Booklet No. 3660

"P.I.C." SLIDE RULES

BOOK OF INSTRUCTIONS

FOR THE

STANDARD, REITZ, AND ENGINEERS' PATTERNS
with LOG LOG SCALES

Price - One Shilling and Sixpence

"P.I.C" SLIDE RULES

BOOK OF INSTRUCTIONS

for the

STANDARD, REITZ, and ENGINEERS' PATTERNS with LOG-LOG SCALES

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INSTRUCTIONS FOR "P.I.C." STANDARD AND REITZ PATTERN SLIDE RULES

1. GENERAL DESCRIPTION

A well-known engineer was heard to say (perhaps seriously), "If there were no slide rules I would give up engineering. The arithmetic would be so tedious. With a slide rule the arithmetic is rather fun."

To really enjoy using the rule one must be free from the worry of wondering whether the result is correct and whether the decimal point is in the right place. The student can gain assurance in these matters by following the procedure advocated in this book. The answer to a long calculation, with its decimal point in the right place, comes out almost automatically without the exercise of alert judgment at each stage.

Besides multiplication and division, which are the slide rule's main operations, calculations of many other kinds can be made with facility. For instance, square roots and cube roots, logarithms and antilogarithms, trigometrical functions of angles and the answers to some problems in engineering can be read off in the simplest manner.

The sliding strip should be capable of being moved easily in the central groove, there being just enough friction to prevent the strip from moving by its own weight. A stiff slider is much more difficult to set with accuracy and speed. The "P.I.C." rules are made with a thin steel strip in the back which keeps the friction constant, notwithstanding changes in humidity. The glass cursor has a spring which keeps the fine transverse line exactly at right angles to the scales. This fine line is referred to in this book as K.

On the front of the rule the most important scales are two identical scales usually referred to as C and D. The scale C is on the lower edge of the sliding strip and the scale D is adjacent to it on the body of the rule, so that when the strip is central the numbers from 1 to 10 on C are exactly opposite the same numbers on D. These numbers are spaced logarithmically as explained below.

On the upper edge of the sliding strip is scale B. This is set out logarithmically to half the size of scale C, so that the figures 1 to 10 on the left occupy only half the length of the rule. The same graduations are set out again on the right-hand end of scale B and are there numbered 10 to 100. The reasons for this will appear later (see Art. 3). Adjacent to scale B and above it on the body of the rule is scale A, which is identical with B.

Sine and tangent scales, marked S and T, are engraved on the back of the sliding strip, and can be read in conjunction with index marks on the sides of the slots on the back of the stock. There is also a scale of logarithms, marked L, which can be read in conjunction with a mark on the side of the smaller slot. It is sometimes convenient to draw out the sliding strip and reverse it, so that the scales on the back can be read directly with the scales on the stock by means of the cursor.

2. DESCRIPTION OF THE LOGARITHMIC SCALES

The beginner will be aided in understanding the spacing of the numbers on scale D if he draws out the sliding strip, reverses it and puts it back into the stock, so that the 0 of the L scale is opposite the 1 on the left-hand end of the D scale. The arrangement of the L scale and the D scale will

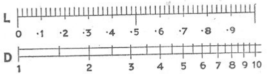


FIG. 1

then be somewhat as shown in Fig. 1. The figures on the L scale should always be read as decimals (less than unity) whether

the decimal is marked or not. Opposite 2 on the D scale is 0·301 on the L scale. Now this is the logarithm of 2. Opposite 3 is 0·477 on the L scale. This is the logarithm of 3. So we see that the numbers of the D scale are spaced so that their distances from the left-hand end is proportional to their logarithms. The length 1 to 2 on the D scale gives us the logarithm of 2.

Now withdraw the L scale, reverse the sliding strip and replace it so that scale C is opposite scale D. These two scales are exactly similar. To multiply 2 by 3 we move the sliding strip until C 1 comes opposite D 2. It will be seen that C 3 is opposite D 6. This is because we have added the logarithm of 2 to the logarithm of 3 and obtained the logarithm of 6. Opposite scale C and on scale D is the two-times table. To get the table for 6, 7, etc., we must move C 10 down to D 2. If we pull out the sliding strip until C 1 is opposite D 3 we have the three-times table. Fig. 2 shows portions of the A and B scales of a slide rule with the subdivision lines omitted.

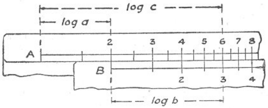


FIG. 2—MULTIPLICATION AND DIVISION

Let $c = ab$, then $\log c = \log a + \log b$

also $a = \frac{c}{b}$ and $\log a = \log c - \log b$

This explains in general terms how the slide rule does its work. For more definite instructions see Arts. 4 and 5.

The student, of course, knows that the "mantissa" of the logarithm is the decimal part only, the whole number part being called the "characteristic." The mantissa of 2, namely 0·30103, is the same as the mantissa of 2000. 2000 may be written 2×10^3, so its characteristic is 3. The logarithm of 2000 is therefore 3·30103. The slide rule adds and substracts the mantissae. The characteristics are looked after independently (see Art. 4).

If we take the scale C immediately in front of us to represent the numbers from 1 to 10, whose characteristic is 0, we may imagine another scale graduated exactly like C, placed immediately to the right of it and representing the numbers from 10 to 100, whose characteristic is unity. Immediately to the right of that we may imagine another similar scale representing numbers from 100 to 1000, whose characteristic is 2, and so on for any number of scales. We can also imagine a similar scale on the left of the rule whose characteristic is minus one. Therefore, the scale C in front of us gives us the mantissa of any number. Scales C and D enable us to multiply and divide any numbers, however great or however small, so long as we look after the characteristics in the way described in Art. 4.

3. HOW TO READ THE SCALES

When learning to read the scales attention must be given to the different kinds of graduations at various parts of the rule. The lengths being logarithmic, there is very much more room for low numbers like 2 than for higher numbers like 9.

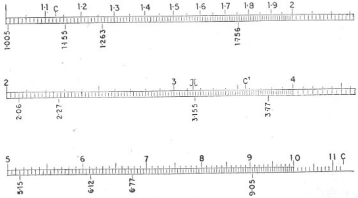


FIG. 3

Many slide rules mark the decimal points between 1 and 2, not 1·1, 1·2, 1·3, etc., as we have done in Fig. 3, but simply 1, 2, 3, etc. This is perhaps good enough when the operator gets used to it, but the beginner would do well to put decimal points before the figures, unless he can always remember that the little 9 between 1 and 2 means 1·9 or 19 or 190, or some number beginning 19, but never a number beginning with a 9.

As seen from Fig. 3, the graduations from 1 to 1·1 have nine times as much room as those from 9 to 9·1. This is as it should be, because 0·1 is one-tenth part of 1, while it is only one-ninetieth part of 9. On a 25 cm. rule there are ten divisions between 1 and 1·1. These represent 1·01, 1·02, etc., up to 1·09 and 1·1. It will be seen that the 1·05 mark is a little longer than the rest to show the half-way house. All the graduations up to 2 on the C and D scales are given in the same manner. At the right-hand end of the rule it is convenient to divide the distance between 9 and 9·1 into only two parts. We have 9, 9·05 and 9·1. Here the 9·05 mark is made shorter than the 9·1 mark. There are similar graduations between 4 and 10. Between 2 and 4 on the C and D scales it is possible to find room for a mark for every 0·02. So we have 2, 2·02, 2·04, 2·06, 2·08 up to 2·1. The beginner soon becomes familiar with the division of the first decimal place into ten parts between 1 and 2, into five parts between 2 and 4, and into only two parts between 4 and 9.

The next step is to be able to estimate the exact spot on the rule that corresponds to a value between two marks. For values between 1 and 2 it is not difficult to get the place right to about four significant figures. We know that, on account of the logarithmic spacing of the rule, 1·005 is not exactly half-way between 1 and 1·01, but the disymmetry is even less than with the 1·05 mark between 1 and 1·1. If we take the 0·005 mark exactly half-way we will not be out by the thickness of a line. It is then necessary to imagine the almost equal halves each divided into five parts, representing 0·001, 0·002, etc. A little practice enables us to do this with fair precision. In Fig. 3 the places representing various values are marked.

Begin by indicating the powers of 10, as shown by the little figures over the top line and under the bottom line.

Bring C 10 down to D 6·8 and K to C 3·141 (marked π on the rule).

The slide sticks out on the left. Put a 1 before 3·141.

Bring C 10 down to K (at D 2·14) and K to C 8·2. Slide sticks out on the left. Put a 1 before 820.

Bring C 1 to K (at 1·751) and K to C 1·3.

Bring C 10 to K (at 2·277) and K to C 5·2.

The slide sticks out at left. Put a 1 before 52. Read off D 1·184.

Product of top line is 1·84 × 10^6.

Now take the lower line.

Bring C 10 down to 7·46 and K to C 2·8.

Put a 1 before the 0·028.

Bring C 1 to K (at 2·09) and K to C 2·54.

Bring C 10 to K (at 5·305) and K to C 8·1.

Put a 1 before 0·81. Read off D 4·3.

Product of lower line is 4·3 × 10.

It is, of course, not necessary to read the positions of K at each stage as we have done above in brackets. The line K keeps the place for us while we move C 1 or C 10. It is then moved and enables us to set off the next figure with precision.

We then deal with the fraction as follows: As the digit above the line is less than that below the line, we multiply it by 10 (of course reducing by one the index power of ten to the right of it). In the example taken we write 11·84 × 10^5 instead of 1·184 × 10^6. The lower digit will now divide into the upper figure. Thus 4·3 goes into 11·84 2·753 times, and the power of ten in our example is 5 — 1 = 4.

7. EXERCISES

In these exercises the student should pay especial attention to the method of fixing the decimal point. In some of the exercises the answer is obvious. One can learn a rule best when it is applied to an obvious case.

In the following three exercises the sum of the logs is less than one. That gives us an answer less than 10. In the next three the sum of the logs is greater than unity, but not as great as two. That gives us an answer between 10 and 100

In the next three the sum of the logs is greater than two but less than three. The answer is then between 100 and 1000. We thus see that the slide rule is simply doing the multiplication by logs and giving the position of the decimal place exactly as the rules for logs gives it to us.

Ex. 1.	1·58 × 2 × 2·51	*Ans.* 7·9
Ex. 2.	1·047 × 3·16 × 1·995	*Ans.* 6·6
Ex. 3.	7·94 × 1·05 × 1·071	*Ans.* 8·9.

In all the above operations the sliding scale sticks out on the right.

Ex. 4.	2·545 × 3·141 × 4	*Ans.* 32
Ex. 5.	3·99 × 5·01 × 3	*Ans.* 60
Ex. 6.	8·23 × 7·12 × 1·56	*Ans.* 91·4

In each of the three exercises above the slider sticks out once on the left, meaning that the answer contains one power of ten.

Ex. 7.	3 × 4 ×5 × 6	*Ans.* 360
Ex. 8.	3·141 × 4·21 × 5·32 × 6·44	*Ans.* 454
Ex. 9.	9·2 × 8·3 × 7·1 × 1·51	*Ans.* 818

In each of the three operations above the slider must be made to stick out twice on the left. This means that the power of ten in the answer is two. In all nine exercises the numbers to be multiplied are digits, the power of ten being zero.

In the following exercises the student must indicate the power of ten in each factor before he begins, and then add an extra power of ten each time the slider sticks out on the left.

Ex. 10.	532 × 746 × 0·0026 × 82·3	*Ans.* 8·5 × 10^4
Ex. 11.	42200 × 0·00081 × 1·53 × 625	*Ans.* 3·27 × 10^4
Ex. 12.	31416 × 0·00421 × 532 × 0·0644	*Ans.* 4·55 × 10^3

8. METHOD OF SAVING TIME

Even when the friction of the slider is light, it is difficult for an operator to move it to the exact spot in an instant of time. He moves it perhaps one-hundredth of an inch too far, and when he moves it back again it goes a little too far in the other direction, by perhaps half as much. In some cases where accuracy is of importance, it may be worth while to spend a little time in getting the sliding scale (or the cursor) into the exact position. But for most engineering work the following plan will be found to be sufficient: Move C 1 or C 10 rapidly to the desired figure. It does not take two seconds

On the part of the C and D scales between 2 and 4, each main division is divided into five, so the 2·01, 2·03, etc., must be judged at points about half-way between 2 and 2·02, 2·02 and 2·04, etc. It is then more difficult to divide each of the imaginary ten divisions into ten again. All that one can do is to guess at the exact place. On this part of the scale we can get, without difficulty, three significant figures and perhaps an approximation to the fourth. In Fig. 3 places representing various values are given on this part of the scale.

On the part of the C and D scales between 4 and 10, the main divisions are divided into two. The 0·05 is marked clearly, so we must imagine the space between the lines to be divided into five to get 0·01, 0·02, etc. This is as far as we can hope to go with any precision.

There are two reasons why the C and D scales are so much better to use than the A and B scales. Four significant figures can be judged with fair approximation on the left half, and three significant figures obtained easily on the right half, whereas on the A and B scales it is difficult to get more than three significant figures even on the most open parts of the scales, and we can hardly work to that precision on the closer parts. But the most important reason is connected with the position of the decimal point. As to this see Art. 4.

After making a close study of Fig. 3 the student should practise setting the cursor line, K, to various values on scales C and D, and have his setting checked by someone who is very familiar with the rule. Many exercises are given below.

After getting proficient on scales C and D he should practise on scales A and B. It will be seen that here the scale between 1 and 2 is divided in the same manner as the part of C or D between 2 and 4, each main division being divided into five parts. The part from 2 to 5 is divided in the same manner as the part from 4 to 10 on the C and D scales. Between 5 and 10 there is only room for one mark for each decimal point. Some users prefer scales A and B, because by their use processes of continued multiplication and division can be carried out by fewer movements of the rule. There are two great drawbacks to using the scales in this way: it is much more

difficult to set the scale accurately, and it requires some alertness to get the decimal point in the right place. The users of scales A and B for multiplication and division have generally so little confidence in their method of fixing the decimal point that they prefer to work out the problem roughly by ordinary arithmetic in order to find where the decimal should come. This is very tedious and not always free from error. The position of the decimal point is as important as any other part of the answer, so the method of fixing it should be simple and free from uncertainty. Scales A and B are, however, useful in quickly finding square roots and cube roots (see Arts. 13 and 14). We therefore strongly recommend the use of scales C and D for multiplication and division, the decimal point being fixed in the way described below.

4. MULTIPLICATION

Multiply 2 by 3.

Put the 1 of scale C opposite 2 on scale D. Then you find that 3 on C is opposite 6 on D.

Again,

Multiply 2000 by 30,000

$$2 \times 10^3 \times 3 \times 10^4 = 2 \times 3 \times 10^7$$

The slide rule gives us the 6, as in the last example, and the addition of the powers of 10 (in this case) gives us the power of 10 in the answer. Note that in this case the sum of the mantissae of the logs is less than 1 (6 is less than 10), so that the answer, 6, can be read off while the sliding scale still sticks out on the right-hand end. We therefore add nothing to the index of the power of 10, which is 7 in this case.

But suppose we multiply 3 by 7. When we put the 1 of scale C opposite 3 D we find that 7 C is off the scale in front of us. Remember (see Art. 2) that there is an imaginary scale D immediately to our right. We therefore pretend that the scale in front of us is this scale to the right. So we move 10 C down to 3 D (that is where it would be with 1 C on 3 on a scale to our left). We now find 7 opposite 2·1, but we must multiply by 10 because we are on the 10 to 100 scale. The answer is therefore 21.

We arrive at the following simple rule: whenever it is necessary to bring 10 C down to a point on the D scale (instead of moving 1 C up to it), we must add 1 to the index of the power of 10 in the answer. This is done almost automatically, as will be seen in the examples below.

Multiply 1800 by 3 by 120 by 15

$$1{\cdot}8 \times 10^3 \times 3 \times 1{\cdot}2 \times 10^2 \times 1{\cdot}5 \times 10^1 = 9{\cdot}72 \times 10^6$$

Here the sum of all the logs of the digits is less than 1, so that there is nothing to add to the index of the power of 10 in the answer. Instead of taking the trouble to write down the tens in practice, we simply put a little figure over each number to indicate the power of 10 contained in it:

$$\overset{3}{1800} \times 3 \times \overset{2}{120} \times \overset{1}{15} = 9{\cdot}72 \times 10^6$$

The little index over each number tells us the number of places that we must move the decimal point to the *left* in order to turn each number into a digit. If the number is a decimal like 0·004 we must move the point three places to the *right* to convert it into a digit, for $0{\cdot}004 = 4 \times 10^{-3}$. In this case we write a little -3 over it, so $\overset{-3}{0{\cdot}004}$.

Multiply 0·004 by 20,000

$$\overset{-3}{0{\cdot}004} \times \overset{5}{200000} = 8 \times 10^2 = 800$$

In this case the sum of the logs is less than 1, so we add nothing to the algebraical sum of the little indices (5 — 3 = 2). To carry out the rule given above, all that is necessary is to put down a little 1 between those numbers which, in the process of adding the logs, require us to move the 10 C down to a figure on the D scale instead of moving C 1 up to it.

Multiply 300 by 7000

$$\overset{2}{300} \overset{1}{\times} \overset{3}{7000} = 2{\cdot}1 \times 10^6$$

Here it is necessary to bring the C 10 down to 3 and make the sliding scale stick out on the left. That means that we must put a little 1 between the 300 and the 7000, and when we add the indices we add on this 1.

After a little practice the putting down of the little 1, whenever the slide sticks out on the left, becomes perfectly automatic with the operator.

$$\text{Multiply } \overset{-3}{0{\cdot}004} \times \overset{1}{20} \overset{1}{\times} \overset{3}{3000} \overset{1}{\times} \overset{-4}{0{\cdot}0006} \times \overset{2}{510} \overset{1}{\times} 7 = 5{\cdot}14 \times 10^2$$

The process is as follows:

First indicate the powers of 10 in each number, as shown.

Bring C 1 to 4.

Bring K to C 2. The slide sticks out on the right (K is at D 8).

Bring C 10 to K (at 8) and bring K to C 3. The slide sticks out on the left. Therefore put a little 1 over the multiplication sign in front of 3000. Again we are compelled to bring down C 10 to K (at 2·4) and move K to C 6. The slide sticks out on the left. Put a little 1 over the multiplication sign in front of 0·0006. Bring C 1 to K (at 1·44). Move K to C 5·1. Slide now sticks out on the right, so we do not add a 1.

Bring C 10 to K (at 7·34) and K to C 7. Slide sticks out on the left. Add a little 1 before the 7.

The reading on D opposite C 7 is 5·14. The sum of the little characteristics is 2, so we have 514 for the answer. The correct answer is 514·08, but we cannot expect to get this accuracy on the slide rule.

It is not necessary to take the reading at K at each stage as we have done in brackets. K keeps the place for us.

5. DIVISION

To divide one number by another we subtract the log of the second from the log of the first.

Divide 3 by 2.

Bring C 2 to D 3 and read off 1·5 at C 1. This subtracts the log length 2 from the log length 3.

When there are several significant figures, the K line helps us to find the exact points on the rule easily. For instance:

Divide 3·49 by 2·11.

Put K to D 3·49.

Put C 2·11 to K. *Ans.* 1·654 at C 1.

When either of the numbers contains several powers of 10 these should be taken out before the operation on the rule and a record kept of them by means of the little figures, as shown below:

Divide 34,900 by 2110

$$\frac{\overset{4}{34900}}{\underset{3}{2110}} = 1{\cdot}654 \times 10^1 = 16{\cdot}54$$

We subtract the 3 from the 4.

Divide 0·0349 by 0·000211

$$\frac{\overset{-2}{0{\cdot}0349}}{\underset{-4}{0{\cdot}000211}} = 1{\cdot}654 \times 10^2$$

Here the -4 is subtracted from the -2, leaving +2.

When (after taking out the powers of 10) the *digit* on the top line is *less than* the digit on lower line, the simplest plan is to multiply it by 10 and reduce the index by 1.

Divide 211 by 349

$$\frac{2{\cdot}11 \times 10^2}{3{\cdot}49 \times 10^2} = \frac{21{\cdot}1 \times 10}{3{\cdot}49 \times 10^2}$$

Here 2·11 is less than 3·49.

We can see from inspection that 3 will go into 21.

Put K to D 2·11 and bring C 3·49 to K.

$$\text{Answer } 6{\cdot}045 \times 10^{-1} = 0{\cdot}6045$$

The reason for doing this is that we can see from inspection that the digit is about 6, and the indices of the powers of 10 can fix the decimal place without hesitation on our part.

Another way (instead of moving the decimal point on the upper line) is to simply divide the upper figures by the lower, and, if we have to read the answer from the C 10 line (instead of from the C 1 line), reduce the power of 10 by 1.

When there are a number of factors on the lower line it is best to multiply them together by the ordinary process of multiplication and then clear the fraction by one operation of division.

Find the value of:

$$\frac{3{\cdot}14 \overset{1}{\times} \overset{3}{6280}}{\underset{1}{51}{\cdot}4 \underset{1}{\times} \underset{-3}{0{\cdot}0028} \underset{1}{\times} \underset{1}{83}} = \frac{1{\cdot}975 \times 10^4}{1{\cdot}194 \times 10^1} = 1{\cdot}652 \times 10^3$$

Take out the powers of 10, as shown by the little characteristics. Multiply out the top line, taking care to add a little 1 whenever the slide sticks out on the left. Multiply out the lower line in exactly the same way. In this case the slide sticks out on the left when we multiply 5·1 by 2·8, so we add a 1 between these figures, and again we multiply by 8·3. The characteristic for the lower line is therefore 4 — 3 = 1. As the 1·975 is greater than 1·194, we simply divide out and subtract the characteristic of the lower line from that of the upper.

6. LONG CALCULATIONS

Engineering and other problems often lead to a long string of figures to be multiplied and divided.

Some users of the rule multiply and divide alternately, using scales A and B for this purpose, because by so doing the number of times the cursor and slider have to be moved is reduced. Our experience, however, is that very few of these users can look after the decimal point when they are doing this. To do so requires the intelligence always on the alert, and it is easy to make a mistake.

It is very much more important to have a simple, certain way of fixing the decimal point and to have the more accurate scale readings on C and D. A few more movements of the slider are of little importance compared with these advantages. We therefore strongly recommend the following course: Use scales C and D. Multiply out all the top line of a long calculation by itself. Multiply all the lower line of the calculation by itself. Clear the fraction by one operation of division.

Example: Find the value of

$$\frac{\overset{3}{6800} \overset{1}{\times} 3{\cdot}141 \overset{1}{\times} \overset{2}{820} \times \overset{-3}{0{\cdot}0013} \overset{1}{\times} \overset{1}{52}}{\underset{2}{746} \underset{1}{\times} \underset{-2}{0{\cdot}028} \times 2{\cdot}54 \underset{1}{\times} \underset{-1}{{\cdot}81}} = \frac{1{\cdot}184 \times 10^6}{4{\cdot}3 \times 10^1}$$

$$= \frac{11{\cdot}84 \times 10^5}{4{\cdot}3 \times 10} = 2{\cdot}753 \times 10^4$$

to get it within $\frac{1}{100}$ of an inch of the mark. If you try to get it nearer it may go to a point $\frac{1}{100}$ of an inch to the other side, and some time may be spent putting it backwards and forwards before you reduce the error to a few thousandths of an inch. Instead of doing this, move the cursor rapidly to the desired mark. Again there will probably be a small error. It is sufficient to have this error in the opposite direction to the last one. The total error is then the difference between the two, and is probably of the order of a few thousandths of an inch. In a long calculation these errors will very nearly cancel if we take care that half of them are positive and half negative. It will generally be found that the line K on the cursor can be more easily moved than the sliding scale C, so that errors in setting C can be easily allowed for. Care should be taken to look at the rule in a way to avoid parallax in the reading of K. Otherwise the errors may be all in the same direction. These remarks are not intended to encourage the careless setting of the rule. The student should practise all the time, setting with the greatest accuracy possible. If the sliding scale is easy to move, adjustments of the order of a few thousandths of an inch can be made by compressing, against the end of the rule, the skin of the finger and thumb that holds the slider. With a little practise you can arrange matters so that the force required to compress the skin is great as compared with the force required to move the slider. In this way a jerky movement is avoided. It will be remembered that in a long multiplication sum it is the line K that is set to the required figure on scale C, and then the line C 1 or C 10 is moved to K. The setting of one line to another line can be done more rapidly than the setting of a line to a figure which we have in our head. The error which appears as a mere thickening of the line can be allowed for in the next setting.

9. RED SCALES AT THE ENDS

Some slide rules are provided with continuations of the scales at both ends, coloured red. These continuations are for the purpose of enabling us to take a reading if we have inadvertently moved C 1 to a point on scale D when we ought to have moved C 10 to that point, or vice versa. The fact that the reading is in red warns us of the danger of forgetting to take note of

the appropriate power of 10. Whether that power is 1 or 0 depends upon the *correct* movement of C 1 or C 10. So if we take the red reading to save ourselves the trouble of moving the scale again, we must remember to indicate the right power of 10.

Example: Multiply 4·8 by 2.

We ought properly to move C 1 to 4·8. But suppose by error we move C 10 to 4·8, we can then read 9·6 on the red scale. The slide sticks out on the left but the red sign tells us we must not multiply by 10.

Again: Multiply 5·2 by 2.

We might inadvertently move C 1 to 5·2. Then the red scale opposite 2 warns us that the reading is 1·04 × 10, or 10·4.

10. FURTHER EXERCISES IN MULTIPLICATION AND DIVISION

2 × 5 = 10

4 × 3 = 12

4·3 × 2·5 = 10·75

4·35 × 2·05 = 8·92

4·2 × 26 × 0·15 = 16·38

3·14 × 78 × 86·1 = 21100

$\frac{1}{8} = 0{\cdot}125.$

$\frac{9}{32} = 0{\cdot}281.$

$\frac{47}{61} = 0{\cdot}771.$

$\frac{47^2}{61} = 36{\cdot}2$

$\frac{47^2}{61^2} = 0{\cdot}594.$

$\frac{47}{61^2} = 0{\cdot}01263.$

$\frac{43 \times 36}{71} = 21{\cdot}8.$

$\frac{0{\cdot}43}{36 \times 71} = 0{\cdot}0001682.$

$\frac{3{\cdot}14 \times 6{\cdot}07 \times 21{\cdot}6}{0{\cdot}471} = 875.$

$\frac{21600}{3{\cdot}14 \times 60{\cdot}7 \times 47{\cdot}1} = 2{\cdot}41.$

−2 1 3 1 4 1 −4

$$\frac{0{\cdot}0625 \times 8600 \times 43200 \times 0{\cdot}00079}{3{\cdot}141 \times 0{\cdot}0034 \times 33000 \times 21} = \frac{1{\cdot}834 \times 10^4}{7{\cdot}4 \times 10^3} = \frac{18{\cdot}34}{7{\cdot}4} = 2{\cdot}478$$

1 −3 4 1

2 1 2 −3 1 1

$$\frac{545 \times 333 \times 0{\cdot}00505 \times 12{\cdot}45}{9450 \times 632 \times 0{\cdot}000133 \times 0{\cdot}16} = \frac{1{\cdot}141 \times 10^4}{1{\cdot}27 \times 10^2} = \frac{11{\cdot}41 \times 10^3}{1{\cdot}27 \times 10^2} = 89{\cdot}8$$

3 1 2 −4 1 −1

3 −1 1 −3

$$\frac{1115 \times 0{\cdot}1257 \times 1{\cdot}732 \times 0{\cdot}0054}{746 \times 0{\cdot}00825 \times 1{\cdot}37 \times 9{\cdot}44} = \frac{1{\cdot}31}{7{\cdot}96 \times 10} = \frac{13{\cdot}1 \times 10}{7{\cdot}96} = 0{\cdot}01647$$

2 1 −3 1

11. SQUARES OF NUMBERS

The length from 1 to 10 on scale A is only half the length from 1 to 10 on scale D. Both are logarithmic scales, but one is half the length of the other. Therefore any particular length, representing, say, the log of 3, on D will represent twice that log on scale A. But if we double the log of a number we get the log of the square of the number. D 3 is therefore opposite A 9. All the numbers on scale A are the squares of the numbers opposite them on D. By means of the line K, we pass readily from any number to its square on scale A. In dealing with numbers greater than 10, one way is to write the number down twice and take the product, so that we may indicate the powers of ten in both factors and also any additional power of ten. Thus we get the accuracy of multiplying on the C and D scales.

Find the square of 8150

$$\overset{3}{8150} \overset{1}{\times} \overset{3}{8150} = 6{\cdot}64 \times 10^7$$

Another way is to take the square with the K line from D to A and remember to multiply the power of ten by two and to add one if K comes between 10 and 100 on the scale A.

12. CUBES OF NUMBERS

On the Reitz Pattern Rule there is a scale of cubes, F, placed near the lower edge and on the face of the rule. It is read in conjunction with the D scale, to which it is referred by the cursor. The scale will be seen to consist of three sections, each exactly alike and each one-third the length of the D scale. It is convenient to regard these as Sections I, II and III.

Section I on the left covers numbers from 1 to 10.

Section II covers numbers from 10 to 100, that is, numbers containing two figures to the left of the decimal. Some users mark the 1 in this section with a 0 after it to make it into 10.

Section III covers numbers from 100 to 1000, that is, numbers containing three figures to the left of the decimal. Some users mark the 1 on the left of this section with 00 to make it into 100.

Given a to find a^3.

Set cursor K to a on D and read a^3 on F.

If the result is read in Section I there will be one figure before the decimal point; if in Section II two figures; and if in Section III three figures.

Example. Find $1{\cdot}76^3$ *Ans.* 5·45

The result is found in Section I of F; hence there is one figure before the decimal point.

Example. Find $4{\cdot}04^3$ *Ans.* 65·9

The result being read in Section II, there are two figures before the decimal point.

Example. Find $6{\cdot}65^3$ *Ans.* 294

The result read in Section III has three figures before the decimal point.

These remarks apply to values of a between 1 and 10. All other numbers are brought into this category by moving the decimal point a sufficient number (say n) of places to bring the first figure into the unit position. After cubing, the decimal point to get the result is moved *three times* the number (that is $3n$) of places, and in the *opposite* direction.

Example. Find 194^3 *Ans.* 7,300,000

The decimal point, moved two places to the left, gives 1·94 and $1{\cdot}94^3 = 7{\cdot}30$. Moving the decimal point $3 \times 2 = 6$ places to the right gives 7,300,000.

Example. Find $0{\cdot}19^3$ *Ans.* 0·00686

Moving the point one place to the right gives 1·9, the cube of which is 6·86. Moving the point $1 \times 3 = 3$ places to the left gives 0·00686.

The rule then is: take the cube as if the original number were a digit (noting whether the cube falls among the tens or the hundreds) and move the decimal place in the answer obtained three times as many places to the right as there are powers of ten in the original number.

Examples: Cube of 2 is 8
Cube of 20 is 8000
Cube of 3·1 is nearly 29·8
Cube of 31 is nearly 29,800
Cube of 7·1 is nearly 358
Cube of 71 is nearly 358,000

From the above we see at once the reason for the rule given on page 24 for dividing a number into groups of three figures before taking the cube-root.

Where there is no cube scale we may proceed as follows:

To get the cube of a number we multiply its square by the number. This can be done conveniently by moving C 1 to the number on the D scale and setting K to the number on the B scale. The cube is then read off on the A scale.

Example. Find the cube of 2. Put C 1 to D 2. Put K to B 2. It then reads 8 on the A scale.

The rules for the decimal point are as follows:

The original number should first of all be considered as a digit; that is to say, the decimal point is moved n places to the left or right to make the number into a digit as explained above. Then find the cube of the digit by the method given. This may consist of one, two or three figures. That is one figure on the left end of A, or two figures on the right end of A. If, when we put C 1 to the number on D, the number on B comes to the right of A 100, it means that it belongs to the imaginary extension of scale A to the right, which covers numbers from 100 to 1000. We must then bring C 10 down to number on D and read at K from the left-hand end of A, but this means that there are three figures in the cube. After cubing, the decimal point in the result is moved *three times* the number (that is $3n$) of places as it was moved to make the original number into a digit, and in the opposite direction.

Example. Find the cube of 550.

Move the decimal point two places to the left to make the digit 5·5. Move C 10 to D 5·5. Move K to B 5·5. K reads A 1·665. This tells us that the cube of 5·5 is 166·5. As we have originally moved the decimal point two places to the left, we now move it 2 × 3 places to the right. *Ans.* 166,500,000.

Another way is to simply multiply out on scales C and D, looking after the decimal point as described in Art. 4.

$$\overset{2}{550} \overset{1}{\times} \overset{2}{550} \overset{1}{\times} \overset{2}{550} = 1{\cdot}665 \times 10^8$$

13. SQUARE ROOTS

Scale A is very useful for taking these roots. The main point to be be careful about is to use the right part of the scale in any particular case. The rule for square roots is as follows:

If the number of figures to the left of the decimal place is odd, use left half of A. If the number of figures in the quantity whose root is to be taken is even, use the right-hand end. This can be remembered because the right-hand end is marked 20, 30, 40, etc. The method is best illustrated by examples.

Find $\sqrt{2}$. There is one figure to the left of the decimal. Therefore use the left of scale A. Bring K to A 2. On scale D read 1·414.

Find the $\sqrt{2000}$. Here we have an even number of figures to the left of the decimal place. Use the right-hand end of A. Bring K to A 20. On scale D read 44·72. The number of figures in the answer to the left of the decimal is always equal to the number of groups of two figures in the original number (half a group being counted as a group).

Find $\sqrt{20000{\cdot}0}$. Divide the figures to the left of the decimal into groups, so, 2/00/00. There is only one figure in the first group, so use the left-hand end of scale A. As there are three groups, there are three figures to the left of the decimal point. *Ans.* 141·4.

Find the $\sqrt{200000{\cdot}0}$. There are two figures in the first group in 20/00/00 and three groups. The answer is 447·2.

It is extraordinary how many mistakes are made in the application of these simple rules.

14. SQUARE ROOTS OF FRACTIONS

Just as the scales A and B can be imagined to be extended on the right to similar scales labelled with higher powers of ten, so on the left they may be imagined to be extended to similar scales labelled with negative powers of ten. Take any decimal quantity, say $\sqrt{0{\cdot}862431}$. Split it up into groups of two figures each starting from the decimal, so, 0·86/24/81.

As there are no zeros after the decimal, the number 0·86 and all that follows it belongs to the imaginary scale immediately to the left of our normal A and B scales. We may therefore move (or imagine we move) the whole rule one full length to the left, and the reading 86 now represents 0·86. Bring K to A 86·25. The reading on scale D is 9·29. To fix the position of the decimal in the answer, observe the following rule: There are as many 0's after the decimal in the answer as there are complete pairs of 0's in the original figure. Here there are no 0's, so the answer is 0·929.

Find $\sqrt{0{\cdot}086248}$. Split up into pairs of figures 0.08/62/48. The 0·08 belongs to the left-hand end of the A and B scales (moved, or imagined to be moved, over one whole length to the left). Put K to 8·625 on the left-hand end of scale A. It reads about 2·94 on D. As there is not a *pair* of 0's to the right of the decimal, the answer is 0·294.

Find $\sqrt{0{\cdot}00008625}$. Split up into pairs of figures 0·00/00/86/25. The 86 now belongs to the right-hand end of the A scale moved three whole lengths to the left. Put K to A 86·25. It reads 9·29 on the D scale. As there are two pairs of 0's after the decimal in the original figure there are two 0's in the answer 0·00929.

15. CUBE ROOTS

Where the rule has a cube scale, as on the Reitz Pattern, the procedure is as follows: Cut off with commas groups of three figures beginning at the decimal point. It may be either to the left, so, 4,096, 41,000, 166,500, or it may be to the right, so, 0·000,166,5, when we are dealing with decimal fraction. The group on the extreme left may consist of one, two or three figures. If there is only one figure in this group we use section I; that is, the part of the cube scale from 1 to 10, which is on the left of that cube scale.

Example. Find the cube root of 4096. Cut off the groups of three, so, 4,096. As there is only one figure in the group on the left, put the cursor just a shade to the left of 4·1 in section I of the cube scale, and it will read 1·6 on the D scale. As there is one group of three figures to the left of the 4 we multiply by one power of 10, getting the answer 16. If there are two groups of figures in the cube, there are two figures in the cube root.

If there are two figures in group on the left, use section II; that is, the part of the cube scale in the middle which represents numbers 10 to 100.

Example. Take the cube root of 41,000. Cut off the group of three figures beginning on the right, so, 41,000. There are two figures in the group on the left. Bring the cursor to 41 in the middle of the cube scale. It reads about 3·45 on the D scale. As there is one group of three figures to the right of 41, the answer is about 34·5. If there are three figures in the group on the left, as there would be for the number 410,000, put the cursor to 410 and read off 7·43. The answer is then about 74·3. In all these cases there are two figures to the left of the decimal in the answer.

Example. Find the cube root of 6,280,000. There are three groups. The first group 6 is less than 10 and so belongs to the part of the cube scale between 1 and 10. Put K to 6·28. It reads 1·845 on scale D. As there are three groups there are three figures to the left of the decimal. *Ans.* 184·5.

Find the cube root of 515,000,000. Again there are three groups, the first of which is 515. This belongs to section III. Put K to 515 in section III. It reads 8·02 on the D scale. As there are three groups we make the answer about 802. The real answer is 801·56.

Where there is no cube scale, cube roots can be found from the A. B. C and D scales by making use of the principle set out in Art. 12 (page 20).

To understand the procedure make the following observations: Pull out the sliding scale to the right until B 1 is opposite A 100. Consider the three lengths 1 to 10, 10 to 100 on A and the additional length 1 to 10 on B. For this purpose call this latter length 100 to 1000 and imagine scale A extended to represent this. The length from A 1 to A 1000 is three times as long as from A 1 to A 10. We know that 10 is the cube root of 1000.

To get the cube root of any number from 1 to 1000 we have only to take one-third of the length of its log. A first approximation can be made by dividing the length into three parts by eye. For all numbers less than 1000 the cube root must be less than 10.

Example. Find the cube root of 27. Judging by eye the length 3 is about a third of the length 27 on scale A. Test this by putting C 1 to D 3 and look to see where B 3 comes to on scale A. It comes exactly to A 27. Therefore 3 is the root required.

Find the cube root of 25. A third of the length of A 25 is less than 3. Put the cursor line K to A 25 (so that the place is clearly marked). Now move C 1 until its reading on scale D is the same as the reading of K on scale B. As nearly as one can read on scale B the root is 2·925.

Take another number less than 1000, say 300. Pull out the sliding scale until B 1 is opposite A 100. The point 3 B now represents 300, because all the B scale in this position represents figures from 100 to 1000. A third of the length 300 is rather less than 7. Try 7. If we put C 1 to D 7 we cannot see where B 7 will come to because there is no A scale there. Put C 10 to D 7. We now see that B 7 comes to about A 343. Note that in this position of the slider the figures 1 to 10 on A represent 100 to 1000.

It is as though instead of moving C 10 down to D 7 we had moved D 7 up to C 10 so as to put scale A 1 to A 10 in the position A 100 to A 1000.

Put K to A 3, which now represents A 300. As the cube of 7 is greater than 300, slide down C 10 below 7 until its reading is the same as that opposite the figure A 3 (representing A 300). When this occurs the reading is a very little less than 6·7. The decimal place is right because the root must be less than 10.

16. CUBE ROOTS OF DECIMAL FRACTIONS

The procedure followed is similar to that adopted for square roots, except that the figures must be divided into groups of

three beginning at the decimal point. As before, we must imagine three half scales in a line, but now 0·005 must be taken as belonging to the digits; 0·050 as belonging to the scale between 10 and 100; and 0·500 as belonging to the scale between 100 and 1000.

Example. Find the cube root of 0·000835. Split up into groups of three, so, 0·000,835. The figure 835 belongs to the part of the A scale between 100 and 1000. The third length is not much less than 10. For convenience we take the left-hand end of A to represent this part of the triple scale. Put K to 8·35. Move C 10 to a little below D 10 until its reading on scale D agrees with the reading of K on scale B. This occurs a little below 9·42. To fix the decimal point observe the rule: There are as many 0's after the decimal point in the answer as there are complete groups of three 0's in the original figure. Here there is only one complete group of 0's, so the answer is 0·0942. Check this on the slide rule just as a matter of practice,

$$\overset{-2}{0{\cdot}0}\overset{1}{942} \times \overset{-2}{0{\cdot}0}\overset{1}{942} \times \overset{-2}{0{\cdot}0942} = 8{\cdot}35 \times 10^{-4}$$

17. FOURTH ROOTS

The square root of a square root is the fourth root. It can be found by means of two successive operations between scale A and scale D.

18. RATIOS

It is sometimes wanted to increase numbers in a certain ratio or to diminish them in a certain ratio. The ratio can be set by one setting of the sliding scale and all the answers read off directly.

Examples. Increase all the numbers 1·8, 2·5, 3, 4·3, 61, 7 and 92 in the ratio $\frac{3}{2}$. Put C 3 to D 2. Read the above numbers on the D scale and opposite them on the C scale are five of the answers required:

2·7 3·75 4·5 6·45 91·5

For the sixth answer scale C is too far to the left. We are warned that the power of ten must be increased by one. Put K to C 10 and then C 1 to K. We can now continue with numbers higher than 6·66. Thus $7 \times \frac{3}{2} = 10·5$. Note that the number of figures to the left of the decimal has been increased by one,

$$92 \times \tfrac{3}{2} = 138$$

The numbers of teeth in a set of change wheels for a lathe range from 15 to 120, in steps of 5. Find whether a pair of wheels can be selected from the set which will give approximately a velocity ratio of (*a*), π; (*b*), $\dfrac{1 \text{ cm.}}{1 \text{ in.}}$

(*a*) Set B π to A 10. Inspect other coinciding pairs of scale readings on B and A and select B 110 and A 35 as giving a suitable choice of wheels.

$$(b)\ \frac{1 \text{ cm.}}{1 \text{ in.}} = \frac{0·393708}{1·0} = \frac{6·29933}{16} = \frac{63}{160} \text{ nearly}$$

No pair of wheels in the set having numbers of teeth near this ratio can be discovered.

$$\text{But since } \frac{63}{160} = \frac{3 \times 21}{8 \times 20} = \frac{30 \times 105}{80 \times 100}$$

it is seen to be possible, with a train of four wheels selected from the set, to cut a metric screw with quite remarkable accuracy, using a guide screw having a pitch based on the inch. The pitch error is in fact only 1 in 9370.

Exercises

1. Solve the following four equations, which are expressed in proportional form.

$\dfrac{x}{6} = \dfrac{11}{29}$ *Ans.* 2.28. $\qquad \dfrac{4·3}{x} = \dfrac{27·3}{0·314}$ *Ans.* 0·0495.

$\dfrac{x}{2·81^2} = \dfrac{14·1}{3·72^2}$ *Ans.* 8·05. $\qquad \dfrac{3·45^2}{17·2} = \dfrac{37}{x}$ *Ans.* 53·5

2. From the set of change wheels specified in the example above, select the pair of wheels which will give a velocity ratio nearest to the ratio of half a lunar month to a week. The mean lunar month is equal to 29·53 days.

Ans. 95 and 45 teeth.

3. An alloy consists of copper, tin and lead in the proportions 17, 29 and 42. A new alloy is made by adding 2 cwt. of copper to each ton of the old alloy. Find the percentage compositions of the new alloy.

$$17 + 29 + 42 = 88$$

The number of lbs. of copper in a ton of alloy is

$$2240 \times \frac{17}{88} = 433 \text{ add } 224 = 657$$

$$2240 \times \frac{29}{88} = 738$$

$$2240 \times \frac{42}{88} = 1070$$

$$2465$$

The percentages work out at 26·7, 29·9 and 43·4.

19. GEOMETRICAL PROGRESSION

If two lines at a constant distance apart be moved over a log scale, the ratio p/q, of any pair of simultaneous readings p and q, is constant. For the distance between the lines is $\log p - \log q$, that is, $\log p/q$, and since the distance is constant so is the ratio p/q.

For the same reason, if a pair of similar log scales like A and B or C and D be set in any relative position, the ratio of any pair of cursor readings is constant. This property enables us to apply arithmetically the rule of three or proportion. See Fig. 2, Art. 2.

It also follows that any set of equidistant readings on a log scale form a geometrical progression. If now the slide be inverted so that the plain scale L is brought to the top, this scale will become available for the setting of the cursor in any required series of equidistant positions.

Where the ratio between successive terms is not far from unity and the logarithm of that ratio gives a simple reading on the log scale, the simplest method of reading off all the terms is that shown in the example below.

Example. Find the seven terms of a geometrical progression whose ratio is 1·26, the first term being 11. Find log 1·26 (see Art. 25). Fortunately the log comes out 0·1. Reverse the L scale and put L 0 opposite D 1·1. The terms can be read off on the D scale opposite the values on the L scale given below.

L scale	0	0·1	0·2	0·3	0·4	0·5	0·6
Terms	11	13·85	17·43	21·95	27·65	34·8	43·8

Where this method is not convenient owing to the equal spacings of the log scale being more difficult to find, the best method is to set the ratio between D 1 and K and set C successively. The last term obtained on C being set to D 1 and the new term being read by K on the C scale.

Example. Find the terms of a geometrical progression whose ratio is 1·34 and whose first term is 122. Set K to D 1·34. Set first term 122 on C scale to D 1. The second term is read off from K on scale C and is 163·5. Set 163·5 on C scale to D 1 and the third term is 219, and so on. Alternatively, we can multiply the terms successively by 1·34.

Exercise

A geometrical progression has seven terms, the first being 16·4 and the sixth 92·2. Determine the progression and its common ratio *r*.

Reverse the logarithm scale and bring L 0 to D 16·4. Opposite D 92·2 read 0·75. As there are to be five operations in going from the first term to the sixth, divide 0·75 by 5. The quotient is 0·15. Set L 0 to D 16·4. Opposite L 0·15 read D 23·2. Set L 0 to 23·2. Opposite L 0·15 read 32·8. Set L 0 to 32·8. Read 46·2. Set L 0 to 46·2. Read 65·2, and so on to 92·2. When we put L 0 to 92·2 we find L 0·15 off the scale, so we must put the *right-hand* end of the L scale to 92·2. Then at L 0·15 we read the seventh term, 130·2. Put L 0 to D 1. At L 0·15 read D 1·413. This is the value of *r*.

For a more general statement of some proportional properties of the rule, let x, y, z be a set of cursor readings on the scales A, B, C, the slide being set in any stationary position, then for all such readings x is proportional to y and to the square of z. Or in symbols,

$$\frac{x}{y} = \frac{x_1}{y_1} = \text{- - - - - - - -} = \text{constant} = \frac{a}{1} = \frac{1}{b}$$

$$\frac{x}{z^2} = \frac{x_1}{z_1^{\,2}} = \text{- - - - - - - -} = \text{constant} = \frac{a}{1} = \frac{1}{c^2}$$

where a and b are the readings on A and B against end lines on B and A and a and c are the readings on A and C against end lines on C and A.

In illustration, if linear dimensions z of similar figures be read on C, proportional areas x will be found on A.

20. THE SCALE OF RECIPROCALS

Above C on the sliding strip will be found, on the Standard Pattern, the scale of reciprocals with red lettering. By putting K opposite any number on C we read off at once the reciprocal of that number. To fix the position of the decimal point it is only necessary to remember that the product of the number and its reciprocal is unity. Thus the reciprocal of 8 is 0·125, while the reciprocal of 80 is 0·0125.

Exercise

Find the reciprocals of π, $\pi/4$, e and log e.

Ans. 0·3183, 1·273, 0·3680, 2·303.

Another use to which the rule may be put is in finding any two factors of a constant quantity. If C 1 is moved to any number on the D scale, then the two numbers one above another, one on the R scale and one on the D scale, read off in any position of K, are the factors of the number opposite C 1. This is useful in calculations relating to hyperbolic expansion curves, pv = a constant.

For instance, let $pv = 1500$. Put C 1 to D 1·5. Let the R scale give p and the D scale give v. When $p = 100$, $v = 15$. When $p = 90$, $v = 16{\cdot}66$. When $p = 50$, $v = 30$, and so on all along the R and D scales.

Some users employ the reciprocal scale when multiplying a long stream of numbers, in order to avoid moving the rule so often. If we divide by the reciprocal of a number we get the same result as multiplying by the number. If we alternately multiply by one and divide by the reciprocal of the next, the settings of the rule are reduced. But we have never known a user who employs this method who could look after the decimal point at the same time. It is, of course, possible to do so by keeping the intelligence sufficiently alert. The author does not recommend this method of dealing with a string of multipliers, but the example below shows how it can be done.

$$\textit{Ex.}\quad 2 \times \underline{3} \overset{1}{\times} \overset{2}{400} \overset{1}{\times} \overset{-4}{\underline{0{\cdot}0005}} \times \overset{2}{600} \overset{1}{\times} \overset{-1}{\underline{0{\cdot}7}} = 504$$

First take out the powers of ten as indicated. The plan is to *divide* by the *reciprocals* of the numbers underlined instead of multiplying by those numbers. It must be remembered, however, that after an operation of *division* the position of the sliding scale must be noted. If it is sticking out on the *right,* put a 1 for an extra power of 10. If it is sticking out on the left, no power of 10 is added. This is opposite to the rule adopted when multiplying, and it is difficult to make it automatic.

Proceed as follows:

Put R 3 to D 2. Reciprocal scale on left, no power of 10.
Move K to C 4. C scale on left indicate 1.
Put R 5 to K. R scale on right indicate 1.
Move K to C 6. C scale on right.
Put R 7 to K. R scale on right indicate 1.

Ans. $5{\cdot}04 \times 10^2$.

Now do it the way we recommend, using C and D only.

C 1 to D 2	K to C 3	No power of ten
C 10 to K	K to 4	Indicate 1
C 10 to K	K to 5	Indicate 1
C 1 to K	K to 6	No power of ten
C 10 to K	K to 7	Indicate 1

The difference in the time involved is about 10 secs. The scale of reciprocals is useful in many trigonometrical calculations.

21. GAUGE MARKS OR CONSTANTS

When a constant comes very frequently into an engineer's work, it is sometimes convenient to have the place of that constant marked on the slide rule with some distinguishing figure. One of the commonest constants is $\pi = 3{\cdot}1416$, the ratio of the circumference of a circle to its diameter. The reciprocal of this, 0·318, is marked with an " M " on the B scale.

The marks " C " and " C¹ " on scales C and D are, by using the cursor, both found to give the reading 1273 on B and A. Now $1{\cdot}273 = \frac{4}{\pi}$ the reciprocal of $\frac{\pi}{4}$. This is useful when working on scales A and B and when we wish to save moving the cursor by dividing by $\frac{4}{\pi}$ instead of multiplying by $\frac{\pi}{4}$.

Example. Find the curved surface and the volume of a cylinder of length l and diameter d.

$$\text{surface} = l\,\pi\,d = \frac{l\,d}{M}$$

$$\text{volume} = l\frac{\pi}{4}d^2 = \frac{d^2}{C}$$

and in each case one setting of the cursor is saved by substituting the divisors M or $1/\pi$ and C or $4/\pi$ for the multipliers π and $\pi/4$.

Exercises

1. Find the curved surface and volume of
   i. A cylinder, diameter 2·25 ft., length 3·62 ft.
      Ans. 25·6 ft.², 14·40 ft.³
   ii. A cone, diameter of base 33 in., height 39 in.
      Ans. Surface $= \pi d \times$ half slant height $= 15{\cdot}24$ ft.²

$$\text{Volume} = \frac{1}{3} \times \frac{\pi}{4}d^2 \times \text{height} = 6{\cdot}43 \text{ ft.}^3$$

2. The diameter of a spherical steel ball measures 2·73 in. Find its surface, volume and weight.
   Ans. Surface $= 4\pi r^2 = 23{\cdot}4$ in.²;

$$\text{Volume} = \frac{4}{3}\pi r^3 = 10{\cdot}65 \text{ in}^3. \quad \text{Weight} = 3{\cdot}02 \text{ lb.}$$

Many users will prefer to mark the constants they most commonly use on their own rules.

A number of useful figures are given on the back of the Standard Pattern Rule.

22. TRIGONOMETRICAL RATIOS ON THE STANDARD PATTERN

On the underside of the sliding strip of the Standard Pattern are two scales, one marked S and the other T. These are scales of sines and tangents (set out logarithmically so as to accord with the scales on the stock of the rule). The angles are figured in degrees and minutes. In the two slots at the ends of the body there are index marks to which the degrees of the angle can be set when it is desired to take a single reading or a few readings on the front of the rule. Alternatively, the sliding strip can be taken out and reversed and inserted exactly in line with the ends of the scales on the stock. The angles of the S scale are then directly opposite the values of the sines of the angles as read on scale A. (See page 23 as to scales on the Reitz Pattern.) The angles of the T scale are then directly opposite the values of the tangents of the angles as read off on scale D.

SINES AND COSECANTS. For any angle the cosecant is the reciprocal of the sine, so both of these ratios can be obtained by one setting of the index in the slot to the angle on the S scale.

**Example*. It is required to find the sine and cosecant of 30°. Push out the sliding strip until 30° on the S scale comes to an index mark in one of the slots (in this case it does not matter which slot). If we have pushed the strip to the left, A 1 will be found opposite B 50. This means that the sine of 30° is 0·5. The decimal point is best fixed by our knowledge of the approximate value of the sine. In this case we move the decimal point two places to the left. The cosecant is read on the A scale opposite B 100. In the example given, cosec 30° = 2. If the strip had been pulled out to the right so as to bring 30° to the index at that end, we would read the value of the sine on the B scale opposite the point A 100. The cosecant in this position is read opposite B 1.

Again find the sine of 5°. Push out the strip until 5° is opposite the index on the left. The reading opposite A 1 is 8·75. We know that the answer is of the order of 0·1. It is therefore 0·0875, not 0·875. Again we have moved the decimal two places to the left. The cosecant of 5° is 11·45 read opposite B 100.

When the sines of a large number of angles are required we reverse the S scale and bring its ends exactly opposite the ends of the A scale. All the values can then be obtained

* See footnote page 36.

direct. The decimal point must be moved two places to the left. The S scale only goes down to 35 minutes of arc because the sines of very small angles would require a very long expansion of the rule. The sines of small angles are almost the same as the number of radians in the angle, and can be obtained approximately from the formula, $\sin \theta° = 0{\cdot}017450\theta°$.

COSINES AND SECANTS. To get $\cos \theta$ and $\sec \theta$, subtract θ from 90° and take the sine and cosecant, because $\sin (90 - \theta) = \cos \theta$ and $\operatorname{cosec} (90 - \theta) = \sec \theta$.

TANGENT AND COTANGENTS. The T scale gives angles from 5·7° to 45°. If it is reversed and its ends lined up with the ends of scale D, we can read off the values of the tangents directly on scale D. We must move the decimal point one place to the left. Thus the tangent of 35° is just a shade over 0·7. To get the cotangents we take the reciprocals of the tangents. If the angle θ whose tangent is required is greater than 45°, we take the cotangent of $(90 - \theta)$. To get the cotangent of θ when it is over 45°, we take the tangent of $(90 - \theta)$.

When we require the values for only one or a few angles, we do not reverse the T scale but set the degrees of the angle to the index mark in the slot of left-hand end. Putting 35° to this mark we see that value of the tangent 0·7 is now given on scale C opposite D 1, while the cotangent 1·428 is given on scale D opposite C 10.

**Example*. Find the six trigonometrical ratios for an angle of 33·3°.

Set S 33·3 against the index mark in one of the end slots. Then read on scale B opposite A 1 sin 33·3 = 0·549. On scale A opposite B 100 cosec 33·3 = 1·82. Repeat the process with the complementary angle 90° — 33·3° or 56·7°.

Cos 33·3° = sin 56·7° = 0·835; sec 33·3° = 1·198.

To get the tangent set 33·3° on the T scale opposite the index mark in the slot on the left.

Opposite D 1 on scale C read tan 33·3° = 0·656.

Opposite C 10 on scale D read cot 33·3° = 1·525.

* See footnote, page 36.

**Example.* A force of 27 lb. acts along a line which makes $41\frac{1}{2}°$ with the horizontal. Find its horizontal and vertical components. *Ans.* 20·2 lb.; 17·88 lb.

We require to find $27 \cos 41\frac{1}{2}°$ and $27 \sin 41\frac{1}{2}°$. That is, $27 \sin 48\frac{1}{2}°$ and $27 \sin 41\frac{1}{2}°$. Invert the slide so that scale S now lies alongside its companion A.

Set the right-hand end line of S against A 27. Take cursor readings on A against the readings $48\frac{1}{2}$ and $41\frac{1}{2}$ on S. These with the decimal points give the answers

$$27 \sin 48\tfrac{1}{2}° = 20{\cdot}2; \qquad 27 \sin 41\tfrac{1}{2}° = 17{\cdot}88.$$

**Example.* Given the three angles α, β, γ of a triangle, find numbers a, b, c which are proportional to its sides.

We have, for any triangle, $\dfrac{\sin \alpha}{a} = \dfrac{\sin \beta}{b} = \dfrac{\sin \gamma}{c}$

Set the inverted slide in any position, and against α, β, γ on S read a, b, c on A.

Example. Given the sides a, b, c of a triangle, to find the angles α, β, γ.

Against a, b, c on A take readings α, β, γ on S. Then adjust the slide, until by trial the sum $\alpha+\beta+\gamma$ of the readings is 180°.

**Example.* The horizontal and vertical components of a vector are 3·27 and 2·46 units. Find the magnitude v of the vector and the angle δ which its line makes with the horizontal. *Ans.* 4·09, 36° 57′.

$$\tan \delta = \frac{2{\cdot}46}{3{\cdot}27} = 0{\cdot}752 \text{ (by slide rule division)}$$

Bring the inverted slide to its middle position in the body. Against 752 on D read, on T, the required angle,

$$\delta = 36° \ 57'.$$

* Note: The instructions marked thus (*) apply only to the ordinary Standard Pattern Rules which include Orthodox Sine and Tangent Scales in their complement.

For the P.I.C. New Series Improved Standard Types (which incorporate patented Direct and Inverse Differential Scales for Trigonometrical computations) **a separate booklet is issued, which fully explains the uses of these special scales and deals with their design.**

For the magnitude v we have

$$v = \frac{\text{hor. comp.}}{\cos \delta} = \frac{3{\cdot}27}{\sin (90 - \delta)} = \frac{3{\cdot}27}{\sin 53° \ 3'}$$

Set 53° 3′ on S against 3·27 on A. Against the end line on S read, on A, the answer,

$$v = 4{\cdot}09 \text{ units.}$$

Exercises

1. Find the sine, cosine and tangent of an angle of 128·5°.

   Ans. 0·783, —0·623, 1·257.

2. Find the six trigonometrical ratios of 1 radian or 57·3°.

   Ans. Sin 1 rdn. = 0·841, cos 1 rdn. = 0·540, tan 1 rdn. = 1·558, cosec 1 rdn. = 1·188, sec 1 rdn. = 1·851, cot 1 rdn. = 0·642.

3. Reduce 10°, 65°, 180° to radians, and $0{\cdot}15\pi$, 0·76, 2·2 radians to degrees.

   Ans. 0·1745, 1·134, 3·142; 27°, 43·5°, 126·1°.

4. Find the sine, tangent and radian measure of 4·5°.

   Ans. 0·0785, 0·0787, 0·0785.

5. Solve the following three equations.

   $\dfrac{x}{\sin 14{\cdot}2°} = \dfrac{3{\cdot}78}{\sin 54{\cdot}6°}$ *Ans.* $x = 1{\cdot}14$.

   $\dfrac{20{\cdot}6\,x}{\tan^2 14{\cdot}2} = \dfrac{3{\cdot}78}{\sin 54{\cdot}6°}$ *Ans.* $x = 0{\cdot}01442$.

   $\dfrac{x \sin 14{\cdot}2°}{20{\cdot}6} = \dfrac{3{\cdot}78}{\sin 54{\cdot}6°}$ *Ans.* $x = 389$.

6. If $a \sin \theta + b \cos \theta = r \sin (\theta + \alpha)$, then $\alpha = \tan^{1} \dfrac{b}{a}$ and $r = \dfrac{b}{\sin \alpha}$
Find α and r when $a = 3{\cdot}26$, $b = 2{\cdot}16$.

   Ans. $\alpha = 33{\cdot}5°$, $r = 3{\cdot}91$.

7. Given $y = 7{\cdot}3 \sin 3x - x \tan 2x$, find y_1 and y_2 corresponding to $x_1 = 0{\cdot}681$ rdn. (39°) and $x_2 = 0{\cdot}698$ rdn. (40°). Hence by linear interpolation deduce an approximate root of the equation $7{\cdot}3 \sin. 3x = x \tan 2x$.

   Ans. 0·11, —0·31. 0·685.

23. TRIGONOMETRICAL RATIOS ON THE REITZ PATTERN RULE

On the underside of the sliding strip of this rule are three scales, one marked S, another T and the third S and T for small angles only. These are scales of sines and tangents (set out logarithmically so as to accord with the scales on the stock of the rule). The angles are figured in degrees and minutes. The method of using these scales is similar to the method described in connection with the Standard Pattern, except that the sine scale is to be read from scale D instead of scale A.

SINES AND COSECANTS. For any angle the cosecant is the reciprocal of the sine, so both of these ratios can be obtained by one setting of the index in the slot to the angle on the S scale.

Example. It is required to find the sine and cosecant of 30° without reversing the slide. Push out the sliding strip until 30° on the S scale comes to an index mark in one of the slots (in this case it does not matter which slot). If we have pushed the strip to the left, D 1 will be found opposite 5. This means that the sine of 30° is 0·5. The decimal point is best fixed by our knowledge of the approximate value of the sine. In this case we move the decimal point one place to the left. The cosecant is read on the D scale opposite C 10. In the example given cosec 30°=2. If the strip had been pulled out to the right so as to bring 30° to the index at that end, we would read the value of the sine from D 10. The cosecant in this position is read opposite D 1.

For small angles (between 35 minutes of arc and 5° 45′) the values of the sines and the tangents are so nearly the same that one scale will do for both.

The scale on the middle of the back of the strip marked S and T is also read in conjunction with scales C and D. For this scale the readings on D must have the decimal point moved two places to the left.

Example. Find sine and cosecant of 3°. Draw out the slider until 3° on the central scale at the back is opposite the datum mark in the slot. On scale C, opposite D 10, read about 5·235. Moving the decimal point two places to the left we get the value 0·05235. The cosecant is the reciprocal of this, about 19·1. The tangent 3° is 0·0524. The error is less than one part in one thousand.

TANGENTS. When the tangent of an angle between 6° and 45° is required we use the scale on the edge of the sliding strip marked T. This is read in conjunction with scales D and C.

Example. Find tangent and cotangent of 22°. Push the sliding strip until 22° on the T scale is opposite the index mark at the left end of the rule. Opposite D 1 on the C scale we read 4·04. We must move the decimal one place to the left. Tan 22° = 0·406. The cotangent is then read on the reciprocal scale. Cotan 22° = 2·48. Where the angle θ is greater than 45° we take the cotangent of (90° — θ).

For small angles use S and T scale just as for finding a sine. When the ratios of a large number of angles are required we reverse the scale and put 45 T opposite D 10. We then have a complete table of ratios before us.

24. TEMPERATURE RESISTANCE SCALE FOR A COPPER CIRCUIT

This scale, as given in the Standard Pattern, uses part of the graduations of the tangent scale which, by a curious coincidence, fit the temperature-resistance coefficient of copper. It is read in conjunction with scale D. A change of resistance of a copper circuit indicated by a distance along scale D will be caused by a change of temperature indicated by the same distance along the temperature scale.

Example 1. At a temperature of 10° C. the resistance of the field windings of a dynamo measures 40 ohms. Find the mean temperature of the circuit when the observed resistance is 45 ohms. *Ans.* 41° C.

Invert the slide so as to bring the tangent scale to the front. Put K on D 40. Bring 10° C. to K. Move K to D 45. Read off the temperature scale 41° C.

Example 2. At 15° C. the resistance of a copper circuit is 12 ohms. Find its resistance when the temperature is 55° C. *Ans.* 13·92.

Put K to D 12. Bring 15° C. to K. Move K to 55° C. and read off the scale D the resistance 13·92.

In cases where we are dealing with temperature rises we simply add the rise to the initial temperature.

Example 3. At 17° C. the resistance of a copper circuit is 14 ohms. What will be its resistance after there has been a temperature rise of 40° C.? Here the final temperature is 17 + 40 = 57° C.

Put K to D 14. Bring 17° C. to K. Move K to 57° C. Read on scale D 16·2 ohms.

25. LOGARITHMS

The scale of logarithms in the case of the Standard Pattern is placed on the underside of the sliding strip. It may be read (without taking out the strip) by means of the index mark in the slot at the back. When taking a reading in this way it is best to turn the whole rule over so that the little slot is to the left, and the numbers on the log scale increase progressively from left to right. One is then less likely to make the mistake of reading 0·47 as 0·53.

When a considerable number of logs are required, the scale may be reversed and placed so that the 0 on the log scale is opposite 1 on the D scale. The mantissae of all the logs can then be read off directly.

Example. Find log 3. On the Standard Pattern put C 1 to D 3. Turn the rule completely over (anti-clockwise) so that the projecting sliding strip points to the left and we can see the index mark in the slot. This reads 0·477. Note there should always be a decimal before the mantissa. If we had merely turned the rule from front to back with the sliding strip pointing to the right, we might inadvertently read the value 0·523 instead of 0·477.

If the number whose logarithm is required is greater than 10 we must add the characteristic. This is always equal to the power of ten in the antilog. Thus the log of 30 (to the base 10) is 1·477 and the log of 30,000 is 4·477. When we are dealing with a decimal fraction the characteristic is negative, and it is usually convenient to keep the mantissa positive.

Thus the log of 0·3 is $\bar{1}$·477. The minus sign is put *over* the characteristic to show that only it is negative. $\bar{1}$·477 is equivalent to —0·523.

The complete minus form is convenient when we wish to multiply the log by any number.

For instance: Find $0{\cdot}03^5$. Log 0·03 = $\bar{2}$·477 = —1·523 — 1·523 × 5 = —7·615 = $\bar{8}$·385. Put K to L 0·385 to find the antilog. Read on scale D 2·43. As the characteristic is —8, we have the answer 0·0000000243.

Example. Calculate $0{\cdot}0465^{-1\cdot42}$ *Ans.* 78·2

First, find log 0·0465, thus: Against 465 on D read the mantissa 0·667 on L (either by an inversion of the slide or by use of the mark in the right-hand slot). Add the characteristic —2 and thus obtain

log 0·0465 = $\bar{2}$·667 = —2 + 0·667 = —1·333.

Now find —1·333 × (—1·42) = 1·893.

This is the logarithm of the answer. To find its antilogarithm:

Against the mantissa 893 on L read 782 on D. Insert the decimal point as indicated by the characteristic 1. We thus obtain, for the final answer, 78·2.

Note. Hyperbolic logarithms are found by multiplying ordinary logarithms by 2·303, or dividing by 0·4343. That is:

$$\log_e = 2{\cdot}303 \log_{10} \text{ or } \frac{\log_{10}}{0{\cdot}4343}$$

On the Reitz Pattern the scale of logarithms is placed near the top edge of the stock, and is read direct by means of the K line of the cursor. The D scale gives the antilog and the log scale gives the mantissa.

Exercises.

1. Obtain $\log_{10}$ and $\log_e$ of the following numbers:

	50	10	2·718	0	0·2	0·1
Ans.	1·699	1·0	0·4343	—∞	$\bar{1}$·301	$\bar{1}$·0
	3·91	2·303	1·0	—∞	—1·610	—2·303

2. Given the following logs find their antilogs:

	1·8	0·4	0·2	0·1	$\bar{1}{\cdot}7$	$\bar{1}{\cdot}5$	$\bar{1}{\cdot}3$
Ans.	63·2	2·512	1·585	1·259	0·501	0·316	0·1995

3. Find

	$32^{1\cdot4}$	$3{\cdot}2^{1\cdot4}$	$0{\cdot}32^{1\cdot4}$	$6{\cdot}31^{2\cdot5}$	$0{\cdot}631^{-2\cdot5}$	$0{\cdot}85^{-1\cdot}$
Ans.	128	5·09	0·203	100	3·16	11·46

4. The expansion curve of the indicator diagram of a gas engine has the equation $pv^{1\cdot3} = 222$. If $p = 100$, find v. If $v = 2$, find p.

Ans. 1·847, 90·4.

"P.I.C." THREE-LINE CURSOR

This cursor is supplied as an alternative to the usual standard single-line cursor. It bears two additional lines to the right and left of the main K line. To distinguish them from the main K line these extra lines are broken into sections, each covering the principal scales of the rule.

The R.H. line on the right is of service when calculations involve the areas of circles whose diameter is given, and vice versa. The distance between the main line and the R.H. line corresponds to the interval 0·7854 — 1 on the A scale. Hence by setting the R.H. line to a diameter on the D scale, the corresponding area of the circle can be read off on A under the main line. Passing from the D scale to the A scale squares the diameter and the R.H. line multiplies by 0·7854. The R.H. line is particularly useful when used the inverse way.

Example. The area of a circle is 120 sq. in.; find the diameter. Set K to 1·2 on scale A. From the R.H. line read D 1·236. As we have moved the decimal point two places to the left in the area, we move it one place to the right in the diameter, and so get 12·36 in. Note that it is important to move the decimal point by cutting off *pairs* of figures as described on page 23, otherwise we will not get the right result when passing from scale A to scale D.

The L.H. line is useful when converting from horse-power to kilowatts. Put K to the number of horse-power on *scale A* and the L.H. line gives us kilowatts on the same scale.

Example. How many kilowatts are equivalent to 150 h.p.? Put K to 1·5 on scale A and read off 1·12 at the L.H. line.

Ans. 112 kW.

ELECTRICAL AND MECHANICAL ENGINEERS' SLIDE RULE

DESCRIPTION

In addition to the scales found on the Standard pattern slide rule, the "P.I.C." Electrical and Mechanical Engineers' pattern slide rule is provided with a log-log scale in two sections, while in the groove under the slide are two scales, one of which facilitates the calculation of the drop in voltage in a copper conductor, while the other is of service in calculating the efficiencies of dynamos and motors.

THE LOG-LOG SCALES

These scales are used for obtaining the powers and roots of numbers, and especially fractional powers and roots. They can also be used to find the logarithm of a number to the base e (that is the base of natural or hyperbolic logarithms, having the value 2·718).

The log-log scales may be mounted of the stock of the rule as in the case of the "Engineers' New Pattern" and of the "Models C and D" slide rules, or they may be mounted on the sliding strip as in the case of the "A.C." rule. When they are mounted on the stock they are fixed with regard to D and used in conjunction with scale C. When they are mounted on the sliding strip they are fixed with regard to C and can be used in conjunction with scale D.

We will here consider the case when the log-log scale is mounted on the stock and read in conjunction with scales C and D. The log-log scales on the "Engineers' New Pattern" are two in number and are labelled E_1 and E_2.

THEORY OF THE LOG-LOG SCALE

A number on scale D is the natural logarithm of the number opposite to it on the log-log scale. If we take E_1 (the upper scale), then the logarithm as read off on scale D must have a *decimal point* in front of it. Thus the 3 on scale D must be read as 0·3 when taken as the logarithm of 1·35 on scale E_1. We see that on the *upper* scale the figure 1·35 is opposite 3 in the scale D. It we take E_2 (the lower scale), then the logarithm as read off on scale D is a digit. Thus 3 on the D scale is read as 3 when taken as the natural logarithm of 20 which stands above it on the E_2 scale. It follows from this that the numbers on scale E_2 are the tenth powers of the numbers opposite them on scale E_1. Thus 30 on scale E_2 is the tenth power of 1·405 just above it on the E_1 scale and the tenth root of 1000 is just a little less than 2.

The figures on the E_1 scale range from 1·105, whose natural log is 0·1, to 2·718, whose natural log is 1. On the E_2 scale they range from 2·718 to 22,000, whose natural log is 10. If the D scale is extended to 11·6 we can have opposite that figure on scale E_2 the antilog 100,000. It will be seen that, as the spacing of the D scale is proportional to the logarithms of the numbers with which it is labelled, the spacing of the E scales are proportional to the logarithms of the logarithms of the numbers with which they are labelled. The student will ask why do we use logarithms to the base e for the E scales while we use logarithms to the base 10 for the D scale. The answer is that the E scales are much more open with the smaller base. If we used the base 10 for both scales the number opposite 10 on the E scale (instead of being $22{,}000 = e^{10}$) would be $10{,}000{,}000{,}000 = 10^{10}$, and all the numbers we are likely to use would be unnecessarily crowded together. As in theory the log-log scale will do what we want it to do, whatever the base employed, we choose the base e because it gives a fairly open scale and carries up to quite a high number on scale E_2. Moreover, it enables us to read off the naperian logarithm of any number when we want it, in addition to the main use. The main use is the raising, with ease, any number to any required power or the taking of any required root.

POWERS AND ROOTS

To find any required power of any number:
Consider the expression

$$a = b^c$$
$$\log a = c \log b$$
$$\text{log-log } a = \text{log-log } b + \log c$$

Scales E_1 and E_2 are divided proportionally to the log-log and scale C is divided proportionally to the log. If, therefore, we add a piece ($= c$) of scale C to the log-log of b we get the log-log of a.

NUMERICAL EXAMPLES

Abbreviation.—As previously noted, a statement such as " 2 on scale C " may be shortened to " C2."

Example 1: Find $1{\cdot}5^2$.

Bring C 1 to E 1·5. Move the cursor to C 2. Read off on E_1 2·25. *Ans.* 2·25.

In this case the slide is sticking out on the right because the log of 2 is not too long to extend beyond the E_1 scale. Therefore we read the answer on scale E_1. If, however, the added log is so great that it will not come within the E_1 scale, we must push the slide to the left. It is then necessary to read the answer on E_2.

Example 2. Find 2^3.

Bring C 1 to E 2. We now find that C 3 is off the scale to the right (that is to say it is opposite the imaginary E_1 continued to the right, which is the same as E_2). It is necessary to bring C 10 to E 2. The scale now sticks out on the left. Bring the cursor to C 3 and read off 8 on scale E_2. (This is the same reading as we would have had on scale E_1 if it had been continued to the right. *Ans.* 8.

Example 3. Find $2{\cdot}2^{4\cdot5}$.

Bring C 10 to E 2·2 (slide sticks out on the left). Bring cursor to C 4·5 and read off as nearly as possible 34·8 on scale E_2. *Ans.* 34·8.

Example 4. Find $4{\cdot}1^{3\cdot2}$.

Bring C 1 to 4·1 on the E_2 scale. Bring cursor to C 3·2 (scale sticks out on the right. Read off 91·5 on the E_2 scale.

Example 5. Find $4{\cdot}1^9$.

Bring C 1 to 4·1 on the E_2 scale. We now find the 9 on the C scale is off on the right. This means that the answer is more than 100,000 and beyond the E_2 scale. This particular log-log scale cannot be used directly for so high a power of 4·1. Several methods of procedure are possible. One of these is to break up the quantity under consideration into factors each of which is small enough to come in the log-log scale. For instance

$$4{\cdot}1^{9} = 4{\cdot}1^{4{\cdot}5} \times 4.1^{4{\cdot}5}.$$

The value of $4{\cdot}1^{4{\cdot}5}$ is found by the log-log scale to be about 572, and this multiplied by itself gives about 327,000. Note that you cannot take the square of 572 on the log-log scale because when we put C 1 to 572 we find C 2 is off the scale. We must not put C 10 to 572 because there is no extension of the E_2 scale to the right *. Another way is to simply proceed by multiplying the log of the number by the index, as shown below. Find the log of 4·1 (see page 40). This is 0·6127 (though it can hardly be read to this accuracy). Multiply this by 9 and get 5·5143. From 0·5143 on the log scale we read off on scale D the number 3·27. The characteristic 5 gives us the index of the power of 10, so the answer is 327,000. On page 52 other methods are given for dealing with numbers out of range.

NEGATIVE INDICES

We know that $b^{-c} = \dfrac{1}{b^c}.$ In dealing with a negative index it is best to proceed as if it were positive and then take the reciprocal of the result thus obtained.

Example 6. Find the value of 3^{-2}. This is the same as $\dfrac{1}{3^2} = \dfrac{1}{9} = 0{\cdot}111.$ *Ans.* 0·111.

Example 7. Find $1{\cdot}3^{-2{\cdot}24}$.

Write it $\dfrac{1}{1{\cdot}3^{2{\cdot}24}}.$ Put C 1 to E 1·3. Opposite C 2·24 read 1·8. Then the answer is $\dfrac{1}{1{\cdot}8} = 0{\cdot}556.$

Example 8. Find $2{\cdot}25^{-4{\cdot}3}$.

Put C 10 to E_1 2·25. Opposite C 4·3 read 33 on E_2.

The answer is $\dfrac{1}{33} = 0{\cdot}0303$

Note that the only cases where we should put C 10 to a figure on the E_2 scale is when we are dealing with fractional powers.

* If there were an extension of the E_2 scale to the right it would extend up to numbers of the order of 10^{14}.

Example. Find $50^{0{\cdot}7}$.

If we put C 1 to E_2 50 we cannot take a reading because C 0·7 is on the imaginary scale to the left of C 1. We therefore put C 10 to E_2 50 and take all the C readings to be decimal parts of 1, so that 7 stands for 0·7 and the number opposite on the E_2 scale is 15·5. If the quantity required had been $50^{{\cdot}07}$, then we would put C 10 to E_2 50 and find the number 1·315 on the E_1 scale opposite 7. The numbers on the E_1 scale are the tenth roots of those on the E scale, so

$$50^{0{\cdot}7} = 15{\cdot}5 \qquad 50^{{\cdot}07} = 1{\cdot}315$$

as nearly as we can read the figures.

ROOTS OF NUMBERS

Consider the equations

$$a = \sqrt[c]{b}$$

$$\log a = \frac{1}{c}\log b$$

$$\text{log-log } a = \text{log-log } b - \log c$$

To find the *c*th root of b we must subtract the log of *c* from the log-log *b* and find the log-log *a*, which on the log-log scale gives us *a* directly.

NUMERICAL EXAMPLES

Example 9. Find $\sqrt{1{\cdot}96}$

Bring C 2 to E 1·96. From C 1 read off 1·4 on scale E.

Ans. 1·4.

Example 10. Find the $\sqrt{16}$

Bring C 2 to 16 on the E_2 scale. We find that C 1 gives us the reading 4 on the E_2 scale. *Ans.* 4.

Example 11. Find the $\sqrt[3]{8}$

Bring C 3 to E_2 8. We find that C 1 is now off on the left. This means that C 1 is on the imaginary continuation of the E_2 scale to the left, which is the same as E_1 scale. We therefore take the reading from C 10 on the E_1 scale and obtain 2. *Ans.* 2.

Example 12. Find $\sqrt{1{\cdot}5}$

Bring C 8 to E 1·5. The slide sticks out on the left, that is below the lowest figure on E_1. These scales therefore cannot be used for finding the 8th root of 1·5. Some slide rules are graduated for the log-log of numbers between 1·01 and 1·105, that is for values for an imaginary continuation of E_1 scale to the left. This scale is so seldom used that the student can be left to work out the 8th root of 1·5 by dividing the log of 1·5 by 8 and finding the antilog. Where, however, the number of which the root is required is sufficiently great, the answer may be found on one or other of the E scales.

Example 13. Find $\sqrt{100}$

Bring C 8 to E_2 100. The scale sticks out on the left—that is C 1 is opposite the imaginary scale E_2 continued to the left, which is the same as scale E_1. We therefore read off from C 10 the figure 1·778. *Ans.* 1·778.

FRACTIONAL POWERS AND ROOTS

As stated above, the numbers on scale E_2 are 10th powers of the numbers opposite to them on scale E_1; and *vice versa* the numbers on scale E_1 are the 10th roots of the numbers opposite to them on scale E_2. These facts can be made use of in taking fractional powers and fractional roots.

Example 14. Find $15^{0·32}$.

Now $15^{0·32} = 15^{0·1 \times 3·2} = \sqrt[10]{15^{3·2}}$

Bring C 1 to E_2 15. Opposite C 3·2 we have E_2 5780 and the tenth root of this is obtained by reading 2·38 on the E_1 scale. *Ans.* 2·38.

Example 15. Find $\sqrt[0·32]{1·5}$

The equivalent is $(\sqrt[3·2]{1·5})^{10}$ We therefore take the 3·2th root and raise it to the tenth power.

Bring C 3·2 to E_1 1·5. Opposite C 1 we have 1·135 on the E_1 scale and 3·55 on the E_2 scale, because 3·55 is the tenth power of 1·135. *Ans.* 3·55.

EXPONENTIAL EQUATIONS

In these equations the unknown quantity is the index of the power of some number which raises it to some other number. The process is the inverse of process in Examples 1 to 8 in this section.

Example 16. Solve the exponential equation $2·25 = 1·5^x$.

Put C 1 to E_1 1·5. From E_1 2·25 read off 2 on scale C. *Ans.* x = 2.

Example 17. Solve $8 = 2^x$.

If we put C 1 to E_1 2 we find that E 8 does not come opposite the C scale, so we must put C 10 to E_1 2. We now find E_2 8 opposite C 3. *Ans.* 3.

In this example we may consider that if the E_1 scale had been moved a whole rule-length to the left, then C 1 would be opposite E_1 2 while E_2 8 would be opposite C 3. We can then see that we are adding the length of log 3 to E_1 2 to get E_2 8. As we are under these circumstances starting from the zero point C 1 the figure 3 is a digit and does not stand for 30. But we cannot always assume that the figure read off scale C is a digit, as we shall see from the next example.

Example 18. Solve $66 = 1·15^x$.

Bring C 1 to E_1 1·15. Bring cursor to E_2 66. The cursor now reads 3 on scale C. Is this to be taken as 3 or 30? Observe that starting from the zero point C 1 the log length 3 added to E_1 1·15 takes us only to E_1 1·52. If we imagine the E_1 scale extended to the right with the same figures as E_2 and the C scale extended to the right beyond C 10 to C 20, C 30, etc., etc., then we would find C 30 opposite the imaginary E_2 66. We can read from E_2 66 direct and get the figure 3, but because we are starting from C 1 opposite the E_1 scale and are taking 66 off the E_2 scale, our reading must be multiplied by 10. *Ans.* 30.

In Example No. 17 we also read from the E_2 scale, but then C 10 was opposite E_1 2 and, as we explained above, the log length 3 was sufficient to take us to E_2 8.

The main uses of the log-log scale outlined above are: The raising of any number to any power, the taking any root of any number, and the solving of exponential equations.

RESULTS BY USE OF CURSOR ALONE

There are several useful results which can be read off with the cursor direct from the scales E_1, E_2 and D. For instance, put the cursor to E_1 1·3. It reads E_2 13·8 and D 2·621. From these three readings we arrive at the following results:

i. *Tenth roots and powers.* The readings on E_1 and E_2 are the tenth roots or the tenth powers, one of the other. In the instance taken:

$$1{\cdot}3 = \sqrt[10]{13{\cdot}8} \quad \text{and } 13{\cdot}8 = 1{\cdot}3^{10}$$

ii. *Hyperbolic logarithms.* The readings on D are the hyperbolic logarithms, those on E_1 and E_2. We must remember that the numbers on D opposite E_1 must have a decimal point put before them. For instance:

$$\log_e 1{\cdot}3 = 0{\cdot}2621.$$

But when we are reading from the E_2 scale the decimal comes after the first digit. For instance:

$$\log_e 13{\cdot}8 = 2{\cdot}621.$$

iii. *Powers of e.* The readings on E_1 scale are the powers of *e*, the indices being read off D with the decimal point in front. For instance:

$$e^{\cdot 2621} = 1{\cdot}3.$$

The readings on E_2 scale are the powers of *e*, the indices being read off D with the decimal point placed after the first digit. For instance:

$$e^{2{\cdot}621} = 13{\cdot}8.$$

LOGARITHMS TO ANY BASE

$$\text{Let } x = \log_b a \text{ or } a = b^x.$$

To find *x* take logs (to any base) of both sides.

$$\log a = x \log b$$

Take logs again.

$$\text{log-log } a = \log x + \text{log-log } b.$$

$$\therefore \log x = \text{log-log } a - \text{log-log } b.$$

This holds whatever system of logs are used. We might have used natural logs in the first case and logs to base 10 in the second. We should then have

$$\log_{10} x = \log_{10}\text{-}\log_e a - \log_{10}\text{-}\log_e b$$

and we can read off *x* direct from the slide rule.

Thus the required logarithm of *a* to the base *b* is obtained by subtracting the log-log of *b* from the log-log of *a*. The difference in lengths along the E scales gives on the C scale the logarithm required.

Example 19. Find $\log_8 12$.

$$12 = 8^x$$

$$\log_e 12 = x \log_e 8$$

$$\log_{10}\text{-}\log_e 12 = \log_{10} x + \log_{10}\text{-}\log_e 8$$

$$\log_{10} x = \log_{10}\text{-}\log_e 12 - \log_{10}\text{-}\log_e 8$$

Bring C 1 to E 8; against E 12 read C 1·195. The length difference between log-log 12 and log-log 8 is equal to the length representing the log of 1·195. *Ans.* 1·195.

Example 20. Find $\log_{12} 8$.

In this case the log-log to be subtracted is greater than the one from which it is taken. The characteristic of the resulting log will be negative, so that the answer will be less than unity. Put C 10 to E 12; opposite E 8 read 8·35. But as this figure is read on the left of C 10 we must take it as 0·835.

Note that when the base is greater than the number whose log is required, both being on E_1 or both on E_2, we must put C 10 opposite the log-log of the base and not C 1, and remember that the answer is less than unity. But consider the case when the two log-logs are on different scales.

Example 21. Find $\log_4 2$.

The easiest way to understand the procedure is to imagine the E_1 scale extended to the right, the numbers there being arranged exactly as on E_2. We have now to take the length log-log 4 from length log-log 2. Put C 10 to E_2 4. Mark the point 7·21 opposite D 1. Put this point opposite D 10, so that C 10 would be opposite the imaginary E_1 4. Take the reading C 5 opposite E 2. The characteristic being —1, we get the answer 0·5.

$$2 = 4^{0{\cdot}5} \quad \text{or} \quad 2 = \sqrt{4}$$

Alternatively we can use the reciprocal scale. Put the 1 on this scale to E 2 and opposite E 4 read off the red figure 5.

Example 22. Find $\log_2 4$.

Imagine the E_2 scale extended to the left with numbers the same as on the E_1 scale. Now we have to take the lesser length log-log 2 from log-log 4 on the extended scale. If we put C 10 opposite the real E_1 2 it will bring C 1 opposite the imaginary E_1 2. Then we read opposite E_2 4 the value C 2. From this we see that the length log 2 when added to log-log 2 gives log-log 4. The answer, therefore, is 2. $4 = 2^2$.

Example 23. Find $\log_2 2{\cdot}5$. *Ans.* 1·32.
Bring C 1 to E 2; read C 1·32 at E 2·5.

Example 24. Find $\log_{2{\cdot}5} 2$. *Ans.* 0·756.
Bring C 10 to E 2·5; read C 7·56 at E 2; move decimal.

Example 25. Solve $6{\cdot}2 = 1{\cdot}3^x$. *Ans.* 6·95.
Bring C 10 to E 1·3; against E 6·2 read C 6·95.

Example 26. Solve $1{\cdot}3 = 6{\cdot}2^x$. *Ans.* 0·1438.
Bring C 1 to E 6·2; against E 1·3 read C 0·1438.

NEGATIVE INDICES

We have $b^{-c} = 1/b^c$. Thus in finding b^{-c} we first obtain b^c and then read off its reciprocal by ordinary slide rule division.

Example 27. Find $1{\cdot}6^{-2{\cdot}3}$. *Ans.* 0·339.

$$\text{First find } 1{\cdot}6^{2{\cdot}3} = 2{\cdot}95\text{; then } \frac{1}{2{\cdot}95} = 0{\cdot}339$$

NUMBERS OUT OF RANGE

As previously stated, the lowest reading on the log-log scale is 1·1 and the highest 100,000. If in the equation $a = b^c$ either a or b fall outside these limts, the calculation for b^c is made indirectly in several steps, one or more of which will be ordinary slide rule multiplication or division. Three methods are available, and these are best shown by numerical examples.

Method 1. By factors.

$$\text{If } b = b_1 \times b_2, \text{ then } b^c = (b_1 \times b_2)^c = b_1^c \times b_2^c$$

Example 28. Find $257^{2{\cdot}1}$. *Ans.* 115,000.

Note that 257 = 10 × 25·7. So

Find $10^{2{\cdot}1} = 125{\cdot}9$, and $25{\cdot}7^{2{\cdot}1} = 910$

Then 125·9 × 910 = 115,000

Method 2. By a preliminary root or power.

$$\text{If } \sqrt[n]{b} = b_1, \text{ and } b_1^c = b_2, \text{ then } b^c = b_2^n.$$

Example 28*a*. Find $257^{2{\cdot}1}$. *Ans.* 115,000.

Find $\sqrt{257} = 16{\cdot}03$, and $16{\cdot}03^{2{\cdot}1} = 339$.

Then $339^2 = 115{,}000$.

Example 29. Find $1{\cdot}05^{20}$. *Ans.* 2·65.

Here $1{\cdot}05^{20} = 1{\cdot}05^{2\times10} = 1{\cdot}102^{10} = 2{\cdot}65$

Method 3. By reciprocals.

We have b^c = reciprocal of $\left(\frac{1}{b}\right)^c$

Example 30. Find $0{\cdot}83^{2{\cdot}9}$. *Ans.* 0·583.

$$\text{Find } \frac{1}{0{\cdot}83} = 1{\cdot}205, \text{ and } 1{\cdot}205^{2{\cdot}9} = 1{\cdot}716.$$

$$\text{Then } \frac{1}{1{\cdot}716} = 0{\cdot}583.$$

Or alternatively, *by factors*. Note that $0{\cdot}83 = \frac{8{\cdot}3}{10}$

Find $8{\cdot}3^{2{\cdot}9} = 463$, and $10^{2{\cdot}9} = 794$

$$\text{Then } \frac{463}{794} = 0{\cdot}583.$$

Miscellaneous Exercises for Practice and Illustration.

1. For various sizes of a certain make of turbine the relation $PN^2 = 2900\,H^{2{\cdot}5}$ holds between power, speed and head. If $P = 50$, calculate and tabulate N when $H = 8$, 12, 20, and H when $N = 100$, 200, 300.

Ans. H	8	12	20	7·84	13·66	18·89
N	102·4	170	324	100	200	300

2. The equation $T_2/T_1 = e^{f\theta}$ refers to a belt when slipping over a pulley or a flexible rope wrapped round a post. Taking $f = 0{\cdot}2$ and 0·4, and $\theta = \pi$, 2π, 4π, and 6π, calculate all the values of the ratio T_2/T_1.

Ans.	π	2π	4π	6π
0·2	1·875	3·52	12·35	43·4
0·4	3·52	12·35	152·5	188·2

3. If £1 becomes £R in n years at r per cent. per annum compound interest, we have $\left(1 + \frac{r}{100}\right)^n = R$ if the interest be added each year; and $e^{\frac{nr}{100}} = R$ if it be added continuously. Using these formulæ, find how much £100 becomes in 20 years if the rate is (*a*) 10% (*b*) 5%.

Ans. (*a*) £673, £739. (*b*) £265·4, £271·8.

4. Find the time required for a sum of money to double itself under each of the conditions of Ex. 3, if the rate of interest is (*a*) 10%, (*b*) 5%.

Ans. (*a*) 7·27, 6·93 years; (*b*) 14·20, 13·86 years.

5. A certain gas having a volume of 6 units at a pressure of 1 atmosphere is compressed adiabatically to a pressure of 5 atmospheres, the law of compression being $pv^{1\cdot4} = c$. Find c and the final volume. Find also the volumes at pressures of 2, 3 and 4 atmospheres, and the pressures when the volumes are 3, 4, and 5 units.

Ans. 12·28, 1·90, and

p	2	3	4	2·64	1·76	1·29
v	3·66	2·74	2·23	3	4	5

6. For an indicator diagram with an expansion curve pv^n constant, we have, for mean forward pressure, p_m the following formulæ:

$$\frac{p_m}{p_1} = \frac{nr^{-1} - r^{-n}}{n - 1}\text{; when } n = 1\text{, take } \frac{1 + \log_e r}{r}.$$

Find all the values of the ratio p_m/p_1 for r = 2, 5, 12 and n = 0·8, 1, and 1·2.

Ans.	2	5	12
0·8	0·872	0·579	0·352
1·0	0·847	0·522	0·290
1·2	0·824	0·475	0·247

7. The curves $y = ae^{bx}$ and $y = cx^n$ both pass through the points (2·0, 31) and (4·2, 55); determine the constants a, b, c, and n. For each curve, find the mid ordinate, *i.e.*, find y when x = 3·1.

Ans. $y = 18{\cdot}41e^{0\cdot261x}$; $y = 18{\cdot}1x^{0\cdot773}$; 41·3, 43·5.

8. Calculate $e^{\frac{pt}{20}} \cos pt$ when t = 0·12 sec. and p = 30 rdn./sec.

Ans. −0·749.

9. For the following hyperbolic functions we have, by definition,

$$\cosh x = \frac{1}{2}(e^x + e^{-x}),\ \sinh x = \frac{1}{2}(e^x - e^{-x}),\ \tanh x = \frac{\sinh x}{\cosh x}.$$

Calculate these when x = 0·19 and 1·9.

Ans. 1·018, 0·191, 0·188 and 3·42, 3·27, 0·956.

10. Evaluate $\cos x \sinh\left(\frac{x}{2} + \frac{\pi}{15}\right)$ when x = 0·384 rdn. (22°).

Ans. 0·382.

GAUGE MARKS FOR ELECTRICAL CALCULATIONS

On the engineering pattern rule gauge marks W and R, are given on scale D. They relate to the weight and resistance of copper conductors.

For the weight of a copper wire we have the formula

$$\text{Weight} = \frac{\text{length} \times (\text{diameter})^2}{(\text{constant})^2}$$

the constant depending on the units of measurement adopted.

This formula, expressed in the form of a proportion, with the units and scales completely specified, takes the following form:

$$\frac{\text{Weight (lbs.), on scale B}}{\text{Dia. (mils.), on scale D}} = \frac{\text{Length (yds.) on scale B}}{\text{Constant W, on scale D}}$$

The corresponding expression for resistance is

$$\frac{\text{Length (yds.) on B}}{\text{Dia. (mils.) on D}} = \frac{\text{Resistance (ohms), on B}}{\text{Constant R, on D}}$$

In the slide rule calculations, the numerical values of the constants are not required, except in locating the decimal points in the answers. Their values are:

$$W^2 = 111{,}000 \qquad R^2 = 30{\cdot}6$$

The method of applying the formulæ is illustrated in the following example:

Example. Find the weight and resistance of a copper conductor 62 yards long and 0·294 inch (294 mils.) diameter.

Ans. 48·2 lbs., 0·022 ohms.

Weight. Set 62 on B against W on D; opposite 294 on D read 486 on B.

Resistance. Set 62 on B against 294 on D; opposite R on D read 220 on B.

These gauge marks are of service to persons who frequently have to make calculations of this sort and who know quite well where the decimal point comes. If they do not know this they would have to work it out roughly by arithmetic, and the advantage of the gauge mark is lost. An alternative method of working out the above exercise, which gives without difficulty the right position of the decimal point, is the following:

$$\text{Weight in lbs.} = \frac{\overset{2}{294} \times \overset{2}{294} \times \overset{1}{62}}{\underset{5}{111000}} = 48{\cdot}2$$

See Art. 4, page 11, for the method of fixing the decimal point.

$$\text{Resistance} = \frac{30{\cdot}6 \times 62}{294 \times 294} = 0{\cdot}022 \text{ ohm.}$$

For any other metal, we first obtain the answers for copper, and then multiply these answers by the ratios of the density and specific resistance of the metal to those of copper.

Thus if the conductor in the above example were made of aluminium, the answers would be:

$$\text{Weight} = 48{\cdot}2 \times \frac{168}{550} = 14{\cdot}7 \text{ lbs.}$$

$$\text{Resistance} = 0{\cdot}022 \times \frac{1{\cdot}6}{1} = 0{\cdot}0352 \text{ ohms.}$$

Gauge points indicating the number of watts in one horse power are found at 746 on the A and B scales.

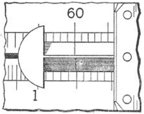
THE ELECTRICAL SCALES

These scales are found under the slide, being engraved on the stock of the rule at the bottom of the groove.

Hitherto, readings of these scales have been taken at the chisel-pointed ends of an index attached to the slide, and in adjusting the slide by pressure of the finger, these chisel points have proved a constant source of annoyance to the operator.

NEW INDEX POINTER

A new pointer, of registered design, is shown in the cut herewith. As will be seen, it presents a smooth, rounded end to the operator's finger, while allowing readings of the two scales to be taken very conveniently at index lines on the bevelled edges of the stem.



It will be evident that the new construction avoids all risk of injury, which with the older form of index always existed when the slide projected to the left, and represents in every way a distinct improvement in slide rule construction.

PRESSURE DROP SCALE

The lower of the two scales, the pressure drop scale, facilitates the calculation of the drop of pressure or voltage in a copper conductor of known length and sectional area carrying a known electrical current. Two patterns of rule are available, one for metric units and the other for English units. In each case the *total length* of the circuit is taken into the calculation.

For metric units:

$$\text{Pressure drop} = \frac{\text{current} \times \text{total length } (m)}{58 \times \text{sectional area}^2 \ (mm^2)}$$

while for English units:

$$\text{Pressure drop} = \frac{\text{current} \times \text{total length } (yds.)}{40{,}900 \times \text{sectional area } (sq.\ in.)}$$

In the first pattern, an inscription "10 Amp." at the left end of the A scale indicates that in calculating pressure drop, the current in amperes is to be read on this scale, and that the figured divisions, 1, 2, 3, etc., are to be taken as 10, 20, 30, etc.

In the same way the marks "10m" and "10mm²" at the left end of the B scale indicate that the total length of the conductor in metres and its sectional area in square millimetres, are to be read on the B scale, the values being again 10 times those figured. The division by the constant is effected by the position allocated to the voltage scale.

With the pattern for English units, the foregoing remarks apply to the inscriptions "10 amps." and "10 yds."; but for the cross-sectional area, the figures 1, 2, 3, etc., are to be read as 0·01, 0·02, 0·03, etc., in square inches as indicated by the inscription "0·01 sq. in." given on scale B. The division by the constant, which with the foregoing modifications becomes 40·9, is again effected by the position assigned to the voltage scale.

Should any of the factors fall outside the limits of the scales, suitable adjustments should be made in the positions of the decimal points with a compensating change in the decimal place in the answer, should this be required.

Example 1. A copper conductor carrying a current of 50 amperes has a total length of 170 metres. If the sectional area of the conductor is 45 square millimetres, what will be the voltage loss? *Ans.* 3·25 volts.

Working. Set 1 on B to 50 on A. Bring cursor K to 170 on B; set 45 on B to the cursor and read 3·25 volts on the volt scale.

Example 2. A copper conductor of 155 metres total length conveys a current of 42 amperes. What is its minimum sectional area if the volts drop is not to exceed 4 volts? *Ans.* 28 mm².

Working. Set 1 on B to 42 on A. Bring K to 155 on B and move the slide so that the indicator reads 4 on the volts scale. Under the cursor line, read 28 mm.² as the required sectional area.

Example 3. A copper conductor of 186 yards total length carries a current of 50 amperes. If the sectional area of the conductor is 0·07 square inches, what is the drop in pressure? *Ans.* 3·24 volts.

Working. Set 1 on B to 50 on A. Bring K to 186 on B. Set 0·07 on B to K, and read 3·24 on the volts scale.

Example 4. A copper conductor 170 yards total length, carries a current of 60 amperes. If the fall in pressure is not to exceed 5 volts, find the necessary sectional area. *Ans.* 0·05 square inches.

Working. Set 1 on B to 60 on A. Bring K to 170 on B and move the slide so that the indicator reads 5 on the volts scale. Under the cursor line read 0·05 square inches as the sectional area required.

EFFICIENCY SCALE FOR DYNAMOS AND MOTORS

This scale, found in the upper part of the recess, is used in conjunction with the scales A and B. It is designed to expedite the calculation of efficiency ratios (power output to input), when the electrical power, delivered or received, is expressed in watts or kilowatts, and the mechanical power in horse power units.

Beginning at the left end, the scale reads upwards from 20 to 100%; *this part applies to dynamos.* It then continues as a *scale for motors,* reading downwards from 100 to 20, thus becoming a scale of reciprocals.

The scale is so placed relative to scale A that a division by 746 is thereby effected (1 horse power = 746 watts). If the slide be set so that the index pointer reads 100, scales A and B become conversion scales, reading kilowatts and horse powers respectively, and they are stamped (at the ends on the right) with the letters KW and HP, by which letters they may be known.

Example 1. Reduce 40 KW to HP and 80 HP to KW.
Ans. 53·7 and 59·5.

Set the pointer to 100 on the efficiency scale.

Against 40 on the KW scale read 53·7 on the HP scale.
do. 80 do. HP do. 59·5 do. KW do.

Example 2. Find the efficiency of a dynamo which absorbs 124 HP and delivers 83 KW.

$$Ans.\ \text{Effy.} = \frac{83}{0{\cdot}746 \times 124} = 0{\cdot}897 \text{ or } 89{\cdot}7\%$$

By slide rule. Set 124 on the HP scale to 83 on the KW scale; on the dynamo scale read 89·7%.

Example 3. Find the horse power required to drive a dynamo with an output of 60 KW and an efficiency of 89%. *Ans.* 90·4 HP.

Set the pointer to 89 on the *left hand* or *dynamo scale;* against 60 on the KW scale read 90·4 on the HP scale.

Example 4. Estimate the efficiency of a motor which receives 29 KW and delivers 34 HP.

$$Ans.\ \text{Effy.} = \text{reciprocal of } \frac{29}{0{\cdot}746 \times 34} = 87{\cdot}5\%$$

By slide rule. Set 34 on the HP scale against 29 on the KW scale; on the motor scale read 87·5%.

Example 5. A motor of 90% efficiency receives 10 amperes at 230 volts; find the horse power given out.

Note. Watts = amperes × volts = 2300. *Ans.* 2·77 HP.

Set the pointer to 90 on the *right hand* or *motor scale*; against 2·3 on the KW scale read 2·77 on the HP scale.

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