PATENT SPECIFICATION

DRAWINGS ATTACHED

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COMPLETE SPECIFICATION

Improvements in Slide Rules

We, PICKETT & ECKEL, INC., a corporation organized under the laws of the State of Illinois, United States of America, of 542, South Dearborn Street, Chicago, Illinois, United States of America, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates to a slide rule having an Ln—L scale arrangement for simplifying the use of a slide rule in computations involving the base e, reducing considerably the number of settings required in comparison to the usual slide rule when this base is used.

The present invention provides a slide rule including the combination of coextensive logarithmic and linear scales in which for one decade of said logarithmic scale there are 2.302585+units of said linear scale, and a cursor movable along said scales and having a hairline against which they may be read enabling the direct finding of powers of e and logarithms to the base e under the hairline of said cursor.

A further object of the invention is to provide an Ln scale which is uniform or linear the same as an L scale, but bears relation to the L scale of the constant log e 10 or 1÷M=2.302585+ for the full length of the Ln scale compared to 1.0 for the full length of the L scale whereby the Ln scale may be used in problems involving the base e in like manner as the L scale is used with problems involving the base 10.

Another object is to provide an Ln scale which may be combined with other slide rule scales such as C and D and which is related to the C and D scales by the constant e or Naperian base (2.7182818+), and may also be combined with the CI and DI scales to find the powers of e, logarithms to the base e, logarithms of proper fractions, powers for negative exponents, and mantissas of logarithm by direct reading across from the Ln and L scales to the C, D, CI and DI scales.

Another object is to provide a slide rule having Ln, L and C and/or D scales bearing the relationship per scale length graduated in the following ratio:—

Ln=2.302585+

L =1.0

C =10.0

D =10.0 and wherein the e points of the C and D scales=2.7182818+and are coincident with 1 cm the Ln scale, the values stated being more accurately identified as:—

2.30258+and

2.71828+respectively, according to the MacMillan Logarithmic and Trigonometric Tables, Page 133.

Still another object is to provide Ln and L scales of linear or uniform progression used in combination with other scales of logarithmic progression involving relative settings of the body and slide of the slide rule whereby multiplication and division with powers, the logarithms of combined operations, the power of other bases, and hyperbolic functions may be determined on a slide rule of our design.

A further object is to provide a slide rule so scaled as to make possible combined operations with the power of e and logarithms to the base e, with the results accurate to three or four significant figures and readable directly without the necessity of reading a value off one scale and then setting it on another one in order to continue with another calculation. Thus a series of calculations involving the base e may be performed without the use of a log logscale and without the necessity of reading off intermediate values and resetting scales, the final result only be directly read on the appropriate scale.

Still a further object it to provide a slide rule having a scale for the base e capable of doing everything that the L scale does for the base 10.

An additional object is to provide a scale for a slide rule which makes possible the reading of e to any exponent between 0 and 2.3 directly on the C or D scales of the rule, or e to any exponent between 0 and −2.3 directly on the CI or DI scales, as well as e to any power between 1 and 10 on the C or D scales, or e to any power between 1 and −10 on the CI or DI scales.

Another additional object is to provide an Ln scale so related to all logarithmic scales of a slide rule that problems involving any logarithmic scale and the base e are greatly simplified, and particularly so in combined operations, in comparison with such problems when worked on a slide rule having no Ln scale.

With these and other objects in view, our invention consists in the construction, arrangement and combination of the various parts of our Ln—L scale slide rule, whereby the objects above contemplated are attained, as hereinafter more fully set forth, pointed out in our claims and illustrated in detail on the accompanying drawings, wherein:—

Figure 1 shows an arrangement of scales for a slide rule wherein our Ln scale is on the slide and arranged back to back with respect to an L scale;

Figure 2 shows another scale arrangement wherein the Ln and L scales are provided on the body of the slide rule, and

Figure 3 shows a complete slide rule with the scales of Figure 1 thereon.

On the accompanying drawings we have used in Figures 1, 2 and 3 the reference numerals 10 and 12 to indicate the body portions of a slide rule, and 14 the slide thereof. The body portions 10 and 12 are connected by end members 16 in the usual way, and a cursor 18 of transparent material is supported by slides 20 and 22 that slide along the top and bottom of the rule so that the hairline 24 of the cursor 18 may be matched with various slide rule graduations as required.

The body 10—12 in Figure 1 is provided with such standard scales as Log-Log (LL1, LL2, LL3 and LL4) and the slide and body are provided with C and D scales respectively. The slide 14 is also provided with linear log scale (L) to which the logarithmic C and D scales are related, and TH and SH scales are also provided on the slide.

The important scale of our invention is the Ln scale which is calibrated in accordance with Napierian or natural logarithms (logarithms to the base e). Napier explained in his book of 1614 the value of e=2,71828 approximately. Accordingly, 1.0 of our Ln scale is coincident with 2.7182818+ of the C scale in Figure 1 and the D scale in Figure 2, and is designated with a mark identified epsilon (ε) throughout the drawings.

The Ln scale is a uniformly divided scale and its length is equal to log 10 to the base e, or 2.302585+. It accordingly bears the relationship of this number to the length 1.0 of the L scale as shown. Thus calibrated, the Ln scale is useful to simplify problems involving the base e when used with the Log scale and/or with any of the other scales that are related (a logarithmic to linear relationship) to the Log scale, as will hereinafter be explained in detail.

Figure 2 shows further scales as follows:—

a D scale folded at π (DF)

a C scale folded at π (CF), and

a CI folded scale (CIF) reciprocal to CF, all related to the Log scale so that they may be used in computations involving the base e through their relation with the Log or L scale and its relation to our L_n scale.

Figure 1 also shows DF/M and CF/M scales folded at the value 2.302585+ which may be used in some computations likewise involving the Ln scale as well as the logarithmic scales of our slide rule.

In general, the Ln scale is used for problems with the base e. In explanation of the use of the designation "Ln", this is a symbol now commonly used in mathematical books, being an abbreviation for "natural logarithm". Log 8 is an expression used for "logarithm of 8 to base 10", and likewise Ln 8 is for "logarithm of 8 to base e".

The range of our Ln scale (from 0 to 2.3) is greater than the range of the L scale (from 0 to 1). By computing a "characteristic" one can use the Ln scale to find any power of e; thus, the effective range for powers of e is from 0 to infinity. Also, for many problems it is more convenient to use an Ln scale than Log-Log scales, and in particular it enables one to solve problems with powers of e in combined operations. Since powers of e are read in the C (or D) scale, accuracy to three or four significant figures is obtained no matter how large or how small the numbers are. Our Ln scale thereby saves steps in many computational problems.

We now propose to use 2.3 as divisor in place of 2.30258. We require the quotient to be q, but get a new remainder which we denote by R, where R>r. Then,

n=2.3q+R, where R<2.3.

Subtracting this from the equation, we have 0=0.00258q+r−R, or r=R−0.00258q.

Thus the error, R−r, in the remainder is 0.00258q.

If this is rounded off to 0.0025q, it expresses one-fouth of 1 per cent of the quotient.

A SHORT CUT IN USING Ln10=2.30258.

To extend the range of Ln the number 2.30258 is needed. Suppose that, to save work, the number 2.3 is used. Some error will of course occur. For example, the remainder in division will be too large. How can we easily correct for this error? The following simple rule will serve:—

Rule. Take 1 per cent of the quotient and divide it by 4. Subtract the result from the remainder to obtain the correct remainder to set on Ln.

Example. Find $e^{17.4}$

Divide:

7.		7.
2.3/17.4	or	2.30258/17.40000
16.1		16.11806
1.3		1.28194

Take 1% of 7; $0.01\times7=0.07$; divide by 4. $0.07\div4=0.02$, approx. Subtract 0.02 from 1.3, to obtain 1.28, the corrected remainder. Then $e^{17.4}=e^{1.28}\times10^{7}$. The basis of this rule is explained below.

Consider $x=e^{n}$. Divide n by 2.30258, and denote the integral part of the quotient by q and the remainder by r.

Then, $n=2.30258q+r$, $r<2.30258$.

When this slide rule is used to divide by 2.30, proceed as follows:—

Set 2.30 of C over 17.4 of D. Under 1 of C read 7.56 on D. The integral part, or "characteristic", is 7. Multiply the decimal fraction 0.56 by 2.3, using the C and D scales. Obtain 1.29 as the reduced exponent of e.

In Figure 1, the quotient may be obtained by merely setting the exponent on DF/M and reading the quotient on D. The relation between readings on the D and the DF/M scales may be indicated symbolically as follows:—

$(D3\times2.30=(DF/M)$ and $(DF/M)\div2.30=(D)$.

For some purposes and for some exponents, this slide rule method is not sufficiently accurate.

The L and Ln can be used in combination with other scales. The methods used when the L scales are on the body (Figure 2) differ from those used when they are on the slide (Figure 1). In solving problems, the numbers are expressed in standard form. The calculations are carried through within the ranges of L and Ln scales, and the decimal points are determined by known rules.

Having described our invention and the use of a number of the disclosed scales in combination with each other, it is believed obvious how our Ln—L scale simplifies problems to the base e, and is operable in combination with many different types of logarithmic scales of a slide rule such as the CF, DF, CF/M, DF/M, CIF, LL, TH and SH scales shown in Figures 1, 2 and 3. These scales are illustrated in their proper relations to the Ln and L scales. Examples are disclosed of logarithmic scales with which the Ln scale can be used, and any other logarithmic scales usable with the L scale can also be used in an obvious manner with the Ln scale for values to the base e instead of the base 10.

WHAT WE CLAIM IS:—

1. A slide rule including the combination of co-extensive logarithmic and linear scales in which one decade of said logarithmic scale there are 2.302585+ units of said linear scale, and a cursor movable along said scales and having a hairline against which they may be read enabling the direct finding of powers of e and logarithms to the base e under the hairline of said cursor.

2. The slide rule according to Claim 1, including a plurality of coextensive logarithmic scales, one of said logarithmic scales being inverted for finding logarithms of proper fractions and the powers for negative exponents.

3. The slide rule according to Claim 1, including a linear L scale having a length of one unit, said linear scale comprising an Ln scale coextensive with said L scale and graduated in equal increments totalling 2.302585+, and at least one of said logarithmic scales having graduations from 1 to 10 coextensive with said Ln and L scales.

4. The slide rule according to Claim 3, wherein one of said logarithmic scales is inverted and coextensive with said Ln scale having graduations from 10 to 1 whereby powers of e for negative exponents, and logarithms of proper fractions to the base e, may be determined from said Ln scale and said inverted logarithmic scale when used in conjunction with each other.

5. The slide rule according to Claim 1, including a plurality of logarithmic scales, some of said logarithmic scales being inverted and others being folded at 2.302585+ for finding the logarithms of proper fractions, the powers for negative exponents, for multiplication and division with powers, logarithms of combined operations and values of hyperbolic functions.

6. The slide rule according to Claim 1, including a plurality of logarithmic scales, some of said logarithmic scales being inverted and others being log log scales for finding the logarithms of proper fractions, the powers for negative exponents, for multiplication and division with powers, logarithms of combined operations and values of hyperbolic functions.

7. The slide rule according to Claim 1, including multiple logarithmic scales and a log scale coextensive with said linear scale and said logarithmic scales and of unit length, some of said logarithmic scales being inverted, others being folded, and at least one being inverted and folded for finding the logarithms of proper fractions, the powers for negative exponents, for multiplication and division

with powers, logarithms of combined operation and values of hyperbolic functions.

8. A slide rule constructed and adapted to operate substantially as herein described with reference to the embodiments illustrated in the accompanying drawings.

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912872 COMPLETE SPECIFICATION

1 SHEET *This drawing is a reproduction of the Original on a reduced scale*

FIG. 1

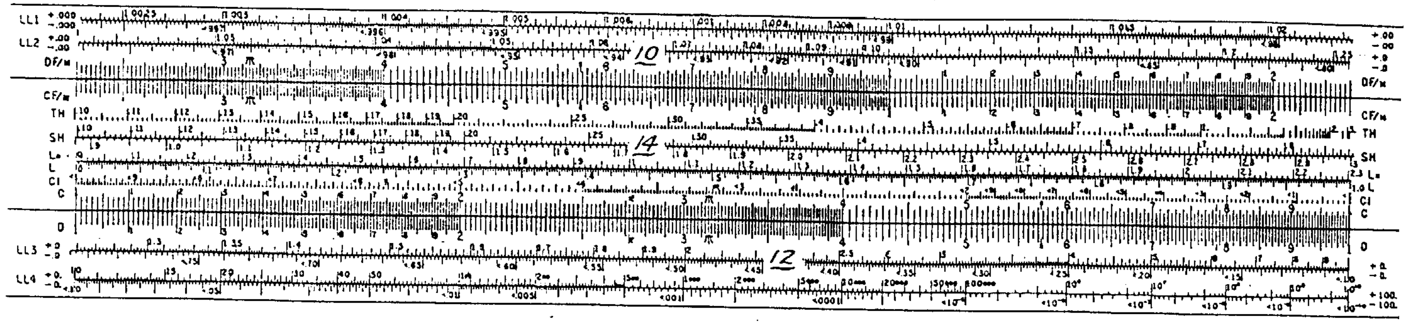

FIG. 2

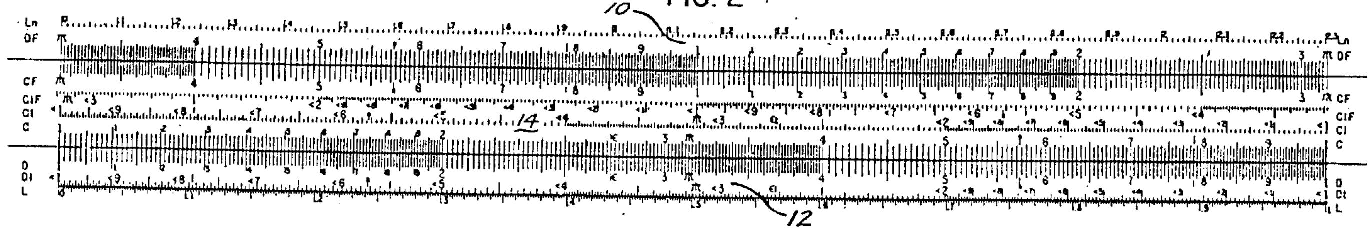

FIG. 3