RICOH

INSTRUCTIONS
FOR THE USE OF YOUR "RICOH" BAMBOO SLIDE RULE

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FOR THE USE OF ELECTRIC SLIDE RULE

1. GENERAL DESCRIPTION

This slide rule has been designed for the expert electrical engineers to simplify in use in calculating problems involving the various electrical phenomena, namely not only the computation of multiplication and division can be done with the A, B, BI, CI, C and D scales, but also the LL_1, LL_2, LL_3, $LL_{\bar{1}}$, $LL_{\bar{2}}$ and $LL_{\bar{3}}$ scales make it possible to obtain the result of $a^{\pm n}$, $e^{\pm x}$ and $\log_e N$, moreover, the P_1, P_2 and Q scales are essential for vector computation, the Sh_1, Sh_2 and Th scales for hyperbolic functions.

2. SCALE ARRANGEMENT AND USAGE

(Front Face) $LL_{\bar{1}}$, $LL_{\bar{2}}$, $LL_{\bar{3}}$, A, B, BI, CI, C, D, LL_3, LL_2, LL_1

(Back Face) Sh_1, Sh_2, P_2, P_1, Q, Sr, Sθ, Th, C, D, T_2, T_1, L

(a) $LL_{\bar{1}}$, $LL_{\bar{2}}$ and $LL_{\bar{3}}$

These are used to find the values of the type form of a^{-n}, e^{-x} and give the natural logarithms of a number.

(b) A and B

These are exactly alike and are used with the C and D scales to find the square and square root.

(c) BI

This is an inverted B scale.

(d) CI

This is an inverted C scale and is used with the C scale in reading directly the reciprocal of a number. And it lets us do multiplication of three factors with just one setting of the slide.

(e) C and D

These are exactly alike and the fundamental scales of the slide rule. And they are used for general fundamental calculations.

(f) LL_1, LL_2 and LL_3

These are used to find the values of the type forms of a^n, e^x and give the natural logarithms of a number.

(g) Sh_1 and Sh_2

These scales make it possible to compute the hyperbolic-sine function, which is frequently necessary in alternating current theory.

(h) P_1, Q and P_2

These are unlogarithmic square scales. The computation of the vector can be conveniently done by the P_1, P_2 and Q scales as ordinary multiplication and division done by the C and D scales.

(i) Sr and $S\theta$

These scales are used to obtain the sine and cosine of an angle, co-operate with the P_1 and Q scales. Angles are graduated at radian in the Sr scale, and degree and its decimal fraction in the $S\theta$ scale. Thus, conversion from degree to radian or its reverse process is made by the use of these scales.

(j) Th

This gives the value of the hyperbolic-tangent function.

(k) T_1 and T_2

These scales are used in the computation of the tangent of an angle from 0.1 to 0.8 radians, and from 0.8 to 1.48 radians respectively.

(l) L

This scale is used with with the C or D scale in finding directly the mantissa of the common logarithms of a number.

3. TORIGONOMETRIC FUNCTIONS

(1) $\sin x$

The sine of an angle, either in radian or in degree can be directly read on the Sr or $S\theta$ scale.

Example 1. $\sin 0.665 = 0.617$
Move hairline to 0.665 on Sr,
under hairline find 6.17 on Q,
read answer as 0.617.

Example 2. $\sin 51.8° = 0.786$
Move hairline to 51.8 on $S\theta$,
under hairline find 7.86 on Q,

read answer as 0.786.

(2) cos x

Example 3. cos 0.665 = 0.787

Move hairline to left index of P_1, set 0.665 on Sr under hairline, opposite right index of Q find 7.87 on P_1,

read answer as 0.787.

Example 4. cos 38.1° = 0.787

Move hairline to 38.1 on Sθ (red),

under hairline find 7.87 on Q,

read answer as 0.787.

(3) Conversion between Degree and Radian

Conversion from degree to radian or its reverse process is made by using the Sr and Sθ scales.

Example 5. 81° = 1.414 radians

Move hairline to 81 on Sθ,

under hairline read answer as 1.414 on Sr.

Example 6. 0.52 radians = 29.8°

Move hairline to 0.52 on Sr,

uuder hairline read answer as 29.8 on Sθ.

In the case of conversion of a small angle, move the decimal point one place to the right, perform the operation as is explained above, and read the answer moving the decimal point one place lower.

Example 7. 0.0935 radians = 5.36°

Move hairline to 0.935 on Sr,

under hairline find 53.6 on Sθ,

read answer as 5.36.

(4) tan x

The T_1 and T_2 scales represent a single scale of angles ranging from 0.1 to 1.48 radians.

When using the T_1 scale to read the value of tan x, read the left index of D as 0.1 and the right index as 1, and using the T_2 scale to read the value of tan x, read the left index of D as 1 and the right index as 10.

Example 8. tan 0.35 = 0.365

Move hairline to 0.35 on T_1,

under hairline find 3.65 on D,

read answer as 0.365.

Example 9. tan 1.17 = 2.36

Move hairline to 1.17 on T_2,

under hainline read answer as 2.36 on D.

4. VECTOR PROBLEMS

(1) The Absolute Value of Vector

The absolute value of vector, represented in the type form of $x+jy$, is equal to $\sqrt{x^2+y^2}$, and by the use of the P_1, Q and P_2 scales this value can be computed very easily in the same operation as ordinary multiplication and division. Namely set zero on the Q scale to x on the P_1 scale, opposite y on the Q scale read $\sqrt{x^2+y^2}$ on the P_1 scale. When y on the Q scale runs off the P_1 scale, then set 10 on the Q scale to x on the P_1 scale, and opposite y on the Q scale read the answer on the P_2 scale.

Exmple 10. $\sqrt{4.81^2+2.35^2}=5.35$

Opposite 4.81 on P_1, set left index of Q,
move hairline to 2.35 on Q.
under hairline read answer as 5.35 on P_1.

Example 11. $\sqrt{0.932^2+0.876^2}=1.279$

Calculate as $\frac{1}{10}\times\sqrt{9.32^2+8.76^2}$.
Opposite 9.32 on P_1, set right index of Q,
move hairline to 8.76 on Q,
under hairline find 12.79 on P_2,
read answer as 1.279,

Example 12. $\sqrt{2.09^2+0.681^2}=2.12$

Calculate as $\frac{1}{3}\times\sqrt{(2.09\times3)^2+(0.681\times3)^2}$

$$=\frac{1}{3}\times\sqrt{6.27^2+2.04^2}.$$

Opposite 6.27 on P_1, set right index of Q,
move hairline to 2.04 on Q,
under hairline find 6.36 on P_1,
read answer as $\frac{6.36}{3}=2.12$

(2) phase Angle of Vector

In the vector of the type form of $x+jy$, the phase angle θ is represented as follows;

$$\theta=\tan^{-1}\frac{y}{x}$$

note: $x>y$............Use the T_1 scale.
$x<y$............Use the T_2 scale.

Example 13. $\tan^{-1}\frac{3.6}{2.5}=0.964$

Move hairline to 3.6 on D,

set 2.5 on C under hairline,
move hairline to left index of C,
under hairline read 0.964 on T_2.

Example 14. $\tan^{-1}\frac{2.5}{3.6}=0.607$

Move hairline to 2.5 on D,
set 3.6 on C under hairline,
move hairline to right index of C,
under hairline read 0.607 on T_1.

(3(Conversion of Coordinate System

From the following relations;

$$x+jy=\sqrt{x^2+y^2}\ \Big|\ \tan^{-1}\frac{y}{x}=R\,\underline{/\theta}$$

here; $R=\sqrt{x^2+y^2}$

$$\theta=\tan^{-1}\frac{y}{x}$$

and $R\,\underline{/\theta}=R\cdot\cos\theta+jR\cdot\sin\theta$,

the conversion of vector in polar co-ordinate system to rectangular coordinate system and its reverse computation can be easily done.

Example 15. $7.50+j6.06=9.64\ \underline{/0.679}$

Opposite 7.5 on P_1, set left index of Q.
move hairline to 6.06 on Q,
under hairline read absolute value as 9.64 on P_1.
Move hairline to 6.06 on D,
set 7.50 on C under hairline,
move hairline to right index of C,
under hairline read answer as 0.679 on T_1.

Example 16. $2.50\ \underline{/52^\circ}=1.54+j\,1.97$

Move hairline to 52 on $S\theta$,
under hairline read sin 52°=0. 788 on Q,
move hairline to 52 on $S\theta$(red),
under hairline read cos 52°=0. 616 on Q,
thus, real part is calculated as
2. 5×0. 616=1. 58 and imaginary part
j 2. 5×0. 788=j 1. 97

(4) **Multiplication and Division of Vectors**

Multiplication and division of two vectors have been computed from the following formulas;

$$R_1\,\underline{/\theta_1}\times R_2\,\underline{/\theta_2}=R_1\cdot R_2\,\underline{/\theta_1+\theta_2}$$

$$\frac{R_1\,\underline{/\theta^1}}{R_2\,\underline{/\theta_2}}=\frac{R^1}{R^2}\,\underline{/\theta_1-\theta_2}$$

Example 17. $\frac{3-j2}{8+j4}=0.404\ \overline{\setminus 1.052}$

$$\frac{3-j2}{8+j4}=\frac{\sqrt{3^2+2^2}\ \underline{\Big/\tan^{-1}\left(\frac{-2}{3}\right)}}{\sqrt{8^2+4^2}\ \underline{\Big/\tan^{-1}\left(\frac{4}{8}\right)}}=\frac{3.61\,\underline{/-0.588}}{8.94\,\underline{/0.464}}$$

$$=0.404\,\underline{/-0.588-0.464}$$

$$=0.404\,\underline{/-1.052}=0.404\,\overline{\setminus 1.052}$$

Example 18. Calculate the currrent $\dot{I}$ in an electric circuit, which impedance is $\dot{Z}=4+j\ 2.6$ and the potentaial difference between its terminals is $\dot{E}=5+j9$.
answer $\dot{I}=1.907+j\ 1.013$

$$\dot{I}=\frac{\dot{E}}{\dot{Z}}=\frac{5+j9}{4+j2.6}=\frac{\sqrt{5^2+9^2}\ \underline{\Big/\tan^{-1}\left(\frac{9}{5}\right)}}{\sqrt{4^2+2.6^2}\ \underline{\Big/\tan^{-1}\left(\frac{2.6}{4}\right)}}$$

$$=\frac{10.29\,\underline{/1.064}}{4.77\,\underline{/0.576}}$$

$$=2.16\,\underline{/0.488}$$

$$=2.16\times\cos 0.488+j\,2.16\times\sin 0.488$$

$$=2.16\times 0.833=j\,2.16\times 0.469$$

$$\therefore\quad \dot{I}=1.907+j\,1.013$$

Example 19, Compute the resultant current $\dot{I}$, of $\dot{I}_1=2+j3$ and $\dot{I}_2=3+j4$ in polar coordinate.
Answer $\dot{I}=8.60\,\underline{/0.951}$

$$\dot{I}=\dot{I}_1+\dot{I}_2=(2+j3)+(3+j4)=5+j7$$

$$=\sqrt{5^2+7^2}\ \underline{\Big/\tan^{-1}\left(\frac{7}{5}\right)}$$

$$=8.60\,\underline{/0.951}$$

5. PROBLEMS OF ALTERNATING CURRENT CIRCUIT

By the use of the gauge mark "f", the problems of an alternating current circuit with inductance and capacitance can be easily done.

(1) **Inductive Reactance**

Example 20. Find the inductive reactance X_L in an alternating current circuit with 30mH inductance under 60 c frequency. Answer 11. 3 Ω

Calculate as $X_L=2\pi fL=2\pi\times 60\times 30\times 10^{-3}$

Move hairline to f on D,
set 6 on C under hairline,
move hairline to 3 on D,
under hairline find 1. 33 on C (extension Part),

read answer as 11.3 Ω.

(2) Capacitive Reactance

Example 21. Find the capacitive reactance X_L in an alternating current circuit with static capacity of 2.5 μF under 60 c frequency Answer 1061 Ω

Calculate as $X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\times 60\times 2.5\times 10^{-6}}$.

Move hairline to f on D,
set 6 on C under hairline,
move hairline to 2.5 on CI,
under hairline find 1.061 on D,
read answer as 1061 Ω.

(3) Resonance Frequency

Example 22. Find the series resonance frequency in an alternating current circuit with 150 mH inductance and 230μF capacitance.

Answer 27.1 c

Calculate as $f_r=\frac{1}{2\pi\sqrt{LC}}$

$$=\frac{1}{2\pi\times\sqrt{150\times 10^{-3}\times 230\times 10^{-6}}}$$

$$=\frac{1}{2\pi\times\sqrt{1.5\times 23\times 10^{-3}}}.$$

move hairline to 1.5 on A left,
set 23 on BI left under hairline,
move hairline to f on D,
under hairline find 2.71 on C,
read answer as 27.1 C.

(4) Surge Impedance

Example 23. Find the surge impedance Z_o, if L=280 mH and C=330 pF are given.

Answer 29.15 kΩ

Calculate as $Z_o=\sqrt{\frac{L}{C}}$

$$=\sqrt{\frac{280\times 10^{-3}}{330\times 10^{-12}}}=\sqrt{\frac{28}{3.3}}\times 10^4.$$

Move hairline to 28 on A right,
set 3.3 on B left under hairline,
opposite left index of C find 2.91 on D,
read answer as 29.1 kΩ.

6. HYPERBOLIC FUNCTIONS

(1) sinh x

The Sh_1 and Sh_2 scales give the value of sinh x. When using the Sh_1 scale to read the value of sinh x, the left index of D as 0.1 and the right index as 1, and using the Sh_2 scale to read the value of sinh x, read the left index of D as 1 and the right index as 10.

Example 24. sinh 0.362=0.370

Move hairline to 0.362 on Sh_1,
under hairline find 3.70 on D,
read answer as 0.370.

Example 25. sinh 2.56=6.42

Move hairline to 2.56 on Sh_2,
under hairline read answer as 6.42 on D.

(2) tanh x

The Th scale gives the value of tanh x. When using the Th scale to read the value of tanh x, read the left index of C as 0.1 and the right index as 1.

Example 26. tanh 0.183 = 0.181

Move hairline to 0.183 on Th,
under hairline find 1.81 on C,
read answer as 0.181.

(3) cosh x

The value of cosh x can be computed from the following formula:

$$\cosh x = \frac{\sinh x}{\tanh x}$$

Example 27. cosh 0.575 = 1.170

Move hairline to 0.575 on Sh_1,
set 0.575 on Th under hairline,
opposite left index of C read answer as 1.170 on D.

7. HYPERBOLIC FUNCTIONS OF COMPLEX QUANTITIES

(1) sinh $(x+jy)$

The hyperbolic sine of the complex quantities may be found from

$$\sinh(x+jy) = \sinh x \cdot \cos y + j \cosh x \cdot \sin y$$
$$= \sqrt{\sinh^2 x + \sin^2 y}\ \underline{/\tan^{-1}\left(\frac{\tan y}{\tanh x}\right)}$$

Example 28. $\sinh(0.43 + j\,0.68) = 0.769\ \underline{/1.106}$

sol. i) cos 0.68 = 0.778
sin 0.68 = 0.629
sinh 0.43 × cos 0.68 = sinh 0.43 × 0.778 = 0.345
Move hairline to 0.43 on Sh_1,
set right index of C under hairline,
move hairline to 7.78 on C,
under hairline find 3.45 on D,
read answer as 0.345.

cosh 0.43 × sin 0.68 = cosh 0.43 × 0.629 = 0.688
Move hairline to 0.43 on Sh_1,
set 0.43 on Th under hairline,
move hairline to 6.29 on C,
under hairline find 6.88 on D,
read answer as 0.688.

Therefore, $\sinh(0.43 + j\,0.68) = 0.345 + j\,0.688$.

Opposite 3.45 on P_1, set left index of Q,
move hairline to 6.88 on Q,
under hairline find 7.69 on P_1,
read answer as 0.769.
Move hairline to 6.88 on D,
set 3.45 on C under hairline,
move hairline to left index of C,
under hairline read answer as 1.106 on T_2.

sol. ii) sinh 0.43 = 0.443
Move hairline to 0.43 on Sh_1,
under hairline find 4.43 on D,
read answer as 0.443.

$\sqrt{\sinh^2 0.43 + \sin^2 0.68} = \sqrt{0.443^2 + \sin^2 0.68} = 0.769$

Opposite 4.43 on P_1, set left index of Sr,
move hairline to 0.68 on Sr,
under hairline find 7.69 on P_1,
read answer as 0.769.

$$\tan^{-1}\left(\frac{\tan 0.68}{\tanh 0.43}\right)=1.106$$

Move hairline to 0.68 on T_1,
set 0.43 on Th under hairline,
move hairline to left index of C,
under hairline read answer as 1.106 on T_2.

$\therefore$ $\sinh(0.43+j0.68)=0.769\,\underline{/1.106}$

$0.769\,\underline{/1.106}=0.769\times\cos 1.106+j\,0.769\times\sin 1.106$

Move hairline to right index of P_1,
set 1.106 on Sr under hainline,
under hairline find 8.94 on Q,
read answer as $0.894=\sin 1.106$,
opposite left index of Q find 4.48 on P_1,
read answer as $0.448=\cos 1.106$,
therefore $0.769\times 0.448=0.345$ and $0.769\times 0.894=0.688$

$\therefore$ $\sinh(0.43+j0.68)=0.769\,\underline{/1.106}=0.345+j0.688.$

Example 29. A telephone line has been series impedance $Z=16.8\,\underline{/1.07}\ \Omega$ per km and shunt admittance $Y=31.2\times 10^{-6}\,\underline{/1.53}$ ℧ per km. The curreut I flowing through a short circuit x km from the source end of the line is given by

$$I=E\cdot\sqrt{\frac{Y}{Z}}\cdot\frac{1}{\sinh(x\cdot\sqrt{Z\cdot Y})}.$$

Find I, when $E=25.8\,\underline{/0}$ V and $x=78$km.

Answer $I=31.7\,\overline{\backslash 1.408}$ mA.

Firstly, calculate $\sqrt{\dfrac{Y}{Z}}$.

$$\sqrt{\frac{Y}{Z}}=\frac{31.2\times10^{-6}/1.53}{16.8\,/1.07}=\sqrt{\frac{31.2\times10^{-6}}{16.8}}\,\Big/\tfrac{1}{2}(1.53-1.07)$$

$$=\sqrt{\frac{31.2}{16.8}}\times10^{-3}\,\Big/\tfrac{1}{2}(1.53-1.07)$$

$$=1.362\times10^{-3}\,/0.23$$

Secondly, calculate $x\cdot\sqrt{Z\cdot Y}$.

$$x\sqrt{Z\cdot Y}=78\times\sqrt{16.8\,/1.07\times 31.2\times10^{-6}\,/1.53}$$

$$=78\times\sqrt{16.8\times31.2\times10^{-6}}\,\Big/\tfrac{1}{2}x(1.07+1.53)$$

$$=78\times\sqrt{524\times10^{-6}}\,\Big/\tfrac{1}{2}\times2.60$$

$$=22.9\times10^{-3}\times78\,/1.30$$

$$=1785\times10^{-3}\,/1.30=1.785\,/1.30$$

$$=0.477+j1.72$$

Therefore,

$$\sinh(x\sqrt{Z\cdot Y})=\sinh(0.477+j1.72)$$

$$=\sinh 0.477\times\cos 1.72+j\cosh 0.477\times\sin 1.72$$

$$=\sinh 0.477\times\cos\left(\frac{\pi}{2}+0.149\right)+j\cosh 0.477\times\sin\left(\frac{\pi}{2}+0.149\right)$$

$$=\sinh 0.477\times(-\sin 0.149)+j\cosh 0.477\times\cos 0.149$$

$$=-\sinh 0.477\times0.149+j\cosh 0.477\times0.989$$

$$=-0.0738+j1.105.$$

$\therefore$ $\sqrt{0.0738^2+1.105^2}=1.107.$

$$\theta=\tan^{-1}\left(\frac{1.105}{-0.0738}\right)^2=\tan^{-1}(-14.97).$$

But, as the absolute value of $\tan\theta$ is larger than 10, you can not read θ on the T_2 scale. However, when angle θ is larger than $\frac{\pi}{2}-\varphi=1.471$, namely when angle φ is less than 0.1, there is the following relation:

$$\tan\theta=\tan\left(\frac{\pi}{2}-\varphi\right)\doteqdot\frac{1}{\sin\varphi}\doteqdot\frac{1}{\varphi}\quad\begin{cases}\theta>1.471\\ \varphi<0.1\end{cases}$$

Thus, $\theta\doteqdot\tan^{-1}\frac{1}{\varphi}$

From the above relation, we can find the value of the tangent angle θ is larger than $\frac{\pi}{2}-\varphi=1.471$.

Calculate as $\frac{1}{\varphi}=14.97$.

Move hairline to 14.97 on C,
under hairline find 6.67 on CI,
read answer as $\varphi=0.0667$.

$$\therefore\quad \theta=\frac{\pi}{2}-\varphi=1.571-0.0667=1.504$$

Thus, $\sinh(-0.0738+j\,1.105)=1.107\,\underline{/\pi-1.504}$.

Calculate as $I=\dfrac{25.8\,\underline{/0}\times1.362\times10^{-3}\,\underline{/0.23}}{1.107\,\underline{/\pi-1.504}}$

$$=\frac{25.8\times1.362\times10^{-3}}{1.107}\,\underline{/0+0.23-\pi+1.504}$$

$$=\frac{25.8\times1.362\times10^{-3}}{1.107}\,\underline{/1.734-\pi}$$

$$=31.7\times10^{-3}\,\underline{/-1.408}$$

$$\therefore\quad I=31.7\,\overline{\backslash 1.408}.$$

(2) cosh $(x+jy)$

The hyperbolic-cosine of the complex quantities may be found from,

$$\cosh(x+jy)=\cosh x\cdot\cos y+j\sinh x\cdot\sin y$$

$$=\sqrt{\sinh^2x+\cos^2y}\,\underline{/\tan^{-1}(\tan y\cdot\tanh x)}$$

Example 30. $\cosh(0.75-j\,1.24)=0.884\,\overline{\backslash 1.075}$

sol. i) $\cos(-1.24)=0.325$

$\sin(1.24)=-0.946$

$\cosh 0.75\times\cos(-1.24)=\cosh 0.75\times0.325=0.421$

Move hairline to 0.75 on Sh_1,
set 0.75 on Th under hairline,
move hairline to 3.25 on C′,
under hairline find 4.21 on D,
read answer as 0.421.

$\sinh 0.75\times\sin(-1.24)=\sinh 0.75\times(-0.946)=-0.777$

Move hairline to 0.75 on Sh_1,
set right index of C under hairline,
move hairline to 9.46 on C,
under hairline find 7.77 on D,
read answer as -0.777.

Therefore, $\cosh(0.75-j\,1.24)=0.421-j\,0.777$.

note: $\sqrt{0.421^2+0.777^2}=0.884$, $\theta=\tan^{-1}\left(\dfrac{-0.777}{0.421}\right)=-1.075$ $\Big\}\rightarrow 0.421-j\,0.777$

$$=0.884\,\underline{/-1.075}$$

sol. ii) $\sinh 0.75=0.822$

$\sqrt{\sin^2 0.75+\cos^2 1.24}=\sqrt{0.822^2+\cos^2 1.24}=0.884$

Move hairline to 8.22 on P_1,
set 1.24 on Sr under hairline,
opposite right index of Q, find 8.84 on P_1,
read answer as 0.884.

$\tan^{-1}(-\tan 1.24\times\tanh 0.75)=-1.075$

Move hairline to 1.24 on T_2,
set right index of C under hairline,
move hairline to 0.75 on Th,
under hairline find 1.075 on T_2,
read answer as -1.075.

note: $\left.\begin{array}{l} 0.884\times\cos(-1.075)=0.421 \\ 0.884\times\sin(-1.075)=-0.777 \end{array}\right\}\rightarrow 0.884\,\underline{/-1.075}$

$$=0.421-j\,0.777$$

Example 31. A transmission line has impedance $Z=5.3+j\,17.9\,\Omega$ per km and admittance $Y=j\,0.00087\,\mho$ per km.
Compute auxiliary constant A is given by $A=\cosh\sqrt{Z\cdot Y}$.

Answer $A=0.992+j\,0.0023$

Firstly, convert to polar coordinate.

$$Z=5.3+j\,17.9=18.67\,\underline{/1.283}$$

$$Y=j\,0.00087=0.00087\,\underline{\Big/\frac{\pi}{2}}$$

$$=0.00087\,\underline{/1.571}$$

$$\therefore\ \sqrt{Z\cdot Y}=\sqrt{18.67\,\underline{/1.283}\times 0.00087\,\underline{/1.571}}$$

$$=\sqrt{18.67\times 0.00087}\,\underline{\Big/\frac{1}{2}(1.283+1.571)}$$

$$=0.1274\,\underline{/1.427}=0.0183+j\,0.126.$$

Thus, $A=\cosh(0.0183+j\,0.126)$

$$=\cosh 0.0183\times\cos 0.126+j\sinh 0.0183\times\sin 0.126$$

$$\fallingdotseq 1\times 0.992+j\,0.0183\times 0.126=0.992+j\,0.0023$$

(3) $\tanh\,(x+jy)$

The type form of $\tanh(x+jy)$ is computed from the following formula:

$$\tanh\,(x+jy)=\frac{\sinh(x+jy)}{\cosh(x+jy)}$$

Example 32. $\tanh(0.56+j\,0.85)=1.08\,\underline{/0.627}$

Calculate as $\tanh(0.56+j\,0.85)=\dfrac{\sinh(0.56+j\,0.85)}{\cosh(0.56+j\,0.85)}$

$$\sinh(0.56+j\,0.85)=0.955\,\underline{/1.151}$$

$$\cosh(0.56+j\,0.85)=0.885\,\underline{/0.524}$$

$$\therefore\ \tanh(0.56+j\,0.85)=\frac{0.955\,\underline{/1.151}}{0.885\,\underline{/0.524}}=\frac{0.955}{0.885}\,\underline{/1.151-0.524}$$

$$=1.08\,\underline{/0.627}$$

note: $\left.\begin{array}{l} 1.08\times\cos\ 0.627=0.875 \\ 1.08\times\sin\ 0.627=0.634 \end{array}\right\}\rightarrow 1.08\,\underline{/0.627}=0.875$

$$+j\,0.634$$

(4) $\tanh^{-1}(x+yj)$

The calculation of the type form of $\tanh^{-1}(x+jy)$ is worked out the following procedures;

Let $\tanh^{-1}(x+jy)=a+jb$,

above relation can be set up as follows;

$$a=\frac{1}{2}\log_e R,\quad b=\frac{\theta}{2}.$$

$$\text{and}\quad \frac{1+(x+jy)}{1-(x+jy)}=R\,\underline{/\theta}.$$

i) calculate $\dfrac{1+(x+jy)}{1-(x+jy)}=R\,\underline{/\theta}$.

ii) calculate $a=\dfrac{1}{2}\log_{10}\ R$.

iii) calculate $b=\dfrac{\theta}{2}$.

Example 33. $\tanh^{-1}(0.734+j\,0.448)$

$=0.618+j\,0.644$

$1+(0.734+j\,0.448)=1.734+j\,0.448$

$=\frac{1}{5}\times(1.735\times5+j\,0.448\times5)=\frac{1}{5}\times(8.67+j\,2.24)$

$=1.791\,\angle 0.253$

$1-(0.734+j\,0.448)=0.226-j\,0.448=0.521\,\angle -1.035$

$\therefore\ \frac{1.791\,\angle 0.253}{0.521\,\angle -1.035}=\frac{1.791}{0.521}\,\angle 0.253-(-1.035)=3.44\angle 1.288$

Thus, $a=\frac{1}{2}\log_e 3.44=\frac{1}{2}\times1.236=0.618$

$b=\frac{1.288}{2}=0.644$

$\therefore\ \tanh^{-1}(0.734+j\,0.448)=0.618+j\,0.644.$

Example 34. Determine the unit line constant per km from the following values, with the line length of 85 km under 60 cycles.

Admittance $Y_0=230.25\times10^{-1}\,\angle 90^\circ$ ℧

Impedance $Z_s=78.5\,\angle 73.6^\circ\,\Omega$

Answer $Z=0.920\,\angle 73.8^\circ\,\Omega$/km

$Y\doteq j2.70\times10^{-6}$ ℧

The values of admittance Y_0 and impedance Z_s are given the following formulas;

$$Y_0=\frac{\tanh\xi D}{R},\quad Z_s=R\tanh\xi D$$

From the above relations, we can find the values of Y and Z by using the following formulas:

$$Y=\frac{\xi}{D},\quad Z=\xi R$$

Firstly, calculate R and ξD.

$$R=\sqrt{\frac{Z_s}{Y_0}}=\sqrt{\frac{78.5\,\angle 73.6^\circ}{230.25\times10^{-6}\,\angle 90^\circ}}$$

$=\sqrt{\frac{78.5}{230.25}}\times10^3\,\angle\frac{73.6^\circ-90^\circ}{2}$

$=0.584\times10^3\,\angle 8.2^\circ$

$\tanh\xi D=\sqrt{Y_0\cdot Z_s}$

$=\sqrt{230.25\times10^{-6}\times78.5\,\angle 90^\circ+73.6^\circ}$

$=\sqrt{2.3025\times78.5\times10^{-4}\,\angle 163.6^\circ}$

$=\sqrt{2.3025\times78.5}\times10^{-2}\,\angle\frac{163.6^\circ}{2}$

$=13.44\times10^{-2}\,\angle 81.8^\circ$

$=0.1344\,\angle 81.8^\circ$

$=0.01918+j\,0.1331$

$\therefore\ \xi D=\tanh^{-1}(0.01918+j\,0.1331)$

$\frac{1+(0.01918+j\,0.1331)}{1-(0.01918+j\,0.1331)}=\frac{1.01918+j\,0.1331}{0.98082-j\,0.1331}$

$=\frac{1.028\,\angle 7.44^\circ}{0.990\,\angle 7.73^\circ}$

$=1.038\,\angle 7.44^\circ+7.73^\circ$

$=1.038\,\angle 15.17^\circ$

$\therefore\ \frac{1}{2}\log_e 1.038=\frac{1}{2}\times0.0373=0.01865$

$\frac{15.17^\circ}{2}=\frac{0.265}{2}=0.1325$

$\therefore\ \xi D=\tanh^{-1}(0.01918+j\,0.1331)$

$=0.01865+j\,0.1325=0.1339\,\angle 82^\circ$

therefore, $\xi=\frac{0.1339\,\angle 82^\circ}{85}1.575\times10^{-3}\,\angle 82^\circ$

from above values;

$Z=\xi R=(1.575\times10^{-3}\,\angle 82^\circ)\times(0.584\times10^3\,\angle -8.2^\circ)$

$=1.575\times0.584\,\angle 82^\circ-8.2^\circ$

$$=0.920\,\angle 73.8^\circ$$
$$=0.0257+j\,0.883\ \Omega/\text{km}$$
$$Y=\frac{\xi}{R}=\frac{1.575\times10^{-3}\,\angle 82^\circ}{0.584\times10^{3}\,\backslash 8.2^\circ}$$
$$=\frac{1.575}{0.584}\times10^{-6}\,\angle 82^\circ+8.2^\circ$$
$$=2.70\times10^{-6}\,\angle 90.2^\circ$$
$$\fallingdotseq j\,2.70\times10^{-6}\ \mho/\text{km}$$

……………… Answers

8. HOW TO USE LL SCALES

(1) Explanation of LL Scales

LL represent that the scale is a logarithm of a logarithm There are two groups of LL scales. One is LL_n scales (LL_1, LL_2 and LL_3) ranging from 1.01 to 10^5 for the computation of the type form of a^{+n} and the other is $LL_{\bar{n}}$ scales ($LL_{\bar{1}}$, $LL_{\bar{2}}$ and $LL_{\bar{3}}$) called Reciprocal LL_n scales ranging from 10^{-5} to 0.99 for the computation of the type form of a^{-n}.

Construction of LL scales { LL_n group…LL_1, LL_2, LL_3 ; $LL_{\bar{n}}$ group…$LL_{\bar{1}}$, $LL_{\bar{2}}$, $LL_{\bar{3}}$ }

And the LL_n and $LL_{\bar{n}}$ scales are in a reciprocal relation respectively. This arrangement can be used very conveniently for calculations of powers and roots of numbers.

(2) Natural Logarithms

Set the hairline to the given number N on the LL scale, $\log_e N$ will be found out under the hairline on the D scale.

Determining the position of the decimal point is as follows:

When N is set on	the LL_3 scale	… one digit at the left	of the decimal point,
	the LL_2 scale	… one digit at the right	
	the LL_1 scare	… two digits at the right	

when N is set on	the $LL_{\bar{3}}$ scale	… one digit at the left	of the decimal point and place the negative sign before the figures.
	the $LL_{\bar{2}}$ scale	… one digit at the right	
	the $LL_{\bar{1}}$ score	… two digits at the right	

Example 35. $\log_e 5=1.609$

Move hairline to 5 on LL_3,
under hairline read answer as 1.609 on D.

Example 36. $\log_e 2=0.693$

Move hairline to 2 on LL_2,
under hairline find 6.93 on D,
read answer as 0.693.

Example 37. $\log_e 1.03=0.0296$

Move hairline to 1.03 on LL_1,
under hairline find 2.96 on D,
read answer as 0.0296.

Example 38. $\log_e 0.23=-1.47$

Move hairline to 0.23 on $LL_{\bar{3}}$,
under hairline find 1.47 on D,
read answer as -1.47.

Exomple 39. $\log_e 0.625=-0.47$

Move hairline to 0.625 on $LL_{\bar{2}}$,
under hairline find 4.7 on D,
read answer as -0.47

Example 40. $\log_e 0.955=-0.0461$

Move hairline to 0.955 on $LL_{\bar{1}}$,

under hairline find 4.61 on D,
read answer as −0.0461.

(3) powers and Roots

The type form of $a^{\pm n}$ or $a^{\pm\frac{1}{n}}$ is simply calculated by the use of the LL scales in an operation similar to multiplication and division.

Example 41. $4.25^{2.12}=21.5$, $4.25^{-2.12}=0.0466$

Move hairline to 4.25 on LL_3,
set left index of C under hairline,
move hairline to 2.12 on C,
under hairline read answer as 21.5 on LL_3,
under hairline read answer as 0.0466 on $LL_{\overline{3}}$.

Example 42. $1.96^{2.3}=4.70$, $1.96^{-2.3}=0.213$

Move hairline to 1.96 on LL_2,
set right index of C under hairline,
move hairline to 2.3 on C,
under hairline read answer as 4.70 on LL_3,
under hairline read answer as 0.213 on $LL_{\overline{3}}$.

Example 43. $1.02^{24.5}=1.624$, $1\ 02^{-24.5}=0.616$

Move hairline to 1.02 on LL_1,
set left index of C under hairline,
move hairline to 2.45 on C,
under hairline read answer as 1.624 on LL_2,
under hairline read answer as 0.616 on $LL_{\overline{2}}$.

Example 44. $11.4^{0.7}=5.50$, $11.4^{-0.7}=0.182$

Move hairline to 11.4 on LL_3,
set 7 on CI under hairline,

move hairline to left index of C,
under hairline read answer as 5.50 on LL_3,
under hairline read answer as 0.182 on $LL_{\overline{3}}$.

Example 45. $330^{\frac{1}{6.2}}=2.55$, $330^{-\frac{1}{6.2}}=0.392$

Move hairline to 330 on LL_3,
set 6.2 on C under hairline,
move hairline to right index of C,
under hairline read answer as 2.55 on LL_2,
under hairline read answer as 0.392 on $LL_{\overline{2}}$.

Example 46. $28.5^{\frac{2.91}{3.41}}=17.4$, $28.5^{-\frac{2.91}{3.41}}=0.0574$

Move hairline to 28.5 on LL_3,
set 3.41 on C under hairline,
move hairline to 2.91 on C,
under hairline read answer as 17.4 on LL_3
under hairline read answer as 0.0574 on $LL_{\overline{3}}$.

Example 47. $0.795^{1.4}=0.725$, $0.795^{-1.4}=1.379$

Move hairline to 0.795 on $LL_{\overline{2}}$
set left index of C under hairline,
move hairline to 1.4 on C,
under hairline read answer as 0.725 on $LL_{\overline{2}}$,
under hairline read answer as 1.379 on LL_2.

Example 48. $0.795^{14}=0.0402$, $0.795^{-14}=24.9$

Move hairline to 0.795 on $LL_{\overline{2}}$,
set left index of C under hairline,
move hairline to 1.4 on C,
under hairline read answer as 0.0402 on $LL_{\overline{3}}$,
under hairline read answer as 24.9 on LL_3.

Example 49, $e^{1.96}=7.10$, $e^{-1.96}=0.1408$

Move hairline to 1.96 on D,

under hairline read answer as 7.10 on LL_3,

under hairline read answer as 0.1408 on $LL_{\overline{3}}$.

Example 50. $e^{0.94}=2.56$, $e^{-0.94}=0.39$

Move hairline to 9.4 on D,

under hairline read answer as 2.56 on LL_2,

under hairline read answer as 0.39 on $LL_{\overline{2}}$.

Example 51. $e^{0.056}=1.0576$, $e^{-0.056}=0.9455$

Move hairline to 5.6 on D,

under hairline read answer as 1.0576 on LL_1,

under hairline read answer as 0.9455 on $LL_{\overline{1}}$.

——THE END——