Howard W. Sams PUBLICATION 20532

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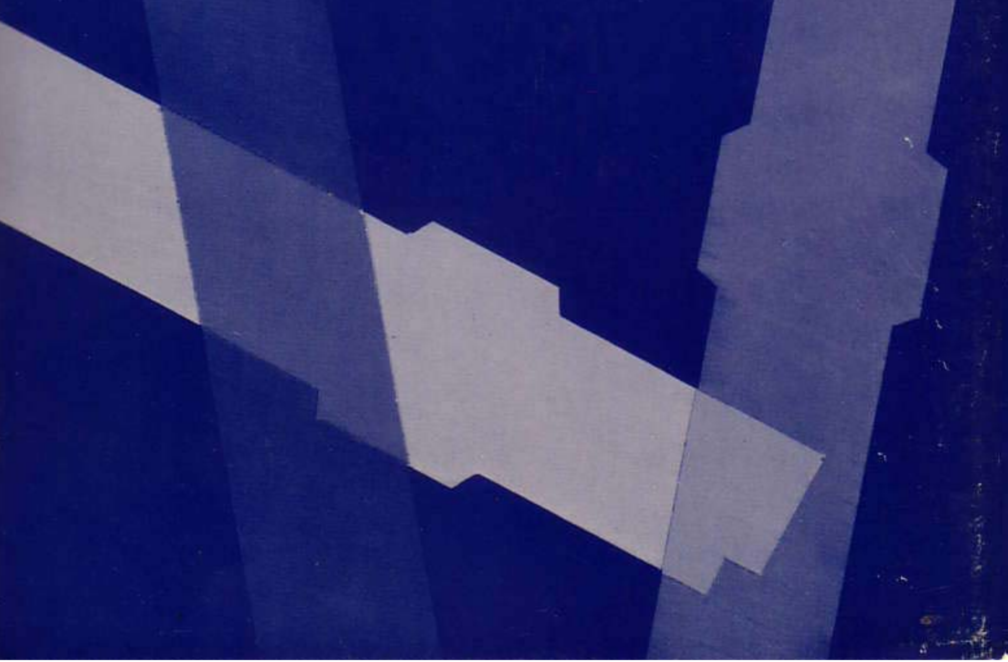
by Don Carper



SLIDE RULE IN ELECTRONICS

by

Don Carper

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California Technical Trade Schools

HOWARD W. SAMS & CO., INC.
THE BOBBS-MERRILL CO., INC.
INDIANAPOLIS • KANSAS CITY • NEW YORK

FIRST EDITION

SECOND PRINTING—1970

Library of Congress Catalog Card Number: 67-18151

Preface

Every student of electronics should know how to use the slide rule. The labor and time spent on such mathematical operations as multiplication, division, squaring, cubing, finding logarithms and antilogarithms, and evaluating trigonometric functions can be greatly reduced.

This book is based on my experience in teaching electronics mathematics at Compton Junior College in Compton, California, the California Technical Trade Schools, and in other specialized courses. It includes the slide-rule methods proved most suitable for electronics mathematics. Certain aspects of slide-rule operation, such as placing the decimal point and understanding the basic mathematics, have received special emphasis. Unless otherwise specified, all instructions and data refer to the ordinary 10-inch slide rule.

Each of the twelve lessons in this book has been divided into sections which, in most cases, contain groups of exercises. These exercises should be completed by the student in order to test his progress before he proceeds to the next section. The answers to the exercises and the odd-numbered examination problems are given in the back of the book. The answers to the even-numbered examination problems are available to qualified instructors from the publisher.

Don Carper

ACKNOWLEDGMENTS

The author wishes to express his appreciation to fellow instructors Edward Marsoobian, David Popp, and Salvatore Duarte, and to Roger Beam, Coordinator of Industrial Technology, and Dean Edgar H. Wilson of Compton College, for their encouragement and cooperation in the development of the course material for the text.

A special vote of thanks goes to Frances Porter of California Technical Trade Schools, for her tireless help in preparing the manuscript.

I also thank the Pickett Co., Keuffel and Esser Co., Dietzgen Co., and the Automatic Electric Co. for the privilege of using their data in the text.

To my wife, Emily, and kids, Judy and John, and, last but not least, to my students who have participated in the "dry run" of the text material, I am forever indebted.

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1

Introduction

SECTION 1

The Slide Rule in General

The slide rule is a fascinating device, and when properly used it will save much time in electronics calculations. It is actually a type of analog computer which has been in existence for a long time. It was in use in the early part of the 17th century, and such notables as Sir Isaac Newton and James Watt had a hand in its development.

About 1850 a young French army officer by the name of Mannheim developed a practical slide rule which provided the basic design and scale standardization for the modern rule. Mannheim's rule is generally referred to as a "general-purpose" rule, and while there are many elaborations, most slide rules in use today utilize the same basic principles as the Mannheim rule.

Slide rules can be obtained in various sizes and types, some for general use in engineering and mathematics calculations and others for specific use in specialized areas. The 10-inch rule is the most popular and practical, while the 5-inch model is handy for making rapid calculations in the field.

For electronics calculations one needs the *decitrig* type, which has trigonometric scales marked in decimal fractions of a degree. Since this type is suitable for most other professional needs, most of the text material is based on it. For the best use of this course your slide rule should have the A, B, C, CI, D, CF, DF, K, L, S, and T scales.

Practice Makes Almost Perfect

Although 100-percent accuracy is not always possible with the slide rule, its use provides answers that are acceptable in most of the mathematics problems of electronics. Even though it is not always possible to obtain a perfectly correct answer from the rule, one can attain a high degree of accuracy and speed through faithful practice. Most of the answers derived are limited to three digits, but there are experts who can approximate answers to four and sometimes five digits.

The answers to all the exercise problems are provided in the back of the book, but you should not confine yourself to just those problems in the text. Make up some of your own problems or work on some which can be found in electronics textbooks.

The lessons are divided into parts called sections. Study each section thoroughly before you go to the exercises and problems that follow. Do not omit these exercises; their completion will enable you to check your progress as you proceed, and this will aid you in retaining the material.

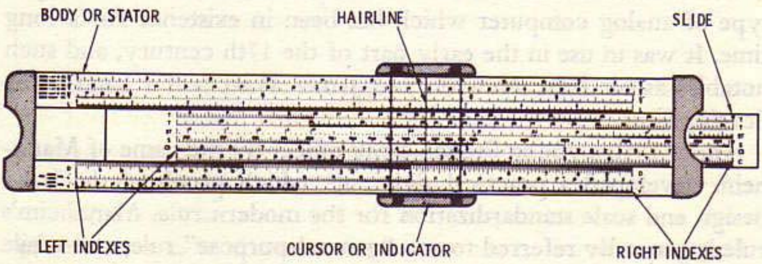


Fig. 1-1. Main parts of the slide rule.

Parts of the Slide Rule

There are three main parts of the slide rule, as shown in Fig. 1-1:

1. The *stationary rules*, or *stator*. This part is called the "body" of the slide rule, and it contains various scales.

2. The *moving rule*, or *slide*. This part includes duplicates of some of the scales found on the body.
3. The *indicator*, or *cursor*. This is the movable marker with the *hairline*, and it is used to line up the numbers involved and to designate the answer.

Exercises

Without referring to Section 1, supply the correct word or words.

1. The part of the slide rule that contains various scales and is movable is called the ______________.
2. The stationary rules are called the ______________ or ______________ and have various ______________ imprinted on them.
3. The part that is used to designate and align the numbers is called the ______________ and has the ______________ running vertically across it.

SECTION 2

The Scales

The scales of a slide rule are not linear but are logarithmic lengths, with the numbers representing the antilogs of these lengths. For example, look at the C or D scale on your slide rule. Note that the numbers are not evenly spaced on it. The length of either of these scales corresponds to the common logarithm of 10, which is 1.

Look at the number 3, which appears at about the middle of the scale. It is actually about 48 percent of the distance from the left end of the scale to the right end, for the logarithm of 3 is 0.4771. The number 4 is about 60 percent of the distance from the left end to the right end, and the logarithm of 4 is 0.6021.

The whole length of the scale can be thought of as representing 100 percent or any of the numbers 1, 10, 100, etc.

The C and D scales are identical. Other identical scales are the A and B scales, and the DF and CF scales.

The Indexes

The number 1's appearing at the left and right ends of the A, B, C, and D scales are called the *indexes*, specifically the *right in-*

dex and the *left index* of each scale. On some slide rules the last number at the right end of the A, B, C, and D scales is 10.

The slide rule is said to be *closed* when the left index of the C scale is directly over the left index of the D scale. This will automatically place the other indexes in matching positions.

Exercises

Without referring to Section 2, supply the correct word or words.

1. The scales of a slide rule are ____________ lengths.
2. The number 3 on the D scale is about ____________ of the length of the scale.
3. The first number on the C and D scales is called the ____________.
4. There are ________ indexes, called the ____________ and ____________ indexes, on the C and D scales.
5. When the slide rule is closed, the left index of the _____ scale should be directly below the left index of the _____ scale.

SECTION 3

Reading the Scales

One of the most important aspects in learning the correct use of the slide rule is in learning to read the scales accurately. This is necessary for you to be able to position the numbers of a problem and then read the answer.

Examine the cursor of your slide rule. Note the hairline. It is used for locating and positioning desired numbers and divisions. Note also that the cursor is generally made of plastic which is sometimes not too transparent. This is sometimes due to scratches caused by accumulations of dust and dirt particles. This "window" should be kept clean, in order to minimize the difficulty in seeing the small divisions through it. A small piece of cotton or soft cloth on the end of a toothpick or match is useful for this purpose.

Note that the C and D scales are identical. Since they are the most widely used scales, we will start by examining them. There are three types of divisions on the scales. Their relations to the digits of the numbers concerned should be memorized:

1. The *primary* or *main divisions.* These represent the first digit of a number.
2. The *secondary divisions.* These are used to represent the second digit of a number.
3. The *tertiary* or *small divisions.* These represent the third digit of a number.

Exercises

Without referring to Section 3, complete the following statements.

1. The second digit of the number 125 is 2. To represent this digit one would place the hairline over a ________ division.
2. The first digit of the number 235 is 2. To represent this digit one would place the hairline over a ________ division.
3. The third digit of the number 436 is 6. To represent this digit the hairline would be placed over a ________ division.

SECTION 4

The Primary Divisions

Locate the D scale. The slide may be removed to avoid confusion, if desired. Since we are primarily concerned with placing numbers on the D scale, and the C and D scales are identical, the procedure for placing the hairline or index over divisions on the C scale is the same as for the D scales.

The large-size numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, ending with 1 (or 10), are the primary divisions on the D scale. These numbers are used to represent the first digit of a number. Thus to represent the number 50, one places the hairline over the large 5, as shown in Fig. 1-2.

To represent the number 5 one must place the hairline in the same position as for 50. In fact, to represent any number that contains 5 as the only nonzero digit (a digit other than zero) the hairline should be placed in the same position.

This procedure brings up an interesting fact concerning the slide rule—it does not show the position of the decimal point.

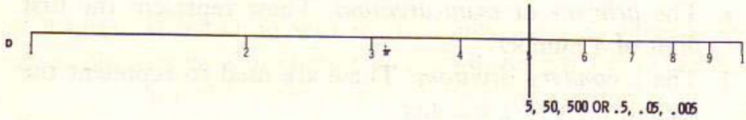


Fig. 1-2. Primary divisions on the D scale.

Only the *digits* of an answer are derived from the slide rule, the decimal point being placed by methods to be discussed later.

Practice placing the hairline over different numbers, as shown before in Fig. 1-2. Be sure that the hairline is *exactly* over the mark of the number desired.

SECTION 5

The Secondary Divisions

Between the primary divisions 1 and 2 there are ten smaller divisions (or marks) called *secondary divisions.* On most rules these divisions are also numbered, but the numbers are smaller in size than those of the primary divisions.

Note that the section between the large 1 and 2 is the only place on the C and D scales where the secondary divisions are numbered. The spaces get smaller as the scales proceed to the right, and there is not enough space to include numbers over the secondary divisions.

The secondary divisions are used to represent the second digit of a number. To form the numbers 15, 150, 1500, etc., one places the hairline over the secondary division numbered 5 as shown by A of Fig. 1-3. (The first digit of 15 is 1, which is represented by the primary division marked with the large 1.) To form the number 25, one places the hairline as shown by B of Fig. 1-3.

Since the hairline cannot be in two places at the same time, the last space or division that it is over represents the last digit of the

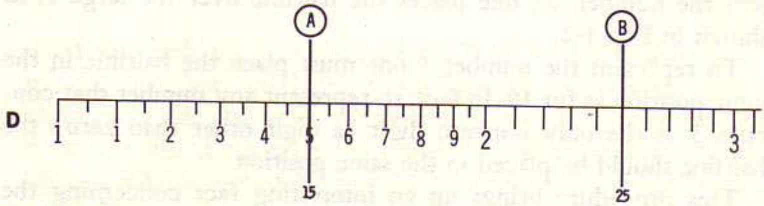
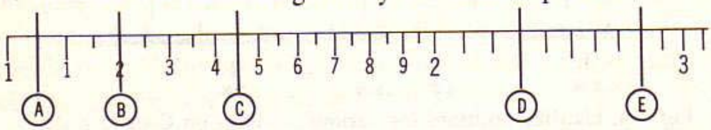


Fig. 1-3. Portion of D scale showing hairline over secondary division marks.

number being formed, which in the case of 15 or 25 (two-digit numbers) is the second digit. The first digit is understood to be the large number to the left of the secondary division.

Exercise

1. Indicate the numbers given by the hairline positions:



SECTION 6

The Tertiary Divisions

In Part A of the exercise following Section 5 you were asked to indicate the number represented by the position of the hairline, which was over a space halfway between the large 1 (primary division) and the small 1 (secondary division).

If your answer was 105, which is the correct answer, you have reasoned correctly that because the first digit of the number under consideration was 1, the second digit would have to be zero, since the hairline was not on a secondary division but was actually over a *tertiary* division.

This tertiary division was the fifth division line between the large 1 and the small 1, and there are ten tertiary divisions between the large 1 and the small 1. In this case the fifth tertiary division therefore represents the number 5 as the third digit, making the given number 105.

The same situation appears in setting C. Again, if you indicate the number formed as being 145, your reasoning is correct. In this instance the second digit is 4, represented by a secondary division, and the third division setting is the fifth unit between the secondary divisions 4 and 5; therefore as the third digit it is worth 5.

Now look at the D- or C-scale section of your slide rule from primary division 1 to primary division 2. Note that there are ten spaces between each pair of consecutive secondary divisions. Note again that the secondary divisions are numbered, but the small marks, which are the tertiaries, are not. Note that the fifth small-division line is slightly longer than the ones preceding and following it. This is for your convenience in reading placements.

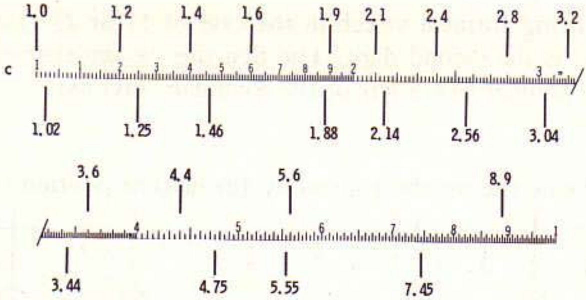


Fig. 1-4. Hairline positions for various numbers on C or D scales.

Now examine the section between primary divisions 2 and 4. Note that here there are only five spaces between the secondary divisions. This makes the value of each of these smaller spaces, which are the tertiary divisions, amount to two units in the third digit of a number.

Now look at the section from 4 to 10. Note that the smaller spaces between the secondary divisions are separated into only two divisions. These divisions are the tertiary divisions, and they are worth five units as the third digit of a number.

Study the example given in Fig. 1-4. Observe that the top row of numbers is formed from the primary and secondary divisions, and the bottom row by adding the tertiary divisions.

In Fig. 1-4 note that in the section of the C scale from 4 to 10, the positions of the hairline are indicated as being over secondary divisions. As an example, look at the last number formed: 8.9. This could also be 89, 890, 0.89, etc. To indicate the number 895 one would simply move the hairline one space to the right and leave it over the tertiary division between the last secondary division and 9, as shown in Fig. 1-5.

Note that when 895 is set on the slide rule, the third digit (which is 5) is represented by a tertiary division, which in this rule section has a value of five units in the third digit of a number.

To indicate the number 8925 or 89.25 one would have to carefully move the cursor so that the hairline rests halfway between the last tertiary division (8.95) and the secondary division 8.9.

Fig. 1-5. Setting the number 895.

This would indicate half the value of the third digit (5), which is 2.5, and thus a four-digit number could be set up.

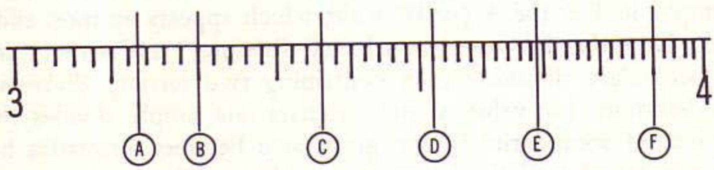
Accuracy of Slide Rule

The slide rule is not 100 percent accurate. In general, it is possible to designate four-digit numbers in only the section from primary 1 to primary 2. In the other sections the tertiary divisions represent too many units to yield more than the third significant digit. With much practice it is possible to set and read four- and even five-digit numbers, but for most problems it is sufficient to be able to set and read three- and four-digit numbers.

In practically all cases the slide rule is adequate for most electronics calculations. Electronic components are generally manufactured to a tolerance of not better than 5 percent, while the slide rule is generally accurate to within 1 percent (an exception is the exponentiation of a number). The accuracy of mathematical operations performed on the slide rule depends not only on which scales are used, but also on which sections of these scales are used.

Exercise

1. Indicate the numbers represented by the position of the hairline in the illustration below.



The student should devote much practice in setting and reading numbers on the slide rule. This is often omitted as being relatively easy or unimportant in comparison with other techniques concerning the use of the rule, but the author has found that many students are deficient in this important consideration.

SECTION 7

Summary of Relations Between the Divisions in the Three Sections of the Rule

As far as the C and D scales are concerned, we can make the following generalizations:

In the section from primary division 1 to 2:

Secondaries are worth one unit in the second digit of a number.

Tertiaries are worth one unit in the third digit of a number.

In the section from primary division 2 to 4:

Secondaries are worth one unit in the second digit of a number.

Tertiaries are worth two units in the third digit of a number.

In the section from primary division 4 to 10:

Secondaries are worth one unit each in the second digit of a number.

Tertiaries are worth five units each in the third digit of a number.

These values also apply to the CF, DF, CI, and DI scales of the slide rule.

It is possible that your rule may not have all the scales previously mentioned, but this will be no major handicap in learning the principles of the use of the rule in various calculations.

By examining the other scales on your rule you should be able to determine the values of the small divisions by first noting how many spaces separate the primary divisions and then figuring out how many of these spaces it takes to make the value of ten. For example, look at the A (or B) scale, which appears on most slide rules. Note that between the primary divisions 3 and 4 there are ten secondary divisions, each containing two tertiary divisions. To determine the value of the tertiaries one simply divides the number of secondaries in any given area between primaries by the number of tertiaries between secondaries. Thus,

$$\frac{10 \text{ Secondaries}}{2 \text{ Tertiaries}} = 5 \text{ Units}$$

This is the value of each of the tertiary divisions between 3 and 4 on the A and B scales.

Examination on Lesson 1

Complete this examination before looking up the answers in the back of the book. The mistakes which you make on this exam will serve you well if they help you in remembering the information the next time.

Circle the T if the statement is true, the F if false. Fill in the correct word or number in the blanks.

1. The slide rule is a type of digital computer. (T, F)
2. No one ever heard of a slide rule before 1900. (T, F)
3. The D scale appears on the body of the rule. (T, F)
4. The cursor is the part that contains the C scale. (T, F)
5. The numbers and divisions on the D scale are the same as those on the C scale. (T, F)
6. The index of the C scale is on the body of the rule. (T, F)
7. The primary divisions are used to indicate the ________ digit of a number.
8. The secondary divisions are used to indicate the ________ digit of a number.
9. The third digit of a number is indicated by a ________ division.
10. To designate the number 6000 the hairline should be placed over the ________ division numbered ________ on the D scale.
11. To designate the number 135 the hairline should be placed over the tertiary division between the secondary divisions of ________ and ________ on the D scale.
12. If there are five tertiary divisions between any two secondaries, each tertiary is worth ________ units.
13. If there are ten tertiary divisions between any two secondaries, each tertiary is worth ________ units.
14. If there are ten secondary divisions between each primary, each secondary is worth ________ units.

2

Multiplication With The Slide Rule

SECTION 1

Functions of the Slide Rule

Some of the calculations that can be done quickly on the slide rule are as follows:

1. Multiplication
2. Division
3. Squaring numbers and extracting square roots
4. Finding the reciprocals of numbers
5. Cubing numbers and extracting cube roots

6. Solving ratios and proportions
7. Raising numbers to powers
8. Finding logarithms
9. Solving many trigonometric problems

The slide rule is not used to add numbers since the scales are not linear but are logarithmic in length. However, this does not decrease the usefulness of the slide rule, since addition can be done mentally or mechanically by calculating devices such as adding machines.

It is best that the student first learn the mechanics of the slide rule as applied to ordinary arithmetic and mathematics problems. Then he can relate these techniques to applications in electronics. We will begin by learning how to use the slide rule for multiplication.

SECTION 2

Multiplication

Multiplication on the slide rule can be done by using any two scales that have identical division markings such as the C and D, A and B, CF and DF, or the CI and DI scales. For most problems

Fig. 2-1. Multiplication of 2 and 2, using C and D scales.

of multiplication the C and D scales are the most practical to use; the principles learned for these scales can be applied to other scales when necessary or desired. Before we proceed with the slide-rule multiplication, however, a review of some of the terms used in multiplication is in order:

Multiplication: A short way to add a number to itself. Adding five 25's yields 125. The same number is derived from multiplying 25 by 5.
Multiplicand: The number to be multiplied.
Multiplier: The number by which the multiplicand is multiplied.
Product: The answer derived from multiplication.

Actually, either number involved in a multiplication problem can be considered the multiplicand or multiplier. These terms simply make it easier to refer to and identify the numbers in any given problem.

Example—Consider $650 \times 2 = 1300$. Here the multiplicand is 650, the multiplier 2, and the product is 1300. This problem can be stated another way:

$$\begin{array}{rl} 650 & \text{(multiplicand)} \\ \times 2 & \text{(multiplier)} \\ \hline 1300 & \text{(product)} \end{array}$$

The symbol $\times$ is used as the "times" sign, indicating that the number preceding it is to be added as many times as indicated by the number following it.

Sometimes a soft-rolled period is used to indicate multiplication. For example,

$$650 \cdot 2 = 1300$$

Note that when a soft-rolled period is used it is placed a little farther from the multiplicand than it would be if it were being used as a decimal point. This is to avoid confusion which could result from careless writing. In electronics mathematics it is customary to omit the times sign. For example, $2\pi fL$ means 2 times pi (π) times frequency (f) times inductance (L).

Exercises

Without referring to Section 2 complete the statements with blanks. Determine which of the remaining statements are true or false (T, F).

1. Multiplication is actually a short way to ________ a number to itself.
2. The number that is multiplied is called the ________.
3. The number that does the multiplying is the ________.
4. In multiplication the answer is called the ________.
5. Either number in a multiplication problem can be considered to be the multiplier, with the same results being obtained. (T, F)

6. When a soft-rolled period is used as the times sign it has the same meaning as a decimal point. (T, F)
7. In many electronics formulas involving multiplication the "times" sign is omitted. (T, F)

SECTION 3

Signed Numbers

In electronics calculations it is often necessary to work with signed numbers, and the electronics student should become familiar with their use.

A number that is preceded by a positive sign (+) or a negative sign (−) is called a *signed number.*

In electronics the term *positive* refers to a point that is less negative than another point when both are considered from the same reference level. If zero is used as a reference, all values above it are positive and all values below it are considered negative. Although any point in a circuit that is lower in potential than another point is considered negative with respect to that higher point, the minus sign is actually not used in such a case; it is understood that a value lower than another is negative with respect to the higher.

The positive sign is generally omitted, but the negative sign is not, so a number that is not positive is therefore negative, and vice versa. As long as the negative sign precedes a number, the omission of it indicates a positive value with respect to zero.

Addition of Signed Numbers

In order to add signed numbers there are two important rules for you to remember:

1. In adding numbers of *like* signs the sum is arithmetic and bears the sign common to both numbers. For example,

$$\begin{array}{r} +5 \\ \underline{+6} \\ +11 \end{array} \qquad \begin{array}{r} -5 \\ \underline{-6} \\ -11 \end{array}$$

2. In adding numbers of *unlike* signs, the sum is algebraic and is the result of subtracting the smaller from the larger and giving it the sign of the larger. For example,

$$\begin{array}{r} +5 \\ \underline{-6} \\ -1 \end{array}$$

Since 6 is the larger number, the 5 is subtracted from it, but the answer bears the sign of the larger, which is a minus sign.

Another way to state an addition of signed numbers is to enclose the signs in parentheses to avoid confusion with the plus sign used to indicate addition:

$$(-6) + (+5) = -1$$

Note that the answer is not in parentheses.

Exercises

Without referring back, complete the following statements.

1. $(-5) + (-45) =$
2. $(+389) + (-899) =$
3. $(+85) + (-85) =$
4. $515 + (-698) =$
5. $350 + 350 + (-700) =$

In the statements below, circle the T if the statement is true, the F if it is false.

6. If point A in an electronic circuit is at 50 volts and point B is at 55 volts, point A is negative with respect to point B. (T, F)
7. Unless a number is preceded by a negative sign it is considered positive with respect to zero. (T, F)
8. In electrical or electronic circuits any point can be used as a reference from which to measure voltage or current. (T, F)

Multiplication of Signed Numbers

In multiplying two signed numbers there are two important rules which must be observed:

1. The product of two numbers with like signs is positive.
2. The product of two numbers with unlike signs is negative.

Example—Consider the following multiplications:

$(-5) \times (-6) = 30$ The product is positive because the numbers have like signs.

$(+5) \times (-6) = -30$ The product is negative because the multiplicand and multiplier have unlike signs.

In multiplication on the slide rule the product is not supplied with the decimal point. Therefore it is necessary that the student become familiar with the common procedures involved in multiplication problems such as the preceding ones, to facilitate placing the decimal point by approximation in problems involving less than three digits per number.

Exercises

Without referring back, answer the following.

1. If two numbers that are both positive are multiplied, the product will be negative. (T, F)
2. If a positive number is multiplied by a negative number, the product is positive. (T, F)
3. Solve: $(-6) \times (-45)$
4. Solve: $(+8) \times (+10)$
5. Solve: $(-75) \times (+2)$

SECTION 4

Using the Slide Rule in Multiplication

When one is multiplying two numbers with the C and D scales of the slide rule the following is the correct procedure:

1. Place hairline over the multiplicand on the D scale.
2. Slide index of the C scale under the hairline.
3. Move the hairline over the multiplier on the C scale.
4. Read the product under the hairline on the D scale.

Example—Find 2×2. The procedure is as follows:

1. Place the hairline over 2 on the D scale.
2. Slide the left index of the C scale under the hairline.
3. Move the hairline over 2 on the C scale.

4. Read answer (4) under the hairline on the D scale (Fig. 2-1).

Example—Find 35×6.5. Here, the procedure is as follows:

1. Place hairline over 35 on the D scale.
2. Slide *right* index of the C scale under hairline.
3. Move hairline over 65 on the C scale.
4. Read digits of answer (2275) under hairline on the D scale.

Note that the product derived from the slide rule consists only of digits, without any clue as to where to place the decimal point. This important consideration will be taken up shortly, but in a problem involving as few digits as the preceding, one can easily see that the product would be a number composed of three digits to the left of the decimal point plus a decimal fraction (.5) to the right. Thus the answer is $35 \times 6.5 = 227.5$.

On the slide rule these digits might appear as either 226, 227, or 228, depending on the accuracy of placement of the hairline and index. This shows the importance of extreme care in placing the hairline over the division marks. It also shows that the slide rule is not always capable of giving a perfect answer.

Exercises

Perform the following multiplications.

1. 67×32
2. 85×1.5
3. 4.5×3
4. 18×235

SECTION 5

Using the Indexes

Refer back to Section 2 of this lesson. Note that in the first problem the left index of the C scale was used, and in the second problem the right index was used. If you experienced the same situation in the practice problems, the question may arise: how does one determine which index to use? This is how:

If the placement of either the left or right index causes the next number in the problem to be forced off the rule, making it impossible to place the hairline over it, simply exchange the places of the indexes; put the other index under the hairline.

This procedure can be illustrated by another easy problem: Find 24×3.

1. Place hairline over 24 on the D scale.
2. Slide *left* index of C scale under hairline.
3. Move hairline over 3 on the C scale.
4. Read product, 72, under hairline on the D scale.

In this problem the left index was used. If the right index had been placed under the hairline in Step 2, it would have been impossible to do Step 3.

This procedure is common and can be quickly grasped by the student. Just use the index that allows the next step to be done. Be careful in sliding the indexes around. If you accidentally move the cursor while doing this, the position of the hairline will be altered, resulting in a wrong answer.

Examination on Lesson 2

Without referring back in Lesson 2 check your progress by taking the quiz below. Circle the T if the statement is true, the F if false.

1. The slide rule can be used to add a column of figures. (T, F)
2. Multiplication can be done with the slide rule. (T, F)
3. The C and D scales are considered to be the most practical for multiplication on the slide rule. (T, F)
4. The answer derived from correct manipulation of the slide rule is always complete with decimal point. (T, F)
5. If the placement of the right index forces the next number in the problem off the rule, simply use the left index. (T, F)

Supply the missing word, number, etc.

6. The following steps are used to multiply on the slide rule:
   a. Place hairline over the ______ on the ______ scale.
   b. Slide index of ______ scale under hairline.
   c. Move ______ over ______ on the C scale.
   d. Read ______ under the ______ on the ______ scale.
7. The number to be multiplied or added to itself a given number of times is called the ____________.

8. The number that indicates how many times the other number is to be multiplied is called the ____________.
9. In multiplication the answer is usually called the ____________.
10. Find the indicated sums:
    a. $(+3) + (-3)$
    b. $(-5) + (-5)$
    c. $(-10) + 12$
    d. $(-1) + (-1) + (+1)$
    e. $(+3) - (+3)$
11. Do the following, using your slide rule:
    a. 45×2.3
    b. $5.6 \times (-78)$
    c. 125×42
    d. 90×7.2
    e. $(-186) \times 8$

3

Division With The Slide Rule

SECTION 1

Division in General

Division is the opposite of multiplication. For division problems on the slide rule the procedure is the opposite of that used to multiply. Before discussing the manipulations involved when one is dividing with the slide rule, a review of some of the terms and principles involved in division is advisable.

To Divide Means to Separate

In division a number is separated into equal parts or groups, each part containing a specified number of units. The number that is being divided is called the *dividend*. The number of units in each part is called the *divisor*. The number of parts obtained is called the *quotient*. If a part having less than the specified number of units remains, it is called the *remainder*. These terms are illustrated in Fig. 3-1, where the number 15 is divided by 4, with a remainder of 3.

As another example, consider the division of 14 by 2:

$$14 \div 2 = 7 \qquad \text{and} \qquad 2\overline{)14}$$

$$14/2 = 7$$

7 ← Quotient
2 ⟌ 14 ← Dividend
2 ← Divisor

In each of these three different ways of writing the problem, the dividend is 14, the divisor is 2, and the quotient is 7. There is no remainder.

Since the quotient represents how many parts the dividend can be divided into by the divisor, and since division is the opposite of multiplication, we can prove the correctness of our answer by multiplying the divisor by the quotient to obtain the dividend:

$$\text{Divisor} \times \text{Quotient} = \text{Dividend}$$

In the preceding example, multiplying 2 and 7 yields 14, so we know that the answer is correct. If we attempt to divide 15 by 2, however, we find that there is a number left over, which is called the remainder:

$$\begin{array}{r} 7 \\ 2\overline{)15} \\ \underline{14} \\ \frac{1}{2} \end{array}$$

The remainder is 1. Placing the divisor under it gives us the fraction that is left over, making the answer 7½, i.e., $7\frac{1}{2} \times 2 = 15$.

Exercises

Without referring to Section 1 circle the T if the statement is true, the F if it is false.

1. Division is the same as multiplication. (T, F)
2. In division problems the number that is being divided is called the *dividend*. (T, F)

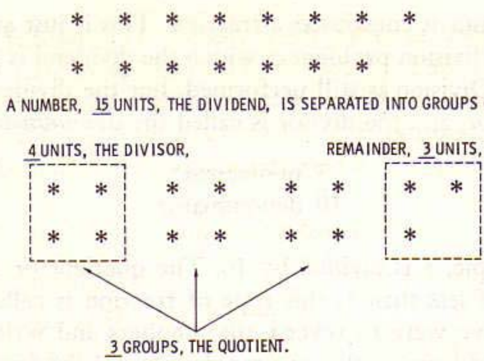


Fig. 3-1. Division as a separation into equal groups.

3. The term *quotient* refers to the part that is left over after the number has been divided. (T, F)
4. A division problem can be proved correct or incorrect by multiplying the quotient by the divisor to yield the dividend. (T, F)
5. If the divisor times the quotient equals the dividend, the division has not been done correctly. (T, F)
6. In division problems changing the order of the divisor and dividend will not change the value of the quotient. (T, F)

SECTION 2

Division of Signed Numbers

A *signed number* is a number that is preceded by a minus (−) or plus (+) sign. Usually a number that is not a negative number needs no sign, and therefore the sign is often omitted. Thus the number 5 is positive, but −5 is negative. There are two basic rules concerning the division of numbers having like and unlike signs:

1. The quotient of two numbers having like signs is positive.
2. The quotient of two numbers having unlike signs is negative.

For example, $10/5 = +2$ and $-10/5 = -2$.

Fractions

When a quotient of a problem of division is not a whole number, the remainder is a part that is left over, and when combined with the divisor it constitutes a *fraction*. This is just another way of writing a division problem in which the dividend is placed over the divisor. Division is still performed, but the dividend is called the *numerator*, and the divisor is called the *denominator*:

$$\frac{5}{10}\begin{array}{l}\text{numerator}\\ \text{denominator}\end{array}$$

In this example, 5 is divided by 10. The quotient or answer will be a value of less than 1; this type of fraction is called a *proper fraction*. If we were to reverse the numbers and write 10/5, the answer would have a value greater than 1, and the fraction would be called an *improper fraction*.

There is one important rule concerning fractions that should be remembered:

> Both the numerator and denominator of a fraction can be multiplied or divided by the same number without changing the value of the fraction.

$$\frac{5}{10}=\frac{5\times 2}{10\times 2}=\frac{5\times 3}{10\times 3}=\frac{5\times 4}{10\times 4}$$

We can write ordinary fractions in many different forms to accommodate a problem. When the numerator and the denominator have both been reduced to the lowest whole numbers, we say that the fraction is in its *lowest form*.

There is no need to go deeply into a discussion of fractions at this time, since the slide rule will easily supply digits in the quotient, leaving the placing of the decimal point to the operator. For this, there are special procedures which will be dealt with in the next lesson.

Exercises

Without referring to Section 2 complete the following statements.

1. A signed number is a number that has either a ________ or a ________ sign preceding it.
2. The sign is usually omitted from ________ numbers.
3. The quotient of two numbers having like signs is ________.

4. The quotient of two numbers having unlike signs is ________.
5. A fraction represents a ________.
6. A proper fraction is a fraction whose value is ________ than 1.
7. An improper fraction is one whose value is ________ than 1.
8. When a division problem is expressed as a fraction the dividend is called the ________.
9. Both the ________ and ________ of a fraction can be multiplied by the same number without changing the value of the fraction.
10. When the numerator and the denominator of a fraction have both been reduced to the lowest possible whole numbers we say that the fraction is in its ________ form.

SECTION 3

Using the Slide Rule for Division

Division can be done quickly by using any two identical scales on the slide rule or by using the CI (C inverted) scale and the D scale. The procedure for using the C and D scales is as follows:

1. Place hairline over dividend on the D scale.
2. Slide divisor on C scale under hairline.
3. Read quotient below index of C scale on the D scale.

Example—Divide 5 by 4 (or find 5/4):

1. Place hairline over 5 on the D scale.
2. Slide 4 of C scale under hairline.
3. Read digits of quotient (125) below left index of C scale.
4. The correct quotient is 1.25.

The slide-rule settings should be the same as is shown in Fig. 3-2.

The same problem can be solved by using the CI and the D scales of the slide rule:

1. Slide C index over 5 on the D scale. (Use right index.)
2. Place hairline over 4 on the CI scale.
3. Read quotient (1.25) under hairline on the D scale.

Note that the CI scale has the same divisions as the C or D scale but that they start at the right end and increase to the left—just the opposite of the C and D scales. The CI scale numbers are the reciprocals of the numbers on the C scale. In a later lesson we will study problems involving reciprocals.

Fig. 3-2. Using C and D scales to divide 5 by 4.

Exercises

Solve the following problems, using the C and D scales first. Then work the same problem, using the CI and D scales.

1. 14/2 = ______
2. 23/4.6 = ______
3. 180/25 = ______
4. 15/9 = ______
5. 324/6 = ______
6. 3.14/4 = ______
7. 25.2/3.14 = ______
8. 75/19 = ______
9. 53/22 = ______
10. 41/8 = ______
11. 6.5 × 34 = ______
12. 3.8 × 2.2 = ______
13. 165 × 32 = ______
14. What is the voltage in a circuit in which the resistance is 5000 ohms and the current is 0.05 ampere?

SECTION 4

Ohm's Law Problems

Every student of electronics should be familiar with Ohm's law. By this law relating voltage, current, and resistance it is possible to find the unknown value in a circuit when the other two are given. The slide rule is handy for such problems. Stated as a formula Ohm's law may be written:

$E = IR$ Voltage equals current times resistance.
$I = E/R$ Current equals voltage divided by resistance.
$R = E/I$ Resistance equals voltage divided by current.

Problem 14 of the preceding exercise problems entailed the application of Ohm's law. Here is another example:

Problem—The voltage drop across a resistor of 150 ohms is 300 volts. What is the current through the resistor?

Solution—To find the current through the resistor use the formula $I = E/R$. First, state the problem in mathematical terms:

$$I = \frac{300 \text{ volts}}{150 \text{ ohms}}$$

Proper manipulation of the slide rule should yield the number 2.00. Therefore, $I = 2$ amperes.

Problem—If the current through a 150-ohm resistor is 2 amperes, what is the voltage across it?

Solution—To find the value of E (voltage) use the formula $E = IR$. First, state the problem:

$$E = 2 \text{ amperes} \times 150 \text{ ohms}$$

Multiplying 150 by 2 with the slide rule yields the number 300. Therefore, $E = 300$ volts.

Since the preceding problems concern the same circuit, each solution proves the validity of the other.

Examination on Lesson 3

Perform the indicated operations.

1. $2564/4.4 =$ ______
2. $3.56/(-2.5) =$ ______
3. $45/1.6 =$ ______
4. $6.28/60 =$ ______
5. $2564 \times 4.4 =$ ______
6. $3.56 \times (-2.5) =$ ______
7. $45 \times 1.6 =$ ______
8. $6.28 \times 60 =$ ______

Using the C and D scales where applicable, and the CI and D scales for division if desired, solve the following Ohm's law problems.

9. A circuit with a resistance of 180 ohms has a current reading of 0.4 ampere. How much voltage is required to produce this current?

10. A 100-volt battery has a lightbulb and an ammeter connected across it in series. What is the resistance of the lightbulb if the meter indicates 5 amperes?
11. An electric blanket with a resistance of 12 ohms is connected across the 117-volt line. How much current is drawn by the blanket?
12. A voltage of 1500 volts is applied across a 1.5-megohm resistor. How much current is there through the resistor?

4

Placing The Decimal Point

SECTION 1

Methods

Since the answer provided by the slide rule consists of digits only, the next problem is that of placing the decimal point. There are three methods that find favor with slide-rule technicians:

1. The approximation method
2. The scientific method (powers of ten)
3. The log method (use of exponents)

We will discuss these methods separately.

The Approximation Method

In this method one merely makes an "educated guess" as to where the decimal point belongs. This is satisfactory in problems involving whole numbers with few digits.

Example—Divide 875 by 250. In this problem the number derived from the slide rule is 35. Common sense tells us that 250 cannot go into 875 that many times, so the correct answer would have to be 3.5.

To be more elaborate, one can round-off the numbers in the problem to provide a simpler one whose solution could be estimated quickly by visual inspection. In this case one can observe that: "If 800/200 equals 4, then 875/250 must equal 3.5." Therefore, 3.5 is the correct answer.

Example—Multiply 65 by 5.5. The slide rule should yield the digits 358 as the answer, which is close enough to the actual product of 375.5 to be valid. To further substantiate this answer one can observe that: "If 60 times 5 equals 300, then 65 times 5.5 must equal 358." Therefore, the answer 358 is correct.

Example—Divide 5600 by 3.5. The slide rule yields the number 160. By approximation one can see that: "If 6000 divided by 3 equals 2000, then 5600 divided by 3.5 must equal 1600."

Exercises

Using the approximation method, place the decimal point in the following problems.

1. 4680/2.2
2. 3.58×0.05
3. 1,500,000/750
4. 1/8
5. 1/70
6. 9.8×38

SECTION 2

The Power of a Number

When a number is multiplied by itself once, twice, or any number of times, the product is called the *power* of that number. The number is called a *factor* of the product. For example,

$$5 \times 5 = 25$$

Here, 5 is a factor, and the product 25 is the *second power* of 5. A shorter way to express this would be $5^2 = 25$.

Some other examples are:

$$4^2 = 16 \quad (4 \times 4)$$
$$8^2 = 64 \quad (8 \times 8)$$
$$6^5 = 7776 \quad (6 \times 6 \times 6 \times 6 \times 6)$$

The number that is placed to the upper right (superscript) of the number being raised to a power is called the *exponent*. Note that it is slightly elevated to avoid confusion. Thus in the equation $2^3 = 8$, the number 2 is called the *base*, the superscript 3 is the exponent, and 8 represents the *third power* of 2.

Powers of Ten

Multiplying 10 by itself we see that:

$$10 \times 10 = 100$$
$$10 \times 10 \times 10 = 1000$$
$$10 \times 10 \times 10 \times 10 = 10{,}000$$
$$10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$$

Expressing these numbers as *powers of ten*, where 10 is the base, yields

$$10^0 = 1$$
$$10^1 = 10$$
$$10^2 = 100$$
$$10^3 = 1000$$
$$10^4 = 10{,}000$$
$$10^5 = 100{,}000$$

These powers of ten will become very important in later sections.

Exercises

Complete the following equations.

1. $10^2 =$ ________	5. $10^6 =$ ________
2. $10^5 =$ ________	6. $10^1 =$ ________
3. $10^0 =$ ________	7. $10^4 =$ ________
4. $10^3 =$ ________	8. $10^8 =$ ________

SECTION 3

Negative Powers of Ten

The preceding examples were those of powers of ten with positive exponents. When the exponent of 10 is positive the decimal point is always moved to the right. When it is negative it is moved to the left. For example,

$$\frac{1.0}{10} = 0.1 \text{ or } 10^{-1}$$

Here, the negative exponent (-1) indicates number of places to left that decimal point is moved.

Note that a positive 1 has the decimal point to the right of the digit 1, and the negative exponent indicates how many places said decimal point should be moved to the left. Here is another example:

$$\frac{1.0}{100} = 0.01 \text{ or } 10^{-2}$$

Again notice that the decimal point has been moved to the left the number of times indicated by the negative exponent. Thus two important facts appear:

1. A *positive* exponent indicates the number of places to move the decimal point to the *right*.
2. A *negative* exponent indicates the number of places to move the decimal point to the *left*.

Exercises

Express in powers of ten:	Change to decimal fractions:
1. 0.001	4. 10^{1}
2. 0.00001	5. 10^{-1}
3. 0.1	6. 10^{-3}

SECTION 4

Multiplication Using Powers of Ten

When a number is multiplied by a power of ten only the position of the decimal changes. For example,

$$6.28 \times 10^2 = 628$$

The digits 6, 2, and 8 are unchanged, but the location of the decimal point is changed, being moved two places to the right.

For a negative exponent of 10, the multiplication is as follows:

$$628 \times 10^{-2} = 6.28$$

Again the digits are not changed, but the location of the decimal point is, moving two places to the *left* because the exponent is negative.

In each case the power of ten used to multiply the digits 6, 2, and 8 is called the *decimal multiplier*, and the digits themselves are called the *significant figures*.

When two or more significant figures are multiplied by powers of ten, the exponents are added arithmetically if they are of like signs, and algebraically if of unlike signs. Here are three examples:

$$2 \times 10^2 \times 3 \times 10^1 = 2 \times 3 \times 10^3 = 6 \times 10^3 = 6000$$

$$5 \times 10^{-2} \times 3 \times 10^3 = 5 \times 3 \times 10^1 = 15 \times 10^1 = 150$$

$$18 \times 10^3 \times 10 \times 10^{-6} = 18 \times 10 \times 10^{-3} = 180 \times 10^{-3} = 0.18$$

Observe that the exponents of 10 are added. Note also how the problems can be rearranged so that the multiplying numbers and the sum of the exponents can be grouped more conveniently.

Exercises

Obtain the products by slide-rule operation and place the decimal point according to the method of powers of ten.

1. $5.8 \times 10^2 \times 45 \times 10^{-5}$
2. $450{,}000 \times 10^6 \times 2$
3. $0.001 \times 10^3 \times 16 \times 10^{-3}$
4. $5.05 \times 10^0 \times 2.5 \times 10^1$
5. $7.07 \times 10^1 \times 1.2 \times 10^{-3}$
6. $57{,}850 \times 0.003$
7. 4506×0.0002

SECTION 5

Standard Notation

When only one significant figure appears to the left of the decimal point a number is said to be expressed in *standard notation.*

This notation gives a convenient way to express mixed decimals or extremely large or small numbers when their original form makes them unwieldy in a problem. It simply involves rewriting the number in standard notation multiplied by a power of ten. For example,

$$4692.13 = 4.69213 \times 10^3$$

Note that the number was changed to a number between 1 and 10, and then by multiplying it in its new form by 10 to the third power it was restored to its original value.

Any number can be expressed in this manner:

$$0.0048 = 4.8 \times 10^{-3}$$

$$0.0002 = 2 \times 10^{-4}$$

$$5{,}689{,}000 = 5.689 \times 10^6$$

This is a useful device when one is working with the slide rule, making it possible to break large numbers down into more handy ones and changing decimal fractions into whole numbers plus decimal fractions. It is also useful in determining the position of the decimal point by approximation when the numbers obtained in the answer by the slide rule do not have as many places as could be obtained by conventional calculations.

In the product 0.052×0.0068 the digits derived from the slide rule could be 3538, 3539, or 354, depending on your interpolation. Expressing the multiplication in standard notation, we have

$$5.2 \times 10^{-2} \times 6.8 \times 10^{-3} = 5.2 \times 6.8 \times 10^{-5}$$

Approximating: If $5 \times 6 \times 10^{-5} = 30 \times 10^{-5} = 0.00030$, then $5.2 \times 6.8 \times 10^{-5}$ must equal 0.00035.

Exercises

Express the following numbers in standard notation.

1. 0.950
2. 6521.76
3. 0.00021
4. 29.95
5. 4.80
6. 100
7. 1000
8. 2,000,000
9. 1.50
10. 360.56
11. 5,890,460
12. 0.00470
13. 0.00068
14. 0.707
15. 346.5
16. 0.075

Find the following products with slide rule and place the decimal point correctly.

17. $65{,}000 \times 1500$
18. 0.047×0.06
19. 9525×0.042
20. 342×65.5
21. 1.414×0.0002
22. 6.28×6.28

SECTION 6

Using Powers of Ten in Division

To divide when one is using the powers of ten, there are two basic rules to be observed:

1. Change the sign of the denominator exponent.
2. Add denominator and numerator exponents algebraically.

Example—Divide 10^3 by 10^2. Expressed as a fraction this division is

$$\frac{10^3 \text{ numerator}}{10^2 \text{ denominator}}$$

The procedure then is:

1. Change sign of denominator exponent: 10^2 to 10^{-2}.
2. Since signs are now unlike, add the exponents algebraically:

$$10^3 \times 10^{-2} = 10^1 \text{ or } \frac{10^3}{10^2} = 10^{3-2} = 10^1$$

When dividing one number by another, using the powers of ten, it is best to express the division as a fraction.

Example—Divide 450 by 30. This division may be expressed as a fraction:

$$\frac{450}{30}$$

Changing to standard notation,

$$\frac{4.5 \times 10^2}{3.0 \times 10^1} = \frac{4.5}{3} \times 10^1 = 1.5 \times 10^1 = 15$$

By using the powers of ten one can place a larger number above the fraction line, making the division easier, and by adding the exponents algebraically the decimal point will be correctly placed.

It is not always advantageous to express both the numerator and the denominator in standard notation. When it is desired to keep the numerator a larger number than the denominator for easier division, it may be best to express only the denominator in standard notation.

Example—Divide 225.75 by 645. Expressed as fraction multiplied by powers of ten this division can be written:

$$\frac{22.575 \times 10^{1}}{6.45 \times 10^{2}} \text{ or } 3.5 \times 10^{-1} = 0.35$$

Note that in the preceding example the numerator was not expressed in standard notation although the denominator was; this expression makes the numerator a larger number than the denominator, for easier division.

Exercises

Use the slide rule to obtain the digits of the quotient. Then place the decimal point by the method of powers of ten.

1. 3140/6.28
2. 7500/0.005
3. 0.045/90
4. 5/0.0001
5. 18,000/3.14
6. 245/0.055
7. 6.35/6.28
8. 0.00015/0.000023
9. 53/68
10. 75/1500

SECTION 7

Electronics Units Expressed in Powers of Ten

Electronics formulas involve basic units of measurement such as the ampere, volt, ohm, farad, and henry. Many times the actual value of a component such as a coil or capacitor is too small to be expressed in basic units. Instead of using cumbersome decimal fractions it is more convenient to express the values in powers of ten.

Micro (μ) means 1/1,000,000 or 0.000001; in powers of ten: 10^{-6}. Thus 5 microfarads can be written as 5×10^{-6} farad.

Pico (p) means 1/1,000,000,000,000 or 0.000000000001; in powers of ten: 10^{-12}. Thus 150 pf (picofarads) can be written as 150×10^{-12} farad.

Kilo (k) means 1000; in powers of ten: 10^3.

Milli (m) means 1/1000 or 0.001; in powers of ten: 10^{-3}.

Mega (M) means 1,000,000; in powers of ten: 10^6.

Exercises

Express the following values in powers of ten of the basic unit.

1. 1.25 megohms
2. 470 picofarads
3. 0.005 microfarads
4. 350 microhenrys
5. 55 kilovolts
6. 200 milliamperes
7. 1600 picofarads
8. 0.047 microfarads
9. 0.0002 microfarads
10. 680 picofarads

SECTION 8

The Log Method

The term "log" is an abbreviation of *logarithm*, which is derived from two Greek words: *logos*, meaning proportion, and *arithmos*, meaning number. The log of any number is the exponent to which a given base must be raised to equal that number. For example, for the base 10 (the *common logarithm*), $10^2 = 100$; therefore 2 is the log of 100. Also, $10^3 = 1000$; so 3 is the log of 1000. To express these it is not necessary to use so many words. Simply write

$$\log 100 = 2$$
$$\log 1000 = 3$$

Since the log of 100 is 2, and the log of 1000 is 3, the log of any given number between 100 and 1000 must fall between the numbers 2 and 3, and it is expressed as an integer plus a decimal quantity. Here are two examples:

$$\log 150 = 2 \text{ plus a decimal quantity}$$
$$\log 950 = 2 \text{ plus a decimal quantity}$$

Since the log of 10 is 1 and the log of 100 is 2, then the log of any given number between 10 and 100 must be 1 plus a decimal fraction. for example,

$$\log 15 = 1 \text{ plus a decimal fraction}$$

There are two parts to a logarithm:

1. The *characteristic*, or part to the left of the decimal point.
2. The *mantissa*, which is the decimal quantity to the right of the decimal point.

Consider the equation

$$\log 15 = 1.1761$$

Here, 1 is the characteristic, and .1761 is the mantissa.

The mantissa is the decimal-fraction part of a logarithm and can be found for a given number in tables prepared for that purpose. It can also be found by using the L scale of the slide rule, but for the present we are primarily concerned with using the characteristic to aid us in placing the decimal point.

There are two rules concerning the placing of the decimal point by the log method wherein the characteristics of the numbers in the problem are added and the sum obtained becomes the characteristic of the answer:

1. For numbers greater than or equal to 1, the characteristic is always one less than the number of digits to the left of the decimal point, and is positive.
2. For numbers less than 1 but greater than zero, the characteristic is negative and is equal to the number of places that the first nonzero digit appears to the right of the decimal point.

Here are two examples:

1. The characteristic of 56.9 is 1, which is one less than the number of digits to the left of the decimal point.
2. The characteristic of 0.005 is -3, which is the number of places that the first nonzero digit (5) is to the right of the decimal point. Note that the characteristic is negative.

Exercises

Give the characteristics for the numbers below. Check your answers by referring to Table 4-1.

1. 0.546
2. 5682.98
3. 1.414
4. 0.707
5. 6.28
6. 5720.5
7. 982.56
8. 0.000089
9. 0.5
10. 367.1

TABLE 4-1. *Numbers and Their Characteristics*

Numbers Greater Than 1			Numbers Less Than 1		
Number	*No. of digits to left*	*Characteristic (+)*	*Number*	*No. of digits to right*	*Characteristic (−)*
1	1	0	0.1	1	−1
10	2	1	0.01	2	−2
100	3	2	0.001	3	−3
1000	4	3	0.0001	4	−4
10,000	5	4	0.00001	5	−5

SECTION 9

Using the Log Method to Place the Decimal Point

The log method of setting the decimal point in the answers derived by slide-rule operation is very accurate when properly done. The procedure for multiplication is as follows:

1. Write the problem down and determine the characteristics of each number as you proceed.
2. Add the characteristics. Their sum is the characteristic of the answer.

There is a very important rule to remember when using this method with the slide rule:

> Each time that the left index of the C scale (or B scale if used) is forced to extend off the rule, a *correction factor of 1 must be added* to the characteristic of the number that caused this situation.

For example, let us multiply 150 by 8.

1. Write the characteristic of each number above it: $\overset{2}{150} \times \overset{0}{8}$.
2. Add the characteristics and the number of correction factors, which is 1 in this case, earned when the left index was forced off to set the number 8. The slide-rule answer indicates the digits 12.

$$\begin{array}{l} 2 + (0 + 1) = 3 \\ 150 \times 8 \end{array}$$

3. The characteristic of 8, a one-digit number, is zero, but with the correction factor of 1 added, the total of the characteristics is 3, which is the characteristic of the answer.
4. The decimal point must therefore be placed so as to accommodate four digits to the left of it, making the correct answer 1200.

Summary

In doing multiplication problems with the slide rule and using the log method to place the decimal point, you must add 1 to the characteristic of each number whose setting causes the left index of the C (or B) scale to extend off the body of the rule. In problems involving more than two numbers, a correction factor of 1 is added to the characteristic of each number obtained when the left index is forced off.

The characteristic of the answer will indicate where to place the decimal point for the digits derived from the slide rule, and it will be equal to the sum of all the characteristics, plus 1 for each number whose setting caused the extension.

Since the left index is off the rule when the right index is used, one can also see that a correction factor of 1 must be added to the characteristic of the number which caused the right index to be used.

Here are some examples:

$$\begin{array}{l} 2 \quad + 1 + 1 = \quad 4 \\ 645 \times 45 = 29025 \end{array}$$

The sum of the characteristic plus 1 is 4, so the answer will have five digits to the left of the decimal point.

$$\overset{-3}{0.0064} \times \overset{+1}{10} = \overset{-2}{0.064}$$

Sum of the characteristics is −2, so the answer will have the first nonzero digit appearing two places to the right of the decimal point. There is no correction factor in this problem. Note that the characteristics were added algebraically.

$$\overset{0}{6.28} \times \overset{+1+1}{16} = \overset{2}{100.48}$$

Sum of characteristics is 2, which becomes the characteristic of the answer, indicating that there must be three digits to the left of the decimal point. There is no correction factor in this problem.

Thus the answer consists of the numbers derived from the slide rule and the decimal point is placed according to the sum of the characteristics and correction factor.

Exercises

Perform the indicated multiplications, using the log method.

1. 0.786×2000
2. 545×6.28
3. 0.005×0.0412
4. $25.6 \times .0045$
5. 8.19×625
6. 4.5×6.28

SECTION 10

Placing the Decimal Point by Using the Log Method in Division Problems

The following rules must be observed for using the log method in divisions:

1. The sign of the characteristic of the divisor is changed and added to the characteristic of the dividend (whose sign is not changed).
2. If the C and D scales are used for division, a correction factor of 1 is added to the characteristic of any number

placed when the left index of the C scale is moved off the rule. If the D and CI scales are used for division, a correction factor of 1 is added to the characteristic of any number placed when the right index of the CI scale is moved off the rule.

3. The characteristic of the quotient results from the addition of the characteristic of the dividend and the characteristic of the divisor but with changed sign.

Here are three examples:

$$\frac{\overset{0}{6.28}}{\underset{0}{2}} = \overset{0}{3.14}$$

In this problem the sum of the characteristics is zero, and there is no correction factor.

$$\frac{\overset{2}{648}}{\underset{1+1}{80}} = \overset{0}{8.1}$$

Here the characteristics are added algebraically. There was a correction factor of 1 earned when the index was forced off to place the number 80.

$$\frac{\overset{0}{4.5}}{\underset{0+1}{9}} = \overset{-1}{0.5}$$

Although the characteristic of 9 is zero, there was a correction factor earned in placing the number.

Exercises

Perform the indicated divisions.

1. $\frac{0.65}{0.00065}$
2. $\frac{54}{200}$
3. $\frac{15.42}{30.2}$
4. $\frac{6482}{32{,}415}$
5. $\frac{0.0075}{37.5}$
6. $\frac{90.6}{5}$
7. $\frac{0.0075}{68}$
8. $\frac{3.1414}{6.28}$
9. $\frac{555}{18}$
10. $\frac{2464}{0.003}$

Examination on Lesson 4

Using the approximation method, place the decimal point in the answers of the following problems.

1. $\frac{6.35}{9.15}$

2. $\frac{30.7}{4.2}$

3. $\frac{6.87}{1.09}$

4. 1.7×8.2

5. 1.73×4.05

6. 3.04×107.3

Write out the following expressions.

7. 3.5×10^{2}

8. 3.5×10^{-2}

9. 8.1×10^{0}

10. 8.1×10^{-0}

11. 67×10^{-1}

12. 1.01×10^{-3}

Express the following numbers in standard notation.

13. 0.67

14. 372.5

15. 0.1414

16. 1.07

Use the powers of ten to work the following problems.

17. $\frac{829{,}000}{510}$

18. 504×17

19. 3.14×314

20. 0.981×0.462

21. $\frac{8.29}{510}$

22. $\frac{0.0163}{524}$

Express in terms of the basic unit:

23. 1.5 microhenrys

24. 2 kilovolts

25. 8 picofarads

26. 3 millivolts

Use the log method to place the decimal point in the following problems. Then put the answer into standard notation.

27. 53×0.17

28. $\frac{92}{0.05}$

29. 1810×0.03

30. $\frac{0.04}{92}$

31. 115×47.3

32. $\frac{18{,}236}{3}$

5

Repeated Multiplication And Division

SECTION 1

Use of C and D Scales

Problems involving combined multiplication and division can be done on the slide rule by using any two identical scales. The C and D scales are the most practical for most problems of this type.

Example—Evaluate the following:

$$\frac{7.5 \times 0.9}{82.5}$$

Solution—Although it is apparent that one can simply multiply 7.5 by 0.9 and then divide the product by 82.5, a good general rule to adopt when using the slide rule is to alternately divide and multiply:

1. Place hairline over 7.5 on the D scale.
2. Slide 82.5 of the C scale under the hairline.
3. Place hairline over 9 on C scale.
4. Read digits of answer (817) under hairline on D scale.

Note that you started on the D scale and ended on the same scale.

Log Method of Placing the Decimal Point in the Problem

Note the characteristics of the numbers involved in the preceding problem:

$$\frac{\overset{0}{7.5} \times \overset{-1+1}{0.9}}{\underset{1+1}{82.5}}$$

Change the sign of the sum of the characteristics in the divisor and add it algebraically to the sum of the characteristics in the dividend:

Divisor characteristics: $1 + 1 = 2$ (correction factor of 1)
Divisor characteristic with changed sign: $= -2$
Dividend characteristics: $0 + (-1) + (+1) = \ \ 0$ (cor. fac. of 1)
Sum: $= -2$

The digits derived by the rule are 817. The characteristic -2 for the answer means that the decimal point is two places to the left of the first nonzero digit. Therefore the correct answer is 0.0817.

SECTION 2

When the Divisor Contains More Factors Than the Dividend

Consider the following example of combined multiplication and division:

$$\frac{6 \times 18}{4.5 \times 2 \times 6}$$

The procedure is as follows:

1. Place hairline over 6 on the D scale.
2. Slide 4.5 of C scale under hairline.
3. Move hairline over 18 on C scale.
4. Slide 2 of C scale under hairline.
5. Move hairline over left index of C scale.
6. Slide 6 of C scale under hairline.
7. Read digit of answer (2) under the right index of the C scale, on the D scale.

In the preceding problem note that we start out by dividing, then multiply, then divide.

The problem can be expressed as a fraction simply by multiplying the factors above and below the fraction line:

$$\frac{6 \times 18}{4.5 \times 2 \times 6} = \frac{108}{54} = 2$$

Placing the decimal point in a problem of this type can be done by approximation unless the factors contain too many digits or decimal fractions, such as the following:

$$\frac{6.28 \times 60 \times 0.001 \times 5}{100 \times 2 \times 40}$$

Here, the procedure is the following:

1. Place hairline over 6.28 on the D scale.
2. Slide 100 (right index) of C scale under hairline.
3. Place hairline over 60 on the C scale.
4. Slide 2 of C scale under hairline.
5. Place hairline over 1 on C scale.
6. Slide 40 of C scale under hairline.
7. Place hairline over 5 on C scale.
8. Read 235 under hairline on the D scale.

The decimal point is placed as follows, using the log method:

Dividend Characteristics	*Divisor Characteristics*
6.28 = 0	100 = 2
60.00 = 1+1 (correction)	2 = 0
0.001 = −3	40 = 1+1 (correction)
5 = 0+1 (correction)	Total = 4
Total = 0	

Change the sign of the divisor-characteristic sum and add it to sum of the characteristics of dividend:

$$-4 + 0 = -4$$

Therefore the correct answer is 0.000235.

Exercises

Perform the indicated operations, using the log method.

1. $\dfrac{0.0045 \times 1500 \times 14 \times 254}{645 \times 3.14 \times 85}$
2. $\dfrac{8.95 \times 32.4 \times 4250}{67.7 \times 8.05}$
3. $\dfrac{381 \times 2220}{955 \times 24 \times 1.59}$
4. $\dfrac{0.0191 \times 18.4 \times 685}{6.3 \times 20.3}$
5. $\dfrac{382 \times 7030 \times 8.96}{1.33 \times 29.8 \times 9.08}$
6. $\dfrac{0.0083 \times 0.0843 \times 1.71}{0.000555 \times 0.678}$

SECTION 3

Extended Multiplication

It is useful to be able to perform extended multiplications on the slide rule. This type of calculation is involved in such electronics applications as computing stage gain and inductive reactance.

Example—Find 2 × 2 × 4. The procedure is as follows:

1. Place hairline over 2 on the D scale.
2. Slide left C index under hairline.
3. Place hairline over 2 on the C scale.
4. Slide right index of C scale under hairline.
5. Place hairline over 4 on the C scale.
6. Read answer on the D scale under the hairline (16).

Example—Find 8 × 5 × 3 × 6. Here, the procedure is the following:

1. Place hairline over 8 on the D scale.
2. Slide right index of C scale under hairline.
3. Place hairline over 5 on the C scale.
4. Slide right C index under hairline.

5. Place hairline over 3 on the C scale.
6. Slide left C index under hairline.
7. Place hairline over 6 on the C scale.
8. Read digits 72 under hairline on the D scale.

Note that in this type of problem one merely continues to multiply by resetting the slide after each hairline setting.

Placing the Decimal Point by the Log Method

Although the placing of the decimal point can be done by approximation in a problem similar to the foregoing problem, this problem will serve to illustrate the advantage of using the log method:

Characteristics in Parentheses:

$$\begin{array}{ccccccccc} (0) & + & (0+1) & + & (0+1) & + & (0) & = & (2) \\ 8 & \times & 5 & \times & 3 & \times & 6 & = & 720 \end{array}$$

Note that there was a correction factor involved for 5 and 3 because the left C index was forced off the rule when these numbers were placed. Since the sum of the characteristics is 2, the answer must be a three-digit number.

Exercises

Perform the indicated multiplications.

1. $16 \times 35 \times 5.6$
2. $3.89 \times 26 \times 0.05$
3. $9 \times 6.28 \times 1000$
4. $3.14 \times 7.05 \times 45$
5. $45{,}000 \times 0.0040 \times 85$
6. $68 \times 25 \times 5.80 \times 3.04 \times 5$

SECTION 4

Problems in Electronics Involving Combined and Repeated Operations

The formula for inductive reactance is

$$X_L = 2\pi fL \quad \text{or} \quad X_L = 6.28 \times f \times L$$

where,

X_L is the inductive reactance in ohms,
π is the constant 3.14159 . . . ,

f is the applied frequency in hertz (cycles per second, abbreviated Hz),
L is the inductance in henrys.

On the slide rule the procedure is as follows:

1. Place hairline over 6.28 ($=2\pi$) on the D scale.
2. Slide C index under hairline.
3. Place hairline over value of f on C scale.
4. Slide C index under hairline.
5. Place hairline over value of L on the C scale.
6. Read digits of answer under the hairline on the D scale.

Example—Find the inductive reactance of a 10-henry choke at 60 Hz.

1. Set up formula: $X_L = 6.28 \times 10 \times 60$.
2. Manipulate slide rule as for extended multiplication, as given in foregoing procedure.
3. Slide rule should yield digits 3765.
4. Establish decimal point by approximation:

$$10 \times 60 = 600$$
$$6 \times 600 = 3600$$

Examination shows that answer must be a four-digit number; so the correct answer therefore is 3765 ohms.

Example—A signal voltage with a frequency of 54 MHz is applied to a coil whose inductance is 20 millihenrys. Find the inductive reactance, X_L.

1. Set up the multiplication $6.28 \times 54 \times 20$.
2. Slide rule should yield the product 6780.
3. To establish the decimal point, use the scientific notation:

$$X_L = 6.28 \times 54 \times 10^6 \times 20 \times 10^{-3} = 6780 \times 10^3 \text{ ohms}$$
$$= 6{,}780{,}000 \text{ ohms}$$

Note that in this problem the values of megahertz and millihenrys were expressed in powers of ten, which placed the decimal point for the final answer.

Resistance in Series Circuits

In the circuit of Fig. 5-1, the voltage (E_1) dropped across R_1 is 56 volts. What is E_2, E_T, and I_T?

Here the correct formula is

$$E_2 = \frac{E_1 \times R_2}{R_1}$$

$$= \frac{56 \times 150}{500} = 16.8 \text{ volts (combined operations)}$$

The value of 16.8 volts indicates the voltage drop across R_2. To find the total voltage (E_T) simply add the two voltage drops (E_1 and E_2) to obtain 72.8 volts.

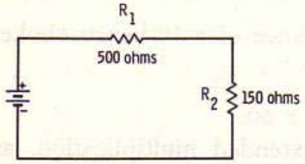


Fig. 5-1. A simple resistive circuit.

The value of 72.8 volts is the circuit voltage. To find the total current (I_T) use Ohm's law:

$$I = \frac{E}{R} = \frac{72.8}{650} = 0.113 \text{ amperes}$$

The decimal point was easily placed by the scientific-notation method:

$$\frac{7.28 \times 10^1}{6.50 \times 10^2} = 1.13 \times 10^{-1} = 0.113$$

Exercises

Work the following problems by writing the formula first, then obtaining the digits of the answer from the slide rule. Do not look up the answers until you have done the problem by yourself, then check your answer and procedure with those in the answer section in the back of the book. Remember that there are electronics formulas involved here and the answers must be decimal-pointed.

1. A voltage with a frequency of 60 Hz is applied to a choke whose inductance is 5 henrys. Find the inductive reactance of the choke.
2. Find the inductive reactance of a coil whose inductance (L) is 200 microhenrys at 5 MHz.

3. Find the inductive reactance of a 5-mh coil at 45.75 kHz.
4. Find the peak value of current through a lamp whose d-c resistance is 250 ohms when connected across a 115-volt a-c line. (To find peak voltage multiply 115 by 1.414.)
5. What is the total gain of a four-stage amplifier whose stage-by-stage gain is:

   Stage No. 1: 50
   Stage No. 2: 100
   Stage No. 3: 200
   Stage No. 4: 150

Examination on Lesson 5

Using the log method, evaluate the following expressions.

1. $\dfrac{3.89}{0.0864 \times 0.0056 \times 8.65}$
2. $\dfrac{0.63 \times 50 \times 291}{0.707 \times 3.14 \times 190}$
3. $\dfrac{261 \times 3.5 \times 41}{8.2 \times 560 \times 0.14}$
4. $\dfrac{38 \times 1.8 \times 102}{7.15 \times 3}$
5. $18.61 \times 5 \times 3 \times 0.8 \times 14$
6. $21 \times 455 \times 645 \times 1500$
7. $1800 \times 3.45 \times 0.00055$
8. $27 \times 5.46 \times 6700 \times 3$
9. A circuit has an inductance of 1.5 henrys across an a-c source of 60 Hz. What is the inductive reactance of the circuit?
10. What is the inductive reactance of a 0.2-henry coil connected across a 60-Hz, 120-volt source?

6

Reciprocals

SECTION 1

Definition of Reciprocal

The *reciprocal* of a number is equal to that number divided into 1. Thus the reciprocal of a number can be expressed as a fraction whose denominator is the number and whose numerator is 1. Let us consider an example.

Example–Find the reciprocal of 20:

1. Expressed as a fraction: $\frac{1}{20}$.
2. Divide 1 by 20.
3. The reciprocal of 20 is equal to 0.05; this is how many times 20 can be divided into 1. *Proof:* $0.05 \times 20 = 1$.

In the foregoing example the reciprocal of a number greater than 1 was found to be less than 1.

Suppose that we find the reciprocal of a number which is less than 1. Let us find the reciprocal of 0.2:

1. Expressed as a fraction: $\frac{1}{0.2}$.
2. Divide 1 by 0.2.
3. The reciprocal of 0.2 is 5 because that is how many times it can be divided into 1.

Thus the reciprocal of a number less than 1 is greater than 1.

How to Use the Slide Rule to Find Reciprocals

This is one of the simplest of procedures for the slide rule. One uses the C and CI scales. Note that the CI scale is an inverted C scale; it starts at the right end and proceeds to the left in the same scale arrangement as the C scale.

The reciprocals of all the numbers on the C scale appear directly above them on the CI scale.

As shown in Fig. 6-1, the procedure for finding the reciprocals of numbers is simple with the slide rule. No moves are required except to place the hairline over the number; the reciprocal will appear under the hairline on the CI scale.

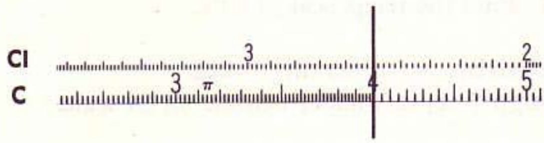


Fig. 6-1. Using C and CI scales to find reciprocal of 4.

Example—Find the reciprocal of 20. The procedure is as follows:

1. Place hairline over 20 on the C scale.
2. Read digit 5 under hairline on the CI scale.

The numbers of the CI scale are usually printed in red on most slide rules, and they are sometimes smaller in size than those on the C or D scales. On some other slide rules the CI scale has italicized numbers.

SECTION 2

Using the Log Method to Place Decimal Point in Reciprocals

For all numbers except 10 and its powers (100, 1000, etc.) the decimal point may be placed as follows:

1. Determine characteristic of number.
2. Add +1 to this characteristic.
3. Change the sign of the sum.
4. The characteristic of the reciprocal will be 1 greater than the characteristic of the number, with the sign of this sum changed.

Example—Place the decimal point in the reciprocal of 20:

1. Characteristic of 20 is 1.
2. Add 1, making the sum 2.
3. Change sign of this sum to −2.
4. The characteristic of the reciprocal is therefore −2.

Thus the reciprocal of 20 is 0.05.

Example—Find the reciprocal of 0.05.

1. Place hairline over 5 on the C scale.
2. The digit 2 appears under hairline on CI scale.

To place the decimal point:

1. The characteristic of 0.05 is −2.
2. Add 1 to this: $(-2) + 1 = -1$.
3. Change sign to +1.

The characteristic of the reciprocal is thus 1, so the correct answer is 20.

To place the decimal point correctly for reciprocals of 10 and its powers it is not necessary to add 1 to the characteristic. Simply change the sign of the characteristic of the number, and this becomes the characteristic of the reciprocal.

Example—Find the reciprocal of 10,000.

1. The characteristic of 10,000 is 4.
2. Change sign, making it −4.
3. This becomes the characteristic of the reciprocal.

The reciprocal of 10,000 is therefore 0.0001.

Exercises

Find the reciprocals of the following numbers. Be sure to place the decimal point correctly.

1. 25
2. 33
3. 50
4. 2.5
5. 0.005
6. 12.5
7. 36,000.5
8. 0.04
9. 8
10. 16.7

SECTION 3

Parallel-Resistance Formulas Easily Handled by the Slide Rule

When two or more resistors are combined in parallel, the total or combined resistance of the combination is less than the smallest value in the combination but not less than half the value of the smallest resistance. This fact can be helpful in placing the decimal point when using the slide rule to solve such problems.

Example—Find the combined resistance of a 25-ohm resistor and a 20-ohm resistor when they are connected in parallel.

The total resistance is given by the formula

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{25 \times 20}{25 + 20} = \frac{500}{45} \text{ ohms}$$

To solve this equation with the slide rule one merely adds the 25 and 20 and uses it as the divisor in an ordinary division problem. The digits 111 should then be obtained. Since the value of the combination must be less than the smaller resistance but not less than half of it, it appears that the correct answer is 11.1 ohms.

The foregoing formula is generally used for only two resistors at a time. When there are three or more resistors in a parallel combination the standard formula is used:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$

where,

R_n is the resistance of the nth resistor.

In the formula the combined resistance (R_T) is the reciprocal of the sum of the reciprocals of each resistor value in the combination.

Example—In the circuit of Fig. 6-2, find the combined resistance of the six resistors.

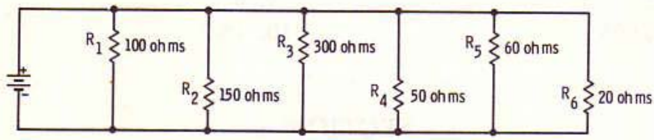


Fig. 6-2. A simple parallel-resistance circuit.

The solution may be found by the following steps:

1. Find each reciprocal.
2. Add all of the reciprocals.
3. Find the reciprocal of the total.

The reciprocals and their sum is given by the following:

Reciprocal of $R_1 = 0.0100$ (R_1 is 100 ohms)
Reciprocal of $R_2 = 0.0066$ (R_2 is 150 ohms)
Reciprocal of $R_3 = 0.0033$ (R_3 is 300 ohms)
Reciprocal of $R_4 = 0.0200$ (R_4 is 50 ohms)
Reciprocal of $R_5 = 0.0165$ (R_5 is 60 ohms)
Reciprocal of $R_6 = \underline{0.0500}$ (R_6 is 20 ohms)

Total of Reciprocals $= 0.1064$

The reciprocal of the sum is:

$$\frac{1}{0.1064} = 9.37 \text{ ohms}$$

By using the log method as explained in Sections 2 and 3 the decimal point can be placed for each reciprocal as one proceeds with the problem; the addition will then be correct. The decimal point can be placed from Step 2 of the same method.

Exercise

1. Solve for the total resistance of the circuit of Fig. 6-3.

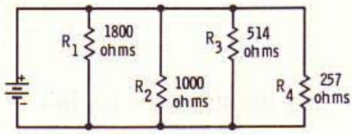


Fig. 6-3. A parallel-circuit problem.

SECTION 4

Capacitive-Reactance Problems

Consider the formula for capacitive reactance in terms of frequency (f) and capacitance (C):

$$X_C = \frac{1}{2\pi f C} = \frac{1}{6.28 \times f \times C}$$

where,

X_C is the capacitive reactance in ohms,
6.28 is 2π,
f is the frequency in hertz (Hz),
C is the capacitance in farads.

The formula shows that to find the capacitive reactance of a certain capacitor at a given frequency one simply finds $6.28 \times f \times C$, and then finds the reciprocal of this product.

Since the term 2π is a constant (6.28), a shorter way to do capacitive-reactance problems is to first obtain the reciprocal of 6.28:

$$\frac{1}{6.28} = 0.159$$

Then the formula can be written

$$X_C = \frac{0.159}{fC}$$

The problem thus becomes a combination division-multiplication problem which is easily handled by the slide rule.

Example—Find the capacitive reactance of a 0.01-microfarad capacitor at 6000 Hz. Substituting these values in the formula,

$$X_C = \frac{0.159}{6000 \times 0.01 \times 10^{-6}}$$
$$= \frac{15.9 \times 10^3}{6}$$

Using the slide rule to find 15.9/6:

$$X_C = 2.65 \times 10^3 = 2650 \text{ ohms}$$

In some electronics texts you will find that the reciprocal of 6.28 is rounded off to 0.16, which simplifies matters when one is using the slide rule.

Exercises

1. Find the capacitive reactance (X_C) of a 100-picofarad capacitor at 1 MHz.
2. What is the capacitive reactance of a 133-microfarad capacitor in a circuit in which the voltage is 117 volts at a frequency of 60 Hz?
3. What is the combined resistance of four parallel resistors with the following values:
   $R_1 = 1500$ ohms,
   $R_2 = 1000$ ohms,
   $R_3 = 600$ ohms,
   $R_4 = 300$ ohms.
4. What is the combined resistance of two 200-ohm resistors connected in parallel?

SECTION 5

Capacitors in Series

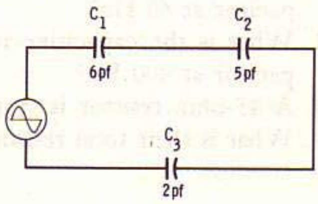
When two or more capacitors are connected in series their total capacitance is equal to the reciprocal of the sum of the reciprocals of the individual capacitances:

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}}$$

This equation is mathematically similar to the parallel-resistance formula, and it can be solved with the slide rule by using the same procedure as for a parallel-resistance circuit problem (see Section 3).

Example—What is the total capacitance of the circuit of Fig. 6-4?

Fig. 6-4. A series-capacitance circuit.



Procedure:

1. Find the reciprocal of 6, or 0.167.
2. Find the reciprocal of 5, or 0.2.
3. Find the reciprocal of 2, or 0.5.
4. Add these to get 0.867. The reciprocal of 0.867 is 1.15. Therefore $C_T = 1.15$ picofarads.

Exercises

1. What is the total capacitance of three capacitors in series whose values are 450, 200, and 540 picofarads respectively?
2. Three 0.05-microfarad capacitors are connected in series. What is their total capacitance?
3. What is the total capacitance of five 0.047-microfarad capacitors connected in parallel?
4. What would be the combined value of the five capacitors of Exercise 3 if they were connected in series?
5. One 47-ohm and three 5600-ohm resistors are connected in series. What is their total resistance?

Examination on Lesson 6

Find the reciprocals of the following numbers.

1. 6500	5. 2.2
2. 0.045	6. 55
3. 7	7. 3
4. 100	8. 1000

Solve the following problems.

9. Two 470-kilohm resistors are connected in parallel. What is their total resistance?
10. Two 470-picofarad capacitors are connected in series. What is the total capacitance of the combination?
11. What is the capacitive reactance of a 2-microfarad capacitor at 60 Hz?
12. What is the capacitive reactance of a 2-microfarad capacitor at 400 Hz?
13. A 45-ohm resistor is in parallel with a 54-ohm resistor. What is their total resistance?

7

Squares And Square Roots

SECTION 1

Definitions

The *square* of a number is simply the product of that number and itself. To indicate that a number is to be squared it is expressed as a power with the exponent 2:

$$5^2 = 25$$

which means: $5 \times 5 = 25$.

The *square root* of a number is that number which when multiplied by itself will equal the given number. To indicate that the square root is desired the number is preceded and covered by a sign called the *radical sign:*

$$\sqrt{25} = 5$$

The extraction of the square root of a two-digit number can sometimes be done mentally, but when three- or more-digit numbers are involved the longhand procedure required is generally unknown to or avoided by many students. Inasmuch as a working knowledge of this procedure is necessary in order to place the decimal point correctly in the answer derived by the slide rule, a discussion of it is in order.

Setting Up a Square-Root Problem—The Hard Way

To extract the square root of a number the number is first separated into groups of two on each side of the decimal point.

Example—Find the square root of 3844:

1. Separate 3844 into groups of two, starting from the decimal point: 38′44.
2. Place the radical sign: $\sqrt{38'44}$.
3. Find the number whose square comes closest to equalling but not exceeding 38, which is comprised of the digits in the first group. This would be 6.
4. Place this number over the line above the first group.
5. Square it and place the square under the first group:

$$\begin{array}{r} 6 \\ \sqrt{38'44} \\ 36 \end{array}$$

6. Draw a line, subtract the square, and bring down the next group.
7. Double the root (6) and place this in a divider position:

$$\begin{array}{rl} & \phantom{\sqrt{}}6 \\ & \sqrt{38'44} \\ & \phantom{\sqrt{}}36 \\ 12 & \overline{\big|\ 2\,44} \end{array}$$

8. Figure out how many times 12 will go into 24, the first two digits of the new dividend (244). This number is 2. Place 2 in the root and annex it to the divisor.
9. Multiply the divisor by the last number in the root so far obtained (2 × 122). Place this product under the dividend:

```
        6  2
      √38'44
       36
122  | 2 44
       2 44
       0 00
```

10. Since there is no remainder, the problem is completed; the square root of 3844 is 62.

Note that the number of digits in the answer was two. This was also the number of groups of two into which the original number was separated. This is an important point to remember, and we shall refer back to it when we discuss placing the decimal point in the answer obtained from the slide rule for the same problem.

Using the Slide Rule to Solve for Roots

In the foregoing problem there were many steps involved in the longhand method. The slide rule can yield the digits of the answer with only one positioning movement—placing the hairline over the number involved (Fig. 7-1).

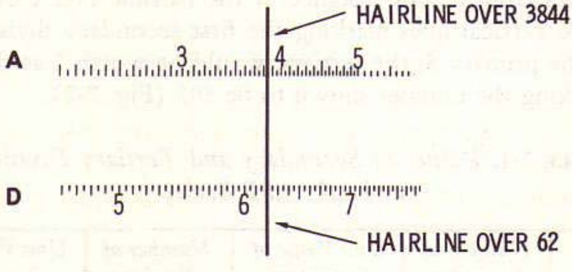


Fig. 7-1. Finding the square root of 3844.

To find the square root of a number, place the hairline over this number on the A scale. The digits of the square root will appear under the hairline on the D scale.

Since the number 3844 is a four-digit number, the placing of the hairline is critical. However, with practice and care it can be done accurately.

SECTION 2

Square Roots and the A Scale

Examine the A scale of your slide rule. It consists of two sections, each of which corresponds to the whole C (or D) scale. Thus the A scale has twice the range of the C and D scales although it has the same physical length.

The B scale is identical to the A scale, except that it is placed on the slide. Since the A and B scales are contracted, there are not as many division markings on them as on the C and D scales. The left section of the A (or B) scale is numbered from 1 to 10, and the right section from 10 to 100. The half to use in extracting the square root of a number depends on the value of the first digit of the derived slide-rule answer. This will be discussed in detail later.

The values of the secondary and tertiary divisions of the A and B scales are summarized in Table 7-1. Note in Table 7-1 that the values of the tertiary divisions vary from 1 to 10 and that between the primary divisions 5 to 10 there are no marked tertiaries, and the value of the third digit of a number can be anywhere from 1 to 9, depending on how accurately one can place the hairline or read values from it. For instance, if the hairline falls exactly between the vertical lines marking the first secondary division following the primary 5, the tertiary would be worth 5 as the third digit, making the number shown to be 505 (Fig. 7-2).

TABLE 7-1. *Values of Secondary and Tertiary Divisions on the A and B Scales*

Primary Range From	*Number of Secondary Divisions*	*Unit Value of Secondary Divisions*	*Number of Tertiary Divisions*	*Unit Value of Tertiary Divisions*
1 to 2	10	1, as second digit.	50	2, as third digit.
2 to 5	10	1, as second digit.	20	5, as third digit.
5 to 10	10	1, as second digit.	none; estimated only.	5, as third digit when halfway between sec. divs.

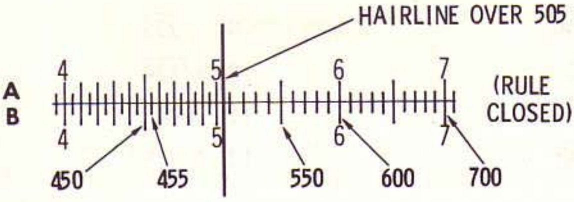


Fig. 7-2. The number 505 on the A and B scales.

SECTION 3

Using the A and D Scales to Square and Extract Square Roots

When the hairline is placed over a number on the D scale, its square is directly under the hairline on the A scale. For example, 5 squared (5^2) equals 25. Conversely, when the hairline is placed over a number on the A scale, its square root is found directly under the hairline on the D scale. For example, the square root of 250 is 15.8+.

In Fig. 7-3 the positions of the hairline are shown for squaring 15.81 and for extracting the root of its square, 250, and for the square of 5 and the square root of 25, depending on which is desired.

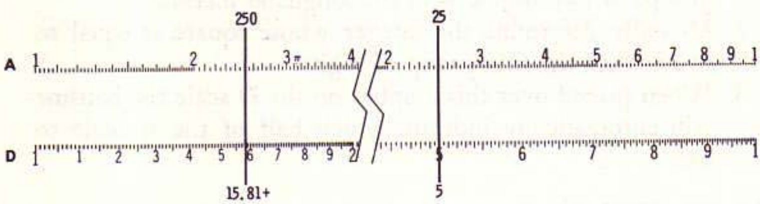


Fig. 7-3. Hairline positions for $\sqrt{250}$ and 5^2.

The arrangement of the A and D scales is pretty handy, requiring no manipulation except to place the hairline. The slide rule is thus especially useful for solving the various problems in electronics concerning impedance, power, and resonance.

Exercises

Perform the indicated operations.

1. $\sqrt{61}$
2. $\sqrt{82}$
3. $\sqrt{55}$
4. $\sqrt{64}$

5. $\sqrt{860}$

6. $\sqrt{52}$

7. $\sqrt{315}$

8. $\sqrt{200}$

9. $\sqrt{225}$

10. $\sqrt{324}$

11. 3844^2

12. 1681^2

SECTION 4

How to Determine Which Half of the A Scale to Use in Square-Root Problems

There are two important considerations in regard to using the A and D scales in finding square roots and in squaring numbers:

1. Knowing which half of the A scale to use in square-root problems.
2. Placing the decimal point in square-root and squaring problems.

To determine the correct section of the A scale to use in setting up a square-root problem it is best to do the following:

1. Write down the number involved and separate it into groups of two digits, as in the longhand method.
2. Mentally determine the integer whose square is equal to or less than the first group of digits.
3. When placed over this number on the D scale the hairline will automatically indicate which half of the A scale to use.

Example—Find the square root of 169.

1. Write the number down and separate it into groups of two. Note the zero used to complete the first group: 01′69.
2. The whole number whose square comes closest to equalling 1 (in the first group) is 1.
3. The hairline over this number on the D scale shows that the left half of the A scale is to be used.

Place the hairline over 169 on the A scale. Under it, on the D scale, 13 is indicated as the square root of 169.

Note that placing the zero in front of the 1 to complete the first group in no way altered the problem. Since 169 is an odddigit number, it was necessary to "pad" it by placing a zero before the first digit.

In separating a number into groups of two, the decimal point becomes the reference from which one counts to the right and left of it.

Example—Find the square root of 24.01.

1. Group it into twos: 24.01. (This has already been done by the decimal point.)
2. The number whose square comes closest to equaling but not exceeding 24, the first group, is 4.
3. Since 4 is on the right side of the D scale, the right side of the A scale must be used.
4. Read answer (4.9) under the hairline on D scale.

In this problem note that the position of the decimal point provided two groups, one on each side of it.

Thus two rules can be made for determining which half of the A scale to use:

1. If the first group of two consists of a zero and one significant figure, the *left* half of the A scale is used.
2. If the first group of two consists of two significant figures, the *right* half of the A scale is used.

Exercises

Indicate, by circling R for right and L for left, which half of the A scale would be used to extract the square root of the following numbers.

1. 243.97	R	L	6. 1.414	R	L
2. 5639	R	L	7. 0.0056	R	L
3. 9.0025	R	L	8. 25,307	R	L
4. 41	R	L	9. 8261.4	R	L
5. 35.116	R	L	10. 594.036	R	L

SECTION 5

Placing the Decimal Point in Squaring and Square-Root Problems

To decimal-point the square of a number one can use the approximation method and the powers of ten.

Example—Square 12.9.

Solution—The square of 12.9 is 166.41. This number is impossible to obtain accurately on the slide rule, so the slide-rule answer will probably not contain more than three digits. The digits derived from the slide rule are 166. To determine where the decimal point should be, proceed as follows:

1. State the problem in scientific notation:

$$12.9^2 = 1.29 \times 10^1 \times 1.29 \times 10^1 = 1.66 \times 10^2 = 166$$

2. The correct answer is therefore 166.

Example—Square 1279.

Solution—Setting the hairline on this number on the D scale yields the digits 164 on the A scale. To place the decimal point:

1. State the square in scientific notation:

$$1279^2 = 1.279 \times 10^3 \times 1.279 \times 10^3 = 1.64 \times 10^6$$

2. Write $1.64 \times 10^6 = 1{,}640{,}000$ (1279^2 is actually 1,635,841).

To place the decimal point in a square-root problem, proceed as follows:

1. Separate the number whose square root is to be extracted into groups of two digits, starting at the decimal point and proceeding to the left and the right.
2. The square root of the number will then have as many digits to the left of the decimal point as there are groups of two, including zeros, to the left of the decimal point in the original number.

Example—Find the square root of 71,400.

1. Separate 71,400 into groups of two: 07′14′00.
2. Placing the hairline over 714 on the A scale (the left side) shows that the answer contains the digits 267.
3. Since there are three groups, the answer is a three-digit number; so the correct answer is 267.

Exercises

Evaluate the following expressions, placing the decimal point correctly after obtaining the digits on the slide rule.

1. $\sqrt{42.25}$
2. $\sqrt{240.25}$
3. $\sqrt{4664.09}$
4. $\sqrt{462.25}$
5. $\sqrt{1036.44}$
6. $\sqrt{25,210.44}$
7. 4.275^2
8. 32.2^2

SECTION 6

Decimal-Pointing the Square Roots of Numbers Less Than 1

If the number whose square root is to be extracted is a decimal fraction, the decimal point in the answer will be placed to the *left* of the first nonzero digit as many places as there are groups of two zeros to the right of the decimal point in the given number.

Example—Find the square root of 0.0000315.

1. Separate the number into groups of two to the right of the decimal point: 0.00′00′31′50. The extra zero after the 5 does not alter the value of the number; it merely completes the grouping.
2. The first group of nonzero digits is 31. The number whose square comes closest to equaling but not exceeding 31 is 5. Placing the hairline over 5 on the D scale shows that the right side of the A scale is to be used.
3. With the hairline placed over 315 on the right side of the A scale, the digits 561 on the D scale are under the hairline.
4. Since there are two groups of zeros to the right of the decimal point in the given number, there should be two zeros to the right of the decimal point in the square root—one zero for each group of zeros.
5. Therefore the square root of 0.0000315 is 0.00561.

Exercises

Find the indicated square roots and place the decimal point correctly.

1. $\sqrt{0.0049}$
2. $\sqrt{0.00036}$
3. $\sqrt{0.0016}$
4. $\sqrt{0.00025}$
5. $\sqrt{0.02}$
6. $\sqrt{0.00064}$

SECTION 7

Problems Involving Squares, Square Roots, and Combined Operations

The following examples show how the slide rule can be used to solve more complicated problems involving squares, square roots, and combination division and multiplication.

Example 1—Find $(20.1 \times 5.3)^2$.

Procedure:

1. Place hairline over 201 on the D scale.
2. Slide right index of C scale under hairline.
3. Move hairline over 53 on the C scale.
4. Under the hairline on the A scale read the digits 113.
5. To place the decimal point note that $20 \times 5 = 100$ and $100^2 = 10{,}000$, so the correct answer must be 11,300.

What has been done here was that 20.1 was first multiplied by 5.3, with the answer being read on the A scale, which automatically shows the square of the number which appears under the hairline on the D scale.

Example 2—Find $\left(\frac{42.3}{3}\right)^2$.

Procedure:

1. Place hairline over 423 on the D scale.
2. Slide 3 of the C scale under the hairline.
3. Read digits 199 on the A scale above the left index of the C scale.

In this example we first disposed of the division, then squared the quotient. To place the decimal point one can approximate: If $45/3 = 15$ and $15^2 = 225$, then the correct answer must be 199.

Example 3—Find the voltage required to produce 200 watts of power across a 50-ohm resistor.

Solution—A derivation from Ohm's law gives us the formula

$$E = \sqrt{PR}$$

where,

E is voltage in volts,
P is power in watts,
R is resistance in ohms.

Therefore, $E = \sqrt{200 \times 50}$. To determine this number:

1. Slide left index of C scale over 2 on the D scale.
2. Place hairline over 5 on the C scale.
3. Read digits 100 under hairline on the A scale.

Here we simply multiplied 200 and 50 and read the square root of the product under the hairline on the A scale. The decimal point is placed by approximation; the answer is 100 volts.

Example 4—If 100 volts is dropped across a 50-ohm resistor, how much power is used?

Solution—The power dissipated is

$$P = \frac{E^2}{R} = \frac{100^2}{50}$$

1. Place hairline over 100 on the D scale (either end).
2. Slide 50 of B scale under hairline.
3. Read digit 2 on the A scale over the B-scale index.

Since 50 will obviously divide into the square of 100 more than twice or 20 times, the right answer is 200 watts.

Example 5—Find the value of the current in the circuit of Example 4.

Solution—Use the formula

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{200}{50}}$$

1. Place hairline over 2 on the right section of the A scale.
2. Slide 5 of B scale under hairline.
3. Read the digit 2 under left index of C scale on the D scale.

To place the decimal point: since $\sqrt{\frac{200}{50}} = \sqrt{4} = 2$, the answer of 2 amperes is correct.

In this problem one must use the correct half of the A scale. Remember that division can also be done using the A and B scales, since they are identical. Here we have done the division first, with the square root of the quotient automatically appearing on the D scale.

Exercises

1. Evaluate $\left(\frac{6.5}{5}\right)^2$.
2. How much voltage is needed to produce 50 watts of power if the circuit resistance is 1500 ohms?
3. How much power is used by a 2000-ohm resistor if a voltage of 150 volts is applied across it?
4. Evaluate $(50 \times 0.65)^2$.

SECTION 8

Impedance Problems and the Slide Rule

The impedance (Z) of an a-c circuit containing resistance and reactance is calculated by the following formulas:

1. For series circuits: $Z = \sqrt{R^2 + X^2}$
2. For parallel circuits: $Z = \dfrac{Z_1Z_2}{\sqrt{Z_1^2 + Z_2^2}}$

where,
Z_1 is the impedance of one branch,
Z_2 is the impedance of the other branch,
all terms are given in ohms.

It makes no difference whether the reactance X is inductive or capacitive—the same formula applies.

Example—Find the impedance of a series combination of a resistor of 1500 ohms and a coil whose inductive reactance is 1000 ohms.

Solution—From the formula for series circuits, we have

$$\begin{aligned} Z &= \sqrt{R^2 + X_L^2} = \sqrt{1000^2 + 1500^2} \\ &= \sqrt{3{,}250{,}000} = \sqrt{3.25 \times 10^6} \\ &= 1800 \text{ ohms} \end{aligned}$$

There are two ways in which the slide rule can be employed to solve this problem:

1. Find the squares of the resistance and reactance by using the slide rule, add them together, and then find the square root of the sum, on the rule.

2. Although the rule cannot be used to add numbers, a system has been devised by which the addition in the problem can also be done in this type of calculation:
   a. Place the C-scale index over 1000 on the D scale.
   b. Place hairline over 1500 on the D scale.
   c. Now look under the hairline at the number appearing on the B scale.
   d. Mentally add 1 *to the first digit of this number*; in this case, it would be 225, and adding 100 to it (the same as adding 1 to the first digit), gives 325.
   e. Place hairline over this number on the B scale.
   f. Read digits of answer on the D scale under the hairline.

In a series-impedance problem the impedance is always less than the sum of the resistance and reactance, but greater than the smaller of these two values. In a parallel-impedance problem the impedance is always less than the smaller value of resistance and reactance in the combination.

Example 2—What would the impedance in the preceding problem be if the resistor and coil were in parallel?

Solution—In this case, $Z_1 = R$ and $Z_2 = X_L$, so that

$$Z = \frac{RX_L}{\sqrt{R^2 + X_L^2}} = \frac{1000 \times 1500}{\sqrt{1800}}$$

$$= \frac{15 \times 10^5}{1.8 \times 10^3} = 8.34 \times 10^2$$

$$= 834 \text{ ohms}$$

In working this problem on the slide rule it is easier if the multiplication is done first, the product jotted down, and then the denominator is evaluated in the manner described earlier in this section. The decimal point is placed by using powers of ten as shown or by approximation, where the answer will be a number smaller than the smallest value in the problem, but not less than half of this value.

Exercises

1. Find the impedance in the circuit of Fig. 7-4.
2. Find the impedance in the circuit of Fig. 7-5.
3. If a capacitor has a reactance of 150 ohms and is shunted

by a resistor of 90 ohms, what is the impedance of the combination?

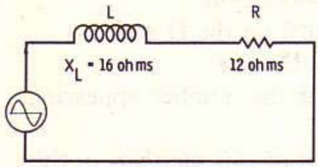


Fig. 7-4. A series *RL* circuit.

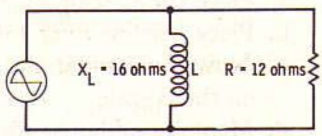


Fig. 7-5. A parallel *RL* circuit.

SECTION 9

Resonant-Frequency Problems

One of the most useful applications of the slide rule is in finding the frequency that causes the inductive and capacitive reactance in a circuit to be equal, thus achieving a state of resonance. The formula is

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

where,

f_r is the resonant frequency in Hz,
2π is 6.28,
L is the inductance in henrys,
C is the capacitance in farads.

The fraction $1/2\pi$ can be expressed as 1/6.28 or its reciprocal, 0.159, for more convenient calculation. The formula then becomes

$$f_r = \frac{0.159}{\sqrt{LC}}$$

Example—Find the resonant frequency for the combination of an 8-microhenry coil and a 10-picofarad capacitor.

Conventional Solution

(a) $f_r = \dfrac{0.159}{\sqrt{LC}}$

(b) $f_r = \dfrac{0.159}{\sqrt{8 \times 10^{-6} \times 10 \times 10^{-12}}}$

(c) $f_r = \dfrac{0.159}{\sqrt{80 \times 10^{-18}}} = \dfrac{0.159}{8.9 \times 10^{-9}}$

(d) $f_r = \frac{15.9 \times 10^{-2}}{8.9 \times 10^{-9}} = 1.78 \times 10^7$

(e) $f_r = 17.8$ MHz

Slide-Rule Solution

Multiply 8 by 10, obtaining 80. Then:

(a) Place hairline over 159 on D scale.
(b) Slide 8 of B scale under hairline (using right side of scale).
(c) Read digits 178 under C index on the D scale.

The decimal point is placed as shown in the conventional-solution method.

SECTION 10

How to Find L *or* C *When Resonance Values are Known*

When the resonant frequency and the value of either L or C are known, then the other value can be calculated from these formulas:

$$C = \frac{1}{(2\pi)^2 f_r^2 L} \quad \text{or} \quad C = \frac{0.0254}{f_r^2 L}$$

$$L = \frac{1}{(2\pi)^2 f_r^2 C} \quad \text{or} \quad L = \frac{0.0254}{f_r^2 C}$$

Example—Find the value of capacitance needed to resonate with an 8-microhenry coil at 17.6 MHz.

Conventional Solution

(a) $C = \frac{0.0254}{f_r^2 L} = \frac{0.0254}{(17.6 \times 10^6)^2 \times 8 \times 10^{-6}}$

(b) $C = \frac{0.0254}{310 \times 10^{12} \times 8 \times 10^{-6}}$

(c) $C = \frac{2.54 \times 10^{-2}}{2480 \times 10^6} = \frac{2.54 \times 10^{-2}}{2.48 \times 10^9}$

(d) $C = 1.01 \times 10^{-11} = 10.1 \times 10^{-12}$
$= 10.1$ picofarads

Slide-Rule Solution

(a) Place hairline over 254 on A scale.
(b) Slide 176 of C scale under hairline.

(c) Place hairline over right B index.
(d) Slide 8 of B scale under hairline.
(e) Over the left B index, read 101 on the A scale.

Place the decimal point as shown in the conventional solution.

Exercise

1. Find the value of inductance needed to resonate with a 25-picofarad capacitor at 5 MHz.

Examination on Lesson 7

Evaluate the following expressions.

1. $\sqrt{6}$
2. 37.5^2
3. $\sqrt{\pi}$
4. $\sqrt{10}$
5. 0.51^2
6. $\sqrt{5000}$
7. 8.05^2
8. 1.415^2
9. $\sqrt{0.0003}$
10. $\sqrt{0.003}$

Solve the following problems.

11. What is the impedance of a circuit containing a 14-microfarad capacitor in series with a resistance of 50 ohms?
12. A 0.2-henry inductor is in series with a 100-ohm resistor. What is the total impedance of the pair?
13. Suppose that the inductor and resistor of Problem 12 are in parallel. What is the total impedance?
14. What is the resonant frequency of a circuit consisting of a 0.32-henry inductor and a 22-microfarad capacitor in series?
15. What is the resonant frequency of a circuit consisting of a 0.22-henry inductor and a 32-microfarad capacitor in parallel?
16. What value of inductance is required in order to resonate with a 44-microfarad capacitor at 60 Hz?
17. What value of series inductance will resonate with a 1-picofarad capacitor at 1 MHz?

8

Ratios, Proportions, And Solving for Unknown Terms

SECTION 1

Ratios

Numbers or expressions of quantity sometimes have no particular meaning unless they are compared to another number or quantity. If a student receives a grade of 80 on a test, the number 80 would have little meaning unless it were compared to some standard, such as the average or norm for the class. Suppose that the average for the class was 60; a grade of 80 would indicate a greater proficiency than the average. The comparison between the given student's grade and the average could be expressed as a ratio:

$$\frac{80}{60}$$

This ratio shows that the student would be 1.3 times as proficient as a student who made the average grade of 60.

The relation between the two numbers involved is shown as a *ratio*. Ratios are fractions and can be expressed as the quotient derived from dividing the numerator by the denominator, or the dividend by the divisor.

In the preceding example the number 1.3 was obtained by dividing 80 by 60, which showed that the grade received by the student was 1.3 times higher than the norm for the class.

The two terms for a ratio are called the *antecedent* and *consequent*. The antecedent, or first term, corresponds to the numerator of a fraction, and the consequent, or second term, corresponds to the denominator of a fraction. In the foregoing example, 80 is the first term, or antecedent, and 60 is the second term, or consequent, of the ratio.

Reducing the Terms of a Ratio

Being a fraction, a ratio can be reduced if there is a common divisor. In the example of 80/60 a common divisor is 20. When 20 is divided into each term it reduces the terms to 4/3, but does not alter the ratio. The ratio can also be written in the following ways: 4:3, 8:6, 80:60, 40:30, etc.

Exercises

Without referring to Section 1 fill in the blank spaces with the appropriate word or words.

1. The number 98 has little meaning unless it is __________ to another number or quantity.
2. When two quantities not necessarily of the same numerical value are compared they are expressed as a __________.
3. In a ratio the first term is called the __________ and corresponds to the __________ of a __________.
4. If the norm for the temperature at this time of year was 60°F, and the thermometer showed 90°F, the ratio could be expressed as _____ to _____.

SECTION 2

Proportions

A *proportion* is an equation involving ratios. When two ratios are compared and are found equal to each other, a proportion is formed. Proportions can be expressed in different ways; here are three of them:

Example—Using the same ratio, 80/60, we can state that:

1. 80:60::4:3, or 80 is to 60 as 4 is to 3.
2. 80/60 = 4/3, or 80 to 60 equals 4 to 3.
3. $\frac{80}{60}=\frac{40}{30}$, or 80 divided by 60 equals 40 divided by 30.

Using the Slide Rule to Find an Unknown Quantity in a Proportion

A slide rule is useful in finding the unknown quantity in a proportion problem. The C and D scales can be used.

Example—Solve for x:

$$\frac{80}{60}=\frac{40}{x}$$

1. Place 80 of C scale over 60 of D scale.
2. Place hairline over 40 on the C scale.
3. Under the hairline on the D scale read the digit 3, or 30.
4. Thus 80/60 = 40/30.

In fact, with the slide rule set as given above, every other ratio equal to 80/60 can be found by simply noting that each number on the C scale is now over a number on the D scale, and this forms a ratio corresponding to 80/60. Also note that the fraction line that separates the numerator and denominator in a fraction can be represented by the slide line separating the C and D scales of the slide rule.

The order in which a proportion problem is stated does not complicate the procedure. For example, let us solve for x in

$$\frac{x}{4}=\frac{6}{3}$$

1. Place 6 of C scale over 3 on the D scale.
2. Place hairline over 4 on the D scale.
3. Under hairline on C scale read digit 8.
4. Therefore $x = 8$.

Exercises

Solve for x.

1. $6/8 = 3/x$
2. $3/4 = x/8$
3. $1/2 = x/6$
4. $x/2 = 2/4$
5. $16/x = 40/5$
6. $2/x = 4/6$
7. $20/35 = x/70$
8. $x/60 = 75/90$
9. $2/18 = x/45$
10. $3/35 = 6/x$

SECTION 3

Ratios Must be Stated in Correct Relation to Each Other

If one ratio in a proportion problem is comparing a greater value to a lesser value, the other ratio must do likewise:

$$\frac{\text{Greater}}{\text{Lesser}} = \frac{\text{Greater}}{\text{Lesser}}$$

In setting up the problem one must determine first whether the unknown value (x) is to be a greater or lesser value than the other term in the ratio.

Example–Consider the equation

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}$$

which states that the turns ratio of a transformer is the same as the voltage ratio. The primary voltage (E_p) divided by the secondary voltage (E_s) equals the number of turns in the primary (N_p) divided by the number of turns in the secondary (N_s).

Suppose that a transformer with 1000 turns in the primary and 2000 turns in the secondary is connected to an a-c source of 50 volts. Let us find the secondary voltage. To do this, the relation of the ratios must be stated:

$$\frac{1000}{2000} = \frac{50}{x}$$

To solve for x:

1. Place the 1 on the C scale over 2 on the D scale.
2. Under 5 on the C scale read the answer 1, or 100.
3. Therefore $x = 100$ volts.

The student must know first that such a turns ratio indicates a step-up transformer and that the secondary voltage will therefore be greater than the primary voltage. Note that the ratio of 1 to 2 is of lesser to greater, so the ratio of 50 to x must also be of lesser to greater, making x a number greater than 50.

Exercises

1. A transformer with 500 turns in the primary and 400 turns in the secondary is connected across an a-c source of 90 volts. What is the secondary voltage?
2. How many turns are required for the secondary of a transformer to obtain 6.5 volts if the primary has 1300 turns and is to be connected across an a-c source of 115 volts?

SECTION 4

Reactance and Frequency

The reactance of a coil, inductive reactance, is directly proportional to the frequency of the current through it. An increase in frequency causes an increase in reactance.

Example—The *inductive reactance* (X_L) of a coil at 50 Hz is 5425 ohms. Calculate its inductive reactance at 60 Hz.

$$\frac{\text{(Lesser)}}{\text{(Greater)}} \quad \frac{50}{60} = \frac{5425}{X_L}$$

1. Slide 5 of C scale over 6 of D scale.
2. Place hairline over 5425 on the C scale.
3. Read digits 6510 under hairline on the D scale.
4. Therefore $X_L = 6510$ ohms.

Note that in this example each ratio consists of like quantities or terms above and below the fraction line. Although the answer would be the same if the terms were interchanged, it is a better policy to state the ratios in this manner. In either case, however, it is necessary to keep the relation of lesser to greater in order.

The reactance of a capacitor, *capacitive reactance* (X_C), is *inversely* proportional to the frequency of the voltage across it. When the frequency increases, the capacitive reactance decreases.

Example—The capacitive reactance of a 5-microfarad capacitor at 50 Hz is 636 ohms. Find its reactance at 60 Hz.

$$\frac{\text{(Greater)}}{\text{(Lesser)}} \quad \frac{60}{50} = \frac{636}{X_C}$$
$$X_C = 530 \text{ ohms}$$

In this problem X_C is to be a lesser value. Note that each ratio consists of like quantities—the first of frequencies, the second of reactances.

Exercises

Solve for the indicated reactances.

1. The inductive reactance of a coil at 60 Hz is 3500 ohms. What is it at 50 Hz?
2. If the capacitive reactance of a capacitor at a frequency of 400 Hz is 8000 ohms, what is it at 600 Hz?

SECTION 5

Placing the Decimal Point in Ratio and Proportion Problems

To place the decimal point in a ratio one merely uses the same method as in division, since a ratio is merely a quotient. In a proportion problem, where it is necessary to place the decimal point correctly in the answer (value of x), there are three methods that are recommended:

The Approximation Method—Consider the following equation:

$$\frac{x}{3.50} = \frac{25.4}{12.7}$$

In this problem note that x must be a number greater than 3.50 because 25.4 is a number greater than 12.7. By changing the numbers slightly to simplify the procedure one could also say that:

$$\frac{24}{12} = 2, \text{ so } \frac{x}{3.50} = 2$$

Therefore x must equal 2×3.50, or 7. The slide rule will show the digit 7 as the value of x which is the correct answer.

The Cross-Products Method—The terms of a proportion should be equal when they are cross multiplied. If $\frac{a}{b} = \frac{c}{d}$, then

$$a \times d = b \times c$$

For example, if $a = 7$, $b = 14$, $c = 21$, and $d = 42$, then

$$\frac{7}{14} = \frac{21}{42} \quad \text{and} \quad 7 \times 42 = 14 \times 21$$

This method also proves the correctness of the calculation.

The Scientific Method (Powers of Ten)—This method is more accurate than the first two methods, and it is preferred by many technicians. In this method, to place the decimal point in proportion problems each term is expressed in scientific notation. The powers of ten are then collected in the numerator on each side of the equation. Thus the exponents of ten must be the same on both sides of the proportion. For example, let us solve for x in

$$\frac{0.150}{600} = \frac{x}{44.1}$$

The slide rule yields the digits 11. To place the decimal point:

1. Change to scientific notation, observing the correct order of lesser to greater:

$$\frac{1.50 \times 10^{-1}}{6.00 \times 10^{2}} = \frac{1.1 \times 10^{?}}{4.41 \times 10^{1}}$$

2. The sum of the exponents in the first ratio which consists of two knowns is -3, so the sum of the exponents in the second ratio must also be -3.
3. To make the total of the exponents in the second ratio also equal to -3 the digits 11 must be expressed as 1.1×10^{-2}. This makes the two exponent totals the same. Thus

$$x = 1.1 \times 10^{-2} \text{ or } x = 0.011$$

Example—Solve for x in

$$\frac{5.64}{80} = \frac{x}{314}$$

Therefore,

$$\frac{5.64 \times 10^0}{8.00 \times 10^1} = \frac{2.215 \times 10^1}{3.14 \times 10^2}$$

To make the sum of the exponents -1 in both ratios, x must equal 2.215×10^1, or 22.15.

Exercises

Solve for x in the following proportions.

1. $\frac{56}{24} = \frac{70}{x}$
2. $\frac{x}{8} = \frac{4}{16}$
3. $\frac{5}{x} = \frac{4}{2}$
4. $\frac{18}{4} = \frac{x}{6}$
5. $\frac{195}{65} = \frac{300}{x}$
6. $\frac{x}{5} = \frac{6}{9}$

Examination on Lesson 8

Solve for x in the following proportions.

1. $\frac{6}{8.5} = \frac{3}{x}$
2. $\frac{7}{13} = \frac{9}{x}$
3. $\frac{12.2}{50} = \frac{x}{4.1}$
4. $\frac{3}{17} = \frac{x}{19}$
5. $\frac{4}{7} = \frac{5}{x}$
6. $\frac{4}{7} = \frac{12}{x}$
7. $\frac{11}{4} = \frac{x}{16.5}$
8. $\frac{37}{93} = \frac{x}{8}$
9. A transformer with a 10:1 turns ratio has an alternating voltage of 120 volts impressed across it. What a-c voltage is induced in the secondary?
10. A step-up transformer has an a-c voltage of 120 volts across the primary coil. If the secondary coil has twice as many turns as the primary, what voltage appears across the secondary?
11. At 60 Hz the inductive reactance of a coil is 300 ohms. What is it at 300 Hz?
12. At 60 Hz the inductive reactance of a coil is 75 ohms. At what frequency is it 100 ohms?

13. The capacitive reactance of a capacitor is 100 ohms at 60 Hz. What is it at 100 Hz?
14. At 60 kHz the capacitive reactance of a capacitor is 52 ohms. At what frequency is it 100 ohms?
15. At 60 Hz an inductor has an inductive reactance of 120 ohms. If the frequency is doubled, does the inductive reactance double?

9

Cubes And Cube Roots

SECTION 1

Definitions

To *cube* a number means to raise it to its third power. A number which is to be cubed bears the exponent 3:

$$4^3 = 4 \times 4 \times 4 = 64$$

To find the *cube root* of a number means to find a number that when used as the only factor three times in multiplication equals that number:

$$\sqrt[3]{64} = 4$$

Note that the radical sign used in square roots is used also for cube roots but in the latter case it has the small 3 nesting in its angle.

While cubes and cube roots do not appear in most formulas of electronics it is useful for the student to know how to use the K scale for cubes and cube roots, which are the main functions of this scale.

The K Scale

Most slide rules have K scales. The K scale covers three times the range of the D and C scales.

Since the other two sections of the K scale are identical to the one shown in Fig. 9-1 it is not necessary to show them. Note that between the primary divisions 1 to 4 there are ten secondary and ten tertiary divisions. This makes each secondary division worth one unit as the second digit of a number and each tertiary division worth five units as the third digit of a number.

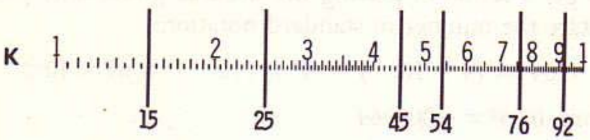


Fig. 9-1. One section of the K scale.

From the primary division marked 4 to the primary division of 10 note that only the secondary divisions are marked, while the tertiary divisions have to be interpolated. From 4 to 10 the secondaries represent one unit as the second digit. When the hairline is placed in the exact center between secondary divisions the number represented is the third digit of a number and is 5.

Exercise

1. Write the numbers indicated by the position of the hairline in Fig. 9-2.

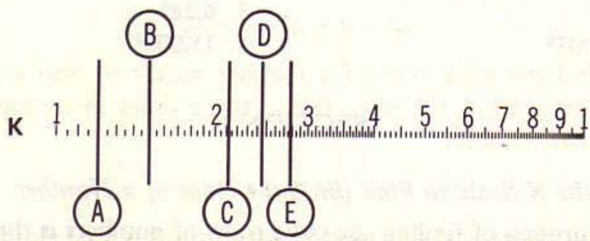


Fig. 9-2. Exercise in hairline reading.

SECTION 2

Using the K Scale to Cube a Number

To cube a number by using the slide rule, simply place the hairline over this number on the D scale and its cube will be found under the hairline on the K scale. For example,

$$4^3 = 64$$

1. Place hairline over 4 on D scale.
2. Read the answer (64) under the hairline on the K scale.

   Here is another example:

$$0.04^3 = ?$$

In this problem the digits of preceding problem are used, but here we get a lesson in placing the decimal point. The procedure is to restate the number in standard notation:

$$0.04^3 = (4 \times 10^{-2})^3 = 4^3 \times (10^{-2})^3 = 64 \times 10^{-6}$$

Therefore, $0.04^3 = 0.000064$.

Example–Find the cube of 3.27:

$$(3.27)^3 = ?$$

The procedure is as follows:

1. Place hairline over 327 on the D scale.
2. Read digits 35 under the hairline on the K scale.

To place the decimal point:

$$(3.27 \times 10^0)^3 = 3.27^3 \times (10^0)^3 = 35 \times 10^0 = 35$$

Exercises

Find the following cubes.

1. 6^3
2. 0.005^3
3. 6.28^3
4. $15{,}000^3$

SECTION 3

Using the K Scale to Find the Cube Root of a Number

The process of finding the cube roots of numbers is the reverse of cubing. The cube roots of numbers on the K scale appear in line with the hairline position on the D scale. There is one detail that must be taken care of, however, and it is to determine which of the three K scales to use.

To do this, one simply states the cube-root problem on paper, grouping the number into groups of three on each side of the decimal point. Then one determines the number whose cube comes closest to equaling but not exceeding the nonzero digit or digits in the first group.

Example—Find the cube root of 35. The procedure is as follows:

1. Separate 35 into groups of three: 035.
2. Determine what number when cubed will most nearly equal 35; that number is 3. (The slide rule can be used for this part of the procedure.)
3. Place hairline over 3 on the D scale and note that it is in the middle K-scale area.
4. Placing hairline over 35 in the middle of the K scale yields 3.27.

Placing the Decimal Point in the Cube Root

Numbers Greater Than 1—If the cube root of a number is a number equal to or greater than 2, the root will contain as many digits to the left of the decimal point as there are groups of three digits to the left of the decimal point in the original number.

Example—Find the cube root of 35.

The process of extracting the cube root of a number is similar to that of extracting the square root except that the number is separated into groups of three:

035.000

There is only one group of three that contains a nonzero digit appearing left of the decimal point, so the cube root will have only one nonzero digit to the left of the decimal point; thus the correct answer is 3.27.

Any digits appearing on the right of the decimal point in such a case would appear as a decimal fraction.

Numbers Less Than 1—Suppose that we want to find the cube root of a number less than 1. This number, expressed as a decimal fraction, can be divided into groups of three digits to the right of the decimal point. Then the cube root will have as many zeros between the decimal point and the first nonzero digit to the right of the decimal point as there are groups of there zeros in the original number.

Example—Find the cube root of 0.000216.

1. Separate the number into groups of three: .000′216.
2. Placing the hairline over the right K-scale section yields the digit 6 on the D scale.
3. Since there is only one complete group of three zeros to the right of the decimal point, there can be only one zero in the cube root; so the correct answer is 0.06.

Table 9-1 can be used as a reference to assist you in determining which K-scale section to use in cube-root problems.

TABLE 9-1. *Sections of K Scale*

If First Group Of Three Digits Is Between	*The K Scale To Use*	*Examples*
1 and 10	Left	5000→005′000
10 and 100	Middle	38→038.000
100 and 1000	Right	275→275.000

Exercises

1. $\sqrt[3]{216}$
2. $\sqrt[3]{0.000000125}$
3. $\sqrt[3]{247}$
4. $\sqrt[3]{25}$
5. $\sqrt[3]{1500}$
6. $\sqrt[3]{0.0428}$

Examination on Lesson 9

Perform the indicated operations.

1. 1.28^3
2. π^3
3. $\sqrt[3]{0.01}$
4. $\sqrt[3]{\pi}$
5. 0.1^3
6. 0.2^3
7. $\sqrt[3]{1000}$
8. $\sqrt[3]{71}$
9. $\sqrt[3]{3.1^3}$
10. 75^3

10

The Folded Scales

SECTION 1

CF and DF Scales

Not all slide rules have the CF and DF scales. While this is no serious handicap, important advantages come with these scales, such as eliminating the necessity of resetting or changing indexes and simplifying problems involving multiplication or division by 3.1416 (π).

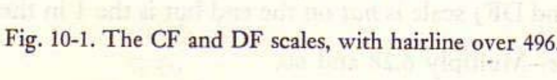


Fig. 10-1. The CF and DF scales, with hairline over 496.

The CF and DF scales are called the *folded* scales because they actually are the C and D scales folded or divided at π (3.1416). In Fig. 10-1 the CF and DF scales are shown with the hairline placed over the digits 496.

The CF and DF scales each consist of two sections, with the left section starting at 3.1416 (π) and ending with 1 (or 10) and the right section starting with this 1 and ending at π.

When it is desired to multiply a number by π, one needs only to place the hairline over this number on the D scale; the product will be under the hairline on the DF scale. For example, let us multiply 157.9 by 3.1416:

1. Place hairline over 1579 on the D scale.
2. Read 495 under hairline on the DF scale.
3. Placing decimal point correctly gives the correct answer as 495.

Exercises

Perform the following multiplications.

1. $9 \times \pi$
2. $10 \times \pi$
3. $2 \times \pi$
4. $15 \times \pi$
5. $25 \times \pi$
6. $3 \times \pi$

SECTION 2

Multiples of π

In electronics it is often necessary to multiply by 2π (6.28). This can be greatly simplified by using the DF scale with the D scale.

Example—Find 6.28×60.

1. Mentally multiply 2 and 60, obtaining 120.
2. Place hairline over 120 on the D scale.
3. Read product 377 under hairline on the DF scale.

The same problem can be solved by using only the CF and DF scales. There is one important point to remember: The index on the CF (and DF) scale is *not* on the end but is the 1 in the middle.

Example—Multiply 6.28 and 60.

1. Place hairline over 6.28 on the DF scale.
2. Slide CF index (1) under hairline.
3. Place hairline over 6 on the CF scale.
4. Read answer (377) under hairline on the DF scale.

In this problem note that the index of the CF scale was used and it was the digit 1 that appears at the center of that scale.

An easy way to locate 6.28 on the DF scale is to simply place the hairline over 2 on the D scale. Since every number on the D scale is automatically multiplied by π when the hairline is placed over it and the answer read on the DF scale, it is sometimes easier to locate 2 on the D scale and then glance along the hairline to the DF scale, where the number 6.28 will be placed accurately.

Inductive-Reactance Problems

The folded scales are useful when one is doing repeated multiplication such as is encountered in inductive-reactance problems. By using these scales it is not necessary to reset the slide when placing numbers on the C or D scales in repeated operations.

Example—Find $6.28 \times 60 \times 1500$.

1. Place hairline over 6.28 on the DF scale.
2. Slide index of the CF scale under the hairline.
3. Place hairline over 6 on the CF scale.
4. Slide CF index under hairline.
5. Place hairline on 15 on the CF scale.
6. Read answer (565) under hairline on the DF scale.

When the decimal point is placed, the answer is found to be 56.5×10^4.

Exercises

1. Find the inductive reactance at 50 Hz of a coil whose inductance is 5 henrys.
2. Find the inductive reactance of a coil with an inductance of 160 microhenrys at 2 MHz.

SECTION 3

How to Divide by π

Since division is the inverse operation of multiplication, one merely places the hairline on the DF scale over the number to be divided by π. Then the answer can be read on the D scale.

Example—Divide 6.28 by π.

1. Place hairline over 6.28 on the DF scale.
2. Read answer (2) under the hairline on the D scale.

Combination Multiplication and Division Problems Involving π and the DF Scales

The CF and DF scales are of the greatest value in problems involving π. Combination problems where π is a factor can also be handled easily by using the DF scale with the C scale.

Example—Evaluate $\frac{150 \times 2\pi}{62}$.

1. Place left index of C scale over 15 on the D scale.
2. Place hairline over 2 on the C scale.
3. Slide 62 of C scale under the hairline.
4. Place hairline over right C index.
5. Read answer (15.15) under hairline on the DF scale.

Placing the Decimal Point in Problems Involving π and the DF Scale

In a problem where a factor is multiplied by π the log method of placing the decimal point can be used. There is one important fact to remember in this connection: *If the product appears to the right of the DF index, add a correction factor of 1.*

Example—Multiply 4.5 by 3.14.

1. Place hairline over 45 on the D scale.
2. Read product (1413) under hairline on the DF scale.

To place the decimal point in this problem write down the characteristics of each number. Since the product appears to the right of the DF index a correction factor is added:

1. The characteristic of 4.5 is zero.
2. The characteristic of 3.14 is zero plus 1 (correction factor).
3. The product's characteristic is therefore 1, making the correct answer 14.13.

In a division problem the characteristic method for placing the decimal point can be employed, provided one remembers to add a correction factor of 1 whenever the number being divided by π appears to the right of the DF index.

Example—Divide 2.1 by π.

1. Place hairline over 2.1 on the DF scale.
2. Read digits 669 under hairline on D scale.

In this problem the quotient appeared under the hairline on the D scale, and since it was to the right of the DF index, a correction factor of 1 was added to the characteristic of π. Thus $0 - (0 + 1) = -1$ is the characteristic of the answer.

If it is desired to divide π by another number, it is convenient to use the C and D scales and treat the problem as an ordinary division.

Exercises

Perform the indicated operations.

1. $\dfrac{9}{\pi}$
2. $\dfrac{12}{\pi}$
3. $\dfrac{25 \times 2\pi}{50}$
4. $\dfrac{2\pi}{3.14}$

SECTION 4

Radian Measure

One *radian* is the angle subtended at the center of a circle by an arc equal in length to the radius (Fig. 10-2). This angle is approximately 57.3° in angular measure. Since the circumference of a circle has a length of $C = 2\pi r$, where r is the length of the radius,

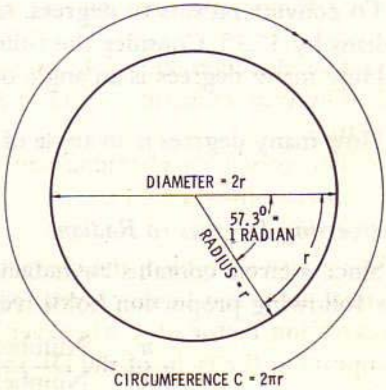


Fig. 10-2. Parts of a circle.

it follows that the circumference subtends 2π radians; i.e., $2\pi \times 57.3° = 360°$.

In electronics, one *cycle* can be represented by a complete rotation of a radius about the center of a circle. In angular measure this is 360° or 2π radians. The student should become familiar with the following conversions between degrees and radians:

$$0° = 0 \text{ radians}$$

$$90° = \frac{\pi}{2} \text{ radians}$$

$$180° = \pi \text{ radians}$$

$$270° = \frac{3\pi}{2} \text{ radians}$$

$$360° = 2\pi \text{ radians}$$

Exercises

1. The radian is an unit of angular measurement indicating ______ degrees.
2. One cycle is composed of ______ radians.
3. The circumference of a circle subtends ______ radians.
4. If the radius of a circle is 2.5 inches, what is the diameter?
5. Find the circumference of a circle whose diameter is 5 inches.

SECTION 5

Converting Radians to Degrees

To convert radians to degrees, simply multiply the number of radians by 57.3°. Consider the following example:

How many degrees is an angle of 5 radians?

Answer: $5 \times 57.3° = 286.5°$

How many degrees is an angle of 6 radians?

Answer: $6 \times 57.3° = 343.8°$

Converting Degrees to Radians

Since a circle contains 2π radians, then π radians are 180° and the following proportion holds true:

$$\frac{\pi}{180°} = \frac{\text{Number of Radians}}{\text{Number of Degrees}}$$

Suppose that a 45° angle must be expressed in radians. From the proportion, we have

$$\frac{\pi}{180^\circ} = \frac{\text{Number of Radians}}{45^\circ}$$

Using the CF and DF scales, solve for the number of radians:

1. Slide 180 of CF scale under π on the right end of the DF scale.
2. Place hairline over 45 on CF scale.
3. Read digits of answer, 786, under hairline on DF scale.

To place the decimal point, use the powers of ten:

$$\frac{3.14 \times 10^0}{1.80 \times 10^2} = \frac{7.86 \times 10^{-1}}{4.5 \times 10^1}$$

When the powers of ten are collected in the numerators, the exponents must be equal. Therefore the correct answer is $7.86 \times 10^{-1} = 0.786$ radian.

Another way to convert degrees to radians is simply to divide the number of degrees by 57.3°:

$$\frac{45^\circ}{57.3^\circ} = 0.786 \text{ radian}$$

Some slide rules have a small *r* placed at 57.3 on the D scale, which is convenient when one is using the C and D scales for a problem involving radians.

Exercises

1. Find the circumference of a circle whose diameter is 2 inches.
2. How many degrees has an angle of 2 radians?
3. $3\pi/2$ radians = _______ degrees.
4. 60° = _______ radians.
5. 117° = _______ radians.
6. 180° = _______ radians.
7. How many radians are in one complete cycle of alternating current?

Examination on Lesson 10

Perform the indicated operations.

1. $8.95 \times \pi$
2. $\pi \times \pi$
3. $1.73 \times \pi$
4. $\frac{8.95}{\pi}$
5. $\frac{70}{\pi}$
6. $\frac{0.615}{\pi}$
7. What is the inductive reactance of a 0.2-henry inductor at 60 Hz?

Convert the following to radians.

8. 10°
9. 41°
10. 127°

Convert the following to degrees.

11. 1.78 radians
12. 1.21 radians
13. 0.68 radian

11

Trigonometric Functions

SECTION 1

Review of Trigonometry

A *right triangle* is defined as a triangle which has one 90° angle. In Fig. 11-1 the right triangle has sides A, B, C, opposite angles a, b, c, respectively. The *trigonometric functions* may be defined with reference to a right triangle.

For any angle $x \neq 90°$ of a right triangle the *sine function* is defined as

$$\sin x = \frac{\text{Side Opposite Angle } x}{\text{Hypotenuse}}$$

For example, in Fig. 11-1,

$$\sin a = \frac{A}{C}, \quad \text{and} \quad \sin b = \frac{B}{C}$$

The sine of 90° is defined as $\sin 90° = 1$.

For any angle $x \neq 90°$ of a right triangle the *cosine function* is defined as

$$\cos x = \frac{\text{Side Adjacent to Angle } x}{\text{Hypotenuse}}$$

In Fig. 11-1,

$$\cos a = \frac{B}{C}, \quad \text{and} \quad \cos b = \frac{A}{C}$$

The cosine of 90° is defined as $\cos 90° = 0$.

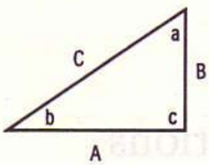


Fig. 11-1. A right triangle.

For any angle $x \neq 90°$ of a right triangle the *tangent function* is defined as

$$\tan x = \frac{\sin x}{\cos x} = \frac{\text{Side Opposite Angle } x}{\text{Side Adjacent to Angle } x}$$

The tangent of 90° is defined as $\tan 90° = \infty$.

There are three other trigonometric functions which are sometimes used. They are:

The secant function: $\sec x = \dfrac{1}{\cos x}$

The cosecant function: $\csc x = \dfrac{1}{\sin x}$

The cotangent function: $\cot x = \dfrac{1}{\tan x}$

These complete the common trigonometric functions.

For a right triangle with sides A, B, and C, where C is the hypotenuse, the following relation holds:

$$A^2 + B^2 = C^2$$

This is the famous *Pythagorean Theorem*, named after the Greek mathematician-philosopher Pythagoras (*circa* 500 B.C.).

Exercises

1. One of the angles of a right triangle is a ______ angle.
2. Prove that $\sin^2 x + \cos^2 x = 1$. (Hint: Draw a right triangle and label it.)
3. The sine of 90° is equal to ______.
4. Show that $\cos(90° - x) = \sin x$. (Hint: Use the fact that the three angles of any triangle total 180°.)
5. The right-triangle equation $A^2 + B^2 = C^2$ is called the ______ ______.
6. What is the tangent of 90°?
7. The longest side of a right triangle is called the ______.
8. Show that $\cos x = \sin(90° - x)$.
9. If the two legs of a right triangle are 3 and 4 units in length, what is the length of the hypotenuse?
10. In Exercise 9, what is the sine of the angle between the hypotenuse and the side of length 3?

SECTION 2

Using the S Scale to Find Sines and Cosines

The sines and cosines of angles can be found in tables of trigonometric functions, but the use of the S and C scales of the slide rule offers a more convenient way when these functions are involved in electronic calculations.

Examine the S scale on your slide rule. Note that there are two sets of numbers, one set increasing in value from left to right, the other increasing from right to left. The numbers are placed on each side of the major division lines. In some cases the numbers to the left of these lines are colored red, while those to the right are colored black.

To find the sine of an angle the numbers to the *right* of the major or primary division lines are used.

To find the cosine of an angle, use the numbers to the *left* of the primary division lines.

While there are variations by different manufacturers of the arrangement of the S scale, the most practical and apparently most popular is that where the values from left to right run from 5.74° to 90°, with the values from right to left running from 0° to 84.25°. In Fig. 11-2 the S scale is shown with the hairline placed over 10°, with the sine value appearing under the hairline on the C scale.

Finding the Sine of an Angle Between 5.74° and 90°

The procedure for finding sine values of angles between 5.74° and 90° is as follows:

1. Place hairline over value of angle on the S scale.
2. Read its sine under the hairline on the C scale. (The value will be between 0.1 and 1.)

Fig. 11-2. S scale with hairline placed over 10°.

Example—Find sin 10°. The procedure is therefore:

1. Place hairline over 10 on the S scale.
2. Read sine value (0.1736) under hairline on the C scale.

To Find the Sine of an Angle Between 0° and 5.74°

For an angle of less than 5.74° the simplest procedure is to ignore the S scale and multiply the angle by 0.01745, using the C and D scales. The sine value will be less than 0.1.

Example—Find sin 3.5°.

1. Place left index of C scale over 35 on the D scale.
2. Move hairline over 1745 on C scale.
3. Read digits 61 under hairline on D scale.

Since the value of the sine is less than 0.1, and $3 \times 0.02 = 0.06$, the correct answer must be 0.061.

To Find the Sine of an Angle Between 0.57° and 5.74°

Some slide rules have an ST scale, which is used for finding sines and tangents of angles between 0.57° and 5.74°. Angles in this range have sine values between 0.01 and 0.1. The procedure is as follows:

1. Align left indexes of D and ST scales.
2. Place hairline over angle value on ST scale.
3. Read answer on D scale. Answer will be between 0.01 and 0.1.

To Find the Cosines of Angles Between 0° and 84.26°

To find the cosine of an angle the numbers to the left of the division lines are used. Actually we are finding the sine of $90° - x$, where x is the angle whose cosine we want.

Example—Find cos 80°.

1. Place hairline over 80 on the S scale (the 80 to the left of 10).
2. Read cosine under the hairline on the C scale (0.1736).

Note that the cosine of 80° is the same as the sine of 10°. The hairline is actually set over the same division mark as it was when finding the sine of 10°, but in this case the numbers to the right of the mark are ignored.

To Find the Cosines of Angles Between 84.26° and 90°

This may not be found directly on all slide rules. Since the cosine of any angle equals the sine of its complementary angle, it is true that:

$$\cos x = \sin (90° - x)$$

Example—Consider $\cos 85° = \sin(90° - 85°) = 5°$. Since 5° lies between 0° and 5.74°, it must be multiplied by 0.01745; thus $\sin 5° = 0.08725$.

Exercises

Find the following sines and cosines by using the slide rule.

1. sin 21°	4. cos 85°	7. sin 2°
2. sin 75°	5. cos 15°	8. cos 85.5°
3. sin 15°	6. sin 4.5°	9. sin 90°

SECTION 3

The Arc Sine and Arc Cosine Functions

The terms arc sine x, arc sin x, and $\sin^{-1}x$ mean, "the angle whose sine is equal to x."

Example—Consider the equation arc sin $0.7071 = 45°$. This means that the angle whose sine is 0.7071 is a 45° angle.

The terms arc cosine x, arc cos x, $\cos^{-1}x$ mean, "the angle whose cosine is equal to x."

Example—Consider the equation arc cos $0.8660 = 30°$. This means that the angle whose cosine is equal to 0.8660 is a 30° angle.

Using the Slide Rule to Find Arc Sines and Arc Cosines

When the sine value is between 0.1 and 1.0, the procedure is as follows:

1. Place hairline over sine value on the C scale.
2. Under the hairline read the angle on the S scale (using the right-hand numbers).

Example—Find arc sin 0.545.

1. Place hairline over 545 on the C scale.
2. Under hairline read 33° on the S scale.

When the sine value is less than 0.1, divide it by 0.01745, using the C and D scales. This will give the value of the angle in degrees.

Example—Find arc sin 0.061.

1. Obtain digits from slide rule by division:
$$\frac{0.061}{0.01745} = \text{digits } 349$$
2. State in scientific notation:
$$\frac{6.1 \times 10^{-2}}{1.745 \times 10^{-2}} = \frac{6.1}{1.745} = 3.49$$
3. Therefore arc sin $0.061 = 3.49°$.

If the slide rule has a ST scale, then sine values between 0.01 and 0.1 may be read:

1. Align ST-scale left index with D-scale left index.
2. Place hairline over sine value on D scale.
3. Read angle in degrees on ST scale.

For example, arc sin $0.02 = 1.15°$.

When the cosine value is between 0.1 and 1, the procedure is as follows:

1. Place hairline over the cosine value on the C scale.
2. Under the hairline, read the angle on the S scale, using the left-hand numbers.

Example—Find arc cos 0.225.

1. Place hairline over 225 on the C scale.
2. Under hairline read digits 77 on the S scale.
3. Thus arc cos $0.225 = 77°$.

When the cosine value is less than 0.1, divide the cosine value by 0.01745 and subtract the result from 90°, obtaining the angle in degrees.

Example—Find arc cos 0.061.

1. Divide 0.061 by 0.1745 with slide rule, obtaining the digits 349.
2. To place the decimal point, express the division in scientific notation:

$$\frac{6.1 \times 10^{-2}}{1.745 \times 10^{-2}} = 3.49$$

3. Subtract 3.49° from 90°: $90° - 3.49° = 86.51°$.
4. Thus arc cos $0.061 = 86.51°$.

Exercises

Find the angles indicated.

1. arc sin 0.45
2. arc sin 0.1736
3. arc sin 0.5150
4. arc sin 0.6820
5. arc cos 0.891
6. arc cos 0.9945
7. arc cos 0.7071
8. arc cos 0.082
9. arc cos 0.043
10. arc sin 0.043

SECTION 4

The Tangent Function

For angles between 5.71° and 45°, the procedure is as follows:

1. Place hairline over the angle value on the T scale.
2. Under the hairline on the C scale, read the tangent value. It will be between 0.1 and 1.0.

Example–Find tan 15°.

1. Place hairline over 15 (right-hand number) on T scale.
2. Read the tangent value under the hairline on the C scale (0.268). So tan 15° = 0.268.

Note that on slide rules with one T scale the T scale generally has two sets of numbers: those which read from left to right, and those which read from right to left. From left to right the T-scale values start at 5.75° and end at 45°, and from right to left the numbers start at 45° and end at 84.25°. Some rules have the left-to-right numbers colored black and right-to-left colored red.

For T scales having only the values from 5.75° to 45°, the relation $\tan x = 1/\tan (90° - x)$ may be used for angles greater than 45° and less than 84.25°. Thus the reciprocal of the tangent of the complementary angle must be found. For example, the tangent of 60° is equal to the reciprocal of the tangent of 30°. Since tan 30° may be read on the C scale, its reciprocal may be read on the CI scale; the reciprocal is 1.732.

For angles between 45° and 84.25°, the procedure for using the T and CI scales is as follows:

1. Place hairline over the angle value on the T scale, using the numbers reading from right to left.
2. Read tangent value on CI scale under the hairline. The value will be between 1 and 10.

Example–Find tan 60°. This is done by the following procedure:

1. Place hairline over 60 on the T scale, using the left-hand number.
2. Read tangent value under hairline on the CI scale. Since the value will be between 1 and 10, the correct answer is 1.732.

For angles between 45° and 84.25° the D scale can be used with the T scale, which in many cases makes it unnecessary to turn the rule over to read the tangent value, as sometimes is the case when one is using the CI scale. The procedure is as follows:

1. Place hairline over right index of D scale.
2. Slide the angle value on the T scale under the hairline.
3. Read tangent value under left index of C scale on D scale.

For angles between 84.25° and 90° the following formula is useful, since these angles cannot be found with the slide rule:

$$\tan x = \frac{57.3^\circ}{90^\circ - x}$$

For example,

$$\tan 85^\circ = \frac{57.3^\circ}{5^\circ} = 11.43$$

The rule can be used for the division.

For an angle between 0° and 5.74° the tangent value cannot be found directly with some slide rules, but it can be found by multiplying the angle in degrees by 0.01745. (For small angles the sine and tangent functions are approximately equal.) Again the slide rule is useful for the multiplication.

The tangent values of angles from 0.57° to 5.74° can be found immediately on slide rules having an ST scale. This scale gives both the sine values and tangent values for small angles. Angles between 0.57° and 5.74° have tangent values between 0.01 and 0.1. The procedure is as follows:

1. Align left indexes of the C and D scales.
2. Place hairline over angle on ST scale.
3. Read answer on D scale.

For example, $\tan 0.9^\circ = 0.057$.

Finding Cotangent Values

The cotangent function is the reciprocal of the tangent function and is equal to the tangent of the complement of an angle. Cotangent is abbreviated "cot." Here are two examples:

$$\tan 60^\circ = \cot 30^\circ \quad \text{and} \quad \tan 10^\circ = \cot 80^\circ$$

To find the cotangent value with the slide rule one may simply find the tangent of the angle and read its reciprocal on the CI or DI scales.

Example—Find the cotangent of 35°.

A. *If rule has one T scale:*

1. Place a hairline over 35 on T scale.
2. Read digits 143 under hairline on DI or CI scale.

B. *If rule has T1 and T2 scales:*

1. Place hairline over 35 on T1 scale.
2. Read 143 under hairline on DI scale.

Cot 35° is equal to tan 55°, and since tan 55° is of an angle greater than 45° the tangent value will be greater than 1; therefore the correct answer is cot 35° = 1.43.

On slide rules with T1 and T2 scales the answer may be read from the cotangent scale, which is the same scale as the tangent scale except that the numbers are read from right to left and appear to the left of the scale marks. Thus, cot 35° appears on the T2 scale at the same mark as tan 55°. Placing the hairline over this value gives a reading of 1.43 on the C scale.

Exercises

Find the following values of the tangent function, using the slide rule.

1. tan 75°
2. tan 80.4°
3. tan 63.5°
4. tan 89°
5. tan 20°
6. tan 10°

SECTION 5

Sines, Cosines, and Tangents of Angles Greater Than 90°

The values for the trigonometric functions of angles greater than 90° cannot be found directly on the slide rules, nor do they generally appear in trig tables. What must be done is to find an equivalent argument value and then the function value of this argument. The following formulas can be helpful:

1. *For angles from 90° to 180°:*
   (a) $\sin x = \sin (180° - x)$.
   Ex. $\sin 150° = \sin (180° - 150°) = \sin 30°$
   (b) $\cos x = -\cos (180° - x)$.
   Ex. $\cos 150° = -\cos (180° - 150°) = -\cos 30°$
   (c) $\tan x = -\tan (180° - x)$.
   Ex. $\tan 120° = -\tan (180° - 120°) = -\tan 60°$
2. *For angles from 180° to 270°:*
   (a) $\sin x = -\sin (x - 180°)$.
   Ex. $\sin 210° = -\sin (210° - 180°) = -\sin 30°$

(b) $\cos x = -\cos (x - 180°)$.

Ex. $\cos 180° = -\cos (180° - 180°) = -\cos 0°$

(c) $\tan x = \tan (x - 180°)$.

Ex. $\tan 190° = \tan (190° - 180°) = \tan 10°$

3. *For angles from 270° to 360°:*

(a) $\sin x = -\sin (360° - x)$.

Ex. $\sin 280° = -\sin (360° - 280°) = -\sin 80°$

(b) $\cos x = \cos (360° - x)$.

Ex. $\cos 280° = \cos (360° - 280°) = \cos 80°$

(c) $\tan x = -\tan (360° - x)$.

Ex. $\tan 280° = -\tan (360° - 280°) = -\tan 80°$

Exercises

Find the following values by using the slide rule.

1. sin 100°
2. cos 100°
3. tan 100°
4. cos 185°
5. tan 210°
6. tan 350°

SECTION 6

The Arc Tangent Function

The term arc tan x means, "the angle whose tangent is equal to x." The slide rule can be employed effectively to find arc tan values. The following rules give the procedure.

When the tangent value is between 0 and 0.1, use the C and D scales and divide the tangent value by 0.01745 to obtain the angle in degrees.

Example—Find arc tan 0.0875. Therefore,

$$\frac{0.0875}{0.01745} = 5 \text{ (approximately)}$$

Thus, arc tan $0.0875 = 5°$.

When the tangent value is between 0.1 and 1, use the T scale with the C scale as follows (if T scale is on body, use D scale with it):

1. Place hairline over tangent value on C scale.
2. Read angle value under hairline on T scale.

Example—Find arc tan 0.4663. Here, the procedure is as follows:

1. Place hairline over 4663 on the C scale (or D scale).
2. Under the hairline read angle value (25) on the T scale.
3. Thus, arc tan 0.4663 = 25°.

When the tangent value is between 1 and 10, use the T scale with the D scale, and proceed as follows:

1. Place hairline over right index of D scale.
2. Slide left index of C scale over tangent value on the D scale.
3. Read angle value under hairline on the T scale, using the numbers running from right to left, i.e., angles between 45° and 84.25°.

Procedure for Finding Arc Tangents
When Slide Rule Has Two T Scales

Some slide rules have two T scales, T1 for tangents of angles from 5.75° to 45°, and T2 for tangents of angles from 45° to 84.25°. When the tangent value is between 1 and 10 use the following procedure:

1. Place hairline over tangent value on the D scale (if T1 and T2 are on body of rule) or the C scale (if T1 and T2 are on slide).
2. Read value of angle under hairline on T1 scale if tangent value is less than 1, or on T2 scale if tangent value is greater than 1.

Example—Find arc tan 1.3270. Two procedures may be given:

A. *If slide rule has one T scale:*

1. Place hairline over right index of D scale.
2. Slide left index of C scale over tangent value 1.327 on the D scale.
3. Read angle (53) under hairline on T scale.

B. *If slide rule has T1 and T2 scales:*

1. Place hairline over 1372 on the C scale (or D scale).
2. Read angle value (53) under hairline on T2 scale.

In Step 2 of Part B above, note that the angle was read on the T2 scale because the tangent value was greater than 1. On the T1 and T2 scales there are generally two sets of numbers. Those running from left to right represent tangent values, while those running from right to left denote cotangent (the reciprocal of tangent) values.

When the tangent value is greater than 10, use the C and D scales and apply the following formula for tangent values greater than 10:

$$\vartheta = \frac{90^\circ \tan x - 57^\circ}{\tan x.}$$

where,

$\vartheta = \text{arc} \tan x.$

Example–Find arc tan 14.30. From the preceding formula we have

$$\vartheta = \frac{(90^\circ \times 14.30) - 57^\circ}{14.30} = 86^\circ$$

Therefore, arc tan 14.30 = 86°.

The answer derived from this formula is an approximate one, as are most of the answers derived from the slide rule in trigonometric-function problems. The rule can be used in this problem for the multiplication and division, and the subtraction by conventional procedures.

Exercises

Find the indicated angles.

1. arc tan 8.1443
2. arc tan 2.0503
3. arc tan 2.7475
4. arc tan 0.5774
5. arc tan 0.8098
6. arc tan 0.0000

SECTION 7

Applications of Trigonometric Functions to Electronics Calculations

When phase angle is a circuit consideration, the sine, cosine, and tangent functions are important in electronics calculations. Referring to the impedance triangle in Fig. 11-3 we find that the following relations are true for series circuits:

$$\phi = \text{arc tan } (X/R) \qquad R = Z \cos \phi$$
$$\sin \phi = X/Z \qquad Z = X/\sin \phi$$
$$X = Z \sin \phi \qquad Z = R/\cos \phi$$

To illustrate the use of the trigonometric functions consider the following examples:

Example 1—A 40-ohm resistor and a coil whose inductive reactance (X_L) is 50 ohms are connected in series across an a-c source. Find the impedance (Z) and the phase angle between the current through the resistor and the current through the coil.

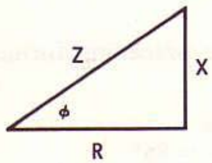


Fig. 11-3. An impedance triangle.

Conventional Solution

1. Find the phase angle, ϕ:
   (a) $\phi = \text{arc tan } (X/R)$
   (b) $\phi = \text{arc tan } (50/40)$
   (c) $\phi = \text{arc tan } 1.25$
   (d) $\phi = 51.4°$

Therefore, $\phi = \text{arc tan } 1.25 = 51.4°$.

2. Solve for the impedance, Z.
   (a) $Z = X/\sin \phi$
   (b) $Z = 50/\sin 51.4°$
   (c) $Z = 50/0.7815$ (from trig table)
   (d) $Z = 63.9$ ohms

Therefore, $Z = 63.9$ ohms.

Slide-Rule Solution

1. Find the phase angle, ϕ:
   (a) Place hairline over 50 on D scale.
   (b) Slide 40 of C scale under hairline.
   (e) Read value of angle under hairline on T scale (left-set.
   (d) Place hairline over right index of D scale.*

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*See Section 6 for procedure in finding tangent value with rules having two T scales.

(e) Read value of angle under hairline on T scale (left-hand number, 51.4).

2. Solve for the impedance, Z.
   (a) Place hairline over 50 on D scale.
   (b) Slide 51.4 of S scale under hairline.
   (c) Read digits 639 under right C index on the D scale.

Example 2—Assume that the components in the series circuit in Example 1 were connected to an a-c source of 80 volts. Find the power factor and the power dissipated by the resistor.

Conventional Solution

1. Find Power Factor:

   The phase angle, which is the difference in phase between the voltage and current in the reactive part of the circuit, is given as 51.4°. Therefore, the power factor (PF) is:
   (a) $PF = \cos \phi$
   (b) $PF = \cos 51.4°$
   (c) $PF = 0.6239$, or 62.39%

Therefore, $PF = \cos 51.4° = 0.624$, or 62.4%.

2. Find Power Dissipated by Resistor:

   The power dissipated by the resistor represents the circuit power and can be found by the formula
   (a) $P = \frac{E^2 \cos \phi}{Z} = \frac{80^2 \times 0.6239}{63.9}$
   (b) $P = \frac{6.4 \times 10^3 \times 6.239 \times 10^{-1}}{6.39 \times 10^1}$
   (c) $P = 62.5$ watts

Thus, the power dissipated by the resistor is 62.5 watts.

Slide-Rule Solution

1. Find Power Factor:
   (a) Align right indexes of S and D scales. Place hairline over 514 on the S scale. Use the left-hand (cosine) numbers.
   (b) Read cosine-value digits 624 under hairline on the D scale.
   (c) Since the angle of 51.4° lies between 0° and 84.25°, its cosine will be a value between 0.1 and 1, hence 0.624.

2. Find Power Dissipated by Resistor:
   (a) Place hairline over 80 (voltage) on the D scale.
   (b) Note digits 64 under hairline on the A scale.
   (c) Place right index of the C scale over 64 on the D scale.
   (d) Place hairline over 624 on C scale.
   (e) Place left index of C scale under hairline.
   (f) Place hairline over 639 on the CI scale.
   (g) Read digits 625 under hairline on D scale.

Thus, the power dissipated by the resistor is 62.5 watts.

Exercises

1. Find the phase angle and the circuit impedance of the circuit in Fig. 11-4.
2. Find the power factor and amount of power dissipated by the resistor in the circuit of Fig. 11-4.

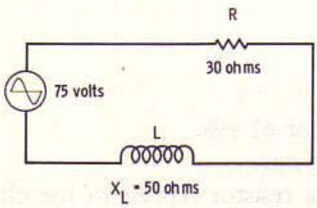


Fig. 11-4. A series *RL* circuit.

SECTION 8

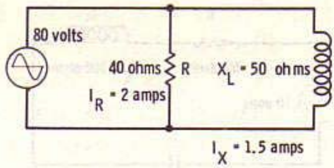
Phase Angle in Parallel A-C Circuits

In a parallel a-c *RL* circuit the phase angle, ϕ, is found by the formula arc tan $\phi = I_X/I_R$, which states that the phase angle is that angle whose tangent is equal numerically to the current through the reactance divided by the current through the resistance.

Example–Find the phase angle and impedance in the parallel *RL* circuit of Fig. 11-5.

The phase angle (ϕ) is the difference in phase between the current and voltage in the reactive part of the circuit. This amounts to the phase angle of the total line current with respect to the voltage, since the phase angle in the resistive part of the circuit is zero. The current lags the voltage in an inductive circuit, so the phase angle is negative.

Fig. 11-5. A parallel *RL* circuit.



Conventional Solution

1. Find the phase angle, ϕ:
   (a) $\phi = -\text{arc tan } (I_X/I_R)$
   (b) $\phi = -\text{arc tan } (1.5/2)$
   (c) $\phi = -\text{arc tan } 0.75$
   (d) $\phi = -36.8°$
2. Find the impedance, Z:
   (a) $Z = E/I_T$
   (b) $Z = 80/2.5$ ohms
   (c) $Z = 32$ ohms

Slide-Rule Solution

1. Find the phase angle, ϕ:
   (a) Place hairline over 15 on D scale.
   (b) Slide 2 of C scale under hairline.
   (c) Under right index of C scale note digits 75 on D scale.
   (d) Place hairline over 75 on C scale.
   (e) Read value of angle under hairline on T scale, using right-hand number (36.8).
2. Find the impedance, Z:
   (a) Place hairline over 80 on the D scale.
   (b) Slide 2.5 of C scale under the hairline.
   (c) Under left index of C scale read value of Z (32) on the D scale.

Note that the total line current (I_T) for the circuit in Fig. 11-5 was given as 2.5 amperes. In an a-c circuit the branch currents must be added vectorially.

Exercise

1. Solve for the impedance (Z) and power (*P*) of the circuit of Fig. 11-6. Assume that the phase angle is 42°.

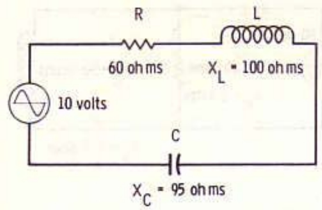


Fig. 11-6. A series *RLC* circuit.

SECTION 9

Instantaneous Voltage Values in a Sine Wave

A sine wave shows the voltage or current waveform of one complete a-c cycle or alternation. Students of electronics will recall that the values of alternating current are instantaneous ones and that the peak values occur in the positive region at 90° and in the negative region at 270°. All the values between peaks are only instantaneous values and can be found for any angle by the formula

$$e = E \sin \phi$$

where,

e = instantaneous value,
E = maximum value of voltage,
ϕ = the angle of rotation.

Example—The sine wave in Fig. 11-7 shows a peak value (E) of +50 volts at 90°. It has a peak value of −50 volts at 270°.

Find the instantaneous voltages at 10°, 45°, 170°, and 270°.

Conventional Solution

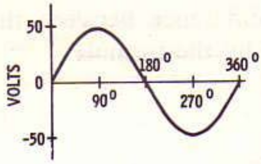
1. At 10°:
   (a) $e = 50 \sin 10°$
   (b) $e = 50 \times 0.1736$
   (c) $e = 8.68$ volts

2. At 45°:
   (a) $e = 50 \sin 45°$
   (b) $e = 50 \times 0.7071$
   (c) $e = 35.3$ volts

3. At 170°:
   (a) $e = 50 \sin 170°$
   (b) $e = 50 \sin 10°$
   (c) $e = 50 \times 0.1736$
   (d) $e = 8.68$ volts

4. At 270°:
   (a) $e = 50\ [-\sin 270°]$
   (b) $e = -50 \sin 90°$
   (c) $e = -50$ volts

Fig. 11-7. One complete sine-wave cycle.



Slide-Rule Solution

1. At 10°:
   (a) Place hairline over 10 on S scale.
   (b) Slide left index of C scale under hairline.
   (c) Place hairline over 50 on the C scale.
   (d) Read 8.68 under hairline on the D scale.
2. At 45°:
   (a) Place hairline over 45 on the S scale.
   (b) Slide right index of C scale under the hairline.
   (c) Place hairline over 50 of C scale.
   (d) Read 35.3 under hairline on the D scale.
3. At 170°:
   (a) Mentally subtract 170° from 180° (see p. 116).
   (b) Proceed as in Step 1.
4. At 270°:
   Close rule by aligning C and D left indexes, and place hairline over 50 on C scale. Read 50 under hairline on D scale. Since the sine function is negative in the third quadrant, $e = -50$ volts.

Exercises

Find value of instantaneous voltage for $E = 100$ volts.

1. At 150°, $e =$ ______ volts.
2. At 180°, $e =$ ______ volts.
3. At 190°, $e =$ ______ volts.
4. At 260°, $e =$ ______ volts.
5. At 360°, $e =$ ______ volts.
6. At 340°, $e =$ ______ volts.
7. At 30°, $e =$ ______ volts.
8. At 100°, $e =$ ______ volts.

SECTION 10

Calculating the Time Factor of the Phase Angle

When there is a *phase difference* between two waves of the same frequency this means that one wave either leads or lags the other. The amplitudes of the waves can be different and measured in different units, as in the case of voltage and current waves. The time difference between the two waves (*time factor*) can be found by the formula

$$t = \frac{\phi}{360} \times \frac{1}{f}$$

where,

t = time taken in seconds by an angular displacement of the wave which is equal to the phase angle,
f = frequency in hertz (Hz),
ϕ = phase angle in degrees.

Example–In Fig. 11-8 wave B is leading wave A by 72°. Each wave has a frequency of 60 Hz. Find the time factor of ϕ.

Conventional Solution

(a) $t = \phi/360 \times 1/f$
(b) $t = 72/360 \times 1/60$
(c) $t = 0.0033$ seconds
(d) $t = 3.3$ milliseconds

Slide-Rule Solution

(a) Place hairline over 72 on D scale.
(b) Slide 360 of C scale under hairline.
(c) Move hairline over left C index.
(d) Slide 6 of C scale under hairline.
(e) Read digits 333 under right C index on the D scale.

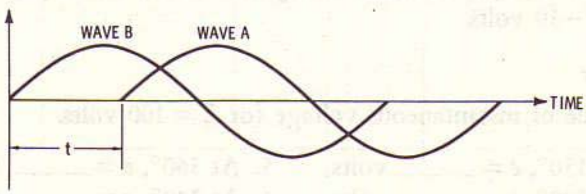


Fig. 11-8. Two waves of the same frequency having a phase difference.

Exercises

1. In an inductive circuit the current is lagging the voltage by 60°. Find the time factor at 120 Hz.
2. In a capacitive circuit the current is leading the voltage by 60°. Find the time factor at 60 Hz.
3. In an a-c circuit where resistance is the only component consideration find the phase angle between the voltage and current at 400 Hz.

SECTION 11

Placing the Decimal Point in Sine, Cosine, and Tangent Values

From 0° to 90° (the first quadrant) the numerical values of the sine function run from 0 to 1, the cosine function from 1 to 0, and the tangent function from 0 to values much greater than 1. If, in placing the decimal point, the scientific method does not easily apply, the characteristic method will prove more practical. Table 11-1 shows the characteristics of the sine, cosine, and tangent values from 0° to 90°.

TABLE 11-1. *Characteristic Method of Decimal-Pointing Trigonometric Functions*

	Characteristic			*Examples*	
Angle	*Sin*	*Cos*	*Tan*	*Function*	*Characteristic*
	Not greater than		Not greater than		
0° to 5.74°	−2	−1	−2	sin 5° = 0.0872	(−2)
5.75° to 45°	−1	−1	−1	sin 10° = 0.1736	(−1)
45° to 84.25°	−1	−1	0	cos 0.6° = 0.9999	(−1)
		Not greater than	Not less than		
84.25° to 90°	−1	−2	1	tan 4° = 0.0699	(−2)

SECTION 12

Complex Numbers

Any impedance Z can be represented mathematically by the equation $Z = R + jX$, where R is a purely resistive component of the impedance, X a purely reactive component, and $j = \sqrt{-1}$. In the preceding equation Z is said to be a *complex number* with *real part* R and *imaginary part* X. The term j can be considered an operator.

The j *Operator*

In a-c circuit analysis j may be considered an operator which effects a rotation of 90°. A positive number with a j as a coefficient represents the value obtained by rotating the point of the number 90° counterclockwise from the positive real axis. In Fig. 11-9 the rotation of the number +10 by the operator j is shown; this results in the value $+j10$, on the positive ordinate. A rotation of this value ($j10$) by the operator j results in the value $j \times j10 = j^2 10$, on the negative abscissa. Since $j = \sqrt{-1}$, then $j^2 = -1$, which means that $j^2 10 = -10$. A further rotation by the operator j yields the value $j \times j^2 10 = j^3 10 = -j10$.

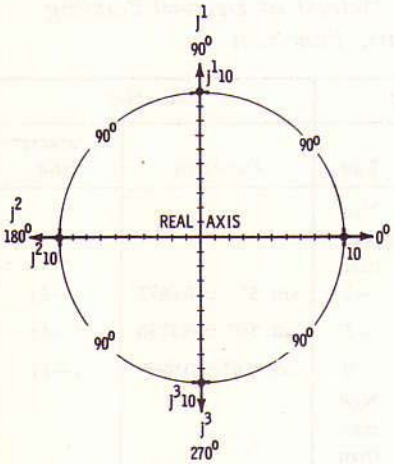


Fig. 11-9. Rotations by the operator j.

Values derived from j operations are generally expressed in powers of j no higher than the first. Thus $j^2 10$ is written as -10, $j^3 10$ as $-j10$, and $j^4 10$ as 10.

The two quantities X and R of an impedance determine the phase angle ϕ by

$$\phi = \text{arc tan}\left(\frac{X}{R}\right)$$

For positive values of X greater than R, the phase angle ϕ is greater than 45°. For positive values of X less than R, the phase angle is less than 45°. For negative values of X whose absolute values are less than R, the phase angle is negative and less than 45°

in magnitude. For negative values of X whose absolute values are greater than R, the phase angle is negative and greater than 45° in magnitude.

Exercises

Fill in the blanks.

1. In a cartesian coordinate system the vertical axis is sometimes called the _____ _______.
2. An impedance has two components: a ______ component and an ______ component.
3. In Fig. 11-6 the horizontal axis is referred to as the ______ axis.
4. The letter j is used to indicate the angle of rotation from the ______ axis.
5. For $Z = 20 + j10$ ohms the phase angle ϕ is ______ than 45°.
6. For $Z = 20 + j20$ ohms, the phase angle is ______ _____ 45°.

SECTION 13

Applications of Complex Numbers

The terms of a complex number can represent quantities such as resistance and reactance, which must be added vectorially to obtain impedance. In a-c circuits the real term represents the resistance value with 0° phase, and the imaginary term is used for reactance, $+j$ being the coefficient for inductive reactance and $-j$ for capacitive reactance.

Inductive reactance is written with a positive j-factor because it denotes that the voltage across a coil is leading the current by 90°, and capacitive reactance is written with a negative j-factor because the voltage lags the current in this component. This principle is true whether the circuit under consideration is series or parallel.

Example—A resistance of 50 ohms has no phase angle and is simply stated as 50. An inductive reactance of 60 ohms is expressed as $j60$, and a capacitive reactance of 60 ohms is $-j60$.

Two Ways to State Impedance

In complex a-c circuits the total impedance (Z) of the circuit is often difficult to calculate by conventional methods. Engineers often use the notation method to state impedance in complex networks. This method takes two general forms:

1. *Polar Form*—In this method the impedance Z is stated in ohms at a given phase angle. For example,

$$Z = 63.9\underline{/51.4^\circ}$$

which states that the total impedance Z is equal to 63.9 ohms at 51.4°.

2. *Rectangular Form*—In this method both the resistive and reactive properties of the circuit are considered to determine the impedance:

$$Z = R + j(X_L - X_C)$$

which states that the total impedance equals the vector sum of the resistance and reactance. For example, $Z = 50 + j60$ means that the impedance Z equals the vector sum of 50 ohms of resistance and 60 ohms of inductive reactance. If the reactive component were 60 ohms capacitive reactance, the equation would be $Z = 50 \ -j60$.

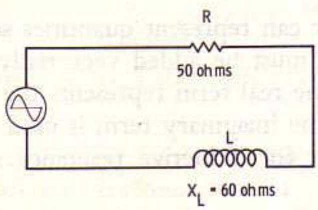


Fig. 11-10. A series-reactance circuit.

Consider the circuit in Fig. 11-10. To analyze this circuit by using complex numbers we could first express it in rectangular form, $Z = 50 + j60$, and then transform to its equivalent polar form.

1. Determine ϕ by the equation which defines it:

$$\begin{aligned}\phi &= \text{arc tan } (X/R) = \text{arc tan } (60/50)\\ &= \text{arc tan } 1.2\\ &= 50.2^\circ\end{aligned}$$

2. Find Z by the following equation:

$$Z = X/\sin\phi$$
$$= 60/\sin 50.2° = 60/0.7683$$
$$= 78 \text{ ohms}$$

Thus $Z = 78\underline{/50.2°}$, expressed in polar form.

Exercises

1. Express the series combination of 45 ohms of resistance and 55 ohms of inductive reactance in rectangular form.
2. A circuit has an impedance of 89 ohms at a phase angle of 67°. Express this in polar form.
3. Using the j operator, express 35 ohms of capacitive reactance.

SECTION 14

Converting from Polar Form to Rectangular Form

Given the polar form $78\underline{/50.2°}$ the conversion to rectangular form is given by the following formula:

$$Z\cos\phi + jZ\sin\phi = 78\cos 50.2° + j78\sin 50.2°$$
$$= (78 \times 0.6401) + j(78 \times 0.7683)$$
$$= 50 + j60 \text{ ohms}$$

When the S scale is on the slide, the slide rule may be used to solve the problem:

1. Place right C index over 78 on the D scale.
2. Place hairline over 50.2 (red or italic numbers) on the cosine part of the S scale.
3. Under hairline on D scale read value of R (50 ohms).
4. Now move hairline over 50.2 (black numbers) on the sine (S) scale.
5. Under hairline read value of X_L (60 ohms).
6. Therefore $78\underline{/50.2°} = 50 + j60$ ohms.

When the S scale is on the body, the slide rule may be used to solve the problem as follows:

1. Place 78 of C scale over right D index.
2. Place hairline over 50.2 on the sine (S) scale.

3. Read X_L value (60) under hairline on the C scale.
4. Move hairline over 39.8 (complement of 50.2) on the sine (S) scale.
5. Read R value (50 ohms) under hairline on the C scale.

Example—Change $30\underline{/25}°$ to rectangular form.

If S scale is on slide:

1. Place right C index over 30 on D scale.
2. Place hairline over red or italic 25 on S scale.
3. Under hairline on D scale read R value (27.2 ohms).
4. Move hairline over black 25 of S scale.
5. Under hairline on D scale read X value (12.7 ohms).

Therefore, $30\underline{/25}° = 27.2 + j12.7$.

If S scale is on body:

1. Place 30 of C scale over D index.
2. Place hairline over 25 (black) on the S scale.
3. Read X value (12.7) under hairline on C scale.
4. Move hairline over 65° (complement of 25°) on sine (S) scale.
5. Under hairline on C scale read R value (27.2 ohms).

Exercises

1. A series circuit has an impedance of 60 ohms at a phase angle of 30°. Find the value of the reactance.
2. What is the value of the resistance in the foregoing problem?
3. Write the impedances of Exercises 1 and 2 in rectangular form.
4. Convert $29\underline{/24}°$ to rectangular form.
5. Convert $16\underline{/45}°$ to rectangular form.

SECTION 15

Total Impedance When Circuit Has Inductive and Capacitive Reactances

In using complex numbers to find the total impedance of a series a-c circuit containing both X_L and X_C, one adds the real and imaginary (j) terms separately. The resistance values are of like sign (+) and are added arithmetically, while the reactance values (j terms) are added algebraically.

Example—Express the impedance in rectangular form for the circuit of Fig. 11-11. From Fig. 11-11 we have

$$Z_T = 25 + 25 \ -j20 + j60$$
$$= 50 + j(60 - 20)$$
$$= 50 + j40$$

Thus $Z_T = 50 + j40$ ohms.

Exercise

1. Convert the rectangular expression $50 + j60$ ohms to polar form.

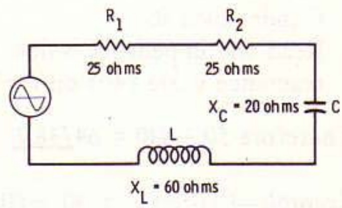


Fig. 11-11. A series resistance-reactance circuit.

SECTION 16

Using the Slide Rule to Change from Rectangular Form to Polar Form

In the preceding section the total impedance was not expressed, merely the rectangular form of the values making up the Z value. To find the actual Z value we can convert the rectangular form to its equivalent polar form. There is a less cumbersome way than the method discussed in Section 13. Here we will use the S and T scales as follows:

1. Slide the C index over the greater of the two values (X and R) on the D scale.
2. Move hairline over the lesser value on the D scale. This gives the value of tan ϕ on CI scale.
3. Read ϕ on T scale. (If X is greater than R, ϕ is greater than 45°. If X is less than R, ϕ is less than 45°.)

4. Find the numerical value of the sine of angle ϕ on the S scale and slide the C index over it.
5. Slide the hairline over the reactance value on the D scale. Read Z under hairline on C scale.

If your rule has two T scales, use the T1 scale, which is the same as the single T on most rules.

Example—Express $50 + j40$ in polar form. For slide rules having the T scale the procedure is as follows:

1. Place right C index over 50 on the D scale.
2. Place hairline over 40 on the D scale.
3. Read value of ϕ (38.7°) under the hairline on the T (or T1) scale.
4. Find sin 38.7° from the S and D scales, and slide the right C index over it.
5. Read the impedance value (64) on the C scale, over the reactance value (40) on the D scale.

Therefore $50 + j40 = 64\underline{/38.7^\circ}$.

Example—Express $Z = 40 - j30$ in polar form, using a slide rule with a T scale.

1. Place right C index over 40 on the D scale.
2. Move hairline over 30 on the D scale.
3. Read ϕ under hairline on the T scale. (Since X is less than *R*, the angle ϕ is less than 45°.) The angle ϕ is −36.9°.
4. Find sin 36.9° on the D scale, from the S scale. Slide right C index over it.
5. Read on C scale the impedance (50) over the reactance value (30).

Therefore $Z = 40 - j30 = 50\underline{/-36.9^\circ}$.

Exercise

1. Express the following expressions in polar form:

(a) $25 - j60$	(e) $125 + j500$
(b) $15 + j15$	(f) $75 - j65$
(c) $26.5 + j11.8$	(g) $47 - j76$
(d) $11.3 - j11.3$	(h) $10 + j14$

SECTION 17

Finding Total Current and Phase Angle by Dividing Polar Forms

Alternating-current circuits can be further analyzed by division of polar forms. When polar forms are divided, the E and Z values are divided and the divisor phase-angle is subtracted from the dividend phase-angle.

Example—From the polar form $Z = 50\underline{/-36.9^\circ}$ ohms find the total current (I_T) and the phase difference between E and I for $E = 100$ volts.

The expression for I_T is

$$I_T = E/Z = \frac{100\ \underline{/0^\circ}}{50\ \underline{/-36.9}} = 2\underline{/0^\circ - (-36.9^\circ)}$$
$$= 2 \text{ amps } \underline{/36.9^\circ}$$

The total current is 2 amperes, and it *leads* the voltage by 36.9°.

The problem is analyzed as follows:

$$\frac{100\underline{/0^\circ}}{50\underline{/-36.9^\circ}} \text{ means } \frac{100 \text{ volts @ } 0^\circ}{50 \text{ ohms @ } -36.9^\circ}$$

The −36.9° indicates that the circuit is predominantly capacitive and the current therefore leads the voltage.

The real numbers, 100 for voltage and 50 for resistance, are divided, yielding the quotient 2. Division is easy with the polar form of complex numbers—the real numbers, which are the magnitudes, are divided, but the divisor angle is subtracted algebraically.

When a quantity is divided into or subtracted from 0° the sign of the divisor or subtrahend is changed, which in this case yields a positive quotient. This answer is correct because the circuit is capacitive and the current leads the voltage.

Exercises

1. Using complex numbers, solve for Z_T, I_T, and ϕ for the circuit of Fig. 11-12.
2. If the voltage source in the previous problem is increased to 100 volts, what are the values of Z_T, I_T, and ϕ?

Examination on Lesson 11

1. A right triangle has legs of 5 and 12 units. What is the length of the hypotenuse?
2. A right triangle has a hypotenuse which is 17 inches in length and a leg which is 15 inches in length. What is the length of the other leg?

Evaluate the following expressions.

3. sin 2.1°
4. sin 21°
5. cos 21°
6. cos 2.1°
7. cos 87°
8. cos 0°

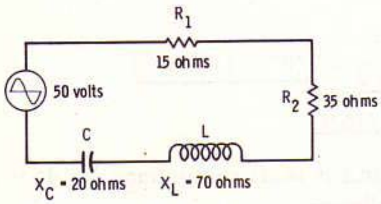


Fig. 11-12. Circuit for Exercise 1.

Find the angles indicated.

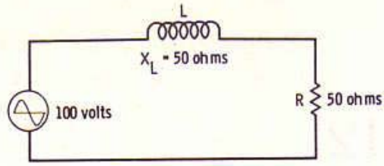
9. arc sin 0.500
10. arc sin 0.050
11. arc cos 0.850
12. arc cos 0.085
13. arc sin 0.707
14. arc cos 0.12

Evaluate the following expressions.

15. tan 20°
16. tan 2.1°
17. tan 75°
18. tan 87°
19. arc tan 0.045
20. arc tan 0.45
21. arc tan 4.50
22. arc tan 45.0

23. A series *RL* circuit contains a 0.5-henry inductor and a 50-ohm resistor. Find the total impedance (Z) and the phase angle (ϕ) of the circuit.
24. What is the instantaneous voltage value at 20° of a sine-wave voltage having a 100-volt peak-to-peak value?
25. Express the total impedance (Z) of the circuit in Fig. 11-13 in polar form and in rectangular form.

Fig. 11-13. Circuit for Problem 25.



26. Convert to rectangular form:
    (a) $Z = 35 - j20$
    (b) $Z = 10 + j15$
27. Convert to polar form:
    (a) $Z = 50\underline{/31^\circ}$
    (b) $Z = 40\underline{/-10^\circ}$

12

Common Logarithms

SECTION 1

The L Scale

Common logarithms of numbers can be found directly by using the L scale with the C or D scale on the slide rule. For a review of the principles of logarithms see Lesson 4, Section 8.

Examine the L scale on your slide rule. (Most rules have this scale.) On some rules the L scale is on the body, with the D scale, while on others it appears on the slide, with the C scale. This should cause no confusion; one merely uses the L scale with the D or the C scale, whichever is on the same part of the slide rule as the L scale. Note that the L scale is the only linear scale on the slide rule. The divisions are evenly spaced and run from 0.0 to 1.0 and are complete with decimal point, representing the mantissas only. Fig. 12-1 shows the C and L scales together.

The procedure for finding the mantissa part of the logarithm with the slide rule is quite simple:

1. Place the hairline over the number on the C and D scale.
2. Under the hairline on the L scale read the mantissa.

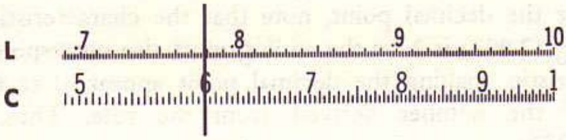


Fig. 12-1. Finding log 60 with L and C scales.

The characteristic is found by inspection of the number, as shown in Lesson 4.

Example—Find the common logarithm of 60, i.e., log 60. The procedure is as follows:

1. Place the hairline over 60 on the C (or D) scale, as in Fig. 12-1.
2. Under the hairline on the L scale read the digits 778.
3. Since the characteristic of 60 is 1, log 60 = 1.778.

Exercises

Find the logs of the following numbers.

1. log 555
2. log 6.5
3. log 945
4. log 101
5. log 256
6. log 27
7. log 10
8. log 549
9. log 0.0549
10. log 0.0067

SECTION 2

Antilogarithms

The number that corresponds to a logarithm is called its *antilogarithm* (or *antilog*). To find the antilog of a log we use the L scale:

1. Place hairline over *mantissa* value on the L scale.
2. Read its antilog under the hairline on the C or D scale.

The characteristic of the given log will indicate the location of the decimal point.

Example—Find the antilogarithm of 2.403, i.e., antilog 2.403.

1. Place hairline over .403 on the L scale.
2. Under the hairline on the C or D scale read the digits 253.

To place the decimal point, note that the characteristic of the given log (2.403) is 2, so the antilog must also correspond to this characteristic, making the decimal point appear after the third digit of the number derived from the rule. Thus, antilog $2.403 = 253$.

Exercises

Find the following antilogs.

1. antilog 3.819
2. antilog 2.332
3. antilog 5.336
4. antilog 1.328
5. antilog 3.356
6. antilog 1.501

SECTION 3

Negative Characteristics

The mantissas of logarithms as they appear on the L scale of a slide rule and in the columns of log tables are invariably positive numbers. However, it can be shown, from the laws of exponents, that negative mantissas and characteristics are perfectly possible. The following table shows what happens to a number and its logarithm as the number is progressively decreased by a factor of 10.

Col. A	*Col. B*	*Col. C*
$400 = 4 \times 10^2$	$\log 400 = \log 4 + 2 \times \log 10$	$= \log 4 + 2$
$40 = 4 \times 10^1$	$\log 40 = \log 4 + 1 \times \log 10$	$= \log 4 + 1$
$4 = 4 \times 10^0$	$\log 4 = \log 4 + 0 \times \log 10$	$= \log 4 + 0$
$0.4 = 4 \times 10^{-1}$	$\log .4 = \log 4 + (-1 \times \log 10)$	$= \log 4 - 1$
$0.04 = 4 \times 10^{-2}$	$\log .04 = \log 4 + (-2 \times \log 10)$	$= \log 4 - 2$

Substituting $\log 4 = .602$ in Column C to obtain Column D:

Col. D

$$\log 400 = 2 + .6021 = 2.602$$
$$\log 40 = 1 + .6021 = 1.602$$

$$\log\ 4 = 0 + .6021 = 0.602$$
$$\log\ .4 = -1 + .6021 = -0.398$$
$$\log\ .04 = -2 + .6021 = -1.398$$

Notice that all numbers greater than 1 have positive characteristics and mantissas, whereas numbers less than 1 have negative characteristics and mantissas. If we followed this system, we would need slide rules with both positive and negative log scales. The following examples show how positive mantissas can be used for numbers less than 1.

Example 1—Consider log 0.04 = 8.602 − 10.

Explanation—The characteristic of 0.04 is −2. If we express it as +2 and add −10 to the log, we obtain 8.602 − 10, which is the equivalent of −2 + .602 or −1.398. (Actually, 8 − 10 is the same as −2.)

Example 2—Find log 0.0019.

Solution—The characteristic of 0.0019 is −3. Adding this to +10 yields +7. This is another way of doing the same thing as we did in Example 1, only here we simply added the negative exponent to a positive 10 and obtained a positive characteristic. By writing −10 at the end of the complete log it is expressed with a positive exponent, which is necessary in order to avoid possible errors that might result if the characteristic were expressed as a negative number.

So log 0.0019 becomes 7.279 − 10.

Exercises

Express the following logarithms with positive exponents.

1. log 0.155
2. log 0.003
3. log 0.0003
4. log 0.251

SECTION 4

Finding Powers and Roots Using Logarithms and Antilogarithms

We have seen that it is relatively simple to square or cube a number, or to extract the square root or cube root of a number, using the A and K scales of the slide rule.

Numbers can be raised to any power, or any root of a number can be extracted, by the use of the L scale and the C and D scales.

Powers—Consider the following equation:

$$\log N^M = M \log N$$

where,

N = number to be raised to a power,
M = power to which N is to be raised.

Example—Solve 67^4, i.e., raise 67 to its fourth power.

Conventional Solution

1. Find log of N and multiply it by M:
   (a) $\log 67^4 = 4 \log 67$
   (b) $\log 67 = 1.826$

Slide-Rule Solution

1. Find the log of N and multiply it by M:
   (a) Place hairline over 67 on the C or D scale.
   (b) Read mantissa (.826) under hairline on the L scale.
   (c) Place hairline over 826 on the D scale.
   (d) Slide right index of C scale under hairline.
   (e) Place hairline over 4 on the C scale.
   (f) Under hairline on the D scale read the product 3.30.
   (g) Add 4 times characteristic of log of 67, i.e., $4 \times 1 = 4$, to 3.30, giving log of $67^4 = 7.30$.
2. Find antilog 7.30:
   (a) Slide .30 of L scale under hairline.*
   (b) Read 1995 under hairline on the C scale.

Since the characteristic is 7, the antilog is expressed as 1.995×10^7. The significant figures that represent the antilog are written as a number between 1 and 10 multiplied by a power of ten whose exponent is equal to the characteristic of the log.

You may notice that as you do the following practice problems your answers may differ considerably with those given in Appendix B. This is due to several reasons: The accuracy of the average slide rule is considered good to only three places; also, any error in reading the L scale is multiplied by the power to

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*If the L scale is on the body of the rule, place hairline over .30 on this scale and read the antilog digits (1995) under hairline on the D scale.

which we are raising the number. Thus, in problems 4 and 5, which follow, any errors will be multiplied eight times.

In the example previously given, when the log of 67 was multiplied by 4, we obtained 7.30, accurate to three places. One digit is lost because it is the characteristic, leaving only .30 to be found on the L scale. A four-place table of logs would give us 7.3043, instead of 7.30 as found on the slide rule. This is the log of 2.015×10^7, a difference of 2 in the third place, when compared to the slide-rule answer. The answers to Problems 1 through 7 are based on four-place logs.

Exercises

Use logarithms on the slide rule to evaluate the following expressions.

1. 88^6
2. 56^5
3. 7.2^4
4. 98^8
5. 60^8
6. 6.9^7
7. 0.05^6 (Hint–Find mantissa on L scale, then express characteristic in form: 8 + mantissa −10. Then multiply by 6.)

SECTION 5

Fractional Exponents

An exponent indicating the power to which a number is raised can be a whole number or a fraction. When the exponent is a fraction it is sometimes convenient to use logarithms. In such cases there are two handy rules to remember:

1. Multiply the log of the number by the fractional exponent.
2. The antilog of the product will be the answer.

Example–Evaluate $(16)^{3/4}$. For this, one must use the equation $\log N^M = M \log N$.

Conventional Solution

(a) $\log\ (16)^{3/4} = \frac{3}{4} \log\ 16$

(b) $\frac{3}{4} \log\ 16 = \frac{3}{4} \times 1.2041$

(c) $\log\ (16)^{3/4} = 0.904$

(d) antilog $0.904 = 8$

Slide-Rule Solution

(a) Place hairline over 16 on D scale if L scale is on the body; C scale if L scale is on slide.
(b) Read mantissa (.204) under hairline on L scale. Add characteristic of 16 (1) to make complete log equal 1.204.
(c) Place hairline over 1204 on D scale.
(d) Slide 4 of C scale under hairline.
(e) Place hairline over right index on C scale.
(f) Set left index of C scale under hairline.
(g) Place hairline over 3 of C scale.
(h) Under hairline on D scale read .904. Now find antilog 0.904.
(i) Place hairline on .904 on L scale.
(j) Read antilog (8) under hairline on D scale (or C scale if L scale is on slide).

Since the characteristic of the given log is zero, the antilog is $8 \times 10^0 = 8$.

Roots

Returning to the equation we have just used for powers, $\log N^M = M \log N$, let us see how it can be used for extracting the root of any number N. If M is a fraction with a numerator of 1, the equation will give the log of a root of N. For example, if M is ½, we obtain the log of the square root of N; if M is ⅓, we obtain the log of the cube root of N; if M is ⅕, we obtain the log of the fifth root of N; and so on.

Example–Solve $(16)^{1/6}$. (Find the sixth root of 16.)

Conventional Solution

1. Find log of N and multiply it by M:
   (a) $\log (16)^{1/6} = \frac{1}{6} \log 16$
   (b) $\log 16 = 1.2041$
   (c) $\log (16)^{1/6} = 1/6 \times 1.2041$
   (d) $\log (16)^{1/6} = .2007$
2. Find the antilog:
   (a) antilog $.2007 = 1.587$
   (b) Therefore $(16)^{1/6} = 1.587$.

Slide-Rule Solution

1. Find log of N and multiply it by M:
   (a) Align C and D indexes and place hairline over 16 on C or D scale.
   (b) Read mantissa (.204) under hairline on L scale.
   (c) Affix characteristic of log 16 (1) to mantissa, getting 1.204.
   (d) Set hairline on 1.204 on D scale.
   (e) Slide 6 on C scale under hairline.
   (f) Read D scale under right index of slide, getting .201.
2. Find antilog of .201:
   (a) Realign C and D indexes and set hairline over .201 on L scale.
   (b) Read antilog of .201 under hairline on C or D scale. The antilog is 1.59.
   (c) Multiply by proper power of 10 (zero power in this case) to set the decimal point.

Exercises

1. $(56)^{1/3}$
2. $(144)^{1/6}$
3. $(25)^{1/9}$
4. $(64)^{1/4}$
5. $(15{,}600)^{1/6}$
6. $(1560)^{1/6}$
7. $(64)^{1/3}$
8. $(0.64)^{1/3}$

More on Fractional Exponents

When the exponent is a fraction with neither numerator nor denominator equal to 1, the resulting calculation is a combination of powers and roots. For example, to find $(12)^{3/5}$ means finding the third power of the fifth root of 12. The procedure is the same as that used for powers and roots. Once the log of 12 is found on the L scale it is multiplied by $\frac{3}{5}$ to get the log of $(12)^{3/5}$.

Exercises

1. $(55)^{6/8}$
2. $(5.5)^{3/4}$
3. $5^{7.5}$
4. $15^{2.2}$

SECTION 6

Decibels

The intensity of sound is measured in terms of units called *decibels* (db), which show the ratio of audio values to electrical values. The log of the ratio of the output power to the input power can be determined by this formula to calculate decibels:

$$\text{db} = 10 \ \log \left(\frac{P_2}{P_1}\right)$$

which states that the decibel power gain or loss is equal to 10 times the log of the output power in watts divided by the input power in watts.

Example—An amplifier stage has an output of 15 watts when 75 milliwatts is applied to the input. Find the decibel gain.

Conventional Solution

(a) db = 10 × log (15/0.075) = 10 log 200
(b) db = 10 × 2.3010
(c) db = 23.01
(d) The correct answer, therefore, is 23 db.

Slide-Rule Solution When L Scale is on Body

(a) Slide left C index over 15 on the D scale.
(b) Place hairline over 75 on the *CI scale.* (See following remarks.)
(c) Under the hairline on the L scale read 0.3 (mantissa of log 200).
(d) Since the characteristic of log 200 is 2, the correct answer is 10 × 2.3 = 23 db.

When the L scale of the slide rule is on the slide, the following procedure can be used:

(a) Place hairline over 15 on the D scale.
(b) Slide 75 of the C scale under the hairline.
(c) Under right C index read digit 2 (quotient of 15/7.5).
(d) Place hairline over right C index. Close rule.
(e) Under hairline on L scale read 0.3, the mantissa of log 200.

In Step (b) of the first procedure the CI scale was used to obtain the reciprocal of 7.5, thus making it possible for the quotient to be under the hairline (multiplication and division are inverse operations). This brings up an important fact in regard to rules with the L scale on the body with the D scale: When the hairline is over the quotient or product, it will also be over the corresponding log on the L scale.

Power Loss

Sometimes a loss in gain occurs through attenuation, which can be either from planning or from malfunction. This can be calculated by using the formula given previously.

Example—It is desired to reduce the signal strength through the use of an attenuation pad. Find the power loss in decibels when a 10-milliwatt signal is applied to the input, resulting in an output of 5 milliwatts.

Solution—From the formula db = 10 log (P_1/P_2) we have

$$\text{db} = 10 \log \left(\frac{5}{10}\right) = 10 \log 0.5 = 10 \times (9.6990 - 10)$$
$$= 10 \times (-0.301)$$
$$= -3$$

Therefore the loss is 3 db.

Voltage and Current Ratios and the Decibel Formula

In terms of voltage and current the db formula can be written as follows:

$$\text{db} = 20 \log \left(\frac{E_2}{E_1}\right)$$
$$\text{db} = 20 \log \left(\frac{I_2}{I_1}\right)$$

Since $P = E^2/R$ or I^2R, and the log of a squared value is equal to twice its log, the log of the power ratio is twice the log of the voltage and current ratios.

Example—Find the db gain of an amplifier when a 5-volt input produces a 25-volt output.

Solution—From the formula in terms of voltage we have

$$\text{db} = 20 \log \left(\frac{25}{5}\right) = 20 \log 5$$
$$= 20 \times 0.6990 = 13.98$$

Therefore the gain is approximately 14 db.

The slide rule can be employed for solving this type of problem in the same manner as shown previously for the power-ratio problem.

Exercises

1. Find the db gain of a stage that produces an output of 50 watts from a 2.5-milliwatt input.
2. A 2-millivolt signal is applied to an amplifier whose stage gain is 1000. Find the voltage increase and the gain in decibels.
3. The input current of an amplifier stage is 150 milliamperes. If the output current is 30 milliamperes, find the loss in decibels.
4. Find the db gain for a stage whose output is 60 watts and whose input is 15 watts.
5. An input of 500 microvolts produces an output of 1.5 millivolts. Find the db loss or gain.

Examination on Lesson 12

Find the logs of the following.

1. 556
2. 0.77
3. 0.00125

Evaluate the following.

4. 8^3
5. 113^5
6. 32^4
7. 0.187^2
8. $(7)^{1/7}$
9. $(25)^{1/5}$
10. $(848)^{1/6}$
11. $(0.187)^{1/12}$
12. $(18)^{5/8}$
13. $(343)^{7/2}$
14. $(116)^{11/26}$

Appendix A. *Scale Function Chart*

Scale	*Description*	*Use*	*Text Reference*
D	The basic scale, on body. Numbers run from left to right, starting with 1 and ending with 10. Spacing is not linear.	Multiplication, division, ratios and proportions. Used with C and other scales.	Lesson 1, pages 11-19 Lesson 2, pages 25-28 Lesson 3, pages 33-36
C	Identical to D scale. On the slide of the rule.	Used with D and other scales.	See above.
CF, DF	Called the "folded" scales. Left section starts at π, ends at 1; right starts at 1, ends at π.	Problems where π is involved. Provides continuous operation without resetting slide. Also for multiplying and dividing.	Lesson 10, pages 99-106
CI, DI	Inverted C and D scales. Numbers run from right to left. CI on slide, DI on body.	To find reciprocals. Also used in trig-function calculations with S and T scales.	Lesson 6, pages 60-66 Lesson 11, pages 114-116
A, B	Has two sections, each half the length of C (or D) scale. A scale related to D scale, and B related to C in problem solving.	To square numbers and find their square roots. Can be used for multiplication and division.	Lesson 7, pages 71-75, 78-84
K	A three-cycle scale; each section is one-third the length of C (or D) scale. Is used with D scale.	To cube numbers and find cube roots.	Lesson 9, pages 95-98
S	Most rules show double numbers: those running from left to right are for sines, and those running from right to left are for cosines.	To find sines and cosines, secants, cosecants, and their inverse (arc) functions. Used in conjunction with D and DI scales.	Lesson 11, pages 109-113

APPENDIX A. *Scale Function Chart* (*cont.*)

Scale	*Description*	*Use*	*Text Reference*
T	Is often double-numbered, with left-to-right numbers covering range of 5.47° to 45°, and right to left from 45° to 84.25°. Some rules instead use two T scales. On some rules T scale is on the slide, on others on the body.	Finding tangents, cotangents, and their inverse (arc) functions. Used with D and DI scales, or C and CI scales.	Lesson 11, pages 113-124
L	The only scale on the rule with a uniform spatial division of numbers from 0 to 1.	To find the mantissas of logarithms. To find antilogarithms. To find powers and roots.	Lesson 12, pages 138-148

APPENDIX B. *Answers to Problems*

LESSON 1

Page 11
1. slide
2. body; stator; scales
3. cursor; hairline

Page 12
1. logarithmic
2. 48 percent
3. left index
4. two; left; right
5. D; C

Page 13
1. secondary
2. primary
3. tertiary

Page 15
1. A = 105
   B = 120
   C = 145
   D = 230
   E = 280

Page 17
1. A = 313
   B = 320
   C = 336
   D = 352
   E = 369
   F = 390

Page 19 (Exam)
1. F
3. T
5. T
7. first
9. tertiary
11. fifth; 13; 14
13. 1

LESSON 2

Pages 22-23
1. add
2. multiplicand
3. multiplier
4. product
5. T
6. F
7. T

Page 24
1. −50
2. −510
3. 0
4. −183
5. 0
6. T
7. T
8. T

Page 25
1. F
2. F
3. 270
4. 80
5. −150

Page 26
1. 2144 (rule showing slightly more than 2140)
2. 127.5 (careful placement should yield this answer exactly)
3. 13.5 (exact)
4. 4230 (third digit could be interpolated as 2.5 or 3, depending on accuracy of placement)

Pages 27-28 (Exam)
1. F
3. T
5. T
7. multiplicand
9. product
11. (a) 103.5; (b) −437; (c) 5250; (d) 648; (e) −1488

LESSON 3

Pages 30-31
1. F
2. T
3. F
4. T
5. F
6. F

Pages 32-33
1. plus; minus
2. positive
3. positive
4. negative
5. division
6. less
7. more
8. numerator
9. numerator; denominator
10. lowest

Page 34
1. 7.00
2. 5.00
3. 7.20
4. 1.67
5. 54.0
6. 0.785
7. 8.02
8. 3.94
9. 2.42
10. 5.12
11. 222
12. 8.36
13. 5280
14. 250 volts

Pages 35-36 (Exam)
1. 585
3. 28.1
5. 11,300
7. 72.0
9. 72.0 volts
11. 9.75 amperes

LESSON 4

Page 38
1. 2127
2. 0.179
3. 2000
4. 0.125
5. 0.0143
6. 372

Page 39
1. 100
2. 100,000
3. 1
4. 1000
5. 1,000,000
6. 10
7. 1000
8. 100,000,000

Page 40
1. 1×10^{-3}
2. 1×10^{-5}
3. 1×10^{-1}
4. 10
5. 0.1
6. 0.001

Page 41
1. 0.261
2. 9×10^{11} or $900{,}000 \times 10^{6}$
3. 0.016
4. 126.2
5. 0.0848
6. 173.5
7. 0.901

Pages 42-43
1. 9.5×10^{-1}
2. 6.52176×10^{3}
3. 2.1×10^{-4}
4. 2.995×10^{1}
5. 4.8 or 4.8×10^{0}
6. 1.00×10^{2}
7. 1×10^{3}
8. 2×10^{6}
9. 1.5×10^{0}
10. 3.6056×10^{2}
11. 5.89046×10^{6}
12. 4.7×10^{-3}
13. 6.8×10^{-4}
14. 7.07×10^{-1}
15. 3.465×10^{2}
16. 7.5×10^{-2}
17. 9.75×10^{7}
18. 0.00282
19. 400
20. 22,401
21. 2.83×10^{-4}
22. 39.4

Page 44
1. 500

2. 1,500,000
3. 0.0005
4. 50,000
5. 5730
6. 4460
7. 1.01
8. 6.525
9. 0.78
10. 0.05

Page 45

1. 1.25×10^6 ohms
2. 470×10^{-12} farad
3. 0.005×10^{-6} farad
4. 350×10^{-6} henry
5. 55×10^3 volts
6. 200×10^{-3} ampere
7. 1600×10^{-12} farad
8. 0.047×10^{-6} farad
9. 0.002×10^{-6} or 2×10^{-10} farad
10. 680×10^{-12} farad

Page 47

1. -1
2. 3
3. 0
4. -1
5. 0
6. 3
7. 2
8. -5
9. -1
10. 2

Page 49

1. 1572
2. 3425
3. 0.000206
4. 0.01122
5. 5120
6. 27.3

Page 50

1. 1000
2. 0.27
3. 0.513
4. 0.200
5. 0.0002
6. 18.1
7. 0.00011
8. 0.5
9. 30.9
10. 822,000

Page 51 (Exam)

1. 0.694
3. 6.30
5. 7.01
7. 350
9. 8.1
11. 6.7
13. 6.7×10^{-1}
15. 1.414×10^{-1}
17. 1.627×10^3
19. 9.75×10^2
21. 1.627×10^{-2}
23. 1.5×10^{-6} henry
25. 8×10^2 farads
27. 9.07
29. 54.4
31. 5.44×10^3

LESSON 5

Page 55

1. 0.14
2. 2260
3. 23.2
4. 1.88
5. 66,900
6. 3.16

Page 56

1. 3136
2. 5.05
3. 56,520
4. 1011
5. 15,300
6. 150,000

Pages 58-59

1. 1875 ohms
2. 6280 ohms
3. 1437 ohms
4. 0.65 ampere
5. 150×10^6 (150,000,000)

Page 59 (Exam)

1. 930
3. 50.4
5. 3130
7. 3.42
9. 552 ohms

LESSON 6

Page 63
1. 0.04
2. 0.0303
3. 0.02
4. 0.4
5. 200
6. 0.08
7. 2.78×10^{-5}
8. 25
9. 0.125
10. 0.06

Page 65
1. 136 ohms

Page 66
1. 1.59 kilohms
2. 20 ohms
3. 150 ohms
4. 100 ohms

Page 67
1. 111 picofarads
2. 0.0167 microfarad
3. 0.235 microfarad
4. 0.0094 microfarad
5. 16,847 ohms

Page 68 (Exam)
1. 0.000154
3. 0.143
5. 0.455
7. 0.333
9. 235 kilohms
11. 1330 ohms
13. 24.5 ohms

LESSON 7

Pages 73-74
1. 7.81
2. 9.01
3. 7.41
4. 8.00
5. 29.3
6. 7.21
7. 17.75
8. 14.14
9. 15.0
10. 18.0
11. 14,800,000
12. 2,820,000

Page 75
1. L
2. R
3. L
4. R
5. R
6. L
7. R
8. L
9. R
10. R

Page 77 (1st set)
1. 6.5
2. 15.51
3. 68.5
4. 21.5
5. 32.1
6. 159
7. 18.2
8. 1040

Page 77 (2nd set)
1. 0.07
2. 0.019
3. 0.04
4. 0.0158
5. 0.1414
6. 0.0253

Page 80
1. 1.69
2. 274 volts
3. 11.25 watts
4. 1058

Pages 81-82
1. 20 ohms
2. 9.6 ohms
3. 77.1 ohms

Page 84
1. $L = 405$ microhenrys

Page 84 (Exam)
1. 2.45
3. 1.772
5. 0.260
7. 64.9
9. 0.0173
11. 195 ohms
13. 60.9 ohms
15. 60 Hz
17. 25.4 millihenrys

LESSON 8

Page 86
1. compared
2. ratio
3. antecedent; numerator; fraction
4. 90; 60

Page 88
1. 4
2. 6
3. 3
4. 1
5. 2
6. 3
7. 40
8. 50
9. 5
10. 70

Page 89
1. 72 volts
2. 73.5 turns

Page 90
1. 2920 ohms
2. 5330 ohms

Page 92
1. 30
2. 2
3. 2.5
4. 27
5. 100
6. 3.33

Pages 92-93 (Exam)
1. 4.25
3. 1
5. 8.75
7. 45.4
9. 12 volts
11. 1500 ohms
13. 60 ohms
15. yes

LESSON 9

Page 95
1. (a) 12
   (b) 15
   (c) 212
   (d) 245
   (e) 277

Page 96
1. 216
2. 125×10^{-9}
3. 247
4. 3.37×10^{12}

Page 98
1. 6
2. 0.005
3. 6.28
4. 2.92
5. 11.45
6. 0.35

Page 98 (Exam)
1. 2.09
3. 0.216
5. 0.001
7. 10
9. 3.1

LESSON 10

Page 100
1. 28.3
2. 31.4
3. 6.28
4. 47.15
5. 78.6
6. 9.42

Page 101
1. 1570 ohms
2. 2010 ohms

Page 103
1. 2.86
2. 3.82
3. 3.14
4. 2.00

Page 104
1. 57.3
2. 6.28 or 2π
3. 6.28 or 2π
4. 5 inches
5. 15.7 inches

Page 105

1. 6.28 inches
2. 104.6
3. 270
4. 0.1047
5. 2.02
6. 3.14
7. 6.28

Page 106 (Exam)

1. 28.1
3. 5.44
5. 22.3
7. 75.4 ohms
9. 0.715 radian
11. 102°
13. 39.0°

LESSON 11

Page 109

1. 90° (*or* right)
2. Refer to Fig. 11-1 (p. 108). Thus $\sin a = A/C$ and $\cos a = B/C$, so that $\sin^2 a + \cos^2 a = A^2/C^2 + B^2/C^2 = (A^2 + B^2)/C^2 = 1$. This is true for any angle $a = x$.
3. 1
4. Refer to Fig. 11-1 (p. 108). Thus $\sin a = A/C$ and $\cos b = A/C$. But $b = 90° - a$, so $\sin a = \cos (90° - a)$. This is true for any angle $a = x$.
5. Pythagorean Theorem
6. ∞
7. hypotenuse
8. Refer to Fig. 11-1 (p. 108). Here $\cos a = B/C$ and $\sin b = B/C$. But $b = 90° - a$, so $\cos a = \sin (90° - a)$. This is true for any angle $a = x$.
9. 5 units
10. 0.8

Page 111

1. 0.358
2. 0.966
3. 0.259
4. 0.08725
5. 0.966
6. 0.0785
7. 0.0349
8. 0.0785
9. 1.00

Page 113

1. 26.7°
2. 10°
3. 31°
4. 43°
5. 27°
6. 6°
7. 45°
8. 85.3°
9. 87.54°
10. 2.46°

Page 116

1. 3.734
2. 5.91
3. 2.01
4. 57.3
5. 0.364
6. 0.1763

Page 117

1. 0.985
2. 0.173
3. 5.67
4. 0.996
5. 0.577
6. 0.1765

Page 119

1. 83°
2. 63.6°
3. 70°
4. 30°
5. 39.2°
6. 0.00°

Page 122

1. $\phi = 50°$; $Z = 58.3$ ohms
2. PF $= 51.5\%$; $P = 50$ watts

Page 123

1. $Z = 60.2$ ohms, $P = 1.23$ watts

Page 125

1. 50 volts
2. 0 volts
3. −17.4 volts
4. −98.5 volts
5. 0 volts
6. −34.2 volts
7. 50 volts
8. 98.5 volts

Page 126

1. 1.39 milliseconds
2. 2.78 milliseconds
3. 0°

Page 129

1. j axis
2. real (*or* resistive); imaginary (*or* reactive)
3. real
4. real
5. less
6. equal to

Page 131

1. $45 + j55$ ohms
2. $89\angle 67°$ ohms
3. $-j35$ ohms

Page 132

1. 30 ohms
2. 51.9 ohms
3. $Z = 51.9 + j30$ ohms
4. $26.5 + j11.8$
5. $11.3 + j11.3$

Page 133

1. $78\angle 50.2°$ ohms

Page 134

1. (a) $65\angle -67.4°$
   (b) $21.2\angle 45°$
   (c) $29\angle 24°$
   (d) $16\angle -45°$
   (e) $512\angle 75.9°$
   (f) $99.3\angle -40.9°$
   (g) $89.5\angle -58.3°$
   (h) $17.2\angle 54.4°$

Page 135

1. $Z_T = 50 + j50$ ohms
   $\phi = 45°$
   $I_T = 1\angle -45°$ ampere
2. $Z_T = 50 + j50$ ohms
   $\phi = 45°$
   $I_T = 2\angle -45°$ amperes

Pages 136-137 (Exam)

1. 13 units
3. 0.0367
5. 0.933
7. 0.0523
9. 30.0°
11. 31.8°
13. 45.0°
15. 0.364
17. 3.73
19. 2.58°
21. 77.5°
23. $Z = 195$ ohms, $\phi = 75.2°$
25. $Z = 70.7\angle 45°$
    $Z = 50 + j50$
27. (a) $Z = 42.1 + j25.8$ ohms
    (b) $Z = 39.4 - j6.95$ ohms

LESSON 12

Page 139

1. 2.744
2. 0.813
3. 2.975
4. 2.004
5. 1.408
6. 1.431
7. 1.000
8. 2.740
9. 8.740 − 10
10. 7.826 − 10

Page 140

1. 6600
2. 215
3. 217,000
4. 21.3
5. 2270
6. 31.7

Page 141

1. 9.190 − 10
2. 7.477 − 10
3. 6.477 − 10
4. 9.400 − 10

Page 143

1. 4.64×10^{11}
2. 5.51×10^{8}
3. 2.69×10^{3}
4. 8.51×10^{15}
5. 1.68×10^{14}
6. 7.45×10^{5}
7. 1.56×10^{-8}

Page 145 (1st set)

1. 3.83
2. 2.29
3. 1.43
4. 2.83
5. 5.00
6. 3.41
7. 4
8. 0.862

Page 145 (2nd set)

1. 20.2
2. 3.6
3. 175,000
4. 387

Page 148
1. 43.01 db
2. 60.2 db
3. −14 db
4. 6 db
5. 9.54 db

Page 148 (Exam)
1. 2.745
3. 7.097 − 10
5. 1.84×10^{10}
7. 1.83×10^{-9}
9. 1.90
11. 0.870
13. 7.47×10^{8}

Index

SLIDE RULE IN ELECTRONICS

by Don Carper

To anyone concerned with rapid, accurate calculations, such as those in electronics, the slide rule is literally worth its weight in gold. It has yet to be surpassed—even by a computer—in light weight, small size, speed and variety of calculations, and simplicity of operation.

The slide rule is especially useful to the electronics technician or student, who can scarcely afford the time required for the vast number of calculations which he often has to make. Luckily, most of the ordinary mathematical operations encountered in electronics can be performed adequately on the slide rule. Calculations can be made for resistance, reactance, impedance, current and voltage relations, frequencies, phase angles, and many other quantities.

This book is designed to teach the most efficient use of the ordinary slide rule in electronics mathematics. It is divided into twelve lessons, each lesson dealing with a particular aspect of slide-rule operation. In these lessons the various functions of the slide rule are thoroughly covered, including explanations of the basic mathematical principles and the applications from electronics. Each lesson, in turn, is divided into sections containing practice problems and exercises for the student. At the end of each lesson is an examination over the contents of the lesson.

The contents of the book are derived from the author's experience in teaching electronics mathematics and have been used in the classroom as a text. The material is suitable for high school, junior college, and technical institute courses, as well as for home study.

ABOUT THE AUTHOR

Don Carper lives a well-filled life: currently he is Director of Training at California Technical Trade Schools, and he also teaches at Compton Junior College in Compton, California, and at the Los Angeles City Schools. A graduate of California State College and former president of the Radio Television Technicians Association, he has written several training manuals on electronics.

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