REVISED EDITION

A COURSE IN THE SLIDE RULE AND LOGARITHMS

E. JUSTIN HILLS

GINN AND COMPANY · PUBLISHERS

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By
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LOS ANGELES CITY SCHOOLS

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PREFACE

The great increase in the use of the slide rule and in the use of logarithms has resulted in the need for a textbook on the subject. The author ventures to hope that this volume will fill that need.

The text is suitable for classroom use or for individual study. Each chapter represents the material for one class period, or one day's study. This arrangement should greatly simplify the teacher's task of planning the course.

Cuts are included of practically all standard slide rules and several special-use rules. Logarithm tables have been carried to five places.

The conscientious study of this textbook will equip the student with a knowledge of how to use the scales on standard slide rules and how to use logarithm tables. Also it will give an understanding of the close relationship between slide-rule procedures and logarithm procedures. For example, the solution of triangles both by slide-rule settings and by logarithms is carefully explained. Many other practical applications are also given.

When this book is used in a class or discussion group, it will be helpful for the leader to use a demonstration slide rule. Both Keuffel and Esser Company and Pickett and Eckel, Inc., have several types available.

The author wishes to express his gratitude to those of his colleagues and students who offered many valuable suggestions while the book was in syllabus and text form.

Also he is grateful to Keuffel and Esser Company, Eugene Dietzgen Company, and Pickett and Eckel, Inc., for the cuts they so kindly contributed.

E. J. H.

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A Course in the Slide Rule and Logarithms

EXPLANATION OF STANDARD SLIDE RULES

The Polyphase Rule (Fig. 1). This is a standard one-faced rule containing sufficient scales to solve problems pertaining to multiplication, division, proportion, squares and square roots, cubes and cube roots, reciprocals, the trigonometric functions, and logarithms. It is often referred to as the Mannheim type rule.

The Pickett Model 200 (Fig. 2).* This is an all metal, pocket size, rule containing all the scales that appear on the polyphase rule. It is half the length of the standard rule.

The Polyphase Duplex Trig Rule (Fig. 3).† This is a duplex (two-faced) type rule containing three folded scales in addition to those of the polyphase type rule. The duplex type rule has many advantages. As a result, most of the rules designed today are the duplex type.

The Log Log Duplex Decitrig Rule (Fig. 4).† This rule is identical in shape to the Polyphase Duplex Trig Rule. The trigonometric scales are marked in degrees and tenths of a degree. It also contains log log scales reading to the natural base (*e*) on the D scale.

The Pickett Model 800 Rule (Fig. 5).* This is a metal rule similar to the Log Log Duplex Decitrig rule. It has C scales on both faces and a reciprocal D scale. Three back-to-back log log scales are used instead of the six LL and LL0 scales, with the reciprocal on LL0 scales directly underneath their respective LL scales.

The Pickett Model 600 Rule (Fig. 6).* This is a six-inch duplex type rule containing the same scales found on the Pickett Model 800 Rule.

The Pickett Model 2 (Fig. 7).* This is a duplex type rule, log log scales reading both to the common log base (10) on the D scale and to the natural log base (*e*) on the DF/M scale. It has C and CI scales on both faces. There is a 20-inch square root scale and a 30-inch cube root scale. It also has a 20-inch tangent scale running from about 5.7° to about 84.3°.

The Pickett Model 4 (Fig. 8).* This rule is the Pickett Model 2 with hyperbolic function scales added.

The Log Log Duplex Vector Rule (Fig. 9).† This is a Log Log Duplex Decitrig Rule with hyperbolic function scales added.

The Phillips Rule (Fig. 10).‡ This rule is similar to the polyphase type rule. It has the inverted scale like the A and B scales instead of like the C and D scales.

The Midget Circular Slide Rule (Fig. 11).‡ This is a typical circular slide rule; answers are obtained by the use of two indicators connected to the center of the rule.

The Rotarule (Fig. 12).‡ This is an elaborate form of a circular slide rule; answers are obtained by the use of revolving discs and a single indicator. It has several spiral scales which permit great accuracy.

* Pickett trademark. † K & E trademark. ‡ Dietzgen trademark.

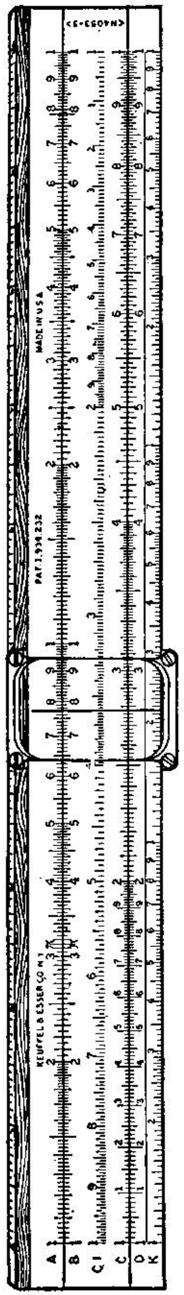

Fig. 1. Polyphase Rule

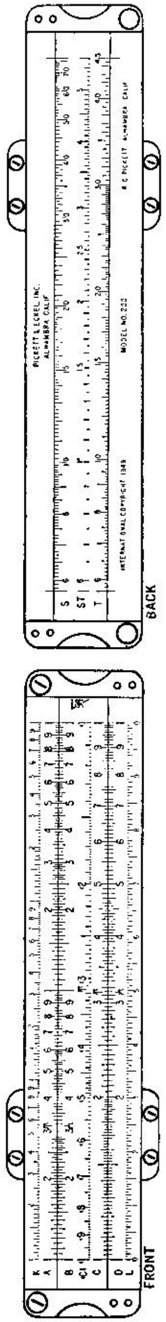

Fig. 2. Pickett Model 200 Rule

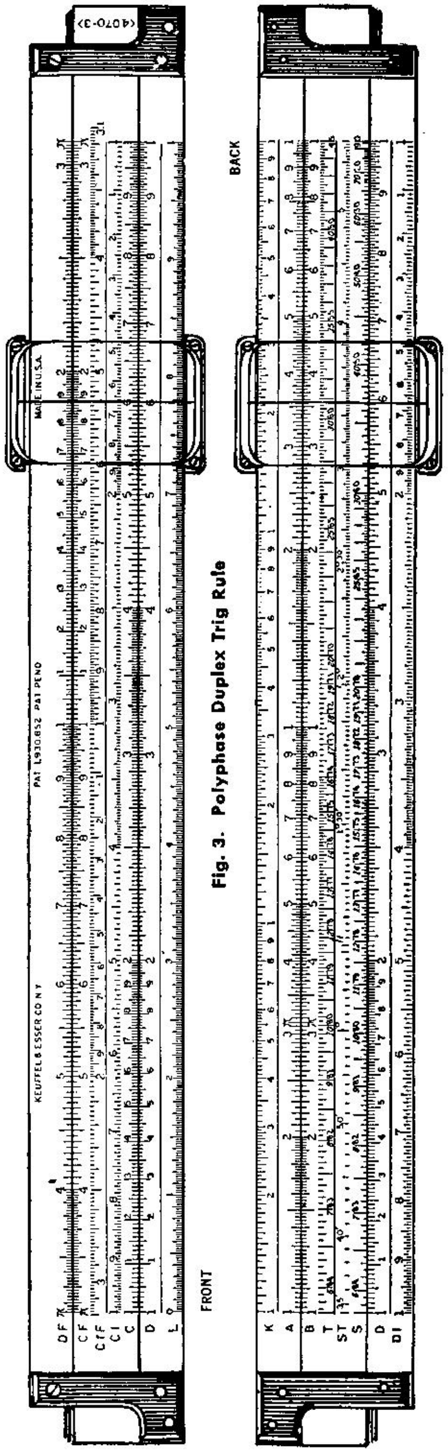

Fig. 3. Polyphase Duplex Trig Rule

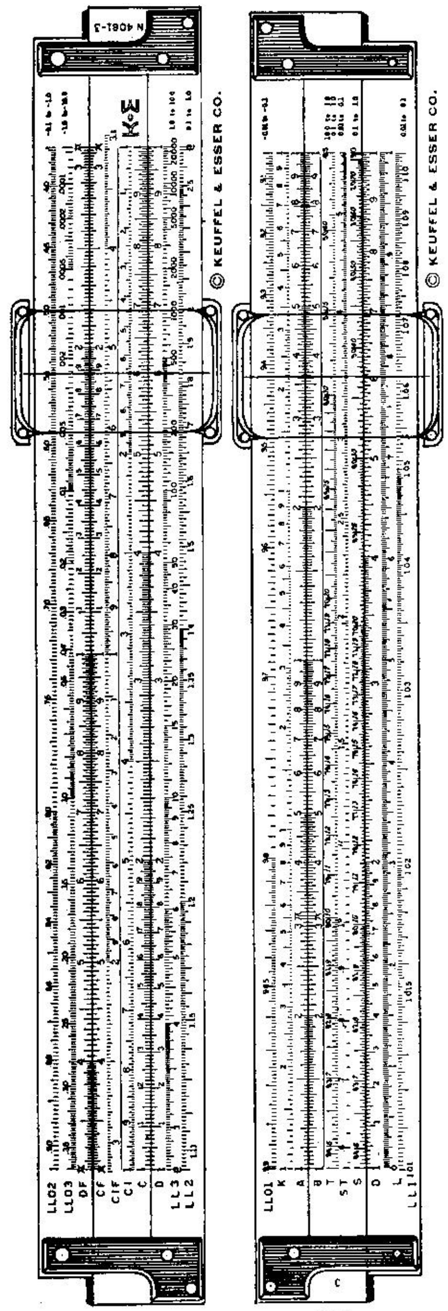

Fig. 4. Log Log Duplex Decitrig Rule

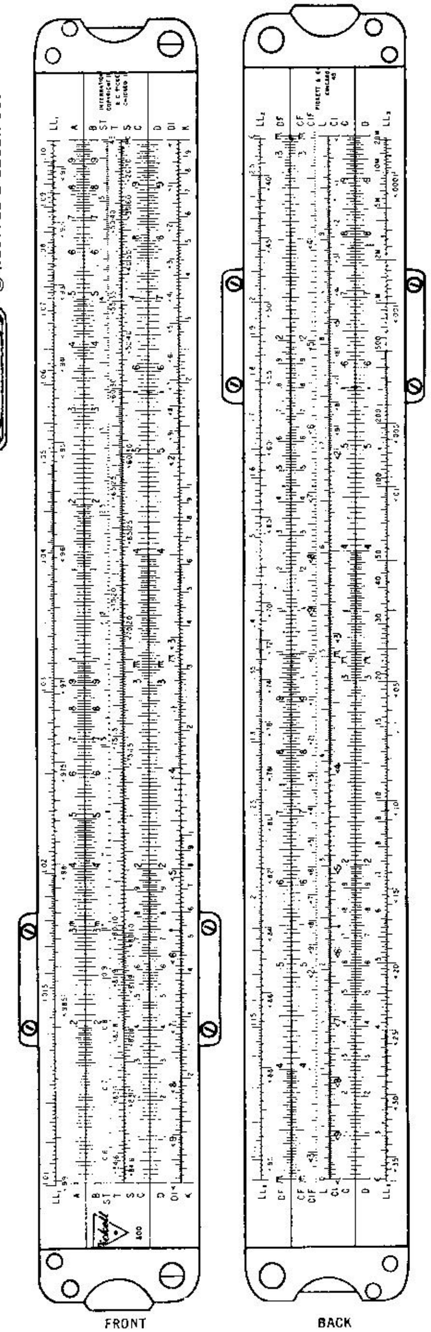

Fig. 5. Pickett Model 800 Rule

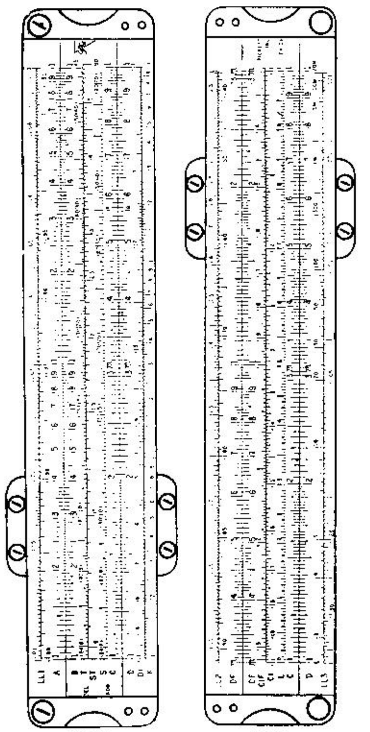

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Fig. 6. Pickett Model 600 Rule

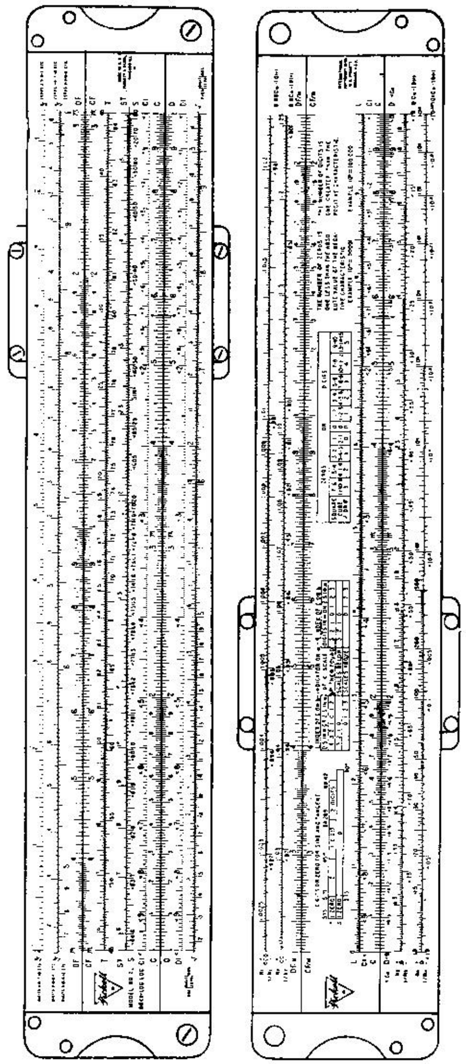

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Fig. 7. Pickett Model 2 Rule

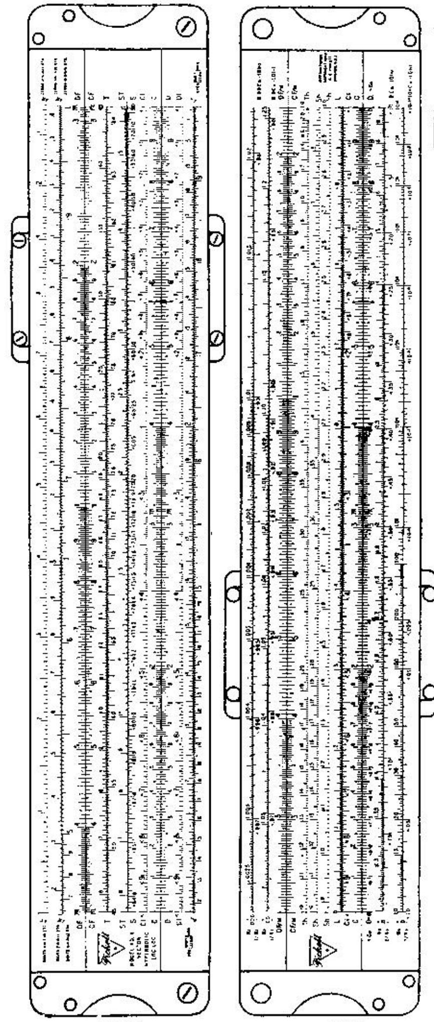

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Fig. 8. The Pickett Model 4 Rule

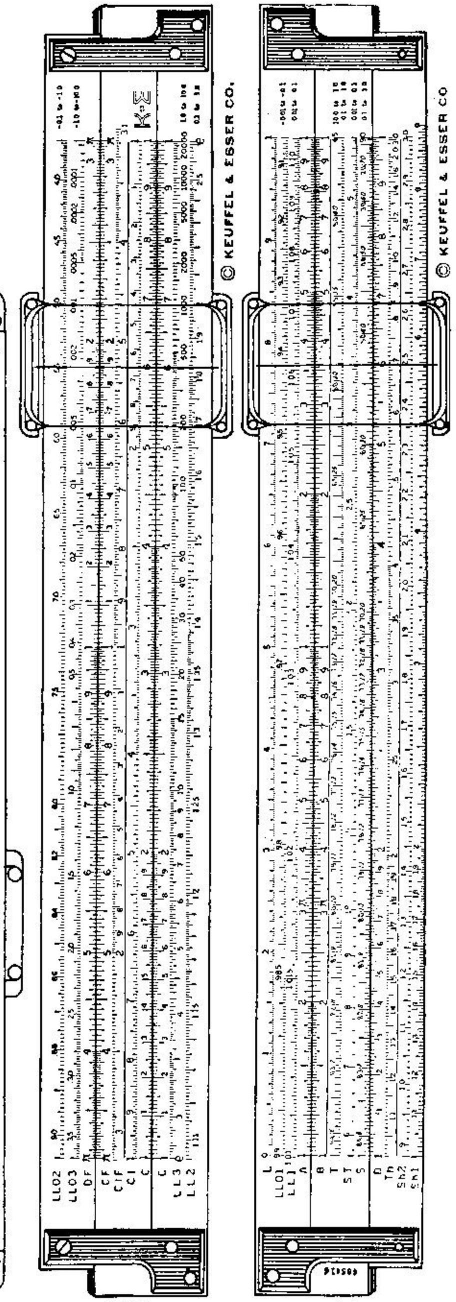

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Fig. 9. Log Log Duplex Vector Rule

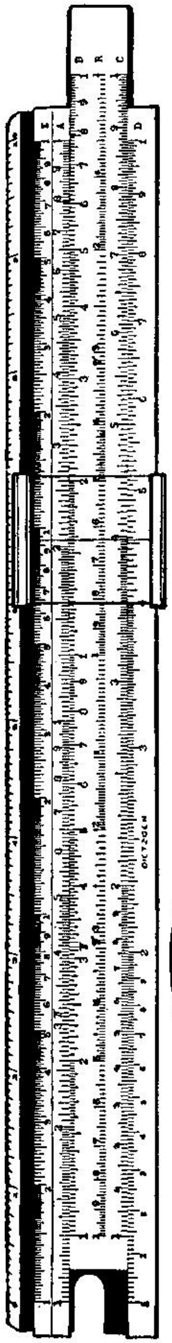


Fig. 10. Phillips Rule

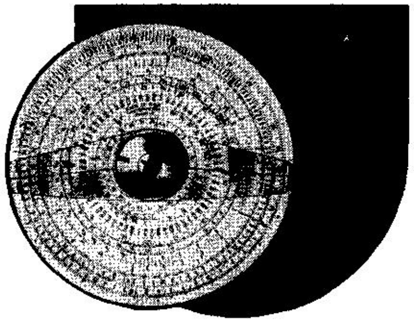


Fig. 11. Midget Circular Slide Rule

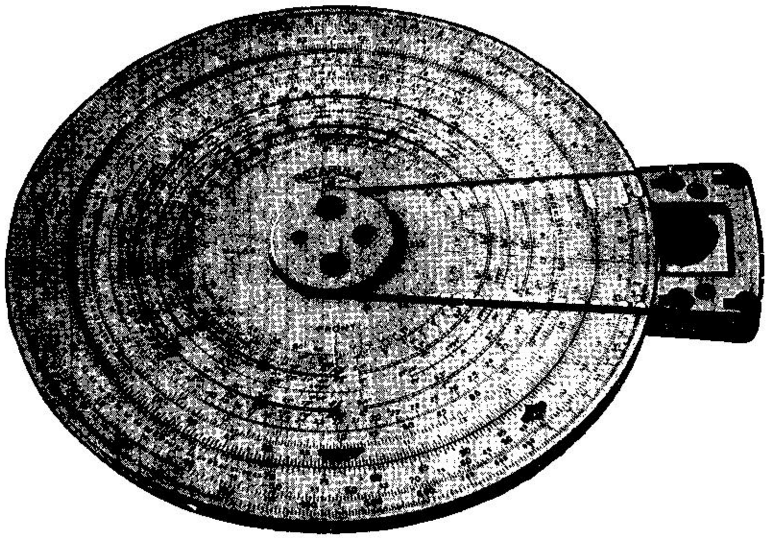


Fig. 12. Rotarule

EXPLANATION OF SPECIAL-USE SLIDE RULES

The Roylance Electrical Rule (Fig. 13).* This is a special polyphase type rule that makes it possible to solve many electrical problems directly by the use of a series of special scales and gauge marks. Note the extra hairlines on the indicator available to calculate circular areas directly.

The Cooke Radio Rule (Fig. 14).* This is a special duplex rule designed to speed up calculations for radio and electronic engineers. It has a 2π scale, which is a D scale folded at $\frac{1}{2\pi}$ or 0.159; also an LC scale that can be used in a formula like $F = 1/(2\pi\sqrt{LC})$, where F is the resonant frequency in kilocycles, L is the inductance in microhenrys, and C is the capacity in micromicrofarads.

The Stadia Rule (Fig. 15).* This is a special stadia surveying rule which is based on the two equations

$$\text{Horizontal distance} = \text{rod reading} \times \cos^2 \alpha$$

$$\text{Vertical height} = \text{rod reading} \times \frac{\sin 2\alpha}{2}$$

The Surveyor's Duplex Rule (Fig. 16).* This special rule reduces complicated spherical-trigonometry calculations of astronomical data essential to surveying to mere mechanical operations.

The Business and Finance Rule (Fig. 17).† This special rule permits one to solve directly any markup problem met in business. It also contains log log scales that permit one familiar with finance formulas to solve such problems directly. The basic formula is $(1 + i)^n$. Many other problems met in business and finance can also be solved directly.

The Business Executive Rule (Fig. 18).† This is a pocket-size rule helpful to all in any field of business. Both the Business and Finance Rule and the Business Executive Rule were designed by the author.

* K & E trademark. † Pickett trademark.

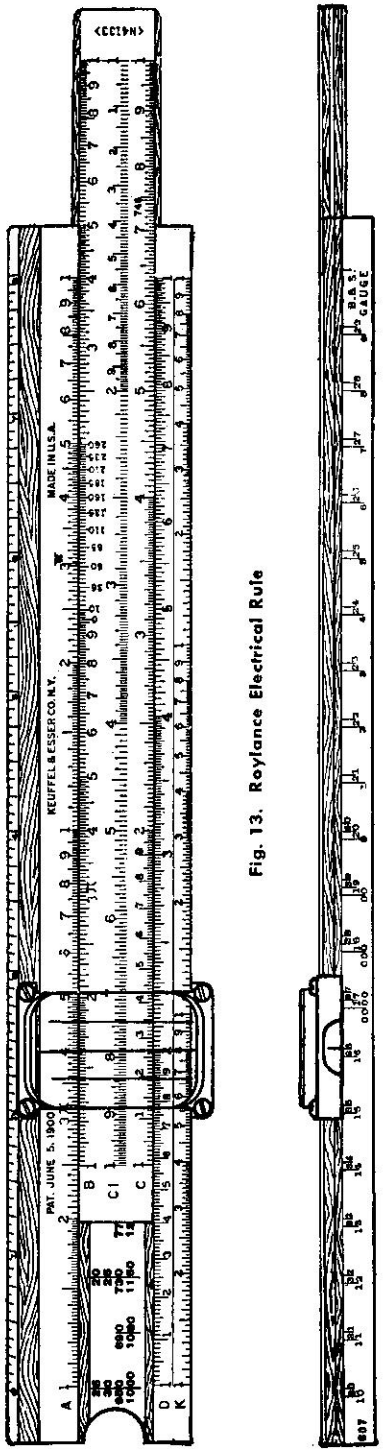


Fig. 13. Roylance Electrical Rule

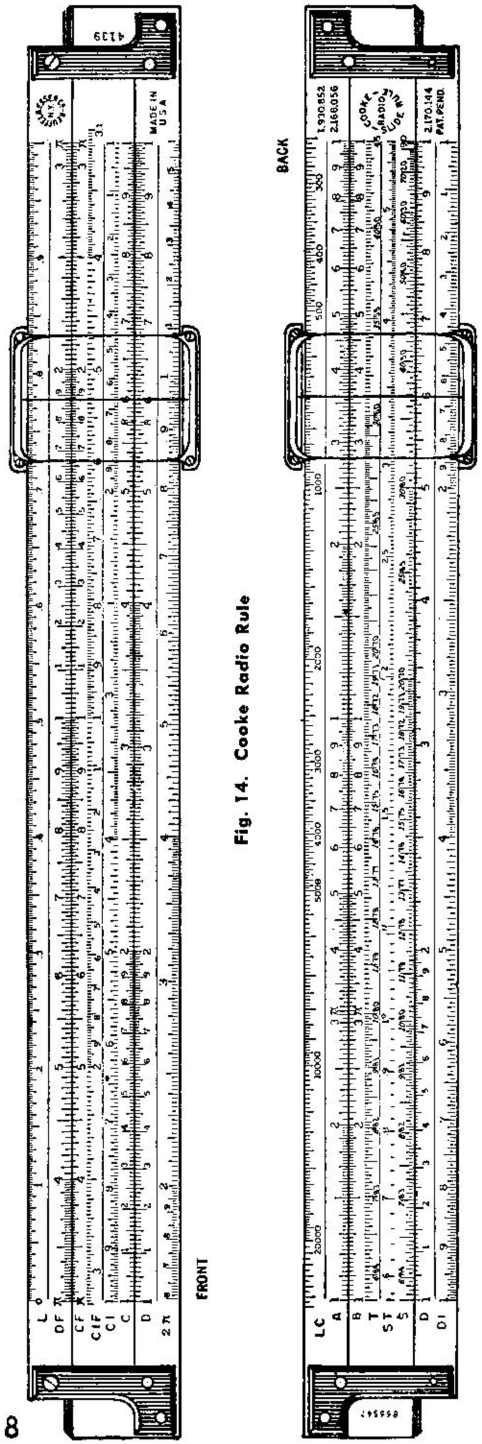


Fig. 14. Cooke Radio Rule

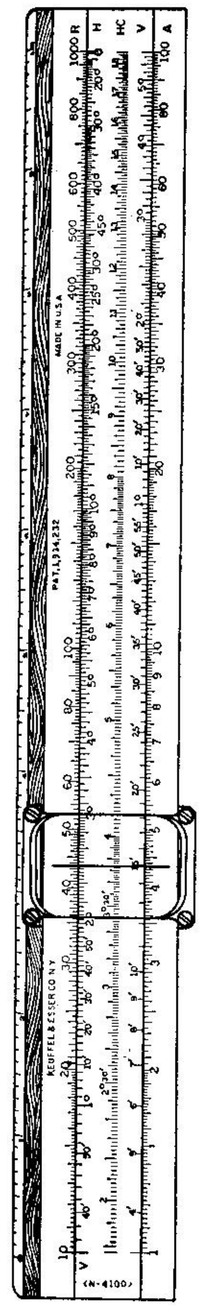


Fig. 15. Stadia Rule

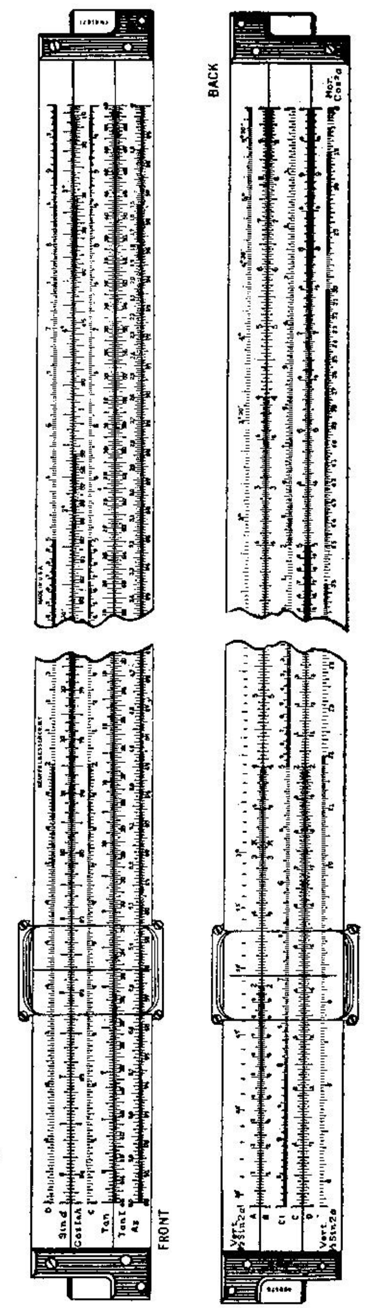


Fig. 16. Surveyor's Duplex Rule

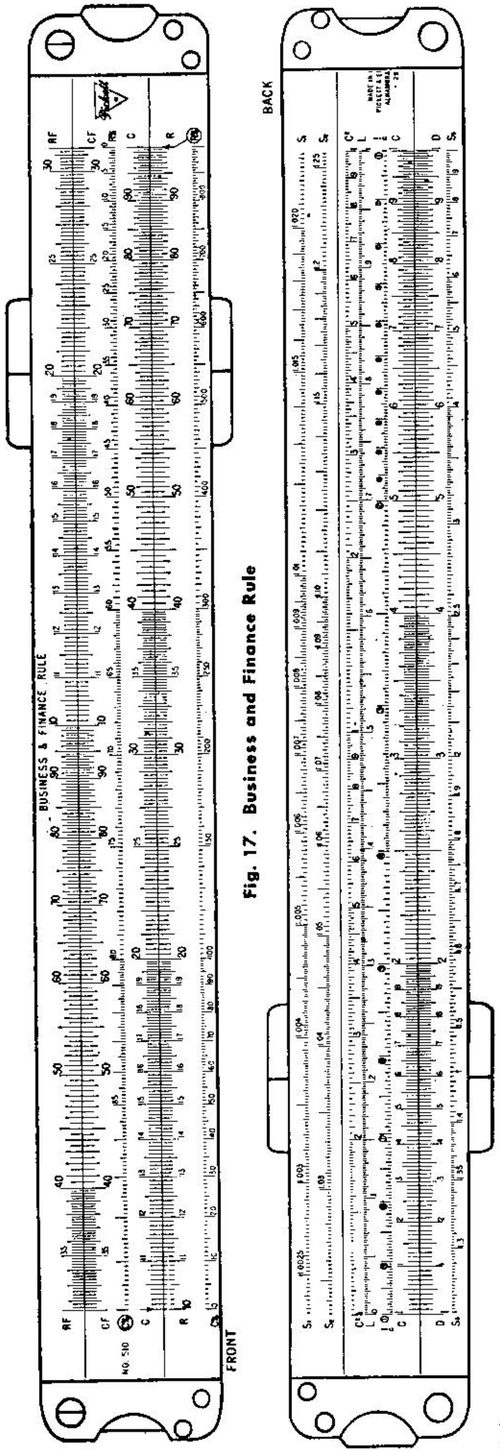


Fig. 17. Business and Finance Rule

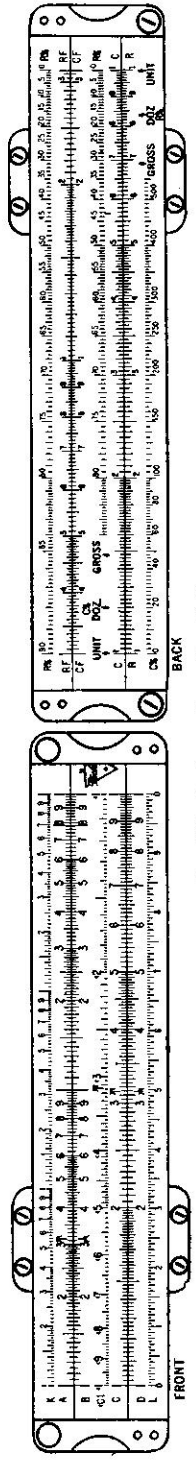


Fig. 18. Business Executive Rule

I · SLIDE RULES AND THEIR USE

What Is a Slide Rule?

A **slide rule** is an instrument used to solve problems involving multiplication, division, raising to a power (involution), and taking a root (evolution). It is based on the association of two or more logarithmically developed scales which, in turn, permits the use of the principle of proportion.

But before going into the theory of the slide rule a brief description will be given of the most common uses of the rule, namely, how to obtain products or quotients of numbers.

There are many **scales** on the slide rule, the basic ones being the C and D scales. The C scale is on the **slider**, the movable part of the rule; and the D scale is on the **stock**, the fixed part of the rule. These scales are alike, and are the ones that are worked together to obtain products or quotients. Note the following problems:

If the 1 at the left end of the C scale is put over the **2** on the D scale, you will find 4 on D under 2 on C, 6 on D under 3 on C, and so on. That is, $2 \times 2 = 4$, $2 \times 3 = 6$, and so on. This setting shows clearly the use of the principle of proportion, namely

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \text{ and so on.}$$

Since $2 \times 3 = 6$, then $6 \div 3 = 2$. That is, the setting for the product 2×3 is the same as the one that would be used for the quotient $6 \div 3$. Thus, put 3 on C over 6 on D, and 2, the answer, is on D under the left 1 on C.

Sometimes the 1 at the right end of the C scale must be used in getting a product. Consider the problem 2×6. In this case the right end of the C scale must be put over the 6 on D, which will give the answer of 12 on D under 2 on C. Try this one, and also 7×2, 7×3, 8×2, and so on.

The Scales and How They Are Used

But what about a product like 2.18×3.26? These are not simple whole numbers, yet exactly the same method of solution can be used as was used for 2×3. But to do so the correct values possible for any position selected on the C and D scales must be known.

There are nine spaces on the scales defined by the figures 1 through 9. Each of these represents the first significant digit of some number. In each space there are many lines not defined yet which represent some digit. The longer lines represent the second digit. Between 1 and 2, since the space is so long, the longer lines are numbered by small figures. Thus the first little 1 defines the number 1.1 or 11 or 0.11, and so on.

Between 2 and 3 the first long line defines the number 2.1 or 21 or 0.21, and so on. The longer lines in every other space have similar meanings; for example, the second long line between 4 and 5 defines 4.2 or 42 or 0.42, and so on. What does the sixth long line between 7 and 8 define? Where are 560 and 78 and 0.67 and 0.0038 on either scale?

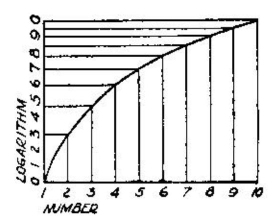
A three-digit number can be read directly, sometimes exactly, while at other times it must be approximated. Note that between 1 and 2 there are 10 parts, each divided into 10 parts; between 2 and 4 there are 10 parts, each divided into 5 parts; and between 4 and 1, at the right end, there are 10 parts, each divided into only two parts. Thus every three-digit number whose first digit is 1 can be determined exactly, such as 137 or 158, and so on. Between 2 and 4 even numbers can be determined exactly, such as 284 or 386, and so on; but odd numbers, such as 2.35, must be approximated. To get this, place the indicator (the glass slider with the hairline) first over 2, then over 2.1, then over 2.2, then over 2.3, then over 2.32 (first short line beyond 2.3), then over 2.34, and finally midway between this short line and the next one, which is about 2.35. Since between 4 and 1, at the right end, there is only one short line between the long lines, all three-digit numbers not ending in five or zero, such as 53.4, must be approximated. Here, put the hairline over 50 (same as 5), then over 51 (same as 5.1), then over 52, then over 53. But now the one short line between 53 and 54 must represent 53.5; so 53.4 is about $\frac{4}{5}$ the way across this first space.

Thus to solve 2.18 × 3.26 (see above) put left end of C scale over 2.18 on D. Then locate 3.26 on C scale and read the answer on the D scale directly below, namely 7.11. Try this.

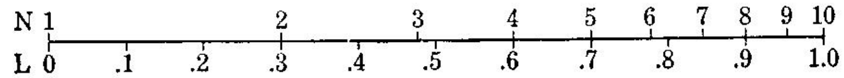
Logarithmic Meaning of the C and D Scales

To understand the true meaning of the scales of a slide rule it is necessary to recognize that the C and D scales, for example, differ from an ordinary scale, such as that on a yardstick, in that the numbers indicating the nine parts are really the **logarithms** of these numbers. Note the numbers read from a three-place logarithm table:

Number	Logarithm	Number	Logarithm
1	0.000	6	0.778
2	0.301	7	0.845
3	0.477	8	0.903
4	0.602	9	0.954
5	0.699	10	1.000



To visualize this comparison note the two scales marked N (number) and L (logarithm). Letting L be equally spaced, the 2 on N is opposite the 0.301 on L, the 3 on N is opposite the 0.477 on L, and so on. The graph may help show this relationship. Note that any value on N can be associated with the corresponding value on L, and vice versa, through the use of the curve shown.



But the N scale is the C or D scale. Thus the 1 at the left end of these scales is really the logarithm of 1, namely 0; the 2, about one third of the way along them, is really the logarithm of 2, namely 0.301; and so on. In like manner all the other lines can be defined, each depending on its position on the rule.

Furthermore, these division numbers can represent any multiple or quotient of them by ten (10), since when compared with logarithms these settings really represent only the **mantissas**, and the **characteristics** must be assumed. Thus 2 not only represents the number 2 but also any of the following: —, 0.002, 0.02, 0.2, 20, 200, —. In like manner 3 represents —, 0.003, 0.03, 0.3, 30, 300, —; and so on for all the other numbers either shown or assumed.

In making your settings, do not think about the decimal point. For example, 14.7 is one-four-seven, not fourteen point seven; 2380 is two-three-eight, not twenty-three hundred eighty; 0.0357 is three-five-seven, not three hundred fifty-seven ten-thousandths. To locate 172 (one-seven-two), put the hairline over the big 1 at the left end (left index) of C or D; then move the hairline over the little 7 between 1 and 2; then move the hairline over the second short line to the right of the little 7. To locate 356 (three-five-six), put the hairline over the big 3; then move it over the fifth long line between 3 and 4; then move it over the third short line beyond the long line just used. To locate 472 (four-seven-two), put the hairline over the big 4; then move it over the seventh long line between 4 and 5; then move it over to a point two-fifths of the way from the long line just used to the next short line.

PROBLEMS

Put the hairline of the indicator over

1. 238, 316, 535, 284, 113, 785, 436, 228, 935, 985.

2. 247, 433, 507, 583, 897, 233, 772, 838, 921, 874.

3. 13.5, 4.31, 0.682, 0.0847, 6350, 0.0852, 88,900.

4. 43,300, 73.3, 1.98, 127.5, 1.863, 998, 106, 7770.

R

5. 100.1 (be careful of this), 101, 110, 100.2, 102, 120.

6. 10.2, 20.3, 80.7, 1.001, 802, 820, 222, 823, 832.

7. Place the indicator over five places between main divisions 1 and 2 and give at least five values to each position represented.

8. Same as problem 7 for five places between 2 and 4.

9. Same as problem 7 for five places between 4 and 10.

Logarithm Tables and L Scale

Before going further in the course, a brief discussion on how to use logarithm tables or the L scale on the slide rule is needed, since logarithms have been referred to as being basic in rule calculations.

The graph on the previous page is that of the exponential equation $x = 10^y$ where x is the number and y is its logarithm. The logarithm table or the L scale of the rule gives the proper y value for the corresponding x value when the latter is between 1 and 10. For example, $3.25 = 10^{0.51188}$ by the logarithm table and $10^{0.512}$ by reading between the D and L scales on the rule. If x is greater than 10 or less than 1, such as 32.5, then it can be found as follows:

$$32.5 = 3.25 \times 10 = 10^{0.51188} \times 10^1 = 10^{1.51188}$$

These expressions can be written in the following form, by definition of a logarithm:

$$\log 3.25 = 0.51188 \text{ (by table)}$$
$$= 0.512 \text{ (by D and L scales on rule)}$$
$$\log 32.5 = 1.5118 \text{ or } 1.512$$

In conclusion, note that by the use of the graph, or the two scales marked N and L that appear on the previous page, it is possible to approximate the logarithm of a number or vice versa. On the other hand, by the use of the D and L scale combination, it is possible to get the logarithm to three-place accuracy; and by the logarithm table it is possible to get the logarithm to five-place accuracy. It will be wise for you to keep in mind the normal relationship between slide-rule procedures and logarithm-table-value procedures.

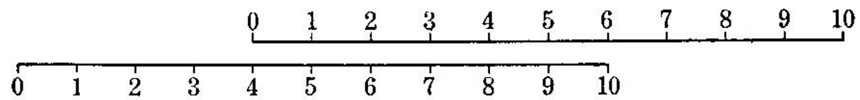
II • MULTIPLICATION AND DIVISION

Logarithms

Since logarithms are exponents they can be used to change multiplication into addition, and division into subtraction. Thus it follows directly that the C and D scales, each based on logarithms, can be used for such operations.

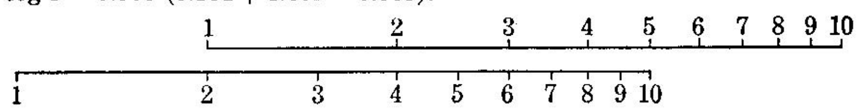
Multiplication-Addition

If two ordinary equal-parts scales are put side by side, addition can be performed. Thus if the 0 of one scale is put over the 4 of the other, the 3 is over the 7, 4 over 8, and so on, since $4 + 3 = 7$, $4 + 4 = 8$, and so on. This scheme depends on the fact that a length of 4 units plus a length of 3 units gives a length of 7 units.



The C and D scales on the slide rule apply in like manner to multiplication.

Example 1. Find 2×4. If the 1 (left index) on C ($\log 1 = 0$) is placed over the 2 (really log 2) on D, the 4 (log 4) on C is over 8 (log 8) on D. That is, a length $\log 2 = 0.301$ plus a length $\log 4 = 0.602$ gives a length $\log 8 = 0.903$ $(0.301 + 0.602 = 0.903)$.



Disregarding the logarithm interpretation, other simple multiplication settings can be made.

Example 2. $2.5 \times 3.7 = 9.25$. Place **left** index of C to 2.5 on D. Place indicator over 3.7 on C and read 9.25 on D.

Example 3. $6.2 \times 4.35 = 27$. Place **right** index of C to 6.2 on D. Place indicator over 4.35 on C and read 27 on D.

Note. Interchanging indexes is the same as multiplying or dividing by 10, and that cannot be noted on the rule, as it affects the **characteristic** only, logarithmically speaking. Thus, if the answer is off the D scale when using the left index, use the right index instead and vice versa.

Example 4. $16.2 \times 21.6 = 350$. Make proper settings and verify the answer. In getting the size of the answer depend on approximation. Since $15 \times 20 = 300$, then 350 is the answer.

Division-Subtraction

Subtraction is the reverse of addition. Thus a length of 7 units minus a length of 3 units gives a length of 4 units, and so on (note above).

In like manner division becomes the taking away of a certain logarithmic length from another logarithmic length. Thus $8 \div 4 = 2$ means a length $\log 8 = 0.903$ minus a length $\log 4 = 0.602$ gives a length $\log 2 = 0.301$ $(0.903 - 0.602 = 0.301)$.

Example 5. $9.85 \div 4.6 = 2.14$. Place indicator to 9.85 on D. Place 4.6 on C to indicator. Find 2.14 under **left** index of C on D.

Example 6. $25 \div 3.76 = 6.65$. Place indicator to 25 on D. Place 3.76 on C to indicator. Find 6.65 under **right** index of C on D.

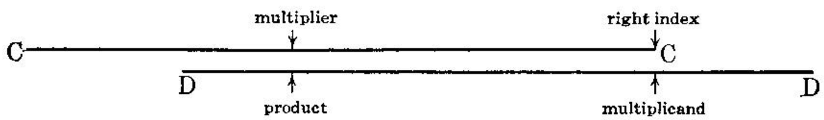
Example 7. $86.5 \div 50 = 1.73$. Make proper setting and verify answer.

Note carefully that division is the exact reverse of multiplication. Division, though, is more direct in that the divisor (always on C) is put over the dividend (always on D) and the quotient (always on D) is always under the index of C lying within the range of D.

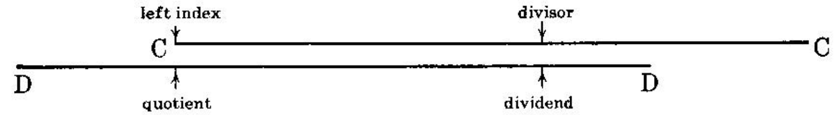
Diagrams of Solution

In a multiplication problem such as $2 \times 3 = 6$, 2 is called the **multiplicand**, 3 is called the **multiplier**, and 6 is called the **product**. The following diagrams show how the C and D scales are used for multiplication.

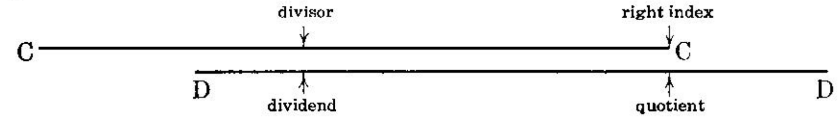
Or



In a division problem such as $8 \div 2 = 4$, 8 is called the **dividend**, **2** is called the **divisor**, and 4 is called the **quotient**. The following diagrams show how the C and D scales are used for division.



Or



Significant Figures

Only the first three significant figures of a number can be read, though frequently a fourth can be approximated. A check of the C and D scales shows also that for some numbers beginning with 2 to 9 the third figure must be approximated; note such numbers as 22.5, 4.07, 0.634, and so on.

PROBLEMS

In the problems that follow, your chief concern is to determine the significant figures of the answer. The symbol ⬭ will be used instead of an equal sign since the magnitude of the answer is not being considered. The position of the decimal point in the answer is very important, but that will not be considered until the next chapter.

1. 212 × 316 ⬭ 670
2. 422 × 127 ⬭ 535
3. 620 × 810 ⬭ 502
4. 432 × 280 ⬭ 121
5. 823 × 333 ⬭ 274
6. 111 × 121 ⬭ 134
7. 846 × 764 ⬭ 645
8. 780 × 126 ⬭ 983
9. 832 × 193 ⬭ 161
10. 473 × 435 ⬭ 206
11. 123 × 265 ⬭ 326
12. 724 × 265 ⬭ 192
13. 368 × 964 ⬭ 355
14. 660 × 173 ⬭ 114
15. 257 × 382 ⬭ 982
16. 279 × 192 ⬭ 535
17. 403 × 527 ⬭ 212
18. 392 × 507 ⬭ 199
19. 228 × 822 ⬭ 187
20. 283 × 238 ⬭ 674
21. 350 ÷ 700 ⬭ 500
22. 810 ÷ 470 ⬭ 172
23. 542 ÷ 224 ⬭ 242
24. 625 ÷ 470 ⬭ 133
25. 128 ÷ 337 ⬭ 380
26. 269 ÷ 123 ⬭ 219
27. 226 ÷ 380 ⬭ 595
28. 686 ÷ 881 ⬭ 779
29. 548 ÷ 134 ⬭ 409
30. 832 ÷ 145 ⬭ 573
31. 738 ÷ 161 ⬭ 459
32. 438 ÷ 121 ⬭ 362
33. 363 ÷ 885 ⬭ 410
34. 888 ÷ 444 ⬭ 200
35. 529 ÷ 713 ⬭ 742
36. 136 ÷ 361 ⬭ 377
37. 528 ÷ 307 ⬭ 172
38. 293 ÷ 562 ⬭ 521
39. 602 ÷ 338 ⬭ 178
40. 125 ÷ 239 ⬭ 523
41. 673 × 887 ⬭ 597
42. 368 ÷ 354 ⬭ 104
43. 358 × 126 ⬭ 451
44. 166 ÷ 363 ⬭ 457
45. 125 × 424 ⬭ 530
46. 784 ÷ 367 ⬭ 214
47. 690 × 935 ⬭ 645
48. 578 ÷ 421 ⬭ 137
49. 265 × 891 ⬭ 236
50. 168 ÷ 987 ⬭ 170
51. 999 × 666 ⬭ 665

III · DETERMINATION OF DECIMAL POINT OF ANSWER IN MULTIPLICATION AND DIVISION; PERCENTAGE

Determination of Decimal Point

Determining the position of the decimal point by inspection is the most universal method used. This position can be determined quickly by following a few suggestions on how decimal points can be shifted. Illustrations will suffice to show what can be done.

Example 1. $12.85 \div 3.37$. The approximate problem could be $12 \div 3$, which equals 4; so the answer must be 3.81, not 38.1 or .381.

Example 2. 42.7×28.5. The approximate problem could be $40 \times 30 = 1200$; so the answer must be 1214, not 121.4 or any other multiple by 10.

Example 3. 0.000874×474. Here one can first move the decimal points as follows: $0.00_{\wedge}0874 \times 4_{\wedge}74$. Then the approximate problem could be $0.09 \times 5 = 0.45$; so the answer must be 0.413.

Example 4. $0.0000279 \times 0.000784$. As in example 3, one can move the decimal points as follows: $0_{\wedge}0000.0000279 \times 0.0007_{\wedge}84$. Then the approximate problem could be $0.000{,}000{,}003 \times 8 = 0.000{,}000{,}024$; so the answer must be 0.000,000,0219.

Example 5. $0.000943 \div 0.00000386$. Again move the decimal points as follows: $0.000943_{\wedge} \div 0.000003_{\wedge}86$. Then the approximate problem could be $900 \div 4 = 225$; so the answer must be 244. +.

Example 6. $7290 \div 0.00337$. Move the decimal point as follows: $7290.000_{\wedge} \div 0.003_{\wedge}37$. Then the approximate problem could be $7{,}200{,}000 \div 3 = 2{,}400{,}000$; so the answer must be 2,160,000.

Example 7. $0.00286 \div 7490$. Move the decimal point as follows: $0_{\wedge}000.00286 \div 7_{\wedge}490$. Then the approximate problem could be $0.000{,}002{,}8 \div 7 = 0.000{,}000{,}4$; so the answer must be 0.000,000,382.

Other examples could be given, but these should suffice to show the simple rules followed. Later, when more than two factors are used, these rules can be easily expanded. Remember that it is just as important to have the proper decimal-point position in the answer as it is to have the correct figures.

Simple Two-Factor Rules

Where only two factors are involved, two rules will be shown by which the position of the decimal point in the answer can always be determined directly.

In Multiplication. If the slider projects to the **left**, when the result is read, the number of digits to the left of the decimal point in the answer is the sum of the number of digits to the left of the decimal points in both terms making up the product. If the slider projects to the **right**, the result has one less place than when it projects to the left.

In Division. If the slider projects to the **left**, subtract the number of places in the divisor from the number of places in the dividend to determine the number of places in the quotient. If the slider projects to the **right**, the result has one more place than when it projects to the **left**.

The following chart should prove helpful. Do not memorize the rules; merely test them out by using simple illustrations, such as 2×3; 4×7; $12 \div 4$; $40 \div 2$; and so on.

	Slider Projects to Left	Slider Projects to Right
Multiplication	Add	Add, subtract one
Division	Subtract	Subtract, add one

In the above rules do not count the digits in the decimal part of a mixed number. But, in the case of decimals **only**, the number of places is considered negative and is equal to the number of zeros between the decimal point and the first significant digit.

Note. These rules also apply when more than two factors are involved, but their primary use is for only two.

Example 8. 42.2×12.7. Place the left index of C over 42.2 on D. Find 5360 on D under 12.7 on C. Since the slider projects to the right the answer must contain $2 + 2 - 1 = 3$ places to the left of the decimal point; thus the answer is 536.0.

Example 9. $16.5 \div 0.245$. Place the indicator over 16.5 on D. Bring 0.245 on C to the hairline. Find quotient under the C index on D, namely 673. Since the slider projects to the left the answer must contain $2 - 0 = 2$ places; thus the answer is 67.3.

Example 10. $0.00655 \div 0.00040$. Place 0.00040 on C over 0.00655 on D. Find the quotient on D, namely 1638. Since the slider projects to the right the answer must contain $-2 - (-3) + 1 = 2$ places to the left of the decimal point; thus the answer is 16.38.

By shifting the decimal point in both dividend and divisor in the latter problem, it could become $65.5 \div 4$. This is simpler, but the other method should be observed.

*CF and DF Scales

Brief reference will be made here to the CF and DF scales that appear on the duplex type of rules. In form they are identical to the C and D scales, except that each scale is folded with the index near the middle. Their first use comes in multiplication in such a problem as: Multiply 4 by 7. If, by chance, the left index on the C scale were put over the 4 on the D scale, then the 7 on the C scale would be off the rule. But note that the 7 on the CF scale is still on the rule. Thus the answer can be read on the DF scale over this value.

Though this method does not always apply when the left index of the C scale is used, it will apply whenever more than half of the slider is available after the first setting (as in the example above), or if either value is less than or equal to 3.14 or any multiple of it by ten.

After a little practice these scales will be found to have much time-saving value. They will have particular importance at a later time when properties and relationships dealing with proportion, equivalent ratios, properties of circles, and so on are discussed.

If the CF and DF scales are used in the **size of number** rules, reverse the words **left** and **right**.

Percentage

There are three types of problems based on the relationship

$$\frac{\text{Percentage}}{\text{Base}} = \text{Rate Per Cent.}$$

1. Percentage = Base times Rate Per Cent ($P = BR$)
2. Rate Per Cent = Percentage divided by Base ($R = P/B$)
3. Base = Percentage divided by Rate Per Cent ($B = P/R$)

To illustrate each, note:

1. What is 37.5% of 160?
   160 × 0.375, namely 60
2. What per cent of $81 is $9?
   9 ÷ 81, namely 0.111 or 11.1%
3. 42 is 7% of what number?
   42 ÷ 0.07, namely 600

Each method is distinct and calls for a certain initial set of facts. Do not attempt to memorize the rules. Ability to reason out the facts and methods is the basic requirement.

PROBLEMS

1. 6.73 × 8.87 = 59.7
2. 3680 ÷ 35.4 = 104.
3. 3580 × 0.0126 = 45.1
4. 0.166 ÷ 3.63 = 0.0457
5. 125 × 42.4 = 5300
6. 7.84 ÷ 0.367 = 21.4
7. 0.690 × 0.0935 = 0.0645
8. 0.00578 ÷ 0.000421 = 13.7
9. 265 × 891 = 236,000
10. 16,800 ÷ 987 = 17.0
11. 99.9 × 0.0666 = 6.65
12. 27.8 ÷ 0.0976 = 285.
13. 0.887 × 0.764 = 0.677
14. 0.154 ÷ 15.7 = 0.00981
15. 0.0458 × 0.0788 = 0.00369
16. 0.00745 ÷ 0.459 = 0.0162
17. 36,000 × 0.00659 = 237.
18. 7040 ÷ 5.08 = 1380
19. 1240 × 77.3 = 95,900
20. 0.0743 ÷ 0.000269 = 276.

21. What is 87% of 45 in.? 67.5% of 8.62 ft.?

22. 56% of $186 is how much? 45.3% of $92.50 is how much?

23. What per cent of 86.5 is 37.9? 56.3? 82.7? 97.3? 115.4?

24. $54 is what per cent of $72? $83.50? $57.30? $38? $16.40?

25. 12 is 8.3% of what number? 45.3 is 27.5% of what number?

26. What number has 31.2 as 2.1% of it? as 0.56% of it?

27. If a man buys stock for $315, which was 42% of the money he had saved, how much had he saved?

28. If a man has a salary of $225 a month, from which he spends $45 for rent, $30 for food, $15 for clothes, and so on, what per cent are each of these of the total salary?

29. If 18% of a barn is filled by 1200 bu. of wheat, how much space is left for other grains? (6670 bu.)

30. If $30,000 is available by a bond issue and 24% is used for foundation, 31% for structural steel, 19% for roadway, and the rest for approaches, how much is spent in building each part of a bridge?

31. What will 42% of 625 bu. of wheat cost at $1.40 per bushel? ($368)

32. A man who has $4520 in the bank withdraws 28% of it. 34% of what was taken out was used to buy a ticket. What was the cost of the ticket?

33. A car travels 280 mi. the first day, 324 mi. the second day, and 196 mi. the third day. What per cent of the whole trip was made each day? (35%, 40.5%, 24.5%)

IV · COMBINED MULTIPLICATION AND DIVISION

Type Formulas

Consider the formulas of the type $R = \frac{X \times Y}{Z}$. In such problems it is most efficient first to divide X by Z and then multiply by Y. For example, $R = \frac{25 \times 48}{32}$. Divide 25 by 32 in the usual way. The result appears on the D scale under the end of the C scale. To multiply this number by 48, find 48 on the C scale, and the answer, 37.5, appears under it on the D scale. The student should keep in mind that in problems of this type it is best to do the division first, since this leaves the rule set for multiplying without moving the slider. All problems of this type can be done with one setting of the rule, although in some cases the second factor will appear off the scale. It is then necessary to change ends of the slide (doing so with the indicator) before doing the multiplication, or use the CF values (duplex type) if possible.

If the problem contains a series of factors in both the numerator and denominator, such as

$$R = \frac{S \times T \times U \times V}{W \times X \times Y \times Z},$$

the result is best obtained by dividing S by W, then multiplying by T, dividing by X, and so on.

In all such problems where more than one operation is necessary, the decimal point in the result is best obtained by approximation. Thus, in the first problem, the result is nearly equal to

$$\frac{25 \times 50}{30}, \quad \text{or} \quad \frac{25 \times 5}{3}, \quad \text{or} \quad 8 \times 5, \text{ or } 40.$$

Thus the result, 37.5, is correct.

An easy way to remember what to do is to move the indicator for every factor **above** the line of the fraction, and to move the slide for every factor **below** the line. Also remember that S is on D, every other factor above or below the line of the fraction is on C, and the answer is on D. Thus, move hairline of indicator to S (above the line) on D; then move the slide so that W (below the line) on C is underneath the hairline; then move the hairline to T (above the line) on C; then move the slide so that X (below the line) on C is underneath the hairline; and so on.

In order to show how the methods discussed in chapter III can be made to apply to a more general problem, note the following example:

$$\frac{25.8 \times 103 \times 3.92}{0.0045 \times 9.35 \times 19.5} = 12700$$

The decimal points in the denominator and numerator could be moved as follows:

$$\frac{25.8 \times 103 \times 3.92_\wedge}{0.004_\wedge 5 \times 9.35 \times 1_\wedge 9.5};$$

so the approximate problem could be

$$\frac{25 \times 100 \times 400}{5 \times 10 \times 2},$$

and by cancellation this reduces to $5 \times 10 \times 200 = 10{,}000$.

While going through the process of getting the answer by the special method, record the direction of projection of the slider after each setting. Check this problem, noting that for the first setting there is Division — with slider to left (D-l), then Multiplication — right (M-r), then D-l, then M-l, and finally D-r. **Add** the digits of the numerator and **subtract** those of the denominator, namely $2 + 3 + 1 - (-2) - 1 - 2 = 5$. Now note that when the slider projects to the right, in multiplication subtract one and in division add one. That is, M-r gives -1 and D-r gives $+1$, and all the others give 0. Thus finally the digit sum of the answer is $5 - 1 + 1 = 5$. In one step, this can be written $2 + 3 + 1 - (-2) - 1 - 2 + 0 - 1 + 0 + 0 + 1 = 5$, where the last five quantities are obtained from the slider projection settings.

If there are more factors in the numerator than in the denominator or vice versa, even them up by using 1's, such as:

$$\frac{25 \times 8 \times 3 \times 7}{4 \times 3} = \frac{25 \times 8 \times 3 \times 7}{4 \times 3 \times 1 \times 1}$$

★Use of CF and DF Scales

Check very carefully the following example. Here the CF and DF scales will be found to be a great timesaver:

$$\frac{28.3 \times 8.38 \times 12.9}{16.7 \times 4.27 \times 5.86} = 7.32$$

Solution. Place the indicator over 28.3 on D scale. Put 16.7 on C scale under hairline. Since 8.38 on C is beyond the D scale, take the 8.38 on CF instead. Answer, so far, is on DF. Now place 4.27 on CF under hairline (this being necessary since the answer prior to this step was on DF). Answer is now on D under the key of C. Put the hairline to 12.9 on C, and finally 5.86 on C to the hairline. Answer is on D scale under right index.

Use the CF and the DF scales whenever you can, since it is only through practice and experiment that their use comes natural.

Approximation of Answers

Often it is not easy to determine the position of the decimal point in the answer to a proportion problem. In the following illustrative example two suggestions are given.

Example 2. $\frac{R}{0.0384} = \frac{0.278}{61.9}$

First suggestion. Shift the decimal points in the ratio that is known, namely 0.278/61.9, so that the denominator agrees favorably with the 0.0384. That is,

$$\frac{R}{0.0384} = \frac{0.000278}{0.0619}$$

Then, since 0.0384 is about two thirds of 0.0619, R is about two thirds of 0.000278; namely $R = 0.0001727$.

Second suggestion. Shift the decimal points of the mean values (in this case), namely 0.0384 and 0.278, in opposite directions so that the 0.278 is fairly close to the 61.9. That is,

$$\frac{R}{0.000384} = \frac{27.8}{61.9}$$

Then, since 27.8 is less than half of 61.9, R must be less than half of 0.000384; namely, $R = 0.0001727$.

Application to Chemistry

It is possible to write the **reacting-weight equation** for a chemical formula and use the principle of proportion to solve for the unknown weight.

Example 3. Determine the weight of oxygen that can be obtained from 20 grams of potassium chlorate ($KClO_3$).

$$2\,KClO_3 = 3\,O_2 + 2\,KCl$$

From the Table of International Atomic Weights, we have

$$2(39.1 + 35.46 + 3 \times 16) = 3 \times 2 \times 16 + 2(39.1 + 35.46)$$
$$245.12 = 96 + 149.12$$

Therefore $245 : 96 = 20 : R$; so $R = 7.84$ grams, which is the weight of oxygen that can be obtained.

Equivalent Ratios

Many times it is necessary to change the kind of unit one is working with. For example, it is frequently necessary in scientific work to convert from metric units to English units, or vice versa. The usual scheme is to find a Table that relates these units. In the Appendix will be found such a conversion table. Note for example that 1 kilogram = 2.2046 pounds.

If one wishes to find on the slide rule the number of pounds equivalent to 2.85 kilograms, an equivalent ratio setting (identical to a proportion setting) can be made as follows: $1 : 2.2046 = 2.85 : R$, from which the proper value can be determined for R.

In order to avoid rather large decimal equivalents, as in the case just mentioned, simple slide-rule settings, relative to the C and D scales, have been worked out. Namely, in the case just referred to, the table shows 176 opposite 388. That is, $176 : 388 = 1 : 2.2046$. With such equivalents of not more than three places the initial setting of the rule can be easily made.

It will be worth while to make a few settings on the C and D scales, knowing the equivalence of two quantities. For example, the circumference of a circle is 3.1416 times the diameter. That is, $c = \pi d = 3.1416\,d$. Place 1, the coefficient of c, on C over 3.1416, and the coefficient of d on D, or vice versa. Though this cannot be done exactly, note that certain division lines can be found on D that continue on to C. Each of these positions can represent a setting. If done accurately, 113 will agree with 355, and so on. That is, $113 : 355 = 1 : 3.1416$. What other ones can you note?

If the above abbreviations are not familiar, refer to the table in the Appendix.

Interpolation

A very important principle based on simple proportion is **interpolation.** It is used when one wishes to find a number not given in mathematical tables, but which does lie between given values. The standard method of solution takes on the form:

$$\frac{x - a}{b - a} = \frac{c - d}{e - d},$$

where x, the value to be determined, lies between table values **a** and **b**; and where **c** is known and also lies between table values **d** and **e**.

Example 4. $(1 + i)^{20} = 2.148$. Find i.

In the formula just given, x equals i and $c = 2.148$. From a finance table, $a = 0.03$, $b = 0.04$, $d = 1.806$, and $e = 2.191$. Thus the formula becomes

$$\frac{i - 0.03}{0.04 - 0.03} = \frac{2.148 - 1.806}{2.191 - 1.806} = \frac{0.342}{0.385} = \frac{R}{0.01},$$

where $R = i - 0.03$. Put 342 on D under 385 on C; then under 0.01 on C read R (=0.00888) on **D.** Since $R = i - 0.03 = 0.00888$, then $i = 0.03888 = 3.888\%$.

Decimals and Fractions

The answers read from a slide rule are always decimal. Frequently there is need to change the decimal answer for a problem to a fraction answer. To do so, the simplest way is to assign some denominator for the fraction and determine the corresponding numerator. The most common denominators used are those found on a standard foot rule, where the inches are divided into eighths, sixteenths, thirty-seconds, sixty-fourths, and so on. Consider the following example.

Example 5. Change 0.418 into eighths, sixteenths, and so on.

$$0.418 = \frac{R_1}{8} = \frac{R_2}{16} = \frac{R_3}{32} = \frac{R_4}{64} = \frac{R_5}{128}$$

Since the R values must not be decimal, take the nearer whole numbers. Therefore $R_1 = 3$, $R_2 = 7$, $R_3 = 13$, $R_4 = 27$, and $R_5 = 53$. That is,

$$0.418 = \tfrac{3}{8} \text{ or } \tfrac{7}{16} \text{ or } \tfrac{13}{32} \text{ or } \tfrac{27}{64} \text{ or } \tfrac{53}{128}$$

or any other fraction with some assigned denominator. Procedure: Put 0.418 on C over either index on D; then each denominator on D gives the corresponding numerator on C.

PROBLEMS

1. $5.87 : 4.56 = 16.2 : R$

2. $1 : 4.68 = 12.8 : R$

3. $3.12 : 67.5 = 1 : R$

4. $388 : 176 = R : 2.85$

5. $113 : 355 = 0.582 : R$

6. $R : 32.6 = 58.2 : 27.9$

7. $23.8 : R = 0.328 : 0.674$

8. $0.452 : R = 45.8 : 78.3$

9. $R : 23.6 = 8.88 : 32.5$

10. $5.45 : 6.53 = R : 40.9$

11. $1 : 6.45 = 31 : R$

12. $R : 26 = 1 : 0.434$

13. $1 : 1.014 = 72 : R$

14. $R : 8.25 = 12 : 276$

15. $41.1 : R = 0.147 : 21$

16. $56.8 : 2.37 = R : 0.0928$

17. $1.01 : R = 4.58 : 12.7$

18. $10.2 : 12.7 = 1.08 : R$

19. $32.8 : 0.0874 = R : 15.6$

20. $0.0835 : 2.78 = R : 0.0534$

21. Find the number of acres in the following areas defined in square miles: 1.5, 0.85, 0.385, 1.75, 3.67.

22. Find the pressure in pounds per square inch for the following depths of water (in feet): 0.5, 2, 4, 8, 9.67.

23. How many feet per second is one going for the following miles per hour: 2, 20, 4.5, 38, 106, 231?

24. Given that 1 B.T.U. = 778 ft.-lb. what are the following equivalents: 435 ft.-lb., 125 B.T.U., 1982 ft.-lb., 1.82 B.T.U.?

25. How many *chevaux-vapeur* are equivalent to the following horse power: 26.5, 12.9, 56.75, 100, 157.5?

26. If 1 kw. = 1.34 h.p. what are the following equivalents: 26.5 kw., 45.1 h.p., 56.6 kw., 78.8 h.p., 3.45 kw.?

27. If 1 ft.-lb. = 1.356 joules, what are the following equivalents: 3.5 ft.-lb., 17.7 joules, 16.3 ft.-lb., 66.8 joules?

28. If an airplane is cruising at 240 mi. per hour, how far will it travel in 2.5 hr., in 4 hr. 42 min., in 8 hr. 28 min., in 11 hr.? (Change all time to minutes.)

29. In Problem 28, how long will it take to travel 156 mi., 280 mi., 625 mi., 1250 mi.?

30. If a pole 3 ft. long casts a shadow 6.3 ft., how tall is a tree that casts a shadow 49 ft.? a tower that casts a shadow 600 ft.? Also how long is the shadow for a telephone pole that is 43 ft. tall? Draw a diagram showing why these values are true. What geometric property applies here?

31. Show how, by rule, the formula $Wh = wH$ can be solved in two distinct ways. Solve for H by both methods when $W = 600$, $w = 250$, and $h = 40$ cm.

32. In an interpolation problem, if $a = 20.8$, $b = 24.6$, $c = 1.37$, $d = 2.38$, and $e = 1.17$, find x.

ANSWERS TO PROBLEMS

1. $R = 12.6$	**7.** $R = 48.8$	**13.** $R = 73.0$	**19.** $R = 5860$
3. $R = 216$	**9.** $R = 6.45$	**15.** $R = 5870$	**20.** $R = 0.00155$
5. $R = 1.83$	**11.** $R = 200$	**17.** $R = 2.80$	

21. 960 acres; 544 acres; 247 acres; 1120 acres; 2350 acres.

24. 0.560 B.T.U.; 97,200 ft.-lb.; 2.55 B.T.U.; 1416 ft.-lb.

27. 4.75 joules; 13.05 ft.-lb.; 22.1 joules; 49.3 ft.-lb.

31. $H = 96$ cm.

R

VI · SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS

Squares and Cubes (A, B, C^2, $\sqrt{\ }$, K, and $\sqrt[3]{\ }$ Scales)

The position of numbers on A are the squares of those on D; those on B or C^2 are the squares of those on C. Find several numbers on the D or C scale, such as 2, 5, 8, 12, and 34, and notice that the squares of these numbers, namely 4, 25, 64, 144, and 1156, appear on the A or B or C^2 scales directly above.

The positions of numbers on K, if such a scale exists on your rule, are the cubes of those on D. Find 3, 4, 5, 7, and 9 on the D scale and read 27, 64, 125, 343, and 729 on the K scale.

On the Pickett rules, Models 2 and 4, the $\sqrt{\ }$ scale is a double length D scale. Any number read on a $\sqrt{\ }$ scale has its square on the D scale. The $\sqrt[3]{\ }$ scale is a triple length D scale. Any value read on a $\sqrt[3]{\ }$ scale has its cube on the D scale.

To find the square or cube of a number, determine the size of the answer by estimating.

Example 1. Find the square of 12.7.

Procedure, using A and D, or B and C, or C^2 and C. Put the hairline over 127 on D. Answer (= 161) is on A under the hairline. Or, put the hairline over 127 on C. Answer is on B or C^2 under the hairline.

Procedure, using D and $\sqrt{\ }$. Put the hairline over 127 on $\sqrt{\ }$. Answer (= 161) is on D under the hairline.

Since $12.7^2 = 12.7 \times 12.7$, the estimated product is $10 \times 10 = 100$. Therefore $12.7^2 = 161$.

Example 2. Find the cube of 0.683.

Procedure, using K and D. Put the hairline over 683 on D. Answer (= 0.318) is on K under the hairline.

Procedure, using D and $\sqrt[3]{\ }$. Put the hairline over 683 on $\sqrt[3]{\ }$. Answer (= 0.318) is on D under the hairline.

The estimated product of 0.683^3 is $0.7 \times 0.7 \times 0.7 = 0.343$.

Square Roots and Cube Roots

To find the square root or the cube root of a number, the scales used are in inverse order to those used in finding the square or the cube of a

number. It will be necessary to determine the part of the A or B or C^2 scale that must be used to get the correct digits of the answer on the D or C scale; or, to determine which part of the $\sqrt{\ }$ or $\sqrt[3]{\ }$ scale will contain the correct digits of the answer. This can be done very simply by noting what is done in the following examples:

Example 3. Find the square root of 23,700. From the decimal point, mark off the digits in blocks of two, as follows: $\sqrt{2'37'00}$. Since there is only one digit in the block farthest to the left, use the **left** half of the A or B or C^2 scale, and read the root on the D or C scale; or, put 237 on D and read the root on the **upper** $\sqrt{\ }$ scale. The result is $\sqrt{\underline{2}'\underline{37}'\underline{00}} = \underline{154}.$ That is, there is a digit in the answer to the left of the decimal point for every block in the number whose root is being taken.

Example 4. Find the square root of 2370. Marking off the digits in blocks of two, the left block contains two digits: $\sqrt{23'70}$. Use the **right** half of the A or B or C^2 scale and read the root on the D or C scale; or, put 237 on D and read the root on the **lower** $\sqrt{\ }$ scale. The result is $\sqrt{\underline{23}'70} = \underline{4}8.7$.

Example 5. Find the square root of 0.00385. Marking off the digits in blocks of two from the decimal point, $\sqrt{0.00'38'5}$, the first block containing at least one significant digit has two of them. Therefore, use the **right** half of the A or B or C^2 scale and read the root on the D or C scale; or, put 385 on D and read the root on the **lower** $\sqrt{\ }$ scale. The result is $\sqrt{0.\underline{00}'\underline{38}'5} = 0.\underline{06}21$. That is, in getting the square root of a decimal, two things must be known, (1) how many zeros between the decimal point and the first significant digit, and (2) what is the first significant digit. In this example, one block contains zeros only; so there is one zero between the decimal point and the first significant digit.

Rule to Determine the Square Root of a Number

1. If the first non-zero block at the left contains **one** significant digit, use the **left** half of the A or B or C^2 scale; or read the answer on the **upper** $\sqrt{\ }$ scale.

2. If the first non-zero block at the left contains **two** significant digits, use the **right** half of the A or B or C^2 scale; or read the answer on the **lower** $\sqrt{\ }$ scale.

Note the following square roots:

$\sqrt{\underline{4}'\underline{38}} = \underline{20}.9$; $\sqrt{\underline{43}'\underline{80}} = \underline{61}.2$; $\sqrt{0.\underline{04}'38} = 0.\underline{2}09$; $\sqrt{0.\underline{00}'\underline{43}'8} = 0.\underline{06}12$

Rule to Determine the Cube Root of a Number

For cube roots, mark off the digits in blocks of three from the decimal point. Note the first block at the left that contains significant digits. If this block contains **one** significant digit, use the **left third** of K, or read the answer on the **top** $\sqrt[3]{\ }$ scale. If the first block contains **two** significant digits, use the **middle third** of K, or read the answer on the **middle** $\sqrt[3]{\ }$ scale. If the first block contains **three** significant digits, use the **right third** of K, or read the answer on the **bottom** $\sqrt[3]{\ }$ scale.

Note the following cube roots:

$\sqrt[3]{4'380} = 16.4$; $\sqrt[3]{43'800} = 35.3$; $\sqrt[3]{438'000} = 75.9$; $\sqrt[3]{0.004'38} = 0.164$

Squares and Square Roots Using the C and D Scales Only (Optional)

Many engineering students avoid the A and B scales when getting squares and square roots.

To get the squares of numbers the method is obvious: merely multiply a quantity by itself.

To get a square root note that the answer is the midpoint of the scale between the index of D and the number whose square root is wanted—where if the number is between 1 and 10, or any product or quotient by 100, the square root is the midpoint using the **left index**; and if between 10 and 100, or any product or quotient by 100, the square root is the midpoint using the **right** index.

Example 6. Find the square root of 8.4. Place the indicator over 8.4 on D. By trial, shift the slider until the value under the **left** index of C is identical with the value on C under the indicator, namely 2.9.

Example 7. Find the square root of 0.76. Place the indicator over 76 on D. By trial, shift the slider until the value under the **right** index of C is identical with the value on C under the indicator, namely 0.872.

This method has one advantage, namely, it permits the use of scales (C and D) that can be read with greater accuracy than is true for the A and B scales. But, do not use a magnifier on the indicator when making the settings, owing to the difficulty of reading the scales.

Use of A and B Scales for Multiplication and Division (Optional)

In simple multiplication and division problems it is just as easy to use the A and B scales as to use the C, D, CF, and DF scales. The only disadvantage is that each half of the A scale is only one half the length of the D scale.

Since the decimal point position in the answer is determined by approximation, there is no need to specify which half of the A or B scale is to be used for a particular setting as that only applies when other scales are used also. (See Chapter VII.)

Try the following problem, solved once before, using only the A and B scales:

$$\frac{28.3 \times 8.38 \times 12.9}{16.7 \times 4.27 \times 5.86} = 7.32$$

Solution. Place the hairline of the indicator over 28.3 on first half of A scale. Put 16.7 on first half of B scale under hairline. Move indicator to 8.38 on first half of B. Now place 4.27 on first half of B scale under hairline. Then move indicator to 12.9 on second half of B scale. Finally, put 5.86 on first half of B scale under the hairline. The answer is over either index of B scale within the range of the A scale. Next try the solution by first placing hairline of the indicator over 28.3 on the second half of A scale and continue by suitable settings.

Remember. **Always keep the slider at least halfway in the stock.**

PROBLEMS

1. Find the squares of 0.35, 10.17, 0.979, 1.265, 5.76, 8.25.

2. Find the squares of 0.0538, 0.684, 2.89, 7.74, 12.67, 8.33.

3. Find the cubes of 3.93, 14.2, 5.71, 1.33, 1.82, 0.662.

4. Find the cubes of 0.0133, 0.052, 0.735, 8.99, 0.992, 893.

5. Find the square roots of 101.7, 125.4, 4.77, 87.9, 0.789.

6. Find the square roots of 0.0925, 0.563, 0.00845, 6.44, 5.9.

7. Find the cube roots of 2.61, 17.8, 1793, 571, 82.7, 40.

8. Find the cube roots of 0.00321, 0.0488, 0.0133, 0.267, 0.882.

9–23. Solve the first fifteen problems in chapter IV, using only the A and B scales.

24–43. Solve the first twenty problems in chapter V, using only the A and B scales.

VII · CALCULATIONS INVOLVING POWERS AND ROOTS

No matter what type of slide rule is available, many problems involving squares and square roots can be solved with only one setting. Below, these simple settings will be stressed and applications made.

Ten Typical Products or Ratios

Products $x^2y,\ x^2y^2,\ x\sqrt{y},\ \sqrt{x}\sqrt{y}$

Ratios $\dfrac{x^2}{y},\ \dfrac{x}{y^2},\ \dfrac{x^2}{y^2},\ \dfrac{\sqrt{x}}{y},\ \dfrac{x}{\sqrt{y}},\ \dfrac{\sqrt{x}}{\sqrt{y}},$

where x and y are known numbers. Four examples to illustrate their use are:

Example 1. $(2.74)^2 \times 3.87 = 29.1$. Put left index of B to 2.74 on D. Answer is on A over 3.87 on B.

Example 2. $30.8\sqrt{0.747} = 26.6$. Put right index of C under 0.747 on A. Answer is on D under 30.8 on C.

Example 3. $(14.8)^2/6.84 = 32.0$. Put 6.84 on B over 14.8 on D. Answer is on A over right index of B.

Example 4. $38.4/\sqrt{77.8} = 4.36$. Put 77.8 on B over 38.4 on D. Answer is on D under right index of C.

It is to be noted that, when a square is involved, the answer is always on A and that, when a square root is involved, it is always on D. Also no problem can be solved directly which involves both a square and a square root. Either the square or the square root must be obtained first.

The equations of motion that pertain to a falling body (and also to an inclined plane) would illustrate these. They are $h = \frac{1}{2}gt^2$, $t = \sqrt{2h}/\sqrt{g}$, $h = v^2/2g$, $g = 2h/t^2$, and so on, where t is time, h is height fallen, v is velocity at any given instant, and g is the acceleration constant.

Type Forms Involving Cubes or Cube Roots

For those that have a K scale, some other type forms are (x^3y^3), $x\sqrt[3]{y}$, $x^{\frac{2}{3}}y$, $(xy)^{\frac{3}{2}}$, $x^{\frac{2}{3}}y^2$, $(x/y)^3$, $\sqrt[3]{x/y}$, $(x/y)^{\frac{3}{2}}$, $x^{\frac{2}{3}}/y^2$, and so on. Two examples for illustration are

Example 5. $3.28\sqrt[3]{284} = 21.5$. Put right index of C over 284 on K. Answer is on D under 3.28 on C.

Example 6. $(14.8)^{\frac{2}{3}}/(2.72)^2 = 0.817$. Put 2.72 on C over 14.8 on K. Answer is on A over right index of C.

Other Problems Involving Squares and Square Roots

Many compound multiplication and division, proportion, and equivalent-ratio problems involve factors that are squares or square roots. If some of the factors are squares, the *principal scales* are A and B. Note the following literal examples:

$$\frac{x^2 \cdot y}{z^2} = ?;\quad \frac{R}{x^2} = \frac{y^2}{z}$$

At first it will be wise for one to assign the scales on which each of these values will be found. That is,

$$\frac{[x^2_{\rightarrow D}]^{\rightarrow A} \cdot y^{\rightarrow B}}{[z^2_{\rightarrow C}]^{\rightarrow B}} = ?^{\rightarrow A};\quad \frac{R^{\rightarrow A}}{[x^2_{\rightarrow C}]^{\rightarrow B}} = \frac{[y^2_{\rightarrow D}]^{\rightarrow A}}{z^{\rightarrow B}}$$

where $[x^2_{\rightarrow D}]^{\rightarrow A}$ means that x on D would give x^2 on A, the principal scale.

If some of the factors are square roots, the *principal scales* are C and D. The scales on which the values are found are shown in the next illustrative examples.

$$\frac{\sqrt{x_{\rightarrow A}}^{\rightarrow D} \cdot y^{\rightarrow C}}{\sqrt{z_{\rightarrow B}}^{\rightarrow C}} = ?^{\rightarrow D};\quad \frac{\sqrt{x_{\rightarrow B}}^{\rightarrow C}}{R^{\rightarrow D}} = \frac{y^{\rightarrow C}}{\sqrt{z_{\rightarrow A}}^{\rightarrow D}}$$

Use these suggestions on the problems that appear at the end of the chapter. Note what was done in the following example:

Example 7. $\dfrac{R}{(3.84)^2} = \dfrac{(18.6)^2}{15.8}$

$$\frac{R^{\rightarrow A}}{(3.84_{\rightarrow C})^{2\rightarrow B}} = \frac{(18.6_{\rightarrow D})^{2\rightarrow A}}{15.8^{\rightarrow B}}$$

Problems from Physics

Example 8. Determine the period of a pendulum whose length is 22 centimeters.

$T = \dfrac{2\pi\sqrt{l}}{\sqrt{g}}$, where T is the period, l is the length of the pendulum in centimeters, and g (the constant of gravitation) is 980 centimeters per second. Thus

$$T = \frac{6.28\sqrt{22}}{\sqrt{980}} = 0.941$$

Procedure. Over 6.28 on D place 980 on B_1 (left half of scale) by use of indicator. The answer is on D under 22 on B_2 (right half).

Example 9. What is the resistance in ohms of a wire 150 feet long whose diameter is 4.5 mils and whose specific resistance is 2 ohms per mil foot?

$R = \frac{Kl}{d^2}$, where R is resistance in ohms, K is specific resistance (measured in ohms per mil-foot), l is length in feet, and d is diameter in mils. Thus

$$R = \frac{2 \times 150}{(4.5)^2} = 14.8$$

Procedure. Under 2 on A_1 place 4.5 on C. Answer is on A above 150 on B.

Example 10. Find the length of a pendulum whose period is 0.82 if, when it has a length of 30 centimeters, it has a period of 1.12. The formula needed is

$$t_1 : t_2 = \sqrt{l_1} : \sqrt{l_2},$$

where t_1 and t_2 are the periods of the pendulums respectively, and l_1 and l_2 are their respective lengths. Thus $1.12 : 0.82 = \sqrt{30} : \sqrt{l_2}$ or $l_2 = 16.10$.

Procedure. Place 1.12 on C over 0.82 on D. Interchange indexes. Answer is on A_1 over 30 on B_2.

PROBLEMS

1. $(17.7)^2 \times 286$
2. $(0.287)^2 \times 4.96$
3. $8.87 \times (1.87)^2$
4. $(3.28)^2 \times (0.674)^2$
5. $(0.298)^2 \times (0.0742)^2$
6. $(387)^2 \times (0.00284)^2$
7. $12.7\sqrt{4.87}$
8. $0.287\sqrt{39.6}$
9. $\sqrt{0.0874} \times 21.7$
10. $\sqrt{2.28} \times \sqrt{47.3}$
11. $\sqrt{0.0782} \times \sqrt{22.9}$
12. $\sqrt{0.713} \times \sqrt{0.00626}$
13. $(14.4)^2/28.5$
14. $(0.0287)^2/0.00397$
15. $(282)^2/1070$
16. $12.9/(3.76)^2$
17. $82.8/(14.7)^2$
18. $1.74/(0.296)^2$
19. $(16.8)^2/(8.87)^2$
20. $(1.87)^2/(0.298)^2$
21. $(6.62)^2/(30.8)^2$
22. $\sqrt{69.9}/3.22$
23. $\sqrt{0.228}/1.07$
24. $\sqrt{826}/15.9$
25. $8.18/\sqrt{19.7}$
26. $0.272/\sqrt{0.0333}$
27. $197/\sqrt{667}$
28. $\sqrt{61.6}/\sqrt{3.97}$
29. $\sqrt{89.7}/\sqrt{163}$
30. $\sqrt{0.0272}/\sqrt{0.383}$
31. $(14.7 \times 0.0773)^3$
32. $(2.79 \times 1.84)^3$
33. $8.16 \times \sqrt[3]{12.4}$
34. $\sqrt[3]{696} \times 0.0917$
35. $6.16 \times (18.7)^{\frac{2}{3}}$
36. $(40.7)^{\frac{2}{3}} \times 0.836$
37. $(2.72 \times 1.07)^{\frac{3}{2}}$
38. $(0.0927 \times 40.3)^{\frac{3}{2}}$
39. $(12.2)^{\frac{2}{3}} \times (1.37)^2$
40. $(0.0681)^{\frac{2}{3}} \times (40.0)^2$
41. $(18.7/3.62)^3$
42. $(0.979/0.0383)^3$
43. $\sqrt[3]{61.6}/4.87$
44. $\sqrt[3]{0.939}/0.174$
45. $(6.26/1.94)^{\frac{3}{2}}$
46. $(12.8/0.0740)^{\frac{3}{2}}$
47. $(9.17)^{\frac{2}{3}}(3.16)^2$
48. $(0.0278)^{\frac{2}{3}}(0.169)^2$
49. $\frac{2.72 \times \sqrt{0.0284}}{37.8}$
50. $\frac{\sqrt{327} \times 0.0738}{\sqrt{18.6}}$
51. $\frac{6.27 \times (17.8)^2}{31.6}$
52. $\frac{(0.238)^2 \times 18.6}{(6.37)^2}$
53. $\frac{\sqrt{47.8} \times \sqrt{0.0236}}{18.6 \times \sqrt{2.94}}$

54. $\sqrt{47.2} : R = 18.6 : \sqrt{37.9}$
55. $R : \sqrt{0.0236} = \sqrt{61.8} : 15.9$
56. $(18.4)^2 : 9.28 = R : (0.236)^2$
57. $(8.28)^2 : (0.316)^2 = 72.9 : R$
58. $\sqrt{R} : 16.9 = \sqrt{0.236} : 3.84$

59. In a falling-body problem $g = 32.2$. How long will it take for an object to fall 200 ft.? to fall 1000 ft.? to fall 1 mi.?

60. How far will an object fall in 8 sec.? in 40 sec.? in 1 min.?

61. How far has an object fallen if its velocity is 60 ft. per second? if its velocity is 300 ft. per second? if 1000 ft. per second?

62. What is the resistance in ohms of a wire 300 ft. long whose diameter is 5 mils and whose specific resistance is 1.72 ohms per mil-foot?

63. Determine the period of a pendulum of length 24.6 cm. if a pendulum of 28 cm. has a period of 1.612.

64. Determine the period of a pendulum of length 30.2 cm. when considering the facts of Problem 63.

65. Determine the length of a pendulum whose period is 0.78 if when it has a length of 42 cm. it has a period of 0.89.

66. Determine the length of a pendulum whose period is 1.42 when considering the facts of Problem 65.

ANSWERS TO PROBLEMS

1. 89,800	**16.** 0.911	**34.** 0.814	**53.** 0.0334
4. 4.90	**19.** 3.59	**37.** 4.95	**56.** 2.02
7. 28.0	**22.** 2.59	**40.** 268	**59.** 3.52; 7.88; 18.1
10. 10.38	**25.** 1.846	**43.** 0.810	**62.** 20.6
13. 7.29	**28.** 3.94	**46.** 2270	**65.** 32.3
	31. 1.46	**49.** 0.0121	

VIII · FIVE SPECIAL APPLICATIONS

There are many other special applications of the slide rule, five of which will be considered at this time. They apply to any type of rule and are of sufficient importance to receive special attention.

Diameter and Area of a Circle

The only special relationship that appears on the face of the rule pertains to the areas of circles. The area of a circle is given by the formula $\pi d^2/4 = 0.7854\, d^2$. Notice that 0.7854 is marked on scales A and B by a longer line; also note π on the same scales.

Example 1. Required the area of a circle whose diameter is 11.6 inches. Put the indicator to 11.6 on D, set the proper index of B to indicator, and over 0.7854 on B read the answer on A, namely 105.7.

If one wishes, the area can also be determined by using πr^2. Try this out; also try to determine other ways to use the 0.7854 lines to get the area.

The diameter or radius of a circle of given area can be determined by reversing these processes.

A single setting can be made with these 0.7854 lines, which should be done if a series of results are needed. Either put the 0.7854 line on A over the right index of B and read from C to A, or put the 0.7854 line on B under the right index of A and read from D to B. Also it is possible to make a single setting using either π value on A or B over or under the left index of these scales. Try both settings and verify your results.

On the **Pickett rules, Models 2 and 4**, put the 0.7854 line on the C scale over the right index of the D scale. Then over a diameter, say 11.6, on $\sqrt{\ }$ read the area (= 105.7) on C. If the formula $A = \pi r^2$ is used, over r on $\sqrt{\ }$ read A on DF. For example, if $r = 5$, $A = 78.5$.

Example 2. Verify the following answers: $d = 2.87$, $A = 6.48$; $d = 0.0516$, $A = 0.00209$; $A = 18.7$, $d = 4.88$; $r = 4.17$, $A = 54.6$.

Diameter and Circumference of a Circle

The duplex type of rule, having the CF and DF scales, has an added feature in that the values on the DF scale are all π times the values on the D scale. Thus π times any relationship can be determined by the use of

the folded scales. Such relationships as $\pi \times X \times Y$ and $\pi X/Y$ can be determined by obtaining first $X \times Y$ or X/Y from the C and D scales and reading the final result on the DF scale.

The relationship between the diameter and the circumference of a circle, namely $c = \pi \times d$, becomes automatic with the folded scales. As was the case for the area of a circle, read the diameter on D. The circumference is then the value on DF, and vice versa.

If π is in the denominator of the problem, make the initial settings with the CF and DF scales and read the answer on the D scale. That is, any value on D is the corresponding value on DF divided by π.

Type Form $Y = X/a$

To solve problems of the type $Y = X/a$, where a is a constant and X or Y takes on successive values, it is possible to do so with only a single setting. By making the setting so that more than half of the slider is in the stock (always possible), one can get, with a duplex rule, every answer by either C on D, or CF on DF. For the polyphase type it may be necessary to interchange indexes on C for some of the settings. Two examples will be used to illustrate.

Example 3. Find what part of 47.6 are 8.32, 3.74, 9.82, and 7.28. Consider the type form to read $Y = X \times 1/47.6$. Then the single setting is 47.6 on C over the right index of D. Then 8.32 on CF gives 0.175 on DF, 3.74 on C gives 0.785 on D, and so on.

Example 4. Given $X = 12.7\ Y$, find X when Y has the values 0.287, 0.667, 0.803, and 1.87. Put left index of C over 12.7 on D; then 0.287 on C gives 3.64 on D, 0.803 on CF gives 10.2 on DF, and so on.

This type form can also be set up as a proportion; namely,

$$\frac{Y}{X} = \frac{1}{a}$$

Put 1 on C over a on D; then any value X on D will be under Y on C. Or put 1 on D under a on C, and so on.

Type Form $Y^2 = pX$

Many problems, such as those in structural design and gravitational attraction, take on the form $Y^2 = pX$, where p is a constant. Graphically, the curve this equation represents is called a *parabola*. This type form can be set up as follows:

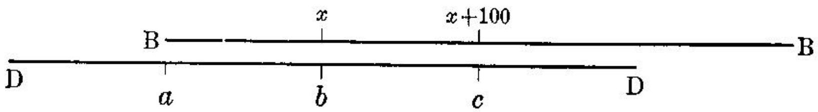
$$\frac{Y^2}{X} = \frac{p}{1}$$

Put p on B under 1 on A; then any value X on A is over Y on C. Or put p on A over 1 on B; then any value X on B is over Y on D.

Example 5. If $p = 42.8$, find Y if X has the values 0.0274, 0.238, 3.17, and 6.28. Put 42.8 on B under 1 on A (right index); then 0.0274 on A is over 1.08 on C, 0.238 on A is over 3.18 on C, and so on.

Type Forms $a^2 + b^2 = c^2$ and $c^2 - a^2 = b^2$

The type form $a^2 + b^2 = c^2$ can be easily solved by using the following diagram, where a is less than b:



Procedure. Put the left index of B over a on D. Move hairline over b on D and read x on B under hairline. Mentally add 100. Then shift hairline to $x + 100$ on B and read c on D under hairline.

If some multiple of 10 lies between a and b, such as $a = 81.6$ and $b = 127$, use right index of C over a and still add 100 to x. If b is over $\sqrt{10}$ (3.16) times a, such as $a = 4.29$ and $b = 21.8$, add only 10 to x. If b is over 10 times a, such as $a = 4.29$ and $b = 51.6$, add only 1 to x.

Procedure on a Pickett rule, Model 2 or 4. Same as above except to use the C scale instead of the B scale; and use a $\sqrt{\ }$ scale instead of the D scale.

Example 6. Prove by slide rule that $3^2 + 4^2 = 5^2$. Put the left index of B over 3 on D. Move hairline over 4 on D and read x (= 178) on B. Change to 278 and move hairline over it on B and read 5 on D under hairline. Or, put the right index of C over 3 on $\sqrt{\ }$. Move hairline over 4 on $\sqrt{\ }$ and read x on C. Move hairline to 278 on C and read 5 on $\sqrt{\ }$.

The type form $c^2 - a^2 = b^2$ can be solved as follows:

$$c^2 - a^2 = (c + a)(c - a); \text{ and } b = \sqrt{(c + a)(c - a)}.$$

Example 7. Find b in $c^2 - a^2 = b^2$ if $c = 25.8$ and $a = 15.4$.
$b = \sqrt{(25.8 + 15.4)(25.8 - 15.4)} = \sqrt{41.2 \times 10.4} = 20.7.$ (Verify this.)

PROBLEMS

1. $8.27 \times 5.87 \times \pi$

2. $\pi \times 9.28 \times 0.274$

3. $\pi \times 40.8/3.72$

4. $\pi \times 1.18/0.0271$

5. $3.84/(2.96 \times \pi)$

6. $8.21 \times 7.38/\pi$

7. $\pi\sqrt{8.45}$

8. $\pi(6.72)^2$

9. $\pi \times \sqrt{15.2}/6.15$

10. $\pi \times 28.4/\sqrt{56.8}$

11. $\pi/(2.67)^2$

12. $(45.7)^2/\sqrt{183}$

13. Find the area of the circles whose diameters are 7.56 in., 2.6 ft., 0.778 in., 12.9 yd., 0.0542 ft.

14. Find the diameters of the circles containing the following areas: 9.74 sq. in., 65.8 sq. ft., 0.884 sq. in., 0.0744 sq. ft.

15. Find the circumferences of the circles in Problem 13.

16. Find the diameters of the circles containing the following circumferences: 45 in., 6.89 ft., 0.574 in., 0.00953 ft.

17. Determine the area of the circles having the following circumferences: 22 in., 5.44 ft., 574 in., 0.00952 ft.

18. What is the volume of water delivered by a 30-inch pipe with a velocity flow of 42 ft. per second? of 60 ft. per second?

19. Given $X = Y/7.84$, find X when Y has the values 3.8, 42.6, 16.67.

20. Given $X = Y/10.7$, find Y when X has the values 4.86, 34.7, 5.78.

21. Solve Problem 19 for Y when X has the values given.

22. Solve Problem 20 for X when Y has the values given.

23. Given $X = Y/8.45$, find X when Y has the values 5.8, 61.2, 18.8.

24. Given $X = Y/67.4$, find X when Y has the values 89.2, 116.2, 14.2.

25. Given $X = Y/\sqrt{116.1}$, find X when Y equals 14.4, 56.3, $\sqrt{88.8}$.

26. Given $X = Y/\sqrt{58.3}$, find X when Y equals 6.8, $\sqrt{45.2}$, $\sqrt{8.3}$.

27. Find what part of 427 are the following: 61.6, 40.8, 83.7, 12.8, 127.

28. Find what part of 87.6 are the following: 32.8, 18.1, 1.96, 0.784.

29. Given $Y^2 = 25.8\,X$, find Y when X has the values 0.0326, 0.874, 1.28, 3.74.

30. Solve Problem 29 for X when Y has the values given.

31. Given $Y = 3.84\sqrt{X}$, find Y when X has the values 0.737, 2.38, 5.24, 8.17.

32. Solve Problem 31 for X when Y has the values given.

33. If $a^2 + b^2 = c^2$, find c if $a = 23.7$, $b = 41.8$; if $a = 4.72$, $b = 7.32$; if $a = 0.384$, $b = 0.772$; if $a = 437$, $b = 562$.

34. If $a^2 + b^2 = c^2$, find c if $a = 31.2$, $b = 37.9$; if $a = 5.24$, $b = 8.12$; if $a = 0.552$, $b = 0.834$; if $a = 88.4$, $b = 107.5$.

ANSWERS TO PROBLEMS

1. 152.4 **4.** 137.0 **7.** 9.15 **10.** 11.88

13. 45 sq. in.; 5.3 sq. ft.; 0.475 sq. in.; 131 sq. yd.; 0.00231 sq. ft.

16. 14.3 in.; 2.19 ft.; 0.182 in.; 0.00303 ft.

19. 0.485; 5.44; 2.12

22. 0.454; 3.24; 0.54

25. 1.333; 5.22; 0.873

28. 37.4%; 20.6%; 2.24%; 0.895%

31. 3.30; 5.93; 8.80; 11.0

34. 49.1; 9.66; 1; 139

IX · THE INVERTED SCALES

The Polyphase rule has an inverted C scale (the CI scale). The duplex rules have both an inverted C scale and an inverted CF scale (the CIF scale). Both of them allow one to read reciprocals directly, can be used to speed up multiplications, and are valuable when applied to certain special problems.

For those that have only a Mannheim-type rule, turn the slider upside down. Having done so, note that the C scale, in this position, is identical with the CI scale on the more elaborate rules; so any problems involving CI and D only, that are given below, can be solved directly. The big disadvantage will be that there is no longer a C scale, so that one loses the chance to solve directly problems involving both C and CI.

Reciprocals

To find the reciprocal of 2, set the indicator to 2 on C and read the result 0.5 on CI. Likewise note that the reciprocals of 4, 5, 8, and 12 are 0.25, 0.20, 0.125, and 0.0833 respectively. The reverse relationship also holds in that any value on CI has its reciprocal on C.

The CI scale can also be used to solve such problems as $1/(3.29)^2$ or $1/(3.29)^3$. Find the value 3.29 on CI and read the result of the first on B, namely 0.0924, and the result of the second on K, namely 0.028. Why are these true?

Those who have the Modern-Duplex types can read reciprocals between the D and DI scales.

Note. Reciprocals can also be read by using the C and D scales only. Put left index of C scale over a number on D scale and read its reciprocal on the C scale over the right index of the D scale. Try this.

Multiplication

Since $4 \div \frac{1}{2}$ is identical with 4×2, the methods of multiplication can be made as simple and direct as division by using the CI scale (reciprocal scale) along with the D scale. Thus, to make the above multiplication, set the indicator to 4 on D; then set 2 on CI to the indicator. Under the right index of C find 8 on D. Thus there is never any question as to which index is to be used.

The proper position of the decimal point in the answer, after having used the CI scale, is best determined by approximation. If the simple two-factor rules given in chapter III are used, it is necessary to interchange the words left and right.

Example 1. Find 23.5×5.76. Put 5.76 on CI over 23.5 on D. The answer is under the left index of C on D, namely 135.5.

Example 2. Find 0.587×1.36. Put 1.36 on CI over 0.587 on D. The answer is under the right index of C on D, namely 0.798.

Important Suggestion

Always use the CI and D scales for multiplication and the C and D scales for division. Get into the habit of using this suggestion, for it will be a timesaver.

Continued Products

The CI and CIF scales have great value in solving problems that involve continued products, such as $R = A \times B \times C \times D \times E$. Consider the problem $R = 2 \times 3 \times 4 \times 5 \times 6 = 720$. Over 2 on D place 3 on CI; move indicator to 4 on C; bring 5 on CI under indicator. Answer is on D under 6 on C. This problem takes on the form of one involving compound multiplication and division by noting that

$$A \times B \times C \times D \times E = A \times \frac{1}{\frac{1}{B}} \times C \times \frac{1}{\frac{1}{D}} \times E,$$

which shows the need of the reciprocal scales. Always make it a point when three or more factors are involved to use alternately the CI and C, or CIF and CF, scales, for then a minimum amount of operation is needed. For comparison work the above problem using the C and D scales only, as would be required on the Mannheim type.

Example 3. Find $2.67 \times 5.83 \times 0.746 \times 13.9 \times 8.35$. Over 2.67 on D put 5.83 on CI. Since 0.746 on C is beyond the D scale, put indicator to 0.746 on CF. Put 13.9 on CIF under hairline. The answer, namely 1349, appears on DF over 8.35 on CF. The CIF settings could have been avoided by interchanging indexes at one point in the procedure, but why do so?

Example 4. Find $5.87 \times 0.483 \times 10.5 \times 22.8 \times 0.0554$. Over 5.87 on D put 0.483 on CI; move indicator to 10.5 on C; put 22.8 on CI under hairline. The answer is on D under 0.0554 on C, namely 37.6.

Do not hesitate to use the CF, DF, and CIF scales whenever you can. It is only through use that one comes to appreciate their value and purpose.

Type Form $X = a/Y$

Solving problems of the type $X = a/Y$, where a is a constant and either X or Y assumes successive values, the CI scale is most useful.

Example 5. Find the resulting field currents for various resistances, as 200, 260, 325, 383, when given a voltage of 110 volts.

$I = E/R = 110/200;\ 110/260;\ 110/325;\ 110/383.$ To 110 on D set 10 on CI. Opposite 200 on CI read 0.55 on D, and so on.

So long as the value on D under 10 on CI is less than or equal to 3.14, any value on CI off the scale can be found on CIF and the result be read on DF instead of D. In the example above, if $R = 105$, then, since 105 on CI is off the scale, over 105 on CIF read the answer 1.047 on DF.

Many ratio and proportion problems can be worked in like manner. Any relationship, where a product of two variables equals a constant, can be solved by the CI scale, as was just illustrated. The following formulas are familiar: Boyle's Law, $P \times V =$ constant; lever problems when given a certain weight and distance from a fulcrum as compared to a varying weight and its needed distance from the same fulcrum — that is, $W_1L_1 = W_2L_2$, where W_2 and L_2 are assumed constant; and so on.

Type Form $1/a + 1/b = 1/c$

If you have a DI scale on your rule, $1/a + 1/b = 1/c$ can be solved by methods similar to those developed for $a^2 + b^2 = c^2$. Put an index of C over a on DI. Move hairline to b on DI and read x on C under hairline. Move hairline to $x + 100$ on C, and read c on DI.

Example 6. Show that $\frac{1}{3} + \frac{1}{2} = \frac{1}{1.2}$. Put left index of C over 3 on DI. Move hairline to 2 on DI and read 150 on C. Now move hairline to 250 on C and read 1.2 on DI. If you do not have a DI scale on your rule, put a on CI (or C inverted) over left index of D. Move hairline over b on CI and read x on D. Over $x + 100$ on D, read c on CI under hairline.

PROBLEMS

1. Find the reciprocals of 7.2, 0.41, 37.8, 389, 17.6.

2. Find the reciprocals of 0.0659, 0.326, 0.00725, 214.

3. Solve, using CI and D scales: $23.9 \times 8.45 = ?$ $34.7 \times 4.65 = ?$ $0.638 \times 445 = ?$ $2.67 \times 0.0638 = ?$ $5.85 \times 78.2 = ?$ $0.0587 \times 23.4 = ?$

4. Solve, using CI and D scales: $50.4 \times 3.32 = ?$ $10.1 \times 34.2 = ?$ $0.00585 \times 0.0478 = ?$ $0.0607 \times 2.03 = ?$ $89.7 \times 3.31 = ?$

5. Find $13.4 \times 9.63 \times 7.46 \times 349$.

6. Find $159 \times 0.834 \times 76.7 \times 9.13 \times 0.0365$.

7. Find $12.7 \times 0.0825 \times 4.46 \times 67.4 \times 2.23 \times 0.00252 \times 18.7$.

8. Find $47.8 \times 3.56 \times 12.9 \times 0.0000466 \times 9.08 \times 4.5 \times 23.7$.

9. Find $2.56 \times 7.83 \times 3.14 \times 4.07 \times 7.22 \times 0.00452 \times 0.43 \times 667 \times 0.0256$.

10. Find $33.3 \times 0.277 \times 0.585 \times 0.0589 \times 1.33 \times 12.6 \times 6.65 \times 5.03 \times 4.01$.

11. $1/(2.38)^2 = ?$ $1/(0.873)^2 = ?$ $1/(18.7)^2 = ?$ $1/(0.0562)^2 = ?$

12. $1/(2.45)^3 = ?$ $1/(0.587)^3 = ?$ $1/(4.13)^3 = ?$ $1/(0.0486)^3 = ?$

13. Given $X = 3.2/Y$, find X when Y takes the values 2.6, 4.48, 12.9, 1.25, 0.956, 0.248, 0.0685, 0.00445.

14. Given $W = 18.5/V$, find W when V takes the values 6.65, 12.4, 16.9, 18.8, 21.5, 42.7, 125, 165.

15. Given $H = 82.7/P$, find H when P takes the values 14.7, 28.5, 45.3, 67.9, 80.1, 87.3, 101, 127, 149, 204.

16. Given $A = 185/B$, find A when B takes the values 44.5, 67.2, 100, 156.5, 178, 194, 265, 487, 905.

17. Given $W = 0.467/V$, find W when V takes the values 0.238, 0.558, 0.895, 1.42, 2.68, 5.45, 0.112, 0.0867.

18. At a pressure of two atmospheres a certain quantity of a certain gas has a volume of 22 cu. ft. Assuming Boyle's Law to hold, find the volume of the gas at pressures of 8, 15.5, 26.2, 30.6, 38.2, and 46.

19. A lever of the first class has the fulcrum 6 in. from the end where a weight of 380 lb. is placed. At what distances from the fulcrum will the weights given here be placed in order to insure equilibrium, one weight acting at a time? The weights are 50, 73.4, 16.93, and 217.

20. Consider Problem 19 when given the following distances to find the proper weights. The distances are 4, 5.5, 7.74, and 12.9.

21. Given that $I = E/R$, where $E = 220$ volts, find the values of the resistance, R, for the following amperages (field currents). They are 1.67, 0.785, 5.88, and 12.75.

22. Consider Problem 21 when given certain resistances to find the field current. The values for R are 666, 8970, 56.8, 127.5, 983, 2150, and 16.7.

23. $\frac{1}{5} + \frac{1}{8} = \frac{1}{?}$; $\frac{1}{3} + \frac{1}{5} = \frac{1}{?}$; $\frac{1}{70} + \frac{1}{50} = \frac{1}{?}$.

ANSWERS TO PROBLEMS

1. 0.139; 2.44; 0.0265; 0.00257; 0.0569

5. 336,000 **7.** 33.0 **10.** 712

13. 1.23; 0.714; 0.248; 2.56; 3.35; 12.9; 46.7; 719

16. 4.16; 2.75; 1.85; 1.18; 1.04; 0.955; 0.7; 0.38; 0.202

19. 45.6; 31.1; 135; 10.53 **23.** 3.08; 1.87; 33.3

* X • CIRCULAR SLIDE RULES

Basic Scales

If the C and D scales of a polyphase type slide rule are put on the circumferences of two circles so that the indexes of each coincide, one would have the basic scales of a simple circular slide rule. Since the scales are continuous, and since there is only one index on each scale, multiplication and division are simple steps. Either of two principles is used to get answers. They are (1) the two circles coincide, with one rotating inside the other, making it possible to read between the **inner** and the **outer** scales (Rotarule on page 6); and (2) only one basic-scale circle is on the rule (usually called C scale), so that two indicators are used to get the answer. (They are connected to the center of the circle. See Midget on page 6.) Only one of the indicators is independent of the other. Note how these principles apply in the examples that follow.

Example 1. $27.8 \cdot 3.78 = ?$

First principle. Place 1 on **inner** scale next to 27.8 on **outer** scale. Next to 3.78 on **inner** scale find answer (= 105) on **outer** scale.

Second principle. Place hairline of **nonindependent** indicator over 1 on basic scale; place hairline of **independent** indicator over 27.8 on basic scale; move hairline of **nonindependent** indicator over 3.78 on basic scale; then find answer on basic scale under hairline of **independent** indicator, namely 105.

Example 2. $27.8/3.78 = ?$

First principle. Place 3.78 on **inner** scale next to 27.8 on **outer** scale. Next to 1 (the index) on **inner** scale find 7.35 on **outer** scale.

Second principle. Place hairline of **nonindependent** indicator over 3.78 (divisor); place hairline of **independent** indicator over 27.8 (dividend); move hairline of **nonindependent** indicator over 1 (the index); then find answer on basic scale under hairline of **independent** indicator, namely 7.35.

General Uses

The fact that the basic scale or scales are continuous has its advantages. For example, there is never any concern about which index to use, since there is only one. Furthermore, the result can never lie beyond the end of the scale, as it commonly does on the ruler type. Note the following **compound multiplication and division** example:

Example 3. $\dfrac{38.2 \cdot 17.8}{77.2} = ?$

First principle. Put 77.2 on **inner** scale next to 38.2 on **outer** scale (division). Then 17.8 on **inner** scale is next to answer (= 8.81) on **outer** scale.

Second principle. Place hairline of **nonindependent** indicator over 77.2; place hairline of **independent** indicator over 38.2 (division); move hairline of **nonindependent** indicator over 17.8; then find answer on basic scale under hairline of **independent** indicator, namely 8.81.

Problems in **simple proportion** and all **equivalent ratio** problems can be solved by **one** setting of the basic scales or indicators, just as would be the case for a duplex slide rule which contains both the C and D scales and the CF and DF scales.

Example 4. $\dfrac{R}{32.7} = \dfrac{61.3}{127}.$

First principle. Put 61.3 on either scale next to 127 on other scale. Then 32.7 on latter scale is next to R (= 15.8) on first scale used.

Second principle. Place hairline of **nonindependent** indicator over 127; place hairline of **independent** indicator over 61.3; put hairline of **nonindependent** indicator over 32.7; then find answer on basic scale under hairline of **independent** indicator, namely $R = 15.8$.

Reciprocals can be found by using the methods noted in the following example. The solution is based on the proportion $1/a = b/1$.

Example 5. What is the reciprocal of 5.28?

First principle. Put the index of either scale next to 5.28 on the other scale. Then its reciprocal is on the latter scale next to the index of the first scale, namely 0.1895.

Second principle. Place hairline of **nonindependent** indicator over index; place hairline of **independent** indicator over 5.28; move **nonindependent** indicator, forcing hairline of **independent** indicator over index; then find answer on basic scale under hairline of **nonindependent** indicator, namely 0.1895.

Many circular slide rules have scales that can be used to find squares or square roots, logarithms or antilogarithms, sines and tangents (discussed later), and even log log scales (discussed later). When these scales are included, one or two indicators are attached to the center of the rule, having a hairline down the middle.

Example 6. What are the square of and the logarithm of 8.32?

Put an indicator hairline over 8.32 on the basic scale that is on the same part of the rule (if necessary) that contains the square scale and the logarithm scale. The answers are 69.3 and 0.920 respectively.

PROBLEMS

Solve the problems given in chapters II, IV, V, VI, and XI.

XI · THE COMMON LOGARITHM SCALE

There are many problems that cannot be solved directly on the standard slide rules. For many of them the use of logarithms is required. The L scale, which is an equal-parts scale, or a logarithm table can be used for this purpose. The use of the L scale will be considered in this chapter, and the use of a logarithm table will be considered in chapter XII.

Definition of a Logarithm

Since $10^1 = 10$ and $10^2 = 100$, any number lying between 10 and 100 can be expressed as 10 to some exponent between 1 and 2. For example, $59.4 = 10^{1.774}$ by use of the L scale and $10^{1.77379}$ by the use of a logarithm table. That is, *the logarithm of a number is the exponent of 10 representing that number.*

Mantissa: The Decimal Part of the Logarithm of a Number (L Scale)

The exponent of 10 representing a number is made up of two parts — the integral part (characteristic) and the decimal part (mantissa). The decimal part can be read directly from the slide rule by the use of the D and L scales.

Procedure: Polyphase Type. Two methods can be used:

1. Turn slider over so that the S, L, and T scales show. Then for any number on the D scale the decimal part of the logarithm is found on the L scale. Use the indicator.

2. Without turning the slider over, move it so that the right index of the D scale is directly under the number on the C scale. The required mantissa is thus obtained by turning the rule over and reading the result on the L scale under the fixed hairline found there.

Note. On some polyphase-type rules the L scale is reversed, making the first method awkward. By the second method the scale reads: Put the left index of the C scale over the number on the D scale. The hairline on the back is over the required mantissa on the L scale.

Procedure: Duplex Types. For any number on the D scale the mantissa of the logarithm is found on the L scale. Use the indicator.

Characteristic: The Integral Part of the Logarithm of a Number

The integral part can be quickly determined by noting that a number between 1 and 10, such as 5.94, can be expressed as 10 to some exponent

between 0 and 1, since $1 = 10^0$. That is, $5.94 = 10^{0.774}$ (by rule-setting). If the digits 5, 9, and 4 are the same, and the decimal point is assigned different positions, the only change that takes place is the characteristic. Note what happens in the following illustrations:

$$594 = 5.94 \times 10^2 = 10^{0.774} \times 10^2 = 10^{2.774}$$
$$59.4 = 5.94 \times 10^1 = 10^{1.774}$$
$$5.94 = 5.94 \times 10^0 = 10^{0.774}$$
$$0.594 = 5.94 \times 10^{-1} = 10^{-1+0.774} \quad \text{or} \quad 10^{9.774-10}$$
$$0.0594 = 5.94 \times 10^{-2} = 10^{-2+0.774} \quad \text{or} \quad 10^{8.774-10}$$

To obtain the characteristic, the following procedure is recommended: *Change any number given, if necessary, to a number between 1 and 10 times 10 to an integral exponent. This integral exponent is the characteristic.* Note the following examples, which have been put down in two forms:

Example 1. What is the logarithm of 3.17?

First form. $3.17 = 10^{0.501}$
Second form. $\log 3.17 = 0.501$

Example 2. What is the logarithm of 2580?

First form. $2580 = 2.58 \times 10^3 = 10^{3.411}$
Second form. $\log 2580 = 3.411$

Example 3. What is the logarithm of 0.00594?

First form. $0.00594 = 5.94 \times 10^{-3} = 10^{-3+0.774}$ or $10^{7.774-10}$
Second form. $\log 0.00594 = -3 + 0.774$ or $7.774 - 10$

Involution and Evolution

The greatest use of the L scale is in solving problems involving involution and evolution. Three types of problems will be considered which cannot be solved by using the scales discussed so far.

First Type. $a^b = X$, where a and b are both known constants.

Example 4. Determine the value of $3.17^{2.58}$

By first form. $3.17^{2.58} = (10^{0.501})^{2.58} = 10^{1.294}$
$= 10^{0.294} \times 10^1 = 1.97 \times 10 = 19.7$

By second form. $\log 3.17^{2.58} = 2.58 \times \log 3.17 = 2.58 \times 0.501$
$= 1.294 = \log 19.7$

Second Type. $a^X = b$, where a and b are both known constants and X is to be determined. Two distinct setups arise as shown in the next two examples. Only the first form will be used in each.

Example 5. $8^X = 56.6$. Find X.

$$8^X = 56.6;\ (10^{0.903})^x = 10^{1.753};\ 0.903\,x = 1.753$$

Therefore $X = 1.945$ (use C and D scales to get answer).

Example 6. $5.43^X = 0.827$. Find X.

$$5.43^X = 0.827;\ (10^{0.734})^X = 10^{-1+0.918} = 10^{-0.082}$$
$$0.734\,X = -0.082$$

Therefore $X = -0.112$.

Third Type. $X^a = b$, where a and b are both known constants and X is to be determined.

Example 7. $X^4 = 12$. Find X.

$$X^4 = 12 = 10^{1.0798};\ X = 10^{1.0798 \div 4} = 10^{0.2698}$$

Therefore $X = 1.860$.

All three of these type forms can be solved directly if log log scales are on the rule. See the discussion given in chapter XXI.

Compound-Interest Formula

The compound-interest formula fits perfectly into the discussion on involution and evolution. The second form of logarithm procedure is the more suitable. The formula is

$$A = P(1 + i)^n,$$

where P is the **principal** (or **present value**), i is the interest rate per interest period, n is the number of interest periods, and A is the **amount.** i is always expressed in hundredths.

When logarithms are applied to this formula, it becomes

$$\log A = \log P + n \times \log (1 + i)$$

Once any three values are known, the fourth can be determined.

Example 8. Find i, given $A = \$500$, $P = \$225$, and $n = 12$.

$$\log 500 = \log 225 + 12 \times \log (1 + i)$$

Then $\log (1 + i) = (\log 500 - \log 225) \div 12$

$$= (2.699 - 2.352) \div 12 = 0.347 \div 12 = 0.0289$$
$$1 + i = 1.07$$
$$i = 0.07 = 7\%$$

Physics and Engineering Formulas

There are many formulas met in physics and engineering that are solved by logarithms, where the accuracy obtained by the use of the D–L scale combination is sufficient. Two examples are considered.

Example 9. What is the factor of safety in a tall standpipe where the water is 25 feet deep?

$$f = h^{\frac{4}{11}} = 25^{\frac{4}{11}} = (10^{1.398})^{\frac{4}{11}} = 10^{0.509} = 3.135$$

Example 10. Find the modulus of torsion of a homogeneous circular steel bar of radius 0.25 inches if the modulus of rigidity (u) of steel is 12×10^6 pounds per square inch.

$$R = \tfrac{1}{2} \times u \times \pi \times r^4 = \tfrac{1}{2} \times 12 \times 10^6 \times \pi \times (0.25)^4$$
$$\log R = \log 12 + 6 + \log 3.14 + 4 \times \log 0.25 - \log 2$$

The completion is left to the student.

Addition and Subtraction

One who has a slide rule with the L scale on the **slider** can add or subtract a series of three-digit numbers if he desires to do so. This is because the L scale is an equal-parts scale. It is not possible to do this on a duplex rule if the L scale is on the **stock.**

Example 11. $3.84 + 5.28 - 4.27 + 8.35 = ?$

Have ends of L scale flush with ends of D scale. Put hairline to 3.84 on L. Move 0 on L scale under hairline. Put hairline to 5.28 on L. Put 4.27 on L under hairline. Put 10 on L under hairline. This must be done since 1 on L under hairline would make 8.35 be out of rule. Put hairline to 8.35. Set L scale flush with ends of D scale and read answer on L scale under hairline. The answer seems to be 3.20. The correct answer is 13.20; namely, the answer can be any multiple of 10 plus the final reading on the L scale.

Example 12. $47.8 - 39.2 = ?$

This example must read $30 - 30 + 17.8 - 9.2$; so answer is 8.6. With L scale in flush position, put hairline to 7.8 (namely $17.8 = 10 + 7.8$). Put 9.2 under hairline. Move hairline to index of L scale which is inside stock. Put L scale in flush position and read answer.

Another method of adding is similar to the method used in solving $a^2 + b^2 = c^2$ and $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$. That is, to solve $a + b = c$, put an index of C over a on D. Move hairline to b on D and read x on C under hairline. Move hairline to $x + 100$ on C, and read c on D. If b is more than 10 times a, add 10 to x. If b is more than 100 times a, add 1 to x.

Example 13. $2.78 + 4.57 = ?$ Put left index of C over 2.78 on D. Move hairline to 4.57 on D and read 164 on C. Move hairline to 264 on C and read the sum, 7.35, on D.

Example 14. 7.35 + 83.4 = ? Put left index of C over 7.35 on D. Move hairline to 83.4 on D and read 113.5 on C. Move hairline to 123.5 on C and read the sum, 90.75, on D.

PROBLEMS

1. Find the logarithms of the following numbers to three places in the mantissa: 218, 131.7, 0.983, 0.0436, 7194, 0.00837, 88.4, 34700, 0.000087.

2. Solve $(1.28)^{\frac{5}{3}}$ by use of logarithms
3. Solve $(8.26)^5$; $(0.845)^4$
4. Solve $(0.00678)^5$; $(0.1015)^5$
5. Solve $(5.89)^{6.2}$; $(80.8)^{0.67}$
6. Solve $(0.0896)^{0.46}$
7. Find y when $12^y = 47.6$
8. Find y when $15.3^y = 8.68$
9. Find y when $0.78^y = 0.834$
10. Find X when $X^8 = 31.9$
11. Find X when $X^{6.2} = 12.4$
12. Find X when $X^{0.69} = 4.58$
13. Find X when $X^{2.7} = 0.581$
14. $4.82^x = 0.397$
15. $0.727^x = 3.27$
16. $27.4^x = 0.793$
17. $1.97^x = 0.0674$
18. Solve the equation $21.6^n = 12.4$ for n.
19. In how many years will $25 amount to $42 when money is compounded at 6% (0.06)?
20. How much must be set aside today in order to have $600 10 yr. hence if compounded at 5%?
21. How long will it take for money to double itself at 6%?
22. $10 today is worth how much 100 yr. hence if money is worth 5%?
23. At what interest rate will $65.50 amount to $92 when compounded for 8 yr.?
24. What is the factor of safety in a tall standpipe where the water is 37.50 ft. deep?
25. Complete Example 10 given on page 51.

ANSWERS TO PROBLEMS

4. 1.43×10^{-11}; 1.073×10^{-5}	10. 1.54	19. 8.9 yr.
7. 1.55	14. − 0.589	22. $1320

XII · TABLE OF COMMON LOGARITHMS

In addition to a knowledge of the basic principles of the slide rule and its use in solving all sorts of practical problems, one should know how to use tables of logarithms. There is a dual need for this, first because the slide rule is based on logarithms and second because many problems require a higher degree of accuracy than is possible on the rule.

The Characteristic and Mantissa of a Logarithm

Read carefully the discussions given in the previous chapter on the definition of a logarithm and the way to determine its **characteristic**. These statements apply whenever a logarithm is asked for or a problem is solved by the use of logarithms.

The **mantissa** can be obtained from the Table of Logarithms in the Appendix. A number, such as 458, can be expressed as 10 to some exponent by the following method:

$$458 = 4.58 \times 10^2 = 10^{2.66087}$$

Find 458 by reading 45 under N and the 8 along the top or bottom of the page. Go horizontally from the 45 and vertically from the 8 to find the mantissa. Try this.

Logarithm-Table Values versus L-Scale Values

Values read from the table can be compared with the values read on the L scale. Note that the N values in the table are the D-scale values and the table values themselves are the L-scale values.

Use of the Logarithm Table

The first requirement is to know how to find the logarithm of any number. Some can be read directly from the table, while others must be approximated. Note the two examples which follow.

Example 1. Express 10.56, 3.97, 0.0653 as 10 to some exponent.

$$10.56 = 10^{1.02366};\ 3.97 = 10^{0.59879};\ 0.0653 = 10^{8.81491-10}$$

See if you can find these values.

Example 2. Express 10.564, 3.976, 0.06538 as 10 to some exponent.

To do this use the cd column at the right of the table or figure the actual differences in the table. For the digits 1056 the logarithm reading is 02366 and for the digits 1057 the reading is 02407. The table difference is 41. Therefore add 0.4 of this difference, namely 16 (16.4 exactly), to the smaller number. Thus

$$10.564 = 10^{1.02382} \text{ (check this)}$$

In like manner $3.976 = 10^{0.59945}$; $0.06538 = 10^{8.81544-10}$ (check these).

The reverse story is equally important; namely, given the logarithm of a number, find what it is. Note the example.

Example 3. $10^{1.92038}$ = what number?

Since the table gives only the decimal part, look up the value 92038 in the table. This does not appear in the table; so the answer must be approximated. The table gives $10^{1.92012} = 83.2$ and a cd value of 52. The fourth digit is therefore 38 − 12, or 26, over 52, namely about ½ of 10, or 5; so the answer is $10^{1.92038} = 83.25$. In like manner show that $10^{8.81541-10} = 0.06538$.

Since $10^x \times 10^y = 10^{x+y}$, $10^x \div 10^y = 10^{x-y}$, $(10^x)^y = 10^{xy}$, and $\sqrt[y]{10^x} = (10^x)^{\frac{1}{y}} = 10^{\frac{x}{y}}$, it is possible to use logarithms to change a product to a sum, a quotient to a difference, a power to a product, and a root to a quotient. This can be shown in one example. Check it over step by step.

Example 4. $\sqrt[3]{\dfrac{27.8 \times (8.27)^2}{0.0347}} = ?$

$$\sqrt[3]{\frac{27.8 \times (8.27)^2}{0.0347}} = \sqrt[3]{\frac{10^{1.44404} \times (10^{0.91751})^2}{10^{8.54033-10}}}$$

$$= 10^{\frac{1.44404 + 1.83502 - 8.54033 + 10}{3}} = 10^{1.57958}$$

$$= 37.98$$

For convenience this can be put into a much more workable form by using the notation log 59.4 = 1.77379. Note page 48. If this is done, a compact layout of the example can be set up before a single value is read from the table.

Layout	*Solution*
log 8.27 =	log 8.27 = 0.91751
2 × log 8.27 =	2 × log 8.27 = 1.83502
log 27.8 = ______	log 27.8 = 1.44404
Add last two =	13.27906 − 10
Subtract log 0.0347 = ______	log 0.0347 = 8.54033 − 10
Difference =	4.73873
Divide by 3 =	÷ 3 = 1.57958
Answer =	Answer = 37.98

Note that the 3.27906 had to be changed to 13.27906 − 10 in order that subtraction might be possible.

If in such a problem one should have the form

$$\frac{7.38264 - 10}{6},$$

change the numerator to read 57.38264 − 60.

PROBLEMS

1. 37.3×0.287

2. 0.00384×0.0237

3. 81.6×428

4. $127.5 \div 0.0386$

5. $0.07123 \div 82.72$

6. $0.00381 \div 0.2174$

7. $(41.7 \times 18.2)^{3/2}$

8. $(138 \div 8.27)^{-3}$

9. $(27.91 \times 0.00328)^4$

10. $(0.0176 \div 14.8)^5$

11. $\sqrt[5]{\dfrac{18.6 \times 0.2734}{427.2}}$

12. $\sqrt[3]{\dfrac{0.278}{14.6 \times 0.0174}}$

13. $\left(\dfrac{127 \times 0.0182}{16.92}\right)^3$

14. $\left(\dfrac{35.9 \times 18.61}{0.717 \times 1.82}\right)^4$

15. $\sqrt[3]{\dfrac{8.28 \times (0.374)^2}{(0.0236)^{\frac{3}{2}}\, 15.92}}$

16. $12^x = 47.6$

17. $15.3^x = 8.68$

18. $X^{0.69} = 4.58$

19. $4.82^x = 0.397$

20. $1.97^x = 0.0674$

ANSWERS TO PROBLEMS

1. 10.7	6. 0.01762	11. 0.4122	16. 1.557
2. 0.0000908	7. 20,900	12. 1.03	17. 0.792
3. 34,900	8. 0.000223	13. 0.002548	18. 9.08
4. 3310	9. 0.00007023	14. 7.05×10^{10}	19. − 0.588
5. 0.0008611	10. 2.38×10^{-15}	15. 2.63	20. − 3.97

XIII · THE TRIGONOMETRIC FUNCTIONS

Polyphase and Simple-Duplex Types

Function Values: Polyphase Types

The trigonometric functions can be found to three significant figures by the use of the S and T scales. They can be determined with the slide in its normal position or reversed, as was the case for logarithms. If read with the slider reversed, the **sine** of the angle, read on the S scale, is given on the A scale when indexes agree; and the **tangent** of the angle, up to 45°, read on the T scale, is evaluated on the D scale in like manner. If read with the slider in the normal position, the angle is put under the hairline on the back. The **sine** value is read on the B scale under the A index, and the tangent value is read on the C scale over the D index.

Function Values: Simple-Duplex Types

If the simple-duplex type of rule is used the **sine** of the angle, given on S, is read on B; and the **tangent** of the angle, given on T, is read on C.

Note. The sine of an angle is always decimal, and the tangent is decimal for angles less than 45°. Thus the first half of the A and B scales denotes the sine values from 0.01 to 0.1, while the second half of these scales denotes values from 0.1 to 1, and the C or D scales denote values from 0.1 to 1.

Cosine-Cosecant-Secant

To find the **cosine** of an angle note the identity, $\cos A = \sin (90° - A)$, by which the S scale can be used. The **secant** demands the identity $\sec A = 1/\cos A = 1/\sin (90° - A)$. The cosecant demands $\csc A = 1/\sin A$.

Special Tangent Relations-Cotangent

There are two special tangent relations:

1. The tangent of an angle greater than 45° cannot be determined directly on these slide rules. It requires the identity $\tan A = 1/\tan (90° - A)$.

Example 1. Find the tangent of 61° 30′. $\tan 61° 30' = 1/\tan 28° 30' = 1/0.543 = 1.84$. That is, having the angle changed to one less than 45°, the tangent can be read directly.

2. The tangent of an angle less than 6° is to be read as if it were the sine of the angle. That is because the sine and the tangent agree to nearly three places up to 6°. Note $\sin 6° = 0.1045$ and $\tan 6° = 0.1051$.

The **cotangent** can be found by noting that it is directly related to the tangent by the formulas $\cot A = 1/\tan A$ or $\cot A = \tan (90° - A)$.

Modern Duplex Types

Function Values

On the modern duplex type of rules both the sine of the angle, given on S, and the tangent of the angle, given on T, are read on C or D. The left index of C or D denotes the value 0.1, and the right index the value 1. In addition, there is a scale marked ST (Sine-Tangent) which gives either the sine or the tangent of small angles less than about 5.7°. For this scale, the left index of C or D denotes the value of 0.01.

Nearly all S, T, and ST scales on these modern rules are graduated in degrees and tenths of a degree. A few rules still have the scales graduated in degrees and minutes. In either case it is easy to change from one measure to the other. For example, 0.6° = 36′, 54′ = 0.9°.

Example 2. Show that $\sin 34.7° = 0.569$; $\tan 27.3° = 0.511$; $\tan 2.4° = 0.0415$. Put hairline over 34.7 on S; over 27.3 on T; over 2.4 on ST; read the answer on C or D.

On the Pickett rules, Models 2 and 4, there is a double T scale. The upper scale runs from about 5.7° to 45°, while the lower scale runs from 45° to about 84.3°. The C or D scale values run from 1 to 10 for this lower scale.

On most duplex type rules the tangent of angles greater than 45° can be read directly between the **red** angles on the T scale and the CI or DI scales; or between the angles with the symbol $<$ (such as < 75) on the T scale and the CI or DI scales.

Example 3. Show that $\tan 63.4° = 1.997$.

The cosine of any angle less than 90° can also be read directly on these modern rules. The **red** angles or the angles with the symbol $<$ (such as < 75) on the S scale are the complements of the angles associated with them.

Example 4. Show that $\cos 42.3° = 0.740$. Put the hairline over the red 42.4 or over < 42.3 on S and read the answer on C or D.

Note. The Table of Natural Trigonometric Functions given in the Appendix can be used to verify the above answers.

Small Angles; Gauge Points (Optional, Really Has Little Value)

For angles less than 40′, two methods can be used to determine their functional value. Note the following example: Determine the sine (or tangent) of 8′ 25″.

First Method. 8′ 25″ is 1/10 of 10 × 8′ 25″, or 1/10 of 80′ 250″, or 1/10 of 1° 24′ 10″, or 1/10 of 0.0245, or 0.00245. That is, the sine or tangent of 8′ 25″ = 0.00245.

This method is based on the idea that the sine or tangent of very small angles can be assumed directly proportional to the angles themselves.

Second Method: Polyphase or Simple-Duplex Types. Note the two gauge points on the S scale; the **second** (″) is to the right of 1° 1′, and the **minute** (′) is to the left of 2°. In addition, keep in mind that sin 1′ = 0.0003 and sin 1″ = 0.000005.

Set **minute** gauge point under 8 on A and read the value on A over the left index of S, namely 233. Now 8 × sin 1′ = 0.0024; so sin 8′ = 0.00233. Next set the **second** gauge point under 25 on A and read the value on A as before, namely 121. Now 25 × sin 1″ = 0.000125; so sin 25″ = 0.000121. Therefore sin or tan 8′ 25″ = 0.00233 + 0.00012 = 0.00245.

Furthermore, on the duplex type of rule, if either gauge point is placed under the left index of A and the number of minutes or seconds is read on B, the result appears on A. This is an equivalent ratio setting in which the right index of B gives the correct reading for 1′ and 1″ on A. Note that these are not quite the values given above. These are more accurate.

★**Second Method: Polyphase or Simple-Duplex Types.** Note the two gauge points on the ST scale; the **second** (″) is to the right of 1° 1′, and the **minute** (′) is to the **left** of 2°. Also, as above, keep in mind that sin 1′ = 0.0003 and sin 1″ = 0.000005. Now if either gauge point is placed over the left index of D and the number of minutes or seconds is read on C, then the result is on D. To illustrate, use the given example. Using **minute** gauge point, find that 8 on C gives 233 on D, or, since sin 1′ = 0.0003, then sin 8′ = 0.00233. Then using **second** gauge point 25 on C gives 121 on D, or, since sin 1″ = 0.000005, then sin 25″ = 0.000121. Therefore sin or tan 8′ 25″ = 0.00233 + 0.00012 = 0.00245.

Consider also the problem, What is the tangent of 89° 51′ 35″? tan 89° 51′ 35″ = 1/tan 8′ 25″ = 1/0.00245 (from above) = 408. This method would apply to any angle greater than 89° 20′ when the tangent is wanted. This naturally does not apply to the sine, for it can be taken directly from the A scale on polyphase and simple-duplex types and from the D scale on modern-duplex types of rules.

Radian Measure

Frequently the algebraic method of measuring an angle is required. This is known as its radian measurement, and is found by dividing the arc (cut off by the angle) by the radius. Thus 2 π radians = 360 degrees; or, as an equivalent ratio, it becomes π : 180.

Four equivalent ratio settings are possible depending on the type and make of rule. They are

1. Some rules have π (= 3.1416) marked on C and D scales. Either put π on C over *180* on D or vice versa.
2. A few rules have *R* gauge marks at 5.73 on C and D scales. Either put *R* on C over an index on D or vice versa. Note that π : 180 = 1 : 57.3.
3. Many rules have π marked on A and B scales only. Either put π on A over *180* on B or vice versa. Do not do this unless necessary.
4. If you have the CF and DF scales on your rule, still another method is possible. Either put *180* on C under π on DF or vice versa.

Logarithms of Function Values

Since on all modern-duplex rules the S and T scales are associated with the C and D scales, and since the L scale is associated with the C and D scales, it is possible to read directly the logarithm of the sine or cosine of any angle less than 90°, and the tangent or cotangent of any angle between 5.7° and 84.3°. If the L scale is on the slide, read directly between the S or T scale and the L scale. If the L scale is on the stock, have the indexes of the S and T lined up with the indexes of the D scale. Then read directly between the S or T scale and the L scale.

Since most function values are decimal, show that the following values are true: log sin 27° 30′ = 9.664 − 10; log tan 31° 18′ = 9.784 − 10; log cos 58° 24′ = 9.719 − 10; log tan 62° 42′ = 0.2872; log cot 71° 48′ = 9.517 − 10; log sin 2° 36′ = 8.657.

PROBLEMS

1. Find sin 32° 14′, sin 85° 40′, tan 19° 20′, tan 42° 30′, sin 16° 12′.
2. Find cos 29° 20′, cos 73° 10′, cot 67° 25′, cot 82°, cos 63° 47′.
3. Find sec 45° 30′, sec 73° 40′, csc 11° 20′, csc 36° 39′.
4. Find tan 63° 45′, tan 77° 15′, cot 23° 30′, cot 12° 50′.
5. Find tan 4° 30′, sin 2° 40′, cos 87°, cot 88° 40′.
6. Find sin 18′ 50″, tan 6′ 22″, sin 18″, tan 1′ 3″.
7. Find tan 89° 30′ 15″, cot 11′ 18″.

8. Prove that $\tan A = \sin A/\cos A$ when $A = 13°$ and when $A = 35'$.

9. What angles have their sine equal to the square of the tangent of 20° and of 42° 30′?

10. Prove that $\sin^2 A + \cos^2 A = 1$ when $A = 37°$ and when $A = 52°$.

11. Prove that $\sin (A + B) = \sin A \times \cos B + \cos A \times \sin B$, assigning values to A and B so that $A + B$ is less than 90°.

12.

Degrees	40°		137°		215°
Radians		0.472		2.72	

13. If arc (a) = radius (r) times angle (A) measured in radians, what is a if $r = 27$ and A is 42°? if $r = 15.8$ and $A = 127°$?

14. Using Problem 13, what is r if $a = 42.8$ and the angle is 27°? if $a = 15.8$ and $A = 68°$?

15. Using Problem 13, what is A if $r = 27$ and $a = 24$? if $r = 12.8$ and $a = 16.7$?

16. Determine the following values: log sin 18° 42′; log tan 27° 6′; log cos 41° 54′; log tan 68° 18′; log cot 32° 30′; log cot 56° 48′; log sin 1° 24′; log tan 2° 12′; log sin 0° 36′.

ANSWERS TO PROBLEMS

1. 0.533; 0.997; 0.331; 0.916; 0.279

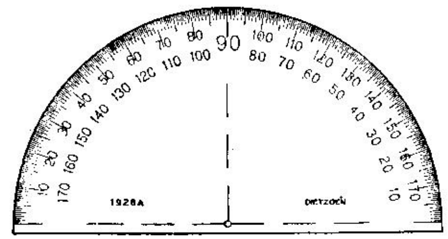
4. 2.03; 4.42; 2.30; 4.39

7. 116; 304 **13.** 19.8; 33.4

16. 9.506 − 10; 9.709 − 10; 9.872 − 10; 0.400; 0.196; 9.816 − 10; 8.388 − 10; 8.585 − 10; 8.020 − 10

XIV · GRAPHICAL SOLUTION OF TRIANGLES

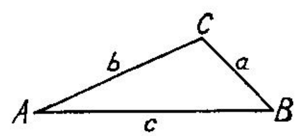
In the next four chapters parts of triangles will be determined by slide-rule procedures when other parts are known. Later, methods will be shown to solve triangles by logarithms. It will be necessary to draw diagrams of the triangles in order to apply the techniques of procedure or to use the formulas.



If the diagrams of the triangles are drawn accurately, using a **rule** and a **protractor** (device to measure angles), one can get fairly accurate graphical solutions of the triangles. Since this graphical procedure is the primary method of solution in navigation, and since good drawings of triangles help materially in their algebraic solutions, the two most common triangle patterns will be considered here.

Given Two Sides and the Included Angle

Assume that side b, side c, and angle A are known, where side b is opposite angle B, and so on. Therefore side a, angle B, and angle C are desired. Using a suitable unit, lay off c. Use a protractor to determine the position of side defined by AC, whose length is b. Then lay off b, using same unit that was used for c. Draw CB and measure a with ruler, using same scale that was used for b and c. Measure B and C with protractor. The values thus found will be fairly accurate answers for the parts unknown.



Given Two Angles and a Side

Assume that A, C, and c are known (note same triangle). Therefore a, b, and B are desired. First find B by using the fact that $A + B + C = 180°$. Then lay off c. Use protractor to determine the positions of AC and BC. a and b can now be measured, and all parts of the triangle are known.

The Two Other Triangle Patterns

Two other triangle patterns can also be considered, namely (a) given two sides and the angle opposite one of them, and (b) given three sides. Both situations require the use of compasses (device to draw circles), but if one follows common-sense procedures they will be very easy to draw.

Note. In any of these procedures one of the angles can be a right angle (90°), which will make it a **right triangle**. Otherwise the triangle is an **oblique triangle**.

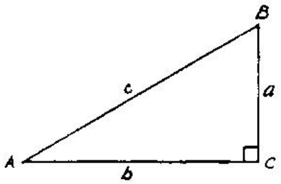
R

XV · ALGEBRAIC SOLUTION OF RIGHT TRIANGLES

Trigonometric Functions

In order to solve a right triangle algebraically, the ratios that define the trigonometric functions are needed. They are

$b/c = \cos A = \sin B$; $c/b = \sec A = \csc B$
$a/c = \sin A = \cos B$; $c/a = \csc A = \sec B$
$a/b = \tan A = \cot B$; $b/a = \cot A = \tan B$



Types of Problems

Three types of problems arise dependent on the parts (side and angles) given and the parts wanted. Let us illustrate these by three examples.

Example 1. To find a side given another side and either acute angle; such as given $A = 40°$, $c = 35$ ft., to find a. In solving the second ratio it gives $a = c \times \sin A = 35 \times \sin 40°$.

★*Procedure: Polyphase type, first method.* Put 40° on S under hairline on back. Read answer on B under 35 on A, namely 22.5 ft.

Procedure: Polyphase type, second method. Reverse the slider; put the right index of S under 35 on A. Answer is on A over 40° on S.

★*Procedure: Simple-duplex types.* Follow the second method given for the Mannheim type, knowing that the slider is literally always reversed.

★*Procedure: Modern-duplex types.* By the use of the hairline on the indicator put the right index of S over 35 on D. Read the answer on D under 40° on S; or put 40° on S over 35 on DI and read the answer on DI under the right index of S.

Example 2. To find either acute angle given two sides; such as, given $a = 35$ ft., $b = 48$ ft.; to find A. The third ratio gives $\tan A = a/b = 35/48$.

★*Procedure: Polyphase type.* Put 35 on C over 48 on D. Answer is on T under the hairline on back, namely 36° 10′. If a had been larger than b, determine B and then A from the identity $A = (90° - B)$.

★*Procedure: Simple-duplex type.* Put 35 on C over 48 on D. Answer is on T under the hairline of the indicator placed at the right index of D.

★*Procedure: Modern-duplex type.* By the use of the DI scale and the trigonometric relationship $a = b \tan A = c \sin A$ this problem can be solved directly. Assume a less than b; then

$$\frac{1}{\frac{1}{a}} = \frac{\tan A}{\frac{1}{b}} = \frac{\sin A}{\frac{1}{c}}; \quad a < b < c$$

Put the right index of S over a (= 35) on DI. Then A (= 36°10′) on T is over b (= 48) on DI. Finally, 36°10′ on S is over c (= 59.2) on DI.

Note. This formula just given can be applied to any right-triangle problem when one has a trig-duplex type of rule.

Example 3. To solve the triangle given an angle and a side; such as, given $b = 20$ in., $A = 26° 20'$; to find c, a, and B. Note that

$$c = \frac{a}{\sin A} = \frac{b}{\sin B};$$

so a proportion relationship exists between the sides and the angles opposite these sides. Here

$$\frac{a}{\sin 26°\, 20'} = \frac{20}{\sin 63°\, 40'} = \frac{c}{1\ (= \sin 90°)}$$

Procedure: Polyphase and simple-duplex types. Put 20 on A over 63° 40′ on S. Over 26° 20′ find $a = 9.9$ in.; over 90° (right index of S) find $c = 22.3$ in. B had been found directly; so the triangle is solved.

★*Procedure: Modern-duplex types.* Put 63° 40′ on S over 20 on D; under right index of S read $c = 22.3$ on D, and under 26°20′ on S read $a = 9.9$ on D; or put the problem in the form

$$\frac{1}{\frac{1}{a}} = \frac{\tan 26°\, 20'}{\frac{1}{20}} = \frac{\sin 26°\, 20'}{\frac{1}{c}}$$

and then put 26° 20′ on T over 20 on DI; then 26° 20′ on S is over c (= 22.3) on DI, and left index of S is over a (= 9.9) on DI.

Please note also that if $b < a < c$ the formula must be

$$\frac{1}{\frac{1}{b}} = \frac{\tan B}{\frac{1}{a}} = \frac{\sin B}{\frac{1}{c}}$$

Check Work

When the problem has been solved, using the methods of Example 1 or 2, primarily 2, the method of the third example is a very good check. The use of the slide rule for checking has been of great value in many fields. This can be seen to be of special value in triangle solution, such as in surveying.

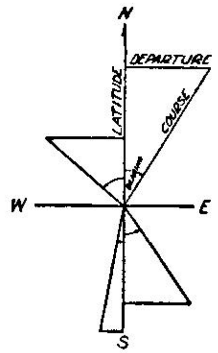
The Table of Logarithms of Trigonometric Functions

The Table of Logarithms of Trigonometric Functions given in the Appendix can be used to solve problems involving triangles. Chapter XIX shows how the table is used. The method of solution is entirely different; since the slide rule permits the use of the principle of proportion, while the table uses the principle of changing products and quotients into sums and differences.

Latitudes and Departures

From the methods of Example 1, latitudes and departures of courses needed in surveying can be worked out. The first is determined by the length of a course times the cosine of bearing (from north or south, note diagram); and the second by the length of a course times the sine of bearing. Furthermore, the bearing itself can be found by the formula

$$\tan \text{bearing} = \text{dep./lat.}$$



Finally, when given the latitude or departure and bearing, the distance can be found by the formulas

$$\text{dist.} = \text{dep.}/\sin \text{bearing} = \text{lat.}/\cos \text{bearing}.$$

PROBLEMS

1. $8.74 \sin 37° 30' = ?$
2. $2.37 \cos 24° 18' = ?$
3. $18.82 \tan 37.5° = ?$
4. $15.8 \tan 61° 42' = ?$
5. $0.728 \cot 31° 30' = ?$
6. $8.28 \cot 54° 30' = ?$
7. $12.9 \sec 41.7° = ?$
8. $20.7 \csc 35° 30' = ?$
9. $15.2 \tan 41° 42' \sin 48° 42' = ?$
10. $8.37 \sec 40° 42' \cos 30° 30' = ?$
11. $\dfrac{13.8 \cos 50° 20'}{\tan 30° 20' \times \sin 15° 30'}$
12. $\dfrac{37.4 \times \cos 47° 30'}{18.8}$
13. $\dfrac{41.8 \times 0.107}{\tan 40.7°}$
14. $\dfrac{\cos 15° 30' \times \sin 41.7°}{\tan 18° 18'}$
15. $\dfrac{\sin 61° 18'}{\cos 33° 30' \times \tan 18.3°}$
16. Given $a = 17.6$, $b = 14.8$, determine A, then c.
17. Given $a = 23.8$ ft., $A = 42° 18'$, solve triangle.
18. Given $b = 18.4$ in., $A = 31° 30'$, determine c.
19. Given $c = 21.6$ in., $B = 72° 18'$, solve triangle.
20. Given $a = 15.2$ ft., $c = 34.6$ ft., solve triangle.
21. Given $a = 4.58$ ft., $b = 28.6$ in., solve triangle.
22. Given $b = 5.83$ in., $c = 1.4$ ft., solve triangle.
23. Given $a = 7.84$ in., $b = 4.12$ in., solve triangle.
24. Given $a = 30.7$ in., $b = 41.8$ in., solve triangle.
25. Given $b = 12.8$ ft., $c = 16.2$ ft., solve triangle.

26. Given a bearing of N 34° 38′ E, and a distance of 287 ft. Find the latitude and departure.
27. Given a latitude of 234 ft. and a departure of 187 ft. Find the bearing and distance.
28. Given a latitude of 56.8 ft. and a bearing of 34°. Find the distance.
29. A ship is traveling S 45° 40′ W. What is its course length when the departure is 82 mi.?
30. A transport is traveling N 68° 30′ W. What is its latitude and departure when it has traveled 357 mi.?

ANSWERS TO PROBLEMS

1. 5.32	8. 35.6	15. 3.18
2. 2.16	9. 10.2	17. $b = 26.2$, $c = 35.4$
3. 11.47	10. 9.40	19. $a = 6.57$, $b = 20.6$
4. 29.3	11. 5.65	21. $A = 9.1°$, $c = 29.0$
5. 1.188	12. 1.343	23. $B = 27° 45'$, $c = 8.86$
6. 5.98	13. 5.20	25. $B = 52° 10'$, $a = 9.95$
7. 16.4	14. 1.94	27. Bearing, 38° 42′; distance, 299 ft.

XVI · ALGEBRAIC SOLUTION OF OBLIQUE TRIANGLES

Types of Solutions

In discussing the algebraic solution of oblique triangles, four distinct type problems are to be considered (note discussion in chapter XIV):

1. *Given two angles and a side.*
2. *Given two sides and an angle opposite one of them.*
3. *Given two sides and the included angle.*
4. *Given three sides.*

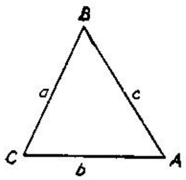
The first two types will be considered in this chapter. Both of them demand the **Law of Sines**, which is almost identical with the proportion given for the third example of chapter XV, namely

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where $A + B + C = 180°$ is the only limitation.

First Type

Consider this example: Given $A = 45°$, $B = 60°$, and $c = 12.5$. To find a, b, and C. First,



$$C = 180° - 45° - 60° = 75°;$$

so
$$\frac{a}{\sin 45°} = \frac{b}{\sin 60°} = \frac{12.5}{\sin 75°}$$

Procedure:* *Polyphase and simple-duplex types. Put 12.5 on A over 75° on S. Then over 60° on S find b on A, namely 11.2; and over 45° on S find a on A, namely 9.17.

★***Procedure:* *Modern-duplex types.*** Put 75° on S over 12.5 on D. Then under 60° on S find b on D, namely 11.2; and under 45° on S find a on D, namely 9.17. It will be necessary to interchange indexes on the S scale before this last reading can be made. Note also that if the triangle is divided into two right triangles the unique method given in chapter XV can be used.

If any angle is greater than 90°, use the trigonometric identity

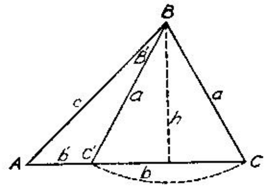
$$\sin X = \sin (180° - X)$$

It does not affect the other angles.

Thus, in the first type, very little work is needed to get a complete solution. As was shown before, this method will be a very good check of the answers of all problems involving triangles.

Second Type

The second type is more difficult since it represents what is sometimes called the **ambiguous case**. Namely, if given two sides and an angle opposite one of them, there is the possibility of **one solution, two solutions**, or **no solution** depending on the length of the line opposite the given angle.



In the construction of such a triangle, this situation is shown clearly. Consider as given c, a, and A. Lay off c and A. At B, with a pair of compasses opened to equal a, strike off an arc with B as center. If a fails to reach b there is **no solution**; if the arc cuts b twice (to the right of A), there are **two solutions**; and if a just reaches b, or a is greater than or equal to c, there is only **one solution**.

Since h (note figure) equals $c \times \sin A$, a direct check on the number of solutions is possible. The following conditions can be used to tell the number of solutions.

1. If $a < h$, **no solution**
2. If $a > h$, and $c > a$, **two solutions**
3. If $a = h$, or if $a > h$, and $a > c$ or $a = c$, **one solution**

Suggestions for Drawing Triangle

Since one will not always have the sides lettered as above, have the base the side that is not given. Construct the given angle at the left end of this line. Measure along the terminal side of the angle the length not opposite the given angle. The point so located will be the center of the arc whose radius is the side opposite the angle.

Always construct the triangle first as a guide in the number of solutions. Note the example which follows.

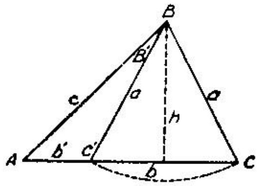
Illustrative example under second type. Consider the example $a = 12.7$, $c = 16.3$, and $A = 40° 30'$. First determine h as a means of verification of type of solution. $h = c \times \sin A = 16.3 \times \sin 40° 30' = 10.6$. Therefore, since a is greater than 10.6, and c is greater than a, there are two distinct solutions. The triangle has been constructed to verify this and to be a guide.

The Law of Sines gives

$$\frac{12.7}{\sin 40° 30'} = \frac{b}{\sin B} = \frac{16.3}{\sin C}$$

Procedure:* *Polyphase and simple-duplex types. Place 12.7 on A over 40° 30′ on S. Under 16.3 on A find the value for C, namely 56° 28′.

★*Procedure: Modern-duplex types.* Place 40° 30′ on S over 12.7 on C. Over 16.3 on D find value for C, namely 56° 28′ on S. But this is not the only value that will satisfy this setting, because of the trigonometric identity mentioned before, $\sin (180° - C) = \sin C$. This other angle is therefore $180° - 56° 28' = 123° 32'$. Note the figure, which shows how the triangles are denoted. Continuing, $B = 180° - 40° 30' - 56° 28' = 83° 2'$. $B' = 180° - 40° 30' - 123° 32' = 15° 58'$.



So in triangle ABC $\quad \dfrac{12.7}{\sin 40° 30'} = \dfrac{b}{\sin 83° 2'}$, or $b = 19.4$;

and in triangle ABC' $\quad \dfrac{12.7}{\sin 40° 30'} = \dfrac{b'}{\sin 15° 58'}$, or $b' = 5.38$.

Thus there are two solutions when given $a = 12.7$, $c = 16.3$, and $A = 40° 30'$; namely,

$A = 40° 30'$, $B = 83° 2'$, $C = 56° 28'$,
$a = 12.7$, $b = 19.4$, $c = 16.3$,

and

$A = 40° 30'$, $B' = 15° 58'$, $C' = 123° 32'$,
$a = 12.7$, $b' = 5.38$, $c = 16.3$.

Note. Since the problems of these types, as well as those considered in the next chapter, involve multiplication and division, they can be solved by the methods illustrated in chapter XIX. Note that the solution by logarithms is more accurate, since five-place table values are used instead of the three-place rule values. But the method is much longer; so when higher accuracy is needed and logarithms are employed, the rule can be used as a quick check of the answers found.

PROBLEMS

Solve triangles 1 to 6.

1. $a = 1419$, $B = 29° 30'$, $C = 16° 20'$
2. $b = 43.2$, $A = 36° 15'$, $C = 61° 45'$
3. $a = 28.4$, $b = 45.8$, $A = 32° 30'$
4. $a = 43.1$, $b = 38.1$, $A = 56° 40'$
5. $a = 56.8$, $A = 12° 20'$, $B = 20° 40'$
6. $b = 27.8$, $c = 36.3$, $B = 42° 30'$

7. It is desired to determine the width of a river. Two points are taken as nearly opposite as possible. A third point is taken 400 ft. from the second. At the second an angle of 110° is read between the first and third. At the third an angle of 18° 30′ is read between the first and second. How wide is the river?

ANSWERS TO PROBLEMS

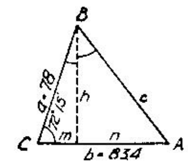
1. $b = 972$; $c = 557$; $A = 134° 10'$
4. $c = 50.0$; $B = 47° 30'$; $C = 75° 50'$

XVII · ALGEBRAIC SOLUTION OF OBLIQUE TRIANGLES (Continued)

Third Type

When two sides and the included angle of a triangle are given, the method of solution on the slide rule is entirely different from that when logarithm tables are used. The latter permits the use of the **Law of Tangents** (see chapter XIX).

If the rule is a **polyphase** or **simple-duplex type,** the method is to divide the triangle into two right triangles as shown in the illustration. Disregarding the numerical values shown, the following formulas are needed to completely solve the triangle:



$$h = a \times \sin C;\ m = a \times \cos C;\ n = b - m;$$
$$\tan A = h/n;\ B = 180° - (C + A);\ c = h/\sin A$$

If the rule is a **modern-duplex type,** the method is the same except that the formulas for solution are more compact, since a DI scale is on the rule. In the illustration shown above, disregarding the numerical values shown, either of two setups will be used. Note them carefully, because great care must be taken all the time that the diagram, the formulas, and the settings are all correct.

First Setup. If the given angle is **less** than 45°, the formulas used are

$$h = a \times \sin C = m \times \tan C = n \times \tan A = c \times \sin A;$$

so

$$\frac{1}{\frac{1}{h}} = \frac{\sin C}{\frac{1}{a}} = \frac{\tan C}{\frac{1}{m}} = \frac{\tan A}{\frac{1}{n}} = \frac{\sin A}{\frac{1}{c}},$$

where C is the included angle, a is known, and $m + n = b$, which is known.

Second Setup. If the given angle is **greater** than 45°, as shown in the illustration, the formulas use the part of the angle B that is in the right triangle made up of a, h, and m; namely, $B' = 90° - C$. Therefore

$$m = a \times \sin B' = h \times \tan B';\ n = h \times \tan B'' = c \times \sin B'';$$

where $B' + B'' = B$. In proportion form this becomes

$$\frac{1}{\frac{1}{m}} = \frac{\sin B'}{\frac{1}{a}} = \frac{\tan B'}{\frac{1}{h}};\ \frac{1}{\frac{1}{n}} = \frac{\tan B''}{\frac{1}{h}} = \frac{\sin B''}{\frac{1}{c}}$$

Example. Solve the triangle given $b = 83.4$, $a = 78$, and $C = 72° 15'$.

Procedure: Polyphase and simple-duplex types. Draw triangle as shown at top of page 69. Drop perpendicular from B onto b, dividing latter into m and n. Then

$$h = 78 \times \sin 72° 15' = 74.3$$
$$m = 78 \times \cos 72° 15' = 78 \times \sin 17° 45' = 23.8$$
$$n = 83.4 - 23.8 = 59.6$$
$$\tan A = 74.3/59.6 = 1.247$$
$$A = 51° 15' \text{ (use } \tan (90° - A) = 1/1.247)$$
$$B = 180° - (51° 15' + 72° 15') = 56° 30'$$
$$c = 74.3/\sin 51° 15' = 74.3/0.780 = 95.3$$

★***Procedure: Modern-duplex types.*** Draw triangle. Drop perpendicular from B to b, dividing the latter into m and n. Then

$$\frac{1}{\frac{1}{m}} = \frac{\sin 17° 45'}{\frac{1}{78}} = \frac{\tan 17° 45'}{\frac{1}{h}},$$

since $B' = 90° - 72° 15' = 17° 45'$. Put 17° 45' (17.75° on decitrig) on S over 78 on DI; then h (= 74.3) is on DI under 17° 45' on T, and m (= 23.8) is on DI under the right index of S (sin 90° = 1). Therefore $n = 83.4 - 23.8 = 59.6$.

Then, in the second right triangle, one has

$$\frac{1}{\frac{1}{59.6}} = \frac{\tan B''}{\frac{1}{74.3}} = \frac{\sin B''}{\frac{1}{c}},$$

where $B' + B'' = B$. Put right index of S over 59.6 on DI; then B'' (= 38° 45') is over 74.3 on DI. Then 38° 45' on S is over c (= 95.3) on DI. Finally, $B = B' + B''$ $= 17° 45' + 38° 45' = 56° 30'$, and $A = 180° - (56° 30' + 72° 15') = 51° 15'$, and the example is completely solved.

Thus the answers, using any type of rule, are

$$A = 51° 15'; B = 56° 30'; \text{ and } c = 95.3$$

Note. If the derived base angle is greater than 45°, a special setup will be needed to get the proper angle. That is, suppose one had

$$\frac{1}{\frac{1}{45.8}} = \frac{\tan A}{\frac{1}{37.9}};$$

then, if right index of S or T were put over 45.8 on DI, one would find that 37.9 on DI would not be under slider. If this occurs, rewrite this proportion in the form

$$45.8 = 37.9 \times \tan A$$

Solve for A, using C, D, and T scales; then reset rule and continue with original setup. Another method can be used that does not require this special setup. Place indicator in original setup to 45° on T; pull slider out of rule, turn it upside down, and place 45° on T (now upside down) under indicator; read answer on T, using *red* angles or the angles with the symbol $<$ (such as < 70), over 37.9 on DI; then reset rule as it was originally and continue with your solution.

If the given angle is greater than 90°, the perpendicular will fall outside the triangle. This will mean that $n = b + m$ (see above illustration) instead of $n = b - m$.

Procedure: Pickett rules, Models 2 and 4. Draw triangle. Use the proportion of the first setup on page 69.

$$\frac{\sin 72.25°}{\frac{1}{78}} = \frac{\tan 72.25°}{\frac{1}{m}} = \frac{\tan A}{\frac{1}{n}} = \frac{\sin A}{\frac{1}{c}}, \text{ where } n = 83.4 - m.$$

Put 72.25 on S over 78 on DI; interchange indexes of the C scale; then 72.25 on T is over m (= 23.8) on DI. $n = 83.4 - 23.8 = 59.6$. Over 59.6 on DI, read 51.25° (or 51° 15') on T. (It is the lower T scale since the drawing shows a large angle.) Interchange indexes of the C scale again since 51.25 on S is beyond the D and DI scales. Finally, under 51.25 on S, read c (= 95.3) on DI.

No matter what procedure you use, remember that there are three things that must always be done: *first, draw the diagram of the triangle; second, draw an altitude from either of the unknown angles; and third, define each part of the diagram by using letters, as was done in the illustration. Having completed these steps, set up the formulas needed and solve the problem.*

PROBLEMS

1. $b = 2.72$, $c = 7.45$, $A = 28° 30'$

2. $a = 26.4$, $b = 34.8$, $C = 34° 20'$

3. $a = 82.3$, $b = 84.6$, $C = 72° 20'$

4. $a = 456$, $c = 312$, $B = 45° 30'$

5. $b = 34.6$, $c = 12.5$, $A = 56° 20'$

6. $a = 4.58$, $b = 1.83$, $C = 64° 40'$

7. If two sides of a triangular lot are 150.6 ft. and 212.8 ft. and the angle between them is 41° 46', what is the length of the third side?

8. If a path 68.8 ft. long makes an angle of 66° 35' with a road, how far is the terminal point, say a spring, from a car 40 ft. down the road?

ANSWERS TO PROBLEMS

1. $B = 14° 22'$; $C = 137° 8'$; $a = 5.40$

4. $A = 91° 20'$; $C = 43° 10'$; $b = 325$

XVIII · ALGEBRAIC SOLUTION OF OBLIQUE TRIANGLES (Concluded)

Fourth Type

The last distinct situation regarding oblique triangles is when three sides are given. The formulas that apply are sometimes called the **half-angle formulas**, namely

$$\sin 1/2 \times A = \sqrt{\frac{(s-b)\,(s-c)}{bc}}, \text{ and so on,}$$

where $s = 1/2 \times (a + b + c)$. In determining B use sides a and c, and so on. Having determined two of the angles, use the relationship $C = 180° - (A + B)$ for the third angle, and check using the **Law of Sines**.

Consider the example $a = 6.7$, $b = 3$, $c = 4.3$. First determine s. $s = 1/2 \times (6.7 + 3 + 4.3) = 7$; so $s - b = 4$, $s - c = 2.7$, and

$$\sin 1/2 \times A = \sqrt{\frac{4 \times 2.7}{3 \times 4.3}} = 0.916;$$

hence $A = 132° \, 40'$.

Procedure. Determine the value, exclusive of the square root, using the first half of the A and B scales. Find the square-root value on the D scale under the indicator. Use the A or D and S scales to determine the angle. In like manner angle B or C can be determined.

Areas of Triangles

Any triangle that can be definitely determined when given three parts has a definite area. The three types and their respective formulas are as follows:

1. *Given two sides and the included angle:*
   $S = 1/2 \times b \times c \times \sin A = 1/2 \times a \times c \times \sin B = 1/2 \times a \times b \times \sin C$
2. *Given two angles and a side:*

$$S = \frac{a^2 \times \sin B \times \sin C}{2 \sin A}, \text{ if given } a, B, \text{ and } C$$

   The others are left to the student.

3. *Given three sides:*

$$S = \sqrt{s(s-a)\,(s-b)\,(s-c)}$$

These are all rather difficult to handle on the rule; so the wisest method is probably to evaluate them step by step, though it is not absolutely essential.

Radii of Inscribed and Circumscribed Circles

In any triangle with angles A, B, and C opposite sides a, b, and c, respectively, the radius, r, of the inscribed circle, and the radius, R, of the circumscribed circle, can be found by the following formulas:

$$r = (s-a) \tan \frac{A}{2} = (s-b) \tan \frac{B}{2} = (s-c) \tan \frac{C}{2}$$

$$R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C}$$

PROBLEMS

1. $a = 25.6$, $b = 35.8$, $c = 30.2$
2. $a = 5.68$, $b = 2.34$, $c = 6.93$
3. $a = 6.83$, $b = 11.4$, $c = 8.73$
4. Find the area of Problem 1.
5. Find the area of Problem 2.
6. Find the area of Problem 3.

Find the area, radius of inscribed circle, and radius of circumscribed circle, given

7. $a = 127$, $b = 238$, $C = 54° \, 42'$
8. $a = 39.7$, $A = 57° \, 30'$, $B = 68° \, 18'$

ANSWERS TO PROBLEMS

1. $A = 44° \, 36'$, $B = 77° \, 6'$, $C = 58° \, 18'$
3. $A = 36.7°$, $B = 93.7°$, $C = 49.6°$
5. $S = 4.78$
7. $r = 39.0$; $R = 119.3$

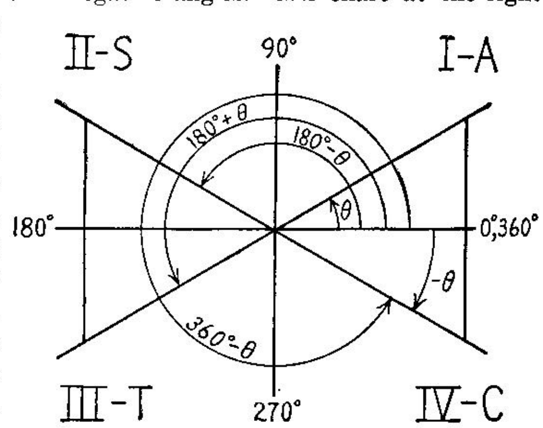
XIX · TRIGONOMETRIC FUNCTION TABLES

Two types of problems will be considered in this chapter, first those involving the **general angle,** and second those involving the **solution of triangles**.

Functions of the General Angle

On page 62 the ratios that define the six trigonometric functions were given. The values of these functions for each degree from 0° to 90° are given in the Natural Trigonometric Function Table. Note that $\sin 37° = 0.6018$; $\cot 42° = 1.1106$; and so on.

Frequently one will have need to know the natural-function values of angles greater than 90° and of negative angles. The chart at the right shows how these can be obtained. Set up a co-ordinate system (x-axis, y-axis). Define any angle in quadrant I as θ, in quadrant II as $180° - \theta$, in quadrant III as $180° + \theta$, and in quadrant IV as $360° - \theta$. Note the four letters S, A, T, and C. They mean that only the sine and its reciprocal the cosecant are positive in quadrant II; that all are positive in quadrant I; that only the tangent and its reciprocal the cotangent are positive in quadrant III; and that only the cosine and its reciprocal the secant are positive in quadrant IV. Thus



$$\sin(180° - \theta) = \sin\theta;\quad \cos(180° + \theta) = -\cos\theta;\ \text{and so on.}$$

Namely, the function values stay the same; the only concern is the sign of the answer.

Example 1. Give the natural values of sin 64°, cos 127°, tan 215°, tan 327°, cos (− 120°).

$\sin 64° = 0.8988$
$\cos 127° = \cos(180° - 53°) = -\cos 53° = -0.6018$
$\tan 215° = \tan(180° + 35°) = \tan 35° = 0.7002$
$\tan 327° = \tan(360° - 33°) = -\tan 33° = -0.6494$
$\cos(-120°) = \cos(360° - 120°) = \cos 240° = -\cos 60° = -0.8660$

Solution of Right Triangles

On page 62 will be found a diagram of a right triangle with the sides a, b, and c opposite the angles A, B, and C, where angle C is a right angle (90°). The formula that can be used to find unknown parts is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{1}$$

Algebraically, if two of these letters are known, one of which must be a side, it is possible to determine the value of the other three letters. The logical procedure is to solve by logarithms, since multiplication or division is the only operation involved. The Table of Logarithms of Trigonometric Functions is to be used when the trigonometric functions are involved. Three illustrative examples will explain the method of procedure to be used.

Example 2. Given $a = 34.7$, $A = 43.6°$, in a right triangle, find B, a, and c.

$$\frac{34.7}{\sin 43.6°} = \frac{b}{\sin 46.4°} = \frac{c}{1};\quad B = 90° - 43.6° = 46.4°;$$

then $c = 34.7 \div \sin 43.6°$, and $b = 34.7 \times \sin 46.4° \div \sin 43.6°$.

$\log 34.7 = 11.54033 - 10$
$\log \sin 43.6° = 9.83861 - 10$
$\log c = 1.70172$
$c = 50.32$

$\log 34.7 = 1.54033$
$\log \sin 46.4° = 9.85984 - 10$
$\text{Sum} = 11.40017 - 10$
$\log \sin 43.6° = 9.83861 - 10$
$\log b = 1.56156$
$b = 36.44$

Example 3. Given $a = 34.7$, $c = 48.5$, in a right triangle, find A, B, and b.

$$\frac{34.7}{\sin A} = \frac{b}{\sin B} = \frac{48.5}{1};$$

then $\sin A = 34.7 \div 48.5$, $B = 90° - A$, and $b = 48.5 \times \sin B$.

$\log 34.7 = 11.54033 - 10$
$\log 48.5 = 1.68574$
$\log \sin A = 9.85459 - 10$
$A = 45.7°$
$B = 90° - A = 44.3°$

$\log 48.5 = 1.68574$
$\log \sin 44.3° = 9.84411 - 10$
$\log b = 1.52985$
$b = 33.87$

Example 4. Given $a = 33.7$, $b = 41.2$, in a right triangle, find A, B, and c.

$$\frac{33.7}{\sin A} = \frac{41.2}{\sin B} = \frac{c}{1}$$

First change the first equality to

$$\frac{\sin A}{\sin B} = \frac{33.7}{41.2}$$

Since $\sin B = \cos(90° - B) = \cos A$, then

$$\frac{\sin A}{\sin B} = \frac{\sin A}{\cos A} = \tan A = \frac{33.7}{41.2}$$

$$\begin{aligned}
\log 33.7 &= 11.52763 - 10 \\
\log 41.2 &= \underline{1.61490} \\
\log \tan A &= 9.91273 - 10 \\
A &= 39.28° \\
B &= 90° - A = 50.72°
\end{aligned}$$

To get c, use

$$\frac{33.7}{\sin 39.28°} = \frac{c}{1}$$

$$\begin{aligned}
\log 33.7 &= 11.52763 - 10 \\
\log \sin 39.28° &= \underline{9.80148 - 10} \\
\log c &= 1.72615 \\
c &= 53.23
\end{aligned}$$

Note that interpolation was used in this last example. It is recommended that the slide rule be used to get values needed.

If one desires to change an angle to degrees, minutes, and seconds, or vice versa, note that there are 60 minutes in a degree and 60 seconds in a minute. Two examples will illustrate.

Example 5. Change 39.28° into degrees, minutes, and seconds.

$$0.28 \times 60' = 16.8' \quad \text{and} \quad 0.8 \times 60'' = 48''$$

Therefore

$$39.28° = 39°\ 16'\ 48''$$

Example 6. Change 37° 22′ 34″ into degrees and hundredths of a degree.

$$22'\ 34'' = \frac{22}{60} + \frac{34}{60^2} = \frac{1320 + 34}{3600} = \frac{1354}{3600} = 0.38$$

Therefore

$$37°\ 22'\ 34'' = 37.38°$$

Solution of Oblique Triangles

On page 66 will be found a diagram of an oblique triangle, namely one that does not have a right angle (90°). If two angles and a side are given, it is possible to solve for the other two sides and angle with the formulas

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{and} \quad A + B + C = 180°$$

If two sides and the included angle are given, it is possible to solve for the other two angles with the formula

$$\frac{a - b}{a + b} = \frac{\tan 1/2(A - B)}{\tan 1/2(A + B)},$$

where it is assumed that a, b, and C are given and a is greater than b. Note that $A + B$ can be obtained from the condition that $A + B + C = 180°$.

This is known as the **Law of Tangents**. Note that if a, c, and B are given and c is greater than a, the formula would become

$$\frac{c - a}{c + a} = \frac{\tan 1/2(C - A)}{\tan 1/2(C + A)}$$

After finding all the angles, it is then possible to solve for the third side by the use of the **Law of Sines**.

Example 7. Given $b = 27.8$, $c = 41.6$, $A = 37.6°$, find B, C, and a.

$c - b = 41.6 - 27.8 = 13.8$; $c + b = 41.6 + 27.8 = 69.4$; and $\frac{1}{2}(C + B) = \frac{1}{2}(180° - 37.6°) = 71.2°$. Thus

$$\frac{13.8}{69.4} = \frac{\tan \frac{1}{2}(C - B)}{\tan 71.2°}$$

$$\begin{aligned}
\log 13.8 &= 1.13988 \\
\log \tan 71.2° &= \underline{0.49341} \\
\text{Sum} &= 11.63329 - 10 \\
\log 69.4 &= \underline{1.84136} \\
\log \tan \tfrac{1}{2}(C - B) &= 9.79193 - 10
\end{aligned}$$

Thus

$$\begin{aligned}
\tfrac{1}{2}(C - B) &= 31.8° \\
\tfrac{1}{2}(C + B) &= 71.2° \\
C &= 103.0° \\
B &= 39.4°
\end{aligned}$$

Now use the **Law of Sines** to complete the problem. The answer is obtained from the following setup:

$$\frac{27.8}{\sin 39.4°} = \frac{c}{\sin 103.0°} = \frac{a}{\sin 37.6°},$$

from general angle analysis.

$$\begin{aligned}
\log 27.8 &= 1.44404 \\
\log \sin 37.6° &= \underline{9.78543 - 10} \\
\text{Sum} &= 11.22947 - 10 \\
\log \sin 39.4° &= \underline{9.80259 - 10} \\
\log a &= 1.42688 \\
a &= 26.73
\end{aligned}$$

The **Law of Tangents** can also be used to solve Example 4. Note also that Examples 2, 3, and 4 were solved by the **Law of Sines**, where $C = 90°$ ($\sin C = 1$).

Example 8. Given $a = 34.7$, $b = 41.2$, in a right triangle.

$$\frac{41.2 - 34.7}{41.2 + 34.7} = \frac{\tan 1/2(B - A)}{\tan 1/2(B + A)} = \frac{\tan 1/2(B - A)}{\tan 45°} = \tan 1/2(B - A),$$

since $\tan 45° = 1$. Complete the problem and show that $A = 39.28°$ and $B = 50.72°$.

General Rule to Solve Any Triangle

There are three sides and three angles in every triangle. Either the **Law of Sines** or the **Law of Tangents** can be used to solve for the unknown parts if at least one side is known along with two other parts, except when all three sides are known (note the half-angle formulas on page 72, which must be used for this rare situation).

Rule. Use **Law of Sines** when given one side and any two angles or two sides and the angle opposite one of them (for the latter note the discussion on the ambiguous case given earlier).

Use **Law of Tangents** when given two sides and the included angle. You must also use the **Law of Sines** in order to get the third side.

In any case, construct an accurate diagram of the triangle. The reasons why this should be done were discussed in Chapter XIV.

PROBLEMS

1. sin 47° 20′; cos 38° 40′; tan 61° 30′; sec 18° 10′; csc 25° 20′.

2. 17.8 × cos 61° 30′; 43.8 × csc 32° 30′; 7.81 × tan 31° 20′.

3. log sin 27° 30′; log tan 18° 20′; log cot 41° 15′.

4. sin $A = 0.3842$; cos $A = 0.7127$; sec $A = 1.272$.

5. log tan $A = 0.1372$; log sin $A = 9.3827 - 10$.

6–30. Solve Problems 6 to 30 on pages 64 and 65.

31–38. Solve Problems 1 to 8 on page 71.

ANSWERS TO PROBLEMS

1. 0.7353; 0.7808; 1.8418; 1.0566; 2.338

4. 22.6°; 44.45°; 38.2°

Note answers on pages 65 and 71. Get answers in degrees and tenths of a degree if you desire.

XX · APPLICATIONS TO BUSINESS, FINANCE, AND STATISTICS

The wide range of applications of the slide rule in everyday life has just begun to be recognized. It is proving its worth in determining values (costs), checking prices, making percentage reports of all kinds, developing graphic trends, helping in statistical work, evaluating profit-and-loss percentages, and so on. Several examples will be shown here to illustrate these possibilities.

Simple Business Applications

1. When given both the cost and selling price, to determine the per cent profit based either on cost or on selling price.

Example 1. Cost 65¢, sells for 90¢. Set 65 on C over 90 on D. Under C index (100%) read 138.5% on D. That is, a profit of 38.5% is made over cost. Over D index (100%) read 72.3%. That is, a profit of 27.7% is made based on sales.

This is an equivalent ratio setting where cost is on the C scale and selling price is on the D scale.

Note. From this last setting, if an article costs $1 instead of 65¢, it would sell for $1.385 instead of 90¢. Also, if an article sold for $1 instead of 90¢, it costs 72.3¢, or 27.7¢ less than sold for.

2. To determine the selling price of an article when a certain net-profit per cent and cost-of-upkeep per cent are given both based on sales.

Example 2. Suppose an establishment finds that 18% of their sales are required for upkeep, and that 12% of their sales is a reasonable net profit, what will be the selling price of an article that costs $2.25? If 100% is sales price per cent, then cost per cent is 100% − 18% − 12%, namely 70%.

In order to continue to read the cost on the C scale and the selling price on the D scale, as was done in the first application, place 70 (the cost per cent) over the right index of D (100%, the selling price per cent). Then under 2.25 on C read 3.21, the required selling price, on D.

3. To determine the per cent profit on sales equivalent to a certain per cent profit on cost, and vice versa.

a. If 100 % is cost per cent, and it is increased 25%, then the equivalent per cent on sales is

$$\frac{25\%}{100\% + 25\%} = 20\%$$

Procedure. Place 125 on C over 25 on D; namely, regular division.

b. If 100% is selling price per cent, and it is discounted 20%, then the equivalent per cent on cost is

$$\frac{20\%}{100\% - 20\%} = 25\%$$

4. To determine the net price (cost) of an article given the list price and a trade-discount series or given a net-cost-rate factor.

Example 3. Suppose an article in a catalogue three years old is listed at \$2.60. Two years ago this price was dropped 10%; a year ago this new price was raised 15%; and now the last price has been dropped 8%. What is the net price and what is the net-cost-rate factor (F)?

$$\begin{aligned}\text{Net price} &= \$2.60(1-0.10)\cdot(1+0.15)\cdot(1-0.08)\\ &= \$2.60 \times 0.90 \times 1.15 \times 0.92 = \$2.48\\ &\text{(using compound-multiplication method)}\\ F &= 2.48/2.60 = 0.953\end{aligned}$$

Note. These applications to business are met so frequently that slide rules have been developed for such everyday calculations. These are the Business and Finance Rule and the Business Executive Rule described on page 7. Illustrations of these two rules appear on page 10. One cannot stress too much how valuable the slide rule is in determining selling prices, profits, etc.; also how valuable it would be in inventory work etc.

Simple Applications to Finance

1. To check on, or apply to, financing plans used in buying on time.

Example 4. In financing a car on an 18 months' contract, quoted at 6% in advance (or 1/2% per month in advance), what must be the monthly payments if \$660 is still owed on the contract, including insurance and so on?

$X = \$660/(1-0.09) = \$660/0.91$; and the monthly payments (P) are $X/18$. By use of C and D scales $X = \$725$, and $P = \$40.30$ a month.

2. To determine the compound amount, or present value, or compound interest on funds on deposit and so on.

Example 5. How much must be set aside today in order to have \$500 ten years hence if money is worth 6%?

$$P(1.06)^{10} = \$500$$

This problem can be solved by the methods discussed in chapter XI, namely log P + 10 × log (1.06) = log 500. The answer is \$279. **Try it.**

3. To determine the periodic payments, or the debt, or the amount of an annuity.

Example 6. What must be the annual payments to amortize a debt of \$3000 in 10 years, if money is worth 6%?

$$3000 = R\,\frac{1-(1.06)^{-10}}{0.06}$$

This must be worked rather carefully. First get $(1.06)^{-10}$ by logarithms (= 0.558). Then

$$R = \frac{3000 \times 0.06}{0.442} = \$407$$

Note. Those that have the log log scales will be able to handle problems like these much more rapidly.

There are many other finance applications possible to consider, such as depreciation, perpetuities, capitalized cost, composite life, and so on.

Simple Applications to Statistics

1. To determine what per cent each part is of the whole.

Example 7. If the ideal student day has 8 hours for sleep, 2 for meals, 4 for classes, 5 for study, and 5 for recreation or work, what per cent is each of the whole? This takes on the form $Y = X/a$; so put 24 on C over 1 on D; then 8 on C gives 33.3% on D, and so on.

2. To determine the result for any one of many statistical formulas. A few of these are

$$\frac{1-r^2}{\sqrt{N}};\quad \frac{S.D.}{\sqrt{2\,N}};\quad \sqrt{1-r^2};\ \text{and so on}$$

Example 8. Given a correlation unit (r) of 0.60 for a sampling of 220 cases (N), what is the reliability factor (x)?

$$x = \frac{1-(0.60)^2}{\sqrt{220}} = \frac{1-0.36}{\sqrt{220}} = \frac{0.64}{\sqrt{220}} = 0.0431$$

Put 220 on B over 0.64 on D. Answer is on D under the index of C.

3. To aid in developing the graphical pictures of the data being used.

Example 9. To develop a circle graph of the data given in 1. If 100% represents 360°, then 33.3% represents 120°, and so on. This data could also be put into a rectangle graph, or a comparative bar graph, and so on.

Note. In any kind of graphing the slide rule will prove to be of tremendous value.

Naturally many other examples could be given. As they are met in practice, the person that can handle a rule easily can speed up all his calculations and find many valuable uses for the rule.

PROBLEMS

1. If the cost of doing business (upkeep) is 16% of sales, and net profit desired is 13% of sales, what will be the selling prices on the following costs: \$2.45, 85¢, 4.6¢, \$26.68, \$1.21, and 45¢?

2. If cost of upkeep is 27% of sales, and net profit desired is 19% of sales, what will be the selling prices on the following costs: \$18.20, \$32.70, \$7.20, \$83.50, \$1.67, 62¢, 37¢?

3. What per cent of sales are equivalent to the following per cents on cost: 21%, 33 1/3%, 42%, 67%, 58.3%, 61.7%?

4. If an article is sold with an increase of 31% on cost, what per cent discount is needed to sell at cost?

5. An article listed at \$3.40 has trade discounts of 10%, then 15%. If it is sold to yield a 30% profit, based on sales, what is the selling price?

6. If a wholesale house has a net-cost-rate factor of 0.92 on stoves that list at \$32, what is the price?

7. If a hardware store pays \$3.50 for an article, what must be the selling price if after allowing a trade discount of 10% a profit of 50% is still made, based on sales?

8. If an article costs \$23.50 and sells for \$37.50, determine the per cent profit based on both cost and selling price.

9. If 1 franc is worth 3.5¢, what are 235 francs worth? What is \$540 worth in francs?

10. If an article that sells for \$50 is discounted 30% to determine its cost, what is the cost?

11. If hats are purchased for \$38.50 a dozen and sold for \$4 each, what is the per cent of gross profit?

12. Determine the monthly cost of a loan on a machine worth \$300, if the rate is 6% in advance and the time is 36 mo. ($X = 300/(1 - 0.18)$)

13. Solve Problem 12 on a 24-months basis.

14. Solve Problem 12 on a 30-months basis.

15. Solve Problem 12 on a 5% basis.

16. Find some loan plan similar to these mentioned above and check on the price paid.

17. How much must be set aside today in order to have \$1000 20 yr. hence if money is worth 6%?

18. Solve Problem 17 if money is worth 4%; compare.

19. What must be the annual payments to build up a sinking fund of \$2500 in 10 yr. if money is worth 6%? Note:

$$2500 = R\ \frac{(1.06)^{10} - 1}{0.06}$$

20. Solve Problem 19 if money is worth 4%; compare.

21. What is R in Problem 19 if the \$2500 is a debt?

22. What is the commission on a sale of \$265 if the rate of commission is 3%? 5%? 3.85%?

23. A certain department of a store has five clerks. What per cent of the total sales for the department were made if the sales for each clerk were as follows: I, \$245; II, \$267; III, \$197.30; IV, \$315; and V, \$169.50?

24. Put the results of Problem 23 into a circle graph.

25. Given that $x = S.D./\sqrt{2N}$, find x when $S.D. = 20.5$ and $N = 145$.

26. Solve Problem 25 if $N = 840$; compare.

27. Given that $x = \sqrt{1 - r^2}$, find x when $r = 0.72$; when $r = 0.28$.

ANSWERS TO PROBLEMS

1. \$3.45, \$1.20, 6.5¢, \$37.60, \$1.71, 64¢
2. \$33.70, \$60.50, \$13.32, \$154.80, \$3.10, \$1.15, 69¢
3. 17.4%, 25%, 29.6%, 40.1%, 36.9%, 38.1%
4. 23.6% **5.** \$3.72 **6.** \$29.40 **7.** \$6.30
8. 59.5%; 37.4% **9.** \$8.23; 15,400 francs
10. \$35 **11.** 19.7% on retail or 24.7% on cost
12. \$10.15 **13.** \$14.23 **14.** \$11.76 **15.** \$9.81
17. \$312.20 **18.** \$456.00; need \$144 more
19. $R = \$190$ **20.** $R = \$208$; need \$18 more per year
21. $R = \$340$ **22.** \$7.95; \$13.25; \$10.20
23. 20.5%; 22.4%; 16.5%, 26.4%, 14.2% **25.** $x = 1.20$
26. $x = 0.5$; less than half the value for six times N
27. $x = 0.694$; $x = 0.960$

Note. In August, 1948, the author devised a slide rule for those interested in business, business statistics, and finance. See Fig. 17 in the front of the book. All the problems given in this chapter can be solved directly with this new rule. It can also be used to solve all the problems in the first six chapters.

XXI · THE LOG LOG SCALES

The Log Log Scales

All the modern duplex type slide rules have log log scales. Some rules have five scales marked LL_1, LL_2, LL_3, LL00, and LL0. Others have six scales marked LL_1, LL_2, LL_3, $LL0_1$, $LL0_2$, $LL0_3$. Still others have scales identical to those just mentioned back to back. All these scales are based on natural logarithms, that is, to the base $e = 2.71828 \cdots$. All these scales are associated with the C scales, except the LL00 and LL0 scales which are associated with the B scale. All the scales are continuous, and are of great value in solving problems involving involution, evolution, exponential functions, natural logarithms, hyperbolic functions, and so on.

On the Pickett rules, Models 2 and 4, there are eight log log scales marked N_1, N_2, N_3, N_4, $1/N_1$, $1/N_2$, $1/N_3$, and $1/N_4$. These scales are based on common logarithms, that is, to the base 10. All these scales are associated with the C scale. If natural logarithms are desired, these log log scales are associated with the scales marked CF/M and DF/M.

Note. All log log scales have the decimal points of all numbers definitely located. For example, 1.25 is on LL_2 or on N_2; 12.5 is on LL_3 or on N_4; and so on. On your rule, these log log scales are the easiest ones to use and to get acquainted with.

Involution and Evolution

Attention will first be given to the use of the log log scales in problems that pertain to raising to a power and to taking a root. Chapter XI shows how the log scale can be used for such problems. But the log log scales greatly speed up the processes, since only one setting is usually needed.

Raising to a Power

Method. An index of C or B is put over or under the **base** on an LL or an N scale. The answer is on an LL or an N scale over or under the **power** on the C scale, or over the **power** on the B scale. Remember, the B scale is used only if you have an LL00 and an LL0 scale on your rule.

Important. If the exponent is between 1 and 10, the answer is on the same LL or N scale as the base if the left index of C is used, and is on the next higher scale (if possible) if the right index of C is used. If the exponent is between 10 and 100, the answer is on the next higher scale if the left index of C is used, and two higher if the right index of C is used. If the exponent is between 0.1 and 1, the answer is on the next lower scale (if possible) if the left index of C is used, and is on the same scale if the right index of C is used.

Example	Base on	Index Used	Power on	Answer on	Answer
$3.6^{4.2}$	LL_3	left	C	LL_3	218
	N_3	right	C	N_3	
1.0385^7	LL_1	right	C	LL_2	1.3025
	N_2	right	C	N_3	
1.0385^{70}	LL_1	right	C	LL_3	14.1
	N_2	right	C	N_4	
$6.82^{0.42}$	LL_3	left	C	LL_2	2.241
	N_3	right	C	N_3	
$0.438^{3.2}$	LL00(LL0)	left	B	LL00(LL0)	0.071
	$LL0_2$ or lower LL_2	right	C	$LL0_3$ or lower LL_3	
	$1/N_3$	right	C	$1/N_4$	

Taking a Root

Method. The method used to take a root is the inverse of that used to raise to a power; namely, the **root** on C or B is over or under the **number** on an LL or N scale. The answer is on an LL or N scale over or under the index of C or over an end index of B.

Example	Number on	Root on	Answer on	Under Index at	Answer
$\sqrt[3.3]{460}$	LL_3	C	LL_3	left	6.4
	N_4	C	N_4	right	
$\sqrt[6.7]{18.6}$	LL_3	C	LL_2	right	1.547
	N_4	C	N_3	right	
$\sqrt[67]{18.6}$	LL_3	C	LL_1	right	1.0446
	N_4	C	N_2	right	
$\sqrt[0.56]{28.4}$	LL_3	C	LL_3	right	392
	N_4	C	N_4	right	
$\sqrt[6.2]{0.083}$	LL00(LL0)	B	LL00(LL0)	left	0.669
	$LL0_2$ or lower LL_2	C	$LL0_2$ or lower LL_2	right	
	$1/N_4$	C	$1/N_3$	right	

Note. If these two sets of problems are carefully considered, the process of solution seems logical. What actually happens is that if $x^a = y$, then log a (on C or B) + log log x (on LL or N) = log log y (also on LL or N).

Observe that squares, square roots, cubes, and cube roots can also be worked directly with these scales.

Power and Exponential Equations (Empirical Equations)

In Chapter XI equations of two kinds were solved that can be worked directly with the LL or N scales. They are

$$x^a = b \quad \text{and} \quad a^x = b,$$

where a and b are known and x is to be determined.

Example 1. $x^{2.84} = 37.8$. Put 2.84 on C over 37.8 on LL_3. Answer is under the left index of C, namely 3.59. Or, put 2.84 on C over 37.8 on N_4. Answer is under right index of C.

Example 2. $x^{1.78} = 0.724$. Put 1.78 on C over or under 0.724 on $LL0_2$ (or lower LL_2). Answer is over or under left index of C on $LL0_2$ (or lower LL_2), namely 0.834. Or, put 1.78 on C over 0.724 on $1/N_3$. Answer is over right index of C on $1/N_2$. Or, put 1.78 on B under 0.724 on LL00 (LL0). Answer is over left index of B.

Example 3. $2.58^x = 40.8$. Put right index of C over 2.58 on LL_2 or N_3. Move hairline to 40.8 on LL_3 or N_4. Answer is on C under hairline, namely 3.91.

Example 4. $0.627^x = 0.428$. Put left index of C over or under 0.627 on $LL0_2$ (or lower LL_2). Answer is on C under hairline, namely 1.82. Or, put left index of C over 0.627 on $1/N_3$. Answer is on C under hairline; Or, put left index of B under 0.627 on LL00 (LL0). Move hairline to 0.428 on LL00 (LL0). Answer is on B under hairline.

Example 5. $5.84^x = 0.256$. x is negative since one of the knowns is greater than 1 and the other is less than 1. Put right index of C over or under 5.84 on LL_3. Answer is on C over or under 0.256 on $LL0_3$ (or lower LL_3), namely -0.772. Or, put right index of C over 5.84 on N_3. Answer is on C over 0.256 on $1/N_3$. Or, if you have the LL00 (LL0) scales, rewrite the problem $5.84^x = 3.91$, where 3.91 is the reciprocal of 0.256.

These are really examples of two type forms of empirical equations used in analyzing experimental facts. Such equations are $x^a = y$ and $a^x = y$, where a is known. Both require merely a single setting of the rule and have an equivalent ratio-setup characteristic. Also the graphs of such equations follow certain natural forms.

Example 6. $y = 5.92^x$. Find y when $x = 0.27, 0.83, 1.42$, and 2.27. Put left index of C over 5.92 on LL_3; then 0.27 on C gives 1.616 on LL_2, 0.83 on C gives 4.38 on LL_3 (after interchanging indexes), 1.42 on C gives 12.5 on LL_3, and so on. Or, put right index of C over 5.92 on N_3 and read the answers on N_3 under the x values on C.

Answer beyond the Limit

If the resulting answer is beyond the limit of the scale applying, then the following schemes can be applied:

$$X^y = (A \times B)^y = A^y \times B^y; \quad \sqrt[y]{X} = \sqrt[y]{A \times B} = \sqrt[y]{A} \times \sqrt[y]{B}$$

That is: Choose quantities whose product (or perhaps quotient) is that of the base such that the power or root of each is within the limits of the scales. With these results final multiplication (or division) can then be made. One quantity could be some multiple of 10.

Use of the Natural-Logarithms Table

Natural logarithms are always needed if the problem involves calculus (differentiation or integration). The table is given here since the LL scales on the slide rule are based on such values. The values assume the base $e(= 2.71828)$ instead of 10.

Example 7. $13 = e$ to what exponent?

$$13 = 1.3 \times 10 = e^{0.26236} \times e^{2.30259} = e^{2.56495}$$

$$\text{or } \ln 13 = 2.56495$$

Since the base is so small, natural logarithms are not convenient for calculations of the type carried out using logarithm tables, which are often called *common logarithms.*

PROBLEMS

1. $2.75^{4.66}$
2. $0.66^{1.79}$
3. $1.458^{0.666}$
4. $\sqrt[8.2]{15.200}$
5. $\sqrt[15.9]{0.125}$
6. $50^{2.48}$
7. $892^{0.265}$
8. $\sqrt[0.333]{1.688}$
9. $1.01125^{40.6}$
10. $0.585^{0.477}$
11. $\sqrt[7.55]{1.254}$
12. $\sqrt[0.555]{0.725}$
13. $2.28^{9.49}$
14. $\sqrt[2.45]{1.6426}$

15. $3.78^x = 82.9$
16. $7.29^x = 2.62$
17. $1.072^x = 40.8$
18. $0.397^x = 0.629$
19. $0.637^x = 0.286$
20. $x^{1.72} = 2.37$
21. $x^{40.8} = 8.82$
22. $x^{0.272} = 1.37$
23. $x^{4.23} = 0.684$
24. $x^{0.487} = 0.528$
25. $4.38^x = 0.286$
26. $0.238^x = 3.84$
27. $3.27^x = 0.0738$
28. $0.0384^x = 1.28$

29. $Y = 6.87^x$. Find Y when $x = 1.77$, 0.748, 0.237, 0.0848.

30. What is the compound amount when $50 is compounded for 28.3 yr. if money is worth 5.52%?

31. What amount today is worth $200 24 yr. hence if money is worth 4.70%?

32. What is the present value of an annuity of $300 a year for 18 yr. if money is worth 6%? (See chapter XX.)

33. What is the compound amount of an annuity of $250 a year for 12 yr. if money is worth 5.6%? (See chapter XX.)

34. $Y = X^{2.34}$. Find Y when $X = 1.82$, 0.887, 0.265, and 0.0997.

ANSWERS TO PROBLEMS

1. 112	**10.** 0.774	**19.** 2.77
2. 0.475	**11.** 1.0304	**20.** 1.651
3. 1.286	**12.** 0.9437	**21.** 1.0548
4. 3.235	**13.** 2500	**22.** 3.18
5. 0.877	**14.** 1.225	**23.** 0.914
6. 17,000	**15.** 3.33	**24.** 0.270
7. 6.03	**16.** 0.485	**25.** − 0.845
8. 4.82	**17.** 55.6	**26.** − 0.939
9. 1.575	**18.** 0.501	**27.** − 2.56

28. − 0.0757 **29.** 58.5, 4.22, 1.578, 1.774 **30.** $230 **31.** $66.40
32. $3245 **33.** $2140 **34.** 4.06, 0.756, 0.0445, 0.0042

XXII · NATURAL FUNCTIONS

Natural Base Exponential Forms and Natural Logarithms

If you have scales marked LL on your rule, they are based on natural logarithms. That is, the left index of LL_3 is e (= 2.718); the left index of LL_2 is $e^{0.1}$ (= 1.105); and the left index of LL_1 is $e^{0.01}$ (= 1.01).

If you have scales marked N on your rule, they are based on common logarithms. That is, the left index of N_4 is 10; the left index of N_3 is $10^{0.1}$ (= 1.26); the left index of N_2 is $10^{0.01}$ (= 1.0232); and the left index of N_1 is $10^{0.001}$ (= 1.0023). If you have scales marked N, you also have scales marked DF/M and CF/M.

No matter which type of log log scales you have, it is possible to get e to any reasonable power by reading directly from D to an LL scale or from DF/M to an N scale.

Example 1. $e^{2.72} = ?$ Put hairline over 2.72 on D and read the answer on LL_3, namely 15.2. Or, put hairline over 2.72 on DF/M and read the answer on N_4. In like manner $e^{0.272} = 1.312$, $e^{0.0272} = 1.0276$.

The natural logarithm (indicated by 'ln') of a number is the exponent of e representing the number. Thus from Example 1, ln 15.2 = 2.72, ln 1.312 = 0.272, ln 1.0276 = 0.0272.

Example 2. ln 28.7 = ? Put hairline over 28.7 on LL_3 or N_4. Answer is on D or DF/M respectively, namely 3.36.

If the exponent of e is negative, the number representing its value must be decimal, and vice versa.

Example 3. $e^{-2.72} = ?$ Put hairline over 2.72 on D and read the answer, 0.066, on $LL0_3$ (or lower LL_3). Or, put hairline over 2.72 on DF/M and read the answer on $1/N_4$. Or, put hairline over 2.72 on A and read the answer on LL00 (LL0).

Example 4. ln 0.384 = ? Put hairline over 0.384 on $LL0_2$ (or lower LL_2). Answer is on D under hairline, namely − 0.959. Or, put hairline over 0.384 on $1/N_3$. Answer is on DF/M under hairline. Or, put hairline over 0.384 on LL00 (LL0). Answer is on A under hairline.

Note that LL_1 and $LL0_1$, LL_2 and $LL0_2$, LL_3 and $LL0_3$, N_1 and $1/N_1$, N_2 and $1/N_2$, N_3 and $1/N_3$, N_4 and $1/N_4$ scale value are reciprocals. Also the upper and lower LL scale values on the Pickett Model 800 are reciprocals. That is, 2 on LL_2 and 0.5 on $LL0_2$ are reciprocals; 2 on N_3 and 0.5 on $1/N_3$ are reciprocals; 2 and 0.5 are back to back on the LL_2 scale of Pickett Model 800.

The Table of Natural Logarithms

The Table of Natural Logarithms given at the back of the text can be used to determine the values of the examples just solved. Interpolation will be required to get the answers found, but the table can be used as a check on your solutions and will show its relationship with the log log scales.

Logarithm System to Any Base

By putting either index of the C scale or the index of the CF/M scale at any value on a log log scale, an exponential and a logarithmic system is set up with the value selected as its base. For you that have the LL scales, put either index of C over 10 on LL_3 in order to have common logarithms. If you have a Pickett rule, Model 2 or 4, read directly between N and D.

Example 5. Find log 25, assuming a base of 10. Put left index of C over 10 on LL_3. The answer, 1.398, is on C over 25 on LL_3. Or, over 25 on N_4 read the answer on D.

Example 6. Find $\log_{12} 25$. Since the base is 12, put the left index of C over 12 on LL_3 or N_4. The answer, 1.295, is on C over 25 on LL_3 or N_4.

Example 7. Find $\log 0.42$, assuming a base of 10. First $\log 0.42 = \log 4.2 - \log 10 = 0.623 - 1$ or $9.623 - 10$.

Example 8. Find $\log 72{,}800$, assuming a base of 10. First $\log 72{,}800 = \log 7.28 + \log 10^4 = 4.863$.

These last two examples make use of a very important observation; that is, any number, such as 4580, can be expressed as 4.58×10^3, so 3 is the characteristic. The reverse is also true. By using this simple principle, there should never be any difficulty about characteristic or decimal point.

Indeterminate Area

Since the log log scales are based on the logarithm of the logarithm, and since the logarithm of 1 is zero, then the logarithm of the logarithm of 1 is minus infinity. This explains why the scales of values less than 1 are not continuous with the scales of values greater than 1.

Hyperbolic Functions

In conclusion note the hyperbolic functions found in higher mathematics:

$$\sinh x = (e^x - e^{-x})/2$$
$$\cosh x = (e^x + e^{-x})/2$$
$$\tanh x = (e^x - e^{-x})/(e^x + e^{-x})$$

Assign any value to x within the limits of the rule, and these functions can be quickly determined.

$y = \cosh x$ is the equation of the catenary, the path taken by a free-hanging chain. The log log vector rule has scales for the hyperbolic functions which greatly speed up this work.

Many problems in calculus, such as the graph of $y = e^{ax}$ or $y = \ln ax$, can be quickly done by the log log rule. No matter what the base, such as $y = 25^x$, the graph of the equation can be rapidly made through assigning values to x, within the limit of the rule, and getting the corresponding values for y.

PROBLEMS

1. Find the natural logarithms of 26.8, 1.735, 14.2, 0.337, 0.085.

2. Find the common logarithms of 268, 12.1, 0.847, 2.305, 1.0174, using the log log scales.

3. What is the value for e^x when $x = 2.6, 3.75, 4.15, 6.87, 9.19$?

4. What is the value for e^{-x} when $x = 2.8, 0.745, 0.082, 0.237, 1.16$?

5. Find the natural antilogarithms of 4.68, 8.25, 0.328, 0.0272, − 0.667.

6. Find the common antilogarithms of the values in Problem 5.

7. Find the value of $\sinh x$ when $x = 2, 1.7, 0.834, 0.238$.

8. Given the catenary, find y when $x = 0.287, 0.872, 1.43, 1.87$.

9. Given $y = e^{2x}$, find y when $x = 0.318, 0.712, 1.04, 1.37$.

10. Given $y = e^{\frac{x}{2}}$, find y when $x = -1.72, -0.248, 0.716, 1.48$.

11. Determine 3^8, 4.7^5, $\log_5 80$, $\log_7 220$.

12. Given $y = 12^x$, find the value for y corresponding to the following values for x: 2.6, 1.84, 1.23, 0.97, 0.43, 0.25, 0.035.

ANSWERS TO PROBLEMS

1. 3.28, 0.550, 2.65, − 1.09, − 2.47

2. 2.45, 1.09, − 0.0721, 0.363, 0.00749

3. 13.9, 42.4, 63.5, 960, 9800

4. 0.058, 0.475, 0.9212, 0.789, 0.314

5. 108, 4250, 1.388, 1.0276, 0.514

6. 47,500, 6.68×10^8, 2.128, 1.0646, 1.184, 0.215

7. 3.632, 3.644, 0.932, 0.240

8. 1.042, 1.648, 2.205, 3.318

9. 1.89, 4.15, 8, 15.5

10. 2.364, 1.132, 0.699, 0.477

11. 6500, 2300, 2.72, 2.77

12. 640, 96.5, 21.3, 11.18, 2.915, 1.862, 1.091

APPENDIX

Ratios and Proportions

If $a : b = c : d$, then $ad = bc$; $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$; $\frac{a}{c} = \frac{b}{d} = \frac{a+b}{c+d}$; $\frac{a+b}{a-b} = \frac{c+d}{c-d}$; $\frac{a+c}{a-c} = \frac{b+d}{b-d}$; $\frac{a^x}{b^x} = \frac{c^x}{d^x}$.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$, then

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = \frac{a+c+e+\cdots}{b+d+f+\cdots} = \frac{ua+vc+we+\cdots}{ub+vd+wf+\cdots}$$

International Atomic Weights $(O = 16)$

Aluminum	Al	27.1	Manganese	Mn	54.93
Antimony	Sb	120.2	Mercury	Hg	200.6
Arsenic	As	74.96	Molybdenum	Mo	96.0
Barium	Ba	137.37	Nickel	Ni	58.68
Bismuth	Bi	208.0	Nitrogen	N	14.01
Boron	B	11.0	Oxygen	O	16.0
Bromine	Br	79.92	Palladium	Pd	106.7
Cadmium	Cd	112.4	Phosphorus	P	31.04
Calcium	Ca	40.07	Platinum	Pt	195.2
Carbon	C	12.0	Potassium	K	39.1
Chlorine	Cl	35.46	Radium	Ra	226.4
Chromium	Cr	52.0	Silicon	Si	28.3
Cobalt	Co	58.97	Silver	Ag	107.88
Copper	Cu	63.57	Sodium	Na	23.0
Fluorine	F	19.0	Strontium	Sr	87.63
Gold	Au	197.2	Sulphur	S	32.07
Hydrogen	H	1.008	Tantalum	Ta	181.5
Iodine	I	126.92	Tin	Sn	119.0
Iridium	Ir	193.1	Titanium	Ti	48.1
Iron	Fe	55.84	Tungsten	W	184.0
Lead	Pb	207.1	Uranium	U	238.5
Lithium	Li	6.94	Vanadium	V	51.0
Magnesium	Mg	24.32	Zinc	Zn	65.37

Table of Equivalents

Linear	C opposite D
1 in. = 2.54 cm.	100 — 254
= 0.1263 links	95 — 12
1 ft. = 0.3048 m.	82 — 25
= 1.515 links	66 — 100
1 yd. = 0.9144 m.	82 — 75
1 mi. = 1.609 km.	87 — 140
= 0.868 knot	38 — 33
Square	
1 sq. in. = 6.452 cm² (sq. cm.)	31 — 200
1 sq. ft. = 0.0929 m²	140 — 13
= 2.2954 sq. links	44 — 101
1 sq. yd. = 0.8361 m²	61 — 51
1 sq. mi. = 2.590 km²	22 — 57
= 640 acres	1 — 640
1 acre = 0.4047 hectare	42 — 17
Cubic	
1 cu. in. = 16.39 cm³	5 — 82
1 cu. ft. = 0.02832 m³	600 — 17
1 cu. yd. = 0.7643 m³	85 — 65
Volumes	
1 cu. ft. = 28.32 liters	6 — 170
= 7.4805 U.S. Gal.	107 — 800
1 U.S. Gal. = 231 cu. in.	1 — 231
= 3.785 liters	14 — 53
= 0.8333 Imperial Gal.	6 — 5
1 cord of wood = 128 cu. ft.	1 — 128
1 perch of masonry = 24.75 cu. ft.	40 — 990
Weight	
1 gram = 15.432 gr.	7 — 108
1 kg. = 2.2046 lb.	176 — 388
1 cu. ft. = 62.43 lb. of water	5 — 312
1 U.S. Ton = 1.0158 metric tons	63 — 64
Pressures	
1 ft. water = 0.4335 lb. per sq. in.	60 — 26
1 lb. per sq. in. = 0.07031 kg. per sq. cm.	640 — 45
= 2.0357 in. of mercury	28 — 57
1 in. of mercury = 1.1333 ft. of water	15 — 17
1 atmosphere = 14.7 lbs. per sq. in.	10 — 147
= 1.0333 kg. per cm²	30 — 31
= 29.798 in. of mercury	99 — 2960
1 kg. per cm² = 10 m. of water	1 — 10

Compound Units	C opposite D
1 lb. per ft. = 1.4882 kg. per m.	43 — 64
1 liter per sec. = 2.119 cu. ft. per min.	42 — 89
1 lb. per sq. yd. = 0.5435 kg. per sq. m.	46 — 25
1 mi. per hr. = 1.4667 ft. per sec.	30 — 44
1 ft. per min. = 0.3049 m. per min.	82 — 25
1 lb. of sea water = 1.0263 lb. of fresh water	38 — 39
1 kg.-m. = 7.233 ft.-lb.	22 — 159
1 ft.-lb. = 1.356 joules	45 — 61
1 h.p.-hour = 0.746 kw.-hour	67 — 50
1 B.T.U. = 778 ft.-lb.	9 — 7000
1 h.p. = 0.707 B.T.U. per sec.	232 — 164
1 h.p. = 1.014 metric h.p. (cheval vapeur)	72 — 73
1 lb. per h.p. = 0.447 kg. per metric h.p.	56 — 25
1 U.S. gallon of water = 8.333 lb.	3 — 25
1 lb. per sq. ft. = 4.8825 kg. per m^2	51 — 249
1 lb. per cu. ft. = 16.0192 kg. per m^3	156 — 2500
1 ft.-lb. = 0.1383 kg.-m.	159 — 22
1 B.T.U. per sq. ft. per min. = 0.122 watts per sq. in.	41 — 5
1 watt per sq. in. = 8.19 B.T.U. per sq. ft. per min.	58 — 475
1 kilowatt = 0.948 B.T.U. per sec.	19 — 18
= 1.34 h.p.	100 — 134
1 watt = 0.7373 ft.-lb. per sec.	80 — 59

Abbreviations

cu. = cubic
ft. = foot
lb. = pound
in. = inch
yd. = yard
sec. = second
min. = minute
hr. = hour
h.p. = horsepower
B.T.U. (British Thermal Units) = heat units
kg.-m. = kilogram-meter
kg. = kilogram
kw. = kilowatt
m. = meter
cm. = centimeter
sq. = square
gal. = gallon
mm. = millimeter
mi. = mile
gr. = grain
r = radius
d = diameter
log = the logarithm of
ln = the natural logarithm of
v = volume
S = area (also A)
c = circumference
km. = kilometer
= = is equal to
> = greater than
< = less than
+ = plus
− = minus
° = degrees
′ = minutes
″ = seconds

N	0	1	2	3	4	5	6	7	8	9	Average cd
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389	43
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817	43
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242	43
103	01284	01326	01368	01410	01452	01494	01536	01578	01620	01662	42
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078	42
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490	41
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898	41
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302	41
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703	40
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100	40
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555	380
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059	350
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301	323
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319	302
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140	282
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789	264
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285	250
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646	235
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885	223
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015	212
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044	202
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984	194
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840	185
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620	178
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975	165
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560	158
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090	153
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567	147
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996	143
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379	138
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720	133
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020	129
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283	126
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509	123
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703	119
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864	116
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995	113
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097	110
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172	108
N	0	1	2	3	4	5	6	7	8	9	

N	0	1	2	3	4	5	6	7	8	9	Average cd
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221	105
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246	102
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246	100
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225	98
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181	95
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117	93
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034	91
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931	90
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810	88
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672	86
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517	84
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346	83
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159	81
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957	80
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741	78
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511	77
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268	76
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012	75
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743	73
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462	72
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169	71
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865	70
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550	68
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224	67
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889	66
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543	65
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187	64
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822	63
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448	62
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065	62
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673	61
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273	60
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864	59
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448	58
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024	58
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593	57
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154	56
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708	55
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255	55
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795	54
N	0	1	2	3	4	5	6	7	8	9	

N	0	1	2	3	4	5	6	7	8	9	Average cd
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855	53
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376	52
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891	51
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399	51
86	93450	93500	93551	93601	93651	93702	93753	93802	93852	93902	50
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399	50
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376	49
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856	48
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332	48
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802	47
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267	47
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727	46
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182	46
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632	45
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078	45
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520	44
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957	44
N	0	1	2	3	4	5	6	7	8	9	

TABLE OF NATURAL LOGARITHMS

N	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
		-10	-10	-10	-10	-10	-10	-10	-10	-10
0	-	7.697	8.391	8.796	9.084	9.307	9.489	9.643	9.777	9.895
1	0.0000	0.0953	0.1823	0.2624	0.3365	0.4055	0.4700	0.5306	0.5878	0.6419
2	0.6932	0.7419	0.7885	0.8329	0.8755	0.9163	0.9555	0.9933	1.0296	1.0647
3	1.0986	1.1314	1.1632	1.1939	1.2239	1.2528	1.2809	1.3083	1.3350	1.3610
4	1.3863	1.4100	1.4351	1.4586	1.4816	1.5041	1.5261	1.5476	1.5686	1.5892
5	1.6094	1.6292	1.6487	1.6677	1.6864	1.7048	1.7228	1.7405	1.7579	1.7750
6	1.7918	1.8083	1.8246	1.8406	1.8563	1.8718	1.8871	1.9021	1.9169	1.9315
7	1.9459	1.9601	1.9741	1.9879	2.0015	2.0149	2.0282	2.0412	2.0541	2.0669
8	2.0794	2.0919	2.1041	2.1163	2.1282	2.1401	2.1518	2.1633	2.1748	2.1861
9	2.1972	2.2083	2.2192	2.2300	2.2407	2.2513	2.2618	2.2721	2.2824	2.2925
10	2.3026									
N	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

Angle	sin	cos	tan	cot	sec	csc	
0°	.0000	1.0000	.0000	-	1.0000	-	90°
1°	.0175	.9998	.0175	57.2900	1.0002	57.2987	89°
2°	.0349	.9994	.0349	28.6363	1.0006	28.6537	88°
3°	.0523	.9986	.0524	19.0811	1.0014	19.1073	87°
4°	.0698	.9976	.0699	14.3007	1.0024	14.3356	86°
5°	.0872	.9962	.0875	11.4301	1.0038	11.4737	85°
6°	.1045	.9945	.1051	9.5144	1.0055	9.5668	84°
7°	.1219	.9925	.1228	8.1443	1.0075	8.2055	83°
8°	.1392	.9903	.1405	7.1154	1.0098	7.1853	82°
9°	.1564	.9877	.1584	6.3138	1.0125	6.3925	81°
10°	.1736	.9848	.1763	5.6713	1.0154	5.7588	80°
11°	.1908	.9816	.1944	5.1446	1.0187	5.2408	79°
12°	.2079	.9781	.2126	4.7046	1.0223	4.8097	78°
13°	.2250	.9744	.2309	4.3315	1.0263	4.4454	77°
14°	.2419	.9703	.2493	4.0108	1.0306	4.1336	76°
15°	.2588	.9659	.2679	3.7321	1.0353	3.8637	75°
16°	.2756	.9613	.2867	3.4874	1.0403	3.6280	74°
17°	.2924	.9563	.3057	3.2709	1.0457	3.4203	73°
18°	.3090	.9511	.3249	3.0777	1.0515	3.2361	72°
19°	.3256	.9455	.3443	2.9042	1.0576	3.0716	71°
20°	.3420	.9397	.3640	2.7475	1.0642	2.9238	70°
21°	.3584	.9336	.3839	2.6051	1.0711	2.7904	69°
22°	.3746	.9272	.4040	2.4751	1.0785	2.6695	68°
23°	.3907	.9205	.4245	2.3559	1.0864	2.5593	67°
24°	.4067	.9135	.4452	2.2460	1.0946	2.4586	66°
25°	.4226	.9063	.4663	2.1445	1.1034	2.3662	65°
26°	.4384	.8988	.4877	2.0503	1.1126	2.2812	64°
27°	.4540	.8910	.5095	1.9626	1.1223	2.2027	63°
28°	.4695	.8829	.5317	1.8807	1.1326	2.1301	62°
29°	.4848	.8746	.5543	1.8040	1.1434	2.0627	61°
30°	.5000	.8660	.5774	1.7321	1.1547	2.0000	60°
31°	.5150	.8572	.6009	1.6643	1.1666	1.9416	59°
32°	.5299	.8480	.6249	1.6003	1.1792	1.8871	58°
33°	.5446	.8387	.6494	1.5399	1.1924	1.8361	57°
34°	.5592	.8290	.6745	1.4826	1.2062	1.7883	56°
35°	.5736	.8192	.7002	1.4281	1.2208	1.7434	55°
36°	.5878	.8090	.7265	1.3764	1.2361	1.7013	54°
37°	.6018	.7986	.7536	1.3270	1.2521	1.6616	53°
38°	.6157	.7880	.7813	1.2799	1.2690	1.6243	52°
39°	.6293	.7771	.8098	1.2349	1.2868	1.5890	51°
40°	.6428	.7660	.8391	1.1918	1.3054	1.5557	50°
41°	.6561	.7547	.8693	1.1504	1.3250	1.5243	49°
42°	.6691	.7431	.9004	1.1106	1.3456	1.4945	48°
43°	.6820	.7314	.9325	1.0724	1.3673	1.4663	47°
44°	.6947	.7193	.9657	1.0355	1.3902	1.4396	46°
45°	.7071	.7071	1.0000	1.0000	1.4142	1.4142	45°
	cos	sin	cot	tan	csc	sec	Angle

By interpolation, answers in degrees and tenths of a degree, or vice versa, are possible except for the first five or six values under cot and csc.

0.0° to 4.5°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
0.0°	-		-		-	10.00000	0	90.0°
.1	7.24188	30103	7.24188	30103	2.75812	10.00000	0	.9
.2	7.54291	17609	7.54291	17609	2.45709	10.00000	1	.8
.3	7.71900	12493	7.71900	12494	2.28100	9.99999	0	.7
.4	7.84393	9691	7.84394	9692	2.15606	9.99999	1	.6
0.5°	7.94084	7918	7.94086	7918	2.05914	9.99998	0	89.5°
.6	8.02002	6694	8.02004	6696	1.97996	9.99998	1	.4
.7	8.08696	5789	8.08700	5800	1.91300	9.99997	1	.3
.8	8.14495	5115	8.14500	5116	1.85500	9.99996	1	.2
.9	8.19610	4576	8.19616	4576	1.80384	9.99995	2	.1
1.0°	8.24186	4138	8.24192	4140	1.75808	9.99993	1	89.0°
.1	8.28324	3779	8.28332	3780	1.71668	9.99992	2	.9
.2	8.32103	3475	8.32112	3478	1.67888	9.99990	1	.8
.3	8.35578	3218	8.35590	3219	1.64410	9.99989	2	.7
.4	8.38796	2996	8.38809	2998	1.61191	9.99987	2	.6
1.5°	8.41792	2802	8.41807	2804	1.58193	9.99985	2	88.5°
.6	8.44594	2632	8.44611	2634	1.55389	9.99983	2	.4
.7	8.47226	2482	8.47245	2484	1.52755	9.99981	2	.3
.8	8.49708	2347	8.49729	2350	1.50271	9.99979	3	.2
.9	8.52055	2227	8.52079	2229	1.47921	9.99976	2	.1
2.0°	8.54282	2118	8.54308	2121	1.45692	9.99974	3	88.0°
.1	8.56400	2019	8.56429	2022	1.43571	9.99971	3	.9
.2	8.58419	1930	8.58451	1933	1.41549	9.99968	3	.8
.3	8.60349	1847	8.60384	1850	1.39616	9.99965	3	.7
.4	8.62196	1772	8.62234	1775	1.37766	9.99962	3	.6
2.5°	8.63968	1702	8.64009	1706	1.35991	9.99959	4	87.5°
.6	8.65670	1638	8.65715	1641	1.34285	9.99955	3	.4
.7	8.67308	1578	8.67356	1582	1.32644	9.99952	4	.3
.8	8.68886	1523	8.68938	1527	1.31062	9.99948	4	.2
.9	8.70409	1471	8.70465	1475	1.29535	9.99944	4	.1
3.0°	8.71880	1423	8.71940	1426	1.28060	9.99940	4	87.0°
.1	8.73303	1377	8.73366	1382	1.26634	9.99936	4	.9
.2	8.74680	1335	8.74748	1339	1.25252	9.99932	4	.8
.3	8.76015	1295	8.76087	1300	1.23913	9.99928	5	.7
.4	8.77310	1258	8.77387	1262	1.22613	9.99923	4	.6
3.5°	8.78568	1221	8.78649	1226	1.21351	9.99919	5	86.5°
.6	8.79789	1189	8.79875	1193	1.20125	9.99914	5	.4
.7	8.80978	1156	8.81068	1162	1.18932	9.99909	5	.3
.8	8.82134	1127	8.82230	1131	1.17770	9.99904	5	.2
.9	8.83261	1097	8.83361	1103	1.16639	9.99899	5	.1
4.0°	8.84358	1071	8.84464	1076	1.15536	9.99894	5	86.0°
.1	8.85429	1045	8.85540	1051	1.14460	9.99889	6	.9
.2	8.86474	1020	8.86591	1025	1.13409	9.99883	5	.8
.3	8.87494	996	8.87616	1002	1.12384	9.99878	6	.7
.4	8.88490	976	8.88618	980	1.11382	9.99872	6	.6
4.5°	8.89464		8.89598		1.10402	9.99866		85.5°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

85.5° to 90°

4.5° to 9.0°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
4.5°	8.89464	953	8.89598	959	1.10402	9.99866	6	85.5°
.6	8.90417	932	8.90557	938	1.09443	9.99860	6	.4
.7	8.91349	912	8.91495	919	1.08505	9.99854	7	.3
.8	8.92261	893	8.92414	899	1.07586	9.99847	6	.2
.9	8.93154	876	8.93313	882	1.06687	9.99841	7	.1
5.0°	8.94030	857	8.94195	865	1.05805	9.99834	6	85.0°
.1	8.94887	841	8.95060	848	1.04940	9.99828	7	.9
.2	8.95728	825	8.95908	831	1.04092	9.99821	7	.8
.3	8.96553	810	8.96739	817	1.03261	9.99814	7	.7
.4	8.97363	794	8.97556	802	1.02444	9.99807	7	.6
5.5°	8.98157	780	8.98358	787	1.01642	9.99800	8	84.5°
.6	8.98937	767	8.99145	774	1.00855	9.99792	7	.4
.7	8.99704	752	8.99919	760	1.00081	9.99785	8	.3
.8	9.00456	740	9.00679	748	0.99321	9.99777	8	.2
.9	9.01196	727	9.01427	735	0.98573	9.99769	8	.1
6.0°	9.01923	716	9.02162	723	0.97838	9.99761	8	84.0°
.1	9.02639	703	9.02885	712	0.97115	9.99753	8	.9
.2	9.03342	692	9.03597	700	0.96403	9.99745	8	.8
.3	9.04034	681	9.04297	690	0.95703	9.99737	9	.7
.4	9.04715	671	9.04987	679	0.95013	9.99728	8	.6
6.5°	9.05386	660	9.05666	669	0.94334	9.99720	9	83.5°
.6	9.06046	650	9.06335	659	0.93665	9.99711	9	.4
.7	9.06696	641	9.06994	649	0.93006	9.99702	9	.3
.8	9.07337	631	9.07643	640	0.92357	9.99693	9	.2
.9	9.07968	621	9.08283	631	0.91717	9.99684	9	.1
7.0°	9.08589	613	9.08914	623	0.91086	9.99675	9	83.0°
.1	9.09202	605	9.09537	613	0.90463	9.99666	10	.9
.2	9.09807	595	9.10150	606	0.89850	9.99656	9	.8
.3	9.10402	588	9.10756	597	0.89244	9.99647	10	.7
.4	9.10990	580	9.11353	590	0.88647	9.99637	10	.6
7.5°	9.11570	572	9.11943	582	0.88057	9.99627	10	82.5°
.6	9.12142	564	9.12525	574	0.87475	9.99617	10	.4
.7	9.12706	557	9.13099	568	0.86901	9.99607	11	.3
.8	9.13263	550	9.13667	560	0.86333	9.99596	10	.2
.9	9.13813	543	9.14227	553	0.85773	9.99586	11	.1
8.0°	9.14356	535	9.14780	547	0.85220	9.99575	10	82.0°
.1	9.14891	530	9.15327	540	0.84673	9.99565	11	.9
.2	9.15421	523	9.15867	534	0.84133	9.99554	11	.8
.3	9.15944	516	9.16401	527	0.83599	9.99543	11	.7
.4	9.16460	510	9.16928	522	0.83072	9.99532	12	.6
8.5°	9.16970	504	9.17450	515	0.82550	9.99520	11	81.5°
.6	9.17474	499	9.17965	510	0.82035	9.99509	12	.4
.7	9.17973	492	9.18475	504	0.81525	9.99497	11	.3
.8	9.18465	487	9.18979	499	0.81021	9.99486	12	.2
.9	9.18952	481	9.19478	493	0.80522	9.99474	12	.1
9.0°	9.19433		9.19971		0.80029	9.99462		81.0°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

81.0° to 85.5°

TABLE OF LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

9.0° to 13.5°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
9.0°	9.19433	476	9.19971	488	0.80029	9.99462	12	81.0°
.1	9.19909	471	9.20459	483	0.79541	9.99450	12	.9
.2	9.20380	465	9.20942	478	0.79058	9.99438	13	.8
.3	9.20845	461	9.21420	473	0.78580	9.99425	12	.7
.4	9.21306	455	9.21893	468	0.78107	9.99413	13	.6
9.5°	9.21761	450	9.22361	463	0.77639	9.99400	12	80.5°
.6	9.22211	446	9.22824	459	0.77176	9.99388	13	.4
.7	9.22657	441	9.23283	454	0.76717	9.99375	13	.3
.8	9.23098	437	9.23737	449	0.76263	9.99362	14	.2
.9	9.23535	432	9.24186	446	0.75814	9.99348	13	.1
10.0°	9.23967	428	9.24632	441	0.75368	9.99335	13	80.0°
.1	9.24395	423	9.25073	437	0.74927	9.99322	14	.9
.2	9.24818	419	9.25510	433	0.74490	9.99308	14	.8
.3	9.25237	415	9.25943	429	0.74057	9.99294	13	.7
.4	9.25652	411	9.26372	425	0.73628	9.99281	14	.6
10.5°	9.26063	407	9.26797	421	0.73203	9.99267	15	79.5°
.6	9.26470	403	9.27218	417	0.72782	9.99252	14	.4
.7	9.26873	400	9.27635	414	0.72365	9.99238	14	.3
.8	9.27273	395	9.28049	410	0.71951	9.99224	15	.2
.9	9.27668	392	9.28459	406	0.71541	9.99209	14	.1
11.0°	9.28060	388	9.28865	403	0.71135	9.99195	15	79.0°
.1	9.28448	385	9.29268	400	0.70732	9.99180	15	.9
.2	9.28833	381	9.29668	396	0.70332	9.99165	15	.8
.3	9.29214	377	9.30064	393	0.69936	9.99150	15	.7
.4	9.29591	375	9.30457	389	0.69543	9.99135	16	.6
11.5°	9.29966	370	9.30846	387	0.69154	9.99119	15	78.5°
.6	9.30336	368	9.31233	383	0.68767	9.99104	16	.4
.7	9.30704	364	9.31616	380	0.68384	9.99088	16	.3
.8	9.31068	362	9.31996	377	0.68004	9.99072	16	.2
.9	9.31430	358	9.32373	374	0.67627	9.99056	16	.1
12.0°	9.31788	355	9.32747	372	0.67253	9.99040	16	78.0°
.1	9.32143	352	9.33119	368	0.66881	9.99024	16	.9
.2	9.32495	349	9.33487	366	0.66513	9.99008	17	.8
.3	9.32844	346	9.33853	362	0.66147	9.98991	16	.7
.4	9.33190	344	9.34215	361	0.65785	9.98975	17	.6
12.5°	9.33534	340	9.34576	357	0.65424	9.98958	17	77.5°
.6	9.33874	338	9.34933	355	0.65067	9.98941	17	.4
.7	9.34212	335	9.35288	352	0.64712	9.98924	17	.3
.8	9.34547	332	9.35640	349	0.64360	9.98907	17	.2
.9	9.34879	330	9.35989	347	0.64011	9.98890	18	.1
13.0°	9.35209	327	9.36336	345	0.63664	9.98872	17	77.0°
.1	9.35536	324	9.36681	342	0.63319	9.98855	18	.9
.2	9.35860	322	9.37023	340	0.62977	9.98837	18	.8
.3	9.36182	320	9.37363	337	0.62637	9.98819	18	.7
.4	9.36502	317	9.37700	335	0.62300	9.98801	18	.6
13.5°	9.36819		9.38035		0.61965	9.98783		76.5°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

76.5° to 81.0°

13.5° to 18.0°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
13.5°	9.36819	314	9.38035	333	0.61965	9.98783	18	76.5°
.6	9.37133	312	9.38368	331	0.61632	9.98765	19	.4
.7	9.37445	310	9.38699	328	0.61301	9.98746	18	.3
.8	9.37755	307	9.39027	326	0.60973	9.98728	19	.2
.9	9.38062	306	9.39353	324	0.60647	9.98709	19	.1
14.0°	9.38368	302	9.39677	322	0.60323	9.98690	19	76.0°
.1	9.38670	301	9.39999	320	0.60001	9.98671	19	.9
.2	9.38971	299	9.40319	317	0.59681	9.98652	19	.8
.3	9.39270	296	9.40636	316	0.59364	9.98633	19	.7
.4	9.39566	294	9.40952	314	0.59048	9.98614	20	.6
14.5°	9.39860	292	9.41266	312	0.58734	9.98594	20	75.5°
.6	9.40152	290	9.41578	309	0.58422	9.98574	19	.4
.7	9.40442	288	9.41887	308	0.58113	9.98555	20	.3
.8	9.40730	286	9.42195	306	0.57805	9.98535	20	.2
.9	9.41016	284	9.42501	304	0.57499	9.98515	21	.1
15.0°	9.41300	282	9.42805	303	0.57195	9.98494	20	75.0°
.1	9.41582	279	9.43108	300	0.56892	9.98474	21	.9
.2	9.41861	279	9.43408	299	0.56592	9.98453	20	.8
.3	9.42140	276	9.43707	297	0.56293	9.98433	21	.7
.4	9.42416	274	9.44004	295	0.55996	9.98412	21	.6
15.5°	9.42690	272	9.44299	293	0.55701	9.98391	21	74.5°
.6	9.42962	271	9.44592	292	0.55408	9.98370	21	.4
.7	9.43233	269	9.44884	290	0.55116	9.98349	22	.3
.8	9.43502	267	9.45174	289	0.54826	9.98327	21	.2
.9	9.43769	265	9.45463	287	0.54537	9.98306	22	.1
16.0°	9.44034	263	9.45750	285	0.54250	9.98284	22	74.0°
.1	9.44297	262	9.46035	284	0.53965	9.98262	22	.9
.2	9.44559	260	9.46319	282	0.53681	9.98240	22	.8
.3	9.44819	258	9.46601	280	0.53399	9.98218	22	.7
.4	9.45077	257	9.46881	279	0.53119	9.98196	22	.6
16.5°	9.45334	255	9.47160	278	0.52840	9.98174	23	73.5°
.6	9.45589	254	9.47438	276	0.52562	9.98151	22	.4
.7	9.45843	252	9.47714	275	0.52286	9.98129	23	.3
.8	9.46095	250	9.47989	273	0.52011	9.98106	23	.2
.9	9.46345	249	9.48262	272	0.51738	9.98083	23	.1
17.0°	9.46594	247	9.48534	270	0.51466	9.98060	24	73.0°
.1	9.46841	245	9.48804	269	0.51196	9.98036	23	.9
.2	9.47086	244	9.49073	268	0.50927	9.98013	24	.8
.3	9.47330	243	9.49341	266	0.50659	9.97989	23	.7
.4	9.47573	241	9.49607	265	0.50393	9.97966	24	.6
17.5°	9.47814	240	9.49872	264	0.50128	9.97942	24	72.5°
.6	9.48054	238	9.50136	262	0.49864	9.97918	24	.4
.7	9.48292	237	9.50398	261	0.49602	9.97894	24	.3
.8	9.48529	235	9.50659	260	0.49341	9.97870	25	.2
.9	9.48764	234	9.50919	259	0.49081	9.97845	24	.1
18.0°	9.48998		9.51178		0.48822	9.97821		72.0°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

72.0° to 76.5°

18.0° to 22.5°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
18.0°	9.48998	233	9.51178	257	0.48822	9.97821	25	72.0°
.1	9.49231	231	9.51435	256	0.48565	9.97796	25	.9
.2	9.49462	230	9.51691	255	0.48309	9.97771	25	.8
.3	9.49692	228	9.51946	254	0.48054	9.97746	25	.7
.4	9.49920	228	9.52200	252	0.47800	9.97721	25	.6
18.5°	9.50148	226	9.52452	251	0.47548	9.97696	26	71.5°
.6	9.50374	224	9.52703	250	0.47297	9.97670	25	.4
.7	9.50598	223	9.52953	249	0.47047	9.97645	26	.3
.8	9.50821	222	9.53202	248	0.46798	9.97619	26	.2
.9	9.51043	221	9.53450	247	0.46550	9.97593	26	.1
19.0°	9.51264	220	9.53697	246	0.46303	9.97567	26	71.0°
.1	9.51484	218	9.53943	244	0.46057	9.97541	26	.9
.2	9.51702	217	9.54187	244	0.45813	9.97515	27	.8
.3	9.51919	216	9.54431	242	0.45569	9.97488	27	.7
.4	9.52135	215	9.54673	242	0.45327	9.97461	26	.6
19.5°	9.52350	213	9.54915	240	0.45085	9.97435	27	70.5°
.6	9.52563	212	9.55155	240	0.44845	9.97408	27	.4
.7	9.52775	211	9.55395	238	0.44605	9.97381	28	.3
.8	9.52986	210	9.55633	237	0.44367	9.97353	27	.2
.9	9.53196	209	9.55870	237	0.44130	9.97326	27	.1
20.0°	9.53405	208	9.56107	235	0.43893	9.97299	28	70.0°
.1	9.53613	206	9.56342	234	0.43658	9.97271	28	.9
.2	9.53819	206	9.56576	234	0.43424	9.97243	28	.8
.3	9.54025	204	9.56810	232	0.43190	9.97215	28	.7
.4	9.54229	204	9.57042	232	0.42958	9.97187	28	.6
20.5°	9.54433	202	9.57274	230	0.42726	9.97159	29	69.5°
.6	9.54635	201	9.57504	230	0.42496	9.97130	28	.4
.7	9.54836	200	9.57734	229	0.42266	9.97102	29	.3
.8	9.55036	199	9.57963	228	0.42037	9.97073	29	.2
.9	9.55235	198	9.58191	227	0.41809	9.97044	29	.1
21.0°	9.55433	197	9.58418	226	0.41582	9.97015	29	69.0°
.1	9.55630	196	9.58644	225	0.41356	9.96986	29	.9
.2	9.55826	195	9.58869	225	0.41131	9.96957	30	.8
.3	9.56021	194	9.59094	223	0.40906	9.96927	29	.7
.4	9.56215	193	9.59317	223	0.40683	9.96898	30	.6
21.5°	9.56408	191	9.59540	222	0.40460	9.96868	30	68.5°
.6	9.56599	191	9.59762	221	0.40238	9.96838	30	.4
.7	9.56790	190	9.59983	220	0.40017	9.96808	30	.3
.8	9.56980	189	9.60203	219	0.39797	9.96778	31	.2
.9	9.57169	189	9.60422	219	0.39578	9.96747	30	.1
22.0°	9.57358	187	9.60641	218	0.39359	9.96717	31	68.0°
.1	9.57545	186	9.60859	217	0.39141	9.96686	31	.9
.2	9.57731	185	9.61076	216	0.38924	9.96655	31	.8
.3	9.57916	185	9.61292	216	0.38708	9.96624	31	.7
.4	9.58101	183	9.61508	214	0.38492	9.96593	31	.6
22.5°	9.58284		9.61722		0.38278	9.96562		67.5°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

67.5° to 72.0°

22.5° to 27.0°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
22.5°	9.58284		9.61722		0.38278	9.96562		67.5°
.6	9.58467	183	9.61936	214	0.38064	9.96530	32	.4
.7	9.58648	181	9.62150	214	0.37850	9.96498	32	.3
.8	9.58829	181	9.62362	212	0.37638	9.96467	31	.2
.9	9.59009	180	9.62574	212	0.37426	9.96435	32	.1
23.0°	9.59188	179	9.62785	211	0.37215	9.96403	32	67.0°
.1	9.59366	178	9.62996	211	0.37004	9.96370	33	.9
.2	9.59543	177	9.63205	209	0.36795	9.96338	32	.8
.3	9.59720	177	9.63414	209	0.36586	9.96305	33	.7
.4	9.59895	175	9.63623	209	0.36377	9.96273	32	.6
23.5°	9.60070	175	9.63830	207	0.36170	9.96240	33	66.5°
.6	9.60244	174	9.64037	207	0.35963	9.96207	33	.4
.7	9.60417	173	9.64243	206	0.35757	9.96174	33	.3
.8	9.60589	172	9.64449	206	0.35551	9.96140	34	.2
.9	9.60761	172	9.64654	205	0.35346	9.96107	33	.1
24.0°	9.60931	170	9.64858	204	0.35142	9.96073	34	66.0°
.1	9.61101	170	9.65062	204	0.34938	9.96039	34	.9
.2	9.61270	169	9.65265	203	0.34735	9.96005	34	.8
.3	9.61438	168	9.65467	202	0.34533	9.95971	34	.7
.4	9.61606	168	9.65669	202	0.34331	9.95937	34	.6
24.5°	9.61773	167	9.65870	201	0.34130	9.95902	35	65.5°
.6	9.61939	166	9.66071	201	0.33929	9.95868	34	.4
.7	9.62104	165	9.66271	200	0.33729	9.95833	35	.3
.8	9.62268	164	9.66470	199	0.33530	9.95798	35	.2
.9	9.62432	164	9.66669	199	0.33331	9.95763	35	.1
25.0°	9.62595	163	9.66867	198	0.33133	9.95728	35	65.0°
.1	9.62757	162	9.67065	198	0.32935	9.95692	36	.9
.2	9.62918	161	9.67262	197	0.32738	9.95657	35	.8
.3	9.63079	161	9.67458	196	0.32542	9.95621	36	.7
.4	9.63239	160	9.67654	196	0.32346	9.95585	36	.6
25.5°	9.63398	159	9.67850	196	0.32150	9.95549	36	64.5°
.6	9.63357	159	9.68044	194	0.31956	9.95513	36	.4
.7	9.63715	158	9.68239	195	0.31761	9.95476	37	.3
.8	9.63872	157	9.68432	193	0.31568	9.95440	36	.2
.9	9.64028	156	9.68626	194	0.31374	9.95403	37	.1
26.0°	9.64184	156	9.68818	192	0.31182	9.95366	37	64.0°
.1	9.64339	155	9.69010	192	0.30990	9.95329	37	.9
.2	9.64494	155	9.69202	192	0.30798	9.95292	37	.8
.3	9.64647	153	9.69393	191	0.30607	9.95254	38	.7
.4	9.64800	153	9.69584	191	0.30416	9.95217	37	.6
26.5°	9.64953	153	9.69774	190	0.30226	9.95179	38	63.5°
.6	9.65104	151	9.69963	189	0.30037	9.95141	38	.4
.7	9.65255	151	9.70152	189	0.29848	9.95103	38	.3
.8	9.65406	151	9.70341	189	0.29659	9.95065	38	.2
.9	9.65556	150	9.70529	188	0.29471	9.95027	38	.1
27.0°	9.65705	149	9.70717	188	0.29283	9.94988	39	63.0°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

63.0° to 67.5°

27.0° to 31.5°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
27.0°	9.65705		9.70717		0.29283	9.94988		63.0°
.1	9.65853	148	9.70904	187	0.29096	9.94949	39	.9
.2	9.66001	148	9.71090	186	0.28910	9.94911	38	.8
.3	9.66148	147	9.71277	187	0.28723	9.94871	40	.7
.4	9.66295	147	9.71462	185	0.28538	9.94832	39	.6
27.5°	9.66441	146	9.71648	186	0.28352	9.94793	39	62.5°
.6	9.66586	145	9.71833	185	0.28167	9.94753	40	.4
.7	9.66731	145	9.72017	184	0.27983	9.94714	39	.3
.8	9.66875	144	9.72201	184	0.27799	9.94674	40	.2
.9	9.67018	143	9.72384	183	0.27616	9.94634	40	.1
28.0°	9.67161	143	9.72567	183	0.27433	9.94593	41	62.0°
.1	9.67303	142	9.72750	183	0.27250	9.94553	40	.9
.2	9.67445	142	9.72932	182	0.27068	9.94513	40	.8
.3	9.67586	141	9.73114	182	0.26886	9.94472	41	.7
.4	9.67726	140	9.73295	181	0.26705	9.94431	41	.6
28.5°	9.67866	140	9.73476	181	0.26524	9.94390	41	61.5°
.6	9.68006	140	9.73657	181	0.26343	9.94349	41	.4
.7	9.68144	138	9.73837	180	0.26163	9.94307	42	.3
.8	9.68283	139	9.74017	180	0.25983	9.94266	41	.2
.9	9.68420	137	9.74196	179	0.25804	9.94224	42	.1
29.0°	9.68557	137	9.74375	179	0.25625	9.94182	42	61.0°
.1	9.68694	137	9.74554	179	0.25446	9.94140	42	.9
.2	9.68829	135	9.74732	178	0.25268	9.94098	42	.8
.3	9.68965	136	9.74910	178	0.25090	9.94055	43	.7
.4	9.69100	135	9.75087	177	0.24913	9.94012	43	.6
29.5°	9.69234	134	9.75264	177	0.24736	9.93970	42	60.5°
.6	9.69368	134	9.75441	177	0.24559	9.93927	43	.4
.7	9.69501	133	9.75617	176	0.24383	9.93884	43	.3
.8	9.69633	132	9.75793	176	0.24207	9.93840	44	.2
.9	9.69765	132	9.75969	176	0.24031	9.93797	43	.1
30.0°	9.69897	132	9.76144	175	0.23856	9.93753	44	60.0°
.1	9.70028	131	9.76319	175	0.23681	9.93709	44	.9
.2	9.70159	131	9.76493	174	0.23507	9.93665	44	.8
.3	9.70288	129	9.76668	175	0.23332	9.93621	44	.7
.4	9.70418	130	9.76841	173	0.23159	9.93577	44	.6
30.5°	9.70547	129	9.77015	174	0.22985	9.93532	45	59.5°
.6	9.70675	128	9.77188	173	0.22812	9.93487	45	.4
.7	9.70803	128	9.77361	173	0.22639	9.93442	45	.3
.8	9.70931	128	9.77533	172	0.22467	9.93397	45	.2
.9	9.71058	127	9.77706	173	0.22294	9.93352	45	.1
31.0°	9.71184	126	9.77877	171	0.22123	9.93307	45	59.0°
.1	9.71310	126	9.78049	172	0.21951	9.93261	46	.9
.2	9.71435	125	9.78220	171	0.21780	9.93215	46	.8
.3	9.71560	125	9.78391	171	0.21609	9.93169	46	.7
.4	9.71685	125	9.78562	171	0.21438	9.93123	46	.6
31.5°	9.71809	124	9.78732	170	0.21268	9.93077	46	58.5°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

58.5° to 63.0°

31.5° to 36.0°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
31.5°	9.71809		9.78732		0.21268	9.93077		58.5°
.6	9.71932	123	9.78902	170	0.21098	9.93030	47	.4
.7	9.72055	123	9.79072	170	0.20928	9.92983	47	.3
.8	9.72177	122	9.79241	169	0.20759	9.92936	47	.2
.9	9.72299	122	9.79410	169	0.20590	9.92889	47	.1
32.0°	9.72421	122	9.79579	169	0.20421	9.92842	47	58.0°
.1	9.72542	121	9.79747	168	0.20253	9.92795	47	.9
.2	9.72663	121	9.79916	169	0.20084	9.92747	48	.8
.3	9.72783	120	9.80084	168	0.19916	9.92699	48	.7
.4	9.72902	119	9.80251	167	0.19749	9.92651	48	.6
32.5°	9.73022	120	9.80419	168	0.19581	9.92603	48	57.5°
.6	9.73140	118	9.80586	167	0.19414	9.92555	48	.4
.7	9.73259	119	9.80753	167	0.19247	9.92506	49	.3
.8	9.73377	118	9.80919	166	0.19081	9.92457	49	.2
.9	9.73494	117	9.81086	167	0.18914	9.92408	49	.1
33.0°	9.73611	117	9.81252	166	0.18748	9.92359	49	57.0°
.1	9.73727	116	9.81418	166	0.18582	9.92310	49	.9
.2	9.73843	116	9.81583	165	0.18417	9.92260	50	.8
.3	9.73959	116	9.81748	165	0.18252	9.92211	49	.7
.4	9.74074	115	9.81913	165	0.18087	9.92161	50	.6
33.5°	9.74189	115	9.82078	165	0.17922	9.92111	50	56.5°
.6	9.74303	114	9.82243	165	0.17757	9.92060	51	.4
.7	9.74417	114	9.82407	164	0.17593	9.92010	50	.3
.8	9.74531	114	9.82571	164	0.17429	9.91959	51	.2
.9	9.74644	113	9.82735	164	0.17265	9.91908	51	.1
34.0°	9.74756	112	9.82899	164	0.17101	9.91857	51	56.0°
.1	9.74868	112	9.83062	163	0.16938	9.91806	51	.9
.2	9.74980	112	9.83225	163	0.16775	9.91755	51	.8
.3	9.75091	111	9.83388	163	0.16612	9.91703	52	.7
.4	9.75202	111	9.83551	163	0.16449	9.91651	52	.6
34.5°	9.75313	111	9.83713	162	0.16287	9.91599	52	55.5°
.6	9.75423	110	9.83876	163	0.16124	9.91547	52	.4
.7	9.75533	110	9.84038	162	0.15962	9.91495	52	.3
.8	9.75642	109	9.84200	162	0.15800	9.91442	53	.2
.9	9.75751	109	9.84361	161	0.15639	9.91389	53	.1
35.0°	9.75859	108	9.84523	162	0.15477	9.91336	53	55.0°
.1	9.75967	108	9.84684	161	0.15316	9.91283	53	.9
.2	9.76075	108	9.84845	161	0.15155	9.91230	53	.8
.3	9.76182	107	9.85006	161	0.14994	9.91176	54	.7
.4	9.76289	107	9.85166	160	0.14834	9.91123	53	.6
35.5°	9.76395	106	9.85327	161	0.14673	9.91069	54	54.5°
.6	9.76501	106	9.85487	160	0.14513	9.91014	55	.4
.7	9.76607	106	9.85647	160	0.14353	9.90960	54	.3
.8	9.76712	105	9.85807	160	0.14193	9.90906	54	.2
.9	9.76817	105	9.85967	160	0.14033	9.90851	55	.1
36.0°	9.76922	105	9.86126	159	0.13874	9.90796	55	54.0°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

54.0° to 58.5°

36.0° to 40.5°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
36.0°	9.76922		9.86126		0.13874	9.90796		54.0°
.1	9.77026	104	9.86285	159	0.13715	9.90741	55	.9
.2	9.77130	104	9.86445	160	0.13555	9.90685	56	.8
.3	9.77233	103	9.86603	158	0.13397	9.90630	55	.7
.4	9.77336	103	9.86762	159	0.13238	9.90574	56	.6
36.5°	9.77439	103	9.86921	159	0.13079	9.90518	56	53.5°
.6	9.77541	102	9.87079	158	0.12921	9.90462	56	.4
.7	9.77643	102	9.87238	159	0.12762	9.90405	57	.3
.8	9.77744	101	9.87396	158	0.12604	9.90349	56	.2
.9	9.77846	102	9.87554	158	0.12446	9.90292	57	.1
37.0°	9.77946	100	9.87711	157	0.12289	9.90235	57	53.0°
.1	9.78047	101	9.87869	158	0.12131	9.90178	57	.9
.2	9.78147	100	9.88027	158	0.11973	9.90120	58	.8
.3	9.78246	99	9.88184	157	0.11816	9.90063	57	.7
.4	9.78346	100	9.88341	157	0.11659	9.90005	58	.6
37.5°	9.78445	99	9.88498	157	0.11502	9.89947	58	52.5°
.6	9.78543	98	9.88655	157	0.11345	9.89888	59	.4
.7	9.78642	99	9.88812	157	0.11188	9.89830	58	.3
.8	9.78739	97	9.88968	156	0.11032	9.89771	59	.2
.9	9.78837	98	9.89125	157	0.10875	9.89712	59	.1
38.0°	9.78934	97	9.89281	156	0.10719	9.89653	59	52.0°
.1	9.79031	97	9.89437	156	0.10563	9.89594	59	.9
.2	9.79128	97	9.89593	156	0.10407	9.89534	60	.8
.3	9.79224	96	9.89749	156	0.10251	9.89475	59	.7
.4	9.79319	95	9.89905	156	0.10095	9.89415	60	.6
38.5°	9.79415	96	9.90061	156	0.09939	9.89354	61	51.5°
.6	9.79510	95	9.90216	155	0.09784	9.89294	60	.4
.7	9.79605	95	9.90371	155	0.09629	9.89233	61	.3
.8	9.79699	94	9.90527	156	0.09473	9.89173	60	.2
.9	9.79793	94	9.90682	155	0.09318	9.89112	61	.1
39.0°	9.79887	94	9.90837	155	0.09163	9.89050	62	51.0°
.1	9.79981	94	9.90992	155	0.09008	9.88989	61	.9
.2	9.80074	93	9.91147	155	0.08853	9.88927	62	.8
.3	9.80166	92	9.91301	154	0.08699	9.88865	62	.7
.4	9.80259	93	9.91456	155	0.08544	9.88803	62	.6
39.5°	9.80351	92	9.91610	154	0.08390	9.88741	62	50.5°
.6	9.80443	92	9.91765	155	0.08235	9.88678	63	.4
.7	9.80534	91	9.91919	154	0.08081	9.88615	63	.3
.8	9.80625	91	9.92073	154	0.07927	9.88552	63	.2
.9	9.80716	91	9.92227	154	0.07773	9.88489	63	.1
40.0°	9.80807	91	9.92381	154	0.07619	9.88425	64	50.0°
.1	9.80897	90	9.92535	154	0.07465	9.88362	63	.9
.2	9.80987	90	9.92689	154	0.07311	9.88298	64	.8
.3	9.81076	89	9.92843	154	0.07157	9.88234	64	.7
.4	9.81166	90	9.92996	153	0.07004	9.88169	65	.6
40.5°	9.81254	88	9.93150	154	0.06850	9.88105	64	49.5°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

49.5° to 54.0°

40.5° to 45.0°

Angle	Logsin -10	d	Logtan -10	cd	Logcot	Logcos -10	d	
40.5°	9.81254	89	9.93150	153	0.06850	9.88105	65	49.5°
.6	9.81343	88	9.93303	154	0.06697	9.88040	65	.4
.7	9.81431	88	9.93457	153	0.06543	9.87975	66	.3
.8	9.81519	88	9.93610	153	0.06390	9.87909	65	.2
.9	9.81607	87	9.93763	153	0.06237	9.87844	66	.1
41.0°	9.81694	87	9.93916	153	0.06084	9.87778	66	49.0°
.1	9.81781	87	9.94069	153	0.05931	9.87712	66	.9
.2	9.81868	87	9.94222	153	0.05778	9.87646	67	.8
.3	9.81955	86	9.94375	153	0.05625	9.87579	66	.7
.4	9.82041	85	9.94528	153	0.05472	9.87513	67	.6
41.5°	9.82126	86	9.94681	153	0.05319	9.87446	68	48.5°
.6	9.82212	85	9.94834	152	0.05166	9.87378	67	.4
.7	9.82297	85	9.94986	153	0.05014	9.87311	68	.3
.8	9.82382	85	9.95139	152	0.04861	9.87243	68	.2
.9	9.82467	84	9.95291	153	0.04709	9.87175	68	.1
42.0°	9.82551	84	9.95444	152	0.04556	9.87107	68	48.0°
.1	9.82635	84	9.95596	152	0.04404	9.87039	69	.9
.2	9.82719	83	9.95748	153	0.04252	9.86970	68	.8
.3	9.82802	83	9.95901	152	0.04099	9.86902	70	.7
.4	9.82885	83	9.96053	152	0.03947	9.86832	69	.6
42.5°	9.82968	83	9.96205	152	0.03795	9.86763	69	47.5°
.6	9.83051	82	9.96357	153	0.03643	9.86694	70	.4
.7	9.83133	82	9.96510	152	0.03490	9.86624	70	.3
.8	9.83215	82	9.96662	152	0.03338	9.86554	71	.2
.9	9.83297	81	9.96814	152	0.03186	9.86483	70	.1
43.0°	9.83378	81	9.96966	152	0.03034	9.86413	71	47.0°
.1	9.83459	81	9.97118	151	0.02882	9.86342	71	.9
.2	9.83540	81	9.97269	152	0.02731	9.86271	71	.8
.3	9.83621	80	9.97421	152	0.02579	9.86200	72	.7
.4	9.83701	80	9.97573	152	0.02427	9.86128	72	.6
43.5°	9.83781	80	9.97725	152	0.02275	9.86056	72	46.5°
.6	9.83861	79	9.97877	152	0.02123	9.85984	72	.4
.7	9.83940	80	9.98029	151	0.01971	9.85912	73	.3
.8	9.84020	78	9.98180	152	0.01820	9.85839	73	.2
.9	9.84098	79	9.98332	152	0.01668	9.85766	73	.1
44.0°	9.84177	78	9.98484	151	0.01516	9.85693	73	46.0°
.1	9.84255	79	9.98635	152	0.01365	9.85620	73	.9
.2	9.84334	77	9.98787	152	0.01213	9.85547	74	.8
.3	9.84411	78	9.98939	151	0.01061	9.85473	74	.7
.4	9.84489	77	9.99090	152	0.00910	9.85399	75	.6
44.5°	9.84566	77	9.99242	152	0.00758	9.85324	74	45.5°
.6	9.84643	77	9.99394	151	0.00606	9.85250	75	.4
.7	9.84720	76	9.99545	152	0.00455	9.85175	75	.3
.8	9.84796	77	9.99697	151	0.00303	9.85100	76	.2
.9	9.84873	76	9.99848	152	0.00152	9.85024	75	.1
45.0°	9.84949		10.00000		0.00000	9.84949		45.0°
	-10 Logcos	d	-10 Logcot	cd	Logtan	-10 Logsin	d	Angle

45.0° to 49.5°

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