The British Journal of Radiology, *72* (1999), 48–54

Radiographic exposure slide rules

J E BURNS, MSc, FInstP

National Physical Laboratory, Teddington, Middlesex TW11 0LW, UK

Abstract. Before automatic exposure control was fitted to diagnostic X-ray sets, radiographers were faced with the problem of choosing the parameters that would give the best radiographic image. For a new X-ray set whose performance was unknown, this was no easy matter, and often required considerable trial and error because of the number of variables involved. To reduce the amount of work, special slide rules were invented which calculated the effect of some of these variables. Five such slide rules, dating from between about 1910 and 1950, are illustrated and discussed, including the light they shed on changes in radiographic practice over the years.

Until their replacement by small electronic calculators about 25 years ago, slide rules were widely used. Anyone entering a science-based career would be expected to have at least a reasonable fluency with the device. Radiology was no exception. As an example of its importance, Kemp [1], in an elementary textbook intended for student radiographers published in 1951, included a 29-page chapter on how to use a slide rule. The basic slide rule in common use was intended for the rapid multiplication and division of numbers to an accuracy of about 0.5%. It had two pairs of logarithmic scales, with the upper pair being the square of the numbers on the lower pair. A sliding cursor enabled one to read between two scales that were not adjacent to each other.

Since their invention in the early part of the 17th century, slide rules had been designed for many specialized purposes. So long as the effect of one variable on another was known, either by a mathematical formula or by measurement, or even by definition (such as tax legislation), then a slide rule could be designed in order to calculate the effect. The manufacture and sale of such a slide rule depended on whether there were sufficient customers who were prepared to purchase it.

As has been well documented, the use of X-radiation for medical radiography increased very rapidly during the early years of the 20th century. It was soon realized that, while it was not difficult to obtain a recognizable image on a photographic plate, for the best results it was necessary to choose the penetrating power of the beam and the exposure time as carefully as possible. With early X-ray equipment this was far from easy, and could involve either considerable calculation or a great deal of trial and error. It is not surprising therefore that a demand arose for a specially designed slide rule to help in these calculations. As a slide rule could soon pay for itself by saving both time and wasted X-ray plates, it was not long before a manufacturer produced such a device. However, in order to understand its use, it is helpful to discuss what medical radiography was like in those early days.

Received 25 June 1998 and in revised form 26 August 1998, accepted 14 September 1998.

The early X-ray department

Compared with a modern department, one important difference of the early X-ray department was the use of gas X-ray tubes in which the gas (air) inside was not an undesirable fault but an essential feature, because the cathode was unheated and the electrons that were accelerated to the target (called the anticathode in those days) were obtained from the ionization of the gas by the high voltage applied to the tube. Another difference was the type of high voltage generator. The first type was an electrostatic generator such as a Wimshurst machine. Unfortunately, the output current (and therefore tube current) was rather small, sometimes necessitating exposures of 15 or 20 minutes, or even longer. This was soon superseded by the induction coil, the larger current from which enabled exposure times to be reduced to less than a couple of minutes or so.

When a gas tube was used over a period of time, the gas was gradually absorbed, so increasing the resistance of the tube. This had two consequences. Firstly, the tube current decreased, until eventually the tube became unusable. Secondly, with the high voltage generators available at that time, the voltage across the tube was increased, *i.e.* the tube became "harder". In consequence, radiologists found it convenient to possess a collection of gas tubes of different hardnesses (or penetrating powers) which could be chosen for different X-ray examinations. These tubes were often kept in a special rack near the high voltage generator.

With the technology available at that time it was

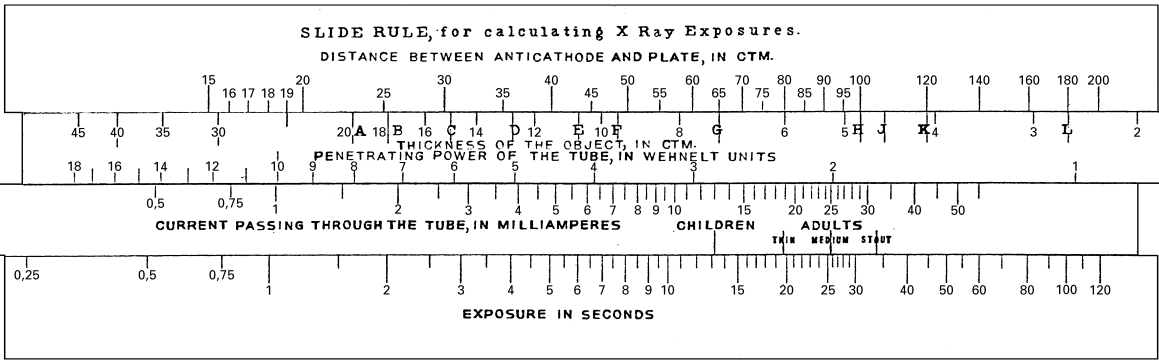

Figure 1. Schall & Son radiographic slide rule (*ca.* 1912).

not convenient to measure the kV applied to the tube, so the penetrating power of the beam was measured by a penetrameter. These came in various types, of which the Wehnelt penetrameter consisted of a plate of a high atomic number metal (silver) adjacent to a range of thicknesses of a low atomic number metal (aluminium). The "Wehnelt number" referred to the thickness of aluminium that attenuated an X-ray beam by the same amount as the silver plate. This was not an easy device to use, so it was eventually superseded by an adjustable spark gap in parallel with the X-ray tube; the minimum gap distance which just prevented an arc depended on the kV, and was taken as a measure of the penetrating power of the beam.

The original recording medium was the ordinary photographic dry plate, with some high atomic number materials added to the emulsion to increase its sensitivity to X-radiation. At this time photographers were changing over from glass plates to film. However, this presented difficulties in radiography. Because of their large size, the films tended to curl up at the edges and corners when being processed; this made it difficult to keep them immersed in the horizontal shallow trays used at that time to contain the developing and fixing fluids. Glass plates therefore continued to be used in some X-ray departments well into the 1930s.

Perhaps the most obvious difference from a modern X-ray department was the fact that the equipment was not shock proof. Bare wires supported by insulators connected the X-ray tube to the generator. Great care had to be taken not to electrocute either the patient or the operator, or both. It did have the advantage that frequent changes of X-ray tubes could be made without difficulty. On the other hand it was often necessary to position a patient much further away from the tube than would otherwise be desired, so preventing any standardization of target–plate distance.

Schall & Son slide rule

The slide rule shown in Figure 1 was included in an illustrated catalogue of electromedical equipment published by Schall & Son of London [2] in 1914. Its invention is unlikely to date back to earlier than about 1910, because the range of exposure times on the scale (0.25–120 s) is appropriate to the use of an early induction coil such as the high voltage generator, rather than an electrostatic generator which would have needed longer exposures.

There are two slides and the operation is very simple, starting with the top scale and ending with the bottom scale. First, the top slide is adjusted so that the thickness of the part of the anatomy being radiographed is set to be adjacent to the anticathode (target) X-ray plate distance used. Next, the lower slide is adjusted so that the tube current is set to be adjacent to the penetrating power of the X-ray tube in Wehnelt units. Finally, the required exposure time is read on the bottom scale adjacent to the index line marked "medium". An alternative mode of operation is to use one of the letters interspersed in the "thickness of object" scale. These correspond to different parts of the anatomy, a list of which is given on the reverse of the slide rule, ranging in decreasing mass thickness from "A" for lumbar vertebrae to "J" for a hand and "L" for lungs. In this case, one of the four index lines should be used corresponding to the size of the patient.

All the scales are logarithmic, with the exposure time being inversely proportional to the tube current and proportional to the square of the target–plate distance, as would be expected from the inverse square law. Less obviously, the exposure time is proportional to the square of the thickness of the object, and inversely proportional to the square of the penetrating power of the tube in Wehnelt units; presumably these latter two scales were derived experimentally. No provision is made for different X-ray plate sensitivities, but on the reverse of the rule it is stated that "The slide rule gives mean values and these are only right when special X-ray dry plates and properly working tubes are used". The importance of eliminating any inverse current through the X-ray tube is also emphasized, highly relevant when using gas tubes.

Another slide rule with the same arrangement and range of scales, but without the letters allow-

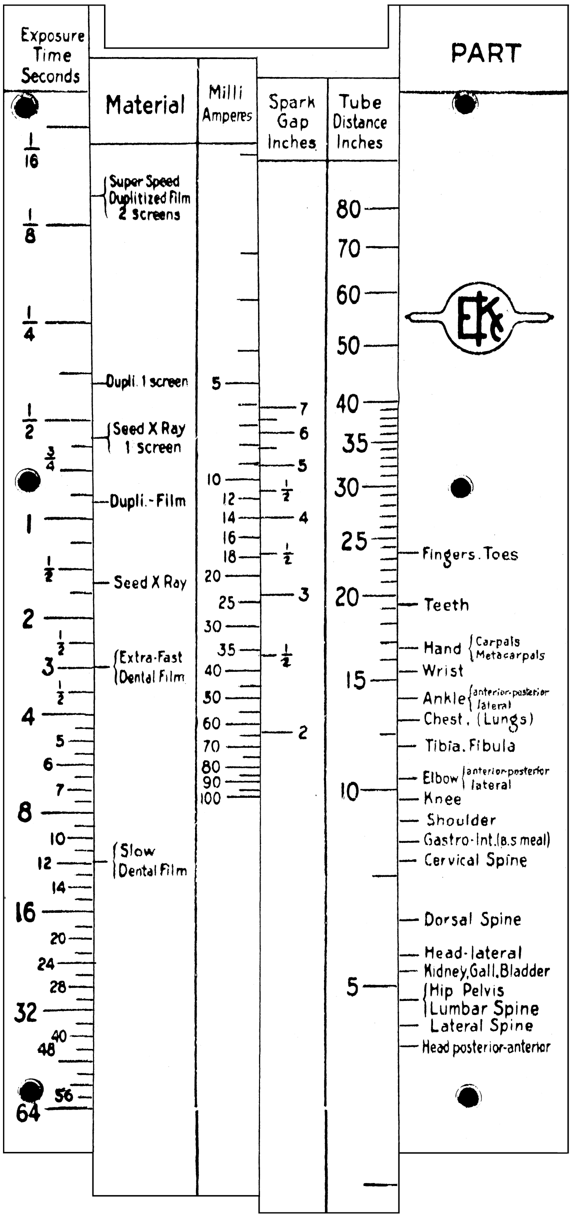

Figure 2. Eastman Kodak radiographic slide rule (1919).

ing the alternative mode of operation described earlier, was illustrated in a catalogue of Cavendish Electrical Co. (London) in about 1912. Slightly earlier versions of both of these slide rules, but with inscriptions in German, were marketed by Reiniger, Gebbert & Schall (Erlangen) and a discussion published by a member of that company [3], later corrected by Christen [4] who took the discussion further.

Eastman Kodak slide rule

The date printed on the reverse of this slide rule is 1919. As shown in Figure 2, there are two slides, and again the operation is very simple, but this time working from right to left. First, the right-hand slide is adjusted to set the target–plate distance adjacent to the name of the part of the anatomy to be radiographed. Next, the left-hand slide is adjusted to set the tube current adjacent to the spark gap separation that just prevents an arc forming. Finally, the required exposure time is indicated on the left-hand scale adjacent to the type of X-ray film or plate being used.

With the exception of the scale of spark gap separation, the numerical scales are logarithmic, with the exposure time being inversely proportional to the tube current and proportional to the square of the target–plate distance. As would be expected, increasing spark gap separation requires shorter exposure times, but the exact numerical relationship was presumably derived experimentally.

The right-hand slide is double sided, and can be removed and turned over, with the target–plate distance and spark gap separation scaled in centimetres instead of inches. No adjustment is provided for variations in the size of the patient, and in the instructions it is stated that the settings relate to a normal subject weighing 150 lbs (68 kg).

The interwar years

The 1920s and 1930s witnessed many improvements in X-ray technology. The gas tube was replaced by the Coolidge tube (as used to the present day). This had a heated cathode providing a beam of electrons by thermionic emission and, when operated under saturation conditions, the tube current could be adjusted to whatever value required, and the kV applied across the tube could be chosen independently of the tube current. After some years of development, the Coolidge tube was capable of coping with much higher beam currents, and therefore much shorter exposure times, than the gas tube. Induction coils were replaced by alternating current (ac) transformers as ac mains supplies became readily available in hospitals. Developments in high voltage technology allowed the kV to be adjusted to any required value, and a pre-reading kV meter enabled the kV during an exposure to be indicated in advance of the actual exposure.

Heavy and expensive glass plates were eventually replaced by X-ray films once departments had equipped themselves with vertical processing tanks, and hangers which kept the film under tension during development and fixing. Safety in an X-ray department was greatly improved by the introduction of shock-proof equipment, with insulated cables replacing bare wires. As a result, it became possible to choose target–film distances to give the best results without any danger of electrocution, and often to choose one standard distance for a wide range of techniques.

The Schall & Son slide rule was replaced in that

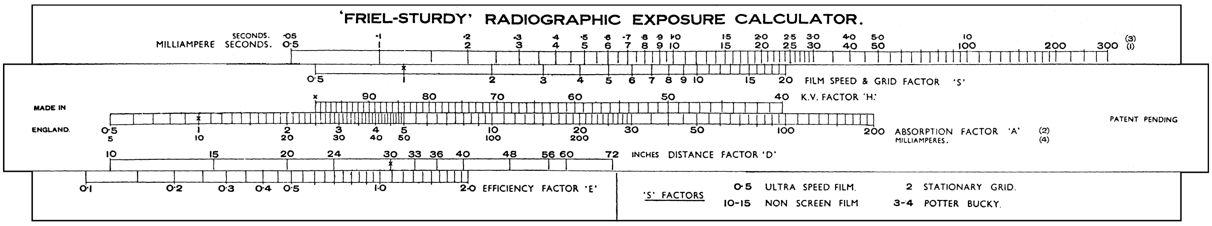

Figure 3. Friel–Sturdy radiographic slide rule (1946).

manufacturer's catalogue, 18th edition in 1925 [5] by the Eastman Kodak slide rule. This slide rule was also advertised in the catalogues of several other suppliers of X-ray equipment and accessories until 1939. The fact that this slide rule still used spark gap separation as an indicator of beam penetration and still referred to glass plates implies that the replacement of old equipment in X-ray departments lagged behind the development of new types of X-ray equipment.

During this period two new radiographic exposure slide rules are known to have been produced, and were marketed by Watson & Sons (London). The earlier is a "Sunic" compact calculator "suitable for the waistcoat pocket" and consisting of four co-axial discs with five scales having the same purpose as the linear scales of the Schall & Son slide rule, but with the beam penetrating power being expressed in Benoist penetrameter units. The later slide rule of about 1930 is of linear design, with two slides and a rectangular attachment with what appears to be a long cursor; the accompanying text claimed that "exposures can be calculated for radiographing any part of the body, and allowances are made not only for kilovoltage and milliamperage, but also for tube distance, speed of film and whether a Potter–Bucky Diaphragm is used or not".

A third slide rule appeared for the first time in the 5th edition (1940) of Schall's textbook on X-ray apparatus [6]. The invention is attributed to Dr Bruce MacLean, and the manufacturer is Schall (London). It consists of a circular disc able to rotate over a flat rectangular plate. On the right-hand side of the disc there is a scale of thickness of the part to be radiographed adjacent to a scale of generating potential on the lower plate, such that an increase of 2 cm in thickness corresponds to an increase of 5 kV. This relation between kV and thickness is based on experimental results by Rhinehart [7]. On the left-hand side of the disc is a scale of focus–film distance; adjacent to this on the lower plate are scales of milliampere-seconds (mAs) and exposure times in a fixed relation corresponding to a tube current of 30 mA. Numerical corrections are provided for the use of grids, screens, different film speeds, and for special types of examinations such as chests. This slide rule also appeared in Schall's 6th edition (1947) but not in the subsequent 7th (1956) and 8th (1961) editions.

The continuous improvement in technology during these years did not permit much standardization to take place, and there were so many possible combinations of equipment, X-ray films, grids and intensifying screens that it was impossible to predict in advance what the best settings should be for a new X-ray set. On the other hand, what also improved was the manufacturer's quality control of equipment, film, etc., so that once suitable settings had been found by experience, these would then stay the same for the lifetime of the X-ray set, despite replacements of X-ray tubes and the regular supply of X-ray film. This allowed tables to be drawn up and displayed adjacent to control panels, listing standard settings and the variations to be employed for patients of different sizes.

Soon after World War II, three more slide rules appeared which were intended to reduce the amount of trial and error involved in drawing up these tables. They all used the same basic principle that the calculations had to be standardized by an initial trial on each X-ray set to find the best kV and exposure time for one or more types of X-ray examinations, but the subsequent procedure differed for the three slide rules.

Friel–Sturdy slide rule

The slide rule shown in Figure 3 was designed by two radiographers, manufactured by "Unique" Slide Rule Co. and reviewed in Radiography in 1947 [8]. In order to use it, the first action required is to derive an Efficiency Factor which allows for differences in X-ray output for given kV and mAs settings, depending on whether the X-ray generator is self-rectified, constant-potential etc. This is carried out by a trial process of adjusting the mAs to give a good radiograph of the hand using 45 kV, usual filter, 30 inch target–film distance, ordinary X-ray film and standard intensifying screens. The Efficiency Factor E for the X-ray set is then given by:

$$E = \text{mAs} \times (\text{kV}/100)^4 = \text{mAs} \times (45/100)^4 = \text{mAs} \times 0.041$$

For this target–film distance and film–screen combination, the slide rule enables the mAs required for any other type of X-ray examination to be

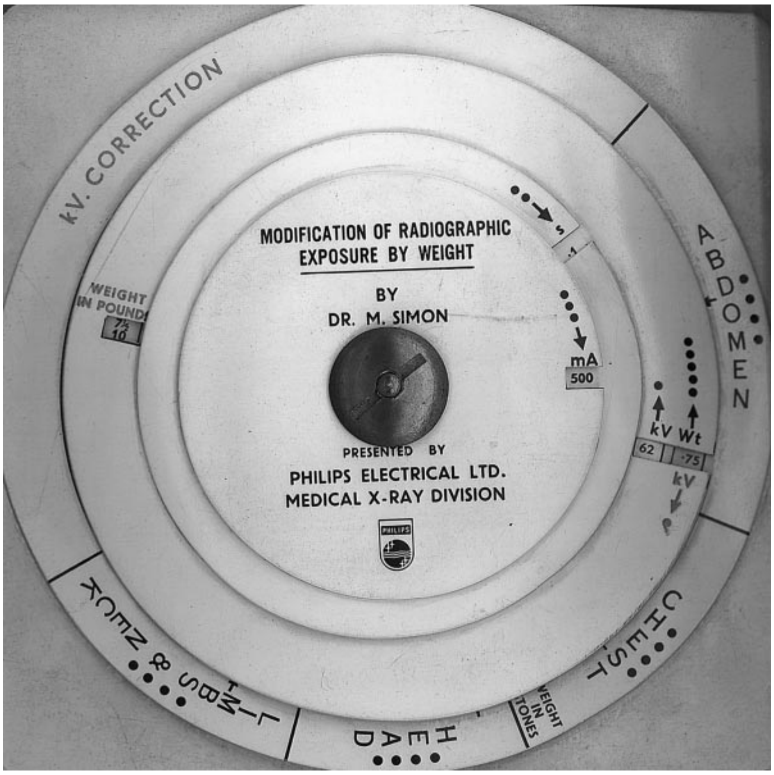

Figure 4. Philips circular radiographic slide rule (*ca.* 1950).

calculated by multiplying the Efficiency Factor E by an Absorption Factor A and a kV Factor H, using the scales marked E, A and H in succession, and reading the mAs on the scale at the top by means of a cursor. Recommended values for the kV and the Absorption Factor A are given on the reverse of the slide rule (and in more detail in an accompanying instruction booklet) for a wide range of X-ray examinations, ranging from 45 kV and $A=0.6$ for fingers to 90 kV and $A=200$ for a lateral pelvis. The values for the Absorption Factor are those for an average adult weighing 8 stones (51 kg). The scale for the KV Factor is marked in kV and is graduated such that, when used as a multiplying factor, $H=(100/\text{kV})^4$; this assumes that, to obtain a constant density of a radiographic image under typical conditions, the mAs should be varied in the inverse proportion of the fourth power of the generating voltage, $(\text{kV})^4$.

For a radiograph taken at a different target–film distance, an adjustment to the Efficiency Factor or the mAs is calculated by using the scale marked D, which is based on the inverse square law, taking 30 inches as the standard distance. If grids or high speed or non-screen films are employed, adjustments can be made to the mAs by means of the scale marked S, values for which are printed in the lower right-hand corner of the face of the slide rule. Two additional scales to convert mAs into the tube current in mA and the exposure time in seconds are also provided. Although it has been removed for clarity in Figure 3, a cursor is fitted, and its use is essential to perform the calculations. Alternative values of kV and the Absorption Factor are provided for small children, but otherwise no adjustment is provided for variations in the size of the patient, which the instructions state "can be made in the usual way".

Philips circular slide rule

It is not known exactly when the device shown in Figure 4 was invented and produced, but it is thought to date from about 1950. It consists of five co-axial discs, and it calculates the changes that need to be made to the kV and mAs if a patient differs from the average weight of an adult, taken to be 10 stones or 60 kg. For each type of X-ray examination and each particular X-ray set, the best combination of kV and mAs for an average adult must be obtained by experience.

For a particular patient the procedure is as follows. First, the top three discs marked •••, ••, and • are rotated so that the combination of mA, exposure time and kV found by experience to give a good radiograph for the required type of examination on an average adult are displayed in the appropriate windows. Next, the disc marked •••• is rotated until the weight 10 stones is shown in the window marked "Wt". Finally, for a patient whose weight differs from 10 stones, the disc marked ••••• is rotated until the patient's weight is shown in the window and, by doing this, revised values of kV

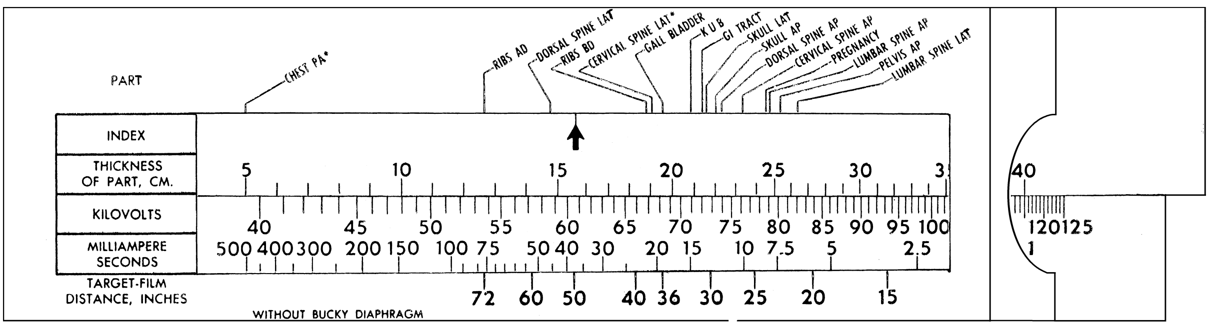

Figure 5. Ansco radiographic slide rule (1950).

and exposure time will appear in the appropriate windows.

Several of the discs are double sided, enabling the weight to be displayed in kilograms and/or the separate indications of tube current and exposure time to be displayed as a single indication of mAs. No provision is made for changes in target–film distance, film speed, grids or intensifying screens, which are assumed to be standardized for each type of X-ray examination.

The numbers on the kV disc are spaced around the disc on the assumption that, to obtain a constant density of a radiographic image under typical conditions, the mAs should be varied in the inverse proportion of $(kV)^5$. This corresponds with experimental results reported by the Philips Laboratories [9].

On the outermost disc marked ••••, the separation of X-ray examinations into four categories—abdomen, chest, head, and limbs and neck—allows for the different variation in the thickness of these regions of the body with total body weight. For example, the thickness of the head depends much less on total body weight than does the thickness of the abdomen. So, if settings of 60 kV and 100 mAs were found to give good radiographs of both the abdomen and head for a 10 stone patient, the calculator indicates that for a 20 stone patient the settings would need to be increased to 80 kV and 400 mAs for the abdomen, but only to 62 kV and 120 mAs for the head.

Ansco slide rule

This slide rule was produced by Ansco, an American manufacturer of X-ray film, in 1950. As shown in Figure 5, there are two slides. Before using it one must have determined the kV, mAs and target–film distance which gives a good radiograph of a part of the body corresponding to one of those printed at the top of the rule, and whose thickness has been measured. The instructions for standardizing the rule are then as follows. First, adjust the lower slide to set the value of the mAs adjacent to the target–film distance. Next, adjust the upper slide so that the thickness of the part is adjacent to the kV setting. Finally, draw an arrow on the upper slide pointing to the line connected to the name of the part of the body used for this standardization procedure. Once standardized for a particular X-ray set and film, the location of the arrow should remain constant and no further adjustment should be necessary for that combination. If the type of film, grid or screen is changed, the position of the arrow will no longer be correct. However, there is nothing to prevent a user from drawing two or more arrows corresponding to different film–screen combinations, with or without a grid, so long as the various arrows can be clearly distinguished, possibly by using different colours.

To use this slide rule for a particular patient, adjust the upper slide so that the arrow points to the line connected to the name of that part of the body being radiographed, then adjust the lower slide so that the mAs is adjacent to the target–film distance used. The appropriate kV setting will then be found adjacent to the thickness of the part being radiographed. If this kV setting is inconvenient, the lower slide may be adjusted until a convenient combination of kV and mAs is found. Scales on the reverse of the slide rule enable the tube current and exposure time to be calculated from the mAs.

Although no numbered scale divisions are marked, the positions of the lines connected to the anatomical names at the top of the slide rule constitute in effect a scale of average body density, taking account of the proportion of air or bone (instead of soft tissue) in that part of the body. Setting the upper slide therefore multiplies the linear thickness of that part of the body by its average density to provide a setting which is proportional to its mass thickness, which is the quantity that is more closely correlated with the attenuation of the beam than the linear thickness.

The kV scale is graduated such that, to obtain a constant density of a radiographic image under typical conditions, the mAs should be varied approximately in the inverse proportion of $(kV)^{5.5}$. If it is assumed that the low end of the kV range is used to radiograph thin parts of the body, and the high end for thick parts, then the thickness scale is graduated such that an increase of 1 cm in thickness requires an increase of about 2 kV in generating potential (keeping the mAs constant).

Conclusions

By examining radiographic slide rules and their modes of operation one can trace the evolution of X-ray technology from the early days when the only way to control radiographic image density was by the timer, to the sophisticated controls available to operators in later years. It is also interesting to see how the designers of the slide rules decided what approximations to make for effects that can only be imperfectly predicted, such as the variation of film density with tube potential in a typical radiographic exposure. Or perhaps requiring a sociological explanation, their differing assumptions about the weight of an average adult.

There is no point in trying to judge which of these slide rules might be "better" than another. Each inventor obviously viewed the problem in a different light and under different conditions. In any case, none of them is now needed. With the introduction of automatic exposure control a radiographer can concentrate on the thing that matters most—the correct positioning of the patient to give the best radiograph for a particular purpose. Now, as always, this is where the knowledge, skill and experience of a radiographer is really required.

In celebrating the centenary of the discovery of X-rays, many books and articles have recently been written on the history of radiology, often discussing the development of X-ray apparatus. However, radiographic exposure slide rules do not appear to have received even a mention. One cannot claim that these devices have been crucial in radiography, but they seem to have served a useful purpose. The object of this paper has been to record their use and the light they shed on changes in radiographic practice over the years. It would be a shame if these examples of human ingenuity in the application of mathematics to routine medical radiography were allowed to disappear into oblivion.

Acknowledgments

I wish to thank Julian Holland of the Macleay Museum of the University of Sydney, Australia, who was the instigator of this article, and who provided the illustration of the Schall & Son slide rule, also Dr A M Thomas who allowed me to photocopy the Eastman Kodak and Ansco slide rules in his possession, and also Dr T Healey for allowing me to examine and photograph his example of the Philips circular slide rule. Thanks are also due to Dr U Busche of the Deutsches Röntgen-Museum for supplying details of the two German slide rules. Particular thanks are due to D J Friel, the co-inventor of the Friel–Sturdy slide rule, for interesting discussions and who loaned me his slide rule and then decided to present it to the Science Museum in London, who until then did not possess a single radiographic slide rule among their very large collection of special slide rules. I also wish to thank D R Guttery who pointed out a number of errors in the first draft of this paper, and who supplied additional information contained in his collection of advertisements and manufacturers' catalogues.

References

1. Kemp LAW. A student's radiological mathematics. Oxford: Blackwell, 1951:179–207.
2. Schall & Son.. Illustrated price list (16th edn). In: Electro-medical instruments and their management. Bristol: John Wright & Sons, 1914:312.
3. Janus F. Der Expositionsmesser, ein neues Hilfsinstrument für Röntgenaufnahmen. Fortschr Geb Röntgenstrah 1909–1910;14:253–6.
4. Christen T. Zur Dosierungsfrage. Fortschr Geb Röntgenstrah 1910;15:31–4.
5. Schall & Son. Illustrated price list (18th edn). In: Schall WE. X-rays; their origin, dosage and practical applications (2nd edn). Bristol: John Wright & Sons 1925:220.
6. Schall WE. X-rays; their origin, dosage and practical applications (5th edn). Bristol: John Wright & Sons, 1940:102, Plate xxxviii.
7. Rhinehart DA. Roentgenographic technique (2nd edn). Philadelphia: Lea & Febiger, 1936:128–30.
8. WW. The Friel–Sturdy radiographic calculator. Radiography 1947;13:54.
9. Bierman A, Boldingh WH. The relation between tension and exposure times in radiography. Acta Radiol 1951;35:22–6.