication. Use the D and C, and the DF and CF scale combinations. Set an index of the C scale at the constant located on the D scale, move the hairline to the various numbers on the C or CF scale, and read the products on the D or DF scale at the hairline.

Example 3.15

Problem: Multiply 1.27 in turn by 3.16, 4.28, 6.55, 8.4, and 9.85.

Operation: Set the left index of C to 1.27 on D.
Move hairline to 3.16 on C.
Read 4.01 on D.

Move hairline to 4.28 on C.
Read 5.44 on D.
Move hairline to 6.55 on C.
Read 8.32 on D.
Move hairline to 8.4 on CF.
Read 10.67 on DF.
Move hairline to 9.85 on CF.
Read 12.51 on DF.

Division. For division of a single factor by a series of numbers, use the D and CI, and the DF and CIF scale combinations. Set an index of the CI scale to the constant located on the D scale, move the hairline to the various numbers on the CI or CIF scale, and read the quotients on the D or DF scale at the hairline.

Example 3.16

Problem: Divide 41.5 in turn by 12.4, 20.8, 44.5 and 92.

Operation: Set right index of CI to 41.5 on D.
Move hairline to 12.4 on CI.
Read 3.35 on D.
Move hairline to 20.8 on CI.
Read 1.995 on D.
Move hairline to 44.5 on CIF.
Read 0.933 on DF.
Move hairline to 92 on CIF.
Read 0.451 on DF.

Likewise, a series of numbers can be divided by a single factor, simply by multiplying each number in the series by the reciprocal of the single factor. As in multiplying a single factor with a series of numbers, the D and C, and DF and CF scale combinations are used.

Example 3.17

Problem: Divide 3.65, 30.5, 95.2, and 6.95, each by .561.

Operation: Set .561 on C to the right index of D.
Move hairline to 3.65 on C.
Read 6.51 on D.
Move hairline to 30.5 on C.
Read 54.4 on D.
Move hairline to 95.2 on CF.
Read 169.7 on DF.
Move hairline to 6.95 on CF.
Read 12.39 on DF.

Exercises in Multiplication and Division of a Series by a Single Factor.

51. Multiply 320 successively by 1.15, 2.42, 3.18, 4.5, 5.42, 6.88, 7.96, 8.05, and 9.6.

52. Divide 7.18 successively by 1.02, 2.15, 3.29, 4.18, 5.67, 6.41, 7.85, 8.76, and 9.34.

53. Divide 107, 181, 257, 294, 352, 671, 707, 775, 988, each by 358.

3.3 PROPORTION

The slide rule is very effective in solving simple equations in the form of a proportion, without the algebraic manipulation of terms. In a proportion such as $\frac{d}{c} = \frac{x}{c'}$ the known value d is located on a D scale (D or DF), the known values of c and c′ are located on a C scale (C or CF), and the unknown x is located on a D scale (D or DF). The folded scales are used for part or all of the proportion to limit slide movement to half its length.

Example 3.18

Problem: $\frac{8}{12} = \frac{x}{21}$; $x = 14$

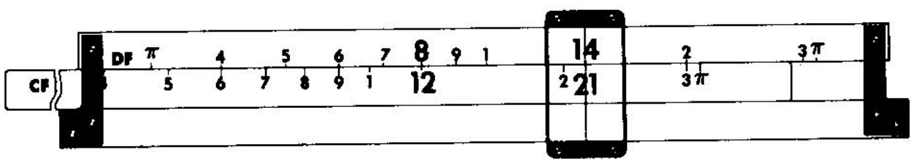


Figure 3.2

Operation: Set hairline to 8 on DF.
Move 12 on CF to hairline.
Move hairline to 21 on CF.
Read 14 on DF at hairline.

In Example 3.18, notice that the physical form of the proportion is exactly duplicated on the slide rule when the DF and CF scales are used. For problems that lend themselves exclusively to the D and C scales for solution, it is sometimes convenient to mentally invert the equation by locating the numerators on the C scale and the denominators on the D scale. Both solutions are shown for the following example.

Example 3.19

Problem: $\frac{639}{725} = \frac{x}{318}$; $x = 280$

Operation (A): Set hairline to 725 on D.
Move 639 on C to hairline.
Move hairline to 318 on D.
Read 280 on C at hairline.

Operation (B): Set hairline to 639 on D.
Move 725 on C to hairline.
Move hairline to 318 on C.
Read 280 on D at hairline.

Example 3.20

Problem: $\frac{8.7}{15.2} = \frac{x}{27.6} = \frac{44.4}{y} = \frac{z}{39.3}$

Operation: Set hairline at 8.7 on DF.
Move 15.2 on CF to hairline.
Move hairline to 27.6 on CF.
Read $x = 15.8$ on DF.
Move hairline to 44.4 on DF.
Read $y = 77.5$ on CF.
Move hairline to 39.3 on C.
Read $z = 22.5$ on D.

A large number of so-called "word problems" are in fact, equivalent expressions and lend themselves to solution by the proportional principle.

Example 3.21

Problem: A speed of 60 mph is equivalent to 88 ft./sec. What is the speed in ft/sec corresponding to a speed of 37 mph?

Operation: The problem is set up in the following form:

$$\frac{60 \text{ mph}}{88 \text{ ft/sec}} = \frac{37 \text{ mph}}{x}; \qquad x = 54.3 \text{ ft/sec.}$$

Set hairline to 60 on DF.
Move 88 on CF to hairline.
Move hairline to 37 on DF.
Read 54.3 on CF at hairline.

Exercises in Proportion.

54. $\frac{21.4}{195} = \frac{x}{12.1}$

55. $\frac{71}{705} = \frac{18.25}{x}$

56. $\frac{7.18}{x} = \frac{32.4}{17.9}$

57. $\frac{356}{51} = \frac{42.5}{x} = \frac{x}{y} = \frac{y}{z}$

3.4 QUADRATIC EQUATION SOLUTION BY FACTORING

The slide rule may be used for the rapid factoring of a quadratic equation. All that is required is a single setting of the slide and mental summation of factors. Any quadratic equation may be reduced to the form: $x^2 + Ax + B = 0$

The factors, or roots, of the equation are designated as r_1 and r_2, and must satisfy the following conditions.

(1) $r_1 \times r_2 = A$

(2) $r_1r_2 = B$

The general procedure of finding two factors, whose sum is A and whose product is B, on the slide rule follows:

(1) Set the appropriate index of C opposite the location of B on the D scale. Now the slide is in such a position that for any setting of the indicator, the *product* of the reading at the hairline on the D and CI scales, or on the DF and CIF scales, is equal to the number B.

(2) Move the hairline so that the *sum* of the readings on the D and CI, or the DF and CIF scales is equal to the number A.

The following examples will clarify this procedure.

Example 3.22

Problem: $x^2 + 10x + 15 = 0$

Operation: Set left index of C at 15 on D.
Move hairline until $r_1 + r_2 = 10$.
(They will be positive as both 10 and 15 are positive.)
This occurs with hairline at 8.15 on CI and 1.84 on D.
Thus the roots are 8.15 and 1.84.

Sum $= A = 10 = 8.15 + 1.84$
Product $= B = 15 = (8.15)(1.84)$

Example 3.23

Problem: $x^2 - 12.2x - 17.2 = 0$

Operation: Set left index of C at 17.2 on D.
Move hairline until $r_1 + r_2 = -12.2$ which is when the hairline is at -13.5 on DF and 1.275 on CIF. (Since the product is negative, one of the factors must be negative. Also, since the sum is negative the larger factor must be negative.) The roots are -13.5 and 1.275.

Sum $= A = -13.5 + 1.275 = -12.225$
Produce $= B = (-13.5)(1.275) = -17.2$

Hence, it is obvious that this method involves trial-and-error in setting the hairline, and a little practice is necessary to master the technique.

Exercises in Solving Quadratic Equations by Factoring.

58. $x^2 - 34.5x + 18 = 0$
59. $x^2 - 21.1x + 32 = 0$
60. $x^2 - 20.2x - 120 = 0$
61. $2x^2 + 82.8x + 840 = 0$
62. $1.2x^2 - 13.38x + 36 = 0$

CHAPTER 4

POWERS AND ROOTS

LOGARITHMS

The Versatrig enables the solution of any power or root of a number. The powers and roots most commonly encountered can be rapidly calculated with just one setting of the hairline. For squares and square root, a choice of scales is available to permit greater speed and accuracy in these calculations. For operations involving exponents other than 2, 3, and 4 (and 1/2, 1/3, and 1/4), the L or logarithm scale is used. Uses of the R_1, R_2, A, K, and L scales are discussed in this chapter.

4.1 SQUARES AND SQUARE ROOT: SIMPLE OPERATIONS

The R_1 and R_2 Scales. These scales are called the "root" scales. Actually, they are the two halves of one 20 inch scale (50 cm.) similar to a D scale, but twice as long. The R_1 scale is 25 cm. long, graduated and numbered from left to right, and ranges from 1 to $\sqrt{10}$, 3.16; while the R_2 scale, also 25 cm. long, and graduated and numbered from left to right, ranges from $\sqrt{10}$, 3.16, to 10. These scales yield unusual accuracy in solving for squares and square roots.

Squares Using the R_1 and R_2 Scales. The squares of numbers on the R_1 and R_2 scales are directly opposite on the D scale. The simple mathematical relationship of the R and D scales may be expressed as; $R^2 = D$.

To square a number whose digits fall between 1 and 3.16;

1. Set the hairline to the number on R_1.
2. Read the square of the number on D at the hairline.

To square a number whose digits fall between 3.16 and 10;

1. Set the hairline to the number of R_2.
2. Read the square of the number on D at the hairline.

Example 4.1

Problem: $3^2 = 9$

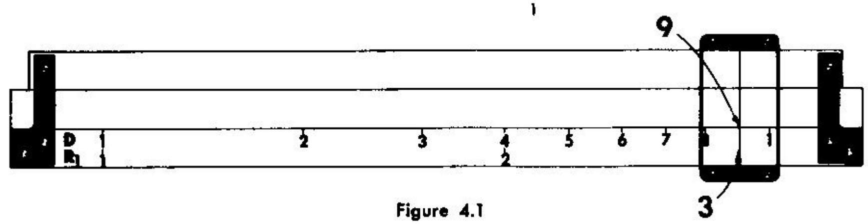


Figure 4.1

Operation: Set hairline to 3 on R_1.
Read square, 9, on D at hairline.

Example 4.2
Problem: $(7.1)^2 = 50.4$

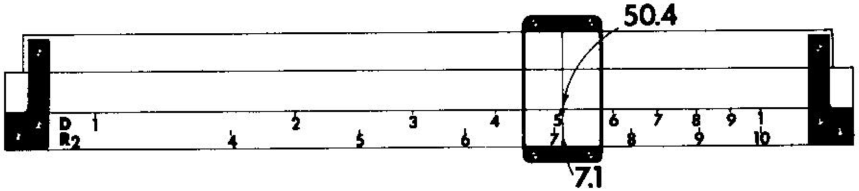


Figure 4.2

Operation: Set hairline to 7.1 on R_2.
Read square, 50.4, on D at hairline.

When squaring larger or smaller numbers, the location of the decimal point requires more consideration. Use of the standard form (as outlined in Section 2.5) is recommended. In squaring numbers in their standard form, $(N \times 10^n)^2$ becomes $N^2 \times 10^{2n}$. It is precise and the occasional user need not learn additional rules. However, a short cut is available for quick placement of the decimal point and it may be well to remember the following:

Squares of numbers greater than 1 (>1);

1. The square of a number greater than 1 on the R_1 scale will have an *odd* number of digits to the left of the decimal point—one less than twice the number of digits to the left of the decimal point in the number being squared.

2. The square of a number greater than 1 on the R_2 scale will have an *even* number of digits to the left of the decimal point—exactly twice the number of digits to the left of the decimal point in the number being squared.

Squares of numbers less than 1 (<1);
Zeros appearing to the right of the decimal point and before the first non-zero digit in a number less than 1 may be defined as significant zeros.

1. The square of a number less than 1 on the R_1 scale will have an *odd* number of significant zeros—one more than twice the number of significant zeros in the number being squared.

2. The square of a number less than 1 on the R_2 scale will have an *even* number of significant zeros—exactly twice the number of significant zeros in the number being squared.

Example 4.3
Problem: $(26.53)^2 = 704$

Operation: Set hairline to 26.53 on R_1.
Read 704 on D at hairline.
Locate decimal point by either;
a) $(2.653 \times 10^1)^2 = 7.04 \times 10^2 = 704.$
b) R_1, $(2 \times 2) - 1 = 3$ digits.

Example 4.4
Problem: $(6,110)^2 = 37,300,000$

Operation: Set hairline to 6,110 on R_2.
Read 37,300,000 on D at hairline.
Locate decimal point by either;
a) $(6.11 \times 10^3)^2 = 37.3 \times 10^6 = 37,300,000$
b) R_2, $2 \times 4 = 8$ digits

Example 4.5
Problem: $(0.1575)^2 = 0.0248$

Operation: Set hairline to .1575 on R_1.
Read .0248 on D at hairline.
Locate decimal point by either;
a) $(1.575 \times 10^{-1})^2 = 2.48 \times 10^{-2} = 0.0248$
b) R_1, $(2 \times 0) + 1 = 1$ significant zero.

Example 4.6
Problem: $(0.00917)^2 = 0.0000841$

Operation: Set hairline to .00917 on R_2.
Read .0000841 on D at hairline.
Locate decimal point by either;
a) $(9.17 \times 10^{-3})^2 = 84.1 \times 10^{-6} = 0.0000841.$
b) R_2, $2 \times 2 = 4$ significant zeros.

Square Root Using the R_1 and R_2 Scales. The square root of a number N is that number whose square is N. For example, if N = 9, then;

$$\sqrt{9} \text{ or } 9^{\frac{1}{2}} = 3 \text{ and } 3^2 = 9.$$

The superscript after 9 is 1/2, the reciprocal of 2. This inverse relationship indicates a reversal of the slide rule procedure. Thus, the square root of numbers on the D scale are directly opposite on the R_1 and R_2 scales. A modified standard form is recommended to correctly perform the operation. The square root required is expressed as $(N \times 10^n)^{\frac{1}{2}}$, when the exponent n must be evenly divisible by 2, while the number N can be between 1 and 100. Using the short cut rules for positioning the decimal point, odd numbers of digits are again associated with the R_1 scale, while even numbers of digits are associated with the R_2 scale. The rules are as follows:

Square roots of numbers greater than 1 (>1);

1. The square root of a number greater than 1 with an *odd* number of digits to the left of the decimal point is found on the R_1 scale. To obtain the number of digits to the left of the decimal point in the square root, add one to the number of digits to the left of the decimal point in the number and divide by two.
2. The square root of a number greater than 1 with an *even* number of digits to the left of the decimal point is found on the R_2 scale. The number of digits to the left of the decimal point in the square root is exactly half the number of digits to the left of the decimal point in the number.

Square roots of numbers less than 1 (<1);

1. The square root of a number less than 1 with an *odd* number of significant zeros to the right of the decimal point is found on the R_1 scale. To obtain the number of significant zeros in the square root, subtract one from the number of significant zeros in the number and divide by two.
2. The square root of a number less than 1 with an *even* number of significant zeros or no significant zeros to the right of the decimal point is found on the R_2 scale. The number of significant zeros in the square root is exactly half the number of significant zeros in the number.

Example 4.7

Problem: $\sqrt{196} = 14$

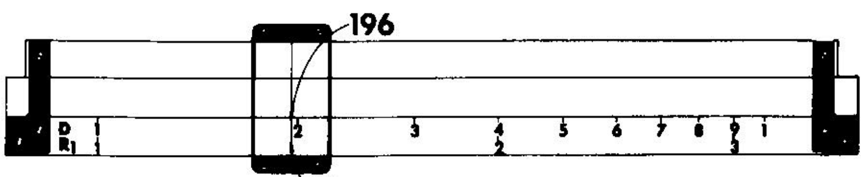


Figure 4.3

Operation: Set hairline to 196 on D.
Read square root, 14, on R_1 at hairline.
Locate decimal point by either;
a) $(1.96 \times 10^2)^{\frac{1}{2}} = 1.4 \times 10^1 = 14$
b) $R_1, \dfrac{3+1}{2} = 2$ digits

Example 4.8

Problem: $\sqrt{124{,}600} = 353$

Operation: Set hairline to 124,600 on D.
Read 353 on R_2 at hairline.
Locate decimal point by either;
a) $12.46 \times 10^4)^{\frac{1}{2}} = 3.53 \times 10^2 = 353.$
b) $R_2, \dfrac{6}{2} = 3$ digits.

Example 4.9

Problem: $\sqrt{0.43} = 0.656$

Operation: Set hairline to .43 on D.
Read .656 on R_2 at hairline.
Locate decimal point by either;
a) $(43. \times 10^{-2})^{\frac{1}{2}} = 6.56 \times 10^{-1} = .656$
b) $R_2, \dfrac{0}{2} = 0$ significant zeros.

Example 4.10

Problem: $\sqrt{0.00097} = 0.03115$

Operation: Set hairline to .00097 on D.
Read .03115 on R_1 at hairline.
Locate decimal point by either;
a) $(9.7 \times 10^{-4})^{\frac{1}{2}} = 3.115 \times 10^{-2} = .03115$
b) $R_1, \dfrac{3-1}{2} = 1$ significant zero.

Selection of the R_1 or R_2 scale on which to read the square root of a number is determined by the number of digits to the left of the decimal point for numbers greater than 1, and the number of significant zeros in numbers less than 1. This is further illustrated by the following examples.

Example 4.11

Problem: $\sqrt{0.5} = 0.707$

Operation: Set hairline to .5 on D.
Read .707 on R_2 at hairline.

Example 4.12

Problem: $\sqrt{0.05} = 0.2236$

Operation: Set hairline to .05 on D.
Read .2236 on R_1 at hairline.

Example 4.13

Problem: $\sqrt{0.005} = 0.0707$

Operation: Set hairline to .005 on D.
Read .0707 on R_2 at hairline.

Example 4.14

Problem: $\sqrt{0.0005} = 0.02236$

Operation: Set hairline to .0005 on D.
Read .02236 on R_1 at hairline.

Example 4.15

Problem: $\sqrt{0.00005} = 0.00707$

Operation: Set hairline to .00005 on D.
Read .00707 on R_2 at hairline.

Example 4.16

Problem: $\sqrt{0.000005} = 0.002236$

Operation: Set hairline to .000005 on D.
Read .002236 on R_1 at hairline.

Exercises in Squares and Square Root. The following exercises are provided to gain proficiency in finding squares and extracting square roots. Pay particular attention to the placement of the decimal point.

63. $(20.4)^2$
64. $(715)^2$
65. $(1{,}070)^2$
66. $(125.4)^2$
67. $(0.85)^2$
68. $(0.000157)^2$
69. $(0.094)^2$
70. $(0.0076)^2$
71. $\sqrt{27}$
72. $\sqrt{925}$
73. $\sqrt{820{,}000}$
74. $\sqrt{1{,}265}$
75. $\sqrt{71{,}500}$
76. $\sqrt{51{,}000{,}000}$
77. $\sqrt{1{,}970{,}000}$
78. $\sqrt{660}$
79. $\sqrt{0.424}$
80. $\sqrt{0.0875}$
81. $\sqrt{0.00097}$
82. $\sqrt{0.00725}$

The A Scale. The A scale can also be used for square and square root calculations. It ranges from 1 to 100. When the greater accuracy provided by the R_1 and R_2 scales is not required, some series of computations are more rapidly solved using the A scale. The A scale is, in effect, two short (12.5 cm.) D scales placed end to end, and that is approximately how it is used. In finding squares and square root, the left half of the A scale is used as a D scale and the left half of the D scale is used as an R_1 scale. Likewise, the right half of the A side is used again as a D scale and the right half of the D scale is used as an R_2 scale. The simple mathematical relationship of the A and D scales may be expressed as; $D^2 = A$.

Squares Using the A Scale. The square of numbers on the D scale are directly opposite on the A scale. The procedure only requires the use of the hairline. The short cut rules for the placement of the decimal point are the same as when using the R_1 and R_2 scales, except that the left half of the A scale is associated with odd numbers of digits (as the R_1 scale) and the right half, with even numbers of digits (as the R_2 scale).

Example 4.17

Problem: $(24.8)^2 = 615$

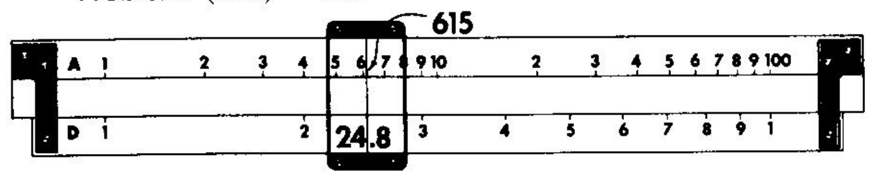


Figure 4.4

Operation: Set hairline to 24.8 on D.
Read 615 on A at hairline.
Locate decimal point by either;
a) $(2.48 \times 10^1)^2 = 6.15 \times 10^2 = 615.$
b) A left, $(2 \times 2) - 1 = 3$ digits.

Example 4.18

Problem: $(417)^2 = 174{,}000$

Operation: Set hairline to 417 on D.
Read 174,000 on A at hairline.
Locate decimal point by either;
a) $(4.17 \times 10^2)^2 = 17.4 \times 10^4 = 174{,}000.$
b) A right, $2 \times 3 = 6$ digits.

Example 4.19

Problem: $(0.0196)^2 = 0.000384$

Operation: Set hairline to .0196 on D.
Read .000384 on A at hairline.
Locate decimal point by either;
a) $(1.96 \times 10^{-2})^2 = 3.84 \times 10^{-4} = 0.000384$.
b) A left, $(2 \times 1) + 1 = 3$ significant zeros.

Example 4.20

Problem: $(0.822)^2 = 0.676$

Operation: Set hairline to .822 on D.
Read .676 on A at hairline.
Locate decimal point by either;
a) $(8.22 \times 10^{-1})^2 = 67.6 \times 10^{-2} = 0.676$.
b) A right, $2 \times 0 = 0$ significant zeros.

Square Root Using the A Scale. The square roots of numbers on the A scale are directly opposite on the D scale. The procedure is essentially the reverse of finding squares using the A scale, but in locating numbers on the A scale, the position of the decimal point is important since the A scale ranges from 1 to 100. For numbers outside of this range, the use of a modified standard form is recommended. The number is rewritten in the form of $N \times 10^n$, where n must be evenly divisible by 2, and N is between 1 and 100. The number N is then located on the A scale, $\sqrt{N}$ read on the D scale, and the number rewritten in its ordinary form.

A short cut rule can also be used. The rule for locating the decimal point is the same as when using the R_1 and R_2 scales, but in locating numbers on the A scale, the following can be used.

> Use the *left* half of the A scale for numbers greater than 1 with an *odd* number of digits to the left of the decimal point and for numbers less than 1 with an *odd* number of significant zeros to the right of the decimal point.
>
> Use the *right* half of the A scale for numbers greater than 1 with an *even* number of digits to the left of the decimal point and for numbers less than 1 with either no significant zeros or an *even* number of significant zeros to the right of the decimal point.

Example 4.21

Problem: $\sqrt{5{,}480} = 74.0$

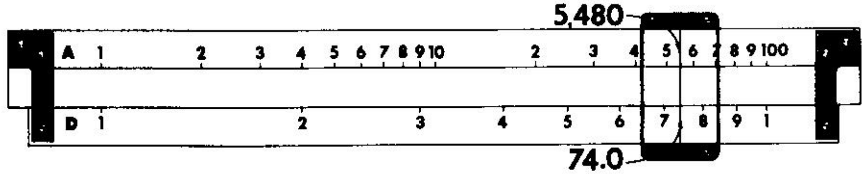


Figure 4.5

Operation: Set hairline to 5,480 on right half of A.
Read 74.0 on D at hairline.
Locate decimal point by either;
a) $(54.80 \times 10^2)^{\frac{1}{2}} = 7.40 \times 10^1 = 74.0$
b) A right, $\frac{4}{2} = 2$ digits.

Example 4.22

Problem: $\sqrt{54{,}800} = 234$

Operation: Set hairline to 54,800 on left half of A.
Read 234 on D at hairline.
Locate decimal point by either;
a) $(5.48 \times 10^4)^{\frac{1}{2}} = 2.34 \times 10^2 = 234$.
b) A left, $\frac{5 + 1}{2} = 3$ digits.

Example 4.23

Problem: $\sqrt{0.0000176} = 0.0042$

Operation: Set hairline to .0000176 on right half of A.
Read .0042 on D at hairline.
Locate decimal point by either;
a) $(17.6 \times 10^{-6})^{\frac{1}{2}} = 4.2 \times 10^{-3} = .0042$.
b) A right, $\frac{4}{2} = 2$ significant zeros.

Example 4.24

Problem: $\sqrt{0.000176} = 0.01327$

Operation: Set hairline to .000176 on left half of A.
Read .01327 on D at hairline.
Locate decimal point by either;
a) $(1.76 \times 10^{-4})^{\frac{1}{2}} = 1.327 \times 10^{-2} = 0.01327$.
b) A left, $\frac{3 - 1}{2} = 1$ significant zero.

Exercises 63 to 82 can be worked again using the A scale for practice. Concentrate on the correct decimal point placement. Notice the difference in accuracy in using the A scale compared to the R_1 and R_2 scales.

4.2 SQUARES AND SQUARE ROOT: COMPOUND OPERATIONS

Compound operations, in the sense used here, are series of operations which include the square or square root of a number. These computations can be rapidly performed since the Versatrig provides alternate scales for finding squares and square roots.

Choice of the R_1 and R_2 or A Scales. For maximum accuracy, use the R_1 and R_2 scales with the D scale for computations involving squares and square root. When the increased accuracy provided by the R_1 and R_2 scales is not required, square roots may be entered directly into a series of computations, or squares read directly following a series of computations without an additional setting by using the A scale. In the following examples, it is assumed that A scale accuracy is sufficient.

Example 4.25

Problem: $\dfrac{\sqrt{47.2} \times 7.85}{13.51 \times 6.11} = 0.653$

Operation: Set hairline to 47.2 on A.
Move right index of C to hairline.
Move hairline to 7.85 on CF.
Move 13.51 on CF to hairline.
Move hairline to 6.11 on CIF.
Read .653 on DF at hairline.

Example 4.26

Problem: $\dfrac{(3.485)^2 \times 9.44}{0.777 \times 3.9} = 37.8$

Operation: Set hairline to 3.485 on R_2.
Move 9.44 on CI to hairline.
Move hairline to .777 on CI.
Move 3.9 on C to hairline.
Read 37.8 on D at right index of C.

Example 4.27

Problem: $\sqrt{\dfrac{9.7 \times 14}{2.35}} = 7.6$

Operation: Set hairline to 9.7 on D.
Move 14 on CI to hairline.
Move hairline to 2.35 on CI.
Read 7.6 on R_2 at hairline.

Example 4.28

Problem: $\left(\dfrac{4.1}{6.95 \times 0.233}\right)^2 = 6.4$

Operation: Set hairline to 4.1 on D.
Move 6.95 on C to hairline.
Move hairline to .233 on CI.
Read 6.4 on A at hairline.

Areas of Circles. Finding the area of a circle when the radius is given is a commonly encountered problem. Solution requires only a single setting of the hairline—another decided advantage of the R_1 and R_2 scales. Set the radius, r, on the R_1 or R_2 scale and read the area, πr^2, on the DF scale. (The value r^2 is available at the hairline on the D scale.)

Example 4.29

Problem: Find the area of a circle whose radius is 4.82 feet.
Area = 73 square feet.

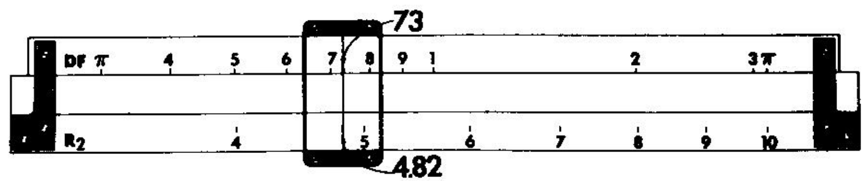


Figure 4.6

Operation: Set hairline to 4.82 on R_2.
Read area, 73, on DF at hairline.

When the area of a circle is known and the radius required, the inverse of the above procedure can be used. Set the area on the DF scale and read the radius on either the R_1 or R_2 scale—whichever is appropriate. The same rules as stated in Section 4.1 apply to the selection of the proper root scale.

Example 4.30

Problem: Find the radius of a circle whose area is 2,670 sq. in.
Radius = 29.15 in.

Operation: Set hairline to 2,670 on DF.
Read 29.15 on R_1 at hairline.

When the diameter of a circle is given, rather than radius, the area can be found by solving for $\frac{\pi d^2}{4}$, where d is the diameter. The operation is similar to Example 4.29, but a setting of the slide is necessary for the division.

Example 4.31

Problem: Find the area of a circle whose diameter is 2.437 in.
Area = 4.66 sq. in.

Operation: Set hairline to 2.437 on R_1.
Move 4 on C to hairline.
Move hairline to left index of C.
Read area, 4.66, on DF at hairline.

An alternate solution to the preceding example is available by using a special unidentified line appearing on the right end of the A scale—approximately at the value 785. This graduation represents $\frac{\pi}{4}$. With a single setting of the slide, the C scale may be used with the A scale and this gauge mark for finding areas of circles whose diameters are known. To use this expedient, set the right index of C opposite the $\frac{\pi}{4}$ gauge mark on A, set the hairline to the diameter of the circle on C, and read the area of the circle on A. Example 4.31 is illustrated again. Notice the difference in accuracy.

Example 4.32

Problem: Find the area of a circle whose diameter is 2.437 in.
Area = 4.66 sq. in.

Operation: Set hairline to gauge mark on A.
Move right index of C to hairline.
Move hairline to approx. 2.437 on C.
Read approx. 4.66 on A at hairline.

Another special gauge mark is located near the left end of the C scale which is on the same face of the rule as the A scale. It is marked by a letter c and represents the constant $\sqrt{\frac{4}{\pi}}$. Using this mark and with a single setting of the slide, it is possible to determine the diameter of a circle when the area is known. The procedure followed is; set "c" mark opposite the left index of D, set the hairline to the area of the circle on A, read the diameter of the circle on C at the hairline.

Example 4.33

Problem: Find the diameter of a circle whose area is 18.4 sq. ft.
Diameter = 4.84 ft.

Operation: Set hairline to left index of A.
Move "c" mark on C to hairline.
Move hairline to 18.4 on A.
Read diameter, 4.84, on C at hairline.

Exercises in Compound Operations Involving Squares and Square Root.

83. $\frac{17\sqrt{676}}{3.19 \times 12}$

84. $\frac{(14)^2 \times 3.2}{225 \times 6.4}$

85. $\sqrt{27^2 \times 41.3}$

86. $\frac{\pi (3.955)^2}{4.1 \times 2.49}$

87. $\left(\frac{11.45 \times 6.8\pi}{1.605 \times 5.35}\right)^2$

88. $\left(\frac{\sqrt{811} \times 4}{2.36}\right)^2$

89. $(\pi (6.69)^2 \times 1.19)^2$

90. Find the area of circles whose radii are known:
a) 6; b) 4.2; c) .0581; d) 31; e) 1.314.

91. Find the area of circles whose diameters are known:
a) 7.1; b) .42; c) 1.09; d) .0495; e) 1,700.

92. Find the radius of circles whose areas are known:
a) 116.5; b) .0491; c) .601; d) 760; e) 80.4.

93. Find the diameter of circles whose areas are known:
a) 5; b) 50; c) 760; d) .0106; e) .601.

4.3 CUBES AND CUBE ROOT

The K Scale. The K scale is used with the D scale for finding cubes and cube root. It is a three segment scale which may be thought of as three short D scales placed end to end. It ranges from 1 to 1,000. The simple mathematical relationship of the D and K scales may be expressed as $D^3 = K$.

Cubes. Cubes of numbers on the D scale are directly opposite on the K scale. For numbers between 1 and 10, the location of the decimal point is indicated by the K scale, since it ranges from 1 to 1,000.

Example 4.34

Problem: $(6.1)^3 = 227$

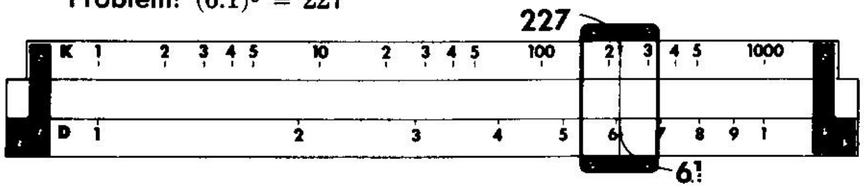


Figure 4.7

Operation: Set hairline to 6.1 on D.
Read 227 on K at hairline.

For numbers larger than 10 and smaller than 1, the location of the decimal point is not as obvious and the use of the standard form is again recommended. Briefly, since $(N \times 10^n)^3 = N^3 \times 10^{3n}$, the K scale is used for finding the cube of N and the power of ten for relocating the decimal point.

Example 4.35

Problem: $(1{,}214)^3 = 1{,}790{,}000{,}000$

Operation: Express problem as $(1.214 \times 10^3)^3$.
Set hairline to 1.214 on D.
Read 1.79 on K at hairline.
Answer; $1.79 \times 10^9 = 1{,}790{,}000{,}000$.

Example 4.36

Problem: $(0.0721)^3 = 0.000375$

Operation: Express problem as $(7.21 \times 10^{-2})^3$.
Set hairline to 7.21 on D.
Read 375. on K at hairline.
Answer; $375 \times 10^{-6} = .000375$.

Cube Root. Cube roots of numbers on the K scale are directly opposite on the D scale. Therefore, the cube root of numbers between 1 and 1,000 range between 1 and 10. For numbers beyond the range of 1 to 1,000, use of a modified standard form is recommended to assure

correct placement of the decimal point. In the modified standard form, the number is expressed as $N \times 10^n$, where n must be evenly divisible by 3 while N can range from 1 to 1,000 (since the K scale ranges from 1 to 1,000).

Example 4.37

Problem: $\sqrt[3]{5.2} = 1.73$

Operation: Set hairline to 5.2 on K.
Read 1.73 on D at hairline.

Example 4.38

Problem: $\sqrt[3]{26{,}400} = 29.8$

Operation: Express problem as $(26.4 \times 10^3)^{1/3}$.
Set hairline to 26.4 on K.
Read 2.98 on D at hairline.
Answer; $2.98 \times 10^1 = 29.8$.

Example 4.39

Problem: $\sqrt[3]{0.0052} = 0.1732$

Operation: Express problem as $(5.2 \times 10^{-3})^{1/3}$.
Set hairline to 5.2 on K.
Read 1.732 on D at hairline.
Answer; $1.732 \times 10^{-1} = .1732$.

Example 4.40

Problem: $\dfrac{\sqrt[3]{0.000475}}{4.6} = 0.01696$

Operation: Express problem as $\dfrac{\sqrt[3]{475.}}{4.6} \times (10^{-6})^{1/3}$.
Set hairline to 475. on K.
Move 4.6 on C to hairline.
Read 1.696 on D at index of C.
Answer; $1.696 \times 10^{-2} = 0.01696$.

Exercises in Cubes and Cube Root. Perform the operations indicated using the K and D scales.

94. $(1.26)^3$	100. $\sqrt[3]{6}$	106. $(0.245)^3$
95. $(2.715)^3$	101. $\sqrt[3]{24}$	107. $(0.036)^3$
96. $(5.85)^3$	102. $\sqrt[3]{270}$	108. $(0.0048)^3$
97. $(41)^3$	103. $\sqrt[3]{1{,}720}$	109. $\sqrt[3]{0.32}$
98. $(750)^3$	104. $\sqrt[3]{29{,}000}$	110. $\sqrt[3]{0.041}$
99. $(3.2)^3$	105. $\sqrt[3]{560{,}000}$	111. $\sqrt[3]{0.0075}$

4.4 OTHER POWERS AND ROOTS

Any power or root of a number can be found using the L scale as is explained in Section 4.5. However, using the R_1, R_2, D, A, and K scales powers of 4, 6, and 3/2 (and 1/4, 1/6 and 2/3) can be found with a single setting of the hairline. As was previously mentioned, the mathematical relationship between these scales may be expressed as: $R^2 = D$, $D^2 = A$, and $D^3 = K$, and therefore, $R^4 = A$, $R^6 = K$ and $A^3 = K^2$.

Powers of 4 and 1/4. The root scales are used with the A scale for powers of 4 and 1/4. The square of numbers on an R scale are directly opposite on the D scale, and the square of numbers on the D scale are directly opposite on the A scale. Therefore, the 4th power of numbers on an R scale are directly opposite on the A scale. Symbolically, $N^4 = (N^2)^2$; $N^{\frac{1}{4}} = (N^{\frac{1}{2}})^{\frac{1}{2}}$.

Example 4.41

Problem: $(22)^4 = 234{,}000$

Operation: Express number as $(2.2 \times 10^1)^4$.
Set hairline to 2.2 on R_1.
Read 23.4 on A at hairline.
Answer, $23.4 \times 10^4 = 234{,}000$.

Example 4.42

Problem: $(0.06)^{\frac{1}{4}} = 0.495$

Operation: Express number as $(600 \times 10^{-4})^{\frac{1}{4}}$.
Set hairline to 600 on A (left half).
Read 4.95 on R_2 at hairline.
Answer, $4.95 \times 10^{-1} = 0.495$.

Powers of 6 and 1/6. The root scales are used with the K scale for finding powers of 6 and 1/6. The 1/3rd power of numbers on the K scale are opposite the D scale, and the square root (1/2 power) of numbers on the D scale are opposite on an R scale. Symbolically, $N^6 = (N^3)^2$; $N^{\frac{1}{6}} = (N^{\frac{1}{3}})^{\frac{1}{2}}$.

Example 4.43

Problem: $(2)^6 = 64$

Operation: Set hairline to 2 on R_1.
Read 64 on K at hairline.

Example 4.44

Problem: $(2{,}000)^{\frac{1}{6}} = 3.55$

Operation: Set hairline to 2,000 on K.
Read 3.55 on R_2 at hairline.

Powers of 2/3 and 3/2. The A and K scales are used since the 1/3rd power of numbers on the K scale are opposite on the D scale and squares of numbers on the D scale are opposite on the A scale. Thus, the 2/3 power of numbers on the K scale are opposite on the A scale. Symbolically, $N^{\frac{2}{3}} = (N^{\frac{1}{3}})^2$; $N = (N^3)^{\frac{1}{2}}$.

Example 4.45

Problem: $0.875^{\frac{2}{3}} = 0.915$

Operation: Set hairline to .875 on K.
Read .915 on A at hairline.

Example 4.46

Problem: $18.2^{\frac{3}{2}} = 78.$

Operation: Set hairline to 18.2 on A.
Read 78. on K at hairline.

4.5 LOGARITHMS

The L Scale. The L scale is a uniformly divided scale, graduated and numbered from left to right, and ranging from 0 to 1. The logarithm of a number is the power to which the base of the logarithm must be raised to equal the number. Logarithms to the base 10, called common logarithms, are read on the L scale. The common logarithm of a number N is designated as $\log_{10}N$ or simply, as log N.

The logarithm consists of two parts; a) the *characteristic* which is to the left of the decimal point, and b) the *mantissa* which is to the right of the decimal point. When a number is expressed in the standard form (see Section 2.5) the power of 10 is the characteristic of the logarithm. The mantissa is located on the L scale opposite the number on the D scale.

Example 4.47

Problem: log 164.3 = 2.216

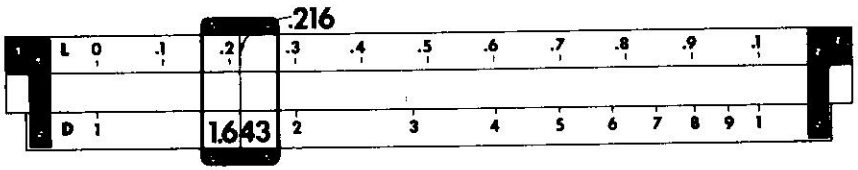


Figure 4.8

Operation: Express number as 1.643×10^2.
Characteristic = 2.
Set hairline to 1.643 on D.
Read mantissa, .216, on L at hairline.

Example 4.48

Problem: log 8.47 = 0.928

Operation: Express number as 8.47×10^0.
Characteristic = 0.
Set hairline to 8.47 on D.
Read mantissa, .928, on L at hairline.

Logarithms of numbers less than one have negative characteristics. Since the mantissa is always positive and less than one (as indicated by the above procedure), negative logarithms are usually expressed in a positive form, as log .001 = log (1×10^{-3}) = 0.000 − 3 or 7.000 − 10.

Example 4.49

Problem: log 0.0718 = 8.856 − 10

Operation: Express number as 7.18×10^{-2}.
Characteristic = − 2, or 8. − 10.
Set hairline to 7.18 on D.
Read mantissa, .856, on L at hairline.

The antilogarithm is found by the inverse procedure.

Example 4.50

Problem: log N = 3.346
N = 2,220

Operation: Set hairline to mantissa, .346, on L.
Read 2.22 on D at hairline.
N = 2.22×10^3, or 2,220

Example 4.51

Problem: log N = 8.015 − 10.
N = 0.01035

Operation: Set hairline to mantissa, .015 on L.
Read 1.035 on D at hairline.
N = 1.035×10^{-2}, or 0.01035

Conversion to Natural Logarithms. The value e, equal to 2.71828 . . . is sometimes used as a base of logarithms rather than 10. The logarithm to the base e, called natural logarithm, of a number N is expressed as $\log_e N$. Since $\log_e N = (\log_e 10)(\log_{10} N)$, and $\log_e 10 = 2.30$, only a simple slide rule computation is required to convert common logs to natural logs.

Example 4.52

Problem: $\log_e$ 33.2 = 3.50

Operation: Express number as 3.32×10^1.
Characteristic = 1.
Set hairline to 3.32 on D.
Read mantissa, .521 on L at hairline.
Set left index of C to $\log_{10}$33.2, 1.521, on D.
Move hairline to 2.30 on C.
Read 3.50 on D at hairline.

When converting from the natural logarithm, first find the common logarithm, then the antilogarithm.

Example 4.53

Problem: $\log_e$ N = 7.33
N = 1,550

Operation: Set hairline to 7.33 on DF.
Move 2.30 on CF to hairline.
Read $\log_{10}$N, 3.19, on D at right index of C.
Set hairline to mantissa, .19, on L.
Read 1.55 on D at hairline.
Answer, 1.55×10^3 = 1,550

Powers and Roots Using Logarithms. Since the relationship $N^n = n \log N$ exists, any number N may be raised to any power n by multiplying the log of the number N by the power n. On the Versatrig, this involves finding the logarithm, multiplying, and finding the antilogarithm.

Example 4.54

Problem: $(28.6)^{1.26} = 68.4$

Operation: Express number as 2.86×10^1.
Characteristic = 1.
Set hairline to 2.86 on D.
Read mantissa, .456, on L at hairline.
Log 28.6 = 1.456.
Product of log and power:
$1.456 \times 1.26 = 1.835$.
Set hairline to 8.35 on L.
Read 6.84 on D at hairline.
Answer, $6.84 \times 10^1 = 68.4$

Example 4.55

Problem: $(0.513)^{0.85} = 0.566$

Operation: Express number as 5.13×10^{-1}.
Characteristic = − 1 or 9. − 10.
Set hairline to 5.13 on D.
Read mantissa, .710, on L at hairline.
Log 0.513 = 9.710 − 10 = − 0.290.
Product of log and power:
$-0.290 \times 0.85 = -0.247 = 9.753 - 10$.
Set hairline to .753 on L.
Read 5.66 on D at hairline.
Answer, $5.66 \times 10^{-1} = 0.566$.

Exercises in Powers Using Logarithms. Find the value of the following:

112. $(1.95)^{2.7}$
113. $(650)^{0.5}$
114. $(31)^{0.845}$
115. $(0.568)^{9.1}$
116. $(0.114)^{0.252}$
117. $(415)^{-0.75}$
118. $(0.877)^{-2.5}$
119. $(1.31)^{-3.2}$
120. $(0.992)^{4.1}$

Exponential Equations. Equations in the form $N^p = A$, in which N and A are known quantities, may be solved for the unknown exponent

p. The problem may be stated as: To what exponent p must N be raised so that the result is A? This can be expressed as $p = \frac{\log A}{\log N}$. An example follows:

Example 4.56

Problem: $(2.4)^p = 180$
$p = 5.94$

Operation: Set hairline to 2.4 on D.
Read mantissa, .380 on L at hairline.
Log 2.4 = 0.380.
Express other number as 1.8×10^2.
Set hairline to 1.8 on D.
Read mantissa, .255 on L at hairline.
Log 180 = 2.255.
Therefore 0.380p = 2.255.
Set hairline to 2.255 on D.
Move 0.380 on C to hairline.
Read p, 5.94, on D at index of C.

Exercises in Exponential Equations. In the following equations, solve for p.

121. $(9.1)^p = 16.4$
122. $(25.5)^p = 17.5$
123. $(3.25)^p = 71.5$
124. $(0.915)^p = 0.614$
125. $(0.425)^p = 0.0174$

CHAPTER 5

TRIGONOMETRIC OPERATIONS

5.1 THE TRIGONOMETRIC FUNCTIONS

There are six basic trigonometric functions, or relations between the sides of a right triangle. Each angular function is expressed as the ratio of a particular pair of sides of the triangle. These six ratios are the sine, cosine, tangent, cotangent, secant, and cosecant of an angle.

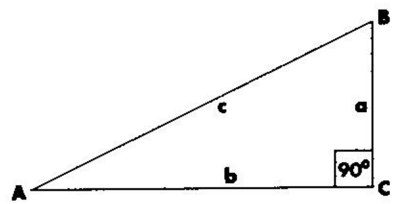


Figure 5.1 The Right Triangle

The six basic trigonometric functions may be written as:

$$\text{sine A} = \sin A = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{1}{\text{cosec A}}$$

$$\text{cosine A} = \cos A = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{1}{\sec A}$$

$$\text{tangent A} = \tan A = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{1}{\cot A}$$

$$\text{cotangent A} = \cot A = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{1}{\tan A}$$

$$\text{secant A} = \sec A = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{1}{\cos A}$$

$$\text{cosecant A} = \csc A = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{1}{\sin A}$$

Note the reciprocal nature between the first and last three functions. The first set of three (sine, cosine, and tangent) find the most frequent application. Consequently scales for finding values of these functions for any given angle are found on the slide rule. Should the other ratios (cotangent, secant, cosecant) be required, they may be obtained through the use of a convenient reciprocal scale.

5.2 THE TRIGONOMETRIC SCALES

Designation. The three trigonometric scales are located on the slide and are designated as Cos S, Sec T ST, and T T. They are divided in degrees and decimals of degrees. Each set of graduations is actually two scales in one, for each is numbered in two directions (left to right, and right to left). The two scale designations appearing at the left end of the slide denote the direction of increase of the angular values corresponding to each symbol. The designation on the left corresponds to the angular values at the left of the graduations, and these values increase toward the left (right to left). Likewise, the designation on the right corresponds to the angular values at the right of the graduations, and these values increase toward the right.

Color Coding. The unique color coding of the trigonometric scales is consistent with all of the other scales on the Versatrig. It is easiest to learn for beginners, and helps experienced users to avoid errors, and speed calculations.

The use of the trigonometric scales can most readily be mastered by noting that they are basically C and CI scales for which values of angles (in accordance with the trigonometric function represented) have been substituted for numerical values. Angles are numbered in black when the scale is to be used like a C scale, which is also numbered in black. Angles are numbered in red when the scale is to be used like a CI scale, which is also numbered in red. Therefore, it is only necessary to match the colors—black with black and red with red—when reading trigonometric functions. And when multiplying or dividing by the angular value, use those numbered in black as though they were in fact C scales, and those numbered in red as though they were in fact CI scales. Such is indeed the fundamental nature of these scales—a simple fact that makes their use quite as simple as the C and CI scales.

The Cos S Scale. Numbered entirely in black, the values of both the sine and cosine are read on the C scale. The Cos designation is on the left, and accordingly, the cosine numerals are on the left of the graduations and the angular values increase to the left. The S designation is on the right and therefore, the sine numerals are on the right of the graduations and the angular values increase to the right.

The sine varies from 0.10 to 1.0 for angles varying from 5.74° to 90°, whereas the cosine varies from 1.0 to 0.10 for angles varying from 0° to 84.3°. Therefore all sine and cosine functions for angles shown on the Cos S scale vary from 0.10 to 1.0.

The T T Scale. The T T scale is exclusively for finding tangents of angles. The left T is red and the angular values, also in red, are at the left of the graduations, and increase in magnitude toward the left from 45° to 84.3°. The red color indicates that values of tangents in this range are found on the CI scale. The right T is black and the angular values, also black, are at the right of the graduations, and increase toward the right from 5.7° to 45°. The black color indicates that values of the tangents of angles in this range are found on the C scale.

The tangent varies from 0.10 to 1.0 for angles ranging from 5.7° to 45°. Therefore, for angles shown in black on the T scale, tangents read on C vary from 0.10 to 1.0. Since tangents vary from 1.0 to 10.0 for angles between 45° and 84.3°, the tangent read on CI for angles shown in red must vary from 1.0 to 10.0.

The Sec T ST Scale. As indicated by the red marking on the left, the values of the secant and tangent are at the left of the graduations, and increase toward the left from 84.26° to 89.43°. Secant and tangent functions in this range are read on the CI scale. The values to the right of the graduations are the sine and tangent for angles from 0.57° to 5.74°. Since the numberals are black and to the right of the graduations, values increase to the right and those functions are read on the C scale.

Sines and tangents vary from 0.01 to 0.10 for angles between 0.57° and 5.74°. Therefore, sines and tangents of angles shown in black on the ST scale vary from 0.01 to 0.10. However, tangents and secants vary from 10 to 100 for angles between 84.26° and 89.43°. Therefore, for angles in this range, the secant and the tangent varies from 10 to 100.

Decimal Point Placement. The problem of locating the decimal point when dealing with trigonometric functions is resolved with the knowledge of the ranges of values of the functions corresponding to the range of angles on the trig scales. These values are summarized in the following table.

Trigonometric Functions	Range: Angular	Range: Numerical	Scales for: Angles	Scales for: Numbers	Color of Numerals
Sine	0.57°- 5.74°	0.01- 0.10	Sec T ST	C	Black
Sine	5.7° -90°	0.10- 1.0	Cos S	C	Black
Cosine	0° -84.3°	0.10- 1.0	Cos S	C	Black
Tangent	0.57°- 5.74°	0.01- 0.10	Sec T ST	C	Black
Tangent	5.7° -45°	0.10- 1.0	T T	C	Black
Tangent	45° -84.3°	1.0 - 10.0	T T	CI	Red
Tangent	84.26°-89.43°	10.0 -100	Sec T ST	CI	Red
Secant	84.26°-89.43°	10.0 -100	Sec T ST	CI	Red

The ranges of values in the above table should be memorized. A review of this section may be desirable before continuing to the next. With the use of the slide rule, note particularly the color coding, numerical placement, direction of scale numbering, and the ranges of the scales as summarized above.

5.3 NATURAL TRIGONOMETRIC FUNCTIONS

As suggested in the previous section, the determination of natural trigonometric functions requires only a setting of the hairline and a little knowledge of the numerical range of the function. The scale upon which the function is read is given by the color of the numerals of the trig scale.

Example 5.1

Problem: sin 35° = 0.574

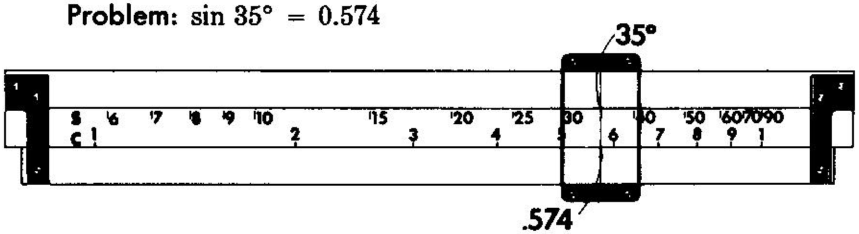


Figure 5.2

Operation: Set hairline to 35° on S.
Read the sine, .574, on C at hairline.

Example 5.2

Problem: sin θ = 0.182
θ = 10.5°

Operation: Set hairline to .182 on C.
Read angle, 10.5°, on S at hairline.

Example 5.3

Problem: cos 16° = 0.961

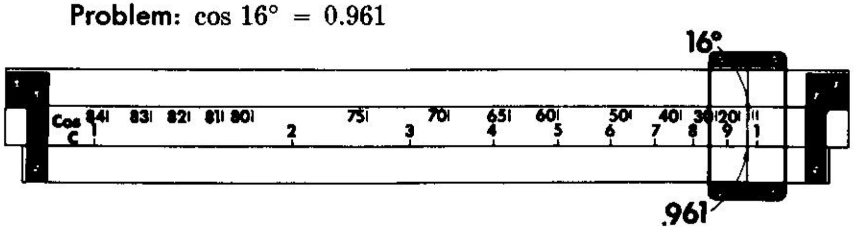


Figure 5.3

Operation: Set hairline to 16° on Cos.
Read cosine, .961, on C at hairline.

Example 5.4

Problem: tan 8° = 0.1405

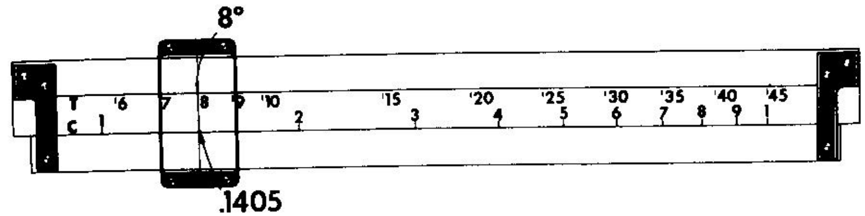


Figure 5.4

Operation: Set hairline to 8° on T.
Read tangent, 0.1405, on C at hairline.

Example 5.5

Problem: tan 85.5° = 12.75

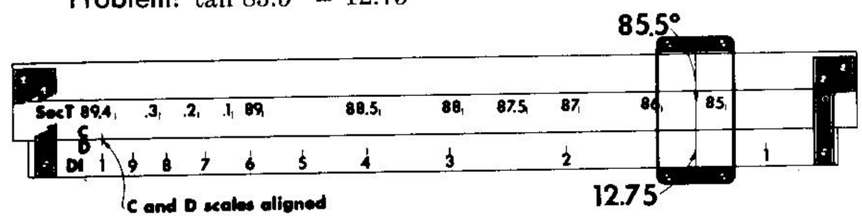


Figure 5.5

Operation: Set hairline to 85.5° on Sec T.
Read tangent, 12.75, on CI at hairline.

As shown in Figure 5.5, if the C and D scales are aligned, the tangent can be read on the DI scale and the rule need not be turned over. Unless many readings are taken, however, it is generally quicker to turn over the rule than to align the C and D scales.

Exercises in Natural Functions. Determine the following natural functions. Angles expressed in minutes and seconds require conversion to decimals of degrees.

126. sin 76°	131. cos 34.5°	136. tan 77.5°
127. sin 54.5°	132. cos 74.7°	137. tan 83.55°
128. sin 15.4°	133. cos 83.5°	138. sec 89° 18′
129. sin 0° 54′	134. tan 15° 42′	139. tan 88.4°
130. sin 3° 51′ 36″	135. tan 49° 18′	140. tan 2° 24′

5.4 LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

The use of logarithms of functions may be required for the solution of particular problems—especially in finding odd powers of the trigonometric functions. The trig scales, used again as though they were C or CI scales, are used with the L scale. The characteristic is easily determined if the ranges of values of each scale are kept in mind. The method of finding the mantissa of logarithms of functions of angles on scales numbered in red differs slightly from the method for scales numbered in black.

For angles on trigonometric scales numbered in black:

1. Align the C and D scales.
2. Set the hairline to the angle on the trigonometric scale.
3. Read the mantissa on the L scale at the hairline.

For angles on trigonometric scales numbered in red:

1. Set the hairline to the angle on the trigonometric scale.
2. Read the function of the angle at the hairline on the CI scale.
3. Set the hairline to the function of the angle on the D scale.
4. Read the mantissa on the L scale at the hairline.

When finding logarithms of angles on red numbered scales, if the C and D scales are aligned, it is, of course, more convenient to use the DI scale instead of the CI scale.

Example 5.6

Problem: log sin 28.2° = 9.674 − 10

Operation: Align C and D scales.
Set hairline to 28.2° on S.
Read mantissa of log sin, .674, on L at hairline.
Characteristic = −1, since sines of angles between 5.7° and 90° range between 0.1 and 1.

Example 5.7

Problem: log cos θ = 9.172 − 10
θ = 81.45°

Operation: Align C and D scales.
Set hairline to .172 on L.
Read θ = 81.45° on Cos at hairline.

Example 5.8

Problem: log tan 15.6° = 9.446 − 10

Operation: Align C and D scales.
Set hairline to 15.6° on T.
Read mantissa of log tan, .446, on L at hairline.
Characteristic = −1, since tangents of angles between 5.7° and 45° range between 0.1 and 1.

Example 5.9

Problem: log tan 79.58° = 0.736

Operation: Set hairline to 79.58° on T.
Read function, 5.44 on CI at hairline, (or if the C and D scales are aligned, on DI at hairline).
Set hairline to 5.44 on D.
Read mantissa of log tan, .736 on L at hairline.
Characteristic = 0, since tangents of angles between 45° and 84.3° range between 1. and 10.

Exercises in Logarithms of Trigonometric Functions. Find the logarithm of the function in exercises 141 through 145, and θ in exercises 146 through 150.

141. log sin 76°
142. log sin 0.9°
143. log cos 34.5°
144. log tan 77.5°
145. log tan 2.4°
146. log sin θ = 9.424 − 10
147. log cos θ = 9.421 − 10
148. log tan θ = 9.449 − 10
149. log tan θ = 1.554
150. log tan θ = 0.654

5.5 COMBINED OPERATIONS

Calculations involving products and quotients of trigonometric functions may be performed using the trigonometric scales without actually reading the functions from the C or CI scales. It is only necessary to remember to use any scale as a C scale when the angles are numbered in black, and as a CI scale when the angles are numbered in red. Examples of this type of computation follow.

Example 5.10

Problem: 9.2 sin 43° cos 70.46° = 2.10

Operation: Set right index of C to 9.2 on D.
Move hairline to 43° on S.
Move right index of C to hairline.
Move hairline to 70.46° on Cos.
Read 2.10 on D at hairline.

Example 5.11

Problem: 10.1 tan 18.5° tan 48° = 3.75

Operation: Set left index of C to 10.1 on D.
Move hairline to 18.5° on T.
Move 48° on T to hairline.
Read 3.75 on D at right index of C.

Example 5.12

Problem: $\dfrac{12.8 \tan 19° \sin 47°}{\cos 25° \tan 32°} = 5.69$

Operation: Set left index of C to 12.8 on D.
Move hairline to 19° on T.
Move right index of C to hairline.
Move hairline to 47° on S.
Move 25° on Cos to hairline.
Move hairline to right index of C.
Move 32° on T to hairline.
Read 5.69 on D at right index of C.

5.6 SOLUTION OF TRIANGLES

In this section, a number of typical trigonometric applications involving right triangles and oblique triangles are illustrated. A familiarity of the trigonometric functions defined in Section 5.1 and a thorough understanding of the numerical ranges of the trigonometric scales as tabled in Section 5.2 is essential.

Right Triangles.

Example 5.13

Problem: Find the length of the hypotenuse (side c) of the triangle in Figure 5.6.

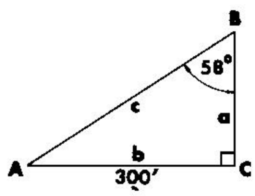


Figure 5.6

Operation: $\sin 58° = \dfrac{300'}{c}$

Set hairline to 300′ on D.
Move 58° on S to hairline.
Read c, 354′ on D at right index of C.

Example 5.14

Problem: Solve the triangle in Figure 5.7 for A, B, and a.

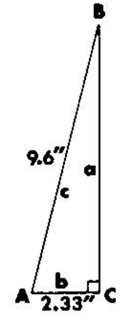


Figure 5.7

Operation: $\sin B = \dfrac{2.33''}{9.6''} = \cos A$

$c \sin A = a$
Set right index of C to 9.6″ on D.
Move hairline to 2.33″ on D.
Read B = 14.05° on S at hairline.
Read A = 75.95° on Cos at hairline.
Move hairline to 75.95° on S.
Read 9.3″ on D at hairline.

Example 5.15

Problem: Solve the triangle in Figure 5.8 for a,b, and B.

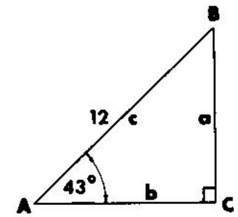


Figure 5.8

Operation: $a = c \sin A$
$b = c \cos A$
$B = 90° - A$
Set left index of C to 12 on D.
Move hairline to 43° on S.
Read a = 8.18 on D at hairline.
Move hairline to 43° on Cos.
Read b = 8.76 on D at hairline.
$B = 90° - A = 90° - 43° = 47°$.

Example 5.16

Problem: Solve the triangle in Figure 5.9 for A,B, and c.

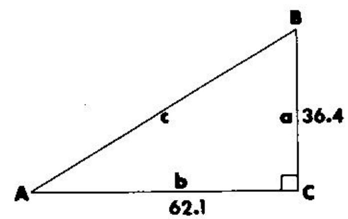


Figure 5.9

Operation (A): $\tan A = \dfrac{36.4}{62.1}$
$B = 90° - A$
$c = \dfrac{36.4}{\sin A}$
Set right index of C to 62.1 on D.
Move hairline to 36.4 on D.
Read A = 30.4° on T (black) at hairline.
Read B = 59.6° on T (red) at hairline.
Move 30.4° on S to hairline.
Read c = 72 on D at right index of C.

Operation (B): a = b Tan A = c sin A can be rewritten as,

$$\frac{1}{\frac{1}{a}} = \frac{\tan A}{\frac{1}{b}} = \frac{\sin A}{\frac{1}{c}}, \text{ therefore}$$

$$\frac{1}{\frac{1}{36.4}} = \frac{\tan A}{\frac{1}{62.1}} = \frac{\sin A}{\frac{1}{c}}$$

Set right index of C to 36.4 on DI.
Move hairline to 62.1 on DI.
Read A = 30.4° on T (black) at hairline.
Read B = 59.6° on T (red) at hairline.
Move hairline to 30.4° on S.
Read c = 72 on DI at hairline.

Example 5.17

Problem: Solve the triangle in Figure 5.10 for A, a, and b.

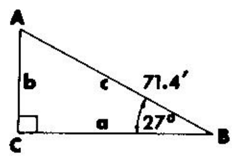


Figure 5.10

Operation: Applying the law of sines, which may be written $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, solve as a proportion.

$$\frac{a}{\sin(90° - 27°)} = \frac{b}{\sin 27°} = \frac{71.4'}{\sin 90°}$$

Set hairline to 71.4′ on D.
Move 90° on S to hairline.
Move hairline to A, 63° (90° − 27°) on S.
Read a = 63.6′ on D at hairline.
Move hairline to 27° on S.
Read b = 32.4′ on D at hairline.

Oblique Triangles. The law of sines, illustrated in Example 5.17, is applicable to any triangle—right or oblique. When the three sides are given, the angles can be determined using the law of cosines, which is:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

While the law of cosines can be written in three forms (one for each angle), it is preferable to use it to find just one angle and use the law of sines, which is easier to calculate, for finding the other angles.

Example 5.18

Problem: Solve the triangle in Figure 5.11 for A, B, and C.

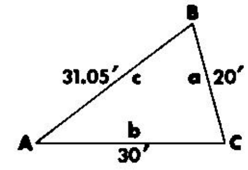


Figure 5.11

Operation: $\text{Cos } A = \frac{900 + 964 - 400}{2(30)31.05} = \frac{1464}{1863}.$

Set right index of C to 1863 on D.
Move hairline to 1464 on D.
Read A = 38.2° on Cos at hairline.

$$\frac{20}{38.2°} = \frac{30}{\sin B} = \frac{31.05}{\sin C}.$$

Set hairline to 20 on D.
Move 38.2° on S to hairline.
Move hairline to 30 on D.
Read B = 68.1° on S at hairline.
Move hairline to 31.05 on D.
Read C = 73.7° on S at hairline.
C can be accurately determined also by 180° − A − B = 73.7°.

Example 5.19

Problem: Solve the triangle in Figure 5.12 for B, C, and a.

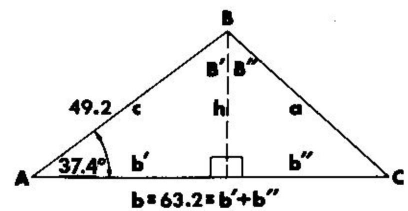


Figure 5.12

Operation: A perpendicular, h, is drawn from B to the base to form two right triangles.

$$\frac{h}{\sin 37.4°} = \frac{49.2}{\sin 90°} = \frac{b'}{\sin B'}.$$

Set right index of C to 49.2 on D.
Move hairline to 37.4° on S.
Read h = 29.8 on C at hairline.
Move hairline to 52.6° on S.
Read b′ = 39.1.
Then, b″ = 63.2 − 39.1 = 24.1.

$$\tan C = \frac{29.8}{24.1}.$$

Set right index of C to 29.8 on D.
Move hairline to 24.1 on D.
Read C = 51° on T.

Then, $\frac{29.8}{\sin 51°} = \frac{24.1}{\sin B''} = \frac{a}{\sin 90°}$.

Set hairline to 29.8 on D.
Move 51° on S to hairline.
Move hairline to 24.1 on D.
Read B″ = 39° on S at hairline.
Read a = 38.4 on D at right index of C (90° on S).
B = 52.6° + 39° = 91.6°.

Exercises in the Solution of Triangles.

151. Determine A, B, and c of the triangle in Figure 5.13.
152. Determine a and c of the triangle in Figure 5.14.
153. Determine A, B, and C of the triangle in Figure 5.15.

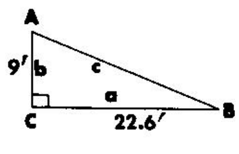


Figure 5.13

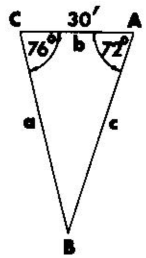


Figure 5.14

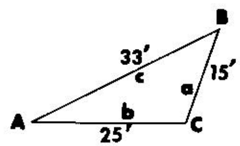


Figure 5.15

5.7 VECTOR ANALYSIS AND COMPLEX NUMBERS

From the standpoint of the mechanics of the computation, vector analysis is essentially an application of right triangle analysis and solution. The theory of complex numbers deals with subject matter only slightly more complicated, but conveniently solved with the Versatrig slide rule.

Vector Analysis. A vector is a quantity having both magnitude and direction. Using rectilinear coordinates, x and y, a vector quantity is completely described by an x component and a y component which are combined (i.e., the vector sum is taken) to form resultant R, oriented at some angle θ with the horizontal.

$$x = R \cos \theta$$
$$y = R \sin \theta$$
$$R = \sqrt{x^2 + y^2}$$
$$\tan \theta = \frac{y}{x}$$

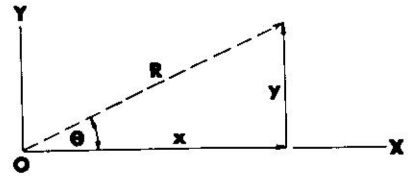


Figure 5.16

Example 5.20

Problem: If R = 8 and θ = 19°, solve for the x and y components.

Operation: Set the right index of C to 8 on D.
Move hairline to 27° on Cos.
Read x = 7.13 on D at hairline.
Move hairline to 27° on S.
Read y = 3.63 on D at hairline.

Complex Numbers. A complex number of the form x + jy where $j = \sqrt{-1}$, may be represented in a complex plane, using a coordinate system, with x being the real axis and jy, the imaginary axis.

The number may be presented graphically in Figure 5.17 by the point R, whose coordinates are x and y.

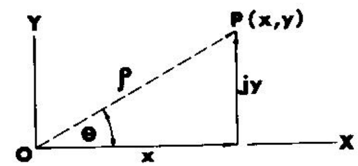


Figure 5.17

It may be shown that $e^{j\theta} = \cos \theta + j \sin \theta$ and therefore $\rho e^{j\theta} = \rho \cos \theta + j\rho \sin \theta$. Here the complex number $\rho e^{j\theta}$ (which is commonly simplified, $\rho e^{j\theta} = \rho\underline{/\theta}$) consists of two parts, $\rho \cos \theta$ being the real part, and $j\rho \sin \theta$, the imaginary part.

The above expression may be simplified by noting that $\rho \cos \theta$ = x and $\rho \sin \theta$ = y. Then $\rho\underline{/\theta}$ = x + jy. The x and y values are determined as vector components of ρ as previously explained. This operation is called changing from exponential form to component form.

Example 5.21

Problem: Change to exponential form: $7.2 + j4.5 = 8.49\underline{/32^\circ}$.

Operation: $\tan\theta = \frac{4.5}{7.2}$

Set right index of C to 7.2 on D.
Move hairline to 4.5 on D.
Read $\theta = 32^\circ$ on T at hairline.

$$\rho = \frac{y}{\sin 32^\circ}$$

Move 32° on S to hairline.
Read $\rho = 8.49$ on D at right index of C.

Example 5.22

Problem: Change to exponential form:

$$17.1 + j29.5 = 34.1\ \underline{/59.9^\circ}.$$

Operation: Set hairline to 17.1 on DI.
Move right index of C to hairline.
Move hairline to 29.5 on DI.
Read $\theta = 59.9^\circ$ on T at hairline (since $jy > x$, $\theta > 45^\circ$).
Move hairline to 59.9° on Cos.
Read $\rho = 34.1$ on DI at hairline.

Example 5.23

Problem: Change to coordinate form: $5.83\ \underline{/39.4^\circ} = 4.5 + j3.7$.

Operation: Set right index of C to 58.3 on D.
Move hairline to 39.4° on S.
Read $y = 3.7$ on D at hairline.
Move hairline to 39.4° on Cos.
Read $x = 4.5$ on D at hairline.

Exercises in Complex Numbers.

154. Determine the x and y components of the vector $R = 16.8$, if $\theta = 54^\circ$.

155. Solve for x and y in the equation $21\underline{/27^\circ} = x + jy$.

156. Change the complex number $14 + j8.9$ to exponential form.

5.8 ANGLES IN RADIANS

Angles in radians may be converted to angles in degrees by use of a multiplication factor. Since one radian is equal to $\frac{180}{\pi} = 57.3^\circ$, the angle in radians multiplied by 57.3 equals the angle in degrees. For convenience, a graduation designated r has been placed at this point on the C and D scales on one face of the rule.

Example 5.24

Problem: Convert to degrees; 0.53, 2.19 and 1.43 radians.

Operation: Set right index of C to r on D.
Move hairline to 0.53 on C.
Read 30.4° on D at hairline.
Move hairline to 2.19 on C.
Read 125.5° on D at hairline.
Move hairline to 1.43 on CF.
Read 82° on DF at hairline.

Example 5.25

Problem: Convert to radians; 30.4°, 125.5°, and 82°.

Operation: Set r on C to right index of D.
Move hairline to 30.4° on C.
Read 0.53 on D at hairline.
Move hairline to 125.5° on C.
Read 2.19 on D at hairline.
Move hairline to 82° on CF.
Read 1.43 on DF at hairline.

5.9 VERY SMALL ANGLES

On the Versatrig ST scale, two special graduations are provided for angles smaller than one degree. Since for very small angles, the sine function, tangent function, and the angle in radians are very nearly equal, these three functions may be used interchangeably.

A special graduation, identifed by a single dot, on the ST scale opposite approximately 3,440 on the C scale is provided for the conversion of minutes to radians. One minute is equal to $\frac{\pi}{180 \times 60} = \frac{1}{3{,}437}$ radians.

Another graduation, identified by a double dot on the ST scale opposite approximately 206,000 on the C scale, is provided for the conversion of seconds to radians. One second is equal to $\frac{\pi}{180 \times 60 \times 60} = \frac{1}{206{,}240}$ radians.

Example 5.26

Problem: Determine the angle in radians of 0°49.2′ or sin 0°49.2′, or tan 0°49.2′ = 0.0143.

Operation: Set hairline to 49.2 on D.
Move the minute mark on ST to the hairline.
Read 0.0143 on D at left index of C.

Example 5.27

Problem: Determine the angle in radians of 0°0′29″, or sin 0°0′29″, or tan 0°0′29″ = 0.0001406.

Operation: Set hairline to 29 on D.
Move second mark on ST to hairline.
Read 0.0001406 on D at index of C.

CHAPTER 6

PRACTICAL APPLICATIONS

In this chapter, the use of the Versatrig in solving problems chosen from the fields of mechanical, electrical and civil engineering, chemistry and statistics, is demonstrated. Equations, where used, are given without derivation. In reviewing these examples, the reader is urged to follow the operations, recognize why each step is employed, and why it accomplishes its objectives. In this way, a more thorough understanding of the rule will be developed, and soon, complete mastery of its operations.

Surveying: Inaccessible Distances. In running a survey line, obstacles may occur on the line of sight. It may be necessary to extend the line by indirect methods. Figure 6.1 illustrates a method for passing an obstacle by use of angular deflections.

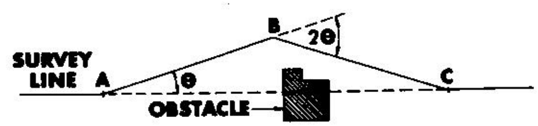


Figure 6.1

Point B is a point visible from A. The procedure then, is to measure distance AB and the angle θ. The angle B is then taken as 2 θ and the length BC as equal to AB. By sighting along BC, point C may be located. Distance AC is then 2(AB) cos θ.

For example, if AB measures 94′ and θ = 21.8°, then AC = 2(94′) cos 21.8° = 174.6′. Set right index of C to 94′ on D, move hairline to 21.8° on Cos, move right index of C to hairline, read 174.6′ on D opposite 2 on C.

Exercise 157. The deflection angle measured by transit at A was 37° and the taped distance AB was 86 ft. Determine the length AC.

Structural Drafting: The Miter Joint. A miter joint is one in which intersecting members meet on a common line of contact. In detailing the top chord members of a bridge truss, and in other cases, it is important to know the angle or bevel of the line of intersection. Figure 6.2 indicates the manner in which this angle, designated ϕ, may be determined. The depths d_1 and d_2 and angles of slope θ_1 and θ_2 are known for the intersecting members. The tangent of angle ø may be determined from the formula.

$$\text{Tan } \phi = \frac{d_1 \cos\theta_2 - d_2 \cos\theta_1}{d_2 \sin\theta_1 - d_1 \sin\theta_2}$$

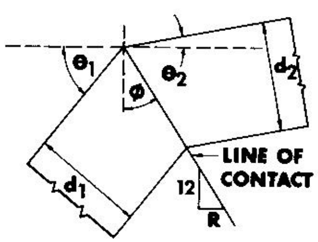


Figure 6.2

As an example: $d_1 = 12\frac{3}{4}''$, $d_2 = 12\frac{1}{2}''$, $\theta_1 = 48°$, and $\theta_2 = 9° - 30' - 45''$ or 9.51°.

Then $\tan \phi = \dfrac{12.75 \cos 9.51° - 12.5 \cos 48°}{12.5 \sin 48° - 12.75 \sin 9.51°} = \dfrac{12.58 - 8.36}{9.28 - 2.10} = \dfrac{4.22}{7.18} = 0.588.$

$\phi = \text{arc tan } 0.588 = 30.4°.$

The bevel R = 12 tan ø = 7.06″ or 7 1/16″. This bevel, calculated by slide rule, was also calculated by 5-place logarithmic tables. Both methods give the same result to the nearest 1/16″.

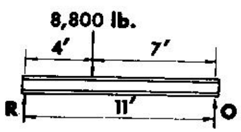
Exercise 158. Determine the bevel R for miter joints having the following properties:

	θ_1	θ_2	d_1	d_2
(a)	40°	11°	14″	12″
(b)	49°	8°	$11\frac{1}{2}''$	$9\frac{1}{2}''$
(c)	52°	0°	$7\frac{1}{2}''$	$7\frac{1}{2}''$

Structural Analysis: A Steel Beam. In determining the stresses in a beam, it is necessary to determine the forces acting; the shear, and the bending moment. The internal stresses in the material are then determined from the theory of strength of materials.

Figure 6.3 illustrates a steel beam designed to carry a concentrated load of 8,800 lbs. on a span of 11′.

Figure 6.3



8WF17
S = 14.1 (in.)³
Aw = 1.84 (in.)²

Its section modulus, S, and web area, Aw, are given for use in determining the moment and shear stresses in the material.

The left reaction force, R, may be determined by proportion, utilizing the distances from point 0 at the right end of the span. $\dfrac{R}{8{,}800} = \dfrac{7'}{11'}$, from which R = 5,600 lbs. This force is the shear acting at the left of the load. It will be resisted primarily by the web area. Hence, the shearing unit stress $\dfrac{5{,}600}{1.84} = 3{,}040$ lbs. per sq. in.

The moment under the 8,800 lb. load will be the product R × 4′ = 5,600(4) = 22,400′ lbs. The unit stress due to this bending moment will be $\dfrac{22{,}400(12)}{14.1} = 19{,}060$ lbs. per sq. in.

Exercise 159. Solve for the end reaction R_L and R_R and for the moment in the beam at each load point. Calculate the maximum bending stress for the loads shown if the section modulus of the beam is 107.8 (in.)³.

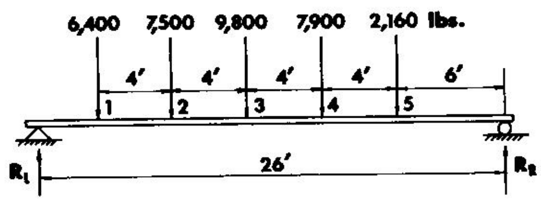


Figure 6.4

Resistance Changes Resulting from Temperature Changes. Resistances of metallic conductors increase with increasing temperature. The formula representing this change is most conveniently expressed as a proportion, $\dfrac{R_2}{R_1} = \dfrac{234.5 + t_2}{234.5 + t_1}$, where R_2 is resistance at centigrade temperature t_2, and R_1 is resistance at t_1. The constant 234.5 is suitable for "standard annealed copper." Other constants are required for other materials. The slide rule C and D scales are very convenient for the solution of any proportion. For example, the field winding of a motor has 56 ohms resistance at an ambient temperature of 25° C. After full load operation for two hours the resistance is found to be 74.3 ohms. What average temperature was reached by the winding?

The slide rule is used to find $234.5 + t_2$ in $\dfrac{74.3}{56} = \dfrac{234.5 + t_2}{259.5}$. The procedure using proportion is to bring 74.3 on the C scale in register with 56 on the D scale. Then set the hairline to 259.5 on D and read $234.5 + t_2 = 344.5$ on C. $t_2 = 344.5 - 234.5 = 110°$C. Any one of the four quantities R_2, R_1, t_2, or t_1 may, of course, be the unknown.

Exercise 160. A 100 watt tungsten filament lamp operating at 2,200°C has a resistance of 132 ohms. What is the resistance just after switching on, before the temperature has had a chance to rise above room temperature of 20°C? The constant for tungsten is 202.

Circular-mil Areas of Rectangular Conductors. The cross-sectional area expressed in circular-mils of a rectangular conductor is found as follows: Area $= \frac{4ab}{\pi}$ circular mils, when a and b are the cross-section dimensions in mils.

Find the cross-section in circular mils of a rectangular rotorconductor in an induction motor that has a cross section $\frac{1}{4}''$ by $\frac{1}{2}''$.

Area $= \frac{4 \times 250 \times 500}{\pi} = 159{,}100$ circular mils. Here the important thing is to make economical use of the π folded scales. After noting that $4 \times 250 = 1{,}000$, it is only necessary in this case, to set the hairline on 500,000 on the DF scale, and read 159,100 on D at the hairline.

Exercise 161. What is the cross-section in circular mils of a bus bar 0.25 inches thick and 3.5 inches wide?

Copper Loss in Wires and Machines when the Current and the Resistance are Known. For a current of 120 amperes in a resistance of 0.076 ohms, to find the power dissipated using $P = I^2R$; set the hairline to 120 on R_1, move 0.076 on CI to the hairline, read the result on D at the index of C. P = 1,094 watts.

Copper Loss in Wires and Machines when the Potential Drop and the Resistance are Known. For a voltage drop of 9.11 volts in a resistance of 0.076 ohms, to find the power dissipated using $P = E^2/R$; set hairline to 9.11 on R, move 0.076 on C to hairline, and read result on D under left index of C. P = 1,092 watts.

Note that in these operations, greater accuracy is possible than with slide rules employing the conventional "A" and "B" scales and setting the decimal point is simplified.

Power Factor for Phase Angles Less than 10 Degrees. We may use the approximation $\cos x = 1 - \frac{x^2}{2}$. The cosine scale on the slide rule is so condensed below 10 degrees that accurate interpolation is diffi-

cult. When a cosine in this range must be accurately known, as is often the case in power factor problems, the approximation given above may be used to advantage.

The phase angle in radians is x. The upper limit at which this approximation should be applied is 10° = 0.1745 radians. Let us calculate cos (0.1745 radians) according to the approximation and compare the result with a five-place table. The error made will be the maximum, since for smaller values of x, the method becomes more accurate.

$x = 0.1745$ radians.

$x^2 = 0.03045$ radians, using the R scale in the usual way with the D scale.

$\frac{x^2}{2} = 0.01523.$

$1 - \frac{x^2}{2} = 0.98477 = \cos 10°$ approximately.

From a five-place table, cos 10° = 0.98481. The difference is 0.00004.

Logarithmic Power Ratios. The L scale is useful for calculation of logarithmic power ratios in terms of decibels by either of the formulae:

$$\text{d.b.} = 10 \log_{10}\frac{P_2}{P_1} \quad \text{or} \quad \text{d.b.} = 20 \log_{10}\frac{V_2}{V_1}.$$

Let $\frac{P_2}{P_1} = 460$. Set the hairline to 460 on D. Read the mantissa of $\log_{10}460 = 0.663$ at the hairline on L. Mentally determine the characteristic of the logarithm and add it to the mantissa, thus: 2.663. Then d.b. = 26.63. If data from the same physical situation had been in terms of voltage ratio, this would have been $\frac{V_2}{V_1} = 21.45$. Proceeding as before, $\log_{10}21.45 = 1.3315$ or d.b. = 26.63.

Sometimes it is necessary to calculate the power ratio corresponding to a known number of decibels change in power level. The relationship is expressed by the equation $\frac{P_2}{P_1} = \text{Log}_{10}^{-1}\left(\frac{\text{d.b.}}{10}\right)$. Let d.b. = 26.63. $\frac{\text{d.b.}}{10} = 2.663 = \log_{10}\frac{P_2}{P_1}$. Set hairline to .663 on L. Read at the hairline on D the digits 460 representing $\frac{P_2}{P_1}$. The decimal point is placed after the third digit, because the characteristic of the logarithm 2.663. is 2.

If the voltage ratio is desired from the above data,

$$\frac{V_2}{V_1} = \sqrt{\frac{P_2}{P_1}} = \sqrt{460} = 21.45$$

may be obtained from the D and R scales in the usual way.

Heat Transfer: Radiation. Heat may be transferred from one surface to another by radiation. The general relation expressing the interchange of heat between two surfaces may be expressed by the equation,

$$Q = 0.173\ F_A F_E A\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right], \text{ where;}$$

- Q = heat transferred by radiation, btu/hr,
- F_A = an angle factor which depends upon the geometry of the surfaces and their relative positions, dimensionless,
- F_E = an emissivity factor which depends upon the ability of the surfaces to absorb and emit energy, dimensionless,
- A = area of one of the surfaces, the choice of which depends upon the method of evaluating F_A, ft^2,
- T_1 = absolute temperature of the warmer surface, deg. R. = 460 + F,
- T_2 = absolute temperature of the cooler surface, deg. R. = 460 + F.

Find the heat transferred per square ft. of surface of one of two parallel plates if the angle factor is unity and if the emissivity factor is 0.154. The temperature of the two surfaces are 400°F and 60°F respectively.

$$Q = 0.173 \times 1 \times 0.154\left[\left(\frac{460+400}{100}\right)^4 - \left(\frac{460+60}{100}\right)^4\right] =$$
$$= 0.02665\ [(8.60)^4 - (5.20)^4]$$
$$= 126.6 \text{ btu/hr ft}^2.$$

The values of $(8.60)^4$ and $(5.20)^4$ are determined by the use of the R and A scales. Set hairline to $(8.60)^4$ on R_2 and read 5,480 on A. Set hairline to $(5.20)^4$ on R_2 and read 730 on A. Then Q = 0.02665(5,480 −730) = 0.02665 × 4,750 = 126.6.

Exercise 162. A bare steam pipe passes through a room whose walls are at a temperature of 70° F. If the surface temperature of the pipe is 325° F, find the rate at which heat is lost to the walls per square ft. of pipe surface as a result of radiation. For this case, assume F_A = 1.00 and F_E = 0.90.

Displacement and Velocity of the Piston of a Reciprocating Engine.

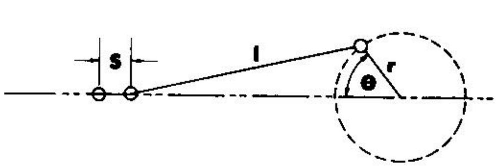


Figure 6.5 Connecting Rod and Crank

A crank and connecting rod similar to that employed on reciprocating engines for the converting of rectilinear motion to rotary motion is shown in Figure 6.5. With this mechanism, two important problems arise. These are the determination of piston displacement and piston velocity as a function of crank angle θ.

The two quantities may be expressed by the equations:

$$S = r\left(1 - \cos\theta + \frac{1}{2}\frac{r}{l}\sin^2\theta + \frac{1}{8}\left(\frac{r}{l}\right)^3\sin^4\theta + \ldots\right), \text{ and}$$

$$V = 2\pi N r\left(\sin\theta + \frac{r}{l}\sin\theta\cos\theta + \frac{1}{2}\left(\frac{r}{l}\right)^3\sin^3\theta\cos\theta + \ldots\right) \text{ where;}$$

- S = piston displacement, ft. or inches,
- l = length of piston rod, same units as S,
- r = radius of crank, same units as S,
- θ = crank angle, degrees,
- V = piston velocity, ft/min or in/min depending on units chosen for l and r,
- N = revolutions per minute.

These equations may be solved with a high degree of accuracy by including the last term, but in general, this may be neglected.

For example, find the piston displacement in inches and piston velocity in ft/min for an internal combustion engine operating at 3000 rpm, if θ = 68 degrees, l = 8 in, and r = 3 in. Answer: 2.38 in., 4,950 ft/min.

$$S = 3\left[1 - \cos 68° + \tfrac{1}{2} \times \tfrac{3}{8}\sin^2 68° + \tfrac{1}{8}\left(\tfrac{3}{8}\right)^3 \times \sin^4 68° + \ldots\right], \text{ and}$$

$$V = \frac{2\pi \times 3000 \times 3}{12}\left[\sin 68° + \tfrac{3}{8}\sin 68° \cos 68° + \tfrac{1}{2}\left(\tfrac{3}{8}\right)^3 \sin^3 68° \cos 68° + \ldots\right].$$

Using the S scale in conjunction with the C scale, both the sin and cos of 68° are found to be 0.934 and 0.3745 respectively. The remaining steps are simple and need not be explained in detail.

$$S = 3[1 - 0.3745 + \tfrac{3}{16} \times 0.934^2 + \tfrac{1}{8}\left(\tfrac{3}{8}\right)^3 \times 0.934^4]$$
$$= 3[1 - 0.3745 + 0.1636 + 0.00406] = 2.38 \text{ in.}$$
$$V = 4615[0.934 + \tfrac{3}{8} \times 0.934 \times 0.3745 + \tfrac{1}{2} \times \left(\tfrac{3}{8}\right)^3 \times 0.934^3 \times 0.3745]$$
$$= 4615[0.934 + 0.1312 + 0.00805] = 4{,}950 \text{ ft/min.}$$

As another example, let θ = 185° in the above example and neglect the last term.

Since sin 185° = −sin 5° and since cos 185° = −cos 5° one may find from the ST and S scales in conjunction with the C scale the following:

Sin 185° = −sin 5° = −0.0871 (from ST scale)
Cos 185° = −cos 5° = −0.996 (from S scale),

then

$$S = 3[1 + 0.996 + \tfrac{1}{2} \times \tfrac{3}{8} \times 0.0871^2] = 5.99 \text{ in.}$$
$$V = 4615[-0.0871 + \tfrac{3}{8} \times 0.0871 \times 0.996] = -252 \text{ ft/min.}$$

The negative sign for velocity in this case simply means that the piston in Figure 6.5 is traveling from right to left.

Exercise 163. Find the piston displacement and velocity for a steam engine operating at 150 rpm, if $\theta = 80°$, 1 = 3 ft., and r = 0.75 ft. Solve the same problem for θ = 12 degrees.

Composition and Resolution of Forces. A group of two or more forces acting upon a body comprise a force system. Each of the forces in a force system may be resolved into two or more components. Commonly, rectangular X and Y axes are used as the basis for resolution computations, and then, for a known force F;

Component in X direction $= F_x = F \cos\theta$
Component in Y direction $= F_y = F \sin\theta$

Where θ is the angle that the force makes with the horizontal or X-axis, which may be defined as; $\theta = \tan^{-1} \frac{F_x}{F_y}$. It may also be necessary to combine a known group of forces into a single resultant force whose action is equivalent in both magnitude and direction to the combined effect of the component forces. This procedure is known as the composition of forces. The following are formulas for the solution of the resultant. $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$\theta = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

As shown in the figure at the right, a body is acted upon by three forces; F_1 = 22 lb, F_2 = 17 lb, and F_3 = 38.5 lb. The forces act at angles with the horizontal; θ_1 = 17°, θ_2 = 0°, θ_3 = 52.3°.

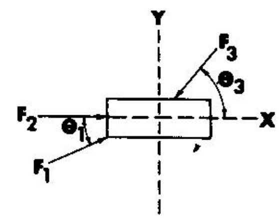
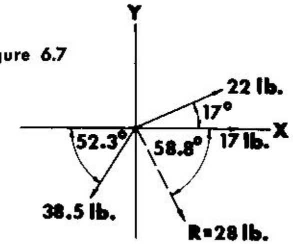


Figure 6.6

To compute the magnitude and direction of the resultant force, Figure 6.7 may be constructed with a set of coordinate axes through the center of gravity of the body, and the following table constructed.

Figure 6.7



Force	Vertical Component		Horizontal Component	
F_1=22 lb	22 sin 17° =	6.43 lb	22 cos 17° =	21.0 lb
F_2=17 lb	sin 0° =	0.00 lb	cos 0° =	17.0 lb
F_3=38.5 lb	−38.5 sin 52.3° =	−30.40 lb	−38.5 cos 52.3° =	−23.5 lb
	ΣF_y	= −23.97 lb	ΣF_x	= +14.5 lb

In each case, the S and Cos scales are used as C scales in multiplying the angular values.

$$\text{Then } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(14.5)^2 + (-23.97)^2}$$
$$R = 28 \text{ lbs. (magnitude)}$$

Set hairline to 14.5 lbs. on R_1 and read 210 lbs., on D, set hairline to −23.94 lbs. on R_1 and read 574 lbs. on D. Opposite 784 on D, read 28 lbs. on R_1.

$$\text{The direction is given by } \tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-23.97}{14.5}$$
$$\theta = -58.8°.$$

Divide −23.97 (approx.) by 14.5 using the D and C scales to find −1.654. Set the hairline to −1.654 on CI and read θ = −58.8° on T at the hairline. The resultant is shown by the dashed line in the figure.

Exercise 164. Find the magnitude and direction of the resultant force in Figure 6.8.

F_1 = 47.3 lb.	θ_1 = 25.6°
F_2 = 19.8 lb.	θ_2 = 31.3°
F_3 = 72.2 lb.	θ_3 = 47.1°
F_4 = 35.5 lb.	θ_4 = 0°

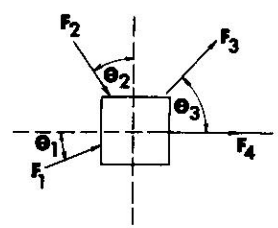


Figure 6.8

Density and Specific Gravity. Density is simply defined as mass per unit volume. Density $= \dfrac{\text{mass of body}}{\text{volume of body}}$. Common units of density are pounds per cubic foot (lb/ft^3) and perhaps most common in the field of chemistry, grams per cubic centimeter, (gm/cm^3).

The specific gravity is an expression of the ratio of the density of a body to that of water under standard conditions.

Specific Gravity $= \dfrac{\text{density of given body}}{\text{density of water at 4°C.}}$

4°C is the temperature at which water has its maximum density, (62.4 lb/ft^3). Since the specific gravity is a ratio, there are no units involved.

To compute the density and specific gravity of a body that weighs 75.6 lbs. and has a volume of 0.82 cu. ft.,

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{75.6}{0.82} = 92.2 \text{ lb/ft}^3$$

Align 75.6 on D with 0.82 on C and read 92.2 at the index of C.

$$\text{Specific Gravity} = \frac{\text{density of body}}{\text{density of water}} = \frac{92.2}{62.4} = 1.48$$

With the slide in the same position, move the hairline to 62.4 on CI, and read 1.48 on D at the hairline.

Exercise 165. The specific gravity of a certain metal is 6.48. Compute its density in English and metric units. What volume will 90 lbs of the metal occupy?

A Gravity Dam. Gravity dams are structures in which the weight of the dam itself is utilized to balance the pressure of water and to prevent overturning. In calculating the pressures on the base and in investigating the stability of such structures, the weight of each part of the dam is calculated separately. Then the moment of all forces about a common point is determined and the resultant force acting on the base of the dam is located.

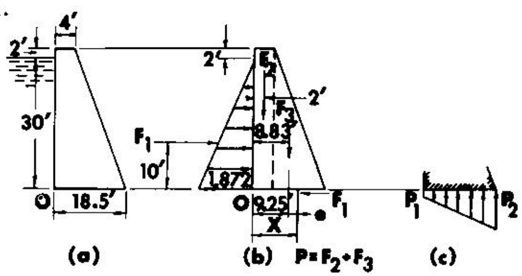


Figure 6.9

In Figure 6.9 (a), a simple dam is shown which is to retain a 30′ head of water weighing 62.4 lb. per cu. ft. For analysis, a width of dam of one foot, perpendicular to the figure, is used. The dam is to be constructed of concrete assumed to weigh 145 lb. per cu. ft. Figure 6.9 (b) indicates the separate forces and their locations. Distance x, locating the resultant vertical component P, is to be determined.

The water pressure acts horizontally and at the bottom of the dam has an intensity of 62.4(30′) = 1,872 lb. per foot. The total force due to water pressure is the area of the force triangle and is termed F_1. $F_1 = \frac{30}{2}(1{,}872) = 28{,}100$ lb. The forces F_2 and F_3 are due to the weight of concrete, F_2 being equal to 145(4)32 = 18,600 lb. and F_3 being $\frac{145(14.5)32}{2} = 33{,}600$ lb. The moment of forces F_1, F_2, and F_3 about point 0 is balanced by the moment $P \cdot x$, shown in Figure 6.9 (b). Therefore (18,600 + 33,600) x = 28,100(10) + 18,600(2) + 33,600(8.83).

$$x = \frac{615{,}000}{52{,}200} = 11.78'.$$

The eccentricity e, measured from the center line of the base, is then 11.78′ − 9.25′ = 2.53′. Pressures p_1 and p_2 indicated in Figure 6.9 (c) may now be determined from the equations $p_1 = \frac{P}{L}\left(1 - \frac{6e}{L}\right)$ $\frac{52{,}200}{18.5}\left(1 - \frac{6(2.53)}{18.5}\right) = 2{,}820(1 - 0.821) = 506$ lb. per sq. ft.; and $p_2 = \frac{P}{L}\left(1 + \frac{6e}{L}\right) = 2{,}820(1 + 0.821) = 5{,}140$ lb. per sq. ft. The resultant is located within the base, since x < 18.5′, which indicates that the dam is stable and will not overturn.

Exercise 166. Determine the base pressures p_1 and p_2 for a dam 30 ft. high if the width is 3 ft. at the top and 14 ft. at the bottom. The depth of water retained is to be 24 ft. and the weight of masonry is 145 lbs./cu. ft.

Statistical Analysis: Normal Distribution. Statistical analysis is a body of methods enabling more informed decisions in the face of uncertainty. The Versatrig can be used to readily solve many problems in this area. In testing the hypothesis that the sample mean $\overline{X}$ is not significantly greater than the population mean μ when the population variance σ^2 is known, it is necessary to solve the following equation for z. $z = \dfrac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}$. If $z > z_{1-\alpha}$, the hypothesis is rejected at the

1 − α level of significance. As an example, a particular type of machine produces an average of 18.5 units per hour. The variance of this output is 2.0 units. A modification becomes available that will improve production, but it will be uneconomical unless production is increased to at least 20 units per hour. Management is willing to take a 20% risk of accepting the modification when it is not economical ($\alpha = .20$). From a table of the Normal Distribution, $z_{.80} = 0.842$. A sample of twelve modified machines is tested and produce a mean of 20.3 units per hour. The hypothesis being tested is that the average output of all modified machines is not greater than 20 units per hour.

$$z = \frac{20.3 - 20}{\sqrt{\frac{2.0}{12}}} = \sqrt{\frac{(0.3)^2(12)}{2.0}} = 0.734$$

Set the hairline at 0.3 on R_1, move 12 on CI to the hairline, move the hairline to 2.0 on CI, and read 0.734 on R_2. Since z, 0.734, is not greater than $z_{.80}$, 0.842, the hypothesis is accepted and the modification is not purchased.

Statistical Analysis: Binomial Distribution. The probability of having r successes in a random sample of N from a population with the parameter P is expressed in the Binomial Distribution as:

$$Pr = C\binom{N}{r}P^r(1 - P)^{N-r}.$$

What is the probability of finding exactly 2 defects in a random sample of 10 machine parts from a lot that contains 5% defective parts? In this case:

N = the sample size
r = the number of defects in the sample
P = the proportion of defects in the population

By substitution, the probability is expressed as:

$$Pr = \frac{10!}{8!\,2!}(0.05)^2(0.95)^8 = 45\ (0.0025)\ (0.664) = 0.0746$$

To solve, set the hairline to .95 on R_2 and read $(.95)^4 = .815$ on A at the hairline. Set the hairline to .815 on R_2 and read $(.95)^8 = (.815)^2 = .664$ on D at the hairline. Set the hairline to .05 on R_2 and read .0025 on D at the hairline. Cancelling 8! in both the numerator and denominator, 9 on D multiplied mentally by 10 and divided by 2 on CI equals 45 on D, slide 0.0025 on CI to hairline and read 0.0746 on D opposite 0.664 on C. Thus the probability of finding exactly 2 defects in this lot is 0.0746, or about a 7½% chance.

CHAPTER 7

THE PRINCIPLE OF THE SLIDE RULE

This chapter briefly describes how the slide rule works. While this knowledge is not essential for efficient use of the rule, it may satisfy one's curiosity or enable one to reason through a correct computation method when the exact procedures may have been forgotten.

Multiplication and Division. Those who have a little facility with mathematics will recognize that the slide rule is based on logarithms. Multiplication and division can be accomplished by the addition and subtraction of logarithms of numbers. $N_1 \times N_2 = \log N_1 + \log N_2$. Using the slide rule to multiply and divide is, in effect, the addition and subtraction of logarithms—actually, the addition and subtraction of logarithmic lengths of the scales. The graduations of the C and D scales are measured lengths (from the left index), proportional to logarithms of numbers between 1 and 10. Or, more generally, the graduations of the C and D scales are lengths proportional to the mantissa of common logarithms of all numbers. Considering just the integers on the C and D scale of a 10 inch slide rule (whose scales are 25 centimeters, or approximately 9.84 inches long), the scale lengths might be tabled as follows:

Number (N)	Logarithm of Number (log N)	Distance in Inches from Left Index (9.84″ log N)
1	0.	0.
2	0.301	2.96
3	0.477	4.70
4	0.602	5.93
5	0.699	6.88
6	0.778	7.66
7	0.845	8.32
8	0.903	8.89
9	0.954	9.39
10	1.	9.84

Several simple examples of the addition and subtraction of logarithmic lengths using the slide rule follow. Similar addition and subtraction can be applied to more involved computations. Use the L scale to verify these examples.

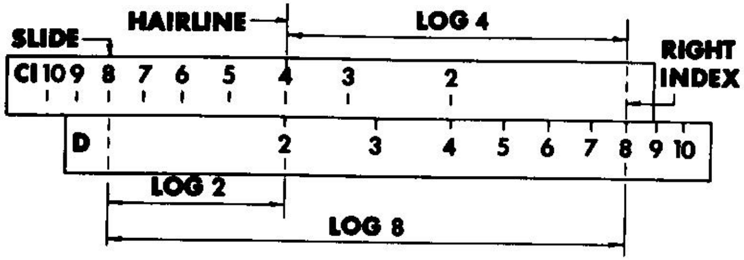


Figure 7.1

Example 7.1

Problem: $2 \times 4 = 8$

Solution: $\log 2 + \log 4 = \log 8$
$0.301 + 0.602 = 0.903$

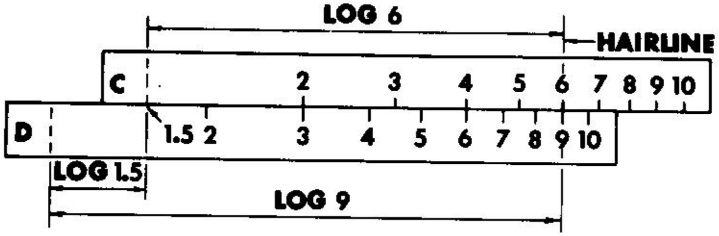


Figure 7.2

Example 7.2

Problem: $1.5 \times 6 = 9$

Solution: $\log 1.5 + \log 6 = \log 9$
$0.176 + 0.778 = 0.954$

Example 7.3

Problem: $\frac{9}{6} = 1.5$

Solution: $\log 9 - \log 6 = \log 1.5$
$0.954 - 0.778 = 0.176$

Since the characteristic of logarithms of numbers on the slide rule is not represented, the operator must keep track of it. Use of the standard form, as described in Section 2.5 is recommended—especially for involved calculations and when working with very large and small numbers.

Construction of the inverted scales, CI and DI, is the same as the C and D scales, except that distances are measured from the right index rather than the left. With a correction of the decimal point, at any setting of the hairline, values M on the DI scale are reciprocals of values N on the D scale. Since $\log N + \log M = \log 10$, the entire scale length, then $\log N = \log \frac{10}{M}$, and $N = \frac{10}{M}$.

The folded scales are simply C and D scales that start at π rather than 1. At any setting of the hairline, N on the D scale is opposite πN on the DF scale.

Powers and Roots. Numbers on the A scale are squares of numbers opposite on the D scale. Therefore, any length log N on the D scale equals $\log N^2$, or 2 log N length on the A scale. Thus, the range of the A scale is 2 log 10, or from 1 to 100.

Example 7.4

Problem: $3^2 = 9$

Solution: $2 \log 3 = \log 9$
$2 \times 0.477 = 0.954$

Similarily, the construction and function of the R_1 and R_2 scales can be more easily understood if they were thought of as one 20 inch scale opposite two 10 inch D scales. The relationship between the R and D scales would then be identical to the relationship between the D and A scales, except the scale length would be 19.685 inches and computations performed with only half the error.

Numbers on the K scale are third powers of numbers opposite on the D scale. Therefore, any length log N on the D scale equals $\log N^3$ or 3 log N length on the K scale. The range of the K scale is 3 log 10, or from 1 to 1,000.

Trigonometric Operations. As described in Chapter 5, the black trigonometric scales are simply C scales renumbered, and the red trigonometric scales, renumbered CI scales. The trigonometric scales are not divided in 10 major divisions, but graduated in degrees and decimals of degrees of the mantissa of the logarithm of the trigonometric function represented. Therefore, any length log N on the C scale, is equal to log sin θ on the S scale, log cos θ on the Cos scale, etc.

Operations involving multiplication and division of angular values on one or more of the trigonometric scales is simply the addition or subtraction of logarithmic lengths. For example, $N \sin \theta = \log N + \log \sin \theta$.

Example 7.5

Problem: $2 \times \cos 60° = 10$

Solution: $\log 2 + \log \cos 60° = \log 10$
$0.301 + 0.699 = 1.000$

Logarithms. The L scale is simply a uniformly graduated scale ranging from 0 to 1. While this scale is called the L or logarithm scale, it is the only scale on the Versatrig that is not logarithmically graduated. The mantissa of common logarithms of values on the appropriate logarithmically graduated scales are directly opposite on the uniformly graduated L scale.

Effects of Errors in Reading the Scale. When the hairline is set incorrectly or the reading is made incorrectly, the effect may be evaluated by use of the scale equation, $x = 9.84 \log_{10} N$, in which x is the distance in inches from the left end of the C or D scale to any number N appearing on the scale. Taking the derivative of both sides with respect to N, the following equation results: $\frac{dN}{N} = 2.30 \left(\frac{dx}{9.84}\right)$. The term $\frac{dN}{N}$ is the relative error in the number N, while $\frac{dx}{9.84}$ is the relative error in reading or setting the hairline. Therefore the relative error in the number is independent of the size of the number or its location on the scale, and is 2.30 times the relative error in reading the scale.

ANSWERS TO EXERCISES

READING THE SCALES—Page 7.

1.	a 3	b 6	c 1.5	d 2.3	e 9.05
2.	a 11.1	b 17.5	c 31	d 79.9	e 99.7
3.	a .01005	b .02	c .051	d .0842	e .0902
4.	a 1.1	b 9.9	c 7	d 1.31	e 8.03
5.	a 150	b 200	c 41	d 499	e 835

MULTIPLICATION—Page 12.

6.	24	12.	12
7.	84	13.	563
8.	8,400	14.	109.5
9.	37.0	15.	1,020
10.	6,510	16.	831
11.	50.7	17.	209

Use D and C scales, exercises 6 to 11.

Use D and CI scales, exercises 12 to 17.

DIVISION—Page 16.

18.	3.02	24.	4.27	30.	1.431
19.	2.89	25.	0.814	31.	1.670
20.	2.84	26.	0.0650	32.	2.11
21.	23.3	27.	0.444	33.	0.840
22.	23.4	28.	43.3	34.	7.87
23.	4.48	29.	20.6	35.	184.0

Use D and C scales, exercises 18 to 23.

Use DF and CF scales, exercises 24 to 29.

Use D and C, or DF and CF scales, exercises 30 to 35.

COMBINED OPERATIONS—Page 28

36.	121.4	44.	80.5
37.	255	45.	.611
38.	5,520,000	46.	280
39.	.0608	47.	1.91
40.	.611	48.	1.25
41.	.506	49.	1.267
42.	.1644	50.	1.507
43.	2.77		

MULTIPLICATION AND DIVISION OF A SERIES BY A SINGLE FACTOR—Page 30

51. 368; 774; 1,018; 1,440; 1,734; 2,200; 2,550; 2,580; 3,070.
52. 7.04; 3.34; 2.18; 1.718; 1.266; 1.120; 0.915; 0.820; 0.769.

53. 0.299; 0.506; 0.718; 0.821; 0.983; 1.874; 1.975; 2.16; 2.76.

PROPORTION—Page 32

54. 1.328
55. 181.2
56. 3.97
57. x = 6.09; y = .872; z = .125

QUADRATIC EQUATIONS BY FACTORING—Page 33

58. 34.0 and 0.53	61. −23.6 and −17.8
59. 19.5 and 1.64	62. 6.6 and 4.55
60. 25.0 and −4.8	

SQUARES AND SQUARE ROOT—Page 40

63. 416	70. .0000578	77. 1,404
64. 511,000	71. 5.20	78. 25.7
65. 1,145,000	72. 30.41	79. 0.651
66. 15,730	73. 906	80. 0.2958
67. .722	74. 35.57	81. 0.03115
68. .0000000246	75. 267.4	82. 0.0851
69. .00884	76. 7,140	

SQUARES AND SQUARE ROOT— COMPOUND OPERATIONS—Page 47

83. 11.53	87. 811
84. .436	88. 2,330
85. 173.5	89. 28,000
86. 4.81	

90.	a) 113.1;	b) 55.4;	c) .0106;	d) 3,020;	e) 5.42.
91.	a) 39.6;	b) .1385;	c) .8;	d) .01924;	e) 2,270,000.
92.	a) 6.09;	b) .125;	c) .4375;	d) 15.55;	e) 5.06.
93.	a) 2.522;	b) 7.98;	c) 31.1;	d) .1162;	e) .875.

CUBES AND CUBE ROOT—Page 49

94. 2	100. 1.817	106. .0147
95. 20	101. 2.88	107. .0000467
96. 200	102. 6.46	108. .000000111
97. 69,000	103. 11.98	109. .684
98. 422,000,000	104. 30.7	110. .345
99. 32.8	105. 82.4	111. .1957

POWERS USING LOGARITHMS—Page 54

112. 6.08	115. 0.0058	118. 1.387
113. 25.5	116. 0.578	119. 0.421
114. 18.2	117. 0.0109	120. 0.968

EXPONENTIAL EQUATIONS—Page 55

121. 1.267	124. 5.50
122. 0.884	125. 4.74
123. 3.62	

NATURAL TRIGONOMETRIC FUNCTIONS—Page 61

126. 0.970	131. 0.824	136. 4.51
127. 0.814	132. 0.264	137. 8.85
128. 0.266	133. 0.1132	138. 81.8
129. 0.0157	134. 0.281	139. 35.8
130. 0.0673	135. 1.163	140. 0.0419

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS—Page 63

141. 9.987 − 10	145. 8.622 − 10	148. 15.7°
142. 8.196 − 10	146. 15.4°	149. 88.4°
143. 9.916 − 10	147. 74.7°	150. 77.5°
144. 0.654		

SOLUTION OF TRIANGLES—Page 68

151. A = 68.3°; B = 21.7°; c = 24.3′.
152. a = 53.8′; c = 54.9′.
153. A = 25.6°; B = 45.8°; C = 108.6°.

COMPLEX NUMBERS—Page 71

154. x = 9.87; y = 13.60
155. x = 18.71; y = 9.54
156. 16.6 $\underline{/32.4°}$.

PRACTICAL APPLICATIONS—Page 73

157. 137.4′
158. a) R = $10\frac{13}{16}$″
b) R = $11\frac{1}{8}$″
c) R = $5\frac{7}{8}$″
159. R_L = 19,430 lbs.; R_R = 14,330 lbs.; Movents in foot pounds are; 77,700; 129,800; 152,000; 134,900; and 86,000 at points 1, 2, 3, 4 and 5 respectively. Maximum bending stress is 16,900 lbs/sq. in.
160. 12.2 ohms.
161. 1,114,000 circular mils.
162. 467 btu/hr.
163. 0.711 ft., 727 ft./min.; 0.0214 ft., 183 ft./min.
164. R = 148.6 lbs.; θ = 22.6°
165. 404 lb/ft^3; 0.223 cu. ft.: 6.48 gm/cm^3; 6.300 cu. cm.
166. p_1 = 680 lb./sq. ft.; p_2 = 4,600 lb./sq. ft.