THE

DIETZGEN

MANIPHASE MULTIPLEX

(TRADE MARK)

DECIMAL TRIG TYPE LOG LOG

SLIDE RULE No. 1732

A Self-Teaching Manual

by

H. LOREN THOMPSON

and

OVID W. ESHBACH, ***Dean***

The Technological Institute

NORTHWESTERN UNIVERSITY

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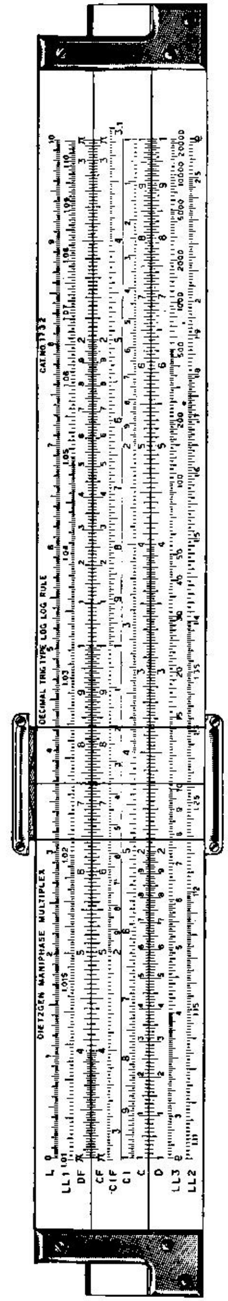
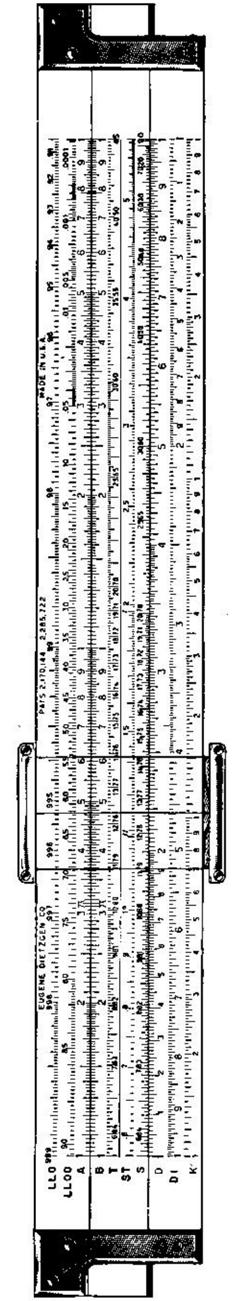
No. 1732

This manual covers instructions on the use of the DECIMAL TRIG TYPE LOG LOG Slide Rule No. 1732 which has the Sine "S," Sine-Tangent "ST," and Tangent "T" scales divided to represent DEGREES and *DECIMALS OF A DEGREE.*

For rules which have the Sine "S," Sine-Tangent "ST," and Tangent "T" scales divided to represent DEGREES and *MINUTES*, see our rule and instruction book on TRIG TYPE LOG LOG Slide Rule No. 1731.



FRONT FACE



REVERSE FACE

PREFACE

To the beginner, even the simplest slide rule may appear very complex and difficult of mastery. Such a viewpoint should be avoided because it is incorrect and because it is a real handicap in learning to use a slide rule. The mistaken notion that a good slide rule is complex is due to the fact that it is equipped with a multiplicity of scales so that it may be used to solve a wide range of problems.

Actually, a slide rule would be a good investment as a timesaver if it had nothing more than the two scales "C" and "D" for the processes of multiplication and division. And this is exactly the proper starting point for learning how to use a slide rule. Until the "C" and "D" scales are mastered, all other scales are completely ignored. Anyone, with nothing more than a background of simple arithmetic, can learn to multiply and divide by the use of the "C" and "D" scales in short order.

One step at a time this self-instruction manual makes clear the purposes of all other scales on the rule and the manner in which problems involving powers, roots, proportions, trigonometrical functions, logarithmical functions and combinations of these various mathematical considerations may be solved.

It is important to note *all* slide rules from the simplest to the most expensive are based on the same fundamentals—that any problem which can be solved on a simple rule is solved in the same manner or even more simply on a better rule. The more expensive rules merely provide *more* scales for solving problems the less expensive rules cannot handle. This is an important consideration in selecting a slide rule . . . because a rule should be purchased with its ultimate use in mind, rather than the extent of the buyer's mathematical training at the time the purchase is made. As slide rules are normally purchased for a lifetime of use, the more expensive rules are usually the best investments as they not only provide the finest in materials and precision construction, but a range of usefulness always adequate for the needs of their owners.

Eugene Dietzgen Co.

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CHAPTER I

GENERAL THEORY AND CONSTRUCTION

1. The Slide Rule is a tool for rapidly making calculations. It is an indispensable aid to the engineer and the scientist as well as to the accountant, statistician, manufacturer, teacher, and student or to ANYONE who has calculations to solve.

The theory of the SLIDE RULE is quite simple and with a little practice proficiency in its operation may rapidly be developed. A knowledge of the few principles which underlie the workings of the Slide Rule will reduce the time required to learn its use as well as give you a feeling of security in the operation—a feeling that makes you know you are doing the right operation for the information you want to obtain.

The beginner should have no difficulty in mastering the use of the Slide Rule if he will study the instructions carefully and practice the various exercises given. GO SLOWLY AND SURELY, and much time will be saved and your errors will be few.

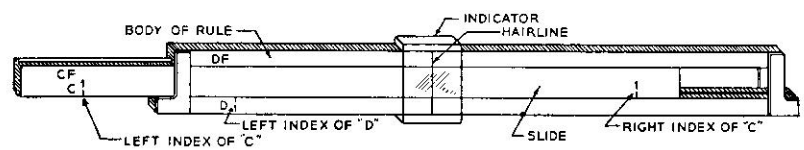


Fig. 1

2. General Description.

The slide rule consists of three main parts, see Figure 1; the BODY which is the fixed part of the rule, the SLIDE which can be moved left or right between the BODY, and the INDICATOR which slides either left or right on the face of the BODY and SLIDE. The INDICATOR has a fine hairline etched on the glass which is used for accuracy of settings and for marking results.

Figure 3 shows the "C" scale. This is just the "plot" of the logarithms of numbers from 1 to 10.

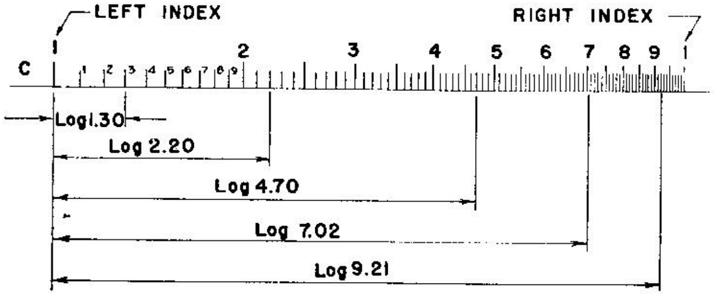


Fig. 3

For numbers above ten or below one the decimal part of the logarithm does not change— only the number to the LEFT of the decimal. The number to the left of the decimal is called the "*Characteristic*" while the numbers to the right are called the "Mantissa" of the logarithm.

THEREFORE, the "C" scale in Figure 3 is a "plot" of the "Mantissas" *of all numbers*— the "characteristic" not shown. The "characteristic" can be obtained by inspection—being defined as follows:

> For numbers greater than "1", it is *positive* and is one less than the number of digits to the *left* of the decimal point.
>
> For numbers less than "1", it is *negative* and is numerically one greater than the number of zeros *immediately following* the decimal point.

Thus, the logarithm of 2 is 0.3010, while the logarithm of 20 is 1.3010—or the logarithm of 200 is 2.3010. Since in 200 there are 3 digits to the LEFT of the decimal—the characteristic is $3 - 1 = 2$. The characteristic of the logarithm of 0.002 is -3. Since in 0.002 there are two zeros IMMEDIATELY FOLLOWING the decimal point, the characteristic is $-(2+1) = -3$, but the mantissa is still $+0.3010$. Therefore, the actual logarithm is $(-3 + 0.3010)$ which equals -2.6990. This last figure is not in the most convenient form with which to work so (for convenience) we write it as $(+7.3010 - 10)$. Since the $(+7 - 10)$ is -3 we still have the actual logarithm. Thus when the number is less than one its characteristic is written as indicated here—as *Log 0.002* $= +7.3010 - 10$.

In Figure 3 the distance from the left "1" (called the LEFT INDEX) to the number 4.70 represents the mantissa of 4.70, 0.470, 470, or 4700 or any decimal multiple of 4.70—the proper characteristic being used in each case.

5. How to Read the Scale.

First, notice that the scale constructed in Article 4 is divided into numbers from 1 to 10— the right 1 (RIGHT INDEX) can be read as 10. Each space is divided into ten parts. These divisions are therefore approximately 1/10th of the space between the large division numbers. These subdivisions are further divided into decimal parts.

There are three sections of the scale where the subdivisions are different—between prime numbers 1 and 2; 2 and 4; and 4 to 10.

The number 14 would be located at the fourth long mark (4th tenth mark) after the prime number "1" (left index). The first short mark after the number 14 would be 141— the second short mark would be 142—the third short mark would be 143, etc. Therefore, each short mark between the first subdivisions represents the third digit of the number.

If the hairline of the indicator is placed as shown below, the reading would be 143.

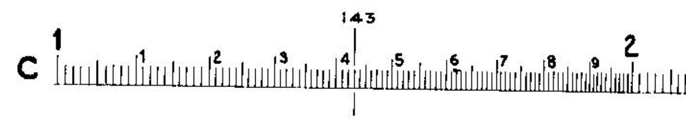


Fig. 4

The fourth digit of a number must be located by *estimation.* Thus, the number 1567 is estimated as

Fig. 5

In locating the fourth digit of any number falling between the prime numbers 1 and 2, the interval between the small divisions can be imagined divided into ten parts and the fourth digit estimated.

From the above, it appears that we may read four figures of a result in this section of the scale. This means an attainable accuracy of, roughly, 1 part in 1000, or one tenth of one percent.

In the second section of the scale, between the prime numbers 2 and 4 (Figure 6), the first subdivision marks (tenths) are *not* numbered. However, the halfway marks (five tenths); namely, 2.5 and 3.5, are, for convenience, longer than the other tenths' marks. There are ten subdivisions between the prime numbers and each of the subdivisions are divided into five parts—each part being 2/10ths of the first subdivision, or 2/100ths of the main division.

Fig. 6

The number 23 would be located at the third long subdivision mark after the prime number 2. The first short mark after the number 23 would be 232—the second short mark would be 234—the third short mark would be 236, etc. Therefore, each short mark between the first subdivisions represents the third digit of the numbers—and only the *even* ones. To obtain the location of the number when the third digit is an *odd* number, as 235, estimate halfway between the short divisions.

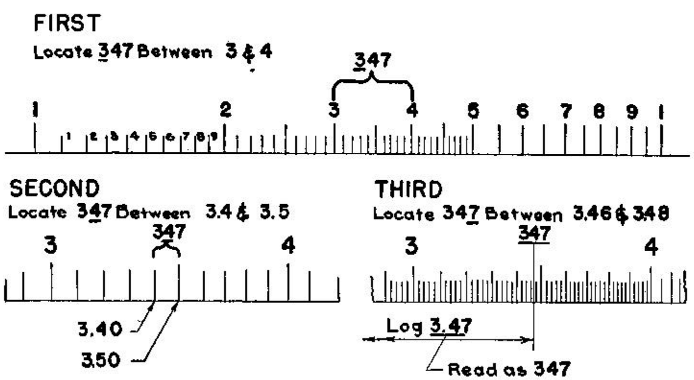


Fig. 7

To locate the number 347, determine the first digit—3 in this case. This indicates the number is between the prime numbers 3 and 4. Therefore, FIRST bring indicator hairline to prime number 3.

SECOND, locate the second digit—4—by bringing the indicator hairline to the fourth long subdivision mark following the prime number 3.

THIRD, locate the third digit—7—by moving the hairline halfway between the third and fourth short mark following this, 346 and 348, which therefore gives you the number 347 as shown in Figure 7.

In locating the fourth digit of any number falling between the prime numbers 2 and 4, the interval between the small divisions (two tenths) can be imagined divided in half, and each of these halves (one tenth each) imagined divided into ten parts, and the fourth digit estimated.

In the third section of the scale, between the prime numbers 4 and 10, the first subdivisions (tenths) are not numbered, see Figure 8. However, the halfway marks (five tenths); namely, 4.5, 5.5, 6.5, etc., are, for convenience, longer than the other tenths' marks. There are ten subdivisions between the prime numbers and each of the subdivisions are divided into two parts—each part being 5/10ths of the first subdivision, or 5/100ths of the main division.

Fig. 8

The number 67 would be located at the seventh long subdivision mark after the prime number 6. The short mark after this would be 675.

To obtain the location of a third digit of any number falling between prime numbers 4 and 10, the interval between the first subdivisions (tenths) can be imagined divided into ten parts (the fifth part being already indicated by a short line), and the third digit estimated.

When there are more digits in a number than can be accurately read, "round off" the number to either four digits (if between prime numbers 1 and 4 on the scale), or three digits (if between prime numbers 4 and 10 on the scale). The number 12346 should be "rounded off" as 12350; the number 56783 should be "rounded off" as 56800.

Exercises

MAKE ALL SETTINGS ON YOUR SLIDE RULE

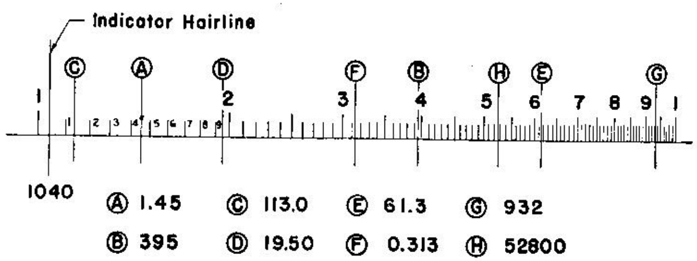
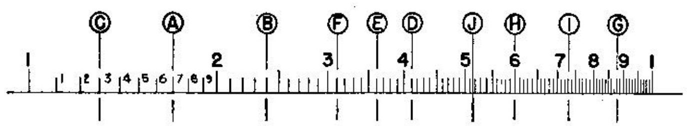


Fig. 9

In Figure 9, the hairline is first placed at 1040. Place the indicator hairline of your slide rule to this and the other placed as shown. Do you read the values given?

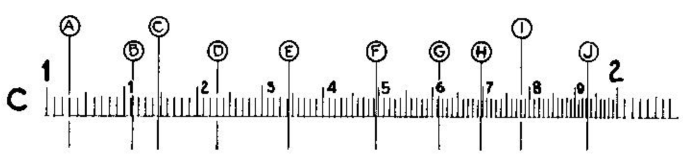
EXERCISE 1.



1. Record the Readings for the Hairlines Indicated.

A.________ C.________ E.________ G.________ I.________
B.________ D.________ F.________ H.________ J.________

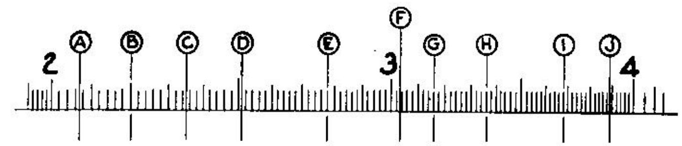
EXERCISE 2.



2. Record the Letter Opposite the Corresponding Number.

1230________ 1.612________ 0.01112________ 1.78________ 1696________
14.94________ 1.030________ 193.2________ 13.42________ 1.147________

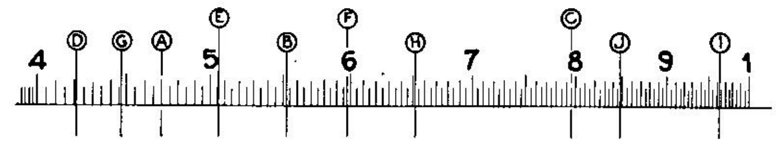
EXERCISE 3.



3. Record the Letter Opposite the Corresponding Number.

2.78________ 2350________ 2.51________ 0.368________ 2.07________
3.035________ 2.20________ 38.9________ 31.6________ 336________

EXERCISE 4.



4. Record the Scale Reading Opposite the Hairline Indicated.

A.________ C.________ E.________ G.________ I.________
B.________ D.________ F.________ H.________ J.________

ANSWERS TO EXERCISES ABOVE

1.	A.	17	C.	13	E.	36	G.	88	I.	73
	B.	24	D.	41	F.	31	H.	6	J.	515

2.	1230	D	1.612	G	0.01112	B	1.78	I	1696	H
	1494	F	1.030	A	193.2	J	13.42	E	1.147	C

3.	2.78	E	2350	C	2.51	D	0.368	I	2.07	A
	3.035	F	2.20	B	38.9	J	31.6	G	336	H

4.	A.	470	C.	795	E.	506	G.	447	I.	963
	B.	553	D.	421	F.	597	H.	652	J.	848

CHAPTER II

MULTIPLICATION AND DIVISION

6. Multiplication.

In the discussion on the theory of the Slide Rule it was stated that in order to multiply by the use of logarithms one added the logarithms of the numbers you intended to multiply.

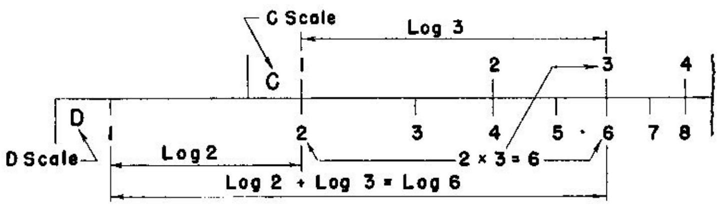


Fig. 10

In Figure 10 is indicated the multiplication of 2 × 3. Set the left index of the "C" scale opposite the "2" on the "D" scale. Move the INDICATOR to "3" on the "C" scale and opposite this on the "D" scale read the answer as 6.

Note what you have actually done. In Figure 10 a distance equal to the logarithm of 2 has been added to a distance equal to the logarithm of 3. The sum of these logarithms is the logarithm of 6 which is the answer. Since the logarithms are not shown but only the numbers they represent, one can read the answer directly.

In Figure 11 the mechanical operation for multiplying 19.30 × 5 is indicated. What is actually done is the addition of the logarithm of 19.30 to the logarithm of 5 which gives you the logarithm of 96.5. This is read directly on the "D" scale.

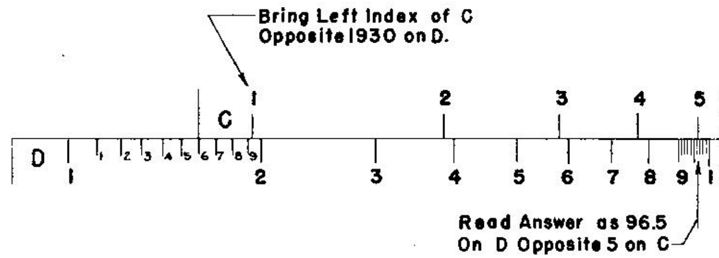


Fig. 11

The mechanical operation is performed as follows: First, set the left index of "C" opposite the number 1930 on "D"; second, move the indicator until it is at 5 on the "C" scale; and third, read the answer **965** *on the "D" scale opposite the 5 on the "C" scale. This does not give you the decimal point. However, 19.30 is about 20 and 5 × 20 is 100. Therefore, the answer is approximately 100. It is obvious then that the answer must be* **96.5**.

7. Accuracy of the Slide Rule.

The "C" and "D" scales of the "10-inch" Slide Rule can be read to three significant figures throughout their length. Between the left index and the primary number "2" one can estimate quite accurately to four significant figures. It is recommended that one not attempt to estimate beyond the third significant figure if the answer is to the right of the primary number "2" and beyond four significant figures when the setting is between the left index and the primary number "2". However, it is possible to make a rough estimate of the fourth significant figure between "2" and "4" but this last place should not be considered as too accurate.

8. Decimal Point.

The decimal point is best obtained by a quick mental calculation. For instance in Figure 12, the number 0.215 is multiplied by 0.229. The Slide Rule gives the answer as 492 which would be the same as if you multiplied any of the other decimal combinations as indicated in the Figure. Obviously the answer for all the possible decimal arrangements could not be 492. Therefore, the decimal point must be located.

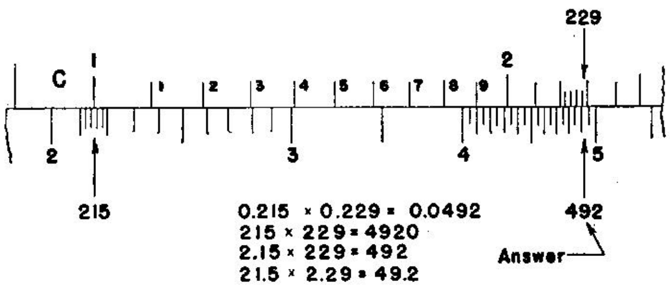


Fig. 12

Since 0.215 is approximately 0.2 and 0.229 is approximately 0.23, you can make the quick mental calculation of $0.2 \times 0.23 = 0.046$. This indicates you should read the slide rule answer as 0.0492.

Likewise, in more complicated problems you can locate the decimal by quick calculations. If you had the following to evaluate $\frac{145 \times 205 \times 68}{89 \times 12.5}$, you might write this as $\frac{150 \times 200 \times 70.}{100 \times 10}$ A quick mental calculation of this would give $2{,}100{,}000 \div 1000$ or 2100. Your answer would be approximately 2100 or four figures to the left of the decimal. The actual answer in this case is 1817. Such quick mental calculations are quite simple and the decimal point can be located accurately by this means.

9. Use of the Right Index in Multiplication.

In using the "C" and "D" scales to multiply numbers, such as 8×5—where one or both of the numbers are on the right end of the scales, the right index can be used.

In Figure 13 is indicated the multiplication of 8×5. Set the right index of the "C" over 8 on the "D" scale. Move the indicator to 5 on the "C" scale and under the hairline read the answer—40—on the "D" scale.

NOTE: If you had used the left index of "C" over 8 on the "D" scale, the answer which is read under the 5 on the "C" scale would have been off the rule.

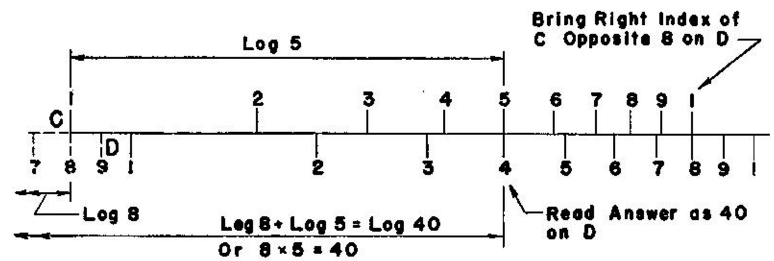


Fig. 13

Therefore, the right index and the left index of any of the scales can be used interchangeably, whichever will place the answer on the rule.

The reason for the above statement is that the "C" and "D" scales can be thought of as being continuous—or that they repeat themselves. In Figure 13 to the left of the LEFT INDEX of "D" is shown in "dotted" the numbers 7, 8, and 9. These are the same numbers and are placed identically as those on the right end of the actual "D" scale. Therefore, you can think of an imaginary scale to the left of the LEFT INDEX of the "D" scale.

In Figure 13, the right index of "C" is brought to 8 on "D". Notice that when this is done the left index of "C" is at 8 on the imaginary or "dotted" portion of "D". Now, the multiplication can be made as with any other numbers using the LEFT INDEX of "C". The answer is on "D" opposite 5 on "C".

Exercises

Fill in the following table, first, with the slide rule reading, and second, with the decimal point located correctly.

Exercise No.	Multiplication	Answer as Read on Slide Rule	Corrected Answer
1	2.45 × 31	760	76.0
2	345 × 3.46	______	______
3	972 × 0.45	______	______
4	1.035 × 0.081	______	______
5	23.1 × 1.03	______	______
6	758 × 123.46	______	______
7	4051 × 7.854	______	______
8	45.78 × 32.98	______	______
9	2.3 × 0.119	______	______
10	3.7 × 6.75	______	______

Multiply the following:

11. 3.5 × 798
12. 3.891 × 9243
13. 1.067 × 2.346
14. 78.9 × 2.453
15. 6.57 × 8.77
16. 45.03 × 77.7
17. 2.1 × 72.3
18. 0.00891 × 0.246
19. 0.0452 × 10089
20. 1.9099 × 103.4

ANSWERS TO EXERCISES ABOVE

Exercise No.	Answer	Exercise No.	Answer
2	1194	11	2790
3	437	12	36000
4	0.0838	13	2.50
5	23.8	14	193.5
6	93600.	15	57.6
7	31800.	16	3500.
8	1510	17	151.8
9	0.274	18	0.00219
10	25.0	19	456
		20	197.5

10. Division.

In dividing by logarithms one subtracts the logarithm of the divisor from the logarithm of the dividend in order to obtain the logarithm of the quotient or answer. This can be done by simple mechanical manipulation on the slide rule.

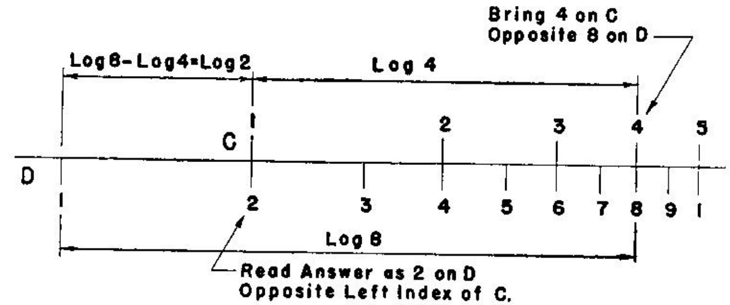


Fig. 14

In Figure 14 is indicated the division of 8 by 4. This is performed mechanically on the slide rule by the subtraction of the logarithm of 4 from the logarithm of 8.

Set the indicator at 8 on the "D" scale. Bring 4 on the "C" scale over 8 on the "D" scale and read the answer opposite the left index *of the "C" scale as 2 on the "D" scale.*

Note what you have actually done. In Figure 14 a distance equal to the logarithm of 8 (dividend) is located on the "D" scale, from which is subtracted a distance equal to the logarithm of 4 (divisor) on the "C" scale, leaving a distance equal to the logarithm of 2 (the quotient, or answer) on the "D" scale.

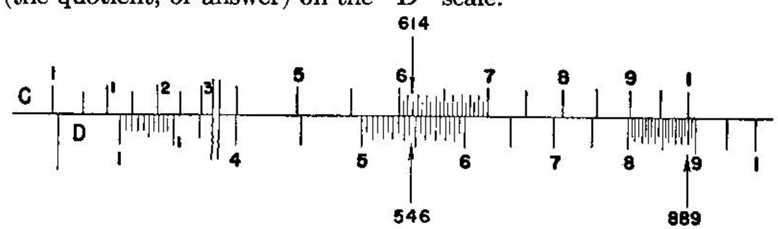


Fig. 15

The division of 546 by 614 is indicated in Figure 15. First, bring the indicator to the dividend, 546, on "D" and, second, bring to the hairline of the indicator 614 on the "C". You can then read your answer as **889** *on "D" opposite the right index on "C". The left index would also be opposite the answer* **but no** *scale of "D" exists at this point.*

You will find that if both the "C" and "D" scales repeated themselves an infinite number of times, the numbers opposite the indices of one would be the same. That is, if in Figure 15 the "C" and "D" scales were continuous and repeated themselves, the number opposite the indices on the "C" scale for this setting would always be 889 on "D" and that opposite the indices on the "D" scale would always be 1125 on "C".

In determining the location of the decimal when dividing 546 by 614, you can again make a quick mental calculation as 500 divided by 600 gives an answer a little less than 1. Therefore, the answer should read as 0.889. Likewise, if the number 546 is to be divided by 6.14, you can make the quick mental calculation as 540 divided by 6 gives 90. Your answer would then be 88.9 in this second case.

Fill in the following table, first, with the slide rule reading, and second, with the decimal point located correctly.

Exercise

Exercise No.	Division to be made	Answer as Read on Slide Rule	Corrected Decimal Point
1.	9.866 ÷ 2	493	4.93
2.	10.34 ÷ 31.4	______	______
3.	44.56 ÷ 1.239	______	______
4.	33.78 ÷ 98.7	______	______
5.	1245 ÷ 1.23	______	______
6.	3.46 ÷ 6.25	______	______
7.	3.3378 ÷ 22.89	______	______
8.	0.00033 ÷ 36.7	______	______
9.	0.0376 ÷ 0.0057	______	______
10.	1.34573 ÷ 6.784	______	______

Divide the following:

11. 87.5 ÷ 35.9	14. 0.0566 ÷ 5.47	17. 0.0348 ÷ 7.43
12. 3.45 ÷ 0.032	15. 3.42 ÷ 3.27	18. 3.142 ÷ 78.0
13. 1025 ÷ 9.71	16. 286 ÷ 3.45	19. 8.96 ÷ 44.5
		20. 1.773 ÷ 0.667

Answers to the above exercises:

2. 0.329	8. 0.00000899	14. 0.01035
3. 36.0	9. 6.60	15. 1.046
4. 0.342	10. 0.1984	16. 82.9
5. 1012	11. 2.44	17. 0.00468
6. 0.554	12. 107.8	18. 0.0403
7. 0.1458	13. 105.6	19. 0.201
		20. 2.66

11. Use of Reciprocals in Division.

The method of dividing 9 by 2 as explained above would be to bring the 2 on "C" opposite the 9 on "D". This could be done, but it requires that you bring the slide over to the right until it is almost out of the body of the rule. This division can be done in an easier manner by using the reciprocal of one of the numbers.

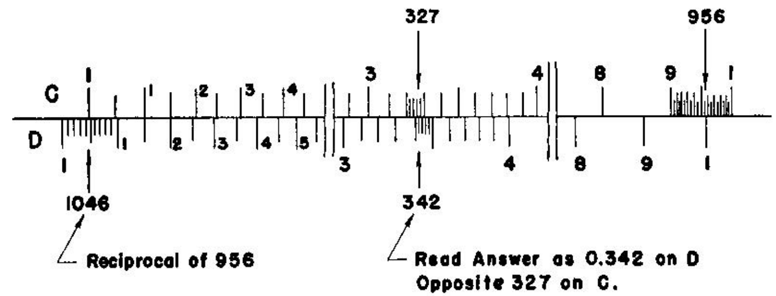


Fig. 16

In figure 16, the number 327 is divided by 956. This division is the same as if you multiplied 327 by $\frac{1}{956}$. Therefore, divide 1 by 956 first and then multiply the answer you obtain by 327.

In the figure, the 956 on "C" is brought opposite the right index on "D". The reciprocal or $\frac{1}{956}$ is read on "D" opposite the left index on "C". Move the indicator until it is at 327 on "C". Opposite this, read the answer as 342 on "D". Determine the decimal by mentally dividing 300 by 1000 giving 0.3. Therefore, the correct answer is 0.342. This manipulation saves the large movement of the slide. Now try the regular method of dividing 327 by 956. You will obtain the same answer. Next, try again the method just explained and notice the saving in time and labor.

AGAIN, THE SLIDE RULE IS A TOOL. Only when you are fully familiar with what it can do for you, can you reduce the amount of effort required in your calculations.

When dividing 277 by 11.24 as in Figure 17, you can use the same scheme as above. First divide 1 by 11.24. This is done by bringing the 1124 opposite the left index on "D". The reciprocal could be read at the right index of "C" on "D" but instead move the indicator to 277 on "C". Opposite this, read 246 on the" D" scale, which is 277 ÷ 11.24.

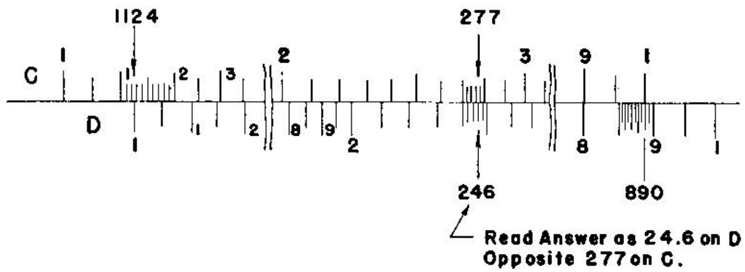


Fig. 17

12. The Folded Scales—"CF" and "DF".

The folded scales "CF" and "DF", in reality, are "C" and "D" scales cut in half at π(=3.1416) and the two halves switched, bringing the left and right indices to the middle as one index and π to each end in alignment with the left and right indices of the "C" and "D" scales. The "CF" and "DF" scales and their location with respect to the "C" and "D" scales are shown in Figure 18.

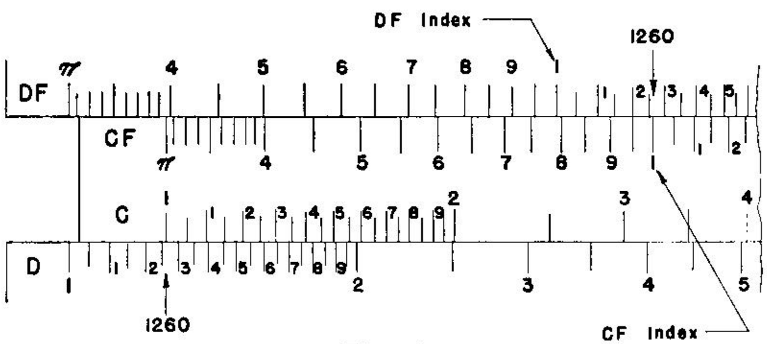


Fig. 18

This arrangement of scales has two distinct advantages:—Any number on the "C" and "D" scales is easily multiplied by π directly

above on the "CF" and "DF" scales. Thus, to multiply any number by π, bring the indicator hairline to the number on "D" and read the answer under the hairline on the "DF" scale. Likewise, one can divide a number by π by bringing the hairline to the number on the "DF" scale and reading the answer under the hairline on the "D" scale. For instance, to multiply π (=3.1416) by 2, bring the indicator hairline to 2 on "D", and above on "DF" read the answer —6.28. To divide 9.42 by π, bring the hairline of the indicator to 9.42 on "DF" and below read the answer 3 on "D".

The other advantage of the folded scales enables factors to be read without resetting the slide, which factors might be beyond the end of the rule when using only the "C" and "D" scales. *In effect, the use of the "CF" and "DF" folded scales is like extending a half scale length to each end of the "C" and "D" scales.*

Looking again to the "DF" and "CF" scales in Figure 18, you will notice that no matter what number the left index of "C" is placed opposite on "D", the middle index (the only index) of "CF" is opposite the same number on "DF". Likewise, wherever the "D" indices are with respect to the "C" scale, the "DF" index is opposite the same number on the "CF" scale. In Figure 18, the left index of "C" is opposite 1260 on "D". Also, the index of "CF" is opposite 1260 on "DF". Set your slide rule as in Figure 18. Notice the numbers opposite the right index of "D" and the index of "DF". These should be the same.

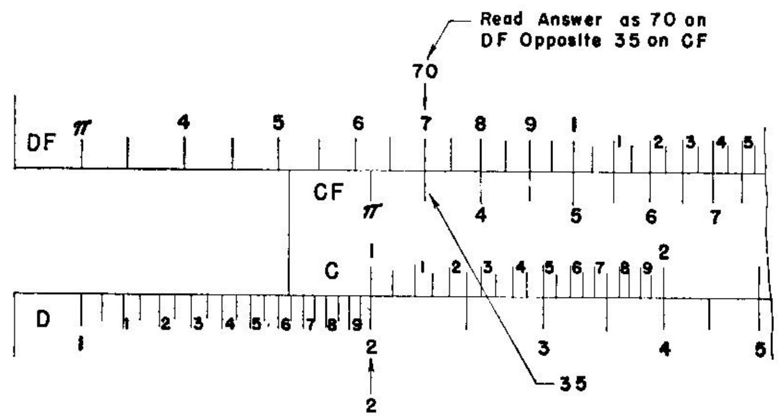


Fig. 19

In order to multiply by using the "CF" and "DF" scales, see the illustrated problem in Figure 19. Here 2 is multiplied by 35. *First set the left index of "C" opposite 2 on "D". The answer can be read on "D" opposite 35 on "C" as in the regular manner. Also, you can read the answer on "DF" opposite the 35 on "CF". Again, the answer is* **70.**

Using this same figure multiply 2 × 9. *The left index of "C" is placed opposite 2 on "D". Since 9 on "C" is off the scale on "D" you must read the answer as* **18** *on "DF" opposite 9 on "CF". This permits you to obtain the answer when it would otherwise be off the regular "D" scale.*

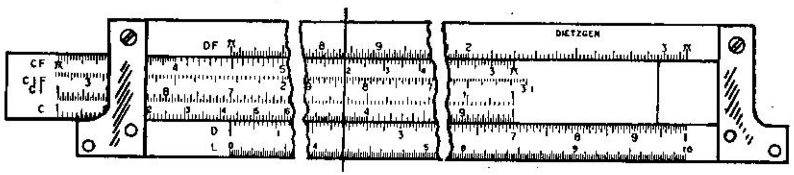


Fig. 20

To multiply 7 × 12 place the right index of "C" opposite 7 on "D", see Figure 20. Opposite the 12 on "CF" read **84** *on "DF", which is the answer.*

13. The Reciprocal Scales—"CI", "DI" and "CIF".

The reciprocal of a number is 1 divided by the number. Thus, the reciprocal of 8 is $\frac{1}{8}$ or 0.125.

The "CI", "DI" and "CIF" scales are reciprocal scales. They are constructed in a similar manner as the "C" and "CF" scales—except, they are subdivided in the opposite direction. The "CI" (or inverted "C" scale) starts with "1" at the right end and is subdivided from 1 to 10 from right to left. The numbers on the inverted "CI", "DI" and

"CIF" scales are in red to help identify them and to make it easier for one to be sure the correct scale is being used.

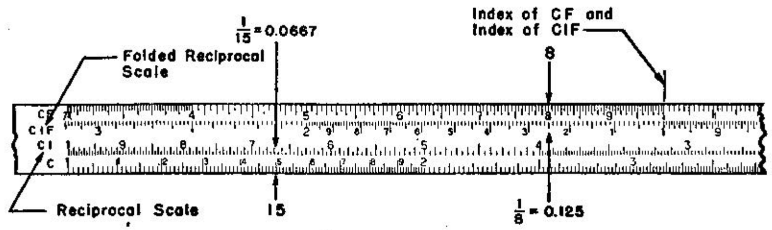


Fig. 21

Since the scales are inverted the indicator hairline can be brought to any number on the "C" scale and the reciprocal of that number can be read on the "CI" scale. A similar relation exists between the "D" and "DI" scales. Likewise, the reciprocal of any number shown on the "CF" scale can be found directly opposite the number on the "CIF" scale.

For instance, determine the reciprocal of 15. *Bring the indicator hairline to 15 on the "C" scale and read directly above on the "CI" scale* **0.0667**, *see Figure 21. Also, for the reciprocal of 8: bring the hairline to 8 on the "CF" scale and directly below this read* **0.125** *on the "CIF" scale.*

These scales are used in connection with the "C", "D", "CF", and "DF" scales in multiplication and division. To multiply 12 × 2 × 15, one must add the logarithms of the three numbers together. This sum will be the logarithm of the answer. To do this on your slide rule, the indicator is first brought to 12 on the "D" scale. Second, the slide is moved until the "2" on the "CIF" is at the indicator. Third, the indicator is moved to the 15 on the "CF" scale. Fourth, read the answer as **360** on "D" directly under the hairline.

What has actually been done? A distance equal to the logarithm of 12 has been added to a distance equal to the logarithm of 2, which would bring you to the index on "CIF". Since the index on the "CF" scale is at the same point, you can then add a distance equal to the logarithm of 15 by sliding the indicator to the right until it is at 15 on the "CF" scale. The answer is then read on the "D" scale under the hairline of the indicator.

Using the same setting of your rule multiply 12 × 2 and divide the result by 6. Set the indicator at 12 on the "D" scale as before. To this, bring the "2" on the "CIF" scale. Slide the indicator to 6 on the "CIF" scale and read the answer **4** on the "D" scale under the hairline.

Note what you have actually done is to multiply 12 × 2 by adding a distance equal to the logarithm of 12 to a distance equal to the logarithm of 2 giving you a distance equal to the logarithm of 24 (the product of 12 × 2) on the "D" scale. From the distance equal to the logarithm of 24 is subtracted a distance equal to the logarithm of 6. This last step ordinarily would be done by moving the indicator to the *left* from the index of "CIF". However, since our answer would be off the rule on the left and since we are dealing with folded scales, where the right section of the "CIF" scale is a continuation of the left section of the "CIF" scale, we can effect this subtraction mechanically by moving the indicator to the *right* to 6 on the "CIF" scale. Under the hairline at this point read the answer as **4** on the "D" scale.

Do the indicated calculations.

1. 3.45 × 54.7 × 106.8
2. 90.8 × 35 ÷ 55.8
3. 77.5 × 45.7 ÷ (3.3 × 3.6)
4. 12.78 × 23.4 ÷ 301.5
5. 145 × 36.5 × 347.0 ÷ (23.1 × 44.7)
6. 23.4 × 1.467 × 5.34 ÷ (1.67 × 6.78)
7. 0.034 ÷ (1.007 × 3.46)
8. 0.965 × 0.1045 ÷ 0.00884
9. 3.36 ÷ (2.33 × 4.05)
10. 56.78 ÷ (0.008 × 12.01)

Repeat the above calculations using not more than TWO settings of the slide.

Use the "C", "D", "CF", "DF", "CI", and "CIF" scales in combination to help you solve the problems.

11. Find (a) 7.67 per cent of 19.45
    (b) 56.4 per cent of 356.9
    (c) 19.4 per cent of 524.8
    (d) 35.2 per cent of 1235.85

12. Solve the following:
    (a) A car travels 524 miles in 9.34 hours. What is the average speed of the car in miles per hour?
    (b) If 60 miles per hour is equivalent to 88 feet per second, what is the car's speed in feet per second in the above problem?
    (c) A train travels at the rate of 38.5 miles per hour for 7.45 hours and 47.9 miles per hour for 3.51 hours. What is the train's average speed for the total distance?
    (d) A business man borrows $5675 for a period of 3.5 years and must pay 6% interest annually on the amount. What does the interest amount to in the given period?

13. What per cent of
    (a) 57 is 18.3?
    (b) 33.6 is 30.4?
    (c) 78.4 is 89.6?
    (d) 445 is 65.8?

14. Using the "D" and "DF" scales, do the following:
    (a) 345.6 × π
    (b) 2.48 × π
    (c) 99.24 ÷ π
    (d) 44.5 ÷ π

ANSWERS to the Exercises above.

Exercise No.	Answer	Exercise No.	Answer
1	20200	12 (a)	56.1 miles per hr.
2	56.9	(b)	82.3 ft. per sec.
3	298	(c)	455 miles
4	0.992		41.5 miles per hr.
5	1779	(d)	$1192
6	16.19		
7	0.00976	13 (a)	32.1%
8	11.41	(b)	90.5%
9	0.356	(c)	114.3%
10	591	(d)	14.79%
11 (a)	1.492	14 (a)	1086
(b)	201	(b)	7.79
(c)	101.8	(c)	31.6
(d)	435	(d)	14.16

CHAPTER III

PROPORTIONS

14. Proportion.

The ratio of two numbers X and Y is the quotient of X divided by Y written as $\frac{X}{Y}$. The ratio of 4 to 12 is $\frac{4}{12}$ or $\frac{1}{3}$. A *proportion* is an equation stating that two ratios are equal. Thus,

$$\frac{4}{12}=\frac{1}{3};\quad \frac{X}{7}=\frac{3.7}{45.0};\quad \text{or}\quad \frac{X}{Y}=\frac{C}{D}$$

are all proportions. These are often read as *"4 is to 12 as 1 is to 3"*, or *"X is to 7 as 3.7 is to 45.0"*, or again *"X is to Y as C is to D"*.

Many problems are solved by means of proportions. Generally only one of the four quantities is unknown as in the case with the second proportion above *"X is to 7 as 3.7 is to 45.0"*.

ILLUSTRATION:

If a 60 miles per hour speed is equivalent to 88 feet per second, how many feet per second is a car traveling that is moving at a speed of 75 miles per hour?

Set the proportion up as follows with "X" as the unknown: 88 feet per second is to 60 miles per hour as "X" is to 75

or $\frac{88}{60}=\frac{\text{"X"}}{75}$

The proportion can often be made as in the above illustration and the equation solved for "X".

15. Use of Proportion.

Proportions can easily be solved on the slide rule because of the following property of the "C" and "D" scales (also "A" and "B", as well as the "CF" and "DF" scales):

With the slide in any position, the ratio of any number on "C" to its opposite number on "D" is the same as the ratio of any other number on "C" to its opposite on "D".

This means that any two numbers on "C" together with their opposites on "D" form a proportion. Thus if 8 on "C" is set opposite

6 on "D", we also have 4 on "C" opposite 3 on "D", and 2 on "C" opposite 1.5 on "D". This is illustrated in Figure 22. The proportions can be read as "8 is to 6 as 4 is to 3 as 2 is to 1.5".

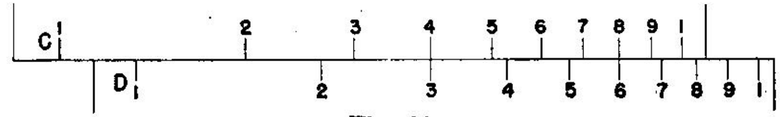


Fig. 22

The above is true of the "A" and "B" scales and the "CF" and "DF" scales as well. These additional scales will be found to be handy in the solution of proportions.

To illustrate the use of the rule for the solution of proportion problems, let us return to the illustration of article 14. Here we wanted to know the number of feet per second to which 75 miles per hour is equivalent. Write the proportion as

$$\frac{X}{75}=\frac{88}{60}$$

and make the following setting of your rule:

To 88 on "D" set 60 on "C"
Opposite 75 on "CF", read **110** *on "DF".*

In the above illustration, you have divided 88 by 60 and then multiplied this result by 75. You may always resort to straight multiplication and division for the solution of proportion problems if you wish.

It will be noted that in this illustration, the "CF" and "DF" scales were used. This is due to the fact that 75 on "C" is "off" the scale on "D"—thus we must use the folded scales.

ILLUSTRATION: A man receives $65.00 for a 40 hour week. How much should he receive for 33.5 hours of work at this rate?

Set the proportion up as "X" *is to 33.5 hours as $65.00 is to 40 hours.* Write this as

$$\frac{X}{33.5}=\frac{65}{40}$$

To 65 on "D", set 40 on "C"
Opposite 33.5 on "C" read the amount **$54.40** *on "D"*

Sometimes it may become necessary to solve a series of unknowns that can be set up as proportions. For example, let us determine the numerical values of the lettered quantities in the following proportion:

$$\frac{34.2}{4.65} = \frac{X}{34.5} = \frac{547.0}{Y} = \frac{2.312}{Z}$$

This proportion can be solved for the unknowns by one setting of the rule as follows:

To 34.2 on "D", set 4.65 on "C"

In this, note that the numerator of the ratio is on the "D" scale while the denominator is read on "C". Likewise, the numerators and denominators of the other ratios in the proportion will be read respectively on the "D" and "C" scales.

Thus, to obtain the unknowns with this setting of the rule
Read X = **254** *on "D" opposite 34.5 on "C"*
Read Y = **74.4** *on "C" opposite 547 on "D"*
Read Z = **0.3145** *on "C" opposite 2.312 on "D"*

The decimal point in each case was determined by approximating the known ratio. Thus, approximately 35 is to 5 as 7 is to 1. Therefore, in each case the ratio is a little less than 7 to 1.

ILLUSTRATION: If there are 16 ounces in 1 pound, how many ounces in 3.45 pounds?

First, set this up into a proportion that reads as follows:

"16" is to 1 as "X" is to 3.45.

To 16 on "D", set 1 on "C"
Opposite 3.45 on "C" read **55.2** *on "D"*

This illustrates the possibility of using the number "1" in a proportion. Often this is of considerable value in making proportion calculations.

Percentage problems can be solved quickly by the use of the proportion principle.

ILLUSTRATION: Find 37% of 1352.

37% of 1352 is the same as $\frac{37}{100}$ of 1352, or

0.37 × 13 2

To 1352 on "D", set left index of "C"
Opposite 0.37 on "C" read **500** *on "D"*

Write the above in a proportion form.

$$\frac{37}{X} = \frac{100}{1352}$$

The setting is the same but in this form we can easily see that if one wanted any other definite percentage of the whole (1352), it could easily be obtained with this one setting.

ILLUSTRATION: A company's total sales in the four states of Illinois, Indiana, Michigan, and Ohio were $186,500. What percentage of the total sales were the sales in the respective states if these were: Illinois, $51,200; Indiana, $35,700; Michigan, $63,100; and Ohio $36,500.

Write the following proportion:

$$\frac{100}{186{,}500} = \frac{X}{51{,}200} = \frac{Y}{35{,}700} = \frac{W}{63{,}100} = \frac{Z}{36{,}500}$$

To 186500 on "D", set left index of "C"
Opposite the sales figures for the four states on "D", read the percentages on "C",

You should read X = **27.46**%, *Y =* **19.14**%, *W =* **33.83**% *and Z =* **19.57**% *respectively.*

To check these percentages—their sum must be 100% which is the case in this illustration.

Exercises

In each of the following exercises, determine the value of the unknown quantities. If the exercise is not set up in proportion form, set it up first in this form before solving the exercise.

1. $\frac{Y}{6.73} = \frac{81}{109}$

2. $Y = \frac{14 \times 0.787}{3.45}$

3. $\frac{X}{2.81} = \frac{3.92}{5.41} = \frac{4.32}{Z} = \frac{Y}{8.92}$

4. $\frac{17}{38} = \frac{X}{9} = \frac{10}{Z}$

5. $407 = \frac{71.2\ X}{48.3}$

6. $\frac{1}{4.28} = \frac{Z}{9.39}$

7. $\frac{33\ Z}{4.58} = 9.78$

8. $Y = \frac{8.71 \times 4.32}{3.21}$

9. $\frac{7.92}{84.32} = \frac{0.695\ X}{392.5}$

10. $4.81\ Y = \frac{0.281 \times 7.45}{3.81} = \frac{Z}{9.1}$

11. A head of a family receives $360 per month for his services and he uses this in the following manner: Clothes 15%, rent 25%, savings 12%, church 5%, recreation 5%, food 23%, car 5%, and miscellaneous 10%. Determine the amount this head of the family spent on each item.

12. A distributing organization had the choice of four railroads to ship their merchandise on and they shipped in one year a total of 2,345,000 tons of merchandise. Railroad A received 540,000 tons, railroad B received 302,000 tons, railroad C received 756,000 tons, and railroad D received 747,000 tons. What percentage did each railroad receive?

Answers to the above exercises.

1. $Y = 5.01$
2. $Y = 3.19$
3. $X = 2.04$, $Z = 5.96$, $Y = 6.46$
4. $X = 4.02$, $Z = 22.35$
5. $X = 277$
6. $Z = 2.19$
7. $Z = 1.356$
8. $Y = 11.73$
9. $X = 53.1$
10. $Y = 0.1142$, $Z = 5.00$
11. 54, 90, 43.25, 18, 18, 82.75, 18, and 36 dollars.
12. 23.05, 12.87, 32.24, and 31.84 per cent.

CHAPTER IV

SQUARES AND SQUARE ROOTS

Using "A" and "B" Scales

16. Squares.

In solving problems, there are many occasions when a number must be multiplied by itself. Thus, the area of a square 4 ft. on each side is 4×4 (or 4^2) which equals 16 sq. ft. This operation is called squaring.

Instead of writing 4×4, or 35×35, or any other number multiplied by itself, the operation is indicated by writing 4^2, or $(35)^2$. This is read 4 squared, or 35 squared—sometimes read as 4 or 35 to the second power.

You will find that it is always possible to square a number by using the "C" and "D" scales. A shorter method is to use the "A" and "D" scales, or the "B" and "C" scales.

THE "A" AND "B" SCALES, which are exactly alike, are what are called two-unit logarithmic scales; that is, two complete logarithmic scales which, when placed end to end, equal the length of the single logarithmic scale "D" or "C", in connection with which they are usually used. You will note by the fact that these two-unit logarithmic scales "A" and "B" are directly above the single-unit logarithmic scale "D" that when the hairline of the indicator is set to a number on the "D" scale, the square of the number is found directly above under the hairline on the "A" scale. Likewise, if the hairline is set to a number on the "C" scale, the square of that number is found under the hairline on the "B" scale.

Note that dual faced rules, having graduations on both sides, have an encircling indicator permitting any one of the scales on one side to be read in connection with any of the scales on the opposite side. Thus, if the hairline of the indicator is set to 2 on "C", the square of 2, namely, 4, will be found under the hairline on the opposite side of the rule on scale "B".

Note also that since the "A" and "B" scales are each two complete logarithmic scales, they can be used for multiplication and division the same as the "C" and "D" scales; as, for example, to multiply 2×3, set either the left or the middle index of "B" under either the 2 on the first unit of "A" or under 2 on the second unit of "A" and above 3 on "B", read the answer **6** on "A" in either the first or the second unit.

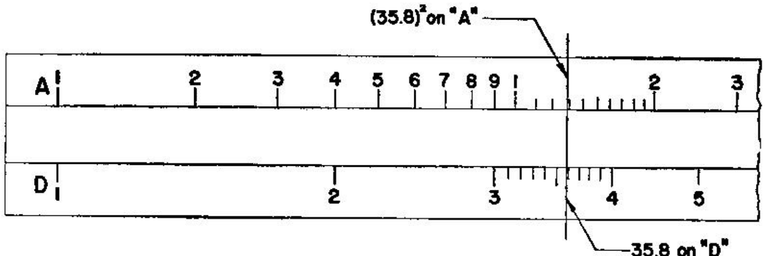


Fig. 23

ILLUSTRATION: What is the square of 35.8 or what is $(35.8)^2$? See Figure 23.

Set indicator at 35.8 on the "D" scale
Read answer on the "A" scale under the hairline as **1282.**
(The fourth digit being estimated)
Obtain the decimal by estimation as 40 × 40 is 1600
Therefore read the answer as **1282.**

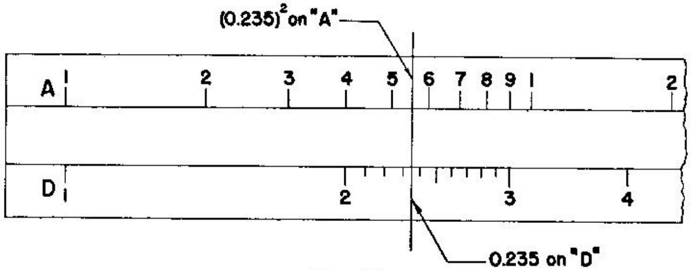


Fig. 24

ILLUSTRATION: What is 0.235 squared? See Figure 24.

Bring indicator to 235 on "D"
Read answer as **552** *on "A" under hairline*
Estimate decimal point by 0.2 × 0.2 which equals 0.04
Read answer as **0.0552.**

17. Applications of Squares.

The area of a circle is given as $\frac{\pi D^2}{4}$. This involves the square of the diameter.

Determine the area of a circle of 12″ diameter. Bring the indicator to 12 on "D", the square of which is 144 and is read immediately above under the hairline on "A". This is then multiplied by π by moving the left index of "B" under the hairline and sliding the indicator to π on "B", the product of which is immediately under the hairline on "A". Hold this product under the hairline on "A" and divide same by 4 which is done by moving 4 on the "B" scale under the hairline and reading the answer **113.0** on "A" immediately above the left index "B".

This operation could have been performed using the "A", "C", "D", and "DF" scales as follows: Use the "D" and "A" scales as above to obtain the square of 12. Since our calculation involves dividing by 4, we can effect this division by multiplying by the reciprocal of 4. Therefore, using the "C" and "D" scales, set 4 on "C" over the right index of "D" and read the reciprocal of 4 under the left index of "C". This reciprocal is then multiplied by $(12)^2$ or 144, by moving the indicator hairline to 144 on "C". This product can then be immediately multiplied by π by reading the answer **113.1** directly above under the hairline on "DF".

In this second method, you will be able to read to four significant figures (since you are between the prime numbers 1 and 2 of the rule), while in the first method, only three significant figures can be read on the "A" scale. It might be well that you do both of these operations again to familiarize yourself with the advantage of one method over the other.

In solving problems involving both multiplication and division, it is not necessary to read intermediate answers of each step in the calculation for all we are interested in is the final result. The best way to approach problems of this kind is to perform alternately—first, division; then multiplication; then division; then multiplication, and continue in this manner until the problem is solved. This minimizes the number of settings of the slide and the movement of the indicator.

ILLUSTRATION: Do the following indicated operation:

$$\left[\frac{45.8 \times 31.9}{5.6}\right]^2$$

Set the indicator at 45.8 on "D"
Bring 5.6 on "C" to hairline
Move indicator to 31.9 on "C"
Under hairline on "A" read **681**

Estimate decimal by 50 × 30 ÷ 5 equals 300
300 squared is 90,000
Therefore, answer should be **68,100**

The area of a circle was given above as $\frac{\pi}{4}$ times the diameter squared. π is 3.14 and $\frac{\pi}{4}$ is 0.785. Therefore, the area of a circle is the constant (0.785) times the diameter squared. Toward the right end of both the "A" and "B" scales is a long mark at 0.785 or $\pi/4$.

To obtain the area of a 12″ circle, bring the 0.785 mark on "B" to the right index of "A". Move the indicator to 12 on "D" and read the answer on "B" under the hairline as 113.0. In this operation, you are multiplying 0.785 by the square of 12. *Thus, to obtain the area of any circle, bring 0.785 mark on "B" to right index of "A". Bring indicator to the diameter on "D" and read answer on "B" under the hairline.*

Exercises

1. Use the slide rule to find the squares of each of the following numbers: 23, 33, 0.31, 87, 3358, 1.334, 6.78, 2.09, 31.9, 0.978, 31×10^3, 0.0065.

2. Determine the area of the circles (perform the operation in at least two ways) having the following diameters: (a) 3.45 ft., (b) 35 in., (c) 2.45, and (d) 12.5 in.

3. Do the following operations and square the answers:

a. $\frac{3.67 \times 7.34}{15.89}$ c. $\frac{0.89 \times 34.24}{1 + 34.1 \times 3.0}$

b. $\frac{67 + 4.5}{2.1 \times 34.5}$ d. $\frac{79.67 \times 3.45}{5.35}$

e. $\frac{3967 + 5280}{12300}$ f. $\frac{5.81 \times 9.89}{689.7}$

ANSWERS TO THE ABOVE EXERCISES.

1. 529, 1089, 0.0961, 7570, 11,270,000, 1.78, 46.0, 4.37, 1018, 0.956, 961×10^6, 42.3×10^{-6}.

2. (a) 9.34 sq. ft. (b) 962 sq. in. (c) 4.71 sq. ft. (d) 122.7 sq. in.

3. (a) 2.87 (d) 2640
(b) 0.974 (e) 0.566
(c) 0.874 (f) 0.00694

18. Square Roots.

The square root of any number is another number whose square is the first number. Five squared is 25 and the square root of 25 is 5. The symbol for the square root is $\sqrt{}$. Thus to indicate the square root of 25 the symbol is used as $\sqrt{25}$.

ILLUSTRATION:

$\sqrt{9} = 3$ $\sqrt{100} = 10$
$\sqrt{16} = 4$ $\sqrt{121} = 11$
$\sqrt{49} = 7$ $\sqrt{169} = 13$

The square root of a number is found on the slide rule by reversing the process used in finding the square of a number; namely, locating the number whose square root is desired on the "A" scale and reading the square root of same under the indicator on the "D" scale.

The "A" scale has two parts that are identical. This scale is divided into divisions from 1 to 10 in one-half the length of the rule and again into divisions from 1 to 10 in the second half of the rule. The "B" scale is identical with the "A" scale. The first half of the "A" and "B" scales will be referred to as A-LEFT or B-LEFT and the other half as either A-RIGHT or B-RIGHT.

In order to find the square root of numbers with an *odd number of digits* to the left of the decimal point, use the A-LEFT scale in conjunction with the "D" scale.

ILLUSTRATION: What is the square root of 9 or 900?

Bring the indicator to 9 on A-LEFT
Under the hairline on "D" read the square root as **3**.

For 900 make the same setting
Read the answer as **30**.

To find the square root of any number with an *even number of digits* to the left of the decimal point, use the A-RIGHT scale in conjunction with the "D" scale.

ILLUSTRATION: What is the square root of 16 or 1600?

Bring the indicator to 16 on the A-RIGHT scale
Under the hairline, read **4** *on the "D" scale*

Or if the number is 1600, the setting is the same.
In this case, read the answer as **40**.

A study of the above two illustrations indicates that your answer should have one place to the left of the decimal for each two digits left of the decimal of the original number if it was an even number. When the original number is odd, add 1 to the number of digits and divide by 2.

To find the decimal point for the $\sqrt{7854}$, add the number of digits and divide by 2. Thus there are four digits and, therefore, the answer should have two digits to the left of the decimal.

To find the decimal point for $\sqrt{785.4}$, add 1 to the number of digits and divide by 2 again. Since there is an odd number of digits, add 1 giving 4 and divide by 2 giving 2 places to the left of the decimal point.

ILLUSTRATION: What is the square root of 7854?
The number has an even number of digits. Therefore:

Bring the indicator to 7850 on the A-RIGHT scale
Read the answer **88.6** *on "D" under the hairline.*

What is the square root of 785.4?
This number has an odd number of digits. Therefore:
Bring the indicator to 785.0 on the A-LEFT scale,
Read the answer as **28.0** *on "D" under the hairline.*

In both of the above cases the number was "rounded off" to three significant figures, to be within the accuracy of the rule.

19. Square Roots of Numbers Less Than Unity.

If the square root of a number less than unity is desired, move the decimal point to the RIGHT an even number of places until you have a number greater than 1. Thus to obtain $\sqrt{0.000347}$, change the number to read the $\sqrt{3.47}$. Obtain the $\sqrt{3.47}$ as before which is 1.864. Since the decimal point was moved 4 places to the right in the first operation, move it back to the left *half* of this amount. You would then read the answer as **0.01864.**

ILLUSTRATION: What is the square root of 0.0956?
Move the decimal 2 places to the right to obtain 9.56

Set the indicator at 9.56 on A-LEFT
Under the hairline read **3.09** *on "D".*

Finally, move the decimal $\frac{1}{2}$ *of 2 places to the left*

Read the answer as **0.309**

ILLUSTRATION: What is the square root of 0.0000158?
Move the decimal 6 places to the right to obtain 15.8

Set indicator at 15.8 on A-RIGHT
Under the hairline read **3.97**

Move the decimal $\frac{1}{2}$ *of 6 places to the left*

Finally the answer should be read as **0.00397**

Exercises

1. Find the square roots of each of the following numbers: 3, 30, 785, 78.5, 9.8, 98, 0.81, 0.081, 0.000152, 0.0000152, 35580, 1210.

2. The area of a circle is $\frac{\pi D^2}{4}$. If $\frac{\pi}{4}$ is 0.785, determine the diameter of the circles having the following areas: (a) 345 sq. ft., (b) 144 sq. in., (c) 0.724 sq. ft., (d) 192,000 sq. ft.

3. Determine the length of the sides of squares having the following areas: (a) 23.56 sq. ft., (b) 324.5 sq. in.. (c) 3,458 sq. in., (d) 1.3786 sq. ft.

ANSWERS TO THE ABOVE EXERCISES.

1. 1.732, 5.48, 28.0, 8.86, 3.13, 9.90, 0.9, 0.285, 0.01233, 0.0039, 188.7, 34.8.
2. (a) 20.95 ft., (b) 13.54 in., (c) 0.96 ft., (d) 494 ft.
3. (a) 4.86 ft., (b) 18.0 in., (c) 58.8 in., (d) 1.175 ft.

20. Combined Operations Involving Squares and Square Roots.

The "B" and "C" scales can be used in the same manner as the "A" and "D" scales to obtain the square roots of numbers. This makes various combined operations easy with the slide rule.

For example, to obtain the result of $4 \times \sqrt{354}$, set the left index of "C" at 4 on "D" and move the indicator to 354 on "B-LEFT". Read the answer as **75.2** on "D" under the hairline. Likewise, to obtain the result of $8.6 \times \sqrt{34.8}$, set the right index of "C" at 8.6 on "D" and move indicator to 34.8 on "B-RIGHT". Read the answer as **50.7** on "D" under the hairline.

For simplicity the following general form will be used for all slide rule settings:

What is the value of $23.4\sqrt{7.86}$?

To 23.4 on "D", set 1 on "C"
Opposite 7.86 on B-LEFT read **65.6** *on "D"*

The same plan is used below for evaluating $94 \div \sqrt{34.9}$.

To 94 on "D", set 34.9 on B-RIGHT
Opposite 1 on "C" read **15.92** *on "D"*

To find the value of $\frac{8.78\sqrt{2.35}}{67.4}$ perform the operation as follows:

To 8.78 on "D", set 67.4 on "C"
Opposite 2.35 on B-LEFT read **0.200** *on "D"*

The reciprocal scale, "CI", can be used for evaluating $x = \frac{4.51}{21.2\sqrt{32.8}}$ as follows:

To 4.51 on "D", set 32.8 on B-RIGHT
Opposite 21.2 on "CI" read **0.0371** *on "D"*

SPECIAL EXAMPLES: Make all indicated operations with your own rule.

Example 1. Evaluate $\frac{0.356\sqrt{0.078} \times 54.3}{\sqrt{46.8}}$

To 0.356 on "D", set 46.8 on B-RIGHT
Move indicator to 0.078 on B-LEFT
Set 54.3 on "CI" to hairline
Read answer as **0.789** *on "D" opposite 1 on "C"*

Reviewing these operations you have done the following: First, 0.356 has been divided by $\sqrt{46.8}$ and, second, this result has been multiplied by $\sqrt{0.078}$. In the third step, you have multiplied by 54.3 by using your "CI" scale. Your slide rule in this third operation adds the logarithm of 54.3 to the logarithm of the result of step 2.

Example 2. Evaluate $\frac{\sqrt{89.5}\,(43.2)^2}{31.6 \times 903}\,(\pi)$

To 89.5 on A-RIGHT, set 903 on "C"
Move indicator to 31.6 on "CI"
Bring 43.2 on "CI" to hairline
Opposite 43.2 on "C" read **1.94** *on "DF"*

Reviewing these operations, you have done the following: First, $\sqrt{89.5}$ has been divided by 903 and, second, this result has been divided by 31.6. Third, the second result has been multiplied by 43.2 by using the "CI" scale and, fourth, this result has been again multiplied by 43.2 using the "C" scale. Finally, you have multiplied by π when the answer is read on the "DF" scale.

Example 3. Evaluate $\frac{\sqrt{4.35} \times \sqrt{54.2}\,(2.3)^2}{(8.4)^2\sqrt{34.9}}$

To 54.2 on A-RIGHT set 8.4 on "C"
Move indicator to 4.35 on B-LEFT
Bring 2.3 on "CIF" to hairline
Move indicator to 2.3 on "CF"
Bring 8.4 on "C" to hairline
Move indicator to left "C" index
Bring 34.9 on B-RIGHT to hairline
Read answer as **0.01947** *opposite right index of "D".*

Reviewing these operations, you have done the following: First, $\sqrt{54.2}$ has been divided by 8.4; second, this result has been multiplied by $\sqrt{4.35}$; third, the second result has been multiplied by 2.3 by using the "CIF" scale (dividing a product by the reciprocal of a number gives the same result as multiplying by the number); fourth, this result has been again multiplied by 2.3 using the "CF" scale; fifth, this result has again been divided by 8.4 using the "C" scale. Sixth, or finally, this result has been divided by $\sqrt{34.9}$ using the "B-RIGHT" scale and the answer **0.01947** is read on the "D" scale.

Exercises

In each of the following exercises perform the indicated operation.

1. $\sqrt{\dfrac{0.932}{0.012}}$
2. $\sqrt{\dfrac{3.26 \times 281}{0.821}}$
3. $\dfrac{3.83\sqrt{81.3}}{0.65}$
4. $\dfrac{(3.18)^2\ (\pi)}{\sqrt{3.91}}$
5. $\dfrac{37.8\ (2.31)^2}{7.31 \times 4.20}\sqrt{\dfrac{4.2 \times 9}{6 \times 7.1}}$
6. $\dfrac{384\sqrt{792}\ (0.945)}{\sqrt{7.2} + 8.3\sqrt{5}}$
7. $\dfrac{21.7\ (7.72)^2\ (6.7)^2}{\sqrt{4.67} \times \sqrt{81}}$
8. $\dfrac{2.39\sqrt{6.3}}{(5.1)^2\sqrt{4.7}}$
9. $\dfrac{\sqrt{89.3}\ (7.81)^2}{\sqrt{75} + 8\sqrt{121}}$
10. $\dfrac{75\ (3.81)^2\sqrt{972}}{\sqrt{0.0079}}$

Answers to the Above Exercises.

1. 8.82
2. 33.4
3. 53.1
4. 16.10
5. 6.18
6. 481
7. 2980
8. 0.1063
9. 5.96
10. 382,000

CHAPTER V

CUBES AND CUBE ROOTS

Using "K" Scale

21. Cubes.

Just as 4^2 means 4×4, so 4^3 (read four-cubed) means $4 \times 4 \times 4$. The small number, 3, to the upper right indicates how many 4's (or whatever the number is) must be multiplied together. This small number is called the exponent or power of the number. To illustrate:

$$10^3 = 10 \times 10 \times 10$$
$$(4.7)^3 = 4.7 \times 4.7 \times 4.7$$

It is always possible to multiply these numbers out on the "C" and "D" scales—and in combined operations for complicated calculations, it is sometimes more convenient. However, the "K" scale on the slide rule is designed to give you the cubes of all numbers directly.

The "K" scale is what is called a three-unit logarithmic scale; that is, three complete logarithmic scales of a length which, when placed end to end, equal the length of the single logarithmic scale "D" with which it is usually used. You will note that this "K" scale is so arranged beneath the "D" scale that when the indicator is set to a number on the "D" scale, the cube of that number is given under the hairline on the "K" scale.

Illustration: What is the cube of 34.5?

Set indicator to 34.5 on "D"
Under hairline on "K" read **41,100**

To carry out this calculation on the full length scales, do the following:

To 34.5 on "D" set 34.5 on "CI"
Move indicator to 34.5 on "C"
Read **41,100** *under the hairline on "D"*

The reciprocal and folded scales are invaluable in shortening various calculations and one who expects to become proficient in the operation of the slide rule should use these scales as often as possible; as, for instance, dividing a product by the reciprocal of a number as illustrated in the above example gives the same result as multiplying by the number. A tool is of value only when it is used.

22. Cube Roots.

The cube root of a number is a number which when multiplied by itself three times gives the original number. Thus, the cube root of 27 is 3, because $3 \times 3 \times 3$ is 27. The symbol of cube root is $\sqrt[3]{}$ and the cube root of 8000 is written as $\sqrt[3]{8000}$.

The "K" scale is a triple scale, consisting of three identical sections, one following the other. In finding the cube roots of numbers, the "K" scale is considered as a single scale.

The first division of the "K" scale will be referred to as K-LEFT; the second division as K-MIDDLE; and the third division as K-RIGHT. To obtain cube roots of numbers, set the hairline on the number on the "K" scale (see unit below) and read the cube root at the hairline on "D" scale, using:

K-LEFT	for numbers between	1 and 10
K-MIDDLE	" " "	10 and 100
K-RIGHT	" " "	100 and 1000

For numbers greater than 1,000 or less than 1 (unity), proceed as follows:

FIRST: Move the decimal point to the left or right three places at a time until a number between 1 and 1000 is obtained.

SECOND: Take the cube root of this number using K-LEFT, K-MIDDLE, or K-RIGHT as explained above. Place the decimal point after the first figure of this reading.

THIRD: Now move the decimal point in the opposite direction one-third as many places as it was moved in (First) above.

ILLUSTRATION: What is the cube root of 34560?

Move the decimal point to the left three places (one group of three), thus obtaining 34.560. Since the part to the left of the decimal point is between numbers 10 and 100, use the K-MIDDLE scale.

Set indicator to 346 on K-MIDDLE and
Read 3.26 under hairline on "D" scale.

Set decimal point one place $\left[\frac{1}{3}(3) = 1\right]$ *to the right to obtain the answer,* **32.6.**

ILLUSTRATION: What is the cube root of 4,567,000?

Move the decimal point to the left six places (two groups of three), thus obtaining 4.567. Since the part to the left of the decimal point is between 1 and 10, use the K-LEFT scale.

Set indicator to 4567 on K-LEFT and
Read 1.658 under the hairline on "D" scale.

Set decimal point two places $\left[\frac{1}{3}(6) = 2\right]$ *to the right to obtain the answer,* **165.8.**

ILLUSTRATION: What is the cube root of 0.0000315?

Move the decimal point to the right six places (two groups of three), thus obtaining 31.5. Since the part to the left of the decimal point is between numbers 10 and 100, use the K-MIDDLE scale.

Set indicator to 31.5 on K-MIDDLE and
Read 3.16 under the hairline on "D" scale.

Set decimal point two places $\left[\frac{1}{3}(6) = 2\right]$ *to the left to obtain the answer,* **0.0316.**

After a little practice, the steps in determining the location of the decimal point, as well as the correct section of the "K" scale to be used, can be easily determined mentally.

ILLUSTRATION: What is the cube root of 0.00315?

Set indicator to 3.15 on K-LEFT and
Read **0.1466** *under the hairline on "D" scale.*

23. Combined Operations.

The "K" scale can be used to advantage with the other scales to obtain results for various combined operations.

Example 1. Evaluate $23.3 \times \sqrt[3]{87.9}$

To 87.9 on K-MIDDLE set 23.3 on "CI"
Opposite "1" on "C" read **103.5** *on "D"*

Example 2. Evaluate $\dfrac{2.45 \times \sqrt[3]{7.8}}{5.67}$

To 7.8 on K-LEFT set 2.45 on "CIF"
Move indicator to 5.67 on "CIF"
Under hairline on "D" read **0.856.**

Reviewing this last example, the cube root of 7.8 is first multiplied by 2.45 using the "CIF" scale (dividing a product by the reciprocal of a number gives the same result as multiplying by the number), and then this result is divided by 5.67 using the "CIF" scale.

Example 3. Evaluate $\dfrac{34.5 \times 7.93 \sqrt[3]{895}}{(2.38)^3}$

To 895 on K-RIGHT set 2.38 on "CF"
Move indicator to 2.38 on "CIF"
Bring 7.93 on "CI" to hairline
Move indicator to "1" on "C"
Bring 2.38 on "C" to hairline
Move indicator to 34.5 on "C"
Read answer as **195.5** *under hairline on "D"*

Example 4. Evaluate $\dfrac{\sqrt{0.78} \times 8.97 \times \sqrt[3]{54.8}}{4.58 \times 82.1}$

To 54.8 on K-MIDDLE set 4.58 on "C"
Move indicator to 0.78 on B-RIGHT
Bring 8.97 on "CIF" to hairline
Move indicator to 82.1 on "CIF"
Read answer as **0.0799** *under hairline on "D"*

Exercises

1. $\pi\,(63.2)^3$
2. $\sqrt[3]{63.2}\,(\pi)$
3. $7.81\,(2.31)^3$
4. $\sqrt[3]{0.0785}$
5. $\sqrt[3]{92756}$
6. $(0.00312)^3$
7. $\dfrac{(81.2)\sqrt[3]{8.1}}{7.2}$
8. $\dfrac{2.45 \times \sqrt[3]{72.8}}{\sqrt{6.3}}$
9. $\dfrac{7.81 + \sqrt[3]{9.71}}{34.2\sqrt[3]{752}}$
10. $\dfrac{9.45 \times \sqrt{96.1}}{\sqrt[3]{\dfrac{831}{5.1}}}$
11. $\dfrac{\sqrt[3]{0.0831} \times \sqrt{81.0}}{\pi\,(3.87)^2}$
12. $\sqrt{(2.78)^2 - \sqrt[3]{5.92}}$
13. $\dfrac{(2.81)^3 - \sqrt{8.1}}{(2.03)^2}$
14. $(0.431)\,(0.003)^2\,\sqrt[3]{87.2}$
15. $\dfrac{(\pi)^2\,(1.815)^2}{\sqrt{\pi + 4.18}}$

ANSWERS

1. 794,000
2. 12.52
3. 96.3
4. 0.428
5. 45.3
6. 30.4×10^{-9}
7. 22.6
8. 4.07
9. 0.0320
10. 16.95
11. 0.830
12. 2.43
13. 4.72
14. 1.715×10^{-5}
15. 12.03

CHAPTER VI

PLANE TRIGONOMETRY

Use of the "S", "T", and "ST" Scales

24. Fundamental Ideas and Formulas of Plane Trigonometry.

A review of a few of the fundamental ideas and formulas of plane trigonometry is given here to help in understanding the explanation of the use of the "S", "T", and "ST" scales on your slide rule.

In the right triangle, Figure 25, the corners or angles are labeled A, B, and C. The triangle is referred to as triangle ABC. The sides are labeled a, b, and c, with a opposite angle A, b opposite angle B, and c opposite angle C. For right triangles the 90° angle is labeled C.

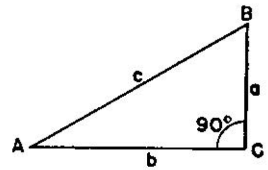


Fig. 25

Referring to this figure, the following definitions and relationships can be written.*

Definitions of the sine, cosine, and tangent:

$$\text{Sine A (written sin A)} = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{Cosine A (written cos A)} = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{Tangent A (written tan A)} = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}$$

Reciprocal relations:

$$\text{Cosecant A (written csc A)} = \frac{c}{a} = \frac{1}{\sin A}$$

$$\text{Secant A (written sec A)} = \frac{c}{b} = \frac{1}{\cos A}$$

$$\text{Cotangent A (written cot A)} = \frac{b}{a} = \frac{1}{\tan A}$$

*See any standard text on Plane Trigonometry.

RELATION BETWEEN FUNCTIONS OF ANGLES LESS THAN 90°:

$$\cos A = \sin (90° - A)$$
$$\cot A = \tan (90° - A)$$

Likewise,

$$\sin A = \cos (90° - A)$$
$$\tan A = \cot (90° - A)$$

From a table of functions of angles, the cosine of 35° is given as 0.819152. Looking up the sine of (90° − 35°) or the sine of 55°, we find that it is again 0.819152. You can check these relationships given above in a similar manner.

Complementary angles have their sum equal to 90°. Thus, in the above example, 35° and 55° are complementary angles since their sum is 90°.

RELATION BETWEEN FUNCTIONS OF ANGLES BETWEEN 90° AND 180°:

The definitions of the trigonometric functions given at the beginning of this article apply only to angles between 0° and 90°. More general definitions applying to angles of any size are given in texts on trigonometry. Since we will have to deal with functions of angles between 90° and 180°, a summary of these relationships only will be given here, and one is referred to any text on trigonometry for a complete statement of these definitions.

If A is an angle between 90°, and 180° then the following relationships hold between the functions of these angles:

$$\sin A = \sin (180° - A)$$
$$\cos A = -\cos (180° - A)$$
$$\tan A = -\tan (180° - A)$$

Thus, if the angle A is 123°, we may write:

$$\sin 123° = \sin (180° - 123°) = \sin 57°$$
$$\cos 123° = -\cos (180° - 123°) = -\cos 57°$$
$$\tan 123° = -\tan (180° - 123°) = -\tan 57°$$

From these relationships, the value of the functions of any angle between 90° and 180° can be obtained. These will be used later for the solution of oblique triangles.

RELATION BETWEEN ANGLES OF TRIANGLES

In a right triangle, the sum of the other two angles is 90°. Referring to Figure 25, the sum of A and B equals 90° and the sum of A, B, and C equals 180°.

In equation form:

For a right triangle:
$$A + B = 90° \text{ (where angle C is 90°)}$$

For any triangles:
$$A + B + C = 180°$$

In any triangle as Figure 26, the relation between the sides and the angles can be expressed as shown below:

Law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\ (\cos A)$$
$$\text{Or } b^2 = a^2 + c^2 - 2ac\ (\cos B)$$
$$\text{Or } c^2 = a^2 + b^2 - 2ab\ (\cos C)$$

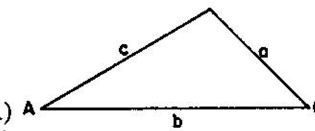


Fig. 26

25. The "S" (Sine) and "ST" (Sine-Tangent) Scales.

The "S" and "ST" scales are two sections of one long scale which, operating with "C", gives the sines of the angles between 0.57° and 90°. The "S" scale represents the scale of sines of angles from 5.74° to 90° (and for cosines of angles from 0° to 84.26°). The "ST" scale represents the scale of sines and tangents of angles from 0.57° to 5.74° and for cosines of angles from 84.26° to 89.43°. Since the value of the sine and tangent of angles below 5.74° is for all practical purposes identical, we can use the same scale for finding either the sine or the tangent for angles below 5.74° and above 0.57°. Thus, the reason for the "ST" scale.

The black numbers on "S" are used for sines and the red numbers for cosines.

The "S" and "ST" scales are so designed and arranged that when the indicator is set to a black number (angle) on the "S" or "ST" scales, the sine of the angle is given under the hairline on the "C" scale, or if the indices of the "C" and "D" scales coincide, the sine of the angle can be read on the "D" scale.

When using the "S" scale to read the value of sines of any angle, read the left index of "C" as 0.1 and the right index as 1.0. When using the "ST" scale to read the value of the sine of any angle, read the left index of "C" as 0.01 and the right index as 0.1. This is illustrated in Figure 27.

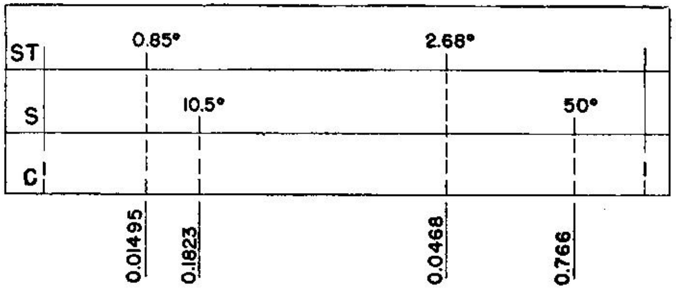


Fig. 27

The "S" scale between the left index (5.74°) and 10° has each degree numbered and the interval between each degree is first divided into ten parts representing 0.1° and each of these ten parts are then divided into two parts—each part representing 0.05°. From 10° to 20°, each degree is numbered and the interval between each degree is divided into ten parts representing 0.1°. Therefore, between the left index (5.74°) and 20° of the "S" scale, you can easily estimate the angles to the nearest 0.01°. From 20° to 30°, the degrees are not numbered except the 25°, but are indicated by a long mark. The interval between the degrees is divided into five parts—each part representing 0.2°. Therefore, between 20° and 30°, you can read to the nearest 0.1°. From 30° to 60°, each ten degrees are numbered and the primary interval between each ten degrees represents 1.0°. Each degree is again divided into two parts by a short mark representing 0.5°. Here you can still estimate to the nearest 0.1°. From 60° to 80°, each ten degrees is marked and numbered and the primary interval between each ten degrees represents 1.0°. With reasonable accuracy, you can estimate to the nearest 0.1°. From 80° to 90°, one can only estimate to the nearest degree.

Example 1. What is the sin 6.75°?
Bring indicator to 6.75 on "S"
Under the hairline read **0.1175** *on "C"*

Example 2. What is the sin 27.584°?
Round this off to either 27.6 or 27.58
The last digit may be estimated fairly well
Set indicator to 27.58 on "S"
Under hairline read **0.463** *on "C"*

Example 3. What is the sin 75.4°?
Set the indicator to 75.4 on "S"
Under the hairline read **0.967** *on "C"*

Example 4. What is the sin 3.45°?
Set indicator to 3.45 on "ST"
Under hairline read **0.0602** *on "C"*

Example 5. What is the sin 0.785
Set indicator to 0.785 on "ST"
Under hairline read **0.0137** *on "C"*

In the above examples the result may be read on the "D" scale if the indices of "C" and "D" coincide (if the rule is *closed*). This will permit the reading of the sine without turning the rule over.

EXERCISES

1. Obtain the sine of each of the following angles:

(a) 23.7°	(c) 13.578°	(e) 54.8°	(g) 75.8°	(i) 37.8°
(b) 30°	(d) 23.45°	(f) 87.0°	(h) 45.735°	(j) 20.59°

2. If the cos A = sin (90° – A), determine the cosine of the angles in exercise 1.

3. The sine of various angles are given below. Obtain the angle represented by each.

(a) 0.776	(d) 0.0652	(g) 0.01125
(b) 0.1235	(e) 0.443	(h) 0.678
(c) 0.985	(f) 0.500	(i) 0.563
		(j) 0.0866

Answers to the above exercises.

1.	a. 0.402	c. 0.2348	e. 0.817	g. 0.969	i. 0.613
	b. 0.500	d. 0.398	f. 0.999	h. 0.716	j. 0.352
2.	a. 0.916	c. 0.972	e. 0.576	g. 0.245	i. 0.790
	b. 0.866	d. 0.917	f. 0.0523	h. 0.698	j. 0.936
3.	a. 50.9°	c. 81°	e. 26.3°	g. 0.644°	i. 34.3°
	b. 7.09°	d. 3.74°	f. 30°	h. 42.7°	j. 4.97°

26. The "T" (Tangent) Scale.

The "T" scale is designed to give directly the tangents and cotangents of angles between 5.72° and 84.28°. When the indicator is set to any black number (angle) on the "T" scale, the tangent of that angle is given on the "C" scale. Also, if the indices of the "C" and "D" scales coincide, the tangent may be read on the "D" scale.

When the indicator is set to any black number (angle) on the "T" scale, the cotangent of that angle can be read under the hairline on the "CI" scale.

For angles between 0.57° and 5.72°, the tangent and the sine are for all practical purposes almost the same. We can therefore use the "ST" (Sine-Tangent) scale in conjunction with the "C" scale to obtain the tangents and cotangents of angles between 0.57° and 5.72°.

Thus, to determine the tangent of the angle 35.2°, set the indicator to 35.2 on "T" and under the hairline on "C", read *0.705* for the tangent. Under the hairline on "CI", read *1.418* as the cotangent.

You will note that the black numbers (angles) on the "T" scale go from 5.72° to 45° and the values of their tangents are read directly on the "C" scale (or "D" with rule closed). For angles greater than 45°, use the relation of cotangent A = tan (90°–A). Therefore, in order to obtain the tangent of 62.5°, determine the cotangent of (90° – 62.5°) or the cotangent of 27.5°. Set the indicator to 27.5 on "T" and under the hairline read *1.921* on "CI". This is the cotangent of 27.5° as well as the tangent of 62.5°. You will observe that the red numbers (angles) on the "T" scale read from right to left (45° to 84.29°) and you could have set the indicator to 62.5 in red on the "T" scale and thus avoided the necessity of subtracting 62.50 from 90°.

Exercises

1. Determine the tangent of each of the following angles:

(a) 34.5°	(c) 6.905°	(e) 67°	(g) 55.5°	(i) 25.9°
(b) 5.85°	(d) 45°	(f) 7.35°	(h) 37.45°	(j) 80°

2. For each of the angles given in Exercise 1, obtain the cotangent.

3. Determine the angle for which the following numbers are their tangents:

(a) 0.1168	(c) 0.652	(e) 1.567	(g) 0.528	(i) 2.1345
(b) 0.978	(d) 0.500	(f) 4.672	(h) 0.120	(j) 0.5438

4. For each of the numbers given in Exercise 3, obtain the angle for which they are the cotangents.

Answers to the above exercises.

1.	a. 0.687	c. 0.1211	e. 2.356	g. 1.455	i. 0.486
	b. 0.1025	d. 1.000	f. 0.1290	h. 0.766	j. 5.67
2.	a. 1.457	c. 8.26	e. 0.425	g. 0.687	i. 2.059
	b. 9.76	d. 1.000	f. 7.76	h. 1.307	j. 0.1763
3.	a. 6.66°	c. 33.1°	e. 57.44°	g. 27.87°	i. 64.87°
	b. 44.44°	d. 26.6°	f. 77.9°	h. 6.85°	j. 28.56°
4.	a. 83.34°	c. 56.9°	e. 32.56°	g. 62.13°	i. 25.13°
	b. 45.56°	d. 63.4°	f. 12.1°	h. 83.15°	j. 61.44°

27. The Red Numbers on the "S" and "T" Scales.

The *red numbers* on the "S" and "T" scales represent the complements of the angles as shown by the corresponding black numbers on these scales. The sum of complementary angles is 90°. Thus, if you set the indicator to the black number 25 (25°) on the "S" or "T" scales, you will also be able to read under the hairline the red number 65 (65°). The sum of these numbers is 90.

From this and the fact that *sin (90° – A) = cos A*, you can read the functions cosine (and cotangent) directly on the "C" scale by using the *red numbers.* Thus, to obtain the cosine of 65°, do the following:

Set indicator to the red 65 on "S"
Under hairline read **0.423** *on "C"*

Therefore, cos 65° = 0.423

Also, determine the cot. 65°

Set indicator to the red 65 on "T"
Under hairline read **0.466** *on "C"*

Cot. 65° = 0.466

The reciprocal function secant (equal to 1/cosA) can be obtained by using the red numbers on "S" and the "CI" scale, since the reciprocal of any number on "C" is given at the hairline on "CI".

ILLUSTRATION: Determine the sec 65°.

Set indicator to the red 65 on "S"
Under hairline read **2.362** *on "CI"*

Sec 65° = 2.362

The reciprocal scale, "CI", can be used to obtain the tangent of angles greater than 45° by using the red numbers on the "T" scale. Since the tangent is the reciprocal of the cotangent, it is always possible to convert from one to the other by using the "C" and "CI" scales.

ILLUSTRATION: Again determine the cot 65°.

Set indicator to the red 65 on "T"
Read the cotangent as 0.466 *on "C"*
Read the tangent as 2.145 *on "CI"*

28. Summary of Settings on "S", "T" and "ST" Scales.

As an aid in reviewing the individual settings for the various trigonometric functions, the following summary is given here:

FOR SINES:

0.57° to 5.74° —Read *black* numbers (angles) on "ST" scale to "C" scale (*black* numbers) giving a value between *0.01, and 0.1. Black to Black.*

5.74° to 90° —Read *black* numbers (angles) on "S" scale to "C" scale (*black* numbers) giving a value between *0.1* and *1.0. Black to Black*

FOR COSINES:

0° to 84.26° —Read *red* numbers (angles) on "S" scale to "C" scale (*black* numbers) giving a value between *0.1* and *1.0. Red to Black*

84.26° to 89.43°—Use the relationship cos A = sin (90° −A). Read (90° −A) on "ST" scale to "C" scale giving values between *0.01* and *0.1.*

FOR TANGENTS:

0.57° to 5.71° —Read *black* numbers (angles) on "ST" scale to "C" scale (*black* numbers) giving a value between *0.01* and *0.1. Black to Black.*

5.71° to 45° —Read *black* numbers (angles) on "T" scale to "C" scale (*black* numbers) giving a value between *0.1* and *1.0. Black to Black.*

45° to 84.29° —Read *red* numbers (angles) on "T" scale to "CI" scale (*red* numbers) giving a value between *1.0* and *10.0. Red to Red*

84.29° to 89.43°—Use the relationship tan A = cot (90°−A). Set (90° −A) on "ST" and read answer on "CI" scale (red numbers) giving a value between *10.0* and *100.0.*

FOR COTANGENTS:

0.57° to 5.71° —Read *black* numbers (angles) on "ST" scale to "CI" scale (*red* numbers) giving a value between *10.0* and *100.0. Black to Red.*

5.71° to 45° —Read *black* numbers (angles) on "T" scale to "CI" scale (*red* numbers) giving a value between *1.0* and *10.0. Black to Red.*

45° to 84.29° —Read *red* numbers (angles) on "T" scale to "C" scale (*black* numbers) giving a value between *0.1* and *1.0. Red to Black.*

84.29° to 89.43°—Use the relationship cot A = tan (90° −A). Read (90° −A) on "ST" scale to "C" scale giving a value between *0.01 and 0.1.*

FOR SECANTS:

0° to 84.26° —Read *red* numbers (angles) on "S" scale to "CI" scale (*red* numbers) giving a value between *1.0* and *10.0. Red to Red.*

84.26° to 89.43°—Use the relationship $\sec A = \frac{1}{\cos A}$ and cos A = sin (90° −A). Read (90° −A) on "ST" scale to "CI" scale giving a value between *10.0* and *100.0.*

FOR COSECANTS:

0.57° to 5.74° —Use the relationship $\csc A = \frac{1}{\sin A}$. Read *black* numbers (angles) on "ST" scale to "CI" scale (*red* numbers) giving a value between *10.0* and *100.0. Black to Red.*

5.74° to 90° —Read *black* numbers (angles) on "S" scale to "CI" scale (*red* numbers) giving a value between *10.0* and *1.0. Black to Red.*

For angles smaller than 0.57° or larger than are shown in the above summary, see article 37 in this chapter covering the functions of small angles.

You will notice that for sine, tangent, and secant (the direct functions), one always reads on like colors, *BLACK* to *BLACK* or *RED* to *RED* numbers (except when you use the relationship of complementary angles). Also, in the same manner for cosine, cotangent, and cosecant (the co-functions), one always reads on opposite colors, *BLACK* to *RED* or *RED* to *BLACK* numbers on the respective scales.

By using the reciprocal relations and the relations between complementary angles as $\cos A = \sin (90° - A)$, any of the six trigonometric functions of an angle can be replaced by a sine or tangent of an angle. Hence, by using these relations, the red scales may be avoided. It is recommended that the student always use the red numbers to avoid subtracting an angle from 90° where possible.

However, if one uses the trigonometric scales infrequently, it is advisable that one employ mainly the sine and tangent.

29. Combined Operations.

Since the "S", "T", and "ST" scales are placed on the "slide" part of the rule, these scales can be used quite conveniently with the other scales of the rule to solve combined multiplication and division, etc., involving trigonometric functions.

The examples given below illustrate the various types of problems that can be solved using the "S", "T", and "ST" scales.

Example 1. Evaluate 4.53 sin 12.5°.

This indicates the multiplication of 4.53 times the sine of 12.5°.

Set left end of "S" to 4.53 on "D"
Bring indicator to 12.5 on "S"
Under hairline read **0.982** *on "D"*

Example 2. Evaluate $\frac{23.5 \sin 34.7°}{\tan 15.3°}$.

To 23.5 on "D" set 15.3 on "T"
Bring indicator to 34.7 on "S"
Under hairline read **48.8** *on "D"*

Example 3. Evaluate $\frac{8.34\sqrt{34} \sin 63.0°}{4.23 \tan 42.4°}$.

To 34 on "A-RIGHT" set 4.23 on "C"
Bring indicator to 8.34 on "CF"
Move slide so 42.4 on "T" is at hairline
Bring indicator to 63.0 on "S"
Under hairline read **11.20** *on "DF"*

What you have done in the above operations for the solution of Example 3 is this: First, you have divided $\sqrt{34}$ by 4.23 and multiplied this by 8.34 (this result would be at the index on "DF"); second, you have divided by tan *42.4°*; and third, you have multiplied by sin *63.0°*. The answer must, of course, be read on "DF" since the last two operations are done with respect to this scale.

Example 4. Evaluate $\frac{67.3 \csc 25° \cos 56°}{\sqrt{5.78} \tan 34.6°}$

Bring 5.78 on "B-LEFT" to left index of "A"
Move indicator to 67.3 on "C"
Bring sin **25°** *(this equals 1/csc 25°) on "S" to hairline*
Move indicator to 56 red *on "S" (this is to* cos **56°** *on "S")*
Bring 34.6 on "T" to hairline
Read **53.7** *on "D" opposite right index*

When you have combinations of trigonometric functions involving the reciprocal functions (cosecant, cotangent, and secant), it may help in their solution to write them as 1/sine, 1/tangent, and 1/cosine. For the cosecant of 25° in Example 4, the csc 25° was used on the slide rule as 1/sin 25°.

Example 5. Evaluate $\frac{3.42 \times 2.67 \times \sqrt{38.9}}{\sin 80° \times \tan 28° \times 4.08}$

To 3.42 on "D" bring 28 on "T"
Move indicator to 38.9 on "B-RIGHT"
Bring 80 on "S" to hairline
Move indicator to 2.67 on "C"
Bring 4.08 on "C" to hairline
Read **26.7** *on "D" at the right index*

The preceding example may be solved in a number of ways using different scales. To illustrate, make the following settings on your rule:

To 38.9 on "A-RIGHT" set 4.08 on "C"
Move indicator to left index of "C"
Bring 2.67 on "CIF" to hairline
Move indicator to 3.42 on "CF"
Bring 28 on "T" to hairline
Move indicator to 90 on "S"
Bring 80 on "S" to hairline
Opposite 90 on "S" read **26.7** *on "D"*

This last method is not necessarily shorter. In all of these illustrations, try on your own part to do them in more than one way. This will give you more familiarity with your rule.

Exercises

Evaluate the following problems:

1. $\dfrac{2.45 \cos 36°}{\sin 61.5°}$
2. $\dfrac{3.17 \tan 60°}{\sin 27°}$
3. $\dfrac{45.2\sqrt{7.81}}{\tan 21.5°}$
4. $\dfrac{7.31\ (\pi)\sqrt{45.8}}{31.9 \cot 45°}$
5. $\dfrac{(0.0121) \sin 67°}{8.01 \tan 2.0°}$
6. $\dfrac{13.12 \sin 12.2°}{\csc 38.1°\sqrt{45.3}}$
7. $\dfrac{4.3 \sec 40.8°}{\sqrt{8.31}\ (\tan 5°)}$
8. $\dfrac{\sqrt[3]{95} \sin 45°}{\sqrt{30.3} \tan 19.75°}$
9. $\dfrac{1.015 \cos 31.8° \sin 31.8°}{\sqrt{4.93} \tan 40.9°}$
10. $\dfrac{8.5 \csc 21° \cot 42°}{\sqrt{95.8} \sin 31° \tan 30°}$
11. $\dfrac{(8.5 \times 10^{-5}) \sin 12.75°}{(3 \times 10^{-6}) \sin 16.5°\ (\tan 60°)}$
12. $\dfrac{(0.92)\ (\sqrt{45}) \cot 27°}{5 \tan 18.5°}$

Answers to the above exercises:

1. 2.26	5. 0.0399	9. 0.236
2. 12.09	6. 0.254	10. 9.05
3. 321.	7. 22.7	11. 12.68
4. 4.87	8. 1.634	12. 7.24

30. Solution of Right Triangles.

In many engineering and scientific calculations, it is necessary to determine certain parts of a right triangle having given sufficient information to define the triangle.

CASE I. Given one side "a" and the hypotenuse "c" of a right triangle, determine the side "b" and the angles A and B. (Side "a" is always opposite angle A, side "*b*" is always opposite angle B, and side "c" is always opposite the angle C, which in this manual is considered as the 90° angle of the right triangle).

Example 1. Find side "b" and angles A and B in a right triangle for which a = 3 and c = 5. See Figure 28.

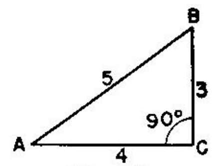


Fig. 28

Solution:

a/c = sin A = cos B.

To 5 on "D" set right index of slide.
Set hairline to 3 on "D".
Under hairline on black "S" scale read A = **36.9°**.
Under hairline on red "S" scale read B = **53.1°**.

b = c sin B

Keep right index of slide still set to 5 on "D".
Opposite 53.1° on black "S" scale read b = **4** *on "D".*

CASE II. Given the hypotenuse "c" and one acute angle B, determine "a", "b", and A.

Example 2. In a right triangle with c = 7.81 and B = 40°, find "a", "b", and A. See Figure 29.

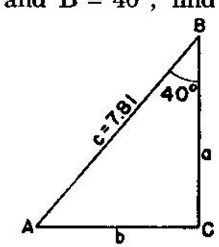


Fig. 29

Solution:

a = c cos B and b = c sin B

To 7.81 on "D" set right index of slide.
Opposite 40° on red "S" scale read a = **5.98** *on "D".*

Opposite 40° on black "S" scale read b = **5.02** *on "D".*
By mental calculation A = 90° − B = **50°**.

CASE III. Given one side "a" and one acute angle A, determine "b" and "c" and the angle B.

Example 3. In a right triangle with a = 17.21 and A = 32.4°, find "b", "c", and B. See Figure 30.

Solution:

By mental calculation
B = 90° − A = *57.6°.*

$$c = \frac{a}{\sin A} \text{ and } b = c \sin B$$

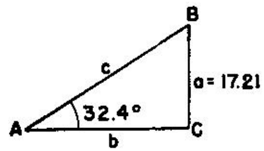


Fig. 30

To 17.21 on "D" set 32.4° on black "S" scale.
Opposite right index of slide read c = **32.1.**
Opposite 57.6° on black "S" scale read b = **27.15.**

CASE IV. Given the two sides "a" and "b", determine "c" and the acute angles A and B.

Example 4. Given a = 4 and b = 7, find "c" and A and B.

See Figure 31.

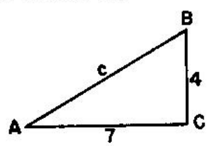


Fig. 31

Solution:

a/b = tan A

Set right index of slide to 7 on "D".
Opposite 4 on "D" read A = **29.8°** *on black "T".*
Scale and read B = **60.2°** *on red "T" scale.*

$$c = \frac{a}{\sin A}$$

Keep hairline set to 4 on "D" as above.
Bring 29.8° on black "S" scale under the hairline.
Opposite right index of slide read c = **8.05** *on "D".*

31. Solution of Right Triangles by the "Law of Sines."

The law of sines applies to all triangles and is given as

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

Or

$$\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$$

The law of sines makes it possible to solve right triangles by proportion on the slide rule. See Chapter 3 on PROPORTION.

Example 1. Given side "a" and angle A as 456 and 34° respectively, determine the hypotenuse and the other leg of the triangle.
See Figure 32.
Write this in the form

$$\frac{456}{\text{Sin } 34°} = \frac{c}{\text{Sin } 90°} = \frac{b}{\text{Sin (90-A)}}$$

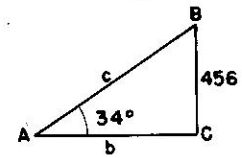


Fig. 32

To 456 on "D" set 34 on "S"
Opposite 90 on "S" read c = **816** *on "D"*
Opposite 56 (90 − 34) on "S" read b = **677** *on "D"*

Example 2. Given a right triangle in which B = 40.8° and c = 78.5 ft., find a, b and A. See Figure 33.

Solution:

A = (90° − B) = 49.2°
To 78.5 on "D" bring 90 on "S"
*Opposite 40.8 on "S" read b =***51.3** *on "D"*
*Opposite 49.2 on "S" read a =***59.6** *on "D"*

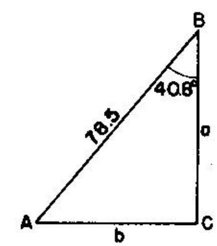


Fig. 33

Reviewing the above two examples—what you have actually done is to divide one number, which is a side of the triangle, by the sine of the angle opposite this side and then multiply this result by the sine of another angle. For instance, the law of sines could be written as follows:

$$c = \frac{a}{\sin A} \times \sin C$$

or any combination of these sides and angles in a similar manner. In words, this means the ratio of one side to the sine of the angle opposite, times the sine of the second, gives the side opposite the second angle. This holds for any triangle.

Sometimes only two of the sides are given and you are to find the other properties of the triangle.

Example 3. Given a = 34.5 and b = 47.2, find c, A, and B. See Figure 34.

Solution:

$$\text{Tan A} = \frac{34.5}{47.2}$$

To 47.2 on "D" set 34.5 on "C"
Move indicator to right index of "D"
Under hairline read A = **36.2°** *on "T"*
Under hairline read B = **53.8°** *on red "T"*
To 34.5 on "D" set 36.2 on "S"
Opposite C = 90° on "S" read c = **58.4** *on "D"*

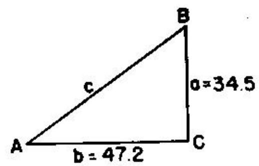


Fig. 34

In the first operation, you have actually obtained the reciprocal of the tangent which could be read opposite the left index of "C" on "D". This would have been the cotangent. However, the reciprocal of the cotangent is the tangent and this would be on "C" opposite the *right* index of "D". Thus, you can look directly under the hairline at this point to obtain the angle corresponding to this tangent, as was done in case IV Art. 30 to obtain the angle.

EXERCISES

In the following exercises for the solution of right triangles C = 90°. Determine the missing parts of the triangle. The law of sines and the proportion principle will be of value in solving these triangles.

1. a = 62.7, A = 30°.	3. b = 200, A = 68°	5. c = 423, A = 30°, B = 60°	7. b = 40.7, c = 59.4	9. a = 51.2, b = 24.8
2. b = 31.7, c = 49.8	4. c = 39.8, a = 12.3	6. a = 42.8, b = 12.3	8. a = 12.34, b = 11.97	10. A = 3.27°, c = 175.8

Answers to the above exercises.

1. B = 60°, b = 108.5, c = 125.4	3. B = 22°, a = 496, c = 534	5. a = 211.5, b = 367	7. A = 46.7°, B = 43.3°, a = 43.2	9. A = 64.1°, B = 25.9°, c = 56.8
2. A = 50.4°, B = 39.6°, a = 38.4	4. A = 18°, B = 72°, b = 37.9	6. A = 73.97°, B = 16.03°, c = 44.6	8. A = 45.85°, B = 44.15°, c = 17.18	10. B = 86.73°, a = 10.05, b = 175.6

32. The Law of Sines Applied to Oblique Triangles.

The same procedure as used for the solution of right triangles by the law of sines can be used for oblique triangles, since the law of sines is applicable to any triangle.

Example 1. Given the oblique triangle in Figure 35 in which c = 43.7 ft., a = 58.9 ft., and A = 35°. Find b, B, and C.

Solution:

$$\frac{c}{\text{Sin C}} = \frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}}$$

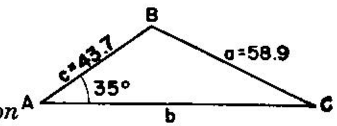


Fig. 35

To 58.9 on "D" set 35 on "S"
Opposite 43.7 on "D" read C = **25.2°** *on "S"*
Since A + B + C = 180°
B = 180° − (35° + 25.2°) = 119.8°
sin 119.8° = sin (180° − 119.8°) = sin **60.2°**.
With your slide rule set as above again
Move indicator to 60.2 on "S"
Read b = **89.1** *ft. on "D" under hairline*

Results: B = *119.8°*, C = *25.2°*, and b = 89.1 ft.

Example 2. Given the oblique triangle in Figure 36 in which b = 50.0 ft., a = 4 ft., and B = 68.5°, determine A, C, and c.

Solution:

To 50.0 on "D" set 68.5 on "S"
Move indicator to 4 on "D"
Under hairline read A = **4.27°** *on "ST"*
Move indicator to 72.77 on "S"
Read c = **51.3** *ft. on "D" under the hairline.*
To obtain C = **107.23** *we use the relation C = 180° − (A + B) but sin 107.23° = sin (180° − 107.23°) = sin 72.77°*
72.77° was used above on "S"

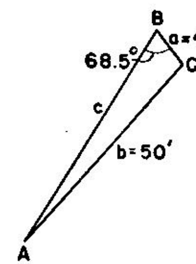


Fig. 36

Results: A = 4.27°, C = 107.23°, and c = 51.3 ft.

33. Law of Sines Applied to Oblique Triangles (Continued).

When the given parts of a triangle are two sides and an angle opposite one of them, and when the *side opposite* the given angle is *less* than the other given side, there may be *two triangles* which have the given parts. In both the cases solved in the previous article, the side opposite the given angle has been *greater* than the other given side.

Example 1. Given the oblique triangle in Figure 37 in which a = 43.7 ft., c = 58.9 ft., and A = 35°, find b, B, and C. The Figure 37 shows the two possible solutions.

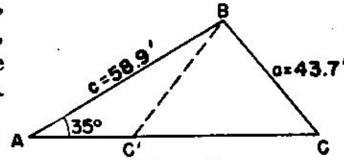


Fig. 37

Solution: (First)

To 43.7 on "D" set 35 on "S"
Opposite 58.9 on "D" read C = **50.5°** *on "S"*
Then B = 180° − (35° + 50.5°) = **94.5°**
With rule set as before
Move indicator to (180° − 94.5°) 85.5 on "S"
Under hairline read b = **76.0** *ft.*

Results of first solution: B = 94.5°, C = 50.5°, and b = 76.0 ft.

Solution: (Second)

The second solution comes in since the sin 50.5° is the same as the sin (180° − 50.5°).
Therefore, in the second solution C = 129.5°.
To 43.7 on "D" set 35 on "S"
Opposite 58.9 on "D" read C = (180° − 50.5°) = **129.5°**.
Now B = 180° − (35° + 129.5°) = **15.5°**
Opposite 15.5 on "S" read b = **20.35** *ft. on "D".*

Results of second solution: B = 15.5°, C = 129.5°, and b = 20.35 ft.

The dotted line in Figure 37 shows the position of the leg "a" for the second solution and for this second solution, the angle C is marked C′.

In this Example 1, it should be noticed that *both solutions* were made with the same setting of the slide rule. This is possible since from trigonometry, we know that *sin A = sin (180° − A)*.

34. Law of Sines Applied to an Oblique Triangle in Which Two Sides and the Included Angle Are Given.

To solve an oblique triangle when two sides and the included angle are given, it is convenient to think of the triangle made up of *two right triangles.* This is illustrated as follows:

Example 1. Given the oblique triangle in Figure 38 in which c = 45.6, b = 67.8, and A = 36.8°, solve the triangle.

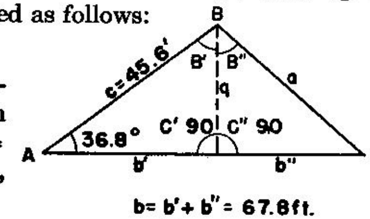


Fig. 38

Solution: The dotted line is drawn from B perpendicular to the base. This forms two right triangles. Call the perpendicular "q".

$$\frac{q}{\sin A} = \frac{45.6}{\sin 90°} = \frac{67.8}{\sin B'} \quad \text{(where } B' = 53.2°\text{)}$$

To 45.6 on "D" set 90 on "S"
Move indicator to 36.8 on "S"
Under hairline read q = **27.3** *on "D"*
Move indicator to 53.2 on "S"
Under hairline read b′ = **36.5** *on "D"*
From the right triangle B″, C, and **C″**
b″ = 67.8 − 36.5 = *31.3* and

$$\tan C = \frac{q}{b''} = \frac{27.3}{31.3}$$

Set to 31.3 on "D" 27.3 on "C"
Opposite right index of "D" read C = **41.1°** *on "T"*
Set to q = 27.3 on "D" 41.1 on "S"
Opposite b″ = 31.3 on "D" read B″ = **48.9** *on "S"*
Opposite 90 on "S" read c″ = **41.5** *on "D"*
B = B′ + B″ = 53.2° + 48.9° = **102.1°**

Results: c″ = a = *41.5*, **B** = *102.1°*, and **C** = *41.1°*

If the given angle is greater than 90°, the perpendicular will fall outside the given triangle, but the solution is essentially the same. See Figure 39.

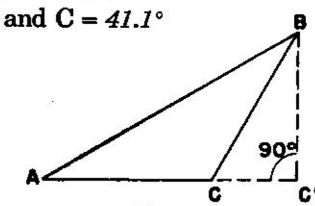


Fig. 39

Exercises

Solve the following oblique triangles. The "ST" scale must be used in Exercises 6, 7 and 10.

1. $c = 75$, $B = 39°$, $C = 105°$
2. $a = 12$, $b = 38$, $A = 7.8°$
   (Hint): Two solutions
3. $b = 7.81$, $c = 19.75$, $C = 97°$
4. $a = 0.7758$, $b = 0.721$, $A = 65°$
5. $b = 90.7$, $c = 82.1$, $B = 49.7°$
6. $a = 4.27$, $A = 3.75°$, $C = 100°$
7. $a = 8$, $b = 120$, $B = 60°$
8. $a = 120$, $b = 91$, $A = 58°$
9. $a = 12.02$, $b = 7.21$, $B = 32.7°$
   (Hint): Two solutions
10. $b = 3.21$, $B = 2.39°$, $C = 103.7°$

Answers to the above problems.

1. $a = 45.6$, $b = 48.9$, $A = 36°$
2. *First Solution* $c = 48.6$, $B = 25.5°$, $C = 146.7°$; *Second Solution* $c = 26.9$, $B = 154.5°$, $C = 17.7°$
3. $a = 17.24$, $A = 59.9°$, $B = 23.1°$
4. $c = 0.722$, $B = 57.4°$, $C = 57.6°$
5. $a = 118.6$, $A = 86.6°$, $C = 43.7°$
6. $b = 63.4$, $c = 64.3$, $B = 76.25°$
7. $c = 123.8$, $A = 3.31°$, $C = 116.69°$
8. $c = 140.2$, $B = 40.0°$, $C = 82.0°$
9. *First Solution* $c = 13.24$, $A = 64.2°$, $C = 83.1°$; *Second Solution* $c = 6.97$, $A = 115.8°$, $C = 31.5°$
10. $a = 73.8$, $c = 74.8$, $A = 73.91°$

35. Law of Cosines Applied to Oblique Triangles in Which Three Sides Are Given.

When the three sides of a triangle are given, we may find the value of one angle by the use of the law of cosines first, and then having one angle known, solve the other angles by means of the law of sines which is easier to use.

Example 1. Given a triangle in which the sides are $a = 34.5$, $b = 52.3$, and $c = 46.3$. Solve the triangle.

Solution:

The law of cosines is

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

From this we get

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}$$

$$\textbf{Cos A} = \sin(90° - A) = \frac{(52.3)^2 + (46.3)^2 - (34.5)^2}{2 \times 52.3 \times 46.3}$$

$$\sin(90° - A) = \frac{3690}{4850}$$

Set to 4850 on "D" 3690 on "C"
Opposite right index of "D" read **49.6** *on "S"*
This is (90° − A)
Therefore, A = **40.4°**
To 34.5 on "D" set 40.4 on "S"
Opposite c = 46.3 on "D" read C = **60.6** *on "S"*
Opposite b = 52.3 on "D" read B = **79** *on "S"*

Results: A = *40.4°*, B = *79.0°*, and C = *60.6°*.
Check: A + B + C = 180°. Thus, 40.4° + 79.0° + 60.6° = 180°.

Exercises

Solve the triangles in the following exercises:

1. a = 20, b = 36.3, c = 39.9	3. a = 0.499, b = 0.751, c = 0.704	5. a = 2.97, b = 61.0, c = 61.4	7. a = 2.19, b = 3.69, c = 3.85	9. a = 469, b = 925, c = 633
2. a = 3.84, b = 9.06, c = 8.54	4. a = 9.75, b = 6.49, c = 5.79	6. a = 38.2, b = 45.8, c = 72.8	8. a = 87.5, b = 46.4, c = 62.6	10. a = 151.0, b = 158.0, c = 123.8

Answers to the above exercises.

1. A = 30°, B = 65°, C = 85°	3. A = 40°, B = 75°, C = 65°	5. A = 2.75°, B = 80.0°, C = 97.25°	7. A = 33.7°, B = 69.2°, C = 77.1°	9. A = 27.8°, B = 113.2°, C = 39.0°
2. A = 25°, B = 85°, C = 70°	4. A = 105°, B = 40°, C = 35°	6. A = 27°, B = 33°, C = 120°	8. A = 106.7°, B = 30.3°, C = 43.0°	10. A = 63.5°, B = 69.3°, C = 47.2°

36. Conversions Between Degrees and Radians.

If an angle is made a central angle of a circle, the number of radians in the angle equals the ratio of the length of the intercepted arc to the length of the radius of the circle. Hence, since an angle of 180° intercepts an arc equal to a semi-circle,

$$180° = \frac{\pi r}{r} = \pi \text{ radians.}$$

Therefore, the following relation can be set up:

$$\frac{\pi}{180} = \frac{\text{R (number of radians)}}{\text{D (number of degrees)}}.$$

Based on this proportion we have the following GENERAL RULE for conversions between degrees and radians:

Set 180 on "CF" to π right on "DF".
Opposite a given number of degrees on "CF" (or "C") read the equivalent number of radians on "DF" (or "D".)
Opposite a given number of radians on "DF" (or "D") read the equivalent number of degrees on "CF" (or "C").
The decimal point is located from a mental estimate.

$$(1 \text{ radian} = \frac{180°}{\pi} = 57.3°).$$

Example 1. How many radians are equivalent to 125.5°?
Set 180 on "CF" to π right on "DF".
Opposite 125.5 on "CF" (or "C") read **2.19** *radians on "DF" (or "D").*

Example 2. How many degrees are equivalent to 5.46 radians?
Set 180 on "CF" to π right on "DF".
Opposite 5.46 on "D" read **313°** *on "C".*

If, in conversions between radians and degrees, an accuracy of ⅙ of 1% is sufficient, then the conversion can be made more simply. For small angles sin A = A (in radians) to a close approximation. Therefore, opposite an angle marked on the "ST" scale, we can read its radian measure A = sin A on the "C" scale. For A = 1° the error in the approximation is 1 part in 200,000. For A = 5.74°, the maximum angle on the "ST" scale, the error is 1 part in 600, or ⅙ of 1%. To this accuracy, then, angles marked in degrees on the "ST" scale have their radian values indicated on the "C" scale, or on the "D" scale if the rule is closed. *Decimal multiples of angles on the "ST" scale will have radian values which are decimal multiples of the values read on the "C" scale.* Hence the following GENERAL RULE:

To convert between degrees and radians, to an accuracy of ⅙ of 1% or better, read radians on "C" opposite degrees on "ST", or vice versa.

Example 3. How many radians are equivalent to 125.5°?
Set hairline to 125.5° (or 1.255°) on "ST".
Under hairline on "C" read **2.19** *radians.*

Example 4. How many degrees are equivalent to 5.46 radians?
Close rule and set hairline to 5.46 on "D".
Under hairline on "ST" read **313°.**

Exercises

1. Express the following angles in radians: 3.45°, 76.5°, 45.6°, 0.8°, 48.2°, 346°, 320°, 201°, 308°, and 57.3°.

2. Express the following angles in degrees: 0.089, 2.345, 6.28, 6.34, 5.24, 0.896, 1.0894, 2.34, and 4.72. All given values are in radians.

Answers to the above exercises.

1. 0.0603, 1.336, 0.797, 0.01396, 0.842, 6.04, 5.58, 3.51, 5.38, and 1.00.

2. 5.1°, 134.3°, 360°, 363°, 300°, 51.3°, 62.3°, 134°, and 270°.

37. Sines and Tangents of Small Angles.

The sines and tangents of angles smaller than those given on the "ST" scale can be found by the following approximation:

$$\sin A = \tan A = A \text{ (in radians).}$$

The error in the above approximations is less than 1 part in 10,000 for angles less than 1°.

Therefore, *to find the sine or tangent of an angle less than 1°, find the value of the angle in radians.* Methods for converting an angle from degrees to radians have been given in Section 36. To locate decimal points recall that $1° = \frac{\pi}{180} = 0.01745$ radians.

Example 1. Find sin 0.2°.
Set hairline to 0.2° (or 2°) on "ST".
Under hairline on "C" read sin 0.2° = **0.00349** *on "D".*

If the small angles whose sines or tangents are to be found are given in terms of minutes or seconds, their values in radians may be found by means of the "minute" and "second" marks on the "ST" scale.

$$1' = \frac{1°}{60} = 0.01667° = 0.0003 \text{ radians (approximately).}$$

$$1'' = \frac{1°}{3600} = 0.0002778° = 0.000005 \text{ radians (approximately).}$$

The "minute" mark is placed on the "ST" scale at 1.667° (or 0.01667°) and the "second" mark on the same scale at 2.778° (or 0.0002778°). Below the marks one can read on the "C" scale the values of 1′ and 1″ in radians— 0.000291 and 0.00000485 respectively. To obtain the radian value for any given angle expressed in minutes or seconds one needs merely to multiply the given number of minutes or seconds times the number of radians in one minute or one second by using the gauge marks. The decimal point is placed by making an approximate mental calculation.

Example 2. Express 16′ in radians.

Set left index of slide opposite 16 on "D".
Opposite "minute" mark on "ST" read
16′ = **0.00466** *radians on "D".*

Example 3. Find tan 23″.

Set right index of slide opposite 23 on "D".
Opposite "second" mark on "ST" read tan
23″ = **0.0001114** *on "D".*

Trigonometric functions for angles very near 90° can also be determined by finding the co-named function of the small complementary angles.

Example 4. Find tan 89.75°.

$$\text{Tan } 89.75° = \cot 0.25° = \frac{1}{\tan 0.25°}.$$

Set hairline to 0.25° (or 2.5°) on "ST".
Under hairline on "CI" read tan 89.75° = **229.**

Exercises

Find the values of the following:

1. sin 3′ 3. cot 0.05° 5. sec 18′ 7. sin 8.6′ 9. cot 0.2′
2. csc 27″ 4. tan 36″ 6. sin 9.8″ 8. tan 0.8′ 10. $\left[\frac{\tan 0.34°}{0.0001237}\right]$

Answers to the above exercises.

1. 0.000873 3. 1146 5. 1.000 7. 0.0025 **9. 17180**
2. 7640 4. 0.0001747 6. 0.0000475 8. 0.000233 **10. 48.0**

38. Trigonometric Applications.

Applications Involving Vectors: In engineering and scientific calculations, there are an infinite number of problems whose solution involves the application of vectors.

A vector is a segment of a straight line with an arrowhead on one end. A vector specifies the magnitude and direction of some quantity. In Figure 40 a vector "R" is shown, and a set of X- and Y-axes has been added. The projection of the vector on the X-axis is called the X-component of the vector. It is denoted by R_x and may be calculated from the formula

$$R_x = R \cos A,$$

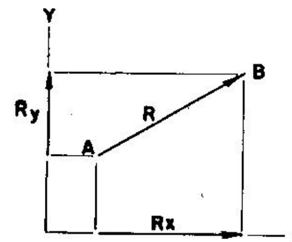


Fig. 40

where R is the magnitude of the vector and A is the angle which the line of the vector would make with the X-axis. Similarly, R_y is the Y-component of the vector, and it may be calculated from the formula

$$R_y = R \sin A.$$

Complex numbers X + jY have two components along perpendicular axes just as do vectors. The magnitude X is the horizontal component of the complex number, and the magnitude Y is the vertical component. The *j* indicates that the magnitude Y is to be laid off along the vertical axis. (Mathematically, $j = \sqrt{-1}$ is the unit imaginary number, and the x- and y-axes are called the real and imaginary axes respectively.)

Example 1. Find the magnitude R and the angle A of the complex number X + jY = 4.3 + j5.7. See Figure 41.

Solution:

$$\frac{X}{Y} = \cot A \text{ and } R = \frac{X}{\sin (90° - A)}$$

Set right index of slide to 5.7 on "D".
Bring hairline to 4.3 on "D".
Under hairline read A = **53.0°** *on red "T" scale (A > 45° because Y > X.)*

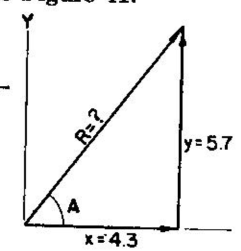


Fig. 41

Note the acute angle 90° − A = 37.0° on black "T" scale

under hairline.
Bring 37.0° on black "S" scale under the hairline.
Opposite right index of slide read R = **7.15** *on "D".*
Hence, $X + jY = R\underline{/A} = 7.15\underline{/53.0°}$.

The process of finding R and A when X and Y are given is called converting from component to polar form for the complex number, or vector. The process occurs so frequently in problems involving vectors that the following general method will be helpful.

GENERAL METHOD. *To find the magnitude and angle of a vector whose components are known:*

1. Set index of slide to the larger component (X or Y) on "D". Use whichever index will bring the smaller component (Y or X) on "D" opposite some point on the slide scales.
2. Set the hairline to the smaller component (Y or X) on "D".
3. Under hairline on the black "T" (or "ST") scale read the value of the acute angle of the triangle in Figure 41. Write it down.
4. Bring the acute angle on the black "S" scale under the hairline.
5. Opposite the index of the rule read the magnitude of the vector R on "D".
6. Take the angle A of the vector as the acute angle found above or as its complementary angle according to whether $Y < X$ or $Y > X$.

Example 2. An electric circuit has resistance R = 4.3 ohms and reactance X = 3.1 ohms in series. Find the magnitude Z and angle A of the impedance: $Z\underline{/A} = R + jX$. See Figure 41a.

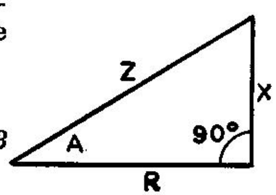


Fig. 41a

Solution:

Set right index of slide to **4.3** *on "D".*
Bring hairline to 3.1 on "D".
Under hairline read A = **35.8°** *on "T".*
Bring 35.8° on black "S" scale under hairline.
Opposite right index of slide read Z = **5.30** *on "D".*
Hence, $R + jX = Z\underline{/A} = \mathbf{5.30}\underline{/\mathbf{35.8}°}$ *ohms.*

APPLICATIONS TO RECTILINEAR FIGURES: The solution of many practical problems is made by working with rectilinear figures. A few typical problems are given below as examples of what can be solved by means of the slide rule.

Example 1. Determine the length of the side CD in the Figure 42.

Solution:

$$\text{Write } \frac{24.7}{\sin 80°} = \frac{BD}{\sin 60°} \text{ and } \frac{BD}{\sin 35°} = \frac{CD}{\sin 45°}$$

To 24.7 on "D" set 80 on "S"
Opposite 60 on "S" read BD = **21.7**
To 21.7 on "D" set 35 on "S"
Opposite 45 on "S" read CD = **26.8**

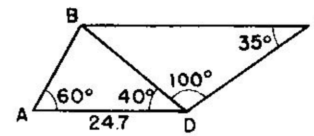


Fig. 42

Or *To 24.7 on "D" set 80 on "S"*
Bring indicator to 60 on "S"
To hairline bring 35 on "S"
Opposite 45 on "S" read CD = **26.8**

In the second method for the solution of Example 1, the intermediate value of BD was not read. This is the only difference in the two methods.

Example 2. A surveyor wants to determine the distance between two inaccessible points A and B and the direction of the line between them. He runs the line CD and finds it 375 ft. in length and bears South 15° East. The angles he measures are as indicated in the Figure 43a.

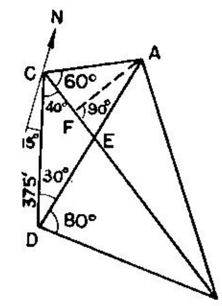


Fig. 43a

Solution: Using the relation between angles in a triangle, determine the various angles of the figure. Next, determine DE and then EB. Next, solve for CE and then EA. Angle AEB is equal to angle CED and thus you can determine two sides (AE and EB) and the included angle.

$\angle$ CED = 180° − (40° + 30°) = 110°
Sin 110° is the same as sin (180° − 110°) = sin 70°
$\angle$ AEB = $\angle$ CED = 110°
$\angle$ DEB = 180° − 110° = 70° which equals $\angle$ CEA
$\angle$ EBD = 180° − (70° + 80°) = 30°
$\angle$ CAE = 180° − (70° + 60°) = 50°

Write:

$$\frac{375}{\sin 110^\circ} = \frac{DE}{\sin 40^\circ} \text{ and } \frac{DE}{\sin 30^\circ} = \frac{EB}{\sin 70^\circ}$$

To 375 on "D" set 70 (180° − 110°) on "S"
Move indicator to 40 on "S"
To hairline bring 30 on "S"
Opposite 70 on "S" read EB = **483** *on "D"*

Likewise:
To 375 on "D" set 70 on "S"
Move indicator to 30 on "S"
To hairline bring 50 on "S"
Opposite 60 on "S" read AE = **225.5** *on "D"*

Drop a perpendicular to CB from A giving AF.

$\angle$ AEF = 70° and sin AEF = $\frac{AF}{225.5}$

From this AF = *212* ft.

$\cos 70^\circ = \frac{EF}{225.5}$ from which

EF = *77.2* ft.

BF = 77.2 + 483 = 560.2 ft.

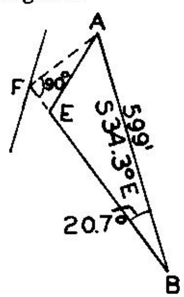


Fig. 43b

Now $\tan FBA = \frac{FA}{FB} = \frac{212}{560.2}$

$\angle$ FBA = *20.7°*

To 212 on "D" set 20.7 on "S"
Opposite 90 on "S" read AB = **599** *on "D"*

To determine the direction, add 15° + 40° and subtract 20.7°. This gives 34.3° off of North. Therefore, AB is South 34.3° East.

Results: AB = *599* ft., and AB is *S34.3°E*. See Figure 43b.

Example 3. The diameter of a circle is the base of a triangle having a 7.23 ft. leg. If the diameter of the circle is 14.34 ft., determine the angles of the triangle and the other side.

Solution: The triangle and the inscribing circle are shown in Figure 44.

The side AB is the diameter and angle C is 90°.

Use the law of sines to solve this triangle.

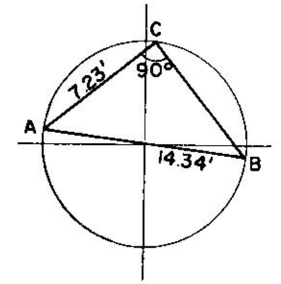


Fig. 44

To 14.34 on "D" set left index of "C"
This is the same as placing 90 on "S" opposite 14.34 on "D"
Opposite 7.23 read B = **30.3°**
Opposite (90° − 30.3°) = 59.7° on "S" read
a = **12.37** *on "D"*

For the last step 59.7 is off the rule with the setting given. To obtain the reading, you must bring the right index of "C" (90 on "S") to 14.34 on "D". Now opposite *59.7°* on "S", you can read a = **12.37** on "D".

Exercises

1. Determine the unknown angles and the unknown magnitudes of the vectors of (a) Fig. 45, (b) Fig. 46, (c) Fig. 47, and (d) Fig. 48.

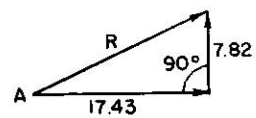


Fig. 45

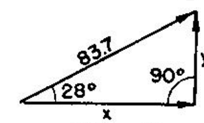


Fig. 46

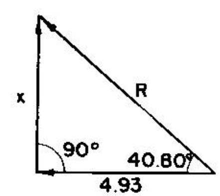


Fig. 47

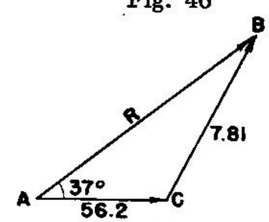


Fig. 48

2. The rectangular components of a vector are + 13.45 feet horizontally and + 7.45 feet vertically. Determine the magnitude of the vector and the angle it makes with the horizontal.

3. Find the horizontal and vertical components of a vector having a magnitude of 56.7 pound, and making an angle of 19.5° with the horizontal.

4. A 34.5 pound vector and an unknown vector "r" have as a resultant a 67.5 pound vector which makes a 32° angle with the 34.5 pound vector. Determine the unknown vector "r". See Figure 49.

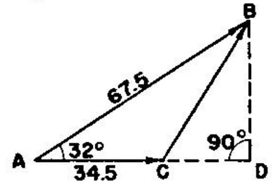


Fig. 49

5. Determine the magnitude and the angle of the vector (measured counter-clockwise from the positive X-axis) representing the complex numbers—3.57 + J 5.67.

6. Determine the length of the unknown side (marked with a letter) in the rectilinear figures shown in (a) Figure 50, (b) Figure 51, and (c) Figure 52.

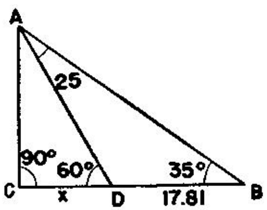


Fig. 50

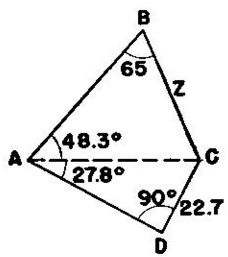


Fig. 51

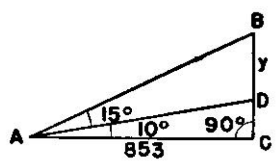


Fig. 52

7. AB is vertical in Figure 53 and represents a tower on a hill. The line CB was measured and found to be 1248′ in length. The angles were measured and are as given in the figure. Determine the height of the tower AB.

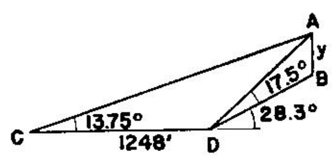


Fig. 53

Answers to the above exercises:

1. (a) A = 24.15° R = 19.14 (b) Y = 39.3 X = 73.8 (c) X = 4.27 R = 6.52 (d) B = 25.65 C = 117.35° R = 11.54

2. R = 15.39 A = 29° 3. Y = 18.86 lb. X = 53.4 lb. 4. R = 42.3 lb. 5. A = 122.2° R = 6.70

6. (a) X = 12.08 (b) Z = 40.1 (c) Y = 248

7. Y = 191 ft.

CHAPTER VII

EXPONENTS, LOGARITHMS, AND THE "L" SCALE

39. Exponents.

In Chapter 4, the squares of numbers were obtained by the use of the following scales: "C", "D", "A", and "B". The notation used was

$$4^2 = 4 \times 4 \text{ or } 16$$

This small number to the upper right of the 4 is called the exponent. If 4 is to be cubed, it is written as 4^3; and in this case, the exponent is 3. Another term used for "*exponent*" is the "*power*" of the number.

Four raised to the second power is 4^2, or four raised to any power "a" is 4^a. The "4" in this case is called by definition the "base". Thus, any number can be a so called "base".

A short table using 10 as a base follows:

$$10^1 = 10 = 10$$
$$10^2 = 10 \times 10 = 100$$
$$10^3 = 10 \times 10 \times 10 = 1{,}000$$
$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$$

From this we see that $100 \times 1{,}000 = 100{,}000$

Since 100 is 10^2 and 1000 is 10^3, we may write

$$10^2 \times 10^3 = 10^{2+3} = 10^5$$

In this manner, we are using the *addition* of the exponents to obtain our results. Thus, in the *multiplication* of exponential terms TO THE SAME BASE, *add* the exponents for the result.

$$100{,}000 \div 100 = 1{,}000 \text{ or } \frac{10^5}{10^2} = 10^{5-2} = 10^3$$

From this and the above table, it is seen that in order to *divide* exponential terms TO THE SAME BASE, it is only necessary to *subtract* their exponents.

ILLUSTRATION: What is the value $\frac{2^7}{2^4}$?

$$\frac{2^7}{2^4} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 2^3$$

$$\text{or } \frac{2^7}{2^4} = 2^{7-4} = 2^3 \text{ (the same as before)}$$

In this illustration, the number "2" is used as the base.

40. Negative Exponents.

If 10^3 is divided by 10^5, the result would be 10^{3-5} or 10^{-2}. This indicates that the result is $\frac{1}{100}$. Thus, using 10 as a base and for any negative exponent, the result can be indicated by $1 \div 10^{+a}$, where "a" was the negative exponent.

ILLUSTRATION: What is $10^2 \div 10^7$?

$10^{2-7} = 10^{-5}$ or this may be written as

$$10^{2-7} = 10^{-5} = \frac{1}{10^5}$$

41. Notation Using the Base "10".

It is often convenient to change a number by either multiplying or dividing it by 10 to some exponent.

ILLUSTRATION: Change the number 30,000,000 to a more convenient form.

Divide this by 10^6 and write the number as 30×10^6

Or divide by 10^7 and write the number as 3×10^7

Change the number 0.000065 to a more convenient form.

Multiply this number by 10^5 and write the number as 6.5×10^{-5}

In the first illustration, the exponent of 10 is positive; and this indicates that the actual number of digits to the right of the number is the same as the exponent of 10. In the second illustration, the actual number of places to the left of the decimal as indicated by 10^{-5} is 5.

In each case, the number of places through which the decimal point moves is equal to the exponent of ten.

ILLUSTRATION: Evaluate $3450 \times 732 \times 0.032$

First, this can be changed to

$$3.45 \times 10^3 \times 7.32 \times 10^2 \times 3.2 \times 10^{-2}$$

Again, write it as

$$3.45 \times 7.32 \times 3.2 \times 10^{3+2-2}$$

To 7.32 on "D" set 3.2 on "CI"
Bring the indicator to 3.45 on "C" and
Read the answer as **80.8** *on "D" under* ***hairline.***
Correct answer is then 80.8 × 10³ or **80,800**

42. Logarithms.

Logarithms are exponents. A base is selected, and the logarithm of a given number to this base is simply the *exponent* of the base that will yield the given number. Tables of common logarithms refer to the base 10.

Since log 25 = 1.398, therefore $10^{1.398} = 25$.
Since log 4 = 0.602, therefore $10^{0.602} = 4$.

When multiplying or dividing numbers, one adds or subtracts their logarithms to find the logarithm of the result. This is true because the logarithms are simply exponents and obey the laws of exponents.

Thus, $4 \times 25 = 10^{0.602} \times 10^{1.398} = 10^{0.602 + 1.398} = 10^2 = 100$.
Or $\log (4 \times 25) = \log 4 + \log 25 = 0.602 + 1.398 = 2$.

Since $1 = 10^0$ and $10 = 10^1$, therefore numbers between 1 and 10 will have logarithms between 0 and 1. Numbers less than 1 will have logarithms less than 0, i.e. negative logarithms. Numbers greater than 10 will have logarithms greater than 1. In general, the logarithm of a given number consists of a whole number, the *characteristic*, plus a decimal portion, the *mantissa*. These two portions are the two exponents obtained when the given number is expressed as the product of a number between 1 and 10 multiplied times an integral power of 10. Thus,

$2{,}500 = 2.5 \times 10^3 = 10^{0.398} \times 10^3 = 10^{3 + 0.398}$, or
$\log 2{,}500 = 3 + 0.398 = 3.398$.

The characteristic is the integral power of 10 and the mantissa is the decimal portion. Similarly,

$0.025 = 2.5 \times 10^{-2} = 10^{0.398} \times 10^{-2} = 10^{-2 + 0.398}$, or
$\log 0.025 = -2 + 0.398 = 8.398 - 10$.

Since the integral power of 10 is always equal to the number of places through which the decimal point must be shifted to change the given number to a number between 1 and 10, therefore the characteristic of the logarithm may be read off by counting this number of places the decimal point shifts.

43. The "L" (Logarithmic) Scale.

The "L" scale is a simple scale of equal parts marked with values ranging from 0 to 1 over a scale length of 25 centimeters. It is the top scale pictured in Figure 2a of this MANUAL. Consequently, values read on the "L" scale are the mantissas of the opposed numbers on the "D" scale.

ILLUSTRATION: What is the logarithm of 456?

$456 = 4.56 \times 10^2$. Characteristic is 2.
Set hairline to 4.56 on "D".
Under hairline on "L" read 0.659, the mantissa.
Therefore, log 456 = **2.659.**

ILLUSTRATION: What is the logarithm of 0.0752?

$0.0752 = 7.52 \times 10^{-2}$. Characteristic is -2.
Set hairline to 7.52 on "D".
Under hairline on "L" read 0.8761, the mantissa.
Therefore, log 0.0752 = $-2 + 0.8761$ = **8.8761 − 10.**

44. Calculations by Logarithms.

The logarithm of a number to the base 10 is defined as the exponent of 10 that will give the number. Thus,

$$10^2 = 100$$

Therefore, the logarithm of 100 is 2 because 10 raised to the second power gives 100.

Likewise, Log 34.5 = 1.5378. This means

$$10^{1.5378} = 34.5$$

Since the logarithms as given on the "L" scale are all to the base ten, one can multiply and divide by obtaining the logarithms of the numbers and then either adding or subtracting the logarithms depending upon whether you want to multiply or divide. The addition and subtraction of the logarithms is the same as the addition or subtraction of exponents as explained in article 39—the base being 10 in this case.

ILLUSTRATION: Evaluate $\dfrac{34.5 \times 9716}{3.24}$

Obtain the log 34.5 = 1.5378
Obtain the log 9716 = 3.9875
Their sum is 5.5253
Obtain the log 3.24 = 0.5106
Their difference is 5.0147
Set the indicator to 0.0147 on "L" scale
Under hairline read **1.034**
Characteristic is 5; therefore, the answer is
1.034×10^5 = 103,400.

This indicates a method of calculating problems as above, but as this can be done easier with the "C", "D", etc., scales, the "L" scale is used primarily when numbers with exponents are to be either multiplied or divided.

ILLUSTRATION: Evaluate $\dfrac{(3.24)^{2.5}\,(45.6)}{(34.5)^{1.35}}$

Analyzing this computation: The log $(3.24)^{2.5}$ is equal to $2.5 \times$ log 3.24 and the log $(34.5)^{1.35}$ is $1.35 \times$ log 34.5. Therefore, obtain the log of these numbers and multiply them by their respective exponents.

SOLUTION: Log 3.24 as read on "L" scale is 0.511
Log 34.5 as read on "L" scale is 1.538
Log 45.6 as read on "L" scale is 1.659

Set 0.511 on "CI" to 2.5 on "D"
Read **1.278** *on "D" opposite 1 on "C"*

Set 1 on "C" to 1.35 on "D"
Opposite 1.538 on "C" read **2.075** *on "D"*

$2.5 \times$ log 3.24 = 2.5×0.511 = 1.278
Log 45.6 = 1.659
Their sum is 2.937
Subtract $1.35 \times$ log 34.5 2.075
This difference is 0.862

Set hairline to 0.862 on "L" scale
Under hairline read **7.27** *on "D".*

The "L" scale, can be used in the same manner as a table of logarithms. This was done in the above illustration.

Exercises

1. By use of the "L" scale, determine the logarithms of the following numbers: 3.45, 34.5, 34500, 52.9, 0.00845, 0.95638, 4.56, 34.92, 5.6638, 0.056638, 78.48×10^{-2}.

2. Evaluate the following problems:

(a) $4^{2.13}$
(b) $3.45 \times (8.4)^{0.8}$
(c) $(23.5)^{2.1} \times \sqrt{3.78}$
(d) $(7.32)^{1/2}\,(34.7)^{0.35}$
(e) $\dfrac{34.5 \times \sqrt{8.1}}{(3.75)^{0.9}}$
(f) $10^{2.50}$
(g) $\dfrac{10^{0.75} \times 10^{4.5}}{10^{5.25}}$
(h) $10^{3.2} \times 10^{-4.2} \times 10^{6.25}$
(i) $(2.34 \times 10^{-5})\,(54.7 \times 10^{3})$
(j) $3^{0.45} \times 3^{-1.07} \times 3^{0.82}$

Answers to the above exercises

1. 0.538, 1.538, 4.538, 1.724, 7.927-10, 9.981-10, 0.659, 1.543, 0.753, 8.753-10, 9.895-10.
2. (a) 19.2 (b) 18.92 (c) 1480 (d) 9.37 (e) 29.8 (f) 316 (g) $10^0 = 1$ (h) $10^{5.25}$ = 178,000 (i) 1.28 (j) $3^{0.2}$ = 1.246

CHAPTER VIII

THE LOG LOG SCALES

45. The "LL" Scales.

The most frequent use of the Log Log (LL) scales is to find the powers and roots of numbers. Engineering and scientific calculations frequently involve non-integral powers and roots of quantities, and they often involve powers of e and logarithms of numbers to the base e, where $e = 2.71828\ldots$ is the base of *natural* logarithms. With the "LL" scales the computation of $(1.23)^{1.84}$ becomes as simple as multiplying 1.23×1.84, and the evaluation of $\log_e 102.5$ becomes as easy as finding $\frac{1}{102.5}$.

Example 1. Find $(1.23)^{1.84}$. (See Figure 54 below).

Set left index of slide opposite 1.23 on "LL2" scale.
Opposite 1.84 on "C" read $(1.23)^{1.84}$ = **1.464** *on "LL2".*

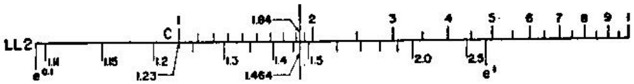


Fig. 54

Example 2. Find $\log_e 102.5$.

Set hairline to 102.5 on "LL3" scale.
Under hairline on "D" scale read $\log_e 102.5$ = **4.63**.

The "LL" scales are designed to solve problems of the following types:

1. $Y = X^n$. Given X and n, find Y.
2. $n = \log_x Y$. Given X and Y, find n.

The above types of problems can be solved with one setting of the slide, or merely by a setting of the hairline if $X = e = 2.71828\ldots$

GENERAL RULES:

1. *To find* $Y = X^n$, *set index of slide opposite X on the appropriate "LL" scale. Opposite n on the "C" (or "B") scale read Y on the appropriate "LL" scale.*

2. *To find* $n = \log_x Y$, *set index of slide opposite X on the appropriate "LL" scale. Opposite Y on the appropriate "LL" scale read n on the "C" (or "B") scale.*

Example 3. Find the amount A of $100 invested for ten years at 5%, compounded semi-annually.

$$A = \$100\left(1 + \frac{0.05}{2}\right)^{20}$$

Set right index of "C" scale to 1.025 on "LL1".
Opposite 20 on "C" read A = **163.86** *on "LL2".*

Example 4. A plate of glass transmits 0.88 of the light incident on it. Find the number of plates n necessary to cut the transmitted light down to 0.50 or less.

$0.50 \geq 0.88^n$

Set left index of "B" scale to 0.88 on "LLOO".
Opposite 0.50 on "LLOO" read 5.42 on "B".
Hence **6** *plates of glass will be used.*

The five "LL" scales may be considered in two groups. First, the "LL1", "LL2", and "LL3" scales cover numbers greater than 1.00 (from 1.010 to 22,026) and are read against the "C" and "D" scales, or the folded and reciprocal (CF, DF and CIF) scales. Second, the "LLOO" and "LLO" scales cover numbers less than 1.00 (from 0.00005 to 0.999) and are read against the "A" and "B" scales. The detailed techniques for using these groups of scales are explained in separate sections below.

46. The "LL1", "LL2", and "LL3" Scales—For Numbers Greater than Unity.

In Figure 55 the "LL1", "LL2", and "LL3" scales are shown as sections of one long scale representing numbers from 1.010 to 22,026. They are aligned with three sections of the "D" scale placed end to end.

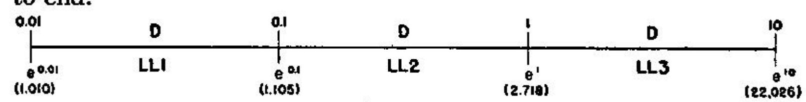


Fig. 55

As shown in Figure 55 the number $e = 2.71828\ldots$ lies at the junction of the "LL2" and "LL3" scales opposite an index of the "D" scale. Because of this alignment of the scales *a number on an "LL" scale is equal to e raised to the power opposite that number on* *the "D" scale.* The range of numbers and of powers of *e* covered by each of the three scales under consideration is indicated in the table below.

Scale	Range of Numbers	Range of Powers of e
LL1	1.010 to 1.105	0.01 to 0.10
LL2	1.105 to 2.718	0.10 to 1.0
LL3	2.718 to 22,026	1.0 to 10

On the actual slide rule the three sections of the "LL" scale have been slid over one another so that they are aligned with the single "D" scale on the body of the rule. Hence, an exponent read on "D" must correspond, in location of the decimal point, to the "LL" scale on which the number is read.

Example 1. Find $e^{0.5}$ and e^5.

Set hairline to 5 on "D"
Under hairline on "LL2" read $e^{0.5}$ = **1.649.**
Under hairline on "LL3" read e^5 = **148.**

A. *Finding powers of numbers greater than unity.*

In Example 1 note that $e^{0.5}$ is less than *e* while e^5 is greater than *e*. These results illustrate a GENERAL RULE which is very helpful in finding powers of numbers. *When any given number greater than unity is raised to a power, it will yield a result which is greater or less than the given number according to whether the exponent is greater or less than 1.00.*

Let us now develop methods for finding powers of any number greater than unity. The construction of the "LL" scales is such that if an index of the "C" scale is placed opposite a given number on the "LL1", "LL2", or "LL3" scale, then a power of the given number may be found on the appropriate "LL" scale opposite the indicated exponent on the "C" scale. Look back at Figure 54, which illustrates the setting of the rule for evaluation of $(1.23)^{1.84} = 1.464$. Since the exponent was greater than 1.00, we worked toward the right along the "LL2" scale and found a result (1.464) which was greater than the given number. If the exponent were less than 1.00, the result would be less than 1.23.

Example 2. Evaluate $(1.23)^{0.8}$

Set right index of "C" opposite 1.23 on "LL2".
Opposite 0.8 on "C" read $(1.23)^{0.8}$ = **1.18** *on "LL2".*

In finding a power of a given number:

(a) if the exponent is greater than 1.00, the result will lie to the right of the given number along the chain of "LL" scales in Figure 55, and

(b) if the exponent is less than 1.00, the result will lie to the left of the given number.

For any two numbers separated by one scale length along the chain of "LL" scales, the number to the right is the 10th power of the number to the left, or the lefthand number is the 0.1 power of the righthand number. On the slide rule such numbers lie opposite each other on different "LL" scale.

Sometimes in finding powers of numbers the result lies off the "LL" scale on which the given number is located. In such a case the "LL" scales are treated as one long scale (Figure 55), and the slide is set to read the result on the proper scale. For instance, let us find $(1.5)^5$. (See Figure 55a below.) If we set the left index of "C" to 1.5 on "LL2" in the usual manner, the value of $(1.5)^5$ would be read (opposite 5 on "C") on the fictitious dotted "LL3" scale extending rightward from the "LL2" scale. Since, on the actual rule the dotted "LL3" scale has been slid left one scale length, we can find the value of $(1.5)^5$ by sliding the "C" scale back one scale length in Figure 55a and reading $(1.5)^5$ on "LL3" opposite 5 on "C".

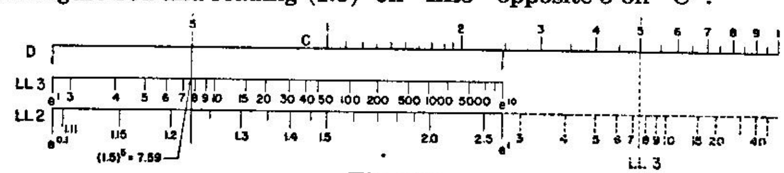


Fig. 55a

Set right index of "C" to 1.5 on "LL2".
Opposite 5 on "C" read $(1.5)^5$ = *7.59 on "LL3".*

Example 3. Evaluate $(7)^{0.12}$ and $(7)^{1.2}$.

Set left index of "C" opposite 7 on "LL3".
Opposite 0.12 on "C" read $(7)^{0.12}$ = **1.263** *on "LL2".*
Opposite 1.2 on "C" read $(7)^{1.2}$ = **10.3** *on "LL3".*

Example 4. Find the amount A of $100 invested for 30years at 4%, compounded semi-annually.

A = $100 $(1 + 0.02)^{60}$

Set right index of "C" opposite 1.02 on "LL1".
Opposite 60 on "C" read A = **$440** *on "LL3".*

If the exponent is given in fractional form, the settings of the slide rule are similar to those used in multiplying a number by a fraction.

Example 5. Evaluate $e^{\frac{1}{2}}$.

Set 2 on "C" to e on "LL2".
Opposite left index of slide read $e^{\frac{1}{2}}$ = **1.649** *on "LL2".*
(The result checks with $e^{0.5} = 1.649$ from Example 1.)

Example 6. Evaluate $(27)^{\frac{2}{3}}$.

Set 3 on "C" opposite 27 on "LL3".
Opposite 2 on "C" read $(27)^{\frac{2}{3}}$ = **9** *on "LL3".*

Example 7. Evaluate $\sqrt[7]{(1.2)^2} = (1.2)^{\frac{2}{7}}$.

Set 7 on "CF" opposite 1.2 on "LL2".
Opposite 2 on "CF" read $(1.2)^{\frac{2}{7}}$ = **1.0535** *on "LL1".*

B. Finding logarithms of numbers greater than unity.

The "LL1", "LL2", and "LL3" scales are well adapted to finding logarithms of numbers to any base, but especially to the base *e*. The logarithm of a number to a given base is simply the exponent to which one must raise the base to yield the number. Thus, if $Y = e^n$, then $n = \log_e Y = \ln Y$. Logarithms to the base *e* are called *natural* logarithms and will be written ln Y to distinguish them from common logarithms (to the base 10), which will be written log Y. If other bases are used, they will be indicated. Thus,

(1) if $Y = e^n$, then $n = \ln Y$,
(2) if $Y = 10^n$, then $n = \log Y$, and
(3) if $Y = X^n$, then $n = \log_x Y$.

Because of the alignment of the "LL" and "D" scales *a value read on "D" is the natural logarithm of the opposed number on the corresponding "LL" scale.*

Example 8. Find ln 21.3.

Set hairline to 21.3 on "LL3".
Under hairline on "D" read ln 21.3 = **3.06.**

The "L" scale is used to determine the mantissas of common logarithms. The characteristics may be found by reading off the exponent of 10 after the given number has been expressed as a product of a number between 1 and 10 multiplied by a power of 10.

Example 9. Find log 230.

$230 = 2.3 \times 10^2$. Characteristic is 2.
Set hairline to 2.3 on "D".
Under hairline on "L" read 0.362.
log 230 = **2.362.**

Example 10. Find log 0.00872.

$0.00872 = 8.72 \times 10^{-3}$. Characteristic is **−3.**
Set hairline to 8.72 on "D".
Under hairline on "L" read 0.940.
log 0.00872 = −3 + 0.940 = **7.940 − 10.**

TO FIND LOGARITHMS TO ANY BASE other than *e* or 10, slide rule is set as in determining the power of a given number.

$Y = X^n$. Given Y and X, find $n = \log_x Y$.
Set index of slide to X on an "LL" scale.
Opposite Y on an "LL" scale read n on "C" and place its decimal point properly.

Example 11. Find $\log_{1.5} 2$.

Set left index of slide opposite 1.5 on "LL2".
Opposite 2 on "LL2" read $\log_{1.5} 2$ = **1.71** *on "C".*

Example 12. How long must a sum of money be invested in order to double itself, if interest is 3%, compounded semi-annually?

$(1.015)^n = 2$.
Set left index of slide to 1.015 on "LL1".
Opposite 2 on "LL2" read n = **46.5** *years on "C".*

Example 13. Find $\log_{20} 1.2$.

$1.2 = 20^n$.
Set right index of "C" opposite 20 on "LL3".
Opposite 1.2 on "LL2" read n = **0.061** *on "C".*

Similar settings may be used to find an unknown base.

$X^n = Y$. Given Y and n, find X.
Set n on "C" to Y on an "LL" scale.
Opposite index on "C" read X on the proper "LL" scale.

Exercises

Evaluate the following expressions. Use the "LL3", "LL2", and "LL1" scales as may be required. Determine "X" in Exercise 9, 10, 11, and 12.

1. $e^{6.3}$
2. $(6.31)^{2.15}$
3. $e^{0.014}$
4. $(3.16)^{0.75}$
5. $(3.16 \times \pi)^{2.7}$
6. $\left(\frac{9.20 \times 3.8}{18.6}\right)^{0.712}$
7. $(1.319)^{\frac{2.75}{3.21}}$
8. $e^{\frac{1.82}{0.5}}$
9. $10.7^{x} = 92.5$
10. $(X)^{2.81} = 1.218$
11. $(1.123)^{x} = 3.27$
12. $(X)^{2.3} = 85.9$
13. $\sqrt[2.7]{81}$
14. ln 67
15. log 0.171
16. ln 1.014
17. $\log_2 9$
18. log 367
19. $\log_3 243$
20. $\log_{\pi} 1.331$

Answers to the above exercises.

1. 545
2. 52.5
3. 1.0141
4. 2.37
5. 490
6. 1.565
7. 1.268
8. 38.2
9. 1.91
10. 1.0726
11. 10.20
12. 6.92
13. 5.09
14. 4.20
15. 9.233 – 10
16. 0.0139
17. 3.17
18. 2.564
19. 5
20. 0.25

47. The "LLO" and "LLOO" Scales—For Numbers Less than Unity.

In Figure 56 the "LLO" and "LLOO" scales are shown as sections of one long scale representing numbers from 0.00005 to 0.999. They are aligned with four scale lengths of the "A" scale placed end to end.

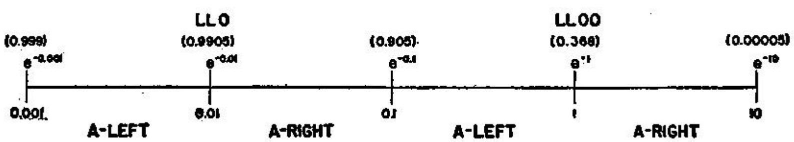


Fig. 56

As shown in Figure 56 the number $e^{-1} = \frac{1}{e} = 0.36788\ldots$ lies at the midpoint of the "LLOO" scale opposite the center index of the

"A" scale. Because of this alignment of the scales *a number on an "LL" scale is equal to e^{-1} raised to the power indicated opposite that number on the "A" scale.* The range of numbers and of powers of e covered by each of the two scales under consideration is indicated in the table below:

Scale	Range of Numbers	Range of Powers of e^{-1}
LLO	0.999 to 0.905	0.001 to 0.10
LLOO	0.905 to 0.00005	0.10 to 10

On the actual slide rule the two sections of the "LL" scale have been slid over each other so that they are aligned with the "A" scales on the body of the rule. Hence, an exponent read on "A" must correspond, in location of the decimal point, to the "LL" scale on which the number is read.

Example 1. Find $e^{-0.5}$, e^{-5}, and $e^{-0.005}$.

Set hairline to 5 on A-LEFT.
Under hairline on "LLOO" read $e^{-0.5}$ = **0.607.**
Under hairline on "LLO" read $e^{-0.005}$ = **0.9950.**
Set hairline to 5 on A-RIGHT.
Under hairline on "LLOO" read e^{-5} = **0.0067.**

A. Finding powers of numbers less than unity.

In finding powers of numbers less than unity remember the following GENERAL RULE. *When any given number less than unity is raised to a power, it yields a result which is less than or greater than the given number according to whether the exponent is greater or less than 1.00.* The "LLO" and "LLOO" scales are laid out with numbers decreasing from left to right. Hence, in finding a power of a given number:

(a) if the exponent is greater than 1.00, the result will lie to the right of the given number along the chain of "LL" scales in Figure 56, and

(b) if the exponent is less than 1.00, the result will lie to the left of the given number.

For any two numbers separated by one length of the "A" scale (the distance between adjacent indices), the number to the right is the tenth power of the number to the left on the "LL" scale; or the lefthand number is the one-tenth power of the righthand number. Numbers on the "LLOO" scale are the hundredth power of the numbers opposite them on the "LLO" scale.

Calculations using the "LL" scale for numbers less than unity are made by settings quite analogous to those described above for the "LL" scales for numbers greater than unity.

Example 2. Evaluate $(0.8)^3$.

Set left index of "B" opposite 0.8 on "LLOO".
Opposite 3 on B-LEFT read $(0.8)^3$ = **0.512** *on "LLOO".*

Example 3. Evaluate $(0.45)^{0.057}$.

Set left index of "B" opposite 0.45 on "LLOO".
Opposite 0.057 on B-RIGHT read $(0.45)^{0.057}$ = **0.9555** *on "LLO".*

Example 4. What amount P invested at the present time at 3% interest, compounded semi-annually, will amount to \$1 in 25 years?

$$P = (1.015)^{-25} = \left(\frac{1}{1.015}\right)^{25}.$$

Set hairline to 1.015 on "C".
Under hairline on "CI" read $\frac{1}{1.015} = 0.985$.
Set right index of "B" to 0.985 on "LLO".
Opposite 25 on B-RIGHT read P = **\$0.686** *on "LLOO".*

Example 5. The current in an electric circuit decreases by a factor of $\frac{1}{e}$ every 2.5 seconds. If the current starts at 10 amperes, what will be its value I after 6 seconds?

$$I = 10\left(\frac{1}{e}\right)^{\frac{6}{2.5}} = 10(e^{-\frac{6}{2.5}}).$$

Set 2.5 on "B" opposite e^{-1} *on "LLOO" (or opposite center index of "A").*
Opposite 6 on "B" read I = **0.91** *amps on "LLOO".*

B. Finding logarithms of numbers less than unity.

Since decile powers of e^{-1} on the "LLO" and "LLOO" scales are placed opposite indices of the "A" scale, therefore the natural logarithms of given numbers on the "LLO" and "LLOO" scales are read on the "A" scale opposite the given numbers. The value of the logarithm will be negative and the decimal point is readily located in the logarithm by referring to the powers of *e* marked on the "LL" scales opposite indices of the "A" scale.

Example 6. Find ln 0.983 and ln 0.18.

Set hairline through 0.983 on "LLO" and through 0.18 on "LLOO".
Under hairline on "A" read ln 0.983 = **−0.0172** *and ln 0.18 =* **−1.72.**

For numbers less than unity common logarithms (to the base 10) are found by use of the "L" scale as explained in Section 46-B.

TO FIND LOGARITHMS TO ANY BASE other than *e* or 10, the slide rule is set as in determining the power of a given number, as explained in Section 46-B. In settings for which the results lie off the end of the "LL" scale on which you start, keep in mind the idea of the chain of "LL" scales pictured in Figure 56.

Example 7. Find $\log_{0.6} 0.25$.

Set left index of "B" to 0.6 on "LLOO".
Opposite 0.25 on "LLOO" read $\log_{0.6} 0.25$ = **2.71** *on B-LEFT.*

Example 8. Find $X = \log_2 0.95$.

$0.95 = 2^x = (\frac{1}{2})^{-x}$.
Set left index of "B" opposite 0.50 $(=\frac{1}{2})$ *on "LLOO".*
Opposite 0.95 on "LLO" read X = **−0.074** *on B-LEFT.*

Example 9. Find $\log_{0.98} 0.0032$.

Set left index of "B" opposite 0.98 on "LLO".
Opposite 0.0032 on "LLOO" read $\log_{0.98} 0.0032$ = **284** *on B-LEFT.*

Exercises

Evaluate the following exercises. If there is an unknown letter value given, determine this unknown. (See Exercise 10).

1. $e^{-3.6}$	6. $\sqrt[7]{0.0108}$	11. $e^{-x} = 0.564$	16. ln 0.1
2. $(0.895)^{4.56}$	7. $(0.018)^{0.6}$	12. $e^{-x} = 0.97$	17. $\log_{0.5} 0.01$
3. $e^{-0.012}$	8. $(0.018)^{0.06}$	13. $(X)^{0.67} = 0.954$	18. $\log_{\pi} 0.92$
4. $(0.563)^{0.97}$	9. $(0.018)^{0.006}$	14. $(X)^{1.50} = 0.67$	19. log 0.0212
5. $\sqrt[5]{0.735}$	10. $e^{-5.67} = X$	15. $(X)^{3000} = 0.002$	20. ln 0.995

Answers to the above exercises:

1. 0.0273	8. 0.786	15. 0.99793
2. 0.603	9. 0.9762	16. − 2.30
3. 0.9881	10. X = 0.0035	17. 6.63
4. 0.573	11. X = − 0.573	18. − 0.0728
5. 0.9402	12. X = − 0.0305	19. 8.326 − 10
6. 0.523	13. X = 0.932	20. − 0.00502
7. 0.09	14. X = 0.766	

48. Readings Beyond the Limits of the "LL" Scales.

If, in calculations involving powers and logarithms, one has to deal with numbers greater than 22,026 (maximum number on "LL3") or less than 0.00005 (minimum number on "LLOO") then one of the following methods may be resorted to:

Method 1. By factoring (splitting) the base, as $28^5 = (4 \times 7)^5 = 4^5 \times 7^5$. When solved in the usual way, $4^5 = 1024$ and $7^5 = 16{,}807$. Multiplying these results together, we obtain 17,210,368.

Method 2. By breaking (splitting) the exponent, as $28^5 = 28^2 \times 28^3$. When solved in the usual way, $28^2 = 784$ and $28^3 = 21{,}952$. Multiplying these results together, we obtain 28^5 equals 17,210,368.

Method 3. By means of common logarithms, using the L Scale, log 28 equals 1.44716, multiplied by 5 = 7.23580. The number whose log is 7.23580, we find to be 17,210,368.

Example 1. Evaluate $(128)^4$.
$(128)^4 = (1.28)^4 \times (100)^4$.
Set left index of "C" to 1.28 on "LL2".
Opposite 4 on "C" read 2.68 on "LL2".
Hence, $(128)^4 =$ **2.68×10^8**.

Example 2. Evaluate $e^{23.2}$ by splitting the exponent
$e^{23.2} = e^{10+10+3.2} = e^{10} \times e^{10} \times e^{3.2}$.
$e^{10} = 22{,}026$.
Set hairline to 3.2 on "D".
Under hairline on "LL3" read $e^{3.2} = 24.5$.
Set right index of "C" to 22, 026 on "D".
Opposite 24.5 on B-RIGHT read $e^{23.2} =$ **11.85×10^9**.

Example 3. Evaluate $(0.0129)^{5.2}$ by factoring the base.
$(0.0129)^{5.2} = (1.29)^{5.2} \times (10^{-2})^{5.2} =$
$(1.29)^{5.2} \times (10^{-2})^5 \times (10^{-2})^{0.2}$.
Set right index of "C" opposite 1.29 on "LL2".
Opposite 5.2 on "C" read $(1.29)^{5.2} = 3.76$ *on "LL3".*
Set right index of "B" opposite 0.01 (= 10^{-2}*) on "LLOO".*

Opposite 0.2 on B-RIGHT read $10^{-0.4} = 0.398$ *on "LLOO".*
Set left index of "C" to 3.76 on "D".
Opposite 0.398 on "CF" read 1.494 on "DF".
Hence, $(0.0129)^{5.2} =$ **1.494×10^{-10}**.

Example 4. Evaluate $(0.0129)^{5.2}$ by use of logarithms.
Set hairline to 1.29 on "D".
Under hairline on "L" read log (1.29) = 0.1104.
Hence, $\log(0.0129) = -2 + 0.1104$.
Set left index of "C" to 0.1104 on "D".
Opposite 5.2 on "C" read 0.574 on "D".
Hence, $\log(0.0129)^{5.2} = -10.4 + 0.574.$
$= -10 + 0.174.$
Set hairline to 0.174 on "L".
Under hairline on "D" read 1.494.
Therefore, $(0.0129)^{5.2} =$ **1.494×10^{-10}**.

If powers of numbers very near 1.00 are needed, then the "LL" scales may also prove inadequate, since the lowest value on "LL1" is 1.01 and the highest value on "LLO" is 0.999. For such calculations one may make use of the binominal expansion.

$$(1 \pm X)^n = 1 \pm nX + \frac{n(n-1)}{2}X^2 \pm \dots$$

If nX is less than 0.05, then the first two terms in the series give the correct value with an error of about 1 part in 1,000, or less.

$$(1 \pm X)^n = 1 \pm nX \text{ (approximately)}$$

If nX is larger than 0.05, then three terms in the series may be used, or another method of calculation may be tried.

Example 5. Evaluate $(1.0032)^{4.32}$.
$(1 + 0.0032)^{4.32} = 1 + 4.32\ (0.0032)$
Set left index of "C" to 0.0032 on "D".
Opposite 4.32 on "CF" read 0.0138 on "DF".
Hence $(1.0032)^{4.32} =$ **1.0138**.

Example 6. Evaluate $(0.9995)^{0.47}$.
$(1 - 0.0005)^{0.47} = 1 - 0.47\ (0.0005)$
$= 1 - 0.000235 =$ **0.999765**.

Exercises

Evaluate the following (using all methods applicable).

1. $(24)^6$
2. $(128)^{4.21} = (128)\ (128)^{1.21}\ (128)^2$
3. $\left[\dfrac{21 \times 8.2}{3.21}\right]^4$
4. $\dfrac{(52 \times 8.134)^3}{\sqrt{42}}$
5. $\left[\dfrac{\sqrt[3]{8.18}\ (51.2)}{\sqrt{6.92}}\right]^4$
6. $(1.0039)^{5.21}$
7. $(1.0085)^{0.398}$
8. $(1.000069)^{9.2}$
9. $\left[(127)\ \dfrac{(0.000124)}{(0.01564\ )}\right]^{0.05}$
10. $[1.00072]^{9.35/6.12}$

Answers to above exercises.

1. 1.91×10^8
2. 7.46×10^8
3. 8.28×10^6
4. 11.64×10^6
5. 2.35×10^6
6. 1.0203
7. 1.00338
8. 1.000635
9. 1.00025
10. 1.0011

49. Theory Underlying Construction of the "LL" Scales.

In the preceding four sections the use of the "LL" scales has been explained in detail, but little has been said regarding their construction. Let us see how the "LL" scales have been laid out to have such useful properties.

In Figure 55 it can be seen that the numbers located at the ends of the "LL" scales are decile powers of *e* and that the opposed numbers on the "D" scales are simply the exponents of *e*. Since these exponents of *e* are, by definition, the natural logarithms of the numbers marked on the "LL" scales, it follows that the numbers marked on the "D" scale are, assuming proper location of their decimal points, the natural logarithms of the opposed numbers marked on the "LL" scale. The relationship is true throughout the length of the scales because of the following facts:

1. The distance from the left index to a given number n marked on the "D" scale is equal to the mantissa of the log n multiplied by a scale factor of 25 centimeters. For example, the mark for 3.2 is located at a distance (log 3.2) $\times$ 25 cm. = 0.505 $\times$ 25 cm. = 12.6 cm. = 4.97 inches from the left index. Measure it!

2. The distance from the left index to a given number X on an "LL" scale is equal to the mantissa of log (1n X) multiplied by a scale factor of 25 centimeters. (Since the natural logarithm of X is itself simply a number, we can take its common logarithm just as we could the common logarithm of any other number.) For example, the mark for 24.5 on "LL3" is located at a distance [log (1n 24.5)] $\times$ 25 cm. = (log 3.2) $\times$ 25 cm. = 0.505 $\times$ 25 cm. = 12.6 cm. = 4.97 inches from the left index. Measure it! (It is because distances on "LL" scales are proportional to log (1n X) that the scales are called Log Log scales.)

3. If a number n on the "D" scale and a number X on an "LL" scale are the same distance from the left index, it follows that
   log (1n X) $\times$ 25 cm. = log n $\times$ 25 cm.
   log (1n X) = log n,
   1n X = n, and
   $X = e^n$.

   Since n = 3.2 on "D" is the same distance from the left index as is Y = 24.5 on "LL3", it follows that 1n 24.5 = 3.2 or that $24.5 = e^{3.2}$. Correct location of the decimal point in n is assumed.

Now let us see how this layout of the "LL" scales permits the easy determination of powers of numbers as described in the preceding sections. Look back at Figure 54. The distance from the left index of "LL2" to the mark for 1.23 is equal to log (1n 1.23) $\times$ 25 cm. The left index of the "C" scale was set to 1.23 on "LL2" and a distance equal to (log 1.84) $\times$ 25 cm. was added to the distance from the left index. If we call Y the number on "LL2" opposite 1.84 on "C", then the equation for its distance from the left index of "LL2" is:

log (1n Y) $\times$ 25 cm. = log (1n 1.23) $\times$ 25 cm. + log 1.84 $\times$ 25 cm.

Dividing by 25 cm. and applying the law for addition of logarithms to the right side, we have:

log (1n Y) = log [1.84 (1n 1.23)], or
1n Y = 1.84 (1n 1.23) = 1n $(1.23)^{1.84}$.

Hence, $Y = 1.23^{1.84}$. On the "LL2" scale we read Y = 1.464.

In general, if $Y = X^n$ then
$\ln Y = n \ln X$, and
$\log(\ln Y) = \log(\ln X) + \log n$.

Add the *n*-distance on "C" to the X-distance on an "LL" scale to obtain the Y-distance on an "LL" scale.

The "LLO" and "LLOO" scales differ only in that the distance from the left index to a number X is proportional to the common logarithm of the power of e^{-1} which would yield X. And the scale factor is 12.5 cm. so that these "LL" scales are read against the "A" and "B" scales, which have the same scale factor. For instance, $0.135 = e^{-2} = (e^{-1})^2$. Hence the distance from the center index of "A" to the mark for 0.135 on "LLOO" is equal to (log 2) × 12.5 cm. = 0.301 × 12.5 cm. = 3.76 cm. = 1.48 inches. Hence, if $Y = X^n$, then

$$\log_{e^{-1}} Y = n \log_{e^{-1}} X, \text{ and}$$
$$\log\left(\log_{e^{-1}} Y\right) = \log\left(\log_{e^{-1}} X\right) + \log n.$$

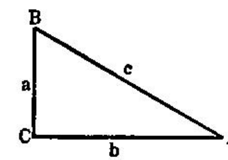
Multiply each term by 12.5 cm. The terms are then converted into distances on the "LLO–OO" and "B" scales. Therefore, if $Y = X^n$, add the *n*-distance on "B" to the X-distance on the "LLO–OO" scale to yield the Y-distance on the "LLO–OO" scale.

CHAPTER IX

MATHEMATICAL FORMULAE

51. Plane Trigonometry.

Right Triangle



$\sin A = \frac{a}{c}$ $\qquad \cos A = \frac{b}{c}$

$\tan A = \frac{a}{b}$ $\qquad \cot A = \frac{b}{a}$

$\sec A = \frac{c}{b}$ $\qquad \text{cosec}\, A = \frac{c}{a}$

$$\sin A = \cos\left(\frac{\pi}{2} - A\right) = -\cos\left(\frac{\pi}{2} + A\right)$$

$$\cos A = \sin\left(\frac{\pi}{2} - A\right) = \sin\left(\frac{\pi}{2} + A\right)$$

$$\tan A = \cot\left(\frac{\pi}{2} - A\right) = -\cot\left(\frac{\pi}{2} + A\right)$$

$$\cot A = \tan\left(\frac{\pi}{2} - A\right) = -\tan\left(\frac{\pi}{2} + A\right)$$

$$\sec A = \text{cosec}\left(\frac{\pi}{2} - A\right) = \text{cosec}\left(\frac{\pi}{2} + A\right)$$

$$\text{cosec}\, A = \sec\left(\frac{\pi}{2} - A\right) = -\sec\left(\frac{\pi}{2} + A\right)$$

$\sin(-A) = -\sin A$ $\qquad \cos(-A) = \cos A$

$\tan(-A) = -\tan A$ $\qquad \cot(-A) = -\cot A$

$\sec(-A) = \sec A$ $\qquad \text{cosec}(-A) = -\text{cosec}\, A$

NUMERICAL VALUES

Angle	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

MATHEMATICAL FORMULAE

Plane Geometrical Figures

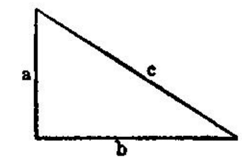
Right Triangle

$c = \sqrt{a^2 + b^2}$

$a = \sqrt{c^2 - b^2}$

$b = \sqrt{c^2 - a^2}$

area $= \frac{1}{2}\, ab$

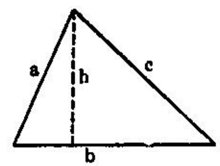


Any Triangle

area $= \frac{1}{2}\, bh$

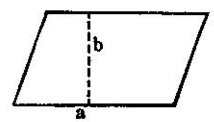
area $= \sqrt{s(s-a)\ (s-b)\ (s-c)}$

$s = \frac{1}{2}\ (a+b+c)$



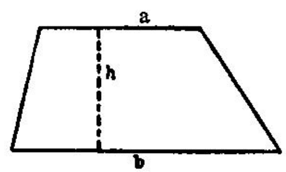
Parallelogram

area $= ab$



Trapezoid

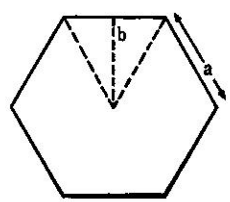
area $= \frac{1}{2}h\ (a+b)$



Regular Polygon

area $= \frac{1}{2}\, abn$

n = number of sides



Parabola

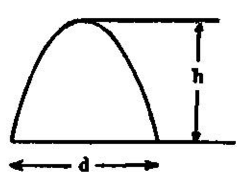
length of arc $= \dfrac{d^2}{8\,h}\Big[\sqrt{c\ (1+c)} +$

$2.0326 \log_{10}(\sqrt{c} + \sqrt{1+c})\Big]$

in which

$c = \left(\dfrac{4\,h}{d}\right)^2$

area $= \frac{2}{3}\, dh$



MATHEMATICAL FORMULAE

Plane Geometrical Figures.

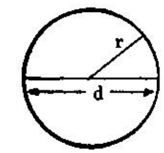
Circle

circumference $= 2\,\pi r$

$= \pi d$

area $= \pi r^2$

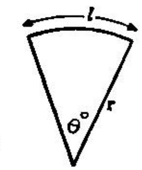
$= \pi \dfrac{d^2}{4}$



Sector of Circle

arc $= l = \pi r \dfrac{\Theta^\circ}{180^\circ}$

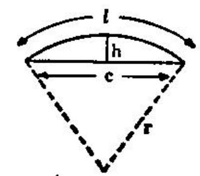
area $= \frac{1}{2}\, rl = \pi r^2 \dfrac{\Theta^\circ}{360^\circ}$



Segment of Circle

chord $= c = 2\sqrt{2\,hr - h^2}$

area $= \frac{1}{2}\, rl - \frac{1}{2}\, c\ (r-h)$



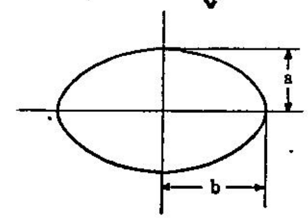
Ellipse

circumference =

$$\pi\ (a+b)\frac{64 - 3\left(\dfrac{b-a}{b+a}\right)^4}{64 - 16\left(\dfrac{b-a}{b+a}\right)^2}$$

(close approximation)

area $= \pi\, ab$

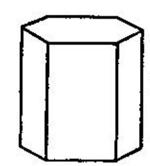


Solid Geometrical Figures.

Right Prism

lateral surface = perimeter of base $\times\ h$

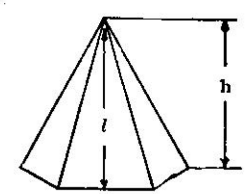
volume = area of base $\times h$



Pyramid

lateral area $= \frac{1}{2}$ perimeter of base $\times l$

volume = area of base $\times \dfrac{h}{3}$



MATHEMATICAL FORMULAE

Solid Geometrical Figures.

Frustum of Pyramid

lateral surface = $\frac{1}{2} l \, (P + p)$

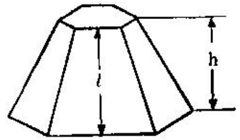
P = perimeter of lower base

p = perimeter of upper base

volume = $\frac{1}{3}h \, [A + a + \sqrt{Aa}]$

A = area of lower base

a = area of upper base

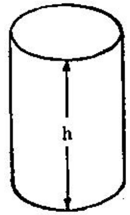


Right Circular Cylinder

lateral surface = $2 \, \pi r h$

r = radius of base

volume = $\pi r^2 h$

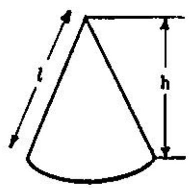


Right Circular Cone

lateral surface = $\pi r l$

r = radius of base

volume = $\frac{1}{3} \, \pi r^2 h$



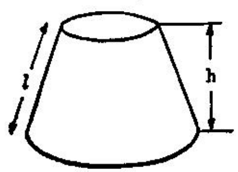
Frustum of Right Circular Cone

lateral surface = $\pi l \, (R + r)$

R = radius of lower base

r = radius of upper base

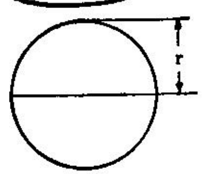
volume = $\frac{1}{3} \, \pi h \, [R^2 + Rr + r^2]$



Sphere

surface = $4 \, \pi r^2$

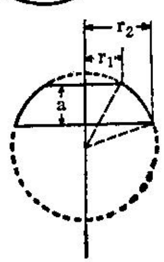
volume = $\dfrac{4}{3} \, \pi r^3$



Segment of Sphere

volume of segment

$$= \frac{1}{6} a \, \pi \, [3 \, (r_1{}^2 + r_2{}^2) + a^2]$$

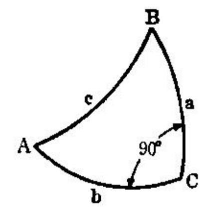


MATHEMATICAL FORMULAE

Spherical Trigonometry.

Right Spherical Triangles

$\cos c = \cos a \cos b$	$\cos A = \tan b \cot c$
$\sin a = \sin c \sin A$	$\cos B = \tan a \cot c$
$\sin b = \sin c \sin B$	$\sin b = \tan a \cot A$
$\cos A = \cos a \sin B$	$\sin a = \tan b \cot B$
$\cos B = \cos b \sin A$	$\cos c = \cot A \cot B$



Oblique Spherical Triangles

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos A = \sin B \sin C \cos a - \cos B \cos C$$

$$\cot a \sin b = \cot A \sin C + \cos C \cos b$$

$$s = \tfrac{1}{2} \, (a + b + c)$$

$$S = \tfrac{1}{2} \, (A + B + C)$$

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin b \sin c}}$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{\sin s \sin (s - a)}{\sin b \sin c}}$$

$$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}}$$

$$\sin\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S \cos (S - A)}{\sin B \sin C}}$$

$$\cos\left(\frac{a}{2}\right) = \sqrt{\frac{\cos (S - B) \cos (S - C)}{\sin B \sin C}}$$

$$\tan\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S \cos (S - A)}{\cos (S - B) \cos (S - C)}}$$

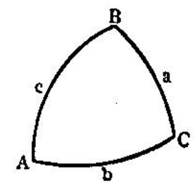
$$\tan \tfrac{1}{2} \, (a - b) = \frac{\sin \frac{1}{2} \, (A - B)}{\sin \frac{1}{2} \, (A + B)} \tan \tfrac{1}{2} c$$

$$\tan \tfrac{1}{2} \, (a + b) = \frac{\cos \frac{1}{2} \, (A - B)}{\cos \frac{1}{2} \, (A + B)} \tan \tfrac{1}{2} c$$

$$\tan \tfrac{1}{2} \, (A - B) = \frac{\sin \frac{1}{2} \, (a - b)}{\sin \frac{1}{2} \, (a + b)} \cot \tfrac{1}{2} C$$

$$\tan \tfrac{1}{2} \, (A + B) = \frac{\cos \frac{1}{2} \, (a - b)}{\cos \frac{1}{2} \, (a + b)} \cot \tfrac{1}{2} C$$

$$\tan \tfrac{1}{2} c = \frac{\sin \frac{1}{2} \, (A + B) \tan \frac{1}{2} \, (a - b)}{\sin \frac{1}{2} \, (A - B)}$$



Multiply	by	to obtain
Abamperes	10	amperes.
“	3×10^{10}	statamperes.
abamperes per sq. cm.	64.52	amperes per sq. inch.
abampere-turns	10	ampere-turns.
“ “	12.57	gilberts.
abampere-turns per cm.	25.40	ampere-turns per inch.
abcoulombs	10	coulombs.
“	3×10^{10}	statcoulombs.
abcoulombs per sq. cm.	64.52	coulombs per sq. inch.
abfarads	10^{9}	farads.
“	10^{15}	microfarads.
“	9×10^{20}	statfarads.
abhenries	10^{-9}	henries.
“	10^{-6}	millihenries.
“	$1/9\times10^{-20}$	stathenries.
abmhos per cm. cube	$10^{5}/\delta$	mhos per meter-gram.
“ “ “ “	1.662×10^{2}	mhos per mil foot.
“ “ “ “	10^{3}	megmhos per cm. cube.
abohms	10^{-15}	megohms.
“	10^{-3}	microhms.
“	10^{-9}	ohms.
“	$1/9\times10^{-20}$	statohms.
abohms per cm. cube	10^{-3}	microhms per cm. cube.
“ “ “ “	6.015×10^{-3}	ohms per mil foot.
“ “ “ “	$10^{-5}\delta$	ohms per meter-gram.
abvolts	$1/3\times10^{-10}$	statvolts.
“	10^{-8}	volts.
acres	43,560	square feet.
“	4047	square meters.
“	1.562×10^{-3}	square miles.
“	5645.38	square varas.
“	4840	square yards.
acre-feet	43,560	cubic-feet.
“ “	3.259×10^{5}	gallons.
amperes	1/10	abamperes.
“	3×10^{9}	statamperes.
amperes per sq. cm.	6.452	amperes per sq. inch.
amperes per sq. inch	0.01550	abamperes per sq. cm.
“ “ “ “	0.1550	amperes per sq. cm.
“ “ “ “	4.650×10^{8}	statamperes per sq. cm.
ampere-turns	1/10	abampere-turns.
“ “	1.257	gilberts.
ampere-turns per cm.	2.540	ampere-turns per inch.
ampere-turns per inch	0.03937	abampere-turns per cm.
“ “ “ “	0.3937	ampere-turns per cm.
“ “ “ “	0.4950	gilberts per cm.
ares	0.02471	acres.
“	100	square meters.
atmospheres	76.0	cms. of mercury.
“	29.92	inches of mercury.
“	33.90	feet of water.
“	10,333	kgs. per square meter.
“	14.70	pounds per sq. inch.
“	1.058	tons per sq. foot
Bars	9.870×10^{-7}	atmospheres.
“	1	dynes per sq. cm.
“	0.01020	kgs. per square meter.
“	2.089×10^{-3}	pounds per sq. foot.
“	1.450×10^{-5}	pounds per sq. inch.
board-feet	144 sq. in. × 1 in.	cubic inches.
British thermal units	0.2520	kilogram-calories.
“ “ “	777.5	foot-pounds.
“ “ “	3.927×10^{-4}	horse-power-hours.
“ “ “	1054	joules.
“ “ “	107.5	kilogram-meters.
“ “ “	2.928×10^{-4}	kilowatt-hours.
B.t.u. per min.	12.96	foot-pounds per sec.
“ “ “	0.02356	horse-power.
“ “ “	0.01757	kilowatts.
“ “ “	17.57	watts.
B.t.u. per sq. ft. per min.	0.1220	watts per square inch.
bushels	1.244	cubic feet.
“	2150	cubic inches.

Multiply	by	to obtain
bushels (cont.)	0.03524	cubic meters.
“	4	pecks.
“	64	pints (dry).
“	32	quarts (dry).
Centares	1	square meters.
centigrams	0.01	grams.
centiliters	0.01	liters.
centimeters	0.3937	inches.
“	0.01	meters.
“	393.7	mils.
“	10	millimeters.
centimeter-dynes	1.020×10^{-3}	centimeter-grams.
“ “	1.020×10^{-8}	meter-kilograms.
“ “	7.376×10^{-8}	pound-feet.
centimeter-grams	980.7	centimeter-dynes.
“ “	10^{-5}	meter-kilograms.
“ “	7.233×10^{-5}	pound-feet.
centimeters of mercury	0.01316	atmospheres.
“ “ “	0.4461	feet of water.
“ “ “	136.0	kgs. per square meter.
“ “ “	27.85	pounds per sq. foot.
“ “ “	0.1934	pounds per sq. inch.
centimeters per second	1.969	feet per minute.
“ “ “	0.03281	feet per second.
“ “ “	0.036	kilometers per hour.
“ “ “	0.6	meters per minute.
“ “ “	0.02237	miles per hour.
“ “ “	3.728×10^{-4}	miles per minute.
cms. per sec. per sec.	0.03281	feet per sec. per sec.
“ “ “ “ “	0.036	kms. per hour per sec.
“ “ “ “ “	0.02237	miles per hour per sec.
circular mils	5.067×10^{-6}	square centimeters.
“ “	7.854×10^{-7}	square inches.
“ “	0.7854	square mils.
cord-feet	4 ft. × 4 ft. × 1 ft.	cubic feet.
cords	8 ft. × 4 ft. × 4 ft.	cubic feet.
coulombs	1/10	abcoulombs.
“	3×10^{9}	statcoulombs.
coulombs per sq. inch	0.01550	abcoulombs per sq. cm.
“ “ “ “	0.1550	coulombs per sq. cm.
“ “ “ “	4.650×10^{8}	statcouls. per sq. cm.
cubic centimeters	3.531×10^{-5}	cubic feet.
“ “	6.102×10^{-2}	cubic inches.
“ “	10^{-6}	cubic meters.
“ “	1.308×10^{-6}	cubic yards.
“ “	2.642×10^{-4}	gallons.
“ “	10^{-3}	liters.
“ “	2.113×10^{-3}	pints (liq.).
“ “	1.057×10^{-3}	quarts (liq.).
cubic feet	2.832×10^{4}	cubic cms.
“ “	1728	cubic inches.
“ “	0.02832	cubic meters.
“ “	0.03704	cubic yards.
“ “	7.481	gallons.
“ “	28.32	liters.
“ “	59.84	pints (liq.).
“ “	29.92	quarts (liq.).
cubic feet per minute	472.0	cubic cms. per sec.
“ “ “ “	0.1247	gallons per sec.
“ “ “ “	0.4720	liters per second.
“ “ “ “	62.4	lbs. of water per min.
cubic inches	16.39	cubic centimeters.
“ “	5.787×10^{-4}	cubic feet.
“ “	1.639×10^{-5}	cubic meters.
“ “	2.143×10^{-5}	cubic yards.
“ “	4.329×10^{-3}	gallons.
“ “	1.639×10^{-2}	liters.
“ “	0.03463	pints (liq.).
“ “	0.01732	quarts (liq.).
cubic meters	10^{6}	cubic centimeters.
“ “	35.31	cubic feet.
“ “	61,023	cubic inches.
“ “	1.308	cubic yards.

Multiply	by	to obtain
cubic meters (cont.)	264.2	gallons.
“ “	10^{3}	liters.
“ “	2113	pints (liq.).
“ “	1057	quarts (liq.).
cubic yards	7.646×10^{5}	cubic centimeters.
“ “	27	cubic feet.
“ “	46,656	cubic inches.
“ “	0.7646	cubic meters.
“ “	202.0	gallons.
“ “	764.6	liters.
“ “	1616	pints (liq.).
“ “	807.9	quarts (liq.).
cubic yards per minute	0.45	cubic feet per second.
“ “ “ “	3.367	gallons per second.
“ “ “ “	12.74	liters per second.
Days	24	hours.
“	1440	minutes.
“	86,400	seconds
decigrams	0.1	grams.
deciliters	0.1	liters.
decimeters	0.1	meters.
degrees (angle)	60	minutes.
“ “	0.01745	radians.
“ “	3600	seconds.
degrees per second	0.01745	radians per second.
“ “ “	0.1667	revolutions per min.
“ “ “	0.002778	revolutions per sec.
dekagrams	10	grams.
dekaliters	10	liters.
dekameters	10	meters.
dollars (U.S.)	5.182	francs (French).
“ “	4.20	marks (German).
“ “	0.2055	pounds sterling (Brit.).
“ “	4.11	shillings (British).
drams	1.772	grams.
“	0.0625	ounces
dynes	1.020×10^{-3}	grams.
“	7.233×10^{-5}	poundals.
“	2.248×10^{-6}	pounds.
dynes per square cm.	1	bars.
Ergs	9.486×10^{-11}	British thermal units.
“	1	dyne-centimeters.
“	7.376×10^{-8}	foot-pounds.
“	1.020×10^{-3}	gram-centimeters.
“	10^{-7}	joules.
“	2.390×10^{-11}	kilogram-calories.
“	1.020×10^{-8}	kilogram-meters.
ergs per second	5.692×10^{-9}	B.t.units per minute.
“ “ “	4.426×10^{-6}	foot-pounds per min.
“ “ “	7.376×10^{-8}	foot-pounds per sec.
“ “ “	1.341×10^{-10}	horse-power.
“ “ “	1.434×10^{-9}	kg.-calories per min.
“ “ “	10^{-10}	kilowatts.
Farads	10^{-9}	abfarads.
“	10^{6}	microfarads.
“	9×10^{11}	statfarads.
fathoms	6	feet.
feet	30.48	centimeters.
“	12	inches.
“	0.3048	meters.
“	.36	varas.
“	1/3	yards.
feet of water	0.02950	atmospheres.
“ “ “	0.8826	inches of mercury.
“ “ “	304.8	kgs. per square meter.
“ “ “	62.43	pounds per sq. ft.
“ “ “	0.4335	pounds per sq. inch.
feet per minute	0.5080	centimeters per sec.
“ “ “	0.01667	feet per second.
“ “ “	0.01829	kilometers per hour.
“ “ “	0.3048	meters per minute.

Multiply	by	to obtain
feet per minute (cont.)	0.01136	miles per hour.
feet per second	30.48	centimeters per sec
“ “ “	1.097	kilometers per hour
“ “ “	0.5921	knots per hour.
“ “ “	18.29	meters per minute
“ “ “	0.6818	miles per hour.
“ “ “	0.01136	miles per minute.
feet per 100 feet	1	per cent grade.
feet per sec. per sec.	30.48	cms. per sec. per sec.
“ “ “ “ “	1.097	kms. per hr. per sec.
“ “ “ “ “	0.3048	meters per sec. per sec
“ “ “ “ “	0.6818	miles per hr. per sec.
foot-pounds	1.286×10^{-3}	British thermal units.
“ “	1.356×10^{7}	ergs.
“ “	5.050×10^{-7}	horse-power-hours.
“ “	1.356	joules.
“ “	3.241×10^{-4}	kilogram-calories.
“ “	0.1383	kilogram-meters.
“ “	3.766×10^{-7}	kilowatt-hours.
foot-pounds per minute	1.286×10^{-3}	B.t.units per minute.
“ “ “ “	0.01667	foot-pounds per sec.
“ “ “ “	3.030×10^{-5}	horse-power.
“ “ “ “	3.241×10^{-4}	kg.-calories per min.
“ “ “ “	2.260×10^{-5}	kilowatts.
foot-pounds per second	7.717×10^{-2}	B.t.units per minute.
“ “ “ “	1.818×10^{-3}	horse-power.
“ “ “ “	1.945×10^{-2}	kg.-calories per min.
“ “ “ “	1.356×10^{-3}	kilowatts.
francs (French)	0.193	dollars (U. S.).
“ “	0.811	marks (German).
“ “	0.03965	pounds sterlg. (Brit.)
furlongs	40	rods.
Gallons	3785	cubic centimeters.
“	0.1337	cubic feet.
“	231	cubic inches.
“	3.785×10^{-3}	cubic meters.
“	4.951×10^{-3}	cubic yards.
“	3.785	liters.
“	8	pints (liq.).
“	4	quarts (liq.).
gallons per minute	2.228×10^{-3}	cubic feet per second
“ “ “	0.06308	liters per second.
gausses	6.452	lines per square inch.
gilberts	0.07958	abampere-turns.
“	0.7958	ampere-turns.
gilberts per centimeter	2.021	ampere-turns per inch.
gills	0.1183	liters.
“	0.25	pints (liq.).
grains (troy)	1	grains (av.).
“ “	0.06480	grams.
“ “	0.04167	pennyweights (troy).
grams	980.7	dynes.
“	15.43	grains (troy).
“	10^{-3}	kilograms.
“	10^{3}	milligrams.
“	0.03527	ounces.
“	0.03215	ounces (troy).
“	0.07093	poundals.
“	2.205×10^{-3}	pounds.
gram-calories	3.968×10^{-3}	British thermal units.
gram-centimeters	9.302×10^{-8}	British thermal units.
“ “	980.7	ergs.
“ “	7.233×10^{-5}	foot-pounds.
“ “	9.807×10^{-5}	joules.
“ “	2.344×10^{-8}	kilogram-calories.
“ “	10^{-5}	kilogram-meters.
grams per cm.	5.600×10^{-3}	pounds per inch.
grams per cu. cm.	62.43	pounds per cubic feet.
“ “ “ “	0.03613	pounds per cubic inch.
“ “ “ “	3.405×10^{-7}	pounds per mil-foot.
Hectares	2.471	acres.
“	1.076×10^{-5}	square feet.

Multiply	by	to obtain
hectograms	100	grams.
hectoliters	100	liters.
hectometers	100	meters.
hectowatts	100	watts.
hemispheres (sol. angle)	0.5	sphere.
" " "	4	spherical right angles.
" " "	6.283	steradians.
henries	10^9	abhenries.
"	10^3	millihenries.
"	$1/9\times10^{-11}$	stathenries.
horse-power	42.44	B.t. units per min.
" "	33,000	foot-pounds per min.
" "	550	foot-pounds per sec.
" "	1.014	horse-power (metric).
" "	10.70	kg.-calories per min.
" "	0.7457	kilowatts.
" "	745.7	watts.
horse-power (boiler)	33,520	B.t.u. per hour.
" " "	9.804	kilowatts.
horse-power-hours	2547	British thermal units.
" " "	1.98×10^6	foot-pounds.
" " "	2.684×10^6	joules.
" " "	641.7	kilogram-calories.
" " "	2.737×10^5	kilogram-meters.
" " "	0.7457	kilowatt-hours.
hours	60	minutes.
"	3600	seconds.
Inches	2.540	centimeters.
"	10^3	mils.
"	.03	varas.
inches of mercury	0.03242	atmospheres.
" " "	1.133	feet of water.
" " "	345.3	kgs. per square meter.
" " "	70.73	pounds per square ft.
" " "	0.4912	pounds per square in.
inches of water	0.002458	atmospheres.
" " "	0.07355	inches of mercury.
" " "	25.40	kgs. per square meter.
" " "	0.5781	ounces per square in.
" " "	5.204	pounds per square ft.
" " "	0.03613	pounds per square in.
Joules	9.486×10^{-4}	British thermal units.
"	10^7	ergs.
"	0.7376	foot-pounds.
"	2.390×10^{-4}	kilogram-calories.
"	0.1020	kilogram-meters.
"	2.778×10^{-4}	watt-hours.
Kilograms	980,665	dynes.
"	10^3	grams.
"	70.93	poundals.
"	2.2046	pounds.
"	1.102×10^{-3}	tons (short).
kilogram-calories	3.968	British thermal units.
" "	3086	foot-pounds.
" "	1.558×10^{-3}	horse-power-hours.
" "	4183	joules.
" "	426.6	kilogram-meters.
" "	1.162×10^{-3}	kilowatt-hours.
kg.-calories per min.	51.43	foot-pounds per sec.
" " " "	0.09351	horse-power.
" " " "	0.06972	kilowatts.
kgs.-cms. squared	2.373×10^{-3}	pounds-feet squared.
" " "	0.3417	pounds-inches sq'd.
kilogram-meters	9.302×10^{-3}	British thermal units.
" "	9.807×10^7	ergs.
" "	7.233	foot-pounds.
" "	9.807	joules.
" "	2.344×10^{-3}	kilogram-calories.
" "	2.724×10^{-6}	kilowatt-hours.
kgs. per cubic meter	10^{-3}	grams per cubic cm.
" " " "	0.06243	pounds per cubic feet.

Multiply	by	to obtain
kgs. per c. meter (cont.)	3.613×10^{-5}	pounds per cubic inch.
" " " "	3.405×10^{-10}	pounds per mil. foot.
kgs. per meter	0.6720	pounds per foot.
kgs. per square meter	9.678×10^{-5}	atmospheres.
" " " "	98.07	bars.
" " " "	3.281×10^{-3}	feet of water.
" " " "	2.896×10^{-3}	inches of mercury.
" " " "	2.2048	pounds per square ft.
" " " "	1.422×10^{-3}	pounds per square in.
kgs. per sq. millimeter	10^6	kgs. per square meter.
kilolines	10^3	maxwells.
kiloliters	10^3	liters.
kilometers	10^5	centimeters.
"	3281	feet.
"	10^3	meters.
"	0.6214	miles.
"	1093.6	yards.
kilometers per hour	27.78	centimeters per sec.
" " "	54.68	feet per minute.
" " "	0.9113	feet per second.
" " "	0.5396	knots per hour.
" " "	16.67	meters per minute.
" " "	0.6214	miles per hour.
kms. per hour per sec.	27.78	cms. per sec. per sec.
" " " " "	0.9113	ft. per sec. per sec.
" " " " "	0.2778	meters per sec. per sec.
" " " " "	0.6214	miles per hr. per sec.
kilometers per min.	60	kilometers per hour.
kilowatts	56.92	B.t. units per min.
"	4.425×10^4	foot-pounds per min.
"	737.6	foot-pounds per sec.
"	1.341	horse-power.
"	14.34	kg.-calories per min.
"	10^3	watts.
kilowatt-hours	3415	British thermal units.
" "	2.655×10^6	foot-pounds.
" "	1.341	horse-power-hours.
" "	6×10^6	joules.
" "	860.5	kilogram-calories.
" "	3.671×10^5	kilogram-meters.
knots	6080	feet.
"	1.853	kilometers.
"	1.152	miles.
"	2027	yards.
knots per hour	51.48	centimeters per se.
" " "	1.689	eet per second.
" " "	1.853	ilometers per hour.
" " "	1.152	miles per hour.
Lines per square cm.	1	gausses.
lines per square inch	0.1550	gausses.
links (engineer's)	12	inches.
links (surveyor's)	7.92	inches.
liters	10^3	cubic centimeters.
"	0.03531	cubic feet.
"	61.02	cubic inches.
"	10^{-3}	cubic meters.
"	1.308×10^{-3}	cubic yards.
"	0.2642	gallons.
"	2.113	pints (liq.).
"	1.057	quarts (liq.).
liters per minute	5.885×10^{-4}	cubic feet per second.
" " "	4.403×10^{-3}	gallons per second.
log^{10} N	2.303	log$_\epsilon$ N or $ln\ N$.
log$_\epsilon$ N or $ln\ N$	0.4343	log$_{10}$ N.
lumens per sq. ft.	1	foot-candles.
Marks (German)	0.238	dollars (U. S.).
" "	1.233	francs (French).
" "	0.04890	pounds sterling (Brit.).
maxwells	10^{-3}	kilolines.
megalines	10^6	maxw lls.

Multiply	by	to obtain
megmhos per cm. cube	10^{-3}	abmhos per cm. cube.
" " " "	2.540	megmhos per in. cube.
" " " "	$10^2/\delta$	mhos per meter-gram.
" " " "	0.1662	mhos per mil foot.
megmhos per inch cube	0.3937	megmhos per cm. cube.
megohms	10^6	ohms.
meters	100	centimeters.
"	3.2808	feet.
"	39.37	inches.
"	10^{-3}	kilometers.
"	10^3	millimeters.
"	1.0936	yards.
meter-kilograms	9.807×10^7	centimeter-dynes.
" "	10^5	centimeter-grams.
" "	7.233	pound-feet.
meters per minute	1.667	centimeters per sec.
" " "	3.281	feet per minute.
" " "	0.05468	feet per second.
" " "	0.06	kilometers per hour.
" " "	0.03728	miles per hour.
meters per second	196.8	feet per minute.
" " "	3.281	feet per second.
" " "	3.6	kilometers per hour.
" " "	0.06	kilometers per min.
" " "	2.237	miles per hour.
" " "	0.03728	miles per minute.
meters per sec. per sec.	3.281	feet per sec. per sec.
" " " " "	3.6	kms. per hour per sec.
" " " " "	2.237	miles per hour per sec.
mhos per meter-gram	$10^{-5}\delta$	abmhos per cm. cube.
" " " "	$10^{-2}\delta$	megmhos p. cm. cube.
" " " "	$2.540\times10^{-2}\delta$	megmhos per in. cube.
" " " "	$1.662\times10^{-3}\delta$	mhos per mil foot.
mhos per mil foot	6.015×10^{-3}	abmhos per cm. cube.
" " " "	6.015	megmhos p. cm. cube.
" " " "	15.28	megmhos per in. cube.
" " " "	$601.5/\delta$	mhos per meter-gram.
microfarads	10^{-15}	abfarads.
"	10^{-6}	farads.
"	9×10^5	statfarads.
micrograms	10^{-6}	grams.
microliters	10^{-6}	liters.
microhms	10^3	abohms.
"	10^{-12}	megohms.
"	10^{-6}	ohms.
"	$1/9\times10^{-17}$	statohms.
microhms per cm. cube	10^3	abohms per cm. cube.
" " " "	0.3937	microhms p. in. cube.
" " " "	$10^{-2}\delta$	ohms per meter-gram.
" " " "	6.015	ohms per mil feet.
microhms per inch cube	2.540	microhms p. cm. cube.
microns	10^{-6}	meters.
miles	1.609×10^5	centimeters.
"	5280	feet.
"	1.6093	kilometers.
"	1760	yards.
"	1900.8	varas.
miles per hour	44.70	centimeters per sec.
" " "	88	feet per minute.
" " "	1.467	feet per second.
" " "	1.6093	kilometers per hour
" " "	0.8684	knots per hour.
" " "	26.82	meters per minute.
miles per hour per sec.	44.70	cms. per sec. per sec.
" " " " "	1.467	feet per sec. per sec.
" " " " "	1.6093	kms. per hour per sec.
" " " " "	0.4470	M. per sec. per sec.
miles per minute	2682	centimeters per sec.
" " "	88	feet per second.
" " "	1.6093	kilometers per min.
" " "	0.8684	knots per minute.
" " "	60	miles per hour.
milliers	10^3	kilograms.

Multiply	by	to obtain
milligrams	10^{-3}	grams.
millihenries	10^6	abhenries.
"	10^{-3}	henries.
"	$1/9\times10^{-14}$	stathenries.
milliliters	10^{-3}	liters.
millimeters	0.1	centimeters.
"	0.03937	inches.
"	39.37	mils.
mils	0.002540	centimeters.
"	10^{-3}	inches.
miner's inches	1.5	cubic feet per min.
minutes (angle)	2.909×10^{-4}	radians.
" (")	60	seconds (angle).
months	30.42	days.
"	730	hours.
"	43,800	minutes.
"	2.628×10^6	seconds.
myriagrams	10	kilograms.
myriameters	10	kilometers.
myriawatts	10	kilowatts.
Ohms	10^9	abohms.
"	10^{-6}	megohms.
"	10^6	microhms.
"	$1/9\times10^{-11}$	statohms.
ohms per meter-gram	$10^5/\delta$	abohms per cm. cube.
" " " "	$10^2/\delta$	microhms p. cm. cuba.
" " " "	$39.37/\delta$	microhms p. in. cube.
" " " "	$601.5/\delta$	ohms per mil foot
ohms per mil foot	166.2	abohms per cm. cube.
" " " "	0.1662	microhms p. cm. cube
" " " "	0.06524	microhms pr. in. cube.
" " " "	$1.662\times10^{-3}\delta$	ohms per meter-gram.
ounces	8	drams.
"	437.5	grains.
"	28.35	grams.
"	0.0625	pounds.
ounces (fluid)	1.805	cubic inches.
" (")	0.02957	liters.
ounces (troy)	480	grains (troy).
" (")	31.10	grams.
" (")	20	pennyweights (troy).
" (")	0.08333	pounds (troy).
ounces per square inch	0.0625	pounds per sq. inch.
Pennyweights (troy)	24	gains (troy).
" (")	1.555	grams.
" (")	0.05'	ounces (troy).
perches (masonry)	24.75	cubic feet.
pints (dry)	33.60	cubic inches.
pints (liq.)	28.87	cubic inches.
poundals	13,826	dynes.
"	14.10	grams.
"	0.03108	pounds.
pounds	444,823	dynes.
"	7000	grains.
"	453.6	grams.
"	16	ounces.
"	32.17	poundals.
pounds (troy)	0.8229	pounds (av.).
pound-feet	1.356×10^7	centimeter-dynes.
" "	13,825	centimeter-grams.
" "	0.1383	meter-kilograms.
pounds-feet squared	421.3	kgs.-cms. squared.
" " "	144	pounds-ins. squared.
pounds-inches squared	2.926	kgs.-cms. squared.
" " "	6.945×10^{-3}	pounds-feet squared.
pounds of water	0.01602	cubic feet.
" " "	27.68	cubic inches.
" " "	0.1198	gallons.
pounds of wat per min.	2.669×10^{-4}	cubic feet per sec.
pounds per cubic foot	0.01602	grams per cubic cm.
" " " "	16.02	kgs. per cubic meter.
" " " "	5.787×10^{-4}	pounds per cubic inch.
" " " "	5.456×10^{-9}	pounds per mil foot.

Multiply	by	to obtain
pounds per cubic inch	27.68	pounds per cubic cm.
" " " "	2.768×10^{4}	kgs. per cubic meter.
" " " "	1728	pounds per cubic foot.
" " " "	9.425×10^{-6}	pounds per mil foot.
pounds per foot	1.488	kgs. per meter.
pounds per inch	178.6	grams per cm.
pounds per mil foot	2.306×10^{6}	grams per cubic cm.
pounds per square foot	0.01602	feet of water.
" " " "	4.882	kgs. per square meter.
" " " "	6.944×10^{-3}	pounds per sq. inch.
pounds per square inch	0.06804	atmospheres.
" " " "	2.307	feet of water.
" " " "	2.036	inches of mercury.
" " " "	703.1	kgs. per square meter.
" " " "	144	pounds per sq. foot.
pounds sterl. (British)	4.8665	dollars (U. S.).
" " (")	25.22	francs (French).
" " (")	20.44	marks (German).
Quadrants (angle)	90	degrees.
" (")	5400	minutes.
" (")	1.571	radians.
quarts (dry)	67.20	cubic inches.
quarts (liq.)	57.75	cubic inches.
quintals	100	pounds.
quires	25	sheets.
Radians	57.30	degrees.
"	3438	minutes.
"	0.637	quadrants.
radians per second	57.30	degrees per second.
" " "	0.1592	revolutions per sec.
" " "	9.549	revolutions per min.
radians per sec. per sec	573.0	revs. per min. per min.
" " " " "	9.549	revs. per min. per sec.
" " " " "	0.1592	revs. per sec. per sec.
reams	500	sheets.
revolutions	360	degrees.
"	4	quadrants.
"	6.283	radians.
revolutions per minute	6	degrees per second.
" " "	0.1047	radians per second.
" " "	0.01667	revolutions per sec.
revs. per min. per min.	1.745×10^{-3}	rads. per sec. per sec.
" " " " "	0.01667	revs. per min. per sec.
" " " " "	2.778×10^{-4}	revs. per sec. per sec.
revolutions per second	360	degrees per second.
" " "	6.283	radians per second.
" " "	60	revs. per minute.
revs. per sec. per sec.	6.283	rads. per sec. per sec.
" " " " "	3600	revs. per min. per. min.
" " " " "	60	revs. per min. per sec.
rods	16.5	feet.
Seconds (angle)	4.848×10^{-6}	radians.
spheres (solid angle)	12.57	steradians.
spherical right angles	0.25	hemispheres.
" " "	0.125	spheres.
" " "	1.571	steradians.
square centimeters	1.973×10^{5}	circular mils.
" "	1.076×10^{-3}	square feet.
" "	0.1550	square inches
" "	10^{-6}	square meters.
" "	100	square millimeters.
sq. cms.-cms. sqd.	0.02402	sq. inches-inches sqd.
square feet	2.296×10^{-5}	acres.
" "	929.0	square centimeters.
" "	144	square inches.
" "	0.09290	square meters
" "	3.587×10^{-8}	square miles.
" "	.1296	square varas.
" "	1/9	square yards.
sq. feet-feet sqd.	2.074×10^{4}	sq. inches-inches sqd.
square inches	1.273×10^{6}	circular mils.

Multiply	by	to obtain
square inches (cont.)	6.452	square centimeters.
" "	6.944×10^{-3}	square feet.
" "	10^{6}	square mils.
" "	645.2	square millimeters.
sq. inches-inches sqd.	41.62	sq. cms.-cms. sqd.
" " " "	4.823×10^{-5}	sq. feet-feet sqd.
square kilometers	247.1	acres.
" "	10.76×10^{6}	square feet.
" "	10^{6}	square meters.
" "	0.3861	square miles.
" "	1.196×10^{6}	square yards.
square meters	2.471×10^{-4}	acres.
" "	10.764	square feet.
" "	3.861×10^{-7}	square miles.
" "	1.196	square yards.
square miles	640	acres.
" "	27.88×10^{6}	square feet.
" "	2.590	square kilometers
" "	3,613,040.45	square varas.
" "	3.098×10^{6}	square yards.
square millimeters	1.973×10^{3}	circular mils.
" "	0.01	square centimeters
" "	1.550×10^{-3}	square inches.
square mils	1.273	circular mils.
" "	6.452×10^{-6}	square centimeters.
" "	10^{-6}	square inches.
square varas	.0001771	acres.
" "	7.716049	square feet.
" "	.0000002765	square miles.
" "	.857339	square yards.
square yards	2.066×10^{-4}	acres.
" "	9	square feet.
" "	0.8361	square meters.
" "	3.228×10^{-7}	square miles.
" "	1.1664	square varas.
statamperes	$1/3\times10^{-10}$	abamperes.
"	$1/3\times10^{-9}$	amperes.
statcoulombs	$1/3\times10^{-10}$	abcoulombs.
"	$1/3\times10^{-9}$	coulombs.
statfarads	$1/9\times10^{-20}$	abfarads.
"	$1/9\times10^{-11}$	farads.
"	$1/9\times10^{-5}$	microfarads.
stathenries	9×10^{20}	abhenries.
"	9×10^{11}	henries.
"	9×10^{14}	millihenries.
statohms	9×10^{20}	abohms.
"	9×10^{5}	megohms.
"	9×10^{17}	microhms.
"	9×10^{11}	ohms.
statvolts	3×10^{10}	abvolts
"	300	volts
steradians	0.1592	hemispheres
"	0.07958	spheres.
"	0.6366	spherical right angles
steres	10^{3}	liters.
Temp. (degs. C.) + 273	1	abs. temp. (degs. C.)
" (" ") + 17.8	1.8	temp. (degs. Fahr.).
temp. (degs. F.) + 460	1	abs. temp. (degs. F.).
" (" ") − 32	5/9	temp. (degs. Cent.).
tons (long)	1016	kilograms.
" (")	2240	pounds.
tons (metric)	10^{3}	kilograms.
" (")	2205	pounds.
tons (short)	907.2	kilograms.
" (")	2000	pounds.
tons (short) per sq. ft.	9765	kgs. per square meter.
" (") " " "	13.89	pounds per sq. inch.
tons (short) per sq. in.	1.406×10^{6}	kgs. per square meter.
" (") " " "	2000	pounds per sq. inch.
Varas	2.7777	feet.
"	33.3333	inches.
"	.000526	miles.

Multiply	by	to obtain
varas (cont.)	.9259	yards.
volts	10^{8}	abvolts
"	1/300	statvolts.
volts per inch	3.937×10^{7}	abvolts per cm.
" " "	1.312×10^{-3}	statvolts per cm.
Watts	0.05692	B.t. units per min.
"	10^{7}	ergs per second.
"	44.26	foot-pounds per min.
"	0.7376	foot-pounds per sec.
"	1.341×10^{-3}	horse-power.
"	0.01434	kg.-calories per min.
"	10^{-3}	kilowatts.
watt-hours	3.415	British thermal units.
" "	2655	foot-pounds.
" "	1.341×10^{-3}	horse-power-hours.
" "	0.8605	kilogram-calories.

Multiply	by	to obtain
watt-hours (cont.)	367.1	kilogram-meters
" "	10^{-3}	kilowatt-hours.
webers	10^{8}	maxwells.
weeks	168	hours.
"	10,080	minutes.
"	604,800	seconds.
Yards	91.44	centimeters.
"	3	feet.
"	36	inches.
"	0.9144	meters.
"	1.08	varas.
years (common)	365	days.
" (")	8760	hours.
years (leap)	366	days.
" (")	8784	hours.