PATENT SPECIFICATION

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PROVISIONAL SPECIFICATION.

Improvements in Slide Rules.

I, JOHN ST. VINCENT PLETTS, of Marconi House, Strand, London, Consulting Engineer, do hereby declare the nature of this invention to be as follows:—

This invention relates to improvements in slide rules whereby most of the mathematical functions of frequent occurrence in practical problems may be evaluated much more easily and therefore more accurately than they can be with existing slide rules.

The principal improvement on the ordinary slide rule (in which the lengths are proportional to the logarithms of the numbers which they represent) was the introduction of an additional log log scale (in which the lengths are proportional to the logarithms of the logarithms of the numbers which they represent), thus enabling any number to be raised to any power or the logarithm of any number to any base to be obtained.

Since however the ordinary or log scale is usually repeated only twice in the length of the rule it follows that the log log scale is necessarily limited in range. To obviate this disadvantage slide rules have been made with two and even three log log scales, one beginning with the number with which the other ends, and thus considerably extending the range. I have found that by taking advantage of the well known properties of characteristics and mantissæ of logarithms to the base 10 I can extend the range of a single log log scale to within .025 of unity in one direction and indefinitely in the other direction.

With this object I arrange the scales so that the numbers on the log scale are the logarithms to the base 10 of the numbers opposite them on the log log scale, and I make 10 the highest number on the log log scale. If now, for example, it is required to raise a number to a power which makes it greater than 10 the result cannot be found in the ordinary way on the log log scale, but the logarithm of the result can be found on the log scale, and if the first figure or characteristic of this logarithm be dropped, the result can be found on the log log scale opposite the remaining mantissa on the log scale, it being of course necessary to move the decimal point according to the characteristic dropped. Similarly for numbers less than unity I take advantage of this property of logarithms to the base 10 by providing a scale the numbers on which are the reciprocals of the numbers opposite them on the log log scale.

It will be seen that though the oft recurring function e^{ax} can thus be readily evaluated the equally common function ae^{x} cannot be evaluated without transferring an intermediate result from one scale to another, and it is evident that each such transferance is a loophole for error besides causing much labour when a series of values have to be determined.

In order to simplify the evaluation of such functions I provide, preferably along the centre of the slide, an equally divided scale the numbers on which are the natural logarithms of the numbers opposite them on the log scale. It follows therefore that ae^{x} or $\log_e ax$ can be readily evaluated.

Moreover since any number on the log scale can either be multiplied by e^{x} or have its natural logarithm determined by this means it follows that any func-

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tion reading on this scale can be so treated. Accordingly I arrange the sine and tangent scales (which are as usual placed on the back of the slide), and may also add other scales as for example hyperbolic sine and cosine scales, in such a manner that they all read appropriately on the log scale. Thus all such functions as $e^x \sin a$ and logcosh x can be evaluated without reference to tables and without writing down or transferring intermediate results.

Dated the 2nd day of October, 1919.

J. St. VINCENT PLETTS.

COMPLETE SPECIFICATION.

Improvements in Slide Rules.

I, John St. Vincent Pletts, of Marconi House, Strand, London, W.C. 2, Consulting Engineer, British subject, do hereby declare the nature of this invention and in what manner the same is to be performed, to be particularly described and ascertained in and by the following statement:—

This invention relates to improvements in slide rules whereby most of the mathematical functions of frequent occurrence in practical problems may be evaluated much more easily and therefore more accurately than they can be with existing slide rules.

The principal improvement on the ordinary slide rule (in which the lengths are proportional to the logarithms of the numbers which they represent) was the introduction of an additional log log scale (in which the lengths are proportional to the logarithms of the logarithms of the numbers which they represent), thus enabling any number to be raised to any power or the logarithm of any number to any base to be obtained. Since, however, the ordinary or log scale usually occurs only twice in the length of the rule, it follows that the log log scale, which does not repeat itself over and over again like the log scale, is necessarily limited in range. To obviate this disadvantage slide rules have been made with the log log scale two or even three times the length of the log scale, one part beginning with the number with which the other ends, and thus considerably extending the range.

Other improvements have been introduced by the provision of equally divided scales to give logarithms, and of scales to give circular and other functions; but such scales have not hitherto been related in such a way as to give, in a simple manner, the natural logarithms of such functions, or the product of such functions and an exponential function, both of which are of frequent occurrence in practical problems.

I have found that by taking advantage of the well known properties of characteristics and mantissæ of logarithms to the base 10, I can indefinitely extend the range of the log log scale without making it longer than the log scale. With this object I arrange the scales so that the numbers on the log scale are the logarithms to the base 10 of the numbers opposite them on the log log scale, and I make 10 the highest number on the log log scale, so that, if the log scale to which it is related occurs twice in the length of the rule, the lowest number on the log log scale is within .025 of unity.

This arrangement is shewn in Figure 1 where A is the usual twice occurring log scale and E the log log scale, arranged so that the numbers on the former are the logarithms to the base 10 of the numbers opposite them on the latter. B is the usual twice occurring log scale on the slide S; and in order to raise any number on scale E to the power n, the left-hand end of scale B is placed opposite the number on scale E (by means of the usual cursor) and the result is read on scale E opposite n on scale B. If the result is greater than 10 it cannot be read on scale E, but by using the right-hand end of scale B the logarithm of the result can be read on scale A opposite n on scale B, and if the first figure or characteristic of this logarithm be dropped the result can be read on scale E opposite the remaining mantissa on scale A, it being of course necesary to move the decimal point according to the characteristic dropped. The numbers on scale F are the reciprocals of those on scale E, and therefore enable numbers less than unity to be similarly dealt with.

Numbers within .025 of unity are not as a rule required, but they may be obtained, though with less accuracy, by the method described in reference to Figure 2, where the scales E and F are related to the once occurring log scale D instead of to scale A, so that they only come to within

.26 of unity. With this arrangement the scale D is extended to 1.1, and the scales E and F are extended to correspond, so as to give the logarithm of any number within 2.6 of 10. It follows therefore that the mantissa of any such logarithm is the logarithm of a tenth of the number, thus enabling the logarithms of all numbers within .26 of unity to be obtained.

I find it advantageous to indicate, as shewn in the figures, the position of the decimal point in the numbers on the log scales corresponding to the E and F scales, and I prefer the arrangement of these scales shewn in Figure 1, to which arrangement alone I hereinafter refer. It will be observed that with this arrangement the position of the decimal point shows at once which part of scales A and B has to be used when extracting square roots, while other advantages will appear later.

It will be seen that though the oft recurring function e^{ax} can thus be readily evaluated the equally common function ae^x cannot be evaluated without transferring an intermediate result from one scale to another, and it is evident that each such transference is a loophole for error besides causing much labour when a series of values have to be determined.

In order to simplify the evaluation of such functions, I provide, preferably along the centre of the slide S, an equally divided scale G, the numbers on which are the natural logarithms of the numbers opposite them on one of the log scales, but preferably on scale B as shewn in Figure 1. It follows therefore that if 0 on scale G is put opposite a on scale A the value of ae^x can be read on scale A opposite x on scale G; while if the right-hand end of scale A be put opposite a on scale B, $\log_e ax$ can be read on scale G opposite x on scale A. As with this arrangement the highest number on scale G is 4.6 I take advantage of the fact that $e^{4\cdot 6}$ is very nearly equal to 100 by numbering the other side of the scale from 4.6 to 9.2 and placing a line l at a distance equal to the difference between 4.6 and the natural logarithm of 100 from each end of the scale, so that the error which would otherwise be introduced may be eliminated by using these lines instead of the ends themselves when employing this other side of the scale.

Now since any number on scale A or B can either be multiplied by e^x or have its natural logarithm determined by this means, it follows that any function reading on these scales can be so treated. I therefore place on the back of the slide scales for the circular and hyperbolic functions arranged so as to read on scale A or B by means of lines fixed as usual to the back of the rule. Figure 3 shows the back of the slide S (turned top for bottom, not end for end), and m m represent the positions of the lines which are preferably drawn on windows let into the back of the rule at each end. The upper edge of K is the sin-cosec scale and the lower edge of K is the cos-sec scale. The left-hand half of L is the tan-cot scale. These scales for the circular functions may be divided into radians or into degrees and minutes, but as the conversion from one to the other by means of the slide rule gives and requires degrees and decimals of a degree, I prefer to divide these scales in this way, while I may add lines for each tenth of a radian, thus rendering conversion unnecessary in many cases. To facilitate conversion I may provide on one of the log scales lines for $\pi/180$ and $180/\pi$, as shewn on scales C and D (Figure 1). The tan-cot scale L runs from below 6° to 45° on the upper edge and from 45° to above 84° on the lower edge. For smaller angles $\tan x=x$ and $\cot x=1/x$, and for larger angles $\tan x=1/\left(\frac{\pi}{2}-x\right)$ and $\cot x=\left(\frac{\pi}{2}-x\right)$ with sufficient accuracy. The sin-cosec scale K runs from below .6° to 90°, below which values $\sin x=x$ and $\operatorname{cosec} x=1/x$ with sufficient accuracy, and the cos-sec scale K runs from 0° to 89.4°, above which $\cos x=\left(\frac{\pi}{2}-x\right)$ and $\sec x=1/\left(\frac{\pi}{2}-x\right)$ with sufficient accuracy. The remaining scales on the back of the slide are for the hyperbolic functions. J and the left-hand half of M is the sinh-cosech scale, the right-hand half of M is the cosh-sech scale, and the right-hand half of L is the tanh-coth scale. The tanh-coth scale L runs from .1 to ∞, below which values $\tanh x=x$ and $\coth x=1/x$ with sufficient accuracy. The sinh-cosech scale runs from .01 to nearly .9 on J and from nearly .9 to 3 on M, below which values $\sinh x=x$ and $\operatorname{cosech} x=1/x$ with sufficient accuracy. The cosh-sech scale M runs from 0 to 3. Above 3, with which both scales on M end, $\sinh x=\cosh x=e^x/2$ and $\operatorname{cosech} x=\operatorname{sech} x=2e^{-x}$ with sufficient accuracy. By taking advantage of all these approximate equalities the scales for all these functions can be arranged so as to occupy no more than four lengths of the slide, one such arrangement being shewn in the figure.

An example will be sufficient to illustrate the use of these scales J to M and their relation to the scales A to G on the front. If x on the upper edge of scale K be set under the right-hand line m then opposite a on B will be found the value of asin x on A, $\sqrt{\text{asin } x}$ on D, 10 asin x on E, and $10^{-\text{asin } x}$ on F, opposite a on A will be found the value of acosec x on B, $\sqrt{\text{acosec } x}$ on C, and logacosec x on G; and opposite a on G will be found the value of $e^{a}\sin x$ on A, $\sqrt{e^{a}\sin x}$ on D, $10^{e^{a}\sin x}$ on E, and $10^{-e^{a}\sin x}$ on F. It will be obvious that with one movement of the slide or cursor the values of logasin x, $1/e^{a}\sin x$, $e^{-a}\sin x$, and so on, can be obtained. If the left-hand line m be used instead of the right-hand the position of the decimal point as marked on the scales may have to be moved, but the readings are otherwise identical. Similarly with all the other scales, each function reading on scale A if it is less than unity and on scale B if it is greater than unity, and therefore being amenable to the treatment described above with reference to the sine function. But owing to the tanh-coth scale L and the cosh-sech scale M occupying the spaces left by the other scales, the middle of the scale A or B has to be taken instead of the ends as the indicator for these functions. Thus if 2 on the cosh-sech scale M be placed under the right-hand line m, it will be found on scale G opposite .1 on scale A that log cosh 2=1.325.

It will of course be understood, although I have only described a straight slide rule, that similar scales may be similarly related on a circular or cylindrical slide rule, and also that the skeleton scales shewn in the figures are so drawn for the sake of clearness and may be further subdivided to any convenient extent.

Having now particularly described and ascertained the nature of my said invention and in what manner the same is to be performed, I declare that what I claim is:—

1. In slide rules having log log scales in addition to the ordinary log scales, arranging the former with respect to the latter so that the numbers on one of the latter are the common logarithms of the numbers opposite them on the former, and so that advantage can be taken of the properties of characteristics and mantissæ of such logarithms to extend the range of the log log scales substantially as described.

2. In slide rules having an equally divided scale in addition to the ordinary log scales, arranging the former with respect to the latter so that the numbers on the former are the natural logarithms of the numbers opposite them on one of the latter, on which last named scale, or on the corresponding scale of the other member of the slide rule, other functions are also arranged to read, substantially as and for the purpose described.

3. In slide rules as claimed in Claim 2, providing scales for the circular and hyperbolic functions arranged to read on the therein last named scale, or on the corresponding scale of the other member of the slide rule, substantially as and for the purpose described.

4. Improvements in slide rules substantially as described and illustrated in the drawings.

Dated this 29th day of March, 1920.

J. St. VINCENT PLETTS.

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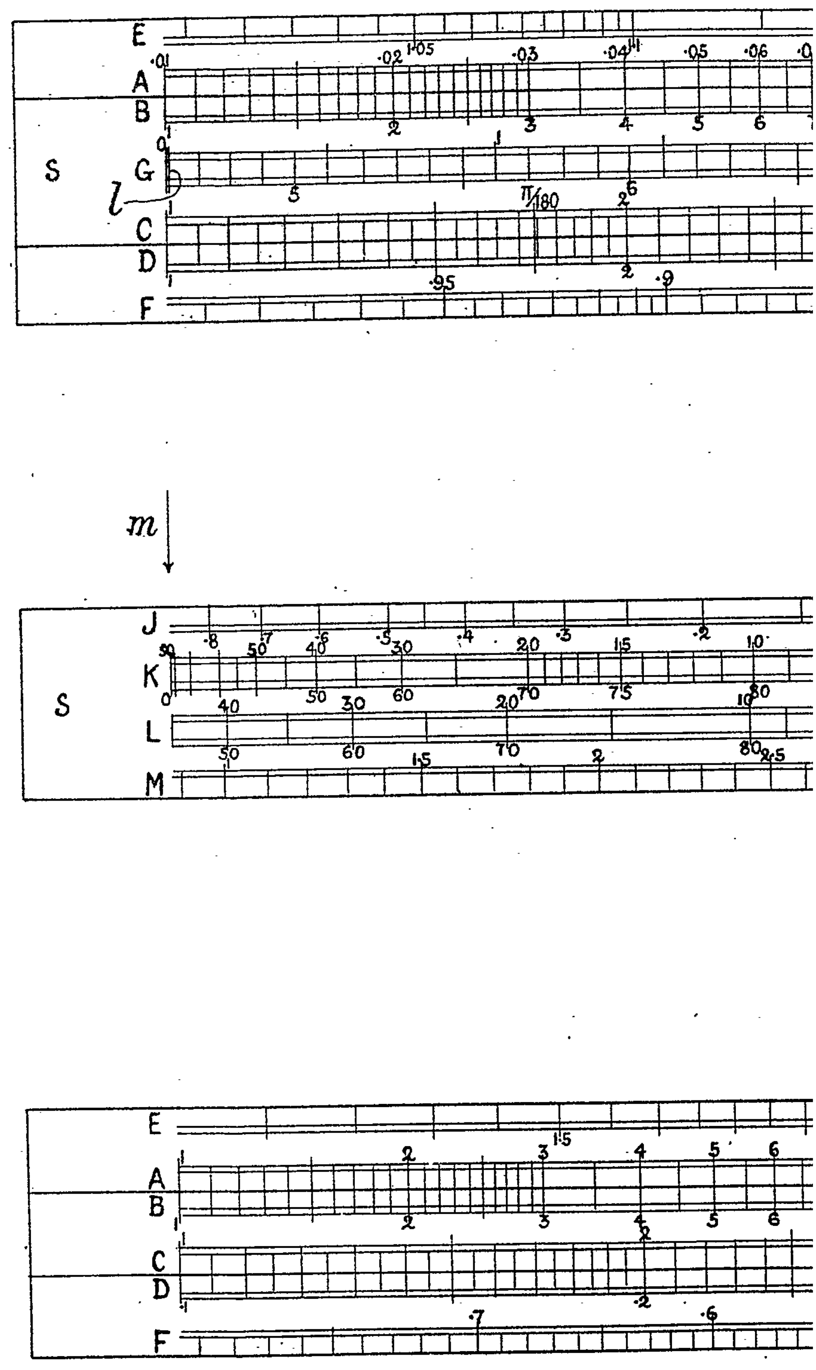
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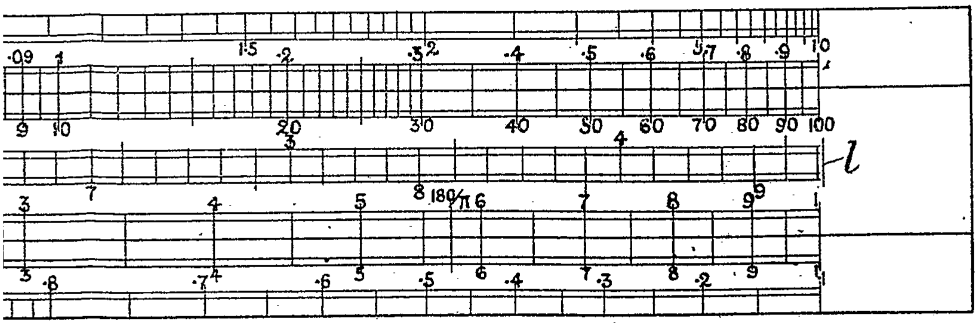

Fig: 1.

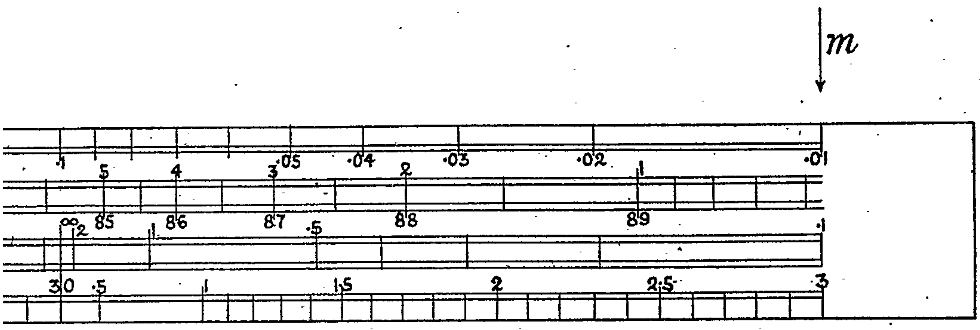

Fig: 3.

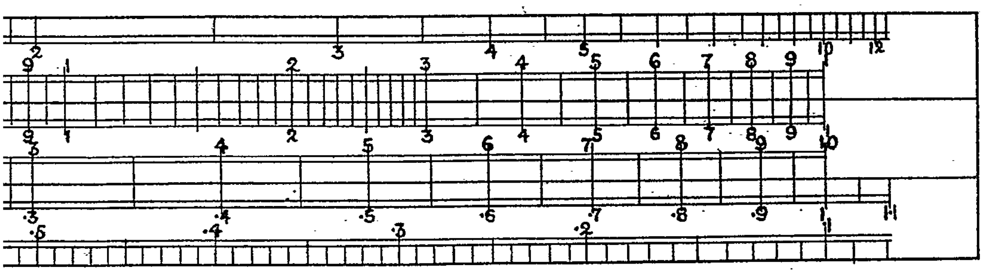

Fig: 2.

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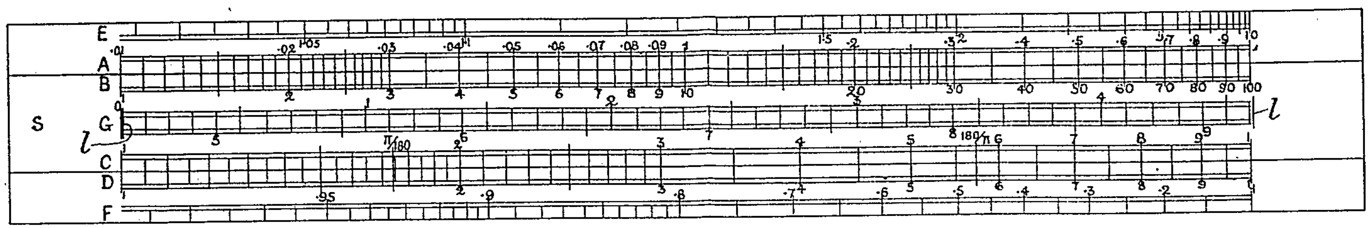

Fig: 1.

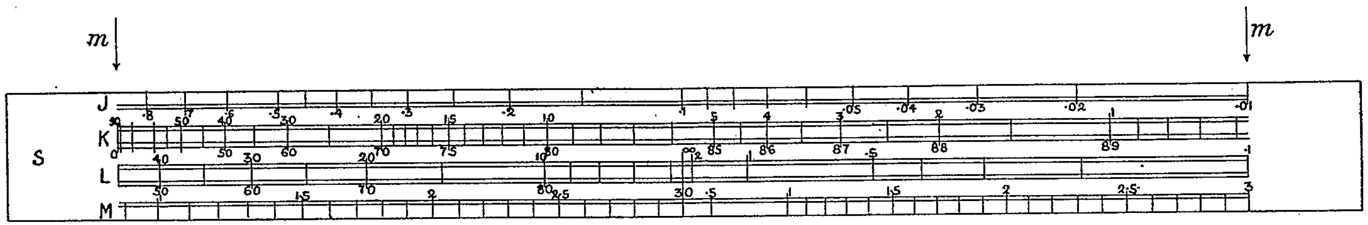

Fig: 3.

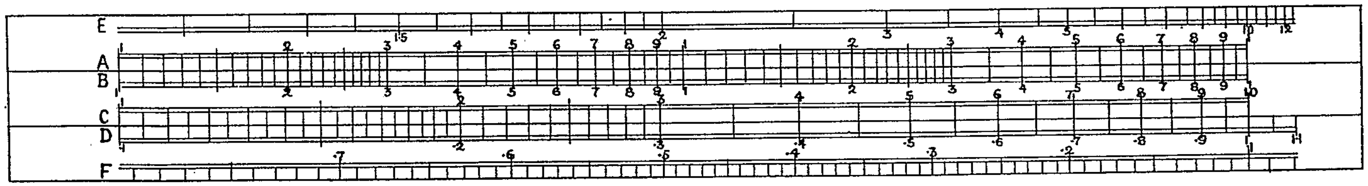

Fig: 2.