# SLIDE RULE MANUAL





MULTILOG

No. 0970

# MANUAL OF

ARISTO®
MULTILOG
SLIDE RULE
No. 970



# Front of Rule

# CONTENTS

1	. The Scales	3
2	Reading the Scales	5
3	The Principle of Slide Rule Calculation	5
4	. Multiplication	6
5	Division	7
6	. Multiplication and Division Combined	8
7	Proportion	8
8	. The Reciprocal Scales CI and CIF	9
9	. Solution of Examples Explained in Diagram Form	10
10	. The Folded Scales CF, DF and CIF	10
11	. The Scales A, B and K	11
	11.1 Squares and Cubes: Square Roots and Cube Roots	11
	11.2 Multiplication and Division with the Scales of Squares	12
12	. Trigonometric Functions	12
	12.1 The Sine Scale	12
	12.2 The Tangent Scale	12
	12.3 Small Angles	13
	12.4 Conversion of Degrees to Radians	14
13	. Trigonometric Solution of Plane Triangles	15
14	. The Log Log Scales LLO-LL3 and LL00-LL03	17
	14.1 The 10th, 100th and 1000th Powers and Roots	17
	14.2 Powers y = ax	17
	14.3 Exceptional Cases of $y = a^x$	19
	14.4 Powers y = e <sup>x</sup>	20
	14.5 Roots $a = \sqrt[4]{y}$	21
	14.6 Logarithms	
	14.7 Solving Proportions with the Log Log Scales	
15	Hyperbolic Functions	
	The Detachable Cursor and its Lines	
10	16.1 The Mark 36	
	16.2 The Marks for Circle Areas	
	16.3 The Marks kW and HP	
	16.4 Detaching the Cursor	
	16.5 Adjustment of the Cursor	
17	Treatment of the ARISTO Slide Rule	

# THE SLIDE RULE ARRISTO-MULTILOG

	=======================================	
	sole:	
rles	of Rules	
e Scales	Front	
The		
-		

Upper panel of body	On slide	Lower panel of body
e -0.01x	m x m x m x 1/m x 1/m x 1/x x	e <sub>0.01</sub> × × × × × × × × × × × × × × × × × × ×
Log Log Scale, Range: .999 .9135.		
LLoz		EFE
Front of Rule:		

						-					_	
						. 0					280	
	100	g-0.001x			3	⊄arc	i v		1		E 0.0011	
		à	"×	"×	× V	A	A	×	×	- 14	0	100
				0	E 0		S	=	-			
		886	30		a V		4		<b>E</b> _		E	
				54	Fee A		y		-		1.011	
		- 3	6	+3	-	0		- 3	E			100
		06		1		ø.			-		10	455
		0	-	-	-5		90	-	-	-	010	
		100	on-	4		4	-	1 1	E			
	VB	- 3		o-	-01		2					100
		166	00	- di			2	-	E .	-	600	
			-	ω-	0		30.60 20.70 80.90	- Oi-	0		0	
	96	X 45		1	-	9	90	in the	E			
	22	1	10		20 40		9			-01		22
	24	992			10						800	100
			ın-			3		00-	0		0	100
	la l			0	ω	9	20		E	m		
	200	1 2				490	97		E			200
	7.7	0		3					E			200
		993	V.	m-	55 35	4	¥.	_	E		100	
	Ť.			T T	55	86	ĕ		E		-	
	65		9					2	E	- MI		
			10			1	0.40	13	E		-	
	-57	1/5		7	2	9.0	20	1 3	E	.00	-	
		766	W		E	0		10	En		900	100
	14	-	04-	8	- 0		10	9	-			72.
		1000	2	ete	60,30	-	5,3	3	E		-	
	9.01	11/2		m -	Em.	1	NO.		E	.00	1	1
	10	345		I's	-			V-	E		-	15
	31	766 966	1	1		13		13	E	.0	=	
		66	17	-	-	0	30	10	E		900	
				10		962 25 673 35 864	9	11)	E	44	-	
		1	1	1	- 10		100	1	Lite Ce Le Ce Le	18		0.0
	NV.	1		-	65,25			1	E		4	
	W	11-	gn.	CV.	-01			4	-		-	
	50	101		Tion.		19		1	F			
	227		00-	1		64	25		F 78			
	2	966	p-	- ill			65	3			14	10
		66		- H				4	- 4		700	
	90		10-					1				933
	339										d l	
	704		w:	1	20			al al				100
		10.3	1		2			1			1	
	1			of the	19 70.20	98	20					
	A			-				0.75		- 6	-	-
	0		d.		9	-	6.	911			1 003	0
	a			-		Φ.	B	E.	E	*		0
		.997		1	1		18				03	8 I
				0	6	h.		m-	m		0	0
					9.	1						ARISTO
	3			8	-m							5
*		1 3										
				1	10		16	li i		JE.		7
	_	D. UILE		7	75 15	10	5 16	multon				Ā
				Tuni mil	75 15	5	7515 16	minimi	minulin			A
	0	, in	Laborator and the second	5 7	7 7 44 7515	5	7515 16	diministra	n lumbundu	7		AA
	0	The state of the s	The Part of the Pa	6 7	6 7 6 15 15	1.5	91 5154 71	dundanatan		7		Ø A
	0			6 7	3 6 7 7515	1.5	16 7515 16	2000		-	7	O A
	0			6 7	13 6 7	51 77	91 91 92 91 6	2000		-	7	O AI
	0			5 6 7	5 13 6 7 75 15	2. 2.	13 14 7515 16	2000		-	7	O Ar
	0			S 6 7	2 13 6 7515	1.3	13 14 7515 16	2000		-	7	A P
	0			rithining 5 6 7	12 13 6 7	12 13 14 15	12 13 14 7515 16	2000		-	7	A P
	0			distribution 5 6 7	\$154 71 Et 21 7	1.2 1.3 1.4	12 13 14 7515 16	2000	2	-	7	A P
	0			things of S 6 7	1 2 13 6 7 14 15 15 15 15 15 15 15 15 15 15 15 15 15	1.2 1.3 1.4	12 13 14 7515 16	2000	2	-	1002	A O
	0			The South of the S	2 2 6 7 7 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1.1 1.2 1.3 1.4	11 12 13 14 7515 16	2000	2 6	-	7	A O
	0			7 6 5 6 7	2 2 2 2 4 2 4 2 4 2 4 4 4 4 4 4 4 4 4 4	1.2 1.3 1.4 1.5	11 12 13 14 7515 16	2000	2 6 8	-	7	A O
	0	856"	6 7 8 9 1	T. 1 5 6 7	S152 73 61 23 74 016	1 11 12 13 14 15	83	2000	8 9 2	-	7	A O
	0	856"	6 7 8 9 1	37, 4, 4, 1, 1, 5, 6, 7	371 4 5 5 6 7	11 12 13 14 15	8010 11 12 13 14 7515 16	2000	7 8 9 2	-	7	A
	9	856"	6 7 8 9 1	37. 4. 5. 6. 7	317 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	1	83	2000	7 8 9 2	-	7	TO STATE OF THE PARTY OF THE PA
	0)	856"	6 7 8 9 1	37. 4. 5. 6. 7	317 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	12 13 17 12	83	2000	6 7 8 9 2	-	7	TO STATE OF THE PARTY OF THE PA
	9	856"	6 7 8 9 1	311 4 5 6 7	3.7. 2. 5. 12. 5. 12. 6. 7. 18. 18. 18. 18. 18. 18. 18. 18. 18. 18	9 691 11 12 13 14 15	83	2000	9 2 8 9 2	-	1,002	A O
	9	856"	6 7 8 9 1	37, 4, 5, 6, 7	3 Tt 2 5 13 6 7 75:15	9 687 11 12 13 14 15 1	83	2000	5 6 7 8 9 2	io e	1,002	A O
	0)	856"	6 7 8 9 1	3 1 2 2 6 7	3.T. 1, 2, 5, 12 6, 7, 75, 15	9 687 11 12 13 14 15	83	2000	5 6 7 8 9 2	io e	1,002	4
	0)	856"	6 7 8 9 1	2 3 3 4 4 5 6 7	2 3 Tt 1 2 5 13 6 7 75 15 5 15 6 7 75 15	9 8891 11 12 13 14 15	S	2000	5 6 7 8 9 2	io e	1,002	4
	0)	856"	6 7 8 9 1	2 37 4 5 6 7	8 9 3 T 11 2 5 13 6 4 75 15	6 9 11 11 12 13 14 15 1	S	2000	4 5 6 7 8 9 2	io e	1,002	A P
	0)	856"	6 7 8 9 1	2 3π 4 5 6 7	8 9 8010 11 4 2 5 13 64 7515	0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	S	2000	4 5 6 7 8 9 2	io e	1,002	A O
	0)	856"	6 7 8 9 1	2 3 3 4 6 7 6 7	2 2 2 2 2 2 3 1	12 13 14 15 1	S	2000	3 4 5 6 7 8 9 2	-	1,002	A O
	0	856"	6 7 8 9 1	2 3 7 6 7	3.17 5 5 5 5 7 1 0.08 6 8 5 5 1 7 5 1 5 1 5 1 5 1 5 1 5 1 5 1 5 1	6 6 71 11 12 13 17 18 1	S	2000	3 4 5 6 7 8 9 2	10	LOG	A O
	0	856"	6 7 8 9 1	3 7 4 5	3.77. 11. 12. 5. 12. 6. 7. 7.5.15.	7 8 9 891 11 12 13 14 15 1	S	3 4 5 6 7 8 9 2	3 4 5 6 7 8 9 2	10	LOG	A O
	0	856"	6 7 8 9 1	3 7 4 5	37. 2 5 6 7 7. 15 15 15 15 15 15 15 15 15 15 15 15 15	7 8 9 891 11 12 13 14 15 1	9 8010	3 4 5 6 7 8 9 2	2 3 4 5 6 7 8 9 2	10	LOG	A B
	0	856"	6 7 8 9 1	3 7 4 5	317. 27. 57. 6. 5. 5. 14. 7.5.15. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5	7 8 9 6691 11 12 13 14 15 1	9 8010	3 4 5 6 7 8 9 2	2 3 4 5 6 7 8 9 2	10	LOG	AL DE
	0	856"	6 7 8 9 1	3 7 4 5	3 TF 75-15	7 8 9 897 11 "12 13 14 15 1	9 8010	3 4 5 6 7 8 9 2	2 3 4 5 6 7 8 9 2	10	LOG	A O
	0	866	6 7 8 9 1	2 3 3 4 4 5	7 8 9 8040 H 12 5 13 6 7 75-15	7 8 9 897 11 12 13 14 15 1	S	2000	1 2 3 4 5 6 7 8 9 2	10	LOG	NA ®
	0	866	6 7 8 9 1	2 3 3 4 4 5	5 5 2 7 1 2 2 13 6 2 2 1 7 1 2 2 13 6 2 2 2 1 2 2 13 6 5 2 1 2 1 3 6 5 2 1 3 6 5 1 3 6 6 5 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 1 3 6 6 6 6	6 9 691 11 12 13 14 15 1	9 8010	7 2 3 4 5 6 7 8 9 2	1 2 3 4 5 6 7 8 9 2	10	LOG	NA ®
	0	866	6 7 8 9 1	2 3 3 4 4 5	2 3 3 4 5 2 14 6 5 5 3 2 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	6 9 11 11 112 13 14 15 1	9 8010	7 2 3 4 5 6 7 8 9 2	1 2 3 4 5 6 7 8 9 2	10	LOG	NA ®
	0	856"	6 7 8 9 1	3 7 4 5	3 T	6 691 11 112 13 14 15 1	9 8010	3 4 5 6 7 8 9 2	1 2 3 4 5 6 7 8 9 2	10	LOG	A O
	0	866	6 7 8 9 1	2 3 3 4 4 5	2 3 5 2 1 7 1 2 2 1 3 6 2 5 9 5 5 5 1 5 1 5 1 5 1 5 1 5 1 5 1 5 1	98 9 13 17 17 18 18 18	9 8010	7 2 3 4 5 6 7 8 9 2	1 2 3 4 5 6 7 8 9 2	10	1,002	AL OF THE PROPERTY OF THE PROP
	0	866	6 7 8 9 1	2 3 3 4 4 5	2 5 5 6 7 7 11 11 11 11 11 11 11 11 11 11 11 11	98 9 13 17 17 18 18	9 8010	7 2 3 4 5 6 7 8 9 2	1 2 3 4 5 6 7 8 9 2	10	LOG	AL OF THE PROPERTY OF THE PROP
	0	866	6 7 8 9 1	2 3 3 4 4 5	55 6 7 13 64 75 15	55 6 9 891 11 12 13 14 15 1	9 8010	7 2 3 4 5 6 7 8 9 2	1 2 3 4 5 6 7 8 9 2	10	LOG	AL OF THE PROPERTY OF THE PROP
	0	866	6 7 8 9 1	2 3 3 4 4 5	8 3 5 5 6 5 7 14 25 8 8 10 10 10 10 10 10 10 10 10 10 10 10 10	55 6 9 891 11 12 13 14 15 1	9 8010	7 2 3 4 5 6 7 8 9 2	9 1 2 3 4 5 6 7 8 9 2	10	LOG	A A
	0	8867 3865	6 7 8 9 1	2 3 3 4 4 5	8 9 5 6 5 5 6 5 5 6 5 5 6 5 5 6 5 5 5 5 5	6 9 897 11 12 13 14 15 1	9 8010	7 2 3 4 5 6 7 8 9 2	9 1 2 3 4 5 6 7 8 9 2	9 6	ARISTO-MULTILOG	NA O
	0	866	6 7 8 9 1	2 3 3 4 4 5	8 6 7 7 8 3 3T	6	9 8010	State   State	9 9 2 8 9 2	9 6	ARISTO-MULTILOG	A A
	0	8867 3865	6 7 8 9 1	2 3 3 4 4 5	2 19 6 1 2 1 1 0 0 0 0 6 8 2 9 5 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	6	9 8010	7 2 3 4 5 6 7 8 9 2	D 9 1 2 3 4 5 6 7 8 9 2	9 6	ARISTO-MULTILOG	NY 0
	0	8867 3865	6 7 8 9 1	2 3 3 4 4 5	8 8 9 7 1 2 5 1 6 5 5 5 1 1 1 2 1 2 1 1 1 1 1 1 1 1 1 1 1	6	9 8010	State   State	0 9 1 2 3 4 5 6 7 8 9 2	9 6	ARISTO-MULTILOG	NY 0
	0	8867 3865	6 7 8 9 1	2 3 3 4 4 5	8 8 9 900 11 2 5 5 6 7 7 10 10 10 10 10 10 10 10 10 10 10 10 10	6	9 8010	State   State	9 1 2 3 4 5 6 7 8 9 2	9 6	ARISTO-MULTILOG	A STATE OF THE STA
	0	8867 3865	6 7 8 9 1	2 3 3 4 4 5	8 8 9 55 5 5 7 1 0 00 8 6 8 2 5 9 55 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	6	9 8010	State   State	9 1 2 3 4 5 6 7 8 9 2	9 6	ARISTO-MULTILOG	NY 0

07219A 0760.1N HILL OCILES

Log Log Scale, Range .999 to .989
Scale of Cubes
Scale of Squares
Scale of Squares
Scale of Tangents and Cotangents
Scale of Small Angles in Radians
Scale of Sines and Cosines
Fundamental Scale

Upper panel of body

-0.001×

Lower panel

of body

Log Log Scale, Range 1.001 to 1.011

Fundamental Scale

A B L L S U Q Q

Reciprocal Scale

On slide

tan

2. Reading the Scales

The following three systems of division reappear in all scales:

- A Read as you would read a scale graduated to full millimeters. The tenths of an interval, i. e. the fourth digit, is easy to locate by estimate within the smallest intervals.
- B The intervals between the shortest lines amount to two tenths of the secondary intervals marked by slightly longer division lines. The odd third, and also the fourth digit is located by estimate.

Back of Rule

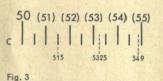
Fig. 2

C Here each interval between the shortest lines is five tenths of the secondary intervals. Intermediate locations for the third digit are estimated.

All numbers printed in brackets in Fig. 3 are not actually printed on the sliderule itselfand must, in practice, be counted off from the preceding number. Similarly, the labels 1.3, 1.4, 1.5, etc. in Fig. 3 A are abbreviated to 3, 4, 5, etc., the first digit 1 being self-evident in this rangé.







The slide rule only takes account of digits in consecutive order. The value 325 may also be read as 3.25, .0325, 3250 etc. The correct place for the decimal point is determined after obtaining the result by a rough approximation. To avoid reading errors it is good policy to think of and pronounce all numbers digit-by-digit. For instance, 235 = two-three-five, not two hundred and thirtyfive.

At first many beginners have some difficulty in locating numbers on these rather unusual scales. Patient and methodical practice by setting and reading various numbers of two and more digits with one slide end or the cursor hairline is, therefore, strongly recommended. Once you have learned to locate each and every value quickly, correctly and without groping, you are well on the road to being a competent slide rule user, because the mechanics employed in solving actual problems by slide rule are easily grasped.

# 3. The Principle of Slide Rule Calculation

Mathematical operations on the slide rule are performed by the mechanical addition or subtraction of two segments of graduated scales. This method is best demonstrated by two millimeter scales sliding lengthwise to each other.

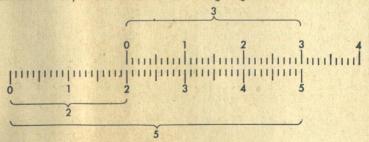


Fig. 4

Back of Rule:

Fig. 4 illustrates the sample problem 2+3=5. When the first line, 0, of the upper scale is shifted to the value 2 on the lower scale, we can add to this line segment, 2, any other line segment of the upper scale, in this case the length 3. Under 3 of the upper scale read the sum 5 on the lower scale. Similarly, too, we read the sum of 2+1=3, and, by counting off on the millimeter intervals, also 20+15=35 etc. etc. The subtraction 5-3=2 is the same process in reverse. When the values 5 and 3 are made to coincide, the remainder, 2, appears under the first line of the upper scale on the lower scale.

In more elaborate form the same basic principle finds application in the slide rule, with the fundamental difference that here the division lines are logarithmically spaced (but labeled with the numbers whose logarithms they represent). The result is that, since the product of two numbers is the sum of their logarithms, we can obtain the product by adding two segments of slide rule scales. Conversely, by subtracting one segment from another we obtain the quotient of one number divided by another.

# 4. Multiplication

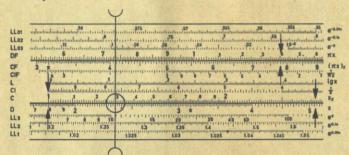


Fig. 5 Example:  $18 \times 13 = 234$  Roughly:  $20 \times 10 = 200$ 

Set the left 1 (called the left index) of scale C to coincide with the value 18 of scale D. Now move the cursor to 13 on scale C and find the product 234 under the hairline on D. One essential feature of the slide rule consists in that we can perform as many other multiplications by 18 as may be required without changing the initial setting.

The arrows in Fig. 5 indicate two such further operations, viz.

$$18 \times .285 = 5.13$$
 Roughly  $20 \times .25 = 5$   
 $18 \times 7.8 = 140.4$  Roughly  $20 \times 7 = 140$ 

It will soon be observed, as in  $18 \times 7.8$ , that sometimes the slide projects so far beyond the end of the body scale that no reading can be taken. The simple remedy then consists of setting the right index of the slide over 18 and shifting the cursor to 7.8.

This end-for-end exchange of the indexes is rather troublesome and it will therefore be appreciated that the upper pair of scales CF and DF enables us to find a better solution. By studying the rule you will find that we can continue reading on these scales where the lower scales break off. The arrows in the example  $18\times7.8$  show a case in point. This procedure is always applicable provided that the slide does not project out of the body by more than half its length.

Examples: 
$$3.04 \times 2.68 = 8.15$$
  $17.38 \times 4.47 = 77.7$   $3.04 \times 4.05 = 12.31$   $10.03 \times .2484 = 2.490$   $3.04 \times 7.27 = 22.10$   $.739 \times .655 = .484$   $3.04 \times 8.025 = 24.40$   $148.3 \times .039 = 5.78$   $9.81 \times 7.05 = 69.2$   $2.208 \times .0213 = .0470$   $4.33 \times 8.95 = 38.8$ 

It is often advantageous to make the first setting with the index of the CF scale placed opposite the multiplier on DF, because in this case there is no need to decide whether to start with the right or the left index. Furthermore, in all settings made with the upper pair of scales no more than half a slide length will ever project beyond the body scales. This means that the product can always be read on either the upper or the lower pair of scales, often on both scales simultaneously. It is advisable to repeat the previous exercises by first starting with C and D and next with CF and DF. In this way you can appreciate by experience which type of setting is the better one.

#### 5. Division

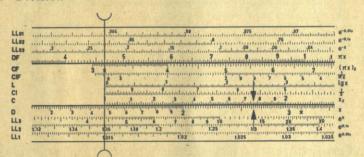


Fig. 6

Example: 2620 ÷ 17.7 = 148

Roughly: 3000 ÷ 20 = 150

Set the cursor over the value 2620 on scale D and draw 17.7 on C into alignment so that both values are opposite each other. The quotient 148 then appears under the left slide index.

Notice: It is worthy of note that when this manipulation is completed the setting of the rule is also identical to that for the multiplication 17.7  $\times$  148 = 2620. The only difference between multiplication and division is the order of the setting and the reading.

In the problem  $582 \div 7.23 = 80.4$  the quotient will be found under the right slide index. It follows that in division no end-for-end switches of the indexes will occur. Later in this text it will be demonstrated how this feature can be usefully employed. The following chapter also contains a reference to the same subject.

Examples:

$$3.78 \div 4.5 = .840$$
  $6.25 \div .1328 = 47.1$   $53 \div 7.45 = 7.12$   $.914 \div 16.29 = .0561$   $31.5 \div 22.8 = 1.382$   $180 \div \pi = 57.3$   $402 \div 26.66 = 15.08$   $\pi \div 180 = .01745$ 

The exercises in division, too, should be done, first with the C and D and then with the CF and DF scales. When the problem is set on the upper pair of scales the values are arranged in the fraction form of notation, with the numerator above the denominator, thus:

$$\frac{3.78}{4.5} = .840$$

The answer can be read both over the index of CF on DF and under the right index of C and D.

# 6. Multiplication and Division Combined

In problems of the type  $\frac{a \times b}{c}$  division usually comes first, followed by the required multiplication because in this order end-for-end changes of the indexes will be relatively seldom.

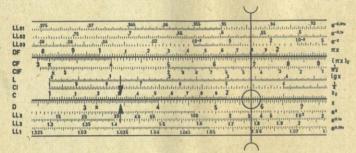


Fig. 7

Example: 
$$\frac{345 \times 22}{132} = 57.5$$

Example: 
$$\frac{345 \times 22}{132} = 57.5$$
 Roughly:  $\frac{300 \times 20}{100} = 60$ 

The intermediate result of the division 345 ÷ 132 = 2.61 can be ignored and the cursor must be moved directly to the value 22 on scale C, opposite which the final result 57.5 appears on scale D.

Examples: 
$$\frac{23.7 \times 4.8}{15} = 7.58$$
  $\frac{9.17 \times 6.55}{5.08} = 11.82$   $\frac{174 \times .63}{21.9} = 5.01$ 

# 7. Proportion

The slide rule is particularly convenient for computations involving proportions of the form  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$  for the reason that with one setting of the given ratio all the required terms can be obtained by simply passing the cursor along the scales. The joint between the scales of the slide and the body can be regarded as the dividing line in a common fraction.

Whenever it is possible to express a problem in the form of a proportion this type of computation should be preferred. So, for instance, the problem in Fig. 7 can be easily rearranged to read:  $\frac{345}{132} = \frac{57.5}{22}$ . A little practice in this direction will make the slide rule user more independent of the orthodox methods of computation. It really makes no difference whether the given ratio is set  $\frac{a}{b}$  or  $\frac{b}{a}$ as long as the other ratios are read in accordance with the first setting.

#### Example:

Conversion of inches to millimeters: How many millimeters are in 3, 5, 7 inches? The initial ratio is known to be 1 in. = 25.4 mm. Hence, expressing our problem in proportional form, we write:  $\frac{1}{25.4} = \frac{3}{x} = \frac{5}{y} = \frac{7}{z}$ 

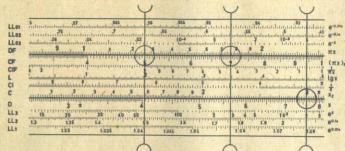


Fig. 8

Once the given ratio 1: 25.4 is set on the rule the other terms can be found by merely moving the cursor as required: x = 76.2 mm, y = 127.0 mm and

Exercises: 
$$\frac{27.0}{43.8} = \frac{x}{25.2} = \frac{63.4}{y} = \frac{z}{97.2}$$
  $\frac{kg}{lbs} = \frac{.454}{1} = \frac{x'}{13} = \frac{31.5}{y'}$ 

Results: x = 15.54; y = 102.9; z = 59.9; x' = 5.90; y' = 69.4

# 8. The Reciprocal Scales CI and CIF

These scales are the counterparts of their respective companions C and CF, except that their graduations and numerations run in the opposite direction. By virtue of this arrangement the reciprocal  $\frac{1}{x}$  of any value x on the fundamental scale C can be directly read on Cl under the cursor line. The utility of this feature consists in being able to convert at will a multiplication to a division and

So, for instance: 
$$4 \times 5 = 4 \div \frac{1}{5}$$
 and  $4 \div 5 = 4 \times \frac{1}{5}$ .

Expressions of the form a  $\times$  b  $\times$  c or  $\frac{a}{b \times c \times d}$  are solved by alternate multi-

plication and division. In the course of a computation we may freely switch from the group of scales C, D and Cl to the group CF. DF and ClF to avoid resetting the slide in multiplication. The following example shows such a situation.

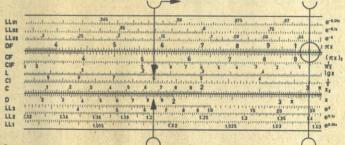


Fig. 9

Example: 
$$185 \times 6 \times .95 = 1054$$
 Roughly:  $200 \times 6 \times 1 = 1200$ 

Rearrange the problem to read:  $\frac{185}{1/6}$  × .95. Use the cursor hairline to bring 185 on scale D into coincidence with 6 on the reciprocal scale Cl, as in any ordinary division. The subsequent multiplication by .95 is best performed by use of scale CF and the product 1054 is then read directly above on scale DF.

Examples: 
$$19.7 \times .875 \times 21.8 = 376.0$$
  $(20 \times 1 \times 20 = 400)$   $6.12 \times 13.82 \times .074 = 6.26$   $(6 \times 10 \times 0.1 = 6)$   $7 \times 8 \times 9 = 504$   $(5 \times 10 \times 10 = 500)$ 

In the last example,  $7\times 8\times 9$ , the computation is simplified if made with DF in conjunction with CIF. In the example

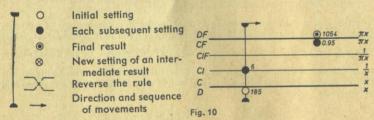
$$\frac{8.5}{69 \times .76} = .1621 \qquad \left(\frac{10}{100 \times 1} = 0,1\right)$$

the first division  $8.5 \div 69$  is done with DF and CF and the second division by .76 with the CI scale. The answer is instantly available on D. Only one setting is involved in this solution.

$$\frac{7.93}{.75 \times 2.98 \times .13} = 27.3 \qquad \frac{39 \times .21 \times 78}{5.3} = 120.5$$

# 9. Solution of Examples Explained in Diagram Form

In the following text an easily memorized method of explanation will be employed, so as to show the step-by-step operations in the respective computation with greater clarity than in the customary form of a facsimile slide rule. Parallel lines bearing their corresponding marginal labels represent the scales and the following symbols will make the diagrams very easy to interpret:



Hairline of the cursor

Fig. 10 shows how the example given in Fig. 9 will appear in the improved form of diagram.

 $185 \times 6 \times .95 = 1054$ 

# 10. The Folded Scales CF, DF and CIF

We have already had occasion to use these scales in multiplication and division, in which class of computations they are principally employed. The folded scales are identical to the fundamental scales, except that any location on one type of scale is laterally displaced by the value  $\pi=3.14159$  relative to the other scale. This means that the value  $\pi$  on the folded scales is exactly matched with the indexes of the fundamental scales. Consequently the graduation runs from  $\pi$  past 1, in about the middle of the rule, to  $\pi$  at the right extreme. There are short extensions of the graduation at both ends for greater convenience.

The effect of this folded arrangement consists in simplifying various computations involving the factor  $\pi$ . It will be clear that any switch-over from D to DF automatically supplies the product of any number set on D multiplied by the factor  $\pi$ . Conversely then the division by  $\pi$  is achieved by following the opposite course.

Typical Problems: Circumference of Circles 
$$C=d \pi$$
Angular Velocity  $\omega=2 f \pi$ 
Area of Circles  $A=r^2 \pi$ 

The first two formulas can be computed with one cursor setting, whereas we must first perform the multiplication  $r \times r$  with the scales C and D to find the circle area (see also chap. 16.2 on p. 25).

## Example:

To find the area for r = 2 in.  ${\rm A}=2^2\pi=2\times2\times\pi=12.56\,{\rm sg.\,in.}$ 

To find the circumference  $C = 4.7 \pi = 14.76$ 

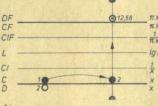
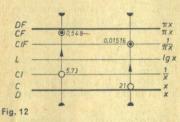


Fig. 11

Examples showing how to use the reciprocal scales:

$$\frac{\pi}{5.73} = .548$$
  $\frac{1}{21 \pi} = .01516$ 



# 11. The Scales A, B and K

# 11.1 Squares and Cubes; Square Roots and Cube Roots

When the cursor hairline is set to any value of x on scale D,  $x^2$  can be read on scale A and  $x^3$  on scale K. In the opposite order the switch-over from K to D furnishes the cube root and from A to D the square root.

# Examples:

$$2^2 = 4$$
  $2^3 =$ 

c) 
$$\sqrt{9} = 3$$
  $\sqrt{27} = 3$ 

d) 
$$\sqrt[2]{51} = 7.14$$
  $\sqrt[3]{364} = 7.14$ 

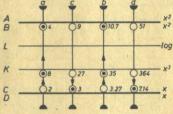


Fig. 13

When it is clear that the root concerned will fall within the range 1 to 10 no special calculating rules need be applied. This is the case when the radicand of a square root lies somewhere between 1 and  $100 = 10^2$  or the radicand of a cube root between 1 and  $1000 = 10^3$ . In all other cases it is good policy to reduce the quantity under the radical sign to the more handy form of a power of ten notation.

#### Examples:

$$\sqrt[2]{3200} = \sqrt[2]{100 \times 32} = \sqrt[2]{10^2 \times 32} = 10 \times \sqrt[2]{32} = 10 \times 5.66 = 56.6$$

$$\sqrt[3]{.270} = \sqrt[3]{\frac{270}{1000}} = \sqrt[3]{\frac{270}{10^3}} = \frac{1}{10} \times \sqrt[3]{270} = \frac{6.46}{10} = .646$$

$$\sqrt[2]{.242} = \frac{1}{10} \times \sqrt[2]{24.2} = .492 \qquad \sqrt[3]{1795} = 10 \times \sqrt[3]{1.795} = 12.15$$

# 11.2 Multiplication and Division with the Scales of Squares

Multiplication and division can also be done by using the scales A and B by the same process as that used for C and D, but the precision obtained will be somewhat less refined. In many problems beginnig with a squaring operation it is an advantage to be able to continue the computation on the A and B scales.

It is recommended to repeat the examples given in the chapters on multiplication and division on pages 5—7 with the scales of squares for practice in the use of these scales and in order to judge the accuracy here obtainable as compared with that of the fundamental scales.

# 12. Trigonometric Functions

The functions of an angle are obtained by adjusting the cursor to the angle value on the appropiate scale, either S, T or ST, and reading the function on the fundamental scale C, or on D provided that the indexes of both scales coincide. Reversing this process we obtain the angle corresponding to the respective function.

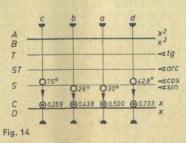
In the setting of angular values it must be borne in mind that the trigonometric scales are divided into degrees and decimals of degrees. Attention must be paid to the varying magnitude of the scale intervals as in using the fundamental scales.

The slide rule only gives the functions of angles in the first quadrant. To reduce angles of other quadrants to the first, consult the following conversion table.

	± α	90° ± α	180° ± α	270° ± α	45° ± α
sin	± sin α	+ cos α	∓ sin α	— cos α	$\cos (45^{\circ} \mp \alpha)$
cos	+ cos α	∓sinα	— cos α	± sin α	$\sin (45^{\circ} \mp \alpha)$
tan	± tan a	∓ cot α	± tan α	$\mp \cot \alpha$	$\cot (45^{\circ} \mp \alpha)$
cot	± cot α	∓ tan α	± cot α	∓ tan α	$tan (45^{\circ} \mp \alpha)$

12.1 The Sine Scale extends from 5.5° to 90° and also contains the cosine values in red numerals, running backwards from right to left. All sines read on the fundamental scales must be given the prefix "0.".

Examples: a. sin 30° = .500 b. sin 26° = .438 c. cos 75° = .259 d. cos 42.8° = .733

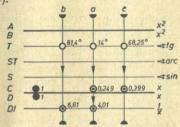


**12.2** The Tangent Scale is numerated from 5.5° to 45° in black and, in opposite direction from 45° to 84.5° in red color. For angular values in black read the tangents on scale C, pre-fixing "0".

The tangents of all angles  $\alpha > 45^\circ$  (red numeration) are found on the CI scale (red numeration) since  $\tan \alpha = \frac{1}{\tan (90 - \alpha)}$ . These values are always > 1. With the slide "in neutral", readings can also be taken from scales D or DI.

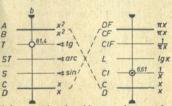
The formula cot  $\alpha=\frac{1}{\tan\alpha}$  governs the rule for finding the cotangent of an angle. Therefore read the cotangents on CI for angles  $\alpha<45^{\circ}$  and on C for angles  $\alpha>45^{\circ}$  (or on DI and D, respectively).

a.  $\tan 14^{\circ} = .249$   $\cot 14^{\circ} = 4.01$  $\tan 81.4^{\circ} = 6.61$ 



ig. 15 Slide in "neutral position"

b. tan 81.4° = 6.61 c. cot 68.25° = .399



Slide in operational position, read on rear face Fig. 16

#### 12.3 Small Angles

The following relation is applicable to angles  $\alpha < 5.5^{\circ}$ :

$$\sin \alpha \approx \tan \alpha \approx \cos (90^{\circ} - \alpha) \approx \cot (90^{\circ} - \alpha) \approx \alpha$$
 in radians

The trigonometric scale ST running from .55° to 6° is graduated to radians and, therefore, enables simultaneous readings of sine, tangent and radian values on scale C.

Example:  $\sin 1^{\circ} \approx \tan 1^{\circ} \approx \cos 89^{\circ} \approx \cot 89^{\circ} \approx \arctan 1^{\circ} = \frac{\pi}{180} = .01745 \text{ radians}$ 

$$\cot 1^{\circ} = \frac{1}{\tan 1^{\circ}} \approx \frac{180}{\pi} = 57.3 \text{ radians (on CI or DI, respectively)}$$

The cosines of small angles and the sines of large angles cannot be computed directly with the slide rule. When these are involved the solution requires the use of the series progression

$$\cos \alpha \approx 1 - \frac{\alpha^2}{2}$$
 ( $\alpha$  in radians).

Example: 
$$\cos 1^{\circ} \approx 1 - \frac{.01745^{2}}{2} = 1 - .000152 = .999848$$

The square of  $\alpha$  in radians is immediately available on scale B over the angle setting on scale ST. To find the angle corresponding to a given cosine reverse this process.

Examples for Cap. 12.1 to 12.3:

sin	51.5°	=	.783	tan	37.4°	=	.764
cos	59.3°	=	.510	tan	63.5°	=	2.006
sin	3.8°	=	.0000	100000	13.7°		
cos	3.70	=	.998	cot	48.2°	=	.894
200	88 20	_	0314	tan	2.740	=	.0478

The inverse trigonometric functions can be read in the reversed order

# 12.4 Conversion of Degrees to Radians

Scale ST is a duplicate of the fundamental scales, modified only in that the one graduation is displaced laterally by the value  $\frac{\pi}{180}$  relative to the other

graduation. Therefore by following the cursor line from scale ST to C we achieve the conversion of degrees to radians, and vice versa. This form of calculation is applicable not only to the small angles discussed above but to large angles as well, by virtue of the decimal subdivision of the degrees.

since the displacement of the graduations by  $\frac{\pi}{180}$  is simply a constant multiplication factor.

Any setting of an angle  $\alpha$  may also be regarded as representing .1  $\alpha$ , 10  $\alpha$ ,  $100 \,\alpha$  etc. and the decimal point in the radian is then placed accordingly.

For instance: 
$$1.0^{\circ} = .001745$$
 radians  $10.0^{\circ} = .1745$  radians  $100.0^{\circ} = 1.745$  radians  $100.0^{\circ} = 1.745$  radians

When small angles are given in terms of minutes and seconds they can be converted to decimal parts of the degree as follows:  $1' = \frac{10}{60}$  and  $1'' = \frac{10}{3600}$ .

The marks ' and " on the scale ST give a means of direct computations of radians, when minutes or seconds are given. They are derived from the conversion factors:

$$\frac{180}{\pi} \times 60 = 3438$$
 for minutes, and  $\frac{180}{\pi} \times 60 \times 60 = 206265$  for seconds arc  $\alpha$  ( $\alpha$  in radians) =  $\frac{\alpha'}{\text{minutes gauge'}}$  or  $\alpha' = \text{arc } \alpha \times \text{minutes gauge'}$ 

 $\operatorname{arc} \alpha (\alpha \text{ in radians}) = \frac{\alpha''}{\operatorname{seconds gauge''}} \text{ or } \alpha'' = \operatorname{arc} \alpha \times \operatorname{seconds gauge''}$ 

$$arc \alpha (\alpha in radians) = \frac{\alpha}{seconds gauge''}$$
 of Examples:

 $380'' = \frac{380}{206265} = .001843 \text{ radians}$ .0045 radians =  $.0045 \times 3438 = 15.47'$ 



The gauge marks for minutes' and seconds' are a useful aid in solving circle sectors for radii r, arc lengths b and central angles  $\alpha$ .

Fig. 18

Fig. 17

$$\alpha = \frac{b}{r} \times \text{gauge mark } b = \frac{\alpha \times r}{\text{gauge mark}}$$

Examples:

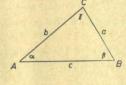
$$\alpha = \frac{.6}{45} \times \text{minutes gauge'} = 45.8'$$

$$b = \frac{48'' \times 67}{\text{seconds gauge''}} = .0156$$



# 13. Trigonometric Solution of Plane Triangles

The law of sines is a convincing example of the efficiency of the slide rule in computations involving proportions.



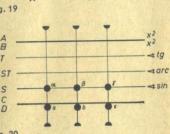


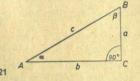
Fig. 20

By setting the given angle on S opposite the given side on D, the other ratios are immediately in coincidence, i. e. the angle corresponding to the given side or the side opposite the given angle are directly available.

In practice this form of computation is mostly concerned with right triangles. In this particular case we obtain  $\dot{\gamma} = 90^{\circ}$ , hence  $\sin \gamma = 1$ . With  $\alpha = 90^{\circ} - \beta$ and  $\beta = 90^{\circ} - \alpha$  the law of sines is therefore rearranged to the formula:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{1} = \frac{a}{\cos \beta} = \frac{b}{\cos \alpha}$$

and further:  $\tan \alpha = \frac{\alpha}{L}$ 

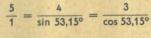


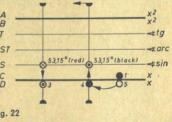
Depending on the given elements there are two basic operations, viz.

A. Given: Any two parts (except

case B). B. Given: The small sides a and b.

Example for A: Given: c = 5, b = 4





Set the slide index (sin 90° = 1) of scale C over the hypothenuse 5 on D. The required values  $\alpha$ ,  $\alpha$  and  $\beta$  can now be read by merely moving the cursor.

Over the side 4 on the D scale find  $\beta=53.15^{\circ}$  on S (black numerals). It makes no difference whether the next operation requires to find the side 3 on the D scale is the setting of the cursor to  $\alpha = 90^{\circ} - 53.15^{\circ} = 36.85^{\circ}$  on scale S with the black numeration, or its direct setting to the cosine 53.15° on the red numeration.

When the given elements are one side and one angle, the first step consists in aligning the side to the opposite angle; the following operations are then as outlined in the above diagram and the hypotenuse is found under the slide index on D.

In some cases it will be found convenient to use the DF scale instead of D to avoid end-for-end changes of the indexes. All sides then appear on the DF scale. The method itself is unchanged.

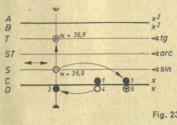
Example for Case B: Given: a=3, b=4 To find: c,  $\alpha$ ,  $\beta$ 

First solve for  $\alpha$  from the given sides:

$$\tan \alpha = \frac{a}{b} = \frac{3}{4}$$

or, better, in the proportional form:

$$\frac{4}{1} = \frac{3}{\tan \alpha}$$



Set the slide index over the value 4 on scale D and move the cursor to 3 on scale D. Read the angle  $\alpha = 36.9^{\circ}$  by following the hairline to scale T. The next step is

to find c by applying the law of sines in  $\frac{c}{1} = \frac{3}{\sin 36.9^{\circ}}$ . Since the cursor is already

located over the value 3, we now shift the slide so as to bring the angle 36.9° in scale S under the hairline. The answer c = 5 can now be read on D under the slide index.

In cases where  $\alpha > 45^{\circ}$ , with a > b, the computation runs as described above and invariably begins with the larger one of the two sides. In this case, however, we must read the complement of the angle (red numerals) on scale T and consequently set the cosine (red numerals) on scale S in the next step.

In order to clarify this kind of computation and avoid mistakes it is useful to visualize the particular triangle in your mind's eye.

Example: Given the right triangle in which:

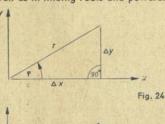
$$a = 10.63$$
  $b = 19.8$   $\alpha = 28.23^{\circ}$   $a = 46.7$   $b = 16.5$   $\alpha = 70.53^{\circ}$   $c = 78.2$   $\alpha = 22.7^{\circ}$   $a = 30.2$ 

b = 72.3The two classes of solutions for right triangles discussed in the preceding text are of particular significance in problems involving coordinates, vectors, or

complex numbers. Such problems invariably require conversions of rectangular coordinates to polar coordinates and vice versa. These conversions occur quite frequently in electrical engineering problems for the reason that, in the component form Z = a + i b, the values are easy

to add or subtract. The exponential form  $Z = r/\varphi$  is equally convenient for performing multiplications and divisions as well as in finding roots and powers.

Coordinates:  $\Delta x$ ,  $\Delta y \leftrightarrow r$ ,  $\varphi$ See example for case B (Fig. 23)



Complex numbers:

$$Z = a + ib = re^{i\varphi} = r/\varphi$$

$$Z = 4.5 + j 1.3 = 4.68/16.13^{\circ}$$

$$Z = 6.7/49^{\circ} = 4.39 + j 5.05$$

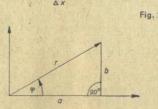


Fig. 25

# 14. The Log Log Scales LL0-LL3 and LL00-LL03

All Log Log scales are used together with the fundamental scales C and D. The e+x-scales LL0, LL1, LL2 and LL3 cover the range 1.001 to 100,000 and the e-x-scales LL00, LL01, LL02 and LL03 the range .00001 to .999.

The e+x- and e-x-scales are reciprocal to each other. Reciprocals of numbers < 2.5 can be read with a higher degree of accuracy than can be expected when using the scales CI and CIF, e. g.: the reciprocal value of 1.0170 is .98328.

Attention: The Log Log scales supply immutable values, either whole numbers or numbers with their fractional parts in decimals. This means that when we read 1.35 on the scale, this is the only value concerned. It is not decimally variable as in dealing with the fundamental scales.

## 14.1 The 10th, 100th and 1000th Powers and Roots

The Log Log scales are mutually coordinated in such a manner that, in passing from one scale to the adjacent scale, the tenth power or the tenth root of a number set on the one scale can be read on the neighbouring scale, depending on the direction in which the reading is made.

The examples depicted in Fig. 26 will make it clear how the tenth and hundredth power or root of a given number can be determined by the simple process of following the cursor hairline to the appropriate scale. For greater clarity the scales LL00 and LL0 are here shown grouped together with the other Log Log scales on one face of the rule. .

Examples:

$$1.015^{.1} = \sqrt[1]{1.015} = 1.00149$$

$$1.015^{1} = 1.015$$

$$1.015^{10} = 1.1605$$

$$1.015^{100} = 4.43$$

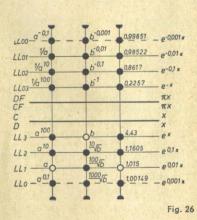
$$\frac{1}{1.015^{100}} = 1.015^{-100} = .2257$$

$$\frac{1}{1.015^{10}} = 1.015^{-10} = .8617$$

$$\frac{1}{1.015^{1}} = 1.015^{-1} = .98522$$

$$\frac{1}{1.015^{.1}} = \frac{1}{10}$$

 $= 1.015^{-.1} = .99851$ 



Other reading examples based on the range of numbers in Fig. 26:

$$100 \text{ } 100 \text{ } 1000 \text{ } 100 \text{ } 1000 \text{ } 100 \text{ } 100 \text{ } 1000$$

Examples such as the above will hardly ever arise in practice. They are used here to make the system governing the Log Log scales easy to comprehend.

# 14.2 Powers $y = a^{x}$

Raising a number to any power is done exactly as multiplications are performed with the fundamental scales.

#### Procedure:

- a) Use the cursor to set the index of scale C to the base "a" on the appropriate LL scale.
- b) Shift the cursor hairline to the value of the exponent on C.
- c) Read the power y under the hairline on the corresponding LL scale. (See the chapter "Reading Rules".)

When the slide is set to the value of the base "a" we obtain a complete table of values corresponding to the function  $y = a^x$ . Fig. 27 depicts the setting of the slide to be made for the function  $y = 3.2^x$  showing the cursor aligned to the exponent 2.5 and its decimal variates.

LL00	- 0,997096	e-0,001 a
LL01	0,97134	e-0,01 x
LL02	0,7476	e-0,1 x
LL03	0.0540	е-х
DF	-	πx πx
C 1	2,5	×
D LL3 — 32	18,3	e ×
LL2	1,338	e 0,1 x
LL1	1,02956	e 0,01 x
		e0,001;
LL0	- 6 1,002912 -	e <sup>0,0</sup>

Examples:

Reading on scale

3 22.5 = 18.3113 3.2.25 = 1.338LL2 3,2.025 = 1.02956LL1 3.2.0025 = 1.002912LLO = .0546 LL03 3.2-.25 = 7476 LL02 3.2-.025 = .97134LL01 3.2-.0025 = .997096Loo

Fig. 27

#### Reading Rules:

- a) For positive exponents set and read in the same group of scales LL0—LL3 or LL00—LL03 i. e. use scales having numerals of uniform color. For negative exponents we must switch over from one group of scales to the other (alternating the colours).
- b) In conformity with the labels given at the right end of each scale, read on the adjoining scale with the inferior label for each place that the decimal point in the exponent is moved to the left. (Cf. example Fig. 27.)
- c) When the base is set with the right slide index, all readings must be taken from the adjoining "higher" labeled scale.

When  $0 < \alpha < 1$  the powers with positive exponents are found in the group LL00-LL03 of the Log Log scales, and with negative exponents in the group LL0-LL3.

Examples to clarify the reading rules:

$$.685^{2.7} = .36$$
  $.685^{-2.7} = 2.78$   $1.46^{2.7} = 2.78$   $1.46^{-2.7} = .36$ 

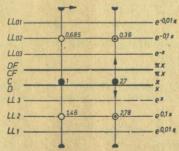
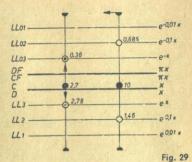


Fig. 28

Figs. 28 and 29 show the same examples, but in Fig. 28 the slide is set with the left index whereas in Fig. 29 the right index is used.



Examples:  $1.021^{2.4} = 1.0512$  $1.162^{-4} = .5485$ 

$$1.162^{-4} = .5485$$
  
 $2.13^{5.3} = 55.0$   
 $2.13^{.53} = 1.493$ 

$$2.13^{-.53} = .6698$$
  
 $8.5^{3.7} = 2740$   
 $25.4^{-2.6} = .000223$ 

$$1.0023^{3.7} = 1.00854$$
 $e^{-\pi} = .0432$ 

$$3^4$$
 = .0081  
 $3^{-4}$  = 123  
 $.49^{2.8}$  = .1357  
 $.49^{1.2}$  = .425  
 $.49^{-12}$  = .918

$$.49^{.12} = .918$$
  
 $.93^{5.1} = 0.691$   
 $.93^{-5.1} = 1.448$ 

 $.93^{-3.1} = 1.448$  $.9625^9 = .709$ 

# 14.3 Exceptional Cases of $y = a^x$

Since the range of the Log Log scales is restricted, cases will arise where the exponent is either too great or too small to permit direct reading of the power.

# y > 100,000 and y < .00001

When the power corresponding to a base with a large exponent is greater than 100,000 or smaller than .00001 the alternative consists in breaking up the exponent into several factors.

Example:

3.14<sup>19</sup> = 
$$3.14^{6+6+7}$$
 =  $(3.14^6)^2 \times 3.14^7$   
=  $.955^2 \times 10^6 \times 3.02 \times 10^3 = 2.76 \times 10^9$ 

For expressions with negative exponents the procedure is, of course, analogous.

#### .999 < y < 1.001

When for a small exponent x, the value of the power is either smaller than 1.001 or greater than .999 the answer is obtained by use of an approximation.

From the series expansion

$$a^{\pm x} = 1 \pm \frac{x}{1!} \log_e a + \frac{x^2}{2!} (\log_e a)^2 \pm \frac{x^3}{3!} (\log_e a)^3 + \cdots$$

can be derived  $a^{\pm x} \approx 1 \pm x \log_e a$  for  $|x| \ll 1$ 

If the index of C is set opposite the base value a on LL by aid of the cursor line,  $\log_e a$  on D (without reading its value) can be multiplied by x. The product added to 1 or, respectively, subtracted from 1 gives the power. The smaller the exponent the more precise will be the result.

Returning to our previous example with the base 3.2 (Fig. 27), we can now continue, e. q.:

$$3.2^{.00025} \approx 1 + .00025 \times log_e 3.2$$
  
 $\approx 1 + .0002908 = 1.0002908$   
 $3.2^{-.00025} \approx 1 - .0002908 = .9997092$ 

When the decimal mutations of the exponent are carried still further, only the number of ciphers or nines after the decimal point will vary in the result:

$$3.2^{.000025} = 1.00002908$$

.999 < a < 1.001

When, in the type of problem  $y=a^x$ , the value of the base is approximately 1 and within the range 0.999 < a < 1.001, the solution is worked out by use of the approximate formula

$$a^{x} = (1 \pm n)^{x} \approx 1 + x \times \log_{e} (1 \pm n)$$
  $\log_{e} (1 \pm n) \approx \pm n \text{ (for } |n| \ll 1)$ 

It is immaterial whether, in the here discussed range  $\log_e(1\pm n)$ , is set in scale LL or the value n in scale D, in consideration of  $\log_e(1\pm n)\approx \pm$  n. The smaller the magnitude of the value n the closer the correctness of the approximation. It follows, then, that where the LL scale breaks off scale D can be used as the continuation of the LL scale, in this case substituting  $\pm$  n for  $1\pm n$ . When the index of the C scale coincides with n on the D scale this setting is practically identical with the setting  $\log_e{(1\pm n)}$  within an imaginary additional LL scale covering the range 1.0001 to 1.001 or, respectively, .9990 to .9999 and so on. The computation then continues by looking up the power as previously discussed. Actually any answer read on the D scale is derived from a simple multiplication but has to be complemented by the addition of "1" or the subtraction from "1", as the case may be. When, with growing exponent, the power falls within the readable range of the LL scales readings are taken directly from these scales.

Examples: 
$$1.00023^{3.7} = (1 + .00023)^{3.7} = 1.000851$$
  
Set on scale D, read on scale D and add 1  
 $1.00023^{37} = 1.00855$   
Set on scale D, read on scale LL0  
 $.99977^{3.7} = (1 - .00023)^{3.7} = .999149$   
Set on scale D, read on scale D and deduct from 1  
 $.99977^{37} = .99154$   
Set on scale D, read on scale LL00  
 $1.19^{.31} = 1.0554$  (Set on LL2, read on LL1)  
 $1.19^{.0031} = 1.00540$  (Set on LL2, read on LL0)  
 $1.19^{.0031} = 1.000538$  (Set on LL2, read 538 on D)  
 $1.0048^{1.9} = 1.00912$  (Set on LL0, read on LL0)  
 $1.00021^{4.2} = 1.000882$  (Set on D, read 882 on D)  
 $.99979^{4.2} = .999118$  (Set 21 on D, read 882 on D)

## 14.4 Powers y = ex

When the indexes of the slide and the body scales coincide the rule is adjusted to the equation  $y = e^x$ . The base e = 2.718 on scale LL3 being always aligned with the index of scale D, it follows that any power of e can be found by corresponding moves of the cursor to the exponent on scale D. The setting used in Fig. 26, for instance, would be correct for the exponent 1.489 and its decimal variations

$$e^{1.489} = 4.43$$
  $e^{.01489} = 1.015$   $e^{.1489} = 1.1605$   $e^{.001489} = 1.001489$ 

With the last example we again arrive at the equivalence of  $e^x \approx 1 + x$ .

14.5 Roots 
$$a = \sqrt[\times]{y}$$

Expressions containing roots are often easier to handle when changed to the form of a power. In this case set the exponent on the CI scale.

$$\sqrt[3.5]{e} = e^{\frac{1}{3.5}} = 1.3307; \frac{1}{.35} = e^{-\frac{1}{.35}} = .0574; \frac{1}{3.5} = e^{-\frac{1}{3.5}} = .7514$$

Inversely to the process of raising a number to a power we can also find the roots of numbers by using the Log Log scales in the same manner as the fundamental scales are used in division.

From 
$$y = a^x$$
 we can derive  $\sqrt[2]{y} = a$ .

Procedure:

- a) Set the radical index on C opposite the radicand y on scale LL.
- b) Read the value of the root under either the left or the right slide index on the appropriate Log Log scale.

The principles of the reading rules on p. 18 are also applicable in this instance. Bear in mind that when the reading is taken under the right slide index, the answer will appear on the next lower labeled Log Log scale LL0—LL3 or LL00—LL03.

$$\sqrt[.77]{\sqrt{21}} = 52.1 \qquad \frac{1}{.77} = .0192$$

$$\sqrt[.77]{\sqrt{21}} = 1.485 \qquad \frac{1}{7.7} = .6734$$

$$\sqrt[.77]{\sqrt{21}} = 1.0403 \qquad \frac{1}{.77} = .96122$$

$$\sqrt[.770]{\sqrt{21}} = 1.00396 \qquad \frac{1}{.770} = .99605$$

$$\sqrt[.700]{21} = 99605$$

# 14.6 Logarithms

Logarithms to any base can be determined with the Log Log scales. The logarithm corresponding to a number is obtained when the process of raising a number to a power is reversed.

$$y = a^{x}$$
  
 $x = log_{a}y$  (Read: Logarithm of y

 $x = log_{\alpha}y$  (Read: Logarithm of y to the base a)

Finding a logarithm is identical to a problem in which the exponent of a power is to be determined from the given number and the base. The course to be followed in solving this class of problems is easy to understand by studying the explanatory diagrams depicting first the process of raising a number to a power (Fig. 31), and then the two inverse operations (Figs. 32 and 33).

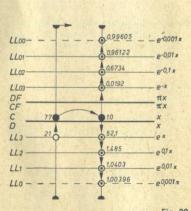
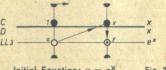
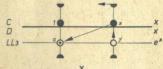


Fig. 30



Initial Equation:  $y = a^X$  Fig. 31



First Reversal:  $\sqrt{y} = a$ 

Fig. 32

#### Procedure:

- a) Place the cursor hairline over the value of the base "a" on the Log Log scale.
- b) Draw the slide index under the hairline.
- c) Shift the cursor to the antilogarithm y on the Log Log scale.
- d) Read the logarithm x under the hairline on scale C.

## Reading Rules:

- a) Each departure from one LL scale to the adjacent LL scale, in the order LL3, LL2, LL1, LL0 or respectively LL03, LL02, LL01, LL00 causes a shift of the decimal point by one place to the left for the reading of the logarithm on scale C and, in the opposite order by one place to the right.
- b) Logarithms are positive when both antilogarithm and base are set on Log Log scales having likecolored numerations, and negative when the colors are unlike.

The place for the decimal point is determined from  $\log_a a = 1$ .

When the slide index is set to the base "a", all values to the right of "a" on scale C are > 1 and all values to the left of "a" are < 1.

The natural logarithms to the base e and the common logarithms to the base 10, i. e. by far the most important in practical problems, are special cases. When these logarithms are involved the base e or 10 is set (Figs. 35 and 36).

When in the course of a computation the right index takes the place of the left index all readings appear to the left of the base; hence they are < 1 so that the decimal point must be moved one place to the left.

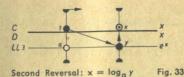
a) 
$$\log_{10}50 = 1.699$$

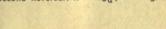
b) 
$$\log_{10} 2 = .3010$$

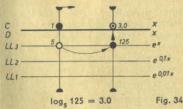
b) 
$$\log_{10}$$
 .5 = - .3010

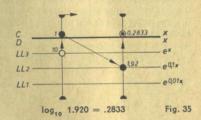
c) 
$$\log_{10} 1.03 = .01284$$

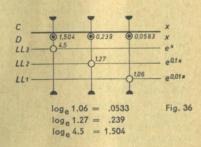
d) 
$$\log_{10}$$
 .015 =  $-1.824$ 

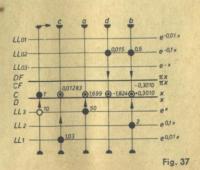








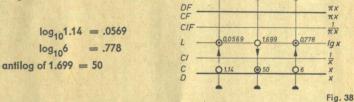




#### Practice Problems:

$$\log_{10}6 = .778$$
  $\log_{2}16 = 4.0$   $\log_{2}52 = .5$   $\log_{10}1.14 = .0569$   $\log_{2}1.02 = .0286$   $\log_{e} .05 = -3.0$   $\log_{10}1.015 = .00647$   $\log_{2} .25 = -2$   $\log_{e} .622 = - .475$ 

The most frequently used decadal logarithms are also contained in the customary mantissa scale L on the slide of the rule. This scale gives the mantissas only. As in using a logarithmic table, the characteristic is determined by use of the rule "number of digits in the antilogarithm minus 1" and prefixed to the mantissa. The logarithm of any number set on scale C can therefore be found directly on scale L and, inversely, one can also find the antilogarithm corresponding to a given logarithm.



# 14.7 Solving Proportions with the Log Log Scales

When the slide index is set to some base value of "a" on a Log Log scale, the powers to any exponent and also the logarithms of any number to this base can be obtained. The base a, when set on the Log Log scale, can therefore be regarded as one of the terms in a proportion.

14.71 
$$y_1 = a^n$$

$$y_2 = a^m$$

$$\log y_1 = n \times \log a$$

$$\log y_2 = m \times \log a$$

$$\frac{\log a}{1} = \frac{\log y_1}{n} = \frac{\log y_2}{m}$$
or 
$$\frac{\log_e}{1} = \frac{\log_e y_1}{n} = \frac{\log_e y_2}{m}$$

When three given terms of a proportion are set, the required fourth term can be read and the rule is at the same time adjusted for reading as many other relations proportional to the first ratio as may be required. Here we have another opportunity for applying the principle of proportion for which the slide rule is so eminently practical to use.

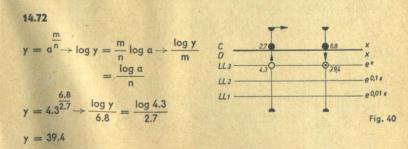


Fig. 39

After setting 2.7 on C opposite 4.3 on scale LL3, the result 39.4 will be found on the LL3 scale under 6.8 of C. Modifications of this problem are, of course, solved analogously:

$$y = \sqrt[2.7]{4.3^{6.8}}$$
 or  $y^{2.7} = 4.3^{6.8}$ 

#### 14.73

The formulas of many laws in the natural sciences can be suitably arranged to permit solution in the manner discussed above when the change in one variable is proportional to the logarithm of the ratio of the other variable.

$$\log \frac{y_2}{y_1} = \text{const} \times (x_2 - x_1)$$

Any change  $x_1$  to  $x_2$  over the increment i entails a change of  $y_1$  to  $y_2$ . When the ratio  $\frac{y_2}{y_1}$  is given the designation r, i. e. the rest of the original whole quantity, the above equation can be written:

$$\frac{\log r}{i} = \text{const} = \frac{\log r_1}{i_1} = \frac{\log r_2}{i_2} = \cdots$$

Example: Radioactive Decay

A substance is known to disintegrate at the rate of 40% in 30 days, leaving a residue of 60%.  $i_4 = 30$ ,  $r_4 = .6$ .

After how many days will 20% be left?  $r_0 = .2$ 

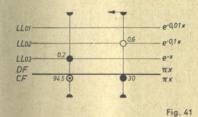
$$\frac{\log .6}{30} = \frac{\log .2}{x}$$
 x = 94.5 days

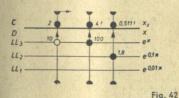
#### 14.74

For the multiplication of a constant factor by a logarithm the constant on C is set opposite to the base of the logarithm on the Log Log scale. Thus a tabulating position is obtained.

$$x = c \times \log_{\alpha} y$$
 or  $\frac{x}{\log_{\alpha} y} = \frac{c}{1} = \frac{c}{\log_{\alpha} a}$ 

Examples:  $2 \times \log_{10} 100 = 4$  $2 \times \log_{10} 1.8 = .511$ 





In electrical engineering it is often necessary to compute the decibel corresponding to a given voltage ratio

$$db = 20 \log \frac{U1}{U2}$$

# 15. Hyperbolic Fonctions

The unique construction of the Log Log scale system enables the formation of hyperbolic functions. The values of  $e^x$  and  $e^{-x}$  can be obtained by one setting of the cursor.

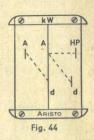
$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

## 16. The Detachable Cursor and its Lines

#### 16.1 The Mark 36

The front face of the cursor contains a short line to the right of the center line and over the folded scales. The lateral distance of this line from the center line corresponds to the factor 36 for readings on scale DF relative to any setting on the fundamental scale D. By virtue of this arrangement the cursor can be used for conversions of:





Hours to Seconds:

1 hour = 3600 seconds

1 meter per second = 3.6 kilometers per hour

Degrees to Seconds: 1° = 3600"

Years to Days: 1 year = 360 days (for interest computations)

#### 16.2 The Marks for Circle Areas

The intervals between the upper left or the lower right line on the one hand and the center line on the other hand are equal to  $\frac{\pi}{4}=.785$  i. e. the factor applicable

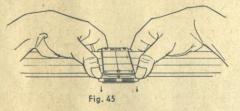
in computations of circle areas or round sections  $A=d^2\frac{\pi}{4}$  (Fig. 44). To find any required circular area, set the lower right or the center hairline to the given diameter d on scale D and read the area under the center line or the upper left line, respectively, on scale A.

#### 16.3 The Marks kW and HP

The interval between the upper right line and the center line is equivalent to the coefficient .746, applicable to conversions of HP to kW (Fig. 44). Hence, when the center hairline is set to 20 kW, on the A scale, then the upper right line indicates the equivalent in HP viz. 26.8. Inversely, when the short right line is set to 7 HP the center line will produce the equivalent 5.22 kW.

# 16.4 Detaching the Cursor

The hairlines of the two cursor windows are precisely matched so that the ser can pass from one face of the rule to the other when required in the course of a problem. The accuracy of this adjustment is not disturbed when the cursor is taken off for cleaning. To remove the cursor, use both thumbs to press the tips of the bar marked with arrows gently downwards. This releases the snap fastener and the cursor can be taken off the slide rule (Fig. 45).



#### 16.5 Adjustment of the Cursor

Even though the cursor hairs are reliably adjusted violent jarring of the rule may throw them out of alignment. In such a case loosen the four screws on the cursor face with the HP mark. Turn the slide rule over and shift the other window about the center snap fastener until the hairline is accurately aligned to the index lines of the scales. Holding the adjusted window firmly in position, turn the slide rule over, and adjust the first window in a similar manner. Tighten all screws carefully to prevent renewed dislocation of the hairlines.

# 17. Treatment of the ARISTO Slide Rule

This instrument is a valuable calculating aid and deserves careful treatment. Scales and cursor should be protected from dirt and scratches, so that the reading accuracy may not suffer. It is advisable to give the rule an occasional treatment with the special cleanser fluid DEPAROL, followed by a dry polishing. Avoid chemical substances of any description as they may spoil the scales.

Do not leave the rule on heated surfaces such as radiators or expose it for a greater length of time to powerful sunlight. Deformations may occur any temperatures above 60° C (140° F). Rules so damaged will not be exchanged free of charge.

All rights reserved, including that of translation.

This booklet, or parts thereof, may not be reproduced without permission.

© 1954 by DENNERT & PAPE · ARISTO-WERKE · HAMBURG

8th Edition · 051061 · Printed in Germany by Borek KG · 1828