Faber-Castell 989 Komplex-Rechenplatte
Richard Smith Hughes

English Translation of the German Manual
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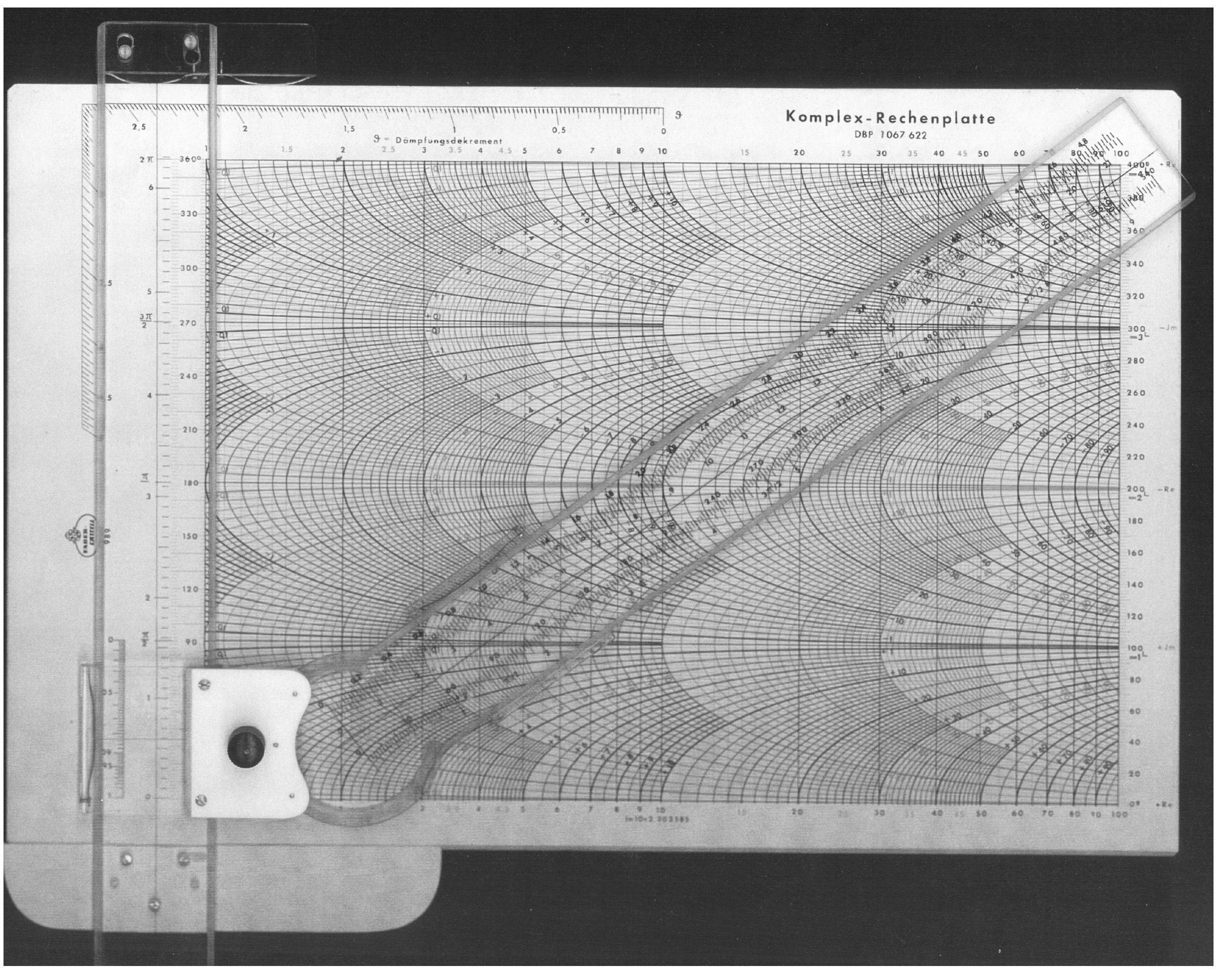
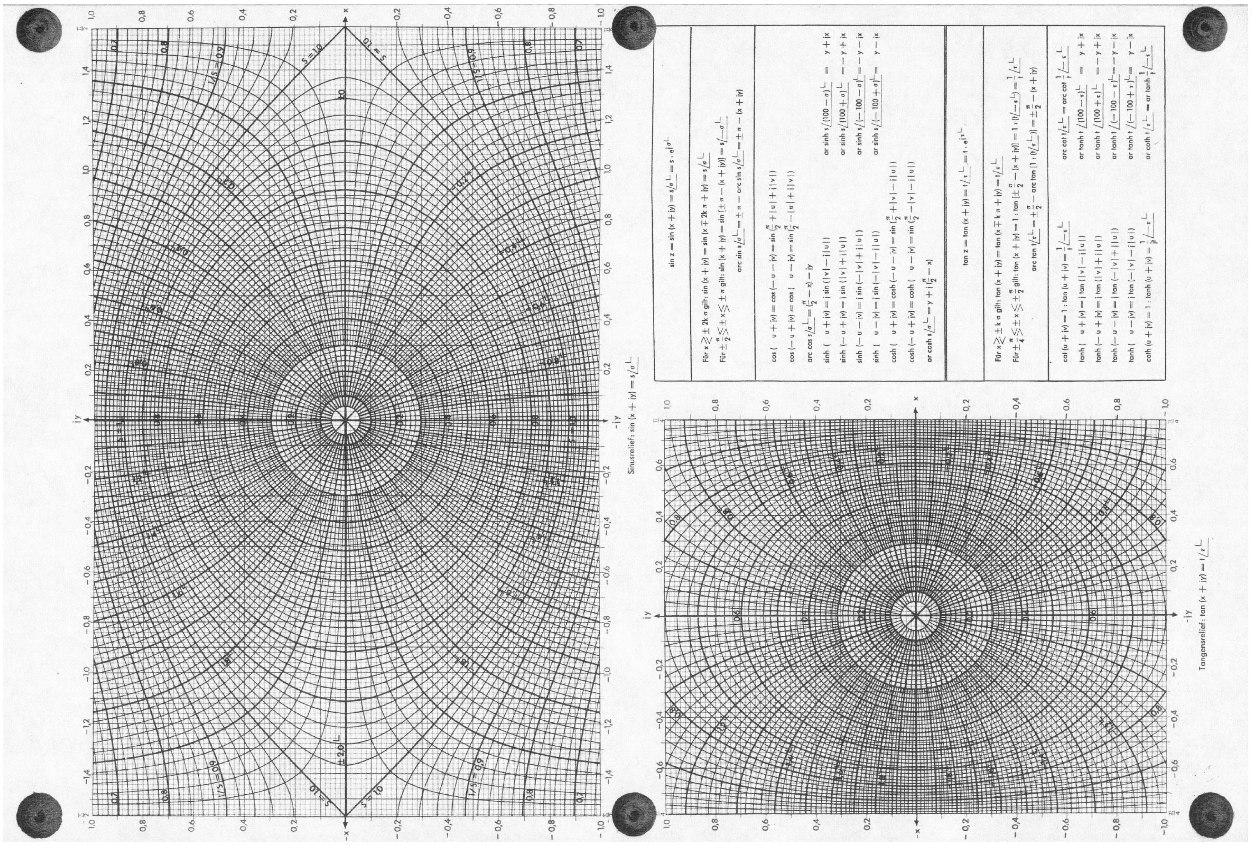




The Complex Number Nomogram

The representation of complex numbers as points on a grid on a Gaussian number plane has found wide application in pure mathematics and also in technology. This is especially true in electronics where calculations involving complex numbers are required to depict the vector of alternating current magnitude. However, the complex number plane is only a theoretical construct, and it is not well suited to practical calculations. The application of the formula

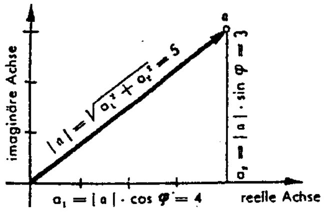
$$\ln(a_1 + j\,a_2) = \ln(|\alpha|\ e^{j\varphi}) = \ln|\alpha| + j\,\varphi.....$$

permits a satisfactory representation of complex numbers on a semi-log grid. This transformation of the complex number plane onto a semi-log grid has provided the foundation for a practical calculating aid, which has the added advantage of preserving completely the clarity of presentation.
This aid, which has been designated the Complex Number Nomogram, shows the above-mentioned semi-log grid in red on a base-plate.In accordance with the formula $\ln|\alpha| + j\varphi$, the angle (Versor) is shown as the linear ordinate and the absolute value (vector magnitude) on the logarithmic abscissa .
In that normal grid there are the transformed grid lines of the Gauß-numerical plane shown as family of curves, for the real component in black and for the imaginary components in blue.
The whole board shows all four quadrants, the first in the lower part, the fourth in the upper part.

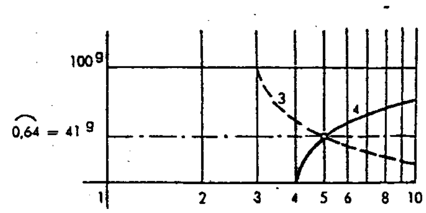
Multiplication, division, powers and roots can be done as on the slide rule by means of the rule appliance, movable by angle setting and parallel setting. Addition and subtraction can be worked out by simple reduction of the normal components.
Because the Complex Number Nomogram includes all four quadrants (the whole radian frequency), harmonically as well as exponentially damped oscillations can be represented and treated mathematically as straight lines.

1. Depiciton of complex numbers

Gauß'sche Zahlenebene



Komplex-Rechenplatte

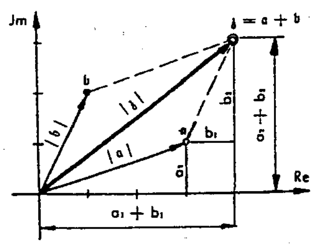


$$a = (a_1 + j\,a_2) = (4 + j\,3) = 5\,e^{j0,64} = 5\,\underline{/41^g}$$

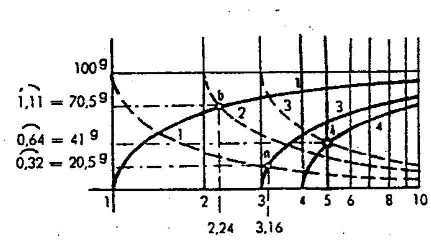
On the Komplex-Rechenplatte one can read for complex numbers the combination between vector and versor and otherwise between imaginary part and real part.

2. Addition of complex numbers

Gauß'sche Zahlenebene



Komplex-Rechenplatte



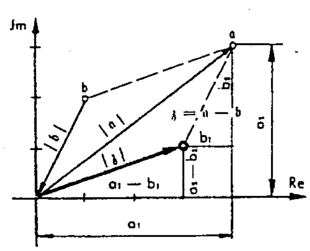
$$a + b = |a|\,e^{j\varphi} + |b|\,e^{j\psi} = a\underline{/\varphi} + b\underline{/\psi} = (a_1+ja_2) + (b_1+jb_2) = (a_1+b_1) + j(a_2+b_2)$$

$$3{,}16\,e^{j0,32} + 2{,}24\,e^{j1,11} = 3{,}16\underline{/20{,}5^g} + 2{,}24\underline{/70{,}5^g} = (3\times j1)+(1+j2) = (4+j3) = 5\,e^{j0,64} = 5\,\underline{/41^g}$$

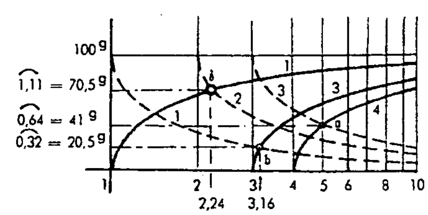
For one can find the real and imaginary components for each of these complex numbers, one only must add them und then is to find the new point, and coincidently one can find vector and versor.

3. Subtraction of complex numbers

Gauß'sche Zahlenebene



Komplex-Rechenplatte



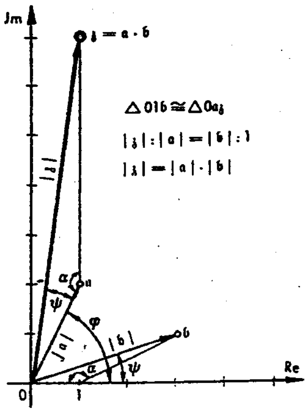
$$a - b = |a|\,e^{j\varphi} - |b|\,e^{j\psi} = a\underline{/\varphi} - b\underline{/\psi} = (a_1+ja_2) - (b_1+jb_2) = (a_1-b_1) + j\,(a_2-b_2)$$

$$5\,e^{j0,64} - 3{,}16\,e^{j0,32} = 5\underline{/41^g} - 3{,}16\underline{/20{,}5^g} = (4+j3) - (3+j1) = (1+j2) = 2{,}24\,e^{j1,11} = 2{,}24\underline{/70{,}5^g}$$

The operation is analogous to addition , but the components are subtracted.

4. Multiplication of complex numbers

Gauß'sche Zahlenebene



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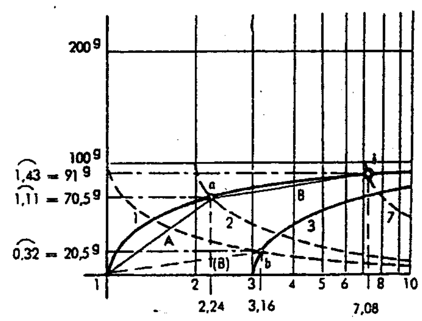


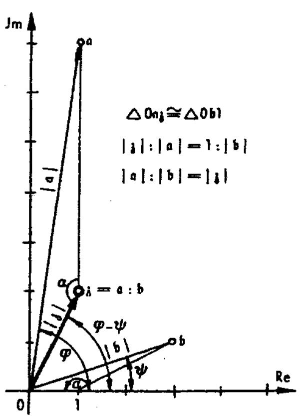
$a \cdot b = |a| e^{j\varphi} \cdot |b| e^{j\psi} = a/\underline{\varphi} \cdot b/\underline{\psi} = (a_1+ja_2)\cdot(b_1+jb_2) = a_1 b_1 - a_2 b_2 + j(a_1 b_2 + a_2 b_1) = |a| \cdot |b|\, e^{j(\varphi+\psi)} = ab/\underline{\varphi+\psi}$

$2{,}24\ e^{j1{,}11} \cdot 3{,}16\ e^{j0{,}32} = 2{,}24/\underline{70{,}5^g} \cdot 3{,}16/\underline{20{,}5^g} = (1+j2)\cdot(3+j1) = (1+j7) = 7{,}08/\underline{91^g} = 7{,}08\ e^{j1{,}43}$

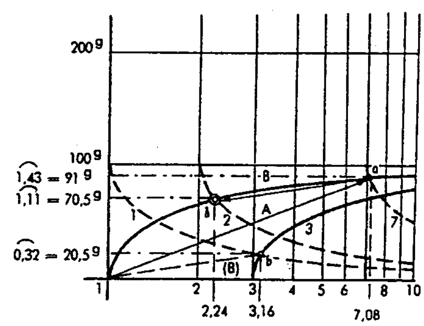
The geometric addition of the lines A and B by means of the movable rule appliance immediately locates the point $\mathfrak{z}$ with the value $7.08/91^g = (1 + j7) = 7{,}08\ e^{j1.43}$.

5. Division of complex numbers

Gauß'sche Zahlenebene



Komplex-Rechenplatte



$a : b = |a|\, e^{j\varphi} : |b|\, e^{j\psi} = a/\underline{\varphi} : b/\underline{\psi} = (a_1+ja_2) : (b_1+jb_2)$

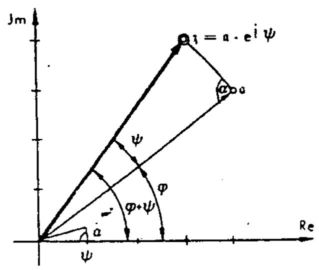
$$= \frac{(a_1+ja_2)\cdot(b_1-jb_2)}{b_1^2 + b_2^2} = \frac{a_1 b_1 + a_2 b_2}{b_1^2 + b_2^2} + j\,\frac{a_2 b_1 - a_1 b_2}{b_1^2 + b_2^2} = \frac{|a|}{|b|}\, e^{j(\varphi-\psi)} = \frac{a}{b}\,/\underline{\varphi-\psi}$$

$7{,}08\ e^{j1{,}43} : 3{,}16\ e^{j0{,}32} = 7{,}08/\underline{91^g} : 3{,}16/\underline{20{,}5^g} = (1+j7) : (3+j1) = (1+j2) = 2{,}24/\underline{70{,}5^g} = 2{,}24\ e^{j1{,}11}$

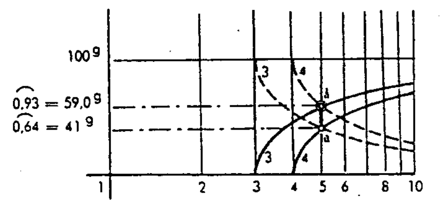
The geometric subtraction of the line B from line A by means of the movable rule appliance immediately locates the point $\mathfrak{z}$ with the value $2{,}24/\underline{70.5^g} = (1 + j2) = 2{,}24\ e^{j1.11}$.

6. Multiplication vs. Division by e^{Ψ}

Gauß'sche Zahlenebene



Komplex-Rechenplatte



$$a \cdot e^{j\psi} = |a|\, e^{j\varphi} \cdot e^{j\psi} = a/\underline{\varphi} \cdot 1/\underline{\psi} = |a|\, e^{j(\varphi+\psi)} = a/\underline{\varphi+\psi}$$

$$5\, e^{j0,64} \cdot e^{j0,29} = 5/\underline{41^g} \cdot 1/\underline{18^g} = 5\, e^{j0,93} = 5/\underline{59,0^g}$$

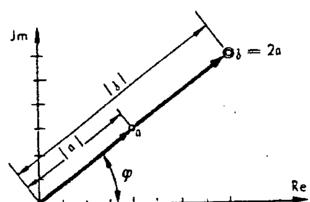
$$(4+j3) \cdot (0,97+j0,29) = (3,88-0,88) + j\,(1,1+2,9) = 3+j4$$

The multiplication with the unit vector $1/\underline{\Psi}$ is done with the Komplex-Rechenplatte by a vertical shift of point α upward to the ordinate $\varphi + \psi$ in point $\mathfrak{z}$. (Note: The Gauß numerical plane shows vector α as a turn around the angle ψ counterclockwise, that means a multiplication of the unit vector $1 / \psi$.

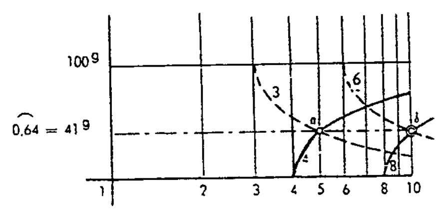
The division in the Gauß numerical plane depends on a turn around clockwise and on the complex device to a vertical shift downwards.

7. Multiplication vs. division of complex number by a real number.

Gauß'sche Zahlenebene



Komplex-Rechenplatte

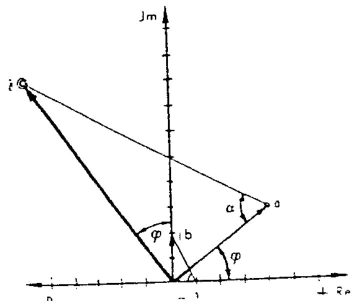


$\cdot\, p = p \cdot |a|\, e^{j\varphi} = p \cdot a/\underline{\varphi} = (a_1+ja_2) \cdot p = (pa_1+jpa_2)$ $\quad 2 \cdot 5\, e^{j0,64} = 2 \cdot 5/\underline{41^g} = (4+j3) \cdot 2 = 10\, e^{j0,64} = 10/\underline{41^g} = (8+j6)$

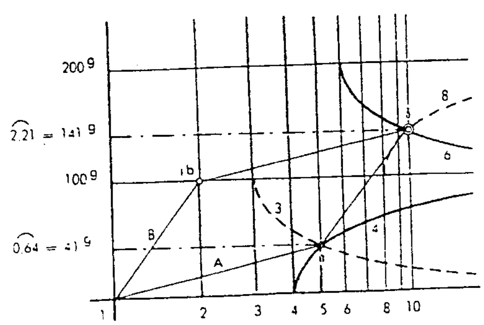
The multiplication of a complex number α by a real number p means an increase of the vector magnitude by a factor of p while the the angle φ remans unchanged. It is shown on the complex device as a horizontal shift of point α to right with distance d to abszissa $|\alpha|$ x p at point $\mathfrak{z}$.
Division corresponds to a decrease in the magnitude of the vector by a factor of p while the angle φ remains unchanged. This is represented on the complex number nomogram by a horizontal shift of point $\mathfrak{z}$ to the left by the distance p to the abscissa value $|\mathfrak{z}|/p$ at point α.

8. Multiplication vs. division of complex number by an imaginary number.

Gauß'sche Zahlenebene



Komplex-Rechenplatte



There follows:

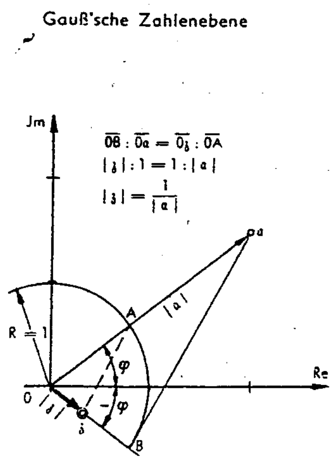
$$e^{j\pi/2} = j = 1/\underline{100^g};\quad e^{j\pi} = -1 = 1/\underline{200^g};\quad e^{-j\pi/2} = -j = 1/\underline{300^g} \text{ usw.}$$

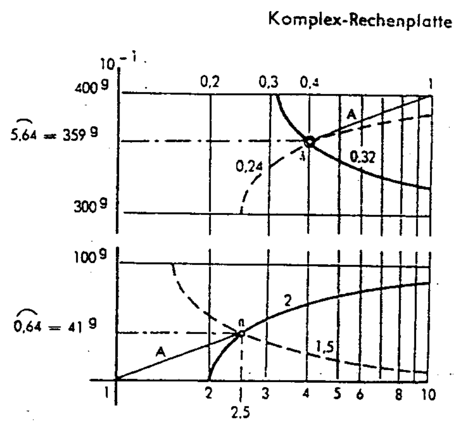
$$a \cdot jb = |a|\, e^{j\varphi} \cdot |b|\, e^{j\pi/2} = a/\underline{\varphi} \cdot b/\underline{100^g} = (a_1+ja_2)\cdot jb = (-a_2 b + j a_1 b) = |a| \cdot b\, e^{j(\varphi+\pi/2)} = ab/\underline{\varphi+100^g}$$

$$5\, e^{j0,64} \cdot 2\, e^{j\pi/2} = 5/\underline{41^g} \cdot 2/\underline{100^g} = (4+j3)\cdot j2 = 10\, e^{j2,21} = (-6+j8) = 10/\underline{141^g}$$

Multiplication is on the complex device same as multiplication of complex numbers, that means a geometrical addition of lines A and B.
Division same as division of complex numbers, that means a geometrical subtraction.

9. Reciprocal value of a complex number





$$\frac{1}{a} = \frac{1}{|a|\, e^{j\varphi}} = \frac{1}{a/\underline{\varphi}} = \frac{1}{a_1+ja_2} = \frac{a_1-ja_2}{a_1^2+a_2^2} = \frac{a_1}{a_1^2+a_2^2} - j\,\frac{a_2}{a_1^2+a_2^2}$$

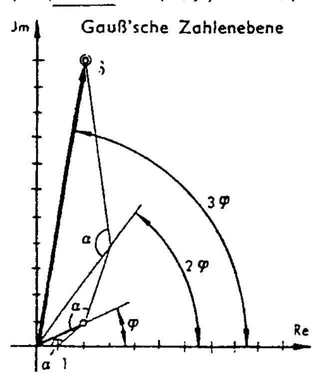
$$\frac{1}{2{,}5\, e^{j0,64}} = \frac{1}{2{,}5/\underline{41^g}} = \frac{1}{2+j1{,}5} = \frac{2}{6{,}25} - j\,\frac{1{,}5}{6{,}25} = 0{,}32 - j0{,}24 = 0{,}4/\underline{-41^g} = 0{,}4/\underline{359^g} = 0{,}4\, e^{j5,64}$$

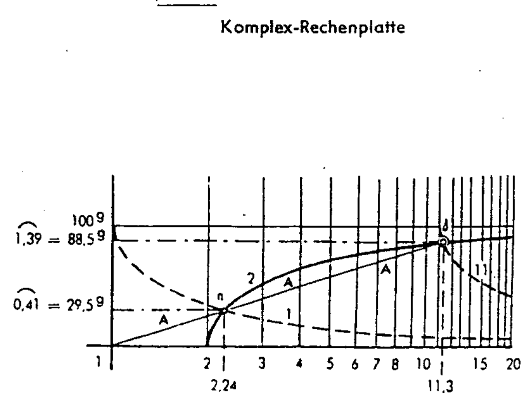
To have a reciprocal value of a complex number means a division into 1 and is done with the complex device as a geometric subtraction beginning from point 1. Note, that the 4th quadrant of the foregoing period is not existing and must be substituted by (a) 4th quadrant, which must be imagined to be shifted by one (or, if necessesary , two) decades to the left.

10. Potentation of a complex number

$$a^n = (|a|\, e^{j\varphi})^n = (a/\underline{\varphi})^n = (a_1+ja_2)^n = |a|^n \cdot e^{jn\varphi} = a^n/\underline{n\varphi} = z/\underline{\psi}$$

$$(2{,}24\, e^{j0,46})^3 = (2{,}24/\underline{29{,}5^g})^3 = (2+j1)^3 = 2+j11 = 11{,}3\, e^{j1,39} = 11{,}3\,/\underline{88{,}5^g}$$





Potentation is accomplished by extending line segment A the number of times indicated by the exponent. A linear scale (e.g. the ln scale on the swiveling ruler is well suited to this purpose).

11. Extracting a root of a complex number

To extract a root of a complex number reverse the procedure outlined for potentiation. Use a slide rule to divide the measured length of line A.

$$\sqrt[n]{\mathfrak{z}} = \sqrt[n]{|\mathfrak{z}| \cdot e^{j\psi}} = \sqrt[n]{z\underline{/\psi}} = \sqrt[n]{z_1 + jz_2} = \sqrt[n]{|\mathfrak{z}|} \cdot e^{j\frac{\psi}{n}} = \sqrt[n]{z}\,\underline{/\psi/n} = a\underline{/\varphi} = |a|\,e^{j\varphi} = a$$

12. Taking the logarithm of complex numbers

$\ln a = \ln(|a|\,e^{j\varphi}) = \ln|a| + j\varphi + j2k\pi$ with $k = 0;\ \pm 1;\ \pm 2\ \ldots$

we have: $\ln j = j\,\pi/2;\ \ln -1 = j\pi;\ \ln(-j) = j\,\frac{3\pi}{2}$

$\ln(4+j3) = \ln(5e^{j0{,}64}) = 1{,}61 + j\,0{,}64 = 1{,}74\underline{/24{,}1^g} = 1{,}74\,e^{j0{,}377}$

The natural logarithm of the absolute value α is read by means of the ln-scale (linear scale up to 4.6.) on the swiveling ruler.
It is also possible to determine powers and roots with exponents. That are not integrals.

$(4+j3)^{1{,}2} = (5e^{j0{,}64})^{1{,}2} = \mathfrak{z};\ \ln\mathfrak{z} = 1{,}2 \cdot \ln(5e^{j0{,}64}) = 1{,}2\,(1{,}61+j0{,}64)$

$\ln\mathfrak{z} = 1{,}93+j0{,}77;\ \mathfrak{z} = 6{,}9\,e^{j0{,}77} = 6{,}9\underline{/49{,}0^g} = 4{,}95+j4{,}81$

13. Harmonic oscillation

A line parallel to the ordinate represents a moving vector of constant magnitude. Reading of the intersecting point at the black graticule curves gives a cosine wave and with the blue ones a sine wave. That depiction also shows the correlation of the two vertical arranged even oscillations of equal amplitude, which complement to a circular oscillation.
The two lower scales at the swiveling ruler (ordinate scales) make possible to show lead and lag in phase oscillations.One puts the rule parallel to the ordinate axis and moves depending to lead or lag in phase.
When adding the individual values one can find as well additive as multiplicative oscillationsuperhets.

14. Exponential damped oscillation

On the complex number nomogramm a line inclined toward the ordinate represents an exponentially damped oscillation with a dampening decrement δ, which is indicated by the angle of inclination. To fix a damping oscillation, take off the swiveling rule (guide part also) and put it reversed again (then the swiveling ruler is positioned left from the vertical rule. The lockable point of rotation then is near to the lower edge of the complex device. Now put the swiveling ruler in this way, that its midline points to the 10 of the lowest real basic scale and to the desired dampening decrement of the scale at the upper edge of complex device. Then move the midline to the original value of vector R.
Now you can read all intermediate values 0 to 2π as well as vector value and versor, and real and imaginary components. To follow the damped oscillation over the 1st period , one transfers the vector value $r_{2\pi}$ (found at 2 π) to the lowest basic scale and then place the midline of the swiveling rule to that new found value. By going on in that way one can find the oscillations optional up to the complete cease oscillation. The exponential damped oscillation relates to:

$$Y = e^{-\frac{\varphi}{2\pi}} \cdot R\,(\cos\varphi + j\sin\varphi) \quad \text{oder} \quad Y = e^{-\frac{\omega t}{2\pi}} \cdot R\left(\cos(\omega t) + j\sin(\omega t)\right)$$

We have also:

$$r = e^{-\frac{\varphi}{2\pi}} \cdot R \text{ und } x = r \cdot \cos\varphi \text{ bzw. } y = r \cdot j\sin\varphi$$

In addition an example of an exponential damped oscillation with an original value of vector R = 10 and the damping decrement δ = 1 .

We have then: Für $\varphi = 2\pi$

$Y = e^{-1} \cdot 10\,(\cos 2\pi + j\sin 2\pi)$

$Y = 10/e \cdot (1 + j0) = 10/e = 3{,}68$

für $\varphi = 4\pi$

$Y = e^{-2} \cdot 10\,(\cos 4\pi + j\sin 4\pi)$

$Y = 10/e^2 = 10/7{,}39 = 1{,}35$

Damping decrement then is:

$$\vartheta = \ln\frac{R}{r_{2\pi}} = \ln\frac{r_{2\pi}}{r_{4\pi}} = \ldots\ldots = \ln\frac{10}{3{,}68} = \ln\frac{3{,}68}{1{,}35} = 1$$

The function´s progress is shown in the following tabell and drawing.

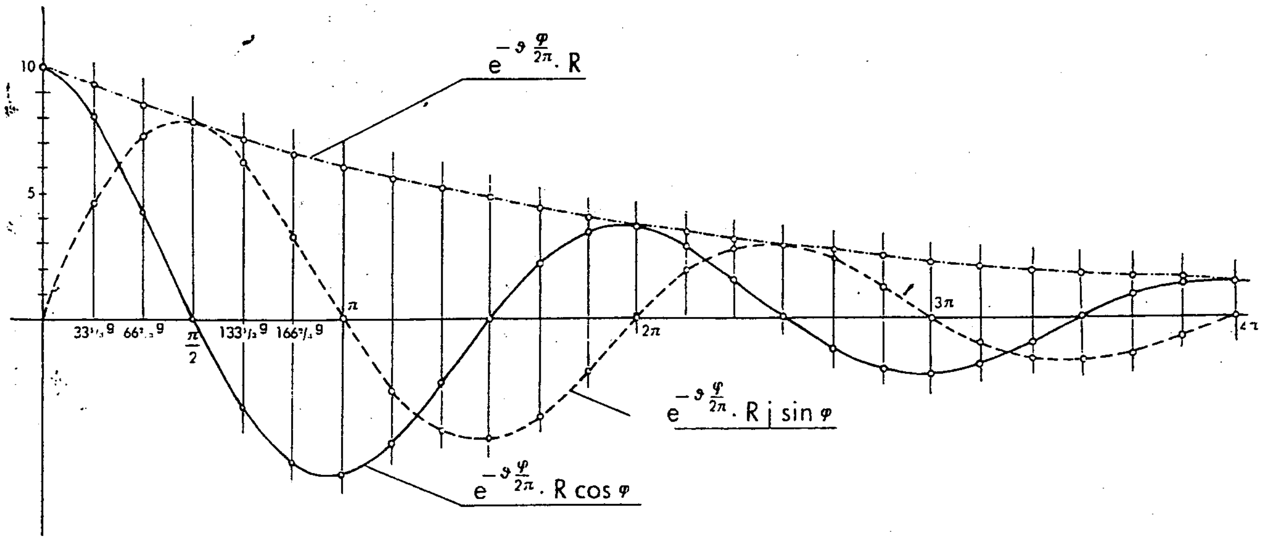
Exponential damped oscillation ($\delta = 1$ and $R = 10$)

$$Y = e^{-\vartheta \frac{\varphi}{2\pi}} R (\cos\varphi + j \sin\varphi) = e^{-1 \frac{\varphi}{2\pi}}\, 10 (\cos\varphi + j \sin\varphi);\; r = e^{-\vartheta \frac{\varphi}{2\pi}} \cdot R;\; x = r \cdot \cos\varphi;\; j\,y = r \cdot j \sin\varphi$$

dampening decrement $= \ln \frac{R}{r_{2\pi}} = \ln \frac{r_{2\pi}}{r_{4\pi}} \ldots = \ln \frac{10}{3{,}68} = \ln \frac{3{,}68}{1{,}35} = \ln e = 1$

Versor $/\varphi$	0	$33{,}3^g$	$66{,}6^g$	$\frac{\pi}{2}$	$133{,}3^g$	$166{,}6^g$	π	$233{,}3^g$	$266{,}6^g$	$\frac{3\pi}{2}$	$333{,}3^g$	$366{,}6^g$	2π
Vektor r	10	9,25	8,50	7,80	7,20	6,65	6,10	5,60	5.15	4,75	4,40	4,00	3,68
r. Komp. x	10	8,00	4,25	0	-3,60	-5,75	-6,10	-4,80	-2,60	0	2,20	3,50	3,68
im. Komp. jy	0	4.60	7,30	7,80	6,25	3,35	0	-2,80	-4,45	-4,75	-3,80	-2,00	0

Versor $/\varphi$	$433{,}3^g$	$466{,}6^g$	$\frac{5\pi}{2}$	$533{,}3^g$	$566{,}6^g$	3π	$633{,}3^g$	$666{,}6^g$	$\frac{7\pi}{2}$	$733{,}3^g$	$766{,}6^g$	4π
Vektor r	3,40	3,12	2,85	2,63	2,43	2,23	2,05	1,90	1,73	1,60	1,47	1,35
r. Komp. x	2,95	1,55	0	-1,32	-2,10	-2,23	-1,78	-0,95	0	0,80	1,28	1,35
im. Komp. jy	1,70	2,70	2,85	2,30	1,20	0	-1,03	-1,65	-1,73	-1,40	-0,74	0



15. Circular functions and Hyperbolic functions of complex arguments

To find circular- and hyperbolic functions of complex arguments find the both graphs of sinus- and tangens reliefs on the reverse side of the complex device.On the Gaußian numerical plane one finds a read rightangled graticule , subdivided in that way, that there is a real part and an imaginary part of the argument at each point. Superimposed on the red grid there is a graticule of orthogonal family of curves (black) for read-ing of the functional value depending on vector value and versor.
To find hyperbolic functions from circular functions you find a table of formulae near the tangent relief.

a. Sinus of complex number

If the argument is fixed with real part and imaginary part (x + jy), you can find the value s and the versor σ^g of the functional value directly from the sinus relief.

$$\sin z = \sin(x + jy) = s \,/\underline{\sigma^g} = s \cdot e^{j\sigma^g}$$

F. ex. :

$$\sin(0{,}9 + j\,0{,}46) = 0{,}92 \,/\underline{21^g}$$
$$\sin(-0{,}38 + j\,0{,}53) = 0{,}67 \,/\underline{144^g}$$
$$\sin(-0{,}20 - j\,0{,}90) = 0{,}94 \,/\underline{-118^g}$$
$$\sin(0{,}8 - j\,0{,}48) = 0{,}88 \,/\underline{-26^g}$$

For $x \gtrless \pm 2k\pi$ gilt: $\sin(x + jy) = \sin(x \mp 2k\pi + jy) = s \,/\underline{\sigma^g}$

$$\sin(6{,}08 - j\,0{,}9) = \sin(6{,}08 - 6{,}28 - j\,0{,}9)$$
$$= \sin(-0{,}2 - j\,0{,}9) = 0{,}94 \,/\underline{-118^g}$$
$$\sin(-6{,}66 + j\,0{,}53) = \sin(-6{,}66 + 6{,}28 + j\,0{,}53)$$
$$= \sin(-0{,}38 + j\,0{,}53) = 0{,}67 \,/\underline{144^g}$$

For $\pm \frac{\pi}{2} \lesseqgtr \pm x \lesseqgtr \pm \pi$ gilt: $\sin(x + jy) = \sin(\pm\pi - (x + jy)) = s \,/\underline{-\sigma^g}$

$$\sin(2{,}24 + j\,0{,}46) = \sin(3{,}14 - (2{,}24 + j\,0{,}46))$$
$$= \sin(0{,}9 - j\,0{,}46) = 0{,}92 \,/\underline{-21^g}$$
$$\sin(-2{,}34 - j\,0{,}48) = \sin(-3{,}14 - (-2{,}34 - j\,0{,}48))$$
$$= \sin(-0{,}8 + j\,0{,}48) = 0{,}88 \,/\underline{174^g}$$

b. To find argument with fixed sinus value

You use the frontside of the complex device: If the sine value is given as a + jb, you must transform to $r / \underline{\varphi^g} = s / \underline{\sigma^g}$. With s and σ you find at the sinus relief the values x and y including signs.

F. ex.:

$\arcsin 0{,}92 \;/\!\underline{-21^g} = 0{,}9 - j\,0{,}46$ or also with $\pi-(x+jy) = 2{,}24 + j\,0{,}46$ *)

$\arcsin 0{,}88 \;/\;174^g = -0{,}8 + j\,0{,}48$ or also with $-\pi-(x+jy) = -2{,}34 - j\,0{,}48$ *)

c. Cosine of a complex number

Cosine is converted to sine by the following formulae:

$$\cos(u + jv) = \cos(-u - jv) = \sin(\tfrac{\pi}{2} + |u| + j\,|v|)$$

$$\cos(-u + jv) = \cos(u - jv) = \sin(\tfrac{\pi}{2} - |u| + j\,|v|)$$

F.ex.: $\cos(0{,}67+j\,0{,}46) = \sin(1{,}57+0{,}67+j\,0{,}46) = \sin(2{,}24 + j\,0{,}46) = 0{,}92 \;/\!\underline{-21^g}$

$\cos(1{,}83 - j\,0{,}3) = \sin(1{,}57-1{,}83+j\,0{,}3) = \sin(-0{,}26 + j\,0{,}3) = 0{,}4 \;/\!\underline{148^g}$

d. To find argument with fixed cosine value

The same as **b.** and to put the got values x and y with sign into the following formula:

$$\arccos s\,/\!\underline{\sigma^g} = (\tfrac{\pi}{2} - x) - jy$$

$\arccos 0{,}4 \;/\!\underline{148^g}$; mit $\sigma = 148^g$ und $s = 0{,}4$ one gets from the sine relief for $x = -0{,}26$ und für $y = 0{,}3$

$\arccos 0{,}4 \;/\!\underline{148^g} = (1{,}57 + 0{,}26) - j\,0{,}3 = 1{,}83 - j\,0{,}3$

$\arccos 0{,}67 \;/\!\underline{-144^g}$; ($x = -0{,}38$ und $y = -0{,}53$)

$\arccos 0{,}67 \;/\!\underline{-144^g} = (1{,}57+0{,}38) + j\,0{,}53 = 1{,}95 + j\,0{,}53$

* If there are ambigous results better to use additionally (also for sake of clarity) the extended sine relief (see page 23).

e. Hyperbolic sine of complex number

Hyperbolic sine is transformed to circular sine by the following formulae:

$\mathfrak{Sin}(u+jv) = j\,\sin(|v| - j\,|u|)$; $\mathfrak{Sin}(-u-jv) = j\,\sin(-|v| + j\,|u|)$

$\mathfrak{Sin}(-u+jv) = j\,\sin(|v| + j\,|u|)$; $\mathfrak{Sin}(u-jv) = j\,\sin(-|v| - j\,|u|)$

F.ex.: $\mathfrak{Sin}(0{,}8-j\,0{,}22) = j\,\sin(-0{,}22-j\,0{,}8) = j\,0{,}91 \;/\!\underline{-121^g}$; to multiply by j means a turnaround $+100^g$

$\mathfrak{Sin}(0{,}8-j\,0{,}22) = 0{,}91 \;/\!\underline{-21^g}$

$\mathfrak{Sin}(0{,}35+j\,6{,}49) = j\,\sin(6{,}49-j0{,}35)$; $6{,}49 - 2\pi = 6{,}49-6{,}28 = 0{,}21$;

$= j\,\sin(0{,}21-j\,0{,}35) = j\,0{,}42 \;/\!\underline{-62^g} = 0{,}42 \;/\!\underline{38^g}$

f. To find argument with fixed Hyperbolic sine

Depending, if σ^g at 1st, 2nd, 3rd, or 4th quadrant, σ^g is to set equal $(100 - \sigma)^g$, $(100 + \sigma)^g$ $(-100 - \sigma)^g$ or $(-100 + \sigma)^g$, and then one reads x and y from the sine relief and finds the argument by the following formulae:

$\mathfrak{Ar}\,\mathfrak{Sin}\, s\,/\!\underline{(100-\sigma)^g} = y+jx$; $\mathfrak{Ar}\,\mathfrak{Sin}\, s\,/\!\underline{(-100-\sigma)^g} = -y-jx$

$\mathfrak{Ar}\,\mathfrak{Sin}\, s\,/\!\underline{(100+\sigma)^g} = -y+jx$; $\mathfrak{Ar}\,\mathfrak{Sin}\, s\,/\!\underline{(-100+\sigma)^g} = y-jx$

F.ex.:

$\mathfrak{Ar}\,\mathfrak{Sin}\, 0{,}91 \;/\!\underline{-21^g}$; $s\,/\!\underline{(-100+\sigma)^g} = 0{,}91 \;/\!\underline{79^g}$; $x=0{,}22$, $y=0{,}8$

$\mathfrak{Ar}\,\mathfrak{Sin}\, 0{,}91 \;/\!\underline{-21^g} = 0{,}8 - j\,0{,}22$

$\mathfrak{Ar}\,\mathfrak{Sin}\, 0{,}42 \;/\!\underline{38^g}$; $s\,/\!\underline{(100-\sigma)^g} = 0{,}42 \;/\!\underline{62^g}$; $x=0{,}22$, $y=0{,}35$

$\mathfrak{Ar}\,\mathfrak{Sin}\, 0{,}42 \;/\!\underline{38^g} = 0{,}35 + j\,0{,}22$

g. Hyperbolic cosine of complex number

The Hyperbolic sine is transformed to circular sine by the following formulae:

$$\mathfrak{Cos}(u+jv) = \mathfrak{Cos}(-u-jv) = \sin(\tfrac{\pi}{2} + |v| - j\,|u|)$$

$$\mathfrak{Cos}(-u+jv) = \mathfrak{Cos}(u-jv) = \sin(\tfrac{\pi}{2} - |v| - j\,|u|)$$

F. ex.:

$\mathfrak{Cos}(-0{,}42+j\,0{,}45) = \sin(1{,}57-0{,}45-j\,0{,}42) = \sin(1{,}12 - j\,0{,}42) = 1{,}0 \;/\!\underline{-12^g}$

$\mathfrak{Cos}(-0{,}46-j\,0{,}67) = \sin(1{,}57+0{,}67-j\,0{,}46) = \sin(2{,}24 - j\,0{,}46) = \sin(\pi-(x+jy)) = \sin(0{,}9 + j\,0{,}46) = 0{,}92 \;/\!\underline{21^g}$

h. To find argument with fixed Hyperbolic cosine

The same as **b.** and to put the got values x and y with sign into the following formulae:

$$s\ \underline{/\sigma^g} = y + j\left(\frac{\pi}{2} - x\right)$$

F. ex.:

$1{,}0\ \underline{/-12^g}$; 4. Quadrant; $x = 1{,}12,\ y = -0{,}42$

$1{,}0\ \underline{/-12^g} = -0{,}42 + j(1{,}57 - 1{,}12) = -0{,}42 + j\ 0{,}45$

but also : $x=2{,}02;\ y=0{,}42$ mit $\pi-(x+jy)$ and then

$1{,}0\ \underline{/-12^g} = 0{,}42 + j(1{,}57 - 2{,}02) = 0{,}42 - j\ 0{,}45$

$0{,}92\ \underline{/21^g}$; 1. Quadrant; $x=0{,}9;\ y=0{,}46$ bzw. $x= 2{,}24;\ y=-0{,}46$

$0{,}92\ \underline{/21^g} = 0{,}46 + j\ 0{,}67$ bzw. $= -0{,}46 - j\ 0{,}67$

i. Tangent of complex number

If argument is given with real part and imaginary part <u>x + jy</u> , one can find the vector value ι and the versor τ^g of the functional value from the tangent relief directly.

$$\mathrm{tg}\ z = \mathrm{tg}\ (x+jy) = \iota\ \underline{/\tau^g} = \iota \cdot e^{j\tau^g}$$

F. ex.: $\mathrm{tg}\ (0{,}36 + j\ 0{,}6) = 0{,}64\ \underline{/74^g}$

$\mathrm{tg}\ (-0{,}44 - j\ 0{,}38) = 0{,}58\ \underline{/-147^g}$

ür $x \lesseqgtr \pm k\pi$ gilt: $\mathrm{tg}(x+jy) = \mathrm{tg}\ (x \mp k\pi + jy) = \iota\ \underline{/\tau^g}$

$\mathrm{tg}\ (-9{,}69 - j\ 0{,}34) = (-x + k\pi - jy) = (-9{,}69+3\pi - j\ 0{,}34) = (-0{,}26 - j\ 0{,}34) = 0{,}42\ \underline{/-138^g}$

ür $\pm\frac{\pi}{4} \lesseqgtr \pm x \lesseqgtr \pm\frac{\pi}{2}$ gilt: $\mathrm{tg}(x+jy) = 1 : \mathrm{tg}(\pm\frac{\pi}{2} - (x+jy)) = 1 : (\iota\ \underline{/-\tau^g}) = \frac{1}{\iota}\ \underline{/\tau^g}$

$\mathrm{tg}\ (1{,}22 - j\ 0{,}86) = 1 : \mathrm{tg}(1{,}57 - 1{,}22 + j\ 0{,}86) = 1 : \mathrm{tg}(0{,}35 + j\ 0{,}86) = 1 : (0{,}76\ \underline{/85^g}) = 1{,}32\ \underline{/-85^g}$

j. To find argument with fixed tangent value

If tangent value is given (a + jb), take the version $r\underline{/\ \sigma^g} = \iota\ \underline{/\ \tau^g}$. With ι and τ use the tangent relief to find the values x and y and their signs.

F.ex.: $\mathrm{arc\ tg}\ 0{,}58\ \underline{/-147^g} = -0{,}44 - j\ 0{,}38$ und auch $= -\frac{\pi}{2} - \mathrm{arc\ tg}\ 1{,}72\ \underline{/147^g} = -1{,}13 - j0{,}38$ *)

$\mathrm{arc\ tg}\ 1{,}32\ \underline{/-85^g} = \frac{\pi}{2} - \mathrm{arc\ tg}\ 0{,}76\underline{/85^g} = 1{,}22 - j0{,}86$ and also $= 0{,}35 - j\ 0{,}86$ *)

k. Cotangent of complex number

Cotangent is transposed to circular tangent by the following formulae:

$$\mathrm{ctg}\ (u+jv) = 1 : \mathrm{tg}(u+jv) = \frac{1}{\iota}\ \underline{/-\tau^g}$$

F.ex.:

$\mathrm{ctg}\ (0{,}5 + j0{,}67) = 1 : \mathrm{tg}(0{,}5+j0{,}67) = 1 : (0{,}76\underline{/72^g}) = 1{,}32\ \underline{/-72^g}$

$\mathrm{ctg}\ (-0{,}38 - j0{,}74) = 1 : \mathrm{tg}(-0{,}38 - j0{,}74) = 1 : (0{,}72\ \underline{/-120^g}) = 1{,}39\ \underline{/120^g}$

l. To find argument with fixed cotangent value

Use the following relationship: $\mathrm{arc\ ctg}\ \iota\underline{/\tau^g} = \mathrm{arc\ tg}\ \frac{1}{\iota}\ \underline{/-\tau^g}$

F.ex.:

$\mathrm{arc\ ctg}\ 2{,}08\ \underline{/-144^g} = \mathrm{arc\ tg}\ \frac{1}{2{,}08}\underline{/144^g} = \mathrm{arc\ tg}\ 0{,}48\ \underline{/144^g} = -0{,}34 + j\ 0{,}35$

$\mathrm{arc\ ctg}\ 0{,}76\ \underline{/85^g} = \mathrm{arc\ tg}\ 1{,}32\ \underline{/-85^g} = 1{,}22 - j\ 0{,}86$

m. Hyperbolic tangent of complex number

Hyperbolic tangent is transformed to circular tangent by the following formulae:

$\mathfrak{T}\mathrm{g}\ (u+jv) = j\ \mathrm{tg}(|v| - j\ |u|)$; $\mathfrak{T}\mathrm{g}\ (-u-jv) = j\ \mathrm{tg}(-|v| + j\ |u|)$

$\mathfrak{T}\mathrm{g}\ (-u+jv) = j\ \mathrm{tg}(|v| + j\ |u|)$; $\mathfrak{T}\mathrm{g}\ (u-jv) = j\ \mathrm{tg}(-|v| - j\ |u|)$

F.ex.: :

$\mathfrak{T}\mathrm{g}\ (-0{,}64+j0{,}22) = j\ \mathrm{tg}(+0{,}22 + j\ 0{,}64) = j\ 0{,}60\ \underline{/84^g}$ to multiply by j means a turnaround by $+100^g$

$\mathfrak{T}\mathrm{g}\ (-0{,}64+j0{,}22) = 0{,}6\ \underline{/184^g}$

$\mathfrak{T}\mathrm{g}\ (-0{,}54-j0{,}44) = j\ \mathrm{tg}\ (-0{,}44 + j\ 0{,}54) = j\ 0{,}66\ \underline{/134^g} = 0{,}66\ \underline{/-166^g}$

- * If there are one-many results better to use additionally (also for sake of clarity) the extended sine relief (see page 23).

n. To find argument with fixed Hyperbolic tangent

Depending, if φ^g at 1st, 2nd, 3rd, or 4th quadrant, φ^g is to set equal $(100-\tau)^g$, $(100+\tau)^g$, $(-100-\tau)^g$ or $(-100+\tau)^g$, and then one reads x and y from the tangent relief and finds the argument by the following formulae:

$$\mathfrak{Ar}\,\mathfrak{Tg}\; t\; \underline{/(100-\tau)^g} = y + jx; \qquad \mathfrak{Ar}\,\mathfrak{Tg}\; t\; \underline{/(-100-\tau)^g} = -y - jx$$
$$\mathfrak{Ar}\,\mathfrak{Tg}\; t\; \underline{/(100+\tau)^g} = -y + jx; \qquad \mathfrak{Ar}\,\mathfrak{Tg}\; t\; \underline{/(-100+\tau)^g} = y - jx$$

F. ex.:

$$\mathfrak{Ar}\,\mathfrak{Tg}\; 0{,}6\; \underline{/184^g};\; s\; \underline{/(100+\tau)^g} = 0{,}6\; \underline{/84^g};\; x = 0{,}22;\; y = 0{,}64$$
$$\mathfrak{Ar}\,\mathfrak{Tg}\; 0{,}6\; \underline{/184^g} = -0{,}64 + j0{,}22$$
$$\mathfrak{Ar}\,\mathfrak{Tg}\; 0{,}66\; \underline{/-166^g};\; s\; \underline{/(-100-\tau)^g} = 0{,}66\; \underline{/-66^g};\; x=0{,}44;\; y=0{,}54$$
$$\mathfrak{Ar}\,\mathfrak{Tg}\; 0{,}66\; \underline{/-166^g} = -0{,}54 - j0{,}44$$

o. Hyperbolic cotangent of complex number

Hyperbolic cotangent is transposed to hyperbolic tangent by the following formula:

$$\mathfrak{Ctg}\,(u+jv) = 1 : \mathfrak{Tg}\,(u+jv) = \frac{1}{jt}\; \underline{/-\tau^g}$$

F. ex.:

$$\mathfrak{Ctg}\,(0{,}76-j0{,}18) = 1 : \mathfrak{Tg}\,(0{,}76-j0{,}18) = 1 : j\,tg(-0{,}18-j0{,}76) = 1 : (j0{,}66\; \underline{/-110^g}) = (1{,}52\; \underline{/110^g}) : j$$

To divide by j means a rotation by -100^g

$$\mathfrak{Ctg}\,(0{,}76-j0{,}18) = 1{,}52\; \underline{/10^g}$$
$$\mathfrak{Ctg}\,(-0{,}18-j0{,}76) = 1 : \mathfrak{Tg}\,(-0{,}18-j0{,}76) = 1 : j\,tg(-0{,}76+j0{,}18)$$
$$= 1 : (j0{,}94\,\underline{/178^g}) = (1{,}06\,\underline{/-178^g}) : j$$
$$\mathfrak{Ctg}\,(-0{,}18-j0{,}76) = 1{,}06\; \underline{/122^g}$$

p. To find argument with Hyperbolic cotangent

Take the correlation

$$\mathfrak{Ar}\,\mathfrak{Ctg}\; t\; \underline{/\tau^g} = \mathfrak{Ar}\,\mathfrak{Tg}\; \frac{1}{t}\; \underline{/-\tau^g}$$

F. ex.:

$$\mathfrak{Ar}\,\mathfrak{Ctg}\; 1{,}52\; \underline{/10^g} = \mathfrak{Ar}\,\mathfrak{Tg}\; 0{,}66\; \underline{/-10^g};\; t\; \underline{/(-100+90)^g};\; x=0{,}18;\; y = 0{,}76$$
$$= 0{,}76 - j\; 0{,}18$$
$$\mathfrak{Ar}\,\mathfrak{Ctg}\; 1{,}06\; \underline{/122^g} = \mathfrak{Ar}\,\mathfrak{Tg}\; 0{,}94\; \underline{/-122^g};\; t\; \underline{/(-100-22)^g};\; x = 0{,}76;\; y = 0{,}18$$
$$= -0{,}18 - j\; 0{,}76$$

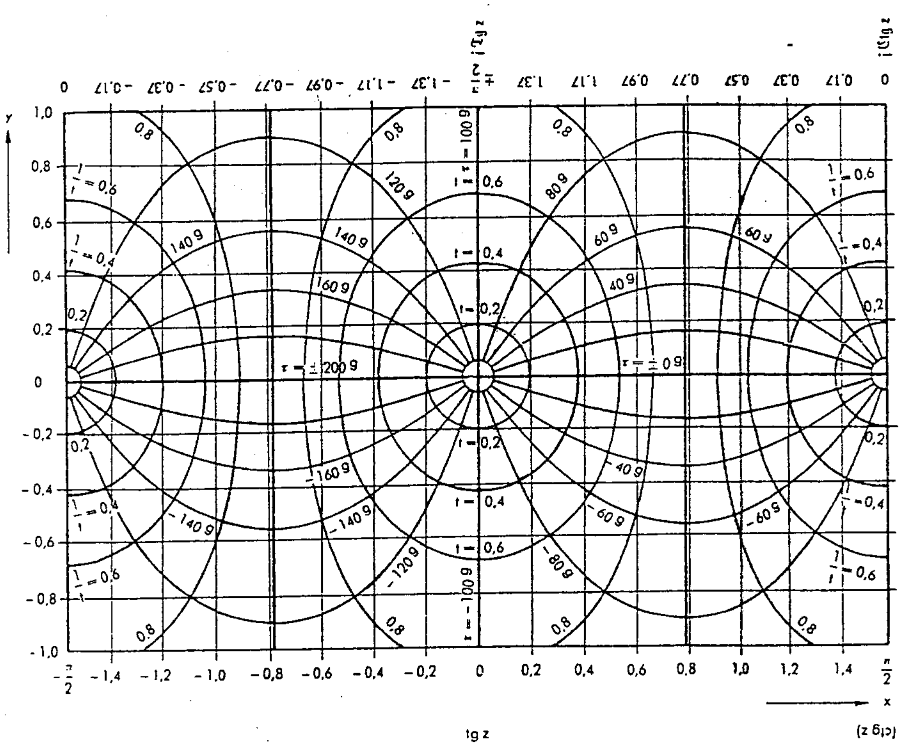
Note: To avoid errors and to find an idea for the possible solutions at ambigous solutions, see pages 22 and 23 (each a extended relief of sine and tangent function!).

Ref. to these reliefs from several views, one gets the functions sin z, cos z, *Cos* z, j *Sin* z or tan z, j *Tan* z, j *Cotan* z, + cotan z, relating to the selected zero point.

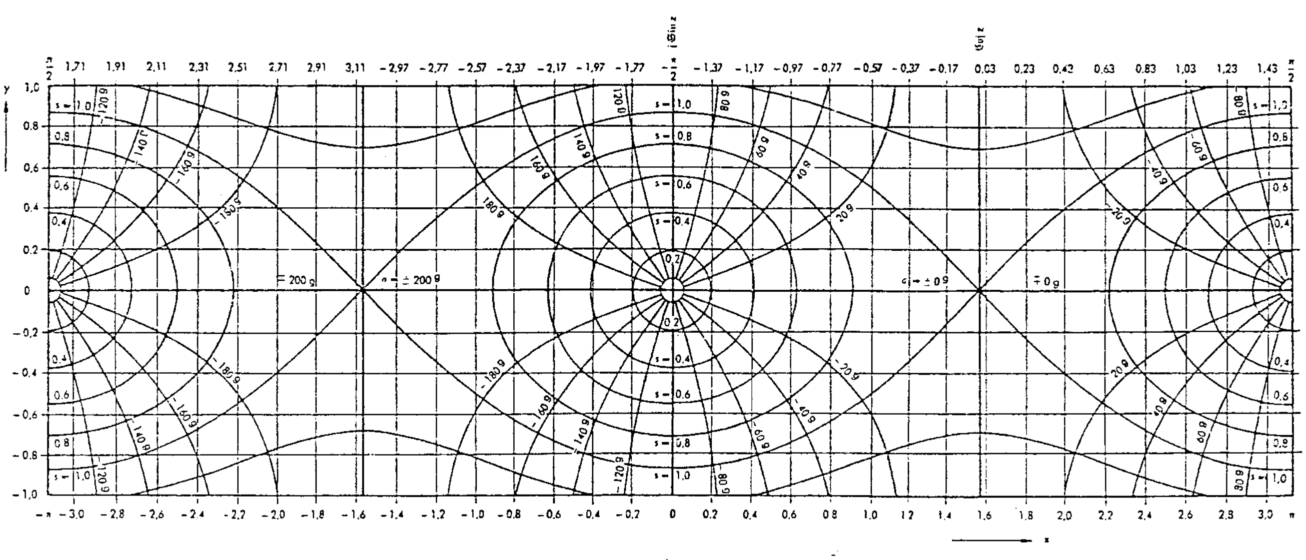
The accurate values, however, will be found in the reliefs on the reverse side of the complex device.

1952 , written by Dipl. Ing. Harald Bachmann in A. W. Faber- Castell (German version)

1999 , translated by Dieter von Jezierski , Bahnhofstr.3 -D 90547 Stein (English version)
1999 , revised by Rodger Shepherd , M.D., M.P.H., Oakland CA, USA (English version)



Tangensrelief: $\operatorname{tg} z = \operatorname{tg}(x + jy) = t\,/\underline{\tau}$



Sinusrelief: $\sin z = \sin(x + jy) = s\,/\underline{\sigma}$