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SUN

INSTRUCTIONS

FOR THE USE OF

THE "HEMMI'S" BAMBOO SLIDE RULES

"UNIVERSAL"

DUPLEX

PUBLISHED BY

THE HEMMI SEISAKUSHO CO., LTD.

TOKIO, JAPAN

Printed in Japan

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Fig. 1 Mechanical Engineer's 10" (Front face).

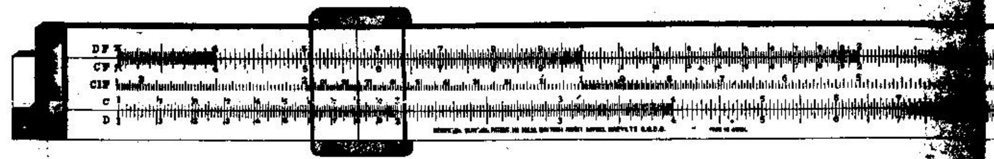


Fig. 3 Electrical Engineer's 10" (Front face).

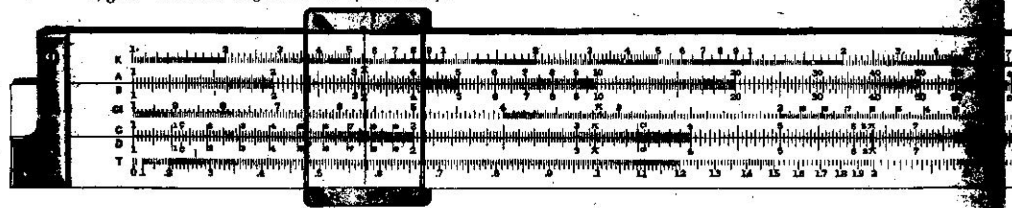


Fig. 5 Electrical Engineer's 10" (Front face).

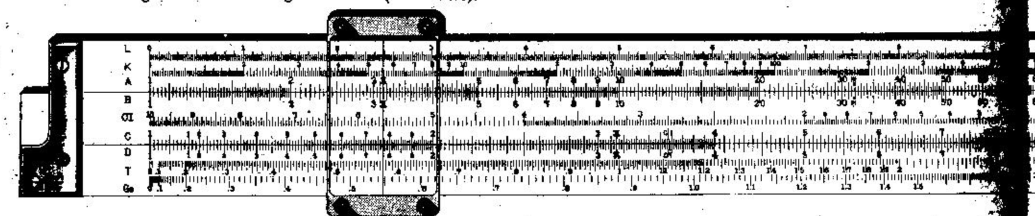


Fig. 7 Electrical Engineer's 20" (Front face).

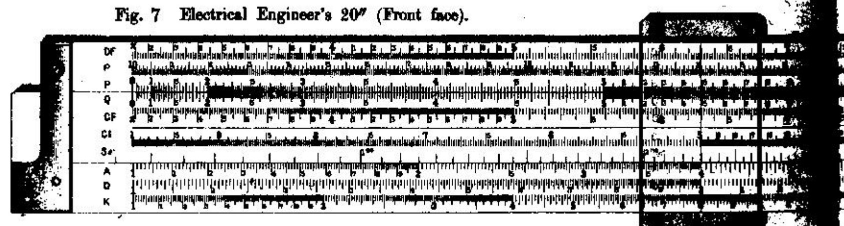
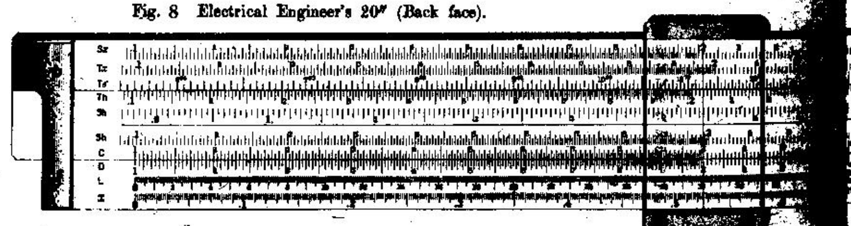


Fig. 8 Electrical Engineer's 20" (Back face).



Hemmi's Bamboo "Universal" Duplex Slide Rule (Reduced Size ½)

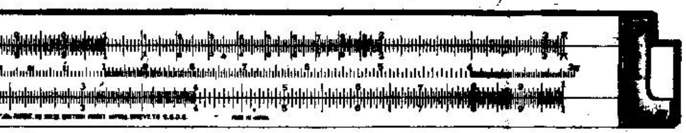
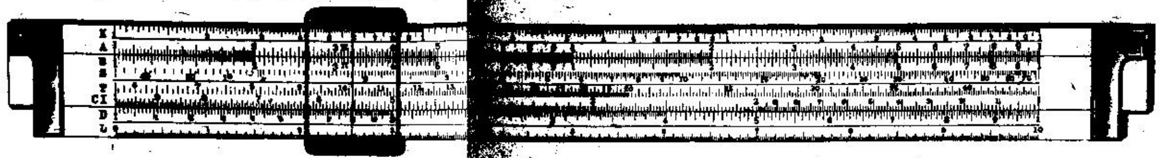


Fig. 2 Mechanical Engineer's 10″ (Back face).



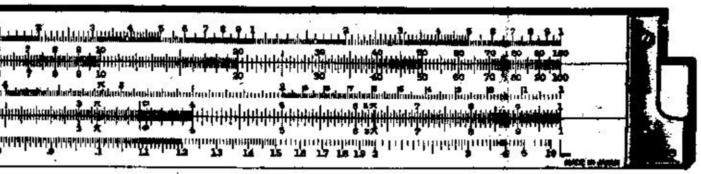
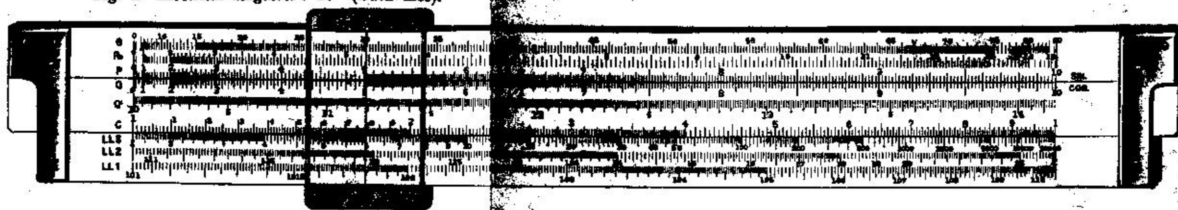


Fig. 4 Electrical Engineer's 10″ (Back face).



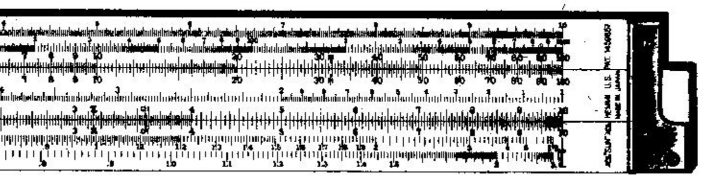
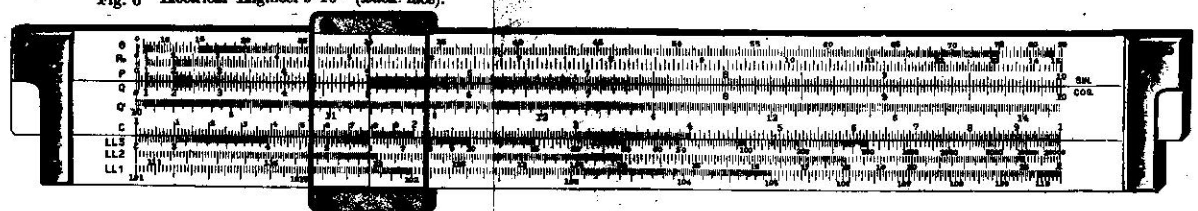
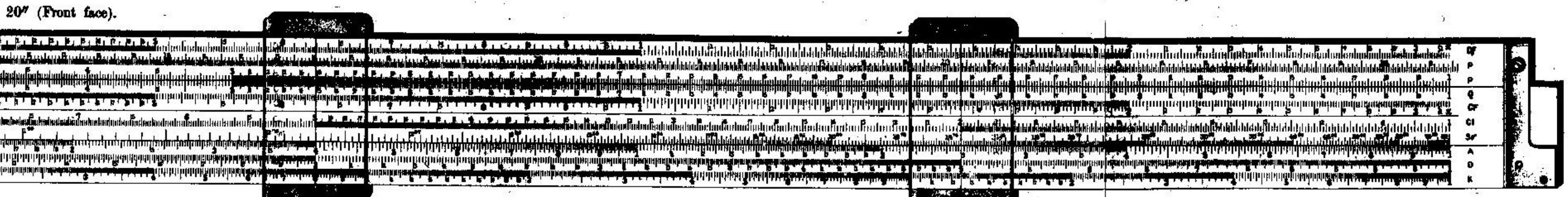


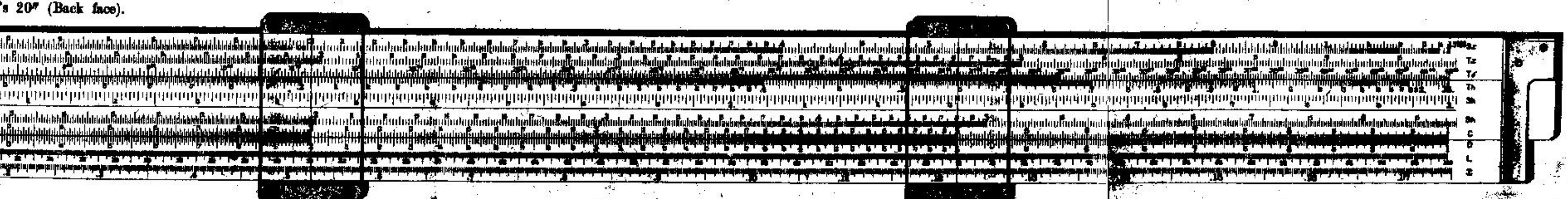
Fig. 6 Electrical Engineer's 10″ (Back face).



20″ (Front face).



's 20″ (Back face).



PREFACE

It is a great pleasure to see that the invention of the slide rule has made it possible to solve a wide variety of complicated arithmetical and other calculations with ease and rapidity. There have been invented also special slide rules for engineers, designers, contractors, statisticians and commercial people. The number of inventions in slide rules may be taken as the measure of the civilization of a country.

The wide use of slide rules has brought forth inventions of different scales, and the variety of scales induced improvements in the construction of the instrument. The " Ritz " and the " Stadia " slide rules are really prominent inventions; and there has finally come the " Duplex " slide rule. This book is to explain the duplex slide rule: Mechanical and Electrical Engineers' " Universal " slide rule, the very best of the type.

(1) Specific Characteristics of the Mechanical Engineer's Slide Rule.

This slide rule has, besides tne ordinary (*A*), (*B*), (*C*) and (*D*) scales, the cube and reciprocal scales; and also three folded scales (*DF*), (*CF*) and (*CIF*) that often dispense with " resetting " for multiplication, division, &c., and also simplify the circle calculations. They are conveniently arranged on both faces of the slide rule.

(2) Special Characteristics of the Electrical Engineer's " Universal " Slide Rule with Patent Vector and Log-Log Scales.

Electrical engineers, especially those who are engaged in research, experiment, or design, find their own work mostly consist of calculations. And these calculations are not limited to multiplication and division, but extend to trigonometrical functions, logarithms, complex numbers, &c. &c. That is why electrical engineer's slide rules, classified as special slide rules, are the most popular among slide rule users. But the old electrical engineer's slide rule does not show much improvement or addition to the ordinary Mechanical Engineer's slide rule. It had the log-log scales which we admit, were of some use; it had the efficiency scale for generaters and motors, and the voltage drop scale; and these were but quite dispensable. The questions of vectors and trigonometrical functions were still to be solved.

Our Electrical Engineer's "Universal" slide rule has been invented for these aims. It calculates the complicated complex numbers, vector functions, circuit calculations, &c. &c. with ease and rapidity. It is of the duplex type. It has our patent non-logarithmic scales of (P), (Q) for complex numbers, an angle scale (θ), a radian scale ($R\theta$), tangent scale (T), and a set of log-log scales in three parts. It is of course for electrical engineers, but at the same time, it is good for vector calculations as in Applied Dynamics, and angle calculations.

(3) Specific Characteristics of the Electrical Engineers "Universal" 20 inch Slide Rule of the Highest Class, with our Patent Vector and Hyperbolic Scales.

This is the invention of Dr. S. Bekku, engineer of the Dept. of Communication patented by him; it is the quintessence of the inventions of slide rules, in Japan, the U.K., the U.S.A. and all other countries. It is the further improvement of the 10 inch slide rule above described and has in place of the log-log scales the hyperbolic sine and tangent scales &c. &c., and so is good for much wider use. It is the most advanced slide rule in the world for electrical engineering.

CHAPTER I.

SLIDE RULE LACONISM AND INTERNATIONAL SLIDE RULE DIAGRAM

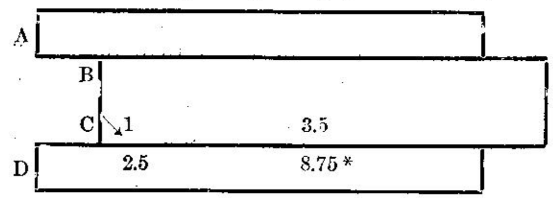
A Chemical Equation, *e.g.* $2H_2+O_2=2H_2O$, or an Arithmetical Equation, *e.g.* $2.5\times3.5=8.75$ is international and laconic; and boys of any nationality understand their meanings quite concretely: even better than the prosaic expression in his own mother tongue. If the readers had a convention, similarly international and laconic as well, given to them to guide them in their study of slide rules, it would no doubt assist them very much. Mr. E. Hirano, of the Third Commercial School of Tokyo Prefecture, has made two such proposals which the author of this book has accepted.

One of the proposals is a Laconism and the other an International Diagram. Either of the two is quite sufficient, and independent of the other. Here we shall give examples of both:—

Example 1. $2.5\times3.5=8.75$

Laconism:—Set 1 C to 2.5 D against 3.5 C read 8.75 D

Diagram

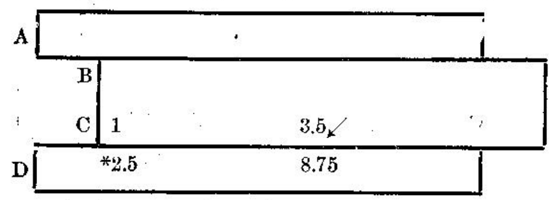


∴ *Ans.* 8.75

Example 2. $8.75\div3.5=2.5$

Laconism:—Set 3.5 C to 8.75 D against 1 C read 2.5 D

Diagram

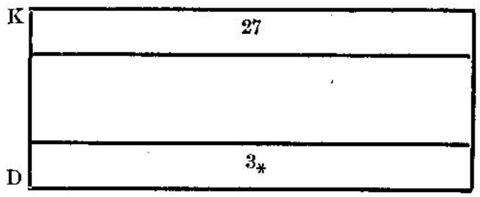


∴ *Ans.* 2.5

Regarding the Laconism, in cases where either "no setting is necessary" or "setting the slide end on end" or "the hairline only is to be employed" the left hand half of the laconism may be entirely omitted.

Example 3. $\sqrt[3]{27}=3$
Laconism: against 27 *K* read 3 *D*

Diagram



∴ *Ans.* 3.

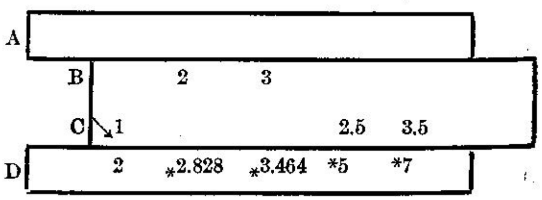
Regarding the Diagram, the lettering on the left side of the Diagram is to denote the names of the scales. But (*A*), (*B*), (*C*), (*D*) are so familar on the ordinary slide rule, that they may be omitted and understood for the sake of simplicity.

The arrow in the diagram points out the point of setting the slide; and the asterisk the answer required.

When one setting is used for more than one reading, all the Laconisms may be combined, also the diagrams as well.

Example 4. $2\times\sqrt{2}$, $2\times\sqrt{3}$, 2×2.5, 2×3.5
Laconism:—Set 1 *C* to 2 *D* against 2 *B* read 2.828 *D*
„ 3 *B* „ 3.464 *D*
„ 2.5 *C* „ 5 *D*
„ 3.5 *C* „ 7 *D*

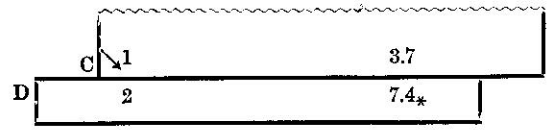
Diagram



∴ *Ans.* 2.828, 3.464, 5, 7 respectively.

When the meaning will be clear and obvious to the reader, part of the diagram might be omitted or cut off.

Example 5. $2\times 37=74$



∴ *Ans.* 74.

For teaching or writing about a slide rule, the Diagram is better as it gives a fuller idea than the Laconism; but for the student to write down on his examination papers, the Laconism would be better as it is simpler. Simple as it is, the Laconism makes it clear, whether or not the student has made a correct calculation, whether or not he has correctly used the slide rule, whether or not he has used it in the wisest way. The teacher can instruct his students to solve their problems specifying the use of such and such scales, rather than others.

If you like to internationalize the Laconism, you might as well write down as:—

1 *C*→2.5 *D*=3.5 *C*→8.75 *D* (Example 1)
3.5 *C*→8.75 *D*= 1 *C*→2.5 *D* („ 2)
27 *K*→ 3 *D* („ 3)
1 *C*→ 2 *D*= 2 *B*→2.828 *D*
= 3 *B*→3.464 *D*
=2.5 *C*→5 *D*
=3.5 *C*→7 *D* („ 4)

CHAPTER II.

MECHANICAL ENGINEER'S "UNIVERSAL" DUPLEX SLIDE RULE

Section A. SCALES DESCRIBED.

The Mechanical Engineer's Duplex Slide Rule consists of (see Figs. 1. and 2.) a couple of bars fastened to each other with a space between, and a slide which is of the same thickness as the bars and has its two faces flush with those of the bars, to run between them. Both the bars and the slide have several scales on them. The cursor consists of two small panes of glass with a hairline on each, fastened to each other to run over the whole slide rule.

Fig. 1. shows the front face of the slide rule, which has (*DF*) and (*D*) scales on the bars, and (*CF*), (*CIF*) and (*C*) scales on the slide. Among the scales, (*C*) and (*D*) are exactly the same as those of the ordinary slide rule. (*CF*) and (*DF*) are what we call folded scales, i.e. (*C*) or (*D*) cut at the point $\pi=3.1416$ and the lefthand part shifted over to the right, to make the graduation continuous from π to 10π. So any point on either of the two, represent the value, π times the reading of the point just opposite on either (*C*) or (*D*). They are good for calculations regarding circles, and also they are used for multiplication and division that would require resetting if (*C*) and (*D*) only were employed.

The scale (*CIF*) is (*CF*) inverted. It is used for calculating the reciprocal of a given number, the product of three factors, &c.

TABLE OF SCALES

Numbers to be referred to Figs. 1. & 2.	Denominations of Scales	Functions according to which the Scales are divided	Limits of x or θ in the Functions	Other Scales to be engaged with mostly	Functions best calculated
1.	*DF*	$\log x$	$\pi - 10\pi$	*CF, C, D*	πab, &c.
2.	*CF*	$\log x$	$\pi - 10\pi$	*DF, C, D*	πab, &c.
3.	*CIF*	$\log\frac{10}{x}$	$\frac{10}{\pi} - \pi$	*CF, DF*	abc, $\frac{1}{a}$, &c.
4.	*C*	$\log x$	$1 - 10$	*D*	$\frac{ab}{c}$, &c.
5.	*D*	$\log x$	$1 - 10$	*C*	$\frac{ab}{c}$, &c.
6.	*K*	$\log x$	$1 - 1000$	*D*	x^3, $\sqrt[3]{x}$, &c.
7.	*A*	$\log x$	$1 - 100$	*D*	x^2, $\sqrt{x}$, &c.
8.	*B*	$\log x$	$1 - 100$	*C*	x^2, $\sqrt{x}$, &c.
9.	*S*	$\log \sin\theta$	$34' - 90°$	*A*	$\sin\theta°$, $\sin^{-1}\theta°$
10.	*T*	$\log \tan\theta$	$5°-43-45°$	*D*	$\tan\theta°$, $\tan^{-1}\theta°$
11.	*CI*	$\log\frac{10}{x}$	$10 - 1$	*C, D*	abc, &c.
12.	*D*	$\log x$	$1 \div 10$	*C*	$\frac{ab}{c}$, &c.
13.	*L*	x	$0 - 10$	*D*	$\log x$, $\log^{-1} x$

Fig. 2. shows the back face of the slide rule. On this face, there are (*K*), (*A*), (*D*) and (*L*) on the bars and (*B*), (*S*), (*T*) and (*CI*) on the slide.

(*A*) and (*B*) are two-section logarithmic scales just as those on the ordinary slide rule. (*D*) is just like the (*D*) on the front face. (*CI*) is the inverted scale of (*C*). What (*CI*) is to (*D*), (*CIF*) is to (*DF*). You can get the reciprocal of a number and the product of three factors between (*CI*) and (*C*), (*D*).

(*L*) has the whole length divided decimally into 10 and decimals. Between (*L*) and (*C*), (*D*) you can get $\log x$ or $\log^{-1} x$.

(*K*) is a three-section logarithmic scale; Between (*K*) and (*D*) you can get x^3 and $\sqrt[3]{x}$.

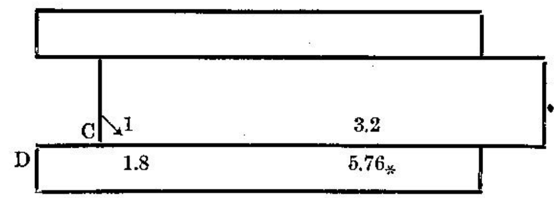
Scales (*S*) and (*T*) are for trigonometric functions: (*S*) is to work with (*A*) and (*B*), while (*T*) with (*C*) and (*D*). These scales are in degrees and *minutes* and not in *decimals*.

Section B. MULTIPLICATION AND DIVISION

[1] THE ORDINARY METHOD BETWEEN (*C*) AND (*D*)

Example 1. $18 \times 32 = 576$

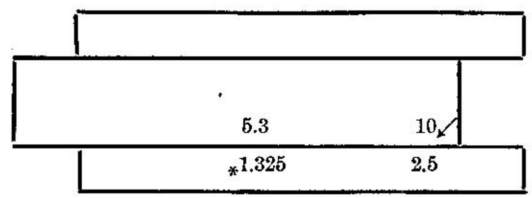
Set 1 *C* to 18 *D*, against 32 *C* read 576 *D*



∴ *Ans.* 576

Example 2. $2.5 \times 5.3 = 13.25$

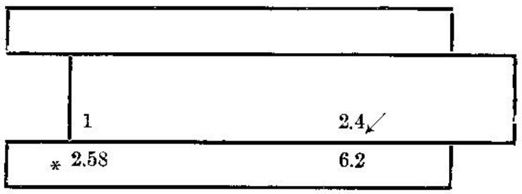
Set 10 *C* to 2.5 *D* against 5.3 *C* read 13.25 *D*



∴ *Ans.* 13.25

Example 3. $62 \div 2.4 = 25.8$

Set 2.4 *C* to 62 *D* against 1 *C* read 25.8 *D*

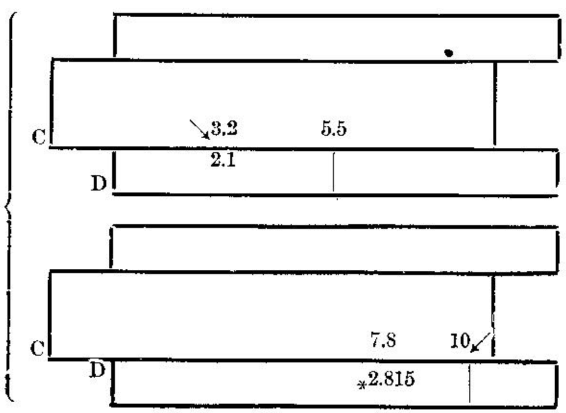


∴ *Ans.* 25.8

Example 4. $\dfrac{2.1 \times 5.5 \times 7.8}{32} = 2.815$

Set 32 *C* to 2.1 *D* at 5.5 *C* put Hairline

Set 10 *C* to Hairline, against 7.8 *C* read 2.815 *D*



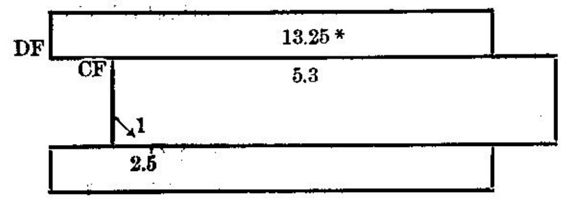
∴ *Ans.* 2.815

[2] THE USE OF (*CF*) AND (*DF*)

In the ordinary slide rules, if you set 1 *C* instead of 10 *C* to 2.5 *D* in the above Example 2. you would find 5.3 *C* off (*D*), and so you would be compelled to "reset" the slide. But if your slide rule has (*CF*) and (*DF*), this resetting is unnecessary; you can at once read 13.25 on (*DF*) against 5.3 *CF*.

Example 2. $2.5 \times 5.3 = 13.25$ (repeated)

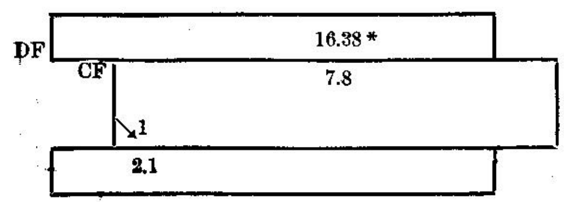
Set 1 *C* to 2.5 *D*, against 5.3 *CF* read 13.25 *DF*



∴ *Ans.* 13.25

Example 5. $2.1 \times 7.8 = 16.38$

Set 1 *C* to 2.1 *D*, against 7.8 *CF* read 16.38 *DF*



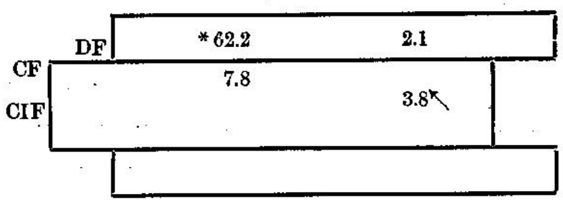
∴ *Ans.* 16.38

Make the shorter projection of the slide, to the right or to the left, then you will surely have the result without scaling out.

[3] THE USE OF (CIF)

Example 6. $2.1 \times 7.8 \times 3.8 = 62.2$

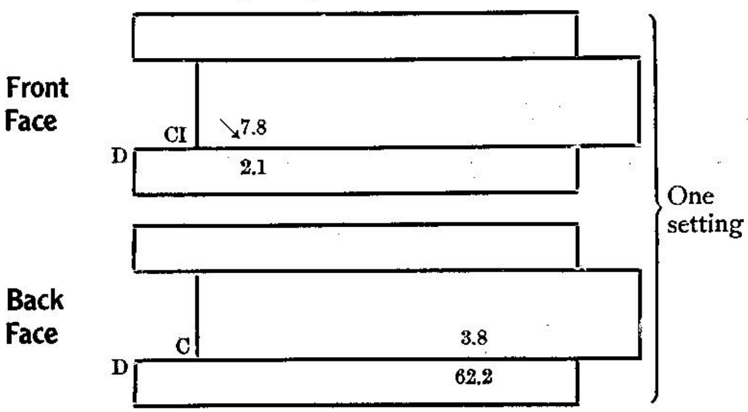
Set 3.8 *CIF* to 2.1 *DF*, against 7.8 *CF* read 62.2 *DF*



∴ *Ans.* 62.2

This example could be solved as well by means of (*C*), (*D*) and (*CI*); but as you see in the diagram, you must use both faces of the slide rule. So the above method of using (*DF*), (*CF*) and (*CIF*) is the better. At any rate:—

Set 7.8 *CI* to 2.1 *D*, turn over the whole slide rule, and against 3.8 *C* read 62.2 *D*



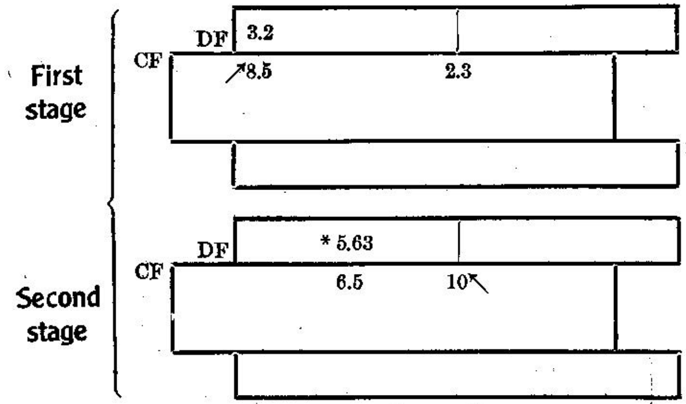
∴ *Ans.* 62.2

Example 7. $\dfrac{3.2 \times 6.5 \times 2.3}{8.5} = 5.63$

This must be attacked in two stages, or by two settings.

Set 8.5 *CF* to 3.2 *DF*, at 2.3 *CF* put Hairline

Set 10 *CF* to Hairline, against 6.5 *CF* read 5.63 *DF*

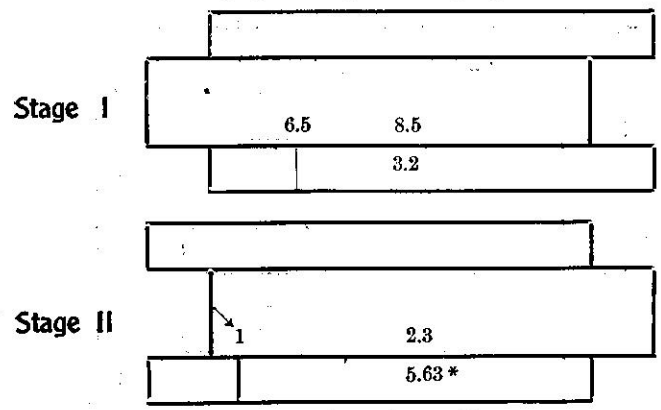


∴ *Ans.* 5.63

You could as well attack this with (*C*) and (*D*)

Set 8.5 *C* to 3.2 *D*, at 6.5 *C* put Hairline.

Set 1 *C* to Hairline, against 2.3 *C* read 5.63 *D*



∴ *Ans.* 5.63

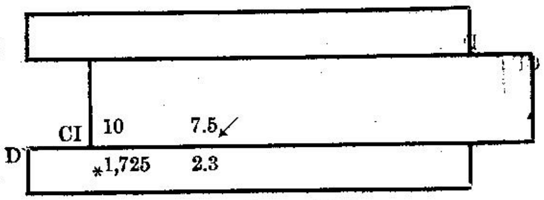
However, in order to explain (*CF*), (*DF*), the 1st method has been included.

[4] THE USE OF (*CI*)

We stated before that (*CI*) is the inverted scale of (*C*); but we shall discuss on this scale further on. Here we will explain only a few applications. We also stated in (2) that (*CF*) and (*DF*) are for avoiding "resetting" in single multiplications; here with (*CI*) "resetting" will never be necessary for a simple multiplication.

Example 8. $23 \times 75 = 1725$

Set 75 *CI* to 23 *D*, against 10 *CI* read 1725 *D*

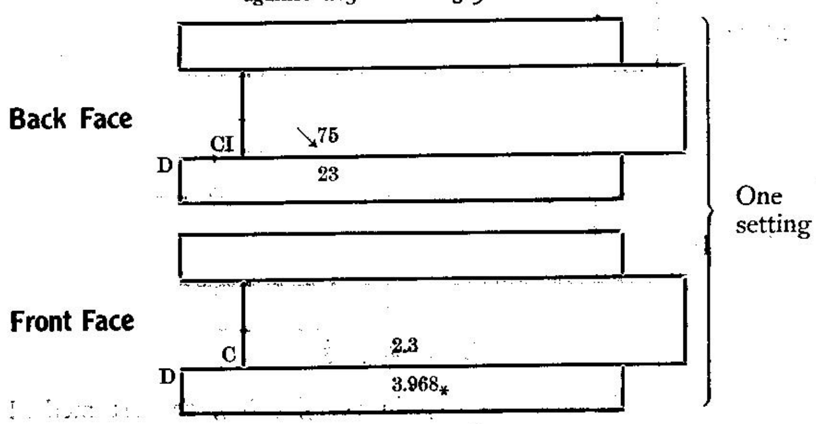


∴ *Ans.* 1725

Example 9. $23 \times 75 \times 2.3 = 3.968$

You can do this at one setting as was shown in Example 6., but you must use both faces of the slide rule.

Set 75 *CI* to 23 *D* then turn over the whole slide rule and against 2.3 *C* read 3.968 *D*



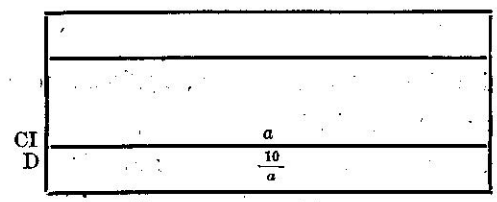
∴ *Ans.* 3.968

[5] FURTHER APPLICATIONS OF SCALES

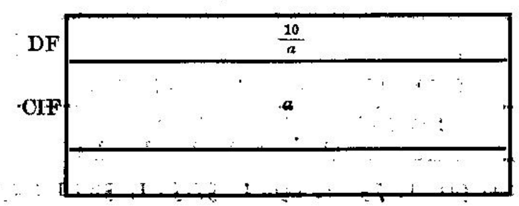
(a) Reciprocals.

You can get reciprocals either between (*CI*) and (*D*) on the back face, or between (*CIF*) and (*DF*) on the front.

Against a *CI* read $\frac{10}{a}D$

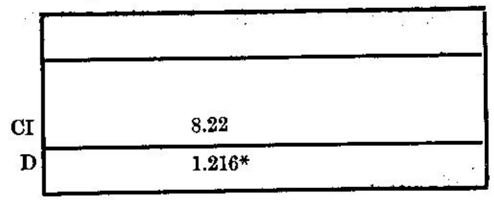


Against a *CIF* read $\frac{10}{a}DF$



Example 10. The reciprocal of 8.22

Against 8.22 *CI* read 0.1216 *D*



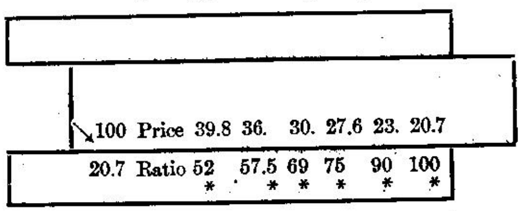
∴ *Ans.* 0.1216

The arithmetical product of the price of silver in New York and the ratio of values of gold and silver is a constant, because the two factors are inversely proportional to each other. And the constant is 2070 (demonstration omitted). So you can calculate the ratio for any price very easily.

Example 11.

If the prices be 20.70 c., 23.00 c., 27.60 c., 30.00 c., 36.00 c., 39.80 c. what are the ratios?

Set 10 *CI* to 207 *D* against 20.7 *CI* read 100 *D*
„ 23 *CI* „ 90 *D*
„ 27.6 *CI* „ 75 *D*
„ 30 *CI* „ 69 *D*
„ 36 *CI* „ 57.5 *D*
„ 39.8 *CI* „ 52 *D*



Ans. 100 : 1, 90 : 1, 75 : 1, 69 : 1, 57.5 : 1, 52 : 1 respectively.

(b) Proportion.

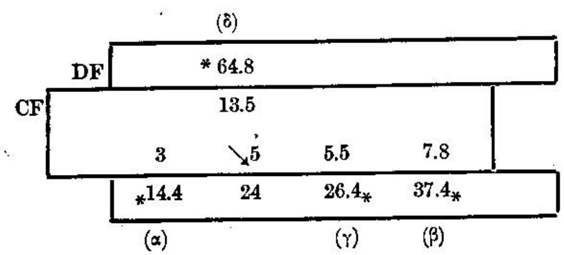
This is an application of combined multiplication and division. With the "Universal" slide rule having the (*CF*) and (*DF*) scales a wider range of problems can be handled easily and rapidly.

Example 12. $5 : 24 = 3. : x$ (α)
$= 7.8 : x$ (β)
$= 5.5 : x$ (γ)
$= 13.5 : x$ (δ)

They are to be transformed:

$\frac{24}{5} \times$ 3.0 = (α)
$\frac{24}{5} \times$ 7.8 = (β)
$\frac{24}{5} \times$ 5.5 = (γ)
$\frac{24}{5} \times$ 13.5 = (δ)

Set 5 *C* to 24 *D*, against 3.0 *C* read 14.4 *D* (α)
„ 7.8 *C* „ 37.4 *D* (β)
„ 5.5 *C* „ 26.4 *D* (γ)
„ 13.5 *CF* „ 64.8 *D* (δ)



Ans. (α) 14.4 (β) 37.4 (γ) 26.4 (δ) 64.8

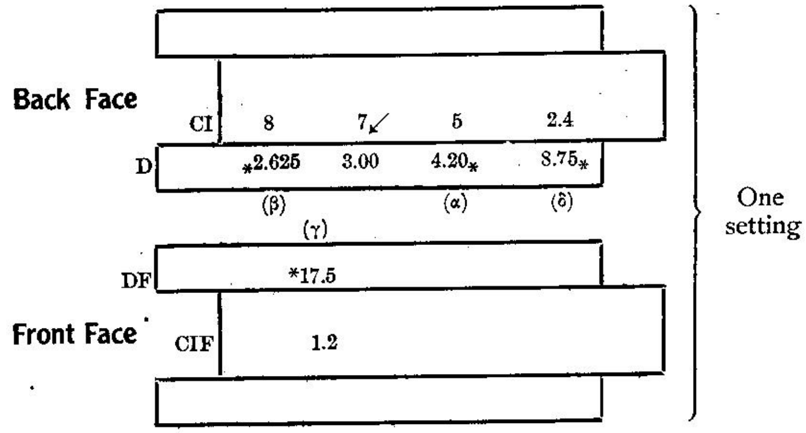
(c) Inverse Proportion.

With the old slide rule, we used to do inverse proportion with the slide inverted; but with the "Universal" slide rule, you can use either (CI) or (CIF) in addition to other scales without inverting the slide.

Example 13. A driving pulley of 7″ diameter is turning at a speed of 300 r. p. m., with what speeds will pulleys of the following diameters be driven by belt from the above driving pulley?
(1) 5″ diameter (2) 8″ diameter (3) 12″ diameter (4) 24″ diameter.
The turning speed of a pulley is inversely proportional to the diameter. The four parts of the problem could be rewritten:

$300 \times 7 \div$
- $5 =$ (α)
- $8 =$ (β)
- $12 =$ (γ)
- $24 =$ (δ)

Set 7 CI to 300 D, against 8 CI read 262.5 D (β)
„ 5 CI „ 420 D (α)
„ 24 CI „ 87.5 D (δ)
Then turn over the whole slide rule and against 12 CIF read 175 DF (γ)



Ans. (1) 420 r.p.m. (2) 262.5 r.p.m. (3) 175 r.p.m. (4) 87.5 r.p.m.

(d) For Statistics.

To convert statistical numbers into percents, is a form of proportion.

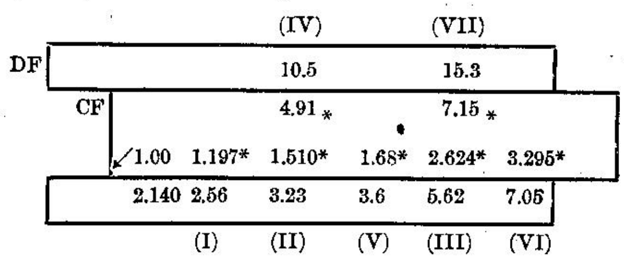
Example 14. Calculate the percentage of each item in the following statistical figures.

	256	(I)
	323	(II)
	562	(III)
	105	(IV)
	36	(V)
	705	(VI)
	153	(VII)
Total	2,140	

It is first to be rewritten :—

$$\left.\begin{matrix}256\\323\\562\\105\\36\\705\\153\end{matrix}\right\} \times 100 \div 2{,}140 = \left\{\begin{matrix}(I)\\(II)\\(III)\\(IV)\\(V)\\(VI)\\(VII)\end{matrix}\right.$$

Set 100 *C* to 2140 *D*, against	256 *D*	read	11.97 *C*	(I)
,,	323 *D*	,,	15.10 *C*	(II)
,,	36 *D*	,,	1.68 *C*	(V)
,,	562 *D*	,,	26.24 *C*	(III)
,,	705 *D*	,,	32.95 *C*	(VI)
,,	105 *DF*	,,	4.91 *CF*	(IV)
,,	153 *DF*	,,	7.15 *CF*	(VII)



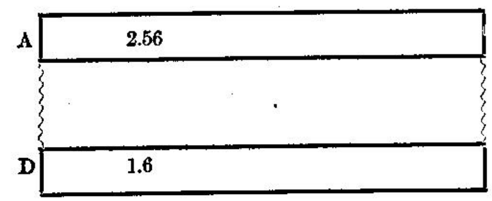
∴ *Ans.*			
	256	11.97%	(I)
	323	15.10 ,,	(II)
	562	26.24 ,,	(III)
	105	4.91 ,,	(IV)
	36	1.68 ,,	(V)
	705	32.95 ,, (shifted)	(VI)
	153	7.15 ,,	(VII)
Total	2,140	100.00%	

Section C. SQUARES AND SQUARE ROOTS

[1] **Squares :**—Squares can be obtained between (*A*) and (*D*) by the help of the hairline alone.

Example 1. $16^2 = 256$

Against 16 *D* read 256 *A*



[2] **Square Roots :**—The extraction of a square root is just the reverse of squaring. The only point that claims your attention is that you must place the given number in the proper section of (*A*).

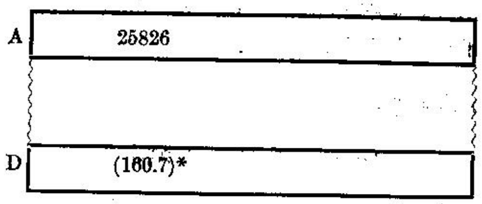
(Rule 1). Take the decimal point of the original number as the origin and mark all the digits into groops of two each, either to the right or to the left as the case may be, until you reach the first useful digit.

(Rule 2). If the first such group be of one digit, place the given number on the left section of (*A*).

(Rule 3). Should the first such group be of two digits, place the given number on the right section of (*A*).

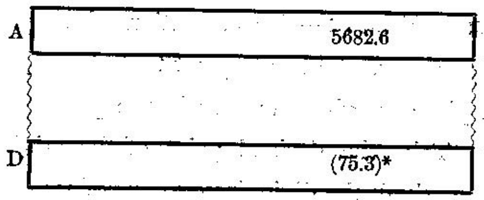
(Rule 4). The square root contains one figure for each group of two digits (or part of a group) contained in the original number.

For instance to get $\sqrt{25826}$, put 2,58,26. As the first group is of one figure, place the number on the left section of (A), put the hairline there and read 1607 on (D).



It must be 160.7 as there were three groups.

Another instance $\sqrt{5682.6}$. Put 56,82.6. As the first group is of two digits, put the hairline at the number on the right section of (A), and read 753 on (D).

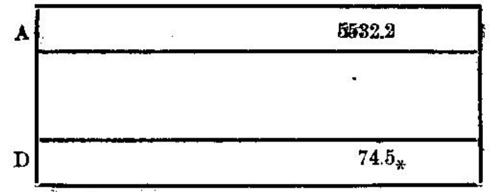


It must be 75.3 as there were two groups in the original number.

A decimal number 0.00065 is marked off as 0.00,06,5. As there is only one useful digit in the first group, place the hairline on the left section of (A), and you will find 256 on (D). In the original number there was one group of two noughts after the decimal point, there must be one nought in the square root. So the result is 0.0256.

Example 2. $\sqrt{5532.2}=74.5$

Mark the digits into groups of two figures each, starting from the decimal point, thus 55,32.2. By Rule 3, place the hairline at 5532.2 on the right section of (A) and read 745 on (D).



As the original number had two groups in the integral part, the square root must have two digits in the integral part. So the square root required is 74.5.

Example 3. $\sqrt{0.00035}=0.0187$.

Mark 0.00,03,5. By Rule 2, put the hairline at 350 on the left section of (A) to get a digit value 187. As the 03 is the second group, the square root required must be 0.0187.

❋

If $\sqrt{n}=a$, then $\sqrt{10\,n}=3.162\,a$

$$\sqrt{100\,n}=10\,a$$

$$\sqrt{1000\,n}=31.62\,a$$

$$\sqrt{10000\,n}=100\,a$$

$$\sqrt{100000\,n}=316.2\,a$$

$$\sqrt{1000000\,n}=1000\,a$$

$$\sqrt{10000000\,n}=3162\,a$$

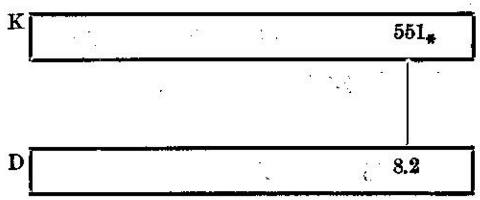
&c.

Section D. CUBES AND CUBE ROOTS

As (*A*) and (*D*) are to find squares and square roots, (*K*) and and (*D*) are used to find cubes and cube roots. (*K*) is to indicate at any point the value of the cube of a number represented by the point on (*D*) just below.

[1] Cubes.

Example 1. $8.2^3 = 551$. Against 8.2 *D* read 551 *K*



∴ *Ans.* 551

Note:—When the original number is of one digit in the integral part, the above method is simply adopted; but when the original number is of more than one place, the result thus obtained is to be multiplied by 1000 for increase of each place in the original. In the above example, if the original number should be 82 instead of 8.2, its cube then must be 551,000 instead of 551. Further should it be 820, then its cube must be 551,000,000.

[2] Cube Roots:—

The extraction of a cube root is just the reverse of finding a cube. The following rules are to be regarded for setting the hairline and decimalization.

(Rule 1). Starting from the decimal point either to the right or to the left, divide the digits by commas into groups of three figures each.

(Rule 2). If the first group be of one useful digit, the setting is to be done on the left section of (*K*) scale.

(Rule 3). If the first group be of two useful digits, the setting is to be done in the middle section of (*K*).

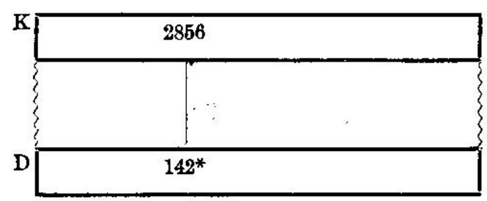
(Rule 4). If the first group be of three useful digits, the setting is to be done on the right section of (*K*).

(Rule 5). For decimalization, the number of groups of three digits and a part, if any, in the original number from the first group to the decimal point is the number of places in the cube root obtained, from the first useful digit to the decimal point.

Example 2. $\sqrt[3]{2856} = 14.2$

Divide the figures by comma into groups of three figures thus 2,856. As the first group is only of one useful digit, put the hairline at 2,857 in the left section of (*K*) according to Rule 2, and read under the hairline 142 on (*D*). By Rule 5, the cubic root required is 14.2.

Against 2856 *K* read 14.2 *D*

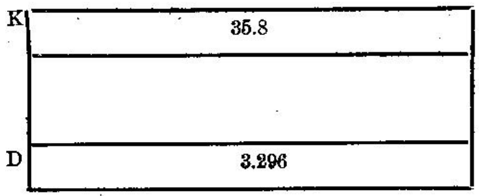


∴ *Ans.* 14.2

Example 3. $\sqrt[3]{0.0358}=0.3296$

Mark the digits into groups of three figures starting from the decimal point, thus 0.035,8. As there are two useful digits in the first group, set the hairline at 358 in the middle section of (*K*) according to Rule 3. By Rule 5, the cube root required must be 0.3296.

Against 0.0358 *K* read 0.3296 *D*

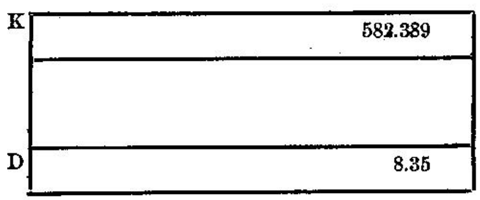


∴ *Ans.* 0.3296

Example 4. $\sqrt[3]{582389}=83.5$

As it is 582,389, so take the original number on the right section of (*K*).

Against 582,389 *K* read 83.5 *D*



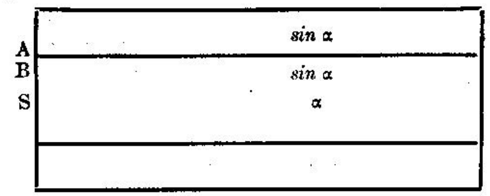
∴ *Ans.* 83.5

Section E. TRIGONOMETRICAL FUNCTIONS

[1] Sines.

Against *α-S* read *sin α-A*

Against *α-S* read *sin α-B*

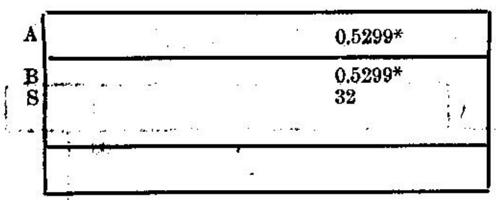


Notice here that the figures read on the lefthand section of either (*A*) or (*B*) start from the second decimal place, and those read on the right-hand section of either (*A*) or (*B*) start from the first decimal place.

Example 1. *sin* 32°=0.5299

Against 32 *S* read 0.5299 *A*

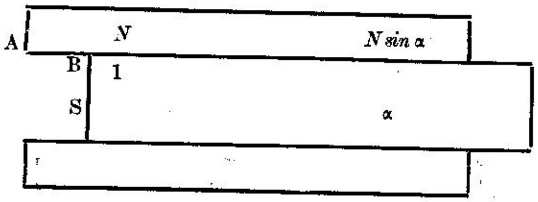
Or Against 32 *S* read 0.5299 *B*



∴ *Ans.* 0.5299

$N \sin \alpha$, $\frac{\sin \alpha}{N}$, $\frac{N}{\sin \alpha}$,

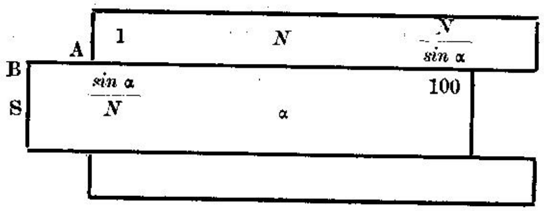
Put 1 B to N-A, against α-S read $N \sin \alpha$-A.



Put α-S to N-A, against 1 A read $\frac{\sin \alpha}{N}$-B

„ 100 B „ $\frac{N}{\sin \alpha}$-A

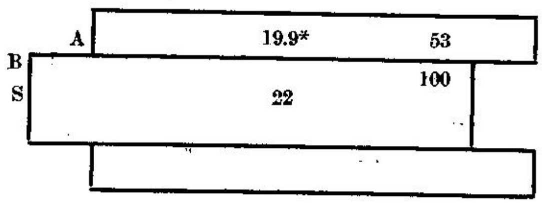
When the slide is projected to the right, 100 A and 1 B are for 1 A and 100 B respectively.



Thus you can calculate the three functions required.

Example 2. $53 \sin 22° = 19.9$

Set 100 B to 53 A, against 22 S read 19.9 A

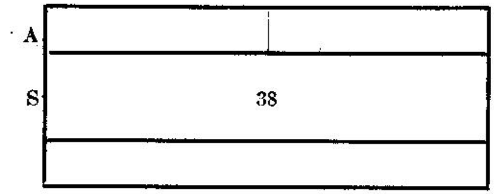


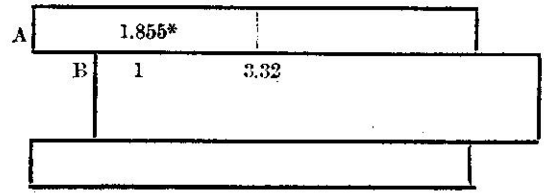
∴ *Ans.* 19.9

Example 3. $\frac{\sin 38°}{3.32} = 0.1855$

Put Hairline at 38 S

Set 3.32 B to Hairline, against 1 B read 0.1855 A

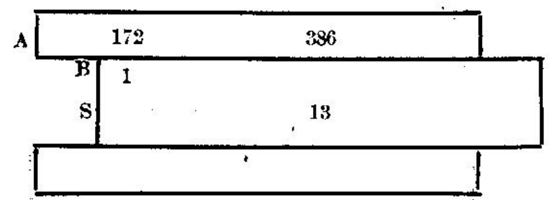




∴ *Ans.* 0.1855

Example 4. $\frac{38.6}{\sin 13°} = 172$

Set 13 S to 38.6 A, against 1 B read 172 A



∴ *Ans.* 172

[2] Cosines.

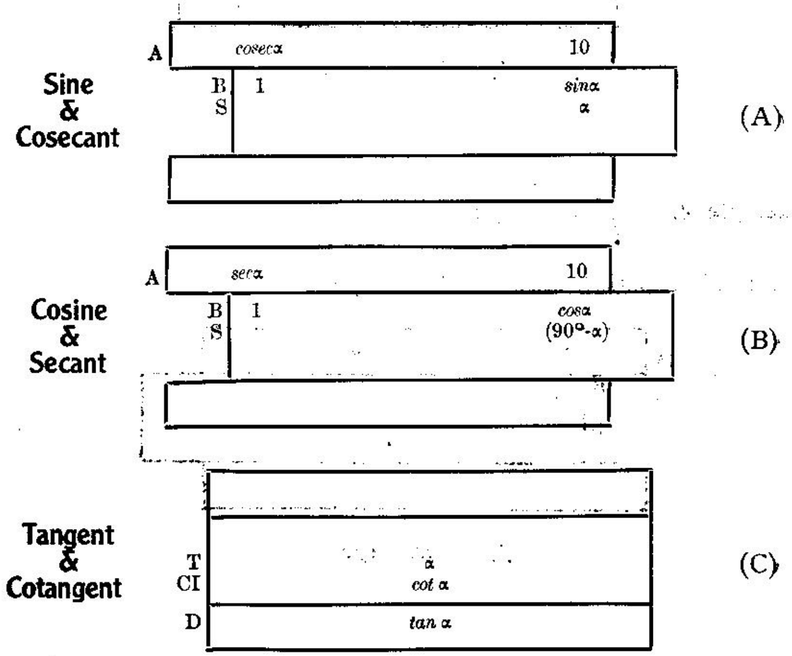
On the Mechanical Engineer's slide rule, there is no scale that gives *cos a* directly. But applying the trigonometrical formula

$$\cos a = \sin(90^\circ - a)$$

where a is a known angle, so $(90^\circ - a)$ is another known angle. Hence you can get $\cos a$ by looking for $\sin(90^\circ - a)$.

(To be continued from page 32.)

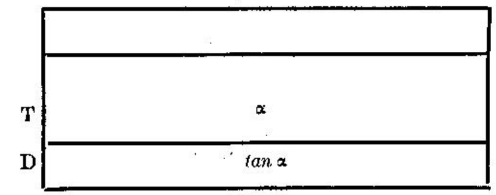
When a is given —



[3] Tangents.

To calculate *tang a*:

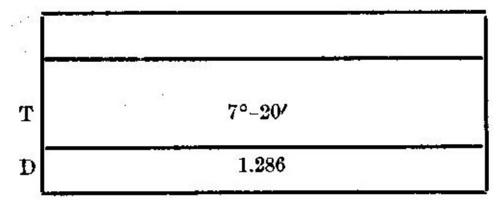
Against a-T read $\tan a$-D



Here the decimal point is to be shifted to the left just one place; *e.g.* 3.52 D should be understood for 0.352 for the value of the tangent.

Example 5. $\tan 7^\circ 20' = 0.1286$

Against $7^\circ 20'$ T, read 0.1286 D



∴ *Ans.* 0.1286

$$N \tan a, \qquad \frac{\tan a}{N}, \qquad \frac{N}{\tan a}$$

These can be had among (T), (C) and (D), just as you can get $N \sin a$, $\frac{\sin a}{N}$ and $\frac{N}{\sin a}$ among (S), (A) and (B).

[4] Cotangents, Secants, Cosecants.

These can be had indirectly as

$$cot\ \alpha = \frac{1}{tan\ \alpha},$$

$$sec\ \alpha = \frac{1}{cos\ \alpha}$$

$$cosec\ \alpha = \frac{1}{sin\ \alpha}$$

However, you can get *cot α* functions directly as per Diagram (C) page 30; and that on the slide only quite independently of the rule, so naturally without setting the slide.

Moreover, *sec α* and *cosec α* can be had very simply as per Diagrams (B) and (A), page 30, respectively.

Note *sin α*, *cos α* and *cot α* are found all on the slide, and do not require any setting of the slide. Only you are to read them out by the help of the hairline.

Again we must remind our readers that scales (S) and (T) on this slide rule are divided as per degrees and *minutes* and not in *decimals*. θ on the 10″ slide rule, stated in Chapter III and ($S\theta°$) and ($T\theta$) on the 20″ slide rule stated in Chapter IV are all in degrees and *decimals* and not in *minutes*.

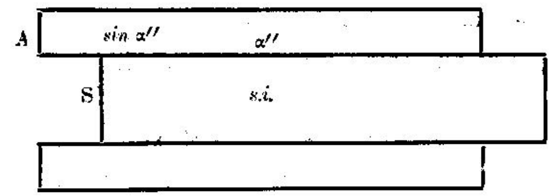
(To be continued on page 30.)

[5] Sines and Tangents of small angles.

For a small angle, say not larger than 6°, the sine is so nearly equal to the tangent that the difference is negligible.

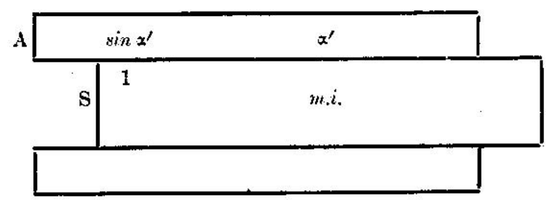
On (S) you will see an index near 1°–10′ which we call the "second index," and another index near 2° which we call the "minute index." These are for either the sine or the tangent of an angle expressed in seconds or in minutes respectively.

Set "second index"-S to α''-A, against 1 B read $sin\ \alpha''$-A
,, 1 B ,, $tan\ \alpha''$-A



Similarly for $tan\ \alpha''$. ∵ $tan\ \alpha'' = sin\ \alpha''$

Set "minute index"-S to α-A, against 1 B read $sin\ \alpha'$-A
,, 1 B ,, $tan\ \alpha'$-A



Similarly for $tan\ \alpha'$. ∵ $tan\ \alpha' = sin\ \alpha'$

Notice $\begin{cases} sin\ 1' = tan\ 1' = 0.0003 \\ sin\ 1'' = tan\ 1'' = 0.000005 \end{cases}$

Section F. CALCULATIONS OF CIRCLES

[1] The Circumference of a Circle.

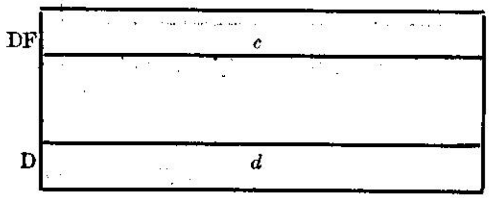
c, circumference of a circle,
d, diameter ,, the ,, , then

$$c=3.1416\,d$$

The constant 3.1416 is usually denoted by π.

You can calculate the circumference by multiplying π into d, as the Mechanical Engineer's Duplex slide rule has π indicated on (C), (D), (A) and (B). But you can do it in another way:

Against d on D read c on DF

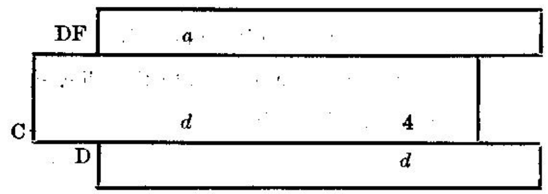


[2] The Area of a Circle.

a, area of a circle,
d, dia. ,, the ,, ,
r, rad. ,, ,, ,, , then

$$a=\frac{\pi}{4}d^2=0.7854\,d^2 \qquad \text{(I)}$$
$$=\pi\ r^2=3.1416\,r^2 \qquad \text{(II)}$$

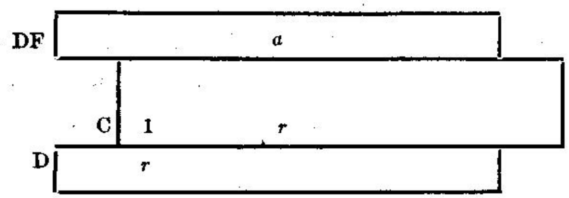
(I) Set 4 C to d-D, against d-C read d-DF



Proof: $4:d=d:\frac{a}{\pi}$

Whence $a=\frac{\pi}{4}d^2$

(II) Set 1 or 10 C to r-D, against r-C read a-DF

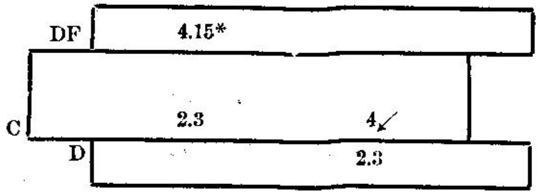


Proof: $1:r=r:\frac{a}{\pi}$

Whence $a=\pi\ r$

Example 1. Area of a circle with dia. 2.3 ft. *Ans.* 4.15 sq. ft.

Set 4 *C* to 2.3 *D*, against 2.3 *C* read 4.15 *DF*



$\therefore$ *Ans.* 4.15 sq. ft.

(*DF*) gives more accurate result than (*A*)

It is self-evident that we can calculate the diameter of a circle whose area is known by trial method.

Example 2. What is the diameter of a circle whose area is 4.15 sq. ft.?

Put Hairline to 4.15.*DF*

Set $\left\{\begin{array}{l}2.3\text{-}C \text{ to } 4.15\text{-}DF\\ 2.3\text{-}D \text{ to } 4\text{-}C\end{array}\right\}$ by trial (see Diagram in Example 1)

But there is a limit, $\pi<\text{area}<10\pi$; i.e. we can not do when $10\pi<\text{area}<100\pi$. Besides the method is too troublesome for practical use. So we advise you to use the other face. As for the area, the above method is preferred for accurancy.

[3] The Volume of a Cylinder.

v, volume of a cylinder
d, dia. „ the „
l, length „ „ „ , then

$$v=\frac{\pi}{4}d^2 l \qquad \text{(I)}$$

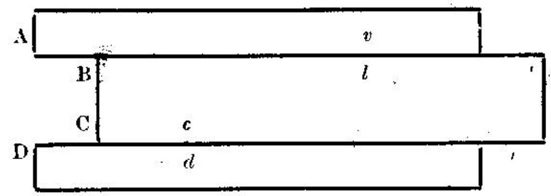
or

$$=al \qquad \text{(II)}$$

where *a* is the area of the cross section.

This is done best with the old slide rule.

(I) Set *c*-*C* to *d*-*D*, against *l*-*CF* read *v*-*DF*

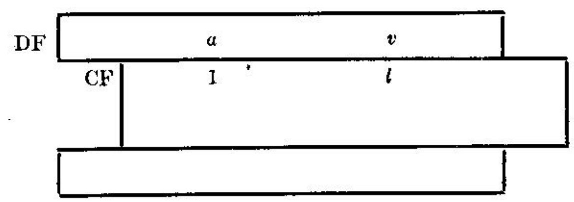


where $c=\sqrt{\frac{4}{\pi}}$ and is indicated on (*C*) and (*D*).

Proof: $c:d=\sqrt{l}:\sqrt{v}$

Whence $v=\frac{1}{c^2}d^2l=\frac{\pi}{4}d^2l.$

(II) Set 1 *CF* to *a*-*DF*, against *l*-*CF* read *v*-*DF*

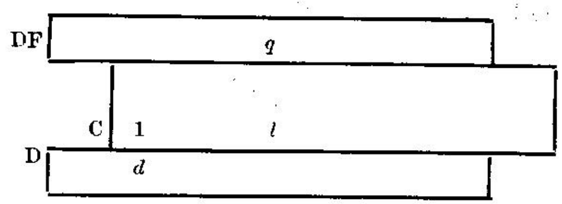


Ans. $v=al$, when *a* is given instead of *d*.

[4] The Area of the Side of a Cylinder.

q, area of the side of a cylinder, then $q=\pi dl$.

Set 1 C to d-D, against l-C read q-DF



Proof: $1 : d = l : \frac{q}{\pi}$

Whence $q=\pi dl$.

Section G. LOGARITHMS

[1] Logarithms.

When $a=10^x$, we say: "x is the logarithm of a on base 10." We write down also $log_{10}a=x$ or simply $log a=x$. The logarithm on base 10, is called the common logarithm. We know that

$$log 10 = 1$$
$$log 100 = 2$$
$$log 1000 = 3$$

Generally a logarithm consists of an integer and a decimal; the former is called the characteristic and the latter the mantissa.

You can get only the mantissa on the slide rule, and the characteristic is always determined by inspection. It is done by the following rules.

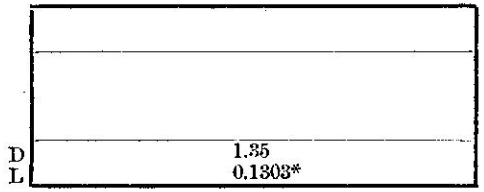
Rule 1. If the number whose logarithm is sought contains one or more integral figures, the characteristic is always one less than the number of integral figures in the original number, and is positive.

Rule 2. If the number is wholly a decimal, the characteristic of its logarithm is the same as the order number of the place from the decimal point which the first significant figure occupies, and is negative. In this case the negative sign is placed over the characteristic, to show that it alone is negative, the mantissa being always positive. For instance, $log 0.00798=\bar{3}.902$.

Example 1. $\log 1.35 = 0.1303$

Set Hairline to 1.35 D, under Hairline read 0.1303 L

or Against 1.35 D „ 0.1303 L

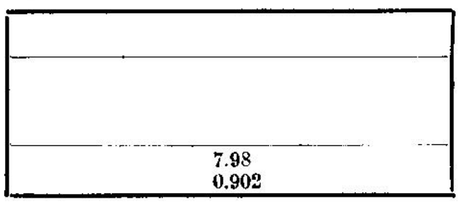


$\therefore$ *Ans.* 0.1303

For decimalization of the mantissa, keep the rule in mind that the whole length of (L) is 1, not 100.

Example 2. What is the number whose logarithm is $\overline{3}.902$?

Against 0.902-L read 7.98 D



$\therefore \quad \log 7.98 = 0.902$

$\therefore$ *Ans.* $\overline{3}.902 = \log 0.00798$

[2] Evolutions and Involutions.

It is well known that involution, a^n, and evolution, $\sqrt[n]{a}$, can be calculated easily by means of logarithms. Get the logarithm, $\log a$, of the original number, a. For a^n, multiply $\log a$ by n; for $\sqrt[n]{a}$, divide $\log a$ by n; and seek for the antilogarithm of the product or of the quotient.

$$a^n = \log^{-1}[n \times \log a]$$

$$\sqrt[n]{a} = \log^{-1}\left[\frac{1}{n} \times \log a\right]$$

Example 3. What is the sum of principal and interest for 5 years at 5% compounded annually?

Ans. 1.273 times the principal.

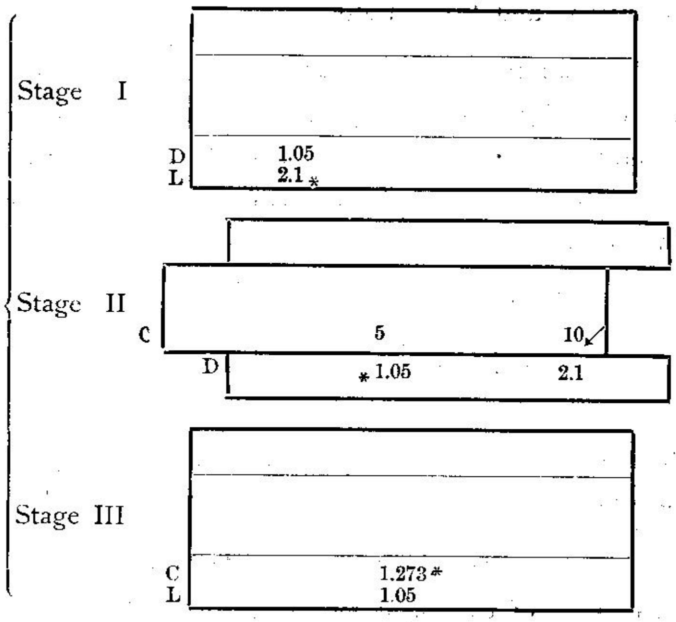
The calculation of compound interest is the typical one of involution of high powers, though you are to get the result only in round numbers. Here it is expressed by the equation

$$x = (1 + 0.05)^5$$
$$= 1.05^5$$

Stage I. Against 1.05 D read 0.021 L

Stage II. Set 10 C to 0.021 D, against 5 C read 0.105 D

Stage III. Against 0.105 L read 1.273 D



∴ *Ans.* $x = 1.273$

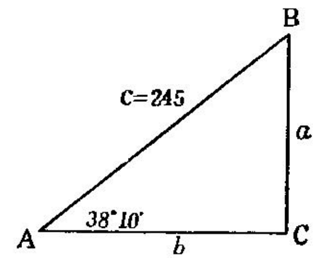
Note: Evolution and involution can be calculated as easily as multiplication and division by means of the "Universal" Electrical Engineer's slide rule, with the *log-log* scales which are to be explained later on.

It must be well understood that a slide rule gives answers in three or four places only. Hence it is perfectly good for all evolutions; but it gives only the round numbers of the answers sought for involutions of large numbers. This sometimes fails a bank clerk. Be satisfied with round numbers.

Section H. SOLUTION OF TRIANGLES

(i) Right Triangles.

Example: Given an acute angle A and the Hypotenuse c.



Let $A = 38°10'$ and $c = 245$.

Solution: $B = 90° - A = 51°50'$.

Use the formulae.

$$\frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}$$

or

$$\frac{245}{\sin 90°} = \frac{a}{\sin 38°10'} = \frac{b}{\sin 51°50'}$$

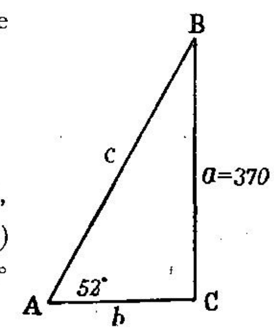
Set the right index on (S) to $c = 245$ on (A), read $a = 15.3$ and $b = 19.27$ on (A) respectively, against $38°10'$ and $51°50'$ on (S).

Example: Given an Acute angle A and the opposite side a.

Let $A = 52°$ and $a = 370$.

Solution: $B = 90° - 52° = 38°$.

Set $52°$ on (S) to 370 on (A), read $c = 470$ and $b = 289$ on (A) respectively against $90°$ and $38°$ on (S).



and then, Set $\frac{A+B}{2}=25°$ on (T) to $a+b=9.5$ on (D). Read $\frac{A-B}{2}=7°$ on (T) against $a-b=2.5$ on (D).

Solving two equations.

$$\frac{A+B}{2}=25° \text{ and } \frac{A-B}{2}=7°$$

Get $A=32°$ and $B=18°$

finally, By the theory of sine proportion as previous manner.

Set $A=320$ on (S) to $a=b$ on (A). Read $c=8.68$ on (A) against $A+B=50°$ on (S).

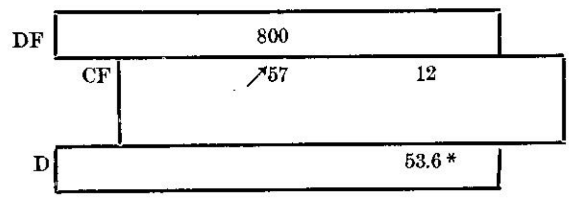
Section I. APPLICATION TO MECHANICS

The slide rule is so extensively used that we can not explain all cases. Here we shall deal with only a few instances.

Example 1. A revolving grinder should best turn at a lineal speed of 800 ft. per min. If the dia. be 57″, what shall be the number, of revolutions per min.?

$$x=800\div\left(\pi\frac{57}{12}\right)=800\times\frac{12}{57}\times\frac{1}{\pi}$$

Set 57 CF to 800 DF, against 12 CF read 53.6 D



$\therefore$ $Ans.=53.6$ r.p.m.

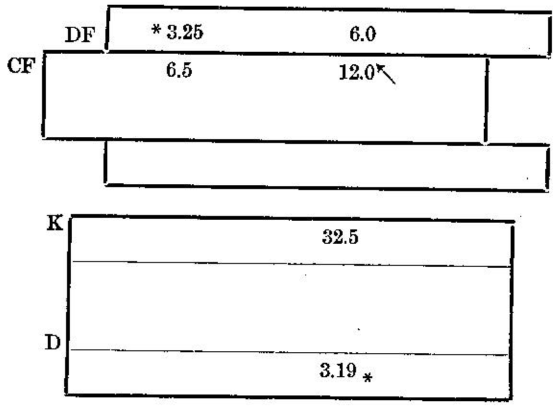
Example 2. To design a shaft to transmit 60 HP at a speed of 120 revolutions per min. What shall be the dia. in inches?

(I) By formula, $d=\sqrt[3]{\frac{65\,h}{n}}$

where d, the dia. required in inches,
h, the horse power to be transmitted,
n, the number of revolutions per min.

$$\therefore \quad d=\sqrt[3]{\frac{65\,h}{n}}=\sqrt[3]{\frac{65\times60}{120}}$$

Set 120 *CF* to 60 *DF*, against 65 *CF* read 32.5 *DF*
„ 32.5 *K* „ 3.19 *D*



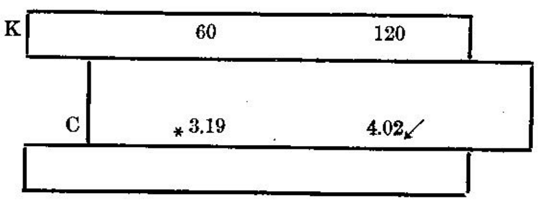
∴ *Ans.* 3.19″ ≒ $3\frac{1}{4}$″ say

(II) There is another way, a better way:
Use the formula transformed

$$d=4.02\sqrt[3]{\frac{h}{u}}$$

For this example $d=4.02\sqrt[3]{\frac{60}{120}}$

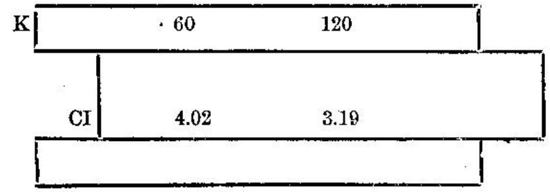
Set 4.02 *C* to 120 *K* against 60 *K* read 3.19 *C* (The slide turned over)



∴ *Ans.* 3.19″ ≒ $3\frac{1}{4}$″

The method II requires turning over either the slide only or the whole slide rule, as you see in the figures, (*K*) is on the back face while (*C*) is on the front. There is another way to do away with this trouble: take (*CI*) in place of (*C*).

(III) Set 4.02 *CI* to 60 *K* against 120 *K* read 3.19 *C*.



∴ *Ans.* 3.19″ = $3\frac{1}{4}$″

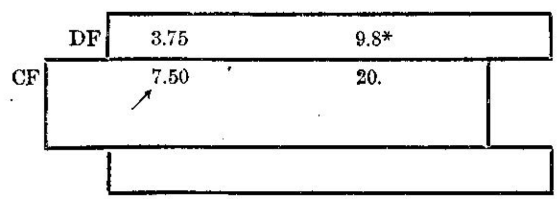
Example 3. A double belt, *w* inches wide runs at a lineal speed of *v* ft. per min. to transmit *h* horse powers, with a relation:

$$w=\frac{375h}{v}$$

If *h*, 20 *HP*,
v, 750 ft.,
what shall be the width?

$$w=\frac{375\times 20}{750}$$

Set 750 *CF* to 375 *DF* against 20 *CF* read 9.8 *DF*



Ans. 10″ (≒ 9.8″)

CHAPTER III.

ELECTRICAL ENGINEER'S "UNIVERSAL" DUPLEX SLIDE RULE WITH PATENT VECTOR AND LOG-LOG SCALES

Section A. SCALES DESCRIBED

The type of this slide rule is Duplex, just like the Mechanical Engineer's Slide Rule; see Figs. 3 and 4. Fig. 3 shows the front face of this slide rule, and Fig. 4 the back face. The cursor is also of the same type as that on the Mechanical Engineer's Slide Rule. As you see in the figures the slide rule has on its front face, (K), (A), (D) and (T) on the rule, and (B), (CI) and (C) on the slide.

Among the seven scales, (T) is entirely a new scale, but other six scales are familiar with old slide rules. (A), (B) and (C), (D) are all logarithmic scales, the former two of two-sections, the latter two of one-section each. They do all serve for Multiplication, Division, Proportion, Squares and Square Roots, Circles, etc. (K) is a logarithmic scale of three-sections; and works with (C) or (D) to give x^3, $\sqrt[3]{x}$ or ax^3, $a\sqrt[3]{x}$.

(CI) is the inverted scale of (C); it works with (D) to give the reciprocal of a number. It also gives abc at one setting when it works with (C) and (D) in cooporation.

(T) is to give $tan\theta$ with respect to $\theta°$ or $R\theta$ on (θ) or on $(R\theta)$ respectively. It is entirely different from the (T) scale that was familiar with the old slide rules. For further particulars, see Section B, [3] "Trigonometrical Functions," p. 54.

On the back face, this slide rule has (θ), $(R\theta)$, (P), (LL_3), (LL_2) and (LL_1) on the rule, and (Q) (Q') and (C) on the slide.

Among the nine scales, (C) is entirely the same as (C) on the front face, a one-section longarithmic scale.

(LL_1), (LL_2), (LL_3) are dived as per *log-log* x, x ranging from 1.01 to 22000; which is cut off into three pieces at $\varepsilon^{0.1}$ and ε. Like the *log-log* scales in the old slide rules they are to serve with (C) to do involution and evolution.

But as they are cut off at $\varepsilon^{0.1}$ and ε, they can give natural logarithms of a given number without having the slide set in any way.

By setting the index of (C) to 10 on (LL_3) you can get the common logarithm of a given number, or $log_{10}x$. Thus you can get the logarithm of any number on any base.

Another advantage of this slide rule is this. The old slide rule with *log-log* scales had its scales divided only from 1.1 and upward, and naturally was of little use for the calculation of compound interest while this slide rule has its *log-log* scales divided for the range of x 1.01–22,000.

(θ), $(R\theta)$, (P), (Q) and (Q') are entirely new scales, and in them the uniqueness of this slide rule does lie.

Here we shall give the outline of the scales, but for particulars we shall dwell upon later.

(P) and (Q) are the most important and prominent scales of all; they are both divided as per x^2, x ranging from 0 to 10. They are non-logarithmic. They are called either "vector scales" or "square scales." (Q') is nothing but the extention of (Q), x ranging from 10. to 14.14.

When you know two sides of a right triangle, you are to

Section B. THE USE OF SCALES

[1] Multiplication, Division, Proportion, Squares and Square Roots, Cubes and Cube Roots.

All these could be disposed of among (*A*), (*B*), (*C*), (*D*), (*K*) and (*CI*) scales just as in the ordinary slide rules. And we shall not dwell upon these subjects here, as we have stated already in Chapter II.

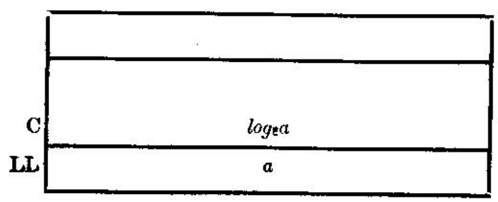
[2] Logarithms.

With the ordinary slide rule, the common logarithm could be obtained by means of the (*L*) scale. With the "Universal" slide rule we could compute out generally the logarithm on any base among (*C*) and (LL_3), (LL_2), (LL_1). Here we shall explain the Natural and the Common Logarithms only.

(a) **Natural Logarithms** (on base, $\varepsilon=2.71828$).

Set the slide end by end with the rule, put the hairline at a number on any of (LL_3), (LL_2) or (LL_1) according to the magnitude of the number, and read the natural logarithm of the number on (*C*).

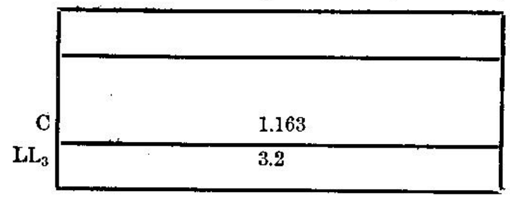
Against *a*-*LL* read $log_\varepsilon a$ *C*



Here keep in mind that the readings on (*LL*) must be taken just as they are described, and not universal to the congruent numbers of similar digit value, as it is the case with the ordinary logarithmic scales. Another point: "*LIC*" or the left index of (*C*) corresponds with 1.0, 0.1, 0.01 on LL_3, LL_2, LL_1 respectively.

Example 1. $log_\varepsilon 3.2=$[1.163 as below]

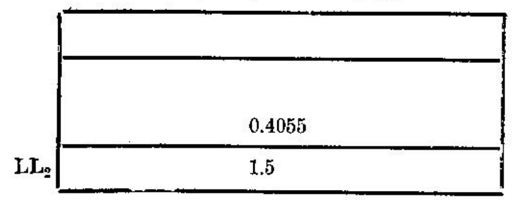
Against 3.2 LL_3 read 1.163 *C*



∴ *Ans.* 1.163

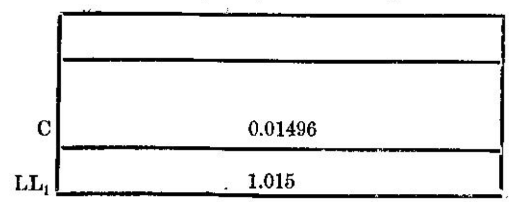
Example 2. $log_\varepsilon 1.5=$[0.4055 as below]

Against 1.5 LL_2 read 0.4055 *C*



Example 3. $log_\varepsilon 1.015=$[0.01496 as below]

Against 1.015 LL_1 read 0.01496 *C*

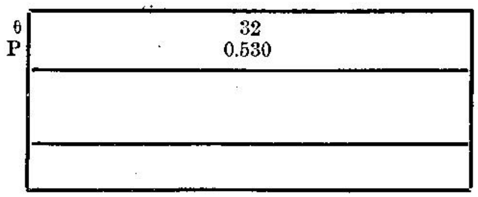


[3] Trigonometrical Functions.

In the old slide rule, you found only the sine scale, (S), and the tangent scale, (T). The cosine, which is exceedingly important for electrical engineers, was calculated indirectly by $\cos\theta = \sin(90° - \theta)$. With this slide rule you can get it all at once with $\sin\theta$, $\tan\theta$. You know electrical engineers are compelled to compute out $\cos\theta$, when you have $\tan\theta$ only. It is really a hard and complicated job with the ordinary slide rule, and naturally inaccuracy is the result. With this slide rule, as we have already stated you can get $\sin\theta$, $\cos\theta$, $\tan\theta$, θ, $R\theta$ all at once or simultaneously if you ever have but one of the five values given. So naturally you can get them with rapidity and accuracy.

Example 7. $\sin 32° = 0.530$

Against 32 θ read 0.530 P

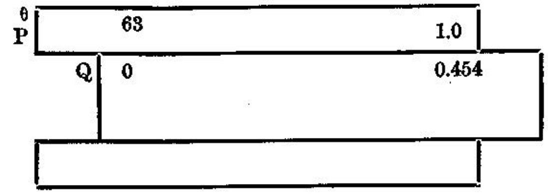


∴ *Ans.* 0.530

Note, for Trigonometrical functions, take the whole length of (P), (Q), (Q') as unit so that the highest figure read on the scale is on the first decimal place.

Example 8. $\cos 63° = 0.454$

Set 0 Q to 63 θ against 1.0 P read 0.454 Q



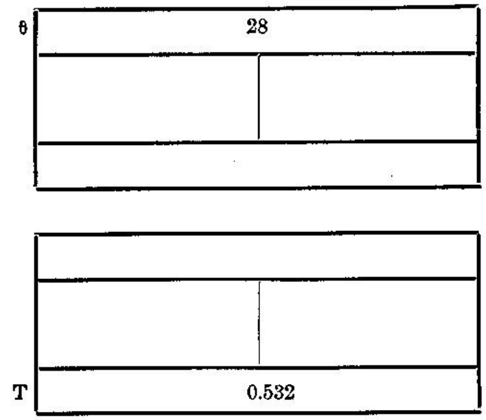
∴ *Ans* 0.454

Example 9. $\tan 28° = 0.532$

Against 28 θ read 0.532 T on the other side of the slide rule

Back Face

Front Face



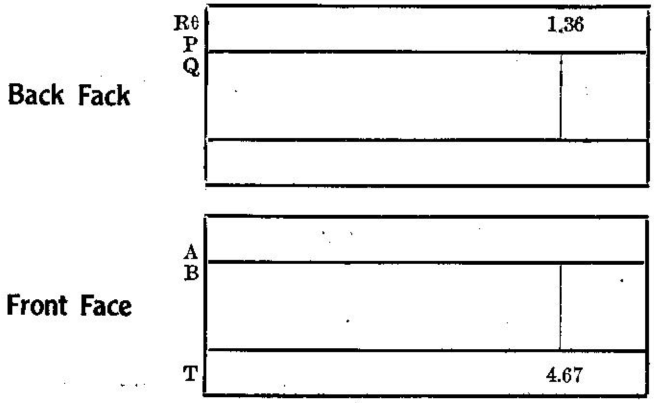
Ans. 0.532

The above cases are for the degrees and decimals of angle, but for angles in radians, only you have got to take ($R\theta$) in place of (θ).

Example 10. $\tan(1.36) = 4.67$

Where ( ) means that the angle is in radians.

Against 1.36 $R\theta$ read 4.67 T on the other side of the slide rule

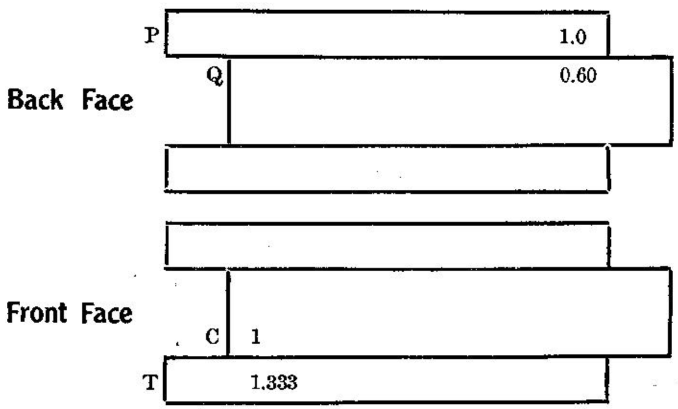


∴ $\tan(1.36)=4.67$

When the angle is obtuse, transform first either $\sin\theta=\cos(\theta-1.57)$ or $\cos\theta=\sin(\theta-1.57)$, as $(1.57)=90°$.

Example 11. To get $\tan\theta$, when $\cos\theta=0.6$

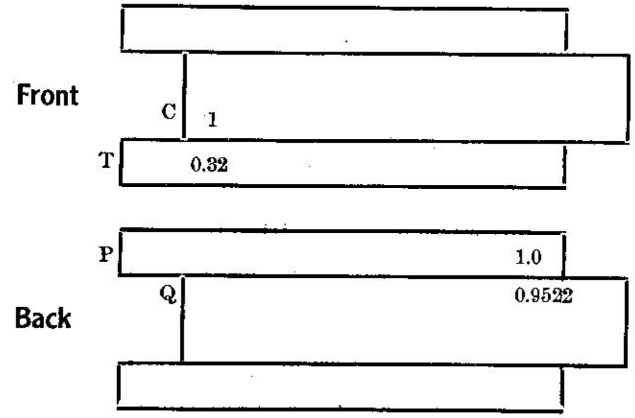
Set 0.6 Q to 1.0 P against 1 C read 1.333 T



∴ *Ans.* $\tan\theta=1.333$

Example 12. To calculate $\cos\theta$, when $\tan\theta=0.32$

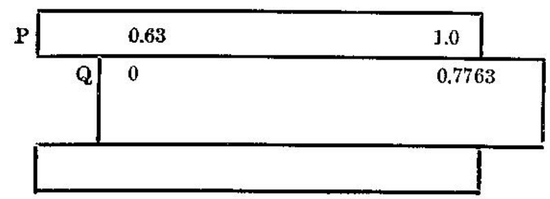
Set 1.0 C to 0.32 T against 1.0 P read 0.9522 Q



∴ $\cos\theta=0.9522$

Example 13. To get $\cos\theta$ when $\sin\theta=0.63$

Set 0 Q to 0.63 P against 1.0 P read 0.7763 Q



∴ $\cos\theta=0.7763$

This slide rule is also good for hyperbolic functions, as what (Q') is to $\cosh\theta$ is (Q) to $\sinh\theta$.

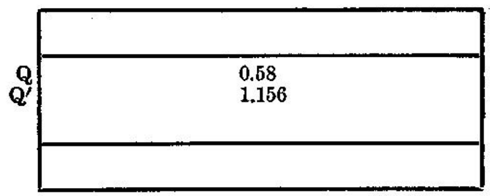
Here you cannot calculate out the angle, but you can get the relation between $\sinh\theta$ and $\cosh\theta$ within the limit of some 0°–80°. In practice, keep in mind the formula :—

$$\cosh^2\theta = 1 + \sinh^2\theta$$

or $$\cosh\theta = \sqrt{1 + \sinh^2\theta}$$

Example 14. To get $\cosh\theta$, when $\sinh\theta = 0.58$

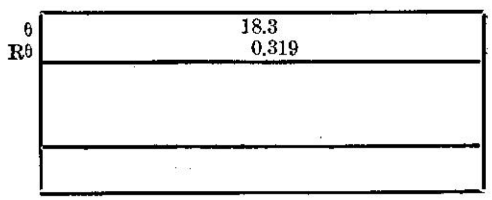
Against 0.58 Q read 1.156 Q'



$\therefore$ $\cosh\theta = 1.156$

Example 15. Convert 18.3° into radians.

Against 18.3 θ read 0.319 $R\theta$



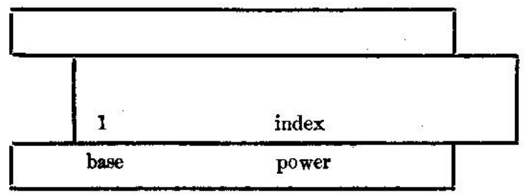
$\therefore$ *Ans.* 0.319 radian.

You could do this between (C) and (D) quite as well.

[4] Involutions and Evolutions.

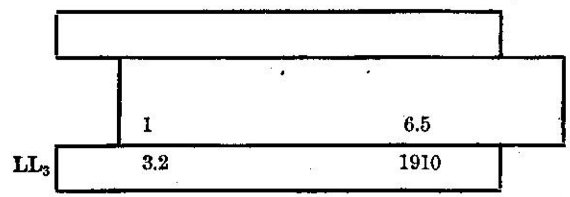
In the old slide rule, we had the *log-log* scales only for a limit 1.1—20,000. This new slide rule has a wide limit of 1.01—22,000. So with the old slide rules you could not calculate such functions as the compound interests when the rate is lower than 10%. Here you can do it with this new slide rule for the rate as low as 1%.

In this slide rule, the *log-log* scale is in three parts; and graduated as per $\log_\varepsilon(\log_{10}x)$ or $\log_\varepsilon(x\text{-}C)$. Here ($x$-$C$) means a reading, x, on the scale (C). Hence, ε^n, which so often occurs in electricians' computation, could be had without setting the slide. Generally :—



Example 16. $3.2^{6.5} = 1910$

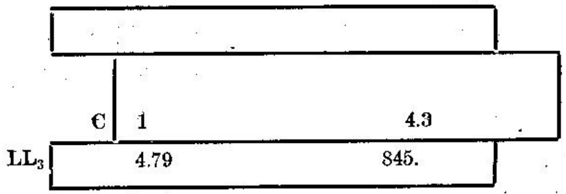
Set 1 C to 3.2 LL_3 against 6.5 C read 1910 LL_3



$\therefore$ *Ans.* 1910

Example 17. $\sqrt[4.3]{845}=4.79$

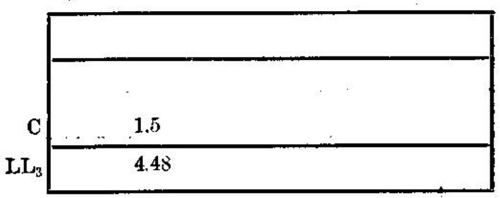
Set 4.3 C to 845 LL_3 against 1 C read 4.79 LL_3



∴ *Ans.* 4.79

Example 18. $\varepsilon^{1.5}=4.48$

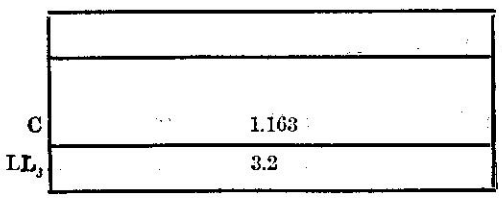
Against 1.5 C read 4.48 LL_3



∴ *Ans.* 4.48

Example 19. log_e 1.163=[3.2 as helow]

Against 1.163 C read 3.2 LL_3



∴ *Ans.* 3.2

For decimalization of ε^n, keep in mind that

Left end of $LL_3=\varepsilon^{1.0}=2.717$

" " " $LL_2=\varepsilon^{0.1}=1.105$

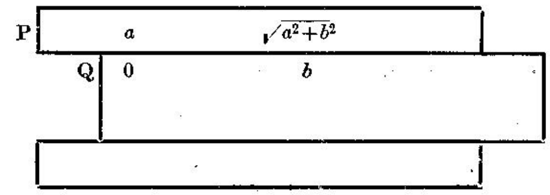
" " " $LL_1=\varepsilon^{0.01}=1.010$

[5] Calculations on a Right Triangle.

The sides of a right triangle, is calculated by $\sqrt{a^2\pm b^2}$ according to the Pythagorian theorem; and for the calculation of such functions there is nothing like this patent slide rule. The calculation is done between (P) and (Q), (Q').

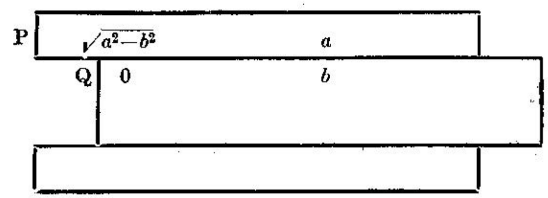
For $\sqrt{a^2+b^2}$.

Set 0 Q to a P against b Q read $\sqrt{a^2+b^2}$ P



For $\sqrt{a^2-b^2}$.

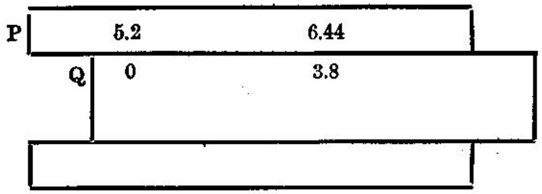
Set b Q to a P against 0 Q read $\sqrt{a^2-b^2}$ P



Sometimes you will have to take (Q') in place of (Q). When the factors and the result are of the same number of places, the above method of using (Q) is all right. But when they are of different numbers, (Q') must be used. See Example 21. Extreme cases require further concideration.

Example 20. $\sqrt{5.2^2+3.8^2}=6.44.$

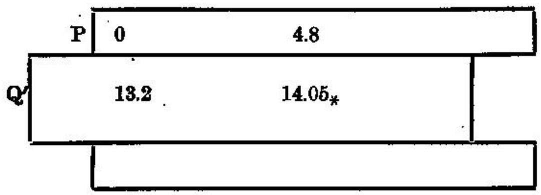
Set o Q to 5.2 P against 3.8 Q read 6.44 P



Ans. 6.44

Example 21. $\sqrt{13.2^2+4.8^2}=14.05.$

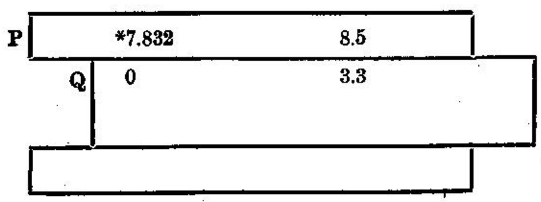
Set 13.2 Q' to o P against 4.8 P read 14.05 Q'



Ans 14.05

Example 22. $\sqrt{8.5^2-3.3^2}=7.832.$

Set 3.3 Q to 8.5 P against o Q read 7.832 P

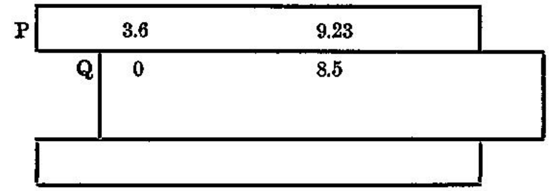


Ans. 7.832

Example 23. In a right triangle, $a=8.5$ ft.; $b=3.6$ ft.; what is the length of the hypotenuse, c in feet?

As $c=\sqrt{a^2+b^2}$.

Set o Q to 3.6 P against 8.5 Q read 9.23 P



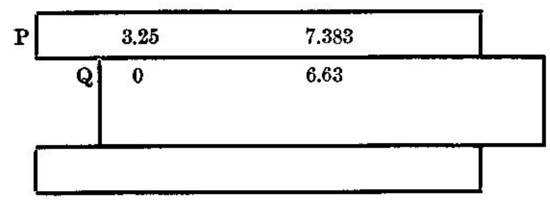
Ans. 9.23 ft.

Example 24. What is the absolute value of the compound number, $3.25+j6.63$ (where $j=\sqrt{-1}$)

The absolute value of $(a+jb)$ is $\sqrt{a^2+b^2}$.

Hence

Set o Q to 3.25 P against 6.63 Q read 7.383 P



Ans. 7.383

Section C. APPLICATIONS TO HIGHER MATHEMATICS

[1] On Compound Numbers.

(a) The absolute value, $\sqrt{a^2+b^2}$, of a compound number, $(a+jb)$.

$0\ Q$ to $a\ P$ against $b\ Q$ read $\sqrt{a^2+b^2}\ P$

just as per Example 23 in the preceding Section.

(b) To convert a compound number $(a\pm jb)$ into the form of $A\,(cos\theta \pm j sin\theta)$.

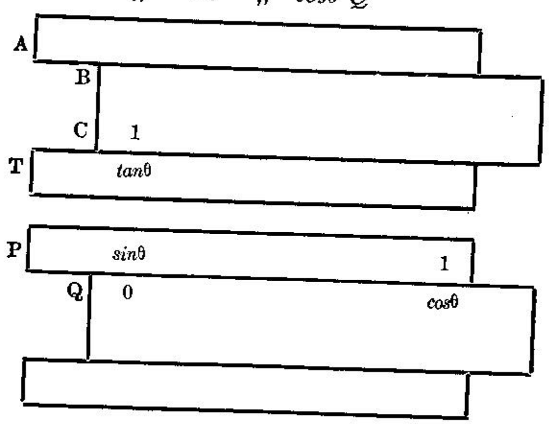
By the formulae, $A=\sqrt{a^2+b^2}$, $tan\theta=\dfrac{b}{a}$

A is obtained by the method already stated in (a).

For $sin\theta$ and $cos\theta$, first get $\dfrac{b}{a}$ or $tan\theta$ between (C) and (D).

Set 1 C to $tan\theta\ T$ against $0\ Q$ read $sin\theta\ P$

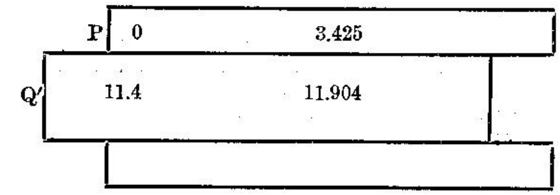
„ 1 P „ $cos\theta\ Q$



Example 1. To convert $(6.85+22.8j)$ into the form of $A\,(cos\theta+j\,sin\theta)$.

(i) As $\sqrt{6.85^2+22.8^2}$ is rather awkward to attack it should be converted first into $\sqrt{4}\,\sqrt{3.425^2+11.4^2}$.

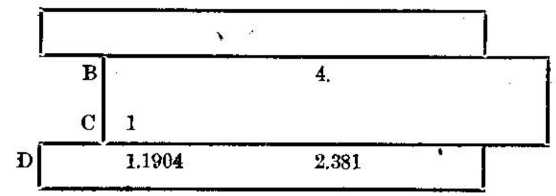
Set 11.4 Q' to 0 P against 3.425 P read 11.904 Q'



$\therefore\ \sqrt{3.425^2+11.4^2}=11.904$ or by inspection $A=23.808$

For $11.904\times\sqrt{4}=A$ on the slide rule :—

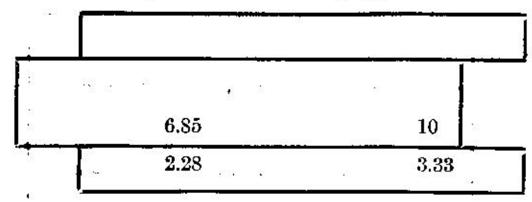
Set 1 C to 11.904 D against 4 B read 23.81 D



$\therefore$ *Ans.* $A=23.81$

(ii) For $\dfrac{22.8}{6.85}$ [or $=tan\theta$]

Set 6.85 C to 22.8 D against 10 C read 3.33 D



$$\therefore\ tan\theta=\frac{22.8}{6.85}=3.33$$

(iii) For $sin\theta$ and $cos\theta$, we are to calcurate $tan^{-1}3.33$:

Against 3.33 T read 73.2 θ on the other side of the slide rule.

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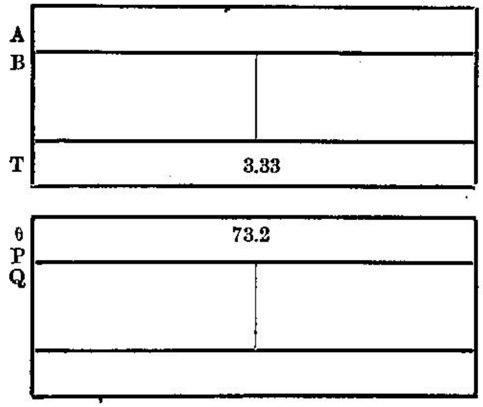


$\therefore$ *Ans.* $23.81\ (cos\ 73.2^\circ + jsin\ 73.2^\circ)$

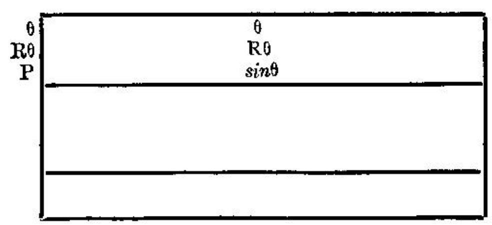
(c) $A(cos\theta \pm jsin\theta)$ to be converted into the form $(a \pm jb)$. This is very simple for $a = A \times cos\theta$, $b = A \times sin\theta$

(d) $A(cos\theta \pm jsin\theta)$ to be converted into the form $A_\varepsilon^{j\theta}$.

(d′) When $cos\theta$ and $sin\theta$ are expressed in terms of θ: You can very simply write down $A_\varepsilon^{j\theta}$.

(d″) When $cos\theta$ or $sin\theta$ are given instead of θ:

Against $sin\theta$ P read θ-θ; and also read Rθ-$R\theta$



Hence you have either θ or Rθ. Then treat exactly as (d′).

(e) To get $(a_1+jb_1)(a_2+jb_2)$

Say $(a_1+jb_1) = A_1 \varepsilon^{j\theta_1}$

$(a_2+jb_2) = A_2 \varepsilon^{j\theta_2}$

Then $(a_1+jb_1)(a_2+jb_2) = A_1 A_2 \times \varepsilon^{j(\theta_1+\theta_2)}$

If you should have the result in the form of a compound number, then you can have

$$(a_1+jb_1)(a_2+jb_2) = A_1A_2[cos(\theta_1+\theta_2) + jsin(\theta_1+\theta_2)]$$

Example 2. $(5.8 + j3.2) \times (6.8 + j2.6)$

Similarly as Example 1 :—

$$5.8 + j3.2 = 6.622(cos\ 28.85^\circ + jsin\ 28.85^\circ)$$

$$6.8 + j2.6 = 7.28(cos\ 20.9^\circ + jsin\ 20.9^\circ)$$

$\therefore$ $A = 6.622 \times 7.28 = 48.2$

$\theta = 28.85^\circ + 20.9^\circ = 49.75^\circ$

$\therefore$ *Ans.* $48.2_\varepsilon{}^{j49.75^\circ}$ Q.E.I.

$48.2\ cos\ 49.75^\circ = 31.2$

$48.2\ sin\ 49.75^\circ = 36.8$

$\therefore$ *Another Ans.* $(31.2 + j36\ 8)$ Q.E.I.

(f) The quotient of complex numbers can easily be calculated by the reverse method.

To get $\dfrac{a_1+jb_1}{a_2+jb_2}$

$$\frac{a_1+jb_1}{a_2+jb_2} = \frac{A_1}{A_2} \times \varepsilon^{j(\theta_1-\theta_2)}$$

Example 3. $\dfrac{(6.336+j5.22)}{(5.8\quad+j0.56)}$

Just as in the previous example :—

$$6.336+j5.22=8.207\varepsilon^{j(0.69)}$$

(The angle here has been computed out in radians)

$$5.8+j0.56=5.83\varepsilon^{j(0.096)}$$

$$A=\frac{8.207}{5.83}=1.407$$

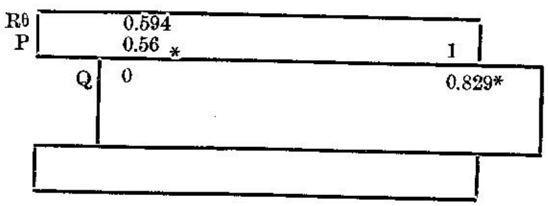
$$\theta=0.69-0.096=(0.594)$$

$\therefore$ *Ans.* $1.407\varepsilon^{j(0.594)}$

To convert this form of a vector into the form of $(a+jb)$:—
This is nothing but (c).

Against 0.594 $R\theta$ read 0.56 P

Set 0 Q to 0.594 $R\theta$ against 1.0 P read 0.829 Q



$\therefore$ $sin\theta=0.56$

$cos\theta=0.829$

$\therefore$ $a\ =1.407\times0.829=1.166$

$b\ =1.407\times0.56\ =0.788$

$\therefore$ *Ans.* $1.166+j0.788$

Thus you can get generally :—

$$(a_1+jb_1)\times(a_2+jb_2)^{\pm1}=A_1A_2^{\pm1}\times\varepsilon^{j(\theta_1\pm\theta_2)}=(a+jb)$$

[2] Examples of Application to Electrical Calculations.

Example 4. There is a voltage wave with higher frequencies.

The effective value of the fundamental wave, $Ee_1=82$,

Ditto of the third hermonic wave, $Ee_3=25$,

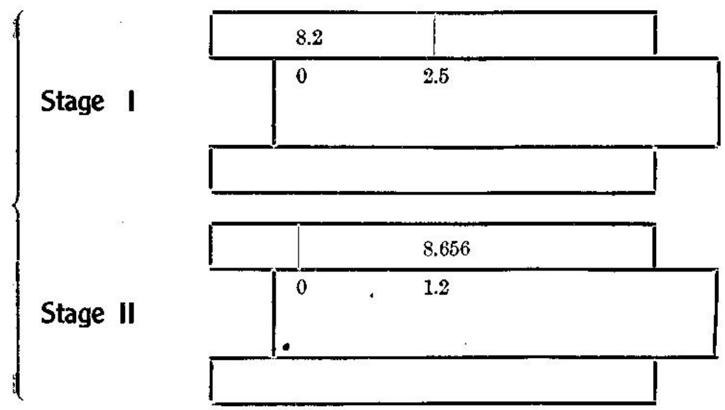
Ditto of the fifth hermonic wave, $Ee_5=12$.

What is the effective value of the distorted wave?

$$Ee_0=\sqrt{Ee_1^2+Ee_3^2+Ee_5^2}$$

$$=\sqrt{82^2+25^2+12^2}$$

Set 0 Q to 82 P against 25 Q put Hairline
Set 0 Q to Hairline against 12 Q read 86.56 P



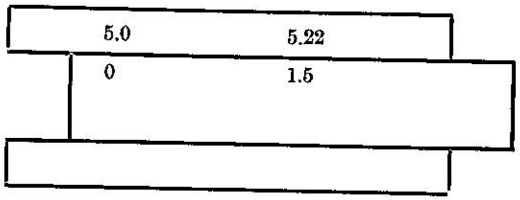
$\therefore$ $Ee_0=86.56$

Example 5. A circuit whose Impedance is $(2+j3.2)$, is subjected to an *E.M.F*, $(50+j15)$ between both ends. What is the current ?

$$\dot{I}=\frac{\dot{E}}{\dot{Z}}=\frac{50+j15}{2+j3.2}$$

(i) $(50+j15)$ to be converted into the form of $A_1|\underline{\theta_1}$.

Set 0 Q to 5 P against 1.5 Q read 5.22 P

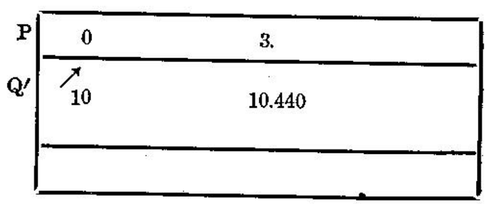


$\therefore\ A_1=5.22$

[Note: for accuracy, the following method is preferred:

$$\tfrac{1}{2}\sqrt{100^2+30^2}$$

Against 3 P read 10.440 Q'

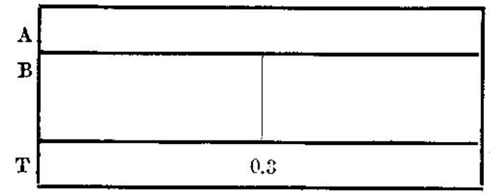


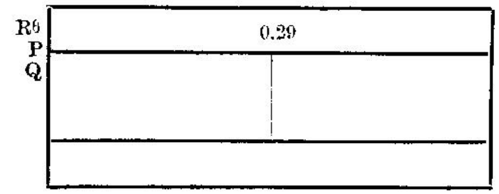
$A_1=104.40\div 2=52.20$

Four places have been made clear.]

As $\tan\theta_1=\frac{15}{50}=0.3$

Against 0.3 T read 0.29 $R\theta$



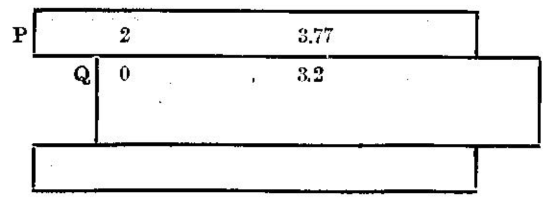


$\therefore\ R\theta=(0.29)$

$\therefore\ 50+j15=52.2|\underline{0.29}$ $\qquad (\dot{E})$

(ii) Similarly we can get $(2+j3.2)=A_2|\underline{\theta_2}$

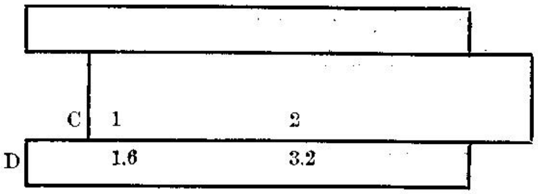
Set 0 Q to 2 P against 3.2 Q read 3.77 P



$\therefore\ A_2=3.77$

To get $tan\theta_2 = \frac{3.2}{2} = 1.6$

Set 2 C to 3.2 D against 1 C read 1.6 D



$$\therefore \quad tan\theta_2 = 1.6$$

To get Rθ:

Against 1.6 T read 1.011 $R\theta$ on the other side of the slide rue

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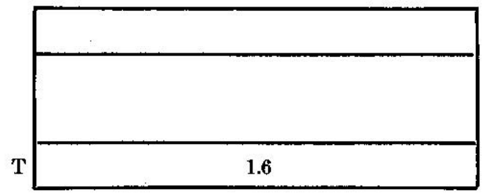

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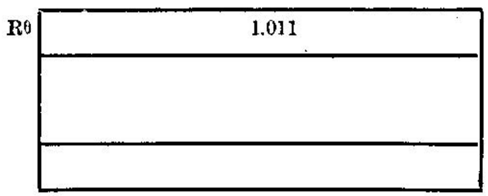


$$\therefore 2 + j3.2 = 3.77 \underline{|1.011} \qquad (\dot{Z})$$

Thus we have the values of $\dot{E}$ and $\dot{Z}$ separately; so now we shall proceed for $\dot{I}$ or $\frac{\dot{E}}{\dot{Z}}$:—

$$\therefore \quad \dot{I} = \frac{2.2 \underline{|0.29}}{3.77 \underline{|1.011}} = A_0 \underline{|\theta_0} \qquad \frac{\dot{E}}{\dot{Z}}$$

$$A_0 = \frac{52.2}{3.77} = 13.85$$

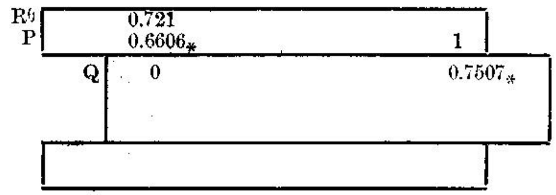
$$\theta_0 = 0.29 - 1.011 = -0.721$$

$\therefore$ The vector required is $13.85\varepsilon^{-j0.721}$.

This can be transformed into the form $(a + jb)$.

$$13.85\varepsilon^{-j0.721} = 13.85[cos(0.721) - jsin(0.721)]$$

Set 0 Q to 0.721 $R\theta$ against 0 Q read 0.6606 P and also
„ 1 P „ 0.7507 Q



$$\therefore \quad sin(0.721) = 0.6606$$

$$cos(0.721) = 0.7507$$

$$\therefore \quad 13.85 \times 0.7507 = 10.4$$

$$13.85 \times 0.6606 = 9.15$$

$$\therefore \quad \dot{I} = 10.4 - j9.15 \qquad \text{Q.E.I.}$$

The calculations look complicated, but in practice they can be done with simplicity, easiness and rapidity.

[3] Supplementary.

(I) $\sqrt{a^2 \pm b^2}$

The vector scales (P) and (Q) are quite different from logarithmic scales, and a and b must not differ too much from each other; or they must be of alike places in digits. When $a=38$, $b=6$, you could not do

Set 0 Q to 38 P against 6 Q read $\sqrt{38^2+6^2}$ P

as you will find $\sqrt{38^2+6^2}$ off (P). If you take 0.6 in place of 6, you will get the result only inaccurately.

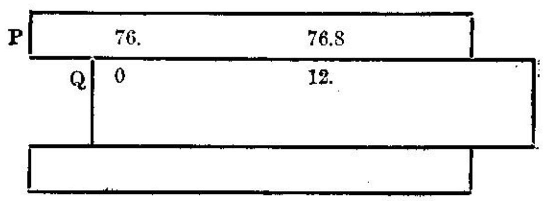
In such a case, you can do in the follywing way:—

$$\sqrt{a^2+b^2}=\frac{1}{n}\sqrt{(na)^2+(nb)^2}$$

where n is to be chosen a very simple integer or a fraction like 2 or $\frac{1}{2}$, &c.

Example 6. $\sqrt{38^2+6^2}=\frac{1}{2}\sqrt{76^2+12^2}$

Set 0 Q to 76 P against 12 Q read 76.8 P



$$\therefore \quad \sqrt{38^2+6^2}=76.8\div 2=38.4$$

[4] Hyperbolic Functions.

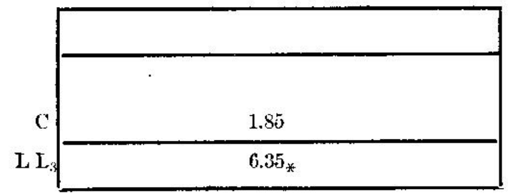
The 20 inch "Universal" slide rule has scales for hyperbolic functions, but the 10 inch one has these destroyed. However with this slide rule, you can calculate the hyperbolic functions in the following way:

(a) To get $\sinh\theta$.

By the formula $\sinh\theta=\frac{1}{2}(\varepsilon^{\theta}-\varepsilon^{-\theta})=\frac{1}{2}\left(\varepsilon^{\theta}-\frac{1}{\varepsilon^{\theta}}\right)$, while ε^{θ} can be had easily by means of the *log-log* scales.

Example 7. $\sinh 1.85=3.096$

Against 1.85 C read 6.35 LL_3



$$\therefore \quad \varepsilon^{1.85}=6.35$$

$$\frac{1}{\varepsilon^{1.85}}=\frac{1}{6.35}=0.158$$

$$\therefore \quad \sinh 1.85=\frac{1}{2}(6.35-0.158)$$

$$=3.096$$

(b) To get $\cosh\theta$.

$$\cosh\theta=\frac{1}{2}(\varepsilon^{\theta}+\varepsilon^{-\theta})=\frac{1}{2}\left(\varepsilon^{\theta}+\frac{1}{\theta^{\theta}}\right).$$

Example 8. $\cosh 1.85=\frac{1}{2}(6.35+0.158)=3.254$

(c) To get $tanh\theta$.

$$tanh\theta = \frac{sinh\theta}{cosh\theta} = \frac{\varepsilon^{\theta}-\varepsilon^{-\theta}}{\varepsilon^{\theta}+\varepsilon^{-\theta}} = \frac{\varepsilon^{2\theta}-1}{\varepsilon^{2\theta}+1}$$

by which you can calculate $tanh\theta$.

Example 9. $tanh\ 1.85 = \frac{\varepsilon^{3.7}-1}{\varepsilon^{3.7}+1}$

$$= \frac{39.4}{41.4} \quad \because \varepsilon^{3.7}=40.4$$

$$=0.951$$

(d) To convert $sinh(a \pm jb)$ into $(x+jy)$

Put $sinh(a \pm jb) = x \pm jy$

As $sinh(a \pm jb) = sinha.\ cosb \pm jcosha.\ sinb$

$$= \frac{1}{2} cosb(\varepsilon^{a}-\varepsilon^{-a}) \pm j\frac{1}{2} sinb(\varepsilon^{a}+\varepsilon^{-a}).$$

$\therefore$ $x = \frac{1}{2} cosb(\varepsilon^{a}-\varepsilon^{-a})$

Also $y = \frac{1}{2} sinb(\varepsilon^{a}+\varepsilon^{-a})$

Example 10. $sinh(0.82+j1.2) = 0.33+j1.263$

$\varepsilon^{0.82} = 2.27$

$\varepsilon^{-0.82} = 0.44$

$cos\ 1.2 = 0.362$

$sin\ 1.2 = 0.932$

$\therefore$ $x = \frac{1}{2} \times 0.362 \times (2.27-0.44) = 0.332$

Also $y = \frac{1}{2} \times 0.932 \times (2.27+0.44) = 1.263.$

$\therefore$ $sinh(0.82+j1.2) = 0.332+j1.263.$

(e) To convert $cosh(a \pm jb)$ into $(x+jy)$

Again put $cosh(a \pm jb) = x \pm jy$

$$cosh(a \pm jb) = cosha.\ cosb \pm jsinha.\ sinb$$

$$= \frac{1}{2} cosb(\varepsilon^{\theta}+\varepsilon^{-\theta}) \pm j\frac{1}{2} sinb(\varepsilon^{\theta}-\varepsilon^{-\theta})$$

$\therefore$ $x = \frac{1}{2} cosb(\varepsilon^{\theta}+\varepsilon^{-\theta})$

Also $y = \frac{1}{2} sinb(\varepsilon^{\theta}-\varepsilon^{-\theta})$

Example 11. Take the above example :—

$$x = \frac{1}{2} \times 0.362 \times (2.27+0.44) = 0.491$$

$$y = \frac{1}{2} \times 0.932 \times (2.27-0.44) = 0.853$$

$\therefore$ $cosh(0.82+j1.2) = 0.491+j0.853$

(f) To convert $tanh(a \pm jb)$ into $(x+jy)$

$$tanh(a \pm jb) = \frac{sinh(a \pm jb)}{cosh(a \pm jb)}$$

You can calculate $sinh(a \pm jb)$ and $cosh(a \pm jb)$; and then $tanh(a \pm jb)$.

(g) To convert $tanh^{-1}(a \pm jb)$ into $(x+jy)$

Put $tanh^{-1}(a \pm jb) = x \pm jy$

$$\frac{\varepsilon^{(x+jy)}-\varepsilon^{-(x+jy)}}{\varepsilon^{(x+jy)}+\varepsilon^{-(x+jy)}} = a+jb$$

$$\frac{\varepsilon^{2(x+jy)}-1}{\varepsilon^{2(x+jy)}+1} = a+jb$$

Whence $\varepsilon^{2(x+jy)} = \frac{1+a+jb}{1-a-jb} = A|\theta$

Then $y = \frac{\theta}{2}$ $\quad x = \frac{1}{2} log_{\varepsilon} A = 1.15\ log_{10} A$

CHAPTER IV

ELECTRICAL ENGINEER'S "UNIVERSAL" DUPLEX SLIDE RULE WITH PATENT VECTOR AND GUDERMANIAN SCALES

Section A. INTRODUCTION

Hemmi's "Universal" Duplex Slide Rule for Electrical Engineers with Patent Vector Scales has won the world's reputation. It does well deserve it. The only missing point, however, was that you had to go into complicated computations for hyperbolic functions by means of the Log-Log Scales.

The development of high voltage, distant transmission of electric power has brought forth the hyperbolic functions to occur so frequently in the daily life of electrical engineers, that we should have one slide rule or another to find out the values of hyperbolic functions of a given hyperbolic angle. The missing link is at last come forth to the solution: we have just invented the "Gudermanian Scale" $(G\theta)$. The addition of the new scale $(G\theta)$ to the slide rule above stated gives you at once *sinh*θ, *tanh*θ, *sech*θ for a given θ, and so also *cosech*θ, *coth*θ, *cosh*θ by taking their reciprocals without interfering at all the nature and use of the old scales. And you can get them in a simple and rapid way. The slide rule is now truly "Universal."

Section B. THE SCALES

The new slide rule has all the scales exactly the same as Hemmi's No. 152 or the "Universal" Duplex 10″ slide Rule for Electrical Engineers, only with the addition of (L) above (K) and the new patent Gudermanian Scale $(G\theta)$ below (T), just as per Fig. 7.

(L) is nothing but the same equi-division scale as you find in the old familiar slide rules. It is ofcourse good for obtaining the logarism of a given number. But (L) in this slide rule is still more valuable, for it facilitates the calculation of functions akin to $\sqrt{a^2 \pm b^2}$, such as $(a^2 \pm b^2)$, $(a^2 \pm b)$, $(a - b^2)$, &c.

The Gudermanian Scale $(G\theta)$ is divided with reference to the Radian Scale $(R\theta)$: a reading x on $(G\theta)$ is equivalent to gdx on $(R\theta)$. So by the Gudermanian theorem:—

$$tanhx = singdx$$
$$sinhx = tangdx$$
$$sechx = cosgdx$$

and each of them is very easy to get.

Section C. FUNCTIONS AKIN TO $\sqrt{a^2 \pm b^2}$

(i) $a^2 \pm b^2$

Set oQ to aP, against bQ read $(a^2 \pm b^2)L$

Set bQ to aP, against oQ read $(a^2 \pm b^2)L$

(ii) $\sqrt{a \pm b^2}$

Set oQ to aL, against bQ read $\sqrt{a+b^2}P$,

also against bQ read $(a+b^2)L$

Set bQ to aL, against oQ read $\sqrt{a-b^2}P$,

also against oQ read $(a-b^2)L$

Set aQ to bL, against oP read $\sqrt{a^2-b}Q$

Here is one point to be put in mind: (a^2-b) cannot be got at one setting. You are to proceed further:—

Against $\sqrt{a^2-b}$ D, read (a^2-b) A

Example 1. $4^2+3^2=25$

Set $0Q$ to $4P$, against $3Q$ read $25L$

Example 2. $\sqrt{5.2+2.5^2}=3.38$

Set $0Q$ to $5.2L$, against $2.5Q$ read $3.38P$

Example 3. $\sqrt{6^2-11}=5$

Set $6Q$ to $11L$, against $0P$ read $5Q$

Example 4. $25+3^2=34$

Set $0Q$ to $25L$, against $3Q$ read $34L$

Section D. CALCULATIONS OF HYPERBOLIC FUNCTIONS

Example 5. $\sinh 0.32=0.325$

Against $0.32G\theta$, read $0.325T$

Example 6. $\tanh 0.83=0.68$

Against $0.83G\theta$, read $0.68P$

Example 7. $\cosh 0.55=1.155$

Method 1.

Against $0.55G\theta$, read $\sinh 0.55=0.578T$

Against $0.578Q$, read $1.155Q'$

Method 2.

Set $0Q$ to $0.55G\theta$, against right end or $1P$ read $\operatorname{sech} 0.55=0.866Q$

Set $0C$ to $0.866D$, against right end D read $\dfrac{1}{0.866}=1.155C$

Thus the addition of $(G\theta)$ to the patent vector scales (P) and

(Q), and the tangent scale (T), has simplified the computations of hyperbolic functions. Also the hyperbolic functions of complex number can be obtained easily by the following formurae:—

$$\sinh(a+jb)=\sqrt{\sinh^2 a+\sin^2 b}\ \Big|\underline{\tan^{-1}\left(\frac{\tan b}{\tanh a}\right)}$$

$$\cosh(a+jb)=\sqrt{\sinh^2 a+\cos^2 b}\ \Big|\underline{\tan^{-1}(\tan b,\ \tanh a)}$$

$$\tanh(a+jb)=\sqrt{\frac{\sinh^2 a+\sin^2 b}{\sinh^2 a+\cos^2 b}}\ \Big|\underline{\tan^{-1}\left(\frac{\sin 2b}{\sinh 2a}\right)}$$

Example 8.

$\sinh(0.32+j\ 1.22)=0.994\ \big|\underline{1.460}$

$\sinh 0.32=0.326$

$\sin 1.22=0.939$

$\tanh 0.32=0.310$

$\tan 1.22=2.73$

$\sinh(0.32+j\ 1.22)=\sqrt{0.326^2+0.939^2}\ \Big|\underline{\tan^{-1}\frac{2.730}{0.310}}$

$=0.994\ \big|\underline{1.460}$

Example 9.

$\cosh(0.257+j\ 0.652)=0.836\ \big|\underline{0.190}$

$\sinh 0.257=0.260$

$\tanh 0.257=0.250$

$\cos 0.652=0.795$

$\tan 0.652=0.763$

$\cosh(0.257+j\ 0.652)=\sqrt{0.260^2+0.795^2}\ \big|\underline{\tan^{-1}(0.250\times 0.763)}$

$=0.836\ \big|\underline{0.190}$

Example 10.

$\tanh(1.25+j\ 0.28)=0.87\ \ \underline{0.0885}$

$\sinh 1.52=1.60$

$\sin 0.28=0.276$

$\cos\ 0.28 = 0.961$

$\sinh\ (2 \times 1.25) = 6.0$

$\sin\ (2 \times 0.28) = 0.531$

$$\tanh\ (1.25 + j\ 0.28) = \sqrt{\frac{1.60^2 + 0.276^2}{1.60^2 + 0.961^2}}\ \angle \tan^{-1}\left(\frac{0.531}{6.0}\right)$$

$$= \sqrt{\frac{0.80^2 + 0.138^2}{0.80^2 + 0.4805^2}}\ \angle \tan^{-1}\ 0.0885$$

$$= 0.87\ \angle 0.0885$$

Example 11.

Calculate $Y_{00} = \dfrac{\tanh (0.511 + j\ 0.04)}{0.88 \times 10^3\ \angle 0.033}$

First obtain the hyperbolic tangent by the following formura :—

$$\tanh\ (a + jb) = \sqrt{\frac{\sinh^2 a + \sin^2 b}{\sinh^2 a + \cos^2 b}}\ \angle \tan^{-1}\left(\frac{\sin\ 2b}{\sinh\ 2a}\right)$$

$\sinh\ a = \sinh\ 0.511 = 0.534$

$\sinh\ 2a = \sinh\ 1.022 = 1.210$

$\sin\ b = \sin\ 0.04 \fallingdotseq 0.04$

$\sin\ 2b = \sin\ 0.08 \fallingdotseq 0.08$

$\cos\ b = \cos\ 0.04 \fallingdotseq 0.9995$

$$\tanh\ (0.511 + j\ 0.04) = \frac{0.536}{1.133}\ \angle \tan^{-1} \frac{0.08}{1.210}$$

$$= 0.473\ \angle 0.066$$

$$Y_{00} = \frac{0.473}{0.88 \times 10^3}\ \angle 0.066 + 0.033 = 0.538\ \angle 0.099 \times 10^{-3}$$

Example 12.

Calculate $V_{00} = \dfrac{\cosh (1.191 + j\ 2.485)}{\cosh (2.230 + j\ 2.551)}$

$\times (-56.9 \times 10^3\ \angle 0.180)$

$\sinh\ 1.191 = 1.492$

$\sinh\ 2.230 = 4.60$

$\cos\ 2.485 = -0.611$

$\cos\ 2.551 = -0.557$

$\tanh\ 1.191 = 0.831$

$\tanh\ 2.230 = 0.977$

$\tan\ 2.485 = -1.293$

$\tan\ 2.551 = -1.490$

$$\cosh\ (1.191 + j\ 2.485) = \sqrt{1.492^2 + 0.611^2}\ \angle \tan^{-1}(1.293 \times 0.831)$$

$$= 1.72\ \angle 0.83 + \pi$$

$$\cosh\ (2.230 + j2.551) = \sqrt{4.60^2 + 0.557^2}\ \angle \tan^{-1}(0.977 \times 1.49)$$

$$= 4.63\ \angle 0.969 + \pi$$

$$V_{00} = \frac{1.72}{4.63} \times (-56.9 \times 10^3)\ \angle 0.82 + \pi - 0.969 - \pi - 0.18$$

$$= -22.13 \times 10^3\ \angle 0.32$$

CHAPTER V

THE HIGHEST CLASS ELECTRICAL ENGINEER'S "UNIVERSAL" 20″ DUPLEX SLIDE RULE, WITH PATENT VECTOR AND HYPERBOLIC SCALES

Section A. SCALES DESCRIBED

This slide rule is of the polyphase type; it has ten scales on each side, making no less than 20 scales in all. Fig. 5 shows its front face, and fig. 6 its back face. Each scale is just 500 mm. The slide rule is provided with two cursors on the scales. For convenience's sake, we shall nominate all the twenty scales.

Scales (7), (8), (19) are each one section logarithmic scale, or one divided as per $\log x$, x ranging 1.0 to 10.0; scales (11), (15) are also one section logarithmic scales, or divided as per $\log x$, only x ranging π to 10π; all extended to the whole length of 500 mm. The former are the familiar things in the ordinary slide rule; and the latter are also found on some other slide rules.

The scale (16) is divided inversely as per $\log x$, or in other words as per $\log \frac{10}{x}$, x ranging from 10.0 to 1.0. It is the inverted slide of (C) or (D), so it is called (CI); and

(1) is divided as per $\log \sin x$, x ranging 0.099 to 1.5708. It works with (8) or (D), and is denominated (Sx). You can get $\sin x$ and $\operatorname{arc} \sin x$ between (Sx) and either (C) or (D).

ON THE FRONT FACE

Numbers to be referred to Figs. 1. & 2.	Denominations of Scales	Functions according to which the Scales are divided	Limits of x or θ in the Functions	Other Scales to be engaged with mostly	Functions best calculated
(1)	Sx	$\log \sin x$	0.099–1.5708	D	$\sin x$, $\operatorname{arc}\sin x$
(2)	Tx	$\log \tan x$	0.098–0.790	D	$\tan x$, $\operatorname{arc}\tan x$
(3)	$T\theta°$	$\log \tan\theta$	5.6° – 45°	D	$\tan\theta$, $\operatorname{arc}\tan x$
	,,	in red	84.4° – 45°	D	$\cot\theta$, $\operatorname{arc}\cot x$
(4)	Th	$\log \tanh x$	0.099–4.0	C	$\tanh x$
(5)	Sh	$\log \sinh x$	0.87 –3.01	C	$\sinh x$
(6)	,,	,,	0.098–0.89	,,	,,
(7)	C	$\log x$	1–10	D	ab, $a \div b$, &c.
(8)	D	$\log x$	1–10	C	ab, $a \div b$, &c.
(9)	L	x	0–100	D	$\log x$, $\log^{-1} x$
(10)	x	x	0–1.750	L	$\theta°$, $R\theta$
ON THE BACK FACE					
(11)	DF	$\log x$	π–10π	C, CF	ab, πab, &c.
(12)	P'	x^2	10–14.14	Q	$\sqrt{a^2 \pm b^2}$, &c.
(13)	P	x^2	0–10	Q	$\sqrt{a^2 \pm b_2}$, &c.
(14)	Q	x^2	0–10	P	$\sqrt{a^2 \pm b^2}$, &c.
(15)	CF	$\log x$	π–10π	D, DF	ab, πab, &c.
(16)	CI	$\log 10/x$	10–1	C, D	abc, &c.
(17)	$S\theta°$	$\log \sin\theta°$	0.55°–90°	A	$\sin\theta$, $\cos\theta°$, $\sin^{-1}\theta°$ &c.
(18)	A	$\log x$	1–100	D	x^2, $\sqrt{x}$, &c.
(19)	D	$\log x$	1–10	A, K, CI, $S\theta$	
(20)	K	$\log x$	1–1000	D	x^3, $\sqrt[3]{x}$, &c.

(2) is divided as per *log tanx*, x ranging 0.0980 to 0.790. It works with (8) or (D); and it is denominated (Tx). Between (Tx) and (D) you can get *tanx* and *arc tanx*.

(3) is divided as per *log tan*$\theta°$, $\theta°$ ranging 5.6° to 45°. Or *log cot*$\theta°$, $\theta°$ ranging between 84.4° to 45°. It is called ($T\theta°$). Between ($T\theta°$) and (D) you can get *tan*$\theta°$ for a given θ and *arc tan*$\theta°$ or $\theta°$ for a known *tan*θ. Also by employing the figures in red, you can get *cot*$\theta°$ for a given $\theta°$, and *arc cot*$\theta°$ or $\theta°$ for a known *cot*θ.

Here only you are to keep in mind that the angles are in degrees and *decimals* instead of in *minutes* and *seconds*.

(4) is divided as per *log tanhx*, x ranging 0.099 to 4.0, to work with (7) or (C). It is denominated (Th). Between (Th) and (C), you can get *tanhx*.

(6) and (5) are divided as per *log sinhx*; in (6) x ranging from 0.098 to 0.89, and in (5) x ranging from 0.87 to 3.01. They are denominated (Sh). Working with (C), they give *sinhx* for a given x and *arc sinhx* or x for a known *sinhx*.

(9) has its whole length of 500 mm divided dicimally, and is denominated (L). Between (L) and (D), you can get the mantissa of *logx* for any value of x. Working either with (Sx) or with (Tx), it gives the mantissa of *log sinx* or of *log tanx* respectively. Also working with ($T\theta°$), (Th), or (Sh), &c., it gives *log tan*$\theta°$, *log tanhx*, *log sinhx*, &c. respectively.

(10) is divided, like (L), into equal parts, only x ranging from 0 to 1.750. It is denominated (x). Let (L) indicate an angle in degrees, then x automatically indicates the angle in circular measure or in radians.

(13), (14) are divided as per x^2, x ranging from 0 to 10.0.

(13) is denominated (P) and (14) (Q). Between them both you can get $\sqrt{x^2 \pm y^2}$, $\sqrt{x^2+y^2-z^2}$.

(12) is divided as per x^2, x ranging from 10 to 14.16. It is to work with (14) or (Q); it is nothing but an extention of (P), and so it is denominated (P'). It sometimes works with (13) or (P) without setting of the slide. See p. 95, Example 5.

(18) is divided as per *logx*, ranging from 1 to 100. It works with (19). It is nothing but the common logarithmic scale of two sections, and is denominated (A). Hence in accordance with (D), it gives x^2 and $\sqrt{x}$, &c.

(17) is divided as per *log sin*$\theta°$, θ ranging from 0.55° to 90° and is denominated ($S\theta°$). Between ($S\theta°$) and (A), you can get *sin*$\theta°$, *cos*$\theta°$ for a given angle $\theta°$. Also it is good for converting $A\underline{|\theta°}$ into a complex number for which we shall discuss later.

(20) is divided as per *logx*, x ranging from 1 to 1000; and it is denominated (K). It works with (19) or (D) to give x^3 and $x^{\frac{1}{3}}$. Also with (A), it gives $x^{\frac{3}{2}}$ and $x^{\frac{2}{3}}$. These were obtainable with old only with difficulty slide rules, but with this slide rule you can also get $sin^{\frac{3}{2}}\theta°$ and $cos^{\frac{3}{2}}\theta°$ &c. very easily between (K) and ($S\theta°$).

Section B. SIMPLER APPLICATIONS

Multiplication, Division, Squares and Square Roots, Cubes and Cube Roots, the Circumferences, Diameters and Radii of Circles.

You can get them all in the same way as in the old slide rules; but this slide rule has (CF), (DF), and (CI), so that you can get all of them with greater ease and rapidity :—

$x=\pi a$ Against $a\ D$ read $\pi a\ DF$

$x=\dfrac{a}{\pi}$ Against $a\ DF$ read $\dfrac{a}{\pi}\ D$

$x=\dfrac{\pi}{a}$ Against $a\ DF$ read $\dfrac{\pi}{a}\ CI$

$x=abc$ Set $a\ CI$ to $b\ D$ against $c\ C$ read $abc\ D$

$x=\pi abc$ Set $a\ CI$ to $b\ D$ against $c\ C$ read $\pi abc\ DF$

$x=\dfrac{ab}{c}$ Set $c\ C$ to $a\ D$ against $b\ C$ read $\dfrac{ab}{c}\ D$

,, $b\ CF$,, $\dfrac{ab}{c}DF$

$x=\dfrac{\pi ab}{c}$ Set $c\ C$ to $a\ D$ against $b\ C$ read $\dfrac{\pi ab}{c}DF$

Section C. LOGARITHMIC FUNCTIONS

As already stated (L) has its whole length as unit, divided decimally into 100 parts, numbered at every other part and further decimals unnumbered. If x on (D) corresponds with l on (L), then

$$\log x : l = (\log 10 - \log 1) : 100$$

or

$$\log x = \frac{(\log 10 - \log 1)\, l}{100}$$

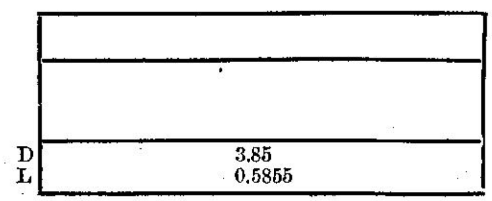
$$= \frac{l}{100}$$

Thus you can get the value of $\log x$ on (L). With the old slide rules, you had to set the slide and then to turn over the whole slide rule. It is not only $\log x$, but also you can as well get the mantissas of $\log \sin x$, $\log \sinh x$, $\log \tan x$, $\log \tanh x$, &c.

Example 1. $\log 385 = 2.5855$

As $\log 385 = 2 + \log 3.85$

Against $3.85\ D$ read $0.5855\ L$

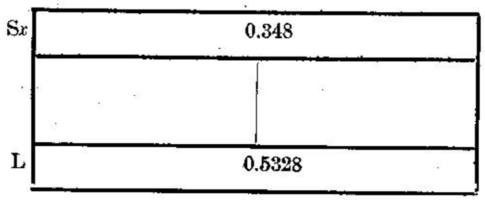


Cf. Fig. 1.

$\therefore$ *Ans.* 2.5855 Q.E.I.

Example 2. *log sin* 0.348 = $\bar{1}$.5328

Against 0.348 *Sx* read 0.5328 *L*

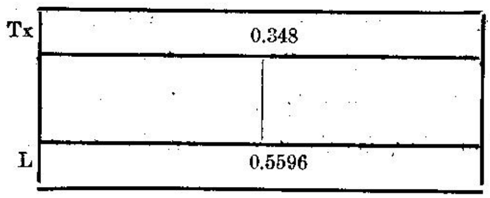


Cf. Fig. 1.

∴ *Ans.* $\bar{1}$.5328 Q.E.I

Example 3. *log tan* 0.348 = $\bar{1}$.5596.

Against 0.348 *Tx* read 0.5596 *L*

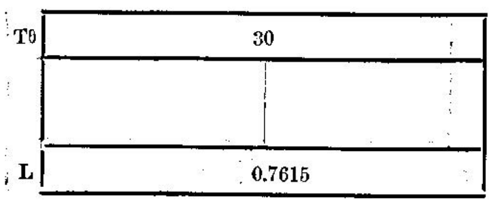


Cf. Fig. 1.

∴ *Ans.* $\bar{1}$.5596 Q.E.I.

Example 4. *log tan* 30° = $\bar{1}$.7615.

Against 30° *Tθ°* read 0.7615 *L*

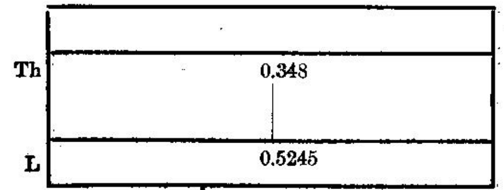


See Fig. 1.

∴ *Ans.* $\bar{1}$.7615 Q.E.I.

Example 5. *log tanh* 0.348 = $\bar{1}$.5245

Against 0.348 *Th* read 0.5245 *L*

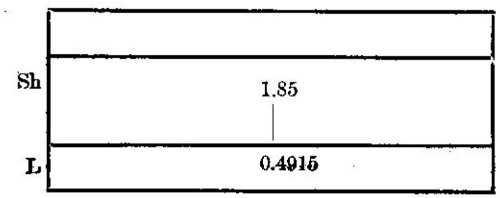


See Fig. 1.

∴ *Ans.* $\bar{1}$.5245 Q.E.I.

Example 6. *log sinh* 1.85 = 0.4915

Against 1.85 *Sh* read 0.4915 *L*

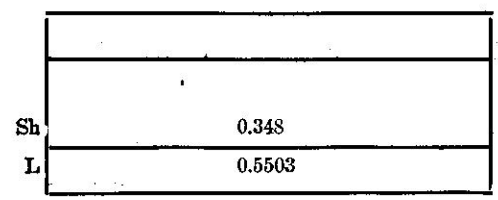


See Fig. 1.

∴ *Ans.* 0.4195 Q.E.I.

Example 7. *log sinh* 0.348 = $\bar{1}$.5503

Against 0.348 *Sh* read 0.5503 *L*



See Fig. 1.

∴ *Ans.* $\bar{1}$.5503 Q.E.I.

Section D. A FUNCTION $f(x)$, OF A VARIABLE x.

When you want the value of $f(x)$, take the value of x on a scale divided as per $log[f(x)]$, then you will have the value of $f(x)$ either on (C) or on (D).

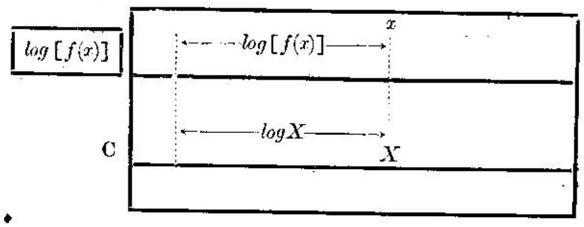
Assume here is a scale, graduated as per $log f(x)$. Take a point on each of this scale and of (C), equidistant from the origin; and let the reading on the former be x, and that on the latter X. Now the point on the former must be at a distance of $log[f(x)]$, and that on (C), $log X$. As they are equidistant from the origin,

$$log[f(x)] = log X$$

$$\therefore \quad f(x) = X \qquad \text{which is sought after.}$$

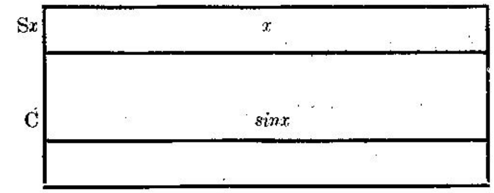
So what you read on (C) is the value of $f(x)$ that you want.

Against x on the scale, $log[f(x)]$, read X on C



For instance, say $\underline{sinx}$ is to be calculated where x is given. Take the value of x on (Sx), then the real distance from the origin must be $log\,sinx$, as the scale is divided as per $log\,sinx$. Now assume that the point on (C) just opposite to that point is graduated X; it must be at a distance from the origin $logX$.

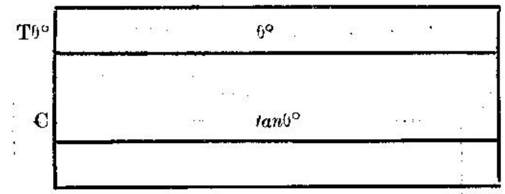
$$\therefore \quad log\,sinx = logX, \quad \therefore \quad X = sinx$$



$\therefore$ $sinx$ is obtained.

Another instance, $\underline{tan\theta^\circ}$.

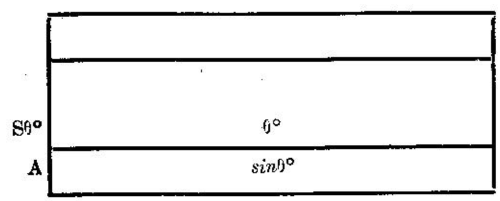
Against θ°-$T\theta^\circ$ read $tan\theta^\circ$-C



$\therefore$ $tan\theta^\circ$ is got.

Another instance, $\underline{sin\theta^\circ}$.

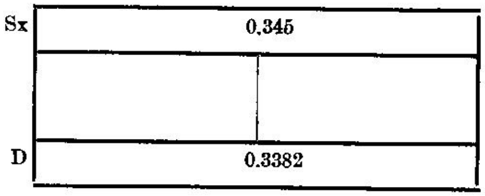
Against θ°-$S\theta^\circ$ read $sin\theta^\circ$-A



$\therefore$ $sin\theta^\circ$ is obtained.

Example 1. *sin* 0.345=0.3382

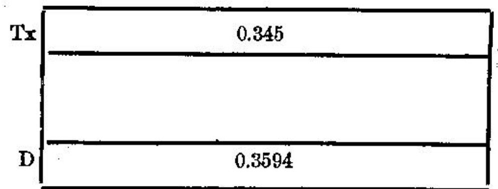
Against 0.345 *Sx* read 0.3382 *D*



∴ *Ans.* 0.3382 Q.E.I.

Example 2. *tan* 0.345=0.3594

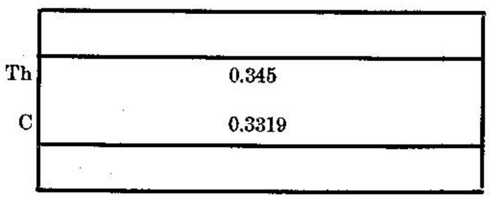
Against 0.345 *Tx* read 0.3594 *D*



∴ *Ans.* 0.3594 Q.E.I.

Example 3. *tanh* 0.345=0.3319

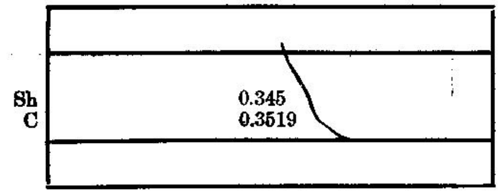
Against 0.345 *Th* read 3.319 *C*



∴ *Ans.* 0.3319 Q.E.I.

Example 4. *sinh* 0.345=0.3519

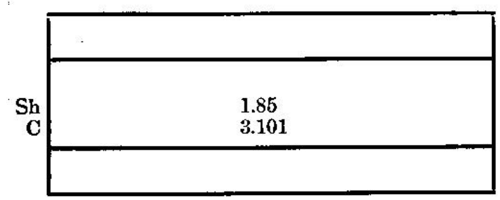
Against 0.345 *Sh* read 0.3519 *C*



∴ *Ans.* 0.3519 Q.E.I.

Example 5. *sinh* 1.85=3.101

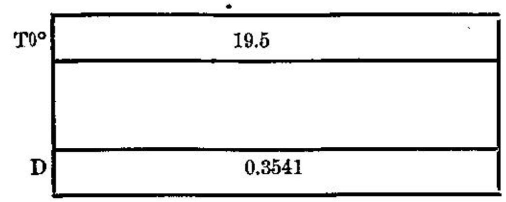
Against 1.85 *Sh* read 3.101 *C*



∴ *Ans.* 3.101 Q.E.I.

Example 6. *tan* 19.5°=0.3541

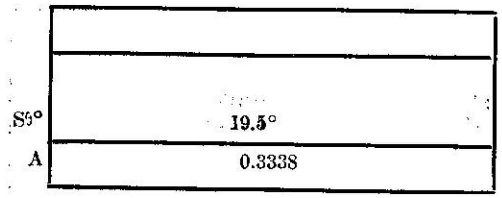
Against 19.5 *Tθ°* read 0.3541 *D*



Ans. 0.3541 Q.E.I.

Example 7. *sin* 19.5°=0.3338

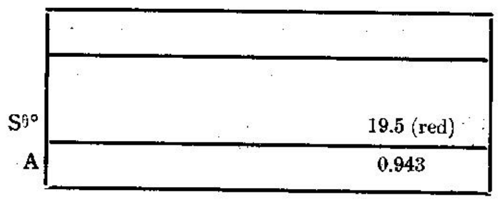
Against 19.5 $S\theta°$ read 0.3338 A



Ans. 0.3338 Q.E.I.

Example 8. *cos* 19.5°=0.943

Against 19.5 in red on $S\theta$ read 0.943 A



∴ *cos* 19.5°=0.943 Q.E.I.

Section E. AN ANGLE IN RADIANS

As already stated the x scale is for converting an angle in degrees into radians. Let the graduation at a point on (x) scale be x, and its corresponding point on (L) be $\theta°$; then it must be:

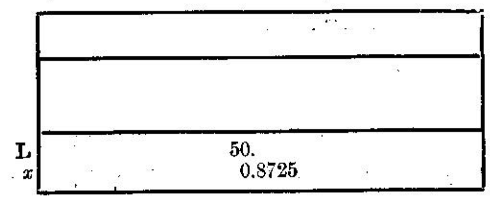
$$\frac{x}{\theta} = \frac{\pi}{180}$$

$$\therefore \quad x = \frac{\pi}{180}\theta$$

which is the formula for converting degrees into radians.

Example 1. 50°=0.8725

Against 50 L read 0.8725 x



∴ *Ans.* 0.8725 Q.E.I.

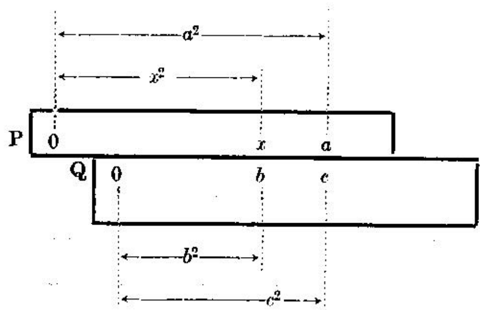
You could as well obtain this between (C) and (D).

Section F. THE SQUARE ROOT OF A SUM OR OF A DIFFERENCE OF DIFFERENT SQUARES

$$\sqrt{a^2 \pm b^2}, \qquad \sqrt{a^2+b^2-c^2}$$

(P) and (Q) are graduated as per x^2, x ranging from 0 to 10; (P') is similar only x ranging from 10 to 14.14.

Now Set $c\ Q$ to $a\ P$ against $b\ Q$ read $x\ P$



As x on (C) or (D) is at a distance of $log x$ from the origin; so are a, b, c, x on (P) or (Q) at a distance of a^2, b^2, c^2, x^2 respectively from the origin. Hence

$$c^2-b^2=a^2-x^2$$

$$\therefore \quad x=a^2+b^2-c^2$$

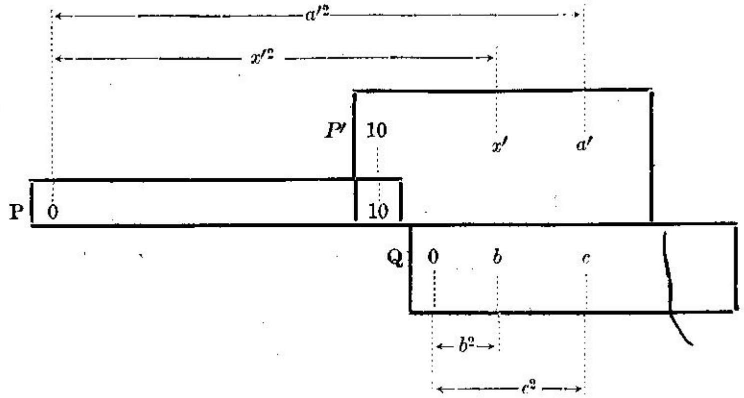
$$\therefore \quad x=\sqrt{a^2+b^2-c^2} \qquad \text{(I)}$$

Take $c=o$, $x=\sqrt{a^2+b^2}$ (II)

„ $b=o$, $x=\sqrt{a^2-c^2}$ (III)

When $a'>10$, $x'>10$, use P'.

Set $c\ Q$ to $a'\ P'$ against $b\ Q$ read $x'\ P'$



$$c^2-b^2= a'^2-x'^2$$

$$x'^2= a'^2+b^2-c^2$$

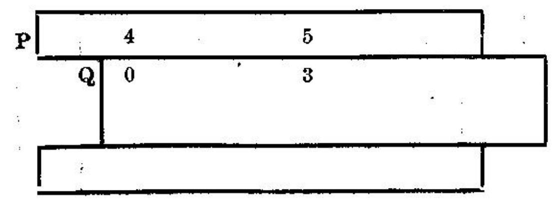
$$x' =\sqrt{a'^2+b^2-c^2} \qquad \text{(IV)}$$

Take $c=o$, $x' =\sqrt{a'^2+b^2}$ (V)

Take $b=o$, $x' =\sqrt{a'^2+c^2}$ (VI)

Example 1. $\sqrt{4^2+3^2}=5$

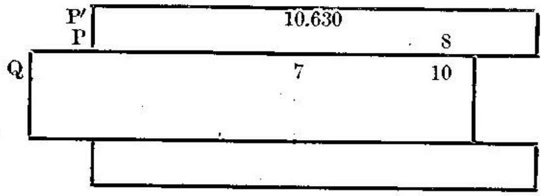
Set 0 Q to 4 P against 3 Q read 5 P



$\therefore$ *Ans.* 5. Q.E.I.

Example 2. $\sqrt{8^2+7^2}=10.63$

Set 10 Q to 8 P against 7 Q read 10.630 P'

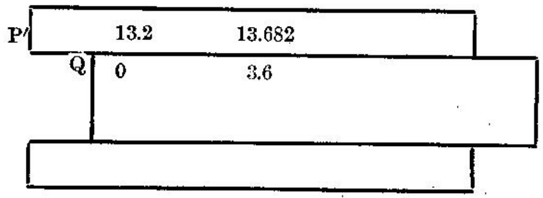


∴ *Ans.* 10.630 Q.E.I.

Note: You can get so far as five figures.

Example 3. $\sqrt{13.2^2+3.6^2}=13.682$

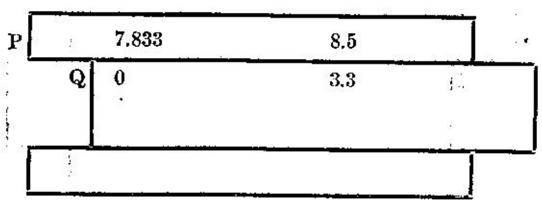
Set 0 Q to 13.2 P' against 3.6 Q read 13.682 P'



Ans. 13.682 Q.E.I.

Example 4. $\sqrt{8.5^2-3.3^2}=7.833$

Set 3.3 Q to 8.5 P against 0 Q read 7.833 P

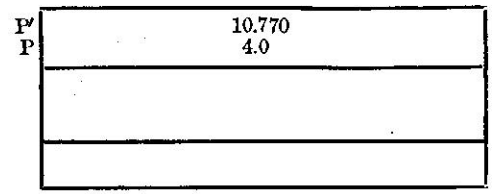


∴ *Ans.* 7.833 Q.E.I.

Example 5. Calculate $coshx$ when $sinhx=0.4$

As $coshx=\sqrt{1+sinh^2x}=\sqrt{1+0.4^2}$

Against 0.4 P read 1.0770 P'

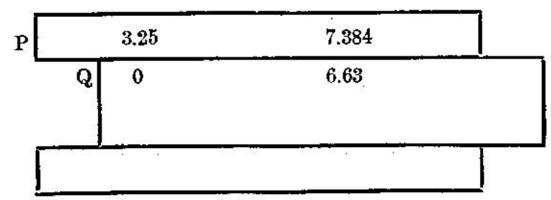


∴ *Ans.* $coshx=1.0770$ Q.E.I.

Example 6. The two sides of a right triangle are 3.25 and 6.63; what is the third side or the hypotenuse?

$$\sqrt{3.25^2+6.63^2}=7.384$$

Set 0 Q to 3.25 P against 6.63 Q read 7.384 P



∴ *Ans.* 7.384 Q.E.I.

Section G. TO CONVERT THE POLAR CO-ORDINATES OF A COMPLEX NUMBER INTO THE CARTESIAN CO-ORDINATES

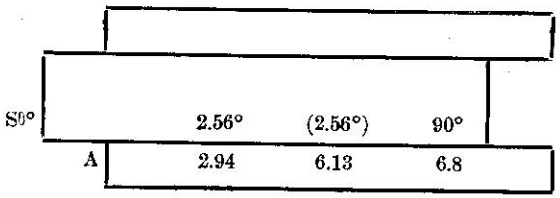
$$A\varepsilon^{j\theta} = A(\cos\theta^\circ + j\sin\theta^\circ)$$

where $A\cos\theta^\circ$ is the real part and $A\sin\theta^\circ$ the imaginary part of the complex number.

Example 1. $6.8\varepsilon^{j25.6^\circ} = 6.13 + j\,2.94$

Set 90° $S\theta^\circ$ to 6.8 A against 25.6°(red) $S\theta^\circ$ read 6.13 A

also against 25.6°(black) $S\theta^\circ$ read 2.94 A



∴ *Ans.* $6.13 + j\,2.94$ Q.E.I.

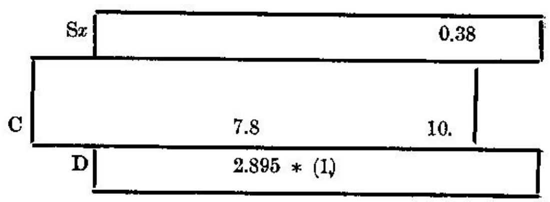
Example 2. $7.8\,\varepsilon^{j0.38} = 7.245 + j\,2.895$

Method (I) $7.8\,\varepsilon^{j0.38} = 7.8\cos 0.38 + j\,7.8\sin 0.38$ ① ②

To calculate $7.8\sin 0.38$

① Set 10 C to 0.38 Sx against 7.8 C read 2.895 D

①



Next to calculate $7.8\cos 0.38$

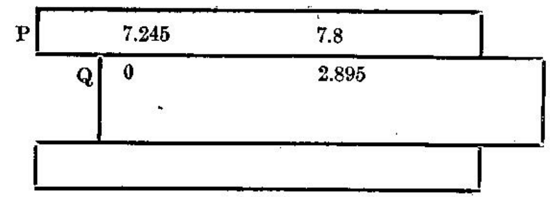
$$7.8\cos 0.38 = 7.8\sqrt{1 - \sin^2 0.38}$$

$$= \sqrt{7.8^2 - 7.8^2 \sin^2 0.38}$$

$$= \sqrt{7.8^2 - 2.895^2}$$

② Set 2.895 Q to 7.8 P against 0 Q read 7.245 P

②



∴ $7.8\,\varepsilon^{j0.38} = 7.245 + j\,2.895$ Q.E.I.

Method (II) There is another way:

First convert the angle 0.38 into degrees.

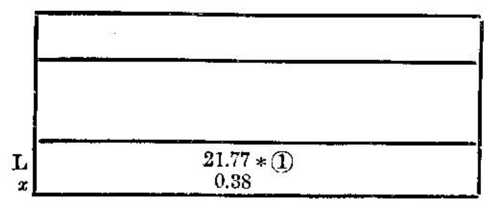
① Against 0.38 x read 21.77 L

Next to get $7.8\sin 21.77^\circ$ and $7.8\cos 21.77^\circ$ at once.

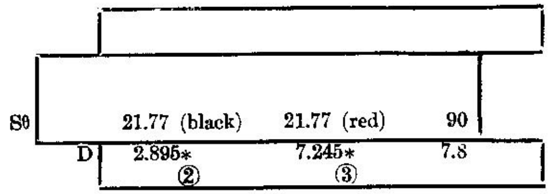
② Set 90° $S\theta$ to 7.8 A against 21.77 (black) $S\theta$ read 2.895 A,

③ and also against 21.77 (red) $S\theta$ read 7.245 A

①



② ③



$\therefore\quad 7.8\,\varepsilon^{j0.38}=7.245+j\,2.895$

However, it is a hard job to get from (*A*) so accurate a result of 4 places, and Method (I) is better for accuracie's sake.

Example 3. $6.9\,\varepsilon^{j1.35}=1.51+j6.73$

$6.9\,\varepsilon^{j1.35}=6.9\cos 1.35+j6.9\sin 1.35$

Method (I) To get $6.9\sin 1.35$

① Set 10 *C* to 1.35 *Sx* against 6.9 *C* read 6.73 *D*

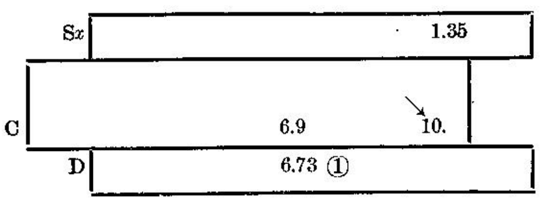
$\therefore\quad 6.9\sin 1.35=6.73$

Next to get $6.9\cos 1.35$

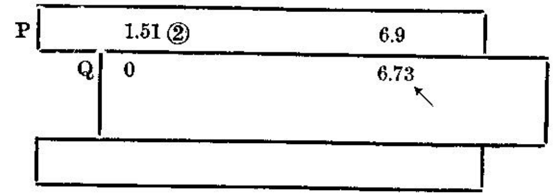
$$6.9\cos 1.35=6.9\sqrt{1-\sin^2 1.35}$$
$$=\sqrt{6.9^2-69\sin^2 1.35}$$
$$=\sqrt{6.9^2-6.73^2}$$

② Set 6.73 *Q* to 6.9 *P* against 0 *Q* read 1.51 *P*

①



②



$\therefore\quad 6.9\,\varepsilon^{j1.35}=1.51+j6.73$ Q.E.I.

Method (II) Going to the other way:—

$$6.9\cos 1.35+j6.9\sin 1.35$$
$$=6.9\sin\left(\frac{\pi}{2}-1.35\right)+j6.9\cos\left(\frac{\pi}{2}-1.35\right)$$
$$=6.9\sin 0.2208+j6.9\cos 0.2208$$
$$\left[\because\ \frac{\pi}{2}=1.5708\right]$$

Next to get $6.9\sin 0.2208$

① Set 10 *C* to 0.2208 *Sx* against 6.9 *C* read 1.512 *D*

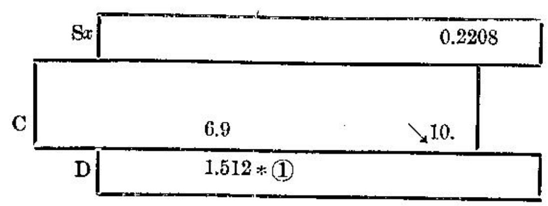
$\therefore\quad 6.9\sin 0.2208=1.512$

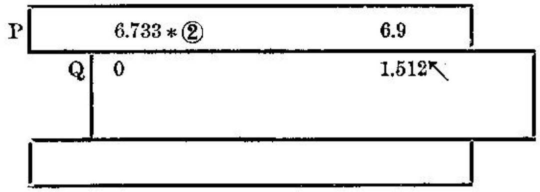
Then to get $6.9\cos(0.2208)$

$$=\sqrt{6.9^2-1.512^2}$$

② Set 1.512 *Q* to 6.9 *P* against 0 *Q* read 6.733 *P*

①



② 

$\therefore\ 6.9\,\varepsilon^{j1.35} = 1.512 + j6.733$ Q.E.I.

Method (III) There is still another way :—

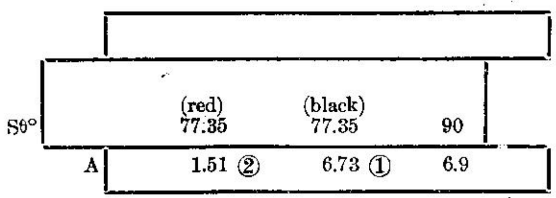
Against 1.35 x read 77.35 L

$\therefore$ 1.35 radians $= 77.35°$

Next to get $6.9\,sin\,77.35°$ and $6.9\,cos\,77.35°$ at once.

① Set 90 $S\theta°$ to 6.9 A against 77.35 (black) $S\theta°$ read 6.73 A,

② also against 77.35 (red) $S\theta°$ read 1.51 A



$\therefore\ 6.9\,\varepsilon^{j1.35} = 1.51 + j6.73$ Q.E.I.

In this case Method (II) gives the most accurate result. That is, when the angle is near to either 0 or $\frac{\pi}{2}$, there is a big difference between $sin\theta$ and $cos\theta$; so calculate first the smaller of the two parts, either real or imaginary, by means of (Sx), (C) and (D). And then the other part by means of (P) and (Q).

Example 4. $3.5\,\varepsilon^{j0.75} = ?$

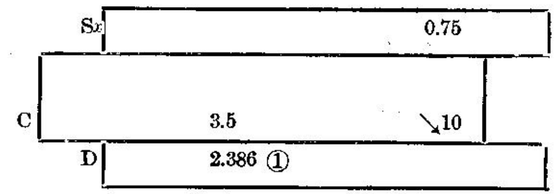
Method (I) $3.5\,\varepsilon^{j0.75} = 3.5\,cos\,0.75 + 3.5\,sin\,0.75$ ②, ①

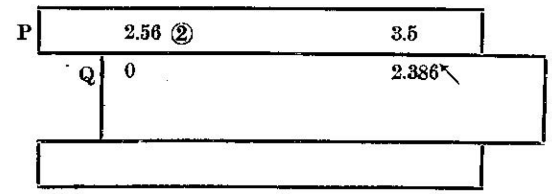
① Set 10 C to 0.75 Sx against 3.5 C read 2.386 D

$\therefore\ 3.5\,sin\,0.75 = 2.386$

Next to calculate $3.5\,cos\,0.75 = \sqrt{3.5^2 - 2.386^2}$

$\therefore$ ② Set 2.386 Q to 3.5 P against 0 Q read 2.56 P

① 

② 

$\therefore\ 3.5\,\varepsilon^{j0.75} = 2.56 + j2.386$ Q.E.I.

Method (II) Take the complementary angle :—

$1.5708 - 0.75 = 0.8208$

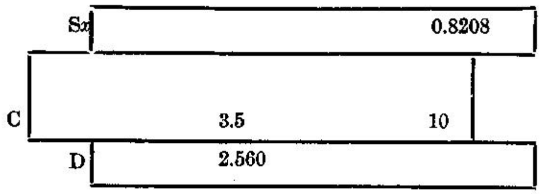
To get the real part,

$3.5\,cos\,0.75 = 3.5\,sin\,0.8208$

① Set 10 C to 0.8208 Sx against 3.5 C read 2.560 D

$$\therefore\ 3.5\ cos\ 0.75 = 2.560$$

For the imaginary part, adopt the same method as before.

① 

$$\therefore\ 3.5\ \varepsilon^{j0.25} = 2.560 + j2.386 \qquad Q.E.I$$

Method (III) Still another way :—

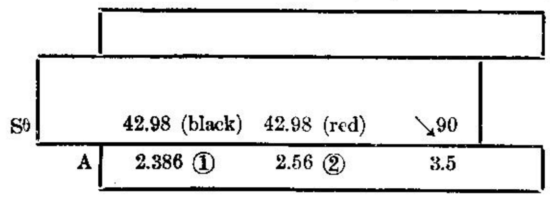
$$0.75\ \text{radian} = 42.98° \qquad x \rightarrow L$$

$$\therefore\ 3.5\ \varepsilon^{j0.75} = 3.5^{j42.98°}$$

$$= 3.5\ cos\ 42.98° + j3.5\ sin\ 42.98°$$

For both terms at once

① Set 90 $S\theta$ to 3.5 A against 42.98° (black) $S\theta$ read 2.386 A,
also against 42.98° (red) $S\theta$ read 2.56 A

① ② 

$$\therefore\ 3.5\ \varepsilon^{j0.75} = 2.56 + j2.386 \qquad Q.E.I.$$

Now you see, when the angle is near to $\frac{\pi}{4}$, either method will do. For simplicitie's sake [rather adopt Method (III), or first convert the angle into degrees and decimals, and then seek for the two terms at once by means of ($S\theta°$) and (A).

Section H. CARTESIAN CO-ORDINATES OF A COMPLEX NUMBER TO BE CONVERTED INTO POLAR CO-ORDINATES

$$a + jb = A\,|\underline{\theta}$$

Where

$$A = \sqrt{a^2 + b^2}$$

$$\theta = tan^{-1}\frac{b}{a} \qquad \text{or} \doteq cot^{-1}\frac{a}{b}$$

We can attack this in three different ways :—

Method (I).

To get A and θ independently.

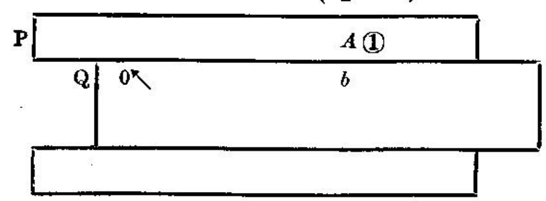
① $A = \sqrt{a^2 + b^2}$

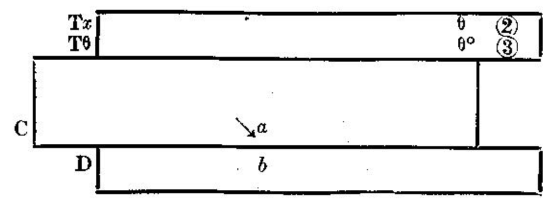
② $\theta = tan^{-1}\frac{b}{a}\left(= cot^{-1}\frac{a}{b} = \frac{\pi}{2} - tan^{-1}\frac{a}{b}\right)$

① Set 0 Q to a P against b Q read A-P

② Set a C to b D against 1 C read $\theta°$-$T\theta$
against 1 C read θ-Tx

Set b C to a D against 1 C read $(90° - \theta°)$ $T\theta$
also against 1 C read $\left(\frac{\pi}{2} - \theta\right)Tx$

① 

② 

Here θ always could be calculated accurately; but A could not be got very accurately when a and b are very small. When both a and b are too small, or too large you are to calculate A by a gradual method :—

$$A = \frac{1}{n}\sqrt{(na)^2+(nb)^2}$$

where n is a suitable and simple integer or a fraction.

When one of the two, a or b, is very small compared to the other, A would be nearly equal to the larger, and you have to calculate the angle only. And you could assume $tanx = x$, where x is very small and is in radian. Here when you want to express the angle in degrees and decimal, you have to convert the radian into degrees and decimals by means of (L) and (x).

Method (II).

By means of (A) and $(S\theta^\circ)$, or to get A and θ at once.

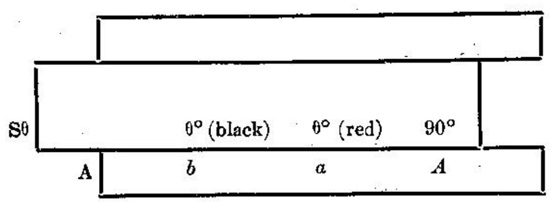
$$a + jb = A(cos\theta^\circ + jsin\theta^\circ)$$

or

$$\frac{a}{cos\theta^\circ} = \frac{b}{sin\theta^\circ} = A$$

Hence A and θ can be had at once, by setting a reading in black on $(S\theta^\circ)$ to b on (A), and at the same time alike reading in red on $(S\theta^\circ)$ to a on (A), and against $90^\circ(S\theta^\circ)$ read A on $(A.)$

Set θ (black) on $S\theta^\circ$ to bA } against $90^\circ S\theta^\circ$ read A on A
and also (red) on $S\theta^\circ$ to aA }



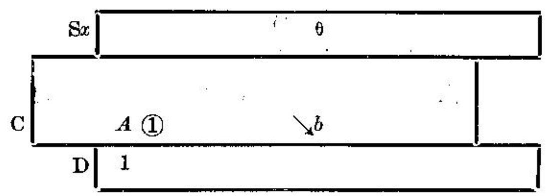
But the above method takes you much labour, as you have to set the slide by trial method, for you do not know what reading on $(S\theta^\circ)$ is to be set to a or b on (A),

So the following method would be better.

Method (III).

Get $\theta = tan^{-1}\frac{b}{a}$ as per in Method I.

Then ② Set $b\,C$ to $\theta\,Sx$ against $1\,D$ read A-C



Thus we have A.

When you have to employ n as stated in Method (I), all the necessary insertions must be made accordingly.

Example 1. $3.45 + j8.24 = 8.933\,\varepsilon^{j67.28^\circ}$

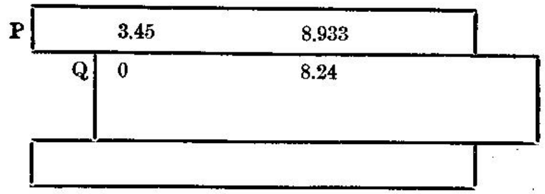
Method (I).

As $A = \sqrt{3.45^2 + 8.24^2}$

① Set $0\,Q$ to $3.45\,P$ against $8.24\,Q$ read $8.933\,P$

Next as $\frac{b}{a} = \frac{8.24}{3.45} > 1$, so prefer $cot^{-1}\theta$ to $tan^{-1}\theta$.

② Set 8.24 C to 3.45 D against 10 C read 67.28 $T\theta$

①


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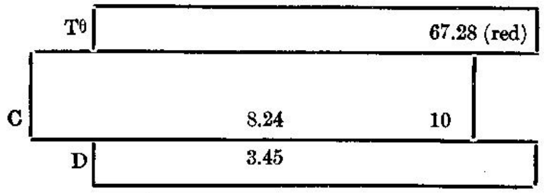
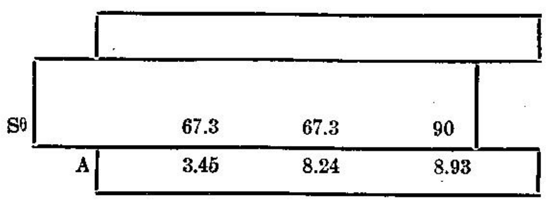


$\therefore \quad 3.45 + j8.24 = 8.933\varepsilon^{j67.28}$ Q.E.I.

By Method (II).

Set a reading θ in red on $(S\theta^\circ)$ to 3.45 on (A), and also at the same time θ in black on $(S\theta^\circ)$ to 8.24 on (A), against 90° on $(S\theta^\circ)$ read 8.93 on (A).

Set θ (red) $S\theta^\circ$ to 3.45 A }
and also θ (black) $S\theta^\circ$ to 8.24 A } against 90° $S\theta$ read 8.93 A



$\therefore \quad 3.45 + j8.24 = 8.93\varepsilon^{j67.3^\circ}$ Q.E.I.

By Method (III).

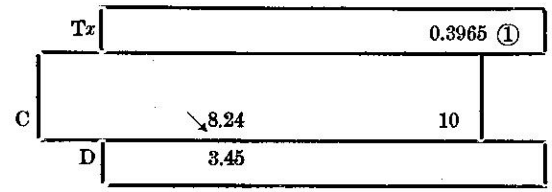
As
$$\tan^{-1}\frac{b}{a} = \theta$$

① Set 82.4 C to 3.45 D against 10 C read 0.3965 Tx

$$\theta = \left(\frac{\pi}{2} - 0.3965\right) = (1.1743) = 67.28^\circ$$

Next for A, as $\dfrac{\sin x}{b} = \dfrac{1}{A}$,

② Set 3.45 C to 0.3965 Sx against 10 D read 8.93 C

①


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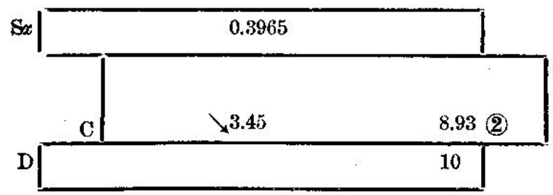


$\therefore \quad A = 8.93$

$\therefore \quad 3.45 + j8.24 = 8.93\varepsilon^{j67.28^\circ}$ Q.E.I.

In such a case as the above example, when either a or b is greater than 6, and the other is not very much smaller, Method (I) gives a more accurate result.

Section I. HYPERBOLIC FUNCTIONS OF A COMPLEX NUMBER

[1] $\mathbf{sinh(a \pm jb) = a_1 \pm jb_1 = A_1 \lfloor \pm \theta_1}$

First to convert it into (a_1+jb_1)

Method (I)

$$sinh\,(a+jb) = sinha\,cosb + jcosha\,sinb$$

$$= sinha\,sin\left(\frac{\pi}{2}-b\right) + j\sqrt{1+sinh^2 a}\,sinb$$

Calculate *sinha* between (Sh) and (C). $a\ Sh \rightarrow sinha\ C$

Calculate *cosha* between (P) and (P). $sinha\ P \rightarrow cosha\ P'$

Calculate *cosb*, by the theorem $cosb = sin\left(\frac{\pi}{2}-b\right)$

$$\therefore \left(\frac{\pi}{2}-b\right)Sx \rightarrow cosb\ D$$

Thus you have the four foctors separately and you can get a_1 and b_1 very easily between (C) and (D).

Method (II)

① But here is a short cut to reach the result: that is to calculate $a_1 = sinha\,sin\left(\frac{\pi}{2}-b\right)$ at once.

④ For $b_1=\sqrt{1+sinh^2 a}\,sinb$, you have to calculate *cosha* for preparation.

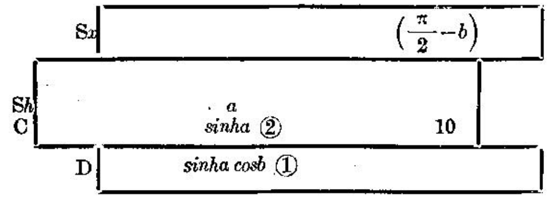
① Set 10 C to $\left(\frac{\pi}{2}-b\right)Sx$ against $a\ Sh$ read $a_1\ D$

② Against $a\ Sh$ read $sinha\ C$

③ „ $sinha\ P$ „ $cosha\ P'$

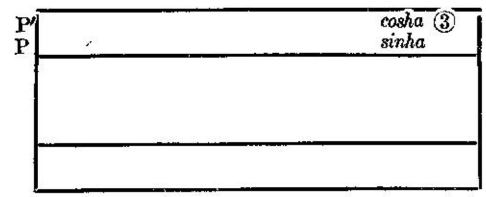
④ Set 10 C to $b\ Sx$ against $cosha\ C$ read $b_1\ D$

①, ②

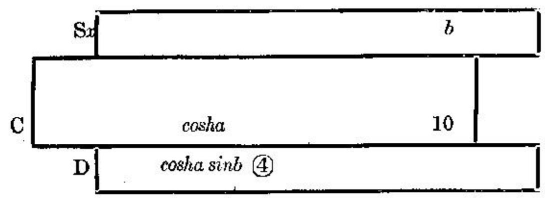


You must take notice that ② is quite unnecessary for a_1, it is necessary only for ③ or b_1 altimately.

③



④



① and ④ are what you want.

To convert (a_1+jb_1) into $A_1 \lfloor \theta_1$, readers are referred to Section H, p. 103.

Example 1. $sinh(0.82+j1.2)$

$$=sinh\,0.82\,cos\,1.2+j\,cosh\,0.82\,sin\,1.2$$

①, ②, ③ $=sinh\,0.82\,sin\,0.3708+j\sqrt{1+sinh^2 0.82}\,sin\,1.2$

④ $=0.9150\,sin\,0.3708+j1.3555\,sin\,1.2$

$$=0.3316+j1.263$$

$$\left(\frac{\pi}{2}-1.2\right)=0.3708$$

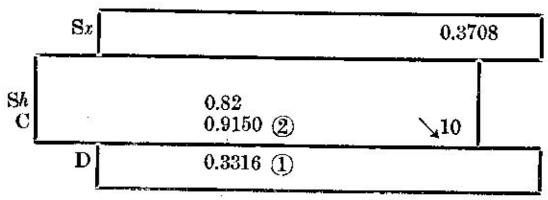
① Set 10 C to 0.3708 Sx against 0.82 Sh read 0.3316 D

② Against 0.82 Sh read 0.9150 C

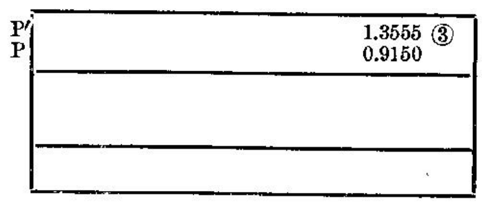
③ Against 0.9150 P read 1.3555 P'

④ Set 10 C to 1.2 Sx against 1.3555 C read 1.263 D

①, ②



③



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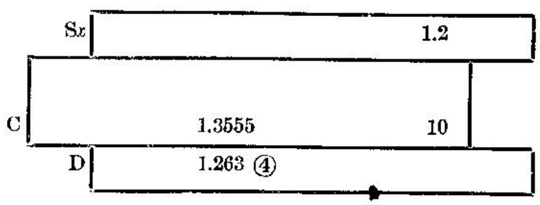


$\therefore\ a_1=0.3316,\quad b_1=1.263$

Again to convert these into the Polar Co-ordinates,

$$sinh\,(0.82+j1.2)=\ 0.3316+j1.263$$

$$=\sqrt{0.3316^2+1.263^2}\ \Big|\ cot^{-1}\frac{0.3316}{1.263}$$

$$=\ 1.3062\ |\ 75.26^\circ$$

$$=\ 1.3062\ |\ 1.3136$$

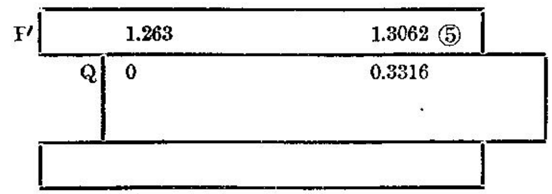
⑤ $A=\sqrt{0.3316^2+1.263^2}=1.3062$

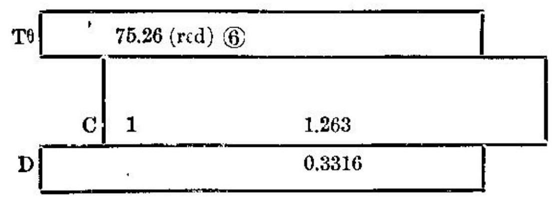
⑥ $\theta=\ cot^{-1}\dfrac{0.3316}{1.263}=75.26^\circ$

⑤ Set 0 Q to 1.263 P' against 0.3316 Q read 1.3062 P'

⑥ Set 1.263 C to 0.3316 D against 1 C read 75.26° (red) $T\theta$

⑦ Against 75.26° L read 1.3136 x





$\therefore\ A_1=1.302,\quad \theta_1=75.26$

[2] $\mathbf{cosh\,(a \pm jb) = a_2 \pm jb_2 = A_2 \,|\, \pm\theta_2}$

$cosh\,(a+jb) = cosha\,cosb + j\,sinha\,sinb$

In preparation to attack the first term, or the real part.

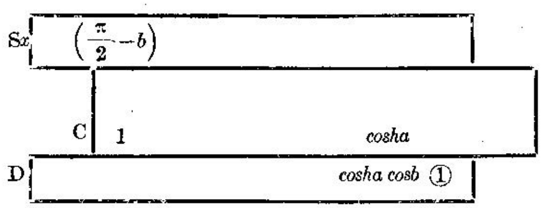
$cosb = sin\left(\frac{\pi}{2} - b\right)$

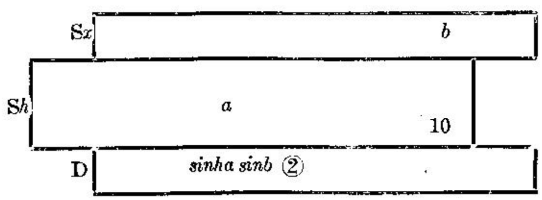
$cosha$ by $sinha\ P \rightarrow cosha\ P'$

For the second term or the imaginary part, no preparation is wanted.

① Set 1 C to $\left(\frac{\pi}{2} - b\right) Sx$ against $cosha\ C$ read $cosha\ cosb\ D$

② Set 10 C to $b\ Sx$ against $a\ Sh$ read $sinha\ sinb\ D$





① and ② in the diagrams are what you want.

To convert $(a_2 + jb_2)$ into $A_2 \,|\, \theta_2$, refer to Section H, p. 103.

Example 2. $cosh\,(0.82 + j1.2) = 0.4912 + j\,0.8528$
$= 0.9841 \,|\, 1.0485$

$cosh\,(0.82 + j\,1.2)$

$= cosh\,0.82\,cos\,1.2 + j\,sinh\,0.82\,sin\,1.2$

$= \sqrt{1 + sinh^2\,0.82}\,sin\,0.3708 + j\,sinh\,0.82\,sin\,1.2$

①, ② $= 1.3555\,sin\,0.3708 + j\,0.9150\,sin\,1.2$

$= 0.4912 + j\,0.8528$

③, ④ $= \sqrt{0.4912^2 + 0.8528^2}\,\Big|\,cot^{-1}\frac{0.4912}{0.8528}$

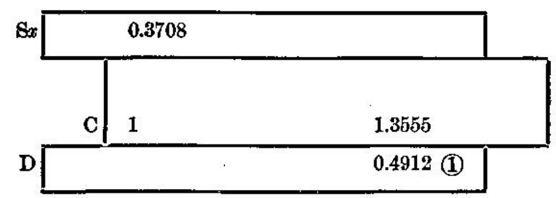
⑤ $= 0.9841 \,|\, 60.08°$

$= 0.9841 \,|\, 1.0485$

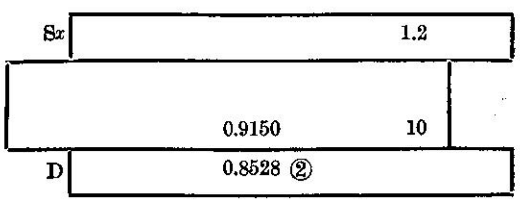
① Set 1 C to 0.3708 Sx against 1.3555 C read 0.4912 D

② Set 10 C to 1.2 Sx against 0.9150 C read 0.8528 D

①



②



$\therefore \quad a_2=0.4912$

$b_2=0.8528$

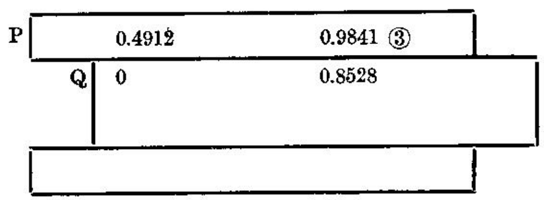
Further for A_2 and θ_2:—

③ Set 0 Q to 0.4912 P against 0.8528 Q read 0.9841 P

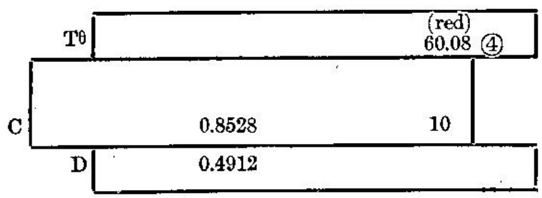
④ Set 0.8528 C to 0.4912 D against 10 C read 60.08° (red) $T\theta°$

③ Against 60.08° L read 1.0485 x

③



④



$\therefore \quad A_2= 0.9841$

$\theta_2=60.08°$

or=(1.0485)

$\therefore \quad cosh\,(0.82+j1.2)=0.4912+j0.8528$ Q.E.I.

$=0.9841\,\underline{|1.0485}$ Q.E.I.

[3] To convert sinh(a±jb) directly into $A_1\,\underline{|\pm\theta_1}$, instead of going through the form $(a_1\pm jb_1)$.

$$sinh\,(a+jb)$$

$$=\sqrt{(sinha\ cosb)^2+(cosha\ sinb)^2}\ \varepsilon^{j\tan^{-1}(tanb/tanha)}$$

$$=\sqrt{cosh^2a-cos^2b}\ \varepsilon^{j\tan^{-1}(tanb/tanha)}$$

$$=\sqrt{sinh^2a+sin^2b}\ \varepsilon^{j\tan^{-1}(tanb/tanha)} \qquad ①,\ ②\ ④$$

or $$=\sqrt{sinh^2a+sin^2b}\ \varepsilon^{j\cot^{-1}\left\{\tan\left(\frac{\pi}{2}-b\right)tanha\right\}} \qquad ③,\ ⑤$$

$$=A_1\varepsilon^{j\alpha_1}$$

① Against a Sh read $sinha$ C

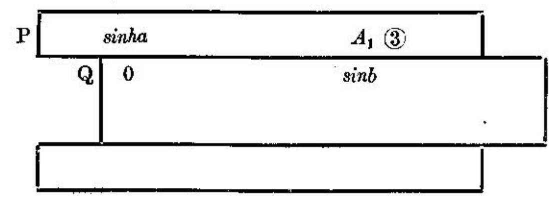
② Against b Sx read $sinb$ D

③ Set 0 Q to $sinha$ P against $sinb$ Q read A_1-P

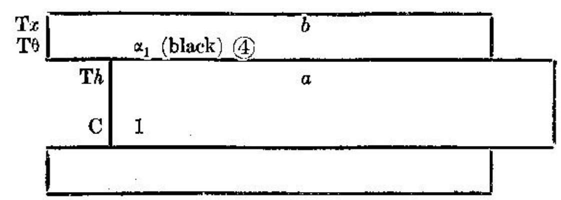
④ Set a Th to b Tx against end-C read a_1 (black) $T\theta$

or Set end-C to $\left(\frac{\pi}{2}-b\right)$ Tx against a Th read a_1 (red) $T\theta$

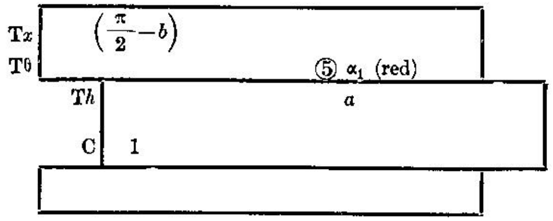
③



④



or



Thus you can get A_1 and α_1.

Example 3. $sin\,(0.82+j1.2)=1.306\,\underline{|75.26°}$

$sinh\,(0.82+j1.2)=\sqrt{sinh^2\,0.82+sin^2\,1.2}\;\varepsilon^{j tan^{-1}(tan\,1.2/tanh\,0.82)}$

① Against 0.82 *Sh* read 0.9150 *C*

② Against 1.2 *Sx* read 0.932 *D*

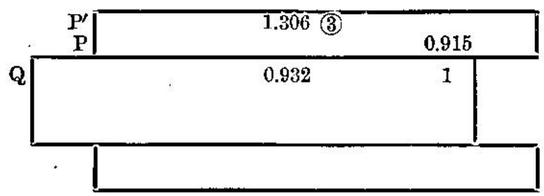
③ Set 1 *Q* to 0.9150 *P* against 0.932 *Q* read 1.306 *P′*

④ $\theta=tan^{-1}\frac{tan\,1.2}{tanh\,0.82}=tan^{-1}\frac{1}{tan\left(\frac{\pi}{2}-1.2\right)tanh\,0.82}$

$=cot^{-1}(tan\,0.3708\;tanh\,0.82)$

∴ Set 10 *C* to 0.3708 *Tx* against 0.82 *Th* read 75.26° (red) *Tθ′*

③



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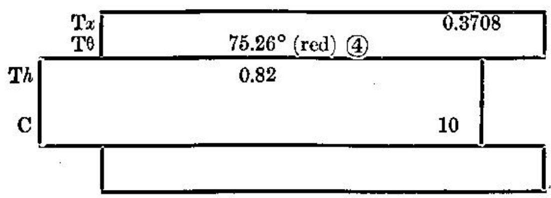


∴ $sin\,(0.82+j1.2)=1.306\,\underline{|75.26°}$

[4] cosh(a ± jb) = $A_2\,\underline{|\pm\theta_2}$ directly.

$cosh\,(a+jb)$

$=\sqrt{cosh^2a-sin^2b}\;\varepsilon^{j tan^{-1}(tanb\,tanha)}$

$=\sqrt{1+sinh^2a-sinb^2}\;\varepsilon^{j tan^{-1}(tanb\,tanha)}$ ①, ②, ③

$=\sqrt{1+sinh^2a-sin^2b}\;\varepsilon^{j cot^{-1}\left\{tan\left(\frac{\pi}{2}-b\right)/tanha\right\}}$ ④

$≑ A_2\varepsilon^{j\alpha_2}$

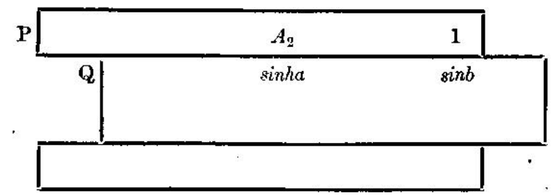
① Set *sinb Q* to 1 *P* against *sinha Q* read A_2 *P*

or Set *sinb Q* to 1 *P′* against *sinha Q* read A_2 *P′*

② Set end *C* to *tanb Tx* against *tanha Th* read α_2(black) *Tθ*

or Set *tanha Th* to $tan\left(\frac{\pi}{2}-b\right)$ *Tx* against end *C* read α_2(red) *Tθ*

①



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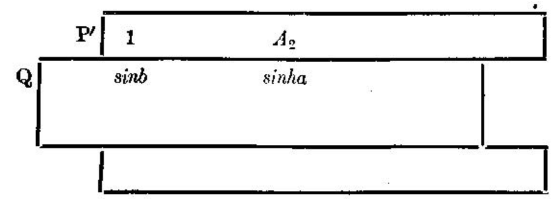

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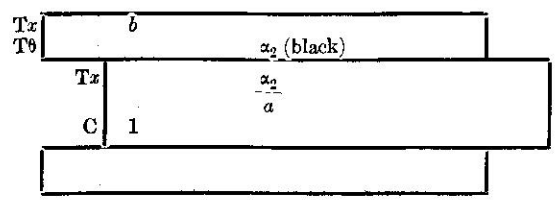

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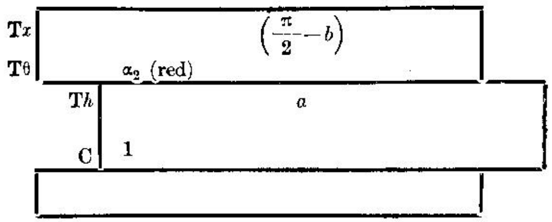


Thus you can get both A_2 and a_2

Example 4. $cosh\,(0.82+j1.2)=0.984\,\underline{|60.06^\circ}$

$$cosh(0.82+j1.2)=\sqrt{1+sinh^2\,0.82-sin^2\,1.2}\;\varepsilon^{j\tan^{-1}(\tan 1.2\,\tanh 0.82)}$$

① ②

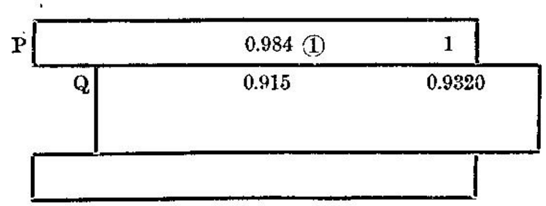
① $sinh\,0.82=0.915,\qquad sin\,1.2=0.9320$ (Example 1)

Set 0.9320 Q to 1 P against 0.915 Q read 0.984 P

② $a_2=tan^{-1}(tan\,1.2\,tanh\,0.82)=cot^{-1}\dfrac{tan\,0.3708}{tanh\,0.82}$

Set 0.82 Th to 0.3708 Tx against 10 C read 60.06 (red) $T\theta^\circ$

①



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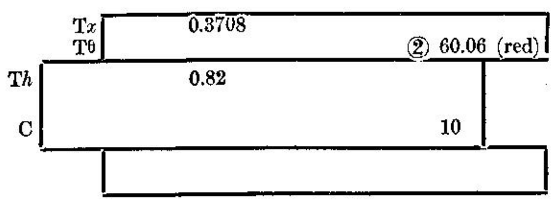


$\therefore\; cosh\,(0.82)+j1.2=0.984\,\underline{|60.06^\circ}$

[5] $\tanh(a\pm jb)=A_3\,\underline{|\pm\theta_3}=a_3\pm jb_3$

$$tanh(a+jb)=\frac{sinh(a+jb)}{cosh(a+jb)}=\frac{a_1+jb_1}{a_2+jb_2}\qquad\frac{(a)}{(b)}$$

$$=\frac{A_1\,\underline{|\theta_1}}{A_2\,\underline{|\theta_2}}=\frac{A_1}{A_2}\underline{|\theta_1-\theta_2}$$

$$=A_3\,\underline{|\theta_3}$$

$$=a_3+jb_3$$

Example 5. $tanh\,(0.82+j1.2)=1.327\,\underline{|15.20^\circ}$

$$tanh\,(0.82+j1.2)=\frac{sinh\,(0.82+j1.2)}{cosh\,(0.82+j1.2)}$$

$sinh\,(0.82+j1.2)=1.306\,\underline{|75.26^\circ}$
$cosh\,(0.82+j1.2)=0.984\,\underline{|60.06^\circ}$ as shown in Examples 1 and 2.

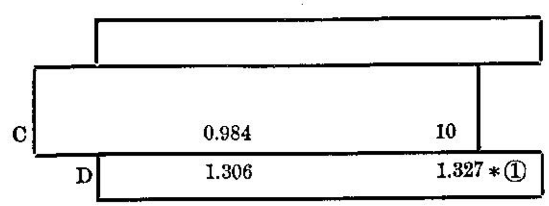
$$tanh(0.82+j1.2)=\frac{1.306\,\underline{|75.26^\circ}}{0.984\,\underline{|60.06^\circ}}$$

$$=\frac{1.306}{0.984}\underline{|75.26^\circ-60.06^\circ}\qquad ①$$

$$=1.327\,\underline{|15.20^\circ}$$

① Set 0.984 C to 1.306 D against 10 C read 1.327 D

①



$\therefore\; tanh\,(0.82+j1.2)=1.327\,\underline{|15.20^\circ}$

Sction J. ANTI-HYPERBOLIC TANGENT OF A COMPLEX NUMBER

$$tanh^{-1}(a\pm jb)=\frac{\mu}{2}\log\sqrt{\frac{(1+a^2)+b^2}{(1-a^2)+b^2}}\pm j\tfrac{1}{2}\left(tan^{-1}\frac{b}{1+a}+tan^{-1}\frac{b}{1-a}\right)$$

The function $tanh^{-1}(a+jb)$ is very important for the calculation of electric circuits; and you can calculate it out in the following way:—

Put $tanh^{-1}(a+jb)=z=x+jy$

Then $tanh z=a+jb$

$$\frac{\epsilon^z-\epsilon^{-z}}{\epsilon^z+\epsilon^{-z}}=a+jb$$

$$\frac{\epsilon^{2z}-1}{\epsilon^{2z}+1}=a+jb$$

$$\epsilon^{2z}=\epsilon^{2(x+jy)}=\frac{1+a+jb}{1-(a+jb)}$$

Say $\frac{1+a+jb}{1-a-jb}=A\underline{|\theta}$

or $\epsilon^{2(x+jy)}=\epsilon^{2x}\,\epsilon^{j2y}=A\underline{|\theta}$

$\therefore$ $\epsilon^{2x}=A$

$$2x=\log_\epsilon A$$

$$x=\tfrac{1}{2}\log_\epsilon A$$

$$=\tfrac{1}{2}\log_{10}A\times\frac{1}{\log_{10}\epsilon}$$

$$=1.1513\log A$$

$(\because\ \frac{1}{\log\epsilon}=2.30258509=\mu$ popularly)

And $2y=\theta$

$$y=\frac{\theta}{2}$$

So by Methods already stated, you convert $(1+a)+jb$, and $(1-a)-jb$, into the forms of Polar Co-ordinates; from which you can get A and θ. Get $\log A$ between (D) and (L), which is to be multiplied by 1.1513 for the value of x.

It is particularly to be noticed that here

$$A\underline{|\theta}\neq tanh^{-1}(a+jb)$$

Let $\begin{cases}1+a+jb=A_1\underline{|\theta_1}\\ 1-a-jb=A_2\underline{|\theta_2}\end{cases}$

$$1+a+jb=(1+a)+jb$$

$$1-a-jb=(1-a)-jb$$

$$A_1=\sqrt{(1+a)^2+b^2}$$

$$A_2=\sqrt{(1-a)^2+b^2}$$

$$\theta_1=\ tan^{-1}\frac{b}{1+a}$$

$$\theta_2=-tan^{-1}\frac{b}{1-a}$$

Then $A=\frac{A_1}{A_2}$

$$=\sqrt{\frac{(1+a)^2+b_2}{(1-a)^2+b^2}}=\sqrt{1+\frac{4a}{(1-a)^2+b^2}} \qquad \text{I}$$

$$\theta=\theta_1-\theta_2$$

$$=tan^{-1}\frac{b}{1+a}+tan^{-1}\frac{b}{1-a} \qquad \text{II A}$$

Now there is another way for getting θ, which method is sometimes preferable for accuracie's sake.

As $A\underline{|\theta} = \dfrac{1+a+jb}{1-a-jb}$

$$= \frac{(1+a+jb)(1-a+jb)}{(1-a)^2+b^2}$$

$$= \frac{1-(a^2+b^2)+j2b}{(1-a)^2+b^2}$$

$$\therefore \quad \theta = tan^{-1}\frac{2b}{1-(a^2+b^2)}$$

or $$= -tan^{-1}\frac{2b}{a^2+b^2-1} \qquad \text{II B}$$

$\therefore \quad tanh^{-1}(a+jb)$

$$= 1.1513\, log\sqrt{\frac{(1+a)^2+b^2}{(1-a)^2+b^2}} + j\tfrac{1}{2}\left(tan^{-1}\frac{b}{1+a} + tan^{-1}\frac{b}{1-a}\right)$$

$$= 0.5756\, log\left\{\frac{(1+a)^2+b^2}{(1-a)^2+b^2}\right\} + j\tfrac{1}{2}\, tan^{-1}\frac{2b}{1-a^2-b^2}$$

Of the forms

$$1.1513\, log\sqrt{\frac{(1+a)^2+b^2}{(1-a)^2+b^2}}, \quad 0.5756\, log\frac{(1+a)^2+b^2}{(1-a)^2+b^2}$$

the former is better for this slide rule, and the latter is preferable for mathematical tables.

Example 1.

$$tanh^{-1}\frac{1281.6\,\overline{|7.89^\circ}}{500\ \underline{|5.0^\circ}} = 0.3966 + j\left(\frac{\pi}{2} + 0.10135\right)$$

$$\frac{1281.6\,\overline{|7.89^\circ}}{500\ \underline{|5.0^\circ}} = 2.563\,\overline{|12.89^\circ}$$

$$= 2.563\, cos\, 12.89^\circ - j2.593\, sin\, 12.89^\circ$$

$$= 2.498 - j0.572 \qquad (6)$$

$$\therefore \quad tanh^{-1}\frac{1281.6\,\overline{|7.89^\circ}}{500\ \underline{|5.0^\circ}} = tanh^{-1}(2.498 - j0.572)$$

$$= 1.1513\, log\sqrt{\frac{(1+2.498)^2+0.572^2}{(1-2.498)^2+0.572^2}} + j\tfrac{1}{2}\, tan^{-1}\frac{-2\times 0.572}{1-2.498^2-0.572^2}$$

①, ②, ④

$$= 1.1513\, log\frac{3.544}{1.603} + j\tfrac{1}{2}\, tan^{-1}\frac{-2\times 0.572}{-4\times 1.3920} \qquad ③, ⑤$$

$$= 1.1513 \times 0.3445 + j\tfrac{1}{2}(\pi + 0.2027)$$

$$= 0.3966 + j\left(\frac{\pi}{2} + 0.10135\right) \qquad \text{Q.E.I.}$$

① $\sqrt{(1+2.498)^2+0.572^2} = \sqrt{3.498^2+0.572^2}$

$= 4\sqrt{0.8745^2+0.1430^2}$

$= 4 \times 0.8864 = 3.544$

Set 0 Q to 0.8745 P against 0.1430 Q read 0.8864 P

② $\sqrt{(1-2.498)^2+0.572^2} = \sqrt{1.498^2+0.572^2}$

$= 4\sqrt{0.3745^2+0.1430^2}$

$= 4 \times 0.4008 = 1.603$

Set 0 Q to 0.3745 P against 0.1430 Q read 0.4008 P

③ $log\dfrac{3.544}{1.603} = log\dfrac{0.8864}{0.4008} = 0.3445$

Set 0.4008 C to 0.8864 D against 1 C read 0.3445 L

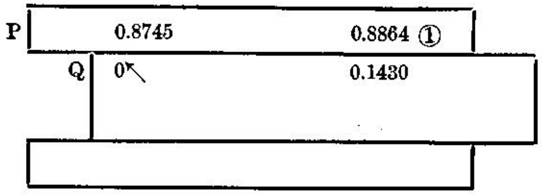
④ $1-2.498^2-0.572^2 = -4\times(1.249^2+0.286^2-0.5^2)$

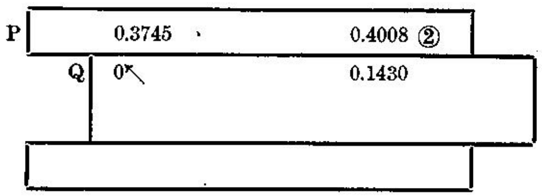
$= -4 \times 1.3920$

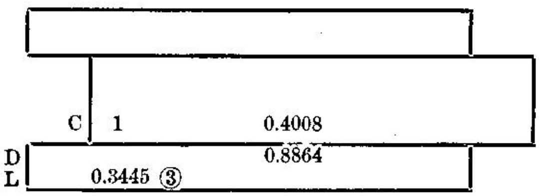
Set 0.5 Q to 1.249 P' against 0.286 Q read (1+)0.3920 L

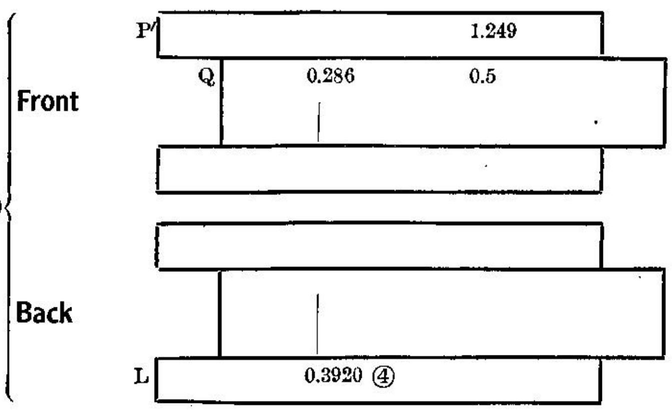
⑤ $\tan^{-1}\dfrac{-2\times 0.572}{-4\times 1.3920}=\tan^{-1}\dfrac{0.572}{2.784}=0.2027$

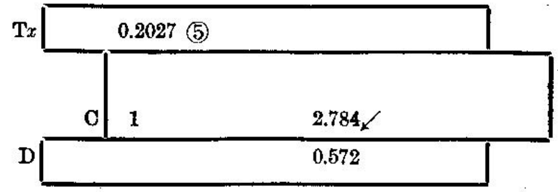
Set 2.784 C to 0.572 D against 1 C read 0.2027 Tx

①


②


③


④


⑤


∴ ① $\sqrt{0.8745^2+0.1430^2}=0.8864$

② $\sqrt{0.3745^2+0.1430^2}=0.4008$

③ $\log\dfrac{0.8864}{0.4008}=0.3445$

④ $(1.249^2+0.286^2-0.5^2)=1.3920$ (0.3920 L)

⑤ $\tan^{-1}\dfrac{0.572}{2.784}=0.2027$

Hence the final result :—

$$\tanh^{-1}\frac{1281.6\,|\,7.89^\circ}{500\,|\,5.0^\circ}=0.3966+j\left(\frac{\pi}{2}+0.10135\right)\qquad \text{Q.E.I.}$$

as already stated on pp. 118–9.

Just for reference, the result that has been reached with a reckoning machine of 18 places, is

$$\tanh^{-1}\frac{1281.6\,|\,7.89^\circ}{500\,|\,5.0^\circ}=0.39663+j\left(\frac{\pi}{2}+0.101345\right)$$

which shows that the errors in the result that had been reached with this slide rule were only less than 0.01% and 0.005%. These are of course quite negligible.

Section K. THE DISTRIBUTION OF THE VOLTAGE AND THE CURRENT IN THE ELECTRIC CIRCUIT

Coefficient of inductance, $\xi = 1.672 \times 10^{-3} \underline{|81.74^\circ}$

$$=(0.2403 + j1.654) \times 10^{-3} \qquad (6)$$

Impedance of the specific characters, $R = 1.2816 \times 10^{-3} \overline{|7.89^\circ} \qquad ②$

Impedance at the second terminus, $Z = 500 \overline{|5.00^\circ}$

What are the distributions of the voltage and the current per 100 km of circuit when the voltage of the main circuit is V_s?

If $\theta = tan^{-1}\frac{R}{Z}$, then at a point x km from the first terminus,

$$I = \frac{sinh\{\xi(D-x)+\theta\}}{cosh(\xi D+\theta)} \frac{V_s}{R} \qquad V = \frac{cosh\{\xi(D-x)+\theta\}}{cosh(\xi D+\theta)} V_s$$

(*a*) The voltage and the current at the second terminus:—

$$V = \frac{cosh\,\theta}{cosh(\xi D+\theta)} V_s$$

Here $\theta = 0.397 + j\left(\frac{\pi}{2} + 0.10135\right)$ as you saw in p. 125, hence

$$V = \frac{cosh\left\{0.397 + j\left(\frac{\pi}{2} + 0.10135\right)\right\}}{cosh\left\{(0.2403 + j1.654) \times 100 \times 10^{-3} + 0.397 + j\left(\frac{\pi}{2} + 0.10135\right)\right\}} V_s$$

$$= \frac{cosh\left\{0.397 + j\left(\frac{\pi}{2} + 0.10135\right)\right\}}{cosh\left\{0.4210 + j\left(\frac{\pi}{2} + 0.2668\right)\right\}} V_s$$

Here $cosh\left\{0.387 + j\left(\frac{\pi}{2} + 0.10135\right)\right\}$

$$= cosh\,0.397\,cos\left(\frac{\pi}{2} + 0.10135\right) + j sinh\,0.397\,sin\left(\frac{\pi}{2} + 0.10135\right)$$

$$= -1.0798\,sin\,0.10135 + j sinh\,0.397\,sin\left(\frac{\pi}{2} - 0.10135\right)$$

$$= -0.10925 + j\,0.4055$$

$$= 0.4198 \underline{\left|\frac{\pi}{2} + 0.2632\right.} \qquad (8)$$

Also $cosh\left\{0.4210 + j\left(\frac{\pi}{2} + 0.2668\right)\right\}$

$$= -cosh\,0.4210\,sin\,0.2668 + j sinh\,0.4210\,sin\,1.3140$$

$$= -0.2875 + j\,0.4193$$

$$= 0.508 \underline{\left|\frac{\pi}{2} + 0.601\right.} \qquad (8)$$

$$\therefore \quad V_r = \frac{0.4168 \underline{\left|\frac{\pi}{2} + 0.2632\right.}}{0.508 \underline{\left|\frac{\pi}{2} + 0.601\right.}} V_s$$

$$= 0.826 \overline{|0.338} V_s$$

$$= (0.7802 - j\,0.2750) V_s \qquad \text{Q.E.I.}$$

Just for reference, we shall mention the result that has been reached with a reckoning machine:—

$$V_r = (0.78018 - j\,0.27498) V_s$$

which proves that the result on the slide rule is accurate enough.

$$I_r=\frac{V_r}{Z}=\frac{0.826\,|\overline{0.338}}{500\,|\overline{5.0^\circ}}V_s=\frac{0.826\,|\overline{0.338}}{500\,|\overline{0.087}}$$

$$=1.652\times10^{-3}\,|\overline{0.425}\,V_s$$

$$=1.652\times10^{-3}\,|\overline{24.21^\circ}\,V_s$$ Q.E.I.

(β) The Current at the first terminus :—

$$I_s=\frac{\sinh(\xi D+\theta)}{\cosh(\xi D+\theta)}\frac{V_s}{R}$$

Here $\sinh(\xi D+\theta)=\sinh\left\{0.4210+j\left(\frac{\pi}{2}+0.2668\right)\right\}$

$$=\sinh 0.4210\cos\left(\frac{\pi}{2}+0.2668\right)+j\cosh 0.4210\sin\left(\frac{\pi}{2}+0.2668\right)$$

$$=-\sin 0.4210\sin 0.2668+j\,1.0899\sin 1.3040$$

$$=-0.1101+j\,1.050$$

$$=1.0580\,\left|\overline{\frac{\pi}{2}+0.1045}\right. \qquad (8)$$

$$\therefore\quad I_s=\frac{1.058\,\left|\overline{\frac{\pi}{2}+0.1045}\right.}{\left(0.508\,\left|\overline{\frac{\pi}{2}+0.601}\right.\right)\times\left(1.2816\times10^3\,|\overline{0.1375}\right)}V_s$$

$$=1.626\times10^{-3}\,|\overline{0.359}\,V_s$$

$$=(1.525-j\,0.568)\times10^{-3}V_s$$ Q.E.I.

Just for reference, the reckoning machine gives

$$I_s=(1.5252-j\,0.5678)\times10^{-3}V_s$$

which shows errors only less than 0.02% or 0.5% respectively.

Section L. THE LINE CONSTANTS

Y_o, Admittance indicated at one of the terminuses, having the other terminus open.

Z_s, Impedance indicated at one of the terminuses, having the other terminuses short.

$$Y_o=\frac{\tanh\xi D}{R}$$

$$Z_s=R\tanh\xi D$$

$$\therefore\quad R=\sqrt{\frac{Z_s}{Y_o}}$$

$$\xi D=\tanh^{-1}\sqrt{Y_o Z_s}$$

y, Admittance per unit length of the circuit.

Z, Impedance per unit length of the circuit.

$$y=\frac{\xi}{R}$$

$$Z=\xi R$$

Example 1.

The admittance, Y_o, and the impedance Z_s have been experimented as follows when the length of the circuit is 77.9 km and the frequency is 60∼. Calculate the constants per 1 km of the circuit.

$Y_o = 212.25 \times 10^{-6} \angle 90^\circ$ mho

$Z_s = 72.61 \angle 71.8^\circ$

$R = \sqrt{\frac{72.61}{212.25}} \times 10^3 \angle 9.1^\circ = 585 \angle 9.1^\circ$

$\sqrt{Y_o Z_s} = \sqrt{72.61 \times 212.25} \times 10^{-3} \angle 80.9$

$= 0.01964 + j\, 0.1220$

$\varepsilon^{2(x+jy)} = \frac{1.01964 + j\, 0.1220}{0.98036 - j\, 0.1220}$

$= \frac{1.026 \angle 6.83^\circ}{0.9875 \angle 7.10^\circ}$

$= 1.039 \angle 13.93$

$y = \frac{13.93^\circ}{2} = 6.965^\circ = 0.12155$

$x = 1.1513 \log 1.039 = 0.01905$

$\xi D = 0.01905 + j\, 0.12155$

$\xi = 1.579 \times 10^{-3} \angle 81.12^\circ$

Whence

$Z = R\xi = 0.924 \angle 72.02^\circ$

$= 0.2852 + j\, 0.8795$ ohms/km Q.E.I.

$y = \frac{\xi}{R} = 2.699 \times 10^{-6} \angle 90.22^\circ$

$= j\, 2.699 \times 10^{-6}$ mho Q.E.I.

APPENDIX

FOR BEGINNERS

(THE ABC OF THE ABCD SLIDE RULE)

THE ABCD SLIDE RULE MEANS THE STUDENT'S RULE WHICH IS VERY POPULAR THROUGHOUT THE WORLD

Section A. GENERAL IDEA

Of Hemmi's Bamboo Slide Rules, there are a number of different sorts on the market, designed for special purposes, and containing suitable scales for their special purposes. But here we shall deal with what we call the "student's rule" or the ABCD slide rule, which is really the basis of all slide rules. With this slide rule we can do multiplication, division, proportion, squaring extraction of square roots, cubes and extraction of cube roots, &c.

The instrument consists of two parts: one a fixed part which we call the "rule," and the other movable between the fixed scales on the rule, and so is called the "slide."

When looking at the face of this slide rule, the lower scale on the rule is the "*D*" scale [hereafter we shall call it simply (*D*)]; the similar scale on the lower edge of the slide is the "*C*" scale [which we shall call hereafter (*C*)], both of which are one-section scales; that is, the figures run from 1.0 to 10.0 throughout the whole length of the scales. The upper scale on the rule is called the "*A*" scale or (*A*), and the similar scale on the upper edge of the slide is called the "*B*" scale or (*B*), both of which are two-section scales; that is, the figures run from 1 to 10. twice throughout the whole length of the scales; or in more precise words, the numbers run from 1.0 to 100.0 throughout the whole length of the scale.

These four scales are divided accurately according to the logarithms of the numbers they represent, and so the divisions get smaller as they proceed to the right. From this it follows that if two sections of these scales are added together, the sum of these sections represent the product of the numbers, represented by those sections (Demonstration omitted). In using these scales or the very slide rule, there is no need to think of logarithms, only the figures being dealt with. This is the great advantage of the slide rule.

It is particularly requested that the readers should throughly understand "the Slide Rule Laconism and the International Slide Rule Equation," pages 3-5 before they proceed any further.

Section B. READING THE SCALES

Each line on (*A*), (*B*), (*C*) or (*D*), represents some number. In the finer divided portions of the scales, some of the lines are omitted to avoid crowding, and thus facilitate reading.

Taking first (*D*), it will be noticed that the figures, given to the lines of the nine main divisions throughout the whole length of the scale, run from 1.0 to 10.0, 1.0 being at the left end, and 10.0 at the right. Each of these nine divisions is in turn divided into ten sub-divisions, those between 1.0 and 2.0 on the left portion of the scale are usually numbered, while those in the remaining main divisions are marked but not numbered. Returning to the left, the sub-divisions between 1.0 and 2.0 are each divided again into ten smaller divisions, while those between 2.0 and 4.0 are divided into five smaller divisions only. But as all the divisions are to be divided decimally into ten parts (either marked by a line or not) the lines marked represent 0.02, 0.04, 0.06 and 0.08; while the odd numbers 0.01, 0.03, 0.05, 0.07 and 0.09 between have to be estimated between the lines marked by the eye. Looking at the ten sub-divisions between 4.0 and 5.0, 5.0 and 6.0, etc., towards the right-hand end of the scale, you see that each of these sub-divisions contains only one line, dividing each into two portions. It therefore follows that as each of these sub-divisions has to be divided into ten parts, four divisions must be read mentally on each side of the division line. The 0.01, 0.02, 0.03 and 0.04 are to be read to the left of the marked line, the line itself representing the 0.05 and 0.06, 0.07, 0.08 and 0.09 are to be read to the right of the line.

It will soon be realised that the mental estimating of the unmarked divisions is a more accurate method of reading, and is less trying to the sight than endeavouring to read overcrowded division lines.

However as any one line, for instance 3.24, is to stand for any number of similar digit value, such as 32.4, 32400, 0.00324, &c., you may ignore decimal points in the numbers read on the scales. We call this a "digit reading." See Figs. 9 and 10.

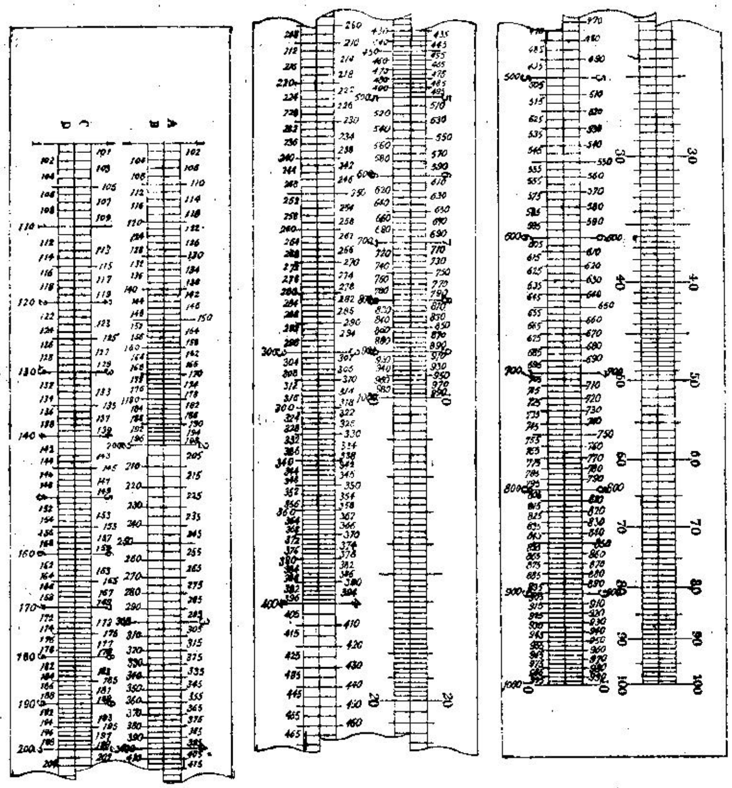


Fig. 9.

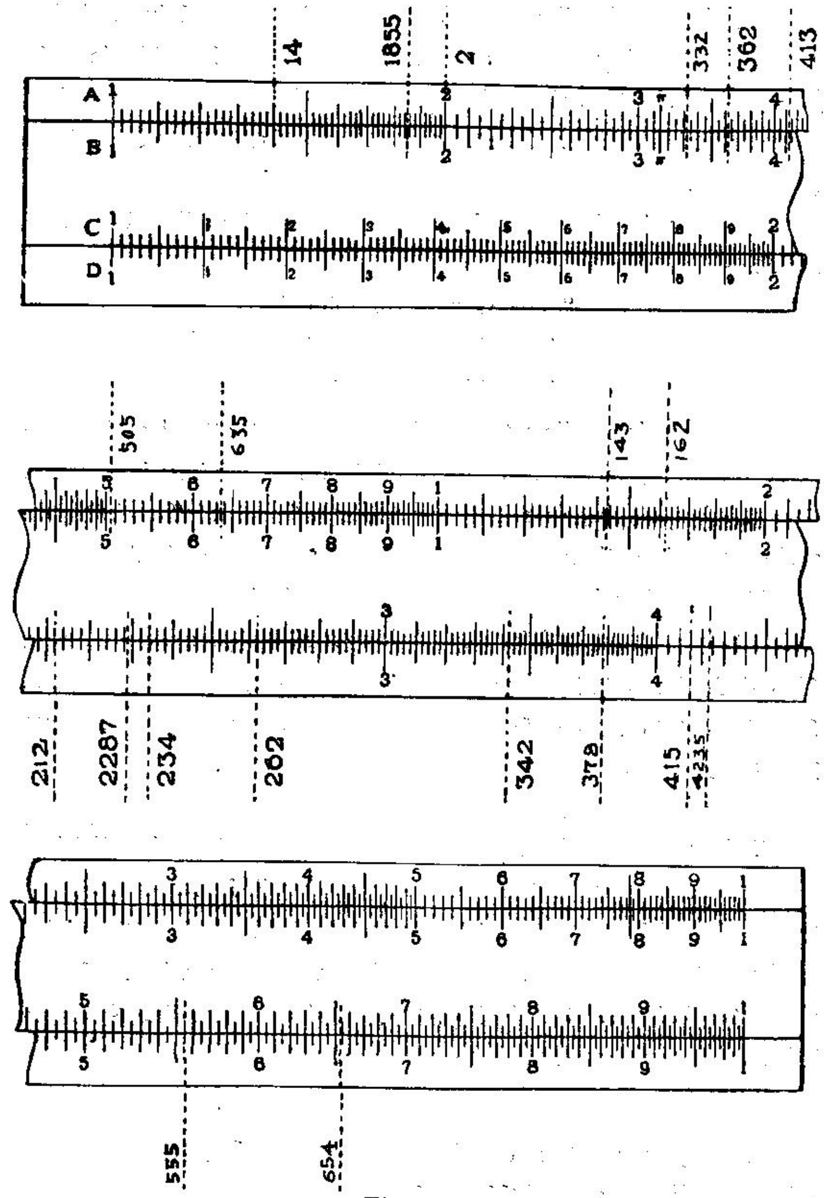


Fig. 10.

(*C*) is exactly like (*D*). (*A*) and (*B*) are read in a similar manner, but owing to the shorter length of the logarithmic unit, just half as short as that of (*C*) and (*D*), fewer lines are marked than on (*C*) and (*D*). The figures run from 1.0 to 100.0 on either (*A*) or (*B*).

Reading is the most important, hence the *C.G.S.* & *3-R* Rule :—

"The Correct Reading is
The Golden Rule of
The Slide Rule."

Really it is reading that claims your best attention, because correct reading means accuracy, and incorrect reading inaccuracy, errors, mistakes. It is not going too far to say: "Training on a slide rule is entirely the training in reading the scales."

For learning how to read the scales, it is the best way to look at the scales in Fig. 8. Follow 212, 2287, 234, 262, 342, 378, 415, 4235, 555 and 654 on (*C*) and (*D*); and 14, 185, 2, 332, 362, 413, 505, 635, 143 and 162 on (*A*) and (*B*). Do not pass away carelessly; follows them most carefully.

For trial, read 234 on (*D*).

This number will be found in the second main division, between 2.0 and 3.0; the second figure "3" is represented by three sub-divisions, after which the hairline is moved to the right, as far as to the second smaller division, when you will read 234 under the hairline. See in Fig. 8, the third dotted line from the left on (*D*) figured 234.

Here you must read **"Laconism and International Diagram,"** pp. 3–5 of this book.

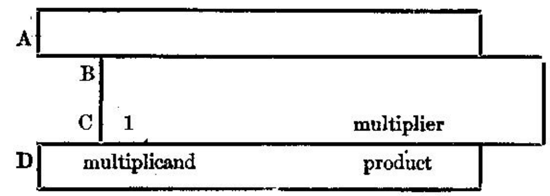
Section C. MULTIPLICATION

[1] Simple Multiplication :—

Rule I. Set 1.0 or sometimes 10.0 on (*C*) to the multiplicand on (*D*), against the multiplier on (*C*) read the product on (*D*). The decimal point can be logically located, but it is usually better to be determined by Rule II.

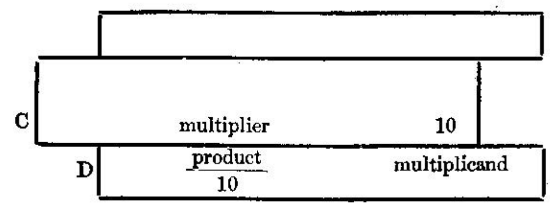
① Set 1 *C* to multiplicand-*D* against multiplier-*C* read product-*D*

①



② Set 10 *C* to multiplicand-*D* against multiplier-*C* read $\frac{\text{product}}{10}$-*D*

②



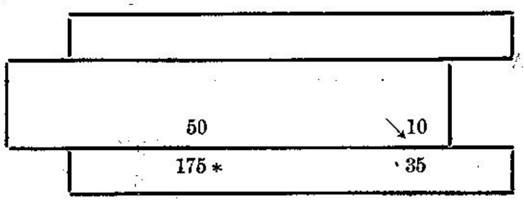
The result in ② is $\frac{1}{10}\times$product instead of the product itself. So it is necessary to shift the decimal point just one place to your right. But if you expect the digit reading (of the product) only, it is quite all right. ② is good only when ① fails.

Rule II. For the determination of the decimal point, take a simple number nearest to the multiplicand, and another nearest to the multiplier; multiply them each other mentally. Now you see the final result required, can not be very far from the result of this multiplication.

Example 1. $35 \times 5 = 175$

First it is a case of ②.

Set 10 *C* to 35 *D* against 50 *C* read 175 *D* (Rule I)



For comparison $40 \times 5 = 200$ (Rule II)

∴ *Ans.* 175.0

Example 2. $35 \times 0.5 = 17.5$

The slide rule gives the digit reading 175 as in Example 1; the answer must not be very far from $30 \times 0.5 = 15$; so it must be 17.5.

Example 3. $3{,}500 \times 5{,}000 = 17{,}500{,}000$

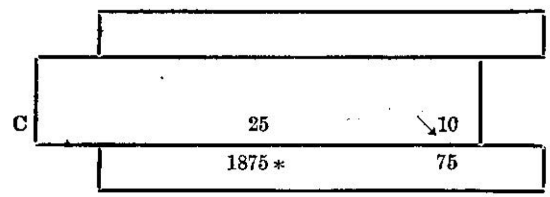
The digit value is 175 as in Example 1; the answer must be something not very far from $3{,}000 \times 5{,}000 = 15{,}000{,}000$. Hence it must be 17,500,000.

Note: You can do alike calculation between (*A*) and (*B*) instead of (*D*) and (*C*); but the former are just half size contraction of the latter and will naturally give you results with less accuracy. With (*A*) and (*B*), however, you will never be necessitated to do "resetting" or "setting over again when you find the multiplier on (*C*) off (*D*)."

Example 4. $7.5 \times 2.5 = 18.75$

First it is a case of ①.

Set 10 *C* to 75 *D* against 25 *C* read 1875 *D* (Rule I)



∴ The digit reading = 1875

But $8 \times 2 = 16$. (Rule II)

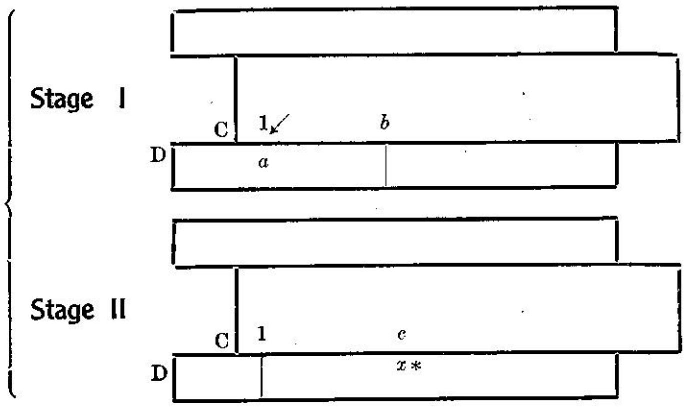
∴ *Ans.* 18.75

[2] Continuous Multiplication.

When there are more than two factors to be multiplied into one, you are only to multiply them gradually :—

$abc = x$ say

{Set 1 *C* to *a*-*D* against *b*-*C* put Hairline
{Set 1 *C* to Hairline against *c*-*C* read *x*-*D*



It is taken for granted that when there are more than three factors, the above method is only to be repeated.

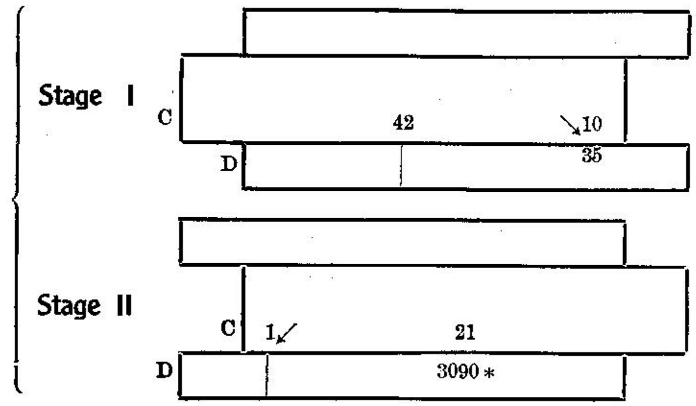
When you find any multiplier on (*C*) off (*D*), the slide is to be projected out to your left just as shown in ②, page 137.

Example 5. $35 \times 420 \times 0.21 = 3{,}090$

For comparison $40 \times 400 \times 0.2 = 3200$; and the coming result must be of four digits in the integral part.

Now by the slide rule.

{Set 10 *C* to 35 *D* against 42 *C* put Hairline
{Set 1 *C* to Hairline against 21 *C* read 3090 *D*



∴ *Ans.* 3.090

In Stage I, you are to see that it is not necessary to read the product $35 \times 420 = 1470$, but you have only to put the hairline there.

Readers are advised to go through Section B. Multiplication and Division of Chapter II, pp. 9–10 of this book, as it is absolutely good for this slide rule. (*C*) and (*D*) on that slide rule are absolutely the same as (*C*) and (*D*) on this slide rule respectively.

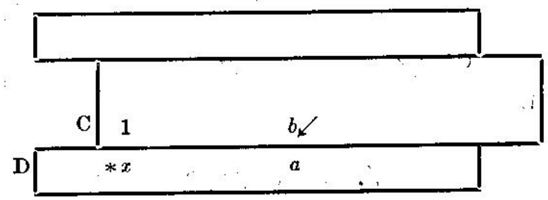
Section C. DIVISION

[1] Simple Division.

Rule III. Set the division on (C) to the dividend on (D), against 1.0 or 10.0 on (C) read the quotient on (D).

$a \div b = x$ say,

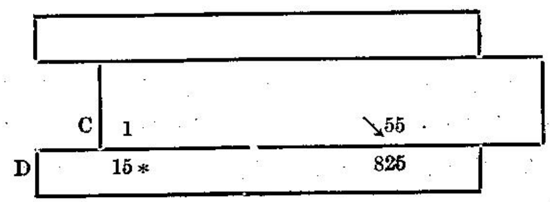
Set b-C to a-D against 1 C read x-D



Example 1. $8.25 \div 5.5 = 1.5$

First $8 \div 5 = 1.6$ Hence the quotient required must be of one place in the integral part.

Next Set 55 C to 825 D against 1 C read 15 D



∴ *Ans.* 1.5

Note: The above operation is just the reverse to the multiplication $1.5 \times 5.5 = 8.25$. It must also be remembered that you could as well do between (A) and (B), instead of (D) and (C).

[2] Continuous Division.

When there are more than one divisor, set the first divisor on (C) to the dividend on (D), put the hairline at 1.0 or 10.0 on (C); next set the second divisor on (C) to the hairline, and against 1.0 or 10.0 on (C), read the second result on (D). If there be still more than two divisors, you are to continue the operation in a similar manner.

In other words, if $\frac{a}{bc} = x$

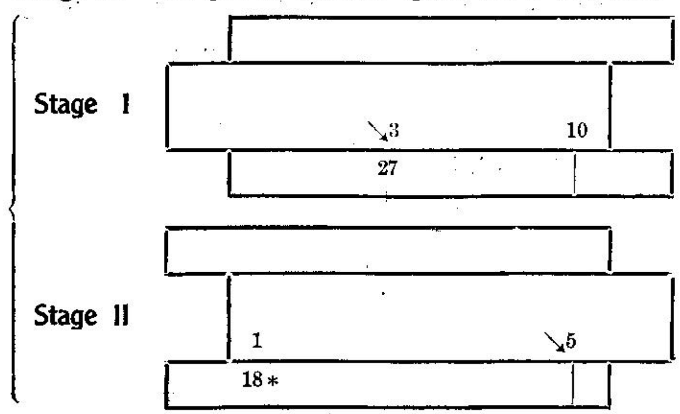
Set b C to a D against 1.0 (or 10.0) C put Hairline
Set c C to Hairline against 1.0 (or 10.0) C read x D

Example 2. $2.7 \div 0.3 \div 5 = 1.8$

For comparison, $3 \div 0.3 \div 5 = 2$.

∴ Previously you understand that the result would be of one digit in the integral part. And by the slide rule:

Stage I Set 3 C to 27 D against 10. C put Hairline
Stage II Set 5 C to Hairline against 1.0 C read 18 D

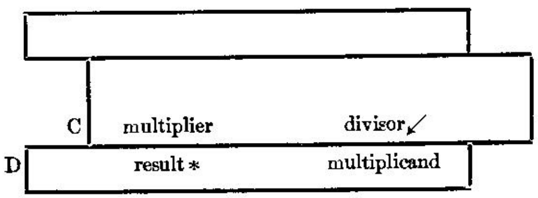


∴ *Ans.* 1.8

Section D. CONTINUOUS MULTIPLICATION AND DIVISION

To calculate $\frac{\text{multiplicand} \times \text{multiplier}}{\text{divisor}}$.

Set divisor-C to multiplicand-D against multiplier-C read result-D



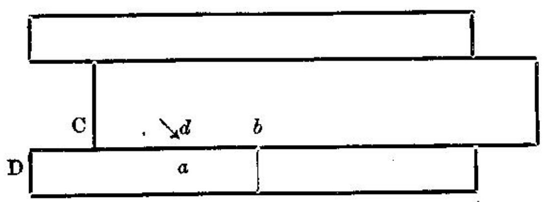
To calculate $\frac{abc}{de}$

Stage I Set d-C to a-D against b-C put Hairline

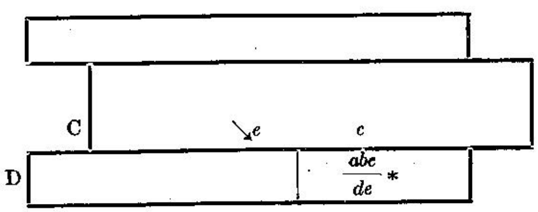
Stage II Set e-C to Hairline against e-C read $\frac{abc}{de}$-D

Multiply a first by $\frac{b}{d}$, and then the intercurrent result by $\frac{c}{e}$

Stage I

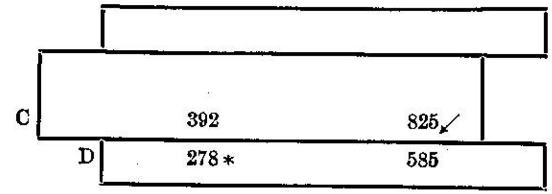


Stage II



Example 1. $\frac{5.85 \times 3.92}{8.25} = 2.78$

Set 825 C to 585 against 392 C read 278 D



∴ The digit value = 278

For comparison of the place of decimal point:

$$\frac{5 \times 4}{8} = 2.5$$

∴ *Ans.* 2.78

Example 2. $\frac{5850 \times 39200}{82500} = 27800$

The digit value to be sought on the slide rule must be 278 as that of Example 1.

Next for comparison

$$\frac{5000 \times 400000}{80000} = 25000$$

∴ The final result must be of five places in the integral part.

∴ *Ans.* 27800

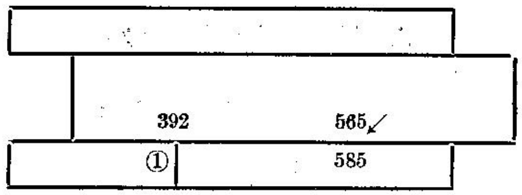
Example 3. $\dfrac{5.85 \times 3.92 \times 9.45}{5.65 \times 8.25} = 4.65$

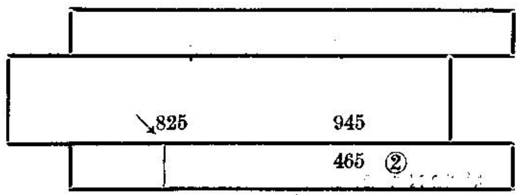
It is to be done in two stages, because

$$\frac{5.85 \times 3.92 \times 9.45}{5.65 \times 8.25} = 5.85 \times \frac{3.92}{5.65} \times \frac{9.45}{8.25}$$

① Set 565 C to 585 D against 392 C put Hairline

② Set 825 C to Hairline against 945 C read 465 D

①


②


∴ The digit result=465

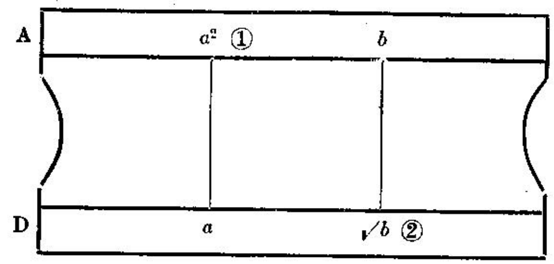
Next for comparison, $\dfrac{6 \times 4 \times 9}{6 \times 9} = 4.$

So the final result must be of one place in the integral part.

∴ *Ans.* 4.65

Section E. SQUARES AND SQUARE ROOTS.

The slide rule has a very strange and very advantageous character; take off the slide and the diagram here below shows the rule only :—



Put the hairline at any place on the rule, the reading under the hairline on (A) is just the square of that on (D). So it follows that the reading under the hairline on (D) is just the square root of that on (A). In the international slide rule laconisms :—

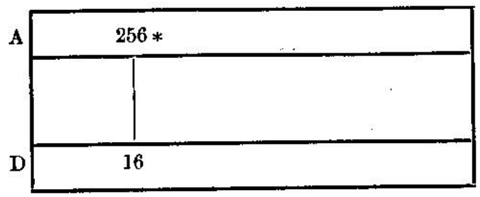
Against a-D read a^2-A ①

Against b-A read $\sqrt{b}$-D ②

Nothing is more simple than these calculations, while ② or the extraction of a square root is what other reckoning instruments often fail to solve. The slide is not wanted, but it is self evident that the slide alone is as good as the rule; only the hairline cannot be set upright without the straight-edge guide of the rule.

Example 1. $16^2=256$

Against 16 *D* read 256 *A*



And for comparison, $10^2=100$

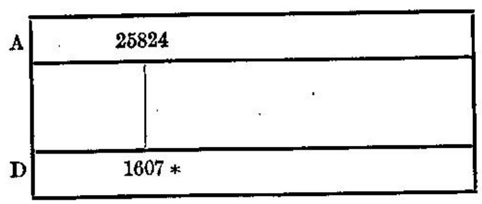
$\therefore\ 16^2=256$

Example 2. $\sqrt{25824}=160.7$

$25824 \equiv 2.5824$ ($\equiv$ means congruent.)

So it is to be read on the left section of (*A*)

Against 25824 *A* read 1607 *D*



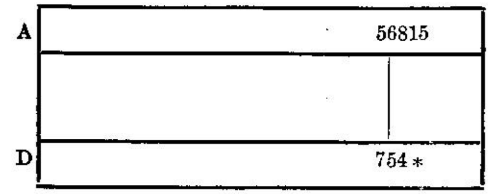
For comparison $\sqrt{10000}=100$, so the final result must be of three places in the integral part.

$\therefore\ \sqrt{25824}=160.7$

Example 3. $\sqrt{5685.15}=75.4$

As $5685.15 \equiv 56.8515$

Against 56.8515 *A* read 7.54 *D*



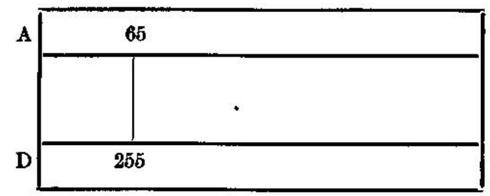
And for comparison, $\sqrt{10000}=100.$

$\therefore\ \sqrt{5685.15}=75.4$

Example 4. $\sqrt{0.00065}=0.0255$

$0.00065 \equiv 6.5$

Against 6.5 *A* read 255 *D*



And for comparison, $\sqrt{0.0001}=0.01$

$\therefore\ \sqrt{0.00065}=0.0255$

Readers are advised to go through pp. 21–23.

Section F. AREAS AND RADII OF CIRCLES

A, area of the circle,

r, radius of the circle,

then we have

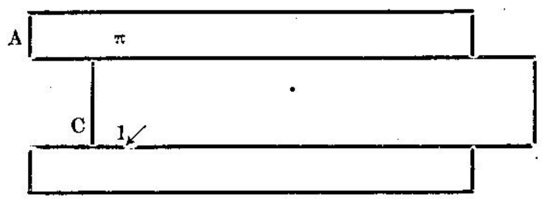
$$A = \pi r^2$$

Put $r = 1$, then $A = \pi$. It means that the area of a circle, whose radius is a unit length, is π or 3.1416 square units. So set 1.0 or 10.0 on (*C*) to π on (*A*), and you can read out the area of a circle of any radius.

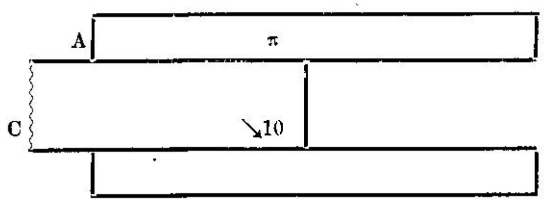
① Set 1.0 *C* to π-*A* against *r*-*C* read *A*-*A*

② Set 10.0 *C* to π-*A* against *r*-*C* read *A*-*A*

①



②



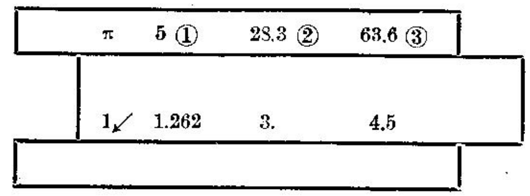
(① and ② are complementary settings.)

Example 1. There are three circles whose radii are 1.262, 3, 4.5 cm.. What are their areas?

① Set 1 *C* to π-*A* against 1.262 *C* read 5 *A*

② also against 3 *C* read 28.3 *A*

③ also against 4.5 *C* read 63.6 *A*



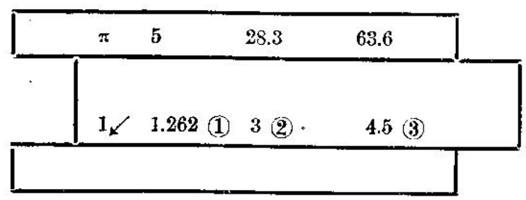
∴ *Ans's* 5, 28.3, 63.6 cm² respectively.

Example 2. There are three circles whose areas are 5, 28.3, 63.6 cm²; what are their radii?

① Set 1 *C* to π-*A* against 5 *A* read 1.262 *C*

② also against 28.3 *A* read 3 *C*

③ also against 63.6 *A* read 4.5 *C*



∴ *Ans's* 1.262, 3, 4.5 cm resp.

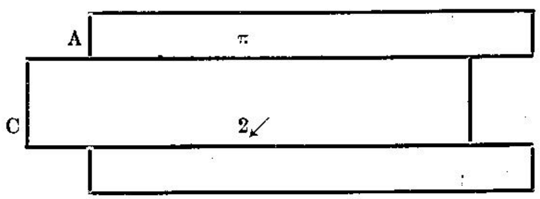
Section G. AREAS AND DIAMETERS OF CIRCLES

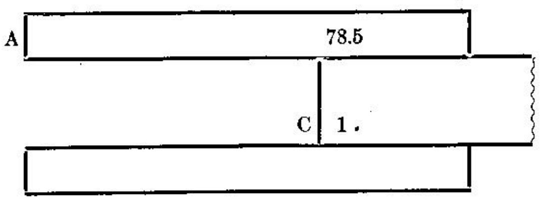
A, area of the circle,
d, dia. of the circle,
then we have

$$A=\frac{\pi}{4}d^2$$

Put $d=2$, then $A=\pi$. It means that the area of a circle, whose diameter is two units of length, is π or 3.1416 square units. So set 2 on (C) to π on (A), and you can read out the area of a rcle of any diameter. Failing this, you can get it by "resetting" the slide, or on its complementary setting.

① Set 2 C to π-A against d-C read A-A
② Set 1 C to 78.5 A against d-C read A-A

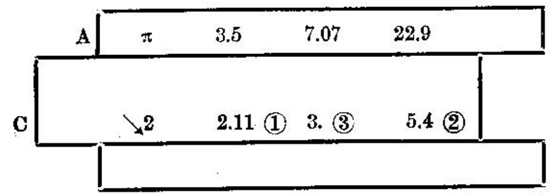
①


②


You can see it very easily that ① and ② are complementary settings to each other.

Example 1. There are three circles whose areas are 3.5, 7.07, 22.9 cm^2; what are their diameters?

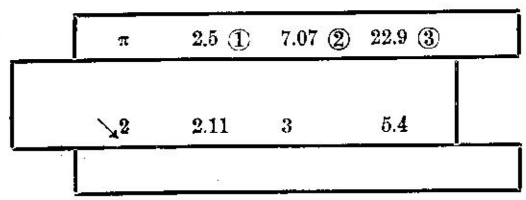
Set 2 C to π-A against 3.5 A read 2.11 C
also against 7.07 A read 3.·C
also against 22.9 A read 5.4 C



∴ *Ans's* 2.11, 3, 5.4 cm respectively.

Example 2. There are three circles whose diameters are 2.11, 3, 5.4 cm.; what are their areas?

Set 2 C to π-A against 2.11 C read 3.5 A
also against 3. C read 7.07 A
also against 5.4 C read 22.9 A



Ans. 3.5, 7.07, 22.9 $cm.^2$

Section H. CUBES AND CUBE ROOTS.

[1] Cubes :—

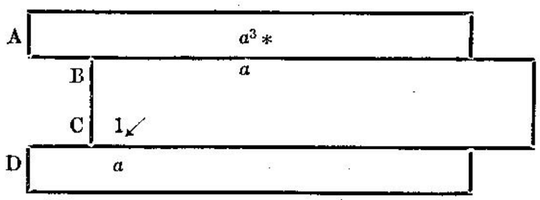
The cubes are simple to get, but the cube roots are somewhat troublesome to make out by the student's rule. If the slide rule had a 'K' scale, the job would be very simple. Here we shall explain how to solve the problem with the student's rule which [redacted] 'K' scale.

Quite unlike the squares and square roots, the areas and diameters &c. of a circle, the edge and the volume of one cube require a setting of their own. That is, if the edge varies a little, you must make a new setting to get the volume.

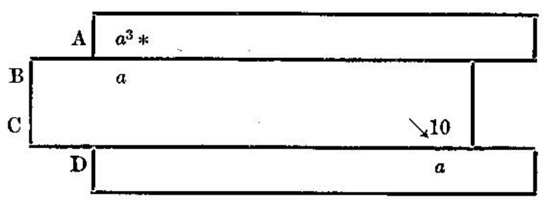
Set 1 C to a-D against a-B read a^3-A

Set 10 C to a-D against a-B read a^3-A

①



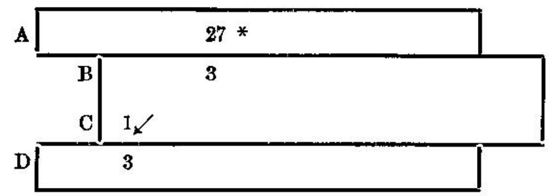
②



Readers will understand very easily, when they handle a slide rule, that if $a<4.642$, then the operation is like ①; and if $a>4.642$, then it is like ②. And ① and ② are a pair of complementary settings.

Example 1. $3^3=27$

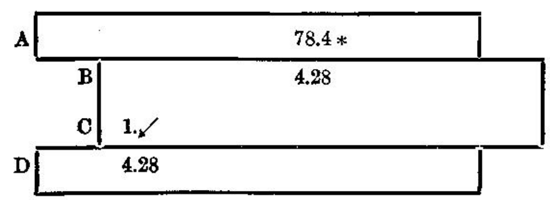
Set 1 C to 3 D against 3 B read 27 A



$\therefore\ 3^3=27.$

Example 2. $4.28^3=78.4$

Set 1 C to 4.28 D against 4.28 B read 78.4 A



$\therefore\ 4.28^3=78.4$

[2] **Cube Roots:—**

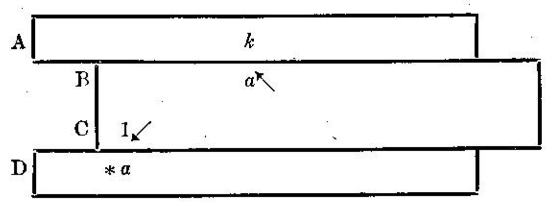
The extraction of a cube root is very hard; but it is just the reverse of the cubing.

Calculate $\sqrt[3]{k}=a$ where k is a known quantity, and a an unknown quantity.

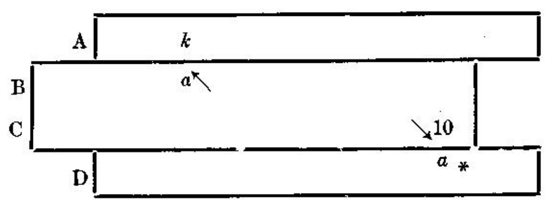
Set a-B to k-A against 1 C read a-D

It is to move the slide slowly to a certain point at which the two readings, one on (B) against the volume, k on (A), which is stationary, and the other on (D) against 1.0 or 10.0 on (C) which is movable, shall happen to be equal to each other. When this troublesome setting is made, each of the very equal readings of two such points, is the edge of the cube.

①



②

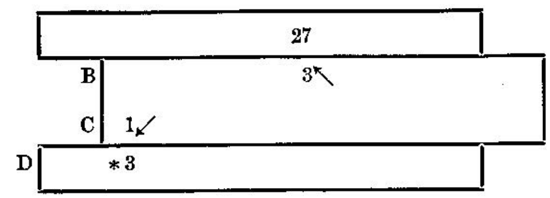


The original numbers are first classified into three absolute readings; those with one or two digits in the integral part of the absolute reading, are dealt with in the way like ①, and those with three digits in the way like ②.

It should be well understood that a-B is an unknown quantity. It is to be equalized with a-D. This can be done only by trial.

Example 3. $\sqrt[3]{27}=3$

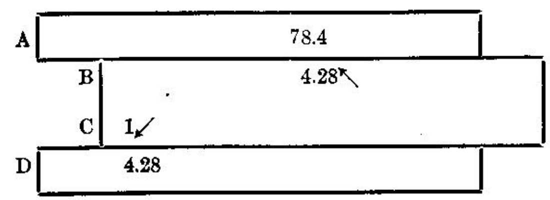
Set 3 B to 27 A against 1 C read 3 D



$\therefore\ \sqrt[3]{27}=3$

Example 4. $\sqrt[3]{78.4}=4.28$

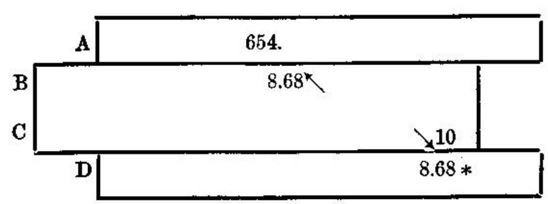
Set 4.28 to 78.4 A against 1 C read 4.28 D



$\therefore\ \sqrt[3]{78.4}=4.28$

Example 5. $\sqrt[3]{654}=8.68$

Set 8.68 B to 654. A against 1 C read 8.68 D



$\therefore\ \sqrt[3]{654}=8.68$