MATHEMATICS MADE EASY

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ARITHMETIC

By far the most used form of Mathematics is Arithmetic, the science of numbers and computations, which you have been using daily in its simpler forms even if only to count your change after buying a package of cigarettes.

Fundamental Operations

The fundamental operations of arithmetic are addition, subtraction, multiplication, division, and the finding of squares, square roots, cubes, and cube roots.

We assume that you know how to add, subtract, multiply, and divide whole numbers. However, it is quite possible that you have forgotten how to perform these operations with fractions, mixed numbers, and decimals, and so, you are asked to give special attention to these items in the following paragraphs, and practice using fractions and decimals until you can handle them with facility.

Symbols

In discussing the arithmetical operations, it will be convenient to use certain symbols and terms. These are therefore given here:

- \(=\) equals
- \(+\) plus
- \(-\) minus
- \(\times\) times
- \(\div\) divided by

The Sum = all the parts added.
The Difference = the Minuend — the Subtrahend.
The Product = the Multiplier \(\times\) the Multiplier.
The Quotient = the Dividend \(\div\) the Divisor.

FRACTIONS

The use of fractions implies that a certain unit has been divided into a number of parts and that one or more of these parts are being considered. Suppose that a board five feet long is sawed into five equal lengths of one foot each. Then, the amount of wood in two such pieces would be two-fifths of the original amount of wood. The two-fifths is a fraction and is usually expressed as \(2/5\) or \(\frac{2}{5}\) where the number below the line (5), called the Denominator, expresses the number of parts into which the whole is divided and the number above the line (2), called the Numerator, expresses the quantity of divisions being considered. The diagonal or horizontal line is called the Fraction Line.

The value of a fraction does not change if both its numerator and its denominator are multiplied or divided by the same amount. Thus

\[
\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}
\]

To reduce a fraction to its lowest terms, divide both numerator and denominator by the greatest number both can be divided by without a remainder, i.e., by the greatest common divisor. Thus to reduce \(\frac{8}{20}\) to its lowest terms, divide both numerator and denominator by 4 (the greatest common divisor) obtaining \(\frac{2}{5}\).

Mixed Numbers

Frequently, fractions are combined with whole numbers. For example, suppose you had two boards, each five feet long and the two one foot lengths that you considered in the preceding paragraphs. Expressing the amount of lumber, then, in terms of the five foot length, you would have two and two-fifths lengths, expressed as \(2 \frac{2}{5}\) lengths. This is known as a Mixed Number because it involves a whole number called the Integer or Integral Part and a fraction. In many of your calculations you will find it desirable to change Mixed Numbers into Pure Fractions. This can be done if we consider that a unit could be expressed as five-fifths, in which case then the above mixed number contains two units of five-fifths each, or ten-fifths, plus two-fifths, making a total of twelve-fifths. You say, then, that \(2 \frac{2}{5}\) is the same as \(\frac{12}{5}\). For measuring purposes, the mixed number is more desirable but for calculations the pure fraction is desirable.

A fraction whose numerator is greater than its denominator is called an Improper Fraction.

To change an improper fraction to a mixed number, simply divide the numerator by the denominator. Thus

\[
\frac{25}{12} = 25 \div 12 = 2 \frac{1}{12}
\]

Adding and Subtracting Fractions

In adding or subtracting pure fractions whose denominators are the same, we add or subtract the numerator of the fraction in the usual manner. Thus \(\frac{3}{8}\) and \(\frac{2}{8}\) are expressed in the same fractional unit (eighths).

To add these two fractions, simply add their numerators and you obtain \(\frac{5}{8}\). In subtracting, subtract the one numerator from the other and you obtain a difference of \(\frac{1}{8}\).

If the fractions to be added or subtracted do not have the same denominator, the denominator must be changed so that they are the same. This may be done by increasing the denominator of one (or both) of the fractions, but it is important for you to remember that whenever you increase the denominator of a fraction, you must also increase the numerator if you do not want to change the value of the fraction. If you had a fraction of \(\frac{3}{4}\) and you wanted to change that fraction so that it would
have a denominator of 8, you would have to multiply the numerator by
the same amount as you would multiply the denominator so that $\frac{3}{4}$
becomes $\frac{6}{8}$.

Now, suppose that you wanted to add the two fractions $\frac{3}{4}$ and
$\frac{7}{8}$ which, as you see, do not have a common denominator. By multiplying
both numerator and denominator of $\frac{3}{4}$ by 2 it becomes $\frac{6}{8}$ which then has
the same denominator as $\frac{7}{8}$ and can be easily added or subtracted. The
sum of $\frac{6}{8}$ and $\frac{7}{8}$ is $\frac{13}{8}$ or $1\frac{5}{8}$ while the difference between $\frac{7}{8}$ and $\frac{3}{4}$ is $\frac{1}{8}$.

Now, suppose you wish to add $\frac{2}{3} + \frac{3}{4} + \frac{4}{5}$. In this case all the
denominators (and numerators) will have to be multiplied by a number
such as to make all three changed denominators alike. You should select
the smallest number which will be a multiple of 3, 4, and 5. In this case
60 is such a number and the fractions will be changed to

$$\frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{133}{60} = \frac{2}{8}$$

In the above addition the 60 is known as the Least Common Denominator.

In adding mixed numbers, simply add the fractions and the integers
separately. If the sum of the fractions results in an improper fraction, this
may be changed to a mixed number whose integer will be added to the
sum of the given integers. Thus

$$\frac{3}{8} + 4\frac{1}{8} + 3\frac{3}{8} = 10\frac{9}{8} = 10 + 1\frac{1}{8} = 11\frac{1}{8}$$

Multiplying and Dividing Fractions

To multiply a fraction by a whole number, the numerator only is
multiplied by that number and the product is then written as the fraction
whose numerator is the product of the original numerator and the whole
number, and whose denominator remains the same. For example, suppose
you multiply $\frac{5}{3}$ by 3. The product has for its numerator $5 \times 3$ or 15, while
the denominator of the product remains as 8; thus, the answer is $\frac{15}{8}$
or $1\frac{7}{8}$.

To multiply one fraction by another fraction, multiply the numerators
together and multiply the denominators together; thus,

$$\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$$

If you were to multiply $\frac{3}{5} \times \frac{5}{3}$ you would get $\frac{15}{15}$ or 1. When the
product of two fractions is 1, one fraction is said to be the reciprocal of
the other.

To divide by a fraction, you need merely multiply by its reciprocal; thus,

$$\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{20}{24} = \frac{5}{6}$$

Since the denominator is actually the number of parts that something
has been divided into, the fraction line represents the same thing as a division sign. The numerator is the same as the dividend while the
denominator is the divisor. For example, $5 \div 2 = \frac{5}{2}$.

Below you see various ways in which an identical calculation is
indicated:

$$7 + 3 \times 4 + 9 = \frac{28}{27} = 1 \frac{1}{27}$$
$$\frac{7}{3} \times \frac{4}{9} = \frac{28}{27}$$

$$\frac{(7 \times 4) + (3 \times 9)}{27} = \frac{28}{27}$$
$$\frac{(7 \times 9)}{3} = \frac{28}{27}$$

$$\frac{7 \times 4 + 3 + 9}{27} = \frac{28}{27}$$
$$\frac{7 \times 3}{9} = \frac{28}{27}$$

$$\frac{3 \times 9}{3} = \frac{28}{27}$$
$$\frac{(7 + 3)(4 \div 9)}{27} = \frac{28}{27}$$

Exercises:

1. Change the following to pure improper fractions.
   (a) $\frac{3}{8}$
   (b) $\frac{4}{3}$
   (c) $\frac{6}{5}$

2. Change the following to mixed numbers.
   (a) $\frac{7}{2}$
   (b) $\frac{19}{15}$
   (c) $\frac{16}{9}$

3. $\frac{3}{4} + \frac{7}{8} + \frac{4}{5} = ?$

4. $\frac{1}{3} + \frac{1}{5} + \frac{4}{16} = ?$

5. (a) $\frac{3}{7} \times \frac{8}{9} = ?$
   (b) $\frac{4}{7} \div \frac{8}{9} = ?$
   (c) $\frac{7}{9} \times 12 \times \frac{56}{8} = ?$

The answers to these exercises will be found on the last page of this book.
DECIMALS

Expressing Fractions with Decimals

Fractions may also be written in decimal form. In our monetary system, we use a decimal system with the dollar as the unit. We signify the dollar by the number 1. If we wish to designate a certain part of this dollar, say 3 cents, it is not as convenient to write it as 5/100 of a dollar as it would be to write .05.

Fractions written in this manner are called decimal fractions, or simply Decimals, and the period placed before the figure is called the decimal point. In this system, all numbers placed to the left of the decimal point are whole numbers, or Integers, and the numbers placed to the right of the decimal point are the Decimals. The decimals form the numerator of the fraction while the denominator is equal to 10 times the number of places behind the decimal point. Thus, \( \frac{7}{10} = .7 \), \( \frac{235}{1000} = .235 \), and \( \frac{3}{10,000} = .0003 \).

To Convert a Common Fraction into a Decimal

Suppose that you wanted to change the fraction \( \frac{7}{13} \) into a decimal correct to three places behind the decimal point. This is the same as converting it to a fraction whose denominator is 1000. Now remember that if you multiply both the numerator and denominator by the same amount, the value of the fraction doesn't change. Therefore, \( \frac{7}{13} = \frac{7 \times 1000}{13 \times 1000} = \frac{7000}{13 \times 1000} \). Now, divide 7000 by 13 and you have the number of thousandths: \( \frac{7000}{13 \times 1000} = \frac{538}{1000} = .538 \).

In connection with measurements on blueprints, you will frequently have occasion to convert fractional parts of an inch into decimals, and vice versa. This can always be done by the method just described. However, it is very handy to have a table of decimal equivalents especially if you are in one of those industries where blueprints are so extensively used. This you will find in Table I of this book. Tables such as these are invaluable for making mathematics easy.

To Add or Subtract Decimal Numbers

In adding and subtracting numbers with decimals, they should be written one below the other with their decimal points in line and the addition or subtraction can then be carried out in the usual way. In the sum or remainder, the decimal point is then placed in line with the decimal points above.

To illustrate this, suppose we wish to add 13.46, 2.625, and 230.8 together. This is written down and carried out as shown at the right:

\[
\begin{align*}
13.46 & \quad 2.625 & \quad 230.8 \\
\hline
16.08 & \quad 5.250 & \quad 254.6 & \quad \text{Total}
\end{align*}
\]

Notice that in this operation the decimal point is retained in its position in line with the rest of the decimal points.

Subtraction is carried out in the same manner and, from this, it should be realized that addition and subtraction of decimals will give no trouble when the numbers are written with the decimal points in line, one below the other.

To Multiply or Divide Decimal Numbers

To multiply two decimal numbers, first carry out the multiplication without regard to the position of either of their decimal points. You must then, however, very carefully determine the location of the decimal point by strict adherence to the following:

**RULE:** The number of places to the right of the decimal point in the product of two decimal numbers equals the sum of the numbers of places to the right of the decimal points in the multiplicand and the multiplier.

As an example, suppose you want to multiply 1.025 by 3.04. Dropping the decimal points, the product is 311600. To determine the location of the decimal point in the product, you observe that the multiplicand has 3 figures to the right of the decimal point and that the multiplier has 2 figures. Applying the above rule, you place the decimal point to the LEFT of the last five figures in the product, and you find the product to be 3.11600, or, what is the same thing, 3.116.

The rule for determining the decimal point position in the quotient of the division is as follows:

**RULE:** Subtract the number of places to the right of the decimal point in the divisor from the number in the dividend to get the number in the quotient.

For example, divide 3.804 by 1.2. Dividing without regard to decimal points you get 317. Now, applying the above rule, you subtract the 1 place in the divisor from the 3 places in the dividend and you get 2 places for the quotient. Therefore, the correct answer is 3.17.

**Percentage**

A method of indicating fractions whose denominators are 100 is by means of percentage. Thus, twenty-seven percent (27%) means \( \frac{27}{100} \) or .27. In all calculations, the percentage is reduced to a decimal after which you may proceed exactly as in the preceding paragraph.

To Multiply Mixed Numbers

Suppose you had to multiply two figures together, such as \( 1 \frac{7}{8} \) by \( 1 \frac{3}{4} \). To perform this multiplication you have two methods available; one is to reduce each of the factors to a common fraction and the other is to change each to decimals.
Using the first method, \( \frac{7}{8} \) becomes \( \frac{15}{8} \) and \( \frac{3}{4} \) becomes \( \frac{7}{4} \).

Then, \( \frac{15}{8} \times \frac{7}{4} = \frac{105}{32} \) or \( \frac{9}{32} \).

Using the second method, \( \frac{7}{8} \) becomes 1.875 and \( \frac{3}{4} \) becomes 1.75.

Then, \( 1.875 \times 1.75 = 3.28125 \).

**Exercises:**

6. Change the following to three place decimals.

(a) \( \frac{17}{63} \)  
(b) \( \frac{8}{9} \)  
(c) \( \frac{7}{8} \)  
(d) \( \frac{9}{13} \)

7. \( 10.31 + 6.98 + .432 + 18.9 = ? \)

8. (a) \( 6.98 \times 4.32 = ? \)
   
   (b) \( 8.92 \div 2.13 = ? \)

9. \( 68\% \) of 439 = ?

10. Multiply \( 6 \frac{2}{8} \) by \( 4 \frac{10}{13} \)

The answers to these exercises will be found on the last page of this book.

**POWERS AND ROOTS**

If a quantity is multiplied by itself one or more times, the product is known as a **Power** of the quantity, which is specified by the number of times the quantity is taken as a factor. The power to which a number is raised is indicated by writing a small figure, called the **Exponent**, to the right of and a little above the number. Thus:

\[ 3^2 = 3 \times 3 \] the second power of 3 or the square of 3.

\[ 3^3 = 3 \times 3 \times 3 \] the third power of 3 or the cube of 3.

\[ 3^4 = 3 \times 3 \times 3 \times 3 \] the fourth power of 3, etc.

The finding of a quantity which when multiplied by itself a given number of times equals the given quantity is known as **Extracting the Root**, and the process is indicated by a radical sign \( \sqrt{\ } \) in the following way:

\( \sqrt[2]{\ } \) indicates square root

\( \sqrt[3]{\ } \) indicates cube root

\( \sqrt[4]{\ } \) indicates fourth root, etc.

You will readily understand the meanings of these from the following examples:

\( \sqrt{9} = 3 \) since \( 3 \times 3 = 9 \)

\( \sqrt[3]{27} = 3 \) since \( 3 \times 3 \times 3 = 27 \)

\( \sqrt[4]{81} = 3 \) since \( 3 \times 3 \times 3 \times 3 = 81 \)

Often the process of extracting a root is indicated by a fractional exponent; thus:

\[ \sqrt[4]{9} = 9^{\frac{1}{4}} \]

\[ \sqrt[8]{1} = 9^{\frac{1}{8}} \]

**Finding the Square Root of a Number**

**Problem:** Find the square root of 1892.25.

<table>
<thead>
<tr>
<th>43.5</th>
<th>1892.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>83</td>
<td>292</td>
</tr>
<tr>
<td>249</td>
<td>865</td>
</tr>
</tbody>
</table>

**Procedure:**

1. Beginning from the decimal point, point off as many groups of 2 digits* as possible both to the right and to the left of the decimal point.

2. Find the greatest number whose square is equal to or less than the first left-hand group and place it as the first figure in the root (in this case 4).

3. Subtract its square (16) from the first group (18) and annex the remainder (2) to the second group (92).

4. Double the first figure of the root for a partial trial divisor (8), divide it into the remainder (29 omitting the right hand digit); and place the integer (3) of the quotient as the next digit of the root.

5. Annex the root digit just found (3) to the trial divisor (8), multiply the complete divisor (29) by the root digit (3), and subtract the product (249) from the dividend; then proceed as before.

**NOTE:** If, in steps 4 and 5, the product of the second figure in the root by the completed divisor is greater than the dividend, erase the second figure both from the root and from the divisor, and substitute the next lower digit until the product can be subtracted from the dividend.

**Exercises:**

11. Find the square root of

(a) 776161  
(b) 85.1929  
(c) 0.555025

The answers to these exercises will be found on the last page of this book.

**RATIO AND PROPORTION**

**Ratio** is the relation between two magnitudes of the same kind. Thus, if a cubic foot of cast iron weighs 450 pounds and a cubic foot of steel weighs 490 pounds, the ratio of the weight of cast iron to the weight of steel is 450 to 490, or 45 to 49. It is frequently written \( \frac{45}{49} \) but it may also be written \( \frac{45}{49} \) since a ratio is, after all, a fraction.

*A digit is any one of the numerals from 0 to 9.*
When two different ratios are expressed as being equal, a Proportion is formed. This equality is expressed either by the ordinary equality sign ( = ) or by a special proportional sign ( : : ).

Thus a proportion may be expressed as

\[ x : 14 :: 45 : 49 \]

or \[ x : 14 = 45 : 49 \]

or \[ \frac{x}{14} = \frac{45}{49} \]

In these expressions the \( x \) and 49 are called the Extremes and the 14 and 45 are the Means. The \( x \) is the symbol for the Unknown Quantity. It appears here as an extreme but it may also be a mean.

A proportion is easily solved for the unknown \( x \) by applying the following rules:

**RULE 1.** The product of the extremes is equal to the product of the means.

**RULE 2.** The product of the extremes divided by either mean gives the other mean.

**RULE 3.** The product of the means divided by either extreme gives the other extreme.

**Example 1:** If a welder can make a five foot weld in 13 minutes, how long will it take him to make 28 feet of weld at the same rate of speed?

*Note:* This is an example of a Direct Proportion since the longer the time taken the greater will be the amount of weld.

**Solution:**

\[ 5 : 28 :: 13 : x \]

From Rule 3, \[ x = \frac{28 \times 13}{5} = 72.8 \text{ minutes.} \]

**Example 2:** If 3 men can lay a roof in 7 days, how long will it take 5 men?

*Note:* This is an example of an Inverse Proportion, since the greater the number of men employed, the less will be the amount of time consumed.

**Solution:**

\[ \frac{3}{5} = \frac{x}{7} \]

From Rule 2, \[ x = \frac{7 \times 3}{5} = 4\frac{1}{5} \text{ days.} \]

**Exercises:**

12. An iron casting weighs 236 lbs. How much would the same casting weigh if made of aluminum, assuming that cast iron weighs 450 lbs per cu. ft. and aluminum weighs 108 lbs per cu. ft.?

13. If a machinist must run a drill at 284 R.P.M. for drilling steel at a cutting speed of 65 feet per minute, at how many R.P.M. must the drill run for the same size hole in aluminum at a cutting speed of 195 feet per minute.

The answers to these exercises will be found on the last page of this book.

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**ARITHMETICAL SHORT-CUTS**

**Cancellation**

A very valuable device for making mathematics easy is the process of Cancellation. This is the "rejecting" or "cancelling out" of factors which are common to both the numerator and denominator, i.e., both the dividend and divisor.

**Example 1.** Divide \( 8 \times 50 \) by \( 4 \times 25 \).

\[ \frac{8 \times 2}{4 \times 25} = \frac{4}{25} \times \frac{1}{1} = \frac{4}{25} \]

Here \( 4 \) is a common factor of \( 8 \) and \( 4 \). Therefore, when both are divided by \( 4 \), \( 8 \) becomes \( 2 \) and \( 4 \) becomes \( 1 \). Similarly, \( 50 \div 25 = 2 \) and \( 25 \div 25 = 1 \).

**Example 2.** Divide \( 96 \times 42 \times 40 \) by \( 84 \times 60 \times 21 \).

\[ \frac{96 \times 42 \times 40}{84 \times 60 \times 21} = \frac{32}{21} \]

Here \( 12 \) is rejected by \( 96 \) and \( 84 \); \( 20 \) from \( 40 \) and \( 60 \); and \( 21 \) from \( 42 \) and \( 21 \).

**To Multiply Numbers between 11 and 99 by 11.**

(a) When the sum of the digits of the multiplicand is 9 or less, the product is written directly by simply placing in between the two digits of the multiplicand their sum. Notice that:

\[ \text{FIRST DIGIT \underline{45} \times \text{SECOND DIGIT}}\]

\[ = \underline{495} \text{ \text{SUM OF 2 DIGITS}}\]

(b) When the sum of the digits of the multiplicand is greater than 9, the second digit of the sum is placed before the second digit of the multiplicand and is preceded by the first digit increased by one. Thus:

\[ \underline{67} \times 11 = \underline{737} \text{ \text{1ST DIGIT PLUS 1}}\]

\[ \underline{737} \text{ \text{2ND DIGIT OF MULTIPLICAND}}\]

\[ \underline{67} \text{ \text{2ND DIGIT OF SUM OF GIVEN DIGITS}}\]

**To Multiply by \( \frac{3}{4} \)**

The ordinary way of multiplying a number by \( \frac{3}{4} \) is to multiply by 3 and divide by 4. You will find it much easier however to write \( \frac{1}{4} \) the number and then \( \frac{1}{2} \) of this half and add the two. Thus to multiply 92 by \( \frac{3}{4} \),

write 46

and 23

and add, obtaining 69

**To Multiply by 25**

The quick way is to multiply by 100 and divide by 4, all of which can be done mentally. Thus,

\[ 68 \times 25 = \frac{6800}{4} = 1700. \]
Of course, this method is also applicable to multiplication by 250, 2\(\frac{1}{2}\), .25, etc., providing you lay off the correct number of places.

**To Multiply by 75**

Here the best way is to multiply by \(100 \times \frac{3}{4}\). Thus, to multiply 86 by 75,

write \[
\begin{array}{c}
4300 \\
\hline
2150 \\
\hline
6450
\end{array}
\]

and add, obtaining 6450.

**To Add Two Numbers when One is just under 100**

In adding a number such as 97 it requires less mental strain to consider it as adding 100 and subtracting 3. Thus you can add 58 to 97 mentally by thinking of it as 158 — 3 = 155.

**To Subtract a Number just Less than 100**

To subtract, say, 97, add 3 and subtract 100. Thus \[185 — 97 = 85 + 3 = 88.\]

You should observe that this method is also applicable to adding or subtracting numbers like 996, 9997, 99.98, etc.

**To Multiply by a Number just under 100**

Multiplying by, say, 98, is the same as multiplying by 100 — 2. Therefore, multiply the multiplicand by 100 and then by 2 (which can be done mentally) and subtract. Thus to multiply 87 by 98,

\[
\begin{align*}
87 \times 100 &= 8700 \\
87 \times 2 &= 174
\end{align*}
\]

Subtracting we get \[8526 = \text{Product}\]

Now see if you can apply this trick to multiply (a) 88 \(\times\) .998 (b) \(47 \times 9.97\) (c) \(79 \times 999\). Check your answers by the long method.

**To Square a Number of Two Digits**

A rule that is very useful for shortening the labor of finding squares is the following:

1. **Square the first digit and place in front of the Square the second digit**
2. **Multiply the product of the digits by 20**
3. **Add the results of 1 and 2**

**Example:** Find the square of 87.

**Solution:**

<table>
<thead>
<tr>
<th>USUAL LONG METHOD</th>
<th>SHORT CUT METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>(8^2) alongside (7^2)</td>
</tr>
<tr>
<td>82</td>
<td>((20 \times 8 \times 7) mentally)</td>
</tr>
<tr>
<td>609</td>
<td>6449</td>
</tr>
<tr>
<td>690</td>
<td>1120</td>
</tr>
<tr>
<td>7569</td>
<td>7569</td>
</tr>
</tbody>
</table>

**To Square a Number just under 100**

\((98)^2\) is the same as \((100 — 2)^2\) which in turn is equal to \((100)^2 + (2)^2 — (2 \times 100 \times 2).\) Thus \((98)^2\) can be done very rapidly by writing \[
\begin{array}{c}
10004 \\
\hline
- 400
\end{array}
\]

and subtracting to get 9604.

Similarly \(998^2 = 1000^2 + 2^2 — 4000\)

\[9.97^2 = 10^2 + .03^2 — .60\]

There are other numbers where this principle can be usefully applied. Examples are:

\[
\begin{align*}
(148)^2 &= 150^2 + 2^2 — 2 \times 2 \times 150 \\
&= 22504 — 600 = 21904
\end{align*}
\]

\[
\begin{align*}
(198)^2 &= 200^2 + 2^2 — 2 \times 2 \times 200 \\
&= 40004 — 800 = 39204
\end{align*}
\]

**UNITS OF MEASUREMENT AND WEIGHT**

If you have ever been employed in construction or mechanical work where blueprint reading and estimating were involved, you will realize how important it is to be thoroughly familiar with geometric shapes and expressions and formulae for their mensuration. It is often necessary to convert lengths given in certain units to lengths in other units. Or, you may be called upon to calculate areas, volumes, and weights from the measurements given on blueprints. Before you can begin to do this you must know the relationship that exists between the various units used for measurement. This important information will now be given to you in a series of tables.

**Long Measure**

The following Long Measure Table expresses the relationship between the usual units of linear measurement:

- 12 inches (") = 1 foot (")
- 3' = 1 yard (yd.)
- 5\(\frac{1}{2}\) yds. or 16\(\frac{1}{2}\)' = 1 rod
- 40 rods = 1 furlong
- 8 furlongs or 320 rods = 1 statute mile

**Surveyor's or Old Land Measure**

In reading blueprints of old Land Maps, you are likely to find lengths expressed in the units of the Surveyor's or Old Land Measure, a table of which is given here:

- 7.92 inches = 1 link
- 25 links = 1 rod
- 4 rods or 66' = 1 chain
- 80 chains = 1 mile

**Note:** Rods are seldom used in this measure, distances being usually in chains and links.
Square Measure:

When the area of a surface is to be indicated, the units of a Square Measure are used. Area represents a product of 2 linear dimensions both of which are in the same unit. The product of the 2 lengths is expressed by the word “square”. Thus:

- inches × inches = square inches (sq. ins.)

The Table of Square Measure follows:

<table>
<thead>
<tr>
<th>Square Measure</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>144 sq. ins.</td>
<td>1 sq. ft.</td>
</tr>
<tr>
<td>9 sq. ft.</td>
<td>1 sq. yd.</td>
</tr>
<tr>
<td>30½ sq. yds.</td>
<td>1 sq. rod</td>
</tr>
<tr>
<td>43,560 sq. ft.</td>
<td>160 sq. rods = 1 acre</td>
</tr>
<tr>
<td>640 acres</td>
<td>1 sq. mile</td>
</tr>
</tbody>
</table>

Surveyor's Square Measure:

Here is the table for Square Measure corresponding to the Old Land Long Measure:

<table>
<thead>
<tr>
<th>Square Measure</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>625 sq. links</td>
<td>1 pole</td>
</tr>
<tr>
<td>16 poles</td>
<td>1 sq. chain</td>
</tr>
<tr>
<td>10 sq. chains</td>
<td>1 acre</td>
</tr>
<tr>
<td>640 acres</td>
<td>1 sq. mile</td>
</tr>
<tr>
<td>36 sq. miles</td>
<td>1 township</td>
</tr>
</tbody>
</table>

Cubic Measure:

The units of this measure are used to express the volume of material in a solid or the space within a container. Volume represents a product of three dimensions, such as length, width, and thickness, either of which may be expressed in any of the linear units, such as inches, provided the units are the same for all three. The word “cubic” in conjunction with the word expressing the linear unit denotes the product of three dimensions using this unit. For example:

- feet × feet × feet = cubic feet

The Cubic Measure Table follows:

<table>
<thead>
<tr>
<th>Cubic Measure</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,728 cubic ins.</td>
<td>1 cubic ft.</td>
</tr>
<tr>
<td>27 cubic ft.</td>
<td>1 cubic yd.</td>
</tr>
<tr>
<td>24¾ cubic ft.</td>
<td>1 perch (of stone masonry)</td>
</tr>
</tbody>
</table>

Liquid Measure:

If, in your vocation, you handle containers for liquids, you should know the Liquid Measure Table:

<table>
<thead>
<tr>
<th>Liquid Measure</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 gills</td>
<td>1 pint</td>
</tr>
<tr>
<td>2 pints</td>
<td>1 quart</td>
</tr>
<tr>
<td>4 quarts</td>
<td>1 gallon</td>
</tr>
<tr>
<td>31½ gallons</td>
<td>1 barrel</td>
</tr>
<tr>
<td>2 barrels</td>
<td>1 hogshead</td>
</tr>
</tbody>
</table>

Note: U. S. gallon = 231 cubic ins.; approximately 7½ such gallons = 1 cubic ft.

Board Measure:

This is the system in general use for measuring lumber. A Board Foot or Foot Board Measure (F.B.M.) equals 1 ft. × 1 ft. × 1 in. Thus, a board 5 ft. long × 18 ins. wide × 3 ins. thick would contain 5 × 1.5 × 3 = 22.5 F.B.M.

Avoirdupois Weight:

Three measures of weight are used in this country, namely Troy, Apothecaries, and Avoirdupois. Only the last mentioned, however, is of sufficient industrial importance to be included here:

- 437.5 grains = 1 ounce
- 16 ounces = 1 pound
- 100 pounds = 1 hundred weight (cwt.)
- 2000 pounds = 1 short ton
- 2240 pounds = 1 long ton

Angular Measure:

An Angle is the space between two intersecting lines and is measured in degrees, minutes, and seconds. If the two lines coincide the angle is zero, while if one of these lines is made to revolve about the point of intersection until it again coincides, the angle generated is 360 degrees. Thus, a degree is \( \frac{1}{360} \) th of this revolution and this, in turn, is further divided into minutes and seconds in accordance with the following table:

<table>
<thead>
<tr>
<th>Angle Measure</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 seconds (60°)</td>
<td>1 minute</td>
</tr>
<tr>
<td>60 minutes (60°)</td>
<td>1 degree</td>
</tr>
<tr>
<td>90 degrees (90°)</td>
<td>1 right angle</td>
</tr>
</tbody>
</table>

THE METRIC SYSTEM

In the Metric System, a length of a number of specified units can be expressed in terms of any other unit by multiplying or dividing by the appropriate power of ten (i.e., 10, 100, 1000, etc.). The same is also true of areas, volumes, and weights. For that reason, calculations are greatly simplified when this system is used. In continental Europe, the Metric System is used universally, whereas in England and the United States it is confined more or less to scientific weights and measurements.

The base of the Metric System is the Meter, which is fixed as one ten-millionth of the distance on the earth's surface from the Equator to either the North or the South Pole.

Tables of Long, Square, and Cubic Measure in the Metric System with the English (or U. S.) equivalents, follow:

**Metric Long Measure Table:**

<table>
<thead>
<tr>
<th>Metric Measure</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Millimeter (mm.) = .001 Meter = .03937 in.</td>
<td></td>
</tr>
<tr>
<td>1 Centimeter (cm.) = .01 Meter = .3937 in.</td>
<td></td>
</tr>
<tr>
<td>1 Decimeter (dm.) = .1 Meter = 3.937 in.</td>
<td></td>
</tr>
<tr>
<td>1 Dekameter = 10 Meter = 32.809 ft.</td>
<td></td>
</tr>
<tr>
<td>1 Hectometer = 100 Meters = 328.09 ft.</td>
<td></td>
</tr>
<tr>
<td>1 Kilometer = 1000 Meters = .62137 mi.</td>
<td></td>
</tr>
<tr>
<td>1 Myriameter = 10000 Meters = 6.2137 mi.</td>
<td></td>
</tr>
</tbody>
</table>
Metric Square Measure Table:

100 sq. mms. = 1 sq. Centimeter = .155 sq. in.
100 sq. cms. = 1 sq. Decimeter = 15.5 sq. ins.
100 sq. dms. = 1 sq. Meter (sq. m.) = 10.7639 sq. ft.
1 Centiare = 1 sq. Meter = 1.19604 sq. yds.
100 Centiares = 1 Are = 119.6034 sq. yds.
100 Ares = 1 Hectare = 2.4711 acres
100 Hectares = 1 sq. Kilometer = .3861 sq. mi.

Metric Cubic Measure Table:

1000 cu. mms. = 1 cu. cm. = .061 cu. in.
1000 cu. cms. = 1 cu. dm. = 61.023 cu. ins.
1000 cu. dms. = 1 cu. meter = 35.314 cu. ft.

Metric Capacity Table:

In the Metric System, the measurement of capacity is the same whether for liquids or solids. The basic unit of this measure is the Liter, which is equal to one cubic decimeter or 1.0567 U.S. liquid quarts. The table follows:

1 Milliliter = 1 cu. cm. = .00845 gill
1 Centiliter = 10 cu. cms. = .0845 gill
1 Deciliter = 100 cu. cms. = .845 gills
1 Liter = 1 cu. dm. = 1.0567 qts.
1 Dekaliter = 10 cu. dms. = 2.6417 gals.
1 Hectoliter = .1 cu. meter = 26.417 gals.
1 Kiloliter = 1 cu. meter = 264.17 gals.
1 Myrialiter = 10 cu. meters = 2641.7 gals.

Metric Weight Measure Table:

The Metric basic unit of weight is the Gram, which is equal to the weight of a cubic centimeter of distilled water. A Gram equals .35273 ounces Avoirdupois. A Kilo, or Kilogram equals 2.20462 pounds Avoirdupois. A Tonneau, or Ton equals 2204.02125 pounds. The complete table follows:

10 Milligrams (mg) = 1 Centigram
10 Centigrams (cg) = 1 Decigram
10 Decigrams (dg) = 1 Gram
10 Grams (g) = 1 Dekagram
10 Dekagrams (Dg) = 1 Hectogram
10 Hectograms (hg) = 1 Kilogram or Kilo
10 Kilograms (kg) = 1 Myriagram
10 Myriagrams (Mg) or 100 Kilograms = 1 Quintal
10 Quintals or 1 Tonneau, or 1000 Kilos = 1 Ton

TABLE I

DECIMALS OF AN INCH FOR EACH 64TH WITH MILLIMETER EQUIVALENTS

<table>
<thead>
<tr>
<th>Fraction</th>
<th>⅛6ths</th>
<th>Decimal</th>
<th>Millimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>1</td>
<td>.015625</td>
<td>0.39688</td>
</tr>
<tr>
<td>⅛</td>
<td>2</td>
<td>.03125</td>
<td>.78375</td>
</tr>
<tr>
<td>3/16</td>
<td>3</td>
<td>.046875</td>
<td>1.19063</td>
</tr>
<tr>
<td>⅛</td>
<td>4</td>
<td>.0625</td>
<td>1.56750</td>
</tr>
<tr>
<td>...</td>
<td>5</td>
<td>.078125</td>
<td>1.95438</td>
</tr>
<tr>
<td>5/16</td>
<td>6</td>
<td>.09375</td>
<td>2.32105</td>
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<tr>
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<td>7</td>
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<td>...</td>
<td>8</td>
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<tr>
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<td>3.95876</td>
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<table>
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<th>Millimeters</th>
</tr>
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<tr>
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<td>64</td>
<td>1</td>
<td>25.40005</td>
</tr>
</tbody>
</table>

Courtesy of the
Bausch & Lomb Co. Inc.
### TABLE II

**DECIMALS OF A FOOT FOR EACH 32ND OF AN INCH**

<table>
<thead>
<tr>
<th>Inch</th>
<th>0&quot;</th>
<th>1&quot;</th>
<th>2&quot;</th>
<th>3&quot;</th>
<th>4&quot;</th>
<th>5&quot;</th>
<th>6&quot;</th>
<th>7&quot;</th>
<th>8&quot;</th>
<th>9&quot;</th>
<th>10&quot;</th>
<th>11&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>¼</td>
<td>.006</td>
<td>.009</td>
<td>.018</td>
<td>.026</td>
<td>.035</td>
<td>.045</td>
<td>.053</td>
<td>.062</td>
<td>.070</td>
<td>.079</td>
<td>.086</td>
<td>.094</td>
</tr>
<tr>
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<td>.022</td>
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<td>.053</td>
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<td>.070</td>
<td>.079</td>
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<td>.072</td>
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<td>.090</td>
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<td>.020</td>
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<td>.034</td>
<td>.043</td>
<td>.052</td>
<td>.061</td>
<td>.070</td>
<td>.079</td>
<td>.088</td>
<td>.097</td>
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<td>.115</td>
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<td>.035</td>
<td>.044</td>
<td>.053</td>
<td>.062</td>
<td>.071</td>
<td>.080</td>
<td>.089</td>
<td>.098</td>
<td>.107</td>
<td>.116</td>
<td>.125</td>
</tr>
<tr>
<td>2½</td>
<td>.036</td>
<td>.043</td>
<td>.052</td>
<td>.061</td>
<td>.070</td>
<td>.079</td>
<td>.088</td>
<td>.097</td>
<td>.106</td>
<td>.115</td>
<td>.124</td>
<td>.133</td>
</tr>
<tr>
<td>3</td>
<td>.045</td>
<td>.053</td>
<td>.061</td>
<td>.070</td>
<td>.079</td>
<td>.088</td>
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<td>.133</td>
<td>.142</td>
</tr>
<tr>
<td>3½</td>
<td>.054</td>
<td>.062</td>
<td>.071</td>
<td>.080</td>
<td>.089</td>
<td>.097</td>
<td>.106</td>
<td>.115</td>
<td>.124</td>
<td>.133</td>
<td>.142</td>
<td>.151</td>
</tr>
</tbody>
</table>

**THE SLIDE RULE**

By means of a Slide Rule, such as the one we have given you, all sorts of problems involving multiplication, division, squares, cubes, and square root and cube root can be correctly solved with much less effort and in much faster time than by the usual methods.

After you read the following paragraphs and learn how to use a Slide Rule through careful study and practice, you will be convinced that it is definitely a most satisfactory device for making mathematical calculations easy.

You can use a Slide Rule to advantage even if you are not familiar with the higher forms of mathematics which follow. All you need is a thorough knowledge of fractions and decimals, which subjects you are strongly urged to review before attempting the study of the Slide Rule.

**Significant Figures**

An important consideration when using a Slide Rule is its degree of accuracy. To understand any discussion of this, you must understand what is meant by Significant Figures. Probably you will best understand this from a discussion of a series of examples:

Suppose you multiply 284 by 346. You will find that the product is 98,264. There are 5 so-called Digits in this product, each of which is considered a Significant Figure. In many multiplication operations it is not necessary to get the product as accurately as this. For instance, the product could be expressed as 98,200 which would be correct to 4 Significant Figures, or as 98,300 which would be correct to 3 Significant Figures, or simply as 98,000 which would be correct to 2 Significant Figures.

In performing the operation just described by the Slide Rule, the answer 98,300 would have been obtained since the Slide Rule is accurate for the most part to only 3 Significant Figures, although in one portion of the Slide Rule accuracy to 4 Significant Figures can be obtained. For most engineering calculations, it is not necessary to work any closer than this.

Assuming, now, that you understand the application of Significant Figures to whole numbers, let us take an example involving decimals. Suppose that we are to multiply 1.36 by 1.26. The correct answer is 1.7136, which is correct to 5 Significant Figures. This would, ordinarily, be read on the Slide Rule, however, as 1.714 (correct to 4 Significant Figures) or 1.71 (correct to 3 Significant Figures).

As a further example, suppose we multiply .000123 by .0032. The product is .0000003936. Although there are 10 Digits here behind the decimal point, the figure is correct to only 4 Significant Figures. In other words, a zero is only a Significant Figure of a decimal when it appears between two other digits. On a Slide Rule, this product would, ordinarily, be read as .000000394 (correct to 3 Significant Figures).

**READING THE C AND D SCALES**

Probably the most difficult part of using a Slide Rule is becoming familiar with the Scales, of which there are six on the Slide Rule you...
possess; namely, the A, B, CI, C, D, and K. The C and D scales are alike, while the CI scale is inverted, or opposite hand to the C and D scales. Since the C and D scales are used more than the others, let us begin by studying one of these.

You will observe that at each end there is a line, or graduation, numbered 1. Since the Slide Rule gives no indication of the number of digits, either before or after the decimal point, the 1 on the left, called the Left Index, may be read as .001, 0.1, 1, 10, 100, 1000, etc. However, when the left hand 1 is read as 1, then the right hand 1, called the Right Index, must be read as 10; in other words, the right-hand 1 is always 10 times as great as the 1 on the left end.

Now, beginning at the left end index, skip over the small, secondary graduations 1, 2, 3 . . . 8, 9, until you come to the next main or prime graduation marked 2 (about 3 inches to the right of 1). Following still further you find additional prime graduations labeled 3, 4 . . . 8, 9, the space between getting smaller and smaller. Taking the graduation 4 as an example, this should be read as 0.4, 4, 40, or 400, depending on whether the left hand 1 is read as 0.1, 1, 10, or 100.

Consider next the secondary divisions labeled 1, 2 . . . 9, between the left index and the prime 2. If the left index is read as 1, these will be read as 1.1, 1.2, 1.3, 1.4 . . . 1.9; while if the left index is called 100, the secondary graduations will be read 110, 120, 130 . . . 190. Note that the secondary graduations to the right of the prime 2 are not numbered. Therefore, to determine their values, you will have to count the spaces from the nearest prime graduation, until, after sufficient practice, you will recognize them at sight.

Now notice that the spaces between the secondary lines or graduations between prime 1 and prime 2 are subdivided into 10 parts each; the secondary spaces between prime 2 and prime 4 are subdivided into five spaces each; and those from prime 4 to the right index into two spaces each.

To read a number in the range between prime 1 and 2 (see Fig. 1), the first digit is taken as 1, the second is the secondary figure to the left of the point being read, the third digit is the number of spaces between the secondary line and the subdivision line nearest to and to the left of the point read, while a fourth digit can be approximately determined by estimating the proportional distance between the subdivision to the left and that to the right. Suppose these digits are 1, 3, 6, and 5. The number could be read as 1.365 or 136.5 or 1,365. However, since the decimal point is usually determined as a later step, the number is best read as merely one, three, six, five and written as either 1,365 or 1.365. Note that in this range, numbers can be accurately read to three Significant Figures, and very closely approximated to the fourth Significant Figure. For examples of reading numbers in other ranges, see Figs. 2 and 3.

In order that you may become familiar with the Slide Rule and have confidence in your ability to use it, let us begin with the multiplication of two simple numbers of which you know the product, such as 2 × 4 = 8. (The 2 and 4 in this operation are known as Factors).

Set the left index of the C scale at the prime 2 line of the D scale. Now, slide the indicator until its hair-line coincides with the prime 4 of the C scale. The product 8 is then read along the indicator hair-line on the D scale.

Division is the reverse of this process. To divide 8 by 4, set the indicator hair-line at 8 on the D scale. Now bring the prime 4 of the C scale to coincide with this, and the quotient, 2, can be read on the D scale opposite the left C index.

Now, for practice, try a few more simple examples of this sort, such as 2 × 3 = 6, 3 × 3 = 9, 9 ÷ 3 = 3, etc.

If you try to find 3 times 4 by setting the left C index at 3, you will find that 4 on the C scale comes off the right end of the rule because the product is greater than ten. In all such cases, you should set the right index of the C scale at 3 of the D scale and move the indicator until the hair-line coincides with C4, whereupon you will read the product, 12, on the D scale.

For further practice in multiplication, find, in the manner just described, the products of 4 × 8, 9 × 5. You will then find that multiplication on the Slide Rule is really a very simple procedure. Now, you will probably want to know when you should set the left index and when the right. This cannot always be known offhand, but by following a rule...
for approximation, which will be given later, and by practicing with the Slide Rule as often as possible, you will soon be able to guess right most of the time.

What has been said in regard to Multiplication and Division on the Slide Rule may be summarized in the two following rules:

A. To Multiply Two Factors Together
1. Set index of C scale adjacent to one of the factors on the D scale.
2. Move indicator hair-line to the other factor on C scale.
3. Read the product on D scale under indicator hair-line.

B. To Divide One Number (the Dividend) by Another (the Divisor)
1. Set the indicator hair-line on the dividend on the D scale.
2. Move the C scale until the divisor on it coincides with the hair-line.
3. Read the quotient on the D scale on line with the index of the C scale.

PLACING THE DECIMAL POINT WHEN USING THE SLIDE RULE

Let us consider the following problems:

(a) $261 \times 3.43 = ?$
(b) $261 \times 3.43 = ?$
(c) $261 \times 343 = ?$
(d) $261 \times .343 = ?$
(e) $2.61 \times 3.43 = ?$
(f) $.261 \times .0343 = ?$

As far as the Slide Rule work is concerned, all of these problems are solved identically. The left index of scale C is set at 261 of the D scale, i.e. prime 2, secondary 6, subj. 1, and the product is read on the D scale opposite 343 (i.e. prime 3, secondary 4, and subj. 3) on the C scale. The result is read as prime 8, secondary 9, subj. 5 (or merely eight, nine, five). But does this mean 895, 89500, 895, or what? Evidently it is different for each of the six problems.

The method of arriving at the correct location of the decimal point is to make a rough calculation, after the Slide Rule work, using round numbers. For instance, in example (a), approximate 3.43 as 3 and 261 as 300. Then, $3 \times 300$ is mentally determined as 900, and the correct answer to problem (a) is 895 because that is nearer 900 than either 89.5 or 8950.

Similarly, the answers to the other problems of the group are:

(b) $.895$
(c) 895.000
(f) $.00895$
(d) 89.5
(e) 8.95

When you have operated the Slide Rule for some time, you will learn to make these approximations mentally and almost instantaneously.

WHICH INDEX SHOULD YOU USE?

In the six preceding examples the left index was used. However, if you were to multiply 3.62 by 458, you would find that by setting the left index at D362, the C458 would come off the right end of the D scale. Therefore, the right index would have to be set at D362 to get the desired result. It isn’t always possible to tell beforehand which is the proper index to use, but the following rule will be found very helpful in most cases:

RULE: If the product of the first digits of the given factors is less than 10, use the left hand index; otherwise, use the right hand index.

To illustrate the above rule, consider $2.36 \times 1.45$. Then, $2 \times 1$ is less than 10 and the left index should be used. In multiplying $3.34 \times 5.14$, $3 \times 5$ is greater than 10 and the right index is used.

You will find exceptions to this rule, such as $3.43 \times 3.12$, where $3 \times 3$ is less than 10 but where the actual product (10.70) is greater than 10. However, you will find the rule a great time-saver in the majority of cases.

Exercises in Multiplication and Division:

14. $1.416 \times .0625 = ?$
15. $891 \times 45 = ?$
16. $*14154 \times 31.2 = ?$
17. $3.14 \times 14 = ?$
18. $.205 \times .317 = ?$
19. $.0023 \times .069 = ?$
20. $81 \times 64 = ?$
21. $649 \div 18 = ?$
22. $.742 \div .152 = ?$
23. $1065 \div .276 = ?$
24. $\frac{1}{36} \times 452 = ?$
25. $1.655 \div 455 = ?$

All of these problems should be worked out on the Slide Rule and at least some of them by ordinary arithmetical multiplication. By doing some of them two ways you will be able to observe:

1. The saving in time by using the Slide Rule.
2. The relative degree of accuracy of the two methods.

You will find the answers to the above problems (to the number of significant figures as determined by the slide rule) on the last page of this book.

MULTIPLICATION OF THREE OR MORE FACTORS

Suppose you had a problem, such as multiplying 3.5 by 642 by .0164. To perform this multiplication, you proceed with the first two factors as you would in your other problems, that is, you will set the C index at the 3.5 of the D scale and then move the indicator until the hair-line coincides with the 642 of the C scale. At this point, however, it is not necessary to read the product of these two numbers since we are only interested in the final result. Then, keeping the indicator as just set, move the C index until it coincides with the hair-line of the indicator. Now, move the indicator to .0164 on the C scale and the digits of the product will be found on the D scale under the hair-line as 369.

* This should be set as 1415 since the 5th Significant Figure is lost on the Slide Rule.
You may determine the position of the decimal point in the usual manner of substituting approximate round numbers. Thus, \(600 \times 4 \times .01 = 24\) from which you know that the final answer is 36.9 since that is closer to 24 than either 369 or 3.69.

Any number of factors can be multiplied together in a similar manner. Later, when you learn to use the C1 scale you will learn of a still quicker method of multiplying three or more factors.

**SOLUTION OF PROBLEMS INVOLVING BOTH MULTIPLICATION AND DIVISION**

Problems involving both Multiplication and Division can be worked out on the Slide Rule with a tremendous saving in time over the ordinary method. In solving a problem of this type, it is not necessary to read the answer for each step when we are interested only in the final result. Take as an example:

\[
\frac{\frac{840 \times 648 \times 426}{790 \times 611}}{790 \times 611} = ?
\]

In the above example, the long fraction line, as you already know, stands for division. This problem could be read as the product of 840 by 648 by 426 divided by the product of 790 by 611 or it could be read as 840 divided by 790 multiplied by 648 divided by 611 multiplied by 426. For Slide Rule calculations it is better to consider the problem as stated in the latter manner, since alternating the processes of division and multiplication on the Slide Rule saves time by requiring fewer settings of the index and indicator.

The problem can best be worked out in the following steps:
1. Set the indicator at 840 on D.
2. Move the Slide until 790 on C coincides with indicator hair-line. (Division).
4. Move 611 on C to indicator line. (Division).
5. Move indicator to 420 on C. (Multiplication).
6. Read 480 on D, which is the answer, not considering the proper number of digits.

The correct number of digits must be determined by the usual method of approximation; thus,

\[
\frac{800 \times 700 \times 400}{800 \times 600} = \text{about 500.}
\]

Therefore, the correct answer is 480 and not 48.0 or 4800.

**Exercises:**

26. \(\frac{248 \times 1.141}{38.3} = ?\)
29. \(.0034 \times \frac{7}{97} = ?\)
27. \(\frac{3.14 \times 19.11 \times 16.42}{9.87 \times 13.14} = ?\)
30. \(\frac{421}{18.4} \times \frac{639}{1412} = ?\)
28. \(\frac{1}{25} \times \frac{3}{83} \times \frac{41}{7} = ?\)
31. \(\frac{181 \times 324}{16.4} = ?\)

The answers to these exercises will be found on the last page of this book.

**THE A AND B SCALES**

If you will now look at your Slide Rule, you will see on the top two scales labeled A and B. Careful observation will point out to you that these scales are the same as the C and D scales except that they are only half-size and that there are two of them in the same space that is occupied by one of the C or D scales. The A and B scales can be used in the same manner as the C and D scales to perform multiplication or division. However, they will not be as accurate for the simple reason that the lengths of them are so much shorter.

There is one advantage in using the A and B scales for multiplication and division, and that is that the left index can always be the one set in performing multiplication. For example, if you were to multiply \(3 \times 4\), you could set the left index at the 3 of the left half of the A scale and when you go to set the indicator at B4 you will find that, instead of coming off at the end of the Slide Rule, as happened when you used the C and D scale, the answer 12 can be read on the right half of the A scale.

The A and B scales are used most advantageously, however, in determining the squares and square roots of numbers and in determining areas of circles.

**TO FIND THE SQUARE OF A NUMBER**

If a number is multiplied by itself, the product is said to be the **Square** of the number. The operation of squaring a number is indicated by a small number 2 to the upper right of the number to be squared, known as the **exponent** of the number. Thus, we can write:

\[2^2 = 4 \quad 3^2 = 9 \quad 4^2 = 16\]

To find the square of 2 on the Slide Rule, set the indicator at 2 on the D scale and read the answer 4 on the A scale, under the indicator hairline.

Now, take another example: Find the square of 4.36 by setting the indicator at 436 on the D scale and reading the answer 190 on the A scale. To determine the correct decimal point in the answer, the following rule will guide you:

**RULE:** The number of places to the left of the decimal point in the answer is equal to TWICE the number of places before the decimal in the original number if the answer is found on the RIGHT HALF of the A scale.

If the answer is found in the LEFT HALF of the A scale, then the number of places to the left of the decimal point is equal to TWICE THE NUMBER OF PLACES IN THE ORIGINAL NUMBER, MINUS 1.

Thus, the correct answer to the problem that we just worked out would be 19.0 since the answer was found on the **Right Half** of the A scale.

If there are no places to the left of the decimal point, the following rule can be used to determine the position of the decimal point with respect to the 1st significant figure:
RULE: If the square is found on the RIGHT HALF of the A scale, then the number of zeros between the decimal point and the 1st significant figure equals TWICE the number of zeros between the decimal point and the 1st Significant Figure of the original number.

If the answer appears on the LEFT HALF of the A scale, then the number of zeros is GREATER BY 1 than that determined by the previous rule.

For example,

\[(0.451)^2 = 0.201\]
\[(0.3)^2 = 0.096\]

TO FIND THE SQUARE ROOT OF A NUMBER

The Square root of a number is that factor which, when multiplied by itself, will give you the number. The process of finding the square root of a number is indicated by the radical sign \(\sqrt{\text{over the number}}\).

For example, \(\sqrt{9} = 3\). Another, but less common, way of indicating this operation is by showing the exponent as a fraction; thus \((9)^{\frac{1}{2}} = 3\). The procedure of carrying out the operation on the Slide Rule is just the reverse of finding the square.

Thus, to find the square root of 7.5, you set the indicator at 7.5 on the A scale and read the root on the D scale as 2.74. In doing this, however, you may be at a loss to know whether you should set the indicator at the left half of the A scale or on the right half of the A scale and, therefore, it would be well for you to remember the following rule:

RULE: If the number of places to the left of the decimal point is EVEN, then the RIGHT HALF of the A scale should be used.

If the number of places to the left of the decimal point is ODD, then the LEFT HALF of the A scale should be used.

If there are NO PLACES to the left of the decimal point, then count the number of zeros between the decimal point and the 1st Significant Figure. If this number of zeros is ODD, then use the LEFT HALF of the A scale. If the number of zeros is EVEN, then use the right half of the A scale.

TO FIND THE AREA OF A CIRCLE

A very important use for the A and B scales is in determining the area of a circle. The formula by which such an area is found can be expressed as:

\[\text{Area} = 3.1416 \times (R)^2\] where \(R\) is the radius of the circle.

Or, \[\text{Area} = .7854 \times (D)^2\] where \(D\) is the diameter of the circle.

Suppose you wanted to find the area of a circle whose radius is 2.58 feet. Set the left index of C at 2.58 on D. Then, the left index of B would be opposite the square of 2.58 (on A). It is not necessary, however, to read this square but, instead, we continue with the multiplication by setting the indicator at 3.14 on the B scale and the required area will then be read on the A scale as 20.8 square feet.

TO FIND THE CUBE OF A NUMBER

If you will now look at the bottom of your Slide Rule you will find a scale marked K which, again, is similar to the other scales we discussed except that there are three such scales within the space occupied by one on the D scale. This K scale indicates the Cube of the numbers on the D scale.

The Cube of a number is the product of the number multiplied by itself twice and is represented by the exponent 3. Thus, \(3^3 = 3 \times 3 \times 3 = 27\).

Now, suppose you want to find the cube of 3.42. You set the indicator at 3.42 on the D scale and read the answer as 40.8 on the K scale. How do you know where to place the decimal point? The rules are as follows:

If the cube falls in the RIGHT HAND THIRD of the K scale, then the number of places to the left of the decimal point will be THREE TIMES that of a given number.

If the cube falls in the MIDDLE THIRD of the K scale, then the number of places to the left of the decimal point is equal to THREE TIMES that of the given figure, MINUS 1.

If the cube falls in the LEFT HAND THIRD of the K scale, then the number of places to the left of the decimal point, is equal to THREE TIMES that of the given figure, MINUS 2.

Where there are no digits to the left of the decimal point, the rules are as follows:

When the cube falls in the RIGHT HAND THIRD of the K scale, the number of zeros between the decimal point and the 1st significant figure is equal to THREE TIMES that of the given figure.

When the cube falls within the MIDDLE THIRD of the K scale, the number of zeros between the decimal point and the 1st significant figure is equal to THREE TIMES that of the given figure, PLUS 1.

When the cube falls on the LEFT HAND THIRD of the K scale, the number of zeros between the decimal point and the 1st significant figure is equal to THREE TIMES that of the given figure, PLUS 2.

TO FIND THE CUBE ROOT OF A NUMBER

The Cube Root of a number is that factor which, when multiplied by itself twice, would produce the given number. The Cube Root is indicated by a very small 3 in the upper portion of the groove of the radical sign. For example, \(\sqrt[3]{27} = 3\) or, sometimes, the cube is written with a fractional exponent as \((27)^{\frac{1}{3}} = 3\). The process of finding the Cube Root is the reverse of finding the Cube.

To find the Cube Root of 8.3, you set the indicator at 8.3 on the left hand third of the K scale and read the root as 2.3 on the D scale. Again, the most difficult part of this operation is in determining which third of the K Scale you are to use and the rules for this follow along the same lines as those given for determining the number of places in a Cube. The rules are as follows:
When the number of digits to the left of the decimal point is a MULTIPLE OF 3 (i.e., 3, 6, 9, 12, etc.) use the RIGHT HAND THIRD of the K scale.

When the number of digits to the left of the decimal point is 1 LESS THAN A MULTIPLE OF 3 (i.e., 2, 5, 8, 11, etc.) use the MIDDLE THIRD of the K scale.

When the number of digits to the left of the decimal point is 2 LESS than a multiple of 3 (i.e., 1, 4, 7, 10, etc.) use the LEFT HAND THIRD of the K scale.

When there are no digits to the left of the decimal point, the following rules apply:

When the number of zeros between the decimal point and the 1st significant figure is a MULTIPLE OF 3, use the RIGHT HAND THIRD of the K scale.

When the number of zeros between the decimal point and the 1st significant figure is 1 MORE THAN A MULTIPLE OF 3, use the MIDDLE THIRD of the K scale.

When the number of zeros between the decimal point and the 1st significant figure is 2 MORE THAN A MULTIPLE OF 3, use the LEFT HAND THIRD of the K scale.

Exercises:

32. \((6.24)^2 = ?\)
33. \((14.18)^2 = ?\)
34. \(\sqrt{81.6} = ?\)
35. \(\sqrt{0.000931} = ?\)
36. \((16.38)^3 = ?\)
37. \(.0029)^1 = ?\)
38. \(\sqrt[3]{187.4} = ?\)
39. What is the area of a circle whose diameter is 28 feet?

The answers to these exercises will be found on the last page of this book.

THE CI SCALE

If you will look at your Slide Rule you will see between the B and C scales another scale on the sliding portion which is labeled CI and, after close inspection, you will realize that the scale is exactly opposite hand to the C scale. In other words, it is an inverted C scale, for which reason it is often called a Reciprocal Scale.

Before you can appreciate the use of such an inverted scale it is necessary for you to know what is meant by the reciprocal of a number. This may be defined as a number which, when multiplied by the given number, produces 1. For example, \(1/5\) is the reciprocal of 5, or \(4\) is the reciprocal of 2.5. Notice that every reading on the CI scale is opposite its reciprocal on the C scale.

Now, of what use could this reciprocal be? It so happens that multiplying by any number is the same as dividing by its reciprocal and we can make use of this relationship to find a shortcut method of multiplying several factors together. As an example, suppose we want to multiply \(2 \times 3 \times 7 \times 16\). This would be the same as \(2 \times 3 \div \frac{1}{7} \times 16\). A quick method of performing this operation is to set the C index at 2 on the D scale, move the indicator to 3 on the C scale, move the slide until the 7 on the CI scale coincides with the hair-line and then move the hair-line to 16 on C. The product is then read on the D scale, the significant figures being 672 and the decimal point being determined by approximation in the usual way, fixing the product at 672. You will notice that in determining this product only 4 settings were required, whereas in the orthodox method that you previously learned, 6 settings would be required.

For practice in using this CI scale, see if you can find the products of the following, using the method just described:

40. \(2.8 \times 32.8 \times 1.615 = ?\)
41. \(.062 \times .274 \times .0067 = ?\)
42. \(10350 \times 645 \times .310 = ?\)
43. \(.0113 \times 42.6 \times .0069 = ?\)
44. \(9.8 \times 23.4 \times .643 = ?\)

The answers to these exercises will be found on the last page of this book.

SOLVING PROBLEMS WITH THE SLIDE RULE

Illustrative Problems:

1. (For the Draftsman). A gear has 42 teeth and a diametral pitch of 1 1/4. What is its pitch diameter?

Solution: According to Page 18, Lesson 8 of “A Home Study Course in Blueprint Reading”:

\[
\text{Diametral Pitch (D.P.)} = \frac{\text{Number of Teeth (n)}}{\text{Pitch Diameter (P. D.)}}
\]

Therefore

\[
\text{Pitch Diameter (P. D.)} = \frac{n}{D.P.} = \frac{42}{1.25} = 33.6 \text{ inches}
\]

2. (For the Machinist). At what speed (R. P. M.) should a 3/8" drill be run for a cutting speed of 65 feet per minute?

Solution:

\[
\text{R.P.M.} = \frac{\text{cutting speed}}{\text{perimeter of drill}} = \frac{65 \times 12}{.875 \times 3.1416} = 284 \text{ R.P.M.}
\]

3. (For the Contractor). What will be the cost of materials on a job requiring 148 cubic yards of concrete at $9.55 per C.Y. and 51,800 Ft. B.M. of lumber at $38.00 per thousand FBM?

Solution:

\[
148 \times 9.55 + 518 \times 38.0 = $1413 + $1968 = $3381
\]

Exercises:

Now see if you can solve the following practical problems with the Slide Rule. Check your answers with those given on the last page of this book.

45. (For the Plumber). A plumber needs 64 feet of 1/4" pipe at .850 pounds per foot, 39 feet of 3/4" pipe at 1.130 lbs./ft. and 43 feet of 1" pipe at 1.678 lbs./ft. If the pipe costs 9.4c per pound, what will be the total cost of the pipe?
46. (For the Sheet Metal Worker) What will be the total weight of 32 sq. ft. of #12 ga. sheet metal at 4.462 pounds per sq. ft. and 48 sq. ft. of #16 ga. at 2.55 pounds per sq. ft.?

47. (For the Electrician). How many kilowatt hours are required to run a D. C. motor developing 5 Horse Power for 19 hours if the efficiency of the motor is 85%? (Note that 1 H. P. = .746 KW.)

\[ \text{K.W.H.} = \frac{100}{85} (5 \times 19 \times .746) \]

48. (For the Carpenter). How many board feet (F. B. M.) in 69---3" \times 8" boards 22' 0" long?

49. (For the Ordnance Man). What is the diameter in inches of the bore of a 75 mm. gun?

50. (For the Welder). What will be the cost of electrodes for 438 feet of a weld between sheets of 10 gauge metal, if .051 pounds of electrode are required per foot of weld, and electrodes cost $.095 per pound?

51. (For the Machine Designer). According to the American Standards Association, a Medium Fit (Class 3) should have an allowance of \( 0.0009 \sqrt{d^2} \) between hole and shaft, where \( d \) is the diameter of the shaft.

For a \( 4\frac{15}{16} \) inch shaft, what should be the allowance?

The answers to these exercises will be found on the last page of this book.

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**GEOMETRY**

**Geometrical Magnitudes and Shapes**

If you have ever read a blueprint, whether of a machine part, tool, or house, you know how important it is to recognize the Geometrical Magnitudes or Shapes shown on these prints. For the purpose of identifying these Shapes and applying formulas for calculations of their lengths, areas, volumes and weights, it is highly desirable that you become familiar with the names and properties of the common geometrical figures.

There are four kinds of Geometrical Magnitudes. First, there is the point which has neither length, breadth, nor thickness. The Line has length but no breadth or thickness. Areas have length and breadth but no thickness. Solids have length, breadth and thickness.

**Lines:**

There are two classes of Lines, namely Straight and Curved. A Straight Line is one which never changes its direction, whereas a Curved Line continually changes its direction.

Two Lines which have exactly the same direction are said to be Parallel.

When one line intersects another, four angles are formed. If these angles are all equal they are called Right Angles and the lines are said to be Perpendicular to each other. A Right Angle is equal to 90°.

**Plane Figures:**

A surface in which a straight line can be placed in any position and lie completely within the surface is known as a Plane. A Plane Figure is a portion of a Plane bounded by either straight or curved lines.

When the bounding lines of a Plane Figure are straight, the figure is known as a Polygon. Polygons are classified according to the number of sides that bound them.

**Triangles** (Fig. 4):

- Polygons which have three sides are known as Triangles.
- An Equilateral Triangle is one with three equal sides.
- An Isosceles Triangle is one with only two of its sides equal.
- A Scalene Triangle has each of its three sides of different length.
- A Right Triangle is one that has a right angle between two of its sides.

**Quadrilaterals** (Fig. 5):

- A polygon with four sides is known as a Quadrilateral.
- A Parallelogram is a quadrilateral having its opposite sides parallel.
- A Rectangle is a parallelogram having four right angles.
- A Square is a rectangle having four equal sides.
- An Oblong is a rectangle with adjacent sides not equal.
- A Rhombus is a parallelogram not having right angles but with all four sides equal.
A Rhomboid is a parallelogram without right angles and with adjacent edges not equal.
A Trapezoid is a quadrilateral with two sides parallel but not equal.
A Right Trapezoid is a trapezoid having two right angles.
A Trapezium is a quadrilateral with no two opposite sides parallel.
A Right Trapezium is a trapezium having at least one right angle.

Additional Polygons:
A Pentagon is a polygon having five sides.
A Hexagon is a polygon having six sides.
A Heptagon is a polygon having seven sides.
An Octagon is a polygon having eight sides.
A Dekagon is a polygon having ten sides.

Plane Figures with Curved Boundaries:
The Circle (Fig. 6) is a plane figure bounded by a curved line, every point of which is equidistant from the center.

The Radius of the circle is the distance from the center to the curved boundary.
The Diameter of the circle is the widest dimension of the circle and is equal to twice the radius.
The Circumference or Periphery of a circle is the length of the boundary.
A Sector of a circle is that part of the circle bounded by two radii and a portion of the periphery, such as the hatched area shown in Fig. 6.
The Segment of a circle is that portion of a sector which remains after the Isosceles Triangle between the two bounding radii has been deducted. The area with double line cross-hatching in Fig. 6 is a Segment.
The Arc of a circle is a portion of the circumference.
The Ellipse is a plane figure bounded by a curve drawn so that the sum of the distances from any point on it to two fixed points, known as the Foci, is constant and equal to the Major Axis of the Ellipse. The Ellipse looks like a flattened circle, the largest diameter of which is called the Major Axis, and the smallest the Minor Axis.

Solids:
Solids are three-dimensional magnitudes bounded by Planes, Single-curved Surfaces, Double-curved Surfaces, or combinations of these. As an example of one bound by a double-curved surface, you have the Sphere, which is like a round ball. Cylinders and Cones have single-curved surfaces but plane bases. Fig. 7 illustrates a number of geometric solids.

A Polyhedron is a solid bound by plane surfaces.
A Prism is a polyhedron that has two parallel and equal polygons as bases and whose other sides (lateral faces) are parallelograms. A
A Prism is known according to the polygon which forms the base. Thus, a prism with a triangle as a base is known as a Triangular Prism and a prism with a hexagon as a base is known as a Hexagonal Prism, etc.

A Lateral Edge of a Prism is the line of intersection between two of the lateral faces.

The Axis of a Prism is a line joining the centers of the bases.

A Right Prism is one whose lateral edges, and therefore axis, are perpendicular to the bases.

An Oblique Prism is one in which the axis is not perpendicular to the bases.

A Pyramid is a polyhedron bounded by a base which is a polygon and by triangular lateral faces which meet at a common point known as the Vertex or Apex.

The Axis of a Pyramid is the line joining the apex with the center of the base.

A Right Pyramid is one whose axis is perpendicular to the base.

An Oblique Pyramid is one whose axis is oblique to the base.

A Frustum of a Pyramid is a portion of a pyramid having two parallel bases.

A Cylinder is a solid with a singly curved surface having parallel elements, and two parallel plane bases. The most common form of cylinder is that having a circle as a base.

A Cone is a solid having a plane base and a singly curved surface with elements intersecting at a point called the Apex or Vertex.

Cylinders and Cones may be Right or Oblique, depending on whether the axis is at a right angle or at an oblique angle with the base.

A Frustum of a Cone is a part of a cone with two parallel bases.

GEOMETRICAL CONSTRUCTIONS

1. To bisect a straight line (Fig. 8).

From the ends A and B as centers, describe arcs of equal radii intersecting at C and D. The line joining C and D bisects AB at E.

2. To erect a perpendicular to a straight line (AB) from a given point (A) in that line.

First Method (Fig. 9)

From a convenient center, C, draw an arc through A intersecting AB at D. Through C and D draw a line intersecting the arc at E. AE is then the required perpendicular.

Second Method (Fig. 10)

With A as a center and a radius of 3 units, describe an arc. On the line AB, lay off AC equal to 4 units. With C as a center and 5 units as a radius, describe an arc intersecting the arc previously drawn at point D. Then the line AD is the required perpendicular.

3. To construct a perpendicular to a straight line (BC) from a point (A) without it (Fig. 11).

With A as a center and any convenient radius greater than the distance from A to BC, describe an arc intersecting the line BC at the points D and E. Now, with D and E as centers and any convenient equal radii, describe arcs intersecting at the point F. Then the line from A to F is perpendicular to BC.

4. To divide a straight line (AB) into a number of equal parts (Fig. 12).

Through A, draw a line AC of length equal to the same number of units as the number of parts into which AB is to be divided. Draw CB and then through the points of division on AC draw lines parallel to CB until they intersect AB. The points of intersection thus determined are the points of division of the line AB.

By a similar process, a line may also be divided unequally, as required.
5. Upon a straight line (AB), to draw an angle (BAC) equal to a
given angle (DEF) (Fig. 13).

With E and A as centers and
any convenient equal radii, draw
arcs intersecting the given sides
at 1, 2, and 3 (see Fig. 13). On
the arc through 3, lay off 3-4
equal to 1-2. Through 4, draw
AC and the required angle BAC
is determined.

6. To bisect an angle (ABC) (Fig. 14).

With B as a center and any convenient
radius, describe an arc intersecting BC at E
and BA at F. Then with E and F as centers
and any convenient equal radii, describe arcs
intersecting at point D. Then BD is the bisection
of the angle ABC.

7. To draw a tangent to a circle from
a given point (A) on it (Fig. 15).

Through A, draw a line through the center
of the circle O. Then at A, erect a perpendicular
to AO which is then the required
tangent.

8. To draw a tangent to a circle
from a point (A) without the circle
(Fig. 16).

Draw a line from A to O (the center
of the circle) and bisect this line
at C. With C as the center and CA as
the radius, describe an arc intersecting
the circle at D. A line joining A with
D is the required tangent.

9. To construct a triangle, knowing the
length of the three sides, AB, BC, and CA
(Fig. 17).

Draw the side of the triangle AB with the
known length. With A as the center and AC as
the radius, describe an arc. With B as the center
and BC as the radius, describe an arc. The intersec-
tion of these two arcs locates the third point
of the triangle.

10. To construct a regular hexagon knowing
its diagonal AD (Fig. 18).

With the diagonal AD as a diameter, construct
a circle. With A and D as centers and the radius
of the circle AO as radii, draw arcs intersecting
the circle at B, C, E, and F. Lines joining A to B,
B to C, etc., form the hexagon.

11. To change a square to an octagon (Fig.
19).

Draw the diagonals of the square BD and AC,
intersecting at E. With A as center and AE as
radius, describe an arc intersecting AB at 2 and
AD at 7. Determine points 1, 4 similarly from B,
3 and 6 from C, and 5 and 8 from D. The lines
joining 1, 2, 3, 4, 5, 6, 7 and 8 form the octagon.

MENSURATION OF PLANE FIGURES

The relationship that exists between lengths, areas, and volumes
of geometrical magnitudes is known as Mensuration. Mensuration
is continually being used for estimating the quantities of
materials and costs of
construction and mechanical work on buildings, bridges, airplanes,
ships, tanks, etc.

The Perimeter of a plane figure is the sum of the lengths of the
bounding lines.

The Area of a plane figure is the number of unit squares that can
be included within the boundaries.

The Base of a polygon is any one of the edges.

The Altitude of a plane figure is the overall distance measured perpen-
dicular to the base.

Triangle:

Area = Base × ½ Altitude (see Fig. 4)

Right Triangle:

Referring to Fig 4(d)

\[ c^2 = a^2 + b^2 \]

where c is the hypotenuse and a and b are the other two legs. From this, it
follows that

\[ c = \sqrt{a^2 + b^2} \]

\[ a = \sqrt{c^2 - b^2} \]

\[ b = \sqrt{c^2 - a^2} \]
MENSURATION OF SOLIDS

The **Volume** of a solid is the number of unit cubes that could be contained within the boundaries.

The **Altitude** of a prism, or cylinder, or frustum of a cone or pyramid, is the perpendicular distance between the bases.

The **Length** of a prism, cylinder, or frustum of a cone or pyramid, is the length of the axis.

In the case of a Right Prism or Cylinder, the Altitude and Length are the same.

The **Altitude** of a cone or pyramid is the perpendicular distance from apex to base.

The **Slant Height** of a Right Regular Pyramid is the distance from the apex to the midpoint of one of the edges of the base.

The **Slant Height** of a Right Circular Cone is the distance from the apex to the boundary of the base.

**Prism and Cylinder:**

- Lateral Surface Area = Length \times Base Perimeter
- Total Surface Area = Lateral Surface + (2 \times Base Area)
- Volume = Altitude \times Base Area

**Pyramid or Cone:**

- Volume = \( \frac{1}{3} \times \text{Altitude} \times \text{Base Area} \)

**Right Pyramid or Cone:**

- Lateral Surface Area = \( \frac{1}{2} \times \text{Slant Height} \times \text{Base Perimeter} \)
- Total Surface Area = Lateral Surface + Base Area

**Frustum of Pyramid or Cone:**

- Volume = \( \frac{1}{3} \times h \times (a + A + \sqrt{aA}) \)
  - where \( h \) = altitude
  - where \( a = \text{area of small base} \)
  - where \( A = \text{area of large base} \)

**Frustum of Right Circular Cone:**

- Volume = \( 0.2618 \times h \times (d^2 + dD + D^2) \)
- Lateral Surface Area = \( 1.5708 \times (D + d) \times \sqrt{\frac{1}{4} (D - d)^2 + h^2} \)
  - where \( h = \text{altitude} \)
  - where \( d = \text{diameter of small base} \)
  - where \( D = \text{diameter of large base} \)

**Hollow Right Cylinder:**

- Volume = \( 0.7854 \times h \times (D^2 - d^2) \)
  - where \( h = \text{altitude} \)
  - where \( D = \text{outside diameter} \)
  - where \( d = \text{inside diameter} \)
Sphere:

Surface Area = \(3.1416 \times d^2\)
Volume = \(0.5236 \times d^3\)
where \(d\) = diameter

Hollow Sphere:

Volume = \(0.5236 \times (D^3 - d^3)\)
where \(D\) = outside diameter
\(d\) = inside diameter

Circular Ring (Torus):

Surface Area = \(9.8696 \times D \times d\)
Volume = \(2.4674 \times D \times d^2\)
where \(D\) = mean diameter of ring
\(d\) = diameter of section

Exercises:

Here are some practical problems involving the formulae just given. See if you can solve them.

52. Find the number of cubic yards of concrete in a footing, if you read on a blueprint that it is 6'10' long, 6'10'' wide, and 2'6'' deep.
Suggestion: Reduce inches to decimals of a foot by means of Table II.
53. Suppose that sand has been dumped in a pile shaped like a right circular cone. The height is 6 feet and the base diameter is 12 feet. Find the number of cubic yards in the pile.
54. A piece of round steel stock 1\(\frac{1}{2}\)'' diameter is 3'0'' long. Find its weight if the steel weighs 490 lbs. per cu. ft.
Note: A piece of this steel 1 in. square \(\times\) 1' 0'' would weigh 490 \(\div\) 144 = 3.4 pounds.
55. A conical piece of a duct of \#20-gauge metal (U.S.S.) is 2'0'' long, has a diameter of 1'0'' at one end, and a diameter of 6'' at the other. Find its weight.
Note: \#20 U.S.S. gauge is equivalent to a weight of 1.5 pounds per square foot.
56. What is the length of diagonal pipe necessary to join two points when their elevations are \(+\ 28'0''\) and 16'5\(\frac{3}{4}\)''\?, and the distance between them is scaled on a plan (horizontal projection) as 13'8''?
57. Find the weight of a hollow cylindrical iron casting (right circular) when the outside diameter is 6'', the inside 4'', and the length 8''. Cast iron weighs 450 lbs. per cu. ft.
58. How many gallons will a cylindrical tank hold whose diameter is 10 feet and whose altitude is 10 feet?
59. Change 13.245 meters to feet and inches.
60. What is the weight of a steel ball 2\(\frac{1}{4}\) inches in diameter if steel weighs 490 lbs. per cu. ft.

ALGEBRA

Literal Numbers

In Algebra, letters as well as Arabic numerals are used to represent numbers. Thus, in the statement that the Area of a rectangle equals the Base times the Altitude, the Area may be expressed by the letter \(A\), the Base by \(b\) and the Altitude by \(h\), and the statement may be expressed as
\[A = b \times h\]

Symbols

The expression \(A = b \times h\) can also be expressed as
\[A = b \cdot h\]
or \(A = bh\)

Division is usually expressed as a fraction in Algebra. Thus,
\[h = \frac{A}{b}\]

Addition, subtraction, and equality symbols are the same as those used in arithmetic, as are also the symbols for powers and roots.

\(\therefore\) means "therefore".
\(\pm\) means "plus or minus".

Coefficients

An Arabic number prefixed to a literal quantity to indicate how many times the quantity is to be taken is called a Coefficient. Thus, \(3x\) means \(ax + ax + ax\), the coefficient being 3.

ALGEBRAIC EXPRESSIONS, FORMULAE AND EQUATIONS

A quantity expressed by one or more literal numbers with symbols to indicate how they are combined is called an Algebraic Expression.

Thus if \(m\) and \(n\) stand for certain numbers, \(7 m, n + 2, m + n, 2 mn\), etc., are Algebraic Expressions.

These Algebraic Expressions can be readily evaluated if the value of the literal numbers are known.

Thus, if \(m = 2\) and \(n = 4\),
\[7 m = 14\]
\[n + 2 = 6\]
\[m + n = 6\]
\[2 mn = 16\]

A Term of an algebraic expression is any combination of symbols and coefficients not separated by a plus or minus sign. Thus, in the expression \(3 bx + 2 ay\), the Terms are \(3 bx\) and \(2 ay\).

Similar or Like Terms are those which differ only in their coefficients. Thus, \(4 ax\) and \(3 ax\) are Like Terms.

The number of literal factors contained in a term determines the Degree of the term. Thus,

\(2a\) is of the first degree
\(3ab\) is of the second degree
\(6a^2b^2\) is of the fifth degree
When one algebraic expression is indicated as being equal to a number or another algebraic expression, there results an Equation or Formula.

Examples of equations are:

\[ m + n = 6 \]
\[ 2mn = a + 2b \]

The formula for the area of a rectangle is:

\[ A = b \cdot h \]

The numbers on the right side of the equality sign comprise the Right Member or Right Side of the equation, while those on the left side make up the Left Member or Left Side of the equation.

Exercises:

Given \( a = 3 \) and \( b = 4 \), find the values of:

61. \( a \cdot b \)
65. \( 3a - b \)
66. \( (a + b)a \)
67. \( a + ba \)
68. \( (b - a)^2 \)
69. \( b^a \)
70. \( a^3 \)

The answers to these exercises will be found on the last page of this book.

Note: In exercise 66 above, the symbol ( ) is called Parentheses. The number inside must be combined before using them in any other addition, subtraction, multiplication, or division. When no parentheses are used, multiplications and divisions should be performed before additions or subtractions. Thus, in exercise 66,

\[ (a + b)a = (3 + 4)3 = 7 \cdot 3 = 21 \]
while in exercise 67,

\[ a + ba = 3 + 4 \cdot 3 = 3 + 12 = 15 \]

Many problems which cannot easily be solved by the ordinary methods of arithmetic can be readily solved by translating the statement into an algebraic equation and then solving the equation for the unknown quantities. That is one reason why Algebra is a valuable tool for making calculations easy.

Before you can solve equations, however, you must learn the significance of the various parts of an equation and how to manipulate them. You must learn how to add, subtract, multiply, divide, factor, and find the powers and roots of algebraic expressions. When you accomplish this you will be surprised how easy the solution of algebraic equations becomes.

ALGEBRAIC MANIPULATION

Positive and Negative Quantities

Positive and Negative numbers are used in Algebra to distinguish between opposite quantities. The + or – signs are used respectively, to denote whether terms are positive or negative.

Examples of the Use of Negative Numbers

1. The number of degrees below zero as recorded on a thermometer may be described as \(-8^\circ\).
2. A $100 loss may be written \(-$100 as contrasted with a $100 gain.
3. The elevation of a certain point on a structure may be indicated as \(+80^\prime 4^\prime\) while that of the cellar level may be indicated as \(-10^\prime 2^\prime\), the ground level being taken as elevation 0.

The minus sign must always be written before a Negative Number, for a number with no sign before it is assumed to be Positive.

The number itself, without regard to the Sign, is known as the Absolute Value of the number. Thus the Absolute Value of either \(+8\) or \(-8\) is 8.

Rules for Adding Positive and Negative Numbers:

1. To add two numbers of LIKE SIGNS, add the Absolute Values and prefix the common sign to the result.
2. To add two numbers of UNLIKE SIGNS, subtract the smaller Absolute Value from the larger and prefix the sign of the LARGER to the result.

Examples:

\[ (+3) + (+5) = +8 \]
\[ (+3) + (-5) = -2 \]
\[ (-3) + (-5) = -8 \]

Rules for Subtracting Positive and Negative Numbers.

In Subtraction, the quantity to be subtracted is the Subtrahend; the quantity from which it is to be subtracted is the Minuend; the result is the Remainder.

RULE: Change the sign of the SUBTRAHEND and add the CHANGED SUBTRAHEND to the Minuend.

Examples:

\[ (+6) - (+4) = (+6) + (-4) = +2 \]
\[ (+5) - (-3) = (+5) + (+3) = +8 \]
\[ (-8) - (+4) = (-8) + (-4) = -12 \]

ADDITION AND SUBTRACTION OF ALGEBRAIC TERMS

If we add three dimes to five dimes we get eight dimes. Expressing this numerically, we have

\[ 3 \times 10 + 5 \times 10 = 8 \times 10 \text{ or } 80 \]

To express this algebraically, let \( d \) equal dime or the number ten.

Then \( 3d + 5d = 8d \)
In that way you can see how easy it is to add algebraic expressions involving Like Terms. In the above expressions, the $a$'s, or the $10$'s are called Common Factors.

You can make the subtraction of Like Terms just as easy by just memorizing the rule, "Change the sign of the Subtrahend and add the Changed Subtrahend to the Minuend".

Example:

Subtract $-8ab$ from $+10ab$

Solution:

$$10ab - (-8ab) = 10ab + 8ab = 18ab$$

Rules for Removing Parentheses:

1. If the parentheses are removed from an expression preceded by a plus sign and the enclosed terms are left with their original signs, the value of the expression is unchanged.

   Thus:

   $$(a + b) + (a - c) + (a - d)$$

   $$= a + b - a - c + a - d$$

   $$= a + b - c - d$$

2. To remove the parentheses from an expression preceded by a minus sign, without changing its value, the signs of the enclosed terms must be changed.

   Thus:

   $$a + b - (a - c) - (a - d)$$

   $$= a + b + a + c - a + d$$

   $$= a + b + c + d$$

In the above examples note how the Like Terms ($a$'s) were combined in the last step.

When parentheses are included between parentheses, you should proceed step by step in removing them. *Brackets* [ ] or *Braces* { } are used instead of parentheses in such cases.

Thus:

$$32 - [3 + 7 - 5\{3 - (5 + 2)\} - 8]$$

$$= 32 - [3 + 7 - 5\{3 - 7\} - 8]$$

$$= 32 - [10 + 20 - 8]$$

$$= 32 - 22$$

$$= 10$$

Addition and Subtraction of Polynomials:

A Monomial is an algebraic expression of one term.

A Polynomial is an expression of two or more terms, such as

$$a^2 + a + ab + 3$$

In adding or subtracting Polynomials it is necessary simply to add all like terms, changing the signs of the terms of those Polynomials which are to be subtracted.

Example: To the expression $a^2 + a + ab + 3$ add $3a^2 + 4a + 15$ and subtract $2a^2 + 3a - 5ab + 4$.

This may be written:

$$(a^2 + a + ab + 3) + (3a^2 + 4a + 15) - (2a^2 + 3a - 5ab + 4)$$

$$= a^2 + a + ab + 3 + 3a^2 + 4a + 15 - 2a^2 - 3a + 5ab - 4$$

$$= a^2 + 3a^2 - 2a^2 + a + 4a - 3a + ab + 5ab + 3 + 15 - 4$$

$$= 2a^2 + 2a + 6ab + 14$$

Multiplication and Division of Positive and Negative Numbers

**RULE**: The PRODUCT of two factors is POSITIVE if the signs of the factors are alike. Otherwise the PRODUCT is NEGATIVE.

The above rule also applies to division which is, as you have already observed, the same as multiplying by the reciprocal.

Examples:

$$(+4) \times (+3) = +12$$

$$(+4) \times (-3) = +12$$

$$(-4) \times (+3) = -12$$

$$(+12) \div (-3) = (+12) \times (-\frac{1}{3}) = -4$$

Exponent, Base, and Power

When a quantity is multiplied by itself one or more times, the number of times it is taken as a factor is called the Exponent, the quantity itself is called the Base, and the result the Power of the number.

The Exponent is written as a small integer to the right of and higher than the base.

Thus $x^4$ is read as "$x$ fourth" or "$x$ fourth power" where $x$ is the base and 4 the exponent.

Every power of a positive number is positive.

Every even power of a negative number is positive, while every odd power is negative.

For example:

$$(+3)^2 = (+3) \cdot (+3) = +9$$

$$(+3)^3 = (+3) \cdot (+3) \cdot (+3) = +27$$

$$(-3)^2 = (-3) \cdot (-3) = +9$$

$$(-3)^3 = (-3) \cdot (-3) \cdot (-3) = -27$$

Rules of Algebraic Multiplication

1. The product of two terms containing both numbers and letters is written as the product of the numerals followed by the product of the letters; for example,

   $$3a \cdot 4b = 12ab$$

2. The exponent of the product of two factors having the same base, is equal to the sum of the exponents of the factors.
You can readily understand this rule by considering the multiplication of \( a^4 \) by \( a^7 \). For then, 
\[
a^4 \cdot a^7 = (a \cdot a \cdot a \cdot a) \times (a \cdot a \cdot a \cdot a \cdot a)
\]
\[
= a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a
\]
\[
= a^{11}
\]

The rule for multiplying Positive and Negative Numbers applies equally well toLiteral Numbers.

For example,
\[
(-a) \cdot (+3b) = -3ab
\]
\[
(-a^4) \cdot (-a^3) = +a^{7}
\]

3. To multiply a POLYNOMIAL by a MONOMIAL, multiply the
Monomial by EACH of the terms of the Polynomial and write the resulting
terms in order with their proper signs.

Thus:
\[
[a^2b^3 + 2ab + b^2 - 3] \cdot [3a]
\]
\[
= 3a \cdot a^2b^3 + 3a \cdot 2ab + 3a \cdot b^2 - 3a \cdot 3
\]
\[
= 3a^3b^3 + 6a^2b + 3ab^2 - 9a
\]

4. To multiply a POLYNOMIAL by another POLYNOMIAL, multiply each term of the one by each term of the other in turn, and add the partial products.

To understand this, first consider the multiplication of \((2 + 5)\) by
\((3 + 6)\). This, of course is \(7 \times 9 = 63\). The multiplication could also
be carried out in this way:
\[
(2 + 5)(3 + 6) = 2(3 + 6) + 5(3 + 6)
\]
\[
= 2 \cdot 9 + 5 \cdot 9
\]
\[
= 18 + 45
\]
\[
= 63
\]

Example: Multiply \(3y^2 - 8y + 6\) by \(y - 4\)
\[
(3y^2 - 8y + 6) \cdot (y - 4) = y(3y^2 - 8y + 6) - 4(3y^2 - 8y + 6)
\]
\[
= 3y^3 - 8y^2 + 6 - 12y^2 + 32y - 24
\]
\[
= 3y^3 - 20y^2 + 38y - 24
\]

Algebraic Division:

Division is indicated by the sign \(\div\) or as a fraction.

Thus: \(8 \div 2 = \frac{8}{2} = 4\)

or \(a \div b = \frac{a}{b}\)

RULE: The Exponent of any letter in a Quotient is Equal to its Exponent in the Dividend MINUS its Exponent in the Divisor.

This is easily seen by considering the division of \(a^8\) by \(a^4\).
\[
a^8 = \frac{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a}
\]
\[
= \frac{a \cdot a \cdot a \cdot a}{a \cdot a \cdot a}
\]
\[
= 1 \cdot 1 \cdot 1 \cdot 1 \cdot a^2
\]
\[
= a^2
\]

When the Dividend and the Divisor both have the Same Exponent, the Quotient is \(1\); e.g.,
\[
a^2 = \frac{a \cdot a}{a \cdot a} = \frac{a}{a} = 1 \cdot 1 = 1
\]

You will probably best understand the method of dividing Monomials by the following examples:

1. Divide \(5a^2\) by \(a^2\).
\[
\frac{5a^2}{a^2} = 5 \cdot \frac{a^2}{a^2} = 5 \cdot a = 5a
\]

2. \(\frac{a^3b^3c^2}{ab^2c} = \frac{a^3}{a} \cdot \frac{b^3}{b^2} \cdot c = a \cdot 1 \cdot c = ac\)

3. \(\frac{48ax^4}{-8bx^4} = \frac{48}{-8} \cdot \frac{a}{b} \cdot \frac{x^4}{x^4} = -6 \cdot \frac{a}{b} = -\frac{6ax^4}{b}\)

4. \(\frac{-27a^2b^2}{-9a^2b^3} = \frac{-27}{-9} \cdot \frac{a^2}{a^2} \cdot \frac{b^2}{b^3} = +3 \cdot a \cdot 1 = 3a\)

The following rule should be used for dividing a Polynomial by a Monomial:

RULE: Divide each term of the Dividend by the Divisor and write the results in succession.

Example: Divide \(9x^3 - 12x^2 + 3x\) by \(-3x\)
\[
\frac{9x^3 - 12x^2 + 3x}{-3x} = \frac{9x^3}{-3x} - \frac{12x^2}{-3x} + \frac{3x}{-3x} = -3x^2 + 4x - 1
\]

The following example will illustrate to you how to divide a Polynomial by another Polynomial. You will see from this that the method is the same as that used in Arithmetic.

Example: Divide \(6x^2 - 22x + 12\) by \(x - 3\)

Solution:

1. \(6x^2 \div x = 6x\) Divisor \(x - 3\)
2. \(6x^2 - 18x = 6x^2 - 18x;\) subtract
3. \(-4x + x = -4\)
4. \((x - 3) - 12 = -4x + 12;\) subtract

Check:
\[
(x - 3)(6x - 12) = 6x^2 - 18x - 4x + 12
\]
\[
= 6x^2 - 22x + 12
\]
Factoring

To Factor an expression is to find other expressions which, when multiplied by each other, result in the given expression.

Example: The factors of $3x^2 + 6x - ax$ are $x$ and $3x + 6 - a$ since $x(3x + 6 - a) = 3x^2 + 6x - ax$.

In general, Factors cannot always be readily found, but you should know of a few types of expressions that are easily factorable:

1. Polynomials with Monomial Factors such as the example just given.
2. The difference of two squares, e.g.,
   $$a^2 - b^2 = (a + b)(a - b)$$
3. The sum of two cubes, e.g.,
   $$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
4. The difference of two cubes, e.g.,
   $$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
5. Trinomial squares, e.g.,
   $$(a + b)^2 = a^2 + 2ab + b^2$$
   $$(a - b)^2 = a^2 - 2ab + b^2$$

A noteworthy example of the use of Factoring in making mathematics easy is in the finding of a leg of a triangle, given the hypotenuse and the other leg. You will recall that $b = \sqrt{c^2 - a^2}$ where $a$ and $b$ are the legs and $c$ the hypotenuse.

Suppose $c = 3.25$ and $a = 1.25$ and you wish to find $b$.

Then $b = \sqrt{(3.25^2 - (1.25)^2}$
Factoring $b = \sqrt{(3.25 - 1.25)(3.25 + 1.25)}$
   $$= \sqrt{2 \times 4.50} = \sqrt{9} = 3$$

Determine for yourself if this isn’t easier than finding first $(3.25)^2$, then $(1.25)^2$ and then subtracting.

SOLVING AN EQUATION

When $x = 4$,
$$3x + 12 = 24$$
since $3 \cdot 4 + 12 = 12 + 12 = 24$.

The number 4 is then said to satisfy the equation $3x + 12 = 24$ or to be the Root of that equation.

Note that the equation is not satisfied when $x$ has any other value.

To solve an equation such as $3x + 10 = 22$ is to find the value of the unknown quantity, (in this case $x$) which will satisfy the equation. The solution of equations is made very easy if you learn the following facts:

1. Both sides of the equation may be multiplied or divided by the same number without changing the equality.
2. If a number is added to one side of the equation, the same number must be added to the other side to preserve the equality.
3. If a number is subtracted from one side of the equation, the same number must be subtracted from the other side to preserve the equality.
4. Both sides of the equation may be raised to the same power or have the same root extracted without destroying the equality.
5. Quantities which are equal to the same quantity are equal to each other.

Examples:

1. Solve the equation $4y = 48$
   Solution: Dividing both sides by 4 we get $y = 12$.

2. Solve the equation $\frac{x}{3} = 11$
   Solution: Multiplying by 3 we get $x = 33$.

3. Solve the equation $x + 16 = 28$
   Solution: Subtracting 16 from each side $x = 28 - 16 = 12$.

The process of transferring the 16 from the left side to the right side is known as Transposition and is done in accordance with the following

RULE: Any term may be transposed from one side of an equation to the other by changing its sign.

4. From the formula $V = \frac{1}{3} \cdot h \cdot b$, find $b$ when $V = 20$ and $h = 10$.
   Solution:
   Substituting $20 = \frac{1}{3} \cdot 10 \cdot b$
   or $\frac{10b}{3} = 20$
   or $b = \frac{2}{3}$
   Dividing by 10,
   $b = \frac{2}{3}$
   Multiplying by 3,
   $b = 6$

5. Solve the equation $\frac{x - \frac{3}{5}}{\frac{5}{3}} = \frac{x}{\frac{8}{3}}$
   Solution: Multiplying by 120, the least common denominator,
   $$\frac{120x}{4} - \frac{360}{5} = \frac{-120x}{3} + \frac{600}{8}$$
   Clearing of fractions $30x - 72 = -40x + 75$
   Transposing $70x = 147$
   $$x = \frac{147}{70} = \frac{2 \frac{7}{10}}{2 \frac{11}{10}} = \frac{2}{10}$$
6. Solve the equation \( ax + 7a = a^2 + 4x + 12 \)

**Solution:**
- Transposing
  \( ax - 4x = a^2 - 7a + 12 \)
- Collecting coefficients of \( x \)
  \( x(a - 4) = a^2 - 7a + 12 \)
- Dividing by \( a - 4 \)
  \( x = \frac{a^2 - 7a + 12}{a - 4} \)
- Performing the division
  \( x = a - 3 \)

**Exercises:**

Solve the following equations for \( x \) or \( y \)

71. \( 15x = 225 \)  
75. \( \frac{1}{2}x - 9 = 5 \)

72. \( \frac{1}{3}y = 4 \)  
76. \( 3ax - 2a = 7a \)

73. \( 2x + 14 = 28 \)  
77. \( 5x - 2a = 10a + 3x \)

74. \( 3y - 4 = 5 \)  
78. \( x^2 + 3a = a^2 - 2x - 10 \)

79. \( ax - bc = bx - ac \)

80. From the Formula \( S = \pi r h \), find \( r \) when \( \frac{S}{4} = \frac{22}{7} \), \( h = 3 \), and \( S = 66 \)

The answers to these exercises will be found on the last page of this book.

**PROBLEMS SOLVED BY EQUATIONS**

Algebra becomes extremely useful in the solution of many types of problems. You can make the solution of these problems very easy if you first translate the wording of the problem into an equation, letting the unknown quantity be represented by some letter, such as \( x \) or \( y \).

**Examples:**

1. The sum of two numbers is 60. The larger is 4 times smaller. What are the numbers?

**Solution:** Let \( x = \) the smaller number
- Then \( 4x = \) the larger number
- \( x + 4x = 60 \)
- Or \( 5x = 60 \)
- Dividing by 5, \( x = 12 \)
- \( 4x = 48 \)

Check: The sum of 48 and 12 is 60 and 48 is 4 times 12.

2. An airplane covered 1000 miles at the rate of 200 miles per hour. How long did it travel?

**Solution:** In problems of this type \( d = r \times t \), where
- \( d = \) distance
- \( r = \) rate
- \( t = \) time
- \( d = 1000 \)
- \( r = \frac{1000}{200} = 5 \) hours

3. Two cars started out at the same time and place, for the same destination. Car A traveled at 50 miles an hour and reached its destination 1 hour before car B which averaged 35 miles an hour. How far did they travel?

**Solution:** Let \( t = \) time taken by Car A  
Then \( t + 1 = \) time taken by Car B  
Then the distance \( 50t = 35 \cdot (t + 1) \)  
\( = 35t + 35 \)
- Transposing \( 15t = 35 \)
- \( \frac{35}{15} = \) \( \frac{7}{3} \)
- Then \( d = 50t = \frac{350}{3} = 116\frac{2}{3} \) miles

4. A square 25 ft. \( \times \) 25 ft. is to be divided into two rectangles so that the area of one is 50 sq. ft. more than the other. What will be the widths of the rectangles?

**Solution:** Let \( x = \) the width of the larger rectangle  
And \( 25 - x = \) the width of the smaller rectangle  
Then \( 25 \cdot x = 25 \cdot (25 - x) = 50 \)  
\( 25x - 625 + 25x = 50 \)  
\( 50x = 675 \)
- \( x = 13.5 \)

Now try to solve the following problems, remembering to first translate the wording into an equation:

81. The perimeter of a rectangle is 150 feet. The length is four times the width. Find the dimensions.

82. What is the area of a triangle whose base is one-half as long as the altitude, and the sum of whose base and altitude is twelve inches.

83. The perimeter of a quadrilateral \( ABCD \) is 168 feet. The side \( CD \) is twice as long as \( AB \); the side \( AD \) is three times as long as \( AB \); the side \( BC \) is one and one-half times as long as \( AD \). How long is each?

84. A, traveling 40 miles per hour, starts at 7 A. M. towards B, who is 570 miles away. At 10 A. M., B sets out to meet A at 50 miles per hour. At what time will they meet?

85. A steel rod 135 inches long is to be cut into three pieces so that the second is 7" longer than the first and the third 13" longer than the second. What will be the lengths?

86. Find a number whose difference from 96 is three times as great as the number is larger than 48.

87. In a triangle whose sides are \( AB, BC, \) and \( CA \), and whose perimeter is 24 inches, \( BC \) is 3 times as large as \( AB \), and \( CA \) is \( \frac{4}{3} \) as large as \( BC \). Find the lengths of the three sides.

The answers to these exercises will be found on the last page of this book.
SIMULTANEOUS EQUATIONS

When the conditions of a problem are stated by two or more separate equations, the equations are said to be simultaneous. Simultaneous equations can be solved only if the number of unknowns does not exceed the number of equations. The solution of simultaneous equations of two unknowns is very important, and you should practice with them until you find that you can solve them readily.

In the solution of 2 simultaneous equations, one of the unknowns is eliminated either by altering one or both of the equations until the numerical coefficient of one of the unknowns is the same in each equation. Then, this unknown is eliminated by either addition or subtraction, depending on the signs.

Examples:

1. Solve the equations:
   
   \[4x + 3y = 18\]  \hspace{1cm} (1)
   
   \[2x - y = 4\]  \hspace{1cm} (2)

   Solution: 
   
   Multiplying equation (2) by 3 and subtracting we get:
   
   \[12x - 3y = 12\]
   \[8x = 10\]
   \[x = 3/2\]

2. Solve the equations:
   
   \[4x + 3y = 25\]  \hspace{1cm} (1)
   
   \[3x - 2y = 6\]  \hspace{1cm} (2)

   Solution: 
   
   Multiplying equation (1) by 2 and equation (2) by 3 and adding we get:
   
   \[8x + 6y = 50\]
   \[9x - 6y = 18\]
   \[17x = 68\]
   \[x = 4\]

   Substituting in equation (1) we get:
   
   \[4(4) + 3y = 25\]
   \[3y = 9\]
   \[y = 3\]

Now see if you can solve the following pairs of simultaneous equations:

88. \[4x + 5y = 84\]
   \[3x + 4y = 64\]

90. \[2x + \frac{y}{3} = -7\]
   \[-3x - 2y = 6\]

89. \[11x - 2y = 69\]
   \[2x - 3y = 2\]

91. \[\frac{1}{4}x + \frac{2}{3}y = \frac{4}{3}\]
   \[\frac{2}{3}x - 3y = 4\]

The answers to these exercises will be found on the last page of this book.

Simultaneous Equation Problems:

Problem: Two weights \(A\) and \(B\) of 15 and 25 pounds respectively balance each other on a lever at unknown distances from the fulcrum. If 5 pounds are added to \(A\), \(B\) must be moved one foot further from the fulcrum to maintain the balance. What was the original distance from the fulcrum to each of the weights?

Solution: Let \(x\) be the distance from \(A\) to the fulcrum and \(y\) the distance from \(B\) to the fulcrum.

Then, according to the Laws of Equilibrium, from the first condition:

\[Ax = By\]
\[15x = 25y\]  \hspace{1cm} (1)

From the second condition:

\[(15 + 5)x = 25(y + 1)\]
\[20x = 25y + 25\]  \hspace{1cm} (2)

Subtracting (1) from (2):

\[5x = 25\]
\[x = 5\] feet

Substituting in (1):

\[15(5) = 25y\]
\[75 = 25y\]
\[y = 3\] feet

QUADRATIC EQUATIONS

A Quadratic Equation is one that contains the square of the unknown quantity but no higher power.

All Quadratic Equations have two possible values of the unknown. Thus, in the equation \(x^2 - 4 = 0\), the roots are \(+2\) and \(-2\) since both of these satisfy the equation.

In the equation \(x^2 + x = 2 = 0\), the roots are easily obtained because the left hand term can be factored into \((x - 1) \cdot (x + 2)\), therefore \(x - 1 = 0\) and \(x + 2 = 0\) and \(x = 1\) or \(-2\).

Not all Quadratics are easily factorable, however. A method employing the fact that \(\pm(x + a)^2 = x^2 + 2ax + a^2\) and \(\pm(x - a)^2 = x^2 - 2ax + a^2\) is available however and is illustrated below:

1. Solve the equation \(3x^2 + 2x - 6 = 0\)

Solution:

Dividing each term by 3 and transposing:

\[x^2 + \frac{2}{3}x = +2\]

Adding \((\frac{1}{2})^2\) to each side to get it into the form \(x^2 + 2ax + a^2\) (a perfect square):

\[x^2 + 2 \cdot \frac{1}{3}x + \frac{1}{9} = +2 + \frac{1}{9} = \frac{19}{9}\]

Taking the square root of each side:

\[x + \frac{1}{3} = \pm \sqrt{\frac{19}{9}}\]
from which

$$x = -\frac{1}{9} \pm \sqrt{\frac{19}{9}}$$

This method is known as "completing the square", the rule for which follows:

RULE: 1. Group all terms in $x^2$ and $x$ on the left side of the equation, and the known quantities on the right.
2. Divide all terms by the coefficient of $x^2$.
3. To both sides of the equation, add the square of $\frac{1}{2}$ the coefficient of the $x$ term.
4. Find the square root of both sides of the equation and then solve as in a simple equation.

Problems Involving Quadratic Equations:

1. Find the dimensions of a rectangle if its perimeter is to be 28 feet and its area 45 square feet.

Solution: Let $$\frac{28-2x}{2} = \text{ length } = 14-x$$

Then

$$x(14-x) = 45$$
$$-x^2 + 14x = 45$$
$$x^2 - 14x = -45$$

$$x^2 - 14x + (\frac{7}{2})^2 = -45 + (\frac{7}{2})^2$$
$$\left(x - \frac{7}{2}\right)^2 = \frac{49}{4}$$

Then

$$x = 7 \pm 2 = 9 \text{ or } 5$$

Therefore

Length = 14 - 9 = 5
or
and Width

Thus the Length should be 9 or 5
and the Width 5 or 9.

Now see if you can solve the following problems, the answers of which will be found on the last page of this book.

92. What must be the lengths of the sides $a$ and $b$ of a right triangle if $a$ plus $b$ are to be 10 feet and the hypotenuse $c$ is to be 7.33 feet?

93. The difference in volume between two cylindrical tanks four feet high is 10.21 cu. ft. The difference between their diameters is 6 inches. Find their diameters.

FRACTIONAL AND NEGATIVE EXPONENTS

You have seen that:

1. $a^m \cdot a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Or, in general terms:

1. $a^m \cdot a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Herefore, you have considered only Positive Whole Number Exponents. It is possible, however, to have Fractional, Zero, and Negative Exponents which satisfy the above conditions.

1. In a Fractional Exponent, the Numerator indicates the power to which the number is raised while the Denominator indicates the root to be extracted. Thus:

$$a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = \sqrt{a} \cdot \sqrt{a} = a^{\frac{1}{2} + \frac{1}{2}} = a$$

$$(a^2)^2 = (\sqrt{a})^2 = a$$

2. Any quantity, regardless of its value, with an exponent of zero, attains a value of 1. Since $a^0 \cdot a^m = a^0 \cdot a^m = a^m$,

$$a^0 \text{ must equal 1}$$

3. Any quantity with a Negative Exponent is equal to the Reciprocal of the quantity raised to the corresponding positive power.

Since $a^{-m} \cdot a^m = a^0 = 1$

$$a^{-m} \text{ must equal } \frac{1}{a^m}$$

LOGARITHMS

If $a^x = b$, then $x$ is known as the logarithm of $b$ to the base $a$ and is expressed as

$$\log_a b$$

The anti-logarithm of $x$, to the base $a$, is $b$.

As an example, consider the equation of $2^4 = 16$. Here 4 is the logarithm of 16, and 16 the anti-logarithm of 4 to the base 2, or $4 = \log_2 16$.

Exercises:

94. $\log_2 32 = ?$
95. $\log_{10} 1000 = ?$
96. $\log_a a^4 = ?$
97. $\log_a 1 = ?$

The answers to these exercises will be found on the last page of this book.

Important Relationships Involving Logarithms

1. The logarithm of the product of two or more numbers equals the sum of the logarithms of these numbers.

For example,

$$\log_{10} 1000 = \log_{10} 100 + \log_{10} 10$$

or

$$\log_a bc = \log_a b + \log_a c$$

56

57
2. The logarithm of the quotient of two numbers equals the difference between the logarithms of the numbers.

For example,
\[
\log_{10} 1000 = \log_{10} 1000 - \log_{10} 100
\]
\[
\text{or } \log_c b = \log_c b - \log_c c
\]

3. The logarithm of a number to the n\text{th} power, equals n times the logarithm of the number.

For example,
\[
\log_{10} 100^3 = 3 \log_{10} 100
\]
\[
\text{or } \log_a b^n = n \log_a b
\]

**COMMON LOGARITHMS**

When logarithms are used for numerical calculations, the base 10 is generally used, in which case they are known as Common Logarithms. In writing common logarithms the base is frequently omitted. Thus
\[
\log_{10} 139 \text{ is written } \log 139.
\]

The common logarithm of 100 is evidently 2, and 1000 is 3. But what about the logarithms of the intermediate values such as the log139? It is extremely difficult to calculate such logarithms, but you needn't worry about that because mathematics has been made easy for you through the medium of logarithm tables. In Table III you will find an abbreviated form of logarithm table, which gives logarithms to four decimal places and can be used for numbers of four significant figures. Volumes giving tables to many more places have been published and should be used where greater accuracy is required.

**USE OF LOG TABLES**

The Logarithms are usually expressed as decimal numbers of which the integer is called the Characteristic and the decimal portion the Mantissa. Thus, in considering 1.4771 which is the approximate logarithm of 30, the characteristic is 1 and the mantissa .4771.

The use of Table III is best illustrated by some examples:

1. Find the logarithm of 683.

Here the mantissa is obtained by looking in the first column for 68 and then over on this row to the column headed by 3. The mantissa is then read as .8344 and represents the log of 6.83.

The characteristic is obtained without the use of the table, but by an inspection of the number itself.

Note that
\[
10^4 = 10,000, \text{ or } \log 10,000 = 4
\]
\[
10^3 = 1,000, \text{ or } \log 1,000 = 3
\]
\[
10^2 = 100, \text{ or } \log 100 = 2
\]
\[
10^1 = 10, \text{ or } \log 10 = 1
\]
\[
10^0 = 1, \text{ or } \log 1 = 0
\]

From the tables of pages 60 and 61, it is obvious that the log of 683 must lie between 2 and 3 and hence the part of the logarithm in front of the decimal, i. e., its characteristic, must be 2. Another way to think of this is that
\[
\log 683 = \log (100 \times 6.83)
\]
\[
= \log 100 + \log 6.83 = 2 + .8344 = 2.8344
\]

You will find that memorizing the rules of characteristics will greatly aid in making the use of logarithms easy to learn.

**RULE:** The characteristic of the logarithm of a number greater than one is one less than the number of digits to the left of the decimal point.

2. Find the logarithm of .00683.

Now note that
\[
10^{-1} = .1, \text{ or } \log .1 = -1
\]
\[
10^{-2} = .01, \text{ or } \log .01 = -2
\]
\[
10^{-3} = .001, \text{ or } \log .001 = -3, \text{ etc.}
\]

Then note that
\[
\log .00683 = \log (6.83 \times .001)
\]
\[
= \log 6.83 + \log .001
\]

But the log of 6.83 is .8344, and hence
\[
\log .00683 = .8344 - 3
\]

Thus the characteristic is \(-3\).

In calculations using logarithms it is convenient to express this as
\[
7 - 10. \text{ Thus:}
\]
\[
\log 6.83 = 7.8344 - 10
\]

The following rule will guide you in determining the logarithms of numbers less than 1:

**RULE:** The characteristic of the logarithm of a number less than 1 is equal to 9—n—10 where n is the number of zeros between the decimal point and the first significant figure. This characteristic is written in two parts. The first part, 9—n, is written at the left of the mantissa, and the \(-10\) at the right.

3. Find the logarithm of 683.4.

Here the characteristic is 2. The mantissa cannot be obtained directly from the table but must be obtained by interpolation.

From the table,
\[
\log 683.0 = 2.8344
\]
\[
684.0 = 2.8351
\]

The difference between these logarithms is \(.0007\), from which we can write
\[
\log 683.2 = 2.8344 + (0.4 \times .0007)
\]
\[
= 2.8344 + 0.00028
\]
\[
= 2.8344 + 0.0003
\]
\[
= 2.8347
\]

Don't write logarithms to five decimal figures when using four-place tables. If the fifth figure is 5, 6, 7, 8 or 9, omit it and increase the fourth figure by 1.
Exercises:

Find the logarithms of the following numbers:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>98</td>
<td>1429</td>
<td>100</td>
<td>.00745</td>
<td>102</td>
</tr>
<tr>
<td>99</td>
<td>.6928</td>
<td>101</td>
<td>428.9</td>
<td>103</td>
</tr>
</tbody>
</table>

The answers to these exercises will be found on the last page of this book.

Antilogarithms

1. Find the antilog 1.3032.

Here the mantissa is found directly in the table corresponding to the number 2.01. But since the characteristic is 1 the number of places in front of the decimal point must be 2.

Hence antilog 1.3032 = 20.1.

2. Find antilog 1.3042.

The mantissa of this logarithm lies between .3032, the log of 2.01 and .3054, the log of 2.02.

Therefore, the antilog 1.3042 must lie between 20.1 and 20.2. The difference between .3032 and .3054 is .0022, and the difference between .3032 and .3042 is .0010. Hence the required antilogarithm is \(\frac{10}{22}\) of the way from 20.1 and 20.2.

Antilog 1.3042 = 20.1 + \(\frac{10}{22}\) (.1)

= 20.1 + .045

= 20.15

RULE: (a) Find two consecutive mantissae in the table between which the given one lies.

(b) Find their difference.

(c) Find the difference between the lower of the two tabular mantissae and the given mantissa and divide this by the difference (b), expressing the quotient to the nearest digit.

(d) Annex this digit as a fourth place to the three digits corresponding to the smaller tabular mantissa.

(e) Place the decimal point as indicated by the characteristic.

Exercises: Find the antilogarithm of the following:

<p>| | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>104</td>
<td>.6561</td>
<td>106</td>
<td>9.7841</td>
<td>10</td>
</tr>
<tr>
<td>105</td>
<td>.37531</td>
<td>107</td>
<td>7.1345</td>
<td>10</td>
</tr>
</tbody>
</table>

The answers to these exercises will be found on the last page of this book.

Multiplication and Division by Logarithms

Logarithms are extensively used for performing multiplication, division raising to a given power, or for extracting roots. The work is less laborious than that involved in the ordinary arithmetical methods but at the same time more accuracy can be obtained than with the slide rule.

Naturally, the more places the logarithm tables are carried to, the greater will be the accuracy of the work.

In all work involving logarithms, you are advised to carefully arrange and lay out the work before looking up any logarithms.

Examples:

1. If \(x = \frac{6.28 \times 39.42}{18.31 \times 42.19}\), find \(x\).

Solution: Here \(x = \log 6.28 + \log 39.42 - \log 18.31 - \log 42.19\)

\[
\begin{align*}
\log 6.28 &= 0.7980 \\
\log 39.42 &= 1.5957 \\
\log 18.31 &= 1.2627 \\
\log 42.19 &= 1.6252
\end{align*}
\]

\[
\begin{align*}
\log 6.28 \times 39.42 &= 2.3937 \\
\log 18.31 \times 42.19 &= 2.8879
\end{align*}
\]

\[
\log x = 9.5058 - 10
\]

\[
x = 0.3205
\]

2. If \(x = \sqrt[4]{681}\), find \(x\).

Solution: Here \(x = 4 \log 3.12 - \frac{1}{5} \log 0.681\)

\[
\begin{align*}
\log 3.12 &= 0.4942 \\
\log 0.681 &= 0.8331 - 50
\end{align*}
\]

\[
\begin{align*}
\log (3.12)^4 &= 1.9768 \\
\log (0.681)^{\frac{1}{5}} &= 0.8331 - 10
\end{align*}
\]

\[
\log x = 2.0102 \\
x = 10.24
\]

In order that you may appreciate the use of logarithms, ask your mathematical friends to solve problem 2 without them.

Exercises:

Find numerical values of \(x\) by logarithms when \(x\) equals:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>108</td>
<td>322.6 \times 14.18</td>
<td>110</td>
<td>(14.28)^4</td>
<td></td>
</tr>
<tr>
<td>162</td>
<td>\times 13.6</td>
<td>(6.29)^a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>109</td>
<td>481.0 \times 35.14 \times 16.7</td>
<td>111</td>
<td>(\sqrt[4]{228})(9.68)</td>
<td></td>
</tr>
<tr>
<td>182.9</td>
<td></td>
<td></td>
<td>16.18</td>
<td></td>
</tr>
</tbody>
</table>

The answers to these exercises will be found on the last page of this book.
TRIGONOMETRY

Trigonometry is that branch of mathematics which deals with the functions of angles called Trigonometric Functions. These functions are called the Sine, Cosine, Tangent, Cotangent, Secant, and Cosecant and for any angle \( A \) are abbreviated \( \sin A, \cos A, \tan A, \cot A, \sec A, \text{ and } \csc A \), respectively.

If \( ABC \) in Fig. 20 be any right angle triangle with the sides \( a \) and \( b \), hypotenuse \( c \), acute angles \( A \) and \( B \), and right angle \( C \), the Trigonometric Functions are defined as follows:

\[
\begin{align*}
\sin A &= \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \\
\cos A &= \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \\
\tan A &= \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \\
\cot A &= \frac{1}{\tan A} = \frac{1}{\frac{b}{a}} = \frac{a}{b} \\
\sec A &= \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b} \\
\csc A &= \frac{1}{\sin A} = \frac{1}{\frac{a}{c}} = \frac{c}{a}
\end{align*}
\]

To these six Functions are sometimes added:
- Versed sine \( \text{vers } A = 1 - \cos A \), written vers \( A \)
- Coversed sine \( \text{covers } A = 1 - \sin A \), written covers \( A \)

In Fig. 20, angle \( B \) is the **Complement** of angle \( A \), i.e., it has the value of \( 90^\circ - A \) (since \( C = 90^\circ \) and the three sides of any triangle add up to \( 180^\circ \)) and hence:

\[
\begin{align*}
\sin A &= \cos (90^\circ - A) \\
\tan A &= \cot (90^\circ - A) \\
\sec A &= \csc (90^\circ - A)
\end{align*}
\]

**Trigonometric Tables:**

The values of the Trigonometric Functions for all angles have been computed to many decimal places and are available in many forms. In Table IV you will find the values of the Sine, Cosine, Tangent, and Cotangent, of every degree from zero to ninety, correct to four decimal places.

**Problem:** In the right triangle \( ABC \) (Fig. 20), \( a \) is 35' and angle \( A \) is 35°. Find the hypotenuse \( c \).

**Solution:**

\[
\begin{align*}
\csc A = \frac{1}{\sin A} = \frac{1}{\frac{a}{c}} = \frac{c}{a}
\end{align*}
\]

Therefore

\[
\begin{align*}
c &= \frac{9}{\sin A} = \frac{35}{\frac{5736}{6101}} = 61.01'
\end{align*}
\]

For functions of angles involving fractions of degrees (i.e., minutes) you will have to interpolate, as you did for Logarithms, unless you have a more complete table available.

**Examples:**

1. Find \( \sin 24^\circ 32' \).

   **Solution:** From the Table, \( \sin 24^\circ \) is .4067 and \( \sin 25^\circ \) is .4226 the difference being \( .0159 \times .32 \).

   \[
   .0159 \times .32 = .0050
   \]

   or .0085 must be added to .4067, obtaining .4152 as the required sine.

2. Find \( a \) if \( \sin A = .6008 \).

   **Solution:** \( \sin A \) lies between .6018 and .6157, the sines of 37° and 38°, respectively.

   The difference between .6157 and .6018 is .0139. Since .6008 is greater than .6018 by .0080, it follows that the required angle is \( .0080 \times 60 = .480 \times 60 = 35 \) minutes greater than 37°.

   Thus the required angle \( A \) is 37° 35'.

   You will find the Slide Rule very helpful in the process of interpolation.

**LOGARITHMIC TABLES OF TRIGONOMETRIC FUNCTIONS**

The solution of trigonometrical problems requires a considerable amount of multiplication and division, for which reason it is usually more convenient to use Logarithms of Functions rather than the functions themselves. In Table V you will find this information in brief form. In using this Table note that the sines and cosines of all angles less than 90° and greater than 0° are less than one, and that the tangents of all angles less than 45° are less than one. The logarithms of these functions are therefore negative and minus 10 is understood after each in the table.

**Problems Involving Right Triangles:**

1. A Parallelogram, as shown in full lines in Fig. 21, is to be cut from sheet metal. What must be the size of the rectangular sheet from which it is cut.
Solution: The dotted lines indicate the rectangular plate from which it will be cut. It is seen that \( a = 16 \sin (90^\circ - 64^\circ) = 16 \sin 26^\circ \) and that \( W = 16 \cos 26^\circ \).

\[
\begin{align*}
\log 16 &= 1.2041 \\
\log \sin 26^\circ &= 0.4648-10 \\
\log a &= 10.8459-10 \\
\log 16 &= 1.2041 \\
\log \cos 26^\circ &= 0.9537-10 \\
\log W &= 11.1578-10 = 1.1578 \\
a &= 7.01 = 7^\circ \\
W &= 14.38 = 14\frac{3}{8}^\circ \\
\end{align*}
\]

Thus the width \( W \) must be \( 14\frac{3}{8}^\circ \) and the length \( L \) must be \( 16 + a = 16^\circ + 7^\circ = 23^\circ \).

2. At two shore observation posts 200 feet apart, the angles \( A \) and \( B \) (Fig. 22) to a ship are observed. How far is the ship from shore? \( A = 59^\circ \), \( B = 51^\circ \).

Solution: Let \( x \) be the distance from the shore

Then \( x \cot A + x \cot B = 200 \)

Or \( x (\cot A + \cot B) = 200 \)

Using natural functions:

\[
\begin{align*}
\cot 59^\circ &= 0.6069 \\
\cot 51^\circ &= 0.8098 \\
\cot 59^\circ + \cot 51^\circ &= 1.4107 \\
\end{align*}
\]

Therefore \( x = \frac{200}{1.4107} = 141.8 \text{ ft.} \)

Note that this is really the solution of an oblique triangle. Almost any oblique triangle can be solved by breaking it into appropriate right triangles. This usually requires, however the addition and subtraction of functions which requires the use of natural functions. Later, you will be given formulae for directly solving oblique triangles.

\[\text{Fig. 22}\]

Now see if you can solve the following problems by yourself, the correct answers to which will be found on the last page of this book.

112. A machinist has to make a shaft having a taper 1 foot long. The large end of the taper is 4 inches in diameter and the small end 3 inches. What is the angle of taper? (Fig. 23).

113. How long must a guy wire be to help brace a mast 60 feet high if the wire is to make an angle of 50° with the mast?

114. Two bevel gears have dimensions as shown in Fig. 24. What is the cone angle \( A \)?

115. At a point on the ground 260 feet from a building, the line of sight to the top of the building from the observer makes an angle of 36° with the horizontal. How high is the building above the elevation of the observer’s eye?

116. Referring to Fig. 25, an aviator (A) observes that his altitude (distance to B) is 1500 feet, that the angle \( BAC \) is 40° and that angle \( BAD \) is 65°. What is the distance from \( C \) to \( D \)?

117. The radius of a circle is 5 inches and the length of a chord is 4 inches. Find the angle subtended by the chord.

118. A rectangle is \( 48 \times 22 \). Find the angle made by a diagonal with the longer side.

Functions of Obtuse Angles:

In the solution of oblique triangles you will occasionally have to find the Functions of Obtuse Angles. This should give you no trouble, however, if you remember these simple relationships:

\[
\begin{align*}
sin A &= -sin (180^\circ - A) & \cot A &= -\cot (180^\circ - A) \\
\cos A &= -\cos (180^\circ - A) & \sec A &= -\sec (180^\circ - A) \\
\tan A &= -\tan (180^\circ - A) & \csc A &= +\csc (180^\circ - A)
\end{align*}
\]
FORMULAE FOR THE SOLUTION OF OBLIQUE TRIANGLES

\[
C = 180^\circ - (A + B) \quad b = \frac{a \cdot \sin B}{\sin A}
\]
\[
c = \frac{a}{\sin A} \cdot \sin (A + B)
\]

\[
\sin B = \frac{\sin A \cdot b}{a} \quad C = 180^\circ - (A + B)
\]
\[
c = \frac{a}{\sin A} \cdot \sin C
\]

\[
\frac{1}{2}(A + B) = 90^\circ - \frac{1}{2} C
\]
\[
\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b}
\]

\[
A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)
\]
\[
B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)
\]

\[
c = (a + b)(\cos \frac{1}{2}(A + B)) - (a - b)(\sin \frac{1}{2}(A + B))
\]
\[
\cos \frac{1}{2}(A - B)
\]
\[
\sin \frac{1}{2}(A + B)
\]
\[
\sin \frac{1}{2}(A - B)
\]

Area
\[
K = \frac{1}{2} \cdot ab \cdot \sin C
\]

\[
\text{Let } s = \frac{1}{2}(a + b + c)
\]
\[
\sin \frac{1}{2}A = \sqrt{s(b - c)}
\]
\[
\cos \frac{1}{2}A = \frac{\sqrt{s(a - c)}}{bc}
\]
\[
\tan \frac{1}{2}A = \frac{\sqrt{s(b - c)}}{s - a}
\]
\[
\sin A = \frac{2\sqrt{s(s - a)(s - b)(s - c)}}{bc}
\]
\[
\text{vers } A = \frac{2(s - b)(s - c)}{bc}
\]

Area
\[
K = \sqrt{s(s - a)(s - b)(s - c)}
\]

\[
K = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}
\]
### TABLE V

<table>
<thead>
<tr>
<th>SIN</th>
<th>COS</th>
<th>TAN</th>
<th>COT</th>
<th>SIN</th>
<th>COS</th>
<th>TAN</th>
<th>COT</th>
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<td>0</td>
<td>9.7033</td>
<td>15°</td>
<td>0.9956</td>
<td>0</td>
<td>0.9999</td>
</tr>
<tr>
<td>0°</td>
<td>0</td>
<td>9.7033</td>
<td>0</td>
<td>15°</td>
<td>0.9956</td>
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<td>15°</td>
<td>0.9956</td>
<td>0</td>
<td>0.9999</td>
</tr>
</tbody>
</table>
ANSWERS

1. (a) $\frac{31}{8}$  (b) $\frac{19}{4}$  (c) $\frac{33}{5}$
2. (a) $\frac{3}{2}$  (b) $\frac{1}{15}$  (c) $\frac{7}{9}$
3. $\frac{17}{40}$
4. $\frac{5}{16}$
5. (a) $\frac{8}{21}$  (b) $\frac{32}{63}$  (c) $\frac{1}{9}$
6. (a) .270  (b) .839  (c) .875  (d) .692
7. 36.622
8. (a) 30.1536  (b) 4.188
9. 298.52
10. $\frac{41}{52}$ or 32.789
11. (a) 881  (b) 9.23  (c) .745
12. 88.11 lbs.
13. 852 R.P.M.
14. .0885
15. 40.100
16. 442,000
17. 44.0
18. .0650
19. .0001587
20. 5.180
21. 36.1
22. 4.88
23. 3.820
24. 12.56
25. .00364
26. 7.39
27. 7.60
28. .00847
29. .000245
30. 10.36
31. 3.580
32. 38.9
33. 201
34. 9.03
35. 0.0965
36. 4.390
37. .0000000244
38. 5.72
39. 610 sq. ft.
40. 148.3
41. .001133
42. 2,070,000
43. .0332
44. 147.5
45. $\$16.04$
46. 265.2 lbs.
47. 83.4 KWH.
48. 3,040 FBM.
49. 2.26 inches
50. $\$2.12$
51. .0026
52. 4.32 C.Y.
53. 8.38 C.Y.
54. 18.0 lbs.
55. 7.13 lbs.
56. 17 ft. 10 $\frac{13}{16}$ in.
57. 32.7 lbs.
58. 5,880 gals.
59. 43° - 5 $\frac{15}{32}$
60. 1.69 lbs.
61. 12
62. $\frac{3}{4}$
63. 34
64. 2
65. 5
66. 21
67. 15
68. 1
69. 48
70. 27
71. 15
72. 12
73. 7
74. 3
75. 28
76. 3
77. $6a$
78. $a = 5$
79. $-c$
80. $3\frac{1}{2}$
81. 15' by 60'
82. 16 sq. in.
83. 16, 32, 48 and 72
84. 3 P. M.
85. 36, 43, 56
86. 60
87. 3, 9, 12
88. $x = 16, y = 4$
89. $x = 7, y = 4$
90. $x = -4, y = 3$
91. $x = 8, y = 4$
92. 6.37 and 3.63
93. 3 ft. and 3 ft. 6 in.
94. 5
95. 3
96. 4
97. 0
98. 3.1550
99. 9.8406 — 10
100. 7.8722 — 10
101. 2.6324
102. 1.7957
103. 2.4783
104. 4.530
105. 5.664
106. .6083
107. .001363
108. 2.076
109. 1543
110. 167.1
111. 9.674
112. $1^\circ 12'$
113. 93.34 ft.
114. 33° 42'
115. 118.9 ft.
116. 395 ft.
117. 47° 9'
118. 24° 37'