Edmund Gunter and the Sector

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January 21, 2003

1 Edmund Gunter (1581-1626)

Edmund Gunter was born in 1581 and became Gresham Professor of Astronomy in 1619. He made major contributions to the science of navigation [2]. Henery Briggs was also at Gresham College until 1620, and in the year they were together they worked closely. The two men played a major part in bringing the use of logarithms into the practice of navigation.

To those who are familiar only with the use of electronic calculators, it is hard to imagine how trigonometrical calculations, such as those associated with navigation, were resolved before Napier's invention of the logarithm. What Briggs did for logarithms of numbers, Gunter did for logarithms of trigonometrical functions. In fact, he introduced the terms cosine, cotangent and cosectant for the sine, tangent and secant of complementary angles.

Gunter's most important book was his *Description and use of the Sector*, which was first published in English in 1623. Cotter, [2], hails this as *"the most important work on the science of navigation to be published in the seventeenth century"*. A sector is a mathematical instrument which consists of two hinged rulers on which there are engraved scales. The scales allow various questions in trigonometry to be resolved by using the property that two similar (equiangular) triangles have sides in a constant ratio.

The issue of who first invented by the sector is not without controversy. Its invention is often attributed to Galilao Galilei in approximately 1597 [6, 7] although [4] claims that it was invented by a Guidobaldo de Monte, who was a friend on Galileo, as early as 1568.

What singles out Gunter's sector is that it is the first mathematical instrument to be inscribed with a logarithmic scale to facilitate the resolution of numerical problems. This is not a slide-rule in any sense of the term; the single logarithmic scale is used in conjunction with a pair of compasses. Such a rule is frequently referred to as a Gunter line. A two foot long boxwood ruler inscribed with a variety of scales was a standard navigator's tool up until the end of the nineteenth century.

The first description of the Gunter's logarithmic scale in the form of a plain scale was published in 1624 in Paris by Edmund Wingate. Wingate included a copperplate engraving, measuring

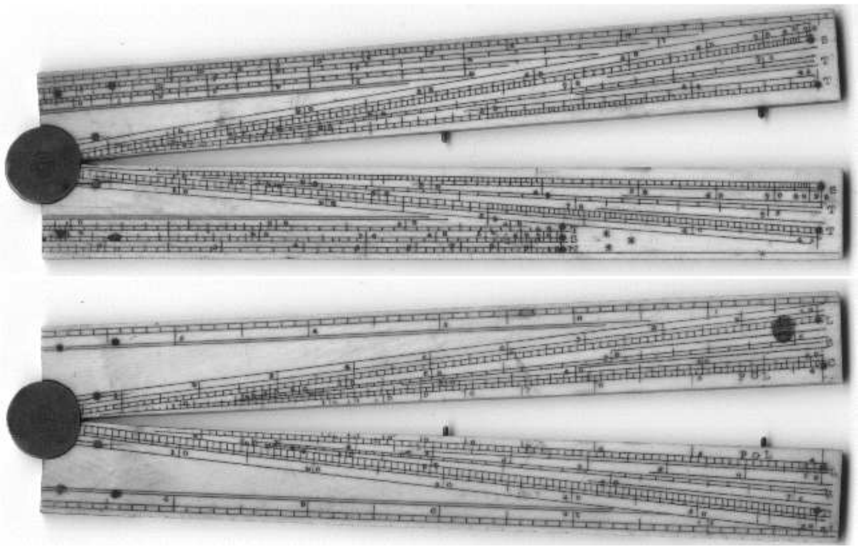

Figure 1: A sector

25 inches in length, of Gunter's scales. In 1628 Wingate published an English translation of this work with the title *The Construction and Use of the Line of Proportion, by the use whereof the hardest Questions of Arithmetique and Geometrie, as well as broken and whole numbers, are resolved by Addition and Subtraction.* This book contains a "double scale", on one side of which is a logarithmic scale, on the other a tabular scale. One simply reads off the logarithm of a number on the logarithmic scale by examining the proportional scale next to it. This is not a slide-rule since it has no moving parts and is not a Gunter line since one does not perform calculations with the aid of compasses: it is simply a diagrammatic substitute for a table of logarithms. Such a scale was published in 1903, [5], but it is unlikely that Knowles was aware of Wingate's book since he makes no reference to it.

There is much controversy over the invention of the slide rule proper. Cajori, [1], originally attributes the invention of the slide-rule proper to Wingate's *Of Natural and Artificial Arithmetic*, London, 1630. However he retracts this assertion in his Addendum, having not read Wingate's book and relied on the report of Favaro. This conclusion is the altered again in his essay contained in [7] where a 30 page pamphlet published by Richard Delemain in 1630 is discussed. Until recently, Florin Cajori's *A history of the slide-rule* [1] published in 1909 was the only book devoted to the history of the slide rule. This gap has now been filled by [4] wherein a more details discussion of the slide rule and the history if its invention can be found along with a very comprehensive bibliography.

2 A typical sector and how to use it

This section is a basic introduction to some of the calculations that are possible using a sector and an explanation of the most commonly appearing scales. An example of a typical sector[1] is shown in Figure 1. A full description of how to use such a sector can be found in [3].

[1]This sector, the property of the author, is of unknown origin. Made of Ivory and brass it is 12 inches long.

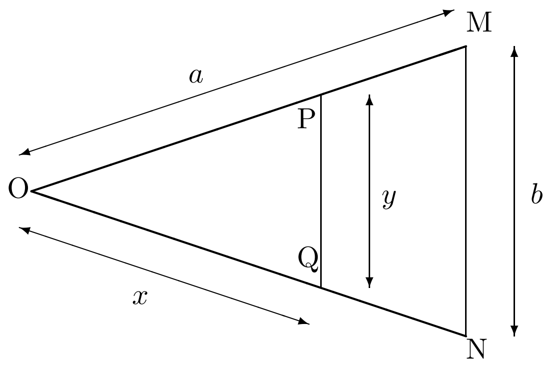

Figure 2: The principle of a sector

2.1 A description of the instrument

A sector is an instrument consisting of two rulers of equal length which are joined by a hinge. A number of scales are inscribed upon the instrument which facilitate various mathematical and trigonometrical calculations.

Open the instrument fully and examine the long outside edge. This edge is marked with a scale divided into ten equal parts, each of these are further subdivided into ten parts. This scale is known as the **scale of decimals**.

On one flat side, close to the outside edge of the sector in a **scale of inches**. Typically a small pocket sector will be twelve inches long when opened out.

On the other side are three long scales parallel to the long outside edge of the sector. These are marked N, S and T. The line N is a Gunter Line, that is to say, a line marked in a logarithmic scale. The other two lines S and T are the lines of sines and tangents respectively. These three scales are usually referred to as the **Lines of Artificial Numbers**.

The other scales all radiate out from the point where the two rules are hinged and are used with the sector open at an angle. Many of these scales are inscribed twice, once on each leg of the sector are are referred to as *double scales*. These will be described individually in more detail below. When the compasses are used on a scale to measure a length from the centre it is called a *lateral distance*. For example distance x in Figure 2. When a measure is taken from any point on the line, to its corresponding point on the line of the same denomination, on the other leg, it is called a *transverse distance*. For example, the distance y in Figure 2.

2.2 Calculations possible and the principle involved

The sector can be used to perform many different numerical and plane or spherical trigonometrical calculations. There are two principal modes of operation;

- fully open as a straight rule,
- partially open with the scales radiating out from the centre.

At all times the sector is used in conjunction with a pair of compasses.

The sector derives its name from the forth proposition of the sixth book of Euclid, where it is demonstrated that similar triangles have their like sides proportional. Consider the two similar triangles shown in Figure 2. We assume that the large outer triangle OMN has a small similar triangle OPQ within it. The line PQ is parallel to MN. The proposition says that $\frac{a}{b} = \frac{x}{y}$. The sector uses this principle when the arms are open at an angle.

2.3 The Gunter Line and associated scales or "Artificial Lines"

First, open the rule out fully so that it is straight and locate the "Gunter Line", marked N, which is divided into unequal parts. The scale runs from a 1 at the left through to 9 and then again from 1 through to 10. Each of the main divisions is subdivided up into ten parts and if the sector is long enough these are further subdivided. As with all logarithmically marked scales the number of divisions may vary on different parts of the scale so careful reading it is needed.

This line is used to solve multiplication and division in conjunction with a pair of compasses. For example to solve 4×3.5 place one compass point on the left hand end of the scale and open the compasses to the length of 3.5. Remove the compasses and without altering their opening place the point on the 4. The other compass point, extending to the right will give the answer. As with all calculations using logarithms, care must be exercised to place the decimal point correctly. Division can be performed in an obvious way using the reverse process.

The line marked S laid in parallel to the scale N give the logarithms of the sines. The scale is marked in angles from 0^0 to 90^0 with the corresponding sine of the angle is read directly from the scale N. For example, locate the number 30 on the scale S. This will be directly along side the second 5 on N representing $\sin(30^0) = 0.5$. The scale T is a scale of tangents of angles and is used in an identical way. These lengths can be used directly in other calculations.

2.4 Use of the sector to perform multiplication and division

Multiplication can also be performed with the arms of the sector open. The double scales are those which appear twice, once on each arm of the instrument. Locate the scales marked L which radiate along the arms from the centre and are marked in ten equal parts. Numbers can be multiplied using these scales by recalling Figure 2 where the lines OM and ON represent the scales marked L on the legs of the sector and O is the point at which the legs pivot. Recall that $\frac{a}{b} = \frac{x}{y}$ or $ay = bx$. The length OM is always fixed at 10 units. We are left with three unknowns, x, y and b. Take a pair of compasses with one end at O open them to a length b on the scale L. Open the sector so that the distance between M and N is the length b by using the compasses. Next locate the points P and Q which are a lateral distance x along the scale. Using the compasses take the transverse length y. This length can be read by placing one point at O and the other point on the line OM and reading directly on the scale. The length y which has been constructed is the answer to $10y = bx$. Division can be performed by an obvious reverse process.

This process is slow, requires a high degree of manual dexterity and is of doubtful accuracy. It is, therefore, of limited practical utility. The only possible use would be during a plane figure construction where it was necessary to construct a line of a given proportion, from one already on the figure. Even here, more accurate ruler and compass methods exists.

2.5 As a protractor to calculate plane angles

The sector can also be used to construct angles using the **Lines of Chords.** This is a double scale, marked C, with scales that run from 0 to 60. To protract an angle of $0 < \Theta < 60$ degrees use the scale C and open the sector so that the transverse distance MN equals the lateral distance $OM = ON$. It is clear that the angle $MON = 60^o$. Using the compasses on the scale measure the lateral distance from O to Θ. Draw a circular arc, centred at O, from M to N and place one point of the compass at N. Where the other point intersects the arc will be marked R. The angle RON is Θ. Angles can be measured using the reverse procedure. An angle, Ψ, greater than sixty degrees can be protracted by repeatedly subtracting 60^0 or 90^0 from Ψ.

2.6 To inscribe polygons inside a given circle

The sector can easily be used to inscribe a regular polygon inside a circle of any given radius using the **Line of Polygons**. This double scale, marked POL, usually on the inside of the sector, is marked from 12, nearest O to 4 at N and M. As usual refer to Figure 2 and let ON represent the scale POL. Take a circle and open the compasses to the radius. Recalling the fact that the lengths of the sides of an inscribed hexagon is the radius of a circle use the compasses to open the sector so that the transverse distance between the 6's on the POL scale equals the radius of the circle. The transverse distance between the figures are the lengths of the sides for an inscribed polygon with that number of sides. For example to inscribe a square measure the transverse distance between the 4's on the POL scale and use this to inscribe a square directly in the circle.

To construct a polygon with a given number of sides and a given side length, as opposed to an inscribed polygon, is also possible. Measure the length of the sides with the compasses and open the sector so that y is the transverse distance between points P and Q, on the POL scale, which correspond to the number of sides in the proposed polygon. The radius of the required circle is the transverse distance between the 6's. This is simply the reverse process although here the sector may not open wide enough.

2.7 Other scales

The above gives some examples of the range and types of calculations that can be performed by the sector. The other scales can be used for a variety of other calculations which I will not detail here.

Although the sector was a standard component in a set of mathematical instruments there is little evidence that they were ever used for practical calculations. In fact, the smaller pocket sectors would have been little or no use because of the difficulty in accurate use. Compass points damage scales with regular use which reduces the accuracy of the instrument. The number of well preserved sectors that exist would add weight to the school of thought which claims they were of little practical utility.

References

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[6] Ad Meskens. Michiel Coignet's Contribution to the Development of the Sector. *Annals of Science*, 54:143–160, 1997.

[7] David Eugene Smith. *A Source Book In Mathematics.* McGraw-Hill, 1929.