Relay

BAMBOO SLIDE RULES

DUPLEX

INSTRUCTION
for
General Calculation

SAN-AI KEIKI CO., LTD.

Advantages of "Relay" Bamboo Duplex Slide Rule

Bamboo, which is a special product of Japan does not shrink or lengthen under any change of atmospheric temperature and humidity. Each part of the "Relay" Bamboo Duplex Slide Rule is composed of 2-5 pieces of well selected and seasoned bamboo.

The graduations of the "Relay" Bamboo Duplex Slide Rule are divided, line by line, by a special machine, so they are very accurate and distinct.

CONTENTS

The Scales and Their Usages

The following is the brief description of the various scales of the "Relay" Bamboo Duplex Slide Rules.

1. **C and D scales.** These fundamental scales are exactly alike and are used for all operations; multiplication and division etc.

2. **CF and DF scales.** These are C and D scales "folded" at π (π=3.1416), and are used with C and D scales in order to decrease the number of operations.

3. **CI scale.** This is an "inverted" C scale, and is used with C scale in reading directly the reciprocal of a number.

4. **CIF scale.** This is a CI scale "folded" at π, and is used with CF scale in the same relation as CI scale with C scale.

5. **S scale.** This scale gives the sines and cosines of angles.

6. **T scale.** This scale gives the tangents and cotangents of angles.

7. **S T scale.** This scale gives the sines and tangents of small angles.

8. **A and B scales.** These scales consist of two half size of C or D scales placed end to end. These scales are used with C and D scales to give squares and square roots.

9. **K scale.** This scale consists of three one-third size of C scale placed end to end, and is used in finding cubes and cube roots.

10. **L scale.** This scale is used with D scale in giving directly the mantissa of the common logarithms of a number.

11. **LL scale.** Most of our duplex slide rules have so-called log log scales, which are used in calculating expressions such as x^y ($x>1$). LL scales also give directly the value of the function e^x and are used in reading the natural logarithms of numbers.

Article No.		Scale Range
651,	LL_1	from $e^{0.1}=1.105$ to $e=2.718$
	LL_2	from $e=2.718$ to $e^{10}=22026$
	with some extension scales on both ends, used with C and D scales.	
100	LL	from $e^{0.1}=1.105$ to $e^{10}=22026$
	with some oxtension scales on both ends, used with A and B scales.	
150	LL_1	from $e^{0.01}=1.010$ to $e^{0.1}=1.105$
	LL_2	from $e^{0.1}=1.105$ to $e=2.718$
	LL_3	from $e=2.718$ to $e^{10}=22026$
	used with C and D scales.	
157	LL_2	from $e^{0.1}=1.105$ to $e=2.718$
	LL_3	from $e=2.718$ to $e^{10}=22026$
	LL'_1	from 1 to 1.1
	used with C and D scales. This LL'_1 scale is deviced by us to substitute LL_1 and LL_0 scales.	

12. **LL_0, LL_{00} and RLL scales.** Some of our slide rules also have LL_0, LL_{00} and RLL scales, which are used with A and B scales in finding powers of numbers x^y smaller than 1

(x<1). These also give directly the values of the functions e^x for negative values of x.

Article No.		Scale Range
100	RLL	from $e^{-0.1}=0.905$ to $e^{-10}=0.0000454$ with some extension scales on both ends.
150	LL_0	from $e^{-0.001}=0.999$ to $e^{-0.1}=0.905$
	LL_{00}	from $e^{-0.1}=0.905$ to $e^{-10}=0.0000454$

Slide Rule Operations

In what follows, the left hand 1 of a scale is called its Left Index, the right hand 1 is called its Right Index.

1. Multiplication

Rule: a Locate one of the factors on D and set the right or left index of C on it.

b Opposite the other factor on C, read the product on D.

Example 1. $24 \times 3 = 72$

a Opposite 24 on D, set the left index of C.

b Opposite 3 on C, read answer 72 on D.

Example 2. $4.5 \times 3.2 = 14.4$

a Opposite 45 on D, set the right index of C.

b Opposite 32 on C, read answer 14.4 on D.

Note in this case that the reading would have been "Off Scale" if the left index had been used.

The decimal point may be fixed by making a rough mental calculation.

We can also operate these calculations by using D and CI scale.

Example 3 $2.3 \times 3.4 = 7.82$

a Opposite 2.3 on D, set 3.4 on CI.

b Opposite right index of C, read answer 7.82 on D.

2. Continuous Multiplication

To multiply three factors, first multiply two of them, and then multiply the result by the third.

Example $1.5 \times 3.2 \times 8 = 38.4$

a Opposite 15 on D, set the left index of C.

b Opposite 32 on C, set the hair line.

c Opposite the hair line, set the right index of C.

d Opposite 8 on C, read answer 38.4 on D.

You need not read the intermediate answer $1.5 \times 3.2 = 4.8$. The decimal point can be determined by a rough mental calculation.

3. Division

Rule: a Locate the dividend on D, set the diviser on C.

b Opposite the index of C, read the quotient on D.

Example $58.5 \div 3 = 19.5$

a Opposite 58.5 on D, set 3 on C.

b Opposite the left index of C, read 19.5 on D.

As you see, this operation is exactly the inverse of multiplication.

4. Mixed Calculation of Multiplication and Division

Example $\frac{1.47 \times 30 \times 4}{3.5 \times 2} = 25.2$

a Opposite 1.47 on D, set 3.5 on C.

b Opposite 3 on C, set the hair line of runner.

c Opposite the hair line, set 2 on C.

d Opposite 4 on C, read 25.2 on D.

5. The Folded Scale CF and DF

CF and DF scales are similar to C and D scales folded at π. As π is very near to $\sqrt{10}$, so 1 of CF and DF scales lie about in the middle and π on both ends of scale. These scales can often be used in calculation in order to avoid resetting when the answer runs off scale.

Example Convert 2 and 6 feet in metres.

As 292 feet = 89 metres

a Opposite 89 on D, set 292 on C.

b Opposite 2 on C, read 0.61 on D (2f=0.61 m).

c Opposite 6 on CF, read 1.83 on DF (6f=1.83 m).

As you see in above example when the slide is in any position with a number x on the D scale appearing opposite a number y on the C scale, then this same number x appears also on the DF scale opposite y on the CF scale. If the reading is off scale on the C-D scale it may be found on the CF-DF scale.

Moreover we can use the CF and DF scales in problems requiring multiplication by π (π=3.142 approximately). Opposite any number on the D scale, read π times of this number on the DF scale. Thus if we take any number on the D scale as diameter of a circle, its circumference can be found on the DF scale.

6. Squares and Square Roots

Opposite any number on the C scale, read its square on the B scale.

Example Opposite 3.5 on C, read $3.5^2=12.35$ on B.

Conversely opposite any number on the B scale, read its square root on the C scale.

Example Opposite 484 on B (left), read $\sqrt{484}=22$ on C.

Opposite 0.64 on B (right), read $\sqrt{0.64}=0.8$ on C.

Use the left or right half of the B scale as shown in the following table.

	left half of B	right half of B
A given number	1 ... 10	10 ... 100
	100 ... 1000	1000 ... 10000
	⋮	⋮
	⋮	1 ... 0.1
	0.1 ... 0.01	0.01 ... 0.001
	0.001 ... 0.0001	0.0001 ... 0.00001

7. Cubes and Cube Roots

Opposite any number on the D scale, read its cube on the K scale. Thus ;

Example Opposite 3.2 on D, read $3.2^3=32.8$ on K.

The decimal point may be fixed by making a rough mental calculation.

Conversely, opposite a number on K scale read its cube on the D scale.

Example

Opposite 5360 on K (left), read $\sqrt[3]{5360}=17.5$ on D.

Opposite 28.6 on K (middle), read $\sqrt[3]{28.7}=3.05$ on D.

Opposite 0.186 on K (right), read $\sqrt[3]{0.186}=0.571$ on D.

Use left, midple or right third of the K scale as shown in the following table.

	left third of K	middle third of K	right third of K
a given number	1...10	10...100	100...1000
	1000...10000	10000...100000	100000...1000000
	⋮	⋮	⋮
	0.01...0.001	0.1...0.01	1...0.1
	0.00001...0.000001	0.0001...0.00001	0.001...0.0001

8. Receiprocal

Opposite any number on the C scale, read its reciprocal on the CI scale. The number on the CI scale is given by the red figures.

Example Opposite 2.5 on C, read $\frac{1}{2.5}=0.4$ on CI.

Opposite 125 on C, read $\frac{1}{125}=0.008$ on CI.

9. Another Fundamental Calculation

a. $a^2b=x$ $1.5^2\times 3.14=7.07$

a Opposite a on D, set left index of C.

b Opposite b on B, read x on A,

b. $a^2b^2=x$ $72^2\times 0.45^2=1050$

a Opposite a on D, set right index of C.

b Opposite b on C, read x on A.

c. $\frac{a^2}{b}=x$ $\frac{11^2}{4.9}=24.7$

a Opposite a on D, set b on B,

b Opposite index of C, read x on A.

d. $\frac{a^2b}{c}=x$ $\frac{8.05^2\times 0.34}{51.5}=0.428$

a Opposite a on D, set c on B.

b Opposite b on B, read x on A.

e. $\sqrt{ab}=x$ $\sqrt{1.83\times 0.26}=0.69$

a Opposite a on A, set index of B.

b Opposite b on B, read x on D.

f. $\frac{a}{\sqrt{b}}=x$ $\frac{79.3}{\sqrt{2.35}}=51.7$

a Opposite a on D, set b on B.

b Opposite index of C, read x on D.

g. $\frac{a\sqrt{b}}{c}=x$ $\frac{31.93\times\sqrt{147}}{3.2}=120.8$

a Opposite a on D, set c on C.

b Opposite b on B, read x on D.

h. $ab^3=x$ $0.65\times 2.3^3=7.91$

a Opposite a on K, set index of C.

b Opposite b on C, read x on K.

i. $\frac{ab^3}{c^3}=x$ $\frac{1.95\times 6.08^3}{3.9^3}=7.39$

a Opposite a on K, set c on C.

b Opposite b on C, read x on K.

j. $\sqrt{a^3b^3}=x$ $\sqrt{9.42^3\times 4.12^3}=242$

a Opposite a on A, set index of B.

b Opposite b on B, read x on K.

As a=9.42, take a on left half of B, and as b=4.12 take b on left half of B.

10. The Sine of an Angle

To get the sine of an angle α, we use

S and C or D scales for No. 100

S and B or A scales for No. 450, 550, 650, 651, 652

Sr (S_θ) and P scales for No. 157

Set the hair line on α (on S scale), read sin α on D scale (No. 150) or on C scale (No. 100).

Example sin 9° 30′ (No. 150)

Set the hair line to 9° 30′ on S, read 0.1650 on D (under the hair line).

11. The Cosine of an Angle

We find the cosine of an angle α by reading the sine of its compliment $90-\alpha$, or

$$\cos 32° = \sin (90° - 32°) = \sin 58°$$

12. The Tangent of an Angle

To get the tangent of an angle α, set the hair line on α (on T scale) and read tan α on D scale (No. 150) or on C scale (No. 100)

Example tan 34°

Set the hair line to 34° on T, read 0.675 on D.

13. The Sine and Tangent of an Angle smaller than 5° 44′

To obtain the value of sine and tangent of an angle between 0° 34′ and 5°44′, the ST scale is used. Note that the value of sine and tangent of these small angles are nearly alike.

Example sin 1° 30′ (≒tan 1° 30′)

Set the hair line to 1° 30′ on ST, read 0.00262 on D (No. 150).

14. Function of Angles Less than 0° 34′

The values of angles less than 0° 34′ can be obtained readily by using the relation.

sin α= tan α = α (in readian) approximately.

0.1° = 0.002 radians

0.1′ = 0.0003 radians

0.1″ = 0.000005 radians

Example sin 4′ = 4× 0.0003 = 0.0012

15. Other Trigonometrical Functions

To get cotangent, secant and cosecant of an angle ,we use the following formula.

$$\cot \alpha = \frac{1}{\tan \alpha}$$

$$\sec \alpha = \frac{1}{\cos \alpha}$$

$$\operatorname{cosec} \alpha = \frac{1}{\sin \alpha}$$

Thus, first take the tangent, cosine and sine of α then get their reciprocals.

16. Logarithms

Slide rule gives only the mantissa or decimal part of the common logarithms of a number, and the characteristic or the integral part can be determined by inspection. We use in this calculation L and C (D) scale.

Example log 38.7=1.588

a Set hair line to 38.7 on D, read 0.588 on L.

b Add characteristic 1, then the answer is 1.558.

Log log Scales.

The log log scales LL_1, LL_2 and LL_3 are used in the calculations involving fractional powers or roots of numbers such as $1.03^{2.67}$ $58.2^{\frac{1}{7.17}}$ and LL_0, LL_{00} for the calculations such as $0.452^{3.78}$ and $0.763^{0.215}$ etc.

17. LL_1, LL_2, LL_3 Scales

These scales are used in case the numbers are greater than one, although the exponents may be either greater or less than one. The standard operation for multiplication and division can be similary applied to exponential expression x^y except the following technique ;

Rule ; a Set the number on the LL (or LL_0) scales instead of the D scale, and the exponent on the CI and C (or CIF and CF) scale.

b Read the answer on the LL or LL_0 scale.

Example $1.024^{3.73}=1.0925$

a Opposite 1.024 on LL_1, set left index of C.

b Opposite 3.73 on C, read 1.0925 on LL_4.

or

a Opposite 1.024 on LL_1, set 3.73 on CI.

b Opposite right index of C, read 1.0925 on LI_4.

The following examples show the various operations according to the exponent.

Example 1. $1.0246^{5.25}=1.136$

The eqponent is between 1 and 10. Sometimes the right

index will be found to be off of the rule, when a scale is followed to the right to read the answer. In such cases read the answer on the higher scale under the left index.

a Opposite 1.0246 on LL_1, set 5.25 on CI.

b Opposite left index of CI, read 1.136 on LL_2.

Example 2. $2.14^{0.643}=1.631$

The exponent is between 1 and 0.1. In this case, as the exponent is less than one the answer is always read on the left of the number.

a Opposite 2.14 on LL_2, set 0.643 on CIF.

b Opposite centre index of CIF, read 1.631 on LL_2.

Example 3. $1.048^{36}=5.41$

The exponent is between 10 and 1000. In this case, reduce the exponent to a number between 1 and 10 perform the operation for this new exponent, and read the answer on the higher scales, depending upon the number of places the decimal point was moved.

a Perform as $1.048^{3.6}$.

b Opposite 1.048 on LL_1, set 3.6 on CI.

c Opposite left index of CI, read 5.41 on LL_3.

Example 4. $9.65^{0.058}=1.1405$

The exponent is between 0.1 and 0.001. The procedure is similar to Example 3, that is, move the decimal point one or two places to the right, perform the operation for this new exponent, and read the answer on one or two scales lower depending upon the numbers of places the decimal point was moved.

a Calculate as $9.65^{0.58}$.

b Opposite 9.65 on LL_3, set 5.8 on CI.

c Opposite left index of CI, read 1.1405 on LI_3.

18. LL_0 LL_{00} Scales

These scales are used with A and B scales, in the case the numbers are less than one, although the exponents may be either greater or less than one. Depending upon the order of exponent use the left or right half of B scale.

exponent			B scale
1—10	100—1000	0.1—0.01	use the left half
10—100	1—0.1	0.01—0.001	use the right half

Example 1. $0.816^{4.41}=0.408$

The exponent is between 1 and 10.

a Opposite 0.816 on LL_{00}, set left index of B.

b Opposite 4.41 on B (left), read 0.408 on LL_{00}.

Example 2. $0.847^{27}=0.0114$

a Opposite 0.847 on LL_{00}, set left index of B.

b Opposite 27 on B (right), read 0.0114 on LL_{00}.

Example 3. $0.9835^{254}=0.0146$

The exponent is between 100 and 1000.

a Perform as $0.9835^{2.54}$.

b Opposite 0.9835 on LL_0, set left index of B.

c Opposite 2.54 on B (left), read 0.0146 on LL_{00} (instead of LL_0).

Example 4. $0.024^{0.21}=0.457$

The exponent is between 1 and 0.1.

a Opposite 0.24 on LL_{00}, set right index of B.

b Opposite 0.21 on B (right), read 0.457 on LL_{00}.

Example 5. $0.028^{0.00885}=0.969$

The exponent is between 0.01 and 0.001.

a Opposite 0.028 on LL_{00}, set right index of B.

b Opposite 0.00885 on B, read 0.969 on LL_0 (instead of LL_{00}).

19. Powers of e

We can read directly e^x from LL scales and e^{-x} from LL_0 scales.

$0.01<x<0.1$	Opposite x on D, read e^x on LL_1.
$0.1<x<1$	Opposite x on D, read e^x on LL_2.
$1<x<10$	Opposite x on D, read e^x on LL_3.
$0.001<x<0.01$	Opposite x on B (left), read e^{-x} on LL_0.
$0.01<x<0.1$	Opposite x on B (right), read e^{-x} on LL_0.
$0.1<x<1$	Opposite x on B (left), read e^{-x} on LL_{00}.
$1<x<10$	Opposite x on B (right), read e^{-x} on LL_{00}.

Example Opposite 3 on D, read $e^{0.3}=1.350$ on LL_2.
Opposite 3 on B (right), read $e^{-3}=0.0498$ on LL_{00}.

20 Natural Logarithms

Logarithms to the base e (e=2.71828) are called natural logarithms. We denote the natural logarithms of a number N by the symbol In N. We can read from LL scales natural logarithms of a number between 1.10 and 22026, from LL_0 scales that of a number between 0.999 and 0.0005.

Example Opposite 1.04 on LL_1, read In 1.04=0.0392 on D.
Opposite 8.4 on LL_3, read In 8.4=2.13 on D.
Opposite 0.94 on LL_0, read In 0.94=−0.0619 on A.

21. LL_1' Scale

As a extension of the C scale there is a very short scale at the right end of the C scale marked from 1 to 1.1 in red. This minute scale called LL'_1 scale is invented in our laboratory to substitute the several lower LL scales in calculation of In x, x^y and e^x etc.

(1) Natural Logarithms

When x is nearly equal to 1, its natural logarithms is approximately equal to $x-1$. But using LL'_1 scale we can obtain its value more precisely. The procedure is ;

a Opposite $(x-1)$ on D, set the hair line.

b Opposite the hair line, set x on LL_1'.

c Opposite right index of C, read answer on D.

Example In $1.05=0.0488$

In ordinary slide rule we use the LL_1 and D scales, and read the result on the LL_1 scale.

a Opposite 0.05 on D, set 1.05 on LL_1'.

b Opposite right index of C, read 0.0488 on D.

(2) Powers of e

When x is nearly equal to zero, the value of e^x is approximately equal to $(1+x)$. Using LL'_1 scale we can obtain it precisely ;

a Opposite x on D, set the right index of C.

b Move the hair line along LL'_1 until the number of decimal part (LL'_1-1) on LL'_1 scale coincide with the number on D (that is approximately equal to $1+x$ on LL'_1).

c Under the hair line read the fractional part of answer.

Example $e^{0.021}=1.0212$

In ordinary slide rule the result is read on the LL_1 scale opposite the D scale.

a Opposite 0.021 on D, set the right index of C.

b Move the hair line until the number of decimal part (LL'_1-1) on LL'_1 scale coincide with the number on D, (that is approximately equal to $1+0.021$).

c Under the hair line, read 0.0212 on D, thus the answer is $1+0.0212=1.0212$.

(3) Computation of x^y

Using the LL'_1 scale we can compute the form x^y without LL_1 and LL_0 scales.

Example 1. $1.06^{2.68}=1.169$

In ordinary slide rule we use the LL_1 and CI scales, and read the answer on the LL_2 scale.

a Opposite 0.06 on D, set 1.06 on LL'_1.

b Opposite 2.63 on C, read 1 169 on LL_2.

Example 2. $1.008^{14.8}=1.125$

Without the ordinary LL_0 scale this computation can be done.

a Opposite 0.008 on D, set 1.008 on LL'_1.

b Opposite 14.8 on C, read 1.125 on LL_2.

Example 3. $1.21^{\frac{1}{7}}=1.0276$

In ordinary slide rule the answer is read on the LL_1 scale.

a Opposite 1.21 on LL_2, set 7 on C.

b Move the hair line until the number of decimal part $(LL'_1 - 1)$ on LL'_1 scale coincide with the number on D (that is approximately equal to 0.027)

c Under the hair line, read the fractional part 0.0276, then the answer is $1 + 0.0276 = 1.0276$

Duplex Slide Rule for Expert Electrical Engineer

(Relay No. 157)

22. General Description

This slide rule has been designed for expert electrical engineer to simplify the various calculations occurred frequently in electricity, namely not only the computation of multiplication and division can be done with A, B, C, D, CF, DF and K scales, but also LL_1, LL_2, and LL'_1 scales make it possible to obtain the result of x^y, e^x and In x. Moreover, the P, Q and P′ scales are essential for vector computation, Sh_1, Sh_2 and Th scales for hyperbolic function.

23. Arrangement and Usage of Scales

Front face ; Sr, S_θ, P′, P, Q, CF, CI, C, D, DF, LL_2, LL_3, LL'_1,

Back face ; Sh_2, Sh_1, A, B, K, Th, C, D, Tr_1, Tr_2, db,

(1) Sr, S_θ Scales

These scales are used to obtain the sine and cosine of an angle, cooperate with P and Q scales. Angles are graduated at degree and its decimal fraction in Sr, and at radian in S_θ scale. Thus conversion from degree to radian or its reverse process is made by the use of these scales.

(2) P, Q and P′ Scales

The computation of the vector can be conveniently done by these scales as ordinary multiplication and division done by C and D scales.

(3) C, D, CI, CF, DF, A, B and K scales

Of these scales, CF and DF scales are C and D scales folded at π, so every number on these scales is equal to the number on D and C scales multiplied by π.

(4) LL'_1, LL_2, LL_3 scales

This slide rule provides so called log log scales LL'_1, LL_2 and LL_3 which are used in computation of x^y, e^x and In x. LL'_1, which as an extension scale of C, is marked in red at the right end of it, substitutes the ordinary LL_1 and LL_0 scales. Its usage is already explained in page 14.

(5) Sh_1, Sh_2 and Th scales

These scales make it possible to compute the hyperbolic function, which is frequently necessary in alternating current theory. Sh_1 and Sh_2 refer to D scale and Th scale refers to C scale.

(6) Tr_1 rnd Tr_2 scales

These scales are used in the computation of tangent of an angle from 0.1 to 0.8 radians, and from 0.785 to 1.472 radians respectively.

(7) db scale

This decibel scale is useful in the computation of e'ectric communication circuit. Moreover this equaly subdivided scale can be used as ordinary log scale to obtain the common logarithms of a number.

24 Trigonometric Function

(1) sin x, cos x

The sine of an angle, either in degree or in radian can

be directly read on S_θ or Sr scale.

Set the hair line of runner to x on S_θ or Sr scale, then the answer can be read on P scale. Moreover, set 10 on Q scale to the hair line, then opposite zero on P scale cos x can be read on Q scale.

Example $\sin 51.8° = 0.786$

$\cos 51.8° = 0.618$

a Set the hair line to 51.8° on S_θ

b Read 0.786 on P (sin x).

c Set 10 on Q to the hair line.

d Opposite 0 on P, read 0.618 on Q (cos x).

Example 2 $\sin 0.665 = 0.617$

$\cos 0.665 = 0.787$

a Set the hair line to 0.665 radians on Sr,

b Read 0.617 on P (sin x).

c Set 10 on Q to the hair line.

d Opposite 0 on P, read 0.787 on Q (cos x).

(2) $\sin^{-1} x$

Opposite x on P scale we can read directly the value of $\sin^{-1} x$ on S_θ scale (in degree) or on Sr scale (in radian). The function of angles smaller than 0.1 radian can be obtained readly by using the relation, namely for small angles

$\sin x = \tan x = x$ (in radians)

Example $\sin^{-1} 0.742 = 0.836$ radians $= 47.9°$

a Set the hair line to 0.742 on P.

b Read 0.836 radians on Sr and 47.9° on S_θ,

(3) $\cos^{-1} x$

Move the slide to the left so that x on Q scale is opposite zero on P scale, then the hair line to 10 on Q scale, read $\cos^{-1} x$ on Sr scale (in radians) and on S_θ scale (in degree).

Example $\cos^{-1} 0.812 = 35.7° = 0.623$ radians

a Set 0.812 on Q to 0 on P.

b Opposite 10 on Q, read 35.7° on S_θ and 0.623 radians on Sr.

(4) tan x, $\tan^{-1} x$

The Tr_1 and Tr_2 scales represent a single scale of angles reading from 0.1 to 1.472 radians, which has been split into two parts, i. e. the Tr_2 scale begins at the left where the Tr_1 scale ends at the right.

	x radians	tan x
Tr_1	0.1 — 0.8	0.1 — 1.0296
Tr_2	0.7854 — 1.472	1 — 10

Example 1 tan 0.35 = 0.365

tan 1.17 = 2.36

Opposite 0.35 on Tr_1 and 1.17 on Tr_2, read 0.365 and 2.36 on D.

Example 2 $\tan^{-1} 5.8 = 1.40$

$\tan^{-1} 0.28 = 0.273$

Opposite 5.8 and 0.28 on D, read 1.40 on Tr_2 and 0.273 on Tr_1.

(5) Conversion between degree and radian

Covnersion from degree to radian or its reverse process is made by using Sr and S_θ scales.

Example 1 53° = 0.925 radians

0.52 radians = 29.8°

Opposite 53° on S_θ, read 0.925 radians on Sr.

Opposite 0.52 radians on Sr, read 29.8° on S_θ.

In the case of conversion of a small angle, move the decimal point one plece to the right, perform the operation as is explained above, and read the answer moving the decimal point one place lower.

Example 2 0.06 radians=3.44°

Opposite 0.6 radians on Sr, read 34.4° on S_θ.

The answer is 3.44°.

25. Vector

(1) The absolute value of vector

The absolute value of vector, represented in the form of a+jb, is equal to $\sqrt{a^2+b^2}$, and by the use of P, Q and P′ scales this value can be computed very easily in the same operation as ordinary multiplication and division. Namely set zero on Q scale to a on P scale, opposite b on Q scale read $\sqrt{a^2+b^2}$ on P scale. When b on Q scale runs off P scale, then set 10 on Q scale to a on P scale, and opposite b on Q scale read the answer on P′ scale.

Example 1 Find the absolute value of 3+j4.

a Opposite 3 on P, set 0 on Q.

b Opposite 4 on Q, read 5 on P.

Example 2. $\sqrt{8.76^2+9.32^2} = 12.79$

a Opposite 8.76 on P, set 10 on Q.

b Opposite 9.32 on Q, read 12.79 on P′.

The computation of form a^2-b^2 can be done in the same way.

Example 3 $\sqrt{10.53^2-5.96^2}=8.63$

a Opposite 10.53 on P′, set 5.96 on Q.

b Opposite 10 on Q, read 8.63 on P.

Example 4 $\sqrt{93.4^2-41.8^2}=83.5$

Calculate as $10\sqrt{9.34^2-4.18^2}=10\times 8.35$

a Opposite 9.34 on P, set 4.18 on Q.

b Opposite 0 on Q, read 8.35 on P.

c The answer is 83.5

(2) Phase angle of vector

In the vector of the form a+jb the phase angle θ between the real part and the absolute value of given vector is represented as follows;

$$\theta=\tan^{-1}\frac{b}{a}$$

Example $\tan^{-1}\frac{3.6}{2.5}=0.964$ radians

a Opposite 3.6 on D, set 2.5 on C.

b Opposite 1 on C, read 0.964 radians on Tr_2

(3) Conversion of coordinate system

From following relations;

$$a+jb=\sqrt{a^2+b^2}\ \angle\tan^{-1}\frac{a}{b}$$

$$R\angle\theta = R\cos\theta + j\ R\sin\theta$$

the conversion of vector in polar coordinate system to rectangular coordinate system and its reverse computation can be easily done.

Example 1 $-7.5+j6.0=9.604\ \angle\pi-0.675$

This is the example of conversion of a+jb to polor coordinate system.

a Opposite 7.5 on P, set zero on Q,

b Opposite 6 on Q, read sbsolute value 9.604 on P.

c Opposite 6.0 on D, set 7.5 on C.

d Opposite right index of C, read 0.675 radians on Tr_1.

Thus the phase angle is equal to $\pi-0.675$.

Example 2. $25\angle 52^\circ=15.4+j19.7$

This is the example of conversion of polar coordinate system to a+jb. The angle is given in degree.

a Opposite 52° on S_θ, set 10 on Q.

b Opposite zero on P, read $\cos\theta=0.616$ on Q.

c Opposite 10 on Q, read $\sin\theta=0.788$ on P.

From these values and using C, D scales calculate the real part $R\cos\theta=25\times 0.616=15.8$ and the imaginary part $jR\sin\theta = j25\times 0.788=j19.7$

(4) Multiplication and division of vectors

Multiplication and division of two vectors have been computed from the following formula;

$$R_1\angle\theta_1 \times R_2\angle\theta_2 = R_1R_2\angle\theta_1+\theta_2$$

$$\frac{R_1\angle\theta_1}{R_2\angle\theta_2}=\frac{R_1}{R_2}\angle\theta_1-\theta_2$$

Example $\frac{5-j4}{-4-j8}=0.718\ \angle 78^\circ$

$$\frac{5-j4}{-4-j8}=\frac{\sqrt{5^2+4^2}\ \angle\tan^{-1}(-4/5)}{\sqrt{4^2+8^2}\ \angle\tan^{-1}(-8/-4)}$$

From P and Q, read $\sqrt{5^2+4^2}=6.40$ and $\sqrt{4^2+8^2}=8.94$

From D, C, Tr_1 and Tr_2. read $\tan^{-1}(-4/5)=0.675$

and $\tan^{-1}(-8/-4)=1.107$

$$\text{Then}\ \frac{5-j4}{-4-j8}=\frac{6.40\ \underline{/-0.675}}{8.94\ \underline{/-\pi+1.107}}$$

$$=0.716\ \underline{/-0.675-(-\pi+1.107)}$$

$$=0.716\ \underline{/1.360}=0.716\ \underline{/78^\circ}$$

26. Hyperbolic Function

The logarithmic scales Sh_1, Sh_2 and Th are used for the computation of hyperbolic function.

(1) sinh x, $\sinh^{-1}$ x

Sh_1 and Sh_2 scales on the stock are used with referense to the D scale. Set the slide exactly in the stock, then the value of sinh x is given merely by the shifting of runner. When x is on Sh_1 scale the value of sinh x is between 0.1 and I, otherwise when x is on Sh_2 scale sinh x is between 1 and 10.

Example 1 sinh 0.362 = 0.370
sinh 2.56 = 6.43

a Set the slide exactly in the stock.

b Opposite 0.362 on Sh_1, and 2.56 on Sh_2, read 0.370 and 6.43 on D.

Example 2 $\sinh^{-1}$ 0.825 = 0.752
$\sinh^{-1}$ 2.06 = 1.470

a Set the slide exactly in the stock.

b Opposite 0.825 and 2.06 on D, read 0.752 on Sh_1, and 1.470 on Sh_2.

(2) tanh x, $\tanh^{-1}$ x

Th scale on the slide is used with reffered to C scale.

Example thah 0.183 = 0.181
$\tanh^{-1}$ 0.705 = 0.827

(3) cosh x

The value of cosh x can be computed from the following formula;

$$\cosh x=\frac{\sinh x}{\tanh x}$$

Example cosh 0.575 = 1.170

a Opposite 0.575 on Sh_1, set 0.575 on Th.

b Opposite the left index of C, read 1.170 on D.

27. Hyperbolic Function of Complex Angle

Hyperbolic function of complex angle is given from the following formula:

(1) $\sinh(a+jb)=\sinh a.\cos b+j\cosh a.\sin b$
$=\sqrt{\sinh^2 a+\sin^2 b}\ \underline{/\tan^{-1}(\tan b/\tanh a)}$

(2) $\cosh(a+jb)=\cosh a.\cos b+j\sinh a.\sin b$
$=\sqrt{\sinh^2 a+\cos^2 b}\ \underline{/\tan^{-1}(\tan b/\tanh a)}$

(3) $\tanh(a+jb)=\sinh(a+jb)/\cosh(a+jb)$
$=\sqrt{\dfrac{\sinh^2 a+\sin^2 b}{\sinh^2 a+\cos^2 b}}\ \underline{/\tan^{-1}\left(\dfrac{\sin 2b}{\sin 2a}\right)}$

(4) $\tanh^{-1}(a+jb)=x+jy$

Here, $x=\frac{1}{2}\log_e R$, $y=\frac{\theta}{2}$

and $\dfrac{1+(a+jb)}{1-(a+jb)}=R\underline{/\theta}$

Example 1 $\sinh(0.43 + j0.68) = 0.7694 \angle 1.106$

$\sinh 0.43 = 0.443$

Opposite 0.43 on Sh_1, read 0.443 on D.

$\sqrt{\sinh^2 0.43 + \sin^2 0.68} = \sqrt{0.443^2 + \sin^2 0.68} = 0.7694$

a Opposite 0.68 on Sr_1, set 0 on Q.

b Opposite 0.443 on Q, read 0.7694 on P.

$\tan^{-1}(\tan 0.68/\tanh 0.43) = 1.106$ radians

a Opposite 0.68 on Tr_1, set 0.43 on Th.

b Opposite 1 on C, read 1.106 on Tr_2.

Example 2 $\cosh(0.75 - j1.24) = 0.884 \angle -1.075$

$\sinh 0.75 = 0.822$

Opposite 0.75 on Sh_1, read 0.822 on D.

$\sqrt{0.822^2 + \cos^2 1.24} = 0.884$

a Opposite 1.24 on Sr, set 10 on Q,

b Opposite 0.822 on P, read 0.884 on Q.

$\tan^{-1}(-\tan 1.24/\tanh 0.75) = -1.075$

a Opposite 1.24 on Tr_2, set the right index of C.

b Opposite 0.75 on Th, read –1.075 on Tr_2.

Example 3 $\tanh^{-1}(0.2 + j\,0.4) = 0.1736 + j\,0.393$

From formula (4);

$$\frac{1 + (0.2 + j\,0.4)}{1 - (0.2 + j\,0.4)} = \frac{1.2 + j\,0.4}{0.8 - j\,0.4}$$

$$= \frac{\sqrt{1.2^2 + 0.4^2}\ \angle \tan^{-1}(0.4/1.2)}{\sqrt{0.8^2 + 0.4^2}\ \angle \tan^{-1}(-0.4/0.8)}$$

$$= \frac{1.265\ \angle 0.322}{0.894\ \angle -0.464} = 1.415\ \angle 0.786$$

The procedure is as follows;

$$1.2 + j0.4 = \frac{1}{10}(12 + j4) = \frac{1}{10}\sqrt{12^2 + 4^2}$$

$$= \frac{12.65}{10} = 1.265$$

When one of these scales become scale off, or the given number are too small or large to acquire a precise result, multiply or divide it by a simple digit as 10 etc., so as to be treated them in the proper range of P, Q scales. Of course this result must be reduced by reverse treatment to get the right answer.

a Multiply 1.2 and 0.4 by 10.

b Opposite 12 on P′, set 4 on Q.

c Opposite 4 on Q, read 12.65 on P′.

d Divide the result by 10.

$\tan^{-1}(0.4/1.2) = 0.322$

a Opposite 4 on D, set 1.2 on C.

b Opposite the left index of C, read 0.322 on Tr_1.

Therefore; $1.2 + j\,0.4 = 1.265 \angle 0.322$

Similarly, $0.8 - j\,0.4 = 0.894 \angle -0.464$

Then,

$$x = \frac{1}{2}\log_e \frac{1.265}{0.894} = \frac{1}{2}\log_e 1.415$$

$$= \frac{1}{2}0.3472 = 0.1736$$

Opposite 1.415 on LL_2, read 0.3472 on D.

$$y = \frac{\theta}{2} = \frac{0.786}{2} = 0.393$$

Thus answer is;

$\tan^{-1}(0.2 + j\,0.3) = 0.1736 + j\,0.393$

28. Decibel Calculation

In the electric communication circuit, let voltage and current at input side be V_1 and I_1 and those at output side be V_2 and I_2, the decibel for voltage ratio db (V) and the decibel for current ratio db (I) are as follows ;

$$\text{db (V)} = 20 \log \frac{V_2}{V_1}$$

$$\text{db ((I)} = 20 \log \frac{I_2}{I_1}$$

When V_2, V_1 or I_2, I_1 are given, the ratio $\frac{V_2}{V_1}$ or $\frac{I_2}{I_1}$ are obtained by using C and D scales, and decibel of these ratio are read on db scale.

Let power ratio of input and output side be W, the decibel of power ratio is ;

$$\text{db (W)} = 10 \log W$$

Find the power ratio W by the use of A and B scale, db is read on db scale opposite the index of the slide.

29. Some Applications on Electrical Problems

Example 1 Calculate the current in an electric circuit, which impedance is $4 + j2.6$ and the potential difference between its terminals is $5 + j9$.

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{5 + j9}{4 + j2.6}$$

Represent both numerator and denominator in a polar coordinate ;

$$5 + j9 = 10.30 \angle 1.064$$

$$4 + j2.6 = 4.77 \angle 0.576$$

and then

$$\dot{I} = \frac{10.30}{4.77} \angle 1.064 - 0.576$$

$$= 2.16 \angle 0.488$$

Or convert this value in a rectangular coordinate using P, Q and Sr scales as follows ;

$$\sin 0.488 = 0.469$$

$$\cos 0.488 = 0.883$$

then real part $2.16 \times 0.469 = 1.013$

and imaginary part $2.16 \times 0.883 = 1.907$

Therefore $\dot{I} = 1.013 + j\,1.907$

Example 2 Compute the resultant current $\dot{I}$, of $\dot{I}_1 = 2 + j3$ and $\dot{I}_2 = 3 + j4$ in polar coodinate.

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = (2 + j3) + (3 + j4) = 5 + j7$$

The absolute value of vector is ;

$$I = \sqrt{5^2 + 7^2} = 8.60$$

a Opposite 5 on P, set 0 on Q.

b Opposite 7 on Q, read 8.60 on P.

The phase angle is

$$\theta = \tan^{-1} \frac{7}{5} = 0.95 \text{ radians}$$

a Opposite 7 on D, set 5 on C.

b Opposite the left index of C, read 0.95 on Tr_2.

Answer is $8.60 \angle 0.95$.

Details of "Relay" Bamboo Slide Rules

Article No.	Length	Scale
403—I(MI)	4″	A.B.CI.C.D/S.L.T.
505—I	5″	A.B.CI.C.D.K/S.L.T.
515—I	5″	A.B.CI.C.D.K/T_2.T_1.L.S.
512—I	5″	DF.CF.CI.C.D.A/S.L.T.
513—I	5″	DF.CF.CI.C.D.A/T_2.T_1.L.S.
602—I	6″	K.A.B.CI.C.D.L/S.S&T.T.
605—I	6″	LL_1.A.B.CI.C.D.LL_2. Volt Dynamo-Motor./S.L.T.
80—I	8″	A.B.CI.C.D.
82—I	8″	A.B.CI.C.D.K/S.L.T.
83—I	8″	K.DF.CF.CI.C.D.A/S.L.T.
84—I	8″	K.DF.CF.CI.C.D.A/S.L.T.
102—I	10″	A.B.CI.C.D.K/S.L.T.
103—I	10″	K.DF.CF.CI.C.D.A/S.L.T.
105—I	10″	K.A.B.CI.C.D.L/S.S&T.T.
112—I	10″	A.DF.CF.CI.C.D.K/S.L.T.
113—I	10″	A.DF.CF.CIF.CI.C.D.K/T_1.T_2.L.S.
114—I	10″	K.DF.CF.CI.C.D.A/T_2.T_1.L.S.
115—I	10″	K.A.B.CI.C.D.L/T_1.T_2.ST.S.
104—I	10″	L.LL.D_1.M_2.M_1.C.D.K.A/S.S&T.T.
107—I	10″	LL_1.A.B.CI.C.D.LL_2. Volt Dynamo-Motor./S.L.T.
120—I	10″	Darmstadt L.K.A.B.CI.C.D.P.S.T/$LL_1$$LL_2$.$LL_3$.
450—I	4″	Duplex DF.CF.CI.C.D/A.S.L.T.D.
550—I	5″	Duplex DF.CF.CI.C.D.L/A.B.S.T.C.D.K.
650—I	6″	Duplex DF.CF.CIF.C.D/K.A.B.S.T.CI.D.L.
651—I	6″	Duplex LL_1.DF.CF.CI.C.D.LL_2/ K.A.B.S.T Dynamo-Motor. Volt.
652—I	6″	Duplex K.DF.CF.CIF.CI.C.D.L/ LL.A.B.S.T.C.D.LL_0.
150—I	10″	Duplex L.LL_1.DF.CF.CIF.CI.C.D.LL_3.LL_2./ LL_0.LL_{00}.A.B.K.CI.C.D.S.ST.T.
151—I	10″	Duplex $LL_{\bar{1}}$.$LL_{\bar{2}}$.$LL_{\bar{3}}$.DF.CF.CIF.CI.C.D.LL_3.LL_2. LL_1./$LL_{\bar{0}}$.L.K.A.B.S.ST.T.C.D.DI.P.LL
152—I	10″	Duplex DF.CF.CIF.CI.C.D.L./ K.A.B.S.ST.T.C.D.DI.
153—I	10″	Duplex L.LL_1.DF.CF.CIF.CI.C.D.LL_3.LL_2/ K.A.B.S.ST.T.C.D.DI.
157—I	10″	Duplex Sr.Sθ.P′.P.Q.CF.CI.C.D.DF.LL_1'.LL_2.LL_3/ Sh_2.Sh_1.A.B.K.Th.C.D.Tr_1.Tr_2.db.
158—I	10″	Duplex Sh_2.Sh_1.Th.A.BI.S.T.CI.C.D.LL_3.LL_2.LL_1/ X_2.X_1.P_2.P_1.Q.Y.L.⌊x.I.I_3.⌊θ.⌊θ.⌊Y.

"Relay" plastic Slide Rules

Article No.	Length	Scale
42—I	4″	A.B.CI.C.D/S.L.T.
53—I	5″	DF.CF.CI.C.D.A/S.L.T.
55—I	5″	A.B.CI.C.D.K/S.L.T.

Remarks I .... with instruction book
M .... with Magnifier

Rules with mangnifier need no carboards in principle.

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