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A. F. PUCHSTEIN

DEVICE FOR MAKING VECTOR CALCULATIONS

Filed May 12, 1921

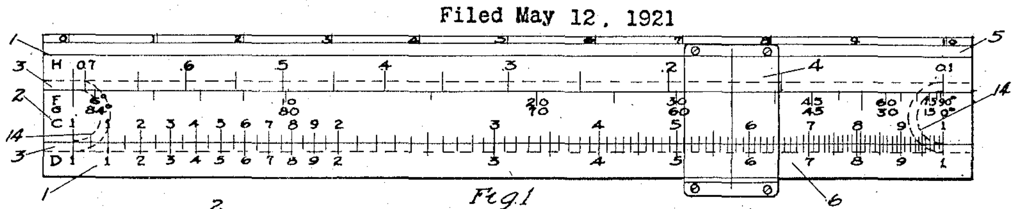

Fig. 1

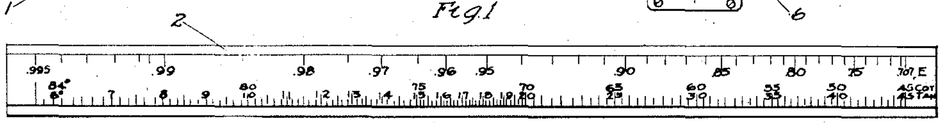

Fig. 2

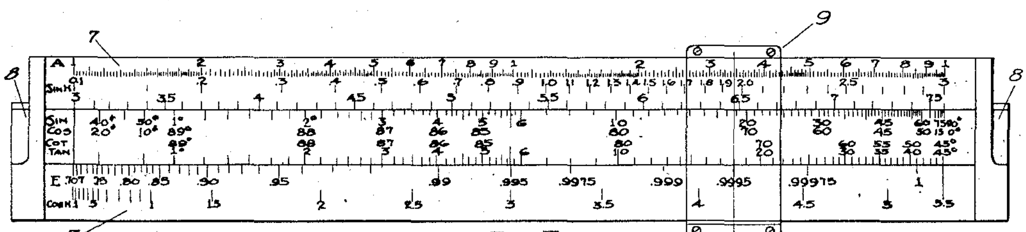

Fig. 3

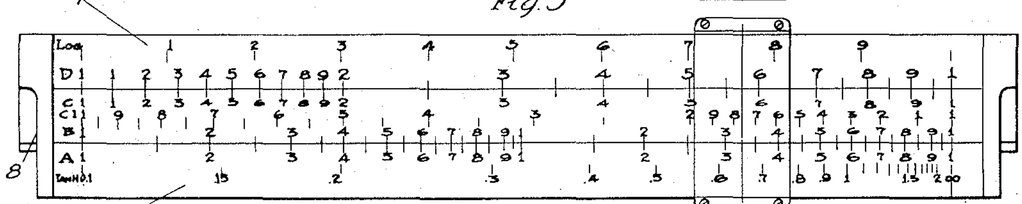

Fig. 4

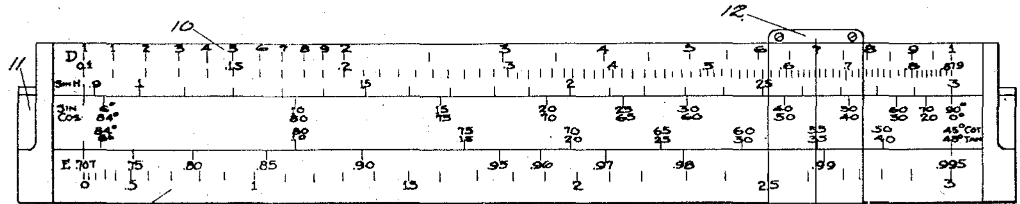

Fig. 5

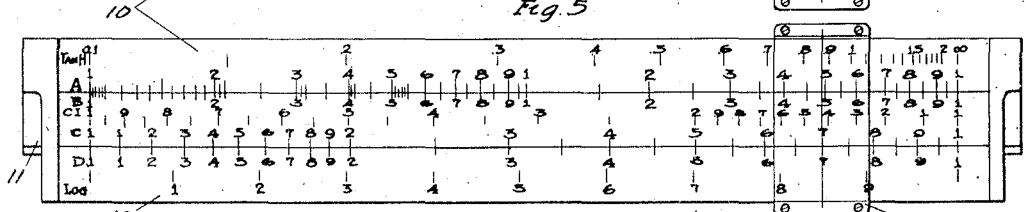

Fig. 6

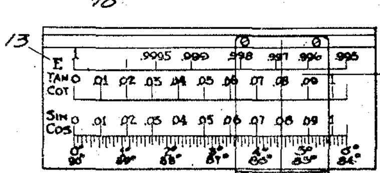

Fig. 7

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# UNITED STATES PATENT OFFICE.

ALBERT F. PUCHSTEIN, OF COLUMBUS, OHIO.

## DEVICE FOR MAKING VECTOR CALCULATIONS.

Application filed May 12, 1921. Serial No. 469,043.

*To all whom it may concern:*

Be it known that I, Albert F. Puchstein, a citizen of the United States of America, residing at Columbus, in the county of Franklin and State of Ohio, have invented certain new and useful Improvements in Devices for Making Vector Calculations, of which the following is a specification.

My invention relates to a device for making vector calculations. It aims to provide a ready means for calculating with both vectors and the functions of vectors. For instance, my device is of such a nature that calculations may be readily made as to the addition, subtraction, multiplication, division, powers and roots of vectors and also as to logarithms, circular sines, cosines, tangents, etcetera, as well as hyperbolic sines, cosines, tangents, etcetera, of vectors. My device is also adapted to make the corresponding inverse calculations.

Heretofore in devices of this character it has been proposed to employ two hinged arms, a protractor and a T square having all of the members graduated, but a cumbersome structure is necessary in order to secure the required degree of accuracy. Prior devices have not been capable of calculating circular or hyperbolic functions of vectors and, furthermore, the scope of these prior devices is quite limited. My invention possesses advantages over these prior devices and avoids the deficiencies above noted, as will be hereinafter pointed out.

My invention is primarily founded on the application of the well-known trigonometrical principle that if we divide the greater component of a vector by the lesser component of such vector, we obtain the tangent or cotangent of its angle of inclination. I have provided a novel scale of such form that when properly set it gives not only the result of this division but also gives the corresponding coefficient. Then, if we reset the scale to divide the greater component by this coefficient, we obtain the vector length. In addition, sine and cosine scales are provided for converting the expression of the vector from polar to rectangular coordinate form.

In addition, I have provided a ready means whereby the product of any circular tangent, cotangent, sine or cosine by the hyperbolic tangent, sine or cosine or the product of any of these by any factor can be determined. The importance of this will be apparent.

The preferred embodiment of my invention is shown in the accompanying drawings wherein similar characters of reference designate corresponding parts and wherein—

Figure 1 is a detail plan view of my plain vector slide rule showing its supplemental slide and indicator in normal relation thereto.

Figure 2 is a detail view of the reverse side of the supplemental slide shown in Figure 1.

Figure 3 is the obverse view of a modified form of my rule which contains in addition hyperbolic and circular scales of long range.

Figure 4 is the reverse view of the structure shown in Figure 3.

Figure 5 is the obverse view of a modified form of my rule which contains in addition hyperbolic scales of short range.

Figure 6 is a reverse view of the rule shown in Figure 5.

Figure 7 is a detail plan view of a supplemental device for covering the range from 84 degrees to 90 degrees and from zero degrees to 6 degrees.

In the drawings, my invention is shown as comprising a plain vector slide rule, a modified form consisting of a combined vector and hyperbolic function slide rule of long range, a second modified form consisting of a combined vector and hyperbolic function slide rule of short range and an extending device adapted to be used either with the plain vector or the hyperbolic rules of long or short range.

The plain vector slide rule is shown in Figures 1 and 2 and consists of a frame 1 adapted to receive a slide 2 graduated on both sides and a standard transparent runner 4. The face of the frame 1 is provided adjacent the top of the slide 2 with the sine and cosine scale H, Figure 1, and adjacent the bottom of the slide 2 with the D scale of the ordinary Mannheim rule.

The slide 2, adapted to slide in the grooves 3 of the frame 1, is provided on its face and at its upper edge with a degree scale F bearing a definite relation to the ordinary Mannheim C scale carried at its bottom edge. Located between the F and C scales of the slide and coinciding with the divisions of said F scale, is the degree scale G. The reverse side of the slide 2 is provided at its top edge with the sine and cosine scale E in opposed relation to the H scale of the frame 1 and along its bottom edge with a tangent and cotangent degree scale.

Attached to and sliding along the edges 5 and 6 of the frame 1 is a standard transparent runner 4 which contains a vertical hairline in the center thereof. The under side of the frame 1 is provided at each end with transparent indicators 14. Each indicator 14 is provided with an index mark which permits reading of the scales on the reverse face of the slide 2 without reversing the slide.

A modified form of my invention is shown in Figures 3 and 4 as applied to a combined vector and hyperbolic function slide rule of long range. This rule is somewhat similar in structure to the standard duplex slide rule and comprises a frame 7, a slide 8 and a runner 9 with a hairline encircling the whole scale. The front face of the frame 7 is provided on its upper edge with the A scale of the ordinary Mannheim rule, at its edge adjacent the top of the slide with a sinh scale reading from 0.1 to 7.5 or more and at its edge adjacent the bottom of the slide with the E scale as used in Figure 2 but carried more closely to 1.000 and at its lower edge with the cosh scale running from 0 to 5.3 or more as desired.

The front face of the slide 8, adapted to slide in the frame 7, is provided with the sin., cos., tan., and cot. scales which are the degree scales corresponding to the values of the A scale from .01 to 1.0.

The reverse side of the frame 7, Figure 4, is provided at its upper edge with the ordinary log. scale, at its edge adjacent the top of the slide 8 with the ordinary Mannheim D scale, and at its edge adjacent the bottom of the slide 8 with the ordinary Mannheim A scale and along its lower edge with the tanh scale, corresponding to values of the D scale from 0.1 to 1.0.

The reverse face of the slide 8, adapted to slide in the frame 7, is provided with the ordinary C, C1 and B scales of the Mannheim rule, the C and B being in a definite relation to the D and H scales at all times on the frame 7.

A further modified form of my invention is shown in Figures 5 and 6 as comprising a combined vector and hyperbolic function slide rule of short range. This rule which consists of a frame 10, a slide 11 and a runner 12 is similar in structure though, as stated, of more limited scale range than the modified rule shown in Figures 3 and 4. The front face of the frame 10 is provided with the ordinary Mannheim D scale in place of the A scale of Figure 3, a sinh scale extending from 0.1 to 3.0 or more, and an E scale of more limited range than that provided in the slide of Figure 3.

The front face of the slide 11, adapted to slide in the frame 10, is provided with the sine, cosine, tangent and cotangent degree scales as provided on the front and back sides of the plain vector rule slide as shown in Figures 1 and 2.

The reverse side of the frame 10 (Figure 6) is provided with a set of four scales, tanh, A, D and logarithmic which are duplicates of the four scales used on the reverse side of the frame 7 (Figure 4), the four scales on the frame 10 being arranged in the reverse order to those of the frame 7.

The reverse side of the slide 11 is also provided with the ordinary C, C1 and B scales of the Mannheim rule as used on the reverse side of the slide 8, but arranged in the reverse order, which order, however, may be changed as desired.

Adapted to work with the plain vector slide rule and the combined vector and hyperbolic function slide rule of short range, is a range extender 13 which may be attached to either of the two rules as a supplemental device, may form a permanent part thereof, of may be an independent and non-attached unit. This range extender 13 is provided with a set of degree scales which cover the range from 0° to 6° and from 84° to 90° and a set of corresponding tangent, cotangent, sine, cosine and E scales whose value extends from 0 to 0.1, with the exception of the E scale which extends from point .995 to 1.000.

It will be understood that the forms of the above-described and illustrated rules and the range, arrangement and number of scales thereon comprise merely the preferred means and forms for making vector calculations and that other ranges, arrangements and different numbers of scales may be used advantageously in the operation of any of the above slide rules.

It should be further understood that the graduations as shown in the drawings are necessarily only approximate, although it is believed that they will be sufficiently accurate, to illustrate the principle involved. It should be noted that the term vector as used is synonymous with complex quantities and that real numbers constitute a particular kind of vector.

I am giving herewith a typical example to illustrate the application of my vector slide rule and the vector features of the hyperbolic rule:

*Example 1.*—Find the value of the expression

$$\sqrt{(2+j1)(1+j4)}.$$

Solution: See Figures 1 and 2. Taking separately each factor, as $(2+j1)$, set slide to divide the greater component 2 by the lesser component 1, that is, set runner to 2 on D; set index on C to hairline on runner, turn rule over and read angle, $\theta_1=26° 30'$ (had the vector been $(1+j2)$, we would have read the other value, $63° 30'$), also read $E_1=0.896$. Move slide so that this value on C comes to hairline on runner at 2 on D. Read vector length,

$$r_1=\frac{2}{0.896}=2.24.$$

In like manner for the factor, $(1+j4)$, set the slide to divide the greater component 4 by the lesser component 1 and read $\theta_2=76°$, $E_2=0.970$, reset slide to give

$$r_2=\frac{4}{0.970}=4.125.$$

The above expression can now be written in a new form as:

$$\sqrt{(2.24/\underline{26° 30'})\times(4.125/\underline{76° 0'})}=3.04/\underline{51° 15'}.$$

To find the rectangular components of this result, set index on slide to 3.04 on D. Move runner consecutively to 51° 15′ on F and G and read 2.37 and 1.90, respectively, on D. The application of this rule, as shown in Figures 3, 4 and 4, 5, depends upon the same principle as those illustrated in this example, though they admit of certain variations.

Note: The class of calculation as herein set forth is used in electrical engineering, mechanics, map making on Mercator's projection, and navigation. Doubtless other uses will be developed.

I am giving herewith some typical examples to illustrate the application of the hyperbolic slide rule:

*Example 2.*—Find the value of sinh $(0.36\pm j0.50)$.

Solution: Multiply the component 0.50 by 57.3 to reduce to degrees $=28.6°=28° 40'$. This step may be eliminated if the graduations are in radians instead of degrees. The expression now is:

$$\sinh(0.36\pm j\,28° 40')=\sinh .36\times\cos 28° 40'\pm j\cosh 0.36\times\sin 28° 40'.$$

Set runner to 0.36 on sinh scale, bring index on slide to hairline on runner, move runner to 28° 40′ at cosine scale on slide, read product 0.322 on A; similarly, for the $j$ item, set runner to 0.36 on cosh scale, bring index on slide to hairline on runner, move runner to 28° 40′ at sine scale on slide; read product 0.510 on A. Result:

$$\sinh(0.36\pm j50)=.322\pm j0.510.$$

By use of vector rule, this also equals

$$.604/\underline{57° 40'}.$$

I am giving herewith some examples to illustrate the application of the hyperbolic rule to circular functions of vectors.

*Example 3.*—Find the value of sin $(0.36\pm j.50)$.

Solution: Reduce 0.36 radians to degrees $57.3\times0.36$ $20.6°=20° 36'$.

$$\sin(20° 36'\pm j.50)=\pm\sinh .50\times\cos 20° 36'-j\cosh 0.50\times\sin 20° 36'.$$

Set rule to these values as in Example No. 2 and Example No. 3. The result is:

$$\pm.521\times.936=\pm.488$$

$$-j1.128\times.352=-j0.397$$

or

$$\sin(0.36\pm j.50)=\pm.488-j0.397$$

I am giving herewith an example of inverse hyperbolic functions of vectors by means of slide rule.

*Example 4.*—Find the value of

$$\sinh^{-1}(.322\pm j.510).$$

Solution: Write:

$$\sinh^{-1}(u\pm jv)=\cosh^{-1}\left(\frac{\sqrt{(1+0.510)^2}+\sqrt{(1+0.510)^2+(0.322)^2}}{2}\right)$$
$$\pm j\sin^{-1}\left(\frac{\sqrt{(1+0.510)^2+(0.322)^2}-\sqrt{(1-0.510)^2+(0.322)^2}}{2}\right)$$

and use the vector part of the rule to evaluate the radicals, thus divide greater by lesser

$$\frac{1.510}{.322}$$

and read $E=.978$;

$$\text{radical}=\frac{1.510}{0.978}=1.546;\quad 1-\frac{.510}{.322}=\frac{.490}{.322},$$

$$E=.8355,\ \text{radical}\ \frac{.49}{.8355}=.5865.$$

$$\cosh^{-1}\frac{1.546+5865}{2}=\cosh^{-1}\frac{2.1325}{2}=\cosh^{-1}1.066.$$

Set runner to this value on A and read result on cosh scale

$$=0.360\pm j\sin^{-1}\frac{1.546-.5865}{2}=\pm j\sin^{-1}\frac{.9595}{2}=\pm j\sin^{-1}.048=\pm j28^\circ 40'=\pm j.50.$$

$$\cosh^{-1}(u\pm jv)=\cosh^{-1}\left(\frac{\sqrt{(1+0.936)^2+(0.176)^2}+\sqrt{(1-0.936)^2+(0.176)^2}}{2}\right)$$

$$\pm j\cos^{-1}\left(\frac{\sqrt{(1+0.936)^2+(0.176)^2}-\sqrt{(1-0.936)^2+(0.176)^2}}{2}\right)$$

Having thus described my invention, what I claim is:

1. A calculating device comprising in combination a stock and an adjustable slide, a logarithmic scale on one edge of said stock, an inverted sine and cosine scale on the opposite edge of said stock, a logarithmic and degree scale on one side of said adjustable slide and a sine and cosine scale on the opposite side of said slide in opposed relation to said first named sine and cosine scale.

2. A calculating device comprising in combination a stock and an adjustable slide, a logarithmic scale on one edge of said stock, a sine and cosine scale on the opposite edge of said stock, a logarithmic and degree scale on one side of said adjustable slide and a sine and cosine scale on the opposite side of said slide and arranged in opposed relation to said first mentioned sine and cosine scale.

3. A calculating device comprising in combination a stock and an adjustable slide, a logarithmic scale on one edge of said stock, a sine and cosine scale on the opposite edge of said stock, a logarithmic and degree scale on one side of said adjustable slide and a reversed sine and cosine, tangent and reversed cotangent scale on the opposite side of said slide.

4. A calculating device comprising in combination a stock and an adjustable slide, a logarithmic scale on one edge of said stock, a sine and cosine scale on the opposite edge of said stock, a logarithmic and degree scale on one side of said adjustable slide and a sine and cosine, tangent and cotangent scale on the opposite side of said slide, said second named sine and cosine scale being arranged in opposed relation to said first named sine and cosine scale.

5. A calculating device comprising in combination a stock and an adjustable slide, logarithmic and trigonometric scales on opposite edges of said stock and sine, cosine, cotangent and tangent scales on one side of said slide correspondingly arranged with one of said logarithmic scales.

6. A calculating device comprising in combination a stock and an adjustable slide, logarithmic and trigonometric scales on opposite edges and opposed sides of said stock and trigonometric and logarithmic scales on opposite sides of said slide, the trigonometric scales on said slide arranged in order corresponding to the logarithmic scales on the adjacent side of said stock and the logarithmic scales on said slide arranged in order corresponding to the trigonometric scales on the adjacent side of said stock.

7. A calculating device comprising in combination a frame, trigonometric and logarithmic scales thereon, a slide relatively adjustable to said frame and a supplemental scale divided correspondingly to said frame and slide.

8. A calculating device comprising in combination a frame, trigonometric and logarithmic scales thereon, a slide relatively adjustable to said frame and a supplemental scale having trigonometric and logarithmic scales thereon and arranged to extend the range of scales on said frame and stock.

9. A calculating device comprising in combination a stock and an adjustable slide, an inverted sine and cosine scale on one edge of said stock and a sine and cosine scale on the opposite side of said slide arranged in opposed relation to said first named scale.

10. A calculating device comprising in combination a stock and an adjustable slide, a logarithmic scale on one edge of such stock and reversed sine and cosine scale on the opposite edge of said stock, logarithmic and degree scales on opposite edges of said slide, a degree scale between said last named logarithmic and degree scale on said slide and tangent and reversed sine and cosine and cotangent scales on the opposite side of said slide.

11. In a slide rule the combination of a stock provided with a logarithmic scale, a reversed trigonometric scale, a runner longitudinally adjustable in said stock provided with a degree scale bearing a definite relation to a logarithmic scale and a reversed degree scale between said last named scales and trigonometric scales on the reverse side of said runner and arranged with a definite relation to the scales on the front side of said stock and runner, all of said scales co-operating for solving a right-angled triangle.

12. In a slide rule the combination of a stock provided with logarithmic scales, trigonometric scales comprising a sinh scale and a cosh scale, and a runner longitudinally adjustable in said stock and provided with sine, cosine, cotangent and tangent scales on one side and logarithmic scales on

the opposite side for changing coordinates from the rectangular to the polar form and vice versa.

13. In a slide rule the combination of a stock provided with logarithmic scales and a runner longitudinally adjustable in said stock and provided with sine, cosine, tangent and cotangent degree scales on one side and with C, C1 and B scales in the order named on the reverse side, said scales on said runner being arranged with reference to said first named scales whereby a function may be read direct or used as a factor for further operation.

In testimony whereof I hereby affix my signature.

ALBERT F. PUCHSTEIN.