Position Line Slide Rules:
Bygrave and Höhenrechenschieber

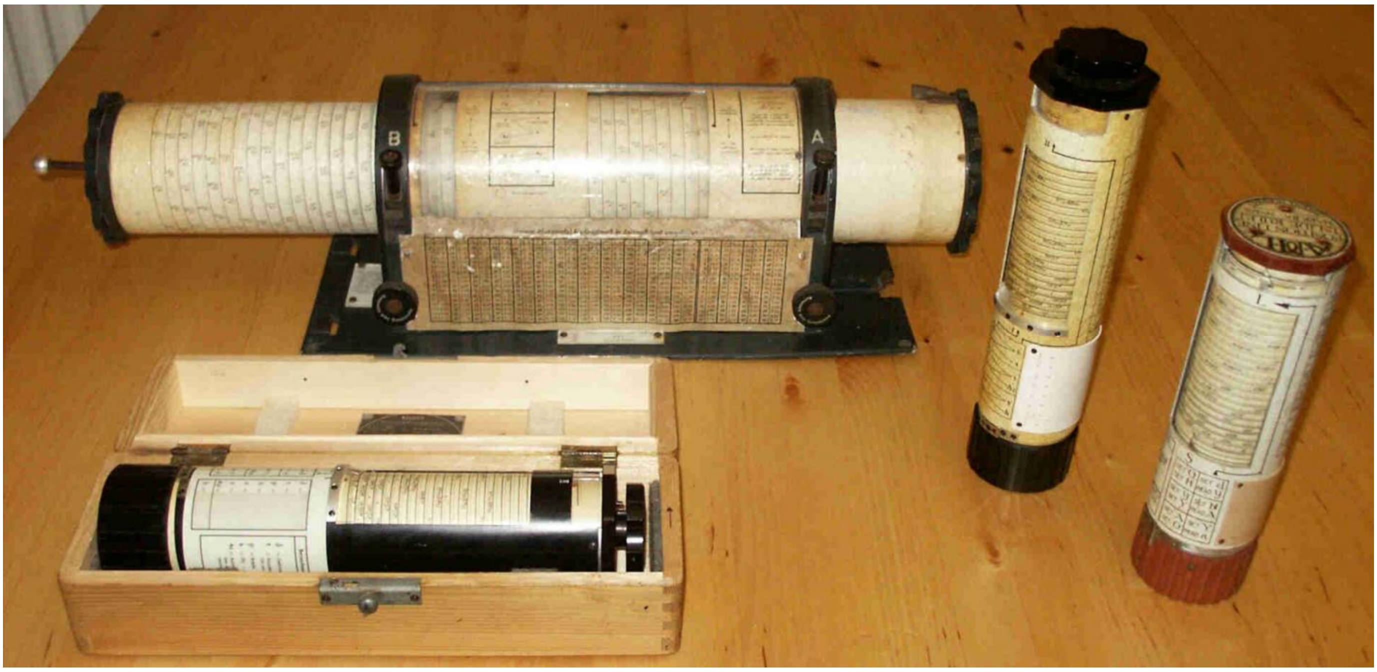

Figure 1: clockwise from bottom left: MHR1 {MC}, HR2 {MC}, HR1 {RvR}, Bygrave Mk. IIA {RvR}[1]

Introduction

In the slide rule literature, ever so often, the Bygrave slide rule is mentioned as one of the most fascinating and elusive slide rules ever, as are its brethren the HR series. These latter have been called "the finest slide rule ever made", but given that they have been developed for a single application only, this is difficult to justify.

The construction of the Bygrave itself left something to be desired, a major flaw that was corrected in the WWII era German derivative, the HR1, its almost identical twin the MHR1 and its larger offspring, the HR2. All these four variants are depicted in Figure 1 (a picture taken in October 2007 when probably for the first time in history they were all gathered in one location) and all four will be described in depth in this article.

Both the Bygrave and HR2 are rare with only a handful of copies known to have survived, some in museums; the HR1 and especially the MHR1 are a bit more common but still, not more than a few dozen seem to have survived. Because of their rarity, several people have constructed cardboard replicas to get a feel of how it was to use them in real navigation.

This article is meant to paint the full picture of these, collectively called "position line slide rules", their application area, development and practical use.

For a proper understanding of these slide rules, it is unavoidable to give some theoretical background on celestial navigation, but rest assured, I will try to keep this to the absolute minimum necessary to understand the principles and operation. For those readers interested in further theoretical treatment of celestial navigation, a multitude of books on the subject is available. My personal favorite is [Admiralty 1938].

As a side note, there has been an even more elusive position line slide rule: the circular one with seven scales mentioned in [Bowditch 1958] and invented by professor Charles L. Poor, but the approach taken by this slide rule is completely different from that of the Bygrave and the HR's, so its discussion falls outside the scope of this paper.

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[1] Curly braces { } are used to indicate collections; a full list of such collection indicators is included at the end of this article.

Celestial Navigation

Navigation by the stars has been an age-old principle for navigation when away from any fixed references. As such it has been the realm of ocean-crossing seafarers for centuries, perfecting the craft with ever more accurate means to measure the position of celestial bodies (sun, moon, planets, stars) and of keeping time, culminating in today's sextants (Figure 3) and precision watches.

The precision of navigation ideally would be in the order of meters, but this is not attainable for celestial navigation. With the best equipment available, an angle can be measured at a precision of about 0.5' (1' is one minute of arc or 1/60th of a degree), time can be measured at a precision of about 1 second. Translated into position errors, these amount to respectively ½ and ¼ nautical mile. Adding variations in density of the atmosphere, instrument errors and personal errors results in a practical lower limit to the precision of celestial navigation in the order of 1 nautical mile (1 nautical mile = 1852 m).

How simple life would be if all we would have to do is look straight up, identify the celestial body that is directly overhead, determine its position in the heavens at the precise moment of observation, and from this derive our position on earth. But out of the billions of stars, it is extremely rare that one can be seen directly overhead within 1' and positively identified using astronomical data. Even then, astronomical data can only be collected for a limited number of stars and these stars have to be bright enough to be easily identifiable even under relatively poor conditions (broken cloud, haze, twilight).

For these reasons, alternate approaches have been developed that have a much more general applicability.

Please note that for the sake of brevity, henceforth, I will use the term "star" instead of "celestial body", although what follows holds equally for objects from our solar system.

Principles

The principle of practical celestial navigation is simple: determine the altitude (angle with the horizontal) of a star and one can draw a circle on the earth where one's position must be. This circle is the intersection of the earth and a cone extending downwards from the star. "Shoot" another star, draw a second circle and one of the two intersections of these circles is your position. Assuming that the two stars are sufficiently far apart in the heaven, using one's estimated position is usually enough to eliminate the ambiguity and positively identify the current position. For added precision, a third star could be included in the process.

As might be expected, life isn't this simple: it turns out to be mathematically difficult to describe the circles on the earth's surface in such a way as to make the calculation of the position a straightforward process. Graphs were developed for different combinations of two stars [Weems 1937] (Figure 2) and recently, computer algorithms have been implemented for these calculations [Walden 2007], but the graphs took a long time to make and this was only feasible for a limited number of star combinations (usually involving the Polar Star and therefore impossible to use in the southern hemisphere) and the algorithms were not practical in the era of the great sea voyages or the early aviators.

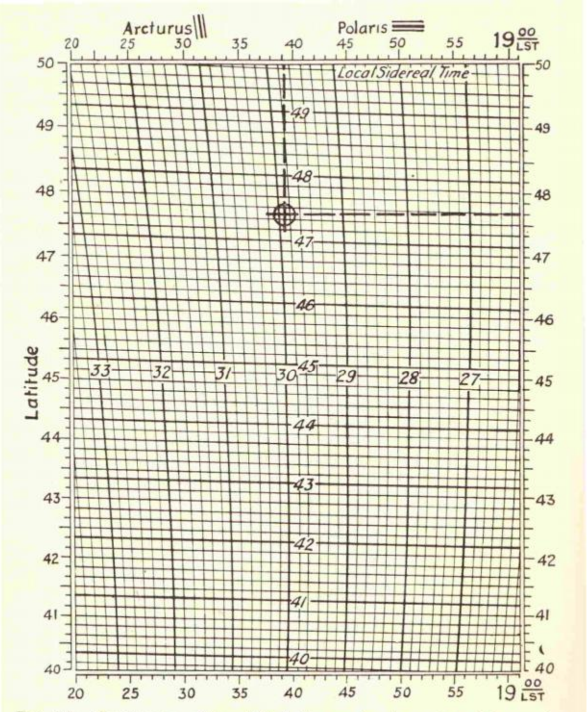


FIG. 186.—Sample page of *Star Altitude Curves*, showing a plotted intersection.

Figure 2: graph of star altitude curves

Position Lines

The method used instead is to measure the altitude of a star, compare that with the altitude calculated from an "assumed" position and determine the correction to be applied to this assumed position.

Let's first of all see how the altitude of a star is determined and how this altitude relates to an observer's position on earth. It is of vital importance here that stars are sufficiently far away that their rays hitting the earth anywhere can be regarded as parallel. The moon is not quite far enough for this assumption to hold, so special corrections need to be made when using the moon and we won't go into this any further.

A sextant is used to measure the angle between the star and the horizon; this angle is called the star's altitude. We will ignore various corrections to be made, such as those concerned with the bending of the star's light through the atmosphere, although in real life these will have to be applied to arrive at the required precision. On board ships, mirror based sextants (Figure 3) are used to measure the star's altitude with respect to the horizon. In an aircraft, this is not practical: either the horizon is below clouds or due to the aircraft's altitude, the horizon is so far below the horizontal that a large correction would need to be applied. Aircraft sextants therefore have some form of artificial horizon, usually a bubble not unlike that found in a carpenter's spirit level, but pendulums and gyroscopes are also found in aircraft sextants. **Error! Reference source not found.** shows a variety of bubble sextants used in the World War II period, clearly demonstrating the lack of standardization of this – then – young technology.

Figure 3: marine sextant {RvR}

Figure 4: World War II bubble sextants {RvR}

Figure 5 shows two observers ($\mathbf{O_1}$ and $\mathbf{O_2}$) respectively at 30º and 45º from the star's ground projection (**G**) respective to the earth's centre **C**. **S** is the direction of the star; $\mathbf{H_1}$ and $\mathbf{H_2}$ are the horizons of $\mathbf{O_1}$ and $\mathbf{O_2}$. Angle $\mathbf{H_1}$ – $\mathbf{O_1}$ – **S** equals 90º minus angle **G** – **C** – $\mathbf{O_1}$. More importantly, the difference between this angle and the angle $\mathbf{H_2}$ – $\mathbf{O_2}$ – **S** is equal to the difference between the corresponding angles at **C**. An observer at $\mathbf{O_1}$ would see the star at an altitude 15º higher than an observer at $\mathbf{O_2}$. Now, since the nautical mile is defined as 1/60th of a degree of a great circle, this means the two observers are 60 × 15 = 900 nm apart. Summarizing: as long as we are measuring along a great circle in the direction of the star's ground projection, the altitude difference in minutes of arc is equal to the distance along the earth's surface in nautical miles.

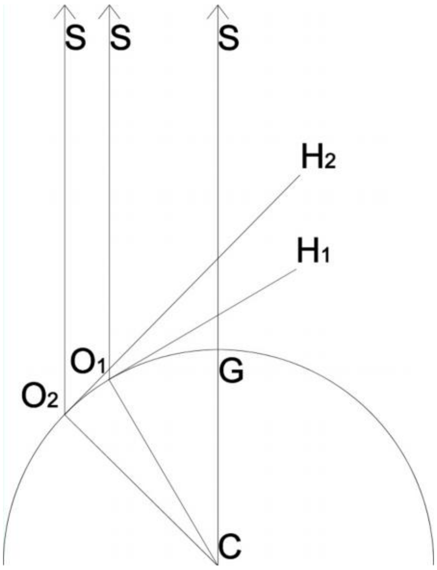


Figure 5: star altitudes

On a map at a large enough scale, we could draw circles centered on the ground projection **G** with the respective radii but in practice these circles are too large to be drawn. If we take a typical navigational map, the part of these circles on the map approximate to two parallel straight lines, perpendicular to a line towards the ground projection of the star (the azimuth line). In this approximation, we explicitly discard the situation where the star would be almost directly overhead and where the circles would have very small radii and thus their segments on the map cannot be represented as straight lines.

If we have only one observer, we can get a very similar result by taking an "assumed position" (either a dead reckoning position or a nearby position chosen for ease of calculations), calculate the altitude for that position and compare this with our measurement. This will result in a distance between the assumed position and the straight line representing the circle segment through the actual position equal to the difference in measured and calculated altitude. If we can also calculate the direction to the ground projection of the star (the azimuth), then we have all data necessary to plot a position line, i.e. a line on which our position must lie. Plotting two or more of these position lines with stars sufficiently far apart results in a unique position at the intersection of these position lines (Figure 6): AP is the Assumed Position, PosLine1 is based on an observation of Capella and PosLine 2 is based on a simultaneous observation of Procyon. The square at the intersection of these position lines is the best estimate for the current position based on these celestial observations.

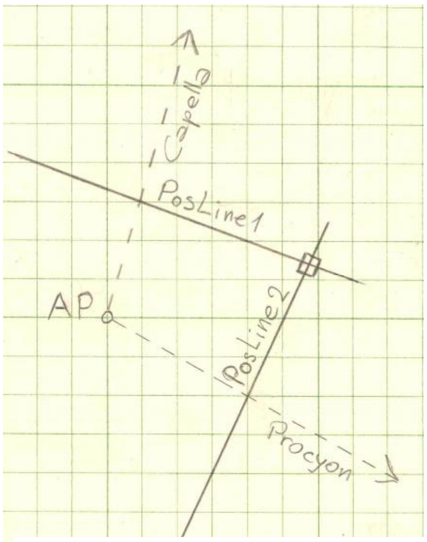


Figure 6: position lines

The Celestial Triangle

The calculations of the altitude and azimuth of the star involve three-dimensional geometry and is commonly referred to as "solving the celestial triangle".

The following description of the solution of the celestial triangle is based on the description found in [Hughes 1938], adapted to make it easier to understand for those not intimate with celestial navigation (Figure 7). Additionally, to avoid unnecessary complications, we will only consider the case where both the observer and the star are located north of the equator and where the star is to the east of the observer. Various other simplifications are also made to keep this description from becoming too lengthy and complex

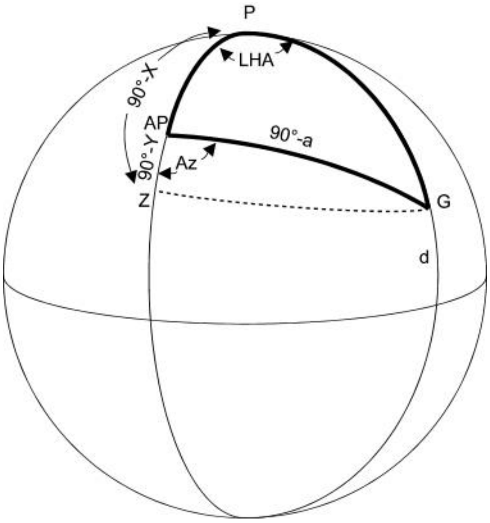


Figure 7: the celestial triangle

Just as a position on the earth is expressed in latitude (**Lat**) and longitude (**Long**), the position of a star in the sky is expressed in declination **d** (degrees north of the celestial equator) and Local Hour Angle **LHA** (degrees west of the Greenwich meridian). Declination varies so little over time that its table lookup value can be used for several years without adjustment; LHA varies per second as the earth rotates in space and has to be calculated based on the position of the star in the heaven and the Greenwich Time of observation.

The observation of a star results in an altitude **a** (degrees above the observer's horizon), its direction is the Azimuth **Az** (degrees from north).

Additionally, we will use the following names for points used in the calculations:

- **P** Pole (here we will always assume the North Pole),
- **AP** Assumed Position of the observer,
- **G** Ground projection of star: the point where a perpendicular from the star intersects the earth's surface.

The thick solid lines in the picture denote our celestial triangle; the dotted line is a perpendicular from **G** to the meridian through **AP**; its intersection with this meridian is called **Z**. **X** and **Y** are angles introduced to simplify the solution. **Az** and **a** are to be calculated. Please bear in mind that distances are of no importance for our calculations; therefore circle segments on the earth's surface are always expressed as angles (as seen from the center of the earth).

Many sets of equations have been developed over time to solve the celestial triangle, depending on the solution method; the one we will use here is based on Napier's Rules of Circular Parts [Perkins 1856] using two right-angled spherical triangles **P** – **Z** – **G** and **AP** – **Z** – **G** to arrive at a set of formulae that only contain multiplications and divisions using cosines and tangents:

Equation 1: calculation of intermediate angle

$$\tan X = \frac{\tan d}{\cos LHA}$$

Equation 2: calculation of azimuth

$$\tan Az = \frac{\tan LHA \cdot \cos X}{\cos Y}$$

Equation 3: calculation of altitude

$$\tan a = \cos Az \cdot \tan Y$$

Later on we will see how these three formulae translate into manipulations of the slide rule.

Marine Navigation versus Aerial Navigation

On board ships, it has been custom to use tables of log trig functions, either simple sine/cosine/tangent or more exotic functions like versine or haversine, depending somewhat on the cultural background of the navigator. Such calculations typically involve looking up multiple log trig values in tables (usually in five decimals, sometimes in six decimals), manually adding and subtracting them, translating them back to angles etc., typically taking a quarter of an hour or more to determine one's position.

This is perfectly acceptable if one steams along at some leisurely pace, but when flying around at hundreds of miles per hour, such calculations take too long. Methods had to be devised to speed up the calculations even at the expense of some accuracy, taking the adage that it is much more useful to know even within about 2 miles where one was 3 or 4 minutes ago than to know within 1 mile where one was 15 minutes ago.

Immediately preceding and during WWII, special sets of tables were developed, called the Air Almanac (with daily changing data) used in combination with the Astronomical Navigation Tables (a set of 14 books with constant star data) to allow relatively fast determination of one's position, but as early as 1920 an equally fast and much simpler procedure had been developed using specialized slide rules.

The Bygrave or A.M.L. Position Line Slide Rule

An obvious problem using slide rules is the required precision: if one wants to be anywhere close to the 1' precision of conventional methods, a maximum error of 1' in 90º is required, or 1 in 5400 (about 0.02 %). Even though [Gallice 1898] describes a method to solve the celestial triangle using an early Mannheim type slide rule of 50 cm length, the required precision is obviously unattainable with normal slide rules, but through the use of spiral scales as in the Fuller type of slide rules, the required precision proved possible.

This is the approach taken by the Air Ministry Laboratories (AML) at Kensington, London where Captain L.C. Bygrave of the R.A.F. developed the Position Line Slide Rule. The previous sentence combines the three names under which the device became known: "Bygrave" after its inventor, "AML" after the government body who initiated and sponsored its development and "Position Line Slide Rule" as the official Royal Air Force designation, with varieties being given successive Mark numbers as is probably better known for aircraft like the Spitfire Mk. I (initial version), the Mk. II (second production version) etc.

Description of the Bygrave

The following describes the A.M.L. Position Slide Rule Mk. IIA with serial number 355 {RvR} in detail (Figure 8). Inspection of other samples has shown no major difference, with the exception of the construction of the outer tube with the cursors on the Mk. II.

Overall length of the Bygrave is 23.1 cm when fully closed; maximum diameter is 6.6 cm. The notched base is 3.4 cm high; the top is octagonal and 0.9 cm thick.

The Bygrave consists of three concentric tubes all made of aluminum:

- Outer tube of 6.4 cm diameter and 18.5 cm length, containing instructions for use, tables of correction factors to be applied to bubble sextant readings as typically used in aircraft and two pointers acting as cursors for the scales on the inner and intermediate tubes.
- Intermediate tube of 5.9 cm diameter and 18.2 cm length, containing further instruction for use and a log cosine scale over 9.9 cm at a pitch of 0.45 cm covering 0º … 89º40’ (and back from 90º20’ to 180º) on a scale spiraling just over 22 times for a total length of 414.3 cm. This scale is very condensed near 0º and spreads out towards the higher angles, as is to be expected when the value of the cosine approaches 0.
- Inner tube of 5.4 cm diameter and 22.2 cm length, containing a log cotangent scale over 19.8 cm at a pitch of 0.45 cm covering 0º20’ … 89º40’ (and back from 90º20’ to 179º40’) on a scale spiraling almost 45 times for a total scale length of 758.4 cm. This scale is condensed around 45º and spreads out towards both the lower and the higher angles, as is to be expected when the value of the cotangent approaches either ∞ or 0. Making allowance for the smaller diameter, this corresponds to a scale exactly twice the length of the log cosine scale on the intermediate tube. Comparing these nearly 8 meters to the 50 centimeters of [Gallice 1898] shows the increased precision obtained by using the spiral scales.

All publications known to me, including the patent, mention that the Bygrave has a log tangent scale on the inner tube, but this is wrong: it is a log **co**tangent scale. The scale runs from 0º20’ … 87º40’ in the same direction as the cosine scale. A log tangent scale would run “reverse” to a log cosine scale. As an additional check on the scales, I have set the cursor at 0º on the log cosine scale and then adjusted the inner tube to make this read 45º, effectively setting cos(0º) = cotan(45º) = 1 (but also tan(45º) = 1). Careful not to move the inner tube, I then set the cursor to 60º on the log cosine scale for the inner tube to now read 63º26’, showing cos(60º) = 0.5 = cotan(63º26’).

Figure 8: Bygrave Mk. IIA {RvR}

Why Bygrave chose a log cotangent scale rather than a log tangent scale and why he has incorrectly described this in his patent, can only be guessed at (it is noteworthy that the drawing in the patent application (Figure 11: sketch from patent application) shows a log cotangent scale, making the patent application itself inconsistent). The most logical explanation seems that by using a log cotangent the orientation of the scale is the same as that of the log cosine, whereas a log tangent scale would have been oriented in a reverse direction. This in itself may not be a big problem, but when interpolating especially at small or large values, one would have to work right-to-left on the log tangent for correct results and this would have been contrary to the interpolation on the log cosine scale, introducing a potential for error of up to 5 miles if done wrongly. It is possible that in his original design, he intended to use a log tangent scale, then found out about this potential error and decided to change to log cotangent for the real thing. Forgetting to update his patent application is easily done especially in the days of the typewriter. It is only curious to note that this error has persisted for almost 90 years until it was noticed by two people almost simultaneously and independently: Gary LaPook identified the correct scale layout at just about the same time that I did [LaPook 2007].

Another question then arises: if log cosine and log cotangent have the same orientation, then log sine and log tangent would have this as well, and even more logical running from 0 to 1. The formulae could have been reworked using sine and tangent, so why was this not done? Two explanations seem plausible. First, the original equations all contain cosines and tangents, so perhaps Bygrave had started off with these two and because the interpolation error was apparent on the log tangent scale, it seemed more logical to adapt that scale while keeping the log cosine. A second explanation, which I find more convincing, could be that the log cosine scale provides for an easy starting point for the calculations: log cosine 0º = 1 is an easier start of moving cursors on cylindrical slide rule scales than would have been log sine 90º = 1 with the cursor all the way up and then going back. Obviously, a log sine scale running backwards would negate any advantage.

Over the full length of the scales a base line is present as is a line parallel to it at a distance half the length of the major markings, which appear at each full degree. In addition, the following markings are present:

- Log cosine scale (Figure 9):

from 3º on	¾ length line at each half degree (30’)
from 10º on	⅔ length line at each 10’
from 20º to 45º	½ length line at each 5’
from 45º to 60º	⅓ length line at each 2’
from 60º on	½ length line at each 5’ and ⅓ length line at each 1’

  Numbers are included at 0º, 5º, 10º, 15º, 20º, then each degree until 75º, then at each 30’ until 84º, then each 15’ until 88º, then each 5’ until the end. From 35º onwards, the complementary angle is also given.

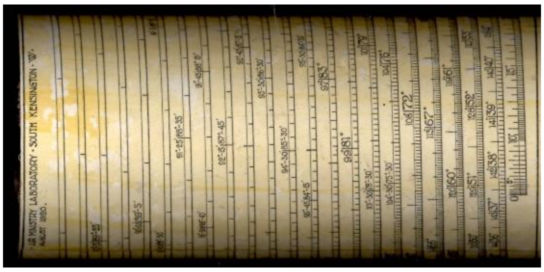


Figure 9: log cosine scale

- Log cotangent scale over the entire scale (**Error! Reference source not found.**):

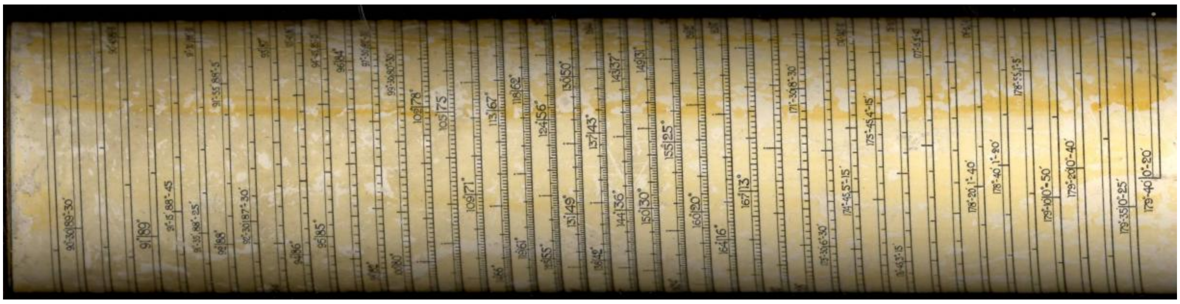


Figure 10: log cotangent scale

  ¾ length line at each half degree (30’)
  ⅔ length line at each 10’
  ½ length line at each 5’
  ⅓ length line at each 1’
  Numbers are included at each degree from 10º until 80º, then at each 30’ from 5º to 10º and from 80º until 85º, then each 15’ until 2º and 88º, then each 5’ until the end on both the high and the low end. Over the entire scale, the complementary angle is also given.

The outer and intermediate tubes contain full directions for use of the slide rule as well as several tables with correction factors to be applied to sightings of celestial bodies. These instructions include the more complex cases than described above (for example, when the observer and/or the observed star is in the southern hemisphere or is to the west of the observer) and for some special cases: the standard operations give very imprecise results when the declination or azimuth is close to 90º.

A small piece of aluminum is screwed onto the intermediate tube to act as a zero stop for the outer (cursor) tube (Figure 8, bottom left). A small rectangular piece of matte plastic material is screwed onto the outer tube (Figure 8, bottom right of cursor) for pencil writing of the initial and intermediate values. This still serves its purpose quite well.

Friction between the tubes is arranged through strips of felt and a slightly bent piece of aluminum strip on the inner tube, but this gives insufficient friction to allow for simple manipulation without fear of the inner tube moving along with the cursor tube. I have even removed a strip of felt between the cursor tube and the inner tube and added a strip of felt between the intermediate tube and the inner tube, but to no avail: the inner tube still moves too easily. It would be tempting to say that this is perhaps due to wear over time, but in contemporary documents [Bennett 1940] mention is made of this as a flaw and a comment is made that it should be improved by providing a locking mechanism.

Using the Bygrave to Solve the Celestial Triangle

To solve the Celestial Triangle using a slide rule with log cosine and log cotangent scales, Equations 1 to 3 need to be reworked into a different form, as follows:

Step 1:

Equation 1 $\tan X = \dfrac{\tan d}{\cos LHA}$ becomes

$$\frac{1}{\cot d} = \frac{\cos LHA}{\cot X}$$

Equation 4: intermediate angle

Set the cursor on the zero stop (cos 0º = 1), set **d** on the inner tube at the cursor (cot *d*) to set the left side of the equation as a proportion. Now move the cursor to **LHA** on the intermediate tube (cos *LHA*) and read the intermediate angle **X** on the inner tube (cot *X*).

Determine **Y** = 90º – **X** + latitude of Assumed Position

Step 2:

Equation 2 $\tan Az = \dfrac{\tan LHA \cdot \cos X}{\cos Y}$ becomes

$$\frac{\cos X}{\cot LHA} = \frac{\cos Y}{\cot Az}$$

Equation 5: azimuth

Set the cursor on **X** on the intermediate tube (cos *X*), set **LHA** on the inner tube at the cursor (cot *LHA*) to set the left side of the equation as a proportion. Now move the cursor to **Y** on the intermediate tube (cos *Y*) and read the Azimuth **AZ** on the inner tube (cot *Az*).

Step 3:

Equation 3 $\tan a = \cos Az \cdot \tan Y$ becomes

$$\frac{\cos Az}{\cot Y} = \frac{1}{\cot a}$$

Equation 6: altitude

Set the cursor on **Az** on the intermediate tube (cos *Az*), set **Y** on the inner tube at the cursor (cot *Y*) to set the left side of the equation as a proportion. Now move the cursor to the zero stop (cos 0º = 1) and read the Altitude **a** on the inner tube (cot *a*).

These instructions are imprinted on the slide rule in condensed form. Additional instructions are given for special cases, such as when the **LHA** or azimuth is close to 90º, in which case the corresponding cotangent cannot be set.

The declination of the star **d** is taken from tables; it is constant enough for practical purposes over several years. The Local Hour Angle **LHA** is calculated based on the longitude of the assumed position and the Greenwich Mean Time of the observation.

Bygrave Patent

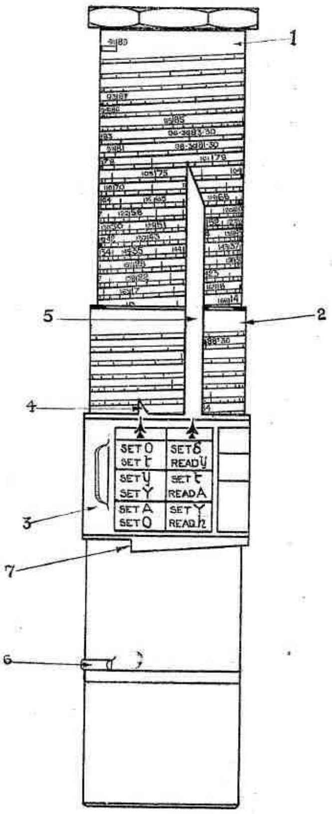


Figure 11: sketch from patent application

UK Patent 162895 describes the AML Position Line Slide Rule under the tile "Improvements in Calculating Apparatus", a cryptic title indeed, especially since the "improvements" themselves are not described in the text, and neither is the original apparatus for which this would provide an improvement.

The inventor is given as Leonard Charles Bygrave. The application is dated 29 March 1920 with the patent having been granted on 12 May 1921.
The patent describes the background of solving the celestial triangle and it shows a sketch of the Bygrave in generally similar form although the scales are only present to 89° (Figure 11). This is explained because of the rapidly expanding scales when the angles approach 90°. To solve the equations more easily for such angles, condensed scales are proposed of one circumference for log cosine and two for the log cotangent. These additional scales would have to be used in conjunction with each other. Needless to say that these condensed scales negate the accuracy advantage of the long spiral scales, probably the reason why these scales were dropped from the production Bygraves.

A second possible configuration is given as two cylinders with equal diameter, on a common spindle and having cursors on a parallel bar (**Error! Reference source not found.**). The operating principle of this alternative configuration is completely different: whereas in the standard Bygrave, the cursors have equal distance and the scales move longitudinally to set the values, here the scales are fixed longitudinally and rotate freely, while the cursors (10 and 11 in **Error! Reference source not found.**) are moved to set the values.

Even though in general configuration this alternate configuration looks somewhat like the later HR2 (which operates largely like the standard Bygrave, see below), it is a substantially different design that was apparently not proceeded with.

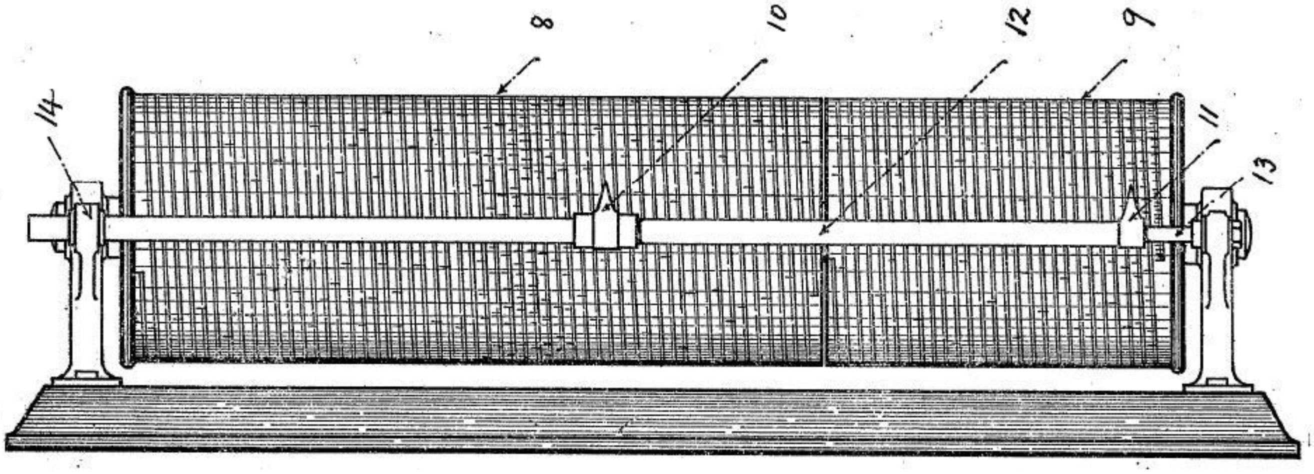


Figure 12: alternative design from patent application

Bygrave Prototype


Figure 13: Bygrave prototype {RAFM}

Even before the above-mentioned patent, the Air Ministry Laboratories of Kensington, London, manufactured what is best described as a proof-of-concept vehicle of the position line slide rule (Figure 13). This particular slide rule is in the collection of the R.A.F. Museum in Stafford, UK. It is much smaller than the production Bygrave, with a total length of 15 cm.

Its design follows closely the sketch from the patent application (Figure 11) with as the only noticeable difference the strengthening of the zero stop on the outer tube.

Figure 14: prototype scales

The scales are made using the negative type of photocopying of the time (Figure 14), with white lettering on a black background. Unfortunately, age and handling has partly erased the printing and the instructions for use are no longer readable at all. The paper itself has partly let loose and is slightly torn. The scales are still readable, however, and they are actually slightly different from those of the production Bygraves; apart from being of different length, there are additional scales, called B scales:

- inner tube: length 15 cm, diameter 4.3 cm
  log cotangent scale with 30 spirals covering 1° … 89°
  B scale with 2 spirals covering 2’ … 89° 58’
- intermediate tube: length 13.8 cm, diameter 4.5 cm
  log cosine scale with 15 spirals covering 0° … 89°
  B scale with 1 spiral covering 0’ … 89° 58’

These B scales are those mentioned in the patent application and are condensed log cosine and log cotangent scales to more easily deal with the cases where either of these values exceeds 89°.

The slide rule has the text “Air Ministry Laboratories” and the date is given as 28 January 1920 (Figure 14), so it actually pre-dates the patent application by about two months.

The most logical explanation for this is that this sample was used in the preparation of the patent application and in the definition of the position line slide rule. It is possible that more of such proof-of-concept slide rules were made, but this is the only one known to exist. The fact that it has survived at all is something of a miracle: it was found in a drawer of a desk in the Air Ministry when the place was cleared out and only by pure luck has it been preserved.

[Wimperis 1920] shows an identical looking Bygrave slide rule (Figure 15). It is described as being 6 inches long (about 15 cm) and 2 inches in diameter (about 5 cm), but it is not clear how exact these measurements were. They seem to be close enough to those of the item in the R.A.F. Museum’s collection that they could be the same item, although the item in [Wimperis 1920] doesn’t have the strengthened zero stop.

Its scales seem identical to those of the patent application and the prototype in the R.A.F. Museum’s collection. The B scale is described as “*of short length (…) for dealing with hour angles between 89° and 91° and declinations less than 1°*”. No further description is given as to their layout or use, however.

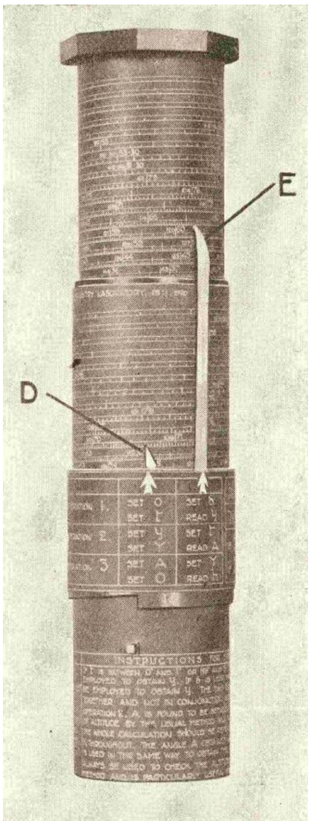


Figure 15: early Bygrave

It is of special interest to note that in the preface of [Wimperis 1920] the author is "*indebted to Mr. L.C. Bygrave, late Captain R.A.F.*", thus giving full credit to Bygrave even though dr. Henry E. Wimperis at the time was the Director of Research at the British Air Ministry [Williams 1994] and thus actually Bygrave's boss in a project like this. Even more noteworthy is that this preface is dated 28 February 1920, a month later than the prototype but again pre-dating the patent application by almost a month, sewing doubt as to whether the patent should have been granted in the first place: one cannot apply for a patent after publication.

Bygrave Manufacture and Varieties

As far as is known, Henry Hughes & Son Ltd was responsible for production of all Bygraves, although the slide rules themselves don't mention any manufacturer. No production records have survived and there are no recollections of Bygraves having been made from 1936 onwards [Harvey 2007]. This same source confirms that at Henry Hughes & Son, initial production batches were often made by hand at the premises, with later production being handed off to outside specialized firms, the product still selling under the Henry Hughes & Son name. Again, no such subcontract records can be located anymore.

Of interest are [Hughes 1938] and [Hughes 1942], respectively a commercial brochure and a catalogue by Henry Hughes & Son Ltd, indicating that the Bygrave was still being offered for sale in those years; whether any were actually sold in this period, is not known, but since there seems not to have been any production any more in these years, it may just have been a matter of clearing out existing stock.

Bygrave Mk. I?

Nothing is known about a Position Line Slide Rule Mk. I, although common sense dictates that this must have been produced. It surely must have had a short production run, if it was produced in any numbers at all, since the scales on the Mk. II and Mk. IIA are dated 18 August 1920, a mere seven months after the above-mentioned prototype.

It is unclear how the Mk. I would have been different from the Mk. II, although one can make an educated guess. My belief is that the additional scales from the prototype and mentioned by [Wimperis 1920] were present on the Mk. I and that they were found unsatisfactory. They were then replaced by a revised procedure for the special cases of the low declinations and hour angles near 90º, these scales were removed and the designation changed to Mk. II. It is also quite possible, and not in violation of R.A.F. naming conventions, that the Mk. I *was* the prototype, although no such designation is visible on the item in the R.A.F. Museum, nor is this mentioned in [Wimperis 1920]. This could then mean that Mk. Is were never produced in any series at all.

Perhaps an object clearly marked as a Position Line Slide Rule Mk. I will turn up some day to fully resolve this issue.

Bygrave Mk. II

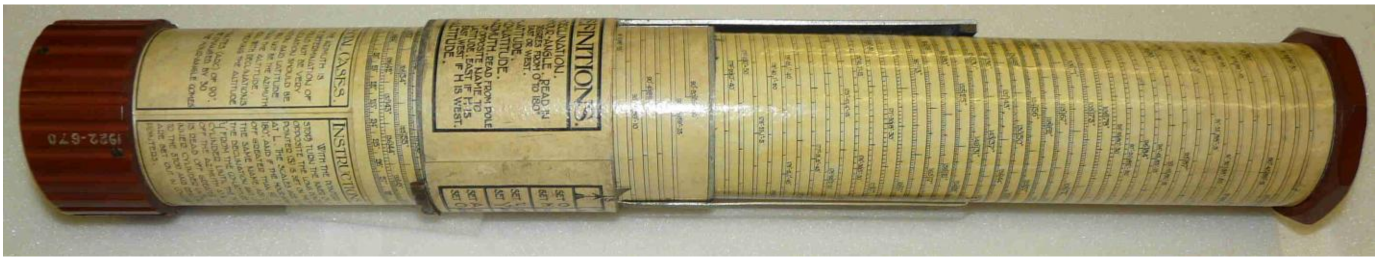


Figure 16: Bygrave Mk. II {SciM}

This is the version of the Bygrave in the collection of the Science Museum in London (Figure 16). Its Museum catalog number is 1922-670, meaning it was entered in the Museum’s collection in 1922. The Science Museum copy has the serial number 105 (Figure 18). This in turn means that since August 1920 in about two years a total of more than 100 Bygraves were produced, a significant output for a complex and presumably expensive item.

Figure 17: Bygrave Mk. II cursor detail

Its outer tube has a short and a long pointer to act as cursors for respectively the log cosine and log cotangent scales, much like the cursors on the Fuller slide rules (Figure 17).

The scales are identical to those described above for the Mk. IIA, but the rear of the cursor tube contains tables for converting time into hour angles instead of the extensive correction tables of the Mk. IIA.

It probably was manufactured from August 1920, the date printed on the scales. The most recent picture of a Mk. II that I have been able to find is in [Eaton 1926], indicating that the Mk. II was manufactured at least until 1926.

Practical use probably showed that the long pointers got hooked in the often thick flying clothes of the pilots or navigators, which is why a redesign was effected, resulting in a new designation being given.

Figure 18: Bygrave Mk. II identification

Bygrave Mk. IIA

This was the improved design, featuring an outer tube with a closed top and two short pointers to set and read the scales, a much more sensible layout for a device to be used in the cramped cockpits of those days and with the aircrew usually wearing heavy clothing.

My Mk. IIA has serial number 355 (Figure 19), suggesting that the switch-over from Mk. II to Mk. IIA was somewhere between serial numbers 105 and 355 if the serial numbers were issued consecutively across these subtypes. The earliest reference to a Mk. IIA that I have been able to find is in [Stewart 1930], indicating that the changeover from Mk. II to Mk. IIA was effected sometime between 1926 and 1930. This would then also indicate that fewer than 355 examples of the Mk. II were produced: about 100 were produced in the first two years, meaning a total of some 300 in six years seems feasible.

Figure 19: Bygrave Mk. IIA identification

This item with serial number 355 was used for most of the pictures included in this paper as well as for the detailed description, unless explicitly otherwise noted. Insofar as the dimensions are concerned, the Mk. II and Mk. IIA are identical.

Japanese version of the Bygrave

A Japanese version of the standard Bygrave Mk. II was built. It has the same dimensions, the same original cursor design and all text seems to be a straightforward translation of the English version.

Figure 20 shows the example from a German collector who wishes to remain anonymous, while [Hems 2004] shows pictures of a Japanese Bygrave coming from an American collector {JB}, identical in lay-out, as is a third Japanese Bygrave known to exist. Differences in small imperfections between these three prove that they do not depict the same sample, so at least three of the Japanese Bygraves have survived. Attempts to contact the owners of two of these three Japanese examples have been unsuccessful so far.

Figure 20: Japanese Bygrave {anon}

This Japanese Bygrave does not show any designation or manufacturer, the only markings on the top of the slide rule (Figure 21) are “NO. 872” (which looks to be a serial number) and the Japanese Katakana character “ ト” in a circle with an anchor sign, which when used in combination was a stamp of acceptance by the Japanese naval forces. This does not necessarily mean that the Japanese used them on board ships: both before and during World War II the Japanese navy had a sizeable air fleet, as large as or even larger than that of the Japanese army (there was no separate Japanese air force). Whether 872 is a serial number in the same series as those of the regular Bygraves, is not known.

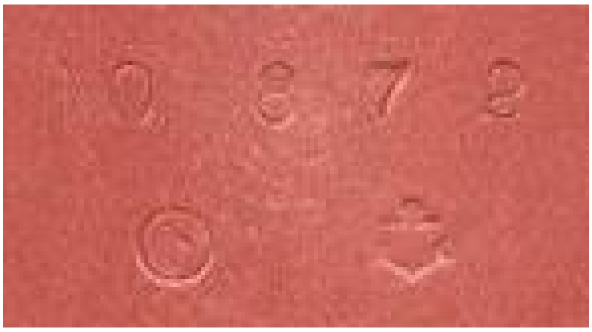


Figure 21: Japanese Bygrave identification

Again, no production details are known, and nothing is known of its history: why the Japanese ordered these, how many were built, if they were actually ever delivered to Japan or used in actual service.

Surviving Bygraves

The following Table 1 shows Bygrave Position Line Slide Rules known to have survived, with type, serial number (when known) and collection:

Type	Serial #	collection
prototype	none	{RAFM}
Mark II	105	{SciM}
Mark IIA	355	{RvR}
Mark IIA	491	eBay auction 2007
Mark IIA	492	{RAFM}
Japanese	??	{JB}
Japanese	??	{anon}
Japanese	872	{anon}

Table 1: surviving Bygraves

Please note that it is not certain that the number 872 on the Japanese Bygrave is a serial number. If it is, it could be either in a separate series (but could there really have been more than 800 built for Japan when possibly fewer than 355 Mk. IIs were built for the UK?) or it could be in the same series as the British Mk. IIs, in which case the serial number 355 for my Mk. IIA would mean that numbering restarted at 1 with the production of this variant. Only more data on Bygrave serial numbers can answer questions like these.

Practical Use of the Bygrave

As far as is known, the Bygrave was only used for aerial navigation. There are no records of naval use. It was used for various headline-making long-range flights in the early nineteen-thirties, the best documented of which is the crossing from New Zealand to Australia in a single-engined floatplane in March to June 1931 by (later sir) Francis Chichester [Chichester 1933]. The navigation for the second leg of this flight, between Norfolk Island and tiny Lord Howe Island in the Tasman Sea on 1 April 1931, is shown in Figure 22 [Chichester 1942]. This leg had a length of 575 statute miles and was flown in an elapsed time of 7 hours and 40 minutes.

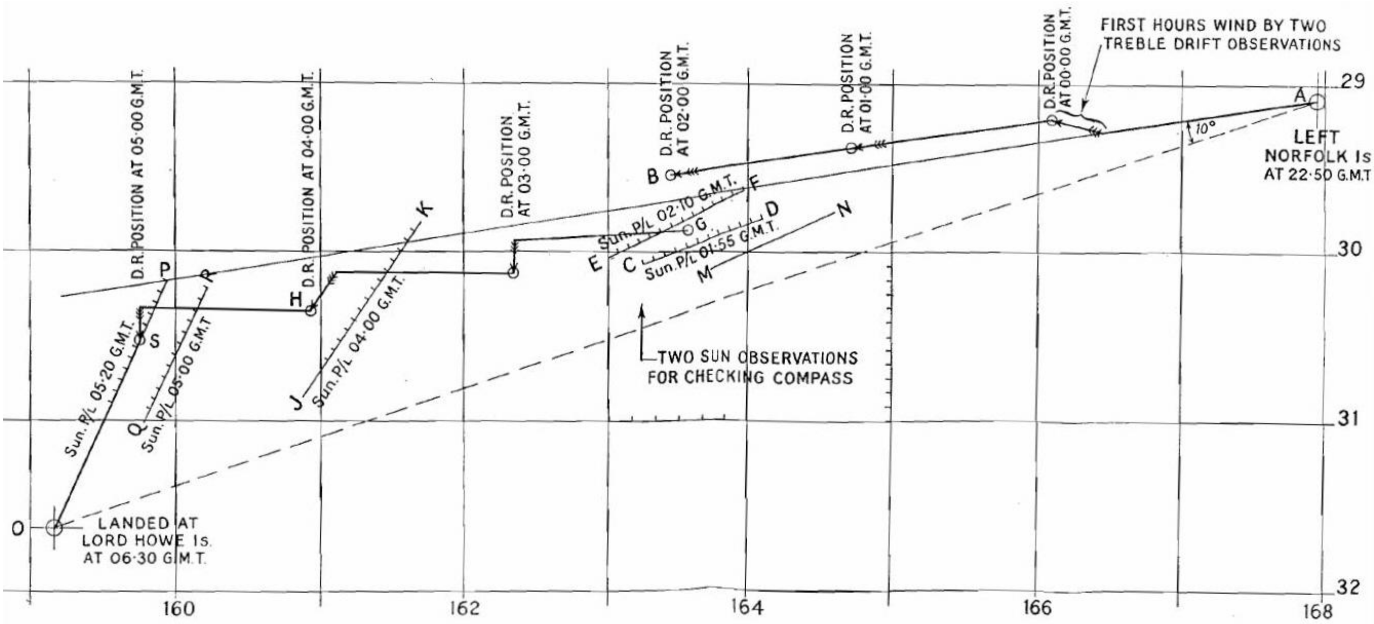


Figure 22: annotated map of Chichester's 1 April 1931 flight

The initial course steered was 10º north of the track to ensure that the arrival near Lord Howe would be to the north even when the wind would be significantly different from the estimates. Immediately upon take-off (point A), a position line was calculated for the arrival near Lord Howe (line O - P), based on the expected time of arrival. Navigation was mostly by dead reckoning ("DR", see [van Riet 2007]), but at various times, sun sightings were made to correct the DR position: position lines C - D and E - F corrected the DR position B to position G, resulting in an important correction in the wind used for DR navigation. Successive position lines J - K and Q - R were used to keep track of progress and when position S was reached, Chichester made a left turn along the position line towards Lord Howe. This guaranteed him that he would end up over the island, although he had no idea how far away he was when turning left. The flight was made at daytime and therefore, only the sun could be used for navigation, eliminating the possibility of determining actual position by two position lines based on different celestial bodies. Given that Chichester was flying a small, slow floatplane, made of wood, fabric and piano wire while performing some of the most difficult navigation and without autopilot or other aids other than a basic compass, airspeed indicator, altimeter and clock, this flight was proof of remarkable airmanship indeed.

Evidence of much later use of the Bygrave is given by Hughes [1946] who states "*this instrument still provides the favorite means of reducing a sight for one R.A.F. navigator, Air Vice Marshall D.C.T. Bennett, the Pathfinder A.O.C.*", indicating that Donald Bennett still used his Bygrave at the end of World War II, making him very likely the last person to ever have used a Bygrave in anger. Bennett was a pathfinder pilot, meaning he had to fly in advance of the main bomber formations and mark the targets. Accurate navigation was of paramount importance and celestial navigation using the Bygrave was the most accurate and fastest navigational method, making it plausible that Bennett did, indeed, use his Bygrave throughout World War II.

Unfortunately, I have never seen any photographs of anyone using a Bygrave; such pictures rank high on my "most wanted" list.

Users of the Bygrave reported that the most important problem in using it was that the inner tube could not be fixed with respect to the intermediate tube. [Bennett 1940] states unambiguously: *"But it has one disadvantage. When old it is liable to slip, and if this is not noticed an undetected and dangerous error may result. It is regrettable that some very positive means of preventing this is not incorporated"*. Given that this is written by one of the greatest proponents of the Bygrave and likely the person who used it for the longest period, this is real criticism indeed. As stated earlier, I can only confirm this as a real problem, so much so that practical use of my Bygrave Mk. IIA in its current state is impossible.

The Dennert & Pape HR series

HR is an abbreviation of “Höhenrechenschieber”, German for “altitude slide rule”, referring to the fact that it is used to calculate the altitude of a celestial body. As such, its designation refers to what is being calculated rather than the application for which this calculation is used, as is the case with the Bygrave.

The HR’s are often referred to as “carbon copies” or “facsimiles” of the Bygrave, but as they provide a locking mechanism, the absence of which was seen as a major flaw in the Bygrave, the HR-series can justifiably be regarded as a major development of the theme giving the HR’s their own place in history if for nothing else.

HR’s are often referred to as HR-1 etc. in literature. Designations engraved on the items themselves as well as the production details (see below), invariably mention HR1 etc. as the spelling convention, so I will stick to the designations without the hyphen.

HR1

The HR1 (Figure 23) is very similar to the Bygrave Mk. IIA in layout: same construction of three concentric tubes, same scale layout (log cosine on the intermediate tube, log cotangent on the inner tube) to the extent that one is forgiven at a first glance to regard it as a carbon copy of the Bygrave, and in fact, in several publications it has been described either as such or as a facsimile. But it is slightly larger and there are small differences to show that it has been redesigned (the scales are slightly longer and have a different font) and it has as a major improvement a mechanism to lock the inner and intermediate tubes with respect to each other, solving the problem inherent to the Bygrave and referred to several times earlier.

Figure 23: HR1 {RvR}

Like the Bygrave, the HR1 was originally meant to be used in aircraft and was given Luftwaffe designations. It was most likely used for marine navigation as well, although the MHR1 variant (see below) is often described as being a dedicated Kriegsmarine variant.

For completeness’ sake, to make this part of the paper useful in its own right, but at the risk of duplicating some information, I will give the full description of the HR1 in the same manner as I have before given that of the Bygrave.

Description of the HR1

Overall length of the HR1 is 28.3 cm when fully closed; maximum diameter is 6.2 cm. The base is 3.7 cm high; the octagonal top is 0.7 cm thick with a knob extending a further 1.6 cm.

The HR1 consists of three concentric tubes all made of aluminum:

- Outer tube of 6.2 cm diameter and 22.2 cm length, containing instructions for use, tables of correction factors to be applied to bubble sextant readings as typically used in aircraft and two plastic strips with red pointers acting as cursors for the scales on the inner and intermediate tubes.
- Intermediate tube of 5.8 cm diameter and 22.5 cm length, containing further instruction for use and a log cosine scale over 9.9 cm at a pitch of 0.45 cm covering 0º … 89º40’ (and back from 90º20’ to 180º) on a scale spiraling just over 22 times for a total length of 407.3 cm. This scale is very condensed near 0º and spreads out towards the higher angles, as is to be expected when the value of the cosine approaches 0.
- Inner tube of 5.4 cm diameter and more than 23.0 cm length (the slide rule cannot be easily disassembled like the Bygrave and therefore no exact length can be measured), containing a log cotangent scale over 19.8 cm at a pitch of 0.45 cm covering 0º20’ … 89º40’ (and back from 90º20’ to 179º40’) on a scale spiraling almost 45 times for a total scale length of 758.4 cm. This scale is condensed around 45º and spreads out towards both the lower and the higher angles, as is to be expected when the value of the cotangent approaches either ∞ or 0. Making allowance for the smaller diameter, this corresponds to a scale exactly twice the length of the log cosine scale on the intermediate tube.

Over the full length of the scales a base line is present as is a line parallel to it at a distance half the length of the major markings, which appear at each full degree. In addition, the following markings are present:

- Log cosine scale:

from 3º on	¾ length line at each half degree (30’)
from 10º on	⅔ length line at each 10’
from 20º to 45º	½ length line at each 5’
from 45º to 60º	⅓ length line at each 2’
from 60º on	½ length line at each 5’ and ⅓ length line at each 1’

  Numbers are included at 0º, 5º, 10º, 15º, 20º, then each degree until 75º, then at each 30’ until 84º, then each 15’ until 88º, then each 5’ until the end. From 35º onwards, the complementary angle is also given.
- Log cotangent scale over the entire scale:
  ¾ length line at each half degree (30’)
  ⅔ length line at each 10’
  ½ length line at each 5’
  ⅓ length line at each 1’
  Numbers are included at each degree from 10º until 80º, then at each 30’ from 5º to 10º and from 80º until 85º, then each 15’ until 2º and 88º, then each 5’ until the end on both the high and the low end. Over the entire scale, the complementary angle is also given.

The outer and intermediate tubes contain full directions for use of the slide rule as well as a table with correction factors to be applied to sightings of celestial bodies. These instructions include the more complex cases than described above (for example, when the observer and/or the observed star is in the southern hemisphere or is to the west of the observer) and for some special cases: the standard operations give very imprecise results when the declination or azimuth is close to 90º.

A small piece of aluminum is screwed on the intermediate tube to act as a zero stop for the outer (cursor) tube.

A small rectangular piece of matte plastic material is screwed on the outer tube for pencil writing of the initial and intermediate values. This still serves its purposes quite well.

One noticeable feature of many, if not all, HR1’s is the pockmarked surface of the tubes. This is more or less even across the entire surface, negating the suggestion that it is caused by corrosion of the aluminum. Such widespread corrosion would be highly unusual anyway, since the uneven surface also appears on samples with largely or entirely intact paper covers. Moreover, on those places where the paper is damaged, the aluminum doesn’t seem extremely corroded. All this taken into account, it seems that the surface was made irregular for a reason and the only one I can think of is to give it a better grip than a smooth surface.

The main improvement feature in the HR1 is the knob on the top that allows the inner and intermediate tube to be locked against each other. The mechanism is probably that of flanges on the inner tube being pressed outside against the intermediate tube, but since I have never seen the innards of an HR1, I have no idea what this looks like exactly. To lock the tubes, the knob at the top is turned counterclockwise, to unlock, it is turned clockwise against a stop. The canister for the HR1 gives instructions on how to adjust the knob in case the locking mechanism has become too loose (Figure 24). There also is an explicit warning not to carry out any other repairs, one reason I haven't tried to take the HR1 apart. The locking of the tubes is a major improvement, making the slide rule much easier to handle and much less error-prone.

Figure 24: top of HR1 canister

The bottom of the HR1 is imprinted with its identification (Figure 25) and gives the following numbers:

Figure 25: HR1 identification

- Gerät-Nummer (Equipment Number): two variants exist:
  - 127-136A distinguishable by two holes in its base. The only reason I can think of for having these holes is to make the tubes move smoother: air can move freely when the tubes are slid together or apart.
  - 127-136A-1 without the holes in the base. My best guess as to why the holes were removed is that too much dust entered through these holes, obstructing the proper operation of the device.

  It is not known when the 127-136A-1 replaced the 127-136A or if the earliest models were designated 127-136, i.e. without the A suffix.
  I haven't noticed any other differences between these two variants other than that the pock markings on the 127-136A seem less pronounced than on the 127-136A-1.
- Werk-Nummer (Construction Number): see the table with production data and Werk-Nummer below for a full overview.
- Anforderzahl (Stock Number) Fl 23892: this number is the same for all HR1's I have seen and follows the standard German wartime Luftwaffe navigation numbering: Fl for Flugzeuggerät (aircraft equipment), 23 indicating it is navigation equipment.

HR2

Although the designations suggest otherwise, the HR2 (Figure 26) predated the MHR1 (see production data below).

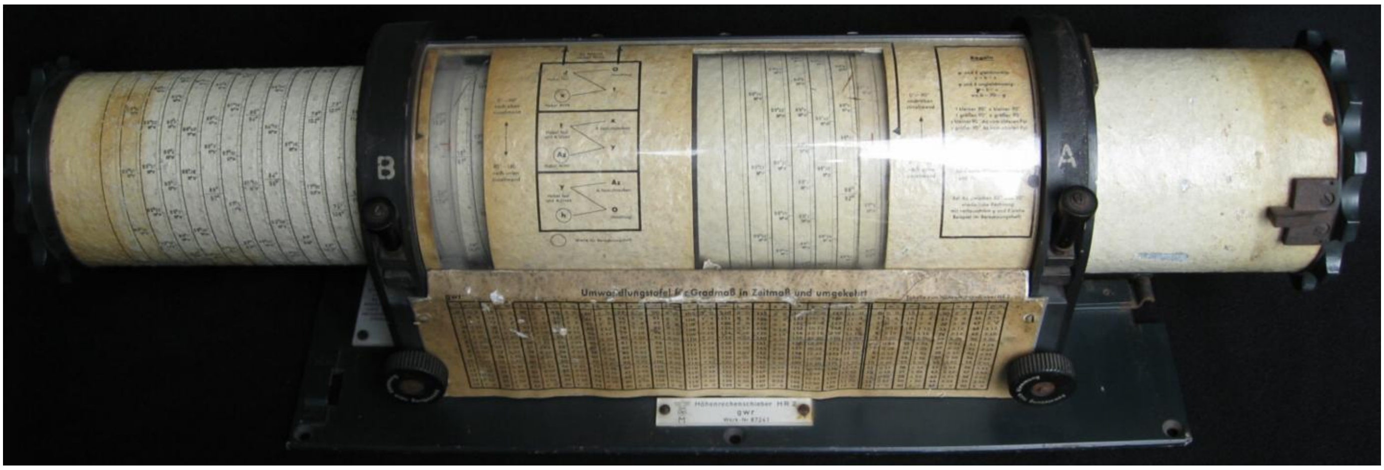


Figure 26: HR2 {MC}

The HR2 is a larger, heavier version of the basic HR1 for use on board submarines. It could be screwed onto a table for stable use even in a heavy swell.

It weighs 4.5 kg. Its base measures 42 x 17 cm and its total height is 15.5 cm. When fully closed it measures 43 cm in length, fully extended its length is 66 cm. The inner and intermediate tubes have circumferences of 29.5 and 31 cm respectively.

Its operation is significantly different from that of the HR1, where the cursor is moved over the intermediate tube to set the cosine, following which the inner tube is moved to set the cotangent value. With the HR2, the cursor tube is fixed to the base and so the intermediate tube is moved left and right to set the cosine value in the rightmost window (Figure 27), following which the inner tube is moved to set the cotangent value in the leftmost window (Figure 28). With this latter window being as narrow as it is, the setting of the cotangent may be awkward, especially on the large or small values, where the distance between individual markings is comparatively large.

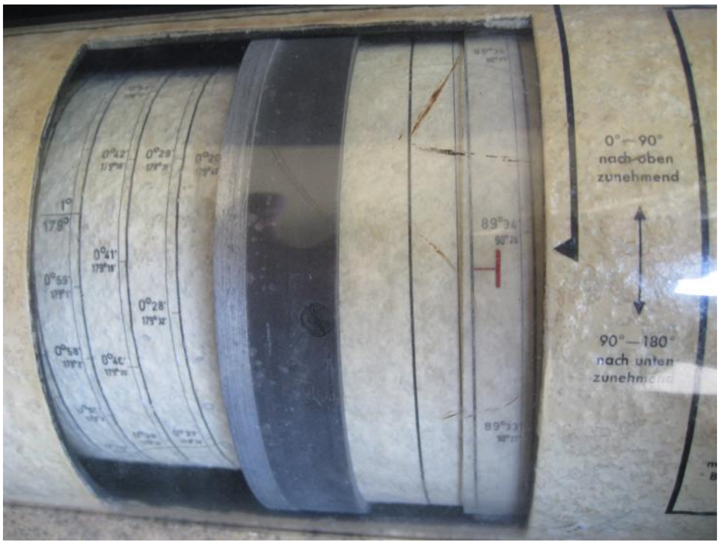


Figure 27: cosine window

Figure 28: cotangent window

The scales (Figure 29 shows a detail of the cotangent scale) are generally similar to those of the HR1 with a pitch of 9 mm, double that of the HR1 to accommodate the horizontal writing of the scale values.

The inner tube has a diameter of 9 cm and 27 spirals, for a total scale length of about 7.60 m and a corresponding length for the cosine scale of about 3.90 m, roughly equal to those of the HR1. It is perhaps surprising that such a large, more complex and thus more expensive item was built in the war years without any justification by increase of precision. The possibility to screw the HR2 onto a table to allow for easier operation in rough seas seems a meager excuse for its construction.

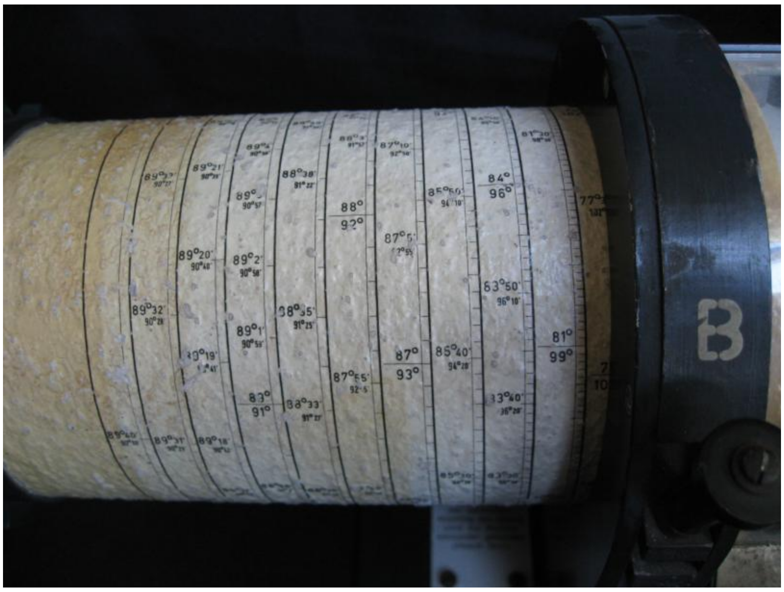


Figure 29: cotangent scale detail

The HR2 has a locking mechanism as has the HR1, but this was more advanced using a handle at the extreme left. When straight, the tubes would move freely; pulling it towards oneself locks the tubes. But with the tubes locked, they can be micro-adjusted using knobs on the left and right below the letters “B” and “A” (Figure 30) and here lies a major improvement in operating the HR2 as compared to the HR1.

Like the HR1, the HR2 contains a scratchpad to write down initial and intermediate values, a number of conversion tables as well as instructions for use, similar to those of the HR1 but adapted to the revised operating method.

Figure 30: micro-adjustment mechanism

The identification markings (Figure 31) of the HR2 are Kriegsmarine as shown by the abstract eagle and swastika over the capital letter M (for Marine). g w r is the standard wartime designation for Dennert & Pape.

Figure 31: HR2 identification plaque

MHR1

The MHR1 (Figure 32) was manufactured as a successor to the HR1. It is generally assumed that MHR1 stands for Marine HR1 (Navy HR1), although it could also stand for Modifizierter HR1 (Modified HR1). It has earlier been suggested that the M stands for Manuell (Manual), but that seems to make little sense, since the HR1 was as much a manual computer as was the MHR1.

Figure 32: MHR1 {RH}

Assuming that "M" stands for Marine, it is strange that MHR1's were produced from May 1943 onwards (see production data below), when the U-Boote were less prominent than in 1941 or early 1942. Perhaps before May 1943, the Kriegsmarine used standard Luftwaffe HR1's: such use is known for the ARG-1 (Astronomisches Rechengerät, another mechanical device to solve the celestial triangle) where the items retained their original Luftwaffe markings and designations even though they were almost exclusively used by the Kriegsmarine.

Later on, I will make a strong case for the "M" in MHR1 standing for Modifiziert and my personal opinion is that this is the real meaning of the abbreviation.

Its general look and feel (and use) is like the HR1 with as the most obvious differences:

- outer tube, which is predominantly black;
- no correction tables on the outer tube;
- the "scratch pad" is integrated in the outer tube instead of being bolted on separately;
- the surface of the tubes is smooth, not pock-marked as on the HR1.

The dimensions of the MHR1 are identical to those of the HR1. The scales are identical to those of the HR1. The bottom of the MHR1 is imprinted with its identification (Figure 33) and gives the wartime military designation of Dennert&Pape (g w r) in addition to the Werk-Nummer.

Some MHR1's were delivered in a metal tube like the HR1 (e.g. Werk-Nummer 86607), others in a wooden box (e.g. Werk-Nummer 90661, see Figure 1).
I would expect that those in the metal tube were delivered to the Luftwaffe, those in the wooden box were destined for the Kriegsmarine.

Figure 33: MHR1 identification

HR Production

In the Company Archive for Dennert & Pape, the Deutsches Museum has a small black notebook containing handwritten entries for the orders received from the German military for items to be produced. The entries start at Werk-Nummer 50000, allotted on 11 January 1938. Whether earlier Werk-Nummer existed is not known: it is quite possible that 50000 was an arbitrary starting number as it seems too much of a coincidence that a new book was started at such a round number and at the beginning of a year.

The same archive also contains a letter written by a major employee of Dennert & Pape after WWII, indicating that the HR1 was developed at the request of the Reichs Luftfahrt Ministerium (RLM: Ministry of Aviation) in early 1938.

These two facts make it highly unlikely that any HR's were built before 1938, so this little black book is supposed to contain the full order list of Höhenrechenschieber until the end of WWII, represented here in tabular format:

	Werk-Nummer					Number Ordered		
date	from	to	Type	designations mentioned		HR1	HR2	MHR1
11-1-1938	50000	50019	HR1	FL23892		20		
11-1-1938	50020	50024	HR2				5	
23-12-1938	51370	51373	HR1	FL23892		4		
10-10-1939	55993	56192	HR1	FL23892	127-136A	200		
12-10-1939	56192	56792	HR1	FL23892	127-136A	601		
3-11-1939	56794	56795	HR1	FL23892	127-136A	2		
1-8-1940	64632	64637	HR1			6		
29-5-1942	81969	81969	HR2				1	
23-6-1942	81976	82225	HR1			250		
2-10-1942	82335	82542	HR1			208		
13-11-1942	82545	82553	HR2				9	
25-11-1942	82919	83278	HR1			360		
8-1-1943	83309	83548	HR1			240		
4-8-1942	83551	83950	HR1			400		
26-1-1943	83951	84000	HR1			50		
26-8-1942	84001	84015	HR1			15		
23-2-1943	84024	84263	HR1			240		
21-5-1943	86450	86689	MHR1					240
21-5-1943	86690	86929	MHR1					240
20-10-1943	86952	87231	MHR1					280
20-10-1943	87232	87631	HR2				400	
6-9-1944	89924	90163	MHR1					240
6-9-1944	90164	90403	MHR1					240
6-9-1944	90404	90643	MHR1					240
6-9-1944	90644	90883	MHR1					240
6-9-1944	90884	91123	MHR1					240
6-9-1944	91124	91363	MHR1					240
6-9-1944	91364	91963	MHR1					600
				totals ordered		2596	415	2800

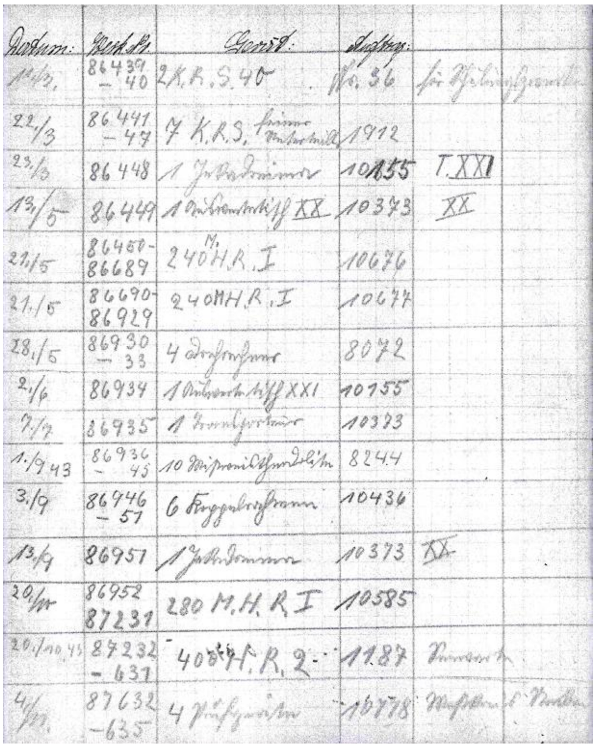
Table 2: HR order details

The explicit mention of “127-136A” for the third to fifth production batches of the HR1 leads to the suggestion that earlier HR1’s were perhaps designated 127-136 (without the “A”), although not having seen any sample from the first two production batches (24 HR1’s in total), this cannot be confirmed at this time. No other military designations for any of the HR’s can be found in this book.

MHR1’s completely replaced the HR1’s on the order list (and thus on the production line) from 21 May 1943. It is noteworthy that for the first two series of MHR1’s ordered, the entry was originally HR1, later changed into MHR1 by adding the “M” (Figure 34), which is why I believe that the “M”, indeed, stood for Modifiziert instead of Marine: it seems highly unlikely that Luftwaffe orders were replaced by Kriegsmarine orders and moreover, why no earlier Kriegsmarine versions and no later Luftwaffe versions?

The large number of MHR1’s ordered late in 1944 is also noteworthy and seems to have been of little practical value; it seems more likely that such production was an excuse for people involved in the manufacture of such “articles of strategic value” to be kept in the relative safety of a factory and not to be sent to the front.

The late production of MHR1’s may well explain why many of the surviving MHR1’s are in a comparatively good state: if they were issued at all, they cannot have been used for a long time and thus had little risk of damage.



Datum:	Werk-Nr.	Gerät:	Auftrag:		
1/3.	86439 – 40	2 K.R.S. 40	No. 36	für [illegible]	
22/3	86441 – 47	7 K.R.S. [illegible]	1912		
23/3	86448	1 [illegible]	10155	T. XXI	
13/5	86449	1 [illegible] XX	10373	XX	
21/5	86450- 86689	240 M. H.R. I	10676		
21/5	86690- 86929	240 MH.R. I	10677		
28/5	86930 – 33	4 [illegible]	8072		
2/6	86934	1 [illegible] XXI	10155		
7/7	86935	1 [illegible]	10373		
1/9 43	86936 – 45	10 [illegible]	8244		
3/9	86946 – 51	6 [illegible]	10436		
13/9	86951	1 [illegible]	10373	XX	
20/10	86952 87231	280 M.H. R I	10585		
20/10 43	87232 – 631	400 H.R. 2	1187	[illegible]	
4/11	87632 –635	4 [illegible]	10778	[illegible]	[illegible]

Figure 34: page from the little black book {DeuM}

The totals are in proportion to the number of items preserved: clearly more HR1’s and MHR1’s are preserved than are HR2’s.

Table 3 lists HR’s known to have survived, with type, Werk-Nummer, collection and additional information:

Type	Werk-Nr	collection	additional notes
HR1	56323	{RAFM}	127-136A
HR1	56395	{RH}	127-136A
HR1	no label	{RAFM}	127-136A-1 (?)
HR1	83411	{SciM}	127-136A-1
HR1	84229	{RvR}	127-136A-1
HR1	84235	{anon}	127-136A-1
HR2	not known	{HP}	
HR2	87241	{MC}	
MHR1	no label	{KlK}	metal tube
MHR1	86607	{HP}	metal tube
MHR1	90015	{JH}	wooden box
MHR1	90026	{DMM}	
MHR1	90033	{RH}	
MHR1	90176	{anon}	
MHR1	90318	{JH}	wooden box
MHR1	90416	{JBr}	wooden box
MHR1	90424	{SS}	wooden box
MHR1	90661	{MC}	wooden box

Table 3: surviving HR’s

There is one more MHR1 that has survived, with a serial number in a completely different sequence and with slight differences in other respects as well, see Figure 35 and Figure 36 {GB}.

It clearly shows the marking Dennert & Pape, Hamburg and has a St.-Nr. 26576. I have no idea what St.-Nr. means, nor in what series this number belongs, but I consider it highly unlikely that would be an earlier one in the same series from the little black book (see above). For one, all evidence indicates that Werknummer below 50000 were not allocated (at least not to HR1's) and the book extends all the way to the end of World War II. It could be a post-WWII version, but then it would be very early post-WWII, since the later models are assumed to have been marked Aristo and would have the type number 510 (see below).

The wooden box seems a little smaller than that of other MHR1's and has a metal label with the text

DENNERT & PAPE
Fabrik Wissenschaftl. Inst.
Hamburg – Altona.

Unfortunately, I have not been able to obtain a clear picture of this label as yet.

Interesting is also, that the center part of the base seems slightly dome-shaped (check the highlight spot and the shadow) rather than flat as in all other (M)HR1's that I have seen.

It seems most likely that this is, indeed, a post-war version, but the nature of the number still is to be resolved.

Figure 35: MHR1 unidentified variant {GB}

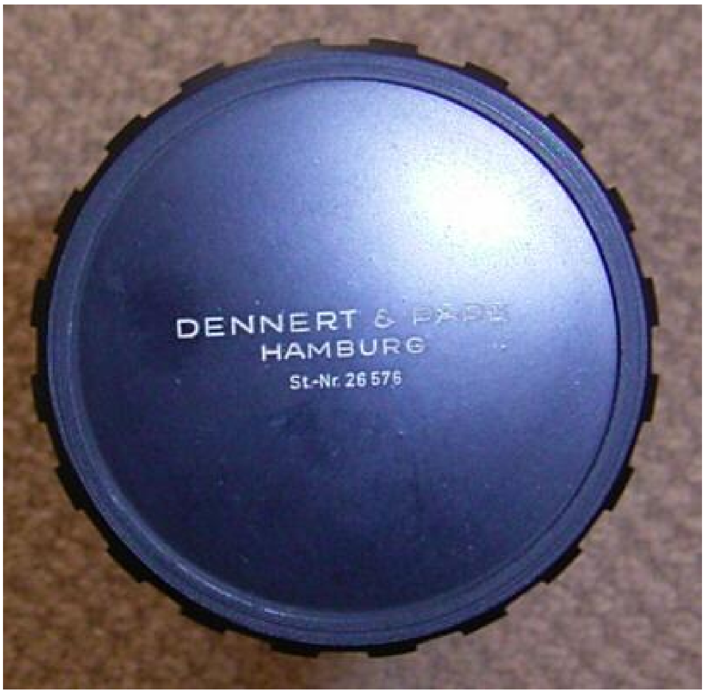


Figure 36: MHR1 unidentified variant serial number (?)

HR1’s post war

The end of World War II did not signal the end of the HR1, very likely not even the end of the production: it seems that HR1’s have been produced from 1951 to 1958 under the designation Aristo 510 [D&P 1994].

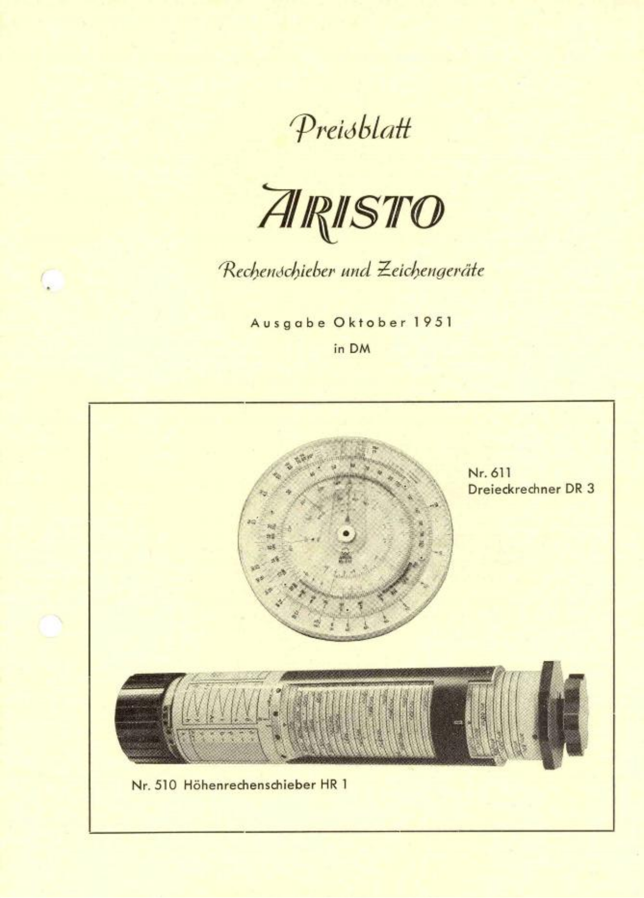
This version of the HR1 was also included in post-war Aristo product brochures [Aristo 1951] (Figure 37) under this same designation 510.

The accompanying picture clearly shows an MHR1, so it is slightly surprising that this item was called HR1.

It was sold in 1951 at the then sizeable price of DM 180 (Figure 38); compare this to a regular Aristo 902 Student slide rule which sold at DM 9.

I have yet to see any other photographs or actual samples produced as Aristo 510.

Finally, the earlier mentioned letter of 1975 explicitly states: “*Thank you for your order of an HR1. (…) HR1’s are no longer being produced and there are few remaining in stock.*” Apparently, as late as 1975, HR1’s were still being sold new by Aristo, 55 years after the first Bygrave must have left the production line at Henry Hughes & Son.



Preisblatt

ARISTO

Rechenschieber und Zeichengeräte

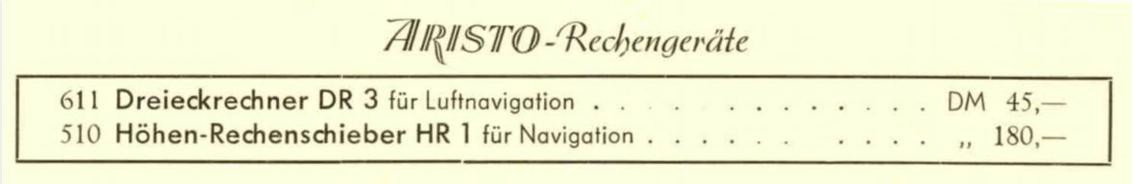
Ausgabe Oktober 1951

in DM

Nr. 611
Dreieckrechner DR 3

Nr. 510 Höhenrechenschieber HR 1

Figure 37: post-war HR1 in 1951 pricelist



ARISTO-Rechengeräte

611	**Dreieckrechner DR 3** für Luftnavigation	DM 45,—
510	**Höhen-Rechenschieber HR 1** für Navigation	„ 180,—

Figure 38: price of HR1 from 1951 pricelist

Modern reproductions

As stated at the beginning of this paper, several people have made reproductions of the Bygrave position slide rule, mostly from cardboard and paper.

Zvi Doron [Doron 2007] wrote a computer program to print the correct scales on paper, fiddling with the scaling to make the printed scales just wide enough to exactly fit the circumference of the cardboard tubes on which they had to be glued. This same program was used – slightly adapted, especially in the scaling of the printout – by both Geoffrey Kolbe [Kolbe 2006] and Gary LaPook [LaPook 2007]. Zvi actually built two, the first one, which he regarded as a prototype, without a zero-stop. He kindly presented this one to me in early 2007 (Figure 39).

Figure 39: Zvi Doron's first cardboard reproduction {RvR}

Gary built three: the first all cardboard, a second one with the inner tube made of metal and the cursor tube made of transparent plastic (Figure 40). His third design is most noteworthy in that it dispenses with a cursor tube: the log cosine scale is printed on transparent plastic, allowing direct alignment of the log cosine and log cotangent scales.

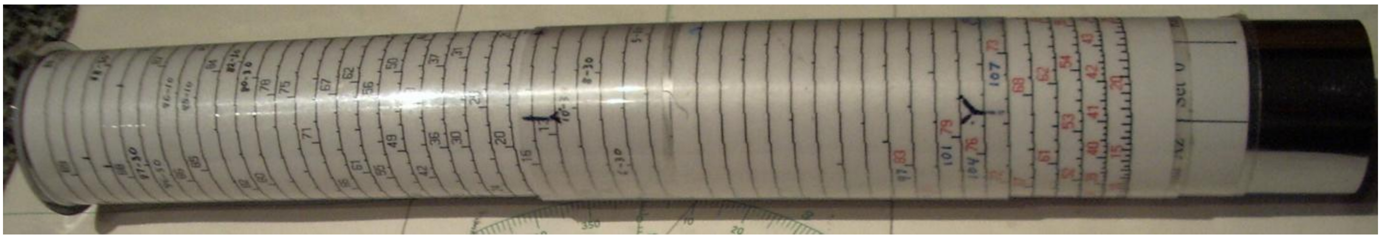


Figure 40: Gary LaPook's second Bygrave reproduction

In his fourth replica, Gary has gone one step further: he constructed a “flat” Bygrave by copying and pasting two cotangent scales next to each other and overlaying this with a transparent cosine scale (Figure 41): by moving the two scales over each other, the calculations can be performed very quickly and efficiently. Interestingly, he still uses the label “tangent scale” even though he has recognized this to be a cotangent scale.

All three homebuilders have tested their reproductions with real star or sun shots, but only Gary seems to have used his while flying an airplane, thus bringing the Bygrave back to the environment for which it was developed now nearly a century ago.

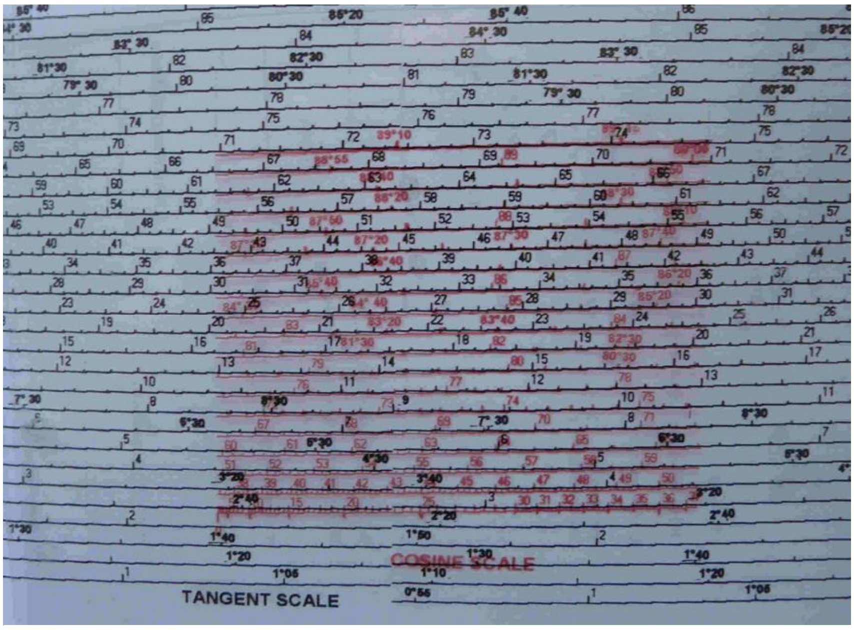


Figure 41: Gary LaPook's “flat” Bygrave reproduction

Conclusion

Some important questions remain even after this, the most comprehensive study of the position line slide rules so far:

1. How many Bygraves were made and in what years?
2. When was the switch made from the Position Line Slide Rule Mk. II to the Mk. IIA?
3. What was the Position Line Slide Rule Mk. I?
4. What is the exact relationship between the black prototype and the production Bygrave and were more similar prototypes made; is the black version described in [Wimperis 1920] the same as the prototype in the R.A.F. Museum?
5. What is the full story behind the Japanese Bygrave?
6. Why were the Germans the only ones to adopt a position line slide rule in any numbers in normal service?

Such questions mean that a study like this will probably never fully end: new discoveries of individual slide rules of the types mentioned here may shed some light on these questions; new documents may give full answers. Research continues, albeit at a somewhat slower pace...

Acknowledgements

I am grateful to all who have contributed in some way to this article, especially Ian Alder of the R.A.F. Museum in Stafford and Graham Wheeldon of the Science Museum in London for allowing me to study their respective Bygrave copies, Max Carran who provided the photographs and dimensions used in writing the section on the HR2 and Klaus Kühn who kindly stepped back from presenting a paper on the HR1 and who provided useful material on the HR series.

A very special thanks goes out to Zvi Doron, who really aroused my interest in celestial navigation and whose gift of one of his cardboard Bygrave copies was the start of the quest to find out all there is to know about the position line slide rules: without Zvi, this paper would not have been written.

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Indicators for Collections

{anon}	indicates a collector who doesn't wish to be named or whose name is not known
{DeuM}	Deutsches Museum, Germany
{DMM}	Deutsches Marine Museum, Germany
{GB}	George Bennett, Australia
{HP}	Henning Pohlmann, Germany; deceased; fate of items in collection not known
{JH}	John Hunt, UK
{JB}	Joe Bechtold, USA
{JBr}	Jim Bready, USA
{KlK}	Klaus Kühn, Germany
{MC}	Max Carran, UK
{RAFM}	R.A.F. Museum, UK
{RH}	Ray Hems, UK
{RvR}	Ronald van Riet, The Netherlands
{SciM}	Science Museum, UK
{SS}	Serge Savoysky, France
{TD}	Tom Dilatush, USA

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