SIMPLIFIED
Mathematics
FOR DAILY USE
AND
HOW TO USE
THE SLIDE RULE

RAPPOLT

DOUBLEDAY
DORAN

*Simplified*

# MATHEMATICS

AND

*How To Use the*

# SLIDE RULE

# *Simplified* MATHEMATICS AND *How To Use the* SLIDE RULE

*By*

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DOUBLEDAY, DORAN AND COMPANY, INC.
GARDEN CITY · · · 1943

# PREFACE

Mathematics is the vital, indispensable tool of production today. The life of industrial America is daily reaching a new highpoint. From one end of our land to the other, new factories; entirely new industries have sprung into being almost overnight. Into these vast new centers of production stream an ever growing number of men and women; people who never before worked in the fields of industrial production; people who must be equipped to meet this new demand for technical skill and labor. One of the main requirements is that the worker be equipped with a sound practical knowledge of mathematics.

The average man is no longer content with knowing simple arithmetic. In the world of today his job and his future depend to a great extent upon his ability to work with mathematics—the instrument for solving a thousand and one problems of everyday work.

This volume has been especially designed as a rapid home-study course. It offers assistance of tremendous value to people who have had little or no previous training in mathematics; and it will prove of equal value to many who have forgotten much of their early training and background in the science of numbers. Thus the book combines the properties of an original self-teaching course and a necessary mathematics refresher to help the average man meet the many technical and industrial problems he faces today.

The author has done his best to destroy the false notion that mathematics is a difficult subject to grasp. Difficulty in learning mathematics is the result of poor and outworn methods of teaching. Actually, mathematics can be made a most fascinating game and learning can be as entertaining and pleasant as reading a good book. The reader will find for himself, after going through just a few of the lessons in this course, that the instruction given here can be understood with a minimum of time and a maximum of pleasure. All explanations have been stated in simple and easily-understood language. All necessary points have been clearly illustrated with expertly drawn diagrams, made especially for the home-study student. Throughout this volume mathematics is related to the practical problems that the average man will meet in his daily work.

As this course is really two volumes in one, it is accordingly divided into two principal parts. The first of these deals with the most important elements of Arithmetic, Geometry, Algebra and Trigonometry. No special training is needed for the complete understanding of these subjects, as they have been presented here. The reader and student is taken, step-by-step, from simple addition and subtraction, through a working knowledge of fractions and decimals. In treating geometry, the author has stressed chiefly those absolutely essential facts which will enable the student to meet and solve the practical problems of his work. He is shown the most rapid way of determining such things as areas, volumes and lengths, and the most rapid and accurate method for doing simple geometric constructions.

Part II contains simplified thorough instructions in the use of the slide rule—that most ingenious mathematical time-saver. Because of the ever increasing importance of the slide rule in modern business and industry, that portion of the book devoted to this instrument is unusually complete



and explicit. For, while the slide rule was at one time merely the tool of the professional engineer, today it is the valued helper of the mechanic, the student, the estimator and the executive.

Throughout each chapter there are carefully selected lists of exercise problems. Practise is indispensable to the student of mathematics and many rules, laws, and formulas which may seem complicated and difficult upon reading, become extremely simple and easy to understand once the student has applied them to the solution of actual problems. At the end of this volume, the student will find complete answers to all problems so that he may check the accuracy of his own calculations.

It is the feeling of the author, based upon many years of teaching and actual engineering experience, that the instruction furnished here will enable the home-study student to become proficient in those branches of mathematics which are most important to the practical man. The purpose of this study course is to teach you to work easily, rapidly and efficiently in the new life of industrial America. Nothing has been spared to make this book a practical and helpful tool that you can use in your daily work.

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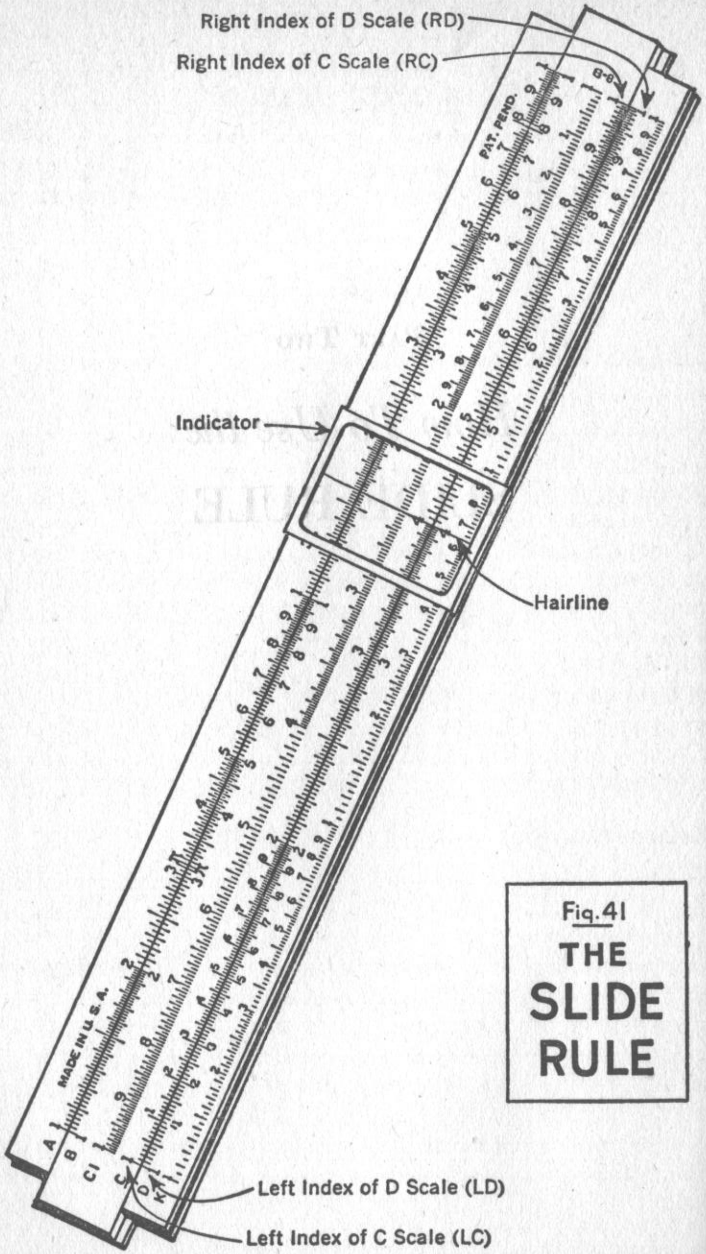


Fig. 41
THE SLIDE RULE

# PART TWO

# HOW TO USE THE SLIDE RULE

## INTRODUCTION

### Description and Purpose of the Slide Rule

The slide rule is a device for saving time and labor in calculations involving multiplication, division, proportion, squares, square roots, cubes or cube roots or any combination of these processes.

In Fig. 41 is portrayed a slide rule of the most common variety, which consists of three parts, namely: the body containing the *A*, *D*, and *K* scales; the slide containing the *B*, *CI* and *C* scales; and the glass Indicator containing a Hairline. Some slides are equipped with scales of sines, tangents, and logarithms on the reverse side.

Although the appearance of a slide rule is very much like that of a measuring rule, its function is entirely different. The scales are not used to measure distances; the markings represent numbers which are used in calculations. Exactly how to read and use the scales will be fully described in subsequent articles.

### Accuracy of the Slide Rule

There are certain limitations to the accuracy obtained with slide rules. With a well constructed slide rule and with care in its use, results accurate to $\frac{1}{10}$th of one per cent can be obtained; while even with less care and poorer equipment it is possible to achieve results within one-half of one per cent of the correct value. When a much higher degree of precision is required, other methods of calculation must be employed. Fortunately, however, most of our technical calculations require no higher precision, and any attempt to attain it would be simply wasteful of time and energy. As a matter of fact, results which appear extremely accurate are frequently misleading, since the given data are obtained approximately.

In one range of the slide rule the first four digits of any number can be read (or set) on the scale, while over the greater portion of the rule only three digits can be read. In other words, the slide rule scale readings have an accuracy to three or four significant figures, depending on the range. The following article will give you a clearer conception of the meaning of significant figures.

### Significant Figures

Every number consists of a series of figures from 0 to 9, known as Digits. In the number 432, the first digit is 4, the second is 3 and the third is 2.

Now let us suppose that the population of a certain city has been recorded as 4,355,256 (a number of 7 significant digits). But there will be a great many inaccuracies in the taking of the census. Furthermore, there are people moving in or out of the city while the census is being taken. Therefore, in speaking of the population, it might be better to "round out" the number to 4,355,000. The latter figure, while still having 7 digits, has only 4 significant figures, for the 3 "naughts" (or "zeros") in this case are only approximate and are put there simply to fix the position of the decimal point.

Now consider the decimal number .135. This has 3 decimal places and has 3 significant figures. The decimal .000135 has 6 decimal places but has only 3 significant figures.

*Examples:*

| NUMBER | SIGNIFICANT FIGURES |
|---|---|
| 308 | 3 |
| 30,800 | 3 |
| 20.002 | 5 |
| 20,000 | 1 |
| .002 | 1 |
| .2002 | 4 |
| 20.2 | 3 |

Note that in the number 20.002, all 5 digits are significant figures, even though 3 of them are "zeros." For in this case the zeros are not put in merely to place the decimal point, but are digits which are used to distinguish the number from any such numbers as 20.012, 21.002, 20.202, etc.



*Example:*

Consider the multiplication

$$284 \times 346.$$

By the ordinary method of multiplication you would obtain a product of 98,624. There are 5 digits in this product, each of which is a significant figure.

In many multiplication operations it is not necessary to get the product as accurately as this. For instance, the product could be expressed as 98,260 which would be correct to 4 significant figures, or as 98,300 which would be correct to 3 significant figures, or simply as 98,000 which would be correct to 2 significant figures.

In performing the operation just described by the slide rule, the answer 98,300 would have been obtained since the slide rule is read for the most part to only 3 significant figures.

*Example:*

A casting weighs 25.1 lbs. How much will 26 similar castings weigh?

*Solution:*

By ordinary arithmetic

$$25.1 \times 26 = 652.6$$

which is given to 4 significant figures, although the weight of the casting is only correct to 3 significant figures.

By the slide rule, we would obtain as the result 653 lbs., a number of 3 significant figures.

Now since the weight of one casting is given only to 3 significant figures the actual weight could be anything from 25.06 up to 25.14.

$$\text{But } 25.06 \times 26 = 651.56$$
$$\text{and } 25.15 \times 26 = 653.64$$

It is thus seen that a result in this case to more than 3 significant figures is actually misleading, and that the slide rule result is sufficiently accurate (and, of course, much more rapidly obtained).

*Example:*

Multiply .000123 × .0032.

Here the product is .0000003936. Although there are 10 digits here behind the decimal point, the figure is correct to only 4 significant figures. A zero is only a significant figure of a decimal when it appears between two other significant digits. On a slide rule, this product would, ordinarily, be read as .000000394 (correct to 3 significant figures).

*Exercises:*

325. How many significant figures are there in the following:

(a) 2940 Ans. ________
(b) 2904 Ans. ________
(c) .135 Ans. ________
(d) .0135 Ans. ________

326. Round out the following to 3 significant figures:

(a) 48,369,290 Ans. ________
(b) 68,345 Ans. ________
(c) .02314 Ans. ________

327. Determine the product of 8.94 and 6.24 to 3 significant figures. Ans. ________

328. Divide 4.86 by 92, expressing the quotient in 3 significant figures. Ans. ________

## HOW TO READ THE SCALES

### The Slide Rule Scales

Before you can begin to calculate with the slide rule, you must become thoroughly familiar with the reading of the scales.

The most common type of slide rule has 6 scales in front; namely, the *A*, *B*, *CI*, *C*, *D* and *K*.

Since the *C* and *D* scales are used more than the others, we will study these first.

While studying this text you will find it necessary to have your slide rule in front of you all the time. So take your slide rule out of the case, and we will begin the reading of the *C* and *D* scales.

### How to Read the *C* and *D* Scales

Your first observation in studying the *C* and *D* scales, will be that they are exactly alike. Therefore, anything that we may say regarding one of them applies equally as well to the other.

Notice that at each end there is a line, or graduation, numbered



1. That on the left is called the *Left Index*, while the one on the right is the *Right Index*.

The slide rule scale does not indicate the position of any digit relative to the decimal point. Hence the left index may be read as .001, 0.1, 1, 10, 100, or 1000. However, if the left index is read as 1, then the right index is read as 10; in other words, the right index is always 10 times as great as the left index.

Now, beginning at the left end index, skip over the small, *secondary* graduations 1, 2, 3, . . . . . 8, 9, until you come to the next main or *prime* graduation marked 2 (about 3 inches to the right of 1). Following still further you find additional prime graduations labeled 3, 4, . . . . . 8, 9, the space between getting smaller and smaller. Taking the graduation 4 as an example, this should be read as 0.4, 4, 40, or 400, depending on whether the left hand 1 is read 0.1, 1, 10, or 100.

Consider next the *secondary* divisions labeled 1, 2, . . . . . 9, between the left index and the prime 2. If the left index is read as 1, these will be read as 1.1, 1.2, 1.3, 1.4, . . . . . 1.9; while if the left index is called 100, the secondary graduations will be read 110, 120, 130, . . . . . 190. Note that the secondary graduations to the right of the prime 2 are not numbered. Therefore, to determine their values, you will have to count the spaces from the nearest prime graduation, until, after sufficient practice, you will recognize them at sight.

Now notice that the spaces between the secondary lines or graduations between prime 1 and prime 2 are subdivided into 10 parts each; the secondary spaces between prime 2 and prime 4 are subdivided into 5 spaces each; and those from prime 4 to the right index into 2 spaces each.

To read a number in the range between prime 1 and 2 (see Fig. 42), the first digit is taken as 1, the second is the secondary figure to the left of the point being read, the third digit is the number of spaces between the secondary line and the subdivision line nearest to and to the left of the point read, while a fourth digit can be approximately determined by estimating the proportional distance between the subdivision to the left and that to the right. Suppose these digits are 1, 3, 6, and 5. The number could be read as 1.365 or 136.5 or 1,365. However, since the decimal point is usually determined as a later step, the number is best read as merely one, three, six, five and written as either 1365 or

1.365. Note that in this range, numbers can be accurately read to 3 significant figures, and very closely approximated to the fourth significant figure. For examples of reading numbers in other ranges, see Figs. 43 and 44.

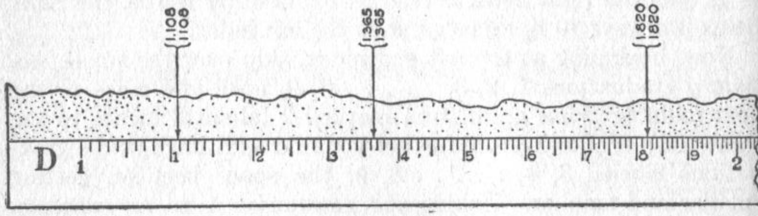


Fig. 42

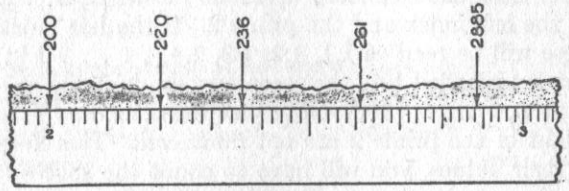


Fig. 43

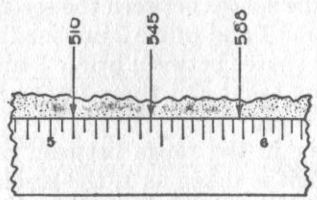


Fig. 44

You should practice reading and setting until you can do so accurately and without hesitation. Then you are ready to give attention to the solution of simple multiplication problems.

*Exercises:*

329. Record the readings for the points indicated in Fig. 45, assuming the left index to be 1.000.



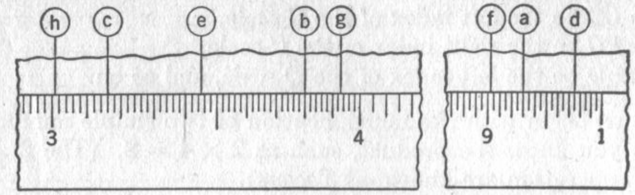


Fig. 45

| | |
|---|---|
| (a) 9.30 | (e) ______ |
| (b) ______ | (f) ______ |
| (c) ______ | (g) ______ |
| (d) ______ | (h) ______ |

330. Write in the correct letter for each number listed below (see Fig. 46):

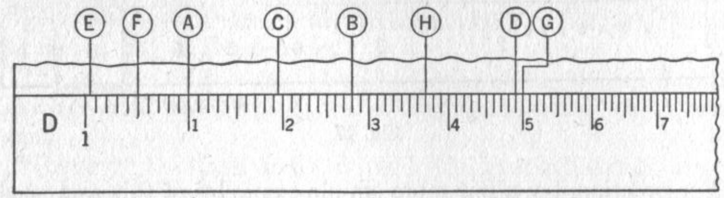


Fig. 46

| | | | |
|---|---|---|---|
| 1.280 | ____B____ | 1.100 | ______ |
| 1.005 | ______ | 1.050 | ______ |
| 1.370 | ______ | 1.500 | ______ |
| 1.195 | ______ | 1.490 | ______ |

## HOW TO MULTIPLY AND DIVIDE WITH THE SLIDE RULE

### Simple Multiplication

To multiply one number by another, either the *C* and *D* scales together or the *A* and *B* scales together may be used.

The *C* and *D* scales are more accurate and are preferred for this purpose, and will, consequently be the ones considered in this article.

It will be convenient in the discussions which follow to use abbreviations in describing the indexes of the various scales.

Let *LC* be the left index of the *C* scale,
*RC* be the right index of the *C* scale,
*LD* be the left index of the *D* scale, and so on.

Let us begin with the multiplication of two simple numbers of which you know the product, such as $2 \times 4 = 8$. (The 2 and 4 in this operation are known as *Factors*).

Set the left index of the *C* scale (*LC*) at the prime 2 line of the *D* scale. (Fig. 47)

Now, slide the indicator until its hair-line coincides with the prime 4 of the *C* scale. The product 8 is then read along the indicator hairline on the *D* scale.

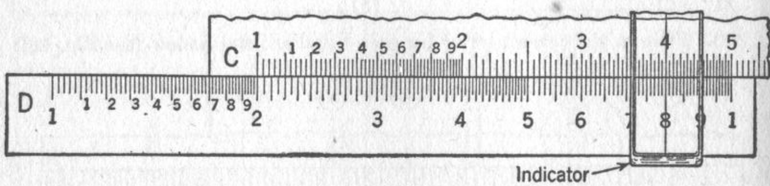


Fig. 47

For practice, try a few more simple examples of this sort, such as $2 \times 3 = 6$ and $3 \times 3 = 9$.

A rule for multiplication on the slide rule may be stated as follows:

*RULE: To Multiply Two Factors Together*

1. *Set index of C scale adjacent to one of the factors on the D scale.*
2. *Move the indicator hair-line to the other factor on the C scale.*
3. *Read the product on the D scale under the indicator hair-line.*

The procedure just described applies just as well when there are two or more digits in each of the factors, as illustrated in the following:

*Example:*

$$1.55 \times 1.95 = ?$$

*Solution* (Fig. 48):

1. Set *LC* at 1.55 on the *D* scale.
2. Move the indicator to 1.95 on the *C* scale.
3. Read, on the *D* scale, the product 3.02 under the hair-line.



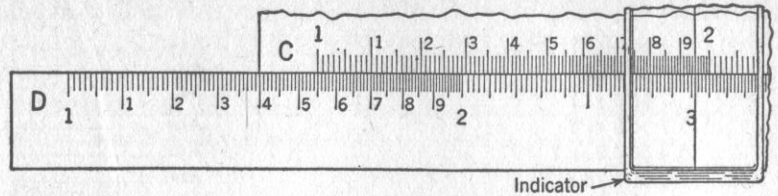


Fig. 48

If you try to find 3 times 4 by setting *LC* at 3, you will find that 4 on the *C* scale comes off the right end of the rule because the product is greater than 10.

In this case, you should set *RC*, instead of *LC*, at 3 on the *D* scale, and move the indicator until the hair-line coincides with *C*4 (Fig. 49). You will then read, under the hairline, on the *D* scale, the digits, 120, of the product.

The product in this case should be read as 12, rather than 1.2, although each factor has only one digit to the left of the decimal point.

Whenever the right index is used, the product should always be 10 times as much as a product obtained when the left index is used. This rule, however, need not be considered when we use for determining the position of the decimal point, a method of estimation to be described in a subsequent article.

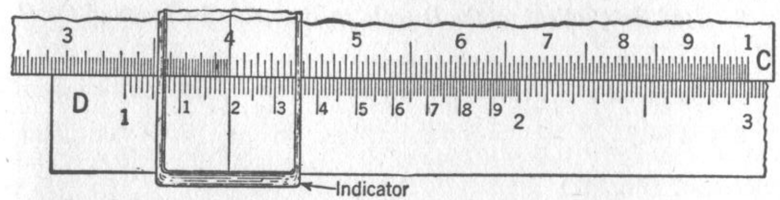


Fig. 49

You will probably now wish to know when you should set the left index and when the right. This cannot always be known offhand, but by following a rule for approximation, which will be given later, and by practicing with the slide rule as often as possible, you will soon be able to guess right most of the time.

*Exercises:*

| | | |
|---|---|---|
| 331. | $4 \times 8 = ?$ | Ans. ________ |
| 332. | $9 \times 5 = ?$ | Ans. ________ |
| 333. | $7 \times 9 = ?$ | Ans. ________ |
| 334. | $1.7 \times 3 = ?$ | Ans. ________ |
| 335. | $1.4 \times 4.2 = ?$ | Ans. ________ |
| 336. | $1.33 \times 1.78 = ?$ | Ans. ________ |
| 337. | $2.1 \times 4.3 = ?$ | Ans. ________ |
| 338. | $7.8 \times 8.3 = ?$ | Ans. ________ |
| 339. | $2.45 \times 3.12 = ?$ | Ans. ________ |
| 340. | $3.14 \times 6.7 = ?$ | Ans. ________ |

**Simple Division**

Division is the reverse of multiplication.

In learning the following rule for division on the slide rule, you are reminded that the dividend is the number to be divided, the divisor is the number by which the dividend is divided, and the quotient is the result of the division.

$$\text{Dividend} \div \text{Divisor} = \text{Quotient}$$

or,

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient}$$

*RULE: To Divide One Number by Another*

*1. Set the indicator hair-line on the dividend on the D scale.*

*2. Move the C scale until the divisor on it coincides with the hair-line.*

*3. Read the quotient on the D scale on line with the index of the C scale.*

*Example:*

$8 \div 4 = ?$

*Solution:* (Fig. 47)

1. Set the indicator at 8 on the *D* scale.
2. Move the slide until 4 on the *C* scale is under the hair-line.
3. Read, on the *D* scale, the quotient, 2, opposite the left index of the *C* scale (*LC*).

*Example:*

$3.02 \div 1.95 = ?$



*Solution* (Fig. 48):

1. Set the indicator at 3.02 on the *D* scale.
2. Move the slide until 1.95 on the *C* scale is under the hairline.
3. Read, on the *D* scale, the quotient, 1.55 opposite the left index of the *C* scale (*LC*).

*Example:*

$63 \div 9 = ?$

*Solution* (Fig. 50):

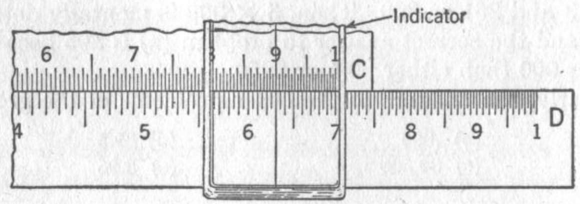


Fig. 50

1. Set the indicator hairline at 63 on the *D* scale.
2. Move the slide until 9 on the *C* scale lies under the hairline.
3. Read, on the *D* scale, the quotient 7 opposite *RC*.

*Exercises:*

| | | |
|---|---|---|
| 341. | $9 \div 3 = ?$ | Ans. ________ |
| 342. | $63 \div 7 = ?$ | Ans. ________ |
| 343. | $28 \div 4 = ?$ | Ans. ________ |
| 344. | $32 \div 8 = ?$ | Ans. ________ |
| 345. | $36 \div 6 = ?$ | Ans. ________ |
| 346. | $7.5 \div 2.5 = ?$ | Ans. ________ |
| 347. | $8.4 \div 2.4 = ?$ | Ans. ________ |
| 348. | $28.7 \div 7.0 = ?$ | Ans. ________ |
| 349. | $8.38 \div 2.16 = ?$ | Ans. ________ |
| 350. | $58.9 \div 6.23 = ?$ | Ans. ________ |

**Placing the Decimal Point When Using the Slide Rule**

Let us consider the following problems:

| | |
|---|---|
| (a) $261 \times 3.43 = ?$ | (d) $261 \times .343 = ?$ |
| (b) $.261 \times 3.43 = ?$ | (e) $2.61 \times 3.43 = ?$ |
| (c) $261 \times 343 = ?$ | (f) $.0261 \times .0343 = ?$ |

As far as the slide rule work is concerned, all of these problems are solved identically. The left index of scale $C$ is set at 261 of the $D$ scale, (i.e. prime 2, secondary 6, subdivision 1), and the product is read on the $D$ scale opposite 343 (i.e. prime 3, secondary 4, and subdivision 3) on the $C$ scale. The result is read as prime 8, secondary 9, subdivision 5 (or merely eight, nine, five). But does this mean 8.95, 895000, 895, or what? Evidently it is different for each of the 6 problems.

The method of arriving at the correct location of the decimal point is to make a rough calculation, following the slide rule work, using round numbers. For instance, in example (a), approximate 3.43 as 3 and 261 as 300. Then, $3 \times 300$ is mentally determined as 900, and the correct answer to problem (a) is 895 because that is nearer 900 than either 89.5 or 8950.

Similarly, the answers to the other problems of the group are:

(b) .895 (d) 89.5
(c) 89,500 (e) 8.95
(f) .000895

*Example:*

Divide 824 by 26.

*Solution:*

1. Set hairline at 824 on $D$ scale.
2. Move slide so that 26 on $C$ scale is under the hairline.
3. Read, opposite $LC$, 317 on $D$ scale.

To find the position of the decimal point, approximate 824 as 800 and 26 as 20.

Then,

$$800 \div 20 = 40$$

Therefore, the correct quotient = 31.7, since this is nearer to 40 than 317 or 3.17.

*Example:*

Multiply $.0423 \times 1.444$.

*Solution:*

$LC$ opposite 1444 on $D$ scale



Indicator hairline $\begin{cases} \text{423 on } C \text{ scale} \\ \text{611 on } D \text{ scale} \end{cases}$

To find the position of the decimal point

$$.04 \times 1.0 = .04$$

Therefore, the correct product = .0611

**Which Index Should You Use?**

Suppose you wish to multiply 362 by 458. If you set $LC$ at $D$ 362, $C$ 458 would come off the right end of the $D$ scale. In such a case the $RC$ will have to be set at $D$ 362 to get the desired result. It isn't always possible to tell beforehand which is the proper index to use, but the following rule will be found very helpful in most cases:

*RULE: If the product of the first digits of the given factors is less than 10, use the left hand index; otherwise, use the right hand index.*

To illustrate the above rule, consider $2.36 \times 1.45$. Then, $2 \times 1$ is *less* than 10 and the *left* index should be used. In multiplying $3.34 \times 5.14$, $3 \times 5$ is *greater* than 10 and the *right* index is used.

You will find exceptions to this rule, such as $3.43 \times 3.12$, where $3 \times 3$ is less than 10 but where the actual product (10.70) is greater than 10. However, you will find the rule a great time-saver in the majority of cases.

*Exercises:*

| | | |
|---|---|---|
| 351. | $1.416 \times .0625 = ?$ | Ans. ________ |
| 352. | $891 \times 45 = ?$ | Ans. ________ |
| 353. | $*14154 \times 31.2 = ?$ | Ans. ________ |
| 354. | $3.14 \times 14 = ?$ | Ans. ________ |
| 355. | $.205 \times .317 = ?$ | Ans. ________ |
| 356. | $.0023 \times .069 = ?$ | Ans. ________ |
| 357. | $81 \times 64 = ?$ | Ans. ________ |
| 358. | $649 \div 18 = ?$ | Ans. ________ |
| 359. | $.742 \div .152 = ?$ | Ans. ________ |
| 360. | $1055 \div .276 = ?$ | Ans. ________ |
| 361. | $\frac{1}{16} \times 452 = ?$ | Ans. ________ |
| 362. | $1.655 \div 455 = ?$ | Ans. ________ |

*** This should be set as 1415 since the 5th significant figure is lost on the slide rule.**

363. $1582 \times 395 = ?$ Ans. ________
364. $9852 \div 16 = ?$ Ans. ________
365. $1.212 \times 35.6 = ?$ Ans. ________
366. $48.2 \times 1.26 = ?$ Ans. ________
367. $\frac{1}{8} \times 214 = ?$ Ans. ________
368. $1.111 \times 1.005 = ?$ Ans. ________
369. $97.9 \div 42.6 = ?$ Ans. ________
370. $2.12 \div 33 = ?$ Ans. ________

All of these problems should be worked out on the slide rule and at least some of them by ordinary arithmetical multiplication. By doing some of them two ways you will be able to observe:

1. The saving in time by using the slide rule.
2. The relative degree of accuracy of the two methods.

You will find the answers to the above problems (to the number of significant figures as determined by the slide rule) on the pages from 205-209.

**Multiplication of Three or More Factors**

Suppose you had a problem, such as multiplying 3.5 by 642 by .0164. To perform this multiplication, you proceed with the first two factors as you would in your other problems, that is, you will set the *C* index at the 350 of the *D* scale and then move the indicator until the hair-line coincides with the 642 of the *C* scale. At this point, however, it is not necessary to read the product of these 2 numbers since we are only interested in the final result. Then, keeping the indicator as just set, move the *C* index until it coincides with the hairline of the indicator. Now, move the indicator to 164 on the *C* scale and the digits of the product will be found on the *D* scale under the hairline as 369.

You may determine the position of the decimal point in the usual manner of substituting approximate round numbers. Thus, $600 \times 4 \times .01 = 24$ from which you know that the final answer is 36.9 since that is closer to 24 than either 369 or 3.69.

Any number of factors can be multiplied together in a similar manner. Later, when you learn to use the *CI* scale you will learn of a still quicker method of multiplying 3 or more factors.

*Example:*

Multiply $1.35 \times 27.9 \times 22.9$



*Solution:*

Step 1. *LC* at 135

Indicator hairline at $\begin{cases} 279 \text{ on } C \\ 377 \text{ on } D \text{ (Don't read)} \end{cases}$

Step 2. *LC* at 377 (where hairline is)
Move hairline to 229 on *C*.
Read hairline at 862 on *D*.

Step 3. To determine decimal point,

$$1 \times 30 \times 20 = 600.$$

Therefore, the correct product is 862.

*Example:*

Multiply $92.4 \times 86.2 \times 4.89$

*Solution:*

Step 1. Set *RC* at 924.
Set hairline at 862 on *C*. The hairline will then be at 797 on *D*, but this value need not be read.

Step 2. Set *RC* where the hairline is, (at 797).
Move hairline to 489 on *C*.
Read, under the hairline, 390 on *D*.

Step 3. To determine the decimal point

$$100 \times 100 \times 5 = 50,000$$

Therefore the correct product is 39,000.

*Exercises:*

371. $3.14 \times 6.93 \times 42.3 = ?$ Ans. ________
372. $.147 \times .123 \times .122 = ?$ Ans. ________
373. $600 \times 425 \times 678 = ?$ Ans. ________
374. $7240 \times 2.5 \times .128 = ?$ Ans. ________
375. $19.6 \times 24.3 \times 968 = ?$ Ans. ________

**Problems Involving Both Multiplication and Division**

Problems involving both multiplication and division can be worked out on the slide rule with a tremendous saving in time over the ordinary method. In solving a problem of this type, it is not necessary to read the answer for each step when we are interested only in the final result. Take as an example:

$$\frac{840 \times 648 \times 426}{790 \times 611} = ?$$

In the above example, the long fraction line, as you already know, stands for division. This problem could be read as the product of 840 by 648 by 426 divided by the product of 790 by 611 or it could be read as 840 divided by 790 multiplied by 648 divided by 611 multiplied by 426. For slide rule calculations it is better to consider the problem as stated in the latter manner, since alternating the processes of division and multiplication on the slide rule saves time by requiring fewer settings of the index and indicator.

The problem can best be worked out in the following steps:

1. Set the indicator at 840 on *D*.
2. Move the slide until 790 on *C* coincides with the indicator hairline. (Division).
3. Move indicator to 648 on *C*. (Multiplication).
4. Move 611 on *C* to indicator line. (Division).
5. Move indicator to 426 on *C*. (Multiplication).
6. Read 480 on *D*, which is the answer, not considering the proper number of digits.

The correct number of digits must be determined by the usual method of approximation; thus,

$$\frac{800 \times 700 \times 400}{800 \times 600} = \text{about } 500.$$

Therefore, the correct answer is 480 and not 48.0 or 4800.

*Exercises:*

376. $\dfrac{248 \times 1.141}{38.3} = ?$ Ans. ________

377. $\dfrac{3.14 \times 19.11 \times 16.42}{9.87 \times 13.14} = ?$ Ans. ________

378. $\frac{1}{25} \times \frac{3}{83} \times \frac{41}{7} = ?$ Ans. ________

379. $.0034 \times \frac{7}{97} = ?$ Ans. ________

380. $\dfrac{421}{18.4} \times \dfrac{639}{1412} = ?$ Ans. ________



381. $\dfrac{181 \times 324}{16.4} = ?$ Ans. ________

382. $\dfrac{148 \times 1.24}{67.7} = ?$ Ans. ________

383. $\dfrac{.00211 \times 14.7}{18.3} = ?$ Ans. ________

### The *A* and *B* Scales and their Use

On the top of your slide rule you will find two scales labeled *A* and *B*. The *A* scale is on the body of the rule while the *B* scale is on the slide. These scales are the same as the *C* and *D* scales except that they are only half-size and that there are two of them in the same space that is occupied by one of the *C* or *D* scales.

The *A* and *B* scales can be used in the same manner as the *C* and *D* scales to perform multiplication or division. However, they will not be as accurate for the simple reason that the lengths of them are so much shorter.

The one advantage in using the *A* and *B* scales for multiplication is that it does not matter whether you use the left or right index of the *B* scale. For example, if you were to multiply 3 × 4, you could set the left index at the 3 of the left half of the *A* scale and when you set the indicator at *B*4 you will find that, instead of coming off at the end of the slide rule, as happened when you used the *C* and *D* scales, the answer 12 can be read on the right half of the *A* scale.

The most important uses for the *A* and *B* scales, however, are in the determination of squares and square roots of numbers, areas of circles, and sines and cosines of angles.

### The *CI* Scale

If you will look at your slide rule you will see between the *B* and *C* scales another scale on the sliding portion which is labeled *CI* and, after close inspection, you will realize that the scale is exactly opposite hand to the *C* scale. In other words, it is an inverted *C* scale, for which reason it is often called a *Reciprocal Scale.*

Before you can appreciate the use of such an inverted scale it is necessary for you to review what is meant by the reciprocal of a number. This was defined in the chapter on Arithmetic as a number which, when multiplied by the given number, produces 1.

For example 1/5 is the reciprocal of 5, or .4 is the reciprocal of 2.5. Notice that every reading on the *CI* scale is opposite its reciprocal on the *C* scale.

Now of what use can this reciprocal scale be? Again reviewing your study of arithmetic, you will remember that dividing by a number is the same as multiplying by its reciprocal. Furthermore, multiplying by a number is the same as dividing by its reciprocal.

These facts lead us to a *shortcut method* of multiplying several factors together, and of shortening the labor of performing a series of divisions.

### The Use of the *CI* Scale in Multiplication

To illustrate how the *CI* scale can be used to save time and labor in multiplication, consider the following:

*Example:*

$2 \times 3 \times 7 \times 16 = ?$

*Solution:*

This is the same as $2 \times 3 \div \frac{1}{7} \times 16$

1. Set *LC* opposite 2 on the *D* scale.
2. Move the indicator to 3 on the *C* scale. (Multiplication by 3)
3. Move the slide until 7 on the *CI* scale coincides with the hairline. (Division by $\frac{1}{7}$)
4. Move the hairline to 16 on the *C* scale. (Multiplication by 16)
5. Read, under the hairline, on the *D* scale, the significant figures of the product, 672.
6. The decimal point is determined in the usual way, fixing the product at 672.

You will notice that in determining this product only 4 settings were required, whereas in the orthodox method that you previously learned, 6 settings would be required.

### The Use of the *CI* Scale in Division

To illustrate the use of the *CI* scale in shortening the work involved in division, consider the following:

*Example:*

$$\frac{168}{.0123 \times 348 \times 14.3 \times .095} = ?$$



*Solution:*

This is the same as $168 \div .0123 \times \frac{1}{348} \div 14.3 \times \frac{1}{.095}$

1. Set the indicator hairline at 168 on the *D* scale.
2. Move the slide until 123 of *C* scale is under the hairline.
3. Move the indicator hairline to 348 on the *CI* scale.
4. Move the slide until 143 of *C* scale is under the hairline.
5. Move the indicator hairline to 950 on the *CI* scale.
6. Read the digits of the answer (289) under the hairline, on the *D* scale.
7. Determine the decimal point by the usual method of estimation:

$$150 \times 100 \times \tfrac{1}{300} \times \tfrac{1}{10} \times 10 = 50$$

The correct answer is 28.9

*Exercises:*

For practice in using the *CI* scale, see if you can evaluate the following expressions, using the methods just described. Check your results by the methods previously learned and note the saving in time and motion by the shortcut.

384. $2.8 \times 32.8 \times 1.615 = ?$ Ans. ________
385. $.062 \times .274 \times .0667 = ?$ Ans. ________
386. $\frac{231}{48.1 \times 3.21 \times .169} = ?$ Ans. ________
387. $\frac{1}{4} \times \frac{1}{7} \times \frac{1}{629} \times \frac{1000}{3.17} = ?$ Ans. ________
388. $10350 \times 645 \times .310 = ?$ Ans. ________
389. $0.113 \times 42.6 \times .0069 = ?$ Ans. ________
390. $\frac{1000}{2.34} \times \frac{1}{168 \times 414} = ?$ Ans. ________
391. $9.8 \times 23.4 \times .643 = ?$ Ans. ________
392. $1.37 \times 3.14 \times 8.42 \times 6.48 = ?$ Ans. ________
393. $4320 \times .0126 \times 2.25 = ?$ Ans. ________

## POWERS AND ROOTS WITH THE SLIDE RULE

### How to Find the Square of a Number

If a number is multiplied by itself, the product is said to be the *Square* of the number. The operation of squaring a number is indicated by a small number 2 to the upper right of the number to be squared, known as the *exponent* of the number. Thus, we can write:

$$2^2 = 4 \qquad 3^2 = 9 \qquad 4^2 = 16$$

It is always possible to square a number using the $C$ and $D$ scales in the usual manner for multiplication, as follows:

*Example:*

$2.45^2 = ?$

*Solution:*

1. Set $LC$ at 245 on the $D$ scale.
2. Set the indicator at 2.45 on the $C$ scale.
3. Read on the $D$ scale, under the hairline, the significant figures of the answer, 600.
4. As a rough calculation for determining the decimal point,

$$2^2 = 4.$$

Hence the correct square is 6.00. (Note that in this case, the two zeros after the decimal point are significant).

The squaring of a number can be done more rapidly, however, even though a little less accurately, by the use of the $A$ scale in conjunction with the $D$ scale.

*RULE: To square a number, set the indicator hairline to the number on the D scale, and read the square under the hairline on the A scale.*

*Example:*

$2.45^2 = ?$

*Solution:*

1. Set the hairline at 245 on the $D$ scale.
2. Read, on the $A$ scale, under the hairline, the digits 600.
3. By rough calculation, the decimal place is fixed behind the 6, giving 6.00 as the correct square of 2.45.

*Example:*

$4.36^2 = ?$



*Solution:*

1. Set the hairline at 436 on the $D$ scale.
2. Read, on the $A$ scale, under the hairline, the digits 190.
3. Knowing that $4^2 = 16$, the correct answer is determined to be 19.0.

*Example:*

Find the square of 268

*Solution:*

$$\text{Indicator hairline at} \begin{cases} 268 \text{ on } D \text{ scale} \\ 718 \text{ on } A \text{ scale} \end{cases}$$

To determine the decimal point,

$$300^2 = 90{,}000$$

Therefore, the correct answer is 71,800.

If you have difficulty in determining the decimal point by estimation, you may resort to the following:

*RULE: The number of places to the left of the decimal point in the answer is equal to TWICE the number of places before the decimal in the original number if the answer is found on the RIGHT HALF of the A scale.*

*If the answer is found in the LEFT HALF of the A scale, then the number of places to the left of the decimal point is equal to TWICE THE NUMBER OF PLACES IN THE ORIGINAL NUMBER, MINUS 1.*

In finding $(4.36)^2$, the answer was found on the *right half* of the $A$ scale and therefore had $2 \times 1 = 2$ places in front of the decimal.

In finding $(2.45)^2$, the answer was found on the *left half* of the $A$ scale and therefore had $2 \times 1 - 1 = 1$ place in front of the decimal.

In finding $(268)^2$, the answer was found on the *left half* of the $A$ scale and therefore would have $3 \times 2 - 1 = 5$ places in front of the decimal.

If there are no places to the left of the decimal point, the following rule can be used to determine the position of the decimal point with respect to the first significant figure:

*RULE: If the square is found on the RIGHT HALF of the A* *scale, then the number of zeros between the decimal point and the first significant figure equals TWICE the number of zeros between the decimal point and the first significant figure of the original number.*

*If the answer appears on the LEFT HALF of the A scale, then the number of zeros is GREATER BY 1 than that determined by the preceding rule.*

*Example:*

$(.0451)^2 = ?$

*Solution:*

$$\text{Indicator hairline at} \begin{cases} 451 \text{ on } D \text{ scale} \\ 203 \text{ on } A \text{ scale} \end{cases}$$

By application of the rule, since the answer is found on the right half of the *A* scale, the number of zeros between the decimal point and the first significant figure is $2 \times 1 = 2$. Hence the correct square is .00203.

*Example:*

$(.31)^2 = ?$

*Solution:*

$$\text{Indicator hairline at} \begin{cases} 310 \text{ on } D \text{ scale} \\ 961 \text{ on } A \text{ scale} \end{cases}$$

The square falls on the left side of the *A* scale, hence the number of zeros between the decimal point and the first significant figure is $2 \times 0 + 1 = 1$. Hence,

$$(0.31)^2 = 0.0961.$$

**How to Find the Square Root of a Number**

The square root of a number is that factor which, when multiplied by itself, will give you the number. The process of finding the square root of a number is indicated by the radical sign $\sqrt{\ }$ over the number.

Thus $\sqrt{9}$ means the square root of 9. Now the number which when squared gives 9, is 3.

Therefore $\sqrt{9} = 3$.



Likewise $\sqrt{36} = 6$, because $6^2 = 36$.

Another, but less common, way of indicating this operation is by showing the exponent as the fraction $\frac{1}{2}$.

Thus

$$(9)^{\frac{1}{2}} = \sqrt{9} = 3$$

and

$$(16)^{\frac{1}{2}} = \sqrt{16} = 4.$$

The procedure of finding the square root on the slide rule is the *reverse of finding the square.*

*RULE: To find the square root of a number, set the indicator hairline at the number on the A scale, and read the square root under the hairline on the D scale.*

But now you notice 2 complete scales on *A* while there is only one at *D*. Which scale will you use? To answer this, you must learn the following:

*RULE 1: If the number of places to the left of the decimal point is EVEN, then the RIGHT HALF of the A scale should be used.*

*RULE 2: If the number of places to the left of the decimal point is ODD, then the LEFT HALF of the A scale should be used.*

*RULE 3: If there are NO PLACES to the left of the decimal point, then count the number of zeros between the decimal point and the first significant figure.*

(a) *If this number of zeros is ODD, then use the LEFT HALF of the A scale.*

(b) *If the number of zeros is EVEN, then use the RIGHT HALF of the A scale.*

*Example:*

Find the square root of 68.4

*Solution:*

From Rule 1, set hairline on 684 of the right half of the *A* scale.
Read under hairline 827 on the *D* scale.
Estimating the decimal point,

$$\sqrt{64} = 8.$$

Therefore, the correct root is 8.27.

*Example:*

Find the square root of 7.51.

*Solution:*

From Rule 2, set the hairline at 751 of the left half of the *A* scale.

Read under the hairline 274 on the *D* scale.

Estimating the decimal point,

$$\sqrt{9} = 3.$$

Therefore the correct root is 2.74.

*Example:*

Find the square root of .00751

*Solution:*

From Rule 3(b) set the hairline on 751 of the right halt of the *A* scale.

Read under the hairline 867 on the *D* scale.

Estimating the decimal point,

$$\sqrt{.0081} = .09.$$

Therefore the correct root is .0867.

*Exercises:*

394. $(6.24)^2 = ?$ Ans. ________
395. $(14.18)^2 = ?$ Ans. ________
396. $(.0148)^2 = ?$ Ans. ________
397. $(.00378)^2 = ?$ Ans. ________
398. $\sqrt{81.6} = ?$ Ans. ________
399. $\sqrt{.0000931} = ?$ Ans. ________
400. $\sqrt{357} = ?$ Ans. ________
401. $\sqrt{.000683} = ?$ Ans. ________

**Finding the Area of a Circle**

A very important use for the *A* and *B* scales is in determining the area of a circle.

In your study of Mensuration you learned the following formula for finding the area of a circle:



$$\text{Area} = \pi r^2 = 3.1416 \times r^2 = .7854 \times d^2$$

where $r$ = the radius
and $d$ = the diameter.

Although the value of $\pi$ is ordinarily expressed as 3.1416, in practical calculations it is sufficient to consider this as 3.14. Likewise the constant .7854 may be taken as .785 for ordinary slide rule calculations.

Both the constants 3.14 and .785 have special markings on the *B* scale of most slide rules. You are advised to scratch these graduations in on your slide rule if they do not already appear.

The following examples will illustrate the use of the *A* and *B* scales in finding areas of circles.

*Example:*

Find the area of a circle whose radius is 2.58 feet

*Solution:*

1. Set *LC* at 258 on the *D* scale.
2. Move hairline to 314 on the *B* scale.
3. Read under the hairline 209 on the *A* scale.
4. Estimating the decimal point

$$(3)^2 \times 3 = 27$$

and the correct area is 20.9 square feet.

Note that at the end of step 1, the left index of the *B* scale is at the square of 258 on the *A* scale. It is not necessary, however, to stop here to read the square. Instead you proceed to multiply this square by 314.

*Example:*

Find the area of a circle whose diameter is 3.58 feet.

*Solution:*

1. Set *RC* at 358 on *D* scale.
2. Move hairline to 785 on *B* scale.
3. Read under hairline 1007 on *A* scale.
4. Estimating the decimal point, the area is 10.07 square feet.

*Exercises:*

402. What is the area of a circle whose diameter is 28 feet? Ans. ________

403. What is the area of a circle whose radius is 7.2 feet? Ans. ________

404. What must be the diameter of a pipe to have a cross-sectional area of 50 square feet? Ans. ________

## How to Find the Cube of a Number

The cube of a number is the product of the number multiplied by itself twice. In other words, it is the product of a number taken 3 times as a factor. The cube is represented by the exponent 3. Thus,

$$3^3 = 3 \times 3 \times 3 = 27$$

There are two methods for finding the cube of a number; one for slide rules without the *K* scale and one involving the use of the *K* scale.

*Method I* (without the *K* scale):

1. Set the index (*LC* or *RC*) at the number on the *D* scale.
2. Set the indicator at the number on the *B* scale.
3. Read the cube under the hairline on the *A* scale.

*Example:*

Find the cube of 14.2.

*Solution:*

1. Set *LC* at 142 on the *D* scale.
2. Set hairline at 142 on the left *B* scale.
3. Read 286 on the *A* scale under the hairline.
4. To locate the decimal,

$$(10)^3 = 1000.$$

Therefore, the correct cube is 2,860.

*Example:*

Find the number of cubic feet in a box whose dimensions are 8.58 ft. × 8.58 ft. × 8.58 ft.

*Solution:*

To do this we find the cube of 8.58.

1. Set *RC* at 858 on the *D* scale.
2. Set hairline at 858 on the right *B* scale.



3. Read 632 on the *A* scale.
4. To locate the decimal,

$$(10)^3 = 1000.$$

Therefore, the correct cube is 632.

*Method II* (using *K* scale):

Most slides have a *K* scale underneath the *D* scale. This is similar to the other scales except that there are 3 such scales within the space occupied by one *D* or *C* scale. This *K* scale indicates the cube of the numbers on the *D* scale.

The decimal point of the cube may be obtained by estimation or by application of the following:

*RULES: 1. If the cube falls in the RIGHT HAND THIRD of the K scale, then the number of places to the left of the decimal point will be THREE TIMES that of a given number.*

*2. If the cube falls in the MIDDLE THIRD of the K scale, then the number of places to the left of the decimal point is equal to THREE TIMES that of the given figure, MINUS 1.*

*3. If the cube falls in the LEFT HAND THIRD of the K scale, then the number of places to the left of the decimal point, is equal to THREE TIMES that of the given figure, MINUS 2.*

When there are no digits to the left of the decimal point in the given number, apply the following:

*RULES: 4. When the cube falls in the RIGHT HAND THIRD of the K scale, the number of zeros between the decimal point and the first significant figure is equal to THREE TIMES that of the given figure.*

*5. When the cube falls within the MIDDLE THIRD of the K scale, the number of zeros between the decimal point and the first significant figure is equal to THREE TIMES that of the given figure, PLUS 1.*

*6. When the cube falls on the LEFT HAND THIRD of the K scale, the number of zeros between the decimal point and the first significant figure is equal to THREE TIMES that of the given figure, PLUS 2.*

*Example:*

Find the cube of 3.44

*Solution:*

1. Set indicator at 344 on the *D* scale.
2. Read 407 on the middle third of the *K* scale.
3. Applying Rule 2, there are $3 \times 1 - 1 = 2$ places before the decimal point in the correct answer, which is therefore 40.7.

*Example:*

Find the cube of 14.2

*Solution:*

1. Set indicator at 142 on the *D* scale.
2. Read 286 on the left hand third of the *K* scale.
3. Applying Rule 3, there are $3 \times 2 - 2 = 4$ places before the decimal point in the correct answer, which is therefore 2860.

*Example:*

Find the cube of .0858.

*Solution:*

1. Set indicator at 858 on the *D* scale.
2. Read 632 on the right hand third of the *K* scale.
3. Applying Rule 4, there are $3 \times 1 = 3$ zeros between the decimal point and the first significant figure of the answer.

The correct answer is therefore .000632.

*Exercises:*

In the following problems, it would be advisable for you to solve first by one method and then check by the other.

405. $(16.38)^3 = ?$ Ans. ________
406. $(.0029)^3 = ?$ Ans. ________
407. $(6.23)^3 = ?$ Ans. ________
408. $(.435)^3 = ?$ Ans. ________
409. $(186)^3 = ?$ Ans. ________
410. $(.764)^3 = ?$ Ans. ________
411. Find the number of cubic feet in a 3 ft. 8 inch cube. Ans. ________

**Finding the Cube Root of a Number**

The cube root of a number is that quantity which, when raised



to the third power, produces the number. Thus, 3 is the cube root of 27, since

$$3^3 = 3 \times 3 \times 3 = 27$$

The cube root is indicated by a very small 3 in the upper portion of the groove of the radical sign. For example

$$\sqrt[3]{27} = 3$$

Sometimes, the cube is written by means of a fractional exponent; for example,

$$(27)^{\frac{1}{3}} = 3$$

The process of finding the cube root is the reverse of finding the cube. There are two methods of finding the cube root on a slide rule, one with and one without the *K* scale.

*Method I* (using *K* scale):

You will now recall that to find the cube of a number, all you had to do was to set the indicator at the number on the *D* scale and read the cube on the *K* scale.

Likewise, to find the cube root of a number, you may set the indicator at the number on the *K* scale and read the root on the *D* scale.

But now you note that the *K* scale is a triple scale. Which third of the *K* scale are you going to use? To decide this, mark off the given number into periods of three digits each, beginning from the decimal point to the right and to the left.

Case I—If the first period has one significant digit, use the left hand third of the *K* scale.

Case II—If the first period has 2 significant digits, use the middle third of the *K* scale.

Case III—If the first period has 3 significant digits, use the right hand third of the *K* scale.

*Example:*

Find the cube root of 2.92.

*Solution:*

1. Marked off into periods of 3 digits each, 2.92 becomes 002′920′000.*

2. Since there is only 1 significant figure in the first period, this falls under case I.

3. Set the indicator hairline at 292 on the left hand third of the *K* scale and read the root digits 143 on the *D* scale under the hairline.

4. Since there is one period before the decimal point, there will be one digit before the decimal point in the root. Hence the correct root is 1.43.

*Example:*

Find the cube root of .0292.

*Solution:*

1. .0292 becomes .029′200′000 (Case II).
2. Set the hairline at 292 of the middle third of the *K* scale.
3. Read the root digits 308 on the *D* scale under the hairline.

The correct root is .308 since there are no significant periods before the decimal point.

*Example:*

Find the cube root of .000000292

*Solution:*

1. This is marked off into periods as follows: .000′000′292

2. Since the first significant period contains 3 significant digits, the problem falls under Case III.

3. Set the hairline at 292 on the right hand third of the *K* scale.

4. Read the root digits 663 on the *D* scale under the hairline.

5. In the given number the third period to the right of the decimal point is the first to contain significant figures. Hence the first significant figure of the root is the third to the right of the decimal point. Hence the correct root is .00663.

* In slide rule work, three periods will generally be used since the accuracy of the slide rule is ordinarily to 3 significant figures.



*Method II* (without the *K* scale):

For those slide rules which are not equipped with *K* scales, the following procedure may be employed for finding cube roots:

Beginning from the decimal point, mark off as many periods of 3 digits each as necessary both to the right and to the left of the decimal point.

The procedure will vary from here on, depending on whether the first period contains 1, 2 or 3 significant digits, as follows:

*Case I—When there is only 1 significant figure in the first significant period:*

(1) Set the hairline at the given number on the left half of the *A* scale.

(2) Move the slide until *LC* is at the same number on the *D* scale as is under the hairline on the *left portion* of the *B* scale.

(3) Place the decimal point of the root so that the first significant digit corresponds to the first significant period of the given number.

*Case II—When there are two significant digits in the first period:*

(1) Set the hairline at the given number on the right half of the *A* scale.

(2) Same as case I.

(3) Same as case I.

*Case III—When there are three significant digits in the first period:*

(1) Same as case I.

(2) Move the slide until *RC* is opposite the same number on the *D* scale as is under the hairline on the *left portion* of the *B* scale.

(3) Same as case I.

*Example:*

Find the cube root of 2.92.

*Solution:*

(1) Divided into periods of three digits each, 2.92 becomes 002.′920′000. Since there is only 1 significant figure in the first period, this falls under Case I.

(2) Set hairline at 292 on *left half* of *A* scale.

(3) Move the slide until *LC* is at the same number on the *D* scale as is under the hairline on the *left half* of the *B* scale; namely, 143.

(4) Since there is one significant period before the decimal point

in the given number, there will be one significant figure before the decimal point in the root. Hence the correct answer is 1.43.

*Example:*

Find the cube root of .0000292.

*Solution:*

(1) Divided into periods of three digits each, the given number becomes .000′029′200 and falls under Case II since there are only 2 significant figures in the first significant digit.

(2) Set the hairline at 292 on the *right half* of the *A* scale.

(3) Move the slide until *LC* is opposite the same number (308) on the *D* scale as is under the hairline on the *left half* of the *B* scale.

(4) Since the first significant period of the given number is the second period to the right of the decimal point, the first significant figure of the root is in the second decimal place. Hence the correct root is .0308.

*Example:*

Find the cube root of 292,000,000,000.

*Solution:*

(1) The number becomes 292′000′000′000, when divided into periods, and falls under Case III.

(2) Set the hairline at 292 on the *left half* of the *A* scale.

(3) Move the slide until *RC* is opposite the same number (663) on the *D* scale as is under the hairline on the *left portion* of the *B* scale.

(4) The correct root is 6,630 since there are 4 significant periods before the decimal point.

*Exercises:*

412. $\sqrt[3]{286} = ?$ Ans. ________
413. $\sqrt[3]{49.8} = ?$ Ans. ________
414. $\sqrt[3]{.00000849} = ?$ Ans. ________
415. $\sqrt[3]{12{,}800{,}000} = ?$ Ans. ________
416. $\sqrt[3]{.0942} = ?$ Ans. ________
417. $\sqrt[3]{614{,}000} = ?$ Ans. ________
418. $\sqrt[3]{3.87} = ?$ Ans. ________



419. $\sqrt[3]{.0073} = ?$ Ans. ________
420. $\sqrt[3]{184{,}000{,}000{,}000} = ?$ Ans. ________
421. $\sqrt[3]{.691} = ?$ Ans. ________

## SOLVING PRACTICAL PROBLEMS WITH THE SLIDE RULE

**Illustrative Problems:**

1. (For the Draftsman). A gear has 42 teeth and a diametral pitch of $1\frac{1}{4}$. What is its pitch diameter?

*Solution:*

You will find in any text on gearing that the diametral pitch is equal to the number of teeth on the gear divided by the pitch diameter.

This should be written in equation form, that is, as an equality:

$$\text{Diametral Pitch (D.P.)} = \frac{\text{Number of Teeth } (n)}{\text{Pitch Diameter (P.D.)}}$$

This the same as saying that

$$\text{Pitch Diameter (P.D.)} = \frac{n}{\text{D.P.}}$$

But since $n = 42$

and $\text{D.P.} = 1\frac{1}{4} = 1.25$

$$\text{P.D.} = \frac{42}{1.25} = 33.6 \text{ inches.}$$

2. (For the Machinist). At what speed (R.P.M.) should a $\frac{7}{8}''$ drill be run for a cutting speed of 65 feet per minute?

*Solution:*

$$\text{R.P.M.} = \frac{\text{cutting speed}}{\text{perimeter of drill}}$$

$$= \frac{65 \times 12}{.875 \times 3.14} = 284 \text{ R.P.M.}$$

3. (For the Contractor). What will be the cost of materials on a job requiring 148 cubic yards of concrete at \$9.55 per C.Y. and 51,800 Feet Board Measure of lumber at \$38.00 per thousand FBM?

*Solution:*

$$148 \times \$9.55 + 51.8 \times \$38.00 = \$1413 + \$1968 = \$3381$$

*Exercises:*

Now see if you can solve the following practical problems with the slide rule. Check your answers with those given on pages 205-209.

422. (For the Plumber). A plumber needs 64 feet of $\frac{1}{2}''$ pipe at .850 pounds per foot, 39 feet of $\frac{3}{4}''$ pipe at 1.130 lbs./ft. and 43 feet of 1″ pipe at 1.678 lbs./ft. If the pipe costs 9.4¢ per pound, what will be the total cost of the pipe? Ans. ________

423. (For the Sheet Metal Worker). What will be the total weight of 32 sq. ft. of No. 12 gage sheet metal at 4.462 pounds per sq. ft. and 48 sq. ft. of No. 16 gage at 2.55 pounds per sq. ft.? Ans. ________

424. (For the Electrician). How many kilowatt hours (K.W.H.) are required to run a D. C. motor developing 5 Horse Power for 19 hours if the efficiency of the motor is 85%? (Note that 1 H.P. = .746 K.W.) Ans. ________

Hint: K.W.H. = $\frac{100}{85}(5 \times 19 \times .746)$

425. (For the Carpenter). How many board feet (F. B. M.) in 69 boards 3 inches thick, 8 inches wide, and 22 feet 0 inches long? Ans. ________

426. (For the Ordnance Man). What is the diameter in inches of the bore of a 75mm. gun? Ans. ________

427. (For the Welder). What will be the cost of electrodes for 438 feet of a weld between sheets of 10 gauge metal, if .051 pounds of electrode are required per foot of weld, and electrodes cost \$.095 per pound? Ans. ________

428. (For the Machine Designer). According to the American Standards Association, a Medium Fit (Class 3) should have an allowance of 0.0009 $\sqrt[3]{d^2}$ between hole and shaft, where $d$ is the diameter of the shaft. For a $4\frac{15}{16}$ inch shaft, what should be the allowance? Ans. ________

## HOW TO USE THE UNDERSIDE OF THE SLIDE

### The Scales on the Underside of the Slide

On the reverse side of the slide of many slide rules will be found three scales (Fig. 51): the sine scale (marked *S* and found at the



upper edge); the tangent scale (marked *T* and found at the lower edge); and the logarithm scale (found in the middle).

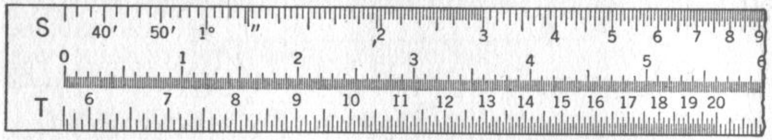


Fig. 51

### How to Read the Sine Scale

The sine scale includes angles from 0° 34′ 23″ to 90°. Between 34′ 23″ and 3°, you can read directly to the nearest 2 minutes and you can approximate to closer values, as shown in Fig. 52.

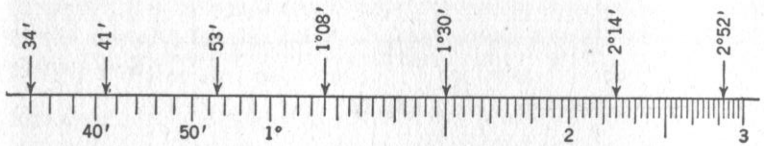


Fig. 52

Between 3° and 10°, you can read directly to the nearest 5 minutes as shown in Fig. 53.

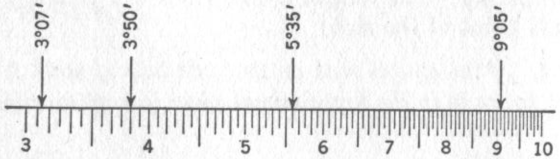


Fig. 53

Between 10° and 20°, each degree is divided into six parts of 10 minutes each (see Fig. 54).

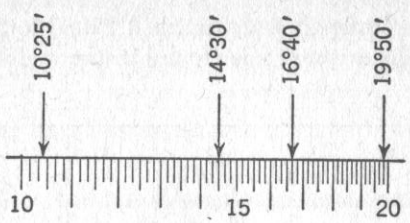


Fig. 54

Between 20° and 40°, the nearest 30′ can be read and between 40° and 70°, the nearest degree. The last line to the right represents 90° and the next to the last 80°. There are five divisions between 70° and 80°, each representing 2°. Fig. 55 demonstrates the reading of the right end of the sine scale.

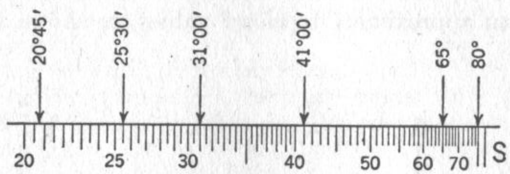


Fig. 55

### How to Find the Sine of an Angle

Set the slide (with the sine scale face up) in the slide rule proper so that the mark at *S* is opposite *RA* and the left index of the *S* scale (representing 0° 34′ 23″) lines up with *LA*.

To find the sine of an angle, set the indicator hairline at the given angle on the *S* scale. The sine is then read on the *A* scale under the hairline. The following two rules will enable you to fix the decimal point of the sine:

*RULE 1: If the sine is read on the right halj of scale A, the first significant figure is in the first decimal place, except sine 90°, which is 1.*

*RULE 2: If the sine is read on the left half of scale A, the first significant figure is in the second decimal place.*

*Example:*

Find the sine of 25° 30′.



*Solution:*

1. Set indicator at 25° 30′ on *S* scale.
2. Under the hairline, read 431 on the *A* scale.
3. Since this reading is on the right half of the *A* scale the sine is 0.431.

*Example:*

Find the sine of 1° 02′.

*Solution:*

1. Set the indicator at 1° 02′ on the *S* scale.
2. Under the hairline, read 180 on the *A* scale.
3. Since this reading is on the left half of the *A* scale, the sine is 0.0180.

### Finding the Cosine of an Angle

The cosine of an angle cannot be read directly on a slide rule. However, if it is remembered that the cosine is equal to the sine of the complement of the angle, a ready means for determining cosines is seen.

*Example:*

Find the cosine of 28°.

*Solution:*

1. Cos 28° = sin (90° − 28°) = sin 62°.
2. Set the indicator at 62° on the *S* scale.
3. Under the hairline, read 883 on the *A* scale.
4. Since this reading is on the right half of the *A* scale, the cosine of 28° = 0.883.

*Exercises:*

429. Find the sine of the following angles:

(a) 30° Ans. ________
(b) 26° Ans. ________
(c) 21° Ans. ________
(d) 1°04′ Ans. ________
(e) 6 30′ Ans. ________
(f) 11° Ans. ________
(g) 20°30′ Ans. ________

(h) 66° Ans. ________
(i) 80° Ans. ________
(j) 4°25′ Ans. ________

430. Find the cosine of the following angles:
(a) 68° Ans. ________
(b) 45° Ans. ________
(c) 89°02′ Ans. ________
(d) 76°05′ Ans. ________
(e) 83° Ans. ________
(f) 44° Ans. ________
(g) 56°30′ Ans. ________
(h) 14° Ans. ________
(i) 80°05′ Ans. ________
(j) 33° Ans. ________

### How to Find an Angle, Given its Sine or Cosine

When the sine is given, the indicator is set at this value on the *A* scale. The angle is read on the *S* scale.

*Example:*

Find $x$ if $\sin x = .342$

*Solution:*

1. Set indicator at 342 on the *right half* of the *A* scale.
2. Read the value of $x$ as 20° 00′ on the *S* scale under the hairline.

*Example:*

Find $x$ if $\sin x = .0407$

*Solution:*

1. Set the indicator at 407 on the *left half* of the *A* scale.
2. Read the value of $x$ as 2° 20′ on the *S* scale under the hairline.

When the cosine is given, proceed as before, but subtract the value of the result obtained from 90° to get the angle required.

*Example:*

Find $x$ if $\cos x = .404$

*Solution:*

1. Set the indicator at 404 on the *right half* of the *A* scale.



2. Read the value 23° 50′ on the *S* scale under the hairline.
3. The complement of 23° 50′ is 66° 10′, which is the required angle.

*Exercises:*

431. Find $x$ if
(a) $\sin x = 0.172$ Ans. ________
(b) $\sin x = 0.0116$ Ans. ________
(c) $\sin x = 0.658$ Ans. ________
(d) $\sin x = 0.866$ Ans. ________
(e) $\sin x = 0.500$ Ans. ________
(f) $\cos x = 0.969$ Ans. ________
(g) $\cos x = 0.191$ Ans. ________
(h) $\cos x = 0.334$ Ans. ________
(i) $\cos x = 0.719$ Ans. ________
(j) $\cos x = 0.638$ Ans. ________

### How to Multiply a Number by the Sine of an Angle

Here the procedure is as follows:

1. Set *RS* (or *LS*) at the given number on *A*.
2. Set the indicator at the given angle on the *S* scale.
3. Read the digits of the product on the *A* scale under the hairline.
4. Determine the decimal point by rough calculation.

*Example:*

Find the value of $30 \times \sin 2° 40′$.

*Solution:*

1. Set *RS* at 300 on the *A* scale.
2. Set the indicator at 2° 40′ on the *S* scale.
3. Read 1395 on the *A* scale.
4. To place the decimal point, note that sin 2° 40′ is a trifle less than .05. Then $30 \times .05 = 1.50$ for the rough calculation, and the correct product is 1.395.

### How to Divide a Number by the Sine of an Angle

Here the procedure is as follows:

1. Set the indicator at the given number on the *A* scale.
2. Move the slide until the given angle on the *S* scale is under the hairline.

3. Read the digits of the quotient on the $A$ scale opposite the index of the $S$ scale.
4. Determine the decimal point by estimation.

*Example:*

Find the value of $\frac{30}{\sin 2° 40'}$

*Solution:*

1. Set indicator at 300 on the $A$ scale.
2. Move the slide until 2° 40′ on the $S$ scale is under the hairline.
3. Read 645 on the $A$ scale at $LS$.
4. To locate the decimal point, note that $\frac{30}{.05} = 600$. Hence the correct quotient is 645.

*Exercises:*

432. $0.329 \times \sin 20°10' = ?$ Ans. ________
433. $425 \times \sin 35° = ?$ Ans. ________
434. $18.2 \times \sin 1°26' = ?$ Ans. ________
435. $72.2 \times \cos 77°40' = ?$ Ans. ________
436. $1.42 \times \cos 35° = ?$ Ans. ________
437. $\frac{22.5}{\sin 42°} = ?$ Ans. ________
438. $\frac{18.4}{\sin 12°20'} = ?$ Ans. ________
439. $\frac{72.2}{\cos 78°} = ?$ Ans. ________
440. $\frac{72.2}{\sin 78°} = ?$ Ans. ________
441. $\frac{0.0543}{\sin 0°50'} = ?$ Ans. ________

**The Tangent Scale**

The scale at the lower edge of the reversed slide is marked $T$ and is known as the tangent scale. This scale has values from 5° 43′ to 45°. Between its left index and 20°, it can be read directly to the nearest 5 minutes. From 20° to 45° the smallest division is a 10 minute unit.



**How to Find the Tangent of an Angle 45° or Less**

With the slide in position so that $RT$ is opposite $RD$, scale $T$ gives readings for angles whose tangents are found opposite on scale $D$. For all values of tangents found on the rule, the first significant figure comes in the first decimal place. The following examples will illustrate how the tangents of angles are found:

*Example:*

Find tan 26°.

*Solution* (Method I):

1. Set slide with $RT$ at $RD$.
2. Move indicator to 26° on $T$ scale.
3. Read 488 on the $D$ scale under the hairline.
4. The required tangent is .488.

*Solution* (Method II):

In this method the slide is inserted into the rule so that the $B$, $C$ and $CI$ scales face up and the $S$ and $T$ scales face down, with the $T$ scale being read from the back. The slide is set so that the 26° reading coincides with a special marker on the back of the rule directly opposite the $RD$ index.

The tangent .488 is read on the $C$ scale opposite the right index of the $D$ scale.

**How to Multiply by the Tangent of an Angle 45° or Less**

*Example:*

Multiply tan 26° × 32.3.

*Solution:*

1. Find tan 26° by method II given above.
2. Move the indicator to 323 on the $D$ scale and read 1575 on the $C$ scale.
3. Since the tangent of 26° is approximately ½, the correct product is 15.75.

*Note:* If in step 2 above, the hairline is to the left of $LC$, the slide must be moved so that the tangent on the $C$ scale is opposite $LD$ instead of $RD$.

*Example:*

Multiply tan 26° × 16.4.

*Solution:*

1. Find tan 26° (= .488) by method II.
2. Move the slide until 488 on the $C$ scale appears opposite $LD$.
3. Move the indicator to 164 on the $D$ scale and read 800 on the $C$ scale.
4. The decimal point is located by estimation, giving the correct product as 8.00.

**The Method for Finding the Tangent of an Angle Greater than 45°**

Although the tangent scale goes only to 45°, we can find the tangent of a larger angle from the relationship

$$\tan A = \frac{1}{\tan (90° - A)}$$

*Example:*

Find the tangent of 70°.

*Solution:*

1. $70° - \frac{1}{\tan (90° - 70°)} = \frac{1}{\tan 20°}$.
2. With the $T$ scale face up, move the slide until 20° is opposite $RD$.
3. Opposite $LT$, read 275 on the $D$ scale.
4. Since tan 20° is approximately .4, the rough calculation for tan 70° is $.\frac{1}{4}$ = 2.5. Therefore the correct value of tan 70° is 2.75.

**Dividing by the Tangent of an Angle Greater than 45°**

Here the problem reduces to dividing by the reciprocal of the tangent of the complement of the angle; or, in other words, multiplying by the tangent of the complement. Once the complement is found, the procedure is the same as in the article "How to Multiply by the Tangent of an Angle 45° or Less."

**How to Divide by the Tangent of an Angle 45° or Less**

Here the procedure is

1. Set the indicator at the value of the dividend on the $D$ scale.



2. Move the slide (with $T$ scale face up) until the given angle on the $T$ scale appears under the hairline.
3. Read the quotient on the $D$ scale opposite the $LT$ or $RT$ index.
4. Determine the decimal point by rough calculation.

*Example:*

Divide 6.24 by tan 20° 30′.

*Solution:*

1. Set the indicator hairline at 624 on the $D$ scale.
2. Move the slide until 20° 30′ is under the hairline.
3. Read opposite $LT$ the digits 167 on the $D$ scale.
4. Since tan 20° is approximately .4, and $\frac{6}{.4} = 15$, the correct quotient is 16.7.

**Multiplying by the Tangent of an Angle Greater than 45°**

Here the problem reduces to finding the complement of the angle and dividing by this complement as in the preceding article.

**How to Find the Tangent of an Angle Less than 5° 43′**

The $T$ scale is usable only for values between 5° 43′ and 45°. Fortunately, however, the sine and the tangent of any angle below 5° 43′ are identical to three significant figures.

Therefore, if the tangent of any angle below 5° 43′ is required, determine its sine as previously described, and use the value of the sine for the tangent.

**How to Find an Angle when its Tangent is Given**

*When the tangent is between 0.1 and 1.0,* proceed as in the following:

*Example:*

Find the angle whose tangent is .423.

*Solution:*

1. Set the slide (with $T$ scale face up) so that $RT$ is opposite $RD$.
2. Set the indicator hairline over 423 (the digits of the given tangent) on the $D$ scale.

3. On the $T$ scale, read, under the hairline, the value of the required angle (22° 55′).

*When the tangent is between 1.0 and 10.0,* proceed as in the following:

*Example:*

Find the angle whose tangent is 4.23.

*Solution:*

1. With the $C$ scale face up, set the left index of $C$ at 423 on the $D$ scale.
2. On the back of the rule read at the special $RD$ marking 13° 20′.
3. The required angle is 90° − 13° 20′ = 76° 40′.

*When the tangent is greater than 10.0,* proceed as in the following:

*Example:*

Find the angle whose tangent is 14.2.

*Solution:*

1. With the $S$ scale face up, set $RS$ opposite 142 on the right half of the $A$ scale.
2. Opposite $LA$, read on the $S$ scale 4° 02′.
3. The required angle is 90° − 4° 02′ = 85° 58′.

**How to Find the Sine or Tangent of an Angle under 34′ 23″**

The readings of the sine scale begin at 34′ 23″. It may on some rare occasion be required to find the sine or tangent of an angle smaller than this. Special marks are provided on the $S$ scale for this purpose. Just to the left of the 2° division is the "minute mark," and near the 1° 10′ division is a "second mark." By setting either of these marks opposite any number on the $A$ scale, the corresponding number of minutes or seconds is read on scale $A$ opposite the index of the $S$ scale.

The decimal point is placed by remembering that

$$\sin 1' = 0.0003$$
$$\sin 1'' = 0.000005$$



Since the tangents and sines of such small angles are practically identical, it goes without saying that the same procedure is followed for finding the tangents of these angles.

*Example:*

Find the sine of 20′.

*Solution:*

1. Opposite 200 on the right half (or left half) of scale $A$, set the "minute mark" of scale $S$.
2. On scale $A$ read 582 opposite $LS$. (If the left half of scale $A$ is used in step 1, the 582 will be opposite $RS$).
3. Since sin 1′ = .0003, sin 20′ is roughly .006. Therefore, the correct value of sin 20° is .00582.

*Example:*

Find the sine of 20″.

*Solution:*

1. Opposite 20 on scale $A$, set the "second mark."
2. On scale $A$ read 969 opposite the index of the $S$ scale.
3. Since 1″ = .000005, sin 20″ is roughly .0001. Therefore, the correct value of sin 20″ is .0000969.

The reverse of the process just indicated can be used for finding an angle, when its sine or tangent is given as less than 0.01.

*Example:*

Find the angle whose sine (or tangent) is 0.00528.

*Solution* (Method I):

1. Set the index of the $S$ scale opposite 528 on the $A$ scale.
2. Move the indicator hairline to the "minute mark" on $S$.
3. Read under the hairline, on the $A$ scale, the digits 182.
4. Since 1 minute = about .0003, a rough calculation gives

$$\frac{.006}{.0003} = 20$$

5. Therefore the correct number of minutes = 18.2 (equal to 18′ 10″).

*Solution* (Method II):

1. Same as Method I.
2. Move the indicator to the "second mark" on *S*.
3. Read, under the hairline, on the *A* scale, the digits 109.
4. Since one second = about .000005, a rough calculation gives

$$\frac{.005}{.000005} = 1000$$

5. Therefore the correct number of seconds = 1090 (equal to 18′ 10″).

*Exercises:*

442. Find the tangents of the following angles:

(a) 44° Ans. ________
(b) 25°40′ Ans. ________
(c) 19°45′ Ans. ________
(d) 6°25′ Ans. ________
(e) 3°10′ Ans. ________
(f) 83°55′ Ans. ________
(g) 87°10′ Ans. ________
(h) 49°20′ Ans. ________
(i) 80°30′ Ans. ________
(j) 58°30′ Ans. ________
(k) 0°23′ Ans. ________
(l) 0°00′45″ Ans. ________

443. Find the angles corresponding to the following tangents:

(a) 0.367 Ans. ________
(b) 0.593 Ans. ________
(c) 0.966 Ans. ________
(d) 0.105 Ans. ________
(e) 1.43 Ans. ________
(f) 2.34 Ans. ________
(g) 4.51 Ans. ________
(h) 17.17 Ans. ________

444. Evaluate the following:

(a) 18.5 × tan 20° = ? Ans. ________
(b) 24.2 × tan 26°10′ = ? Ans. ________
(c) 46.2 ÷ tan 80°30′ = ? Ans. ________
(d) 172 ÷ tan 72°30′ = ? Ans. ________
(e) 425 × tan 56° = ? Ans. ________
(f) 3.21 × tan 87° = ? Ans. ________



(g) 42.3 ÷ tan 13°10′ = ? Ans. ________
(h) 17.4 ÷ tan 2°20′ = ? Ans. ________

After you become proficient in finding sines, cosines, and tangents on the slide rule, and in using these values as factors, dividends or divisors, it would be well for you to try the exercises given in the chapter on Trigonometry in Part I. You will find that you can get results with the slide rule in a small fraction of the time consumed either with longhand multiplication and division, or with logarithms.

In checking your slide rule results of trigonometric problems, however, with the answers given in the back of this book, you can only expect the first three significant figures of a quantity to agree. As was pointed out before, however, this is sufficiently precise for many practical problems.

## Logarithms on the Slide Rule

Between the *T* and *S* scales will be found a scale of equal parts, called the logarithm scale, by means of which the logarithm of a number may be found.

The following examples will illustrate how to use this scale in determining logarithms and in finding powers and roots.

*Example:*

Find log 230.

*Solution:*

1. Set the slide so that *RT* is opposite *RD*.
2. Set the indicator at 230 on the *D* scale.
3. Read under the hairline 362 on the logarithm scale.
4. This 362 is the mantissa. You will recall that the characteristic is one less than the number of figures to the left of the decimal point. Since 230 has three such figures, the characteristic is 2 and the logarithm of 230 is 2.362.

*Example:*

Evaluate $(1.42)^{2.3}$.

*Solution:*

You will recall from your study on logarithms in the chapter on Algebra that

$$\log (1.42)^{2.3} = 2.3 \log 1.42.$$

From the log scale, $\log 1.42 = .152$

and $.152 \times 2.3 = .350$

The antilogarithm of $.350 = 2.24$, the required value.

*Example:*

Evaluate $\sqrt[5]{.681}$.

*Solution:*

Here $\log \sqrt[5]{.681} = \frac{1}{5} \log .681.$

From the log scale, $\log .681 = 49.833 - 50.$

Therefore, $\frac{1}{5} \log .681 = 9.967 - 10.$

Then, antilog $9.967 - 10 = 0.927$, the required value.

*Exercises:*

445. Find the logarithm of the following:

(a) 1.38 Ans. ________

(b) 26.4 Ans. ________

(c) .0243 Ans. ________

(d) 535. Ans. ________

446. Evaluate the following:

(a) $(13.2)^5$ Ans. ________

(b) $(14.8)^{3.2}$ Ans. ________

(c) $\sqrt[3]{89.4}$ Ans. ________

(d) $(139)^{4/5}$ Ans. ________

# ANSWERS

1. $\frac{7}{24}$
2. $\frac{5}{12}$
3. $\frac{3}{7}$
4. $\frac{12}{13}$
5. $\frac{3}{81}$
6. $\frac{128}{16}$
7. $\frac{27}{3}$
8. $\frac{31}{8}$
9. $\frac{19}{4}$
10. $\frac{33}{5}$
11. $3\frac{1}{2}$
12. $1\frac{4}{15}$
13. $1\frac{7}{9}$
14. $\frac{24}{64}$
15. $\frac{28}{32}$
16. $\frac{520}{1000}$
17. $\frac{7}{16}$
18. $\frac{7}{20}$
19. $\frac{1}{8}$
20. $\frac{3}{8}$
21. $2\frac{17}{40}$
22. $8\frac{5}{16}$
23. $\frac{15}{64}$
24. $\frac{39}{64}$
25. $4\frac{17}{64}$
26. $\frac{32}{63}$
27. $\frac{45}{56}$
28. $\frac{9}{64}$
29. $2\frac{9}{20}$
30. $2\frac{70}{81}$
31. 10 ft. 4 in.
32. $\frac{7}{8}$
33. $\frac{21}{32}$
34. 3.0025
35. 71.91
36. 5.7471
37. 2.333
38. .28125
39. .875
40. .6923
41. .69231
42. 36.622
43. 30.1536
44. 4.18
45. 10.532
46. 31.68
47. 10.6875 sq. ft.
48. 298.52
49. 1670.142
50. (a) 10.84%
    (b) 9.78%
51. 81
52. 125
53. 457.96
54. 881
55. 9.23
56. .745
57. 22.8
58. $2\frac{2}{3}$
59. 88.11 lbs.
60. 852 R.P.M.
61. $2\frac{1}{4}$ hours
62. $\frac{3}{14}$
63. $\frac{2}{9}$
64. $\frac{1}{3}$
65. 3
66. 396
67. 748
68. 96
69. 2450
70. 6600
71. 720
72. 19,000
73. 162
74. 169
75. 87.824
76. 269.19
77. 78,921
78. 2304
79. 9801
80. 99.6004
81. 126 qts.
82. 2700 minutes
83. $\frac{1}{3}$ mile
84. 49 cu. yds.
85. 1683 gals.
86. 9842.7 feet
87. 25.4 mm.
88. 17.01 grams
89. 43 ft. $5\frac{15}{32}$ in.
90. 4.32 cu. yds.
91. 8.38 cu. yds.
92. 18.0 lbs.
93. 7.13 lbs.
94. 17 ft. $10\frac{13}{16}$ in.
95. 32.72 lbs.
96. 5,875 gals.
97. 1.69 lbs.
98. 12
99. $\frac{3}{4}$
100. 34
101. 2
102. 5
103. 21
104. 15
105. 1
106. 48
107. 27
108. 0
109. 9
110. 5
111. −6
112. 8
113. 5
114. −8
115. 1
116. −18
117. 16
118. 16
119. −6

334. 5.1
335. 5.88
336. 2.37
337. 9.03
338. 64.7
339. 7.64
340. 21.0
341. 3
342. 9
343. 7
344. 4
345. 6
346. 3
347. 3.5
348. 4.1
349. 3.88
350. 9.45
351. .0885
352. 40,100
353. 442,000
354. 44.0
355. .0650
356. .0001587
357. 5180
358. 36.1
359. 4.88
360. 3820
361. 12.6
362. .00364
363. 625,000
364. 616
365. 43.1
366. 60.7
367. 26.7
368. 1.117
369. 2.30
370. 0.0642
371. 920
372. .00221
373. 172,900,000
374. 2320
375. 461,000
376. 7.39
377. 7.60
378. .00847
379. .000245
380. 10.36
381. 3580
382. 2.71
383. .001695
384. 148.4
385. .001133
386. 8.85
387. .0179
388. 2,070,000
389. .0332
390. .00614
391. 147
392. 235
393. 122.5
394. 38.9
395. 201
396. .000219
397. .0000143
398. 9.03
399. .00965
400. 18.9
401. .0261
402. 616 sq. ft.
403. 162.9 sq. ft.
404. 7.98 feet
405. 4,390
406. .0000000244
407. 242
408. .0823
409. 6,430,000
410. .446
411. 49.3 cu. ft.
412. 6.59
413. 3.68
414 .0204
415. 234
416. .455
417. 85.0
418. 1.57
419. .194
420. 5690
421. .884
422. $16.04
423. 265.2 lbs.
424. 83.4 KWH
425. 3,040 FBM
426. 2.95 inches
427. $2.12
428. .0026 in.
429. (a) 0.500
(b) 0.438
(c) 0.358
(d) 0.0186
(e) 0.113
(f) 0.191
(g) 0.350
(h) 0.914
(i) 0.985
(j) 0.0770
430. (a) 0.375
(b) 0.707
(c) 0.0169
(d) 0.241
(e) 0.1219
(f) 0.719
(g) 0.552
(h) 0.970
(i) 0.1722
(j) 0.839
431. (a) 9°55′
(b) 0°40′
(c) 41°10′
(d) 60°00′
(e) 30°00′
(f) 14°
(g) 79°
(h) 70°30′
(i) 44°0′
(j) 50°20′
432. 0.1135
433. 244
434. 0.455
435. 15.45
436. 1.16
437. 33.6
438. 86.1



439. 347
440. 73.8
441. 3.73
442. (a) .966
(b) .481
(c) .359
(d) .1125
(e) .0553
(f) 9.38
(g) 20.2
(h) 1.164
(i) 5.98
(j) 1.632
(k) .00669
(l) .000218
443. (a) 20°10′
(b) 30°40′
(c) 44°0′
(d) 6°00′
(e) 55°0′
(f) 66°50′
(g) 77°30′
(h) 86°40′
444. (a) 6.73
(b) 11.89
(c) 7.73
(d) 54.2
(e) 630.
(f) 61.2
(g) 181
(h) 427
445. (a) 0.1399
(b) 1.422
(c) 8.386 − 10
(d) 2.728
446. (a) 401,000
(b) 5,560
(c) 1.900
(d) 51.8