INDEX TO MAJOR TOPICS

USE THE THUMB TABS

IMPORTANT REFERENCE TABLES

Mechanical Engineers' Handbook

PREPARED
BY A STAFF OF SPECIALISTS

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PREFACE

This Handbook is intended to supply both the practicing engineer and the student with a reference work which is authoritative in character and which covers the field of mechanical engineering in a comprehensive manner. It is no longer possible for a single individual or a small group of individuals to have so intimate an acquaintance with any major division of engineering as is necessary if critical judgment is to be exercised in the statement of current practice and the selection of engineering data. Only by the co-operation of a considerable number of specialists is it possible to obtain the desirable degree of reliability. This Handbook represents the work of fifty specialists.

Each contributor is to be regarded as responsible for the accuracy of his section. The number of contributors required to ensure sufficiently specialized knowledge for all the topics treated is necessarily large. It was found desirable to enlist the services of thirteen specialists for an adequate handling of the "Properties of Engineering Materials." Such topics as "Automobiles," "Aeronautics," "Illumination," "Patent Law," "Cost Accounting," "Industrial Buildings," "Corrosion," "Air Conditioning," "Fire Protection," "Prevention of Accidents," etc., though occupying relatively small spaces in the book, demanded each a separate writer.

A number of the contributions which deal with engineering practice, after examination by the Editor-in-Chief, were submitted by him to one or more specialists for criticism and suggestions. Their co-operation has proved of great value in securing greater accuracy and in ensuring that the subject matter does not embody solely the practice of one individual but is truly representative. The Editor-in-Chief begs to acknowledge his indebtedness for services of this character to the following gentlemen:

S. R. Bartlett (Lockwood, Greene & Co.), *Building Construction.* W. H. Bassett, Technical Superintendent and Metallurgist, American Brass Co., *Non-ferrous Metals.* F. N. Bushnell, Vice-president, Stone & Webster Engineering Corporation, *Cost of Power.* R. I. Clegg, Editor of Woodcraft, *Woodworking machines.* J. M. Darke (General Electric Company), *Miscellaneous Non-metallic Materials.* H. B. Emerson, Superintendent of Mechanical Department, Arlington Mills, Lawrence, *Electric Drives.* E. H. Fish, Associate Editor, American Machinist, *Machine Shop Practice.* H. J. Freyn, Consulting Engineer, *Internal Combustion Engines.* A. M. Houser, Mechanical Expert, Crane Company, *Pipe and Pipe Fittings.* G. B. Haven, Professor of Machine Design, Mass. Institute of Technology, *Machine Elements.* D. S. Jacobus, President, A. S. M. E., Advisory Engineer Babcock & Wilcox Co., *Steam Boilers.* D. D. Kimball (Richard D. Kimball Company), *Heating and Ventilation.* Albert Kingsbury, Consulting Engineer, *Friction.* H. S. McDewell, Instructor, University of Illinois, *Internal Combustion Engines.* Charles T. Main, Consulting Engineer, *Building Construction.* E. D. Meier (Deceased) Past President, A. S. M. E., President Heine Boiler Co., *Steam Boilers.* F. Nagler, Hydraulic Engineer, Allis-Chalmers Co., *Hydraulic Turbines.* W. S. Schwanhausser, Chief and Consulting Engineer, International Steam Pump Co., *Reciprocating Pumps.* F. N. Speller, Metallurgical Engineer, National Tube Co., *Pipe and Pipe Fittings.* F. R. Still, Chief Engineer, American Blower Co., *Centrifugal Fans.* G. W. Swett, Assistant Professor of Machine Design, Mass. Institute of Technology, *Machine Elements.* F. W. Taylor (Deceased), Past President, A. S. M. E., Consulting Engineer, *Industrial Management.* E. Touceda, Metallurgical Engineer, *Iron and Steel Castings.* J. F. Vaughan, Consultant with Stone & Webster Engineering Corporation, *Hydraulic Turbines.* W. H. Walker, Professor of Chemical Engineering, Mass. Institute of Technology, *Corrosion.* W. R. Webster, General Superintendent Bridgeport Brass Co., *Non-ferrous Metals.* Gardner S. Williams, Consulting Engineer, *Hydraulic Turbines.*

An accuracy of four significant figures has been assumed as the desirable limit; figures in excess of this number have been deleted, except in special cases. In the mathematical tables only four significant figures have been kept.

The Editor-in-Chief desires to express here his appreciation of the spirit of co-operation shown by the Contributors and of their patience in submitting to modifications of their sections. He wishes also to thank the Publishers for giving him complete freedom and hearty assistance in all matters relating to the book from the choice of contributors to the details of typography. And finally, he desires to express his indebtedness to his Editorial Assistant, Mr. Charles M. Sames, for numerous valuable suggestions, for the preparation of certain of the staff contributions, and for aiding him throughout in editing and styling manuscripts, correcting proofs, and compiling the index.

LIONEL S. MARKS.

CAMBRIDGE, MASS.
April 23, 1916

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SYMBOLS AND ABBREVIATIONS

For symbols of chemical elements, see p. 452; for electric and magnetic symbols, see p. 1568; for abbreviations applying to metric weights and measures, see p. 72.

Pairs of parentheses, brackets, etc., are frequently used in this work to indicate corresponding values. For example, the statement that "the cost per kw. of a 30,000-kw. plant is $53; of a 15,000-kw. plant, $62; and of a 8000-kw. plant, $72," is condensed as follows: The cost per kw. of a 30,000 [15,000] (8000)-kw. plant is $53 [62] (72).

abs.	absolute
a.c.	alternating current
a.h.p.	air horse power
amp.	amperes
Am. Iron & Steel Inst.	American Iron and Steel Institute
Am. Ry. Eng. Assn.	American Railway Engineering Association
antilog	antilogarithm of
approx.	approximate
A. S. M. E.	American Society of Mechanical Engineers
A. S. T. M.	American Society for Testing Materials
atm., atmos.	atmospheres (atmospheric)
aux.	auxiliary
avg.	average
avoir.	avoirdupois
bar.	barometer
B. & S.	Brown & Sharpe (gage)
bbl.	barrels
Bé.	Baumé (degrees)
B.G.	Birmingham gage (hoop and sheet)
b.h.p.	brake horse power
Birm.	Birmingham
B.M.	Board measure
B.t.u.	British thermal units
bu.	bushels
Bull.	Bulletin
B.W.G.	Birmingham wire gage
C.	centigrade
c.c.	cubic centimeters
cal.	calories
cent.	centigrade
C.F.	centrifugal force
c.f.m.	cubic feet per minute
C.G.	center of gravity
c. g. s.	centimeter-gram-second
C.I.	cast iron
cir.	circular
cm.	centimeters
c.m.	circular mils
coeff.	coefficient
c. of g.	center of gravity
col.	column(s)
colog	cologarithm of
comp.	compound
conc.	concentrated
cond.	condensing
const.	constant
cos	cosine of
$\cos^{-1}$	arc whose cosine is, anti-cosine of, inverse cosine of
cosec	cosecant of
cosh	hyperbolic cosine of
$\cosh^{-1}$	inverse hyperbolic cosine of
cot	cotangent of
$\cot^{-1}$	arc whose cotangent is (see $\cos^{-1}$)
coth	hyperbolic cotangent of
$\coth^{-1}$	inverse hyperbolic cotangent of
covers	coversed sine of
c.p.	candle power, circular pitch, center of pressure
csc	cosecant of
$\csc^{-1}$	arc whose cosecant is (see $\cos^{-1}$)

csch	hyperbolic cosecant of
$csch^{-1}$	inverse hyperbolic cosecant of
c. to c.	center to center
cu.	cubic
cyl.	cylinder
d.c.	direct current
def.	definition(s)
deg.	degrees
diam.	diameter
diff.	difference
dist.	distance, distributed
d.p.	diametral pitch, double pole
e	base of Napierian logarithmic system (= 2.7182 +)
E	modulus of elasticity in tension
effy.	efficiency
e.h.p.	effective horse power
E.L.	elastic limit
elec.	electric
e.m.f.	electromotive force
evap.	evaporation
exp	exponential function of
exsec	exterior secant of
fahr.	Fahrenheit
f.h.p.	friction horse power
F.O.B.	free on board (cars)
F.S.	factor of safety
ft.	feet
ft.-lb.	foot-pounds
g	acceleration due to gravity (= 32.16 ft. per sec.^{2})
g.	grams
gal.	gallons
g.-cal.	gram-calories
gd	Gudermannian of
gr.	grains
gyr.	gyration
Hg	mercury
hor. (horiz.)	horizontal
h.p.	horse power
h.-p.	high-pressure
h.p.-hr.	horse-power-hours
hr.	hours
i.h.p.	indicated horse power
imp.	imperial
in.	inches
inc.	inclusive
Ind. Eng. Soc.	Indiana Engineering Society
in.-lb.	inch-pounds
Inst. M. E.	Institution of Mechanical Engineers (London)
Int. Cong. Appl. Chem.	International Congress of Applied Chemistry
Int. Soc. Test. Mat.	International Society for Testing Materials
i.-p.	intermediate-pressure
isoth.	isothermal
kg.	kilograms
kg.-cal.	kilogram-calories
kg.-m.	kilogram-meters
km.	kilometers
kva.	kilovolt-amperes
kw.	kilowatts
kw.-hr.	kilowatt-hours
l.	liters
lb.	pounds
lin.	linear
lim.	limit
loco.	locomotive
log	common logarithm of
log.	logarithm
$\log_e$	Napierian logarithm of
$\log_{10}$	common logarithm of
l.-p.	low-pressure
m.	meters
max.	maximum
m.e.p.	mean effective pressure
m.h.c.p.	mean horizontal candle power
m.l.h.c.p.	mean lower hemispherical candle power
mi.	miles
min.	minutes, minimum
misc.	miscellaneous
mm.	millimeters
m.m.f.	magnetomotive force
m.s.c.p.	mean spherical candle power
N	number (in mathematical tables)
nat.	natural
Nat. Dist. Htg. Assn.	National District Heating Association
No. (Nos.)	number(s)
O.D.	outside diameter (pipes)
O.H.	open-hearth (steel)
op. cit.	work already cited
oz.	ounces

p. (pp.)	page (pages)
p. ct.	per cent.
perp.	perpendicular
p.f.	power factor
press.	pressure
Proc.	Proceedings
pt.	point, pint
qt.	quarts
q.v.	which see
rad	radian measure of angle
rad.	radius
rect.	rectangular
rev.	revolutions
R.M.S.	square root of mean square
r.p.m.	revolutions per minute
ry.	railway
S. A. E.	Society of Automobile Engineers
sat.	saturated
sec.	seconds
sec	secant of
$\sec^{-1}$	arc whose secant is (see $\cos^{-1}$)
sech	hyperbolic secant of
sech^{-1}	inverse hyperbolic secant of
segm.	segment
sin	sine of
$\sin^{-1}$	arc whose sine is (see $\cos^{-1}$)
sinh	hyperbolic sine of
$\sinh^{-1}$	inverse hyperbolic sine of
sol.	solution
sp.	specific
sp. gr.	specific gravity
sq.	square
std.	standard
S.W.G.	Standard (British) wire gage
tan	tangent of
$\tan^{-1}$	arc whose tangent is (see $\cos^{-1}$)
tanh	hyperbolic tangent of
$\tanh^{-1}$	inverse hyperbolic tangent of
temp.	temperature
Trans.	Transactions
T.S.	tensile strength
ult.	ultimate
U. S. S.	United States Standard
vel.	velocity
vers	versed sine of
vert.	vertical
vol.	volume
volum.	volumetric
vs.	versus
W.I.	wrought iron
wr't.	wrought
wt.	weight
yd.	yards
yr.	year(s)

ABBREVIATIONS OF TITLES OF PERIODICAL PUBLICATIONS

Am. Engr. American Engineer (New York).

Am. Mach. American Machinist (New York).

Bull. Am. Ry. Eng. Assn. Bulletin of the American Railway Engineering Association (Chicago).

Bull. Bureau of Standards. Bulletin of the Bureau of Standards (Washington, D. C.).

Elec. Jour. Electric Journal (Pittsburgh).

El. Ry. Jour. Electric Railway Journal (New York).

El. Wld. Electrical World (New York).

Eng. Digest. Engineering Digest (New York).

Eng. Mag. Engineering Magazine (New York).

Eng. News. Engineering News (New York).

Eng. Rec. Engineering Record (New York).

Engg. Engineering (London).

Htg. and Vent. Mag. Heating and Ventilating Magazine (New York).

Jour. Am. Chem. Soc. Journal of the American Chemical Society (Easton, Pa.).

Jour. Am. Soc. Nav. Engrs. Journal of the American Society of Naval Engineers (Washington, D. C.).

Jour. A. S. M. E. Journal of the American Society of Mechanical Engineers (New York).

Jour. Ind. & Eng. Chem. Journal of Industrial and Engineering Chemistry (Easton, Pa.).

Jour. Inst. E. E. Journal of the Institution of Electrical Engineers (London).

Jour. Iron and Steel Inst. Journal of the Iron and Steel Institute (London).

Jour. N. E. Water Works Assn. Journal of the New England Water Works Association (Boston).

Jour. West. Soc. Engrs. Journal of the Western Society of Engineers (Chicago).

Machy. Machinery (New York).

Mitteil. über Forschungsarbeiten. Mitteilungen über Forschungsarbeiten (Berlin).

Min. Proc. Inst. C. E. Minutes of the Proceedings of the Institution of Civil Engineers (London).

Phil. Mag. Philosophical Magazine (London).

Proc. A. I. E. E. Proceedings of the American Institute of Electrical Engineers (New York).

Proc. Am. Ry. Eng. Assn. Proceedings of the American Railway Engineering Association (Chicago).

Proc. Am. Ry. M. M. Assn. Proceedings of the American Railway Master Mechanics' Association (Chicago).

Proc. A. S. R. E. Proceedings of the American Society of Refrigerating Engineers (New York).

Proc. Am. Soc. Test. Mat. (A. S. T. M.). Proceedings of the American Society for Testing Materials (Philadelphia).

Proc. Inst. Auto. Eng. Proceedings of the Institute of Automobile Engineers (London).

Proc. Inst. C. E. Proceedings of the Institution of Civil Engineers (London).

Proc. Inst. M. E. Proceedings of the Institution of Mechanical Engineers (London).

Proc. Inst. of Metals. Proceedings of the Institute of Metals (London).

Proc. M. C. B. Assn. Proceedings of the Master Car Builders' Association.

Proc. Roy. Soc. Proceedings of the Royal Society (London).

R. R. Gaz. Railroad Gazette (New York).

Schweiz. Bauzeitung. Schweizerische Bauzeitung (Zürich).

Stahl u. Eisen. Stahl und Eisen (Düsseldorf).

Stevens Inst. Ind. Stevens Institute Indicator (Hoboken, N. J.).

St. Ry. Jl. Street Railway Journal (New York).

Trans. Am. Ry. Eng. Assn. Transactions of the American Railway Engineering Association (Chicago).

Trans. Am. Soc. C. E. Transactions of the American Society of Civil Engineers (New York).

Trans. A. S. H. & V. E. Transactions of the American Society of Heating and Ventilating Engineers (New York).

Trans. A. S. M. E. Transactions of the American Society of Mechanical Engineers (New York).

Trans. Inst. Naval Architects. Transactions of the Institution of Naval Architects (London).

Trans. Soc. Nav. Arch. & Mar. Eng. Transactions of the Society of Naval Architects and Marine Engineers (New York).

Trans. S. Wales Inst. Engrs. Transactions of the South Wales Institution of Engineers (London).

Zeit. f. Inst. Zeitschrift für Instrumentenkunde (Berlin).

Zeit. Oest. Ing. u. Arch. Ver. Zeitschrift des österreichischen Ingenieur- und Architekten-Vereins (Vienna).

Zeit. Ver. Deutsch. Ing. (Z. V. D. I.). Zeitschrift des Vereines deutscher Ingenieure (Berlin).

MATHEMATICAL SIGNS AND SYMBOLS

$+$ plus (sign of addition)
$+$ positive
$-$ minus (sign of subtraction)
$-$ negative
$\pm$ ($\mp$) plus or minus (minus or plus)
$\times$ times, by (multiplication sign)
$\cdot$ multiplied by
$\div$ sign of division
$/$ divided by
$:$ ratio sign, divided by, is to
$::$ equals, as (proportion)
$<$ less than
$>$ greater than
$=$ equals
$\approx$ approximately equals
$\leq$ equal to or less than
$\geq$ equal to or greater than
$+$ $\neq$ not equal to
$\doteq$ approaches
$\propto$ varies as
∞ infinity
$\sqrt{4}$ square root of
$\sqrt[3]{\ }$ cube root of
$\therefore$ therefore
$\parallel$ parallel to
( ) [ ] { } parentheses, brackets and braces; quantities enclosed by them to be taken together in multiplying, dividing, etc.
$\overline{AB}$ length of line from A to B
π pi, $= 3.14159+$
μ microns $= 0.001$ mm.
$\mu\mu$ micromillimeters $= 0.001\ \mu$
$^\circ$ degrees
$'$ minutes
$''$ seconds
$\angle$ angle
dx differential of x
Δ (delta) difference
Δx increment of x
$\partial u/\partial x$ partial derivative of u with respect to x
$\int$ integral of
$\int_a^b$ integral of, between limits a and b
Σ (sigma) summation of
$f(x)$, $F(x)$ functions of x
$4!$ factorial $4 = 1 \times 2 \times 3 \times 4$
$|x|$ absolute value of x

SECTION 1

MATHEMATICAL TABLES

AND

WEIGHTS AND MEASURES

BY

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CONTENTS

MATHEMATICAL TABLES

BY E. V. HUNTINGTON

WEIGHTS AND MEASURES

BY LOUIS A. FISCHER

SQUARES OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00	1.000	1.002	1.004	1.006	1.008	1.010	1.012	1.014	1.016	1.018	2
1	1.020	1.022	1.024	1.026	1.028	1.030	1.032	1.034	1.036	1.038	
2	1.040	1.042	1.044	1.047	1.049	1.051	1.053	1.055	1.057	1.059	
3	1.061	1.063	1.065	1.067	1.069	1.071	1.073	1.075	1.077	1.080	
4	1.082	1.084	1.086	1.088	1.090	1.092	1.094	1.096	1.098	1.100	
1.05	1.102	1.105	1.107	1.109	1.111	1.113	1.115	1.117	1.119	1.121	
6	1.124	1.126	1.128	1.130	1.132	1.134	1.136	1.138	1.141	1.143	
7	1.145	1.147	1.149	1.151	1.153	1.156	1.158	1.160	1.162	1.164	
8	1.166	1.169	1.171	1.173	1.175	1.177	1.179	1.182	1.184	1.186	
9	1.188	1.190	1.192	1.195	1.197	1.199	1.201	1.203	1.206	1.208	
1.10	1.210	1.212	1.214	1.217	1.219	1.221	1.223	1.225	1.228	1.230	
1	1.232	1.234	1.237	1.239	1.241	1.243	1.245	1.248	1.250	1.252	
2	1.254	1.257	1.259	1.261	1.263	1.266	1.268	1.270	1.272	1.275	
3	1.277	1.279	1.281	1.284	1.286	1.288	1.290	1.293	1.295	1.297	
4	1.300	1.302	1.304	1.306	1.309	1.311	1.313	1.316	1.318	1.320	
1.15	1.322	1.325	1.327	1.329	1.332	1.334	1.336	1.339	1.341	1.343	
6	1.346	1.348	1.350	1.353	1.355	1.357	1.360	1.362	1.364	1.367	
7	1.369	1.371	1.374	1.376	1.378	1.381	1.383	1.385	1.388	1.390	
8	1.392	1.395	1.397	1.399	1.402	1.404	1.407	1.409	1.411	1.414	
9	1.416	1.418	1.421	1.423	1.426	1.428	1.430	1.433	1.435	1.438	
1.20	1.440	1.442	1.445	1.447	1.450	1.452	1.454	1.457	1.459	1.462	
1	1.464	1.467	1.469	1.471	1.474	1.476	1.479	1.481	1.484	1.486	
2	1.488	1.491	1.493	1.496	1.498	1.501	1.503	1.506	1.508	1.510	
3	1.513	1.515	1.518	1.520	1.523	1.525	1.528	1.530	1.533	1.535	
4	1.538	1.540	1.543	1.545	1.548	1.550	1.553	1.555	1.558	1.560	
1.25	1.562	1.565	1.568	1.570	1.573	1.575	1.578	1.580	1.583	1.585	3
6	1.588	1.590	1.593	1.595	1.598	1.600	1.603	1.605	1.608	1.610	
7	1.613	1.615	1.618	1.621	1.623	1.626	1.628	1.631	1.633	1.636	
8	1.638	1.641	1.644	1.646	1.649	1.651	1.654	1.656	1.659	1.662	
9	1.664	1.667	1.669	1.672	1.674	1.677	1.680	1.682	1.685	1.687	
1.30	1.690	1.693	1.695	1.698	1.700	1.703	1.706	1.708	1.711	1.713	
1	1.716	1.719	1.721	1.724	1.727	1.729	1.732	1.734	1.737	1.740	
2	1.742	1.745	1.748	1.750	1.753	1.756	1.758	1.761	1.764	1.766	
3	1.769	1.772	1.774	1.777	1.780	1.782	1.785	1.788	1.790	1.793	
4	1.796	1.798	1.801	1.804	1.806	1.809	1.812	1.814	1.817	1.820	
1.35	1.822	1.825	1.828	1.831	1.833	1.836	1.839	1.841	1.844	1.847	
6	1.850	1.852	1.855	1.858	1.860	1.863	1.866	1.869	1.871	1.874	
7	1.877	1.880	1.882	1.885	1.888	1.891	1.893	1.896	1.899	1.902	
8	1.904	1.907	1.910	1.913	1.915	1.918	1.921	1.924	1.927	1.929	
9	1.932	1.935	1.938	1.940	1.943	1.946	1.949	1.952	1.954	1.957	
1.40	1.960	1.963	1.966	1.968	1.971	1.974	1.977	1.980	1.982	1.985	
1	1.988	1.991	1.994	1.997	1.999	2.002	2.005	2.008	2.011	2.014	
2	2.016	2.019	2.022	2.025	2.028	2.031	2.033	2.036	2.039	2.042	
3	2.045	2.048	2.051	2.053	2.056	2.059	2.062	2.065	2.068	2.071	
4	2.074	2.076	2.079	2.082	2.085	2.088	2.091	2.094	2.097	2.100	
1.45	2.102	2.105	2.108	2.111	2.114	2.117	2.120	2.123	2.126	2.129	
6	2.132	2.135	2.137	2.140	2.143	2.146	2.149	2.152	2.155	2.158	
7	2.161	2.164	2.167	2.170	2.173	2.176	2.179	2.182	2.184	2.187	
8	2.190	2.193	2.196	2.199	2.202	2.205	2.208	2.211	2.214	2.217	
9	2.220	2.223	2.226	2.229	2.232	2.235	2.238	2.241	2.244	2.247	

Moving the decimal point ONE place in *N* requires moving it TWO places in body of table (see p. 6).

SQUARES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.50	2.250	2.253	2.256	2.259	2.262	2.265	2.268	2.271	2.274	2.277	3
1	2.280	2.283	2.286	2.289	2.292	2.295	2.298	2.301	2.304	2.307	
2	2.310	2.313	2.316	2.320	2.323	2.326	2.329	2.332	2.335	2.338	
3	2.341	2.344	2.347	2.350	2.353	2.356	2.359	2.362	2.365	2.369	
4	2.372	2.375	2.378	2.381	2.384	2.387	2.390	2.393	2.396	2.399	
1.55	2.402	2.406	2.409	2.412	2.415	2.418	2.421	2.424	2.427	2.430	
6	2.434	2.437	2.440	2.443	2.446	2.449	2.452	2.455	2.459	2.462	
7	2.465	2.468	2.471	2.474	2.477	2.481	2.484	2.487	2.490	2.493	
8	2.496	2.500	2.503	2.506	2.509	2.512	2.515	2.519	2.522	2.525	
9	2.528	2.531	2.534	2.538	2.541	2.544	2.547	2.550	2.554	2.557	
1.60	2.560	2.563	2.566	2.570	2.573	2.576	2.579	2.582	2.586	2.589	
1	2.592	2.595	2.599	2.602	2.605	2.608	2.611	2.615	2.618	2.621	
2	2.624	2.628	2.631	2.634	2.637	2.641	2.644	2.647	2.650	2.654	
3	2.657	2.660	2.663	2.667	2.670	2.673	2.676	2.680	2.683	2.686	
4	2.690	2.693	2.696	2.699	2.703	2.706	2.709	2.713	2.716	2.719	
1.65	2.722	2.726	2.729	2.732	2.736	2.739	2.742	2.746	2.749	2.752	
6	2.756	2.759	2.762	2.766	2.769	2.772	2.776	2.779	2.782	2.786	
7	2.789	2.792	2.796	2.799	2.802	2.806	2.809	2.812	2.816	2.819	
8	2.822	2.826	2.829	2.832	2.836	2.839	2.843	2.846	2.849	2.853	
9	2.856	2.859	2.863	2.866	2.870	2.873	2.876	2.880	2.883	2.887	
1.70	2.890	2.893	2.897	2.900	2.904	2.907	2.910	2.914	2.917	2.921	
1	2.924	2.928	2.931	2.934	2.938	2.941	2.945	2.948	2.952	2.955	
2	2.958	2.962	2.965	2.969	2.972	2.976	2.979	2.983	2.986	2.989	
3	2.993	2.996	3.000	3.003	3.007	3.010	3.014	3.017	3.021	3.024	
4	3.028	3.031	3.035	3.038	3.042	3.045	3.049	3.052	3.056	3.059	
1.75	3.062	3.066	3.070	3.073	3.077	3.080	3.084	3.087	3.091	3.094	4
6	3.098	3.101	3.105	3.108	3.112	3.115	3.119	3.122	3.126	3.129	
7	3.133	3.136	3.140	3.144	3.147	3.151	3.154	3.158	3.161	3.165	
8	3.168	3.172	3.176	3.179	3.183	3.186	3.190	3.193	3.197	3.201	
9	3.204	3.208	3.211	3.215	3.218	3.222	3.226	3.229	3.233	3.236	
1.80	3.240	3.244	3.247	3.251	3.254	3.258	3.262	3.265	3.269	3.272	
1	3.276	3.280	3.283	3.287	3.291	3.294	3.298	3.301	3.305	3.309	
2	3.312	3.316	3.320	3.323	3.327	3.331	3.334	3.338	3.342	3.345	
3	3.349	3.353	3.356	3.360	3.364	3.367	3.371	3.375	3.378	3.382	
4	3.386	3.389	3.393	3.397	3.400	3.404	3.408	3.411	3.415	3.419	
1.85	3.422	3.426	3.430	3.434	3.437	3.441	3.445	3.448	3.452	3.456	
6	3.460	3.463	3.467	3.471	3.474	3.478	3.482	3.486	3.489	3.493	
7	3.497	3.501	3.504	3.508	3.512	3.516	3.519	3.523	3.527	3.531	
8	3.534	3.538	3.542	3.546	3.549	3.553	3.557	3.561	3.565	3.568	
9	3.572	3.576	3.580	3.583	3.587	3.591	3.595	3.599	3.602	3.606	
1.90	3.610	3.614	3.618	3.621	3.625	3.629	3.633	3.637	3.640	3.644	
1	3.648	3.652	3.656	3.660	3.663	3.667	3.671	3.675	3.679	3.683	
2	3.686	3.690	3.694	3.698	3.702	3.706	3.709	3.713	3.717	3.721	
3	3.725	3.729	3.733	3.736	3.740	3.744	3.748	3.752	3.756	3.760	
4	3.764	3.767	3.771	3.775	3.779	3.783	3.787	3.791	3.795	3.799	
1.95	3.802	3.806	3.810	3.814	3.818	3.822	3.826	3.830	3.834	3.838	
6	3.842	3.846	3.849	3.853	3.857	3.861	3.865	3.869	3.873	3.877	
7	3.881	3.885	3.889	3.893	3.897	3.901	3.905	3.909	3.912	3.916	
8	3.920	3.924	3.928	3.932	3.936	3.940	3.944	3.948	3.952	3.956	
9	3.960	3.964	3.968	3.972	3.976	3.980	3.984	3.988	3.992	3.996	

$\pi^2 = 9.86960$ $1/\pi^2 = 0.101321$ $e^2 = 7.38906$

SQUARES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Ave. diff.
2.00	4.000	4.004	4.008	4.012	4.016	4.020	4.024	4.028	4.032	4.036	4
1	4.040	4.044	4.048	4.052	4.056	4.060	4.064	4.068	4.072	4.076	
2	4.080	4.084	4.088	4.093	4.097	4.101	4.105	4.109	4.113	4.117	
3	4.121	4.125	4.129	4.133	4.137	4.141	4.145	4.149	4.153	4.158	
4	4.162	4.166	4.170	4.174	4.178	4.182	4.186	4.190	4.194	4.198	
2.05	4.202	4.207	4.211	4.215	4.219	4.223	4.227	4.231	4.235	4.239	
6	4.244	4.248	4.252	4.256	4.260	4.264	4.268	4.272	4.277	4.281	
7	4.285	4.289	4.293	4.297	4.301	4.306	4.310	4.314	4.318	4.322	
8	4.326	4.331	4.335	4.339	4.343	4.347	4.351	4.356	4.360	4.364	
9	4.368	4.372	4.376	4.381	4.385	4.389	4.393	4.397	4.402	4.406	
2.10	4.410	4.414	4.418	4.423	4.427	4.431	4.435	4.439	4.444	4.448	
1	4.452	4.456	4.461	4.465	4.469	4.473	4.477	4.482	4.486	4.490	
2	4.494	4.499	4.503	4.507	4.511	4.516	4.520	4.524	4.528	4.533	
3	4.537	4.541	4.545	4.550	4.554	4.558	4.562	4.567	4.571	4.575	
4	4.580	4.584	4.588	4.592	4.597	4.601	4.605	4.610	4.614	4.618	
2.15	4.622	4.627	4.631	4.635	4.640	4.644	4.648	4.653	4.657	4.661	
6	4.666	4.670	4.674	4.679	4.683	4.687	4.692	4.696	4.700	4.705	
7	4.709	4.713	4.718	4.722	4.726	4.731	4.735	4.739	4.744	4.748	
8	4.752	4.757	4.761	4.765	4.770	4.774	4.779	4.783	4.787	4.792	
9	4.796	4.800	4.805	4.809	4.814	4.818	4.822	4.827	4.831	4.836	
2.20	4.840	4.844	4.849	4.853	4.858	4.862	4.866	4.871	4.875	4.880	
1	4.884	4.889	4.893	4.897	4.902	4.906	4.911	4.915	4.920	4.924	
2	4.928	4.933	4.937	4.942	4.946	4.951	4.955	4.960	4.964	4.968	
3	4.973	4.977	4.982	4.986	4.991	4.995	5.000	5.004	5.009	5.013	
4	5.018	5.022	5.027	5.031	5.036	5.040	5.045	5.049	5.054	5.058	
2.25	5.062	5.067	5.072	5.076	5.081	5.085	5.090	5.094	5.099	5.103	5
6	5.108	5.112	5.117	5.121	5.126	5.130	5.135	5.139	5.144	5.148	
7	5.153	5.157	5.162	5.167	5.171	5.176	5.180	5.185	5.189	5.194	
8	5.198	5.203	5.208	5.212	5.217	5.221	5.226	5.230	5.235	5.240	
9	5.244	5.249	5.253	5.258	5.262	5.267	5.272	5.276	5.281	5.285	
2.30	5.290	5.295	5.299	5.304	5.308	5.313	5.318	5.322	5.327	5.331	
1	5.336	5.341	5.345	5.350	5.355	5.359	5.364	5.368	5.373	5.378	
2	5.382	5.387	5.392	5.396	5.401	5.406	5.410	5.415	5.420	5.424	
3	5.429	5.434	5.438	5.443	5.448	5.452	5.457	5.462	5.466	5.471	
4	5.476	5.480	5.485	5.490	5.494	5.499	5.504	5.508	5.513	5.518	
2.35	5.522	5.527	5.532	5.537	5.541	5.546	5.551	5.555	5.560	5.565	
6	5.570	5.574	5.579	5.584	5.588	5.593	5.598	5.603	5.607	5.612	
7	5.617	5.622	5.626	5.631	5.636	5.641	5.645	5.650	5.655	5.660	
8	5.664	5.669	5.674	5.679	5.683	5.688	5.693	5.698	5.703	5.707	
9	5.712	5.717	5.722	5.726	5.731	5.736	5.741	5.746	5.750	5.755	
2.40	5.760	5.765	5.770	5.774	5.779	5.784	5.789	5.794	5.798	5.803	
1	5.808	5.813	5.818	5.823	5.827	5.832	5.837	5.842	5.847	5.852	
2	5.856	5.861	5.866	5.871	5.876	5.881	5.885	5.890	5.895	5.900	
3	5.905	5.910	5.915	5.919	5.924	5.929	5.934	5.939	5.944	5.949	
4	5.954	5.958	5.963	5.968	5.973	5.978	5.983	5.988	5.993	5.998	
2.45	6.002	6.007	6.012	6.017	6.022	6.027	6.032	6.037	6.042	6.047	
6	6.052	6.057	6.061	6.066	6.071	6.076	6.081	6.086	6.091	6.096	
7	6.101	6.106	6.111	6.116	6.121	6.126	6.131	6.136	6.140	6.145	
8	6.150	6.155	6.160	6.165	6.170	6.175	6.180	6.185	6.190	6.195	
9	6.200	6.205	6.210	6.215	6.220	6.225	6.230	6.235	6.240	6.245	

Moving the decimal point ONE place in *N* requires moving it TWO places in body of table (see p. 6).

SQUARES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. dif.
2.50	6.250	6.255	6.260	6.265	6.270	6.275	6.280	6.285	6.290	6.295	5
1	6.300	6.305	6.310	6.315	6.320	6.325	6.330	6.335	6.340	6.345	
2	6.350	6.355	6.360	6.366	6.371	6.376	6.381	6.386	6.391	6.396	
3	6.401	6.406	6.411	6.416	6.421	6.426	6.431	6.436	6.441	6.447	
4	6.452	6.457	6.462	6.467	6.472	6.477	6.482	6.487	6.492	6.497	
2.55	6.502	6.508	6.513	6.518	6.523	6.528	6.533	6.538	6.543	6.548	
6	6.554	6.559	6.564	6.569	6.574	6.579	6.584	6.589	6.595	6.600	
7	6.605	6.610	6.615	6.620	6.625	6.631	6.636	6.641	6.646	6.651	
8	6.656	6.662	6.667	6.672	6.677	6.682	6.687	6.693	6.698	6.703	
9	6.708	6.713	6.718	6.724	6.729	6.734	6.739	6.744	6.750	6.755	
2.60	6.760	6.765	6.770	6.776	6.781	6.786	6.791	6.796	6.802	6.807	
1	6.812	6.817	6.823	6.828	6.833	6.838	6.843	6.849	6.854	6.859	
2	6.864	6.870	6.875	6.880	6.885	6.891	6.896	6.901	6 906	6.912	
3	6.917	6.922	6.927	6.933	6.938	6.943	6.948	6.954	6.959	6.964	
4	6.970	6.975	6.980	6.985	6.991	6.996	7.001	7.007	7.012	7.017	
2.65	7.022	7.028	7.033	7.038	7.044	7.049	7.054	7.060	7.065	7.070	
6	7.076	7.081	7.086	7.092	7.097	7.102	7.108	7.113	7.118	7.124	
7	7.129	7.134	7.140	7.145	7.150	7.156	7.161	7.166	7.172	7.177	
8	7.182	7.188	7.193	7.198	7.204	7.209	7.215	7.220	7.225	7.231	
9	7.236	7.241	7.247	7.252	7.258	7.263	7.268	7.274	7.279	7.285	
2.70	7.290	7.295	7.301	7.306	7.312	7.317	7.322	7.328	7.333	7.339	
1	7.344	7.350	7.355	7.360	7.366	7.371	7.377	7.382	7.388	7.393	
2	7.398	7.404	7.409	7.415	7.420	7.426	7.431	7.437	7.442	7.447	
3	7.453	7.458	7.464	7.469	7.475	7.480	7.486	7.491	7.497	7.502	
4	7.508	7.513	7.519	7.524	7.530	7.535	7.541	7.546	7.552	7.557	
2.75	7.562	7.568	7.574	7.579	7.585	7.590	7.596	7.601	7.607	7.612	6
6	7.618	7.623	7.629	7.634	7.640	7.645	7.651	7.656	7.662	7.667	
7	7.673	7.678	7.684	7.690	7.695	7.701	7.706	7.712	7.717	7.723	
8	7.728	7.734	7.740	7.745	7.751	7.756	7.762	7.767	7.773	7.779	
9	7.784	7.790	7.795	7.801	7.806	7.812	7.818	7.823	7.829	7.834	
2.80	7.840	7.846	7.851	7.857	7.862	7.868	7.874	7.879	7.885	7.890	
1	7.896	7.902	7.907	7.913	7.919	7.924	7.930	7.935	7.941	7.947	
2	7.952	7.958	7.964	7.969	7.975	7.981	7.986	7.992	7.998	8.003	
3	8.009	8.015	8.020	8.026	8.032	8.037	8.043	8.049	8.054	8.060	
4	8.066	8.071	8.077	8.083	8.088	8.094	8.100	8.105	8.111	8.117	
2.85	8.122	8.128	8.134	8.140	8.145	8.151	8.157	8.162	8.168	8.174	
6	8.180	8.185	8.191	8.197	8.202	8.208	8.214	8.220	8.225	8.231	
7	8.237	8.243	8.248	8.254	8.260	8.266	8.271	8.277	8.283	8.289	
8	8.294	8.300	8.306	8.312	8.317	8.323	8.329	8.335	8.341	8.346	
9	8.352	8.358	8.364	8.369	8.375	8.381	8.387	8.393	8.398	8.404	
2.90	8.410	8.416	8.422	8.427	8.433	8.439	8.445	8.451	8.456	8.462	
1	8.468	8.474	8.480	8.486	8.491	8.497	8.503	8.509	8.515	8.521	
2	8.526	8.532	8.538	8.544	8.550	8.556	8.561	8.567	8.573	8.579	
3	8.585	8.591	8.597	8.602	8.608	8.614	8.620	8.626	8.632	8.638	
4	8.644	8.649	8.655	8.661	8.667	8.673	8.679	8.685	8.691	8.697	
2.95	8.702	8.708	8.714	8.720	8.726	8.732	8.738	8.744	8.750	8.756	
6	8.762	8.768	8.773	8.779	8.785	8.791	8.797	8.803	8.809	8.815	
7	8.821	8.827	8.833	8.839	8.845	8.851	8.857	8.863	8.868	8.874	
8	8.880	8.886	8.892	8.898	8.904	8.910	8.916	8.922	8.928	8.934	
9	8.940	8.946	8.952	8.958	8.964	8.970	8.976	8.982	8.988	8.994	

$\pi^2 = 9.86960$ $1/\pi^2 = 0.101321$ $e^2 = 7.38906$

SQUARES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
3.00	9.000	9.006	9.012	9.018	9.024	9.030	9.036	9.042	9.048	9.054	6
1	9.060	9.066	9.072	9.078	9.084	9.090	9.096	9.102	9.108	9.114	
2	9.120	9.126	9.132	9.139	9.145	9.151	9.157	9.163	9.169	9.175	
3	9.181	9.187	9.193	9.199	9.205	9.211	9.217	9.223	9.229	9.236	
4	9.242	9.248	9.254	9.260	9.266	9.272	9.278	9.284	9.290	9.296	
3.05	9.302	9.309	9.315	9.321	9.327	9.333	9.339	9.345	9.351	9.357	
6	9.364	9.370	9.376	9.382	9.388	9.394	9.400	9.406	9.413	9.419	
7	9.425	9.431	9.437	9.443	9.449	9.456	9.462	9.468	9.474	9.480	
8	9.486	9.493	9.499	9.505	9.511	9.517	9.523	9.530	9.536	9.542	
9	9.548	9.554	9.560	9.567	9.573	9.579	9.585	9.591	9.598	9.604	
3.10	9.610	9.616	9.622	9.629	9.635	9.641	9.647	9.653	9.660	9.666	
1	9.672	9.678	9.685	9.691	9.697	9.703	9.709	9.716	9.722	9.728	
2	9.734	9.741	9.747	9.753	9.759	9.766	9.772	9.778	9.784	9.791	
3	9.797	9.803	9.809	9.816	9.822	9.828	9.834	9.841	9.847	9.853	
4	9.860	9.866	9.872	9.878	9.885	9.891	9.897	9.904	9.910	9.916	
3.15	9.922	9.929	9.935	9.941	9.948	9.954	9.960	9.967	9.973	9.979	
6	9.986	9.992	9.998	10.005							6
3.1							9.99	10.05	10.11	10.18	6
2	10.24	10.30	10.37	10.43	10.50	10.56	10.63	10.69	10.76	10.82	
3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	7
4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	
6	12.96	13.03	13.10	13.18	13.25	13.32	13.40	13.47	13.54	13.62	
7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	8
8	14.44	14.52	14.59	14.67	14.75	14.82	14.90	14.98	15.05	15.13	
9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65	16.73	
1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	
2	17.64	17.72	17.81	17.89	17.98	18.06	18.15	18.23	18.32	18.40	
3	18.49	18.58	18.66	18.75	18.84	18.92	19.01	19.10	19.18	19.27	9
4	19.36	19.45	19.54	19.62	19.71	19.80	19.89	19.98	20.07	20.16	
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	
6	21.16	21.25	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	
7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75	22.85	22.94	10
8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	
9	24.01	24.11	24.21	24.30	24.40	24.50	24.60	24.70	24.80	24.90	

$\pi^2 = 9.86960$ $(\pi/2)^2 = 2.46740$ $1/\pi^2 = 0.101321$

Explanation of Table of Squares (pp. 2–7).

This table gives the value of N^2 for values of N from 1 to 10, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure).

To find the square of a number N outside the range from 1 to 10, note that moving the decimal point **one** place in column N is equivalent to moving it **two** places in the body of the table. For example:

$(3.217)^2 = 10.35$; $(0.03217)^2 = 0.001035$; $(3217)^2 = 10350000$

This table can also be used inversely, to give square roots.

SQUARES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	25.00	25.10	25.20	25.30	25.40	25.50	25.60	25.70	25.81	25.91	10
1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	
2	27.04	27.14	27.25	27.35	27.46	27.56	27.67	27.77	27.88	27.98	
3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05	11
4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	30.03	30.14	
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25	
6	31.36	31.47	31.58	31.70	31.81	31.92	32.04	32.15	32.26	32.38	
7	32.49	32.60	32.72	32.83	32.95	33.06	33.18	33.29	33.41	33.52	
8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	12
9	34.81	34.93	35.05	35.16	35.28	35.40	35.52	35.64	35.76	35.88	
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	
1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32	
2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	
3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83	13
4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	
6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	
7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	
8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.33	47.47	14
9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	50.13	50.27	
1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	51.55	51.70	
2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	53.00	53.14	
3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	15
4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.95	56.10	
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.46	57.61	
6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	
7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.53	60.68	
8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	62.09	62.25	16
9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45	
1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08	
2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	
3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39	17
4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08	
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	
6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52	
7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	
8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03	18
9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.64	80.82	
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63	
1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	
2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	85.93	86.12	86.30	
3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.98	88.17	19
4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06	
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	
6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	
7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45	95.65	95.84	
8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	20
9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	
10.0	100.0										

Moving the decimal point ONE place in *N* requires moving it TWO places in body of table (see p. 6).

CUBES OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00	1.000	1.003	1.006	1.009	1.012	1.015	1.018	1.021	1.024	1.027	3
1	1.030	1.033	1.036	1.040	1.043	1.046	1.049	1.052	1.055	1.058	
2	1.061	1.064	1.067	1.071	1.074	1.077	1.080	1.083	1.086	1.090	
3	1.093	1.096	1.099	1.102	1.106	1.109	1.112	1.115	1.118	1.122	
4	1.125	1.128	1.131	1.135	1.138	1.141	1.144	1.148	1.151	1.154	
1.05	1.158	1.161	1.164	1.168	1.171	1.174	1.178	1.181	1.184	1.188	
6	1.191	1.194	1.198	1.201	1.205	1.208	1.211	1.215	1.218	1.222	
7	1.225	1.228	1.232	1.235	1.239	1.242	1.246	1.249	1.253	1.256	
8	1.260	1.263	1.267	1.270	1.274	1.277	1.281	1.284	1.288	1.291	4
9	1.295	1.299	1.302	1.306	1.309	1.313	1.317	1.320	1.324	1.327	
1.10	1.331	1.335	1.338	1.342	1.346	1.349	1.353	1.357	1.360	1.364	
1	1.368	1.371	1.375	1.379	1.382	1.386	1.390	1.394	1.397	1.401	
2	1.405	1.409	1.412	1.416	1.420	1.424	1.428	1.431	1.435	1.439	
3	1.443	1.447	1.451	1.454	1.458	1.462	1.466	1.470	1.474	1.478	
4	1.482	1.485	1.489	1.493	1.497	1.501	1.505	1.509	1.513	1.517	
1.15	1.521	1.525	1.529	1.533	1.537	1.541	1.545	1.549	1.553	1.557	
6	1.561	1.565	1.569	1.573	1.577	1.581	1.585	1.589	1.593	1.598	
7	1.602	1.606	1.610	1.614	1.618	1.622	1.626	1.631	1.635	1.639	
8	1.643	1.647	1.651	1.656	1.660	1.664	1.668	1.672	1.677	1.681	
9	1.685	1.689	1.694	1.698	1.702	1.706	1.711	1.715	1.719	1.724	
1.20	1.728	1.732	1.737	1.741	1.745	1.750	1.754	1.758	1.763	1.767	
1	1.772	1.776	1.780	1.785	1.789	1.794	1.798	1.802	1.807	1.811	
2	1.816	1.820	1.825	1.829	1.834	1.838	1.843	1.847	1.852	1.856	
3	1.861	1.865	1.870	1.875	1.879	1.884	1.888	1.893	1.897	1.902	
4	1.907	1.911	1.916	1.920	1.925	1.930	1.934	1.939	1.944	1.948	5
1.25	1.953	1.958	1.963	1.967	1.972	1.977	1.981	1.986	1.991	1.996	
6	2.000	2.005	2.010	2.015	2.019	2.024	2.029	2.034	2.039	2.044	
7	2.048	2.053	2.058	2.063	2.068	2.073	2.078	2.082	2.087	2.092	
8	2.097	2.102	2.107	2.112	2.117	2.122	2.127	2.132	2.137	2.142	
9	2.147	2.152	2.157	2.162	2.167	2.172	2.177	2.182	2.187	2.192	
1.30	2.197	2.202	2.207	2.212	2.217	2.222	2.228	2.233	2.238	2.243	
1	2.248	2.253	2.258	2.264	2.269	2.274	2.279	2.284	2.290	2.295	
2	2.300	2.305	2.310	2.316	2.321	2.326	2.331	2.337	2.342	2.347	
3	2.353	2.358	2.363	2.369	2.374	2.379	2.385	2.390	2.395	2.401	
4	2.406	2.411	2.417	2.422	2.428	2.433	2.439	2.444	2.449	2.455	
1.35	2.460	2.466	2.471	2.477	2.482	2.488	2.493	2.499	2.504	2.510	6
6	2.515	2.521	2.527	2.532	2.538	2.543	2.549	2.554	2.560	2.566	
7	2.571	2.577	2.583	2.588	2.594	2.600	2.605	2.611	2.617	2.622	
8	2.628	2.634	2.640	2.645	2.651	2.657	2.663	2.668	2.674	2.680	
9	2.686	2.691	2.697	2.703	2.709	2.715	2.721	2.726	2.732	2.738	
1.40	2.744	2.750	2.756	2.762	2.768	2.774	2.779	2.785	2.791	2.797	
1	2.803	2.809	2.815	2.821	2.827	2.833	2.839	2.845	2.851	2.857	
2	2.863	2.869	2.875	2.881	2.888	2.894	2.900	2.906	2.912	2.918	
3	2.924	2.930	2.936	2.943	2.949	2.955	2.961	2.967	2.974	2.980	
4	2.986	2.992	2.998	3.005	3.011	3.017	3.023	3.030	3.036	3.042	
1.45	3.049	3.055	3.061	3.068	3.074	3.080	3.087	3.093	3.099	3.106	
6	3.112	3.119	3.125	3.131	3.138	3.144	3.151	3.157	3.164	3.170	
7	3.177	3.183	3.190	3.196	3.203	3.209	3.216	3.222	3.229	3.235	
8	3.242	3.248	3.255	3.262	3.268	3.275	3.281	3.288	3.295	3.301	7
9	3.308	3.315	3.321	3.328	3.335	3.341	3.348	3.355	3.362	3.368	

Moving the decimal point ONE place in *N* requires moving it THREE places in body of table (see p. 10).

CUBES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. dif.
1.50	3.375	3.382	3.389	3.395	3.402	3.409	3.416	3.422	3.429	3.436	7
1	3.443	3.450	3.457	3.464	3.470	3.477	3.484	3.491	3.498	3.505	
2	3.512	3.519	3.526	3.533	3.540	3.547	3.554	3.561	3.568	3.575	
3	3.582	3.589	3.596	3.603	3.610	3.617	3.624	3.631	3.638	3.645	
4	3.652	3.659	3.667	3.674	3.681	3.688	3.695	3.702	3.709	3.717	
1.55	3.724	3.731	3.738	3.746	3.753	3.760	3.767	3.775	3.782	3.789	
6	3.796	3.804	3.811	3.818	3.826	3.833	3.840	3.848	3.855	3.863	
7	3.870	3.877	3.885	3.892	3.900	3.907	3.914	3.922	3.929	3.937	
8	3.944	3.952	3.959	3.967	3.974	3.982	3.989	3.997	4.005	4.012	8
9	4.020	4.027	4.035	4.042	4.050	4.058	4.065	4.073	4.081	4.088	
1.60	4.096	4.104	4.111	4.119	4.127	4.135	4.142	4.150	4.158	4.166	
1	4.173	4.181	4.189	4.197	4.204	4.212	4.220	4.228	4.236	4.244	
2	4.252	4.259	4.267	4.275	4.283	4.291	4.299	4.307	4.315	4.323	
3	4.331	4.339	4.347	4.355	4.363	4.371	4.379	4.387	4.395	4.403	
4	4.411	4.419	4.427	4.435	4.443	4.451	4.460	4.468	4.476	4.484	
1.65	4.492	4.500	4.508	4.517	4.525	4.533	4.541	4.550	4.558	4.566	
6	4.574	4.583	4.591	4.599	4.607	4.616	4.624	4.632	4.641	4.649	
7	4.657	4.666	4.674	4.683	4.691	4.699	4.708	4.716	4.725	4.733	
8	4.742	4.750	4.759	4.767	4.776	4.784	4.793	4.801	4.810	4.818	
9	4.827	4.835	4.844	4.853	4.861	4.870	4.878	4.887	4.896	4.904	9
1.70	4.913	4.922	4.930	4.939	4.948	4.956	4.965	4.974	4.983	4.991	
1	5.000	5.009	5.018	5.027	5.035	5.044	5.053	5.062	5.071	5.080	
2	5.088	5.097	5.106	5.115	5.124	5.133	5.142	5.151	5.160	5.169	
3	5.178	5.187	5.196	5.205	5.214	5.223	5.232	5.241	5.250	5.259	
4	5.268	5.277	5.286	5.295	5.304	5.314	5.323	5.332	5.341	5.350	
1.75	5.359	5.369	5.378	5.387	5.396	5.405	5.415	5.424	5.433	5.442	
6	5.452	5.461	5.470	5.480	5.489	5.498	5.508	5.517	5.526	5.536	
7	5.545	5.555	5.564	5.573	5.583	5.592	5.602	5.611	5.621	5.630	10
8	5.640	5.649	5.659	5.668	5.678	5.687	5.697	5.707	5.716	5.726	
9	5.735	5.745	5.755	5.764	5.774	5.784	5.793	5.803	5.813	5.822	
1.80	5.832	5.842	5.851	5.861	5.871	5.881	5.891	5.900	5.910	5.920	
1	5.930	5.940	5.949	5.959	5.969	5.979	5.989	5.999	6.009	6.019	
2	6.029	6.039	6.048	6.058	6.068	6.078	6.088	6.098	6.108	6.118	
3	6.128	6.139	6.149	6.159	6.169	6.179	6.189	6.199	6.209	6.219	
4	6.230	6.240	6.250	6.260	6.270	6.280	6.291	6.301	6.311	6.321	
1.85	6.332	6.342	6.352	6.362	6.373	6.383	6.393	6.404	6.414	6.424	
6	6.435	6.445	6.456	6.466	6.476	6.487	6.497	6.508	6.518	6.529	
7	6.539	6.550	6.560	6.571	6.581	6.592	6.602	6.613	6.623	6.634	11
8	6.645	6.655	6.666	6.677	6.687	6.698	6.708	6.719	6.730	6.741	
9	6.751	6.762	6.773	6.783	6.794	6.805	6.816	6.827	6.837	6.848	
1.90	6.859	6.870	6.881	6.892	6.902	6.913	6.924	6.935	6.946	6.957	
1	6.968	6.979	6.990	7.001	7.012	7.023	7.034	7.045	7.056	7.067	
2	7.078	7.089	7.100	7.111	7.122	7.133	7.144	7.156	7.167	7.178	
3	7.189	7.200	7.211	7.223	7.234	7.245	7.256	7.268	7.279	7.290	
4	7.301	7.313	7.324	7.335	7.347	7.358	7.369	7.381	7.392	7.403	
1.95	7.415	7.426	7.438	7.449	7.461	7.472	7.484	7.495	7.507	7.518	12
6	7.530	7.541	7.553	7.564	7.576	7.587	7.599	7.610	7.622	7.634	
7	7.645	7.657	7.669	7.680	7.692	7.704	7.715	7.727	7.739	7.751	
8	7.762	7.774	7.786	7.798	7.810	7.821	7.833	7.845	7.857	7.869	
9	7.881	7.892	7.904	7.916	7.928	7.940	7.952	7.964	7.976	7.988	

$\pi^3 = 31.0063 \qquad 1/\pi^3 = 0.0322515+$

CUBES (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
2.00	8.000	8.012	8.024	8.036	8.048	8.060	8.072	8.084	8.096	8.108	12
1	8.121	8.133	8.145	8.157	8.169	8.181	8.194	8.206	8.218	8.230	
2	8.242	8.255	8.267	8.279	8.291	8.304	8.316	8.328	8.341	8.353	
3	8.365	8.378	8.390	8.403	8.415	8.427	8.440	8.452	8.465	8.477	
4	8.490	8.502	8.515	8.527	8.540	8.552	8.565	8.577	8.590	8.603	
2.05	8.615	8.628	8.640	8.653	8.666	8.678	8.691	8.704	8.716	8.729	13
6	8.742	8.755	8.767	8.780	8.793	8.806	8.818	8.831	8.844	8.857	
7	8.870	8.883	8.895	8.908	8.921	8.934	8.947	8.960	8.973	8.986	
8	8.999	9.012	9.025	9.038	9.051	9.064	9.077	9.090	9.103	9.116	
9	9.129	9.142	9.156	9.169	9.182	9.195	9.208	9.221	9.235	9.248	
2.10	9.261	9.274	9.287	9.301	9.314	9.327	9.341	9.354	9.367	9.381	
1	9.394	9.407	9.421	9.434	9.447	9.461	9.474	9.488	9.501	9.515	
2	9.528	9.542	9.555	9.569	9.582	9.596	9.609	9.623	9.636	9.650	14
3	9.664	9.677	9.691	9.704	9.718	9.732	9.745	9.759	9.773	9.787	
4	9.800	9.814	9.828	9.842	9.855	9.869	9.883	9.897	9.911	9.925	
2.15	9.938	9.952	9.966	9.980	9.994	10.008					14
2.1						9.94	10.08	10.22	10.36	10.50	14
2	10.65	10.79	10.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01	15
3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	13.48	13.65	16
4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44	18
2.5	15.62	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37	20
6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47	21
7	19.68	19.90	20.12	20.35	20.57	20.80	21.02	21.25	21.48	21.72	23
8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14	24
9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	26.46	26.73	26
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50	28
1	29.79	30.08	30.37	30.66	30.96	31.26	31.55	31.86	32.16	32.46	30
2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61	32
3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96	34
4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51	36
3.5	42.88	43.24	43.61	43.99	44.36	44.74	45.12	45.50	45.88	46.27	39
6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24	40
7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44	42
8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86	44
9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52	47
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42	49
1	68.92	69.43	69.93	70.44	70.96	71.47	71.99	72.51	73.03	73.56	52
2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	78.95	54
3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60	58
4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52	59
4.5	91.12	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70	62
6	97.34	97.97	98.61	99.25	99.90	100.54					64
6						100.5	101.2	101.8	102.5	103.2	7
7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9	7
8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9	7
9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3	7

Explanation of Table of Cubes (pp. 8–11).

This table gives the value of N^3 for values of N from 1 to 10, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the cube of a number N outside the range from 1 to 10, note that moving the decimal point **one** place in column N is equivalent to moving it **three** places in the body of the table. For example:

$(4.852)^3 = 114.2$; $(0.4852)^3 = 0.1142$; $(485.2)^3 = 114200000$

This table may also be used inversely, to give cube roots.

CUBES *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9	8
1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8	
2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0	
3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.9	155.7	156.6	9
4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5	
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7	
6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2	10
7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1	
8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3	
9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9	
6.0	216.0	217.1	218.2	219.3	220.3	221.4	222.5	223.6	224.8	225.9	11
1	227.0	228.1	229.2	230.3	231.5	232.6	233.7	234.9	236.0	237.2	
2	238.3	239.5	240.6	241.8	243.0	244.1	245.3	246.5	247.7	248.9	12
3	250.0	251.2	252.4	253.6	254.8	256.0	257.3	258.5	259.7	260.9	
4	262.1	263.4	264.6	265.8	267.1	268.3	269.6	270.8	272.1	273.4	
6.5	274.6	275.9	277.2	278.4	279.7	281.0	282.3	283.6	284.9	286.2	13
6	287.5	288.8	290.1	291.4	292.8	294.1	295.4	296.7	298.1	299.4	
7	300.8	302.1	303.5	304.8	306.2	307.5	308.9	310.3	311.7	313.0	14
8	314.4	315.8	317.2	318.6	320.0	321.4	322.8	324.2	325.7	327.1	
9	328.5	329.9	331.4	332.8	334.3	335.7	337.2	338.6	340.1	341.5	
7.0	343.0	344.5	345.9	347.4	348.9	350.4	351.9	353.4	354.9	356.4	15
1	357.9	359.4	360.9	362.5	364.0	365.5	367.1	368.6	370.1	371.7	
2	373.2	374.8	376.4	377.9	379.5	381.1	382.7	384.2	385.8	387.4	16
3	389.0	390.6	392.2	393.8	395.4	397.1	398.7	400.3	401.9	403.6	
4	405.2	406.9	408.5	410.2	411.8	413.5	415.2	416.8	418.5	420.2	17
7.5	421.9	423.6	425.3	427.0	428.7	430.4	432.1	433.8	435.5	437.2	
6	439.0	440.7	442.5	444.2	445.9	447.7	449.5	451.2	453.0	454.8	18
7	456.5	458.3	460.1	461.9	463.7	465.5	467.3	469.1	470.9	472.7	
8	474.6	476.4	478.2	480.0	481.9	483.7	485.6	487.4	489.3	491.2	
9	493.0	494.9	496.8	498.7	500.6	502.5	504.4	506.3	508.2	510.1	19
8.0	512.0	513.9	515.8	517.8	519.7	521.7	523.6	525.6	527.5	529.5	
1	531.4	533.4	535.4	537.4	539.4	541.3	543.3	545.3	547.3	549.4	20
2	551.4	553.4	555.4	557.4	559.5	561.5	563.6	565.6	567.7	569.7	
3	571.8	573.9	575.9	578.0	580.1	582.2	584.3	586.4	588.5	590.6	21
4	592.7	594.8	596.9	599.1	601.2	603.4	605.5	607.6	609.8	612.0	
8.5	614.1	616.3	618.5	620.7	622.8	625.0	627.2	629.4	631.6	633.8	22
6	636.1	638.3	640.5	642.7	645.0	647.2	649.5	651.7	654.0	656.2	
7	658.5	660.8	663.1	665.3	667.6	669.9	672.2	674.5	676.8	679.2	23
8	681.5	683.8	686.1	688.5	690.8	693.2	695.5	697.9	700.2	702.6	24
9	705.0	707.3	709.7	712.1	714.5	716.9	719.3	721.7	724.2	726.6	
9.0	729.0	731.4	733.9	736.3	738.8	741.2	743.7	746.1	748.6	751.1	25
1	753.6	756.1	758.6	761.0	763.6	766.1	768.6	771.1	773.6	776.2	
2	778.7	781.2	783.8	786.3	788.9	791.5	794.0	796.6	799.2	801.8	26
3	804.4	807.0	809.6	812.2	814.8	817.4	820.0	822.7	825.3	827.9	
4	830.6	833.2	835.9	838.6	841.2	843.9	846.6	849.3	852.0	854.7	27
9.5	857.4	860.1	862.8	865.5	868.3	871.0	873.7	876.5	879.2	882.0	
6	884.7	887.5	890.3	893.1	895.8	898.6	901.4	904.2	907.0	909.9	28
7	912.7	915.5	918.3	921.2	924.0	926.9	929.7	932.6	935.4	938.3	
8	941.2	944.1	947.0	949.9	952.8	955.7	958.6	961.5	964.4	967.4	29
9	970.3	973.2	976.2	979.1	982.1	985.1	988.0	991.0	994.0	997.0	
10.0	1000.0										

$\pi^3 = 31.0063$ $\qquad 1/\pi^3 = 0.0322515+$

Moving the decimal point ONE place in *N* requires moving it THREE places in body of table (see p. 10).

SQUARE ROOTS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044	5
1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	
2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136	4
3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179	
4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261	
6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300	
7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338	
8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375	
9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446	
1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480	3
2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	
3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	
4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578	
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	
6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640	
7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670	
8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700	
9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729	
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758	
1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786	
2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	
3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841	
4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868	
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895	
6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921	
7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947	
8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972	
9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997	
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022	
1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047	2
2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	
3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095	
4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119	
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142	
6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	
7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189	
8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211	
9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234	

$\sqrt{\pi} = 1.77245+$ $1/\sqrt{\pi} = 0.56419$ $\sqrt{\pi/2} = 1.25331$ $\sqrt{e} = 1.64872$

Explanation of Table of Square Roots (pp. 12–15).

This table gives the values of $\sqrt{N}$ for values of N from 1 to 100, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the square root of a number N outside the range from 1 to 100, divide the digits of the number into blocks of two (beginning with the decimal point), and note that moving the decimal point **two** places in N is equivalent to moving it **one** place in the square root of N. For example:

$$\sqrt{2.718} = 1.648; \quad \sqrt{271.8} = 16.48; \quad \sqrt{0.0002718} = 0.01648;$$

$$\sqrt{27.18} = 5.213; \quad \sqrt{2718} = 52.13; \quad \sqrt{0.002718} = 0.05213.$$

SQUARE ROOTS *(continued)*

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256	2
1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278	
2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300	
3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322	
4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343	
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364	
6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385	
7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	
8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427	
9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447	
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468	
1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	
2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	
3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	
4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567	
6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587	
7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	
8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625	
9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663	
1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	
2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	
3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718	
4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737	
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755	
6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	
7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	
8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809	
9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827	
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844	
1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862	
2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	
3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897	
4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914	
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	
6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	
7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965	
8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982	
9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998	
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015	
1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032	
2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048	
3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064	
4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081	
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097	
6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113	
7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129	
8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145	
9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161	

Moving the decimal point TWO places in *N* requires moving it ONE place in body of table **(see p. 12).**

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
10.	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302	16
1.	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450	15
2.	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592	14
3.	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728	
4.	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860	13
15.	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987	
6.	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111	12
7.	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231	
8.	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347	
9.	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461	11
20.	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572	
1.	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680	
2.	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785	
3.	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	10
4.	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990	
25.	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089	
6	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	
7.	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282	
8.	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	9
9.	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468	
30.	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559	
1.	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648	
2.	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736	
3.	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822	
4.	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908	8
35.	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992	
6.	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	6.066	6.075	
7.	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156	
8.	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237	
9.	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317	
40.	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395	
1.	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473	
2.	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550	
3.	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626	
4.	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	
45.	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775	7
6.	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	
7.	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921	
8.	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993	
9.	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064	

SQUARE ROOTS OF CERTAIN FRACTIONS

N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$	N	$\sqrt{N}$
1/2	0.7071	3/5	0.7746	4/7	0.7559	1/9	0.3333	5/12	0.6455	9/16	0.7500
1/3	0.5774	4/5	0.8944	5/7	0.8452	2/9	0.4714	7/12	0.7638	11/16	0.8292
2/3	0.8165	1/6	0.4082	6/7	0.9258	4/9	0.6667	11/12	0.9574	13/16	0.9014
1/4	0.5000	5/6	0.9129	1/8	0.3536	5/9	0.7454	1/16	0.2500	15/16	0.9682
3/4	0.8660	1/7	0.3780	3/8	0.6124	7/9	0.8819	3/16	0.4330	1/32	0.1768
1/5	0.4472	2/7	0.5345	5/8	0.7906	8/9	0.9428	5/16	0.5590	1/64	0.1250
2/5	0.6325	3/7	0.6547	7/8	0.9354	1/12	0.2887	7/16	0.6614	1/50	0.1414

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9
50.	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134
1.	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204
2.	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273
3.	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342
4.	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409
55.	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477
6.	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543
7.	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609
8.	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675
9.	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740
60.	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804
1.	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868
2.	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931
3.	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994
4.	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056
65.	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118
6.	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179
7.	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240
8.	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301
9.	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361
70.	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420
1.	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479
2.	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538
3.	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597
4.	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654
75.	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712
6.	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769
7.	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826
8.	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883
9.	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939
80.	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994
1.	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050
2.	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105
3.	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160
4.	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214
85.	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268
6.	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322
7.	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375
8.	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429
9.	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482
90.	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534
1.	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586
2.	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638
3.	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690
4.	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742
95.	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793
6.	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844
7.	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894
8.	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945
9.	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995

$\sqrt{\pi} = 1.77245+$ $1/\sqrt{\pi} = 0.56419$ $\sqrt{\pi/2} = 1.25331$ $\sqrt{e} = 1.6487$

Moving the decimal point TWO places in N requires moving it ONE place in bo table (see p. 12).

CUBE ROOTS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	1.000	1.003	1.007	1.010	1.013	1.016	1.020	1.023	1.026	1.029	3
1	1.032	1.035	1.038	1.042	1.045	1.048	1.051	1.054	1.057	1.060	
2	1.063	1.066	1.069	1.071	1.074	1.077	1.080	1.083	1.086	1.089	
3	1.091	1.094	1.097	1.100	1.102	1.105	1.108	1.111	1.113	1.116	
4	1.119	1.121	1.124	1.127	1.129	1.132	1.134	1.137	1.140	1.142	
1.5	1.145	1.147	1.150	1.152	1.155	1.157	1.160	1.162	1.165	1.167	2
6	1.170	1.172	1.174	1.177	1.179	1.182	1.184	1.186	1.189	1.191	
7	1.193	1.196	1.198	1.200	1.203	1.205	1.207	1.210	1.212	1.214	
8	1.216	1.219	1.221	1.223	1.225	1.228	1.230	1.232	1.234	1.236	
9	1.239	1.241	1.243	1.245	1.247	1.249	1.251	1.254	1.256	1.258	
2.0	1.260	1.262	1.264	1.266	1.268	1.270	1.272	1.274	1.277	1.279	
1	1.281	1.283	1.285	1.287	1.289	1.291	1.293	1.295	1.297	1.299	
2	1.301	1.303	1.305	1.306	1.308	1.310	1.312	1.314	1.316	1.318	
3	1.320	1.322	1.324	1.326	1.328	1.330	1.331	1.333	1.335	1.337	
4	1.339	1.341	1.343	1.344	1.346	1.348	1.350	1.352	1.354	1.355	
2.5	1.357	1.359	1.361	1.363	1.364	1.366	1.368	1.370	1.372	1.373	
6	1.375	1.377	1.379	1.380	1.382	1.384	1.386	1.387	1.389	1.391	
7	1.392	1.394	1.396	1.398	1.399	1.401	1.403	1.404	1.406	1.408	
8	1.409	1.411	1.413	1.414	1.416	1.418	1.419	1.421	1.423	1.424	
9	1.426	1.428	1.429	1.431	1.433	1.434	1.436	1.437	1.439	1.441	
3.0	1.442	1.444	1.445	1.447	1.449	1.450	1.452	1.453	1.455	1.457	
1	1.458	1.460	1.461	1.463	1.464	1.466	1.467	1.469	1.471	1.472	
2	1.474	1.475	1.477	1.478	1.480	1.481	1.483	1.484	1.486	1.487	
3	1.489	1.490	1.492	1.493	1.495	1.496	1.498	1.499	1.501	1.502	
4	1.504	1.505	1.507	1.508	1.510	1.511	1.512	1.514	1.515	1.517	
3.5	1.518	1.520	1.521	1.523	1.524	1.525	1.527	1.528	1.530	1.531	
6	1.533	1.534	1.535	1.537	1.538	1.540	1.541	1.542	1.544	1.545	1
7	1.547	1.548	1.549	1.551	1.552	1.554	1.555	1.556	1.558	1.559	
8	1.560	1.562	1.563	1.565	1.566	1.567	1.569	1.570	1.571	1.573	
9	1.574	1.575	1.577	1.578	1.579	1.581	1.582	1.583	1.585	1.586	
4.0	1.587	1.589	1.590	1.591	1.593	1.594	1.595	1.597	1.598	1.599	
1	1.601	1.602	1.603	1.604	1.606	1.607	1.608	1.610	1.611	1.612	
2	1.613	1.615	1.616	1.617	1.619	1.620	1.621	1.622	1.624	1.625	
3	1.626	1.627	1.629	1.630	1.631	1.632	1.634	1.635	1.636	1.637	
4	1.639	1.640	1.641	1.642	1.644	1.645	1.646	1.647	1.649	1.650	
4.5	1.651	1.652	1.653	1.655	1.656	1.657	1.658	1.659	1.661	1.662	
6	1.663	1.664	1.666	1.667	1.668	1.669	1.670	1.671	1.673	1.674	
7	1.675	1.676	1.677	1.679	1.680	1.681	1.682	1.683	1.685	1.686	
8	1.687	1.688	1.689	1.690	1.692	1.693	1.694	1.695	1.696	1.697	
9	1.698	1.700	1.701	1.702	1.703	1.704	1.705	1.707	1.708	1.709	

$\sqrt[3]{\pi} = 1.46459 \qquad 1/\sqrt[3]{\pi} = 0.682784$

Explanation of Table of Cube Roots (pp. 16–21).

This table gives the values of $\sqrt[3]{N}$ for all values of N from 1 to 1000, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the cube root of a number N outside the range from 1 to 1000, divide the digits of the number into blocks of three (beginning with the decimal point), and note that moving the decimal point three places in column N is equivalent to moving it one place in the cube root of N. For example:

$$\sqrt[3]{2.718} = 1,396; \quad \sqrt[3]{2718} = 13.96; \quad \sqrt[3]{0.000002718} = 0.01396.$$
$$\sqrt[3]{27.18} = 3.007; \quad \sqrt[3]{27180} = 30.07; \quad \sqrt[3]{0.00002718} = 0.03007.$$
$$\sqrt[3]{271.8} = 6.477; \quad \sqrt[3]{271800} = 64.77; \quad \sqrt[3]{0.0002718} = 0.06477.$$

CUBE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	1.710	1.711	1.712	1.713	1.715	1.716	1.717	1.718	1.719	1.720	1
1	1.721	1.722	1.724	1.725	1.726	1.727	1.728	1.729	1.730	1.731	
2	1.732	1.734	1.735	1.736	1.737	1.738	1.739	1.740	1.741	1.742	
3	1.744	1.745	1.746	1.747	1.748	1.749	1.750	1.751	1.752	1.753	
4	1.754	1.755	1.757	1.758	1.759	1.760	1.761	1.762	1.763	1.764	
5.5	1.765	1.766	1.767	1.768	1.769	1.771	1.772	1.773	1.774	1.775	
6	1.776	1.777	1.778	1.779	1.780	1.781	1.782	1.783	1.784	1.785	
7	1.786	1.787	1.788	1.789	1.790	1.792	1.793	1.794	1.795	1.796	
8	1.797	1.798	1.799	1.800	1.801	1.802	1.803	1.804	1.805	1.806	
9	1.807	1.808	1.809	1.810	1.811	1.812	1.813	1.814	1.815	1.816	
6.0	1.817	1.818	1.819	1.820	1.821	1.822	1.823	1.824	1.825	1.826	
1	1.827	1.828	1.829	1.830	1.831	1.832	1.833	1.834	1.835	1.836	
2	1.837	1.838	1.839	1.840	1.841	1.842	1.843	1.844	1.845	1.846	
3	1.847	1.848	1.849	1.850	1.851	1.852	1.853	1.854	1.855	1.856	
4	1.857	1.858	1.859	1.860	1.860	1.861	1.862	1.863	1.864	1.865	
6.5	1.866	1.867	1.868	1.869	1.870	1.871	1.872	1.873	1.874	1.875	
6	1.876	1.877	1.878	1.879	1.880	1.881	1.881	1.882	1.883	1.884	
7	1.885	1.886	1.887	1.888	1.889	1.890	1.891	1.892	1.893	1.894	
8	1.895	1.895	1.896	1.897	1.898	1.899	1.900	1.901	1.902	1.903	
9	1.904	1.905	1.906	1.907	1.907	1.908	1.909	1.910	1.911	1.912	
7.0	1.913	1.914	1.915	1.916	1.917	1.917	1.918	1.919	1.920	1.921	
1	1.922	1.923	1.924	1.925	1.926	1.926	1.927	1.928	1.929	1.930	
2	1.931	1.932	1.933	1.934	1.935	1.935	1.936	1.937	1.938	1.939	
3	1.940	1.941	1.942	1.943	1.943	1.944	1.945	1.946	1.947	1.948	
4	1.949	1.950	1.950	1.951	1.952	1.953	1.954	1.955	1.956	1.957	
7.5	1.957	1.958	1.959	1.960	1.961	1.962	1.963	1.964	1.964	1.965	
6	1.966	1.967	1.968	1.969	1.970	1.970	1.971	1.972	1.973	1.974	
7	1.975	1.976	1.976	1.977	1.978	1.979	1.980	1.981	1.981	1.982	
8	1.983	1.984	1.985	1.986	1.987	1.987	1.988	1.989	1.990	1.991	
9	1.992	1.992	1.993	1.994	1.995	1.996	1.997	1.997	1.998	1.999	
8.0	2.000	2.001	2.002	2.002	2.003	2.004	2.005	2.006	2.007	2.007	
1	2.008	2.009	2.010	2.011	2.012	2.012	2.013	2.014	2.015	2.016	
2	2.017	2.017	2.018	2.019	2.020	2.021	2.021	2.022	2.023	2.024	
3	2.025	2.026	2.026	2.027	2.028	2.029	2.030	2.030	2.031	2.032	
4	2.033	2.034	2.034	2.035	2.036	2.037	2.038	2.038	2.039	2.040	
8.5	2.041	2.042	2.042	2.043	2.044	2.045	2.046	2.046	2.047	2.048	
6	2.049	2.050	2.050	2.051	2.052	2.053	2.054	2.054	2.055	2.056	
7	2.057	2.057	2.058	2.059	2.060	2.061	2.061	2.062	2.063	2.064	
8	2.065	2.065	2.066	2.067	2.068	2.068	2.069	2.070	2.071	2.072	
9	2.072	2.073	2.074	2.075	2.075	2.076	2.077	2.078	2.079	2.079	
9.0	2.080	2.081	2.082	2.082	2.083	2.084	2.085	2.085	2.086	2.087	
1	2.088	2.089	2.089	2.090	2.091	2.092	2.092	2.093	2.094	2.095	
2	2.095	2.096	2.097	2.098	2.098	2.099	2.100	2.101	2.101	2.102	
3	2.103	2.104	2.104	2.105	2.106	2.107	2.107	2.108	2.109	2.110	
4	2.110	2.111	2.112	2.113	2.113	2.114	2.115	2.116	2.116	2.117	
9.5	2.118	2.119	2.119	2.120	2.121	2.122	2.122	2.123	2.124	2.125	
6	2.125	2.126	2.127	2.128	2.128	2.129	2.130	2.130	2.131	2.132	
7	2.133	2.133	2.134	2.135	2.136	2.136	2.137	2.138	2.139	2.139	
8	2.140	2.141	2.141	2.142	2.143	2.144	2.144	2.145	2.146	2.147	
9	2.147	2.148	2.149	2.149	2.150	2.151	2.152	2.152	2.153	2.154	

Moving the decimal point THREE places in N requires moving it ONE place in body of table (see p. 16).

CUBE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. dif.
10.	2.154	2.162	2.169	2.176	2.183	2.190	2.197	2.204	2.210	2.217	7
1.	2.224	2.231	2.237	2.244	2.251	2.257	2.264	2.270	2.277	2.283	6
2.	2.289	2.296	2.302	2.308	2.315	2.321	2.327	2.333	2.339	2.345	
3.	2.351	2.357	2.363	2.369	2.375	2.381	2.387	2.393	2.399	2.404	
4.	2.410	2.416	2.422	2.427	2.433	2.438	2.444	2.450	2.455	2.461	
15.	2.466	2.472	2.477	2.483	2.488	2.493	2.499	2.504	2.509	2.515	5
6.	2.520	2.525	2.530	2.535	2.541	2.546	2.551	2.556	2.561	2.566	
7.	2.571	2.576	2.581	2.586	2.591	2.596	2.601	2.606	2.611	2.616	
8.	2.621	2.626	2.630	2.635	2.640	2.645	2.650	2.654	2.659	2.664	
9.	2.668	2.673	2.678	2.682	2.687	2.692	2.696	2.701	2.705	2.710	
20.	2.714	2.719	2.723	2.728	2.732	2.737	2.741	2.746	2.750	2.755	4
1.	2.759	2.763	2.768	2.772	2.776	2.781	2.785	2.789	2.794	2.798	
2.	2.802	2.806	2.811	2.815	2.819	2.823	2.827	2.831	2.836	2.840	
3.	2.844	2.848	2.852	2.856	2.860	2.864	2.868	2.872	2.876	2.880	
4.	2.884	2.888	2.892	2.896	2.900	2.904	2.908	2.912	2.916	2.920	
25.	2.924	2.928	2.932	2.936	2.940	2.943	2.947	2.951	2.955	2.959	
6.	2.962	2.966	2.970	2.974	2.978	2.981	2.985	2.989	2.993	2.996	
7.	3.000	3.004	3.007	3.011	3.015	3.018	3.022	3.026	3.029	3.033	
8.	3.037	3.040	3.044	3.047	3.051	3.055	3.058	3.062	3.065	3.069	
9.	3.072	3.076	3.079	3.083	3.086	3.090	3.093	3.097	3.100	3.104	
30.	3.107	3.111	3.114	3.118	3.121	3.124	3.128	3.131	3.135	3.138	3
1.	3.141	3.145	3.148	3.151	3.155	3.158	3.162	3.165	3.168	3.171	
2.	3.175	3.178	3.181	3.185	3.188	3.191	3.195	3.198	3.201	3.204	
3.	3.208	3.211	3.214	3.217	3.220	3.224	3.227	3.230	3.233	3.236	
4.	3.240	3.243	3.246	3.249	3.252	3.255	3.259	3.262	3.265	3.268	
35.	3.271	3.274	3.277	3.280	3.283	3.287	3.290	3.293	3.296	3.299	
6.	3.302	3.305	3.308	3.311	3.314	3.317	3.320	3.323	3.326	3.329	
7.	3.332	3.335	3.338	3.341	3.344	3.347	3.350	3.353	3.356	3.359	
8.	3.362	3.365	3.368	3.371	3.374	3.377	3.380	3.382	3.385	3.388	
9.	3.391	3.394	3.397	3.400	3.403	3.406	3.409	3.411	3.414	3.417	
40.	3.420	3.423	3.426	3.428	3.431	3.434	3.437	3.440	3.443	3.445	
1.	3.448	3.451	3.454	3.457	3.459	3.462	3.465	3.468	3.471	3.473	
2.	3.476	3.479	3.482	3.484	3.487	3.490	3.493	3.495	3.498	3.501	
3.	3.503	3.506	3.509	3.512	3.514	3.517	3.520	3.522	3.525	3.528	
4.	3.530	3.533	3.536	3.538	3.541	3.544	3.546	3.549	3.552	3.554	
45.	3.557	3.560	3.562	3.565	3.567	3.570	3.573	3.575	3.578	3.580	
6.	3.583	3.586	3.588	3.591	3.593	3.596	3.599	3.601	3.604	3.606	
7.	3.609	3.611	3.614	3.616	3.619	3.622	3.624	3.627	3.629	3.632	
8.	3.634	3.637	3.639	3.642	3.644	3.647	3.649	3.652	3.654	3.657	2
9.	3.659	3.662	3.664	3.667	3.669	3.672	3.674	3.677	3.679	3.682	

CUBE ROOTS OF CERTAIN FRACTIONS

N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$	N	$\sqrt[3]{N}$
1/2	.7937	3/5	.8434	4/7	.8298	1/9	.4807	5/12	.7469	9/16	.8255
1/3	.6934	4/5	.9283	5/7	.8939	2/9	.6057	7/12	.8355	11/16	.8826
2/3	.8736	1/6	.5503	6/7	.9499	4/9	.7631	11/12	.9714	13/16	.9331
1/4	.6300	5/6	.9410	1/8	.5000	5/9	.8221	1/16	.3969	15/16	.9787
3/4	.9086	1/7	.5228	3/8	.7211	7/9	.9196	3/16	.5724	1/32	.3150
1/5	.5848	2/7	.6586	5/8	.8550	8/9	.9615	5/16	.6786	1/64	.2500
2/5	.7368	3/7	.7539	7/8	.9565	1/12	.4368	7/16	.7591	1/50	.2714

CUBE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
50.	3.684	3.686	3.689	3.691	3.694	3.696	3.699	3.701	3.704	3.706	2
1.	3.708	3.711	3.713	3.716	3.718	3.721	3.723	3.725	3.728	3.730	
2.	3.733	3.735	3.737	3.740	3.742	3.744	3.747	3.749	3.752	3.754	
3.	3.756	3.759	3.761	3.763	3.766	3.768	3.770	3.773	3.775	3.777	
4.	3.780	3.782	3.784	3.787	3.789	3.791	3.794	3.796	3.798	3.801	
55.	3.803	3.805	3.808	3.810	3.812	3.814	3.817	3.819	3.821	3.824	
6.	3.826	3.828	3.830	3.833	3.835	3.837	3.839	3.842	3.844	3.846	
7.	3.849	3.851	3.853	3.855	3.857	3.860	3.862	3.864	3.866	3.869	
8.	3.871	3.873	3.875	3.878	3.880	3.882	3.884	3.886	3.889	3.891	
9.	3.893	3.895	3.897	3.900	3.902	3.904	3.906	3.908	3.911	3.913	
60.	3.915	3.917	3.919	3.921	3.924	3.926	3.928	3.930	3.932	3.934	
1.	3.936	3.939	3.941	3.943	3.945	3.947	3.949	3.951	3.954	3.956	
2.	3.958	3.960	3.962	3.964	3.966	3.968	3.971	3.973	3.975	3.977	
3.	3.979	3.981	3.983	3.985	3.987	3.990	3.992	3.994	3.996	3.998	
4.	4.000	4.002	4.004	4.006	4.008	4.010	4.012	4.015	4.017	4.019	
65.	4.021	4.023	4.025	4.027	4.029	4.031	4.033	4.035	4.037	4.039	
6.	4.041	4.043	4.045	4.047	4.049	4.051	4.053	4.055	4.058	4.060	
7.	4.062	4.064	4.066	4.068	4.070	4.072	4.074	4.076	4.078	4.080	
8.	4.082	4.084	4.086	4.088	4.090	4.092	4.094	4.096	4.098	4.100	
9.	4.102	4.104	4.106	4.108	4.109	4.111	4.113	4.115	4.117	4.119	
70.	4.121	4.123	4.125	4.127	4.129	4.131	4.133	4.135	4.137	4.139	
1.	4.141	4.143	4.145	4.147	4.149	4.151	4.152	4.154	4.156	4.158	
2.	4.160	4.162	4.164	4.166	4.168	4.170	4.172	4.174	4.176	4.177	
3.	4.179	4.181	4.183	4.185	4.187	4.189	4.191	4.193	4.195	4.196	
4.	4.198	4.200	4.202	4.204	4.206	4.208	4.210	4.212	4.213	4.215	
75.	4.217	4.219	4.221	4.223	4.225	4.227	4.228	4.230	4.232	4.234	
6.	4.236	4.238	4.240	4.241	4.243	4.245	4.247	4.249	4.251	4.252	
7.	4.254	4.256	4.258	4.260	4.262	4.264	4.265	4.267	4.269	4.271	
8.	4.273	4.274	4.276	4.278	4.280	4.282	4.284	4.285	4.287	4.289	
9.	4.291	4.293	4.294	4.296	4.298	4.300	4.302	4.303	4.305	4.307	
80.	4.309	4.311	4.312	4.314	4.316	4.318	4.320	4.321	4.323	4.325	
1.	4.327	4.329	4.330	4.332	4.334	4.336	4.337	4.339	4.341	4.343	
2.	4.344	4.346	4.348	4.350	4.352	4.353	4.355	4.357	4.359	4.360	
3.	4.362	4.364	4.366	4.367	4.369	4.371	4.373	4.374	4.376	4.378	
4.	4.380	4.381	4.383	4.385	4.386	4.388	4.390	4.392	4.393	4.395	
85.	4.397	4.399	4.400	4.402	4.404	4.405	4.407	4.409	4.411	4.412	
6.	4.414	4.416	4.417	4.419	4.421	4.423	4.424	4.426	4.428	4.429	
7.	4.431	4.433	4.434	4.436	4.438	4.440	4.441	4.443	4.445	4.446	
8.	4.448	4.450	4.451	4.453	4.455	4.456	4.458	4.460	4.461	4.463	
9.	4.465	4.466	4.468	4.470	4.471	4.473	4.475	4.476	4.478	4.480	
90.	4.481	4.483	4.485	4.486	4.488	4.490	4.491	4.493	4.495	4.496	
1.	4.498	4.500	4.501	4.503	4.505	4.506	4.508	4.509	4.511	4.513	
2.	4.514	4.516	4.518	4.519	4.521	4.523	4.524	4.526	4.527	4.529	
3.	4.531	4.532	4.534	4.536	4.537	4.539	4.540	4.542	4.544	4.545	
4.	4.547	4.548	4.550	4.552	4.553	4.555	4.556	4.558	4.560	4.561	
95.	4.563	4.565	4.566	4.568	4.569	4.571	4.572	4.574	4.576	4.577	
6.	4.579	4.580	4.582	4.584	4.585	4.587	4.588	4.590	4.592	4.593	
7.	4.595	4.596	4.598	4.599	4.601	4.603	4.604	4.606	4.607	4.609	
8.	4.610	4.612	4.614	4.615	4.617	4.618	4.620	4.621	4.623	4.625	
9.	4.626	4.628	4.629	4.631	4.632	4.634	4.635	4.637	4.638	4.640	

Moving the decimal point THREE places in *N* requires moving it ONE place in body of table (see p. 16).

CUBE ROOTS (*continued*)

N	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Avg. diff.
10	4.642	4.657	4.672	4.688	4.703	4.718	4.733	4.747	4.762	4.777	15
1	4.791	4.806	4.820	4.835	4.849	4.863	4.877	4.891	4.905	4.919	14
2	4.932	4.946	4.960	4.973	4.987	5.000	5.013	5.027	5.040	5.053	13
3	5.066	5.079	5.092	5.104	5.117	5.130	5.143	5.155	5.168	5.180	
4	5.192	5.205	5.217	5.229	5.241	5.254	5.266	5.278	5.290	5.301	12
15	5.313	5.325	5.337	5.348	5.360	5.372	5.383	5.395	5.406	5.418	
6	5.429	5.440	5.451	5.463	5.474	5.485	5.496	5.507	5.518	5.529	11
7	5.540	5.550	5.561	5.572	5.583	5.593	5.604	5.615	5.625	5.636	
8	5.646	5.657	5.667	5.677	5.688	5.698	5.708	5.718	5.729	5.739	10
9	5.749	5.759	5.769	5.779	5.789	5.799	5.809	5.819	5.828	5.838	
20	5.848	5.858	5.867	5.877	5.887	5.896	5.906	5.915	5.925	5.934	
1	5.944	5.953	5.963	5.972	5.981	5.991	6.000	6.009	6.018	6.028	9
2	6.037	6.046	6.055	6.064	6.073	6.082	6.091	6.100	6.109	6.118	
3	6.127	6.136	6.145	6.153	6.162	6.171	6.180	6.188	6.197	6.206	
4	6.214	6.223	6.232	6.240	6.249	6.257	6.266	6.274	6.283	6.291	
25	6.300	6.308	6.316	6.325	6.333	6.341	6.350	6.358	6.366	6.374	8
6	6.383	6.391	6.399	6.407	6.415	6.423	6.431	6.439	6.447	6.455	
7	6.463	6.471	6.479	6.487	6.495	6.503	6.511	6.519	6.527	6.534	
8	6.542	6.550	6.558	6.565	6.573	6.581	6.589	6.596	6.604	6.611	
9	6.619	6.627	6.634	6.642	6.649	6.657	6.664	6.672	6.679	6.687	
30	6.694	6.702	6.709	6.717	6.724	6.731	6.739	6.746	6.753	6.761	7
1	6.768	6.775	6.782	6.790	6.797	6.804	6.811	6.818	6.826	6.833	
2	6.840	6.847	6.854	6.861	6.868	6.875	6.882	6.889	6.896	6.903	
3	6.910	6.917	6.924	6.931	6.938	6.945	6.952	6.959	6.966	6.973	
4	6.980	6.986	6.993	7.000	7.007	7.014	7.020	7.027	7.034	7.041	
35	7.047	7.054	7.061	7.067	7.074	7.081	7.087	7.094	7.101	7.107	
6	7.114	7.120	7.127	7.133	7.140	7.147	7.153	7.160	7.166	7.173	6
7	7.179	7.186	7.192	7.198	7.205	7.211	7.218	7.224	7.230	7.237	
8	7.243	7.250	7.256	7.262	7.268	7.275	7.281	7.287	7.294	7.300	
9	7.306	7.312	7.319	7.325	7.331	7.337	7.343	7.350	7.356	7.362	
40	7.368	7.374	7.380	7.386	7.393	7.399	7.405	7.411	7.417	7.423	
1	7.429	7.435	7.441	7.447	7.453	7.459	7.465	7.471	7.477	7.483	
2	7.489	7.495	7.501	7.507	7.513	7.518	7.524	7.530	7.536	7.542	
3	7.548	7.554	7.560	7.565	7.571	7.577	7.583	7.589	7.594	7.600	
4	7.606	7.612	7.617	7.623	7.629	7.635	7.640	7.646	7.652	7.657	
45	7.663	7.669	7.674	7.680	7.686	7.691	7.697	7.703	7.708	7.714	5
6	7.719	7.725	7.731	7.736	7.742	7.747	7.753	7.758	7.764	7.769	
7	7.775	7.780	7.786	7.791	7.797	7.802	7.808	7.813	7.819	7.824	
8	7.830	7.835	7.841	7.846	7.851	7.857	7.862	7.868	7.873	7.878	
9	7.884	7.889	7.894	7.900	7.905	7.910	7.916	7.921	7.926	7.932	

AUXILIARY TABLE OF TWO-THIRDS POWERS AND THREE-HALVES POWERS (see pp. 22–23)

(To assist in locating the decimal point)

N	$N^{2/3}(=\sqrt[3]{N^2})$	$N^{3/2}(=\sqrt{N^3})$	
.0001	.002154	.000001	For complete table of three-halves powers, see pp. 22–23. That table, used inversely, provides a complete table of two-thirds powers.
.001	.01	.00003162	
.01	.0464	.001	
.1	.2154	.03162278	
1.	1.	1.	
10.	4.64	31.62278	
100.	21.54	1000.	
1000.	100.	31622.78	
10000.	464.16	1000000.	

CUBE ROOTS (*continued*)

N	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
50	7.937	7.942	7.948	7.953	7.958	7.963	7.969	7.974	7.979	7.984
1	7.990	7.995	8.000	8.005	8.010	8.016	8.021	8.026	8.031	8.036
2	8.041	8.047	8.052	8.057	8.062	8.067	8.072	8.077	8.082	8.088
3	8.093	8.098	8.103	8.108	8.113	8.118	8.123	8.128	8.133	8.138
4	8.143	8.148	8.153	8.158	8.163	8.168	8.173	8.178	8.183	8.188
55	8.193	8.198	8.203	8.208	8.213	8.218	8.223	8.228	8.233	8.238
6	8.243	8.247	8.252	8.257	8.262	8.267	8.272	8.277	8.282	8.286
7	8.291	8.296	8.301	8.306	8.311	8.316	8.320	8.325	8.330	8.335
8	8.340	8.344	8.349	8.354	8.359	8.363	8.368	8.373	8.378	8.382
9	8.387	8.392	8.397	8.401	8.406	8.411	8.416	8.420	8.425	8.430
60	8.434	8.439	8.444	8.448	8.453	8.458	8.462	8.467	8.472	8.476
1	8.481	8.486	8.490	8.495	8.499	8.504	8.509	8.513	8.518	8.522
2	8.527	8.532	8.536	8.541	8.545	8.550	8.554	8.559	8.564	8.568
3	8.573	8.577	8.582	8.586	8.591	8.595	8.600	8.604	8.609	8.613
4	8.618	8.622	8.627	8.631	8.636	8.640	8.645	8.649	8.653	8.658
65	8.662	8.667	8.671	8.676	8.680	8.685	8.689	8.693	8.698	8.702
6	8.707	8.711	8.715	8.720	8.724	8.729	8.733	8.737	8.742	8.746
7	8.750	8.755	8.759	8.763	8.768	8.772	8.776	8.781	8.785	8.789
8	8.794	8.798	8.802	8.807	8.811	8.815	8.819	8.824	8.828	8.832
9	8.837	8.841	8.845	8.849	8.854	8.858	8.862	8.866	8.871	8.875
70	8.879	8.883	8.887	8.892	8.896	8.900	8.904	8.909	8.913	8.917
1	8.921	8.925	8.929	8.934	8.938	8.942	8.946	8.950	8.955	8.959
2	8.963	8.967	8.971	8.975	8.979	8.984	8.988	8.992	8.996	9.000
3	9.004	9.008	9.012	9.016	9.021	9.025	9.029	9.033	9.037	9.041
4	9.045	9.049	9.053	9.057	9.061	9.065	9.069	9.073	9.078	9.082
75	9.086	9.090	9.094	9.098	9.102	9.106	9.110	9.114	9.118	9.122
6	9.126	9.130	9.134	9.138	9.142	9.146	9.150	9.154	9.158	9.162
7	9.166	9.170	9.174	9.178	9.182	9.186	9.189	9.193	9.197	9.201
8	9.205	9.209	9.213	9.217	9.221	9.225	9.229	9.233	9.237	9.240
9	9.244	9.248	9.252	9.256	9.260	9.264	9.268	9.272	9.275	9.279
80	9.283	9.287	9.291	9.295	9.299	9.302	9.306	9.310	9.314	9.318
1	9.322	9.326	9.329	9.333	9.337	9.341	9.345	9.348	9.352	9.356
2	9.360	9.364	9.368	9.371	9.375	9.379	9.383	9.386	9.390	9.394
3	9.398	9.402	9.405	9.409	9.413	9.417	9.420	9.424	9.428	9.432
4	9.435	9.439	9.443	9.447	9.450	9.454	9.458	9.462	9.465	9.469
85	9.473	9.476	9.480	9.484	9.488	9.491	9.495	9.499	9.502	9.506
6	9.510	9.513	9.517	9.521	9.524	9.528	9.532	9.535	9.539	9.543
7	9.546	9.550	9.554	9.557	9.561	9.565	9.568	9.572	9.576	9.579
8	9.583	9.586	9.590	9.594	9.597	9.601	9.605	9.608	9.612	9.615
9	9.619	9.623	9.626	9.630	9.633	9.637	9.641	9.644	9.648	9.651
90	9.655	9.658	9.662	9.666	9.669	9.673	9.676	9.680	9.683	9.687
1	9.691	9.694	9.698	9.701	9.705	9.708	9.712	9.715	9.719	9.722
2	9.726	9.729	9.733	9.736	9.740	9.743	9.747	9.750	9.754	9.758
3	9.761	9.764	9.768	9.771	9.775	9.778	9.782	9.785	9.789	9.792
4	9.796	9.799	9.803	9.806	9.810	9.813	9.817	9.820	9.824	9.827
95	9.830	9.834	9.837	9.841	9.844	9.848	9.851	9.855	9.858	9.861
6	9.865	9.868	9.872	9.875	9.879	9.882	9.885	9.889	9.892	9.896
7	9.899	9.902	9.906	9.909	9.913	9.916	9.919	9.923	9.926	9.930
8	9.933	9.936	9.940	9.943	9.946	9.950	9.953	9.956	9.960	9.963
9	9.967	9.970	9.973	9.977	9.980	9.983	9.987	9.990	9.993	9.997
100	10.00									

Moving the decimal point THREE places in *N* requires moving it ONE place in of table (see p. 16).

THREE-HALVES POWERS OF NUMBERS (see also p. 20)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.	1.000	1.154	1.315	1.482	1.657	1.837	2.024	2.217	2.415	2.619	183
2.	2.828	3.043	3.263	3.488	3.718	3.953	4.192	4.437	4.685	4.939	237
3.	5.196	5.458	5.724	5.995	6.269	6.548	6.831	7.117	7.408	7.702	280
4.	8.000	8.302	8.607	8.917	9.230	9.546	9.866	10.190			313
4.								10.19	10.52	10.85	33
5.	11.18	11.52	11.86	12.20	12.55	12.90	13.25	13.61	13.97	14.33	35
6.	14.70	15.07	15.44	15.81	16.19	16.57	16.96	17.34	17.73	18.12	38
7.	18.52	18.92	19.32	19.72	20.13	20.54	20.95	21.37	21.78	22.20	41
8.	22.63	23.05	23.48	23.91	24.35	24.78	25.22	25.66	26.11	26.55	44
9.	27.00	27.45	27.90	28.36	28.82	29.28	29.74	30.21	30.68	31.15	46
10.	31.62	32.10	32.58	33.06	33.54	34.02	34.51	35.00	35.49	35.99	49
1.	36.48	36.98	37.48	37.99	38.49	39.00	39.51	40.02	40.53	41.05	51
2.	41.57	42.09	42.61	43.14	43.66	44.19	44.73	45.26	45.79	46.33	53
3.	46.87	47.41	47.96	48.50	49.05	49.60	50.15	50.71	51.26	51.82	55
4.	52.38	52.95	53.51	54.08	54.64	55.21	55.79	56.36	56.94	57.51	57
15.	58.09	58.68	59.26	59.85	60.43	61.02	61.62	62.21	62.80	63.40	59
6.	64.00	64.60	65.20	65.81	66.41	67.02	67.63	68.25	68.86	69.48	61
7.	70.09	70.71	71.33	71.96	72.58	73.21	73.84	74.47	75.10	75.73	63
8.	76.37	77.00	77.64	78.28	78.93	79.57	80.22	80.87	81.51	82.17	65
9.	82.82	83.47	84.13	84.79	85.45	86.11	86.77	87.44	88.10	88.77	66
20.	89.44	90.11	90.79	91.46	92.14	92.82	93.50	94.18	94.86	95.55	68
1.	96.23	96.92	97.61	98.30	99.00	99.69	100.38				69
1.							100.4	101.1	101.8	102.5	7
2.	103.2	103.9	104.6	105.3	106.0	106.7	107.4	108.2	108.9	109.6	7
3.	110.3	111.0	111.7	112.5	113.2	113.9	114.6	115.4	116.1	116.8	7
4.	117.6	118.3	119.0	119.8	120.5	121.3	122.0	122.8	123.5	124.3	7
25.	125.0	125.8	126.5	127.3	128.0	128.8	129.5	130.3	131.0	131.8	8
6.	132.6	133.3	134.1	134.9	135.6	136.4	137.2	138.0	138.7	139.5	8
7.	140.3	141.1	141.9	142.6	143.4	144.2	145.0	145.8	146.6	147.4	8
8.	148.2	149.0	149.8	150.5	151.3	152.1	152.9	153.8	154.6	155.4	8
9.	156.2	157.0	157.8	158.6	159.4	160.2	161.0	161.9	162.7	163.5	8
30.	164.3	165.1	166.0	166.8	167.6	168.4	169.3	170.1	170.9	171.8	8
1.	172.6	173.4	174.3	175.1	176.0	176.8	177.6	178.5	179.3	180.2	8
2.	181.0	181.9	182.7	183.6	184.4	185.3	186.1	187.0	187.8	188.7	9
3.	189.6	190.4	191.3	192.2	193.0	193.9	194.8	195.6	196.5	197.4	9
4.	198.3	199.1	200.0	200.9	201.8	202.6	203.5	204.4	205.3	206.2	9
35.	207.1	208.0	208.8	209.7	210.6	211.5	212.4	213.3	214.2	215.1	9
6.	216.0	216.9	217.8	218.7	219.6	220.5	221.4	222.3	223.2	224.2	9
7.	225.1	226.0	226.9	227.8	228.7	229.6	230.6	231.5	232.4	233.3	9
8.	234.2	235.2	236.1	237.0	238.0	238.9	239.8	240.8	241.7	242.6	9
9.	243.6	244.5	245.4	246.4	247.3	248.3	249.2	250.1	251.1	252.0	9
40.	253.0	253.9	254.9	255.8	256.8	257.7	258.7	259.7	260.6	261.6	10
1.	262.5	263.5	264.5	265.4	266.4	267.3	268.3	269.3	270.2	271.2	10
2.	272.2	273.2	274.1	275.1	276.1	277.1	278.0	279.0	280.0	281.0	10
3.	282.0	283.0	283.9	284.9	285.9	286.9	287.9	288.9	289.9	290.9	10
4.	291.9	292.9	293.9	294.9	295.9	296.9	297.9	298.9	299.9	300.9	10
45.	301.9	302.9	303.9	304.9	305.9	306.9	307.9	308.9	310.0	311.0	10
6.	312.0	313.0	314.0	315.0	316.1	317.1	318.1	319.1	320.2	321.2	10
7.	322.2	323.2	324.3	325.3	326.3	327.4	328.4	329.4	330.5	331.5	10
8.	332.6	333.6	334.6	335.7	336.7	337.8	338.8	339.9	340.9	342.0	10
9.	343.0	344.1	345.1	346.2	347.2	348.3	349.3	350.4	351.4	352.5	11

This table gives $N^{3/2}$ from $N = 1$ to $N = 100$. Moving the decimal point TWO places in N requires moving it THREE places in body of table. Thus:

$(7.23)^{3/2} = 19.44$; $(723.)^{3/2} = 19440$; $(0.0723)^{3/2} = 0.01944$

$(72.3)^{3/2} = 614.8$; $(7230.)^{3/2} = 614800$; $(0.723)^{3/2} = 0.6148$

Used inversely, table gives $M^{2/3}$ from $M = 1$ to $M = 1000$. Thus: $(0.6148)^{2/3} = 0.7230$.

THREE-HALVES POWERS (*continued*) (See also p. 20)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
50.	353.6	354.6	355.7	356.7	357.8	358.9	359.9	361.0	362.1	363.1	11
1.	364.2	365.3	366.4	367.4	368.5	369.6	370.7	371.7	372.8	373.9	11
2.	375.0	376.1	377.1	378.2	379.3	380.4	381.5	382.6	383.7	384.8	11
3	385.8	386.9	388.0	389.1	390.2	391.3	392.4	393.5	394.6	395.7	11
4.	396.8	397.9	399.0	400.1	401.2	402.3	403.4	404.6	405.7	406.8	11
55.	407.9	409.0	410.1	411.2	412.3	413.5	414.6	415.7	416.8	417.9	11
6.	419.1	420.2	421.3	422.4	423.6	424.7	425.8	426.9	428.1	429.2	11
7.	430.3	431.5	432.6	433.7	434.9	436.0	437.2	438.3	439.4	440.6	11
8.	441.7	442.9	444.0	445.1	446.3	447.4	448.6	449.7	450.9	452.0	11
9.	453.2	454.3	455.5	456.6	457.8	459.0	460.1	461.3	462.4	463.6	12
60.	464.8	465.9	467.1	468.2	469.4	470.6	471.7	472.9	474.1	475.3	12
1.	476.4	477.6	478.8	479.9	481.1	482.3	483.5	484.6	485.8	487.0	12
2.	488.2	489.4	490.6	491.7	492.9	494.1	495.3	496.5	497.7	498.9	12
3.	500.0	501.2	502.4	503.6	504.8	506.0	507.2	508.4	509.6	510.8	12
4.	512.0	513.2	514.4	515.6	516.8	518.0	519.2	520.4	521.6	522.8	12
65.	524.0	525.3	526.5	527.7	528.9	530.1	531.3	532.5	533.8	535.0	12
6.	536.2	537.4	538.6	539.8	541.1	542.3	543.5	544.7	546.0	547.2	12
7.	548.4	549.6	550.9	552.1	553.3	554.6	555.8	557.0	558.3	559.5	12
8.	560.7	562.0	563.2	564.5	565.7	566.9	568.2	569.4	570.7	571.9	12
9.	573.2	574.4	575.7	576.9	578.1	579.4	580.6	581.9	583.2	584.4	13
70.	585.7	586.9	588.2	589.4	590.7	591.9	593.2	594.5	595.7	597.0	13
1.	598.3	599.5	600.8	602.1	603.3	604.6	605.9	607.1	608.4	609.7	13
2.	610.9	612.2	613.5	614.8	616.0	617.3	618.6	619.9	621.2	622.4	13
3.	623.7	625.0	626.3	627.6	628.8	630.1	631.4	632.7	634.0	635.3	13
4.	636.6	637.9	639.2	640.4	641.7	643.0	644.3	645.6	646.9	648.2	13
75.	649.5	650.8	652.1	653.4	654.7	656.0	657.3	658.6	659.9	661.2	13
6.	662.6	663.9	665.2	666.5	667.8	669.1	670.4	671.7	673.0	674.4	13
7.	675.7	677.0	678.3	679.6	680.9	682.3	683.6	684.9	686.2	687.6	13
8.	688.9	690.2	691.5	692.9	694.2	695.5	696.8	698.2	699.5	700.8	13
9.	702.2	703.5	704.8	706.2	707.5	708.8	710.2	711.5	712.9	714.2	13
80.	715.5	716.9	718.2	719.6	720.9	722.3	723.6	725.0	726.3	727.7	13
1.	729.0	730.4	731.7	733.1	734.4	735.8	737.1	738.5	739.8	741.2	14
2.	742.5	743.9	745.3	746.6	748.0	749.3	750.7	752.1	753.4	754.8	14
3.	756.2	757.5	758.9	760.3	761.6	763.0	764.4	765.8	767.1	768.5	14
4.	769.9	771.2	772.6	774.0	775.4	776.8	778.1	779.5	780.9	782.3	14
85.	783.7	785.0	786.4	787.8	789.2	790.6	792.0	793.4	794.8	796.1	14
6.	797.5	798.9	800.3	801.7	803.1	804.5	805.9	807.3	808.7	810.1	14
7.	811.5	812.9	814.3	815.7	817.1	818.5	819.9	821.3	822.7	824.1	14
8.	825.5	826.9	828.3	829.7	831.1	832.6	834.0	835.4	836.8	838.2	14
9.	839.6	841.0	842.5	843.9	845.3	846.7	848.1	849.5	851.0	852.4	14
90.	853.8	855.2	856.7	858.1	859.5	860.9	862.4	863.8	865.2	866.7	14
1.	868.1	869.5	870.9	872.4	873.8	875.2	876.7	878.1	879.6	881.0	14
2.	882.4	883.9	885.3	886.8	888.2	889.6	891.1	892.5	894.0	895.4	14
3.	896.9	898.3	899.8	901.2	902.7	904.1	905.6	907.0	908.5	909.9	15
4.	911.4	912.8	914.3	915.7	917.2	918.6	920.1	921.6	923.0	924.5	15
95.	925.9	927.4	928.9	930.3	931.8	933.3	934.7	936.2	937.7	939.1	15
6.	940.6	942.1	943.5	945.0	946.5	948.0	949.4	950.9	952.4	953.9	15
7.	955.3	956.8	958.3	959.8	961.3	962.7	964.2	965.7	967.2	968.7	15
8.	970.2	971.6	973.1	974.6	976.1	977.6	979.1	980.6	982.1	983.5	15
9.	985.0	986.5	988.0	989.5	991.0	992.5	994.0	995.5	997.0	998.5	15
100.	1000.0										

Moving the decimal point TWO places in N requires moving it THREE places in body of table (see also auxiliary table on p. 20).

RECIPROCALS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00		.9990	.9980	.9970	.9960	.9950	.9940	.9930	.9921	.9911	−10
1	.9901	.9891	.9881	.9872	.9862	.9852	.9843	.9833	.9823	.9814	
2	.9804	.9794	.9785	.9775	.9766	.9756	.9747	.9737	.9728	.9718	
3	.9709	.9699	.9690	.9681	.9671	.9662	.9653	.9643	.9634	.9625	−9
4	.9615	.9606	.9597	.9588	.9579	.9569	.9560	.9551	.9542	.9533	
1.05	.9524	.9515	.9506	.9497	.9488	.9479	.9470	.9461	.9452	.9443	
6	.9434	.9425	.9416	.9407	.9398	.9390	.9381	.9372	.9363	.9355	
7	.9346	.9337	.9328	.9320	.9311	.9302	.9294	.9285	.9276	.9268	
8	.9259	.9251	.9242	.9234	.9225	.9217	.9208	.9200	.9191	.9183	−8
9	.9174	.9166	.9158	.9149	.9141	.9132	.9124	.9116	.9107	.9099	
1.10	.9091	.9083	.9074	.9066	.9058	.9050	.9042	.9033	.9025	.9017	
1	.9009	.9001	.8993	.8985	.8977	.8969	.8961	.8953	.8945	.8937	
2	.8929	.8921	.8913	.8905	.8897	.8889	.8881	.8873	.8865	.8857	
3	.8850	.8842	.8834	.8826	.8818	.8811	.8803	.8795	.8787	.8780	
4	.8772	.8764	.8757	.8749	.8741	.8734	.8726	.8718	.8711	.8703	
1.15	.8696	.8688	.8681	.8673	.8666	.8658	.8651	.8643	.8636	.8628	
6	.8621	.8613	.8606	.8598	.8591	.8584	.8576	.8569	.8562	.8554	−7
7	.8547	.8540	.8532	.8525	.8518	.8511	.8503	.8496	.8489	.8482	
8	.8475	.8467	.8460	.8453	.8446	.8439	.8432	.8425	.8418	.8410	
9	.8403	.8396	.8389	.8382	.8375	.8368	.8361	.8354	.8347	.8340	
1.20	.8333	.8326	.8319	.8313	.8306	.8299	.8292	.8285	.8278	.8271	
1	.8264	.8258	.8251	.8244	.8237	.8230	.8224	.8217	.8210	.8203	
2	.8197	.8190	.8183	.8177	.8170	.8163	.8157	.8150	.8143	.8137	
3	.8130	.8123	.8117	.8110	.8104	.8097	.8091	.8084	.8078	.8071	−6
4	.8065	.8058	.8052	.8045	.8039	.8032	.8026	.8019	.8013	.8006	
1.25	.8000	.7994	.7987	.7981	.7974	.7968	.7962	.7955	.7949	.7943	
6	.7937	.7930	.7924	.7918	.7911	.7905	.7899	.7893	.7886	.7880	
7	.7874	.7868	.7862	.7855	.7849	.7843	.7837	.7831	.7825	.7819	
8	.7812	.7806	.7800	.7794	.7788	.7782	.7776	.7770	.7764	.7758	
9	.7752	.7746	.7740	.7734	.7728	.7722	.7716	.7710	.7704	.7698	
1.30	.7692	.7686	.7680	.7675	.7669	.7663	.7657	.7651	.7645	.7639	
1	.7634	.7628	.7622	.7616	.7610	.7605	.7599	.7593	.7587	.7582	
2	.7576	.7570	.7564	.7559	.7553	.7547	.7541	.7536	.7530	.7524	
3	.7519	.7513	.7508	.7502	.7496	.7491	.7485	.7479	.7474	.7468	
4	.7463	.7457	.7452	.7446	.7440	.7435	.7429	.7424	.7418	.7413	
1.35	.7407	.7402	.7396	.7391	.7386	.7380	.7375	.7369	.7364	.7358	−5
6	.7353	.7348	.7342	.7337	.7331	.7326	.7321	.7315	.7310	.7305	
7	.7299	.7294	.7289	.7283	.7278	.7273	.7267	.7262	.7257	.7252	
8	.7246	.7241	.7236	.7231	.7225	.7220	.7215	.7210	.7205	.7199	
9	.7194	.7189	.7184	.7179	.7174	.7168	.7163	.7158	.7153	.7148	
1.40	.7143	.7138	.7133	.7128	.7123	.7117	.7112	.7107	.7102	.7097	
1	.7092	.7087	.7082	.7077	.7072	.7067	.7062	.7057	.7052	.7047	
2	.7042	.7037	.7032	.7027	.7022	.7018	.7013	.7008	.7003	.6998	
3	.6993	.6988	.6983	.6978	.6974	.6969	.6964	.6959	.6954	.6949	
4	.6944	.6940	.6935	.6930	.6925	.6920	.6916	.6911	.6906	.6901	
1.45	.6897	.6892	.6887	.6882	.6878	.6873	.6868	.6863	.6859	.6854	
6	.6849	.6845	.6840	.6835	.6831	.6826	.6821	.6817	.6812	.6807	
7	.6803	.6798	.6793	.6789	.6784	.6780	.6775	.6770	.6766	.6761	
8	.6757	.6752	.6748	.6743	.6739	.6734	.6729	.6725	.6720	.6716	
9	.6711	.6707	.6702	.6698	.6693	.6689	.6684	.6680	.6676	.6671	

$1/\pi = 0.318310$ $\quad 1/e = 0.367879$

Moving the decimal point in either direction in *N* requires moving it in the OPPOSITE direction in body of table (see p. 26).

RECIPROCALS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.50	.6667	.6662	.6658	.6653	.6649	.6645	.6640	.6636	.6631	.6627	−4
1	.6623	.6618	.6614	.6609	.6605	.6601	.6596	.6592	.6588	.6583	
2	.6579	.6575	.6570	.6566	.6562	.6557	.6553	.6549	.6545	.6540	
3	.6536	.6532	.6527	.6523	.6519	.6515	.6510	.6506	.6502	.6498	
4	.6494	.6489	.6485	.6481	.6477	.6472	.6468	.6464	.6460	.6456	
1.55	.6452	.6447	.6443	.6439	.6435	.6431	.6427	.6423	.6418	.6414	
6	.6410	.6406	.6402	.6398	.6394	.6390	.6386	.6382	.6378	.6373	
7	.6369	.6365	.6361	.6357	.6353	.6349	.6345	.6341	.6337	.6333	
8	.6329	.6325	.6321	.6317	.6313	.6309	.6305	.6301	.6297	.6293	
9	.6289	.6285	.6281	.6277	.6274	.6270	.6266	.6262	.6258	.6254	
1.60	.6250	.6246	.6242	.6238	.6234	.6231	.6227	.6223	.6219	.6215	
1	.6211	.6207	.6203	.6200	.6196	.6192	.6188	.6184	.6180	.6177	
2	.6173	.6169	.6165	.6161	.6158	.6154	.6150	.6146	.6143	.6139	
3	.6135	.6131	.6127	.6124	.6120	.6116	.6112	.6109	.6105	.6101	
4	.6098	.6094	.6090	.6086	.6083	.6079	.6075	.6072	.6068	.6064	
1.65	.6061	.6057	.6053	.6050	.6046	.6042	.6039	.6035	.6031	.6028	
6	.6024	.6020	.6017	.6013	.6010	.6006	.6002	.5999	.5995	.5992	
7	.5988	.5984	.5981	.5977	.5974	.5970	.5967	.5963	.5959	.5956	
8	.5952	.5949	.5945	.5942	.5938	.5935	.5931	.5928	.5924	.5921	
9	.5917	.5914	.5910	.5907	.5903	.5900	.5896	.5893	.5889	.5886	
1.70	.5882	.5879	.5875	.5872	.5869	.5865	.5862	.5858	.5855	.5851	−3
1	.5848	.5845	.5841	.5838	.5834	.5831	.5828	.5824	.5821	.5817	
2	.5814	.5811	.5807	.5804	.5800	.5797	.5794	.5790	.5787	.5784	
3	.5780	.5777	.5774	.5770	.5767	.5764	.5760	.5757	.5754	.5750	
4	.5747	.5744	.5741	.5737	.5734	.5731	.5727	.5724	.5721	.5718	
1.75	.5714	.5711	.5708	.5705	.5701	.5698	.5695	.5692	.5688	.5685	
6	.5682	.5679	.5675	.5672	.5669	.5666	.5663	.5659	.5656	.5653	
7	.5650	.5647	.5643	.5640	.5637	.5634	.5631	.5627	.5624	.5621	
8	.5618	.5615	.5612	.5609	.5605	.5602	.5599	.5596	.5593	.5590	
9	.5587	.5583	.5580	.5577	.5574	.5571	.5568	.5565	.5562	.5559	
1.80	.5556	.5552	.5549	.5546	.5543	.5540	.5537	.5534	.5531	.5528	
1	.5525	.5522	.5519	.5516	.5513	.5510	.5507	.5504	.5501	.5498	
2	.5495	.5491	.5488	.5485	.5482	.5479	.5476	.5473	.5470	.5467	
3	.5464	.5461	.5459	.5456	.5453	.5450	.5447	.5444	.5441	.5438	
4	.5435	.5432	.5429	.5426	.5423	.5420	.5417	.5414	.5411	.5408	
1.85	.5405	.5402	.5400	.5397	.5394	.5391	.5388	.5385	.5382	.5379	
6	.5376	.5373	.5371	.5368	.5365	.5362	.5359	.5356	.5353	.5350	
7	.5348	.5345	.5342	.5339	.5336	.5333	.5330	.5328	.5325	.5322	
8	.5319	.5316	.5313	.5311	.5308	.5305	.5302	.5299	.5297	.5294	
9	.5291	.5288	.5285	.5283	.5280	.5277	.5274	.5271	.5269	.5266	
1.90	.5263	.5260	.5258	.5255	.5252	.5249	.5247	.5244	.5241	.5238	
1	.5236	.5233	.5230	.5227	.5225	.5222	.5219	.5216	.5214	.5211	
2	.5208	.5206	.5203	.5200	.5198	.5195	.5192	.5189	.5187	.5184	
3	.5181	.5179	.5176	.5173	.5171	.5168	.5165	.5163	.5160	.5157	
4	.5155	.5152	.5149	.5147	.5144	.5141	.5139	.5136	.5133	.5131	
1.95	.5128	.5126	.5123	.5120	.5118	.5115	.5112	.5110	.5107	.5105	
6	.5102	.5099	.5097	.5094	.5092	.5089	.5086	.5084	.5081	.5079	
7	.5076	.5074	.5071	.5068	.5066	.5063	.5061	.5058	.5056	.5053	−2
8	.5051	.5048	.5045	.5043	.5040	.5038	.5035	.5033	.5030	.5028	
9	.5025	.5023	.5020	.5018	.5015	.5013	.5010	.5008	.5005	.5003	

Moving the decimal point in either direction in N requires moving it in the OPPOSITE direction in body of table (see p. 26).

RECIPROCALS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
2.0	.5000	.4975	.4950	.4926	.4902	.4878	.4854	.4831	.4808	.4785	– 24
1	.4762	.4739	.4717	.4695	.4673	.4651	.4630	.4608	.4587	.4566	– 21
2	.4545	.4525	.4505	.4484	.4464	.4444	.4425	.4405	.4386	.4367	– 20
3	.4348	.4329	.4310	.4292	.4274	.4255	.4237	.4219	.4202	.4184	– 18
4	.4167	.4149	.4132	.4115	.4098	.4082	.4065	.4049	.4032	.4016	– 17
2.5	.4000	.3984	.3968	.3953	.3937	.3922	.3906	.3891	.3876	.3861	– 15
6	.3846	.3831	.3817	.3802	.3788	.3774	.3759	.3745	.3731	.3717	– 14
7	.3704	.3690	.3676	.3663	.3650	.3636	.3623	.3610	.3597	.3584	– 13
8	.3571	.3559	.3546	.3534	.3521	.3509	.3497	.3484	.3472	.3460	– 12
9	.3448	.3436	.3425	.3413	.3401	.3390	.3378	.3367	.3356	.3344	– 12
3.0	.3333	.3322	.3311	.3300	.3289	.3279	.3268	.3257	.3247	.3236	– 11
1	.3226	.3215	.3205	.3195	.3185	.3175	.3165	.3155	.3145	.3135	– 10
2	.3125	.3115	.3106	.3096	.3086	.3077	.3067	.3058	.3049	.3040	– 10
3	.3030	.3021	.3012	.3003	.2994	.2985	.2976	.2967	.2959	.2950	– 9
4	.2941	.2933	.2924	.2915	.2907	.2899	.2890	.2882	.2874	.2865	– 8
3.5	.2857	.2849	.2841	.2833	.2825	.2817	.2809	.2801	.2793	.2786	– 8
6	.2778	.2770	.2762	.2755	.2747	.2740	.2732	.2725	.2717	.2710	– 8
7	.2703	.2695	.2688	.2681	.2674	.2667	.2660	.2653	.2646	.2639	– 7
8	.2632	.2625	.2618	.2611	.2604	.2597	.2591	.2584	.2577	.2571	– 7
9	.2564	.2558	.2551	.2545	.2538	.2532	.2525	.2519	.2513	.2506	– 6
4.0	.2500	.2494	.2488	.2481	.2475	.2469	.2463	.2457	.2451	.2445	– 6
1	.2439	.2433	.2427	.2421	.2415	.2410	.2404	.2398	.2392	.2387	– 6
2	.2381	.2375	.2370	.2364	.2358	.2353	.2347	.2342	.2336	.2331	– 6
3	.2326	.2320	.2315	.2309	.2304	.2299	.2294	.2288	.2283	.2278	– 5
4	.2273	.2268	.2262	.2257	.2252	.2247	.2242	.2237	.2232	.2227	– 5
4.5	.2222	.2217	.2212	.2208	.2203	.2198	.2193	.2188	.2183	.2179	– 5
6	.2174	.2169	.2165	.2160	.2155	.2151	.2146	.2141	.2137	.2132	– 5
7	.2128	.2123	.2119	.2114	.2110	.2105	.2101	.2096	.2092	.2088	– [illegible]
8	.2083	.2079	.2075	.2070	.2066	.2062	.2058	.2053	.2049	.2045	– 4
9	.2041	.2037	.2033	.2028	.2024	.2020	.2016	.2012	.2008	.2004	– 4

$1/\pi = 0.318310 \qquad 1/e = 0.367879$

Explanation of Table of Reciprocals (pp. 24–27).

This table gives the values of $1/N$ for values of N from 1 to 10, correct to four figures. (Interpolated values may be in error by 1 in the fourth figure.)

To find the reciprocal of a number N outside the range from 1 to 10, note that moving the decimal point any number of places in either direction in column N is equivalent to moving it the same number of places in the **opposite direction** in the body of the table. For example:

$$\frac{1}{3.217} = 0.3108; \quad \frac{1}{3217.} = 0.000\,3108; \quad \frac{1}{0.003217} = 310.8$$

RECIPROCALS (*continued*)

N	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	.2000	.1996	.1992	.1988	.1984	.1980	.1976	.1972	.1969	.1965	−4
.1	.1961	.1957	.1953	.1949	.1946	.1942	.1938	.1934	.1931	.1927	
.2	.1923	.1919	.1916	.1912	.1908	.1905	.1901	.1898	.1894	.1890	
.3	.1887	.1883	.1880	.1876	.1873	.1869	.1866	.1862	.1859	.1855	
.4	.1852	.1848	.1845	.1842	.1838	.1835	.1832	.1828	.1825	.1821	−3
5.5	.1818	.1815	.1812	.1808	.1805	.1802	.1799	.1795	.1792	.1789	
.6	.1786	.1783	.1779	.1776	.1773	.1770	.1767	.1764	.1761	.1757	
.7	.1754	.1751	.1748	.1745	.1742	.1739	.1736	.1733	.1730	.1727	
.8	.1724	.1721	.1718	.1715	.1712	.1709	.1706	.1704	.1701	.1698	
.9	.1695	.1692	.1689	.1686	.1684	.1681	.1678	.1675	.1672	.1669	
6.0	.1667	.1664	.1661	.1658	.1656	.1653	.1650	.1647	.1645	.1642	
.1	.1639	.1637	.1634	.1631	.1629	.1626	.1623	.1621	.1618	.1616	
.2	.1613	.1610	.1608	.1605	.1603	.1600	.1597	.1595	.1592	.1590	
.3	.1587	.1585	.1582	.1580	.1577	.1575	.1572	.1570	.1567	.1565	−2
.4	.1563	.1560	.1558	.1555	.1553	.1550	.1548	.1546	.1543	.1541	
6.5	.1538	.1536	.1534	.1531	.1529	.1527	.1524	.1522	.1520	.1517	
.6	.1515	.1513	.1511	.1508	.1506	.1504	.1502	.1499	.1497	.1495	
.7	.1493	.1490	.1488	.1486	.1484	.1481	.1479	.1477	.1475	.1473	
.8	.1471	.1468	.1466	.1464	.1462	.1460	.1458	.1456	.1453	.1451	
.9	.1449	.1447	.1445	.1443	.1441	.1439	.1437	.1435	.1433	.1431	
7.0	.1429	.1427	.1425	.1422	.1420	.1418	.1416	.1414	.1412	.1410	
.1	.1408	.1406	.1404	.1403	.1401	.1399	.1397	.1395	.1393	.1391	
.2	.1389	.1387	.1385	.1383	.1381	.1379	.1377	.1376	.1374	.1372	
.3	.1370	.1368	.1366	.1364	.1362	.1361	.1359	.1357	.1355	.1353	
.4	.1351	.1350	.1348	.1346	.1344	.1342	.1340	.1339	.1337	.1335	
7.5	.1333	.1332	.1330	.1328	.1326	.1325	.1323	.1321	.1319	.1318	
.6	.1316	.1314	.1312	.1311	.1309	.1307	.1305	.1304	.1302	.1300	
.7	.1299	.1297	.1295	.1294	.1292	.1290	.1289	.1287	.1285	.1284	
.8	.1282	.1280	.1279	.1277	.1276	.1274	.1272	.1271	.1269	.1267	
.9	.1266	.1264	.1263	.1261	.1259	.1258	.1256	.1255	.1253	.1252	
8.0	.1250	.1248	.1247	.1245	.1244	.1242	.1241	.1239	.1238	.1236	
.1	.1235	.1233	.1232	.1230	.1229	.1227	.1225	.1224	.1222	.1221	
.2	.1220	.1218	.1217	.1215	.1214	.1212	.1211	.1209	.1208	.1206	
.3	.1205	.1203	.1202	.1200	.1199	.1198	.1196	.1195	.1193	.1192	
.4	.1190	.1189	.1188	.1186	.1185	.1183	.1182	.1181	.1179	.1178	−1
8.5	.1176	.1175	.1174	.1172	.1171	.1170	.1168	.1167	.1166	.1164	
.6	.1163	.1161	.1160	.1159	.1157	.1156	.1155	.1153	.1152	.1151	
.7	.1149	.1148	.1147	.1145	.1144	.1143	.1142	.1140	.1139	.1138	
.8	.1136	.1135	.1134	.1133	.1131	.1130	.1129	.1127	.1126	.1125	
.9	.1124	.1122	.1121	.1120	.1119	.1117	.1116	.1115	.1114	.1112	
9.0	.1111	.1110	.1109	.1107	.1106	.1105	.1104	.1103	.1101	.1100	
.1	.1099	.1098	.1096	.1095	.1094	.1093	.1092	.1091	.1089	.1088	
.2	.1087	.1086	.1085	.1083	.1082	.1081	.1080	.1079	.1078	.1076	
.3	.1075	.1074	.1073	.1072	.1071	.1070	.1068	.1067	.1066	.1065	
.4	.1064	.1063	.1062	.1060	.1059	.1058	.1057	.1056	.1055	.1054	
9.5	.1053	.1052	.1050	.1049	.1048	.1047	.1046	.1045	.1044	.1043	
.6	.1042	.1041	.1040	.1038	.1037	.1036	.1035	.1034	.1033	.1032	
.7	.1031	.1030	.1029	.1028	.1027	.1026	.1025	.1024	.1022	.1021	
.8	.1020	.1019	.1018	.1017	.1016	.1015	.1014	.1013	.1012	.1011	
.9	.1010	.1009	.1008	.1007	.1006	.1005	.1004	.1003	.1002	.1001	

Moving the decimal point in either direction in *N* requires moving it in the OPPOSITE direction in body of table (see p. 26).

CIRCUMFERENCES OF CIRCLES BY HUNDREDTHS

(For circumferences by eighths, see p. 32)

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	3.142	3.173	3.204	3.236	3.267	3.299	3.330	3.362	3.393	3.424	31
.1	3.456	3.487	3.519	3.550	3.581	3.613	3.644	3.676	3.707	3.738	
.2	3.770	3.801	3.833	3.864	3.896	3.927	3.958	3.990	4.021	4.053	
.3	4.084	4.115	4.147	4.178	4.210	4.241	4.273	4.304	4.335	4.367	
.4	4.398	4.430	4.461	4.492	4.524	4.555	4.587	4.618	4.650	4.681	
1.5	4.712	4.744	4.775	4.807	4.838	4.869	4.901	4.932	4.964	4.995	
.6	5.027	5.058	5.089	5.121	5.152	5.184	5.215	5.246	5.278	5.309	
.7	5.341	5.372	5.404	5.435	5.466	5.498	5.529	5.561	5.592	5.623	
.8	5.655	5.686	5.718	5.749	5.781	5.812	5.843	5.875	5.906	5.938	
.9	5.969	6.000	6.032	6.063	6.095	6.126	6.158	6.189	6.220	6.252	
2.0	6.283	6.315	6.346	6.377	6.409	6.440	6.472	6.503	6.535	6.566	
.1	6.597	6.629	6.660	6.692	6.723	6.754	6.786	6.817	6.849	6.880	
.2	6.912	6.943	6.974	7.006	7.037	7.069	7.100	7.131	7.163	7.194	
.3	7.226	7.257	7.288	7.320	7.351	7.383	7.414	7.446	7.477	7.508	
.4	7.540	7.571	7.603	7.634	7.665	7.697	7.728	7.760	7.791	7.823	
2.5	7.854	7.885	7.917	7.948	7.980	8.011	8.042	8.074	8.105	8.137	
.6	8.168	8.200	8.231	8.262	8.294	8.325	8.357	8.388	8.419	8.451	
.7	8.482	8.514	8.545	8.577	8.608	8.639	8.671	8.702	8.734	8.765	
.8	8.796	8.828	8.859	8.891	8.922	8.954	8.985	9.016	9.048	9.079	
.9	9.111	9.142	9.173	9.205	9.236	9.268	9.299	9.331	9.362	9.393	
3.0	9.425	9.456	9.488	9.519	9.550	9.582	9.613	9.645	9.676	9.708	
.1	9.739	9.770	9.802	9.833	9.865	9.896	9.927	9.959	9.990	10.022	31
1										10.02	3
.2	10.05	10.08	10.12	10.15	10.18	10.21	10.24	10.27	10.30	10.34	
.3	10.37	10.40	10.43	10.46	10.49	10.52	10.56	10.59	10.62	10.65	
.4	10.68	10.71	10.74	10.78	10.81	10.84	10.87	10.90	10.93	10.96	
3.5	11.00	11.03	11.06	11.09	11.12	11.15	11.18	11.22	11.25	11.28	
.6	11.31	11.34	11.37	11.40	11.44	11.47	11.50	11.53	11.56	11.59	
.7	11.62	11.66	11.69	11.72	11.75	11.78	11.81	11.84	11.88	11.91	
.8	11.94	11.97	12.00	12.03	12.06	12.10	12.13	12.16	12.19	12.22	
.9	12.25	12.28	12.32	12.35	12.38	12.41	12.44	12.47	12.50	12.53	
4.0	12.57	12.60	12.63	12.66	12.69	12.72	12.75	12.79	12.82	12.85	
.1	12.88	12.91	12.94	12.97	13.01	13.04	13.07	13.10	13.13	13.16	
.2	13.19	13.23	13.26	13.29	13.32	13.35	13.38	13.41	13.45	13.48	
.3	13.51	13.54	13.57	13.60	13.63	13.67	13.70	13.73	13.76	13.79	
.4	13.82	13.85	13.89	13.92	13.95	13.98	14.01	14.04	14.07	14.11	
4.5	14.14	14.17	14.20	14.23	14.26	14.29	14.33	14.36	14.39	14.42	
.6	14.45	14.48	14.51	14.55	14.58	14.61	14.64	14.67	14.70	14.73	
.7	14.77	14.80	14.83	14.86	14.89	14.92	14.95	14.99	15.02	15.05	
.8	15.08	15.11	15.14	15.17	15.21	15.24	15.27	15.30	15.33	15.36	
.9	15.39	15.43	15.46	15.49	15.52	15.55	15.58	15.61	15.65	15.68	

Explanation of Table of Circumferences (pp. 28–29)

This table gives the product of π times any number D from 1 to 10; that is, it is a table of multiples of π. (D = diameter.)

Moving the decimal point **one** place in column D is equivalent to moving it **one** place in the body of the table.

$$\text{Circumference} = \pi \times \text{diam.} = 3.141593 \times \text{diam.}$$

Conversely,

$$\text{Diameter} = \frac{1}{\pi} \times \text{circumf.} = 0.31831 \times \text{circumf.}$$

CIRCUMFERENCES BY HUNDREDTHS (*continued*)

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	15.71	15.74	15.77	15.80	15.83	15.87	15.90	15.93	15.96	15.99	3
.1	16.02	16.05	16.08	16.12	16.15	16.18	16.21	16.24	16.27	16.30	
.2	16.34	16.37	16.40	16.43	16.46	16.49	16.52	16.56	16.59	16.62	
.3	16.65	16.68	16.71	16.74	16.78	16.81	16.84	16.87	16.90	16.93	
.4	16.96	17.00	17.03	17.06	17.09	17.12	17.15	17.18	17.22	17.25	
5.5	17.28	17.31	17.34	17.37	17.40	17.44	17.47	17.50	17.53	17.56	
.6	17.59	17.62	17.66	17.69	17.72	17.75	17.78	17.81	17.84	17.88	
.7	17.91	17.94	17.97	18.00	18.03	18.06	18.10	18.13	18.16	18.19	
.8	18.22	18.25	18.28	18.32	18.35	18.38	18.41	18.44	18.47	18.50	
.9	18.54	18.57	18.60	18.63	18.66	18.69	18.72	18.76	18.79	18.82	
6.0	18.85	18.88	18.91	18.94	18.98	19.01	19.04	19.07	19.10	19.13	
.1	19.16	19.20	19.23	19.26	19.29	19.32	19.35	19.38	19.42	19.45	
.2	19.48	19.51	19.54	19.57	19.60	19.63	19.67	19.70	19.73	19.76	
.3	19.79	19.82	19.85	19.89	19.92	19.95	19.98	20.01	20.04	20.07	
.4	20.11	20.14	20.17	20.20	20.23	20.26	20.29	20.33	20.36	20.39	
6.5	20.42	20.45	20.48	20.51	20.55	20.58	20.61	20.64	20.67	20.70	
.6	20.73	20.77	20.80	20.83	20.86	20.89	20.92	20.95	20.99	21.02	
.7	21.05	21.08	21.11	21.14	21.17	21.21	21.24	21.27	21.30	21.33	
.8	21.36	21.39	21.43	21.46	21.49	21.52	21.55	21.58	21.61	21.65	
.9	21.68	21.71	21.74	21.77	21.80	21.83	21.87	21.90	21.93	21.96	
7.0	21.99	22.02	22.05	22.09	22.12	22.15	22.18	22.21	22.24	22.27	
.1	22.31	22.34	22.37	22.40	22.43	22.46	22.49	22.53	22.56	22.59	
.2	22.62	22.65	22.68	22.71	22.75	22.78	22.81	22.84	22.87	22.90	
.3	22.93	22.97	23.00	23.03	23.06	23.09	23.12	23.15	23.18	23.22	
.4	23.25	23.28	23.31	23.34	23.37	23.40	23.44	23.47	23.50	23.53	
7.5	23.56	23.59	23.62	23.66	23.69	23.72	23.75	23.78	23.81	23.84	
.6	23.88	23.91	23.94	23.97	24.00	24.03	24.06	24.10	24.13	24.16	
.7	24.19	24.22	24.25	24.28	24.32	24.35	24.38	24.41	24.44	24.47	
.8	24.50	24.54	24.57	24.60	24.63	24.66	24.69	24.72	24.76	24.79	
.9	24.82	24.85	24.88	24.91	24.94	24.98	25.01	25.04	25.07	25.10	
8.0	25.13	25.16	25.20	25.23	25.26	25.29	25.32	25.35	25.38	25.42	
.1	25.45	25.48	25.51	25.54	25.57	25.60	25.64	25.67	25.70	25.73	
.2	25.76	25.79	25.82	25.86	25.89	25.92	25.95	25.98	26.01	26.04	
.3	26.08	26.11	26.14	26.17	26.20	26.23	26.26	26.30	26.33	26.36	
.4	26.39	26.42	26.45	26.48	26.52	26.55	26.58	26.61	26.64	26.67	
8.5	26.70	26.73	26.77	26.80	26.83	26.86	26.89	26.92	26.95	26.99	
.6	27.02	27.05	27.08	27.11	27.14	27.17	27.21	27.24	27.27	27.30	
.7	27.33	27.36	27.39	27.43	27.46	27.49	27.52	27.55	27.58	27.61	
.8	27.65	27.68	27.71	27.74	27.77	27.80	27.83	27.87	27.90	27.93	
.9	27.96	27.99	28.02	28.05	28.09	28.12	28.15	28.18	28.21	28.24	
9.0	28.27	28.31	28.34	28.37	28.40	28.43	28.46	28.49	28.53	28.56	
.1	28.59	28.62	28.65	28.68	28.71	28.75	28.78	28.81	28.84	28.87	
.2	28.90	28.93	28.97	29.00	29.03	29.06	29.09	29.12	29.15	29.19	
.3	29.22	29.25	29.28	29.31	29.34	29.37	29.41	29.44	29.47	29.50	
.4	29.53	29.56	29.59	29.63	29.66	29.69	29.72	29.75	29.78	29.81	
9.5	29.85	29.88	29.91	29.94	29.97	30.00	30.03	30.07	30.10	30.13	
.6	30.16	30.19	30.22	30.25	30.28	30.32	30.35	30.38	30.41	30.44	
.7	30.47	30.50	30.54	30.57	30.60	30.63	30.66	30.69	30.72	30.76	
.8	30.79	30.82	30.85	30.88	30.91	30.94	30.98	31.01	31.04	31.07	
.9	31.10	31.13	31.16	31.20	31.23	31.26	31.29	31.32	31.35	31.38	
10.0	31.42										

Moving the decimal point ONE place in *D* requires moving it ONE place in body of table (see p. 28).

AREAS OF CIRCLES BY HUNDREDTHS

(For areas by eighths, see p. 32)

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	0.785	0.801	0.817	0.833	0.849	0.866	0.882	0.899	0.916	0.933	16
.1	0.950	0.968	0.985	1.003	1.021	1.039	1.057	1.075	1.094	1.112	18
.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307	20
.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517	21
.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744	23
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986	24
.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243	26
.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516	27
.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806	29
.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110	31
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431	32
.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767	34
.2	3.801	3.836	3.871	3.906	3.941	3.976	4.011	4.047	4.083	4.119	35
.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486	37
.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870	38
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269	40
.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683	42
.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114	43
.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560	45
.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022	46
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499	48
.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992	49
.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501	51
.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026	53
.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566	54
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.010			56
.5								10.01	10.07	10.12	6
.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69	6
.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28	
.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88	
.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50	
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14	7
.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79	
.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45	
.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14	
.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83	
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55	
.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28	
.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02	
.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78	8
.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56	

Explanation of Table of Areas of Circles (pp. 30–31)

Moving the decimal point **one** place in column D is equivalent to moving it **two** places in the body of the table. (D = diameter.)

$$\text{Area of circle} = \frac{\pi}{4} \times (\text{diam.}^2) = 0.785398 \times (\text{diam.}^2)$$

Conversely,

$$\text{Diam.} = \sqrt{\frac{4}{\pi}} \times \sqrt{\text{area}} = 1.128379 \times \sqrt{\text{area}}$$

AREAS OF CIRCLES BY HUNDREDTHS (*continued*)

D	0	1	2	3	4	5	6	7	8	9
5.0	19.63	19.71	19.79	19.87	19.95	20.03	20.11	20.19	20.27	20.35
.1	20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16
.2	21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98
.3	22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82
.4	22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67
5.5	23.76	23.84	23.93	24.02	24.11	24.19	24.28	24.37	24.45	24.54
.6	24.63	24.72	24.81	24.89	24.98	25.07	25.16	25.25	25.34	25.43
.7	25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33
.8	26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25
.9	27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.09	28.18
6.0	28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13
.1	29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09
.2	30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07
.3	31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.87	31.97	32.07
.4	32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08
6.5	33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11
.6	34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15
.7	35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21
.8	36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28
.9	37.39	37.50	37.61	37.72	37.83	37.94	38.05	38.16	38.26	38.37
7.0	38.48	38.59	38.70	38.82	38.93	39.04	39.15	39.26	39.37	39.48
.1	39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60
.2	40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74
.3	41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89
.4	43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06
7.5	44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25
.6	45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45
.7	46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66
.8	47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89
.9	49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14
8.0	50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40
.1	51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68
.2	52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98
.3	54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29
.4	55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61
8.5	56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95
.6	58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31
.7	59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.68
.8	60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07
.9	62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48
9.0	63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90
.1	65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33
.2	66.48	66.62	66.77	66.91	67.06	67.20	67.35	67.49	67.64	67.78
.3	67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25
.4	69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73
9.5	70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23
.6	72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75
.7	73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28
.8	75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82
.9	76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38

Moving the decimal point ONE place in *D* requires moving it TWO places in b of table (see p. 30).

CIRCUMFERENCES AND AREAS OF CIRCLES BY EIGHTHS, ETC.

(For tenths, see p. 28)

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
			7/8	2.749	.6013	4	12.57	12.57	9	28.27	63.62
1/64	.04909	.00019	57/64	2.798	.6230	1/16	12.76	12.96	1/8	28.67	65.40
1/32	.09817	.00077	29/32	2.847	.6450	1/8	12.96	13.36	1/4	29.06	67.20
3/64	.1473	.00173	59/64	2.896	.6675	3/16	13.16	13.77	3/8	29.45	69.03
1/16	.1963	.00307	15/16	2.945	.6903	1/4	13.35	14.19	1/2	29.85	70.88
5/64	.2454	.00479	61/64	2.994	.7135	5/16	13.55	14.61	5/8	30.24	72.76
3/32	.2945	.00690	31/32	3.043	.7371	3/8	13.74	15.03	3/4	30.63	74.66
7/64	.3436	.00940	63/64	3.093	.7610	7/16	13.94	15.47	7/8	31.02	76.59
1/8	.3927	.01227	1	3.142	.7854	1/2	14.14	15.90	10	31.42	78.54
9/64	.4418	.01553	1/16	3.338	.8866	9/16	14.33	16.35	1/8	31.81	80.52
5/32	.4909	.01917	1/8	3.534	.9940	5/8	14.53	16.80	1/4	32.20	82.52
11/64	.5400	.02320	3/16	3.731	1.108	11/16	14.73	17.26	3/8	32.59	84.54
3/16	.5890	.02761	1/4	3.927	1.227	3/4	14.92	17.72	1/2	32.99	86.59
13/64	.6381	.03241	5/16	4.123	1.353	13/16	15.12	18.19	5/8	33.38	88.66
7/32	.6872	.03758	3/8	4.320	1.485	7/8	15.32	18.67	3/4	33.77	90.76
15/64	.7363	.04314	7/16	4.516	1.623	15/16	15.51	19.15	7/8	34.16	92.89
1/4	.7854	.04909	1/2	4.712	1.767	5	15.71	19.63	11	34.56	95.03
17/64	.8345	.05542	9/16	4.909	1.917	1/16	15.90	20.13	1/8	34.95	97.21
9/32	.8836	.06213	5/8	5.105	2.074	1/8	16.10	20.63	1/4	35.34	99.40
19/64	.9327	.06922	11/16	5.301	2.237	3/16	16.30	21.14	3/8	35.74	101.6
5/16	.9817	.07670	3/4	5.498	2.405	1/4	16.49	21.65	1/2	36.13	103.9
21/64	1.031	.08456	13/16	5.694	2.580	5/16	16.69	22.17	5/8	36.52	106.1
11/32	1.080	.09281	7/8	5.890	2.761	3/8	16.89	22.69	3/4	36.91	108.4
23/64	1.129	.1014	15/16	6.087	2.948	7/16	17.08	23.22	7/8	37.31	110.8
3/8	1.178	.1104	2	6.283	3.142	1/2	17.28	23.76	12	37.70	113.1
25/64	1.227	.1198	1/16	6.480	3.341	9/16	17.48	24.30	1/8	38.09	115.5
13/32	1.276	.1296	1/8	6.676	3.547	5/8	17.67	24.85	1/4	38.48	117.9
27/64	1.325	.1398	3/16	6.872	3.758	11/16	17.87	25.41	3/8	38.88	120.3
7/16	1.374	.1503	1/4	7.069	3.976	3/4	18.06	25.97	1/2	39.27	122.7
29/64	1.424	.1613	5/16	7.265	4.200	13/16	18.26	26.53	5/8	39.66	125.2
15/32	1.473	.1726	3/8	7.461	4.430	7/8	18.46	27.11	3/4	40.06	127.7
31/64	1.522	.1843	7/16	7.658	4.666	15/16	18.65	27.69	7/8	40.45	130.2
1/2	1.571	.1963	1/2	7.854	4.909	6	18.85	28.27	13	40.84	132.7
33/64	1.620	.2088	9/16	8.050	5.157	1/8	19.24	29.46	1/8	41.23	135.3
17/32	1.669	.2217	5/8	8.247	5.412	1/4	19.63	30.68	1/4	41.63	137.9
35/64	1.718	.2349	11/16	8.443	5.673	3/8	20.03	31.92	3/8	42.02	140.5
9/16	1.767	.2485	3/4	8.639	5.940	1/2	20.42	33.18	1/2	42.41	143.1
37/64	1.816	.2625	13/16	8.836	6.213	5/8	20.81	34.47	5/8	42.80	145.8
19/32	1.865	.2769	7/8	9.032	6.492	3/4	21.21	35.78	3/4	43.20	148.5
39/64	1.914	.2916	15/16	9.228	6.777	7/8	21.60	37.12	7/8	43.59	151.2
5/8	1.963	.3068	3	9.425	7.069	7	21.99	38.48	14	43.98	153.9
41/64	2.013	.3223	1/16	9.621	7.366	1/8	22.38	39.87	1/8	44.37	156.7
21/32	2.062	.3382	1/8	9.817	7.670	1/4	22.78	41.28	1/4	44.77	159.5
43/64	2.111	.3545	3/16	10.01	7.980	3/8	23.17	42.72	3/8	45.16	162.3
11/16	2.160	.3712	1/4	10.21	8.296	1/2	23.56	44.18	1/2	45.55	165.1
45/64	2.209	.3883	5/16	10.41	8.618	5/8	23.95	45.66	5/8	45.95	168.0
23/32	2.258	.4057	3/8	10.60	8.946	3/4	24.35	47.17	3/4	46.34	170.9
47/64	2.307	.4236	7/16	10.80	9.281	7/8	24.74	48.71	7/8	46.73	173.8
3/4	2.356	.4418	1/2	11.00	9.621	8	25.13	50.27	15	47.12	176.7
49/64	2.405	.4604	9/16	11.19	9.968	1/8	25.53	51.85	1/8	47.52	179.7
25/32	2.454	.4794	5/8	11.39	10.32	1/4	25.92	53.46	1/4	47.91	182.7
51/64	2.503	.4987	11/16	11.58	10.68	3/8	26.31	55.09	3/8	48.30	185.7
13/16	2.553	.5185	3/4	11.78	11.04	1/2	26.70	56.75	1/2	48.69	188.7
53/64	2.602	.5386	13/16	11.98	11.42	5/8	27.10	58.43	5/8	49.09	191.7
27/32	2.651	.5591	7/8	12.17	11.79	3/4	27.49	60.13	3/4	49.48	194.8
55/64	2.700	.5800	15/16	12.37	12.18	7/8	27.88	61.86	7/8	49.87	197.9

CIRCUMFERENCES AND AREAS BY EIGHTHS—(*continued*)

Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area	Diam.	Circum.	Area
16	50.27	201.1	19 ½	61.26	298.6	23	72.26	415.5	29	91.11	660.5
⅛	50.66	204.2	⅝	61.65	302.5	⅛	72.65	420.0	¼	91.89	672.0
¼	51.05	207.4	¾	62.05	306.4	¼	73.04	424.6	½	92.68	683.5
⅜	51.44	210.6	⅞	62.44	310.2	⅜	73.43	429.1	¾	93.46	695.1
½	51.84	213.8	20	62.83	314.2	½	73.83	433.7	30	94.25	706.9
⅝	52.23	217.1	⅛	63.22	318.1	⅝	74.22	438.4	¼	95.03	718.7
¾	52.62	220.4	¼	63.62	322.1	¾	74.61	443.0	½	95.82	730.6
⅞	53.01	223.7	⅜	64.01	326.1	⅞	75.01	447.7	¾	96.60	742.6
17	53.41	227.0	½	64.40	330.1	24	75.40	452.4	31	97.39	754.8
⅛	53.80	230.3	⅝	64.80	334.1	¼	76.18	461.9	¼	98.17	767.0
¼	54.19	233.7	¾	65.19	338.2	½	76.97	471.4	½	98.96	779.3
⅜	54.59	237.1	⅞	65.58	342.2	¾	77.75	481.1	¾	99.75	791.7
½	54.98	240.5	21	65.97	346.4	25	78.54	490.9	32	100.5	804.2
⅝	55.37	244.0	⅛	66.37	350.5	¼	79.33	500.7	¼	101.3	816.9
¾	55.76	247.4	¼	66.76	354.7	½	80.11	510.7	½	102.1	829.6
⅞	56.16	250.9	⅜	67.15	358.8	¾	80.90	520.8	¾	102.9	842.4
18	56.55	254.5	½	67.54	363.1	26	81.68	530.9	33	103.7	855.3
⅛	56.94	258.0	⅝	67.94	367.3	¼	82.47	541.2	¼	104.5	868.3
¼	57.33	261.6	¾	68.33	371.5	½	83.25	551.5	½	105.2	881.4
⅜	57.73	265.2	⅞	68.72	375.8	¾	84.04	562.0	¾	106.0	894.6
½	58.12	268.8	22	69.12	380.1	27	84.82	572.6	34	106.8	907.9
⅝	58.51	272.4	⅛	69.51	384.5	¼	85.61	583.2	¼	107.6	921.3
¾	58.90	276.1	¼	69.90	388.8	½	86.39	594.0	½	108.4	934.8
⅞	59.30	279.8	⅜	70.29	393.2	¾	87.18	604.8	¾	109.2	948.4
19	59.69	283.5	½	70.69	397.6	28	87.96	615.8	35	110.0	962.1
⅛	60.08	287.3	⅝	71.08	402.0	¼	88.75	626.8	¼	110.7	975.9
¼	60.48	291.0	¾	71.47	406.5	½	89.54	637.9	½	111.5	989.8
⅜	60.87	294.8	⅞	71.86	411.0	¾	90.32	649.2	¾	112.3	1003.8

AREAS OF CIRCLES. Diameters in Feet and Inches, Areas in Square Feet

Feet	Inches											
	0	1	2	3	4	5	6	7	8	9	10	11
0	.0000	.0055	.0218	.0491	.0873	.1364	.1963	.2673	.3491	.4418	.5454	.6600
1	.7854	.9218	1.069	1.227	1.396	1.576	1.767	1.969	2.182	2.405	2.640	2.885
2	3.142	3.409	3.687	3.976	4.276	4.587	4.909	5.241	5.585	5.940	6.305	6.681
3	7.069	7.467	7.876	8.296	8.727	9.168	9.621	10.08	10.56	11.04	11.54	12.05
4	12.57	13.10	13.64	14.19	14.75	15.32	15.90	16.50	17.10	17.72	18.35	18.99
5	19.63	20.29	20.97	21.65	22.34	23.04	23.76	24.48	25.22	25.97	26.73	27.49
6	28.27	29.07	29.87	30.68	31.50	32.34	33.18	34.04	34.91	35.78	36.67	37.57
7	38.48	39.41	40.34	41.28	42.24	43.20	44.18	45.17	46.16	47.17	48.19	49.22
8	50.27	51.32	52.38	53.46	54.54	55.64	56.75	57.86	58.99	60.13	61.28	62.44
9	63.62	64.80	66.00	67.20	68.42	69.64	70.88	72.13	73.39	74.66	75.94	77.24
10	78.54	79.85	81.18	82.52	83.86	85.22	86.59	87.97	89.36	90.76	92.18	93.60
11	95.03	96.48	97.93	99.40	100.9	102.4	103.9	105.4	106.9	108.4	110.0	111.5
12	113.1	114.7	116.3	117.9	119.5	121.1	122.7	124.4	126.0	127.7	129.4	131.0
13	132.7	134.4	136.2	137.9	139.6	141.4	143.1	144.9	146.7	148.5	150.3	152.1
14	153.9	155.8	157.6	159.5	161.4	163.2	165.1	167.0	168.9	170.9	172.8	174.8

If given diameter is not found in this table, reduce diameter to feet and decimals of a foot by aid of the following auxiliary table, and then find area from pp. 30–31.

From Inches and Fractions of an Inch to Decimals of a Foot

Inches	1	2	3	4	5	6	7	8	9	10	11
Feet	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167
Inches	⅛	¼	⅜	½	⅝	¾	⅞				
Feet	.0104	.0208	.0313	.0417	.0521	.0625	.0729				

Example. 5 ft. 7⅜ in. = 5.0 + 0.5833 + 0.0313 = 5.6146 ft.

SEGMENTS OF CIRCLES, GIVEN h/c

Given: h = height; c = chord. (For explanation of this table, see p. 38)

$\frac{h}{c}$	$\frac{\text{Diam.}}{c}$	Diff.	$\frac{\text{Arc}}{c}$	Diff.	$\frac{\text{Area}}{h \times c}$	Diff.	Central angle, v	Diff.	$\frac{h}{\text{Diam.}}$	Diff.
.00			1.000	0	.6667	0	0.00°	458	.0000	4
1	25.010	12490	1.000	1	.6667	2	4.58	458	.0004	12
2	12.520	*4157	1.001	1	.6669	2	9.16	457	.0016	20
3	8.363	*2073	1.002	2	.6671	4	13.73	457	.0036	28
4	6.290	*1240	1.004	3	.6675	5	18.30	454	.0064	35
.05	5.050	*823	1.007	3	.6680	6	22.84°	453	.0099	43
6	4.227	*586	1.010	3	.6686	7	27.37	451	.0142	50
7	3.641	*436	1.013	4	.6693	8	31.88	448	.0192	58
8	3.205	*337	1.017	4	.6701	9	36.36	446	.0250	64
9	2.868	*268	1.021	5	.6710	10	40.82	442	.0314	71
.10	2.600	*217	1.026	6	.6720	11	45.24°	439	.0385	77
1	2.383	*180	1.032	6	.6731	12	49.63	435	.0462	83
2	2.203	*150	1.038	6	.6743	13	53.98	432	.0545	88
3	2.053	*127	1.044	7	.6756	14	58.30	427	.0633	94
4	1.926	*109	1.051	8	.6770	15	62.57	423	.0727	99
.15	1.817	*94	1.059	8	.6785	16	66.80°	418	.0826	103
6	1.723	*82	1.067	8	.6801	17	70.98	413	.0929	107
7	1.641	*72	1.075	9	.6818	18	75.11	409	.1036	111
8	1.569	*63	1.084	10	.6836	19	79.20	403	.1147	115
9	1.506	56	1.094	9	.6855	20	83.23	398	.1262	117
.20	1.450	50	1.103	11	.6875	21	87.21°	392	.1379	120
1	1.400	44	1.114	10	.6896	22	91.13	387	.1499	123
2	1.356	39	1.124	12	.6918	23	95.00	381	.1622	124
3	1.317	35	1.136	11	.6941	24	98.81	375	.1746	127
4	1.282	32	1.147	12	.6965	24	102.56	370	.1873	127
.25	1.250	28	1.159	12	.6989	25	106.26°	364	.2000	128
6	1.222	26	1.171	13	.7014	27	109.90	358	.2128	130
7	1.196	23	1.184	13	.7041	27	113.48	352	.2258	129
8	1.173	21	1.197	14	.7068	28	117.00	345	.2387	130
9	1.152	19	1.211	14	.7096	29	120.45	341	.2517	130
.30	1.133	17	1.225	14	.7125	29	123.86°	334	.2647	130
1	1.116	15	1.239	15	.7154	31	127.20	328	.2777	129
2	1.101	13	1.254	15	.7185	31	130.48	322	.2906	128
3	1.088	13	1.269	15	.7216	32	133.70	316	.3034	128
4	1.075	11	1.284	16	.7248	32	136.86	311	.3162	127
.35	1.064	10	1.300	16	.7280	34	139.97°	305	.3289	125
6	1.054	8	1.316	16	.7314	34	143.02	299	.3414	124
7	1.046	8	1.332	17	.7348	35	146.01	293	.3538	123
8	1.038	7	1.349	17	.7383	36	148.94	288	.3661	122
9	1.031	6	1.366	17	.7419	36	151.82	282	.3783	119
.40	1.025	5	1.383	18	.7455	37	154.64°	277	.3902	119
1	1.020	5	1.401	18	.7492	38	157.41	271	.4021	116
2	1.015	4	1.419	18	.7530	38	160.12	266	.4137	115
3	1.011	3	1.437	18	.7568	39	162.78	261	.4252	112
4	1.008	2	1.455	19	.7607	40	165.39	256	.4364	111
.45	1.006	3	1.474	19	.7647	40	167.95°	251	.4475	109
6	1.003	1	1.493	19	.7687	41	170.46	245	.4584	107
7	1.002	1	1.512	19	.7728	41	172.91	241	.4691	105
8	1.001	1	1.531	20	.7769	42	175.32	237	.4796	103
9	1.000	0	1.551	20	.7811	43	177.69	231	.4899	101
.50	1.000		1.571		.7854		180.00°		.5000	

* Interpolation may be inaccurate at these points.

SEGMENTS OF CIRCLES, GIVEN h/D

Given: h = height; D = diameter of circle. (For explanation of this table, see p. 38)

$\frac{h}{D}$	$\frac{\text{Arc}}{D}$	Diff.	$\frac{\text{Area}}{D^2}$	Diff.	Central angle, v	Diff.	$\frac{\text{Chord}}{D}$	Diff.	$\frac{\text{Arc}}{\text{Circumf.}}$	Diff.	$\frac{\text{Area}}{\text{Circle}}$	Diff.
.00	0.0000	2003	.0000	13	0.00°	2296	.0000	*1990	.0000	*638	.0000	17
1	.2003	*835	.0013	24	22.96	*956	.1990	*810	.0638	*265	.0017	31
2	.2838	*644	.0037	32	32.52	*738	.2800	*612	.0903	*205	.0048	39
3	.3482	*545	.0069	36	39.90	*625	.3412	*507	.1108	*174	.0087	47
4	.4027	*483	.0105	42	46.15	*553	.3919	*440	.1282	*154	.0134	53
.05	.4510	*439	.0147	45	51.68°	*504	.4359	*391	.1436	*139	.0187	58
6	.4949	*406	.0192	50	56.72	*465	.4750	*353	.1575	*130	.0245	63
7	.5355	*380	.0242	52	61.37	*435	.5103	*323	.1705	121	.0308	67
8	.5735	*359	.0294	56	65.72	*411	.5426	*298	.1826	114	.0375	71
9	.6094	*341	.0350	59	69.83	*391	.5724	*276	.1940	108	.0446	74
.10	.6435	*326	.0409	61	73.74°	*374	.6000	*258	.2048	104	.0520	79
1	.6761	*314	.0470	64	77.48	*359	.6258	*241	.2152	100	.0599	81
2	.7075	*302	.0534	66	81.07	*347	.6499	*227	.2252	96	.0680	84
3	.7377	*293	.0600	68	84.54	*335	.6726	*214	.2348	93	.0764	87
4	.7670	*284	.0668	71	87.89	*326	.6940	*201	.2441	91	.0851	90
.15	.7954	276	.0739	72	91.15°	316	.7141	*191	.2532	88	.0941	92
6	.8230	270	.0811	74	94.31	309	.7332	*181	.2620	86	.1033	94
7	.8500	263	.0885	76	97.40	302	.7513	*171	.2706	83	.1127	97
8	.8763	258	.0961	78	100.42	295	.7684	162	.2789	82	.1224	99
9	.9021	252	.1039	79	103.37	289	.7846	154	.2871	81	.1323	101
.20	0.9273	248	.1118	81	106.26°	284	.8000	146	.2952	79	.1424	103
1	0.9521	243	.1199	82	109.10	279	.8146	139	.3031	77	.1527	104
2	0.9764	240	.1281	84	111.89	274	.8285	132	.3108	76	.1631	106
3	1.0004	235	.1365	84	114.63	271	.8417	125	.3184	75	.1737	109
4	1.0239	233	.1449	86	117.34	266	.8542	118	.3259	74	.1846	109
.25	1.0472	229	.1535	88	120.00°	263	.8660	113	.3333	73	.1955	111
6	1.0701	227	.1623	88	122.63	260	.8773	106	.3406	72	.2066	112
7	1.0928	224	.1711	89	125.23	256	.8879	101	.3478	72	.2178	114
8	1.1152	222	.1800	90	127.79	254	.8980	95	.3550	70	.2292	115
9	1.1374	219	.1890	92	130.33	251	.9075	90	.3620	70	.2407	116
.30	1.1593	217	.1982	92	132.84°	249	.9165	85	.3690	69	.2523	117
1	1.1810	215	.2074	93	135.33	247	.9250	80	.3759	69	.2640	119
2	1.2025	214	.2167	93	137.80	245	.9330	74	.3828	68	.2759	119
3	1.2239	212	.2260	95	140.25	242	.9404	70	.3896	67	.2878	120
4	1.2451	210	.2355	95	142.67	241	.9474	65	.3963	67	.2998	121
.35	1.2661	209	.2450	96	145.08°	240	.9539	61	.4030	67	.3119	122
6	1.2870	208	.2546	96	147.48	238	.9600	56	.4097	66	.3241	123
7	1.3078	206	.2642	97	149.86	237	.9656	52	.4163	66	.3364	123
8	1.3284	206	.2739	97	152.23	235	.9708	47	.4229	65	.3487	124
9	1.3490	204	.2836	98	154.58	235	.9755	43	.4294	65	.3611	124
.40	1.3694	204	.2934	98	156.93°	233	.9798	39	.4359	65	.3735	125
1	1.3898	203	.3032	98	159.26	233	.9837	34	.4424	65	.3860	126
2	1.4101	202	.3130	99	161.59	231	.9871	31	.4489	64	.3986	126
3	1.4303	202	.3229	99	163.90	232	.9902	26	.4553	64	.4112	126
4	1.4505	201	.3328	100	166.22	230	.9928	22	.4617	64	.4238	126
.45	1.4706	201	.3428	99	168.52°	230	.9950	18	.4681	64	.4364	127
6	1.4907	201	.3527	100	170.82	230	.9968	14	.4745	64	.4491	127
7	1.5108	200	.3627	100	173.12	230	.9982	10	.4809	64	.4618	127
8	1.5308	200	.3727	100	175.42	229	.9992	6	.4873	63	.4745	128
9	1.5508	200	.3827	100	177.71	229	.9998	2	.4936	64	.4873	127
.50	1.5708		.3927		180.00°		1.0000		.5000		.5000	

* Interpolation may be inaccurate at these points.

VOLUMES OF SPHERES BY HUNDREDTHS

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	.5236	.5395	.5556	.5722	.5890	.6061	.6236	.6414	.6596	.6781	173
.1	.6969	.7161	.7356	.7555	.7757	.7963	.8173	.8386	.8603	.8823	208
.2	.9048	.9276	.9508	.9743	.9983	1.0227					236
.2						1.023	1.047	1.073	1.098	1.124	25
.3	1.150	1.177	1.204	1.232	1.260	1.288	1.317	1.346	1.376	1.406	29
.4	1.437	1.468	1.499	1.531	1.563	1.596	1.630	1.663	1.697	1.732	33
1.5	1.767	1.803	1.839	1.875	1.912	1.950	1.988	2.026	2.065	2.105	38
.6	2.145	2.185	2.226	2.268	2.310	2.352	2.395	2.439	2.483	2.527	43
.7	2.572	2.618	2.664	2.711	2.758	2.806	2.855	2.903	2.953	3.003	48
.8	3.054	3.105	3.157	3.209	3.262	3.315	3.369	3.424	3.479	3.535	54
.9	3.591	3.648	3.706	3.764	3.823	3.882	3.942	4.003	4.064	4.126	60
2.0	4.189	4.252	4.316	4.380	4.445	4.511	4.577	4.644	4.712	4.780	66
.1	4.849	4.919	4.989	5.060	5.131	5.204	5.277	5.350	5.425	5.500	73
.2	5.575	5.652	5.729	5.806	5.885	5.964	6.044	6.125	6.206	6.288	80
.3	6.371	6.454	6.538	6.623	6.709	6.795	6.882	6.970	7.059	7.148	87
.4	7.238	7.329	7.421	7.513	7.606	7.700	7.795	7.890	7.986	8.083	94
2.5	8.181	8.280	8.379	8.479	8.580	8.682	8.785	8.888	8.992	9.097	102
.6	9.203	9.309	9.417	9.525	9.634	9.744	9.855	9.966	10.079		110
.6									10.08	10.19	11
.7	10.31	10.42	10.54	10.65	10.77	10.89	11.01	11.13	11.25	11.37	12
.8	11.49	11.62	11.74	11.87	11.99	12.12	12.25	12.38	12.51	12.64	13
.9	12.77	12.90	13.04	13.17	13.31	13.44	13.58	13.72	13.86	14.00	14
3.0	14.14	14.28	14.42	14.57	14.71	14.86	15.00	15.15	15.30	15.45	15
.1	15.60	15.75	15.90	16.06	16.21	16.37	16.52	16.68	16.84	17.00	16
.2	17.16	17.32	17.48	17.64	17.81	17.97	18.14	18.31	18.48	18.65	17
.3	18.82	18.99	19.16	19.33	19.51	19.68	19.86	20.04	20.22	20.40	18
.4	20.58	20.76	20.94	21.13	21.31	21.50	21.69	21.88	22.07	22.26	19
3.5	22.45	22.64	22.84	23.03	23.23	23.43	23.62	23.82	24.02	24.23	20
.6	24.43	24.63	24.84	25.04	25.25	25.46	25.67	25.88	26.09	26.31	21
.7	26.52	26.74	26.95	27.17	27.39	27.61	27.83	28.06	28.28	28.50	22
.8	28.73	28.96	29.19	29.42	29.65	29.88	30.11	30.35	30.58	30.82	23
.9	31.06	31.30	31.54	31.78	32.02	32.27	32.52	32.76	33.01	33.26	25
4.0	33.51	33.76	34.02	34.27	34.53	34.78	35.04	35.30	35.56	35.82	26
.1	36.09	36.35	36.62	36.88	37.15	37.42	37.69	37.97	38.24	38.52	27
.2	38.79	39.07	39.35	39.63	39.91	40.19	40.48	40.76	41.05	41.34	28
.3	41.63	41.92	42.21	42.51	42.80	43.10	43.40	43.70	44.00	44.30	30
.4	44.60	44.91	45.21	45.52	45.83	46.14	46.45	46.77	47.08	47.40	31
4.5	47.71	48.03	48.35	48.67	49.00	49.32	49.65	49.97	50.30	50.63	33
.6	50.97	51.30	51.63	51.97	52.31	52.65	52.99	53.33	53.67	54.02	34
.7	54.36	54.71	55.06	55.41	55.76	56.12	56.47	56.83	57.19	57.54	35
.8	57.91	58.27	58.63	59.00	59.37	59.73	60.10	60.48	60.85	61.22	37
.9	61.60	61.98	62.36	62.74	63.12	63.51	63.89	64.28	64.67	65.06	38

Explanation of Table of Volumes of Spheres (pp. 36–37).

Moving the decimal point **one** place in column *D* is equivalent to moving it **three** places in the body of the table. (*D* = diameter.)

$$\text{Volume of sphere} = \frac{\pi}{6} \times (\text{diam.}^3) = 0.523599 \times (\text{diam.}^3)$$

Conversely,

$$\text{Diam.} = \sqrt[3]{\frac{6}{\pi}} \times \sqrt[3]{\text{volume}} = 1.240701 \times \sqrt[3]{\text{volume}}$$

VOLUMES OF SPHERES *(continued)*

D	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	65.45	65.84	66.24	66.64	67.03	67.43	67.83	68.24	68.64	69.05	40
.1	69.46	69.87	70.28	70.69	71.10	71.52	71.94	72.36	72.78	73.20	42
.2	73.62	74.05	74.47	74.90	75.33	75.77	76.20	76.64	77.07	77.51	43
.3	77.95	78.39	78.84	79.28	79.73	80.18	80.63	81.08	81.54	81.99	45
.4	82.45	82.91	83.37	83.83	84.29	84.76	85.23	85.70	86.17	86.64	47
5.5	87.11	87.59	88.07	88.55	89.03	89.51	90.00	90.48	90.97	91.46	48
.6	91.95	92.45	92.94	93.44	93.94	94.44	94.94	95.44	95.95	96.46	50
.7	96.97	97.48	97.99	98.51	99.02	99.54	100.06				52
.7							100.1	100.6	101.1	101.6	5
.8	102.2	102.7	103.2	103.8	104.3	104.8	105.4	105.9	106.4	107.0	5
.9	107.5	108.1	108.6	109.2	109.7	110.3	110.9	111.4	112.0	112.5	6
6.0	113.1	113.7	114.2	114.8	115.4	115.9	116.5	117.1	117.7	118.3	6
.1	118.8	119.4	120.0	120.6	121.2	121.8	122.4	123.0	123.6	124.2	
.2	124.8	125.4	126.0	126.6	127.2	127.8	128.4	129.1	129.7	130.3	
.3	130.9	131.5	132.2	132.8	133.4	134.1	134.7	135.3	136.0	136.6	
.4	137.3	137.9	138.5	139.2	139.8	140.5	141.2	141.8	142.5	143.1	7
6.5	143.8	144.5	145.1	145.8	146.5	147.1	147.8	148.5	149.2	149.8	
.6	150.5	151.2	151.9	152.6	153.3	154.0	154.7	155.4	156.1	156.8	
.7	157.5	158.2	158.9	159.6	160.3	161.0	161.7	162.5	163.2	163.9	
.8	164.6	165.4	166.1	166.8	167.6	168.3	169.0	169.8	170.5	171.3	
.9	172.0	172.8	173.5	174.3	175.0	175.8	176.5	177.3	178.1	178.8	8
7.0	179.6	180.4	181.1	181.9	182.7	183.5	184.3	185.0	185.8	186.6	
.1	187.4	188.2	189.0	189.8	190.6	191.4	192.2	193.0	193.8	194.6	
.2	195.4	196.2	197.1	197.9	198.7	199.5	200.4	201.2	202.0	202.9	
.3	203.7	204.5	205.4	206.2	207.1	207.9	208.8	209.6	210.5	211.3	
.4	212.2	213.0	213.9	214.8	215.6	216.5	217.4	218.3	219.1	220.0	9
7.5	220.9	221.8	222.7	223.6	224.4	225.3	226.2	227.1	228.0	228.9	
.6	229.8	230.8	231.7	232.6	233.5	234.4	235.3	236.3	237.2	238.1	
.7	239.0	240.0	240.9	241.8	242.8	243.7	244.7	245.6	246.6	247.5	
.8	248.5	249.4	250.4	251.4	252.3	253.3	254.3	255.2	256.2	257.2	10
.9	258.2	259.1	260.1	261.1	262.1	263.1	264.1	265.1	266.1	267.1	
8.0	268.1	269.1	270.1	271.1	272.1	273.1	274.2	275.2	276.2	277.2	
.1	278.3	279.3	280.3	281.4	282.4	283.4	284.5	285.5	286.6	287.6	
.2	288.7	289.8	290.8	291.9	292.9	294.0	295.1	296.2	297.2	298.3	11
.3	299.4	300.5	301.6	302.6	303.7	304.8	305.9	307.0	308.1	309.2	
.4	310.3	311.4	312.6	313.7	314.8	315.9	317.0	318.2	319.3	320.4	
8.5	321.6	322.7	323.8	325.0	326.1	327.3	328.4	329.6	330.7	331.9	
.6	333.0	334.2	335.4	336.5	337.7	338.9	340.1	341.2	342.4	343.6	12
.7	344.8	346.0	347.2	348.4	349.6	350.8	352.0	353.2	354.4	355.6	
.8	356.8	358.0	359.3	360.5	361.7	362.9	364.2	365.4	366.6	367.9	
.9	369.1	370.4	371.6	372.9	374.1	375.4	376.6	377.9	379.2	380.4	13
9.0	381.7	383.0	384.3	385.5	386.8	388.1	389.4	390.7	392.0	393.3	
.1	394.6	395.9	397.2	398.5	399.8	401.1	402.4	403.7	405.1	406.4	
.2	407.7	409.1	410.4	411.7	413.1	414.4	415.7	417.1	418.4	419.8	
.3	421.2	422.5	423.9	425.2	426.6	428.0	429.4	430.7	432.1	433.5	14
.4	434.9	436.3	437.7	439.1	440.5	441.9	443.3	444.7	446.1	447.5	
9.5	448.9	450.3	451.8	453.2	454.6	456.0	457.5	458.9	460.4	461.8	
.6	463.2	464.7	466.1	467.6	469.1	470.5	472.0	473.5	474.9	476.4	15
.7	477.9	479.4	480.8	482.3	483.8	485.3	486.8	488.3	489.8	491.3	
.8	492.8	494.3	495.8	497.3	498.9	500.4	501.9	503.4	505.0	506.5	16
.9	508.0	509.6	511.1	512.7	514.2	515.8	517.3	518.9	520.5	522.0	
10.0	523.6										

Moving the decimal point ONE place in *D* requires moving it THREE places in body of table (see p. 36).

SEGMENTS OF SPHERES

(h = height of segment; D = diam. of sphere)

$\frac{h}{D}$	$\frac{\text{Vol. segm.}}{D^3}$	Diff.	$\frac{\text{Vol. segm.}}{\text{Vol. sphere}}$	Diff.
0.00	0.0000	2	0.0000	3
1	0.0002	4	0.0003	9
2	0.0006	8	0.0012	14
3	0.0014	10	0.0026	21
4	0.0024	14	0.0047	26
0.05	0.0038	16	0.0073	31
6	0.0054	19	0.0104	36
7	0.0073	22	0.0140	42
8	0.0095	25	0.0182	46
9	0.0120	27	0.0228	52
0.10	0.0147	29	0.0280	56
1	0.0176	32	0.0336	61
2	0.0208	34	0.0397	66
3	0.0242	37	0.0463	70
4	0.0279	39	0.0533	74
0.15	0.0318	41	0.0607	79
6	0.0359	44	0.0686	83
7	0.0403	45	0.0769	86
8	0.0448	47	0.0855	91
9	0.0495	50	0.0946	94
0.20	0.0545	51	0.1040	98
1	0.0596	53	0.1138	101
2	0.0649	55	0.1239	105
3	0.0704	56	0.1344	108
4	0.0760	58	0.1452	110
0.25	0.0818	60	0.1562	114
6	0.0878	61	0.1676	117
7	0.0939	63	0.1793	120
8	0.1002	64	0.1913	122
9	0.1066	65	0.2035	125
0.30	0.1131	67	0.2160	127
1	0.1198	67	0.2287	130
2	0.1265	69	0.2417	131
3	0.1334	70	0.2548	134
4	0.1404	71	0.2682	135
0.35	0.1475	72	0.2817	138
6	0.1547	73	0.2955	139
7	0.1620	74	0.3094	141
8	0.1694	74	0.3235	142
9	0.1768	75	0.3377	143
0.40	0.1843	76	0.3520	145
1	0.1919	76	0.3665	145
2	0.1995	77	0.3810	147
3	0.2072	77	0.3957	147
4	0.2149	78	0.4104	148
0.45	0.2227	78	0.4252	149
6	0.2305	78	0.4401	150
7	0.2383	78	0.4551	149
8	0.2461	78	0.4700	150
9	0.2539	79	0.4850	150
0.50	0.2618		0.5000	

Explanation of Table on this page

Given, h = height of segment,
D = diam. of sphere.

To find the volume of the segment, form the ratio h/D and find from the table the value of (vol./D^3); then, by a simple multiplication,

vol. segment = $D^3 \times$ (vol./D^3)

The table gives also the ratio of the volume of the segment to the entire volume of the sphere.

NOTE. Area of zone = $\pi \times h \times D$. (Use Table of Multiples of π, p. 28)

Explanation of Table on p. 34

Given, h = height of segment,
c = chord.

To find the diam. of the circle, the length of arc, or the area of the segment, form the ratio h/c, and find from the table the value of (diam./c), (arc/c), or (area/hc); then, by a simple multiplication,

diam. = $c \times$ (diam./c),
arc = $c \times$ (arc/c),
area = $h \times c \times$ (area/hc).

The table gives also the angle subtended at the center, and the ratio of h to D. See p. 106.

Explanation of Table on p. 35

Given, h = height of segment,
D = diam. of circle.

To find the chord, the length of arc, or the area of the segment, form the ratio h/D, and find from the table the value of (chord/D), (arc/D), or (area/D^2); then, by a simple multiplication,

chord = $D \times$ (chord/D),
arc = $D \times$ (arc/D),
area = $D^2 \times$ (area/D^2).

The table gives also the angle subtended at the center, the ratio of the arc of the segment to the whole circumference, and the ratio of the area of the segment to the area of the whole circle. See p. 106.

NOTE. Vol. segm. = $\frac{1}{6}\pi h^2 (3D-2h)$.

REGULAR POLYGONS

n = number of sides;
v = $360°/n$ = angle subtended at the center by one side;

a = length of one side $= R\left(2 \sin \frac{v}{2}\right) = r\left(2 \tan \frac{v}{2}\right)$;

R = radius of circumscribed circle $= a\left(\frac{1}{2} \csc \frac{v}{2}\right) = r\left(\sec \frac{v}{2}\right)$;

r = radius of inscribed circle $= R\left(\cos \frac{v}{2}\right) = a\left(\frac{1}{2} \cot \frac{v}{2}\right)$;

$$\text{Area} = a^2\left(\tfrac{1}{4}\, n \cot \frac{v}{2}\right) = R^2\left(\tfrac{1}{2}\, n \sin v\right) = r^2\left(n \tan \frac{v}{2}\right).$$

n	v	$\frac{\text{Area}}{a^2}$	$\frac{\text{Area}}{R^2}$	$\frac{\text{Area}}{r^2}$	$\frac{R}{a}$	$\frac{R}{r}$	$\frac{a}{R}$	$\frac{a}{r}$	$\frac{r}{R}$	$\frac{r}{a}$
3	120°	0.4330	1.299	5.196	0.5774	2.000	1.732	3.464	0.5000	0.2887
4	90°	1.000	2.000	4.000	0.7071	1.414	1.414	2.000	0.7071	0.5000
5	72°	1.721	2.378	3.633	0.8507	1.236	1.176	1.453	0.8090	0.6882
6	60°	2.598	2.598	3.464	1.0000	1.155	1.000	1.155	0.8660	0.8660
7	51°.43	3.634	2.736	3.371	1.152	1.110	0.8678	0.9631	0.9010	1.038
8	45°	4.828	2.828	3.314	1.307	1.082	0.7654	0.8284	0.9239	1.207
9	40°	6.182	2.893	3.276	1.462	1.064	0.6840	0.7279	0.9397	1.374
10	36°	7.694	2.939	3.249	1.618	1.052	0.6180	0.6498	0.9511	1.539
12	30°	11.20	3.000	3.215	1.932	1.035	0.5176	0.5359	0.9659	1.866
15	24°	17.64	3.051	3.188	2.405	1.022	0.4158	0.4251	0.9781	2.352
16	22°.50	20.11	3.062	3.183	2.563	1.020	0.3902	0.3978	0.9808	2.514
20	18°	31.57	3.090	3.168	3.196	1.013	0.3129	0.3168	0.9877	3.157
24	15°	45.58	3.106	3.160	3.831	1.009	0.2611	0.2633	0.9914	3.798
32	11°.25	81.23	3.121	3.152	5.101	1.005	0.1960	0.1970	0.9952	5.077
48	7°.50	183.1	3.133	3.146	7.645	1.002	0.1308	0.1311	0.9979	7.629
64	5°.625	325.7	3.137	3.144	10.19	1.001	0.0981	0.0983	0.9988	10.18

BINOMIAL COEFFICIENTS

(For table giving binomial coefficients for fractional values of n, see p. 116).

$(n)_0 = 1$; $(n)_1 = n$; $(n)_2 = \frac{n(n-1)}{1 \times 2}$; $(n)_3 = \frac{n(n-1)(n-2)}{1 \times 2 \times 3}$; etc.; in general,

$(n)_r = \frac{n(n-1)(n-2)\ .\ .\ .\ (n-[r-1])}{1 \times 2 \times 3\ .\ .\ .\ \times r}$. Another notation: $\binom{n}{r} = (n)_r$.

n	$(n)_0$	$(n)_1$	$(n)_2$	$(n)_3$	$(n)_4$	$(n)_5$	$(n)_6$	$(n)_7$	$(n)_8$	$(n)_9$	$(n)_{10}$	$(n)_{11}$	$(n)_{12}$	$(n)_{13}$
1	1	1	......	......	......	......	......	......	......	......	......	......	......	......
2	1	2	1	......	......	......	......	......	......	......	......	......	......	......
3	1	3	3	1	......	......	......	......	......	......	......	......	......	......
4	1	4	6	4	1	......	......	......	......	......	......	......	......	......
5	1	5	10	10	5	1	......	......	......	......	......	......	......	......
6	1	6	15	20	15	6	1	......	......	......	......	......	......	......
7	1	7	21	35	35	21	7	1	......	......	......	......	......	......
8	1	8	28	56	70	56	28	8	1	......	......	......	......	......
9	1	9	36	84	126	126	84	36	9	1	......	......	......	......
10	1	10	45	120	210	252	210	120	45	10	1	......	......	......
11	1	11	55	165	330	462	462	330	165	55	11	1	......	......
12	1	12	66	220	495	792	924	792	495	220	66	12	1	......
13	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105

For $n = 14$, $(n)_{14} = 1$; for $n = 15$, $(n)_{14} = 15$, and $(n)_{15} = 1$.

COMMON LOGARITHMS (*special table*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.00	0.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	4
1.01	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	
1.02	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	
1.03	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	
1.04	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	
1.05	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	
1.06	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	
1.07	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	
1.08	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	
1.09	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	
1.10	0.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	
1.11	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	
1.12	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	
1.13	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	
1.14	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	
1.15	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	
1.16	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	
1.17	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	
1.18	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	
1.19	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	
1.20	0.0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	
1.21	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	
1.22	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	
1.23	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	
1.24	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	
1.25	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	3
1.26	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	
1.27	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	
1.28	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	
1.29	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	
1.30	0.1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	
1.31	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	
1.32	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	
1.33	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	
1.34	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	
1.35	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	
1.36	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	
1.37	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	
1.38	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	
1.39	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	
1.40	0.1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	
1.41	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	
1.42	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	
1.43	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	
1.44	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	
1.45	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	
1.46	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	
1.47	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	
1.48	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	
1.49	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	

Moving the decimal point n places to the right [or left] in the number requires adding $+n$ [or $-n$] in the body of the table (see p. 42).

COMMON LOGARITHMS (*special table, continued*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.50	0.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	3
1.51	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	
1.52	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	
1.53	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	
1.54	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	
1.55	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	
1.56	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	
1.57	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	
1.58	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	
1.59	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	
1.60	0.2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	
1.61	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	
1.62	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	
1.63	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	
1.64	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	
1.65	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	
1.66	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	
1.67	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	
1.68	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	
1.69	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	
1.70	0.2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	
1.71	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	
1.72	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	
1.73	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	
1.74	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2
1.75	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	
1.76	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	
1.77	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	
1.78	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	
1.79	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	
1.80	0.2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	
1.81	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	
1.82	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	
1.83	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	
1.84	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	
1.85	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	
1.86	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	
1.87	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	
1.88	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	
1.89	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	
1.90	0.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	
1.91	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	
1.92	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	
1.93	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	
1.94	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	
1.95	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	
1.96	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	
1.97	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	
1.98	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	
1.99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	

COMMON LOGARITHMS

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	0.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	See pages 40–41
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	
1.3	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	
1.4	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	
1.5	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	
1.6	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	
1.7	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	
1.8	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	
1.9	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	
2.0	0.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
2.1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
2.2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
2.3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18
2.4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	17
2.5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
2.6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
2.7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
3.0	0.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
3.2	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
3.3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
3.4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
3.5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
3.6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
3.7	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
3.8	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
3.9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
4.0	0.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
4.1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10
4.2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
4.3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
4.4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
4.5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
4.6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	10
4.7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
4.8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
4.9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9

$\log \pi = 0.4971$ $\log \pi/2 = 0.1961$ $\log \pi^2 = 0.9943$ $\log \sqrt{\pi} = 0.2486$
$\log e = 0.4343$ $\log (0.4343) = 0.6378 - 1$

These two pages give the common logarithms of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right [or left] in the number is equivalent to adding n [or $-n$] to the logarithm. Thus, $\log 0.017453 = 0.2419 - 2$, which may also be written $\bar{2}.2419$ or $8.2419 - 10$. See p. 91. Graphs, p. 174.

$$\log (ab) = \log a + \log b \qquad \log (a^N) = N \log a$$

$$\log \left(\frac{a}{b}\right) = \log a - \log b \qquad \log \left(\sqrt[N]{a}\right) = \frac{1}{N} \log a$$

COMMON LOGARITHMS (*continued*)

Number	0	1	2	3	4	5	6	7	8	9	Avg. dif.
5.0	0.6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9
5.1	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	8
5.2	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
5.3	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
5.4	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8
5.5	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
5.6	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8
5.7	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8
5.8	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7
5.9	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7
6.0	0.7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
6.1	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7
6.2	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7
6.3	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
6.4	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7
6.5	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7
6.6	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7
6.7	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	6
6.8	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	6
6.9	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	6
7.0	0.8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	6
7.1	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	6
7.2	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
7.3	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6
7.4	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6
7.5	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
7.6	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6
7.7	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6
7.8	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	6
7.9	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	5
8.0	0.9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5
8.1	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	5
8.2	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	5
8.3	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	5
8.4	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	5
8.5	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
8.6	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
8.7	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
8.8	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5
8.9	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5
9.0	0.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5
9.1	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5
9.2	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5
9.3	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	5
9.4	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	5
9.5	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5
9.6	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	4
9.7	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	4
9.8	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	4
9.9	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	4

DEGREES AND MINUTES EXPRESSED IN RADIANS (See also p. 69)

Degrees						Hundredths				Minutes	
1°	.0175	61°	1.0647	121°	2.1118	0°.01	.0002	0°.51	.0089	1′	.0003
2	.0349	2	1.0821	2	2.1293	2	.0003	2	.0091	2′	.0006
3	.0524	3	1.0996	3	2.1468	3	.0005	3	.0093	3′	.0009
4	.0698	4	1.1170	4	2.1642	4	.0007	4	.0094	4′	.0012
5°	.0873	65°	1.1345	125°	2.1817	.05	.0009	.55	.0096	5′	.0015
6	.1047	6	1.1519	6	2.1991	6	.0010	6	.0098	6′	.0017
7	.1222	7	1.1694	7	2.2166	7	.0012	7	.0099	7′	.0020
8	.1396	8	1.1868	8	2.2340	8	.0014	8	.0101	8′	.0023
9	.1571	9	1.2043	9	2.2515	9	.0016	9	.0103	9′	.0026
10°	.1745	70°	1.2217	130°	2.2689	0°.10	.0017	0°.60	.0105	10′	.0029
1	.1920	1	1.2392	1	2.2864	1	.0019	1	.0106	11′	.0032
2	.2094	2	1.2566	2	2.3038	2	.0021	2	.0108	12′	.0035
3	.2269	3	1.2741	3	2.3213	3	.0023	3	.0110	13′	.0038
4	.2443	4	1.2915	4	2.3387	4	.0024	4	.0112	14′	.0041
15°	.2618	75°	1.3090	135°	2.3562	.15	.0026	.65	.0113	15′	.0044
6	.2793	6	1.3265	6	2.3736	6	.0028	6	.0115	16′	.0047
7	.2967	7	1.3439	7	2.3911	7	.0030	7	.0117	17′	.0049
8	.3142	8	1.3614	8	2.4086	8	.0031	8	.0119	18′	.0052
9	.3316	9	1.3788	9	2.4260	9	.0033	9	.0120	19′	.0055
20°	.3491	80°	1.3963	140°	2.4435	0°.20	.0035	0°.70	.0122	20′	.0058
1	.3665	1	1.4137	1	2.4609	1	.0037	1	.0124	21′	.0061
2	.3840	2	1.4312	2	2.4784	2	.0038	2	.0126	22′	.0064
3	.4014	3	1.4486	3	2.4958	3	.0040	3	.0127	23′	.0067
4	.4189	4	1.4661	4	2.5133	4	.0042	4	.0129	24′	.0070
25°	.4363	85°	1.4835	145°	2.5307	.25	.0044	.75	.0131	25′	.0073
6	.4538	6	1.5010	6	2.5482	6	.0045	6	.0133	26′	.0076
7	.4712	7	1.5184	7	2.5656	7	.0047	7	.0134	27′	.0079
8	.4887	8	1.5359	8	2.5831	8	.0049	8	.0136	28′	.0081
9	.5061	9	1.5533	9	2.6005	9	.0051	9	.0138	29′	.0084
30°	.5236	90°	1.5708	150°	2.6180	0°.30	.0052	0°.80	.0140	30′	.0087
1	.5411	1	1.5882	1	2.6354	1	.0054	1	.0141	31′	.0090
2	.5585	2	1.6057	2	2.6529	2	.0056	2	.0143	32′	.0093
3	.5760	3	1.6232	3	2.6704	3	.0058	3	.0145	33′	.0096
4	.5934	4	1.6406	4	2.6878	4	.0059	4	.0147	34′	.0099
35°	.6109	95°	1.6581	155°	2.7053	.35	.0061	.85	.0148	35′	.0102
6	.6283	6	1.6755	6	2.7227	6	.0063	6	.0150	36′	.0105
7	.6458	7	1.6930	7	2.7402	7	.0065	7	.0152	37′	.0108
8	.6632	8	1.7104	8	2.7576	8	.0066	8	.0154	38′	.0111
9	.6807	9	1.7279	9	2.7751	9	.0068	9	.0155	39′	.0113
40°	.6981	100°	1.7453	160°	2.7925	0°.40	.0070	0°.90	.0157	40′	.0116
1	.7156	1	1.7628	1	2.8100	1	.0072	1	.0159	41′	.0119
2	.7330	2	1.7802	2	2.8274	2	.0073	2	.0161	42′	.0122
3	.7505	3	1.7977	3	2.8449	3	.0075	3	.0162	43′	.0125
4	.7679	4	1.8151	4	2.8623	4	.0077	4	.0164	44′	.0128
45°	.7854	105°	1.8326	165°	2.8798	.45	.0079	.95	.0166	45′	.0131
6	.8029	6	1.8500	6	2.8972	6	.0080	6	.0168	46′	.0134
7	.8203	7	1.8675	7	2.9147	7	.0082	7	.0169	47′	.0137
8	.8378	8	1.8850	8	2.9322	8	.0084	8	.0171	48′	.0140
9	.8552	9	1.9024	9	2.9496	9	.0086	9	.0173	49′	.0143
50°	.8727	110°	1.9199	170°	2.9671	0°.50	.0087	1°.00	.0175	50′	.0145
1	.8901	1	1.9373	1	2.9845					51′	.0148
2	.9076	2	1.9548	2	3.0020					52′	.0151
3	.9250	3	1.9722	3	3.0194					53′	.0154
4	.9425	4	1.9897	4	3.0369					54′	.0157
55°	.9599	115°	2.0071	175°	3.0543					55′	.0160
6	.9774	6	2.0246	6	3.0718					56′	.0163
7	.9948	7	2.0420	7	3.0892					57′	.0166
8	1.0123	8	2.0595	8	3.1067					58′	.0169
9	1.0297	9	2.0769	9	3.1241					59′	.0172
60°	1.0472	120°	2.0944	180°	3.1416					60′	.0175

Arc 1° = 0.0174533 Arc 1′ = 0.000290888 Arc 1″ = 0.00000484814
1 radian = 57°.295780 = 57° 17′.7468 = 57° 17′ 44″.806

RADIANS EXPRESSED IN DEGREES

0.01	0°.57	.64	36°.67	1.27	72°.77	1.90	108°.86	2.53	144°.96
2	1°.15	.65	37°.24	8	73°.34	1	109°.43	4	145°.53
3	1°.72	6	37°.82	9	73°.91	2	110°.01	2.55	146°.10
4	2°.29	7	38°.39	1.30	74°.48	3	110°.58	6	146°.68
.05	2°.86	8	38°.96	1	75°.06	4	111°.15	7	147°.25
6	3°.44	9	39°.53	2	75°.63	1.95	111°.73	8	147°.82
7	4°.01	.70	40°.11	3	76°.20	6	112°.30	9	148°.40
8	4°.58	1	40°.68	4	76°.78	7	112°.87	2.60	148°.97
9	5°.16	2	41°.25	1.35	77°.35	8	113°.45	1	149°.54
.10	5°.73	3	41°.83	6	77°.92	9	114°.02	2	150°.11
1	6°.30	4	42°.40	7	78°.50	2.00	114°.59	3	150°.69
2	6°.88	.75	42°.97	8	79°.07	1	115°.16	4	151°.26
3	7°.45	6	43°.54	9	79°.64	2	115°.74	2.65	151°.83
4	8°.02	7	44°.12	1.40	80°.21	3	116°.31	6	152°.41
.15	8°.59	8	44°.69	1	80°.79	4	116°.88	7	152°.98
6	9°.17	9	45°.26	2	81°.36	2.05	117°.46	8	153°.55
7	9°.74	.80	45°.84	3	81°.93	6	118°.03	9	154°.13
8	10°.31	1	46°.41	4	82°.51	7	118°.60	2.70	154°.70
9	10°.89	2	46°.98	1.45	83°.08	8	119°.18	1	155°.27
.20	11°.46	3	47°.56	6	83°.65	9	119°.75	2	155°.84
1	12°.03	4	48°.13	7	84°.22	2.10	120°.32	3	156°.42
2	12°.61	.85	48°.70	8	84°.80	1	120°.89	4	156°.99
3	13°.18	6	49°.27	9	85°.37	2	121°.47	2.75	157°.56
4	13°.75	7	49°.85	1.50	85°.94	3	122°.04	6	158°.14
.25	14°.32	8	50°.42	1	86°.52	4	122°.61	7	158°.71
6	14°.90	9	50°.99	2	87°.09	2.15	123°.19	8	159°.28
7	15°.47	.90	51°.57	3	87°.66	6	123°.76	9	159°.86
8	16°.04	1	52°.14	4	88°.24	7	124°.33	2.80	160°.43
9	16°.62	2	52°.71	1.55	88°.81	8	124°.90	1	161°.00
.30	17°.19	3	53°.29	6	89°.38	9	125°.48	2	161°.57
1	17°.76	4	53°.86	7	89°.95	2.20	126°.05	3	162°.15
2	18°.33	.95	54°.43	8	90°.53	1	126°.62	4	162°.72
3	18°.91	6	55°.00	9	91°.10	2	127°.20	2.85	163°.29
4	19°.48	7	55°.58	1.60	91°.67	3	127°.77	6	163°.87
.35	20°.05	8	56°.15	1	92°.25	4	128°.34	7	164°.44
6	20°.63	9	56°.72	2	92°.82	2.25	128°.92	8	165°.01
7	21°.20	1.00	57°.30	3	93°.39	6	129°.49	9	165°.58
8	21°.77	1	57°.87	4	93°.97	7	130°.06	2.90	166°.16
9	22°.35	2	58°.44	1.65	94°.54	8	130°.63	1	166°.73
.40	22°.92	3	59°.01	6	95°.11	9	131°.21	2	167°.30
1	23°.49	4	59°.59	7	95°.68	2.30	131°.78	3	167°.88
2	24°.06	1.05	60°.16	8	96°.26	1	132°.35	4	168°.45
3	24°.64	6	60°.73	9	96°.83	2	132°.93	2.95	169°.02
4	25°.21	7	61°.31	1.70	97°.40	3	133°.50	6	169°.60
.45	25°.78	8	61°.88	1	97°.98	4	134°.07	7	170°.17
6	26°.36	9	62°.45	2	98°.55	2.35	134°.65	8	170°.74
7	26°.93	1.10	63°.03	3	99°.12	6	135°.22	9	171°.31
8	27°.50	1	63°.60	4	99°.69	7	135°.79	3.00	171°.89
9	28°.07	2	64°.17	1.75	100°.27	8	136°.36	1	172°.46
.50	28°.65	3	64°.74	6	100°.84	9	136°.94	2	173°.03
1	29°.22	4	65°.32	7	101°.41	2.40	137°.51	3	173°.61
2	29°.79	1.15	65°.89	8	101°.99	1	138°.08	4	174°.18
3	30°.37	6	66°.46	9	102°.56	2	138°.66	3.05	174°.75
4	30°.94	7	67°.04	1.80	103°.13	3	139°.23	6	175°.33
.55	31°.51	8	67°.61	1	103°.71	4	139°.80	7	175°.90
6	32°.09	9	68°.18	2	104°.28	2.45	140°.37	8	176°.47
7	32°.66	1.20	68°.75	3	104°.85	6	140°.95	9	177°.04
8	33°.23	1	69°.33	4	105°.42	7	141°.52	3.10	177°.62
9	33°.80	2	69°.90	1.85	106°.00	8	142°.09	1	178°.19
.60	34°.38	3	70°.47	6	106°.57	9	142°.67	2	178°.76
1	34°.95	4	71°.05	7	107°.14	2.50	143°.24	3	179°.34
2	35°.52	1.25	71°.62	8	107°.72	1	143°.81	4	179°.91
3	36°.10	6	72°.19	9	108°.29	2	144°.39	3.15	180°.48

Interpolation

.0002	0°.01
04	.02
06	.03
08	.05
.0010	0°.06
12	.07
14	.08
16	.09
18	.10
.0020	0°.11
22	.13
24	.14
26	.15
28	.16
.0030	0°.17
32	.18
34	.19
36	.21
38	.22
.0040	0°.23
42	.24
44	.25
46	.26
48	.28
.0050	0°.29
52	.30
54	.31
56	.32
58	.33
.0060	0°.34
62	.36
64	.37
66	.38
68	.39
.0070	0°.40
72	.41
74	.42
76	.44
78	.45
.0080	0°.46
82	.47
84	.48
86	.49
88	.50
.0090	0°.52
92	.53
94	.54
96	.55
98	.56

Multiples of π

1	3.1416	180°
2	6.2832	360°
3	9.4248	540°
4	12.5664	720°
5	15.7080	900°
6	18.8496	1080°
7	21.9911	1260°
8	25.1327	1440°
9	28.2743	1620°
10	31.4159	1800°

NATURAL SINES AND COSINES

Natural Sines at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											0.0000	90°	
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	0523	87	17
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	0698	86	17
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	0.0872	85	17
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	1045	84	17
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	1219	83	17
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	1392	82	17
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	1564	81	17
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	0.1736	80°	17
10°	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	1908	79	17
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2079	78	17
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2250	77	17
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2419	76	17
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	0.2588	75	17
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2756	74	17
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2924	73	17
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3090	72	17
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3256	71	17
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	0.3420	70°	16
20°	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3584	69	16
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3746	68	16
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3907	67	16
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	4067	66	16
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	0.4226	65	16
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	4384	64	16
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	4540	63	16
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	4695	62	16
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	4848	61	15
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	0.5000	60°	15
30°	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	5150	59	15
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	5299	58	15
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	5446	57	15
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	5592	56	15
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	0.5736	55	14
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	5878	54	14
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	6018	53	14
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	6157	52	14
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	6293	51	14
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	0.6428	50°	13
40°	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	6561	49	13
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	6691	48	13
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	6820	47	13
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	6947	46	13
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	0.7071	45°	12
45°	0.7071												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

(For graphs, see p. 174.)

Natural Cosines

NATURAL SINES AND COSINES (*continued*)

Natural Sines at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 —(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											0.7071	45°	
45°	0.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	7193	44	12
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	7314	43	12
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	7431	42	12
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	7547	41	12
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	0.7660	40°	11
50°	0.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	7771	39	11
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	7880	38	11
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	7986	37	11
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	8090	36	10
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	0.8192	35	10
55	0.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	8290	34	10
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	8387	33	10
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	8480	32	9
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	8572	31	9
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	0.8660	30°	9
60°	0.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	8746	29	9
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	8829	28	8
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8910	27	8
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	8988	26	8
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	0.9063	25	7
65	0.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	9135	24	7
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	9205	23	7
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	9272	22	7
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	9336	21	6
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	0.9397	20°	6
70°	0.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	9455	19	6
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	9511	18	6
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	9563	17	5
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	9613	16	5
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0.9659	15	5
75	0.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	9703	14	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	9744	13	4
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	9781	12	4
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	9816	11	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0.9848	10°	3
80°	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	9877	9	3
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	9903	8	3
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	9925	7	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	9945	6	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0.9962	5	2
85	0.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	9976	4	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	9986	3	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	9994	2	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0.9998	1	0
89	0.9998	9999	9999	9999	9999	0000	0000	0000	0000	0000	1.0000	0°	0
90°	1.0000												
		°.9 —(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural Cosines

NATURAL TANGENTS AND COTANGENTS

Natural Tangents at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 —(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											0.0000	90°	
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	0524	87	17
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	0699	86	18
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	0.0875	85	18
5	0.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	1051	84	18
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	1228	83	18
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	1405	82	18
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	1584	81	18
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	0.1763	80°	18
10°	0.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	1944	79	18
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	2126	78	18
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	2309	77	18
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	2493	76	18
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	0.2679	75	19
15	0.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	2867	74	19
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3057	73	19
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3249	72	19
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3443	71	19
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	0.3640	70°	20
20°	0.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3839	69	20
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	4040	68	20
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	4245	67	21
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	4452	66	21
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	0.4663	65	21
25	0.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4877	64	21
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	5095	63	22
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	5317	62	22
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	5543	61	23
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	0.5774	60°	23
30°	0.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	6009	59	24
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	6249	58	24
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	6494	57	25
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	6745	56	25
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	0.7002	55	26
35	0.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	7265	54	26
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	7536	53	27
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	7813	52	28
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	8098	51	28
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	0.8391	50°	29
40°	0.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	8693	49	30
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	9004	48	31
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	9325	47	32
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	0.9657	46	33
44	0.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	1.0000	45°	34
45°	1.0000												
		°.9 —(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

(For graphs, see p. 174.)

Natural Cotangents

NATURAL TANGENTS AND COTANGENTS (*continued*)

Natural Tangents at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 —(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1.0000	45°	
45°	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	0355	44	35
46	0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	0724	43	37
47	0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	1106	42	38
48	1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	1504	41	40
49	1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	1.1918	40°	41
50°	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	2349	39	43
51	2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	2799	38	45
52	2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	3270	37	47
53	3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	3764	36	49
54	3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	1.4281	35	52
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	4826	34	55
56	4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	5399	33	57
57	5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	6003	32	60
58	6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	6643	31	64
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	1.7321	30°	67
60°	1.732	1.739	1.746	1.753	1.760	1.767	1.775	1.782	1.789	1.797	1.804	29	7
61	1.804	1.811	1.819	1.827	1.834	1.842	1.849	1.857	1.865	1.873	1.881	28	8
62	1.881	1.889	1.897	1.905	1.913	1.921	1.929	1.937	1.946	1.954	1.963	27	8
63	1.963	1.971	1.980	1.988	1.997	2.006	2.014	2.023	2.032	2.041	2.050	26	9
64	2.050	2.059	2.069	2.078	2.087	2.097	2.106	2.116	2.125	2.135	2.145	25	9
65	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	2.246	24	10
66	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	2.356	23	11
67	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	2.475	22	12
68	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	2.605	21	13
69	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	2.747	20°	14
70°	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	2.904	19	16
71	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	3.078	18	17
72	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.251	3.271	17	19
73	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	3.487	16	22
74	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.706	3.732	15	24
75	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	4.011	14	28
76	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	4.331	13	32
77	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	4.705	12	37
78	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	5.145	11	44
79	5.145	5.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	5.671	10°	53
80°	5.671	5.730	5.789	5.850	5.912	5.976	6.041	6.107	6.174	6.243	6.314	9	
81	6.314	6.386	6.460	6.535	6.612	6.691	6.772	6.855	6.940	7.026	7.115	8	
82	7.115	7.207	7.300	7.396	7.495	7.596	7.700	7.806	7.916	8.028	8.144	7	
83	8.144	8.264	8.386	8.513	8.643	8.777	8.915	9.058	9.205	9.357	9.514	6	
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	11.43	5	
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	14.30	4	
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46	19.08	3	
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	28.64	2	
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	57.29	1	
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°	
90°	∞												
	=	°.9 (54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural Cotangents

NATURAL SECANTS AND COSECANTS

Natural Secants at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 —(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1.0000	90°	
0°	1.0000	0000	0000	0000	0000	0000	0001	0001	0001	0001	0002	89	0
1	0002	0002	0002	0003	0003	0003	0004	0004	0005	0006	0006	88	0
2	0006	0007	0007	0008	0009	0010	0010	0011	0012	0013	0014	87	1
3	0014	0015	0016	0017	0018	0019	0020	0021	0022	0023	0024	86	1
4	0024	0026	0027	0028	0030	0031	0032	0034	0035	0037	1.0038	85	1
5	1.0038	0040	0041	0043	0045	0046	0048	0050	0051	0053	0055	84	2
6	0055	0057	0059	0061	0063	0065	0067	0069	0071	0073	0075	83	2
7	0075	0077	0079	0082	0084	0086	0089	0091	0093	0096	0098	82	2
8	0098	0101	0103	0106	0108	0111	0114	0116	0119	0122	0125	81	3
9	0125	0127	0130	0133	0136	0139	0142	0145	0148	0151	1.0154	80°	3
10°	1.0154	0157	0161	0164	0167	0170	0174	0177	0180	0184	0187	79	3
11	0187	0191	0194	0198	0201	0205	0209	0212	0216	0220	0223	78	4
12	0223	0227	0231	0235	0239	0243	0247	0251	0255	0259	0263	77	4
13	0263	0267	0271	0276	0280	0284	0288	0293	0297	0302	0306	76	4
14	0306	0311	0315	0320	0324	0329	0334	0338	0343	0348	1.0353	75	5
15	1.0353	0358	0363	0367	0372	0377	0382	0388	0393	0398	0403	74	5
16	0403	0408	0413	0419	0424	0429	0435	0440	0446	0451	0457	73	5
17	0457	0463	0468	0474	0480	0485	0491	0497	0503	0509	0515	72	6
18	0515	0521	0527	0533	0539	0545	0551	0557	0564	0570	0576	71	6
19	0576	0583	0589	0595	0602	0608	0615	0622	0628	0635	1.0642	70°	7
20°	1.0642	0649	0655	0662	0669	0676	0683	0690	0697	0704	0711	69	7
21	0711	0719	0726	0733	0740	0748	0755	0763	0770	0778	0785	68	7
22	0785	0793	0801	0808	0816	0824	0832	0840	0848	0856	0864	67	8
23	0864	0872	0880	0888	0896	0904	0913	0921	0929	0938	0946	66	8
24	0946	0955	0963	0972	0981	0989	0998	1007	1016	1025	1.1034	65	9
25	1.1034	1043	1052	1061	1070	1079	1089	1098	1107	1117	1126	64	9
26	1126	1136	1145	1155	1164	1174	1184	1194	1203	1213	1223	63	10
27	1223	1233	1243	1253	1264	1274	1284	1294	1305	1315	1326	62	10
28	1326	1336	1347	1357	1368	1379	1390	1401	1412	1423	1434	61	11
29	1434	1445	1456	1467	1478	1490	1501	1512	1524	1535	1.1547	60°	11
30°	1.1547	1559	1570	1582	1594	1606	1618	1630	1642	1654	1666	59	12
31	1666	1679	1691	1703	1716	1728	1741	1753	1766	1779	1792	58	13
32	1792	1805	1818	1831	1844	1857	1870	1883	1897	1910	1924	57	13
33	1924	1937	1951	1964	1978	1992	2006	2020	2034	2048	2062	56	14
34	2062	2076	2091	2105	2120	2134	2149	2163	2178	2193	1.2208	55	15
35	1.2208	2223	2238	2253	2268	2283	2299	2314	2329	2345	2361	54	15
36	2361	2376	2392	2408	2424	2440	2456	2472	2489	2505	2521	53	16
37	2521	2538	2554	2571	2588	2605	2622	2639	2656	2673	2690	52	17
38	2690	2708	2725	2742	2760	2778	2796	2813	2831	2849	2868	51	18
39	2868	2886	2904	2923	2941	2960	2978	2997	3016	3035	1.3054	50°	19
40°	1.3054	3073	3093	3112	3131	3151	3171	3190	3210	3230	3250	49	20
41	3250	3270	3291	3311	3331	3352	3373	3393	3414	3435	3456	48	21
42	3456	3478	3499	3520	3542	3563	3585	3607	3629	3651	3673	47	22
43	3673	3696	3718	3741	3763	3786	3809	3832	3855	3878	3902	46	23
44	3902	3925	3949	3972	3996	4020	4044	4069	4093	4118	1.4142	45°	24
45°	1.4142												
		°.9 —(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

(For graphs, see p. 174.)

Natural Cosecants

NATURAL SECANTS AND COSECANTS (*continued*)

Natural Secants at intervals of 0°.1, or 6′. (For 10′ intervals, see pp. 52–56)

Deg.	°.0 —(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg. diff.
											1.4142	45°	
45°	1.4142	4167	4192	4217	4242	4267	4293	4318	4344	4370	4396	44	25
46	4396	4422	4448	4474	4501	4527	4554	4581	4608	4635	4663	43	27
47	4663	4690	4718	4746	4774	4802	4830	4859	4887	4916	4945	42	28
48	4945	4974	5003	5032	5062	5092	5121	5151	5182	5212	5243	41	30
49	5243	5273	5304	5335	5366	5398	5429	5461	5493	5525	1.5557	40°	31
50°	1.5557	5590	5622	5655	5688	5721	5755	5788	5822	5856	5890	39	33
51	5890	5925	5959	5994	6029	6064	6099	6135	6171	6207	6243	38	35
52	6243	6279	6316	6353	6390	6427	6464	6502	6540	6578	6616	37	37
53	6616	6655	6694	6733	6772	6812	6852	6892	6932	6972	7013	36	40
54	7013	7054	7095	7137	7179	7221	7263	7305	7348	7391	1.7434	35	42
55	1.7434	7478	7522	7566	7610	7655	7700	7745	7791	7837	7883	34	45
56	7883	7929	7976	8023	8070	8118	8166	8214	8263	8312	8361	33	48
57	8361	8410	8460	8510	8561	8612	8663	8714	8766	8818	8871	32	51
58	8871	8924	8977	9031	9084	9139	9194	9249	9304	9360	1.9416	31	54
59	1.9416	9473	9530	9587	9645	9703	9762	9821	9880	9940	2.0000	30°	58
60°	2.000	2.006	2.012	2.018	2.025	2.031	2.037	2.043	2.050	2.056	2.063	29	6
61	2.063	2.069	2.076	2.082	2.089	2.096	2.103	2.109	2.116	2.123	2.130	28	7
62	2.130	2.137	2.144	2.151	2.158	2.166	2.173	2.180	2.188	2.195	2.203	27	7
63	2.203	2.210	2.218	2.226	2.233	2.241	2.249	2.257	2.265	2.273	2.281	26	8
64	2.281	2.289	2.298	2.306	2.314	2.323	2.331	2.340	2.349	2.357	2.366	25	8
65	2.366	2.375	2.384	2.393	2.402	2.411	2.421	2.430	2.439	2.449	2.459	24	9
66	2.459	2.468	2.478	2.488	2.498	2.508	2.518	2.528	2.538	2.549	2.559	23	10
67	2.559	2.570	2.581	2.591	2.602	2.613	2.624	2.635	2.647	2.658	2.669	22	11
68	2.669	2.681	2.693	2.705	2.716	2.729	2.741	2.753	2.765	2.778	2.790	21	12
69	2.790	2.803	2.816	2.829	2.842	2.855	2.869	2.882	2.896	2.910	2.924	20°	13
70°	2.924	2.938	2.952	2.967	2.981	2.996	3.011	3.026	3.041	3.056	3.072	19	15
71	3.072	3.087	3.103	3.119	3.135	3.152	3.168	3.185	3.202	3.219	3.236	18	16
72	3.236	3.254	3.271	3.289	3.307	3.326	3.344	3.363	3.382	3.401	3.420	17	18
73	3.420	3.440	3.460	3.480	3.500	3.521	3.542	3.563	3.584	3.606	3.628	16	21
74	3.628	3.650	3.673	3.695	3.719	3.742	3.766	3.790	3.814	3.839	3.864	15	24
75	3.864	3.889	3.915	3.941	3.967	3.994	4.021	4.049	4.077	4.105	4.134	14	27
76	4.134	4.163	4.192	4.222	4.253	4.284	4.315	4.347	4.379	4.412	4.445	13	31
77	4.445	4.479	4.514	4.549	4.584	4.620	4.657	4.694	4.732	4.771	4.810	12	36
78	4.810	4.850	4.890	4.931	4.973	5.016	5.059	5.103	5.148	5.194	5.241	11	43
79	5.241	5.288	5.337	5.386	5.436	5.487	5.540	5.593	5.647	5.702	5.759	10°	52
80°	5.759	5.816	5.875	5.935	5.996	6.059	6.123	6.188	6.255	6.323	6.392	9	
81	6.392	6.464	6.537	6.611	6.687	6.765	6.845	6.927	7.011	7.097	7.185	8	
82	7.185	7.276	7.368	7.463	7.561	7.661	7.764	7.870	7.979	8.091	8.206	7	
83	8.206	8.324	8.446	8.571	8.700	8.834	8.971	9.113	9.259	9.411	9.567	6	
84	9.567	9.728	9.895	10.07	10.25	10.43	10.63	10.83	11.03	11.25	11.47	5	
85	11.47	11.71	11.95	12.20	12.47	12.75	13.03	13.34	13.65	13.99	14.34	4	
86	14.34	14.70	15.09	15.50	15.93	16.38	16.86	17.37	17.91	18.49	19.11	3	
87	19.11	19.77	20.47	21.23	22.04	22.93	23.88	24.92	26.05	27.29	28.65	2	
88	28.65	30.16	31.84	33.71	35.81	38.20	40.93	44.08	47.75	52.09	57.30	1	
89	57.30	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°	
90°	∞												
		°.9 —(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg.	

Natural Cosecants

TRIGONOMETRIC FUNCTIONS (at intervals of 10′)
Annex −10 in columns marked *. (For 0.°1 intervals, see pp. 46–51)

Degrees	Radians	Sines Nat.	Sines Log.*	Cosines Nat.	Cosines Log.*	Tangents Nat.	Tangents Log.*	Cotangents Nat.	Cotangents Log.		
0° 00′	0.0000	.0000	∞	1.0000	0.0000	.0000	∞	∞	∞	1.5708	90° 00′
10	0.0029	.0029	7.4637	1.0000	.0000	.0029	7.4637	343.77	2.5363	1.5679	50
20	0.0058	.0058	.7648	1.0000	.0000	.0058	.7648	171.89	.2352	1.5650	40
30	0.0087	.0087	.9408	1.0000	.0000	.0087	.9409	114.59	.0591	1.5621	30
40	0.0116	.0116	8.0658	0.9999	.0000	.0116	8.0658	85.940	1.9342	1.5592	20
50	0.0145	.0145	.1627	.9999	.0000	.0145	.1627	68.750	.8373	1.5563	10
1° 00′	0.0175	.0175	8.2419	.9998	9.9999	.0175	8.2419	57.290	1.7581	1.5533	89° 00′
10	0.0204	.0204	.3088	.9998	.9999	.0204	.3089	49.104	.6911	1.5504	50
20	0.0233	.0233	.3668	.9997	.9999	.0233	.3669	42.964	.6331	1.5475	40
30	0.0262	.0262	.4179	.9997	.9999	.0262	.4181	38.188	.5819	1.5446	30
40	0.0291	.0291	.4637	.9996	.9998	.0291	.4638	34.368	.5362	1.5417	20
50	0.0320	.0320	.5050	.9995	.9998	.0320	.5053	31.242	.4947	1.5388	10
2° 00′	0.0349	.0349	8.5428	.9994	9.9997	.0349	8.5431	28.636	1.4569	1.5359	88° 00′
10	0.0378	.0378	.5776	.9993	.9997	.0378	.5779	26.432	.4221	1.5330	50
20	0.0407	.0407	.6097	.9992	.9996	.0407	.6101	24.542	.3899	1.5301	40
30	0.0436	.0436	.6397	.9990	.9996	.0437	.6401	22.904	.3599	1.5272	30
40	0.0465	.0465	.6677	.9989	.9995	.0466	.6682	21.470	.3318	1.5243	20
50	0.0495	.0494	.6940	.9988	.9995	.0495	.6945	20.206	.3055	1.5213	10
3° 00′	0.0524	.0523	8.7188	.9986	9.9994	.0524	8.7194	19.081	1.2806	1.5184	87° 00′
10	0.0553	.0552	.7423	.9985	.9993	.0553	.7429	18.075	.2571	1.5155	50
20	0.0582	.0581	.7645	.9983	.9993	.0582	.7652	17.169	.2348	1.5126	40
30	0.0611	.0610	.7857	.9981	.9992	.0612	.7865	16.350	.2135	1.5097	30
40	0.0640	.0640	.8059	.9980	.9991	.0641	.8067	15.605	.1933	1.5068	20
50	0.0669	.0669	.8251	.9978	.9990	.0670	.8261	14.924	.1739	1.5039	10
4° 00′	0.0698	.0698	8.8436	.9976	9.9989	.0699	8.8446	14.301	1.1554	1.5010	86° 00′
10	0.0727	.0727	.8613	.9974	.9989	.0729	.8624	13.727	.1376	1.4981	50
20	0.0756	.0756	.8783	.9971	.9988	.0758	.8795	13.197	.1205	1.4952	40
30	0.0785	.0785	.8946	.9969	.9987	.0787	.8960	12.706	.1040	1.4923	30
40	0.0814	.0814	.9104	.9967	.9986	.0816	.9118	12.251	.0882	1.4893	20
50	0.0844	.0843	.9256	.9964	.9985	.0846	.9272	11.826	.0728	1.4864	10
5° 00′	0.0873	.0872	8.9403	.9962	9.9983	.0875	8.9420	11.430	1.0580	1.4835	85° 00′
10	0.0902	.0901	.9545	.9959	.9982	.0904	.9563	11.059	.0437	1.4806	50
20	0.0931	.0929	.9682	.9957	.9981	.0934	.9701	10.712	.0299	1.4777	40
30	0.0960	.0958	.9816	.9954	.9980	.0963	.9836	10.385	.0164	1.4748	30
40	0.0989	.0987	.9945	.9951	.9979	.0992	.9966	10.078	.0034	1.4719	20
50	0.1018	.1016	9.0070	.9948	.9977	.1022	9.0093	9.7882	0.9907	1.4690	10
6° 00′	0.1047	.1045	9.0192	.9945	9.9976	.1051	9.0216	9.5144	0.9784	1.4661	84° 00′
10	0.1076	.1074	.0311	.9942	.9975	.1080	.0336	9.2553	.9664	1.4632	50
20	0.1105	.1103	.0426	.9939	.9973	.1110	.0453	9.0098	.9547	1.4603	40
30	0.1134	.1132	.0539	.9936	.9972	.1139	.0567	8.7769	.9433	1.4574	30
40	0.1164	.1161	.0648	.9932	.9971	.1169	.0678	8.5555	.9322	1.4544	20
50	0.1193	.1190	.0755	.9929	.9969	.1198	.0786	8.3450	.9214	1.4515	10
7° 00′	0.1222	.1219	9.0859	.9925	9.9968	.1228	9.0891	8.1443	0.9109	1.4486	83° 00′
10	0.1251	.1248	.0961	.9922	.9966	.1257	.0995	7.9530	.9005	1.4457	50
20	0.1280	.1276	.1060	.9918	.9964	.1287	.1096	7.7704	.8904	1.4428	40
30	0.1309	.1305	.1157	.9914	.9963	.1317	.1194	7.5958	.8806	1.4399	30
40	0.1338	.1334	.1252	.9911	.9961	.1346	.1291	7.4287	.8709	1.4370	20
50	0.1367	.1363	.1345	.9907	.9959	.1376	.1385	7.2687	.8615	1.4341	10
8° 00′	0.1396	.1392	9.1436	.9903	9.9958	.1405	9.1478	7.1154	0.8522	1.4312	82° 00′
10	0.1425	.1421	.1525	.9899	.9956	.1435	.1569	6.9682	.8431	1.4283	50
20	0.1454	.1449	.1612	.9894	.9954	.1465	.1658	6.8269	.8342	1.4254	40
30	0.1484	.1478	.1697	.9890	.9952	.1495	.1745	6.6912	.8255	1.4224	30
40	0.1513	.1507	.1781	.9886	.9950	.1524	.1831	6.5606	.8169	1.4195	20
50	0.1542	.1536	.1863	.9881	.9948	.1554	.1915	6.4348	.8085	1.4166	10
9° 00′	0.1571	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS (*continued*)

Annex −10 in columns marked *. (For 0.°1 intervals, see pp. 46–51)

Degrees	Radians	Sines Nat.	Sines Log.*	Cosines Nat.	Cosines Log.*	Tangents Nat.	Tangents Log.*	Cotangents Nat.	Cotangents Log.		
9° 00′	0.1571	.1564	9.1943	.9877	9.9946	.1584	9.1997	6.3138	0.8003	1.4137	81° 00′
10	0.1600	.1593	.2022	.9872	.9944	.1614	.2078	6.1970	.7922	1.4108	50
20	0.1629	.1622	.2100	.9868	.9942	.1644	.2158	6.0844	.7842	1.4079	40
30	0.1658	.1650	.2176	.9863	.9940	.1673	.2236	5.9758	.7764	1.4050	30
40	0.1687	.1679	.2251	.9858	.9938	.1703	.2313	5.8708	.7687	1.4021	20
50	0.1716	.1708	.2324	.9853	.9936	.1733	.2389	5.7694	.7611	1.3992	10
10° 00′	0.1745	.1736	9.2397	.9848	9.9934	.1763	9.2463	5.6713	0.7537	1.3963	80° 00′
10	0.1774	.1765	.2468	.9843	.9931	.1793	.2536	5.5764	.7464	1.3934	50
20	0.1804	.1794	.2538	.9838	.9929	.1823	.2609	5.4845	.7391	1.3904	40
30	0.1833	.1822	.2606	.9833	.9927	.1853	.2680	5.3955	.7320	1.3875	30
40	0.1862	.1851	.2674	.9827	.9924	.1883	.2750	5.3093	.7250	1.3846	20
50	0.1891	.1880	.2740	.9822	.9922	.1914	.2819	5.2257	.7181	1.3817	10
11° 00′	0.1920	.1908	9.2806	.9816	9.9919	.1944	9.2887	5.1446	0.7113	1.3788	79° 00′
10	0.1949	.1937	.2870	.9811	.9917	.1974	.2953	5.0658	.7047	1.3759	50
20	0.1978	.1965	.2934	.9805	.9914	.2004	.3020	4.9894	.6980	1.3730	40
30	0.2007	.1994	.2997	.9799	.9912	.2035	.3085	4.9152	.6915	1.3701	30
40	0.2036	.2022	.3058	.9793	.9909	.2065	.3149	4.8430	.6851	1.3672	20
50	0.2065	.2051	.3119	.9787	.9907	.2095	.3212	4.7729	.6788	1.3643	10
12° 00′	0.2094	.2079	9.3179	.9781	9.9904	.2126	9.3275	4.7046	0.6725	1.3614	78° 00′
10	0.2123	.2108	.3238	.9775	.9901	.2156	.3336	4.6382	.6664	1.3584	50
20	0.2153	.2136	.3296	.9769	.9899	.2186	.3397	4.5736	.6603	1.3555	40
30	0.2182	.2164	.3353	.9763	.9896	.2217	.3458	4.5107	.6542	1.3526	30
40	0.2211	.2193	.3410	.9757	.9893	.2247	.3517	4.4494	.6483	1.3497	20
50	0.2240	.2221	.3466	.9750	.9890	.2278	.3576	4.3897	.6424	1.3468	10
13° 00′	0.2269	.2250	9.3521	.9744	9.9887	.2309	9.3634	4.3315	0.6366	1.3439	77° 00′
10	0.2298	.2278	.3575	.9737	.9884	.2339	.3691	4.2747	.6309	1.3410	50
20	0.2327	.2306	.3629	.9730	.9881	.2370	.3748	4.2193	.6252	1.3381	40
30	0.2356	.2334	.3682	.9724	.9878	.2401	.3804	4.1653	.6196	1.3352	30
40	0.2385	.2363	.3734	.9717	.9875	.2432	.3859	4.1126	.6141	1.3323	20
50	0.2414	.2391	.3786	.9710	.9872	.2462	.3914	4.0611	.6086	1.3294	10
14° 00′	0.2443	.2419	9.3837	.9703	9.9869	.2493	9.3968	4.0108	0.6032	1.3265	76° 00′
10	0.2473	.2447	.3887	.9696	.9866	.2524	.4021	3.9617	.5979	1.3235	50
20	0.2502	.2476	.3937	.9689	.9863	.2555	.4074	3.9136	.5926	1.3206	40
30	0.2531	.2504	.3986	.9681	.9859	.2586	.4127	3.8667	.5873	1.3177	30
40	0.2560	.2532	.4035	.9674	.9856	.2617	.4178	3.8208	.5822	1.3148	20
50	0.2589	.2560	.4083	.9667	.9853	.2648	.4230	3.7760	.5770	1.3119	10
15° 00′	0.2618	.2588	9.4130	.9659	9.9849	.2679	9.4281	3.7321	0.5719	1.3090	75° 00′
10	0.2647	.2616	.4177	.9652	.9846	.2711	.4331	3.6891	.5669	1.3061	50
20	0.2676	.2644	.4223	.9644	.9843	.2742	.4381	3.6470	.5619	1.3032	40
30	0.2705	.2672	.4269	.9636	.9839	.2773	.4430	3.6059	.5570	1.3003	30
40	0.2734	.2700	.4314	.9628	.9836	.2805	.4479	3.5656	.5521	1.2974	20
50	0.2763	.2728	.4359	.9621	.9832	.2836	.4527	3.5261	.5473	1.2945	10
16° 00′	0.2793	.2756	9.4403	.9613	9.9828	.2867	9.4575	3.4874	0.5425	1.2915	74° 00′
10	0.2822	.2784	.4447	.9605	.9825	.2899	.4622	3.4495	.5378	1.2886	50
20	0.2851	.2812	.4491	.9596	.9821	.2931	.4669	3.4124	.5331	1.2857	40
30	0.2880	.2840	.4533	.9588	.9817	.2962	.4716	3.3759	.5284	1.2828	30
40	0.2909	.2868	.4576	.9580	.9814	.2994	.4762	3.3402	.5238	1.2799	20
50	0.2938	.2896	.4618	.9572	.9810	.3026	.4808	3.3052	.5192	1.2770	10
17° 00′	0.2967	.2924	9.4659	.9563	9.9806	.3057	9.4853	3.2709	0.5147	1.2741	73° 00′
10	0.2996	.2952	.4700	.9555	.9802	.3089	.4898	3.2371	.5102	1.2712	50
20	0.3025	.2979	.4741	.9546	.9798	.3121	.4943	3.2041	.5057	1.2683	40
30	0.3054	.3007	.4781	.9537	.9794	.3153	.4987	3.1716	.5013	1.2654	30
40	0.3083	.3035	.4821	.9528	.9790	.3185	.5031	3.1397	.4969	1.2625	20
50	0.3113	.3062	.4861	.9520	.9786	.3217	.5075	3.1084	.4925	1.2595	10
18° 00′	0.3142	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	1.2566	72° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS (*continued*)

Annex −10 in columns marked *. (For 0.°1 intervals, see pp. 46–51)

Degrees	Radians	Sines Nat.	Sines Log.*	Cosines Nat.	Cosines Log.*	Tangents Nat.	Tangents Log.*	Cotangents Nat.	Cotangents Log.		
18° 00′	0.3142	.3090	9.4900	.9511	9.9782	.3249	9.5118	3.0777	0.4882	1.2566	72° 00′
10	0.3171	.3118	.4939	.9502	.9778	.3281	.5161	3.0475	.4839	1.2537	50
20	0.3200	.3145	.4977	.9492	.9774	.3314	.5203	3.0178	.4797	1.2508	40
30	0.3229	.3173	.5015	.9483	.9770	.3346	.5245	2.9887	.4755	1.2479	30
40	0.3258	.3201	.5052	.9474	.9765	.3378	.5287	2.9600	.4713	1.2450	20
50	0.3287	.3228	.5090	.9465	.9761	.3411	.5329	2.9319	.4671	1.2421	10
19° 00′	0.3316	.3256	9.5126	.9455	9.9757	.3443	9.5370	2.9042	0.4630	1.2392	71° 00′
10	0.3345	.3283	.5163	.9446	.9752	.3476	.5411	2.8770	.4589	1.2363	50
20	0.3374	.3311	.5199	.9436	.9748	.3508	.5451	2.8502	.4549	1.2334	40
30	0.3403	.3338	.5235	.9426	.9743	.3541	.5491	2.8239	.4509	1.2305	30
40	0.3432	.3365	.5270	.9417	.9739	.3574	.5531	2.7980	.4469	1.2275	20
50	0.3462	.3393	.5306	.9407	.9734	.3607	.5571	2.7725	.4429	1.2246	10
20° 00′	0.3491	.3420	9.5341	.9397	9.9730	.3640	9.5611	2.7475	0.4389	1.2217	70° 00′
10	0.3520	.3448	.5375	.9387	.9725	.3673	.5650	2.7228	.4350	1.2188	50
20	0.3549	.3475	.5409	.9377	.9721	.3706	.5689	2.6985	.4311	1.2159	40
30	0.3578	.3502	.5443	.9367	.9716	.3739	.5727	2.6746	.4273	1.2130	30
40	0.3607	.3529	.5477	.9356	.9711	.3772	.5766	2.6511	.4234	1.2101	20
50	0.3636	.3557	.5510	.9346	.9706	.3805	.5804	2.6279	.4196	1.2072	10
21° 00′	0.3665	.3584	9.5543	.9336	9.9702	.3839	9.5842	2.6051	0.4158	1.2043	69° 00′
10	0.3694	.3611	.5576	.9325	.9697	.3872	.5879	2.5826	.4121	1.2014	50
20	0.3723	.3638	.5609	.9315	.9692	.3906	.5917	2.5605	.4083	1.1985	40
30	0.3752	.3665	.5641	.9304	.9687	.3939	.5954	2.5386	.4046	1.1956	30
40	0.3782	.3692	.5673	.9293	.9682	.3973	.5991	2.5172	.4009	1.1926	20
50	0.3811	.3719	.5704	.9283	.9677	.4006	.6028	2.4960	.3972	1.1897	10
22° 00′	0.3840	.3746	9.5736	.9272	9.9672	.4040	9.6064	2.4751	0.3936	1.1868	68° 00′
10	0.3869	.3773	.5767	.9261	.9667	.4074	.6100	2.4545	.3900	1.1839	50
20	0.3898	.3800	.5798	.9250	.9661	.4108	.6136	2.4342	.3864	1.1810	40
30	0.3927	.3827	.5828	.9239	.9656	.4142	.6172	2.4142	.3828	1.1781	30
40	0.3956	.3854	.5859	.9228	.9651	.4176	.6208	2.3945	.3792	1.1752	20
50	0.3985	.3881	.5889	.9216	.9646	.4210	.6243	2.3750	.3757	1.1723	10
23° 00′	0.4014	.3907	9.5919	.9205	9.9640	.4245	9.6279	2.3559	0.3721	1.1694	67° 00′
10	0.4043	.3934	.5948	.9194	.9635	.4279	.6314	2.3369	.3686	1.1665	50
20	0.4072	.3961	.5978	.9182	.9629	.4314	.6348	2.3183	.3652	1.1636	40
30	0.4102	.3987	.6007	.9171	.9624	.4348	.6383	2.2998	.3617	1.1606	30
30	0.4131	.4014	.6036	.9159	.9618	.4383	.6417	2.2817	.3583	1.1577	20
50	0.4160	.4041	.6065	.9147	.9613	.4417	.6452	2.2637	.3548	1.1548	10
24° 00′	0.4189	.4067	9.6093	.9135	9.9607	.4452	9.6486	2.2460	0.3514	1.1519	66° 00′
10	0.4218	.4094	.6121	.9124	.9602	.4487	.6520	2.2286	.3480	1.1490	50
20	0.4247	.4120	.6149	.9112	.9596	.4522	.6553	2.2113	.3447	1.1461	40
30	0.4276	.4147	.6177	.9100	.9590	.4557	.6587	2.1943	.3413	1.1432	30
40	0.4305	.4173	.6205	.9088	.9584	.4592	.6620	2.1775	.3380	1.1403	20
50	0.4334	.4200	.6232	.9075	.9579	.4628	.6654	2.1609	.3346	1.1374	10
25° 00′	0.4363	.4226	9.6259	.9063	9.9573	.4663	9.6687	2.1445	0.3313	1.1345	65° 00′
10	0.4392	.4253	.6286	.9051	.9567	.4699	.6720	2.1283	.3280	1.1316	50
20	0.4422	.4279	.6313	.9038	.9561	.4734	.6752	2.1123	.3248	1.1286	40
30	0.4451	.4305	.6340	.9026	.9555	.4770	.6785	2.0965	.3215	1.1257	30
40	0.4480	.4331	.6366	.9013	.9549	.4806	.6817	2.0809	.3183	1.1228	20
50	0.4509	.4358	.6392	.9001	.9543	.4841	.6850	2.0655	.3150	1.1199	10
26° 00′	0.4538	.4384	9.6418	.8988	9.9537	.4877	9.6882	2.0503	0.3118	1.1170	64° 00′
10	0.4567	.4410	.6444	.8975	.9530	.4913	.6914	2.0353	.3086	1.1141	50
20	0.4596	.4436	.6470	.8962	.9524	.4950	.6946	2.0204	.3054	1.1112	40
30	0.4625	.4462	.6495	.8949	.9518	.4986	.6977	2.0057	.3023	1.1083	30
40	0.4654	.4488	.6521	.8936	.9512	.5022	.7009	1.9912	.2991	1.1054	20
50	0.4683	.4514	.6546	.8923	.9505	.5059	.7040	1.9768	.2960	1.1025	10
27° 00′	0.4712	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	1.0996	63° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS *(continued)*

Annex −10 in columns marked *. **(For 0°.1 intervals, see pp. 46–51)**

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
27° 00′	0.4712	.4540	9.6570	.8910	9.9499	.5095	9.7072	1.9626	0.2928	1.0996	63° 00′
10	0.4741	.4566	.6595	.8897	.9492	.5132	.7103	1.9486	.2897	1.0966	50
20	0.4771	.4592	.6620	.8884	.9486	.5169	.7134	1.9347	.2866	1.0937	40
30	0.4800	.4617	.6644	.8870	.9479	.5206	.7165	1.9210	.2835	1.0908	30
40	0.4829	.4643	.6668	.8857	.9473	.5243	.7196	1.9074	.2804	1.0879	20
50	0.4858	.4669	.6692	.8843	.9466	.5280	.7226	1.8940	.2774	1.0850	10
28° 00′	0.4887	.4695	9.6716	.8829	9.9459	.5317	9.7257	1.8807	0.2743	1.0821	62° 00′
10	0.4916	.4720	.6740	.8816	.9453	.5354	.7287	1.8676	.2713	1.0792	50
20	0.4945	.4746	.6763	.8802	.9446	.5392	.7317	1.8546	.2683	1.0763	40
30	0.4974	.4772	.6787	.8788	.9439	.5430	.7348	1.8418	.2652	1.0734	30
40	0.5003	.4797	.6810	.8774	.9432	.5467	.7378	1.8291	.2622	1.0705	20
50	0.5032	.4823	.6833	.8760	.9425	.5505	.7408	1.8165	.2592	1.0676	10
29° 00′	0.5061	.4848	9.6856	.8746	9.9418	.5543	9.7438	1.8040	0.2562	1.0647	61° 00′
10	0.5091	.4874	.6878	.8732	.9411	.5581	.7467	1.7917	.2533	1.0617	50
20	0.5120	.4899	.6901	.8718	.9404	.5619	.7497	1.7796	.2503	1.0588	40
30	0.5149	.4924	.6923	.8704	.9397	.5658	.7526	1.7675	.2474	1.0559	30
40	0.5178	.4950	.6946	.8689	.9390	.5696	.7556	1.7556	.2444	1.0530	20
50	0.5207	.4975	.6968	.8675	.9383	.5735	.7585	1.7437	.2415	1.0501	10
30° 00′	0.5236	.5000	9.6990	.8660	9.9375	.5774	9.7614	1.7321	0.2386	1.0472	60° 00′
10	0.5265	.5025	.7012	.8646	.9368	.5812	.7644	1.7205	.2356	1.0443	50
20	0.5294	.5050	.7033	.8631	.9361	.5851	.7673	1.7090	.2327	1.0414	40
30	0.5323	.5075	.7055	.8616	.9353	.5890	.7701	1.6977	.2299	1.0385	30
40	0.5352	.5100	.7076	.8601	.9346	.5930	.7730	1.6864	.2270	1.0356	20
50	0.5381	.5125	.7097	.8587	.9338	.5969	.7759	1.6753	.2241	1.0327	10
31° 00′	0.5411	.5150	9.7118	.8572	9.9331	.6009	9.7788	1.6643	0.2212	1.0297	59° 00′
10	0.5440	.5175	.7139	.8557	.9323	.6048	.7816	1.6534	.2184	1.0268	50
20	0.5469	.5200	.7160	.8542	.9315	.6088	.7845	1.6426	.2155	1.0239	40
30	0.5498	.5225	.7181	.8526	.9308	.6128	.7873	1.6319	.2127	1.0210	30
40	0.5527	.5250	.7201	.8511	.9300	.6168	.7902	1.6212	.2098	1.0181	20
50	0.5556	.5275	.7222	.8496	.9292	.6208	.7930	1.6107	.2070	1.0152	10
32° 00′	0.5585	.5299	9.7242	.8480	9.9284	.6249	9.7958	1.6003	0.2042	1.0123	58° 00′
10	0.5614	.5324	.7262	.8465	.9276	.6289	.7986	1.5900	.2014	1.0094	50
20	0.5643	.5348	.7282	.8450	.9268	.6330	.8014	1.5798	.1986	1.0065	40
30	0.5672	.5373	.7302	.8434	.9260	.6371	.8042	1.5697	.1958	1.0036	30
40	0.5701	.5398	.7322	.8418	.9252	.6412	.8070	1.5597	.1930	1.0007	20
50	0.5730	.5422	.7342	.8403	.9244	.6453	.8097	1.5497	.1903	0.9977	10
33° 00′	0.5760	.5446	9.7361	.8387	9.9236	.6494	9.8125	1.5399	0.1875	0.9948	57° 00′
10	0.5789	.5471	.7380	.8371	.9228	.6536	.8153	1.5301	.1847	0.9919	50
20	0.5818	.5495	.7400	.8355	.9219	.6577	.8180	1.5204	.1820	0.9890	40
30	0.5847	.5519	.7419	.8339	.9211	.6619	.8208	1.5108	.1792	0.9861	30
40	0.5876	.5544	.7438	.8323	.9203	.6661	.8235	1.5013	.1765	0.9832	20
50	0.5905	.5568	.7457	.8307	.9194	.6703	.8263	1.4919	.1737	0.9803	10
34° 00′	0.5934	.5592	9.7476	.8290	9.9186	.6745	9.8290	1.4826	0.1710	0.9774	56° 00′
10	0.5963	.5616	.7494	.8274	.9177	.6787	.8317	1.4733	.1683	0.9745	50
20	0.5992	.5640	.7513	.8258	.9169	.6830	.8344	1.4641	.1656	0.9716	40
30	0.6021	.5664	.7531	.8241	.9160	.6873	.8371	1.4550	.1629	0.9687	30
40	0.6050	.5688	.7550	.8225	.9151	.6916	.8398	1.4460	.1602	0.9657	20
50	0.6080	.5712	.7568	.8208	.9142	.6959	.8425	1.4370	.1575	0.9628	10
35° 00′	0.6109	.5736	9.7586	.8192	9.9134	.7002	9.8452	1.4281	0.1548	0.9599	55° 00′
10	0.6138	.5760	.7604	.8175	.9125	.7046	.8479	1.4193	.1521	0.9570	50
20	0.6167	.5783	.7622	.8158	.9116	.7089	.8506	1.4106	.1494	0.9541	40
30	0.6196	.5807	.7640	.8141	.9107	.7133	.8533	1.4019	.1467	0.9512	30
40	0.6225	.5831	.7657	.8124	.9098	.7177	.8559	1.3934	.1441	0.9483	20
50	0.6254	.5854	.7675	.8107	.9089	.7221	.8586	1.3848	.1414	0.9454	10
36° 00′	0.6283	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	0.9425	54° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

TRIGONOMETRIC FUNCTIONS (*continued*)

Annex −10 in columns marked *. (For 0°.1 intervals, see pp. 46–51)

Degrees	Radians	Sines		Cosines		Tangents		Cotangents			
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
36° 00′	0.6283	.5878	9.7692	.8090	9.9080	.7265	9.8613	1.3764	0.1387	0.9425	54° 00′
10	0.6312	.5901	.7710	.8073	.9070	.7310	.8639	1.3680	.1361	0.9396	50
20	0.6341	.5925	.7727	.8056	.9061	.7355	.8666	1.3597	.1334	0.9367	40
30	0.6370	.5948	.7744	.8039	.9052	.7400	.8692	1.3514	.1308	0.9338	30
40	0.6400	.5972	.7761	.8021	.9042	.7445	.8718	1.3432	.1282	0.9308	20
50	0.6429	.5995	.7778	.8004	.9033	.7490	.8745	1.3351	.1255	0.9279	10
37° 00′	0.6458	.6018	9.7795	.7986	9.9023	.7536	9.8771	1.3270	0.1229	0.9250	53° 00′
10	0.6487	.6041	.7811	.7969	.9014	.7581	.8797	1.3190	.1203	0.9221	50
20	0.6516	.6065	.7828	.7951	.9004	.7627	.8824	1.3111	.1176	0.9192	40
30	0.6545	.6088	.7844	.7934	.8995	.7673	.8850	1.3032	.1150	0.9163	30
40	0.6574	.6111	.7861	.7916	.8985	.7720	.8876	1.2954	.1124	0.9134	20
50	0.6603	.6134	.7877	.7898	.8975	.7766	.8902	1.2876	.1098	0.9105	10
38° 00′	0.6632	.6157	9.7893	.7880	9.8965	.7813	9.8928	1.2799	0.1072	0.9076	52° 00′
10	0.6661	.6180	.7910	.7862	.8955	.7860	.8954	1.2723	.1046	0.9047	50
20	0.6690	.6202	.7926	.7844	.8945	.7907	.8980	1.2647	.1020	0.9018	40
30	0.6720	.6225	.7941	.7826	.8935	.7954	.9006	1.2572	.0994	0.8988	30
40	0.6749	.6248	.7957	.7808	.8925	.8002	.9032	1.2497	.0968	0.8959	20
50	0.6778	.6271	.7973	.7790	.8915	.8050	.9058	1.2423	.0942	0.8930	10
39° 00′	0.6807	.6293	9.7989	.7771	9.8905	.8098	9.9084	1.2349	0.0916	0.8901	51° 00′
10	0.6836	.6316	.8004	.7753	.8895	.8146	.9110	1.2276	.0890	0.8872	50
20	0.6865	.6338	.8020	.7735	.8884	.8195	.9135	1.2203	.0865	0.8843	40
30	0.6894	.6361	.8035	.7716	.8874	.8243	.9161	1.2131	.0839	0.8814	30
40	0.6923	.6383	.8050	.7698	.8864	.8292	.9187	1.2059	.0813	0.8785	20
50	0.6952	.6406	.8066	.7679	.8853	.8342	.9212	1.1988	.0788	0.8756	10
40° 00′	0.6981	.6428	9.8081	.7660	9.8843	.8391	9.9238	1.1918	0.0762	0.8727	50° 00′
10	0.7010	.6450	.8096	.7642	.8832	.8441	.9264	1.1847	.0736	0.8698	50
20	0.7039	.6472	.8111	.7623	.8821	.8491	.9289	1.1778	.0711	0.8668	40
30	0.7069	.6494	.8125	.7604	.8810	.8541	.9315	1.1708	.0685	0.8639	30
40	0.7098	.6517	.8140	.7585	.8800	.8591	.9341	1.1640	.0659	0.8610	20
50	0.7127	.6539	.8155	.7566	.8789	.8642	.9366	1.1571	.0634	0.8581	10
41° 00′	0.7156	.6561	9.8169	.7547	9.8778	.8693	9.9392	1.1504	0.0608	0.8552	49° 00′
10	0.7185	.6583	.8184	.7528	.8767	.8744	.9417	1.1436	.0583	0.8523	50
20	0.7214	.6604	.8198	.7509	.8756	.8796	.9443	1.1369	.0557	0.8494	40
30	0.7243	.6626	.8213	.7490	.8745	.8847	.9468	1.1303	.0532	0.8465	30
40	0.7272	.6648	.8227	.7470	.8733	.8899	.9494	1.1237	.0506	0.8436	20
50	0.7301	.6670	.8241	.7451	.8722	.8952	.9519	1.1171	.0481	0.8407	10
42° 00′	0.7330	.6691	9.8255	.7431	9.8711	.9004	9.9544	1.1106	0.0456	0.8378	48° 00′
10	0.7359	.6713	.8269	.7412	.8699	.9057	.9570	1.1041	.0430	0.8348	50
20	0.7389	.6734	.8283	.7392	.8688	.9110	.9595	1.0977	.0405	0.8319	40
30	0.7418	.6756	.8297	.7373	.8676	.9163	.9621	1.0913	.0379	0.8290	30
40	0.7447	.6777	.8311	.7353	.8665	.9217	.9646	1.0850	.0354	0.8261	20
50	0.7476	.6799	.8324	.7333	.8653	.9271	.9671	1.0786	.0329	0.8232	10
43° 00′	0.7505	.6820	9.8338	.7314	9.8641	.9325	9.9697	1.0724	0.0303	0.8203	47° 00′
10	0.7534	.6841	.8351	.7294	.8629	.9380	.9722	1.0661	.0278	0.8174	50
20	0.7563	.6862	.8365	.7274	.8618	.9435	.9747	1.0599	.0253	0.8145	40
30	0.7592	.6884	.8378	.7254	.8606	.9490	.9772	1.0538	.0228	0.8116	30
40	0.7621	.6905	.8391	.7234	.8594	.9545	.9798	1.0477	.0202	0.8087	20
50	0.7650	.6926	.8405	.7214	.8582	.9601	.9823	1.0416	.0177	0.8058	10
44° 00′	0.7679	.6947	9.8418	.7193	9.8569	.9657	9.9848	1.0355	0.0152	0.8029	46° 00′
10	0.7709	.6967	.8431	.7173	.8557	.9713	.9874	1.0295	.0126	0.7999	50
20	0.7738	.6988	.8444	.7153	.8545	.9770	.9899	1.0235	.0101	0.7970	40
30	0.7767	.7009	.8457	.7133	.8532	.9827	.9924	1.0176	.0076	0.7941	30
40	0.7796	.7030	.8469	.7112	.8520	.9884	.9949	1.0117	.0051	0.7912	20
50	0.7825	.7050	.8482	.7092	.8507	.9942	.9975	1.0058	.0025	0.7883	10
45° 00′	0.7854	.7071	9.8495	.7071	9.8495	1.0000	0.0000	1.0000	0.0000	0.7854	45° 00′
		Nat.	Log.*	Nat.	Log.*	Nat.	Log.*	Nat.	Log.		
		Cosines		Sines		Cotangents		Tangents		Radians	Degrees

EXPONENTIALS [e^n and e^{-n}]

n	e^n	Diff.	n	e^n	Diff.	n	e^n	n	e^{-n}	Diff.	n	e^{-n}	n	e^{-n}
0.00	1.000	10	0.50	1.649	16	1.0	2.718*	0.00	1.000	−10	0.50	.607	1.0	.368*
.01	1.010	10	.51	1.665	17	.1	3.004	.01	0.990	−10	.51	.600	.1	.333
.02	1.020	10	.52	1.682	17	.2	3.320	.02	.980	−10	.52	.595	.2	.301
.03	1.030	11	.53	1.699	17	.3	3.669	.03	.970	−9	.53	.589	.3	.273
.04	1.041	10	.54	1.716	17	.4	4.055	.04	.961	−10	.54	.583	.4	.247
0.05	1.051	11	0.55	1.733	18	1.5	4.482	0.05	.951	−9	0.55	.577	1.5	.223
.06	1.062	11	.56	1.751	17	.6	4.953	.06	.942	−10	.56	.571	.6	.202
.07	1.073	10	.57	1.768	18	.7	5.474	.07	.932	−9	.57	.566	.7	.183
.08	1.083	11	.58	1.786	18	.8	6.050	.08	.923	−9	.58	.560	.8	.165
.09	1.094	11	.59	1.804	18	.9	6.686	.09	.914	−9	.59	.554	.9	.150
0.10	1.105	11	0.60	1.822	18	2.0	7.389	0.10	.905	−9	0.60	.549	2.0	.135
.11	1.116	11	.61	1.840	19	.1	8.166	.11	.896	−9	.61	.543	.1	.122
.12	1.127	12	.62	1.859	19	.2	9.025	.12	.887	−9	.62	.538	.2	.111
.13	1.139	11	.63	1.878	18	.3	9.974	.13	.878	−9	.63	.533	.3	.100
.14	1.150	12	.64	1.896	20	.4	11.02	.14	.869	−8	.64	.527	.4	.0907
0.15	1.162	12	0.65	1.916	19	2.5	12.18	0.15	.861	−9	0.65	.522	2.5	.0821
.16	1.174	11	.66	1.935	19	.6	13.46	.16	.852	−8	.66	.517	.6	.0743
.17	1.185	12	.67	1.954	20	.7	14.88	.17	.844	−9	.67	.512	.7	.0672
.18	1.197	12	.68	1.974	20	.8	16.44	.18	.835	−8	.68	.507	.8	.0608
.19	1.209	12	.69	1.994	20	.9	18.17	.19	.827	−8	.69	.502	.9	.0550
0.20	1.221	13	0.70	2.014	20	3.0	20.09	0.20	.819	−8	0.70	.497	3.0	.0498
.21	1.234	12	.71	2.034	20	.1	22.20	.21	.811	−8	.71	.492	.1	.0450
.22	1.246	13	.72	2.054	21	.2	24.53	.22	.803	−8	.72	.487	.2	.0408
.23	1.259	12	.73	2.075	21	.3	27.11	.23	.795	−8	.73	.482	.3	.0369
.24	1.271	13	.74	2.096	21	.4	29.96	.24	.787	−8	.74	.477	.4	.0334
0.25	1.284	13	0.75	2.117	21	3.5	33.12	0.25	.779	−8	0.75	.472	3.5	.0302
.26	1.297	13	.76	2.138	22	.6	36.60	.26	.771	−8	.76	.468	.6	.0273
.27	1.310	13	.77	2.160	21	.7	40.45	.27	.763	−7	.77	.463	.7	.0247
.28	1.323	13	.78	2.181	22	.8	44.70	.28	.756	−8	.78	.458	.8	.0224
.29	1.336	14	.79	2.203	23	.9	49.40	.29	.748	−7	.79	.454	.9	.0202
0.30	1.350	13	0.80	2.226	22	4.0	54.60	0.30	.741	−8	0.80	.449	4.0	.0183
.31	1.363	14	.81	2.248	22	.1	60.34	.31	.733	−7	.81	.445	.1	.0166
.32	1.377	14	.82	2.270	23	.2	66.69	.32	.726	−7	.82	.440	.2	.0150
.33	1.391	14	.83	2.293	23	.3	73.70	.33	.719	−7	.83	.436	.3	.0136
.34	1.405	14	.84	2.316	24	.4	81.45	.34	.712	−7	.84	.432	.4	.0123
0.35	1.419	14	0.85	2.340	23	4.5	90.02	0.35	.705	−7	0.85	.427	4.5	.0111
.36	1.433	15	.86	2.363	24			.36	.698	−7	.86	.423		
.37	1.448	14	.87	2.387	24	5.0	148.4	.37	.691	−7	.87	.419	5.0	.00674
.38	1.462	15	.88	2.411	24	6.0	403.4	.38	.684	−7	.88	.415	6.0	.00248
.39	1.477	15	.89	2.435	25	7.0	1097.	.39	.677	−7	.89	.411	7.0	.000912
0.40	1.492	15	0.90	2.460	24	8.0	2981.	0.40	.670	−6	0.90	.407	8.0	.000335
.41	1.507	15	.91	2.484	25	9.0	8103.	.41	.664	−7	.91	.403	9.0	.000123
.42	1.522	15	.92	2.509	26	10.0	22026.	.42	.657	−6	.92	.399	10.0	.000045
.43	1.537	16	.93	2.535	25	$\pi/2$	4.810	.43	.651	−7	.93	.395	$\pi/2$	.208
.44	1.553	15	.94	2.560	26	$2\pi/2$	23.14	.44	.644	−6	.94	.391	$2\pi/2$	.0432
0.45	1.568	16	0.95	2.586	26	$3\pi/2$	111.3	0.45	.638	−7	0.95	.387	$3\pi/2$	.00898
.46	1.584	16	.96	2.612	26	$4\pi/2$	535.5	.46	.631	−6	.96	.383	$4\pi/2$	.00187
.47	1.600	16	.97	2.638	26	$5\pi/2$	2576.	.47	.625	−6	.97	.379	$5\pi/2$	.000388
.48	1.616	16	.98	2.664	27	$6\pi/2$	12392.	.48	.619	−6	.98	.375	$6\pi/2$	.000081
.49	1.632	17	.99	2.691	27	$7\pi/2$	59610.	.49	.613	−6	.99	.372	$7\pi/2$	.000017
0.50	1.649		1.00	2.718		$8\pi/2$	286751.	0.50	0.607		1.00	.368	$8\pi/2$	.000003

* **Note**: Do not interpolate in this column.

$e = 2.71828$ $1/e = 0.367879$ $\log_{10} e = 0.4343$ $1/(0.4343) = 2.3026$

$\log_{10}(0.4343) = \bar{1}.6378$ $\log_{10}(e^n) = n(0.4343)$

For table of multiples of 0.4343, see p. 62. Graphs, p. 174.

HYPERBOLIC LOGARITHMS

	n	n (2.3026)	n (0.6974–3)
These two pages give the natural (hyperbolic, or Napierian) logarithms ($\log_e$) of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right [or left] in the number is equivalent to adding n times 2.3026 [or n times $\bar{3}.6974$] to the logarithm. Base $e = 2.71828+$	1	2.3026	0.6974–3
	2	4.6052	0.3948–5
	3	6.9078	0.0922–7
	4	9.2103	0.7897–10
	5	11.5129	0.4871–12
	6	13.8155	0.1845–14
	7	16.1181	0.8819–17
	8	18.4207	0.5793–19
	9	20.7233	0.2767–21

Number	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	0.0000	0100	0198	0296	0392	0488	0583	0677	0770	0862	95
1.1	0953	1044	1133	1222	1310	1398	1484	1570	1655	1740	87
1.2	1823	1906	1989	2070	2151	2231	2311	2390	2469	2546	80
1.3	2624	2700	2776	2852	2927	3001	3075	3148	3221	3293	74
1.4	3365	3436	3507	3577	3646	3716	3784	3853	3920	3988	69
1.5	0.4055	4121	4187	4253	4318	4383	4447	4511	4574	4637	65
1.6	4700	4762	4824	4886	4947	5008	5068	5128	5188	5247	61
1.7	5306	5365	5423	5481	5539	5596	5653	5710	5766	5822	57
1.8	5878	5933	5988	6043	6098	6152	6206	6259	6313	6366	54
1.9	6419	6471	6523	6575	6627	6678	6729	6780	6831	6881	51
2.0	0.6931	6981	7031	7080	7129	7178	7227	7275	7324	7372	49
2.1	7419	7467	7514	7561	7608	7655	7701	7747	7793	7839	47
2.2	7885	7930	7975	8020	8065	8109	8154	8198	8242	8286	44
2.3	8329	8372	8416	8459	8502	8544	8587	8629	8671	8713	43
2.4	8755	8796	8838	8879	8920	8961	9002	9042	9083	9123	41
2.5	0.9163	9203	9243	9282	9322	9361	9400	9439	9478	9517	39
2.6	9555	9594	9632	9670	9708	9746	9783	9821	9858	9895	38
2.7	0.9933	9969	*0006	*0043	*0080	*0116	*0152	*0188	*0225	*0260	36
2.8	1.0296	0332	0367	0403	0438	0473	0508	0543	0578	0613	35
2.9	0647	0682	0716	0750	0784	0818	0852	0886	0919	0953	34
3.0	1.0986	1019	1053	1086	1119	1151	1184	1217	1249	1282	33
3.1	1314	1346	1378	1410	1442	1474	1506	1537	1569	1600	32
3.2	1632	1663	1694	1725	1756	1787	1817	1848	1878	1909	31
3.3	1939	1969	2000	2030	2060	2090	2119	2149	2179	2208	30
3.4	2238	2267	2296	2326	2355	2384	2413	2442	2470	2499	29
3.5	1.2528	2556	2585	2613	2641	2669	2698	2726	2754	2782	28
3.6	2809	2837	2865	2892	2920	2947	2975	3002	3029	3056	27
3.7	3083	3110	3137	3164	3191	3218	3244	3271	3297	3324	27
3.8	3350	3376	3403	3429	3455	3481	3507	3533	3558	3584	26
3.9	3610	3635	3661	3686	3712	3737	3762	3788	3813	3838	25
4.0	1.3863	3888	3913	3938	3962	3987	4012	4036	4061	4085	25
4.1	4110	4134	4159	4183	4207	4231	4255	4279	4303	4327	24
4.2	4351	4375	4398	4422	4446	4469	4493	4516	4540	4563	23
4.3	4586	4609	4633	4656	4679	4702	4725	4748	4770	4793	23
4.4	4816	4839	4861	4884	4907	4929	4951	4974	4996	5019	22
4.5	1.5041	5063	5085	5107	5129	5151	5173	5195	5217	5239	22
4.6	5261	5282	5304	5326	5347	5369	5390	5412	5433	5454	21
4.7	5476	5497	5518	5539	5560	5581	5602	5623	5644	5665	21
4.8	5686	5707	5728	5748	5769	5790	5810	5831	5851	5872	20
4.9	5892	5913	5933	5953	5974	5994	6014	6034	6054	6074	20

$\log_e x = (2.3026) \log_{10} x$ $\qquad$ $\log_{10} x = (0.4343) \log_e x$

where $2.3026 = \log_e 10$ and $0.4343 = \log_{10} e$ (see p. 62). For graphs, see p. 174.

HYPERBOLIC LOGARITHMS (*continued*)

Number	0	1	2	3	4	5	6	7	8	9
5.0	1.6094	6114	6134	6154	6174	6194	6214	6233	6253	6273
5.1	6292	6312	6332	6351	6371	6390	6409	6429	6448	6467
5.2	6487	6506	6525	6544	6563	6582	6601	6620	6639	6658
5.3	6677	6696	6715	6734	6752	6771	6790	6808	6827	6845
5.4	6864	6882	6901	6919	6938	6956	6974	6993	7011	7029
5.5	1.7047	7066	7084	7102	7120	7138	7156	7174	7192	7210
5.6	7228	7246	7263	7281	7299	7317	7334	7352	7370	7387
5.7	7405	7422	7440	7457	7475	7492	7509	7527	7544	7561
5.8	7579	7596	7613	7630	7647	7664	7681	7699	7716	7733
5.9	7750	7766	7783	7800	7817	7834	7851	7867	7884	7901
6.0	1.7918	7934	7951	7967	7984	8001	8017	8034	8050	8066
6.1	8083	8099	8116	8132	8148	8165	8181	8197	8213	8229
6.2	8245	8262	8278	8294	8310	8326	8342	8358	8374	8390
6.3	8405	8421	8437	8453	8469	8485	8500	8516	8532	8547
6.4	8563	8579	8594	8610	8625	8641	8656	8672	8687	8703
6.5	1.8718	8733	8749	8764	8779	8795	8810	8825	8840	8856
6.6	8871	8886	8901	8916	8931	8946	8961	8976	8991	9006
6.7	9021	9036	9051	9066	9081	9095	9110	9125	9140	9155
6.8	9169	9184	9199	9213	9228	9242	9257	9272	9286	9301
6.9	9315	9330	9344	9359	9373	9387	9402	9416	9430	9445
7.0	1.9459	9473	9488	9502	9516	9530	9544	9559	9573	9587
7.1	9601	9615	9629	9643	9657	9671	9685	9699	9713	9727
7.2	9741	9755	9769	9782	9796	9810	9824	9838	9851	9865
7.3	1.9879	9892	9906	9920	9933	9947	9961	9974	9988	*0001
7.4	2.0015	0028	0042	0055	0069	0082	0096	0109	0122	0136
7.5	2.0149	0162	0176	0189	0202	0215	0229	0242	0255	0268
7.6	0281	0295	0308	0321	0334	0347	0360	0373	0386	0399
7.7	0412	0425	0438	0451	0464	0477	0490	0503	0516	0528
7.8	0541	0554	0567	0580	0592	0605	0618	0631	0643	0656
7.9	0669	0681	0694	0707	0719	0732	0744	0757	0769	0782
8.0	2.0794	0807	0819	0832	0844	0857	0869	0882	0894	0906
8.1	0919	0931	0943	0956	0968	0980	0992	1005	1017	1029
8.2	1041	1054	1066	1078	1090	1102	1114	1126	1138	1150
8.3	1163	1175	1187	1199	1211	1223	1235	1247	1258	1270
8.4	1282	1294	1306	1318	1330	1342	1353	1365	1377	1389
8.5	2.1401	1412	1424	1436	1448	1459	1471	1483	1494	1506
8.6	1518	1529	1541	1552	1564	1576	1587	1599	1610	1622
8.7	1633	1645	1656	1668	1679	1691	1702	1713	1725	1736
8.8	1748	1759	1770	1782	1793	1804	1815	1827	1838	1849
8.9	1861	1872	1883	1894	1905	1917	1928	1939	1950	1961
9.0	2.1972	1983	1994	2006	2017	2028	2039	2050	2061	2072
9.1	2083	2094	2105	2116	2127	2138	2148	2159	2170	2181
9.2	2192	2203	2214	2225	2235	2246	2257	2268	2279	2289
9.3	2300	2311	2322	2332	2343	2354	2364	2375	2386	2396
9.4	2407	2418	2428	2439	2450	2460	2471	2481	2492	2502
9.5	2.2513	2523	2534	2544	2555	2565	2576	2586	2597	2607
9.6	2618	2628	2638	2649	2659	2670	2680	2690	2701	2711
9.7	2721	2732	2742	2752	2762	2773	2783	2793	2803	2814
9.8	2824	2834	2844	2854	2865	2875	2885	2895	2905	2915
9.9	2925	2935	2946	2956	2966	2976	2986	2996	3006	3016
10.0	2.3026									

Moving the decimal point *n* places to the right [or left] in the number requires ac *n* times 2.3026 [or *n* times (0.6974–3)] in the body of the table. See auxiliary tab multiples on top of the preceding page.

HYPERBOLIC SINES [sinh $x = \frac{1}{2}(e^x - e^{-x})$]

x	0	1	2	3	4	5	6	7	8	9	Avg. diff.
0.0	.0000	.0100	.0200	.0300	.0400	.0500	.0600	.0701	.0801	.0901	100
1	.1002	.1102	.1203	.1304	.1405	.1506	.1607	.1708	.1810	.1911	101
2	.2013	.2115	.2218	.2320	.2423	.2526	.2629	.2733	.2837	.2941	103
3	.3045	.3150	.3255	.3360	.3466	.3572	.3678	.3785	.3892	.4000	106
4	.4108	.4216	.4325	.4434	.4543	.4653	.4764	.4875	.4986	.5098	110
0.5	.5211	.5324	.5438	.5552	.5666	.5782	.5897	.6014	.6131	.6248	116
6	.6367	.6485	.6605	.6725	.6846	.6967	.7090	.7213	.7336	.7461	122
7	.7586	.7712	.7838	.7966	.8094	.8223	.8353	.8484	.8615	.8748	130
8	.8881	.9015	.9150	.9286	.9423	.9561	.9700	.9840	.9981	1.012	138
9	1.027	1.041	1.055	1.070	1.085	1.099	1.114	1.129	1.145	1.160	15
1.0	1.175	1.191	1.206	1.222	1.238	1.254	1.270	1.286	1.303	1.319	16
1	1.336	1.352	1.369	1.386	1.403	1.421	1.438	1.456	1.474	1.491	17
2	1.509	1.528	1.546	1.564	1.583	1.602	1.621	1.640	1.659	1.679	19
3	1.698	1.718	1.738	1.758	1.779	1.799	1.820	1.841	1.862	1.883	21
4	1.904	1.926	1.948	1.970	1.992	2.014	2.037	2.060	2.083	2.106	22
1.5	2.129	2.153	2.177	2.201	2.225	2.250	2.274	2.299	2.324	2.350	25
6	2.376	2.401	2.428	2.454	2.481	2.507	2.535	2.562	2.590	2.617	27
7	2.646	2.674	2.703	2.732	2.761	2.790	2.820	2.850	2.881	2.911	30
8	2.942	2.973	3.005	3.037	3.069	3.101	3.134	3.167	3.200	3.234	33
9	3.268	3.303	3.337	3.372	3.408	3.443	3.479	3.516	3.552	3.589	36
2.0	3.627	3.665	3.703	3.741	3.780	3.820	3.859	3.899	3.940	3.981	39
1	4.022	4.064	4.106	4.148	4.191	4.234	4.278	4.322	4.367	4.412	44
2	4.457	4.503	4.549	4.596	4.643	4.691	4.739	4.788	4.837	4.887	48
3	4.937	4.988	5.039	5.090	5.142	5.195	5.248	5.302	5.356	5.411	53
4	5.466	5.522	5.578	5.635	5.693	5.751	5.810	5.869	5.929	5.989	58
2.5	6.050	6.112	6.174	6.237	6.300	6.365	6.429	6.495	6.561	6.627	64
6	6.695	6.763	6.831	6.901	6.971	7.042	7.113	7.185	7.258	7.332	71
7	7.406	7.481	7.557	7.634	7.711	7.789	7.868	7.948	8.028	8.110	79
8	8.192	8.275	8.359	8.443	8.529	8.615	8.702	8.790	8.879	8.969	87
9	9.060	9.151	9.244	9.337	9.431	9.527	9.623	9.720	9.819	9.918	96
3.0	10.02	10.12	10.22	10.32	10.43	10.53	10.64	10.75	10.86	10.97	11
1	11.08	11.19	11.30	11.42	11.53	11.65	11.76	11.88	12.00	12.12	12
2	12.25	12.37	12.49	12.62	12.75	12.88	13.01	13.14	13.27	13.40	13
3	13.54	13.67	13.81	13.95	14.09	14.23	14.38	14.52	14.67	14.82	14
4	14.97	15.12	15.27	15.42	15.58	15.73	15.89	16.05	16.21	16.38	16
3.5	16.54	16.71	16.88	17.05	17.22	17.39	17.57	17.74	17.92	18.10	17
6	18.29	18.47	18.66	18.84	19.03	19.22	19.42	19.61	19.81	20.01	19
7	20.21	20.41	20.62	20.83	21.04	21.25	21.46	21.68	21.90	22.12	21
8	22.34	22.56	22.79	23.02	23.25	23.49	23.72	23.96	24.20	24.45	24
9	24.69	24.94	25.19	25.44	25.70	25.96	26.22	26.48	26.75	27.02	26
4.0	27.29	27.56	27.84	28.12	28.40	28.69	28.98	29.27	29.56	29.86	29
1	30.16	30.47	30.77	31.08	31.39	31.71	32.03	32.35	32.68	33.00	32
2	33.34	33.67	34.01	34.35	34.70	35.05	35.40	35.75	36.11	36.48	35
3	36.84	37.21	37.59	37.97	38.35	38.73	39.12	39.52	39.91	40.31	39
4	40.72	41.13	41.54	41.96	42.38	42.81	43.24	43.67	44.11	44.56	43
4.5	45.00	45.46	45.91	46.37	46.84	47.31	47.79	48.27	48.75	49.24	47
6	49.74	50.24	50.74	51.25	51.77	52.29	52.81	53.34	53.88	54.42	52
7	54.97	55.52	56.08	56.64	57.21	57.79	58.37	58.96	59.55	60.15	58
8	60.75	61.36	61.98	62.60	63.23	63.87	64.51	65.16	65.81	66.47	64
9	67.14	67.82	68.50	69.19	69.88	70.58	71.29	72.01	72.73	73.46	71
5.0	74.20										

If $x > 5$, sinh $x = \frac{1}{2}(e^x)$ and $\log_{10}$ sinh $x = (0.4343)x + 0.6990 - 1$, correct to four significant figures. **For table of multiples of 0.4343, see p. 62. Graphs, p. 174.**

HYPERBOLIC COSINES [$\cosh x = \frac{1}{2}(e^x + e^{-x})$]

x	0	1	2	3	4	5	6	7	8	9	Avg. diff.
0.0	1.000	1.000	1.000	1.000	1.001	1.001	1.002	1.002	1.003	1.004	1
1	1.005	1.006	1.007	1.008	1.010	1.011	1.013	1.014	1.016	1.018	2
2	1.020	1.022	1.024	1.027	1.029	1.031	1.034	1.037	1.039	1.042	3
3	1.045	1.048	1.052	1.055	1.058	1.062	1.066	1.069	1.073	1.077	4
4	1.081	1.085	1.090	1.094	1.098	1.103	1.108	1.112	1.117	1.122	5
0.5	1.128	1.133	1.138	1.144	1.149	1.155	1.161	1.167	1.173	1.179	6
6	1.185	1.192	1.198	1.205	1.212	1.219	1.226	1.233	1.240	1.248	7
7	1.255	1.263	1.271	1.278	1.287	1.295	1.303	1.311	1.320	1.329	8
8	1.337	1.346	1.355	1.365	1.374	1.384	1.393	1.403	1.413	1.423	10
9	1.433	1.443	1.454	1.465	1.475	1.486	1.497	1.509	1.520	1.531	11
1.0	1.543	1.555	1.567	1.579	1.591	1.604	1.616	1.629	1.642	1.655	13
1	1.669	1.682	1.696	1.709	1.723	1.737	1.752	1.766	1.781	1.796	14
2	1.811	1.826	1.841	1.857	1.872	1.888	1.905	1.921	1.937	1.954	16
3	1.971	1.988	2.005	2.023	2.040	2.058	2.076	2.095	2.113	2.132	18
4	2.151	2.170	2.189	2.209	2.229	2.249	2.269	2.290	2.310	2.331	20
1.5	2.352	2.374	2.395	2.417	2.439	2.462	2.484	2.507	2.530	2.554	23
6	2.577	2.601	2.625	2.650	2.675	2.700	2.725	2.750	2.776	2.802	25
7	2.828	2.855	2.882	2.909	2.936	2.964	2.992	3.021	3.049	3.078	28
8	3.107	3.137	3.167	3.197	3.228	3.259	3.290	3.321	3.353	3.385	31
9	3.418	3.451	3.484	3.517	3.551	3.585	3.620	3.655	3.690	3.726	34
2.0	3.762	3.799	3.835	3.873	3.910	3.948	3.987	4.026	4.065	4.104	38
1	4.144	4.185	4.226	4.267	4.309	4.351	4.393	4.436	4.480	4.524	42
2	4.568	4.613	4.658	4.704	4.750	4.797	4.844	4.891	4.939	4.988	47
3	5.037	5.087	5.137	5.188	5.239	5.290	5.343	5.395	5.449	5.503	52
4	5.557	5.612	5.667	5.723	5.780	5.837	5.895	5.954	6.013	6.072	58
2.5	6.132	6.193	6.255	6.317	6.379	6.443	6.507	6.571	6.636	6.702	64
6	6.769	6.836	6.904	6.973	7.042	7.112	7.183	7.255	7.327	7.400	70
7	7.473	7.548	7.623	7.699	7.776	7.853	7.932	8.011	8.091	8.171	78
8	8.253	8.335	8.418	8.502	8.587	8.673	8.759	8.847	8.935	9.024	86
9	9.115	9.206	9.298	9.391	9.484	9.579	9.675	9.772	9.869	9.968	95
3.0	10.07	10.17	10.27	10.37	10.48	10.58	10.69	10.79	10.90	11.01	11
1	11.12	11.23	11.35	11.46	11.57	11.69	11.81	11.92	12.04	12.16	12
2	12.29	12.41	12.53	12.66	12.79	12.91	13.04	13.17	13.31	13.44	13
3	13.57	13.71	13.85	13.99	14.13	14.27	14.41	14.56	14.70	14.85	14
4	15.00	15.15	15.30	15.45	15.61	15.77	15.92	16.08	16.25	16.41	16
3.5	16.57	16.74	16.91	17.08	17.25	17.42	17.60	17.77	17.95	18.13	17
6	18.31	18.50	18.68	18.87	19.06	19.25	19.44	19.64	19.84	20.03	19
7	20.24	20.44	20.64	20.85	21.06	21.27	21.49	21.70	21.92	22.14	21
8	22.36	22.59	22.81	23.04	23.27	23.51	23.74	23.98	24.22	24.47	23
9	24.71	24.96	25.21	25.46	25.72	25.98	26.24	26.50	26.77	27.04	26
4.0	27.31	27.58	27.86	28.14	28.42	28.71	29.00	29.29	29.58	29.88	29
1	30.18	30.48	30.79	31.10	31.41	31.72	32.04	32.37	32.69	33.02	32
2	33.35	33.69	34.02	34.37	34.71	35.06	35.41	35.77	36.13	36.49	35
3	36.86	37.23	37.60	37.98	38.36	38.75	39.13	39.53	39.93	40.33	39
4	40.73	41.14	41.55	41.97	42.39	42.82	43.25	43.68	44.12	44.57	43
4.5	45.01	45.47	45.92	46.38	46.85	47.32	47.80	48.28	48.76	49.25	47
6	49.75	50.25	50.75	51.26	51.78	52.30	52.82	53.35	53.89	54.43	52
7	54.98	55.53	56.09	56.65	57.22	57.80	58.38	58.96	59.56	60.15	58
8	60.76	61.37	61.99	62.61	63.24	63.87	64.52	65.16	65.82	66.48	64
9	67.15	67.82	68.50	69.19	69.89	70.59	71.30	72.02	72.74	73.47	71
5.0	74.21										

If $x > 5$, $\cosh x = \frac{1}{2}(e^x)$ and $\log_{10} \cosh x = (0.4343)x + 0.6990 - 1$, correct to four significant figures. For table of multiples of 0.4343, see p. 62. Graphs, p. 174.

HYPERBOLIC TANGENTS [$\tanh x = (e^x - e^{-x})/(e^x + e^{-x}) = \sinh x/\cosh x$]

x	0	1	2	3	4	5	6	7	8	9	Avg. diff.
0.0	.0000	.0100	.0200	.0300	.0400	.0500	.0599	.0699	.0798	.0898	100
1	.0997	.1096	.1194	.1293	.1391	.1489	.1587	.1684	.1781	.1878	98
2	.1974	.2070	.2165	.2260	.2355	.2449	.2543	.2636	.2729	.2821	94
3	.2913	.3004	.3095	.3185	.3275	.3364	.3452	.3540	.3627	.3714	89
4	.3800	.3885	.3969	.4053	.4136	.4219	.4301	.4382	.4462	.4542	82
0.5	.4621	.4700	.4777	.4854	.4930	.5005	.5080	.5154	.5227	.5299	75
6	.5370	.5441	.5511	.5581	.5649	.5717	.5784	.5850	.5915	.5980	67
7	.6044	.6107	.6169	.6231	.6291	.6352	.6411	.6469	.6527	.6584	60
8	.6640	.6696	.6751	.6805	.6858	.6911	.6963	.7014	.7064	.7114	52
9	.7163	.7211	.7259	.7306	.7352	.7398	.7443	.7487	.7531	.7574	45
1.0	.7616	.7658	.7699	.7739	.7779	.7818	.7857	.7895	.7932	.7969	39
1	.8005	.8041	.8076	.8110	.8144	.8178	.8210	.8243	.8275	.8306	33
2	.8337	.8367	.8397	.8426	.8455	.8483	.8511	.8538	.8565	.8591	28
3	.8617	.8643	.8668	.8693	.8717	.8741	.8764	.8787	.8810	.8832	24
4	.8854	.8875	.8896	.8917	.8937	.8957	.8977	.8996	.9015	.9033	20
1.5	.9052	.9069	.9087	.9104	.9121	.9138	.9154	.9170	.9186	.9202	17
6	.9217	.9232	.9246	.9261	.9275	.9289	.9302	.9316	.9329	.9342	14
7	.9354	.9367	.9379	.9391	.9402	.9414	.9425	.9436	.9447	.9458	11
8	.9468	.9478	.9488	.9498	.9508	.9518	.9527	.9536	.9545	.9554	9
9	.9562	.9571	.9579	.9587	.9595	.9603	.9611	.9619	.9626	.9633	8
2.0	.9640	.9647	.9654	.9661	.9668	.9674	.9680	.9687	.9693	.9699	6
1	.9705	.9710	.9716	.9722	.9727	.9732	.9738	.9743	.9748	.9753	5
2	.9757	.9762	.9767	.9771	.9776	.9780	.9785	.9789	.9793	.9797	4
3	.9801	.9805	.9809	.9812	.9816	.9820	.9823	.9827	.9830	.9834	4
4	.9837	.9840	.9843	.9846	.9849	.9852	.9855	.9858	.9861	.9863	3
2.5	.9866	.9869	.9871	.9874	.9876	.9879	.9881	.9884	.9886	.9888	2
6	.9890	.9892	.9895	.9897	.9899	.9901	.9903	.9905	.9906	.9908	2
7	.9910	.9912	.9914	.9915	.9917	.9919	.9920	.9922	.9923	.9925	2
8	.9926	.9928	.9929	.9931	.9932	.9933	.9935	.9936	.9937	.9938	1
2.9	.9940	.9941	.9942	.9943	.9944	.9945	.9946	.9947	.9949	.9950	1
3.	.9951	.9959	.9967	.9973	.9978	.9982	.9985	.9988	.9990	.9992	4
4.	.9993	.9995	.9996	.9996	.9997	.9998	.9998	.9998	.9999	.9999	1
5.	.9999										

If $x > 5$, $\tanh x = 1.0000$ to four decimal places. Graphs, p. 174.

MULTIPLES OF 0.4343 ($0.43429448 = \log_{10} e$)

x	0	1	2	3	4	5	6	7	8	9
0.	0.0000	0.0434	0.0869	0.1303	0.1737	0.2171	0.2606	0.3040	0.3474	0.3909
1.	0.4343	0.4777	0.5212	0.5646	0.6080	0.6514	0.6949	0.7383	0.7817	0.8252
2.	0.8686	0.9120	0.9554	0.9989	1.0423	1.0857	1.1292	1.1726	1.2160	1.2595
3.	1.3029	1.3463	1.3897	1.4332	1.4766	1.5200	1.5635	1.6069	1.6503	1.6937
4.	1.7372	1.7806	1.8240	1.8675	1.9109	1.9543	1.9978	2.0412	2.0846	2.1280
5.	2.1715	2.2149	2.2583	2.3018	2.3452	2.3886	2.4320	2.4755	2.5189	2.5623
6.	2.6058	2.6492	2.6926	2.7361	2.7795	2.8229	2.8663	2.9098	2.9532	2.9966
7.	3.0401	3.0835	3.1269	3.1703	3.2138	3.2572	3.3006	3.3441	3.3875	3.4309
8.	3.4744	3.5178	3.5612	3.6046	3.6481	3.6915	3.7349	3.7784	3.8218	3.8652
9.	3.9087	3.9521	3.9955	4.0389	4.0824	4.1258	4.1692	4.2127	4.2561	4.2995

MULTIPLES OF 2.3026 ($2.3025851 = 1/0.4343$)

x	0	1	2	3	4	5	6	7	8	9
0.	0.0000	0.2303	0.4605	0.6908	0.9210	1.1513	1.3816	1.6118	1.8421	2.0723
1.	2.3026	2.5328	2.7631	2.9934	3.2236	3.4539	3.6841	3.9144	4.1447	4.3749
2.	4.6052	4.8354	5.0657	5.2959	5.5262	5.7565	5.9867	6.2170	6.4472	6.6775
3.	6.9078	7.1380	7.3683	7.5985	7.8288	8.0590	8.2893	8.5196	8.7498	8.9801
4.	9.2103	9.4406	9.6709	9.9011	10.131	10.362	10.592	10.822	11.052	11.283
5.	11.513	11.743	11.973	12.204	12.434	12.664	12.894	13.125	13.355	13.585
6.	13.816	14.046	14.276	14.506	14.737	14.967	15.197	15.427	15.658	15.888
7.	16.118	16.348	16.579	16.809	17.039	17.269	17.500	17.730	17.960	18.190
8.	18.421	18.651	18.881	19.111	19.342	19.572	19.802	20.032	20.263	20.493
9.	20.723	20.954	21.184	21.414	21.644	21.875	22.105	22.335	22.565	22.796

STANDARD DISTRIBUTION OF RESIDUALS (p. 121)

a = any positive quantity;
y = the number of residuals which are numerically $< a$;
r = the probable error of a single observation;
n = number of observations.

$\frac{a}{r}$	$\frac{y}{n}$	Diff.
0.0	.000	54
1	.054	53
2	.107	53
3	.160	53
4	.213	51
0.5	.264	50
6	.314	49
7	.363	48
8	.411	45
9	.456	44
1.0	.500	42
1	.542	40
2	.582	37
3	.619	36
4	.655	33
1.5	.688	31
6	.719	29
7	.748	27
8	.775	25
9	.800	23
2.0	.823	20
1	.843	19
2	.862	17
3	.879	16
4	.895	13
2.5	.908	13
6	.921	10
7	.931	10
8	.941	9
9	.950	7
3.0	.957	6
1	.963	6
2	.969	5
3	.974	4
4	.978	4
3.5	.982	3
6	.985	2
7	.987	3
8	.990	1
9	.991	2
4.0	.993	6
5.0	.999	

FACTORS FOR COMPUTING PROBABLE ERROR (p. 121)

n	Bessel		Peters	
	$\frac{0.6745}{\sqrt{(n-1)}}$	$\frac{0.6745}{\sqrt{n(n-1)}}$	$\frac{0.8453}{\sqrt{n(n-1)}}$	$\frac{0.8453}{n\sqrt{n-1}}$
2	.6745	.4769	.5978	.4227
3	.4769	.2754	.3451	.1993
4	.3894	.1947	.2440	.1220
5	.3372	.1508	.1890	.0845
6	.3016	.1231	.1543	.0630
7	.2754	.1041	.1304	.0493
8	.2549	.0901	.1130	.0399
9	.2385	.0795	.0996	.0332
10	.2248	.0711	.0891	.0282
11	.2133	.0643	.0806	.0243
12	.2034	.0587	.0736	.0212
13	.1947	.0540	.0677	.0188
14	.1871	.0500	.0627	.0167
15	.1803	.0465	.0583	.0151
16	.1742	.0435	.0546	.0136
17	.1686	.0409	.0513	.0124
18	.1636	.0386	.0483	.0114
19	.1590	.0365	.0457	.0105
20	.1547	.0346	.0434	.0097
21	.1508	.0329	.0412	.0090
22	.1472	.0314	.0393	.0084
23	.1438	.0300	.0376	.0078
24	.1406	.0287	.0360	.0073
25	.1377	.0275	.0345	.0069
26	.1349	.0265	.0332	.0065
27	.1323	.0255	.0319	.0061
28	.1298	.0245	.0307	.0058
29	.1275	.0237	.0297	.0055
30	.1252	.0229	.0287	.0052
31	.1231	.0221	.0277	.0050
32	.1211	.0214	.0268	.0047
33	.1192	.0208	.0260	.0045
34	.1174	.0201	.0252	.0043
35	.1157	.0196	.0245	.0041
36	.1140	.0190	.0238	.0040
37	.1124	.0185	.0232	.0038
38	.1109	.0180	.0225	.0037
39	.1094	.0175	.0220	.0035
40	.1080	.0171	.0214	.0034
45	.1017	.0152	.0190	.0028
50	.0964	.0136	.0171	.0024
55	.0918	.0124	.0155	.0021
60	.0878	.0113	.0142	.0018
65	.0843	.0105	.0131	.0016
70	.0812	.0097	.0122	.0015
75	.0784	.0091	.0113	.0013
80	.0759	.0085	.0106	.0012
85	.0736	.0080	.0100	.0011
90	.0715	.0075	.0094	.0010
95	.0696	.0071	.0089	.0009
100	.0678	.0068	.0085	.0008

COMPOUND INTEREST. AMOUNT OF A GIVEN PRINCIPAL

The amount A at the end of n years of a given principal P placed at compound interest to-day is $A = P \times x$ or $A = P \times y$ or $A = P \times z$, according as the interest (at the rate of r per cent. per annum) is compounded annually, semi-annually, or quarterly; the factor x or y or z being taken from the following tables.

Values of x. (Interest compounded annually; $A = P \times x$.)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236	1.1449
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910	1.2250
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.1925	1.2155	1.2625	1.3108
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382	1.4026
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185	1.5007
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036	1.6058
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938	1.7182
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.4861	1.5513	1.6895	1.8385
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.5530	1.6289	1.7908	1.9672
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.6239	1.7103	1.8983	2.1049
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	1.7959	2.0122	2.2522
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329	2.4098
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.3519	1.9799	2.2609	2.5785
15	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2.3966	2.7590
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404	2.9522
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928	3.1588
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543	3.3799
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.3079	2.5270	3.0256	3.6165
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071	3.8697
25	1.6406	1.8539	2.0938	2.3632	2.6658	3.0054	3.3864	4.2919	5.4274
30	1.8114	2.0976	2.4273	2.8068	3.2434	3.7453	4.3219	5.7435	7.6123
40	2.2080	2.6851	3.2620	3.9593	4.8010	5.8164	7.0400	10.286	14.974
50	2.6916	3.4371	4.3839	5.5849	7.1067	9.0326	11.467	18.420	29.457
60	3.2810	4.3998	5.8916	7.8781	10.520	14.027	18.679	32.988	57.946

This table is computed from the formula $x = [1 + (r/100)]^n$.

Values of y. (Interest compounded semi-annually; $A = P \times y$.)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0201	1.0252	1.0302	1.0353	1.0404	1.0455	1.0506	1.0609	1.0712
2	1.0406	1.0509	1.0614	1.0719	1.0824	1.0931	1.1038	1.1255	1.1475
3	1.0615	1.0774	1.0934	1.1097	1.1262	1.1428	1.1597	1.1941	1.2293
4	1.0829	1.1045	1.1265	1.1489	1.1717	1.1948	1.2184	1.2668	1.3168
5	1.1046	1.1323	1.1605	1.1894	1.2190	1.2492	1.2801	1.3439	1.4106
6	1.1268	1.1608	1.1956	1.2314	1.2682	1.3060	1.3449	1.4258	1.5111
7	1.1495	1.1900	1.2318	1.2749	1.3195	1.3655	1.4130	1.5126	1.6187
8	1.1726	1.2199	1.2690	1.3199	1.3728	1.4276	1.4845	1.6047	1.7340
9	1.1961	1.2506	1.3073	1.3665	1.4282	1.4926	1.5597	1.7024	1.8575
10	1.2202	1.2820	1.3469	1.4148	1.4859	1.5605	1.6386	1.8061	1.9898
11	1.2447	1.3143	1.3876	1.4647	1.5460	1.6315	1.7216	1.9161	2.1315
12	1.2697	1.3474	1.4295	1.5164	1.6084	1.7058	1.8087	2.0328	2.2833
13	1.2953	1.3812	1.4727	1.5700	1.6734	1.7834	1.9003	2.1566	2.4460
14	1.3213	1.4160	1.5172	1.6254	1.7410	1.8645	1.9965	2.2879	2.6202
15	1.3478	1.4516	1.5631	1.6828	1.8114	1.9494	2.0976	2.4273	2.8068
16	1.3749	1.4881	1.6103	1.7422	1.8845	2.0381	2.2038	2.5751	3.0067
17	1.4026	1.5256	1.6590	1.8037	1.9607	2.1308	2.3153	2.7319	3.2209
18	1.4308	1.5639	1.7091	1.8674	2.0399	2.2278	2.4325	2.8983	3.4503
19	1.4595	1.6033	1.7608	1.9333	2.1223	2.3292	2.5557	3.0748	3.6960
20	1.4889	1.6436	1.8140	2.0016	2.2080	2.4352	2.6851	3.2620	3.9593
25	1.6446	1.8610	2.1052	2.3808	2.6916	3.0420	3.4371	4.3839	5.5849
30	1.8167	2.1072	2.4432	2.8318	3.2810	3.8001	4.3998	5.8916	7.8781
40	2.2167	2.7015	3.2907	4.0064	4.8754	5.9301	7.2096	10.641	15.676
50	2.7048	3.4634	4.4320	5.6682	7.2446	9.2540	11.814	19.219	31.191
60	3.3004	4.4402	5.9693	8.0192	10.765	14.441	19.358	34.711	62.064

Formula: $y = [1 + (r/200)]^{2n}$.

Values of z. (Interest compounded quarterly; $A = P \times z$; see opposite page)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0202	1.0252	1.0303	1.0355	1.0406	1.0458	1.0509	1.0614	1.0719
2	1.0407	1.0511	1.0616	1.0722	1.0829	1.0936	1.1045	1.1265	1.1489
3	1.0617	1.0776	1.0938	1.1102	1.1268	1.1437	1.1608	1.1956	1.2314
4	1.0831	1.1048	1.1270	1.1496	1.1726	1.1960	1.2199	1.2690	1.3199
5	1.1049	1.1327	1.1612	1.1903	1.2202	1.2508	1.2820	1.3469	1.4148
6	1.1272	1.1613	1.1964	1.2326	1.2697	1.3080	1.3474	1.4295	1.5164
7	1.1499	1.1906	1.2327	1.2763	1.3213	1.3679	1.4160	1.5172	1.6254
8	1.1730	1.2206	1.2701	1.3215	1.3749	1.4305	1.4881	1.6103	1.7422
9	1.1967	1.2514	1.3086	1.3684	1.4308	1.4959	1.5639	1.7091	1.8674
10	1.2208	1.2830	1.3483	1.4169	1.4889	1.5644	1.6436	1.8140	2.0016
11	1.2454	1.3154	1.3893	1.4672	1.5493	1.6360	1.7274	1.9253	2.1454
12	1.2705	1.3486	1.4314	1.5192	1.6122	1.7108	1.8154	2.0435	2.2996
13	1.2961	1.3826	1.4748	1.5731	1.6777	1.7891	1.9078	2.1689	2.4648
14	1.3222	1.4175	1.5196	1.6288	1.7458	1.8710	2.0050	2.3020	2.6420
15	1.3489	1.4533	1.5657	1.6866	1.8167	1.9566	2.1072	2.4432	2.8318
16	1.3760	1.4900	1.6132	1.7464	1.8905	2.0462	2.2145	2.5931	3.0353
17	1.4038	1.5276	1.6621	1.8083	1.9672	2.1398	2.3274	2.7523	3.2534
18	1.4320	1.5661	1.7126	1.8725	2.0471	2.2378	2.4459	2.9212	3.4872
19	1.4609	1.6056	1.7645	1.9389	2.1302	2.3402	2.5705	3.1004	3.7378
20	1.4903	1.6462	1.8180	2.0076	2.2167	2.4473	2.7015	3.2907	4.0064
25	1.6467	1.8646	2.1111	2.3898	2.7048	3.0609	3.4634	4.4320	5.6682
30	1.8194	2.1121	2.4514	2.8446	3.3004	3.8285	4.4402	5.9693	8.0192
40	2.2211	2.7098	3.3053	4.0306	4.9138	5.9892	7.2980	10.828	16.051
50	2.7115	3.4768	4.4567	5.7110	7.3160	9.3693	11.995	19.643	32.128
60	3.3102	4.4608	6.0092	8.0919	10.893	14.657	19.715	35.633	64.307

Formula: $z = [1 + (r/400)]^{4n}$.

AMOUNT OF AN ANNUITY

The amount S accumulated at the end of n years by a given annual payment Y set aside at the end of each year is $S = Y \times v$, where the factor v is to be taken from the following table. (Interest at r per cent. per annum, compounded annually.)

Values of v

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0200	2.0250	2.0300	2.0350	2.0400	2.0450	2.0500	2.0600	2.0700
3	3.0604	3.0756	3.0909	3.1062	3.1216	3.1370	3.1525	3.1836	3.2149
4	4.1216	4.1525	4.1836	4.2149	4.2465	4.2782	4.3101	4.3746	4.4399
5	5.2040	5.2563	5.3091	5.3625	5.4163	5.4707	5.5256	5.6371	5.7507
6	6.3081	6.3877	6.4684	6.5502	6.6330	6.7169	6.8019	6.9753	7.1533
7	7.4343	7.5474	7.6625	7.7794	7.8983	8.0192	8.1420	8.3938	8.6540
8	8.5830	8.7361	8.8923	9.0517	9.2142	9.3800	9.5491	9.8975	10.260
9	9.7546	9.9545	10.159	10.368	10.583	10.802	11.027	11.491	11.978
10	10.950	11.203	11.464	11.731	12.006	12.288	12.578	13.181	13.816
11	12.169	12.483	12.808	13.142	13.486	13.841	14.207	14.972	15.784
12	13.412	13.796	14.192	14.602	15.026	15.464	15.917	16.870	17.888
13	14.680	15.140	15.618	16.113	16.627	17.160	17.713	18.882	20.141
14	15.974	16.519	17.086	17.677	18.292	18.932	19.599	21.015	22.550
15	17.293	17.932	18.599	19.296	20.024	20.784	21.579	23.276	25.129
16	18.639	19.380	20.157	20.971	21.825	22.719	23.657	25.673	27.888
17	20.012	20.865	21.762	22.705	23.698	24.742	25.840	28.213	30.840
18	21.412	22.386	23.414	24.500	25.645	26.855	28.132	30.906	33.999
19	22.841	23.946	25.117	26.357	27.671	29.064	30.539	33.760	37.379
20	24.297	25.545	26.870	28.280	29.778	31.371	33.066	36.786	40.995
25	32.030	34.158	36.459	38.950	41.646	44.565	47.727	54.865	63.249
30	40.568	43.903	47.575	51.623	56.085	61.007	66.439	79.058	94.461
40	60.402	67.403	75.401	84.550	95.026	107.03	120.80	154.76	199.64
50	84.579	97.484	112.80	131.00	152.67	178.50	209.35	290.34	406.53
60	114.05	135.99	163.05	196.52	237.99	289.50	353.58	533.13	813.52

Formula: $v = \{[1 + (r/100)]^n - 1\} \div (r/100)$
$= (z - 1) \div (r/100)$.

PRINCIPAL WHICH WILL AMOUNT TO A GIVEN SUM

The principal P, which, if placed at compound interest to-day, will amount to a given sum A at the end of n years is $P = A \times x'$ or $P = A \times y'$ or $P = A \times z'$, according as the interest (at the rate of r per cent. per annum) is compounded annually, semi-annually, or quarterly: the factor x' or y' or z' being taken from the following tables.

Values of x'. (Interest compounded annually; $P = A \times x'$)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	.98039	.97561	.97087	.96618	.96154	.95694	.95238	.94340	.93458
2	.96117	.95181	.94260	.93351	.92456	.91573	.90703	.89000	.87344
3	.94232	.92860	.91514	.90194	.88900	.87630	.86384	.83962	.81630
4	.92385	.90595	.88849	.87144	.85480	.83856	.82270	.79209	.76290
5	.90573	.88385	.86261	.84197	.82193	.80245	.78353	.74726	.71299
6	.88797	.86230	.83748	.81350	.79031	.76790	.74622	.70496	.66634
7	.87056	.84127	.81309	.78599	.75992	.73483	.71068	.66506	.62275
8	.85349	.82075	.78941	.75941	.73069	.70319	.67684	.62741	.58201
9	.83676	.80073	.76642	.73373	.70259	.67290	.64461	.59190	.54393
10	.82035	.78120	.74409	.70892	.67556	.64393	.61391	.55839	.50835
11	.80426	.76214	.72242	.68495	.64958	.61620	.58468	.52679	.47509
12	.78849	.74356	.70138	.66178	.62460	.58966	.55684	.49697	.44401
13	.77303	.72542	.68095	.63940	.60057	.56427	.53032	.46884	.41496
14	.75788	.70773	.66112	.61778	.57748	.53997	.50507	.44230	.38783
15	.74301	.69047	.64186	.59689	.55526	.51672	.48102	.41727	.36245
16	.72845	.67362	.62317	.57671	.53391	.49447	.45811	.39365	.33873
17	.71416	.65720	.60502	.55720	.51337	.47318	.43630	.37136	.31657
18	.70016	.64117	.58739	.53836	.49363	.45280	.41552	.35034	.29586
19	.68643	.62553	.57029	.52016	.47464	.43330	.39573	.33051	.27651
20	.67297	.61027	.55368	.50257	.45639	.41464	.37689	.31180	.25842
25	.60953	.53939	.47761	.42315	.37512	.33273	.29530	.23300	.18425
30	.55207	.47674	.41199	.35628	.30832	.26700	.23138	.17411	.13137
40	.45289	.37243	.30656	.25257	.20829	.17193	.14205	.09722	.06678
50	.37153	.29094	.22811	.17905	.14071	.11071	.08720	.05429	.03395
60	.30478	.22728	.16973	.12693	.09506	.07129	.05354	.03031	.01726

Formula: $x' = [1 + (r/100)]^{-n} = 1/x.$

Values of y'. (Interest compounded semi-annually; $P = A \times y'$)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	.98030	.97546	.97066	.96590	.96117	.95647	.95181	.94260	.93351
2	.96098	.95152	.94218	.93296	.92385	.91484	.90595	.88849	.87144
3	.94205	.92817	.91454	.90114	.88797	.87502	.86230	.83748	.81350
4	.92348	.90540	.88771	.87041	.85349	.83694	.82075	.78941	.75941
5	.90529	.88318	.86167	.84073	.82035	.80051	.78120	.74409	.70892
6	.88745	.86151	.83639	.81206	.78849	.76567	.74356	.70138	.66178
7	.86996	.84037	.81185	.78436	.75788	.73234	.70773	.66112	.61778
8	.85282	.81975	.78803	.75762	.72845	.70047	.67362	.62317	.57671
9	.83602	.79963	.76491	.73178	.70016	.66998	.64117	.58739	.53836
10	.81954	.78001	.74247	.70682	.67297	.64082	.61027	.55368	.50257
11	.80340	.76087	.72069	.68272	.64684	.61292	.58086	.52189	.46915
12	.78757	.74220	.69954	.65944	.62172	.58625	.55288	.49193	.43796
13	.77205	.72398	.67902	.63695	.59758	.56073	.52623	.46369	.40884
14	.75684	.70622	.65910	.61523	.57437	.53632	.50088	.43708	.38165
15	.74192	.68889	.63976	.59425	.55207	.51298	.47674	.41199	.35628
16	.72730	.67198	.62099	.57398	.53063	.49065	.45377	.38834	.33259
17	.71297	.65549	.60277	.55441	.51003	.46930	.43191	.36604	.31048
18	.69892	.63941	.58509	.53550	.49022	.44887	.41109	.34503	.28983
19	.68515	.62372	.56792	.51724	.47119	.42933	.39128	.32523	.27056
20	.67165	.60841	.55126	.49960	.45289	.41065	.37243	.30656	.25257
25	.60804	.53734	.47500	.42003	.37153	.32873	.29094	.22811	.17905
30	.55045	.47457	.40930	.35313	.30478	.26315	.22728	.16973	.12693
40	.45112	.37017	.30389	.24960	.20511	.16863	.13870	.09398	.06379
50	.36971	.28873	.22563	.17642	.13803	.10806	.08465	.05203	.03206
60	.30299	.22521	.16752	.12470	.09289	.06925	.05166	.02881	.01611

Formula: $y' = [1 + (r/200)]^{-2n} = 1/y.$

Values of z'. (Interest compounded quarterly; $P = A \times z'$; see opposite page)

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	.98025	.97539	.97055	.96575	.96098	.95624	.95152	.94218	.93296
2	.96089	.95138	.94198	.93268	.92348	.91439	.90540	.88771	.87041
3	.94191	.92796	.91424	.90074	.88745	.87437	.86151	.83639	.81206
4	.92330	.90512	.88732	.86989	.85282	.83611	.81975	.78803	.75762
5	.90506	.88284	.86119	.84010	.81954	.79952	.78001	.74247	.70682
6	.88719	.86111	.83583	.81132	.78757	.76453	.74220	.69954	.65944
7	.86966	.83991	.81122	.78354	.75684	.73107	.70622	.65910	.61523
8	.85248	.81924	.78733	.75670	.72730	.69908	.67198	.62099	.57390
9	.83564	.79908	.76415	.73079	.69892	.66849	.63941	.58509	.53550
10	.81914	.77941	.74165	.70576	.67165	.63923	.60841	.55126	.49960
11	.80296	.76022	.71981	.68159	.64545	.61126	.57892	.51939	.46611
12	.78710	.74151	.69861	.65825	.62026	.58451	.55086	.48936	.43486
13	.77155	.72326	.67804	.63570	.59606	.55893	.52415	.46107	.40570
14	.75631	.70546	.65808	.61393	.57280	.53447	.49874	.43441	.37851
15	.74137	.68809	.63870	.59291	.55045	.51108	.47457	.40930	.35313
16	.72673	.67115	.61989	.57260	.52897	.48871	.45156	.38563	.32946
17	.71237	.65464	.60164	.55299	.50833	.46733	.42967	.36334	.30737
18	.69830	.63852	.58392	.53405	.48850	.44687	.40884	.34233	.28676
19	.68451	.62281	.56673	.51576	.46944	.42732	.38903	.32254	.26754
20	.67099	.60748	.55004	.49810	.45112	.40862	.37017	.30389	.24960
25	.60729	.53630	.47369	.41845	.36971	.32670	.28873	.22563	.17642
30	.54963	.47347	.40794	.35154	.30299	.26120	.22521	.16752	.12470
40	.45023	.36903	.30255	.24810	.20351	.16697	.13702	.09235	.06230
50	.36880	.28762	.22438	.17510	.13669	.10673	.08337	.05091	.03113
60	.30210	.22417	.16641	.12358	.09181	.06823	.05072	.02806	.01555

Formula: $z' = [1 + (r/400)]^{-4n} = 1/z$.

ANNUITY WHICH WILL AMOUNT TO A GIVEN SUM (SINKING FUND)

The annual payment, Y, which, if set aside at the end of each year, will amount with accumulated interest to a given sum S at the end of n years is $Y = S \times v'$, where the factor v' is given below. (Interest at r per cent. per annum, compounded annually.)

Values of v'

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
2	.49505	.49383	.49261	.49140	.49020	.48900	.48780	.48544	.48309
3	.32675	.32514	.32353	.32193	.32035	.31877	.31721	.31411	.31105
4	.24262	.24082	.23903	.23725	.23549	.23374	.23201	.22859	.22523
5	.19216	.19025	.18835	.18648	.18463	.18279	.18097	.17740	.17389
6	.15853	.15655	.15460	.15267	.15076	.14888	.14702	.14336	.13980
7	.13451	.13250	.13051	.12854	.12661	.12470	.12282	.11914	.11555
8	.11651	.11447	.11246	.11048	.10853	.10661	.10472	.10104	.09747
9	.10252	.10046	.09843	.09645	.09449	.09257	.09069	.08702	.08349
10	.09133	.08926	.08723	.08524	.08329	.08138	.07950	.07587	.07238
11	.08218	.08011	.07808	.07609	.07415	.07225	.07039	.06679	.06336
12	.07456	.07249	.07046	.06848	.06655	.06467	.06283	.05928	.05590
13	.06812	.06605	.06403	.06206	.06014	.05828	.05646	.05296	.04965
14	.06260	.06054	.05853	.05657	.05467	.05282	.05102	.04758	.04434
15	.05783	.05577	.05377	.05183	.04994	.04811	.04634	.04296	.03979
16	.05365	.05160	.04961	.04768	.04582	.04402	.04227	.03895	.03586
17	.04997	.04793	.04595	.04404	.04220	.04042	.03870	.03544	.03243
18	.04670	.04467	.04271	.04082	.03899	.03724	.03555	.03236	.02941
19	.04378	.04176	.03981	.03794	.03614	.03441	.03275	.02962	.02675
20	.04116	.03915	.03722	.03536	.03358	.03188	.03024	.02718	.02439
25	.03122	.02928	.02743	.02567	.02401	.02244	.02095	.01823	.01581
30	.02465	.02278	.02102	.01937	.01783	.01639	.01505	.01265	.01059
40	.01656	.01484	.01326	.01183	.01052	.00934	.00828	.00646	.00467
50	.01182	.01026	.00887	.00763	.00655	.00560	.00478	.00344	.00238
60	.00877	.00735	.00613	.00509	.00420	.00345	.00283	.00188	.00121

Formula: $v' = (r/100) + [[1 + (r/100)]^n - 1] = 1/s$.

PRESENT WORTH OF AN ANNUITY

The capital C, which, if placed at interest to-day, will provide for a given annual payment Y for a term of n years before it is exhausted is $C = Y \times w$, where the factor w is given below. (Interest at r per cent. per annum, compounded annually.)

Values of w

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
1	0.9804	0.9756	0.9709	0.9662	0.9615	0.9569	0.9524	0.9434	0.9346
2	1.9416	1.9274	1.9135	1.8997	1.8861	1.8727	1.8594	1.8334	1.8080
3	2.8839	2.8560	2.8286	2.8016	2.7751	2.7490	2.7232	2.6730	2.6243
4	3.8077	3.7620	3.7171	3.6731	3.6299	3.5875	3.5460	3.4651	3.3872
5	4.7135	4.6458	4.5797	4.5151	4.4518	4.3900	4.3295	4.2124	4.1002
6	5.6014	5.5081	5.4172	5.3286	5.2421	5.1579	5.0757	4.9173	4.7665
7	6.4720	6.3494	6.2303	6.1145	6.0021	5.8927	5.7864	5.5824	5.3893
8	7.3255	7.1701	7.0197	6.8740	6.7327	6.5959	6.4632	6.2098	5.9713
9	8.1622	7.9709	7.7861	7.6077	7.4353	7.2688	7.1078	6.8017	6.5152
10	8.9826	8.7521	8.5302	8.3166	8.1109	7.9127	7.7217	7.3601	7.0236
11	9.7868	9.5142	9.2526	9.0016	8.7605	8.5289	8.3064	7.8869	7.4987
12	10.575	10.258	9.9540	9.6633	9.3851	9.1186	8.8633	8.3838	7.9427
13	11.348	10.983	10.635	10.303	9.9856	9.6829	9.3936	8.8527	8.3577
14	12.106	11.691	11.296	10.921	10.563	10.223	9.8986	9.2950	8.7455
15	12.849	12.381	11.938	11.517	11.118	10.740	10.380	9.7122	9.1079
16	13.578	13.055	12.561	12.094	11.652	11.234	10.838	10.106	9.4466
17	14.292	13.712	13.166	12.651	12.166	11.707	11.274	10.477	9.7632
18	14.992	14.353	13.754	13.190	12.659	12.160	11.690	10.828	10.059
19	15.678	14.979	14.324	13.710	13.134	12.593	12.085	11.158	10.336
20	16.351	15.589	14.877	14.212	13.590	13.008	12.462	11.470	10.594
25	19.523	18.424	17.413	16.482	15.622	14.828	14.094	12.783	11.654
30	22.396	20.930	19.600	18.392	17.292	16.289	15.372	13.765	12.409
40	27.355	25.103	23.115	21.355	19.793	18.402	17.159	15.046	13.332
50	31.424	28.362	25.730	23.456	21.482	19.762	18.256	15.762	13.801
60	34.761	30.909	27.676	24.945	22.623	20.638	18.929	16.161	14.039

Formula: $w = [1 - [1 + (r/100)]^{-n}] \div [r/100] = v/x.$

ANNUITY PROVIDED FOR BY A GIVEN CAPITAL

The annual payment Y provided for for a term of n years by a given capital C placed at interest to-day is $Y = C \times w'$. (Interest at r per cent. per annum, compounded annually; the fund supposed to be exhausted at the end of the term.)

Values of w'

Years	$r = 2$	2½	3	3½	4	4½	5	6	7
2	.51505	.51883	.52261	.52640	.53020	.53400	.53780	.54544	.55309
3	.34675	.35014	.35353	.35693	.36035	.36377	.36721	.37411	.38105
4	.26262	.26582	.26903	.27225	.27549	.27874	.28201	.28859	.29523
5	.21216	.21525	.21835	.22148	.22463	.22779	.23097	.23740	.24389
6	.17853	.18155	.18460	.18767	.19076	.19388	.19702	.20336	.20980
7	.15451	.15750	.16051	.16354	.16661	.16970	.17282	.17914	.18555
8	.13651	.13947	.14246	.14548	.14853	.15161	.15472	.16104	.16747
9	.12252	.12546	.12843	.13145	.13449	.13757	.14069	.14702	.15349
10	.11133	.11426	.11723	.12024	.12329	.12638	.12950	.13587	.14238
11	.10218	.10511	.10808	.11109	.11415	.11725	.12039	.12679	.13336
12	.09456	.09749	.10046	.10348	.10655	.10967	.11283	.11928	.12590
13	.08812	.09105	.09403	.09706	.10014	.10328	.10646	.11296	.11965
14	.08260	.08554	.08853	.09157	.09467	.09782	.10102	.10758	.11434
15	.07783	.08077	.08377	.08683	.08994	.09311	.09634	.10296	.10979
16	.07365	.07660	.07961	.08268	.08582	.08902	.09227	.09895	.10586
17	.06997	.07293	.07595	.07904	.08220	.08542	.08870	.09544	.10243
18	.06670	.06967	.07271	.07582	.07899	.08224	.08555	.09236	.09941
19	.06378	.06676	.06981	.07294	.07614	.07941	.08275	.08962	.09675
20	.06116	.06415	.06722	.07036	.07358	.07688	.08024	.08718	.09439
25	.05122	.05428	.05743	.06067	.06401	.06744	.07095	.07823	.08581
30	.04465	.04778	.05102	.05437	.05783	.06139	.06505	.07265	.08059
40	.03656	.03984	.04326	.04683	.05052	.05434	.05828	.06646	.07467
50	.03182	.03526	.03887	.04263	.04655	.05060	.05478	.06344	.07238
60	.02877	.03235	.03613	.04009	.04420	.04845	.05283	.06188	.07121

Formula: $w' = [r/100] \div [1 - [1 + (r/100)]^{-n}] = 1/w = x' + (r/100).$

DECIMAL EQUIVALENTS

From minutes and seconds into decimal parts of a degree

0′	0°.0000	0″	0°.0000
1	.0167	1	.000[illegible]
2	.0333	2	.0006
3	.05	3	.0008
4	.0667	4	.0011
5′	.0833	5″	.0014
6	.10	6	.0017
7	.1167	7	.0019
8	.1333	8	.0022
9	.15	9	.0025
10′	0°.1667	10″	0°.0028
1	.1833	1	.0031
2	.20	2	.0033
3	.2167	3	.0036
4	.2333	4	.0039
15′	.25	15″	.0042
6	.2667	6	.0044
7	.2833	7	.0047
8	.30	8	.005
9	.3167	9	.0053
20′	0°.3333	20″	0°.0056
1	.35	1	.0058
2	.3667	2	.0061
3	.3833	3	.0064
4	.40	4	.0067
25′	.4167	25″	.0069
6	.4333	6	.0072
7	.45	7	.0075
8	.4667	8	.0078
9	.4833	9	.0081
30′	0°.50	30″	0°.0083
1	.5167	1	.0086
2	.5333	2	.0089
3	.55	3	.0092
4	.5667	4	.0094
35′	.5833	35″	.0097
6	.60	6	.01
7	.6167	7	.0103
8	.6333	8	.0106
9	.65	9	.0108
40′	0°.6667	40″	0°.0111
1	.6833	1	.0114
2	.70	2	.0117
3	.7167	3	.0119
4	.7333	4	.0122
45′	.75	45″	.0125
6	.7667	6	.0128
7	.7833	7	.0131
8	.80	8	.0133
9	.8167	9	.0136
50′	0°.8333	50″	0°.0139
1	.85	1	.0142
2	.8667	2	.0144
3	.8833	3	.0147
4	.90	4	.015
55′	.9167	55″	.0153
6	.9333	6	.0156
7	.95	7	.0158
8	.9667	8	.0161
9	.9833	9	.0164
60′	1.00	60″	0°.0167

From decimal parts of a degree into minutes and seconds (exact values)

0°.00	0′	0°.50	30′
1	0′ 36″	1	30′ 36″
2	1′ 12″	2	31′ 12″
3	1′ 48″	3	31′ 48″
4	2′ 24″	4	32′ 24″
0°.05	3′	0°.55	33′
6	3′ 36″	6	33′ 36″
7	4′ 12″	7	34′ 12″
8	4′ 48″	8	34′ 48″
9	5′ 24″	9	35′ 24″
0°.10	6′	0°.60	36′
1	6′ 36″	1	36′ 36″
2	7′ 12″	2	37′ 12″
3	7′ 48″	3	37′ 48″
4	8′ 24″	4	38′ 24″
0°.15	9′	0°.65	39′
6	9′ 36″	6	39′ 36″
7	10′ 12″	7	40′ 12″
8	10′ 48″	8	40′ 48″
9	11′ 24″	9	41′ 24″
0°.20	12′	0°.70	42′
1	12′ 36″	1	42′ 36″
2	13′ 12″	2	43′ 12″
3	13′ 48″	3	43′ 48″
4	14′ 24″	4	44′ 24″
0°.25	15′	0°.75	45′
6	15′ 36″	6	45′ 36″
7	16′ 12″	7	46′ 12″
8	16′ 48″	8	46′ 48″
9	17′ 24″	9	47′ 24″
0°.30	18′	0°.80	48′
1	18′ 36″	1	48′ 36″
2	19′ 12″	2	49′ 12″
3	19′ 48″	3	49′ 48″
4	20′ 24″	4	50′ 24″
0°.35	21′	0°.85	51′
6	21′ 36″	6	51′ 36″
7	22′ 12″	7	52′ 12″
8	22′ 48″	8	52′ 48″
9	23′ 24″	9	53′ 24″
0°.40	24′	0°.90	54′
1	24′ 36″	1	54′ 36″
2	25′ 12″	2	55′ 12″
3	25′ 48″	3	55′ 48″
4	26′ 24″	4	56′ 24″
0°.45	27′	0°.95	57′
6	27′ 36″	6	57′ 36″
7	28′ 12″	7	58′ 12″
8	28′ 48″	8	58′ 48″
9	29′ 24″	9	59′ 24″
0°.50	30′	1°.00	60′

0°.000	0″.0
1	3″.6
2	7″.2
3	10″.8
4	14″.4
0°.005	18″
6	21″.6
7	25″.2
8	28″.8
9	32″.4
0°.010	36″

Common fractions

8 ths	16 ths	32 nds	64 ths	Exact decimal values
			1	.01 5625
		1	2	.03 125
			3	.04 6875
	1	2	4	.06 25
			5	.07 8125
		3	6	.09 375
			7	.10 9375
1	2	4	8	.12 5
			9	.14 0625
		5	10	.15 625
			11	.17 1875
	3	6	12	.18 75
			13	.20 3125
		7	14	.21 875
			15	.23 4375
2	4	8	16	.25
			17	.26 5625
		9	18	.28 125
			19	.29 6875
	5	10	20	.31 25
			21	.32 8125
		11	22	.34 375
			23	.35 9375
3	6	12	24	.37 5
			25	.39 0625
		13	26	.40 625
			27	.42 1875
	7	14	28	.43 75
			29	.45 3125
		15	30	.46 875
			31	.48 4375
4	8	16	32	.50
			33	.51 5625
		17	34	.53 125
			35	.54 6875
	9	18	36	.56 25
			37	.57 8125
		19	38	.59 375
			39	.60 9375
5	10	20	40	.62 5
			41	.64 0625
		21	42	.65 625
			43	.67 1875
	11	22	44	.68 75
			45	.70 3125
		23	46	.71 875
			47	.73 4375
6	12	24	48	.75
			49	.76 5625
		25	50	.78 125
			51	.79 6875
	13	26	52	.81 25
			53	.82 8125
		27	54	.84 375
			55	.85 9375
7	14	28	56	.87 5
			57	.89 0625
		29	58	.90 625
			59	.92 1875
	15	30	60	.93 75
			61	.95 3125
		31	62	.96 875
			63	.98 4375

WEIGHTS AND MEASURES

BY

LOUIS A. FISCHER

In the United States the measures of weight and length commonly employed are identical with the corresponding English units, but the capacity measures differ from those now in use in the British Empire, the U. S. gallon being defined as 231 cu. in. and the bushel as 2150.42 cu. in., whereas the corresponding British imperial units are, respectively, 277.418 cu. in., and 2219.344 cu. in. (1 imp. gal. = 1.2 U. S. gal., approx.; 1 imp. bu. = 1.03 U. S. bu., approx.).

The metric system of weights and measures was legalized and its use made permissive in the United States by an Act of Congress, passed in 1866. In 1872, by the concurrent action of the principal governments of the world, it was agreed to establish an International Bureau of Weights and Measures near Paris.

Prior to 1891 the British imperial yard was regarded as the real standard of the United States. In 1891, the Office of Weights and Measures (now Bureau of Standards) fixed the value of the United States yard in terms of the international meter, according to the ratio: one yard = 3600/3937 meters. At the same time, the pound was fixed in terms of the international kilogram, according to the relation: one pound = 453.59243 grams.

U. S. Customary Weights and Measures

Measures of Length

12 inches	= 1 foot
3 feet	= 1 yard
5½ yards = 16½ feet	= 1 rod, pole or perch
40 poles = 220 yards	= 1 furlong
8 furlongs = 1760 yards = 5280 feet	= 1 mile
3 miles	= 1 league
4 inches	= 1 hand
9 inches	= 1 span

Nautical Units

6080.2 feet	= 1 nautical mile
6 feet	= 1 fathom
120 fathoms	= 1 cable length
1 nautical mile per hr.	= 1 knot

Surveyor's or Gunter's Measure

7.92 inches	= 1 link
100 links = 66 ft. = 4 rods	= 1 chain
80 chains	= 1 mile
33⅓ inches	= 1 vara (Texas)

Measures of Area

144 square inches	= 1 square foot
9 square feet	= 1 square yard
30¼ square yards	= 1 square rod, pole or perch
160 square rods = 10 square chains = 43,560 sq. ft. = 5645 sq. varas (Texas)	= 1 acre
640 acres = 1 square mile	= 1 "section" of U. S. Govt. surveyed land
1 circular inch = area of circle 1 inch in diameter	= 0.7854 sq. in.
1 square inch	= 1.2732 cir. in.
1 circular mil	= area of circle 0.001 in. in diam.
1,000,000 cir. mils	= 1 cir. in.

Measures of Volume

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard
1 cord of wood	= 128 cu. ft.
1 perch of masonry	= 16½ to 25 cu. ft.

U. S. Customary Weights and Measures—*(continued)*

Measures of Volume

Liquid or Fluid Measure

4 gills	= 1 pint
2 pints	= 1 quart
4 quarts	= 1 gallon
7.4805 gallons	= 1 cubic foot

(There is no standard liquid "barrel.")

Apothecaries' Liquid Measure

60 minims	= 1 liquid dram or drachm
8 drams	= 1 liquid ounce
16 ounces	= 1 pint

Water Measure

The **Miner's Inch** is the quantity of water that will pass through an orifice 1 sq. in. in cross-section under a head of from 4 to 6½ in., as fixed by statutes, and varies from 1/40 cu. ft. to 1/50 cu. ft. per sec. The units now most in use are 1 cu. ft. per sec. and 1 gal. per sec., the U. S. Reclamation Service employing the former. See p. 260.

Dry Measure

2 pints	= 1 quart
8 quarts	= 1 peck
4 pecks	= 1 bushel

Shipping Measure

1 Register ton	= 100 cu. ft.
1 U. S. shipping ton	= 40 cu. ft.
	= 32.14 U. S. bu. / 31.14 imp. bu.
1 British shipping ton	= 42 cu. ft.
	= 32.70 imp. bu. / 33.75 U. S. bu.

Board Measure

1 board foot = 144 cu. in. = volume of board 1 ft. sq. and 1 in. thick.

No. of board feet in a log = $[\frac{1}{4}(d - 4)]^2L$, where d = diam. of log (usually taken inside the bark at small end), in., and L = length of log, ft. The 4 in. deducted are an allowance for slab. This rule is variously known as the Doyle, Conn. River, St. Croix, Thurber, Moore and Beeman, and the Scribner rule.

Weights
(The grain is the same in all systems)

Avoirdupois Weight

16 drams = 437.5 grains	= 1 ounce
16 ounces = 7000 grains	= 1 pound
100 pounds	= 1 cental
2000 pounds	= 1 short ton
2240 pounds	= 1 long ton
Also (in Great Britain):	
14 pounds	= 1 stone
2 stone = 28 lb.	= 1 quarter
4 quarters = 112 lb.	= 1 hundredweight (cwt.)
20 hundredweight	= 1 long ton

Troy Weight

24 grains	= 1 pennyweight (dwt.)
20 pennyweights = 480 grains	= 1 ounce
12 ounces = 5760 grains	= 1 pound

1 **Assay Ton** = 29,167 milligrams, or as many milligrams as there are troy ounces in a ton of 2000 lb. avoirdupois. Consequently, the number of milligrams of precious metal yielded by an assay ton of ore gives directly the number of troy ounces that would be obtained from a ton of 2000 lb. avoirdupois.

Apothecaries' Weight

20 grains	= 1 scruple ℈
3 scruples = 60 grains	= 1 dram ʒ
8 drams	= 1 ounce ℥
12 ounces = 5760 grains	= 1 pound

Weight for Precious Stones

1 carat = 200 milligrams

(Adopted by practically all important nations.)

Circular Measure

60 seconds	= 1 minute
60 minutes	= 1 degree
90 degrees	= 1 quadrant
360 degrees	= circumference
57.2957795 degrees (= 57° 17′44.806″)	= 1 radian (or angle having arc of length equal to radius)

METRIC SYSTEM

The fundamental unit of the metric system is the **meter**—the unit of length, from which the units of volume (**liter**) and of mass (**gram**) are derived. All other units are the decimal subdivisions or multiples of these. These three units are simply related: one cubic decimeter equals one liter, and one liter of water weighs one kilogram. The metric tables are formed by combining the words "meter," "gram," and "liter" with numerical prefixes.

All lengths, areas, and cubic measures in the following conversion tables are derived from the international meter. The customary weights are likewise derived from the kilogram. All capacities are based on the practical equivalent: 1 cubic decimeter equals 1 liter. (The liter is defined as the volume occupied by the mass of 1 kilogram of water under a pressure of 76 cm. of mercury and at the temperature of 4 deg. cent. According to the best information, 1 liter = 1.000027 cubic decimeters.)

The customary weights derived from the international kilogram are based on the value 1 avoirdupois lb. = 453.59243 grams. The value of the troy lb. is based on the same relation and also the equivalent 5760/7000 avoirdupois lb. equals 1 troy lb.

Metric Measures

Length			Area		
Unit	Symbol	Value in meters	Unit	Symbol	Value in sq. meters
Micron	μ	0.000001			
Millimeter	mm.	0.001	Sq. millimeter	$mm.^2$	0.000001
Centimeter	cm.	0.01	Sq. centimeter	$cm.^2$	0.0001
Decimeter	dm.	0.1	Sq. decimeter	$dm.^2$	0.01
Meter (unit)	m.	1.0	Sq. meter (centiare)	$m.^2$	1.0
Dekameter	dkm.	10.0	Sq. dekameter (are)	a.	100.0
Hectometer	hm.	100.0	Hectare	ha.	10,000.0
Kilometer	km.	1,000.0	Sq. kilometer	$km.^2$	1,000,000.0
Myriameter	Mm.	10,000.0			
Megameter		1,000,000.0			

Volume			Cubic measure		
Unit	Symbol	Value in liters	Unit	Symbol	Value in cubic meters
Milliliter	ml. or $cm.^3$	0.001	Cubic kilometer	$km.^3$	10^9
Liter (unit)	l. or $dm.^3$	1.0	Cubic hectometer	$hm.^3$	10^6
Kiloliter	kl. or $m.^3$	1,000.0	Cubic dekameter	$dkm.^3$	10^3
Also					
Centiliter	cl.	0.01	Cubic meter	$m.^3$	1
Deciliter	dl.	0.1	Cubic decimeter	$dm.^3$	10^{-3}
Dekaliter	dkl.	10.0	Cubic centimeter	$cm.^3$	10^{-6}
Hectoliter	hl.	100.0	Cubic millimeter	$mm.^3$	10^{-9}
			Cubic micron	μ^3	10^{-18}

Weight					
Unit	Symbol	Value in grams	Unit	Symbol	Value in grams
Microgram		0.000001	Dekagram	dkg.	10.0
Milligram	mg.	0.001	Hectogram	hg.	100.0
Centigram	cg.	0.01	Kilogram	kg.	1,000.0
Decigram	dg.	0.1	Myriagram	Mg.	10,000.0
Gram (unit)	g.	1.0	Quintal	q.	100,000.0
			Ton	t.	1,000,000.0

SYSTEMS OF UNITS

The principal units of interest to mechanical engineers can all be derived from the three fundamental units of **force, length,** and **time.** These three fundamental units may be chosen at pleasure; each such choice gives rise to a "system" of units. The following table gives the units of the four "systems" most often met with in the literature.

The precise definitions of the fundamental units in these systems are as follows. (In these definitions the "standard pound body" and the "standard kilogram body" refer to two special lumps of metal, carefully preserved at London and Paris, respectively; the "standard locality" means sea level, 45 deg. latitude; or, more strictly, any locality in which the acceleration due to gravity has the value 980.665 cm. per sec.^{2} = 32.1740 ft. per sec.^{2}, which may be called the **standard acceleration.**

The **pound** (*force*) is the force required to support the standard pound body against gravity, *in vacuo*, in the standard locality; or, it is the force which, if applied to the standard pound body, supposed free to move, would give that body the "standard acceleration." The word "pound" is used for the unit of both force and mass, and consequently is ambiguous. To avoid uncertainty it is desirable to call the units "pound force" and "pound mass," respectively.

The **kilogram** (*force*) is the force required to support the standard kilogram against gravity, *in vacuo*, in the standard locality; or, it is the force which, if applied to the standard kilogram body, supposed free to move, would give that body the "standard acceleration." The word "kilogram" is used for the unit of both force and mass and consequently is ambiguous. To avoid uncertainty it is desirable to call the units "kilogram force" and "kilogram mass," respectively.

The **poundal** is the force which, if applied to the standard pound body, would give that body an acceleration of 1 ft. per sec.^{2}; that is, 1 poundal = 1/32.1740 of a pound force.

The **dyne** is the force which, if applied to the standard gram body, would give that body an acceleration of 1 cm. per sec.^{2}; that is, 1 dyne = 1/980.665 of a gram force.

Systems of Units

Name of unit	Dimensions of units in terms of F, L, T	British "gravitational" system, or "foot-pound-second" system	Metric "gravitational" system, or "kilogram-meter-second" system	Metric "absolute" system, or "C. G. S." system	British "absolute" system (little used)
Force	F	1 lb.	1 kg.	1 dyne	1 poundal
Length	L	1 ft.	1 m.	1 cm.	1 ft.
Time	T	1 sec.	1 sec.	1 sec.	1 sec.
Velocity	L/T	1 ft. per sec.	1 m. per sec.	1 cm. per sec.	1 ft. per sec.
Acceleration	L/T^2	1 ft. per sec.^{2}	1 m. per sec.^{2}	1 cm. per sec.^{2}	1 ft. per sec.^{2}
Pressure	F/L^2	1 lb. per ft.^{2}	1 kg. per m.^{2}	1 dyne per cm.^{2}	1 pdl. per ft.^{2}
Impulse or momentum	FT	1 lb.-sec.	1 kg.-sec.	1 dyne-sec.	1 pdl.-sec.
Work or energy	FL	1 ft.-lb.	1 kg.-m.	1 dyne-cm. = 1 "erg."	1 ft.-pdl.
Power	FL/T	1 ft.-lb. per sec.	1 kg.-m. per sec.	1 dyne-cm. per sec.	1 ft.-pdl. per sec.
Mass	$F/(L/T^2)$	1 lb. per (ft. per sec.^{2}) = 1 "slug."	1 kg. per (m. per sec.^{2}) = 1 "metric slug."	1 dyne per (cm. per sec.^{2}) = 1 gram mass.	1 pdl. per (ft. per sec.^{2}) = 1 pound mass.

NOTE. The "slug" (also called the "geepound," or the "engineer's unit of mass"), the "metric slug," and the "poundal" are never used in practice.

Other common units are as follows:

Work: 1 joule = 10^7 ergs = 10,000,000 dyne-cm.
1 kilowatt-hour = 3,600,000 joules = 3600×10^{10} dyne-cm.

Power: 1 horse power = 550 ft.-lb. per sec.
1 poncelet = 100 kg.-m. per sec.
1 force de cheval = 75 kg.-m. per sec.
1 watt = 1 joule per sec. = 10,000,000 dyne-cm. per sec.
1 kilowatt = 1000 watts = 10^{10} dyne-cm. per sec.

A new horse power of 550.220 ft.-lb. per sec., or 746 watts, has been proposed, but has not been accepted by mechanical engineers.

The *weight* of a body (in a given locality) always means a *force*, namely, the force, required to support the body against gravity (in that locality). When no particular locality is specified, the standard locality may be assumed. Thus, the "standard weight" of the pound body is 1 lb.; the "standard weight" of the kilogram body is 1 kg.

Heat Units. The units of heat commonly used are (1) the quantity of heat required to raise the temperature of 1 gram of water 1 deg. cent. at a mean temperature of 15 deg. cent., or (2) the heat required to raise the temperature of 1 lb. of water 1 deg. fahr. The former quantity is called the **gram-calorie** (small calorie), while the latter is known as the **British thermal unit** or B.t.u. The **kilogram-calorie** (large calorie), which is equal to 1000 g.-cal., is largely used in engineering work in metric countries. See also p. 295.

Force Equivalents

Dynes $\times 10^6$	Kilograms	Pounds	Poundals
1	1.020 0.00848	2.248 0.03518	72.33 1.85933
0.9807 $\bar{1}.99149$	1	2.205 0.34334	70.93 1.85084
0.4448 $\bar{1}.64819$	0.4536 $\bar{1}.65667$	1	32.17 1.50750
0.01383 $\bar{2}.14067$	0.01410 $\bar{2}.14916$	0.03108 $\bar{2}.49249$	1

CONVERSION TABLES

Length Equivalents

Centimeters	Inches	Feet	Yards	Meters	Chains	Kilometers	Miles
1	0.3937 $\bar{1}.59517$	0.03281 $\bar{2}.51598$	0.01094 $\bar{2}.03886$	0.01 $\bar{2}.00000$	$0.0_3 4971$ $\bar{4}.69644$	10^{-5} $\bar{5}.00000$	$0.0_5 6214$ $\bar{6}.79335$
2.540 0.40483	1	0.08333 $\bar{2}.92082$	0.02778 $\bar{2}.44370$	0.0254 $\bar{2}.40483$	0.001263 $\bar{3}.10127$	$0.0_4 254$ $\bar{5}.40483$	$0.0_4 1578$ $\bar{5}.19818$
30.48 1.48402	12 1.07918	1	0.3333 $\bar{1}.52288$	0.3048 $\bar{1}.48402$	0.01515 $\bar{2}.18046$	$0.0_3 3048$ $\bar{4}.48402$	$0.0_3 1895$ $\bar{4}.27761$
91.44 1.96114	36 1.55630	3 0.47712	1	0.9144 $\bar{1}.96114$	0.04545 $\bar{2}.65758$	$0.0_3 9144$ $\bar{4}.96114$	$0.0_3 5682$ $\bar{4}.75449$
100 2.00000	39.37 1.59517	3.281 0.51598	1.0936 0.03886	1	0.04971 $\bar{2}.69644$	0.001 $\bar{3}.00000$	$0.0_3 6214$ $\bar{4}.79335$
2012 3.30356	792 2.89873	66 1.81954	22 1.34242	20.12 1.30356	1	0.02012 $\bar{2}.30356$	0.0125 $\bar{2}.09691$
100000 5.00000	39370 4.59517	3281 3.51598	1093.6 3.03886	1000 3.00000	49.71 1.69644	1	0.6214 $\bar{1}.79335$
160935 5.20665	63360 4.80182	5280 3.72263	1760 3.24551	1609 3.20665	80 1.90309	1.609 0.20665	1

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Conversion of Lengths

	Inches to millimeters	Millimeters to inches	Feet to meters	Meters to feet	Yards to meters	Meters to yards	Miles to kilometers	Kilometers to miles
1	25.40	0.03937	0.3048	3.281	0.9144	1.094	1.609	0.6214
2	50.80	0.07874	0.6096	6.562	1.829	2.187	3.219	1.243
3	76.20	0.1181	0.9144	9.842	2.743	3.281	4.828	1.864
4	101.60	0.1575	1.219	13.12	3.658	4.374	6.437	2.485
5	127.00	0.1968	1.524	16.40	4.572	5.486	8.047	3.107
6	152.40	0.2362	1.829	19.68	5.486	6.562	9.656	3.728
7	177.80	0.2756	2.134	22.97	6.401	7.655	11.27	4.350
8	203.20	0.3150	2.438	26.25	7.315	8.749	12.87	4.971
9	228.60	0.3543	2.743	29.53	8.230	9.842	14.48	5.592

Mechanical Equivalent of Heat. See p. 311.* The value most commonly accepted among American engineers as the work equivalent of 1 **mean B.t.u.** is 777.5 ft.-lb., and the **mean gram-calorie** = 4.183 joules, which are the values used throughout this book. The U. S. Bureau of Standards does not recommend any special value; for its own purposes it takes the 59 deg. fahr. B.t.u. as 778.2 ft.-lb. and the 68 deg. B.t.u. as 777.5 ft.-lb. The 15 deg. calorie = 4.187 joules; 20 deg. calorie = 4.183 joules. There is an uncertainty of about 1 part in 1000 in these values.

Conversion of Lengths: Inches and Millimeters

Common fractions of an inch to millimeters (From 1/64 to 1 in.)

64ths	Millimeters	64ths	Millimeters	64ths	Millimeters	64ths	Millimeters	64ths	Millimeters	64ths	Millimeters
1	0.397	13	5.159	25	9.922	37	14.684	49	19.447	57	22.622
2	0.794	14	5.556	26	10.319	38	15.081	50	19.844	58	23.019
3	1.191	15	5.953	27	10.716	39	15.478	51	20.241	59	23.416
4	1.588	16	6.350	28	11.113	40	15.875	52	20.638	60	23.813
5	1.984	17	6.747	29	11.509	41	16.272	53	21.034	61	24.209
6	2.381	18	7.144	30	11.906	42	16.669	54	21.431	62	24.606
7	2.778	19	7.541	31	12.303	43	17.066	55	21.828	63	25.003
8	3.175	20	7.938	32	12.700	44	17.463	56	22.225	64	25.400
9	3.572	21	8.334	33	13.097	45	17.859				
10	3.969	22	8.731	34	13.494	46	18.256				
11	4.366	23	9.128	35	13.891	47	18.653				
12	4.763	24	9.525	36	14.288	48	19.050				

Decimals of an inch to millimeters. (From 0.01 in. to 0.99 in.)

	0	1	2	3	4	5	6	7	8	9
.0		0.254	0.508	0.762	1.016	1.270	1.524	1.778	2.032	2.286
.1	2.540	2.794	3.048	3.302	3.556	3.810	4.064	4.318	4.572	4.826
.2	5.080	5.334	5.588	5.842	6.096	6.350	6.604	6.858	7.112	7.366
.3	7.620	7.874	8.128	8.382	8.636	8.890	9.144	9.398	9.652	9.906
.4	10.160	10.414	10.668	10.922	11.176	11.430	11.684	11.938	12.192	12.446
.5	12.700	12.954	13.208	13.462	13.716	13.970	14.224	14.478	14.732	14.986
.6	15.240	15.494	15.748	16.002	16.256	16.510	16.764	17.018	17.272	17.526
.7	17.780	18.034	18.288	18.542	18.796	19.050	19.304	19.558	19.812	20.066
.8	20.320	20.574	20.828	21.082	21.336	21.590	21.844	22.098	22.352	22.606
.9	22.860	23.114	23.368	23.622	23.876	24.130	24.384	24.638	24.892	25.146

Millimeters to decimals of an inch. (From 1 to 99 mm.)

	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0		0.0394	0.0787	0.1181	0.1575	0.1969	0.2362	0.2756	0.3150	0.3543
1	0.3937	0.4331	0.4724	0.5118	0.5512	0.5906	0.6299	0.6693	0.7087	0.7480
2	0.7874	0.8268	0.8661	0.9055	0.9449	0.9843	1.0236	1.0630	1.1024	1.1417
3	1.1811	1.2205	1.2598	1.2992	1.3386	1.3780	1.4173	1.4567	1.4961	1.5354
4	1.5748	1.6142	1.6535	1.6929	1.7323	1.7717	1.8110	1.8504	1.8898	1.9291
5	1.9685	2.0079	2.0472	2.0866	2.1260	2.1654	2.2047	2.2441	2.2835	2.3228
6	2.3622	2.4016	2.4409	2.4803	2.5197	2.5591	2.5984	2.6378	2.6772	2.7165
7	2.7559	2.7953	2.8346	2.8740	2.9134	2.9528	2.9921	3.0315	3.0709	3.1102
8	3.1496	3.1890	3.2283	3.2677	3.3071	3.3465	3.3858	3.4252	3.4646	3.5039
9	3.5433	3.5827	3.6220	3.6614	3.7008	3.7402	3.7795	3.8189	3.8583	3.8976

*See Marks' Mechanical Engineers' Handbook.

Area Equivalents

(For conversion table see p. 77)

Square meters	Square inches	Square feet	Square yards	Square rods	Square chains	Roods	Acres	Square miles or sections
1	1550 3.19033	10.76 1.03197	1.196 0.07773	0.0395 $\bar{2}.59699$	0.002471 $\bar{3}.39288$	$0.0_3 9884$ $\bar{3}.99494$	$0.0_3 2471$ $\bar{4}.39288$	$0.0_6 3861$ $\bar{7}.58670$
$0.0_3 6452$ $\bar{4}.80967$	**1**	0.006944 $\bar{3}.84164$	$0.0_3 7716$ $\bar{4}.88740$	$0.0_4 2551$ $\bar{5}.40667$	$0.0_5 1594$ $\bar{6}.20255$	$0.0_6 6377$ $\bar{7}.80461$	$0.0_6 1594$ $\bar{7}.20255$	$0.0_9 4910$ $\overline{10}.3963$
0.09290 $\bar{2}.96803$	144 2.15836	**1**	0.1111 $\bar{1}.04576$	0.003673 $\bar{3}.56503$	$0.0_3 2296$ $\bar{4}.36091$	$0.0_4 9184$ $\bar{5}.96297$	$0.0_4 2296$ $\bar{4}.36091$	$0.0_7 3587$ $\bar{8}.55473$
0.8361 $\bar{1}.92227$	1296 3.11260	9 0.95424	**1**	0.03306 $\bar{2}.51927$	0.002066 $\bar{3}.31515$	$0.0_3 8264$ $\bar{4}.91721$	0.0002066 $\bar{4}.31515$	$0.0_6 3228$ $\bar{7}.50898$
25.29 1.40300	39204 4.59333	272.25 2.43497	30.25 1.48072	**1**	0.0625 $\bar{2}.79588$	0.02500 $\bar{2}.39794$	0.00625 $\bar{3}.79588$	$0.0_5 9766$ $\bar{6}.98970$
404.7 2.60712	627264 5.79745	4356 3.63909	484 2.68484	16 1.20412	**1**	0.4 $\bar{1}.60206$	0.1 $\bar{1}.00000$	0.0001562 $\bar{4}.19382$
1012 3.00506	1568160 6.19539	10890 4.03703	1210 3.08278	40 1.60206	2.5 0.39794	**1**	0.25 $\bar{1}.39794$	$0.0_3 3906$ $\bar{4}.59176$
4047 3.60712	6272640 6.79745	43560 4.63909	4840 3.68484	160 2.20412	10 1.00000	4 0.60206	**1**	0.001562 $\bar{3}.19382$
$2589_8 8$ 6.41330		27878400 7.44527	3097600 6.49102	102400 5.01030	6400 3.80618	2560 3.40824	640 2.80618	**1**

(1 hectare = 100 ares = 10,000 centiares or square meters)

Volume and Capacity Equivalents

(For conversion table see p. 77)

Cubic inches	Cubic feet	Cubic yards	U. S. Apothecary liquid ounces	U. S. quarts		U. S. gallons		Bushels U. S.	Liters (l)
				Liquid	Dry	Liquid	Dry		
1	$0.0_3 5787$ $\bar{4}.76246$	$0.0_4 2143$ $\bar{5}.33109$	0.5541 $\bar{1}.74360$	0.01732 $\bar{2}.23845$	0.01488 $\bar{2}.17263$	$0.0_2 4329$ $\bar{3}.63639$	$0.0_2 3720$ $\bar{3}.57057$	$0.0_3 4650$ $\bar{4}.66748$	0.01639 $\bar{2}.21450$
1728 3.23754	**1**	0.03704 $\bar{2}.56864$	957.5 2.98114	29.92 1.47599	25.71 1.41017	7.481 0.87393	6.429 0.80811	0.8036 $\bar{1}.90602$	28.32 1.45205
46656 4.66891	27 1.43136	**1**	25853 4.41251	807.9 2.90736	694.3 2.84153	202.0 2.30530	173.6 2.23948	21.70 1.33638	764.6 2.88341
1.805 0.25640	0.001044 $\bar{3}.01886$	$0.0_4 3868$ $\bar{5}.58749$	**1**	0.03125 $\bar{2}.49485$	0.02686 $\bar{2}.42903$	0.007813 $\bar{3}.89279$	0.006714 $\bar{3}.82697$	$0.0_3 8392$ $\bar{4}.92388$	0.02957 $\bar{2}.47091$
57.75 1.76155	0.03342 $\bar{2}.52401$	0.001238 $\bar{3}.09264$	32 1.50515	**1**	0.8594 $\bar{1}.93418$	0.25 $\bar{1}.39794$	0.2148 $\bar{1}.33212$	0.02686 $\bar{2}.42903$	0.9464 $\bar{1}.97606$
67.20 1.82737	0.03889 $\bar{2}.58983$	0.001440 $\bar{3}.15847$	37.24 1.57097	1.164 0.06582	**1**	0.2909 $\bar{1}.46376$	0.25 $\bar{1}.39794$	0.03125 $\bar{2}.49485$	1.101 0.04188
231 2.36361	0.1337 $\bar{1}.12607$	0.004951 $\bar{3}.69470$	128 2.10721	4 0.60206	3.437 0.53624	**1**	0.8594 $\bar{1}.93418$	0.1074 $\bar{1}.03109$	3.785 0.57812
268.8 2.42943	0.1556 $\bar{1}.19189$	0.005761 $\bar{3}.76053$	148.9 2.17303	4.655 0.66788	4 0.60206	1.164 0.06582	**1**	0.125 $\bar{1}.09691$	4.405 0.64394
2150 3.33252	1.244 0.09498	0.04609 $\bar{2}.66362$	1192 3.07612	37.24 1.57097	32 1.50515	9.309 0.96891	8 0.90309	**1**	35.24 1.54703
61.02 1.78550	0.03531 $\bar{2}$ 54795	0.001308 $\bar{3}.11659$	33.81 1.52909	1.057 0.02394	0.9081 $\bar{1}.95812$	0.2642 $\bar{1}.42188$	0.2270 $\bar{1}.35606$	0.02838 $\bar{2}.45297$	**1**

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Conversion of Areas

	Sq. in. to sq. cm.	Sq. cm. to sq. in.	Sq. ft. to sq. m.	Sq. m. to sq. ft.	Sq. yd. to sq. m.	Sq. m. to sq. yd.	Acres to hectares	Hectares to acres	Sq. mi. to sq. km.	Sq. km. to sq. mi.
1	6.452	0.1550	0.0929	10.76	0.8361	1.196	0.4047	2.471	2.590	0.3861
2	12.90	0.3100	0.1858	21.53	1.672	2.392	0.8094	4.942	5.180	0.7722
3	19.35	0.4650	0.2787	32.29	2.508	3.588	1.214	7.413	7.770	1.158
4	25.81	0.6200	0.3716	43.06	3.345	4.784	1.619	9.884	10.360	1.544
5	32.26	0.7750	0.4645	53.82	4.181	5.980	2.023	12.355	12.950	1.931
6	38.71	0.9300	0.5574	64.58	5.017	7.176	2.428	14.826	15.540	2.317
7	45.16	1.085	0.6503	75.35	5.853	8.372	2.833	17.297	18.130	2.703
8	51.61	1.240	0.7432	86.11	6.689	9.568	3.237	19.768	20.720	3.089
9	58.06	1.395	0.8361	96.87	7.525	10.764	3.642	22.239	23.310	3.475

Conversion of Volumes or Cubic Measure

	Cu. in. to cu. cm.	Cu. cm. to cu. in.	Cu. ft. to cu. m.	Cu. m. to cu. ft.	Cu. yd. to Cu. m.	Cu. m. to cu. yd.	Gallons to cu. ft.	Cu. ft. to gallons
1	16.39	0.06102	0.02832	35.31	0.7646	1.308	0.1337	7.481
2	32.77	0.1220	0.05663	70.63	1.529	2.616	0.2674	14.96
3	49.16	0.1831	0.08495	105.9	2.294	3.924	0.4011	22.44
4	65.55	0.2441	0.1133	141.3	3.058	5.232	0.5348	29.92
5	81.94	0.3051	0.1416	176.6	3.823	6.540	0.6685	37.41
6	98.32	0.3661	0.1699	211.9	4.587	7.848	0.8022	44.89
7	114.7	0.4272	0.1982	247.2	5.352	9.156	0.9359	52.36
8	131.1	0.4882	0.2265	282.5	6.116	10.46	1.070	59.85
9	147.5	0.5492	0.2549	317.8	6.881	11.77	1.203	67.33

Conversion of Volumes or Capacities

	Liquid ounces to cu. cm.	Cu. cm. to liquid ounces	Pints to liters	Liters to pints	Quarts to liters	Liters to quarts	Gallons to liters	Liters to gallons	Bushels to hectoliters	Hectoliters to bushels
1	29.57	0.03381	0.4732	2.113	0.9464	1.057	3.785	0.2642	0.3524	2.838
2	59.15	0.06763	0.9464	4.227	1.893	2.113	7.571	0.5283	0.7048	5.676
3	88.72	0.1014	1.420	6.340	2.839	3.170	11.36	0.7925	1.057	8.513
4	118.3	0.1353	1.893	8.453	3.785	4.227	15.14	1.057	1.410	11.35
5	147.9	0.1691	2.366	10.57	4.732	5.283	18.93	1.321	1.762	14.19
6	177.4	0.2029	2.839	12.68	5.678	6.340	22.71	1.585	2.114	17.03
7	207.0	0.2367	3.312	14.79	6.625	7.397	26.50	1.849	2.467	19.86
8	236.6	0.2705	3.785	16.91	7.571	8.453	30.28	2.113	2.819	22.70
9	266.2	0.3043	4.259	19.02	8.517	9.510	34.07	2.378	3.172	25.54

Conversion of Masses

	Grains to grams	Grams to grains	Ounces (avoir.) to grams	Grams to ounces (avoir.)	Pounds (avoir.) to kilograms	Kilograms to pounds (avoir.)	Short tons (2000 lb.) to metric tons	Metric tons (1000 kg.) to short tons	Long tons (2240 lb.) to metric tons	Metric tons to long tons
1	0.06480	15.43	28.35	0.03527	0.4536	2.205	0.907	1.102	1.016	0.984
2	0.1296	30.86	56.70	0.07055	0.9072	4.409	1.814	2.205	2.032	1.968
3	0.1944	46.30	85.05	0.1058	1.361	6.614	2.722	3.307	3.048	2.953
4	0.2592	61.73	113.40	0.1411	1.814	8.818	3.629	4.409	4.064	3.937
5	0.3240	77.16	141.75	0.1764	2.268	11.02	4.536	5.512	5.080	4.921
6	0.3888	92.59	170.10	0.2116	2.722	13.23	5.443	6.614	6.096	5.905
7	0.4536	108.03	198.45	0.2469	3.175	15.43	6.350	7.716	7.112	6.889
8	0.5184	123.46	226.80	0.2822	3.629	17.64	7.257	8.818	8.128	7.874
9	0.5832	138.89	255.15	0.3175	4.082	19.84	8.165	9.921	9.144	8.857

Velocity Equivalents

(For conversion table see p. 80)

Centimeters per sec.	Meters per sec.	Meters per min.	Kilometers per hour	Feet per sec.	Feet per min.	Miles per hour	Knots
1	**0.01**	**0.6** $\bar{1}$.77815	**0.036** $\bar{2}$.55630	**0.03281** $\bar{2}$.51598	**1.9685** 0.29414	**0.02237** $\bar{2}$.34965	**0.01942** $\bar{2}$.28825
100 2.00000	**1**	**60** 1.77815	**3.6** 0.55630	**3.281** 0.51598	**196.85** 2.29414	**2.237** 0.34965	**1.942** 0.28825
1.667 0.22184	**0.01667** $\bar{2}$.22184	**1**	**0.06** $\bar{2}$.77815	**0.05468** $\bar{2}$.73783	**3.281** 0.51598	**0.03728** $\bar{2}$.57150	**0.03237** $\bar{2}$.51018
27.78 1.44370	**0.2778** $\bar{1}$.44370	**16.67** 1.22184	**1**	**0.9113** $\bar{1}$.95968	**54.68** 1.73783	**0.6214** $\bar{1}$.79335	**0.53960** $\bar{1}$.73207
30.48 1.48402	**0.3048** $\bar{1}$.48402	**18.29** 1.26217	**1.097** 0.04032	**1**	**60** 1.77815	**0.6818** $\bar{1}$.83367	**0.59209** $\bar{1}$.77238
0.5080 $\bar{1}$.70586	**0.005080** $\bar{3}$.70586	**0.3048** $\bar{1}$.48402	**0.01829** $\bar{2}$.26217	**0.01667** $\bar{2}$.22185	**1**	**0.01136** $\bar{2}$.05553	**0.00987** $\bar{3}$.99423
44.70 1.65035	**0.4470** $\bar{1}$.65035	**26.82** 1.42850	**1.609** 0.20670	**1.467** 0.16633	**88** 1.94448	**1**	**0.86839** $\bar{1}$.93871
51.497 1.71178	**0.51497** $\bar{1}$.71178	**30.898** 1.48993	**1.8532** 0.26793	**1.68894** 0.22761	**101.337** 2.00577	**1.15155** 0.06128	**1**

Mass Equivalents

(For conversion table see p. 77)

Kilograms	Grains	Ounces		Pounds		Tons		
		Troy and apoth.	Avoirdupois	Troy and apoth.	Avoirdupois	Short	Long	Metric
1	**15432** 4.18843	**32.15** 1.50719	**35.27** 1.54745	**2.6792** 0.42801	**2.205** 0.34333	**0.0$_2$1102** $\bar{3}$.04230	**0.0$_3$9842** $\bar{4}$.99309	**0.001** $\bar{3}$.00000
0.0$_4$6480 $\bar{5}$.81157	**1**	**0.0$_2$2083** $\bar{3}$.31876	**0.0$_2$2286** $\bar{3}$.35902	**0.0$_3$1736** $\bar{4}$.23958	**0.0$_3$1429** $\bar{4}$.15490	**0.0$_7$7143** $\bar{8}$.85387	**0.0$_7$6378** $\bar{8}$.80465	**0.0$_7$6480** $\bar{8}$.81157
0.03110 $\bar{2}$.49281	**480** 2.68124	**1**	**1.09714** 0.04026	**0.08333** $\bar{2}$.92082	**0.06857** $\bar{2}$.83614	**0.0$_4$3429** $\bar{5}$.53511	**0.0$_4$3061** $\bar{5}$.48590	**0.0$_4$3110** $\bar{5}$.49281
0.02835 $\bar{2}$.45255	**437.5** 2.64098	**0.9115** $\bar{1}$.95974	**1**	**0.07595** $\bar{2}$.88056	**0.0625** $\bar{2}$.79588	**0.0$_4$3125** $\bar{5}$.49485	**0.0$_4$2790** $\bar{5}$.44563	**0.0$_4$2835** $\bar{5}$.45255
0.3732 $\bar{1}$.57199	**5760** 3.76042	**12** 1.07918	**13.17** 1.11944	**1**	**0.8229** $\bar{1}$.91532	**0.0$_3$4114** $\bar{4}$.61429	**0.0$_3$3673** $\bar{4}$.56508	**0.0$_3$3732** $\bar{4}$.57199
0.4536 $\bar{1}$.65667	**7000** 3.84510	**14.58** 1.16386	**16** 1.20412	**1.215** 0.08468	**1**	**0.0005** $\bar{4}$.69897	**0.0$_3$4464** $\bar{4}$.64975	**0.0$_3$4536** $\bar{4}$.65667
907.2 2.95770	**140$_6$** 7.14613	**29167** 4.46489	**320$_3$** 4.50515	**2431** 3.38571	**2000** 3.30103	**1**	**0.8929** $\bar{1}$.95078	**0.9072** $\bar{1}$.95770
1016 3.00691	**15680$_4$** 7.19535	**326$_3$** 4.51411	**35840** 4.55437	**2722** 3.43492	**2240** 3.35025	**1.12** 0.04922	**1**	**1.016** 0.00691
1000 3.00000	**15432356** 7.18843	**32151** 4.50719	**35274** 4.54745	**2679** 3.42801	**2205** 3.34333	**1.102** 0.04230	**0.9842** $\bar{1}$.99309	**1**

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Pressure Equivalents

(For conversion table see p. 80)

Megabars or megadynes per sq. cm.	Kilograms per sq. cm. (Metric atmospheres)	Pounds per sq. in.	Short tons per sq. ft.	Atmospheres	Columns of mercury at temperature 0° C.		Columns of water at temperature 15° C.		
					Meters	Inches	Meters	Inches	Feet
1	1.0197 0.00848	14.50 1.16148	1.044 0.01882	0.9869 $\bar{1}.99427$	0.7500 $\bar{1}.87508$	29.53 1.47025	10.21 1.00886	401.8 2.60402	33.48 1.52484
0.9807 $\bar{1}.99152$	1	14.22 1.15300	1.024 0.01034	0.9678 $\bar{1}.98579$	0.7355 $\bar{1}.86660$	28.96 1.46177	10.01 1.00038	394.0 2.59555	32.84 1.51636
0.06895 $\bar{2}.83852$	0.07031 $\bar{2}.84700$	1	0.072 $\bar{2}.85733$	0.06804 $\bar{2}.83279$	0.05171 $\bar{2}.71360$	2.036 0.30878	0.7037 $\bar{1}.84738$	27.70 1.44254	2.309 0.36336
0.9576 $\bar{1}.98119$	0.9765 $\bar{1}.98966$	13.89 1.14267	1	0.9450 $\bar{1}.97545$	0.7182 $\bar{1}.85627$	28.28 1.45143	9.773 0.99004	384.8 2.58521	32.06 1.50603
1.0133 0.00573	1.0333 0.01421	14.70 1.16722	1.058 0.02955	1	0.76 $\bar{1}.88081$	29.92 1.47598	10.34 1.01459	407.2 2.60976	33.93 1.53058
1.3333 0.12492	1.3596 0.13340	19.34 1.28640	1.392 0.14373	1.316 0.11919	1	39.37 1.59517	13.61 1.13378	535.7 2.72894	44.64 1.64976
0.03386 $\bar{2}.52975$	0.03453 $\bar{2}.53823$	0.4912 $\bar{1}.69124$	0.03536 $\bar{2}.54857$	0.03342 $\bar{2}.52402$	0.02540 $\bar{2}.40484$	1	0.3456 $\bar{1}.53861$	13.61 1.13378	1.134 0.05460
0.09798 $\bar{2}.99114$	0.09991 $\bar{2}.99962$	1.421 0.15262	0.1023 $\bar{1}.00996$	0.09670 $\bar{2}.98541$	0.07349 $\bar{2}.86622$	2.893 0.46139	1	39.37 1.59517	3.281 0.55198
0.002489 $\bar{3}.39598$	0.002538 $\bar{3}.40446$	0.03610 $\bar{2}.55746$	0.002599 $\bar{3}.41479$	0.002456 $\bar{3}.39024$	0.001867 $\bar{3}.27106$	0.07349 $\bar{2}.86622$	0.02540 $\bar{2}.40484$	1	0.08333 $\bar{2}.92082$
0.02986 $\bar{2}.47516$	0.03045 $\bar{2}.48364$	0.4332 $\bar{1}.63664$	0.03119 $\bar{2}.49397$	0.02947 $\bar{2}.46942$	0.02240 $\bar{2}.35024$	0.8819 $\bar{1}.94540$	0.3048 $\bar{1}.48402$	12 1.07918	1

Energy or Work Equivalents

(For conversion table see p. 80)

Joules = 10^7 ergs	Kilogram-meters	Foot-pounds	Kilo-watt-hours	Cheval-vapeur-hours	Horse-power-hours	Liter-atmospheres	Kilo-gram-calories	British thermal units
1	0.10197 $\bar{1}.00848$	0.7376 $\bar{1}.86780$	0.0_62778 $\bar{7}.44370$	0.0_63777 $\bar{7}.57711$	0.0_63725 $\bar{7}.57113$	0.009869 $\bar{3}.99427$	0.0_32390 $\bar{4}.37848$	0.0_39486 $\bar{4}.97709$
9.80665 0.9915207	1	7.233 0.85932	0.0_52724 $\bar{6}.43522$	0.0_537037 $\bar{6}.56863$	0.0_53653 $\bar{6}.56265$	0.09678 $\bar{2}.98579$	0.002344 $\bar{3}.37000$	0.009302 $\bar{3}.96861$
1.356 0.13220	0.1383 $\bar{1}.14068$	1	0.0_63766 $\bar{7}.57590$	0.0_651206 $\bar{7}.70932$	0.0_650505 $\bar{7}.70333$	0.01338 $\bar{2}.12647$	0.0_33241 $\bar{4}.51068$	0.001286 $\bar{3}.10929$
3.6×10^6 6.55630	3.671×10^5 5.56478	2.655×10^6 6.42410	1	1.3596 0.13342	1.341 0.12743	35528 4.55057	860.5 2.93478	3415 3.53339
2.648×10^6 6.42288	270000. 5.43136	1.9529×10^6 6.29068	0.7355 $\bar{1}.86658$	1	0.9863 $\bar{1}.99401$	26131. 4.41715	632.9 2.80135	2512 3.39996
2.6845×10^6 6.42887	2.7375×10^5 5.43735	1.98×10^6 6.29667	0.7457 $\bar{1}.87257$	1.0139 0.00598	1	26494 4.42314	641.7 2.80735	2547 3.40595
101.33 2.00573	10.333 1.01421	74.73 1.87353	0.0_42815 $\bar{5}.44943$	0.0_43827 $\bar{5}.58284$	0.0_43774 $\bar{5}.57686$	1	0.02422 $\bar{2}.38425$	0.09612 $\bar{2}.98281$
4183 3.62153	426.6 2.63000	3086 3.48932	0.001162 $\bar{3}.06522$	0.001580 $\bar{3}.19864$	0.001558 $\bar{3}.19265$	41.29 1.61579	1	3.968 0.59861
1054 3.02291	107.5 2.03139	777.52 2.89071	0.0_32928 $\bar{4}.46661$	0.0_33981 $\bar{4}.60003$	0.0_33927 $\bar{4}.59405$	10.40 1.01719	0.25200 $\bar{1}.40139$	1

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Linear and Angular Velocity Conversion Factors

	Cm. per sec. to feet per min.	Feet per min. to cm. per sec.	Cm. per sec. to miles per hour	Miles per hour to cm. per sec.	Feet per sec. to miles per hour	Miles per hour to feet per sec.	Radians per sec. to rev. per min.	Rev. per min. to radians per sec.
1	1.97	0.508	0.0224	44.7	0.682	1.47	9.55	0.1047
2	3.94	1.016	0.0447	89.41	1.364	2.93	19.10	0.2094
3	5.91	1.524	0.0671	134.1	2.046	4.40	28.65	0.3142
4	7.87	2.032	0.0895	178.8	2.727	5.87	38.20	0.4189
5	9.84	2.540	0.1118	223.5	3.409	7.33	47.75	0.5236
6	11.81	3.048	0.1342	268.2	4.091	8.80	57.30	0.6283
7	13.78	3.556	0.1566	312.9	4.773	10.27	66.85	0.7330
8	15.75	4.064	0.1789	357.6	5.455	11.73	76.39	0.8378
9	17.72	4.572	0.2013	402.3	6.136	13.20	85.94	0.9425

Conversion of Pressures

	Pounds per sq. in. to kilograms per sq. cm.	Kilograms per sq. cm. to pounds per sq. in.	Atmospheres to pounds per sq. in.	Pounds per sq. in. to atmospheres	Atmospheres to kilograms per sq. cm.	Kilograms per sq. cm. to atmospheres
1	0.0703	14.22	14.70	0.0680	1.033	0.9678
2	0.1406	28.45	29.39	0.1361	2.067	1.936
3	0.2109	42.67	44.09	0.2041	3.100	2.903
4	0.2812	56.89	58.79	0.2722	4.133	3.871
5	0.3515	71.12	73.48	0.3402	5.166	4.839
6	0.4218	85.34	88.18	0.4082	6.200	5.807
7	0.4922	99.56	102.9	0.4763	7.233	6.774
8	0.5624	113.8	117.6	0.5443	8.266	7.742
9	0.6328	128.0	132.3	0.6124	9.300	8.710

Conversion of Energy, Work, Heat

	Ft.-lb. to kilogram-meters	Kilogram-meters to ft.-lb.	Ft.-lb. to B.t.u.	B.t.u. to ft.-lb.	Kilogram-meters to large calories	Large calories to kilogram-meters	Joules to small calories	Small calories to joules
1	0.1383	7.233	0.001286	777.5	0.002344	426.6	0.2390	4.183
2	0.2765	14.47	0.002572	1555.0	0.004688	853.2	0.4780	8.367
3	0.4148	21.70	0.003858	2333.0	0.007033	1280.0	0.7170	12.55
4	0.5530	28.93	0.005144	3110.0	0.009377	1706.0	0.9560	16.73
5	0.6913	36.16	0.006431	3888.0	0.01172	2133.0	1.195	20.92
6	0.8295	43.40	0.007717	4665.0	0.01407	2560.0	1.434	25.10
7	0.9678	50.63	0.009003	5443.0	0.01641	2986.0	1.673	29.28
8	1.106	57.86	0.01029	6220.0	0.01875	3413.0	1.912	33.47
9	1.244	65.10	0.01157	6998.0	0.02110	3839.0	2.151	37.65

Conversion of Power

	Horse powers to kilowatts	Kilowatts to horse powers	Metric horse powers to kilowatts	Kilowatts to metric horse powers	Horse powers to metric horse powers	Metric horse powers to horse powers
1	0.7457	1.341	0.7354	1.360	1.014	0.9863
2	1.491	2.682	1.471	2.719	2.028	1.973
3	2.237	4.023	2.206	4.079	3.042	2.959
4	2.983	5.364	2.942	5.439	4.056	3.945
5	3.728	6.705	3.677	6.799	5.069	4.932
6	4.474	8.046	4.413	8.158	6.083	5.918
7	5.220	9.387	5.148	9.518	7.097	6.904
8	5.965	10.73	5.884	10.88	8.111	7.890
9	6.710	12.07	6.619	12.24	9.125	8.877

Power Equivalents

(For conversion table see p. 80)

Horse power 550 standard ft.-lb. per sec.	Kilowatts (1000 joules per sec.)	Cheval-vapeur (metric h.p.)	Poncelets	M.-kg. per sec.	Ft.-lb. per sec.	Kg.-cal. per sec.	B.t.u per sec.
1	**0.7457** $\bar{1}.87256$	**1.014** 0.00599	**0.7604** $\bar{1}.88106$	**76.04** 1.88106	**550** 2.74036	**0.1783** $\bar{1}.25104$	**0.7074** $\bar{1}.84965$
1.341 0.12743	**1**	**1.360** 0.13343	**1.020** 0.00848	**102.0** 2.00848	**737.6** 2.86780	**0.2390** $\bar{1}.37848$	**0.9486** $\bar{1}.97709$
0.9863 $\bar{1}.99402$	**0.7355** $\bar{1}.86659$	**1**	**0.75** $\bar{1}.87506$	**75** 1.87506	**542.3** 2.73438	**0.1758** $\bar{1}.24506$	**0.6977** $\bar{1}.84367$
1.315 0.11896	**0.9807** $\bar{1}.99152$	**1.333** 0.12498	**1**	**100** 2.00000	**723.3** 2.85932	**0.2344** $\bar{1}.37000$	**0.9303** $\bar{1}.96861$
0.01315 $\bar{2}.11896$	**0.009807** $\bar{3}.99152$	**0.01333** $\bar{2}.12493$	**0.01** $\bar{2}.00000$	**1**	**7.233** 0.85932	**0.002344** $\bar{3}.37000$	**0.009303** $\bar{2}.96861$
0.00182 $\bar{3}.25946$	**0.001356** $\bar{3}.13219$	**0.00184** $\bar{3}.26562$	**0.00138** $\bar{3}.14067$	**0.1383** $\bar{1}.14067$	**1**	**0.0_33241** $\bar{4}.51068$	**0.001286** $\bar{3}.10929$
5.610 0.74896	**4.183** 0.62153	**5.688** 0.75494	**4.266** 0.63000	**426.6** 2.63000	**3086** 3.48932	**1**	**3.968** 0.59861
1.414 0.15035	**1.054** 0.02291	**1.433** 0.15632	**1.075** 0.03139	**107.5** 2.03139	**777.5** 2.89071	**0.2520** $\bar{1}.40138$	**1**

The equivalents are given in the heavier type. Logarithms of the equivalents are given immediately below.

Subscripts after any figure, 0_3, 9_4, etc., mean that that figure is to be repeated the indicated number of times.

Density Equivalents and Conversion Factors

Equivalents					Conversion factors				
Grams per cu. cm.	Lb. per cu. in.	Lb. per cu. ft.	Short tons (2000 lb.) per cu. yd.	Lb. per U. S. gal.		Grams per cu. cm. to lb. per cu. ft.	Lb. per cu. ft. to grams per cu. cm.	Grams per cu. cm. to short tons per cu. yd.	Short tons per cu. yd. to grams per cu. cm.
1	**0.03613** $\bar{2}.55787$	**62.43** 1.79539	**0.8428** $\bar{1}.92572$	**8.345** 0.92143	1 2	62.43 124.90	0.01602 0.03204	0.8428 1.6860	1.186 2.373
27.68 1.44217	**1**	**1728** 3.23754	**23.33** 1.36792	**231** 2.36361	3 4	187.30 249.70	0.04806 0.06407	2.5280 3.3710	3.600 4.746
0.01602 $\bar{2}.20466$	**0.0_35787** $\bar{4}.76245$	**1**	**0.0135** $\bar{2}.13033$	**0.1337** $\bar{1}.12613$	5 6	312.40 374.60	0.08009 0.09611	4.2140 5.0570	5.933 7.119
1.186 0.07428	**0.04286** $\bar{2}.63205$	**74.07** 1.86964	**1**	**9.902** 0.99572	7 8	437.00 499.40	0.11210 0.12820	5.9000 6.7420	8.306 9.492
0.1198 $\bar{1}.07855$	**0.004329** $\bar{3}.63639$	**7.481** 0.87396	**0.1010** $\bar{1}.00432$	**1**	9 10	561.90 624.30	0.14420 0.16020	7.5850 8.4280	10.680 11.870

Conversion of Heat Transmission and Conduction

	Small calories per sq. cm. to B.t.u. per sq. ft.	B.t.u. per sq. ft. to small calories per sq. cm.	Small calories per sq. cm. per cm. to B.t.u. per sq. ft. per in.	B.t.u. per sq. ft. per in. to small calories per sq. cm. per cm.	Small calories per sec. per sq. cm. per 1 deg. cent. per cm. thick, to B.t.u. per hr. per sq. ft. per 1 deg. fahr. per in. thick	B.t.u. per hr. per sq. ft. per 1 deg. fahr. per in. thick to small calories per sec. per sq. cm. per 1 deg. cent. per cm. thick
1	3.687	0.2712	1.451	0.6892	2.903×10^3	$0.0_3 3445$
2	7.374	0.5424	2.902	1.378	5.806×10^3	$0.0_3 6890$
3	11.06	0.8136	4.353	2.068	8.709×10^3	$0.0_2 1034$
4	14.75	1.085	5.804	2.757	11.61×10^3	$0.0_2 1378$
5	18.44	1.356	7.255	3.446	14.52×10^3	$0.0_2 1722$
6	22.12	1.627	8.706	4.135	17.42×10^3	$0.0_2 2067$
7	25.81	1.898	10.16	4.824	20.32×10^3	$0.0_2 2412$
8	29.50	2.170	11.61	5.514	23.22×10^3	$0.0_2 2756$
9	33.18	2.441	13.06	6.203	26.13×10^3	$0.0_2 3100$

NOTE. 1 gram-calorie per sq. cm. = 3.687 B.t.u. per sq. ft.
1 gram-calorie per sq. cm. per cm. = 1.451 B.t.u. per sq. ft. per in.
1 gram-calorie per sec. per sq. cm. for a temp. grad. of 1 deg. cent. per cm.
= 360 kilogram-calories per hour per sq. m. for a temp. grad. of 1 deg. cent. per m.
= 2.903×10^3 B.t.u. per hour per sq. ft. for a temp. grad. of 1 deg. fahr. per in.

Values of Foreign Coins

(Legal standards: (*G*) = gold; (*S*) = silver)

Country	Monetary unit	Value in terms of U. S. money	Country	Monetary unit	Value in terms of U. S. money
Argentina (*G*)	Peso	$0.9647	Great Britain (*G*)	Pound sterling.	$4.8665
Austria-Hungary (*G*)	Crown	0.2026	Greece (*G* and *S*)	Drachma	0.1929
Belgium (*G* and *S*)	Franc	0.1929	Haiti (*G*)	Gourde	0.9647
Bolivia (*G*)	Boliviano	0.3893	India (British) (*G*)	Rupee	0.3244
Brazil (*G*)	Milreis	0.5463	Italy (*G* and *S*)	Lira	0.1929
British colonies in Australasia and Africa (*G*).	Pound sterling.	4.8665	Japan (*G*)	Yen	0.4984
			Liberia (*G*)	Dollar	1.0000
			Mexico (*G*)	Peso	0.4984
Canada (*G*)	Dollar	1.0000	Netherlands (*G*)	Florin	0.4019
Central American States:			Norway (*G*)	Crown	0.2679
			Panama (*G*)	Balboa	1.0000
Costa Rica (*G*)	Colon	0.4653	Persia (*G* and *S*)	Kran	Variable
British Honduras (*G* or *S*).	Dollar	1.0000	Peru (*G*)	Libra	4.8665
			Philippine Islands (*G*)	Peso	0.5000
Guatemala (*S*)	Peso	0.4446	Portugal (*G*)	Escudo	1.0805
Honduras (*S*)	Peso	0.4446	Roumania (*G*)	Leu	0.1929
Salvador (*S*)	Peso	0.4446	Russia (*G*)	Ruble	0.5145
Nicaragua (*S*)	Cordoba	1.0000	Santo Domingo (*G*)	Dollar	1.0000
Chile (*G*)	Peso	0.3649	Servia (*G*)	Dinar	0.1929
China (*S*)	Yuan	0.4777	Siam (*G*)	Tical	0.3708
Colombia (*G*)	Pound	4.8665	Spain (*G* and *S*)	Peseta	0.1929
Denmark (*G*)	Crown	0.2680	Straits Settlement (*G*)	Dollar	0.5677
Ecuador (*G*)	Sucre	0.4866	Sweden (*G*)	Crown	0.2679
Egypt (*G*)	Pound	4.9429	Switzerland (*G*)	Franc	0.1929
Finland (*G*)	Markka	0.1929	Turkey (*G*)	Piaster	0.0439
France (*G* or *S*)	Franc	0.1929	Uruguay (*G*)	Peso	1.0340
German Empire (*G*)	Mark	0.2381	Venezuela (*G*)	Bolivar	0.1929

TIME

Kinds of Time. Three kinds of time are recognized by astronomers, viz., sidereal, apparent solar, and mean solar time. The **sidereal day** is the interval between two consecutive transits of some fixed celestial object across any given meridian, or it is the interval required by the earth to make one complete revolution on its axis. This interval is constant but it is inconvenient as a time unit because the noon of the sidereal day occurs at all hours of the day and night. The **apparent solar day** is the interval between two consecutive transits of the sun across any given meridian. On account of the variable distance between the sun and earth, the variable speed of the earth in its orbit, the effect of the moon, etc., this interval is not constant and consequently cannot be kept by any simple mechanism, such as clocks or watches. To overcome the objection noted above, the **mean solar day** was devised. The mean solar day is the length of the average apparent solar day. Like the sidereal day it is constant, and like the apparent solar day its noon always occurs at approximately the same time of day. The astronomical day begins at mean solar noon and the hours run from one to twenty-four, while the civil day (mean solar) begins 12 hours earlier, at midnight, and the hours run from one to twelve, and then repeat from noon to midnight.

The Year. There are three different kinds of year used, the sidereal, the tropical, and the anomalistic. The **sidereal year** is the time taken by the earth to complete one revolution around the sun from a given star to the same star again. Its length is 365 days, 6 hours, 9 minutes, and 9 seconds. The **tropical year** is the time included between two successive passages of the vernal equinox by the sun, and since the equinox moves westward 50.″2 of arc a year, the tropical year is shorter by 20′23.″5 in time than the sidereal year. As the seasons depend upon the earth's position with respect to the equinox, the tropical year is the year of civil reckoning. The **anomalistic year** is the interval between two successive passages of the perihelion, namely, the time of the earth's nearest approach to the sun. The anomalistic year is only used in special calculations in astronomy.

The Calendar. The month depended originally upon the changes of the moon. The Mohammedan nations still use a lunar calendar with years of twelve lunar months, which alternately contain 355 and 356 days. According to their method of reckoning the same month falls in different seasons, and their calendars gain 1 year on ours every 33 years. The **Julian Calendar** (established 45 B. C.) discards all consideration of the moon and adopts 365¼ days as the true length of the year. It is still used in Russia and generally by the Greek Church. **Gregorian Calendar:** The true length of the tropical year is 365 days, 5 hr., 48 min., 45.5 sec., a difference of 11 min., 14.5 sec. by which the Julian year is too long. This amounts to a little more than 3 days in 400 years. To correct for this, those century years are made leap years which are divisible by 400 without remainder.

Standard Time. Prior to 1883 each city of the U. S. had its own time, which was determined by the time of passage of the sun across the local meridian. A system of standard time is used at present, according to which the United States, which extends from 65 deg. to 125 deg. West longitude, is divided into four sections, each of 15 deg. of longitude. The first or eastern section includes all territory between the Atlantic coast and an irregular line drawn from Detroit, Mich., through Pittsburg to Charleston, S. C., its most southern point. The time of this section is that of the 75-deg. meridian, which is 5 hr. slower than Greenwich time. The second (central) section includes all territory between the line mentioned, and an irregular line drawn from Bismarck, N. D., to the mouth of the Rio Grande. The third (mountain) section includes all territory between the last-named line and a line which passes through the western part of Idaho, Utah and Arizona. The fourth (Pacific) section covers the rest of the country to the Pacific Ocean. Standard time is uniform in each of these sections, but the time in one section differs by exactly 1 hr. from the section next to it. In cities situated on the border line of two sections, as, say, Pittsburg and Atlanta, the standard times of both sections are used, and in such cities when the time is given, it should be specified as eastern, central, etc. The system of standard time has been adopted in almost all civilized countries. All continental Europe, except Russia, uses a time 1 hr. faster than that of Greenwich; in Japan and Australia the time is 9 hr. faster.

TERRESTRIAL GRAVITY

By **standard gravity** is meant any locality where g_0 = 980.665 cm. per sec. per sec., or 32.1740 ft. per sec. per sec. This value, g_0, is assumed to be the value of g at sea level and latitude 45 deg.

Acceleration of Gravity

(U. S. Coast and Geodetic Survey, 1912)

Latitude, deg.	g		g/g_0	Latitude, deg.	g		g/g_0
	Cm./sec.²	Ft./sec.²			Cm./sec.²	Ft./sec.²	
0	978.0	32.088	0.9973	50	981.1	32.187	1.0004
10	978.2	32.093	0.9975	60	981.9	32.215	1.0013
20	978.6	32. 08	0.9979	70	982.6	32.238	1.0020
30	979.3	32.130	0.9986	80	983.1	32.253	1.0024
40	980.2	32.158	0.9995	90	983.2	32.258	1.0026

Correction for altitude above sea level: − 0.3 cm. per sec.² for each 1000 meters; − 0.003 ft. per sec.² for each 1000 feet.

SPECIFIC GRAVITY AND DENSITY

The **specific gravity of a solid or liquid** is the ratio of the mass of the body to the mass of an equal volume of water at some standard temperature. At the present time a temperature of 4 deg. cent. (39 deg. fahr.) is commonly used by physicists, but the engineer uses 60 deg. fahr. The specific gravity **of gases** is usually expressed in terms of hydrogen or air.

The **density** of a body is its mass per unit volume. If the gram is used as the unit of mass and the milliliter as the unit of volume, the figures representing the density are the same as the specific gravity of the body referred to water at 4 deg. cent. as unity. The customary unit is pounds per cu. ft.

The specific gravity of liquids is usually measured by means of an hydrometer (see p. 254).* Special arbitrary hydrometer scales are used in various trades and industries. The most common of these are the Baumé, Twaddell and Beck. Twaddell's hydrometer is used for liquids heavier than water. The number of degrees, N, which it indicates may be converted to specific gravities, G, by the formula $G = (5N + 1000)/1000$. The formula for the Beck hydrometer is $G = 170/(170 \pm N)$; for the Brix hydrometer $G = 400/(400 \pm N)$. In both of these the + sign is to be used for liquids lighter than water, the − sign for heavier liquids. For the salinometer (salometer), see p. 1734.* The specific gravities corresponding to the indications of the Baumé hydrometer are given in the following tables.

*See Marks' MECHANICAL ENGINEERS' HANDBOOK.

Specific Gravities at $\frac{60°}{60°}$ Fahr. Corresponding to Degrees Baumé for Liquids Lighter than Water

[Calculated from the formula, specific gravity $\frac{60°}{60°}$ fahr. $= \frac{140}{130 + \text{Deg. Bé.}}$]

Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity
10	1.0000	25	0.9032	40	0.8235	55	0.7568	70	0.7000	85	0.6512
11	0.9929	26	0.8974	41	0.8187	56	0.7527	71	0.6965	86	0.6482
12	0.9859	27	0.8917	42	0.8140	57	0.7487	72	0.6931	87	0.6452
13	0.9790	28	0.8861	43	0.8092	58	0.7447	73	0.6897	88	0.6422
14	0.9722	29	0.8805	44	0.8046	59	0.7407	74	0.6863	89	0.6393
15	0.9655	30	0.8750	45	0.8000	60	0.7368	75	0.6829	90	0.6364
16	0.9589	31	0.8696	46	0.7955	61	0.7330	76	0.6796	91	0.6335
17	0.9524	32	0.8642	47	0.7910	62	0.7292	77	0.6763	92	0.6306
18	0.9459	33	0.8589	48	0.7865	63	0.7254	78	0.6731	93	0.6278
19	0.9396	34	0.8537	49	0.7821	64	0.7216	79	0.6699	94	0.6250
20	0.9333	35	0.8485	50	0.7778	65	0.7179	80	0.6667	95	0.6222
21	0.9272	36	0.8434	51	0.7735	66	0.7143	81	0.6635	96	0.6195
22	0.9211	37	0.8383	52	0.7692	67	0.7107	82	0.6604	97	0.6167
23	0.9150	38	0.8333	53	0.7650	68	0.7071	83	0.6573	98	0.6140
24	0.9091	39	0.8284	54	0.7609	69	0.7035	84	0.6542	99	0.6114
										100	0.6087

Specific Gravities at $\frac{60°}{60°}$ Fahr. Corresponding to Degrees Baumé for Liquids Heavier than Water

[Calculated from the formula, specific gravity $\frac{60°}{60°}$ fahr. $= \frac{145}{145 - \text{Deg. Baumé}}$]

Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity	Degrees Baumé	Specific gravity
0	1.0000	12	1.0902	24	1.1983	36	1.3303	48	1.4948	60	1.7059
1	1.0069	13	1.0985	25	1.2083	37	1.3426	49	1.5104	61	1.7262
2	1.0140	14	1.1069	26	1.2185	38	1.3551	50	1.5263	62	1.7470
3	1.0211	15	1.1154	27	1.2288	39	1.3679	51	1.5426	63	1.7683
4	1.0284	16	1.1240	28	1.2393	40	1.3810	52	1.5591	64	1.7901
5	1.0357	17	1.1328	29	1.2500	41	1.3942	53	1.5761	65	1.8125
6	1.0432	18	1.1417	30	1.2609	42	1.4078	54	1.5934	66	1.8354
7	1.0507	19	1.1508	31	1.2719	43	1.4216	55	1.6111	67	1.8590
8	1.0584	20	1.1600	32	1.2832	44	1.4356	56	1.6292	68	1.8831
9	1.0662	21	1.1694	33	1.2946	45	1.4500	57	1.6477	69	1.9079
10	1.0741	22	1.1789	34	1.3063	46	1.4646	58	1.6667	70	1.9333
11	1.0821	23	1.1885	35	1.3182	47	1.4796	59	1.6860	......	....

Mohs's Scale of Hardness

1. Talc. 2. Gypsum. 3. Calc spar. 4. Fluor spar. 5. Apatite.
6. Feldspar. 7. Quartz. 8. Topaz. 9. Sapphire. 10. Diamond.

SECTION 2

MATHEMATICS

BY

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CONTENTS

MATHEMATICS

BY

EDWARD V. HUNTINGTON

ARITHMETIC

NUMERICAL COMPUTATION

Number of Significant Figures. In any engineering computation, the data are ordinarily the results of measurement, and are correct only to a limited number of significant figures. Each of the numbers 3.840 and 0.003840 is said to be given "correct to four figures;" the true value lies in the first case between 3.8395 and 3.8405; in the second case, between 0.0038395 and 0.0038405. The **absolute error** is less than 0.001 in the first case, and less than 0.000001 in the second; but the **relative error** is the same in both cases, namely, an error of less than "one part in 3840."

If a number is written as 384000, the reader is left in doubt whether the number of correct significant figures is 3, 4, 5, or 6. This doubt can be removed by writing the number as 3.84×10^5 or 3.840×10^5 or 3.8400×10^5 or 3.84000×10^5.

In any numerical computation, the possible or desirable degree of accuracy should be decided on and the computation should then be so arranged that the required number of significant figures, and no more, is secured. Carrying out the work to a larger number of places than is justified by the data, is to be avoided, (1) because the form of the results leads to an erroneous impression of their accuracy, and (2) because time and labor are wasted in superfluous computation. The labor of working with six-place tables is nearly three times as great as that with four-place tables. In computations involving several steps, it is desirable to retain one extra figure until just before the final result is reached, in order to protect the last figure against the possible cumulative effect of small tabular errors. In **discarding superfluous figures,** if the first discarded figure is 5 or more, increase the preceding figure by 1. Thus, 3.14159, written correct to four figures, is 3.142; correct to three figures, 3.14. Again, 6.1297, correct to four figures, is 6.130.

Addition. In adding numbers, note that a doubtful final figure in any one number will render doubtful the whole column in which that figure lies; hence all figures to the right of that column are superfluous, and contribute nothing to the accuracy of the result.

0.2056x
2.572xx
14.25xxx
576.1xxxx
593.1

Subtraction. The "Austrian" or "shop" method is recommended. The mental process is as follows, the figures here printed in boldface type being the only ones written down:

[3 plus how many is 12?] 3 plus **9** is 12; 1 to carry.	14752
[7 plus how many is 15?] 7 plus **8** is 15; 1 to carry.	8463
5 plus **2** is 7. 8 plus **6** is 14.	**6289**

This method is especially useful when it is desired to subtract from a given number the sum of several other numbers.

7 plus 1 is 8; plus 5 is 13; plus 9 is 22; 2 to carry.	14752
5 plus 0 is 5; plus 2 is 7; plus 8 is 15; 1 to carry.	3125
3 plus 1 is 4; plus 1 is 5; plus 2 is 7.	101
5 plus 3 is 8; plus 6 is 14.	5237
	6289

The use of a wavy line to indicate subtraction is also recommended, as it will minimize the danger of adding when subtraction is intended.

Multiplication. In long examples in multiplication, the arrangement of work here illustrated is recommended, since it facilitates the abbreviation of the work by the omission, in practice, of all the figures on the right of the vertical line.

4956	
8372	
39648	
1486	8
346	92
9	912
41492	xxx

The **position of the decimal point** should be determined by reference to the first, or left-hand, figures of the numbers, rather than by "pointing off" so-and-so many places from the right-hand end. For the right-hand figures of a number are the least important ones, and in many cases are entirely unknown (especially when the slide rule or a computing machine is used). The mental process for determining the decimal point is as follows:

(*a*) If the multiplier is a number like 3.1416, with only one figure preceding the decimal point, think of this number as "a little over 3;" then the product must be "a little over three times the number which is being multiplied;" and this gives the position of the decimal point at once, by inspection.

(*b*) If the multiplier is a number like 3141.6 [or 0.000 003 141 6], think of this number as "about 3, with the point moved three places to the right" [or "about 3, with the point moved six places to the left"]; then think what the answer would be if the multiplier were simply "about 3," and shift the decimal point accordingly.

Multiplication Tables. Crelle's large volume (Berlin, G. Reimer) gives the product of every three-figure number by every three-figure number; Peters's (Berlin, G. Reimer), of every four-figure number by every two-figure number. The smaller table of H. Zimmermann (Berlin, Wm. Ernst) gives the product of every three-figure number by every two-figure number.

Division. In long division, where the numbers are given only approximately, the work can be much abbreviated without loss of accuracy by "cutting off" one figure of the divisor at each step, instead of "bringing down" a doubtful zero in the dividend. Thus, 3.1416 ÷ 2.3026 = 1.3644.

23026)	31416(1
	23026
2303)	8390(3
	6909
230)	1481(6
	1380
23)	101(4
	92
2)	9(4

To determine the **position of the decimal point** in a problem of fractional division, shift the point (mentally) in both numerator and denominator (the same number of places in each) until the denominator is a number in the "standard form," that is, a number with only one figure preceding the decimal point. (This will not change the value of the fraction.) Then estimate the approximate magnitude of the quotient by inspection. Thus:

$$\frac{0.2718}{3141.6} = \frac{0.000\ 2718}{3.1416} = \text{"about } 0.000\ 09\text{"} = 0.000\ 08652;$$

$$\frac{31.416}{0.002718} = \frac{31\ 416.}{2.718} = \text{"about } 10\ 000\text{"} = 11\ 558.$$

Reciprocals. The reciprocal of N is $1/N$. Instead of dividing by a long number N, it is often better to multiply by the reciprocal of N. The table of reciprocals on pp. 24–27 gives the reciprocal of any number, correct to four figures. Barlow's Table (Spon & Chamberlain, New York) gives the reciprocal of every four-figure number correct to seven figures (but without facilities for interpolation). The reciprocals of numbers having more than four figures are best found by the use of a large table of logarithms.

Reciprocals of 1 ± x when x is Small.

$$1/(1+x) = 1 - x + [\text{error} < x^2, \text{ if } x \text{ is between 0 and 1}],$$
$$= 1 - x + x^2 - [\text{error} < x^3, \text{ if } x \text{ is between 0 and 1}].$$
$$1/(1-x) = 1 + x + [\text{error} < x^2 + 2x^3, \text{ if } x \text{ is between 0 and } \tfrac{1}{2}],$$
$$= 1 + x + x^2 + [\text{error} < x^3 + 2x^4, \text{ if } x \text{ is between 0 and } \tfrac{1}{2}].$$

NOTE. $1/(a \pm b) = (1/a)[1/(1 \pm x)]$, where $x = b/a$.

Notation by Powers of 10. All questions concerning the position of the decimal point are readily answered if each number is expressed in the "standard form," that is, as the product of two factors, one of which is a number with only one figure preceding the decimal point, while the other is a positive or negative power of 10. Thus, 3.1416×10^3 means 3.1416 with the point moved three places to the right, that is, 3141.6. Again, 3.1416×10^{-6} means 3.1416 with the point moved six places to the left, that is, 0.000 003 1416. This notation by powers of 10 should always be used in dealing with very large or very small numbers. Among electrical engineers its use is very general, even for numbers of moderate size.

Square Root. (*a*) If four figures of the root are sufficient, take the answer directly from the table of square roots, pp. 12–15. (*b*) To obtain a root of six or seven figures from the table, use the formula: $\sqrt{N} = a + [(N - a^2)/2a]$ (approx.), where a is the nearest value of $\sqrt{N}$ obtainable from the table, with three or four ciphers annexed. Here a^2 must be found exactly, by direct multiplication, so that at least three significant figures of the difference $N - a^2$ shall be known correctly; but this done, the division of $N - a^2$ by $2a$ should be carried to only three figures (logarithms or slide rule may be used).

NOTE. The simplest way to obtain any root of a seven-figure number correct to seven figures is to use a seven-place table of logarithms, if such a table is at hand.

Square Roots of 1 ± x when x is Small.

$$(1+x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}x - [\text{error less than } \tfrac{1}{8}x^2 \text{ if } 0 < x < 1]$$
$$= 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + [\text{error} < \tfrac{1}{16}x^3 \text{ if } 0 < x < 1]$$
$$(1-x)^{\frac{1}{2}} = 1 - \tfrac{1}{2}x - [\text{error} < \tfrac{1}{8}x^2 + \tfrac{1}{10}x^3 \text{ if } 0 < x < \tfrac{1}{2}]$$
$$= 1 - \tfrac{1}{2}x - \tfrac{1}{8}x^2 - [\text{error} < \tfrac{1}{16}x^3 + \tfrac{1}{10}x^4 \text{ if } 0 < x < \tfrac{1}{2}]$$

NOTE. $\sqrt{a+b} = \sqrt{a}\,(1+x)^{\frac{1}{2}}$, where $x = b/a$.

Cube Root. (*a*) If four figures of the root are sufficient, take the answer directly from the table of cube roots, pp. 16–21. (*b*) To obtain a root of six or seven figures from the table, use the formula: $\sqrt[3]{N} = a + [(N - a^3)/3a^2]$ (approx.), where a is the nearest value of $\sqrt[3]{N}$ obtainable from the table, with three or four ciphers annexed. Here a^3 must be found correct to seven or eight figures, by direct multiplication, so that at least three significant figures of the difference $N - a^3$ shall be known; but this done, the division of $N - a^3$ by $3a^2$ should be carried to only three or four figures (logarithms or the slide rule may be used).

NOTE. The simplest way to obtain any root of a seven-figure number correct to seven figures is to use a seven-place table of logarithms, if such a table is at hand.

Cube Roots of $1 \pm x$ when x is Small.

$$(1+x)^{1/3} = 1 + \tfrac{1}{3}x - [\text{error} < \tfrac{1}{9}x^2 \text{ if } 0 < x < 1],$$
$$= 1 + \tfrac{1}{3}x - \tfrac{1}{9}x^2 + [\text{error} < \tfrac{1}{16}x^3 \text{ if } 0 < x < 1],$$
$$(1-x)^{1/3} = 1 - \tfrac{1}{3}x - [\text{error} < \tfrac{1}{9}x^2 + \tfrac{1}{10}x^3 \text{ if } 0 < x < \tfrac{1}{2}],$$
$$= 1 - \tfrac{1}{3}x - \tfrac{1}{9}x^2 - [\text{error} < \tfrac{1}{16}x^3 + \tfrac{1}{18}x^4 \text{ if } 0 < x < \tfrac{1}{2}].$$

NOTE. $\sqrt[3]{a+b} = \sqrt[3]{a}(1+x)^{1/3}$, where $x = b/a$.

LOGARITHMS

Tables of Logarithms. The use of a table of logarithms greatly reduces the labor of multiplication, division, raising to powers, and extracting roots. The table on pp. 42–43 is carried out to four significant figures, and the following explanations should be sufficient to permit the use of the table readily, even by one without previous experience. For algebraic theory, see p. 113.

If more than four-figure accuracy is required, recourse must be had to a larger table. Five-place tables are available in great variety; the Macmillan Tables, 1913, are perhaps as convenient as any. If more than five figures are required, use Bremiker's six-place table, or proceed at once to a seven-place table: Schrön (Vieweg und Sohn, Braunschweig); Bruhns; Vega-Bremiker. If extreme accuracy is required, use the eight-place table by Bauschinger and Peters (Engelmann, Leipzig). Logarithmic paper, see p. 176.

To Find the Logarithm of Any Given (Positive) Number.

(*a*) WHEN THE GIVEN NUMBER IS BETWEEN 1 AND 10.

An inspection of the table on pp. 42–43 shows that as the number increases from 1 to 9.99. . . the logarithm of that number increases continuously from 0 to 0.999. . . For example, log 2.97 = 0.4728; log 2.98 = 0.4742.

If the given number contains four significant figures, it is necessary to interpolate between the tabulated values, as follows:

To find log 2.973, notice that this number is $\frac{3}{10}$ of the way from 2.97 to 2.98; hence its logarithm will be (approximately) $\frac{3}{10}$ of the way from 0.4728 to 0.4742. The difference here is 14 units, and $\frac{3}{10}$ of this difference is 4 (to the nearest unit); hence, by adding this 4 to 4728, log 2.973 = 0.4732. This process of interpolating should be performed mentally; the step of finding the tabular difference will be facilitated by a glance at the last column on the right, which gives, for each line of the table, the average of the differences along that line.

Again, to find log 4.098: From table, log 4.09 = 0.6117; adding $\frac{8}{10}$ of the difference (11), or about 9, gives: log 4.098 = 0.6126. Or better, since $\frac{8}{10}$ of the way forward is equal to $\frac{2}{10}$ of the way back, find in table log 4.10 = 0.6128, and *subtract* $\frac{2}{10}$ of 11, or 2, giving log. 4.098 = 0.6126. It should be noted that any interpolated value may be in error by 1 in the last place.

If the given number contains more than four significant figures, it should be cut down to four figures (see p. 88), since the later figures will not affect the result in four-place computations.

(*b*) WHEN THE GIVEN NUMBER IS LESS THAN 1 OR MORE THAN 10, it is simply necessary to notice that every such number can be regarded as obtainable from some number between 1 and 10 by merely shifting the decimal point (see p. 90); and that according to the rule at the foot of the table, moving the decimal point n places to the right [or left] in the number-column is equivalent to adding n [or $-n$] to the logarithm in the body of the table.

For example, to find log 2973. Here $2973 = 2.973 \times 10^3$ (*i.e.*, 2.973 with the decimal point moved 3 places to the right). From the table, log 2.973 = 0.4732. Hence, log 2973 = 0.4732 + 3, which may be written as 3.4732.

Again, to find log 0.0002973. Here $0.0002973 = 2.973 \times 10^{-4}$ (*i.e.*, 2.973 with the decimal point moved 4 places to the left). From the table, log 2.973 = 0.4732. Hence, log 0.0002973 = 0.4732 − 4. (This may be written as $\bar{4}.4732$, if desired, and is equal of course, to − 3.5268; this latter form, however, is not convenient in practice.)

It is thus evident that the logarithm of every positive number may be regarded as consisting of two parts: a decimal fraction, which is always positive (or zero); and a whole number, which may be positive, negative, or zero. The fractional part is called the **mantissa**, and is found from the table; the whole-number part is called the **characteristic**, and is determined by inspection.

To Find the Number Corresponding to a Given Logarithm.

(*a*) WHEN THE GIVEN LOGARITHM IS A POSITIVE DECIMAL FRACTION (CHARACTERISTIC ZERO), simply reverse the process for finding the logarithm of a number between 1 and 10.

For example, given log N = 0.4732; to find N. In the body of the table it is seen that 0.4732 lies a little beyond 0.4728; hence N must lie a little beyond 2.97. By taking differences it is found that 4728 is in fact $4/14$ of the way from 0.4728 to the next higher logarithm; therefore N must be $4/14$ of the way from 2.97 to the next higher number. But $4/14$ of 1 is 0.3 (to the nearest tenth), hence N = 2.973.

Again, given log N = 0.6126; to find N. Here, 0.6126 is $9/11$ of the way from 0.6117 to the next higher logarithm; therefore N must be $9/11$ of the way from 4.09 to the next higher number. But $9/11$ of 1 is 0.8 (to the nearest tenth), hence N = 4.098.

(*b*) WHEN THE GIVEN LOGARITHM HAS ANY GIVEN VALUE (CHARACTERISTIC NOT ZERO), proceed as follows: First, be sure the given logarithm is in the "standard form," that is, a positive decimal fraction (mantissa) plus a positive or negative whole number (characteristic). For example, if log N is originally given in the form log N = − 3.5268, this must first be reduced to the (equivalent) form log N = 0.4732 − 4 (or $\bar{4}.4732$), before entering the table. Having the logarithm given in the standard form, suppose for the moment that the characteristic is zero, and find in the table the number corresponding to the given mantissa; then move the decimal point to the right or left according as the value of the characteristic is positive or negative.

For example, given log N = 0.4732 + 3; to find N. From the table, the number corresponding to 0.4732 is 2.973. The characteristic (+ 3) directs that the decimal point be moved 3 places to the right; hence $N = 2.973 \times 10^3 = 2973$.

Again, given log N = 0.4732 − 4; to find N. From the table, the number corresponding to 0.4732 is 2.973. The characteristic (− 4) indicates that the decimal point is to be moved 4 places to the left; hence $N = 2.973 \times 10^{-4} = 0.0002973$.

The number corresponding to a given logarithm is called its **antilogarithm.** Thus, if log 2973 = 0.4732 + 3, then 2973 = antilog (0.4732 + 3).

NOTE 1. In most tables of logarithms the decimal point is omitted, the tables being in fact not tables of logarithms, but tables of mantissas. This omission is of no consequence to the experienced computer, but is often perplexing to one who makes only occasional use of such tables.

NOTE 2. Many computers prefer to write negative characteristics in the form of some positive number minus some multiple of 10; thus, 0.4732 − 4 = 6.4732 − 10; 0.4732 − 13 = 7.4732 − 20; etc.

Fundamental Properties of Logarithms. The usefulness of logarithms in computation depends on the following properties:

$$(1)\ \log(ab) = \log a + \log b; \quad (3)\ \log(a^n) = n \log a;$$
$$(2)\ \log(a/b) = \log a - \log b; \quad (4)\ \log \sqrt[n]{a} = (1/n) \log a;$$
$$(5)\ \log 10^n = n$$

It is to be noted also that log 1 = 0, log 10 = 1, and log $(1/n)$ = −log n

To Multiply by Logarithms. Find from the table the log. of each factor, and add; the result will be the log. of the product. Then find the product itself from the table.

EXAMPLE. To find $x = (4.098)(0.0002973)(72.1)$.

Answer: $x = 8.784 \times 10^{-2} = 0.08784$

$$\begin{array}{lll} \log\ 4.098 & = 0.6126 & \\ \log\ 0.0002973 & = 0.4732 - 4 & \\ \log\ 72.1 & = \underline{0.8579 + 1} & \\ \log\ x & = 1.9437 - 3 = 0.9437 - 2. & \end{array}$$

To Divide by Logarithms. First Method: Find from the table the log. of the numerator and the log. of the denominator, and subtract the second from the first; the result will be the logarithm of the quotient. Then find the quotient itself from the table.

EXAMPLE. To find $x = \dfrac{4.098}{0.0002973}$

Answer: $x = 1.378 \times 10^4 = 13780$

$$\begin{array}{ll} \log\ 4.098 & = 0.6126 \\ \log\ 0.0002973 & = \underline{0.4732 - 4} \\ \log\ x & = 0.1394 + 4 \end{array}$$

In order **to avoid negative mantissas** in cases where a larger mantissa would have to be subtracted from a smaller, modify the upper logarithm by adding and subtracting 1.

EXAMPLE. To find $x = \dfrac{0.0291}{63.4}$.

Answer: $x = 4.590 \times 10^{-4} = 0.0004590.$

$$\begin{array}{ll} \log 0.0291 & = 0.4639 - 2 = 1.4639 - 3 \\ \log 63.4 & = 0.8021 + 1 = \underline{0.8021 + 1} \\ \log x & \qquad\qquad\qquad\ = 0.6618 - 4 \end{array}$$

But if the logarithms are written with the characteristics in front, and the "shop method" of subtraction is used (see p. 88), then no such special device is here required. Thus:

$$\begin{array}{ll} \log\ 0.0291 & = \bar{2}.4639 \\ \log\ 63.4 & = \underline{1.8021} \\ \log\ x & = \bar{4}.6618 \end{array}$$

To Divide by Logarithms. Second Method: Instead of subtracting the log. of a number, it is often convenient to add the **cologarithm** of that number; the colog. of N being defined by: $\operatorname{colog} N = \log (1/N) = -\log N$.

To find the colog. of a number, write the log. of the number in the standard form, and subtract it from $1.0000 - 1$, as in the following examples:

$$\begin{array}{rl} & 1.0000 - 1 \\ \log 69.5 = & \underline{0.8420 + 1} \\ \operatorname{colog} 69.5 = & 0.1580 - 2 \end{array} \qquad \begin{array}{rl} & 1.0000 - 1 \\ \log 0.0002973 = & \underline{0.4732 - 4} \\ \operatorname{colog} 0.0002973 = & 0.5268 + 3 \end{array}$$

This subtraction should be performed mentally. Thus, to subtract the mantissa, subtract each digit from 9 until the last non-zero digit is arrived at, and subtract this from 10; to subtract the characteristic, follow the regular rule of algebra ("reverse the sign and add"). Hence, if the logarithm itself is already written down, or can be read off from the table without interpolation, the cologarithm can be written down at once, by inspection. The use of cologarithms is not essential in logarithmic computation, but it often facilitates a compact arrangement of the work, especially in cases where the denominator of a fraction is itself the product of two or more factors.

To Find the nth Power of a Number by Logarithms. Find from the table the log. of the number, and multiply it by n; the result will be the logarithm of the nth power of that number. Then find the power itself from the tables.

EXAMPLE 1. Find $x = (0.0291)^3$

Answer: $x = 2.464 \times 10^{-5} = 0.00002464.$

$$\begin{array}{ll} \log 0.0291 & = 0.4639 - 2 \\ & \qquad\qquad\ \underline{\ \ 3\ \ } \\ \log x & = 1.3917 - 6 = 0.3917 - 5. \end{array}$$

EXAMPLE 2. Find $x = (0.0291)^{1.41}$

Answer: $x = 6.825 \times 10^{-3}$

$= 0.006825$

$$\begin{array}{rrr} \log 0.0291 & = 0.4639 - 2 = & -1.5361 \\ & & 1.41 \\ \hline & & 15361 \\ & & 61444 \\ & & 15361 \\ \hline \log x & = & -2.1659 \\ & = & 0.8341 - 3 \end{array}$$

To Find the nth Root of a Number by Logarithms. Find from the table the log. of the number, and divide it by n; the result will be the log. of the nth root of that number. Then find the root itself from the table.

EXAMPLE. Find $x = \sqrt[3]{4.098}$ $\qquad \log 4.098 = 0.6126$

Answer: $x = 1.600$ $\qquad \log x = 0.2042$

In order **to avoid fractional characteristics,** if the characteristic is not divisible by n, make it so divisible by adding and subtracting a suitable number before dividing.

EXAMPLE. Find $x = \sqrt[3]{0.0004590}.$ $\qquad \log 0.0004590 = 0.6618 - 4$

Answer: $x = 7.714 \times 10^{-2}$ $\qquad 3)2.6618 - 6$

$= 0.07714$ $\qquad \log x = 0.8873 - 2$

But if the characteristic is positive, it is simpler to write it in front of the mantissa, and then divide directly.

THE SLIDE RULE

The slide rule is an indispensable aid in all problems in multiplication, division, proportion, squares, square roots, etc., in which a limited degree of accuracy is sufficient. The ordinary 10-in. Mannheim rule (see below) costs \$3 to \$4.50 and gives three significant figures correctly; the 20-in. rule (\$12.50) gives from three to four figures; the Fuller spiral rule (\$30) or the Thacher cylindrical rule (\$35) gives from four to five figures. For many problems the slide rule gives results more rapidly than a table of logarithms; it requires, however, more care in placing the decimal point in the answer. In all work with the slide rule, the position of the decimal point should be determined by inspection (see p. 89), only the sequence of digits being obtained from the instrument itself. Rapidity in the use of the instrument depends mainly on the skill with which the eye can estimate the values of the various divisions on the scale; expertness in this respect comes only with practice. The following explanations should be sufficient to permit the use of the ordinary slide rule successfully without previous experience and without knowledge of logarithms.

Multiplication and Division with a (Theoretical) Complete Logarithmic Scale. Consider a *complete* logarithmic scale (D, Fig. 1), assumed to extend indefinitely in both directions, only the main section, from 1 to 10, however, being usually available. Note that the divisions within the several sections are indentical, except that the numeral attached to each division of any one section is ten times the numeral attached to the corresponding division in the preceding section. [The distances laid off from 1 are proportional to the logarithms of the corresponding numbers, the distance from 1 to 10 being taken as unity.] Consider also a duplicate scale, C, numbered from 1 to 10, and arranged to slide along the fixed scale D as in the figures. By means of such a scale D, and slide C, any two numbers between 1 and 10 (and hence any two numbers whatever, with proper attention to the decimal point) can be multiplied or divided, as in the following examples.

To MULTIPLY 4 BY 6. In Fig. 1, starting with point 1 of the fixed scale, run the eye along from 1 to 4; then set the 1 of the slide opposite this point 4, and *run the eye forward along the slide from* 1 *to* 6; the point thus reached on the fixed scale is 24, which is equal to 4 × 6. This process gives the distance from 1 to 4 *plus* the distance from 1 to 6, and is, in fact, a mechanical method of adding the logarithms of these numbers; hence the result is the product of the numbers. Conversely,

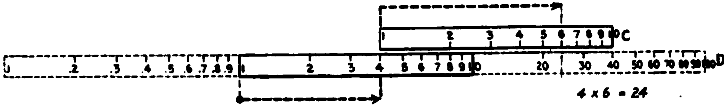


FIG. 1.

To DIVIDE 4 BY 6. In Fig. 2, starting with the point 1 of the fixed scale, run the eye along from 1 to 4; then set the 6 of the slide opposite the point 4, and *run the eye backward along the slide from* 6 *to* 1; the point thus reached on the fixed scale is 0.667, which is equal to 4 ÷ 6. This process gives the distance from 1 to 4 *minus* the distance from 1 to 6; and is, in fact, a mechanical method of subtracting the logarithms of these numbers; hence the result is their quotient.

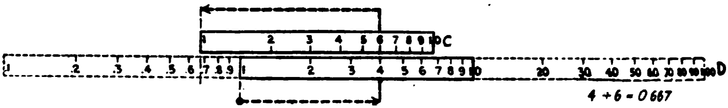


FIG. 2.

Multiplication and Division, Using Only a Single Section of the Scale. If only the main section of scale D is available (as is usually the case in practice), the result of multiplication may fall beyond the scale, as it does in Fig. 1. In such cases *divide the first factor by* 10 *before beginning to multiply*; this will bring the result within the scale, without affecting the sequence of digits.

For example, to multiply 4 by 6. Having found that the setting shown in Fig. 1 is not successful, *reset the slide as in Fig.* 3, *with* 10 *instead of* 1 *opposite* 4; run the eye backward along the slide from 10 to 1, thus reaching the (unrecorded) point corresponding to 4 ÷ 10; then, continuing from this point, run the eye forward along the slide from 1 to 6, as before; the point finally reached on the main scale is 2.4, which has the same sequence of digits as the required value 24. After a little practice, this preliminary step of dividing by 10 will be performed almost intuitively. Whether or not this step is necessary in any given case, can be determined only by trial.

The **general rule for multiplication** may be stated as follows, if preferred: To find the product of two factors, find one factor on the fixed scale; opposite this, set (tentatively) point 1 of the slide; on the slide find the second factor, and opposite this read the product on the main scale, if possible. If the product falls beyond the scale, begin over again, using point 10 of the slide instead of point 1.

In division also, the result may fall beyond the main section of the scale, as it does in Fig. 2. In such cases, it suffices merely to *multiply the result by* 10 in order to bring it within the scale; this will not affect the sequence of digits.

For example, to divide 4 by 6, set the slide as in Fig. 4, and follow out mentally the steps indicated by the arrows. It will be noticed that the supplementary step of multiplying by 10 is performed by simply running the eye along the slide from 1 to 10 without resetting the slide; for this reason, division on the slide rule is slightly easier than multiplication.

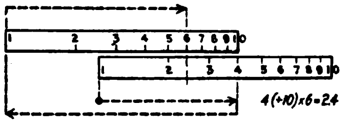


Fig. 3.

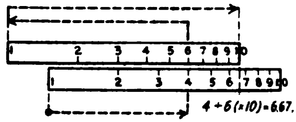


Fig. 4.

The Ordinary Mannheim Slide Rule has four scales, A, B, C, D, as shown in Fig. 5. Scales C and D are essentially the same as the C and D scales described above, and the principle just explained shows how they are used in multiplication and division. The fact that the D scale covers only the main section from 1 to 10 (all decimal points being omitted) is practically no restriction on the scope of the scale, as is seen in the preceding examples. A runner is provided, so that intermediate positions reached in the course of an extended computation may be indicated temporarily on the scale without the necessity of reading off their numerical values. The best runners are those which have no side frame to obscure the numerals.

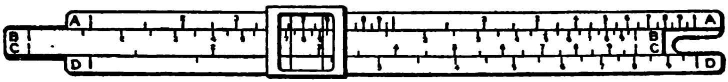


Fig. 5.

In problems involving successive multiplications and divisions, arrange the work so that multiplication and division are performed alternately.

For example, to calculate $\frac{a \times b \times c}{d \times e}$, divide the product $a \times b$ by d; multiply this quotient by c; and divide this product by e. Each operation will require only one shifting either of the slide (for multiplication) or of the runner (for division).

To multiply a number of different quantities by a *constant multiplier*, x, set the point 1 of slide opposite x, and read, by aid of the runner, the products of x by all the quantities which do not fall beyond the scale; then reset the slide, setting 10 instead of 1 opposite x, and read the products of x by all the remaining quantities.

To divide a number of different quantities by a *constant divisor*, y, first find (by the slide rule) the quotient $1 \div y$, and then use this as a constant multiplier.

Scales A and B are exactly like scales C and D, except that they cover two sections of the complete logarithmic scale, the graduations being only half as fine. Either pair of scales may be used for multiplication and division; C and D give more accurate readings, but have the disadvantage that in the case of multiplication the slide must often be shifted to the other end in order to keep the result on the scale—an inconvenience which is not present when the less accurate scales A and B are employed.

By the use of both pairs of scales, problems in squares and square roots may be readily solved; for every number on A, except for the decimal point, is the square of the number directly below it on D (use the runner).

A scale of sines, tangents, and logarithms is often printed on the back of the slide. For further details concerning the use of the slide rule in various problems, see the instruction books furnished with each instrument: Wm. Cox, "Manual of the Mannheim Slide Rule;" F. A. Halsey, "Manual of the Slide Rule;" etc.

Other Types of Slide Rules. The **duplex slide rule** ($5 to $18 according to length) shows on one face the regular *A, B, C, D* scales, and on the other face the scales *A, B′, C′, D* (where *B′* and *C′* are the same as *B* and *C*, only numbered in the reverse order), with a runner encircling the whole scale. This arrangement makes possible the solution of more complicated problems with fewer settings of the slide, but if the rule is to be used only for simple problems, the multiplicity of scales is rather confusing. Less complicated is the **polyphase rule**, which is like a Mannheim rule with the addition of a single inverted scale, *C′*, printed in the middle of the slide. The **log log duplex slide rule** (10 in., $8) is especially adapted for handling complex problems involving fractional powers or roots, hyperbolic logarithms, etc. A number of **circular slide rules** are on the market, the best of which are operated by a milled thumbnut, like the stem wind of a watch. The advantage of the circular rule, aside from its compact size (some models are scarcely larger than a watch), lies in the fact that the scale is endless, so that the slide never has to be reset in order to bring the result within the scale. A disadvantage is found in the necessity of reading the figures in oblique positions, or else continually turning the instrument as a whole in the hand. The **Fuller and Thacher rules** already mentioned are invaluable for problems requiring greater accuracy than can be obtained with the ordinary rules. There are also many **special slide rules**, adapted to various special types of computation, such as calculating discharge of water through pipes, horse power of engines, dimensions of lumber, stadia measurements, etc. One of the most recent devices of this kind is the **Ross meridiograph** (L. Ross, San Francisco, Cal.), which is a circular slide rule for solving certain cases of spherical triangles. The **Eichhorn trigonometrical slide rule** solves any plane triangle.

COMPUTING MACHINES

For certain purposes computing machines have ceased to be luxuries and have become almost necessities; but they are expensive, and should be selected with reference to the special work which is to be done. The machines may be classified roughly into three groups, as follows:

Adding Machines, Non-listing. Of the machines of this kind, the most convenient in the hands of a careful operator is the well-known **Comptometer** (Felt & Tarrant Co., Chicago, Ill.; $250 to $350 according to size), or the recent **Burroughs non-listing** adding machine (Detroit, Mich., $175). To add a number, simply press a key in the proper column; the result appears on the dials in front of the keyboard. Multiplication as well as addition can be performed on this machine with great rapidity, and division also after a little practice. Weight, about 15 lb. Much less rapid, but less expensive and requiring somewhat less skill in operation, is the **Barrett adding machine** (Philadelphia, Pa.) with multiplying attachment. Other key-operated machines are the **Mechanical Accountant** (Providence, R. I.), and the **Austin** (Baltimore, Md.). The **American adding machine** (American Can Co., Chicago, Ill.; $39.50) is operated by pulling up a finger-lever for each digit. Small machines, operated by the use of a stylus, are the **Rapid computer** (Benton Harbor, Mich., $25); the **Gem** (Automatic Adding Machine Co., New York; $10), the **Arithstyle** (New York, $36) and the **Triumph** (Brooklyn, N. Y., $35). These machines, while much less rapid than the key-operated machines, are useful in simple addition. The **Underwood typewriter** is now supplied with a complete electrically driven adding machine attached, and the **Wahl adding attachment** is supplied on the Remington and other typewriters. **Ray Subtracto-Adder** (Richmond, Va., $25).

Adding and Listing Machines. The machines of this group not only add, but also print the items, totals and sub-totals. The **Burroughs** (Detroit, Mich.), the **Wales** (Adder Machine Co., Wilkes-Barre, Pa.), the **Comptograph** (Chicago, Ill.) and the **White** (New Haven, Conn.), resemble each other in having an 81-key keyboard; the **Dalton** (Cincinnati, Ohio) and the **Commercial** (White Adding Machine Co., New Haven, Conn.) have a 10-key and a 9-key keyboard respectively, admitting of operation by the touch method. On all these machines, in order to add a number, first depress the proper keys and then pull a handle (or, in the case of electrically driven machines, press a button) to record the item. Multiplication cannot be performed conveniently, except on the Dalton. Subtraction can be performed only by adding the complement, except on the Commercial and on one type of the Burroughs. The prices range from $125 to $600, according to size and style, new models being constantly devised for special commercial purposes. A new and more portable machine of the 81-key type is the **Barrett** adding and listing machine (Philadelphia, Pa., $250). A cheaper machine, with a 10-key keyboard, is the **Standard** (St. Louis, Mo.). The new **American** adding and listing machine (American Can Co., Chicago, Ill.), operated by pulling up a finger-lever for each digit, costs only $88. The **Ellis** (Newark, N. J.) is an elaborate adding and listing machine having a complete typewriter incorporated with it. **The Elliott-Fisher bookkeeping machine** (Harrisburg, Pa.) and the **Moon-Hopkins billing machine** (St. Louis, Mo.) are intended primarily for commercial use; the latter is a complicated electric machine ($750) which combines many of the features of an adding and listing machine with those of a calculating machine.

Calculating Machines (so-called). Machines of this third group are intended primarily for multiplication and division; the types which have a keyboard can be used effectively for addition and subtraction also. They are all non-listing. The earliest commercially successful types were the Thomas and the Brunsviga. In both these types the multiplicand is set up by moving pegs in slots, or (in the newest models) by depressing keys, and the multiplication is effected by turning a handle for each digit of the multiplier—twice for a digit 2, three times for a digit 3, etc.; the result then appears on the dials. In the Thomas type the handle always turns in the same direction, the change from multiplication to division being effected by a shift key. In the Brunsviga type the handle is turned forward for multiplication and backward for division. Among the best examples of the Thomas type now on the American market are the **Tim**, with a single row of dials, the **Unitas**, with a double row of dials (both sold by Oscar Müller Co., New York City; also with keyboard and electric drive), and the **Reuter** (Philadelphia, Pa.). Prices, $300 upward. Another machine of this type, with keyboard, is the **Record** (U. S. Adding Machine Co., New York City). The **Brunsviga** is represented by Carl H. Reuter, Philadelphia, Pa.; various models. Of somewhat similar type are the **Triumphator** (New York City; $250), and **Colt's calculator** (Culmer Engineering Co., New York City). A new machine, on the same principle, but with keyboard, is the **Monroe** (made in Orange, N. J.; $250). The **Millionaire** (W. A. Morschhauser, New York City; $400), is from the mechanical point of view, the only true multiplying machine on the market (except the Moon-Hopkins). After the multiplicand is set up on the pegs, the digits of the multiplier are indicated successively by moving a pointer, the handle being turned only once for each digit. Further, the movement of the carriage is automatic. The newest models have keyboard and electric drive. The **Ensign electric calculating machine** (Boston, Mass.; $400) is a new machine with an 81-key keyboard on which it adds like an adding machine, and a secondary 10-key keyboard by means of which it multiplies and divides quite as rapidly as any of the calculating machines, the proper key being pressed just once for each digit of the multiplier. The **National calculator** (New York), and the **Lamb calculator** (Calculator Mfg. Co., New York) are less expensive machines devised for figuring payrolls and labor costs. A still simpler device for the same purpose is the **Calculacard** (New York). The machine called the **Calculagraph** (New York) is a time clock which automatically computes labor costs.

For graphical methods of computation, see pp. 106, 119, 170, 173–185.

FINANCIAL ARITHMETIC

For the facts which are commonly required in regard to compound interest, sinking funds, etc., see the headings of the tables on pp. 64–68.

ELEMENTARY GEOMETRY AND MENSURATION

GEOMETRICAL THEOREMS

(For geometrical constructions, see p. 101)

Right Triangles. $a^2 + b^2 = c^2$. (See Fig. 1). $\angle A + \angle B = 90°$. $p^2 = mn$. $a^2 = mc$. $b^2 = nc$. See also p. 105 and p. 132.

Oblique Triangles. (See also pp. 105, 134.) Sum of angles = 180°. An exterior angle = sum of the two opposite interior angles. (Fig. 1.)

The medians, joining each vertex with the middle point of the opposite side, meet in the center of gravity G (Fig. 2), which trisects each median.

The altitudes meet in a point called the orthocenter, O.

The perpendiculars erected at the midpoints of the sides meet in a point C, the center of the circumscribed circle. [In any triangle G, O, and C lie in line, and G is two-thirds of the way from O to C.]

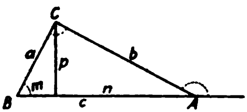


FIG. 1.

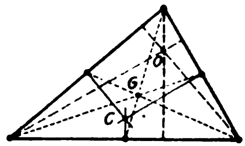


FIG. 2.

The bisectors of the angles meet in the center of the inscribed circle (Fig. 3).

The largest side of a triangle is opposite the largest angle; it is less than the sum of the other two sides, and greater than their difference.

FIG. 3.

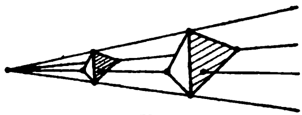


FIG. 4.

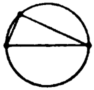
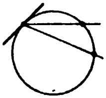
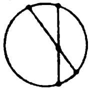
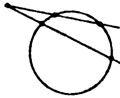
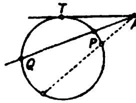
Similar Figures. Any two similar figures, in a plane or in space, can be placed in "perspective," that is, so that straight lines joining corresponding points of the two figures will pass through a common point (Fig. 4). That is, of two similar figures, one is merely an enlargement of the other. Assume that each length in one figure is k times the corresponding length in the other; then each area in the first figure is k^2 times the corresponding area in the second, and each volume in the first figure is k^3 times the corresponding volume in the second. If two lines are cut by a set of parallel lines (or parallel planes), the corresponding segments are proportional.

The Circle. (See also pp. 106, 137.) An angle inscribed in a semicircle is a right angle (Fig. 5). An angle inscribed in a circle, or an angle between a chord and a tangent, is measured by half the intercepted arc (Fig. 6). An angle formed by any two lines which meet a circle is measured by half the sum or half the difference of the intercepted arcs, according as the point of intersection of the lines lies inside (Fig. 7) or outside the circle (Fig. 8).

A tangent is perpendicular to the radius drawn to the point of contact.

If a variable line through A (Figs. 9 and 10) cuts a circle in P and Q, then

$\overline{AP} \times \overline{AQ}$ is constant; in particular, if A is an external point, $\overline{AP} \times \overline{AQ} = \overline{AT}^2$, where $\overline{AT}$ is the tangent from A.

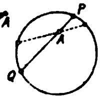


FIG. 5. FIG. 6. FIG. 7. FIG. 8. FIG. 9. FIG. 10.

The radical axis (Fig. 11) of two circles is a straight line such that the tangents drawn from any point of this line to the two circles are of equal length. If the two circles intersect, the radical axis passes through their points of intersection. In any case, the radical axis bisects the common tangents of the two circles. The three radical axes of a set of three circles meet in a common point. (For equations, see p. 137.)

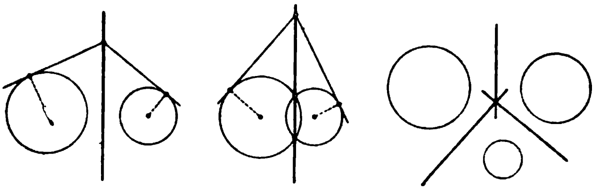


FIG. 11.

Dihedral Angles. The dihedral angle between two planes is measured by a plane angle formed by two lines, one in each plane, perpendicular to the edge (Fig. 12). (For solid angles, see p. 110.)

In a **tetrahedron**, or triangular pyramid, the four medians, joining each vertex with the center of gravity of the opposite face, meet in a point, the center of gravity of the tetrahedron; this point is 3/4 of the way from any vertex to the center of gravity of the opposite face. The four perpendiculars erected at the circumcenters of the four faces meet in a point, the center of the circumscribed sphere. The four altitudes meet in a point called the orthocenter of the tetrahedron. The planes bisecting the six dihedral angles meet in a point, the center of the inscribed sphere.

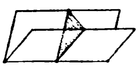
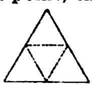
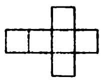
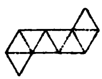
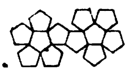
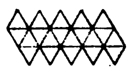







FIG. 12. FIG. 13. FIG. 14. FIG. 15. FIG. 16. FIG. 17.

Regular Polyhedra (see also p. 110): Regular tetrahedron (Fig. 13), bounded by four equilateral triangles; cube (Fig. 14), bounded by six squares; octahedron (Fig. 15), bounded by eight equilateral triangles; dodecahedron (Fig. 16), bounded by twelve regular pentagons; icosahedron (Fig. 17), bounded by twenty equilateral triangles. Figs. 13–17 show how these solids can be made by cutting the surface out of paper and folding it together.

The Sphere. (See also p. 109.) If AB is a diameter, any plane perpendicular to AB cuts the sphere in a circle, of which A and B are called the poles. A great circle on the sphere is formed by a plane passing through the center. A spherical triangle is bounded by arcs of great circles (see p.

134). In two polar triangles, each angle in one is the supplement of the corresponding side in the other. In two symmetrical triangles, the sides and angles of one are equal to the corresponding sides and angles of the other, but arranged in the reverse order (like right-handed and left-handed gloves).

GEOMETRICAL CONSTRUCTIONS

To Bisect a Line AB (Fig. 18). (*a*) From A and B as centers, and with equal radii, describe arcs intersecting in P and Q, and draw PQ, which will bisect AB in M.

(*b*) Lay off $AC = BD$ = approximately half of AB, and then bisect CD.

To Draw a Parallel to a Given Line l Through a Given Point A (Fig. 19). With point A as center draw an arc just touching the line l; with any point O of the line as center, draw an arc BC with the same radius. Then a line through A touching this arc will be the required parallel. Or, use a straight edge and triangle. Or, use a sheet of celluloid with a set of lines parallel to one edge and about ¼ in. apart ruled upon it.

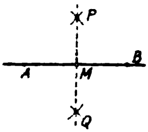


FIG. 18.

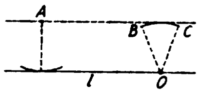


FIG. 19.

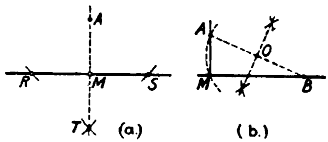


FIG. 20.

To Draw a Perpendicular to a Given Line from a Given Point A Outside the Line (Fig. 20). (*a*) With A as center, describe an arc cutting the line in R and S, and bisect RS in M. Then M is the foot of the perpendicular. (*b*) If A is nearly opposite one end of the line, take any point B of the line and bisect AB in O; then with O as center, and OA or OB as radius, draw an arc cutting the line in M. Or, (*c*) use a straight edge and triangle.

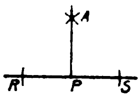


FIG. 21.

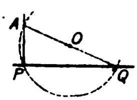


FIG. 22.

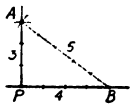


FIG. 23.

To Erect a Perpendicular to a Given Line at a Given Point P. (*a*) Lay off $PR = PS$ (Fig. 21), and with R and S as centers draw arcs intersecting at A. Then PA is the required perpendicular. (*b*) If P is near the end of the line, take any convenient point O (Fig. 22) above the line as center, and with radius OP draw an arc cutting the line in Q. Produce QO to meet the arc in A; then PA is the required perpendicular. (*c*) Lay off $PB = 4$ units of any scale (Fig. 23); from P and B as centers lay off $PA = 3$ and $BA = 5$; then APB is a right angle.

To Divide a Line AB into n Equal Parts (Fig. 24). Through A draw a line AX at any angle, and lay off n equal steps along this line. Connect the last of these divisions with B, and draw parallels through the other divisions. These parallels will divide the given line into n equal parts. A similar method may be used to divide a line into parts which shall be proportional to any given numbers.

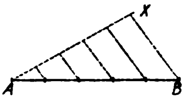


FIG. 24.

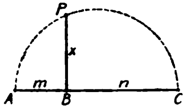


FIG. 25.

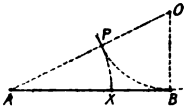


FIG. 26.

To Construct a Mean Proportional (or Geometric Mean) Between Two Lengths, m and n (Fig. 25). Lay off $AB = m$ and $BC = n$ and construct a semicircle on AC as diameter. Let the perpendicular erected at B meet the circumference at P. Then $BP = \sqrt{mn}$. (See p. 115.)

To Divide a Line AB in Extreme and Mean Ratio (the "golden section"). At one end, B, of the given line (Fig. 26), erect a perpendicular, BO, equal to half AB, and join OA. Along OA lay off $OP = OB$, and along AB lay off $AX = AP$. Then X is the required point of division; that is, $\overline{AX}^2 = AB \times BX$. Numerically, $AX = \frac{1}{2}(\sqrt{5} - 1)(AB) = 0.618(AB)$.

To Bisect an Angle AOB (Fig. 27). Lay off $OA = OB$. From A and B as centers, with any convenient radius, draw arcs meeting in M; then OM is the required bisector.

To draw the bisector of an angle when the vertex of the angle is not accessible (Fig. 28). Parallel to the given lines a, b, and equidistant from them, draw two lines a', b' which intersect; then bisect the angle between a' and b'.

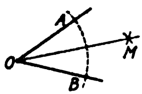


FIG. 27.

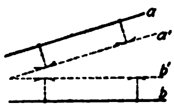


FIG. 28.

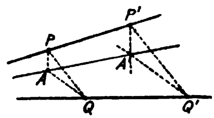


FIG. 29.

To Draw a Line Through a Given Point A and in the Direction of the Point of Intersection of Two Given Lines, when this point of intersection is inaccessible (Fig. 29). Draw any two parallel lines PQ and $P'Q'$ as in the figure; through P' draw a line parallel to PA, and through Q' draw a line parallel to QA; let these lines intersect in A', and draw the line AA'. This line AA' will (if produced) pass through the intersection of the two given lines.

To Construct, Approximately, the Length of a Circular Arc (Rankine). In Fig. 30 draw a tangent at A. Prolong the chord BA to C, making $AC = \frac{1}{2} AB$. With C as center, and radius CB, draw arc cutting the tangent in D. Then AD = arc AB, approximately (error about 4 min. in an arc of 60 deg.). Conversely, to find an arc AB on a given circle to equal a given length AD, take E one-fourth of the way from A to D, and with E as center and radius ED draw an arc cutting the circumference in B. Then arc $AB = AD$, approximately.

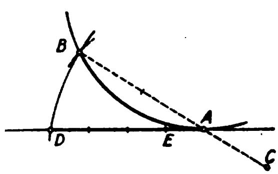


FIG. 30.

To Inscribe a Hexagon in a Circle (Fig. 31). Step around the circumference with a chord equal to the radius. Or, use a 60-deg. triangle.

To Circumscribe a Hexagon About a Circle (Fig. 32). Draw a chord AB equal to the radius. Bisect the arc AB in T. Draw the tangent at T (parallel to AB), meeting OA and OB in P and Q. Then draw a circle with radius OP or OQ and inscribe in it a hexagon, one side being PQ.

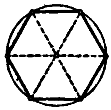


Fig. 31.

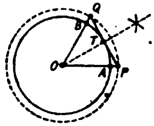


Fig. 32.

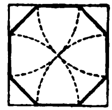


Fig. 33.

To Inscribe an Octagon in a Square (Fig. 33). From the corners as centers, and with radius equal to half the diagonal, draw four arcs, cutting the sides in eight points. The points will be the vertices of the octagon.

To Inscribe an Octagon in a Circle. Draw two perpendicular diameters, and bisect each of the quadrant arcs.

To Circumscribe an Octagon About a Circle. Draw a square about the circle, and draw the tangents to the circle at the points where the circle is cut by the diagonals of the square.

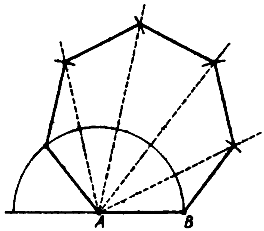


Fig. 34.

To Construct a Polygon of n Sides, One Side AB being Given (Fig. 34). With A as center and AB as radius, draw a semicircle, and divide it into n parts, of which $n - 2$ parts (counting from B) are to be used. Draw rays from A through these points of division, and complete the construction as in the figure (in which $n = 7$). Note that the center of the polygon must lie in the perpendicular bisector of each side.

To Draw a Tangent to a Circle from an external point A (Fig. 35). Bisect AC in M; with M as center and radius MC, draw arc cutting circle in P; then P is the required point of tangency.

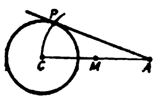


Fig. 35.

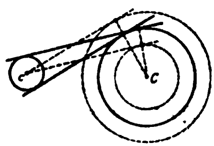


Fig. 36.

To Draw a Common Tangent to Two Given Circles (Fig. 36). Let C and c be the centers and R and r the radii ($R > r$). From C as center, draw two concentric circles with radii $R + r$ and $R - r$; draw tangents to these circles from c; then draw parallels to these lines at distance r. These parallels will be the required common tangents.

To Draw a Circle Through Three Given Points A, B, C, or to find the center of a given circular arc (Fig. 37). Draw the perpendicular bisectors of AB and BC; these will meet in the center, O.

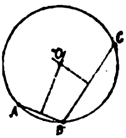


Fig. 37.

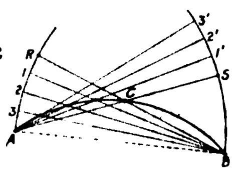


Fig. 38.

To Draw a Circular Arc Through Three Given Points When the Center is not Available (Fig. 38). With A and B as centers, and chord

AB as radius, draw arcs, cut by *BC* in *R* and by *AC* in *S*. Divide *RA* into *n* equal parts, 1, 2, 3, . . . Divide *BS* into the same number of equal parts, and continue these divisions at 1′, 2′, 3′, . . . Connect *A* with 1′, 2′, 3′, . . . and *B* with 1, 2, 3, . . . Then the points of intersection of corresponding lines will be points of the required arc. (Cônstruction valid only when $CA = CB$.)

To Draw a Circle Through Two Given Points, A, B, and Touching a Given Line, l (Fig. 39). Let *AB* meet line *l* in *C*. Draw any circle through *A* and *B*, and let *CT* be tangent to this circle from *C*. Along *l*, lay off *CP* and *CQ* equal to *CT*. Then either *P* or *Q* is the required point of tangency. (Two solutions.) Note that the center of the required circle lies in the perpendicular bisector of *AB*.

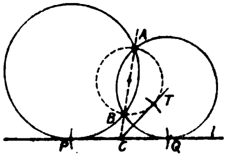


FIG. 39.

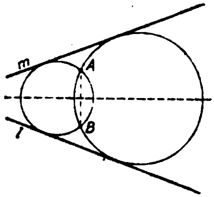


FIG. 40.

To Draw a Circle Through One Given Point, A, and Touching Two Given Lines, l and m (Fig. 40). Draw the bisector of the angle between *l* and *m*, and let *B* be the reflection of *A* in this line. Then draw a circle through *A* and *B* and touching *l* (or *m*), as in preceding construction. (Two solutions.)

To Draw a Circle Touching Three Given Lines (Fig. 41). Draw the bisectors of the three angles; these will meet in the center *O*. (Four solutions.) The perpendiculars from *O* to the three lines give the points of tangency.

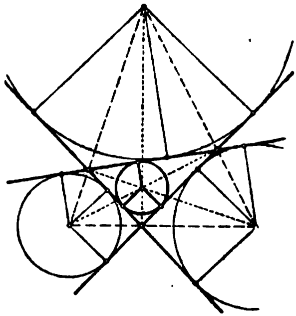


FIG. 41.

To Draw a Circle Through Two Given Points A, B, and Touching a Given Circle (Fig. 42). Draw any circle through *A* and *B*, cutting the given circle in *C* and *D*. Let *AB* and *CD* meet in *E*, and let *ET* be tangent from *E* to the circle just drawn. With *E* as center, and radius *ET*, draw an arc cutting the given circle in *P* and *Q*. Either *P* or *Q* is the required point of contact. (Two solutions.)

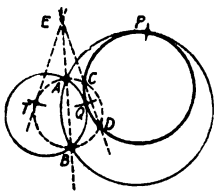


FIG. 42.

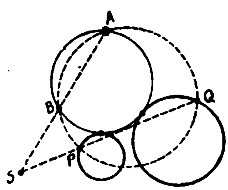


FIG. 43.

To Draw a Circle Through One Given Point, A, and Touching Two Given Circles (Fig. 43). Let *S* be a center of similitude for the two given circles, that is, the point of intersection of two external (or internal) common tangents. Through *S* draw any line cutting one circle in two points, the nearer of which shall be called *P*, and the other in two points, the more remote of which shall be called *Q*. Through *A*, *P*, *Q* draw a circle cutting SA in B. Then draw a circle through A and B and touching one of the given circles (see preceding construction). This circle will touch the other given circle also. (Four solutions.)

To Draw an Annulus Which Shall Contain a Given Number of Equal Contiguous Circles (Fig. 44). (An annulus is a ring-shaped area enclosed between two concentric circles.) Let $R + r$ and $R - r$ be the inner and outer radii of the annulus, r being the radius of each of the n circles. Then the required relation between these quantities is given by $r = R \sin(180°/n)$, or $r = (R + r)[\sin(180°/n)]/[1 + \sin(180°/n)]$.

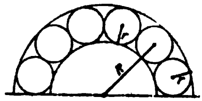


FIG 44.

For **methods of constructing ellipses and other curves,** see pp. 139–156.

LENGTHS AND AREAS OF PLANE FIGURES

Right Triangle (Fig. 45). $a^2 + b^2 = c^2$.
Area $= \frac{1}{2}ab = \frac{1}{2}a^2 \cot A = \frac{1}{2}b^2 \tan A = \frac{1}{4}c^2 \sin 2A$.

Equilateral Triangle (Fig. 46). Area $= \frac{1}{4}a^2\sqrt{3} = 0.43301a^2$.

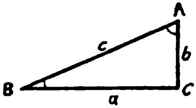


FIG. 45.

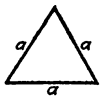


FIG. 46.

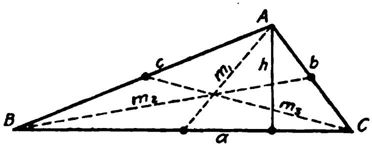


FIG. 47.

Any Triangle (Fig. 47). $s = \frac{1}{2}(a + b + c)$, $t = \frac{1}{2}(m_1 + m_2 + m_3)$,

$r = \sqrt{(s - a)(s - b)(s - c)/s}$ = radius inscribed circle,

$R = \frac{1}{2}a/\sin A = \frac{1}{2}b/\sin B = \frac{1}{2}c/\sin C$ = radius circumscribed circle;

Area $= \frac{1}{2}$ base $\times$ altitude $= \frac{1}{2}ah = \frac{1}{2}ab \sin C = rs = abc/4R$

$= \sqrt{s(s - a)(s - b)(s - c)} = \frac{4}{3}\sqrt{t(t - m_1)(t - m_2)(t - m_3)}$

$= r^2 \cot \frac{1}{2}A \cot \frac{1}{2}B \cot \frac{1}{2}C = 2R^2 \sin A \sin B \sin C$

$= \pm\frac{1}{2}\{(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\}$, where (x_1, y_1), (x_2, y_2), (x_3, y_3) are co-ordinates of vertices. See also p. 134.

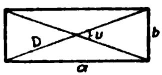


FIG. 48.

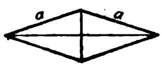


FIG. 49.

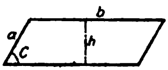


FIG. 50.

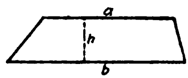


FIG. 51.

Rectangle (Fig. 48). Area $= ab = \frac{1}{2}D^2 \sin u$. [$u$ = angle between diagonals D, D.]

Rhombus (Fig. 49). Area $= a^2 \sin C = \frac{1}{2}D_1D_2$. [$C$ = angle between two adjacent sides; D_1, D_2 = diagonals.]

Parallelogram (Fig. 50). Area $= bh = ab \sin C = \frac{1}{2}D_1D_2 \sin u$. [$u$ = angle between diagonals D_1 and D_2; $D_1^2 + D_2^2 = 2(a^2 + b^2)$].

Trapezoid (Fig. 51). Area $= \frac{1}{2}(a + b)h = \frac{1}{2}D_1D_2 \sin u$. [Bases a and b are parallel; u = angle between diagonals D_1 and D_2.]

Quadrilateral Inscribed in a Circle (Fig. 52). Area $= \frac{1}{2}D_1D_2 \sin u = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \frac{1}{2}(ac+bd)\sin u$; $s = \frac{1}{2}(a+b+c+d)$.

Any Quadrilateral (Fig. 53). Area $= \frac{1}{2}D_1D_2 \sin u$.

NOTE. $a^2+b^2+c^2+d^2 = D_1^2+D_2^2+4m^2$, where m = distance between midpoints of D_1 and D_2.

Polygons. See table, p. 39.

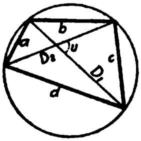


FIG. 52.

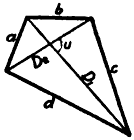


FIG. 53.

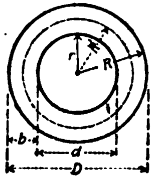


FIG. 54.

FIG. 55.

Circle. Area $= \pi r^2 = \frac{1}{2}Cr = \frac{1}{4}Cd = \frac{1}{4}\pi d^2 = 0.785398d^2$ (table, p. 30). Here r = radius, d = diam., C = circumference $= 2\pi r = \pi d$ (table, p. 28).

Annulus (Fig. 54). Area $= \pi(R^2 - r^2) = \pi(D^2 - d^2)/4 = 2\pi R'b$, where R' = mean radius $= \frac{1}{2}(R+r)$, and $b = R - r$.

Sector (Fig. 55). Area $= \frac{1}{2}rs = \pi r^2(A/360°) = \frac{1}{2}r^2$ rad A, where rad A = radian measure of angle A, and s = length of arc $= r$ rad A (table, p. 44).

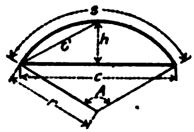


FIG. 56.

Segment (Fig. 56). Area $= \frac{1}{2}r^2$ (rad $A - \sin A$) $= \frac{1}{2}[r(s-c)+ch]$, where rad A = radian measure of angle A (table, pp. 34–35, 44). For small arcs, $s = \frac{1}{3}(8c' - c)$, where c' = chord of half the arc. (Huygens's approximation.) NOTE. $c = 2\sqrt{h(d-h)}$; $c' = \sqrt{dh}$ or $d = c'^2/h$, where d = diameter of circle; $h = r(1 - \cos \frac{1}{2}A)$, $s = 2r$ rad $\frac{1}{2}A$.

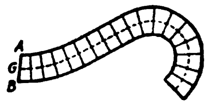


FIG. 57.

Ribbon bounded by two parallel curves (Fig. 57). If a straight line AB moves so that it is always perpendicular to the path traced by its middle point G, then the **area** of the ribbon or strip thus generated is equal to the length of AB times the length of the path traced by G. (It is assumed that the radius of curvature of G's path is never less than $\frac{1}{2}AB$, so that successive positions of the generating line will not intersect.)

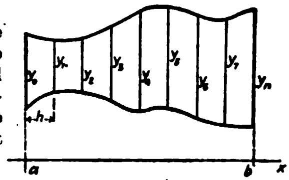


FIG. 58.

Simpson's Rule (Fig. 58). Divide the given area into n panels (where n is some even number) by means of $n+1$ parallel lines, called ordinates, drawn at constant distance h apart; and denote the lengths of these ordinates by $y_0, y_1, y_2, \ldots, y_n$. (Note that y_0 or y_n may be zero.) Then Area $= \frac{1}{3}h[(y_0+y_n)+4(y_1+y_3+y_5\ldots)+2(y_2+y_4+y_6\ldots)]$, approx. The greater the number of divisions, the more accurate the result. Note: Taking $y = f(x)$, where x varies from $x = a$ to $x = b$, and $h = (b-a)/n$, then the error $= -\frac{1}{180}\frac{(b-a)^5}{n^4}f''''(X)$, where $f''''(X)$ is the value of the fourth derivative of $f(x)$ for some (unknown) value, $x = X$, between a and b.

Ellipse (Fig. 59; see also p. 140). Area of ellipse $= \pi ab$. Area of shaded segment $= xy + ab \sin^{-1}(x/a)$. Length of perimeter of ellipse $= \pi(a + b)K$, where $K = [1 + \frac{1}{4}m^2 + \frac{1}{64}m^4 + \frac{1}{256}m^6 + \ldots]$, $m = (a - b)/(a + b)$.

For m =	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K =	1.002	1.010	1.023	1.040	1.064	1.092	1.127	1.168	1.216	1.273

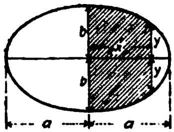


FIG. 59.

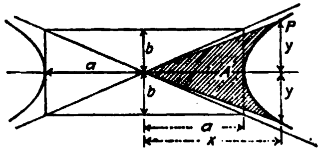


FIG. 60.

Hyperbola (Fig. 60; see also p. 144). In any hyperbola, shaded area $A = ab \log_e\left(\frac{x}{a} + \frac{y}{b}\right)$. In an equilateral hyperbola $(a = b)$, area $A = a^2 \sinh^{-1}(y/a) = a^2 \cosh^{-1}(x/a)$. For tables of hyperbolic functions, see p. 60. Here x and y are co-ordinates of point P.

Parabola (Fig. 61; see also p. 138). Shaded area $A = \frac{2}{3}ch$. In Fig. 62, length of arc $OP = s = \frac{1}{2}PT + \frac{1}{2}p \log_e \cot \frac{1}{2}u$. Here c = any chord; p = semi-latus rectum; PT = tangent at P. Note: $OT = OM = x$.

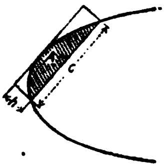


FIG. 61,

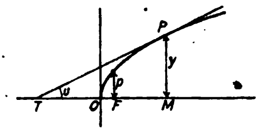


FIG. 62.

Other Curves. For lengths and areas, see pp. 147–156.

SURFACES AND VOLUMES OF SOLIDS

Regular Prism (Fig. 63). Volume $= \frac{1}{2}nrah = Bh$. Lateral area $= nah = Ph$. Here n = number of sides; B = area of base; P = perimeter of base.

Right Circular Cylinder (Fig. 64). Volume $= \pi r^2 h = Bh$. Lateral area $= 2\pi rh = Ph$. Here B = area of base; P = perimeter of base.

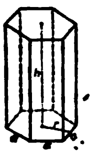


FIG. 63.

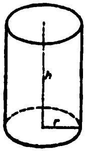


FIG. 64.

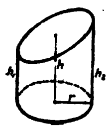


FIG. 65.

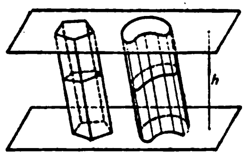


FIG. 66.

Truncated Right Circular Cylinder (Fig. 65). Volume $= \pi r^2 h = Bh$. Lateral area $= 2\pi rh = Ph$. Here h = mean height $= \frac{1}{2}(h_1 + h_2)$; B = area of base; P = perimeter of base.

Any Prism or Cylinder (Fig. 66). Volume $= Bh = Nl$. Lateral area $= Ql$. Here l = length of an element or lateral edge; B = area of base; N = area of normal section; Q = perimeter of normal section.

Any Truncated Prism or Cylinder (Fig. 67). Volume $= Nl$. Lateral area $= Qk$. Here l = distance between centers of gravity of areas of the two bases; k = distance between centers of gravity of perimeters of the two bases; N = area of normal section; Q = perimeter of normal section. For a truncated triangular prism with lateral edges a,b,c, $l = k = \frac{1}{3}(a + b + c)$. Note: l and k will always be parallel to the elements.

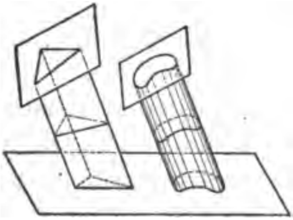


FIG. 67.

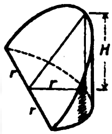


FIG. 68.

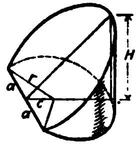


FIG. 69.

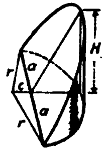


FIG. 70.

Special Ungula of a right circular cylinder. (Fig. 68.). Volume $= \frac{2}{3}r^2H$. Lateral area $= 2rH$. r = radius. (Upper surface is a semi-ellipse.)

Any Ungula of a right circular cylinder. (Figs. 69 and 70.) Volume $= H(\frac{2}{3}a^3 \pm cB)/(r \pm c) = H[a(r^2 - \frac{1}{3}a^2) \pm r^2c \text{ rad } u]/(r \pm c)$. Lateral area $= H(2ra \pm cs)/(r \pm c) = 2rH(a \pm c \text{ rad } u)/(r \pm c)$. If base is greater (less) than a semicircle, use $+$ $(-)$ sign. r = radius of base; B = area of base; s = arc of base; u = half the angle subtended by arc s at center; rad u = radian measure of angle u (see table, p. 44).

Hollow Cylinder (right and circular). Volume $= \pi h(R^2 - r^2) = \pi hb(D - b) = \pi hb(d + b) = \pi hbD' = \pi hb(R + r)$. Here h = altitude; $r,R(d,D)$ = inner and outer radii (diameters); b = thickness $= R - r$; D' = mean diam. $= \frac{1}{2}(d + D) = D - b = d + b$.

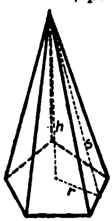


FIG. 71.

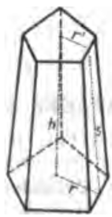


FIG. 72.

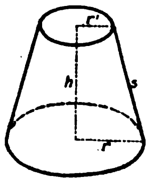


FIG. 73.

Regular Pyramid (Fig. 71). Volume $= \frac{1}{3}$ altitude $\times$ area of base $= \frac{1}{6}hran$. Lateral area $= \frac{1}{2}$ slant height $\times$ perimeter of base $= \frac{1}{2}san$. Here r = radius of inscribed circle; a = side (of regular polygon); n = number of sides; $s = \sqrt{r^2 + h^2}$. Vertex of pyramid directly above center of base.

Right Circular Cone. Volume $= \frac{1}{3}\pi r^2h$. Lateral area $= \pi rs$. Here r = radius of base; h = altitude; s = slant height $= \sqrt{r^2 + h^2}$.

Frustum of Regular Pyramid (Fig. 72).

Volume $= \frac{1}{6}hran[1 + (a'/a) + (a'/a)^2]$.

Lateral area = slant height $\times$ half sum of perimeters of bases = slant height $\times$ perimeter of mid-section $= \frac{1}{2}sn(r + r')$. Here r,r' = radii of inscribed circles; $s = \sqrt{(r - r')^2 + h^2}$; a, a' = sides of lower and upper bases; n = number of sides.

Frustum of Right Circular Cone (Fig. 73). Volume = $\frac{1}{3}\pi r^2 h[1 + (r'/r) + (r'/r)^2] = \frac{1}{3}\pi h(r^2 + rr' + r'^2) = \frac{1}{4}\pi h[(r + r')^2 + \frac{1}{3}(r - r')^2]$. Lateral area $= \pi s(r + r')$; $s = \sqrt{(r - r')^2 + h^2}$.

Any Pyramid or Cone. Volume $= \frac{1}{3}Bh$. B = area of base; h = perpendicular distance from vertex to plane in which base lies.

Any Pyramidal or Conical Frustum (Fig. 74). Volume = $\frac{1}{3}h(B + \sqrt{BB'} + B') = \frac{1}{3}hB[1 + (P'/P) + (P'/P)^2]$. Here B, B' = areas of lower and upper bases; P, P' = perimeters of lower and upper bases.

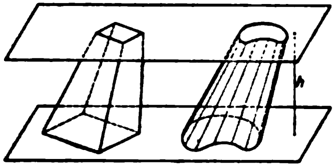


FIG. 74.

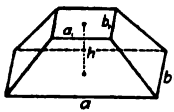


FIG. 75.

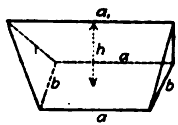


FIG. 76.

Obelisk (Frustum of a rectangular pyramid. Fig. 75).
Volume $= \frac{1}{6}h[(2a + a_1)b + (2a_1 + a)b_1] = \frac{1}{6}h[ab + (a + a_1)(b + b_1) + a_1b_1]$.

Wedge (Rectangular base; a_1 parallel to a, a and at distance h above base. Fig. 76). Volume $= \frac{1}{6}hb(2a + a_1)$.

Sphere. Volume $= V = \frac{4}{3}\pi r^3 = 4.188790r^3 = \frac{1}{6}\pi d^3 = 0.523599d^3$ (table, p. 36) = $\frac{2}{3}$ volume of circumscribed cylinder. Area $= A = 4\pi r^2$ = four great circles (table, p. 30) $= \pi d^2 = 3.14159d^2$ = lateral area of circumscribed cylinder. Here r = radius; $d = 2r$ = diameter $= \sqrt[3]{6V/\pi} = 1.24070\sqrt[3]{V} = \sqrt{A/\pi} = 0.56419\sqrt{A}$.

Hollow Sphere, or spherical shell. Volume = $\frac{4}{3}\pi(R^3 - r^3) = \frac{1}{6}\pi(D^3 - d^3) = 4\pi R_1^2 t + \frac{1}{3}\pi t^3$. Here R, r = outer and inner radii; D, d = outer and inner diameters; t = thickness $= R - r$; R_1 = mean radius = $\frac{1}{2}(R + r)$.

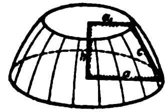


FIG. 77.

Spherical Segment of One Base. Zone (spherical "cap" of Fig. 78). Volume $= \frac{1}{6}\pi h(3a^2 + h^2) = \frac{1}{3}\pi h^2(3r - h)$ (table, p. 38). Lateral area (of zone) $= 2\pi rh = \pi(a^2 + h^2)$. Note: $a^2 = h(2r - h)$, where r = radius of sphere.

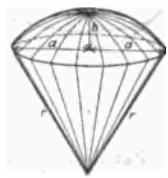


FIG. 78.

Any Spherical Segment. Zone (Fig. 77). Volume $= \frac{1}{6}\pi h(3a^2 + 3a_1^2 + h^2)$. Lateral area (zone) $= 2\pi rh$. Here r = radius of sphere. If the inscribed frustum of a cone be removed from the spherical segment, the volume remaining is $\frac{1}{6}\pi hc^2$, where c = slant height of frustum $= \sqrt{h^2 + (a - a_1)^2}$.

Spherical Sector (Fig. 78). Volume $= \frac{1}{3}r \times$ area of cap $= \frac{2}{3}\pi r^2 h$. Total area = area of cap + area of cone $= 2\pi rh + \pi ra$. Note: $a^2 = h(2r - h)$.

Spherical Wedge bounded by two plane semicircles and a **lune.** (Fig. 79.) Volume of wedge ÷ volume of sphere = $u/360°$. Area of lune ÷ area of sphere = $u/360°$. u = dihedral angle of the wedge.

Spherical Triangle bounded by arcs of three great circles. (Fig. 80.) Area of triangle = $\pi r^2 E/180°$ = area of octant $\times E/90°$. E = spherical excess = $180° - (A + B + C)$, where A, B, and C are angles of the triangle. See also p. 134.

Solid Angles. Any portion of a spherical surface subtends what is called a **solid angle** at the center of the sphere. If the area of the given portion of spherical surface is equal to the square of the radius, the subtended solid angle is called a **steradian,** and this is commonly taken as the unit. The entire solid angle about the center is called a **steregon,** so that 4π steradians = 1 steregon. A so-called "solid right angle" is the solid angle subtended by a quadrantal (or trirectangular) spherical triangle, and a "spherical degree" (now little used) is a solid angle equal to $1/90$ of a solid right angle. Hence 720 spherical degrees = 1 steregon, or π steradians = 180 spherical degrees. If u = the angle which an element of a cone makes with its axis, then the solid angle of the cone contains $2\pi(1 - \cos u)$ steradians.

FIG. 79.

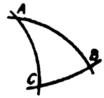


FIG. 80.

Regular Polyhedra. A = area of surface; V = volume; a = edge.

Name of solid (see p. 100)	Bounded by	A/a^2	V/a^3
Tetrahedron	4 triangles	1.7321	0.1179
Cube	6 squares	6.0000	1.0000
Octahedron	8 triangles	3.4641	0.4714
Dodecahedron	12 pentagons	20.6457	7.6631
Icosahedron	20 triangles	8.6603	2.1817

Ellipsoid (Fig. 81). Volume = $\frac{4}{3}\pi abc$, where a, b, c = semi-axes.

Spheroid (or ellipsoid of revolution). The volume of any segment made by two planes perpendicular to the axis of revolution may be found accurately by the prismoidal formula (p. 111).

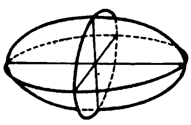


FIG. 81.

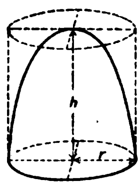


FIG. 82.

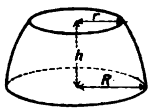


FIG. 83.

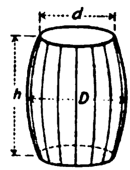


FIG. 84.

Paraboloid of Revolution (Fig. 82). Volume = $\frac{1}{2}\pi r^2 h$ = $\frac{1}{2}$ volume of circumscribed cylinder.

Segment of Paraboloid of Revolution (Bases perpendicular to axis, Fig. 83). Volume of segment = $\frac{1}{2}\pi(R^2 + r^2)h$.

Barrels or Casks (Fig. 84). Volume = $\frac{1}{12}\pi h(2D^2 + d^2)$ approx. for circular staves. Volume = $\frac{1}{15}\pi h(2D^2 + Dd + \frac{3}{4}d^2)$ exactly for parabolic staves.

For a standing cask, partially full, compute contents by the prismoidal formula, p. 111. Roughly, the number of gallons, G, in a cask is given by $G = 0.0034n^2h$, where n = number of inches in the mean diameter, or $\frac{1}{2}(D + d)$, and h = number of inches in the height.

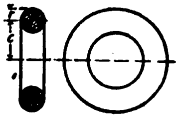


FIG. 85.

Torus, or Anchor Ring (Fig. 85). Volume $= 2\pi^2cr^2$. Area $= 4\pi^2cr$ (Proof by theorems of Pappus).

Theorems of Pappus. 1. Assume that a plane figure, area A, revolves about an axis in its plane but not cutting it; and let s = length of circular arc traced by its center of gravity. Then volume of the solid generated by A is $V = As$. For a complete revolution, $V = 2\pi rA$, where r = distance from axis to center of gravity of A.

2. Assume that a plane curve, length l, revolves about an axis in its plane but not cutting it; and let s = length of circular arc traced by its center of gravity. Then area of the surface generated by l is $S = ls$. For a complete revolution, $S = 2\pi rl$, where r = distance from axis to center of gravity of l.

NOTE. If V_1 or S_1 about any axis is known, then V_2 or S_2 about any parallel axis can be readily computed when the distance between the axes is known.

Generalized Theorems of Pappus. Consider any curved path of length s. If (1) a plane figure, area A [or (2) a plane curve, length l] moves so that its center of gravity slides along this curved path (Fig. 86), while the plane of A [or l] remains always perpendicular to the path, then (1) the volume generated by A is $V = As$ [and (2) the area generated by l is $S = ls$]. The path is assumed to curve so gradually that successive positions of A [or l] will not intersect.

FIG. 86.

The Prismoidal Formula (Fig. 87). Volume $= \frac{1}{6}h(A + B + 4M)$, where h = altitude, A and B = areas of bases and M = area of a plane section midway between the bases. This formula is exactly true for any solid lying between two parallel planes and such that the area of a section at distance x from one of these planes is expressible as a polynomial of not higher than the third degree in x. It is approximately true for many other solids.

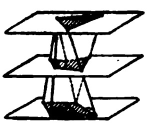


FIG. 87.

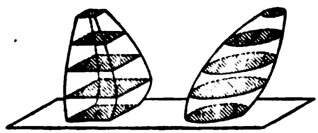


FIG. 88.

Simpson's Rule may be applied to finding volumes, if the ordinates y_1, y_2, be interpreted as the areas of plane sections, at constant distance h apart (p. 106).

Cavalieri's Theorem. Assume two solids to have their bases in the same plane. If the plane section of one solid at every distance x above the base is equal in area to the plane section of the other solid at the same distance x above the base, then the volumes of the two solids will be equal. See Fig. 88.

ALGEBRA

FORMAL ALGEBRA

Notation. The main points of separation in a simple algebraic expression are the + and − signs. Thus, $a + b \times c - d \div x + y$ is to be interpreted as $a + (b \times c) - (d \div x) + y$. In other words, the range of operation of the symbols $\times$ and $\div$ extends only so far as the next + or − sign. As between the signs $\times$ and $\div$ themselves, $a \div b \times c$ means, properly speaking, $a \div (b \times c)$; that is, the $\div$ sign is the stronger separative; but this rule is not always strictly followed, and in order to avoid ambiguity it is better to use the parentheses.

The range of influence of exponents and radical signs extends only over the next adjacent quantity. Thus, $2ax^3$ means $2a(x^3)$, and $\sqrt{2}ax$ means $(\sqrt{2})\,(ax)$. Instead of $\sqrt{2}ax$, it is safer, however, to write $\sqrt{2}\cdot ax$, or, better, $ax\sqrt{2}$.

Any expression within parentheses is to be treated as a single quantity. A horizontal bar serves the same purpose as parentheses.

The notation $a \cdot b$, or simply ab, means $a \times b$; and $a:b$, or a/b, means $a \div b$.

The symbol $|a|$ means the "absolute value of a," regardless of sign; thus, $|-2| = |+2| = 2$.

The symbol $n!$ (where n is a whole number) is read: "n factorial," and means the product of the natural numbers from 1 to n, inclusive. Thus $1! = 1$; $2! = 1 \times 2$; $3! = 1 \times 2 \times 3$; $4! = 1 \times 2 \times 3 \times 4$; etc.

The symbol $\neq$ or $\ne$ means "not equal to"; $\pm$ means "plus or minus."

The symbol $\approx$ is sometimes used for "approximately equal to."

Addition and Subtraction. $a + b = b + a$.

$(a + b) + c = a + (b + c)$. $a - (-b) = a + b$. $a - a = 0$.

$a + (x - y + z) = a + x - y + z$. $a - (x - y + z) = a - x + y - z$.

A minus sign preceding a parenthesis operates to reverse the sign of every term within, when the parentheses are removed.

Multiplication and Simple Factoring. $ab = ba$. $(ab)c = a(bc)$. $a(b + c) = ab + ac$. $a(b - c) = ab - ac$. Also, $a \times (-b) = -ab$, and $(-a) \times (-b) = ab$; "unlike signs give minus; like signs give plus."

$(a + b)(a - b) = a^2 - b^2$.

$(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$.

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$; etc.

(See table of binomial coefficients, p. 39; also p. 114.)

$a^2 - b^2 = (a - b)(a + b)$, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + ab^{n-2} + b^{n-1})$.

$a^n + b^n$ is factorable by $a + b$ only when n is odd; thus,

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$,

$a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$; etc.

The following transformation is sometimes useful:

$$ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2\right].$$

Fractions. If m is not zero, $\dfrac{ma + mb + mc}{mx + my} = \dfrac{a + b + c}{x + y}$; that is, both numerator and denominator of a fraction may be multiplied or divided by any quantity different from zero, without altering the value of the fraction.

To add two fractions, reduce each to a common denominator, and add the numerators: $\frac{a}{b}+\frac{x}{y}=\frac{ay}{by}+\frac{bx}{by}=\frac{ay+bx}{by}$.

To multiply two fractions: $\frac{a}{b}\times\frac{x}{y}=\frac{ax}{by}$; $\frac{a}{b}\times x=\frac{a}{b}\times\frac{x}{1}=\frac{ax}{b}$.

To divide one fraction by another, invert the divisor and multiply:

$$\frac{a}{b}\div\frac{x}{y}=\frac{a}{b}\times\frac{y}{x}=\frac{ay}{bx};\quad \frac{a}{b}\div x=\frac{a}{b}\times\frac{1}{x}=\frac{a}{bx}.$$

Ratio and Proportion. The notation $a:b::c:d$, which is now passing out of use, is read: "a is to b as c is to d," and means simply $(a/b)=(c/d)$, or $ad=bc$. a and d are called the "extremes," b and c the "means," and d the "fourth proportional" to a, b, and c. The "mean proportional" between two numbers is the square root of their product; also called the "geometric mean" of the numbers (p. 115). If $a/b=c/d$, then $(a+b)/b=(c+d)/d$, and $(a-b)/b=(c-d)/d$; whence also, $(a+b)/(a-b)=(c+d)/(c-d)$. If $a/x=b/y=c/z=\ .\ .\ .=r$, then

$$(a+b+c+\ .\ .\ .)/(x+y+z+\ .\ .\ .)=r.$$

Variation. The notation $x \propto y$ is read: "x varies directly as y," or "x is directly proportional to y," and means $x=ky$, where k is some constant. To determine the constant k, it is sufficient to know any pair of values, as x_1 and y_1, which belong together; then $x_1=ky_1$, and hence $x/x_1=y/y_1$, or $x=(x_1/y_1)y$. The expression "x varies inversely as y," or "x is inversely proportional to y," means that x is proportional to $1/y$, or $x=k/y$.

Exponents. $a^{m+n}=a^ma^n$. $a^{m-n}=a^m/a^n$. $a^0=1$ (if $a\neq 0$). $a^{-m}=1/a^m$. $(a^m)^n=a^{mn}$. $a^{1/n}=\sqrt[n]{a}$. Thus: $a^{1/2}=\sqrt{a}$, and $a^{1/3}=\sqrt[3]{a}$. $a^{m/n}=\sqrt[n]{a^m}$. Thus: $a^{2/3}=\sqrt[3]{a^2}$ and $a^{3/2}=\sqrt{a^3}$. $(\sqrt[n]{a})^n=a$. $(ab)^n=a^nb^n$. $(a/b)^n=a^n/b^n$. $(-a)^n=a^n$ if n is even. $(-a)^n=-a^n$ if n is odd. If n is positive and increases indefinitely, a^n becomes infinite if $a>1$, and approaches 0 if $a<1$ (a being always positive). Graphs, p. 174; series, p. 160.

Radicals. Except in the simple cases of square root and cube root, radical signs should always be replaced by fractional exponents: $\sqrt[n]{a}=a^{1/n}$. $(\sqrt[n]{a})^n=(a^{1/n})^n=a$. If n is odd, $\sqrt[n]{-a}=-\sqrt[n]{a}$; but if n is even, $\sqrt[n]{-a}$ is imaginary. Every positive number a has two square roots, one positive and the other negative; but the notation $\sqrt{a}$ always means the positive root; thus, $\sqrt{9}=3$; $-\sqrt{9}=-3$. If the denominator of a fraction is of the form $\sqrt{a}\pm\sqrt{b}$, it is possible to "rationalize the denominator" by multiplying both numerator and denominator by $\sqrt{a}\mp\sqrt{b}$. Thus:

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\frac{a+b+2\sqrt{ab}}{a-b}$$

Logarithms. (For the use of logarithms in numerical computation, see p. 91.) The logarithm of a (positive) number N is the exponent of that power to which the base (10 or e) must be raised to produce N. Thus, $x=\log_{10}N$ means that $10^x=N$, and $x=\log_e N$ means that $e^x=N$. Logarithms to base 10 are called **common, denary**, or **Briggsian** logarithms. For table of 4-place common logarithms see pp. 40–43.

Logarithms to base e are called **hyperbolic, natural,** or **Napierian** logarithms. Here $e = 1 + 1 + 1/2! + 1/3! + 1/4! + \ldots = 2.718281828459\ldots$ For table of 4-place hyperbolic logarithms see pp. 58, 59.

If the subscript 10 or e is omitted, the base must be inferred from the context, the base 10 being used in numerical computation, and the base e in theoretical work. In either system,

$\log (ab) = \log a + \log b$	$\log (a^n) = n \log a$	$\log 0 = -\infty$
$\log (a/b) = \log a - \log b$	$\log (\sqrt[n]{a}) = (1/n) \log a$	$\log 1 = 0$
$\log (1/n) = -\log n$	$\log (\text{base}) = 1$	$\log \infty = \infty$

The two systems are related as follows:

$\log_{10} e = M = 0.4342944819\ldots$; $\log_e 10 = 1/M = 2.3025850930\ldots$
$\log_{10} x = 0.4343 \log_e x$; $\log_e x = 2.3026 \log_{10} x$.

For tables of multiples of M and $1/M$, see p. 62. For graphs of the logarithmic and exponential functions, see p. 174; series, p. 160.

The Binomial Theorem. (For table of binomial coefficients, see p. 39 and p. 116.)

Let $(n)_1 = n$, $(n)_2 = \dfrac{n(n-1)}{1 \times 2}$, $(n)_3 = \dfrac{n(n-1)(n-2)}{1 \times 2 \times 3}$,

$$(n)_4 = \frac{n(n-1)(n-2)(n-3)}{1 \times 2 \times 3 \times 4}, \ldots$$

Then, for any value of n, provided $|x| < 1$,

$$(1+x)^n = 1 + (n)_1 x + (n)_2 x^2 + (n)_3 x^3 + (n)_4 x^4 + \ldots$$

(If n is a positive integer, the series breaks off with the term in x^n, and is valid without restrictions on x, see p. 112.)

The most useful **special cases** are the following:

$$\sqrt{1+x} = (1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \ldots \quad (|x| < 1)$$

$$\sqrt[3]{1+x} = (1+x)^{1/3} = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \frac{10}{243}x^4 + \ldots \quad (|x| < 1)$$

$$\frac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots \quad (|x| < 1)$$

$$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \ldots \quad (|x| < 1)$$

$$\frac{1}{\sqrt[3]{1+x}} = (1+x)^{-1/3} = 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \frac{35}{243}x^4 - \ldots \quad (|x| < 1)$$

$$\sqrt{(1+x)^3} = (1+x)^{3/2} = 1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3 + \frac{3}{128}x^4 - \ldots \quad (|x| < 1)$$

$$\frac{1}{\sqrt{(1+x)^3}} = (1+x)^{-3/2} = 1 - \frac{3}{2}x + \frac{15}{8}x^2 - \frac{35}{16}x^3 + \frac{315}{128}x^4 - \ldots \quad (|x| < 1)$$

with corresponding formulæ for $\sqrt{1-x}$, etc., obtained by reversing the signs of the odd powers of x. Also, provided $|b| < |a|$:

$$(a+b)^n = a^n\left(1 + \frac{b}{a}\right)^n = a^n + (n)_1 a^{n-1}b + (n)_2 a^{n-2}b^2 + (n)_3 a^{n-3}b^3 + \ldots$$

where $(n)_1$, $(n)_2$, etc., have the values given above.

Arithmetical Progression. In an arithmetical progression, a; $a + d$; $a + 2d$; $a + 3d$; . . ., each term is obtained from the preceding term by adding a constant, called the constant difference, d. If n is the number of terms, the last term is $l = a + (n-1)d$; the "average" term is $\frac{1}{2}(a + l)$;

and the sum of the n terms is n times the average term, or $S = \frac{1}{2}n(a + l)$. The **arithmetical mean** between a and b is $(a + b)/2$.

Geometrical Progression. In a geometrical progression, a; ar; ar^2; ar^3; . . ., each term is obtained from the preceding term by multiplying by a constant, called the constant ratio, r. The nth term is ar^{n-1}. The sum of the first n terms is $S = a(r^n - 1)/(r - 1) = a(1 - r^n)/(1 - r)$. If r is a positive or negative fraction, that is, if $-1 < r < +1$, then r^n will approach zero as n increases, and the sum of n terms will approach $a/(1 - r)$ as a limit. The **geometric mean** between a and b is $\sqrt{ab}$; also called the **mean proportional** between a and b (p. 113; construction, p. 102).

The **harmonic mean** between a and b is $2ab/(a + b)$.

Summation of Certain Series by Second and Third Differences. Let $a_1, a_2, a_3, \ldots a_n$ be any series of n numbers, as in the first column of the adjoining scheme. By subtracting each number from the next following, form the column of "first differences," and by repeating this process, form the columns of second, third, etc., differences. If the kth differences are all equal, so that subsequent differences are all zero, the original series is called an arithmetical series of the kth order. In this special case the series can be summed as follows: Denote the numbers which stand at the head of the successive columns of differences by $D', D'', D''', \ldots$ Then the nth term of the series is a_n, and the sum of the first n terms is S_n, where

Numbers	1st diff.	2nd diff.	3rd diff.
−64	37	.	
−27	19	−18	
−8	7	−12	6
−1	1	−6	6
0	1	0	6
1	7	6	6
8	.	.	.
.			

$$a_n = a_1 + (n-1)D' + \frac{(n-1)(n-2)}{1 \times 2}D'' + \frac{(n-1)(n-2)(n-3)}{1 \times 2 \times 3}D''' + \ldots$$

$$S_n = na_1 + \frac{n(n-1)}{1 \times 2}D' + \frac{n(n-1)(n-2)}{1 \times 2 \times 3}D'' + \frac{n(n-1)(n-2)(n-3)}{1 \times 2 \times 3 \times 4}D''' + \ldots$$

If the series is, for example, of the third order, each of these formulæ will stop with the term involving D'''; and only a few terms of the series are required for the computation of the D's. (Differentials, p. 159.)

Sum of the Squares or Cubes of the First n Natural Numbers.

$$1 + 2 + 3 + \ldots + (n - 1) + n = \tfrac{1}{2}n(n + 1).$$
$$1^2 + 2^2 + 3^2 + \ldots + (n - 1)^2 + n^2 = \tfrac{1}{6}n(n + 1)(2n + 1).$$
$$1^3 + 2^3 + 3^3 + \ldots + (n - 1)^3 + n^3 = [\tfrac{1}{2}n(n + 1)]^2.$$

Formula for Interpolation by Second Differences. In any ordinary table giving a quantity y as a function of a variable x, let it be required to find the value of y corresponding to a value of x which is not given directly in the table, but which lies between two tabulated values, as x_1 and x_2. If $x = x_1 + md$, where $d = x_2 - x_1$ = the constant interval between two successive x's, and m is some proper fraction, then the corresponding value of y will be given by the formula

$$y = y_1 + mD' + \frac{m(m-1)}{1 \times 2}D'' + \frac{m(m-1)(m-2)}{1 \times 2 \times 3}D''' + \ldots$$

where $D', D'', D''', \ldots$ are the first, second, third, . . . differences in the series of y's which begins with y_1 (see above), provided the function is of such a nature that the differences of higher orders become negligibly small.

The coefficients of D', D'', D''', . . . in the formula are the binomial coefficients for fractional values of m (see following table). The several terms of the formula (with careful attention to sign) are the successive corrections which must be added to y_1; the sum of these corrections should be rounded out to the nearest unit of the last significant place before adding. If $D'' < 4$, the term involving D'', and later terms, can be neglected; the formula then reduces to $y = y_1 + mD'$, which is the familiar formula for ordinary, or "linear," interpolation. If $D''' < 8$ (or $D''''' < 12$, or $D'''''' < 16$), the term involving D''' (or D'''', or D''''') can be neglected.

Binomial Coefficients for Fractional Values of m

m	$(m)_2$	$(m)_3$	$(m)_4$	$(m)_5$
0.0	− 0.0000	0.0000	− 0.0000	0.0000
0.1	− 0.0450	0.0285	− 0.0207	0.0161
0.2	− 0.0800	0.0480	− 0.0336	0.0255
0.3	− 0.1050	0.0595	− 0.0402	0.0297
0.4	− 0.1200	0.0640	− 0.0416	0.0300
0.5	− 0.1250	0.0625	− 0.0391	0.0273
0.6	− 0.1200	0.0560	− 0.0336	0.0228
0.7	− 0.1050	0.0455	− 0.0262	0.0173
0.8	− 0.0800	0.0320	− 0.0176	0.0113
0.9	− 0.0450	0.0165	− 0.0087	0.0054

Here $(m)_2 = \frac{m(m-1)}{1 \times 2}$, $(m)_3 = \frac{m(m-1)(m-2)}{1 \times 2 \times 3}$, $(m)_4 = \frac{m(m-1)(m-2)(m-3)}{1 \times 2 \times 3 \times 4}$, etc. Compare p. 39.

Permutations. The number of possible permutations or arrangements of n different elements is $1 \times 2 \times 3 \times \ldots \times n = n!$ (read: "n factorial").

If among the n elements there are p equal ones of one sort, q equal ones of another sort, r equal ones of a third sort, etc., then the number of possible permutations is $(n!)/(p! \times q! \times r! \times \ldots)$, where $p + q + r + \ldots = n$.

Combinations. The number of possible combinations or groups of n elements taken r at a time (without repetition of any element within any one group), is $[n(n-1)(n-2)(n-3) \ldots (n-r+1)]/(r!) = (n)_r$. (See table of binomial coefficients, p. 39.) If repetitions are allowed, so that a group, for example, may contain as many as r equal elements, then the number of combinations of n elements taken r at a time is $(m)_r$, where $m = n + r - 1$. NOTE: $(n)_1 + (n)_2 + \ldots + (n)_n = 2^n - 1$.

SOLUTION OF EQUATIONS IN ONE UNKNOWN QUANTITY

Roots of an Equation. An equation containing a single variable x will in general be true for some values of x and false for other values. Any value of x for which the equation is true is called a **root** of the equation. To "solve" an equation means to find all its roots. Any root of an equation, when substituted therein for x, will "satisfy" the equation. An equation which is true for all values of x, like $(x+1)^2 = x^2 + 2x + 1$, is called an **identity** [often written $(x+1)^2 \equiv x^2 + 2x + 1$].

Types of Equations.

(*a*) Algebraic Equations:
of the first degree (linear), *e.g.*, $2x + 6 = 0$ (root: $x = -3$);
of the second degree (quadratic), *e.g.*, $x^2 - 2x - 3 = 0$ (roots: $-1, 3$);
of the third degree (cubic), *e.g.*, $x^3 - 6x^2 + 5x + 12 = 0$ (roots: $-1, 3, 4$).

(*b*) Transcendental Equations:
exponential equations, *e.g.*, $2^x = 32$ (root: $x = 5$); $2^x = -32$ (no root); trigonometric equations, *e.g.*, $10 \sin x - \sin 3x = 4$ (roots: 30°, 150°).

Legitimate Operations on Equations. An equation which is true for a particular value of x will remain true for that value of x after any one of the following operations is performed:

Adding any quantity to both sides; subtracting any quantity from both sides; transposing any term from one side to the other, provided its sign be changed; multiplying or dividing both sides by any quantity which is not zero; changing the signs of all the terms; raising both sides to any positive integral power; extracting any odd root of both sides; extracting any even root of both sides, provided the $\pm$ sign is used; taking the logarithms of both sides (both sides being positive); taking the sin, cos, tan, etc., of both sides.

Notice, however, that the new equation obtained by some of these operations may possess "additional roots" which did not belong to the original equation. This occurs especially when both sides are squared; thus, $x = -2$ has only one root, namely, -2; but $x^2 = 4$, obtained by squaring, has not only the root -2 but also another root, $+2$.

Equations of the First Degree (Linear Equations). Solution: Collect all the terms involving x on one side of the equation, thus: $ax = b$, where a and b are known numbers. Then divide through by the coefficient of x, obtaining $x = b/a$ as the root.

Equations of the Second Degree (Quadratic Equations). Solution: Throw the equation into the standard form $ax^2 + bx + c = 0$. Then the two roots are:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The roots are real-and-distinct, coincident, or imaginary, according as $b^2 - 4ac$ is positive, zero, or negative. The sum of the roots is $x_1 + x_2 = -b/a$; the product of the roots is $x_1x_2 = c/a$.

Graphical Solution. Write the equation in the form $x^2 = px + q$, and plot the parabola $y_1 = x^2$, and the straight line $y_2 = px + q$. The abscissæ of the points of intersection will be the roots of the equation. If the line does not cut the parabola, the roots are imaginary.

Equations of the Third Degree with Term in x^2 Absent. Solution: After dividing through by the coefficient of x^3, any equation of this type can be written $x^3 = Ax + B$. Let $p = A/3$ and $q = B/2$. The general solution is as follows:

Case 1. $q^2 - p^3$ positive. One root is real, namely

$$x_1 = \sqrt[3]{q + \sqrt{q^2 - p^3}} + \sqrt[3]{q - \sqrt{q^2 - p^3}};$$

the other two roots are imaginary.

Case 2. $q^2 - p^3 =$ zero. Three roots real, but two of them equal.

$$x_1 = 2\sqrt[3]{q}, \quad x_2 = -\sqrt[3]{q}, \quad x_3 = -\sqrt[3]{q}.$$

Case 3. $q^2 - p^3$ negative. All three roots real and distinct. Determine an angle u between 0 and 180°, such that $\cos u = q/(p\sqrt{p})$. Then

$$x_1 = 2\sqrt{p}\cos(u/3), \quad x_2 = 2\sqrt{p}\cos(u/3 + 120°), \quad x_3 = 2\sqrt{p}\cos(u/3 + 240°).$$

Graphical Solution. Plot the curve $y_1 = x^3$, and the straight line $y_2 = Ax + B$. The abscissæ of the points of intersection will be the roots of the equation.

Equations of the Third Degree (General Case). Solution: The general cubic equation, after dividing through by the coefficient of the highest power, may be written $x^3 + ax^2 + bx + c = 0$. To get rid of the term in x^2, let $x = x_1 - a/3$. The equation then becomes $x_1^3 = Ax_1 + B$, where $A = 3(a/3)^2 - b$, and $B = -2(a/3)^3 + b(a/3) - c$. Solve this equation for x_1, by the method above, and then find x itself from $x = x_1 - (a/3)$.

Graphical Solution. Without getting rid of the term in x^2, write the equation in the form $x^3 = -a[x + (b/2a)]^2 + [a(b/2a)^2 - c]$, and solve by the graphical method.

General Properties of Algebraic Equations. An algebraic equation of the nth degree in x is an equation of the type

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0$$

where the a's are any given numbers (a_0 not zero), the expression on the left being called a **polynomial** of the nth degree in x. Such an equation will, in general, have n roots; but some of these n roots may be equal, and some may be imaginary. **Imaginary roots** always occur in pairs.

If the equation is written in the form: (a polynomial in x) $= 0$, then (1) if a is a root of the equation, $x - a$ is a factor of the polynomial; (2) if the polynomial can be factored in the form $(x - p)(x - q)(x - r) \ldots = 0$, each of the quantities $p, q, r, \ldots$ is a root of the equation; (3) if x is very large (either positive or negative), the higher powers of x are the most important; (4) if x is very small, the higher powers may be neglected.

Short Method of Substitution in a Polynomial. To find the value of $4x^4 - 14x^3 + 23x - 26$ when $x = 3$, for example, first arrange the terms in order of descending powers of x, and write the detached coefficients, with their signs, in a row, taking care to supply a zero coefficient for any missing term, including the constant term. Then, beginning at the left, bring down the first coefficient; multiply this by 3, and add to the second coefficient; multiply this result by 3 again, and add to the third coefficient; and so on. The final result, -11, is the value of the polynomial when $x = 3$.

$$\begin{array}{rrrrrl} 4 & -14 & 0 & 23 & -26 & (3 \\ & 12 & -6 & -18 & 15 & \\ \hline 4 & -2 & -6 & 5 & -11 & \end{array}$$

Short Method of Dividing a Polynomial by x − a. The device just explained gives not only the value of the polynomial when $x = 3$, but also the result of dividing the polynomial by $x - 3$. Thus, in the case illustrated, the quotient is $4x^3 - 2x^2 - 6x + 5$ and the remainder is -11. That is, $4x^4 - 14x^3 + 0x^2 + 23x - 26 = (x - 3)(4x^3 - 2x^2 - 6x + 5) - 11$.

Exponential Equations. To solve an equation of the form $a^x = b$, take the logarithms of both sides: $x \log a = \log b$, whence $x = (\log b)/(\log a)$. For example, if $3^x = 0.4$, $x = \log 0.4/\log 3 = (0.6021 - 1)/0.4771 = -0.3979/0.4771 = -0.8340$. Notice that the complete logarithm must be taken, not merely the mantissa.

Trigonometric Equations. (1) To solve $a \cos x + b \sin x = c$, where a and b are positive: Find the acute angle u for which $\tan u = b/a$, and the angle v (between 0 and 180°) for which $\cos v = c/\sqrt{a^2 + b^2}$. Then $x_1 = u + v$ and $x_2 = u - v$ are roots of the equation.

(2) To solve $a \cos x - b \sin x = c$, where a and b are positive: Find u and v as above. Then $x_1 = -(u + v)$ and $x_2 = -(u - v)$ are roots of the equation.

General Method of Solution by Trial and Error. This method is applicable to a numerical equation of any form, and can be carried out to any desired degree of approximation. It is especially useful when a first approximation to a root is already known. Write the equation in the form

$f(x) = 0$, where $f(x)$ means any function of x, and plot the curve $y = f(x)$ for a sufficient number of values of x to obtain a general idea of the shape of the curve. Then pick out the regions in which the curve appears to cross the axis of x, and plot the curve more accurately in each of these regions. Thus, by successive approximations, plotting the important parts of the curve on a larger and larger scale, determine as accurately as necessary the points where the curve crosses the axis—that is, the values of x which make $f(x)$ equal to zero.

Thus, suppose that $f(x) = 3.0$ when $x = 2.6$ and -5.0 when $x = 2.7$ (see Fig. 1). Then the curve must cross the axis somewhere between $x = 2.6$ and $x = 2.7$; and since it will not vary greatly from a straight line between those points, it is seen that it must

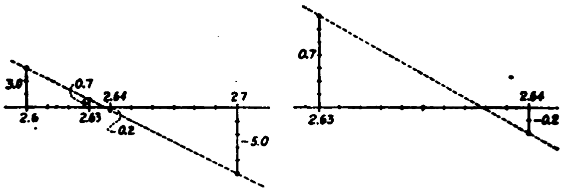


FIG. 1.

cross near 2.64. Suppose the value of $f(x)$ when computed for $x = 2.64$, is -0.2, and when computed for $x = 2.63$ is $+0.7$; then the root lies between $x = 2.63$ and 2.64. Plotting this section on the larger scale, it is seen that the next guess should be about 2.638; and so on.

Instead of writing the original equation with all the terms on the left-hand side, it is often better to divide the expression into two parts, say $f_1(x)$ and $f_2(x)$, writing the equation in the form $f_1(x) = f_2(x)$. If then the two curves $y_1 = f_1(x)$ and $y_2 = f_2(x)$ be plotted separately, on the same diagram, the value of x corresponding to their point of intersection will be the desired root.

SOLUTION OF SIMULTANEOUS EQUATIONS

The Meaning of a System of Simultaneous Equations. To solve a system of n simultaneous equations in n unknowns, means to find all the sets of values of the unknowns (if any) which, when substituted in the given equations, will satisfy all the equations at the same time. If a system of equations has no solution, the equations are "inconsistent;" if it has an infinite number of solutions, the equations are "not all independent."

Simultaneous Equations of the First Degree in Two Unknowns.

$$\begin{array}{ll|c|c|}
 & & \multicolumn{2}{c}{\text{Factors}} \\
(1) & a_1x + b_1y = c_1 & b_2 & -a_2 \\
(2) & a_2x + b_2y = c_2 & -b_1 & a_1 \\
\hline
\end{array}$$

$$(a_1b_2 - a_2b_1)x = b_2c_1 - b_1c_2 \qquad \therefore x = (b_2c_1 - b_1c_2)/(a_1b_2 - a_2b_1)$$
$$(a_1b_2 - a_2b_1)y = a_1c_2 - a_2c_1 \qquad \therefore y = (a_1c_2 - a_2c_1)/(a_1b_2 - a_2b_1)$$

Here (1) is multiplied by b_2, (2) by $-b_1$, and the products added so as to eliminate y; again, (1) is multiplied by $-a_2$, (2) by a_1, and the products added so as to eliminate x. (The process is most conveniently performed as follows: Write the multipliers, as b_2 and $-b_1$, at the right of the equations; multiply the first term of each equation by its proper multiplier and add; then multiply the second term of each equation by its proper multiplier, and add; and so on. This is simpler than the common practice of multiplying out each equation separately before adding.) If $a_1b_2 - a_2b_1 = 0$, the equations have no solution when $c_1 \neq c_2$, and an infinite number of solutions when

$c_1 = c_2$. The following **special solution** is possible when the sum and difference of the two unknowns are given:

$$\begin{array}{lll} & \text{Let } x + y = m & (1) \\ & \text{and } x - y = n & (2) \\ \hline (1) + (2): & 2x = m + n & \therefore x = \tfrac{1}{2}(m+n) \\ (1) - (2): & 2y = m - n & \therefore y = \tfrac{1}{2}(m-n) \end{array}$$

Simultaneous Equations of the Second Degree in Two Unknowns.

(a) When the product of the unknowns, and their sum or difference, are given:

$$\begin{array}{lll} & x + y = 5 & (1) \\ & xy = 4 & (2) \\ \hline \text{Squaring (1),} & x^2 + 2xy + y^2 = 25 & \\ \text{From (2),} & -4xy = -16 & \\ \hline \text{Adding,} & x^2 - 2xy + y^2 = 9 & \\ \hline \text{Hence,} & x - y = 3 \text{ or } -3 & \\ \text{But} & x + y = 5 \text{ or } 5 & \\ \hline \text{Therefore} & x = 4,\ y = 1 \text{ or } x = 1,\ y = 4 & \end{array}$$

$$\begin{array}{ll} x - y = 3 & (1) \\ xy = 4 & (2) \\ \hline x^2 - 2xy + y^2 = 9 & \\ 4xy = 16 & \\ \hline x^2 + 2xy + y^2 = 25 & \\ \hline x + y = 5 \text{ or } -5 & \\ x - y = 3 \text{ or } 3 & \\ \hline x = 4,\ y = 1 \text{ or } x = -1,\ y = -4 & \end{array}$$

(b) When the product and the sum of the squares are given:

$$\begin{array}{lll} & xy = 5 & (1) \\ & x^2 + y^2 = 26 & (2) \\ \hline \text{From (1),} & 2xy = 10 & (3) \\ (2)+(3): & x^2 + 2xy + y^2 = 36 & (4) \\ (2)-(3): & x^2 - 2xy + y^2 = 16 & (5) \end{array}$$

$$\begin{array}{l} \sqrt{(4)}: x + y = 6 \text{ or } 6 \text{ or } -6 \text{ or } -6 \\ \sqrt{(5)}: x - y = 4 \text{ or } -4 \text{ or } 4 \text{ or } -4 \\ \hline \therefore x = 5 \text{ or } 1 \text{ or } -1 \text{ or } -5 \\ \therefore y = 1 \text{ or } 5 \text{ or } -5 \text{ or } -1 \end{array}$$

(c) When the sum or difference, and the sum of the squares, are given:

$$\begin{array}{lll} & x + y = 5 & (1) \\ & x^2 + y^2 = 17 & (2) \\ (1)^2: & x^2 + 2xy + y^2 = 25 & \\ (2): & x^2 + y^2 = 17 & \\ \hline (1)^2 - (2): & 2xy = 8 & \\ & xy = 4 & \end{array}$$

Then proceed as in case (*a*), above.

$$\begin{array}{lll} & x - y = 3 & (1) \\ & x^2 + y^2 = 17 & (2) \\ (1)^2: & x^2 - 2xy + y^2 = 9 & \\ (2): & x^2 + y^2 = 17 & \\ \hline (1)^2 - (2): & -2xy = -8 & \\ & xy = 4 & \end{array}$$

Then proceed as in case (*a*), above.

(d) When one equation is of the first degree and the other of the second, as $ax + by = c$, and $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$: Solve the first equation for y in terms of x, and substitute in the second. This will give a quadratic equation in x. Solve this quadratic for the two values of x, and for each of these values of x find the corresponding value of y by substituting in the equation of the first degree.

Simultaneous Equations of the First Degree in n Unknowns. For example:

		Factors		
(*a*)	$2x - y + 3z + 5w = 29$	3	1	2
(*b*)	$5x + 2y - 2z + 3w = 15$	−5		
(*c*)	$3x - 4y + 7z - w = 12$		5	
(*d*)	$4x + 3y - 5z + 2w = 3$			−5

(*e*)	$-19x - 13y + 19z = 12$	−2	−31
(*f*)	$17x - 21y + 38z = 89$	1	
(*g*)	$-16x - 17y + 31z = 43$		19

(*h*)	$55x + 5y = 65$	16
(*i*)	$285x + 80y = 445$	−1

$$
\begin{array}{llr}
(j) \quad 595x = 595; & & \therefore x = 1; \\
5y = 65 - 55x = 65 - 55 = 10; & & \therefore y = 2; \\
19z = 12 + 19x + 13y = 12 + 19 + 26 = 57; & & \therefore z = 3; \\
2w = 3 - 4x - 3y + 5z = 3 - 4 - 6 + 15 = 8; & & \therefore w = 4.
\end{array}
$$

Here w is eliminated from (a) and (b), obtaining (e); from (a) and (c), obtaining (f); and from (a) and (d), obtaining (g). Then z is eliminated from (e) and (f), obtaining (h), and from (e) and (g), obtaining (i). Then y is eliminated from (h) and (i), obtaining (j), which contains only the single variable x. Hence $x = 1$. Now substituting this value of x in either (h) or (i), y is found; substituting these values of x and y in either (e), (f), or (g), z is found; and so on. (Solution by determinants, see p. 123.)

Approximate Solution of a Set of Simultaneous Equations of the First Degree When the Number of Equations is Greater Than the Number of Unknowns. (Method of Least Squares.)

Case 1. Single Unknown Quantity. Given n equations in one unknown x; for example, n equally careful, independent measurements of some physical quantity:

$$x = x_1, \; x = x_2, \; . \; . \; . \; x = x_n.$$

As the "best" value of x, take the arithmetic mean, x_0, of the several determinations, namely, $x_0 = (x_1 + x_2 + \; . \; . \; . \; + x_n)/n$. The quantities $v_1 = x_0 - x_1$, $v_2 = x_0 - x_2$, . . . $v_n = x_0 - x_n$ are called the **residuals** of the observed values with respect to x_0, and their absolute values (that is, their numerical values without regard to sign) are denoted by $|v_1|$, $|v_2|$, . . . $|v_n|$. [It can be shown that the sum of the squares of the residuals with respect to x_0 is smaller than the sum of the squares of the residuals with respect to any other value x'_0; hence the name of the method: "least squares."]

The quantities r and r_0, defined exactly by Bessel's formulæ:

$$r = \frac{0.6745}{\sqrt{n-1}}\sqrt{v_1^2 + v_2^2 + \; . \; . \; . \; + v_n^2},$$

$$r_0 = \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{v_1^2 + v_2^2 + \; . \; . \; . \; + v_n^2},$$

or given approximately by the simpler formulæ of Peters:

$$r = \frac{0.8453}{\sqrt{n(n-1)}}\left(|v_1| + |v_2| + \; . \; . \; . \; + |v_n|\right),$$

$$r_0 = \frac{0.8453}{n\sqrt{n-1}}\left(|v_1| + |v_2| + \; . \; . \; . \; + |v_n|\right),$$

are called the **probable error of a single observation** (r), and the **probable error of the mean** (r_0), for the given series of observations. Note that $r_0 = r/\sqrt{n}$. For tables of the coefficients, see p. 63. This quantity r (or r_0) is best regarded as merely a conventional means of recording the relative precision of different sets of observations. If r is small, it may be inferred that most errors of the "accidental" class have been eliminated; but it should be especially noted that the smallness of r gives no information in regard to "constant" or "systematic" errors.

A statement like "x is equal to 2.36 with a probable error of 0.02," is written: $x = 2.36 \pm 0.02$, and is usually understood to mean that the true value of x, as far as can be told, is just as likely to lie inside as outside the interval from 2.34 to 2.38.

To test the **distribution of residuals**, arrange the residuals in order of magnitude, without regard to sign, and count the number, y, of residuals which are numerically less than some assigned value a; divide y by n, the total number of observations, and divide a by r, the probable error of a single observation. Do this for various values of a, and compare the results with the table on p. 63, which gives the standard distribution of residuals, as found from experience from a large number of different series of observations. In particular, the number of residuals numerically less than r should be about equal to the number numerically greater than r (if n is large). If any large discrepancy appears, the series of observations should be regarded as unsatisfactory.

NOTE. The "mean square error" sometimes met with is equal to the probable error divided by 0.6745.

Case 2. Several Unknown Quantities. Assume that there have been obtained by measurement or observation n different equations of the first degree involving, say, three unknown quantities, x, y, z. There are then n simultaneous equations in three unknowns, and if $n > 3$ there will be, in general, no set of values of x, y, z which will satisfy all these n equations exactly. In such a case, the "best" set of values, x_0, y_0, z_0, may be found by the method of least squares as follows. (The process usually involves a large amount of labor; the use of a computing machine is advisable.)

Given Equations

$$\begin{aligned} a_1x + b_1y + c_1z &= p_1 \\ a_2x + b_2y + c_2z &= p_2 \\ \cdot \quad \cdot \quad \cdot \quad &\cdot \\ a_nx + b_ny + c_nz &= p_n \end{aligned}$$

First, arrange the n given equations in the form indicated, being careful not to modify any of them by multiplication or division. (Any of the coefficients may of course be zero.)

Next, form the three "normal equations" as follows: (1) Multiply each of the given equations by the coefficient of x in that equation, and add; the result will be the first normal equation. (2) Multiply each of the given equations by the coefficient of y in that equation, and add; the result will be the second normal equation. (3) Similarly for the third. {Notation: $[aa] = a_1^2 + a_2^2 + \ . \ . \ . + a_n^2$; $[ab] = a_1b_1 + a_2b_2 + \ . \ . \ . + a_nb_n$; $[ap] = a_1p_1 + a_2p_2 + \ . \ . \ . + a_np_n$; etc.}

Normal Equations

$$\begin{aligned} [aa]x_0 + [ab]y_0 + [ac]z_0 &= [ap] \\ [ba]x_0 + [bb]y_0 + [bc]z_0 &= [bp] \\ [ca]x_0 + [cb]y_0 + [cc]z_0 &= [cp] \end{aligned}$$

Finally, solve the three normal equations for the three unknowns in the usual way.

The quantities $v_1 = a_1x_0 + b_1y_0 + c_1z_0 - p_1$, etc., are called the **residuals** with respect to x_0, y_0, z_0. [It can be shown that the sum of the squares of the residuals with respect to x_0, y_0, z_0 is smaller than the corresponding quantity with respect to any other set of values, x'_0, y'_0, z'_0; this relation is taken as the criterion for the "best" set of values of x, y, z.]

The **probable error of a single observation** is

$$r = \frac{0.6745}{\sqrt{n-m}}\sqrt{v_1^2 + v_2^2 + \ . \ . \ . + v_n^2}, \text{ or approximately,}$$

$$r = \frac{0.8453}{\sqrt{n(n-m)}}(|v_1| + |v_2| + \ . \ . \ . + |v_n|),$$

where m = the number of unknown quantities (here $m = 3$).

DETERMINANTS

Determinants are used chiefly in formulating theoretical results; they are seldom of use in numerical computation.

Evaluation of Determinants:

Of the second order:

$$\begin{vmatrix} a_1 b_1 \\ a_2 b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Of the third order:

$$\begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 c_2 \\ b_3 c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 c_1 \\ b_3 c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 c_1 \\ b_2 c_2 \end{vmatrix}$$

$$= a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1) + a_3(b_1 c_2 - b_2 c_1)$$

Of the fourth order:

$$\begin{vmatrix} a_1 b_1 c_1 d_1 \\ a_2 b_2 c_2 d_2 \\ a_3 b_3 c_3 d_3 \\ a_4 b_4 c_4 d_4 \end{vmatrix} = a_1 \begin{vmatrix} b_2 c_2 d_2 \\ b_3 c_3 d_3 \\ b_4 c_4 d_4 \end{vmatrix} - a_2 \begin{vmatrix} b_1 c_1 d_1 \\ b_3 c_3 d_3 \\ b_4 c_4 d_4 \end{vmatrix} + a_3 \begin{vmatrix} b_1 c_1 d_1 \\ b_2 c_2 d_2 \\ b_4 c_4 d_4 \end{vmatrix} - a_4 \begin{vmatrix} b_1 c_1 d_1 \\ b_2 c_2 d_2 \\ b_3 c_3 d_3 \end{vmatrix}$$

etc. In general, to evaluate a determinant of the nth order, take the elements of the first column with signs alternately plus and minus, and form the sum of the products obtained by multiplying each of these elements by its corresponding **minor**. The minor corresponding to any element a_1 is the determinant (of next lower order) obtained by striking out from the given determinant the row and column containing a_1.

Properties of Determinants.

1. The columns may be changed to rows and the rows to columns:

$$\begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} = \begin{vmatrix} a_1 a_2 a_3 \\ b_1 b_2 b_3 \\ c_1 c_2 c_3 \end{vmatrix}$$

2. Interchanging two columns changes the sign of the result.
3. If two columns are equal, the determinant is zero.
4. If the elements of one column are m times the elements of another column, the determinant is zero.
5. To multiply a determinant by any number m, multiply all the elements of any one column by m.
6. $\begin{vmatrix} a_1 + p_1 + q_1, & b_1 & c_1 \\ a_2 + p_2 + q_2, & b_2 & c_2 \\ a_3 + p_3 + q_3, & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} + \begin{vmatrix} p_1 b_1 c_1 \\ p_2 b_2 c_2 \\ p_3 b_3 c_3 \end{vmatrix} + \begin{vmatrix} q_1 b_1 c_1 \\ q_2 b_2 c_2 \\ q_3 b_3 c_3 \end{vmatrix}$
7. $\begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} = \begin{vmatrix} a_1 + mb_1, & b_1 & c_1 \\ a_2 + mb_2, & b_2 & c_2 \\ a_3 + mb_3, & b_3 & c_3 \end{vmatrix}$

Solution of Simultaneous Equations by Determinants.

If
$$\begin{aligned} a_1 x + b_1 y + c_1 z &= \mathbf{p}_1 \\ a_2 x + b_2 y + c_2 z &= \mathbf{p}_2 \\ a_3 x + b_3 y + c_3 z &= \mathbf{p}_3 \end{aligned} \qquad \text{where } D = \begin{vmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ a_3 b_3 c_3 \end{vmatrix} \neq 0,$$

then
$$\begin{aligned} x &= D_1/D, \\ y &= D_2/D, \\ z &= D_3/D, \end{aligned} \quad \text{where } D_1 = \begin{vmatrix} \mathbf{p}_1 b_1 c_1 \\ \mathbf{p}_2 b_2 c_2 \\ \mathbf{p}_3 b_3 c_3 \end{vmatrix},\ D_2 = \begin{vmatrix} a_1 \mathbf{p}_1 c_1 \\ a_2 \mathbf{p}_2 c_2 \\ a_3 \mathbf{p}_3 c_3 \end{vmatrix},\ D_3 = \begin{vmatrix} a_1 b_1 \mathbf{p}_1 \\ a_2 b_2 \mathbf{p}_2 \\ a_3 b_3 \mathbf{p}_3 \end{vmatrix}$$

Similarly for a larger (or smaller) number of equations.

THE ALGEBRA OF IMAGINARY OR COMPLEX QUANTITIES

In the algebra of imaginary or complex quantities, the objects on which the operations of the algebra are performed are not numbers in any ordinary sense of the word, but are best thought of as **points in a plane** (or as **vectors** drawn from a fixed origin to these points). The "**complex plane**" is determined by three fundamental points, O, U, i, arranged as in Fig. 2 and called the **zero point**, the **unit point**, and the **imaginary unit point**, respectively. All points on the line through O and U are called **real points**—positive if on the right of O, negative if on the left. All the remaining points in the plane are called **imaginary points**—those on the line through O and i being called the **pure imaginary points**.

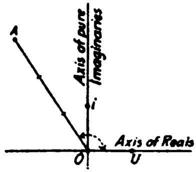


FIG. 2.

The position of any point A in the plane may be determined by the *distance* from the origin O, measured in terms of OU as the unit length, and the *angle* φ which OA makes with the positive direction of the axis of reals. The distance r is sometimes called the modulus or absolute value of the point; the angle φ is sometimes called the amplitude or argument of the point. The notation $A = (3, \angle 120°)$ means the point whose *distance*, r, is 3 times OU, and whose *angle*, φ, is 120°. The development of the algebra depends wholly on the definitions of three fundamental operations denoted by $A + B$, $A \times B$, and e^A, as follows.

Addition and Subtraction. The **sum**, $A + B$, of two points A and B is defined as the point reached by starting from A and performing a journey equal in length and direction to the journey from O to B. That is, the vector from O to $A + B$ is the vector sum of the vectors OA and OB. In case A and B are not in line with O, the point $A + B$ is the fourth vertex of a parallelogram of which OA and OB are the sides (Fig. 3). Conversely, if any two points A and B are given, there is a definite point X such that $A = B + X$; this point X is called the **remainder**, A minus B, and is denoted by $A - B$. The point $O - B$ is denoted for brevity by $-B$. With these definitions of $A + B$ and $A - B$, all the ordinary laws of addition and subtraction that hold in the algebra of real numbers hold also in the algebra of complex quantities. In particular, the zero point O has all the formal properties of the number zero, and is denoted by 0.

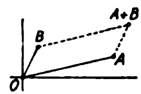


FIG. 3.

[Note: If A and B are "real" points, $A + B$ and $A - B$ will also be real.

Repeated Addition. Multiples and Submultiples. The point $A + A + A + \ldots + A$ to n terms is called the **nth multiple of A** and is denoted by nA. The points U, $2U$, $3U$, . . . are denoted, for brevity, by 1, 2, 3, ... Conversely, if any point A, and any positive integer n are given, there is a definite point X such that $nX = A$; this point X is called the **nth submultiple of A**, and is denoted by A/n. The points $U/2$, $U/3$, . . . are denoted, for brevity, by ½, ⅓, ...

Multiplication and Division. The **product**, $A \times B$, or $A \cdot B$, or AB, of two points A and B is defined as the point whose angle is the sum of the angles of the given points, and whose distance is the product of the distances. (See Fig. 4.) Thus, if $A = (5, \angle 120°)$ and $B = (2, \angle 270°)$, then $AB = (10, \angle 30°)$. Conversely, if any two points A and B are given, provided B is not zero, there is a definite point X such that

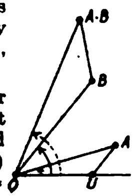


FIG. 4.

$A = BX$. This point X is called the **quotient,** A divided by B, and is denoted by A/B (where $B \neq 0$). Thus, the point A/B is a point whose angle is the angle of A minus the angle of B, and whose distance is the distance of A divided by the distance of B. The point U/B ($B \neq 0$) is called the **reciprocal** of the point B, and is denoted by $1/B$. (See Fig. 5.) With these definitions of AB and A/B the elementary laws of multiplication and division that hold in the algebra of real numbers hold also in the algebra of complex quantities. In particular, the point U has all the formal properties of the number unity, and is denoted by 1.

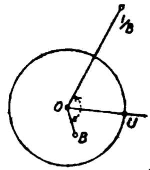


FIG. 5.

[Note: If A and B are real, AB and A/B will also be real.]

Repeated Multiplication. Powers and Roots. The point $A \times A \times A \times \ldots \times A$ to n factors is called the nth power of A and is denoted by A^n (Fig. 6). Conversely, if any point A (not 0) and any positive integer n are given, there will be n distinct points X such that $X^n = A$; each of these points is called an **nth root of A,** some one of them, usually the one with the smallest positive angle, being denoted by $\sqrt[n]{A}$ or $A^{1/n}$. Thus, the point $\sqrt[n]{A}$ is a point whose distance is the nth root of the distance of A, and whose angle is $1/n$th of the angle of A. All the nth roots of A will lie on the circumference of a circle about O as center, and will divide that circumference into n equal parts (Fig. 7). Every point A (not 0) has two square roots, three cube roots, etc. Hence the theorem "If $A^n = B^n$ then $A = B$" does not hold in this algebra, and the ordinary rules for radical signs must be applied with caution. For example, if A and B are positive reals, $\sqrt{-A} \cdot \sqrt{-B} = -\sqrt{AB}$ and not $\sqrt{(-A)(-B)}$, which would give $+\sqrt{AB}$.

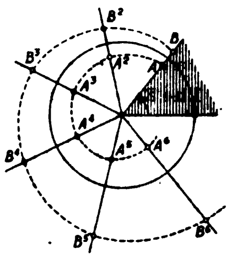


FIG. 6.

[Note: If A is real and positive, $\sqrt[n]{A}$ will be real and positive; if A is real and negative, $\sqrt[n]{A}$ will be real if n is odd and imaginary if n is even.]

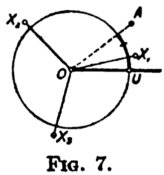


FIG. 7.

Properties of i. The point i is the point whose distance is 1 and whose angle is 90 deg. It follows from the definition above that **multiplying any point A by i has the effect of rotating the point through an angle of** $+ 90°$ without changing its distance from O. In particular,

$i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i$, etc.; $i = \sqrt{-1}$, $-i = -\sqrt{-1}$; where "1" denotes not the *number* one, but the *point* U.

Similarly, multiplying any point A by -1 has the effect of rotating the point through 180 deg.

First Standard Form for a Complex Quantity (Fig. 8). Any point A can be expressed in the form $x + iy$, where x and y are real points. For example, the three cube roots of 1 are 1, $-\frac{1}{2} + \frac{1}{2}i\sqrt{3}$, and $-\frac{1}{2} - \frac{1}{2}i\sqrt{3}$.

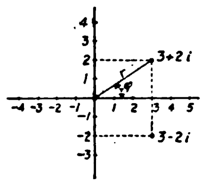


FIG. 8.

In general, $(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2)$;

$$(x_1 + iy_1)(x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_2y_1 + x_1y_2);$$

$$\frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\,\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}.$$

If two complex quantities are equal, their real parts must be equal, and the coefficients of their pure imaginary parts must also be equal. That is, if $x_1 + iy_1 = x_2 + iy_2$, then $x_1 = x_2$ and $y_1 = y_2$. Thus a single equation between complex quantities is equivalent to two equations between real quantities.

Conjugate Imaginaries. Two points $A = x + iy$ and $B = x - iy$ are called conjugate imaginaries. Two such points are symmetrically situated with regard to the axis of reals. The sum and product of two conjugate imaginaries will be real.

Second Standard Form for a Complex Quantity. Since $x = r \cos \varphi$ and $y = r \sin \varphi$, any point $A = x + iy$ can be expressed $A = r (\cos \varphi + i \sin \varphi)$, where r is real and positive (namely, the distance of A), and φ is real (namely the angle of A). For example, the three cube roots of 1 are 1, $\cos 120° + i \sin 120°$, and $\cos 240° + i \sin 240°$. In general,

$[r_1 (\cos \varphi_1 + i \sin \varphi_1)] [r_2(\cos \varphi_2 + i \sin \varphi_2)] = r_1r_2[(\cos (\varphi_1 + \varphi_2) + i \sin (\varphi_1 + \varphi_2)]$;

$[r(\cos \varphi + i \sin \varphi)]^n = r^n[\cos(n\varphi) + i \sin (n\varphi)]$ (**De Moivre's Theorem**).

The Exponential Function, e^A, or $\exp A$, of any point $A = x + iy$ is defined as the point whose distance is e^x and whose angle (measured in radians) is y. That is, $e^{x+iy} = e^x(\cos y + i \sin y)$. Here e^x means the ordinary exponential function of the real quantity x, where $e = 2.718$.

From this definition, the usual formal laws of exponents can be deduced: $e^Ae^B = e^{A+B}$, $(e^A)^n = e^{nA}$, $e^{-A} = 1/e^A$; $e^1 = e$, $e^0 = 1$.

The function e^A is a periodic function with a pure imaginary period $2\pi i$; that is, $e^{A \pm k2\pi i} = e^A$, where k is any positive integer.

If A is made to move along a line parallel to the axis of reals [or axis of pure imaginaries], the corresponding point e^A will move along a straight line through O [or along a circle about O as center].

Properties of $e^{i\varphi}$. The point $e^{i\varphi}$ is a point whose distance is 1 and whose angle is φ. It follows from the definitions above that **multiplying any point A by $e^{i\varphi}$ has the effect of rotating the point through an angle** φ, without changing its distance from O. In particular, $e^{i\pi} = -1$, $e^{-i\pi} = -1$; $e^{i\pi/2} = i$; $e^{-i\pi/2} = -i$; $e^{2\pi i} = 1$.

Third Standard Form for a Complex Quantity. Any point A can be expressed in the form $A = re^{i\varphi}$, where r is the distance and φ the angle of the point. For example, the three cube roots of 1 are 1, $e^{\frac{2}{3}\pi i}$, $e^{\frac{4}{3}\pi i}$. In general,

$$(r_1e^{i\varphi_1})(r_2e^{i\varphi_2}) = (r_1r_2)e^{i(\varphi_1 + \varphi_2)}; \quad (re^{i\varphi})^n = (r^n)e^{in\varphi}.$$

If $x + iy = re^{i\varphi}$, then $r = \sqrt{x^2 + y^2}$, $\sin \varphi = \dfrac{y}{r}$, $\cos \varphi = \dfrac{x}{r}$, $\tan \varphi = \dfrac{y}{x}$.

If two complex quantities are equal, their distances will be equal, and their angles will differ at most by some multiple of 2π. Thus, if $r_1e^{i\varphi_1} = r_2e^{i\varphi_2}$ then $r_1 = r_2$ and $\varphi_1 = \varphi_2$ or $\varphi_2 \pm k2\pi$. Here again a single equation between complex quantities is equivalent to two equations between real quantities.

Definition of A^B. Let $A = re^{i\varphi}$; then $A^B = \exp[(\log_e r + i\varphi)B]$.

For example, $i^i = e^{-\pi/2}$ where $i = \sqrt{-1}$.

If a is a positive real, $a^{x+iy} = a^x [\cos(y \log_e a) + i \sin(y \log_e a)]$.

Trigonometric and Hyperbolic Functions of a Complex Variable. If A is any point, then, by definition,

$$\sin A = \frac{e^{iA} - e^{-iA}}{2i}, \quad \cos A = \frac{e^{iA} + e^{-iA}}{2}, \quad \tan A = \frac{\sin A}{\cos A} \quad (\cos A \neq 0);$$

$$\sinh A = \frac{e^{A} - e^{-A}}{2}, \quad \cosh A = \frac{e^{A} + e^{-A}}{2}, \quad \tanh A = \frac{\sinh A}{\cosh A}.$$

Hence the formulæ that hold for these functions in the real case (p. 131; p. 135; p. 161) hold also for the complex case. Further:

$$\begin{aligned}
\sin(x+iy) &= \sin x \cosh y + i \cos x \sinh y, & \sin iy &= i \sinh y;\\
\cos(x+iy) &= \cos x \cosh y - i \sin x \sinh y, & \cos iy &= \cosh y;\\
\sinh(x+iy) &= \sinh x \cos y + i \cosh x \sin y, & \sinh iy &= i \sin y;\\
\cosh(x+iy) &= \cosh x \cos y + i \sinh x \sin y, & \cosh iy &= \cos y;
\end{aligned}$$

where $\sin x$, $\sinh x$, etc., are the ordinary trigonometric and hyperbolic functions of the real variables x and y. The functions $\sin A$ and $\cos A$ are periodic with a real period 2π. The functions $\sinh A$ and $\cosh A$ are periodic with a pure imaginary period $2\pi i$.

Logarithmic and Other Inverse Functions of a Complex Variable. If any point A is given, there will be an infinite number of points X such that $e^X = A$; any one of these points may be called a logarithm of A, and be denoted by $\log A$. All the values of the logarithm of A may be obtained from any one value by adding multiples of $2\pi i$.

If $x + iy = re^{i\varphi}$, then $\log_e(x + iy) = \log_e r + i\varphi \pm k \cdot 2\pi i$.

If any point A is given, there will be an infinite number of points X such that $\sin X = A$; any one of these may be denoted by $\sin^{-1} A$. The functions $\cos^{-1} A$, $\sinh^{-1} A$, etc., are defined in a similar way.

The elementary laws of operation which hold for these functions in the algebra of reals hold also, in a general way, in the algebra of complex quantities; but caution must be used, on account of the ambiguity in the symbols $\log A$, $\sin^{-1} A$, etc., which denote many-valued functions.

Differentiation of Functions of a Complex Variable. If $w = f(z)$, the derivative of w with respect to z is defined as

$$dw/dz = \lim \{[f(z + \Delta z) - f(z)]/\Delta z\} \text{ when } \Delta z \text{ approaches } 0.$$

It can be shown that $\lim \{[\exp \Delta z - 1]/\Delta z\} = 1$; hence $d(e^z) = e^z dz$, $d(\sin z) = \cos z\, dz$, etc., so that the formulæ for differentiation here are the same as in the case of a real variable (p. 157).

NOTE. For the algebra of vector analysis, which differs in important respects from the algebra of complex quantities, see p. 185.

TRIGONOMETRY

FORMAL TRIGONOMETRY

Angles, or Rotations. An **angle** is generated by the rotation of a ray, as Ox, about a fixed point O in the plane. Every angle has an **initial line** (OA) from which the rotation started (Fig. 1), and a **terminal line** (OB) where it stopped; and the counterclockwise direction of rotation is taken as positive. Since the rotating ray may revolve as often as desired, angles of any magnitude, positive or negative, may be obtained. Two angles are **congruent** if they may be superposed so that their initial lines coincide and their terminal lines coincide. That is, two congruent angles are either equal or differ by some multiple of 360 deg. Two angles are **complementary** if their sum is 90 deg.; **supplementary** if their sum is 180 deg. (The acute angles of a right-angled triangle are complementary.) If the initial line is placed so that it runs horizontally to the right, as in Fig. 2, then the angle is said to be an angle in the 1st, 2nd, 3rd, or 4th **quadrant** according as the terminal line lies across the region marked I, II, III, or IV. The angles 0 deg., 90 deg., 180 deg., 270 deg. are called the quadrantal angles.

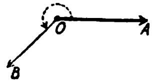


FIG. 1.

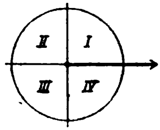


FIG. 2.

Units of Angular Measurement.

(1) SEXAGESIMAL MEASURE. (360 degrees = 1 revolution.) 1 degree = 1° = $1/90$ of a right angle. The degree is usually divided into 60 equal parts called minutes ($'$), and each minute into 60 equal parts called seconds ($''$); while the second is subdivided decimally. But for many purposes it is more convenient to divide the degree itself into decimal parts, thus avoiding the use of minutes and seconds. (See tables, pp. 46–51.)

(2) CENTESIMAL MEASURE, used chiefly in France. (400 grades = 1 revolution.) 1 grade = $1/100$ of a right angle. The grade is always divided decimally, the following terms being sometimes used: 1 "centesimal minute" = $1/100$ of a grade; 1 "centesimal second" = $1/100$ of a centesimal minute. In reading Continental books it is important to notice carefully which system is employed.

(3) RADIAN, OR CIRCULAR, MEASURE. (π radians = 180 degrees.) 1 radian = the angle subtended by an arc whose length is equal to the length of the radius. The radian is constantly used in higher mathematics and in mechanics, and is always divided decimally. Table, pp. 44–45.

1 radian $= 57^\circ.30- = 57^\circ.2957795131 = 57^\circ\ 17'\ 44''.806247 = 180^\circ/\pi$.

$1^\circ = 0.01745\ldots$ radian $= 0.01745\,32925$ radian.

$1' = 0.00029\,08882$ radian. $1'' = 0.00000\,48481$ radian.

(For 10-place conversion tables, see the Smithsonian Tables of Hyperbolic Functions, Washington, D. C.)

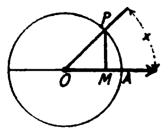


FIG. 3.

Definitions of the Trigonometric Functions. Let x be any angle whose initial line is OA and terminal line OP (see Fig. 3). Drop a perpendicular from P on OA or OA produced. In the right triangle OMP, the three sides are MP = "side opposite" O (positive if running upward); OM = "side adjacent" to O (positive if running to the right); OP = "hypothenuse" or "radius" (may always be taken as positive); and the six ratios between these sides are the principal trigonometric functions of the angle x; thus:

$$\begin{aligned}
\text{sine of } x &= \sin x = \text{opp/hyp} = MP/OP;\\
\text{cosine of } x &= \cos x = \text{adj/hyp} = OM/OP;\\
\text{tangent of } x &= \tan x = \text{opp/adj} = MP/OM;\\
\text{cotangent of } x &= \cot x = \text{adj/opp} = OM/MP;\\
\text{secant of } x &= \sec x = \text{hyp/adj} = OP/OM;\\
\text{cosecant of } x &= \csc x = \text{hyp/opp} = OP/MP.
\end{aligned}$$

The last three are best remembered as the reciprocals of the first three:

$$\cot x = 1/\tan x;\ \sec x = 1/\cos x;\ \csc x = 1/\sin x.$$

Other functions in use are the versed sine, the coversed sine, and the exterior secant:

$$\text{vers } x = 1 - \cos x;\ \text{covers } x = 1 - \sin x;\ \text{exsec } x = \sec x - 1.$$

For graphs, see p. 174; series, p. 161.

Signs of the Trigonometric Functions

If x is in quadrant	I	II	III	IV
$\sin x$ and $\csc x$ are	+	+	−	−
$\cos x$ and $\sec x$ are	+	−	−	+
$\tan x$ and $\cot x$ are	+	−	+	−

vers x and covers x are always positive.

Variations in the Functions as x Varies from 0 deg. to 360 deg. are shown in the accompanying table. The variations in the sine and cosine are

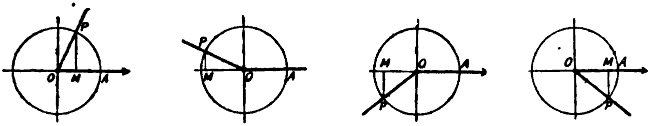


FIG. 4.

best remembered by noting the changes in the lines MP and OM (Fig. 4) in the "unit circle" (that is, a circle with radius $= OP = 1$), as P moves around the circumference.

x	0° to 90°	90° to 180°	180° to 270°	270° to 360°	Values at 30°	45°	60°
sin x	$+0$ to $+1$	$+1$ to $+0$	-0 to -1	-1 to -0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$
csc x	$+\infty$ to $+1$	$+1$ to $+\infty$	$-\infty$ to -1	-1 to $-\infty$	2	$\sqrt{2}$	$\frac{2}{3}\sqrt{3}$
cos x	$+1$ to $+0$	-0 to -1	-1 to -0	$+0$ to $+1$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$
sec x	$+1$ to $+\infty$	$-\infty$ to -1	-1 to $-\infty$	$+\infty$ to $+1$	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2
tan x	$+0$ to $+\infty$	$-\infty$ to -0	$+0$ to $+\infty$	$-\infty$ to -0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
cot x	$+\infty$ to $+0$	-0 to $-\infty$	$+\infty$ to $+0$	-0 to $-\infty$	$\sqrt{3}$	1	$\frac{1}{3}\sqrt{3}$
vers x	$+0$ to $+1$	$+1$ to $+2$	$+2$ to $+1$	$+1$ to $+0$			
covers x	$+1$ to $+0$	$+0$ to $+1$	$+1$ to $+2$	$+2$ to $+1$			

$\sqrt{2} = 1.4142$; $\frac{1}{2}\sqrt{2} = 0.7071$; $\sqrt{3} = 1.7321$; $\frac{1}{2}\sqrt{3} = 0.8660$; $\frac{1}{3}\sqrt{3} = 0.5774$; $\frac{2}{3}\sqrt{3} = 1.1547$

Trigonometrical Tables. The tables on pp. 46–56 give the values of the principal trigonometric functions and of their logarithms, correct to four places of decimals, the angle advancing either by tenths of a degree (p. 46) or by 10 min. (p. 52). These tables will be found adequate for most computations in which an accuracy of 1 part in 1000 is sufficient. If much computing is to be done, it is advisable to use a separate volume of tables, containing more facilities for interpolation, and printed in larger type, such as the four-place tables of E. V. Huntington (Harvard Coöperative Society, Cambridge, Mass.), with convenient marginal tabs; the five-place tables published by Macmillan or many others; the six-place tables of Bremiker; the standard seven-place tables of Schrön, Vega, or Bruhns (angles advancing by 10 sec.); or the great eight-place of Bauschinger and Peters (angles advancing at intervals of 1 sec. from 0 deg. to 90 deg.). The larger tables give only the logarithms of the functions, not the natural values.

To Find Any Function of a Given Angle. (Reduction to the first quadrant.) It is often required to find the functions of any angle x from a table that includes only angles between 0 deg. and 90 deg. If x is not already between 0 deg. and 360 deg., first "reduce to the first revolution" by simply adding or subtracting the proper multiple of 360 deg.; [for any function of (x) = the same function of $(x \pm n \times 360°)$]. Next **reduce to the first quadrant** as follows:

If x is between	90° and 180°	180° and 270°	270° and 360°
Subtract	90° from x	180° from x	270° from x
Then sin x	$=+\cos(x-90°)$	$=-\sin(x-180°)$	$=-\cos(x-270°)$
csc x	$=+\sec(x-90°)$	$=-\csc(x-180°)$	$=-\sec(x-270°)$
cos x	$=-\sin(x-90°)$	$=-\cos(x-180°)$	$=+\sin(x-270°)$
sec x	$=-\csc(x-90°)$	$=-\sec(x-180°)$	$=+\csc(x-270°)$
tan x	$=-\cot(x-90°)$	$=+\tan(x-180°)$	$=-\cot(x-270°)$
cot x	$=-\tan(x-90°)$	$=+\cot(x-180°)$	$=-\tan(x-270°)$
vers x	$=1+\sin(x-90°)$	$=1+\cos(x-180°)$	$=1-\sin(x-270°)$
covers x	$=1-\cos(x-90°)$	$=1+\sin(x-180°)$	$=1+\cos(x-270°)$

The "reduced angle" ($x - 90°$, or $x - 180°$, or $x - 270°$) will in each case be an angle between 0° and 90°, whose functions can then be found in the table.

[NOTE. The formulæ for sine and cosine are best remembered by aid of the unit circle.]

To Find the Angle When One of Its Functions is Given. In general, there will be two angles between 0 deg. and 360 deg. corresponding to any given function. The following tabulated rules show how to find these angles.

Given	First find from the tables an *acute* angle x_0 such that	Then the required angles x_1 and x_2 will be
$\sin x = +a$	$\sin x_0 = a$	x_0 and $180° - x_0$
$\cos x = +a$	$\cos x_0 = a$	x_0 and $[360° - x_0]$
$\tan x = +a$	$\tan x_0 = a$	x_0 and $[180° + x_0]$
$\cot x = +a$	$\cot x_0 = a$	x_0 and $[180° + x_0]$
$\sin x = -a$	$\sin x_0 = a$	$[180° + x_0]$ and $[360° - x_0]$
$\cos x = -a$	$\cos x_0 = a$	$180° - x_0$ and $[180° + x_0]$
$\tan x = -a$	$\tan x_0 = a$	$180° - x_0$ and $[360° - x_0]$
$\cot x = -a$	$\cot x_0 = a$	$180° - x_0$ and $[360° - x_0]$

The angles enclosed in brackets lie outside the range from 0 deg. to 180 deg., and hence cannot occur as angles in a triangle.

For solution of trigonometric equations, see p. 118.

Relations Between the Functions of a Single Angle. (See Fig. 5.)

$\sin^2 x + \cos^2 x = 1$; $\tan x = \dfrac{\sin x}{\cos x}$; $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$;

$1 + \tan^2 x = \sec^2 x = \dfrac{1}{\cos^2 x}$; $1 + \cot^2 x = \csc^2 x = \dfrac{1}{\sin^2 x}$;

$$\sin x = \sqrt{1 - \cos^2 x} = \frac{\tan x}{\sqrt{1 + \tan^2 x}} = \frac{1}{\sqrt{1 + \cot^2 x}};$$

$$\cos x = \sqrt{1 - \sin^2 x} = \frac{1}{\sqrt{1 + \tan^2 x}} = \frac{\cot x}{\sqrt{1 + \cot^2 x}}.$$

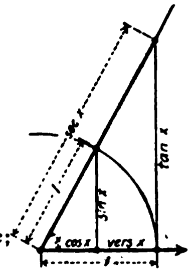


FIG. 5.

Functions of Negative Angles. $\sin(-x) = -\sin x$; $\cos(-x) = \cos x$; $\tan(-x) = -\tan x$.

Functions of the Sum and Difference of Two Angles.

$\sin(x + y) = \sin x \cos y + \cos x \sin y$;
$\cos(x + y) = \cos x \cos y - \sin x \sin y$;
$\tan(x + y) = [\tan x + \tan y]/[1 - \tan x \tan y]$;
$\cot(x + y) = [\cot x \cot y - 1]/[\cot x + \cot y]$;

$\sin(x - y) = \sin x \cos y - \cos x \sin y$;
$\cos(x - y) = \cos x \cos y + \sin x \sin y$;
$\tan(x - y) = [\tan x - \tan y]/[1 + \tan x \tan y]$;
$\cot(x - y) = [\cot x \cot y + 1]/[\cot y - \cot x]$;

$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$;
$\sin x - \sin y = 2 \cos \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$;
$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x + y) \cos \tfrac{1}{2}(x - y)$;
$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x + y) \sin \tfrac{1}{2}(x - y)$;

$\tan x + \tan y = \dfrac{\sin(x + y)}{\cos x \cos y}$; $\cot x + \cot y = \dfrac{\sin(x + y)}{\sin x \sin y}$;

$\tan x - \tan y = \dfrac{\sin(x - y)}{\cos x \cos y}$; $\cot x - \cot y = \dfrac{\sin(y - x)}{\sin x \sin y}$;

$\sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x = \sin(x + y) \sin(x - y)$;
$\cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x = \cos(x + y) \cos(x - y)$;
$\sin(45° + x) = \cos(45° - x)$; $\tan(45° + x) = \cot(45° - x)$;
$\sin(45° - x) = \cos(45° + x)$; $\tan(45° - x) = \cot(45° + x)$.

In the following transformations, a and b are supposed to be positive, $c = \sqrt{a^2 + b^2}$, A = the positive acute angle for which $\tan A = a/b$, and B = the positive acute angle for which $\tan B = b/a$:

$a \cos x + b \sin x = c \sin(A + x) = c \cos(B - x)$;
$a \cos x - b \sin x = c \sin(A - x) = c \cos(B + x)$.

Functions of Multiple Angles and Half Angles.

$\sin 2x = 2 \sin x \cos x$; $\sin x = 2 \sin \tfrac{1}{2}x \cos \tfrac{1}{2}x$;
$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$;

$\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$; $\cot 2x = \dfrac{\cot^2 x - 1}{2 \cot x}$;

$\sin 3x = 3 \sin x - 4 \sin^3 x$; $\tan 3x = \dfrac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$;

$\cos 3x = 4 \cos^3 x - 3 \cos x$;

$$\sin (nx) = n \sin x \cos^{n-1} x - (n)_3 \sin^3 x \cos^{n-3} x + (n)_5 \sin^5 x \cos^{n-5} x - \ldots;$$
$$\cos (nx) = \cos^n x - (n)_2 \sin^2 x \cos^{n-2} x + (n)_4 \sin^4 x \cos^{n-4} x - \ldots,$$
where $(n)_2$, $(n)_3$, . . . are the binomial coefficients (see p. 39).

$$\sin \tfrac{1}{2} x = \pm \sqrt{\tfrac{1}{2}(1 - \cos x)};\; 1 - \cos x = 2 \sin^2 \tfrac{1}{2}x;$$
$$\cos \tfrac{1}{2}x = \pm \sqrt{\tfrac{1}{2}(1 + \cos x)};\; 1 + \cos x = 2 \cos^2 \tfrac{1}{2}x;$$
$$\tan \tfrac{1}{2}x = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x};$$
$$\tan \left(\frac{x}{2} + 45^\circ\right) = \pm \sqrt{\frac{1 + \sin x}{1 - \sin x}}.$$

Here the + or − sign is to be used according to the sign of the left-hand side of the equation.

Relations Between Three Angles Whose Sum is 180°.

$$\sin A + \sin B + \sin C = 4 \cos \tfrac{1}{2}A \cos \tfrac{1}{2}B \cos \tfrac{1}{2}C;$$
$$\cos A + \cos B + \cos C = 4 \sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \sin \tfrac{1}{2}C + 1;$$
$$\sin A + \sin B - \sin C = 4 \sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \cos \tfrac{1}{2}C;$$
$$\cos A + \cos B - \cos C = 4 \cos \tfrac{1}{2}A \cos \tfrac{1}{2}B \sin \tfrac{1}{2}C - 1;$$
$$\sin^2 A + \sin^2 B + \sin^2 C = 2 \cos A \cos B \cos C + 2;$$
$$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C;$$
$$\tan A + \tan B + \tan C = \tan A \tan B \tan C;$$
$$\cot \tfrac{1}{2}A + \cot \tfrac{1}{2}B + \cot \tfrac{1}{2}C = \cot \tfrac{1}{2}A \cot \tfrac{1}{2}B \cot \tfrac{1}{2}C;$$
$$\cot A \cot B + \cot A \cot C + \cot B \cot C = 1;$$
$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C;$$
$$\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C.$$

Inverse Trigonometric Functions. The notation $\sin^{-1}x$ (read: anti-sine of x, or inverse sine of x; sometimes written arc sin x) means the principal angle whose sine is x. Similarly for $\cos^{-1}x$, $\tan^{-1}x$, etc. (The principal angle means an angle between −90° and +90° in case of $\sin^{-1}$ and $\tan^{-1}$, and between 0° and 180° in the case of $\cos^{-1}$.) For graphs, see p. 174.

SOLUTION OF PLANE TRIANGLES

The "parts" of a plane triangle are its three sides, a, b, c, and its three angles A, B, C (A being opposite a, B opposite b, C opposite c, and $A + B + C = 180°$). A triangle is, in general, determined by any three parts (not all angles). To "solve" a triangle means to find the unknown parts from the known. The fundamental formulæ are:

Law of sines: $\dfrac{a}{b} = \dfrac{\sin A}{\sin B}$. Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.

Right Triangles. Use the definitions of the trigonometric functions, selecting for each unknown part a relation which connects that unknown with known quantities; then solve the resulting equations. Thus, in Fig. 6, if $C = 90°$, then $A + B = 90°$, $c^2 = a^2 + b^2$,

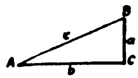


Fig. 6.

$$\sin A = a/c,\ \cos A = b/c,\ \tan A = a/b,\ \cot A = b/a.$$

If A is very small, use $\tan \tfrac{1}{2}A = \sqrt{(c - b)/(c + b)}$.

Oblique Triangles. There are four cases. It is highly desirable in all these cases to draw a sketch of the triangle approximately to scale before commencing the computation, so that any large numerical error may be readily detected.

Case 1. Given Two Angles (provided their sum is < 180 deg.), and One

SIDE (say a, Fig. 7). The third angle is known, since $A + B + C = 180°$. To find the remaining sides, use $b = \dfrac{a \sin B}{\sin A}$, $c = \dfrac{a \sin C}{\sin A}$. Or, drop a perpendicular from either B or C on the opposite side, and solve by right triangles.

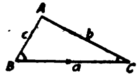


FIG. 7.

Check: $c \cos B + b \cos C = a$.

Case 2. GIVEN TWO SIDES (say a and b), AND THE INCLUDED ANGLE (C); and suppose $a > b$. Fig. 8.

First Method: Find c from $c^2 = a^2 + b^2 - 2ab \cos C$ [or $c^2 = (a - b)^2 + 2ab \text{ vers } C$]; then find the smaller angle, B, from $\sin B = (b/c) \sin C$; and finally, find A from $A = 180° - (B + C)$. Check: $a \cos B + b \cos A = c$.

Second Method: Find $\tfrac{1}{2}(A - B)$ from the law of tangents:

$$\tan \tfrac{1}{2}(A - B) = [(a - b)/(a + b)] \cot \tfrac{1}{2}C,$$

and $\tfrac{1}{2}(A + B)$ from $\tfrac{1}{2}(A + B) = 90° - C/2$; hence $A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B)$ and $B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B)$. Then find c from $c = a \sin C/\sin A$ or $c = b \sin C/\sin B$. Check: $a \cos B + b \cos A = c$.

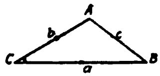


FIG. 8.

Third Method: Drop a perpendicular from A to the opposite side, and solve by right triangles.

Case 3. GIVEN THE THREE SIDES (provided the largest is less than the sum of the other two), Fig. 9.

First Method: Find the largest angle A (which may be acute or obtuse) from $\cos A = (b^2 + c^2 - a^2)/2bc$ {or $\text{vers } A = [a^2 - (b - c)^2]/2bc$} and then find B and C (which will always be acute) from $\sin B = b \sin A/a$ and $\sin C = c \sin A/a$. Check: $A + B + C = 180°$.

Second Method: Find A, B, and C from $\tan \tfrac{1}{2}A = r/(s - a)$, $\tan \tfrac{1}{2}B = r/(s - b)$, $\tan \tfrac{1}{2}C = r/(s - c)$, where $s = \tfrac{1}{2}(a + b + c)$, and $r = \sqrt{(s - a)(s - b)(s - c)/s}$. Check: $A + B + C = 180°$.

Third Method: If only one angle, say A, is required, use

$$\sin \tfrac{1}{2}A = \sqrt{(s - b)(s - c)/bc} \quad \text{or}$$

$$\cos \tfrac{1}{2}A = \sqrt{s(s - a)/bc},$$

according as $\tfrac{1}{2}A$ is nearer 0° or nearer 90°.

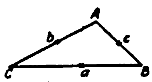


FIG. 9.

Case 4. GIVEN TWO SIDES (say b and c) AND THE ANGLE OPPOSITE ONE OF THEM (B). This is the "ambiguous case" in which there may be two solutions, or one, or none (see Fig. 10).

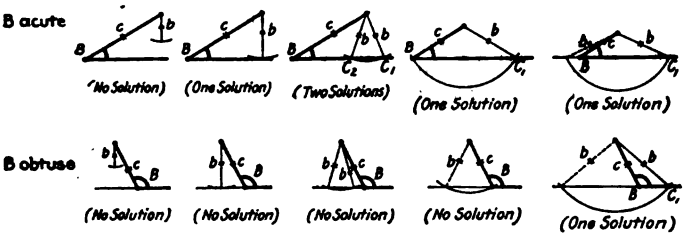


FIG. 10.

First, try to find C from $\sin C = c \sin B/b$. If $\sin C > 1$, there is no solution. If $\sin C = 1$, $C = 90°$ and the triangle is a right triangle. If $\sin C < 1$, this determines two angles C, namely, an acute angle C_1, and an obtuse angle $C_2 = 180° - C_1$. Then C_1 will yield a solution when and only when

$C_1 + B < 180°$ (see Case 1); and similarly C_2 will yield a solution when and only when $C_2 + B < 180°$ (see Case 1).

Other Properties of Triangles. (See also p. 99 and p. 105.)

Area $= \frac{1}{2}ab \sin C = \sqrt{s(s-a)(s-b)(s-c)} = rs$, where $s = \frac{1}{2}(a+b+c)$, and $r =$ radius of inscribed circle $= \sqrt{(s-a)(s-b)(s-c)/s}$.

Radius of circumscribed circle $= R$, where

$$2R = a/\sin A = b/\sin B = c/\sin C;\ r = 4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{abc}{4Rs}.$$

The length of the bisector of the angle C is

$$z = \frac{2\sqrt{abs(s-c)}}{a+b} = \frac{\sqrt{ab[(a+b)^2 - c^2]}}{a+b}.$$

The median from C to the middle point of c is $m = \frac{1}{2}\sqrt{2(a^2+b^2) - c^2}$.

SOLUTION OF SPHERICAL TRIANGLES

For the occasional solution of a spherical triangle the following formulæ will be sufficient. For a detailed discussion, see any text-book on spherical trigonometry.

Let a, b, c be the sides of the spherical triangle, that is, portions of arcs of great circles of the sphere; and let A, B, C be the angles of the triangle, that is, the angles made by tangents drawn to the sides at their points of intersection on the sphere. The sum of the angles will always be greater than two right angles, and may be nearly six right angles. The angle $E = A + B + C - 180°$ is called the **spherical excess** of the triangle. (See also p. 100.)

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B};\quad \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C};\quad \frac{\sin c}{\sin C} = \frac{\sin a}{\sin A}.$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

with similar formulæ for $\cos b$ and $\cos c$.

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a,$$

with similar formulæ for $\cos B$ and $\cos C$.

In the special case of a right spherical triangle, in which $C = 90°$,

$$\cos c = \cos a \cos b = \cot A \cot B;\ \cos a = \frac{\cos A}{\sin B};\ \cos b = \frac{\cos B}{\sin A};$$

$$\sin A = \frac{\sin a}{\sin c};\ \cos A = \frac{\tan b}{\tan c};\ \tan A = \frac{\tan a}{\sin b}.$$

$$\frac{\text{The area of a spherical triangle}}{\text{area of a great circle}} = \frac{\text{spherical excess}}{180°}.$$

HYPERBOLIC FUNCTIONS

The **hyperbolic sine, hyperbolic cosine,** etc., of any number x, are functions of x which are closely related to the exponential e^x, and which have formal properties very similar to those of the trigonometric functions, sine, cosine, etc. Their definitions and fundamental properties are as follows (see also p. 127; graphs, p. 175; table, p. 60; series, p. 161):

$\sinh x = \frac{1}{2}(e^x - e^{-x})$; $\cosh x = \frac{1}{2}(e^x + e^{-x})$; $\tanh x = \sinh x/\cosh x$;

$\operatorname{csch} x = 1/\sinh x$; $\operatorname{sech} x = 1/\cosh x$; $\coth x = 1/\tanh x$;

$\cosh^2 x - \sinh^2 x = 1$; $1 - \tanh^2 x = \operatorname{sech}^2 x$; $1 - \coth^2 x = -\operatorname{csch}^2 x$;
$\sinh(-x) = -\sinh x$; $\cosh(-x) = \cosh x$; $\tanh(-x) = -\tanh x$;

$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$;
$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$;
$\tanh(x \pm y) = (\tanh x \pm \tanh y)/(1 \pm \tanh x \tanh y)$;

$\sinh 2x = 2 \sinh x \cosh x$; $\cosh 2x = \cosh^2 x + \sinh^2 x$;
$\tanh 2x = (2 \tanh x)/(1 + \tanh^2 x)$;
$\sinh \frac{1}{2}x = \sqrt{\frac{1}{2}(\cosh x - 1)}$; $\cosh \frac{1}{2}x = \sqrt{\frac{1}{2}(\cosh x + 1)}$;
$\tanh \frac{1}{2}x = (\cosh x - 1)/(\sinh x) = (\sinh x)/(\cosh x + 1)$.

The **inverse hyperbolic sine** of y, denoted by $\sinh^{-1} y$, is the number whose hyperbolic sine is y; that is, the notation $x = \sinh^{-1} y$ means $\sinh x = y$. Similarly for $\cosh^{-1} y$, $\tanh^{-1} y$, etc. These functions are closely related to the logarithmic function, and are especially valuable in the integral calculus. For graphs, see p. 175.

$\sinh^{-1}(y/a) = \log_e(y + \sqrt{y^2 + a^2}) - \log_e a$;
$\cosh^{-1}(y/a) = \log_e(y + \sqrt{y^2 - a^2}) - \log_e a$;

$$\tanh^{-1}\frac{y}{a} = \frac{1}{2}\log_e \frac{a + y}{a - y};\quad \coth^{-1}\frac{y}{a} = \frac{1}{2}\log_e \frac{y + a}{y - a}.$$

The **anti-gudermannian** of an angle u, denoted by $\operatorname{gd}^{-1} u$, is a number defined by $\operatorname{gd}^{-1} u = \log_e \tan(\frac{1}{4}\pi + \frac{1}{2}u) = \int \sec u\, du$. When u is small, $\operatorname{gd}^{-1} u = u + \frac{1}{6}u^3 + \frac{1}{24}u^5 + \frac{61}{5040}u^7 + \ldots$.

ANALYTICAL GEOMETRY

THE POINT AND THE STRAIGHT LINE

Rectangular Co-ordinates (Fig. 1). Let $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$. Then, distance $P_1P_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$; slope of $P_1P_2 = m = \tan u = (y_2 - y_1)/(x_2 - x_1)$; co-ordinates of mid-point are $x = \frac{1}{2}(x_1 + x_2)$, $y = \frac{1}{2}(y_1 + y_2)$; co-ordinates of point $(1/n)$th of the way from P_1 to P_2 are $x = x_1 + (1/n)(x_2 - x_1)$, $y = y_1 + (1/n)(y_2 - y_1)$.

Let m_1, m_2 be the slopes of two lines; then, if the lines are parallel, $m_1 = m_2$; if the lines are perpendicular to each other, $m_1 = -1/m_2$.

Equations of a Straight Line.

1. Intercept Form (Fig. 2): $\dfrac{x}{a} + \dfrac{y}{b} = 1$. ($a$, b = intercepts of the line on the axes.)

2. Slope Form (Fig. 3): $y = mx + b$. ($m = \tan u$ = slope; b = intercept on the y-axis; see also Fig. 2, p. 174.)

3. Normal Form (Fig. 4): $x \cos v + y \sin v = p$. (p = perpendicular from origin to line; v = angle p makes with the x-axis.)

4. Parallel-intercept Form (Fig. 5): $\dfrac{y - b}{c - b} = \dfrac{x}{k}$. ($b$, c = intercepts on two parallels at distance k apart.)

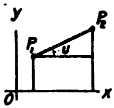


FIG. 1.

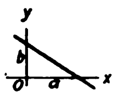


FIG. 2.

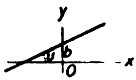


FIG. 3.

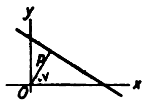


FIG. 4.

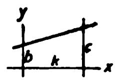


FIG. 5.

5. General Form: $Ax + By + C = 0$. [Here $a = -C/A$, $b = -C/B$, $m = -A/B$, $\cos v = A/R$, $\sin v = B/R$, $p = -C/R$, where $R = \pm\sqrt{A^2 + B^2}$ (sign to be so chosen that p is positive).]

6. Line Through (x_1, y_1) with Slope m: $y - y_1 = m(x - x_1)$.

7. Line Through (x_1, y_1) and (x_2, y_2): $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

8. Line Parallel to x-axis: $x = a$; to y-axis: $y = b$.

Angles and Distances.

If u = angle between two lines whose slopes are m_1, m_2, then

$$\tan u = \frac{m_2 - m_1}{1 + m_2 m_1}.$$

If parallel, $m_1 = m_2$.
If perpendicular, $m_1 m_2 = -1$.

If u = angle between the lines $Ax + By + C = 0$ and $A'x + B'y + C' = 0$, then

$$\cos u = \frac{AA' + BB'}{\pm\sqrt{(A^2 + B^2)(A'^2 + B'^2)}}.$$

If parallel, $A/A' = B/B'$.
If perpendicular, $AA' + BB' = 0$.

The equations of the bisectors of the angles between the two lines just mentioned are

$$\frac{Ax + By + C}{\sqrt{A^2 + B^2}} \mp \frac{A'x + B'y + C'}{\sqrt{A'^2 + B'^2}} = 0.$$

The equation of a line through (x_1, y_1) and meeting a given line $y = mx + b$ at an angle u, is

$$y - y_1 = \frac{m + \tan u}{1 - m \tan u}(x - x_1).$$

The distance from (x_0, y_0) to the line $Ax + By + C = 0$ is

$$D = \left|\frac{Ax_0 + By_0 + C}{\sqrt{A^2 + B^2}}\right|$$

where the vertical bars mean "the absolute value of."

The distance from (x_0, y_0) to a line which passes through (x_1, y_1) and makes an angle u with the x-axis, is

$$D = (x_0 - x_1)\sin u - (y_0 - y_1)\cos u.$$

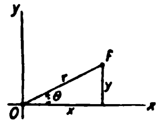


FIG. 6.

Polar Co-ordinates (Fig. 6). Let (x, y) be the rectangular and (r, θ) the polar co-ordinates of a given point P. Then $x = r\cos\theta$; $y = r\sin\theta$; $x^2 + y^2 = r^2$.

Transformation of Co-ordinates. If origin is moved to point (x_0, y_0), the new axes being parallel to the old, $x = x_0 + x'$, $y = y_0 + y'$.

If axes are turned through the angle u, without change of origin,

$$x = x'\cos u - y'\sin u, \quad y = x'\sin u + y'\cos u.$$

THE CIRCLE

(See also pp. 99, 103–105, 106)

Equation of Circle with center (a,b) and radius r:

$$(x - a)^2 + (y - b)^2 = r^2.$$

If center is at the origin, the equation becomes $x^2 + y^2 = r^2$. If circle goes through the origin and center is on the x-axis at point $(r, 0)$, equation becomes $x^2 + y^2 = 2rx$. The **general equation** of a circle is

$x^2 + y^2 + Dx + Ey + F = 0$; it has center at $(-D/2, -E/2)$, and radius $= \sqrt{(D/2)^2 + (E/2)^2 - F}$ (which may be real, null, or imaginary).

The **equation of the radical axis** of two circles, $x^2 + y^2 + Dx + Ey + F = 0$ and $x^2 + y^2 + D'x + E'y + F' = 0$, is $(D - D')x + (E - E')y + (F - F') = 0$. The tangents drawn to two circles from any point of their radical axis are of equal length. If the circles intersect, the radical axis passes through their points of intersection (see p. 100).

The **equation of the tangent** to $x^2 + y^2 = r^2$ at (x_1, y_1) is $x_1x + y_1y = r^2$. The tangent to $x^2 + y^2 + Dx + Ey + F = 0$ at (x_1, y_1) is $x_1x + y_1y + \frac{1}{2}D(x + x_1) + \frac{1}{2}E(y + y_1) + F = 0$. The line $y = mx + b$ will be tangent to the circle $x^2 + y^2 = r^2$ if $b = a\sqrt{1 + m^2}$.

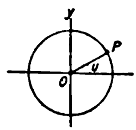


FIG. 7.

Equations of Circle in Parametric Form. It is sometimes convenient to express the co-ordinates x and y of the moving point P (Fig. 7) in terms of an auxiliary variable, called a **parameter**. Thus, if the parameter be taken as the angle u which the radius OP makes with the x-axis, then the equations of the circle in parametric form will be $x = a\cos u$; $y = a\sin u$. For every value of the parameter u, there corresponds a point (x, y) on the circle. The ordinary equation $x^2 + y^2 = a^2$ can be obtained from the parametric equations by eliminating u.

THE PARABOLA

The **parabola** (see also p. 107) is the locus of a point which moves so that its distance from a fixed line (called the **directrix**) is always equal to its distance from a fixed point F (called the **focus**). See Fig. 8. The point half-way from focus to directrix is the **vertex**, O, The line through the focus, perpendicular to the directrix, is the **principal axis**. The breadth of the curve at the focus is called the **latus rectum**, or **parameter**, $= 2p$, where p is the distance from focus to directrix. (Compare also Fig. 3, p. 174.)

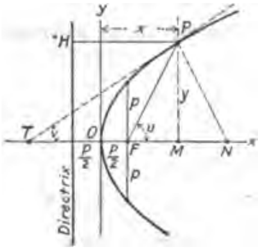


Fig. 8.

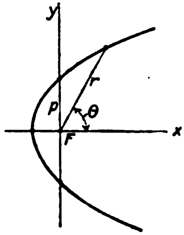


Fig. 9.

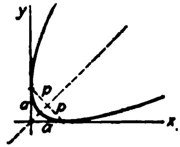


Fig. 10.

Any section of a right circular cone made by a plane parallel to a tangent plane of the cone will be a parabola.

Equation of Parabola, origin at vertex (Fig. 8): $y^2 = 2px$.

Polar Equation of Parabola, referred to F as origin and Fx as axis (Fig. 9): $r = p/(1 - \cos\theta)$.

Equation Referred to the Tangents at the ends of the latus rectum as axes (Fig. 10): $x^{1/2} + y^{1/2} = a^{1/2}$, where $a = p\sqrt{2}$.

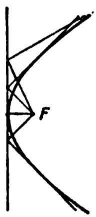


Fig. 11.

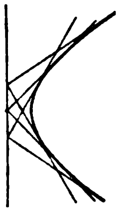


Fig. 12.

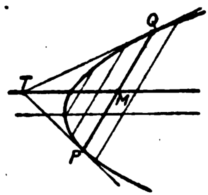


Fig. 13.

Equation of Tangent to $y^2 = 2px$ at (x_1, y_1): $y_1 y = p(x + x_1)$. The line $y = mx + b$ will be tangent to $y^2 = 2px$ if $b = p/(2m)$.

The **tangent** PT at any point P bisects the angle between PF and PH (Fig. 8). A ray of light from F, reflected at P, will move off parallel to the principal axis. The **subtangent**, TM, is bisected at O. The **subnormal**, MN, is constant, and equal to p. The locus of the foot of the perpendicular from the focus on a moving tangent is the tangent at the vertex (Fig. 11). The locus of the point of intersection of perpendicular tangents is the directrix (Fig. 12). The locus of the mid-points of a set of parallel chords whose slope is m is a straight line parallel to the principal axis at a distance p/m

and is called a **diameter** (Fig. 13). If M is the mid-point of a chord PQ, and if T is the point of intersection of the tangents at P and Q, then TM is parallel to the principal axis, and is bisected by the curve (Fig. 13).

To Construct a Tangent to a Given Parabola. (1) At a given point of contact, P (Fig. 14): Find T so that $OT = OM$, or $FT = FP$. Then TP is the tangent at P. Or, make $MN = p = 2(OF)$; then PN is the normal at P.

(2) From a given external point, Q (Fig. 15): With Q as center and radius QF draw circle cutting the directrix in H; draw HP parallel to principal axis; then P is required point of contact. As check, note that QP is the perpendicular bisector of FH.

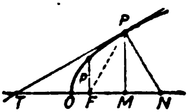


FIG. 14.

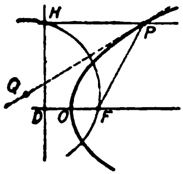


FIG. 15.

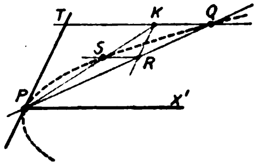


FIG. 16.

To Construct a Parabola. 1. GIVEN ANY TWO POINTS, P AND Q, THE TANGENT PT AT ONE OF THEM, AND THE DIRECTION OF THE PRINCIPAL AXIS OX. In Fig. 16, let K be a variable point on a line through Q parallel to OX. Draw KR parallel to PT (meeting PQ in R), and draw RS parallel to OX (meeting PK in S); then S is a point of the curve. NOTE. A line through P parallel to the principal axis bisects all chords parallel to the tangent PT.

2. GIVEN THE VERTEX O AND FOCUS F. (a) In Fig. 17 draw Oy perpendicular to OF, and slide the vertex of a right angle along Oy so that one side always passes through F; then the other side will always be a tangent to the parabola.

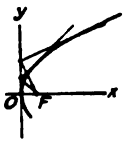


FIG. 17.

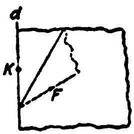


FIG. 18.

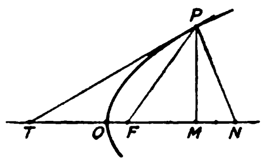


FIG. 19.

(b) Take a piece of paper (Fig. 18) with a straight edge, d, and mark a point F near the edge. Let K be a variable point of the edge, and fold the paper so that K coincides with F. The crease will be a tangent to the parabola which has focus F and directrix d.

(c) In Fig. 19, let M be a variable point of the principal axis, and lay off $MN = 2(OF) = p$. With F as center and radius FN draw a circle, cutting the perpendicular at M in P. Then P is a point of the curve, and PT and PN are the tangent and normal at P.

3. GIVEN TWO TANGENTS AND THEIR POINTS OF CONTACT, P AND Q (Fig. 20). Divide TP and QT into any number of equal parts (here 4). Then the lines 11, 22, 33, . . . will be tangents to the parabola. This method is especially advantageous in drawing rather flat arcs.

The Radius of Curvature of $y^2 = 2px$ at a point $P = (x,y)$ is $R = (p + 2x)^{3/2}/\sqrt{p}$, or $R = p/\sin^3 v$, where v = the angle which the tangent at P makes with PF (Fig. 21). At the vertex, $R = p$. To construct the radius of curvature at any point P, lay off $PR = 2(PF)$ parallel to the principal axis, and draw RC perpendicular to the axis, meeting the normal, PN, in C. Then C is the center of curvature for the point P, and a circle about C with radius CP will coincide closely with the parabola in the neighborhood of P.

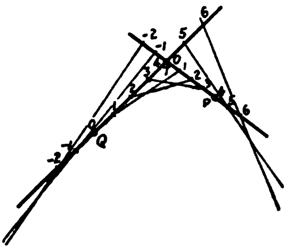


FIG. 20.

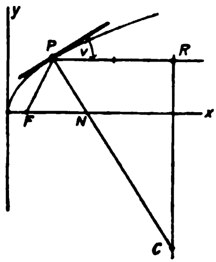


FIG. 21.

THE ELLIPSE

The **ellipse** (see also p. 107) has two **foci**, F and F' (Fig. 22), and two **directrices**, DH and $D'H'$. If P is any point of the curve, $PF + PF'$ is constant, $= 2a$; and PF/PH (or PF'/PH') is also constant, $= e$, where e is the **eccentricity** ($e < 1$). Either of these properties may be taken as the definition of the curve. The relations between e and the semi-axes a and b are as shown in Fig. 23. Thus, $b^2 = a^2(1 - e^2)$, $ae = \sqrt{a^2 - b^2}$, $e^2 = 1 - (b/a)^2$. The **semi-latus rectum** $= p = a(1 - e^2) = b^2/a$. Note that b is always less than a, except in the special case of the circle, in which $b = a$ and $e = 0$.

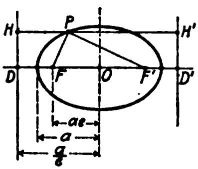


FIG. 22.

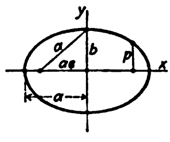


FIG. 23.

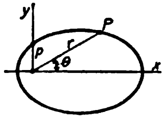


FIG. 24.

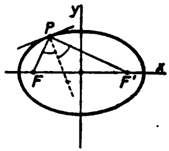


FIG. 25.

Any section of a right circular cone made by a plane which cuts all the elements of one nappe of the cone will be an ellipse; if the plane is perpendicular to the axis of the cone, the ellipse becomes a circle.

Equation of Ellipse, center as origin:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{or} \quad y = \pm \frac{b}{a}\sqrt{a^2 - x^2}.$$

If $P = (x, y)$ is any point of the curve, $PF = a + ex$, $PF' = a - ex$.

Equations of the Ellipse in Parametric Form: $x = a \cos u$, $y = b \sin u$, where u is the eccentric angle of the point $P = (x,y)$. See Fig. 28.

Polar Equation, focus as origin, axes as in Fig. 24: $r = p/(1 - e \cos \theta)$.

Equation of the Tangent at (x_1,y_1): $b^2x_1x + a^2y_1y = a^2b^2$.

The line $y = mx + k$ will be a tangent if $k = \pm\sqrt{a^2m^2 + b^2}$. The normal at any point P bisects the angle between PF and PF' (Fig. 25). The locus of the foot of the perpendicular from the focus on a moving tangent is the circle on the major axis as diameter (Fig. 26). The locus of the point of intersection of perpendicular tangents is a circle with radius $\sqrt{a^2 + b^2}$ (Fig. 27).

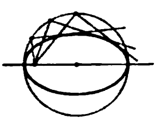


Fig. 26.

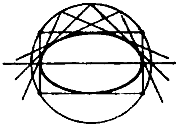


Fig. 27.

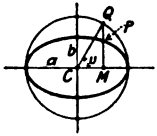


Fig. 28.

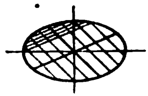


Fig. 29.

Ellipse as a Flattened Circle. Eccentric Angle. If the ordinates in a circle are diminished in a constant ratio, the resulting points will lie on an ellipse (Fig. 28). If Q traces the circle with uniform velocity, the corresponding point P will trace the ellipse, with varying velocity. The angle u in the figure is called the eccentric angle of the point P.

Conjugate Diameters are lines through the center, each of which bisects all the chords parallel to the other (Fig. 29). If m_1 and m_2 are the slopes, then $m_1m_2 = -b^2/a^2$. One pair of conjugate diameters are the diagonals of the rectangle circumscribing the ellipse. The eccentric angles of the ends of two conjugate diameters differ by 90 deg. Thus (Fig. 30), if CQ and CQ' are perpendicular radii in the circle, CP and CP' will be conjugate semi-diameters in the ellipse. A parallelogram formed by tangents drawn parallel to a pair of conjugate diameters has a constant area, $= 4ab$ (Fig. 31). Also, if a',b' are conjugate semi-diameters, and w the angle between them, then $a'^2 + b'^2 = a^2 + b^2$ and $a'b' = ab/\sin w$.

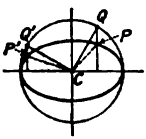


Fig. 30.

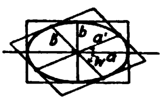


Fig. 31.

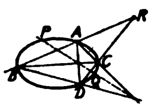


Fig. 32.

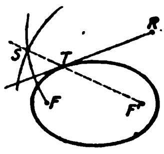


Fig. 33.

To Construct a Tangent to a Given Ellipse. (1) At a Given Point of Contact, P. Bisect the angle between the focal radii PF and PF' (Fig. 25).

(2) From a Given External Point, R. (*a*) Through R draw any two lines cutting the ellipse, one in A and B, the other in C and D (Fig. 32). Through the point of intersection of AD and BC and the point of intersection of AC and BD, draw a line cutting the ellipse in P and Q. Then P and Q are the required points of contact. (*b*) With R as a center and radius RF, draw an arc; with F' as center and radius $2a$ draw an arc, intersecting the first in S; and let SF' meet the curve in T. Then T is the point of contact (Fig. 33).

To Construct an Ellipse, Given a and b. (1) In Fig. 34, with O as center, draw circles with radii a and b (and also a third circle with radius $a + b$). Let a variable ray through O cut these circles in J, K (and S); through J and K draw parallels to the axes, meeting in P. Then P is a point of the ellipse (and SP is the normal at P).

(2) In Fig. 35, let P divide a line AB so that $PA = a$ and $PB = b$. Then if A and B slide on the axes, P will describe an ellipse.

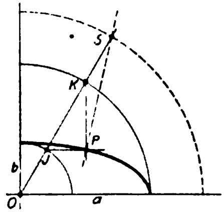


FIG. 34.

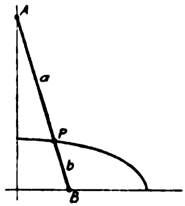


FIG. 35.

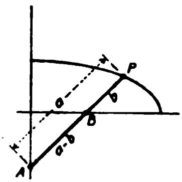


FIG. 36.

(3) In Fig. 36, let PBA be a straight line such that $PA = a$ and $PB = b$. Then if A and B slide on the axes, P will trace an ellipse. (Use a strip of paper, with the points P, B, and A marked on it.)

(4) Find the foci, F and F', by striking an arc of radius a with center at B (Fig. 37). Drive pins at F, F', and B, and adjust a loop of thread around them. Then remove the pin at B, and replace it by a pencil point; by moving the pencil so as to keep the string taut, the complete ellipse can be drawn at one sweep. Or, use a mechanical ellipsograph.

(5) and (6). Apply methods (1) and (2) of the following paragraph to the special case in which OP and OQ are perpendicular semi-axes.

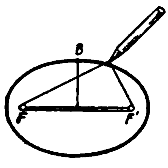


FIG. 37.

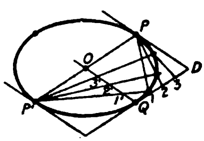


FIG. 38.

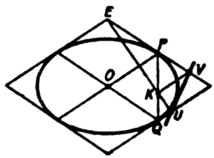


FIG. 39.

To Construct an Ellipse, Given a Pair of Conjugate Semi-diameters, OP and OQ. (1) Complete the parallelogram, as in Fig. 38. Divide QD and QO into n equal parts, 1, 2, 3, . . . and 1′, 2′, 3′, . . . Connect P with 1, 2, 3, . . . and P' with 1′, 2′, 3′ . . .. The points of intersection of corresponding lines will be points of the ellipse.

(2) Take any point K on PQ (Fig. 39). Draw EKU, and draw KV parallel to OP. Then UV will be a tangent. By varying K along PQ as many tangents may be drawn as desired, thus "enveloping" the ellipse.

(3) Through P (Fig. 40), draw a perpendicular PT to OQ, and lay off $PR = PS = OQ$. Then if the line RPT is made to slide with one end on OR and the other on OQ, P will trace the ellipse. Further, the bisectors of the angle ROS show the directions of the principal axes, and $OR + OS = 2a$ and

$OR - OS = 2b$. Also, if a line through P perpendicular to RS (and therefore tangent to the ellipse at P) meets the minor axis in M, a circle with M as center and MR or MS as radius will cut the major axis in the two foci.

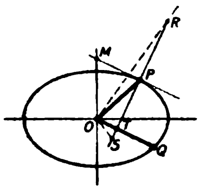


FIG. 40.

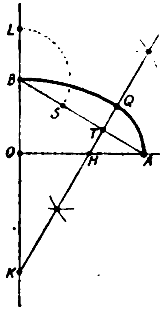


FIG. 41.

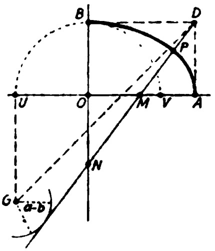


FIG. 42.

To Construct an Ellipse Approximately by Circular Arcs. [Methods (1) and (2) employ two radii, (3) and (4) employ three radii.] (1) In Fig. 41, lay off $OL = OA$ and $BS = BL = a - b$. Bisect SA in T, and draw THK perpendicular to BA. Then H is one center, with radius HA, and K is the other center, with radius KB. The junction point Q of the two arcs will fall a little outside the true ellipse.

(2) In Fig. 42, lay off $OU = OV = OB = b$. Draw UG perpendicular to the axis and DG at 45°. With G as center draw an auxiliary arc with radius

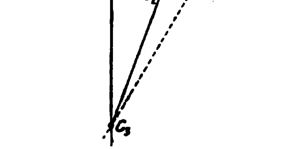


FIG. 43.

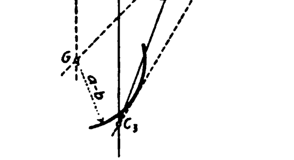


FIG. 44.

$= AV = a - b$, and through D draw DMN just touching this arc. Then M is one center (with radius MA) and N is the other center (with radius NB). The junction point P of the two arcs will be a true point of the ellipse. [E. V. Huntington.]

(3) Through D (Fig. 43) draw DC_1C_3 perpendicular to AB. Call $C_1A = r_1$ and $C_3B = r_3$. Lay off $BE = BO$ ($= b$), and on ED as diameter draw a semicircle cutting the minor axis in W; then $BW = \sqrt{ab} = r_2$. Lay off $AZ =$

BW. From C_1 with radius $C_1Z(=r_2-r_1)$, and from C_3 with radius C_3W $(=r_3-r_2)$, draw arcs intersecting in C_2. Draw C_3C_2 extended and C_2C_1 extended. Then draw in the three arcs, with centers at C_1, C_2, C_3 and radii r_1, r_2, r_3. NOTE. Since r_1 and r_3 are the radii of curvature of the ellipse at A and B, this construction gives a curve which is a little too sharp at A and a little too flat at B. A more accurate construction is the following:

(4) In Fig. 44, lay off $BE = BH = BO = b$. Through the mid-point X of BE draw XG perpendicular to the axis, and through D draw DG at an angle of 45 deg. From G as center draw an auxiliary arc with radius $= DH$ $(= a - b)$, and through D draw DC_1C_3 just touching this arc. Take C_1A as r_1 and C_3B as r_3. On DE as diameter draw a semi-circle cutting the minor axis in W, and take $BW(=\sqrt{ab})$ as r_2. Lay off $AZ = BW$. From C_1 with radius $C_1Z(=r_2-r_1)$, and from C_3 with radius $C_3W(=r_3-r_2)$, draw arcs intersecting in C_2. Then C_1, C_2, C_3 are the required centers. [E. V. Huntington.]

Radius of Curvature of Ellipse at Any Point $P = (x, y)$ is $R = a^2b^2(x^2/a^4 + y^2/b^4)^{3/2} = p/\sin^3 v$, where v is the angle which the tangent at P makes with PF or PF'. At end of major axis, $R = b^2/a = MA$; at end of minor axis, $R = a^2/b = NB$ (see Fig. 45). To construct the radius of curvature at any other point P (Fig. 46), draw the normal at P (by bisecting the angle between PF and PF') and let it meet the major axis in N. At N draw a perpendicular to PN meeting PF in H. At H draw a perpendicular to PH meeting PN in C. Then C is the center of curvature for the point P, and a circle about C with radius CP will coincide closely with the ellipse in the neighborhood of P. [Note. If the circle of curvature meets the ellipse in Q, then the tangent at P and the line PQ are equally inclined to the axis.]

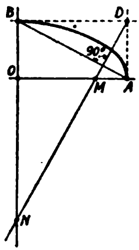


FIG. 45.

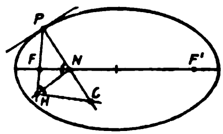


FIG. 46.

THE HYPERBOLA

The **hyperbola** (see also p. 107) has two **foci**, F and F', at distances $\pm ae$ from the center, and two **directrices**, DH and $D'H'$, at distances $\pm a/e$ from

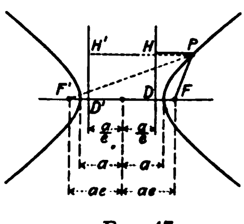


FIG. 47.

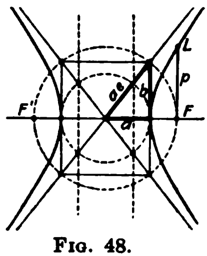


FIG. 48.

the center (Fig. 47). If P is any point of the curve, $|PF - PF'|$ is constant, $= 2a$; and PF/PH (or PF'/PH') is also constant, $= e$ (called the **eccentricity**), where $e > 1$. Either of these properties may be taken as the definition of the curve. The curve has two branches which approach more and more nearly two straight lines called the **asymptotes.** Each asymptote makes with the principal axis an angle whose tangent is b/a. The relations between e, a, and b are shown in Fig. 48: $b^2 = a^2(e^2 - 1)$, $ae = \sqrt{a^2 + b^2}$, $e^2 = 1 + (b/a)^2$. The semi-latus rectum, or ordinate at the focus, is $p = a(e^2 - 1) = b^2/a$.

Any section of a right circular cone made by a plane which cuts both nappes of the cone will be a hyperbola. (Compare also Fig. 3, p. 174.)

Equation of the Hyperbola, center as origin:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ or } y = \pm \frac{b}{a}\sqrt{x^2 - a^2}.$$

If $P = (x,y)$ is on the right-hand branch, $PF = ex - a$, $PF' = ex + a$. If P is on the left-hand branch, $PF = -ex + a$, $PF' = -ex - a$.

Equations of Hyperbola in Parametric Form. (1) $x = a \cosh u$, $y = b \sinh u$. (For tables of hyperbolic functions, see pp. 60 and 61.) Here u may be interpreted as A/a^2, where A is the area shaded in Fig. 49.

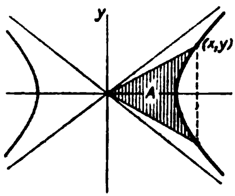


FIG. 49.

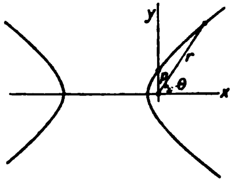


FIG. 50.

(2) $x = a \sec v$, $y = b \tan v$, where v is an auxiliary angle of no special geometric interest.

Polar Equation, referred to focus as origin, axes as in Fig. 50:

$$r = p/(1 - e \cos \theta).$$

Equation of the Tangent at (x_1,y_1): $b^2x_1x - a^2y_1y = a^2b^2$.

The line $y = mx + k$ will be a tangent if $k = \pm\sqrt{a^2m^2 - b^2}$. The tangent at any point P (Fig. 51) bisects the angle between PF and PF'. The locus of the foot of the perpendicular from the focus on a moving tangent is the circle on the principal axis as diameter (Fig. 52). The locus of the point of intersection of perpendicular tangents is a circle with radius $\sqrt{a^2 - b^2}$, which will be imaginary if $b > a$ (Fig. 53).

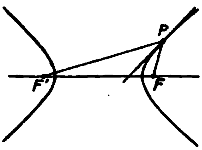


FIG. 51.

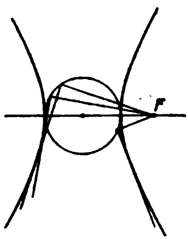


FIG. 52.

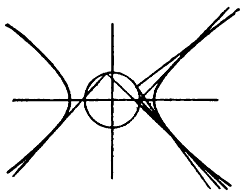


FIG. 53.

Properties of the Asymptotes. (Fig. 54.) If P is any point of the curve, the product of the perpendicular distances from P to the two asymptotes is constant, $= a^2b^2/(a^2 + b^2)$. Also, the product of the oblique distances (the distance to each asymptote being measured parallel to the other) is constant, and equal to $\frac{1}{4}(a^2 + b^2)$. If a line cuts the hyperbola and its asymptotes, the parts of the line intercepted between the curve and the asymptotes are equal. The part of a tangent intercepted between the asymptotes is bisected by the point of contact. The triangle bounded by the asymptotes and a variable tangent is of constant area, $= ab$. If a line through Q perpendicular to the principal axis meets the asymptotes in R and S (see Fig. 54), then $\overline{QR} \times \overline{QS} = b^2$. If a line through Q parallel to the principal axis meets the asymptotes in U and V, then $\overline{QU} \times \overline{QV} = a^2$.

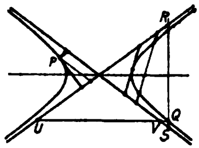


FIG. 54.

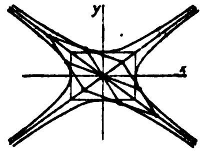


FIG. 55.

Conjugate Hyperbolas are two hyperbolas having the same asymptotes with semi-axes interchanged (Fig. 55). The equation of the hyperbola conjugate to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

Conjugate Diameters are lines through the center, each of which bisects all the chords parallel to the other—a chord which does not meet the given hyperbola being understood to be terminated by the conjugate hyperbola (Fig. 55). If m_1 and m_2 are the slopes, then $m_1m_2 = b^2/a^2$. Each asymptote, regarded as a diameter, is its own conjugate. If a parallelogram is formed by tangents drawn parallel to a pair of conjugate diameters, its vertices will lie on the asymptotes, and its area will be constant $= 4ab$. If a', b' are conjugate semi-diameters, and w the angle between them, then $a'^2 - b'^2 = a^2 - b^2$, and $a'b' = ab/\sin w$.

Equilateral Hyperbola ($a = b$). Equation referred to principal axes (Fig. 56): $x^2 - y^2 = a^2$. NOTE. $p = a$. Equation referred to asymptotes as axes (Fig. 57): $xy = a^2/2$. (See also Fig. 3, p. 174.)

Asymptotes are perpendicular. Eccentricity $= \sqrt{2}$. Any diameter is equal in length to its conjugate diameter.

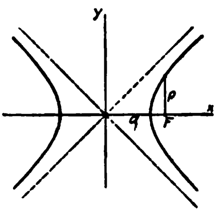


FIG. 56.

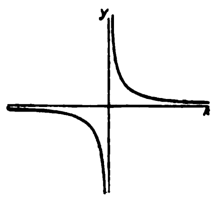


FIG. 57.

To Construct a Tangent at any given point P of a hyperbola. In Fig. 58, draw PA and PB parallel to the asymptotes, and take $OS = 2(OA)$ and $OT = 2(OB)$. Then ST is the tangent at P.

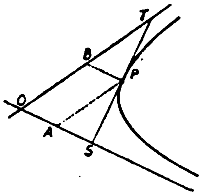


FIG. 58.

To Construct a Hyperbola, given the asymptotes and any point P. (1) In Fig. 59 let TPT' be a variable line through P, and lay off $T'P' = TP$; then P' is a point of the curve.

(2) In Fig. 60, draw PA and PB parallel to the asymptotes. Lay off $OA' = n(OA)$ and $OB' = (1/n)(OB)$, where n is any number; and through A' and B' draw parallels to the axes; these will meet in a point P' of the curve.

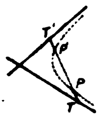


FIG. 59.

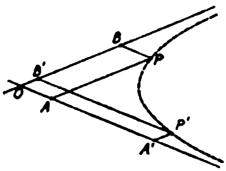


FIG. 60.

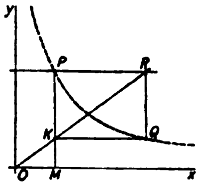


FIG. 61.

(3) (Fig. 61.) Take any point K in the ordinate PM, and draw OK meeting the line through P parallel to the x-axis in R. Draw a parallel to the x-axis through K and a parallel to the y-axis through R, meeting in Q. Then Q is a point of the curve.

THE CATENARY

The catenary is the curve in which a flexible chain or cord of uniform density will hang when supported by the two ends. Let w = weight of the chain per unit length; T = the tension at any point P; and T_h, T_v = the horizontal and vertical components of T. The horizontal component T_h is the same at all points of the curve.

The length $a = T_h/w$ is called the **parameter** of the catenary, or the distance from the lowest point O to the **directrix** DQ (Fig. 62). When a is very large, the curve is very flat. For methods of finding a in any given case, see problems 1–6 below.

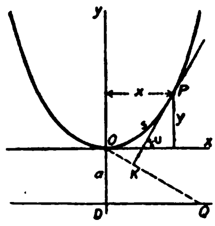


FIG. 62.

The rectangular **equation**, referred to the lowest point as origin, is $y = a\ [\cosh\ (x/a) - 1]$. (For table of hyperbolic functions, see p. 60.) In case of very flat arcs (a large), $y = \dfrac{x^2}{2a} + \ldots$; $s = x + \frac{1}{6}\dfrac{x^3}{a^2} + \ldots$, approximately, so that in such a case the catenary closely resembles a parabola.

If the perpendicular from O to the tangent at P meets the directrix in Q, then $DQ = \text{arc}\, OP = s$ and $OQ = y + a$. The **radius of curvature** at P is $R = (y + a)^2/a$, which is equal in length to the portion of the normal intercepted between P and the directrix.

Problems on the Catenary (Fig. 62). When any two of the four quantities x, y, s, T/w are known, the remaining two, and also the parameter a, can be found, as follows:

(1) GIVEN x AND y. Compute y/x, and find from Table 1 the value of the auxiliary variable z. Then compute $a = x/z$, $s = a \sinh z$, and $T = wa \cosh z$. Or, having z, find s/x and wx/T by using Tables 3 and 2 inversely, and hence (since x is known) compute s and T/w without the use of a.

TABLE 1. GIVING z WHEN y/x IS KNOWN. THEN $a = x/z$.

y/x	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0200	0.0400	0.0600	0.0800	0.0999	0.1199	0.1398	0.1597	0.1795
0.1	0.1993	0.2191	0.2389	0.2586	0.2782	0.2978	0.3173	0.3368	0.3562	0.3756
0.2	0.3948	0.4140	0.4332	0.4522	0.4712	0.4901	0.5089	0.5276	0.5463	0.5648
0.3	0.5833	0.6016	0.6199	0.6381	0.6561	0.6741	0.6919	0.7097	0.7274	0.7449
0.4	0.7623	0.7797	0.7969	0.8140	0.8311	0.8480	0.8647	0.8814	0.8980	0.9145
0.5	0.9308	0.9471	0.9632	0.9792	0.9951	1.0109	1.0266	1.0422	1.0576	1.0730
0.6	1.0883	1.1034	1.1184	1.1334	1.1482	1.1629	1.1775	1.1920	1.2064	1.2207

NOTE. $y/x = (\cosh z - 1)/z$.

(2) GIVEN x AND T/w. Compute wx/T, and find from Table 2 the value of the auxiliary variable z. Then compute $a = x/z$, $y = a\,(\cosh z - 1)$ and $s = a \sinh z$. Or, having z, find y/x and s/x by using Tables 1 and 3 inversely, and hence (since x is known) compute y and s without the use of a.

TABLE 2. GIVING z WHEN wx/T IS KNOWN. THEN $a = x/z$

wx/T	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0100	0.0200	0.0300	0.0400	0.0501	0.0601	0.0702	0.0803	0.0904
0.1	0.1005	0.1107	0.1209	0.1311	0.1414	0.1517	0.1621	0.1725	0.1830	0.1936
0.2	0.2042	0.2149	0.2256	0.2365	0.2474	0.2584	0.2695	0.2807	0.2920	0.3035
0.3	0.3150	0.3267	0.3385	0.3505	0.3626	0.3749	0.3874	0.4000	0.4129	0.4259
0.4	0.4392	0.4528	0.4666	0.4806	0.4950	0.5097	0.5248	0.5403	0.5562	0.5726
0.5	0.5894	0.6068	0.6249	0.6436	0.6632	0.6836	0.7051	0.7277	0.7517	0.7775
0.6	0.8053	0.8357	0.8695	0.9082	0.9541	1.0132	1.1110	.....	......	......

NOTE. $wx/T = z/\cosh z$. For every value of wx/T there are two values of z, one less than 1.200 and one greater than 1.200. Only the smaller of these values is tabulated.

(3) GIVEN x AND s. Compute s/x, and find from Table 3 the value of the auxiliary variable z. Then compute $a = x/z$, $y = a\,(\cosh z - 1)$, and $T = wa \cosh z$. Or, having z, find y/x and wx/T by using Tables 1 and 2 inversely, and hence (since x is known) compute y and T/w without the use of a.

TABLE 3. GIVING z WHEN s/x IS KNOWN. THEN $a = x/z$

s/x	0	1	2	3	4	5	6	7	8	9
1.000	......	0.0245	0.0346	0.0424	0.0490	0.0548	0.0600	0.0648	0.0693	0.0735
1	0.0774	0.0812	0.0848	0.0883	0.0916	0.0948	0.0980	0.1010	0.1039	0.1067
2	0.1095	0.1122	0.1149	0.1174	0.1200	0.1224	0.1249	0.1272	0.1296	0.1319
3	0.1341	0.1363	0.1385	0.1407	0.1428	0.1448	0.1469	0.1489	0.1509	0.1529
4	0.1548	0.1567	0.1586	0.1605	0.1623	0.1642	0.1660	0.1678	0.1696	0.1713
1.005	0.1731	0.1748	0.1765	0.1782	0.1799	0.1815	0.1831	0.1848	0.1864	0.1880
6	0.1896	0.1911	0.1927	0.1942	0.1958	0.1973	0.1988	0.2003	0.2018	0.2033
7	0.2047	0.2062	0.2076	0.2091	0.2105	0.2119	0.2133	0.2147	0.2161	0.2175
8	0.2188	0.2202	0.2215	0.2229	0.2242	0.2255	0.2269	0.2282	0.2295	0.2308
9	0.2321	0.2334	0.2346	0.2359	0.2372	0.2384	0.2397	0.2409	0.2421	0.2434
1.01	0.2446	0.2565	0.2678	0.2787	0.2892	0.2993	0.3091	0.3186	0.3278	0.3367
2	0.3454	0.3539	0.3621	0.3702	0.3781	0.3859	0.3934	0.4009	0.4082	0.4153
3	0.4224	0.4293	0.4361	0.4428	0.4494	0.4559	0.4623	0.4686	0.4748	0.4809
4	0.4870	0.4930	0.4989	0.5047	0.5105	0.5162	0.5218	0.5274	0.5329	0.5383
1.05	0.5437	0.5490	0.5543	0.5595	0.5647	0.5698	0.5749	0.5799	0.5849	0.5898
6	0.5947	0.5996	0.6044	0.6091	0.6139	0.6186	0.6232	0.6278	0.6324	0.6369
7	0.6414	0.6459	0.6504	0.6548	0.6591	0.6635	0.6678	0.6721	0.6763	0.6806
8	0.6848	0.6889	0.6931	0.6972	0.7013	0.7053	0.7094	0.7134	0.7174	0.7213
9	0.7253	0.7292	0.7331	0.7369	0.7408	0.7446	0.7484	0.7522	0.7559	0.7597
1.10	0.7634	......	......	......	......	......	NOTE: $s/x = \sinh z/z$			

(4) GIVEN y AND s. Then $\dfrac{T}{w} = \dfrac{s^2}{2y} + \dfrac{y}{2}$, $x = \left(\dfrac{s^2}{y} - y\right)\tanh^{-1}\left(\dfrac{y}{s}\right)$, $a = \dfrac{s^2}{2y} - \dfrac{y}{2}$. Or, if y/s is small, $x = s\left[1 - \dfrac{2}{3}\left(\dfrac{y}{s}\right)^2 - \dfrac{2}{15}\left(\dfrac{y}{s}\right)^4 - \cdots\right]$.

(5) GIVEN y AND T/w. Then $a = \dfrac{T}{w} - y$, $x = \left(\dfrac{T}{w} - y\right)\cosh^{-1}\dfrac{T/w}{(T/w) - y}$, $s = \sqrt{2y(T/w) - y^2}$. Or, if $y/(T/w)$ is small,

$$x = \sqrt{\frac{2yT}{w}}\left[1 - \frac{7}{12}\frac{wy}{T} - \cdots\right], \quad \frac{s - x}{s} = \frac{1}{3}\frac{wy}{T}, \text{ approximately,}$$

$$s = \sqrt{\frac{2yT}{w}}\left[1 - \frac{1}{4}\frac{wy}{T} - \frac{1}{32}\left(\frac{wy}{T}\right)^2 - \frac{1}{128}\left(\frac{wy}{T}\right)^3 - \cdots\right].$$

(6) GIVEN s AND T/w. Then $x = \dfrac{T}{w}\sqrt{1 - \left(\dfrac{ws}{T}\right)^2}\tanh^{-1}\left(\dfrac{ws}{T}\right)$,

$y = \dfrac{T}{w} - \dfrac{T}{w}\sqrt{1 - \left(\dfrac{ws}{T}\right)^2}$, $a = \dfrac{T}{w}\sqrt{1 - \left(\dfrac{ws}{T}\right)^2}$. Or, if ws/T is small,

$$x = s\left[1 - \frac{1}{6}\left(\frac{ws}{T}\right)^2 - \frac{11}{120}\left(\frac{ws}{T}\right)^4 - \cdots\right], \quad y = s\left[\frac{1}{2}\left(\frac{ws}{T}\right) + \frac{1}{8}\left(\frac{ws}{T}\right)^3 + \cdots\right]$$

$$s = \frac{T}{w}\left[1 - \frac{1}{2}\left(\frac{ws}{T}\right)^2 - \frac{1}{8}\left(\frac{ws}{T}\right)^4 - \cdots\right].$$

Given the Length 2L of a Chain Supported at Two Points A and B not in the Same Level, to find a. (See Fig. 63; b and c are supposed known.) Let $(\sqrt{L^2 - b^2})/c = s/x$; enter Table 3 with this value of s/x, and find the corresponding value of the auxiliary variable z. Then $a = c/z$, Note. The co-ordinates of the mid-point M of AB (see Fig. 63) are $x_0 = a \tanh^{-1}(b/L)$, $y_0 = (L/\tanh z) - a$, so that the position of the lowest point is determined.

Correction for Sag in Chaining Uphill (Fig. 64). Let l = length of tape (corrected for stretch and temperature), w = weight per unit length of tape, A = angle between the chord AB and the horizontal.

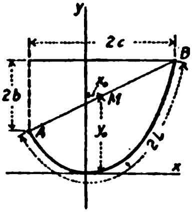


Fig. 63.

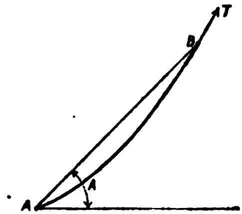


Fig. 64.

If the tension P at the upper end is known, compute wl/P and find k from Table 4. If the tension Q at the lower end is known, compute wl/Q and find k from Table 5. In either case, chord $AB = l - kl$.

Table 4. Giving k

$\frac{wl}{P}$	$A = 0°$	10°	20°	30°	40°	50°	60°	70°	80°
.01	.00000	000	000	000	000	000	000	000	000
.02	002	002	001	001	001	001	000	000	000
.03	004	004	003	003	002	002	001	000	000
.04	007	006	006	005	004	003	002	001	000
.05	011	010	009	008	006	004	003	001	000
.06	.00015	015	013	012	009	006	004	002	000
.07	020	020	018	016	012	009	005	003	001
.08	027	026	024	021	016	012	007	003	001
.09	034	033	031	026	021	015	009	004	001
.10	042	041	038	033	026	019	011	005	001
.11	.00051	050	046	040	032	023	014	007	002
.12	060	060	055	048	038	027	017	008	002
.13	070	070	065	057	045	032	020	009	002
.14	082	081	076	066	053	038	023	011	003
.15	094	094	087	076	061	044	027	013	003
.16	.00107	107	100	087	070	050	031	015	004
.17	121	121	113	099	079	057	035	017	004
.18	136	136	128	112	090	065	040	019	005
.19	151	152	143	125	101	073	045	021	006
.20	168	168	159	140	113	082	050	024	006

Table 5. Giving k

$\frac{wl}{Q}$	$A = 0°$	10°	20°	30°	40°	50°	60°	70°	80°
.01	.00000	000	000	000	000	000	000	000	000
.02	002	002	001	001	001	001	000	000	000
.03	004	004	003	003	002	001	001	000	000
.04	007	006	006	005	004	003	002	001	000
.05	011	010	009	008	006	004	002	001	000
.06	.00015	014	013	011	008	006	004	002	000
.07	020	020	018	015	011	008	005	002	001
.08	027	026	023	019	015	011	006	003	001
.09	034	032	029	024	019	013	008	004	001
.10	042	040	036	030	023	016	010	004	001
.11	.00051	048	043	036	028	019	011	005	001
.12	060	057	051	043	033	023	014	006	002
.13	070	067	060	050	038	026	016	007	002
.14	082	078	069	057	044	030	018	008	002
.15	094	089	079	066	050	035	021	010	002
.16	.00107	101	090	074	057	039	022	011	003
.17	121	114	101	084	064	044	026	012	003
.18	136	128	113	092	071	049	029	013	003
.19	151	142	125	103	079	054	032	015	004
.20	168	157	138	114	087	060	035	016	004

Note. $k = 1 - \{[1 - \sqrt{1 - 2m \sin u + m^2}]/[m \sin A]\}$, where $m = wl/P$ and u is given by

$$[1 - \sqrt{1 - 2m \sin u + m^2}] \sec u = [\sinh^{-1}(\tan u) - \sinh^{-1}(\tan u - m \sec u)] \tan A.$$

Also, $Q = P - wl(1 - k) \sin A$, where k is the value in Table 4 corresponding to the given values of P and A.

Correction for Stretch in Chaining Uphill. Let L = unstretched length of tape at working temperature, w = weight per unit length of tape, A = angle between chord AB and the horizontal, F = area of cross-section, E = Young's modulus of elasticity (for steel, E = 29,000,000 lb. per sq. in.), l = stretched length (along curve).

If the tension P at the upper end is known, compute wL/P and find m from Table 6. Then $l = L + (LP/FE)(1 - m)$.

If the tension Q at the lower end is known, compute wL/Q and find n from Table 7. Then $l = L + (LQ/FE)(1 + n)$.

TABLE 6. GIVING m

$\frac{wL}{P}$	A = 0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
.00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	.001	.010	.018	.026	.033	.039	.044	.047	.049	.050
.20	.003	.021	.038	.053	.067	.078	.088	.094	.099	.100

TABLE 7. GIVING n

$\frac{wL}{Q}$	A = 10°	20°	30°	40°	50°	60°	70°	80°	90°
.00	.000	.000	.000	.000	.000	.000	.000	.000	000
.10	.008	.016	.024	.032	.038	.043	.047	.049	.050
.20	.014	.031	.047	.062	.075	.086	.094	.099	.100

OTHER USEFUL CURVES

The Cycloid is traced by a point on the circumference of a circle which rolls without slipping along a straight line. **Equations** of cycloid, in parametric form (axes as in Fig. 65): $x = a\,(\text{rad}\,u - \sin u)$, $y = a(1 - \cos u)$, where a is

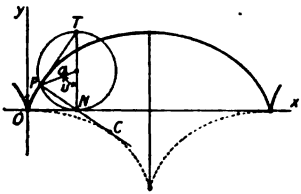


FIG. 65.

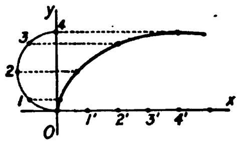


FIG. 66.

the radius of the rolling circle, and rad u is the radian measure of the angle u through which it has rolled. The tangent and normal at any point pass through the highest and lowest points of the corresponding position of the generating circle. The **radius of curvature** at any point P is $PC = 4a \sin(u/2) = 2\sqrt{2ay}$ = twice the length of the normal, PN. The **evolute**, or locus of centers of curvature, is an equal cycloid. **To construct** a cycloid (Fig. 66), divide the semi-circumference of the generating circle into n equal parts (here 4) and lay off these arcs along the base (from O to $4'$). Describe arcs with centers at $1'$, $2'$, . . . and radii equal to the chords $O1$, $O2$, . . ., and sketch the cycloid as a curve tangent to all of these arcs. Or, on horizontal lines through 1, 2, . . . lay off distances equal to $O1'$, $O2'$, etc.; the points thus reached will lie on the cycloid.

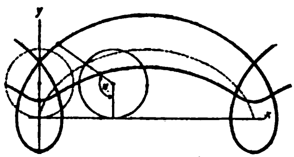
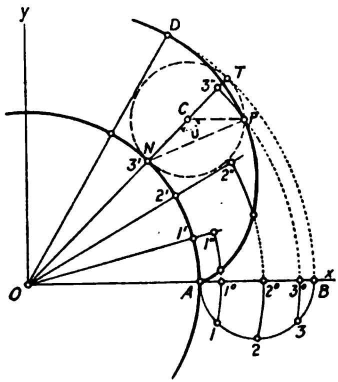


FIG. 67.

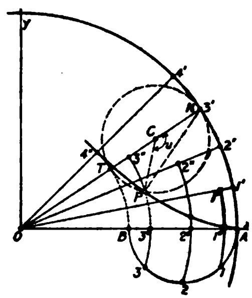
The area of one arch $= 3\pi a^2$, length of arc of one arch $= 8a$. Area bounded by the ordinate of the point P corresponding to any value of u is $a^2\,(\tfrac{3}{2}\,\text{rad}\,u - 2 \sin u + \tfrac{1}{4} \sin 2u) = \tfrac{3}{2}\,ax - \tfrac{1}{2}\,y\sqrt{(2a - y)y}$. Length of arc $OP = 4a\,(1 - \cos \tfrac{1}{2} u) = 4a - 2\sqrt{2a(2a - y)}$.

The Trochoid is a more general curve, traced by any point on a radius of the rolling circle, at distance b from the center (Fig. 67). It is a prolate trochoid if $b < a$, and a curtate or looped trochoid if $b > a$. The equations in either case are $x = a \text{ rad } u - b \sin u, \quad y = a - b \cos u$.

The Epicycloid (or **Hypocycloid**) is a curve generated by a point on the circumference of a circle of radius a which rolls without slipping on the outside (or inside) of a fixed circle of radius c. For the **equations**, put $b = a$ in the equations of the epi- (or hypo-) trochoid, below. The normal at any point P passes through the point of contact N of the corresponding position of the rolling circle. To construct the curve (Figs. 68 and 69),



Epicycloid.
FIG. 68.



Hypocycloid.
FIG. 69.

divide the semi-circumference of the rolling circle into n equal parts, by points 1, 2, 3 . . ., and lay off these arcs ($A1$, $A2$, $A3$) along the circumference of the base circle, as $A1'$, $A2'$, $A3'$, ... Describe circles with centers at $1'$, $2'$, $3'$, . . . and radii equal to the chords $A1$, $A2$, $A3$, . . .; then the required curve will be tangent to all these circles. Or, with O as center, draw arcs through 1, 2, 3, . . ., meeting the radius OA in 1^0, 2^0, 3^0, . . ., and the radii $O1'$, $O2'$, $O3'$, . . . in $1''$, $2''$, $3''$, . . .; then from $1''$, $2''$, $3''$, . . . lay off arcs equal to 1^01, 2^02, 3^03, . . . respectively; the points thus reached will be points of the curve.

The area $OAP = \dfrac{a(c \pm a)(c \pm 2a)}{2c}(\text{rad } u - \sin u)$, where the upper sign applies to the epicycloid, the lower to the hypocycloid, and rad u = the radian measure of the angle u shown in Figs. 68 and 69. Arc $AP = (4\,a/c)(c \pm a)(1 - \cos \tfrac{1}{2}u)$; arc $AD = (4a/c)(c \pm a)$. [In Fig. 69, $D = 4''$.]

Radius of curvature at any point P is $R = \dfrac{4a(c \pm a)}{c \pm 2a} \sin \tfrac{1}{2}u$; at A, $R = 0$; at D, $R = \dfrac{4a(c \pm a)}{c \pm 2a}$.

Special Cases. If $a = \tfrac{1}{2}c$, the hypocycloid becomes a straight line, diameter of the fixed circle (Fig. 70). In this case the hypotrochoid traced by any

point rigidly connected with the rolling circle (not necessarily on the circumference) will be an ellipse. If $a = \frac{1}{4}c$, the curve generated will be the four-cusped hypocycloid, or **astroid,** (Fig. 71), whose equation is $x^{2/3} + y^{2/3} = c^{2/3}$. If $a = c$, the epicycloid is the **cardioid,** whose equation in polar coordinates (axes as in Fig. 72) is $r = 2c(1 + \cos\theta)$. Length of cardioid $= 16c$.

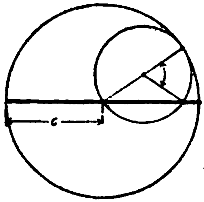
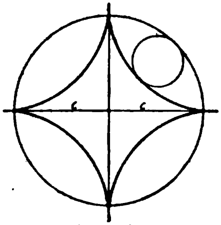
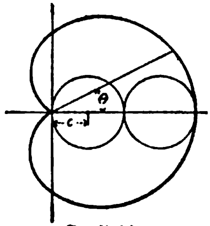


FIG. 70.



Astroid.
FIG. 71.

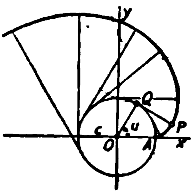


Cardioid.
FIG. 72.

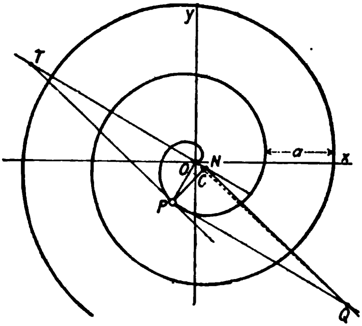
The Epitrochoid (or Hypotrochoid) is a curve traced by any point rigidly attached to a circle of radius a, at distance b from the center, when this circle rolls without slipping on the outside (or inside) of a fixed circle of radius c. The equations are $x = (c \pm a)\cos\left(\frac{a}{c}u\right) \mp b\cos\left[\left(1 \pm \frac{a}{c}\right)u\right]$,

$y = (c \pm a)\sin\left(\frac{a}{c}u\right) - b\sin\left[\left(1 \pm \frac{a}{c}\right)u\right]$, where u = the angle which the moving radius makes with the line of centers; take the upper sign for the epi- and the lower for the hypo-trochoid. The curve is called prolate or curtate according as $b < a$ or $b > a$. When $b = a$, the special case of the epi- or hypocycloid arises.

The Involute of a Circle is the curve traced by the end of a taut string which is unwound from the circumference of a fixed circle, of radius c. If QP



Involute of Circle.
FIG. 73.



Spiral of Archimedes.
FIG. 74.

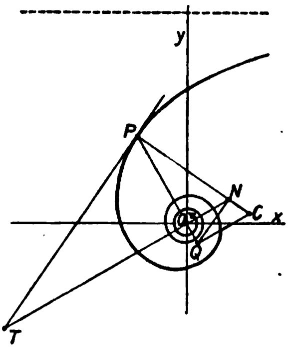
is the free portion of the string at any instant (Fig. 73), QP will be tangent to the circle at Q, and the length of QP = length of arc QA; hence the construction of the curve. The equations of the curve in parametric form (axes as in figure) are $x = c(\cos u + \text{rad } u \sin u)$, $y = c(\sin u - \text{rad } u \cos u)$, where rad u is the radian measure of the angle u which OQ makes with the x-axis. Length of arc $AP = \frac{1}{2}c(\text{rad } u)^2$; radius of curvature at P is QP.

The Spiral of Archimedes (Fig. 74) is traced by a point P which, starting from O, moves with uniform velocity along a ray OP, while the ray itself revolves with uniform angular velocity about O. Polar equation: $r = k \text{ rad } \theta$, or $r = a\,(\theta/360°)$. Here $a = 2\pi k =$ the distance, measured along a radius, from each coil to the next.

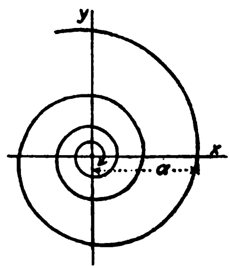
In order to construct the curve, draw radii $O1, O2, O3, \ldots$ making angles $\frac{1}{n}(360°)$, $\frac{2}{n}(360°)$, $\frac{3}{n}(360°)$, . . . with Ox, and along these radii lay off distances equal to $\frac{1}{n}a$, $\frac{2}{n}a$, $\frac{3}{n}a$, . . .; the points thus reached will lie on the spiral. The figure shows one-half of the curve, corresponding to positive values of θ.

Construction for tangent and normal: Let PT and PN be the tangent and normal at any point P, the line TON being perpendicular to OP. Then $OT = r^2/k$, and $ON = k$, where $k = a/(2\pi)$. Hence the construction.

The radius of curvature at P is $R = (k^2 + r^2)^{3/2}/(2k^2 + r^2)$. To construct the center of curvature, C, draw NQ perpendicular to PN and PQ perpendicular to OP; then OQ will meet PN in C. Length of arc $OP = \frac{1}{2}k\,[\text{rad } \theta \sqrt{1 + (\text{rad } \theta)^2} + \sinh^{-1}(\text{rad } \theta)]$. After many windings, arc $OP = \frac{1}{2}r^2/k$, approximately.



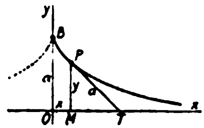
Hyperbolic Spiral.
FIG. 75.



Logarithmic Spiral.
FIG. 76.

The Hyperbolic Spiral is the curve whose polar equation is $r = a/\text{rad } \theta$. To construct the curve, take a series of points along Ox (Fig. 75); through each of these points, with center at O, draw an arc extending into the upper half of the plane; and along each of these arcs lay off a length $= a$. The points thus reached will lie on the curve. A line parallel to the x-axis, at distance a, is an asymptote of the curve. The curve winds around and around the point O without ever reaching it (asymptotic point). The figure shows one-half of the curve, corresponding to positive values of θ. If PT and PN are the tangent and normal at any point P, the line TON being perpendicular to OP, then $OT = a$, and $ON = r^2/a$. Hence a construction for the tangent and normal. Radius of curvature at P is $R = r/\sin^3 v$, where v = angle between OP and the tangent at P. Construction: At N draw a perpendicular to PN, meeting PO in Q; at Q draw a perpendicular to PQ, meeting PN in C; then C is the center of curvature for the point P.

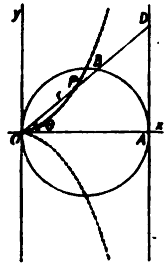
The Logarithmic Spiral (Fig. 76), is a curve which cuts the radii from O at a constant angle v, whose cotangent is m. Polar equation: $r = ae^{m \text{ rad } \theta}$. Here a is the value of r when $\theta = 0$. For large negative values of θ, the curve winds around O as an asymptotic point. If PT and PN are the tangent and normal at P, the line TON being perpendicular to OP (not shown in fig.), then $ON = rm$, and $PN = r\sqrt{1 + m^2} = r/\sin v$. Radius of curvature at P is PN. The evolute of the spiral is an equal spiral whose axis makes an angle $\frac{1}{2}\pi - (\log_e m)/m$ with the axis of the given spiral. Area swept out by the radius r from $r = 0$ (where $\theta = -\infty$) to $r = r$, is $A = r^2/(4m)$ = half the triangle OPT. Length of arc from O to $P = s = r/\cos v = PT$.



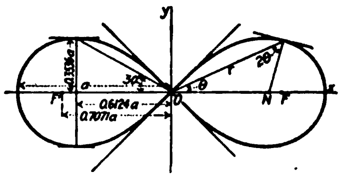
Tractrix.
FIG. 77.

The Tractrix, or Schiele's Anti-friction Curve (Fig. 77), is a curve such that the portion PT of the tangent between the point of contact and the x-axis is constant $= a$. Its equation is $x = \pm a \left[\cosh^{-1}\frac{a}{y} - \sqrt{1 - \left(\frac{y}{a}\right)^2}\right]$, or, in parametric form, $x = \pm a\,[t - \tanh t]$, $y = a/\cosh t$. (For tables of hyperbolic functions, see p. 60.) The x-axis is an asymptote of the curve. Length of arc $BP = a \log_e (a/y)$. The evolute (locus of centers of curvature) is the catenary whose lowest point is at B, and whose directrix is Ox.

The Cissoid (Fig. 78) is the locus of a point P such that OP, laid off on a variable ray from O, is equal to BD, the portion of the ray lying between a fixed circle through O and a fixed tangent at the point A opposite O. If a is the radius of the circle, the polar equation is $r = 2a \sin^2 \theta / \cos \theta$. Rectangular equation, $y^2(2a - x) = x^3$.



Cissoid.
FIG. 78.

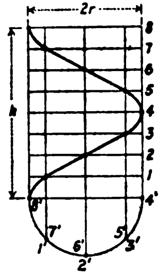


Lemniscate.
FIG. 79.

The Lemniscate (Fig. 79) is the locus of a point P the product of whose distances from two fixed points F, F' is constant, equal to $\frac{1}{2} a^2$. The distance $FF' = a\sqrt{2}$. Polar equation is $r = a\sqrt{\cos 2\theta}$. Angle between OP and the normal at P is 2θ. The two branches of the curve cross at right angles at O.

Maximum y occurs when $\theta = 30°$ and $r = a/\sqrt{2}$, and is equal to $\frac{1}{4} a\sqrt{2}$.
Area of one loop $= a^2/2$.

The Helix (Fig. 80) is the curve of a screw thread on a cylinder of radius r. The curve crosses the elements of the cylinder at a constant angle, v. The pitch, h, is the distance between two coils of the helix, measured along an element of the cylinder; hence $h = 2\pi r \tan v$. Length of one coil $= \sqrt{(2\pi r)^2 + h^2} = 2\pi r/\cos v$. To construct the projection of a helix on a plane containing the axis of the cylinder, draw a rectangle, breadth $2r$ and height h, to represent the plane, with a semicircle below it, as in the figure, to represent the base of the cylinder. Divide h into equal parts (here 8), numbered from 1 to 8; think of the circumference as also divided into 8 equal parts, represented on the semicircle by numbers from $1'$ to $4'$ and back again from $4'$ to $8'$. Then the point of intersection of a horizontal line through 1, 2, . . . with a vertical line through $1'$, $2'$, . . . will be a point of the required projection. If the cylinder is rolled out on a plane, the development of the helix will be a straight line, with slope equal to $\tan v$.



Helix.
FIG. 80.

DIFFERENTIAL AND INTEGRAL CALCULUS

DERIVATIVES AND DIFFERENTIALS

Derivaties and Differentials. A **function** of a single variable x may be denoted by $f(x), F(x)$, etc. The value of the function when x has the value x_0 is then denoted by $f(x_0), F(x_0)$, etc. The **derivative** of a function $y = f(x)$ may be denoted by $f'(x)$, or by dy/dx. The value of the derivative at a given point $x = x_0$ is the **rate of change** of the function at that point; or, if the function is represented by a curve in the usual way (Fig. 1), the value of the derivative at any point shows the **slope of the curve** (that is, the slope of the tangent to the curve) at that point (positive if the tangent points upward, and negative if it points downward, moving to the right).

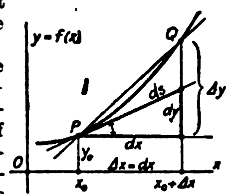


FIG. 1.

The **increment** Δy (read: "delta y"), in y is the change produced in y by increasing x from x_0 to $x_0 + \Delta x$; that is, $\Delta y = f(x_0 + \Delta x) - f(x_0)$. The **differential**, dy, of y is the value which Δy would have if the curve coincided with its tangent. (The differential, dx, of x is the same as Δx when x is the independent variable.) Note that the derivative depends only on the value of x_0, while Δy and dy depend not only on x_0 but also on the value of Δx. The ratio $\Delta y/\Delta x$ represents the slope of the secant, and dy/dx the slope of the tangent (see Fig. 1). If Δx is made to approach zero, the secant approaches the tangent as a limiting position, so that the derivative $= f'(x) =$

$$\frac{dy}{dx} = \lim_{\Delta x \doteq 0}\left[\frac{\Delta y}{\Delta x}\right] = \lim_{\Delta x \doteq 0}\left[\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}\right].$$ Also, $dy = f'(x)\,dx$.

The symbol "lim" in connection with $\Delta x \doteq 0$ means "the limit, as Δx approaches 0, of . . ." [A constant c is said to be the **limit** of a variable u if, whenever any quantity m has been assigned, there is a stage in the variation-process beyond which $|c - u|$ is always less than m; or, briefly, c is the limit of u if the difference between c and u can be made to become and remain as small as we please.]

To find the derivative of a given function at a given point: (1) If the function is given only by a curve, measure graphically the slope of the tangent at the point in question; (2) if the function is given by a mathematical expression, use the following rules for differentiation. These rules give, directly, the differential, dy, in terms of dx; to find the derivative, dy/dx, divide through by dx.

Rules for Differentiation. (Here $u, v, w, \ldots$ represent any functions of a variable x, or may themselves be independent variables. a is a constant which does not change in value in the same discussion; $e = 2.71828$.)

1. $d(a + u) = du$. 2. $d(au) = adu$.
3. $d(u + v + w + \ldots) = du + dv + dw + \ldots$
4. $d(uv) = udv + vdu$.
5. $d(uvw\ldots) = (uvw\ldots)\left(\frac{du}{u} + \frac{dv}{v} + \frac{dw}{w} + \ldots\right)$
6. $d\frac{u}{v} = \frac{udv - vdu}{v^2}$.
7. $d(u^m) = mu^{m-1}du$.

Thus, $d(u^2) = 2udu$; $d(u^3) = 3u^2du$; etc.

8. $d\sqrt{u} = \dfrac{du}{2\sqrt{u}}$. 9. $d\left(\dfrac{1}{u}\right) = -\dfrac{du}{u^2}$.

10. $d(e^u) = e^u du$. 11. $d(a^u) = (\log_e a)a^u du$.

12. $d \log_e u = \dfrac{du}{u}$. 13. $d \log_{10} u = (\log_{10} e)\dfrac{du}{u} = (0.4343\ldots)\dfrac{du}{u}$.

14. $d \sin u = \cos u\, du$. 15. $d \csc u = -\cot u \csc u\, du$.

16. $d \cos u = -\sin u\, du$. 17. $d \sec u = \tan u \sec u\, du$.

18. $d \tan u = \sec^2 u\, du$. 19. $d \cot u = -\csc^2 u\, du$.

20. $d \sin^{-1} u = \dfrac{du}{\sqrt{1-u^2}}$. 21. $d \csc^{-1} u = -\dfrac{du}{u\sqrt{u^2-1}}$.

22. $d \cos^{-1} u = -\dfrac{du}{\sqrt{1-u^2}}$. 23. $d \sec^{-1} u = \dfrac{du}{u\sqrt{u^2-1}}$.

24. $d \tan^{-1} u = \dfrac{du}{1+u^2}$. 25. $d \cot^{-1} u = -\dfrac{du}{1+u^2}$.

26. $d \log_e \sin u = \cot u\, du$. 27. $d \log_e \tan u = \dfrac{2du}{\sin 2u}$.

28. $d \log_e \cos u = -\tan u\, du$. 29. $d \log_e \cot u = -\dfrac{2du}{\sin 2u}$.

30. $d \sinh u = \cosh u\, du$. 31. $d \operatorname{csch} u = -\operatorname{csch} u \coth u\, du$.

32. $d \cosh u = \sinh u\, du$. 33. $d \operatorname{sech} u = -\operatorname{sech} u \tanh u\, du$.

34. $d \tanh u = \operatorname{sech}^2 u\, du$. 35. $d \coth u = -\operatorname{csch}^2 u\, du$.

36. $d \sinh^{-1} u = \dfrac{du}{\sqrt{u^2+1}}$. 37. $d \operatorname{csch}^{-1} u = -\dfrac{du}{u\sqrt{u^2+1}}$.

38. $d \cosh^{-1} u = \dfrac{du}{\sqrt{u^2-1}}$. 39. $d \operatorname{sech}^{-1} u = -\dfrac{du}{u\sqrt{1-u^2}}$.

40. $d \tanh^{-1} u = \dfrac{du}{1-u^2}$. 41. $d \coth^{-1} u = \dfrac{du}{1-u^2}$.

42. $d(u^v) = (u^{v-1})(u \log_e u\, dv + v\, du)$.

Derivatives of Higher Orders. The derivative of the derivative is called the second derivative; the derivative of this, the third derivative; and so on. Notation: if $y = f(x)$,

$$f'(x) = D_x y = \frac{dy}{dx};\quad f''(x) = D_x^2 y = \frac{d^2y}{dx^2};\quad f'''(x) = D_x^3 y = \frac{d^3y}{dx^3};\quad \text{etc.}$$

NOTE. If the notation d^2y/dx^2 is used, this must not be treated as a fraction, like dy/dx but as an inseperable symbol, made up of a symbol of operation, d^2/dx^2, and an operand y

The geometric meaning of the second derivative is this: if the original function $y = f(x)$ is represented by a curve in the usual way, then at any point where $f''(x)$ is *positive*, the curve is *concave upward*, and at any point where $f''(x)$ is *negative*, the curve is *concave downward* (Fig. 2). When $f''(x) = 0$, the curve usually has a **point of inflection.**

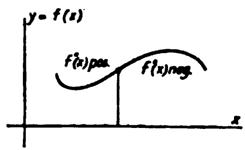


FIG. 2.

Differentials of Higher Orders. The differential of the differential is called the second differential; the differential of this, the third differential; etc. These quantities are of little importance except in the case where dx = a constant. In this case

$$dy = f'(x)dx; \quad d^2y = f''(x)\cdot(dx)^2; \quad d^3y = f'''(x)\cdot(dx)^3; \ . \ . \ .$$

The first, second, third, etc., differentials are close approximations to the first, second, third, etc., differences (p. 115), and are therefore sometimes useful in constructing tables. Thus, denoting the first, second, third, etc., differences by D', D'', D''', etc., and, assuming always that dx = a constant,

$$D' = dy + \tfrac{1}{2} d^2y + \tfrac{1}{6} d^3y + \tfrac{1}{24} d^4y + \ . \ . \ .; \quad d^3y = D''' - \tfrac{3}{2} D'''' + \ . \ . \ .$$
$$D'' = d^2y + d^3y + \tfrac{7}{12} d^4y + \ . \ . \ .; \quad d^2y = D'' - D''' + \tfrac{11}{12} D'''' + \ . \ . \ .$$
$$D''' = d^3y + \tfrac{3}{2} d^4y + \ . \ . \ .; \quad dy = D' - \tfrac{1}{2} D'' + \tfrac{1}{3} D''' - \tfrac{1}{4} D'''' + \ . \ . \ .$$

Functions of Two or More Variables may be denoted by $f(x, y, \ . \ . \ .)$, $F(x, y, \ . \ . \ .)$, etc. The derivative of such a function $u = f(x, y, \ . \ . \ .)$ formed on the assumption that x is the only variable (y, . . . being regarded for the moment as constants) is called the **partial derivative of u with respect to x**, and is denoted by $f_x(x,y)$, or D_xu, or $\dfrac{d_xu}{dx}$, or $\dfrac{\partial u}{\partial x}$. Similarly, the partial derivative of u with respect to y is $f_y(x,y)$, or D_yu, or $\dfrac{d_yu}{dy}$, or $\dfrac{\partial u}{\partial y}$.

NOTE. In the third notation, d_xu denotes the differential of u formed on the assumption that x is the only variable. If the fourth notation, $\partial u/\partial x$, is used, this must not be treated as a fraction like du/dx; the $\partial/\partial x$ is a symbol of operation, operating on u, and the "∂x" must not be separated.

Partial derivatives of the second order are denoted by f_{xx}, f_{xy}, f_{yy}, or by D_x^2u, $D_x(D_yu)$, D_y^2u, or by $\dfrac{\partial^2 u}{\partial x^2}$, $\dfrac{\partial^2 u}{\partial x \partial y}$, $\dfrac{\partial^2 u}{\partial y^2}$, the last symbols being "inseparable." Similarly for higher derivatives. Note that $f_{xy} = f_{yx}$.

If increments Δx, Δy, (or dx, dy) are assigned to the independent variables x, y, the increment, Δu, produced in $u = f(x,y)$ is

$$\Delta u = f(x + \Delta x, y + \Delta y) - f(x,y);$$

while the **differential**, du, that is, the value which Δu would have if the partial derivatives of u with respect to x and y were constant, is given by

$$du = (f_x)\cdot dx + (f_y)\cdot dy.$$

Here the coefficients of dx and dy are the values of the partial derivatives of u at the point in question.

If x and y are functions of a third variable t, then the equation

$$\frac{du}{dt} = (f_x)\,\frac{dx}{dt} + (f_y)\,\frac{dy}{dt}$$

expresses the rate of change of u with respect to t, in terms of the separate rates of change of x and y with respect to t.

For the graphical representation of $u = f(x,y)$, see p. 178.

Implicit Functions. If $f(x,y) = 0$, either of the variables x and y is said to be an implicit function of the other. To find dy/dx, either (1) solve for y in terms of x, and then find dy/dx directly; or (2) differentiate the equation through as it stands, remembering that both x and y are variables, and then divide by dx; or (3) use the formula $dy/dx = -(f_x/f_y)$, where f_x and f_y are the partial derivatives of $f(x,y)$ at the point in question.

MAXIMA AND MINIMA

A Function of One Variable, as $y = f(x)$, is said to have a **maximum** at a point $x = x_0$, if at that point the slope of the curve is zero and the concavity downward (see Fig. 3); a sufficient condition for a maximum is $f'(x_0) = 0$ and $f''(x_0)$ negative. Similarly, $f(x)$ has a **minimum** if the slope is zero and the concavity upward; a sufficient condition for a minimum is $f'(x_0) = 0$ and $f''(x_0)$ positive. If $f'(x_0) = 0$ and $f''(x_0) = 0$ and $f'''(x_0) \neq 0$, the point x_0 will be a **point of inflection.** If $f'(x_0) = 0$ and $f''(x_0) = 0$ and $f'''(x_0) = 0$, the point x_0 will be a maximum if $f''''(x_0) < 0$, and a minimum if $f''''(x_0) > 0$. It is usually sufficient, however, in any practical case, to find the values of x which make $f'(x) = 0$, and then decide, from a general knowledge of the curve, which of these values (if any) give maxima or minima, without investigating the higher derivatives.

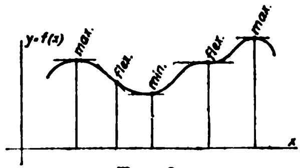


Fig. 3.

A Function of Two Variables, as $u = f(x,y)$, will have a **maximum** at a point (x_0,y_0) if at that point $f_x = 0$, $f_y = 0$, and $f_{xx} < 0$, $f_{yy} < 0$; and a **minimum** if at that point $f_x = 0$, $f_y = 0$, and $f_{xx} > 0$, $f_{yy} > 0$; provided, in each case, $(f_{xx})(f_{yy}) - (f_{xy})^2$ is positive. If $f_x = 0$ and $f_y = 0$, and f_{xx} and f_{yy} have opposite signs, the point (x_0,y_0) will be a "saddle point" of the surface representing the function (p. 178).

EXPANSION IN SERIES

The range of values of x for which each of the series is convergent is stated at the right of the series.

Arithmetical and Geometrical Series, and the **Binomial Theorem.** See p. 114.

Exponential and Logarithmic Series.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots; \qquad -\infty < x < +\infty.$$

$$a^x = e^{mx} = 1 + \frac{m}{1!}x + \frac{m^2}{2!}x^2 + \frac{m^3}{3!}x^3 + \cdots; \quad a > 0, \quad -\infty < x < +\infty,$$

where $m = \log_e a = (2.3026)(\log_{10} a)$.

$$\log_e(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots; \qquad -1 < x < +1.$$

$$\log_e(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots; \qquad -1 < x < +1.$$

$$\log_e\left(\frac{1 + x}{1 - x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right); \qquad -1 < x < +1.$$

$$\log_e\left(\frac{x + 1}{x - 1}\right) = 2\left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \frac{1}{7x^7} + \cdots\right); \qquad x < -1 \text{ or } +1 < x.$$

$$\log_e x = 2\left[\frac{x - 1}{x + 1} + \frac{1}{3}\left(\frac{x - 1}{x + 1}\right)^3 + \frac{1}{5}\left(\frac{x - 1}{x + 1}\right)^5 + \cdots\right]; \qquad 0 < x < \infty.$$

$$\log_e(a + x) = \log_e a + 2\left[\frac{x}{2a + x} + \frac{1}{3}\left(\frac{x}{2a + x}\right)^3 + \frac{1}{5}\left(\frac{x}{2a + x}\right)^5 + \cdots\right]; \quad \begin{cases} 0 < a < +\infty \\ -a < x < +\infty \end{cases}$$

Series for the Trigonometric Functions. In the following formulæ, *all angles must be expressed in radians.* If D = the number of degrees in the angle, and x = its radian measure, then $x = 0.017453\,D$.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots; \qquad -\infty < x < +\infty.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots; \qquad -\infty < x < +\infty.$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \ldots; \qquad -\pi/2 < x < +\pi/2.$$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} - \ldots; \qquad -\pi < x < +\pi.$$

$$\sin^{-1} y = y + \frac{y^3}{6} + \frac{3y^5}{40} + \frac{5y^7}{112} + \ldots; \qquad -1 \leqq y \leqq +1.$$

$$\tan^{-1} y = y - \frac{y^3}{3} + \frac{y^5}{5} - \frac{y^7}{7} + \ldots; \qquad -1 \leqq y \leqq +1.$$

$\cos^{-1} y = \frac{1}{2}\pi - \sin^{-1} y; \qquad \cot^{-1} y = \frac{1}{2}\pi - \tan^{-1} y.$

Series for the Hyperbolic Functions (x a pure number).

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots; \qquad -\infty < x < \infty.$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots; \qquad -\infty < x < \infty.$$

$$\sinh^{-1} y = y - \frac{y^3}{6} + \frac{3y^5}{40} - \frac{5y^7}{112} + \ldots; \qquad -1 < y < +1.$$

$$\tanh^{-1} y = y + \frac{y^3}{3} + \frac{y^5}{5} + \frac{y^7}{7} + \ldots; \qquad -1 < y < +1.$$

General Formulæ of Maclaurin and Taylor. If $f(x)$ and all its derivatives are continuous in the neighborhood of the point $x = 0$ (or $x = a$), then, for any value of x in this neighborhood, the function $f(x)$ may be expressed as a power series arranged according to ascending powers of x (or of $x - a$), as follows:

$$(1)\ f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots$$
$$+ \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + (P_n)x^n. \quad \text{(Maclaurin.)}$$

$$(2)\ f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$
$$+ \frac{f^{(n-1)}(a)}{(n-1)!}(x-a)^{n-1} + (Q_n)(x-a)^n. \quad \text{(Taylor.)}$$

Here $(P_n)x^n$, or $(Q_n)(x-a)^n$, is called the **remainder term**; the values of the coefficients P_n and Q_n may be expressed as follows:

$P_n = \{f^{(n)}(sx)\}/n! = \{(1-t)^{n-1}\ f^{(n)}(tx)\}/(n-1)!$

$Q_n = \{f^{(n)}[a + s(x-a)]\}/n! = \{(1-t)^{n-1} f^{(n)}[a + t(x-a)]\}/(n-1)!$

where s and t are certain unknown numbers between 0 and 1; the s-form is due to Lagrange, the t-form to Cauchy.

The error due to neglecting the remainder term is less than $(\bar{P}_n)x^n$, or

$(\bar{Q}_n)(x-a)^n$, where $\bar{P}_n$, or $\bar{Q}_n$, is the largest value taken on by P_n, or Q_n, when s or t ranges from 0 to 1. If this error, which depends on both n and x, approaches 0 as n increases (for any given value of x), then the general-expression-with-remainder becomes (for that value of x) a convergent infinite series.

The sum of the first few terms of Maclaurin's series gives a good approximation to $f(x)$ for values of x near $x = 0$; Taylor's series gives a similar approximation for values near $x = a$.

Fourier's Series. Let $f(x)$ be a function which is finite in the interval from $x = -c$ to $x = +c$ and has only a finite number of discontinuities in that interval (see note below), and only a finite number of maxima and minima. Then, for any value of x between $-c$ and c,

$$f(x) = \tfrac{1}{2} a_0 + a_1 \cos\frac{\pi x}{c} + a_2 \cos\frac{2\pi x}{c} + a_3 \cos\frac{3\pi x}{c} + \ldots$$

$$+ b_1 \sin\frac{\pi x}{c} + b_2 \sin\frac{2\pi x}{c} + b_3 \sin\frac{3\pi x}{c} + \ldots$$

where the constant coefficients are determined as follows:

$$a_n = \frac{1}{c}\int_{-c}^{c} f(t)\cos\frac{n\pi t}{c}\,dt, \quad b_n = \frac{1}{c}\int_{-c}^{c} f(t)\sin\frac{n\pi t}{c}\,dt.$$

In case the curve $y = f(x)$ is symmetrical with respect to the origin, the a's are all zero, and the series is a sine series. In case the curve is symmetrical with respect to the y-axis, the b's are all zero, and a cosine series results. (In this case, the series will be valid not only for values of x between $-c$ and c, but also for $x = -c$ and $x = c$.) A Fourier's series can be integrated term by term; but the result of differentiating term by term will in general not be a convergent series.

NOTE. If $x = x_0$ is a point of discontinuity, $f(x_0)$ is to be defined as $\tfrac{1}{2}[f_1(x_0) + f_2(x_0)]$, where $f_1(x_0)$ is the limit of $f(x)$ when x approaches x_0 from below, and $f_2(x_0)$ is the limit of $f(x)$ when x approaches x_0 from above.

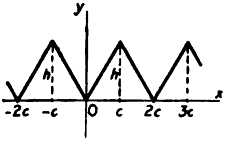


FIG. 4.

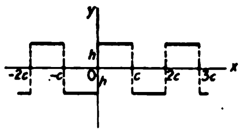


FIG. 5.

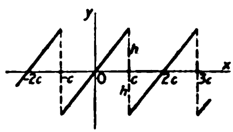


FIG. 6.

Examples of Fourier's Series.

1. If $y = f(x)$ is the curve in Fig. 4,

$$y = \frac{h}{2} - \frac{4h}{\pi^2}\left(\cos\frac{\pi x}{c} + \frac{1}{9}\cos\frac{3\pi x}{c} + \frac{1}{25}\cos\frac{5\pi x}{c} + \ldots\right)$$

2. If $y = f(x)$ is the curve in Fig. 5,

$$y = \frac{4h}{\pi}\left(\sin\frac{\pi x}{c} + \frac{1}{3}\sin\frac{3\pi x}{c} + \frac{1}{5}\sin\frac{5\pi x}{c} + \ldots\right)$$

3. If $y = f(x)$ is the curve in Fig. 6,

$$y = \frac{2h}{\pi}\left(\sin\frac{\pi x}{c} - \frac{1}{2}\sin\frac{2\pi x}{c} + \frac{1}{3}\sin\frac{3\pi x}{c} - \ldots\right)$$

INDETERMINATE FORMS

In the following paragraphs, $f(x)$, $g(x)$ denote functions which approach 0; $F(x)$, $G(x)$ functions which increase indefinitely; and $U(x)$ a function which approaches 1; when x approaches a definite quantity a. The problem in each case is to find the limit approached by certain combinations of these functions when x approaches a. The symbol $\doteq$ is to be read "approaches."

CASE 1. "$\frac{0}{0}$." To find the limit of $f(x)/g(x)$ when $f(x) \doteq 0$ and $g(x) \doteq 0$, use the theorem that $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$, where $f'(x)$ and $g'(x)$ are the derivatives of $f(x)$ and $g(x)$. This second limit may be easier to find than the first. If $f'(x) \doteq 0$ and $g'(x) \doteq 0$, apply the same theorem a second time: $\lim \frac{f'(x)}{g'(x)} = \lim \frac{f''(x)}{g''(x)}$; and so on.

CASE 2. "$\frac{\infty}{\infty}$." If $F(x) \doteq \infty$ and $G(x) \doteq \infty$, then $\lim \frac{F(x)}{G(x)} = \lim \frac{F'(x)}{G'(x)}$, precisely as in Case 1.

CASE 3. "$0 \cdot \infty$." To find the limit of $f(x) \cdot F(x)$ when $f(x) \doteq 0$ and $F(x) \doteq \infty$, write $\lim\ [f(x) \cdot F(x)] = \lim \frac{f(x)}{1/F(x)}$, or $= \lim \frac{F(x)}{1/f(x)}$; then proceed as in Case 1 or Case 2.

CASE 4. "0^0." If $f(x) \doteq 0$ and $g(x) \doteq 0$, find $\lim\ [f(x)]^{g(x)}$ as follows: let $y = [f(x)]^{g(x)}$, and take the logarithm of both sides thus:

$$\log_e y = g(x) \log_e f(x);$$

next, find $\lim\ [g(x) \log_e f(x)]$, $= m$, by Case 3; then $\lim y = e^m$.

CASE 5. "1^∞." If $U(x) \doteq 1$ and $F(x) \doteq \infty$, find $\lim\ [U(x)]^{F(x)}$ as follows: let $y = [U(x)]^{F(x)}$, and take the logarithm of both sides, as in Case 4.

CASE 6. "∞^0." If $F(x) \doteq \infty$ and $f(x) \doteq 0$, find $\lim\ [F(x)]^{f(x)}$ as follows: let $y = [F(x)]^{f(x)}$, and take the logarithm of both sides, as in Case 4.

CASE 7. "$\infty - \infty$." If $F(x) \doteq \infty$ and $G(x) \doteq \infty$, write $\lim\ [F(x) - G(x)]$

$$= \lim \frac{\frac{1}{G(x)} - \frac{1}{F(x)}}{\frac{1}{F(x) \cdot G(x)}};$$

then proceed as in Case 1. Sometimes it is shorter to expand the functions in series. It should be carefully noticed that expressions like $0/0$, ∞/∞, etc., do not represent mathematical quantities.

CURVATURE

The **radius of curvature** R of a plane curve at any point P (Fig. 7) is the distance, measured along the normal, on the concave side of the curve, to the **center of curvature,** C, this point being the limiting position of the point of intersection of the normals at P and a neighboring point Q, as Q is made to approach P along the curve. If the equation of the curve is $y = f(x)$,

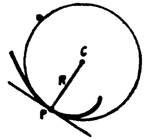


FIG. 7.

$$R = \frac{ds}{du} = \frac{[1 + (y')^2]^{3/2}}{y''}$$

where $ds = \sqrt{dx^2 + dy^2}$ = the differential of arc, $u = \tan^{-1}[f'(x)]$ = the angle which the tangent at P makes with the x-axis, and $y' = f'(x)$ and $y'' = f''(x)$ are the first and second derivatives of $f(x)$ at the point P. Note that $dx = ds \cos u$ and $dy = ds \sin u$. The **curvature**, K, at the point P, is $K = 1/R = du/ds$; that is, the curvature is the rate at which the angle u is changing with respect to the length of arc s. If the slope of the curve is small, $K = f''(x)$.

If the equation of the curve in polar co-ordinates is $r = f(\theta)$, where r = radius vector and θ = polar angle, then

$$R = \frac{[r^2 + (r')^2]^{3/2}}{r^2 - rr'' + 2(r')^2},$$

where $r' = f'(\theta)$ and $r'' = f''(\theta)$.

The **evolute** of a curve is the locus of its centers of curvature. If one curve is the evolute of another, the second is called the **involute** of the first.

INDEFINITE INTEGRALS

An integral of $f(x)dx$ is any function whose differential is $f(x)dx$, and is denoted by $\int f(x)dx$. All the integrals of $f(x)dx$ are included in the expression $\int f(x)dx + C$, where $\int f(x)dx$ is any particular integral, and C is an arbitrary constant. The process of finding (when possible) an integral of a given function consists in recognizing by inspection a function which, when differentiated, will produce the given function; or in transforming the given function into a form in which such recognition is easy. The most common integrable forms are collected in the following brief table; for a more extended list, see B. O. Peirce's "Table of Integrals" (Ginn & Co.).

General Formulæ

1. $\int a\,du = a\int du = au + C$ 2. $\int (u + v)dx = \int u\,dx + \int v\,dx$

3. $\int u\,dv = uv - \int v\,du$ 4. $\int f(x)dx = \int f[F(y)]F'(y)dy,\ x = F(y)$

5. $\int dy \int f(x,y)dx = \int dx \int f(x,y)dy.$

Fundamental Integrals

6. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$; when $n \neq -1$

7. $\int \frac{dx}{x} = \log_e x + C = \log_e cx$ 8. $\int e^x dx = e^x + C$

9. $\int \sin x\,dx = -\cos x + C$ 10. $\int \cos x\,dx = \sin x + C$

11. $\int \frac{dx}{\sin^2 x} = -\cot x + C$ 12. $\int \frac{dx}{\cos^2 x} = \tan x + C$

13. $\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x + C = -\cos^{-1} x + c$

14. $\int \frac{dx}{1 + x^2} = \tan^{-1} x + C = -\cot^{-1} x + c$

Rational Functions

15. $\int (a + bx)^n dx = \frac{(a + bx)^{n+1}}{(n + 1)b} + C$

16. $\int \frac{dx}{a+bx} = \frac{1}{b}\log_e (a+bx) + C = \frac{1}{b}\log_e c(a+bx)$

17. $\int \frac{1}{x^2}dx = -\frac{1}{x} + C$ 18. $\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)} + C$

19. $\int \frac{dx}{1-x^2} = \frac{1}{2}\log_e \frac{1+x}{1-x} + C = \tanh^{-1} x + C,$ when $x < 1$

20. $\int \frac{dx}{x^2-1} = \frac{1}{2}\log_e \frac{x-1}{x+1} + C = -\coth^{-1} x + C,$ when $x > 1$

21. $\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}\left(\sqrt{\frac{b}{a}}x\right) + C$

22. $\int \frac{dx}{a-bx^2} = \frac{1}{2\sqrt{ab}}\log_e \frac{\sqrt{ab}+bx}{\sqrt{ab}-bx} + C$

$= \frac{1}{\sqrt{ab}}\tanh^{-1}\left(\sqrt{\frac{b}{a}}x\right) + C$

when $a > 0,\quad b > 0$

23. $\int \frac{dx}{a+2bx+cx^2} = \frac{1}{\sqrt{ac-b^2}}\tan^{-1}\frac{b+cx}{\sqrt{ac-b^2}} + C$ when $ac - b^2 > 0$;

$= \frac{1}{2\sqrt{b^2-ac}}\log_e \frac{\sqrt{b^2-ac}-b-cx}{\sqrt{b^2-ac}+b+cx} + C$

$= -\frac{1}{\sqrt{b^2-ac}}\tanh^{-1}\frac{b+cx}{\sqrt{b^2-ac}} + C,$

when $b^2 - ac > 0$;

24. $\int \frac{dx}{a+2bx+cx^2} = -\frac{1}{b+cx} + C,$ when $b^2 = ac$

25. $\int \frac{(m+nx)dx}{a+2bx+cx^2} = \frac{n}{2c}\log_e (a+2bx+cx^2) + \frac{mc-nb}{c}\int \frac{dx}{a+2bx+cx^2}$

26. In $\int \frac{f(x)dx}{a+2bx+cx^2}$, if $f(x)$ is a polynominal of higher than the first degree, divide by the denominator before integrating.

27. $\int \frac{dx}{(a+2bx+cx^2)^p} = \frac{1}{2(ac-b^2)(p-1)} \times \frac{b+cx}{(a+2bx+cx^2)^{p-1}}$

$+ \frac{(2p-3)c}{2(ac-b^2)(p-1)}\int \frac{dx}{(a+2bx+cx^2)^{p-1}}$

28. $\int \frac{(m+nx)dx}{(a+2bx+cx^2)^p} = -\frac{n}{2c(p-1)} \times \frac{1}{(a+2bx+cx^2)^{p-1}}$

$+ \frac{mc-nb}{c}\int \frac{dx}{(a+2bx+cx^2)^p}$

29. $\int x^{m-1}(a+bx)^n dx = \frac{x^{m-1}(a+bx)^{n+1}}{(m+n)b}$

$- \frac{(m-1)a}{(m+n)b}\int x^{m-2}(a+bx)^n dx$

$= \frac{x^m(a+bx)^n}{m+n} + \frac{na}{m+n}\int x^{m-1}(a+bx)^{n-1} dx$

IRRATIONAL FUNCTIONS

30. $\int \sqrt{a+bx}\,dx = \frac{2}{3b}(\sqrt{a+bx})^3 + C$

31. $\int \frac{dx}{\sqrt{a+bx}} = \frac{2}{b}\sqrt{a+bx} + C$

32. $\int \frac{(m+nx)dx}{\sqrt{a+bx}} = \frac{2}{3b^2}(3mb - 2an + nbx)\sqrt{a+bx} + C$

33. $\int \frac{dx}{(m+nx)\sqrt{a+bx}}$; substitute $y = \sqrt{a+bx}$, and use 21 and 22

34. $\int \frac{f(x, \sqrt[n]{a+bx})}{F(x, \sqrt[n]{a+bx})}dx$; substitute $\sqrt[n]{a+bx} = y$

35. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C = -\cos^{-1}\frac{x}{a} + c$

36. $\int \frac{dx}{\sqrt{a^2+x^2}} = \log_e[x + \sqrt{a^2+x^2}] + C = \sinh^{-1}\frac{x}{a} + c$

37. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log_e[x + \sqrt{x^2-a^2}] + C = \cosh^{-1}\frac{x}{a} + c$

38. $\int \frac{dx}{\sqrt{a+2bx+cx^2}} = \frac{1}{\sqrt{c}}\log_e[b + cx + \sqrt{c}\sqrt{a+2bx+cx^2}] + C,$ when $c > 0$;

$= \frac{1}{\sqrt{c}}\sinh^{-1}\frac{b+cx}{\sqrt{ac-b^2}} + C,$ when $ac - b^2 > 0$;

$= \frac{1}{\sqrt{c}}\cosh^{-1}\frac{b+cx}{\sqrt{b^2-ac}} + C,$ when $b^2 - ac > 0$;

$= \frac{-1}{\sqrt{-c}}\sin^{-1}\frac{b+cx}{\sqrt{b^2-ac}} + C,$ when $c < 0$

39. $\int \frac{(m+nx)dx}{\sqrt{a+2bx+cx^2}} = \frac{n}{c}\sqrt{a+2bx+cx^2}$

$+ \frac{mc-nb}{c}\int \frac{dx}{\sqrt{a+2bx+cx^2}}$

40. $\int \frac{x^m dx}{\sqrt{a+2bx+cx^2}} = \frac{x^{m-1}X}{mc} - \frac{(m-1)a}{mc}\int \frac{x^{m-2}dx}{X}$

$- \frac{(2m-1)b}{mc}\int \frac{x^{m-1}dx}{X},$ where $X = \sqrt{a+2bx+cx^2}$

41. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log_e(x + \sqrt{a^2+x^2}) + C$

$= \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a} + c$

42. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

43. $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log_e\left(x + \sqrt{x^2 - a^2}\right) + C$

$= \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a} + c$

44. $\int \sqrt{a + 2bx + cx^2}\, dx = \frac{b + cx}{2c}\sqrt{a + 2bx + cx^2}$

$+ \frac{ac - b^2}{2c}\int \frac{dx}{\sqrt{a + 2bx + cx^2}} + C$

Transcendental Functions

45. $\int a^x dx = \frac{a^x}{\log_e a} + C$

46. $\int x^n e^{ax}\, dx = \frac{x^n e^{ax}}{a}\left[1 - \frac{n}{ax} + \frac{n(n-1)}{a^2x^2} - \ldots \pm \frac{n!}{a^n x^n}\right] + C$

47. $\int \log_e x\, dx = x \log_e x - x + C$

48. $\int \frac{\log_e x}{x^2}\, dx = -\frac{\log_e x}{x} - \frac{1}{x} + C$

49. $\int \frac{(\log_e x)^n}{x}\, dx = \frac{1}{n+1}(\log_e x)^{n+1} + C$

50. $\int \sin^2 x\, dx = -\tfrac{1}{4}\sin 2x + \tfrac{1}{2}x + C = -\tfrac{1}{2}\sin x \cos x + \tfrac{1}{2}x + C$

51. $\int \cos^2 x\, dx = \tfrac{1}{4}\sin 2x + \tfrac{1}{2}x + C = \tfrac{1}{2}\sin x \cos x + \tfrac{1}{2}x + C$

52. $\int \sin mx\, dx = -\frac{\cos mx}{m} + C$ 53. $\int \cos mx\, dx = \frac{\sin mx}{m} + C$

54. $\int \sin mx \cos nx\, dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C$

55. $\int \sin mx \sin nx\, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C$

56. $\int \cos mx \cos nx\, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C$

57. $\int \tan x\, dx = -\log_e \cos x + C$ 58. $\int \cot x\, dx = \log_e \sin x + C$

59. $\int \frac{dx}{\sin x} = \log_e \tan\frac{x}{2} + C$ 60. $\int \frac{dx}{\cos x} = \log_e \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C$

61. $\int \frac{dx}{1 + \cos x} = \tan\frac{x}{2} + C$ 62. $\int \frac{dx}{1 - \cos x} = -\cot\frac{x}{2} + C$

63. $\int \sin x \cos x\, dx = \tfrac{1}{2}\sin^2 x + C$ 64. $\int \frac{dx}{\sin x \cos x} = \log_e \tan x + C$

65.* $\int \sin^n x\, dx = -\frac{\cos x \sin^{n-1} x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$

66.* $\int \cos^n x\, dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$

* If n is an odd number, substitute $\cos x = z$ or $\sin x = z$.

67. $\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$

68. $\int \cot^n x\,dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\,dx$

69. $\int \frac{dx}{\sin^n x} = -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2} x}$

70. $\int \frac{dx}{\cos^n x} = \frac{\sin x}{(n-1)\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} x}$

71.* $\int \sin^p x \cos^q x\,dx = \frac{\sin^{p+1} x \cos^{q-1} x}{p+q} + \frac{q-1}{p+q}\int \sin^p x \cos^{q-2} x\,dx$

$= -\frac{\sin^{p-1} x \cos^{q+1} x}{p+q} + \frac{p-1}{p+q}\int \sin^{p-2} x \cos^q x\,dx$

72.* $\int \sin^{-p} x \cos^q x\,dx = -\frac{\sin^{-p+1} x \cos^{q+1} x}{p-1} + \frac{p-q-2}{p-1}\int \sin^{-p+2} x \cos^q x\,dx$

73.* $\int \sin^p x \cos^{-q} x\,dx = \frac{\sin^{p+1} x \cos^{-q+1} x}{q-1} + \frac{q-p-2}{q-1}\int \sin^p x \cos^{-q+2} x\,dx$

74. $\int \frac{dx}{a + b\cos x} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\frac{a-b}{a+b}} \tan \tfrac{1}{2}x\right) + C$, when $a^2 > b^2$

$$\left.\begin{aligned} &= \frac{1}{\sqrt{b^2 - a^2}} \log_e \frac{b + a\cos x + \sin x\sqrt{b^2 - a^2}}{a + b\cos x} + C, \\ &= \frac{2}{\sqrt{b^2 - a^2}} \tanh^{-1}\left(\sqrt{\frac{b-a}{b+a}} \tan \tfrac{1}{2}x\right) + C, \end{aligned}\right\}\ \text{when } a^2 < b^2$$

75. $\int \frac{\cos x\,dx}{a + b\cos x} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a + b\cos x} + C$

76. $\int \frac{\sin x\,dx}{a + b\cos x} = -\frac{1}{b} \log_e (a + b\cos x) + C$

77. $\int \frac{A + B\cos x + C\sin x}{a + b\cos x + c\sin x}\,dx = A\int \frac{dy}{a + p\cos y}$

$+ (B\cos u + C\sin u)\int \frac{\cos y\,dy}{a + p\cos y} - (B\sin u - C\cos u)\int \frac{\sin y\,dy}{a + p\cos y},$

where $b = p\cos u$, $c = p\sin u$ and $x - u = y$.

78. $\int e^{ax} \sin bx\,dx = \frac{a\sin bx - b\cos bx}{a^2 + b^2} e^{ax} + C$

79. $\int e^{ax} \cos bx\,dx = \frac{a\cos bx + b\sin bx}{a^2 + b^2} e^{ax} + C$

80. $\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1 - x^2} + C$

81. $\int \cos^{-1} x\,dx = x\cos^{-1} x - \sqrt{1 - x^2} + C$

82. $\int \tan^{-1} x\,dx = x\tan^{-1} x - \tfrac{1}{2}\log_e (1 + x^2) + C$

83. $\int \cot^{-1} x\,dx = x\cot^{-1} x + \tfrac{1}{2}\log_e (1 + x^2) + C$

* If p or q is an odd number, substitute $\cos x = z$ or $\sin x = z$

84. $\int \sinh x\,dx = \cosh x + C$
85. $\int \tanh x\,dx = \log_e \cosh x + C$
86. $\int \cosh x\,dx = \sinh x + C$
87. $\int \coth x\,dx = \log_e \sinh x + C$
88. $\int \operatorname{sech} x\,dx = 2\tan^{-1}(e^x) + C$
89. $\int \operatorname{csch} x\,dx = \log_e \tanh(x/2) + C$
90. $\int \sinh^2 x\,dx = \tfrac{1}{2}\sinh x \cosh x - \tfrac{1}{2}x + C$
91. $\int \cosh^2 x\,dx = \tfrac{1}{2}\sinh x \cosh x + \tfrac{1}{2}x + C$
92. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$
93. $\int \operatorname{csch}^2 x\,dx = -\coth x + C$

DEFINITE INTEGRALS

The definite integral of $f(x)dx$ from $x = a$ to $x = b$, denoted by $\int_a^b f(x)dx$, is the limit (as n increases indefinitely) of a sum of n terms:

$$\int_a^b f(x)dx = \lim_{n=\infty} [f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \ldots + f(x_n)\Delta x],$$

built up as follows: Divide the interval from a to b into n equal parts, and call each part Δx, $= (b - a)/n$; in each of these intervals take a value of x (say $x_1, x_2, \ldots x_n$), find the value of the function $f(x)$ at each of these points, and multiply it by Δx, the width of the interval; then take the limit of the sum of the terms thus formed, when the number of terms increases indefinitely, while each individual term approaches zero.

Geometrically, $\int_a^b f(x)dx$ is the area bounded by the curve $y = f(x)$, the x-axis, and the ordinates $x = a$ and $x = b$ (Fig. 8); that is, briefly, the "area under the curve, from a to b." The **fundamental theorem** for the evaluation of a definite integral is the following:

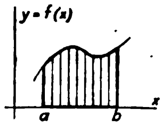


FIG. 8.

$$\int_a^b f(x)dx = \left[\int f(x)dx\right]_{x=b} - \left[\int f(x)dx\right]_{x=a};$$

that is, the definite integral is equal to the difference between two values of any one of the indefinite integrals of the function in question. In other words, the limit of a sum can be found whenever the function can be integrated.

Properties of Definite Integrals.

$$\int_a^b = -\int_b^a; \quad \int_a^c + \int_c^b = \int_a^b.$$

THE MEAN-VALUE THEOREM FOR INTEGRALS.

$$\int_a^b F(x)\ f(x)dx = F(X)\int_a^b f(x)dx,$$

provided $f(x)$ does not change sign from $x = a$ to $x = b$; here X is some (unknown) value of x intermediate between a and b.

THEOREM ON CHANGE OF VARIABLE. In evaluating $\int_{x=a}^{x=b} f(x)dx$, $f(x)dx$ may be replaced by its value in terms of a new variable t and dt, and $x = a$ and $x = b$ by the corresponding values of t, provided that throughout the interval the relation between x and t is a one-to-one correspondence (that is, to each value of x there corresponds one and only one value of t, and to each value of t there corresponds one and only one value of x).

Differentiation with Respect to the Upper Limit. If b is variable, then $\int_a^b f(x)dx$ is a function of b, whose derivative is

$$\frac{d}{db}\int_a^b f(x)dx = f(b).$$

Differentiation with Respect to a Parameter.

$$\frac{\partial}{\partial c}\int_a^b f(x,c)dx = \int_a^b \frac{\partial f(x,c)}{\partial c}dx.$$

Functions Defined by Definite Integrals. The following definite integrals have received special names, and their values have been tabulated; see, for example, B. O. Peirce's "Table of Integrals."

1. Elliptic integral of the first kind $= F(u, k) = \int_0^u \frac{dx}{\sqrt{1-k^2\sin^2 x}} (k^2 < 1)$

2. Elliptic integral of the second kind $= E(u, k) = \int_0^u \sqrt{1 - k^2 \sin^2 x}\, dx$ $(k^2 < 1)$

3, 4. Complete elliptic integrals of the first and second kinds; put $u = \pi/2$ in (1) and (2).

5. The Probability integral $= \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx$

6. The Gamma function $= \Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$

Approximate Methods of Integration. Mechanical Quadrature.

(1) Use Simpson's rule. See p. 106.

(2) Expand the function in a power series, and integrate term by term.

(3) Plot the area under the curve $y = f(x)$ from $x = a$ to $x = b$ on squared paper and measure this area roughly by "counting squares," or more accurately, by the use of a planimeter ($14 to $35; instruction for use with each instrument).

(4) Coradi's Mechanical Integraph ($240) provides a means of drawing on paper the curve $y = \int f(x)dx$, when the curve $y = f(x)$ is given, and can be used to facilitate the solution of certain differential equations. Full instructions for use with each instrument.

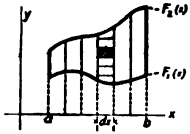


Fig. 9.

Double Integrals. The notation $\int\int f(x, y)dy\,dx$ means $\int\{\int f(x, y)dy\}dx$, the limits of integration in the inner, or first, integral being functions of x (or constants).

Example. To find the weight of a plane area whose density, w, is variable, say $w = f(x, y)$. The weight of a typical element, $dx\,dy$, is $f(x, y)dx\,dy$. Keeping x and dx constant, and summing these elements from, say, $y = F_1(x)$ to $y = F_2(x)$, as determined by the shape of the boundary, the weight of a typical strip perpendicular to the x-axis is $dx\int_{y=F_1(x)}^{y=F_2(x)} f(x,y)dy$. Finally, summing these strips from, say, $x = a$ to $x = b$, the weight of the whole area is $\int_{x=a}^{x=b}\{dx\int_{y=F_1(x)}^{y=F_2(x)} f(x, y)dy\}$, or, briefly, $\int\int f(x, y)dy\,dx$.

DIFFERENTIAL EQUATIONS

An **ordinary differential equation** is one which contains a single independent variable, or argument, and a single dependent variable, or function, with its derivatives of various orders. A **partial differential equation** is one which contains a function of several independent variables, and its partial derivatives of various orders. The order of a differential equation is the order of the highest derivative which occurs in it. A solution of a differential equation is any relation between the variables, which, when substituted in the given equation, will satisfy it. The general solution of an ordinary differential equation of the nth order will contain n arbitrary constants. A differential equation is usually said to be solved when the problem is reduced to a simple quadrature, that is, an integration of the form $y = \int f(x)dx$.

Methods of Solving Ordinary Differential Equations

Differential Equations of the First Order

(1) If possible, separate the variables; that is, collect all the x's and dx on one side, and all the y's and dy on the other side; then integrate both sides, and add the constant of integration.

(2) If the equation is homogeneous in x and y, the value of dy/dx in terms of x and y will be of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$. Substituting $y = xt$ will enable the variables to be separated. Solution: $\log_e x = \int \frac{dt}{f(t) - t} + C$.

(3) The expression $f(x,y)dx + F(x,y)dy$ is an *exact differential* if $\frac{\partial f(x,y)}{\partial y} = \frac{\partial F(x,y)}{\partial x} (= P$, say). In this case the solution of $f(x,y)dx + F(x,y)dy = 0$ is

$$\int f(x,y)dx + \int [F(x,y) - \int P dx]dy = C$$

or

$$\int F(x,y)dy + \int [f(x,y) - \int P dy]dx = C$$

(4) Linear differential equation of the first order: $\frac{dy}{dx} + f(x)\cdot y = F(x)$.

Solution: $y = e^{-P}\left\{\int e^P F(x)dx + C\right\}$, where $P = \int f(x)dx$.

(5) Bernoulli's equation: $\frac{dy}{dx} + f(x)\cdot y = F(x)\cdot y^n$. Substituting $y^{1-n} = v$ gives $\frac{dv}{dx} + (1 - n)f(x)\cdot v = (1 - n)F(x)$, which is linear in v and x.

(6) Clairaut's equation: $y = xp + f(p)$, where $p = dy/dx$. The solution consists of the family of lines given by $y = Cx + f(C)$, where C is any constant, together with the curve obtained by eliminating p between the equations $y = xp + f(p)$ and $x + f'(p) = 0$, where $f'(p)$ is the derivative of $f(p)$.

Differential Equations of the Second Order

(7) $\frac{d^2y}{dx^2} = -n^2y$. Solution: $y = C_1 \sin(nx + C_2)$

or $y = C_3 \sin nx + C_4 \cos nx$

(8) $\frac{d^2y}{dx^2} = +n^2y$. Solution: $y = C_1 \sinh (nx + C_2)$

or $y = C_3 e^{nx} + C_4 e^{-nx}$

(9) $\frac{d^2y}{dx^2} = f(y)$. Solution: $x = \int \frac{dy}{\sqrt{C_1 + 2P}} + C_2$, where $P = \int f(y)\, dy$.

(10) $\frac{d^2y}{dx^2} = f(x)$. Solution: $y = \int P dx + C_1 x + C_2$, where $P = \int f(x) dx$

or $y = xP - \int x f(x) dx + C_1 x + C_2$

(11) $\frac{d^2y}{dx^2} = f\left(\frac{dy}{dx}\right)$. Putting $\frac{dy}{dx} = z$, $\frac{d^2y}{dx^2} = \frac{dz}{dx}$, $x = \int \frac{dz}{f(z)} + C_1$ and $y = \int \frac{z dz}{f(z)} + C_2$; then eliminate z from these two equations.

(12) The equation for damped vibration: $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = 0$.

Case I. If $a^2 - b^2 > 0$, let $m = \sqrt{a^2 - b^2}$. Solution:

$y = C_1 e^{-bx} \sin (mx + C_2)$ or $y = e^{-bx}[C_3 \sin (mx) + C_4 \cos (mx)]$

Case II. If $a^2 - b^2 = 0$, solution is $y = e^{-bx}[C_1 + C_2 x]$.

Case III. If $a^2 - b^2 < 0$, let $n = \sqrt{b^2 - a^2}$. Solution:

$y = C_1 e^{-bx} \sinh (nx + C_2)$ or $y = C_3 e^{-(b+n)x} + C_4 e^{-(b-n)x}$

(13) $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = c$. Solution: $y = \frac{c}{a^2} + y_1$, where y_1 = the solution of the corresponding equation with second member zero [see (12) above].

(14) $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = c \sin(kx)$. Solution:

$y = R \sin(kx - S) + y_1$, where $R = c/\sqrt{(a^2 - k^2)^2 + 4b^2k^2}$,

$\tan S = \frac{2bk}{a^2 - k^2}$, and y_1 = the solution of the corresponding equation with second member zero [see (12) above].

(15) $\frac{d^2y}{dx^2} + 2b\frac{dy}{dx} + a^2y = f(x)$. Solution: $y = y_0 + y_1$, where y_0 = any particular solution of the given equation, and y_1 = the general solution of the corresponding equation with second member zero [see (12) above].

If $b^2 > a^2$, $y_0 = \frac{1}{2\sqrt{b^2 - a^2}}\left\{ e^{m_1 x}\int e^{-m_1 x} f(x)dx - e^{m_2 x}\int e^{-m_2 x} f(x)dx \right\}$.

where $m_1 = -b + \sqrt{b^2 - a^2}$ and $m_2 = -b - \sqrt{b^2 - a^2}$.

If $b^2 < a^2$, let $m = \sqrt{a^2 - b^2}$; then $y_0 =$

$\frac{1}{m}e^{-bx}\left\{ \sin (mx)\int e^{bx} \cos (mx)\cdot f(x)dx - \cos (mx)\int e^{bx} \sin (mx)\cdot f(x)\, dx \right\}$.

If $b^2 = a^2$, $y_0 = e^{-bx}\left\{ x\int e^{bx} f(x)\, dx - \int x\cdot e^{bx} f(x)\, dx \right\}$.

GRAPHICAL REPRESENTATION OF FUNCTIONS

For graphical methods in statistics, etc., see W. C. Brinton's "Graphical Methods for Presenting Facts"

EQUATIONS INVOLVING TWO VARIABLES

The Curve $y = f(x)$. To represent graphically any function, y, of a single variable, x, lay off the values of x as **abscissae** along a uniformly graduated horizontal axis, whose positive direction (as usually chosen) runs to the right, and at each point on this x-axis erect a perpendicular (called an **ordinate**) whose length represents the value of y at that point. The unit of measurement for the y-scale, whose positive direction (as usually chosen) runs upward, need not be the same as the unit for the x-scale. Draw a smooth curve through the extremities of the ordinates; this is the **graph** of the given function in rectangular co-ordinates, or the **curve** of the function.

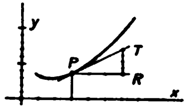


FIG. 1.

To measure graphically the rate of change of the function at any point P (Fig. 1), draw the tangent at P; then **rate of change** at $P = RT/PR$, where RT and PR are measured in units of the y-axis and x-axis, respectively. This ratio, which is positive if RT runs upward, negative if RT runs downward, is equal to the derivative of the function at the point P (see p. 157).

Graphs of Important Functions. Figs. 2–9 show the graphs (in rectangular co-ordinates) of the most important elementary functions, namely:

The **linear function,** $y = mx + b$ (Fig. 2).

The **power functions,** $y = x^n$ [n positive (parabolic type); n negative (hyperbolic type)] (Fig. 3).

The **exponential function,** $y = 10^x$ or $y = e^x$, and the **logarithmic function,** $y = \log_{10} x$ or $y = \log_e x$ (Fig. 4).

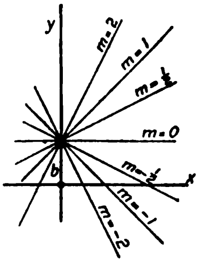
The **trigonometric functions** (Fig. 5), and the inverse trigonometric functions (Fig. 6).

The **hyperbolic functions** (Figs. 7 and 8) and the inverse hyperbolic functions (Fig. 9).

Various **special functions** (Figs. 10–12).

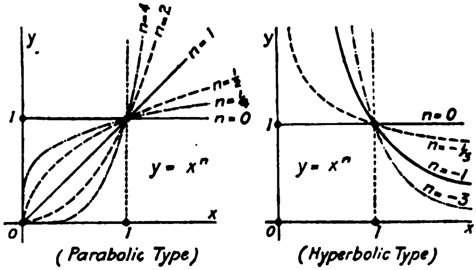
By a slight modification, each of these diagrams may be made to represent a somewhat more general function than that for which it is primarily intended. For, if x is replaced by $x - a$ in the equation, this merely requires re-numbering the x-axis so that each number is moved a units to the left; and similarly, if y is replaced by $y - b$ in the equation, this merely requires re-numbering the y-axis so that each number is moved b units downward. (Such a change is called a translation of the curve to the right, or upward.) Further, if x is replaced by x/c [or y by y/c] in the equation, it is merely necessary to multiply each of the numbers written along the x-axis [or y-axis] by c, in order to adapt the graph to the new equation. (Such a change is called a "stretching" of the curve along one of the axes.)

Empirical Curves. Any set of values of two variables x and y can be represented by plotting the points (x,y) on rectangular co-ordinate paper, and drawing a smooth curve through these points. The points which correspond to actual data should be clearly indicated by small circles or crosses, intermediate points being spoken of as interpolated points. While this process of graphically interpolating a continuous series of points *between* given values is usually fairly safe, the process of extrapolation—that is, extending the curve *beyond* the range of the given values, is dangerous.



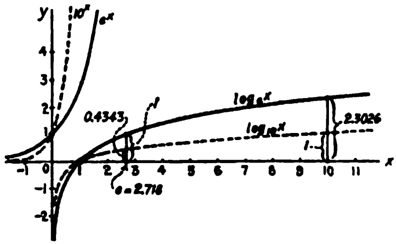
Linear function, $y = mx + b$.

FIG. 2.



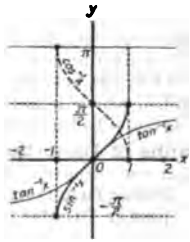
Power function, $y = x^n$.

FIG. 3.



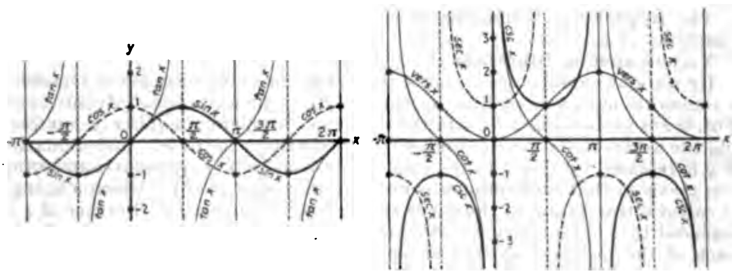
Exponential function (10^x or e^x).
Logarithmic function ($\log_{10} x$ or $\log_e x$).

FIG. 4.



Inverse trigonometric functions.

FIG. 6.

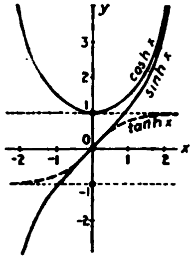


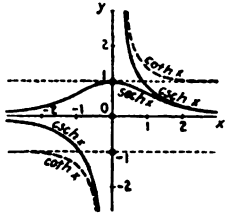
Trigonometric functions.

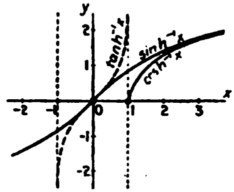
FIG. 5.

To Find a Mathematical Equation to Fit a Given Empirical Curve. This problem is one which in general requires much patience and ingenuity. Only the simplest cases can be mentioned here.

CASE 1. If the given empirical curve is a straight line, then the law connecting the given values of x and y is $y = mx + b$, where m = the slope of the line, and b = the value of y at the point where the line crosses the y-axis. If







Hyperbolic functions and inverse hyperbolic functions.

FIG. 7. FIG. 8. FIG. 9.

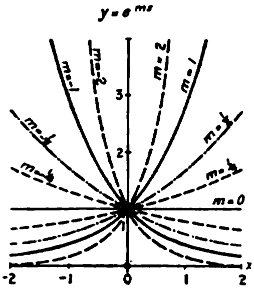


FIG. 10.

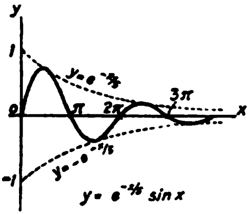


FIG. 11.

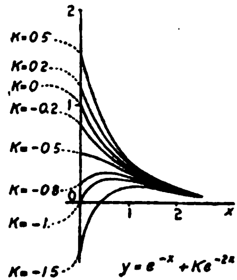


FIG. 12.

the points lie only approximately on a straight line, the best position for this line can usually be found by stretching a black thread among the points; or, assume a law of the form $y = mx + b$, and, by substituting in this formula n pairs of values of x and y, obtain n equations connecting the coefficients m and b; various pairs of these equations may then be solved for m and b, and the average of the results taken. Or, if great accuracy is required, all n of the equations may be solved for m and b by the method of least squares (p. 121).

If any law of the form $f(x,y) = m \cdot F(x,y) + b$ is suspected, where $f(x,y)$ and $F(x,y)$ are any expressions involving either x or y or both x and y, such a law may be tested by plotting $F(x,y)$ instead of x, and $f(x,y)$ instead of y, on rectangular cross-section paper, and seeing whether or not the points lie on a straight line. If they do, the form of the law is verified, and the values of m and b can be read from the figure as before. For example, if $y^2 = mxy + b$, a straight line will be obtained by plotting y^2 against xy. Again, if $xy = bx + my$, a straight line will be obtained by plotting y against y/x, since the equation may be written $y = b + m\,(y/x)$.

CASE 2. If a law of the form $y = cx^n$ is suspected, plot the points (x,y) on logarithmic paper (see below).

CASE 3. If a law of the form $y = c \cdot 10^{mx}$ [or $y = c \cdot e^{mx}$] is suspected, plot the points (x,y) on semi-logarithmic paper (see below).

CASE 4. If the given curve resembles the logarithmic curve, $y = \log x$, interchange x and y and proceed as in Case 3.

CASE 5. If the given curve is a wavy line, resembling a sine or cosine curve, try an equation of the form $y = a \sin bx$ or $y = a \cos bx$. If the heights of the waves diminish as x increases, try an equation of the form $y = ae^{-nx} \sin bx$. [NOTE. Any periodic function (satisfying certain simple conditions) can be expressed by a Fourier's series (p. 162)].

CASE 6. A great variety of functions can be represented approximately by a polynomial of the form $y = a + bx + cx^2 + dx^3 + ex^4 + \ldots$, the first three or four terms being usually sufficient. To determine the coefficients $a, b, c, \ldots$, most accurately, substitute in the formula all the given pairs of values of x and y, and solve the resulting equations for $a, b, c, \ldots$ by the method of least squares (p. 121).

CASE 7. Many simple curves can be represented approximately by an equation of the hyperbolic form, $xy = c + bx + ay$, where a, b, and c are determined by substituting the co-ordinates of three conspicuous points of the curve. The lines $x = a$ and $y = b$ are the asymptotes of the hyperbola. The equation may also be written $(x - a)(y - b) = k$, where $k = ab + c$.

Logarithmic Cross-section Paper. In this form of cross-section paper (Fig. 13), the distance from the origin to any point on the x- or y-axis is equal to the logarithm of the number written against that point. Thus, in Fig. 13 the distances (shown for clearness on two auxiliary scales X and Y) are the logarithms of the numbers written along x and y.

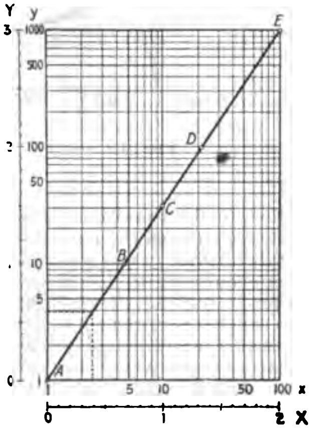


FIG. 13.

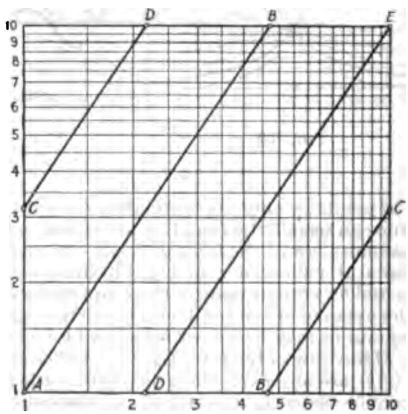


FIG. 14.

Accurately made logarithmic paper can be obtained from the principal dealers in draftmen's supplies. Logarithmic paper can be easily constructed, in case of need, by copying the logarithmic scale from any ordinary slide rule. The actual figures along the x- and y-axes are usually left for the user to insert; in so doing, notice that the numbers . . ., 0.01, 0.1, 1, 10, 100, . . ., or such of them as may be needed to cover any given range of values, must be placed at the points of division which separate the main squares. It is often convenient, however, to omit the decimal point, numbering each square independently from 1 to 10. The length of the side of one square is called the *unit* or *base* of the logarithmic paper; the larger the unit, the finer the possible subdivisions of the scale.

To plot a point (x,y) on logarithmic paper, for example, the point (3,5), means to find the point of intersection of the vertical line marked $x = 3$ and the horizontal line marked $y = 5$. In interpolating between two lines, account should be taken of the fact that the divisions are not of uniform length.

Any equation of the form $y = cx^n$ when plotted on logarithmic paper will be represented by a straight line whose slope is n. For, if $y_1 = cx_1^n$ and $y_2 = cx_2^n$, then $y_1/y_2 = (x_1/x_2)^n$, or $(\log y_1 - \log y_2)/(\log x_1 - \log x_2) = n$. The slope must be measured by aid of an auxiliary *uniform* scale.

EXAMPLE. Let $y = x^{3/2}$. When $x = 1$, $y = 1$; plot this point A on the logarithmic paper, and draw the straight line AE with a slope equal to $3/2$ (Fig. 13). By the aid of this line, the value of y for any value of x between 1 and 100 can be read off directly; for example, if $x = 2.50$, $y = 3.95$, as shown by dotted lines, so that $(2.50)^{3/2} = 3.95$. To find the value of y for any value of x outside this range, note that moving the decimal point 2 places in x is equivalent to moving it 3 places in y. The line shown in Fig. 13 is thus equivalent to a complete table of three-halves powers.

It will be noticed that this line crosses four squares of the logarithmic paper. By superposing these four squares the whole diagram may be condensed into a single square (Fig. 14), in which, however, the scales for x and y now give only the sequence of digits in the answer, the position of the decimal point having to be determined by inspection.

To determine whether a given set of values, x and y, satisfies a law of the form y = cx^n, plot the values on logarithmic paper, and see whether they lie on a straight line; if they do, then the given values satisfy a law of this form; moreover, the slope of the line gives the value of n, and the value of y when $x = 1$ gives the value of c.

If the plotted points fail to lie exactly in line, but form a curve slightly concave upward, try subtracting some constant b from all the y's, that is, move each point downward a distance equal to b units of the y-scale at that point. If it proves possible to choose b so that the resulting points lie in line, then the original values obey a law of the form $y - b = cx^n$, where n is again the slope of the line, and c is the value of $y - b$ when $x = 1$. (Conversely, if the curve is concave *downward*, try *adding* b to all the y's; that is, move each point *upward;* if the new points lie in line, the original values obey a law of the form $y + b = cx^n$.) Another method of "straightening" the curve consists of adding some constant, $\pm a$, to all the values of x, which has the effect of shifting all the points to the right or left (by varying amounts); if this method succeeds, the original values obey a law of the form $y = c(x + a)^n$.

Semi-logarithmic Cross-section Paper*. This form of paper (Fig. 15) has a logarithmic scale along y and a uniform scale along x. The "**scale value,**" k, of the paper is the number which stands, on the x-axis, at a distance from the origin equal to the width of one of the main horizontal strips. Thus, in Fig. 15, each number shown along the auxiliary scale Y is the logarithm of the corresponding number along y, and each number shown along the auxiliary scale X is $1/k$th of the corresponding number along x (here $k = 5$). The number k, which may be chosen at pleasure, should be taken equal to some simple integer, as 1, 2, or 5, or some integral power of 10.

In preparing the paper for use it is important to notice that the numbers . . ., 0.01, 0.1, 1, 10, 100, . . . (or such of them as may be needed in any given case) must be placed along the y-axis at the points which mark the main lines of division between the horizontal strips; while the numbers . . ., $-2k$, $-k$, 0, $+k$, $+2k$, . . . (or such of them as may be needed) must be placed along the x-axis at uniform intervals, each interval (from 0 to k, from k to $2k$, etc.) being equal to the width of one of the main horizontal strips. The width of one of these strips is called the *unit* or *base* of the semi-

*Made by the Educational Exhibition Co., 26 Custom House St., Providence, R. I.

logarithmic paper; the larger the unit, the finer the possible subdivisions of the scale.

To plot a point (x,y), as $x = 3$, $y = 5$, on semi-logarithmic paper means to find the point of intersection of the vertical line marked $x = 3$ with the horizontal line marked $y = 5$.

Any equation of the form $y = c\cdot 10^{mx}$ [or $y = c\cdot e^{mx}$] when plotted on semi-logarithmic paper with scale value k, will be represented by a straight line whose slope is km [or $0.4343\ km$.]. By a suitable choice of the scale value k, any given range of values of x can be brought within the size of the paper. Note that $e = 10^{0.4343}$.

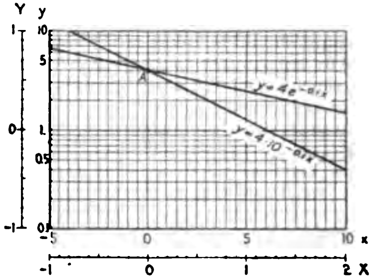


FIG. 15.

EXAMPLE. Given $y = 4\cdot 10^{-0.1x}$ [or $y = 4\cdot e^{-0.1x}$]. In Fig. 15, when $x = 0$, $y = 4$. By plotting this point (A) on the semi-logarithmic paper, with scale value 5, and drawing through it a straight line with slope equal to -0.5 [or -0.217] a graphical representation is obtained from which, for any value of x, the corresponding value of y can be read off. If it is desired to condense the figure, several horizontal strips may be superposed on a single strip; this of course renders the decimal point in the y-scale undetermined (unless a separate y-scale is provided for each section of the graph).

In order to determine whether a given set of values of x and y satisfy a law of the form $y = c\cdot 10^{mx}$ [or $y = c\cdot e^{mx}$], plot the values of x and y on semi-logarithmic paper, with a suitable scale value k, and see whether they lie on a straight line; if they do so, the law is satisfied, and the values of m and c may be found as follows: m = the slope of the line divided by k [or the slope of the line divided by $0.4343k$], and c = the value of y when $x = 0$.

If the plotted points fail to lie exactly in line, but form a curve slightly concave upward, try subtracting some constant b from all the y's, and plot the values thus modified; if b can be so chosen that the revised points lie in line, then the original values obey a law of the form $y - b = c\cdot 10^{mx}$ [or $y - b = c\cdot e^{mx}$], where m and c are to be found as before. If the curve is concave downward, add b, instead of subtracting; and replace $y - b$ by $y + b$ in the law.

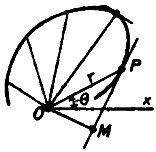


FIG. 16.

Curves in Polar Co-ordinates. Any function, r, of a single variable, θ, can be represented by a curve in polar co-ordinates (p. 137). Lay off the given values of θ as angles, the initial line Ox running toward the right, and the counterclockwise direction about the origin being taken as positive. Along the terminal side of each angle θ, lay off the corresponding value of r, forward if r is positive, backward if r is negative; and pass a smooth curve through the points thus determined.

The rate of change of r with respect to θ at a given point P is represented graphically as follows (Fig. 16): On the tangent at P drop a perpendicular OM from the origin; then $r(MP/OM)$ represents the rate of change, $dr/d\theta$, provided θ is measured in radians. Specially ruled polar co-ordinate paper is supplied by dealers in drafting supplies.

EQUATIONS INVOLVING THREE VARIABLES

The Surface $z = f(x, y)$. Any function, z, of two variables, x and y, may be represented by a surface, as follows: Plot the given pairs of values of x and y as points in a horizontal x, y plane, called the base plane; at each of these points erect an ordinate, parallel to a vertical axis z, and representing by its length the value of z at that point. Then conceive a smooth surface passed through the extremities of these ordinates: this surface is said to represent the function. In practice, the ordinates may be made by implanting stiff vertical rods in a horizontal board of soft wood which serves as the base plane; the surface may then be constructed by filling in the spaces with plaster of Paris. Or, more simply, pieces of cardboard may be cut out to represent parallel plane sections of the surface, and then stood on edge in slots cut in the board to receive them. The units employed along x, y, and z need not be equal to each other.

Contour-line Charts. All the points of a surface $z = f(x, y)$ which are at any given height above the base plane form a curve on the surface, called a contour line of the surface. If each of these contour lines be projected on the base plane, and each labeled with the value of z to which it corresponds, a complete representation of the function $z = f(x, y)$ is obtained, all in one plane. A topographical map, with contour lines showing elevations above the sea, and a weather map, with contour lines showing barometric pressure, are familiar examples. If there are several values of z corresponding to any given point (x, y), there will be several contour lines whose projections pass through that point.

Contour-line Charts for Simultaneous Equations [of the form $z = f(x,y)$, $w = F(x,y)$]. In Fig. 17, plot the function $z = f(x,y)$ by contour lines on an x,y plane, and plot the function $w = F(x,y)$ by contour lines on the same x,y plane. Then every point on the diagram (either directly or by interpolation) is the intersection of four curves—an x-curve, a y-curve, a z-curve, and a w-curve. Here, by "curve" is meant any line, straight or curved. By the aid of such a diagram, when the values of any two of these four variables are given, the values of the other two can be found. The method of use consists simply in entering the diagram along the two given curves (or lines), tracing them to their point of intersection, and then coming out again along the two curves (or lines) whose values are required. The best manner of numbering the curves is indicated in the figure.

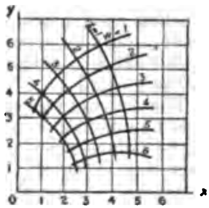


FIG. 17.

Alignment Charts for Three Variables, t, u, v. Any relation between three variables, t, u, v, which can be thrown into one of the forms listed in later paragraphs, can be represented graphically by a very convenient form of diagram called an alignment chart. In the simplest form of an alignment chart for three variables there are three scales (straight or curved), along which the values of the three variables, t, u, v, are marked in such a way that any three values of t, u, v which satisfy the given equation are represented by three points which lie in line. Hence, if the values of any two of the variables are given, the corresponding value of the third can be found by simply drawing a straight line through the two given points and reading the value of the point where it crosses the third scale.

The most important methods of constructing alignment charts for three variables are described below. Where several methods are applicable in a given case, the best one must be determined largely by trial. For further information see M. d'Ocagne, "Traité de Nomographie" (Gauthier-Villars, Paris); Carl Runge, "Graphical Methods" (Columbia University Press); J. B. Peddle, "Construction of Graphical Charts" (McGraw-Hill); see also page 185.

Notation. In each of the equations which follow, U stands for any function of u alone, V for any function of v alone, and $F_1(t)$, $F_2(t)$ for any functions of t alone. Any of these functions may reduce to a constant. The axes of x, y, and y' which are mentioned are of merely temporary use in constructing the diagram, and the letters x, y, y' should not be written on the chart. It is not necessary that the axes be at right angles, provided the x of a point is always measured parallel to the x-axis, and its y parallel to the y-axis.

Method 1. Given, an equation which can be thrown into the form

$$U \cdot F_1(t) + V \cdot F_2(t) = 1,$$

where, for the given range of values of u and v, the largest variations in U and V are less than a certain number m.

Draw a pair of (temporary) x,y axes (Fig. 18), and through the point $x = 1$ draw a third axis, which may be called the axis of y', parallel to the axis of y. In ordinary cases, the unit of measurement along x should be nearly equal to the full width of the paper. Now choose a unit for y and y' such that m times this unit will about equal the height of the paper, and plot, in the usual way, the points (x,y) given by

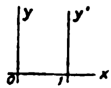


FIG. 18.

$$x = \frac{F_2(t)}{F_1(t) + F_2(t)}, \quad y = \frac{1}{F_1(t) + F_2(t)},$$

labeling each point with the value of t to which it corresponds. Connect these points by a smooth curve, which gives the t-scale of the diagram. [If $F_1(t)/F_2(t)$ = a constant, the t-scale will prove to be a straight line parallel to the y-axis.]

Then, using the same units as above, plot along y the points given by $y = U$, labeling each point with the corresponding value of u; and plot along y' the points given by $y' = V$, labeling each of these points with the corresponding value of v. This gives the u- and v-scales of the diagram. The three scales being thus constructed, the x-axis may now be erased, and the diagram is ready for use. Any three points t, u, v which lie in line correspond to three values of t, u, v, which satisfy the given equation. The numbering on each scale should be shown at sufficiently frequent intervals to permit of easy interpolation.

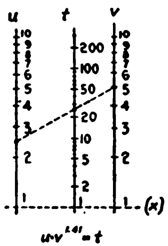


FIG. 19.

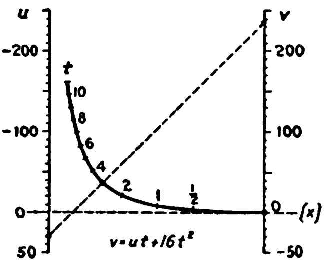


FIG. 20.

EXAMPLE 1 (Fig. 19). Let $uv^{1.41} = t$. By taking the logarithm of both sides, and dividing through by $\log t$, reduce the equation to the form $(\log u)\,(1/\log t) + (\log v) \times (1.41/\log t) = 1$. Here $U = \log u$, $V = \log v$, $F_1(t) = 1/\log t$, $F_2(t) = 1.41/\log t$, and $x = 1.41/2.41 = 0.585$, $y = (1/2.41)\log t$.

EXAMPLE 2 (Fig. 20). Let $v = ut + 16t^2$, which reduces to the form $(-u/16)(1/t) + (v/16)(1/t^2) = 1$. Here $U = -u/16$, $V = v/16$, $F_1(t) = 1/t$, $F_2(t) = 1/t^2$ and $x = 1/(1+t)$, $y = t^2/(1+t)$.

NOTE. If $m = \infty$, values of u and v which give large values of U and V cannot be shown within the limits of the paper. In such cases, the chart may be supplemented by a second chart, made according to Method 2, below.

Method 2. Given, an equation which can be thrown into the form

$$\frac{F_1(t)}{U} + \frac{F_2(t)}{V} = 1,$$

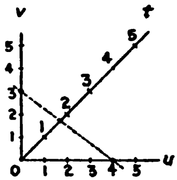
where, for the given range of values of u and v, the largest variation in U is less than a certain number m. and the largest variation in V is less than a certain number n.

Draw a pair of temporary x,y axes, and having chosen a unit for the x-axis equal to about $(1/m)$th of the width of the paper, and a unit for the y-axis equal to about $(1/n)$th of the height, plot the points (x,y) given by

$$x = F_1(t), \quad y = F_2(t),$$

labeling each point of this curve with the value of t to which it corresponds. Connect these points by a smooth curve, which gives the t-scale of the diagram. [If $F_1(t)/F_2(t) =$ a constant, the t-scale will be a straight line through the origin.]

Then, using the same units as above, plot along x the values of U, labeling each point with the corresponding value of u; and plot along y the values of V, labeling each point with the corresponding value of v. This gives the u- and v-scales of the diagram. On the chart as thus completed, any three points t, u, v which lie in line correspond to three values of t, u, v which satisfy the given equation.



$t = uv/(u+v)$

FIG. 21.

EXAMPLE (Fig. 21). Let $t = (uv)/(u+v)$, which may be written in the form $t/u + t/v = 1$. Here $U = u$, $V = v$, $F_1(t) = t$, $F_2(t) = t$.

NOTE. If $m = \infty$ and $n = \infty$, values of u and v which give large values of U and V cannot be shown within the limits of the paper. In such cases the chart may be supplemented by a second chart, made according to Method 1, above.

Method 3. Given, an equation which can conveniently be thrown into the form

$$F_2(t) = V \cdot F_1(t) + U,$$

where, for the given range of values of t, the largest variation in $F_1(t)$ is less than a certain number m, and the largest variation in $F_2(t)$ is less than a certain number n.

Draw a pair of temporary x,y axes, and, having chosen a unit for x equal to about $(1/m)$th of the width of the paper and a unit for y equal to about $(1/n)$th of the height, plot the points (x,y) given by

$$x = F_1(t), \quad y = F_2(t),$$

labeling each point of the curve with the value of t to which it corresponds. Connect these points by a smooth curve, which forms the t-scale. Next, using the same unit for y as above, plot along the y-axis the values of U, labeling each point with the corresponding value of u. This gives the u-scale. Finally, with the origin as center, and any convenient radius, draw a circle cutting the x-axis in A. Along this circular arc, starting from A in the counterclockwise direction, lay off the angles whose slopes are equal to V, labeling each point of the arc with the value of v to which it corresponds.

This gives the v-scale, which in this case, however, plays a peculiar rôle, since, in using this form of chart, two straight lines are required instead of one. Thus:

In order to determine whether three values, t, u, v, satisfy the given equation, lay one straight line through the points t and u, and another straight line through the point v and the origin; if these lines are parallel, the three values of t, u, v satisfy the equation. It will be noticed that the function of the v-scale here is to measure, in a certain sense, the slope of the line joining t and u. A chart of this type may be called "an alignment chart with a sliding scale for one of the variables."

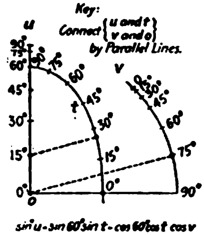


FIG. 22.

EXAMPLE (Fig. 22). Let $\sin u = \sin 60° \sin t - \cos 60° \cos t \cos v$, which may be put in the form

$$(\sin 60° \sin t) = \cos v\,(\cos 60° \cos t) + \sin u.$$

Here $F_1(t) = \cos 60° \cos t$, $F_2(t) = \sin 60° \sin t$, $U = \sin u$, $V = \cos v$.

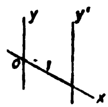


FIG. 23.

Method 4. Given, an equation which can be reduced to the form

$$U \cdot F(t) + V = 0,$$

where, for the given range of values of u and v, the largest variations in U and V are less than a certain number m.

In Fig. 23, draw temporary axes x, y, and y', and choose the units as in Method 1. To construct the t-scale, which will now coincide with the x-axis, plot along x the points for which

$$x = \frac{1}{1 + F(t)},$$

labeling each point with the value of t to which it corresponds. The u-scale, along the axis of y, and v-scale, along the axis of y', are constructed exactly as in Method 1, and the finished chart is used in the same way.

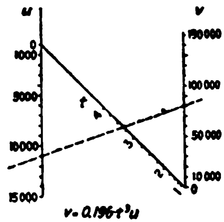


FIG. 24.

EXAMPLE (Fig. 24). Let $v = 0.196\, t^2 u$, where u is to range from 0 to 15,000 and v from 0 to 150,000. The equation may be written in the form $(-10\,u)(0.0196t^2) + v = 0$. Here $U = -10\,u$, $V = v$, $F(t) = 0.0196t^2$.

NOTE. If $m = \infty$, values of u and v which give large values of U and V cannot be shown within the limits of the paper.

EQUATIONS INVOLVING FOUR VARIABLES

[For simultaneous equations of the form $z = f(x,y)$, $w = F(x,y)$, see p. 179.]

Alignment Charts for Four Variables. The extension of the methods of the alignment chart to the case of four variables, say r, s, u, v, consists essentially in replacing the t-scale of the earlier diagram by a network of two scales, one for r and one for s. The point where a curve $r = r_1$ and a curve $s = s_1$ intersect may be spoken of as the point (r_1, s_1). In the following equations, U denotes as before any function of u alone, V any function of v alone; while $F_1(r,s)$ and $F_2(r,s)$ represent any functions of r and s.

Method 1a. Given, an equation of the form

$$U \cdot F_1(r,s) + V \cdot F_2(r,s) = 1.$$

Draw axes x, y, and y' as in Method 1, and plot the network of curves given by the equations

$$x = \frac{F_2(r,s)}{F_1(r,s) + F_2(r,s)}, \qquad y = \frac{1}{F_1(r,s) + F_2(r,s)}.$$

[To do this (Fig. 25), find the point (x,y) that corresponds to each given pair of values of r and s, by direct substitution in the equations for x and y. Connect all the points for which $r = 1$ by a curve, and label it $r = 1$; connect all the points for which $r = 2$ by another curve, and label it $r = 2$; etc. This gives the family of r-curves. Similarly, through all the points for which $s = 1$ draw a curve labeled $s = 1$; through all the points for which $s = 2$ draw a curve labeled $s = 2$; etc. This gives the family of s-curves, intersecting the family of r-curves. Note, however, that if it is possible to eliminate s (or r) from the equations that give x and y, the resulting equation in x, y, and r (or x, y, and s) can often be plotted directly for each given value of r (or of s).]

Next, construct the u- and v-scales along the axes of y and y' as in Method 1. [The letters x, y, and y', and the units used in plotting along these axes, should be omitted from the finished diagram, as should also the axis of x.]

In the chart, as thus completed, any three points, (r,s), u, and v which lie in a straight line, correspond to values of r, s, u, v which satisfy the given equation. Hence, when any three of these four values are given, the fourth can be found from the chart.

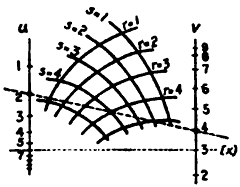


FIG. 25.

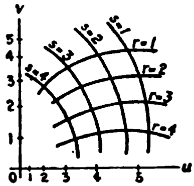


FIG. 26.

Method 2a. Given, an equation of the form

$$\frac{F_1(r,s)}{U} + \frac{F_2(r,s)}{V} = 1.$$

Draw axes of x and y as in Method 2, and plot the network of curves given by

$$x = F_1(r,s), \quad y = F_2(r,s).$$

To do this, follow the plan outlined for a similar case under Method 1a, labeling each curve of the r-family (Fig. 26) with the corresponding value of r, and each curve of the s-family with the corresponding value of s. Next, construct the u- and v-scales along the x- and y-axes, precisely as in Method 2. Then any three points, (r,s), u, and v, which lie in a straight line correspond to values of r, s, u, v which satisfy the given equation.

Method 3a. Given, an equation of the form

$$F_2(r,s) = V \cdot F_1(r,s) + U.$$

Draw axes of x and y, as in Method 3, and plot the network of curves given by $x = F_1(r,s)$, $y = F_2(r,s)$, following the plan outlined for a similar case under Method 1a, and labeling each curve of the r-family (or s-family) with the value of r (or s) to which it corresponds. Next, construct the u-scale along the y-axis, and the v-scale along a circular arc, precisely as in Method 3. Then any three points, (r,s) u, and v, which are so related that the line through (r,s) and u is parallel to the line joining v with the origin, will correspond to values of r, s, u, v which satisfy the given equation.

EXAMPLE for Method 3a (Fig. 27). Let $\cot v = \cot r \cos s + \csc r \sin s \cot u$, which may be written $(\cos r \cot s) = \cot v (\sin r \csc s) - \cot u$. Here $U = -\cot u$, $V = \cot v$, $F_1(r,s) = \sin r \csc s$, $F_2(r,s) = \cos r \cot s$, whence $\dfrac{x^2}{\csc^2 s} + \dfrac{y^2}{\cot^2 s} = 1$, $\dfrac{x^2}{\sin^2 r} - \dfrac{y^2}{\cos^2 r} = 1$, so that the s-curves are ellipses and the r-curves hyperbolas.

Parallel Charts, or Proportional Charts, for Four Variables. In the following methods of representation there are four scales, one for each of the four variables, and the method of using the diagram consists in connecting two pairs of points by parallel lines.

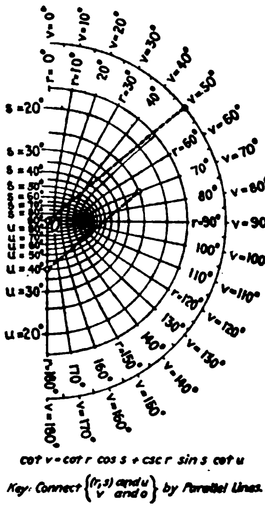


FIG. 27.

Method A. Given, an equation of the form

$$R - S = U - V$$

where R, S, U, V are any functions of the variables r, s, u, v, respectively. [It will be noted that any proportion $R/S = U/V$ can at once be thrown into this form by taking the logarithm of both sides.]

In Fig. 28, draw four vertical axes, y_1, y_2, y'_1, y'_2, such that the distance between y_1 and y'_1 (which may be zero) is equal to the distance beween y_2 and y'_2, and so that the four zero points lie in line. Along these axes, using the same unit for all, plot the points given by $y_1 = R$, $y'_1 = S$, $y_2 = U$, $y'_2 = V$, and label each point with the value of r, s, u, or v to which it corresponds. (The letters y_1, y_2, y'_1, y'_2 are temporary, and should not appear on the diagram.) Then if the line joining two points r and u is parallel to the line joining two points s and v, the four values of r, s, u, v will satisfy the given equation. In this and the following methods, a parallel ruler, or a pair of draftman's triangles, will be useful in reading the chart. A "key" stating which points are to be joined with which, should be clearly given on the diagram.

EXAMPLE (Fig. 28). Let $32.2\, vr = us^2$, or $\log r - 2 \log s = \log u - \log (32.2\, v)$. Here $R = \log r$, $S = 2 \log s$, $U = \log u$, $V = \log (32.2\, v)$.

Method B. Given, an equation of the form

$$\frac{R}{S} = \frac{U}{V}$$

In Fig. 29, draw a pair of axes, x,y, and parallel to them (or coinciding with them) a second pair of axes, x_1,y_1. Using any convenient horizontal unit, plot along x and x_1 the points given by $x = R$, $x_1 = U$, and using any convenient vertical unit, plot along y and y_1 the points given by $y = S$, $y_1 = V$. Label each point with the value of r, s, u, v, to which it corresponds. (The letters x, y, x_1, y_1 should not appear on the diagram.) Then if the line joining two points r and s is parallel to the line joining two points u and v, the four values r, s, u, v will satisfy the given equation.

Method C. Given, an equation of the form

$$R - S = \frac{V}{U}.$$

In Fig. 30, take a pair of axes, x,y, and through the point $x = 1$ draw a third axis, y', parallel to y. Also, take a second pair of axes, x_2,y_2, parallel to (or coinciding with) the axes of x and y. Having chosen a suitable unit for x and x_2, and a suitable unit for y, y', and y_2, lay off the values of R and

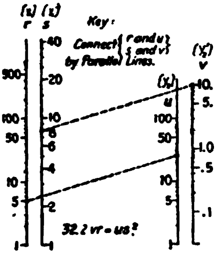


FIG. 28.

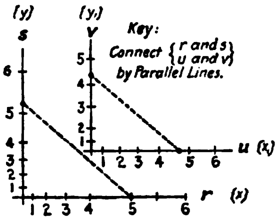


FIG. 29.

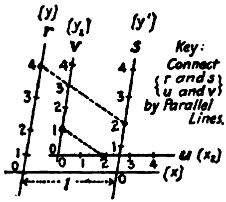


FIG. 30.

S along y and y', respectively, labeling each point with the value of r or s to which it corresponds; and lay off the values of U and V along x_2 and y_2, labeling each point with the value of u or v to which it corresponds. Then if the line joining two points r and s is parallel to the line joining two points u and v, the four values r, s, u, v will satisfy the given equation. This form of chart is sometimes called a "Z-chart."

For further examples, see R. C. Strachan, "Nomographic Solutions for Formulas of Various Types," Trans. Am. Soc. Civil Engineers, vol. 78, 1915.

VECTOR ANALYSIS

Many problems involving directed magnitudes can be advantageously treated by the methods of vector analysis. The following is a brief summary of the principal definitions and formulæ.

A set of arrows, each arrow having a given *length* and pointing in a given *direction*, is called a set of **vectors**, provided they combine by addition according to the parallelogram law (see below). Notation: $\mathbf{a}$ or $\bar{a}$ for a vector; a or $|\mathbf{a}|$ for its length. Two "free" vectors are equal if they have the same length and point in the same direction; two "sliding" vectors are equal if they have the same length and direction, and also lie in the same line.

A **scalar** is any real number, positive, negative, or zero.

Addition of vectors.—If an arrow $\mathbf{a}$ is immediately followed, tip to tail, by a second arrow $\mathbf{b}$, then the arrow which runs from the beginning of $\mathbf{a}$ to the end of $\mathbf{b}$ is called the **sum** of $\mathbf{a}$ and $\mathbf{b}$, denoted by $\mathbf{a} + \mathbf{b}$. Conversely, if $\mathbf{a} + \mathbf{x} = \mathbf{b}$, then $\mathbf{x} = \mathbf{b} - \mathbf{a}$. The laws of operation for $+$ and $-$ are the same as in ordinary algebra (pp. 112, 124). If m is a scalar, then $m\mathbf{a}$ means a vector having the same direction as $\mathbf{a}$, and m times its length.

Multiplication of vectors is of two kinds, as follows:

The **scalar product,** or dot product, of two vectors $\mathbf{a}$ and $\mathbf{b}$, denoted by $\mathbf{a}\cdot\mathbf{b}$—or sometmes by $S\mathbf{ab}$, or by $(\mathbf{ab})$ in round parentheses—is defined as the scalar quantity $ab \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

EXAMPLE. If $\mathbf{F}$ is a force whose point of application moves along a vector distance $\mathbf{x}$, then $\mathbf{F}\cdot\mathbf{x}$ = *work* done by $\mathbf{F}$ during this displacement.

Peculiarities of scalar products: (1) Since $\mathbf{a}\cdot\mathbf{b}$ is not a vector, expressions like $(\mathbf{a}\cdot\mathbf{b})\cdot\mathbf{c}$ will not occur; (2) from $\mathbf{a}\cdot\mathbf{x} = \mathbf{a}\cdot\mathbf{y}$ we cannot infer that $\mathbf{x} = \mathbf{y}$, hence, quotients will not occur; (3) from $\mathbf{a}\cdot\mathbf{b} = 0$, it follows that $\mathbf{a}$ is **perpendicular** to $\mathbf{b}$ (unless $\mathbf{a}$ or $\mathbf{b}$ is zero).

On the other hand, scalar products are like ordinary products in the following respects: $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$, and $(\mathbf{a} + \mathbf{b})\cdot(\mathbf{c} + \mathbf{d}) = \mathbf{a}\cdot\mathbf{c} + \mathbf{a}\cdot\mathbf{d} + \mathbf{b}\cdot\mathbf{c} + \mathbf{b}\cdot\mathbf{d}$; also, $m(\mathbf{a}\cdot\mathbf{b}) = (m\mathbf{a}\cdot\mathbf{b}) = \mathbf{a}\cdot(m\mathbf{b})$, where m is any scalar.

The **vector product,** or cross product, of two vectors $\mathbf{a}$ and $\mathbf{b}$, denoted by $\mathbf{a}\times\mathbf{b}$—or sometimes by $V\mathbf{ab}$, or by $[\mathbf{ab}]$ in square brackets—is defined as the *vector* whose length is $ab \sin\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, and whose direction is perpendicular to the plane of $\mathbf{a}$ and $\mathbf{b}$ (in such a sense that a right-handed screw advancing along $\mathbf{a}\times\mathbf{b}$ would turn $\mathbf{a}$ toward $\mathbf{b}$).

EXAMPLE. If $\mathbf{F}$ is a force acting on a particle whose radius vector is $\mathbf{r}$, then $\mathbf{r}\times\mathbf{F}$ = the *torque* of $\mathbf{F}$ about the origin.

Peculiarities of vector products: (1) $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$, so that the order of the factors is always important; (2) $\mathbf{a}\times\mathbf{a} = 0$; (3) it is not true that $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\times\mathbf{c}$; (4) from $\mathbf{a}\times\mathbf{x} = \mathbf{a}\times\mathbf{y}$ it does not follow that $\mathbf{x} = \mathbf{y}$; hence, quotients will not occur; (5) from $\mathbf{a}\times\mathbf{b} = 0$, it follows that $\mathbf{a}$ and $\mathbf{b}$ are **parallel** (unless $\mathbf{a}$ or $\mathbf{b}$ is zero).

On the other hand, as in ordinary algebra

$$(\mathbf{a} + \mathbf{b})\times(\mathbf{c} + \mathbf{d}) = \mathbf{a}\times\mathbf{c} + \mathbf{a}\times\mathbf{d} + \mathbf{b}\times\mathbf{c} + \mathbf{b}\times\mathbf{d},$$

provided the order of factors in each product is preserved; also, $m(\mathbf{a}\times\mathbf{b}) = (m\mathbf{a})\times\mathbf{b} = \mathbf{a}\times(m\mathbf{b})$, where m is any scalar. Further laws are:

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}); \text{ and } \mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}.$$

Vector Differentiation. If $\mathbf{r} = \mathbf{f}(t)$ gives a vector $\mathbf{r}$ as a function of a scalar t, then $d\mathbf{r}/dt = \lim\{[\mathbf{f}(t + \Delta t) - \mathbf{f}(t)]/\Delta t\}$ as Δt approaches zero.

$$d(\mathbf{a} + \mathbf{b}) = d\mathbf{a} + d\mathbf{b}, \quad d(m\mathbf{a}) = m(d\mathbf{a}) + (dm)\mathbf{a},$$

$$d(\mathbf{a}\cdot\mathbf{b}) = (d\mathbf{a})\cdot\mathbf{b} + \mathbf{a}\cdot(d\mathbf{b}), \quad d(\mathbf{a}\times\mathbf{b}) = (d\mathbf{a})\times\mathbf{b} + \mathbf{a}\times(d\mathbf{b}).$$

EXAMPLE. If $\mathbf{r} = \mathbf{f}(t)$ gives the position-vector of a moving particle as a function of the time t, then $d\mathbf{r}/dt$ = its vector velocity, $\mathbf{v}$, and $d\mathbf{v}/dt$ = its vector acceleration, $\mathbf{a}$. If $\mathbf{m}$ and $\mathbf{n}$ are unit vectors in the direction of the tangent and normal to the path at the time t, then $\mathbf{v} = v\mathbf{m}$, where $v = ds/dt$ = the (scalar) path-velocity, and $d\mathbf{m} = [(ds/R)]\mathbf{n}$, where R = the (scalar) radius of curvature of the path. Then

$$\mathbf{a} = \frac{d(v\mathbf{m})}{dt} = \frac{dv}{dt}\mathbf{m} + v\frac{d\mathbf{m}}{dt} = \frac{dv}{dt}\mathbf{m} + \frac{v^2}{R}\mathbf{n}.$$

Here dv/dt and v^2/R are the familiar expressions for the components of acceleration along the tangent and normal.

SECTION 3

MECHANICS

OF SOLIDS AND LIQUIDS

BY

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MECHANICS OF RIGID BODIES

BY

HARRISON W. HAYWARD

KINEMATICS

Motion in a Straight Line—Rectilinear Motion

The position of a moving point at any instant may be stated by giving its distance and direction from some fixed point in its path which is taken as an origin. This distance s can be taken as a function of the time t, giving the equation of position $s = f(t)$.

Velocity. The velocity v of a moving point is the rate at which the distance s changes with respect to the time t, and may be uniform or varying.

$$v = ds/dt; \quad s = \int v dt.$$

A Space-Time Curve offers a convenient means for the study of the motion of a point. The **slope of the curve** at any point will represent the **velocity** at that time. In Fig. 1(a) the slope is constant, as the graph is a straight line; the velocity is therefore uniform. In Fig. 1(b) the slope of the curve varies from point to point, so the velocity must vary also. At p and q the slope is zero, therefore the velocity of the point at the corresponding times must also be zero.

The Acceleration a of a moving point is the rate at which its velocity v changes with respect to the time t, and may be uniform or varying, positive or negative.

For rectilinear motion, $a = dv/dt = d^2s/dt^2$; $s = \int\int a dt^2 = \int v dt$.

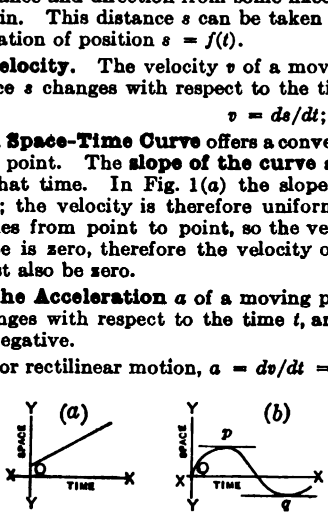

FIG. 1.

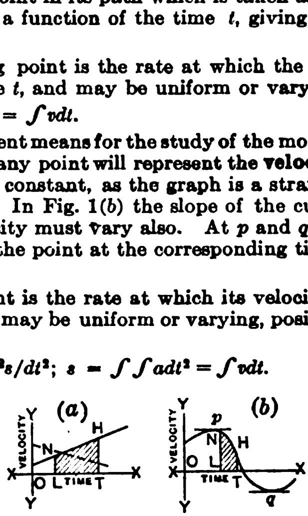

FIG. 2.

A Velocity-Time Curve offers a convenient means for the study of acceleration. The **slope of the curve** at any point will represent the **acceleration** at that time. In Fig. 2(a) the slope is constant, so the acceleration must be constant. In the case represented by the full line, the acceleration is positive, so the velocity is increasing. The dotted line shows a negative acceleration and therefore a decreasing velocity. In Fig. 2(b) the slope of the curve varies from point to point, so the acceleration must also vary. At p and q the slope is zero, therefore the acceleration of the point at the corresponding times must also be zero and the velocity uniform. The area under the velocity-time curve between any two ordinates such as NL and HT will represent the distance moved in time interval LT. In the case of the uniformly accelerated motion shown by the full line in Fig. 2(a), the area $LNHT$ is $\frac{1}{2}(NL + HT) \times (OT - OL)$ = mean velocity multiplied by the time interval = space passed over during this time interval. In Fig. 2(b) the mean velocity can be obtained from the equation of the curve by means of the calculus, or graphically by use of instruments.

An Acceleration-Time Curve (Fig. 3) may be constructed by plotting accelerations as ordinates, and times as abscissæ. The area under this curve between any two ordinates will represent the total increase in velocity during the time interval. The area $ABCD$ represents the total increase in velocity between time t_1 and time t_2.

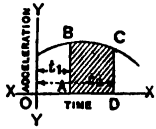

FIG. 3.

General Expressions Showing the Relations Between Space, Time, Velocity and Acceleration, for Rectilinear Motion

Given $s = f(t)$; $v = ds/dt$, and $a = dv/dt = d^2s/dt^2$.

Given $v = f(t)$; $s = s_0 + \int_0^t vdt$, and $a = dv/dt$.

Given $v = f(s)$; $t = \int_{s_0}^s ds/v$, and $a = vdv/ds$.

Given $a = f(t)$; $v = v_0 + \int_0^t adt$, and $s = s_0 + \int_0^t vdt$.

Given $a = f(s)$; $v = \sqrt{v_0^2 + 2\int_{s_0}^s ads}$, and $t = \int_{s_0}^s ds/v$.

Given $a = f(v)$; $s = s_0 + \int_{v_0}^v \frac{v}{a}dv$, and $t = \int_{v_0}^v dv/a$

Special Motions

Uniform Motion. If the **velocity** is **constant** the **acceleration** must be **zero**, and the point has uniform motion. The space-time curve becomes a straight line inclined toward the time axis [Fig. 1(a)]. The velocity-time curve becomes a straight line parallel to the time axis. For this motion $a = 0$, $v =$ constant, and $s = s_0 + vt$.

Uniformly Accelerated or Retarded Motion. If the **velocity** is **not uniform** but the **acceleration** is **constant,** the point has uniformly accelerated motion; the acceleration may be either positive or negative. The space-time curve becomes a parabola and the velocity-time curve becomes a straight line inclined toward the time axis, Fig. 2(a). The acceleration-time curve becomes a straight line parallel to the time axis. For this motion $a =$ constant, $v = v_0 + at$, $s = s_0 + v_0t + \frac{1}{2}at^2$.

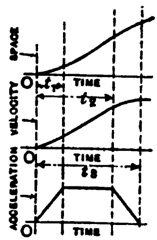

FIG. 4.

If the Point Starts from Rest, $v_0 = 0$. Care should be taken concerning the sign + or − for acceleration.

Example. Starting motion of a street car or hauling engine —see Fig. 4 During the period of time O to t_1, the acceleration increases from 0 to a maximum approximately in a straight line, the velocity-time curve is a parabola and the space-time curve one of the third degree. From t_1 to t_2 the acceleration is constant, the velocity increases uniformly, and the space-time curve is a parabola. From t_2 to t_3 the acceleration decreases from its maximum to 0 approximately in a straight line, and the space-time and velocity-time curves are like those from O to t_1. This concludes the start and a condition of equilibrium follows, with the acceleration zero, the velocity constant, and the space increasing uniformly.

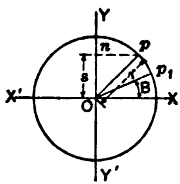

FIG. 5.

Periodic Motion (Simple Harmonic Motion). If a point travels uniformly in a circle, the motion of its projection upon any diameter is a simple harmonic one. In Fig. 5, the line Op revolves about O with a uniform angular velocity ω; n is the projection of point p upon diameter YY'. As p moves about O its projection moves from O to Y, back to O, then to Y' and back to O again. This cycle of motion may be repeated indefinitely. If ωt is the variable angle XOp, the displacement s of n from O is $r \sin \omega t$, where $r = Op$. Velocity $v = ds/dt$, $\therefore v = \omega r \cos \omega t$. Acceleration $a = dv/dt$, $\therefore a = -\omega^2 r \sin \omega t = -\omega^2 s$. These equations assume that Op starts from position OX. If there is a lead angle $XOp' = B$, or a negative angle known as a lag, the expression will become $s = r \sin (\omega t + B)$, $v = \omega r \cos (\omega t + B)$, $a = -\omega^2 r \sin (\omega t + B) = -\omega^2 s$. When ωt has increased by 2π, *i.e.*, after the time $t = 2\pi/\omega$ has elapsed, s, v, and a regain their original values. t is called the **period** of the motion. The angle $(\omega t + B)$ is the **phase angle.**

Composition and Resolution of Velocities

A velocity can be represented by a **vector,** which is a straight line having an arrow representing the direction of the motion and a length representing its magnitude.

Resultant. A velocity is said to be the resultant of two other velocities when it is represented by a vector that is the geometric sum of the vectors representing the other two velocities. This is the **parallelogram of motion.** In Fig. 6, v_r is the resultant of v_1 and v_2 and is represented by the diagonal of a parallelogram of which v_1 and v_2 are the sides; or it is the third side of a triangle of which v_1 and v_2 are the other two sides.

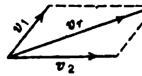

FIG. 6.

Polygon of Motion. The parallelogram of motion may be extended to the polygon of motion. Let v_1, v_2, v_3, v_4 [Fig. 7(*a*)] show the directions of four velocities imparted in the same plane to point O. If the lines v_1, v_2, v_3, v_4 [Fig. 7(*b*)] are drawn parallel to and proportional to the velocities imparted to point O, v_r will represent the resultant velocity imparted to O. It will make no difference in what order the velocities are taken in constructing the motion polygon. As long as the arrows showing the direction of the motion follow each other in order about the polygon, the resultant velocity of the point will be represented in magnitude by the closing side of the polygon, but opposite in direction.

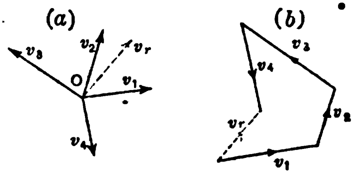

FIG. 7.

Resolution of Velocities. Velocities may be resolved into **component velocities** in the same plane, as shown by Fig. 8. Let the velocity of point O be v_r. In Fig. 8(*a*) this velocity is resolved into two components in the same plane as v_r and at right angles to each other.

$$v_r = \sqrt{(v_1)^2 + (v_2)^2}$$

In Fig. 8(*b*) the components are in same plane as v_r, but are not at right angles to each other. In this case,

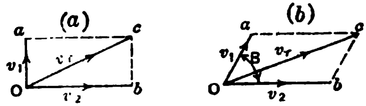

FIG. 8.

$$v_r = \sqrt{(v_1)^2 + (v_2)^2 + 2v_1v_2 \cos B.}$$

If the components v_1 and v_2 and angle B are known, the direction of v_r can be determined. $\sin boc = (v_1/v_r)\sin B$. $\sin coa = (v_2/v_r)\sin B$. Where v_1 and v_2 are at right angles to each other, $\sin B = 1$.

Composition and Resolution of Accelerations. Accelerations may be combined and resolved in the same manner as velocities, but in this case the lines or vectors represent accelerations instead of velocities. Velocities and accelerations may be resolved into components *not* in the same plane by what is known as the **parallelopiped of motion.** In this case the **resultant of three motions not in the same plane** is the diagonal of a parallelopiped whose sides are lines whose length and direction represent the motions. See Fig. 9. Oa, Oc and Ob represent three velocities or accelerations; then Od represents the resultant velocity or acceleration. When the velocities or accelerations are at right angles to each other, the angles that the resultant Od makes with these axes are A, B and C, respectively. Then $\cos A = Oa/Od$, $\cos B = Ob/Od$, $\cos C = Oc/Od$, and

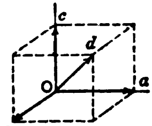

FIG. 9.

$$Od = \sqrt{Oa^2 + Ob^2 + Oc^2}.$$

Curvilinear Motion

The Linear Velocity $v = ds/dt$ is the same as for rectilinear motion, and its direction is tangent to the path of the point. In Fig. 10(a), let $p_1p_2p_3$ be the path of a moving point, and v_1, v_2, v_3 represent its velocity at points p_1, p_2, p_3, respectively. If O be taken as a pole [Fig. 10(b)] and vectors v_1, v_2, v_3 representing the velocities of the point at p_1, p_2, and p_3 be drawn, the curve connecting the terminal points of these vectors is known as the **hodograph** of the motion. This velocity diagram is applicable only to motions all in the same plane.

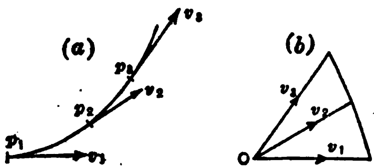

FIG. 10.

Acceleration. Tangents to the curve, Fig. 10(b), indicate the directions of the **momentary accelerations.** The direction of the tangents does not, as a rule, coincide with the direction of the velocities as represented by tangents to the path. If the acceleration a at some point in the path is resolved by means of a parallelogram into components tangent and normal to the path, the normal acceleration $a_n = v^2/r$, where r = radius of curvature of the path at the point in question, and the tangential acceleration $a_t = dv/dt$, where v = velocity tangent to the path at the same point. $a = \sqrt{a_n^2 + a_t^2}$. The normal acceleration is constantly directed toward the center of the path.

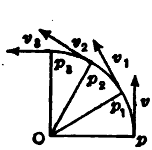

FIG. 11.

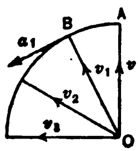

FIG. 12.

Example. Find the acceleration of a point moving in a circle with uniform velocity. In Fig. 11, p, p_1, p_2, p_3 is the circular path of a point. v, v_1, v_2, v_3 represent the uniform velocities at p, p_1, p_2, p_3. Construct the hodograph of the motion, as in Fig. 12. At point p_1 the acceleration is in the direction of the tangent to the hodograph at B, shown by line a_1. Let $pp_1 = s$, $AB = s_1$ and $Op = r$; then $s/r = s_1/v$. $\therefore s_1 = sv/r$. $ds_1/dt = (ds/dt)(v/r) = v^2/r$ and as $ds_1/dt = a_n$, $a_n = v^2/r$ and is in a direction toward the center of the circle. $a_t = 0$.

Motion of a Particle in a Circular Path. If a point moves in a circle of radius r, its angular velocity being ω, angular acceleration a_a, and linear velocity v, $v = \omega r$, $a_t = a_a r$, $a_n = v^2/r = r\omega^2$.

Uniform Rotary Motion. In the case of uniform rotary motion, equal circular paths are traveled in equal intervals of time. ∴ Angular acceleration $a_a = 0$ and $a_t = a_a r = 0$. $a_n = v^2/r$. The **angular velocity** is usually **expressed in radians per sec.**, and when the number (N) of revolutions per min. (r.p.m.) is known, the angular velocity is $\omega = 2\pi N/60 = 0.10472N$; linear velocity $v = \omega r$.

Composition of Motions

A point may have several motions imparted to it at the same time by different means, in which case its motion is the resultant of the different component motions. Each of these components, by what is known as the **principle of independence**, exerts its full influence. The polygon or parallelopiped of motion may be used for the composition of motions.

Many **examples of combined motion** occur, *e.g.*, velocity given by gravity to a body having a horizontal motion; combined rotation and rectilinear motion of a point on a rolling wheel; a point whose motion is governed by the action of two or more cams; the movement of persons or machinery on board trains or boats; movement of a projectile.

Relative Motions. In order that a motion shall be fully determined, it is necessary that the base to which it is referred be carefully stated. The majority of engineering problems assume the earth as the reference base.

Motion of Rigid Bodies

A body is said to be rigid when the distances between all of its particles are invariable. Theoretically, rigid bodies do not exist, but materials used in engineering are practically rigid under their working stresses.

Elementary Motion. If a body moves so that a straight line connecting any two of its particles remains fixed in direction, it is said to have a motion of **translation.** If the motion is indefinitely small—distance $= ds$ and time $= dt$—it may be considered to have **elementary translation**; $ds/dt =$ velocity of translation. The elementary translation is fully determined by a line drawn from any point in the rigid body, representing the velocity in magnitude and direction.

If a body moves so that all its particles describe circles whose planes are at right angles to a straight line XX which contains simultaneously the centers of all the circles, it is said to have a **rotary motion** about the axis XX. If the motion is indefinitely small—angle of rotation $= d\theta$ and time $= dt$—it is called **elementary rotation.** The path ds of a point distant r from the axis of rotation is $ds = r\,d\theta$, and $d\theta/dt =$ elementary angular velocity of rotation. Elementary rotation is completely determined by a straight line whose length is equal to ω drawn on the axis of rotation with an arrow so placed that when looking in the direction of the arrow rotation takes place in a plane at right angles to that line and in the direction of the hands of a clock.

The motion of a body in space is determined by the motion of three points of the body which do not lie in the same straight line. **The instantaneous axis** is the axis about which a body is revolving at a given instant. If a body is being translated at a given instant, the instantaneous axis is at an infinite distance.

Motion in a Plane

A Plane Motion is one in which all points of the moving body remain at constant distances from a fixed plane.

Angular Displacement. The angular displacement of any body moving in a plane is the angle described by any line drawn in the body parallel to the plane of the motion.

Angular Velocity is the rate at which the angular displacement θ varies with respect to the time t. Angular velocity $\omega = d\theta/dt$.

Angular Acceleration is the rate at which the angular velocity ω varies with respect to the time t. Angular acceleration $a_a = d\omega/dt = d^2\theta/dt^2$.

Instantaneous Axis. When the axis about which any body may be considered to rotate changes its position, any one position is known as an instantaneous axis, and the line through all positions of the instantaneous axis in space as the **centrode.**

When the motion of two points in the same plane of a rigid body having plane motion is known, the instantaneous axis for the body will be at the intersection of the lines drawn from each point and perpendicular to its line of motion. See Fig. 13, in which A and B are two points on the rod AB, v_1 and v_2 representing their velocities. O is the instantaneous axis for AB; $\therefore$ point C will move as shown in a line perpendicular to OC.

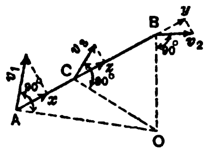

FIG. 13.

Linear Velocities of points in a body rotating about an instantaneous axis are proportional to their distances from this axis. In Fig. 13, $v_1 : v_2 : v_3 = AO : OB : OC$. If the motions of A and B were parallel, the lines OA and OB would also be parallel and there would be no instantaneous axis. The motion of the rod would be translation, and all points would be moving with the same velocity in parallel straight lines.

If a body has plane motion, the components of the velocities of any two points in the body along the straight line joining them must be equal. Ax must be equal to By and Cz in Fig. 13.

Centrode. If the path of the instantaneous center be plotted, the centrode for the motion of the body is obtained. See Fig. 14. Consider A_0B_0 to be fixed and let C_0D_0 rotate about it. Its motion being determined by the links AD and BC, the path of the instantaneous center for CD can be plotted as the curve xy. CD can be considered to be fixed and the centrode for AB found at eo. If the bars AB and CD are fixed to these centrodes and they are rolled upon each other, the relative motion of the two bars A_0B_0 and C_0D_0 will be the same as if they were connected by the links AD and BC. This principle is made use of in the **design of elliptic gears for quick-return motions.** If a wheel rolls along a track, the centrode is the surface of the track.

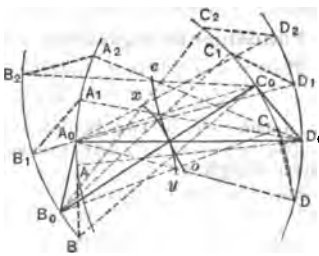

FIG. 14.

The angular velocity of a body at any time can be found by dividing the linear velocity of any point by the distance of the point from the instantaneous center. See Fig. 15, in which the wheel rolls along the track XX. Point A has a linear velocity of v ft. per sec.; O is the instantaneous center. v/AO = angular velocity of wheel. If v_1 is the velocity of point B, the angular velocity of wheel $= v_1/BO = v/AO$.

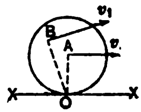

FIG. 15.

For **linkages**, see p. 652.

PHYSICAL MECHANICS

General Laws

Every particle of matter remains either at rest or moves uniformly in a straight line unless acted upon by some external influence. All variations of velocity, either in magnitude or direction, require the action of **force**.

Laws of Gravity. A body falling in a vacuum moves with a uniformly accelerated motion, the acceleration of the fall being the same for all bodies at the same place. Common values taken for this acceleration (usually designated as g) are 32.16 ft. per sec. per sec. or 9.81 meters per sec. per sec.

$g = 32.1721 - 0.08211 \cos 2a - 0.000003h$, where a is the latitude in degrees, and h the height in feet above sea level.

If a body is suspended by an elastic cord, the latter is elongated by the action of what is known as a force, in this case by the force of gravity. The suspended body is not only acted upon by gravity, but by a force exerted upon it by the cord, which imparts to it the same acceleration upward as gravity does downward, thereby maintaining the body at rest. The two forces are equal in magnitude.

Action is Equal to Reaction. The forces exerted by two bodies upon each other act in the same straight line, are equal in magnitude, and are opposite in direction.

Relations Between Mass, Acceleration and Force. When acted upon by a constant unbalanced force, a body will move with a **uniformly accelerated motion. The accelerations produced in any body by different forces are proportional to the forces,** so that $a_1/p_1 = a_n/p_n$ = constant = $1/m$, where p_1, p_n are the forces applied and a_1, a_n are the resulting accelerations. The constant ratio m is known as the **mass** of the body.

Fundamental Equation: Force = Mass × Acceleration.

Mass = w/g, where w = weight and g = falling acceleration due to gravity.

$$F = (w/g)a = ma = mdv/dt = md^2s/dt^2.$$

In the case of two bodies at the same place, $w_1 = m_1g$, $w_2 = m_2g$ and $m_1/m_2 = w_1/w_2$; *i.e.*, **the masses of the two bodies are proportional to their weights.**

Example. If an unbalanced force of 5 lb. acts upon a body weighing 10 lb., what will be the acceleration of the body? $5 = 10a/g$; $\therefore a = 5g/10 = g/2$.

Forces, like velocities and accelerations, are quantities in which **direction is a factor,** *i.e.*, their complete determination requires not only a statement of their magnitude, but also of their line of action and direction.

Law of the Conservation of Mass. The mass of a body remains unchanged by any physical or chemical change to which it may be subjected. The **momentum** of a body is the product of its mass and its velocity, $= mv$.

Technical Systems of Measurement

In the **absolute systems**, see p. 73, the units of length, mass and time are arbitrarily taken, the unit of force being derived.

In the **gravitation systems**, see p. 73, the units of force are weights of absolute units of mass, and therefore depend upon the force of gravity and are gravitation units. The word **weight** is then used to denote both the mass of the body and the force with which it is attracted by the earth.

Absolute units are used for most scientific work and **gravitation units for engineering work** other than electrical. w and g both change as a body is moved from place to place, (**see p. 84,**) but the number of units of mass in the body $= w/g =$ constant. w and g vary very little at ordinary altitudes.

STATICS OF RIGID BODIES

General Considerations

If the forces acting on a rigid body do not produce any acceleration, they must neutralize each other, that is, form a **system of forces in equilibrium.** Equilibrium is said to be **stable** when the body with the forces acting upon it returns to its original position after being displaced a very small amount from that position; **unstable** when the body tends to move still further from its original position than the very small displacement; and **neutral** when the forces retain their equilibrium when the body is in its new position.

External and Internal Forces. The forces by which the individual particles of a body act on each other are known as internal forces. All other forces are called external forces. If a body is supported by other bodies while subject to the action of forces, deformations and forces will be produced at the points of support or contact and these internal stresses will be distributed throughout the body until equilibrium exists and the body is said to be in a state of tension, compression or shear. The forces exerted by the body on the supports are known as **reactions.** They are equal in magnitude and opposite in direction to the forces with which the supports act on the body, known as **supporting forces.** The supporting forces are external forces applied to the body.

In considering a body at a definite section, it will be found that all the internal forces act in pairs, the two forces being equal and opposite. The external forces act singly.

General Law. When a body is at rest, the forces acting externally to it must form an equilibrium system. This law will hold for any part of the body, in which case the forces acting at any section of the body become external forces when the part on either side of the section is considered alone. In the case of a **rigid body,** any two forces of the same magnitude but acting in opposite directions in any straight line, may be added or removed without change in the action of the forces acting on the body, providing the strength of the body is not affected.

Composition, Resolution and Equilibrium of Forces

(For graphical methods, see p. 200)

The **resultant** of several forces acting at a point is a force which will produce the same effect as all the individual forces acting together.

Forces Acting on a Body at the Same Point. The resultant R of two forces F_1 and F_2 applied to a rigid body at the same point is represented in magnitude and direction by the diagonal of the parallelogram formed by F_1 and F_2. See Figs. 16 and 17.

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos a}$; $\sin a_1 = (F_2 \sin a)/R$; $\sin a_2 = (F_1 \sin a)/R$.

When $a = 90$ deg., $R = \sqrt{F_1^2 + F_2^2}$, $\sin a_1 = F_2/R$ and $\sin a_2 = F_1/R$;

$\left.\begin{array}{l}\text{When } a = 0 \text{ deg., } R = F_1 + F_2 \\ \text{When } a = 180 \text{ deg., } R = F_1 - F_2\end{array}\right\}$ Forces act in same straight line.

A force R may be resolved into two component forces intersecting anywhere on R and acting in the same plane as R, by the reverse of the operation shown by Figs. 16 and 17; and by repeating the operation with the components, R may be resolved into any number of component forces intersecting R at the same point and in the same plane.

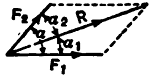

FIG. 16.

Resultant of Any Number of Forces Applied to a Rigid Body at the Same Point. Resolve each of the given forces F into components along three rectangular co-ordinate axes. If A, B and C are the angles made with XX, YY and ZZ, respectively, by any force F, the components will be $F \cos A$ along XX, $F \cos B$ along YY, $F \cos C$ along ZZ; add the components of all the forces along each axis algebraically and obtain $\Sigma F \cos A = \Sigma X$ along XX, $\Sigma F \cos B = \Sigma Y$ along YY, and $\Sigma F \cos C = \Sigma Z$ along ZZ.

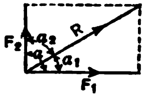

FIG. 17.

The resultant $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2 + (\Sigma Z)^2}$. The angles made by the resultant with the three axes are A_r with XX, B_r with YY, C_r with ZZ, where

$$\cos A_r = \Sigma X/R, \quad \cos B_r = \Sigma Y/R, \quad \cos C_r = \Sigma Z/R.$$

The **direction of** the **resultant** can be determined by plotting the algebraic sums of the components.

If the forces are all in the same plane the components of each of the forces along one of the three axes (say, ZZ) will be 0, that is, angle $C_r = 90°$ and $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$, $\cos A_r = \Sigma X/R$, and $\cos B_r = \Sigma Y/R$.

For equilibrium, it is necessary that $R = 0$; that is to say, ΣX, ΣY, and ΣZ must each be equal to zero.

General Law. In order that a number of forces acting at the same point shall be in equilibrium, the algebraic sum of their components along any *three* co-ordinate axes must each be equal to zero. When the forces all act in the same plane, the algebraic sum of their components along any *two* co-ordinate axes must each equal zero.

When the Forces Form a System in Equilibrium. Three unknown forces can be determined if the lines of action of the forces are *all* known and are in different planes. If the forces are all in the same plane, the lines of action being known, only *two* unknown forces can be determined. If the lines of action of the unknown forces are *not* known, only *one* unknown force can be determined in either case.

Couples and Moments

Couple. Two forces of equal magnitude, Fig. 18, which act in opposite and parallel directions form a couple. **A couple cannot be reduced to a single force.**

Displacement and Change of a Couple. The forces forming a couple may be moved about and their magnitude and direction changed, provided that they always remain parallel to each other, remain either in the original plane or one parallel to it, and provided that the product of one of the forces and the perpendicular distance between the two is constant and the direction of rotation remains the same.

Moment of a Couple. The moment of a couple is the product of the magnitude of one of the forces and the perpendicular distance between the lines of action of the forces. Fa = moment of couple; a = arm of couple. If the forces are measured in pounds and the distance a in feet, the **unit of rotation moment** is the foot-pound. If the force is measured in kilograms and the distance in meters, the unit is the meter-kilogram. In the C. G. S. system the unit of rotation moment is 1 cm.-dyne.

Rotation Moments of Couples acting in the same plane are considered to be **positive or negative**, according to whether they appear to rotate in the direction of the hands of a clock or in the reverse direction. The couple shown in Fig. 18 is positive. The magnitude, direction and sense of rotation of a couple are completely determined by its moment axis, or moment vector, which is a line drawn perpendicular to the plane in which the couple acts, with an arrow indicating the direction from which the couple will appear to have right-handed rotation; the length of the line represents the magnitude of the moment of the couple. See Fig. 19, in which AB represents the magnitude of the moment of the couple. Looking along the line in the direction of the arrow, the couple will have right-handed rotation in any plane perpendicular to the line.

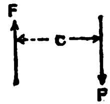

FIG. 18.

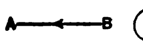

FIG. 19.

Composition of Couples. Couples may be combined by adding their moment vectors geometrically, in accordance with the parallelogram rule, in the same manner in which forces are combined.

Couples lying in the same or parallel planes are added algebraically. Let $+40$, -60, and $+100$ ft.-lb. be the moments of three couples in the same or parallel planes; their resultant is a single couple lying in the same or in a parallel plane, whose moment is $\Sigma M = +40 - 60 + 100 = +80$ ft.-lb.

If the polygon formed by the moment vectors of several couples closes itself, the couples form an equilibrium system. Two couples will balance each other when they lie in the same or parallel planes, and have the same moment in magnitude, but opposite in sign.

Combination of a Couple and a Single Force in the Same Plane. (See Fig. 20.) Given a force $F = 20$ lb. acting as shown distant x from YY, and a couple whose moment is -60 ft.-lb. in the same or a parallel plane, to find the resultant. A couple may be changed to any other couple in the same or a parallel plane having the same moment and same sign. Let the couple consist of two forces of 20 lb. each and let the arm be 3 ft. Place the couple in such a manner that one of its forces is opposed to the given force at p. This force of the couple and the given force being of the same magnitude and opposite in direction, will neutralize each other, leaving the other force of the couple acting at a distance of 3 ft. from p and parallel and equal to the given force $F = 20$.

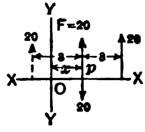

FIG. 20.

General Rule. The resultant of a couple and a single force lying in the same or parallel planes is a single force, equal in magnitude, in the same direction and parallel to the single force, and acting at a distance from the line of action of the single force equal to the moment of the couple divided by the single force. The moment of the resultant force about any point on the line of action of the given single force must be of the same sense as that of the couple, positive if the moment of the couple is positive, and negative if moment of couple is negative. If the moment of the couple in Fig. 20 had been $+ 60$ instead of $- 60$, the resultant would have been a force of 20 lb. acting in the same direction and parallel to F, but at a distance of 3 ft. to the left of it (shown dotted), making the moment of the resultant about any point on F positive.

To Effect a Parallel Displacement of a Single Force F over a distance a, a couple whose moment is Fa must be added to the system. The sense of the couple will depend upon which way it is desired to displace force F.

The Moment of a Force with Respect to a Point is the product of the force F and the perpendicular distance from the point to the line of action of the force.

The Moment of a Force with Respect to a Straight Line. If the force is resolved into components parallel and perpendicular to the given line, the moment of the force with respect to the line is the product of the magnitude of the perpendicular component and the distance from its line of action to the given line.

Forces with Different Points of Application

Composition of Forces. If each force F is resolved into components parallel to three rectangular co-ordinate axes XX, YY and ZZ, the magnitude of the resultant is $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2 + (\Sigma Z)^2}$, and its line of action makes angles A_r, B_r and C_r with axes XX, YY and ZZ, where $\cos A_r = \Sigma X/R$, $\cos B_r = \Sigma Y/R$, and $\cos C_r = \Sigma Z/R$; and there are three couples which may be combined by their moment vectors into a single resultant couple having the moment $M_r = \sqrt{(M_x)^2 + (M_y)^2 + (M_z)^2}$, whose moment vector makes angles of A_m, B_m and C_m with axes XX, YY and ZZ, such that $\cos A_m = M_x/M_r$, $\cos B_m = M_y/M_r$, and $\cos C_m = M_z/M_r$. If this single resulting couple is in the same plane as the single resulting force at the origin or a plane parallel to it, the system may be reduced to a single force R acting at a distance from $R = M_r/R$. If the couple and force are not in the same or parallel planes, it is impossible to reduce the system to a single force. If $R = 0$, that is, if ΣX, ΣY and ΣZ all equal zero, the system will reduce to a single couple whose moment is M_r. If $M_r = 0$, that is, if M_x, M_y and M_z all equal zero, the resultant will be a single force R.

When the Forces are All in the Same Plane, one of the angles A_r, B_r or $C_r = 0$, say, $C_r = 90°$. Then $R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$, $M_r = \sqrt{M_x^2 + M_y^2}$ and the final resultant is a force equal and parallel to R, acting at a distance from R equal to M_r/R.

A system of forces in the same plane can always be replaced by either a couple or a single force. If $R = 0$ and $M_r > 0$, the resultant is a couple. If $M_r = 0$ and $R > 0$, the resultant is a single force.

A rigid body is in equilibrium when acted upon by a system of forces whenever $R = 0$ and $M_r = 0$, *i.e.*, when the following six conditions hold true: $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma Z = 0$, $M_x = 0$, $M_y = 0$ and $M_z = 0$. When the system of forces is in the same plane, equilibrium prevails when the following three conditions hold true: $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$.

Forces Applied to Support Rigid Bodies

The external forces in equilibrium acting upon a body may be statically determinate or indeterminate according to the number of unknown forces existing. When the forces are all in the same plane and act at a common point, two unknown forces may be determined if their lines of action are known, one if unknown.

When the forces are all in the same plane and are parallel, two unknown forces may be determined if the lines of action are known, one if unknown.

When the forces are anywhere in the same plane, three unknown forces may be determined if their lines of action are known, if they are not parallel or do not pass through a common point; if the lines of action are unknown, only one unknown force can be determined.

If the forces all act at a common point but are in different planes, three unknown forces can be determined if the lines of action are known, one if unknown.

If the forces act in different planes but are parallel, three unknown forces can be determined if their lines of action are known, one if unknown.

The first step in the solution of problems in statics is the determination of the supporting forces. The following data are required for the complete knowledge of supporting forces: magnitude, direction, and point of application. According to the nature of the problem, none, one, or two of these quantities are known.

One Fixed Support. The point of application, direction, and magnitude of the load are known. See Fig. 21. As the body on which the forces act is in equilibrium, the supporting force P must be equal in magnitude and opposite in direction to the resultant of the loads L.

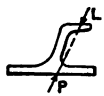

FIG. 21.

In the case of a **rolling surface,** the point of application of the support is obtained from the center of the connecting bolt A (Fig. 22), both the direction and magnitude being unknown. The point of application and line of action of the support at B are known, being determined by the rollers.

When three forces acting in the same plane on the same rigid body are in equilibrium, their lines of action must pass through the same point O. The load L is known in magnitude and direction. The line of action of the support at B is known on account of the rollers. The point of application of the support at A is known. The three forces are in equilibrium and are in the same plane, and therefore the lines of action must meet at the point O.

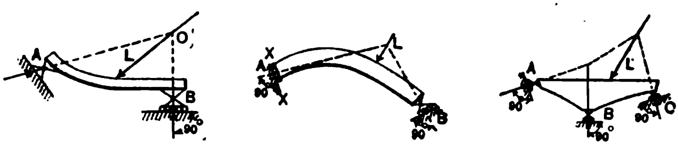

FIG. 22. FIG. 23. FIG. 24.

In the case of the rolling surfaces shown in Fig. 23, the direction of the support at A is known, the magnitude and point of application unknown. The line of action and point of application of the supporting force at B are known, its magnitude unknown. The lines of action of the three forces must meet in a point, and the supporting force at A must be perpendicular to the plane XX. In the case shown in Fig. 24, the directions and points of application of the supporting forces are known, and the magnitudes unknown. The lines of action of resultant of supports A and B, the support at C and load L must meet at a point. Resolve the resultant of supports at A and B into components at A and B, their direction being determined by the rollers.

If a member of a truss or frame in equilibrium is pinned at two points and loaded at these two points only, the line of action of the forces exerted on the member or by the member at these two points must be along a line connecting the pins.

If the external forces acting upon a rigid body in equilibrium are all in the same plane, the equations $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$ must be satisfied. When trusses, frames and other structures are under discussion, these equations are usually used as $\Sigma V = 0$, $\Sigma H = 0$, $\Sigma M = 0$, where V and H represent vertical and horizontal components, respectively.

The **supports** are said to be **determinate statically** when the laws of equilibrium are sufficient for their determination. When the conditions are not sufficient for the determination of the supports or other forces, the structure is said to be **statically indeterminate** and the unknown forces can only be determined by considerations involving the elasticity of the material.

When several bodies are so connected to one another as to make up a rigid structure, the forces at the points of connection must be considered as internal forces and are not taken into consideration in the determination of the supporting forces for the structure as a whole.

The distortion of any practically rigid structure under its working loads is so small as to be negligible when determining supporting forces. When the forces acting at the different joints in a built-up structure cannot be determined by dividing the structure up into parts, the structure is said to be **statically indeterminate internally**. A structure may be statically indeterminate internally and still be statically determinate externally.

Fundamental Problems in Graphical Statics

A force may be represented by a straight line in a determined position, and its magnitude by the length of the straight line. The direction in which it acts may be indicated by an arrow.

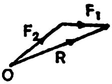

FIG. 25.

Polygon of Forces. The parallelogram of two forces intersecting each other (see Fig. 8) leads directly to the graphic composition by means of the triangle of forces. In Fig. 25, R is called the **closing side**, and represents the resultant of the forces F_1 and F_2 in magnitude and direction. Its position is given by the point of application O. By means of repeated use of the triangle of forces and by omitting the closing sides of the individual triangles, the magnitude and direction of the resultant R of any number of forces in the same plane and intersecting at a single point can be found. In Fig. 26 the lines representing the forces start from point O, and in the force polygon, Fig. 27, they are joined in any order, the arrows showing their directions following around the polygon in the same direction. The magnitude of the resultant at the point of application of the forces is represented by the closing side R of the force polygon; its direction, as shown by the arrow, is counter to that in the other sides of the polygon.

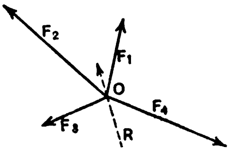

FIG. 26.

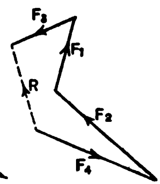

FIG. 27.

If the forces are in equilibrium, R must equal zero, *i.e.*, **the force polygon must close.**

If in a closed polygon one of the forces is reversed in direction, this force becomes the resultant of all the others.

If the **forces** do **not** all lie **in the same plane,** the diagram becomes a polygon in space. If the projections of the forces be drawn on any two planes, say horizontal and vertical, the closing lines of the projected polygons are the projections of the closing line of the force diagram in space.

Funicular Polygon

Composition of Forces in a Plane. In Fig. 28, to determine the resultant R of the forces F_1, F_2, F_3 and F_4—all in the same plane, construct the force polygon shown at (*b*). R will represent the magnitude and direction of the resultant force, and its location in (*a*) may be determined as follows: In Fig. 28(*b*) assume any convenient point, O, as a **pole**, draw the straight lines $O0$, $O1$, $O2$, $O3$ and $O4$, and in Fig. 28(*a*) draw the lines s_0, s_1, s_2, s_3, s_4, parallel to the similarly lettered lines in Fig. 28(*b*). The location of s_0 in Fig. 28(*a*) is optional, so long as it intersects the line of action of F_1. R must pass through the intersection of s_0 and s_4 and be parallel to R in Fig. 28(*b*).

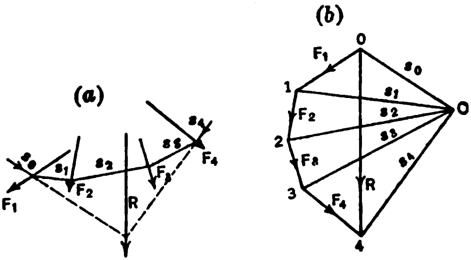

FIG. 28.

The figure $O01234$ is called the **polygon of forces** or **force polygon,** the lines s_0, s_1, s_2, s_3, s_4 in Fig. 28(*b*) are **rays,** and the series of lines s_0, s_1, s_2, s_3, s_4, in Fig. 28(*a*) is called a **string polygon** or **funicular polygon,** s_0, s_1, s_2, s_3, s_4 in (*a*) being **known as strings.** The following three cases may be met:

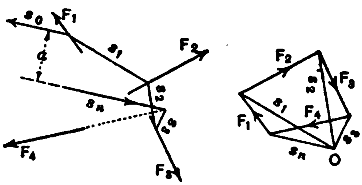

FIG. 29.

(*a*) The general case, where the starting point of the polygon of forces does not coincide with the terminal point. The resultant is a single force, as shown by Fig. 28. (*b*) The polygon of forces closes and the end strings of the funicular polygon are parallel. In this case (Fig. 29) the resultant is a couple whose moment is $s_0a = s_na$. (*c*) The polygon of forces closes and the funicular polygon also closes, the result being **equilibrium.** See Fig. 30. The two coinciding terminal sides of the funicular polygon are called the **closing sides** or strings.

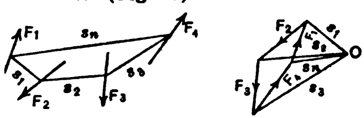

FIG. 30.

Let a polygon of forces and a funicular polygon be drawn for the system of forces shown in Fig. 31 with poles at both O and O_1. The intersections of the corresponding strings of the funicular polygons all lie in the same straight line A_0A_n parallel to OO_1. This line is the **polar axis** of both funicular polygons.

Let the starting point of the force polygon be taken as a pole; then the sides of the funicular polygon will always give the line of action of the resultants of all the forces preceding. The last side will coincide with the resultant of all the forces.

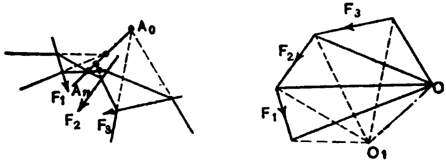

FIG. 31.

To determine the supporting forces for a rigid body when the external forces are all in the same plane and are statically determinate, see Fig. 32. Let F_1 and F_2 be two forces acting on beam which is supported at A by a roller and at B by a pin. Construct a force polygon, Fig. 32(b), using any convenient point O for a pole, and the funicular polygon shown in Fig. 32(a). This latter must start at B, as this is the only point on the line of action of the unknown support at B that is known. Draw the strings parallel to the corresponding rays of the force polygon and find that the last string intersects the line of action of the left-hand support at P. If the forces are in equilibrium, both force and funicular polygons must close. The closing side of the funicular polygon is PB. OX, Fig. 32(b), is drawn parallel to PB until it meets a line drawn from 1 parallel to the supporting force at A. The line $X3$ closes the force polygon and gives the magnitude and direction of the supporting force at B. It is necessary that either the lines of action of both unknown supports, or the line of action of one and the point of application of the other, be known in order to use this construction.

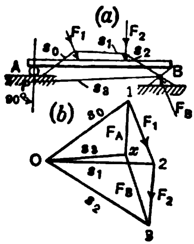

FIG. 32.

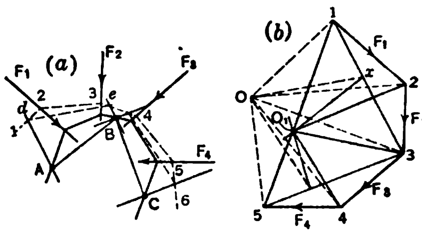

FIG. 33.

To draw a **funicular polygon through three points,** see Fig. 33. Given the forces F_1, F_2, F_3, and F_4, Fig. 33(a), let it be required to draw a funicular polygon passing through points A, B and C. Construct a force polygon, Fig. 33(b), using O as a pole, and a corresponding funicular polygon, 1, 2, 3, 4, 5, 6, Fig. 33(a). Consider the forces between points A and B; if they are to be balanced by two forces each parallel to their resultant and acting through A and B, these two forces can be determined by drawing lines parallel to 1 3, Fig. 33(b), through A and B, Fig. 33(a). The points where these lines are intersected by the funicular polygon are d and e; the closing side of a funicular polygon for these two forces is de and the magnitude of the balancing forces can be determined as $1x$ and $x3$, Fig. 33(b), by drawing from O a line parallel to *de* and intersecting 1 3 which is parallel to the forces at *d* and *e*. If the funicular polygon had passed through A and B, the same balancing forces for F_1 and F_2 parallel to 1 3 must be found so that the locus of the poles of all the funicular polygons for F_1 and F_2 that will pass through A and B must lie on a line drawn from x and parallel to AB. In the same manner the locus of the poles of all the funicular polygons for F_3 and F_4 can be determined, and the intersection of these two loci will be the pole O_1 required.

Moment of Any System of Forces in a Plane. The resultant moment M of the forces F_1, F_2, F_3 (Fig. 34) with reference to point A can be determined as follows: Fig. 34(b) is a force polygon for F_1, F_2, F_3, using any convenient point O as a pole. Fig. 34(a) shows a corresponding funicular polygon. Through points A and P of (a) draw lines parallel to the resultant of the forces R in (b); point P is the intersection of the end strings of the funicular polygon, and m and q are the points where the line drawn through point A intersects the end strings. Consider the shaded triangles; the homologous sides are parallel, the triangles are therefore similar, and $mq/x = R/p$. $\therefore (mq)p = Rx$; but Rx is the moment of the resultant of the forces with respect to A; $\therefore (mq)p$ must also be the same moment. mq is known as the **intercept** and p is known as the **pole distance.** In the case of parallel forces, the pole distance will be a constant; therefore, the moment at different points will vary as the intercept.

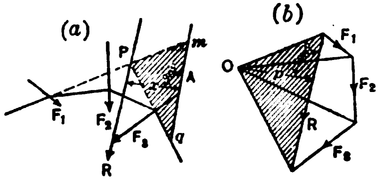

FIG. 34.

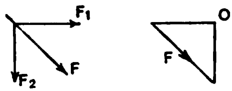

FIG. 35.

Resolution of Forces. Forces may be resolved into components in the same plane by means of the funicular polygon. See Fig. 35.

The funicular polygon is of great service in the solution of **stone arch and dam problems,** in determining the line of resistance at the joints. Bending moments and shearing forces for beams and other structures may also be determined graphically by its use.

Determination of Stresses in Members of a Statically Determinate Plane Structure with Loads at Rest

It will be assumed that the loads are applied at the joints of the structure, *i.e.*, at the points where the different members are connected, and that the connections are pins with no friction. The stresses in the members must then be along lines connecting the pins, unless any member is loaded at more than two points by pin connections. If the members are straight, the forces exerted on them or by them must coincide with the axes of the members. In other words, there shall be no bending stresses in any of the members of the structure.

Equilibrium. In order that the whole structure shall be in equilibrium, it is necessary that the external forces (loads and supports) shall form a balanced system. Graphical and analytical methods are both of service.

Supporting Forces. When the supporting forces are to be determined, it is not necessary to pay any attention to the make-up of the structure under consideration so long as it is practically rigid; the loads may be taken as they occur, or the resultant of the loads may be used instead. When the stresses in the members of the structure are being determined, the loads *must* be distributed at the joints where they belong.

Method of Joints. When all the external forces have been determined, any joint at which there are not more than two unknown forces may be taken and these unknown forces determined by the methods of the stress polygon, resolution or moments. In Fig. 36, let O be the joint of a structure and F be the only known force; but let $O1$ and $O2$ be two members of the structure joined at O. Then the lines of action of the unknown forces are known and their magnitude may be determined (*a*) by a **stress polygon** which, for equilibrium, must close; (*b*) by resolution into H and V components, using the condition of equilibrium $\Sigma H = 0$, $\Sigma V = 0$; or (*c*) by moments, using any convenient point on the line of action of $O1$ or $O2$ and the condition of equilibrium $\Sigma M = 0$. No more than *two* unknown forces can be determined.

FIG. 36.

In this manner, proceeding from joint to joint, the stresses in all the members of the truss can usually be determined if the structure is statically determinate internally. The structure may be divided into two parts by passing a section through it cutting some of its members; one part may then be treated as a rigid body and the external forces acting upon it determined. Some of these forces will be the stresses in the members themselves. For example, let xx (Fig. 37) be a section taken through a truss loaded at P_1, P_2 and P_3, and supported on rollers at S. As the whole truss is in equilibrium, any part of it must be also, and consequently the part shown to the left of xx must be in equilibrium under the action of the forces acting externally to it. Three of these forces are the stresses in the members aa, bb, and bc, and are the unknown forces to be determined. They can be determined by applying the condition of equilibrium for forces acting in the same plane but not at the same point. $\Sigma H = 0, \Sigma V = 0, \Sigma M = 0$. The three unknown forces can be determined only if they are not parallel or do not pass through the same point; if, however, the forces are parallel or meet in a point, two unknown forces only can be determined. The conditions of equilibrium when using the funicular polygon construction are that both the funicular and force polygons must close. Sections may be passed through a structure cutting members in any convenient manner; as a rule, however, cutting not more than three members.

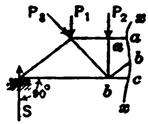

FIG. 37.

For the determination of stresses in framed structures, see p. 224.

Center of a Set of Parallel Forces; Center of Gravity

The center of a set of parallel forces is the point through which the resultant of the forces always passes, no matter how the forces are turned, providing that they always remain parallel and their points of application remain in the same relation to each other.

The resultant R of the two parallel forces F_1 and F_2 (Fig. 38) divides the line connecting the points of application A_1, A_2 into two segments a_1 and a_2 at the point S. $F_1a_1 = F_2a_2$. The point S is independent of the direction of the parallel forces, provided they always remain parallel and applied at A_1 and A_2. The point S, through which the resultant always passes, is called **the center of the parallel forces** F_1 **and** F_2.

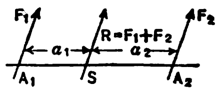

FIG. 38.

The **static moment** of a line, surface or solid with respect to an axis is the length of the line, the area of the surface or the weight of the solid multiplied by the distance from the axis to the center of gravity of the line, surface or solid.

Centers of Gravity of Lines. To find the center of gravity of a line, divide it into an indefinitely large number of indefinitely short lengths, to be known as elementary divisions. Multiply the length of each elementary division by its perpendicular distance from some reference axis in a plane containing the line. When the line is all on one side of the axis, add these products (or moments, as they are usually designated), but when the line is on both sides of

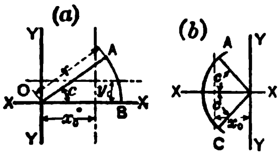

FIG. 39.

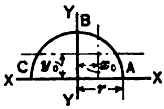

FIG. 40.

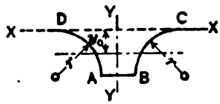

FIG. 41.

the axis, subtract the moments of the part on one side from the moments of the part on the other. Divide this sum or difference by the total length of the line, and the quotient will be the perpendicular distance from the axis to the center of gravity of the line, and should be measured on the side of the axis that has the greater moment if the line is on both sides of the axis.

Centers of Gravity of Plane Areas. Proceed in the same manner as for lines, dividing the area into elementary areas. If the plane area has an axis of symmetry, the center of gravity lies on this axis.

Centers of Gravity of Technically Important Lines, Areas and Solids

CENTERS OF GRAVITY OF LINES

Straight Line. The center of gravity is at its middle point.

Circular Arc AB, Fig. 39(a): $x_0 = r \sin c/\text{rad } c$; $y_0 = 2r \sin^2 \frac{1}{2}c/\text{rad } c$. (rad c = angle c measured in radians; see p. 44.)

Circular Arc AC, Fig. 39(b): $x_0 = r \sin c/\text{rad } c$; $y_0 = 0$.

Quadrant, AB, Fig. 40: $x_0 = y_0 = 2r/\pi = 0.6366\, r$.

Semi-circumference, AC, Fig. 40: $y_0 = 2r/\pi = 0.6366r$; $x_0 = 0$.

Combination of Arcs and Straight Line, Fig. 41: AD and BC are two quadrants of radius r. $y_0 = \{(AB)r + 2[0.5\pi r(r - 0.6366r)]\} + [AB + 2\,(0.5\pi r)]$.

CENTERS OF GRAVITY OF PLANE AREAS

Triangle. Center of gravity (C. G.) lies at the intersection of the lines joining the vertices with the mid-points of the sides, and at a distance from any side equal to one-third of the corresponding altitude.

Parallelogram. C. G. lies at the point of intersection of the diagonals.

Trapezoid, Fig. 42. C. G. lies on the line joining the middle points m and n of the parallel sides. The distances h_a and h_b are

$$h_a = h(a + 2b)/3\,(a + b);\ h_b = h(2a + b)/3(a + b).$$

Draw $BE = a$ and $CF = b$; EF will then intersect mn at C. G.

Any Quadrilateral. The C. G. of any quadrilateral may be determined by the general rule for areas, or graphically by dividing it into two sets of triangles by means of the diagonals. Find the C. G. of each of the four triangles and connect the C. G's. of the triangles belonging to the same set. The intersection of these lines will be C. G. of area. Thus, in Fig. 43, O, O_1, O_2 and O_3 are respectively the centers of gravity of the triangles ABD, ABC, BDC and ACD. The intersection of O_1O_3 with OO_2 gives C. G.

Segment of a Circle, Fig. 44: $x_0 = \frac{2}{3}r \sin^3 c/(\text{rad}\, c - \cos c \sin c)$. A segment may be considered to be a sector from which a triangle is subtracted, and the general rule applied.

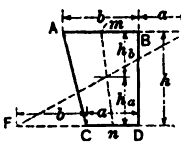

FIG. 42.

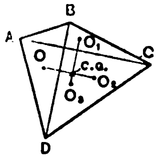

FIG. 43.

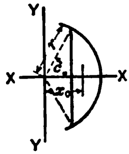

FIG. 44.

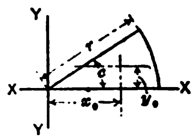

FIG. 45.

Sector of a Circle, Fig. 45: $x_0 = \frac{2}{3}r \sin c/\text{rad}\, c$; $y_0 = \frac{4}{3}r \sin^2 \frac{1}{2}c/\text{rad}\, c$.

Semi-circle, $x_0 = \frac{4}{3}r/\pi = 0.4244r$; $y_0 = 0$.

Quadrant (90-deg. sector): $x_0 = y_0 = \frac{4}{3}r/\pi = 0.4244r$.

Parabolic Half Segment, Fig. 46, Area ABO: $x_0 = \frac{3}{5}x_1$; $y_0 = \frac{3}{8}y_1$.

Parabolic Spandrel, Fig. 46, Area AOC: $x'_0 = \frac{3}{10}x_1$; $y'_0 = \frac{3}{4}y_1$.

Quadrant of an Ellipse, Fig. 47, Area OAB: $x_0 = \frac{4}{3}(a/\pi)$; $y_0 = \frac{4}{3}(b/\pi)$.

The center of gravity of a figure such as that shown in Fig. 48 may be determined as follows: Divide the area $OABC$ into a number of parts by lines drawn perpendicular to the axis XX, *e.g.*, 11, 22, 33, etc. These parts

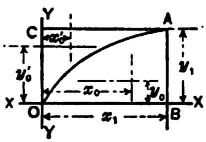

FIG. 46.

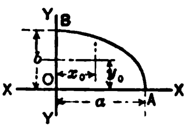

FIG. 47.

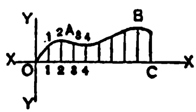

FIG. 48.

will be approximately either triangles, rectangles or trapezoids. The area of each division may be obtained by taking the product of its mean height and its base. The center of gravity of each area may be obtained as previously shown. The sum of the moments of all the areas about XX and YY, respectively, divided by the sum of the areas will give approximately the distances from the center of gravity of the whole area to the axes XX and YY. The greater the number of areas taken the more nearly exact the result.

CENTERS OF GRAVITY OF SOLIDS

Prism or Cylinder With Parallel Bases. C. G. lies in the center of the line connecting the centers of gravity of the bases.

Oblique Frustum of a Right Circular Cylinder, Fig. 49. Let 1 2 3 4 be the plane of symmetry. The distance from the base to the C. G. is

$\frac{1}{2}h + (r^2 \tan^2 c/8h)$, where c is the angle of inclination of the oblique section to the base. The distance of the C. G. from the axis of the cylinder is $\frac{1}{4} r^2 \tan c/h$.

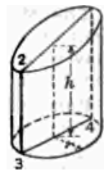

FIG. 49.

Pyramid or Cone. C. G. lies in the line connecting the C. G. of base with the vertex and at a distance of one-fourth of the altitude above the base.

Truncated Pyramid. If h is the height of the truncated pyramid and A and B the areas of its bases, the distance of its C. G. from the surface of A is

$$h(A + 2\sqrt{AB} + 3B)/4(A + \sqrt{AB} + B).$$

Truncated Circular Cone. If h is the height of the frustum and R and r the radii of the bases, the distance from the surface of the base whose radius is R to the center of gravity is $h(R^2 + 2Rr + 3r^2)/4(R^2 + Rr + r^2)$.

Segment of a Sphere, Fig. 50, volume ABC: $x_0 = 3(2r - h)^2/4(3r - h)$.

Hemisphere. $x_0 = 3r/8$.

Hollow Hemisphere. $x_0 = 3(R^4 - r^4)/8(R^3 - r^3)$, where R and r are respectively the outer and inner radii.

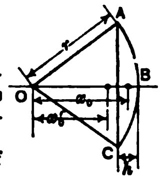

FIG. 50.

Sector of a Sphere, Fig. 50, volume $OABCO$: $x'_0 = \frac{3}{8}(2r - h)$.

Ellipsoid, with semi-axes a, b and c. For each octant, distance from center of gravity to each of the bounding planes $= \frac{3}{8} \times$ length of semi-axis perpendicular to the plane considered.

The formulæ given for the determination of the centers of gravity of lines and areas can be used to determine the areas and volumes, of surfaces and solids of revolution, respectively by employing the theorems of Pappus, p. 111.

Determination of Center of Gravity of a Body by Experiment. The center of gravity may be determined by hanging the body up from different points and plumbing down; the point of intersection of the plumb lines will give the center of gravity. The C. G. may also be determined as shown in Fig. 51. The body is placed on knife edges which rest on platform scales. The sum of the weights registered on the two scales $(w_1 + w_2)$ must equal the weight (w) of the body. Taking a moment axis at either end (say, O), $w_2A/w = x_0 =$ distance from O to plane containing the center of gravity.

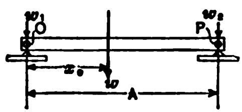

FIG. 51.

Graphical Determination of the Centers of Gravity of Plane Areas. See p. 211 and Fig. 62.

Moment of Inertia

The Moment of Inertia of a Solid Body with respect to a given axis is the limit of the sum of the products of the weights or masses of each of the elementary particles into which the body may be conceived to be divided and the square of their distance from the given axis.

If dw or dm represents the weight or mass of an elementary particle and y its distance from an axis, the moment of inertia I of the body about this axis will be $I_w = \int y^2 dw$ in weight units, and $I_m = \int y^2 dm$ in mass units.

If the lengths are measured in feet and the weights in pounds, the unit for I_w is the lb. ft^2.

If $I_w = r^2w$ or $I_m = r^2m$, the quantity r is called the **radius of gyration** or the **radius of inertia.** Also $I_w = I_m \times g$.

If a body is considered to be composed of a number of parts, its moment of inertia about an axis is equal to the sum of the moments of inertia of the several parts about the same axis, or $I = I_1 + I_2 + I_3 + \ldots\ldots I_n$.

The Moment of Inertia of an Area with respect to a given axis is the limit of the sum of the products of the elementary areas into which the area may be conceived to be divided and the square of their distance (y) from the axis in question. $I = \int y^2dA = r^2A$, where r = radius of gyration.

Relation Between the Moments of Inertia of an Area and a Solid. The moment of inertia of a solid of elementary thickness about an axis is equal to the moment of inertia of the area of one face of the solid about the same axis multiplied by the weight per unit volume of the solid times the elementary thickness of the solid.

Moments of Inertia About Parallel Axes. The moment of inertia of an area or solid about any given axis is equal to the moment of inertia about a parallel axis through the center of gravity plus the square of the distance between the two axes times the area or weight.

In Fig. 52(a), the moment of inertia of the area $ABCD$ about axis YY is equal to I_0 (or the moment of inertia about Y_0Y_0 through the center of gravity of the area and parallel to YY) plus x_0^2A, where A = area of $ABCD$. In Fig. 52(b), the moment of inertia of the weight w about $YY = I_0 + x_0^2w$. Y_0Y_0 passes through C. G. of weight and is parallel to YY. If mass units are used, $I = I_0 + x_0^2m$.

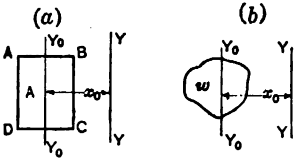

FIG. 52.

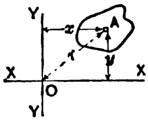

FIG. 53.

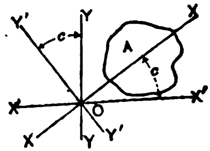

FIG. 54.

Polar Moment of Inertia. The polar moment of inertia, Fig. 53, is taken about an axis perpendicular to the plane of the area. Referring to Fig. 53, if I and J be the moments of inertia of the area A about YY and XX, respectively, then the polar moment of inertia of area about $O = I + J$, or the polar moment of inertia is equal to the sum of the moments of inertia about any two axes at right angles to each other in the plane of the area and intersecting at the pole.

Moment of Deviation, or Product of Inertia. This quantity will be represented by K and is $\int\int xydydx$, where x and y are the co-ordinates of any elementary part into which the area may be conceived to be divided. K may be positive or negative, depending upon the position of the area with respect to the co-ordinate axes XX and YY.

Relation Between Moments of Inertia About Axes Inclined to Each Other. Referring to Fig. 54, let I and J be the moments of inertia of the area A about YY and XX, respectively, I_1 and J_1 the moments about $Y'Y'$ and $X'X'$, and K and K_1 the products of inertia for XX and YY, and $X'X'$ and $Y'Y'$, respectively. Also, let c be the angle between the respective pairs of axes, as shown. Then,

$$I_1 = I\cos^2 c + J\sin^2 c + 2K\cos c\sin c,$$
$$J_1 = I\sin^2 c + J\cos^2 c - 2K\cos c\sin c,$$
$$K_1 = (J - I)\cos c\sin c + K(\cos^2 c - \sin^2 c)$$

Principal Moments of Inertia. In every plane area, a given point being taken as the origin, there is at least one pair of rectangular axes in the plane of the area about one of which the moment of inertia is a maximum, and a minimum about the other. These moments of inertia are called the **principal moments of inertia**, and the axes about which they are taken are the **principal axes of inertia.** One of the conditions for principal moments of inertia is that the product of inertia K shall equal zero. **Axes of symmetry** of an area are always principal axes of inertia.

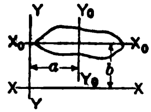

FIG. 55.

Relation Between Products of Inertia for Parallel Axes. In Fig. 55, X_0X_0 and Y_0Y_0 pass through the center of gravity of the area parallel to the given axes XX and YY. If K be the product of inertia for XX and YY, and K_0 that for X_0X_0 and Y_0Y_0, then $K = K_0 + ab\int dA$.

Ellipse of Inertia. In Fig. 56, let $I = \int x^2dA$ and $J = \int y^2dA$ be the moments of the area about the principal axes XX and YY. The moment of inertia about axis $AA = I\sin^2 c + J\cos^2 c$, as $K = 0$. The polar equation of an ellipse constructed on the semi-axes a and b will be $a^2\sin^2 c + b^2\cos^2 c = (a^2b^2)/r^2$, r being the radius vector of any point on the ellipse and c the angle that it makes with XX. Lay off $a = \sqrt{I}$ and $b = \sqrt{J}$; then the moment of inertia about $AA = (a^2b^2)/r^2 = (IJ)/r^2$.

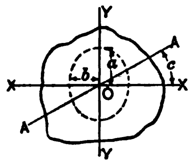

FIG. 56.

The moment of inertia of the area about any axis in the plane and passing through O will be inversely proportional to the square of the semi-diameter cut by that axis from an ellipse constructed with $a = \sqrt{I}$ and $b = \sqrt{J}$, a and b being the semi-major and semi-minor axes, respectively.

The following **conclusions** may be drawn: If the principal moments of inertia are equal, the ellipse of inertia becomes a circle and the moments of inertia about all axes in the plane and passing through O are the same. If the moments of inertia about more than two axes in the plane and passing through O are equal, the moments of inertia about all axes in the plane and passing through O are the same. There are two axes in the plane making equal and opposite angles with a principal axis and passing through O, about which the moments of inertia are equal.

The Moment of Inertia of Any Area may be considered to be made up of the sum or difference of the known moments of inertia of simple figures. For example, the dimensioned figure shown in Fig. 57 represents the section of a rolled shape with hole *oprs*, and may be divided into the semi-circle *abc*, rectangle *edkg*, and triangles *mfg* and *hkl*, from which the rectangle *oprs* is to be subtracted. Referring to axis XX,

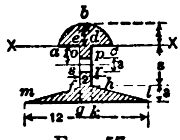

FIG. 57.

$$I_{xx} = \pi 4^4/8 \text{ for semi-circle } abc; \quad = (2 \times 11^3)/3 \text{ for rectangle } edkg;$$
$$= 2\,[(5 \times 3^3)/36 + 10^2(5 \times 3)/2] \text{ for the two triangles } mfg \text{ and } hkl.$$

From the sum of these there is to be subtracted $I_{xx} = [(2 \times 3^3)/12 + 4^2(2 \times 3)]$ for the rectangle *oprs*.

If the moment of inertia of the whole area is required about an axis parallel to XX, but passing through the center of gravity of the whole area, $I_0 = I_{xx} - x_0^2A$, where x_0 = distance from XX to center of gravity. The **moments of inertia of built-up sections** used in structural work may be found in the same manner, the moments of inertia of the different **rolled sections** being given on pp. 1289 to 1297.

Moments of Inertia of Solids. For moments of inertia of solids about parallel axes, $I_x = I_0 + x_0^2w$ or $I_x = I_0 + x_0^2m$, see p. 208. If I_m = moment of inertia in mass units and I_w = moment of inertia in weight units, $I_w = I_m \times g$, and $I_m = I_w/g$.

Moment of Inertia with Reference to Any Axis. Let a mass particle dm of a body have x, y and z as co-ordinates, XX, YY and ZZ being the co-ordinate axes and O the origin. Let $X'X'$ be any axis passing through the origin and making angles of A, B and C with XX, YY and ZZ, respectively. The moment of inertia with respect to this axis then becomes =

$$I_x' = \cos^2 A\int(y^2 + z^2)dm + \cos^2 B\int(z^2 + x^2)dm + \cos^2 C\int(x^2 + y^2)dm$$
$$-2\cos B\cos C\int yzdm - 2\cos C\cos A\int zxdm - 2\cos A\cos B\int xydm.$$

Let the moment of inertia about $XX = I_x = \int(y^2 + z^2)dm$, about $YY = I_y = \int(z^2 + x^2)dm$, and about $ZZ = I_z = \int(x^2 + y^2)dm$.

Let the products of inertia about the three co-ordinates axes be

$$K_{yz} = \int yzdm, \quad K_{zx} = \int zxdm, \text{ and } K_{xy} = \int xydm;$$

then the moment of inertia I_x' becomes equal to

$$I_x\cos^2 A + I_y\cos^2 B + I_z\cos^2 C - 2K_{yz}\cos B\cos C - 2K_{zx}\cos C\cos A - 2K_{xy}\cos A\cos B.$$

The moment of inertia of any solid may be considered to be made up of the sum or difference of the moments of inertia of simple solids of which the moments of inertia are known.

Moments of Inertia of Important Solids (Homogeneous)

w = weight per unit of volume of the body. W = total weight of body.
r = radius. $I = I_w$ = moment of inertia in weight units.

Solid Circular Cylinder about its axis:
$I = \pi r^4wa/2 = Wr^2/2$. (a = length of axis of cylinder.)

Solid Circular Cylinder about an axis through the center of gravity and perpendicular to axis of cylinder: $I = W[r^2 + (a^2/3)]/4$.

Hollow Circular Cylinder about its axis:
$I = \pi wa(r_1^4 - r_2^4)/2$. ($r_1$ and r_2 = outer and inner radii; a = length.)

Thin Hollow Circular Cylinder about its axis: $I = Wr^2$.

Solid Sphere about a diameter: $I = 8w\pi r^5/15 = 2Wr^2/5$.

Thin Hollow Sphere about a diameter: $I = 2Wr^2/3$.

Thick Hollow Sphere about a diameter:
$I = 8w\pi(r_1^5 - r_2^5)/15$. ($r_1$ and r_2 are outer and inner radii.)

Rectangular Prism about an axis through center of gravity and perpendicular to a face whose dimensions are a and b: $I = W(a^2 + b^2)/12$.

Solid Right Circular Cone about an axis through its apex and perpendicular to its axis:
$I = 3W[(r^2/4) + h^2]/5$. (h = altitude of cone, r = radius of base.)

Solid Right Circular Cone about its axis of revolution: $I = 3Wr^2/10$.

Ellipsoid with semi-axes a, b, and c: I about diameter $2c$ (z-axis) =
$I = 4w\pi abc\,(a^2 + b^2)/15$.
[Equation of ellipsoid: $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$].

Ring with Circular Section, Fig. 58:

$I_{yy} = \frac{1}{4}w\pi^2 Ra^2(4R^2 + 3a^2)$;

$I_{xx} = w\pi^2 Ra^2[R^2 + (5a^2/4)]$.

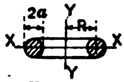

FIG. 58.

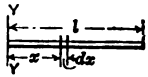

FIG. 59.

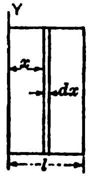

FIG. 60.

Approximate Moments of Inertia of Solids. In order to determine the moment of inertia of a solid, it is necessary to know all its dimensions. In the case of a rod of weight W, Fig. 59, and length l, with shape and size of the cross-section unknown, making the approximation that the weight is all concentrated along the axis of the rod, the moment of inertia about YY will be $I_{yy} = \int_0^l (W/l)x^2dx = Wl^2/3$.

A Thin Plate may be treated in the same way (Fig. 60):

$$I_{yy} = \int_0^l (W/l)x^2dx$$

Here the weight of the plate is assumed as concentrated at its middle layer.

Thin Ring, or Cylinder (Fig. 61): Assume the weight, W, of the ring or cylinder to be concentrated at a distance r from O. The moment of inertia about an axis through O perpendicular to plane of ring or along axis of cylinder will be $I = Wr^2$; this will be greater than the exact moment of inertia, and r is sometimes taken as the distance from O to the center of gravity of the cross-section of the rim.

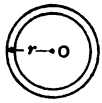

FIG. 61.

Graphical Determination of the Centers of Gravity and Moments of Inertia of Plane Areas. Required, to find the center of gravity of the area MNP, Fig. 62, and its moment of inertia about any axis XX.

Draw any line SS parallel to XX and at a distance d from it. Draw a number of lines such as AB and EF across the figure parallel to XX. From E and F draw ER and FT perpendicular to SS. Select as a pole any point O on XX, preferably the point nearest the area, and draw OR and OT, cutting EF at E' and F'. If the same construction is repeated, using other lines parallel to XX, a number of points will be obtained, which, if connected by a smooth curve, will give the area $M'N'P'$. Project E' and F' on to SS by lines $E'R'$ and $F'T'$. Join F' and T' with O, obtaining E'' and F'', connect the points obtained using other lines parallel to XX and obtain an area $M''N''P''$. The area $M'N'P' \times d$ = moment of area MNP about the line XX, and the distance from XX to the center of gravity of MNP = area $M'N'P' \times d$/area MNP. Also, area $M''N''P'' \times d^2$ = moment of inertia of MNP about XX. The areas $M'N'P'$ and $M''N''P''$ can best be obtained by use of a planimeter.

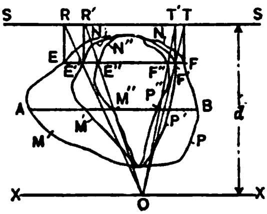

FIG. 62.

DYNAMICS OF RIGID BODIES

Motion Under Unbalanced Forces

If a body is acted upon by a system of forces forming a balanced system, it will either remain at rest or move uniformly in a straight line. Only unbalanced forces change the motion of a body.

Uniformly Accelerated or Retarded Motion requires the action of a constant unbalanced force. A common example of **uniformly accelerated**

motion is the action of gravity on a **falling body.** If the action of gravity is wholly unresisted, the velocity v at the end of time t (in sec.), starting from rest, will be $v = gt$, where g = acceleration due to gravity. If the body has an initial velocity v_0, $v = v_0 + gt$. An example of **uniformly retarded motion** is that of a body projected into the air vertically with a velocity v_0; its velocity after any time t will be $v = v_0 - gt$. If s = space passed through, $s = \frac{1}{2}gt^2$ for a falling body starting from rest, and $s = v_0t + \frac{1}{2}gt^2$ if starting with an initial velocity v_0. For a body projected vertically upward, $s = v_0t - \frac{1}{2}gt^2$.

General Formulæ for the Motion of a Body Under the Action of a Constant Unbalanced Force

s = space in ft., a = acceleration in ft. per sec. per sec., v = velocity in ft. per sec., v_0 = initial velocity in ft. per sec., h = height in ft., F = force, m = mass, w = weight, g = acceleration due to gravity.

Initial velocity = 0:	Initial velocity = v_0:
$F = ma = (w/g)a.$	$F = ma = (w/g)a.$
$v = at.$	$v = v_0 + at.$
$s = \frac{1}{2}at^2 = \frac{1}{2}vt.$	$s = v_0t + \frac{1}{2}at^2 = v_0t + \frac{1}{2}vt.$

$v = \sqrt{2as} = \sqrt{2gh}$ (falling freely from rest).

If a body is to be moved in a straight line by a force, the line of action of this force must pass through its center of gravity.

General Rule for the Solution of Problems When the Forces Are Constant in Magnitude and Direction. Resolve all the forces acting on the body into two components, one in the direction of the body's motion and one at right angles to it. Add the components along the body's motion algebraically and find the **unbalanced force,** if any exists.

Examples. (*a*) The body in Fig. 63 weighs 100 lb., is subjected to external forces F and F_1, and the coefficient of friction between the body and the inclined plane is 0.1. Required, the velocity of the body at the end of five (5) sec. if it starts from rest.

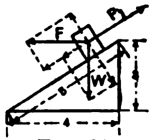

FIG. 63.

First, determine all of the forces acting externally to the body. These are F = 40 lb., F_1 = 80 lb., W = 100 lb., and the force with which the plane reacts upon the body. Resolve the forces into components along the plane and normal to it. The components along the plane are 32 lb. and 60 lb. acting down, 80 lb. acting up, and that component of the plane's resistance which acts against motion. This component of the plane's resistance is the normal component of the pressure between the surfaces multiplied by 0.1, which is the coefficient of friction. The components normal to the plane are 24 lb. acting away and 80 lb. acting toward the plane. The normal pressure is 80 − 24 = 56 lb., and the friction is 56 × 0.1 = 5.6 lb. The unbalanced force acting on the body along the plane is 60 + 32 − 80 − 5.6 = 6.4 lb. and acts downward.

$$F = ma = (100/g)a \quad \therefore a = 0.064g$$

The body is acted upon by constant forces and starts from rest, therefore $v = at$ and $s = \frac{1}{2}at^2$. If $t = 5$, $v = 0.064g \times 5 = 0.32g$, $s = (1/2)0.32g \times 5 = 0.8g$; taking g = 32.16, $v = 0.32 \times 32.16 = 10.29$ ft. per sec., $s = 0.8 \times 32.16 = 25.73$ ft.

(*b*) Find the constant force necessary to start from rest a train weighing 100 tons and give it a speed of 30 miles per hour in 1 min.; track to be level and straight, train resistance to be constant and equal to 12 lb. per ton. The external forces acting upon the train are the resistance of 100 × 12 = 1200 lb., the constant force F pulling the train along the level track, the force of gravity equal to weight of train, and the reaction of the track. The forces normal to the track form a balanced system, but, as the train is to be started from rest and given a velocity, the forces acting along the track must form an unbalanced system. A velocity of 30 miles per hour = 44 ft. per sec.; to acquire this velocity in 60 sec. requires an acceleration of 44/60, as $a = v/t$. The unbalanced force to give the train this acceleration is $F = ma = (100 \times 2000/g)(44/60) = 4580$ lb. The total tractive force required is therefore $4580 + 1200 = 5780$ lb. If the train is on a grade, the problem is the same, with the addition of another force acting along the track, that is, the component of the weight along the track.

The so-called **laws of inclined planes** are simply derived by the above method. In Fig. 64 the weight W slides down the plane without friction. The unbalanced component of the forces acting along the plane is $W \sin B$, $\therefore a = g \sin B$, $v = (g \sin B)t$, and $s = (\frac{1}{2}g \sin B)t^2$ as the body starts from rest. The time necessary to pass over a distance s is $t = \sqrt{2s/g \sin B}$, and $v = at = \sqrt{2gs \sin B} = \sqrt{2gH}$, showing that if the body slides without friction from T to A, its velocity will be the same as if it fell through the same vertical distance TN. With initial velocity v_0, $v = v_0 \pm (g \sin B)t$, and $s = v_0 t + (\frac{1}{2}g \sin B)t^2$.

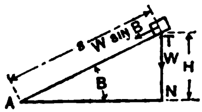

FIG. 64.

Tension in a Rope Connected to a Moving Body. The force with which the rope acts on the body is equal and opposite to the force with which the body acts on the rope, and each is equal to the tension in the rope. In Fig. 65, neglecting the weight of pulley A and the cord, find the tension in the cord. The unbalanced force acting on the two weights is $20 - 5 = 15$ lb. and acts on the two weights. $\therefore 15 = (25/g)a$, and $a = 3g/5$. w_1 is accelerated in an upward direction and w_2 downward. $T_1 = w_1 +$ the unbalanced force necessary to give w_1 an upward acceleration of $(3g/5) = 5 + (5/g) \times (3g/5) = 8$ lb. $T_2 = w_2 -$ the unbalanced force necessary to give w_2 a downward acceleration of $(3g/5) = 20 - (20/g) \times (3g/5) = 8$ lb. If a body is moved uniformly upward by a rope, the tension in the rope is equal to the weight of the body, without regard to the velocity.

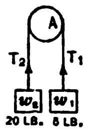

FIG. 65.

If a weight is to be pulled up an inclined plane by means of a rope, the tension in the rope is equal to the component of the weight along the plane plus the normal pressure between the surfaces times the coefficient of friction, plus the unbalanced force necessary to give the weight the acceleration that it has. If the weight is to move uniformly up the plane there will be no acceleration, and the tension in the rope must simply balance all resistance to motion.

Harmonic Motion

Forces Necessary to Cause Harmonic Motion. In Fig. 66, point A moves uniformly in the circle $X'YXY'$, and point P moves to and fro along $X'CX$.

Let ω be the angular velocity of point A. Consider that it starts from X' and moves to A in time t; then the angle $B = \omega t$. If $CA = r$, $X'P = r - CP = r - r \cos \omega t = s$. The velocity v of the point P will equal $ds/dt = \omega r \sin \omega t$, and the acceleration $a = dv/dt = -\omega^2 r \cos \omega t$. The unbalanced force acting on the body P at any time $= F = ma = m\omega^2 r \cos \omega t = (w/g)\omega^2 r \cos \omega t = m\omega^2 CP$, showing that the force varies as the distance of the body from the center of its path, being 0 at the center and $(w/g)\ \omega^2 r$ at the ends of the path. The velocity is 0 at the ends and a maximum at the middle, where $v = \omega r$.

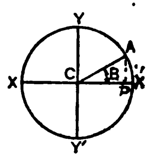

FIG. 66.

When a body is acted upon by varying forces, methods similar to those adopted for constant forces are used, but, in general, more simple solutions can be found by using the principles of work and energy.

Work and Energy

Work. When a body is displaced against resistance or accelerated, work must be done upon it. The work done by a constant force is the force F multiplied by the distance through which the force moves, or work $= Fs$. The work done by a force that varies $= \int F ds$ when the force F is expressed as a function of the space s. In the **work diagram,** Fig. 67, the ordinates are forces at different times and the abscissæ are spaces passed over. The area under the curve represents the work done, which is equal to $\int F ds$.

Units of Work. When the force of 1 lb. acts through the distance of 1 ft., **1 ft.-lb.** of work is done. The **foot-ton** unit is sometimes used. In countries using the metric system the unit employed is the **meter-kilogram.**

Energy. A body is said to possess energy when it can do work. A body may possess this capacity through its **position** or **condition.** When a body is so held that it can do work, if released, it is said to possess energy of position or **potential energy.** When a body is moving with some velocity, it is said to possess energy of motion or **kinetic energy.** An example of potential energy is a body held suspended by a rope; the position of the body is such that if the rope be removed work can be done by the body.

Energy is expressed in the same units as work. The kinetic energy of a body is expressed by the formula $E = \frac{1}{2}mv^2 = \frac{1}{2}(w/g)v^2$.

If a force which varies acts through a space on a body of mass m, the work done is $\int_{s_1}^{s_2} F ds$, and if the work is all used in giving kinetic energy to the body it is equal to $\frac{1}{2}m(v_2^2 - v_1^2) =$ **change in kinetic energy,** where v_2 and v_1 are the velocities at distances s_2 and s_1 respectively. For the kinetic energy of rotation, see p. 217.

If a force which varies acts for a certain time on a body of mass m, the quantity $\int_{t_0}^{t_1} F dt = m(v_1 - v_0)$, = the **change in momentum** of the body.

Certain problems in which the velocity of a body at any point in its straight-line path when acted upon by varying forces is required, can be easily solved by the use of a **work diagram.**

In Fig. 67, let a body weighing 100 lb. start from rest at A and be acted upon by a force that varies in accordance with the diagram $AFGBA$. Let the resistance to motion be a constant force $= x$. Find the velocity of the body at point B. The area $AFGBA$ represents the work done upon the body and the area $AEDBA$ ($=$ force $x \times$ distance AB) represents the work that must be done to overcome resistance. The difference of these areas, or $EFGDE$, will represent work done in excess of that required to overcome resistance, and consequently is equal to the increase in kinetic energy. Equating the work represented by the area $EFGDE$ to $\frac{1}{2}wv^2/g$ and solving for v, will give the required velocity at B. If the body did not start from rest this area would represent the change in kinetic energy, and the velocity could be obtained by the formula: Work $= \frac{1}{2}(w/g)(v_1^2 - v_0^2)$, v_1 being the required velocity.

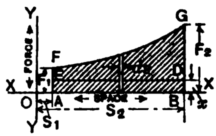

FIG. 67.

General Rule for Rectilinear Motion. Resolve each force acting on the body into components, one of which acts along the line of motion of the body and the other at right angles to the line of motion. Take the sum of all the components acting in the direction of the motion and multiply this sum by the distance moved through for constant forces. (Take the average force times distance for forces that vary.) This product will be the total work done upon the body. If there is no unbalanced component there will be no change in velocity and consequently no change in kinetic energy. If there is an unbalanced component the change in kinetic energy will be this unbalanced component multiplied by the distance moved through.

Conservation of Energy. The sum of the kinetic and potential energies of a body acted on by gravity alone is constant.

If a body is at a certain height h above the ground and is motionless, its potential energy is equal to the work done in raising it to that height. If the body falls to the ground its velocity v when it strikes will be $\sqrt{2gh}$, and its kinetic energy will be $wv^2/2g$. The potential energy before starting to drop is, therefore, the same as its kinetic energy when it reaches the earth. At any point during its fall its total energy, obtained by adding the kinetic and potential energies, will be the same $wv^2/2g$, where v is the velocity that the body would have if it fell freely to the earth from the original height.

The work done by a system of forces acting on a body is equal to the algebraic sum of the work done by each force taken separately.

Power is the rate at which work is performed, or the number of units of work performed in unit time. The unit of power employed by engineers is the **horse power,** or 33,000 ft.-lb. per min. = 550 ft.-lb. per sec.

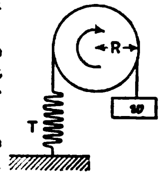

FIG. 68.

Friction Brake. In Fig. 68 a pulley revolves under the band and in direction of the arrow, exerting a pull of T on the spring. The friction of the band on the rim of the pulley is $(T - w)$, where w is the weight attached to one end of the band. Let the pulley make N r.p.m.; then the work done per minute against friction by the rim of the pulley is $2\pi RN(T-w)$, and the horse power absorbed by brake $= 2\pi RN(T-w)/33{,}000$.

Centrifugal Force

Centrifugal and Centripetal Forces. When a body revolves about an axis, some connection must exist capable of applying force enough to the body to constantly deviate it toward the axis. This deviating force is known as **centripetal force.** The equal and opposite resistance offered by the body to the connection is called a **centrifugal force.** The acceleration toward the axis necessary to keep a particle moving in a circle about that axis is v^2/r, therefore the force necessary is $ma = mv^2/r = wv^2/gr$. This force is constantly directed toward the axis.

The centrifugal force of a solid body revolving about an axis is the same as if the whole mass of the body were concentrated at its center of gravity. Centrifugal force $= wv^2/gr = mv^2/r = w\omega^2r/g$, where w and m are the weight and mass of the whole body, r is the distance from the axis about which the body is rotating to the center of gravity of the body, ω the angular velocity of the body about the axis in radians, and v the linear velocity of the center of gravity of the body.

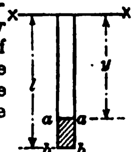

FIG. 69.

Examples (Uniform angular velocity). (1) In Fig. 69 a homogeneous rod of length l revolves about XX at a speed of N r.p.m.; find the pressure on the axis XX due to centrifugal force (C. F.). As the whole body is deviated toward XX, the force necessary must be C. F. $= (w/g)(2\pi N/60)^2(l/2)$ w being the weight deviated, $2\pi N/60$ = the angular velocity and $l/2$ the distance from axis to center of gravity.

(2) Find the tension in the rod at aa, distant y from the axis. The force with which the length y of the rod acts on the length ab is the deviating force necessary to keep $aabba$ constantly accelerated toward the axis. This force $= \frac{w'}{g}\left(\frac{2\pi N}{60}\right)^2\left[y + \frac{(l-y)}{2}\right]$, w' being the weight of $aabba$, and $\left[y + \frac{(l-y)}{2}\right]$ being the distance from XX to the center of gravity of $aabba$.

(3) In Fig. 70 find the pressure on the axis YY and maximum tension due to centrifugal force. Determine the center of gravity of the body; let it be r_0 from the axis YY; then the pressure on the axis will be $(w/g)(2\pi N/60)^2 r_0$. To determine the maximum tension due to centrifugal force, determine which part of the body on one side of the axis has the greater moment about the axis; the maximum tension will be on this side of the axis, and the pressure on the axis will be the difference between the tensions on the two sides.

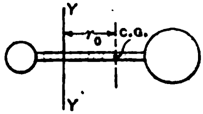

FIG. 70.

The tension in a belt due to centrifugal force is wv^2/g, where w = weight of material per cu. ft., and v = velocity in ft. per sec. It should be noted that the tension **is independent of the diameter of the pulley** upon which the belt runs.

A body revolving about an axis passing through its center of gravity exerts no pressure on the axis due to centrifugal force.

Conical Pendulum. In the simple revolving pendulum shown in Fig. 71 a ball weighing w lb. revolves about YY at a speed of N r.p.m.; find the angle a. The forces acting upon the mass of the ball are the weight w and the tension T in the cord. As the ball stands out at a constant angle, the vertical component of T must equal w, leaving the horizontal component of T unbalanced. This unbalanced component is the deviating force necessary to keep the weight revolving about YY. This deviating force is the centripetal force.

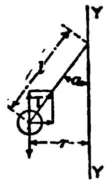

FIG. 71.

$T \sin a =$ C. F. and $T \cos a = w$, but C. F. $= w\omega^2 r/g$; $\therefore T \sin a = w\omega^2 r/g$, $T = w\omega^2 r/g \sin a$, and $\sin a = w\omega^2 r/gT$; $r = l \sin a$, and $\omega = 2\pi N/60$. From these formulæ the tension and angle can be found if N is known, or the tension and number of revolutions per minute can be determined if the angle is known. It should be noted that $\omega^2 l$ must be greater than g.

Balancing

A rotating body is said to be in **standing balance** when its center of gravity coincides with the axis upon which it revolves. Standing balance may be obtained by resting the axis carrying the body upon two horizontal plane surfaces, as in Fig. 72. If the center of gravity of the wheel A coincides with the center of the shaft B, there will be no movement, but if the center of gravity does not coincide with the center of the shaft, the shaft will roll until the center of gravity of the wheel comes directly under the center of the shaft. The center of gravity may be brought to the center of the shaft by adding or taking away weight at proper points on the diameter passing through the center of gravity and the center of the shaft. Weights may be added to or subtracted from any part of the wheel so long as its center of gravity is brought to the center of the shaft.

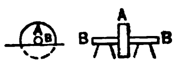

FIG. 72.

A rotating body may be in standing balance and not in **running balance.** In Fig. 73, AA and BB are two disks whose centers of gravity are at o and p, respectively. The shaft and the disks are in standing balance if the disks are of the same weight and the distances of o and p from the center of the shaft are equal, and o and p lie in the same axial plane but on opposite sides of the shaft. Let the weight of each disk be w and the distances of o and p from the center of the shaft each be equal to r. The force exerted on the shaft by AA is equal to $w\omega^2r/g$, where ω is the angular velocity of shaft. Also, the force exerted on shaft by $BB = w\omega^2r/g$. These two equal and opposite parallel forces act at a distance x apart, and constitute a couple with a moment tending to rotate the shaft, as shown by the arrows, of $(w\omega^2r/g)x$. A couple cannot be balanced by a single force, so two forces at least must be added to or subtracted from the system to get running balance.

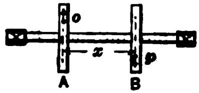

FIG. 73.

For an example of balancing by counterweights, see p. 1207.

Curvilinear Motion

An Unresisted Projectile has a motion compounded of the vertical motion of a falling body, and of the horizontal motion due to the horizontal component of the velocity of projection. In Fig. 74 the only force acting after the projectile starts is gravity, which causes an acceleration downward. The horizontal component of the original velocity v_0 is not changed by gravity. The projectile will rise until the velocity given to it by gravity is equal to the vertical component of the starting velocity v_0 and the equation $v_0 \sin a = gt$, gives the time t required to reach the highest point in the curve. The same time will be taken in falling if the surface XX is level, and the projectile will therefore be in flight $2t$ sec. The distance $s = v_0 \cos a \times 2t$, and the maximum height of ascent $h = (v_0 \sin a)^2/2g$. The expressions for the co-ordinates of any point on the path of the projectile are: $x = (v_0 \cos a)t$, and $y = (v_0 \sin a)t - \frac{1}{2} gt^2$, giving $y = x \tan a - (gx^2/2v_0^2 \cos^2 a)$ as the equation for the curve of the path. The radius of curvature of the highest point may be found by using the general expression C. F. $= wv^2/gr$ and solving for r, v being taken equal to $v_0 \cos a$.

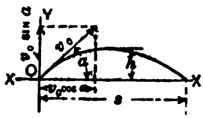

FIG. 74.

Simple Circular Pendulum. The time of a single oscillation in one direction $= t = \pi \sqrt{l/g}$, where l is the length of the pendulum and the length of the swing is not great as compared to l.

Rotation of Solid Bodies About Axes

If a body rotates about an axis under the action of a constant force F, the relations between the angular acceleration a_a, the angular velocity ω, the angle moved through θ, and the kinetic energy of rotation E, are $Fl = a_a I/g$; $\omega = a_a t$, or $= \omega_0 + a_a t$; $\theta = \frac{1}{2} a_a t^2 = \frac{1}{2}\omega t$, or $= \omega_0 t + \frac{1}{2} a_a t^2$; $E = \omega^2 I/2g$; where Fl is the unbalanced turning moment, I is the moment of inertia of the body in weight units, and ω_0 is the initial angular velocity.

General Rule for Rotating Bodies. Determine all the external forces acting and their moments about the axis of rotation. If these moments are balanced, there will be no change of motion. If the moments are unbalanced, this unbalanced moment or **torque** will cause an angular acceleration about the axis.

Rotation About an Axis Passing Through the Center of Gravity. The rotation of a body about its center of gravity can only be caused or changed by a **couple.** See Fig. 75. If a single force F is applied to the rim of the wheel, the axis immediately acts on the wheel with an equal force to prevent translation, and the result is a couple (moment Fr) acting on the body and causing rotation about its center of gravity. **The work done by the couple causing rotation is $M\theta$,** where M is the moment of the couple and θ is the angular space passed over, in radians. If there is no unbalanced moment there must exist either rest or uniform rotation.

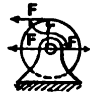

FIG. 75.

In calculations pertaining to **hoisting machinery**, it is sometimes necessary to determine the total weight to be accelerated. This consists of the combined weight of the load to be hoisted, the rope or chain and the cage or skip (if used) *plus* a certain weight w_1 equivalent to that of the hoisting drum if concentrated at a point in its circumference. This latter may be determined as follows: Energy in ft.-lb. of rotating drum $= E = I\omega^2/2 = w\omega^2r^2/2g$, where r = radius of gyration of drum, in ft.; ω = angular velocity, and w = weight in lb. The product wr^2 is sometimes known as the **flywheel effect.** Let R = radius of drum from shaft axis to rope center, in ft. Then, considering the weight concentrated at radius R, for the same quantity of energy, $E = w_1\omega^2R^2/2g$, whence $w_1R^2 = wr^2$, or $w_1 = wr^2/R^2$. For example, in a large electric mine hoist, w = 16,000 lb., r = 6.4 ft. and R = 8 ft. $\therefore$ equivalent weight at $R = w_1 = (16{,}000 \times 6.4 \times 6.4)/(8 \times 8) = 10{,}240$ lb.

Center of Percussion

Combined Rotation and Translation. Any body that is revolving about an axis not passing through its center of gravity may be considered to have a motion of translation and a motion of rotation about its center of gravity combined. The body shown in Fig. 76 is revolving about C with an angular velocity ω, but this motion may be considered to be made up of a translation with the velocity ωl of the center of gravity of the body about C and a revolution about the center of gravity with the same angular velocity (ω) as that with which the body rotates about C.

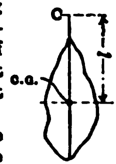

FIG. 76.

The kinetic energy of the body is $\omega^2 I/2g$, in which I is the moment of inertia of the body with respect to C. This is the same as $\frac{1}{2}wv^2/g + \omega^2 I_0/2g$, where $v = \omega l$ and I_0 = moment of inertia of the body with respect to an axis passing through its center of gravity and parallel to the axis at C. The motion of translation requires a single force applied at the center of gravity of the body, and the motion of rotation about the center of gravity requires the action of a couple, therefore the combined motion must be caused by the resultant of a single force and a couple.

The point of application of this resultant is known as the **center of percussion,** and may be **defined** as the point of application of all of the forces tending to cause a body to rotate about a certain axis. It is the point at which a suspended body may be struck without causing any pressure on the axis passing through the point of suspension.

Center of Percussion. The distance from the axis of suspension to the center of percussion is $l = I/wx_0$, where I = moment of inertia of the body about its axis of suspension, w = total weight of the body and x_0 = distance from the axis of suspension to the center of gravity of the body.

Examples. (1) Find the center of percussion of the homogeneous rod (Fig. 77) of length L and weight w, suspended at XX.

$$l = \frac{I}{wx_0}; \quad I(\text{approx.}) = \frac{w}{L}\int_0^L x^2dx; \quad x_0 = \frac{L}{2}; \quad \therefore l = 2\int_0^L x^2dx = 2L/3.$$

(2) Find the center of percussion of a solid cylinder, of weight w, resting on a horizontal plane. In Fig. 78, the instantaneous center of the cylinder is at A. The center of percussion will therefore be at a height above the plane equal to $l = I/wx_0$. Since $I = (wr^2/2) + wr^2$ and $x_0 = r$, $l = 3r/2$.

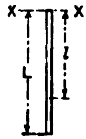

FIG. 77.

(3) In Fig. 79, a solid cylinder weighing 100 lb. is moved along a horizontal plane by a horizontal force of 12 lb. applied at the axis. If the cylinder rolls without slipping, how far may it be moved in 10 sec. and what must be the friction?

The forces acting are F, gravity, and the reaction of the plane. The vertical component of the reaction of the plane balances gravity, leaving the horizontal component of the reaction of plane and F as the forces causing rotation about the instantaneous center O. The resultant R of the 12-lb. force and the friction must be applied at the center of percussion, which is at the distance I/wx_0 from O or 3 ft. The resultant can be found by taking moments about O. $3R = 2 \times 12$; $\therefore R = 8$ lb. The friction must be $12 - 8 = 4$ lb. acting to the left, because 8 lb. is the resultant of $F = 12$ lb. and the friction. This resultant of 8 lb. at a distance of 3 ft. from O has the same effect as the two forces of 12 lb. and 4 lb. acting together. This single force of 8 lb. must be also the resultant of a couple and a single force at the axis, see Fig. 79(b). The single force of 8 lb. acting to the right at the axis causes translation, and the couple whose moment is 8×1 causes rotation about the center of gravity. The linear acceleration along the plane is found by using the 8-lb. force. $F = ma$ $\therefore 8 = 100a/g$, $a = 8g/100$, and $v = (8g/100) \times 10 = (4g/5)$ ft. per sec. $S = ½ (80g/100)$.

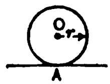

FIG. 78.

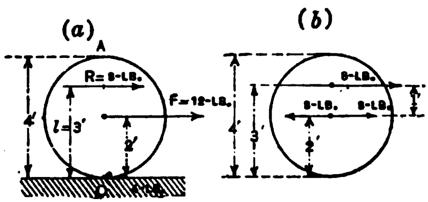

FIG. 79.

Wheel or Cylinder Rolling Down a Plane. In this case the component of the weight along the plane tends to make it roll down, and is treated as a force causing rotation. The forces acting on the body should be resolved into components along the line of motion and perpendicular to it. If the forces are all known, their resultant is at the center of percussion. If one force is to be determined (the exact conditions as regards slipping or not slipping must be known), the center of percussion can be determined and the unknown force found.

When weights act as shown in Fig. 80, where the pulley over which the ropes run has appreciable weight, it is necessary to take into account the moment of the force required to accelerate the pulley itself.

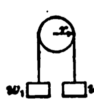

FIG. 80.

In Fig. 80, the unbalanced force $w_1 - w_2$ is accelerating the pulley as well as the two weights. Therefore, $w_1 - w_2 = (w_1 + w_2)a/g + a_aI/gr$, where $(w_1 + w_2)a/g$ is the unbalanced force necessary to give the two weights the acceleration a, and a_aI/g is the unbalanced moment necessary to give the pulley the angular acceleration a_a.

Relation Between the Center of Percussion and Radius of Gyration. $l = I/wx_0 = r^2/x_0 \therefore r^2 = x_0 l$, where r = radius of gyration. Therefore the radius of gyration is a mean proportional between the distance from the axis of oscillation to the center of percussion and the distance from the same axis to the center of gravity.

Interchangeability of Center of Percussion and Axis of Oscillation. If a body is suspended from an axis, the center of percussion for that axis can be found. If the body be suspended from this center of percussion as an axis, the original axis of suspension will then become the center of percussion. The center of percussion is sometimes known as the **center of oscillation.**

Time of Oscillation of a Compound Pendulum. The length of an equivalent simple pendulum is the distance from the axis of suspension to the center of percussion of the body in question. To find the **time of oscillation** of a body about a given axis, find the distance $l = I/wx_0$ from that axis to the center of percussion of the swinging body. The length of the simple pendulum that will oscillate in the same time is this distance l. The time of oscillation for the equivalent simple pendulum is $t = \pi\sqrt{l/g}$. This is the time necessary for a single swing, not over and back.

Determination of Moment of Inertia by Experiment. To find the moment of inertia of a body, suspend it from some axis not passing through the center of gravity and, by swinging it, determine the time of a single oscillation in seconds. The known values will then be t = time of single oscillation, x_0 = distance from axis to center of gravity, and w = weight of rod. The length of the equivalent simple pendulum is $l = I/wx_0$. Substituting this value of l in $t = \pi\sqrt{l/g}$ gives $t = \pi\sqrt{I/wx_0g}$, from which $t^2 = \pi^2 I/wx_0g$, or $I = wx_0gt^2/\pi^2$.

Impulse and Momentum

Impulse of a force $= F(t_2 - t_1)$ when the force is constant and $t_2 - t_1$ is the time interval during which it acts. If the force is not constant in magnitude but always acts in the same direction, impulse $= \int_{t_1}^{t_2} F dt$.

Impulses may be added algebraically by means of a polygon, or an impulse may be resolved into components by means of a parallelogram. The moment of an impulse may be found in the same manner as the moment of a force by representing the impulse by a line and taking the product of the magnitude of the impulse by the perpendicular distance from the line representing it to the point about which the moment is to be taken.

Momentum. The momentum of a particle is mv, where m = mass of the particle and v = its velocity. Momentums can be represented by lines (vectors) and added and resolved in the same manner as forces by means of polygon and parallelogram. The moment of momentum can be determined by the same methods as used for the moment of a force or moment of an impulse, the momentum being represented by a line. **Angular momentum** is the product of the component of momentum at right angles to the radius, and the radius.

Impact

The straight line perpendicular to the common contact plane of two colliding bodies and passing through the point of contact, is called the **line of impact.** If the centers of gravity of the bodies lie on this line, the impact is called **central impact**; in any other case, **eccentric impact.** If the directions of the motions of the two bodies coincide with the line of impact, the impact is said to be **direct**; in any other case, **oblique.**

Direct Central Impact. When two masses m_1 and m_2 having velocities u_1 and u_2 and moving in the same line meet each other, the relation between the velocities u_1 and u_2 before and v_1 and v_2 after collision is expressed by the equation: $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$. If $v' =$ velocity of m_1, $v'' =$ velocity of m_2 at any time during impact, and $v =$ common velocity at instant of greatest compression, $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 = m_1v' + m_2v''$, and $v = (m_1u_1 + m_2u_2)/(m_1 + m_2)$. **The sum of the momentums of the two masses before, after and during impact is the same.**

A second relation between the velocities is $e = (v_2 - v_1)/(u_1 - u_2)$, e being called the **coefficient of impact**, or the **coefficient of restitution.** Its value depends on the elastic or plastic properties of the bodies which come in collision, being 0 for inelastic bodies (nearly 0 for soft substances such as putty or lead) and 1 for perfectly elastic bodies.

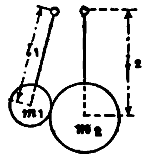

FIG. 81.

To determine the coefficient of restitution between two materials, let m_2 be so large in comparison to m_1 that it may be considered as infinite, *e.g.*, a foundation weighing many tons and a ball weighing a few ounces. Let m_2 be stationary, making u_2 and $v_2 = 0$; the formula $e = (v_2 - v_1)/(u_1 - u_2)$ will then reduce to $e = -v_1/u_1$. Drop the ball from the height H upon m_2 and note the rebound h. Then $u_1 = \sqrt{2gH}$ downward and $v_1 = \sqrt{2gh}$ upward, and $e = -v_1/u_1 = \sqrt{h/H}$. For central impact when the bodies are swinging (see Fig. 81), $v_1 + eu_1 = v_2 + eu_2$, and also if I_1 and $I_2 =$ moments of inertia of m_1 and m_2, $(I_1v_1/gl_1^2) + (I_2v_2/gl_2^2) = (I_1u_1/gl_1^2) + (I_2u_2/gl_2^2)$.

Impact of Perfectly Inelastic Bodies. The two masses move after the impact with a common velocity $u = u_1 = u_2 = (m_1v_1 + m_2v_2)/(m_1 + m_2)$, and the **loss of kinetic energy due to impact** $= E = \frac{1}{2}m_1m_2(v_1 - v_2)^2/(m_1 + m_2)$.

If $v_2 = 0$, $u = m_1v_1/(m_1 + m_2)$ and the loss of energy $= \frac{1}{2}m_1m_2v_1^2/(m_1 + m_2)$.

Perfectly Elastic Impact. In this case, the loss of energy $E = 0$. The velocities after impact become: $u_1 = v_1 - [2m_2(v_1 - v_2)/(m_1 + m_2)]$, and $u_2 = v_2 + [2m_1(v_1 - v_2)/(m_1 + m_2)]$. For $v_2 = 0$, $u_1 = (m_1 - m_2)v_1/(m_1 + m_2)$, and $u_2 = 2m_1v_1/(m_1 + m_2)$; there is no loss of energy and the body m_1 retraces its path.

Imperfectly Elastic Impact. If the coefficient of impact is e, $u_1 = v_1 - [m_2(1 + e)(v_1 - v_2)/(m_1 + m_2)]$; $u_2 = v_2 + [m_1(1 + e)(v_1 - v_2)/(m_1 + m_2)]$. Loss of energy $= m_1m_2(1 - e^2)(v_1 - v_2)^2/2(m_1 + m_2)$.

When a jet of water strikes a flat plate perpendicularly to its surface the force exerted by the water on the plate is wv/g, where w is the weight of water striking the plate in a unit of time and v is the velocity. When the jet is inclined to the surface by an angle A, the pressure is $(wv/g) \cos A$.

Fields of Force—Attraction

The space within which the action of a physical force comes into play on bodies lying within its boundaries is called the **field of the force.**

The **strength** or **intensity of the field** at any given point is the relation between a force F acting on a mass m at that point and the mass. Intensity of field $= i = F/m$; $F = mi$.

The **unit of field intensity** is the same as the unit of acceleration, *i.e.*, 1 ft. per sec. per sec., or 1 m. per sec. per sec. The intensity of a field of force may be represented by a line (or **vector**).

A field of force is said to be **homogeneous** when the intensity at all points is uniform and in the same direction.

A field of force is called a **central field of force** with a center O, if the direction of the force acting on the mass particle m in every point of the field passes through O and its magnitude is a function only of the distance r from O to m. A line so drawn through the field of force that its direction coincides at every point with that of the force prevailing at that point, is called a **line of force.**

Law of Gravitation. Two particles attract each other with a force F proportional to their masses m_1 and m_2 and inversely proportional to the square of the distance r between them, or $F = km_1m_2/r^2$, in which k is the **gravitation constant,** or the force with which two spheres of unit mass attract each other through unit distance. If pounds, feet and seconds are used, $k = 3.31 \times 10^{-11}$; if grams, centimeters and seconds, $k = 6.66 \times 10^{-8}$. The **density of a body** is its mass per unit of volume. Density $= d =$ mass/volume.

EXAMPLES IN ATTRACTION

1. Attraction at any point, O Fig. 82, **due to the thin rod** mn $= F = \dfrac{2kda}{x} \sin \dfrac{A + B}{2}$, where d = density of body, and a = cross-sectional area of the rod. The angle which this force makes with axis $XX = (A - B)/2$.

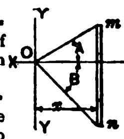

FIG. 82.

2. Attraction due to a **thin circular ring** at a point O on its axis $=$ $F = km \cos A/(r^2 + b^2)$, where b = distance from point to center of ring, r = radius of ring, and A = angle made by a line drawn from circumference of ring to its axis at point O with the axis.

3. Attraction due to a **thin circular plate** at a point O on its axis $=$ $F = 2\pi k(m/A)\ (1 - \cos B)$, where A = area of plate and B = angle made with the axis by a line drawn from circumference of the plate to the axis at point O.

4. Attraction at a point due to a **spherical shell.** If the point is outside of shell, $F = km/b^2$, where m = mass of shell and b = distance from center of shell to the point. If the point is inside of shell, $F = 0$.

5. Attraction at a point due to a **solid sphere.** If point is outside the sphere, $F = km/b^2$, where m = mass of sphere and b = distance from center of sphere to the point. If point is inside of sphere, $F = km'/b^2$, where m' = mass of that part of the sphere that is nearer to the center than the point.

THE GYROSCOPE

By E. V. HUNTINGTON

The elementary facts about gyroscopic forces may be briefly illustrated by the following example. Given, a wheel or disk, of weight w and radius R, spinning rapidly about its axis OA (Fig. 1), and mounted in gimbal rings in such a way that its center is stationary, while its axis is free to point in any direction in space. (See Fig. 2.) Let k = the radius of gyration of the wheel about its axis. (If all the weight is in the rim, $k^2 = R^2$; if the wheel is a homogeneous disk, $k^2 = R^2/2$.) Now suppose the axle end A is constrained to move between two smooth fixed guides, forming a slot as in Fig. 1, and suppose that a force Q is applied to pull the point A along the slot. Then, (1) as far as the motion of A along the slot is concerned, it makes no difference whether the disk is rotating or not; in fact, $Qb = (w/g)(k^2/2)(dv/bdt)$, where Qb is the moment of Q about the center O, and dv/dt is the (linear) acceleration of the point A along the slot; but (2) if the wheel is rotating, then the mere motion of A along the slot will call into play a strong lateral pressure P (called a gyroscopic reaction) against one of the guides.

Which one of the two guides the force P will act against depends on the direction of spin, and is best determined by the following rule (E. V. Huntington, *Eng. News*, July 21, 1910): If the applied force Q is thought of as due to the pressure of a flat board against the side of the revolving axle (the board being perpendicular to Q as shown in dotted line in Fig. 1), then the axle will strive to move in the direction in which it would naturally roll if the board were rough. Thus, in Fig. 1, the axle-end A will push against the *upper* guide, and would actually move in the direction of the P-arrow if the guide were not present.

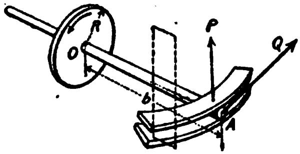

FIG. 1.

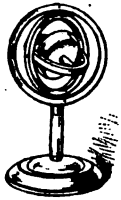
FIG. 2.

The magnitude of the gyroscopic force P at any instant is proportional to the spinning velocity, and also to the velocity of A along the slot, being given by

$$Pb = (w/g)k^2(\pi N/30)(v/b),$$

where Pb = the moment of P about the center, v = the linear velocity of the point A along the slot at the instant in question, N = r.p.m. in the spinning velocity of the wheel, and g = 32.17 ft. per sec. per sec.

It should be noted that the gyroscopic reaction P against the fixed guide does not depend on the magnitude of the accelerating force Q, except in so far as Q is necessary to create the velocity v, in case A starts from rest; when once the velocity v has been established, Q can be made zero, and both v and P will (neglecting friction) continue constant. On the other hand, if the guide is movable, and exerts a pressure $P' = P$ against the axle at A, this force will itself maintain the velocity v (called the precessional velocity) of the point A along the guide. In brief, *the end of the axle of a rotating gyroscope always tends to move at right angles to any force impressed upon it.*

For example, suppose an aeroplane is driven by a right-handed propeller (turning like a right-handed screw when moving forward); if a gust of wind or other force turns the machine to the *left*, the gyroscopic action of the propeller will make the forward end of the shaft strive to *rise;* if the wing surface is large, this motion will be practically prevented by the resistance of the air, and the gyroscopic forces become effective merely as internal stresses, whose maximum value can be computed by the formula above. Similarly, if the aeroplane is dipped *downward*, the gyroscopic action will make the forward end of the shaft strive to turn to the *left*.

In the Brennan monorail car, the slot is parallel to the rail, so that a force Q, applied automatically at the axle end when the car starts to tip, will be converted gyroscopically into a strong righting moment Pb, which forces the car back into a position of lateral equilibrium. Numerical case: If $w = 3200$ lb., $k = 1.4$ ft., $b = 2$ ft., and spinning velocity $= 3000$ r.p.m., then a force Q of 50 lb. acting for ½ sec. will produce at the end of that time a precessional velocity of 1 ft. per sec. along the slot, with a resulting lateral force against the guide of nearly 16,000 lb. The displacement of the point A along the slot will be about 6 deg. An essential feature of the apparatus is a device for bringing the axle back into the ready position after each period of activity.

Other applications are the Whitehead torpedo, the Schlick ship-steadying device, the Griffin grinding mill, the Anschütz gyro-compass, the Sperry gyro-compass, and the Sperry stabiliser for aeroplanes. For the mathematical theory of the gyroscope, see H. Crabtree, "Spinning Tops and Gyroscopic Motion" (Longmans).

STRESSES IN FRAMED STRUCTURES

BY

HARRISON W. HAYWARD

Framed structures may be divided into **two classes**: (*a*) Structures that are **statically determinate** and (*b*) structures that are **statically indeterminate**. The stresses in the members of structures of the first class may be determined by the methods of statics (see p. 195), but structures of the second class require the use of flexure formulæ.

In what follows, structures of the first class only will be considered. The structures will be assumed to be made up of rigid members pinned together at their ends. The forces acting externally to the structure will be assumed as acting in the same plane and at the pin joints only. If these assumptions are made, the stresses in all the members will be either direct tension or compression. The design of the compression members to resist column stresses will be neglected and the weights of the members will be assumed as acting at the joints. Where distributed loads are to be considered, the bending stresses will be neglected and the loads considered to be concentrated at the joints. After the direct stresses are determined, the bending stresses may be found by reference to the theory of beams, pp. 397 to 420.

Two general methods are useful for the determination of the stresses in statically determinate structures: the **method of sections** and the **method of joints**. Most statically determinate structures can be solved by either method, although in some cases the method of joints will fail. That method should be used which will be most convenient for the particular problem at hand, and in many cases both methods may be advantageously used in the same problem.

General Procedure. First determine all the external or outer forces and then determine the inner forces by the most convenient method.

In the simple crane, Fig. 1(*a*), let it be required to determine the stresses in the members AB and AC due to the weight W at A. Let F_1 and F_2 represent the stresses in AB and AC, respectively. The forces acting at point A are F_1, F_2 and W. These forces act at a common point and are all in the same plane; there are two unknown forces of which the line of action is known (see p. 196), therefore the case is statically determinate. Apply the condition of equilibrium (see p. 198) that the algebraic sum of the horizontal and vertical components of the forces acting at a point must equal zero when the forces form an equilibrium system, and obtain:

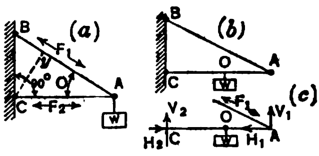

FIG. 1.

$[(F_1 \times BC)/AB] - W = 0$, or $F_1 - [(W \times AB)/(BC] = 0$, from which F_1 (tension) $= W \times AB/BC$; $F_2 - [(W \times AB)/(BC)]\,(AC/AB) = 0$, or $F_2 - [(W \times AC)/BC] = 0$, from which F_2(compression) $= W \times AC/BC$. The same results may be obtained by drawing a polygon of forces for the forces acting at point A (see p. 200).

The method of moments may be used as follows: The algebraic sum of the moments of the forces acting at A taken about any point in their plane must be zero.

$\therefore F_1 \times Cy = W \times AC$, from which $F_1 = (W \times AC)/Cy = (W \times AB)/BC$;
$F_2 \times Bc = W \times AC$, from which $F_2 = (W \times AC)/Bc$.

It should be noted that if a framed structure is to be of any value, the forces acting externally to any part of it must form a system in equilibrium. If the weight W is attached somewhere between A and C, as in Fig. 1(b), the stresses in the members AB and AC cannot both be determined by statics. The stress in AB can be determined and the forces acting on AC can be found. Consider the member AC, Fig. 1(c). This member must be in equilibrium under the action of the forces acting externally to it. Taking moments at C, $W \times CO = V_1 \times AC$, or $V_1 = (W \times CO)/AC$. The line of action of the stress in BA is along AB, therefore when V_1 is determined, H_1 must equal $(V_1 \times AC)/BC$ and $F_1 = (V_1 \times AB)/BC$. $V_2 = W - V_1$ and $H_2 = H_1$. This will allow the stress in AC to be determined by flexure formulæ.

Crane with Single Guy Rope. In Fig. 2, let the stresses in AC, AB and EP be respectively F_1, F_2 and F_3. First determine the stresses F_1 and F_2 by considering the forces acting at point A and applying the conditions for equilibrium for forces in the same plane and acting at the same point. F_1 is compression and F_2 is tension. To determine F_3, consider the forces acting upon the member DE of the structure. Assume a moment axis at point D and apply the condition of equilibrium that the algebraic sum of the moments of forces in the same plane about any point in this plane must be zero; then $(F_2 \times OD) - (F_1 \times TD) = F_3 \times yD$, whence $F_3 = [(F_2 \times OD) - (F_1 \times TD)/yD$, and is tension. The stresses in the member ED cannot be determined by statics, as there are bending stresses.

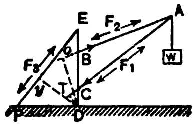

FIG. 2.

Triangular Frame. In Fig. 3, let a single concentrated vertical load W be applied at A. Assume a moment axis at C. Then $(W \times Cd) - (S_2 \times CB) = 0$, or $S_2 = (W \times Cd)/CB$, and $S_1 = W - S_2$. Or, taking a moment axis at B, $(W \times Bd) - (S_1 \times CB) = 0$, and $S_1 = (W \times Bd)/CB$. To determine F_1 and F_3, consider the forces acting at point C and apply conditions for equilibrium for forces in same plane and at the same point. Then $F_1 = (S_1 \times AC)/Ad$ and $F_3 = (F_1 \times Cd)/AC$; also, using point B, $F_2 = (S_2 \times AB)/Ad$ and $F_3 = (F_2 \times Bd)/AB$. If $Cd = Bd$, $F_1 = F_2$ and $S_1 = S_2$. Stresses F_1 and F_2 are compression. Stress F_3 is tension.

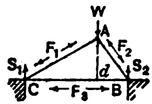

FIG. 3.

Symmetrical King-post Truss. In Fig. 4, the load W is uniformly distributed along CD; one-half of the load therefore may be considered as acting at B. Consider the forces acting at B: stress in AB will be $W/2$, tension in Fig. 4(a) and compression in Fig. 4(b). The stress in AC will equal that in AD and will equal $(W/4) \times AC/AB$, being compression in Fig. 4(a) and tension in Fig. 4(b). The direct stresses in CB and BD can be found by considering the forces acting at points C or D and imposing the condition of equilibrium for forces in same plane and at the same point. These stresses will be the stress in AC or AD multiplied by (BC/AC). (See also p. 1281.)

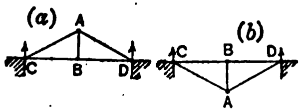

FIG. 4.

Shear Legs and Guy. In Fig. 5, assume that the two legs AD and AC are replaced by a single member Ag which will be in the same plane as AW and AB. The stress in the guy AB will be the same as if the

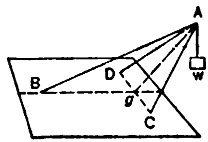

FIG. 5.

two legs were acting and may be determined by considering point A and imposing the condition of equilibrium for the forces W, gA and BA. This will give the stress in the guy AB, and the stress that would be in Ag, if it existed. The two legs AD and AC must support the stress found in Ag, supposing that this member existed alone. Resolve this stress along AD and AC and find the stresses in these members. AD, AC, and Ag are all in the same plane.

Analytical Solution of Trusses

Howe Truss. In Fig. 6, assume the truss to be loaded at the lower panel points as shown, these loads including the weight of the truss. A moment axis at point A will allow the determination of supporting force S_2:

$$S_2 = [(P_1 \times AB) + (P_2 \times AC) + (P_3 \times AD) + (P_4 \times AE) + (P_5 \times AF)]/AG;$$
$$S_1 = P_1 + P_2 + P_3 + P_4 + P_5 - S_2.$$

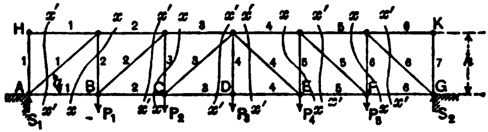

Fig. 6.

Each of the concentrated loads may be divided into parts proportional to the distances from the points A and G to the point of application of the load, and the support for each load found. The total support for all the loads may be obtained from the sums of the partial supporting forces.

Stresses in the Verticals. Pass planes xx through the truss and use method of shears.

In vertical No. 2, stress $= S_1$ (tension)
In vertical No. 3, stress $= S_1 - P_1$ (tension)
In vertical No. 5, stress $= S_2 - P_5$ (tension)
In vertical No. 6, stress $= S_2$ (tension)
In vertical No. 1, stress $= 0$, using joint H
In vertical No. 7, stress $= 0$, using joint K
In vertical No. 4, stress $= P_3$, using joint D, (tension)

Stresses in the Diagonals. Pass planes $x'x'$ through the truss and use method of shears.

In diagonal No.	1	2	3	4	5	6
Vertical component of stress =	S_1,	$S_1 - P_1$,	$S_1 - P_1 - P_2$,	$S_2 - P_5 - P_4$,	$S_2 - P_5$,	S_2

If these vertical components are divided by sin a the stresses in the diagonals can be determined. The stresses will be compression in this case.

Stresses in Upper Chord Members. Pass planes $x'x'$ through the truss and use method of moments.

In upper chord member No. 1, stress $= 0$
In upper chord member No. 2, stress $= (S_1 \times AB)/h$ (compression)
In upper chord member No. 3, stress $= [(S_1 \times AC) - (P_1 \times BC)]/h$ (compression)
In upper chord member No. 4, stress $= [(S_2 \times EG) - (P_5 \times FG)]/h$ (compression)
In upper chord member No. 5, stress $= (S_2 \times FG)/h$ (compression)
In upper chord member No. 6, stress $= 0$

The stresses are all compressions.

Stresses in Lower Chord Members. Use section plane xx. The stresses in the lower chord members Nos. 1, 2, 5 and 6 are equals to the stresses in the upper chord members Nos. 2, 3, 4 and 5, respectively. Stresses in lower chord members 3 and 4 are equal, as can be shown from consideration of joint $D(\Sigma H = 0)$.

To determine the stress in lower chord member No. 3, use section plane $x'x'$ and a moment axis at upper middle panel point.

Stress in No. 3 $= [(S_1 \times AD) - (P_1 \times BD) - (P_2 \times CD)]/h$ (tension).

Whether members are in tension or compression can be determined by methods shown on p. 230.

Warren Truss. In Fig. 7, first determine the two supporting forces S_1 and S_2. Then the stresses in members 1 and 2, and 10 and 11 may be determined by the method of joints, using joint O for 1 and 2 and joint N for 10 and 11. Imposing the conditions of equilibrium for forces in the same plane and acting at the same point,

Stress in member No. 1 $= S_1/\sin a$ (compression)

Stress in member No. 10 $= S_2/\sin a$ (compression)

Stress in member No. 2 = stress in member No. 1 $\times \cos a = S_1 \cot a$ (tension)

Stress in member No. 11 = stress in member No. 10 $\times \cos a = S_2 \cot a$ (tension)

The stresses in the upper chord members 4 and 8 may be determined by passing section planes XX through the truss and taking moments about the joints in the lower chord.

Stress in member No. 4 $= (S_1 \times p)/h$ (compression)

Stress in member No. 8 $= (S_2 \times p)/h$ (compression)

The stresses in member No. 6 may be determined by passing section plane XX through the truss and taking moments about joint C.

Stress in member No. 6 $= [(S_1 \times \frac{3}{2} p) - (P_1 \times p/2)]/h$

$= (S_2 \times \frac{3}{2} p) - (P_2 \times p/2)]/h$, and is tension.

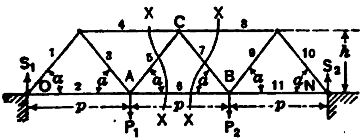

FIG. 7.

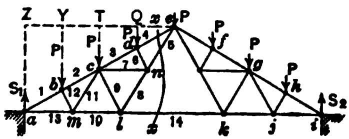

FIG. 8.

Roof Truss. A method of determining the stresses in the members of a roof truss is shown in Fig. 8. After solving for the supporting forces S_1 and S_2, which will be the same and equal to $7P/2$, a stress polygon may be constructed for the forces acting at joint a, which will determine the stresses 1 and 13. A polygon may then be constructed for the forces acting at joint b, which will determine stresses 2 and 12. A polygon for joint m will allow the determination of stresses 10 and 11. The other side of the truss may be treated in the same manner. It is then impossible to go on with the graphical solution, using the method of joints, as more than two unknown forces act at all the other joints. The stress 14 in member lk may be determined by passing the section plane xx through the truss and using the method of sections (see p. 226).

This stress 14 $= [(S_1 \times eZ) - (P \times eY) - (P \times eT) - (P \times eQ)]/S$ (tension). Knowing this stress, a polygon may be constructed for joint e and the solution of the truss completed by the method of joints. A complete analytical solution may be carried out for the joints, using moments or resolution of forces.

Determination of the Diagonal for Tension or Compression. Let the truss in Fig. 9 be loaded with the vertical loads P_1, P_2 and P_3, as shown.

The supporting forces S_1 and S_2 can be determined by taking moments at point a or e for the forces acting externally to the whole truss:

$$S_2 = [(P_1 \times ab) + (P_2 \times ac) + (P_3 \times ad)]/ae;$$
$$S_1 = [(P_3 \times ed) + (P_2 \times ec) + (P_1 \times eb)]/ae.$$

In order that the truss shall be stable, diagonals must be placed in the panels A and B.

Assume that the diagonals are to be tension members. Pass plane XX through the truss and consider the forces acting on the part of the truss to the left of the section. These forces are the load P_1, supporting force S_1, and the stresses in bc, in hg, and in the diagonal that is used.

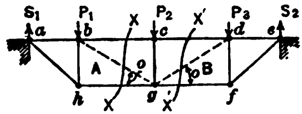

FIG. 9.

Use the method of shears; then the resultant of S_1 and P_1 must be balanced by the vertical component of the stress in the diagonal, bc and hg having no vertical components. The resultant of S_1 and P_1 acts upward, therefore the vertical component of the diagonal must act downward, showing that bg is the required diagonal.

To determine the diagonal for panel B, pass the plane $X'X'$ through the truss and consider the forces acting on the part of the truss to the right. Then the vertical component of the stress in the diagonal must act downward and be equal to $S_2 - P_3$, showing the diagonal gd to be in tension. The stresses in the diagonals may be determined by dividing the vertical components by the sine of the angle o. If compression members are required, use diagonals hc and cf.

When the upper chords are not horizontal and parallel to the lower chord, use the method shown in Fig. 10. The truss in Fig. 10 is symmetrically loaded at the upper panel points. To determine the diagonals for panels A and B, pass plane XX through the truss and consider the forces acting on the portion of the truss to the left of it. Take a moment axis at joint a; then the moment of the stress in the diagonal that is used must be equal and opposite in sense to the moment of P_1. The moment P_1 is $P_1 \times ah$ and is right-handed; therefore, the moment of the stress in the diagonal about a must be left-handed. If the diagonal is to be in compression, use bg; if in tension, use ch. In the same manner, using section plane $X'X'$ and joint e, the diagonal in panel B can be determined. gd will be the compression diagonal and cf the tension member.

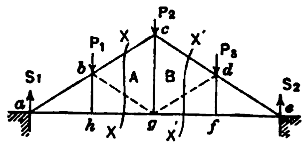

FIG. 10.

Counterbracing. When a truss is to withstand moving loads, it is often necessary to use two diagonals in one panel, or else so design a diagonal as to be able to withstand both tension and compression. In wooden structures the diagonals are designed to resist compression, and in steel structures to withstand tension or compression.

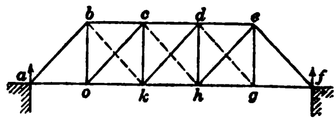

FIG. 11.

In the truss shown in Fig. 11, when the load P is at o, the diagonals oc, kd and he will be in tension; if the load moves to k, the tension diagonals become bk, kd and he; with the load at g, the tension diagonals are bk, ch and dg. As the load P moves from o to g, the stresses in any one set of diagonals

will change from tension to compression, or *vice versa.* In such cases two diagonals must be placed in each panel where the reversal of stress occurs.

Consider the stresses in the members of the truss shown in Fig. 12. This truss is supported on rollers at *a* and fixed at *h*. Vertical loads, as shown, are applied at the points *b, c, e, f* and *g*. A horizontal force of 5000 lb. may be applied at *b*, and act to the right, or at *g* and act to the left. This 5000 lb. may be considered to be wind pressure acting now on one side and now on the other.

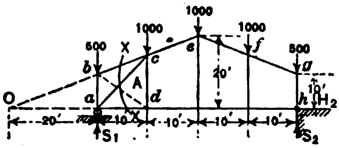

FIG. 12.

First assume the horizontal force to act at point *b*. Then, by taking moments at point *a* for all the forces acting on the truss, $S_2 = (50{,}000 + 10{,}000 + 20{,}000 + 30{,}000 + 20{,}000)/40 = 3250$ upward, and $S_1 = 4000 - 3250 = 750$ upward. H_2 must equal 5000 and act to the left.

Pass plane *XX* through the truss and consider the forces acting upon the part of the truss to the *left* of this section. Take a moment axis at point *O*. Then the moment of the stress in the unknown diagonal must be equal and opposite to the resultant of the moments of all the other forces acting; therefore, the moment of this diagonal must be $50{,}000 + 10{,}000 - 15{,}000 = 45{,}000$ and must be left-handed. *ac* will be the tension diagonal and *bd* the compression diagonal.

Now, assume that the 5000-lb. horizontal force acts at point *g* and to the left. Then $S_2 = (10{,}000 + 20{,}000 + 30{,}000 + 20{,}000 - 50{,}000)/40 = 750$ upward, and $S_1 = 4000 - 750 = 3250$ upward. Pass plane *XX* through the truss and take the forces acting on the part to the left. Take moments about point *O* as before. Then the moment of the unknown diagonal must be $(3250 - 500) \times 20$ and must be right-handed. *bd* will be the tension diagonal and *ac* the compression diagonal. The other panels may be treated in the same manner. This shows that **counterbracing** is sometimes **needed to resist wind pressure** from different sides.

Graphical Solution of Trusses

When the external forces acting upon a truss are all in the same plane and may be assumed to act at the joints of the structure, it is often convenient to make use of graphical constructions to determine the stresses in the members. A series of stress polygons may be constructed for the forces acting at the different joints, using the methods shown on p. 204, but a combination diagram, using what is known as "Bow's Notation," greatly simplifies the work.

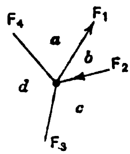

FIG. 13.

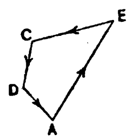

FIG. 14.

The forces F_1, F_2, F_3 and F_4 (Fig. 13) are all in the same plane and form a system in equilibrium. Their lines of action all pass through the same point (*o*). Letters are so placed as to bring each force between two letters, these letters being usually read in a right-handed or clockwise direction around the joint. F_1, F_2, F_3 and F_4 will then be designated *ab*, *bc*, *cd*, and *da*, respectively. A stress polygon (Fig. 14) is then drawn for the forces. Assume that the magnitudes, lines of action and directions of F_1 and F_2 are known and that the lines of action only of F_3 and F_4 are known. The forces must be taken in order and the letters must be so placed that when the forces are read right-handed about the point *o* (Fig. 13) the sequence of the letters (in Fig. 14) will indicate the direction in which the forces act upon point *o*. The manner of using Bow's notation is illustrated by the following problem.

Bow's Notation Applied to a Truss. The truss shown in Fig. 15 is loaded with a uniformly distributed dead load which may be considered to be concentrated at the joints as shown. First plot the polygon of external forces *ABCDEFGA* (Fig. 16). As the forces all act vertically, the sides of the polygon fall in the straight line *AF*. The supporting forces *GA* and *FG* are equal in this case, each being one-half the total load, so that no special construction for their determination is necessary. Start at any joint in the truss where there are not more than two unknown forces, *e.g.*, at the left end of the truss. The stress polygon for this joint is *ABHGA*. Reading the letters in a right-handed direction about the joint, the stress in the upper chord member is *BH*. This sequence of letters in the stress polygon indicates that this member acts downward and to the left on the joint, and therefore is in compression. The stress in the lower chord member is *HG*, and this sequence of letters indicates that this member acts to the right on the joint and therefore is in tension.

At the joint in the middle of the upper chord there are now two unknown stresses. Draw the stress polygon *HBCKH* for the second joint. The stress *HB* is of the same magnitude as for the lower joint but acts in the opposite direction. The stress in the member *kh* is *KH* and is therefore compression.

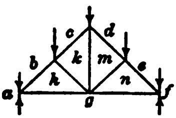

FIG. 15.

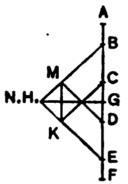

FIG. 16.

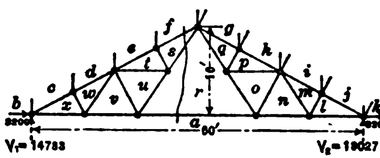

FIG. 17.

Problem. The truss shown in Fig. 17 is loaded with a dead load of 21,760 lb. uniformly distributed over the upper chord, together with a wind load of 13,600 lb. on the right side. Both ends of the truss are fixed and the horizontal components of the supporting forces are assumed to be equal. The supporting forces may be determined graphically by first assuming a roller under one end of the truss, or by the funicular polygon construction (see p. 201). They are often calculated and the polygon of external forces plotted from the results of the calculation. This is a desirable method, as it offers a check on this most important part of the graphical work.

The horizontal component of the wind load is $(16/34)13{,}600 = 6400$ lb. The *H* component of each supporting force is therefore $6400/2 = 3200$ lb. The vertical component of the wind load is $(30/34)13{,}600 = 12{,}000$ lb. Taking moments about the right end of the truss, $(21{,}760 \times 30) + (13{,}600 \times 17) = 60V_1 \quad \therefore V_1 = 14{,}733$; $V_2 = 21{,}760 + 12{,}000 - 14{,}733 = 19{,}027$.

The polygon of external forces can now be constructed, as in Fig. 18. The dotted part of the diagram is the combination of the dead and wind loads, assuming that they are each concentrated. The dotted line *BK* is the resultant of these loads. The supporting forces *KA* and *AB* are determined by plotting to scale their horizontal and vertical components as calculated. The polygon of external forces for the truss is *BCDEFGHIJKAB*, and must check with the polygon shown dotted. The forces *GH*, *HI*, *IJ*, and *JK* are the resultants of the forces acting at the joints on the right side of the truss.

When supporting forces are to be determined the loads *may* be concentrated at the lines of action of their resultants, but when internal stresses are to be determined the loads *must* be distributed at the various joints. Start with some joint of the truss where there are only two unknown forces and draw the stress diagram, Fig. 18. The magnitudes of the stresses in the different members are determined directly from the lengths of the corresponding lines and the scale used in the construction of the diagram. The nature of the stress (tension or compression) is determined by the use of Bow's notation.

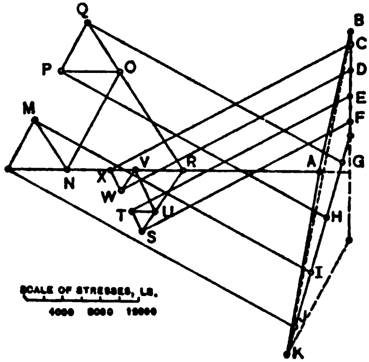

FIG. 18.

A difficulty often arises that is illustrated by the truss of Fig. 17. It is impossible by the usual graphical procedure to complete the stress diagram, as it will be found that, after obtaining the stresses in the members meeting at the left end and at the next joint on both upper and lower chords, no joint with less than three unknown forces is available. To overcome this difficulty, it is necessary to calculate some unknown stress, place it in the diagram and proceed with the graphical solution.

The stress which should be calculated in this case is that in the middle member of the lower chord. Taking moments about a point at the middle of the upper chord, $16RA = (14{,}733 \times 30) - (3200 \times 16) - [2720(22\frac{1}{2} + 15 + 7\frac{1}{2})] - (1360 \times 30) = 227{,}600$ $\therefore RA = 14{,}225$ (tension). Place this stress in the diagram and proceed.

FRICTION

BY

HOWARD D. HESS

REFERENCES: Goodman, "Mechanics Applied to Engineering," Longmans, Green & Co. · Hancock, "Applied Mechanics for Engineers," Macmillan. Kimball and Barr, "Elements of Machine Design," Wiley. Thurston, "Friction and Lost Work," Wiley. Archbutt and Deeley, "Lubrication and Lubricants," Griffin. Alford, "Bearings," McGraw-Hill. Hermann-Smith, "Graphical Statics of Mechanism," Van Nostrand.

COEFFICIENTS OF FRICTION

Friction of Rest

If one body be pressed against another by a force P, as in Fig. 1, the first body will not move provided the angle a_0 included between the line of action of the force and a normal to the surfaces in contact does not exceed a certain value which depends upon the nature of the surfaces. The supporting force R acting through the second body has the same magnitude and line of action as the force P. In Fig. 1, R is resolved into two components: a force N normal to the surfaces in contact, and a force F_r parallel to the surfaces in contact. From the above statement it follows that

$$F_r \leq N \tan a_0 \leq Nf_0,$$

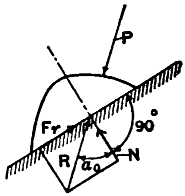

FIG. 1.

where $f_0 = \tan a_0$ is called the **coefficient of friction of rest** (or of static friction) and a_0 is the **angle of friction of rest** (or angle of repose). Values of the coefficients of friction of rest for various materials are given in Table 1.

Table 1. Angles of Repose and Coefficients of Friction of Building Materials

(Rankine's "Applied Mechanics")

Materials	a_0 deg.	$f = \tan a_0$	$\frac{1}{\tan a_0}$
Dry masonry and brickwork	31 to 35	0.6 to 0.7	1.67 to 1.4
Masonry and brickwork with damp mortar	36½	0.74	1.35
Timber on stone	22	about 0.4	2.5
Iron on stone	35 to 16⅔	0.7 to 0.3	1.43 to 3.3
Timber on timber	26½ to 11⅓	0.5 to 0.2	2.0 to 5.0
Timber on metals	31 to 11⅓	0.6 to 0.2	1.67 to 5
Metals on metals	14 to 8½	0.25 to 0.15	4.0 to 6.67
Masonry on dry clay	27	0.51	1.96
Masonry on moist clay	18¼	0.33	3.0
Earth on earth	14 to 45	0.25 to 1.0	4.0 to 1.0
Earth on earth, dry sand, clay, and mixed earth.	21 to 37	0.38 to 0.75	2.63 to 1.33
Earth on earth, damp clay	45	1.0	1.0
Earth on earth, wet clay	17	0.31	3.23
Earth on earth, shingle and gravel	39 to 48	0.81	1.23 to 0.9

Sliding Friction

If the applied force P of Fig. 1, which may be either a single force (*e.g.*, that due to its weight) or the resultant of several forces, makes an angle with the normal to the surfaces in contact exceeding the angle of friction of rest a_0, the frictional resistance F_r will no longer prevent the body from sliding. If a body sliding upon another is acted upon by a force N normal to the surfaces in contact, then a force acting in a direction opposing the motion is being overcome. This force is the **frictional resistance** F.

If the bodies have a common surface of contact, the relative motion is sliding and the resistance is that of sliding friction; if the bodies meet in line contact instead of surface contact, then the relative motion with reference to the line of contact may be either sliding or rolling. Rolling motion is opposed by rolling friction, see p. 236.

The value of the frictional resistance F is commonly expressed in terms of N, as $F = fN$, where f is the **coefficient of sliding friction.** The coefficient of sliding friction f varies with the materials, the character of the surfaces in contact, the unit pressure, the velocity of relative sliding and the temperature. It is less than the coefficient of friction of rest, f_0, except for very low velocities.

Table 2. Coefficients of Sliding Friction

(Condensed from the tables of Morin and from other sources. From Rankine's "Applied Mechanics")

Surfaces	α_0, deg.	f	$1/f$
Wood on wood, dry	14 to 26½	0.25 to 0.5	4.0 to 2.0
Wood on wood, soapy	11½	0.2	5.0
Metals on oak, dry	26½ to 31	0.5 to 0.6	2.0 to 1.67
Metals on oak, wet	13½ to 14½	0.24 to 0.26	4.17 to 3.85
Metals on oak, soapy	11½	0.2	5.0
Metals on elm, dry	11½ to 14	0.2 to 0.25	5.0 to 4.0
Hemp on oak, dry	28	0.53	1.89
Hemp on oak, wet	18½	0.33	3.0
Leather on oak	15 to 19½	0.27 to 0.38	3.7 to 2.86
Leather on metals, dry	29½	0.56	1.79
Leather on metals, wet	20	0.36	2.78
Leather on metals, greasy	13	0.23	4.35
Leather on metals, oily	8½	0.15	6.67
Metals on metals, dry	8½ to 11½	0.15 to 0.2	6.67 to 5.0
Metals on metals, wet	16½	0.3	3.33
Smooth surfaces, occasionally greased	4 to 4½	0.07 to 0.08	14.3 to 12.5
Smooth surfaces, continually greased	3	0.05	20.0
Do., best results	1¾ to 2	0.03 to 0.036	33.3 to 27.6

Table 3. Coefficients of Sliding Friction According to Rennie

(The surfaces were greased and then wiped so that the surfaces in contact were only slightly greasy)

Surface pressure, lb. per sq. in.	Wrought iron on wrought iron	Cast iron on wrought iron	Steel on cast iron	Brass on cast iron	Surface pressure, lb. per sq. in.	Wrought iron on wrought iron	Cast iron on wrought iron	Steel on cast iron	Brass on cast iron
	Values of f					Value of f			
125	0.14	0.17	0.17	0.16	485	0.40	0.37	0.36	0.22
186	0.25	0.28	0.30	0.23	523	0.41	0.37	0.36	0.22
224	0.27	0.29	0.33	0.22	560	Abraded.	0.37	0.36	0.23
260	0.29	0.32	0.34	0.21	600	Abraded.	0.37	0.36	0.23
298	0.30	0.33	0.34	0.21	634	Abraded.	0.37	0.37	0.24
336	0.31	0.33	0.35	0.22	672	Abraded.	0.38	0.40	0.23
373	0.35	0.35	0.35	0.21	710	Abraded.	0.43	Abraded.	0.23
390	0.38	0.36	0.35	0.21	784	Abraded.	Abraded.	Abraded.	0 23
448	0.40	0.37	0.35	0.21	820	Abraded.	Abraded.	Abraded.	0 27

Table 4. Coefficients of Friction for Dry or Slightly Lubricated Surfaces*

(Kimball and Barr, "Elements of Machine Design," 1st ed.)

Materials:	Values of f
Wood on wood, static or at very low velocity	0.3 to 0.5
Wood on metals, static or at very low velocity	0.2 to 0.6
Leather on metals, static or at very low velocity	0.3 to 0.6
Leather on wood, static or at very low velocity	0.3 to 0.5
Metal on metal, static or at very low velocity	0.3
Cast iron on steel, vel. = 440 ft. per min.	0.32
Cast iron on steel, vel. = 2640 ft. per min.	0.2
Cast iron on steel, vel. = 5280 ft. per min.	0.06

* The above values of f must be looked upon as approximate average values only, and should be used with judgment because of variations which come with slight changes of conditions. There are no experimental data giving the decrease in the value of f at high speeds for combinations such as wood or leather on metals. The data for cast iron on steel will, however, serve as a rough guide to what may be expected to occur. In designing brake shoes or other friction machinery where great velocities are involved, allowance must be made for the decrease in the value of the coefficient.

Friction Between Well-lubricated Surfaces. The following five laws (see Alford, "Bearings," p. 19) are based upon the laws of Goodman and the results of Lasche's experiments (see also p. 239):

1. The coefficient of friction for well-lubricated surfaces is practically independent of the materials composing the bearings and journals, but is dependent upon the nature of the lubricant.

2. The coefficient of friction for well-lubricated surfaces is from 1/6 to 1/10 that for unlubricated or scantily lubricated surfaces.

3. The coefficient of friction for moderate pressures and speeds varies approximately inversely as the normal pressure; *i.e.*, the frictional resistance per unit area is a constant, assuming the speed constant. The total frictional resistance varies as the area.

4. The coefficient of friction for well-lubricated surfaces for a given pressure is very high for low rubbing velocities, but decreases rapidly as the velocity increases. For velocities from 100 to 500 ft. per min. it decreases approximately as the square roots of the velocities; for velocities from 500 to 1600 ft. per min. it decreases as the fifth roots of the velocities; above 1600 ft. per min. the coefficient is practically independent of the velocity. (This law does not hold for all bearings; for example, in simple sleeve bearings the coefficient may vary directly as the speed for high speeds if the temperature is kept constant.)

5. The coefficient of friction varies approximately inversely as the temperature up to a point just before abrasion occurs. (This law, however, does not hold good for imperfect lubrication, as Tower's experiments attest.)

With good lubrication and high speeds the coefficient of friction is proportional to the viscosity of the oil and to the speed, and is inversely proportional to the pressure and the thickness of the oil film. At constant temperature the thickness of the oil film is inversely proportional to the square root of the pressure.

Frictional Resistance of Compound Sliding. A body sliding across another may be deflected from its direction of motion by any force; the force resisting motion will then alter its direction so that its component in the direction of the deflecting force equals the latter. A closely fitting plug gage can be freely moved through the ring gage if rotated while pushed axially, whereas the resistance to axial movement alone is very great.

Coefficients of Sliding Friction for Special Cases

Wrought-iron Tires on Dry Wrought-iron Rails (Poirée). Based on tests of cars weighing from 7500 to 18,500 lb.:

Speed, miles per hour	10.3	16.4	19.7	32	44.7	49
Values of f	0.209	0.206	0.171	0.145	0.136	0.112

Steel Tires on Steel Rails (Galton):

Speed, miles per hour	Start	6.8	13.5	27.3	40.9	54.4	60
Values of f	0.242	0.088	0.072	0.07	0.057	0.038	0.027

Cast-iron Brake Shoes on Steel Tires (Galton and Westinghouse):

Avg. speed, miles per hour	10	20	30	40	50	60
Value of f:						
First 3 seconds	0.320	0.205	0.184	0.134	0.100	0.062
5 to 7 seconds	0.209	0.175	0.111	0.100	0.070	0.054
12 to 16 seconds		0.128	0.098	0.080	0.056	0.048

For a uniform speed of v miles per hour, Wichert's tests gave values of the sliding friction between cast-steel brake blocks and steel tires of $f = B(1 + 0.018v)/(1 + 0.097v)$, in which $B = 0.45$ for dry surfaces and 0.25 for wet surfaces. If a train is to be brought to rest from an initial speed v, then an average coefficient of friction f_1 may be assumed for the entire braking period. The following values have been calculated from the formula for various speeds. The values for f_1 are for the unfavorable condition of wet rails.

Speed, miles per hr.	0	6.2	12.4	18.6	24.9	31.1	37.3	43.5	49.8	56.0
Dry surfaces, $f =$	0.450	0.313	0.250	0.215	0.192	0.176	0.164	0.154	0.146	0.141
Wet surfaces, $f =$	0.250	0.174	0.139	0.119	0.107	0.098	0.091	0.086	0.082	0.078
Average value, $f_1 =$		0.201	0.164	0.142	0.128	0.117	0.109	0.103	0.098	0.090

The specifications of the Master Car Builders' Association require that brake shoes upon cast-iron wheels in effecting a stop from an initial velocity of 40 miles per hour shall show a mean coefficient of friction of not less than 0.22 (0.16) where the pressure on the brake shoes is 2808 lb. (6840 lb.) For shoes on steel-tired wheels with an initial velocity of 65 miles per hour the mean coefficient of friction shall be not less than 0.125 (0.11) where the pressure on the shoe is 6840 lb. (12,000 lb.). No limitation is placed on the rise in the coefficient at the end of the stop.

Brakes. According to L. Klein, f is practically constant for velocities from 200 to 4000 ft. per min. and for pressures from 7 to 142 lb. per sq. in. The following values of f are for wood on lengthwise fiber, brake blocks carefully machined:

	Beech	Oak	Poplar	Elm	Willow
Cast iron	0.29–0.37	0.30–0.34	0.35–0.40	0.36–0.37	0.46–0.47
Wrought iron	0.54	0.51–0.40	0.65–0.60	0.60–0.49	0.63–0.60

The higher values apply for cast iron when the brake wheel is cleaned with gasoline, the lower when it is only wiped clean; the reverse holds for wrought iron.

Hydraulic Hoists. According to H. Lang, for bronze or lignum vitæ sliding surfaces on bronze, f is constant for slow, reversing motion and for pressures of from 30 to 1500 lb. per sq. in. For surfaces continuously lubricated, $f = 0.06$; for surfaces wet with water through numerous slots, 0.10; for surfaces running dry and creaking, up to 0.30.

Stuffing Boxes packed with hemp, cotton or leather: f is constant for hydraulic pressures between 15 and 750 lb. per sq. in. For **hemp or cotton packing**, loose or woven, soaked in hot tallow, smooth piston, box not set up too tightly so that the packing is still elastic, usual dimensions—even after months of use, $f = 0.06$ to 0.11 (see also *Am. Mach.* Feb. 3, 1898). When the packing is rendered difficult by unfavorable conditions, $f =$ up to 0.25. For well-made **leather packing rings** of soft leather $f = 0.03$ to 0.07; if of hard, stiff tanned leather, 0.10 to 0.13; under unfavorable conditions such as rough piston, dirty water, etc., up to 0.20. The coefficient of friction is found to be inversely as the diameter of the cylinder; the depth of the leather does not influence friction.

Stuffing Boxes of Steam Engines. Experiments made by C. H. Benjamin (*Trans. A. S. M. E.*, vol. 21, p. 292) with an engine having a 6 × 13-in. cylinder and 2-in. piston rod, using soft rubber, asbestos and graphite packings, braided flax packings lubricated with graphite and paraffin, and square duck packings without lubrication, showed that the power loss varies almost directly with the steam pressure for the harder packings while it is practically constant for the soft kinds. Oiling the rod will greatly reduce the friction with any packing. The friction loss may be made to assume serious proportions by injudiciously tightening the stud nuts. The loss occasioned by the 2-in. rod at a speed of 140 ft. per min. varied between 0.036 h.p. and 0.4 h.p. at 50 lb. steam pressure, with a safe average value for the softer classes of packing of 0.07 h.p.

Grindstones. The coefficient of friction between coarse-grained sandstone and cast iron is $f = 0.21$ to 0.24; for steel, 0.29; for wrought iron, 0.41 to 0.46, according as the stone is freshly trued or dull; for fine-grained sandstone (wet grinding) $f = 0.72$ for cast iron, 0.94 for steel, and 1.0 for wrought iron.

Vehicles on Roads (wheels with iron tires). Values of f:

Smooth granite flags	0.006	Macadam, dust covered	0.028
Avg. street railway tracks	0.006–0.008	Poor stone pavement	0.033
Good asphalt	0.010	Macadam, mud covered, rutted	0.035
Very good stone pavement	0.015	Dirt road, very good	0.045
Macadam in good condition	0.016	Macadam of very poor quality	0.050
Good wood block pavement	0.018	Dirt roads, good to poor	0.08–0.16
Good stone pavement	0.020	Loose sand	0.15–0.30
Macadam in good condition	0.023		

For values of f for **rubber tires**, see p. 1191.

The **resistance of ordinary vehicles on roads** and pavements varies widely according to the type and condition of the vehicle and of the road. The range of values commonly accepted is as follows:

Kind of Road or Pavement	Net Resistance, lb. per ton	Kind of Road or Pavement	Net Resistance, lb. per ton
Asphalt	30 to 70	Plank roads	30 to 50
Brick	15 to 40	Sand roads, ordinary condition	100 to 200
Cobblestones	50 to 100	Stone block	30 to 80
Earth roads, ordinary condition	50 to 200	Wood block, rectangular	30 to 50
Gravel roads	50 to 100	Wood block, cylindrical	40 to 80
Macadam	20 to 100		

Sleds. For **unshod wooden runners** on smooth wood or stone surfaces, $f = 0.07$ (0.15) when tallow (dry soap) is used as a lubricant (= 0.38 when not lubricated); on snow and ice, $f = 0.035$. For **runners with metal shoes** on snow and ice, $f = 0.02$.

Rolling Friction

If a cylinder rolls without sliding upon a flat horizontal surface and carries a load L acting normally to the flat surface, Fig. 2, the rotation about the line of support is due to a couple whose moment is $M = Lf_r$. f_r, which is the arm of the friction couple and is called the **coefficient of friction of rolling motion,** is dependent upon the hardness of the materials and smoothness of the surfaces, and has the following values: For lignum vitæ on lignum vitæ, $f_r = 0.0185$ in.; for elm on lignum vitæ, $f_r = 0.0319$ in.; for iron on iron (or steel on steel), $f_r = 0.0197$ in.

There is a scarcity of data on rolling friction, and its laws have not been definitely established. For railroad tracks, Thurston gives $f_r = 0.003$ ft. (ordinary construction) to 0.001 ft. (best possible construction). For iron on iron, Tredgold gives $f_r = 0.002$ ft.

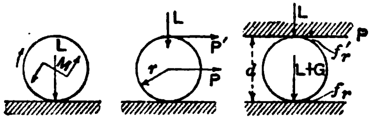

FIG. 2. FIG. 3. FIG. 4.

If the force P which causes rolling acts horizontally through the center of the cylinder (Fig. 3), then $P \times r = M = Lf_r$. If a force P' acts horizontally at the top of the cylinder (Fig. 3), then $P' \times 2r = M = Lf_r$. If a load L be moved on a roller (Fig. 4), and if f'_r and f_r are the respective coefficients of rolling friction and G is the weight of the roller, then $Pd = M = Lf'_r + (L + G)f_r$.

A cylinder of radius r will start to roll down an inclined plane when the angle a the plane makes with the horizontal is such that $\tan a = f_r/r$; whence $f_r = r \tan a$.

FRICTION OF MACHINE ELEMENTS

Work of Friction—Efficiency. In a simple machine or assemblage of two elements, the work done by an applied force P acting through the distance p is measured by the product Pp. The useful work done is less, and is measured by the product Ll of the resistance L by the distance l through which it acts. The **efficiency** e of the machine is the ratio of the useful work performed to the total work received, or $e = Ll/Pp$. The **work expended in friction,** W_f, is the difference between the total work received and the useful work, or $W_f = Pp - Ll$. The **lost-work** ratio $= V = W_f/Ll$, and $e = 1/(1 + V)$.

If a machine consist of a train of mechanisms having the respective efficiencies $e_1, e_2, e_3 \ldots\ldots e_n$, the combined efficiency of the machine is equal to the product of these efficiencies.

Efficiencies of Machines and Machine Elements. The following values for machine elements are from "Elements of Machine Design," by Kimball and Barr. Those for machines are from Goodman's "Mechanics Applied to Engineering." The quantities given are percentage efficiencies.

Common bearing (singly)	96–98	Belting	96–98
Do., long lines of shafting	95	Pin-connected chains (bicycle)	95–97
Roller bearings	98	High-grade transmission chains	97–99
Ball bearings	99	Weston pulley block (½ ton)	20–25
Spur gear, including bearings:		Epicycloidal pulley block	40–45
Cast teeth	93	1-ton steam hoist or windlass	50–70
Cut teeth	96	Hydraulic windlass	60–80
Bevel gear, including bearings:		Hydraulic jack	80–90
Cast teeth	92	Cranes (steam)	60–70
Cut teeth	95	Overhead traveling cranes	30–50
Worm gear: varies with thread angle	..	Locomotives (drawbar h.p./i.h.p.)	65–75

The efficiencies given for single bearings represent average values; the loss in a shaft bearing is not necessarily connected with the power transmitted and may be due chiefly to the load on the shaft.

Thurston found the **friction of a steam engine** to be constant at all outputs and to have the following approximate percentage distribution: main bearings, 35–47; piston and rod, 21–33; crank pin, 5–7; cross head and wrist pin, 4–5; valve and rod, 2.5 (balanced), 22 (unbalanced); eccentric strap, 4–5; link and eccentric, 9.

Wedges

Sliding in V Guides. If a wedge-shaped slide having an angle $2b$ is pressed into a V guide by a force P, Fig. 5, the pressure normal to the wedge faces will be $N = P/\sin b$, and the frictional force opposing motion along the axis of the wedge is $F = fN = fP/\sin b = f'P$, where $f' = f/\sin b$ is the coefficient of friction for V guides. In these formulæ the fact that the elasticity of the materials permits an advance of the wedge into the guide under the load P, has been neglected. If it be taken into account, then $F = fP/(\sin b + f \cos b)$. The efficiency for V guides is $e = 0.88$ to 0.90.

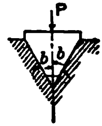

FIG. 5.

Taper Keys. In Fig. 6, if the key be moved in the direction of the force P, the force H must be overcome. The supporting pressures K_1, K_2, and K_3, together with the required force P may be obtained by drawing the force polygon, Fig. 7. The friction angles of these faces are a_1, a_2 and a_3, respectively. In Fig. 7 draw AB parallel to H in Fig. 6, and lay it off to scale to represent H. From the point A draw AC parallel to K_1, that is, making the angle $b + a_1$ with AB; from the other extremity of AB draw BC parallel to K_2 in Fig. 6. AC and CB then give the magnitudes of K_1 and K_2 respectively. Now through C draw CD parallel to K_3 to its intersection with AD which has been drawn through A parallel to P. The magnitudes of K_3 and P are then given by the lengths of CD and DA.

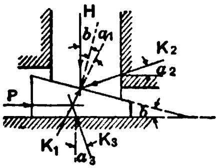

FIG. 6.

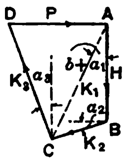

FIG. 7.

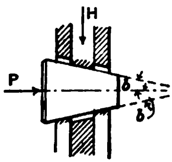

FIG. 8.

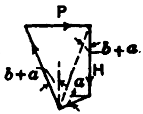

FIG. 9.

By calculation, $K_1/H = \cos a_2/\cos (b + a_1 + a_2)$.

$$P/K_1 = \sin (b + a_1 + a_3)/\cos a_3.$$

$$P/H = \cos a_2 \sin (b + a_1 + a_3)/\cos a_3 \cos (b + a_1 + a_2).$$

If $a_1 = a_2 = a_3 = a$, then $P = H \tan (b + 2a)$, and efficiency $e = \tan b/\tan (b + 2a)$. Force required to loosen the key $= P_1 = H \tan (2a - b)$. In order for the key not to slide out when force P is removed, it is necessary that $b < 2a < (a_1 + a_3)$.

The forces acting upon the taper key of Fig. 8 may be found in a similar way (see Fig. 9).

$$P = 2H \cos a \sin (b + a)/\cos (b + 2a)$$
$$P = 2H \tan (b + a)/[1 - \tan a \tan (b + a)]$$
$$= 2H \tan (b + a) \text{ approximately.}$$

The force to loosen the key is $P_1 = 2H \tan (a - b)$ approximately, and the efficiency $e = \tan b/\tan (b + a)$. The key will be self-locking when $b < a$, or, more generally, when $2b < (a_1 + a_2)$.

Journals and Bearings

The usual methods of calculating journal friction assume a fixed coefficient of friction and a definite distribution of pressure dependent upon whether the bearings are new or well-worn. The actual results may differ considerably. Calculations should be limited to cylindrical journals and flat thrust or step bearings, as only upon these have tests supplying sufficient data been made.

Friction of Journal Bearings. If P = total load on journal in lb., l = journal length in in., and $2r$ = journal diam. in in., then $p = P/2rl$ = mean normal pressure in lb. per sq. in. of the projected area of the journal. Also, if f_1 be the **coefficient of journal friction,** the **moment of journal friction** for a cylindrical journal is $M = f_1Pr$ in in.-lb. The **work expended in friction** at a speed of n r.p.m. is $W_f = 2\pi Mn = 6.283\, f_1Prn$ in.-lb. per min. For the **conical bearing,** Fig. 10, the mean radius $r_m = (r + R)/2$ is to be used.

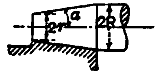

FIG. 10.

Journal friction may be **determined graphically** by assuming that the journal reaction does not pass through the journal axis but is tangent to a circle called the journal friction circle, whose radius is $f_1 \times r$.

Values of the Coefficient of Journal Friction, f_1. The value of f_1 is dependent upon the journal load, the circumferential speed of the journal, the temperature, the lubricant used and the design of the bearing. Due to the unavoidable inaccuracies of manufacture, it is higher for new journals than for those that have worn to a bearing. The experiments of Thurston, Tower, Stribeck, Lasche, Heimann, Kimball, Moore and others, lead to the following conclusions for well-lubricated journals:

1. For solid bearings that permit an uninterrupted distribution of pressure around the shaft, the total friction (or $f_1 \times P$) is independent of the load, as the coefficient of journal friction f_1 varies inversely as the unit pressure on the journal. In the case of bearings with surface interruptions and therefore without a continuous distribution of the pressure around the shaft, this law does not apply. Here the friction (with equal pressure) increases with the size of the friction surface.

2. The coefficient of friction of rest is independent of the pressure. The coefficient of friction of motion decreases with increasing velocity, more rapidly under low than under high pressures. After reaching a minimum value, which is approximately the same for all permissible pressures, the coefficient of friction then increases, increasing more rapidly for the lower pressures. At a velocity of about 2000 ft. per min. a point is reached beyond which the coefficient of friction seems to be independent of velocity. See Fig. 11.

3. With a temperature rise within the limits 32 to 104 deg. fahr., f_1 varies approximately inversely with the temperature above 32 deg. fahr.

4. The total friction ($= f_1P$) is approximately inversely proportional to the thickness of the oil film, that is, to the journal clearance.

Influence of the Method of Lubrication. Tower found the following values of f_1 for a journal of steel running in a bronze bearing 4 in. in diam. and 6 in. long, at a circumferential velocity of 160 ft. per min., and lubricated with rape-seed oil.

With an oil bath and p = 260 lb. per sq. in., f_1 = 0.00139.
With wick lubrication and p = 240 lb. per sq. in., f_1 = 0.00980.
With pad lubrication and p = 270 lb. per sq. in., f_1 = 0.00900.

Values of f_1 for Various Oils as determined by the German National

Bureau of Tests by means of the Martens oil-testing machine (having a steel journal 4 in. in diam. × 2¾ in. long), are given in Table 5.

Table 5. Values of f_1 for Various Lubricating Oils
(German National Bureau of Tests)

Lubricant	Pressure, lb. per sq. in. 142			356			569		
Speed, ft. per min.	98	197	394	98	197	394	98	197	394
Raw rape-seed oil	0.011	0.013	0.019	0.0060	0.0062	0.0096	0.0050	0.0057	0.0068
Raw rape-seed oil (taken from the journals).	0.053	0.025	0.057	......	0.021	0.037	......	......	......
Raw rape-seed oil (filtered)	0.014	0.016	0.023	0.0085	0.0096	0.014	0.0068	0.0076	0.010
Purified rape-seed oil	0.0076	0.014	0.014	0.0064	0.0078	0.011	0.0055	0.0064	0.008
American mineral oil, light	0.0050	0.0076	0.010	0.0046	0.0053	0.0061	0.0038	0.0039	0.0045
Russian mineral oil, light	0.0076	0.013	0.016	0.0062	0.0082	0.010	0.0049	0.0057	0.0076
German cylinder oil, heavy	0.080	0.068	0.059	0.041	0.036	0.031	0.031	0.026	0.021
Gas-engine oil, (compounded)	0.0070	0.0092	0.014	0.0059	0.0075	0.0095	0.0048	0.0062	0.0071

The **dependence of the coefficient of friction on the velocity and pressure** between the surfaces is shown in the values found by Stribeck, Lasche and Tower, given in Fig. 11 and Tables 6 and 7.

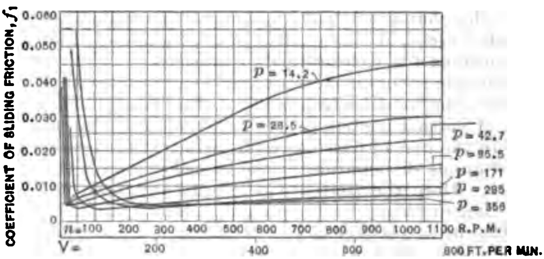

FIG. 11.—Dependence of the Coefficient of Journal Friction on the Velocity and the Pressure Between the Surfaces.

Table 6. Values of f_1 (Stribeck)
[Results of tests of a 2¾ × 9¼-in. ring-oiled babbitted bearing lubricated with gas-engine oil (at 77 deg. fahr.)]

Speed, ft. per min.	Pressure, lb. per sq. in. 14	32	57	128	228	356	512	697
1515	....	0.057	0.04	0.021	0.014	0.011	....	....
790	0.067	0.048	0.036	0.02	0.013	0.010	0.0087	0.0077
547	0.050	0.039	0.029	0.017	0.011	0.0091	0.008	0.0071
274	0.042	0.030	0.021	0.013	0.0085	0.007	0.0063	0.0059
138	0.028	0.021	0.016	0.0091	0.0064	0.0052	0.0048	0.0045
45	0.018	0.011	0.0081	0.0051	0.0035	0.0030	0.0027	0.0026
24	0.013	0.0072	0.0052	0.0032	0.0025	0.0021	0.0020	0.0020
8.7	0.0095	0.0046	0.0031	0.0020	0.0019	0.0017	0.0017	0.0025
5.7	0.0074	0.0039	0.0036	0.0018	0.0016	0.0017	0.0023	0.0058
3.5	0.0067	0.0034	0.0033	0.0017	0.0016	0.0019	0.0031	0.0089
0.0	....	0.21	0.21	0.21	0.21	0.22	0.22	0.23

Table 7. Values of f_1 (Lasche)

[Results of tests on a 4⅜ × 9½-in. babbitted bearing, lubricated with Imperial 0 oil (at 122 deg. fahr.); pressure, 92.5 lb. per sq. in.]

Speed, ft. per min.	590	984	1968	2952	3936	4526
Values of f_1	0.0057	0.0067	0.0083	0.0104	0.0100	0.0102

The **influence of bearing temperature** is shown by Stribeck in Fig. 12 for the journal of Fig. 11. The speed was 1100 r.p.m., corresponding to a sliding velocity of 13.2 ft. per sec.

Table 8 gives values of f_1 obtained by Tower from tests of a 4-in. journal of steel 6 in. long, fitted on its upper side with a gun-metal brass, or half box, embracing somewhat less than one-half the journal circumference, and when running kept at a constant temperature of 90 deg. fahr. Lubrication, a bath of rape oil.

Table 8. Values of f_1 (Tower)

Speed, ft. per min.	Pressure, lb. per sq. in.						
	573	520	415	363	258	153	100
105	....	....	....	....	0.0011	0.0016	0.0028
157	0.0010	0.00055	0.00093	0.00084	0.0014	0.0020	0.0036
209	0.0011	0.0011	0.0011	0.00096	0.0016	0.0024	0.0042
262	0.0012	0.0012	0.0012	0.0011	0.0018	0.0027	0.0050
314	0.0013	0.0013	0.0013	0.0012	0.0019	0.003	0.0058
366	0.0013	0.0013	0.0014	0.0013	0.0021	0.0033	0.0062
419	0.0014	0.0014	0.0015	0.0015	0.0023	0.0037	0.0066
471	....	0.0015	0.0016	0.0016	0.0024	0.0040	0.0071

Professor Kingsbury, from tests of a bearing 1⅜ in. diameter, 2 in. long, 120 deg. contact and with bath lubrication, gives the following average **coefficients of friction for sperm oil** under a pressure of 340 lb. per sq. in. at a temperature of 90 deg. fahr. (*Trans. A. S. M. E.*, vol. 24, p. 149): At 42 ft. per min., 0.000906; at 64 ft. per min., 0.001153; at 101 ft. per min., 0.00151. He also found in tests of a **journal lubricated by air** only, that $f_1 = 0.00075$ (*Jour. Am. Soc. Naval Engrs.*).

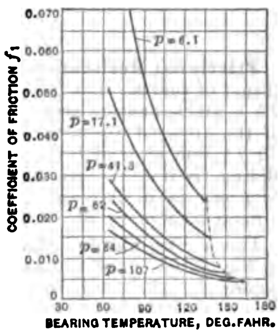

Fig. 12.—Influence of Bearing Temperature on the Coefficient of Journal Friction.

Table 9 gives results of **tests on large shaft bearings** (Kingsbury, *Trans. A. S. M. E.*, 1905, vol. 27, p. 431). In these, a horizontal shaft was supported in two 9 × 30-in. end bearings; a third bearing (15 × 40 in.), midway between the other two, was pressed upward against the shaft by a weighted lever, subjecting the shaft to a pressure of from 25 to 50 tons. The shaft was driven by an electric motor. The journals were flooded with paraffin oil for the lower pressures and heavy machine oil for the higher pressures. The values for the friction h.p. and f_1 in the table are for the three bearings.

Table 9. Tests of Large Shaft Bearings (Kingsbury)

Pressure		Velocity		Friction	
tons	lb. per sq. in.	r.p.m.	ft. per min.	h.p.	f_1
25.0	83	309	1215	12.6	0.0045
25.0	83	506	1990	21.7	0.0048
25.0	83	180	708	6.43	0.0040
25.0	83	179	704	5.12	0.0032
25.0	83	301	1180	10.1	0.0037
33.6	112	454	1785	16.0	0.0029
42.3	141	480	1890	17.9	0.0024
47.0	157	946	3720	41.9	0.0025
47.0	157	1243	4900	47.8	0.0022
50.5	168	1286	5050	52.3	0.0022

For **approximate calculations** where it is preferred to err on the safe side, the coefficient of friction may be assumed at 0.06 for lubrication by rape-seed oil, mineral grease, mineral oil or their compounds for accurately machined journals running in bronze bearings. With machine oil, in common practice, it is often not more than 0.006 and sometimes as low as 0.003; under the very best conditions it may be as low as 0.0006. Where the lubrication is poor and in the open, as for rope drives, turntables, transfer tables, etc., take f_1 = 0.08 to 0.10.

With very **low velocities and high pressures** the coefficient for well-lubricated bearings approaches that for greasy or unlubricated surfaces, a minimum value for which may be taken as 0.15. Tests of new, well-fitted bearings have given values of 0.30 and more. These values should be used in estimating the starting moments for heavy machinery. **For** purposes of **design of ordinary machinery** with pressures from 50 to 500 lb. and velocities from 50 to 500 ft. per min., a fair average range of the coefficient of friction is from 0.02 to 0.008. In extraordinary cases, with perfect lubrication and moderate velocity, the coefficient may be taken as low as 0.001.

Thrust Bearings

Frictional Resistance. Step bearings or pivots are used to resist the end thrust of shafts. Let L = total load in lb. in the direction of the shaft axis, dA = an elementary area of the thrust-bearing surface in sq. in., y = distance of the area dA from its axis of revolution, in in., p = pressure on dA due to load L, in lb. per sq. in., and f = coefficient of sliding friction. Then, **moment of thrust friction** = M = $fp \int y dA$ in in.-lb.; and the **work expended in friction** per min. at n r.p.m. = W_f = $2\pi Mn$ in in.-lb.

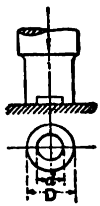

Fig. 13.

For a **ring-shaped flat step bearing** such as that shown in Fig. 13 (or a **collar bearing**), $M = \frac{1}{3}fL(D^3 - d^3)/(D^2 - d^2)$, where D and d are in in. For a **flat circular step bearing**, $d = 0$, and $M = \frac{1}{3}fLD$.

For a **conical pivot bearing,** $M = \frac{1}{3}fLD/\sin a$, where D = diam. of shaft in in. and a = the half angle of the cone or the angle of the slope with the axis. For a **truncated conical pivot bearing,** $M = \frac{1}{3}fL(D^3 - d^3)/(D^2 - d^2) \sin a$, where d = smaller diam. of the frustum in in.

For a **hemispherical step bearing** (end of shaft rounded to a hemisphere of diam. D), $M = \frac{1}{3}fLD$.

For **Schiele's "anti-friction" pivot bearing** (Fig. 14), $M = \frac{1}{2}fLD$. The outline of this bearing is in the form of a tractrix (see p. 155), but the term "anti-friction" applied to it is a misnomer, because its friction is much greater than that of a flat step bearing of the same diameter. It maintains its shape as it wears and is self-adjusting, but it is now seldom used.

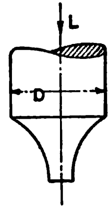

FIG. 14.

A nearly **frictionless step bearing** may be obtained by floating the bearing upon mercury. (See *Engg.*, July 14, 1893, p. 41.)

Values of the Coefficient of Friction f. The general laws of journal friction apply in the case of thrust bearings. The coefficient of friction decreases directly with the fluidity of the lubricant, *i.e.*, decreasing the more slowly the greater the pressure. The **influence of pressure and temperature** upon the coefficient of friction for step bearings as determined by Woodbury is given in Table 10. The ring-shaped flat steel pivot tested was 2.28 in. in outside diam. and 1.43 in. in inside diam. Speed, 300 r.p.m.; lubricant, paraffin oil.

Table 10. Influence of Pressure and Temperature on the Coefficients of Friction of Step Bearings (Woodbury)

Pressure per sq. in., lb.	Temperature, deg. fahr.				
	60	70	80	90	100
1	0.34	0.27	0.21	0.17	0.14
5	0.09	0.074	0.0620	0.052	0.044
10	0.055	0.047	0.040	0.035	0.030
15	0.044	0.038	0.033	0.028	0.026
20	0.038	0.033	0.029	0.026	0.023
25	0.034	0.030	0.027	0.024	0.021
30	0.031	0.027	0.025	0.022	0.020
35	0.029	0.026	0.023	0.021	0.019
40	0.027	0.024	0.022	0.020	0.018

The **influence of pressure and velocity** is shown by the following results of tests by Tower of a flat steel step 3 in. in diam. running on a manganese-bronze bearing and thoroughly lubricated with mineral oil; pressure range, 20 to 160 lb. per sq. in. (at higher pressures seizing occurred):

Speed, r.p.m.		50	128	194	290	353
Range of values of f	from	0.0181	0.0053	0.0051	0.0044	0.0053
	to	0.0221	0.0113	0.0102	0.0178	0.0167

On account of the difficulty in properly lubricating a step bearing it is not easy to assign a definite value to f. Experiments show that 0.01 is a safe average value for ordinary machine design.

Ball Bearings

Frictional Resistance. Let L = total load on a ball race, lb., S = sum of all the individual ball loads, lb.; b = angle between the supporting normals of two adjacent balls; d = ball diam., in.; r = radius of shaft, in.; D_0 = diam. of the circle passing through the ball centers, in. $= 2r + d$; B = number of balls in the race; f_r = the coefficient of rolling friction; f_i = coefficient of friction for ball bearings (referred to the radius of the journal) $= Sf_rD_0/Lrd$, and N = r.p.m. Then, according to Stribeck (see Fig. 15), the friction torque $M = Sf_rD_0/d = f_iLr$ in in.-lb.; the friction work $W_f = 2\pi NM =$

$2\pi NSf_rD_0/d$ in in.-lb. per min.; $S = L_0 + 2L_1 + 2L_2 + \ldots\ldots 2L_n$; $L_1 = L_0 \cos^{5/2} b$; $L_2 = L_0 \cos^{5/2} 2b$; etc.; $L = L_0 (1 + 2\cos^{5/2} b + 2\cos^{5/2} 2b + \ldots\ldots 2\cos^{5/2} nb)$. When the number of balls in a race $B = 10$ to 20, the greatest load on any one ball $= L_0 = 5L/B$ (approx.) and $S = 1.2L$ (approx.); also, $f_r = f_i r/1.2(D_0/d)$. The frictional work W_f decreases with a decrease in the ratio D_0/d, and is therefore lower the smaller the number of balls the race contains.

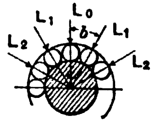

FIG. 15.

Coefficient of Friction of Ball Bearings. Within wide limits f_i is independent of the speed of rotation and the temperature; it decreases, however, with an increase in the load. Stribeck's investigations show that the value of f_i is materially influenced by the design of the bearing. For a bearing like that shown in Fig. 16, having $r = 1\frac{3}{4}$ in., $D_0 = 4$ in. and $d = \frac{7}{8}$ in. (races of steel, carefully ground, ball tracks with radius of curvature $= \frac{2}{3} \times$ ball diam., this design being best for heavy loads), the following values of f_i were found:

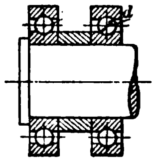

FIG. 16.

Load L, lb.	838	1874	2425	3483	4520	6614	10800
f_i at 65 r.p.m.	0.0033	0.0020	0.0017	0.0016	0.0015	0.0015	......
f_i at 385 r.p.m.	0.0035	0.0021	0.0018	0.0016	0.0015	0.0013	......
f_i at 780 r.p.m.	0.0037	0.0022	0.0019	0.0017	0.0015	0.0013	0.0011

The following values of f_i and f_1 give an idea of the **comparative friction of ball bearings and plain bearings** in large sizes:

Ball bearings, plain cylindrical races, 2-point bearing: $f_i = 0.0012$ to 0.0018; ball thrust bearings, flat races, 2-point bearing: $f_i = 0.0008$ to 0.0012; ball thrust bearings, 1 flat, 1 V-race, 3-point bearing: f_i (mean) $= 0.0018$; ball thrust bearings, 2 V-races, 4-point bearing: $f_i = 0.0055$ (all by Goodman).

Cylindrical gun-metal bearing: with bath lubrication, $f_1 = 0.001$; with ordinary lubrication, $f_1 = 0.01$ (Tower).

Gun-metal step or collar bearings, well lubricated, $f_1 = 0.03$ (Tower).

Roller Bearings

Let L = total load on a bearing, lb., l = length of roller, in., d = diam. of roller, in., B = number of rollers, $D_0 = 2r + d$ = diam. of the cylinder passing through the roller axes, in., $p = 5L/Bld$ = maximum load per sq. in. of projected area of one roller, lb. Then, according to Stribeck, using the same notation as for ball bearings (p. 243), $M = 1.2Lf_rD_0/d = f_iLr$ in in.-lb., and maximum load on one roller for $B = 10$ to 20 is $L_0 = 5L/B$.

Coefficients of Friction for Roller Bearings. In roller bearings the friction of motion is practically independent of the speed and differs but slightly from the friction of rest. It follows, therefore, that the starting resistance is low. The **coefficient of rolling friction** f_r decreases with increasing load and also with increasing temperature. Tests of roller bearings of various designs have shown that

for p =	43	71	103	142	213
f_r (in.) =	0.0018	0.0013	0.0011	0.0009	0.0007

The **coefficient of friction of roller bearings** $= f_i = 1.2D_0f_r/rd$.

The following values of f_i, which are stated to be good averages for an ordinary roller bearing, are given by Goodman in "Mechanics Applied to Engineering:"

Total load, lb.	2000	4000	6000	8000	10,000
At 40 r.p.m.:					
Value of f_t	0.0131	0.0094	0.0082	0.0076	0.0072
End thrust, lb.	82	147	212	276	340
At 400 r.p.m.:					
Values of f_t	0.0053	0.0035	0.0029	0.0026	0.0024
End thrust, lb.	51	89	128	166	205

The end thrust is the force required to prevent the bearing traveling along the shaft.

C. H. Benjamin (*Machinery*, Oct., 1905) gives the following **comparative values of the coefficient of friction of a plain cylindrical bearing and** two types of **roller bearings,** running at a speed of 560 ft. per min.:

Diam. of journal, in.	Hyatt roller bearing. Max.	Min.	Avg.	McKeel roller bearing. Max.	Min.	Avg.	Plain babbitted bearing. Max.	Min.	Avg.
$1\frac{15}{16}$	0.032	0.012	0.018	0.033	0.017	0.022	0.074	0.029	0.043
$2\frac{3}{16}$	0.019	0.011	0.014	...	...	...	0.088	0.078	0.082
$2\frac{7}{16}$	0.042	0.025	0.032	0.028	0.015	0.021	0.114	0.083	0.096
$2\frac{15}{16}$	0.029	0.022	0.025	0.039	0.019	0.027	0.125	0.089	0.107

According to Henry Hess, the **coefficient of friction of** a well-made annular **ball bearing** may be taken as 0.001 to 0.002, **and** that of a good **roller bearing** from 0.0035 to 0.014.

For further discussions on ball and roller bearings see translation and amplifications of the work of Professor Stribeck by Henry Hess (*Trans. A. S. M. E.*, vol. 29, pp. 367–472); "Ball Bearings," by Henry Hess (*Trans. A. S. M. E.*, vol. 27, p. 431, and vol. 31, p. 923); *Engg.*, Apr. 12, 1901, Dec. 26, 1902, and Feb. 20, 1903; *Am. Mach.*, Mar. 4, 1909.

Screws

Screws with Square Threads (Fig. 17). Let r = mean radius of the thread = ½ (radius at root + outside radius), and l = pitch (or lead of a single-threaded screw), both in in.; b = angle of inclination of thread to a line at right angles to the axis of screw ($\tan b = l/2\pi r$); and f = coefficient of sliding friction = $\tan a$ (see p. 232). Then, for a screw in uniform motion (friction of the root and outside surfaces being neglected) there is required a force P acting at right angles to the axis at the distance r. $P = L \tan (b \pm a) = L(l \pm 2\pi rf)/(2\pi r \mp fl)$, where the upper signs are for motion in a direction opposed to that of L and the lower for motion in the same direction as that of L. When $b \leq a$, the screw will not "overhaul" (or move under the action of the load L).

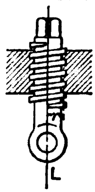

FIG. 17.

The **efficiency** for motion opposed to direction in which L acts $= e = \tan b/\tan (b + a)$; for motion in the same direction in which L acts, $e = \tan (b - a)/\tan b$.

The value of e is a maximum when b = 45 deg. − ½a; for example, e_{max} = 0.81 for b = 42 deg. and f = 0.1. Since e increases rapidly for values of b up to 20 deg., this angle is generally not exceeded; for b = 20 deg., and f_1 = 0.10, e = 0.74. In presses, where the mechanical advantage is required to be great, b is taken down to 3 deg., for which value e = 0.34.

Kingsbury found the following values of f for square-threaded screws from experiments with screws 1.426 in. in outside diam., and of 0.33 in. pitch, running slowly in loose-fitting cast-iron, steel, brass and wrought-iron nuts. The axial thrust on screw varied up to 14,000 lb. (= 14,000 lb per sq. in. of projected bearing surface of thread). No marked difference was noted at different pressures or with different metals.

	Max.	Min.	Avg.
Lard oil	0.25	0.09	0.11
Heavy mineral machinery oil	0.19	0.11	0.14
Heavy oil and graphite	0.15	0.03	0.07

Screws with V Threads. Let c = half the angle between the faces of a thread. Then, using the same notation as for square-threaded screws, for a screw in motion (neglecting friction of root and outside surfaces),

$$P = L\,[\tan b \pm f \cos b\sqrt{1 + \tan^2 b + \tan^2 c}]/(1 \mp f \sin b\sqrt{1 + \tan^2 b + \tan^2 c})$$

As b compared with c is usually small, $\sqrt{1 + \tan^2 b + \tan^2 c} = 1/\cos c$, approximately If then $f \cos b/\cos c$ be placed equal to $\tan a'$, $P = L \tan (b \pm a')$. The upper sign is for axial motion of the screw opposed to the direction of the load L, while the lower sign is for motion in the opposite direction. The screw will not overhaul when $b \overline{\gtrless} a'$, the value of a' corresponding to that for the angle of friction of rest.

The efficiencies are, $e = \tan b/\tan (b + a')$, for motion opposed to L, and $e = \tan (b - a')/\tan b$, for motion with L. The efficiency of a V thread is lower than that of a square thread, since $a' > a$. e is a maximum for $c = 0$.

Example: Assume b = 2 deg. 10 min.; c = 27 deg. 30 min.; $f = \tan a = 0.16$; also a = 9 deg. 5 min. 30 sec. Then $0.16 \times \cos 2°10'/\cos 27°30' = 0.180$. Also, $a' = 10°10'$, and $P = L \tan (b + a') = L \tan 12° 20' = 0.219\,L$; $e = \tan 2°10'/\tan 12° 20' = 0.173$. For a square thread, $P = L \tan, (b + a) = 0.199\,L$; $e = 0.190$. Both of these threads are self-locking, *i.e.*, they will not overhaul.

For a **V-threaded screw and nut,** let D = outside diam. of thread; d = diam. at root of thread—both in in.; and m = width across the flats of nut, in in. Then $r = (D+d)/4$; $\tan b = 2l/\pi(D+d)$; $\tan a' = f \cos b/\cos c$; and r_0 = mean radius of the nut seat $= (D + m)/4 = 1.4r$ (approx.).

To tighten up the nut a turning moment M is required: $M = Pr + Lr_0f' = [\tan(a' + b) + 1.4f']Lr$. **To loosen,** $M = [\tan (a' - b) + 1.4f']Lr$. In these equations f' is the coefficient of friction between the nut and its seat.

Example. For a screw with a U. S. standard thread, let D = ¾ in.; then d = 0.62 in., l = 0.10 in., $2c$ = 60 deg., r = 0.342 in., $\tan b$ = 0.04656, and b = 2 deg. 40 min. 3 sec. Taking $f = f' = 0.41$, a' = 25 deg. 18 min. 36 sec. Tightening the nut on the bolt requires a moment of M = [tan 27 deg. 58 min. 39 sec. + $(1.4 \times 0.41)]Lr = 1.105\,Lr = 0.3779L$ in.-lb. Loosening the nut will require a moment of M = [tan 22 deg. 38 min. 33 sec. + $(1.4 \times 0.41)]Lr = 0.9911Lr = 0.3390L$ in.-lb.

Kingsbury, from tests on U. S. standard bolts, finds efficiencies for tightening up nuts from 0.06 to 0.12, depending upon the roughness of the contact surfaces and the character of the lubrication.

Toothed Gearing

Spur Gears. DETERMINATION OF THE LOST-WORK RATIO V. If two teeth are in contact at E, Fig. 18, and ES be the tooth pressure normal to the teeth, M_1 and M_2 the two gear centers, then the lost-work ratio (or ratio of frictional work to useful work) for the instantaneous position may be derived graphically as follows: At a distance f draw a line parallel to ES. The two radii M_1E and M_2E (projected) will determine a length AB on this parallel, which, measured to the same scale as f, gives the value of the lost-work ratio V. (f is the coefficient of sliding friction.)

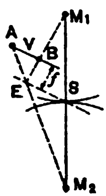

FIG. 18.

For the usual tooth profiles, V is approximately proportional to the lengths of the arcs of contact, q, measured on each side of the center line M_1M_2. As long as only one tooth is in contact, an average value of V for an arc of contact q is therefore $\frac{1}{2} V_1$, if V_1 is the value of V at the end of the arc of contact q_1. If the arcs of contact on both sides of the center line are equal, then this value may be taken as the average lost-work ratio V_m of the pair of gears. If the arcs of contact on the two sides of the center line are unequal, say q_1 and q_2, and the values V_1 and V_2 are those at the ends of the arcs of contact, then, according to Grashof, the average value is $V_m = (V_1q_1 + V_2q_2)/2(q_1 + q_2)$.

Let t_1 and t_2 be the number of teeth in a pair of gears, p = the circular pitch of the teeth, $r_1 = q_1/p$, $r_2 = q_2/p$, and $r = r_1 + r_2$ = arc through which the teeth are in contact; then

$$V = \pi f\left(\frac{1}{t_1} \pm \frac{1}{t_2}\right)\frac{r_1^2 + r_2^2}{r_1 + r_2} = \pi f\left(\frac{1}{t_1} \pm \frac{1}{t_2}\right)\frac{r}{2} \text{ (approx).}$$

the + sign being for external gears, and the − sign for internal gears. Considering that for an arc of contact r there are alternately 1 and 2 teeth in contact, then, more accurately,

$$V = \pi f\left(\frac{1}{t_1} \pm \frac{1}{t_2}\right)(r_1^2 + r_2^2 - r_1 - r_2 + 1)$$

or, approximately,

$$V = \pi f\left(\frac{1}{t_1} \pm \frac{1}{t_2}\right)\left(\frac{r^2}{2} - r + 1\right) = \pi f\left(\frac{1}{t_1} \pm \frac{1}{t_2}\right)u.$$

f ranges from 0.1 to 0.3 according to the material and the surface conditions of the teeth; for well-cut gears that have worn to a good bearing this value is smaller, while for rough teeth it will be higher. The average value $f = 0.16$ gives $\pi f = 0.5$, and

for r = 1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
u = 0.5	0.505	0.520	0.545	0.58	0.625	0.68	0.745	0.82	0.905	1.0

For the usual proportions of cycloidal and involute teeth, r_1 and r_2 are dependent on the numbers of teeth t_1 and t_2, as follows:

Cycloidal	t_1 =	7	10	15	20	30	40	50	∞
	r_1 =	0.58	0.60	0.64	0.66	0.68	0.69	0.71	0.75
Involute	t_1 =	20	25	30	40	50	60	∞	
	r_1 =	0.84	0.90	0.92	0.97	0.99	1.02	1.24	

In the **graphical determination of the friction losses** for a pair of gears, the loss by tooth friction may be found by representing the line of action of the tooth pressure not on a line passing through the point of contact S of the two-pitch circles, but through a point on the center line that is nearer the center of the driven gear by an amount $E = fup/2$. The total efficiency of a pair of gears with machine-molded teeth, including the journal losses, is 0.92 to 0.95.

Bevel Gears. The formulæ for spur gears apply if $(1/t_1) + (1/t_2)$ be replaced by $\sqrt{(1/t_1^2) + (1/t_2^2) + 2 \cos b/t_1t_2}$, where b = angle included between the axes of the two shafts.

Helical Gears. In these, in addition to the tooth friction proper there is the usually much greater friction due to the transverse sliding of the teeth on each other. Let c and c_1 be the angles of inclination of the two helical gears, then the lost-work ratio due to this sliding is $V_s = [\sin c_1 \sin (c + a)/\sin c \sin (c_1 - a)] - 1$. For the special case of the **worm** with $c_1 = 90° - c$,

$$V_s = [\tan (c + a)/\tan c] - 1 = [(1 + 2\pi r f/p)/(1 - pf/2\pi r)] - 1,$$

in which p = lead or pitch of worm thread, r = mean worm radius, and f = $\tan a$ = coefficient of friction for sliding. In this case the tooth friction loss is $\pi fu/t$, t being the number of teeth in the worm wheel; for u, see spur gears. For values of f see Table 3; note also the remarks on p. 234 concerning the dependence of f on certain conditions.

Stribeck gives the following values of f (at 140 deg. fahr.) for a single-threaded cast-iron worm and gear (worn to bearing) for various speeds and pressures between the teeth (Worm: outside diam., 3¾ in.; root diam., 2⅜ in. Wheel: pitch diam., 9.46 in.; number of teeth, 30; width of face, 3 in.; pitch of teeth, 1 in.); v = speed at pitch circle, ft. per min.; P = pressure in lb.:

v =	100	200	300	400	600	800
P =	1100	1100	1100	880	550	352
f =	0.060	0.051	0.047	0.040	0.030	0.025

Friction Gearing

For values of f between driving wheels of millboard, leather, wood, and various fiber boards and driven wheels of cast iron and other metals, see "Machine Elements," p. 735.

Tension Elements

Frictional Resistance. In Fig. 19 let T and t be the tensions with which a rope, belt, chain or brake band is strained over a drum, pulley or sheave, and let the rope or belt be on the point of slipping from t toward T by reason of the difference of tension $T - t$. Then $T - t$ = circumferential force P transferred by friction, must be equal to the frictional resistance W of the belt, rope or band on the drum or pulley. Also, let $b = 2\pi b°/360°$ = angle subtending the arc of contact between the drum and tension element, measured in radians. Then, disregarding centrifugal forces,

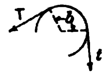

FIG. 19.

$$T = te^{fb} \text{ and } P = (e^{fb} - 1)T/e^{fb} = (e^{fb} - 1)t = W$$

where e = base of the Napierian system of logarithms = 2.718+.

Table 11. Values of e^{fb}

$\frac{b°}{360°}$	f = 0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0.1	1.06	1.1	1.13	1.17	1.21	1.25	1.29	1.33	1.37
0.2	1.13	1.21	1.29	1.37	1.46	1.55	1.65	1.76	1.87
0.3	1.21	1.32	1.45	1.60	1.76	1.93	2.13	2.34	2.57
0.4	1.29	1.46	1.65	1.87	2.12	2.41	2.73	3.10	3.51
0.425	1.31	1.49	1.70	1.95	2.23	2.55	2.91	3.33	3.80
0.45	1.33	1.53	1.76	2.03	2.34	2.69	3.10	3.57	4.11
0.475	1.35	1.56	1.82	2.11	2.45	2.84	3.30	3.83	4.45
0.5	1.37	1.60	1.87	2.19	2.57	3.00	3.51	4.11	4.81
0.525	1.39	1.64	1.93	2.28	2.69	3.17	3.74	4.41	5.20
0.55	1.41	1.68	2.00	2.37	2.82	3.35	3.98	4.74	5.63
0.6	1.46	1.76	2.13	2.57	3.10	3.74	4.52	5.45	6.59
0.7	1.52	1.93	2.41	3.00	3.74	4.66	5.81	7.24	9.02
0.8	1.65	2.13	2.73	3.51	4.52	5.81	7.47	9.60	12.35
0.9	1.76	2.34	3.10	4.11	5.45	7.24	9.60	12.74	16.90
1.0	1.87	2.57	3.51	4.81	6.59	9.02	12.35	16.90	23.14
1.5	2.57	4.11	6.59	10.55	16.90	27.08	43.38	69.49	111.32
2.0	3.51	6.59	12.35	23.14	43.38	81.31	152.40	285.68	535.49
2.5	4.81	10.55	23.14	50.75	111.32	244.15	535.49	1174.5	2575.9
3.0	6.59	16.90	43.38	111.32	285.68	733.14	1881.5	4828.5	12391
3.5	9.02	27.08	81.31	244.15	733.14	2199.90	6610.7	19851	59608
4.0	12.35	43.38	152.40	535.49	1881.5	6610.7	23227	81610	286744

e^{π} = 23.1407. $\log e^{\pi}$ = 1.3643764.

f is the coefficient of friction of repose (f_0) when there is no slip of the belt or band on the drum, and the coefficient of sliding friction (f) when slip takes place. In addition to the values given below, see p. 233.

Average values of f_0 for belts, ropes and brake bands are as follows: For leather belt on slightly greasy wood pulley 0.47. For leather belt on cast-iron pulley, very greasy, 0.12; slightly greasy, 0.28; moist, 0.38. For hemp rope on cast-iron drum, 0.25; on wooden drum, 0.40; on rough wood, 0.50; on polished wood, 0.33. For iron brake bands on cast-iron pulleys, 0.18.

Lost Work Due to the Stiffness of Tension Elements. Let d = the rope diam., the diam. of the chain stock or the diam. of the pin in link chains, and R = the radius of the pitch circle in which the tension element travels, both in in. The internal frictional rigidity of the tension element causes a shortening at the driving end T of the lever arm R an amount h_1 in inches, and at the following end a lengthening of the lever arm R an amount = h_2 in. Then, for simultaneous winding on and off, $T(R - h_1) = t(R + h_2)$. Approximately, $h_1 = h_2 = h$, whence $h_1 + h_2 = 2h$ (approx.) and $T = [1 + (2h/R)]t$. If the tension element is only wound on the drum, $h_1 = 0$ and $T = [1 + (h/R)]t$.

For **chains** the coefficient of friction between the link faces or pivots is f = 0.2 to 0.3; and when d = the diam. of the link pin, $h = fd/2$.

For **hemp ropes** $h = 0.03d^2$ to $0.09d^2$ according to the construction, material and condition of the rope. In the absence of reliable values for **wire ropes,** those for chains may be tentatively used.

The **elastic rigidity** of the material, *i.e.*, the work performed in changing the shape of the tension element, is not a factor in simultaneous winding on and off, since the lever arm R is increased equally at the points of winding on and off; the work expended in bending the tension element as it is wound on is recovered as it straightens out in unwinding. But if there is only winding on, then the lost work due to the bending is to be taken into account.

Lost Work Due to Creeping. Creeping is due to the elastic elongation of the tension element and must not be confused with the **slip** due to insufficient frictional grip on the pulley or drum.

Owing to a change in tension from the winding-on to the winding-off end, the length of the tension element varies in such a way that the driving pulley winds on a greater length than it winds off, the reverse being true for the driven pulley. This causes a loss of relative velocity which is equal to the lost-work ratio V. Assuming a uniform distribution of the tension over each cross-section of the tension element, there results for the entire drive system, *i.e.*, for the driving plus the driven pulley, $V = k(T - t)/A = kP/A$, where P is in lb., A = area of cross-section of element in sq. in., and $k = 1/E$, E being the modulus of elasticity of the tension element in lb. per sq. in. Since P is transmitted at the inner face of the tension element (belt), the tension (and therefore the elongation) is materially greater on the side of the belt in contact with the pulley than Tk/A, also $V > kP/A$. It is further to be considered that k decreases with increasing tension, thus reducing the value of V.

Bach recognizes the lack of uniform tension distribution and the variation in k by the use of an empirical constant m, thus: $V = mkP/A = mP/AE = mp/E$, where $p = P/A$. He gives the following values of m, p, E and V.

	m	p, lb. per sq. in.	E, lb. per sq. in.	$V = mp/E$, per cent.
Leather belt, new.....	2.00	140	17,800	1.6
Leather belt, used.....	2.00	140	32,000	0.9
Hemp rope*..........	1.25	137	107,000	0.16
Wire rope, new*......	1.50	4300	10,000,000	0.065

* p and E are based on the actual cross-section of the strands and wires, *i.e.*, for hemp rope, $A = 0.66\pi d^2/4$, and for wire rope $A = 0.42\pi d^2/4$, where d = diam. of rope in inches.

Efficiency of Rope and Chain Sheaves, at low speeds, including journal friction (180-deg. contact):

For fixed sheaves, chain and wire rope, e = 0.94 to 0.96.
For floating sheaves, chain and wire rope, e = 0.97.
Hemp rope sheaves:

Rope diam., in.	5/8	1	1 1/2	1 3/4	2
Fixed sheaves: e =	0.95–0.96	0.91–0.96	0.89–0.93	0.84–0.92	0.85–0.91
Floating sheaves e =	0.97	0.96	0.95	0.94	0.93

HYDRAULICS

BY

ERNEST W. SCHODER

REFERENCES: Church, "Mechanics of Engineering," J. Wiley & Sons. Hughes and Safford, "Hydraulics," Macmillan Co. Merriman, "Treatise on Hydraulics," J. Wiley & Sons.

General Properties of Liquids

A **liquid** is a substance that **has a definite volume,** but whose particles move relatively to each other so readily that, when unconfined laterally, the action of gravity causes it to flow and seek the lowest possible level. Hence a liquid **conforms to the shape of the containing vessel** or reservoir and, when at rest, presents a level upper surface unless restrained by the rigidity of the walls of a completely filled container.

When two or more liquids that do not mix (*e.g.*, water, oil and mercury) are contained in the same vessel, the heaviest liquid occupies the lowest position and the surfaces of division between the liquids are level. (But see p. 283 for capillarity, cohesion, adhesion and surface tension as affecting glass column gages.) In any continuous body of liquid at rest, the pressures at all points in a horizontal plane are equal. For a body of liquid moving at a uniform rate in a straight line, the above-stated principles are also true, but they are subject to modification for motion in a curved path or for accelerated motion other than vertical.

Compressibility and Elasticity of Liquids. A pressure of 1 lb. per sq. in. compresses liquids in volume as follows: Water, 1 part in about 300,000; mercury, 1 part in about 4,700,000; ether, 1 part in about 120,000. For water in an iron pipe this corresponds to a compression of 2 in. per mile length for a pressure of 10 lb. per sq. in. Because of this small compressibility, **liquids are** said to be **practically incompressible.** Liquids are perfectly elastic, *i.e.*, they regain their former volume upon removal of the pressure. As a consequence of the small compressibility and decided elasticity of liquids, pressure waves, exactly analogous to sound waves in air, are transmitted through liquids with high velocities—for water, over four times as fast as sound in air.

HYDROSTATICS

Liquid Pressure may mean either **total pressure** or **intensity of pressure.**

Gage pressure is the pressure indicated by a gage, and shows pressure above atmospheric pressure. **Absolute pressure** commonly means **intensity of pressure referred to vacuum as zero.** Pressures less than atmospheric are always referred to by some distinct phrase, *e.g.*, "10 lb. **suction,**" "20 in. (mercury) **vacuum,**" "5 lb. per sq. in. **absolute** pressure."

Pressure Head. Liquid pressure is often caused by the weight of the overlying liquid, and all liquid pressures can be thought of as being so caused. Special names are given to this real or imaginary height, as **"pressure head," "static head,"** or, simply, **"head."**

General Considerations Regarding Liquid Pressure

1. Pressure head is the height to the real or imaginary free liquid surface corresponding to the pressure of the liquid.

2. In any continuous body of liquid at rest, the **intensity of pressure** increases directly with the depth of the liquid. $p = hw$, or $h = p/w$, where p is the increase of pressure per unit area, h the increase of depth (or head) and w the weight per unit volume of the liquid. This law is true whether the liquid has a free upper surface open to the atmosphere or is confined and subjected to mechanical, gaseous or other pressure.

To find

Pressure of water in lb. per sq. ft., multiply head in ft. by 62.4.

Pressure of water in lb. per sq. in., multiply head in ft. by 0.433. (See Conversion Scales, Fig. 1.)

Vacuum head of water in ft., multiply "in. of mercury" by 1.13.

Vacuum pressure in lb. per sq. in., multiply "in. of mercury" by 0.49.

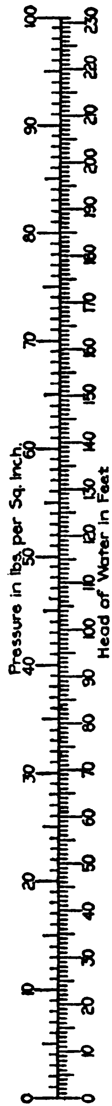

FIG. 1.—Conversion Scale for Water. Head in Feet to Pressure in Lb. per Sq. In.

Hydrostatic Paradox. In the cases shown in Fig. 2 the **intensity of pressure** in lb. per sq. in. is the same at the bottoms of the variously shaped liquid containers. But the total liquid pressures against the various bottoms are proportional to the areas of the bottoms. The amount of liquid or its total weight makes no difference in either the intensity of pressure or the total pressure as long as the head, h, is the same. The fact that the total liquid pressure (bursting pressure) against the bottom may be many times greater or less than the total weight of the liquid, is termed the **hydrostatic paradox.**

3. Liquid pressure is exerted with equal intensity in all directions.

4. **Liquid pressure acts perpendicularly to surfaces** in contact with the liquid. For curved surfaces the pressure at any point acts in the direction of the normal to the surface at that point.

5. **The total liquid pressure against a submerged plane area** equals the product of the average intensity of pressure by the area, *i.e.*, the product of the intensity of pressure at the center of gravity of the area by the area, or, as often stated, it equals the weight of a liquid column with a base equal to the area and height equal to the head on the center of gravity of the area. For a vertical **rectangular** plane extending to surface of liquid, with depth h and width b, the formula becomes: Total pressure $= P = \frac{1}{2}wbh^2$, where w is the weight of a unit volume of liquid. See **special calculating scale** for total pressures of water against vertical planes 1 ft. wide and 1 to 100 ft. deep, Fig. 3.

6. **The component in any direction** of the total liquid pressure on any submerged plane surface equals the product of the area of the projection of the surface on a plane perpendicular to the direction by the intensity of pressure at the center of gravity of the surface. For curved or non-planar surfaces, the principle may be applied to small portions of the surface, each regarded as plane, and the summation taken. Such cases, however, as **hollowed-out pump plungers and dished tank ends** have the same total pressures in the direction of the axis of the cylinder or tank as if the plunger or end were plane, *i.e.*, $P = Ap$, where A is the cross-sectional area of tank and p the average pressure. For submerged and floating bodies, see Buoyancy and Flotation, p. 253.

7. The **center of pressure** of a plane area subject to liquid pressure, *i.e.*, the point where the application of a single support will balance the liquid pressure, is located by the formula:

$$x_{c.p.} = \text{Moment of inertia of area/Area} \times x_{c.g.}$$

where $x_{c.p.}$ is the distance in the plane from the water surface to the center of pressure, $x_{c.g.}$ is the corresponding distance to the center of gravity of the area, and the moment of inertia is referred to line where the plane cuts the water surface. In Fig. 4, the point G represents the center of gravity of the area and point C the center of pressure. C is always below G, but with relatively small areas submerged under large heads they are very close together and for most purposes may be considered coincident. For common shapes, calling the distance to the upper edge s_1 and to the lower edge s_2 (see Fig. 4), the values of $x_{c.p.}$ are:

Rectangle, $\frac{2(s_2^3 - s_1^3)}{3(s_2^2 - s_1^2)}$; Triangle, vertex up, $\frac{1}{2}\frac{3s_2^2 + 2s_1s_2 + s_1^2}{2s_2 + s_1}$; Triangle, vertex down, $\frac{1}{2}\frac{3s_1^2 + 2s_1s_2 + s_2^2}{2s_1 + s_2}$; Circle, $x_{c.g.} + \frac{1}{4}\frac{r^2}{x_{c.g.}}$. If the upper edge is at the surface, the above formulæ become:

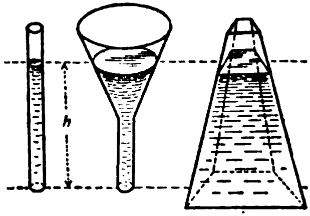

FIG. 2.

Rectangle, $2s_2/3$; Triangle: vertex up, $3s_2/4$; vertex down, $s_2/2$; Circle, $5r/4$.

The maximum bending moment on a vertical rectangular beam subjected throughout its full length to liquid pressure, supported at top and bottom, and with the top support at the surface, occurs 0.09 s_2 above the center of pressure or $\frac{1}{12}$ the span below the middle.

Example. Required the total pressure against a vertical gate 3 ft. wide and 5 ft. high at the bottom of a tank 12 ft. deep; also the position of the center of pressure.

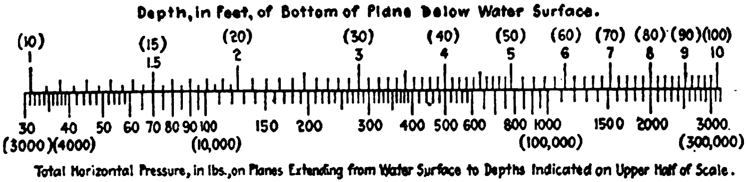

FIG. 3.

The center of gravity of the gate face is $(7 + \frac{5}{2} =)9\frac{1}{2}$ ft. below the surface, and according to (5), total pressure against gate $= 9\frac{1}{2} \times 62.4 \times (5 \times 3) = 8892$, say, 8900 lb. The total pressure scale, Fig. 3, gives 4500 and 1530 lb. respectively for depths of 12 and 7 ft. The difference, 2970 lb., is the pressure per ft. of width, or 8910 lb. total. The center of pressure [according to (7), for a submerged rectangle] is $\frac{2}{3}(12^3 - 7^3)/(12^2 - 7^2) = 9.72$ ft., or only 0.22 ft. below the center of gravity. Graphically, the depth of the center of pressure is the same as that of the center of gravity of the trapezoid forming the bottom 5 ft. of the "triangle of pressures."

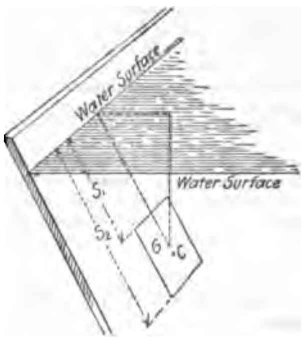

FIG. 4.

Buoyancy and Flotation. The total **upward pressure** of a liquid against a floating or a submerged body is the same as if the portion of the body below the surface were replaced by the liquid. In the latter case there would be simply a mass of liquid at rest, *i.e.*, with the upward pressure just **equal to the weight of** the liquid volume in question, which is the **volume displaced** by the floating or submerged body. Obviously, for a floating body, this weight and upward pressure equals the weight of the body. Hence, for any floating body, $P_u = W = Vw$, where P_u is the total upward pressure, W the weight of the body, V the volume of the displaced liquid (= "the displacement"), and w the weight of a unit volume of the liquid.

The **draft or depth of flotation** may be calculated when the weight, size and shape of the floating body are known, for in all cases $W = Vw$.

The **center of buoyancy** is at the center of gravity of the displaced volume of liquid. It shifts its position, both vertically and sidewise, as the floating body tips. The total upward pressure of the liquid may be regarded as concentrated through the center of buoyancy, about which it balances.

Stability, or tendency to float upright, demands that, when the floating body is tipped, the vertical through the new center of buoyancy shall pass above the center of gravity of the body. When it does so pass, both the weight and buoyancy tend to right the floating body. When it passes below the center of gravity, both the weight and buoyancy act to overturn it. It is not necessary for stability that the center of gravity be below the center of buoyancy. In ships it is usually above. The point where the vertical through the new center of buoyancy cuts the original vertical (now inclined), through the center of gravity, is called the **metacenter.** If, then, the metacenter is above the center of gravity, there is stability; if below, instability. The further above, the greater the stability.

Example. (Float control of a mechanism.) It is desired to obtain a force of 100 lb. from a float when the water surface rises or falls 1 in. Required to find size of float necessary. From $P_w = Vw$, $100 = Vw$ = water-line area $\times$ $1/12$ $\times$ 62.4, or area = 19.25 sq. ft., calling for, *e.g.*, a circular float 4.95 ft. in diam.

For submerged bodies, the "weight in water" (or other liquid) = ($W - V_w$), where V_w is the "loss of weight." The so-called "loss of weight" is not a real loss, but only apparent by reason of the buoyant force of the liquid. Thus, a block of granite weighing 170 lb. per cu. ft. can be supported under water by an upward force of (170 − 62.4 =) 107.6 lb. per cu. ft. of granite. The difference, 62.4 lb. per cu. ft., is borne by the water, and whatever supports the water supports the difference also. Thus, when a ship moves into a drydock and displaces water from the dock, the bottom of the dock is not relieved of any weight at all.

The Hydrometer. A floating body rides higher in a heavy liquid than in a light one. The density and specific gravity, therefore, may be found by noting the depth to which a specially prepared float sinks. Such a device is called a **hydrometer.** The most common type is made of glass, consisting of a graduated stem above a hollow bulb, below which is a smaller bulb containing mercury to make the whole instrument float upright. By properly proportioning the weight and volume any desired degree of sensitiveness may be obtained. Hydrometers to suit various needs are obtainable in the market. The scale is graduated to read either specific gravity (referred to water) directly or according to some arbitrary scale. Of the latter the most widely used is the **Baumé scale.** (See p. 85 for equivalents.) **Twaddell's hydrometer** is used in England for liquids denser than water. See p. 84 for several special hydrometer scales. Fahrenheit's and Nicholson's hydrometers are arranged so that weights may be added to sink them to a standard mark on the stem. For details see text-books on physics. For precise calculations the temperature of the liquid should be measured and corrections made to the hydrometer indications. Nearly all liquids expand and contract much more than water for equal temperature changes near 60 deg. fahr.; see p. 293.

Determination of Specific Gravity and Volume of Solids by Immersion. By weighing an insoluble solid heavier than water in air and then in water, the specific gravity referred to water may be readily found, the specific gravity being (weight in air)/(loss of weight). Also, the volume of the solid = (loss of weight)/(weight of unit volume of water). The method is at once a convenient field method and one of precision when weights are accurately measured.

THE FLOW OF LIQUIDS

General Considerations Regarding Flow

Steady, Unsteady, Uniform and Non-uniform Flows Defined. Flow is called **steady** when conditions in the stream (*i.e.*, in any cross-section of the flowing liquid) remain unchanged, especially the quantity flowing per second, the velocity and the size of the stream. If, also, the size and shape of the stream remain nearly constant from point to point along the course, the flow is called **uniform.** If the size of the stream and the velocity change from point to point along the course, even though the flow be steady, it is called **non-uniform.**

Examples of **steady uniform flow** are the common flows **in pipes, hose, canals and rivers.** This sort of flow is seen to be associated with long conduits, and is controlled by the laws of fluid friction. **Pressure and head** are consumed (hence **"lost"**) in forcing the liquid along against the resistances caused by the portions of the conduit in contact with the liquid.

Steady non-uniform flow is exemplified by the flow **through orifices, nozzles, Venturi meters,** revolving water **turbines, weirs,** etc. It is seen to be associated with cases of changing velocity, where flow starts from a nearly quiet body of liquid or where a flowing stream changes its size and its velocity. For these cases **pressure and head** are not consumed and lost (except to a small extent), but are **converted into moving energy.**

Unsteady Flow proceeds **in waves,** pronounced pulsations **or surges,** or under rapidly changing conditions, *e.g.*, breaking ocean waves, pressure and velocity surges in a pipe line supplying a hydraulic ram, etc. No flow is steady or uniform when started. A short time elapses, of necessity, before steady conditions can be established. No ordinary flow is ever perfectly steady, there always being some pulsations and tremblings in velocity and pressure. The flow through a long pipe line carrying the discharge from a good double-acting plunger or piston pump with air chambers in good order is considered practically steady as far as flow calculations are concerned, although a pressure gage shows fluctuations of several per cent.

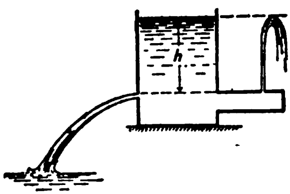

Fig. 5.

Interrelations of Heads. Potential, Pressure and Velocity Heads. When water spouts through an orifice in the side of a tank (Fig. 5), the jet has a velocity $V = \sqrt{2gh}$, nearly, where h is the head at the level of the orifice in the body of water well away from the orifice, and g is the acceleration constant of gravity. As a result of liquid friction at the orifice the actual velocity is 1 to 2 per cent. less than that given by the above formula. A jet issues in any direction according to the same law. If issuing nearly vertically, as in Fig. 5, the jet will rise nearly to the height of h ft. above the orifice. For great heights the friction of the air and the breaking-up of the jet reduce the height considerably. See p. 277. For a liquid flowing with a velocity V, the name **velocity head** is given to the quantity $V^2/2g$, which corresponds to the static or pressure head h that could cause or could be caused by the velocity V.

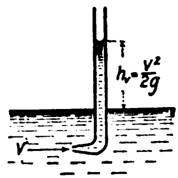

Fig. 6.

Fig. 6 shows the water surface of an open stream flowing with velocity V, and against the current of which is directed the opening of a **Pitot tube.** The water rises in the tube to a height of h ft. above the surface of the stream, such that $h = V^2/2g$ or $V = \sqrt{2gh}$; *i.e.*, the Pitot tube indicates the velocity head of the flowing water. If a tube of this sort is inserted in a pipe with water flowing under pressure, see Fig. 7, the water rises to a height h_v above the height in an open water column attached to a hole in the wall of the pipe; *i.e.*, the Pitot tube shows the sum of the pressure head and the velocity head of the water at the point of the tube, demonstrating that both sorts of head co-exist. The **Venturi meter** also illustrates the relation between pressure and velocity heads. This device (Fig. 8) consists of a conical, nozzle-like reducer followed by a more gradual enlargement to the original size, which is that of the pipe line in which the meter is laid. At 1, 2, and 3 the pressure heads in the pipe are shown as they appear when measured by open water columns. Experiment shows that h_2 is less than h_1 by very nearly the difference in velocity heads $[(V_2^2/2g) - (V_1^2/2g)]$, and that at 3, where the velocity is again V_1, the pressure head is nearly h_1 again, the diminution being accounted for by loss due to friction of the flowing water.

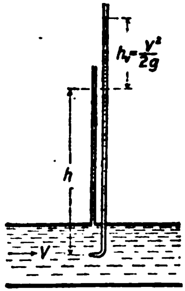

FIG. 7.—Pitot Tube.

If the pressure in the pipe is changed, but the rate of flow kept the same, *e.g.*, by manipulating valves to the left of 1 and right of 3, all the pressure heads rise or fall equal amounts, *i.e.*, the **differences in heads remain unchanged.** If the pipe is not level, Fig. 9, allowance is made as follows: All points are referred to any convenient reference level, *O-O*. The difference in level of the tops of the open water columns is $H_1 - H_2 = (h_1 + Z_1) - (h_2 + Z_2)$, where Z_1 and Z_2 are the heights of 1 and 2 above the datum level, and are called the **potential heads** of these points. With no flow, water would stand at the same level in columns 1, 2 and 3, for both the level and sloping pipes. Hence the differences $h_1 - h_2$ in Fig. 8, and $H_1 - H_2$, in Fig. 9, are due solely to the flow of the water. It is to be noted that the difference in pressure heads of the water in the pipe is $h_1 - h_2$ in both Figs. 8 and 9, but in Fig. 9 this difference is due partly to the hydrostatic effect of the difference of level. Consequently it is seen that a difference in pressure heads at two points along a stream (the velocity being different at these two points) may be due not only to a difference of velocity heads, but also in part to a difference in potential heads according to the hydrostatic law. Hence,

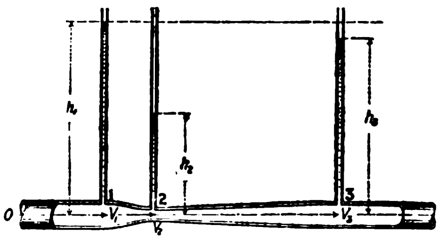

FIG. 8.—Venturi Meter.

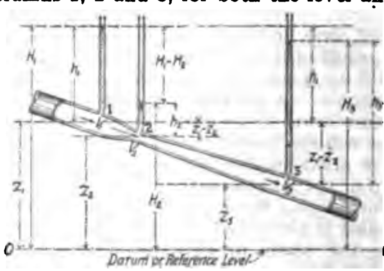

FIG. 9.

$$(h_1 + Z_1) - (h_2 + Z_2) = (V_2^2/2g) - (V_1^2/2g),$$
$$\text{that is, } h_1 + Z_1 + (V_1^2/2g) = h_2 + Z_2 + (V_2^2/2g)$$

This formula is the mathematical expression of **Bernoulli's Theorem.**

To summarize:

1. The **total head** present in a mass of flowing liquid is made up of potential head, pressure head and velocity head, mutually convertible, each into the other's form.

2. **Bernoulli's Theorem.** The total head in a particle of a continuous mass of flowing liquid at any one point in its stream line (*i.e.*, the path along which the liquid flows) is equal to the total head at any other position of its stream line, *provided* that there is no loss between the two positions due to friction, the giving up of work, etc., and no gain due to the application of outside work.

Flow Through Orifices

Flow of Water Through Orifices. The general law of flow is $V = \sqrt{2gh}$, in which V is the velocity of the jet at its smallest section near or in the plane of the opening, h is the head referred to the level of the center of the stream cross-section where the filaments of the issuing jet first become parallel.

The Standard Orifice for measuring or regulating purposes is the **sharp-edged orifice,** also called "orifice in thin plate." The jet contracts in size (Fig. 10*a*) just after coming through the opening of such an orifice. The area of the cross-section of the jet, at a distance out from the opening about one-half its diameter, is about 0.62 of the area of the opening. The average velocity past this contracted section is about 0.98 to 0.99 of $\sqrt{2gh}$. Hence, calling A the area of the opening, the **discharge of a standard sharp-edged orifice** is $Q = 0.61A\sqrt{2gh}$, Q being in cu. ft. per sec. when A is in sq. ft., h in ft. and g in ft. per sec. per sec. The value 0.62 is called the **contraction coefficient;** 0.98 to 0.99 is the **velocity coefficient,** and their product, or 0.61, is the **discharge coefficient.**

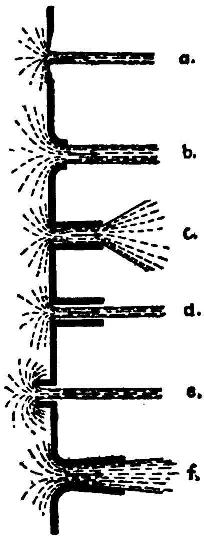

FIG. 10.—Types of Orifices.

For Values of $\sqrt{2gh}$ for any head, see Velocity Head Scale, Fig. 11, also Conversion Scales (cu. ft. per sec. to gal. per min. and per 24 hr.), Fig. 12.

Example. For a circular sharp-edged orifice, 1 in. in diam., under 50 ft. head, Q = 0.189 cu. ft. per sec. = 85 gal. per min.

Large Orifices with Low Heads, *e.g.*, sluice gates (see also Submergence). Unless the orifice is in a horizontal plane, *e.g.*, in the bottom of a tank, the top may be so much nearer the surface of the liquid than the bottom that the head at the level of center of orifice does not correspond to the real average velocity of all the water flowing out, but to more than the average, *i.e.*, the actual discharge is less than that shown by the formula. (For theoretical principle, see Weirs, p. 263.) When the head above the center is equal to the vertical dimension of the orifice, the discharge is only about 1 per cent. less, and when the head is twice the vertical dimension the diminution is

negligible, except for the most precise sort of investigations, for which a special calibration should be made to determine the exact coefficient. For lower heads, if h_c represents the head above the center and O the height of opening, Table 1 gives the percentage reduction in discharge and the corrected discharge coefficient for circular and rectangular orifices. The value of the coefficient, however, is not certain within from 1 to 3 per cent. without special calibration of the particular orifice.

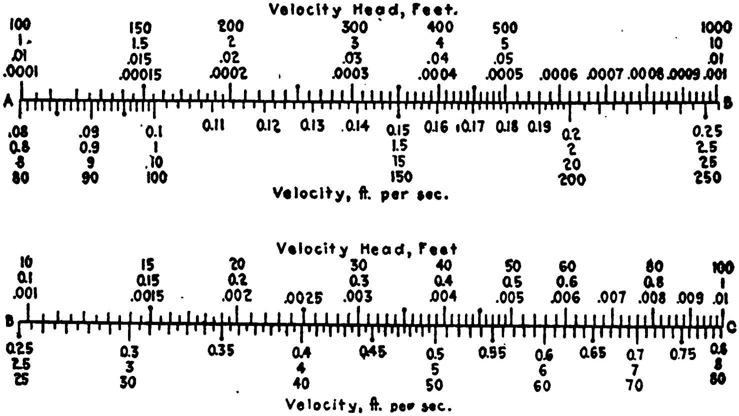

FIG. 11.—Velocity-Velocity Head Conversion Scale.

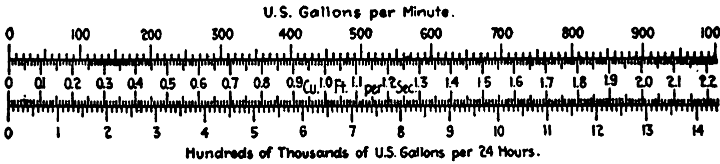

FIG. 12.—Conversion Scale. Cu. Ft. per Sec. to Gal. per Min. and per 24 Hr.

Table 1. Corrections for Rectangular and Circular Orifices

$\frac{h_c}{O}$	Rectangular Orifices: Reduction in discharge, per cent.	Rectangular Orifices: Corrected coefficient of discharge	$\frac{h_c}{O}$	Circular Orifices: Reduction in discharge, per cent.	Circular Orifices: Corrected coefficient of discharge
0.50	5.7	0.575	0.50	4.0	0.585
0.62	3.2	0.590	0.55	3.2	0 590
0.87	1.6	0.600	0.73	1.6	0.600
1.00	1.1	0.603	1.00	0.8	0.605

Hamilton Smith's Coefficients. Relying chiefly on his own precise experiments of 1885 and earlier, and on those of Poncelet and Lesbros, 1827–1835, Smith published tables of coefficients for sharp-edged orifices with square and round openings. The accompanying diagram, Fig. 13, shows the range and variations with head and diameter of these coefficients. The writer has found corresponding variations with small circular sharp-edged orifices ½ to 1 in. in diameter. It will be noted that there is little change in the coefficient for heads above 2 ft. and diameters greater than ½ in.

Variously Shaped Orifices. Typical shapes are shown in Fig. 10. (*a*) is a **sharp-edged orifice**; (*b*) is a **rounded-approach orifice**; (*c*) is a **short pipe** or **orifice in thick wall** (length = 2.5 to 3 × diam.), flowing full; (*d*) is a short pipe flowing like a sharp-edged orifice (see below); (*e*) is a **re-entrant short pipe**, or a **Borda mouthpiece** (when length < diam.); (*f*) is a **Venturi orifice**.

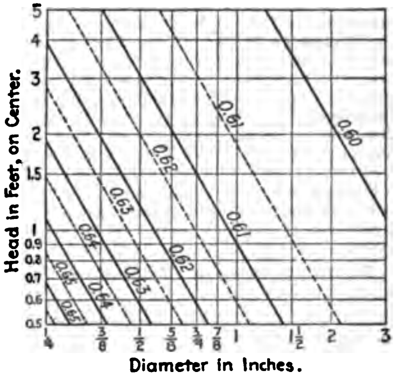

FIG. 13.—Coefficients for Sharp-edged Orifices. (Hamilton Smith.)

For (*b*), when the curve is not too abrupt and there is a short length (= ¼ to ½ diam.) of uniform bore where the jet issues, there is no contraction and the coefficients of discharge and of velocity are equal. The value for an average good, smooth, rounded orifice is 0.97; for the very best, 0.99; for one "well rounded and smooth" but with poor curvature, causing contraction and cross currents, the coefficient may be as low as 0.90.

For (*c*), at low heads, the tube flows full at the downstream end and the discharge is "broomy," *i.e.*, agitated and divergent, while at high heads, above 40 to 50 ft., the jet jumps free at the upstream corner as from a sharp-edged orifice, as shown in Fig. 10(*d*). If the jet is once made to jump free by a high head, the head may be lowered to only a few times the orifice diameter without bringing back the "broomy" flow condition. For "broomy" flow, the discharge coefficient is about 0.82; for clear flow it is 0.61, as for the sharp-edge. If the interior of the tube is greasy, the broomy flow will not occur, even at a low head. Obviously, this is an uncertain measuring device unless conditions are known.

For (*e*), when the length is 2.5 to 3 times the diameter, the above statements for (*c*) and (*d*) apply. The discharge coefficient is about 0.72 for "broomy" and 0.53 for clear discharge. For Borda's mouthpiece the coefficient is 0.53.

For (*f*), if the divergence angle is not greater than 7½ deg. nor the length more than 10 nor less than 5 times the smallest diameter, the tube will flow full, and the velocity through the narrow "throat" will be greater than $\sqrt{2gh}$, because the pressure is less than atmospheric at that point, causing a suction and making the effective head greater than h. Values of the discharge coefficient as high as 1.55 have been found by experiment. For the downstream end of the tube, the coefficient is always less than 1. With high heads the jet may jump free and discharge as (*b*).

Rounding or beveling the sharp upstream edge, even slightly, increases the discharge of an orifice. Placing a sharp-edged orifice close to a side wall or a corner "suppresses" the contraction and increases the discharge. For each side of a rectangular orifice on which contraction is suppressed, the discharge is increased about 3 per cent.

Attainable Precision in estimating the flow through a well-made sharp-edged or rounded-approach orifice depends on several factors. The orifice should be located at least 3 diameters away from any side wall and should be screened from the eddying effects of in-rushing water. If the head is low, guide vanes may be necessary to prevent whirlpools or vortices that obstruct the discharge. The size of the opening must be accurately measured. An error or uncertainty of a small percentage in the diameter makes twice that error in the discharging area; and a small percentage change in the head affects the discharge by half that percentage. **With accurate measurements** made with a knowledge of the various effects above mentioned, the engineer may feel assured that the **discharge will be within 3 per cent. of the calculated value.** By calibrating an orifice under the exact conditions of use, a future precision of 0.5 per cent. may be assured. Such an accuracy is seldom necessary except for precise testing or investigation. For an assured **precision within 1 per cent.**, maintenance of the orifice in "sharp-edge" condition is essential. A very slight dulling of the edge increases the discharge as much as 1 to 2 per cent. As to the effects of a little corrosion, the nature and extent of the change in the coefficient depend on those of the defect. If a corroded edge is tuberculated, the discharge is decreased. If the rust accumulation is brushed or washed off, the effect is the same as dulling the edge. Published tables that give **coefficients to three places**, *e.g.*, 0.615, **are deceptive,** in that they suggest a precision of 1 in 600, when 1 in 60 would be nearer to the accuracy expected by an experienced engineer for routine cases.

Submergence, or Discharge Under Water (*e.g.*, headgates). The coefficients already given apply in this case, but the value of h in the formula is the difference in water levels upstream and downstream from the opening. In the case of headgates there is liable to be uncertainty as to the effective area of the opening and of the amount of contraction. An uncertainty of from 5 to 10 per cent. is to be expected.

Several openings side by side (*e.g.*, headgates, intake screens and baffles) cause an increase in the discharge, if submerged on the downstream side. For openings close together increases of from 5 to 10 per cent. have been measured. Hence the engineer is somewhat safeguarded in his calculations by this fact. Results of a series of experiments on 4-ft.-square openings and tubes with differences of water levels of only a few inches, and with various lengths and entrance conditions, are given in *Bulletin* No. 216, Univ. of Wis., and in *Eng. News*, Jan. 9, 1908.

Velocity of Approach. If, some distance upstream from the orifice, there is a high velocity of approach due to a small approach channel or passageway or due to peculiar flow conditions, this **velocity head,** $V^2/2g$, should be added to the static head to obtain h for the formula. The percentage effect is seldom large unless the channel is so small as to interfere with the flow by reducing contraction, etc., and then, of course, there will be considerable uncertainty. (See Nozzles.)

The Miner's Inch. In some of the western states, water for mining and irrigation is sold by the **Miner's Inch,** which is the quantity of water that will flow through 1 sq. in. of an opening in a plank under a head of (usually) 6 in. The arrangement of the opening (from 1 to 4 in. high), the thickness of the plank (from 1 to 3 in.), the datum level for the measurement of the head (the top, center, or bottom of opening), as well as the head itself (from 3 to 9 in.), are largely matters of local custom and of state law. In Cal. and Mont., 40; in Col., 38.4; and in Ariz., Idaho, Nev. and Utah, 50 miner's inches = 1 cu. ft. per sec. In some old deeds in the eastern states water rights were sold by **inches,** but this commonly meant the discharge through 1 sq. in. of some portion of an old-fashioned water wheel under the head at the particular power site, and, of course, is an extremely variable unit, the subject of many controversies in courts.

Oils, if not too tarry or viscous, **and mercury have** been found to have practically **the same coefficients as water.**

Time Required to Empty Tanks through Orifices. An open tank whose cross-sectional area is uniform throughout its depth (*i.e.*, a vertical cylindrical or prismatic tank) will empty itself through an orifice, if there is no inflow, in just twice the time required to discharge the same amount of water under the initial head. For other shapes of reservoirs, the time may be computed by dividing the volume into five to ten horizontal layers and computing the time for each layer to discharge under the average head for that layer.

Nozzles. Fig. 14 shows typical forms of nozzles. The **orifice formula,** modified to include velocity of approach, **applies to** the flow from **nozzles.**

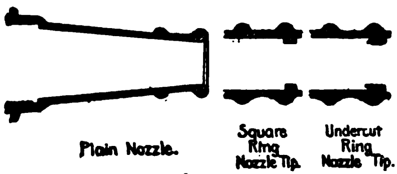

FIG. 14.—Types of Nozzles.

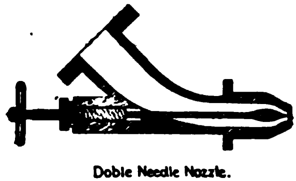

FIG. 15.

Ring and needle nozzles (Figs. 14 and 15) cause contraction of the jet, the amount depending on the shape. A plain conical nozzle (called a **smooth nozzle**) causes no contraction. The velocity coefficient of a **good nozzle is** from 0.97 to 0.99, and this is also the discharge coefficient for smooth nozzles. A **square-ring nozzle** is really a sharp-edged orifice with high velocity of approach. The discharge coefficient depends on the proportion of the area taken up by the ring (see curve in Fig. 16, determined by J. R. Freeman from his experiments). The use of ring and undercut-ring nozzles arose from the mistaken notion that a better fire stream could be thrown from these than from a smooth nozzle. **Undercut-ring nozzles** have discharge coefficients of from 0.58 to 0.72, varying as for square-ring nozzles. The discharge coefficient of a well-finished **Doble needle nozzle** (Fig. 15) referred to the smallest area of passageway (surface of frustum of cone) between needle and nozzle proper, is from about 0.82 to 0.95, depending on the "opening" or position of the "needle." When nearly closed the coefficient is lowest; it is highest at about a ¾-open or full-load operating position, for which the curves are especially designed.

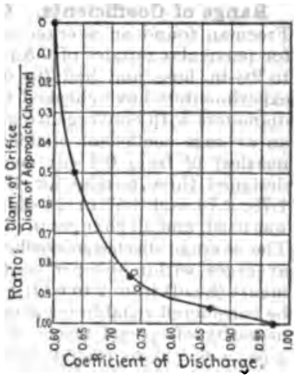

FIG. 16.—Discharge Coefficients for Square-ring Nozzles.

The **nozzle formula** is:

$$Q = CA_2\sqrt{\frac{1}{1-(A_2/A_1)^2}}\sqrt{2gh} = CA_2\sqrt{\frac{1}{1-(d_2/d_1)^4}}\sqrt{2gh}$$

where A_1 and A_2 are the cross-sectional areas at the base of the nozzle (*i.e.*, where the pressure head is measured) and at the discharge tip, respectively, and d_1 and d_2 the corresponding diameters. C is the discharge coefficient and h is the pressure head in the pipe or hose just upstream from the convergence, referred to the level of the center of the discharge tip. (For ordinary purposes, h is usually measured by a spring steam gage, or, more precisely, by an open water-column gage for low heads and a mercury gage for high heads. See p. 284.) h is sometimes called the **gage pressure head,** and $[h + (V_1^2/2g)]$ the **theoretical or total head.**

Effect of Neglecting the Velocity of Approach. The value of the expression $\sqrt{\frac{1}{1-(d_2/d_1)^4}}$, or its equal, $\sqrt{\frac{1}{1-(A_2/A_1)^2}}$, which corrects for the velocity of approach, is as follows for the range of probable values of d_2/d_1:

Table 2. Correction Factors for Nozzles

$\frac{d_2}{d_1}$	$\sqrt{\frac{1}{1-(d_2/d_1)^4}}$	$\frac{d_2}{d_1}$	$\sqrt{\frac{1}{1-(d_2/d_1)^4}}$
0.1	1.00005	0.5	1.033
0.2	1.0008	0.6	1.072
0.25	1.002	0.67	1.116
0.3	1.004	0.7	1.147
0.33	1.006	0.75	1.210
0.4	1.013	0.8	1.301

The correction factor is seen to be practically unity for any ratio d_2/d_1 less than 0.3, and therefore for such nozzles the following simpler formula may be used: $Q = CA_2\sqrt{2gh}$.

Range of Coefficients. Certainty. For smooth fire-hose nozzles, J. R. Freeman found an average coefficient of 0.977, with variations in averages for particular nozzles of 0.5 per cent. each way. The nozzles were attached to 2½-in. hose and had tip diameters of from ¾ to 1⅜ in. Several other experimenters have checked this value closely. For 1¾-, 2- and 2½-in. tip diameters with convergence angles of from 10 to 15 deg., Mr. Freeman found an average coefficient of 0.995, with a variation (in averages for individual nozzles) of from 0.4 per cent. above to 0.8 per cent. below. The writer designed three nozzles for 4-, 6- and 8-in. pipes, the tips being respectively 1.78, 2.74 and 3.57 in. in diameter; a common convergence angle of 25 deg. was used, and all changes in direction were made by easy curves of 9 in. radius. The average discharge coefficient was found to be 0.986, with variations in averages within 0.3 per cent. and extreme variations in individual experiments (heads from 2 to 60 ft.) of about 1 per cent. **A coefficient of 0.98** may be considered reliable for good, smooth nozzles within 1½ per cent., without necessity of special tests. The writer tested, as a nozzle on a 6-in. W. I. pipe, a 6 × 3-in. rusted, riveted-steel reducer, 24 in. long, with projecting flat rivet heads at both ends and along a longitudinal seam. The discharge coefficient was 0.91 under heads of from 2 to 50 ft., allowance being made for rivet heads reducing the tip area. This may be taken as the low limit for an **extra rough conical nozzle.** In case of such a nozzle the coefficient 0.91 means that 17.2 per cent. of the head is "lost," *i.e.*, used up in overcoming friction.

Fire-extinguishing Nozzle Streams: Range and Capacity. See p. 276.

The Venturi Meter. This device was invented by Clemens Herschel in 1887, and named after Venturi, who observed the principle in 1791. The general formula of this useful flow meter is discussed on p. 256. Referring to Fig. 9, the practical formula connecting the measured difference of pressure heads, $H_1 - H_2$, with the mean velocity of flow in the pipe line, is

$$V_1 = C\sqrt{\frac{1}{(d_1/d_2)^4 - 1}}\sqrt{2g(H_1 - H_2)}$$

The value of the coefficient C has usually been taken as 0.97. It appears from recent tests that a value of 0.98 may be safely assumed for the Venturi meters now on the market. Very precise tests made by the writer on new 12-in. meters, with throat diameters (d_2) of 4, 5 and 6 in., showed an average coefficient of 0.99 for throat velocities from 5 to 60 ft. per sec. The departure from the average value was mostly less than 0.5 per cent., and only in a few cases as much as 1 per cent. Precise tests on large Venturi meters after being in service for several years are lacking.

The **nominal size** of a Venturi meter is that of the pipe line in which it is to be placed. The throat diameter usually is made some size between 0.25 and 0.50 of the upstream diameter, depending on the rate of flow to be expected and on the pressure in the line. The throat velocity with the low rates of flow should be great enough so that the difference of pressure heads (often called **the Venturi difference**) will cause a measurable indication on the register or gage, but should not be so great as to cause the throat pressure to drop below atmospheric, lest there be trouble with air leaking into the gage and register connections. The upstream convergence angle of a Venturi meter may be from 25 to 30 deg., but the downstream divergence angle should not be greater than 7½ deg. The pressure connections usually are made to **ring piezometers**, consisting of circumferential passageways communicating to the interior of the meter by four or more small holes equally spaced around the pipe. The object is to assure the obtaining of the average pressure by avoiding dependence on a single piezometer hole subject to possible local disturbances. The ring piezometers are cast as part of the meter tube. The meters are usually made of cast iron and in several sections, bolted together. The throat is bronze-lined, very accurately bored to size and very smoothly finished. Venturi meters have been made of concrete and wood staves, the throats being lathe-finished bronze castings.

The **loss of head** caused by the presence of a Venturi meter in a pipe line, *i.e.*, $H_1 - H_3$, Fig. 9, is from $\frac{1}{6}$ to $\frac{1}{10}$ of the "Venturi difference" $H_1 - H_2$. Expressed in terms of the equivalent additional length (L) of pipe line of diameter d_1, *i.e.*, for same loss of head with same quantity flowing, V_1 being about from 2 to 5 ft. per sec., Venturi meters give: for ratio $d_2/d_1 = 0.5$, $L = 80d_1$; for $d_2/d_1 = 0.4$, $L = 170d_1$; for $d_2/d_1 = \frac{1}{3}$, $L = 430d_1$, and for $d_2/d_1 = 0.25$, $L = 1300d_1$. Improper angles of convergence and divergence cause higher losses.

FLOW OVER DAMS AND WEIRS

Weirs. A weir is a **bulkhead or dam** over which water flows, or a **notch** in the top of such a structure through which water flows. An orifice becomes a weir if the water surface upstream falls below its top edge. The velocity of approach is sometimes considerable, as for a low dam in a river at flood time, or it may be practically negligible, as for a small notch in the side of a large tank.

The theoretical discharge over a weir of width l is $Q = \frac{2}{3}lh\sqrt{2gh}$. This is two-thirds of the theoretical discharge through a rectangular orifice l ft. long and h ft. high under a head h. Actually, for a sharp-crested weir, contraction occurs at the crest. The under surface of the falling sheet of water rises as it leaves the sharp crest, just as for an orifice. The discharge coefficient is about 0.62, *i.e.*, $Q = 0.62\ (\frac{2}{3}lh\sqrt{2gh})$. This formula, for which the product of the constants $= 3.33$, may be expressed as $Q = 3.33lh^{3/2}$, and as such is known as the **Francis formula** for sharp-crested weirs. Q is in cu. ft. per sec., and l and h are in ft. (See Weir Discharge Scale, Fig. 17.)

Accuracy of Francis Formula. This formula is very widely used by engineers. It is **reliable within from 1 to 3 per cent. for heads above 0.3 ft.**, provided that (1) the weir bulkhead has a vertical upstream face and occupies the full width of the channel, (2) the crest is level, (3) the channel of approach is deep (*i.e.*, the weir bulkhead is high, so that the velocity of approach is small—see below for corrections), (4) there is free access of the air to the space below the falling sheet of water (*i.e.*, between it and the downstream face of the weir), (5) the head is measured a distance upstream from the weir of at least 4 times the head, (6) the side walls extend downstream from the weir (above the crest level) to prevent a lateral spreading as the water passes over the crest, and (7) due precautions in measuring are taken.

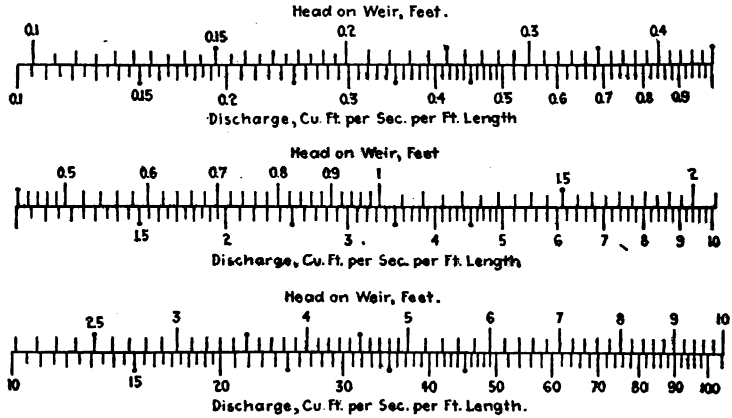

FIG. 17.—Weir Discharge Scale. Francis Formula.

For very low heads Francis formula results for discharge must be increased 3 per cent. when $h = 0.2$ ft., and 7 per cent. when $h = 0.1$ ft.

By a **sharp crest** is meant one with a sharp upstream corner from which the water springs. The crest itself may be of considerable thickness (but see under broad, flat crests below), and is frequently made of a steel plate or angle planed off and sometimes beveled on the downstream side to give a top flat portion from $\frac{1}{16}$ to $\frac{1}{8}$ in. wide.

Effects of Errors or Uncertainties. A small percentage error in head causes $1\frac{1}{2}$ times that error in discharge. Conversely, the allowable percentage error in head is $\frac{2}{3}$ that in discharge. Considerable errors in the measured head may be made by careless methods of referencing the crest level to the head gage scale, and by assuming that a non-level crest is level without knowing the average level. The mean level of the water surface must be measured, not the crests of the small waves or surges existing in all flowing water.

For a **rectangular notch** with vertical sharp-edged ends, contraction occurs at the ends as well as at the crest, and the discharge is reduced approximately as though the length of the weir were decreased by 0.1 h for each end contraction. Hence, the **Francis formula for a sharp-crested weir with two end contractions** is

$$Q = 3.33\,(l - 0.2h)h^{\frac{3}{2}}$$

This formula must not be used unless l is at least 2 times h. (See Narrow Rectangular Notches below.)

Cippoletti Trapezoidal Weir. Cippoletti found that a trapezoidal notch with end slopes 1 horizontal to 4 vertical (Fig. 18) just compensated for the reduction due to end contraction in a rectangular notch of the same crest length, and, accordingly, the simple Francis formula, $Q = 3.33lh^{3/2}$, may be used for such a trapezoidal notch. The crest length l must be at least 2 times h, and the crest should be 2 or 3 times h above the bottom of the channel of approach to avoid a velocity-of-approach correction (see Narrow Trapezoidal Notches below). The Cippoletti weir has become popular in the western states for measuring irrigation water.

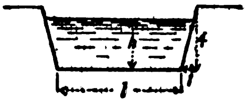

FIG. 18.—Cippoletti Weir.

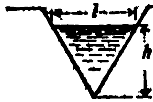

FIG. 19.—Triangular Notch Weir.

Triangular Notch. There are certain advantages in the triangular notch for measuring discharges that vary from a moderate maximum to a very small minimum, *e.g.*, 1 gal. per min., and where about the same degree of precision is desired whether the discharge is high or low. The formula for a sharp-edged notch (Fig. 19) is $Q = C(\frac{4}{15} lh\sqrt{2gh})$, where l is the width of the notch a distance h above the vertex, and C is a coefficient with a value of from 0.57 to 0.60, based on experiments with heads up to 3 ft. and vertex angles of 28, 60, 90 and 120 deg. The coefficient 0.57 was found for the 60- and 90-deg. weirs, and 0.60 for the 28- and 120-deg. weirs. Prof. Thomson found for 90 deg., 0.60, and for 127 deg., 0.62, heads 0.2 to 0.8 ft. Hence the discharge through a triangular notch (for which $l = 2h \tan \frac{\text{notch angle}}{2}$) is from 38 to 40 per cent. of that given by the discharge scale (Fig. 17).

Very precise experiments on 90-deg. V notches, with heads up to 10 in., have been made by James Barr (London *Engineering*, Apr. 8 and 15, and Oct. 28, 1910). With smooth upstream face and wide and deep tank, the coefficients at the same heads on several weirs were found constant within ½ of 1 per cent. for heads ranging from 3 to 10 in., and within 1 per cent. for heads as low as 1½ in. The tests show noticeable changes in discharge due to variations in roughness of upstream face of the notch plate. A surface like coarse emery increases the discharge over that for smooth brass some 2 per cent. (for a head of 0.3 ft.). It was found also that the tank upstream from the notch should be seven to eight times the head in width and three to four times the head in depth below the vertex to avoid, respectively, increases and decreases up to 1 to 2 per cent. in discharge with narrow and shallow tanks. As above, $Q = C(\frac{4}{15} lh\sqrt{2gh})$. For a 90 deg. notch $l = 2h$ and $Q = C(\frac{8}{15}\sqrt{2g})h^{5/2} = Kh^{5/2}$. With sharp-edged notches in smooth brass plates the formula $K = (0.2907 + 0.028/\sqrt{h})$, proposed by T. P. Strickland, applies. This is for Q in cu. ft. per min. and h in in. For Q in cu. ft. per sec. and h in ft., $K = (2.42 + 0.067/\sqrt{h})$ and $C = (0.565 + 0.016/\sqrt{h})$.

Narrow Rectangular and Trapezoidal Sharp-edged Notches with Crest Length Less than 2 Times h. Experiments on narrow rectangular notches beyond the applicability of the Francis formula, covering heads up to 3 ft. and widths down to 2.7 in., have shown that the formula $Q = 0.56\,(\frac{2}{3} lh\sqrt{2gh})$, *i.e.*, $Q = 3.00\,lh^{3/2}$, is reliable when h is as great as or greater than l. Hence for such cases we may take 90 per cent. of the discharge given by Fig. 17. For notches with l between h and $2h$ the coefficient increases and merges into values given by the Francis formula for two end contractions. For narrow trapezoidal notches the discharge may be expressed as the sum of that through a central rectangle and two triangles at the ends (see above formulæ). A discharge coefficient of from 0.58 to 0.60 has been found to apply.

The inverted notch, with discharge proportional to head, allows very simple regulating and recording devices to be used with it. It is constructed with curved sides such that the width of notch above the straight level crest decreases just enough to keep the discharge proportional to the head. The rate of flow is given by the formula, $Q = C \times 1.57\sqrt{2g}(l\sqrt{h})h$, where Q is in cu. ft. per sec., l and h are notch width and head, respectively, in ft. With $C = 0.60$, this becomes $Q = 7.55(l\sqrt{h})h$. The product $l\sqrt{h}$ is constant, and this gives the relation for the curves of the sides. Starting with some desired or convenient head for a value of the discharge, say near the probable maximum to be expected, the value of the notch width at that height above the crest is calculated from the above formula for Q. These values of width and head give the constant value $l\sqrt{h}$. It is not necessary to continue the side curves of the notch closer to the crest than about 1/32 to 1/8 in. (depending on the range of heads); they may terminate in short verticals to the crest. The ratio of the discharge shut off by reason of such an abrupt termination of the curves to the full opening discharge (if it could be secured) is $0.64\sqrt{h_1/h}$, where h_1 is the height of the end verticals and h is the head of water. For values of $h/h_1 = 400$, 100, 25 and 10, respectively, the percentage reductions in discharge are 3, 6, 13 and 20. Unless the notch curves are accurately constructed, large errors in estimated discharge are to be expected. For results of tests see *Eng. News*, Nov. 25, 1915.

Velocity-of-approach Effects have been variously expressed in the formulæ adopted by different engineers. Probably the simplest one, and yet one entirely adequate for our present knowledge, is a modification of the

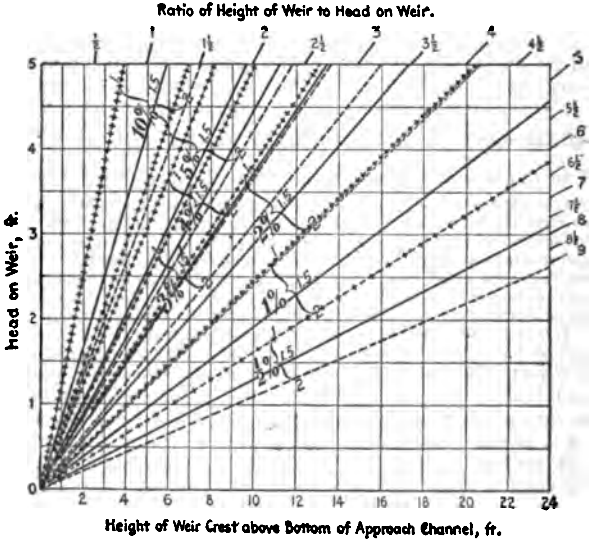

FIG. 20.—Correction Chart for Velocity of Approach at Weirs.

Francis formula, viz.: $Q = 3.33\, l[h + a\,(V^2/2g)]^{3/2}$, in which V is the mean velocity of approach (*i.e.*, $V = Q/A$, where A is the cross-sectional area of the stream in the approach channel upstream from the weir), and a is a factor dependent on the ratio of the head to the height of the crest above the bottom, and also on the distribution of velocities in the approach channel (see below).

For practical purposes a **diagram**, Fig. 20, has been prepared by the writer. This gives the percentage by which results from the Francis formula ($Q = 3.33\ lh^{3/2}$; see Discharge Scale, Fig. 17), are to be increased. The proper percentage may be read off directly when the height of weir crest and the head are known, for values of the factor a of 1.00, 1.50 and 2.00, and for intermediate values by interpolation. For assistance in selecting the proper value for a, a short discussion is given herewith, but it may be stated (see diagram) that when the height of crest above the bottom of the approach channel exceeds 4 times the head, the correction will be about 1 per cent. or less, practically negligible in many cases in view of other uncertainties. A 3 per cent. addition to the discharge corresponds to a ratio of height to head of about 2½ or 3. Only in cases where the head is about as great as the height does the correction reach 10 per cent.

For very low weirs with high heads, the value of the factor a approaches 1.00, while for deep channels with conditions causing high surface velocities upstream from the weir, the value of a is as high as 2.00 or even higher. Most cases lie between these extremes. Basin found average values of a to be 2.21, 1.70, 1.56 and 1.38 for weirs of heights 2.47, 1.64, 1.16 and 0.79 ft., respectively, the heads ranging from 0.3 to 1.4 ft. For **Bazin's formula** the results from the lowest weir were ignored and the value 1.67 used for a, but the velocity of approach is most important for low weirs. **Fteley and Stearns** found values of 1.59, 1.78 and 1.66 for weir heights of 0.50, 1.00 and 1.70 ft., heads from 0.23 to 0.92 ft. They used the value 1.50 for a in their formula, but this employs 3.31 instead of the Francis' 3.33, *i.e.*, for the main coefficient. Based on additional experiments, Fteley and Stearns used 2.05 for a for weirs with two end contractions. The writer's experimental experience covers cases where the value of a is as low as 1.15 and as high as 2.82, the former value for high heads over a low weir, *e.g.*, a 2-ft. head over a weir 1 ft. high, and the latter value for a case of high surface velocity and very low bottom velocity with heads up to 4 ft. over a weir 10 ft. high, as well as many intermediate cases.

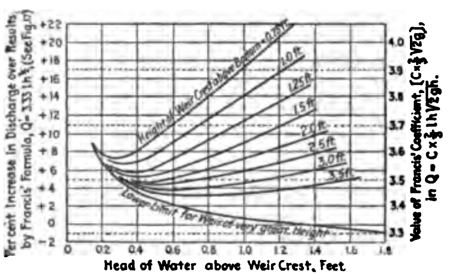

FIG. 21.—Results of Bazin's Experiments on Flow over Sharp-crested Weirs, Expressed in Terms of Francis's Coefficient.

The Fteley and Stearns Formula for sharp-crested weirs without end contractions is $Q = 3.31l[h + 1.5(V^2/2g)]^{3/2} + 0.007l$. For weirs with end contractions they recommended the factor 2.05 instead of 1.5. The units are ft. and cu. ft. per sec. They experimented with weirs from 5 to 19 ft. long (see *ante* for range of heights and heads). The formula is seen to be nearly the same as the modified Francis formula given above. **Hamilton Smith** suggested as average values for the velocity-of-approach multiplier, 1⅓ for weirs without end contraction and 1.4 for weirs with end contraction.

Bazin's Formula for sharp-crested weirs without end contraction is

$$Q = mlh\sqrt{2gh}, \text{ where } m = \left(0.405 + \frac{0.0098}{h}\right)\left[1 + 0.55\left(\frac{h}{p+h}\right)^2\right]$$

This formula eliminates the necessity of a separate velocity-of-approach correction by introducing the height of weir crest above the bottom of the approach channel, p. On this account it has been used to a considerable extent by engineers (see remarks above on the range of Bazin's experiments, etc.). The diagram, Fig. 21, shows the range of the experiments and the results in terms of the Francis coefficient so that discharge computations may be made in connection with the Francis weir discharge diagram, Fig. 17, without the necessity of using a table of Bazin's coefficient, m. It will be seen from the diagram, Fig. 21, that Bazin's formula gives higher discharges for very low heads and lower discharges for heads above 1.3 ft. (for a weir of great height) than does Francis's formula. For the low heads, from 0.1 to 0.3 ft., it is undoubtedly very near to the truth, as numerous recent measurements by several experimenters have shown, thus checking Bazin, Fteley and Stearns, Hamilton Smith and others. The so-called Bazin formula is only an approximation to his precise experimental data. Fig. 21 is prepared from his data, and, excepting the lower limit line, fairly represents Bazin's experiments.

Recommendations for the use of the diagram (Fig. 20) in connection with Francis's formula (see Discharge Scale, Fig. 17): Choose values of the velocity of approach factor a as follows: For cases where the head is greater than the height of crest, 1.00 to 1.3; for intermediate cases, 1.5; for cases where high surface and low bottom velocities prevail upstream from the weir, 2.00 or more. In important work the distribution of velocities should be measured by current meter or Pitot tube (see pp. 282 and 285). In this way one can approximate to the ratio of the mean velocity head for the water above the crest level to that for the full depth of channel. The value of this ratio may be taken as the value of a, with the assurance that the result will be, at least, on a basis firmer than if a compromise formula (*e.g.*, Fteley and Stearns's, Bazin's or Hamilton Smith's) is used for an extreme case. But refinement is unnecessary in a correction that amounts to only a small percentage of the total discharge. Thus, with a 2-ft. head over a weir 3 ft. high, the percentage additions to the Francis formula results are bout 4.5 per cent. for $a = 1.0$, and about 7.5 per cent. for $a = 1.5$. For this ase 6 per cent. would be chosen. The diagram and scale (Figs. 20 and 17) ive results accurate enough for weir measurements.

Flow of Water over Dams varies from about 20 per cent. less to 20 er cent. more than for a sharp crest of the same length and with the same head. For **broad, flat-crested dams,** the flat top wider than the head, the coefficient 2.64, instead of Francis's 3.33, applies, or, with sufficient accuracy, the discharge is 80 per cent. of that given by the Francis formula. If the upstream corner is rounded, the discharge may be greater. **Dams with steeply sloping upstream faces** (about 1 to 1) may have coefficients nearly as high as 4, as will also a thin vertical **bulkhead with a rounded upstream corner** (radius = 2 to 8 in. = thickness of bulkhead). **A very gradually sloping approach,** *e.g.*, 5 horizontal to 1 vertical, or a **rounding crest of large radius** introduces an appreciable friction effect, and the discharge may be no greater than for a sharp-crested weir. Non-vertical upstream faces on sharp-crested dams increase the discharge if inclined downstream and decrease it if inclined upstream, the coefficient being 3.10 for a 1 to 1 upstream and 3.73 for a 2 horiz. to 1 vert. downstream inclination, coefficients for intermediate inclinations being between these values. **If air is not allowed free access under the falling sheet** of water at the crest, the **discharge over** any **narrow-top weir** or dam **is increased,** but is also made less certain due to the tendency of the partial vacuum so formed to break at intervals and cause pulsating flow.

Submerged Weirs and Dams, where the water surface downstream from the dam is at a higher level than the crest, do not show much reduction in discharge, as compared with the unsubmerged condition, until the downstream head, or backwater, is nearly one-half the upstream head above the crest. This is especially true for broad-crested dams, whether flat or rounded or with easy upstream sloping faces, since in these cases there is a considerable surface drop to the water and a decided increase in velocity even before the water leaves the crest. This condition remains practically unchanged with submergence until the downstream backwater head is over half the upstream head, *i.e.*, high enough to smother the flow.

For experimental results on weirs, see "Weir Experiments, Coefficients and Formulas," by R. E. Horton (U. S. Geol. Survey W. S. & I. Paper No. 200).

Flow in Pipes

Flow of Water in Pipes. (See also Pitot Tube, p. 285, and Very Small Tubes, p. 275.) In any pipe or conduit through which water is flowing there is a continuous **loss of head.** This loss of head, also called **friction head,** limits the discharge that a certain reservoir head or pump pressure can cause through a pipe line. Fig. 22 shows a portion of a pipe line in which water is being forced uphill from A toward B. With no flow, *i.e.*, only standing water pressure, the head at B would be h ft. less than at A. But, owing to the flow, there is loss of head and the head at B is $(h + h_f)$ ft. less than at A, where h_f designates the friction loss of head. **The amount of loss of head depends on** (1) the velocity of the water; (2) the roughness of the interior surface of the conduit; (3) the diameter or size of the pipe or conduit; and (4) the length of the conduit. The pressure available at the end of a long pipe and the amount of flow depend largely on the last three items. The pressure of the water in the pipe has no effect, by itself, on the loss of head. For any particular pipe, experimental measurements indicate a law, $h_f = mV^n$, where h_f is the loss of head, V the mean velocity of the water and m and n are constants for a particular pipe of a particular roughness. The loss of head is found to vary directly with the length, and, for simplicity, h_f usually represents the loss of head per 100 or 1000 ft. of length. The value of n depends on the roughness of the interior surface of the pipe and varies from about 1.75 for very smooth pipes to about 2.00 for very rough pipes (see Very Small Tubes, p. 275). The value of m depends on both the diameter and the roughness.

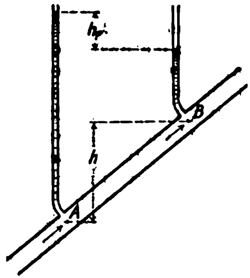

FIG. 22.

In arriving at a reasonably reliable formula for the ordinary pipes of engineering practice, there is considerable difficulty, chiefly for the following reasons: (1) There exists no standard of roughness; (2) the degree of roughness of a pipe's interior surface does not remain constant in service. The **experimental data do not indicate precise coefficients** but show a rather wide range of values from which the investigator must select limits and averages according to his judgment.

In one case a few months' rusting of a 4-in. wrought-iron pipe increased the friction 20 per cent. In another, incrustations due to 17 years' service increased the friction of a 48-in. cast-iron pipe 70 per cent. An unlined linen hose causes over twice the resistance of a high-grade rubber-lined hose. One year's growth of slime in a large aqueduct increased the friction 20 per cent. In these cases the discharges were reduced about half as much as the above-stated percentages for the same head.

Attainable Precision. A pipe, when new, may possibly discharge from 1 to 10 per cent., more or less, than did some other pipe line apparently exactly like it. This difference in performance may be expected to change appreciably after water has been flowing and standing in the pipes for several months, and quite pronouncedly after several years, especially with different waters and in different climates. Nevertheless, data are available that serve to indicate approximately how much such service effects as "a slight layer of rust," "a thin deposit of lime," "a growth of slime," "considerable tuberculation," "heavy incrustation," etc., may be expected to reduce the discharge through a pipe line or increase the loss of head if the same discharge is maintained. The formulæ and diagrams given herewith make due allowances for such of these effects as are stated with them.

Pipe-flow Diagrams. The diagrams in Figs. 23 and 24 give any two of the four quantities: discharge, velocity, diameter and loss of head, when the other two are known or assumed. Any point on the diagram corresponds to the intersection of four lines representing values of these quantities, as indicated. Fig. 23 is for general designing purposes for **average cast-iron and wrought-iron** (not riveted) **pipes in fair interior condition** (not tuberculated). It is based on the formula $h_f = 0.38\, V^{1.86}/d^{1.25}$,* in which h_f is the loss of head in ft. per 1000 ft. of length, V the velocity in ft. per sec. and d the diameter in ft.

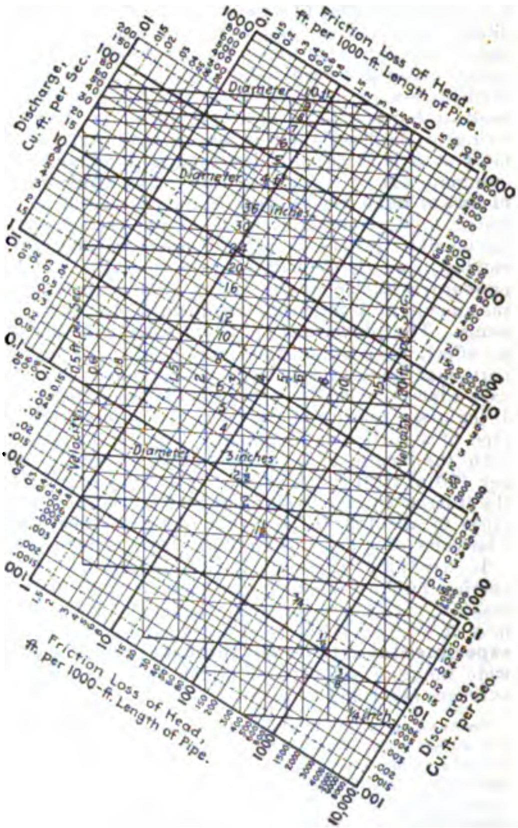

Fig. 23.—Pipe-flow Diagram for Cast-iron and Wrought-iron Pipes in Fair Interior Condition.

Reliability of Diagrams. The diagram probably will not give too high a discharge or velocity for a given friction head and should be within 10 per cent. for a pipe 5 years in service, excepting for very corrosive or highly mineralized waters, and assuming a sufficient velocity to prevent mud deposits. It may be taken as safe for cases where rapid growth of demand or other conditions would probably call for new or additional pipes within about 10 years. To safely assure discharges as great as calculated and friction heads not higher than calculated at the end of some 10 years of service, the diagram in Fig. 24 should be used. For new pipes (not riveted) Fig. 23 will give too low discharges and too high friction heads, *i.e.*, the results are on the safe side. These statements are based on many comparisons with actual experimental measurements covering a wide range of diameters and conditions. The diagram in Fig. 24 is for riveted and rough pipes (see p. 269).

* *Eng. Rec.*, Sept. 3, 1904, p. 281, and *Trans. Am. Soc. C. E.*, vol. 51, 1903, p. 308.

To show how results from these formulæ and diagrams compare with those from the Chézy formula (p. 273), Table 3 gives values of the Chézy coefficient *C* that will produce the same results.

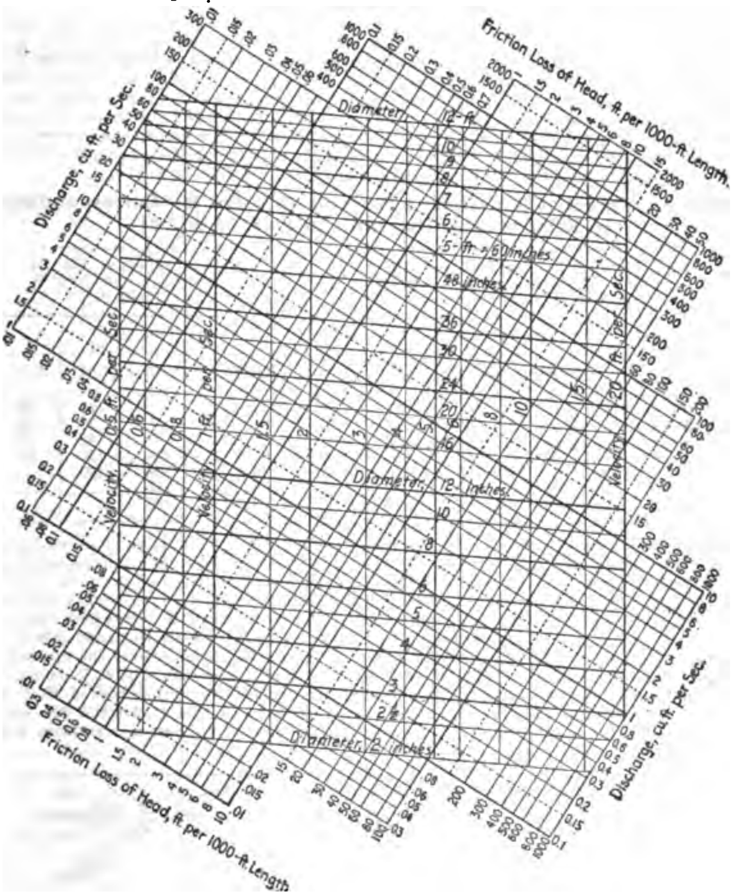

Fig. 24.—Pipe-flow Diagram for Riveted and Rough Pipes, *e.g.*, Cast-iron and Wrought-iron Pipes After 10 Years of Service.

Plain Pipe Problem. A reservoir is to be supplied by a pump through a wrought- or cast-iron pipe line 2800 ft. long containing several elbows, tees, etc., and making the total equivalent length (see p. 275) 3000 ft. The difference of elevation is 100 ft. and the rate of pumping is to be 200 gal. per min. What size of pipe should be laid, and for what working pressure should the pump be specified? Solution: 200 gal. per min. = 0.446 cu. ft. per sec. (see Conversion Scale, Fig. 12). Fig. 23 shows lines for pipes from

$2\frac{1}{2}$ to 12 in. in diam. crossing this discharge value, and the corresponding friction losses of head range from 320 ft. to 0.14 ft. per 1000 ft. of length. The problem, then, is to select a pipe large enough to avoid great friction but not so large as to cause too great a first cost. The friction head for an 8-in. pipe, for $Q = 0.446$ cu. ft. per sec., is seen to be 1 ft. per 1000 ft., or a total of 3 ft. for the 3000 ft. of pipe line. A 6-in. pipe causes about $(4 \times 3 =)$ 12 ft. total loss; a 5-in. pipe $(10 \times 3 =)$ 30 ft.; a 4-in. pipe, $(30 \times 3 =)$ 90 ft., etc. **The pump must force water against a head of 100 ft. plus the friction loss of head.** It would be unwise to use smaller than a 6-in. pipe on account of the large loss of head and consequent wasted work by the pump (see Most Economical Diameter, p. 274). It is seen that, for a given discharge, the loss of head changes much for a small change in diameter. Approximately, h_f varies inversely as d^5 for a given Q; for example, doubling the diameter reduces the loss of head to about $\frac{1}{32}$. See Useful Relations, p. 273, and Percentage Effects, p. 274.

Table 3. Values of C in Chézy's Formula ($V = C\sqrt{Rs}$) Corresponding to Results from Diagrams in Figs. 23 and 24

Value of d in ft.	Formula for diagram, Fig. 23: $h_f = 1000s = 0.38\, v^{1.86}/d^{1.25}$, or $V = 174\, R^{0.67} s^{0.54}$ — Values of s				Formula for diagram, Fig. 24: $h_f = 1000s = 0.50 v^{1.95}/d^{1.25}$ or $V = 117\, R^{0.64} s^{0.51}$ — Values of s			
	0.0001	0.001	0.01	0.1	0.0001	0.001	0.01	0.1
0.1	....	....	77	85	....	....	67	68
0.333	....	86	94	103	....	77	79	80
1	95	104	114	125	88	90	92	94
2	107	117	128	140	97	99	101	103
3	114	125	137	....	102	104	107	....
5	125	137	....	....	110	112	....	....
10	141	154	....	....	121	128	....	....

Compound Pipe Problem. A pipe line is to be made up of sections of several different diameters, the same Q flowing through all. Required to find the discharge Q for assumed diameters and lengths. Solution: Find such portions of the total available head for each pipe as correspond to equal values of Q. Approximate results may be quickly obtained by noting that, for a fixed Q, h_f varies nearly as l/d^5, where l is the length of pipe of a certain diameter. Thus, if the total available friction head is 80 ft., and there are 3000 ft. of 12-in. pipe and 2000 ft. of 8-in. pipe, the ratio of the two friction heads is $(3000/12^5) \div (2000/8^5) = \frac{3}{2} \div (\frac{12}{8})^5 = 0.198$, or, say, $\frac{1}{5}$, i.e., $\frac{1}{5}$ as much loss of head in the 12-in. pipe as in the 8-in. pipe. Hence, the friction head of 80 ft. is divided into 13.3 ft. and 66.7 ft., or, per 1000 ft., into 4.4 and 33.4 ft. for the 12- and 8-in. pipes, respectively. Fig. 23 shows that $Q = 2.8$ cu. ft. per sec. satisfies the friction requirements of both pipes.

Branching Pipe Problem. It is sometimes necessary to design several branching pipes, fed by a single pipe, to discharge certain definite quantities. Such problems are readily solved by trial, using the diagrams. The pressure head where the branching occurs is assumed (or else the loss of head to the branching), and then for the corresponding friction heads the discharges for the several branches are found from the diagrams. The sum of these discharges must, of course, equal the discharge of the main pipe. If it does not, a second trial is made. In Fig. 25 the reservoir A is to supply reservoirs C and D, respectively 10 and 30 ft. lower than A. A–B = 4000 ft., B–C = 2000 ft., and B–D = 5000 ft. What sizes of pipes are necessary to supply 20 cu. ft. per sec. to C and 5 cu. ft per sec. to D? As a trial, assume 5 ft. loss from A to B, leaving 5 ft. for B–C and 25 ft. for B–D, or 2.5 and 5 ft. per 1000 ft., respectively. The discharge Q for A–B is $25 + 5 = 30$ cu. ft. per sec., and the assumed friction head is $\frac{5}{4} = 1.25$ ft. per 1000 ft. On the diagram the intersection of lines for 30 cu. ft. per sec. and 1.25 ft.

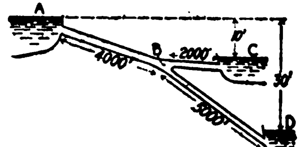

FIG. 25.

friction head gives a diam. of 38 in. Also, 20 cu. ft. per sec. and 2.5 ft. give a diam. of 28 in.; and 5 cu. ft. per sec. and 5 ft. give a diam. of 14 in. These are the desired sizes, but, of course, practically there would be valves located at the reservoirs to adjust the discharges to the needs from time to time, this being a matter for which the design of the pipe sizes does not provide. If it is desired to make the pipes in *A–B* and *B–C* of the same diameter, more loss of head must be allowed from *A* to *B*, say 8 ft. This gives about a 34-in. pipe for both *A–B* and *B–C*, and a 24-in. pipe from *B* to *D*. On the other hand, if it be planned to limit the diameter for *B–D* to 12 in., the loss of head in *B–D* for this diameter and the required discharge (5 cu. ft. per sec.) is first investigated. Fig. 23 shows that the loss of head would be ($12 \times 5 =$) 60 ft., and, since there is only a total head of 30 ft. available, a 12-in. pipe is not large enough for *B–D*.

Looping Pipe Problem. When a pipe branches into two or more pipes which again come together, or are cross-connected (Fig. 26), the arrangement is known as a "loop." In such a case, there is the same total loss of head for each branch, although the loss per 1000 ft. may be different because of different lengths. For example, for fire protection purposes it is desired to have available at *B* a supply of 2500 gal. per min. and 100 lb. per sq. in. hydrant pressure with water flowing. What pressure must be maintained at *A* with this volume flowing and this pressure maintained? Solution: To facilitate computation, note at once that, for a fixed h_f, Q varies nearly as $\sqrt{d^5/l}$. Hence, the ratio of the Q in the 10-in. pipe to that in the 8-in. pipe is about $\sqrt{10^5/1000} + \sqrt{8^5/1800} = \sqrt{1.25^5/0.55}$ = about 2.35. 2500 gal. per min. = 5.57 cu. ft. per sec. (see Conversion Scale, Fig. 12), and dividing this into two parts, one 2.35 times the other, gives 3.91 and 1.66 cu. ft. per sec. In Fig. 23, at the intersection of 3.91 cu. ft. per sec. and 10 in. diam., it is seen that $h_f = 19$ ft. per 1000 ft., and for 1.66 cu. ft. per sec. and an 8-in. pipe, that $h_f = 11$ ft. per 1000 ft., or 20 ft. for the 1800-ft. length. This is a close enough check, and the loss of head from *A* to *B* may therefore be taken as 20 ft., or ($20 \times 0.433 = 8.7$), say 9 lb. per sq. in. drop in pressure, *i.e.*, the pressure at *A* must be 109 lb. per sq. in. If the whole discharge flowed through the 10-in. pipe alone, the loss of head would be 36 ft., or 15.6 lb. per sq. in. drop in pressure (for losses in elbows, tees and hydrant and slight corrections to above, see p. 275).

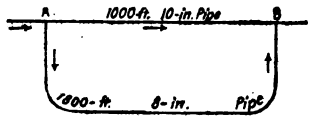

FIG. 26.

Approximate Pipe Formulæ. The well-known **Chézy formula** is $V = C\sqrt{Rs}$, and this is applicable to both pipes and open channels. V is the mean velocity (= discharge ÷ cross-sectional area = Q/A); C is a coefficient; R is the **hydraulic radius** (= cross-sectional area ÷ wetted perimeter = $d/4$ for a circular pipe); s is the **hydraulic slope** [= loss of head (or the surface fall, for open channels) in ft. per ft. of length]. The value of C usually ranges from about 50 to 150 (see Table 3). For offhand estimates, 100 is taken for moderate roughness.

The **Fanning formula** is $h_f = 4f(l/d)(V^2/2g)$ where h_f is the total loss of head, f a friction coefficient, l the length of the pipe and d its diameter, V the mean velocity, and g the acceleration constant of gravity. The value of f usually ranges between 0.004 and 0.010. For offhand estimates, 0.006 is taken for moderate roughness. The Chézy and Fanning formulæ are essentially the same, for $s = h_f/l$ and $R = d/4$, hence $C = \sqrt{2g/f}$ and $f = 2g/C^2$. They are usually accompanied by tables giving values of f and C, which are not constant but vary with V and d. The use of these formulæ, however, is being superseded by the use of those with a constant coefficient and odd decimal exponents.

Useful Relations. From the Fanning formula, $h_f = 4f(l/d)(V^2/2g)$, it is seen that h_f varies as $(l/d)V^2$ if f be assumed constant, which, for a moderate range of values, is approximately true. Introducing into this relation the discharge, Q ($= \pi d^2 V/4$), it is apparent that h_f varies as $(l/d^5)Q^2$. Also, Q varies as $d^{5/2}\sqrt{h_f/l}$, d varies as $Q^{2/5}/(h_f/l)^{1/5}$, and l varies as $h_f d^5/Q^2$. These relations enable one, at first trial, to make close estimates in cases where it is necessary to compare the (unknown) values of Q, h_f, d or l for two or more pipes in terms of the other (known or assumed) quantities.

Percentage Effects due to a change, uncertainty or error in any one factor: The above relations show that, for any pipe line under consideration, the other factors remaining the same, a small percentage change in available friction head causes ½ that per cent. change in the discharge Q, or corresponds to ⅕ that per cent. change in diam. (in opposite way). A small change in Q corresponds to twice that per cent. change in h_f, or ⅖ that in d; in d corresponds to 2½ times that per cent. change in Q, or an opposite change of 5 times that per cent. in h_f; in l corresponds to ⅕ that per cent. change in d, or an opposite change of ½ that per cent. in Q.

The considerable effect of small variations in diameter on the discharge and friction head are thus emphasized. To illustrate, if the diameters of two nominal 6-in. pipe lines are actually 5⅞ and 6⅛ in., a difference of 4 per cent., the discharge of the latter, with the same h_f, will be 10 per cent. more and the friction head, with the same Q, will be 20 per cent. less than the former, due solely to the difference in diameters. If the smaller diameter is in whole or part the result of rusting, tuberculation or incrustation, the increased roughness will still further decrease the discharge or increase the friction head if the same Q is forced through the pipe.

Siphons are arrangements of pipe or hose to cause liquids to flow from one level, *e.g.*, A in Fig. 27, to a lower level C, over an intermediate summit B. Before flow will start the pipe must be filled with the liquid, either by pumping from the end or by pouring in at the summit B. Valves are located, usually, at A and C. B cannot be more than 34 ft. above A (at sea level) *less* the friction head from A to B *less* the velocity head in the pipe. With this limitation calculations are made as for any pipe, the head being the difference in level between A and C. If the pressure at B is low, the dissolved air in the water is liberated and may accumulate in sufficient quantity to stop the flow, unless there is provision for removing it. A low summit, a high velocity and a small pipe tend to counteract the accumulation of air.

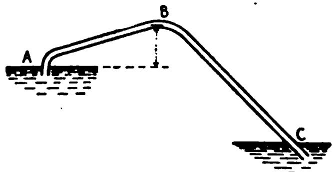

Fig. 27.—Siphon.

The Loss of Head Due to Curves, Elbows, Meters, Etc., may be expressed as equal to the loss in certain additional lengths of straight pipe. When these lengths are expressed in **pipe diameters** instead of in feet, it has been found that the value is nearly enough constant for a certain source of resistance so that a single average value may be used as sufficiently exact for designing purposes. Actually, these resistances are not entirely concentrated in the fitting itself. They create a disturbed, abnormal flow that does not return to normal until some distance downstream. The values given in Table 4 are for the total extra loss.

The Most Economical Diameter of a Pipe Line where pumping is required or where water power is to be delivered, *i.e.*, where it is desired to reduce the friction head as much as possible without increasing too much the diameter and thus the first cost, may be calculated by the following rule:

The sum of the annual value of the power lost by friction and the annual cost of the pipe line must be a minimum. The friction head for each of several diameters delivering the desired volume (in cu. ft. per sec.), may be found from the pipe diagrams, Figs. 23 and 24. The corresponding theoretical wasted h.p. is $Qh_f/8.8$. For pumps this should be increased 20 to 40 per cent., and for a water-power plant it should be reduced 20 per cent. because of the efficiency factor of the machinery. The **annual cost of the pipe line,** or rather that portion of it that would be different for different sizes of pipe, **includes interest** on the cost of pipe and construction **and depreciation.** Obviously, in deciding on the annual value of 1 h.p. and the per cent. annual depreciation in value of a pipe line, economic as well as engineering considerations must be taken into account. (See *Trans. Am. Soc. C. E.*, vol. 59, 1907, pp. 173–194.)

Table 4. Equivalent Resistances of Curves, Elbows, Meters, Etc., Expressed in Lengths of Straight Pipe

Nature of Resistance	Length of straight pipe to cause equal loss of head, expressed in pipe diameters
Square-edged entry. Upstream end of pipe flush with inside face of reservoir wall	20
Entry like Borda's mouthpiece (see Orifices)	40
Rounded entry or very large radius bends	None
90-deg. curves, smooth,* same inside diam. as pipe:	
Center-line radius = diam. of pipe	20
Center-line radius = 2 to 8 diams	10
90-deg. elbows, common screw end, short turn (experiments on ½- to 6-in. ells.)	30
Tees, common screw end, full size branch (experiments on 1- to 4-in. tees)	60
Square elbow (intersection of two cylinders)	50
Water meters †	
Disk, or wobble type	135 to 400
Rotary	400
Reciprocating piston	600
Turbine wheel type	200 to 300

For Venturi water meters, see p. 263, and for hydrants for fire hose, see p. 277.

* *Trans. Am. Soc. C. E.*, vol. 62, 1909, pp. 67–112. Also Bull. No. 403, Univ. of Wis., Madison, Wis.

† Different makes of meters and different sizes of the same make vary considerably.

Flow in Very Small Tubes follows a different law from that for ordinary-size pipes. The friction head varies as the velocity, and is very sensitive to the temperature of the water. The experiments of Poiseuille (1846), Hagen (1854), Reynolds (1883), Coker and Clement (1903), and Saph and Schoder (1903) on glass, brass and lead tubes with internal diameters ranging from 0.001 to 0.631 in. show an average value of about 500 for C in the formula $V = C(h_f/l)d^2(t + 10)/60$, where h_f and l are friction head and length, respectively, both expressed in same units, d the diam. in in., t the temperature of the water in deg. fahr., and V the velocity in ft. per sec. For the larger tubes this formula holds only at very low velocities, viz., the velocity must be less than about $V = 0.37/d^{0.85}$, V in ft. per sec., d in in. This gives roughly the "**critical velocity,**" above which the flow changes from "stream-line" to "eddy" motion and the friction head varies as a higher power of V. The critical velocities from above formula for various diameters are: 1 in., 0.37 ft. per sec.; ½ in., 0.66; ¼ in., 1.2; ⅛ in., 2.2; 1/16 in., 4.0; 1/32 in., 7.1.

Flow of Oil in Pipes. The resistance to the flow of oils lighter than about 30 deg. Baumé, sp. gr., 0.875) is not much different from that of water. The value 96 for C in the Chézy formula ($V = C\sqrt{Rs}$) is used in the oil fields in designing pipe lines for crude oil (42–43 deg. Baumé, sp. gr. 0.814 to 0.819). One per cent. is added for each 3 deg. Baumé, *i.e.*, lighter oils flow more easily. As pumping is universal, diagrams based on the more practical form of the formula, viz., $Q = 1.125\, d^{2.5}\sqrt{p/l}$, are used, in which

Q is expressed in bbl. (of 42 U. S. gal.) per hour and d in in.; p is the necessary pump pressure in lb. per sq. in. and l the length of line in miles. Friction increases with cold oils in winter and decreases in summer. Deposits of paraffin in the pipes conveying crude oil reduce the effective diameter and increase the friction. "Scrapers" are therefore driven through the pipes periodically to clean away the paraffin. Exceedingly viscous asphalt-base oils having a specific gravity of about 0.97 (14 deg. Baumé) may be pumped through a long line by using a "rifled" pipe and mixing 10 per cent. of water with the oil. (*Eng. News*, June 7, 1906, and *Eng. Rec.*, May 23, 1908.)

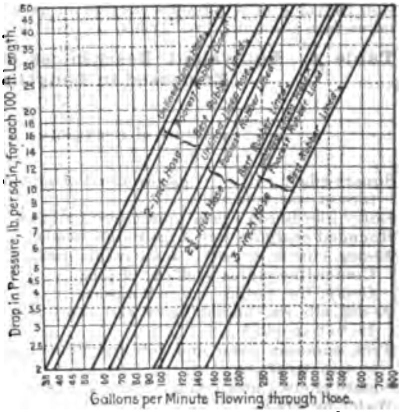

FIG. 28.—Loss of Pressure in 100 Ft. of Fire Hose at Various Rates of Flow.

Fire Hose and Fire-extinguishing Streams. The roughest sort of hose is the unlined linen or cotton woven, and the smoothest is the heavy rubber-lined hose with the fabric interstices so well filled with rubber that the inside remains smooth even under high pressures. A poor grade of rubber lining, or a poor fabric, or both, allow the rubber to be forced into the fabric, when under water pressure, and the interior of the hose may become nearly as rough as if there were no rubber lining at all. In purchasing what purports to be high-grade hose, it should be tested not only for strength to resist bursting but also for loss of head. The diagram in Fig. 28 shows the **loss of pressure** per 100 ft. of length for various rates of flow **in various grades of** 2-, 2½- and 3-in. internal diameter **hose.** Fig. 29 shows the **discharge from** various sizes of **smooth nozzles** for various pressures in the hose close to the nozzle. These diagrams are based on J. R. Freeman's experiments, *Trans. Am. Soc. C. E.*, 1889. Fig. 30 is a diagram showing the ranges, horizontal and vertical, of "good fire streams" and of "average of main body of stream," for various pressures in the hose line at nozzle, By **a good fire stream** is meant one that, at the range in question, has not broken into spray and that projects about three-fourths of the water through a 10-in. circle and nine-tenths of it through a 15-in. circle. The "main body of stream" and the "extreme drops" carry much farther, but the water is too much spread out to be able to penetrate a hot fire and reach the burning materials before evaporating. **A first-class fire stream** is one from a nozzle of at least 1¼ in. tip diameter with at least 40 lb. per sq. in. pressure in the hose

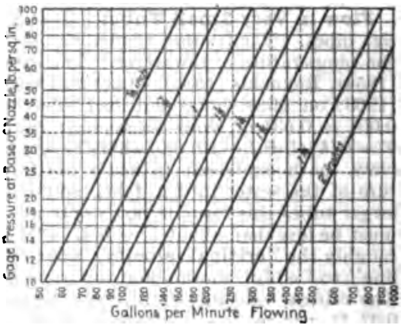

FIG. 29.—Discharge from Fire-hose Nozzles at Various Pressures.

close to the nozzle. For 1¼ in. and 40 lb. the discharge from a smooth nozzle is 240 gal. per min. Such a stream (see Figs. 29 and 30) will be effective as a good fire stream for a height of 60 ft. above the nozzle or for a horizontal distance of about 57 ft., the nozzle being pointed at angles above the horizontal of about 75 deg. and 32 deg., respectively, for farthest throw.

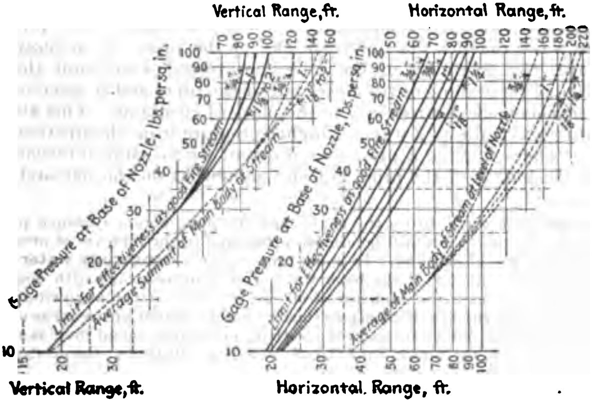

FIG. 30.—Vertical and Horizontal Ranges of Fire Streams.

Extra Losses of Pressure. With as many streams flowing as there are connections on the **hydrant**, the loss of pressure ranges from about 2 to 10 lb. per sq. in., but the total loss from street main to hose connection need not exceed 5 lb. with ample-size waterways in the pipe connections and a well-designed hydrant. **Projecting washers** at the hose couplings cause an extra loss of head; *e.g.*, a single washer 5/16 in. thick with a hole 2.28 in. in diam. used in a 2½ in. hose was found to cause an average loss of 0.56 lb. per sq. in., and a 2-in. washer caused a loss of 3.11 lb. per sq. in., the rate of flow being 240 gal. per min., or a velocity of about 16 ft. per sec. From this an idea may be gained of the resistance caused in small wrought-iron pipes by failure to remove **inside burrs** due to cutting off with wheel cutters. Failure to open fully hydrant valves causes much loss of pressure.

Practical Recommendations for Fire-protection Installations. Hydrants should be located at such intervals that not over 300 ft. of hose need be laid in any one line, and so that about ten first-class fire streams can be concentrated on any large building. The drop will amount to 14 lb. per sq. in. for each 100 ft. of the very best 2½-in. hose, and as high as 30 lb. for a rough quality hose (when 240 gal. per min. are flowing). A main pressure of 100 lb. per sq. in. during a fire is desirable, but, if the hose lines are rough and 500 ft. long, fire protection cannot be assured unless a pressure of 200 lb. is maintained at the hydrant. For important buildings the mains and laterals to hydrants should be at least 6 in. in diameter and as short as possible.

Water Hammer is the **shock produced by** the arrival of a **pressure wave** at an obstruction in a pipe line. The obstruction may be a closed end ("dead end"), the entrance to a smaller pipe, a partially closed valve, etc. The pressure wave may be caused by the operation of a reciprocating pump, by a pressure engine taking water intermittently from a main, by the sudden closure or partial closure of a valve or faucet, or, in general, by any sudden change of pressure at any point of a pipe line full of water. The **pressure wave** so caused **travels** through the water in the pipe at about four times the velocity of sound in air. In the case of a valve or water-wheel gate at the end of a long pipe line with flowing water suddenly closed, wholly or partly, there is a sudden increase of pressure at the place of checking due to the compression of the water suddenly arrested. This increase in pressure is considerable if the rate of checking the flow is rapid and the change of flow is large. The wave or surge of increased pressure travels upstream at high speed, *i.e.*, the flow is not changed instantly throughout the pipe length. When the wave reaches the upstream end of the pipe, *e.g.*, a reservoir or a larger main, the elasticity of the now compressed water and that of the distended pipe force some of the water out of the pipe, and a wave of reduced pressure travels back downstream to the place of checking. This abnormally low pressure wave allows a wave of higher pressure from the upstream end to follow it. There is thus a pressure vibration whose time interval depends chiefly on the length of the pipe, but also somewhat on the size and material of the pipe.

If a high-pressure wave, in its travel through the pipe, enters a branch pipe with a closed, or "dead," end, there will be almost a doubling in the increase of pressure when the wave strikes the closed end. In some pipe systems **dangerous water-hammer pressures** are built up, for, if the back wave from a branch pipe with dead end has access to another branch the high pressure may receive further augmentation. The intensity of the first pressure shock is practically as great farther upstream as at the place of checking. Because of friction and the elasticity of the pipe metal there is some dying away in intensity, but the notion that the effect is concentrated at the downstream end of the arrested column of water is incorrect.

The **intensity of** the **excess pressure** in the "hammer" wave depends on the amount of "extinguished" velocity. Thus, practically the same excess pressure is produced by suddenly reducing the velocity from 7 to 4 ft. per sec. as by entirely stopping a velocity of 3 ft. per sec. If the flow is not checked rapidly, so that the first pressure wave has time to travel upstream to the end and back again several times while the checking is in progress, the excess pressure is very much reduced. Hence, the wisdom of using slow-closing valves on long pipe lines. (See Relief Devices below.)

For **small pipes**, 2 to 6 in. in diameter, Joukovsky's experiments show an **increase of pressure** of about 60 lb. per sq. in. for each ft. per sec. of extinguished velocity. The speed of the pressure wave was found to be about 4200 ft. per sec. For a 24-in. cast-iron pipe the increase of pressure was about 45 lb. per sq. in. for each extinguished ft. per sec., and the speed of transmission about 3300 ft. per sec. These experiments agree well with theory, which gives the formulæ

$p = V\sqrt{Ew/g}$; $S = \sqrt{Eg/w}$, and also $p = V\sqrt{(w/g)\ (EE't)/(tE' + dE)}$; $S = \sqrt{(g/w)\ (EE't)/(tE' + dE)}$. [See Church's "Hydraulic Motors" (J. W. Wiley & Sons), and *Proc. Am. Water Works Assn.*, 1904.] In these formulæ p is the excess pressure intensity and S the speed of transmission of the pressure wave through the water in the pipe. The first two simpler formulæ consider the pipe as perfectly inelastic. The last two formulæ take into account the elasticity of the metal of the pipe. V is the extinguished velocity in ft. per sec., w the weight of 1 cu. ft. of water, $g = 32.2$, E the bulk modulus of elasticity of water = about 300,000 lb. per sq. in., E' the linear modulus of the pipe metal = about 30,000,000 lb. per sq. in. for steel, t the thickness of the pipe metal, and d internal diameter of the pipe. The same system of units should be used throughout. If the ft., lb., sec. system is used, the above values for E and E' must be multiplied by 144.

Relief Devices. Adequately proportioned **air chambers** on pumping mains and surge tanks on water-power supply pipes serve to **absorb** almost entirely the **shock of water hammer.** Relief or safety valves with adjustable springs are not so good for **water-hammer shocks, and are more likely to be out of order.**

Flow in Open Channels

Flow of Water in Open Channels. Open channels may include canals, flumes, rivers, etc., and also any closed conduit, *e.g.*, a large pipe or a tunnel, when it flows only partly full. For open channels, instead of a loss of pressure head in a unit of length, its equivalent, or the surface slope or surface fall of the stream, is taken. The flow in an open channel, if long and of uniform cross-section, tends to adjust itself to a steady uniform flow, so that the surface is parallel to the bottom.

Formulæ: Chézy, Kutter, Bazin. Because of the multitude of different shapes, the dimensions of the cross-section do not appear directly in the general formula, but indirectly in the **hydraulic radius,** or "hydraulic mean depth," R ($= A/w$, *i.e.*, cross-sectional area ÷ wetted perimeter). The **Chézy formula** ($V = C\sqrt{Rs}$) is known as the **Kutter formula** when for C is substituted an expression proposed by Ganguillet and Kutter and depending on the roughness of the channel, the slope and the hydraulic radius; and as the **Bazin formula** when another expression, proposed by Bazin, is used for C.

The Kutter expression for C is $\dfrac{41.6 + (0.0028/s) + (1.81/n)}{1 + [41.6 + (0.0028/s)]\,(n/\sqrt{R})}$ (ft., sec. units), in which n is the **Kutter coefficient of roughness.** The Bazin expression for C is $158/[1 + (N/\sqrt{R})]$, where N is the **Bazin coefficient of roughness.** (See *Eng. News*, Aug. 22, 1912, p. 351, for a comparison of these formulæ.).

Table 5. Values of Kutter's and Bazin's Coefficients of Roughness for Open Channels

	Coefficients Kutter's n	Coefficients Bazin's N
ARTIFICIAL CHANNELS OF UNIFORM CROSS-SECTION:		
Sides and bottom lined with well-planed timber evenly laid	0.009	
Neat cement plaster, smoothest pipes....................	0.010	0.11
Cement plaster (3 cement to 1 sand), smooth iron pipes....	0.011	
Unplaned timber evenly laid, ordinary iron pipes.........	0.012	
Ashlar masonry, best brickwork, well-laid new sewer pipe..	0.013	0.29
(This last value should be used for the previous categories, if in doubt as to the excellence of construction and the maintenance free from slime, rust or other growths and deposits.)		
Average brickwork, foul planks, foul iron pipes, ordinary sewer pipes..	0.015	
Good rubble masonry, concrete laid in rough forms, poor brickwork, heavily incrusted iron pipes..................	0.017	0.83
CHANNELS SUBJECT TO NON-UNIFORMITY OF CROSS-SECTION:		
Excellent clean canals in firm gravel, of fairly uniform section; rough rubble, "dry paving".........................	0.020	1.54
Ordinary earth canals and rivers in good order, free from large stones and heavy weeds.........................	0.025	2.35
Canals and rivers with many stones and weeds............	0.03 to 0.04	3.2

Degree of Roughness. Certainty. Open concrete, masonry, metal or timber flumes can be cleaned periodically, and the removal of slime, weeds, silt deposits, etc., considerably increases their capacity. (See *Eng. News*, Aug. 29, 1912, p. 416.) For earth channels this is not readily done and for pipes it is more difficult and expensive—often impracticable. Open flumes are subject to rapid ice formation in winter in northern latitudes, to upstream retarding wind action, and to very rapid vegetable growths in many localities. These cause a rapid drop in discharging capacity, and should be allowed for by increased cross-section or slope, or both. Channels in earth are also subject to loss of water by seepage, sometimes so considerable as to require special investigation. The state of uniformity of cross-section is as important as mere local roughness. A channel with frequent minute changes of cross-section must be regarded as a rough channel even if the sides and bottom are smooth in appearance.

Erosion. All canals should be designed to have velocities high enough to prevent silt deposits and to retard vegetable growths. Earth channels, however, must not have such high velocities as to scour loose the material of the sides and bottom. Roughly approximate values of eroding velocities, in ft. per sec., are: for clay, ⅓; fine sand, ½; medium sand, 1; coarse sand and fine gravel, 2; gravel, 2 to 4.

Table 6. Values of the Hydraulic Radius, R, for Various Cross-sections
(R = Cross-sectional Area ÷ Wetted Perimeter)

Form of Cross-section:	Value of R
Semi-circle[1]	$r/2$ or $d/4$
Square (depth = width = d)[1]	$d/3$
Half square (width = $2d$, depth = d)[1]	$d/2$
Trapezoidal Channels (bottom width = b; depth = d):	
Half regular hexagon, side slopes 60 deg[1]	$d/2$
Channel with 45 deg. side slopes	$(bd + d^2)/(b + 2.83d)$
Channel with side slopes 1½ hor. to 1 vert	$(bd + 1.5d^2)/(b + 3.61d)$
Channel with side slopes 2 hor. to 1 vert	$(bd + 2d^2)/(b + 4.48d)$
Wide, shallow stream[2]	d (approx.)

[1] Running full. [2] Hence the term "hydraulic mean depth," sometimes used for R.

The Open Channel Flow Diagrams prepared by Prof. I. P. Church* and by Karl R. Kennison† are very convenient for general use, rendering the use of the somewhat complicated formulæ quite simple. For the Kennison diagram, see Fig. 31.

Open Channel Problem. It is necessary to carry 150 cu. ft. of water per sec. in an unplaned timber flume whose width is to be twice the depth of water ($R = d/2$). What are the required dimensions for various slopes of the flume? Solution: For unplaned timber a value of $n = 0.013$ is safe. For assumed values of d = 3, 4, 5, and 6 ft., the cross-sectional areas of stream are 18, 32, 50 and 72 sq. ft. and the mean velocities for 150 cu. ft. per sec. are 8.3, 4.7, 3.0 and 2.1 ft. per sec., respectively. Noting in lower half of Fig. 31 the intersections of the lines for the corresponding values of R (1.5, 2, 2.5 and 3 ft.) with a line interpolated for $n = 0.013$, and looking vertically above for the values of the slopes at the desired velocities, they are found to be 2.9, 0.65, 0.20 and 0.08 ft. per 1000 ft. slope, respectively, for the above assumed sizes. If it is required to find the dimensions of a canal with some desired shape of cross-section for a given discharge and fixed slope, the procedure is to assume trial dimensions, calculate the mean velocity (= Q/area) and see if, on the diagram (at the proper n line vertically below the intersection of lines for assumed velocity and given slope) the value of R agrees closely with the R for the assumed dimensions. If not, new trials are made until there is close agreement.

Measurements of Flow in Open Channels. Where a weir or a dam does not exist, the velocity of flow is generally determined by the use of **floats** or by a **current meter**. **Headgates and water wheels**, considered as orifices, are sometimes used, and are fairly accurate if calibrated for various gate openings by weir, floats or current meter. **Calculation** based on open channel flow formulæ is **sometimes resorted to in** cases **such as extreme flood flows where only high-water marks are left,** in **backwater problems, etc.,** this method being practically the only one available.

* "Diagrams of Mean Velocity of Uniform Motion of Water in Open Channels," (11 diagrams covering the different degrees of roughness from $n = 0.009$ to 0.035), J. Wiley & Sons.

† "A New Kutter Formula Diagram," *Eng. News*, June 20, 1912. Published, 1913, by Karl R. Kennison, 815 Grosvenor Bldg., Providence, R. I., as "Diagrams, Kutter and Bazin Formulæ."

Floats. By observing the time required for floating objects in a stream to pass over a measured distance, a good estimate of the velocity of the water in the path of the float is obtainable. By properly distributing floats across the

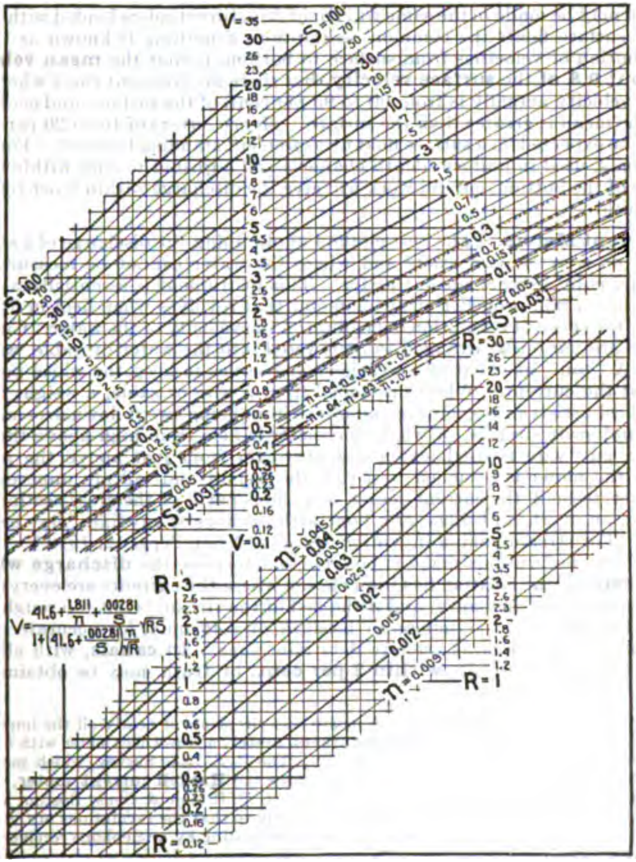

Fig. 31.—Kennison's Open Channel Flow Diagram for the Kutter Formula.
(The units are: *S*, ft. per 1000 ft.; *V*, ft. per sec.; *R*, ft.)

stream, a fair estimate of the total discharge is obtainable if the width and depths have been accurately measured. The use of float measurements is commonly restricted to **surface floats** for rough approximations in reconnaissance, checking other measurements, and in times of high floods when there is too much ice or débris to allow the use of a current meter, and to **rod float measurements** in channels of fairly uniform depth where current meters or weirs are not available. For the surface floats any small pieces of wood will serve for one or two measurements. For more systematic work use 2 × 2 × 12-in. wooden rods, weighted with iron or lead so they will float upright with an inch or so above water. Long rods reaching nearly to the bottom of the stream may be made in the same way, or of 2-in. tin cylinders loaded with sand.

For surface floats the common assumption, if nothing is known as to the distribution of velocities from surface to bottom, is that the **mean velocity is about 0.8 of** the **surface velocity,** but there are frequent cases where the mean velocity actually is from 90 to 95 per cent. of the surface, and occasionally the mean is greater than the surface. Hence, errors of 10 to 20 per cent. are to be expected on a surface float measurement standing by itself. For long rod floats, used in uniform-depth channels and adjusted to sink within a few inches of the bottom, careful work will give the discharge within 5 per cent. of the truth.

Current Meters. The best method of obtaining the discharge of a stream where no weir or dam exists and where the water cannot be measured by volume, is by using a current meter. This instrument is essentially a small wheel like a fan, propeller or anemometer that revolves when held in flowing water, the speed of revolution depending on the velocity of the water. Current meters are variously arranged to be lowered into the water by cable, wire or rod. A "tail" like that of a weather vane serves to keep the wheel pointed against the current. When suspended from a wire or cable, a weight below the wheel assists in preventing the meter from being swept downstream. In deep and swift streams, or where the meter is held from high above the surface, a guy wire is used, extending upstream from just above the meter. When the meter is attached to a rod, the weight and tail are unnecessary. In order to hold the current meter in various parts of the cross-section of a stream, an overhead bridge, or a boat with anchors or guy lines, may be utilized. (For stream gaging in winter, see *Eng. News*, Sept. 12, 1912.)

Careful observations make it possible to determine the **discharge within 5 per cent.** if the location has been chosen where the currents are everywhere fairly parallel to the banks, and if the bed of the stream is not too rough with large boulders, or too indefinite because of soft mud, for accurate depth measurements as well as velocity determinations. **In canals,** with all conditions favorable, results **within 2 per cent.** of truth may be obtained by experienced observers.

The U. S. Geological Survey, which makes measurements of nearly all the important streams in the country, uses the **Price current meter,** a cup-wheel meter with a vertical shaft (W. & L. E. Gurley, Troy, N. Y.). The U. S. Lake Survey, which measures the large rivers connecting the Great Lakes, uses the **Haskell current meter,** a propeller-type instrument having a horizontal shaft (E. S. Ritchie & Sons, Boston, Mass.). Both these current meters are arranged commonly so that the revolutions of the wheel operate a make-and-break in an electric circuit, indicating by a telephone receiver or by a counter register.

A very complete discussion of **stream measurements** is given in Hoyt and Grover's "River Discharge" (J. Wiley & Sons). A new method of **plotting and interpreting data** is given in *Eng. Rec.*, Aug. 3, 1912.

Pressure Due to Deviated Flow

Force or Pressure Due to Deviated Flow. Pressure of a Stream Against Stationary Solids. When a flowing liquid is turned from its course by a body such as a curved vane or a pipe bend, a pressure is exerted on that body. The total force exerted in any direction is equal to the mass of water being deviated, multiplied by the change of velocity in that direction. If the angle turned is B (see Figs. 32 and 33), the pressure or thrust in the direction before the turning is $P_x = (QwV/g)(1 - \cos B)$, where Q is the quantity flowing per sec., w the weight of unit volume of liquid, V the velocity of the liquid, and g the gravity acceleration constant. If A is the cross-section of the stream before turning, the equation may be written $P_x = 2Aw(1 - \cos B)V^2/2g$, thus introducing the velocity head in the flowing liquid. Taking Q in cu. ft. per sec. and V in ft. per sec. the formula becomes, for water, $P_x = 1.94\ QV(1 - \cos B)$. For $B = 90$ deg. and 180 deg., respectively (Figs. 35 and 36), it becomes $P_x = 1.94QV$ and $3.88QV$. The pressure in a direction at right angles to the original direction is $P_y = (QwV/g) \sin B$, or $2Aw \sin BV^2/2g$, or, for water and ft., lb., sec. units, $1.94QV \sin B$. With solids shaped as in Figs. 33 and 37, the components of the pressure in directions other than the original direction neutralize each other due to the divided stream. The **total resultant pressure** due to turning the water aside (P in Fig. 32) is $P = (QwV/g)\sqrt{2(1 - \cos B)}$ and its direction is 90 deg. from midway between the original and final directions of the flowing liquid, *i.e.*, angle C in Fig. 32 $= 90$ deg. $- \frac{1}{2}B$. If a stream impinges squarely against a wall, as in Fig. 34, the effect is the same as it would be in Fig. 33 with $B = 90$ deg.

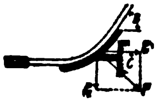
FIG. 32.

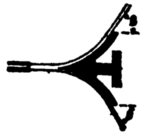
FIG. 33.

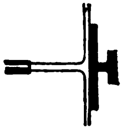
FIG. 34.

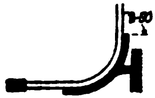
FIG. 35.

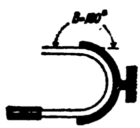
FIG. 36.

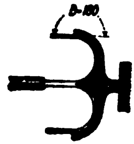
FIG. 37.

GAGES FOR MEASURING PRESSURES, VELOCITIES AND LEVELS

The Open Liquid Column (discussed on p. 256) furnishes the most direct means of measuring and indicating the pressure head or the height of water in tanks, standpipes, boilers, etc. (see Figs. 7 and 50). The arrangement of Fig. 38 may be used for measuring low gas or oil heads, using water instead of mercury.

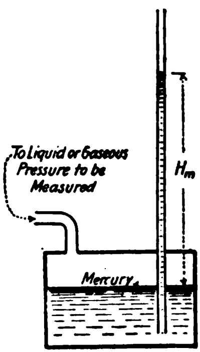

FIG. 38.—Open Liquid Column.

Capillary Attraction. Cohesion, Adhesion and Surface Tension. The exception to the otherwise general statement that the upper surface of a free body of liquid at rest is level, consists in the condition at the edges of the surface area, close to a bounding solid. If the liquid wets the solid (*e.g.*, water and clean glass), it is because there is a greater attraction between the liquid and the solid than between particles of the liquid, or adhesion is stronger than cohesion, and conditions are as shown by Fig. 39 (*a*). On the other hand, if the liquid does not wet the solid (*e.g.*, mercury and glass), cohesion is stronger than adhesion and conditions are as shown by (*b*) Fig. 39. The curved upper surface is called a **meniscus.**

If small-bore glass tubes are used in gages, the effects of capillarity will cause water to stand higher and mercury lower than with large glass tubes, and even with the large tubes there is a curved surface where the liquid touches the glass. For water, the extra height is about $0.046/d$ in., where d is the internal diameter of the tube in in. Thus, if a differential water-air gage (see below) has two glass columns nominally both ⅛ in. in internal diam., but actually one 0.01 in. larger and the other 0.01 in. smaller, the water will stand about 1/16 in. higher in the smaller tube than in the larger, when a common hydrostatic pressure would lead one to expect them to stand at exactly the same level. In experimental work this possible difficulty may usually be avoided by using larger glass tubes, say of ⅜ in. internal diam. **All readings should be taken at the level of the middle of the meniscus,** *i.e.*, the bottom of the curve for water and the top for mercury. These positions are away from the maximum effects of capillary attraction and are nearest to the proper level.

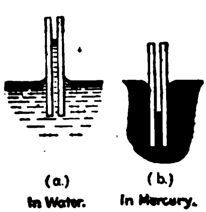

FIG. 39.—Capillary Attraction.

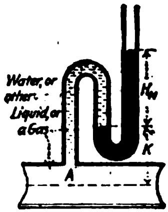

FIG. 40.—Open U-tube Gage.

The Open U-tube Gage (Fig. 40) is much used to measure air, gas and liquid pressures, either above or below atmospheric. Various liquids are used, *e.g.*, mercury, carbon bisulphide colored with iodine, water, kerosene, dilute alcohol, etc. **For gas, air or condenser pressures,** this arrangement will allow pressures ranging from vacuum to several atmospheres to be measured, using mercury; or, using water or kerosene, a more sensitive gage for gas-holder pressures, ventilating pressures, etc., is obtained.

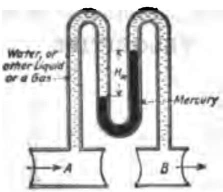

FIG. 41.

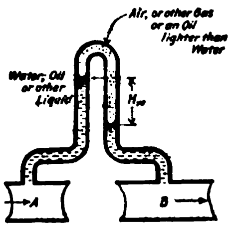

FIG. 42.

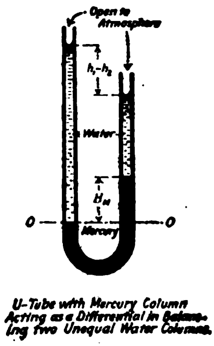

FIG. 43.

FIGS. 41–43.—Differential U-tube Gages.

The Differential U-tube Gage, shown in principle in Figs. 41 and 42, has both branches subject to liquid or gaseous pressures. **The principle of the indications** is shown by Fig. 43, where a liquid column acts as a differential in balancing unequal pressures (equivalent to unequal heads of some other liquid). For the arrangement shown in Figs. 41 and 43, $(h_1 - h_2) = H_M \times [(w_m/w_w) - 1]$, where $(h_1 - h_2)$ is the difference of pressure heads, H_M the gage difference, and (w_m/w_w) the ratio of the heaviness of the liquid in the U of the gage to that of the other liquid. For the case shown in Fig. 42,

$(h_1 - h_2) = Hw[1 - (w_o/w_w)]$, where w_o is the weight per unit volume of the lighter liquid (or of air in the gage, if no second liquid is used).

For mercury and water, $(h_1 - h_2) = 12.57\ H_M$
For carbon bisulphide and water, $(h_1 - h_2) = 0.26\ H_M$
For kerosene and water, $(h_1 - h_2) = 0.21\ H_W$

These ratios vary with the temperature of the liquids.

For gas pressures, unless the pressure is many atmospheres, the value $[(w_m/w_w) - 1]$ may be taken as practically equal to w_m/w_w; and **for the water-air differential gage** (Fig. 42), the value $[1 - (w_o/w_w)]$ may be taken practically equal to 1 in most cases. If the liquid pressure, however, is 100 lb. per sq. in. or over, the value of w_o/w_w will be about 0.01 or more, and the correction then becomes practically appreciable.

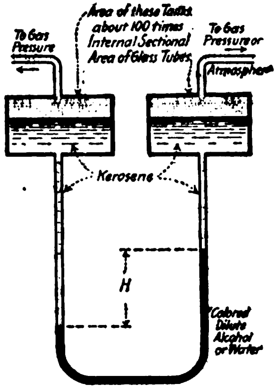

FIG. 44.—Sensitive Differential U-tube Gage.

An Extremely Sensitive Differential (or Open) U-tube Gage for gas pressures is shown in Fig. 44. One of the cisterns may be open to the atmosphere, thus making still another type of **multiplying open U-tube gage.** The heavier liquid may be either water or alcohol diluted with water to make its specific gravity only slightly greater than that of the lighter liquid (kerosene). The alcohol may be colored with an aniline dye that is insoluble in the kerosene. If the specific gravity of the dilute alcohol is 0.83 and that of the kerosene is 0.79, then the difference of gas pressures is $Hw\ (0.83 - 0.79) = 0.04Hw$, where w is the weight of a unit volume of water, *i.e.*, the gage shows a difference 25 times greater than a U-tube with water. A small correction may be necessary for the difference in level of the surfaces in the two cisterns, but by making the cisterns large this correction may be avoided. The glass columns should not be over 3/16 in. in internal diameter to avoid irregular meniscus.

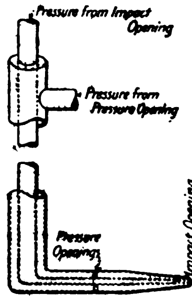

FIG. 45.—Pitot Tube.

The Pitot Tube. See p. 255 and Figs. 6 and 7. Fig. 45 shows one form of Pitot tube. The side "pressure" holes give the pressure head in the water. Such a tube is not readily adapted for insertion through a small hole in the wall of a pipe. Figs. 46 and 47 illustrate representative tubes of more compact type. Such tubes have coefficients less than 1 because of the suction effect at the pressure openings, due to the shape of the tube causing the flowing water to be swerved away from the openings instead of flowing perpendicularly past them as for the side openings of the tube in Fig. 45 or the wall opening in Fig. 7. For tubes as in Fig. 47, $V = 0.84\sqrt{2gh}$, *i.e.*, the indicated water column difference h is about $(1/0.84)^2$ or 42 per cent. greater than the h_v of Fig. 7. Very slight changes in the shape of a compact-type side-pressure-opening tube as in Fig. 46, or in the location of the pressure openings, cause a change in the coefficient of the tube. For Fig. 46, $V = 0.84$ to $0.88\sqrt{2gh}$, depending on location of side openings and shape of tube; for precise work such tubes should be rated. The use of a single-opening tube in connection with a "wall piezometer" (*i.e.*, either a single hole as Fig. 7, a pair of diametrically opposite holes, or a ring piezometer, see p. 263) that transmits only the pressure head of the water, affords the combination of the simplest and most easily constructed instrument with the greatest certainty of indications without necessity of special calibration in case the tube is constructed a little differently from some model. No coefficient is necessary, *i.e.*, $V = \sqrt{2gh}$, for a tube that has its impact opening projecting a little upstream from the body of the tube (see Fig. 48). The pressure holes in the pipe should be about 1 in. upstream from the impact point of the tube, so that the flow past them will be undisturbed by the presence of the tube in the pipe.

For abnormal flow in curved passages, or wherever a wall opening is subject to centrifugal action due to the flow not proceeding parallel to the wall, a two-opening tube is most convenient, as also where many tests in different pipes are to be made.

A sheath, or elongated stuffing box (Fig. 48), into which the point of the tube can be drawn back out of the pipe, is very convenient and is practically essential for high pressures where the flow in the pipe cannot be shut off. A corporation cock (½, ¾, 1 in. or larger, to suit the tube) may be permanently tapped into the pipe. The sheath should have a union coupling adapted for connection to the corporation cock. This gives a shut-off that allows a ready insertion of the tube at any future time. A ¼-in. tap and a gage cock are suitable for the pressure holes when a single-opening tube is used.

FIG. 46.

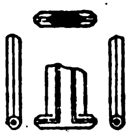

FIG. 47.

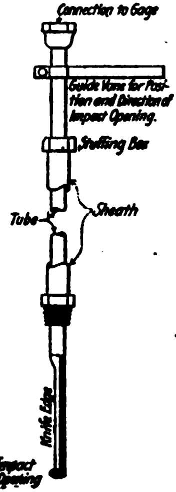

FIG. 48.

FIGS. 46–48.—Pitot Tubes for Insertion in Pipes.

Use of the Pitot Tube in Pipes. The distribution of velocities in the cross-section of a pipe with flowing water is affected by upstream disturbances and by the roughness of the pipe's interior surface. If some 50 diameters' length of straight pipe without tees, valves, etc., extends upstream from the place of measurement, normal distribution prevails. For this the velocity at the wall of the pipe is about one-half the center velocity, and the ratio of the mean velocity to the center velocity, (*the pipe coefficient*), averages about 0.83, the **velocity curve** being approximately a semi-ellipse with the points of mean velocity at about one-fourth the radius from the wall toward the center (see Fig. 49).

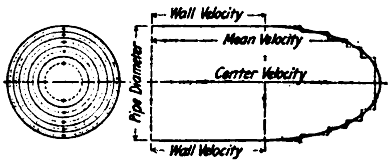

FIG. 49—Distribution of Velocities in a Pipe Cross-section by the 10-point Method.

Discharge Measurements. The Traverse. Various local effects cause departures from the above-stated normal conditions. If the mean velocity deduced from $0.83 \times V_{cen.}$ does not agree with that found at the normal mean velocity points within some 3 per cent., or if the greatest possible precision is required, a complete **traverse** should be made, *i.e.*, the velocity should be found at points distributed along the whole diameter. If this is done for various mean velocities and for two diameters perpendicular to each other, the ratio of mean to center velocities for the particular pipe and cross-section may be determined precisely, and thereafter observations with the Pitot tube set on the center only will suffice for accurate discharge measurements.

The 10-point Method of making a traverse consists in observing with the Pitot tube impact opening set on each of the intersections of a diameter with the circumferences of the odd numbered of 10 concentric circles dividing the cross-section into 10 equal areas. The inner wall of the pipe is the tenth circle (see Fig. 49). Since each point of observation controls one-tenth of the cross-sectional area, it is only necessary to add the calculated velocities and divide by 10 to obtain the mean velocity. The so-calculated mean velocity is theoretically only 0.30 per cent. too high and may be considered practically exact for most measurements. If 20 points are used, the error in summation is theoretically only 0.10 per cent. too high. (See *Proc. A. S. M. E.*, May, 1908, p. 517.)

The Float Gage consists of a hollow float of non-corrosive metal or other substance, either spherical or cylindrical with convex or conical top and bottom. A light, rigid stem extends vertically upward, having an index mark that is guided along the edge of a graduated scale or is attached to an automatic recording mechanism. For cases of **great range of levels**, a graduated tape attached to the float and passing over a pulley or system of pulleys, and with a suitable counterweight, makes a sensitive indicator. The tape need be marked only every foot, if it passes the edge of a fixed auxiliary scale with subdivisions.

For precise work the float should be confined against lateral movement by a vertical pipe with internal diameter ¼ or ½ in. larger than the float. **For the greatest accuracy**, where errors of 0.001 to 0.004 ft. are inadmissible, the float should be made at least 1 in. smaller than the pipe and should be loosely guided (at the stem or by rounded radial fins on the float itself) to keep it away from the pipe so as **to avoid capillary lifting.** To lessen oscillations due to surges and waves, the lower end of the float pipe is capped and one or more small openings are made near the bottom to allow equalization of levels inside and outside. The area of the openings may be some 1 to 3 per cent. of the float-pipe section. For standing liquids in open tanks or reservoirs the float pipe may be placed either inside or outside the container, as is most convenient. For flowing water in open channels it is placed outside the channel and communication is made by a small pipe, which must end perpendicularly to the inside wall of the channel and without projecting into the channel; *i.e.*, its end should be flush with the inside wall, so as to avoid suction effect (see Fig. 50).

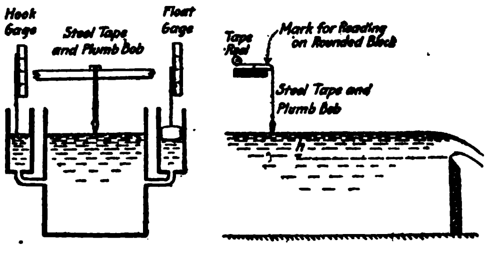

FIG. 50.—Hook, Plumb-bob and Float Gages.

The Hook Gage, Point Gage, Plumb-bob Gage (Fig. 50). These all are devices for locating the height of a liquid surface by observing its contact with the "point" attached to the movable graduated scale of the gage. The point gage has a sharpened point directed downward in place of the hook of the hook gage. For these gages the scale usually is engraved on a square brass rod. A steel or bronze tape is used for the **plumb-bob gage**. The "point" should not be a "needle point," but a conical point with a vertex angle of 90 to 120 deg., so as to produce a maximum of optical effect for slight variations from perfect contact of the point with the liquid surface. The use of a **stilling box**, such as the float pipe above described, is desirable especially for the hook gage (see Fig. 50).

In use, the **hook-gage** point is moved up from beneath until it just pierces the liquid surface. The **point gage** and **plumb-bob gage** are manipulated by lowering from above until a "bubble" shows contact with the liquid surface. If with flowing water or a large surface of a tank or reservoir, where there are waves or surges, and where a stilling box is not available, the gage should be set so that the average time durations during which the point is alternately submerged and exposed shall be as nearly equal as the observer can estimate. Even with a stilling box there may be surges of small amplitude, and several sets of observations may be necessary to assure a determination within 0.001 ft.

The **float gage** is the **best** of the gages mentioned **for sensitiveness and accuracy**. It automatically shows the small variations of water surface level. Whenever there is a perceptible vibration of the water surface, maximum and minimum readings should be taken, corresponding to the crests and roughts of the waves. The average will give the mean level.

SECTION 4

HEAT

BY

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CONTENTS

HEAT

BY

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REFERENCES: Preston, "Theory of Heat," Macmillan. Zeuner, "Technical Thermodynamics," Van Nostrand. Goodenough, "Principles of Thermodynamics," Holt. Marks and Davis, "Steam Tables and Diagrams," Longmans Green. Bryan, "Thermodynamics," Teubner. Peabody, "Thermodynamics of the Steam Engine," Wiley. Lucke, "Engineering Thermodynamics," McGraw-Hill. Bulletins 30, 40, 66, 75, Engineering Experiment Station, Univ. of Illinois.

THERMAL PROPERTIES OF BODIES

The Measurement of Temperature

Thermometers. The scale of the constant-volume **hydrogen thermometer** is taken as the standard temperature scale. Within the limits of ordinary use the scale of the mercury thermometer agrees closely with the standard scale, but above 500 deg. fahr. the divergence between the two scales may be appreciable.

The ordinary **mercury thermometer** may be used to about 600 deg. fahr.; this limit may be extended to 1000 deg. fahr. if the capillary tube above the mercury is filled with nitrogen or carbon dioxide under high pressure. The lower temperature limit for the mercury thermometer is -39 deg. fahr. For lower temperatures, alcohol, pentane or petroleum ether may be used as the thermometric substance. For stem exposure corrections, see "Temperature Measurements," p. 1670. Very high temperatures are measured by various forms of **pyrometers**. See pp. 1671–1674.

Thermometer Scales. Let F and C denote the readings on the Fahrenheit and Centigrade (or Celsius) scales, respectively, for the same temperature; then

$$C = \tfrac{5}{9}(F-32), \qquad F = \tfrac{9}{5}C + 32$$

Table 1 gives corresponding readings on the two scales.

Fixed Temperatures. The following fixed temperatures, adopted by the U. S. Bureau of Standards, are useful in the calibration of pyrometers:

	Deg. fahr.	Deg. cent.
Liquid tin solidifies at	449	232
Liquid lead solidifies at	621	327
Liquid zinc solidifies at	787	419.4
Liquid sulphur boils at	832.5	444.7
Liquid antimony solidifies at	1167	630.5
Liquid aluminum (97.7 per cent. pure) solidifies at	1216	658
Solid gold melts at	1947	1064
Liquid copper solidifies at	1983	1084
Solid nickel melts at	2615	1435
Solid palladium melts at	2815	1546
Solid platinum melts at	3187	1753

High Temperatures and Color. High temperatures may be judged approximately by color, though the estimate of an experienced observer is likely to be 100 deg. fahr. from the true value. The following table associating color and temperature of iron or steel is due to White and Taylor.

	Deg. fahr.		Deg. fahr.
Dark blood red, black red......	990	Orange, free scaling heat.........	1650
Dark red, blood red, low red....	1050	Light orange....................	1725
Dark cherry red...............	1175	Yellow..........................	1825
Medium cherry red............	1250	Light yellow....................	1975
Cherry, full red................	1375	White...........................	2200
Light cherry, light red.........	1550		

Table 1. Conversion of Thermometer Readings

DEGREES CENTIGRADE TO DEGREES FAHRENHEIT

C	F	C	F	C	F	C	F	C	F	C	F
−40	−40.0	+5	+41.0	+40	+104.0	+175	+347	+350	+662	+750	+1382
−38	−36.4	6	42.8	41	105.8	180	356	355	671	800	1472
−36	−32.8	7	44.6	42	107.6	185	365	360	680	850	1562
−34	−29.2	8	46.4	43	109.4	190	374	365	689	900	1652
−32	−25.6	9	48.2	44	111.2	195	383	370	698	950	1742
−30	−22.0	10	50.0	45	113.0	200	392	375	707	1000	1832
−28	−18.4	11	51.8	46	114.8	205	401	380	716	1050	1922
−26	−14.8	12	53.6	47	116.6	210	410	385	725	1100	2012
−24	−11.2	13	55.4	48	118.4	215	419	390	734	1150	2102
−22	− 7.6	14	57.2	49	120.2	220	428	395	743	1200	2192
−20	− 4.0	15	59.0	50	122.0	225	437	400	752	1250	2282
−19	− 2.2	16	60.8	55	131.0	230	446	405	761	1300	2372
−18	− 0.4	17	62.6	60	140.0	235	455	410	770	1350	2462
−17	+ 1.4	18	64.4	65	149.0	240	464	415	779	1400	2552
−16	3.2	19	66.2	70	158.0	245	473	420	788	1450	2642
−15	5.0	20	68.0	75	167.0	250	482	425	797	1500	2732
−14	6.8	21	69.8	80	176.0	255	491	430	806	1550	2822
−13	8.6	22	71.6	85	185.0	260	500	435	815	1600	2912
−12	10.4	23	73.4	90	194.0	265	509	440	824	1650	3002
−11	12.2	24	75.2	95	203.0	270	518	445	833	1700	3092
−10	14.0	25	77.0	100	212.0	275	527	450	842	1750	3182
− 9	15.8	26	78.8	105	221.0	280	536	455	851	1800	3272
− 8	17.6	27	80.6	110	230.0	285	545	460	860	1850	3362
− 7	19.4	28	82.4	115	239.0	290	554	465	869	1900	3452
− 6	21.2	29	84.2	120	248.0	295	563	470	878	1950	3542
− 5	23.0	30	86.0	125	257.0	300	572	475	887	2000	3632
− 4	24.8	31	87.8	130	266.0	305	581	480	896	2050	3722
− 3	26.6	32	89.6	135	275.0	310	590	485	905	2100	3812
− 2	28.4	33	91.4	140	284.0	315	599	490	914	2150	3902
− 1	30.2	34	93.2	145	293.0	320	608	495	923	2200	3992
0	32.0	35	95.0	150	302.0	325	617	500	932	2250	4082
+ 1	33.8	36	96.8	155	311.0	330	626	550	1022	2300	4172
2	35.6	37	98.6	160	320.0	335	635	600	1112	2350	4262
3	37.4	38	100.4	165	329.0	340	644	650	1202	2400	4352
4	39.2	39	102.2	170	338.0	345	653	700	1292	2450	4442

TABLE OF VALUES FOR INTERPOLATION IN THE ABOVE TABLE

Degrees centigrade............	1	2	3	4	5	6	7	8	9
Degrees fahrenheit............	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2

Expansion of Bodies by Heat

Coefficients of Expansion. The **coefficient of linear expansion** of a solid is defined as the increment of length in a unit of length for a rise in temperature of 1 deg. Likewise, the **coefficient of cubical expansion** of a solid, liquid, or gas is the increment of volume of a unit volume for a

Table 1. Conversion of Thermometer Readings.—*Continued*

DEGREES FAHRENHEIT TO DEGREES CENTIGRADE*

F	*C*	*F*	*C*	*F*	*C*	*F*	*C*	*F*	*C*	*F*	*C*
−40	−40.00	+30	−1.11	+80	+26.67	+250	+121.11	+500	+260.00	+900	+482.22
−38	−38.89	31	−0.56	81	27.22	255	123.89	505	262.78	910	487.78
−36	−37.78	32	0.00	82	27.78	260	126.67	510	265.56	920	493.33
−34	−36.67	33	+0.56	83	28.33	265	129.44	515	268.33	930	498.89
−32	−35.56	34	1.11	84	28.89	270	132.22	520	271.11	940	504.44
−30	−34.44	35	1.67	85	29.44	275	135.00	525	273.89	950	510.00
−28	−33.33	36	2.22	86	30.00	280	137.78	530	276.67	960	515.56
−26	−32.22	37	2.78	87	30.56	285	140.55	535	279.44	970	521.11
−24	−31.11	38	3.33	88	31.11	290	143.33	540	282.22	980	526.67
−22	−30.00	39	3.89	89	31.67	295	146.11	545	285.00	990	532.22
−20	−28.89	40	4.44	90	32.22	300	148.89	550	287.78	1000	537.78
−18	−27.78	41	5.00	91	32.78	305	151.67	555	290.55	1050	565.56
−16	−26.67	42	5.56	92	33.33	310	154.44	560	293.33	1100	593.33
−14	−25.56	43	6.11	93	33.89	315	157.22	565	296.11	1150	621.11
−12	−24.44	44	6.67	94	34.44	320	160.00	570	298.89	1200	648.89
−10	−23.33	45	7.22	95	35.00	325	162.78	575	301.67	1250	676.67
− 8	−22.22	46	7.78	96	35.56	330	165.56	580	304.44	1300	704.44
− 6	−21.11	47	8.33	97	36.11	335	168.33	585	307.22	1350	732.22
− 4	−20.00	48	8.89	98	36.67	340	171.11	590	310.00	1400	760.00
− 2	−18.89	49	9.44	99	37.22	345	173.89	595	312.78	1450	787.78
0	−17.78	50	10.00	100	37.78	350	176.67	600	315.56	1500	815.56
+ 1	−17.22	51	10.56	105	40.55	355	179.44	610	321.11	1550	843.33
2	−16.67	52	11.11	110	43.33	360	182.22	620	326.67	1600	871.11
3	−16.11	53	11.67	115	46.11	365	185.00	630	332.22	1650	898.89
4	−15.56	54	12.22	120	48.89	370	187.78	640	337.78	1700	926.67
5	−15.00	55	12.78	125	51.67	375	190.55	650	343.33	1750	954.44
6	−14.44	56	13.33	130	54.44	380	193.33	660	348.89	1800	982.22
7	−13.89	57	13.89	135	57.22	385	196.11	670	354.44	1850	1010.00
8	−13.33	58	14.44	140	60.00	390	198.89	680	360.00	1900	1037.78
9	−12.78	59	15.00	145	62.78	395	201.67	690	365.56	1950	1065.56
10	−12.22	60	15.56	150	65.56	400	204.44	700	371.11	2000	1093.33
11	−11.67	61	16.11	155	68.33	405	207.22	710	376.67	2050	1121.11
12	−11.11	62	16.67	160	71.11	410	210.00	720	382.22	2100	1148.89
13	−10.56	63	17.22	165	73.89	415	212.78	730	387.78	2150	1176.67
14	−10.00	64	17.78	170	76.67	420	215.56	740	393.33	2200	1204.44
15	− 9.44	65	18 33	175	79.44	425	218.33	750	398.89	2250	1232.22
16	− 8.89	66	18.89	180	82.22	430	221.11	760	404.44	2300	1260 00
17	− 8.33	67	19.44	185	85.00	435	223.89	770	410.00	2350	1287.78
18	− 7.78	68	20.00	190	87.78	440	226.67	780	415.56	2400	1315.56
19	− 7.22	69	20.56	195	90.55	445	229.44	790	421.11	2450	1343.33
20	− 6.67	70	21.11	200	93.33	450	232.22	800	426.67	2500	1371.11
21	− 6.11	71	21.67	205	96.11	455	235.00	810	432.22	2550	1398.89
22	− 5.56	72	22.22	210	98.89	460	237.78	820	437.78	2600	1426.67
23	− 5.00	73	22.78	215	101.67	465	240.55	830	443.33	2650	1454.44
24	− 4.44	74	23.33	220	104.44	470	243.33	840	448.89	2700	1482.22
25	− 3.89	75	23.89	225	107.22	475	246.11	850	454.44	2750	1510.00
26	− 3.33	76	24.44	230	110.00	480	248.89	860	460.00	2800	1537.78
27	− 2.78	77	25.00	235	112.78	485	251.67	870	465.56	2850	1565.56
28	− 2.22	78	25.56	240	115.56	490	254.44	880	471.11	2900	1593.33
29	− 1.67	79	26.11	245	118.33	495	257.22	890	476.67	2950	1621.11

TABLE OF VALUES FOR INTERPOLATION IN THE ABOVE TABLE

Degrees fahrenheit	1	2	3	4	5	6	7	8	9
Degrees centigrade*	0.56	1.11	1.67	2.22	2.78	3.33	3.89	4.44	5.00

* All decimals in the table are repeating decimals; 37.78 is really 37.777 . . . . .

Table 2. Coefficients of Expansion

(For pure metals, see p. 521)

Coefficients of Linear Expansion
(Mean values of 10,000a' between 32 and 212 deg. fahr.)

METALS	
Aluminum bronze	0.094
Brass, cast	0.104
Brass, wire	0.107
Bronze	0.100
Constantan (60 Cu, 40 Ni)	0.095
German silver	0.102
Iron:	
Cast	0.059
Soft forged	0.063
Wire	0.080
Magnalium (86 Al, 13 Mg)	0.133
Phosphor bronze	0.094
Solder	0.134
Speculum metal	0.107
Steel:	
Bessemer, rolled hard	0.056
Bessemer, rolled soft	0.063
Nickel (10% Ni)	0.073
Type metal	0.108
OTHER MATERIALS	
Brick	0.031
Caoutchouc	0.372
Carbon—coke	0.030
Cement, neat	0.060
Concrete	0.080
Ebonite	0.468
Glass:	
Thermometer	0.045
Hard	0.033
Plate and crown	0.050
Granite	0.048
Graphite	0.044
Gutta percha	1.100
Limestone	0.014
Marble	0.065
Masonry	0.025 to 0.050
Porcelain	0.017
Rubber	0.428
Vulcanite	0.400
Wood (∥ to fiber):	
Ash	0.053
Chestnut and maple	0.036
Oak	0.027
Pine	0.030
Across the fiber:	
Chestnut and pine	0.019
Maple	0.027
Oak	0.030

Coefficients of Cubical Expansion
(Mean values of 1000a''' at ordinary room temperatures)

LIQUIDS	
Acetic acid	0.80
Alcohol (ethyl)	0.61
Alcohol (methyl)	0.80
Benzene	0.77
Benzol	0.70
Calcium chloride ($CaCl_2$); 5 to 50% solution	0.28
Chloroform	0.77
Ether	1.20
Glycerine	0.28
Hydrochloric acid	0.27
Hydrochloric acid, 50% solution	0.52
Mercury	0.10
Olive oil	0.41
Petroleum	0.55
Phenol (C_6H_6O)	0.50
Rape-seed oil	0.50
Sodium chloride:	
1.6% solution	0.60
26% solution	0.24
Sulphuric acid	0.27
Sulphuric acid, 50% solution	0.45
Turpentine	0.56
Water	0.10
SOLIDS	
Fluorspar	0.035
Ice (4 to 30 deg. fahr.)	0.62
Paraffin wax	0.61
Rock salt	0.67
Sulphur	0.40
Wood (beech)	0.016
Wood (pine)	0.028

rise of temperature of 1 deg. Denoting these coefficients by a' and a''', respectively,

$$a' = \frac{1}{l}\frac{dl}{dt} \qquad a''' = \frac{1}{V}\frac{dV}{dt}$$

in which l denotes length, V volume, and t temperature. For homogeneous solids $a''' = 3a'$, and the **coefficient of superficial expansion** $a'' = 2a'$.

The coefficients of expansion are in general dependent upon the temperature, but for ordinary ranges of temperature constant mean values may be taken. If lengths, areas, and volumes at 32 deg. fahr. (0 deg. cent.) be taken as standard, then these magnitudes at other temperatures t_1 and t_2 are related as follows:

$$\frac{l_1}{l_2} = \frac{1+a't_1}{1+a't_2} \qquad \frac{A_1}{A_2} = \frac{1+a''t_1}{1+a''t_2} \qquad \frac{V_1}{V_2} = \frac{1+a'''t_1}{1+a'''t_2}$$

Since for solids and liquids the expansion is small, the preceding formulæ for these bodies become approximately

$$l_2 - l_1 = a'l_1(t_2 - t_1). \quad A_2 - A_1 = a''A_1(t_2 - t_1). \quad V_2 - V_1 = a'''V_1(t_2 - t_1)$$

For certain metals the variation of the coefficient of expansion with temperature has been investigated by Holborn and Day and by Dittenberger. Denoting by l_0 the length at 32 deg. fahr., and by l the length at temperature t, the following relation is obtained.

$$l = l_0 \left[1 + a\left(\frac{t-32}{1000}\right) + b\left(\frac{t-32}{1000}\right)^2\right]$$

The following table gives values of the constants.

Metal	1000 a	1000 b	Temperature range, deg. fahr.
Aluminum	13.380	2.182	32–1130
Cast iron	5.441	1.747	32–1160
Ingot iron	6.375	1.636	32–1380
Malleable iron	6.503	1.622	32– 930
Ingot steel	6.212	1.623	32–1380
Copper	9.278	1.244	32–1160
Nickel	7.652	1.023	32–1830

The tensile or compressive **stress set up** in a prismatic bar by a **temperature change** of t deg. is $P = a'EAt$, in which E denotes the modulus of elasticity and A the area of the cross-section.

The **linear shrinkage of castings** is approximately as follows:

Bar iron, rolled	1:55	Cast iron	1:96	Steel, puddled	1:72
Bell metal	1:65	Gun metal	1:134	Steel, wrought	1:64
Bismuth	1:265	Iron, fine grained	1:72	Tin	1:128
Brass	1:65	Lead	1:92	Zinc, cast	1:62
Bronze	1:63	Steel castings	1:50	8 Cu + 1Sn (by wt.)	1:184

Table 3. Volume, Density and Specific Heat of Water at Saturation Pressure

(From Marks and Davis's Steam Tables)

Temp., deg. fahr.	Pressure, lb. per sq. in.	Specific volume, cu. ft. per lb.	Density, lb. per cu. ft.	Specific heat	Temp., deg. fahr.	Pressure, lb. per sq. in.	Specific volume, cu. ft. per lb.	Density, lb. per cu. ft.	Specific heat
20	0.06	0.01603	62.37	1.0168	240	24.97	0.01692	59.11	1.012
30	0.08	0.01602	62.42	1.0098	250	29.82	0.01700	58.83	1.015
40	0.12	0.01602	62.43	1.0045	260	35.42	0.01708	58.55	1.018
50	0.18	0.01602	62.42	1.0012	270	41.85	0.01716	58.26	1.021
60	0.26	0.01603	62.37	0.9990	280	49.18	0.01725	57.96	1.023
70	0.36	0.01605	62.30	0.9977	290	57.55	0.01735	57.65	1.026
80	0.51	0.01607	62.22	0.9970	300	67.00	0.01744	57.33	1.029
90	0.70	0.01610	62.11	0.9967	310	77.67	0.01754	57.00	1.032
100	0.95	0.01613	62.00	0.9967	320	89.63	0.01765	56.66	1.035
110	1.27	0.01616	61.86	0.9970	330	103.0	0.01776	56.30	1.038
120	1.69	0.01620	61.71	0.9974	340	118.0	0.01788	55.94	1.041
130	2.22	0.01625	61.55	0.9979	350	135.0	0.01800	55.57	1.045
140	2.89	0.01629	61.38	0.9986	360	153.0	0.01812	55.18	1.048
150	3.71	0.01634	61.20	0.9994	370	173.0	0.01825	54.78	1.052
160	4.74	0.01639	61.00	1.0002	380	196.0	0.01839	54.36	1.056
170	5.99	0.01645	60.80	1.0010	390	220.0	0.01854	53.94	1.060
180	7.51	0.01651	60.58	1.0019	400	247.0	0.0187	53.5	1.064
190	9.34	0.01657	60.36	1.0029	410	276.0	0.0189	53.0	1.068
200	11.52	0.01663	60.12	1.0039	420	308.0	0.0190	52.6	1.072
210	14.13	0.01670	59.88	1.0050	430	343.0	0.0192	52.2	1.077
220	17.19	0.01677	59.63	1.007	440	381.0	0.0194	51.7	1.082
230	20.77	0.01684	59.37	1.009					

Density of Liquefied Gases at Various Temperatures

Gas	Temp., deg. fahr.	Density, lb. per cu. ft.	Gas	Temp., deg. fahr.	Density, lb. per cu. ft.	Gas	Temp., deg. fahr.	Density, lb. per cu. ft.
Hydrogen	−432.0	4.82	Carbon monoxide	−337.0	54.62	Carbon dioxide (see also p. 337).	−110*	97.74
	−422.5	4.47		−310.0	50.66		−76	76.08
Argon	−301.0	89.44		−301.0	49.32		−22	68.35
Oxygen	−346.0	81.45	Acetylene	−11.2	33.15		32	59.09
	−295.6	72.95		68.0	25.55		68	49.32
	−247.0	62.28	Chlorine	−20.8	99.34	Sulphur dioxide.	−22	96.40
Nitrogen	−337.0	54.55		32.0	93.84		32	91.67
	−321.0	51.93	Ammonia (see also p. 333).	−22.0	42.93		86	86.62
	−301.0	48.68		32.0	40.76		212	70.91
Ethylene	−5.8	26.45		86.0	38.14			
	42.8	18.90		212.0	29.70			

* Solid.

Specific Heat

Units of Heat. The mean **British thermal unit** (B.t.u.) is defined as the $\frac{1}{180}$ part of the heat required to raise the temperature of 1 lb. of water from 32 deg. to 212 deg. fahr. This is substantially equal to the heat required to raise 1 lb. of water from 63 deg. to 64 deg. fahr.

The **mean calorie** is $\frac{1}{100}$ of the heat required to raise the temperature of 1 g. of water from 0 deg. to 100 deg. cent. It is practically the same as the 17½-deg. calorie, that is the heat required to raise 1 g. of water from 17 deg. to 18 deg. cent. The 15-deg. calorie is also used extensively. Because of the variation of the heat capacity of water, this is slightly larger than the mean or 17½-deg. calorie. The present tendency is toward the mean calorie (and mean B.t.u.) as the standard heat unit.

In countries which have adopted the metric system, engineers employ the **kilogram calorie** (or "large calorie") as the unit in heat measurements. 1 kilogram calorie = 1000 g. calories = 3.968 B.t.u. (1 B.t.u. = 0.252 kilogram calorie). This is the "wärmeeinheit" (WE) mentioned in German texts.

Specific Heat. The specific heat of a substance is the ratio of the heat required to raise the temperature of unit weight of the substance 1 deg. to the heat required to raise the temperature of unit weight of water 1 deg. at some specified temperature. The specific heat is thus numerically equal to the quantity of heat required to raise the temperature of a unit weight of the substance by 1 deg.

Denoting by c the specific heat, the heat required to raise the temperature of M lb. of a substance from t_1 to t_2 is $Q = Mc(t_2 - t_1)$, provided c is a constant.

In general, c varies with the temperature, though for moderate temperature ranges a constant mean value may be taken. If, however, c is taken as variable, then $Q = M\int_{t_1}^{t_2} c\,dt$. The *mean* specific heat from 0 deg. to t deg. is given by $c_m = \frac{1}{t}\int_0^t c\,dt$. If $c = a_1 + a_2t + a_3t^2 + \ldots$,

$$c_m = a_1 + \tfrac{1}{2}a_2t + \tfrac{1}{3}a_3t^2 + \ldots$$

Specific Heat of Water. The specific heat of water between 32 deg. and 212 deg. fahr. has been investigated by Barnes, Ludin, Dieterici, and others (see Marks and Davis's "Steam Tables," p. 88). For this range of temperatures, two distinct values of the specific heat may be noted, according as (1) the pressure on the water is the saturation pressure corresponding to the temperature, or (2) constant atmospheric pressure. In Table 3 the values of the specific heat are according to the first system. For a full discussion of this topic see Marks and Davis's "Steam Tables," p. 90.

Table 4. Mean Specific Heats of Various Solids and Liquids Between 32 and 212 Deg. Fahr.

(For gases, see p. 316; for pure metals, p. 521)

Substance	Specific heat
SOLIDS	
Alloys:	
Bismuth-tin	0.040–0.045
Bell metal	0.086
Brass, yellow	0.0883
Brass, red	0.090
Bronze	0.104
Constantan	0.098
D'Arcet's metal	0.050
German silver	0.095
Lipowitz's metal	0.040
Nickel steel	0.109
Rose's metal	0.050
Solders (Pb and Sn)	0.040–0.045
Type metal	0.0388
Wood's metal	0.040
40 Pb + 60 Bi	0.0317
25 Pb + 75 Bi	0.030
Asbestos	0.20
Ashes	0.20
Basalt (lava)	0.20
Borax	0.229
Brick	0.22
Carbon–coke	0.203
Chalk	0.215
Charcoal	0.20
Cinders	0.18
Coal	0.314
Concrete	0.27
Cork	0.485
Corundum	0.198
Dolomite	0.222
Ebonite	0.33
Glass:	
Normal	0.199
Crown	0.16
Flint	0.12
Gneiss	0.18
Granite	0.195
Graphite	0.201
Gypsum	0.259
Hornblende	0.195
Humus (soil)	0.44
Ice	0.504
India rubber (Para)	0.27–0.48
Kaolin	0.224
Limestone	0.217
Marble	0.210
Oxides:	
Alumina (Al_2O_3)	0.183
Cu_2O	0.111
Lead oxide (PbO)	0.055
Lodestone	0.156
Magnesia	0.222
Magnetite (Fe_3O_4)	0.168
Silica	0.191
Soda	0.231
Zinc oxide (ZnO)	0.125
Paraffin wax	0.69
Porcelain	0.255
Quartz	0.17–0.28
Quicklime	0.217
Salt, rock	0.21
Sand	0.195
Sandstone	0.22
Serpentine	0.25
Sulphur	0.180
Talc	0.209
Tufa	0.33
Vulcanite	0.331
Wood:	
Fir	0.65
Oak	0.57
Pine	0.67
LIQUIDS	
Acetic acid	0.51
Alcohol (absolute)	0.70
Aniline	0.514
Benzol	0.43
Chloroform	0.23
Ether	0.503
Fusel oil	0.56
Gasoline	0.70
Glycerine	0.576
Hydrochloric acid	0.60
Kerosene	0.50
Naphthalene	0.31
Machine oil	0.40
Mercury	0.033
Olive oil	0.35
Paraffin oil	0.52
Petroleum	0.498
Sulphuric acid	0.336
Sea water	0.94
Toluene	0.40
Turpentine	0.472
Molten metals:	
Bismuth (535–725 deg. fahr.)	0.036
Lead (590–680 deg. fahr.)	0.041
Sulphur (246–297 deg. fahr.)	0.235
Tin (460–660 deg. fahr.)	0.058

Table 5. Mean Specific Heat of Iron (c_m) Between 32 and t Deg. Fahr. (Oberhoffer)

t	600	800	1000	1200	1400
c_m	0.127	0.133	0.139	0.148	0.167
t	1600	1800	2000	2250	2500
c_m	0.170	0.169	0.168	0.167	0.167

Table 6. Specific Heat of Salt (NaCl) Solutions

Parts of NaCl by weight in 100 parts of the solution	Temperature, deg. fahr.										
	0	15	32	50	60	70	80	90	100	120	140
0	......	......	1.006	1.002	1.000	0.999	0.998	0.999	1.000	1.001	1.003
2	......	......	0.966	0.969	0.971	0.973	0.976	0.978	0.980	0.983	0.986
4	......	......	0.944	0.947	0.949	0.951	0.954	0.956	0.958	0.961	0.964
6	......	......	0.923	0.927	0.929	0.930	0.932	0.934	0.936	0.939	0.942
8	......	......	0.904	0.907	0.909	0.910	0.912	0.914	0.916	0.919	0.922
10	......	......	0.885	0.889	0.891	0.892	0.894	0.896	0.899	0.901	0.903
12	......	......	0.869	0.872	0.874	0.875	0.877	0.879	0.881	0.883	0.885
14	......	0.851	0.854	0.857	0.859	0.860	0.861	0.863	0.865	0.867	0.870
16	......	0.838	0.840	0.843	0.844	0.845	0.847	0.849	0.851	0.853	0.855
18	......	0.825	0.827	0.830	0.831	0.832	0.833	0.835	0.837	0.839	0.841
20	......	0.812	0.814	0.816	0.817	0.818	0.820	0.822	0.823	0.824	0.827
22	0.795	0.798	0.801	0.804	0.805	0.806	0.807	0.809	0.810	0.812	0.814
24	......	0.787	0.789	0.791	0.792	0.793	0.794	0.796	0.798	0.799	0.801
26	......	0.776	0.778	0.780	0.781	0.781	0.782	0.784	0.785	0.787	0.788

The **specific heat of salt solutions** increases slowly with rising temperature. Table 6, due to Gröber, gives the specific heat of solutions of NaCl for various temperatures and concentrations. The values are probably more accurate than those given by Siebel in Table 11.

Temperature of Mixtures

The temperature of a mixture of weight $M_1 + M_2$ lb. consisting of M_1 lb. of a substance at temperature t_1 and with specific heat c_1 and M_2 lb. of a substance at temperature t_2 with a specific heat c_2, is given by the equation

$t_m = (M_1c_1t_1 + M_2c_2t_2)/(M_1c_1 + M_2c_2)$. In general, $t_m = \Sigma Mct/\Sigma Mc$.

If it be required to raise the temperature of M_1 lb. of a substance having the specific heat c_1 from the temperature t_1 to t_m, the weight M_2 required of a second substance at temperature t_2 and having a specific heat c_2 is

$$M_2 = M_1c_1(t_m - t_1)/c_2(t_2 - t_m)$$

If M_1 lb. of a gas (corresponding to V_1 cu. ft.) at t deg. be mixed with M_2 lb. of the same gas (corresponding to V_2 cu. ft.) the pressure being constant, then the temperature of the mixture is $t_m = (M_1t_1 + M_2t_2)/(M_1 + M_2) = \{(V_1 + V_2)/[(V_1/T_1) + (V_2/T_2)]\} - 459.6$, in which T_1 and T_2 are the absolute temperatures.

Changes of State Effected by Heat

Table 7. Freezing Mixtures

(The lowest temperature that may be produced by a freezing mixture is the freezing point of the solution)

Mixtures	Composition, parts by weight	Reduction in temperature, deg. fahr. from	to
Sodium phosphate Sal ammoniac Diluted nitric acid	9 6 4	34.3	21.2
Sodium sulphate Ammonium nitrate Diluted nitric acid	6 5 4	50.0	−13.0
Sal ammoniac Saltpeter Water	5 5 16	50.0	10.4
Sodium carbonate Ammonium nitrate Water	1 1 1	50.0	6.8
Sodium sulphate Diluted nitric acid	3 2	50.0	−2.2
Sodium sulphate Sal ammoniac Saltpeter Nitric acid	6 4 2 4	50.0	−9.4
Ammonium nitrate Water	1 1	50.0	3.2
Sodium phosphate Diluted nitric acid	9 4	59.0	15.8
Potassium hydrate Snow	4 3	32.0	−38.3
Sulphuric acid Nitric acid Snow	1 1 2	−2.2	−40.0
Sodium chloride Snow	1 3	32.0	−1.6
Saltpeter Sal ammoniac Water	1 1 1	46.4	−11.2
Diluted nitric acid Snow	1 1	57.2	−31.0
Sodium sulphate Hydrochloric acid	8 5	50.0	−0.4
Sodium sulphate Saltpeter Sal ammoniac Water	8 5 5 16	50.0	5.0
Sodium sulphate Diluted nitric acid	5 4	50.0	3.2
Sodium chloride Snow	1 1	32.0	−0.4
Diluted sulphuric acid Snow	1 1	23.0	−41.8
Calcium chloride Snow	3 2	32.0	−27.4
Calcium chloride Snow	2 1	32.0	−43.6

Table 8. Melting Points of Various Solids, Deg. Fahr.

(For pure metals, see p. 521; for refractories see p. 625)

Alloys:		18 Bi + 36 Pb + 46 Sn	305	Zinc	504.0
Bismuth solder	200-262	10 Bi + 40 Pb + 50 Sn	324	Enamel colors	1760.0
Brass and bronze (about)	1650	Tin solder	275-350	India rubber	257.0
80 Cu + 20 Zn	1845	Blast-furnace slag	2370-2600	Paraffin	129.2
50 Cu + 50 Zn	1615	Borax	1040	Phosphorus	111.2
20 Cu + 80 Zn	1300	Cast iron, gray	2200	Porcelain	2820.0
Delta metal	1742	Cast iron, white	2070	Potassium	143.6
20 Sn + 80 Pb	530			Sodium	204.8
50 Sn + 50 Pb	400	Chlorides:		Spermaceti	120.2
80 Sn + 20 Pb	388			Stearine	122.0
Fusible alloys:		Calcium	1330	Steel	2370-2550
		Potassium	1350	Wrought iron	2460-2640
33 Bi + 33 Pb + 33 Sn	250	Sodium	1422		

Table 9. Melting Points of Alloys

(Smithsonian Physical Tables; see also p. 534)

Alloys of Lead, Tin and Bismuth

	Per cent.									
Lead	32.0	25.8	25.0	43.0	33.3	10.7	50.0	35.8	20.0	70.9
Tin	15.5	19.8	15.0	14.0	33.3	23.1	33.0	52.1	60.0	9.1
Bismuth	52.5	54.4	60.0	43.0	33.3	66.2	17.0	12.1	20.0	20.0
Solidifies at (deg. fahr.)	204.8	213.8	257.0	262.4	293.0	298.4	321.8	357.8	359.6	453.2

Low-melting-point Alloys

(See also Table of Fusible Alloys, p. 552)

	Per cent.						
Cadmium	10.8	10.2	14.8	13.1	6.2	7.1	6.7
Tin	14.2	14.3	7.0	13.8	9.4	....	....
Lead	24.9	25.1	26.0	24.3	34.4	39.7	43.4
Bismuth	50.1	50.4	52.2	48.8	50.0	53.2	49.9
Solidifies at (deg. fahr.)	150.0	153.5	155.3	155.3	169.7	194.0	203.0

Table 10. Freezing Points of Liquids at Atmospheric Pressure

(Deg. fahr.)

Alcohol (absolute)	−148.0	Calcium chloride (sat. sol.)	− 40	Rape-seed oil	25.7
Ammonia	−108.4	Ether	−180	Turpentine	14.0
Aniline	21.2	Glycerine	− 4	Sulphuric acid	−105
Benzol	41.0	Naphthalene	176	Salt (NaCl) sol., sat.	− 0.4
Carbon bisulphide	−171.0	Linseed oil	− 4	Sea water	27.5
Carbon dioxide	−110.2			Toluene	−148
Chloroform	− 83.0				

Mixtures of glycerine and water (Bolley)

Per cent. by weight of glycerine	Specific gravity	Freezing point, deg. fahr.
10	1.0245	30.2
20	1.0498	27.5
30	1.0771	20.8
40	1.1045	1.0
45	1.1183	−15.2
50	1.1320	−25.6
60	1.1582	Below −31.0

Mixtures of alcohol and water (F. Beilstein)

Per cent. by weight of alcohol	Freezing point, deg. fahr.	Per cent. by weight of alcohol	Freezing point, deg. fahr.
2.58	30.2	21.7	10.4
5.22	28.4	23.8	6.8
7.36	26.6	26.0	3.2
9.58	24.8	28.0	− 0.4
11.50	23.0	30.0	− 4.0
13.27	21.2	33.5	−11.2
16.53	17.6	37.3	−18.4
19.09	14.0	41.2	−25.6

Table 11. Freezing Point, Density, and Specific Heat of Salt Solutions

CALCIUM CHLORIDE SOLUTIONS
(U. S. Bureau of Standards)

Per cent. of $CaCl_2$ by weight	Specific Gravity at 39 deg. fahr.	Freezing point, deg. fahr.	Specific heat at 32 deg. fahr.	Per cent. of $CaCl_2$ by weight	Specific Gravity at 39 deg. fahr.	Freezing point, deg. fahr.	Specific heat at 32 deg. fahr.
14.88	1.12	15.8	0.799	23.03	1.20	−11.2	0.714
16.97	1.14	8.6	0.775	24.89	1.22	−20.2	0.690
19.07	1.16	3.2	0.753	26.77	1.24	−29.2	0.679
21.13	1.18	− 4.0	0.733	28.55	1.26	−40.0	0.668

SODIUM CHLORIDE SOLUTIONS
(From Siebel's "Compend of Mechanical Refrigeration")

Per cent. of NaCl by weight	Specific Gravity at 39 deg. fahr.	Freezing point, deg. fahr.	Specific heat at 32 deg. fahr.	Per cent. of NaCl by weight	Specific Gravity at 39 deg. fahr.	Freezing point, deg. fahr.	Specific heat at 32 deg. fahr.
1.0	1.007	30.5	0.992	8	1.061	21.2	0.919
2.0	1.015	29.3	0.984	9	1.068	19.9	0.905
2.5	1.019	28.6	0.980	10	1.076	18.7	0.892
3.0	1.023	27.8	0.976	12	1.091	16.0	0.874
3.5	1.026	27.1	0.972	15	1.115	12.2	0.855
4.0	1.030	26.6	0.968	20	1.155	6.1	0.829
5.0	1.037	25.2	0.960	24	1.187	1.2	0.795
6.0	1.045	23.9	0.946	25	1.196	0.5	0.783
7.0	1.053	22.5	0.932	26	1.204	−1.1	0.771

Table 12. Boiling Points (Deg. Fahr.) at Atmospheric Pressure

(See also p. 323)

Zinc	1680	Glycerine	554	Turpentine	320.0
Sulphur	823	Phosphorus	554	Toluene	230.0
Mercury	675	Naphthalene	424	Sodium chloride (sat. sol.)	226.4
Linseed oil	538	Aniline	363	Helium	−450.0
Paraffin	572	Calcium chloride (sat. sol.)	356		

Table 13. Boiling Points of Water Corresponding to Barometric Heights in Inches of Mercury

(From Marks and Davis's Steam Tables)

Press., in. of Hg.	Temp., fahr.	Press., in. of Hg.	Temp., fahr.	Press., in. of Hg.	Temp., fahr.	Press., in. of Hg.	Temp., fahr.	Press., in. of Hg.	Temp., fahr.
22.0	196.95	25.0	203.10	28.0	208.67	29.5	211.27	31.0	213.80
0.2	197.37	0.2	203.48	0.1	208.85	0.6	211.44	0.1	213.96
0.4	197.79	0.4	203.86	0.2	209.03	0.7	211.62	0.2	214.13
0.6	198.21	0.6	204.24	0.3	209.20	0.8	211.79	0.3	214.29
0.8	198.63	0.8	204.62	0.4	209.37	0.9	211.96	0.4	214.46
23.0	199.05	26.0	205.00	28.5	209.55	30.0	212.13	31.5	214.62
0.2	199.47	0.2	205.38	0.6	209.73	0.1	212.30	0.6	214.79
0.4	199.89	0.4	205.75	0.7	209.91	0.2	212.47	0.7	214.95
0.6	200.31	0.6	206.12	0.8	210.08	0.3	212.64	0.8	215.11
0.8	200.72	0.8	206.49	0.9	210.25	0.4	212.81	0.9	215.27
24.0	201.13	27.0	206.86	29.0	210.42	30.5	212.97	32.0	215.43
0.2	201.54	0.2	207.23	0.1	210.59	0.6	213.13	0.2	215.75
0.4	201.94	0.4	207.59	0.2	210.76	0.7	213.30	0.4	216.08
0.6	202.33	0.6	207.95	0.3	210.93	0.8	213.46	0.6	216.40
0.8	202.72	0.8	208.31	0.4	211.10	0.9	213.63	0.8	216.72

Heat of Fusion. The heat of fusion of a solid is the heat required in B.t.u. to convert 1 lb. of the substance from the solid to the liquid state, without change of temperature.

Table 14. Heat of Fusion, B.t.u. per Lb.

Aluminum	138.0	Sodium	57.1	Lipowitz's metal	12.3
Bismuth	22.8	Tin	25.0	Rose's metal	12.3
Blast-furnace slag	90.0	Zinc	50.6	Wood's metal	14.0
Cadmium	24.6	Alloys:		Zinc and bismuth	20-23
Copper	77.4	30.5 Pb+69.5 Sn	21.0	Benzol (C_6H_6)	55.0
Iron, gray cast	40.0	36.9 Pb+61.3 Sn	28.0	Calcium chloride sol. ($CaCl_2$ + $6H_2O$)	72.5
Iron, white	60.0	63.7 Pb+36.3 Sn	21.0	Glycerine	76.5
Lead	10.6	77.8 Pb+22.2 Sn	17.0	Ice	144.0
Mercury	5.08	78.4 Sn+21.6 Zn	42.3	Naphthalene($C_{10}H_8$)	64.1
Nickel	8.35	93.56 Sn+6.44 Zn	31.8	Phosphorus	9.0
Palladium	65.0	97.32 Sn+2.68 Zn	27.2	Phenol (C_6H_6O)	45.0
Platinum	49.0	Brittania metal (9 Sn + 1 Pb)	13.7	Sulphur	16.9
Potassium	28.09	D'Arcet's metal	10.4		
Silver	36.2				

Table 15. Latent Heat of Vaporization at Atmospheric Pressure, B.t.u. per Lb.

Alcohol	385	Methyl chloride	175	Hydrogen	222.
Aniline	198	Mercury	122	Nitrogen	81 5
Benzol	169	Sulphur	650	Oxygen	92
Chlorine	112	Carbon bisulphide	152.5	Water	970.4
Chloroform	110	Turpentine	126		
Ether	162	Acetone	233		

Table 16. Solution of Gases in Water

t (deg. fahr.) =	32	68	212	t (deg. fahr.) =	32	68	212
Air	0.032	0.020	0.012	Hydrogen	0.023	0.020	0.018
Acetylene	1.89	1.12	.....	Hydrogen sulphide	5.0	2.8	0.87
Ammonia	1250	700	.....	Hydrochloric acid	560	480	.....
Carbon dioxide	1.87	0.96	0.26	Nitrogen	0.026	0.017	0.0105
Carbon monoxide	0.039	0.025	.....	Oxygen	0.053	0.034	0.0185
Chlorine	5.0	2.5	0.00	Sulphuric acid	87	43	.....

Table 17. Solubility of Ammonia in Water (H. Mollier)

One pound of water at the given pressures and temperatures absorbs the following weights of ammonia:

Pressure, lb. per sq. in.	Temperatures, deg. fahr.													
	32	60	80	100	120	140	160	180	200	220	240	260	280	300
2	0.29	.154	.086	0.040	.002	.....	.....	.....	.....	.....	.....	.....	.....	.....
5	0.47	.296	.200	0.128	.071	0.028	.....	.....	.....	.....	.....	.....	.....	.....
10	0.70	.462	.327	0.243	.164	0.102	.056	.016	.....	.....	.....	.....	.....	.....
15	0.90	.607	.426	0.334	.223	0.168	.106	.057	.020	.....	.....	.....	.....	.....
20	1.15	.726	.539	0.423	.302	0.215	.151	.093	.048	.010	.....	.....	.....	.....
30	1.71	.920	.698	0.539	.408	0.310	.228	.158	.100	.050	.014	.....	.....	.....
40	2.31	.....	.862	0.663	.511	0.398	.302	.215	.162	.090	.046	.007	.....	.....
50	.....	.....	.992	0.778	.604	0.465	.358	.271	.195	.128	.079	.035	.001	.....
60	.....	.....	.....	0.890	.686	0.528	.410	.314	.235	.162	.126	.058	.021	.....
70	.....	.....	.....	1.003	.776	0.606	.470	.363	.275	.197	.135	.080	.040	.003
80	.....	.....	.....	.....	.894	0.660	.512	.402	.309	.227	.162	.106	.062	.021
100	.....	.....	.....	.....	.....	0.794	.620	.486	.381	.288	.213	.150	.099	.053
120	.....	.....	.....	.....	.....	0.926	.727	.563	.427	.323	.262	.188	.132	.081
140	.....	.....	.....	.....	.....	1.063	.822	.636	.509	.394	.304	.224	.163	.111

Heat of Vaporization (Table 15). The latent heat of vaporization is the quantity of heat in B.t.u. required to convert 1 lb. of a liquid into vapor at the same temperature under a constant external pressure. The latent heat depends upon the temperature at which the process takes place.

For the **latent heats of steam** at different pressures, see p. 324; and for the **latent heats of refrigerating media**, see pp. 333, 336 and 337.

One cubic foot of water at atmospheric pressure and at the temperature t will dissolve the volumes of gas (in cu. ft. at 32 deg. fahr. and at atmospheric pressure) given in Table 16.

TRANSMISSION OF HEAT

Preliminary Statements. There are three methods by which heat may be propagated or conveyed from one place to another.

1. By **Conduction.** In this method heat passes from one body to another or from one part of a body to another by contact. It is assumed that the warmer molecules impart heat to the colder ones.

2. By **Convection.** Heat is carried from place to place by the medium with which it is associated. Thus, in a hot-water heating system, heat is carried from the furnace through the building by the water in the system; in a boiler, heat is carried through the mass of water by the currents produced by circulation.

3. By **Radiation.** A source of heat, as the sun or a fire, gives off energy in the form of radiant heat. This energy is assumed to be propagated in all directions as a wave motion in the ether, and radiation falling upon a body is to some extent absorbed by it. According to the modern **theory of exchanges,** all bodies whose molecules are in vibration are sources of radiation. If two bodies, one hotter than the other, are placed within an enclosure, there is a continual interchange of energy between them. The hotter body A radiates more energy than it absorbs, the colder body B absorbs more than it radiates. The result is an equalization of temperature, but even after the equilibrium of temperature the process of radiation continues, each body radiating and absorbing energy.

Transmission of Heat by Convection and Conduction

Phenomena of Heat Transmission. In the cases of heat transmission that usually occur in practice—in boilers, condensers, the cooling of engine cylinders, etc.—heat is transmitted from one fluid to another through a wall separating the two. The character of the process is shown in Fig. 1. At the surface of the plate there is a film of the hot fluid of indefinite thickness. This film offers a considerable resistance to the transmission, as shown by the temperature difference $t_1 - t'_1$ through it. A corresponding film on the other side of the plate offers resistance measured by the temperature drop $t'_2 - t_2$. Let Q denote the total heat in B.t.u. transmitted in z hours through a plate area of A sq. ft. Then, for film f_1, $Q = k_1Az(t_1 - t'_1)$; for the plate, $Q = (K/b)\,Az\,(t'_1 - t'_2)$; for the film f_2, $Q = k_2Az\,(t'_2 - t_2)$.

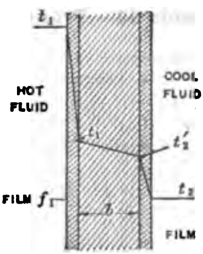

FIG. 1.

The coefficients k_1 and k_2 are the **conductances** of the films f_1 and f_2, respectively, K is the thermal conductivity of the plate and, analogously, (K/b) is the conductance of the plate. The reciprocals of these, viz., $1/k_1$, $1/k_2$, b/K, are the **resistances** of the two films and plate, respectively. The total resistance R to the flow is the sum of the separate resistances, $R = (1/k_1) +$

$(1/k_2) + (b/K)$, and the conductance of the two films and plate is $k_0 = 1/R$. Hence, $Q = k_0 Az\ (t_1 - t_2)$.

If the wall is composed of several layers of different materials having thicknesses b', b'', b''', etc., and thermal conductivities K', K'', K''', etc., the total resistance is $R = (1/k_1) + (1/k_2) + (b'/K') + (b''/K'') + (b'''/K''') + \ldots$, and $k_0 = 1/R$.

The **resistance of the fluid film** depends upon the kind of fluid and the character of the motion of the fluid along the wall. The resistance of a film of hot gas is very high. It has been estimated that in certain cases 98 per cent. of the available temperature drop is required to force the heat through the gas film. If the hot fluid is saturated steam, condensation insures a film of water in contact with the plate and the resistance is relatively small compared with that of a gas film. The film resistance is considerably influenced by the motion of the fluids along the plates. If the hot gas, as in a boiler tube, is given a high velocity the resulting sweeping action partially destroys the film and decreases the resistance. The same is true on the water side of the wall. It is well established that the **rate of heat transmission is dependent on** the **velocity of** the **fluid** along the plate.

A layer of soot or scale on a plate has the effect of making a composite wall. The resistance is increased by b'/K', where b' is the thickness and K' the thermal conductivity of the layer. If k_0 is the total conductance for the clean plate, the ratio of the heat transmitted to that transmitted through the clean plate is

$$Q'/Q = \frac{1}{k_0}\Big/\left(\frac{1}{k_0} + \frac{b'}{K'}\right) = 1\Big/\left(1 + \frac{b'k_0}{K'}\right)$$

The experiments of Clement and Garland (*Bulletin* No. 40, Engineering Experiment Station, University of Illinois) afford valuable data on the **transmission of heat from steam to water.** A stream of water was made to flow at various velocities through a steel tube of 0.985 in. inside diam. and with walls 0.134 in. thick. The tube was surrounded by steam, and the temperature of the outer wall of the tube was measured by thermo-couples. The temperature of the inner wall was calculated from the known conductivity of the metal composing the tube. The results of one series of tests are shown in Table 18.

Table 18. Transmission of Heat from Steam to Water through Steel Tubes

No. of test	Temperatures, deg. fahr.				Velocity of water, ft. per sec.	B.t.u. transmitted per min. per sq. ft.	Conductances, B.t.u. per sq. ft. per sec.		
	of steam	of outside wall	of inside wall	of water, mean			Steam film	Water film	Both films and wall
1	330.2	220.0	165	69.0	17.13	3995	0.606	0.694	0.255
2	330.0	229.4	177	70.8	14.05	3756	0.621	0.591	0.246
3	330.0	233.1	185	74.6	10.39	3455	0.594	0.524	0.225
4	330.2	241.8	197	78.07	8.06	3260	0.614	0.457	0.215
5	330.2	257.4	220	89.5	4.25	2672	0.611	0.340	0.185
6	330.2	267.1	234	110.2	2.31	2370	0.625	0.318	0.179

It appears from these results that the velocity of flow has a marked effect on the conductance of the water film and upon the rate of transmission from the steam to the water.

Conductance of Fluids. As stated in a preceding paragraph, the rate of transmission of heat between a fluid and a metal surface depends upon the conductance of the fluid film. If h denotes the difference of temperature between the fluid and the plate wall, and k the conductance, then $Q = kzAh$. The value of k depends upon the nature of the fluid and also upon the velocity of the fluid along the surface. The following values may be taken for the conditions stated (k = B.t.u. per sq. ft. per hour per deg. fahr. difference of temperature):

1. For boiling water .... $k = 800$ to 1200
2. For condensing steam........................ $k = 2000$

The experiments of Clement and Garland show a mean value of $0.61 \times 3600 = 2200$, approx.

3. Water, not boiling:

If the water is at rest,........................ $k = 100$.

If the water is in motion, k will depend upon the velocity. In the experiments of Clement and Garland k varied from 730 with a velocity of 1.45 ft. per sec. to 2500 with a velocity of 17.13 ft. per sec.

4. Air at Rest. Take h as the difference between the temperature of the surface and the mean temperature of the air in the room. Then, for vertical surfaces,

$k = 0.62 + 0.009h$ for $h < 20$ deg, and $k = 0.385 \sqrt[4]{h}$ for $h > 20$ deg., or

for h =	0	5	10	25	50	100	200	300	400
k =	0.62	0.665	0.71	0.86	1.04	1.22	1.45	1.60	1.72

5. Transmission of Heat from the Surface of a Rotating Cylinder to Air, as in flywheels, armatures, etc. According to the experiments of Hinlein, for

Smooth Surfaces (polished copper).

Velocity w, ft. per sec...........	0	20	40	60	80	100
Conductance k.................	0.47	1.97	2.79	3.14	3.30	3.38

Rough surfaces (dull black, varnished).

w =	0	20	40	60	80	100
k =	0.47	2.49	3.32	3.95	4.50	5.10

Table 19. Thermal Conductivities of Metals

Substance	Temp., deg. fahr.	K	Substance	Temp., deg. fahr.	K
Metals:			Magnesium..............	32–212	92.0
Aluminum..............	64	116.0	Mercury................	32	3.6
Aluminum..............	212	119.0	Mercury................	122	4.6
Antimony..............	32	10.6	Nickel.................	64	34.4
Antimony..............	212	9.7	Nickel.................	212	33.4
Bismuth...............	64	4.7	Platinum...............	64	40.2
Bismuth...............	212	3.9	Platinum...............	212	41.9
Cadmium...............	64	53.7	Silver.................	64	244.0
Cadmium...............	212	52.2	Silver.................	212	240.0
Copper................	64	222.0	Tin....................	64	37.6
Copper................	212	220.0	Tin....................	212	35.0
Gold..................	64	169.0	Zinc...................	64	64.1
Gold..................	212	170.0	Zinc...................	212	63.5
Iron, pure............	64	39.0	Alloys:		
Iron, pure............	212	36.6	Brass..................	63	63.0
Iron, wrought.........	64	34.9	Constantan (60 Ca, 40 Ni)	64	13.1
Iron, wrought.........	212	34.6	Constantan (60 Ca, 40 Ni)	212	15.5
Iron, cast............	129	27.6	German silver..........	32	16.9
Iron, cast............	216	26.8	German silver..........	212	21.5
Steel (1 per cent. C).....	64	26.2	Manganin {84 Cu, 4 Ni, 12 Mn}	64	12.8
Steel (1 per cent. C).....	212	25.9			
Lead..................	64	20.1		212	15.2
Lead..................	212	19.8	Platinoid..............	64	14.5

Thermal Conductivity. The resistance of the plate to heat transmission is b/K, the conductance K/b. The number K in this expression is called the **thermal conductivity** of the material of the plate. The thermal conductivity of a substance may be defined as the quantity of heat (B.t.u.) that flows in a unit of time (1 hr.) through unit area of plate (1 sq. ft.) of unit thickness (1 ft.) having unit difference of temperature (1 deg. fahr.) between its faces. This definition follows from the relation

$$Q = KAz(t'_1 - t'_2)/b, \text{ or } K = Qb/Az(t'_1 - t'_2).$$

It will be observed that the conductivity K involves an element of length, while the conductance k does not.

The thermal conductivity of different materials varies greatly. For metals and alloys K is high, while for certain insulating materials, as asbestos, cork, and silk, K is very low. In general, K **varies with the temperature.** In the case of metals K usually decreases with rising temperature, while for most other substances the reverse is true. Tables 19 to 24, collected from various sources, give values of thermal conductivity.

Table 20. Thermal Conductivities of Miscellaneous Solid Substances

(See also Table 21)

(The values of K in the table below are to be regarded as rough average values for the temperature range indicated)

Substance	Temp. range, deg. fahr.	K	Substance	Temp. range, deg. fahr.	K
Cardboard		0.120	Glass:		
Cement		0.170	Crown, window		0.6
Cotton wool		0.010	Flint, window		0.48
Ebonite		0.100	Jena, window		0.25–0.5
Felt		0.022	Soda, window		0.3–0.44
Gas carbon		2.400	Firebrick	32–2400	0.75
Graphite		2.900	Alumina brick	32–1300	0.5
Marble, white		1.720	Clinker, granular	32–1300	0.27
Mica		0.440	Pumice stone	70–150	0.135
Paper		0.075	Carborundum, coarse	70–212	0.122
Paraffin wax		0.145	Carborundum, fine	70–212	0.121
Porcelain		0.600	Magnesia, fused	70–212	0.114
Quartz:			Magnesia, powder		0.04
Parallel to axis		7.250	Magnesia, calcined	70–212	0.109
Perpendicular to axis		3.900	Coke, powdered	32–212	0.106
Rubber, Para		0.109	Graphite, powdered	70–212	0.097
Sand		0.31	Coarse brickdust	32–212	0.094
Sawdust	70–275	0.037	Silicate enamel	70–208	0.097
Silicate cotton		0.046	Fused quartz	70–212	0.094
Slate		1.140	Infusorial earth	32–1200	0.092
Blast-furnace slag	75–260	0.064	Serpentine (Cornwall)		1.07
Cork stone (asphalted)	50–135	0.041	Strawboard		0.08
Lime	70–212	0.070	Vulcanite		0.21
Firebrick, powdered	70–212	0.068	Beeswax		0.022
Chalk		0.480	Haircloth		0.01
Charcoal, powdered	32–212	0.053	Leather (cowhide)		0.10
Wood ashes	32–212	0.041	Linen		0.05
Lava		0.019	Mill shavings		0.05
Granite		1.300	Hair felt		0.03
Magnesia bricks	32–2400	1.5	Mineral wool		0.035

For steam-pipe coverings, see also pp. 840 to 842.

Table 21. Conductivities of Insulating Materials (Nusselt)

Material	Weight, lb. per cu. ft.	Temperatures, deg. fahr.						
		32	100	200	300	400	600	800
Asbestos	36.0	0.087	0.097	0.110	0.117	0.121	0.125	0.130
Burnt infusorial earth for pipe coverings	12.5	0.043	0.046	0.052	0.057	0.062	0.073	0.085
Insulating composition (loose)	25.0	0.040	0.046	0.050	0.053	0.055	......	......
Same mixed with water and dried	42.5	......	......	......	0.067	0.081 (at 428°)	......	......
Cotton	5.0	0.032	0.035	0.039	......	......	......	......
Silk hair	9.1	0.026	0.030	0.034	......	......	......	......
Silk	6.3	0.025	0.028	0.034	......	......	......	......
Wool	8.5	0.022	0.027	0.033	......	......	......	......
Pulverized cork	10.0	0.021	0.026	0.032	......	......	......	......
Infusorial earth (loose)	22.0	0.035	0.039	0.045	0.047	0.050	0.053	
Same mixed with water and dried	36.0	......	......	......	0.056	......	......	(at 662°) 0.083

Table 22. Conductivities of Insulating Materials at Low Temperatures (Gröber)

Material	Weight, lb. per cu. ft.	Temperatures, deg. fahr.				
		32	−50	−100	−200	−300
Asbestos	44.0	0.1350	0.1320	0.1300	0.1250	0.1000
Asbestos	29.0	0.0894	0.0860	0.0820	0.0720	0.0545
Cotton	5.0	0.0325	0.0302	0.0276	0.0235	0.0198
Silk	6.3	0.0290	0.0256	0.0235	0.0196	0.0155

Table 23. Conductivities of Various Building and Insulating Materials

(Poensgen, *Zeit. Ver. Deutsch. Ing.*, Oct. 12, 1912)

Materials	Wt., lb. per cu. ft	*K* at 68 deg. fahr.	Materials	Wt., lb. per cu. ft	*K* at 68 deg. fahr.
BUILDING MATERIALS					
Linoleum	75	0.107	Artificial stone No. 1 (of fine material)	104.0	0.390
Pine, perpendicular to fiber	34	0.087	Artificial stone No. 2 (of coarse material)	124.0	0.580
Pine, parallel to fiber	34	0.200	Firebrick	107.0	0.340
Teak, perpendicular to fiber	40	0.100	Rough cast	105.0	0.455
			INSULATING MATERIALS		
Teak, parallel to fiber	....	0.215	Cork plate No. 1	3.8	0.024
Oak, perpendicular to fiber	51	0.120	Cork plate No. 5	11.2	0.028
Oak, parallel to fiber	....	0.210	Cork plate No. 10	13.4	0.031
Asbestos sheets	111	0.128	Cork plate No. 14	16.6	0.034
			Cork plate No. 16	21.8	0.038
Gypsum plates enclosing cork	43	0.170	Charcoal, bound in plates	12.7	0.032
Gypsum	78	0.250			
Asphalt	132	0.400	Burnt infusorial earth No. 1	18.5	0.039
Brickwork	115	0.285			
Brickwork, hollow	....	0.190	Burnt infusorial earth No. 2	20.8	0.046
Brick, hand-made	96	0.230			
Brick, machine-made	104	0.300	Burnt infusorial earth No. 3	22.8	0.044
Sandstone, natural, dried	140	0.750	Burnt infusorial earth No. 4	28.2	0.050
Sandstone, natural, fresh-cut	143	0.970	Cork linoleum	33.4	0.046
Concrete No. 1 (1 : 4, dry)	136	0.435			
Concrete No. 2 (1 : 12, fresh)	128	0.470	Cork stone 2.6 in. thick with 0.2 in. of cement on each side	27.8	0.040

For building materials, see also p. 1341.

Table 24. Conductivities of Liquids and Gases

Substance	Temp., deg. fahr.	K	Substance	Temp., deg. fahr.	K
Alcohol	77	0.104	Ammonia	32.0	0.0111
Aniline	54	0.099	Ammonia	212.0	0.0172
Glycerine	77	0.165	Carbon monoxide	32.0	0.0121
Benzol	41	0.081	Carbon monoxide	44.6	0.0123
Ether	48-59	0.073	Carbon dioxide	32.0	0.0079
Oil, olive	....	0.096	Carbon dioxide	212.0	0.0122
Oil, castor	....	0.103	Ethylene	32.0	0.0096
Oil, paraffin	63	0.085	Helium	32.0	0.0082
Oil, petroleum	55	0.086	Hydrogen	32.0	0.0775
Oil, turpentine	55	0.079	Hydrogen	212.0	0.0895
Vaseline	77	0.106	Methane	46.0	0.0156
Water	63	0.032	Nitrogen	45.0	0.0127
Water	52	0.036	Nitrous oxide	32.0	0.0085
Water	77	0.033	Nitrous oxide	212.0	0.0122
Air	32	0.0126	Nitric oxide	46.0	0.0101
Argon	32	0.0094	Oxygen	45.0	0.0136

The conductivities of air and steam are given as functions of the temperature by the relations: K (for air) $= 0.01221\ (1 + 0.00132\ t)$ and K (for steam) $= 0.00882\ (1 + 0.00219\ t)$.

For t (deg. fahr.) =	32	50	100	200	300
K (for air) =	0.0127	0.0130	0.0138	0.0154	0.0170
K (for steam) =	0.0095	0.0098	0.0107	0.0127	0.0146
For t (deg. fahr.) =	400	500	600	700	800
K (for air) =	0.0187	0.0203	0.0219	0.0235	0.0250
K (for steam) =	0.0165	0.0184	0.0204	0.0223	0.0242

Heat Transmission Between Fluids Separated by a Plate. The following are important examples of heat transmission through plates or tubes. ucke's "Engineering Thermodynamics," p. 550):

luid giving up heat	Fluid receiving heat	Examples of heat transmission
Liquid	(a) Liquid	Liquid heat exchangers, etc.
	(b) Gas	Hot-water radiators; cooling-tower surfaces.
	(c) Boiling liquid	Brine coolers; hot-liquid evaporators.
2. Gas	(a) Liquid	Brine coolers in cold-storage rooms; air coolers with water or brine coils; economizers.
	(b) Gas	Steam superheaters; air-cooled motors.
	(c) Boiling liquid	Steam boilers; direct-expansion ammonia coils.
3. Condensing vapor	(a) Liquid	Condensers; feed-water heaters.
	(b) Gas	Steam radiators.
	(c) Boiling liquid	Vacuum evaporators.

Let Q = heat transmitted per hour, k_0 = coefficient of heat transfer = conductance of wall, with scale, soot, and fluid films, A = area in sq. ft. through which transfer is in process, and h_m = mean temperature difference for the process. Then $Q = k_0Ah_m$.

If one of the fluids is condensing or boiling under constant pressure the temperature remains constant during the heat transfer. In case 3(c) both fluids remain at constant temperature and h_m is simply the difference. In all other cases the temperature of one fluid or of both fluids changes during the process. If h_1 is the initial temperature difference and h_2 the final temperature difference, then the mean temperature difference for the process is given by $h_m = (h_1 - h_2)/\log_e(h_1/h_2)$.

Coefficient of Heat Transmission

The **value of** the **coefficient** k_0 depends on a number of conditions, viz.: the character of the fluids, the velocity of the fluids along the separating surfaces, the condition of the surface, and the shape of the surface. In general, k_0 is small when one of the fluids is a gas. The following rough values are given by Lucke for the cases stated in the preceding paragraph:

Case	Fluids	Values of k_0 (B.t.u. per hour per sq. ft. per deg. fahr.)
1 (a)	Liquid—Liquid	50–75
1 (b)	Liquid—Gas	2–6
1 (c)	Liquid—Boiling liquid	10–100
2	Gas—Liquid, gas, or boiling liquid	2–5
3 (a)	Condensing vapor—Liquid	150–350, 1000 under special conditions
3 (b)	Condensing vapor—Gas	2–4
3 (c)	Condensing vapor—Boiling liquid	400–600

For some of the most important cases the variation of k_0 with external conditions has been investigated. The following is a résumé of some of the results.

1. Transmission Between Steam and Water (Condensers, etc.). According to the exhaustive experiments of Orrok (*Trans. A. S. M. E.*, vol. 32) the value of k_0 depends upon the following factors:

(a) k_0 is approximately proportional to the square root of the velocity of the cooling water.

(b) k_0 is inversely proportional to the eighth root of the mean difference of steam and water temperatures.

(c) k_0 depends upon the material of the tube and upon the cleanliness of the tube.

(d) k_0 is reduced in a marked degree by the presence of air in the condenser. Thus if p_s denotes the pressure of the steam alone and p_t the total pressure ($= p_s +$ pressure of air), then k_0 varies as $(p_s/p_t)^5$.

Orrok (1914) gives the following equation as representing the results of **tests on condensers** under various conditions: $k_0 = 350\, c_1c_2(p_s/p_t)^2 \times \sqrt{w}$, in which c_1 ($= 0.5$ to 1.0) is the cleanliness coefficient, and c_2 a coefficient depending on the material of the tubes. For copper, $c_2 = 1.0$; for Admiralty mixture, 0.98 (oxidised, 0.97); for Muntz metal, 0.95; for aluminum bronze, 0.92. The exponent for (p_s/p_t) is now taken as 2 instead of 5. w denotes the velocity of the water in ft. per sec.

The variation of k_0 with the water velocity has received attention from a number of experimenters. The following are some of the relations obtained (*Trans. A. S. M. E.*, vol. 32):

Ser, $k_0 = 520 \sqrt[3]{w}$; Hagemann, $282 \sqrt{w}$; Josse, $487 \sqrt[3]{w}$; Allen, $220 \sqrt{w}$; Stanton, $340 \sqrt[3]{w}$; Orrok, $308 \sqrt{w}$; Clement and Garland, $270 \sqrt[3]{w}$.

Carrier (*Trans. A. S. M. E.*, vol. 33), gives the following formula as representing the results of tests on condensers: $k_0 = 1/[0.000394 + (0.00255/w)]$.

2. Steam and Boiling Liquids. Where **steam coils** are **used for evaporation** of water or other liquids the following coefficients of transmission may be used (Hausbrand, "Evaporating, Condensing and Cooling Apparatus"):

Let d = diameter of pipe and l = length, both in feet; then $k_0 = 1250/\sqrt{dl}$. This value holds for copper pipe, when the boiling liquid is water. For wrought-iron pipe take 0.75 and for cast-iron pipe 0.5 of this value. In practice, ½ of these values should be assumed.

In the case of thick, viscous liquids, the size of pipe has little influence on the rate of transmission, and the following values of k_o may be taken: For long heating coils, 130–150; for short heating coils, 160–180; for thin steam pipes, 200.

3. Steam to Air. Carrier (*Trans. A. S. M. E.*, vol. 33) deduces the following formula for the coefficient of transmission under the conditions that obtain in **hot-blast heating** systems: $k_0 = 1/[a + (b/w)]$, in which w denotes the velocity of the air, ft. per min., and a and b are constants depending on the type of heater. Tests on a Buffalo standard heater with 50 lb. steam pressure, gave $a = 0.0447$, $b = 50.66$. For Vento cast-iron heaters the test showed $a = 0.047$, $b = 61.0$.

Conduction of Heat through Steam Pipes.—Pipe Coverings. The coefficient of heat transmission through bare steam pipes varies with the steam pressure. Assuming the air to be still, k_0 may be taken as about 2.1 for low steam pressure (5 or 6 lb. gage), and 3.0 or 3.1 for a pressure of 90 or 100 lb. gage. Table 25 gives the results of tests by the Johns-Manville Co. on heat conduction through bare pipe and pipe with various insulating coverings. Outside temperature, 72 deg. fahr.

Table 25. Heat Conduction through Bare and Covered Steam Pipes

(Johns-Manville Co., New York)

Nature and thickness of covering	Conductance, B.t.u. per sq. ft. per hr. per deg. fahr.	Heat transmitted in B.t.u. per sq. ft. per hour at steam pressures (per sq. in.) of			
Low-pressure Steam Pipe					
		0 lb.	25 lb.	50 lb.	
Asbestocel, 1 in.	0.564	77.4	109.8	127.2	
Air cell, 1 in.	0.798	111.6	156.0	179.4	
Asbestos, molded, 1 in.	0.696	97.2	135.6	156.6	
Asbestos, indent, 1 in.	0.684	96.0	133.2	154.2	
Bare pipe	3.000	360.0	576.0	876.0	
High-pressure Steam Pipe					
		100 lb.	150 lb.	200 lb.	250 lb.
Asbestos:					
Sponge-felted, 1 in.	0.4524	120.6	133.8	142.8	151.8
Sponge-felted, 2 in.	0.354	94.2	104.4	111.6	118.2
Sponge-felted, 3 in.	0.324	85.2	95.4	102.6	108.6
Magnesia, J. M., 1 in.	0.498	132.6	147.0	156.0	166.2
Magnesia, J. M., 1½ in.	0.450	119.4	132.6	142.2	150.0
Magnesia, J. M., 2 in.	0.378	100.2	111.6	119.4	126.6
Bare pipe	3.000	1170.0	1284.0	1380.0	....
Superheated Steam					
		At 500 deg. fahr.	At 600 deg. fahr.		
Asbestos:					
Sponge-felted, 1 in.	0.452	193.2	238.8		
Sponge-felted, 2 in.	0.354	151.2	187.2		
Sponge-felted, 3 in.	0.324	138.6	171.0		
Magnesia, J. M., 1 in.	0.498	211.4	261.6		
Magnesia, J. M., 1½ in.	0.450	192.0	237.6		
F. F. and A. S. F. 2 in.	0.378	161.4	200.0		
Bare pipe	3.000	2000.0	3120.0		

Transmission of Heat by Radiation

The **absorption capacity** of a body for radiation is the ratio of the heat absorbed to the entire radiation received on the surface of the body. The **absolute "black body"** absorbs all the radiation received; hence its absorption capacity is 1. Bodies that are good absorbers have also high capacity for radiation. Bodies that are good reflectors, as polished steel or silver, have correspondingly small absorption capacity and radiating power. Table 26 gives the relative absorbing, radiating and reflecting properties of certain bodies.

Table 26. Relative Radiating and Reflecting Capacities

Substance	Absorbing or radiating capacity	Reflecting power	Substance	Absorbing or radiating capacity	Reflecting power
Water	100	....	Steel, polished	17	83
Marble	93–98	7–2	Tin	15	85
Glass	90	10	Brass, dead polished	11	89
Ice	85	15	Brass, bright	7	93
Cast iron, polished	25	75	Copper, hammered	7	93
Wrot. iron, polished	23	77	Gold	5	95
Mercury	23	77	Silver, polished	3	97
Zinc	19	81			

The Stefan-Boltzmann Law. The radiating power of a body is proportional to the fourth power of the absolute temperature of the body. This law holds exactly for an absolute black body; for other bodies it holds with sufficient exactness for practical purposes.

Let A = area of radiating surface, sq. ft., z = time in hours, T = absolute temperature on the Fahrenheit scale, and Q = heat radiated in B.t.u. Then

$$Q = CAz\,(T/100)^4$$

The radiation constant C depends on the substance and on the character of the radiating surface. For the absolute black body, C = 0.1618. Values of C for various substances are given in Table 27.

Table 27. Radiation Constants of Various Materials

Material	Temp. range of experiment, deg. fahr.	C	Material	Temp. range of experiment, deg. fahr.	C
Glass, smooth	70	0.154	Red sandstone*	140–400	0.10
Brass, dull	100–660	0.0362	Italian marble*	140–400	0.095
Lampblack	32–100	0.154	Granite*	140–400	0.0745
Copper, slightly polished	100–540	0.0278	Dolomite*	140–400	0.0685
Wrought-iron, dull, oxidized	70–670	0.154	Clay	140–400	0.065
Wrought-iron, clean, bright	85–225	0.0562	Field soil	140–400	0.063
Wrought-iron, highly polished	105–480	0.0467	Chalk	140–400	0.051
Zinc, dull	120–545	0.034	Gravel	140–400	0.0481
Cast iron, rough, highly oxidized	105–480	0.157	Water	140	0.112
Lime plaster, rough, white	50–195	0.151	Ice	32	0.106
Basalt*	140–400	0.120	Gold plate, shining but not polished	70	0.082
Slate*	140–400	0.115			
Humus	140–400	0.110			

* Finished smooth, but not shining.

The simplest case of heat radiation is that in which a body having an external surface A of uniform temperature t_1 is enclosed within a second surface having a uniform temperature t_2. Let C_1, C_2 and C be the radiation constants for the two surfaces and for the absolute black body, respectively. Then the heat transmitted by radiation in z hours is

$$Q = Az[(T_1/100)^4 - (T_2/100)^4]/[(1/C_1) + (1/C_2) - (1/C)]$$

The following example (Dalby, *Inst. M. E.*, Oct., 1909) shows an application of this formula to steam boilers. Let the temperature of the furnace be 3000 deg. and that of the boiler plate or tubes, 800 deg. fahr., abs. The constant C_1 of the incandescent carbon may be taken as equal to the constant C of the black body, and C_2 for iron may be taken as 0.154. The heat radiated per sq. ft. of the boundary per hour is consequently

$$0.154[(3000/100)^4 - (800/100)^4] = 124{,}100 \text{ B.t.u.}$$

Ray and Kreisinger (Breckenridge, "Study of Four Hundred Steaming Tests," U. S. Geol. Survey, 1907) have studied the **relations between heat transmitted by radiation and by convection** in a Heine boiler. It was found that the heat transmitted by radiation increased with the temperature, as indicated by the Stefan law. The ratio of heat received by radiation to that received by convection varied from 3 to 15 per cent.

GENERAL PRINCIPLES OF THERMODYNAMICS

Notation:

Q, q = quantity of heat absorbed, B.t.u.
p = absolute pressure, lb. per sq. ft.
M = weight of substance under consideration, lb.
V, v = volume, cu. ft.
t = temperature, deg. fahr.
$T = t + 459.6$ = absolute temperature.
U, u = internal energy, B.t.u.
I, i = heat content at constant pressure, B.t.u.
S, s = entropy.
$J = 777.7$ = mechanical equivalent of heat.
$A = 1/J$ = reciprocal of mechanical equivalent.
c_p = specific heat at constant pressure.
c_v = specific heat at constant volume.
W = external work performed during a change of state.

In this notation small letters denote magnitudes referred to unit weight of the substance, capital letters corresponding magnitudes referred to M units of weight. Thus, v denotes the volume of 1 lb., $V = Mv$, the volume of M lb. Similarly, $U = Mu$, $S = Ms$, etc. Subscripts are used to indicate different states; thus, p_1, v_1, T_1, u_1, s_1, refer to the initial state, p_2, v_2, T_2, u_2, s_2 refer to the final state. Q_{12} is used to denote the heat absorbed by a body during the change from state 1 to state 2, and W_{12} denotes the external work done during the same change.

State of a Substance.—Characteristic Equations. Assuming that the substance under consideration (as air or steam) is homogeneous and of uniform density and temperature, and that it is subjected to a uniform pressure, the state of the substance is given by the values of p, v, and T. In general, two of these three variables may be changed independently, and the value of the third will then depend upon the values assigned to these two. That is, if v and T are taken to vary independently, then $p = f(v, T)$. The equation that expresses the relation between p, v, and T is called the **characteristic equation** of the substance in question.

For **perfect gases** the characteristic equation has the simple form

$$pv = RT.$$

This is approximately the equation of air, oxygen, nitrogen, and hydrogen. For certain gases, the equation of van der Waals

$$p = \frac{BT}{v - b} - \frac{a}{v^2}$$

represents quite accurately the states of the substance. The empirical equation $v - c = (BT/p) - [(1 + 3ap^{1/2})\, m/T^n]$ applies to **superheated steam.**

Any magnitude that depends upon the state of the substance may be used as a variable to describe the state. In addition to p, v, and T, the entropy s, energy u, and heat content i may be so used.

Fundamental Laws of Thermodynamics

Transformations of Energy are subject to two general laws: (1) The law of **conservation of energy,** which may be stated as follows: The total energy of an isolated system remains constant and cannot be increased or diminished by any physical process whatever. (2) The law of **degradation of energy,** according to which the result of any transformation of energy is a reduction in the quantity of energy available for transformation into useful work.

The following are familiar examples of degradation: Work transformed into heat through friction; electrical energy transformed into heat in the conducting system; flow of heat from a body of higher temperature to one of lower temperature; throttling or wire-drawing processes.

The First Law of Thermodynamics is the conservation law applied to the transformation of heat into work, or *vice versa*. When work is expended in producing heat, the quantity of heat generated is equivalent to the work done; conversely, when heat is employed to do work, a quantity of heat precisely equivalent to the work done disappears.

The first law is expressed symbolically by the equation $W = JQ$, in which W denotes the work and J the **mechanical equivalent of heat.** The following are the values of the constant J in various units:

$$1 \text{ gram-calorie} = 4.184 \text{ joules}$$
$$1 \text{ kg.-calorie} = 426.65 \text{ m.-kg.}$$
$$1 \text{ B.t.u.} = 777.64 \text{ ft.-lb.}$$

For ordinary calculations the values 427 and 778 are sufficiently accurate. Another useful relation is

$$1 \text{ h.p.-hr.} = 2546 \text{ B.t.u.}$$

In writing the equations of thermodynamics it is frequently convenient to use the reciprocal of J, which is denoted by A; thus $A = 1/J$.

The Energy Equation. The first law applied to a change of state of a substance is expressed by the equation

$$JQ_{12} - W_{12} = E_2 - E_1$$

in which E_1 denotes the total energy of the substance in the initial state and E_2 the total energy in the final state. The total energy E is made up of the internal energy U and the external kinetic energy. The latter comes into consideration in the flow of fluids. In case the kinetic energy remains constant or $= 0$, the energy equation becomes

$$Q_{12} = U_2 - U_1 + AW_{12}$$

If the work W arises from the overcoming of fluid pressure, then $W = \int pdV$ and the equation takes the form

$$dQ = dU + ApdV, \text{ or } Q_{12} = U_2 - U_1 + A\int_{V_1}^{V_2} pdV$$

The change of energy depends upon the initial and final states of the system only. The external work depends, however, on the relation between p and V during the change of state, that is, upon the path; hence the heat absorbed also depends upon the path.

If a system passes through a closed cycle of processes and returns to its initial state, the change of energy for the cycle is zero; hence for a closed cycle the heat absorbed by the system is the equivalent of the external work. This statement is expressed symbolically by the equation $J(Q) = (W)$, where (Q) denotes the heat absorbed for the cycle and (W) the net work performed. If the change of state is adiabatic, the heat absorbed is zero and the external work is gained at the expense of the intrinsic energy of the system. That is, $W_{12} = U_1 - U_2$.

The Second Law of Thermodynamics is essentially the law of degradation of energy. While the first law gives a relation that must be satisfied in any transformation of energy, it is the second law that gives information regarding the possibility of transformation and the availability of a given form of energy for transformation into work. The following is a general statement of the second law: **No change in a system of bodies that takes place of itself can increase the available energy of a system.**

A more concrete statement is that of Kelvin, namely: It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of surrounding objects. In effect, Kelvin's statement denies the possibility of deriving work directly from the heat contained in the atmosphere.

The availability of a given quantity of heat energy for transformation into work is given by the efficiency of the **ideal Carnot engine.** Thus, if T denote the temperature of the source of heat and T_0 that of the refrigerator, or coldest body available, then the efficiency is $e = (T - T_0)/T = 1 - (T_0/T)$. If heat Q is taken from the source the part $Q[1 - (T_0/T)]$ may be transformed into work under ideal conditions, but the part $Q(T_0/T)$ at least must be rejected.

Entropy. Experience shows that every actual physical process is irreversible and accompanied by frictional effects. As a result the actual irreversible process is accompanied by a decrease of the quantity of energy available for transformation into work, or, what is the same thing, an increase of unavailable energy. The increase of unavailable energy is the product of two factors: one is T_0, the lowest absolute temperature available (usually the temperature of the atmosphere), the other is a term of the form Q/T or $\int dQ/T$. To this second factor the name **increase of entropy** is given.

When the conception of increase of entropy is applied to the system composed of all the bodies involved in a change, that is, to an isolated system, it appears that the increase of entropy is a measure of the thermodynamic degeneration produced by the change. According to the law of degradation every natural change is accompanied by thermodynamic degeneration, therefore it is accompanied by an increase of entropy. The following important principle is evident: The direction of a process, physical or chemical, that occurs of itself is such as will bring about an increase of entropy of the system. This principle is the foundation of the application of thermodynamics to chemistry.

The conception of increase of entropy may be extended to a body not isolated but in thermal communication with other bodies. In this case the change of entropy is given by the relation

$$S_2 - S_1 = \int_{T_1}^{T_2} \frac{dQ}{T} + \int_{T_1}^{T_2} \frac{dH}{T}$$

in which Q denotes heat absorbed by the body from outside and H heat generated within the system through friction. For a reversible frictionless change, the increase of entropy is simply $\int_{T_1}^{T_2} \frac{dQ}{T}$.

The entropy of the body, as thus defined, depends on the state only; hence S may be used with p, V, and T as a variable defining the state. For reversible frictionless changes, the defining equation gives $dQ = TdS$; hence the energy equation may be written $TdS = dU + ApdV$.

General Thermodynamic Relations. The first law gives the **energy equation**

$$dQ = dU + ApdV \tag{1}$$

the second law gives the **entropy equation**

$$dQ = TdS \tag{2}$$

A third general equation is obtained by the introduction of a magnitude analogous to the energy u and defined by the relation

$$i = u + Apv \quad (I = U + ApV)$$

This is called the **heat content at constant pressure,** for the reason that $I_2 - I_1 = Q_{12}$ for a change at constant pressure. By the introduction of I, Eq. (1) takes the form

$$dQ = dI - AVdp \tag{3}$$

By various transformations the following relations are derived from these three fundamental equations:

1. **Maxwell's thermodynamic relations:**

$$\left(\frac{\partial T}{\partial v}\right)_s = -A\left(\frac{\partial p}{\partial s}\right)_v \qquad \left(\frac{\partial s}{\partial v}\right)_T = A\left(\frac{\partial p}{\partial T}\right)_v$$

$$\left(\frac{\partial T}{\partial p}\right)_s = A\left(\frac{\partial v}{\partial s}\right)_p \qquad \left(\frac{\partial s}{\partial p}\right)_T = -A\left(\frac{\partial v}{\partial T}\right)_p$$

2. **Relations involving specific heats:**

$$c_v = \left(\frac{\partial q}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v \qquad c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

$$c_p = \left(\frac{\partial q}{\partial T}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p \qquad c_p = \left(\frac{\partial i}{\partial T}\right)_p$$

$$c_p - c_v = AT\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial T}\right)_v$$

$$\left(\frac{\partial c_v}{\partial v}\right)_T = AT\left(\frac{\partial^2 p}{\partial T^2}\right)_v \qquad \left(\frac{\partial c_p}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

3. **Relations involving q, u, i, and s:**

$$dq = c_v dT + AT\left(\frac{\partial p}{\partial T}\right)_v dv \qquad dq = c_p dT - AT\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$du = c_v dt + A\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv, \quad di = c_p dT - A\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]dp$$

$$ds = c_v\frac{dT}{T} + A\left(\frac{\partial p}{\partial T}\right)_v dv = c_p\frac{dT}{T} - A\left(\frac{\partial v}{\partial T}\right)_p dp$$

The derivatives involved in these relations are found from the characteristic equation of the substance under investigation. Thus, for a perfect gas,

$pv = BT$, whence $\left(\frac{\partial p}{\partial T}\right)_v = \frac{B}{v}$, $\left(\frac{\partial v}{\partial T}\right)_p = \frac{B}{p}$. Substitution of these expressions in the preceding equations gives the following results for gases:

$$c_p - c_v = AB$$

$$dq = c_v dT + Apdv \qquad dq = c_p dT - Avdp$$

$$du = c_v dT \qquad di = c_p dT$$

$$ds = c_v \frac{dT}{T} + AB\frac{dv}{v} = c_p\frac{dT}{T} - AB\frac{dp}{p}.$$

As an example of the use of the Clausius relation $\left(\frac{\partial c_p}{\partial p}\right)_T = -AT\left(\frac{\partial^2 v}{\partial T^2}\right)_p$, consider the case of superheated steam, the equation of which is $v - c = \frac{BT}{p} - (1+3ap^{1/2})\frac{m}{T^n}$.

$\left(\frac{\partial^2 v}{\partial T^2}\right)_p = -\frac{mn(n+1)}{T^{n+2}}(1+3ap^{1/2})$, whence $\left(\frac{\partial c_p}{\partial p}\right)_T = \frac{Amn(n+1)}{T^{n+1}}(1+3ap^{1/2})$, and

$c_p = \frac{Amn(n+1)}{T^{n+1}}p(1+2ap^{1/2}) + f(T)$.

It follows that c_p for superheated steam varies with the temperature and also with the pressure.

Graphical Representation. The change of state of a substance may be shown graphically by taking any two of the six variables p, V, T, S, U, I, as independent co-ordinates and drawing a curve to represent the successive values of these two variables as the change proceeds. While any pair may be chosen, there are three systems of graphical representation that are specially useful.

1. p and V. The curve (Fig. 2) represents the simultaneous values of p and V during the change from state 1 to state 2. The area between the curve and the axis OV is given by the integral $\int_{V_1}^{V_2} pdV$ and therefore represents the external work W_{12} done by the gas during the change. The area included by a closed cycle represents the work of the cycle (as in the indicator diagram of the steam engine).

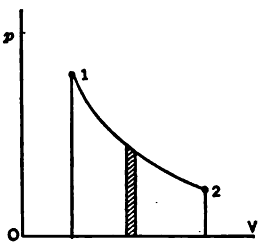

FIG. 2.

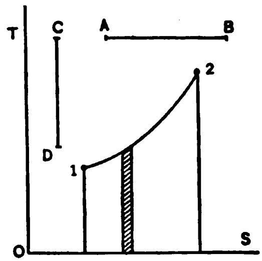

FIG. 3.

2. T and S (Fig. 3). The absolute temperature T is taken as the ordinate; the entropy S as the abscissa. The area between the curve of change of state and the S-axis is given by the integral $\int_{S_1}^{S_2} TdS$, and it therefore represents the heat Q_{12} absorbed by the substance from external sources provided there are no irreversible frictional effects. On the T-S diagram an isothermal is a straight line, as AB, parallel to the S-axis; a reversible adiabatic is a straight line, as CD, parallel to the T-axis.

In the case of internal generation of heat through friction, as in steam turbines, the increase of entropy is given by $\int_{T_1}^{T_2} \frac{dH}{T}$ (see p. 312) and the area under the curve represents the heat H thus generated. In this case an adiabatic is *not* a straight line parallel to the T-axis.

3. I and S. In the system of representation devised by Dr. Mollier, the heat content I is taken as the ordinate and the entropy S as the abscissa. If on this diagram (Fig. 4) a line of constant pressure, as 12, be drawn, the heat absorbed during the change at constant pressure, is given by $Q_{12} = I_2 - I_1$, and this is represented by the line segment 23. The **Mollier diagram** is specially useful in problems that involve the flow of fluids, throttling, and the action of steam in turbines. The advantage lies in the fact that heat and work are represented by line segments instead of areas. A Mollier diagram for steam is given on p. 330, and for ammonia on p. 334.

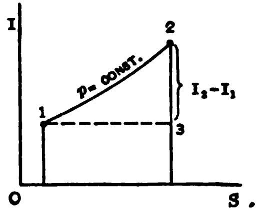

FIG. 4.

PERFECT GASES

Laws of Gases. The so-called perfect gases are those that obey very closely the laws of Boyle and Gay-Lussac. The two laws are combined in the characteristic equations $pv = RT$, $pV = MRT$, in which R is a constant for any gas. The value of R for a gas is inversely proportional to the molecular weight m of the gas and is given by the relation $1544/m$.

Since for a perfect gas $\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v} = \frac{T}{p}$, the general equation (p. 313) becomes $\left(\frac{\partial u}{\partial v}\right)_T = AT\left(\frac{\partial p}{\partial T}\right)_v - Ap = 0$; that is, the internal energy u is independent of the volume and depends on the temperature only.

Specific Heats. According to the experiments of Regnault and others the specific heats c_p and c_v of diatomic gases (oxygen, nitrogen, etc.) are practically constant for considerable ranges of temperature, and in ordinary applications that involve only moderate ranges of temperature they may be taken as constant. Other gases, as superheated steam, ammonia, and carbon dioxide, show a considerable variation of specific heat with the temperature. The values of c_p and c_v in Table 28 are therefore to be considered as approximately correct for the range 32 deg.–400 deg. fahr. For further discussion of specific heats of various gases and of superheated steam, see pp. 365, and 327.

The difference of the specific heats c_p and c_v is given by the relation $c_p - c_v = AR = 1.9855/m$, or approximately $2/m$. The ratio c_p/c_v is denoted by k, and the value of k for diatomic gases is about 1.4.

Specific Weight and Specific Volume. Referred to standard atmospheric pressure and a temperature of 32 deg. fahr., the weight of a cubic foot of gas is 0.002788 m lb. and the volume of one pound in cubic feet is $358.65/m$. The **mol** is frequently taken as the unit of weight for gases. The mol is defined as m lb. where m denotes the molecular weight; thus, for oxygen, 1 mol = 32 lb. For any gas, therefore, the volume of 1 mol at 32 deg. fahr. and atmospheric pressure is 358.65 cu. ft. If M' denotes the weight in mols, then $M = M'm$, and the characteristic equation becomes

$$pV = 1544\ M'T, \quad \text{or} \quad ApV = 1.9855\ M'T$$

The specific heats referred to the mol as the unit of weight are mc_p and mc_v, respectively, and $mc_p - mc_v = 1.9855$ ($= 2$, approx.); $mc_p = 1.9855k/(k - 1)$; $mc_v = 1.9855/(k - 1)$.

Taking $k = 1.4$ for diatomic gases, then approximately $mc_p = 7$, $mc_v = 5$.

Table 28. Properties of Gases

Gas	Chemical symbol	Number of atoms	Molecular weight		Weight in lb. of 1 cu. ft. at atmos. pressure		Density relative to air	Gas constant, R	Specific* heat per lb.		Specific heat per cu. ft. at atmos. pressure and 62 deg. fahr.		$k = \frac{c_p}{c_v}$
			Approximate	Exact, $O_2 = 32$	At 62 deg. fahr.	At 32 deg. fahr.			c_p	c_v	c_p	c_v	
Helium...	He	1	4.0	4.0	0.0105	0.0112	0.137	386.0	1.25	0.75	0.0131	0.0079	1.66
Argon....	Ar	1	40.0	39.9	0.1048	0.1112	1.378	38.70	0.124	0.075	0.0131	0.0079	1.66
Air.......	....	..	29.0	28.95	0.0761	0.0807	1	53.34	0.241	0.171	0.0183	0.0130	1.40
Oxygen...	O_2	2	32.0	32	0.0840	0.0892	1.105	48.25	0.217	0.155	0.0182	0.0130	1.40
Nitrogen..	N_2	2	28.0	28.08	0.0737	0.0783	0.970	54.99	0.247	0.176	0.0182	0.0130	1.40
Hydrogen	H_2	2	2.0	2.016	0.00529	0.00562	0.0696	765.86	3.42	2.44	0.0181	0.0129	1.40
Nitric oxide.....	NO	2	30.0	30.04	0.0789	0.0838	1.038	51.40	0.231	0.165	0.0183	0.0130	1.40
Carbon monoxide	CO	2	28.0	28.00	0.0734	0.0780	0.968	55.14	0.243	0.172	0.0180	0.0126	1.41
Hydrochloric acid.....	HCl	2	36.5	36.45	0.0958	0.1017	1.260	42.35	0.191	0.136	0.0183	0.0130	1.40
Carbon dioxide..	CO_2	3	44.0	44.00	0.1156	0.1227	1.520	35.09	0.210	0.160	0.0243	0.0185	1.31
Nitrous oxide....	N_2O	3	44.0	44.08	0.1157	0.1229	1.522	35.03	0.221	0.171	0.0256	0.0198	1.26
Sulphur dioxide..	SO_2	3	64.0	64.06	0.1684	0.1786	2.213	24.10	0.154	0.123	0.0260	0.0207	1.25
Ammonia.	NH_3	4	17.0	17.06	0.04483	0.0476	0.590	90.50	0.523	0.399	0.0234	0.0178	1.31
Acetylene.	C_2H_2	4	26.0	26.02	0.0684	0.0725	0.899	59.34	0.350	0.270	0.024	0.0185	1.28
Methyl chloride..	CH_3Cl	5	50.5	50.47	0.1326	0.1407	1.744	30.59	0.24	0.20	0.032	0.0265	1.20
Methane..	CH_4	5	16.0	16.03	0.0421	0.0447	0.554	96.31	0.593	0.450	0.025	0.019	1.32
Ethylene..	C_2H_4	6	28.0	28.03	0.0738	0.0780	0.969	55.08	0.40	0.33	0.029	0.024	1.20

* For more accurate values of specific heats, see pp. 365, 1514.

Gas Mixtures. Let V denote the total volume of the mixture, M_1, M_2, M_3 . . . the weights of the constituent gases, R_1, R_2, R_3 . . . the corresponding gas constants, and R_m the constant for the mixture. The **partial pressures** of the constituents, that is, the pressures that the constituents would have if occupying the total volume V, are $p_1 = M_1R_1T/V$, $p_2 = M_2R_2T/V$, etc.

According to Dalton's law, the **total pressure** p **of the mixture** is the sum of the partial pressures; that is, $p = p_1 + p_2 + p_3 + \ldots$ Let $M = M_1 + M_2 + M_3 + \ldots$ denote the total weight of the mixture; then $pV = MR_mT$ and $R_m = \Sigma(M_iR_i)/M$. Also $p_1/p = M_1R_1/MR_m$, $p_2/p = M_2R_2/MR_m$, etc.

Let V_1, V_2, V_3, . . . denote the volumes that would be occupied by the constituents at pressure p and temperature T (these are given by the volume composition of the gas). Then $V = V_1 + V_2 + V_3 + \ldots$ and the apparent molecular weight m_m of the mixture is $m_m = \Sigma(m_iV_i)/V$. Then $R_m = 1544/m_m$.

Volume of 1 lb. at 32 deg. fahr. and atmos. pressure $= 358.65/m_m$.

Weight of 1 cu. ft. at 32 deg. fahr. and atmos. pressure $= 0.002788m_m$.

The **specific heats of the mixture** are, respectively

$$c_p = \Sigma(M_ic_{pi})/M, \qquad c_v = \Sigma(M_ic_{vi})/M.$$

Energy, Heat Content, and Entropy. If a gas changes from an initial state p_1, V_1, T_1, to a final state p_2, V_2, T_2, the following equations hold:

$$U_2 - U_1 = Mc_v\,(T_2 - T_1) = A(p_2V_2 - p_1V_1)/(k-1)$$

$$I_2 - I_1 = Mc_p\,(T_2 - T_1) = Ak(p_2V_2 - p_1V_1)/(k-1).$$

$$S_2 - S_1 = M\left[c_v \log_e \frac{T_2}{T_1} + AR \log_e \frac{V_2}{V_1}\right] = M\left[c_p \log_e \frac{T_2}{T_1} - AR \log_e \frac{p_2}{p_1}\right].$$

$$= M\left[c_p \log_e \frac{V_2}{V_1} + c_v \log_e \frac{p_2}{p_1}\right].$$

In general, the energy per unit weight is $u = c_vT + u_0$,
the heat content is $i = c_pT + i_0$,
and the entropy is $s = c_v \log T + AR \log v + s_0$
$= c_p \log T - AR \log p + s'_0 = c_p \log v + c_v \log p + s''_0$

The two fundamental equations for gases are

$$dq = c_vdT + Apdv, \qquad dq = c_pdT - Avdp.$$

Special Changes of State

In the following formulæ the subscripts 1 and 2 refer to the initial and final states, respectively.

1. Constant Volume: $p_2/p_1 = T_2/T_1$. $Q_{12} = U_2 - U_1 = Mc_v\,(t_2 - t_1)$. $Q_{12} = A\,V(p_2 - p_1)/(k-1)$. $W = 0$. $s_2 - s_1 = Mc_v \log_e(T_2/T_1)$.

2. Constant Pressure: $V_2/V_1 = T_2/T_1$. $W_{12} = p(V_2 - V_1) = MR\,(t_2 - t_1)$. $Q_{12} = Mc_p\,(t_2 - t_1) = AkW_{12}/(k-1)$. $s_2 - s_1 = Mc_p \log_e(T_2/T_1)$.

3. Isothermal. (Constant Temperature): $p_2/p_1 = V_1/V_2$. $U_2 - U_1 = 0$. $W_{12} = MRT \log_e(V_2/V_1) = p_1V_1 \log_e(V_2/V_1)$. $Q_{12} = AW_{12}$. $s_2 - s_1 = Q_{12}/T = MAR \log_e(V_2/V_1)$.

4. Adiabatic: $T_2/T_1 = (V_1/V_2)^{k-1} = (p_2/p_1)^{(k-1)/k}$. $p_1V_1^k = p_2V_2^k$.

$$AW_{12} = U_1 - U_2 = Mc_v(t_1 - t_2). \quad Q_{12} = 0. \quad s_2 - s_1 = 0.$$

$$W_{12} = (p_1V_1 - p_2V_2)/(k-1) = p_1V_1[1 - (p_2/p_1)^{(k-1)/k}]/(k-1).$$

5. Polytropic. This name is given to the change of state which is represented by the equation pV^n = const. The polytropic curve usually represents sufficiently well actual expansion and compression curves in motors and air compressors. By giving n different values the preceding changes are made special cases of the polytropic change, thus,

for $n = 1$,	pv = const.	isothermal
$n = k$,	pv^k = const.	adiabatic
$n = 0$,	p = const.	constant pressure
$n = \infty$,	v = const.	constant volume

For a polytropic change the specific heat is constant and is given by the relation $c_n = c_v(n-k)/(n-1)$; hence for $1 < n < k$, c_n is negative. This is the case in **air compression.** The following are the principal formulæ:

$p_1V_1^n = p_2V_2^n$. $\quad T_2/T_1 = (V_1/V_2)^{n-1} = (p_2/p_1)^{(n-1)/n}$.

$$W_{12} = (p_1V_1 - p_2V_2)/(n-1) = p_1V_1[1 - (p_2/p_1)^{(n-1)/n}]/(n-1).$$

$$Q_{12} = Mc_n(t_2 - t_1).$$

$$AW_{12} : U_2 - U_1 : Q_{12} = k - 1 : 1 - n : k - n$$

Table 29. Adiabatic and Polytropic Expansion

$\frac{p_1}{p_2}$	$n =$				$n =$			
	1.4 (Adiabatic)	1.3	1.2	1.1	1.4 (Adiabatic)	1.3	1.2	1.1
	$V_2/V_1 = (p_1/p_2)^{1/n} =$				$T_1/T_2 = (p_1/p_2)^{(n-1)/n} =$			
1.1	1.070	1.076	1.083	1.090	1.028	1.022	1.016	1.009
1.2	1.139	1.151	1.164	1.180	1.053	1.043	1.031	1.017
1.3	1.206	1.224	1.244	1.269	1.078	1.062	1.045	1.024
1.4	1.271	1.295	1.323	1.358	1.101	1.081	1.058	1.031
1.5	1.336	1.366	1.401	1.445	1.123	1.098	1.070	1.038
1.6	1.399	1.436	1.479	1.533	1.144	1.115	1.081	1.044
1.7	1.461	1.504	1.557	1.620	1.164	1.130	1.092	1.050
1.8	1.522	1.571	1.633	1.706	1.183	1.145	1.103	1.055
1.9	1.581	1.638	1.706	1.791	1.201	1.160	1.113	1.060
2.0	1.641	1.705	1.782	1.879	1.219	1.174	1.123	1.065
2.5	1.924	2.023	2.145	2.300	1.299	1.235	1.165	1.087
3.0	2.193	2.330	2.498	2.715	1.369	1.289	1.201	1.105
3.5	2.449	2.624	2.842	3.126	1.431	1.336	1.232	1.121
4.0	2.692	2.907	3.177	3.505	1.487	1.378	1.260	1.134
4.5	2.926	3.187	3.500	3.925	1.526	1.415	1.285	1.147
5.0	3.156	3.449	3.824	4.320	1.583	1.449	1.307	1.157
5.5	3.378	3.712	4.142	4.710	1.627	1.482	1.328	1.167
6.0	3.598	3.970	4.447	5.100	1.668	1.512	1.348	1.177
6.5	3.809	4.218	4.760	5.483	1.707	1.540	1.366	1.186
7.0	4.012	4.467	5.058	5.861	1.742	1.566	1.383	1.194
7.5	4.217	4.710	5.360	6.250	1.778	1.591	1.399	1.201
8.0	4.415	4.950	5.650	6.620	1.811	1.616	1.414	1.208
9.0	4.800	5.420	6.240	7.370	1.873	1.660	1.442	1.221
0.0	5.188	5.885	6.820	8.120	1.931	1.701	1.468	1.233

Construction of Polytropic Curves (Fig. 5). Through the origin O a line OA is drawn at any convenient angle a with the V-axis. Then a second line OB is drawn making the angle b with the p-axis, where angle b is determined by the relation $(1 + \tan b) = (1 + \tan a)^n$. From the initial point 1 (p_1, V_1) locate points C and D on the axes. Then, by drawing 45-deg. lines between the axes and the auxiliary lines, as shown in the figure, the successive points on the curve, 2, 3, 4, etc., are obtained.

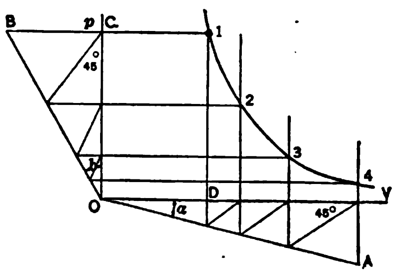

Fig. 5.—Graphical Construction for Polytropic Curves.

Determination of Exponent n. Lay off successive values of p and V, measured at chosen points on the curve under investigation, on logarithmic cross-section paper; or, lay off values of log p and log V on ordinary cross-section paper. If n is a constant, the points will lie in a straight line, and the slope of the line gives the value of n.

If two representative points (p_1, V_1 and p_2, V_2) be chosen, then

$$n = (\log p_1 - \log p_2)/(\log V_1 - \log V_2).$$

Several pairs of points should be used in order to test the constancy of n.

Changes of State with Variable Specific Heat. In case of a considerable range of temperature, as in the internal-combustion motor, the assumption of constant specific heat is not permissible, and the equations referring to changes of state must be suitably modified. Experiments on the specific heat of various gases show that the specific heat may be taken as a linear function of the temperature: thus, $c_v = a + bT$; $c_p = a' + bT$.

From the fundamental equation for gases, $dq = c_v dT + Apdv$, the following expressions are found for the change of energy and entropy, respectively:

$$U_2 - U_1 = M[a(T_2 - T_1) + 0.5b\,(T_2^2 - T_1^2)]$$

$$S_2 - S_1 = M[a \log_e (T_2/T_1) + b(T_2 - T_1) + AR \log_e (V_2/V_1)].$$

For an adiabatic change,

$$W_{12} = J(U_1 - U_2)$$

$$AR \log_e (p_2/p_1) = (a + AR) \log_e (T_2/T_1) + b(T_2 - T_1)$$

$$AR \log_e (V_1/V_2) = a \log_e (T_2/T_1) + b(T_2 - T_1)$$

Ideal Cycles with Perfect Gases

Gases are used as heat media in several important types of motors. In air compressors, air engines, and air refrigerating machines, atmospheric air is the medium. In the internal-combustion engine the medium is a mixture of products of combustion. Motors using gases are operated in certain well-defined cycles, which are described in the following sections. In the analyses given ideal conditions that cannot be attained by actual motors are assumed. However, conclusions derived from such analyses are usually valid for the modified actual cycle.

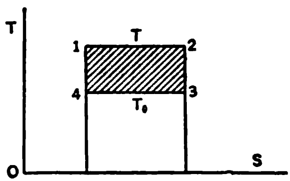

FIG. 6.—Carnot Cycle.

In the following the subscripts 1, 2, 3, etc., refer to corresponding points shown in the figures. The work of the cycle is denoted by (W) and the net heat absorbed by (Q).

Carnot's Cycle. The Carnot cycle (Fig. 6) has historical interest only. It consists of two isothermals and two adiabatics. The heat absorbed along the upper isothermal 12 is $Q_{12} = MART \log_e (V_2/V_1)$, and the heat transformed into work, represented by the cycle area is $A(W) = Q_{12}[1 - (T_0/T)]$.

Hence $\quad (W) = MR(T - T_0) \log_e (V_2/V_1)$

If the cycle is traversed in the reverse sense, $Q_{43} = MART_0 \log_e (V_3/V_4)$ is the heat absorbed from the cold body (brine), and the ratio $Q_{43} : A(W) = T_0 : (T - T_0)$ is the **coefficient of performance** of the refrigerating machine.

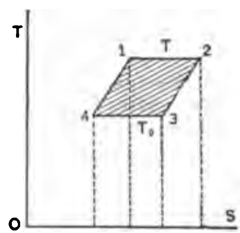

FIG. 7.—Stirling Cycle.

Stirling and Ericsson Cycles. In the Stirling air engine (Fig. 7) the adiabatics of the Carnot cycle are replaced by constant-volume curves; in the Ericsson engine by constant-pressure curves. By the use of a regenerator the heat Q_{23} rejected during the operation 23 is stored and is given back to the medium during the operation 41. In the ideal case, $Q_{23} = Q_{41}$, hence the heat absorbed from the source is $Q_{12} = MART \log_e V_2/V_1$, as in the Carnot cycle, and the efficiency is identical with that of the Carnot cycle.

Air engines of the Stirling and Ericsson type, in which the medium is separated from the furnace by a metal wall, have been failures, and have been replaced by the internal-combustion type, in which the air comes into direct contact with the fuel inside of the working cylinder. The rapid chemical action supported by the medium itself makes possible the rapid heating of large quantities of air to a very high temperature. And by proper cooling of the outside surface of the metal walls, the deterioration of the metal is prevented even at high temperatures.

The ideal cycles usually employed for internal-combustion engines may be classified in two groups: 1. Explosive—Otto. 2. Non-explosive—Diesel, Joule.

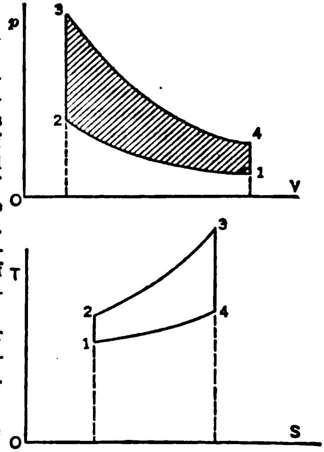

FIG. 8.—Otto Cycle.

The Otto Cycle (Fig. 8). Adiabatic (or polytropic) compression 12 is followed by ignition and rapid heating at constant volume, 23. This is followed by adiabatic expansion, 34. Assuming compression and expansion to be adiabatic, the following relations hold:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{p_3}{p_4}\right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2}\right)^{k-1}.$$

$$Q_{23} = Mc_v(T_3 - T_2).$$

$$(W) = JQ_{23}[1 - (T_1/T_2)] = JMc_v(T_3 - T_4 - T_2 + T_1)$$

$$\text{Efficiency} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{k-1} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}.$$

If the compression and expansion curves are polytropics with the same value of n, replace k by n in the first relation above. In this case,

$$(W) = [(p_3V_3 - p_4V_4) - (p_2V_2 - p_1V_1)]/(n-1) = MR(T_3 - T_4 - T_2 + T_1)/(n-1).$$

The **mean effective pressure** of the diagram is given by

$$p_m = ap_1[(p_3/p_2) - 1],$$

where a has the values given in the following table.

	$p_2/p_1 =$	3	4	5	6	8	10	12	14	16
(n = 1.4)..........	$a =$	1.70	1.94	2.13	2.31	2.62	2.88	3.10	3.31	3.50
(n = 1.3)..........	$a =$	1.69	1.92	2.11	2.28	2.57	2.81	3.08	3.22	3.39
(n = 1.2)..........	$a =$	1.68	1.90	2.08	2.25	2.51	2.74	2.94	3.12	3.27

The Joule Cycle (Fig. 9) consists of two adiabatics and two constant-pressure lines. The following relations hold:

$$V_3/V_2 = V_4/V_1 = T_3/T_2 = T_4/T_1.$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{k-1} = \left(\frac{V_4}{V_3}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

$(W) = JMc_p(T_3 - T_2 - T_4 + T_1)$. Efficiency $= (W)/JQ_{23} = 1 - (T_1/T_2)$.

For the use of the Joule cycle in refrigeration, see p. 346.

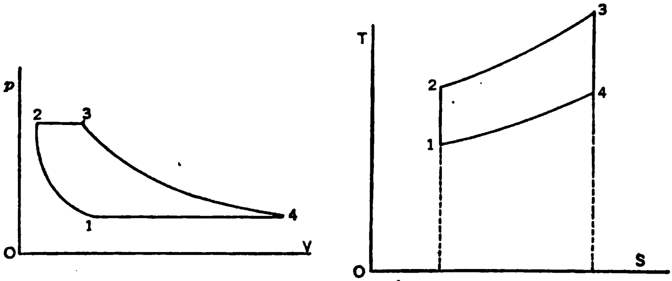

Fig. 9.—Joule Cycle.

The Diesel Cycle. In the Diesel oil engine air is compressed to a pressure of about 500 lb. per sq. in. Fuel is then injected into the air and, as the temperature is above the ignition point, burns at nearly constant pressure (23, in Fig. 10). Adiabatic expansion of the products of combustion is followed by exhaust and suction of fresh air, as in the Otto cycle.

The work obtained is

$$(W) = JM[c_p(T_3 - T_2) - c_v(T_4 - T_1)],$$

and the efficiency of the ideal cycle is

$$1 - [(T_4 - T_1)/k(T_3 - T_2)].$$

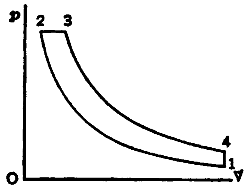

Fig. 10.—Diesel Cycle.

Air Compression. It is assumed that the compressor works under ideal conditions without clearance and without friction losses. If the compression from p_1 to p_2 (Fig. 11*a*) follows the law pV^n = const., the work represented by the indicator diagram is

$$W = n(p_2V_2 - p_1V_1)/(n - 1) = np_1V_1[(p_2/p_1)^{(n-1)/n} - 1]/(n - 1).$$

The temperature at the end of compression is given by $T_2/T_1 = (p_2/p_1)^{(n-1)/n}$. The work W is smaller the smaller the value of n, and to reduce n from the adiabatic value 1.4 is the office of the water jacket. Under usual working conditions, n is about 1.3.

When the pressure p_2 is high it is advantageous to divide the process into two or more stages and cool the air between the cylinders. The saving effected is best shown on the T-S plane (Fig. 11*b*). With single-stage compression, 12 represents the compression from p_1 to p_2, and if the constant-pressure line 23 is drawn cutting the isothermal through point 1 in point 3, the area 1′1233′ represents the work W. When two stages are used, 14 represents the compression from p_1 to an intermediate pressure p', 45 cooling at constant pressure in the intercooler between the cylinders, and 56 the compression in the second stage. The area under 14563 represents the work of the two stages and the area 2456 the saving effected by compounding. This saving is a maximum when $T_4 = T_6$, and this is the case when the intermediate pressure p' is given by $p' = \sqrt{p_1 p_2}$.

The total work in two-stage compression is

$$n\, p_1 V_1 \left[(p'/p_1)^{(n-1)/n} + (p_2/p')^{(n-1)/n} - 2\right] /(n-1).$$

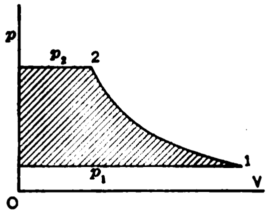

FIG. 11*a*.

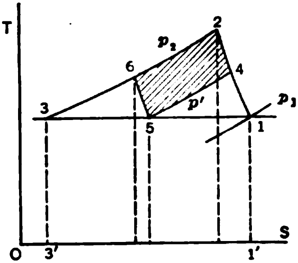

FIG. 11*b*.

Air-compressor Cycle.

VAPORS

General Characteristics of Vapors. Let a gas be compressed at constant temperature; then, provided this temperature does not exceed a certain critical value, the gas begins to liquefy at a definite pressure, which depends upon the temperature. At the beginning of liquefaction, a unit weight of gas will also have a definite volume v'', depending on the temperature. In Fig. 12, AB represents the compression and the point B gives the **saturation** pressure and volume. If the compression is continued the pressure remains constant with the temperature, as indicated by BC, until at C the substance is in the liquid state with the volume v'.

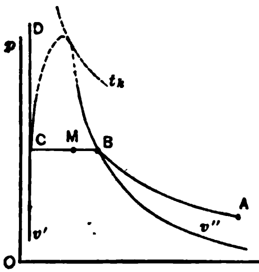

FIG. 12.

The curves v' and v'' giving the volumes for various temperatures at the end and beginning of liquefaction, respectively, may be called the **limit curves.** A point B on curve v'' represents the state of **saturated vapor**; a point C on the curve v' represents the liquid state; and a point M between B and C represents a mixture of vapor and liquid of which the part $x = MC/BC$ is vapor and the part $1 - x = BM/BC$ is liquid. The ratio x is called the **quality of the mixture.** The region between the curves v' and v'' is thus the region of liquid and vapor mixtures. The region to the right of curve v'' is the region of **superheated vapor.** The curve v'' dividing these regions represents the so-called **saturated vapor.**

For saturated vapor or a mixture of vapor and liquid, the pressure is a function of the temperature only, and the volume of the mixture depends upon the temperature and quality x. That is, $p = f(t)$, $v = F(t, x)$.

For the vapor in the superheated state the volume depends on pressure and temperature $[v = F_1(p, t)]$, and these may be varied independently.

Critical States. If the temperature of the gas lies above a definite temperature t_k called the **critical temperature,** the gas cannot be liquefied by compression alone. The saturation pressure corresponding to t_k is the **critical pressure** and is denoted by p_k. At the critical state the limit curves v' and v'' merge; hence for temperatures above t_k it is impossible to have a mixture of vapor and liquid. Table 30 gives the critical data for various gas; also the boiling temperature t_b corresponding to atmospheric pressure.

Table 30. Critical Data for Various Gases

Substance	t_b, deg. fahr.	t_k, deg. fahr.	p_k, atmospheres	Substance	t_b, deg. fahr.	t_k, deg. fahr.	p_k, atmospheres
Acetylene, C_2H_2..	−117.4	95.0	68.0	Ethylene.........	−157.0	50.0	54
Air..............	....	−220.0	39.0	Helium..........	−450.4	−448.6	3
Alcohol, C_2H_6O..	172.4	421.0	65.0	Hydrogen...	−423.0	−402.0	20
Ammonia, NH_3..	− 27.4	266.0	115.0	Hydrogen chloride	−112.0	125.6	87
Benzol, C_6H_6....	176.0	554.0	50.0	Hydrogen sulphide	− 61.6	212.0	94
Bromine.........	142.0	512.0	....	Methane, CH_4....	−263.0	−115.6	57
Carbon dioxide...	−110.0	88.2	77.0	Nitric oxide, NO	−238.0	−137.2	73
Carbon monoxide	−310.0	−222.0	35.9	Nitrous oxide, N_2O	−134.0	96.8	80
Carbon disulphide	115.0	468.0	78.1	Nitrogen.........	−321.0	−236.0	35
Chloroform......	141.0	500.0	54.9	Oxygen..........	−297.0	−180.4	50
Chlorine.........	− 27.4	289.0	92.0	Pentane, C_5H_{12}...	96.8	386.6	34
Ether, $C_4H_{10}O$...	95.0	381.2	37.0	Sulphur dioxide...	14.0	314.6	80
Ethane..........	−135.0	31.0	45.2	Water...........	212.0	706.1	218

Thermal Properties of Saturated Vapors and of Vapor and Liquid Mixtures

Notation:

v' = volume in cu. ft. of 1 lb. of liquid.
v'' = volume in cu. ft. of 1 lb. of saturated vapor, at given temperature.
c' = specific heat of liquid.
c'' = specific heat of saturated vapor, at given temperature.
r = latent heat, or heat required to vaporize 1 lb. of liquid at given constant pressure and temperature.
$AL = Ap(v'' - v')$ = external latent heat, *i.e.*, the heat equivalent of the external work performed during vaporization.
$l = r - Ap(v'' - v')$ = internal latent heat.
i', i'' = heat content of liquid and saturated vapor, respectively.
u', u'' = internal energy of liquid and saturated vapor, respectively.
s' = entropy of the liquid.
s'' = entropy of saturated vapor.

For a unit weight (1 lb.) of saturated vapor the following relations exist:

$$i'' = i' + r. \qquad u'' = u' + l. \qquad s' = \int_{491.6}^{T} c' dT/T. \qquad s'' = s' + (r/T).$$

$r/T = A\, dp/dt\, (v'' - v')$ (Clapeyron's equation), the derivative dp/dt being determined from the relation $p = f(t)$ between temperature and pressure at saturation.

For a unit weight of mixture of vapor and liquid, quality x, these relations become $i = i' + xr, \quad u = u' + xl, \quad s = s' + (xr/T)$;
also $\qquad v = v' + x(v'' - v')$.

Tables of Thermal Properties of Vapors

Water Vapor. The experimental data relating to the properties of water vapor have been correlated by Marks and Davis; and the values given in the Marks and Davis tables may be accepted as the most accurate that have yet been published.

Table 31. Properties of Saturated Steam

Condensed from the Steam Tables of Marks and Davis by permission of the publishers, Longmans, Green and Co.

Abs. pres., lb., p	Temp., deg. fahr., t	Sp. vol., cu. ft. per lb., v''	Density, lb. per cu. ft., $1/v''$	Heat of the liquid, i'	Latent heat of evap., r	Heat content of steam, i''	Internal energy (B.t.u.) evap., l	Entropy: Water, s'	Entropy: Evap., r/T
1	101.83	333.0	0.00300	69.8	1034.6	1104.4	972.9	0.1327	1.8427
2	126.15	173.5	0.00576	94.0	1021.0	1115.0	956.7	0.1749	1.7431
3	141.52	118.5	0.00845	109.4	1012.3	1121.6	946.4	0.2008	1.6840
4	153.01	90.5	0.01107	120.9	1005.7	1126.5	938.6	0.2198	1.6416
5	162.28	73.33	0.01364	130.1	1000.3	1130.5	932.4	0.2348	1.6084
6	170.06	61.89	0.01616	137.9	995.8	1133.7	927.0	0.2471	1.5814
7	176.85	53.56	0.01867	144.7	991.8	1136.5	922.4	0.2579	1.5582
8	182.86	47.27	0.02115	150.8	988.2	1139.0	918.2	0.2673	1.5380
9	188.27	42.36	0.02361	156.2	985.0	1141.1	914.4	0.2756	1.5202
10	193.22	38.38	0.02606	161.1	982.0	1143.1	910.9	0.2832	1.5042
11	197.75	35.10	0.02849	165.7	979.2	1144.9	907.8	0.2902	1.4895
12	201.96	32.36	0.03090	169.9	976.6	1146.5	904.8	0.2967	1.4760
13	205.87	30.03	0.03330	173.8	974.2	1148.0	902.0	0.3025	1.4639
14	209.55	28.02	0.03569	177.5	971.9	1149.4	899.3	0.3081	1.4523
14.7	212.00	26.79	0.03732	180.0	970.4	1150.4	897.6	0.3118	1.4447
15	213.0	26.27	0.03806	181.0	969.7	1150.7	896.8	0.3133	1.4416
16	216.3	24.79	0.04042	184.4	967.6	1152.0	894.4	0.3183	1.4311
17	219.4	23.38	0.04277	187.5	965.6	1153.1	892.1	0.3229	1.4215
18	222.4	22.16	0.04512	190.5	963.7	1154.2	889.9	0.3273	1.4127
19	225.2	21.07	0.04746	193.4	961.8	1155.2	887.8	0.3315	1.4045
20	228.0	20.08	0.04980	196.1	960.0	1156.2	885.8	0.3355	1.3965
21	230.6	19.18	0.05213	198.8	958.3	1157.1	883.9	0.3393	1.3887
22	233.1	18.37	0.05445	201.3	956.7	1158.0	882.0	0.3430	1.3811
23	235.5	17.62	0.05676	203.8	955.1	1158.8	880.2	0.3465	1.3739
24	237.8	16.93	0.05907	206.1	953.5	1159.6	878.5	0.3499	1.3670
25	240.1	16.30	0.0614	208.4	952.0	1160.4	876.8	0.3532	1.3604
26	242.2	15.72	0.0636	210.6	950.6	1161.2	875.1	0.3564	1.3542
27	244.4	15.18	0.0659	212.7	949.2	1161.9	873.5	0.3594	1.3483
28	246.4	14.67	0.0682	214.8	947.8	1162.6	872.0	0.3623	1.3425
29	248.4	14.19	0.0705	216.8	946.4	1163.2	870.5	0.3652	1.3367
30	250.3	13.74	0.0728	218.8	945.1	1163.9	869.0	0.3680	1.3311
31	252.2	13.32	0.0751	220.7	943.8	1164.5	867.6	0.3707	1.3257
32	254.1	12.93	0.0773	222.6	942.5	1165.1	866.2	0.3733	1.3205
33	255.8	12.57	0.0795	224.4	941.3	1165.7	864.8	0.3759	1.3155
34	257.6	12.22	0.0818	226.2	940.1	1166.3	863.4	0.3784	1.3107
35	259.3	11.89	0.0841	227.9	938.9	1166.8	862.1	0.3808	1.3060
36	261.0	11.58	0.0863	229.6	937.7	1167.3	860.8	0.3832	1.3014
37	262.6	11.29	0.0886	231.3	936.6	1167.8	859.5	0.3855	1.2969
38	264.2	11.01	0.0908	232.9	935.5	1168.4	858.3	0.3877	1.2925
39	265.8	10.74	0.0931	234.5	934.4	1168.9	857.1	0.3899	1.2882
40	267.3	10.49	0.0953	236.1	933.3	1169.4	855.9	0.3920	1.2841
41	268.7	10.25	0.0976	237.6	932.2	1169.8	854.7	0.3941	1.2800
42	270.2	10.02	0.0998	239.1	931.2	1170.3	853.6	0.3962	1.2759
43	271.7	9.80	0.1020	240.5	930.2	1170.7	852.4	0.3982	1.2720
44	273.1	9.59	0.1043	242.0	929.2	1171.2	851.3	0.4002	1.2681
45	274.5	9.39	0.1065	243.4	928.2	1171.6	850.3	0.4021	1.2644
46	275.8	9.20	0.1087	244.8	927.2	1172.0	849.2	0.4040	1.2607
47	277.2	9.02	0.1109	246.1	926.3	1172.4	848.1	0.4059	1.2571
48	278.5	8.84	0.1131	247.5	925.3	1172.8	847.1	0.4077	1.2536
49	279.8	8.67	0.1153	248.8	924.4	1173.2	846.1	0.4095	1.2502

Table 31. Properties of Saturated Steam—(*continued*)

Condensed from the Steam Tables of Marks and Davis by permission of the publishers, Longmans, Green and Co.

Abs. pres., lb., p	Temp., deg. fahr., t	Sp. vol. cu. ft. per lb., v''	Density, lb. per cu. ft., $1/v''$	Heat of the liquid, i'	Latent heat of evap., r	Heat content of steam, i''	Internal energy (B.t.u.) evap., l	Entropy Water, s'	Entropy Evap., r/T
50	281.0	8.51	0.1175	250.1	923.5	1173.6	845.0	0.4113	1.2468
51	282.3	8.35	0.1197	251.4	922.6	1174.0	844.0	0.4130	1.2435
52	283.5	8.20	0.1219	252.6	921.7	1174.3	843.1	0.4147	1.2402
53	284.7	8.05	0.1241	253.9	920.8	1174.7	842.1	0.4164	1.2370
54	285.9	7.91	0.1263	255.1	919.9	1175.0	841.1	0.4180	1.2339
55	287.1	7.78	0.1285	256.3	919.0	1175.4	840.2	0.4196	1.2309
56	288.2	7.65	0.1307	257.5	918.2	1175.7	839.3	0.4212	1.2278
57	289.4	7.52	0.1329	258.7	917.4	1176.0	838.3	0.4227	1.2248
58	290.5	7.40	0.1350	259.8	916.5	1176.4	837.4	0.4242	1.2218
59	291.6	7.28	0.1372	261.0	915.7	1176.7	836.5	0.4257	1.2189
60	292.7	7.17	0.1394	262.1	914.9	1177.0	835.6	0.4272	1.2160
61	293.8	7.06	0.1416	263.2	914.1	1177.3	834.8	0.4287	1.2132
62	294.9	6.95	0.1438	264.3	913.3	1177.6	833.9	0.4302	1.2104
63	295.9	6.85	0.1460	265.4	912.5	1177.9	833.1	0.4316	1.2077
64	297.0	6.75	0.1482	266.4	911.8	1178.2	832.2	0.4330	1.2050
65	298.0	6.65	0.1503	267.5	911.0	1178.5	831.4	0.4344	1.2024
66	299.0	6.56	0.1525	268.5	910.2	1178.8	830.5	0.4358	1.1998
67	300.0	6.47	0.1547	269.6	909.5	1179.0	829.7	0.4371	1.1972
68	301.0	6.38	0.1569	270.6	908.7	1179.3	828.9	0.4385	1.1946
69	302.0	6.29	0.1590	271.6	908.0	1179.6	828.1	0.4398	1.1921
70	302.9	6.20	0.1612	272.6	907.2	1179.8	827.3	0.4411	1.1896
71	303.9	6.12	0.1634	273.6	906.5	1180.1	826.5	0.4424	1.1872
72	304.8	6.04	0.1656	274.5	905.8	1180.4	825.8	0.4437	1.1848
73	305.8	5.96	0.1678	275.5	905.1	1180.6	825.0	0.4449	1.1825
74	306.7	5.89	0.1699	276.5	904.4	1180.9	824.2	0.4462	1.1801
75	307.6	5.81	0.1721	277.4	903.7	1181.1	823.5	0.4474	1.1778
76	308.5	5.74	0.1743	278.3	903.0	1181.4	822.7	0.4487	1.1755
77	309.4	5.67	0.1764	279.3	902.3	1181.6	822.0	0.4499	1.1732
78	310.3	5.60	0.1786	280.2	901.7	1181.8	821.3	0.4511	1.1710
79	311.2	5.54	0.1808	281.1	901.0	1182.1	820.6	0.4523	1.1687
80	312.0	5.47	0.1829	282.0	900.3	1182.3	819.8	0.4535	1.1665
81	312.9	5.41	0.1851	282.9	899.7	1182.5	819.1	0.4546	1.1644
82	313.8	5.34	0.1873	283.8	899.0	1182.8	818.4	0.4557	1.1623
83	314.6	5.28	0.1894	284.6	898.4	1183.0	817.7	0.4568	1.1602
84	315.4	5.22	0.1915	285.5	897.7	1183.2	817.0	0.4579	1.1581
85	316.3	5.16	0.1937	286.3	897.1	1183.4	816.3	0.4590	1.1561
86	317.1	5.10	0.1959	287.2	896.4	1183.6	815.6	0.4601	1.1540
87	317.9	5.05	0.1980	288.0	895.8	1183.8	815.0	0.4612	1.1520
88	318.7	5.00	0.2001	288.9	895.2	1184.0	814.3	0.4623	1.1500
89	319.5	4.94	0.2023	289.7	894.6	1184.2	813.6	0.4633	1.1481
90	320.3	4.89	0.2044	290.5	893.9	1184.4	813.0	0.4644	1.1461
91	321.1	4.84	0.2065	291.3	893.3	1184.6	812.3	0.4654	1.1442
92	321.8	4.79	0.2087	292.1	892.7	1184.8	811.7	0.4664	1.1423
93	322.6	4.74	0.2109	292.9	892.1	1185.0	811.0	0.4674	1.1404
94	323.4	4.69	0.2130	293.7	891.5	1185.2	810.4	0.4684	1.1385
95	324.1	4.65	0.2151	294.5	890.9	1185.4	809.7	0.4694	1.1367
96	324.9	4.60	0.2172	295.3	890.3	1185.6	809.1	0.4704	1.1348
97	325.6	4.56	0.2193	296.1	889.7	1185.8	808.5	0.4714	1.1330
98	326.4	4.51	0.2215	296.8	889.2	1186.0	807.9	0.4724	1.1312
99	327.1	4.47	0.2237	297.6	888.6	1186.2	807.2	0.4733	1.1295

Table 31. Properties of Saturated Steam—(*continued*)

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Abs. pres. lb., p	Temp., deg. fahr., t	Sp. vol. cu. ft. per lb., v''	Density lb. per cu. ft., $1/v''$	Heat of the liquid, i'	Latent heat of evap., r	Heat content of steam, i''	Internal energy (B.t.u.) evap., l	Entropy Water, s'	Entropy Evap., r/T
100	327.8	4.429	0.2258	298.3	888.0	1186.3	806.6	0.4743	1.1277
102	329.3	4.347	0.2300	299.8	886.9	1186.7	805.4	0.4762	1.1242
104	330.7	4.268	0.2343	301.3	885.8	1187.0	804.2	0.4780	1.1208
106	332.0	4.192	0.2386	302.7	884.7	1187.4	803.0	0.4798	1.1174
108	333.4	4.118	0.2429	304.1	883.6	1187.7	801.9	0.4816	1.1141
110	334.8	4.047	0.2472	305.5	882.5	1188.0	800.7	0.4834	1.1108
112	336.1	3.978	0.2514	306.9	881.4	1188.4	799.6	0.4852	1.1076
114	337.4	3.912	0.2556	308.3	880.4	1188.7	798.5	0.4869	1.1045
116	338.7	3.848	0.2599	309.6	879.3	1189.0	797.4	0.4886	1.1014
118	340.0	3.786	0.2641	311.0	878.3	1189.3	796.3	0.4903	1.0984
120	341.3	3.726	0.2683	312.3	877.2	1189.6	795.2	0.4919	1.0954
122	342.5	3.668	0.2726	313.6	876.2	1189.8	794.2	0.4935	1.0924
124	343.8	3.611	0.2769	314.9	875.2	1190.1	793.1	0.4951	1.0895
126	345.0	3.556	0.2812	316.2	874.2	1190.4	792.0	0.4967	1.0865
128	346.2	3.504	0.2854	317.4	873.3	1190.7	791.0	0.4982	1.0837
130	347.4	3.452	0.2897	318.6	872.3	1191.0	790.0	0.4998	1.0809
132	348.5	3.402	0.2939	319.9	871.3	1191.2	789.0	0.5013	1.0782
134	349.7	3.354	0.2981	321.1	870.4	1191.5	788.0	0.5028	1.0755
136	350.8	3.308	0.3023	322.3	869.4	1191.7	787.0	0.5043	1.0728
138	352.0	3.263	0.3065	323.4	868.5	1192.0	786.0	0.5057	1.0702
140	353.1	3.219	0.3107	324.6	867.6	1192.2	785.0	0.5072	1.0675
142	354.2	3.175	0.3150	325.8	866.7	1192.5	784.1	0.5086	1.0649
144	355.3	3.133	0.3192	326.9	865.8	1192.7	783.2	0.5100	1.0624
146	356.3	3.092	0.3234	328.0	864.9	1192.9	782.2	0.5114	1.0599
148	357.4	3.052	0.3276	329.1	864.0	1193.2	781.3	0.5128	1.0574
150	358.5	3.012	0.3320	330.2	863.2	1193.4	780.4	0.5142	1.0550
152	359.5	2.974	0.3362	331.4	862.3	1193.6	779.4	0.5155	1.0525
154	360.5	2.938	0.3404	332.4	861.4	1193.8	778.5	0.5169	1.0501
156	361.6	2.902	0.3446	333.5	860.6	1194.1	777.6	0.5182	1.0477
158	362.6	2.868	0.3488	334.6	859.7	1194.3	776.7	0.5195	1.0454
160	363.6	2.834	0.3529	335.6	858.8	1194.5	775.8	0.5208	1.0431
162	364.6	2.801	0.3570	336.7	858.0	1194.7	775.0	0.5220	1.0409
164	365.6	2.769	0.3612	337.7	857.2	1194.9	774.1	0.5233	1.0387
166	366.5	2.737	0.3654	338.7	856.4	1195.1	773.2	0.5245	1.0365
168	367.5	2.706	0.3696	339.7	855.5	1195.3	772.4	0.5257	1.0343
170	368.5	2.675	0.3738	340.7	854.7	1195.4	771.5	0.5269	1.0321
172	369.4	2.645	0.3780	341.7	853.9	1195.6	770.7	0.5281	1.0300
174	370.4	2.616	0.3822	342.7	853.1	1195.8	769.8	0.5293	1.0278
176	371.3	2.588	0.3864	343.7	852.3	1196.0	769.0	0.5305	1.0257
178	372.2	2.560	0.3906	344.7	851.5	1196.2	768.2	0.5317	1.0235
180	373.1	2.533	0.3948	345.6	850.8	1196.4	767.4	0.5328	1.0215
182	374.0	2.507	0.3989	346.6	850.0	1196.6	766.6	0.5339	1.0195
184	374.9	2.481	0.4031	347.6	849.2	1196.8	765.8	0.5351	1.0174
186	375.8	2.455	0.4073	348.5	848.4	1196.9	765.0	0.5362	1.0154
188	376.7	2.430	0.4115	349.4	847.7	1197.1	764.2	0.5373	1.0134
190	377.6	2.406	0.4157	350.4	846.9	1197.3	763.4	0.5384	1.0114
192	378.5	2.381	0.4199	351.3	846.1	1197.4	762.6	0.5395	1.0095
194	379.3	2.358	0.4241	352.2	845.4	1197.6	761.8	0.5405	1.0076
196	380.2	2.335	0.4283	353.1	844.7	1197.8	761.1	0.5416	1.0056
198	381.0	2.312	0.4325	354.0	843.9	1197.9	760.3	0.5426	1.0038

Table 31. Properties of Saturated Steam—(*continued*)

Condensed from the Steam Tables of Marks and Davis by permission of the publishers, Longmans, Green and Co.

Abs. pres., lb., p	Temp., deg. fahr., t	Sp. vol., cu. ft. per lb., v''	Density lb. per cu. ft., $1/v''$	Heat of the liquid, i'	Latent heat of evap., r	Heat content of steam, i''	Internal energy (B.t.u.) evap., I	Entropy Water, s'	Entropy Evap., r/T
200	381.9	2.290	0.437	354.9	843.2	1198.1	759.5	0.5437	1.0019
205	384.0	2.237	0.447	357.1	841.4	1198.5	757.6	0.5463	0.9973
210	386.0	2.187	0.457	359.2	839.6	1198.8	755.8	0.5488	0.9928
215	388.0	2.138	0.468	361.4	837.9	1199.2	754.0	0.5513	0.9885
220	389.9	2.091	0.478	363.4	836.2	1199.6	752.3	0.5538	0.9841
225	391.9	2.046	0.489	365.5	834.4	1199.9	750.5	0.5562	0.9799
230	393.8	2.004	0.499	367.5	832.8	1200.2	748.8	0.5586	0.9758
235	395.6	1.964	0.509	369.4	831.1	1200.6	747.0	0.5610	0.9717
240	397.4	1.924	0.520	371.4	829.5	1200.9	745.4	0.5633	0.9676
245	399.3	1.887	0.530	373.3	827.9	1201.2	743.7	0.5655	0.9638
250	401.1	1.850	0.541	375.2	826.3	1201.5	742.0	0.5676	0.9600
260	404.5	1.782	0.561	378.9	823.1	1202.1	738.9	0.5719	0.9525
270	407.9	1.718	0.582	382.5	820.1	1202.6	735.8	0.5760	0.9454
280	411.2	1.658	0.603	386.0	817.1	1203.1	732.7	0.5800	0.9385
290	414.4	1.602	0.624	389.4	814.2	1203.6	729.7	0.5840	0.9316
300	417.5	1.551	0.645	392.7	811.3	1204.1	726.8	0.5878	0.9251
350	431.9	1.334	0.750	408.2	797.8	1206.1	713.3	0.6053	0.8949
400	444.8	1.17	0.86	422.0	786.0	1208.0	701.0	0.621	0.868
450	456.5	1.04	0.96	435.0	774.0	1209.0	690.0	0.635	0.844
500	467.3	0.93	1.08	448.0	762.0	1210.0	678.0	0.648	0.822
660	486.6	0.76	1.32	469.0	741.0	1210.0	658.0	0.670	0.783

The **relation between pressure and temperature** is most satisfactorily represented by Marks's equation

$$\log p = 10.515354 - (4873.71/T) - 0.00405096T + 0.000001392964T^2.$$

For the **heat content** i'' **of saturated steam** Davis has deduced the equation

$$i'' = 1150.3 + 0.3745(t - 212) - 0.00055(t - 212)^2.$$

This applies to the range 212 deg. to 400 deg. fahr.

The **heat content** i' **of water between 32 deg. and 212 deg. fahr.** is deduced from the experiments of Barnes and Dieterici, and above 212 deg. from the experiments of Dieterici and Regnault. With these fundamental data the remaining properties are readily calculated. For an exhaustive discussion of sources and methods see Marks and Davis, "Steam Tables and Diagrams," pp. 87–106. For density and specific heat of water at saturation pressures see Table 3.

For **condenser calculations** a more detailed table of the properties of saturated steam at temperatures from 50 deg. to 130 deg. fahr. is given in Table 32, which is calculated from the Marks and Davis tables. Vacuums are stated with reference to a 30-in. barometer; the standard atmosphere is equal to 30 in. of mercury when the mercury is at 58.4 deg. fahr. Absolute pressures are given in lb. per sq. in. and also in in. of mercury at 32 deg. fahr.

Properties of Superheated Steam. An experimental basis for the properties of superheated steam is furnished by the researches carried on in the Munich laboratory. Volume measurements have been given by Knoblauch, Linde, and Klebe; specific-heat measurements by Knoblauch and

Jakob and by Knoblauch and Mollier. These two sets of measurements are connected by the Clausius thermodynamic relation

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -AT\frac{\partial^2 v}{\partial T^2}.\quad \text{(See p. 313.)}$$

Table 32. Steam Table for Use in Condenser Calculations
(Calculated from the Steam Tables of Marks and Davis)

Temperature, deg. fahr. t	Vacuum in in. of mercury referred to a 30 in. bar. (Mercury at 58.4 deg. fahr.)	Pressure, lb. per sq. in. absolute p	Pressure, inches of mercury, with mercury at 32 deg. fahr.	Specific volume, cu. ft. per lb. v''	Heat of the liquid i'	Total heat of the steam i''	Internal energy of evaporation l	Entropy of water s'	Entropy of steam s''
50	29.637	0.1780	0.363	1702.0	18.08	1081.4	1007.3	0.0361	2.1226
52	29.609	0.1917	0.390	1586.0	20.08	1082.3	1006.0	0.0401	2.1164
54	29.579	0.2063	0.420	1480.0	22.08	1083.2	1004.6	0.0440	2.1100
56	29.547	0.2219	0.452	1381.0	24.08	1084.1	1003.3	0.0478	2.1037
58	29.513	0.2385	0.486	1291.0	26.08	1085.0	1002.0	0.0517	2.0975
60	29.477	0.2562	0.522	1208.0	28.08	1085.9	1000.7	0.0555	2.0913
62	29.439	0.2749	0.560	1130.0	30.08	1086.8	999.3	0.0593	2.0851
64	29.398	0.2949	0.601	1058.0	32.07	1087.6	998.0	0.0631	2.0791
66	29.354	0.3161	0.644	991.0	34.07	1088.5	996.7	0.0669	2.0731
68	29.308	0.3386	0.690	928.0	36.07	1089.4	995.4	0.0707	2.0672
70	29.259	0.3626	0.739	871.0	38.06	1090.3	994.0	0.0745	2.0613
72	29.208	0.3880	0.790	817.0	40.05	1091.2	992.7	0.0783	2.0556
74	29.153	0.4148	0.845	767.0	42.05	1092.1	991.4	0.0821	2.0499
76	29.095	0.4432	0.903	720.0	44.04	1093.0	990.1	0.0858	2.0443
78	29.034	0.4735	0.964	677.0	46.04	1093.9	988.7	0.0895	2.0386
80	28.968	0.505	1.029	636.8	48.03	1094.8	987.4	0.0932	2.0330
82	28.899	0.539	1.098	598.7	50.03	1095.6	986.1	0.0969	2.0275
84	28.826	0.575	1.171	562.9	52.02	1096.5	984.8	0.1005	2.0220
86	28.749	0.613	1.248	529.5	54.01	1097.4	983.4	0.1041	2.0165
88	28.666	0.654	1.331	498.4	56.01	1098.3	982.1	0.1078	2.0112
90	28.580	0.696	1.417	469.3	58.00	1099.2	980.8	0.1114	2.0058
92	28.489	0.741	1.508	442.2	60.00	1100.1	979.4	0.1151	2.0007
94	28.392	0.789	1.605	417.0	61.99	1101.0	978.1	0.1187	1.9954
96	28.290	0.838	1.706	393.4	63.98	1101.8	976.8	0.1223	1.9903
98	28.183	0.891	1.813	371.4	65.98	1102.8	975.5	0.1259	1.9851
100	28.070	0.946	1.926	350.8	67.97	1103.6	974.1	0.1295	1.9800
102	27.951	1.005	2.045	331.5	69.96	1104.5	972.8	0.1330	1.9750
104	27.825	1.066	2.171	313.3	71.96	1105.3	971.5	0.1365	1.9700
106	27.692	1.131	2.303	296.4	73.95	1106.2	970.1	0.1401	1.9651
108	27.550	1.199	2.443	280.5	75.95	1107.1	968.8	0.1436	1.9602
110	27.404	1.271	2.589	265.5	77.94	1108.0	967.5	0.1471	1.9553
112	27.250	1.346	2.740	251.4	79.93	1108.8	966.2	0.1506	1.9506
114	27.088	1.426	2.904	238.2	81.93	1109.7	964.8	0.1541	1.9458
116	26.919	1.509	3.073	225.8	83.92	1110.6	963.5	0.1576	1.9412
118	26.739	1.597	3.252	214.1	85.92	1111.5	962.2	0.1611	1.9366
120	26.553	1.689	3.438	203.1	87.91	1112.3	960.8	0.1645	1.9319
122	26.355	1.785	3.635	192.8	89.91	1113.2	959.5	0.1679	1.9273
124	26.149	1.886	3.841	183.1	91.90	1114.1	958.2	0.1713	1.9228
126	25.931	1.992	4.057	173.9	93.90	1115.0	956.8	0.1747	1.9183
128	25.706	2.103	4.282	165.3	95.89	1115.8	955.5	0.1781	1.9139
130	25.48	2.219	4.52	157.1	97.89	1116.7	954.1	0.1816	1.9095

Goodenough (*Bulletin* No. 75, Engr. Exper. Station, Univ. of Ill.) gives the following equations for volume and specific heat, which satisfy this relation and at the same time represent accurately the results of the experiments:

$$v - 0.017 = 0.59465\,(T/p) - (1 + 0.0513p^{1/2})\,(m/T^4). \quad \log m = 10.82500.$$

$$c_p = 0.320 + 0.000126T + (23{,}583/T^2) + p(1 + 0.0342p^{1/2})\,(C'/T^5),$$

where p = lb. per sq. in. Also

$$i = 0.320T + 0.000063T^2 - (23{,}583/T) - p(1 + 0.0342p^{1/2})\,(C''/T^4) + 948.7.$$

$$u = 0.2099T + 0.000063T^2 - (23{,}583/T) - p(1 + 0.02992p^{1/2})\,(C'''/T^4) + 948.7.$$

$$s = 0.73683 \log T + 0.000126T - (11{,}792/T^2) - 0.254 \log p - (C^{iv}p/T^5)\,(1 + 0.0342p^{1/2}) - 0.0809.$$

$$\log C' = 11.39361;\ \log C'' = 10.79155;\ \log C''' = \log C^{iv} = 10.69464.$$

The quotient $\int_{t_s}^{t} c_p dt/(t - t_s)$ gives the **mean specific heat of superheated steam** between the saturation temperature t_s and any chosen higher temperature. Table 33 gives values of c_{pm} thus calculated for various pressures and superheats.

Table 33. Mean Specific Heat of Superheated Steam

Pressure, lb. per sq. in.	Superheat, deg. fahr.							
	0	50	100	200	300	400	500	600
1	0.476	0.471	0.466	0.462	0.461	0.460	0.462	0.465
2	0.474	0.470	0.465	0.461	0.460	0.459	0.461	0.464
3	0.475	0.470	0.465	0.461	0.460	0.461	0.462	0.465
5	0.477	0.472	0.467	0.463	0.461	0.462	0.464	0.467
10	0.483	0.477	0.471	0.467	0.465	0.466	0.468	0.470
15	0.490	0.483	0.476	0.471	0.469	0.470	0.471	0.474
20	0.496	0.488	0.480	0.475	0.473	0.473	0.475	0.477
25	0.500	0.493	0.486	0.480	0.477	0.477	0.478	0.480
30	0.506	0.498	0.490	0.483	0.480	0.479	0.481	0.483
40	0.515	0.506	0.498	0.490	0.486	0.484	0.485	0.487
50	0.524	0.515	0.506	0.497	0.491	0.489	0.490	0.491
60	0.534	0.523	0.513	0.503	0.497	0.494	0.494	0.495
80	0.550	0.536	0.525	0.512	0.505	0.502	0.501	0.501
100	0.566	0.550	0.538	0.523	0.514	0.509	0.507	0.507
125	0.582	0.566	0.553	0.535	0.524	0.517	0.515	0.514
150	0.600	0.581	0.566	0.546	0.533	0.525	0.522	0.521
200	0.627	0.606	0.588	0.564	0.549	0.540	0.535	0.533
250	0.653	0.629	0.609	0.581	0.564	0.553	0.547	0.543
300	0.676	0.650	0.627	0.597	0.578	0.565	0.558	0.553

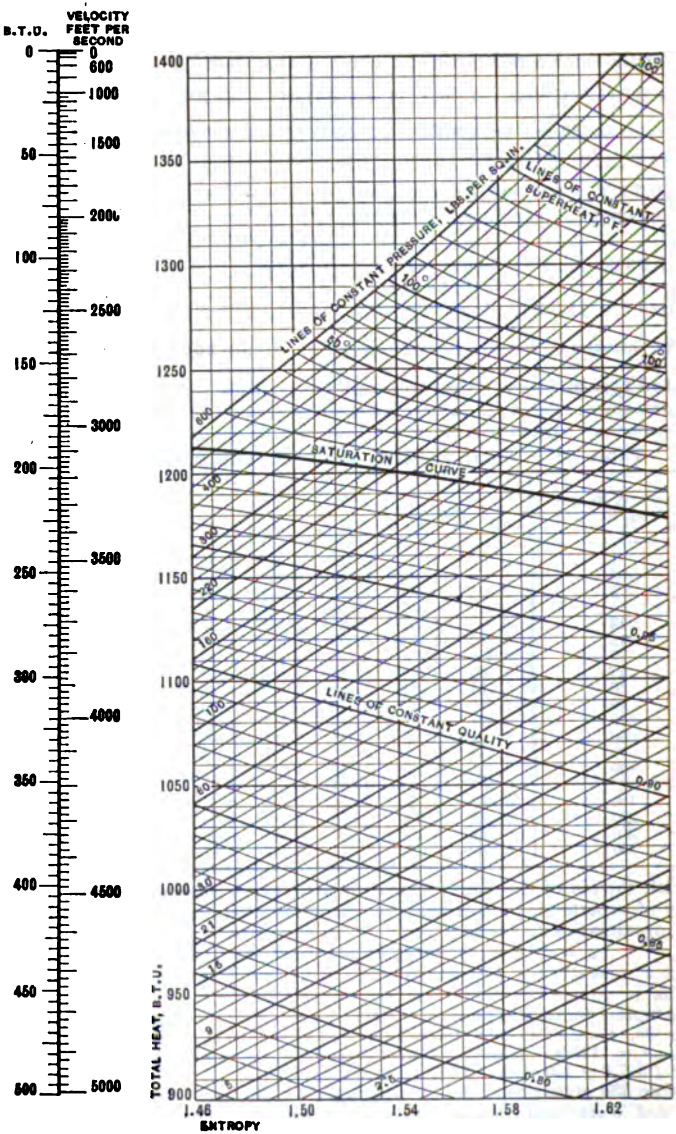
With the aid of the table of mean specific heats, the heat content and entropy of superheated steam may be calculated easily and with a good degree of approximation. Thus,

$$i = i'' + c_{pm}(t - t_s), \qquad s = s'' + c_{pm} \log_e (T/T_s),$$

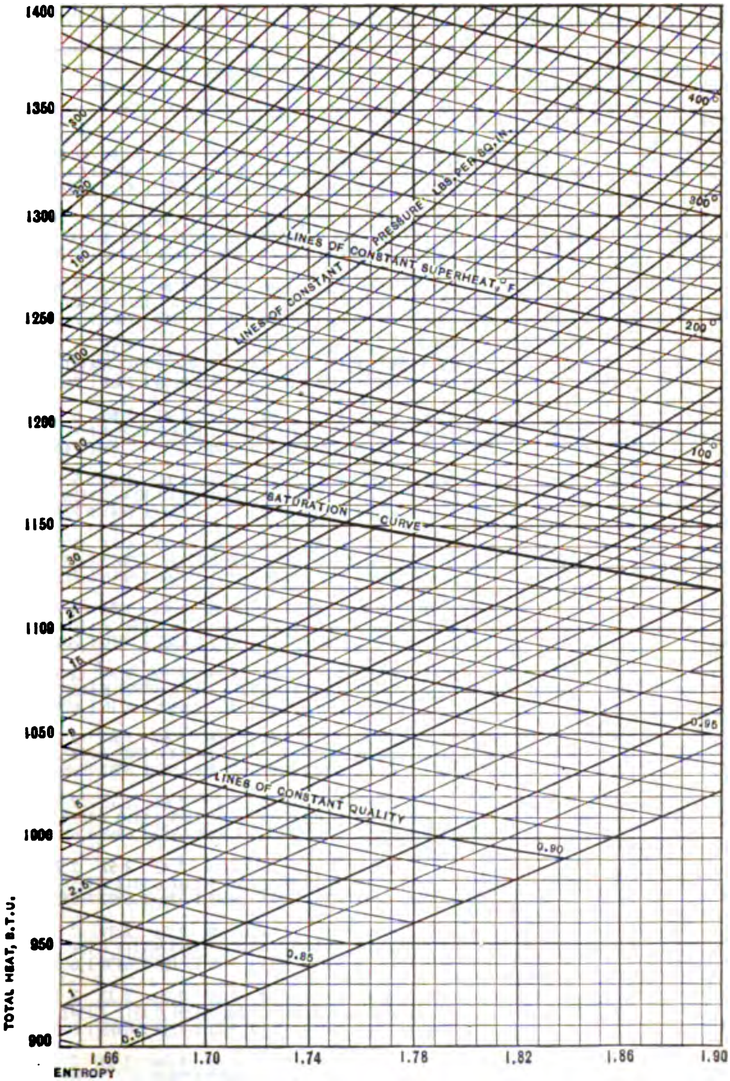
where i'' and s'' [$= s' + (r/T)$] are saturation values taken from the table on p. 325.

Example. Steam at a pressure of 100 lb. per sq. in. is superheated 240 deg. From Table 33, $c_{pm} = 0.519$, and from Table 31, $i'' = 1186.3$, $s'' = 1.6020$, $t_s = 327.8$. Hence, $i = 1186.3 + 0.519 \times 240 = 1310.9$ B.t.u., $s = 1.6020 + 0.519 \log_e [(567.8 + 459.6)/(327.8 + 459.6)] = 1.7402$.

Table 33*a* gives properties of superheated steam condensed from the steam tables of Marks and Davis. A Mollier diagram for steam is given on the following pages.



Total-heat-Entropy Diagram



(Mollier Diagram) for Steam.

Table 33a. Properties of Superheated Steam
(Condensed from Marks and Davis's "Steam Tables and Diagrams")
v = Specific volume in cu. ft. per lb.; i = Total heat in B.t.u. per lb.;
s = Entropy

Pressure, lb. per sq. in. abs.	Temp. of saturated steam, deg. fahr.		Superheat, deg. fahr.								
			20	40	60	80	100	150	200	250	300
		v	20.73	21.37	22.01	22.63	23.25	24.80	26.33	27.85	29.37
20	228.0	i	1165.7	1175.2	1184.6	1194.1	1203.5	1227.1	1250.6	1274.1	1297.6
		s	1.7456	1.7587	1.7716	1.7840	1.7961	1.8251	1.8524	1.8781	1.9026
		v	10.83	11.16	11.50	11.82	12.13	12.93	13.70	14.48	15.25
40	267.3	i	1179.3	1189.1	1198.9	1208.7	1218.4	1242.4	1266.4	1290.3	1314.1
		s	1.6895	1.7025	1.7151	1.7273	1.7392	1.7674	1.7940	1.8189	1.8427
		v	7.40	7.63	7.86	8.08	8.30	8.84	9.36	9.89	10.41
60	292.7	i	1187.3	1197.5	1207.7	1217.7	1227.6	1252.1	1276.4	1300.4	1324.3
		s	1.6568	1.6698	1.6823	1.6944	1.7062	1.7342	1.7603	1.7849	1.8081
		v	5.65	5.83	6.00	6.18	6.34	6.75	7.17	7.56	7.95
80	312.0	i	1193.0	1203.6	1214.0	1224.2	1234.3	1259.0	1283.6	1307.8	1331.9
		s	1.6338	1.6469	1.6594	1.6716	1.6833	1.7110	1.7368	1.7612	1.7840
		v	4.58	4.72	4.86	5.00	5.14	5.47	5.80	6.12	6.44
100	327.8	i	1197.5	1208.4	1219.1	1229.5	1239.7	1264.7	1289.4	1313.6	1337.8
		s	1.6160	1.6294	1.6420	1.6541	1.6658	1.6933	1.7188	1.7428	1.7656
		v	3.85	3.98	4.10	4.22	4.33	4.62	4.89	5.17	5.44
120	341.3	i	1201.1	1212.4	1223.3	1233.8	1244.1	1269.3	1294.1	1318.4	1342.7
		s	1.6016	1.6152	1.6279	1.6400	1.6517	1.6789	1.7041	1.7280	1.7505
		v	3.32	3.44	3.54	3.65	3.75	4.00	4.24	4.48	4.71
140	353.1	i	1204.3	1215.8	1226.8	1237.5	1248.0	1273.3	1298.2	1322.6	1346.9
		s	1.5894	1.6031	1.6159	1.6280	1.6395	1.6666	1.6916	1.7152	1.7376
		v	2.93	3.03	3.12	3.21	3.30	3.53	3.74	3.95	4.15
160	363.6	i	1207.0	1218.8	1230.0	1240.8	1251.3	1276.8	1301.7	1326.2	1350.6
		s	1.5789	1.5928	1.6056	1.6177	1.6292	1.6561	1.6810	1.7043	1.7266
		v	2.62	2.71	2.80	2.88	2.96	3.16	3.35	3.54	3.72
180	373.1	i	1209.4	1221.5	1232.8	1243.8	1254.3	1279.9	1304.8	1329.5	1353.9
		s	1.5697	1.5838	1.5767	1.6088	1.6201	1.6468	1.6716	1.6948	1.7169
		v	2.37	2.45	2.53	2.61	2.68	2.86	3.04	3.21	3.38
200	381.9	i	1211.6	1223.9	1235.5	1246.5	1257.1	1282.6	1307.7	1332.4	1357.0
		s	1.5614	1.5757	1.5886	1.6007	1612.0	1.6385	1.6632	1.6862	1.7082
		v	2.16	2.24	2.31	2.38	2.45	2.62	2.78	2.94	3.10
220	389.9	i	1213.6	1226.2	1237.9	1248.9	1259.6	1285.2	1310.3	1335.1	1359.8
		s	1.5541	1.5686	1.5816	1.5936	1.6049	1.6312	1.6558	1.6787	1.7005
		v	1.99	2.06	2.13	2.20	2.26	2.42	2.57	2.71	2.85
240	397.4	i	1215.4	1228.3	1240.1	1251.3	1261.9	1287.6	1312.8	1337.6	1362.3
		s	1.5476	1.5623	1.5753	1.5873	1.5985	1.6246	1.6492	1.6720	1.6937
		v	1.84	1.91	1.97	2.04	2.10	2.24	2.39	2.52	2.65
260	404.5	i	1217.1	1230.3	1242.3	1253.4	1264.1	1289.9	1315.1	1340.0	1364.7
		s	1.5416	1.5564	1.5695	1.5815	1.5926	1.6186	1.6430	1.6658	1.6874
		v	1.72	1.78	1.84	1.90	1.95	2.09	2.22	2.35	2.48
280	411.2	i	1218.7	1232.2	1244.3	1255.5	1266.2	1291.9	1317.2	1342.2	1367.0
		s	1.5362	1.5512	1.5643	1.5762	1.5873	1.6133	1.6375	1.6603	1.6818
		v	1.60	1.66	1.72	1.78	1.83	1.96	2.09	2.21	2.33
300	417.5	i	1220.2	1234.1	1246.2	1257.4	1268.2	1294.0	1319.3	1344.3	1369.2
		s	1.5310	1.5462	1.5594	1.5713	1.5824	1.6082	1.6323	1.6550	1.6765

Ammonia Vapor. The tables of Goodenough and Mosher (*Bulletin* No. 66, Engineering Exper. Station, University of Illinois), represent the latest and probably the most accurate values of the thermal properties of saturated vapor of ammonia. See Table 34.

Table 34. Properties of Ammonia

Temp., deg. fahr.	Pressure, lb. per sq. in., abs.	Specific volume		Heat content		Heat of vaporization	Internal energy of vaporization	Entropy	
		of liquid, cu. ft. per lb.	of sat. vapor, cu. ft. per lb.	of liquid	of sat. vapor			of liquid	of vaporization
t	p	v'	v''	i'	i''	r	l	s'	r/T
−40	10.12	0.0234	25.45	−75.3	526.6	601.9	554.2	−0.1653	1.4343
−35	11.74	0.0235	22.14	−70.2	528.2	598.3	550.2	−0.1531	1.4090
−30	13.56	0.0236	19.35	−65.0	529.8	594.7	546.2	−0.1410	1.3842
−25	15.61	0.0238	16.95	−59.8	531.3	591.1	542.1	−0.1290	1.3598
−20	17.91	0.0239	14.89	−54.6	532.8	587.4	538.0	−0.1171	1.3360
−15	20.46	0.0240	13.15	−49.4	534.3	583.6	533.9	−0.1054	1.3126
−10	23.30	0.0241	11.63	−44.2	535.7	579.9	529.8	−0.0938	1.2896
− 5	26.46	0.0242	10.32	−38.9	537.1	576.1	525.6	−0.0824	1.2671
0	29.95	0.0244	9.19	−33.7	538.5	572.2	521.4	−0.0709	1.2449
5	33.79	0.0245	8.20	−28.4	539.9	568.3	517.1	−0.0595	1.2231
10	38.02	0.0246	7.34	−23.2	541.2	564.4	512.9	−0.0483	1.2017
15	42.67	0.0248	6.583	−17.9	542.5	560.4	508.6	−0.0372	1.1806
20	47.75	0.0249	5.920	−12.6	543.7	556.3	504.2	−0.0262	1.1599
25	53.30	0.0250	5.336	− 7.3	545.0	552.2	499.8	−0.0153	1.1395
30	59.39	0.0252	4.820	− 1.9	546.2	548.1	495.4	−0.0044	1.1194
35	65.91	0.0253	4.364	+ 3.5	547.4	543.9	491.0	+0.0065	1.0996
40	73.03	0.0255	3.959	8.9	548.5	539.7	486.5	0.0173	1.0801
45	80.75	0.0256	3.599	14.3	549.7	535.3	481.9	0.0280	1.0609
50	89.09	0.0258	3.278	19.8	550.8	531.0	477.3	0.0387	1.0419
55	98.03	0.0259	2.992	25.3	551.9	526.5	472.7	0.0494	1.0231
60	107.7	0.0261	2.734	30.9	552.9	522.0	468.0	0.0601	1.0046
65	118.1	0.0263	2.503	36.5	554.0	517.5	463.3	0.0708	0.9863
70	129.2	0.0264	2.296	42.1	555.0	512.8	458.5	0.0813	0.9683
75	141.1	0.0266	2.109	47.8	556.0	508.1	453.7	0.0919	0.9504
80	153.9	0.0268	1.940	53.6	557.0	503.4	448.8	0.1025	0.9328
85	167.4	0.0270	1.788	59.4	557.9	498.5	443.9	0.1132	0.9153
90	181.8	0.0271	1.650	65.3	558.9	493.5	438.9	0.1238	0.8980
95	197.3	0.0273	1.524	71.3	559.8	488.5	433.9	0.1344	0.8808
100	213.8	0.0275	1.408	77.3	560.7	483.4	428.7	0.1450	0.8638
105	231.2	0.0277	1.305	83.4	561.6	478.2	423.5	0.1557	0.8469
110	249.6	0.0280	1.210	89.6	562.5	472.9	418.3	0.1664	0.8302
115	269.2	0.0282	1.122	95.9	563.3	467.4	412.9	0.1772	0.8135
120	289.9	0.0284	1.042	102.2	564.2	461.9	407.5	0.1881	0.7969
125	311.6	0.0286	0.970	108.7	565.0	456.3	402.0	0.1990	0.7805

The relation between pressure and temperature is derived from the vapor-pressure law, using as a basis the Marks equation (p. 327) connecting the pressure and temperature of saturated steam. For a given pressure p, let T_w denote the saturation temperature (absolute) of water vapor and T_a the corresponding temperature of ammonia vapor. Then

$$T_a = 1/[(1.70356/T_w) - 0.0002242].$$

Below 160 deg. fahr. the volume v' of liquid ammonia is given by

$$v' = 0.06335 - 0.016 \log (273.2 - t).$$

The difference $v'' - v'$ is found from the Clapeyron relation (p. 323), and from this v'', the specific volume of the saturated vapor, is determined. The latent heat is calculated from the empirical formula

$$\log r = 1.856064 + 0.37 \log (273.2 - t).$$

Superheated Ammonia. The specific volume of superheated ammonia is given by the characteristic equation of Mosher, viz.:

$$v + 0.10 = 0.6321\ (T/p) - (79{,}433 \times 10^8/T^5).$$

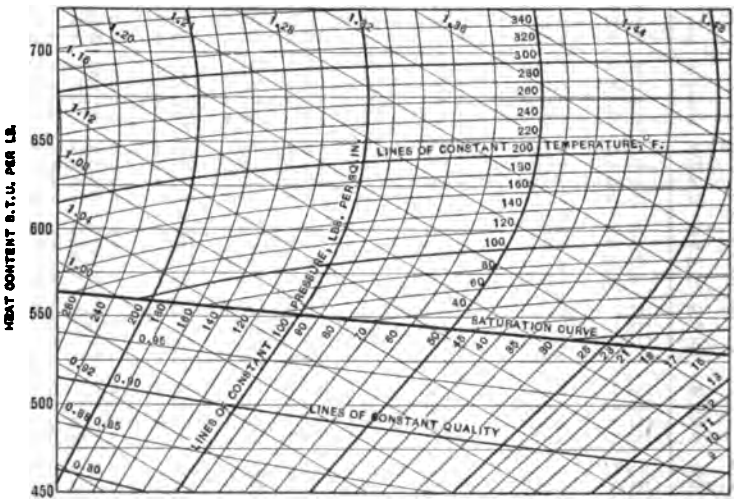
The specific heat at constant pressure is given by the empirical formula

$$c_p = 0.382 + 0.000174T + (Cp/T^6). \quad \log C = 13.644705.$$

For the heat content and entropy the following formulæ are deduced from the preceding:

$$i = 0.382T + 0.000087T^2 - (C'p/T^5) - 0.0185p + 358. \quad \log C' = 12.945734.$$

$$s = 0.8796 \log T + 0.000174T - 0.2695 \log p - (C''p/T^6) - 0.8266.$$
$$\log C'' = 12.866554.$$



Total-heat-Entropy Diagram (Mollier Diagram) for Ammonia.

Table 35 gives these properties of superheated NH_3 for various pressures and degrees of superheat. A Mollier diagram for ammonia is given above. In using this diagram it should be noted that the constant-entropy lines are diagonals (not verticals), and that adiabatics have the same slope as these diagonals.

Sulphur Dioxide (SO_2). The properties of saturated vapor of SO_2 given in Table 36 are based on the researches of Cailletet and Mathias. Certain of these properties are given by the following empirical formulæ:

$$c' = 0.3194 + 0.00065(t - 32). \qquad v' = 0.0113.$$

$$s' = 0.00065(t - 32). \qquad r/T = 0.3327 - 0.00129\,(t - 32).$$

Table 35. Properties of Superheated Ammonia

(Condensed from the Tables of Goodenough and Mosher)

v = Specific volume in cubic feet per pound; i = Total heat in B.t.u. per pound; s = Entropy

Pressure, lb. per sq. in. Abs.	Temp. of saturated vapor, deg. fahr.		Superheat, deg. fahr. 20	40	60	80	100	150	200	250	300
20	−15.9	v	14.2	14.9	15.6	16.3	16.9	18.6	20.2	21.8	23.4
		i	545.2	556.1	566.7	577.1	587.4	612.7	638.0	663.4	689.1
		s	1.2339	1.2569	1.2784	1.2986	1.3178	1.3625	1.4033	1.4413	1.4771
40	12.2	v	7.40	7.76	8.12	8.46	8.80	9.64	10.46	11.26	12.07
		i	553.9	565.5	576.8	587.7	598.3	624.6	650.6	676.6	702.7
		s	1.1742	1.1974	1.2188	1.2390	1.2581	1.3021	1.3422	1.3795	1.4145
50	30.5	v	5.04	5.30	5.54	5.77	6.01	6.57	7.12	7.66	8.20
		i	559.1	571.2	582.9	594.2	605.2	632.1	658.6	684.9	711.3
		s	1.1396	1.1629	1.1846	1.2047	1.2238	1.2676	1.3074	1.3442	1.3787
80	44.5	v	3.84	4.04	4.22	4.40	4.57	5.00	5.42	5.83	6.23
		i	562.8	574.3	587.4	599.0	610.3	637.7	664.6	691.2	717.9
		s	1.1154	1.1389	1.1606	1.1808	1.1999	1.2436	1.2830	1.3196	1.3538
100	56.0	v	3.10	3.26	3.41	3.56	3.70	4.05	4.38	4.71	5.03
		i	565.7	578.6	590.9	602.8	614.3	642.2	669.4	696.3	723.2
		s	1.0968	1.1204	1.1422	1.1625	1.1815	1.2251	1.2645	1.3008	1.3348
120	65.8	v	2.61	2.74	2.87	2.99	3.11	3.40	3.68	3.96	4.23
		i	568.2	581.4	593.9	605.9	617.6	646.0	673.4	700.6	727.6
		s	1.0818	1.1055	1.1274	1.1477	1.1667	1.2103	1.2494	1.2856	1.3195
140	74.5	v	2.25	2.37	2.48	2.58	2.69	2.94	3.17	3.41	3.65
		i	570.2	583.6	596.4	608.7	620.6	649.3	676.9	704.3	731.5
		s	1.0693	1.0931	1.1150	1.1354	1.1544	1.1979	1.2370	1.2730	1.3067
160	82.3	v	1.98	2.08	2.18	2.27	2.36	2.58	2.79	3.00	3.21
		i	572.0	589.7	598.6	611.1	623.2	652.1	680.1	707.6	735.0
		s	1.0585	1.0824	1.1043	1.1247	1.1438	1.1872	1.2263	1.2622	1.2957
180	89.4	v	1.77	1.86	1.94	2.03	2.11	2.31	2.49	2.68	2.86
		i	573.6	587.5	600.6	613.2	625.4	654.7	682.8	710.5	738.1
		s	1.0491	1.0731	1.0950	1.1154	1.1346	1.1779	1.2168	1.2527	1.2861
200	95.9	v	1.59	1.67	1.75	1.83	1.90	2.08	2.25	2.41	2.57
		i	575.0	589.1	602.4	615.2	627.5	657.0	685.4	713.3	741.0
		s	1.0407	1.0648	1.0868	1.1072	1.1263	1.1697	1.2085	1.2442	1.2776
250	110.1	v	1.28	1.35	1.41	1.47	1.53	1.68	1.81	1.95	2.08
		i	578.0	592.4	606.1	619.3	631.9	661.9	690.8	719.0	747.0
		s	1.0232	1.0474	1.0695	1.0899	1.1090	1.1524	1.1911	1.2266	1.2598
300	122.4	v	1.07	1.13	1.18	1.23	1.28	1.41	1.52	1.63	1.74
		i	580.5	595.3	609.3	622.7	635.5	666.1	695.4	723.9	752.1
		s	1.0093	1.0335	1.0556	1.0761	1.0953	1.1386	1.1772	1.2125	1.2455

Carbon Dioxide (CO_2). The properties of CO_2 vapor have been investigated experimentally by Amagat. The results are given in Table 37. The following are the empirical formulæ devised by Mollier:

Critical temperature, T_k = 547.83 deg. fahr., absolute.

$$p = 42.2056\,[(T/180) - 1]^{4.525}\;;\qquad r = 0.6816T^{0.43}(T_k - T)^{0.43}$$

$$c' = 0.000185T + 0.285\,(r/T) + 0.215\,r/(T_k - T)$$

$$s' = 0.10155 + 0.000185(t - 32) - r/2T$$

Table 36. Properties of Sulphur Dioxide

Temp., deg. fahr. t	Pressure, lb. per sq. in. p	Specific volume of sat. vapor v''	Heat content of liquid i'	Heat content of vapor i''	Heat of vaporization r	Internal energy of vaporization l	Entropy of liquid s'	Entropy of vaporization r/T
−25	5.02	16.250	−17.14	159.32	176.46	163.10	−0.0368	0.4057
−20	5.92	12.980	−15.69	159.94	175.63	162.04	−0.0338	0.3996
−15	6.86	10.910	−14.28	160.52	174.80	161.04	−0.0308	0.3935
−10	7.90	9.510	−12.85	161.02	173.87	160.00	−0.0274	0.3869
−5	9.03	8.410	−11.39	161.50	172.89	158.90	−0.0240	0.3803
0	10.35	7.490	−9.907	161.92	171.83	157.70	−0.0208	0.3739
5	11.80	6.660	−8.388	162.29	170.68	156.43	−0.0176	0.3675
10	13.40	5.910	−6.842	162.59	169.43	154.95	−0.0141	0.3608
15	15.15	5.210	−5.321	162.87	168.19	153.59	−0.0110	0.3546
20	17.09	4.635	−3.780	163.12	166.90	152.26	−0.0079	0.3483
25	19.17	4.135	−2.212	163.30	165.51	150.84	−0.0046	0.3418
30	21.46	3.716	−0.622	163.43	164.05	149.36	−0.0012	0.3351
35	24.04	3.350	0.968	163.51	162.54	147.75	0.0020	0.3287
40	26.82	3.005	2.565	163.55	160.99	146.11	0.0051	0.3224
45	29.90	2.700	4.156	163.53	159.37	144.40	0.0083	0.3160
50	33.38	2.433	5.850	163.45	157.60	142.68	0.0117	0.3094
55	36.96	2.210	7.510	163.33	155.82	140.92	0.0150	0.3029
60	40.53	2.003	9.203	163.19	153.99	138.95	0.0182	0.2965
65	44.92	1.817	10.900	162.99	152.09	137.12	0.0215	0.2900
70	49.56	1.650	12.580	162.73	150.15	135.29	0.0248	0.2835
75	54.33	1.499	14.310	162.44	148.13	133.28	0.0280	0.2771
80	59.58	1.362	16.090	162.09	146.00	131.16	0.0312	0.2707
85	65.25	1.238	17.830	161.70	143.87	129.04	0.0345	0.2642
90	71.25	1.137	19.640	161.25	141.61	126.85	0.0377	0.2578
95	77.66	1.041	21.420	160.74	139.32	124.63	0.0410	0.2513
100	84.75	0.950	23.170	160.19	137.02	122.39	0.0443	0.2442
105	92.18	0.867	24.960	159.60	134.64	120.02	0.0475	0.2380

Properties of Other Refrigerating Fluids. In addition to the three uids whose properties have been given in the preceding tables, the following uids have been used to some extent:

Ethyl chloride (C_2H_5Cl); methyl chloride (CH_3Cl); nitrous oxide (N_2O) and Pictet's fluid, a mixture of SO_2 and CO_2 in the proportion of 64 parts SO_2 to 44 parts CO_2 by weight.

The **vapor pressures** of these fluids are given in Table 38. The other thermal properties are not at present well known and the values here given must be regarded as rude approximations. Nitrous oxide closely resembles carbon dioxide in its behavior: the vapor pressure and the density of the two fluids are nearly identical.

The **latent heat** of ethyl chloride is given as 174.5 B.t.u., and that of methyl chloride as 174 B.t.u., temperature not specified, but presumably under atmospheric pressure. Cailletet and Mathias give the following formula for the latent heat of N_2O: $r^2 = 237.15\,(97.5 - t) - 0.928(97.5 - t)^2$.

From this formula the following values are found:

for $t =$	−20	0	20	40	60	80	deg. fahr.
$r =$	122.7	120.4	113.2	102.8	87.1	62.2	

The **specific heat** of N_2O vapor at constant pressure is 0.225. The specific heat of the vapor of ethyl chloride is given by Regnault as 0.274 between 66 deg. and 340 deg. fahr.; the specific heat of the liquid is 0.4276 between −18 deg. and 40 deg. fahr.

The **density** of N_2O may be taken the same as that of CO_2. According to Regnault, the density of ethyl chloride gas is 2.2268 times the density of air.

Table 37. Properties of Carbon Dioxide

Temp. deg. fahr. t	Pressure, lb. per sq. in. p	Specific volume: of liquid, cu. ft. per lb. v'	Specific volume: of vapor, cu. ft. per lb. v''	Heat content: of liquid i'	Heat content: of vapor i''	Heat of vaporisation r	Internal energy of vaporisation l	Entropy: of liquid s'	Entropy: of vaporisation r/T
−25	203.4	0.01551	0.4575	−26.91	100.22	127.13	110.8	−0.0561	0.2925
−20	221.0	0.01556	0.4173	−24.75	100.50	125.25	109.0	−0.0513	0.2851
−15	240.5	0.01565	0.3810	−22.72	100.74	123.46	107.3	−0.0467	0.2778
−10	261.8	0.01578	0.3481	−20.56	100.88	121.44	105.4	−0.0419	0.2702
− 5	284.1	0.01594	0.3185	−18.31	100.99	119.30	103.4	−0.0372	0.2626
0	308.0	0.01612	0.2918	−16.00	101.00	117.00	101.3	−0.0325	0.2549
5	334.2	0.01631	0.2672	−13.73	100.97	114.70	99.2	−0.0276	0.2470
10	362.5	0.01652	0.2450	−11.36	100.89	112.35	96.9	−0.0227	0.2391
15	391.0	0.01675	0.2244	− 8.94	100.70	109.64	94.3	−0.0176	0.2309
20	421.6	0.01696	0.2060	− 6.40	100.43	106.83	91.8	−0.0126	0.2228
25	454.7	0.01720	0.1882	− 3.74	100.08	102.82	88.4	−0.0074	0.2143
30	488.8	0 01747	0.1724	− 1.04	99.43	100.47	86.7	−0.0021	0.2014
35	525.5	0.01776	0.1580	+ 1.74	99.00	97.26	83.6	+0.0032	0.1968
40	564.5	0.01805	0.1444	4.36	98.25	93.89	80.4	0.0087	0.1876
45	606.0	0.01835	0.1323	7.54	97.32	89.78	77.05	0.0145	0.1780
50	650.0	0.01870	0.1205	10.76	96.30	85.54	73.31	0.0205	0.1679
55	696.0	0.01918	0.1090	14.18	95.00	80.82	69.16	0.0268	0.1572
60	744.0	0.01986	0.0986	17.85	93.54	75.69	64.90	0.0334	0.1461
65	794.0	0.02052	0.0890	21.50	91.67	70.17	60.08	0.0406	0 1334
70	847.0	0.02136	0.0816	26.02	89.35	63.33	54.03	0.0483	0.1201
75	906.0	0.02230	0.0706	30.96	86.36	55.40	47.20	0.0576	0.1037
80	965.0	0.02365	0.0614	36.80	82.80	46.00	39.16	0.0684	0.0843
85	1026.0	0.02620	0.0500	44.67	76.60	30.23	26.80	0.0828	0.0559
87	1052.0	0.02782	0.0440	48.98	71.80	23.82	18.25	0.0923	0.0393
88.43	1071.0	0.0346	0.0346	61.45	61.45	0.00	0.00	0.1120	0.000

Table 38. Vapor Pressures of Refrigerating Fluids (Lb. per Sq. In.)

Temp., deg. fahr.	Ethyl chloride C_2H_5Cl	Methyl chloride CH_3Cl	Nitrous oxide N_2O	Pictet's fluid 64 SO_2+ 44 CO_2	Temp., deg. fahr.	Ethyl chloride C_2H_5Cl	Methyl chloride CH_3Cl	Nitrous oxide N_2O	Pictet's fluid 64 SO_2+ 44 CO_2
−20	2.26	11.73	....	11.7	75	22.0	79.8	881	55.8
−15	2.64	13.25	296	12.7	80	24.2	86.7	933	60.1
−10	3.03	14.86	315	13.5	85	26.6	94.2	988	64.5
− 5	3.50	16.69	336	14.4	90	29.1	102.0	1046	69.2
0	4.04	18.73	358	15.6	95	31.8	110.1	1107	74.2
5	4.65	20.87	381	17.0	100	34.7	....	1171	79.5
10	5.31	23.24	405	18.6	105	37.8	....	1239	84.7
15	6.02	25.86	431	20.3	110	41.1	....	....	89.6
20	6.79	28.68	458	22.2	115	44.6	....	....	94.4
25	7.63	31.78	487	24.2	120	48.3	....	....	98.9
30	8.58	35.13	518	26.3	125	52.3	....	....	....
35	9.62	38.83	550	28.6	130	56.5	....	....	....
40	10.76	42.7	584	31.6	135	61.0	....	....	....
45	12.04	47.0	620	34.5	140	65.8	....	....	....
50	13.35	51.5	658	37.6	145	70.7	....	....	....
55	14.73	56.4	698	41.0	150	76.1	....	....	....
60	16.33	61.7	740	44.4	155	81.6	....	....	....
65	18.15	67.3	785	47.8	160	87.6	....	....	....
70	20.02	73.3	832	51.6					

Special Changes of State. Mixtures of Vapor and Liquid

1. Isothermal or Constant Pressure: t = const. p = const. If the initial and final qualities are x_1 and x_2, respectively, then

$$Q = Mr(x_2 - x_1). \qquad U_2 - U_1 = Ml(x_2 - x_1).$$
$$L = p(V_2 - V_1) = Mp\,(v'' - v')(x_2 - x_1).$$

2. Constant Volume:

$$x_2 = x_1\,(v''_1 - v'_1)/(v''_2 - v'_2) = x_1\,v''_1/v''_2, \text{ approx.}$$
$$Q = U_2 - U_1 = M\,[(i'_2 + x_2l_2) - (i'_1 + x_1l_1)]$$

3. Adiabatic: s = const.

$$s'_1 + (x_1r_1/T_1) = s'_2 + (x_2r_2/T_2). \qquad Q = 0.$$
$$L = J(U_1 - U_2) = JM[(i'_1 + x_1l_1) - (i'_2 + x_2l_2)].$$

The relation between p and v during an adiabatic change may be represented approximately by the equation pv^n = const. The exponent n is not constant, however, but varies with the initial quality and initial pressure, as shown by Table 39.

Table 39. Values of n (Water Vapor)

Initial quality	Initial pressure, lb. per sq. in. absolute											
	20	40	60	80	100	120	140	160	180	200	220	240
1.00	1.131	1.132	1.133	1.134	1.136	1.137	1.138	1.139	1.141	1.142	1.143	1.145
0.95	1.127	1.128	1.129	1.130	1.131	1.131	1.132	1.133	1.134	1.135	1.136	1.137
0.90	1.123	1.123	1.124	1.124	1.125	1.125	1.126	1.126	1.127	1.127	1.128	1.129
0.85	1.119	1.119	1.119	1.119	1.120	1.120	1.120	1.120	1.120	1.120	1.120	1.121
0.80	1.115	1.115	1.114	1.114	1.114	1.114	1.113	1.113	1.113	1.113	1.112	1.112
0.75	1.111	1.110	1.110	1.109	1.109	1.108	1.107	1.106	1.106	1.105	1.104	1.104

The volume at the end of expansion (or compression) is $V_2 = V_1\,(p_1/p_2)^{1/n}$, and the external work

$$W = (p_1V_1 - p_2V_2)/(n - 1) = p_1V_1[1 - (p_2/p_1)^{(n-1)/n}]/(n - 1)$$

4. Constant Quality: For a change of state with the quality x constant, an approximate relation between p and v for water vapor, is

$$pv^{1.0631} = 484.2\,x^{1.0631}.$$

For dry steam $x = 1$, and the relation becomes $p^{0.9406}v'' = 327.7$.

This formula may be used for calculating approximately the volume v'' of saturated steam for a given pressure p.

Mixture of Gases and Vapors

Moisture in the Atmosphere. Atmospheric air always contains a certain amount of water vapor mixed with it. The pressure of the vapor cannot exceed the saturation pressure corresponding to the temperature of the atmosphere. Generally, the vapor is in a superheated state, and the pressure is less than the saturation pressure. For a mixture of water vapor and air it may be assumed that Dalton's law holds. Thus, the pressure p_a indicated by the barometer is the sum of p', the pressure of the vapor, and p'', the pressure of the air. On the T-S plane, Fig. 13, let A denote the state of superheated water vapor at the temperature t of the atmosphere, and let m denote the weight of the vapor per

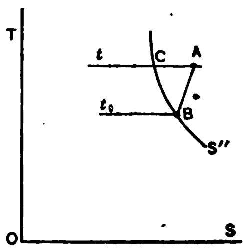


Fig. 13.

cubic foot. Point C represents vapor in the saturated condition at the same temperature t. The density or weight per cubic foot is greater in state C than in state A. Denoting by m_1 the weight per cubic foot in state C, the ratio m/m_1 is the **relative humidity** of the air under the given conditions. If the mixture be cooled at constant pressure, the curve AB represents the change of state of the vapor, and at some lower temperature t_0 the vapor becomes saturated and upon further cooling begins to condense. The temperature t_0 is called the **dew point** corresponding to the state A. If p'_B and p'_C denote the vapor pressures corresponding to the temperatures t_0 and t, respectively (at points B and C), then the relative humidity is approximately the ratio p'_B/p'_C.

Properties of Moist Air.

Let t = temperature of air, deg. fahr.
$T = t + 459.6$, the corresponding absolute temperature.
h = relative humidity.
B = barometric pressure, in inches of mercury.
e = pressure of *saturated* vapor corresponding to temperature t, in inches of mercury.
D' = weight per cu. ft. of saturated vapor at temperature t, lb.
D'' = weight per cu. ft. of dry air at temperature t and pressure B, lb.
D = weight per cu. ft. of mixture at temperature t, and with relative humidity h, lb.
R = 53.34, the characteristic constant of dry air.
R_m = characteristic constant of mixture.

The **partial pressure of** the **vapor** is he, that of the air $B - he$. Hence, by **volume**, the mixture has he/B parts of vapor, and $(B - he)/B$ parts of air; that is, the addition of moisture reduces the effective volume of air 100 he/B per cent.

The constant R_m of the mixture is given by $R_m = 53.34/[1 - 0.378(he/B)]$.

The **weight per cu. ft.** of the mixture is $D = hD' + [D''(B - he)/B]$, and this reduces to $D = 1.326(B/T) - 0.501(he/T)$.

These formulæ apply only when the vapor pressure is small relative to the total pressure. Table 40 facilitates the use of the preceding equations.

Example. The temperature of the air is 84 deg. fahr., the relative humidity 0.70, and the barometric pressure 29.94 in.

From Table 40, the vapor pressure e for 84 deg. is 1.171 in. of mercury and weight of 1 cu. ft. of saturated vapor is 12.37 gr. Hence the air contains $12.37 \times 0.70 = 8.69$ gr. of moisture per cu. ft. and can absorb $12.37 - 8.69 = 3.68$ gr. more before condensation ensues. If the air is cooled the air will become saturated at the pressure $he = 1.171 \times 0.70 = 0.82$ in. of mercury, which corresponds to the temperature 73.5 deg. fahr. This is the dew point. The weight of the mixture per cu. ft. is obtained from the factors in the last two columns of the table: thus, $D = (29.94 \times 0.002439) - (0.70 \times 0.00108) = 0.07226$ lb. The volume of 1 lb. is therefore $1/0.07226 = 13.84$ cu. ft.

Measurement of Humidity. Two methods are available for the determination of the relative humidity of air. 1. The **dew-point method,** in which the dew point is found directly by observing the temperature at which moisture forms on a cooled surface. 2. The **evaporative method,** in which the degree of humidity is deduced from the cooling effect due to the evaporation of moisture in air partially saturated. In the ordinary **wet-bulb psychrometer** the bulb of one thermometer is covered with cloth or wick saturated with water and the bulb of the second thermometer is bare. Evaporation from the wet bulb causes a lowering of temperature, and the difference of the

Table 40. Properties of Air Mixed with Water Vapor

Temp., deg. fahr.	Vapor pressure		Weight of water vapor per cu. ft.		$\frac{1.326}{T} \times 1000$	$0.501\frac{e}{T} \times 1000$
t	Inches of mercury e	Pounds per sq. in.	Grains	Pounds D'		
32	0.1804	0.0886	2.129	0.000304	2.697	0.184
34	0.1955	0.0960	2.297	0.000328	2.686	0.198
36	0.2117	0.1040	2.477	0.000354	2.676	0.214
38	0.2290	0.1125	2.669	0.000381	2.665	0.231
40	0.2477	0.1217	2.872	0.000410	2.654	0.248
42	0.2677	0.1315	3.091	0.000442	2.644	0.267
44	0.2891	0.1420	3.322	0.000475	2.633	0.287
46	0.3119	0.1532	3.570	0.000510	2.623	0.309
48	0.3366	0.1653	3.831	0.000547	2.612	0.332
50	0.3627	0.1781	4.110	0.000587	2.602	0.357
52	0.3905	0.1918	4.411	0.000630	2.592	0.383
54	0.4202	0.2064	4.720	0.000674	2.582	0.410
56	0.4518	0.2219	5.054	0.000722	2.572	0.439
58	0.4856	0.2385	5.410	0.000773	2.562	0.470
60	0.5217	0.2562	5.785	0.000827	2.552	0.503
62	0.5598	0.2749	6.043	0.000883	2.542	0.538
64	0.6005	0.2949	6.604	0.000943	2.532	0.575
66	0.644	0.3161	7.048	0.001007	2.523	0.614
68	0.689	0.3386	7.52	0.001074	2.513	0.655
70	0.738	0.3625	8.02	0.001145	2.504	0.698
72	0.789	0.3879	8.54	0.001220	2.494	0.744
74	0.844	0.4148	9.10	0.001300	2.485	0.793
76	0.903	0.4433	9.68	0.001383	2.476	0.845
78	0.964	0.4735	10.30	0.001471	2.466	0.899
80	1.029	0.5054	10.95	0.001564	2.457	0.956
82	1.098	0.539	11.64	0.001663	2.448	1.016
84	1.171	0.575	12.37	0.001768	2.439	1.080
86	1.248	0.613	13.15	0.001879	2.430	1.146
88	1.329	0.653	13.98	0.001997	2.422	1.216
90	1.415	0.695	14.85	0.002121	2.413	1.290
92	1.506	0.739	15.77	0.002253	2.404	1.368
94	1.602	0.787	16.74	0.002391	2.395	1.450
96	1.704	0.837	17.76	0.002537	2.387	1.537
98	1.812	0.890	18.79	0.002690	2.378	1.629
100	1.925	0.946	19.95	0.002850	2.370	1.724
102	2.044	1.004	21.12	0.00 017	2.361	1.825
104	2.171	1.066	22.35	0.003193	2.353	1.931
106	2.303	1.131	23.63	0.003376	2.344	2.041
108	2.441	1.199	24.96	0.003566	2.336	2.155
110	2.588	1.271	26.36	0.003765	2.328	2.274
120	3.439	1.689	34.42	0.004916	...	...
130	4.518	2.219	44.45	0.00635	...	...
140	5.874	2.885	56.86	0.00812	...	...
150	7.56	3.714	72.0	0.01029	...	...
160	9.64	4.737	90.5	0.01293	...	...
170	12.19	5.988	112.7	0.01611	...	...
180	15.28	7.506	139.4	0.01991	...	...
190	20.01	9.335	171.0	0.02443	...	...
200	23.46	11.523	208.3	0.02976	...	...
210	28.75	14.122	252.1	0.03601	...	...
212	29.92	14.697	261.7	0.03738	...	...

readings of the two thermometers is the **wet-bulb depression.** The psychrometer with stationary wet bulb is considered unreliable and the **sling psychrometer,** in which the wet bulb is kept in motion, is advocated by the U. S. Weather Bureau.

In the determination of humidity by the evaporative method two temperatures and three vapor pressures come under consideration. These are:

t = dry-bulb temperature.
t' = wet-bulb temperature.
e = vapor pressure corresponding to t.
e' = vapor pressure corresponding to t'.
e_0 = vapor pressure corresponding to dew point t_0.

Various formulæ connecting these temperatures and pressures have been proposed. W. H. Carrier (*Trans. A. S. M. E.*, vol. 33, p. 1005) has deduced rationally the following formula, which may be used with confidence:

$$e_0 = e' - [(B - e')(t - t')/(2800 - 1.3t')].$$

The humidity is given immediately by the relation $h = e_0/e$. According to Carrier, the wet-bulb depression $t - t'$ indicated by the sling psychrometer is subject to a mean radiation error of about 1.6 per cent.; therefore, the observed depression should be increased by this amount before insertion in the formula.

The chart, Fig. 14, prepared by Mr. Carrier on the basis of his psychrometric formula, is convenient in the solution of most problems in hygrometry. The horizontal scale gives dry-bulb temperatures, and the inclined lines wet-bulb temperatures. The relative humidity is read directly from the intersection of these constant-temperature lines. Thus, if the dry-bulb temperature is 78 deg. and the wet-bulb temperature is 65 deg., the humidity is found to be 0.50. The main vertical scale gives the weight of vapor in grains per lb. of dry air. In the case just given, this weight is 71 gr.

The curve marked "total heat" is convenient in calculating the heat that must be removed in cooling air and extracting moisture from it. The ordinates of the curve give the sum of the sensible heat in the air above 0 deg. fahr. and the latent heat of the contained vapor. The following example (from Carrier) illustrates the use of the curve.

Required the refrigeration necessary to cool 1 lb. of air containing 98 gr. of moisture, with a dry-bulb temperature of 95 deg. to a final temperature of 40 deg. saturated. Taking 95 on the horizontal scale and 98 on the main vertical scale, it is found that the wet-bulb temperature is 75 deg. Following the inclined line marked 75 deg. to the saturation curve (*i.e.*, dry-bulb temperature = 75 deg.), and then vertically to the total-heat curve, the total heat corresponding to this condition is found to be 37.8 B.t.u. The total heat at 40 deg. is found to be 15.3 B.t.u., and the difference 22.5 B.t.u. is the heat that must be abstracted per pound of air cooled.

Evaporation and Drying

Boiling in a Vacuum. Multiple Effects. Evaporation in a vacuum is employed in the **evaporation of** sugar and salt **solutions,** milk and other liquids containing water. The same process may be used in drying grain or other wet granular materials. The advantage of evaporation in a vacuum lies in the fact that the liquid will boil at a lower temperature than under atmospheric pressure; hence the range of temperature between the two fluids is increased and with it the rate of heat transmission. The lower temperature of boiling also permits the use of exhaust steam in certain cases. Furthermore, a low temperature of evaporation is imperative in the case of certain organic substances, as milk, gelatine, etc.

Economy of operation demands the use of multiple-effect evaporating systems. In these, the steam arising from the boiling liquid in the first

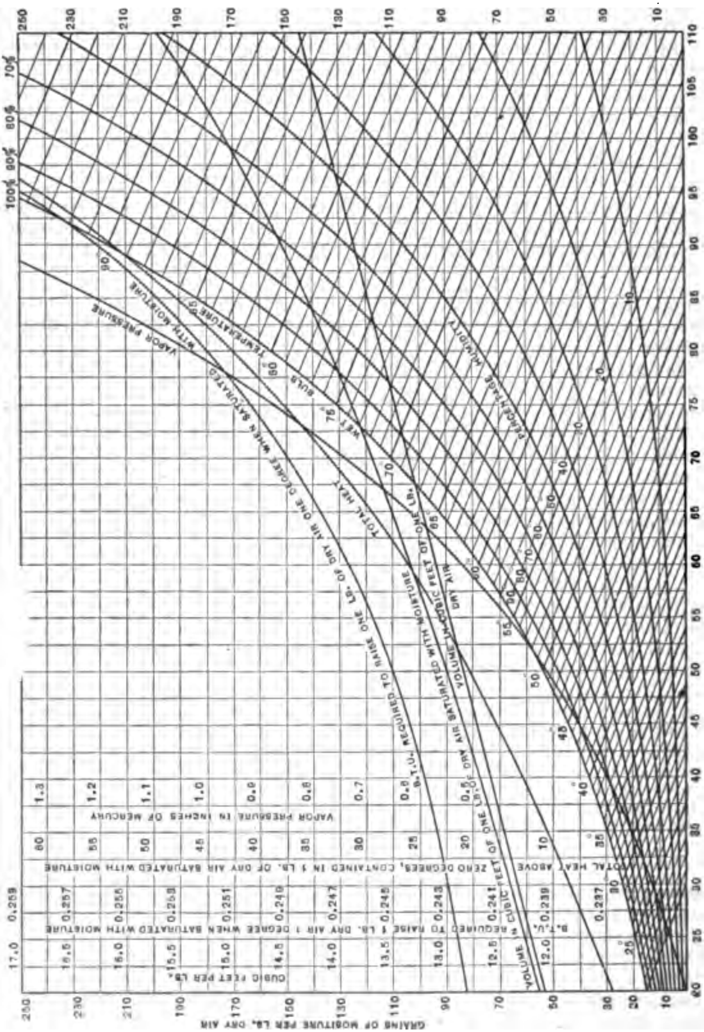


FIG. 14.—Humidity and Psychrometer Chart (Carrier).

vessel is carried to a second vessel in which the boiling temperature is 30 deg. or more lower than in the first vessel. Similarly, steam from the boiling liquid in the second vessel is used as a heating medium in a third vessel, and so on.

An exhaustive treatment of vacuum evaporation is given in Hausbrand's "Evaporating, Condensing and Cooling Apparatus." A good discussion is contained in Creighton's "Steam Engine," chap. xvii.

Theory of Evaporation in a Vacuum.

Let M = weight of liquid introduced in first vessel.
M_1 = water evaporated from liquid in first vessel.
M_2, M_3, etc. = water evaporated from liquid in second, third, etc., vessels.
M_0 = weight of liquid withdrawn from last vessel.
G = weight of steam introduced into first vessel.
t_0 = temperature of liquid entering first vessel.
t, t_2, t_3, etc. = mean temperatures of boiling liquid in first, second, third, etc., vessels.
i = heat content of steam G, r = latent heat of same.
i_1, i_2, i_3, etc. = heat contents of vapor in successive vessels.
r_1, r_2, r_3, etc. = latent heats of vapor in successive vessels.

The heat equations are deduced by the aid of two principles: 1. The heat supplied to a vessel is equal to the heat given out by it. 2. The weight of heating steam used in a vessel is equal to the weight of condensed water formed in the vessel.

For the first vessel, the heat equation is:

$$G(i - t_1 + 32) = M(t_1 - t_0) + M_1r_1$$

For the second vessel:

$$Mi(i_1 - t_2 + 32) = M_2r_2 - (M - M_1)(t_1 - t_2)$$

For the third vessel:

$$M_2(i_2 - t_3 + 32) = M_3r_3 - (M - M_1 - M_2)(t_2 - t_3)$$

For a triple-effect system, $M - M_1 - M_2 = M_0$; hence with M and M_0 known, M_1, M_2, and M_3 can be calculated provided the temperatures and pressures within the vessels are known.

Hausbrand points out that the temperatures t_1, t_2, etc., depend upon the form and size of the heating surfaces, the density of the boiling liquid, and other factors, and that these temperatures are therefore not predetermined. By assuming various values for the intermediate temperatures and calculating the effects in the separate vessels, he arrives at the following conclusions:

1. The smallest amount of heating steam required to produce a certain amount of evaporation is used in all multiple evaporators, when the fall in temperature is the same in each vessel.

2. However the fall of temperature in the separate vessels be arranged, the weight of heating steam to be supplied to the first vessel varies only within narrow limits. Hence the economy of the system is but slightly affected by the temperature distribution between the vessels.

3. The ratio of the evaporation in the first vessel to the total evaporation averages as follows:

For the double-effect........................	$M_1 = 0.466\ (M - M_0)$
For the triple-effect........................	$M_1 = 0.300\ (M - M_0)$
For the quadruple-effect....................	$M_1 = 0.216\ (M - M_0)$

The probable range of values is:

For the double-effect $M_1/(M - M_0) = 0.434$ to 0.484; for the triple-effect, 0.2777 to 0.3152; for the quadruple-effect, 0.1926 to 0.2335.

4. The evaporation is in all cases the least in the first vessel, but the increase

in the following vessels is not great, at most 4 per cent. As a mean, the following ratios may be taken:

	1st	2nd	3d	4th
Double-effect	1	1.045		
Triple-effect	1	1.01	1.04	
Quadruple-effect	1	1.005	1.012	1.02

5. The ratio of the evaporation in the last vessel to the total evaporation is

For the double-effect	0.534
For the triple-effect	0.3703
For the quadruple-effect	0.284

Heating Surface. The heat Q transmitted from steam to boiling fluid in a vessel is given by $Q = kAh$, in which A is the heating surface in sq. ft., h the difference in temperature between the steam and boiling fluid and k the coefficient of heat transmission in B.t.u. per sq. ft. per hr. per deg. fahr. difference of temperature. The value of k is not the same for the different vessels of a multiple-effect system. From the experiments of Claassen the ratios for different vessels may be taken as follows:

	1st	2nd	3d	4th
Double-effect	1	0.66		
Triple-effect	1	0.70	0.33	
Quadruple-effect	1	0.91	0.75	0.55

The value of k depends on many conditions and is difficult to ascertain with accuracy. Creighton ("Steam Engines," p. 524) suggests the following values for cane-sugar evaporators: First vessel, $k = 385$; second, $k = 300$; third, $k = 205$; fourth, $k = 110$. If the fall of temperature in each effect is the same, the heating surfaces must be made larger in the successive vessels. It is usually considered better practice to keep the heating surfaces practically the same and make the temperature drops larger in the successive vessels.

The water evaporated per hr. per sq. ft. of surface varies considerably with the conditions of operation. Hausbrand gives the following figures as the result of practical experience.

Ordinary vertical evaporators with brass heating tubes of 3 ft. long and over evaporate from liquids that present no obstacles to evaporation, as follows:

With single-effect	14.0–16.0 lb. per sq. ft. per hr.
With double-effect	6.0– 7.2 lb. per sq. ft. per hr.
With triple-effect	4.0– 5.0 lb. per sq. ft. per hr.
With quadruple-effect	3.5– 4.2 lb. per sq. ft. per hr.

With iron tubes the evaporation is decreased 10 to 15 per cent. In the case of liquids that evaporate with difficulty, the evaporation may be very much smaller.

The Steam Engine

The Rankine Cycle. The ideal Rankine cycle is generally employed by engineers as a standard of reference by which the performance of steam engines and steam turbines is measured. Fig. 15 shows this cycle on the T-S and p-V planes. AB represents the heating of the water in the boiler, BC represents evap-

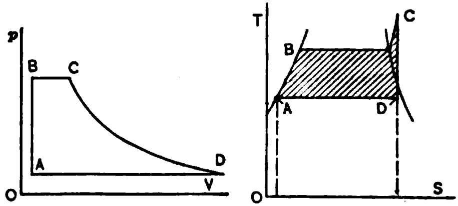


FIG. 15.—Rankine Cycle.

oration (and superheating if there is any), CD the assumed adiabatic expansion in the engine cylinder, and DA condensation in the condenser.

Let i_a, i_b, i_c, i_d represent the heat content per lb. of steam in the four states A, B, C, and D, respectively. Then the heat transformed into work, represented by the area $ABCD$, is $Q_a = i_c - i_d$.

The heat expended on the fluid is $i_c - i_a$; hence the thermodynamic **efficiency of the cycle** is $E_t = (i_c - i_d)/(i_c - i_a)$.

The **steam consumption** of the ideal Rankine engine in lb. per h.p.-hr. is $N_r = 2546/Q_a = 2546/(i_c - i_d)$. For steam consumptions per kw-hour (for use in steam turbine calculations) see pp. 980 and 981.

The performance of an engine is frequently stated in terms of the heat used per h.p.-hr. For the ideal Rankine engine this is

$$Q_r = 2546/E_t = 2546\,(i_c - i_a)/(i_c - i_d).$$

Efficiency of the Actual Engine. Let Q denote the heat transformed into work per lb. of steam by the actual engine; then if Q_1 is the heat furnished by the boiler per lb. of steam, the thermodynamic **efficiency of the engine** is $E_t = Q/Q_1$.

The efficiency thus defined is however misleading, as it takes no account of the conditions of operation, as regards boiler and condenser pressures, superheat, or quality of steam. It is customary therefore to define the efficiency as the ratio Q/Q_a, where Q_a is the **available heat**, or the heat that might possibly be transformed under ideal conditions. This efficiency for engines and turbines ranges from 0.50 to 0.80. The efficiency may also be expressed in terms of steam consumed; thus, if N_a is the steam consumption of the actual engine and N_r is the steam consumption of the ideal Rankine engine under similar conditions, then $E = N_r/N_a$.

Example. Suppose the boiler pressure to be 180 lb. per sq. in. absolute, superheat 150 deg., and the condenser pressure 3 in. of mercury. From the steam tables or diagram the following values are found: $i_c = 1279.9$, $i_d = 940$, $i_a = 82.9$. The available heat is $Q_a = 1279.9 - 940 = 339.9$ B.t.u., and the thermodynamic efficiency of the cycle is $339.9/(1279.9 - 82.9) = 0.284$. The steam consumption per h.p.-hr. is $2546/339.9 = 7.5$ lb., and the heat used per h.p.-hr. is $2546/0.284 = 8960$ B.t.u. If an actual engine working under the same conditions has a steam consumption of 11.4 lb. per h.p.-hr., its efficiency is $7.5/11.4 = 0.658$, and its heat consumption per h.p.-hr. is $8960/0.658 = 13{,}600$ B.t.u.

Calorimetric Analysis of the Steam Engine. The first attempt to determine quantitatively the thermal actions in the steam-engine cylinder is due to Hirn. By **Hirn's method**, equations are derived for the heat interchanges between the steam and the cylinder walls, and the quantities involved are such as may be determined from an accurate test. In Fig. 16, points 1, 2, 3 and 0 represent cut-off, release, compression, and admission, respectively. Let U_0, U_1, U_2, U_3 represent the energy per lb. of fluid at the corresponding points. Let M denote the weight of fluid supplied per stroke, and M_0 the weight of fluid caught in the clearance space. Let Q_a, Q_b, Q_c, Q_d denote respectively the quantities of heat absorbed by the cylinder walls from the fluid during the processes represented by a, b, c, and d on the diagram; and let W_a, W_b, W_c, and W_d denote the external work performed during the corresponding parts of the cycle. Let Q denote the heat brought into the cylinder per stroke by the working fluid; thus, $Q = Mi$, where i is the heat content of

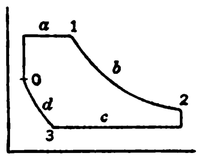


FIG. 16.

the entering steam. Finally, let i' denote the heat content of the liquid at the pressure of the exhaust steam, G the weight of condensing water used per stroke, and t', t'' the initial and final temperatures of the cooling water. Then the heat interchanges are expressed by the following equations:

$$Q_a = Q + U_0 - U_1 - AW_a. \qquad Q_b = U_1 - U_2 - AW_b.$$
$$Q_c = U_2 - U_3 - Mi - G(t'' - t') + AW_c. \qquad Q_d = U_3 - U_0 + AW_d.$$

In getting the expressions for U_0, U_1, etc., there are five unknown quantities, namely M_0, and the four qualities x_0, x_1, x_2, x_3. Between these there are four relations. Let V_0 denote the volume of the clearance space, and V_1, V_2, V_3 the spaces swept through by the piston up to the points 1, 2, 3. Also let $s = v'' - v'$, the difference between the volume of vapor and liquid. Then

$$V_0 = M_0(x_0s_0 + v'). \qquad V_0 + V_1 = (M + M_0)(x_1s_1 + v').$$
$$V_0 + V_2 = (M + M_0)(x_2s_2 + v'). \qquad V_0 + V_3 = M_0(x_3s_3 + v').$$

If one quality be known or assumed, these four equations give M_0 and the remaining qualities. It is usually assumed that the steam in the cylinder is nearly or quite dry at the beginning of compression, hence $x_3 = 1$.

For an exhaustive treatment of the calorimetric analysis of steam engines, see Peabody's "Thermodynamics," 5th ed., chap. xi.

REFRIGERATION

Air Refrigeration. When air is used as a medium for refrigeration, the reversed Joule cycle is employed (see p. 321). The refrigerating machine has four essential organs: 1. A compressor in which the air is compressed. 2. A cooling coil surrounded by water. 3. An expansion cylinder. 4. A brine coil. Fig. 17 represents ideal p-V and T-S diagrams. Point A represents the state of the air entering the compressor from the brine coil or cold room, AB represents the compression, BC the cooling of the air at constant pressure, CD the expansion of air in the expansion cylinder, and DA the absorption of heat by the air during the passage through the brine coils. In the p-V diagram $ABEF$ represents the indicator diagram of the compressor, $ECDF$ that of the expansion cylinder. The difference, area $ABCD$, represents the work that must be furnished from external sources. In the T-S diagram area B_1BCC_1 represents the heat absorbed from the air by the cooling water, area C_1DAB_1, the heat absorbed by the air from the brine, and area $ABCD$ the heat equivalent of the work required to drive the machine. The temperature at point A must be somewhat lower than the brine temperature, the temperature at C somewhat above the temperature of the cooling water. With the open-cycle machine the lower pressure p_1 is atmospheric pressure; with a closed-cycle machine, as the Allen dense-air machine, p_1 may be 40 to 60 lb. per sq. in. and p_2 as high as 200 lb. per sq. in.

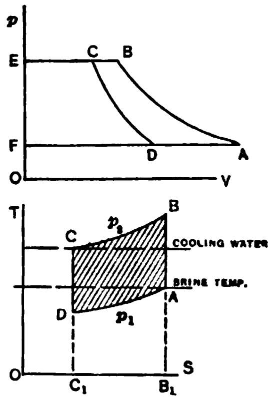


FIG. 17.—Air Refrigeration Cycle.

Let Q = heat in B.t.u. absorbed from brine (or cold room) per min.
M = weight of air circulated per min., lb.
c_p = 0.24, specific heat of air at constant pressure
W = work required per min., ft.-lb.

V_c = displacement volume of compressor cylinder, cu. ft.
V_e = displacement volume of expansion cylinder, cu. ft.
n = number of working strokes per min.

With assumed adiabatic compression and expansion the following relations hold:

$$T_b/T_a = T_c/T_d = (p_2/p_1)^{(k-1)/k}. \qquad M = Q/c_p(T_a - T_d).$$
$$W = 778Q(T_b - T_a)/T_a. \qquad \text{H.P.} = W/33{,}000.$$
$$V_c = MBT_a/144p_1n. \qquad V_e = MBT_d/144p_1n.$$

Vapor Compression Machines. The essential organs of a vapor compression system are the same as in the system using air, except that the expansion cylinder is replaced by an expansion valve through which the liquefied medium flows from the high-pressure condensing coils to the low-pressure brine coils. The cycle of operation is best shown on the T-S plane (Fig. 18). The point B represents the state of the refrigerating medium leaving the brine coils and entering the compressor. Usually in this state the fluid is nearly dry saturated vapor, that is, point B is near the saturation curve S''. BC represents the adiabatic compression, during which the fluid is usually superheated. In the state C the superheated vapor passes into the cooling coils and is cooled at constant pressure, as indicated by CD, and then condensed at temperature T_2 as shown by DE. The liquid now flows through the expansion valve into the brine coils. This is a throttling process and the final-state point A is located on the T_1-line in such a position as to make the heat content for state A (= area $OHGAA_1$) equal to the initial heat content at E (= area $OHEE_1$). The mixture of liquid and vapor now absorbs heat from the brine and vaporizes, as indicated by AB.

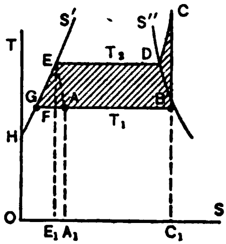


Fig. 18.—Vapor Compression Refrigeration Cycle.

The **heat absorbed from the brine,** represented by area A_1ABC_1, is

$$Q_1 = i_b - i_a = i_b - i_e.$$

The **heat rejected to the cooling water,** represented by area C_1CDEE_1, is

$$Q_2 = i_c - i_e = c_p(T_c - T_d) + r_2$$

where r_2 denotes the latent heat at the upper temperature T_2, and c_p the specific heat of the superheated vapor. The **work** that must be **supplied per pound of fluid circulated** is $W = J(Q_2 - Q_1) = J(i_c - i_b)$.
The ratio $JQ_1/W = (i_b - i_e)/(i_c - i_b)$ is sometimes called the **coefficient of performance.**

If Q denotes the heat that is required to be absorbed from the brine per hr., then the **weight of fluid circulated per hour** is $M = Q/(i_b - i_a)$; or, if B is taken on the saturation curve, $M = Q/(i''_1 - i'_2)$.

The work per hr. is $W = JM(i_c - i''_1) = JQ\,(i_c - i''_1)/(i''_1 - i'_2)$ ft.-lb., and the **h.p. required** is $H = Q(i'_c - i''_1)/2546(i''_1 - i'_2)$.

If v''_1 is the volume of the saturated vapor at the temperature T_1 in the brine coils, and n the number of working strokes per min., the **displacement volume of the compressor cylinder** is $V = Mv''_1/60n$.

The values of i''_1, i_2 in the preceding formulæ are found in the tables of saturated vapors. The heat content i_c of the superheated vapor may be determined in the case of ammonia from Table 35 for superheated ammonia.

Vapors Used in Refrigeration. The three vapors that are principally used for refrigeration are **ammonia** (NH_3), **carbon dioxide** (CO_2) and **sulphur dioxide** (SO_2). The temperatures of the working fluid in the condensing and brine coils are fixed by the temperature of cooling water and the temperature at which the brine is to be kept, respectively. Since for a vapor the pressure depends on the temperature only, the discharge and suction pressures that must be used are dependent on the properties of the fluid employed. Another consideration in the choice of a fluid is the volume of fluid required for a given amount of refrigeration. In general, the fluid for which the ratio (latent heat : specific volume) is greatest requires the smallest cylinder displacement per unit of refrigeration.

Taking 70 deg. fahr. as the temperature of the fluid in the condensing coils and 15 deg. as that in the cooling coils, the following are the conditions for the available vapors:

Pressures, lb. per sq. in.:	NH_3	CO_2	SO_2	Ethyl chloride	Methyl chloride	Nitrous oxide	Pictet's fluid
Suction	42.7	391	15.2	6.0	25.9	431	20.3
Discharge	129.2	847	49.6	20.0	73.3	832	51.6
Relative volume per unit weight of refrigeration	4.4	1	12.0	....	....	...	....

With SO_2 the pressures are low but the volume of medium is comparatively large, while with CO_2 small volume is accompanied by extremely high pressures. In the case of ammonia the pressures are reasonable and the volume of fluid required is not excessive; hence ammonia is preferred for most classes of service.

Refrigeration Produced by Ammonia and Carbon Dioxide. Tables 41 and 42 give the refrigeration produced in B.t.u. per h.p.-hr. under different conditions. It is assumed that there are no losses in the cycle, except that inherent in the use of the expansion valve. The temperature at the top of the column is that of the vapor in the brine coil, while at the side is given the temperature of the condensed fluid as it enters the expansion valve. For example, under a pressure p of 150 lb. with a temperature t_0 of 20 deg. fahr. in the brine coils and a temperature of 70 deg. fahr. at the expansion valve, the refrigeration produced in an ammonia machine without losses is 17,800 B.t.u. per h.p.-hr.; this is 90 per cent. of the refrigeration that would be produced if the liquid temperature were reduced to 20 deg., the temperature in the brine coils.

For other tables of the theoretical performance of ammonia compression machines, with the liquid assumed to arrive at the expansion valve at its saturation temperature, see pp. 1715 and 1716.

Absorption System of Refrigeration. The essential organs of a vapor absorption system are:

1. The **generator** or still, in which ammonia gas is driven off from a solution of ammonia in water through the agency of heat. Usually a steam coil is used for this purpose. The generator takes the place of the compressor in the compression system. 2. The **condenser**, in which the ammonia gas is condensed. 3. The **expansion valve.** 4. The **brine coil**, in which the ammonia by vaporizing absorbs heat from the brine. 5. The **absorber**, in which the ammonia returning from the brine coil is absorbed by a weak solution of ammonia in water. 6. The **pump**, which returns the rich liquor from the absorber to the generator.

Between the generator and absorber there are two channels of communication: one through the condenser, expansion valve, brine coil, absorber, and pump, as noted above; through the other, the weak solution in the generator passes directly to the absorber and there absorbs the gas coming from the brine coil. As in the compression system, there is a region of high pressure including the generator and condenser, and a region of low pressure including the brine coil and absorber. The pump is used to force the strong solution from the lower pressure of the absorber against the higher pressure in the generator.

For efficient operation of an absorption system certain auxiliary organs are required. The **analyzer** forms the upper part of the generator and receives the rich solution from the pump. The solution descends over a series of disks or trays until it meets the boiling liquid in the still. The vapor rising from the still thus comes in intimate contact with the descending liquid and is thus enriched in ammonia and deprived of water.

The **rectifier** is placed between the analyzer and condenser. It consists of a coil surrounded by cooling water, and its function is to remove water vapor from the mixture of water and ammonia driven off from the generator.

The **exchanger** is placed between the generator and absorber. It is evidently desirable that the rich solution entering the still should be at as high a temperature as possible and the weak solution entering the absorber from the still should be as cool as possible. The hot weak solution passing from the generator to the absorber gives up heat to the cooler strong solution passing from the pump to the generator.

Table 41. Refrigeration Produced by Ammonia in B.t.u. per H.p.-Hour

Temp. at expansion cock, deg. fahr. t'	Vapor temperature t_0 in deg. fahr. 30	20	10	0	−10	−20
	CONDENSER PRESSURE p = 100 LB. PER SQ. IN.					
t_0	45,400	30,900	23,350	21,000	17,150	12,900
50	43,600	29,300	21,950	19,100	15,500	11,350
	p = 150 LB.					
t_0	25,000	19,900	15,900	14,400	12,400	10,100
50	24,100	18,700	14,900	13,050	11,000	8,850
70	23,300	17,800	14,250	12,450	10,650	8,500
	p = 200 LB.					
t_0	18,300	15,200	13,050	11,700	10,350	8,600
50	17,800	14,400	12,200	10,650	9,150	7,550
70	17,000	13,800	11,600	10,100	8,750	7,250
90	16,100	13,000	11,150	9,650	8,250	6,850
	p = 250 LB.					
t_0	15,100	12,900	11,400	10,250	9,150	7,800
50	14,600	12,100	10,600	9,300	8,050	6,900
70	14,000	11,500	10,050	8,800	7,700	6,500
90	13,300	11,000	9,650	8,450	7,350	6,200
	p = 300 LB.					
t_0	13,400	11,500	10,200	9,250	8,300	7,150
50	13,000	10,950	9,450	8,450	7,400	6,300
70	12,500	10,400	8,995	8,050	7,100	6,000
90	11,900	10,000	8,650	7,650	6,750	5,700

Table 42. Refrigeration Produced by Carbon Dioxide in B.t.u. per H.p.-Hour

Temp. at expansion cock, deg. fahr. t^a	Vapor temperature t_0 in deg. fahr.					
	30	20	10	0	−10	−20
	Condenser Pressure p = 800 Lb. per Sq. In.					
t_0	32,600	26,200	21,750	18,650	15,600	13,300
50	29,650	23,650	19,000	16,150	13,450	11,050
60	27,600	21,600	17,150	14,350	11,650	9,550
	p = 1000 lb.					
t_0	22,600	18,800	16,050	13,750	12,200	10,700
50	20,800	17,300	14,250	12,000	10,250	8,650
60	19,350	15,850	12,900	10,750	9,150	7,500
70	17,800	14,750	12,050	10,050	8,400	6,950
	p = 1200 lb.					
t_0	16,600	14,700	13,050	11,550	10,350	9,250
50	15,350	13,550	11,500	9,950	8,750	7,500
60	14,450	12,500	10,600	9,050	7,850	6,500
70	13,600	11,700	9,850	8,550	7,250	6,150
80	12,650	10,800	9,150	8,000	6,750	5,600
90	11,450	9,750	8,300	7,100	6,050	5,150
	p = 1400 lb.					
t_0	14,250	12,700	11,400	10,350	9,250	8,250
50	13,200	11,650	10,200	8,950	7,900	6,850
60	12,400	10,900	9,350	8,100	7,050	6,150
70	11,750	10,150	8,750	7,600	6,600	5,650
80	10,900	9,500	8,200	7,200	6,200	5,400
90	10,000	8,750	7,550	6,600	5,650	5,000
	p = 1600 lb.					
t_0	13,150	11,600	10,250	9,550	8,500	7,550
50	12,200	10,600	9,300	8,300	7,400	6,500
60	11,450	10,000	8,600	7,500	6,600	5,850
70	10,700	9,250	8,150	7,000	6,250	5,500
80	9,800	8,750	7,650	6,700	5,850	5,200
90	9,150	8,200	7,150	6,250	5,850	4,950

Heat Balance. An exact thermodynamic analysis of the absorption system is difficult because the properties of ammonia solutions are imperfectly known. The first law may, however, be applied, and from it may be derived an equation giving the heat balance for the system.

For 1 lb. of anhydrous ammonia passing through the condenser, expansion valve, and brine coil, G lb. of rich solution must be circulated by the pump and $G - 1$ lb. of weak solution enters the absorber. The weight G depends on the strengths of the strong and weak solutions. Taking x_1 as the strength of the former (x_1 = lb. of NH_3 in 1 lb. of solution) and x_2 as the strength of the latter, $G = (1 - x_2)/(x_1 - x_2)$.

For example, if the strength of the strong liquor is 0.25, and that of the weak liquor 0.10, $G = 0.90/(0.25 - 0.10) = 6$. Hence, for 1 lb. of ammonia passing through the expansion valve, 6 lb. of strong liquor must be circulated by the pump and 5 lb. of weak liquor will reach the absorber.

For 1 lb. of ammonia passing the expansion valve, the following quantities of heat will be absorbed or rejected at various points in the system:

Q_1 = heat imparted to the fluid in the generator.
Q_2 = heat absorbed by the fluid in the brine coils.
Q_3 = heat rejected to the cooling water as fluid passes through condenser.

Q_4 = heat withdrawn from fluid in absorber.
Q_5 = heat equivalent of work of pump.
Q_6 = heat loss by radiation, etc.

The following equation expresses the **heat balance** for the system:

$$Q_1 + Q_2 + Q_5 = Q_3 + Q_4 + Q_6.$$

Some of the heat quantities are readily calculated. The **heat Q_2 absorbed in the brine coils,** as in the compression system, is $Q_2 = i''_1 - i'_2$, where i''_1 is the heat content of the saturated vapor corresponding to the temperature of the fluid in the brine coil, and i'_2 is the heat content of the liquid corresponding to the temperature of the liquid entering the expansion valve.

If it is assumed that the fluid enters the condenser in the dry saturated state, the heat Q_3 is simply the latent heat r corresponding to the temperature of the fluid in the condenser.

The pump forces G lb. of rich liquor from the pressure p_1 in the absorber to the pressure p_2 in the generator. Hence if v is the volume of 1 lb. of the rich liquor, $Q_5 = Gv(p_2 - p_1)/778$.

The **heat Q_4 withdrawn from the absorber** is found as follows: Let t_1 denote the temperature of the gas in the brine coils, t_0 the temperature of the fluid in the absorber, and t' the temperature of the weak solution entering the absorber. In general, $t_0 > t_1$, and $t_0 < t'$. The entering vapor (1 lb.) receives heat $c\ (t_0 - t_1)$, c being the specific heat of the vapor, which may be taken as 0.5. The entering weak solution gives up the heat $(G - 1)\ c'\ (t' - t_0)$, where c' denotes the specific heat of the weak liquor; this may be taken as 1. The absorption of 1 lb. of gas by the weak solution develops the heat of absorption Q_a, the magnitude of which depends upon the concentrations of the weak and strong solutions, as shown in the following paragraph. Hence,

$$Q_4 = Q_a + (G - 1)\ (t' - t_0) - 0.5\ (t_0 - t_1).$$

Heat of Absorption. Let the weak solution of ammonia in water be composed of n lb. of water to 1 lb. of NH_3. The concentration of the solution is $x = 1/(1 + n)$. If now the solution absorb m lb. of ammonia gas, the strength of the resulting solution is $x_2 = (1 + m)/(1 + n + m)$. Let x denote the mean concentration, that is, $x = \frac{1}{2}(x_1 + x_2)$. Then, according

Table 43. Ammonia Absorption Machine.—Heats of Absorption Q_a and Weights G of Strong Liquor Circulated
(Q_a in B.t.u.; G in lb.)

Concentration of weak solution, x_2		Concentration of strong solution, x_1								
		0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.35	0.40
0.10	Q_a =	825	821	816	811	806	801	796	788	774
	G =	9.0	7.5	6.43	5.63	5.0	4.5	4.09	3.6	3.0
0.12	Q_a =	821	816	811	806	801	796	791	783	769
	G =	11.0	8.8	7.33	6.29	5.5	4.9	4.4	3.83	3.14
0.14	Q_a =	816	811	806	801	796	791	786	777	763
	G =	14.3	10.75	8.6	7.17	6.14	5.19	4.8	4.1	3.31
0.15	Q_a =	8.14	809	804	799	793	788	783	774	761
	G =	17.0	12.14	9.44	7.73	6.54	5.67	5.0	4.25	3.4
0.16	Q_a =	811	806	801	796	791	786	780	772	758
	G =	21.0	14.0	10.5	8.4	7.0	6.0	5.25	4.32	3.5
0.18	Q_a =	806	801	796	791	786	780	775	766	752
	G =	....	20.5	13.67	10.25	8.2	6.83	5.7	4.82	3.73
0.20	Q_a =	....	796	791	786	780	775	769	761	746
	G =	....	40.0	20.0	13.3	10.0	8.0	6.7	5.3	4.0

to the experiments of Hilde Mollier (*Mitteil. über Forschungsarbeiten*, Heft 63), the heat of absorption Q_a is given by the formula

$$Q_a = 887 - 350x - 400x^2.$$

For example, let the weak solution be composed of 1 lb. NH_3 to 9 lb. of water, and let 2 lb. of NH_3 gas be added. Then $x_1 = 1/(1 + 9) = 0.10$, $x_2 = (1 + 2)/(1 + 9 + 2) = 0.25$, $x = 0.175$. Then $Q_a = 887 - (350 \times 0.175) - [400 \times (0.175)^2] = 814$ B.t.u. The formula holds good for values of x up to about 0.60. For this value and higher values Q_a remain constant and is 540 B.t.u.

For various concentrations of the strong and weak solutions Table 43 gives values of Q_a and of G, the weight of strong solution circulated.

Linde's Regenerative Refrigeration Process. The Joule-Thomson effect (see p. 361) has been used effectively by Linde for the **liquefaction of air** and other gases. Referring to Fig. 19, air is compressed to a pressure of about 65 atmospheres and is discharged into a pipe leading to the chamber *c*. The air in its passage to *c* is cooled by water. From *c* the air passes through an expansion valve into a vessel *d*, in which a pressure of about 22 atmospheres is maintained. The throttling in the expansion valve causes a drop in temperature given by the equation

$$t_2 - t_3 = k(p_1 - p_2)/T_2^2.$$

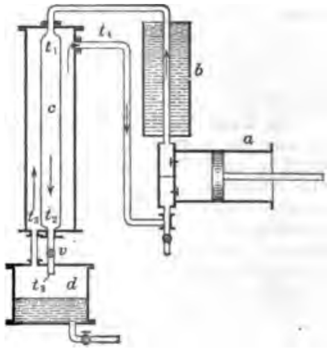


Fig. 19.—Linde Regenerative Refrigeration Process.

The air now passes back to the compressor and in so doing cools the air in *c*. In the actual machine the counter-current apparatus, which in the figure is shown as the chamber *c* and the space surrounding it, consists of two concentric spiral pipes about 330 ft. long. By the cooling action of the return current of air the temperature t_2 gradually sinks, and, as shown by the preceding equation, the temperature t_3 sinks more rapidly. Ultimately t_3 becomes lower than the critical temperature of air (− 220 deg. fahr.), and liquefaction begins.

By the use of the regenerative process liquid air or other gas may be produced continuously and in relatively large quantities by the simple mechanical process of compression. The necessary lowering of temperature is effected by repeated throttling of the gas and a second refrigerating medium is not required.

FLOW OF COMPRESSIBLE FLUIDS

References. An exhaustive discussion of the flow of fluids is given in Zeuner's "Technical Thermodynamics," vol. 1, pp. 225–271; vol. 2, pp. 153–196. The general theory is given in Lucke's "Engineering Thermodynamics," pp. 1083–1127; in Peabody's "Thermodynamics," 5th ed., chap. vxii; in Goodenough's "Principles of Thermodynamics," chap. xiii, and in Stodola's "Steam Turbines," pp. 4 and 45. For experimental results, consult Gutermuth, *Zeit. Ver. Deutsch Ing.*, vol. 48; Buchner, *Zeit. Ver. Deutsch Ing.*, vol. 49; Rateau, "Flow of Steam through Nozzles"; Rosenhain, *Proc. Inst. C. E.*, vol. 140; Wilson, *Engg.* (London), vol. 13.

Important **examples of** the **flow** of compressible fluids are the following: 1. The flow of air and steam through orifices and short tubes or nozzles, as in the steam turbine. 2. The flow of compressed air, steam, and illuminating gas in long mains. 3. The flow of low-pressure gases, as furnace gases in ducts and chimneys or air in ventilating ducts (see pp. 923 and 1359). 4. The flow of gases in moving channels, as in the centrifugal fan (see p. 1531).

Notation.

Let M = weight in lb. of fluid flowing past a given section per sec.
F = area of section, sq. ft.
w = mean velocity in ft. per sec. at the given section.
h = height of cross-section above an assumed datum.
v = volume per unit weight (cu. ft. per lb.).
p = pressure of fluid at given section. lb. per sq. ft.
u = internal energy of fluid, B.t.u. per lb.
i = heat content, B.t.u. per lb.
Q_{12} = heat entering the flowing fluid between sections F_1 and F_2.
R_{12} = energy expended in overcoming internal and external friction between sections F_1 and F_2.

The cross-sections of the tube or channel are denoted by F_1, F_2, etc., Fig. 20, and the various magnitudes pertaining to these sections are denoted by corresponding subscripts. Thus, at section F_1, the velocity, volume, pressure, energy, are respectively w_1, v_1, p_1, u_1; at section F_2 they are w_2, v_2, p_2, u_2.

FIG. 20.

Fundamental Equations. The **condition of continuity** is expressed by the equation

$$M = F_1w_1/v_1 = F_2w_2/v_2, \quad \text{or} \quad dv/v = (dF/F) + (dw/w).$$

The principle of conservation of energy leads to the equation

$$[(w_2^2 - w_1^2)/2g] + J(u_2 - u_1) + p_2v_2 - p_1v_1 + h_2 - h_1 = JQ_{12};$$

or, introducing the expression for heat content $i = u + Apv$,

$$[(w_2^2 - w_1^2)/2g] + J(i_2 - i_1) + h_2 - h_1 = JQ_{12}.$$

If the sections are taken an infinitesimal distance apart, this energy equation takes the form

$$(wdw/g) + Jdi + dh = JdQ.$$

A second fundamental equation is obtained by the further application of the energy equation, taking the friction work R into account. It is

$$dQ + AdR = du + Apdv = di - Avdp.$$

Combining these last equations,

$$(wdw/g) + vdp + dR + dh = 0.$$

Usually $h_1 - h_2$ and Q_{12} are so small as to be negligible. In this case the two fundamental equations become

$$(w_2^2 - w_1^2)/2g = J(i_1 - i_2); \quad (w_2^2 - w_1^2)/2g = -\int_1^2 vdp - R_{12}$$

For gases, $i_1 - i_2 = c_p(T_1 - T_2) = Ak(p_1v_1 - p_2v_2)/(k - 1)$; hence

$$[(w_2^2 - w_1^2)/2g] = Jc_p(T_1 - T_2) = k(p_1v_1 - p_2v_2)/(k - 1).$$

For a mixture of vapor and liquid, $i = i' + xr$; therefore

$$[(w_2^2 - w_1^2)/2g] = J[i'_1 + x_1r_1 - (i'_2 + x_2r_2)].$$

Flow Through Orifices

A case of special importance is the **flow** of a gas or vapor from a reservoir into a region of lower pressure **through an orifice or short tube.** In Fig. 21 the section F_1 is taken within the reservoir where the pressure is p_1, and the section F_2 is taken at the plane of the orifice. The pressure

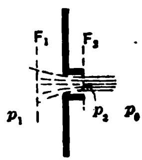


FIG. 21.

at this section is taken as p_2, and the pressure in the region into which the jet is flowing is taken as p_0. The initial velocity w_1 is so small that w_1^2 may be neglected in comparison with w_2^2. Hence

$$w_2^2/2g = J(i_1 - i_2), \text{ and } w_2 = 223.7\sqrt{i_1 - i_2}$$

The character of the flow depends on the ratio p_0/p_1, and two distinct cases may be noted. Let the law of frictionless adiabatic expansion of the fluid in question be $p_1v_1^n = pv^n = \text{const}$. For gases, $n = k$, for steam n has the value given in Table 39, p. 338, and for superheated steam or ammonia n may be taken as 1.3. Take $p_m = p_1[2/(n+1)]^{n/(n-1)}$; then p_m is the **critical pressure.** For gases $p_m/p_1 = 0.53$, approx.; for saturated steam the ratio is about 0.575; and for moderately superheated steam, about 0.55.

CASE 1. $p_0 > p_m$. Then $p_2 = p_0$, and the discharge M is given by the equation

$$M = \frac{F_2}{v_2}\sqrt{2gp_1v_1\frac{n}{n-1}\left[1 - \left(\frac{p_0}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

Thus the weight flowing per second depends upon the two pressures p_1 and p_0.

CASE 2. $p_0 \lesseqgtr p_m$. Then $p_2 = p_m$, that is, p_2 remains constant whatever lower value the pressure p_0 may take. In this case the discharge is

$$M = F_2\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\sqrt{2g\frac{n}{n+1}\frac{p_1}{v_1}}.$$

Hence the discharge depends upon p_1 only and is not influenced by the pressure p_0.

Formulæ for Discharge of Air. When the back pressure p_0 exceeds the critical pressure $p_m = 0.53\ p_1$,

$$M = 2.05\ Fp_0\sqrt{\frac{1}{T}\left(\frac{p}{p_0}\right)^{0.286}\left[\left(\frac{p}{p_0}\right)^{0.286} - 1\right]}$$

in which p and T refer to the air in the reservoir. Values $(p/p_0)^{0.286}$ are given on p. 318. $[(n-1)/n = 0.286$ when $n = 1.4]$

When p_0 is equal to or less than p_m, $M = 0.53\,Fp/\sqrt{T}$.

For a small difference of pressure, $M = 1.1\,F\sqrt{(p/T)(p - p_0)}$

In these three formulæ F is to be taken in sq. in. when p is taken in lb. per sq. in. The discharge as calculated by the formula should be multiplied by a **coefficient of discharge** to take account of friction, contraction, etc. From the experiments of Zeuner and others, the following mean values of this coefficient are obtained:

Orifices in a thin plate	0.64
Short cylindrical pipe, inner corners not rounded	0.75 to 0.84
Well-rounded mouthpiece	0.98

Formulæ for Discharge of Saturated Steam. When the back pressure p_0 is less than the critical pressure p_m, the discharge depends upon the area of orifice F and reservoir pressure p. There are three formulæ widely used to express the discharge G in terms of F and p, as follows:

1. Napier's equation, $M = Fp/70$.
2. Grashof's formula, $M = 0.0165F\,p^{0.97}$.
3. Rateau's formula, $M = Fp(16.367 - 0.96 \log p)/1000$.

In these formulæ F is to be taken in square inches, p in lb. per sq. in. Napier's formula is merely convenient as a rough check. The coefficient of discharge may be taken as 1; that is, no correction is required. For flow of saturated and superheated steam see also p. 983.

Table 44. Values of $p^{0.97}$ for Use in Grashof's Formula

p	$p^{0.97}$	p	$p^{0.97}$	p	$p^{0.97}$	p	$p^{0.97}$
15	13.8	50	44.5	110	95.5	225	191.2
20	18.3	55	48.8	120	104.0	250	212.0
25	22.7	60	53.1	130	112.4	275	232.0
30	27.1	70	61.6	140	120.7	300	253.0
35	31.5	80	70.1	150	129.1		
40	35.8	90	78.6	175	150.0		
45	40.1	100	87.1	200	170.6		

When the back pressure p_0 is greater than the critical pressure p_m the velocity and discharge are found most conveniently from the general formulæ of flow. From the steam tables (p. 324) or from the **Mollier chart** (p. 330) find the initial heat content i_1 and the final heat content i_0 after adiabatic expansion; also the specific volume v_0 in the final state. See Fig. 22. Then

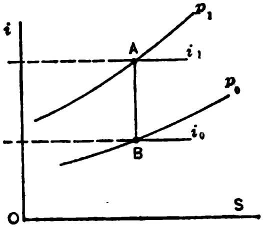


FIG. 22.

$$w = 223.7\sqrt{i_1 - i_0} \qquad \text{and} \qquad M = Fw/v_0.$$

The same method is used in the case of steam initially superheated.

Example. Required the discharge through an orifice ½ in. in diameter of steam at 140 lb. per sq. in. super heated 110 deg.; back pressure, 90 lb. per sq. in.

From the Mollier chart, $i_1 = 1253$, and $i_0 = 1213$. Also $v_0 = 5.26$ cu. ft.

$$w = 223.7\sqrt{1253 - 1213} = 1413 \text{ ft. per sec.}$$
$$F = 0.1964 \text{ sq. in.} = (0.1964/144) \text{ sq. ft.},$$
$$M = Fw/v_0 = (0.1964/144) \times (1413/5.26) = 0.366 \text{ lb. per sec.}$$

Flow Through Diverging Nozzles. At the throat, or smallest cross-section of the nozzle (Fig. 23) the pressure takes the value $p_m = 0.57\ p_1$. The weight discharged is fixed by the area F_m of the throat and the reservoir pressure p_1, and may be found from Grashof's or Rateau's formula. The diverging part of the nozzle permits further expansion to the back pressure p_0, the velocity of the jet meanwhile increasing from w_m, the critical velocity at the throat, to w_0 given by the fundamental equation $w_0 = 223.7\sqrt{i_1 - i_0}$.

The frictional resistances in the nozzle have the effect of decreasing the jet energy $w_0^2/2g$ and correspondingly increasing the energy and heat content of the flowing fluid. Thus, if i_0 is the heat content in the final state with frictionless expansion, $i'_0(>i_0)$ is the heat content when friction is taken into account; hence $w'_0{}^2/2g = J(i_1 - i'_0)$ is less than $w_0^2/2g = J(i_1 - i_0)$. The loss of kinetic energy in B.t.u. is $i'_0 - i_0$, and the ratio of this loss to the available kinetic energy, that is $(i'_0 - i_0)/(i_1 - i_0)$, is denoted by y. For values of y, see p. 356.

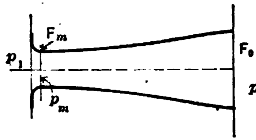


FIG. 23.

The **design of a nozzle for a given discharge** M with pressure p_1 and p_0 is most conveniently effected with the aid of the Mollier chart. Determine p_m, the critical pressure, and i_1, i_m, i_0, assuming frictionless flow. Then

$$w_m = 223.7\sqrt{i_1 - i_m}, \quad \text{and} \quad w'_0 = 223.7\sqrt{(1 - y)(i_1 - i_0)}.$$

Next find v_m and v'_0. Then, from the equation of continuity,

$$F_m = Gv_m/w_m, \text{ and } F'_0 = Gv'_0/w'_0.$$

The following example illustrates the method.

Example. Required the throat and end sections of a nozzle to deliver 0.7 lb. of steam per sec. The initial pressure is 160 lb., the back pressure 15 lb., and the steam is initially superheated 100 deg.; $y = 0.15$.

The critical pressure is $160 \times 0.55 = 88$ lb. On the Mollier chart (Fig. 24) the point A representing the initial state is located, and line of constant entropy (a frictionless adiabatic) is drawn from A. This cuts the curves $p = 88$ and $p = 15$ in the points B and C, respectively. The three values of i are found to be $i_1 = 1251$, $i_m = 1198$, $i_0 = 1065$. Of the available drop in heat content, $i_1 - i_0 = 186$ B.t.u., 15 per cent. or 27.9 B.t.u. is lost through friction. Hence, $CD = 27.9$ is laid off and D is projected horizontally to point C' on the curve $p = 15$. Then C' represents the final state of the steam, and the quality is found to be $x = 0.94$. The specific volume in the state C' is $26.27 \times 0.94 = 24.7$ cu. ft. Likewise the specific volume for the state B is found to be 5.2 cu. ft.

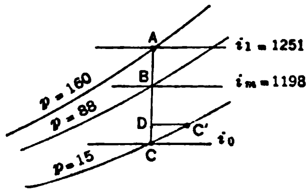


FIG. 24.

For the velocities at throat and end sections,

$$w_m = 223.7\sqrt{1251 - 1198} = 1629 \text{ ft. per sec.}$$
$$w_0 = 223.7\sqrt{186 - 27.9} = 2813 \text{ ft. per sec.}$$
$$F_m = (0.7 \times 5.2)/1629 = 0.00223 \text{ sq. ft.} = 0.322 \text{ sq. in.}$$
$$F_0 = (0.7 \times 24.7)/2813 = 0.0061 \text{ sq. ft.} = 0.88 \text{ sq. in.}$$

The diameters are $d_m = 0.64$ in. and $d_0 = 1.06$ in.

Velocity Coefficients. Loss of Energy y. On account of friction losses the actual velocity w attained by the jet is less than the velocity w_0 calculated under ideal conditions. That is, $w = xw_0$, where x (<1) is a velocity coefficient. Evidently the coefficient x is connected with the coefficient y (see p. 355), giving the loss of energy by the relation $y = 1 - x^2$.

For **orifices in thin plates,** the experiments of Zeuner on air and of Gutermuth and Bendemann on saturated and superheated steam indicate that x has the value of 0.97 to 0.975.

For **flow through nozzles,** Stodola states that the energy loss varies from $y = 0.05$ for short tubes to $y = 0.15$ for long tubes. The most comprehensive experiments in this field are those of Briling (*Mitteil, über Forsch.-arb.*, 68) and of Josse and Christlein (*Zeit. Ver. Deutsch. Ing.*, 1911, p. 2081). Briling used slightly superheated steam and two nozzles, one cylindrical with a diam. of 0.254 in. and length of 1.18 in., the other a slightly converging nozzle with a minimum diam. of 0.34 in. For the cylindrical tube he found $x = 0.95$; for the convergent tube x increased from 0.92 to 0.965 as the velocity increased. These results indicate an energy loss of 10 per cent. in the cylindrical tube and of 7 to 16 per cent. in the convergent tube. The experiments of Josse and Christlein were made on 6 De Laval nozzles having ratios of maximum to minimum sectional areas ranging from 1.185 to 12.94. Under the most favorable conditions, with a tube properly proportioned for the pressure drop, the experiments showed a value of $x = 0.94$ for a velocity of 2000 ft. per sec. and a value around 0.95 for velocities of 2500 to 3500 ft. per sec. Taking $x = 0.945$ as a mean value, the energy loss is,

$$y = 1 - x^2 = 1 - 0.945^2 = 0.107, \text{ or } 10.7 \text{ per cent.}$$

Josse and Christlein also made experiments on the friction loss in a set of **three guide vanes,** which take the place of nozzles in turbines of the Rateau type. Under favorable conditions with slightly superheated steam the mean value $x = 0.923$ was found. The corresponding loss of energy is $y = 0.148$ or 14.8 per cent. With high superheat and a velocity of 2200 ft. per sec., the value $x = 0.95$ was obtained, but at about 800 ft. per sec. x dropped to about 0.85. Rosenhain's experiments give values of x for several types of nozzles. Table 45 gives data and results (see Peabody's "Thermodynamics," p. 442).

Table 45. Values of the Velocity Coefficient x for Various Types of Nozzles

(Rosenhain, *Proc. Inst. C. E.*, vol. 140, p. 199)

Ratio of greatest to least diameter	Taper	Initial pressure	Velocity coefficient, x	Loss of energy, y
1.56	1:20	150.0	0.946	0.105
1.96	1:12	275.0	.....	.....
1.36	1:12	97.5	0.972	0.045
1.28	1:12	80.0	0.903	0.185
1.39	1:30	105.0	0.913	0.166
1.32	1:30	90.0	0 929	0.137
1.26	1:30	77.5	0.901	0.188
1.19	1:30	62.5	0.914	0.165
1.12	1:30	50.0	0.914	0.165

Lewicki's experiments on **small divergent nozzles** showed values of x lying around 0.955 to 0.96. The loss of energy was therefore 8 to 9 per cent.

Flow of Gases and Vapors in Mains

General Equations of Flow. In addition to the notation of the preceding section, let L = length of pipe in feet, d = diam. in *feet*, assumed to be constant, p_1 and p_2 = initial and final pressures in main, lb. per sq. in., and D = weight of fluid, lb. per cu. ft.

The general equations of flow (p. 353) are applicable. It is assumed (1) that the pipe is horizontal, whence $dh = 0$; (2) that there is no transmission of heat, whence $dQ = 0$; (3) that the temperature does not change, or $di = 0$. With these assumptions the general equation of flow is $dR = -vdp$. When the velocity of flow exceeds a certain **critical velocity**, as it does in all practical cases, the frictional resistance is found to be approximately proportional to the square of the velocity and inversely proportional to the hydraulic mean depth of the channel. For a circular pipe, therefore, the resistance varies inversely as the diameter and $dR = cw^2dL/d$. Hence $vdp + (cw^2dL/d) = 0$. The integration of this equation leads to two sets of formulæ that under various disguises are in common use.

1. When the **drop in pressure is small** relative to the initial pressure p_1. In this case integration gives the approximate equation

$$p' = cw^2L/vd, \qquad (a)$$

in which $p' = p_1 - p_2$, the drop in pressure.

Taking V as the volume of air flowing through the main per minute, $V = \frac{1}{4}\pi d^2w \times 60$. Substituting this in the formula, and taking $v = 1/D$, the resulting expression is $V = k\sqrt{p'd^5/DL}$, which is the equation used by Cox, Halsey, and others (*Am. Mach.*, Aug. 20, 1896).

2. When the **drop in pressure is considerable**. In this case integration of the general equation gives the following:

$$p_1^2 - p_2^2 = (2/9\pi^2)(cM^2RTL/d^5) \qquad (b)$$

in which M is the weight of air flowing in lb. per sec.; d and L are taken in ft.. and p_1 and p_2 in lb. per sq. in.

Let V_a = volume of air at atmospheric pressure (*i.e.*, free air) flowing per min. Then the preceding formula may be given the form $p_1^2 - p_2^2 = k'V_a^2L/d^5$. This form of equation is used by Frank Richards (*Power*, vol. 32, p. 2138). Professor C. R. Richards (*Bulletin* No. 63, Univ. of Ill. Eng. Exp. Station) gives the same equation in the form $V_a = 3.061\sqrt{d^5p_1^2(1 - r^2)/KL}$, in which $r = p_2/p_1$, the ratio of final to initial pressure.

Of the many equations that have been proposed for the flow of gas in mains, all that have any claim to validity are deduced by obvious transformations from either eq. (*a*) or eq. (*b*).

Coefficients of Friction. The coefficient c in the general equation $dR = cw^2dL/d$ should be a constant if the frictional resistance, as assumed, varies as the square of the velocity and inversely as the diam. of the pipe. The coefficients k, k', and K in the derived formulæ involve the coefficient c with certain other constants depending on the units employed. Thus in the Cox-Halsey formula $V = k\sqrt{p'd^5/DL}$ with d in inches, the relation between the coefficients is $k^2c = 3600\pi^2/(16 \times 12^5)$. Halsey takes $k = 58$, basing this value on the experiments in connection with the Mt. Cenis tunnel. Stockalper's experiments on the pipe lines of the St. Gotthard tunnel indicate values of k between 63 and 73. If k be given the mean constant value 65, the corresponding constant value of c is 0.00000213.

In the equation $p_1^2 - p_2^2 = k'V_a^2L/d^5$, with d in *inches*, the relation between k' and c is $c = 0.00406k'$. The coefficient k' is usually taken as 0.0006 for compressed air, though Richards (*Power*, vol. 32, p. 2138) assumes the value 0.0005. The corresponding values of c are respectively 0.00000243 and 0.00000203.

Unwin ("Compressed Air Transmission"), on the strength of the experiments of Riedler and Stockalper, finds that the coefficient of friction decreases as the size of the pipe increases and gives the formula $K = 0.0027[1 + (3/10d)]$, d in *feet*. This expression for the coefficient is appropriate for the Unwin formula, which is the one used by Professor C. R. Richards. However, the expression $K = 0.003[1 + (3.6/d)]$, d in *inches*, is used by Professor Richards and others. Martin [*Engg.* (London), Mar. 19, 1897] makes use of Unwin's law of the variation of K with the pipe diameter and suggests the following expression for k (in which d is in *inches*) to replace the constant 58 used by Halsey: $k = \sqrt{7000/[1 + (3.6/d)]}$.

The formula therefore becomes $V = \sqrt{7000p'd^5/DL[1 + (3.6/d)]}$, or $p' = w^2L[1 + (3.6/d)]/7000Dd^5$.

Probably the most trustworthy experiments on the friction of compressible fluids in pipes are those of Fritzsche (*Mitt. über Forschungsarbeit.*, 60). According to these experiments the coefficient c varies with the pipe diam., the velocity of flow and the density of the fluid. With the system of units here employed (p in lb. per sq. in., d and L in ft., etc.) the coefficient c is given by

$$c = 0.0000022/d^{0.269}(wD)^{0.148}$$

Taking $D = \dfrac{1}{v} = \dfrac{144p}{RT}$, the expression for c takes the form

$$c = 0.0000022(R/144)^{0.148}(T/pw)^{0.148}d^{-0.269}$$

For air, $R = 53.34$; for steam, $R = 85.7$.

Table 46 gives for air values of 1,000,000 c, corresponding to various values of the ratio T/pw and for pipe diameters varying from 1 in. to 48 in. Intermediate values are readily found by interpolation. For steam, increase the tabular values by 7 per cent.

Table 46. Values of 1,000,000 c (For Air)

Diam. in inches	Values of the ratio T/pw														
	0.03	0.04	0.05	0.06	0.08	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
1	2.22	2.32	2.40	2.46	2.57	2.64	2.82	2.94	3.03	3.12	3.19	3.26	3.31	3.36	3.40
2	1.84	1.92	1.99	2.04	2.13	2.19	2.34	2.44	2.52	2.59	2.65	2.71	2.75	2.79	2.82
3	1.65	1.73	1.78	1.83	1.91	1.97	2.10	2.19	2.26	2.32	2.38	2.43	2.47	2.51	2.54
4	1.53	1.60	1.65	1.70	1.77	1.82	1.94	2.02	2.09	2.15	2.20	2.24	2.28	2.32	2.34
5	1.44	1.50	1.55	1.60	1.67	1.71	1.83	1.91	1.97	2.02	2.07	2.11	2.15	2.18	2.21
6	1.37	1.43	1.48	1.52	1.59	1.63	1.74	1.82	1.88	1.93	1.97	2.01	2.05	2.08	2.18
8	1.27	1.33	1.37	1.41	1.47	1.51	1.61	1.68	1.74	1.78	1.82	1.86	1.90	1.93	1.95
10	1.19	1.25	1.29	1.32	1.38	1.42	1.52	1.58	1.63	1.68	1.72	1.75	1.78	1.81	1.83
12	1.14	1.19	1.23	1.26	1.32	1.36	1.45	1.51	1.56	1.60	1.64	1.67	1.70	1.73	1.75
14	1.09	1.14	1.18	1.21	1.26	1.30	1.39	1.45	1.49	1.53	1.57	1.60	1.63	1.66	1.68
16	1.05	1.10	1.14	1.17	1.22	1.25	1.34	1.40	1.44	1.48	1.52	1.55	1.57	1.60	1.62
18	1.02	1.06	1.10	1.13	1.18	1.22	1.30	1.35	1.40	1.44	1.47	1.50	1.52	1.54	1.56
20	0.99	1.03	1.07	1.10	1.15	1.18	1.26	1.32	1.36	1.40	1.43	1.46	1.48	1.50	1.52
24	0.94	0.99	1.02	1.05	1.10	1.12	1.20	1.25	1.30	1.33	1.36	1.39	1.41	1.43	1.45
30	0.89	0.93	0.96	0.99	1.03	1.06	1.13	1.18	1.22	1.25	1.28	1.31	1.33	1.35	1.36
36	0.85	0.88	0.92	0.94	0.98	1.01	1.08	1.12	1.16	1.19	1.22	1.25	1.27	1.29	1.30
42	0.81	0.85	0.88	0.90	0.94	0.97	1.03	1.08	1.11	1.15	1.17	1.20	1.22	1.24	1.25
48	0.78	0.82	0.85	0.87	0.91	0.93	0.99	1.04	1.07	1.10	1.13	1.15	1.17	1.19	1.20

Table 47 gives a comparison between the coefficients of friction suggested by various authorities. Two cases are assumed: (1) 6-in. pipe, 80 lb. pressure; (2) 36-in. pipe, 80 lb. pressure.

Table 47. Comparison of Coefficients of Pipe Friction of Various Authorities

Velocity ft. per sec.	Values of 1,000,000c for 6-in. pipe				Values of 1,000,000c for 36-in. pipe			
	Fr.	U	U_1	M	Fr.	U	U_1	M
20	1.95	1.86	2.07	2.04	1.21	1.28	1.43	1.40
30	1.84	1.86	2.07	2.04	1.14	1.28	1.43	1.40
40	1.76	1.86	2.07	2.04	1.09	1.28	1.43	1.40
50	1.70	1.86	2.07	2.04	1.06	1.28	1.43	1.40

Column marked "Fr." gives Fritzsche's values. These decrease as the velocity increases. Column U gives values from the equation $K = 0.0027 \times [1 + (3.6/d)]$, while column U_1 gives the modified Unwin values from $K = 0.003 \times [1 + (3.6/d)]$. Column M gives Martin's values from $k^2 = 7000/[1 + (3.6/d)]$.

Unwin recommends the constant value $K = 0.003$ for all pipes above 12 in. in diam. This corresponds to the value $1{,}000{,}000c = 1.30$. Inspection of the table of Fritzsche's coefficients shows that this value is a fair average for pipes larger than 12 in. in diam. However, for small values of pw, low pressure or small velocity the coefficient may be considerably higher than 1.30, and for large values of pw it may be much lower. It is probable that **Fritzsche's coefficients** should be given the preference in accurate calculations.

Summary of Formulæ. In the calculation of the flow of a gas the desired result is usually (1) the drop in pressure for flow under given conditions, or (2) the diam. of pipe required for a predetermined drop in pressure. The known or assumed data are: L, length of pipe in ft.; M, weight flowing past any cross-section per sec.; T, the temperature; D, the weight per cu. ft., and w, the velocity.

CASE 1. The loss of pressure is relatively small. Denoting the drop $p_1 - p_2$ by p',

$$p' = c\frac{w^2D}{d}L = c\frac{w^2}{vd}L = 1.273c\frac{Mw}{d^3}L = 1.62\,c\frac{M^2}{Dd^5}L$$

If V, the volume flowing per sec., is taken instead of M, the last formula becomes $p' = 1.62cV^2DL/d^5$, whence $V = 0.786\sqrt{p'd^5/cDL}$. For the pipe diameter,

$$d = 1.1\sqrt[5]{cM^2L/Dp'} = 1.1\sqrt[5]{cV^2DL/p'}.$$

CASE 2. When the drop of pressure is considerable,

$$p_1^2 - p_2^2 = 625cM^2L/d^5 = 625cV^2D^2L/d^5.$$

If V_a is the volume of free air per sec., the formula becomes $p_1^2 - p_2^2 = 3.63cV_a^2L/d^5$, whence $V_a = 0.525\sqrt{d^5(p_1^2 - p_2^2)/cL}$ and $d = 1.294\sqrt[5]{cV_a^2L/(p_1^2 - p_2^2)}$.

The ratio T/pw, which is used in finding the value of c, may be expressed in terms of M or V_a, as follows: Taking 27,740 as the mean value of the product RT *for air*, $T/pw = 2.12d^2/M = 27.7\,d^2/V_a$.

Example. Required the diam. of a pipe 10,000 ft. in length for a flow of 50 cu. ft. of free air per sec.; initial pressure, 120 lb. per sq. in.; permissible drop in pressure 10 per cent., or 12 lb.

Since d is unknown, a trial value of c must be assumed. Let this be 1.6×10^{-6}. Then $d = 1.294\sqrt[5]{\dfrac{1.6\times10^{-6}\times 50^2\times 10{,}000}{120^2 - 108^2}} = 0.556$ ft. $= 6.67$ in. Taking a 7-in. pipe, $T/pw = 0.188$. From Table 46, $c = 1.68 \times 10^{-6}$. Hence $p_1^2 - p_2^2 = 3.63 \times 1.68 \times 10^{-6} \times 50^2 \times 10{,}000 \div \left(\frac{7}{12}\right)^5 = 2257$, and $p_2 = 110.8$ lb.

Flow of Steam in Pipes. Since for the relatively short pipes ordinarily used to convey steam the drop in pressure is small, the approximate formula $p' = cw^2L/vd$ is applicable. In the case of saturated steam the product pv'' (v'' = specific volume) remains nearly constant for a considerable range of pressure. Taking a mean value for this product, the equation of flow takes the forms $p' = KcM^2L/pd^5$, $M = \sqrt{p'pd^5/KcL}$. Evidence regarding the value of the coefficient c is conflicting. The experiments of Eberle indicate that c is practically constant for all conditions. On the other hand, experiments conducted for the Bavarian Revision-Verein seem to show that the empirical formulæ given by Fritzsche for air apply also to saturated and superheated steam. In the formulæ given by Martin, Hawksley, and Hurst, c is taken as constant; the formulæ of Babcock and Unwin make c vary with the diam. of pipe according to the law $c = K[1 + (3.6/d)]$, with d in inches. A comparison of different formulæ is made by Professor Gebhardt in *Power*, June, 1907. See also Lucke's "Engineering Thermodynamics," p. 1117.

Eberle's value of the coefficient c is 2.225×10^{-6} with d in ft. and 2.667×10^{-5} with d in in. Fritzche's values of c are those for air (p. 359) increased by 7 per cent.; hence $p' = 1.07cw^2L/vd$, where c is to be taken from Table 46.

In the second formula, $p' = KcM^2L/pd^5$, take c from the table of air coefficients. K has the following values: When p is less than 60 lb. per sq. in., $K = 685$. When p is greater than 60 lb. per sq. in., $K = 780$. The formula used by Martin and others is $p' = KM^2L/Dd^5$, and with M in lb. per min. and d in in., k has the following values: Martin, 0.0003133; Hawksley, 0.0003370; Hurst, 0.0003126.

Babcock's formula is $p' = k\,[1 + (3.6/d)](M^2L/Dd^5)$, with $k = 0.0001321$ (M in lb. per min. and d in in.).

As an example, take saturated steam at a pressure of 80 lb. per sq. in. flowing in a main 500 ft. long and 6 in. in diam. Under these conditions $T/pw = 0.129$, and from the table of coefficients $c = 1.69 \times 10^{-6}$. The specific volume is 5.47 cu. ft. per lb. and $D = 0.1829$ lb. per cu. ft. $M = 2.7$ lb. per sec. $= 162$ lb. per min. Then by the first formula, $p' = 1.07cw^2L/vd = 1.86$ lb. per sq. in.

By the approximate formula $p' = 780cM^2L/pd^5 = 1.92$ lb. per sq. in.

By Babcock's formula, $p' = 1.96$ lb.; by Eberle's $p' = 2.28$ lb., and by Martin's formula, $p' = 2.89$ lb.

The discrepancy due to the different assumed coefficients is evident. In the light of the best experimental evidence available at the present time, preference should be given to Fritzche's coefficients.

Throttling

Throttling or Wire-drawing. When a fluid flows from a region of higher pressure into a region of lower pressure through a valve or constricted passage, it is said to be throttled or wire-drawn. Examples are seen in the passage of steam through pressure-reducing valves, in the flow through ports and passages in the steam engine, and in the expansion valve of the refrigerating machine.

The **general equation applicable to throttling processes** is derived from the general equation for the flow of fluids, namely,

$$(w_2^2 - w_1^2)/2g = (i_1 - i_2)J.$$

The velocities w_2 and w_1 are practically equal, and it follows that $i_1 = i_2$; that is, in a throttling process the initial and final values of the heat content are equal.

For a mixture of liquid and vapor, $i = i' + xr$, hence the equation of throttling is $i'_1 + x_1r_1 = i'_2 + x_2r_2$. In the case of a perfect gas, $i = c_pT + i_0$, hence the equation of throttling is $c_pT_1 + i_0 = c_pT_2 + i_0$, or $T_1 = T_2$.

Joule-Thomson Effect. The investigations of Joule and Lord Kelvin showed that an actual gas experiences a drop in temperature when throttled. The ratio of the observed drop in temperature to the drop in pressure, that is, dT/dp, is the **Joule-Thomson coefficient.** According to these investigators, this coefficient varies inversely as the square of the absolute temperature, but is independent of the pressure. More recent experiments indicate, however, that the coefficient also varies with the pressure.

The **cooling effect produced by throttling** has been applied by Linde to the liquefaction of gases. See p. 352.

Loss Due to Throttling. A throttling process in a cycle of operations always introduces a loss of efficiency. If T_0 is the temperature corresponding to the back pressure, the loss of available energy is the product of T_0 and the increase of entropy during the throttling process. The following example illustrates the calculation in the case of ammonia passing through the expansion valve of a refrigerating machine.

Example. The liquid ammonia at a temperature of 70 deg. fahr. passes through the valve into the brine coil in which the temperature is 20 deg. and the pressure is 47.75 lb. per sq. in. The initial heat content of the liquid ammonia (see Table 24) is $i'_1 = 42.1$, and therefore the final heat content is $i'_2 + xr_2 = -12.6 + 556.3x_2 = 42.1$, whence $x_2 = (42.1 + 12.6)/556.3 = 0.0985$. From the table the initial entropy is $s'_1 = 0.0813$. The final entropy is $s'_2 + (xr_2/T_2) = -0.0262 + 0.0985 \times 1.1599 = 0.0880$. $T_0 = 20 + 460 = 480$; hence the loss of refrigerating effect is $480 \times (0.0880 - 0.0813) = 3.2$ B.t.u.

COMBUSTION

Fuels. For special properties of various fuels, see p. 594. In general, fuels may be classed under three heads: 1. Gaseous fuels. 2. Liquid fuels. 3. Solid fuels.

The combustible elements that characterize fuels are carbon and hydrogen. Sulphur is also a possible constituent. The complete combustion of carbon gives as a product carbon dioxide, CO_2; the combustion of hydrogen gives water, H_2O.

Combustion of Gaseous Fuels

Combustion Equations. The approximate molecular weights of the important elements and compounds are as follows:

Gas	H_2	O_2	N_2	CO	CO_2	H_2O	CH_4	C_2H_4	C_2H_6O
Molecular weight	2	32	28	28	44	18	16	28	46

For the elements C and H, the equations of complete combustion are:

$$\begin{cases} C + O_2 = CO_2 \\ 12 \text{ lb.} + 32 \text{ lb.} = 44 \text{ lb.} \end{cases} \qquad \begin{matrix} H_2 + \tfrac{1}{2}O_2 = H_2O \\ 2 \text{ lb.} + 16 \text{ lb.} = 18 \text{ lb.} \end{matrix}$$

For a combustible compound, as CH_4, the equation may be written

$$CH_4 + x \cdot O_2 = y \cdot CO_2 + z \cdot H_2O.$$

Comparing the two members of the equation, with respect to equal numbers of atoms, it is seen that

$$y = 1, z = 2, \; 2x = 2y + z, \text{ or } x = 2$$

Hence
$$\begin{cases} CH_4 + 2O_2 = CO_2 + 2H_2O \\ 16 \text{ lb.} + 64 \text{ lb.} = 44 \text{ lb.} + 36 \text{ lb.} \end{cases}$$

The coefficients in the combustion equation give the combining volumes of the gaseous components. Thus, in the last equation 1 cu. ft. of CH_4 requires for combustion 2 cu. ft. of oxygen and the resulting products of combustion are 1 cu. ft. of CO_2 and 2 cu. ft. of H_2O. The coefficients multiplied by the corresponding molecular weights give the combining weights. These are conveniently referred to 1 lb. of the fuel. In the combustion of CH_4, for example, 1 lb. of CH_4 requires $64/16 = 4$ lb. of oxygen for complete combustion and the products are $44/16 = 2.75$ lb. of CO_2 and $36/16 = 2.25$ lb. of H_2O.

Air Required for Combustion. The composition of air is approximately 0.23 O_2 and 0.77 N_2 by weight, or 0.21 O_2 and 0.79 N_2 by volume. For exact analyses it may be sometimes necessary to take account of the water vapor mixed with the air, but ordinarily this is neglected.

The minimum weight of air required for the combustion of 1 lb. of a fuel is the weight of oxygen required, as found from the combustion equation, divided by 0.23. Likewise, the minimum volume of air required for the combustion of 1 cu. ft. of a fuel gas is the volume of oxygen divided by 0.21. For example, in the combustion of CH_4 the weight of air required per lb. of CH_4 is $4/0.23 = 17.4$ lb. and the volume of air per cu. ft. of CH_4 is $2/0.21 = 9.54$ cu. ft. Ordinarily in practice more air is provided than is required for complete combustion. Let a denote the minimum weight required and xa the weight of air admitted; then $x - 1$ is the **excess coefficient.**

Products of Combustion. The products arising from the complete combustion of a fuel are CO_2 and H_2O. Accompanying these are the nitrogen brought in with the air and the oxygen in the excess of air. Hence the products of combustion contain principally CO_2, H_2O, N_2, and O_2. The **presence of CO indicates incomplete combustion.** The composition of the products of combustion is readily calculated from the combustion equations, as shown by the following illustrative example.

Example. A producer gas having the volume composition given is burned with 20 per cent. excess of air; required the volume composition of the exhaust gases.

	V	Coefficients in reaction equations O_2	CO_2	H_2O	Coefficients multiplied by V O_2	CO_2	H_2O
H_2	0.08	0.5	0	1	0.04	0	0.08
CO	0.22	0.5	1	0	0.11	0.22	0
CH_4	0.024	2	1	2	0.048	0.024	0.048
CO_2	0.066	0	1	0	0	0.066	0
N_2	0.61	0	0	0	0	0	0
	1.0				0.198	0.31	0.128

For 1 cu. ft. of the producer gas, 0.198 cu. ft. of O_2 is required for complete combustion. The minimum volume of air required is 0.198/0.21 = 0.943 cu. ft. and with 20 per cent. excess the air supplied is 0.943 × 1.2 = 1.132 cu. ft. Of this 0.238 cu. ft. is oxygen and 0.894 cu. ft. is N_2. Consequently, for 1 cu. ft. of the fuel gas, the exhaust gas contains

			or		
CO_2		0.31 cu. ft.		CO_2	15.7 per cent.
H_2O		0.128 cu. ft.		H_2O	6.5 per cent.
N_2	0.61 + 0.894 =	1.504 cu. ft.		N_2	75.8 per cent.
O_2 (excess)	0.238 − 0.198 =	0.040 cu. ft.		O_2	2.0 per cent.
		1.982 cu. ft.			100.0 per cent.

Volume Contraction. As a result of chemical action there is usually a change of volume; for example, in the reaction $2H_2 + O_2 = 2H_2O$, three volumes (two of H_2 and one of O_2) contract to two volumes of water vapor. In the example just given the volume of producer gas and air supplied is 1 cu. ft. gas + 1.132 cu. ft. air = 2.132 cu. ft., and the corresponding volume of the exhaust gas is 1.982 cu. ft., showing a contraction of about 7 per cent. For a hydrocarbon having the composition C_mH_n, the relative volume contraction is $[1 - (n/4)]$; thus for CH_4 and C_2H_4 there is no change of volume, for C_2H_2 the contraction is ½ volume, and for C_2H_6 there is an increase of ½ in volume.

The change of volume accompanying a chemical reaction causes a corresponding change in the gas constant R. Let R' denote the constant for the mixture of gas and air (1 lb. of gas and xa lb. of air) before combustion, and R'' the constant of the mixture of resulting products of combustion. Then, if y is the resulting contraction of volume, $R''/R' = (1 + xa - y)/(1 + xa)$.

Heating Value. A chemical change is accompanied by the generation or absorption of heat. The union of a combustible with oxygen produces heat, and the heat thus generated when 1 lb. of combustible is completely burned is called the **heating value** or the **calorific power** of the combustible, Heating values are determined experimentally by various forms of calorimeters. See p. 603.

For a gaseous fuel, an accurate definition of the heating value requires specification of the external conditions under which the combustion proceeds. Two heating values may thus be distinguished:

1. The **heating value at constant volume** (H_v) is the quantity of heat rejected to external surroundings when the temperature and volume of the combustion products are brought to the temperature and volume, respectively, of the gaseous mixture before burning.

2. The **heating value at constant pressure** (H_p) is the quantity of heat rejected when the temperature and pressure of the products are brought back to the temperature and pressure, respectively, of the gaseous mixture before burning.

If there is no change of volume due to the combustion, the heat values H_p and H_v are the same. When there is a contraction of volume H_p exceeds H_v by the heat equivalent of the work done on the gas during the contraction. For example, in the burning of CO according to the equation $CO + \frac{1}{2}O_2 = CO_2$, there is a contraction of $\frac{1}{2}$ volume. Taking 62 deg. fahr. as the temperature, the volume of 1 lb. CO at atmospheric pressure is 13.6 cu. ft.; hence the equivalent of the work done at atmospheric pressure is $\frac{1}{2} \times 13.6 \times 2116/778 = 18.5$ B.t.u., which is about 0.4 per cent. of the heating value of CO. Since the difference between H_p and H_v is so small in most fuels, it is usually neglected.

Strictly speaking, the **heating value varies with the temperature of the original gaseous mixture** (to which temperature the products of combustion are finally reduced). Thus if H_p^0 denote the heating value at 0 deg. and c'_p and c''_p the specific heats of the gaseous mixture and combustion products, respectively; then at temperature t, $H_p = H_p^0 + m(c'_p - c''_p)t$. The specific heats c'_p and c''_p are usually slightly different, and therefore H_p depends on the temperature at which the process is carried on. As a rule, the variation of H_p with t is not taken into consideration; if a standard temperature is required, however, 62 deg. fahr. is usually taken.

Lower Heating Value. The heating values defined in the preceding paragraphs are known as the **higher heating values.** With combustibles containing hydrogen, it is customary to base calculations relating to gas-engine efficiencies on another value called the **lower heating value of the fuel.** When in a calorimetric determination of the heating value of a fuel gas the products of combustion are cooled to the original temperature of the mixture, say 62 deg. fahr., most of the water vapor is condensed, and the measured quantity of heat H_p thus includes the latent heat recovered during this condensation. If the heat recovered by condensation is subtracted from H_p, the remainder is the so-called lower heating value of the combustible. Let H'_p denote the lower heating value and w the weight of H_2O produced by the combustion of 1 lb. of the fuel; then $H'_p = H_p - wr$.

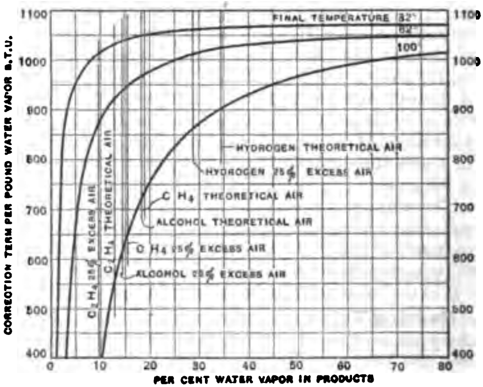


FIG. 25.—Correction Factors for Determining the Lower Heating Value of Fuels.

The factor r is frequently but incorrectly assumed to be the latent heat of steam at 212 deg. (970.4 B.t.u.) or the latent heat at 32 deg. (1073 B.t.u.). As a matter of fact, r varies with the conditions and depends upon the following factors: 1. Percentage of hydrogen in fuel; 2. Weight of air used; 3. Moisture in air and fuel; 4. Temperature adopted as a standard. The curves shown in Fig. 25 have been worked out by Mr. John A. Dent. They show for three temperatures, 32 deg., 62 deg., and 100 deg. fahr., values of r corresponding to various proportions of H_2O by volume in the products of combustion. As an example, suppose CH_4 to be burned with 25 per cent. excess of air. The H_2O formed is 15.5 per cent. of the volume of the combustion products; and for 62 deg. fahr., $r = 945$, while for a standard temperature of

100 deg., $r = 650$. From the combustion equation $w = 2.25$ lb. Taking the higher heating value of CH_4 as 23,840, the lower heating value for a standard temperature of 62 deg., under the conditions stated, is

$$H'_p = 23{,}840 - (2.25 \times 945) = 21{,}704 \text{ B.t.u.}$$

Heating Value per Unit Volume. Since the consumption of a fuel gas is more easily measured by its volume than by weight, it is convenient to express heating values in terms of volumes. For this purpose a standard pressure and temperature must be assumed. It is customary to take atmospheric pressure and 62 deg. fahr.; then the heating value per lb. multiplied by the weight of 1 cu. ft. of the gas under these conditions gives the **heating value of a cubic foot.** For example, the weight of 1 cu. ft. of CO at 14.7 lb. pressure and at 62 deg. fahr. is 0.0734 lb., and the heating value per lb. is 4380 B.t.u.; hence the heating value per cu. ft. is $4380 \times 0.0734 = 322$ B.t.u.

If the gas has some other pressure and temperature, say p_1 and t_1, the heating value per cu. ft. is found by multiplying the heating value per cu. ft. under standard conditions by $35.5p_1/(t_1 + 459.6)$.

Table 48 gives data relating to the combustion of gaseous compounds. In calculating the low heating value it is assumed that the theoretical weight of air is used and 62 deg. fahr. is taken as the standard temperature.

Table 48. Combustion of Gases

Gas	Chemical symbol	Molecular weight	Weight of 1 cu. ft. at 62 deg. fahr. and atmospheric pressure, lb.	Required weight of: O_2 per lb. of gas, lb.	Required weight of: Air per lb. of gas, lb.	Required volume of: O_2 per cu. ft. of gas, cu. ft.	Required volume of: Air per cu. ft. of gas, cu. ft.	Volume contraction per cu. ft. gas burned, cu. ft.	H_2O formed per lb. gas, lb.	H_2O by volume in products of combustion, per cent.	Heating value per lb. of gas: High H B.t.u.	Heating value per lb. of gas: Low H' B.t.u.	Heating value per cu. ft. at 62 deg. fahr.: High B.t.u.	Heating value per cu. ft. at 62 deg. fahr.: Low B.t.u.
Carbon monoxide.	CO	28	0.0734	0.572	2.477	0.5	2.38	0.5	0.0	0.0	4,380	4,380	322	322
Hydrogen...	H_2	2	0.0053	8.0	26.64	0.5	2.38	0.5	9.0	34.5	62,100	52,920	329	281
Methane....	CH_4	16	0.0421	4.0	17.32	2.0	9.52	0.0	2.25	18.8	23,850	21,670	1003	913
Ethane.....	C_2H_6	30	0.0790	3.733	16.16	3.5	16.7	−0.5	1.8	17.1	22,230	20,500	1755	1619
Propane....	C_3H_8	44	0.1158	3.636	15.79	5.0	23.8	−1.0	1.636	15.5	21,600	20,055	2501	2322
Butane.....	C_4H_{10}	58	0.1526	3.586	15.60	6.5	31.0	−1.5	1.55	15.0	21,240	19,780	3242	3020
Ethylene....	C_2H_4	28	0.0737	3.429	14.85	3.0	14.3	0.0	1.286	13.0	21,600	20,420	1591	1504
Propylene...	C_3H_6	42	0.1105	3.429	14.85	4.5	21.4	−0.5	1.286	13.0	21,330	20,150	2356	2226
Butylene...	C_4H_8	56	0.1473	3.429	14.85	6.0	28.6	−1.0	1.286	13.0	20,880	19,700	3076	2902
Acetylene...	C_2H_2	26	0.0684	3.077	13.32	2.5	11.9	0.5	0.692	8.0	21,600	21,020	1477	1437

Heating Value per Unit Volume of Mixture. Let a denote the volume of air required for the combustion of 1 cu. ft. of fuel gas and xa the value of air actually admitted, $x - 1$ being therefore the excess. Then the volume of the mixture of fuel gas and air is $1 + xa$, and the quotient $H/(1 + xa)$ may be called the heating value per cu. ft. of mixture. This magnitude is useful in comparing the relative volumes of mixture required with different fuel gases. Thus a lean gas, as blast-furnace gas or producer gas, has a low heating value H but the value of a is correspondingly low. On the other hand, a rich gas, like natural gas, has a high heating value but requires a large volume of air for combustion.

Specific Heat of Gases. The calculation of the temperature attained upon the combustion of a fuel under given conditions requires a knowledge of the variation of the specific heats of gases. At the present time the laws governing this variation are not very accurately known. It is known, however, that in the case of **simple gases,** as air, nitrogen, oxygen, CO, etc., the specific heat varies with the temperature, while if the law $pv = RT$ is closely obeyed, the specific heat is independent of the pressure. For a second group of gases like water vapor, H_2O, and CO_2, which are **moderately superheated vapors,** the specific heat varies with both the temperature and pressure. For air at moderate temperatures see p. 1513.

Equations connecting c_p or c_v with the temperature t have been given by Langen, Pier, and Holborn and Henning. The following give the mean **molecular specific heat** at constant pressure between 0 deg. cent. and t deg. cent. The molecular specific heat is the specific heat multiplied by the molecular weight.

(*a*) For diatomic gases. Air, O_2, H_2, N_2, CO:

Langen	$6.8 + 0.0006t$
Pier	$6.9 + 0.00045t$
Holborn and Henning	$6.58 + 0.000532t$

(*b*) For CO_2:

Langen	$8.7 + 0.0026t$
Pier	$8.8 + 0.0033t - 0.00000095t^2 + 0.0000000001t^3$
Holborn and Henning	$8.84 + 0.003267t - 0.000000792t^2$

(*c*) For H_2O:

Langen	$7.9 + 0.00215t$
Pier	$8.065 + 0.0005t + 0.0000000002t^3$
Holborn and Henning	$8.43 - 0.0003815t + 0.000000792t^2$

From these expressions for mean specific heat, expressions for the **actual** (or instantaneous) **specific heat** are derived by multiplying the coefficient of t by 2, the coefficient of t^2 by 3, and so on. To get the **specific heat per lb.,** divide by the molecular weight; thus, for oxygen, divide by 32; for CO_2, by 44, etc.

The linear equations of Langen are most convenient and are probably as rustworthy as any. For temperatures on the Fahrenheit scale, **Langen's ormulæ for actual specific heat** become respectively:

(*a*) For simple gases (m = molecular weight):

$$\left.\begin{aligned} c_v &= \frac{1}{m}(4.77 + 0.000667t) \\ &= \frac{1}{m}(4.48 + 0.000667T) \\ c_p &= \frac{1}{m}(6.75 + 0.000667t) \\ &= \frac{1}{m}(6.46 + 0.000667T) \end{aligned}\right\}$$

(*b*) For CO_2:

$$\left.\begin{aligned} c_v &= 0.15 + 0.000066t \\ &= 0.12 + 0.000066T \end{aligned}\right\} \qquad \left.\begin{aligned} c_p &= 0.195 + 0.000066t \\ &= 0.165 + 0.000066T \end{aligned}\right\}$$

(*c*) For H_2O:

$$\left.\begin{aligned} c_v &= 0.324 + 0.000133t \\ &= 0.263 + 0.000133T \end{aligned}\right\} \qquad \left.\begin{aligned} c_p &= 0.435 + 0.000133t \\ &= 0.374 + 0.000133T \end{aligned}\right\}$$

Investigations on the specific heats of other important gases are lacking. For CH_4 and C_2H_4 the following may be taken as rough estimates of the *mean* specific heats:

For CH_4

$$\left.\begin{aligned} c_p &= 0.481 + 0.00028t \\ &= 0.225 + 0.00028T \end{aligned}\right\} \qquad \left.\begin{aligned} c_v &= 0.356 + 0.00028t \\ &= 0.100 + 0.00028T \end{aligned}\right\}$$

For C_2H_4

$$\left.\begin{aligned} c_p &= 0.335 + 0.00021t \\ &= 0.118 + 0.00021T \end{aligned}\right\} \qquad \left.\begin{aligned} c_v &= 0.264 + 0.00021t \\ &= 0.047 + 0.00021T \end{aligned}\right\}$$

An expression for the **specific heat of a gaseous mixture** may be found from the formulæ for the specific heats of the components. Let M', M'', M''', etc., be the weights of the several components, and suppose the specific heats to be given by $a' + b'T$, $a'' + b''T$, $a''' + b'''T$, etc. Then the expression for the specific heat of the mixture is $a + bT$, in which

$$a = \frac{M'a' + M''a'' + M'''a''' + \ldots}{M' + M'' + M''' + \ldots}, \qquad b = \frac{M'b' + M''b'' + M'''b''' + \ldots}{M' + M'' + M''' + \ldots}$$

Table 49 contains values of the mean specific heat of various gases from 0 deg. to t deg. fahr. For the diatomic gases, the tabular values are to be divided by the molecular weight of the gas. Thus, the mean c_v per lb. of oxygen between 0 deg. and 800 deg. fahr. is $5.04/32 = 0.1575$.

Table 49. Mean Specific Heat of Gases (Langen)

Temp., deg. fahr.	Diatomic gases, H_2, N_2, O_2, CO		CO_2		H_2O		Temp., deg. fahr.	Diatomic gases, H_2, N_2, O_2, CO		CO_2		H_2O	
	mc_p	mc_v	c_p	c_v	c_p	c_v		mc_p	mc_v	c_p	c_v	c_p	c_v
0	6.75	4.77	0.1950	0.1500	0.4350	0.3240	2500	7.58	5.60	0.2783	0.2333	0.6017	0.4907
50	6.77	4.79	0.1967	0.1517	0.4383	0.3273	2600	7.62	5.64	0.2817	0.2367	0.6083	0.4973
100	6.78	4.80	0.1983	0.1533	0.4417	0.3307	2700	7.65	5.67	0.2850	0.2400	0.6150	0.5040
200	6.82	4.84	0.2017	0.1567	0.4483	0.3373	2800	7.68	5.70	0.2883	0.2433	0.6217	0.5107
300	6.85	4.87	0.2050	0.1600	0.4550	0.3440	2900	7.72	5.74	0.2917	0.2467	0.6283	0.5173
400	6.88	4.90	0.2083	0.1633	0.4617	0.3507	3000	7.75	5.77	0.2950	0.2500	0.6350	0.5240
500	6.92	4.94	0.2117	0.1667	0.4683	0.3573	3100	7.78	5.80	0.2983	0.2533	0.6417	0.5307
600	6.95	4.97	0.2150	0.1700	0.4750	0.3640	3200	7.82	5.84	0.3017	0.2567	0.6483	0.5373
700	6.98	5.00	0.2183	0.1733	0.4817	0.3707	3300	7.85	5.87	0.3050	0.2600	0.6550	0.5440
800	7.02	5.04	0.2217	0.1767	0.4883	0.3773	3400	7.88	5.90	0.3083	0.2633	0.6617	0.5507
900	7.05	5.07	0.2250	0.1800	0.4950	0.3840	3500	7.92	5.94	0.3117	0.2667	0.6683	0.5573
1000	7.08	5.10	0.2283	0.1833	0.5017	0.3907	3600	7.95	5.97	0.3150	0.2700	0.6750	0.5646
1100	7.12	5.14	0.2317	0.1867	0.5083	0.3973	3700	7.98	6.00	0.3183	0.2733	0.6817	0.5707
1200	7.15	5.17	0.2350	0.1900	0.5150	0.4040	3800	8.02	6.04	0.3217	0.2767	0.6883	0.5773
1300	7.18	5.20	0.2383	0.1933	0.5217	0.4107	3900	8.05	6.07	0.3250	0.2800	0.6950	0.5840
1400	7.22	5.24	0.2417	0.1967	0.5283	0.4173	4000	8.08	6.10	0.3283	0.2833	0.7017	0.5907
1500	7.25	5.27	0.2450	0.2000	0.5350	0.4240	4100	8.12	6.14	0.3317	0.2867	0.7083	0.5973
1600	7.28	5.30	0.2483	0.2033	0.5417	0.4307	4200	8.15	6.17	0.3350	0.2900	0.7150	0.6040
1700	7.32	5.34	0.2517	0.2067	0.5483	0.4373	4300	8.18	6.20	0.3383	0.2933	0.7217	0.6107
1800	7.35	5.37	0.2550	0.2100	0.5550	0.4440	4400	8.22	6.24	0.3417	0.2967	0.7283	0.6173
1900	7.38	5.40	0.2583	0.2133	0.5617	0.4507	4500	8.25	6.27	0.3450	0.3000	0.7350	0.6240
2000	7.42	5.44	0.2617	0.2167	0.5683	0.4573	4600	8.28	6.30	0.3483	0.3033	0.7417	0.6307
2100	7.45	5.47	0.2650	0.2200	0.5750	0.4640	4700	8.32	6.34	0.3517	0.3067	0.7483	0.6373
2200	7.48	5.50	0.2683	0.2233	0.5817	0.4707	4800	8.35	6.37	0.3550	0.3100	0.7550	0.6440
2300	7.52	5.54	0.2717	0.2267	0.5883	0.4773	4900	8.38	6.40	0.3583	0.3133	0.7617	0.6507
2400	7.55	5.57	0.2750	0.2300	0.5950	0.4840	5000	8.42	6.44	0.3616	0.3167	0.7673	0.6573

Temperature of Combustion. In calculating the temperature attained at the end of combustion of a given fuel mixture it is assumed: 1. That there is no loss of heat through the boundary walls. 2. That there is no dissociation. 3. That the combustion is complete. 4. That the inert gases do not take part in the reactions.

The principle employed is expressed by the equation: Energy of fuel mixture above 0 deg. fahr. + heat generated = energy of products above 0 deg. fahr. Taking t_1 as the initial temperature and t_2 as the final temperature, this equation becomes

$$\left.\begin{array}{l}\text{Sum of terms } Mc_vt_1\\ \text{for constituents of fuel mixture}\end{array}\right\} + \text{heat generated} = \left\{\begin{array}{l}\text{Sum of terms } Mc_vt_2\\ \text{of products,}\end{array}\right.$$

in which c_v denotes the *mean* specific heat.

As an example, consider the blast-furnace gas having the following composition, and assume that 15 per cent. excess of air is mixed with the fuel and that the temperature at the beginning of combustion is 600 deg. fahr. Then the composition of the fuel mixture, the mean specific heats, and the energies of the constituents are as follows:

	Composition of Fuel		Lower H_v per lb.	MH_v	Composition of fuel mixture, M	Mean sp. ht. 0° to 600° $c_v]_{0°}^{600°}$	$M \cdot c_v]_0^{600}$
	By volume	By weight, M					
H_2........	0.05	0.0037	52,920	195.8	0.0037	$\frac{1}{2} \times 4.97$	0.009195
CO.......	0.27	0.2814	4,380	1232.6	0.2814	$\frac{1}{28} \times 4.97$	0.049971
CH_4.....	0.015	0.0089	21,670	192.9	0.0089	0.524	0.004664
O_2........	0.085	0.1013			0.2447	$\frac{1}{32} \times 4.97$	0.038005
N_2........	0.58	0.6046			1.0848	$\frac{1}{28} \times 4.97$	0.192550
				1621.3	1.6235		0.294385

Energy of fuel mixture = 0.294385 × 600 = 178.63 B.t.u.

The temperature t_2 must now be assumed and the energy of the products calculated. Then the preceding equation will give a value of t_2 which may be used in a second trial. Taking a trial value t_2 = 4200 deg. fahr., the following results:

	Products M	Mean specific heat $c_v]_0^{4200}$	$M \cdot c_v]_0^{4200}$
CO_2......................	0.4667	0.2900	0.135343
H_2O......................	0.0533	0.6040	0.032193
N_2......................	1.0848	$\frac{1}{28} \times 6.17$	0.239044
O_2......................	0.0187	$\frac{1}{32} \times 6.17$	0.003606
			0.408186

Hence $t_2 = (178.63 + 1621.3)/0.408186 = 4410$ deg. Now taking $t_2 = 4400$, the value of $M \cdot c_v]_0^{4400}$ becomes 0.41676, and $t_2 = (178.63 + 1621.3)/0.41676 = 4320$. Hence t_2 is approximately 4360 deg. fahr.

Comparison of Ideal and Actual Temperatures of Combustion. The observed pressure following combustion in gas-engine cylinders is found to be considerably smaller than the pressure calculated according to the preceding method. It follows that **the actual temperature** attained **is lower than** the **calculated temperature.** The causes of the discrepancy are the following:

1. Flow of Heat through Walls of Vessel. The actual process is not adiabatic, as assumed, but in the case of a water-cooled cylinder a considerable part of the heat generated during combustion flows through the walls. Levin estimates that from 5 to 10 per cent. of the heat liberated is thus lost.

2. Dissociation. The two important combustion reactions, namely:

$$2H_2 + O_2 = 2H_2O, \quad 2CO + O_2 = 2CO_2$$

are reversible. When read from left to right, the second indicates the combination of CO and O_2 to form CO_2, and when read from right to left it indicates the dissociation of CO_2 into the constituents CO and O_2. The reaction does not necessarily proceed until all the CO and O_2 are combined, but under definite conditions of pressure and temperature halts at a definite equilibrium position. Nernst and von Wartenburg give the following data concerning the dissociation of H_2O and CO_2, respectively:

DISSOCIATION OF WATER VAPOR (PER CENT.)

Temperature, deg. cent.	Pressure in atmospheres 10	1	0.1	0.01
1000	1.4×10^{-5}	3.1×10^{-5}	6.7×10^{-5}	1.4×10^{-4}
1500	1.1×10^{-2}	2.2×10^{-2}	4.8×10^{-2}	0.11
2000	0.26	0.56	1.2	2.6
2500	1.6	3.4	7.2	14.7

DISSOCIATION OF CO_2 (PER CENT.)

Temperature, deg. cent.	Pressure in atmospheres 10	1	0.1	0.01
1000	9.8×10^{-6}	2.1×10^{-5}	4.5×10^{-5}	9.8×10^{-5}
1500	1.9×10^{-2}	4×10^{-2}	8.6×10^{-2}	0.19
2000	0.74	1.6	3.5	7.3
2500	6.20	13.0	25.5	46.0

Under the conditions that obtain in the gas-engine cylinder, *i.e.*, with a temperature of perhaps 1500 deg. cent. and a pressure of 20 to 30 atm., it is apparent that the dissociation of both H_2O and CO_2 is extremely small; that is, the combustion reaction is practically completed. Hence dissociation plays a negligible rôle in reducing the temperature of combustion.

3. FORMATION OF ENDOTHERMIC COMPOUNDS. Nernst has shown that at the high temperature arising from the combustion of a gaseous fuel compounds may be formed that could not exist at lower temperatures. Thus the nitrogen brought in with air may unite with oxygen; and when an excess of oxygen is present, water vapor is partly oxidised into hydrogen peroxide, H_2O_2. Any reaction which results in a compound that becomes more stable the higher the temperature is **endothermic**, that is, it absorbs heat. Hence the presence of such endothermic reactions must cause a reduction in the final temperature attained. The extent of the reduction from this cause is at present unknown.

Combustion of Solid Fuels

For **properties of solid fuels**, heating values, etc., see p. 594.

Air Required for Combustion. Let c, h, and o, denote respectively the parts by weight of carbon, hydrogen and oxygen in 1 lb. of the fuel. Then the **minimum weight of oxygen** required for complete combustion is $2.67c + 8h - o$ lb. and the **minimum weight of air** required is $a = (2.67c + 8h - o)/0.23 = 11.6\,[c + 3\,(h - o/8)]$ lb.

With air at 62 deg. fahr. and at atmospheric pressure, the minimum volume of air required is $v_m = 147\,[c + 3(h - o/8)]$ cu. ft. In practice an excess of air over that required for combustion is admitted to the furnace. The actual weight admitted per lb. of fuel may be denoted by xa. Then x = weight admitted ÷ minimum weight.

Combustion Products. If v_m is the minimum volume of air required for complete combustion and xv_m the actual volume supplied, then the products will contain per lb. of fuel $O_2 = 0.21\,v_m(x - 1)$ cu. ft., $N_2 = 0.79\,xv_m$ cu. ft.

From the reaction equation $C + O_2 = CO_2$, the volume of CO_2 formed is equal to the volume of oxygen required for the carbon constituent alone; hence volume of $CO_2 = 0.21v_m\,c/[c + 3\,(h - 0.125o)]$.

Of the *dry* gaseous products (*i.e.*, without water) the CO_2 content by volume is therefore given by the expression

$$CO_2 = 0.21c/[xc + (x - 0.21)3(h - 0.125o)].$$

The combined CO_2 and O_2 content is

$$CO_2 + O_2 = 0.21\{1 - 0.79/[x + cx/3\,(h - 0.125o) - 0.21]\}.$$

If the fuel is all carbon the combined CO_2 and O_2 is by volume 21 per cent. of the gaseous products. The more hydrogen contained in the fuel, the smaller is the $CO_2 + O_2$ content. The CO_2 content depends in the first instance on the excess of air. Thus, for pure carbon, it is $CO_2 = 0.21/x$.

The excess of air may be calculated from the composition of the gases and that of the fuel. Thus $x = 0.21\left[\frac{c}{[CO_2]} + 3(h - 0.125o)\right]\Big/[c + 3\,(h - 0.125o)]$, in which $[CO_2]$ denotes the per cent. by volume of the CO_2 in the dry gas.

The temperature of combustion is calculated by the same method as was employed with gaseous fuels. (See p. 367.) However, as the process usually occurs at constant pressure, c_p instead of c_v should be used.

Gas Producers

(See also pp. 1053 to 1059).

Simple CO Producer. The gas producer is used to convert a solid fuel into a gaseous fuel. In its simplest form the producer is a closed retort in which carbon is burned with a limited supply of oxygen. In the lower level of the producer air introduced through the grate comes in contact with carbon and combustion proceeds in accordance with the reaction

$$C + O_2 = CO_2$$

The heat liberated is 14,650 B.t.u. per lb. of carbon burned. The CO_2 thus formed passes through a bed of incandescent carbon and combines with it. The reaction is

$$CO_2 + C = 2CO$$

and in this reaction 10,220 B.t.u. are absorbed for each lb. of carbon burned. This quantity of heat is the heat of combustion of the CO formed; the efficiency of the process is $10{,}220 \div 14{,}650 = 0.698$. The remaining heat $14{,}650 - 10{,}220 = 4430$ B.t.u. is in part carried off as sensible heat in the gas leaving the producer, in part radiated from the producer.

The gas derived from this simple producer will consist of CO and N_2.

Producer Using Steam. The loss of heat inherent in the simple CO producer may be obviated in part by the introduction of steam into the fuel bed. The steam may be supplied by an external boiler heated by the gases leaving the producer, or it may be formed in a water jacket surrounding the fuel bed. The union of steam with carbon may be expressed by the reactions

$$2H_2O + C = 2H_2 + CO_2, \qquad H_2O + C = H_2 + CO$$

For temperatures between 1000 deg. and 1600 deg. fahr. the first reaction predominates, while for temperatures above 1600 deg. the reverse is the case. In general, high temperature of fuel bed is necessary for the production of CO, while with a low temperature the gas will show an excessive CO_2 content.

Assuming that no CO_2 is formed and that the producer operates under ideal conditions, the theoretical weight of steam required per lb. of carbon is found as follows: 1 lb. C burned to CO liberates 4430 B.t.u.; 1 lb. H_2 burned to H_2O liberates 62,100 B.t.u.; $\frac{1}{9}$ lb. H_2 burned to 1 lb. H_2O liberates 6900 B.t.u.

The heat liberated by the burning of C (4430 B.t.u.) is available for the decomposition of H_2O. Since to decompose 1 lb. H_2O requires 6900 B.t.u., the weight of H_2O supplied must be $4430/6900 = 0.628$ lb.

The gas formed has for its constituents CO, H_2, and N_2. For 1 lb. carbon the weights of constituents are as follows:

	Vol. at 62 deg. and 14.7 lb.	Theoretical composition by volume	Lower heating value (B.t.u. per cu. ft. at 62 deg. fahr.)
$CO = 28/12 = 2.33$ lb.	31.8 cu. ft.	0.396	127.5
$H_2 = 1/9 \times 0.628 = 0.07$ lb.	13.2 cu. ft.	0.165	46.4
O_2 required $= 1.333$ lb.		.....	......
O_2 from $H_2O = 0.628 - 0.07 = 0.558$ lb.		.....	......
O_2 from air $= 1.333 - 0.558 = 0.775$ lb.		.....	......
$N_2 = 0.775$ lb. $\times 77/23 = 2.595$ lb.	35.2 cu. ft.	0.439	0.0
Total volume of gas	80.2 cu. ft.		173.9

Heat Balance. Regarding the producer as a unit, heat is supplied to it from outside, heat is generated and absorbed within it by the various chemical processes, and heat passes out with the gaseous product withdrawn. The fuel, air, and steam furnished the producer bring in a certain amount of sensible heat above some assumed standard temperature, say 62 deg. fahr. Within the producer heat is generated by the combustion of C to CO and to CO_2, and heat is absorbed by the decomposition of H_2O into H_2 and O_2. Heat is also absorbed in raising the temperature of the fresh fuel to that of the producer. Some heat is lost by radiation, and some is carried out of the producer as sensible heat of the gaseous product. Assuming the producer to be maintained at a constant temperature,

$$\left\{\begin{array}{l}\text{heat received}\\ \text{by producer}\end{array}\right\} + \left\{\begin{array}{l}\text{heat generated}\\ \text{in producer}\end{array}\right\} = \left\{\begin{array}{l}\text{heat absorbed}\\ \text{in producer}\end{array}\right\} + \left\{\begin{array}{l}\text{heat carried out}\\ \text{of producer.}\end{array}\right.$$

It is convenient to refer these heat quantities to 1 lb. of carbon burned in the producer.

Let x = part of 1 lb. C that burns to CO_2. $1 - x$ = part of 1 lb. C that burns to CO. y = weight of steam introduced per lb. C. Q_s = heat furnished by entering steam. Q_a = heat furnished by entering air. Q_r = heat lost by radiation. Q_g = heat carried out by gas as sensible heat. Q_f = heat required to raise temp. of fuel to that of producer. The heat generated by the combustion of carbon to CO_2 is $14{,}650x$, that generated by the combustion of C to CO is $4430\,(1 - x)$. If y lb. of steam is decomposed, $\frac{1}{9}y$ lb. of H_2 is formed and the heat absorbed in the decomposition is $\frac{1}{9}y \times 62{,}100 = 6900y$. Hence the heat balance is expressed by the following equation:

$$Q_s + Q_a + 14{,}650x + 4430(1 - x) = 6900y + Q_r + Q_f + Q_g$$

From this equation the CO_2 content of the producer gas is determined when the weight of steam y is fixed.

Efficiency. For 1 lb. of carbon burned in the producer, $2\frac{1}{3}\,(1 - x)$ lb. of CO and $y/9$ lb. of H_2 are formed. The heating value of the gas is $10{,}220\,(1 - x) + 6900y$. Add to this the sensible heat Q_g and the sum is the heat output of the producer per lb. of C. The heat supplied per lb. of C is $Q_s + Q_a + 14{,}650$; hence the efficiency of the producer is

$$e = [10{,}220(1 - x) + 6900y + Q_g]/(Q_s + Q_a + 14{,}650)$$

When the gas is used in furnaces the sensible heat Q_g may be usefully employed and the preceding definition of efficiency holds. If the gas is used in a motor, it must be cooled, and the heat Q_g is wholly or in large part lost. The efficiency in this case, the so-called "cold-gas efficiency," is

$$e' = [10{,}220(1 - x) + 6900y]/(Q_s + Q_a + 14{,}650)$$

In best practice the value of e' may reach 0.85.

Weight of Steam Supplied. The heat quantities Q_s, Q_a, Q_g, etc., in the heat-balance equation depend upon the conditions of operation. The following may be taken as reasonable values under usual conditions:

$$Q_s + Q_a = 1000 \text{ B.t.u.} \quad Q_r = 900. \quad Q_f = 350. \quad Q_g = 950$$

Substituting these values in the equation,

$$14{,}650x + 4430(1 - x) = 6900y + 1200$$

For various values of x (part of carbon burned to CO_2) the weights of water decomposed per lb. of carbon are therefore as follows:

x =	0	0.10	0.20	0.30	0.40
y(lb.) =	0.468	0.616	0.764	0.913	1.061

If the weight of steam supplied exceeds 0.47 lb., there must necessarily be CO_2 in the gas formed.

The temperature of the fuel bed, however, determines largely the per cent. of CO_2 formed. According to the experiment of Clement (*Bulletin* No. 30, Engineering Exper. Station, Univ. of Illinois), the reaction $CO_2 + C = 2CO$ requires a temperature of 2372 deg. fahr. for the formation of 90 per cent. of CO from CO_2 and anthracite coal. With lower temperatures the per cent. of CO falls rapidly. If a small amount of steam is supplied the temperature of the fuel bed is high, thus permitting the formation of maximum CO. If more steam is admitted the temperature is lowered, the reaction between CO_2 and C is incomplete, and CO_2 is formed.

The effect of the introduction of steam is discussed by Clement and Adams (*Bulletin* No. 7, U. S. Bureau of Mines). The following is a condensation of this discussion.

The object of adding steam is threefold—to reduce the temperature of the fuel bed and thus avoid or hinder the fusion of the ash and the formation of clinker; to prevent the destruction of the furnace walls from fusion or slagging, and to reduce the heat losses in the gases leaving the producer. With increasing weight of steam there is a gradual decrease of efficiency.

A considerable amount of steam is necessary, with coals high in ash and where the ash is readily fusible, to prevent the formation of large clinkers.

The rate of gasification increases rapidly with rise in temperature. This suggests the possibility of operating producers at very high temperatures —2700 deg. fahr. or above—by providing for the removal of slag in a liquid state, as in blast-furnace practice. A producer embodying this principle would be adapted to coals which, because they clinker badly, have no market at present.

The higher the temperature, the better will be the quality of the gas and the greater the capacity of the producer.

Composition of Producer Gas. Taking x as the part of the carbon burned to CO_2 and y as the weight of steam supplied per lb. of carbon, the following expressions give, respectively, the weight in lb. and the volume in cu. ft. (at 62 deg. fahr. and 14.7 lb.) of the constituents of the gas formed from 1 lb. of carbon:

Constituent	Weight (lb.)	Volume (cu. ft.)
CO	$\frac{7}{3}(1 - x)$	$31.8(1 - x)$
CO_2	$1\frac{1}{3}x$	$31.7x$
H_2	$\frac{1}{9}y$	$21y$
N_2	$3.35\,[\frac{4}{3}(1 + x) - \frac{8}{9}y]$	$60.6(1 + x) - 40.4y$
	Total weight, $6.8 + 5.8x - 2.87y$.	Total volume, $92.4 + 60.5x - 19.4y$.

Table 50 gives the composition of producer gas for various values of x and the corresponding values of y as calculated in a preceding paragraph.

Table 50. Composition of Producer Gas

Per cent. of carbon burned to CO_2, 100x.	Steam supplied per lb. C, lb.	Volume of gas per lb. carbon, cu. ft.				Total volume, cu. ft.	Total weight, lb.	Composition by volume, per cent.				Lower heating value per cu. ft. at 62 deg. fahr. and 14.7 lb., B.t.u.
		CO	CO_2	H_2	N_2			CO	CO_2	H_2	N_2	
0	0.468	31.80	0.0	9.83	41.69	83.32	6.67	38.2	0.0	11.8	50.0	156
10	0.616	28.62	3.17	12.94	41.77	86.50	7.20	33.1	3.7	15.0	48.3	149
20	0.764	25.44	6.34	16.05	41.84	89.69	7.74	28.4	7.1	17.9	46.6	142
30	0.913	22.26	9.51	19.16	41.92	92.85	8.28	24.0	10.2	20.6	45.1	135
40	1.061	19.08	12.68	22.27	42.00	96.03	8.81	19.9	13.2	23.2	43.7	129

For **actual analyses of producer gas,** see pp. 1057 to 1059.

Surface Combustion. If, after forcing combustible gas through a porous mass of refractory material, igniting, and allowing it to burn in the air with the usual flame, air is forced in with the gas so that the resulting explosive mixture flows with too great velocity for backfiring to occur, the flame becomes non-luminous and, as the surface of the refractory mass is heated up, gradually disappears from the surface of the material, which glows under the action of the surface combustion. The combustion, which is supported by the oxygen admitted with the gas and is not influenced by the oxygen in front of the refractory mass, is perfect with a minimum of excess air. In the case of a porous slab or diaphragm, the combustion takes place within a distance of from ⅛ to ¼ in. from the surface. This method of heating has been applied to muffles and crucibles, where the porous refractory material, in the form of small lumps, is closely packed in the interspaces. Liquids may be evaporated from their surfaces by an inverted diaphragm placed directly over the containing vessel. Muffles can be kept steadily at a temperature of 2700 deg. fahr. (3600 deg. attainable) with the use of only one-half the amount of gas required in the older method of firing. The advantages of this method of combustion, according to Prof. W. A. Bone (*Engg.*, Apr. 14, 1911), are that combustion is accelerated by the incandescent surface, the heat developed can be concentrated just where it is required, very high temperatures are attainable without the use of regenerators, and that the energy is converted into radiant form which is transmitted very rapidly to the object exposed to it.

Surface combustion has been applied to boilers, the tubes being packed throughout their length with pieces of firebrick, the air-gas mixture entering each tube through a ¼-in. hole in a fireclay plug or nozzle (which serves to protect the joints in the tube sheet) at the front end. For gas firing, 3- to 6-in. tubes are used, and 9 to 12 in. for oil firing, the oil being first vaporized in a chamber at the firing end of the tube. Tests on a 10-ft. boiler (4 ft. long, with 110 3-in. tubes) at the Skinningrove Iron Works, in connection with an economiser, using coke-oven gas, after 5 months' use showed that of the heat supplied 92.7 per cent. was utilized, 3 per cent. escaping with the waste gases, and the remainder being lost through radiation, etc., but none in unburnt gas. Allowing for the power required to drive the draft fan, the overall efficiency was practically 90 per cent. The evaporation from and at 212 deg. was about 19 lb. per hour per sq. ft. of heating surface. The combustion was confined to about 4 in. next to the admission nozzles, the rest of the refractory material serving merely to baffle the gases; the heat was greatest in the cores of the material, the parts in contact with the tube walls not being very hot. In this boiler it was found that 70 (22) [8] per cent. of the total evaporation took place over the first (2nd) [3rd] third of the length of the tubes (*Engg.*, May 10, 1912). The length of the boiler is not increased for greater capacities, the increased capacity being obtained by an increase in the diameter of the boiler and the use of a greater number of tubes. Therefore in large units the distortion strains would be negligible because of the short length. In an experimental boiler with ten 3-in. tubes 3 ft. long the following results were obtained: 100 lb. per sq. in. gage was obtained in 20 min. after starting from cold; the evaporation from and at 212 was 21.6 per sq. ft. per hour; the gases left at only 126 deg. fahr. above the steam temperature without the use of an economiser; the CO_2 in the exhaust was 10.6 per cent. as against a theoretical percentage of 11; the free oxygen in the exhaust amounted to only 1.6 per cent. An evaporation of 17 lb. from and at 212 deg. has been obtained per pound of liquid fuel.

SECTION 5

STRENGTH OF MATERIALS

BY

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STRENGTH OF MATERIALS

BY

WILLIAM KENDRICK HATT

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MAIN SYMBOLS

STRESS	
Stress, Apparent	S
Pure Shearing	S_v
True (ideal) Stress	T
Proportional Elastic Limit	S_p
Yield Point	S_y
Ultimate Strength, Tension	S_M
Ultimate Compression	S_c
Vertical Shear in Beams	S_v
Modulus of Rupture	S_R
MOMENT	
Bending	M
Torsion	M_t
EXTERNAL ACTION	
Force	P
Weight of Body	G
Weight of Load	W
External Shear	V
MODULUS OF ELASTICITY	
Longitudinal	E
Shearing	G
Bulk	K
Modulus of Resilience	U_p
Ultimate Resilience	U_R
GEOMETRICAL	
Length	l
Area	A
Volume	V
Velocity	v
Radius of Gyration	r
Rectangular Moment of Inertia	I
Polar Moment of Inertia	I_P
DEFORMATION	
Gross, Longitudinal	e
Unit, Longitudinal	s
Angular	d
Lateral	s'
Poisson's Ratio	m
Reciprocal of Poisson's Ratio	n
Radius	r
Deflection	f

DEFINITIONS AND SIMPLE STRESSES

Deformations are changes in form produced by external forces or loads that act on non-rigid bodies. Deformations are **longitudinal,** e, a lengthening (+) or shortening (−) of the body; and **angular,** d, a change of angle between the faces.

Unit Deformation (abstract number) is the deformation in unit distance. Unit longitudinal deformation, $s = e/l$ (Fig. 1). Unit angular deformation, tan d, equals d approx. (Fig. 2).

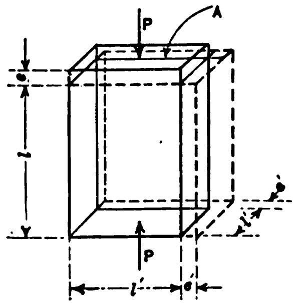


FIG. 1.

Accompanying a longitudinal deformation e, is a **lateral** deformation e', (Fig. 1). The ratio of s'/s is **Poisson's ratio,** m. Values of m are: Glass, 0.244; brass, 0.333; copper, 0.333; cast iron, 0.270; wrought iron, 0.278; steel, 0.303; lead, 0.430; concrete, 0.10 to 0.20 at working stresses and 0.25 at higher stresses.

The weight of a bar and also centrifugal forces are often important deforming forces. If the sides of a bar are restrained, important modifications result (see p. 391).

Stress is an internal distributed force; it is the internal mechanical reaction of the material accompanying deformation. Stresses always occur in pairs. Stresses are: **longitudinal**, S [tensile stress (+) and compressive stress (−)]; and **tangential**, or **shearing**, S_v.

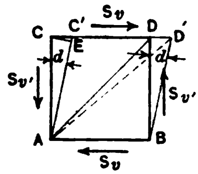


FIG 2.

Uniformly Distributed Longitudinal Stress is produced by a load acting along the geometric axis of a bar.

Intensity of Stress, or **Unit Stress**, S (lb. per sq. in.) is the amount of force per unit of area of surface (Fig. 3). P is the load acting through the center of gravity of the bar. The uniformly distributed normal stress is

$$S = P/A$$

When the stress is not uniformly distributed, $S = dP/dA$.

A **long rod** will stretch under its own weight G and a terminal load P. (See Fig. 4.) The total elongation e is that due to terminal load plus that due to one-half the weight of the rod considered as acting at the end.

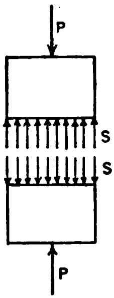


FIG. 3.

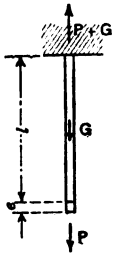


FIG. 4.

$$e = [Pl + (Gl/2)]/AE$$

The maximum stress is at the upper end.

A **solid of uniform strength in tension** or **compression** has curved boundaries (Fig. 5). Let A = area at any section at the distance x from the end; A_0 = area of base; P = load on base; E = base of Napierian logarithmic system; y = weight of material per unit volume; and S = uniform stress. The stress S on the increase of area dA, in height dx, carries the weight dG. $dG = yAdx = SdA$. Then $ydx/S = dA/A$, or

$$A = A_0E^{yx/S}$$

The weight of any portion of height x is $P_0E^{yx/S}$.

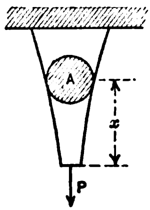


FIG. 5.

When the deformation arising from change of temperature is prevented, **temperature stresses** arise that are proportional to the amount of deformation that is prevented. Let a = coefficient of expansion per degree of temperature, l_1 = length of bar at temperature t_1, and l_2 = length at temperature t_2. Then

$$l_2 = l_1[1 + a(t_2 - t_1)]$$

If, subsequently, the bar is cooled to a temperature t_1, the proportionate deformation is $s = a(t_2 - t_1)$, and the corresponding unit stress $S = Ea \times (t_2 - t_1)$. For **coefficients of expansion**, see p. 293. In the case of steel, a change of temperature of 12 deg. fahr. will cause a unit stress of 2240 lb. per sq. in.

Shearing Stresses, S_v (Fig. 2), act tangentially to the surface of contact and do not change length of sides of elementary volume; they **change the**

angle between faces, and the **length of diagonal.** Two pairs of shearing

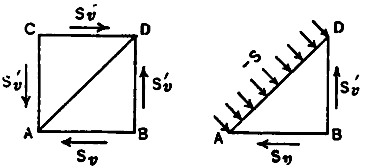


FIG. 6.

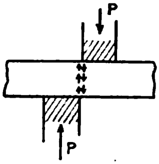


FIG. 7.

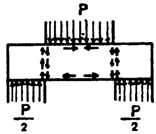


FIG. 8.

stresses must act together. **Shearing stresses are of equal magnitude on all four faces of an element.** $S_v = S_v'$ (Fig. 6).

In the presence of **pure shear** on external faces (Fig. 6), the **resultant stress** S on one diagonal plane at 45 deg. is pure tension, and on the other diagonal plane pure compression; $S = S_v = S'_v$. S on diagonal plane is called "diagonal tension" by writers on reinforced concrete. Failure under pure shear is difficult to produce experimentally. Fig. 7 shows an ideal case, and Fig. 8 a common form of test piece that introduces bending stresses.

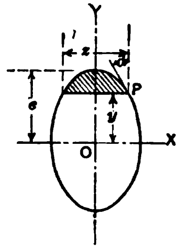


FIG. 9.

Let Fig. 9 represent the section of area A on which a shearing force Q acts. Then, if pure shear should exist, $S_v = Q/A$. This would be uniformly distributed over the area A. When **shear is accompanied by bending (horizontal shear in beams)** the unit shear S_v increases from the extreme fiber to the neutral axis OX, where it is a maximum. The unit shear parallel to axis OY at any point y distant from the neutral axis as at P (Fig. 9), is

$$S_v = \left(Q\int_y^e yzdy\right)/Iz$$

where I is the moment of inertia of the cross-section.

For a **rectangular cross-section** (Fig. 10a),

$$S_v = \frac{3}{2}\frac{Q}{bh}\left[1 - \left(\frac{2y}{h}\right)^2\right]; \qquad S_v\,(\text{max.}) = \frac{3}{2}\frac{Q}{bh} = \frac{3}{2}\frac{Q}{A}, \text{ for } y = 0.$$

For a **circular cross-section** (Fig. 10b),

$$S_v = \frac{4}{3}\frac{Q}{\pi r^2}\left[1 - \left(\frac{y}{r}\right)^2\right]; \qquad S_v\,(\text{max.}) = \frac{4}{3}\frac{Q}{\pi r^2} = \frac{4}{3}\frac{Q}{A}, \text{ for } y = 0.$$

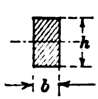


FIG. 10.—a.

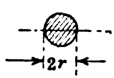


FIG. 10.—b.

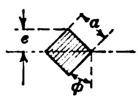


FIG. 10.—c.

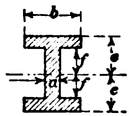


FIG. 10.—d.

For a **circular ring** (thickness small in comparison with the major diameter), S_v (max.) $= 2\,Q/A$, for $y = 0$.

For a **square cross-section** (diagonal vertical, Fig. 10c),

$$S_v = \frac{Q\sqrt{2}}{a^2}\left[1 + \frac{y\sqrt{2}}{a} - 4\left(\frac{y}{a}\right)^2\right]. \qquad S_v\,(\text{max.}) = 1.591\frac{Q}{A}, \text{ for } y = \frac{e}{4}.$$

For an **I-shaped cross-section** (Fig. 10*d*),

$$S_v \text{ (max.)} = \frac{3}{4}\frac{Q}{a}\frac{be^2 - (b-a)f^2}{be^3 - (b-a)f^3}, \text{ for } y = 0.$$

Stress-Strain Diagrams are drawn from results of tests of materials. Gradually increasing loads are applied and the resulting deformations are measured. In Figs. 11 and 12, unit deformations are shown as abscissæ and stresses as ordinates [tension (+) and compression (−)]. The word **strain,** as used here, means deformation.

Figs. 13 and 14 shows several such diagrams. The stresses are not actual but nominal, because the load is divided by the *original,* and not the actual deformed, cross-sectional area.

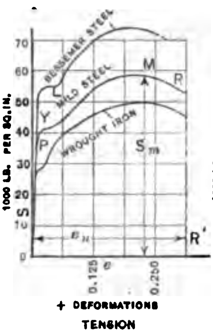


FIG. 11.

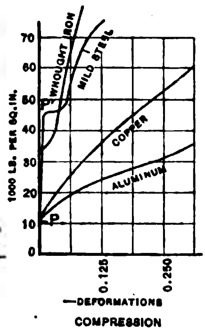


FIG. 12.

Elasticity is the tendency of deformed bodies to resume their former shape. The **elastic limit** is the limit of stress within which the deformation completely disappears after the removal of the stress; that is, no set remains. As measured in tests and used in design this term refers to the **proportional elastic limit,** S_p (Fig. 13), which is the stress within which stresses and deformations are directly proportional. At the **commercial elastic limit** or **yield point,** S_y, some materials experience a sudden and large increase of deformation without increase of stress. S_p ranges from 0.75 S_y (hot rolled steel) to 0.90 S_y (annealed steel). Location of S_p depends upon speed of application of stress.

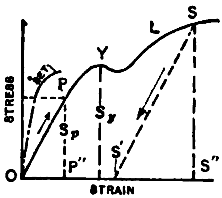


FIG. 13.

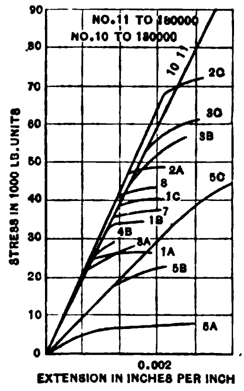


FIG. 14.—Typical Stress-strain Diagrams for Various Metals, Up To the Elastic Limit.

1A, 1B and 1C are for soft O.H. steel; 2A and 2C for axle steel; 3A, 3B and 3C for O.H. steel, forged; 4B for heavy steel casting; 5A, 5B and 5C for copper; 7 for wrought iron; 8 for light steel casting; 10 for chrome tungsten steel; 11 for vanadium steel.

Hooke's Law states that, within the elastic limit, the deformation produced is proportional to the stress. (Unless modified, the deduced formulæ of mechanics apply only within the elastic limit. Beyond this they are modified by experimental coefficients, as, for instance, the modulus of rupture.)

The Modulus of Elasticity (lb. per sq. in.) is the ratio of the increment of unit stress to increment of unit deformation within the elastic limit.

The Modulus of Elasticity in Tension, or Young's Modulus, E, is measured graphically by the slope of OP (Fig. 13). The compression modulus is similarly measured (Fig. 12). The reciprocals of the values of E for several materials express the relative deformations of these materials under the same stress. **E is a measure of stiffness.**

$$E = S/s = Pl/Ae$$

Modulus of Elasticity in Shear (lb. per sq. in.), or coefficient of rigidity, G (Fig. 2):

$$G = S_v/d$$

where d is expressed in radians. Theoretically, $G = En/2(n + 1)$, where n is the reciprocal of Poisson's ratio.

The Bulk Modulus of Elasticity, K, is the ratio of longitudinal stress, applied to all six faces of a cube, to the change of volume. Theoretically, $K = (1/3)En/(n - 2)$. When $n = 3$, $K = E$.

Change of Volume Under Longitudinal Stress. Let l, d, and b represent length, width, and thickness; m = Poisson's ratio; s = unit deformation. Then deformed volume $= (1 + s)l(1 - ms)b(1 - ms)d = (1 + s - 2ms)lbd$. Fractional change of volume $= (1 - 2m)s$. When m is less than ½, the volume is increased in tension and decreased in compression. For steel ($m = 1/3$), change of volume is about 1/3000-part at the elastic limit.

A bar of steel under stress does not at once assume the length due to its stress. Deformation proceeds for days and weeks. These phenomena of **residual elasticity** or **elastic afterworking** do not seem to be of practical importance.

When a load is carried by several paths to a support, the different paths take portions of the load in proportion to their stiffness, which is controlled by material (E) and by design.

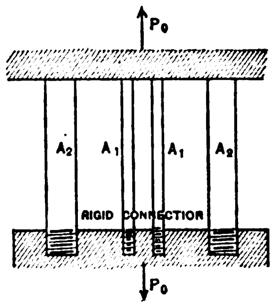


FIG. 15.

Example. Two pairs of bars rigidly connected (with the same elongation) carry a load P_0 (Fig. 15). A_1, A_2 and E_1, E_2 and P_1, P_2 and S_1, S_2 are cross-sections, moduli of elasticity, loads and stresses of the bars, respectively; e = elongation.

$$e = P_1l/(E_1A_1) = P_2l/(E_2A_2); \qquad P_0 = 2P_1 + 2P_2;$$

$$S_2 = P_2/A_2 = \tfrac{1}{2}[P_0E_2/(E_1A_1 + E_2A_2)] \text{ and } S_1 = \tfrac{1}{2}[P_0E_1/(E_1A_1 + E_2A_2)].$$

Resilience, U (in.-lb.), is the potential energy stored up in a deformed body. The amount of resilience is equal to the work required to deform the body from zero stress to stress S, when S does not exceed the elastic limit. For longitudinal stress, P = load; e = deformation; S = stress; A = cross-section; V = volume. Then Resilience = Work of deformation = Average force times deformation $= \tfrac{1}{2}Pe = \tfrac{1}{2}AS \times Sl/E = \tfrac{1}{2}S^2V/E$.

Modulus of Resilience, U_p (in.-lb. per cu. in.), or **Unit Resilience,** is the elastic energy stored up in a cubic inch of material at the elastic limit. For longitudinal stress U_p is graphically expressed by the area OPP'', Fig. 13.

$$U_p = \tfrac{1}{2}\, S_p^2/E$$

The unit resilience for any other kind of stress, as shearing, bending, torsion, is a constant times one-half the square of the stress divided by the appropriate modulus of elasticity. For values, see Table 1.

The **resilience** stored up at a stress **beyond** the **elastic limit** is measured by the area of the triangle $S''S\,S'$ (Fig. 13). The line SS' represents the release of a stress from S to zero load at S', leaving a set OS'.

Table 1. Resilience per Unit of Volume, U_p

(S = Longitudinal stress; S_s = Shearing stress; E = Tension modulus of elasticity; G = Shearing modulus of elasticity)

		TORSION	
Tension or compression....	$\frac{1}{2}S^2/E$	Solid, circular...........	$\frac{1}{4}S_s^2/G$
Shear....................	$\frac{1}{2}S_s^2/G$	Hollow, radii R_1 and R_2..	$\frac{(R_1^2 + R_2^2)}{R_1^2}\frac{1}{4}\frac{S_s^2}{G}$
BEAMS (free ends)		SPRINGS	
Rectangular section, bent in arc of circle no shear.....	$\frac{1}{6}S^2/E$	Carriage...............	$\frac{1}{6}S^2/E$
Ditto, circular section.....	$\frac{1}{8}S^2/E$	Flat spiral, rect. section.	$\frac{1}{24}S^2/E$
Concentrated center load; rectangular cross-section..	$\frac{1}{18}S^2/E$	Helical: axial load, circular wire................	$\frac{1}{4}S_s^2/G$
Ditto, circular cross-section.	$\frac{1}{24}S^2/E$	Ditto, axial twist........	$\frac{1}{8}S^2/E$
Uniform load, rectangular cross-section.............	$\frac{4}{45}S^2/E$	Ditto, axial twist, rect. section................	$\frac{1}{6}S^2/E$
I-beam section, concentrated center load.........	$\frac{3}{32}S^2/E$		

Unit Rupture-work, U_R, sometimes called **ultimate resilience,** is measured by the area of the stress-deformation diagram to rupture, $0PYMRR'$ (Fig. 11). For steel

$$U_R = \tfrac{1}{3}e_u(S_y + 2S_M), \text{ approximately}$$

For structural steel, for instance, $U_R = \frac{1}{3} \times \frac{27}{100} \times [35{,}000 + (2 \times 60{,}000)] = 13{,}950$ in.-lb. per cu. in.

Example 1. A load $P = 40{,}000$ (lb.) compresses a wooden block of cross-sectional area $A = 10$ (sq. in.) and length $= 10$ (in.), an amount $e = \frac{5}{100}$ (in.). Stress $S = \frac{1}{10} \times 40{,}000 = 4000$ (lb. per sq. in.). Unit elongation $s = \frac{5}{100} \div 10 = \frac{1}{250}$ (abstract number). Modulus of elasticity $E = 4000 \div \frac{1}{250} = 1{,}000{,}000$ (lb. per sq. in.). Unit resilience $U_p = \frac{1}{2} \times 4000 \times 4000/1{,}000{,}000 = 8$ (in.-lb. per cu. in.)

Example 2. A weight $G = 5000$ (lb.) falls through a height $h = 2$ (ft.); V = number of cubic inches required to absorb the shock without exceeding a stress of 4000 (lb. per sq. in.). Neglect compression of block. Work done by falling weight $= Gh = 5000 \times 2 \times 12$ (in.-lb.). Resilience of block $= V \times 8$ (in.-lb.) $= 5000 \times 2 \times 12$. Therefore, $V = 15{,}000$ (cu. in.).

MATERIAL STRESSED BEYOND THE ELASTIC LIMIT

Beyond the elastic limit in tension S_p (Fig. 13), ductile materials like steel enter a semi-plastic stage. The material stretches at Y without increase of stress, and the scale beam of the testing machine "drops" and the hard mill scale falls from bar. Steel shows both S_p and S_y; cast iron, S_y but no S_p; wood and hardened steel, S_p, but no S_y.

The shape of the diagram from S_p to S_y depends upon *speed of stress* (Fig. 11). After L, semi-plastic and semi-elastic deformations proceed. One or several contractions of cross-section occur, depending upon homogeneity of metal. The **maximum load** is reached at M (Fig. 11), when the metal at one of these contractions begins to flow. The contraction or "neck" proceeds until bar ruptures at R. The character of the metal changes under this excessive deformation, and therefore the actual stress on the ruptured section is not of practical importance, and is not observed.

Ultimate Strength S_M = maximum load divided by original area. **Per cent. of ultimate elongation** $e'_u = (l' - l)100/l$. **Per cent. of contraction** $c'_u = (A - A')100/A$. Here l and l' are original and deformed lengths, and A and A' are original and deformed areas, respectively. In testing wire, it is common to measure the elongation just before rupture. On account of elasticity the latter is notably greater than elongation after rupture. If the load is released at intervals to observe the set, a **set curve** may be constructed (Fig. 13).

The Character of the Fracture Under Tension indicates quality of material, but is influenced by the speed and the method of producing fracture, and by the shape of the test piece. A metal that is tough and fibrous may appear crystalline if broken quickly at a nicked section. Contraction is greatest in tough and ductile, and least in brittle, materials. The **shape** of fracture is usually a central flat surface of failure in tension, surrounded by a rim on which the metal shears. The extent of the rim is more pronounced when the ratio of shearing to tensile strength is less; is more developed in soft steels; becomes a complete cone in very soft materials; and vanishes in cast iron. The color, grain and shear of the fracture are significant. **Forms of fracture** are shown in Fig. 16. Terms describing fracture are: silky, dull, granular, crystalline, fibrous.

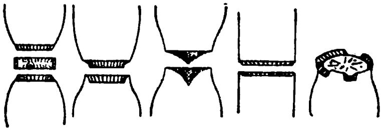


FIG. 16.—Typical Metal Fractures in Tension.

Failure Under Compression (Fig. 17) depends on material, slenderness of specimen, and restraint at ends or sides. **Short blocks of brittle materials** in compression like cast iron, stone, cement, when unconfined at the sides, fail by sliding along inclined planes. The angle of these planes is a function of the shearing stress and of the coefficient of friction of the material. Internal cones with their bases at the pressure heads of the testing machines, and pyramids, form in cylindrical and rectangular specimens, respectively.

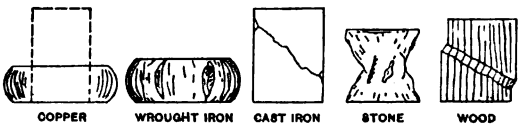


FIG. 17.—Typical Compression Fractures.

Ductile or Soft Materials, like lead, copper and soft steel, cannot be ruptured in compression. They bulge, increase in diameter under increasing stress, and finally become plastic at the **stress of fluidity**. Unwin quotes following values for **pressure of fluidity** in lb. per sq. in.: Mild steel, 112,000; copper, 54,000; lead, 1700. Experiments by Hatt give 76,000 for wrought iron.

The available strength in compression of short blocks **of materials of medium ductility**, like steel or wrought iron, is generally to be taken as the **yield point** of the material. The strength of longer columns depends on the ratio of length to diameter of specimen. The compressive strength of stone and concrete is about 10 times the tensile strength. Fracture forms for several materials in compression are shown in Fig. 17.

Permanent Deformation of metals beyond the elastic limit is accompanied by an increased hardness, an **increased strength** and a **decreased ductility.**

If steel is strained to or beyond the yield point, unloaded, and again immediately loaded, (*a*) the ultimate elongation is diminished, and the ultimate strength increased; and (*b*) the elastic limit is lowered, sometimes to zero, and the modulus of elasticity is diminished, there no longer being a rectilinear relation between stress and strain; and (*c*) the yield point is raised to a stress corresponding to the previous load. If, after the first loading, a period of quiescence is allowed before the second loading, (*a*) the elastic limit will rise above its original value, and (*b*) the yield point rises gradually above the stress corresponding to the previous load. The recovery of the elastic limit, reduced by overstrain, can be hastened by heating the bar for 3 to 4 min. in boiling water. Annealing an overstrained bar at 750 deg. cent., restores its original properties.

The primitive elastic limit of steel observed under first loading, is a limit artificially raised by processes of manufacture. By applying reversals of stress from tension to compression, and *vice versa*, through a range of stress less than the primitive elastic limit, a new and lowered elastic limit in tension is established. There is a range of reversal of stress within which the material is perfectly elastic, and the limiting stress in this case has been called the **natural elastic limit.** This is evidence that this elastic range is the same range to which a bar can be subjected an indefinite number of times without breaking under fatigue. (Abstracted from Unwin.)

Fatigue and Repetitive Stress. Numerous researches by Wöhler, Spangenberg and Martens, Bauschinger and others have shown that when a metal is subjected to cycles of varying stress, not applied in impact but gradually, it breaks at loads varying from one-half to two-thirds the ultimate breaking strength. The greater the range through which the stress varies and the greater the number of repetitions, the lower is the breaking strength. Fig. 18, from Unwin's "Testing the Materials of Construction," illustrates the laws of fatigue. The shaded area represents the range of stress which, applied an indefinite number of times, will not cause failure. For instance, a range of reversed stress less than one-third the ultimate strength for soft steel and one-fourth for high carbon steel will not cause failure when applied an unlimited number of times. An unlimited number of repetitions of a range of stress in tension from *nd* to *ne* will not cause failure. *am* = reversal limit of stress. Two cases arise: 1. The stress is repetitive, of same sign. 2. The stress varies from compression to tension, a reversal of stress.

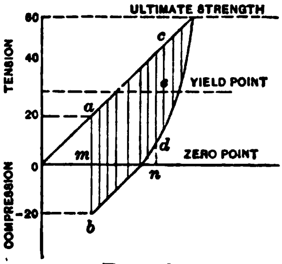


FIG. 18.

Gerber's Formula is an expression of experimental laws. Let S_R = the range of stress = $S_{max} \mp S_{min}$. The upper sign is taken when the stresses are of same sign; and the lower where they are of different sign. Let S_M = statical breaking strength. Then

$$S_{max} = \tfrac{1}{2} S_R + \sqrt{S_M^2 - nS_RS_M}$$

where n is a constant for a given material, ranging from 1.5 in ductile iron and steel to 2 in harder qualities. This equation fixes the maximum strength S_{max} against unlimited repetition when the range of stress S_R and the statical strength S_M are known.

Bars breaking under repetitive and reversed stresses exhibit no ductility. Microscopic examination shows that slippage along the cleavage planes of the crystalline constituents of the metal under repetitive stress ultimately develops into a crack.

STRENGTH OF MATERIALS

Technical Qualities of Materials may be grouped as: **Technological,** having to do with manufacturing requirements, such as malleability, fusibility, forgeability, bending to shape; **physical,** such as specific gravity, homogeneity, structural characteristics; **mechanical** properties examined in tests. The latter, together with criteria commonly used, are as follows:

QUALITY	SERVICE	CRITERIA	EXAMPLE
Strength.	To carry dead load.	Ultimate strength.	Piano wire.
Elasticity.	To undergo deformation and return to shape.	Amount of deformation.	Rubber.
Resilience.	To absorb shock without permanent deformation.	Modulus of resilience.	Hickory.
Stiffness.	To carry load without deformation.	Modulus of elasticity.	Steel.
Hardness.	To (a) withstand wear; (b) to resist penetration.	Scratch test; abrasion test; ball test.	Manganese steel.
Toughness.	Various conceptions; to endure large permanent deformations; to withstand large energy without rupture.	Various.	Rivet steel, hickory.
Endurance.	To withstand repetition of stress and small shocks.	....	Vanadium steel.

General Strength Values. The following tables exhibit the general range of values to be expected in various materials when subjected to various kinds of loading. For more detailed data, see Section 6.

Table 2. Tensile Strength of Iron and Steel*
(Range of Averages)

Material	Specific gravity	Ultimate strength, lb. per sq. in.	Elastic limit, lb. per sq. in.	Yield point, lb. per sq. in.	Ultimate elongation, per cent.	Modulus of elasticity, 1000 lb. per sq. in.
Cast iron	7.207					
Gray iron		15,000–18,000	5,000–6,000		0.1–1.0 in 2 in.	12,000 14,000
Better grade		20,000–30,000	10,000–24,000			
Malleable cast iron		25,000–48,000	12,000–22,000	12,500–19,000	1.0–3.0 in 2 in.	
Wrought iron	7.78	42,000–52,000	21,000–26,000	28,000–34,000	22.0–30.0 in 8 in.	26,000 29,000
Steel						
Extra soft		45,000–55,000	22,500–27,500	30,000–33,000	27.0–33.0 in 8 in.	
Soft (C.–0.08–0.15)	7.833	50,000–60,000	25,000–30,000		25.0–30.0 in 8 in.	
Medium (C.–0.15–0.30)		60,000–70,000	30,000–35,000	37,000–44,000	22.0–25.0 in 8 in.	
Hard (C.–0.30 up)		70,000–80,000	35,000–40,000	38,000–45,000	18.0–22.0 in 8 in.	
Rail (C.–0.55–0.75)		101,000–115,000	50,000–57,000	50,000–57,000	12.0–15.0 in 2 in.	
Ingot iron (C.–0.01–0.15)		42,000–60,000			18.0–36.0 in 8 in.	Modulus of elasticity of all steels varies from 28,000,000 to 31,000,000; average, 30,000,000.
Steel castings	7.917					
Soft		60,000–72,000	30,000–35,000	40,000–46,000	22.0–up in 2 in.	
Medium		72,000–78,000	36,000–39,000		16.0–22.0 in 2 in.	
Hard		78,000 up	39,000 up		12.0–16.0 in 2 in.	
Steel forgings		75,000–90,000	37,000–45,000		22.0–24.0 in 2 in.	
Spring steel						
Untempered		101,000–135,000	50,000–67,000		3.0–6.5 in 8 in.	
Tempered		130,000–200,000	110,000–170,000		0.5–2.1 in 8 in.	
Nickel steel						
Structural (3.5 per cent. N)		100,000–120,000	50,000–60,000	58,000–70,000	16.0–20.0 in 8 in.	
Forging (annealed)		80,000	40,000		30.0 in 2 in.	
Forging (oil-tempered)		98,000	75,000		25.0 in 2 in.	
Vanadium steel						
Annealed		54,000–96,000	27,000–48,000		22.0–24.0 in 2 in.	
Oil-tempered		125,000–232,000	100,000–180,000		11.0–21.0 in 2 in.	

* OTHER STRENGTH FUNCTIONS. Compressive strength of cast iron = 6 × T. S. (Tensile Strength). Compressive strength of wrought iron and steel to be taken as the yield point. Shearing strength of cast iron = 1.10 × T. S.; of wrought iron = 0.85 × T. S.; of hard and soft steels = 0.75 × T. S. Bending strength or modulus of rupture of cast iron = 2 × T. S.; of wrought iron = T. S.; of steel, to be taken as the yield point. Coefficient of expansion for change of 1 deg. fahr. is 0.0000065 for steel; and 0.0000062 for cast iron.

Table 3. Mechanical Properties of Non-ferrous Metals and Their Alloys
(Range of Averages)

Metal	Specific gravity	Tensile elastic limit, lb. per sq. in.	Tensile ultimate strength, lb. per sq. in.	Tensile modulus of elasticity, in 1000 lb. per sq. in.	Compressive strength, lb. per sq. in.
Copper, cast*	8.6–8.9	11,000–15,000	22,000–28,000	10,000–15,000	39,000–48,000
Copper, plate*	8.8–8.9	....	29,000–36,000	12,500–16,800	58,000
Copper wire (hard-drawn)	....	....	49,000–67,000	14,500–17,000	....
Copper wire (annealed)	....	....	32,000–35,000	....	....
Aluminum, cast	2.60	4,000– 6,500	11,000–16,500	8,000–11,000	12,000
Aluminum, rolled	2.72	12,500–14,000	16,500–22,400	9,700–10,000	....
Zinc, cast	7.1	....	4,000– 6,000	12,000–17,000	....
Tin	7.3–7.4	....	4,000– 5,000	3,000– 6,000	6,400
Gun metal	8.6	....	25,000–50,000	10,000	....
Phosphor bronze	8.7	20,000	36,000–40,500	12,000–14,000	....
Manganese bronze (cast)	....	30,000	65,000–85,000	....	....
Tobin bronze (cast)	....	51,000–56,000	66,000–80,000	4,500	....
Aluminum bronze, 90% Cu., 10% Al. (forged)	7.7	60,000	100,000	16,800	....
Delta metal	8.4	20,000	52,000	11,000	....

* Modulus of rupture: Cast copper, 20,000–40,000; plate copper, 30,000–60,000.

Table 4. Mechanical Properties of Stone and Brick*

	Specific gravity	Compressive strength, lb. per sq. in.	Modulus of elasticity, lb. per sq. in.	Absorption of water, parts	Coefficient of expansion per deg. fahr.	Modulus of rupture, lb. per sq. in.
Granite	2.67	19,400	7,300,000	1/150	0.0000040	1,850
Limestone	2.53	9,500	8,460,000	1/38	0.0000045	1,400
Limestone, oolithic	2.48	6,700	7,000,000	1/23	0.0000045	....
Marble	2.72	12,700	8,000,000	1/300	0.0000045	1,400
Sandstone	2.22	9,300	3,000,000	1/24	0.0000055	1,400
Trap	2.96	20,000	12,000,000	....	....	....
Slate	2.77	14,000	14,000,000	1/430	0.0000058	7,700
Brick						
Common	2.00	4,000	2,000,000	1/8	....	....
Hard-burned	2.10	8,000	4,000,000	1/6	....	....
Paving	2.42	10,000	7,000,000	1/100	....	....
Sand-lime	1.85	3,500	1,000,000	1/10	....	....
Brick masonry						
In lime mortar		0.14 × compressive strength of brick				
In cement mortar		0.23 × compressive strength of brick				

* OTHER STRENGTH FUNCTIONS. Shearing strength of brick and stone is from 10 to 20 per cent. of the compressive strength; tensile strength is 4 per cent. of compressive strength; modulus of rupture is 15 per cent. of compressive strength. Poisson's ratio is ¼.

Tensile Strength of Wire depends upon its diameter. The ultimate strength in lb. per sq. in. $= S_M = (c/d) + S_0$, for any diameter d in in. (for diameters not less than 0.01 in.) where S_0 represents the ultimate strength ($d = \infty$) and c is a constant for each material.

Table 5. Tensile Strength of Wire
(Values of c and S_0)

Kind of wire	Not annealed		Annealed	
	c	S_0 lb. per sq. in.	c	S_0 lb. per sq. in.
Best iron wire	894	90,600	213	47,080
Ordinary iron wire	1282	65,140	358	40,820
Steel wire	1495	90,600	213	81,500
Zinc wire	123	18,060	.....	.....
Copper wire	538	49,790	.....	33,570
Brass wire	571	77,940	392	40,820
Hard lead wire	.....	3,130	.....	.....
Soft lead wire	.....	2,420	.....	.....
Platinum wire	678	31,720	538	26,310
Bronze wire	1041	71,540	.....	.....

For average values of the tensile strength, see Section 6. Allowable stresses are ¼ to ⅙ the ultimate stress.

Relation Between Tensile Strength and Elongation. Tetmajer's criterion for a good quality of steel is: The product of percentage elongation and tensile strength in lb. per sq. in., equals a constant varying from 1,000,000 to 1,500,000.

Standard Tests and Test Pieces

Forms of Test Bars. In addition to those shown in Fig. 19 and on pp. 506 and 508, the following are recommended by the American Society for Testing Materials. **Compressive test of metals:** Specimen 1 in. in diameter and from 2.5 to 4 diameters high. **Transverse test of metals:** For cast metals a specimen 1¼ in. in diameter, long enough to furnish a span of at least 15 times the diameter, cast on end, and not machined before testing.

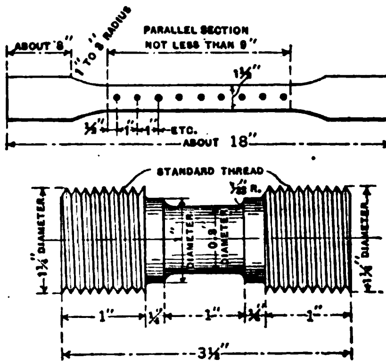


FIG. 19.—Dimensions of Standard Tension Test Pieces.

Impact Tests on notched bars, to detect **brittleness,** are favored abroad. The test piece used is a bar 30 mm. by 30 mm. in cross-section and 160 mm. long, with a notch at center of lower face 15 mm. deep, having a radius at bottom of 2 mm.; bar is tested on a 120-mm. span. Striking edge of hammer to be rounded to radius of 2 mm. Rupture to be effected by a single blow.

In the ordinary commercial tensile test the speed of application of load will not affect the results appreciably when the test ranges between 1 and 6 minutes.

The relation between strength, elongation and shape of tension test bar is expressed by **Barba's law of proportionality** thus: Test bars that are geometrically similar deform similarly and yield equal unit strengths and percentages of elongation and contraction. Thus, for cylindrical test bars the ratio of gage length to diameter is constant for similar bars; in plate specimens the ratio of gage length to square root of cross-sectional area is constant.

The change in elastic limit and other properties with a change in length of stem of test piece is shown in the following table. (See also p. 468.) The material was from the same bar in all cases, and the stem was turned to a uniform diameter of 0.798 in. (Jas. E. Howard, Engg. Congress, 1893, Sec. G.)

Description of stem	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.	Contraction of area, per cent.
1.00 in. long	64,900	94,400	49.0
0.50 in. long	65,320	97,800	43.4
0.25 in. long	68,000	102,420	39.6
Semicircular groove, 0.4 in. radius	75,000	116,380	31.6
Semicircular groove, ⅛ in. radius	86,000 (about)	134,960	23.0
V-shaped groove	90,000 (about)	117,000	Indeterminate

To obtain ductility in case bar breaks outside the middle third for an 8-in. gage, or outside gage for a 2-in. gage, a new test bar may be substituted. To calculate the *approximate* percentage of elongation when fracture is outside the middle third, assume that the ductility is symmetrical on both sides of fracture.

Hardness Tests. Hardness as resistance to permanent distortion is best measured by the Brinell ball test, in which a hardened-steel sphere 1 cm. in diameter is pressed into the surface of the material. Brinell's hardness number is the quotient of the p essure (3000 kg. for hard metals and 500 kg. for soft metals) and the area of surface of indentation. Thus N = hardness number, P = pressure, t = depth of impression, R' = radius of curvature of indentation. $N = P/2\pi tR'$.

Since the relation between load P and depth of impression t is constant for any one metal, Martens suggests the ratio of P/t as a measure of hardness; and the **hardness number** as the load P in kg. necessary to indent the material an amount t equal to 0.05 mm. Devries (*Proc. Am. Soc. Test. Mat.*, 1911, p. 725) gives the following values (Table 6):

Table 6. Hardness Numbers of Metals

Metal	Shore scleroscope	Brinell test $\frac{P}{2\pi tR'}$	Brinell test $\frac{P}{t}$
High-carbon steel	86.1	641.0	4550
Manganese steel	29.5	179.0	641
Cast iron (1)	33.3	149.0	538
Cast iron (2)	32.9	172.3	590
Bessemer steel	26.6	188.0	428
Tool steel	37.8	289.0	1230
Copper	15.0	89.5	235

A fairly close **relation** exists **between** the **hardness number and** the **ultimate tensile strength** of metals. Thus, Devries (Int. Soc. Test. Mat. Congress, 1912) gives results for alloy steels as follows: Tensile strength, S_M (lb. per sq. in.) = 20,000 + (hardness number × 103), in which the hardness

Table 7. Allowable Unit Stresses and Loads
(In Accordance with the Building Laws of Various Cities)

Allowable unit stresses for steel and iron	Revised to 1912			
	New York	Chicago	Philadelphia	Boston
	Lb. per sq. in.			
Compression: Rolled steel, mild	16,000	14,000	14,500	....
Rolled steel, medium	16,000	14,000	16,250	....
Cast steel	16,000	16 000	....	16,000
Wrought iron	12,000	10,000	12,500	12,000
Cast iron (in short blocks)	16,000	10,000	17,500	....
Steel pins and rivets (bearing)	20,000	20,000	....	18,000
Wrought-iron pins and rivets (bearing)	15,000	....	....	15,000
Tension: Rolled steel, mild	16,000	16,000	14,500	16,000
Rolled steel, medium	16,000	16,000	16,250	16,000
Cast steel	16,000	16,000	....	....
Wrought iron	12,000	12,000	12,500	12,000
Cast iron	3,000	....	....	....
Extreme Fiber Stress—Bending:				
Rolled steel beams	16,000	16,000	....	16,000
Rolled steel pins, rivets and bolts	20,000	25,000	....	22,500
Riveted steel beams (net flange sec.)	14,000	16,000	....	....
Rolled wrought-iron beams	12,000	12,000	....	12,000
Rolled wr't-iron pins, rivets and bolts	15,000	....	....	18,000
Riveted wr't-iron beams (net flange sec.)	12,000	12,000	....	....
Cast iron—compression side	16,000	10,000	....	16,000
Cast iron—tension side	3,000	3,000	3,750	3,000
Compression in flanges of built beams, steel	....	....	....	16,000
Compression in flanges of built beams, wrought iron	....	....	....	12,000
Shear: Steel web plates, mild	9,000	10,000	8,750	10,000
Steel web plates, medium	9,000	10,000	10,000	10,000
Steel shop rivets and pins, mild	10,000	12,000	8,750	10,000
Steel shop rivets and pins, medium	10,000	12,000	10,000	10,000
Steel field rivets and pins, mild	8,000	10,000	8,750	10,000
Steel field rivets and pins, medium	8,000	10,000	10,000	10,000
Steel field bolts and pins, mild	7,000	....	8,750	8,000
Steel field bolts and pins, medium	7,000	....	10,000	8,000
Wrought-iron web plates	6,000	....	7,500	9,000
Wrought-iron shop rivets and pins	7,500	....	7,500	9,000
Wrought-iron field rivets	6,000	10,000	7,500	9,000
Wrought-iron field bolts	5,500	....	7,500	7,200
Cast iron	3,000	2,000	....	....
Columns: Mild steel	$15{,}200-58\frac{L}{R}$	$16{,}000-70\frac{L}{R}$	$\frac{14{,}500}{1+\frac{L^2}{13{,}500R^2}}$	$\frac{16{,}000}{1+\frac{L^2}{30{,}000\,R^2}}$
Medium steel	$15{,}200-58\frac{L}{R}$	$16{,}000-70\frac{L}{R}$	$\frac{16{,}250}{1+\frac{L^2}{11{,}000R^2}}$	$\frac{16{,}000}{1+\frac{L^2}{30{,}000\,R^2}}$
Wrought iron	$14{,}000-80\frac{L}{R}$	$12{,}000-60\frac{L}{R}$	$\frac{12{,}500}{1+\frac{L^2}{15{,}000R^2}}$	$\frac{12{,}000}{1+\frac{L^2}{20{,}000\,R^2}}$
Cast iron	$11{,}300-30\frac{L}{R}$	$10{,}000-40\frac{L}{R}$	$\frac{11{,}700}{1+\frac{L^2}{400D^2}}$	....

L = Length of column, R = length of radius of gyration, D = diameter of column, all in inches.

number is the load P (in kg.) required to indent the material to a depth of 0.1 mm.

The **Shore scleroscope** is used to indicate hardness by the rebound of a steel ball from the surface of a metal. The elasticity of the material enters into this rebound, which may be the same for a copper-tin alloy and tool steel. For any one class of material, however, the indications for hardness agree with those obtained in the Brinell tests.

Ability to resist **abrasion** involves toughness as well as hardness and cohesion, and is not necessarily indicated by the hardness number.

WORKING STRESSES

Allowable unit stresses for steel structures according to the building laws of various cities are given in Table 7. For the stresses recommended by the American Bridge Co., see p. 1285.

Working Stresses for Machine Parts, according to Bach, are given in Table 8. For stresses varying frequently but without shock from zero to a maximum, use two-thirds the tabulated working stresses. For stresses varying frequently but without shock from a negative maximum to an equal positive maximum, use one-third of the tabulated working stresses.

Table 8. Working Stresses for Machine Parts, Lb. per Sq. In.
(Dead Load)

Kind of stress	Tension S_t	Compression	Bending S_r	Shear	Torsion S
Wrought iron*	12,800	12,800	12,800	10,200	5,100
Low-carbon steel†	12,800 to 17,000	12,800 to 17,000	12,800 to 17,100	10,200 to 13,600	8,500 to 12,000
Medium-carbon steel†	17,000 to 21,300	17,000 to 21,300	17,100 to 21,300	13,600 to 17,100	12,800 to 17,100
Cast steel	8,500 to 12,800	12,800 to 17,000	10,600 to 15,000	6,800 to 12,000	6,900 to 12,000
Cast iron	4,300	12,800	‡	4,300	§
Rolled sheet copper	8,500**	....	....	....	....

(For varying load and shock, see p. 390.)

* When resulting changes in dimensions are not otherwise objectionable, for good wrought iron these values may be increased ⅓.

† Higher values are applicable only for absolutely reliable material. For wire the allowable tensile strength may be increased ⅙ to ⅓. Greater stress is allowable in special steels.

‡ For malleable cast iron the allowable stress in bending $S_r = nS_t\sqrt{e/z_0}$, where $n = 1.2$ to 1.33, S_t is the allowable tensile strength, e is the distance from the neutral axis to the extreme fiber in tension and z_0 is the distance from the neutral axis to the center of gravity of the area subjected to tension. Tests on various sections give the following: Rectangular, $S_r = 1.7S_t$; circular, $S_r = 2.05S_t$; I-sections, $S_r = 1.45S_t$. For plain cast iron these become: Rectangular, $S_r = 1.4S_t$, circular, $S_r = 1.7S_t$, and I-sections, $S_r = 1.2S_t$.

§ For malleable cast iron the allowable torsional shear for the various sections is as follows: Circular, $S =$ (amply) S_t; hollow circular and elliptical, $S = 0.8$ to 1.0 S_t; elliptical, $S = 1.0$ to 1.25 S_t; rectangular, $S = 1.4$ S_t; triangular and trapezoidal, $S = 1.4$ to 1.6 S_t; hollow rectangular, $S = 1.0$ to 1.25 S_t; I-, channel, cruciform and angle sections, $S = 1.4$ to 1.6 S_t. The influence of the hardened surface of the casting is less here than in bending.

** Air chambers of large fire engines, $S_t \leq 11{,}400$. For centrifugal machinery, $S_t = 7100$.

Hardened spring steel for bending: $S_r = 106{,}000$ and torsional shear $= 85{,}300$. **Special steel for cylindrical helical springs:** $S_r = 142{,}200$, and torsional shear $= 92{,}500$.

Factors of Safety for various materials and methods of loading, according to Unwin, are given in Table 9.

Table 9. Factors of Safety for Different Materials and Loadings

Material	Factors of safety for			
	Dead load	Live or varying load		Structures subject to shock
		Stress of one kind	Reversed stress	
Cast iron	4	6	10	15
Wrought iron and steel	3	5	8	12
Timber	7	10	15	20

IMPACT ON BARS

A static load is one at rest. **Dynamic loads** arise from impact. A **suddenly applied load** arises when a load that is just touching a bar is suddenly released. The velocity of approach is zero, the load is constant throughout the entire deformation, and the internal force in the bar increases from zero to some value AS. The stresses and deformations are double those due to an equal static load. G = weight producing a suddenly applied load. e = maximum elongation. P = internal force accompanying e. Work of weight Ge equals internal resilience $\frac{1}{2}Pe$. Therefore $P = 2G$. The bar springs back and oscillates.

An **impact loading** occurs when a moving weight strikes the end of a bar. The velocity of impact is v. If the elastic limit is not exceeded, the external work of the moving weight falling from height h equals Gh, and also equals the resilience developed. P, S, and e are respectively dynamic force, stress and elongation in bar. $G(h + e) = \frac{1}{2}Pe$. Let e' and S' be the elongation and stress which would accompany G if it acted as a static load. $G/e' = P/e$. Then

$$S = S' + S'[1 + (2h/e')]^{\frac{1}{2}}$$
$$e = e' + e'[1 + (2h/e')]^{\frac{1}{2}}$$

The value of e' is small, and dynamic stresses and deformations are usually large and may exceed the elastic limit. It is here assumed that the entire energy of the weight produces elastic deformation in the bar only, that is, that the supports are rigid and there is no friction.

Materials show a higher elastic limit of deformation under impact than under static loads. E appears to be unchanged (*Proc. Roy. Soc.*, Feb., 1905).

As on axial bars, **impact on beams** causes severe dynamic stresses. A weight W falling from height h on the center of a simple beam, produces a maximum dynamic deflection f, and a dynamic fiber stress S. Let f' and S' be the deflection and stress caused by W acting as a static load. By similar reasoning,

$$S = S' + S'[1 + (2h/f')]^{\frac{1}{2}} \quad (1)$$
$$f = f' + f'[1 + (2h/f')]^{\frac{1}{2}} \quad (2)$$

W must be large compared to the weight of the beam; otherwise, on account of inertia of beam, the energy will be used in local damage. Supports are rigid, and there is no friction.

The Effect of Inertia may be computed by the laws of collision of bodies. Assume impact **entirely inelastic.** nWh is the fractional part of the energy producing elastic deformation in the beam or bar. W' = weight of beam or bar; W = weight of falling body; $m = W'/W$.

For **longitudinal impact** on a bar, $n = (1 + \frac{1}{3}m)/(1 + \frac{1}{2}m)^2$.

For **center impact** on a simple beam, $n = (1 + \frac{17}{35}m)/(1 + \frac{5}{8}m)^2$. If beam is fixed at the ends, use $\frac{13}{35}$ instead of $\frac{17}{35}$ and $\frac{1}{2}$ instead of $\frac{5}{8}$. For a cantilever struck at the end, use $\frac{33}{140}$ instead of $\frac{17}{35}$, and $\frac{3}{8}$ instead of $\frac{5}{8}$. **To compute stresses and deflections,** the value of h in (1) and (2) should be multiplied by n.

COMBINED STRESSES

The stresses treated here are *in one plane.* N is the unit normal stress, T the unit tangential stress on an internal inclined plane, S the unit stress on a plane normal to the geometric axis, P the external force, and R the resultant unit stress.

Stresses acting on different planes cannot be resolved or compounded like forces. The area upon which the stress acts must appear in the equation.

The resultant of several stresses acting together **on the same plane** is the geometric sum of their individual actions

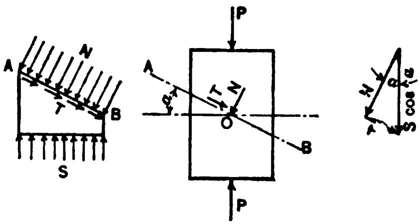


FIG. 20.

Case I. Force Along One Axis (Fig. 20). The force P acts alone. The resultant stress on diagonal plane AB is resolved into a **normal stress** N, and a **tangential stress** T.

$$N = S\cos^2 a \qquad T = S\sin a\cos a$$

The maximum value of T equals $\frac{1}{2}S$ when a equals 45 deg. The stresses on the diagonal plane CD (see Fig. 21) when $a + a' = \pi/2$, are $N' = S\sin^2 a$ and $T' = S\sin a\cos a$.

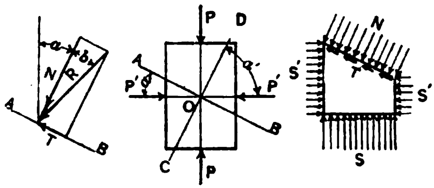


FIG. 21.

Case II. Forces Along Two Right-angled Axes (Fig. 21). Here the forces P and P' act.

$$N = S\cos^2 a + S'\sin^2 a \qquad (1)$$

$$T = (S - S')\sin a\cos a \qquad (2)$$

$N_{max} = S$ or S' and $T_{max} = \frac{1}{2}(S - S')$ when $a = 45$ deg. If $a + a' = 90$ deg., $N + N' = S + S'$ and $T = T'$. The **resultant stress** R is the resultant of N and T. $R = \sqrt{N^2 + T^2}$. The direction angle of R is b, where $\sin b = \sin 2a\,(S - S')/2R$.

Special Cases under Case II. (*a*) If $+S = +S'$ (Fig. 22), $T = 0$, $R = S = S'$, and direction of R is normal. (*b*) If $+S = -S'$ (Fig. 23), $R = S = S'$, and direction angle of $R = 2a$. S bisects the angle nOr.

Example. The metal in a steam boiler is subjected to a unit stress of 8000 lb. per sq. in. in a circumferential direction, and 4000 lb. per sq. in. in a longitudinal direction. Find the stresses on a plane the normal to which makes an angle of 30 deg. with the direction of the 8000-lb. stress. From (1) and (2)

Normal stress $N = 8000 \cos^2 30° + 4000 \sin^2 30° = 7000$ lb. per sq. in.
Tangential stress $T = (8000 - 4000) \sin 30° \cos 30° = 1730$ lb. per sq. in.
Resultant stress $R = \sqrt{7000^2 + 1730^2} = 7210$ lb. per sq. in.

Sin $b = \sin (2 \times 30°) (8000 - 4000)/(2 \times 7210) = 0.24023$, whence angle b, or the direction angle of R, is 13° 54′. For graphical solution, see Fig. 24.

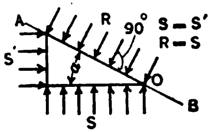


FIG. 22.

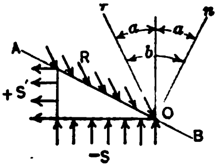


FIG. 23.

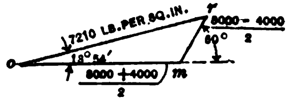


FIG. 24.

A block (Fig. 25) is acted upon by three pairs of normal stresses S_x, S_y, and S_z. The deformation e_x along axis X is due to S_x modified by the lateral effects of S_y and S_z (see p. 376). When the stresses are all of the same sign, $e_x = (S_x/E) - [(S_y + S_z)/nE]$, where n = reciprocal of Poisson's ratio, whence

$$Ee_x = S_x - [(S_y + S_z)/n]$$
$$Ee_y = S_y - [(S_x + S_z)/n]$$
$$Ee_z = S_z - [(S_x + S_y)/n]$$

When the stresses are of different sign, corresponding changes occur in the formulæ. The product Ee is regarded as a stress. The **apparent stress** is S, but the **true stress** $T = Ee$.

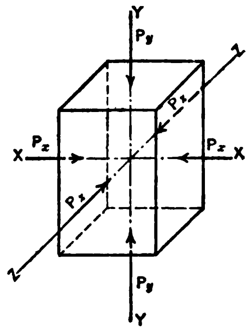


FIG. 25.

Elastic Strength Under Compound Stress is fixed, by three separate theories, as a certain value of either (*a*) the maximum principal apparent stress; (*b*) the true stress accompanying maximum principal deformation; or (*c*) the maximum shearing stress.

For example, metal in a boiler shell is under a hoop tension S_x and a longitudinal tension $S_y = \frac{1}{2}S_x$. According to (*a*), metal will begin to fail when S_x equals the elastic limit of the metal, regardless of the presence of S_y. According to (*b*), the necessary stress at failure is the true stress $T_x = Ee_x = S_x - S_x/n$. When $n = 3$, $T_x = \frac{5}{6}S_x$. That is, a hoop tension one-fifth greater than the elastic limit under simple stress, as shown in a testing machine, is required to cause failure. This method (*b*) is commonly used. According to (*c*), failure will begin when shearing stress $= \frac{1}{2}(S_x - S_y)$ is reached. Recent researches (1904–1908) by Hancock, Guest and Coker point to truth of theory (*c*). This discussion is for elastic stresses and must not be extended to ultimate strength.

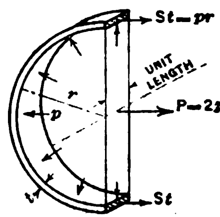


FIG. 26.

CYLINDERS AND SPHERES; TUBES

Thin Cylinders. In a thin envelope or ring (Fig. 26) subjected to an internal unit pressure p, the force of tension in ring P (hoop tension) in a unit length is constant and equal to pr. The uniform tensile stress S in the ring equals pr/t, or $t = pr/S$. An increase in t must be made to compensate for rivet holes in joints.

A portion of any circular curved ring under normal pressure carries a force $P = pr$. Internal pressure yields tension; external pressure yields compression.

If the **cylinder** is **closed at its ends**, a longitudinal stress S' acts. $S' = \frac{1}{2}rp/t = \frac{1}{2}S$. When $n = 3$ (see Poisson's ratio) the true stress $T = S - \frac{1}{3}S' = \frac{5}{6}S$.

Collapsing Pressure of Tubes. Tubes under external load may collapse. The following notation is used in the various formulæ given for collapsing pressure:

P = collapsing pressure, lb. per sq. in.
E = modulus of elasticity.
m = Poisson's ratio.
t = thickness of tube, in.
D = outside diam. of tube, in.
d = inside diam. of tube, in.
l = length of tube, in.

For collapsing pressure, where l is not greater than $6D$, **Fairbairn's empirical formula** (1858) is: $P = 9{,}672{,}000(t^{2.19}/lD)$, or, more simply, $P = 9{,}675{,}600(t^2/lD)$.

For **large iron pipe flues,** diameters 30 in. to 50 in., **D. K. Clark's formula** is: $P = 200{,}000\ t^2/D^{1.75}$.

In 1906, two series of experiments were made by Prof. Reid T. Stewart on Bessemer steel lap-welded tubes (see *Trans. A. S. M. E.*, vol. 27, p. 730). In one series the tubes tested were 8 5/8 in. in outside diameter and of varying thickness and length, while in the other series they were 20 ft. long and of varying thickness and diameter. The tests showed that **all** of the **old formulæ** were **inapplicable** to the wide range of conditions found **in modern practice.** Prof. Stewart found that the length of tube between transverse joints tending to hold it to a circular form, has no practical influence upon the collapsing pressure of a commercial lap-welded steel tube, so long as this length is not less than about six diameters of tube.

As based upon his researches, **Stewart's formulæ for** the **collapsing pressure of** modern **lap-welded Bessemer steel tubes** are as follows:

$$P = 1000[1 - \sqrt{1 - 1600(t^2/D^2)}] \qquad (a)$$
$$P = 86{,}670(t/D) - 1386 \qquad (b)$$

Formula (*a*) is for values of P less than 581 lb., or for values of (t/D) less than 0.023, while formula (*b*) is for values greater than these.

These formulæ are correct for tubes that are 20 ft. in length between transverse joints tending to hold them to a circular form, and at the same time are substantially correct for all lengths greater than about six diameters. They have been tested for seven diameters, ranging from 3 to 10 in., in all obtainable thicknesses of wall. The apparent fiber stress under which the different tubes failed varied from about 7000 lb. per sq. in. for the relatively thinnest to 35,000 lb. per sq. in. for the relatively thickest walls. Since the average yield point of the material was 37,000 and the tensile strength 58,000 lb. per sq. in., it would appear that the strength of a tube subjected to a collapsing fluid pressure is not dependent alone upon either the elastic limit or ultimate strength of the material constituting it. Prof. Stewart therefore suggests as a substitute for formula (*a*) the following:

$$P = 50{,}210{,}000(t/D)^3 \qquad (c)$$

The Experiments by Stewart are considered to be the most reliable.

Carman (University of Illinois Engineering Experiment Station *Bulletin* No. 5, June 1, 1906) states that the portion affected by collapse from hydraulic pressure is generally not greater in length than $12D$, that for greater lengths the collapsing pressure is independent of the length, and that the often-quoted law of Fairbairn type, in which the collapsing pressure varies inversely as the length, is only true for short tubes from $4D$ to $6D$ in length.

Carman's formulæ are as follows:

For thin cold-drawn seamless steel tubes: $P = 50{,}200{,}000\ (t/D)^3$.
For thin brass tubes: $P = 25{,}150{,}000\ (t/D)^3$.
For thick cold-drawn seamless steel tubes: $P = 95{,}520\ (t/D) - 2090$.
For thick lap-welded steel tubes: $P = 83{,}270\ (t/D) - 1025$.
For thick brass tubes: $P = 93{,}365\ (t/D) - 2474$.
Limits of (t/D) are below 0.025 for thin tubes and above 0.03 for thick tubes.
When (t/D) is less than 0.06, **Carman's approximate formulæ** are;
For cold-drawn seamless steel tubing, $P = 1{,}000{,}000(t/D)^2$.
For lap-welded steel, $P = 1{,}125{,}000(t/D)^2$.

These approximate formulæ are stated to give satisfactory rough values for tubes of the most common commercial thicknesses.

A. E. H. Love gives the following **rational formulæ**: $P = [2E/(1 - m^2)] \times (t/D)^3$ **for thin tubes,** and $P = 2S_c[(t/D) - (t/D)^2]$ (special case of Lamé's general formula—see below) **for thick tubes,** where S_c is the ultimate compressive strength (yield point) in lb. per sq. in. The average values of these constants for steel are: $E = 30{,}000{,}000$; $m = 0.295$; $S_c = 40{,}000$; and for brass: $E = 14{,}000{,}000$; $m = 0.357$; $S_c = 11{,}000$. Hence,

For thin steel tubes: $P = 65{,}720{,}000(t/D)^3$.
For thick steel tubes: $P = 80{,}000\,[(t/D) - (t/D)^2]$.
For thin brass tubes: $P = 32{,}090{,}000(t/D)^3$.
For thick brass tubes: $P = 22{,}000\,[(t/D) - (t/D)^2]$.

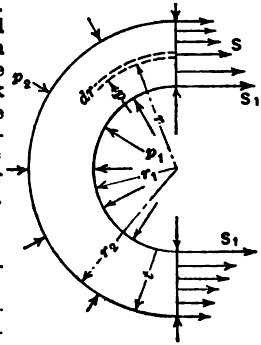


FIG. 27.

The **correction factor** for ellipticity and variation in thickness is $C = (D_{\min}/D_{\max})^3(t_{\min}/t_{\max})^3$. **Love's formula** becomes $P = [2CE/(1 - m^2)](t/D)^3$ **for thin tubes,** in which $C = 0.69$ for Stewart's lap-welded steel flues, t = average thickness and D = maximum outside diam., both in inches. **Lamé's formula** becomes $P = 2CS_c(t/D)[1 - C(t/D)^2]$ **for thick tubes,** in which $C = 0.8$.

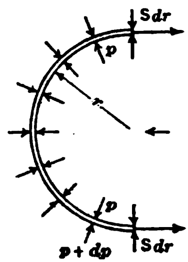


FIG. 28.

Hollow Sphere with normal pressure P. Pressure on meridian plane $= \pi r^2 P$. Tension or compression per lineal unit of shell $= \pi r^2 P/2\pi r = Pr/2$. Unit stress $S = Pr/2t$. The material carries an equal stress at right angles. True stress $T = S - (S/n) = Pr/3t$.

In thick cylinders (Fig. 27) **the stress is not uniformly distributed.** The hoop tension in the cylinder under internal pressure is greatest at the interior and diminishes toward the exterior. An interior hoop, from Fig. 27, is shown in Fig. 28. Cylinder unit pressure at bore $= p_1$. Radial internal compressive unit stress $= p$. External unit pressure $= p_2$. Hoop unit tension $= S$. Longitudinal unit stress $= S_L$.

In **Barlow's formula** it is assumed that the volume of the metal does not change during expansion of the cylinder. Therefore S varies inversely as the radius squared. $Sr^2 = S_1r_1^2$.

The total internal pressure $2r_1p_1 = 2\int Sdr$, or $p = S_1t/(r_1 + t)$.

In **Lamé's formula** the cylinder is assumed to stretch longitudinally so that cross-sections remain plane surfaces; the longitudinal deformation e_L is therefore constant. e_L is so connected with S and p that $Ee_L = S_L - [(S - p)/n]$. Therefore

$$S - p = \text{constant} = K \qquad (a)$$

Considering equilibrium of the internal hoop (Fig. 28),

$$2rp - [(r + dr)2(p \mp dp)] = 2Sdr, \text{ or}$$

$$d(pr)/dr = S \qquad (b)$$

Algebraic development of (a) and (b) leads to Lamé's formula

$$S = [r_1^2p_1 - r_2^2p_2 + (p_1 - p_2)r_1^2r_2^2/r^2]/(r_2^2 - r_1^2)$$

When $r = r_1$ and $p_2 = 0$, $S = S_1$ the hoop tension at the bore, or

$$S_1 = p_1(r_2^2 + r_1^2)/(r_2^2 - r_1^2) \qquad (1)$$

or

$$r_2 = r_1[(S_1 + p_1)/(S_1 - p_1)]^{1/2}$$

When $r = r_2$ and $p_2 = 0$, $S = S_2$ the hoop tension at exterior, or $S_2 = 2p_1r_1^2/(r_2^2 - r_1^2)$. If $r_2 = 2r_1$, $S_1 = 5p_1/3$ and $S_2 = 2/3 p_1$.

These stresses are apparent stresses. The true unit hoop stresses T are found by methods of p. 391.

Example. The variation of stresses in a numerical case is shown in Fig. 29 (from Morley's "Strength of Materials"). In a cylinder 6 in. in internal diam. with walls 2 in. thick, subjected to a pressure of 1000 lb. per sq. in. gage, the stresses [from (1)] would be $S_1 = 1000(5^2 + 3^2)/(5^2 - 3^2) = 2125$ lb. per sq. in.; and $S_2 = 2 \times 1000 \times 3^2/(5^2 - 3^2) = 1125$ lb. per sq. in.

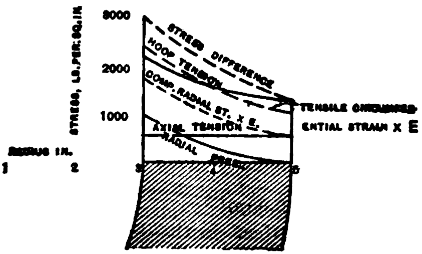


FIG. 29.

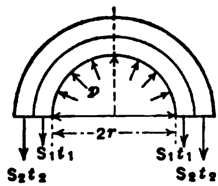


FIG. 30.

For **true stresses,** "Hütte" states that $r_2 = r_1[(T + 0.4p)/(T - 1.3p)]^{1/2}$, where T is the working tensile strength. When p is external this becomes $r_2 = r_1[T/(T - 1.7p)]^{1/2}$ when $p < T/1.7$ only. For the example just given, $T = 2256$ lb. per sq. in. is virtually the same as S_1 from Lamé's formula.

Thick Cylinders with Shrinkage Rings. In these it can be assumed that the stress under internal pressure is uniformly distributed over each ring, provided the ratio r/t is great.

Referring to Fig. 30, let t = sectional area (in sq. in.) of the cylinder for unit length in an axial direction; f = sectional area of the shrinkage rings per unit length of cylinder; S_1 = tensional hoop stress in hollow cylinder, lb. per sq. in.; S_2 = tensional hoop stress in shrinkage rings, lb. per sq. in.; p = interior pressure, lb. per sq. in., and r = interior radius of cylinder, in. Then

$$f = rp/[S_1(t/f) + S_2]$$

If the rings are shrunk on and R = outer radius of the inner cylinder; $R(1 - K)$ = inner radius of the rings before shrinking; $C_1 = 1/E_1$ for the inner cylinder; $C_2 = 1/E_2$ for the shrinkage rings; e_1 and e_2 = unit strains respectively in the inner cylinder and shrinkage rings when cold, then $K = e_1 + e_2 = C_2S_2 - C_1S_1$, and $e_1 = K/[1 + (C_2t/C_1f)]$.

The compressive stress in the cylinder before the pressure p is applied is $S' = e_1/C_1$; and the tensional stress in the shrinkage ring before the pressure p is applied is $S'' = e_2/C_2$. The pressure p causes a tensional stress of $S_1 + S'$ in the cylinder and of $S_2 - S''$ in the shrinkage rings.

The difference K found in this way is a minimum. This difference has to be increased, however, depending upon the condition of the surfaces of the cylinder and rings. If the rings are made of tough material K may be increased considerably, as the effect is only to produce permanent expansion in the rings, while the inner cylinder will be compressed, thereby diminishing its working tensional stress.

The difference in temperature when the rings are forced on should be $d = K/c_w$, where c_w is the coefficient of expansion of the shrinkage ring.

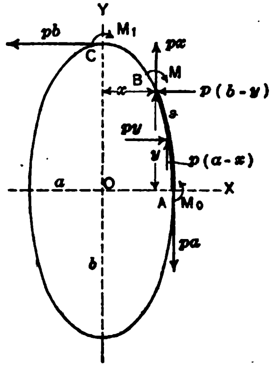


FIG. 31.

Oval Hollow Cylinders. In Fig. 31 let a and b be the semi-minor and semi-major axes. The bending moments at A and C will then be

$$M_0 = (pa^2/2) - (pI_x/2S) - (pI_y/2S)$$
$$M_1 = M_0 - p(a^2 - b^2)/2$$

where I_x and I_y are the moments of inertia of the arc AC about the x and y axes, respectively. The **bending moment at any point** will be

$$M = M_0 - (pa^2/2) + (px^2/2) + (py^2/2).$$

Thick Hollow Spheres. With an **internal pressure** p, where $p < T/0.65$,

$$r_2 = r_1[(T + 0.4p)/(T - 0.65p)]^{1/3}$$

The maximum tensile stress is on the inner surface, in the direction of the circumference. With an **external pressure** p, where $p < T/1.05$,

$$r_2 = r_1[T/(T - 1.05p)]^{1/3}$$

In both cases T is the true stress (see p. 392).

PRESSURE BETWEEN BODIES WITH CURVED SURFACES

Two Spheres. The radius A (in in.) of the compressed area is obtained from the formula $A^3 = 0.68P(c_1 + c_2)/[(1/r_1) + (1/r_2)]$, in which P is the compressing force in lb., c_1 and c_2 ($= 1/E_1$ and $1/E_2$) are reciprocals of the respective moduli of elasticity, and r_1 and r_2 are the radii in in. (Reciprocal of Poisson's ratio assumed to be $n = 10/3$.) The **greatest compressive stress** (lb. per sq. in.) in the middle of the compressed surface will be $S_{max} = 1.5(P/\pi A^2)$, and

$$S_{max}^3 = 0.235P[(1/r_1) + (1/r_2)]^2/(c_1 + c_2)^2$$

The **total deformation** of the two spheres (in in.) will be Y, which is obtained from

$$Y^3 = 0.46P^2(c_1 + c_2)^2[(1/r_1) + (1/r_2)]$$

For $c_1 = c_2 = 1/E$, *i.e.*, two spheres with the same modulus of elasticity, it follows that $A^3 = 1.36\,P/E[(1/r_1) + (1/r_2)]$, $S_{max}^3 = 0.059PE^2[(1/r_1) + (1/r_2)]^2$, and $Y^3 = 1.84P^2[(1/r_1) + (1/r_2)]/E^2$. If the radii of these spheres are also equal, $A^3 = 0.68Pr/E = 0.34Pd/E$; $S_{max}^3 = 0.235PE^2/r^4 = 0.94PE^2/d^2$; and $Y^3 = 3.68P^2/E^2r = 7.36P^2/E^2d$.

Sphere and Flat Plate. In this case $r_1 = r$ and $r_2 = \infty$, and the above formulæ become $A^3 = 0.68Pr(c_1 + c_2) = 1.36Pr/E$, and

$$S_{max}^3 = 0.235P/r^2(c_1 + c_2)^2 = 0.059PE^2/r^2$$

Also $$Y^3 = 0.46P^2(c_1 + c_2)^2/r = 1.84P^2/Er.$$

Two Cylinders. The width b (in in.) of the rectangular pressure surface is obtained from $(b/4)^2 = 0.29P(c_1 + c_2)/l[(1/r_1) + (1/r_2)]$, where r_1 and r_2 are the radii, and l the length, all in in.

$$S_{max}^2 = (4P/\pi bl)^2 = 0.35P[(1/r_1) + (1/r_2)]/l(c_1 + c_2)$$

For cylinders with the same moduli of elasticity, $c_1 = c_2 = 1/E$, and $(b/4)^2 = 0.58P/El[(1/r_1) + (1/r_2)]$; and $S_{max}^2 = 0.175PE[(1/r_1) + (1/r_2)]/l$. When $r_1 = r_2 = r$, $(b/4)^2 = 0.29Pr/El$, and $S_{max}^2 = 0.35PE/lr$.

Cylinder and Flat Plate. Here $r_1 = r$, $r_2 = \infty$, and the above formulæ reduce to $(b/4)^2 = 0.29Pr(c_1 + c_2)/l = 0.58Pr/El$, and

$$S_{max}^2 = 0.35P/lr(c_1 + c_2) = 0.175PE/lr$$

BEAMS

For Properties of Commercial Wooden Beams, see p. 1274.
For Properties of Structural Steel, see pp. 1288–1304.

Notation

R	= Reaction	S_v	= Vertical shearing unit stress
M	= Moment	Z	= Horizontal shearing unit stress
W	= Total distributed load	I	= Rectangular moment of inertia
w	= Unit load	I_p	= Polar moment of inertia
P	= Concentrated load	r	= Radius of curvature
V or Q	= Total vertical shear	i	= Slope
S	= Unit normal apparent stress	f	= Deflection

A **simple beam** is a bar resting on supports near its ends. A **cantilever beam** projects out beyond a support. To compute reactions and moments, a distributed load may be replaced by its resultant acting at the center of gravity (c. of g.) of the load area.

Simple Beams

The **reactions** (in lb.) at the support are computed by the principle of moments. Thus, in Fig. 32, $\frac{1}{2}Wl = R_1l$, or $R_1 = \frac{1}{2}W$; $R_2 = W - R_1 = \frac{1}{2}W$. In Fig. 33, $\frac{1}{3}Wl = R_1l$, or $R_1 = \frac{1}{3}W$; $R_2 = W - R_1 = \frac{2}{3}W$. In general, the weight of the beam must be considered.

The **external moment** (in ft.-lb. or in.-lb.) **at any section** is the algebraic sum of the moments of the external forces on one side only of the section, or $M = \Sigma\,(Px)$

Examples: Uniform Load (Fig. 32). $M_A = \Sigma\,(Px)$ to left of point A at which moment is to be found. $M_A = \frac{1}{2}Wx - (wx \times \frac{1}{2}x)$. **Uniformly Varying Load** (Fig. 33). (Weight of beam neglected.) $M = \Sigma(Px) = \frac{1}{3}Wx - \frac{1}{2}kwx^2\frac{1}{3}x$, where k is a constant $(= h/l)$ and w is the weight of the loading per unit of volume.

A **moment** that bends a beam convex downward is **positive** (+), and convex upward **is negative** (−).

The **external shear** V (in lb.) is the algebraic sum of the forces on one side only of the section, or $V = \Sigma P$. In Fig. 32, $V = \frac{1}{2}W - wx$. In Fig. 33, $V = \frac{1}{3}W - \frac{1}{2}kwx^2$.

The moment and shear may be expressed by an ordinate drawn to scale at the section of the beam under consideration. A **moment diagram** is a series of such ordinates throughout the beam showing the variation of moment. A **shear diagram** is similarly constructed. Table 10 shows moment and shear diagrams for common cases of flexure, and gives values of reactions (R), moments (M), shears (Q) and deflections (f).

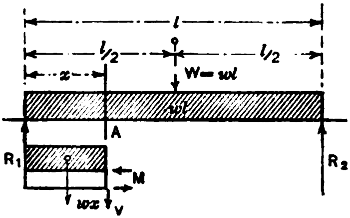


FIG. 32.

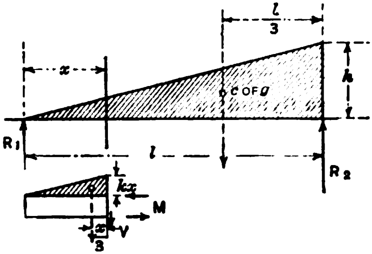


FIG. 33.

The external moment, M, is in equilibrium with the **internal moment** which is supplied by the stresses acting over the cross-section of the beam. The external shear is likewise in equilibrium with the internal shearing stresses.

Table 10. Beams of Uniform Cross-section, Loaded Transversely

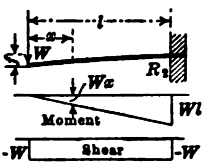
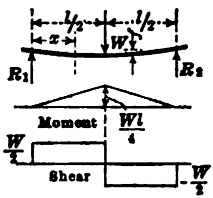
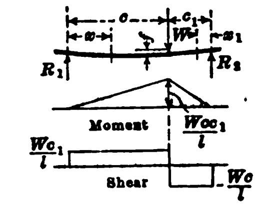
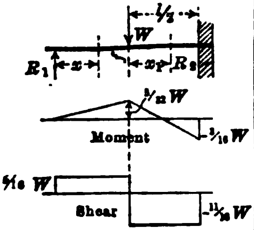
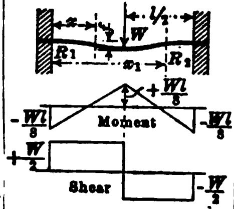
		
$R_2 = W$ $M_x = -Wx$ $M_{max} = -Wl,\ (x = l)$ $Q_x = -W$ $f = \frac{Wl^3}{3EI}$ (max.)	$R_1 = \frac{W}{2},\ R_2 = \frac{W}{2}$ $M_x = \frac{Wx}{2}$ $M_{max} = \frac{Wl}{4},\ \left(x = \frac{l}{2}\right)$ $Q_x = \pm\frac{W}{2}$ $f = \frac{W}{EI}\frac{l^3}{48}$ (max.)	$R_1 = \frac{Wc_1}{l},\ R_2 = \frac{Wc}{l}$ $M_x = \frac{Wc_1x}{l},\ M_{x_1} = \frac{Wcx_1}{l}$ $M_{max} = \frac{Wcc_1}{l},$ $(x_1 = c_1$ or $x = c)$ $Q_x = \frac{Wc_1}{l},\ Q_{x_1} = \frac{Wc}{l}$ $Q_{max} = \frac{Wc}{l},\ c > c_1$ $Q_{max} = \frac{Wc_1}{l},\ c_1 > c$ $f = \frac{W}{EI}\frac{l^3}{3}\frac{c^2}{l^2}\frac{c_1^2}{l^2}$

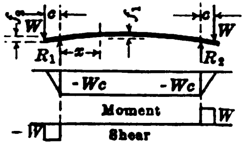
Table 10. Beams of Uniform Cross-section, Loaded Transversely— *(continued)*



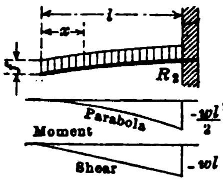
$$R_1 = \frac{5}{16}W, \quad R_2 = \frac{11}{16}W$$
$$M_x = \frac{5}{16}Wx$$
$$M_{x1} = Wl\left(\frac{5}{32} - \frac{11}{16}\frac{x_1}{l}\right)$$
$$M_{\max} = -\frac{3}{16}Wl, \left(x_1 = \frac{l}{2}\right)$$
$$Q_x = +\frac{5}{16}W, \quad Q_{x1} = -\frac{11}{16}W$$
$$Q_{\max} = -\frac{11}{16}W,$$
$$\left(x = \frac{l}{2} \text{ to } x = l\right)$$
$$f = \frac{W}{EI}\frac{7l^3}{768}$$



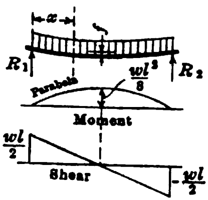
$$R_1 = \frac{W}{2}, \quad R_2 = \frac{W}{2}$$
$$M_x = \frac{Wl}{2}\left(\frac{x}{l} - \frac{1}{4}\right)$$
$$M_{x1} = \frac{Wl}{2}\left(\frac{x}{l} - \frac{3}{4}\right)$$
$$M_{\max} = \frac{Wl}{8}, \left(x = \frac{l}{2}\right)$$
$$Q_x = \frac{W}{2} \quad Q_{x1} = -\frac{W}{2}$$
$$f = \frac{W}{EI}\frac{l^3}{192} \text{ (max.)}$$



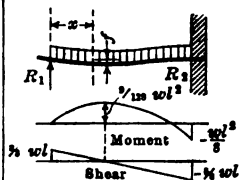
$$R_1 = W$$
$$R_2 = W$$
$$M_x = -Wc = \text{Const.}$$
$$Q_{W \text{ to } R_1} = -W$$
$$Q_{R_1 \text{ to } R_2} = 0$$
$$Q_{R_2 \text{ to } W} = +W$$
$$f_1 = \frac{W}{EI}\frac{l^3}{8}\frac{c}{l} \text{ (max.)}$$
$$f_2 = \frac{W}{EI}\frac{c^2}{3}\left(c + \frac{3l}{2}\right) \text{ (max.)}$$



$$R_2 = W = wl$$
$$M_x = -\frac{wx^2}{2}$$
$$M_{\max} = -\frac{wl^2}{2}, \quad (x = l)$$
$$Q_x = -wx$$
$$Q_{\max} = -wl, \ (x = l)$$
$$f = \frac{W}{EI}\frac{l^3}{8} \text{ (max.)}$$

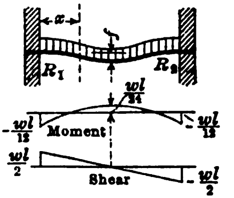


$$R_1 = \frac{W}{2} = \frac{wl}{2}$$
$$R_2 = \frac{W}{2} = \frac{wl}{2}$$
$$M_x = \frac{wx}{2}(l - x)$$
$$M_{\max} = \frac{wl^2}{8}, \ (x = \tfrac{1}{2}l)$$
$$Q_x = \frac{wl}{2} - wx$$
$$Q_{\max} = \frac{wl}{2} \ (x = 0)$$
$$f = \frac{W}{EI}\frac{5l^3}{384} \text{ (max.)}$$



$$R_1 = \frac{3}{8}W = \frac{3}{8}wl,$$
$$R_2 = \frac{5}{8}W = \frac{5}{8}wl$$
$$M_x = \frac{wx}{2}\left(\frac{3}{4}l - x\right)$$
$$M_{\max} = \frac{9}{128}wl^2, \left(x = \frac{3}{8}l\right)$$
$$M_{\max} = -\frac{wl^2}{8}, \ (x = l)$$
$$Q_x = \frac{3}{8}wl - wx,$$
$$Q_{\max} = -\frac{5}{8}wl$$
$$f = \frac{W}{EI}\frac{l^3}{185} \text{ (max.)}$$

Table 10. Beams of Uniform Cross-section, Loaded Transversely— *(continued)*



$R_1 = \frac{W}{2} = \frac{wl}{2},\quad R_2 = \frac{W}{2} = \frac{wl}{2}$

$M_x = -\frac{wl^2}{2}\left(\frac{1}{6} - \frac{x}{l} + \frac{x^2}{l^2}\right)$

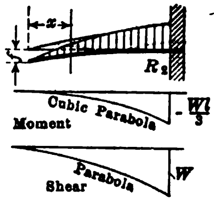
$M_{\max} = -\frac{1}{12}wl^2,$

$(x = 0, \text{ or } x = l)$

$Q_x = \frac{wl}{2} - wx$

$Q_{\max} = \pm\frac{wl}{2}$

$f = \frac{W}{EI}\frac{l^3}{384}$ (max.)



$R_2 = W =$ total load

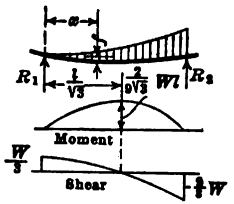
$M_x = -\frac{W}{3}\frac{x^3}{l^2}$

$M_{\max} = -\frac{Wl}{3}$

$Q_x = -\frac{Wx^2}{l^2}$

$Q_{\max} = -W$

$f = \frac{W}{EI}\frac{l^3}{15}$ (max.)



$R_1 = \frac{1}{3}W,\quad R_2 = \frac{2}{3}W$

$M_x = \frac{Wx}{3}\left(1 - \frac{x^2}{l^2}\right)$

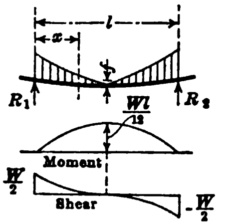
$M_{\max} = \frac{2}{9\sqrt{3}}Wl,$

$\left(x = \frac{l}{\sqrt{3}}\right)$

$Q_x = W\left(\frac{1}{3} - \frac{x^2}{l^2}\right)$

$Q_{\max} = -\frac{2}{3}W,\quad (x = l)$

$f = 0.01304\,\frac{Wl^3}{EI}$ (max.)



$R_1 = \frac{W}{2},\quad R_2 = \frac{W}{2}$

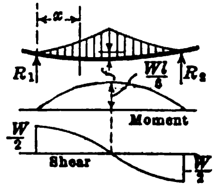
$M_x = Wx\left(\frac{1}{2} - \frac{x}{l} + \frac{2x^2}{3l^2}\right)$

$M_{\max} = \frac{Wl}{12},\ \left(x = \frac{1}{2}l\right)$

$Q_x = W\left(\frac{1}{2} - \frac{2x}{l} + \frac{2x^2}{l^2}\right)$

$Q_{\max} = \pm\frac{W}{2},\ (x = 0)$

$f = \frac{W}{EI}\frac{3l^3}{320}$ (max.)



$R_1 = \frac{W}{2},\quad R_2 = \frac{W}{2}$

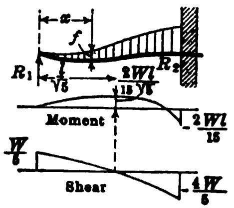
$M_x = Wx\left(\frac{1}{2} - \frac{2}{3}\frac{x^2}{l^2}\right)$

$M_{\max} = \frac{Wl}{6},\ \left(x = \frac{1}{2}l\right)$

$Q_x = W\left(\frac{1}{2} - \frac{2x^2}{l^2}\right)$

$Q_{\max} = \pm\frac{W}{2},\ (x = 0)$

$f = \frac{W}{EI}\frac{l^3}{60}$ (max)



$R_1 = \frac{W}{5},\quad R_2 = \frac{4W}{5}$

$M_x = Wx\left(\frac{1}{5} - \frac{x^2}{3l^2}\right)$

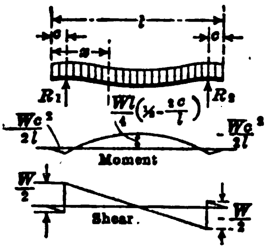
$M_{\max} = -\frac{2}{15}Wl$ at support 2

$Q_x = W\left(\frac{1}{5} - \frac{x^2}{l^2}\right)$

$Q_{\max} = -\frac{4W}{5}$

$f = \frac{16Wl^3}{1500\sqrt{5}EI} = \frac{0.00477\,Wl^3}{EI}$ (max.)

Table 10. Beams of Uniform Cross-section, Loaded Transversely— *(continued)*



$$R_1 = \frac{W}{2} = \frac{wl}{2},\quad R_2 = \frac{W}{2} = \frac{wl}{2}$$

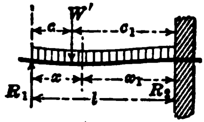
$$M_x = \frac{Wx}{2}\left(1 - \frac{c}{x} - \frac{x}{l}\right),\ (x > c)$$

$$M_x = -\frac{Wx^2}{2l},\ (x \leqq c)$$

$$M_{\max} = \frac{Wl}{4}\left(\frac{1}{2} - \frac{2c}{l}\right),\ c \leqq \left(\frac{\sqrt{2}-1}{2}\right)l$$

$$Q_x = \frac{W}{2} - wx,\ (x > c)$$

$$Q_x = -wx,\ (x \leqq c)$$



Concentrated Load W'
Uniformly Dist. Load $W = wl$

$$R_1 = W'\frac{c_1^2(3c+2c_1)}{2l^3} + \frac{3}{8}W$$

$$R_2 = W'\frac{(2c^2+6cc_1+3c_1^2)c}{2l^3} + \frac{5}{8}W$$

$$M_2 = W'\frac{cc_1(2c+c_1)}{2l^3} + W\frac{l}{8}$$

$$M_{W'} = W'\frac{cc_1^2(3c+2c_1)}{2l^3} + W\frac{(3c_1-c)c}{8l}$$

$$\text{(a)}\ \frac{W'}{W} < \frac{l^2}{4c_1^2}\ \frac{5c-3c_1}{3c+2c_1}$$

$$M_{c\,\max} = \frac{R_1^2}{2W}l,\quad \left(x = \frac{R_1 l}{W}\right)$$

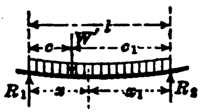
$$\text{(b)}\ \frac{W'}{W} < \frac{l^2(3c_1-5c)}{4c(2c^2+6cc_1+3c_1^2)}$$

$$M_{c_1\,\max} = W'c + \frac{(R_1-W')^2}{2W}l,$$

$$\left(x = \frac{R_1 - W'}{W}l\right)$$

Deflection under W'

$$f = \frac{W'}{EI}\ \frac{c^3c_1^3(4c+3c_1)}{12l^3} + \frac{W}{EI}\ \frac{cc_1^2(3c+c_1)}{48l}$$



Concentrated Load W'
Uniformly Dist. Load $W = wl$; $c < c_1$

$$R_1 = W'\frac{c_1}{l} + \frac{W}{2}$$

$$R_2 = W'\frac{c}{l} + \frac{W}{2}$$

$$\text{(a)}\ \frac{W'}{W} < \frac{c_1 - c}{2c}$$

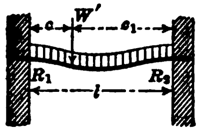
$$M_{\max} = R_2\frac{x_1}{2} = \frac{R_2^2 l}{2W},\quad \left(x_1 = \frac{R_2 l}{W}\right)$$

$$\text{(b)}\ \frac{W'}{W} > \frac{c_1 - c}{2c}$$

$$M_{\max} = \left(W' + \frac{W}{2}\right)\frac{cc_1}{l},\quad (x_1 = c_1)$$

Deflection of beam under W':

$$f = \left(W' + \frac{l^3 + cc_1}{8cc_1}W\right)\frac{c^2c_1^2}{3EIl}$$



$c < c_1$

$$R_1 = W'\frac{(3c+c_1)c_1^2}{l^3} + \frac{W}{2}$$

$$R_2 = W'\frac{(c+3c_1)c^2}{l^3} + \frac{W}{2}$$

$$M_{\max} = M_1 = W'\frac{cc_1^2}{l^2} + \frac{Wl}{12}$$

Deflection under W'

$$f = \frac{1}{EI}\left(W'\frac{c^3c_1^3}{3l^3} + W\frac{c^2c_1^2}{24l}\right)$$

Table 11. Uniformly Distributed Loads on Rectangular Beams 1 In. Wide*

(Calculated for unit fiber stress of 1000 lb. per sq. in.—Jones & Laughlin)

TOTAL LOAD IN POUNDS, INCLUDING WEIGHT OF BEAM

Span in ft.	Depth of beam in inches										
	6	7	8	9	10	11	12	13	14	15	16
5	800	1090	1420	1800	2220	2690	3200	3750	4350	5000	5690
6	670	910	1180	1500	1850	2240	2670	3130	3630	4170	4740
7	570	780	1010	1290	1590	1920	2280	2680	3110	3570	4060
8	500	680	890	1120	1390	1680	2000	2350	2720	3130	3560
9	440	600	790	1000	1230	1490	1780	2090	2420	2780	3160
10	400	540	710	900	1110	1340	1600	1880	2180	2500	2840
11	360	490	650	820	1010	1220	1450	1710	1980	2270	2590
12	330	450	590	750	930	1120	1330	1560	1810	2080	2370
13	310	420	550	690	850	1030	1230	1440	1680	1920	2190
14	290	390	510	640	790	960	1140	1340	1560	1790	2030
15	270	360	470	600	740	900	1070	1250	1450	1670	1900
16	250	340	440	560	690	840	1000	1170	1360	1560	1780
17	230	320	420	530	650	790	940	1100	1280	1470	1670
18	220	300	400	500	620	750	890	1040	1210	1390	1580
19	210	290	380	470	590	710	840	990	1150	1320	1500
20	200	270	360	450	560	670	800	940	1090	1250	1420
22	180	250	320	410	500	610	730	850	990	1140	1290
24	160	230	290	370	460	560	670	780	910	1040	1180
26	150	210	270	340	420	520	610	720	840	960	1090
28	140	190	250	320	390	480	570	670	780	890	1010
30	130	180	240	300	370	450	530	630	730	830	950

* This table is convenient for wooden beams. For any other fiber stress, S', multiply the values in table by $S'/1000$. See p. 1274 for properties of wooden beams of commercial sizes.

Maximum Safe Load on Steel Beams. To obtain maximum safe load (or maximum deflection under maximum safe load) for any of the conditions of loading given in Table 12*b*, multiply the corresponding coefficient in that table by the greatest safe load (or deflection) for distributed load for the particular section under consideration as given in Table 12*a*.

Table 12a. Approximate Greatest Safe Loads in Pounds on Steel Beams

(Pencoyd Iron Works)

Allowable fiber stress for steel, 16,000 lb per sq. in. (basis of table); for iron, 14,000 per sq. in., *i.e.*, reduce values given in table by one-eighth. Beams supported at both ends.

L = Distance between supports in ft.
A = Sectional area of beam in sq. in.
D = Depth of beam in in.
a = Interior area in sq. in.
d = Interior depth in in.
w = Total working load in net tons.

Shape of section	Greatest safe load, pounds		Deflection, inches	
	Load in middle	Load distributed	Load in middle	Load distributed
Solid rectangle......	$\frac{890AD}{L}$	$\frac{1780AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Hollow rectangle...	$\frac{890(AD-ad)}{L}$	$\frac{1780(AD-ad)}{L}$	$\frac{wL^3}{32(AD^2-ad^2)}$	$\frac{wL^3}{52(AD^2-ad^2)}$
Solid cylinder.......	$\frac{667AD}{L}$	$\frac{1333AD}{L}$	$\frac{wL^3}{24AD^2}$	$\frac{wL^3}{38AD^2}$
Hollow cylinder.....	$\frac{667(AD-ad)}{L}$	$\frac{1333(AD-ad)}{L}$	$\frac{wL^3}{24(AD^2-ad^2)}$	$\frac{wL^3}{38(AD^2-ad^2)}$
Even-legged angle or tee.	$\frac{885AD}{L}$	$\frac{1770AD}{L}$	$\frac{wL^3}{32AD^2}$	$\frac{wL^3}{52AD^2}$
Channel or Z-bar...	$\frac{1525AD}{L}$	$\frac{3050AD}{L}$	$\frac{wL^3}{53AD^2}$	$\frac{wL^3}{85AD^2}$
Deck beam.........	$\frac{1380AD}{L}$	$\frac{2760AD}{L}$	$\frac{wL^3}{50AD^2}$	$\frac{wL^3}{80AD^2}$
I-beam............	$\frac{1695AD}{L}$	$\frac{3390AD}{L}$	$\frac{wL^3}{58AD^2}$	$\frac{wL^3}{93AD^2}$

Factors for Reducing Load When Beams Are Long in Comparison to Breadth. (Supported)

Ratio of unsupported (lateral) length in in. to flange width or breadth in in.........	20	30	40	50	60	70
Ratio of greatest safe load to calculated load	10:10	9:10	8:10	7:10	6:10	5:10

Table 12b. Coefficients for Correcting Values in Table 12a for Various Methods of Support and of Loading

Conditions of loading	Maximum relative safe load	Maximum relative deflection under max. relative safe load
Beam Supported at Ends		
Load uniformly distributed over span..........	1.0	1.0
Load concentrated at center of span...........	½	0.80
Two equal loads symmetrically concentrated....	$l/4c$	.....
Load increasing uniformly to one end..........	0.974	0.976
Load increasing uniformly to center...........	¾	0.96
Load decreasing uniformly to center...........	3/2	1.08
Beam Fixed at One End, Cantilever		
Load uniformly distributed over span..........	¼	2.40
Load concentrated at end......................	⅛	3.20
Load increasing uniformly to fixed end.........	⅜	1.92
Beam Continuous over Two Supports Equidistant from Ends		
Load uniformly distributed over span:		
1. If distance $a > 0.2071l$..................	$l^2/4a^2$	.....
2. If distance $a < 0.2071l$..................	$\frac{l}{l-4a}$	.....
3. If distance $a = 0.2071l$..................	5.83	.....
Two equal loads concentrated at ends..........	$l/4a$	.....

l = length of beam; c = distance from support to nearest concentrated load; a = distance from support to end of beam.

Theory of Flexure

A bent beam is shown in Fig. 34. The concave side is in compression and the convex side in tension. These are divided by the **neutral plane** of zero stress $A'B'BA$. The intersection of the neutral plane with the face of the beam is the **neutral line** or **elastic curve** AB. The intersection of the neutral plane with the cross-section is the **neutral axis** NN'.

It is assumed that a beam is prismatic, of a length at least 10 times its depth, and that the external forces are all at right angles to the axis of the beam and in a plane of symmetry, and that flexure is slight. Other assumptions are: (1) That the material is homogeneous, and obeys Hooke's law.

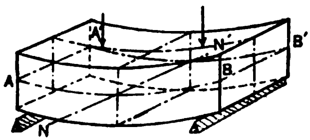


FIG. 34.

(2) That **stresses are within the elastic limit.** (3) That every layer of material is free to expand and contract longitudinally and laterally under stress as if separate from other layers. (4) That the tensile and compressive moduli of elasticity are equal. (5) That the cross-section remains a plane surface. (The assumption of plane cross-sections is only strictly true when the shear is constant or zero over the cross-section, and when the shear is constant throughout the length of the beam.)

It follows then that: (1) The internal forces are in horizontal balance. (2) The **neutral axis contains the center of gravity** of the cross-section. (3) The stress intensity varies directly with the distance from the neutral axis.

The moment of the elastic forces about the neutral axis, that is, the **internal stress-moment** or **moment of resistance,** is $M = SI/c$, where S is an elastic unit stress at outer fiber whose distance from the neutral axis is c; and I is the rectangular moment of inertia about the neutral axis.

$$M = SI/c$$

This formula is for the **strength of beams.** For rectangular beams, $M = \frac{1}{6}Sbh^2$, where b = breadth, and h = depth. That is, the elastic **strength of beam sections** varies as follows:

(*a*) For equal width, as the square of the depth. (*b*) For equal depth, directly as the width. (*c*) For equal depth and width, directly as the strength of the material. (*d*) If span varies, then for equal depth, width and material, inversely as the span.

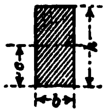
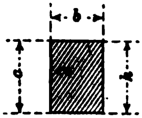
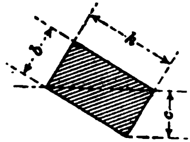
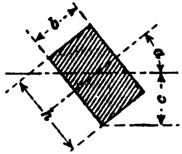
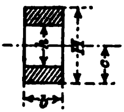
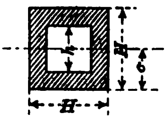
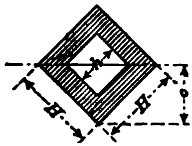
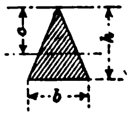
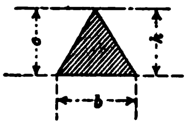
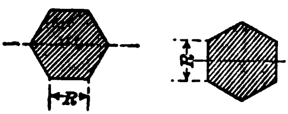
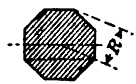
If a beam is cut in halves horizontally, the two halves laid side by side will carry only one-half as much as the original beam.

The term **section modulus** is given to the value of I/c, where c is the distance to the fiber carrying greatest stress. Area of cross-section = A.

Tables 13, 14 and 15 give the properties of various beam cross-sections For properties of structural-steel shapes, see pp. 1288 to 1304.

Table 13. Properties of Various Beam Cross-sections

(I = Moment of inertia; I/c = section modulus; $r = \sqrt{I/A}$ = radius of gyration)

				
$I =$	$\frac{bh^3}{12}$	$\frac{bh^3}{3}$	$\frac{b^3h^3}{6(b^2+h^2)}$	$\frac{bh}{12}(h^2\cos^2 a + b^2\sin^2 a)$
$\frac{I}{c} =$	$\frac{bh^2}{6}$	$\frac{bh^2}{3}$	$\frac{b^2h^2}{6\sqrt{b^2+h^2}}$	$\frac{bh}{6}\left(\frac{h^2\cos^2 a + b^2\sin^2 a}{h\cos a + b\sin a}\right)$
$r =$	$\frac{h}{\sqrt{12}} = 0.289h$	$\frac{h}{\sqrt{3}} = 0.577h$	$\frac{bh}{\sqrt{6(b^2+h^2)}}$	$\sqrt{\frac{h^2\cos^2 a + b^2\sin^2 a}{12}}$
				
$I =$	$\frac{b}{12}(H^3 - h^3)$	$\frac{H^4-h^4}{12}$	$\frac{H^4-h^4}{12}$	$\frac{bh^3}{36}$; $c = \frac{2}{3}h$
$\frac{I}{c} =$	$\frac{b}{6}\,\frac{H^3-h^3}{H}$	$\frac{1}{6}\,\frac{H^4-h^4}{H}$	$\frac{\sqrt{2}}{12}\,\frac{H^4-h^4}{H} = 0.1179\frac{H^4-h^4}{H}$	$\frac{bh^2}{24}$
$r =$	$\sqrt{\frac{H^3-h^3}{12(H-h)}}$	$\sqrt{\frac{H^2+h^2}{12}}$	$\sqrt{\frac{H^2+h^2}{12}}$	$\frac{h}{\sqrt{18}} = 0.236h$
				
$I =$	$\frac{bh^3}{12}$	$\left(\frac{5\sqrt{3}}{16}R^4 = 0.5413R^4\right)$		$\frac{1+\sqrt{2}}{6}R^4 = 0.6381R^4$
$\frac{I}{c} =$	$\frac{bh^2}{12}$	$\frac{5}{8}R^3$;	$0.5413R^3$	$0.6906R^3$
$r =$	$\frac{h}{\sqrt{6}} = 0.408h$	$\left(\sqrt{\frac{5}{24}}R = 0.456R\right)$		$0.475R$

Square, axis same as first rectangle, side $= h$: $I = h^4/12$; $I/c = h^3/6$; $r = 0.289h$.
Square, diagonal taken as axis: $I = h^4/12$; $I/c = 0.1179h^3$; $r = 0.289h$.

Table 13. Properties of Various Beam Cross-sections—(*continued*)

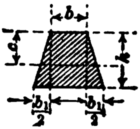
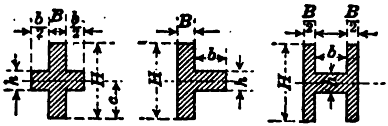
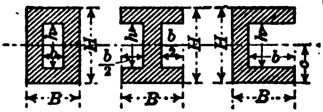
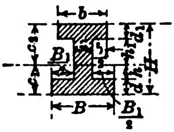
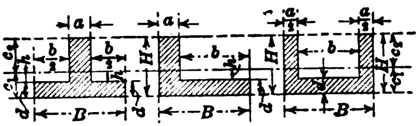
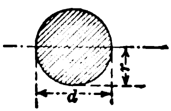
Section	Moment of inertia	Section modulus	Radius of gyration
Equilateral Polygon A = area, (see p. 39) R = rad. circumscribed circle r = rad. inscribed circle n = no. sides a = length of side Axis as in preceding section of octagon.	$I=\frac{A}{24}(6R^2-a^2)$ $=\frac{A}{48}(12r^2+a^2)$ $=\frac{AR^2}{4}$ (approx.)	$\frac{I}{c}=\frac{I}{r}$ $=\frac{I}{R\cos\frac{180°}{n}}$ $=\frac{AR}{4}$ approx.	$\sqrt{\frac{6R^2-a^2}{24}}=\frac{R}{2}$ $\sqrt{\frac{12r^2+a^2}{48}}$
	$I=\frac{6b^2+6bb_1+b_1^2}{36(2b+b_1)}h^3$ $c=\frac{1}{3}\,\frac{3b+2b_1}{2b+b_1}h$	$\frac{I}{c}=\frac{6b^2+6bb_1+b_1^2}{12(3b+2b_1)}h^2$	$\frac{h\sqrt{12b^2+12bb_1+2b_1^2}}{6(2b+b_1)}$
		$I=\frac{BH^3+bh^3}{12}$ $\frac{I}{c}=\frac{BH^3+bh^3}{6H}$	$\sqrt{\frac{BH^3+bh^3}{12(BH+bh)}}$
		$I=\frac{BH^3-bh^3}{12}$ $\frac{I}{c}=\frac{BH^3-bh^3}{6H}$	$\sqrt{\frac{BH^3-bh^3}{12(BH-bh)}}$
	$I=\frac{1}{3}(Bc_1^3-B_1h^3+bc_2^3-b_1h_1^3)$ $c_1=\frac{1}{2}\,\frac{aH^2+B_1d^2+b_1d_1(2H-d_1)}{aH+B_1d+b_1d_1}$		$\sqrt{\frac{I}{(Bd+bd_1)+a(h+h_1)}}$
			$I=\frac{1}{3}(Bc_1^3-bh^3+ac_2^3)$ $c_1=\frac{1}{2}\,\frac{aH^2+bd^2}{aH+bd}$ $c_2=H-c_1$ $r=\sqrt{\frac{I}{[Bd+a(H-d)]}}$
	$I=\frac{\pi d^4}{64}=\frac{\pi r^4}{4}=\frac{A}{4}r^2$ $=0.05d^4$ approx.	$\frac{I}{c}=\frac{\pi d^3}{32}=\frac{\pi r^3}{4}=\frac{A}{4}r$ $=0.1d^3$ approx.	$\frac{r}{2}=\frac{d}{4}$

Table 13. Properties of Various Beam Cross-sections—(*continued*)

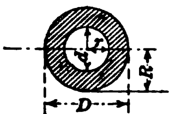
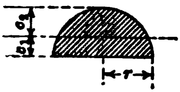
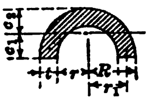
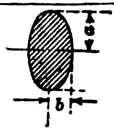
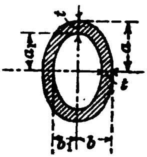
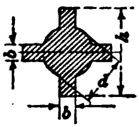
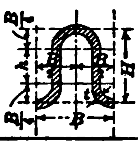
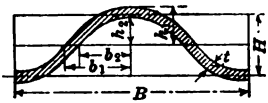
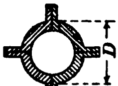
Section	Moment of inertia	Section modulus	Radius of gyration
 $d_m = \frac{1}{2}(D+d)$ $s = \frac{1}{2}(D-d)$	$I = \frac{\pi}{64}(D^4 - d^4)$ $= \frac{\pi}{4}(R^4 - r^4)$ $= \frac{1}{4}A(R^2 + r^2)$ $= 0.05(D^4 - d^4)$ approx.	$\frac{I}{c} = \frac{\pi}{32}\frac{D^4 - d^4}{D}$ $= \frac{\pi}{4}\frac{R^4 - r^4}{R}$ $= 0.8 d_m^2 s$, approx., when $\frac{s}{d_m}$ is very small	$\frac{\sqrt{R^2 + r^2}}{2} = \frac{\sqrt{D^2 + d^2}}{4}$
	$I = r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$ $= 0.1098 r^4$	$\frac{I}{c_2} = 0.1908 r^3$ $\frac{I}{c_1} = 0.2587 r^3$ $c_1 = 0.4244 r$	$\frac{\sqrt{9\pi^2 - 64}}{6\pi} r = 0.264 r$
	$I = 0.1098(R^4 - r^4) - \frac{0.283 R^2 r^2 (R - r)}{R + r}$ $= 0.3 t r_1^3$, approx. when $\frac{t}{r_1}$ is very small	$c_1 = \frac{4}{3\pi}\frac{R^2 + Rr + r^2}{R + r}$ $c_2 = R - c_1$	$\sqrt{\frac{2}{\pi}\frac{I}{(R^2 - r^2)}} =$ $0.31 r_1$ (approx.)
	$I = \frac{\pi a^3 b}{4} = 0.7854 a^3 b$	$\frac{I}{c} = \frac{\pi a^2 b}{4} = 0.7854 a^2 b$	$\frac{a}{2}$
	$I = \frac{\pi}{4}(a^3 b - a_1^3 b_1)$ $= \frac{\pi}{4} a^2 (a + 3b) t$ approx.	$\frac{I}{c} = \frac{\pi}{4} a (a + 3b) t$ approx.	$\sqrt{\frac{I}{(\pi ab - a_1 b_1)}} =$ $\frac{a}{2}\sqrt{\frac{a + 3b}{a + b}}$ approx.
	$I = \frac{1}{12}\left[\frac{3\pi}{16} d^4 + b(h^3 - d^3) + b^3(h - d)\right]$	$\frac{I}{c} = \frac{1}{6h}\left[\frac{3\pi}{16} d^4 + b(h^3 - d^3) + b^3(h - d)\right]$	$\sqrt{\frac{I}{\pi\frac{d^2}{4} + 2b(h - d)}}$ approx.
	$I = \frac{t}{4}\left(\frac{\pi B^3}{16} + B^2 h + \frac{\pi B h^2}{2} + \frac{2}{3} h^3\right)$ $h = H - \frac{1}{2}B$	$\frac{I}{c} = \frac{2I}{H + t}$	$\sqrt{\frac{I}{2\left(\frac{\pi B}{4} + h\right) t}}$

Table 13. Properties of Various Beam Cross-sections—(*continued*)

Section	Moment of inertia and section modulus	Radius of gyration
 Corrugated sheet iron, parabolically curved	$I = \frac{64}{105}(b_1h_1^3 - b_2h_2^3)$, where $h_1 = \frac{1}{2}(H+t)$, $b_1 = \frac{1}{4}(B+2.6t)$; $h_2 = \frac{1}{2}(H-t)$, $b_2 = \frac{1}{4}(B-2.6t)$; $\frac{I}{c} = \frac{2I}{H+t}$	$r = \sqrt{\frac{3I}{t(2B+5.2H)}}$

(Approximate values of *least* radius of gyration r)


Phoenix Column

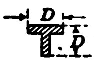

Carnegie Z-Bar Column


I-Beam

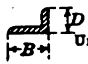

Channel


Deck Beam

	Phoenix Column	Carnegie Z-Bar Column	I-Beam	Channel	Deck Beam
$r =$	$0.3636D$	$0.295D$	$D/4.58$	$D/3.54$	$D/6$


T-Beam


Angle Equal legs


Angle Unequal legs


Cross

	T-Beam	Angle Equal legs	Angle Unequal legs	Cross
$r =$	$D/4.74$	$D/5$	$BD/2.6(B+D)$	$D/4.74$

Table 14. Moments of Inertia of Rectangles
(Unit widths; height = h; axis at middle of depth)

h	I	h	I	h	I	h	I	h	I
1	0.08333	13	183.1	25	1302	37	4221	49	9,804
2	0.6667	14	228.7	26	1465	38	4573	50	10,420
3	2.250	15	281.3	27	1640	39	4943	51	11,050
4	5.333	16	341.3	28	1829	40	5333	52	11,720
5	10.42	17	409.4	29	2032	41	5743	53	12,410
6	18.00	18	486.0	30	2250	42	6174	54	13,120
7	28.58	19	571.6	31	2483	43	6626	55	13,860
8	42.67	20	666.7	32	2731	44	7099	56	14,630
9	60.75	21	771.8	33	2995	45	7594	57	15,430
10	83.33	22	887.3	34	3275	46	8111	58	16,260
11	110.9	23	1014.0	35	3573	47	8652	59	17,110
12	144.0	24	1152.0	36	3888	48	9216	60	18,000

For **horizontal** or **longitudinal shear** in beams, see p. 378.

Relation of Moment and Shear. The shear V is the first differential of the moment M with respect to x. $dM/dx = V$. When M is a maximum, V is zero. In Fig. 32, $dM/dx = \frac{1}{2}W - wx$, which equals V.

Table 15. Moments of Inertia of Circular Cross-sections

I = Moment of inertia about a diam. d, $= \pi d^4/64$
c = Distance from outer fiber to neutral axis
I/c = Section modulus $= \pi d^3/32$
(For polar moment of inertia, multiply values in table by 2.)

d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$	d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$	d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$	d	$\frac{1}{1000} I$	$\frac{1}{1000} I/c$
10	.4909	.09817	35	73.66	4.209	60	636.2	21.21	85	2,562	60.29
11	.7187	.1307	36	82.45	4.580	61	679.7	22.28	86	2,685	62.45
12	1.018	.1696	37	92.00	4.973	62	725.3	23.40	87	2,812	64.65
13	1.402	.2157	38	102.4	5.387	63	773.3	24.55	88	2,944	66.90
14	1.886	.2694	39	113.6	5.824	64	823.6	25.74	89	3,080	69.21
15	2.485	.3313	40	125.7	6.283	65	876.2	26.96	90	3,221	71.57
16	3.217	.4021	41	138.7	6.766	66	931.4	28.23	91	3,366	73.98
17	4.100	.4823	42	152.7	7.274	67	989.2	29.53	92	3,517	76.45
18	5.153	.5726	43	167.8	7.806	68	1,050	30.87	93	3,672	78.97
19	6.397	.6734	44	184.0	8.363	69	1,113	32.25	94	3,833	81.54
20	7.854	.7854	45	201.3	8.946	70	1,179	33.67	95	3,998	84.17
21	9.547	.9092	46	219.8	9.556	71	1,247	35.14	96	4,169	86.86
22	11.50	1.045	47	239.5	10.19	72	1,319	36.64	97	4,346	89.60
23	13.74	1.194	48	260.6	10.86	73	1,394	38.19	98	4,528	92.40
24	16.29	1.357	49	283.0	11.55	74	1,472	39.78	99	4,715	95.26
25	19.18	1.534	50	306.8	12.27	75	1,553	41.42	100	4,909	98.18
26	22.43	1.726	51	332.1	13.02	76	1,638	43.10	101	5,108	101.2
27	26.09	1.932	52	358.9	13.80	77	1,726	44.82	102	5,313	104.2
28	30.17	2.155	53	387.3	14.62	78	1,817	46.59	103	5,525	107.3
29	34.72	2.394	54	417.4	15.46	79	1,912	48.40	104	5,743	110.4
30	39.76	2.651	55	449.2	16.33	80	2,011	50.27	105	5,967	113.7
31	45.33	2.925	56	482.8	17.24	81	2,113	52.17	106	6,197	116.9
32	51.47	3.217	57	518.2	18.18	82	2,219	54.13	107	6,434	120.3
33	58.21	3.528	58	555.5	19.16	83	2,330	56.14	108	6,678	123.7
34	65.60	3.859	59	594.8	20.16	84	2,444	58.19	109	6,929	127.1

Internal Moment Beyond the Elastic Limit

Ordinarily, the expression $M = SI/c$ is used for stresses above the elastic limit, in which case S becomes an experimental coefficient S_R, the **modulus of rupture,** and the formula is empirical. The true relation is obtained by applying to the cross-section a stress-strain diagram from a tension and compression test, as in Fig. 35. Fig. 35 shows the side of a beam of depth d under flexure beyond its elastic limit; line 1–1 shows the distorted cross-section; line 3–3, the usual rectilinear relation of stress to strain; and line 2–2, an actual stress-strain diagram (see Figs. 11 and 12) applied to the cross-section of the beam, compression above and tension below. The neutral axis is then below the gravity axis. The **outer material** may be expected to develop **greater ultimate strength** than in simple stress, on account of the reinforcing action of material nearer the neutral axis that is not yet overstrained. This leads to **an equalization of stress** over the cross-section. For values of S_R, see Table 2. S_R exceeds the ultimate strength S_M in tension as follows: For cast

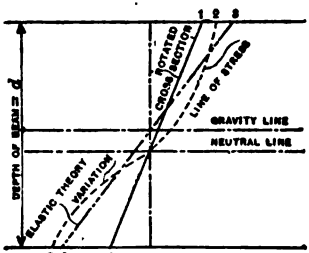


FIG. 35.

iron, $S_R = 2S_M$; for sandstone, $S_R = 3S_M$; for concrete, $S_R = 2.2S_M$; for wood (green), $S_R = 2.3S_M$.

In the case of steel I-beams, failure practically begins when the elastic limit in the compression flange is reached.

On account of support of adjoining material, the **elastic limit in flexure** S_p is also greater than in tension, depending upon the relation of breadth to depth of section. For the same breadth the difference decreases with increase of height. No difference will occur in the case of an I-beam, nor with hard materials. Bauschinger quotes for soft steel plates, 1.27; Considère, 1.37; Hatt, 1.5 (*R. R. Gas.*, 1899).

Wide plates will not expand and contract freely, and the value of E will be increased on account of side constraint. As a consequence of lateral contraction of the fibers of the tension side of a beam and lateral swelling of fibers at the compression side, the cross-section becomes distorted to a trapezoidal shape, and the neutral axis is at the c. of g. of the trapezoid. Strictly, this shape is one with a curved perimeter, the radius being r/m, where r is the radius of the neutral line of the beam, and m is Poisson's ratio.

Deflection of Beams

When a beam is subjected to an external moment, the fibers on one side elongate, while the fibers on the other side shorten (Fig. 36). These changes in length cause the beam to deflect. All points on the beam except those directly over the support fall below their original position, as shown by Fig. 34.

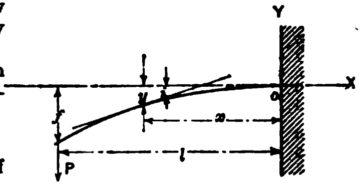


FIG. 36.

The **elastic curve** is the curve taken by the neutral axis. The radius of curvature at any point is

$$r = EI/M$$

A beam bent to a **circular curve** of constant radius has a constant moment and a constant stress.

Replacing r in the equation by its approximate geometrical value, $1/r = d^2f/(dx)^2$, the fundamental equation from which the elastic curve of a bent beam can be developed and the deflection of any beam can be obtained is,

$$M = EId^2f/(dx)^2 \text{ (approximate)}$$

This is a formula for **stiffness of beams.**

Substituting the value of M, in terms of x, and integrating once, gives the value of the angle i of the tangent to the elastic curve of the beam at point x, y; $i = df/dx = \int_0^x Mdx/EI$. A second integration gives the vertical deflection of any point of the elastic curve from its original position.

Example. In the cantilever beam shown in Fig. 36, the bending moment at any section $= -P(l - x) = EId^2f/(dx)^2$. Integrate and determine constant by the condition that when $x = 0$, $df/dx = 0$. Then $EIdf/dx = -Plx + \frac{1}{2}Px^2$. Integrate again and determine constant by the condition that when $x = 0$, $f = 0$. Then $EIf = -\frac{1}{2}Plx^2 + Px^3/6$. This is the equation of the elastic curve. When $x = l$, $f = -Pl^3/3EI$.

Deflection in general, f, may be expressed by the equation $f = Pl^3/mEI$, where m is a coefficient. See Tables 10 and 12 for values of f for beams of various sections and loadings. For coefficients of deflection of wooden beams, see pp. 1274 and 1276; for structural steel shapes, see pp. 1288 and 1289.

Since I varies as the cube of the depth, the **stiffness,** or inverse deflection, of various **beams** varies, other factors remaining constant, inversely as the load, inversely as the cube of the span, and directly as the cube of the depth.

This deflection is due to bending, but is increased by that due to vertical shear, especially in deep beams. For a **rectangular beam** with a center load P, the deflection due to vertical shear is $f' = Pl/4GA$, where G is the modulus of rigidity and A the area of cross-section. The shear is assumed to be uniformly distributed. If the shear S_v is distributed according to the parabolic law, $f' = 3Pl/10GA$. If the **load is distributed**, the deflection due to vertical shear is one-third that due to same load concentrated at the center.

Usually, deflection due to shear is unimportant, but in case of **I-beam girders**, where the vertical shear in the web is large, f' may become important. Here the deflection due to shear $= Pl/4GA$, when the shear due to P is considered to act entirely on the web of area A.

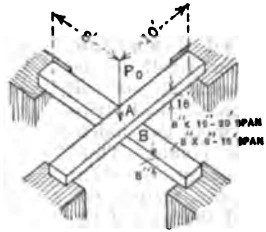


FIG. 37.

Design of beams may be fixed either by consideration of **strength**, or of **stiffness**, when deformations must be low. Stiffness is controlled by design or by material (E).

When a load may pass by two paths to a support, the different paths take parts of the load in proportion to their stiffness.

Example (Fig. 37). Two wooden stringers—one (A) 8 × 16 in. in cross-section and 20 ft. in span, the other (B) 8 in. × 8 in. × 16 ft.—carry the center load $P_0 =$ 22,000 lb. Required, the load carried by each stringer. The deflections, f, of the two stringers must be equal. Load on $A = P_1$, and on $B = P_2$. $f = P_1l_1^3/48EI_1 = P_2l_2^3/48EI_2$. Then $P_1/P_2 = l_2^3I_1/l_1^3I_2 = 4$. $P_0 = P_1 + P_2 = 4P_2 + P_2$, whence $P_2 =$ 22,000/5 = 4400 lb. and $P_1 = 4 \times 4400 =$ 17,600 lb.

Relation Between Deflection and Stress

Combine the formula $M = SI/c = Pl/n$, where n is a constant, P = load, and l = span, with formula $f = Pl^3/mEI$, where m is a constant. Then

$$f = C''Sl^2/Ec$$

where C'' is a new constant $= n/m$. Other factors remaining the same, the **deflection varies directly as the stress and inversely as E.** If the span is constant, a shallow beam will submit to greater deformations than a deeper beam without exceeding a safe stress. If depth is constant, a beam of double span will attain a given deflection with only one-quarter the stress. Values of n, m and C'' are as follows (for other values, see Table 10):

Beam	Load	n	m	C''
Cantilever	Concentrated at end	1	3	1/3
Cantilever	Uniform	2	8	1/4
Simple	Concentrated at center	4	48	1/12
Simple	Uniform	8	384/5	5/48
Fixed ends	Concentrated at center	8	192	1/24
Fixed ends	Uniform	12	384	1/32
One end fixed, One end supported	Concentrated at center	16/3	768/7	7/144
One end fixed, One end supported	Uniform	128/9	185	1/8
Simple	Uniformly varying, maximum at center	6	60	1/10

Graphical Relations

Referring to Fig. 38, the shear V acting at any section is equal to the total load on the right of the section, or

$$V = \int w dx$$

Since wdx is the product of w, a loading intensity (which is expressed as a vertical height in the load diagram), by dx, an elementary length along the horizontal, evidently wdx is the area of a small vertical strip of the **load diagram.** Then $\int wdx$ is the summation of all such vertical strips between two indefinite points. Thus, to obtain the shear in any section mn, find the area of the load diagram up to that section, and draw a second diagram called the **shear diagram,** any ordinate of which is proportional to the shear, or to the area in the load diagram to the right of mn. Since $V = dM/dx$,

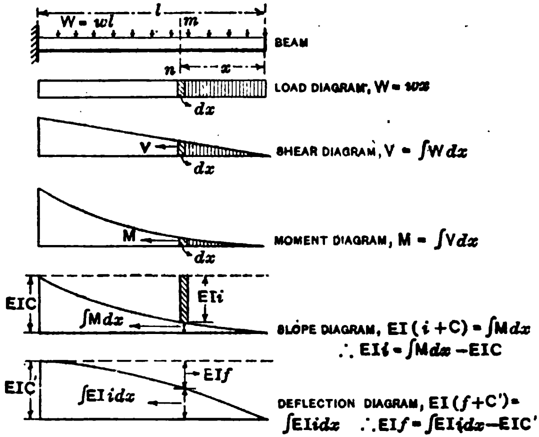


Fig. 38.

$$\int V dx = M$$

By similar reasoning, a **moment diagram** may be drawn, such that the ordinate at any point is proportional to the area of the shear diagram to the right of that point. Since $M = EId^2f/(dx)^2$,

$$\int M dx = EI[(df/dx) + C] = EI(i + C)$$

if I is constant. Here C is a constant of integration. Thus i, the slope or grade of the elastic curve at any point, is proportional to the area of the moment diagram $\int M dx$ up to that point; and a **slope diagram** may be derived from the moment diagram in the same manner as the moment diagram was derived from the shear diagram.

If I is not constant, draw a new curve whose ordinates are M/I and use these M/I ordinates just as the M ordinates were used in the case where I was constant; that is, $\int (M/I)dx = E(i + C)$. The ordinate at any point of the slope curve is thus proportional to the area of the M/I curve to the right of that point. Again, since $iE = Edf/dx$,

$$\int iE dx = \int E df = E(f + C')$$

and thus the ordinate f to the elastic curve at any point is proportional to the area of the slope diagram, $\int i dx$, up to that point. The equilibrium polygon may be used in drawing the **deflection curve** directly from the M/I diagram.

Thus, the five curves of load, shear, moment, slope, and deflection are so related that each curve is derived from the previous one by a process of graphical integration, and with proper regard to scales the deflection is thereby obtained.

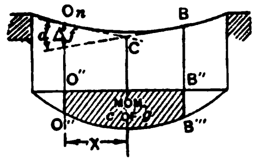


FIG. 39.

The vertical displacement of any point O_n (Fig. 39) in the elastic curve of a bent beam from the tangent line at any other point B, is Δf, and equals the area of the moment diagram between O'' and B'' times the distance X of the center of gravity of the area from O''. Or,

$$\Delta f = \text{Area } O''B''B'''O'''O'' \times X/EI$$

The angle d between the tangents at O_n and B equals the area of the moment diagram between O'' and B'' divided by EI. The intersection of the tangents at O_n and B is in the same vertical line as the center of gravity of the moment area between O'' and B''.

Resilience of Beams

The external work of a load gradually applied to a beam, and which increases from zero to P, is $\frac{1}{2}Pf$, and equals the **resilience** U. But, from formulæ under "Relation Between Deflection and Stress," p. 411, $P = nSI/cl$, and $f = nSl^2/mcE$, where n and m are constants that depend upon loading and supports, S = fiber stress, c = distance from neutral axis to outer fiber, and l = length of span. Substitute for P and f, and

$$U = \frac{n^2}{m}\left(\frac{k}{c}\right)^2 \cdot \frac{S^2V}{2E}$$

where k is the radius of gyration, and V the volume of the beam. For values of U, see Table 1.

The resilience of beams of similar cross-section at a given stress is proportional to their volumes. The **internal resilience**, or the elastic deformation energy in the material of a beam in a length x, is dU, and

$$U = \tfrac{1}{2}\int M^2dx/EI = \tfrac{1}{2}\int Mdi$$

where M is the moment at any point x, and di is the angle between the tangents to the elastic curve at the ends of dx. The values of resilience and deflection in special cases are easily developed from this equation.

Rolling Loads

Rolling or Moving Loads are those loads which may change their position on a beam. Fig. 40 represents a beam with two equal concentrated moving loads, such as two wheels on a crane girder, or the wheels of a wagon on a bridge. Since the maximum moment occurs where the shear is zero, it is evident from the shear diagram that the maximum moment will occur under a wheel. $x < a/2$:

$$R_1 = P\left(1 - \frac{2x}{l} + \frac{a}{l}\right) \qquad M_2 = \frac{Pl}{2}\left(1 - \frac{a}{l} + \frac{2x}{l}\frac{a}{l} - \frac{4x^2}{l^2}\right)$$

$$R_2 = P\left(1 + \frac{2x}{l} - \frac{a}{l}\right) \qquad M_1 = \frac{Pl}{2}\left(1 - \frac{a}{l} - \frac{2a^2}{l^2} + \frac{2x}{l}\frac{3a}{l} - \frac{4x^2}{l^2}\right)$$

M_2 max. when $x = \frac{1}{4}a$, $\qquad M_1$ max. when $x = \frac{3}{4}a$.

$$M_{\max} = \frac{Pl}{2}\left(1 - \frac{a}{2l}\right)^2 = \frac{P}{2l}\left(l - \frac{a}{2}\right)^2$$

Example. Two wheel loads of 3000 lb. each, spaced on 5-ft. centers, move on a span of $l = 15$ ft.; $x = 1.25$ ft. and $R_2 = 2500$ lb. ∴ Moment $= 6.25 \times 2500 = 15{,}600$ ft.-lb.

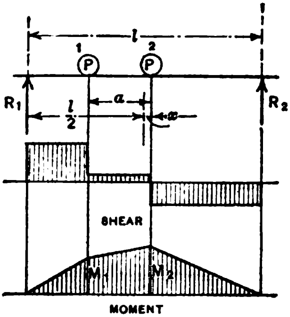


Fig. 40.

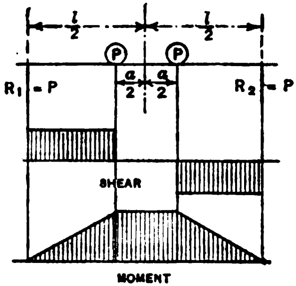


Fig. 41.

Fig. 41 shows the condition when two equal loads are equally distant on opposite sides of the center. The moment is equal under the two loads.

If the **two moving loads are of unequal weight,** the condition for **maximum moment** is that the maximum moment will occur under the heavy wheel when the center of the beam bisects the distance between the center of gravity and the heavy wheel. Fig. 42 shows this position and the shear and moment diagrams.

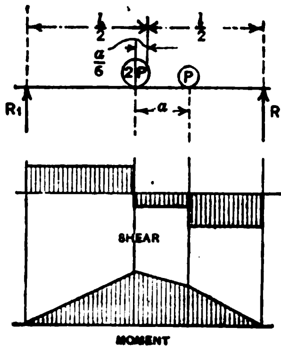


Fig. 42.

When **several wheel loads** constituting a system occur, the several suspected wheels must be examined in turn to determine which will cause the greatest moment. The **position for the greatest moment** that can occur under a given wheel is, as stated, when the center of the span bisects the distance between the wheel in question and the resultant of all the loads then on the span. The **position for maximum shear** at the support will be when one wheel is passing off the span.

Constrained Beams

Constrained beams are those so held or "built in" at one or both ends that the tangent to the elastic curve remains fixed in direction. These beams are held at the ends in such a manner as to allow free horizontal motion, as illustrated by Fig. 43. A constrained beam is stiffer than a simple beam of the same material, on account of the modification of the moment by an end resisting moment. Fig. 44 shows the two most common cases of constrained beams. See also Table 10.

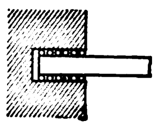


Fig. 43.

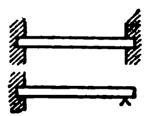


Fig. 44.

Continuous Beams

A continuous beam is one resting upon several supports which may or may not be in the same horizontal plane. The general discussion for beams holds for continuous beams. $S_sA = V$, $SI/c = M$, and $d^2f/dx^2 = M/EI$. The **shear** at any section is equal to the sum of all the reactions on one side of the section minus all the loads on the same side of the section. The internal moment at any section is equal to the moment of all reactions on one

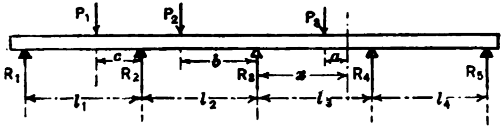


Fig. 45.

side of the section minus the moment of all the loads on the same side of the section. The relations stated above between shear and moment diagrams hold true for continuous beams. The internal bending moment at any section is equal to the bending moment at any other section, plus the shear at that section times its arm, plus the product of all the intervening external forces times their respective arms. To illustrate (Fig. 45):

$$V_x = R_1 + R_2 + R_3 - P_1 - P_2 - P_3$$
$$M_x = R_1(l_1 + l_2 + x) + R_2(l_2 + x) + R_3x - P_1(l_2 + c + x) - P_2(b + x) - P_3a$$
$$M_x = M_3 + V_3x - P_3a$$

Table 16 gives the value of the moment at the various supports of a uniformly loaded continuous beam over equal spans, and it also gives the values of the shears on each side of the supports. Note that the shear is of opposite sign on either side of the supports, and that the sum of the two shears is equal to the reaction.

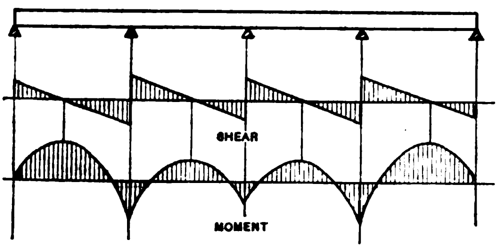


Fig. 46.

Fig. 46 shows the relation between the moment and shear diagrams for a uniformly loaded continuous beam of four equal spans (see Table 16). Table 16 also gives the **maximum bending moment** which will occur **between supports**, and in addition the position of this moment and the points of inflection (see Fig. 47).

Fig. 47 shows the values of the functions for a uniformly loaded continuous beam resting on three equal spans with four supports.

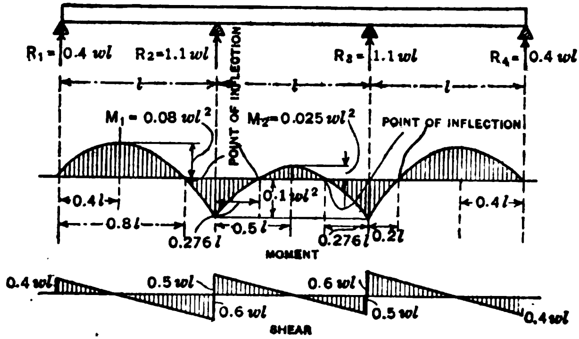


FIG. 47.

Table 16. Uniformly Loaded Continuous Beams Over Equal Spans*

(Uniform load per unit length = w; length of each span = l)

Number of supports	Notation of support or of span	Shear on each side of support. L = left, R = right. Reaction at any support is $L + R$: L	R	Moment over each support	Maximum moment in each span	Distance to point of maximum moment, measured to right from support	Distance to point of inflection, measured to right from support
2	1 or 2	0	1/2	0	0.125	0.500	none
3	1	0	3/8	0	0.0703	0.375	0.750
	2	5/8	5/8	1/8	0.0703	0.625	0.250
4	1	0	4/10	0	0.080	0.400	0.800
	2	6/10	5/10	1/10	0.025	0.500	0.276, 0.724
5	1	0	11/28	0	0.0772	0.393	0.786
	2	17/28	15/28	3/28	0.0364	0.536	0.266, 0.806
	3	13/28	13/28	2/28	0.0364	0.464	0.194, 0.734
6	1	0	15/38	0	0.0779	0.395	0.789
	2	23/38	20/38	4/38	0.0332	0.526	0.268, 0.783
	3	18/38	19/38	3/38	0.0461	0.500	0.196, 0.804
7	1	0	41/104	0	0.0777	0.394	0.788
	2	63/104	55/104	11/104	0.0340	0.533	0.268, 0.790
	3	49/104	51/104	8/104	0.0433	0.490	0.196, 0.785
	4	53/104	53/104	9/104	0.0433	0.510	0.215, 0.804
8	1	0	56/142	0	0.0778	0.394	0.789
	2	86/142	75/142	15/142	0.0338	0.528	0.268, 0.788
	3	67/142	70/142	11/142	0.0440	0.493	0.196, 0.790
	4	72/142	71/142	12/142	0.0405	0.500	0.215, 0.785
Values apply to		wl	wl	wl^2	wl^2	l	l

* The numerical values given are coefficients of the expressions at the foot of each column.

Continuous beams are stronger and much stiffer than simple beams. However, a small, unequal subsidence of piers will cause serious changes in sign and magnitude of the bending stresses, reactions and shears.

Maxwell's Theorem. When a number of loads rest upon a beam, the deflection at any point is equal to the sum of the deflections at this point due to each of the loads taken separately. Maxwell's Theorem states that if unit loads rest upon a beam at two points, A and B, the deflection at A due to the unit load at B equals the deflection at B due to the unit load at A.

Castigliano's Theorem states that the deflection of the point of application of an external force acting on a beam is equal to the partial derivative of the work of deformation with respect to this force. Thus, if P be the force, f the deflection, and U the work of deformation, which equals the resilience,

$$dU/dP = f$$

According to the **principle of least work,** the deformation of any structure takes place in such a manner that the work of deformation is a minimum.

Beams of Uniform Strength

Beams of uniform strength so vary in section that the unit stress S remains constant, and I/c varies as M. For **rectangular beams,** of breadth b and depth d, $I/c = bd^2/6$; and $M = Sbd^2/6$. Thus, for a cantilever beam of rectangular cross-section, under a load P, $Px = Sbd^2/6$. If b is constant, d^2 varies with x, and the profile of the shape of the beam will be a parabola, as Fig. 48. If d is constant, b will vary as x and the beam will be triangular in plan, as shown in Fig. 49.

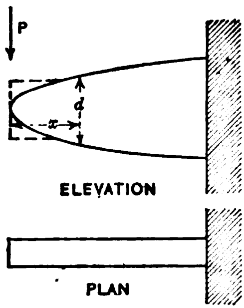


FIG. 48.

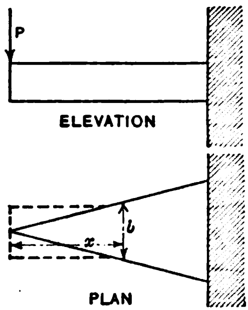


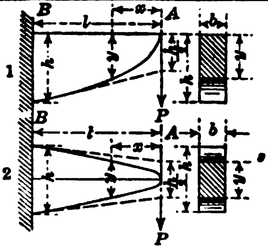
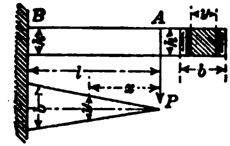
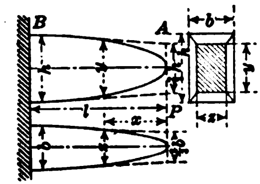
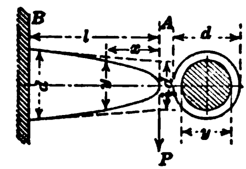
FIG. 49.

Shear at the end of a beam necessitates a modification of the forms determined above. The area required to resist shear will be P/S_v in a cantilever, and R/S_v in a simple beam. The dotted extensions in Figs. 48 and 49 show the changes necessary to enable these cantilevers to resist shear. The waste in material and extra cost in fabricating, however, make many of the forms impractical, except for cast iron.

Table 17 shows some of the simple **sections of uniform strength.** In none of these, however, is shear taken into account.

Table 17. Beams of Uniform Strength (in Bending)

1. Fixed at One End, Load P Concentrated at Other End

Beam	Cross-section	Elevation and plan	Formulæ
	Rectangle: width (b) constant, depth (y) variable.	Elevation: 1, top, straight line; bottom, parabola. 2, complete parabola. Plan: Rectangle	$y^2=\frac{6P}{bS_s}x$ $h=\sqrt{\frac{6Pl}{bS_s}}$ Deflection at A: $f=\frac{8P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Triangle	$y=\frac{6P}{h^2S_s}x$ $b=\frac{6Pl}{h^2S_s}$ Deflection at A: $f=\frac{6P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (z) variable, depth (y) variable: $\frac{z}{y}=k$ (const.)	Elevation: Cubic parabola Plan: Cubic parabola	$y^3=\frac{6P}{kS_s}x$ $z=ky$ $h=\sqrt[3]{\frac{6Pl}{kS_s}}$ $b=kh$
	Circle: diam. (y) variable	Elevation: Cubic parabola Plan: Cubic parabola	$y^3=\frac{32P}{\pi S_s}x$ $d=\sqrt[3]{\frac{32Pl}{\pi S_s}}$

2. Fixed at One End, Load P Uniformly Distributed over l

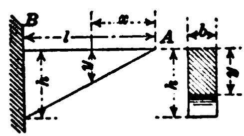
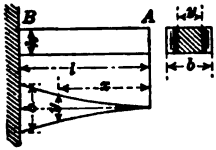
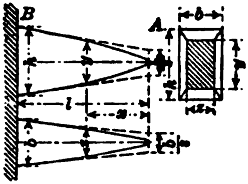
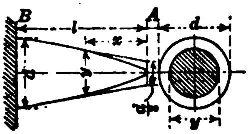
	Rectangle: width (b) constant, depth (y) variable	Elevation: Triangle Plan: Rectangle	$y=x\sqrt{\frac{3P}{blS_s}}$ $h=\sqrt{\frac{3Pl}{bS_s}}$ $f=6\frac{P}{bE}\left(\frac{l}{h}\right)^3$

Table 17. Beams of Uniform Strength (in Bending)—(*continued*)

2. FIXED AT ONE END, LOAD P UNIFORMLY DISTRIBUTED OVER l

Beam	Cross-section	Elevation and plan	Formulæ
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Two parabolic curves with vertices at free end	$y = \frac{3P}{lS_s}\frac{x^2}{h^2}$ $b = \frac{3Pl}{S_s h^2}$ Deflection at A: $f = \frac{3P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (z) variable, depth (y) variable, $\frac{z}{y} = k$	Elevation: Semi-cubic parabola Plan: Semi-cubic parabola	$y^3 = \frac{3P}{kS_s}\frac{x^2}{l}$ $z = ky$ $h = \sqrt[3]{\frac{3Pl}{kS_s}}$ $b = kh$
	Circle: diam. (y) variable	Elevation: Semi-cubic parabola Plan: Semi-cubic parabola	$y^3 = \frac{16P}{\pi l S_s}x^2$ $d = \sqrt[3]{\frac{16Pl}{\pi S_s}}$

3. SUPPORTED AT BOTH ENDS, LOAD P CONCENTRATED AT POINT C

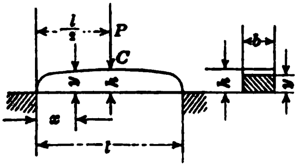
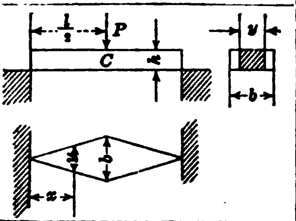
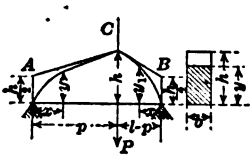
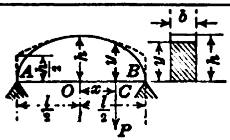
	Rectangle: width (b) constant, depth (y) variable	Elevation: Two parabolas, vertices at points of support Plan: Rectangle	$y = \sqrt{\frac{3P}{S_s b}x}$ $h = \sqrt{\frac{3Pl}{2bS_s}}$ $f = \frac{P}{2Eb}\left(\frac{l}{h}\right)^3$
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Two triangles, vertices at points of support	$y = \frac{3P}{S_s h^2}x$ $b = \frac{3Pl}{2S_s h^2}$ $f = \frac{3Pl^3}{8Ebh^3}$

Table 17. Beams of Uniform Strength (in Bending)—(*continued*)

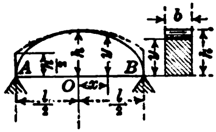
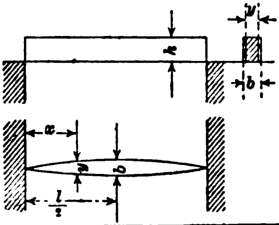
3. Supported at Both Ends, Load P Concentrated at Point C

Beam	Cross-section	Elevation and plan	Formulæ
	Rectangle: width (b) constant, depth (y or y_1) variable	Elevation: Two parabolas, vertices at points of support Plan: Rectangle	$y^2 = \frac{6P(l-p)}{blS_s}x$ $y_1^2 = \frac{6Pp}{blS_s}x_1$ $h = \sqrt{\frac{6P(l-p)p}{blS_s}}$

Load P Moving Across Span

	Rectangle; width (b) constant, depth (y) variable	Elevation: Ellipse, Major axis $= l$ Minor axis $= 2h$ Plan: Rectangle	$\frac{x^2}{\left(\frac{l}{2}\right)^2} + \frac{y^2}{\frac{3Pl}{2bS_s}} = 1$ $h = \sqrt{\frac{3Pl}{2bS_s}}$

4. Supported at Both Ends, Load P Uniformly Distributed over l

	Rectangle: width (b) constant, depth (y) variable	Elevation: Ellipse Plan: Rectangle	$\frac{x^2}{\left(\frac{l}{2}\right)^2} + \frac{y^2}{\frac{3Pl}{4bS_s}} = 1$ $h = \sqrt{\frac{3Pl}{4bS_s}}$ Deflection at O: $f = \frac{1}{64}\frac{Pl^3}{EI}$ $= \frac{3}{16}\frac{P}{bE}\left(\frac{l}{h}\right)^3$
	Rectangle: width (y) variable, depth (h) constant	Elevation: Rectangle Plan: Two parabolas with vertices at center of span	$y = \frac{3P}{S_s h^2}\left(x - \frac{x^2}{l}\right)$ $b = \frac{3Pl}{4S_s h^2}$

STRENGTH OF PLATES

Notation.

P = Total load, lb.
p = load per unit area, lb. per sq. in.
t = thickness of plate, in.
S_{max} = maximum stress, lb. per sq. in.
S_s = safe stress, lb. per sq. in.
E = modulus of elasticity.
f = deflection, in.
K_1 and K_2 = coefficients.

The values of K_1 and K_2 are dependent upon the method of support and upon the initial force required to give a tight joint (to prevent leakage) before the load is applied. Cases 3 and 6 follow Grashof; the others, C. Bach. The formulæ apply only within the elastic limit. For tests of flat plates by Prof. Bryson, see *Rensselaer Polytechnic Bulletin.*

Flat Plates

(1) Circular Plate Subjected to Uniformly Distributed Load.

$$S_{max} = K_1 r^2 p/t^2 \leqq S_s, \text{ and } f = K_2 r^4 p/t^3 E.$$

For cast iron, $K_1 = 0.8$ (fixed edge) to 1.2 (freely supported); $K_2 = 0.17$ (fixed edge) to 0.60 (freely supported). For mild steel, $K_1 = 0.50$ (not < 0.45) with fixed edge to 0.75 (not < 0.67) when freely supported.

FIG. 50.

In Fig. 50, S_{max} is at edge. In Fig. 51, S_{max} is at center. If the fixed ends deflect sufficiently to make the center and circumferential stresses equal, $K_1 = 0.38$, or not < 0.33.

FIG. 51.

(2) Circular Plate Loaded at the Center and Freely Supported at the Circumference (Fig. 52); P uniformly distributed over area πr_0^2.

$$S_{max} = \frac{3}{\pi} K_1 \left(1 - \frac{2}{3}\frac{r_0}{r}\right)\frac{P}{t^2} \leqq S_s; \quad f = K_2 \frac{r^2}{t^3}\frac{P}{E}.$$

For cast iron, $K_1 = 1.5$; $K_2 = 0.4$ to 0.5.

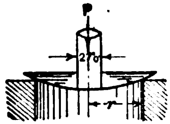


FIG. 52.

(3) Circular Plate Loaded as in (2) but Fixed at the Circumference (Fig. 53).

$$S_{max} = \frac{1.365}{\pi}\frac{P}{t^2}\log_e \frac{r}{r_0} \leqq S_s;$$

$$f = \frac{0.6825}{\pi}\frac{r^2}{t^3}\frac{P}{E} = 0.22\frac{r^2}{t^3}\frac{P}{E}.$$

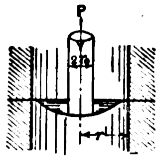


FIG. 53.

(4) Elliptical Plate Subjected to a Uniformly Distributed Load p. (See Figs. 50 and 51).

Major axis $= 2a$; minor axis $= 2b$; ratio of axes $= b/a = c$.

$$S_{max} = K_1 \frac{b^2}{t^2}\frac{2p}{(1+c^2)} \leqq S_s$$

For cast iron, $K_1 = 0.67$ (fixed) to 1.13 (freely supported).

For mild steel, $K_1 = 0.42$ (fixed) to 0.71 (freely supported).

For $c = 1$ (circle), $S_{max} = K_1 b^2 p/t^2$.

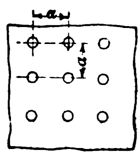


FIG. 54.

(5) Elliptical Plate with Load P **at the Center.** Notation as in (4).

$$S_{max} = \frac{8}{5\pi} K_1 \frac{8 + 4c^2 + 3c^4}{3 + 2c^2 + 3c^4} c \frac{P}{t^2} \leqq S_s.$$

For cast iron, $K_1 = 1.50$ (fixed) to 1.67 (freely supported).

For $c = 1$ (circle), $S_{max} = 3K_1 P/\pi t^2$.

(6) Infinitely Large Plate Subjected to a Uniformly Distributed Load and supported at points distant a from each other (Fig. 54). For each single square,

$$S_{max} = 0.2275\, a^2 p/t^2 \leqq S_s, \quad \text{and} \quad f = 0.0284\, a^4 p/t^3 E.$$

(7) Rectangular Plate Supported at the Periphery and Subjected to a Uniformly Distributed Load. (a and b = length and breadth of plate; $b/a = c$.) For the calculation of stress along the diagonal, as in case (4) of an ellipse,

$$S_{max} = K_1 \frac{(b/2)^2}{t^2} \frac{2p}{(1 + c^2)} \lesseqgtr S_s$$

When $b = a$, $c = 1$ (square plate), and $S_{max} = 0.25K_1a^2p/t^2 \lesseqgtr S_t$.

For cast iron, $K_1 = 0.75$ (fixed) to 1.13 (freely supported).

For mild steel, $K_1 = 0.48$ (fixed) to 0.72 (freely supported).

If the plate is supported on two sides ($b = \infty$), the coefficient of a^2p/t^2 will be 0.50 (fixed) to 0.75 (freely supported).

Note. In cases (4) and (7), values of K_1 for mild steel are: for elliptical plates, 0.43 to 0.55 (max. = 0.86); rectangular plates, 0.56 to 0.75 (max. = 1.13). The maxima, which correspond to values for free support, are to be rarely used. The minimum value of K_1 corresponds to the case where the moment at the center equals the moment at the support, $M_{max} = pl^2/16$.

Tests on rectangular cast-iron plates at Case School of Applied Science (1896–97), with load applied at center, give the following values for the breaking load W. For plates supported at the edges, $W = 276St^2/(l^2 + b^2)$; for plates with fixed edges, $W = 442\,St^2/(l^2 + b^2)$. The plates tested were 10 × 15 in., ½ to 1⅛ in. thick; modulus of rupture of the cast iron, 33,000 lb. per sq. in.

(8) Rectangular Plate Supported at the Periphery and Subjected to a Concentrated Load at the Center. (a and b = length and breadth, and $b/a = c$.) With the assumption made in (7),

$$S_{max} = 1.5K_1 \frac{c}{(1 + c^2)} \frac{P}{t^2} \lesseqgtr S_s.$$

K = 1.75 to 2.00 for cast iron. For $b = a$, $c = 1$ (square plate), and $S_{max} = 0.75Kp/t^2 \lesseqgtr S_s$.

Deflection of Square Plates with Different Methods of Support. Deflection under uniformly distributed load $= f = 15b^4pK/\pi^5t^3E$. Values of K are as follows:

Method of supporting plate	Center of plate K	Middle of unsupported edge K
2 edges supported, 2 rigidly fixed	0.134	.....
3 edges supported, 1 rigidly fixed	0.212	.....
4 edges supported, 0 rigidly fixed	0.310	.....
2 edges supported, 1 rigidly fixed, 1 not supported	0.430	0.825
3 edges supported, 1 not supported	0.602	0.915
2 edges supported, 2 not supported	0.994	1.110

Trapezoidal Plates. Calculations for these are made by assuming equivalent rectangular plates.

Flat Cylinder Heads with Flanged Edges (Fig. 55).

$$S_{max} = p\left\{K_1\frac{R}{t} + K_2\left[\frac{r - 0.5R\left(1 + \frac{R}{r}\right)}{t}\right]^2\right\}$$

in which R = radius of curvature of the flange and r = radius of head or attached cylinder, both in in.; p = internal pressure, in lb. per sq. in.

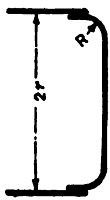


FIG. 55.

For steel heads riveted to the cylinder $K_1 = 0.5$, and $K_2 = 0.33$ to 0.38; for cast-iron heads integral with the cylinder, $K_1 = K_2 = 0.8$.

Curved Plates with Openings (Bach and Pfleiderer). With a single flue (Figs. 56 and 57) riveted into the plate, the greatest stress due to bending will be found at the flanges, and

$$S_{\max} = 0.45\,p\,\frac{r_a - r_i}{t^2}\left(r_a - r_i - 2e + \frac{5e^2}{h + 2e}\right).$$

With two flues (Figs. 58 and 59),

$$S_{\max} = 22p\,\frac{r_a - 1.5r_i}{t^2}\left(r_a - r_i - 2e + \frac{5e^2}{h + 2e}\right).$$

In computing h (Fig. 57), assume $v = 1.5$ to $2t$.

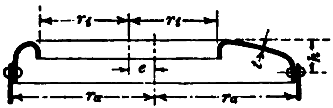


FIG. 56.

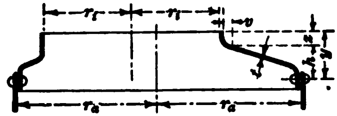


FIG. 57.

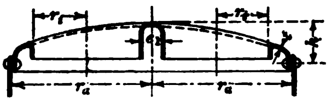


FIG. 58.

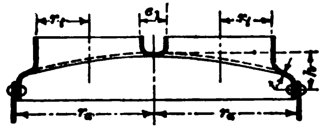


FIG. 59.

TORSION

Under torsion, a bar (Fig. 60) is twisted by a couple of the value Pp. Elements of the surface become helices of angle d, and a radius rotates through an angle a in a length l, both d and a being expressed in radians. S_v = shearing unit stress at distance r from center; I_P = polar moment of inertia; G = shearing modulus of elasticity. It is assumed that the cross-sections remain plane surfaces. The strain on the cross-section is wholly tangential, and is zero at the center of the section. $ld = ra$.

In the case of a circular cross-section, the stress S_v increases directly as the distance of the strained element from the center.

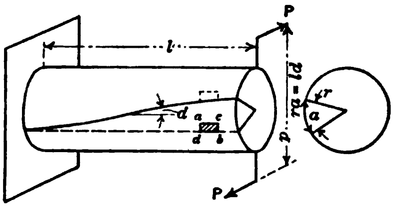


FIG. 60.

The polar moment of inertia I_P for any section may be obtained from $I_P = I_1 + I_2$, where I_1 and I_2 are the rectangular moments of inertia of the section about any two lines at right angles to each other, through the center of gravity.

The external twisting moment M_t is balanced by the internal resisting moment.

For strength, $M_t = S_v I_P / r$.

For stiffness, $M_t = aGI_P / l$.

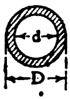
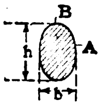
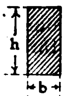
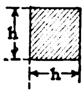
The torsional resilience $U = \frac{1}{2} Ppa = S_v^2 I_P l / 2r^2 G = a^2 G I_P / 2l$.

The state of stress on an element taken from the surface of the shaft, as in Fig. 61, is pure shear. Pure tension exists at right angles to one 45-deg. helix and pure compression at right angles to the opposite helix.

Reduced **formulæ for shafts of various sections** are given in Table 18.

Table 18. Torsion of Shafts of Various Cross-sections

(For allowable stresses, see Table 8, p. 389. For strength and stiffness of shafts, see pp. 439 and 688)

Cross-section	Torsional resisting moment M_t	Angular deflection, a_1 (length = 1 in., radius = 1 in.)		Work of torsion (V = volume)
		In terms of torsional moment	In terms of max. shear	
	$\frac{\pi}{16} d^3 S_s$	$\frac{M_t}{GI_P} = \frac{32}{\pi d^4}\frac{M_t}{G}$	$2\frac{S_s \max}{G}\frac{1}{d}$	$\frac{1}{4}\frac{S_s^2 \max}{G}V$ (Note 1)
	$\frac{\pi}{16}\frac{D^4-d^4}{D}S_s$	$\frac{32}{\pi(D^4-d^4)}\frac{M_t}{G}$	$2\frac{S_s \max}{G}\frac{1}{D}$	$\frac{1}{4}\frac{S_s^2 \max}{G}\frac{D^2+d^2}{D^2}V$ (Note 2)
	$\frac{\pi}{16}b^2hS_s$ ($h > b$)	$\frac{16}{\pi}\frac{b^2+h^2}{b^3h^3}\frac{M_t}{G}$	$\frac{S_s \max}{G}\frac{b^2+h^2}{bh^2}$	$\frac{1}{8}\frac{S_s^2 \max}{G}\frac{b^2+h^2}{h^2}V$ (Note 3)
	$\frac{2}{9}b^2hS_s$ ($h > b$)	$3.6\frac{b^2+h^2}{b^3h^3}\frac{M_t}{G}$*	$0.8\frac{S_s \max}{G}\frac{b^2+h^2}{bh^2}$*	$\frac{4}{45}\frac{S_s^2 \max}{G}\frac{b^2+h^2}{h^2}V$ (Note 4)
	$\frac{2}{9}h^3S_s$	$7.2\frac{1}{h^4}\frac{M_t}{G}$	$1.6\frac{S_s \max}{G}\frac{1}{h}$	$\frac{8}{45}\frac{S_s^2 \max}{G}V$ (Note 5)
	$\frac{b^3}{20}S_s$	$46.2\frac{1}{b^4}\frac{M_t}{G}$	$2.31\frac{S_s \max}{G}\frac{1}{b}$	
	$\frac{b^3}{1.09}S_s$	$0.967\frac{1}{b^4}\frac{M_t}{G}$	$0.9\frac{S_s \max}{G}\frac{1}{b}$	

* When h/b =	1	2	4	8
Coefficient 3.6 becomes	= 3.56	3.50	3.35	3.21 and
Coefficient 0.8 becomes	= 0.79	0.78	0.74	0.71

NOTES.—1. S_s max at circumference. 2. S_s max at outer circumference. 3. S_s max at A; $S_{sB} = 16M_t/\pi bh^2$. 4. S_s max at middle of side h; in middle of b, $S_s = 9M_t/2bh^2$. 5. S_s max at middle of side.

Failure under torsion in brittle materials is a tensile failure at right angles to a helical element of the surface. Plastic materials twist off squarely. Fibrous materials separate in long strips.

Torsion of Non-circular Sections. When a section is not circular, the unit stress no longer varies directly as the distance from the center. Cross-sections become warped, and the greatest unit stress usually occurs at a point on the perimeter of the cross-section *nearest* the axis of twist. There is no stress at the corners of square and rectangular sections, and the analyses become complex.

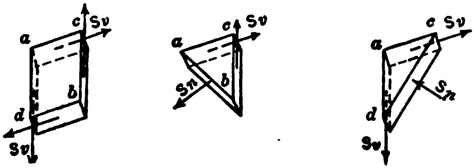


FIG. 61.

SPRINGS

It is assumed in the following formulæ that the springs are in no case stressed beyond the elastic limit (*i.e.*, that they are perfectly elastic), and that they are subject to Hooke's Law.

Notation:

P = Safe load, lb.
f = Deflection for a given load, P, in.
l = Length of spring, in.
V = Volume of spring, cu. in.
S_s = Safe stress (due to bending), lb. per sq. in.
S_v = Safe shearing stress, lb. per sq. in.
U = Resilience in in.-lb.

For values of E and G, see p. 384. Benjamin and Hoffman (in "Machine Design") recommend the following values for S_v, G and E: For spring brass wire, S_v = 45,000 to 60,000; G = 6,000,000; E = 9,000,000. For spring steel, tempered, S_v = 75,000 to 115,000; G = 12,000,000 to 15,000,000; E = 30,000,000.

The **work** in in.-lb. performed **in deflecting a spring** from 0 to f (spring duty) is $U = Pf/2 = S_s^2V/CE$. This is based upon the assumption that the deflection is proportional to the load. C is a constant dependent upon the shape of the springs.

The **time of vibration** T (in seconds) **of a spring** (weight not considered) is equal to that of a simple circular pendulum whose length l_0 equals the deflection f (in ft.) that is produced in the spring by the load P. $T = \pi\sqrt{l_0/g}$, where g = acceleration of gravity in ft. per sec.^2

Springs Subjected to Bending

(See first case in Table 10, p. 398)

1. **Rectangular Plate Spring** (Fig. 62).
$P = bh^2S_s/6l$; $I = bh^3/12$; $U = Pf/2 = VS_s^2/18E$;
$f = Pl^3/3EI = 4Pl^3/bh^3E = 2l^2S_s/3hE$.

2. **Triangular Plate Spring** (Fig. 63). The elastic curve is a circular arc.
$P = bh^2S_s/6l$; $I = bh^3/12$; $U = Pf/2 = S_s^2V/6E$;
$f = Pl^3/2EI = 6Pl^3/bh^3E = l^2S_s/hE$.

3. **Rectangular Plate Spring with End Tapered** in the form of a cubical parabola (Fig. 64). The elastic curve is a circular arc. P, I and f same as for triangular plate spring (Fig. 63). $U = Pf/2 = S_s^2V/9E$.

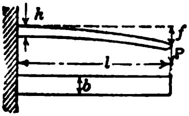


FIG. 62.

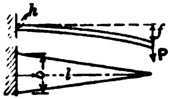


FIG. 63.

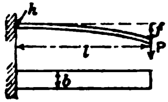
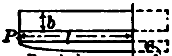
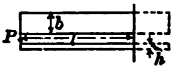
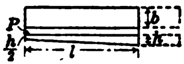
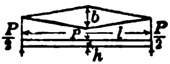
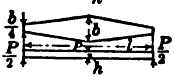
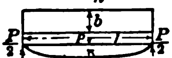
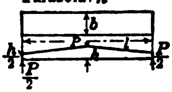
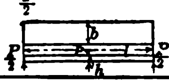


FIG. 64.

The strength and deflection of **single-leaf flat springs** of various forms are given (R. A. Bruce, *Am. Mach.*, July 19, 1900) by the formulæ $h = al^2/f$ and $b = cPl/h^2$. The volume of the spring is given by $V = vlbh$. The values of the constants a and c and the resilience in in.-lb. per cu. in. are given in Table 19, both in terms of the safe stress S_s and for stated specific values of S_s and E. Values of v are given also.

Table 19. Strength and Deflection of Single-leaf Flat Springs

Plans and elevations of springs	General				$S_s = 60{,}000$; $E = 30{,}000{,}000$		
	a	c	U	v	a	c	U
	Load P applied at end of spring						
	$\frac{S_s}{E}$	$\frac{6}{S_s}$	$\frac{S_s^2}{6E}$	$\frac{1}{2}$	$\frac{2}{1000}$	$\frac{1}{10{,}000}$	20.0
 Parabola	$\frac{4S_s}{3E}$	$\frac{6}{S_s}$	$\frac{S_s^2}{6E}$	$\frac{2}{3}$	$\frac{8}{3}\times\frac{1}{1000}$	$\frac{1}{10{,}000}$	20.0
	$\frac{2S_s}{3E}$	$\frac{6}{S_s}$	$\frac{0.33S_s^2}{6E}$	1	$\frac{4}{3}\times\frac{1}{1000}$	$\frac{1}{10{,}000}$	6.66
	$\frac{0.87S_s}{E}$	$\frac{6}{S_s}$	$\frac{0.70S_s^2}{6E}$	$\frac{5}{8}$	$\frac{1.75}{1000}$	$\frac{1}{10{,}000}$	14.0
	$\frac{1.09S_s}{E}$	$\frac{6}{S_s}$	$\frac{0.725S_s^2}{6E}$	$\frac{3}{4}$	$\frac{2.18}{1000}$	$\frac{1}{10{,}000}$	14.52
	Load P applied at center of spring						
	$\frac{S_s}{4E}$	$\frac{6}{4S_s}$	$\frac{S_s^2}{6E}$	$\frac{1}{2}$	$\frac{1}{2}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	20.0
	$\frac{0.87S_s^2}{4E}$	$\frac{6}{4S_s}$	$\frac{0.70S_s^2}{6E}$	$\frac{5}{8}$	$\frac{7}{16}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	14.0
 Parabola	$\frac{S_s}{3E}$	$\frac{6}{4S_s}$	$\frac{S_s^2}{6E}$	$\frac{2}{3}$	$\frac{2}{3}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	20.0
	$\frac{1.09S_s}{4E}$	$\frac{6}{4S_s}$	$\frac{0.725S_s^2}{6E}$	$\frac{3}{4}$	$\frac{0.54}{1000}$	$\frac{1}{40{,}000}$	14.52
	$\frac{S_s}{6E}$	$\frac{6}{4S_s}$	$\frac{0.33S_s^2}{6E}$	1	$\frac{1}{3}\times\frac{1}{1000}$	$\frac{1}{40{,}000}$	6.66

4. Compound (Leaf or Laminated) Springs. If several springs of rectangular section are combined, the resulting compound spring should (1) form a beam of uniform strength that (2) does not open between the joints while bending (*i.e.*, elastic curve must be a circular arc). Only the type immediately following meets both requirements, the others meeting only the second requirement.

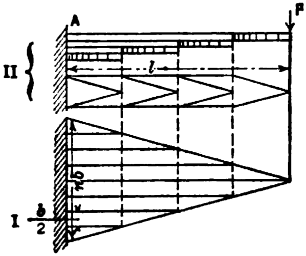


FIG. 65.

5. Laminated Triangular Plate Spring (Fig. 65). If the triangular plate spring shown at I be cut into an even number ($=2n$) of strips of equal width (in this case 8 strips of width $b/2$), and these strips be combined, a laminated spring will be formed whose carrying capacity will equal that of the original uncut spring; or $P = nbh^2S_s/6l$; $n = 6Pl/bh^2S_s$.

6. Laminated Rectangular Plate Spring with Leaf Ends Tapered in the form of a cubical parabola (Fig. 66); see (3) *ante*.

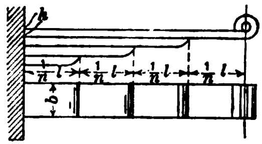


FIG. 66.

7. Laminated Trapezoidal Plate Spring with Leaf Ends Tapered. (Fig. 67.) The ends of the leaves are trapezoidal in form and are tapered according to the formula

$$z = h\Big/\left[1+\sqrt[3]{\frac{b_1}{b}\left(\frac{a}{x}-1\right)}\right]$$

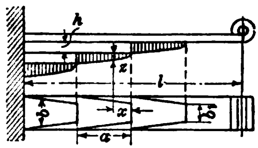


FIG. 67.

8. Semi-elliptic Springs (for locomotives, horse-drawn and automobile trucks, etc.). Referring to Fig. 68, the load $2P$ (lb.) acting on the spring center band produces a tensional stress $P/\cos a$ in each of the inclined shackle links. This is resolved into the vertical force P and the horizontal force $P \tan a$, which together produce a bending moment $M = P(l + p \tan a)$. The equations given in (1), (2) and (3) apply to curved as well as straight springs. The bearing force $= 2P = (2nbh^2/6)[S_s/(l + p \tan a)]$, and the deflection $= (6l^2/nbh^3)P(l + p \tan a)/E = l^2S_s/hE$.

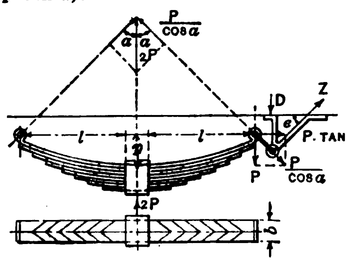


FIG. 68.

In addition to the bending moment, the leaves are subjected to the tensional force $P \tan a$ and the transverse force P, which produce in the upper leaf an additional stress $S = P \tan a/bh$, as well as a shearing stress to be calculated according to p. 412.

In determining the number of leaves n in a given spring, allowance should be made for an excess load on the spring caused by the vibration. This is usually made by decreasing the allowable stress about 15 per cent.

The foregoing does not take account of initial stresses caused by the band. For more detailed information, see "The Design of Automobile Springs," by E. R. Morrison. *Machinery*. Jan., 1910.

9. Elliptic Springs. Safe load P(lb.) $= nbh^2S_s/6l$, where l = ½ distance between bolt eyes (less ½ length of center band, where used); deflection f (in.) $= 4l^2S_sK/hE$, where

$$K = \frac{1}{(1-r)^3}\left[\frac{1-r^2}{2} - 2r(1-r) - r^2\log_e r\right]$$

r being the number of full-length leaves $\div$ total number (n) of leaves in the spring. All dimensions in in. For semi-elliptic springs the deflection is only half as great. Safe load $= nbh^2S_s/3l$. (J. B. Peddle, *Am. Mach.*, Apr. 17, 1913.)

Coiled Springs. In these the load is applied as a couple, Pr, which turns the spring while winding or holds it in place when wound up. If the spindle is not to be subjected to bending moment, P must be replaced by two equal and opposite forces ($P/2$) acting at the circumference of a circle of radius r. The formulæ are the same in both cases. The springs are assumed to be fixed at one end and free at the other. The bending moment acting on the section of least resistance is always Pr. The length of the straightened spring $= l$. See "Experiments on Helical Springs", by Benjamin and French, *Trans. A. S. M. E.*, vol. 23, p. 298.

10. Spiral Coiled Spring of Rectangular Cross-section (Fig. 69).

$P = bh^2S_s/6r$; $I = bh^3/12$; $U = Pf/2 = S_s^2V/6E$;

$f = ra = Plr^2/EI = 12Plr^2/Ebh^3 = 2rlS_s/hE$.

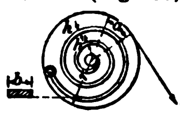


FIG. 69.

11. Cylindrical Helical Spring of Circular Cross-section (Fig. 70).

$P = \pi d^3S_s/32r$; $I = \pi d^4/64$; $U = Pf/2 = S_s^2V/8E$;

$f = ra = Plr^2/EI = 64Plr^2/\pi Ed^4 = 2rlS_s/dE$.

12. Cylindrical Helical Spring of Rectangular Cross-section (Fig. 71).

$P = bh^2S_s/6r$; $I = bh^3/12$; $U = Pf/2 = S_s^2V/6E$;

$f = ra = Plr^2/EI = 12Plr^2/Ebh^3 = 2rlS_s/hE$.

Springs Subjected to Torsion

The statements made concerning coiled springs subjected to bending apply also to (13) and (14).

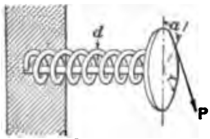


FIG. 70.

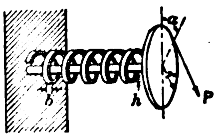


FIG. 71.

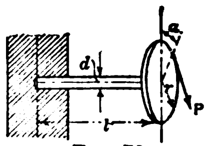


FIG. 72.

13. Straight Bar Spring of Circular Cross-section (Fig. 72).

$P = \pi d^3S_v/16r = 0.1963d^3S_v/r$; $U = Pf/2 = S_v^2V/4G$;

$f = ra = 32r^2lP/\pi d^4G = 2rlS_v/dG$.

14. Straight Bar Spring of Rectangular Cross-section (Fig. 73).

$P = 2b^2hS_v/9r$; $K = b/h$; $U = Pf/2 = 4S_v^2V(K^2+1)/45G$ (maximum when $K = 1$);

$f = ra = 3.6r^2lP(b^2+h^2)/b^3h^3G = 0.8rlS_v(b^2+h^2)/bh^2G$.

Springs Loaded Axially Either in Tension or Compression

[**Note.** For springs Nos. 15 to 18 inclusive, r = mean radius of coil; n = number of coils.]

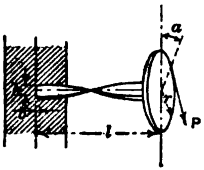
15. Cylindrical Helical Spring of Circular Cross-section (Fig. 74).

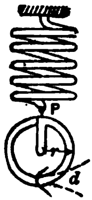
$P = \pi d^3S_v/16r = 0.1963d^3S_v/r$; $U = Pf/2 = S_v^2V/4G$;

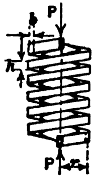
$f = 64nr^3P/d^4G = 4\pi nr^2S_v/dG$.

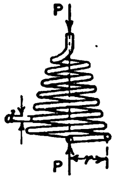
16. Cylindrical Helical Spring of Rectangular Cross-section (Fig. 75).
$P = 2b^2hS_v/9r$; $K = b/h$; $U = Pf/2 = 4S_v^2V(K^2+1)/45G$ (maximum when $K = 1$);
$f = 7.2\pi nr^3P(b^2 + h^2)/b^3h^3G = 1.6\pi nr^2S_v(b^2 + h^2)/bh^2G$.

17. Conical Helical Spring of Circular Cross-section (Fig. 76).
l = length of developed spring; d = diameter of wire; r = maximum mean radius of coil.
$P = \pi d^3S_v/16r = 0.1963d^3S_v/r$; $U = Pf/2 = S_v^2V/8G$;
$f = 16r^2lP/\pi d^4G = 16nr^3P/d^4G = rlS_v/dG = \pi nr^2S_v/dG$.









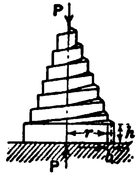


FIG. 73. FIG. 74. FIG. 75. FIG. 76. FIG. 77.

18. Conical Helical Spring of Rectangular Cross-section (Fig. 77).
$K = b/h$; $P = 2b^2hS_v/9r$; $U = Pf/2 = 2S_v^2V(K^2 + 1)/45G$ (maximum when $K = 1$);
$f = 1.8r^2lP(b^2 + h^2)/b^3h^3G = 1.8\pi nr^3P(b^2 + h^2)/b^3h^3G$.
$= 0.4rlS_v(b^2 + h^2)/bh^2G = 0.4\pi nr^2S_v(b^2 + h^2)/bh^2G$.

19. Truncated Conical Springs (Types 17 and 18). The formulæ under (17) and (18) apply for truncated springs. In calculating deflection f, however, it is necessary to substitute $(r_1^2 + r_2^2)$ for r^2, and $\pi n(r_1 + r_2)$ for πnr, r_1 and r_2 being respectively the greatest and least mean radii of the coils.

Note. The preceding formulæ for various forms of coiled springs are sufficiently accurate when the dimensions of the wire cross-section are small in comparison with the radius of the coil, and when the pitch is small. Springs (15) to (19) are for use either in tension or compression.

Safe Working Loads and Deflections of Cylindrical Helical Springs of round steel wire, in tension or compression, are given in Table 20. The values for steel of ¼ in. diam. and larger are from a table calculated by J. Begtrup (*Am. Mach.*, Aug. 18, 1892); while those for Roebling and Washburn & Moen wire-gage sizes (Nos. 000 to 28) are from a table due to F. D. Howe (*Am. Mach.*, Dec. 20, 1906). Both of these tables are based on the formulæ given in (15) *ante:*

When designing a spring for continuous work, as a car spring, a smaller value of S_s should be chosen than that used in calculating the table; for intermittent working, as in a steam-engine governor or a safety valve, the tabular values are to be used

Examples of Use of Table 20. (1) Required the safe load (P) for a spring of ⅜-in. round steel with an outside diam. (D) of 3 in. In the line headed D, under 3, is given the value of P, or 473 lb. To determine the number of coils this spring should have to deflect (say) 3 in. under a load of (say) 400 lb., take the value of f under 473, or 0.0610. This is the deflection of one coil for a load of 100 lb.; therefore $0.0610 \times 400/100 = 0.244$ in. is the deflection of one coil under a 400-lb. load, and $3/0.244 = 12.3$, say 13, = number of coils required. The spring will therefore be 4⅞ in. long when closed (13 × ⅜), counting the working coils only, and will stretch to 7⅞ in. under the 400-lb. load.

Table 20. Safe Working Loads and Deflections of Cylindrical Helical Steel Springs of Circular Cross-section

d = diameter of steel, in. P = safe working load, lb.
D = outside diameter of coil, in. f = deflection of 1 coil by load of 100 lb., in.

Table based on $S_s = 60,000$ lb. per sq. in. and $G = 12,000,000$. For any other value of G, multiply value of f in table by 12,000,000 and divide by the value of G chosen.

For square steel, multiply values of P by 1.2, and values of f by 0.59.

For brass, take $S_s = 10,000$ to 20,000, and multiply values of f by 2 (Howe).

Wire gage	d, in.		Values of D, P and f										
No. 28	0.016	D	0.20	0.25	0.3125	0.375	0.4375	0.500	0.5625	0.625	0.75	0.875	1.00
		P	0.524	0.41	0.31	0.27	0.23	0.20	0.175	0.16	0.13	0.11	0.098
		f	6.32	13.02	30.2	47.	76.	115	166	230	402	695	942
No. 24	.0225	D	0.25	0.3125	0.375	0.4375	0.500	0.5625	0.625	0.75	0.875	1.00	1.125
		P	1.18	0.92	0.76	0.45	0.56	0.50	0.45	0.37	0.31	0.28	0.24
		f	2.78	6.31	11.35	18.57	28.2	40.8	56.9	97.5	166	242	346
No. 22	0.028	D	0.25	0.3125	0.375	0.4375	0.500	0.5625	0.625	0.75	0.875	1.00	1.125
		P	2.35	1.84	1.49	1.26	1.095	0.96	0.865	0.715	0.61	0.53	0.47
		f	1.19	2.5	4.53	7.42	11.4	16.5	23.1	40.8	66	99.5	142
No. 20	0.035	D	0.25	0.3125	0.375	0.4375	0.50	0.5625	0.625	0.75	0.875	1.00	1.125
		P	4.7	3.64	2.97	2.5	2.18	1.92	1.72	1.42	1.20	1.05	0.93
		f	0.451	0.952	1.75	2.9	4.47	6.51	9.14	16.3	26.4	40	57.5
No. 18	0.047	D	0.25	0.3125	0.375	0.4375	0.500	0.625	0.75	0.875	1.00	1.125	1.25
		P	12.05	9.2	7.45	6.57	5.4	4.23	3.48	2.95	2.85	2.27	2.02
		f	0.1158	0.294	0.488	0.824	1.32	1.87	3.96	7.85	12.6	17.5	24.2
No. 16	0.063	D	0.25	0.375	0.500	0.625	0.75	0.875	1.00	1.125	1.25	1.50	1.75
		P	31.5	18.8	13.8	10.5	8.57	7.25	6.28	5.04	4.96	4.10	3.50
		f	0.026	0.122	0.310	0.704	1.29	2.33	3.27	4.76	6.60	11.8	19
No. 14	0.080	D	0.375	0.500	0.625	0.75	0.875	1.00	1.125	1.25	1.50	1.75	2.00
		P	41	28.8	22.2	18.1	15.2	13.15	11.6	10.35	8.52	7.25	6.3
		f	0.0418	0.128	0.342	0.572	0.820	1.27	1.86	2.6	5.48	7.57	11.6
No. 12	0.105	D	0.625	0.75	0.875	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
		P	52.5	42.25	35.4	30.4	23.8	19.5	16.6	14.4	12.7	11.4	10.3
		f	0.069	0.148	0.262	0.395	0.830	1.49	2.45	3.74	5.45	7.34	10.2
No. 10	0.135	D	0.875	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25
		P	77	67	52	42.5	36	31	27	24	22	20	18.5
		f	0.081	0.135	0.276	0.512	0.846	1.295	1.91	2.66	3.58	4.75	6.08
No. 8	0.162	D	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
		P	120	98.5	76	64	55.5	48.8	43.5	39	36	33	30.5
		f	0.057	0.124	0.199	0.354	0.597	0.880	1.26	1.68	2.20	2.85	3.64
No. 6	0.192	D	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00
		P	158	128	107	92.5	81	72	65	59.5	55.5	50	44
		f	0.0572	0.108	0.185	0.284	0.420	0.590	0.802	1.07	1.38	1.74	2.66
No. 5	0.205	D	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50
		P	155	131	113	99	88.5	80	70	67	61.5	53.5	47.3
		f	0.082	0.139	0.218	0.321	0.412	0.6175	0.820	1.60	1.34	2.22	2.98
No. 4	0.225	D	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50
		P	210	175	150	132	118	106	97	89	82	71	63
		f	0.0536	0.093	0 147	0.220	0.303	0.412	0.652	0.715	0.910	1.30	2.02
No. 3	0.242	D	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50
		P	256	225	195	170	152	136	125	114	105	88	80
		f	0.0376	0.0515	0.0813	0.120	0.198	0.297	0.391	0.510	0.650	0.864	1.16

Table 20. Safe Working Loads and Deflections of Cylindrical Helical Steel Springs of Circular Cross-section—(*continued*)

Wire gage	d, in.		Value of D, P and f										
No. 2	0.263	D	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50
		P	345	290	250	215	192	175	156	146	134	115	100
		f	0.0264	0.0458	0.073	0.109	0.154	0.214	0.274	0.371	0.469	0.720	1.06
No. 1	0.283	D	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50	5.00
		P	360	310	270	240	215	195	180	165	145	127	113
		f	0.0328	0.055	0.0778	0.112	0.155	0.208	0.270	0.344	0.530	0.775	1.28
No. 0	0.307	D	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50	5.00
		P	470	400	350	310	280	250	230	212	185	162	145
		f	0.0308	0.038	0.0548	0.0788	0.109	0.149	0.199	0.244	0.327	0.550	0.770
No. 00	0.331	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50	5.00	5.50
		P	510	445	390	350	320	290	270	230	205	183	165
		f	0.0289	0.0388	0.0564	0.078	0.105	0.137	0.176	0.273	0.414	0.562	0 750
No. 000	0.362	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.50	5.00	5.50
		P	700	610	540	480	435	400	365	315	280	250	225
		f	0.0102	0.0158	0.0229	0.032	0.073	0.0563	0.0722	0.113	0.166	0.234	0.319
	¼	D	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	......
		P	368	294	245	210	184	164	147	134	123	113	......
		f	0.0171	0.0333	0.0576	0.0914	0.1365	0.1944	0.2665	0.3548	0.4607	0.5859	......
	5/16	D	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00
		P	605	500	426	371	329	295	267	245	226	209	195
		f	0.0117	0.0207	0.0336	0.0508	0.0732	0.1012	0.1357	0.1771	0.2263	0.2839	0.3505
	⅜	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4 50
		P	765	663	589	523	473	433	398	368	343	321	301
		f	0.0145	0.0222	0.0323	0.0452	0.0610	0.0801	0.1029	0.1297	0.1606	0.1963	0.2367
	7/16	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50
		P	1263	1089	957	853	770	702	644	596	544	518	486
		f	0.0069	0.0108	0.0160	0.0225	0.0306	0.0405	0.0529	0.0661	0.0823	0.1088	0.122
	½	D	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00
		P	1963	1683	1472	1309	1178	1071	982	906	841	736	654
		f	0.0036	0.0057	0.0085	0.0121	0.0167	0.0222	0.0288	0.0366	0.0457	0.0683	0.0972
	9/16	D	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	5.00	5.50
		P	2163	1916	1720	1560	1427	1315	1220	1137	1065	945	849
		f	0.0048	0.0070	0.0096	0.0129	0.0169	0.0216	0.0271	0.0334	0.0406	0,0582	0.0801
	⅝	D	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	5.00	5.50
		P	3068	2707	2422	2191	2001	1841	1704	1587	1484	1315	1180
		f	0.0029	0.0042	0.0058	0.0079	0.0104	0.0133	0.0168	0.0208	0.0254	0.0366	0.0506
	11/16	D	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50
		P	3311	2988	2723	2500	2311	2151	2009	1885	1776	1678	1591
		f	0.0037	0.0050	0.0066	0.0086	0.0108	0.0135	0.0165	0.0200	0.0239	0.0283	0.0333
	¾	D	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50
		P	4418	3976	3615	3313	3058	2840	2651	2485	2339	2209	2093
		f	0.0024	0.0033	0.0044	0.0057	0.0072	0.0090	0.0111	0.0135	0.0162	0.0192	0.0226
	⅞	D	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
		P	6013	5490	5051	4676	4354	4073	3826	3607	3413	3237	3080
		f	0.0018	0.0024	0.0030	0.0038	0.0047	0.0058	0.0070	0.0083	0.0098	0.0115	0.0134
	1	D	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	6.00	6.50
		P	9425	8568	7854	7250	6732	6283	5890	5544	5236	4712	4284
		f	0.0010	0.0014	0.0018	0.0023	0.0028	0.0035	0.0043	0.0051	0.0061	0.0083	0.0111

(2) A $\frac{7}{16}$-in. steel spring of $3\frac{1}{4}$ in. outside diameter has its coils in close contact How much can it be extended without exceeding the limit of safety? The maximum safe load for this spring is found to be 702 lb. and the deflection of one coil for 100 lb. load 0.0405 in.; $(702/100) \times 0.0405 = 0.284$ in. is therefore the greatest admissible opening between any two coils. In this way it is possible to ascertain whether or not a spring is overloaded, without knowledge of the load carried.

ECCENTRIC LOADS

When **short blocks** are **loaded eccentrically** in compression or in tension, *i.e.*, not through the center of gravity (c. of g.), a combination of axial and bending stress results. The maximum unit stress S_m is the algebraic sum of these two unit stresses.

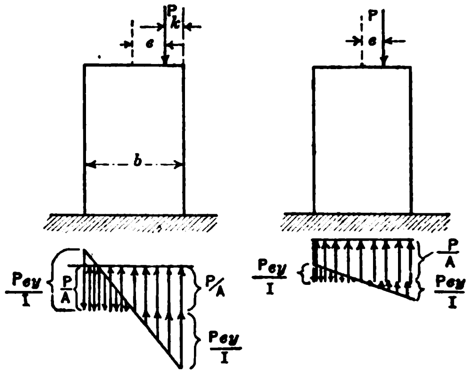


FIG. 78.

In Fig. 78 a load P acts in a line of symmetry at the distance e from c. of g.; r = radius of gyration. The unit stresses are (1) S_c, due to P, as if it acted through c. of g., and (2) S_b, due to the bending moment of P acting with a leverage of e about c. of g. Thus unit stress S at any point $y = S_c \pm S_b$

$$= P/A \pm Pey/I$$

$$= S_c(1 \pm ey/r^2).$$

y is positive for points on the same side of c. of g. as P, and negative on the opposite side. For a **rectangular cross-section** of width b, the maximum stress $S_m = S_c(1 + 6e/b)$. When P is outside the middle third of width b and is a compressive load, tensile stresses occur.

For a **circular cross-section** of diameter d, $S_m = S_c(1 + 8e/d)$. The stress due to the weight of the solid will modify these relations.

Note. In these formulæ e is measured from the gravity axis, and gives tension when e is greater than $\frac{1}{6}$ the width measured in the same direction as e.

If, as in certain classes of masonry nstruction, the **material cannot ithstand tensile stress** and thus no nsion can occur, the center of moments Fig. 79) is taken at the center of stress. For a **rectangular section,** P acts at distance k from the nearest edge. Length under compression $= 3k$, and $S_m = \frac{2}{3}P/hk$. For a **circular section,** $S_m = [0.372 + 0.056(k/r)]P/k\sqrt{rk}$, where r = radius and k = distance of P from circumference. For a **circular ring,** S = average compressive stress on cross-section produced by P; e = eccentricity of P; z = length of diameter under compression (Fig. 80). Values of z/r and of the ratio of S_{max} to average S are given in Tables 21 and 22.

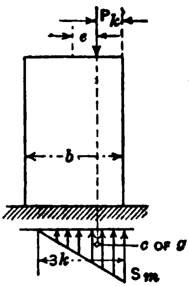


FIG. 79.

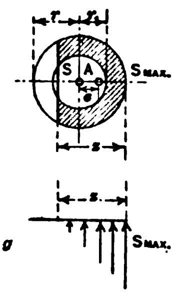


FIG. 80.

Table 21. Values of the Ratio z/r (Fig. 80)

$\frac{e}{r}$	$\frac{r_1}{r}$ 0.0	0.5	0.6	0.7	0.8	0.9	1.0	$\frac{e}{r}$
0.25	2.00	....	....	....	....	....	....	0.25
0.30	1.82	....	....	....	....	....	....	0.30
0.35	1.66	1.89	1.98	....	....	....	....	0.35
0.40	1.51	1.75	1.84	1.93	....	....	....	0.40
0.45	1.37	1.61	1.71	1.81	1.90	....	....	0.45
0.50	1.23	1.46	1.56	1.66	1.78	1.89	2.00	0.50
0.55	1.10	1.29	1.39	1.50	1.62	1.74	1.87	0.55
0.60	0.97	1.12	1.21	1.32	1.45	1.58	1.71	0.60
0.65	0.84	0.94	1.02	1.13	1.25	1.40	1.54	0.65
0.70	0.72	0.75	0.82	0.93	1.05	1.20	1.35	0.70
0.75	0.59	0.60	0.64	0.72	0.85	0.99	1.15	0.75
0.80	0.47	0.47	0.48	0.52	0.61	0.77	0.94	0.80
0.85	0.35	0.35	0.35	0.36	0.42	0.55	0.72	0.85
0.90	0.24	0.24	0.24	0.24	0.24	0.32	0.49	0.90
0.95	0.12	0.12	0.12	0.12	0.12	0.12	0.25	0.95

Table 22. Values of the Ratio $S_{max}/S_{average}$

(NOTE. In determining $S_{average}$ use load P divided by total area of cross-section)

$\frac{e}{r}$	$\frac{r_1}{r}$ 0.0	0.5	0.6	0.7	0.8	0.9	1.0	$\frac{e}{r}$
0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
0.05	1.20	1.16	1.15	1.13	1.12	1.11	1.10	0.05
0.10	1.40	1.32	1.29	1.27	1.24	1.22	1.20	0.10
0.15	1.60	1.48	1.44	1.40	1.37	1.33	1.30	0.15
0.20	1.80	1.64	1.59	1.54	1.49	1.44	1.40	0.20
0.25	2.00	1.80	1.73	1.67	1.61	1.55	1.50	0.25
0.30	2.23	1.96	1.88	1.81	1.73	1.66	1.60	0.30
0.35	2.48	2.12	2.04	1.94	1.85	1.77	1.70	0.35
0.40	2.76	2.29	2.20	2.07	1.98	1.88	1.80	0.40
0.45	3.11	2.51	2.39	2.23	2.10	1.99	1.90	0.45
0.50	3.55	2.80	2.61	2.42	2.26	2.10	2.00	0.50
0.55	4.15	3.14	2.89	2.67	2.42	2.26	2.17	0.55
0.60	4.96	3.58	3.24	2.92	2.64	2.42	2.26	0.60
0.65	6.00	4.34	3.80	3.30	2.92	2.64	2.42	6.65
0.70	7.48	5.40	4.65	3.86	3.33	2.95	2.64	0.70
0.75	9.93	7.26	5.97	4.81	3.93	3.33	2.89	0.75
0.80	13.87	10.05	8.80	6.53	4.93	3.96	3.27	0.80
0.85	21.08	15.55	13.32	10.43	7.16	4.50	3.77	0.85
0.90	38.25	30.80	25.80	19.85	14.60	7.13	4.71	0.90
0.95	96.10	72.20	62.20	50.20	34.60	19.80	6.72	0.95
1.00	∞	∞	∞	∞	∞	∞	∞	1.00

Chimney Problem. Weight of chimney = 563,000 lb.; e = 1.56 ft.; outside diam. of chimney = 10 ft. 8 in.; inside diam. = 6 ft. 6½ in. Overturning moment = Pe = 878,000 ft.-lb. r_1/r = 0.6. e/r = 0.29. This gives $(z/r) > 2$. Therefore, the entire area of the base is under compression. Area under compression = 55.8 sq. ft.; I = 546; $S = (563{,}000/55.8) \pm (878{,}000 \times 5.33)/546$ = 18,700 (max.) and 1500 (min.) lb. compression per sq. ft. From Table 22, by interpolation, $S_{max}/S_{avg} = 1.85 \therefore S_{max} = (563{,}000/55.8) \times 1.85$ = 18,685 lb. per sq. ft.

28

The **kern** is the area around the c. of g. of a cross-section within which any load applied will produce stress of only one sign throughout the entire cross-section. Outside the kern a load produces stresses of different sign. Fig. 81 shows kerns (shaded) for various sections.

For a **circular ring** the radius of the kern $r = D[1 + (d/D)^2]/8$.

For a **hollow square** (H and h = lengths of outer and inner sides) the kern is a square similar to Fig. 81(a), where

$$r_{\min} = \frac{H}{6}\frac{1}{\sqrt{2}}\left[1 + \left(\frac{h}{H}\right)^2\right] = 0.1179H\left[1 + \left(\frac{h}{H}\right)^2\right]$$

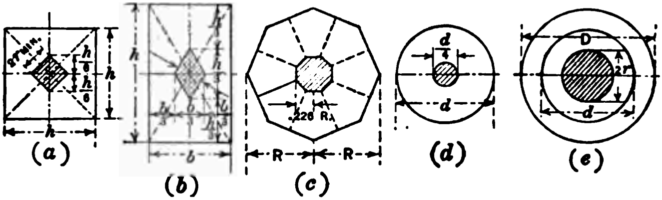
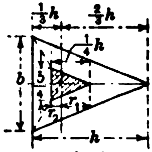


FIG. 81.



For a **hollow octagon** [R_a and R_i = radii of circles circumscribing the outer and inner sides; thickness of wall = 0.9239 ($R_a - R_i$)] the kern is an octagon similar to Fig. 81(c), where 0.2256R becomes

$$0.2256R_a[1 + (R_i/R_a)^2].$$

COLUMNS

A column differs from a tension bar in that any non-uniform yielding in the cross-section brings about further yielding. This non-uniform yielding is not serious in short blocks, but is serious in columns.

Columns are divided for analysis into **long columns** and **short columns,** in both of which initial inequalities introduce serious bending.

Long Columns fail by buckling at a load less than the elastic limit of the material. **Buckling** is the sudden collapse of a long column at or above the **critical load** at which equilibrium no longer obtains. The ratio of length to radius of gyration, or **slenderness ratio,** at which a column begins to fail by buckling, is between 100 and 120. Such columns are computed by Euler's formula. Few structural columns fail as long columns.

Short Columns with values of l/r less than 100 begin to fail when the ombination of direct stress and bending stress reaches the yield point of the aterial. The actual failure is dependent upon the homogeneity of the marial, the straightness of the column, and the eccentricity of loading, all of hich control bending stresses. Failure of built-up columns begins with a cal crippling at some part of the column. Such elements are not susceptible of calculation, and short columns of this kind are to be computed by empirical formulæ which are, however, modeled on rationally derived forms.

Failure of columns also depends upon the condition of their ends, as shown in Fig. 82: (a) **Free-ended** column, as a mast with a terminal load; (b) **round-ended,** as a spherical bearing; (c) **fixed-ended,** when rigidly connected to a rigid support; (d) **square-ended,** when a square-ended column abuts upon a support; and (e) a combination of rounded and fixed ends as a pin. The theoretical curves of flexure are shown by dotted lines. Columns of types (*c*), (*d*) and (*e*) are common. Pure conditions are only obtained practically in free-ended columns. Structural columns have various degrees of freedom. The elements controlling the strength of columns are so indeterminate and have been so little studied in detail, that refined analysis is misapplied.

Long Columns

Notation. A = Area of cross-section; P = load; l = length; r = radius of gyration = $\sqrt{I/A}$; f = deflection; I = rectangular moment of inertia; S = unit axial stress.

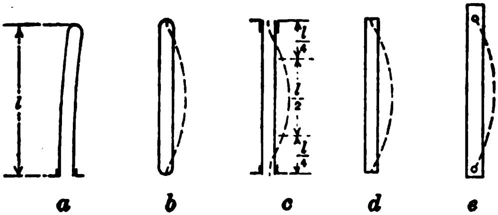


FIG. 82.

Euler's Formula. A perfect or ideal column with a straight axis, of uniform material, and loaded in the direction of its axis, would fail in direct compression like a short block. On account of imperfections, a sudden lateral deflection will occur under some critical load on a long column, which will then be no longer in equilibrium. The **critical load,** acting with a continually increasing leverage, will continue to bend the column until it fails by buckling. This critical load, P, for rounded ends, is given by Euler's formula (published in 1759) as follows:

$$P = \pi^2 EI/l^2$$

The assumptions made in deriving this formula are: (1) That the shortening of the column may be neglected; (2) that the bent length of arc is equal to the length of the vertical projection; and (3) that shearing forces may be neglected. To take account of end conditions, Euler's formula includes a coefficient n.

$$P = n\pi^2 EI/l^2$$

Table 23. Strength of Round-ended Columns According to Euler's Formula

P = allowable load, lb. l = length of column, in. b = smallest dimension of a rectangular section, in. d = diameter of a circular section, in. r = least radius of gyration of section.

Material	Cast iron	Wrought iron	Low-carbon steel	Medium-carbon steel	Wood (fir)
Ultimate compressive strength, lb. per sq. in.	107,000	53,400	62,600	89,000	4,000
Allowable compressive stress, lb. per sq. in.	7,100	15,400	17,000	20,000	850
Modulus of elasticity	14,200,000	28,400,000	30,600,000	31,300,000	1,400,000
Factor of safety	8	5	5	5	6 to 10
Smallest I allowable at worst section, in.^{4}	$\frac{Pl^2}{123{,}000}$	$\frac{Pl^2}{395{,}000}$	$\frac{Pl^2}{425{,}000}$	$\frac{Pl^2}{435{,}000}$	$\frac{Pl^2}{16{,}200 \text{ to } 9{,}700}$
Limit of ratio, $l/r <$	50.0	60.6	59.4	55.	40.5
Rectangle ($r = b\sqrt{1/12}$), $l/b <$	14.4	17.5	17.2	16.0	11.7
Circle ($r = \frac{1}{4}d$), $l/d <$	12.5	15.2	14.9	13.9	10.1
Circular ring of small thickness ($r = d\sqrt{1/8}$), $l/d <$	17.6	21.4	21.1	19.7	....

Values of n: Both ends rounded, n = 1; both ends fixed, n = 4; one rounded, one fixed, n = 2; one end fixed, one end free n = ¼. These values are theoretical. Experiments by Kirsch (*Zeit. Ver. Deutsch. Ing.*, 1905, p. 907) show that when $l/r = 100$ the relation of the first three coefficients, instead of being 1 : 4 : 2, is 1 : 1.13 : 1.05, and when $l/r = 200$, it is 1 : 3.0: 1.78 in the case of wrought-iron bars 20 mm. (0.79 in.) thick.

Euler's formula is proved to be reliable for long columns that fail by buckling within the elastic limit by Tetmajer's experiments (*Zeit. Ver. Deutsch. Ing.*, 1896, p. 1404). The value of the ratio (l/r) at which failure of round-ended columns by buckling occurred was as follows: Wrought iron, 112; soft steel, 105; medium steel, 90; cast iron, 80; wood, 100.

The use of Euler's formula is more common in Germany than elsewhere. Table 23, for columns with rounded ends $(n = 1)$, is based on this formula.

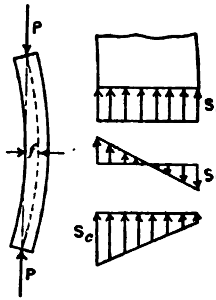


Fig. 83.

Short Columns

Rankine's Formula applies to short columns, which fail under the stress S_c (Fig. 83), this stress being the sum of (1) the stress S_0, a uniform compression due to P acting as an axial load, and (2) S_B in the outer fiber due to moment M of P acting with a lever arm f around the gravity axis of the section. $S_0 = P/A$ and $S_B = Mc/I = Pfc/Ar^2$; $P = S_cA/[1 + (fc/r^2)]$, where r is the radius of gyration and c the distance from the outer fiber to the gravity axis. Here f is unknown and is replaced by Kl^2/c, giving Rankine's formula

$$P = S_cA/[1 + K(l^2/r^2)]$$

This is largely used in design. Values of K have been derived experimentally, and those recommended by Merriman are given in Table 24. Rankine's formula is to be used for values of $l/r = 20$ to 120. The value of S_c should be the allowable unit stress for designing, and should be the ultimate compressive strength where ultimate load is required.

Table 24. Values of the Coefficient K in Rankine's Formula

Material	Both ends fixed	Fixed and rounded	Both ends rounded	Fixed and free
Timber	1/3000	1.95/3000	4/3000	16/3000
Cast iron	1/5000	1.95/5000	4/5000	16/5000
Wrought iron	1/36000	1.95/36000	4/36000	16/36000
Steel	1/25000	1.95/25000	4/25000	16/25000

Tetmajer's Experiments on round-ended columns (*Zeit. Ver. Deutsch. Ing.*, 1896, p. 1404) show that K varies with the material of the column and with the value of l/r. For wrought iron, K varied from 0.000448 to 0.000136; for steel, from 0.000370 to 0.000130. Tetmajer's recommended values for K are as follows:

	l/r	K
Cast iron	20 to 150	0.00070
Wrought iron	20 to 250	0.00016
Soft steel	20 to 250	0.00014
Wood	20 to 200	0.00023

Tetmajer's Formula for ultimate loads on round-ended columns is

$$S = S_c[1 - a(l/r) + b(l/r)^2]$$

Table 25. Values to be Used in Tetmajer's Formula for Short Columns

Materials	S_c	a	b	Limits of l/r Min.	Limits of l/r Max.
Cast iron	110,000	0.01546	0.00007	5.0	80
Wrought iron	43,000	0.00426	0.0	10.0	112
Hard steel	48,000	0.00185	0.0	.....	90
Soft steel	44,000	0.00368	0.0	10.0	105
Wood	4,200	0.00662	0.0	1.8	100

Straight-line Formula for Columns. Plotted results of tests yield a straight-line relation between the load at failure and the slenderness ratio l/r. T. H. Johnson's formula is

$$P/A = S - C(l/r)$$

where S is the unit strength of a very short column, and C is a constant obtained by drawing the straight line tangent to a curve representing Euler's formula at a point where $l/r = 150$. The values recommended for use in this formula are given in Table 26. For designing, S and C should be divided by a proper factor of safety.

Table 26. Constants for Use in Johnson's Straight-line Column Formula

Kind of column	S lb. per sq. in.	C	Limit of l/r
Wrought iron:			
Flat ends	42,000	128	218
Hinged ends	42,000	157	178
Round ends	42,000	203	138
Steel:			
Flat ends	52,500	179	195
Hinged ends	52,500	220	159
Round ends	52,500	284	123
Cast iron:			
Flat ends	80,000	438	122
Hinged ends	80,000	537	99
Round ends	80,000	693	77
Oak:			
Flat ends	5,400	28	128

Table 27. Allowable Unit Stresses (lb. per sq. in.) on Steel Columns According to Various Formulæ

$\frac{l}{r}$	American Bridge Co. $19000-100\frac{l}{r}$ 13000 max.	A. R. E. Assn. and Chicago $16000-70\frac{l}{r}$ 14000 max.	Gordon $\frac{12500}{1+\frac{l^2}{36000r^2}}$	New York $15200-58\frac{l}{r}$	Philadelphia $\frac{16250}{1+\frac{l^2}{11000r^2}}$	Boston $\frac{16000}{1+\frac{l^2}{20000r^2}}$
10	13,000	14,000	12,460	14,620	16,100	15,920
20	13,000	14,000	12,365	14,040	15,680	15,690
	13,000	13,900	12,195	13,460	15,020	15,310
40	13,000	13,200	11,970	12,880	14,185	14,815
50	13,000	12,500	11,690	12,300	13,240	14,220
60	13,000	11,800	11,365	11,720	12,240	13,560
70	12,000	11,100	11,000	11,140	11,240	12,850
80	11,000	10,400	10,615	10,560	10,275	12,120
90	10,000	9,700	10,205	9,980	9,360	11,390
100	9,000	9,000	9,785	9,400	8,510	10,670
110	8,000	8,300	9,355	8,820	7,740	9,970
120	7,000	7,600	8,930	8,240	7,035	9,300

Table 27 gives the allowable unit stresses for **steel columns** as deduced from the American Bridge Co.'s formula, and also as deduced from the American Railway Engineering Association formula (A. R. E. Ass'n); the Gordon formula, and the formulæ from the building codes of the cities of Chicago, New York, Philadelphia and Boston.

For tables of structural-steel columns see pp. 1297 to 1302.
For tables of steel-pipe columns see p. 1279.

For columns carrying traveling machinery, cranes, etc., add 25 per cent. to the live load to allow for impact and vibration.

W. H. Burr's tests on **cast-iron columns** at Phoenixville (*Eng. News*, June 30, 1898) gave a wide range of results. The **average ultimate strength** (lb. per sq. in.) for round columns is expressed by $P/A = 30{,}500 - 165(l/d)$, where d and l are expressed in the same units. A large factor of safety, 4, must be used with cast-iron columns. Maximum slenderness ratio is 70. For tables of cast-iron columns, see p. 1278.

For **wood,** the formula recommended by Am. Ry. Eng. Assn., 1909, is: $S_1 = S\ (1 - l/60d)$, in which S_1 = safe working stress; S = safe stress in compression along grain (see p. 1277); l = length of column, d = least side. l and d are expressed in same unit. Table 28 is calculated from this formula. For l/d less than 15, use S. No column should have a value of l/d greater than 60.

Table 28. Safe Loads for Square Wooden Columns in Units of 1000 Lb.

(Based on safe end-bearing compression of 1000 lb. per sq. in.)

Unbraced length in feet	Size of column in inches						
	4 × 4	6 × 6	8 × 8	10 × 10	12 × 12	14 × 14	16 × 16
4	16.0						
6	11.2	36.0					
8	9.6	26.4					
10	8.0	24.0	64.0				
12	6.4	21.6	44.8	100.0			
14	4.8	19.2	41.6	72.0	144.0		
16		16.8	38.4	68.0	105.6	196.0	
18		14.4	35.2	64.0	100.8	145.6	256.0
20		12.0	32.0	60.0	96.0	140.0	192.0
22		9.6	28.8	56.0	91.2	134.4	185.6
24			25.6	52.0	86.4	128.8	179.2
26			22.4	48.0	81.6	123.2	172.8
28			19.2	44.0	76.8	117.6	166.4
30				40.0	72.0	112.0	160.0
32				36.0	67.2	106.4	153.6
34				32.0	62.4	100.8	147.2
36					57.6	95.2	140.8
38					52.8	89.6	134.4
40					48.0	84.0	128.0
42						78.4	121.6
44						72.8	115.2
46						67.2	108.8
48							102.4
50							96.0

For values of S_1 for common woods, see pp. 590 and 591.

COMBINED STRESSES

Combined Flexure and Torsion arise when a twisted shaft is bent by belts or other forces. Elements of the shaft become helices of variable pitch. An element of the surface (Fig. 84) is subjected to a flexural unit stress S due to bending moment M, and a shearing unit stress S_v, due to torsional moment

M_t. $S = My/I = 8Pl/\pi d^3$, and $S_v = 16M_t/\pi d^3$ for a circular shaft. S and S_v combine (Fig. 84) to produce internal normal (S_n) and tangential (S_t) unit stresses on an interior plane (see p. 000). In Fig. 85, S_n and S_t vary with the

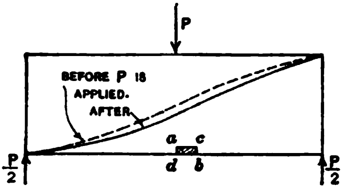


FIG. 84.

angle d. S_n is a maximum when $\cot 2d = -S/2S_v$, and S_t is a maximum when $\tan 2d = +S/2S_v$. The maximum apparent stresses are:

$$S_n = \tfrac{1}{2}S \pm \sqrt{S_v^2 + (\tfrac{1}{2}S)^2} \qquad S_t = \sqrt{S_v^2 + (\tfrac{1}{2}S)^2}$$

These formulæ also apply to any case of **normal stress combined with shear**, as when a bolt is under tension and shear, or when the material of a beam is under tension (or compression) and shear. When S is tension, use (+) sign in order to find the maximum tensile stress S_n, and use (−) sign to find the maximum compressive stress S_n. For **strength under combined stresses**, see p. 391.

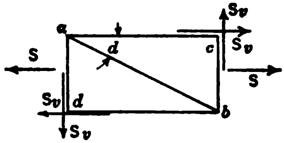


FIG. 85.

Combined Torsion and Longitudinal Force

A common case is the shaft of a vertical turbine. The longitudinal force is then compression. If W be the weight carried on the shaft in pounds, d the shaft diameter in inches, H the horse power transmitted, and n the r.p.m.,

$$S_n = \frac{2W}{\pi d^2} + \sqrt{\frac{321{,}000^2\, H^2}{n^2 d^6} + \frac{16W^2}{\pi^2 d^4}}$$

Combined Flexure and Longitudinal Force

Fig. 86 shows a bar under flexure due to transverse and longitudinal loads. The maximum fiber stress S (Fig. 86) is made up of S_0, due to the direct action of load P, and S_b, due to the entire bending moment M. M is the algebraic sum of two bending moments, M_1 due to longitudinal load (+ for compression and − for tension), and M_2 due to transverse load. $M = M_2 \pm M_1$. Here $M_1 = Pf$ and $f = C''S_0 l^2/Ec$.

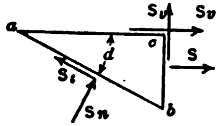


FIG. 86.

$$\frac{S_b I}{c} = M_2 \pm \frac{PC''S_0 l^2}{Ec}, \quad \text{or} \quad S_b = \frac{M_2 c}{I \pm \dfrac{Pl^2}{CE}}$$

$C = 10$ for hinged-end columns, 24 for one end hinged and other end fixed, and 32 for both ends fixed. M is the total eccentric moment. The + sign should be used when P is compression, and − when P is tension.

A common method for **computing stresses on eccentrically loaded**

columns consists in adding to the uniform stress, P/A, the additional stresses due to column action, $(P/A)(Kl^2/r^2)$, as given by Rankine's formula, and that due to eccentricity, $(P/A)(ey/r^2)$. Thus,

Total stress $= \frac{P}{A}\left(1 + \frac{ey}{r^2} + \frac{Kl^2}{r^2}\right)$ when $S_b = \frac{P}{A}\left(1 + \frac{ey}{r^2}\right)$ = stress due to flexure, and $S_c = \frac{P}{A}\left(1 + \frac{Kl^2}{r^2}\right)$ = stress due to longitudinal load.

BENDING OF CURVED BEAMS

(From Morley's "Strength of Materials")

Simple beam equations apply when the radius of curvature of a curved beam is large (8 to 10 times) compared to dimensions of its cross-section. This is not the case in hooks, links, and rings, when some modification of formulæ is necessary. Plane cross-sections, normal to the axis of curvature, are assumed. The deformation (e) is proportional to the distance from the gravity axis, but the unit deformation (s) is not proportional, since the lengths of the fibers vary. The elongation of any curved fiber is evaluated, and the stress determined. E is used. The neutral axis is not at the c. of g., but is near the center of curvature. The stress variation is not rectilinear (Fig. 87).

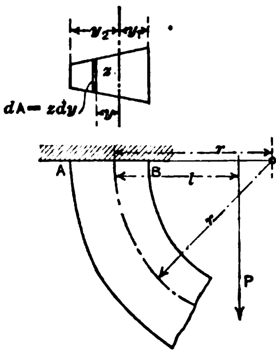


FIG. 87.

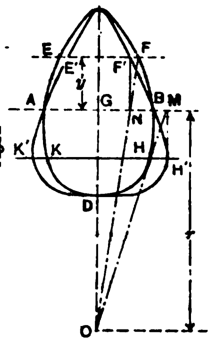


FIG. 88.

In Fig. 87, P = load; l = leverage of load about axis through c. of g. of cross-section; M = moment due to $P = Pl$; A = area of cross-section; r = radius of curvature of undeformed beam from center of curvature to c. of g. of cross-section. Stress due to combination of direct load P (taken as acting at c. of g.) and moment $M = S$, or the normal unit stress at distance y from axis through c. of g. The equations which follow are for pure bending, but apply with little error to ordinary bending, including shear.

$$S_1 \text{ (at } B) = \frac{M}{r(A' - A)}\left(\frac{r}{r - y_1} - \frac{A'}{A}\right) + \frac{P}{A} \text{ (tension)}$$

$$S_2 \text{ (at } A) = \frac{M}{r(A' - A)}\left(\frac{A'}{A} - \frac{r}{r + y_2}\right) - \frac{P}{A} \text{ (compression)}$$

where $$A' = r\Sigma\left(\frac{dA}{y + r}\right)$$

The neutral axis is at a distance y' from c. of g., where $y' = r\left(\frac{A' - A}{A'}\right)$.

A' may be considered as a "modified" area, and is obtained graphically (Fig. 88) as follows: To find $A' = r\Sigma\left(\frac{dA}{y + r}\right)$, first find the c. of g. of the original cross-section. Lay off O so that $OG = r$. Change each width in the ratio of $r/(y+r)$. For example, draw OF intersecting AB in N. Draw NF' perpendicular to AB, giving F'. Then $E'F'$ is the modified width. Similarly, KH becomes $K'H'$. The new area A' is evaluated with a planimeter.

VALUES OF A'. Rectangle (height $= h$, width $= b$): $A' = bd[1 + (h/r)^2/12 + (h/r)^4/80 + \ldots]$. Circle (radius $= R$): $A' = 2\pi r[r - \sqrt{r^2 - R^2}]$. Any Section: $A' = \Sigma\{[1 - (y/r) + (y^2/r^2) - (y^3/r^3) + (y^4/r^4) - \text{etc.}]\,dA\}$. Symmetrical Sections: $A' = A + (I/r^2) + (1/r^4)\Sigma(y^4da) + \text{etc.}$, where I is the moment of inertia of the section about the axis from which y is measured.

Example—Stress in Hooks (Fig. 89). The central horizontal section AB is a trapezoid; area $= 9.855$ sq. in. Radius of curvature $r = 2.5 + 1.843 = 4.343$ in. Eccentricity of load $= 4.343$ in. $A' = 10.673$ sq. in. $y' = 0.33$ in. $M = 20{,}000 \times 4.343$. Substituting in the above equations,

$$S_1 = \frac{20{,}000 \times 4.343}{4.343 \times 0.818}\left(\frac{4.343}{2.5} - \frac{10.673}{9.855}\right) + \frac{20{,}000}{9.855}$$

$= 17{,}700$ lb. per sq. in. tension.

$$S_2 = \frac{20{,}000 \times 4.343}{4.343 \times 0.818}\left(\frac{10.673}{9.855} - \frac{4.343}{7}\right) - \frac{20{,}000}{9.855}$$

$= 9120$ lb. per sq. in. compression.

By the ordinary theory (p. 404), tension at $B = 12{,}700$ lb. per sq. in., and compression at $A = 13{,}380$ lb. per sq. in.

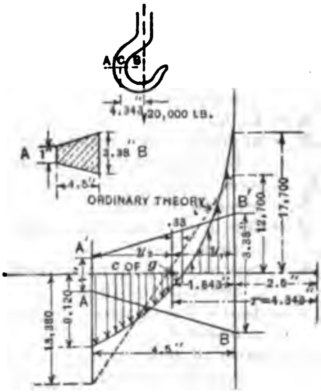


FIG. 89.

Crane Hooks. Experiments by Rautenstrauch (*Am. Mach.*, Oct. 7, 1909) yield the results given in Table 29. They show that the capacity of a hook calculated according to the ordinary theory is in excess of its real strength. See also p. 761.

Table 29. Elastic Limit of Crane Hooks

A = area, sq. in. I = moment of inertia. l = distance from load line to gravity axis, in. y = distance from inner fiber to gravity axis, in.

Nominal capacity, tons	Material	Cross-section dimensions A	I	l	y	Load on hook at elastic limit, lb. per sq. in.	Elastic limit, lb. per sq. in.
30	C. steel	23.35	111.6	7.25	3.36	56,000	34,000
20	C. steel	14.48	.....	5.90	2.75	30,000	28,500
15	C. steel	13.92	.....	5.13	2.23	48,000	62,750
15	W. iron	8.40	11.9	5.00	1.87	16,000	30,000
10	C. steel	8.72	.....	4.30	2.05	18,000	27,500
10	W. iron	6.08	6.5	4.00	1.50	16,000	30,000
5	C. steel	5.69	.....	3.25	1.42	18,000	53,500
5	W. iron	4.80	3.8	3.47	1.35	14,000	30,000
3	C. steel	3.50	.....	2.89	1.16	8,500	40,800
2	C. steel	2.03	.....	2.03	0.88	4,700	45,900

REINFORCED-CONCRETE DESIGN

For reinforced-concrete construction and tables of dimensions of construction units, see pp. 1305 to 1316.

Standard Notation

RECTANGULAR BEAMS:

f_s = tensile unit stress in steel.
f_c = compressive unit stress in concrete.
E_s = modulus of elasticity of steel.
E_c = modulus of elasticity of concrete.
$n = E_s/E_c$.
M = moment of resistance, or bending moment in general.
A_s = steel area.
b = breadth of beam.
d = depth of beam to center of steel.
k = ratio of depth of neutral axis to effective depth d.
z = depth of resultant compression below top.
j = ratio of lever arm of resisting couple to depth d.
$jd = d - z$ = arm of resisting couple.
p = steel ratio (not percentage).
R = coefficient in formula $M = Rbd^2$.

T-BEAMS:

b = width of flange.
b' = width of stem.
t = thickness of flange.

BEAMS REINFORCED FOR COMPRESSION:

A' = area of compressive steel.
p' = steel ratio for compressive steel.
f'_s = compressive unit stress in steel.
C = total compressive stress in concrete.
C' = total compressive stress in steel.
d' = depth to center of compressive steel.
z = depth to resultant of C and C'.

SHEAR AND BOND:

V = total shear.
v = shearing unit stress.
u = bond stress per unit area of bar.
o = circumference or perimeter of bar.
Σo = sum of the perimeters of all bars.

COLUMNS:

A = total net area.
A_s = area of longitudinal steel.
A_c = area of concrete.
P = total safe load.

Reinforced Concrete Under Axial Stress

The following formulæ may be used for (*a*) columns with longitudinal steel only and (*b*) columns with longitudinal steel and circular hoops or spirals by adopting proper values of f_c, as suggested on p. 1306.

$$P = f_c(A_c + nA_s) = f_cA[1 + (n - 1)p] \qquad (1)$$

The spirals are not figured directly, but their effect is considered by increasing the value of f_c. In figuring the total net area A and the area of concrete A_c, the outside shell composing the fireproofing must not be included. For columns with spirals, A is the area of the concrete within the spirals.

For dimensions of round and square columns, see p. 1315.

Columns with Rigid Structural Shapes, when the cross-section of steel exceeds 4 per cent., may be considered as equal in strength to those of the structural shapes (figured according to usual formula for structural steel columns) plus the strength of the concrete within the structural shapes. The unit working stress for this concrete core may be accepted at 75 per cent. of f_c used for columns with longitudinal steel only. The outer shell which is required as fireproofing should not be considered as adding to the strength of the column.

Reinforced Concrete Under Flexure

Beams of concrete are reinforced in the tension flange to prevent failure in tension; and more rarely in the compression flange; and at the ends, either by stirrups, or by bending the longitudinal reinforcement, or both, to prevent failure by diagonal tension produced by the resultant of horizontal and vertical shear. (See p. 1309.) In the following formulæ certain assumptions are made in addition to those of the theory of flexure, namely, that the concrete adheres perfectly to the steel and carries no tensile stress, that the compressive stress-strain relation is rectilinear, and that there are no initial stresses.

Rectangular Beams (Fig. 1):
Position of neutral axis,

$$k = \sqrt{2pn + (pn)^2} - pn \tag{2}$$

Arm of resisting couple,

$$j = 1 - k/3 \tag{3}$$

[For f_s = 15,000 to 16,000 and f_c = 600 to 650, k may be taken at 3/8.]
Fiber stresses,

$$f_s = M_s/A_sjd = M_s/pjbd^2,\ A_s = M_s/f_sjd,\ d = \sqrt{M_s/f_spjb} \tag{4}$$

$$f_c = 2M_c/jkbd^2 = 2pf_s/k \tag{5}$$

Steel ratio, for balanced reinforcement,

$$p = \tfrac{1}{2} \times 1 \Big/ \frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right) \tag{6}$$

M_s = Moment of resistance in terms of steel stress $= f_sA_sjd$ (7)
M_c = Moment of resistance in terms of concrete stress $= \tfrac{1}{2}f_ckjbd^2$ (8)

For usual beams of 1 : 2 : 4 Portland cement concrete, the approximate value of j is 7/8, k is 3/8, and p is 0.75/100 to 1/100. The value of j varies from 0.9 at 0.5 per cent. reinforcement to 0.82 for 2 per cent. reinforcement. A given percentage of reinforcement fixes the ratio of f_s to f_c. In determining the sections, make the bending moment equal to the moments of resistance from formulæ (7) or (8). When the moments of resistance from the two formulæ are not equal, use the smaller value.

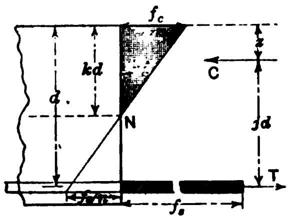


FIG. 1.

At early loads on reinforced-concrete beams rich concrete under tensile stress supplies nearly half the resisting moment. At a stress in the steel of 5000 lb. per sq. in. the concrete in tension begins to crack and throws more stress on the steel. The beam attains its ultimate strength when the yield point of the steel is reached. With weak concrete and a high percentage of steel, failure in compression may result. Deep beams with high shear may fail at ends, and show a diagonal crack.

T-beams. (Fig. 2.) When a slab joins a girder, a portion of the slab, of width not greater than one-fourth the span length of the beam and not overhanging the beam more than 4 times the thickness of the slab, is usually counted upon to withstand compression integrally with the girder, forming a "T" beam. Here $p = A_s/bd$. When the flange is thick ($t > d/4$) and the neutral axis lies in the flange, use the formulæ for rectangular beams. When the neutral axis is in the stem and the compression in stem is neglected, the following formulæ apply:

Table 1. Factors for Rectangular Reinforced-concrete Beams

(Portland cement concrete. $n = 15$)

f_c	f_s	p	k	j	$R = M/bd^2$
500	14,000	0.00623	0.349	0.884	77.1
500	16,000	0.00498	0.319	0.894	71.3
500	18,000	0.00408	0.294	0.902	66.3
500	20,000	0.00341	0.273	0.909	62.0
550	16,000	0.00585	0.340	0.887	83.0
550	18,000	0.00480	0.314	0.895	77.4
550	20,000	0.00402	0.292	0.903	72.7
600	16,000	0.00675	0.360	0.880	95.0
600	18,000	0.00555	0.333	0.889	88.8
600	20,000	0.00466	0.311	0.897	83.5
650	16,000	0.00768	0.378	0.874	107.3
650	18,000	0.00634	0.351	0.883	100.7
650	20,000	0.00533	0.328	0.891	95.0
700	16,000	0.00867	0.396	0.868	120.4
700	18,000	0.00718	0.369	0.877	113.3
700	20,000	0.00602	0.344	0.885	106.5
750	16,000	0.00968	0.413	0.862	133.3
750	18,000	0.00802	0.385	0.872	125.8
750	20,000	0.00675	0.360	0.880	118.7
800	16,000	0.01072	0.429	0.857	147.0
800	18,000	0.00889	0.400	0.867	138.7
800	20,000	0.00750	0.375	0.875	131.2

Position of neutral axis,

$$kd = (2ndA_s + bt^2)/(2nA_s + 2bt) \tag{9}$$

Position of resultant compression,

$$z = (t/3) \times (3kd - 2t)/(2kd - t) \tag{10}$$

Arm of resisting couple,

$$jd = d - z. \tag{11}$$

Fiber stresses,

$$f_s = M/A_s jd \tag{12}$$

$$f_c = Mkd/[bt(kd - \tfrac{1}{2}t)jd] = (f_s/n) \times k/(1 - k) \tag{13}$$

(For approximate results the formulæ for rectangular beams may be used.)

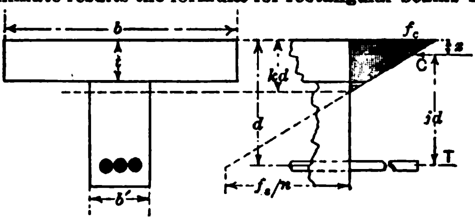


FIG. 2.

The following formulæ take into account the compression in the stem; they are recommended where the flange is small compared with the stem:

Position of neutral axis,

$$kd = \sqrt{\frac{2ndA_s + (b - b')t^2}{b'} + \left(\frac{nA_s + (b - b')t}{b'}\right)^2} - \frac{nA_s + (b - b')t}{b'} \tag{14}$$

Position of resultant compression,

$$z = \frac{(kdt^2 - \frac{2}{3}t^3)b + \{(kd - t)^2[t + \frac{1}{3}(kd - t)]\}b'}{t(2kd - t)b + (kd - t)^2b'} \tag{15}$$

Arm of resisting couple,

$$jd = d - z. \tag{16}$$

Fiber stresses,

$$f_s = M/A_sjd \tag{17}$$

$$f_c = 2Mkd/\{[(2kd - t)bt + (kd - t)^2b']jd\} \tag{18}$$

Shearing stresses must be considered in the webs of T-beams, and compressive stresses in stems of continuous T-beams.

For dimensions of T-beam for uniform loading, see p. 1311.

Beams Reinforced for Compression (Fig. 3):

Position of neutral axis,

$$k = \sqrt{2n[p + p'(d'/d)] + n^2(p + p')^2} - n(p + p') \tag{19}$$

Position of resultant compression,

$$z = \frac{\frac{1}{3}k^3d + 2p'nd'[k - (d'/d)]}{k^2 + 2p'n[k - (d'/d)]} \tag{20}$$

Arm of resisting couple,

$$jd = d - z. \tag{21}$$

Fiber stresses,

$$f_c = 6M/bd^2 \left[3k - k^2 + \frac{6p'n}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)\right] \tag{22}$$

$$f_s = M/pjbd^2 = nf_c(1 - k)/k \tag{23}$$

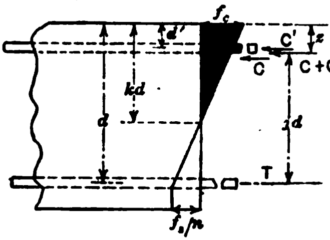


FIG. 3.

$$f'_s = nf_c[k - (d'/d)]/k \tag{24}$$

Bond Stress Between Steel and Concrete, and Shearing Stress. Let V = total vertical shear, u = bond stress per sq. in. between rod and concrete, v = maximum unit shearing stress, o = perimeter of bar, Σo = sum of perimeters of all bars constituting tension reinforcement. Then

For rectangular beams, $v = V/bjd$ (25)

For T-beams, $v = V/b'jd$ (26)

For rectangular and T-beams, $u = V/jd\Sigma o$ (27)

Stresses in Stirrups. A vertical stirrup is computed to carry a load equal to a portion of the vertical shear per sq. in. times the width of the beam times the spacing of the stirrups. The portion of the shear carried by the stirrup is taken to be the shear in excess of 40 lb. per sq. in. (for 1:2:4 concrete).

$$P = mvsb = \frac{mVsb}{\frac{7}{8}db} = \frac{mVs}{\frac{7}{8}d} \tag{28}$$

where P = total load on stirrup, s = horizontal spacing, v = unit shearing stress, V = total vertical shear in beam, and m = fractional part of shear carried on stirrup. The amount of shear carried by inclined stirrups or bent rods is

$$P = 0.7mVs/\tfrac{7}{8}d \tag{29}$$

and

$$\text{the area of cross-section of stirrup} = P/2f_s \tag{30}$$

m may be taken as ⅔, thus allowing ⅓ of the shear to be carried in the concrete.

Deflection of Reinforced-concrete Beams. Tests by Humphrey and Losse (Tech. Papers, Bureau of Standards, No. 2) show that centrally loaded and freely supported beams of 1:2:4 concrete with a stress of 16,000 lb. per sq. in. in steel deflect 1/1000 part of the span, and 1/720 part of the span for 0.5 per cent. and 1.6 per cent. reinforcement, respectively. Tests of buildings designed under the Chicago Ordinance with an imposed load of twice the load for which the buildings were designed, yielded a deflection of 1/800 span for the beams and 1/4000 span for the girders. The measured stress in the steel varied from 8000 to 11,000 lb. per sq. in. Here the concrete adjoining the member under test assists in carrying the load through arch action and slab action.

Flat-slab Floors (Fig. 4) are without beams or girders. They are continuous from one panel to another and are usually supported on enlarged column heads. The system of reinforcement may be either four-way or two-way. A **four-way reinforcement** system consists of two diagonal bands and two cross bands. **Two-way reinforcement** runs crossways from column to column, in bands in which the reinforcement varies in quantity, and in other bands which run at right angles to these. Tests of completed structures show that the greatest moment is over the column capital, and the least in the center of the panel. The circle of inflection is distant approximately ¼ of the span from center to center of columns. The greater moment over the column head is best met by increasing the thickness of the slab around the column. The following proportions for column heads will yield conservative designs (see *Eng. News*, Sept. 24, 1914):

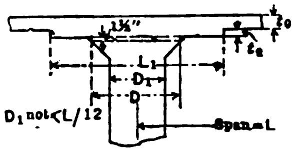


FIG. 4.

$$t_0 \geq L/32 \qquad t_2 \leq 0.4t_0$$
$$D = 0.225L \qquad L_1 \geq 0.4L$$

where L = span center to center of columns along side of panel; t_0 = thickness of slab at center; t_2 = increased thickness of slab; L_1 = width of band, D = diam. of column capital measured where its vertical thickness is at least 1½ in. (Fig. 4). The flare of the column capital should not make an angle greater than 45 deg. with the vertical.

Tests of completed buildings show that conservative design results when the following external bending moments are provided for in case of square panels of four-way systems:

Moment over column = $WL/15$; moment in center of panel = $WL/25$, where W = total live and dead load on panel, and L = span between column centers along side of panel. The reinforcing steel supplying these moments is divided into four bands. It would be theoretically possible to so increase the thickness of the slab over the column that the same steel would meet both $WL/15$ and $WL/25$. However, (1) the crossing of four layers of rods over the column cap cuts down the effective depth, and (2) it is necessary to lap some of the rods over the column. The latter rods when computed as adding to the reinforcement should lap over into the next panel to nearly the ¼ point. The rods over the column should continue in the top of the slab to nearly the ¼ point of the span. An example will explain the method of design.

For dimensions of slabs for different spans and loads see p. 1308.

Design of Flat-slab Square Panel. Example. Span $L = 20$ ft. Total load = 360 lb. per sq. ft. (= 250 lb. live load + 110 lb. dead load). Assume $f_s = 18{,}000$ and $f_c = 700$, whence $R = 113$. To obtain the approximate depth d of slab, employ the usual beam formula $M = Rbd^2$, in which $M = WL/25 = [(360 \times 20) \times 20 \times 12/25] = 69{,}120$ in.-lb. for a beam 12 in. wide and of 20 ft. span; whence $d = \sqrt{M/Rb} = \sqrt{69{,}120/(113 \times 12)} = 7.15$ in. Adding 0.75 in. for steel protection, $d = 7.9$ in., say, an 8-in. slab.

To Design Steel at Center: Load moment $= WL/25 = 360 \times 20^3 \times 12/25 = 1{,}382{,}400$ in.-lb. Area of steel [from (7)] $= A_s = M/f_s jd$, or, taking $j = 7/8$ (approx.), $A_s = 1{,}382{,}400/(18{,}000 \times 7/8 \times 7.15) = 12$ sq. in. ∴ Steel in each of the four bands $= 12/4 = 3$ sq. in., or area of sixteen ½-in. round rods (3.14 sq. in.). Width of band $= 0.4L = 8$ ft. $= 96$ in. Spacing of rods $= 96/(16 - 1) = 6.4$ in. center to center.

To Design Steel over Column. Thickness of slab over column $= t_3 = t_0 + 0.4t_0 = 8 + (0.4 \times 8) = 11.2$ in., say 11.5 in. Allowing ½ in. of concrete over the steel, the distance to the center of gravity of the four layers of rods is $11.5 - (½ + 1) = 10$ in. Load moment over column $= WL/15 = 360 \times 20^3 \times 12/15 = 2{,}304{,}000$ in.-lb. Steel required in 4 bands over column $= 2{,}304{,}000/(7/8 \times 18{,}000 \times 10) = 14.63$ sq. in. Steel at center of panel $= 4 \times 3.14 = 12.56$ sq. in. Additional steel required over column $= 14.63 - 12.56 = 2.07$ sq. in. To obtain this, lap 10 rods. The rods should be bent from bottom to top of slab near the point of inflection.

To obtain the **compressive stresses**, assume that all the steel is in one band, and that the components of two diagonal bands in this direction act on a plane tangent to the edge of the column cap. Thus, in the example, the steel acting to compress the concrete in a direction at right angles to any band has a cross-section of $3.65 + (2 \times 3.65 \cos 45 \text{ deg.}) = 3.65 + (3.65 \times 1.4) = 8.76$ sq. in., which at 18,000 lb. stress $= 157{,}680$ lb. This pull acts on an area equal to the width of the band times the depth to the neutral axis $= 96 \times 3/8 \times 10 = 360$ sq. in., and the maximum stress $f_c = 2 \times 157{,}680/360 = 876$ lb. per sq. in. In slabs of uniform thickness the width of compression area may be taken as the width of band plus three times the thickness of the slab.

Reinforcement for shrinkage placed in the top of the slab throughout the central portion of the area directly between columns inside the lines of inflection, will serve also to stiffen the slab and carry eccentric loads to adjoining panels. Rectangular panels, in which the long side is not more than ⅓ greater than the short side, may be designed by the following rule: Design a square panel whose diagonal equals that of the rectangular panel. The amount of steel in the long side of a rectangular panel equals that in the side of the square panel times L_1^3/L_0^3, and the amount of steel in short side of rectangular panel equals that in the side of square panel times L_0^3/L_2^3, where L_0 = side of equivalent square, L_1 = long side of rectangular panel, and L_2 = short side of rectangular panel. In no case should the amount of the steel in the short side be less than two-thirds of that required for the long side.

Coefficients for bending moments at the middle of **wall panels** should be increased 20 per cent. over those required for interior panels. **Shearing stresses** around column capital and other critical sections should not exceed 120 lb. per sq. in.

SECTION 6

MATERIALS OF ENGINEERING

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CONTENTS

MATERIALS OF ENGINEERING

GENERAL PROPERTIES OF MATERIALS

BY

LOUIS A. FISCHER

CHEMISTRY

A **substance** which cannot be separated into two or more recognizably different substances by any physical means, is known as a **chemically pure substance.** A chemically pure substance is considered to consist of a great number of very small particles, all exactly alike, known as **molecules.** If a chemically pure substance cannot by any available means be separated into two or more different substances, it is known as a **chemical element.** Chemically pure substances which can be separated or decomposed into different elements by chemical means, or which can be made by combining different elements, are known as **chemical compounds.**

The molecules are aggregates of still smaller particles. These smaller particles are known as **atoms,** and the atoms of a given element are regarded as all precisely alike in every respect. The atoms are thus to be considered as the smallest mass-elements which occur separately in the structure of molecules of either compound or elementary substances, so far as can be determined by ordinary chemical analysis. The molecule of an element consists of a definite (usually small) number of its atoms. The molecule of a compound consists of one or more atoms of each of its several elements, the members of the various kinds of atoms and their arrangement being definite and fixed, and determining the character of the compound. This notion of molecules and their constituent atoms is useful for interpreting the observed fact that chemical reactions—*e.g.*, the analysis of a compound into its elements; the synthesis of a compound from the elements, or the changing of one or more compounds into one or more different compounds—take place so that the masses of the various substances concerned in a given reaction stand in definite and fixed ratios.

It appears from recent researches that some substances which cannot by any available means be decomposed into simpler substances and which must therefore be defined as elements, are continually undergoing spontaneous changes or **radioactive transformation** into other substances which can be recognized as physically and chemically different from the original substance. Radium is an element by the definition given and may be considered as made up of atoms. But it is assumed that these atoms, so called because they resist all efforts to break them up and are therefore apparently indivisible, nevertheless split up spontaneously, at a rate which chemists have not been able to influence in any way, into other atoms, thus forming other elementary substances of totally different properties.

The view generally accepted at present is that the atoms of all the chemical elements, including those not yet known to be subject to radioactive transformation, consist of aggregates of large numbers of still smaller particles known as corpuscles or **electrons;** and that these electrons are all alike, the differences between the atoms of different chemical elements being due to the different numbers and arrangements of these smaller particles composing them. An ordinary atom is conceived to be a stable system of such electrons revolving in closed orbits about their common center of mass like the planets of the solar system round the sun. The number of these electrons is, however, very great, about 2000 in one hydrogen atom and about 450,000 in one radium atom. The larger the number of electrons forming a given independent and self-contained system (atom), the greater is the probability that accidental perturbations of the orbits will result in momentary instability, the ejection of one or more electrons from the system and a con-

sequent rearrangement of the remainder into a new stable state of steady motion. This working hypothesis agrees with the fact that the heaviest known atoms—those of uranium, thorium and radium—are the most subject to radioactive transformation, and with a number of other physical observations, but should not at present be regarded as more than a convenient scheme for co-ordinating a number of phenomena about which little is known.

Chemical Elements[1]

Element	Symbol	Atomic weight	Valence	Element	Symbol	Atomic weight	Valence
Aluminum	Al	27.1	3	Molybdenum	Mo	96.0	3, 4, 5, 6, 8
Antimony	Sb	120.2	3, 5	Neodymium	Nd	144.3	3
Argon[2]	A	39.88	0	Neon[2]	Ne	20.2	0
Arsenic[3]	As	74.96	3, 5	Nickel	Ni	58.68	2, 3, 4
Barium	Ba	137.37	2	Niton[9] (radium emanation)	Nt	224.4	....
Bismuth	Bi	208.0	3, 5	Nitrogen[5]	N	14.01	3, 5
Boron[3]	B	11.0	3	Osmium	Os	190.9	2, 3, 4, 8
Bromine[4]	Br	79.92	1, 3, 5	Oxygen[5]	O	16.00	2, 4, 6
Cadmium	Cd	112.4	2	Palladium	Pd	106.7	2, 4
Caesium	Cs	132.81	1	Phosphorus[3]	P	31.04	3, 5
Calcium	Ca	40.07	2	Platinum	Pt	195.2	2, 4
Carbon[3]	C	12.0	4	Potassium	K	39.10	1
Cerium	Ce	140.25	3, 4, 6	Praseodymium	Pr	140.6	3
Chlorine[5]	Cl	35.46	1, 3, 5, 7	Radium	Ra	226.4	2
Chromium	Cr	52.0	2, 3, 6	Rhodium	Rh	102.9	3
Cobalt	Co	58.97	2, 3	Rubinium	Rb	85.45	1
Columbium (Niobium)	Cb	93.5	2, 3, 4, 5	Ruthenium	Ru	101.7	2, 4, 8
Copper	Cu	63.57	1, 2	Samarium	Sa	150.4	3
Dysprosium	Dy	162.5	....	Scandium	Sc	44.1	3
Erbium	Er	167.7	3	Selenium[3]	Se	79.2	2, 4, 6
Europium	Eu	152.0	....	Silicon[3]	Si	28.3	4
Fluorine[6]	F	19.0	1	Silver	Ag	107.88	1
Gadolinium	Gd	157.3	3	Sodium	Na	23.00	1
Gallium	Ga	69.9	2, 3	Strontium	Sr	87.63	2
Germanium	Ge	72.5	2, 4	Sulphur[3]	S	32.07	2, 4, 6
Glucinum (Beryllium)	Gl	9.1	2	Tantalum	Ta	181.5	4, 5
Gold	Au	197.2	1, 3	Tellurium[3]	Te	127.5	2, 4, 6
Helium[2]	He	3.99	0	Terbium	Tb	159.2	....
Holmium[9]	Ho	163.5	....	Thallium	Tl	204.0	1, 3
Hydrogen[7]	H	1.008	1	Thorium	Th	232.4	4
Indium	In	114.8	1, 2, 3	Thulium	Tm	168.5	....
Iodine[3]	I	126.92	1, 3, 5, 7	Tin	Sn	119.0	2, 4
Iridium	Ir	193.1	2, 3, 4	Titanium	Ti	48.1	2, 3, 4, 6
Iron	Fe	55.84	2, 3, 6	Tungsten	W	184.0	4, 5, 6
Krypton[2]	Kr	82.92	0	Uranium	U	238.5	4, 6, 8
Lanthanum	La	139.0	2	Vanadium	V	51.0	1, 2, 3, 4, 5
Lead	Pb	207.10	2, 4	Xenon[2]	Xe	130.2	0
Lithium[8]	Li	6.94	1	Ytterbium (Neoytterbium)	Yb	172.0	3
Lutecium	Lu	174.0	....	Yttrium	Yt	89.0	3
Magnesium	Mg	24.32	2	Zinc	Zn	65.37	2
Manganese	Mn	54.93	2, 3, 4, 6, 7	Zirconium	Zr	90.6	4
Mercury	Hg	200.6	1, 2	....			

[1] All the elements are metals, except as otherwise indicated.
[2] Inert gas. [3] Metalloid. [4] Liquid. [5] Gas.
[6] Most active gas. [7] Lightest gas. [8] Lightest metal. [9] Not placed.

Calculation of the Percentage Composition of Substances. Add together the atomic weights of the elements in the compound to obtain its molecular weight. Multiply the atomic weight of the element to be calculated by the number of atoms present (indicated in the formula by a subscript number) and by 100, and divide by the molecular weight of the compound. For example, hematite iron ore (Fe_2O_3) contains 69.94 per cent. of iron by weight, determined as follows: Molecular weight of $Fe_2O_3 = (55.84 \times 2) + (16 \times 3) = 159.68$. Percentage of iron in compound $= (55.84 \times 2) \times 100/159.68 = 69.94$.

Solubility of Inorganic Salts in Water

(Number of grams of the anhydrous salt soluble in 1000 grams of water)

Salt	Composition	Temperature, deg. cent. 0	50	100
Aluminum sulphate	$Al_2(SO_4)_3$	313	521	891
Potassium alum	$Al_2K_2(SO_4)_4$	30	....	1540
Barium chloride	$BaCl_2$	316	436	588
Barium nitrate	$Ba(NO_3)_2$	50	171	342
Calcium chloride	$CaCl_2$	595	....	1590
Copper nitrate	$Cu(NO_3)_2$	818	....	....
Copper sulphate	$CuSO_4$	149	336	735
Ferrous chloride	$FeCl_2$	....	820	1060
Ferrous sulphate	Fe_2SO_4	156	486	....
Potassium carbonate	K_2CO_3	1050	1210	1560
Potassium chloride	KCl	285	429	566
Potassium nitrate	KNO_3	133	855	2460
Potassium sulphate	K_2SO_4	74	165	241
Magnesium chloride	$MgCl_2$	528	....	730
Magnesium sulphate $(H_2O)_6$	$MgSO_4$ (6 aq)	408	504	738
Ammonium chloride	NH_4Cl	297	504	773
Ammonium bicarbonate	NH_4HCO_3	119	....	....
Ammonium nitrate	NH_4NO_3	1183	3540	8710
Ammonium sulphate	$(NH_4)_2SO_4$	706	844	1033
Sodium carbonate $(H_2O)_7$	Na_2CO_3 (7 aq)	204	475	452
Sodium chloride	$NaCl$	356	367	391
Sodium bicarbonate	$NaHCO_3$	69	145	....
Sodium nitrate	$NaNO_3$	730	1140	1755
Sodium hydroxide	$NaOH$	420	1450	....
Sodium sulphate $(H_2O)_{10}$	Na_2SO_4(10 aq)	50	468	427
Nickel chloride	$NiCl_2$	....	760	....
Nickel sulphate	$NiSO_4$	272	502	776
Lead nitrate	$Pb(NO_3)_2$	365	787	1270
Strontium nitrate	$Sr(NO_3)_2$	395	926	1011
Zinc nitrate	$Zn(NO_3)_2$	948	....	....
Zinc sulphate	$ZnSO_4$	....	768	785

Density of Water at Atmospheric Pressure (in grams per milliliter)

Temp., deg. cent.	Density	Temp., deg. cent.	Density	Temp., deg. cent.	Density	Temp., deg. cent.	Density	Temp., deg. cent.	Density	Temp., deg. cent.	Density
0	0.99987	10	0.99973	20	0.99823	30	0.99567	40	0.99224	50	0.98807
1	0.99993	11	0.99963	21	0.99802	31	0.99537	41	0.99186	51	0.98762
2	0.99997	12	0.99952	22	0.99780	32	0.99505	42	0.99147	52	0.98715
3	0.99999	13	0.99940	23	0.99756	33	0.99473	43	0.99107	53	0.98669
4	1.00000	14	0,99927	24	0.99732	34	0.99440	44	0.99066	54	0.98621
5	0.99999	15	0.99913	25	0.99707	35	0.99406	45	0.99024	55	0.98573
6	0.99997	16	0.99897	26	0.99681	36	0.99371	46	0.98982	56	0.98524
7	0.99993	17	0.99880	27	0.99654	37	0.99336	47	0.98940	57	0.98478
8	0.99988	18	0.99862	28	0.99626	38	0.99299	48	0.98896	58	0.98425
9	0.99981	19	0.99843	29	0.99597	39	0.99262	49	0.98852	59	0.98375

Approximate Specific Gravities and Weights

(Water at 4 deg. cent. and normal atmospheric pressure taken as unity)

For more detailed data on any material, see the section dealing with the properties of that material.

Substance	Specific gravity	Avg. weight, pounds per cu. ft.
Metals, Alloys, Ores		
Aluminum, cast-hammered	2.55–2.80	165
Aluminum, bronze	7.7	481
Brass, cast-rolled	8.4–8.7	534
Bronze, 7.9 to 14% Sn	7.4–8.9	509
Bronze, phosphor	8.88	554
Copper, cast-rolled	8.8–9.0	556
Copper ore, pyrites	4.1–4.3	262
German silver	8.58	536
Gold, cast-hammered	19.25–19.35	1205
Gold coin (U. S.)	17.18–17.2	1073
Iridium	21.78–22.42	1383
Iron, gray cast	7.03–7.13	442
Iron, cast, pig	7.2	450
Iron, wrought	7.6–7.9	485
Iron, spiegel-eisen	7.5	468
Iron, ferro-silicon	6.7–7.3	437
Iron ore, hematite	5.2	325
Iron ore, limonite	3.6–4.0	237
Iron ore, magnetite	4.9–5.2	315
Iron slag	2.5–.30	172
Lead	11.34	710
Lead ore, galena	7.3–7.6	465
Manganese	7.2–8.0	475
Manganese ore, pyrolusite	3.7–4.6	259
Mercury	13.6	849
Nickel	8.30–8.90	537
Nickel monel metal	8.8–9.0	556
Platinum, cast-hammered	21.2–21.7	1330
Silver, cast-hammered	10.4–10.6	656
Steel, cold-drawn	7.83	489
Steel, machine	7.80	487
Steel, tool	7.70–7.73	481
Tin, cast-hammered	7.2–7.5	459
Tin ore, cassiterite	6.4–7.0	418
Tungsten	19.22	1200
Zinc, cast-rolled	6.9–7.2	440
Zinc ore, blende	3.9–4.2	253
Various Solids		
Cereals, oats, bulk	0.51	26
Cereals, barley, bulk	0.62	39
Cereals, corn, rye, bulk	0.73	45
Cereals, wheat, bulk	0.77	48
Cork	0.22–0.26	15
Cotton, flax, hemp	1.47–1.50	93
Fats	0.90–0.97	58
Flour, loose	0.40–0.50	28
Flour, pressed	0.70–0.80	47
Glass, common	2.40–2.80	162
Glass, plate or crown	2.45–2.72	161
Glass, crystal	2.90–3.00	184
Glass, flint	3.2–4.7	247
Hay and straw, bales	0.32	20
Leather	0.86–1.02	59

Substance	Specific gravity	Avg. weight, pounds per cu. ft.
Paper	0.70–1.15	58
Potatoes, piled	0.67	42
Rubber, caoutchouc	0.92–0.96	59
Rubber goods	1.0–2.0	94
Salt, granulated, piled	0.77	48
Saltpeter	1.07	67
Starch	1.53	96
Sulphur	1.93–2.07	125
Wool	1.32	82
Timber, U. S. Seasoned (Moisture, 15 to 20%)		
Apple	0.66–0.74	44
Ash, white-red	0.62–0.65	40
Birch	0.51	32
Cedar, white-red	0.32–0.38	22
Cherry	0.70	44
Chestnut	0.66	41
Cypress	0.48	30
Fir, Douglas spruce	0.51	32
Fir, eastern	0.40	25
Elm, white	0.72	45
Hemlock	0.42–0.52	29
Hickory	0.60–0.93	48
Locust	0.67–0.73	46
Mahogany	0.56–0.85	44
Maple, hard	0.68	43
Maple, white	0.53	33
Oak, chestnut	0.86	54
Oak, live	0.95	59
Oak, red, black	0.65	41
Oak, white	0.74	46
Pine, Oregon	0.51	32
Pine, red	0.48	30
Pine, white	0.41	26
Pine, yellow, long-leaf	0.70	44
Pine, yellow, short-leaf	0.61	38
Poplar	0.35–0.50	27
Redwood, California	0.42	26
Spruce, white, black	0.40–0.46	27
Teak, African	0.98	62
Teak, Indian	0.66–0.88	48
Walnut, black	0.64–0.70	42
Walnut, white	0.41	26
Willow	0.40–0.60	31
Various Liquids		
Alcohol, ethyl (100%)	0.789	49
Alcohol, methyl (100%)	0.796	50
Acid, muriatic, 40%	1.20	75
Acid, nitric, 91%	1.50	94
Acid, sulphuric, 87%	1.80	112
Chloroform	1.526	95
Ether	0.736	46
Lye, soda, 66%	1.70	106
Oils, vegetable	0.91–0.94	58
Oils, mineral, lubricants	0.90–0.93	57
Turpentine	0.861–0.867	54

Approximate Specific Gravities and Weights—(*continued*)

Substance	Specific gravity	Avg. weight, pounds per cu. ft.
Various Liquids		
Water, 4° C., max. density	1.0	62.428
Water, 100° C.	0.9584	59.830
Water, ice	0.88–0.92	56
Water, snow, fresh fallen	0.125	8
Water, sea water	1.02–1.03	64
Gases, see p. 365.		
Ashlar Masonry		
Granite, syenite, gneiss	2.3–3.0	165
Limestone, marble	2.3–2.8	160
Sandstone, bluestone	2.1–2.4	140
Mortar Rubble Masonry		
Granite, syenite, gneiss	2.2–2.8	155
Limestone, marble	2.2–2.6	150
Sandstone, bluestone	2.0–2.2	130
Dry Rubble Masonry		
Granite, syenite, gneiss	1.9–2.3	130
Limestone, marble	1.9–2.1	125
Sandstone, bluestone	1.8–1.9	110
Brick Masonry		
Pressed brick	2.2–2.3	140
Common brick	1.8–2.0	120
Soft brick	1.5–1.7	100
Concrete Masonry		
Cement, stone, sand	2.2–2.4	144
Cement, slag, etc.	1.9–2.3	130
Cement, cinder, etc.	1.5–1.7	100
Various Building Mat'ls		
Ashes, cinders	0.64–0.72	40–45
Cement, Portland, loose	1.44	90
Cement, Portland, set	2.7–3.2	183
Lime, gypsum, loose	0.85–1.00	53–64
Mortar, set	1.4–1.9	103
Slags, bank slag	1.1–1.2	67–72
Slags, bank screenings	1.5–1.9	98–117
Slags, machine slag	1.5	96
Slags, slag sand	0.8–0.9	49–55
Earth, etc., Excavated		
Clay, dry	1.0	63
Clay, damp, plastic	1.76	110
Clay and gravel, dry	1.6	100
Earth, dry, loose	1.2	76
Earth, dry, packed	1.5	95
Earth, moist, loose	1.3	78
Earth, moist, packed	1.6	96
Earth, mud, flowing	1.7	108
Earth, mud, packed	1.8	115
Riprap, limestone	1.3–1.4	80–85
Riprap, sandstone	1.4	90
Riprap, shale	1.7	105
Sand, gravel, dry, loose	1.4–1.7	90–105
Sand, gravel, dry, packed	1.6–1.9	100–120
Sand, gravel, wet	1.9	118–120
Excavations in Water		
Sand or gravel	0.96	60
Sand or gravel and clay	1.00	65
Clay	1.28	80

Substance	Specific gravity
River mud	1.44
Soil	1.12
Stone riprap	1.00
Minerals	
Asbestos	2.1–2.8
Barytes	4.50
Basalt	2.7–3.2
Bauxite	2.55
Borax	1.7–1.8
Chalk	1.8–2.6
Clay, marl	1.8–2.6
Dolomite	2.9
Feldspar, orthoclase	2.5–2.6
Gneiss, serpentine	2.4–2.7
Granite, syenite	2.5–3.1
Greenstone, trap	2.8–3.2
Gypsum, alabaster	2.3–2.8
Hornblende	3.0
Limestone	2.46–2.86
Marble	2.5–2.8
Magnesite	3.0
Phosphate rock, apatite	3.2
Porphyry	2.6–2.9
Pumice, natural	0.37–0.90
Quartz, flint	2.5–2.8
Sandstone, bluestone	2.2–2.5
Shale, slate	2.6–2.9
Soapstone, talc	2.6–2.8
Stone, Quarried, Piled	
Basalt, granite, gneiss	1.5
Limestone, marble, quartz	1.5
Sandstone	1.3
Shale	1.5
Greenstone, hornblende	1.7
Bituminous Substances	
Asphaltum	1.1–1.5
Coal, anthracite	1.4–1.8
Coal, bituminous	1.2–1.5
Coal, lignite	1.1–1.4
Coal, peat, turf, dry	0.65–0.85
Coal, charcoal, pine	0.28–0.44
Coal, charcoal, oak	0.47–0.57
Coal, coke	1.0–1.4
Graphite	1.9–2.3
Paraffin	0.87–0.91
Petroleum	0.87
Petroleum, refined (kerosene)	0.78–0.82
Petroleum, benzine	0.73–0.75
Petroleum, gasoline	0.70–0.75
Pitch	1.07–1.15
Tar, bituminous	1.20
Coal and Coke, Piled	
Coal, anthracite	0.75–0.93
Coal, bituminous, lignite	0.64–0.87
Coal, peat, turf	0.32–0.42
Coal, charcoal	0.16–0.23
Coal, coke	0.37–0.51

Compressibility of Metals

Metal	Coefficient of compressibility per unit volume per			Metal	Coefficient of compressibility per unit volume per		
	Kilogram per sq. cm. $\times 10^6$	Atmosphere $\times 10^6$	Pound per sq. in. $\times 10^6$		Kilogram per sq. cm. $\times 10^6$	Atmosphere $\times 10^6$	Pound per sq. in. $\times 10^6$
Aluminum	1.4	1.3	0.09	Lead	2.5	2.3	0.16
Bismuth	3.0	2.7	0.19	Magnesium	3.0	2.7	0.19
Brass	1.0	0.9	0.06	Manganin	0.8	0.7	0.05
Bronze	1.1	1.1	0.08	Nickel	0.6	0.5	0.04
Cadmium	2.0	1.8	0.12	Palladium	0.6	0.5	0.04
Constantan	0.6	0.5	0.04	Platinum	0.4	0.4	0.02
Copper	0.8	0.7	0.05	Quicksilver	3.8	3.5	0.24
Gold	0.7	0.6	0.04	Silver	1.0	0.9	0.06
Iron, gray cast	1.3	1.2	0.08	Tin	1.9	1.7	0.12
Steel	0.6	0.5	0.04	Zinc	1.2	1.1	0.08
Wrought	0.6	0.5	0.04				

Compressibility of Liquids

If v_1 and v_2 are the volumes of the liquids at pressures of p_1 and p_2 atmospheres respectively and at a temperature of t deg. cent., the coefficient of compressibility b is given by the equation

$$b = \frac{1}{v_1} \times \frac{v_1 - v_2}{p_1 - p_2}.$$

Values of b for water are given in the following table:

Coefficient of Compressibility of Water

(Values of $b \times 10^7$)

Temp., deg. cent.	Pressure range in atmospheres							
	1-25	1-100	1-500	500-1000	1000-1500	1500-2000	2000-2500	2500-3000
0	525	511	475	416	358	324	292	261
20	491	468	434	....	338	....	....	....
50	......	449	416	....	325	....	....	254

The value of $b \times 10^6$ for oils at low pressures at about 20 deg. cent. varies from about 55 to 80; for mercury at 0 deg. cent. it is 3.9; for chloroform at 0 deg. cent. it is 100 and increases with the temperature to 200 at 60 deg. cent.; for ethyl alcohol it increases from about 100 at 0 deg. cent. and low pressures to 125 at 40 deg.; for glycerine it is about 24 at room temperature and low pressure.

Surface Tension of Liquids

Liquid	Surface tension (in dynes per cm.) of liquid in contact with		
	Air	Water	Mercury
Water	75.0	0.0	392
Mercury	513.0	392.0	0
Bisulphide of carbon	30.5	41.7	387
Chloroform	31.8	26.8	415
Ethyl alcohol	24.1	....	364
Olive oil	34.6	18.6	317
Turpentine	28.8	11.5	241
Petroleum	29.7	28.9	271
Hydrochloric acid	72.9	....	392

IRON AND STEEL

BY

H. M. BOYLSTON AND H. V. WILLE

REFERENCES: Thomas Turner, "The Metallurgy of Iron," Lippincott. Harbord and Hall, "The Metallurgy of Steel," Griffin & Co. Campbell, "The Manufacture and Properties of Iron and Steel," McGraw-Hill. Becker, "High Speed Steel," McGraw-Hill. Sauveur, "The Metallography and Heat-Treatment of Iron and Steel," Sauveur & Boylston. Brearley, "The Heat Treatment of Tool Steel," Longmans, and "The Case-hardening of Steel," Van Nostrand.

CLASSIFICATION OF IRON AND STEEL

At the Brussels Congress of the International Association for Testing Materials held in 1906, the following **definitions** of the most important forms of iron and steel were adopted. The definitions of steel and of wrought iron are those proposed to the New York Congress (1912) by the International Committee on Nomenclature.

Steel of Fluid Origin is iron cast from a molten state into a mass which is usefully malleable (initially, at least) in some one range of temperature. Such metal is steel whether it can be hardened or not, whether it contains much or little or even no carbon, and for that matter even if it is chemically pure iron. These steels have been divided into: (*a*) **Ingot iron,** or steel with too little carbon to harden usefully on rapid cooling. The name is misleading but is used by some as a trade name; and (*b*) **ingot steel,** or steel with enough carbon (say, 0.30 per cent. or more) to harden usefully on rapid cooling. Steel made by melting in a crucible is called "crucible steel;" that made in an electric furnace is called "electric steel."

Steel of Plastic Origin is iron which is aggregated from pasty particles without subsequent fusion; is malleable at least in some one range of temperature; and contains enough carbon (say, 0.30 per cent. or more) to harden usefully on rapid cooling from above its critical range. Blister steel and its derivatives and a few other high-carbon steels which are unimportant to-day, are the only present steels covered by this definition.

Blister Bar, Cement Bar, Converted Bar, are names given to steel of plastic origin made by cementing wrought iron with carbonaceous matter; also, commercially, to such steel when heated and worked into merchant sizes. Most writers have used "blister steel" in the former and broader sense. In Sheffield it is used solely in the latter and narrower sense. The blister steel of commerce is made by cementing very pure wrought iron with charcoal. The term "cemented bar" may be applied in a general sense to any wrought-iron bar which has been subjected to a process of cementation.

Blister Steel (or Converted Steel) is the name given to bars rolled or forged from blister bar. Many writers have used "blister steel" in the sense of "blister bar."

Plated Bar is blister steel in the form of bars which have been rolled or hammered while hot. This treatment, which is usually applied to such bars broken to convenient lengths, flattens down their blisters and toughens the metal somewhat.

Single-shear Steel is shear steel made by welding a pile of plated bars into a fagot. The name applies also to the bars and other merchant shapes made by rolling or hammering such a fagot.

Double-shear Steel is shear steel made by piling, hammering, and thus welding bars of single-shear steel into a bloom. The name applies also to the bars and other merchant shapes made by rolling or hammering such a bloom.

Carbon Steel is steel which owes its distinctive properties chiefly to the carbon (as distinguished from the other elements) which it contains. Though among the alloy steels some are but moderately malleable, among the carbon steels industrial usage confirms the name "steel" to products malleable enough to be rolled or forged into merchant shapes.

Alloy Steel is steel which owes its distinctive properties chiefly to some element or elements other than carbon, or jointly to such other elements and carbon. Some of the alloy steels necessarily contain an important percentage of carbon, even as much as 1.25 per cent. There is no agreement as to where the line between the alloy steels and the carbon steels shall be drawn.

Semi-steel is a vague trade name for various products near the border line between steel and cast iron.

Bessemer Steel, Crucible (or Cast) Steel, and Open-hearth Steel, are steels made by the Bessemer, crucible and open-hearth processes, irrespective of carbon content.

Puddled Iron (Steel) is wrought iron (steel) made by the puddling process. Puddled steel is necessarily slag-bearing.

Wrought Iron is malleable iron which is aggregated from pasty particles of metallic iron without subsequent fusion, and contains so little carbon that it does not harden usefully when cooled rapidly. Also known formerly as **weld iron.**

Prices of Iron and Steel, 1910–1915. The accompanying table gives the range of monthly average prices for carload lots during 1910–1914, as well as prices current Aug. 1, 1915. Pig-iron prices are in dollars per ton of 2240 lb.; all others in cents per lb.

Pig Iron:	1910	1911	1912	1913	1914	Aug. 1915
Bessemer*..	15.90–19.90	15.00–15.90	14.90–18.15	15.77–18.15	14.59–15.09	15.20
Basic*.....	13.15–16.87	12.25–13.75	12.35–16.50	12.71–16.41	12.48–13.19	13.95
No. 2 So. Fdry.†....	14.25–17.25	13.20–14.25	13.25–17.25	13.75–16.95	12.50–13.88	13.25
No. 2 Fdry.‡	16.00–19.00	14.00–15.50	14.00–18.00	14.60–17.90	12.50–14.25	13.50
No. 2X Fdry.§....	15.50–19.00	14.85–15.50	14.85–18.25	15.30–18.25	14.25–15.00	14.50
Steel bars and plates*.....	1.40–1.55	1.08–1.40	1.10–1.60	1.20–1.85	1.05–1.20	1.25
Wire nails*...	1.70–1.85	1.53–1.80	1.57–1.72	1.55–1.80	1.52–1.60	1.60
Black sheets, No. 28*.....	2.15–2.40	1.83–2.20	1.80–2.25	1.90–2.35	1.80–1.95	1.80

* Pittsburgh. † Cincinnati. ‡ Chicago. § Philadelphia. For freight rates to other points, see p. 1304.

Prices for plates are for those 6 to 100 in. wide and over ¼ in. thick. Boiler and flange steel ordinarily 0.10 cent higher in price than plates, ordinary fire-box steel 0.2 cent, locomotive fire-box steel 0.5 cent, and boiler rivets 0.5–0.6 cent. Wire-nail prices are for kegs of 100 lb.; cut nails usually 5 cents higher per keg. New York jobbers' warehouse prices for plates, bars and machinery steel are usually from 0.65 to 0.75 cent per lb. higher than the tabular prices for bars and plates.

For prices on structural shapes, see p. 1304.

SPECIFICATIONS FOR IRON AND STEEL

Tables 1 and 2 are condensed from the 1915 specifications of the American Society for Testing Materials, and Table 3 from those of the Society of Automobile Engineers.

Table 1. Strength Specifications for Steel and Iron
(American Society for Testing Materials, Yearbook, 1915)

Metal	Tensile strength, 1000 lb. per sq. in.		Minimum elongation, per cent.		Reduction of area, per cent.	Requirements as to chemical composition (percentages not to be exceeded)					Notes
	Ultimate (*U*)	Yield point (or elastic limit, *E. L.*)	In 8 in.	In 2 in.		C	Mn	P acid	P basic	S	
Steel											
BRIDGES											
Structural steel	55–65	½*U*	1500/*U*	22	......	......	......	0.06	0.04	0.05	1, 2, 3, 34
Rivet steel	46–56	½*U*	1500/*U*	......	......	......	......	0.04	0.04	0.045	4
STRUCTURAL NICKEL STEEL	(Nick	el not to	be under	3.25	per ce	nt.)					
Plates, shapes and bars	85–100	50	1500/*U*	......	25						5, 6, 35
Eyebars and rollers, unannealed	95–110	55	1500/*U*	16	25	0.45	0.70	0.05	0.04	0.05	3, 5,
Eyebars and pins, annealed	90–105	52	20	20	35						3, 5,
Rivets	70– 80	45	1500/*U*	......	40	0.30	0.60	0.04	0.03	0.045	4
BUILDINGS											
Structural steel	55– 65	½*U*	1400/*U*	22	......	......	(Bess.	0.10)	{0.06 O.H.}	......	1, 3, 34
Rivet steel	46– 56	½*U*	1400/*U*	......	......	......	......	......	{0.06 O.H.}	0.045	4
BOILERS											
Flange steel	55– 65	½*U*	1500/*U*	......	......	......	0.3–0.6	0.05	0.04	0.05	7, 21, 37
Firebox steel	52– 62	½*U*	1500/*U*	......	......	0.12–0.25	0.3–0.5	0.04	0.035	0.04	(Cu, 0.05) 7, 21, 37
Rivet steel	45– 55	½*U*	(1500/*U*	≦ 30%)	......	......	0.3–0.5	(0.04	O. H.)	0.045	4, 22
Tubes (lap-welded and seamless)	......	......	......	......	......	0.08–0.18	0.3–0.5	(0.04	O. H.)	0.045	27, 41
Wrought pipe: steel	50	30	18	......	......	......	......	......	......	......	......
" " iron	45	24	12	......	......	......	......	......	......	......	......
LOCOMOTIVES											
Structural steel	55– 65	½*U*	1500/*U*	......	......	......	......	(0.05	O. H.)	0.05	1, 34
Billets for forgings:											
Class A, for welding and case-hard	ening	......	......	......	......	0.08–0.18	0.3–0.5	0.045 open hearth			
Class B, for case-hardening when	subs	equently	heat-tre	ated.	......	0.15–0.25	0.3–0.5				
Class C, for special purposes	......	......	......	......	......	0.25–0.38	0.4–0.6		......	0.05	
Class D, for axles, shafts, connecti	ng ro	ds, etc.			......	0.38–0.52	0.4–0.6				
Class E, for Class D forgings when	they	are to be	heat-tre	ated...	......	0.45–0.60	0.45–0.7				

Table 1. Strength Specifications for Steel and Iron—(*continued*)

Metal	Tensile strength, 1000 lb. per sq. in.		Minimum elongation, per cent.		Reduction of area, per cent.	Requirements as to chemical composition (percentages not to be exceeded)					Notes
	Ultimate (U)	Yield point (or elastic limit, $E.L.$)	In 8 in.	In 2 in.		C	Mn	P acid	P basic	S	
Axles, shafts and other carbon-steel	forg	ings, que	nched an	d temp	ered (	d = outside	diam.,	in.; t	= thi	cknes	s, in.; w =
thickness of wall, in.):			(in 2 in.)								
d or t < 4; w ≦ 2	90	($E.L.$ 55)	2100/U	≧ 20.5	≧ 39	0.25–0.60					8
d or t, 4–7; w ≦ 3½	85	($E.L.$ 50)	2000/U	≧ 20.5	≧ 39	0.35–0.60	0.4–0.7	(0.05	by any process)	0.05	8
d or t, 7–10; w ≦ 5	85	($E.L.$ 50)	1900/U	≧ 19.5	≧ 37	0.35–0.65					9
d or t, 10–20; w, 5–8	82.5	($E.L.$ 48)	1800/U	≧ 19	≧ 36	0.35–0.70					9
Carbon steel forgings, untreated:			(in 2 in.)								
d or t ≦ 8 in.	75	½U	1600/U	≧ 18	≧ 24						
d or t = 8–12 in. incl.	75	½U	1500/U	≧ 17	≧ 22						
Carbon steel forgings, annealed:			(in 2 in.)			....	0.4–0.7	(0.05	by any process)	0.05	
d or t ≦ 8 in.	80	½U	1800/U	≧ 20	≧ 32						
d or t = 8–12 in. incl.	80	½U	1725/U	≧ 19	≧ 30						
d or t = 12–20 in. incl.	80	½U	1650/U	≧ 18	≧ 28						
Structural steel for cars	50–65	½U	1500/U	(pla	tes fo	r cold flang	ing)	0.06	0.04	0.05	1, 34
Car rivets	48–58	½U	1500/U	(pla	tes fo	r cold flang	ing)	0.04	0.04	0.045	1, 34
Carbon steel car and tender axles (o	pen-h	earth)	....	....	......	0.35–0.55	0.70	0.05	....	0.06	32
Cold-rolled axles	70	($E.L.$ 60)	....	18	35	0.40	0.4–0.8	0.05	0.05	0.05	10
Wheels, forged and rolled, forged, or	rolle	d; solid	carbon	steel:							
Acid O. H.	......	....	Si0.15–0.35	....	......	0.6 –0.8	0.55–0.8	0.05	....	0.05	
Basic O. H.	......	....	Si0.10–0.30	....	......	0.65–0.85	0.55–0.8		0.05	0.05	
Tires (A, driving, passenger engine;	B, dr	iving, fre	ight engi	ne, and	all ot	her wheels	except	C; C,	drivin	g, sw	itch engine):
Class A	105			12	16						
Class B	115	(Si, ≦ 0.35)	....	10	14	....	0.75	(0.05	O.H.)	0.05	33
Class C	125		....	8	12						
Springs, elliptic and helical (open-	heart	h, crucib	le or elec	tric)	......	0.9– 1.10	0.5	(0.05)	....	0.05	
Springs, helical (open-hearth, crucible,	or ele	ctric)	....	....	......	0.95–1.15	0.5	(0.05)	....	0.05	
Cold-drawn Steel (Bessemer auto	matic	screw sto	ck)	....	......	0.8– 0.16	0.6–0.8	0.09–0.13	....	0.075–0.15	
			(in 8 in.)								
(O. H. automatic screw stock)	48– 58	½U	1500/U			0.15–0.25	0.6–0.9	0.06		0.075–0.15	
Concrete Reinforcement Bars:											
Billet steel plain	55– 70	33	1400/U	struct	ural	For Besse	mer, P	≦ 0.10	......	......	11, 36
Billet steel, deformed	55– 70	33	1250/U	steel g	rade	For O. H.	, P ≦ 0.0	5	......	......	12, 36
Billet steel, cold twisted	......	55	5						......	......	13

Table 1. Strength Specifications for Steel and Iron—(*concluded*)

Metal	Tensile strength, 1000 lb. per sq. in.		Minimum elongation, per cent.	Reduction of area, per cent.	Notes
	Ultimate (U)	Yield point (or elastic limit, *E. L.*)	In 8 in.		
CONCRETE REINFORCEMENT BARS:					
Rail steel, plain	80	50	$1200/U$	......	14, 36
Rail steel, deformed	80	50	$1000/U$	......	15, 36
Wrought Iron					
Lap-welded boiler tubes (of knobbled, hammered charcoal iron)				.....	26, 28, 42
Stay-bolt iron*	49–53	$0.6U$	30	48	16, 23, 29
Engine-bolt iron (puddled iron)	50–54	$0.6U$	25	40	17, 24, 30, 38
Refined W. I. bars†	48	25	22	......	18, 25, 31, 39
Wrought-iron plates:‡					
A, 6–24 in. (24–90 in.)	49(48)	(*E. L.* 26)	16(12)	......	19, 30, 40
B, 6–24 in. (24–90 in.)	48(47)	(*E. L.* 26)	14(10)	......	20, 31, 40

* Puddled or knobbled charcoal iron.
† Puddled muck bar with or without scrap.
‡ Class A, made from puddled pig and scrap from plate rolling; Class B, from puddled pig or a mixture of pig with cast and wrought scrap; widths given are inclusive.

Notes to Table 1

COLD BEND TESTS (Specimens to bend through 180 deg. without cracking on the outer bent portion):

1. Plates, shapes and bars: For material of thickness $t \leqq \frac{3}{4}$ in., flat on itself; for $t = \frac{3}{4}$ to $1\frac{1}{4}$ in., around pin of diam. t; for $t > 1\frac{1}{4}$ in., around pin of diam. $= 2t$.
2. Eyebar flats: For $t \leqq \frac{3}{4}$ in., around pin of diam. $= t$; for $t = \frac{3}{4}$ to $1\frac{1}{4}$ in., around pin of diam. $= 2t$; for $t > 1\frac{1}{4}$ in., around pin of diam. $= 3t$.
3. Pins, rollers and other bars: Around a 1-in. pin without cracking on outside bent portion.
4. Specimen to bend flat on itself without cracking on outside bent portion.
5. Plates, shapes and bars: For $t \leqq \frac{3}{4}$ in., around pin of diam. $= t$; for $t > \frac{3}{4}$ in., around pin of diam. $= 2t$.
6. Punched rivet holes pitched 2 diam. from a planed edge, to stand drifting until diam. is enlarged 50 per cent., without cracking the metal.
7. Specimen when $t \leqq 1$ in. to bend flat on itself; when $t > 1$ in., around pin of diam. $= t$.
8. Specimen to bend around a 1-in. flat mandrel having a rounded edge of $\frac{1}{2}$-in. radius.
9. Specimen to bend around a $1\frac{1}{2}$-in. flat mandrel having a rounded edge of $\frac{3}{4}$-in. radius.
10. Specimen to bend around a 1-in. pin or mandrel.
11. Around pin of diam. $= t$.
12. For $t < \frac{3}{4}$ in., 180 deg. around pin of diam. $= t$; for $t \geqq \frac{3}{4}$ in., 180 deg. around pin of diam. $= 2t$.
13. For $t < \frac{3}{4}$ in., 180 deg. around pin of diam. $= 2t$; for $t \geqq \frac{3}{4}$ in., 180 deg. around pin of diam. $= 3t$.
14. For $t < \frac{3}{4}$ in., 180 deg. around pin of diam. $= 3t$; for $t \geqq \frac{3}{4}$ in., 90 deg. around pin of diam. $= 3t$.
15. For $t < \frac{3}{4}$ in., 180 deg. around pin of diam. $= 4t$; for $t \geqq \frac{3}{4}$ in., 90 deg. around pin of diam. $= 4t$.
16. 180 deg. flat on itself in both directions without fracture of outside bent portion.
17. 180 deg. around pin of same diam. as specimen.
18. For bars < 4 sq. in. section, 180 deg. around pin of diam. $= 2t$.
19. 90 deg. around pin of diam. $= 1.5t$.
20. 90 deg. around pin of diam. $= 3t$.

QUENCH AND HOT BEND TESTS

21. When heated to light cherry red as seen in the dark ($\geqq 1200$ deg. fahr.), and quenched suddenly in water at 80–90 deg. fahr., material to stand test of Note 7.

22. When treated as in Note 21, material to stand test of Note 4. Head to flatten to diam. = 2½ × diam. of shank without cracking.

23. When heated to a yellow heat and quenched at once in water of 80–90 deg. fahr., to stand test of Note 4.

24. When heated to a bright cherry red, to stand test of Note 4.

25. When heated to 1700–1800 deg. fahr.; for round bars < 2 sq. in. in section, specimen to bend flat on itself; for larger bars, round or flat, specimen to bent 180 deg. around pin of diam. = t of specimen.

26. Strip ½ × 6 in. planed lengthwise from tube to stand bending of ends flat on itself in opposite directions after being heated to a cherry red and quenched in water at 80 deg. fahr.

Flanging Tests

27. Test specimen to stand flanging at right angles to body of tube without showing cracks or flaws; flange to be ⅜ in. wide for tubes ≦ 2½ in. O.D., and ½ in. for tubes > 2½ in.; also to stand flattening until inside walls are in contact without cracking.

Nick Bend Tests

28. Strip of Note 26 when nicked around and broken by light blows to show a wholly fibrous structure.

29. Specimen, when nicked 25 per cent. around with a 60-deg.-edge cutting tool to a depth from 8 to 16 per cent. of its diam., and broken, to show a clean fiber, entirely free from crystallization.

30. Specimen treated as in Note 29 to show a wholly fibrous structure on breaking.

31. Specimen treated as in Note 29 to show not more than 10 per cent. of fractured surface to be crystalline.

Drop Tests

32. Axles to be held on supports 3 ft. apart and struck at mid-length with a 1640-lb. tup; axle to be turned over after first and third blows; axle to stand 5 blows (7 and 9 for two largest sizes) without fracture. Maximum deflections after first blow as follows:

Axle diam. at center, in....	4¼	4⅜	4 7/16	4⅝	4¾	5⅜	5⅞	6 7/16
Height of drop, ft..........	24	26	28½	31	34	43	43	43
Deflection, in.............	8¼	8¼	8¼	8	7¾	6½	5	3½

33. Tire placed vertically on an anvil weighing at least 10 tons to stand successive blows from a 2240-lb. tup falling from heights of 10 to 20 ft. or over until a minimum deflection = $D^2/(40T^2 + 2D)$ is obtained, where D = internal diam. and T = thickness of tire at center of tread, both in inches.

Modifications in Elongation

34. For t > ¾ in., subtract 1 per cent. from col. 3 for each increase of ⅛ in. above ¾ in. to a minimum of 18. For t < 5/16 in., subtract 2½ per cent. from col. 3 for each decrease of 1/16 in. below 5/16 in.

35. For t > 1 in. subtract 1 per cent. from cols. 3 and 4 for each increase of ⅛ in. above 1 in. to a minimum of 14.

36. For t > ¾ in., subtract 1 per cent. from col. 3 for each increase of ⅛ in. above ¾ in. For t ≦ 7/16 in., subtract 1 per cent. from col. 3 for each decrease of 1/16 in. below 7/16 in.

37. For t > ¾ in., subtract 0.5 per cent. from col. 3 for each increase of ⅛ in. above ¾ in. For t ≦ ¼ in., elongation to be measured on a gage length = $24t$.

38. For section > 1¼ sq. in., subtract 2000 lb. per sq. in. from U(col. 2).

39. For flat bars to be reduced in width, subtract 1000 lb. per sq. in. from U(col. 2).

40. For t < 7/16 in., same as in Note 36.

Hydraulic Tests

41. Tubes < 5 in. diam. to stand an internal hydraulic pressure of 1000 lb. per sq. in.; tubes > 5 in., 800 lb.

42. Tubes to stand an internal hydraulic pressure of between 500 and 750 lb. per sq. in.

Table 2. Specifications for Steel Forgings
(American Society for Testing Materials, Yearbook, 1915)

Class	Treatment	Size	Tensile strength, min. (except Class A), 1000 lb. per sq. in. *U*	Yield point (or elastic limit, *E.L.*) 1000 lb. per sq. in.	Elongation in 2 in., min., per cent.		Reduction of area, min., per cent.	
					Inverse ratio	Not under	Inverse ratio	Not under
A	None	All sizes	47–60	½*U*	1500/*U*	......	2500/*U*	......
B	None	(*a*), (*b*)	60	½*U*	1550/*U*	22	2400/*U*	35
		(*c*)	60	½*U*	1480/*U*	21	2220/*U*	32
C	Annealed	(*a*), (*b*)	60	½*U*	1700/*U*	25	2700/*U*	38
		(*c*)	60	½*U*	1600/*U*	24	2520/*U*	36
D	None	(*a*)	75	½*U*	1600/*U*	18	2200/*U*	24
		(*b*)	75	½*U*	1500/*U*	17	2000/*U*	22
		(*c*)	75	½*U*	1400/*U*	16	1800/*U*	20
E	Annealed	(*a*)	75	½*U*	1800/*U*	20	2800/*U*	33
		(*b*)	75	½*U*	1725/*U*	19	2640/*U*	31
		(*c*)	75	½*U*	1650/*U*	18	2400/*U*	29
F	Annealed	(*a*)	80	½*U*	1800/*U*	20	2800/*U*	32
		(*b*)	80	½*U*	1725/*U*	19	2640/*U*	30
		(*c*)	80	½*U*	1650/*U*	18	2400/*U*	28
G	Quenched and tempered.	(*d*)	90	(*E.L.* 55)	2100/*U*	20.5	4000/*U*	39
		(*e*)	85	(*E.L.* 50)	2000/*U*	20.5	3800/*U*	39
		(*f*)	85	(*E.L.* 50)	1900/*U*	19.5	3600/*U*	37
		(*g*)	82.5	(*E.L.* 48)	1800/*U*	19	3400/*U*	36
H (nickel steel).	Annealed	(*a*), (*b*)	80	(*E.L.* 50)	2000/*U*	22	3600/*U*	40
		(*c*)	80	(*E.L.* 50)	1900/*U*	21	3400/*U*	38
I (nickel steel).	Quenched and tempered.	(*d*)	100	(*E.L.* 70)	2200/*U*	20	4500/*U*	41
		(*e*)	100	(*E.L.* 65)	2100/*U*	20	4300/*U*	41
		(*f*)	90	(*E.L.* 60)	2000/*U*	20	4100/*U*	41
		(*g*)	85	(*E.L.* 55)	1900/*U*	20	3900/*U*	41

Notes to Table 2

USES OF STEELS LISTED

Class A, for forgings which may be welded or case-hardened.

Class B, for mild-steel forgings for structural purposes, for minor ship fittings, etc.

Class C, for mild-steel forgings for structural purposes, for ships, etc.

Classes D, E, F, G, H, and I, for various machinery forgings, choice depending upon design and upon the stresses and services to be imposed.

PERCENTAGE CHEMICAL COMPOSITION

Class A: Mn, 0.30–0.55; P, $\leqq$0.05; S, $\leqq$0.05.

Classes B, C, D, E, F, and G: Mn, 0.40–0.80; P, $\leqq$0.05; S, $\leqq$0.05.

Classes H and I: Mn, 0.40–0.80; P, $\leqq$0.04; S, $\leqq$0.05; Ni, $\geqq$3.00.

SIZES

(*a*) Not over 8 in. in outside diameter or overall thickness.

(*b*) Over 8 to 12 in., inclusive. (*c*) Over 12 to 20 in., inclusive.

(*d*) Up to 4 in., 2-in. max. wall. (*e*) Over 4 to 7 in., 3½-in. max. wall.

(*f*) Over 7 to 10 in., 5-in. max. wall. (*g*) Over 10 to 20 in., 5- to 8-in. max. wall.

Specifications for Classes A, B, C, D, E, F, H are for forgings whose max. outside diameter or overall thickness is not over 20 in. Specifications for Classes G, I, are for forgings whose max. outside diameter or thickness is not over 10 in. when solid, and not over 20 in. when bored.

S. A. E. Specifications for Steel. The chemical specifications in Table 3 were adopted by the Society of Automobile Engineers, in June, 1914. Each specification is given a four- (or five-) figure number, the first two digits of which indicate the class or quality of the steel, the last two (or three) the desired percentage carbon content. Thus, a nickel steel of 0.40 per cent. carbon content would have 2340 as its specification number, 23 being the class number assigned to nickel steels and 40 indicating the carbon percentage. The notes and data on physical characteristics, heat treatments and uses (Jan., 1915) are solely for the information of users; they form no part of the specifications and should not be included therein when ordering.

Table 3. S. A. E. Specifications for Steel

*Percentage, min.-max. (desired)		Physical characteristics					
Carbon	Manganese	Yield point,* 1000 lb. per sq. in.		Reduction of area, per cent.		Elongation in 2 in., per cent.	
		Annealed	Heat-treated	Annealed	Heat-treated	Annealed	Heat-treated
CARBON STEELS (Class 10) Max. phosphorus, 0.045 (0.04 for 0.95 carbon steel); max. sulphur, 0.05							
(a) 0.05-0.15(0.10)	0.30-0.60(0.45)	28-36	40- 60[1]	65-55	55-45[1]	40-30	....
(b) 0.15-0.25(0.20)	0.30-0.60(0.45)	30-40	40- 60[2]	60-45	60-30[2]	35-25	35-15[2]
(c) 0.20-0.30(0.25)	0.50-0.80(0.65)	30-40	38- 65[3]	55-40	60-30[3]	30-20	30-10[3]
(d) 0.30-0.40(0.35)	0.50-0.80(0.65)	35-45	40- 80[4]	55-40	60-30[4]	30-20	30-10[4]
(e) 0.40-0.50(0.45)	0.50-0.80(0.65)	40-50	50- 90[6]	50-40	55-25[6]	25-20	25- 5[6]
(f) 0.90-1.05(0.95)	0.25-0.50(0.35)	....	90-180[6]	....	....	....	....
(g) 0.08-0.20(0.14)	0.30-0.80	Phosphorus (max.), 0.12; sulphur, 0.06 to 0.12 [screw stock]					
NICKEL STEELS (Class 23) Phosphorus (max.), 0.04; sulphur (max.), 0.045; nickel, 3.25 to 3.75 (3.50 desired)							
(h) 0.10-0.20(0.15)	0.50-0.80 (0.65)	35-45	40- 80[7]	65-45	65-40[7]	35-25	35-15[7]
(h) 0.15-0.25(0.20)		40-50	50-125[7]	65-40	65-40[7]	30-20	25-10[7]
(i) 0.25-0.35(0.30)		40-50	60-130[8]	60-40	60-30[8]	30-20	25-10[8]
(i) 0.30-0.40(0.35)		45-55	65-160[8]	55-35	55-25[8]	25-15	25-10[8]
....0.35-0.45(0.40)		55-65	70-200[8]	50-30	55-15[8]	25-15	20- 5[8]
NICKEL CHROMIUM STEELS (Classes 31-33) Phosphorus (max.), 0.04; sulphur (max.), 0.04(=0.045 for 1.25 nickel content)							
	Class 31:						
(h) 0.15-0.25(0.20)	Mn, 0.5 -0.8 (0.65) Ni, 1.0 -1.5 (1.25) Cr, 0.45-0.75(0.60)	30-40	40-100[9]	55-40	65-40[9]	35-25	25-15[9]
(i) 0.20-0.30(0.25)		40-55	50-125[5]	50-35	55-25[6]	30-20	25-10[6]
(i) 0.25-0.35(0.30)		40-55	50-125[6]	50-35	55-25[6]	30-20	25-10[6]
(i) 0.30-0.40(0.35)		45-60	55-150[6]	45-30	50-25[6]	25-15	20- 5[6]
(i) 0.35-0.45(0.40)		45-60	55-150[6]	45-30	50-25[6]	25-15	20- 5[6]
	Class 32:						
(k) 0.15-0.25(0.20)	Mn, 0.3 -0.6 (0.45) Ni, 1.5 -2.0 (1.75) Cr, 0.9 -1.25(1.10)	35-50	45-120[9]	60-45	65-30[9]	25-20	20- 5[9]
(i) 0.25-0.35(0.30)		40-50	60-175[10]	55-40	60-30[10]	25-15	20- 5[10]
(l) 0.35-0.45(0 40)		45-60	65-200[10]	50-40	50-20[10]	25-15	15- 2[10]
(m) 0.45-0.55(0.50)		50-60	150-250[11]	0-4	25-15[11]	25-15	15- 2[11]
	Class X33:						
(n) 0.10-0.20(0.15)	Mn, 0.45-0.75(0.60) Ni, 2.75-3.25(3.0) Cr, 0.6 -0.95(0.8)	35-45	40-100[12]	60-45	65-30[12]	25-20	20- 5[13]
(p) 0.30-0.40(0.35)		45-55	60-175[13]	55-40	60-30[13]	25-15	20- 5[12]
(m) 0.45-0.55(0.50)		50-60	150-250[14]	50-40	25-15[14]	25-15	15- 2[14]
	Class 33:						
(k) 0.15-0.25(0.20) (q) 0.35-0.45(0.40)	Mn, 0.30-0.60(0.45) Ni, 3.25-3.75(3.50) Cr, 1.25-1.75(1.50)	Alternative for No. 3220; use heat treatment *L*. Alternative for Nos. 3240 and 3250; use heat treatments *P* or *R*.					
CHROMIUM STEELS (Classes 51 and 52) Phosphorus (max.), 0.04; sulphur (max.), 0.03(=0.045 for 0.75 Cr content)							
	Class 51:						
A. 0.15-0.25(0.20) B. 0.35-0.45(0.40) 0.60-0.70(0.65) 0.90-1.05(0.95) 1.10-1.30(1.20)	Mn, 0.35(Si<0.2) or Mn, 0.7(Si 0.15-0.5) Cr, 0.65-0.85(0.75) Mn, 0.20-0.45(0.35) Cr, 0.90-1.10(1.00)	A.—Similar to Nos. 2320 and 3120, but a better case-hardening grade; treatment *B*. B.—Similar to No. 3140; for high-duty shafting; use treatment *H* or *D*. Use restricted almost entirely to ball-bearing cups and cones, where extreme hardness is the desideratum; treatments *P* and *R* generally followed.					
	Class 52						
0.90-1.05(0.95) 1.10-1.30(1.20)	Mn, 0.20-0.45(0.35) Cr, 1.10-1.30(1.20)						

Table 3. S. A. E. Specifications for Steel—(*concluded*)

Chromium Vanadium Steels (Class 61)
Phosphorus (max.), 0.04; sulphur (max.), 0.04; vanadium desired, 0.18 (0.12 min.)

Percentages, min.-max. (desired)		Physical characteristics					
Carbon	Manganese	Yield point,* 1000 lb. per sq. in.		Reduction of area, per cent.		Elongation in 2 in., per cent.	
		Annealed	Heat-treated	Annealed	Heat-treated	Annealed	Heat-treated
(*r*) 0.15–0.25(0.20)		40–50	55–100[15]	65–50	65–45[15]	30–20	25–10[15]
(*r*) 0.20–0.30(0.25)		40–50	55–100[15]	65–50	65–45[15]	30–20	25–10[15]
(*s*) 0.25–0.35(0.30)		45–55	60–150[16]	60–50	55–25[16]	25–20	15– 5[16]
(*l*) 0.30–0.40(0.35)	Mn, 0.5–0.8(0.65)	45–55	60–150[16]	60–50	55–25[16]	25–20	15– 5[16]
(*t*) 0.35–0.45(0.40)	Cr, 0.7–1.1(0.90)	50–60	65–175[16]	55–45	50–15[16]	25–15	15– 2[16]
(*u*) 0.40–0.50(0.45)		55–65	150–200[17]	55–40	25–10[17]	25–15	10– 2[17]
(*v*) 0.45–0.55(0.50)		60–70	150–225[18]	50–35	35–15[18]	20–15	10– 2[18]
0.90–1.05(0.95)	Mn, 0.20–0.45(0.35)	Cr, 0.90	–1.10(1.00)	.....	....	.....	.....

Silico-manganese Steel (Class 92)
Phosphorus (max.), 0.04; sulphur (max.), 0.04; silicon, 1.75–2.00 (1.90)

(*p*) 0.45–0.55(0.50)	0.60–0.80(0.70)	55–65	60–180[19]	45–30	40–10[19]	25–20	20– 5[19]

* Yield point or elastic limit for all except the first nine steels listed.

Notes to Table 3

Uses of Steels Listed

(*a*) Soft basic open-hearth steel; used for seamless tubing and pressed parts; does not machine freely.
(*b*) Machine steel; used for forged, machined and case-hardened parts where strength is not paramount, also for tubes and pressed parts.
(*c*) For frames and drop forgings requiring moderate ductility but not unusually high strength
(*d*) For axles, driving shafts, steering pivots and other forgings.
(*e*) For crank, driving and propeller shafts; not to be case-hardened; machines well.
(*f*) For springs.
(*g*) Screw stock, Class 11; permits rapid removal of metal and finishes smoothly.
(*h*) For case-hardened gears and other parts requiring a very tough steel with hardened exterior; also for structural parts.
(*i*) For heat-treated structural parts requiring strength and toughness, as axles, shafts, etc.
(*k*) For case-hardened parts and for structural work.
(*l*) For structural parts requiring great strength.
(*m*) For gears where extreme strength and hardness are necessary.
(*n*) For case-hardened parts.
(*p*) For crank shafts, axles, spindles, transmission shafts, etc.
(*q*) For gears to be hardened without carbonising.
(*r*) For case-hardened shafts, gears and the like, and for structural parts.
(*s*) For structural parts; should not be case-hardened.
(*t*) For high-duty shafts and where great strength and fatigue-resisting qualities are demanded.
(*u*) For gears and springs. [For gears, to be annealed after forging by operations (1) and (2) of heat-treatment *U*.]
(*v*) For springs. Operation (5) of treatment *U* should run from 700 to 1100 deg. fahr., according as light spiral or heavy flat springs are being treated.
(*w*) For springs; used somewhat for gears and also structural parts.

Heat-treatments to be Used

[1] Quench at 1500 deg. fahr. (material cold-rolled or cold-drawn). [2] *H*. (for case-hardening, *A* or *B*). When cold-rolled or cold-drawn, yield point = 40–75; reduction of

area, 35–30 per cent. [3] *H*, or with better results, *D*. [4] *H*, *D* or *E*; for highest strength, quench in brine. [5] *H*, *D* or *E*. [6] *F* (Elastic limit values are for transverse test). [7] *H* or *K* (for case-hardening, *G*). [8] *H* or *K*. [9] *H* or *D* (For case-hardening, *G*). [10] *H* or *D*. [11] *M* or *Q*. [12] *G*. [13] *P* or *R*. [14] *R*. [15] *T* for structural parts, *S* for case-hardening. [16] *T*. [17] *U* (for high-strength structural parts, *T*). [18] *U*. [19] *V* for structural parts; for springs, operation (3) to be at 800–900 deg. fahr.

Heat-treatments for Steels

(All temperatures in deg. fahr.)

A. After forging or machining: (1) Carbonize at a temperature between 1600 and 1750 deg. (1650–1700 desired). (2) Cool slowly or quench. (3) Reheat to 1450–1500 deg. and quench.

B. After forging or machining: (1) Carbonize same as for *A*. (2) Cool slowly in carbonizing mixture. (3) Reheat to 1550–1625 deg. (4) Quench. (5) Reheat to 1400–1450 deg. (6) Quench. (7) Draw in hot oil of temperature between 300 and 450 deg., depending on degree of hardness desired.

D. After forging or machining: (1) Heat to 1500–1550 deg. (2) Quench. (3) Reheat to 1450–1500 deg. (4) Quench. (5) Reheat to 600–1200 deg. and cool slowly.

E. (1), (4) and (5) same as for *D*. (2) Cool slowly. (3) Reheat to 1400–1450 deg.

F. After shaping or coiling: (1) Heat to 1425–1475 deg. (2) Quench in oil. (3) Reheat to 400–900 deg. according to temper desired, and cool slowly.

G. After forging or machining: (1), (2), same as for *B*. (3) Reheat to 1500–1550 deg. (4) Quench. (5) Reheat to 1300–1400 deg. (6) Quench. (7) Reheat to a temperature between 250 and 500 deg. according to necessities of case, and cool slowly.

H. After forging or machining: (1) Heat to 1500–1550 deg. (2) Quench. (3) Reheat to 600–1200 deg. and cool slowly.

K. After forging or machining: (1) Heat to 1500–1550 deg. (2) Quench. (3) Reheat to 1300–1400 deg. (4) Quench. (5) Reheat to 600–1200 deg. and cool slowly.

L. After forging or machining: (1) and (2) same as for *B*. (3) Reheat to 1400–1500 deg. (4) Quench. (5) Reheat to 1300–1400 deg. (6) Quench. (7) Reheat to 250–500 deg. and cool slowly.

M. After forging or machining: Heat to 1450–1500 deg. (2) Quench. (3) Reheat to a temperature between 500 and 1250 deg. and cool slowly.

P. After forging or machining: (1) Heat to 1450–1500 deg. (2) Quench. (3) Reheat to 1375–1425 deg. (4) Quench. (5) Reheat to between 500 and 1250 deg. and cool slowly.

Q. After forging: (1) Reheat to 1475–1525 deg., and hold there for ½ hr. (2) Cool slowly. (3) Reheat to 1450–1500 deg. (4) Quench. (5) Reheat to 250–500 deg. and cool slowly.

R. After forging: (1) Heat to 1500–1550 deg. (2) Quench in oil. (3) Reheat to 1200–1300 deg. and maintain temperature 3 hr. (4) Cool slowly. (5) Machine. (6) Reheat to 1350–1450 deg. (7) Quench in oil. (8) Reheat to 250–500 deg. and cool slowly.

S. After forging or machining: (1) and (2) same as for *B*. (3) Reheat to 1650–1750 deg. (4) Quench. (5) Reheat to 1475–1550 deg. (6) Quench. (7) Reheat to 250–500 deg. and cool slowly.

T. After forging or machining: (1) Heat to 1650–1750 deg. (2) Quench. (3) Reheat to some temperature between 500 and 1300 deg. and cool slowly.

U. After forging: (1) Heat to 1525–1600 deg. and hold for about ½ hr. (2) Cool slowly. (3) Reheat to 1650–1700 deg. (4) Quench. (5) Reheat to 350–550 deg. and cool slowly.

V. After forging or machining: (1) Heat to 1650–1750 deg. (2) Quench. (3) Reheat to between 500 and 1200 deg. and cool slowly. The temperature suited to the thickness of piece, and the character of the quenching medium, must be determined experimentally.

WROUGHT IRON

Manufacture. Iron ore, consisting essentially of Fe_2O_3 or Fe_3O_4 with silica, phosphorus, sulphur, manganese, etc., as impurities, is heated in a blast furnace at a high temperature in intimate contact with coke and limestone (flux for the silica), and the resulting molten product is called **pig iron.** This iron contains about 3.5 per cent. C and considerable Si, Mn, P, and S which have been reduced with the iron. The pig iron is then heated in a puddling furnace at a temperature somewhat above its melting point, with the addition of fettling material in the form of iron ore or iron oxides (mill scale, etc.), the lining of the furnace bottom being generally composed of this material. The puddling furnace is a reverberatory furnace and the oxidising flame plays over the bath of molten metal. Air is allowed to enter the furnace, which further promotes the oxidisation. The impurities are gradually burnt out of the iron, and its melting point is thereby raised so that the resulting purer metal forms in globules which are collected together by means of long iron rods manipulated by the puddler. This purer iron is not molten, but comes from the furnace in a pasty condition in the form of balls (sometimes called **blooms**), and contains semi-molten slag (silicate of iron) mechanically included. The ball or bloom is then put through a squeezer or hammered with a steam hammer to remove a large portion of the slag. It finally passes through a rolling mill and is then known as muckbar. **Muckbar** contains too much slag to render the metal useful. The bars are therefore sheared, piled crosswise and the pile is reheated and rerolled, the purer iron product being called **merchant bar.** This is the wrought iron of commerce. When merchant bar is sheared, piled and rerolled in a similar manner, the resulting material is called **double-refined iron.** If a charge of iron scrap or of pig iron is heated in a so-called "knobbling" furnace with charcoal, and air is forced into the furnace through tuyères, the product, after being subjected to the mechanical treatment described above, is known as **charcoal knobbled iron.**

Fagoted Iron is produced by piling pieces of wrought-iron scrap, such as bolts, bars, structural material, etc., in a boxlike shape, using old boiler plates or new muckbar as sides and covers. These so-called "box piles" are then heated to a welding heat and rolled. When the scrap used is strictly wrought iron, the finished bars so made are of good quality, showing higher ductility than those made from ordinary muckbars; but it is difficult to get strictly wrought-iron scrap without an admixture of steel scrap, and, while the mixed-in soft steel does not reduce the strength or ductility of the metal, the latter frequently fails to weld, its resistance to corrosion is reduced, and it should not be used in the manufacture of such products as corrugated iron sheets, etc. **Busheled iron** is made from miscellaneous junk scrap (generally small pieces), sometimes mixed with iron and steel turnings, swarf, etc., by heating these to a welding heat in a furnace similar to a puddling furnace. The material is formed into lumps and treated like a puddled ball. Owing to the irregular composition of the scrap, which is often mixed with high-carbon and alloy steels, the material so produced is very unreliable as to its physical qualities and should never be used where either strength or longevity is an object.

Uses. Wrought iron was formerly used to a large extent for making crucible steel, and is still employed by some manufacturers for that purpose, but it is now chiefly used in the form of staybolts, rivets, corrugated siding sheets, roofing sheets, water pipes, steam pipes, boiler tubes, and by blacksmiths for horseshoes and general forging purposes, especially where welding plays a part. Bars and plates are made of single-refined iron, staybolts of double-refined iron and boiler tubes of charcoal knobbled iron.

Shapes of Finished Iron. The more usual shapes include bars having the following sections: Round, half-round, square, flat, round-edged flat, oval, half-oval, octagon, hexagon, bevel-edge, tapered, together with leveled and bulb iron, and rods; tee (or T-shaped) iron, tee with round top or edges, D-iron; angle (or L-shaped) iron, angle iron with unequal sides or round back, square-root angles; channel iron, H-iron, Z-iron; sheets, plates, bands, hoops and agricultural shapes; rails, including single-headed, double-headed, and flange; and horseshoe iron, which is rolled single-grooved, double-grooved, or concave.

Imperfections in Finished Iron. **Rough edges,** when not due to imperfections in the rolls or careless working, are a sign of red-shortness and are particularly noticeable in flat bars or strips. Red-shortness may be due to an excess of carbon or to the presence of sulphur, particularly if copper is also present. **Spilly places** are spongy or irregularly spotted parts not infrequently noticed in sheets and occasionally in all kinds of wrought iron, and are generally due to imperfect puddling, whereby one part of the iron has been oxidized more than another. **Blisters,** often met with in sheets, are attributed to a reaction between carbon and oxide of iron in wrought iron of inferior quality. **Sand holes** are pits noticed especially on the outside of pipes which have been reheated (*e.g.*, for welding) in a furnace with a sand bottom. The sand unites with some of the metal to form slag, which is more fusible than iron, and this causes the pit, which generally has considerable silicate (slag) attached to it.

Physical Properties of Wrought Iron. Table 4 (due to Turner) gives data on the physical properties of wrought irons of different qualities, which also show the great importance of ductility tests. A typical percentage **analysis** of Swedish bar iron is as follows: C, 0.05; Si, 0.037; S, 0.006; P, 0.012; Mn, 0.108; Cu, trace; As, 0.007.

Table 4. Physical Properties of Wrought Iron

Variety of iron	Quality	Form	Tensile strength, lb. per sq. in.	Elastic limit, lb. per sq. in.	Contraction of area, per cent.	Elongation, per cent. —in.
Swedish charcoal...	Very good	1 in. square....	43,904	27,440	72.18	56.0 on 3⅛
Best Yorkshire*.....		1⅛ in. round...	50,848	30,688	55.00	29.0 on 10
Very common......	Very bad	1 in. square....	46,995	30,800	5.29	1.5 on 3⅛
Puddled iron.......		¼ in. plate.....	41,664	30,912	4.50	3.0 on 10

* Bowling.

The **influence of the length of test bar on the ductility** of iron and steel, as shown by tests made with iron and steel bridge eyebars, is as follows: Percentage elongation in 12 in.: iron, 23; steel, 39; in 18 ft., iron, 15.22; steel, 14.4.

Table 5 is based on the British Standard Specifications, which are also representative of American iron.

Table 5. Physical Properties of Wrought Iron

Shapes.............	Rounds and squares							Flats, angles and tees,	Plates ¼–⅞ in. thick
Dimensions, in.... ≦	⅜	9/16	¾	1½	2	3½	4	all sizes	
T.S., 1000 lb. per sq. in.............	49–56	49–56	47–54	47–53	47–53	47–53	47–53	47–54	47–54†
Yield point, per cent. of T.S.............	56	56	56	56	56	50	50	50	......
Elongation in 8 in., per cent...........	27	28	29	29	26	23–35	22	24–26*	17‡

* 24 for angles and tees. † || to grain; 45 (min.) ⊥ to grain. ‡ || to grain; 12 ⊥ to grain.

Effect of High and Low Temperatures on the Physical Properties. Extreme cold increases the elastic limit of wrought iron, but does not affect the tensile strength appreciably. It increases the ductility very slightly, and decreases the resistance to impact by about 3 per cent. The tensile strength increases with temperature from 0 deg. fahr. up to a maximum at from 400 to 600 deg. fahr., the increase being from 8000 to 10,000 lb. per sq. in., and then decreases steadily until the strength of only 6000 lb. per sq. in. is shown at 1500 deg. fahr. The comparative strengths, taking strength at 68 deg. fahr. as 100, are given by one observer as follows:

Temp., deg. fahr.........................	300	500	700	900	1100	1300	1500
Tensile strength (comparative)	108	116	103	79	43	34	15

Effect of Rolling Temperature. Tests on a high grade of stay-bolt iron of two sizes and finished at various rolling temperatures showed that bars rolled considerably colder than usual in both 1⅜-in. and ⅞-in. sections gave higher tensile strengths than those rolled at the usual temperature or higher, while with the larger bars the low rolling temperature gave the highest elastic limit, and also the greatest elongation and contraction. The greatest difference in elastic limit in either size, however, was only about 5 per cent., in tensile strength 2 per cent., in elongation about 4 per cent., and in contraction about 3 per cent. of the average figures. There was a marked increase in the elastic limit and tensile strength and a slight decrease in elongation, with a slight increase in contraction in the case of the smaller bars, as compared with the larger ones.

Effect of Repeated Reheating. (Turner.) Puddled iron is much improved in quality by being cut up, piled, reheated, and rolled or hammered, but indefinite repetition of this is detrimental. In practice it is advantageous only in special cases to reheat puddled iron more than once. The results given below show the effect of repeated working. The metal began to deteriorate seriously after six workings, and no advantage is seen after the third working when the extra fuel and labor expended and the waste incurred are taken into account.

Working	Original bar	2nd	4th	6th	8th	10th	12th
T. S., lb. per sq. in.	43,900	52,900	59,600	61,800	57,300	54,100	43,900

Effect of Work upon Wrought Iron (Campbell, 1907). Table 6 gives results obtained from plates rolled in a three-high train, and in a 25-in. universal mill. The better figures for the latter mill are said to be due to the continuous rolling in one direction. The width was alike for similar thicknesses and no difference was found in the universal plates whether they were 9 or 42 in. in width.

Table 6. Physical Properties of Wrought-iron Plates from Shear and Universal Mills

Thickness, in.	Sheared plates from three-high train					Plates from 25-in. universal mill				
	Number of tests	Elastic limit, lb. per sq. in.	Ultimate strength, lb. per sq. in.	Elongation in 8 in., per cent.	Reduction of area, per cent.	Number of tests	Elastic limit, lb. per sq. in.	Ultimate strength, lb. per sq. in.	Elongation in 8 in., per cent.	Reduction of area, per cent.
1/4	1	32,400	51,800	11.2	18.9	1	32,100	51,000	13.0	19.9
3/8	......	......	......	......	......	2	31,050	50,650	14.6	21.6
1/2	5	31,180	49,760	14.2	22.0	3	31,100	50,530	17.3	26.2
5/8	4	30,775	50,200	15.5	22.5	3	30,500	50,830	17.2	24.6
3/4	2	30,400	49,050	16.0	22.4	3	31,470	52,570	19.0	26.2

Table 7. Variations in the Composition and Physical Properties of Wrought Irons

(Variations in specimens submitted to the United States Board for Testing Chain Cables)

	Same iron		All irons	
	Min.	Max.	Min.	Max.
Carbon, per cent	0.026 0.042	0.064 0.512	0.015	0.512
Phosphorus, per cent................	0.065 0.095	0.232 0.250	0.065	0.317
Silicon, per cent....................	0.028 0.182	0.182 0.321	0.028	0.321
Manganese, per cent................	tr. 0.021	0.059 0.097	tr.	0.097
Slag, per cent.......................	0.674 1.248	1.738 2.262	0.192	2.262
Ultimate strength, lb. per sq. in......	56,201 47,478	69,779 57,367	47,478	69,779
Elongation in 8 in., per cent.........	11.7 14.1	20.6 32.5	6.5	32.7
Reduction of area, per cent..........	27.7 16.0	59.8 31.5	7.7	59.8

According to Thurston (1894), good iron, when drawn into No. 10 wire (0.134 in. diam.), has a strength of about 90,000 lb., and Nos. 15 and 20 (0.072 and 0.035 in.) have a tensile strength respectively of about 100,000 and 111,000 lb. per sq. in.

Heterogeneity of Wrought Iron. Holley found great variation in the chemical composition of materials submitted. See Table 7.

Influence of Chemical Composition upon the Welding Properties. It has been believed that slag would facilitate welding, but the work by Holley does not bear this out, his conclusion being that, while "slag should theoretically improve welding like any flux, its effect in these experiments could not be definitely traced." The iron highest in slag (2.26 per cent.) "welded less soundly than any other bar of the same iron, and below average as compared with the other irons." He concluded that, "although most of the irons under consideration are alike in composition, the hardening effects of phosphorus and silicon can be traced, and that of carbon is obvious. P up to 0.20 per cent. does not harm and probably improves iron containing Si not above 0.15 per cent. and C not above 0.03 per cent. None of the ingredients, except carbon in the proportions present, seem to very notably affect welding by ordinary methods."

Microstructure of Wrought Iron. Sections taken across or parallel to the direction of rolling show under the microscope symmetrical, polyhedral grains of ferrite (see p. 494), with slag appearing as rounded areas in the transverse section and as strings or fibers in the longitudinal section. The iron itself shows no indication whatever of being fibrous, since except for the appearance of the slag, which constitutes not more than 5 per cent. of the entire area, the microstructure is identical in transverse and longitudinal sections. If there is any appreciable amount of carbon in the iron, it shows at the junctions of the ferrite polyhedra as dark, irregular particles of pearlite (see p. 494), the amount of this constituent varying from zero to about 12 per cent. of the area as the carbon content varies from zero to 0.1 per cent.

Fracture. The fracture of wrought iron depends to a very great extent upon the method employed in breaking the metal. A sudden break causes the production of a so-called "crystalline" or "granular" fracture, while a gradual rupture produces a "fibrous" fracture.

Test for Distinguishing Wrought Iron from Steel. A section ground flat and polished with two grades of emery paper is immersed in a bath containing 9 parts of water, 3 parts of H_2SO_4 (conc.) and 1 part HCl, added in the order named. After 20 to 40 min. immersion, remove the piece and wash off the acid. If the piece is steel, the section will present a bright, solid, unbroken surface, while if made of wrought iron, it will show faint ridges (or, in a pipe section, rings like the age rings in a tree) showing the different layers of iron and streaks of cinder. This test will also show on a section of welded metal whether it has been lap-welded or butt-welded.

STEEL

Steel Manufacture

Open-hearth Process (Siemens-Martin Process). Steel made by this process is called either acid or basic. In either process the product is low in carbon and must be recarbonized by means of proper agents. The processes may be carried on in stationary or tilting furnaces. From 15 to 80 tons are made in one heat, and some special furnaces have a capacity up to 200 tons. The duration of the heat is from 6 to 12 hr.

Acid Open-hearth Steel. The charge consists of pig iron and ore or pig iron and scrap having a low phosphorus content, and is melted in an open-hearth furnace with an acid or siliceous lining. The process consists in removing the impurities in the pig iron to a great extent by means of an oxidizing flame brought about by the union of producer gas with preheated air in a reverberatory regenerative furnace. The process is similar to the puddling process for making wrought iron, but is carried on at a much higher temperature, the products, both metal and slag, being molten.

Basic Open-hearth Steel. The charge of either melted or solid pig iron or a mixture of pig iron and low-carbon scrap is heated in a furnace similar to the acid furnace. The lining in this case is of dolomite, lime, magnesite or other basic material. Ore may or may not be used.

The Bessemer Process may be acid or basic, but no basic Bessemer steel is made in the United States. The production of acid Bessemer steel is rapidly diminishing, giving way to open-hearth, electric, and duplex steel. From 8 to 20 tons of steel are made in one heat which lasts from 10 to 15 min. The furnace or converter is pear-shaped and mounted on trunnions so as to be tilted easily for charging and pouring.

Acid Bessemer Steel is made by blowing air through liquid pig iron, the heat being obtained and the impurities removed by the combustion of the silicon, manganese and carbon. Some of the iron is also oxidized and these oxides must be reduced by the addition of suitable agents. When the reaction is completed the metal is recarburized to the desired extent. The lining of the converter is of siliceous material called ganister.

Basic Bessemer Steel is made in a similar manner, but the lining of the converter is magnesite or other basic material which combines with a large proportion of the phosphorus in the pig, thus eliminating it. The process requires a pig iron very high in phosphorus, whereas the basic open-hearth process can use pig iron containing phosphorus in all but the highest amounts.

Tropenas Steel is made in a converter similar to the Bessemer vessel, but the air is blown over the surface of the metal instead of through it. The furnace generally handles not more than 2 tons to a heat and the process is restricted to making steel castings.

Table 8. Typical Percentage Analyses of Bessemer and Open-hearth Steels (Harbord)

Constituents	Acid				Basic		
	Bessemer*		Open-hearth (Siemens)		Bessemer†		Open-hearth‡
	Soft steel	Rail steel	Soft steel	Medium and hard steel	Soft steel	Rail steel	Soft steel
Carbon (combined)	0.10–0.15	0.32–0.55	0.12–0.20	0.20–1.50	0.08–0.15	0.32–0.55	0.10–0.18
Silicon	0.02–0.06	0.04–0.08	0.04–0.08	0.04–0.35	trace	tr. –0.02	trace
Sulphur	0.03–0.08	0.05–0.08	0.02–0.06	0.02–0.06	0.03–0.08	0.05–0.08	0.02–0.06
Phosphorus	0.04–0.08	0.06–0.08	0.02–0.06	0.02–0.06	0.04–0.08	0.06–0.08	0.03–0.06
Manganese	0.40–0.80	0.60–1.00	0.40–0.60	0.40–1.00	0.40–0.80	0.60–1.00	0.40–0.80
Arsenic§	0.02–0.06	0.02–0.08	0.02–0.06	0.02–0.06	0.02–0.06	0.02–0.60	0.02–0.06

* Some special steels made from selected pig iron will be as low as 0.03 per cent. in phosphorus and sulphur.

† These are simply typical analyses of average steels and it is not unusual to find the impurities exceeding the upper limits given. For exceptional purposes, however, steel is made with less than the minimum impurities given.

‡ In England high-carbon steel is rarely made in basic open-hearth furnaces working with phosphoric pig iron, but in the United States practically all O. H. steel is now basic.

§ Steel made in the United States contains no arsenic.

Electric Steel. The charge is either cold or melted, and the furnace may be of the induction or of the arc type. The induction furnace produces steel of similar quality to crucible steel from a charge of pure scrap steel and pig iron. The arc furnace produces a high-grade steel from a charge of fluid or cold steel scrap covered with a slag containing a high percentage of lime and brought to a high temperature by means of the arc from an electric current in a furnace with a basic lining. The reaction eliminates the impurities such as phosphorus, manganese and carbon, and the proper composition is obtained by the introduction of the necessary alloys.

Arc Furnaces. The **Heroult furnace** consists of a shallow bath of dolomite, similar to the open-hearth, incased in a steel shell. Two carbon electrodes project into the furnace through the roof. The electric current arcs between each of the electrodes (which are connected in series) and the charge, thus passing through the charge. The furnace may be tilted. The **Girod furnace** differs chiefly from the Heroult in having a hearth which conducts the electric current. It is especially satisfactory in the refining of cold scrap steel because of the slight fluctuations in power demand. The furnace has a shallow conducting hearth of dolomite with pieces of soft steel embedded in it near the periphery, and a carbon electrode passing through the roof. The current passes through the carbon electrode and through the steel bath, which touches the tops of the steel poles embedded in the hearth. These steel electrodes are water-cooled and project above the bottom of the hearth a short distance. The **Stassano furnace** differs from other electric steel furnaces in that the electric current does not pass through the metal or slag. All heating is by radiation from three horizontal, water-jacketed arcs. It has a circular hearth with a cylindrical melting chamber above.

Induction Furnaces. The **Kjellin furnace** is really a transformer in which the bath of molten metal forms the secondary circuit. The magnetic circuit is built up of laminated sheet iron like the core of a transformer. The primary circuit is a coil consisting of a number of turns of insulated copper wire or tubing surrounding the magnetic circuit. A ring-shaped crucible, made of suitable refractory materials, also surrounds the magnetic circuit and when filled with molten metal forms the secondary circuit of the transformer. The current passing through the coils excites a variable magnetic flux in the iron core and the variation in the magnetic flux induces a current in the closed circuit formed by the molten metal in the crucible. Before starting the furnace an iron ring must be placed in the crucible and melted down to form a bath, or the crucible must be filled with molten metal taken from another source. The **Röchling-Rodenhauser furnace** is adapted to heat the metal in a hearth of the form of a figure 8 in place of an annular crucible, and differs from the Kjellin in the use of extra secondary coils surrounding the primary coils. These secondary coils are connected to metallic plates covered by an electrically conducting mixture of lining material and the circuit is completed through the metal bath. It combines two methods of heating: (*a*) By currents induced in the bath; (*b*) by currents passed through the bath.

Table 9. Physical Properties of Electric Steel

Percentage composition					Tensile strength, lb. per sq. in.	Elongation in 8 in., per cent.	Reduction of area, per cent.	Uses
C	Mn	P	S	Si				
0.13	0.54	trace	0.014	0.16	58,000	30.0	65.6	Angles.
0.13	0.70	0.018	0.020	0.08	61,100	31.0	62.3	Angles.
0.19	0.56	trace	0.010	0.13	66,849	31.0	55.0	Angles.
0.22	0.90	0.010	0.017	0.25	82,495	27.5	52.8	Angles.
0.25	0.88	trace	0.017	0.26	85,339	23.5	49.1	I-beams.
0.33	0.74	0.010	0.010	0.20	92,450	19.0	42.0	Engine and machine parts.
0.64	0.62	0.010	trace	0.47	122,319	13.2	23.5	Square blocks.
0.53	0.68	0.008	0.010	0.37	123,000	12.8	30.3	Flat steel bars.
0.57	0.70	trace	0.008	0.40	127,165	10.7	21.4	Flat steel bars.
0.72	0.42	0.020	0.010	0.34	138,107	6.0	23.8	Dies.
0.82	0.48	0.006	0.010	0.25	141,520	5.8	20.1	Punches.

The number of **electric furnaces in use** and in construction (1914) and their capacities are as follows (those in construction being in parentheses): Heroult, 31 (20), up to 25 tons; Girod, 16 (5), 2½ to 12 tons; Stassano, 16 (1), up to 2 tons; Kjellin, 9 (0), up to 8½ tons; Röchling-Rodenhauser, 18 (4), up to 10 tons.

The average ultimate strength and elongation of electric and open-hearth plate steels are given in Bulletin 67, U. S. Bureau of Mines, as follows:

Carbon content, per cent	0.08	0.12	0.16	0.20	0.24
Electric, ult. strength, lb. per sq. in.	59,194	64,080	69,220	72,853	69,540
O. H., ult. strength, lb. per sq. in.	51,690	56,510	52,901	58,294	63,560
Electric, elongation, in 2 in., per cent	27.25	26.05	25.25	22.82	23.12
O. H., elongation in 2 in., per cent	32.00	29.70	28.61	28.82	26.25

Defects in Steel Ingots. When steel cools in the mold, **piping** takes place, a cavity or "pipe" being usually found at the central upper portion. Large ingots are usually cast in "bottle-neck" shape, the large lower portion being formed in an iron mold, and the upper in sand. The pipe formed in the rapid-cooling bottom is fed by the hot metal from the upper portion; the volume of waste is thus reduced. Ingots cast wholly in sand pipe more extensively but are not so susceptible to external cracks. Molds are sometimes poured from the bottom by means of suitable ducts; this aids in preventing piping by forcing the lower portion of the ingot to solidify last. Casting ingots large end up will reduce the depth of the pipe, but this method requires special mechanical devices for lifting the ingot from the mold.

In low-carbon steel, deep-seated **blow holes,** if not too numerous, can be welded together during rolling; such blow holes are sometimes purposely allowed to form, since they tend to prevent piping. In most cases the blow holes are eliminated by the addition of silicon or aluminum or both. Manganese and titanium probably aid this elimination, but all of them increase the tendency to pipe. The **segregation** or concentration of impurities in the central upper portion of the ingot produces a porous or spongy mass, resulting in lack of homogeneity. It can be retarded by quick cooling, or remedied by discarding the portion of the ingot most affected, which can be discovered in a finished product by etching a section in hot acids. During rolling and shearing off ends, the worst is discarded, but even then the central portion is not always uniform with the outside of the ingot. Plates rolled directly from ingots are the most troublesome. The top surface of an ingot is usually solid, with the pipe existing beneath it. When rolled, it is possible to shear the plate so that the inner cavity or pipe is not reopened after apparent welding during the rolling, and the finished plate then has an area of lamination and an area of segregation, which defects are near one end of the plate. In plates rolled from slabs streaks of segregation often run through the central axis, but there is no centralization of impurities such as occurs in the older method of manufacture. Segregation and piping can be more easily controlled in the case of large guns and shafting, the latter generally being hollow to allow more even heat-treatment. The pipe and segregated portions are then bored out when the gun or shaft is hollowed.

Influence of Chemical Composition on the Physical Properties of Steel—Alloy Steels

Alloy steels composed of iron, carbon, and one other special element are known as **ternary steels.** This class includes nickel steel, manganese steel, chrome steel, tungsten steel, molybdenum steel, and silicon steel. Several other ternary steels have been investigated and used to a small extent, such as boron steels, cobalt steels, etc. **Quaternary,** or four-part, **steels** consist of iron, carbon, and two other special elements. The commonest and most important of these are nickel-chromium steels, tungsten-manganese, tungsten-chromium, nickel-manganese, manganese-silicon, tungsten-molybdenum, tungsten-nickel, nickel-vanadium steels, etc. The manufacture of alloy steels is usually very simple, as a general thing the alloying element being added like the recarburizer.

Carbon. The influence of carbon upon the physical properties cannot be stated with accuracy on account of the extreme sensitiveness of steel to heat-treatment and the failure of published tests to record the heat-treatment. Carbon increases the hardness and elastic limit of steel. Up to 0.85 per cent. (annealed steel; about 1.20 per cent. in forged steel) it increases the tensile strength and the ability to harden. Beyond that it decreases those properties. Carbon decreases the ductility, toughness, and weldability. It increases the resistance to fatigue, but decreases resistance to shock. Table 10 indicates the uses of carbon steels of varying carbon content.

It is extremely difficult to state the **tensile strength** corresponding to different percentages of carbon in commercial steels, as the strength of the material for the same carbon content will vary according to the process of manufacture, the size of the finished section, and the percentages of other

Table 10. Uses of Steel According to Carbon Content

Carbon, per cent.	Uses
0.05–0.10	Wire, drawn tubing, pressed work, nails.
0.10–0.15	Rivets, screws, machine parts to be case-hardened.
0.15–0.20	Same as for 0.10–0.15; ordinary forgings.
0.20–0.25	Ordinary forgings, boiler plate, structural steel, crane steel.
0.25–0.35	Forgings, structural steel, wearing parts as valves, crank pins, and gears.
0.35–0.45	Strong forgings, shafts and axles, crucible machinery steel.
0.45–0.55	Crank pins and other parts subjected to shocks and heavy stress reversals.
0.60–0.70	Bolt-heading and drop-forging dies, set screws.
0.70–0.80	Anvil facings, band saws, cold chisels, smithing hammers, pickaxes, shear blades, vise jaws, wrenches.
0.80-0.90	Punches and dies, rock drills, machinists' chisels, circular saws.
0.90–1.00	Springs, machinists' hammers, punches and dies.
1.00–1.10	Springs, lathe centers; lathe, planer, shaper and slotter tools; taps, mandrels, granite chisels.
1.10–1.20	Woodworking machine knives, ball-bearing races, taps, thread-cutting dies, reamers, wood chisels, milling cutters, twist drills, metal-cutting dies, stone-channeling bits.
1.20–1.30	Files.
1.30–1.40	Wire-drawing dies, granite-turning tools, paper knives, engravers' tools.
1.40–1.50	Wire-drawing dies, tools for turning chilled iron rolls.
1.50–1.60	Saws for cutting steel.

impurities present. The following table, compiled by Harbord from a large number of results, shows roughly what may be expected from open-hearth steel of good quality in which the manganese does not exceed 0.8 per cent. When the manganese is above 0.8, the tensile strength will be considerably higher. Acid Bessemer and basic Bessemer steels will not differ greatly from open-hearth steel in tensile strength, especially in the low-carbon steels, and comparatively little high-carbon steel is made by these processes, except for rails.

Carbon, per cent.........	0.05–0.10	0.10–0.20	0.20–0.30	0.30–0.40	0.40–0.50	0.50–0.60	0.60–0.70	0.70–0.80	0.80–0.90
T. S., 1000 lb. per sq. in.:									
Acid O. H...	47–61	54–72	65–78	69–92	87–108	105–123	116–134	130–143	137–151
Basic O. H..	45–56	52–65	58–72	65–78	76–96				

Following are **formulæ connecting chemical composition with the tensile strength of steel** (T. S., lb. per sq. in.):

T. S. = 50,000 + 90,000 C......(1) T. S. = 142,000 − 20,600 C.....(2)

where C = percentage of carbon. For $C < 0.83$ per cent., use formula (1), for $C > 0.83$ per cent., use (2). These formulæ (by Sauveur) hold good only for annealed or pearlitic steels. Campbell gives the following formulæ for acid and basic open-hearth steels:

$$\text{T.S. (acid)} = 40{,}000 + 1000C + 1000P + x\text{Mn} + R$$
$$\text{T.S. (basic)} = 41{,}500 + 770C + 1000P + y\text{Mn} + R$$

where C, P and Mn are respectively the contents of carbon (determined by combustion), phosphorus and manganese in hundredths of 1 per cent., R is a variable depending upon the heat treatment the steel has received, and x and y have the following values:

Carbon, per cent....................	0.05	0.10	0.20	0.30	0.40	0.50	0.60
x (0.4 per cent. Mn)*................		80	160	240	320	400	480
y (0.3 per cent. Mn)*................	110	130	170	210	250		

* Beginning only with the percentage of manganese stated.

Sulphur causes **red-shortness,** especially when insufficient manganese is present, or when there is copper in the steel. With manganese it forms manganese sulphide, which if present in large amounts is detrimental, producing sulphide flaws. It is especially deleterious in welding steel. Sulphur is usually limited to 0.05 per cent. in good grades of steel. High sulphur in structural steel is dangerous because it is likely to cause minute cracks in rolling, which are not discovered and make the material break under shock or alternate stresses in service. High-sulphur steel containing 0.10 per cent. sulphur is largely employed for parts to be threaded, especially those made in automatic screw-making machines, since the chips crumble and do not curl.

Phosphorus produces brittleness (**cold-shortness**), particularly under shock. High-carbon steel is affected more distinctly in this way than softer steel. Phosphorus, up to 0.1 per cent., increases the tensile strength and then decreases it, its effect in this respect being about equal to that of carbon, but it renders steel brittle in the case of vibratory stresses and sudden shocks. It also reduces the ductility much more rapidly than carbon. For structural purposes phosphorus should be not over 0.06 per cent. and it is customary to limit it to 0.05 per cent. for important details.

Manganese is always added to Bessemer and open-hearth steel in the proportion of from 0.3 to 0.5 per cent. to prevent the red-shortness generally considered to be due to iron oxides or iron sulphides. More than this is detrimental, although sometimes as much as 0.8 per cent. is used. It increases the strength but not so markedly as carbon. High-carbon steel, however, is affected more distinctly than low-carbon. Additions of 2 to 7 per cent. of manganese make steel very brittle, at least when the carbon content is high, but with larger amounts the ductility again returns. Commercial **manganese steel** containing from 1.25 to 2 per cent. carbon and from 12 to 13 per cent. manganese has a high tensile strength and is very hard in the sense of resistance to wear. It has a low elastic ratio, but possesses great ductility and toughness.

Forged or rolled manganese steel contains less carbon. Until recently it has been difficult to forge or roll manganese steel, but this is now being done, rails especially being manufactured in this manner. The high carbon content has been necessary hitherto, because ferro-manganese contains much carbon, which therefore unavoidably finds its way into the steel. A low-carbon manganese steel due to recently available low-carbon manganese metal is now being developed and is giving evidence of having new and useful properties, being more easily treated and worked.

Cast manganese steel is almost as brittle as glass and practically impossible to machine except with grinding wheels. Heating it to a temperature of more than 1832 deg. fahr. and rapidly cooling by plunging it into water renders it tough and ductile. Its wear-hardness is remarkable, although its resistance to indentation (as in the Brinell test) is no greater than that of ordinary annealed tool steel of the same carbon content.

The chief **uses** of manganese steel are for jaws and wearing parts of rock-crushing machinery and similar apparatus, for railroad frogs and crossings, for railroad rails on curves, mine-car wheels, dredger-bucket teeth, and burglar-proof safes. Its crystalline structure is austenitic when heat-treated, but this austenite (see p. 494) is surrounded by networks of free carbide or manganiferous cementite in the raw casting, causing great brittleness.

Silicon added to fluid steel helps to eliminate blow holes, but increases the shrinkage and the tendency to formation of pipes. It is almost universal practice to add 0.25 per cent. silicon to all steel for castings. Up to 4 per cent. it increases the magnetic permeability and reduces its retentivity. Over 6 per cent. of silicon renders steel exceedingly brittle like cast iron. **Silicon steel** has high magnetic permeability with high electrical resistance, making the steel valuable for electromagnets, electric generators, and motors. The silicon content varies between 1 and 5 per cent., but usually is 2.75 per cent., with smallest possible amounts of C, Mn and other impurities. Before the steel is ready for use, it is subjected to a double heat-treatment by first heating to between 1652 and 2012 deg. fahr. and cooling quickly, then reheating to between 1292 and 1562 deg. fahr. and allowing to cool very slowly. In some cases the second cooling extends over several days. Hadfield obtained best results by heating first to 1958 deg. fahr., cooling quickly to atmospheric temperature and then heating to 1832 deg. fahr. and cooling slowly, after which he sometimes again reheats to 1472 deg. fahr. and then cools slowly.

Silicon and manganese in moderate amounts give steel great resistance to shock. Such steel is used chiefly for springs and gears. A typical percentage analysis of **silicon-manganese steel** is as follows: C, 0.47; Si, 1.83; Mn, 0.70; P, 0.012; S, 0.004.

Nickel increases hardness, toughness, elastic ratio and tensile strength of steel, and decreases the ductility only slightly. The ratio of elastic limit to tensile strength increases with increasing nickel. It decreases corrosion and checks segregation, but it does not prevent blow holes. Over 1 per cent. gives bad welding results. It is often used in conjunction with chromium. In **nickel steels**—the most important of the alloy steels—the nickel generally ranges from 1.50 to 4.50 per cent. and usually between 2 and 3.75 per cent., while the carbon ranges from 0.20 to 0.50, usually 0.30 per cent. A typical percentage analysis of electric-furnace nickel steel is as follows: C, 0.20–0.25; Si, 0.18; Mn, 0.69; P, 0.007; S, 0.013; Ni, 3.59. The chief **uses** are for structural work in bridges, railroad rails (especially on curves), steel castings, ordnance, engine forgings, shafting (especially marine shafting), frame and engine parts for automobiles, wire cables, axles (especially for automobiles and railroad cars), etc. The crystalline structure of nickel steel is finer than that of ordinary carbon steel and cracks develop in it relatively slowly. Castings are relatively free from blow holes. The metal has a lower melting point than carbon steel, and runs more easily in the molds. Nickel steel is less liable to segregation than carbon steel.

Table 11 gives results obtained by Hadfield on nickel steels with nickel varying from 0.27 to 29.07 per cent., the carbon being kept at a low and practically constant value throughout the series. Table 12, due to Harbord, compares the physical properties of low-carbon nickel steel and best soft (flange) carbon steel.

Nickel steel containing as much as 30 per cent. nickel, according to Harbord, can be drawn into **wire** as easily as ordinary steel, and wire of this class contains sufficient nickel to make the non-corroding properties of the nickel prominent. It is well adapted to hawsers and cable service in salt water. A sample of nickel steel (0.116 in. diam.) containing 0.4 per cent. carbon and 27.8 per cent. nickel, used as torpedo defense netting by the U. S. Navy, gave the following physical test: Breaking stress, 198,700 lb. per sq. in.; elongation in 2 in., 6.25 per cent.; reduction of area, 16.5 per cent. The high tensile strength of this wire with the comparative small reduction in elongation and contraction of area below that of ordinary carbon steel of the same dimensions, indicates extreme toughness; at the same time, the material is not acted upon, or but very slightly, by salt water.

Table 11. Tests of Low-carbon Nickel Steels of Varying Nickel Contents

Analysis			Unannealed test bars			
C	Mn	Ni	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.	Elongation, per cent. in 2 in.	Reduction of area, per cent.
0.19	0.79	0.27	42,560	69,440	35	56
0.14	0.75	0.51	44,800	67,200	36	62
0.13	0.72	0.95	56,000	73,920	31	53
0.14	0.72	1.92	58,240	76,160	33	55
0.19	0.65	3.82	62,720	82,880	30	54
0.18	0.65	5.81	62,720	91,840	27	40
0.17	0.68	7.65	69,440	109,760	26	42
0.16	0.86	9.51	94,080	190,400	9	18
0.18	0.93	11.39	145,600	210,560	12	24
0.23	0.93	15.48	123,200	210,560	3	2
0.19	0.93	19.64	105,280	203,840	7	6
0.16	1.00	24.51	71,680	172,480	13	14
0.14	0.86	29.07	56,000	85,120	33	44

Table 12. Comparative Physical Properties of Low-carbon Nickel and Carbon Steels

Composition (per cent.)					Tensile strength, lb. per sq. in.	Elastic limit, lb. per sq. in.	Elongation in 8 in., per cent.	Reduction of area, per cent.
C	Ni	Mn	S	P				
0.08	2.69	0.36	0.038	0.045	65,950	47,080	24.50	51.5
0.10	....	0.27	0.039	0.040	54,430	34,910	27.42	55.4

Chromium is used in the production of chrome steel, either alone or in conjunction with vanadium or nickel. Such steels have a high elastic ratio. **Chromium steels** generally contain from 1 to 2 per cent. Cr and from 0.80 to 2 per cent. C, and are used in the hardened state. They are particularly adapted for armor-piercing projectiles on account of their hardness and very high elastic limit. Also used for armor plate, for parts of machinery, and for very hard steel plate for making 3-ply and 5-ply plate for plows and burglar-proof safes, the plates being alternated with plates of wrought iron for this purpose. Steel with 0.50 to 2 per cent. Cr is also used for automobile parts where hardness is required, *e.g.*, gears and other special parts, although a nickel-chrome steel is more generally used. Typical percentage analyses of chrome-vanadium crucible steel and of chrome-nickel electric steel are as follows:

	C	Mn	Si	P	S	Cr	V	Ni
Chrome-vanadium steel......	0.49	0.86	0.15	0.009	0.021	1.18	0.19	....
Chrome-nickel steel.........	0.48	0.44	0.15	0.009	0.009	0.98	....	2.02

Vanadium is not a good commercial deoxidizing agent, as formerly supposed (G. L. Norris). It undoubtedly has considerable deoxidizing power, but is too expensive to be used in this way. It is used as an alloy to improve the physical properties of steel, and should be added to the steel under such conditions that a minimum is lost by oxidation. It increases the elastic ratio and resistance to shock, and is largely used in conjunction with chromium. **Vanadium steels** contain generally between 0.15 and 0.25 per cent. of vanadium, and present knowledge indicates that for structural steels, *i.e.*, forging and machinery steels, there is little gained by adding higher percentages. They must be heated gradually, but are forged without difficulty, although they must be worked carefully at first. Chrome-vanadium or nickel-vanadium steels are used more than plain vanadium steels, though recent developments indicate that there will be in the future a strong trend to simple carbon-vanadium steel.

Table 13. Properties of Carbon-Vanadium Steels (Norris)

Steel	Analysis per cent.					Treatment	Yield point, 1000 lb. per sq. in.	Elastic limit, 1000 lb. per sq. in.	Ultimate strength, 1000 lb. per sq. in.	Elong. in 2 in., per cent.	Contr. of area, per cent.
	C	Mn	P	S	Va						
A..	0.47	0.90	0.012	0.020	0.15	None.....	71-92	68-85	117-131	16-17	28.5-44
						N	56-71	52-67	90-105	22-24	47-52
						O	85-92	80-86	112-123	20.5-22	50-55
B..	0.34	0.87	......	......	0.13	None.....	84.5	70	102	23.0	51.0
						N	70.5	63	86	27.0	56.5
						O	90.0	82.5	102.5	22.0	57.5

N = annealed at 1450 deg. fahr.; O = quenched at 1600 deg. fahr. and tempered at 1160 deg. (steel A), or 1100 deg. (steel B).

Table 14. Influence of Vanadium on Ordinary Mild Steel and on Mild Nickel Steel (Harbord)

Percentage analyses					Elastic limit, lb. per sq. in.	Maximum tensile strength, lb. per sq. in.	Elongation in 2 in., per cent.	Reduction of area, per cent.
Carbon	Vanadium	Nickel	Silicon	Manganese				
0.22	None	None	0.083	0.24	57,615	68,650	33.5	60.1
0.20	0.27	None	0.092	0.48	83,776	105,500	22.0	51.4
0.25	None	3.35	0.084	0.46	73,024	94,528	26.5	52.8
0.24	0.28	3.38	0.091	0.48	112,650	152,678	17.0	36.3

Table 15. Influence of Vanadium on High-carbon Steels

Percentage analyses			Mechanical results			
Carbon	Vanadium	Aluminum	Elastic limit, lb. per sq. in.	Maximum tensile strength, lb. per sq. in.	Elongation in 2 in., per cent.	Reduction of area, per cent.
1.06	0.14	0.07	96,320	152,100	6.5	6.9
1.02	0.29	0.09	96,770	170,690	8.5	10.0
1.00	0.58	0.36	145,150	191,520	7.0	7.6
1.04	0.77	0.21	131,710	188,160	7.5	9.3
0.05	0.85	0.05	45,700	58,460	37.0	72.0
0.80	1.11	0.45	120,960	172,700	10.0	17.6

Table 14 shows the influence of vanadium on ordinary mild steel and on mild nickel steel. The bars were tested as rolled, being ¾-in. rounds. Table 15 shows the influence of vanadium on high-carbon steels supplied by Wiener and tested by Harbord, a bar of pure iron with 0.85 per cent. vanadium being included in the tests for comparison. Test bars were ¾-in. rounds. Table 16 (Harbord) illustrates the effect of vanadium, both alone and when alloyed with chromium, in raising the tensile strength and the yield point without very appreciably reducing the elongation or reduction of area.

Table 16. Effect of Adding Vanadium and Chromium on the Physical Properties of Various Steels

Steels	Yield point, lb., per sq. in.	Ultimate stress, lb. per sq. in.	Elastic ratio	Elongation, per cent. on 2 in.	Reduction of area, per cent.
Crucible steels (carbon, 0.22 to 0.25 per cent.):					
Plain carbon, manganese	35,840	60,480	0.592	35.0	60.0
Plain carbon + 0.5 per cent. Cr.	51,300	76,160	0.673	33.0	60.6
Plain carbon + 1.0 per cent. Cr.	56,000	85,570	0.654	30.0	57.3
Plain carbon + 0.1 per cent. V.	63,840	77,950	0.819	31.0	60.0
Plain carbon + 0.15 per cent. V.	68,100	81,760	0.833	26.0	59.0
Plain carbon + 0.25 per cent. V.	76,380	88,030	0.867	24.0	59.0
Plain carbon + 1.0 per cent. Cr. + 0.15 per cent. V.	81,090	108,860	0.745	24.0	56.6
Plain carbon + 1.0 per cent. Cr. + 0.25 per cent. V.	110,660	135,300	0.818	18.5	46.3
Nickel steel	58,240	94,080	0.619	24.0	50.0
Nickel steel + 0.2 per cent. V	116,700	129,700	0.899	20.5	52.4
Open-hearth steels (carbon, 0.25 to 0.27 per cent.):					
Plain carbon, manganese	39,650	72,130	0.549	34.0	52.6
Plain carbon + 1.0 per cent. Cr. + 0.15 per cent. V.	77,060	117,820	0.654	25.0	55.5

Chrome-vanadium steels combine the good effects of chromium and vanadium in the same steel. Table 17 (Stoughton) gives the composition of five types of such steels, all of which should contain as little sulphur and phosphorus as possible ($S \leq 0.035$ per cent.). With $P = 0.02$ per cent., Si may be 0.15 per cent. in *D* and 0.10 in *A*, *B* and *C*. With P at 0.03 per cent., Si should not exceed 0.05–0.06 per cent. in *A*, *B* and *C*, or 0.10 in *D*.

Table 17. Types of Chrome-vanadium Steel

Percentage composition	Type *A*	Type *B*	Type *C*	Type *D*	Type *E*
Carbon	0.25–0.30	0.20	0.20	0.45–0.55	0.12–0.15
Manganese	0.40–0.50	0.30–0.40	0.40	0.80–1.00	0.20
Chromium	1.00	0.50	0.80	1.25	0.30
Vanadium	0.16–0.18	0.12	0.16	0.18	0.12

Type *A* is designed for use in light axles, connecting rods, side and main rods, driving axles and piston rods, and should be annealed at 1472 deg. fahr. for 1 or 2 hr. and then cooled in air or ashes according to the nature of piece. It may be used for crankshafts, transmission parts and crank pins after quenching from 1652 deg. fahr. in lard or fish oil, annealing at 1022 deg. fahr. for ½ to 2 hr. and then cooling in air. Also for gears in constant mesh when not unduly pressed in service, after quenching from 1742 deg. in lard oil, tempering at 680 deg. for 15 to 30 min. (preferably in a lead bath) and cooling in air.

Type *B* is used in axles, hammer rods, bolts and in places where great resistance to torsion is demanded. Type *C* is an intermediate steel useful for car axles, holding bolts, etc.

Type *D* is used for solid wheels for railway use, gun barrels, crank pins, etc.; it should be annealed for 1 hr. at 1472 deg., and cooled slowly, taking great care not to chill or to pass from 1472 deg. to 1112 deg. too quickly. It may also be used for automobile, carriage and locomotive springs when quenched in oil from 1652 deg., tempered at from 752 to 842 deg. (preferably in a lead bath), and cooled in air.

Type *E* is a steel for all engine and machine parts that are to be case-hardened. Treatment: ordinary case-hardening process.

Table 18 gives some results of mechanical tests of typical chrome-vanadium and other steels for automobile purposes. Steel No. 3, designated as Type *A*, No. 1, is the Type *A* steel given in Table 17. Steel No. 4, designated as Type *A*, No. 2, is the same steel with the heat-treatment recommended for crank shafts. No. 5 steel, designated as Type *A*, No. 3, is the same steel with heat treatment recommended for gears in constant mesh.

Table 18. Results of Mechanical Tests of Typical Chrome-vanadium and Other Steels (Stoughton)

Test	No. 1. Carbon "axle" steel	No. 2. Nickel "axle" steel	No. 3. Vanadium crank-shaft steel. (Type *A*, No. 1)	No. 4. Vanadium crank-shaft steel. (Type *A*, No. 2)	No. 5. Vanadium gear steel, continual mesh. (Type *A*, No 3)
Yield point, lb. per sq. in.	41,330	49,270	63,570	110,100	224,000
Ultimate tensile strength, lb. per sq. in.	65,840	87,360	96,080	127,800	232,750
Elastic ratio, per cent.	62	56	66	87	96
Elongation on 2 in., per cent.	42	34	33	20	11
Contraction of area, per cent.	61	58	61	58	39
Torsional twists	2.6	3.2	4.2	2.5	1.8
Alternating bends	10	12	18	10	6
Pendulum impact, ft.-lb.	12.3	14	16.5	12	6
Alternating impact, number of stresses	960	800	2,700	1,850	800
Falling weight on notched bar, number of blows	25	35	69	76	....
Rotary vibrations, number of revolutions	6,200	10,000	67,500	....	....

Molybdenum increases the hardness of steel by keeping the carbon in solution. It is used especially in conjunction with tungsten in the production of high-speed cutting tools.

Tungsten also increases the hardness of steel by keeping the carbon in solution. It is largely used in the production of high-speed cutting tools. It increases the magnetic retentivity of high-carbon steel. Steel of approximately the following (percentage) analysis is largely used for permanent magnets: C, 0.60 to 0.75; Si, 0.15; Mn, 0.32; P, 0.015; S, 0.015; W, 5.0.

Aluminum in small quantities (about 0.10 per cent.) increases the fluidity of steel, reduces the iron oxide present, and tends to the elimination of blow holes, but greatly increases the tendency to pipe. Over 0.85 per cent. reduces toughness.

Titanium removes nitrogen and oxides from steel and goes chiefly into the slag as TiO_2. About 0.1 per cent. is usually added when used at all.

Copper increases the red-shortness of steel when much sulphur is present, but otherwise small amounts are not deleterious. It is said to increase the resistance of steel to corrosion.

Oxygen in the form of iron oxide dissolved in iron makes the metal both red-short and cold-short; in combination with manganese this effect is not so noticeable. Ledebur states that iron containing combined oxygen up to 0.10 per cent. can be worked, but that above this the metal is bad.

Nitrogen makes steel brittle, but little is known about it in a quantitative way.

Table 19 gives a résumé of the known effects of the chemical composition upon the physical properties of steel. A plus sign indicates that the element increases the value of the physical property, a minus sign that it decreases it, and a zero that there is no effect, so far as known.

Table 19. Influence of Chemical Composition on the Physical Properties of Steel

	Tensile strength	Elastic ratio	Ductility	Hardness	Hardening power	Brittleness	Resistance to shock	Resistance to vibratory stresses	Forgeability, absence of red-shortness	Weldability
Carbon	(1)	+	−	+	(2)	+	(12)	+	−	−
Phosphorus	(13)		(3)	0		(4)	−	−	0	
Manganese	+	−		+					(5)	
Sulphur				0					−	−
Silicon			(6)	0					(7)	
Aluminum			(8)	0						−
Vanadium	+	+	(9)				+	+		
Chromium	+	+		+			+	+		
Molybdenum	+			+						
Tungsten	+	+		+						
Nickel	+	+	(9)	+			+	+		(10)
Copper				0					(11)	
Oxygen						+			−	
Nitrogen						+				

(1) + up to 1 per cent., then −; (2) + up to 0.85 per cent.; (3) 0 up to 0.1 per cent.; then −; (4) + over 0.1 per cent.; (5) + up to 0.5 per cent.; (6) − above 1.5 per cent.; (7) − over 6 per cent.; (8) − over 0.85 per cent.; (9) − slightly; (10) − over 1 per cent.; (11) − from 2 to 7 per cent.; (12) + up to 0.3 per cent., then − ; (13) + up to 0.1 per cent., then −.

Self-hardening Steel is steel which is hard without being subjected to any quenching treatment or other hardening process. It can be made soft enough for machining only by long and careful annealing. It is often called **air-hardening steel,** because when it cools in the air from a red heat or above it is not soft like ordinary steel but is hard and capable of cutting other metals. All self-hardening steels are austenitic and non-magnetic. A typical analysis of self-hardening **Mushet steel** shows 9 per cent. tungsten, 2.50 per cent. Mn, and 1.85 per cent. C. Steels of this type are used very largely at the present time for very heavy or deep cuts and especially for cutting extra-hard metal, such as roughing cuts on armor plate and other hard alloys. The cutting speed is not much if any greater than that for ordinary carbon tool steel, but the economy of its use is due to the fact that it will take very deep cuts and requires less frequent regrinding. The 2.50 per cent. of manganese in Mushet steel can be replaced by 1 or 2 per cent. of chromium, giving a self-hardening tool steel which has the same advantageous properties. The 9 per cent. of tungsten may be replaced by from 4 to 6 per cent. of molybdenum, this latter increasing the toughness.

High-speed Steels (see also pp. 1430 and 1458) are alloy tool steels of remarkable cutting power. The following table (mainly deduced from Hibbard, Bull. 100, Bureau of Mines) shows the composition of high-speed steels made since 1913. Steel A represents the variation in composition in 16 American steels from nine different makers. Steel B represents the average analysis from one important American maker; C the average of two English samples, and D the average of three German samples.

Analyses of High-speed Steels (Percentage Composition)

Steel	C	Mn	Si	S	P	Cr	W	V	Co
A	0.64–0.77	0.14–0.45	0.07–0.40	0.02–0.04	0.01–0.05	3.30–5.28	13.30–19.10	0.45–1.35	2.70–5.28
B	0.65	0.31	0.27	0.013	0.007	2.99	16.87	0.85–1.0	
C	0.68	0.22	0.26	0.025	0.02	2.95	18.46	1.10	
D	0.66	0.11	0.18	0.023	0.027	4.30	15.73	0.78	4.05

In one sample of steel A, Mn was 2.29 and in another V was 2.50; in a few of the steels Ni from 0.17 to 0.28 occurs (probably added accidentally); in steel A the cobalt was found in only 5 samples. Mo was found in 2 of the German steels, D, 0.67 and 0.72 respectively.

In recent high-speed steels, carbon generally varies between 0.40 and 0.80 per cent., the majority of steels containing about 0.65 per cent. For cutting hot steel of comparatively low carbon content, the cutting tool should have about 0.45 per cent. carbon. Chromium varies between 2 and 6 per cent. in American steels, while sometimes reaching 9 per cent. in Europe. Molybdenum gives a fine cutting edge (its effect is about twice that of tungsten) but its use is being discontinued since tools containing it are liable to crack in quenching and to develop physical imperfections. The vanadium content usually varies from 0.8 to 1.75 per cent.; like chromium, it increases the red hardness of the cutting edge and the life of the tool before regrinding. Cobalt up to about 4 per cent. is said to increase the ability to hold the cutting edge in work and to increase the red hardness.

High-speed steel can be worked satisfactorily under hammer or press at a temperature of about 2150 deg. fahr. (1180 deg. cent.). It may be annealed by machining by heating carefully to 1472 deg. fahr. with extreme slowness and cooling very slowly. The modern heat treatment for high-speed tools is to heat carefully to a temperature near the melting point and quench in oil. Cooling by air blast, and double treatment, formerly recommended, are now uncommon. Milling cutters and similar tools are generally tempered. High-speed steels are austenitic after heat treatment and become harder in use or after moderate tempering.

Gledhill found that a certain high-speed steel after being annealed 12 or 18 hours at 1400 deg. fahr. had the following physical properties: E. L., 98,600 lb. per sq. in.; T. S., 129,200; elong. 18 per cent. in 2 in.; contr. of area, 35 per cent.

In addition to the large use of high-speed steels for cutting tools they have given excellent results in exhaust valves for automobile engines and for the dies used in the manufacture of extruded brass which is near its fusion point when it is forced through the die.

Influence of Mechanical Treatment on the Physical Properties of Steel

Hot Work consists of mechanical treatment such as forging, rolling, pressing, etc., at temperatures above the thermal critical range. It is applied to ingot metal either shortly after solidification or after cooling and reheating to the working temperature of about 2150 deg. fahr. (1180 deg. cent.) It increases tensile strength and ductility, and especially resistance to shocks and vibratory stresses. The finishing temperature is important and should be as near as practicable to the thermal critical range and above it, the exact temperature depending upon the carbon content. If the carbon is low enough, blow holes are closed and probably welded together, thus increasing the soundness. It also corrects fragility due to unequal cooling stresses in the ingot. In large sections the finishing temperature should be so regulated that the central portions will not suffer from the weakening influence of too high a finishing temperature, while at the same time the outside will not suffer unduly from the effect of cold-working. This is especially important for rails or other sections of uneven thickness.

Both wrought iron and steel are rendered exceedingly brittle in the cold state by working at a "blue heat" (430–600 deg. fahr., or the temperature range of the color scale for tempering carbon steel), soft steel being the least affected. Boilersmiths commonly desist from working a plate when it has cooled below a temperature at which the mark made by rubbing it with a piece of wood will glow.

The **effect of rolling** is to stretch steel at the same time it is compressed. **Hammering** is more effective than rolling, but the metal at the interior is not as effectually worked as that on the exterior. This is especially true when the hammer is not large or heavy enough for forging. Drop-forgings are usually heated to high temperatures and the amount of reduction on the forgings is comparatively small. For this reason only well-worked blooms or bars of soft steel should be employed for this purpose. **Pressing** more effectively works the steel than either rolling or hammering, and, as all of the energy of the press is utilized in the compression of the steel and the metal is worked to a greater depth, this method is the best for large sections.

Rolling, hammering, and pressing all reduce the size of the grain of the steel and, if thoroughly done, effectively break up the ingotism or coarse crystallization of a cast ingot as well as the coarseness of grain which may have resulted from heating the forging or bloom to too high a temperature during some previous operation. It is usual to roll blooms from ingots having at least four times the cross-section of the resulting bloom, and blooms are hammered or pressed into forgings with reductions varying in the proportion of from 100 to 75 to 100 to 50. Increasing the amount of mechanical hot work beyond certain moderate limits has little influence on the physical properties. The effects of rolling are shown in the comparison of plates of different gages. Taking ½ in. as a basis, there will be the following changes in the physical properties for every increase of ¼ in. in thickness: (1) A decrease in ultimate strength of 1000 lb. per sq. in.; (2) a decrease in elongation of 1 per cent. when measured in an 8-in. parallel section; (3) a decrease in reduction of area of 2 per cent.

Cold Work is work done on the metal while its temperature is below the thermal critical range. It includes cold-rolling, cold-pressing, twisting and wire-drawing. It increases greatly the elastic limit and tensile strength, especially the former, but greatly increases the brittleness. Tensile strengths after **wire-drawing** are substantially as given in Table 20.

The modulus of elasticity E_1 of wire ropes is less than that of the wires composing them, as follows: Single-strand (1 twist), $E_1 = 0.6E$; strands twisted about around a core—ordinary wire rope (double twist), $E_1 = 0.36\,E$; wire ropes twisted into cable (triple twist), $E_1 = 0.216\,E$. Special qualities of tempered and improved cast-steel wire may attain to 300,000 to 340,000 lb. per sq. in. A test on a piano wire of 0.0284 in. diam. developed a tensile strength of 462,870 lb. per sq. in. Steel used for wire-drawing generally contains from 0.05 to 0.90 per cent. carbon. The effects of **cold-twisting** steel bars are shown in Table 21.

Table 20. Properties of Iron and Steel Wire

	Tensile strength, lb. per sq. in.	Elastic limit, lb. per sq. in.	E, lb. per sq. in.
Iron wire, hard-drawn	80,000–100,000	60,000	28,400,000
Iron wire, annealed*	57,000	28,400	28,400,000
Bessemer steel wire, hard-drawn	92,000	74,000	....
Bessemer steel wire, annealed	57,000– 85,000	32,000	30,500,000
Crucible steel wire	128,000–270,000	142,000	30,500,000
Crucible steel wire for farm plow traction	360,000	....	....
Crucible steel wire: "plow-steel" wire	256,000	....	....
Crucible steel wire: conveying cables	163,000–185,000	....	....
Galvanized steel telegraph wire	192,000	....	....

* Galvanized iron telegraph wire.

Table 21. Increase of Strength of Steel Bars Produced by Cold-twisting

(J. J. Shuman, *A. S. T. M.*, 1907)

	Soft Bessemer bars, ½-in. sq.* (T. S. of plain bar, 60,400 lb. per sq. in.)					0.25 carbon Bessemer bars, ½-in. sq.* (T. S. of plain bar, 75,000 lb. per sq. in.)				
No. of turns per lin. ft.	3.0	4¾	5.0	5¾	5⅞	3.0	4½	4⅞	5	5½
Yield point, 1000 lb. per sq. in.	65.6	72.4	84.8	84.8	80.8	83.6	83.2	88.8	84.2	84.2
Ult. tensile strength, 1000 lb. per sq. in.	83.2	89.6	92.0	90.0	88.8	99.6	99.2	104.0	102.0	100.8
Elongation in 8 in., per cent.	10.0	5.75	6.25	7.50	3.75	8.0	4.5	4.0	5.75	6.0
Soft Bessemer Steel Bars, Different Sizes										
Size, in. sq.	¼	⅜	½	⅝	¾	⅞	1	1¼	1¼	
No. of turns per lin. ft.	4	3½	3	2¼	1½	1¼	1	⅞	¾	
Yield point increase, per cent.	111.0	82.0	64.0	83.0	85.5	77.0	82.0	64 0	59.0	
Ult. strength increase, per cent.	37.0	38.6	41.0	33.5	34.3	29.7	22.8	20.1	28.9	

* Bars twisted off when given more turns than stated.

Effect of Repeated Stresses. Little information is available on the effect of repeated stresses on the physical properties of steel. J. B. Kommers, experimenting with the Landgraf-Turner machine, obtained the results given in Table 22, which illustrate the influence of increasing carbon content, of increasing phosphorus content, and the effect of cold work on the resistance to repeated stresses. He suggests a "quality factor," which is the product of the unit stress to which the specimen is subjected, multiplied by the number of cycles required for rupture and divided by 10,000.

E. Nusbaumer (Carnegie Scholarship Memoirs, Vol. 6, 1914, *Jour. Iron and Steel Inst.*) finds for the steels commonly employed in industrial practice, as follows:

1. In the annealed state the resistance to rotary bending stresses, to repeated shocks, or to alternating bending (vibrations) is proportional to the percentage of carbon in steels when lower than 0.25 to 0.30 per cent. Above this percentage the ratio between the percentage of carbon and the resistance becomes inverse as regards repeated shocks, but remains as it was in regard to rotary bending and alternating bending (vibrations).

Steels with a low percentage of nickel generally display a higher resistance to repeated shocks and to alternating bending (vibrations) than carbon steels. High-nickel steels resist repeated shocks of small intensity remarkably well in an untreated condition. Nickel-chromium steels offer a yet higher resistance than nickel steels to repeated shocks and to alternating bending (vibrations), the dead-mild steel displaying the maximum resistance.

2. Quenching, followed by annealing, raises the resistance of plain carbon steels and of steels containing small percentages of nickel to repeated shocks and to alternating bending (vibrations). It tends to diminish the resistance of nickel-chromium steels. Quench-

Table 22. Results of Tensile Tests and Repeated-stress Tests

(J. B. Kommers, Int. Assn. for Test. Matls., VI Congress, 1912)

Kind of material	Percentage composition				Yield point, lb. per sq. in.	Ultimate strength, lb. per sq. in.	Elongation, per cent.	Reduction of area, per cent.	Average repeated-stress tests	Quality factor
	C	P	S	Mn						
Wrought iron	....	......	......	....	36,200	50,700	26.0	40.6	148	625
Carbon steel	0.10	0.019	0.035	0.44	36,200	49,800	41.0	66.8	579	2130
Steel*	0.22	0.007	0.026	0.47	40,700	66,900	39.0	60.0	481	2130
Carbon steel	0.28	0.016	0.039	0.47	43,900	66,400	35.0	63.3	742	3410
Carbon steel	0.48	0.029	0.04	0.44	49,100	79,100	23.5	42.3	726	4430
Carbon steel	0.71	0.03	0.027	0.39	58,600	120,200	15.5	24.5	442	4350
Cold-rolled steel†	....	......	......	....	91,000	100,200	......	40.5	248	2355
Cold-rolled steel‡	....	......	......	....	39,500	61,300	34.2	62.8	461	1920
Phosphorus steel§	....	0.045	......	....	42,200	58,200	26.5	59.2	550	2450
Phosphorus steel§	....	0.088	......	....	40,600	58,000	34.5	72.7	374	1517
Phosphorus steel§	....	0.153	......	....	43,750	64,400	35.5	70.7	443	1940
Phosphorus steel§	....	0.198	......	....	42,100	65,000	33.0	69.2	390	1650

* Silicon content, 0.12 per cent.
† Not annealed.
‡ Annealed at red heat.
§ Specially prepared from very-low-carbon steel and phosphorus, annealed at red heat.

Table 23. Endurance of Rotating Shafts

Percentage composition		Physical properties				Treatment	Fiber stress, lb. per sq. in.	Number of rotations
C	Ni	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.	Elongation, per cent.	Contraction, per cent.			
0.26	3.282	51,500	81,370	28.75	61.09	Annealed	40,000	1,847,500
0.26	3.282	66,950	90,640	26.90	65.04	O. Q. & A*	40,000	1,815,200
0.25	4.514	61,610	95,490	24.80	57.13	Annealed	40,000	2,366,000
0.25	4.514	101,860	120,190	20.80	60.05	O. Q. & A.	40,000	3,296,700
0.29	5.661	80,090	108,840	22.50	58.71	Annealed	40,000	4,388,400
0.29	5.661	117,610	131,410	19.65	58.88	O. Q. & A.	40,000	3,795,200
0.539	27.353	48,060	104,820	47.50	63.10	Annealed	40,000	2,495,600
0.539	27.353	46,850	97,780	43.35	60.80	O. Q. & A.	40,000	1,088,200
0.24	....	40,560	71,240	32.30	59.81	Annealed	40,000	229,300
0.24	....	45,170	74,440	33.15	69.93	O. Q. & A.	40,000	348,000
0.42	....	44,290	80,885	23.00	56.70	Annealed	40,000	225,900
0.42	....	55,000	92,180	26.05	57.22	O. Q. & A.	40,000	655,600
0.46	....	48,060	94,600	21.15	47.65	Annealed	40,000	976,600
0.46	....	61,110	102,880	23.05	51.27	O. Q. & A.	40,000	1,657,500
0.66	....	65,205	124,200	7.15	17.28	Annealed	40,000	3,689,000
0.66	....	92,040	154,920	13.50	31.48	O. Q. & A.	40,000	4,323,600
1.094	....	....	....	....	....	....	30,000	50,000,000
0.733	....	....	....	....	....	....	30,000	12,547,600
0.824	....	....	....	....	....	....	30,000	16,336,200
0.824	....	....	....	....	....	....	35,000	13,871,000
0.94	....	....	....	....	....	....	35,000	19,152,300

* O. Q. & A. = Oil-quenched and annealed.

ing not followed by annealing raises the resistance of mild carbon steels, and particularly of steels with small percentages of nickel, to repeated shocks. Nickel-chromium steels, particularly the softer grades, also have their resistance increased to a notable degree. It diminishes the resistance of semi-hard nickel-chromium steels, and still more of hard carbon steels with a high percentage of nickel. Quenching not followed by annealing diminishes considerably the resistance of plain carbon steels and of steels with small percentages of nickel to alternating bending (vibrations). It raises, on the other hand, the resistance of nickel-chromium steels.

3. The resistance of a metal to repeated stresses is proportional to its resistance to the simple impact test for notched bars (Frémont test). Austenite is the particular microscopic constituent in steel which offers the greatest resistance to repeated shocks; ferrite offers a far lower degree of resistance, hence the poor results obtained with very soft steels. Martensite is the one whose resistance is lowest, and, generally speaking, very close to zero.

Table 23 gives results of experiments made by the United States Government at the Watertown Arsenal on the endurance of rotating shafts about 1 in. in diameter. It probably gives the best results that can be obtained for the various grades of steel, and shows that a great increase in endurance follows the use of material having a high elastic limit.

Influence of Heat Treatment on the Physical Properties of Steel

The curves given in Figs. 1 and 2 were obtained by Kollmann and Howard, respectively. They show the tensile strength of wrought iron and steels of various carbon percentages at different temperatures, as follows: *aa*, Bessemer steel (0.23 C); *bb*, steel (0.62 C); *cc*, wrought iron (0.1 C); *dd*, steel (0.97 C, 0.8 Mn); *ee*, steel (0.37 C, 0.7 Mn); *ff*, steel (0.09C, 0.11Mn); *gg*, wrought iron. See also p. 543.

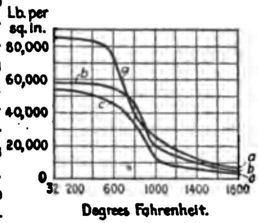

FIG. 1.

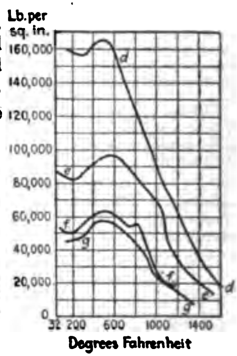

FIG. 2.

Annealing. Steel castings are annealed for the purpose of refining the very coarse microstructure (ingotism) characteristic of all metals cooled slowly through and below the solidification point. The treatment also removes strains due to uneven cooling. Annealing increases the tensile strength and elongation, and particularly resistance to shock. It reduces the hardness. The practice recommended by the American Society for Testing Materials (A. S. T. M. yearbook, 1915, adopted 1914) for the annealing of **carbon-steel castings** is as follows: Castings should be heated slowly and uniformly to temperatures as below.

Carbon content, per cent	up to 0.16	0.16–0.34	0.35–0.54	0.55–0.79
Temperatures, deg. fahr. (cent.)	1697(925)	1607(875)	1562(850)	1526(830)

Temperatures of 122 deg. fahr. (and in special cases 212 deg.) higher than those just given are allowable when necessary. Castings should be kept at maximum temperature sufficiently long to insure the refining of the grain. In general, the heavier the sections the longer must be the time of exposure to maximum temperature. Castings should be cooled slowly and uniformly in the furnace when maximum softness is desired, but may be cooled at an accelerated rate when it is desired that the steel possess rather higher tensile strength and elastic limit than can be procured by very slow cooling. The cooling must be so conducted as to leave the steel reasonably free from cooling stresses.

Objects of **rolled and forged carbon steel** are annealed principally to remove existing coarseness of grain, especially in large sections where the work has not penetrated the metal sufficiently or where the finishing temperature is high. The practice recommended by the A. S. T. M. (yearbook, 1915, adopted 1911) is as follows:

Carbon content, per cent	< 0.12	0.12–0.29	0.30–0.49	0.50–1.00
Annealing temperature, deg. fahr	1607–1697	1544–1598	1499–1544	1454–1499
Annealing temperature, deg. cent	875–925	840–870	815–840	790–815

Heating should be slow so as to allow all parts of metal to reach the annealing temperature without heating the outside needlessly high, which tends to recoarsen the grain. An exposure of 1 hr. at the maximum temperature should be long enough for pieces 12 in. thick. The rate of cooling depends on the carbon content and on the size of the metal. The higher the carbon the slower should be the cooling, and the larger the section the more the bulk will naturally retarded the cooling. The slower the cooling the softer and more ductile the metal will be and the lower will be its tensile strength, elastic limit, and yield point. The greatest softness and ductility are obtained at a certain sacrifice of strength and elasticity, and *vice versa*. For most purposes, neither of these extremes is desired, and the cooling must be so regulated as to give the desired results in any particular case.

Cold-rolled or cold-drawn objects, especially **wire**, are often annealed to remove the great brittleness induced by the mechanical treatment. The A. S. T. M. recommends that the object be heated to about 1427 deg. fahr. (775 deg. cent.) and cooled with a slowness which should increase with the thickness. Thin plates may be heated merely to 1337 deg. fahr. (725 deg. cent.), followed immediately by slow cooling. In the case of thick forgings, the temperature of 1427 deg. fahr. may be maintained for several hours, though always at the cost of superficial decarburization. Steel containing less than 0.15 per cent. carbon should be annealed at 1652 deg. fahr. (900 deg. cent.). Great brittleness may be caused by annealing very-low-carbon steel in the neighborhood of 1290 deg. fahr. (700 deg. cent.) after cold-working.

Ordinary low-carbon steel is unsuitable for manufacture of screws or other inexpensive materials made chiefly by automatic machines. A special grade of steel high in phosphorus and sulphur is therefore indicated for **screw stock**, because the chips are brittle and clear themselves readily from the tool. (See S. A. E. and A. S. T. M. specifications, pp. 460 and 464.) This material is easily machined and cheap, but lacks strength and toughness and is not safe for vital parts. Screws made from hot-rolled bars of this material should be heat-treated and not used in an annealed condition. Screws made from cold-rolled or cold-drawn bars are much stronger, but the best results, in either case, are obtained by the following heat-treatment: After machining, heat to 1500 deg. fahr.; quench; reheat from 600 to 1300 deg. fahr., and cool slowly.

Hardening consists in heating steel above its thermal critical range and cooling suddenly (quenching) in water, oil, or other cooling medium which absorbs heat rapidly. Quenching steel increases its hardness, more especially as the carbon content increases. It also raises the elastic limit and tensile strength and reduces the ductility. It induces cooling strains and the metal is apt to be very brittle, especially with carbon 0.5 per cent. or over. The **effect of carbon on the physical properties of hardened steel** is shown in Table 24.

Table 24. Effect of Carbon on the Physical Properties of Hardened Steels

Carbon, per cent.	Tensile strength in soft state			Tensile strength when hardened		
	Ultimate, lb. per sq. in.	Elastic limit, lb. per sq. in.	Elongation in 2 in., per cent.	Ultimate, lb. per sq. in.	Elastic limit, lb. per sq. in.	Elongation in 2 in., per cent.
0.10	60,300	36,300	29	66,400	40,300	24
0.14	61,500	35,200	27	73,100	39,600	22
0.23	66,500	41,200	26	99,400	54,000	14
0.52	97,800	52,600	20	132,100	81,400	9
0.60	116,400	66,500	14	153,400	102,100	4
0.72	130,700	75,800	9	180,100	105,500	0

The purpose of hardening is to adapt high-carbon steel for cutting-tool purposes, or to increase the elastic limit and tensile strength in low-carbon steel, but it is seldom applied without subsequent tempering or annealing. Hardening refines markedly the microstructure of steel, and for this reason should be done on a rising temperature. In order to obtain maximum results, the temperature of quenching should be above the line *GOS* in the iron-carbon diagram described on p. 493. This line is often called Ac_3.

Tempering consists in reheating hardened or quenched steel to temperatures below the critical range for the purpose of restoring partially its ductility and softness. The rate of cooling is unimportant, although the material is generally quenched in water as a matter of convenience. The elasticity and tensile strength in tempered steels are reduced somewhat below the values for hardened steel, although they are greater than those for the original rolled or forged metal. It is generally necessary to use a pyrometer or other accurate means of measuring temperatures for annealing and hardening, but tempering temperatures are easily controlled by the examination of the surface of the metal, which has previously been made bright. A thin film of oxide forms upon the steel, the color of this film varying with the tempering temperature.

The **tempering temperatures and colors of oxide films for various tools,** arranged approximately in their order on the color scale are as follows: Scrapers for brass, steel-engraving tools, light turning tools, hammer faces, planer tools for steel, planer tools for iron, wood-engraving tools, drills, milling cutters: 437 to 455 deg. fahr. (225 to 235 deg. cent.); light straw to straw. Wire-drawing plates, boring cutters, screw-cutting dies, taps, rock drills, mill chisels and picks, punches and dies, reamers, shear blades, half-round bits, planer knives and molding cutters (to be ground), gouges, plane irons: 456 to 482 deg. fahr. (236 to 250 deg. cent.); dark straw to yellow-brown. Twist drills, flat drills, wood-boring cutters, drifts, cup tools, edging cutters: 483 to 527 deg. fahr. (251 to 275 deg. cent.); yellow-brown to dark purple. Wood bits and augers, cold chisels for steel, axes and adzes, gimlets, cold chisels for cast iron, needles, wood chisels, hack saws, cold chisels for wrought iron, planer knives and molding cutters (to be filed), circular saws for metal, screwdrivers, springs, saws for wood: 528 to 572 deg. fahr. (276 to 300 deg. cent.); dark purple to full blue.

Combined Hardening and Tempering. In tools like chisels where the edge of the tool only is to be hardened, this portion is first heated just above the critical range and then the extreme point only is quenched in water until it is black, after which it is withdrawn and rubbed bright upon a piece of sandpaper or a brick. This is done merely to give a bright surface on which to observe the play of the temper colors. The heat from the shank now begins to creep down into the point, which shows the various temperature colors in order. At the proper time the whole tool is plunged into water to prevent more heat from coming down to the tempered point. It has nothing to do with the tempering operation itself.

Double Quenching. When steel of any carbon content is quenched from above the critical range and then reheated to a point just below the range and quenched again, it is said to have been subjected to "double quenching." The operation is sometimes called "double annealing," but it is in reality a quenching and prolonged tempering operation, since the term annealing refers strictly to comparatively slow cooling from *above* the critical range. It is used principally on steels with 0.5 per cent. carbon or less, but may be applied to steels with higher carbon for the purpose of increasing the resistance to wear while giving a moderate combination of elasticity, tensile strength, and ductility. In the lower-carbon steels, it greatly increases the toughness, elasticity, tensile strength, and resistance to shock, while retaining for the metal considerable ductility. It has been applied successfully to crank shafts as large as 8 in. in diameter, and, although expensive, is coming more and more into use for high-grade mechanical parts. Great care must be observed, however, in carrying out the operations, otherwise incipient quenching cracks will be formed, which later lead to fractures.

The **quenching temperature** should be well above the critical range, and the heating should be slow up to and through the critical range. The quenching may be done in water for very-low-carbon steel, or in oil for higher-carbon, and the piece should be removed from the quenching bath before it reaches 212 deg. fahr. (100 deg. cent.), and preferably above 320 deg. fahr. (160 deg. cent.). This lessens the danger of cracking. The second heating should begin a very few hours after the first quenching, and if possible without ever allowing the piece to cool below 212 deg. fahr. For very high elastic limit and tensile strength, the second heating should be made at from 932 to 1202 deg. fahr. (500 to 650 deg. cent.). In this case the ductility will be low. Some steels, such as watch springs and shafting, are annealed at 662 deg. fahr. (350 deg. cent.). Little commercial annealing is done below 932 deg. fahr. (500 deg. cent.). For tensile strength, elastic limit, and ductility best suited to the majority of cases, the second heating should be at from 1112 to 1202 deg. fahr. (600 to 650 deg. cent.). For the greatest ductility with great strength and elastic limit, the second heating should be at from 1337 to 1382 deg. fahr. (725 to 750 deg. cent.). The object should be held at the second heating temperature long enough not only to allow its interior to reach that temperature, but also to relieve the stress given in the water- or oil-quenching. For pieces of moderate thickness, a 4-hr. exposure should suffice. The above practice is that recommended by the American Society for Testing Materials.

Factors in the Hardening of Tool Steel. Mathews (*Trans. A. S. M. E.*, vol. 36) states that the **carbon content** of tool steels (carbon steels made by the crucible or electric process) ranges from 0.60 to 1.50 per cent. (0.75 to 1.35 per cent. for nine-tenths of the steels used). If the **time of heating** is too short, the temperature of the piece is not uniform and the grain size may be variable. If the piece is held for too long a time above the critical range, the large grain size thus produced is retained on quenching and the result is a weak or cracked piece. The martensitic structure desired in a hardened steel requires **quick cooling** through the thermal critical range. As regards commercial **quenching media,** pure water has a fairly constant quenching rate up to a temperature of 100 deg. fahr., where it begins to fall off. Brine solutions have a quicker rate of cooling and the quenching rate does not begin to fall off seriously below 150 deg. fahr. At 70 deg. fahr. a brine solution quenches about 10 per cent. faster than water. Oils generally have a more constant rate of cooling throughout a larger temperature range than water or brine, but are slower in their quenching powers.

To produce the same hardness a larger section must be heated hotter than a smaller one. Mathews found that a 3/16-in. round bar of a particular steel would harden at 1395 deg. fahr., while a 3/4-in. bar required a temperature of 1450 deg. It is necessary to draw the temper of a tool steel *promptly* after hardening, otherwise the tool may burst and fly apart. The **effect of tempering on hardness** was found by Heyn for a particular steel to be as follows (results in per cent. of original hardness):

Tempering temp., deg fahr..........	212	392	572	752	932	1112
Per cent. decrease in hardness.......	2.5	14	41	70	87.5	97.5

The **effect of time on the tempering process** is shown by Mathews in the following table [Standard 1/2-in. round A. S. T. M. test pieces were quenched in oil from a constant temperature (1550 deg. fahr.) and the temper drawn in the same salt bath at 800 deg. for the number of minutes indicated]:

Time, min.	Elastic limit, lb. per sq. in.	Max. strength, lb. per sq. in.	Elongation, per cent.	Reduction of area, per cent.	Brinell hardness number
8	228,750	260,137	2.5	.....	425
20	201,125	214,562	11.6	45.40	390
40	175,000	183,187	12.0	49.35	340

The following table (Mathews) shows the **influence of mass on the tempering operation.** It also shows that the greater the initial hardness of the piece, the more marked is the effect of drawing the temperature.

Table 25. Hardness of Tool Steel (Brinell Numbers)

Steel No.	Untempered				Tempered at 600 deg. fahr.				Tempered at 1200 deg. fahr.			
	Size of test piece, inches											
	1/2	1	2	3	1/2	1	2	3	1/2	1	2	3
1	400	270	235	215	330	230	215	195	230	205	190	180
2	465	442	407	402	340	339	331	329	216	210	205	204
3	542	494	438	404	420	400	380	368	299	288	273	252
4	591	505	391	350	435	422	376	325	355	342	311	289

Percentage Composition:	C	P	Si	S	Mn	Ni	Cr	V
Steels Nos. 1 and 2........	0.25	0.01	0.09	0.012	0.67	3.47	....	
Steel No. 3...............	0.50	0.01	0.16	0.011	0.44	2.02	1.00	......
Steel No. 4...............	0.49	0.007	0.14	0.010	0.74	......	1.18	0.18

Heating temperature for Nos. 1 and 2, 1600 deg. fahr.; No. 3, 1550 deg.; No. 4, 1700 deg. Quenching medium for No. 2, water; for all others, oil.

Case-hardening. Where steel with a very hard surface is desired, as in gears, bearings, armor plate, etc., a combination of hard exterior with soft and ductile core is obtained by introducing carbon into the skin or shell of the metal. Stock of 0.10 to 0.20 per cent. carbon is generally used, and sometimes nickel steel, chrome steel, or chrome-nickel steel. Ordinarily the steel should not contain over 0.25 per cent. Mn, lest the case be too brittle, although in special cases as much as 1.30 per cent. Mn may be present, with the object of making the case very hard. The elements which exist as double carbides favor the absorption of carbon, *e.g.*, manganese, tungsten, chromium, molybdenum, while those which form solid solutions—nickel, silicon, and aluminum—oppose it.

The **operations** in case-hardening are as follows: 1. Carbonize in case-hardening material. 2. Allow the piece to cool in the box or in air to black in daylight. 3. Reheat to above the critical range of the low-carbon core and quench in oil or water. 4. Reheat to above the critical range of the high-carbon case and quench in oil or water. These four treatments result in a fine, silky core and a hard, close-grained case capable of withstanding heavy shocks and continued wear. Sometimes treatments 3 and 4 are omitted or the steel is quenched directly from the carburizing temperature, but both methods are bad practice.

The **materials used** in case-hardening may be solid, liquid, or gaseous, although solid materials are used most extensively, the most important being charcoal (both wood and bone), charred leather, crushed bone, horn, mixtures of barium carbonate and charcoal, or of salt and charcoal; and for quick or very superficial hardening, powdered potassium cyanide and potassium ferrocyanide, or mixtures of potassium ferrocyanide and potassium bichromate. A molten bath of potassium cyanide heated to 1562 deg. fahr. (850 deg. cent.), in which the steel articles are immersed, produces quickly superficial but hard and even cases. The poisonous character of the escaping gases is a serious objection to the use of cyanide. The carburizing of iron may also be performed at the proper temperature by means of gases, such as illuminating or other coal or oil gases rich in hydrocarbons. At the Krupp works in Germany, gases are used for carburizing the faces of armor plates. The relative merits of wood charcoal, charred leather, and a mixture of barium carbonate and wood charcoal are shown in Fig. 3 (Shaw-Scott). See also Heathcote (*Jour. Iron and Steel Inst.*, 1914) on the relative values of various case-hardening mixtures.

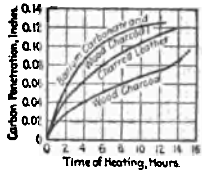

FIG. 3.

Many so-called secret **mixtures** are offered for sale as case-hardening substances, for which extraordinary virtues are claimed, the usual statement being that by their use steel of ordinary or inferior quality may be converted into high-grade metal comparable to the best crucible steel. On investigation they are generally found to be chiefly mixtures of carbonaceous and cyanogen compounds possessing the well-known carburizing properties of those substances.

The **carburizing temperature** will vary somewhat with the results desired, but generally lies between 1652 deg. fahr. (900 deg. cent.) and 1742 deg. fahr. (950 deg. cent.).

The **time required for carburizing** varies with the thickness of the case to be obtained. Fig. 4 (Guillet) shows the influence of temperature and of time on the depth of penetration of carbon. Material used was not stated. The full line represents relative penetrations at 1832 deg. fahr. (1000 deg. cent.) after different lengths of time, while the broken line represents the depths of penetration resulting from heating for 8 hr. at different temperatures.

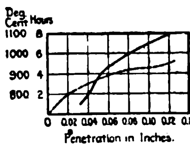

FIG. 4.

The easy way of controlling the case-hardening process is to make microscopic examination of test pieces. The following **case-hardening practice** is recommended by the American Society for Testing Materials (yearbook, 1915, adopted 1914).

1. When hardness of case only is desired and lack of toughness or even brittleness is unimportant, the carburized objects may be quenched from

carburizing temperature, as, for instance, by emptying the contents of the boxes in cold water or oil. Both the core and the case are then coarsely crystalline.

2. In order to reduce the hardening stresses and to decrease the danger of distortion and cracking in the quenching bath, the objects may be removed from the box and allowed to cool before quenching to a temperature slightly exceeding the critical range of the case, namely, 1472 to 1517 deg. fahr. (800 to 825 deg. cent.). Both the core and case remain coarsely crystalline.

3. To refine the case and increase its toughness, the carburized objects should be allowed to cool slowly in the carburizing box within the furnace or outside to 1202 deg. fahr. or below, and should then be reheated to a temperature slightly exceeding the lower critical point of the case (in the majority of instances a temperature varying in accordance with the carbon content and thickness of the case between 1427 and 1517 deg. fahr. will be suitable), and quenched in water, or, for greater toughness but less hardness, in oil. The objects should be removed from the quenching bath before their temperature has fallen below 212 deg. fahr. This treatment is more especially to be recommended when the carburizing temperature has not exceeded 1652 deg. fahr. It refines the case but not the core.

4. To refine the core and the case and to increase their toughness, the objects should be allowed to cool slowly from the carburizing temperature to 1202 deg. fahr. or below and should then be (*a*) reheated to a temperature exceeding the critical point of the core, which will generally be from 1652 to 1742 deg. fahr., followed by quenching in water or in oil; and (*b*) before they have cooled below 212 deg. fahr. they should be reheated to a temperature slightly exceeding the lower critical point of the case (in the majority of instances a temperature varying in accordance with the carbon content and thickness of the case between 1427 and 1517 deg. fahr. will be suitable), and again quenched in water or oil.

The objects should be removed from the quenching bath before they have cooled below 212 deg. fahr. in order to lessen the danger of cracking, and they should be placed in the reheating furnace while still at a temperature of at least 212 deg. fahr. likewise to lessen the danger of cracking, it being inadvisable (*a*) to allow steel to cool completely in the quenching bath and (*b*) to place hardened steel in a hot furnace. Obviously, if the furnace is cold the hardened steel may likewise be cold when placed in it for reheating.

5. In order to reduce the hardening stresses created by quenching, the objects, as a final treatment, may be tempered by reheating them to a temperature not exceeding 392 deg. fahr.

The Special Brittleness of Low-carbon Steel. The special brittleness exhibited by steel with less than 0.15 per cent. of carbon after being subjected to cold work and subsequent annealing, as in thin sheets, wire, etc., is known as "Stead's brittleness." The coarse-grained steel thus produced may be refined by heating to 1652 deg. fahr. or a little higher, that is to say, above the critical range of the steel in question, and cooling slowly.

Burning, Overheating, and Restoring Steel. Overheating is more injurious to high-carbon steels than to those having a low carbon content, but the effects of this treatment can be removed by a proper annealing, provided the steel has not been overheated too much. Overheating may either change the physical properties and the microstructure, the ductility being decreased by an increase in the grain size, or if the temperature is sufficiently high, an actual disintegration of the grains may take place. In the former case it is possible to restore the steel by reworking or by annealing, but no treatment short of remelting will efface the injury to steel heated to temperatures sufficient to cause the separation of the grains, *i.e.*, to temperatures above the curve *AE* of the iron-carbon diagram (see p. 493). For a given steel, the grain size increases progressively with the increase of temperature above the critical range; cooled high-carbon steels are affected more than low-carbon steels by overheating to a given temperature.

because the range through which they must cool before reaching the critical transformation point is greater. The following table (Howe) shows that a 1.20 per cent. carbon steel can be refined when heated to 2400 deg. fahr., but that the steel is burned and the crystals separated by heating above this temperature.

Temperature to which steel was heated, deg. fahr.		2400	2525	2650	2775
Angle through which a 5/16-in. square bar was bent, deg.	steel not refined	14	7	4	3
	steel refined	144	21	10	0

Surface Decarburization of Steel. When high-carbon steel is heated for a long time above the critical range, the surface or skin becomes decarburized, even when packed in sealed iron tubes containing charcoal. This is a common occurrence in the manufacture of tool steel, and the carbon in the skin may be reduced to zero, rendering the outside of the steel useless for cutting purposes. The remedies are either to anneal the steel below the critical point, or to machine off the decarburized skin or "bark."

Special Annealing for Tool Steel. To prevent decarburization and to obtain the free cementite in a globular or spheroidized form (sometimes called granular pearlite), high-carbon steel is often annealed at about 1250 deg. fahr. for several hours, preferably in a neutral packing (cast-iron filings or chips, ashes, or sand). This treatment makes the steel more easily machined and prevents the formation of needlelike crystals or plates of free cementite in the hardened steel.

Formation of Graphite in Steel. When high-carbon steel high in silicon is heated for a long time above the critical range, small rounded particles of graphite (amorphous temper carbon similar to that found in malleable cast iron) are formed, ruining the steel both by the presence of soft spots and the reduction of the combined carbon, sometimes down to 0.4 or 0.5 per cent. The only remedy is remelting. This trouble is especially liable to occur in the manufacture of files.

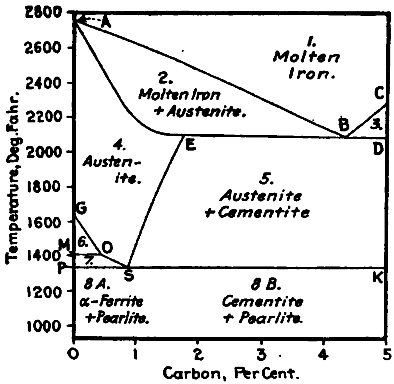

FIG. 5.

Metallography of Iron and Steel

(See Sauveur's "Metallography of Iron and Steel," 1912)

Metallography is the study of the microstructure and the thermal curves of metals. **Critical temperatures** are temperatures at which retardations occur in the heating and cooling curves of iron and steel. The critical temperatures on heating generally occur some 30 to 40 deg. cent. above those on cooling, but indicate reversals of the same phenomena.

Iron-Carbon Equilibrium Diagram. When the critical points (or temperatures) of iron and steel are plotted with temperature and carbon content as co-ordinates, as in Fig. 5, the so-called equilibrium diagram is obtained. This is also called the Roberts-Austen-Roozeboom diagram. The temperatures plotted in the figure are those obtained on heating, since these are of the greater importance in the heat-treatment of steel. They are only approximate. The line *PSK* is often called A_1; *SK*, $A_{3.2.1}$; the line *MO* is called A_2; *GO* is called A_3; the line *OS*, $A_{3.2}$; and the line *SE*, A_{cm}. The diagram represents the pure iron-carbon system. All commercial steels have varying amounts of impurities, and these affect to a considerable extent the positions of the curves and especially the lateral positions of points *S* and *B*; hence it is impracticable to draw a diagram for any actual commercial steel. The **critical range** is the region bounded by *GOS*, *PG*, *PS* and the line *SK*.

The **annealing** of steel is based on the fact that slow cooling through the critical range allows transformation to take place thoroughly and the steel acquires maximum ductility and toughness with fair tensile strength and elasticity. The **hardening** of steel is based on the fact that quick cooling through the critical range serves to retain more or less completely the structures and hence the physical properties normal above that range. Commercial rates of cooling do not prevent the change from austenite to martensite; hence the great hardness of hardened steel.

Microscopic Constituents of Iron and Steel. Specimens of iron and steel, as well as other metals, when polished and suitably etched with weak acids or other reagents, show certain **characteristic crystalline formations** under the microscope corresponding to the various fields on the iron-carbon diagram. In the following list of these constituents are stated their constitution, occurrence, microstructure, and normal position in the iron-carbon diagram.

Alpha (α) **Iron** is the common, strongly magnetic and soft form of iron which is stable for temperatures below *MO*, *OS* and *SK*. It has little or no solvent power for C and Fe_3C. (Known metallurgically as ferrite, alpha ferrite and pearlite ferrite.)

Beta (β) **Iron** is a feebly magnetic, intensely hard and brittle allotropic form of iron, stable (as free beta ferrite) between *GO* and *MO*.

Gamma (γ) **Iron** is the allotropic, non-magnetic iron normal above *G*. When associated with carbon it is stable above *GO*, *OS* and *SK*, and probably to some extent (as austenite) in regions 6 and 7. As regards hardness, it lies between alpha and beta irons.

Austenite is a solid solution of iron carbide in gamma iron. It occurs above *GOSK* (and probably in regions 6 and 7) and is retained when cold in some high-carbon special steels, especially after quick cooling. It appears as polyhedric grains.

Martensite (constitution disputed) is harder than austenite and is the principal constituent of hardened steels. It is normal along *PSK*. Under the microscope it appears as a needlelike structure arranged indistinctly in equilateral triangles.

Troostite is softer than martensite, with which it occurs in hardened steels. It is the principal constituent of steels tempered to 750 deg. fahr. It has no place in the diagram, being a transition form between martensite and sorbite. It etches black or dark brown, the arrangement being either in rosette forms or indistinctly granular at high magnifications.

Sorbite is a transition stage between troostite and pearlite and is softer than troostite. It is the principal constituent of air-cooled (annealed) steels and of hardened steels reheated to 1100 deg. fahr. (double-quenched steel). It has no place in the diagram. It appears as dark grains consisting of extremely fine black and white dots under high magnification.

Pearlite is a mechanical mixture of alpha ferrite and cementite. It is the principal constituent of slowly cooled (annealed) steels. It is always associated with an excess of ferrite or of cementite except when steel contains from 0.70 to 0.90 per cent. carbon. It is normal in regions *8A* and *8B*. It appears as dark grains at low magnification, and as alternate curved white and black laminæ at high magnifications.

Ferrite is nearly pure iron. It may contain a little P and Si. It is the softest constituent of iron and steel and the principal one of wrought iron and low-carbon steels. It also forms 87½ per cent. by weight of pearlite. It is found in regions 6, 7, *8A*, *8B*. It appears under the microscope as polyhedric grains or networks when free, or as the dark laminæ in pearlite.

Cementite is definite iron carbide (Fe_3C) and the hardest constituent. It occurs free in steels containing more than 0.90 per cent. carbon, and also forms 12½ per cent. by weight of pearlite. It is normal in regions 3, 5, *8A* and *8B*. It appears as white, brilliant masses, networks, or needles when free or as the light laminæ in pearlite.

Graphite is the crystallised carbon found in gray and mottled cast iron. It is normal in regions 3, 5, *8A* and *8B* when very slowly cooled and especially when much Si is present. It appears as long, black, wormlike particles in gray cast iron, straight or curved, and as round black dots in malleable cast iron where it occurs in the amorphous form and is known as "temper" graphite.

Steadite is a mechanical mixture of ferrite and iron phosphide (Fe_3P) occurring in cast irons high in P. It appears as fine black dots on a brilliant white background, the latter surrounded by fine black border.

Manganese Sulphide (MnS) occurs in iron and steel containing more than a trace of S, appearing as round or lenticular dove-gray spots.

Effect of Mechanical Treatment on the Microstructure of Steel. Hot work, that is, work performed above the line *GOSK* in the iron-carbon diagram, crushes the coarse crystals in the casting due to slow cooling and prevents recrystallization if the finishing temperature is close to the temperature *GOSK*. If the finishing temperature is much higher than this, the steel is free to recrystallize between the finishing temperature and the critical temperature *GOSK* and coarse-grained steel results, which can be refined only by repeated hot work or annealing. Cold work, or work performed below the critical temperature *GOSK* and more especially below the critical temperature *PSK*, causes distortion of the grains in the direction of the work, with consequent increase of elastic limit and tensile strength and great loss of ductility. This distorted structure can be made symmetrical again by annealing and the ductility restored.

Relation Between Microstructure and Physical Properties. Everything else being equal, a fine granular structure means greater strength, greater ductility, greater resistance to shock, and greater resistance to wear. Martensitic structures are sought when intense hardness is desired; sorbitic structures are strong, have great wearing qualities, and are moderately ductile if fine-grained; pearlitic structures are sought where maximum softness and ductility are desired, but the elastic limit and tensile strength are lower than in sorbitic or martensitic steels. Any of these structures may be fine or coarse, depending on the maximum temperature at which the steel is cooled and the rate of cooling, both above and through the critical range. The higher the maximum temperature from which the steel cools, the coarser the grain; and the slower the rate of cooling above the critical range, as well as through the critical range, the coarser the grain.

Sauveur gives the following **formulæ connecting the microstructure of steel with its physical properties,** which hold good only for slowly cooled or pearlitic steels.

The tensile strength (T. S., lb. per sq. in.) of any hypoeutectoid steel (with less than 0.83 per cent. carbon) can be determined from T. S. $= 500F + 1250P = 50{,}000 + 750P$, where F is the per cent. of ferrite, and P the per cent. of pearlite.

The tensile strength of any hypereutectoid steel (containing more than 0.83 per cent. carbon) may be obtained from T. S. $= 1250P + 50Cm = 5000 + 1200P$, where Cm is the per cent. of cementite.

The ductility of any hypoeutectoid steel in the pearlitic condition is determined by the formula $D = 40 - 0.3P$.

WEIGHTS OF STEEL WIRE, SHEETS AND BARS

Properties of Steel Wire

(Breaking stress = 100,000 lb. per sq. in.)

No., Roebling gage	Breaking stress, lb.	Weight, lb. per 1000 ft.	No., Roebling gage	Breaking stress, lb.	Weight, lb. per 1000 ft.	No., Roebling gage	Breaking stress, lb.	Weight, lb. per 1000 ft.	No., Roebling gage	Breaking stress, lb.	Weight, lb. per 1000 ft.
6-0	16,619	558.4	6	2,895	97.3	17	229	7.70	27	23.0	0.763
5-0	14,522	487.9	7	2,461	82.7	18	174	5.83	28	20.0	0.676
4-0	12,130	407.6	8	2,061	69.3	19	132	4.44	29	18.0	0.594
3-0	10,292	345.8	9	1,720	57.8	20	96	3.23	30	15.0	0.517
2-0	8,605	289.1	10	1,431	48.1	21	80	2.70	31	14.0	0.481
0	7,402	248.7	11	1,131	38.0	22	62	2.07	32	13.0	0.446
1	6,290	211.4	12	866	29.1	23	49	1.65	33	9.5	0.319
2	5,433	182.5	13	665	22.3	24	42	1.40	34	7.9	0.264
3	4,676	157.1	14	503	16.9	25	31	1.06	35	7.1	0.238
4	3,976	133.6	15	407	13.7	26	25	0.855	36	6.4	0.214
5	3,365	113.1	16	312	10.5	......	......	......	......	......	......

Weights of Flat Rolled Steel, Pounds per Linear Foot*

(For iron, subtract 2 per cent.)

Width, in.	Thickness, in. 1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
1/4	0.053	0.106	0.159	0.213	0.27	0.32	0.37	0.43	0.48	0.53	0.58	0.64	0.69	0.74	0.80	0.85
1/2	0.106	0.213	0.319	0.425	0.53	0.64	0.74	0.85	0.96	1.06	1.17	1.28	1.38	1.49	1.59	1.70
3/4	0.159	0.319	0.478	0.638	0.80	0.96	1.12	1.28	1.43	1.59	1.75	1.91	2.07	2.23	2.39	2.55
1	0.213	0.425	0.638	0.850	1.06	1.23	1.49	1.70	1.91	2.13	2.34	2.55	2.76	2.98	3.19	3.40
2	0.425	0.850	1.275	1.700	2.13	2.55	2.98	3.40	3.83	4.25	4.68	5.10	5.53	5.95	6.38	6.80
3	0.638	1.275	1.913	2.550	3.19	3.83	4.46	5.10	5.74	6.38	7.01	7.65	8.29	8.93	9.56	10.20
4	0.850	1.700	2.550	3.400	4.25	5.10	5.95	6.80	7.65	8.50	9.35	10.20	11.05	11.90	12.75	13.60
5	1.063	2.125	3.188	4.250	5.31	6.38	7.44	8.50	9.56	10.63	11.69	12.75	13.81	14.88	15.94	17.00
6	1.275	2.550	3.825	5.100	6.38	7.65	8.93	10.20	11.48	12.75	14.03	15.30	16.58	17.85	19.13	20.40
7	1.488	2.975	4.463	5.950	7.44	8.93	10.41	11.90	13.39	14.88	16.36	17.85	19.34	20.83	22.31	23.80
8	1.700	3.400	5.100	6.800	8.50	10.20	11.90	13.60	15.30	17.00	18.70	20.40	22.10	23.80	25.50	27.20
9	1.913	3.825	5.738	7.650	9.56	11.48	13.39	15.30	17.21	19.13	21.04	22.95	24.86	26.78	28.69	30.60
10	2.125	4.250	6.375	8.500	10.63	12.75	14.88	17.00	19.13	21.25	23.38	25.50	27.63	29.75	31.88	34.00
20	4.25	8.50	12.75	17.00	21.25	25.50	29.75	34.00	38.25	42.50	46.80	51.0	55.30	59.50	63.8	68.00
30	6.38	12.75	19.13	25.50	31.88	38.25	44.63	51.00	57.38	63.75	70.10	76.5	82.90	89.30	95.6	102.00
40	8.50	17.00	25.50	34.00	42.50	51.00	59.50	68.00	76.50	85.00	93.50	102.0	110.50	119.00	127.5	136.00

* For other widths the weights are obtainable by addition; for example, 54 × 3/4 in. = [(10 × 5) + 4] × 3/4 in., and weight = (10 × 12.75) + 10.20 = 137.7 lb. Similarly, for greater thicknesses, the weights are obtainable by addition.

Weights of Rolled Sheet Steel

Gage number, B.W.G.	Thickness, in.	Lb. per sq. ft.	Gage number, B.W.G.	Thickness, in.	Lb. per sq. ft.	Gage number, B.W.G.	Thickness, in.	Lb. per sq. ft.	Gage number, B.W.G.	Thickness, in.	Lb. per sq. ft.
	1/2	20.4	......	1/4	10.2	......	7/64	4.46	23	......	1 02
	15/32	19.1	4	......	9.71	12	......	4.45	24	......	0.898
0000	......	18.5	......	15/64	9.56	13	......	3.88	25	......	0.816
	7/16	17.9	5	......	8.98	......	3/32	3.83	26	......	0.734
000	......	17.3	......	7/32	8.93	14	......	3.39	27	......	0.652
	13/32	16.6	......	13/64	8.29	......	5/64	3.19	......	1/64	0.638
00	......	15.5	6	......	8.28	15	......	2.94	78	......	0.571
	3/8	15.3	......	3/16	7.65	16	......	2 65	29	......	0 530
	11/32	14.0	7	......	7.34	......	1/16	2.55	30	......	0.490
0	......	13.9	......	11/64	7.01	17		2.37			0.408
	5/16	12.8	8	......	6.73	18	......	2.00	32	......	0.367
1	......	12.2	......	5/32	6.38	......	3/64	1.91	33	......	0.326
	19/64	12.1	9	......	6.04	19	......	1.71	......	1/128	0.319
2	......	11.6	......	9/64	5.74	20	......	1.43	34	......	0.286
	9/32	11.5	10	......	5.47	21	......	1.31	35	......	0.204
	17/64	10.8	......	1/8	5.1	......	1/32	1.28	36	......	0.163
3	......	10.6	11	......	4.90	22	......	1.14	......	1/256	0.159

Weights of Square and Round Steel Bars

(For iron, subtract 2 per cent.)

Size, in.	Weight, lb. per lin. ft.		Size, in.	Weight, lb. per lin. ft.		Size, in.	Weight, lb. per lin. ft.		Size, in.	Weight, lb. per lin. ft.	
	Square	Round		Square	Round		Square	Round		Square	Round
0	......	......	3	30.60	24.03	6	122.4	96.1	9	275.4	216.3
1/16	0.013	0.010	1/16	31.89	25.05	1/16	125.0	98.2	1/16	279.2	219.3
1/8	0.053	0.042	1/8	33.20	26.08	1/8	127.6	100.2	1/8	283.1	222.4
3/16	0.120	0.094	3/16	34.54	27.13	3/16	130.2	102.2	3/16	287.0	225.4
1/4	0.213	0.167	1/4	35.91	28.21	1/4	132.8	104.3	1/4	290.9	228.5
5/16	0.332	0.261	5/16	37.31	29.30	5/16	135.5	106.4	5/16	294.9	231.6
3/8	0.478	0.376	3/8	38.73	30.42	3/8	138.2	108.5	3/8	298.8	234.7
7/16	0.651	0.511	7/16	40.18	31.55	7/16	140.9	110.7	7/16	302.8	237.8
1/2	0.850	0.668	1/2	41.65	32.71	1/2	143.7	112.8	1/2	306.9	241.0
9/16	1.076	0.845	9/16	43.15	33.89	9/16	146.4	115.0	9/16	310.9	244.2
5/8	1.328	1.043	5/8	44.68	35.09	5/8	149.2	117.2	5/8	315.0	247.4
11/16	1.607	1.262	11/16	46.23	36.31	11/16	152.1	119.4	11/16	319.1	250.6
3/4	1.913	1.502	3/4	47.81	37.55	3/4	154.9	121.7	3/4	323.2	253.9
13/16	2.245	1.763	13/16	49.42	38.81	13/16	157.8	123.9	13/16	327.4	257.1
7/8	2.603	2.044	7/8	51.05	40.10	7/8	160.7	126.2	7/8	331.6	260.4
15/16	2.988	2.347	15/16	52.71	41.40	15/16	163.6	128.5	15/16	335.8	263.7
1	3.400	2.670	4	54.40	42.73	7	166.6	130.9	10	340.0	267.0
1/16	3.838	3.015	1/16	56.11	44.07	1/16	169.6	133.2	1/16	344.3	270.4
1/8	4.303	3.380	1/8	57.85	45.44	1/8	172.6	135.6	1/8	348.6	273.8
3/16	4.795	3.766	3/16	59.62	46.83	3/16	175.6	137.9	3/16	352.9	277.1
1/4	5.313	4.172	1/4	61.41	48.23	1/4	178.7	140.4	1/4	357.2	280.6
5/16	5.857	4.600	5/16	63.23	49.66	5/16	181.8	142.8	5/16	361.6	284.0
3/8	6.428	5.049	3/8	65.08	51.11	3/8	184.9	145.2	3/8	366.0	287.4
7/16	7.026	5.518	7/16	66.95	52.58	7/16	188.1	147.7	7/16	370.4	290.9
1/2	7.650	6.008	1/2	68.85	54.07	1/2	191.3	150.2	1/2	374.9	294.4
9/16	8.301	6.519	9/16	70.78	55.59	9/16	194.5	152.7	9/16	379.3	297.9
5/8	8.978	7.051	5/8	72.73	57.12	5/8	197.7	155.3	5/8	383.8	301.5
11/16	9.682	7.604	11/16	74.71	58.67	11/16	200.9	157.8	11/16	388.4	305.0
3/4	10.413	8.178	3/4	76.71	60.25	3/4	204.2	160.4	3/4	392.9	308.6
13/16	11.170	8.773	13/16	78.74	61.85	13/16	207.5	163.0	13/16	397.5	312.2
7/8	11.953	9.388	7/8	80.80	63.46	7/8	210.9	165.6	7/8	402.1	315.8
15/16	12.763	10.024	15/16	82.89	65.10	15/16	214.2	168.2	15/16	406.7	319.5
2	13.600	10.681	5	85.00	66.76	8	217.6	170.9	11	411.4	323.1
1/16	14.463	11.359	1/16	87.14	68.44	1/16	221.0	173.6	1/16	416.1	326.8
1/8	15.353	12.058	1/8	89.30	70.14	1/8	224.5	176.3	1/8	420.8	330.5
3/16	16.270	12.778	3/16	91.49	71.86	3/16	227.9	179.0	3/16	425.5	334.2
1/4	17.213	13.519	1/4	93.71	73.60	1/4	231.4	181.8	1/4	430.3	338.0
5/16	18.182	14.280	5/16	95.96	75.36	5/16	234.9	184.5	5/16	435.1	341.7
3/8	19.178	15.062	3/8	98.23	77.15	3/8	238.5	187.3	3/8	439.9	345.5
7/16	20.201	15.866	7/16	100.53	78.95	7/16	242.1	190.1	7/16	444.8	349.3
1/2	21.250	16.690	1/2	102.85	80.78	1/2	245.7	192.9	1/2	449.7	353.2
9/16	22.326	17.534	9/16	105.20	82.62	9/16	249.3	195.8	9/16	454.6	357.0
5/8	23.428	18.400	5/8	107.58	84.49	5/8	252.9	198.7	5/8	459.5	360.9
11/16	24.557	19.287	11/16	109.98	86.38	11/16	256.6	201.5	11/16	464.4	364.8
3/4	25.713	20.195	3/4	112.41	88.29	3/4	260.3	204.5	3/4	469.4	368.7
13/16	26.895	21.123	13/16	114.87	90.22	13/16	264.0	207.4	13/16	474.4	372.6
7/8	28.103	22.072	7/8	117.35	92.17	7/8	267.8	210.3	7/8	479.5	376.6
15/16	29.338	23.042	15/16	119.86	94.14	15/16	271.6	213.3	15/16	484.5	380.5

Wire and Sheet Metal Gages

(Diameters and thicknesses in decimal parts of an inch)

Gage No.	American wire gage, or Brown & Sharpe (for copper wire)	Steel wire gage, or Washburn & Moen or Roebling (for steel wire)	Birmingham wire gage (B.W.G.) or Stubs' iron wire (for steel wire or sheets)	Stubs' steel wire gage	British Imperial standard wire gage (S.W.G.)	U. S. standard gage for sheet metal (iron and steel)	Trenton Iron Co.	Standard Birmingham sheet and hoop (B. G.)
0000000	...	0.4900	...	...	0.500	0.500	...	...
000000	...	0.4615	...	...	0.464	0.469	...	...
00000	...	0.4305	...	...	0.432	0.438	0.450	...
0000	0.460	0.3938	0.454	...	0.400	0.406	0.400	...
000	0.410	0.3625	0.425	...	0.372	0.375	0.360	0.5000
00	0.365	0.3310	0.380	...	0.348	0.344	0.330	0.4452
0	0.325	0.3065	0.340	...	0.324	0.312	0.305	0.3964
1	0.289	0.2830	0.300	0.227	0.300	0.281	0.285	0.3532
2	0.258	0.2625	0.284	0.219	0.276	0.266	0.265	0.3147
3	0.229	0.2437	0.259	0.212	0.252	0.250	0.245	0.2804
4	0.204	0.2253	0.238	0.207	0.232	0.234	0.225	0.2500
5	0.182	0.2070	0.220	0.204	0.212	0.219	0.205	0.2225
6	0.162	0.1920	0.203	0.201	0.192	0.203	0.190	0.1981
7	0.144	0.1770	0.180	0.199	0.176	0.188	0.175	0.1764
8	0.128	0.1620	0.165	0.197	0.160	0.172	0.160	0 1570
9	0.114	0.1483	0.148	0.194	0.144	0.156	0.145	0.1398
10	0.102	0.1350	0.134	0.191	0.128	0.141	0.130	0.1250
11	0.091	0.1205	0.120	0.188	0.116	0.125	0.1175	0.1113
12	0.081	0.1055	0.109	0.185	0.104	0.109	0.105	0.0991
13	0.072	0.0915	0.095	0.182	0.092	0.094	0.0925	0.0882
14	0.064	0.0800	0.083	0.180	0.080	0.078	0.080	0.0785
15	0.057	0.0720	0.072	0.178	0.072	0.070	0.070	0.0699
16	0.051	0.0625	0.065	0.175	0.064	0.062	0.061	0.0625
17	0.045	0.0540	0.058	0 172	0.056	0.056	0.0525	0.0556
18	0.040	0.0475	0.049	0.168	0.048	0.050	0.045	0.0495
19	0.036	0.0410	0 042	0.164	0.040	0.0438	0.040	0.0440
20	0.032	0.0348	0.035	0.161	0.036	0.0375	0.035	0.0392
21	0.0285	0.0317	0.032	0.157	0.032	0.0344	0.031	0.0349
22	0.0253	0.0286	0.028	0.155	0.028	0.0312	0.028	0.0313
23	0.0226	0.0258	0.025	0.153	0.024	0.0281	0.025	0.0278
24	0.0201	0.0230	0.022	0.151	0.022	0.0250	0.0225	0.0248
25	0.0179	0.0204	0.020	0.148	0.020	0.0219	0.020	0.0220
26	0.0159	0.0181	0.018	0.146	0.018	0.0188	0.018	0.0196
27	0.0142	0.0173	0.016	0.143	0.0164	0.0172	0.017	0.0175
28	0.0126	0.0162	0.014	0.139	0.0148	0.0156	0.016	0.0156
29	0.0113	0.0150	0.013	0.134	0.0136	0.0141	0.015	0.0139
30	0.0100	0.0140	0.012	0.127	0.0124	0.0125	0.014	0.0123
31	0.0089	0.0132	0.010	0.120	0.0116	0.0109	0.013	0.0110
32	0.0080	0.0128	0.009	0.115	0.0108	0.0102	0.012	0.0098
33	0 0071	0.0118	0.008	0.112	0.0100	0.0094	0.011	0 0087
34	0.0063	0.0104	0.007	0.110	0.0092	0.0086	0.010	0.0077
35	0.0056	0.0095	0.005	0.108	0.0084	0.0078	0.0095	0.0069
36	0.0050	0.0090	0.004	0.106	0 0076	0.0070	0.009	0.0061
37	0.0045	0.0085	...	0.103	0.0068	0.0066	0.0085	0.0054
38	0 0040	0.0080	...	0.101	0.0060	0 0062	0.008	0.0048
39	0.0035	0.0075	...	0.099	0.0052	...	0.0075	...
40	0.0031	0.0070	...	0.097	0.0048	...	0.007	...
41	...	0.0066	...	0.095	0.0044	...	...	...
42	...	0.0062	...	0.092	0.0040	...	...	...
43	...	0.0060	...	0.088	0.0036	...	...	...
44	...	0.0058	...	0.085	0 0032	...	...	...
45	...	0.0055	...	0.081	0.0028	...	...	...
46	...	0.0052	...	0.079	0.0024	...	...	...
47	...	0.0050	...	0.077	0.0020	...	...	...
48	...	0.0048	...	0.075	0.0016	...	...	...
49	...	0.0046	...	0.072	0.0012	...	...	...
50	...	0.0044	...	0.069	0.0010	...	...	...

IRON AND STEEL CASTINGS

BY

RICHARD MOLDENKE

REFERENCES: West, "American Foundry Practice" and "Molder's Text Book," Wiley. Stoughton, "Metallurgy of Iron and Steel," McGraw-Hill. Hall, "The Steel Foundry," McGraw-Hill. Hatfield, "Cast Iron," Griffin & Co. Moldenke, "Production of Malleable Castings." Trans. Am. Foundrymen's Assn. Trans. British Foundrymen's Assn.

CLASSIFICATION OF CASTINGS

In selecting the variety of casting best suited for a given purpose, the following classification of the regular iron foundry product will be of assistance.

Gray-iron Castings. Three types of castings are made in the ordinary gray-iron jobbing foundry. (*a*) **Soft gray-iron machinery castings** are made from an iron which can be machined readily, is not specially strong, and is sound and reliable; they should be used for the ordinary run of machine construction work. Three weights are to be considered, namely, light, medium, and heavy. Medium machinery mixtures (see Table 1) are used for ordinary castings satisfactorily, but it is not safe to make heavy castings from light machinery mixtures, as they will be very weak, and very light castings from heavy machinery mixtures will be quite hard.

(*b*) **Strong Gray-iron Machinery Castings** are adapted for medium and heavy work where a greater strength than that of ordinary cast iron is desired, but not the special strength of the steel casting. They are made by adding from 10 to 40 per cent. of steel scrap to the ordinary mixtures in which the required silicon has been provided to carry the steel burden safely. There is much variation in the strength of such castings, however, and where extreme care must be exercised, it is safer to use steel castings.

(*c*) **Chilled-iron Castings.** Such castings as crusher jaws, grinding plates, etc., requiring chilled-iron surfaces and soft bodies, are made in the jobbing foundry from special low-silicon irons. Chilled rolls, car wheels (chilled treads), and work of a similar nature are better obtained from roll or car-wheel foundries, as these work to service guarantees and are specially careful in the selection of their melting stock.

Special Gray-iron Castings. Certain classes of castings can best be made in foundries giving special attention to a given line of work. Thus, (*d*) **cylinder and pump castings,** whether for steam, compressed air, or ammonia, requiring a close-grained, dense iron, free from shrinkage spots, are made in pump works, locomotive, and engine shop foundries. This iron, particularly for cylinders, should be fairly hard, though just machining properly, fine in grain, and in wearing very slowly acquires a high polish. Very intricate and light castings requiring fair strength should be obtained from foundries specializing on automobile cylinders. (*e*) **Dynamo-frame castings** are made from very soft iron in which the combined carbon content (though not necessarily total carbon) is as low as possible, to prevent retention of residual magnetism. Certain foundries specialize in this line of work. (*f*) **Stoves, radiators, and ornamental castings** are made from high-phosphorus (and because of being very thin work) high-silicon iron, which is very fluid when molten, in order to fill the finest lines in the mold. This iron is brittle, and, unless carefully produced, inclined to run to hard corners and edges. (*g*) **Gun iron** is the most reliable and highest-grade cast iron made. It is the product of the air furnace (all the castings previously described being made of cupola iron), is improved by steel scrap additions as in (*b*), and for the same mixtures is regularly better than the corresponding cupola iron. Formerly used for cannon and mortars, it is now employed in the construction of high-grade engines, turbines, rolls for the finest finishing of brass and steel, tin plate, etc., and in all work where its comparatively higher cost is no objection. Many other varieties of castings, such as pipe, agricultural implement parts, pipe fittings, flywheels, etc., fall within the above given classes so far as mixtures for making them are concerned.

Malleable Castings range in strength between gray iron and the steel castings, and hence are used where gray iron is too weak and steel too expensive. They are made ordinarily only in sections less than ¾ in. thick, the pieces not much over 4 ft. in length, and the weight not over 300 lb. The usual weights are a few ounces for light, 15 lb. for medium, and 150 lb. for heavy castings. The metal will bend, twist, resist shock remarkably well—its special characteristic—better even than cast steel, is cheap, easily made and specially adapted to repetition work, and therefore in general use for car castings, pipe fittings, agricultural work, hardware, etc. Care must be taken in designing malleable castings to avoid sharp corners and to have as nearly the same thickness of section throughout as possible; round sections are not as good as rectangular. These castings should not be machined, as the interior is not as strong as the metal at and near the surface. Tensile strength, 35,000 to 48,000 lb. per sq. in. European malleable cast iron, made by a somewhat different process, is not as sensitive to machining; the castings, which are thin only, are practically decarbonized in the annealing process; whereas in the American "black-heart" malleable iron, only the skin is decarbonized, the metal adjacent for about ¼ in. partially so, and the central portions contain the full carbon percentage of the original hard white casting.

Chilled castings in which an all-white fracture is wanted, can be made in the malleable casting foundry, as this class of iron forms the basis of the malleable casting process. This iron is very weak and crystalline, difficult to produce in heavy sections unless specially chilled while being poured, and is only useful for its extreme hardness. This metal can be used for slabs for grinding marble surfaces smooth, and for stars for cleaning softer castings in the rumbling barrel.

Steel Castings are made commercially by four methods at the present time: By the **crucible process**, which makes very fine but expensive and only small castings; by the **open-hearth-process**, which makes castings from a few ounces to any tonnage desired; by the **Bessemer process**, which is used principally for small castings, and by the **electric furnace**, which is just coming into use in this country and makes the highest grade of castings of all the methods.

The steel casting should be selected where **strength and reliability** are essential. It should always be annealed, so that the original casting strains are removed and the grain structure improved. It is a difficult casting to make and great skill in molding is required, as the shrinkage is even greater than in white iron, and hence strains, cracks and shrinkages are easily formed.

A variety of compositions can be made, as, for instance, the several carbon steels (for 0.20 and below, up to tool-steel compositions), vanadium, nickel, chromium, titanium and other alloy steels, so that the range is quite wide, and the designing engineer can give special attention to wearing qualities. For ordinary work, however, attention is confined to seeing that important parts are free from blowholes, and that steel castings are only specified where gray-iron or malleable ones will not do.

Miscellaneous Castings. Under this head would be included castings made from "mitis" metal (melted wrought iron)—which is no longer available—hybrids such as "ductile iron castings," "malleable steel," etc. All of them had better be left alone, as their uniformity cannot be depended upon.

Table 1. Suggested Analyses for Various Kinds of Iron and Steel Castings
(Porter)

	Silicon, per cent.	Sulphur, per cent. (below)	Phosphorus, per cent. (below)	Manganese, per cent. (below)	Total carbon, per cent.
Agricultural machinery	2.00–2.50	0.08	0.500–0.800	0.60	3.50
Annealing boxes	0.65	0.08	0.250	0.50	2.25–3.25
Automobile cylinders	1.75–2.00	0.08	0.400–0.500	0.80	3.00–3.25
Bed plates	1.25–1.75	0.12	0.400–0.600*	0.80	3.25–3.75
Car castings	1.50–2.25	0.08	0.400–0.600*	0.80	3.25–3.75
Car wheels	0.60–0.80	0.09	0.400	0.80	3.50–3.75
Chilled castings	0.60–1.25	0.08	0.250	0.60	3.50–3.75
Cylinders: air, gas, ammonia, hydraulic	1.00–1.75	0.10	0.500	0.80	3.00–3.50
Dynamo frames	2.00–2.50	0.08	0.600	0.40	3.25–3.50
Grate bars	2.00–2.25	0.06	0.200	0.60	3.25–3.50
Gun iron	1.00–1.25	0.05	0.300	0.40	2.75–3.25
Hardware	2.25–2.75	0.08	0.800	0.80	3.25–3.75
Heat-resistant iron	1.25–2.25	0.05	0.200	0.40	3.00–3.50
Machinery castings, light	2.15–2.40	0.06	0.700	0.60	3.25–3.75
Machinery castings, medium	1.75–2.15	0.08	0.800	0.80	3.25–3.50
Machinery castings, heavy	1.25–1.75	0.10	0.800	1.00	3.00–3.25
Malleable casting (in the hard)	0.45–1.25	0.10	0.225	0.30	2.50–3.50
Ornamental castings	2.50–3.00	0.06	1.000–1.250*	0.40	3.75–4.00
Pipe fittings	2.00–2.50	0.08	0.400	0.60	3.25–3.75
Piston rings	1.50–2.00	0.08	0.500	0.60	3.25–3.50
Plow points, chilled	0.75–1.25	0.08	0.300	1.00	3.50–4.00
Rolls, chilled	0.60–0.85	0.08	0.400	0.80	3.00–3.25
Steel castings, soft	0.25–0.35	0.05	0.050	0.50–0.75*	0.17–0.20
Steel castings, medium	0.25–0.35	0.05	0.050	0.50–0.75*	0.20–0.30
Steel castings, hard	0.20–0.30	0.05	0.050	0.75–1.00*	0.30–0.40
Stove plate	2.25–2.75	0.06	0.800–1.250*	0.60	3.50–4.00
White-iron castings	0.50–0.90	0.10–0.25	0.800	0.80	4.00†

* Limits. † Below 4.00.

Composition of Castings. The analyses given in Table 1 are intended to serve as a general guide, the limits being fairly close. It is possible to make good castings by exceeding the limits given for silicon, manganese, etc., but this must be left to the judgment of the foundry selected to do the work. It is not always possible to obtain pig irons of just the composition required.

The lower limits for silicon should be used for the heaviest classes of castings, and the higher figures for the lightest. The variation in the total carbon is accomplished by adding steel to the mixtures.

No analysis is given for the "annealed" malleable casting, as the only change from the composition of the "hard" is in the total carbon (all combined), which is reduced by the annealing process from the full amount in the center of thick castings to about 0.20 in the skin and converted from the combined form to an amorphous variety of graphite called "temper carbon."

No subdivision of the total carbon into graphite and combined carbon is given in Table 1, as this division is a function of the silicon content, the casting temperature, and the rate of cooling after pouring determined by the thickness of section of the castings in question.

CHEMISTRY OF CAST IRON

Cast iron is a structure of steel with mechanically mixed graphite flakes between the crystals. This steel, with its high silicon and phosphorus, is a very poor material, and the graphite still further weakens the strength. The low combined carbon content and the abundance of graphite scattered throughout the mass, indicate it as a metal that may be easily machined without the use of a tool lubricant. With an increase in the combined carbon and a corresponding decrease in the graphite, the metal would no longer be coarse-flaked and machining would be hard on the tools and the casting. It is the object, in producing the strong cast-iron grades by steel additions, to reduce the planes of weakness due to graphite by lowering the total carbon, and yet to keep down the combined carbon as low as possible to facilitate machining. This is accomplished by silicon additions to the mixture, leaving a casting with proper silicon and a low total carbon; and in the case of heavy steel additions, by the use of ferro-manganese in the ladle as a deoxidizer of the iron.

Carbon. Beginning with the smallest percentages of carbon, there are the three commercial grades of steel castings. Just beyond a 2 per cent. content will be found the hard castings making tough (when good) malleables. From 2.50 per cent. on, good malleables are obtained, and from 2.75 per cent. upward the strong cast-iron varieties. When 3.75 per cent. is reached, the limit for ordinary cast irons ends, and only the low-silicon charcoal irons used for chilled-casting work run higher (up to 4.25 per cent., or almost the theoretical maximum).

The carbon in iron is of two general kinds: (*a*) **Combined carbon**—as carbides of iron, and in forms and distribution interesting only metallographically for ordinary foundry purposes, though important in heat-treatment work for steel castings; (*b*) **graphite**, or uncombined carbon, present as a mechanical admixture. This form of carbon is subdivided into two classes: (1) The crystalline graphite present in gray iron, produced in the "setting" of the molten metal, and varying in size from the big-flaked "kish" thrown off as excess carbon from blast-furnace casts to the very finely crystallized graphite of the soft cast irons made to set very quickly by means of chills; (2) graphite found in malleable castings, and resulting from the heat-treatment of white irons of the proper composition. Instead of separating out in the molten mass while setting this graphite is formed from the solid, the amorphous particles reposing in the interstices of the crystalline iron structure which are opened up by the long-continued high-temperature application. It is amorphous in shape and is called **temper carbon,** from the German "tempern."

In cast-iron founding, where heat-treatment subsequent to casting is rarely used, the total carbon present may be either all in the combined state, as one extreme, making white-fracture, hard, brittle and almost useless castings, or all graphitic, as the other extreme, making a black-fracture, soft, easily machined and highly useful iron. This range is due largely to the percentage of the other elements present, notably silicon; and in smaller measure to the pouring temperature and to the rate of cooling in the mold after pouring, which latter is a function of the thickness of section.

Silicon. Silicon has a **powerful softening effect,** its presence in cast iron reducing the ability of the iron to retain carbon in chemical combination. Thus, with silicon almost entirely absent, the iron will retain all its carbon in combination, making white iron. With about 3 per cent. silicon present almost no carbon can be held in chemical combination, and gray iron results. Beyond 3 per cent., silicon begins to act as a hardener; the Scotch or silvery irons, the high-percentage-silicon pig irons, and finally the ferro-silicons up to metallic silicon, are brittle substances.

In the case of the steel casting, where there is very little carbon present, silicon acts in a different manner, as it has no graphite to form. Here it deoxidizes the iron and promotes sound castings. In larger quantities and with special subsequent heat-treatment the steel made will show peculiar magnetic values.

In gray-iron work the proper selection of the silicon content in the mixture is the most important factor. Within given limits of the other elements every change desired can be brought out in the castings, so far as their strength, machining qualities, etc., are concerned, by variation in the silicon.

In malleable-casting work the silicon is even more important, as the range is quite small for a casting of given section. Thus, in heavy malleable castings the silicon should run between 0.35 and 0.50 per cent.; for medium sections. about 0.65 (which is the range of general car-castings requirements). For light work, such as pipe fittings, the silicon should be 0.75 to 1.00, and for extremely light work may be 1.25. Rarely can a foundry get satisfactory results when making both light and heavy work together.

Sulphur. Sulphur is **an objectionable impurity** in iron castings of practically all kinds. It probably forms a series of weak iron-sulphur compounds which surround the crystals of the iron and iron compounds that are strong and under other conditions strongly coherent, and the already unhomogeneous mass is made still more undependable. Moreover, sulphur and manganese form combinations with iron which make hard spots near the surface of castings and give trouble in the machine shop. Sulphur therefore is to be avoided where possible. Unfortunately, with every remelt more sulphur is taken up from the fuel, and hence the constant raising of the sulphur percentage in the foundry product through its dependence upon scrap for cheapening the mixtures. Well-made pig irons are fairly low in sulphur.

Sulphur **hardens iron** by counteracting silicon in its power to form graphite. Hence, where high sulphur is present, proportionally more silicon must be charged. Sulphur is advantageous **in cylinder work.** Here the peculiar granular iron-sulphur formations make for a fine grain which polishes highly in service. The only other work in which the presence of sulphur is advantageous, is **work** which has **to be threaded.** Here, again, the granular structure allows the cutting of fine, clean threads, a better material giving torn and rough-looking results.

Manganese. Manganese promotes the retention of carbon in the combined form, and hence **counteracts silicon.** This effect, however, is not specially noticeable until the percentage of Mn passes 0.80, which limit is usually set for gray-iron castings. In the case of malleable castings the effect of over 0.40 Mn is noticeable in the annealing process, where it retards the opening up of the structure of the metal to allow the separating out of the "temper carbon" and for that reason is injurious. For heavy sections manganese is not objectionable up to 1 per cent. Where the sulphur is high manganese will combine with it and if, after tapping, with extremely hot iron, the metal is allowed to stand for a while, manganese sulphide rises and can be skimmed off with what slag has come up also, thus often reducing the sulphur of the metal by one-half. If manganese is to be added to a mixture in the cupola, it should be in form of high-manganese pig iron. Ferro-manganese should never be charged anywhere but in the ladle, for the reason that manganese is easily oxidized and expensive. Where much steel scrap is charged in the cupola, and consequently the total carbon is low in the resulting metal, two things occur: first, the low-carbon metal has a higher freezing point, and hence must be melted much hotter than ordinary gray iron and cast at a higher temperature; second, from the nature of the melting process, the metal is bound to be heavily oxidized, and the addition of ferro-manganese in the ladle is necessary to deoxidize it—otherwise the castings will be porous and not sound. At the higher temperatures produced under the conditions necessary to melt the steel, the manganese added also has the power to deoxidize.

Phosphorus. The great value of phosphorus is to make iron very fluid when molten. Hence its use for art work, stoves, radiators, and similar thin and not necessarily strong castings. **Phosphorus is a hardener,** and requires the presence of the proper amount of silicon. The castings made with phosphorus irons being thin, there must be enough silicon present to prevent the retention of combined carbon, as thin sections cool at a rapid rate in the sand after casting.

Oxygen. This is probably the most serious element to contend with in the gray-iron and malleable foundry. In steel-casting establishments, the silicon and manganese added after melting remove any oxygen present from the melting; otherwise the castings would not be serviceable. As the presence of oxygen (probably as dissolved iron oxide) cannot be determined by ordinary chemical means, it is recognized (1) by the loss of "life" in molten iron, the freezing point having gone up; and (2) by the numerous pinholes found in the planed surface of a casting on its cope side. Here prevention is the only proper cure, and the charging and melting practice of a foundry making such defective work should be corrected.

Other Elements. The effects of nickel, chromium, titanium, vanadium, aluminum, copper, etc., need not be touched upon here, as these elements are either seldom found in pig iron in sufficient quantity to be considered, or else are additions which will be discussed under "Ferro-alloys." The iron made from Cuban ores forms an exception, however, as it contains both nickel and chromium, but its expense makes it of interest only to the specializing foundry and the steel trade.

PHYSICS OF CAST IRON

Shrinkage and Contraction. The term shrinkage is usually employed where contraction is really meant. **Contraction in a casting is the** reduction in dimensions due to the cooling from the temperature at the moment of set to the ordinary temperature of the atmosphere. **Shrinkage,** on the other hand, is the separation of the portions of metal from each other, due to an insufficient supply of liquid metal to fill up the spaces left by the reduction in volume as the iron sets. This forms spongy spots and actual cavities, the walls of which may be lined with pine-tree crystals of iron. Shrinkages greatly weaken the interior structure of the metal, and are highly dangerous through their concealed position.

Irons which throw out graphite on setting will not contract nearly as much as those in which this does not occur. The **ordinary "shrinkage"**—or rather contraction—of gray iron is ⅛ in. per ft. That of white iron and steel is about ¼ in. per ft., but varies with differences in the rate of cooling, the composition of the metal, and the shape of the pattern. In the case of malleable castings, where the original hard-iron casting should contract ¼ in. per ft. (one-half of which is restored in the annealing process), some parts do not contract at all, while others show a contraction of 3/16 in. per ft. This indicates that some portions are held tight in the mold and are actually stretched in the setting, while others readily pull away from the walls of the mold.

Segregation. The most frequent difficulties consist of hard spots of manganese and sulphur-iron compounds which interfere with machining where the two elements are too high; and a segregation of iron phosphide, accompanied by increased hardness in the center of a heavy section made with too-high-phosphorus iron. Further, through improper cooling, there may be an artificial chilling of the metal in spots, with consequent hardening effects. Annealing the casting will generally overcome this.

Strength of Cast Iron. A cast iron with a tensile strength of 20,000 lb. per sq. in. is considered a very good metal. By the addition of steel scrap in the melting charges to cut down the total carbon, the tensile strength is raised, so that, in rather small sections, it is not uncommon to get a value of 44,000 lb. per sq. in. Where special strength is wanted, an iron not under 30,000 lb. per sq. in. T. S. should be specified. The tensile test for cast iron is not to be recommended, as the results obtained are of uncertain value, unless the greatest care is taken with the preparation and testing of the specimen. American specifications make the tensile test optional and at the expense of the purchaser.

The **transverse strength** of cast iron is the one most readily obtainable for specification purposes. The so-called **arbitration bar** of the American Society for Testing Materials (which is the universally adopted standard of the country) is 1¼ in. in diam., 15 in. long, and is broken transversely on supports 12 in. apart. For the strength requirements, see p. 506. A breaking strength of from 2500 to 3000 lb. for the above-mentioned bar, with a deflection of 0.1 in. or over, indicates a good iron for ordinary use.

The **elastic limit** of cast iron is close to its ultimate breaking-strength. It is only in the case of very-low-total-carbon irons made by steel additions and carefully deoxidized by titanium or vanadium additions, that an elastic limit somewhat below the ultimate strength is obtainable. Elongation and reduction in area of specimen are not considered in making strength tests of cast iron. The **crushing strength** of cast iron is probably its most important characteristic, and gives it its value for structural use. It runs from 80,000 to 140,000 lb. per sq. in. The **resistance to shock** of cast iron is not high. On the other hand, the malleable casting has this as its prime characteristic. **Hardness** in ordinary cast irons is a condition which may be brought about artificially if desired **by chilling** the molten metal in the molds. Only the lower-silicon irons are amenable. Chilling is the basis of car-wheel manufacture, the casting of hardened-face jaws for rock breakers, stamps, and the like. Ordinarily, hardness in castings is a thing not desired in machine practice, nor is it good to have hard castings for ordinary classes of work, as the danger of brittleness is greater.

SPECIFICATIONS FOR CASTINGS

Standard Specifications for Gray-iron Castings

(American Society for Testing Materials)

1. Unless furnace iron is specified, all gray castings are understood to be made by the cupola process.

2. The sulphur contents to be as follows: Light castings, not over 0.08 per cent.; medium castings, not over 0.10 per cent.; heavy castings not over 0.12 per cent.

3. In dividing castings into classes, the following standards have been adopted: **Light castings**, or those having any section less than ½ in. thick; **heavy castings**, or

those in which no section is less than 2 in. thick; and medium castings, or those not included in the foregoing definitions.

4. **Transverse Test.** The minimum breaking strength of the "Arbitration Bar" under transverse load shall be not under 2500 lb. for light castings; 2900 lb. for medium castings; 3300 lb. for heavy castings. In no case shall the deflection be under 0.10 in. **Tensile Test.** Where specified, this shall not run less than 18,000 lb. per sq. in. for light castings; 21,000 lb. for medium castings; 24,000 lb. for heavy castings.

5. The quality of the iron going into castings under specification shall be determined by means of the "Arbitration Bar," see p. 505.

6. Two sets of two bars shall be cast from each heat, one set from the first and the other set from the last iron going into the castings. Where the heat exceeds 20 tons, an additional set of two bars shall be cast for each 20 tons or fraction thereof above this amount. In case of a change of mixture during the heat, one set of two bars shall also be cast for every mixture other than the regular one. Each set of two bars is to go into a single mold. The bars shall not be rumbled or otherwise treated, being simply brushed off before testing.

7. The transverse test shall be made on all the bars cast, with supports 12 in. apart, load applied at the middle, and the deflection at rupture noted. One bar of every two of each set made must fulfill the requirements to permit acceptance of the castings represented.

8. In making the mold the pattern shall not be rapped before withdrawing. The flask is to be rammed up with green molding sand, a little damper than usual, well mixed and put through a No. 8 sieve, with a 1:12 mixture of bituminous facing. The mold shall be rammed evenly and fairly hard, thoroughly dried and not cast until it is cold. The test bar shall not be removed from the mold until cold enough to be handled.

9. The rate of application of the load shall be 30 sec. for a deflection of 0.10 in.

10. Borings from the broken pieces of the "Arbitration Bar" shall be used for the sulphur determinations. One determination for each mold made shall be required. In case of dispute, the standards of the U. S. Bureau of Standards shall be used for comparison.

11. Castings shall be true to pattern, free from cracks, flaws and excessive shrinkage. In other respects they shall conform to whatever points may be specially agreed upon.

12. The inspector shall have reasonable facilities afforded him by the manufacturer to satisfy him that the finished material is furnished in accordance with these specifications. All tests and inspections shall, as far as possible, be made at the place of manufacture prior to shipment.

Standard Specifications for Malleable Castings

(American Society for Testing Materials)

These specifications are best adapted for the softer varieties of malleable castings, such as for railroad car construction. Special specifications are being proposed for the harder inelastic malleables suitable for conveyor chains, etc.

1. Malleable castings may be made by either air, open-hearth or electric furnace processes.

2. The test specimen for tensile strength and elongation shall be a round bar of the dimensions shown in Fig. 1. The test specimens for transverse strength and deflection shall be 14 in. in length, 1 in. in width and either ½, ⅝ or ¾ in. in thickness. The thickness of the test specimen selected shall be in proportion to the thickness of the casting which it represents. All test specimens shall be cast without chills, and with ends perfectly free in the mold. Four test specimens (two for each tensile and transverse test) shall be cast in each mold with risers of sufficient height at each end to insure sound bars. When the full heat is used for castings which are subject to specification, two molds shall be poured within five minutes after tapping into the first ladle, and two molds from the last iron of the heat. When only a part of the heat is required, there shall be cast two molds from the first ladle of iron used and two molds after the required iron has been tapped.

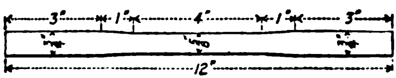

Fig. 1.

The test specimens from one mold each from the first and from the last of the heat shall be annealed in hottest part of the annealing oven and the remaining specimens shall be annealed in the coldest part. From the test specimens from each of the four molds required to represent each heat, there shall be selected for testing one specimen of each type for tensile strength with elongation, and for transverse strength with deflection.

3. The tensile strength of the test specimen shall not be less than 38,000 lb. per sq. in., with an elongation of not less than 5.0 per cent. measured in 2 in.

4. The transverse strength of the standard test specimen tested with cope side up on supports 12 in. apart, pressure being applied at the center, shall not be less than: 900 lb. with 1.25 in. deflection in ½-in. test specimen; 1,400 lb. with 1 in. deflection in ⅝-in. test specimen; 2,000 lb. with 0.75 in. deflection in ¾-in. test specimen.

5. Failure to reach the required limit for the tensile strength with elongation, also transverse strength with deflection on the part of more than one bar from each of the two molds annealed in the two points in the oven specified, rejects the casting from that heat.

6. In addition to the tests above provided for, the inspector shall have the right to satisfy himself of the suitability of the metal used for malleable castings made under these specifications by breaking a reasonable number of castings from the sand to examine for excessive mottling or graphite spots. In the case of castings of special design or importance, he may also require test lugs of such size as will bear proper relation to the thickness of the casting, but not exceeding ⅝ × ¾ in. section. At least one of these lugs shall be left on the casting for final inspection.

7. Castings shall be true to pattern, free from blemishes, scale and shrinkage cracks. A variation of 3/32 in. per ft. shall be permissible.

Standard Specifications for Steel Castings

(American Society for Testing Materials)

1. These specifications cover two classes of castings, namely: *Class A*, ordinary castings for which no physical requirements are specified; and *Class B*, castings for which physical requirements are specified. These are of three grades: hard, medium, and soft.

2. Patterns shall be made so that sufficient finish is allowed to provide for all variations in shrinkage, and be painted three colors to represent metal, cores, and finished surfaces. It is recommended that core prints shall be painted black and finished surfaces red.

3. The steel may be made by the open-hearth, crucible, or by any other approved process.

4. Class A castings need not be annealed unless so specified. Class B castings shall be allowed to become cold; shall then be reheated to the proper temperature to refine the grain, and allowed to cool slowly.

5. The steel shall conform to the following requirements as to chemical composition:

	Class A	Class B
Carbon	not over 0.30 per cent.	
Phosphorus	not over 0.08 per cent.	not over 0.05 per cent.
Sulphur		not over 0.05 per cent.

6. To determine whether the material conforms to the requirements specified in Sec. 5, an analysis shall be made by the manufacturer from a test ingot taken during the pouring of each melt. Drillings for analysis shall be taken not less than ¼ in. beneath the surface of the test ingot. A copy of this analysis shall be given to the purchaser or his representative.

7. A check analysis of Class B castings may be made by the purchaser from a broken tension or bend test specimen, in which case an excess of 20 per cent. above the requirements as to phosphorus and sulphur specified in Sec. 5 shall be allowed. Drillings for analysis shall be taken not less than ¼ in. beneath the surface.

8. The steel for each grade of Class B castings shall conform to the following minimum requirements as to tensile properties:

	Hard	Medium	Soft
Tensile strength, lb. per sq. in.	80,000	70,000	60,000
Yield point, lb. per sq. in.	36,000	31,500	27,000
Elongation in 2 in., per cent	15	18	22
Reduction of area, per cent	20	25	30

The yield point shall be determined by the drop of the beam of the testing machine.

9. The test specimen for soft castings shall bend cold through 120 deg. and for medium castings through 90 deg., around a 1-in. pin, without fracture on the outside of the bent portion.

10. In the case of small or unimportant castings, a test to destruction on three castings from a lot may be substituted for the tension and bend tests. This test shall show the material to be ductile, free from injurious defects, and suitable for the purpose intended. A lot shall consist of all castings from the same melt, annealed in the same furnace charge.

11. Test bars shall be attached to Class B castings weighing 500 lb. or over, provided the design of the castings will permit. If the castings weigh less than 500 lb., or are of such a nature that test bars cannot be attached, two test bars shall be cast to represent each melt; or the quality of the castings shall be determined by tests to destruction as specified in Sec. 10. All test bars shall be annealed with the castings they represent. The manufacturer and purchaser shall agree whether test bars can be attached to castings, and also on the location of the bars on the castings and the method of casting unattached bars. Tension test specimen shall be of the form and dimensions shown in Fig. 2. Bend test specimens shall be 1 × 1½ in. in section.

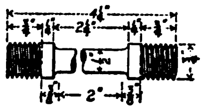

FIG. 2.

12. One tension and one bend test shall be made from each melt. If any test specimen shows defective machining or develops flaws, or if a tension test specimen breaks outside the gage length, it may be discarded; and the manufacturer and the purchaser or his representative shall agree upon the selection of another specimen in its stead.

13. The castings shall substantially conform to the sizes and shapes of the patterns, and shall be made in a workmanlike manner.

14. The castings shall be free from all injurious defects. The castings offered for inspection shall not be painted or covered with any substance that will hide defects, nor rusted to such an extent as to hide defects.

15. Castings which show injurious defects before or after machining will be rejected, and the manufacturer shall be notified.

FOUNDRY MATERIALS

Pig Iron. Pig irons may be generally classified as follows: Charcoal irons, cold blast and hot blast (often called "warm blast"); anthracite irons; coke irons, and electrically made pig irons. This classification is based upon the fuel used in the production. Charcoal gives the best pig iron—also the most expensive. With cold blast the very best results are obtained. Where the ordinary coke pig irons cost $17 a ton, the finest cold-blast charcoal pig iron may cost $45. Hot-blast charcoal iron is a cheaper variety which was formerly used much for malleable castings, car wheels and other work requiring chilling in whole or in part. To-day it is used only for chilled rolls, crusher jaws, and castings which require chilled parts and great strength.

Coke and anthracite irons are all made using a very hot blast. Electrically made pig iron is too new to find much application in the foundry, but is a probable future factor in the foundry business.

Pig iron is now bought almost entirely on its composition. Those who specify by grade number in place of analysis get the iron which could not be shipped to fill chemical specifications.

The requirements of the foundry trade being sharply defined by the classes of castings made, the product of the blast furnace readily divides itself on the phosphorus content. Thus, high-phosphorus pig irons go to the stove trade, while very-low-phosphorus irons, everything else being acceptable, go to the steel foundry. In a lesser degree the sulphur plays a part, the very-high-sulphur irons being unavailable for the better grades of castings. High sulphur means a low selling price. Manganese rarely bothers the foundryman unless a furnace is so situated that high-manganese ores are near by and cheap. The silicon content is the determining factor in the purchase, once the available brands have been gone over and the right ones selected. The foundryman specifies his silicon, and usually also his maximum sulphur.

In order that there may be uniformity in quotations, the following percentages and variations are used. These specifications do not advise that all five elements be specified in all contracts for pig iron, but do recommend that when these elements are specified the following percentages be used:

Silicon (0.25 allowed either way)	Sulphur (maximum)	Total Carbon (minimum)	Manganese (0.20 either way)	Phosphorus (0.15 either way)
1.00	0.04	3.00	0.20	0.20
1.50	0.05	3.20	0.40	0.40
2.00	0.06	3.40	0.60	0.60
2.50	0.07	3.60	0.80	0.80
3.00	0.08	3.80	1.00	1.00
	0.09		1.25	1.25
	0.10		1.50	1.50

Percentages of any element may be specified one-half way between the above. In case of phosphorus and manganese, the percentages may be used as maximum or minimum figures, but unless so specified they are considered to include the variations above given.

In testing pig iron to see whether it will suit for the general work of the foundry, the best method is to use it in regular service for a week to test it out.

Scrap is of two kinds, **domestic**, or that made in the foundry itself, and which remains and is used in the daily heats; and **foreign**, or scrap which is bought in the open market, and consists of several grades of heavy and light scrap machinery, pipe, stove plate, etc. The founder will know how to care for his own scrap, but that which he buys he must watch closely for burnt grate bars, furnace sections, and material which shows evidence of having been subjected to the oxidizing action of continued red heat. Such material is dangerous to add to mixtures and had better go into sash weights only.

Steel when used in foundry practice should be neither too light nor too heavy in section. Clippings from structural steel, rail ends, boiler plate, etc., are best. If too light, the proportion of surface to weight is too great, and oxidation is unavoidable. If too heavy, it will be too low in the bed before being completely melted. Steel should be charged directly on the fuel, pig iron over it, and then scrap cast iron. In this way all portions of the mixture melt at about the same time and good mixing results. In air-furnace practice heavier pieces of steel may be used, though rail, fish plates and the like are preferable.

Softeners, so-called, are the higher-silicon pig irons which are added to the mixture to increase the general silicon content. These irons, also known as **Scotch irons,** run from 4 to 8 per cent. silicon, and should be used with caution, as unless the charges are small and the bull ladle into which the iron is tapped is nearly twice the capacity of the metal in a single charge, there is apt to be hard and soft iron alternately instead of a good mixture. It is far better to charge pig irons of nearly the right silicon than to use extremes.

Ferro-silicon. When the silicon of a pig iron runs above 8 per cent., it begins to be classed as a ferro-silicon. Usually the ferro-silicon pig irons contain about 12 to 14 per cent. of silicon. Above 14 per cent. little is made until the 50 and 75 per cent. ferro-silicons are reached, and the percentage can run still higher than this, though it becomes unavailable for the foundry on account of the expense.

Ferro-manganese, see p. 503, which is used to some extent in the foundry, usually contains about 80 per cent. manganese, and finds its application in car-wheel work, and particularly in cases where much steel scrap is melted. Pig irons with 2 per cent. manganese and over are available and can be charged in the cupola directly. Ferro-manganese should be added to the molten metal in the ladle, and preferably sprinkled on the running stream right at the tap hole.

There are other alloys, such as **silico-spiegel**—a high-silicon ferro-manganese, **ferro-titanium** and **ferro-vanadium,** which also act as deoxidizers. The last-mentioned (see also p. 477), if added in sufficient quantity, leaves some vanadium in the iron. It is expensive, however, and used only for special purposes, as in machinery wearing surfaces. Ferro-titanium is excellent for deoxidation in the case of badly melted iron, but where precautions are taken with the melting practice, its use is not necessary. Both the titanium and vanadium alloys require further investigation, and while they can already be specified for steel, they are of unascertained value for cast iron.

Fluxes. As the pig iron and scrap charged into the cupola carry with them sand and rust, and as the ash of the fuel collects in the melting zone, it is essential for large heats that a flux be added to properly collect this material into a slag that can be drained off through a slag hole just below the bottom of the tuyères. Usually, **slagging off** a cupola is not resorted to for heats smaller than 5 to 10 tons. **Limestone** is the universal flux used, as it is most available. The limestone should contain the maximum possible amount of carbonate of lime, as with much clay or other mineral in it an equivalent amount of lime is used up to flux these substances, and consequently less is available for cupola fluxing. Where available, **oyster shells** form an excellent flux. **Fluor-spar** is another flux which in addition thins the slag very much. It should be used sparingly and in connection with limestone on account of its expense and powerful action on the cupola lining.

Ordinarily, the weight of the flux used is about 2 per cent. of the weight of the metal. In cases of very rusty material, such as stove plate, or all-sand-cast pig iron, which is very dirty, as much as 4 per cent. of limestone may be necessary. On the other hand, with machine-cast pig iron and clean scrap for the mixture, 1 per cent. may be enough. The flux is distributed on the coke charges evenly and away from the lining of the cupola.

Fire Brick for use about a foundry are of two grades. **First-grade fire brick** are intended for the highest temperatures. For the cupola the bricks or blocks used should be rather fine grained to resist abrasion of the charges and chemical action of the slag. For open-hearth and air furnaces, coarse-grained brick are more desirable. In either case they should be made of the highest grade of fire clay to resist the temperatures.

Second-grade fire brick, usually made of the same clay but not finished to size as carefully, are used for annealing-oven work, the upper lining of cupolas, and wherever the temperatures are lower. For gas flues, chimney linings, and where temperatures fall below 1000 deg. fahr., a still lower grade, called **flue brick,** is used.

Fire Sand is a very refractory sand. Sea sand or ordinary building sand, containing a low percentage of fluxing minerals, may be used for furnace bottoms, where a caking-together under heat is wanted; but for mixing with fire clay, as for cupola daubing, or for places where the direct sweep of the flame is to be encountered, the sand must be over 99 per cent. silica, and hence of a highly refractory nature. **Silica brick** (see p. 624) is rarely used in the gray-iron foundry, but is extensively used for the crowns of furnaces in open-hearth malleable casting practice, and entirely so for steel casting work.

Magnesite is used for the bottoms of open-hearth furnaces where basic steel is made, in contradistinction to the sand bottom in the acid open-hearth steel process. Where magnesite is used, and the sides and crowns of the furnaces are of silica brick, it is necessary to interpose a layer of **chrome brick** (see p. 624) and also to patch out the hearth at the slag line with **chromite,** as this is a material neutral to either acid or basic slags. **Dolomite** is also much used as a basic refractory.

Molding Sands, which are refractories so far as being containers for the molten iron for castings is concerned, are treated on p. 1399.

Foundry Facings embrace a variety of substances. The principal one, called **sea coal,** is simply a very finely ground high-grade bituminous gas coal. It is mixed with molding sand usually in proportion of eight sand to one sea coal, and riddled on the patterns in making the molds. These are then backed with ordinary molding sand. The purpose is to present to the running metal a surface which prevents its entrance. The coal generates gases instantly, which, being high in hydrocarbons, decompose with deposition of free carbon. The open structure of the sand allows these gases to escape rapidly, resulting in smooth-surfaced castings. Where the castings are very light and no particularly fine surface is required—as for malleable castings—facings are not used at all. Stove work, however, and other work where fine surfaces are essential, require not only this facing, but often have an application of **graphite** put on in addition.

To prevent **cutting of** the **sand** in larger castings as the consequence of a continued flow of molten metal, the mold surfaces are carefully rubbed with **graphite,** thus giving a very smooth surface. Good grades of this mineral should be used, as the poorer, adulterated ones—particularly if soapstone is present in large quantities—will suffer from the high temperature, and occasionally come off with some sand adhering, leaving bad spots in the surface of the casting. For small, light castings soapstone is not so dangerous, as the metal is in the mold and set before this mineral can melt. Another **blacking,** principally used for a **core wash,** is ground coke. This fills the pores of the core surface sufficiently to make it smooth without spoiling proper venting. Dry and wet **core binders** are dealt with on p. 1400.

MELTING PROCESSES AND MIXTURE MAKING

The melting may be done in crucibles, in the cupola, in the air furnace, the open-hearth furnace (acid or basic) or the electric furnace. Also, in making a special line of steel castings, a small converter is used after preliminary cupola melting of the charge.

The Crucible Process is now obsolete for cast iron, but is still used in making small high-grade steel castings. The absence of contact between metal and fuel makes this an ideal process, but, owing to mechanical difficulties, the output is small and the fuel cost is high. The crucible process, either with coke or oil firing, is the standard one for brass and bronze melting. Where the oil flame is made to play within the crucible—and this is made quite large in that case—the process really becomes that of the hearth furnace. Good brass and bronze castings are made in this way, but for gray iron the process takes too long to make really good work, the oxidation being too marked.

The Cupola is the most generally used melting process for cast iron, the fuel economy being highest, and the ease of manipulation greatest. Iron is melted continuously during the heat and can be poured as melted. The cupola is a shaft furnace, into which a fuel bed is introduced and upon which alternate layers of metal and fuel are charged. Air for combustion is supplied through tuyères near the bottom, placed just high enough to allow the space below to serve as a storage for the required quantity of molten iron. The following conditions must be considered carefully for satisfactory results.

It takes roughly 30,000 cu. ft. of air to melt 1 ton of iron, and the **melting rate** depends upon the rate of air supply. Variation in the melting rate in a given cupola is bad practice, in that it changes the position of the melting zone. Experience has shown that certain definite sizes of cupolas are best adapted for given melting rates per hour. There is some leeway in the following table, but it is not wise to exceed greatly the rates of melting indicated, as the quality of the metal produced will suffer.

Diam. of cupola inside lining, in...	18	24	30	36	42	48	54	60	66	72	78	84	90
Melting rate, tons per hour..........	0.5	1.5	3	4.5	6	8	10	13	16	19	22	26	30

A cupola 30 in. in diam. inside the lining is about as small as should be used in **commercial practice.** On the other hand, while there are cupolas 10 ft. in internal diam. in daily use in the United States, the European practice of using a **series of cupolas,** none of which exceed 48 in. in diam. is to be preferred, as not only is the melting accomplished with smaller variations in temperature, etc., but it is possible to run several mixtures in the shop safely. Ordinarily, American cupolas run between 42 in. and 72 in. diameter inside the lining.

In **charging,** the bed of fuel comes first and is usually made about 20 to 24 in. above the top of the tuyères. If there are upper tuyères, the fuel should go above the top of these. The upper tuyères have a very small area, as they are intended only to give a little extra air to burn the cupola gases and not to start a new melting zone. The cross-section of the lower tuyères, measured at the wind-box entrance, should run from one-fifth the cupola area for small cupolas, to not less than one-tenth for very large ones.

The **height of the fuel bed** is most accurately adjusted by timing the first iron after the blast is put on. It is assumed that the bed has been burned through well, and the charges put on up to the door and allowed to stand for some time to warm through. From the time when the blast is put on to the time at which iron runs from the spout sufficiently to require stopping up, there should elapse from 8 min. for heavy casting work, to 10 min. for very light castings. If less than 8 min., the bed should be raised the next day; if more than 10 min., the bed was too high. Melting within the cupola actually starts in about 5 min., as seen through the peepholes, but the stream should begin to come as above indicated.

The melting zone in the cupola ranges from about 12 in. above the tuyères to some 4 ft. above this, the temperatures rising rapidly from the tuyères to a maximum at about 24 in. above them. The CO_2 then begins to take up carbon from the incandescent fuel, if there is more to be had above this point of maximum temperature, and the formation of CO begins in quantity, with a resultant lowering of the temperature. If the iron is melted at any point other than that of the maximum temperature, the molten iron is "cold" and difficult to handle well. If melted below the zone of maximum temperature, there is also the danger of oxidation, with resulting pinholes, shrinkage spots, etc., in the castings.

As the melting is all done by the upper 4 in. of the fuel bed, the entire charging system should be so regulated that this 4 in. is replaced by the intermediate fuel charges, so that in melting iron the position of the bed may remain unchanged. To determine the size of the charges, place on the charging platform a ring of cupola brick 4 in. high and of the same diameter as the cupola. Fill with fuel, weigh this and make the metal charges 10 times this weight (with variations for carbon content in fuel, temperature of the cupola during melt, etc., as experience indicates and the scoring of the cupola lining demands). The practice of a double first charge is entirely wrong, and the volume of the air blown in should never change during the melting.

The **scoring** in the cupola should be observed from day to day. When from 4 to 6 in. are affected, the cupola is properly charged. If the cutting extends to several feet, there is a fluctuation of the melting position, which should be attended to in the charging.

In **charging** the cupola, care should be taken to charge in very even layers. The coke should be leveled, the pig iron of the charges spread uniformly on this (not all around the edges), and the scrap on the pig iron. Steel, if any is used (and it should not be thinner than ¼ in.), is spread directly on the coke, below the pig iron. The ladle into which the iron is tapped should be sufficient to hold at least 1½ charges to insure a good mixture. The metal charged should not vary widely in composition, as there is apt to be a separation which results in hard and soft castings from the same mixture.

In **the repairs of** the **cupola** only the best fire clay should be used. **Mica-schist** in flakes is often used to patch out the lining at the melting zone. If so, care should be exercised that heating up is slow, so that the moisture is removed without blowing out the patches.

The Air Furnace. Iron made in the air furnace, as well as in the open-hearth furnace, is known as **gun iron,** all the early ordnance having been cast from such metal. The furnace consists of a horizontal chamber with the firing space at one end and the stack at the other. The hearth is sloping, the deep portion being at the fire end. It is necessary to reduce excess air to a minimum so as to get the **maximum temperature** of combustion. Not more than 25 per cent. excess air should be allowed over and above that necessary for combustion. It is essential that the fire should be kept uniform. **Melting** is accomplished not so much by the direct contact of the long flame with the charge of metal as **by radiation** from the incandescent walls and roof of the furnace. The success of an air-furnace heat for a given tonnage in a given furnace, should be measured by the time required between charging and tapping. The longer the metal remains in the furnace the more it is damaged through oxidation from the excess air. The temperature of the bath in the air furnace or open-hearth furnace should be brought up to the desired point, and the bath tapped. Overheating, tapping, and allowing to cool before pouring lead to trouble from spongy iron, etc.

In melting with the air furnace, the bottom is first prepared by spreading sharp sand evenly, placing boards upon this and rolling in the heavy pieces of scrap. The small scrap is next charged so that it comes about in the center of the hearth. The pig iron is then carefully piled at the end, and the big end door closed and luted up. Firing now commences. The flames passing over the hearth first melt the light scrap, the pig and heavy scrap gradually go next, and pigs are then rolled one by one into the molten bath instead letting the mass settle down and melt extremely slowly. As the melting is practically all done by radiation from the crown and sides, the importance of keeping the brickwork of the furnace interior in strict repair is evident.

As soon as the metal is entirely melted, the slag is skimmed off, to afford the metal an opportunity to "refine" or become oxidized to a limited extent.

This reduces the silicon and carbon and at the same time raises the temperature. Recent investigations have shown that this is not desirable. It is better to start with clean iron, so that a minimum of slag is made, to have this thin, so that the heat may penetrate through it, and to obtain the reduction in the total carbon by steel additions. In the open-hearth furnace there is no difficulty in melting with a slag cover, hence the superiority of open-hearth iron over the product of the air furnace, in which this difficulty exists.

The **bath of metal** should be **well rabbled** from time to time to thoroughly mix the metal for composition, as well as to remove the hottest metal from the feather edge of the bath at the far end and promote uniformity in temperature. The old method of rabbling with green hickory poles is still resorted to in some foundries, and is a benefit in so far that it helps to mix the metal well.

The **advantage of** the **air furnace** (and also of the open-hearth) is that it gives a large amount of high-grade metal at one tap. The absence of direct contact of metal and fuel makes possible the very considerable reduction in total carbon by steel additions, without danger of picking up carbon from the fuel afterward; and also the use of heavy pieces of scrap which could not go into a cupola, or even into the open-hearth furnace as usually constructed.

Many attempts have been made to preheat the air used for combustion by recovery from the stack. There are successful furnaces of this nature in use, but it is generally better to resort to the open-hearth furnace as solving the problem in a more scientific and commercially economic way. The cupola does its melting with a ratio of 1 fuel to 8 metal, ordinarily. The air furnace, when run at the highest economy, with several heats a day, can do no better than 1 to 4, while the open-hearth furnace does it with a ratio of 1 to 6.

The Open-hearth Furnace is used for steel principally, for malleable in a limited way, and but very little for cast iron. This furnace, which is really a simple hearth furnace so arranged that the direction of the flame can be reversed at will, and which is provided with means for preheating the air and gas used for combustion, is second only to the electric furnace as regards quality of output.

Gas is used generally, or else oil or tar gasified as it enters the hearth. A gas-producer system is therefore installed for gasifying bituminous coal wherever neither natural gas nor oil is available. The preheated air and gas enter the furnace nearly 1000 deg. fahr. hotter than the room temperature. A 10-ton heat in the air-furnace takes about 4 hr.; the same tonnage in the open-hearth furnace takes only 2½ hr. Hence there is less chance for oxidation, and a better grade of metal results.

As the open-hearth furnace must be kept heated all the time, its use is limited to works which are very steady in running. There is no difficulty in forcing out three heats within the range of daily working hours, though two heats give more assurance of success. The heating of cast iron in an open-hearth furnace differs from the heating of steel in that melting only is to be done, not refining. The hottest temperature of the furnace is required for the shortest possible period. Where, in making a steel heat, the melting only is done in, say, 3 hr., for cast iron it should be cut to less than 2 hr., otherwise the comparatively high temperatures may result in the charge running away, or "Bessemerizing" right in the furnace.

The Bessemer Process is used for malleable to a limited extent in Continental Europe. The **electric furnace** is used only for making small quantities of high-grade steel.

Mixture Making

In making the mixtures for melting, allowance must be made for the burning out of the silicon and manganese, as well as for the absorption of sulphur and carbon from the fuel. Furthermore, the percentage of remelted stock plays an important part, as well as the additions of steel scrap. If the **manganese** does not drop below 0.50 per cent. in the resulting metal, the melting process has been carried out properly. If, however, an initial content of over 0.80 per cent. should be cut down to 0.40 per cent., or even lower, attention must be given to the melting process. In the case of air-furnace or open-hearth malleable practice, these figures do not hold, as it is desirable to finish up with manganese between 0.15 and 0.25.

Silicon is oxidized out by cupola melting to the extent of 0.25 per cent. ordinarily. With high fuel percentages this may drop to 0.20 per cent. The mixtures should always be calculated to contain 0.25 per cent. more silicon than is wanted in the castings. In the case of air-furnace melting the silicon loss is 0.30 and in bad cases even 0.35 per cent.

Sulphur is taken up from the fuel. An initial content of, say, 0.08 per cent. sulphur in the mixture, is often brought up to 0.15 in melting. With proper melting practice it should not exceed 0.10; melting at a high temperature drives off the sulphur from the fuel before much of it goes into the metal.

Steel Additions cut down the total carbon of a mixture only in part, as contact of this low-carbon material with the fuel always results in its absorbing more or less carbon. With special precautions, as high as 50 per cent. of steel can be used. Ordinarily, from 10 to 20 per cent. additions greatly improve the strength of the metal.

The **percentage of scrap gray-iron additions** is important in mixture making. In any case, the gates, sprues, etc., of a foundry plant should be used up as produced. This scrap will amount to from 10 to 50 per cent. of the output, depending upon the grade of castings made. In addition to this, cast-iron scrap bought in the open market is added to cheapen the mixture, and between the two as much as 60 per cent. of scrap may go into it.

Important castings should contain no foreign scrap. In castings which are to have considerable machining, no scrap other than the best selected grade should be used, as oxidized material will ruin the machined surfaces through pin holes and spongy spots. Some castings, as ingot molds, require all pig iron.

Calculations in Mixture Making. In mixture making, it is only necessary to calculate for silicon, if all other elements are known from the pig-iron analyses to be within the requirements. Assume 2000 lb. of iron is required to contain 2.15 per cent. silicon. Assume 20 per cent. of the mixture to be shop sprues, defective castings, gates, etc., containing 2.15 per cent. silicon. Assume also, that 20 per cent. of the mixture consists of purchased scrap containing 2 per cent. of silicon.

The best policy in a foundry is to provide three or more brands of pig iron, each having several silicon analyses, or possibly a dozen or more piles from which to select, and to make the bulk of the mixture from irons of which there are the largest stocks on hand, so that a plentiful variety of compositions may be always remaining. The range of composition should not be very wide, unless a great variety of casting classes are made, since it is not good to mix very-high-silicon with very-low-silicon irons, on account of the danger of uneven melting and mixing and the consequent making of alternate hard and soft castings from the same mixture.

Assume that 500 lb. of an iron with 2.75 silicon be added, and also the same weight of another iron with 2.50 silicon, leaving 200 lb. of the mixture to be disposed of later. The remaining 200 lb. of pig iron should contain 5.2 lb. of silicon or 2.60 per cent.

in order to give 2.40 per cent. silicon for the mixture. If this particular iron is not available, a readjustment of the first two irons is necessary, and by trial calculations this is readily effected. The mixture would then stand:

Sprues, etc.	400 lb. with	8.6 lb. Si
Purchased Scrap	400 lb. with	8.0 lb. Si
No. 1 Pig Iron	500 lb. with	13.7 lb. Si
No. 2 Pig Iron	500 lb. with	12.5 lb. Si
No. 3 Pig Iron	200 lb. with	5.2 lb. Si
	2000 lb. with	48.0 lb. Si

This on melting will yield 2.15 per cent. silicon in the castings made. Mixtures for the air furnace are dealt with in a similar manner.

MALLEABLE CASTINGS

Cupola **melting practice** for malleable cast iron is the same as for gray iron, except that more fuel is used to avoid possibility of oxidation. The allowance for burning out of silicon is 0.30 per cent. for the mixture.

The **composition of** an average-weight **malleable casting** is about as follows: Silicon, 0.65; manganese, 0.18; phosphorus, 0.180; sulphur, 0.07. The total carbon will vary from 0.20 at the skin to the full amount originally in the hard casting at the center, if over ½ in. thick; the original total carbon may have varied from 2.50 to 3.25 for ordinary practice.

The **silicon** should vary with the thickness of the castings. As the casting is white in fracture when coming from the sand, the mixture must be adjusted so that with a given silicon the sand can chill the metal sufficiently. Thus, with 0.65 Si in the iron ready to pour, every casting less than ½ in. in thickness will be good, whereas above that thickness there is danger of mottled or gray iron, which gives poor results when subjected to prolonged heating in the anneal. On the other hand, where all castings are ¼ in. or less in thickness, as in small fittings, the silicon can safely go up to 1.00 and even over. Where castings having parts ¾ in. thick are made, the silicon must be dropped down to 0.45, and it may even be necessary to apply chills to the thick places to get them white in fracture.

Mixtures for Malleable run normally half pig iron, the other half being made up of the sprues, annealed scrap, and some steel (but not over 5 per cent. of steel in any case). This scrap percentage can be varied either way considerably.

In air-furnace and open-hearth practice **the essential point in melting** is to get out the heats rapidly to reduce oxidation to a minimum.

The specifications for **pig irons** used **for making malleable castings** are as follows: Silicon, 0.75 to 1.50, as may be specified, delivery to have an allowance of 0.10 either way from the percentage asked for; manganese, not over 0.60; sulphur, not over 0.50, and phosphorus, not over 0.20. The best way to specify is to call for a given tonnage of the above with 0.75 Si, then for more tonnage with 1.00, more with 1.25, and finally with 1.50. Of these four specified silicon percentages most malleable works use 1.00 and 1.25 per cent. in largest quantity. The small-castings plants use more of the 1.25 and 1.50 per cent., and the heavy-castings plants more of the two lower percentages. All require the other elements as given above.

It is important in the **annealing process**—in which the castings should preferably be packed in an iron oxide of some kind, such as hammer scale, puddle scale, magnetite, hematite, etc.—that the castings be raised fairly rapidly to the full temperature of about 1350 deg. fahr., that this temperature be then maintained steadily for at least 60 hr., and finally, that the castings be allowed to cool very slowly, so that the carbon change effected by annealing may not be disturbed. It is injurious to heat a malleable casting to straighten it; this should always be done cold.

As a general rule, a cupola-iron **malleable casting costs** ½ cent more per lb. than a corresponding gray-iron casting. In the case of air-furnace or open-hearth furnace iron, the advance is about 1 cent per lb.

The **annealing** of the original hard casting converts the combined carbon into "temper carbon." In addition, there is considerable carbon removed from the skin of the casting. It is best not to machine malleable castings, as the toughest portion—the skin—is thus removed. Moreover, normal malleable castings are all subject to an interior shrinkage, due to heavy contraction, poor feeding possibilities of metal with so low a silicon percentage, and irregularities in section. Hence the normal strength of the interior portion is not to be depended upon if the skin is machined away.

In Europe, malleable castings are also made by the converter process, as the first part of the regular converter steel castings process. The best castings, however, are made by the crucible process. This process is too expensive in America, and is only used for high-grade steel castings at the present time.

STEEL CASTINGS

Steel castings may be classified on the basis of quality, from good to best, as follows: (1) Acid Bessemer; (2) basic open-hearth; (3) acid open-hearth; (4) crucible, and (5) electric furnace. In addition, there are the basic Bessemer steel casting, which is not on the market in this country but is used extensively in Europe; and the hybrid steels—in reality malleable castings which have been heated and quenched—which are called steel castings in the trade, and are only used for small edged tools. Their quality cannot stand in comparison with regularly made steel castings.

Basic Bessemer Steel Castings have rather poor physical qualities. The ductility is low, there is a greater chance for occluded gases and slag, and the tendency toward brittleness is specially marked.

Acid Bessemer Steel Castings (usually made in small quantities from steel melted in the "Baby" Bessemer side-blown converter) are much better, as they start with a higher-grade material. The pig iron and steel scrap used are melted in the ordinary foundry cupola, run into the converter—which usually is of 2 tons capacity, blown to low-carbon steel, the proper amounts of silicon and manganese added, and cast.

From the nature of the process there is liable to remain a slight oxidation of the steel, which is not good, as well as a rather high sulphur content. The castings from this process are usually small and unimportant. Their composition will run somewhere between the following limits: Carbon, 0.12 to 0.22; silicon, 0.25 to 0.40; manganese, 0.55 to 0.85; sulphur, 0.03 to 0.08; phosphorus, 0.03 to 0.06. The physical properties are about as follows: Ultimate strength, 70,000 lb. per sq. in.; elastic limit, 40,000 lb.; elongation in 2 in., 30 per cent.; reduction in area, 55 per cent.

Basic Open-hearth Steel Castings are the cheapest to make, and hence lead in tonnage produced. In pouring, the metal is usually somewhat "wild," and a rather high percentage of silicon has to be added to overcome this tendency. The silicon gradually exchanges with the phosphorus of the slag, causing the percentage of the latter to rise, and the former to drop as the steel is poured. The result is spongy parts in the castings. The proper use for this grade of castings is for very heavy and large pieces which require but few portions to be machined. Their composition is usually as follows: Carbon, 0.20 to 0.30; silicon, 0.20 to 0.40; manganese, 0.60 to 0.80; sulphur, not over 0.03; phosphorus, not over 0.04. They have about the same physical properties as those of the acid converter steel castings given above.

Acid Open-hearth Steel Castings are those best adapted for good work which must not be too expensive. They are used for all kinds of work which must be machined extensively, and are quite reliable. The segregation is not serious, except in very bulky pieces, and castings up to 50 tons in weight can be made readily. Like all steel castings, however, they should receive careful attention in the processes of manufacture. For the best classes of work, the castings should always be annealed. The composition of these castings is practically the same as that of the basic variety, except that both the manganese and silicon can be held lower. The physical properties are but little different from those of the castings so far discussed.

A recent development of the acid steel casting process is the introduction of the **"baby" acid open-hearth furnace**, usually of about 2 tons capacity. Here the hearth is swung out of the furnace structure, and the metal poured directly into the molds. The possibilities of the smaller open-hearth units in the acid process for making the higher carbon ranges of cast steel, has brought about a demand for such material; 0.40 carbon-steel castings are now being specified more frequently. Here the ultimate strength may run as high as 90,000 lb., with the elastic limit a little over half as large. The elongation is correspondingly lower, or below 20 per cent., and the reduction in area between 20 and 30 per cent.

Crucible-steel Castings are among the best made. The expense of the process necessarily confines the work to very-high-class and small-weight articles. Chemically, there is little variation from the figures already given, except that the carbon often runs up to 0.50. The elongation may run above 30 per cent., and the reduction in area often exceeds 50 per cent.

Electric-furnace Steel Castings. The melting is non-oxidising and the resulting metal is pure, uniform and quite hot. The castings produced cannot be equaled even by the crucible process. The physical strength runs high, and the reduction in area occasionally exceeds 60 per cent. Chemically, the electric-furnace method of melting makes it possible to keep the sulphur under 0.02 and the phosphorus under 0.015, results which cannot be achieved by any other process.

Alloy-steel Castings are coming into extended use, manganese-, vanadium-, titanium-, nickel- and chrome-steel castings being specified quite frequently. **Vanadium** seems to make for greater wearing quality. **Titanium** purifies by removing oxygen and nitrogen from the steel. **Nickel** will raise the tensile strength up to 100,000 lb. per sq. in., while **chromium** toughens the castings. **Manganese**-steel castings are very hard.

Miscellaneous

Patterns are now made of metal as far as possible. Where wooden patterns must be made, such hard woods as mahogany are selected for the purpose. Allowance for shrinkage is ⅛ in. per ft. for both gray iron and malleable, though in making repetition work the first castings are carefully measured over and the pattern corrected accordingly, as the straining action of the molds and cores upon the setting iron, which in that condition is capable of being stretched, will often nullify the entire so-called shrinkage.

After the metal is cast, care should be taken to see that the molds are not shaken out too soon, as the contact of red-hot castings with the air gives rise to interior strains. Moreover, the surfaces will be chilled somewhat, and this is detrimental to the machining qualities.

Cleaning Castings. Castings are either cleaned by hand with the scratch brush, or else go to the tumbling barrels or into the pickling tanks. Modern practice, in the case of castings allowing it, is rapidly adopting the **sand blast** for cleaning the foundry product. This process gives a very clean surface, removing not only the adhering sand, but also much of the scale due to oxidized iron, slag, and blacking from the mold surface. When well done, sand blasting may even replace the preliminary pickling necessary for subsequent galvanizing and tinning operations.

Foundry Layout. Foundries should be so designed that **transportation** is facilitated as much as possible. For large castings, the flat car should be run directly under the cranes in the foundry proper, or into the cleaning room, if this is thought more desirable. With tracks running directly into the foundry, sand may be brought in direct, saving double handling.

The **carrying of molten metal** is performed preferably in crane ladles, and the ideal foundry of large tonnage has two crane runways, one over the other. These are provided with one to three cranes, with one or two hoists each, depending upon the magnitude of the establishment. The sides are further provided with running jib cranes in any number desired, the radius of the crane action being smaller than half the width of the shop, so that the main cranes can pass while work is suspended from the jib cranes.

The **buildings** should be preferably of brick or structural steel, with steel roof trusses. If these materials are not used, everything about the cupolas should be fireproof. Lean-tos should also be provided with traveling cranes and floor trackage allowing the transfer of ladles from the main hall to the lean-to. Where smaller quantities of molten metal are to be used—not to exceed 1000 lb. at a time, the **overhead track** system is best, as the floor space is left clear of obstructions by trackage. Only as a last resort should metal be transported in ladles on floor track.

In the foundry, **light** is always best obtained through side windows. Skylights either in the slope of the roof or on louvres, are not efficient, as they soon become covered with dust and rusty particles of iron. **Heat** should also be provided in the foundry so that the men may work in comfort in the early winter mornings.

Fire protection is essential. Wooden flasks should not be stored directly against wooden lean-tos or buildings, nor should they be stored where there is a possibility of sparks from the cupola.

Foundry Costs. In the standard system of apportioning foundry costs adopted by the American Foundrymen's Association in 1908, the total expenditures are divided broadly into the actual foundry costs and the commercial costs. Foundry costs are subdivided into cost of metal, surcharges on metal (distributed to classes on a weight basis), cost of applied labor charged to product, and surcharges on applied labor costs (distributed on a percentage basis).

Metal Costs. These comprise the costs of pig iron (freight included), iron and steel scrap purchased, defective castings from foundry, cupola droppings, gates, sprues, etc.

Surcharges on Metal. The main accounts are for cupola expenses (including repairs, labor, materials, supplies, labor for loading and tending, fuel, lime, manganese, etc.); ladle expenses (labor, materials, renewals); foundry castings (charged to foundry at pound rates); molding supplies (sands, gravel, facings, core-room supplies, chaplets, core-oven fuels, lumber for renewing flasks, and labor on these items); general foundry work (*e.g.*, recovering scrap, cleaning up shop, cartage and handling, freight demurrage, power for yard and cupola service, measured or estimated, and charged for at a fixed rate per h.p.-hr.).

Applied Labor Costs include wages paid to molders, helpers, apprentices, core makers and their helpers, cleaners, and chippers. Whenever it is impossible to charge the wages of core makers, cleaners, and chippers to individual jobs, these items must be treated as surcharge labor costs, and distributed on a percentage basis.

Surcharges on Applied Labor. These include the salaries of superintendents, foremen, chemists and foundry clerks; charges for foundry office supplies, for a proportion of the general office expenses, for rent (arbitrary charge per sq. ft. of ground and of floor space), for power, light and heat (measured or estimated, at a fixed rate per h.p.-hr.), for maintenance and renewal of equipment, general miscellaneous work, sundry supplies, and for the wages of pattern makers, carpenters and blacksmiths. (Occasionally it may be possible to treat these wage amounts as applied labor.)

Each of the above accounts and items may be still further subdivided in order to permit distribution of costs to departments or to foremen, so that the progressive cost of iron from its purchase up to its appearance in the form of castings may be shown.

The **commercial costs** embrace all expenditures for administration and for making sales. They are preferably distributed in varying proportions to the different classes of castings, according to judgment rather than on a tonnage, labor or total-cost basis.

From the foundry accounts thus kept the total cost and net weight of good castings produced during the year may be ascertained, and from these the average cost per pound. The output is then divided into a number of classes of work ranging from the heaviest low-grade castings to the lightest and most intricate pieces, and the net price per lb. determined for each. To these are then added their proper shares of the commercial costs. A comparison of the average cost per lb. of all castings with the cost per lb. of each class of work will quickly show the classes in which it will pay the foundry to specialize. Without such a subdivision of costs, and basing estimates on work submitted upon the average cost per lb., a foundry may find itself loaded up with unprofitable work.

NON-FERROUS METALS AND ALLOYS

BY

GUILLIAM HENRY CLAMER

REFERENCES: Roberts-Austen, "Introduction to the Study of Metallurgy," Lippincott. Fulton, "Principles of Metallurgy," McGraw-Hill. Hofman, "General Metallurgy," McGraw-Hill. Law, "Alloys and Their Industrial Applications," Lippincott. Gulliver, "Metallic Alloys," Lippincott. Brannt, "The Metallic Alloys," Baird & Co.

Table 1. Physical Constants of Metals

Metal	Symbol	Atomic weight	Specific gravity	Specific heat	Melting point, deg. fahr.	10,000 × coefficient of linear expansion per deg. fahr.	Electrical conductivity (copper = 100)
Aluminum	Al	27.1	2.56	0.218	1216	0.1280	60.5
Antimony	Sb	120.2	6.71	0.051	1166	0.0583	4.4
Arsenic	As	75.0	5.67	0.081	1472	0.0306	...
Barium	Ba	137.4	3.78	0.047	1562	...	...
Bismuth	Bi	208.0	9.80	0.031	518	0.0740	12.0
Cadmium	Cd	112.4	8.60	0.056	610	0.1700	15.6
Cæsium	Cs	132.8	1.87	0.048	79	...	...
Calcium	Ca	40.1	1.57	0.170	1481	...	...
Cerium	Ce	140.2	6.68	0.045	1152	...	...
Chromium	Cr	52.1	6.50	0.120	2741	...	...
Cobalt	Co	59.0	8.50	0.103	2714	0.0684	...
Columbium	Cb	93.5	12.70	0.071	...	...	...
Copper	Cu	63.6	8.93	0.093	1981	0.0928	100.0
Gallium	Ga	69.9	5.90	0.079	86	...	...
Glucinum	Gl	9.1	1.93	0.621	...	...	...
Gold	Au	197.2	19.32	0.031	1945	0.0800	71.8
Indium	In	114.8	7.42	0.057	311	0.2320	...
Iridium	Ir	193.1	22.42	0.033	4172	0.0389	...
Iron	Fe	55.8	7.86	0.110	2768	0.0672	17.4
Lanthanum	La	139.0	6.20	0.045	1490	...	...
Lead	Pb	207.1	11.37	0.031	621	0.1624	7.8
Lithium	Li	6.9	0.54	0.941	367	...	...
Magnesium	Mg	24.3	1.74	0.250	1204	0.1495	35.8
Manganese	Mn	54.9	8.00	0.120	2237	...	...
Mercury	Hg	200.6	13.59	0.032	−38	0.3390	1.7
Molybdenum	Mo	96.0	8.60	0.072	4532	...	...
Nickel	Ni	58.7	8.80	0.108	2642	0.0706	12.6
Osmium	Os	190.9	22.48	0.031	4530	0.0361	...
Palladium	Pd	106.7	11.50	0.059	2822	0.0651	15.4
Platinum	Pt	195.2	21.50	0.032	3191	0.0495	14.2
Potassium	K	39.1	0.86	0.170	144	0.4680	...
Rhodium	Rh	102.9	12.10	0.058	3452	0.0472	...
Rubidium	Rb	85.5	1.53	0.077	100	...	...
Ruthenium	Ru	101.7	12.26	0.061	3270	0.0534	...
Silver	Ag	107.9	10.53	0.056	1762	0.1067	106.2
Sodium	Na	23.0	0.97	0.290	207	0.3950	...
Strontium	Sr	87.6	2.54	...	1472	...	...
Tantalum	Ta	181.5	10.80	0.036	5252	0.0439	...
Tellurium	Te	127.5	6.25	0.049	825	0.0929	...
Thallium	Tl	204.0	11.85	0.033	578	0.1680	...
Thorium	Th	232.4	11.10	0.028	...	...	...
Tin	Sn	119.0	7.29	0.055	450	0.1240	12.0
Titanium	Ti	48.1	3.54	0.130	3362	...	...
Tungsten	W	184.0	19.10	0.034	5432	...	...
Uranium	U	238.5	18.70	0.028	4352	...	...
Vanadium	V	51.0	5.50	0.125	3182	...	...
Yttrium	Yt	89.0	3.80	...	...	...	...
Zinc	Zn	65.4	7.15	0.094	786	0.1620	27.2
Zirconium	Zr	90.6	4.15	0.066	2700	...	...

NON-FERROUS METALS

Copper

Copper is distinguished from all other metals by a peculiar **red color,** which is pinkish or yellowish on the fresh fracture of the pure metal, but inclines to purple in the case of copper containing cuprous oxide. The **fracture** of cast copper is hackly granular; in forged or rolled copper it is fibrous and shows a pale red silky luster. Copper crystallizes in the cubic system. **Specific gravity:** Pure crystalline copper, 8.940; electrodeposited copper, 8.914; cast copper, 8.921; rolled and hammered copper, 8.952. Ordinary commercial copper is more or less porous, and its specific gravity varies between 8.2 and 8.5.

Copper is **malleable** and **ductile.** It becomes hard by rolling and drawing, but is readily annealed. It is completely softened by being held at a temperature of 350 deg. fahr. for 72 hr., or almost instantly when raised to a temperature of 600 deg. fahr.

Mechanical Properties: $E = 15{,}000{,}000$ (about); tensile strength, lb. per sq. in., hammered, 38,400; drawn, 44,800; fire-box (rolled and annealed), 32,700; special stay-bolt (annealed), 38,400. Elongation, 35–38 per cent.; reduction of area, 45–50 per cent. Shrinkage of castings, 0.1875 in. per ft. Compressive strength (cast), 40,000 lb. per sq. in.

There are **three standard grades** of copper on the market, namely, Lake, Electrolytic and Casting.

Lake Copper is obtained from the ores in the region principally bordering on Lake Michigan by wet concentrating methods. The concentrates are then smelted, and, after rabbling and poling, are cast directly into the form of copper ingots or bars. It is usually sold at a slight advance over electrolytic copper.

Electrolytic Copper is produced in great part from the Western United States sulphide ores, which are first matted and then blown in a Bessemer converter to eliminate most of the iron and sulphur; the metal is then cast into slabs, known as "blister" copper, which usually carry about 96 per cent. of copper. The blister copper is melted in large reverberatory furnaces and cast into anodes; these are dissolved in an electrolytic bath and deposited on cathodes, which are melted in reverberatory furnaces; after refining to eliminate impurities taken up in melting, it is cast into wire bars, cakes, slabs, ingots and billets.

Casting Copper is obtained from ores which do not carry sufficient gold and silver to warrant the expense of electrolytic refining, and also from brass foundry by-products known to the trade as "copper-bearing material." It is produced by smelting and oxidizing the ore or by-products in reverberatory furnaces until the impurities have been largely eliminated. Casting copper may range from 99 to 99¾ per cent. pure, depending on the raw material used and the degree of refining. It is considered inferior to electrolytic and lake copper, and sells at a slightly lower figure than electrolytic copper. **Secondary casting copper,** or that recovered from scrap copper, alloys and copper-bearing residues which occur as by-products in brass foundries and manufacturing plants where copper and copper alloys are used, runs from 98½ to 99¾ per cent. pure, depending on the raw material and the degree of refining.

Standard Specifications for Copper Wire Bars, Cakes, Slabs, Billets, Ingots and Ingot Bars were adopted by the American Society for Testing Materials, Aug. 25, 1913, substantially as follows:

METAL CONTENTS: The copper in all shapes to have a purity of at least 99.88 per cent. as determined by electrolytic assay, silver being counted as copper. (In the case of high-resistance Lake copper, silver and arsenic are counted as copper.)

RESISTIVITY. Low-resistance Lake copper wire bars (ingots and ingot bars) to have a resistivity not to exceed 0.1553 (0.15694) international ohm per meter-gram at 20 deg. (0 deg.) cent., annealed. Electrolytic copper in wire bars (ingots and ingot bars) to have a resistivity not to exceed 0.15535 (0.15694) international ohm per meter-gram at 20 deg. cent., annealed. High-resistance Lake copper has a resistivity greater than 0.15694 international ohm per meter-gram at 20 deg. cent.

Wire bars, cakes, slabs and billets to be substantially free from shrink holes, cold sets, pits, sloppy edges, concave tops and similar defects in set or casting. Five per cent. variation in weight or ¼ in. variation in any dimension from refiner's published list or purchaser's specifications may be considered good delivery.

Standard Specifications for Hard-drawn Copper Wire (Abstract of specifications adopted by the A. S. T. M., Aug. 21, 1911). Wire to be free from all surface imperfections not consistent with the best commercial practice. Necessary brazes to show at least 95 per cent. of the tensile strength of unbrazed wire. Specific gravity assumed as 8.89.

Size to be expressed as the diameter of the wire in decimal parts of an inch (to three places). Permissible size variations: For wire 0.100 in. in diam. and larger, ± 1 per cent.; for wire less than 0.100 in. in diam., 1 mil (= 0.001 in.). Coils to be gaged at both ends and near the middle. If, two points being within limits, the third is off more than 2 per cent. (for wire 0.064 and larger) or 3 per cent. (for wire smaller than 0.064), coil may be rejected.

Wire to have a tensile strength and elongation at least equal to the values given in the following table, as determined on fair samples.

Table 2. Specifications for Hard-drawn Copper Wire

Diam., in.	Tensile strength, lb. per sq. in.	Elongation in 10 in., per cent.	Diam., in.	Tensile strength, lb. per sq. in.	Elongation in 60 in., per cent.	Diam., in.	Tensile strength, lb. per sq. in.	Elongation in 60 in., per cent.
0.460	49,000	3.75	0.165	62,000	1.140	0.081	65,700	0.92
0.410	51,000	3.25	0.162	62,100	1.090	0.080	65,700	0.91
0.365	52,800	2.80	0.144	63,000	1.176	0.072	65,900	0.90
0.325	54,500	2.40	0.134	63,400	1.020	0.065	66,200	0.89
0.289	56,100	2.17	0.128	63,700	1.000	0.064	66,200	0.87
0.258	57,600	1.98	0.114	64,300	1.000	0.057	66,400	0.86
0.229	59,000	1.79	0.104	64,800	0.970	0.051	66,600	0.85
....	....	in 60 in.	0.102	64,900	0.970	0.045	66,800	....
0.204	60,100	1.24	0.092	65,400	0.950	0.040	67,000	....
0.182	61,200	1.18	0.091	65,400	0.940	....	....	....

Resistivity to be determined by resistance measurements on fair samples at 68 deg. fahr., and not to exceed 900.77 lb. per mile-ohm for wire from 0.460 to 0.325 in. in diam. or 910.15 lb. per mile-ohm for wire from 0.324 to 0.040 in. in diam.

Copper Castings. Sound copper castings may be made in sand or metal molds by adding about 1 per cent. of boron suboxide flux (containing 0.08 to 0.10 per cent. of boron suboxide) to the molten metal. Tensile strength of such castings, 24,000 lb. per sq. in.; elastic limit, 11,500 lb.; elongation, 48.5 per cent.; reduction of area, 74.5 per cent.; electrical conductivity, as high as 97 per cent.

Weights and Dimensions of Brass and Copper Tubes are given on p. 810. Weights and thicknesses of **brass and copper sheets and bars** are given in Tables 3 to 6. For properties of copper wire, see pp. 1588 and 1589.

Table 3. Weights of Copper and Brass Sheets and Plates

B. & S. gage No.	Lb. per sq. ft. Copper	Lb. per sq. ft. Brass	B. & S. gage No.	Lb. per sq. ft. Copper	Lb. per sq. ft. Brass	B. & S. gage No.	Lb. per sq. ft. Copper	Lb. per sq. ft. Brass
0000	21.30	20.40	12	3.740	3.580	27	0.657	0.629
000	19.00	18.10	13	3.330	3.190	28	0.585	0.560
00	16.90	16.20	14	2.970	2.840	29	0.521	0.499
0	15.00	14.40	15	2.640	2.530	30	0.464	0.444
1	13.40	12.80	16	2.350	2.250	31	0.413	0.395
2	11.90	11.40	17	2.100	2.000	32	0.368	0.352
3	10.60	10.20	18	1.870	1.790	33	0.328	0.314
4	9.46	9.05	19	1.660	1.590	34	0.292	0.279
5	8.42	8.06	20	1.480	1.420	35	0.260	0.249
6	7.50	7.18	21	1.320	1.260	36	0.232	0.221
7	6.68	6.39	22	1.170	1.120	37	0.206	0.197
8	5.95	5.69	23	1.050	1.000	38	0.184	0.176
9	5.30	5.07	24	0.931	0.890	39	0.164	0.156
10	4.72	4.51	25	0.829	0.793	40	0.146	0.139
11	4.20	4.02	26	0.738	0.706	....	....	....
Thickness, in.			**Thickness, in.**			**Thickness, in.**		
1/16	2.89	2.77	3/4	34.7	33.2	1 7/16	66.6	63.7
1/8	5.79	5.54	13/16	37.6	36.0	1 1/2	69.5	66.4
3/16	8.68	8.30	7/8	40.5	38.8	1 9/16	72.4	69.2
1/4	11.60	11.10	15/16	43.4	41.5	1 5/8	75.2	72.0
5/16	14.50	13.80	1	46.3	44.3	1 11/16	78.1	74.7
3/8	17.40	16.60	1 1/16	49.2	47.1	1 3/4	81.0	77.5
7/16	20.30	19.40	1 1/8	52.1	49.8	1 13/16	83.9	80.3
1/2	23.20	22.10	1 3/16	55.0	52.6	1 7/8	86.8	83.0
9/16	26.00	24.90	1 1/4	57.9	55.4	1 15/16	89.7	85.8
5/8	28.90	27.70	1 5/16	60.8	58.1	2	92.6	88.6
11/16	31.80	30.40	1 3/8	63.7	60.9	....	....	....

Table 4. Thicknesses of Standard Copper Sheets Rolled to Weight

(American Brass Co.)

Lb. per sq. ft.	Thickness, in.	Lb. per sq. ft.	Thickness, in.	Lb. per sq. ft.	Thickness, in.	Lb. per sq. ft.	Thickness, in.	Lb. per sq. ft.	Thickness, in.	Lb. per sq. ft.	Thickness, in.
16	0.3456	9 1/2	0.2052	6 1/2	0.1404	3 1/2	0.0756	1 3/4	0.0378	3/4	0.0162
15	0.3240	9	0.1944	6	0.1296	3	0.0648	1 1/2	0.0324	5/8	0.0135
14	0.3024	8 1/2	0.1836	5 1/2	0.1188	2 3/4	0.0594	1 1/4	0.0270	1/2	0.0108
13	0.2808	8	0.1728	5	0.1080	2 1/2	0.0540	1 1/8	0.0243	3/8	0.0081
12	0.2592	7 1/2	0.1620	4 1/2	0.0972	2 1/4	0.0486	1	0.0216	1/4	0.0054
11	0.2376	7	0.1512	4	0.0864	2	0.0432	7/8	0.0189	1/8	0.0027
10	0.2160										

Table 5. Weights of Drawn Copper Bars, Standard Rectangular Sizes

(Weights in lb. per lin. ft.)

Size, in.	Lb.
1/16 × 1/2	0.121
1/16 × 5/8	0.151
1/16 × 3/4	0.181
1/16 × 7/8	0.211
1/16 × 1	0.241
1/16 × 1 1/4	0.301
1/16 × 1 1/2	0.362
1/8 × 1/2	0.241
1/8 × 5/8	0.301
1/8 × 3/4	0.362
1/8 × 7/8	0.422
1/8 × 1	0.482
1/8 × 1 1/4	0.603
1/8 × 1 1/2	0.724
1/8 × 1 3/4	0.844
1/8 × 2	0.965
1/8 × 2 1/4	1.090
1/8 × 2 1/2	1.210
1/8 × 2 3/4	1.330
1/8 × 3	1.450
3/16 × 1/2	0.362
3/16 × 5/8	0.452
3/16 × 3/4	0.543
3/16 × 7/8	0.633
3/16 × 1	0.724
3/16 × 1 1/4	0.904
3/16 × 1 1/2	1.090
3/16 × 1 3/4	1.270
3/16 × 2	1.450
3/16 × 2 1/4	1.630
3/16 × 2 1/2	1.810
3/16 × 2 3/4	1.990
3/16 × 3	2.170
1/4 × 1/2	0.482
1/4 × 5/8	0.603
1/4 × 3/4	0.724
1/4 × 7/8	0.844
1/4 × 1	0.965
1/4 × 1 1/4	1.210
1/4 × 1 1/2	1.450
1/4 × 1 3/4	1.690
1/4 × 2	1.930
1/4 × 2 1/4	2.170
1/4 × 2 1/2	2.410
1/4 × 2 3/4	2.650
1/4 × 3	2.890
3/8 × 1	1.450
3/8 × 1 1/4	1.810
3/8 × 1 1/2	2.170
3/8 × 1 3/4	2.530
3/8 × 2	2.890
3/8 × 2 1/4	3.260
3/8 × 2 1/2	3.620
3/8 × 2 3/4	3.980
3/8 × 3	4.340
3/8 × 3 1/4	4.700
3/8 × 3 1/2	5.060
3/8 × 3 3/4	5.430
3/8 × 4	5.790
3/8 × 4 1/4	6.150
3/8 × 4 1/2	6.510
3/8 × 4 3/4	6.870
3/8 × 5	7.24
1/2 × 1	1.93
1/2 × 1 1/4	2.41
1/2 × 1 1/2	2.89
1/2 × 1 3/4	3.38
1/2 × 2	3.86
1/2 × 2 1/4	4.34
1/2 × 2 1/2	4.82
1/2 × 2 3/4	5.31
1/2 × 3	5.79
1/2 × 3 1/4	6.27
1/2 × 3 1/2	6.75
1/2 × 3 3/4	7.24
1/2 × 4	7.72
1/2 × 4 1/4	8.20
1/2 × 4 1/2	8.68
1/2 × 4 3/4	9.16
1/2 × 5	9.65
1/2 × 5 1/4	10.10
1/2 × 5 1/2	10.60
1/2 × 5 3/4	11.10
1/2 × 6	11.60
3/4 × 1	2.89
3/4 × 1 1/4	3.62
3/4 × 1 1/2	4.34
3/4 × 1 3/4	5.06
3/4 × 2	5.79
3/4 × 2 1/4	6.51
3/4 × 2 1/2	7.24
3/4 × 2 3/4	7.96
3/4 × 3	8.68
3/4 × 3 1/4	9.41
3/4 × 3 1/2	10.10
3/4 × 3 3/4	10.90
3/4 × 4	11.60
3/4 × 4 1/4	12.30
3/4 × 4 1/2	13.00
3/4 × 4 3/4	13.80
3/4 × 5	14.50
3/4 × 5 1/4	15.20
3/4 × 5 1/2	15.90
3/4 × 5 3/4	17.40
3/4 × 6	16.60
1 × 1	3.86
1 × 1 1/4	4.82
1 × 1 1/2	5.79
1 × 1 3/4	6.75
1 × 2	7.72
1 × 2 1/4	8.68
1 × 2 1/2	9.65
1 × 2 3/4	10.60
1 × 3	11.60
1 × 3 1/4	12.50
1 × 3 1/2	13.50
1 × 3 3/4	14.50
1 × 4	15.40
1 × 4 1/4	16.40
1 × 4 1/2	17.40
1 × 4 3/4	18.30
1 × 5	19.30
1 × 5 1/4	20.30
1 × 5 1/2	21.20
1 × 5 3/4	22.20
1 × 6	23.20

Table 6. Weights of Round and Square Brass Rods

(Lb. per lin. ft.)

Diam., in.	Round	Square	Diam., in.	Round	Square	Diam., in.	Round	Square	Diam., in.	Round	Square
1/16	0.0113	0.0144	11/16	1.37	1.74	1 5/16	4.99	6.36	1 15/16	10.9	13.9
1/8	0.0458	0.0576	3/4	1.63	2.08	1 3/8	5.48	6.97	2	11.6	14.8
3/16	0.1020	0.1300	13/16	1.91	2.44	1 7/16	5.99	7.62	2 1/8	13.1	16.7
1/4	0.1810	0.2300	7/8	2.22	2.83	1 1/2	6.52	8.30	2 1/4	14.7	18.7
5/16	0.2830	0.3600	15/16	2.55	3.24	1 9/16	7.07	9.01	2 3/8	16.3	20.8
3/8	0.4070	0.5190	1	2.90	3.69	1 5/8	7.65	9.74	2 1/2	18.1	23.1
7/16	0.5550	0.7060	1 1/16	3.27	4.16	1 11/16	8.25	10.50	2 5/8	20.0	25.4
1/2	0.7240	0.9220	1 1/8	3.67	4.67	1 3/4	8.87	11.30	2 3/4	21.9	27.9
9/16	0.9170	1.1700	1 3/16	4.09	5.20	1 13/16	9.52	12.10	2 7/8	24.0	30.5
5/8	1.1300	1.4400	1 1/4	4.53	5.76	1 7/8	10.20	13.00	3	26.0	33.2

Table 7. Strength of Sheet Copper at Various Temperatures

(In per cent. of the strength at about + 50 deg. fahr.)

Temperature, deg. fahr.	122	212	392	482	545	693	844	1033
Tensile strength	98	95	91	85	79	75	66	51

Lead

Lead possesses a characteristic bluish-gray color and a dull metallic luster, which is lost on exposure to the air, the surface becoming dull gray. It is soft enough to be indented with the nail and can be cut with a knife, the softness of the metal increasing with its purity. It can be rolled into thin sheets, and, by previous heating, can be "squirted" into rods and tubes; it cannot, however, be drawn into fine wire on account of its want of ductility and its low tensile strength. The **specific gravity**, between 11.254 and 11.395, is only very slightly increased by rolling (according to Knab, 11.352 for cast lead and 11.358 for rolled lead). The specific gravity is lowered by the presence of other metals, so that it furnishes an indication of the softness and purity of the lead.

Lead crystallizes in the cubical system. At a bright red heat, in the presence of air, it volatilizes perceptibly, but only to a very slight extent if air be excluded; at a white heat (2900–3560 deg. fahr.) it boils. Antimony, arsenic, copper and zinc diminish the softness of lead when present to any considerable extent, antimony and arsenic rendering it brittle, hard and easily fusible.

Mechanical Properties: E in lb. per sq. in. = 700,000 for rolled or cast lead (= 1,000,000 for lead wire). Tensile strength in lb. per sq. in. = 1780 for rolled or cast lead (= 3130 for hard and 2420 for soft wire). Shrinkage of castings, 0.3125 in. per ft.

Practically all the lead on the market is obtained from the **sulphide of lead** ore, known as **galena.** Practically all sulphide ores carry gold and silver, and crude lead is therefore subjected to further treatment for the extraction of the precious metals, and finally to an oxidising refining treatment which renders the lead extremely pure.

Table 8. Composition of American Pig Leads

a—Southeast Missouri Undesilverized.
b—Southeast Missouri Desilverized.
c—Southwest Missouri Undesilverized.
d—Ordinary Common.
e—Ordinary Corroding—or Refined.

	a Per cent.	*b* Per cent.	*c* Per cent.	*d* Per cent.	*e* Per cent.
Silver	0.0070	0.0004	0.0005	0.0005	0.0005
Arsenic and zinc	trace	trace	trace	trace	trace
Antimony	0.0030	0.0030	0.0020	0.0100	0.0050
Bismuth	0.0030	0.0030	0.0030	0.0800	0.0500
Copper	0.0600	0.0003	0.0190	0.0006	0.0006
Iron	0.0015	0.0015	0.0015	0.0015	0.0015
Cobalt and nickel	0.0080	none	0.0018	none	none
Lead	99.9175	99.9918	99.9722	99.9074	99.9424

Specifications for Pig Lead. The Pennsylvania Railroad Specification No. 43–A, adopted Dec. 21, 1908, specifies that No. 1 grade shall contain not less than 99½ per cent. of metallic lead, and No. 2 grade (used only for counterbalancing) not less than 97½ per cent.

Chemical Lead. The lead sold in the United States under this name is obtained in the Flat River District in southeast Missouri, from disseminated ores which are contaminated with copper, cobalt, nickel and other impurities. This lead is alloyed naturally with certain percentages of other ingredients which render it more impervious to acid attacks than other brands of lead. The Hoyt Metal Co., of St. Louis, manufacture a standardized chemical lead known as "**Hoyt process" lead,** which is furnished in the form of sheet and pipe. **Hoyt metal** is a lead carrying from 6 to 10 per cent. of antimony, and in the form of sheets and pipes is extensively used in chemical works and other industries where acids are made or employed. The 10 per cent. Hoyt metal weighs 8 per cent. less than chemical lead, its tensile strength is double that of the latter, and it is much more rigid. It is claimed that it will not buckle, creep, stretch, tear or sag in use, and that it is much superior to chemical lead in acid-resisting qualities.

Lead Wool is lead in a fibrous or shredded form, which is used for calking the joints of water and gas pipes.

Lead Pipe. For tables of sizes and weights of lead pipe and tubing, see pp. 811 and 812.

Table 9. Maximum Sizes in Which Sheet Lead Can Be Furnished

Weight per sq. ft., lb.	Max. size in ft.	Weight per sq. ft., lb.	Max. size in ft.
1	8 × 20	10	11½ × 40
1½	8 × 20	12	11 × 40 or 11½ × 35
2	7 × 45	14	11½ × 40 or 11¾ × 30
2½	9 × 45	16	11½ × 40 or 11¾ × 30
3	10 × 45	20	11½ × 40 or 11¾ × 38
3½	10 × 45	20	11¾ × 36
4	10 × 45	24	11¾ × 30
5	10 × 43	24	11 × 34 or 11 ½ × 32
6	10 × 43 or 11 × 40	30	11 × 27 or 11½ × 25½
6	11½ × 30 or 11¾ × 25	30	11½ × 24½ or 12 × 16
8	10 × 40 or 11½ × 35	40	11 × 24 or 12 × 16
10	11½ × 30 or 11 × 40 or 10 × 48	60	12 × 12

Zinc

Zinc is a bluish-white metal having a **specific gravity** of about 7.1. It boils at about 930 deg., so that it can be readily distilled; there is always a sensible loss when it is used in the manufacture of alloys. The metal in fine shavings or vapor burns readily with an intense bluish-white flame, forming dense clouds of white zinc oxide (philosophers' wool). It is malleable and ductile through a limited range of temperature only, and is largely **used** for rolling into sheets **for roofing** and other purposes. It oxidizes only slightly on exposure to the air, with the formation of a basic carbonate.

Zinc is marketed in the form of **rolled sheets,** and also in cast cakes of about 1 in. thick, known as **spelter.** The cakes are very brittle, and break with a more or less crystalline fracture. If the metal be nearly pure the crystal faces are large, bright and smooth; if there be a small quantity of iron present dull spots appear on the crystal faces; with an increased quantity of iron, as in dross spelter, the **fracture** becomes granular. The amount of iron present can be fairly judged from the appearance of the fracture. Its **tensile strength is low**: 27,000 lb. per sq. in. along the grain; 36,000 across grain, for rolled sheets; spelter, 4000 (coarsely crystalline) to 14,000 (fine-grained). Modulus of rupture, 8000 to 22,000 lb. per sq. in., increasing with fineness of grain. Compressive strength (cast), 20,000 lb. per sq. in. E (average) = 13,700,000; elongation, 12 to 38 per cent.; reduction of area, 23 to 56 per cent.

Zinc casts well, contracts but little on solidifying, and is largely used for the manufacture of statuettes and other ornamental castings which are usually coated with bronze or brass by electrodeposition. Shrinkage of castings, 0.3125 in. per ft. Its **chief uses** are in galvanizing and in the manufacture of brass and other alloys.

Zinc is never pure, the principal **impurities** being iron, lead, tin, copper, arsenic, and cadmium. When zinc is used for galvanizing, a hard zinc which contains several per cent. of iron accumulates in the vats. Good commercial spelter should not contain more than 0.05 per cent. of iron, and this is about the maximum allowable for alloy making.

Lead is invariably present in spelter in larger or smaller quantity. For making brass or other alloys, a spelter containing more than 1.5 per cent. of lead should be rejected. The other impurities are rarely present except in unobjectionable quantities.

Table 10. Typical Analyses of Various Grades of Virgin Spelter

Grade	Iron %	Lead %	Cadmium %	Zinc %
High grade	0.02(0.03)	0.05(0.07)	(0.05)	99.930
Intermediate	0.025(0.03)	0.15(0.20)	(0.50)	99.825
Brass special	0.025(0.04)	0.30–0.60(0.75)	0–0.50(0.75)	balance
Prime Western	0.08(0.08)	1.50(1.50)	0.50	97.920
Sheet zinc*	0.015	0.27	0.29	99.400

* Matthiessen & Hegeler Zinc Co.

High-grade spelter is used mainly for brass for spinning and drawing; for the highest grades of alloys, such as manganese bronze, and for galvanizing telegraph and telephone wires that have to withstand sharp bending; also for artistic castings. The **intermediate** grade is used in brasses and bronzes where the very highest quality is not essential, and also for the better grades of casting-brass. The **brass special** grade is used for the better grades of alloys where high ductility is not required. **Prime Western** spelter is used mainly for the ordinary galvanizing of sheet, wire and miscellaneous articles, also in red-brass alloys and free-cutting yellow brass.

Secondary Spelter is recovered by refining zinc dross—an iron-zinc alloy formed in the process of galvanizing. It carries from 3 to 5 per cent. of iron and from 2 to 3 per cent. of lead. This is refined either by distillation or by a liquating method. If refined by the former method a product of fairly high purity results, and if refined by the latter method a very inferior quality results, carrying a high percentage of iron and lead. This latter product is given a marketable appearance by the addition of a very small percentage of aluminum.

Specifications were adopted by the American Society for Testing Materials, Aug. 21, 1911. Limiting percentages of impurities are specified for the four grades of virgin spelter (see values in parentheses in Table 10), methods of sampling, analyzing, etc. High, intermediate and brass special grades are to be free from aluminum, and their combined contents of iron, lead and cadmium are not to exceed 0.10, 0.5, and 1.20 per cent., respectively.

Spelter for slush castings should be free from cadmium and should not contain more than 0.1 per cent. of lead. Sound castings from less pure metal can be obtained only by making them excessively heavy. (Stone, Trans. A. I. Metals, vol. 8.)

Table 11. Weight of Sheet Zinc
(Matthiessen & Hegeler Zinc Co.)

Zinc gage No.	Thickness, in.	Lb. per sq. ft.	Zinc gage No.	Thickness, in.	Lb. per sq. ft.	Zinc gage No.	Thickness, in.	Lb. per sq. ft.
3	0.006	0.22	12	0.028	1.05	21	0.080	3.00
4	0.008	0.30	13	0.032	1.20	22	0.090	3.37
5	0.010	0.37	14	0.036	1.35	23	0.100	3.75
6	0.012	0.45	15	0.040	1.50	24	0.125	4.70
7	0.014	0.52	16	0.045	1.68	25	0.250	9.40
8	0.016	0.60	17	0.050	1.87	26	0.375	14.00
9	0.018	0.67	18	0.055	2.06	27	0.500	18.75
10	0.020	0.75	19	0.060	2.25	28	1.000	37.50
11	0.024	0.90	20	0.070	2.62	....	....	....

Tin

Tin has an almost silvery whiteness with a slightly bluish tinge and a brilliant luster which depends largely on the pouring temperature; if this is too high the surface will show iridescent colors, while if too low the surface will be dull. The admixture of small quantities of lead, arsenic, antimony, iron, or bismuth also diminishes the luster of tin.

The structure of tin is distinctly crystalline. **Specific gravity:** Cast tin, 7.291; rolled tin, 7.299; electrically deposited tin, from 7.143 to 7.178.

Mechanical Properties. Tin is harder than lead but softer than gold. At ordinary temperatures it can be beaten and rolled into thin leaves (sheet tin and tinfoil). **At higher temperatures** its extensibility diminishes, until at 392 deg. fahr. it **is** so **brittle** that it breaks to pieces when hammered, and can be powdered. If the temperature at which tin is cast be either too high or too low, it will be "short," *i.e.*, brittle. The addition of 1 to 2 per cent. of copper or lead increases its hardness and tenacity. Tin is **ductile** but possesses little tenacity (T.S. = 5000 to 5700 lb. per sq. in.; $E = 5{,}700{,}000$); it is most ductile at about 212 deg. fahr.; a wire 0.08 in. in diam. breaks under a load of 54 lb. For cast tin, tensile strength is 3500 lb. per sq. in. and compressive strength 6000 lb. per sq. in.

Impurities and Their Effects. Iron, if present in considerable quantities, makes the tin hard and brittle; arsenic, antimony and bismuth, if present to the extent of 0.5 per cent., reduce its tenacity; copper and lead (1 to 2 per cent.) make it harder and increase its strength, but make it less malleable; tungsten and molybdenum render it less easily fusible; stannous oxide reduces its tenacity; sulphur is said to render it "short."

Table 12. Typical Percentage Analyses of Various Grades of Pig Tin

Grade	Tin	Antimony	Arsenic	Lead	Bismuth	Copper	Iron	Silver	Sulphur
Banca	99.95	0.007	....	trace	....	0.018	0.045	....	trace
Penang	99.939	trace	0.013	trace	....	0.016	0.028	....	....
Singapore	99.87	0.008	0.045	0.034	0.003	0.052	0.003	0.006	0.005
Range of impurities in other grades.	......	0 to 0.569	0 to 0.065	0 to 4.00 (China)	0 to 0.055	0.018 to 0.445	0 to 0.028	traces	0 to 0.013

Sources. The principal **tin ore** is cassiterite, or tin stone—an oxide of tin. The lowest grades of pig tin come from China and Bolivia, and contain as little as 94 per cent. metallic tin. **Banca tin** has the highest reputation of any on the market, and next to this is Straits tin, from Malacca. Tin is now recovered from tin-plate clippings by an electrolytic or "detinning" process in the form of a fine powder, which is then smelted in a reverberatory furnace and cast into pigs. This **by-product tin** usually runs from 97 to 99 per cent. of tin and carries lead, copper and antimony. For the production of the finest grades of solder, Babbitt metal and high-grade alloys subject to physical tests, Banca, Straits or similar grades of tin should be used. For all other uses the lower grades of tin are suitable, due consideration being given to their actual tin content.

Block Tin Pipe. For table of weights and dimensions, see p. 812.

Antimony

Antimony is characterized by its great brilliancy and by its color—silver white with a slight tinge of blue, the latter being increased by the presence of **impurities,** such as sulphur, arsenic, lead, copper and iron.

When pure molten antimony is allowed to solidify slowly and without disturbance under a layer of slag, a fern-like arrangement of raised lines radiating from the center appears on the surface of the metal (the so-called "antimony star"). This is generally regarded as an indication of the purity of the metal.

Sources. Antimony glance (known also as gray antimony ore, antimonite and stibnite) is the most important ore of antimony. It is a sulphide ore (containing 71.77 per cent. of antimony in the pure state), and is commonly associated with quartz, calc-spar, heavy spar, and spathic iron ore. Zinc blende and galena frequently occur intimately mixed with it.

Production. When the ores exist as pure sulphide of 90 per cent. or over, they are usually roasted direct to form an oxide, which is afterward reduced by carbonaceous matter. If the ores are lean the antimony sulphide is liquated to free it of the gangue, and then roasted and reduced in the same manner. By another method the antimony sulphide is directly reduced by iron, which, having a greater affinity for sulphur than antimony, combines with the sulphur and liberates the antimony. The antimony produced by both methods must afterward be refined.

A typical analysis of commercial antimony shows the following percentage composition: Silver, 0.001; arsenic, 0.120; tin, 0.006; lead, 0.089; copper, 0.076; iron, trace; zinc, none; nickel and cobalt, 0.046; manganese, none; sulphur, 0.008; antimony (by diff.), 99.654.

Uses. Antimony is used in making type metal, Babbitts and other bearing metals, its presence in alloys conferring the properties of hardness and of expanding on solidification.

Nickel

Nickel possesses an almost silvery white luster with a steel-gray tinge and great brilliancy. With **great hardness** and capacity for taking polish it combines great **malleability**; it can be easily hammered, rolled, or drawn into wire. Sheets 0.0008 in. thick and wire 0.0004 in. in diameter may be made from it. Its tensile strength surpasses that of iron: for sheets of pure nickel 0.05 in. thick, rolled hard (annealed), 92,000 lb. (76,000 lb.) per sq. in. and 11 (35) per cent. elongation in 2 in.; for 0.065-in. diam. hard-drawn wire, 150,000 to 160,000 lb. per sq. in.

Nickel is attracted by a magnet, and then becomes **magnetic** itself; it loses this property at 662 deg. fahr. It **can** not only **be welded** to itself at a white heat, but it can also be welded to iron and certain alloys. Fleitmann's method of manufacturing nickel-plated wares consists in welding pure nickel to iron and steel or alloys of copper and nickel, and then hammering or rolling into sheets. In welding, it is necessary to completely exclude the air from the surfaces to be welded.

Commercial nickel is exceedingly impure, containing substances of which even traces affect its valuable properties. The most harmful **impurities** are arsenic, sulphur, oxide of nickel and chlorine. Carbon is dissolved by molten nickel, and seems not to affect its good qualities so long as oxides are not present at the same time. It makes nickel slightly more fusible. According to Ledebur, nickel absorbs its own monoxide, and this injures its tenacity and malleability.

Nickel **does not tarnish** in dry air at ordinary temperatures. When heated in contact with air it is slightly oxidised into nickelo-nickelic oxide. At a red heat it decomposes steam very slightly. It is very little acted upon by hydrochloric or sulphuric acid in the cold, but is readily dissolved by dilute nitric acid and aqua regia.

Production. The chief **ores** of nickel are the silicate ores of New Caledonia, known as garnierite, and magnetic pyrites containing nickel and copper found in Ontario. The silicate ores can be treated by direct reduction, although they are usually treated in connection with sulphide ores. The magnetic pyrite ores are first smelted to form matte. This matte is Bessemerized to oxidize the greater portion of the iron and sulphur, with the production of a nickel-copper matte which is afterward fused with sodium sulphate and coal, forming an easily fusible matte with the copper sulphide which is of less specific gravity than the nickel sulphide, and a fairly complete separation can be made because of the difference in the specific gravity of the two mattes. By repeating the process, practically pure nickel sulphide is produced, which is then roasted to form oxide and finally reduced to metallic nickel. Mond's method consists in roasting the Canadian mattes, eliminating the copper by sulphuric acid, and then exposing the resulting product to the reducing action of producer gas at about 660 deg. fahr. The reduced metal is then exposed to the action of CO at 176 deg. fahr. in a "volatilizer," the nickel carbonyl so formed being received in a chamber heated to about 390 deg. fahr., where it decomposes, the nickel being deposited and the CO returned to the volatilizer.

Nickel as ordinarily reduced by carbon is brittle, but by the addition of a small percentage of magnesium or manganese, it can be made ductile.

A typical percentage composition after the addition of magnesium is as follows: Nickel, 98.24; cobalt, 1.09; iron, 0.36; copper, 0.10; silicon, 0.06; magnesium, 0.11.

Uses. In coinage, plating, crucibles; in the preparation of German silver, nickel steels, resistance alloy wires, etc.

Aluminum*

Aluminum (aluminium)† is a white metal closely resembling tin in color, produced electrolytically in large quantities from alumina prepared from the mineral bauxite—a hydrated oxide of the metal. Its **specific gravity** at 62 deg. fahr. ranges as follows: Pure aluminum, 2.56; pure aluminum sheets and wire unannealed, 2.68; pure aluminum sheets and wire annealed, 2.66; aluminum alloy sheets and wire unannealed or annealed (3 S), 2.75; aluminum casting alloys, 2.82 to 2.99. The alloys contain small percentages of Cu, Ni, W, Mn, Cr, Ti, Zn or Sn to produce hardness, rigidity and strength.

Ordinary commercial aluminum has about the **hardness** of copper. By pressing, rolling, forging, stamping or other similar working it may be made very hard and rigid in the finished shapes, whereas the soft annealed metal would be too weak for the purpose intended. This is especially true of aluminum alloyed with other metals in amounts not to exceed 5 or 6 per cent. Greater percentages render the alloys non-malleable.

Aluminum is very **ductile.** It can be rolled into sheets as thin as 0.0007 in. and then be beaten into leaf; drawn into wire as small as 0.004 in. in diameter; spun, stamped or extruded in various shapes. It is susceptible of a high degree of finish by polishing or burnishing. It becomes **hardened by working,** and sheets for stamping or spinning must be annealed.

At ordinary temperatures aluminum is **highly resistant to** all inorganic **acids** except hydrochloric, is little acted upon by salt water and not at all by sulphur below a red heat, by CO or CO_2. Solutions of caustic alkalies, chlorine, bromine, iodine and fluohydric acid rapidly corrode the metal. A clean surface tarnishes in damp air, an almost invisible coating of oxide being formed, which, however, is very permanent and prevents further attack.

* Much of this information has been obtained from the publications of the Aluminum Co. of America.

† Trade usage in North America sanctions the use of the word aluminum.

As the common metals are electronegative to aluminum in a voltaic couple, care should be taken that aluminum exposed to water or aqueous solutions shall not come in contact with any other metal that will cause **galvanic action** to be set up. For example, tubing can be thoroughly insulated by the use of rubber gaskets, washers or nuts. Aluminum sheets and shapes can be insulated from other metal by the liberal use of good heavy paint between the joints.

Commercial aluminum is sold in three **grades**: No. 1 (pure), No. 2, and extra pure. No. 1 has approximately the following composition: Silicon, 0.30 per cent.; iron, 0.15 per cent.; aluminum, 99.55 per cent. For No. 2 the percentages are respectively 2, 2 and 96. Silicon and iron are the only **impurities** commonly found in aluminum.

The **tensile and compressive strengths** of aluminum (99 per cent. pure) average as follows:

	Tension		
	Elastic limit, lb. per sq. in.	Ultimate strength, lb. per sq. in.	Reduction of area, per cent.
Castings	8,500	12,000–14,000	15
Sheet	12,500–25,000	24,000–40,000	20–30
Wire	16,000–33,000	25,000–55,000	40–60
Bars	14,000–23,000	28,000–40,000	30–40
	Compression		
Short cast columns (height = 2 × diam.)	3,500	12,000	

E for cast aluminum = 9,000,000.

Pure aluminum in castings is a metal somewhat open in texture, and when used for cylinders to stand pressure an increase in thickness over that calculated by ordinary formulæ should be made to allow for its **porosity**. **Shrinkage** of castings, 0.2031 in. per ft.

Under transverse tests the metal will bend nearly double before breaking. The strength of aluminum is greatly improved by forging or pressing the ingots at a temperature of about 600 deg. fahr. Weight for weight, aluminum is only exceeded in tensile strength by the best cast steel.

Aluminum Pipes are made in the dimensions given on p. 813.

Extruded Aluminum can be obtained in shapes possible to manufacture no other way, and in any continuous lengths desired. It is possible to extrude shapes in compositions ranging from pure aluminum to very hard alloys, some of which cannot be rolled. The tools used are much less expensive than are the rolls necessary for rolled shapes, making it commercially possible to furnish smaller quantities of special shapes than rolling-mill costs would justify. The maximum sectional dimension of an extruded shape should not exceed 6 in., and, in general, walls under 1/8 in. in thickness should not be specified.

Aluminum Bronze Powder, or aluminum finely powdered by beating in stamp mills, is largely used as a metallic paint. For this purpose it is mixed with a light coach varnish or with "banana oil," a lacquer made by dissolving gun cotton in amyl acetate.

For S. A. E. specifications for **aluminum alloys,** see pp. 542 and 1197.

BRASSES AND BRONZES

Copper-Tin Alloys (Bronzes)

The term **bronze** should be applied only to those alloys of copper in which tin is the predominating metal affecting or modifying the properties of the copper. A **true bronze** is an **alloy of copper and tin** only, in which copper is in predominating proportion, the most useful alloys being those containing from 4 to 25 per cent. of tin. After 12 per cent. of tin is exceeded, the ductility of the alloys is so diminished that they are subject to breakage by shock. Modifications are often made by the addition of relatively small proportions of lead and zinc, and occasionally of other metals.

Table 13. Properties of Various Copper-Tin Alloys

(From Report of U. S. Test Board, 1879–1881)

Composition by analysis		Specific gravity	Color	Fracture	Tensile strength, lb. per sq. in.	Elastic limit, lb. per sq. in.	Modulus of rupture, lb. per sq. in.	Elongation in 5 in., per cent.	Deflection, in bar 1 in. sq., 22 in. long	Crushing strength, lb. per sq. in.	Max. torsion moment, ft.-lb.	Angle of torsion, deg.
Copper	Tin											
100.00	0.00	8.791	Copper red...	Fibrous...	27800	14000	29848	6.47	bent	42000	143	153.0
97.89	1.90	8.564	Red.........	Vesicular..	24580	10000	.....	13.33	....	34000	150	317.0
96.06	3.76	8.649	Reddish-yellow	Vesicular..	32000	16000	33232	14.29	bent	42048	157	247.0
90.27	9.58	8.669	Grayish-yellow	Earthy....	26860	15750	49400	3.66	bent	38000	175	114.0
87.15	12.73	8.681	Mottled.....	Finely vesicular	29430	20000	34531	3.33	4.00	53000	182	100.0
80.95	18.84	8.740	Reddish-gray	Finely granular	32980	.....	56715	0.04	0.49	78000	190	16.0
76.64	23.24	8.565	....	Smooth....	22010	22010	32210	0.00	0.19	114000	122	3.4
69.94	29.88	8.932	White.......	Conchoidal	5585	5585	12076	0.00	0.06	147000	18	1.5
68.58	31.26	8.938	White.......	Conchoidal	1620	.....	9152	0.00	0.04	......	....	.....
65.34	34.47	8.947	Bluish-gray..	....	2201	2201	4776	0.00	0.02	84700	16	1.0
56.70	43.17	8.682	Light gray....	Stony.....	1455	1455	2126	0.00	0.02	......	....	.....
44.52	55.28	8.312	Grayish-white	....	3010	3010	4776	0.00	0.03	35800	23	1.0
34.20	65.80	8.013	Grayish-white	Crystalline	3371	3371	5384	0.00	0.04	19600	17	2.0
23.35	76.29	7.835	Grayish-white	Finely crystalline	6775	6775	12408	0.00	0.27	......	....	.....
15.08	84.62	7.657	Grayish-white	Crystalline	6520	.....	9063	.....	0.86	6500	23	25.0
11.49	88.47	7.552	Grayish-white	Crystalline	6380	3500	10706	4.10	5.85	10100	23	62.0
8.57	91.39	7.490	Grayish-white	Granular..	6450	3500	5305	6.87	bent	9800	23	132.0
3.72	96.31	7.360	Grayish-white	Granular..	4780	2750	6925	12.32	bent	9800	23	220.0
0.00	100.00	7.293	Grayish-white	Fibrous...	3505	.....	3740	35.51	bent	6400	12	557.0

Test pieces for compression tests were 2 in. long, 5/8 in. diam.
Test pieces for torsion were 1 in. long, 5/8 in. diam.

Gun Metal is a copper-tin alloy (8 to 11 per cent. of tin) used before the introduction of steel in the manufacture of ordnance. Occasionally it contains small percentages of iron, zinc and lead. The **U. S. Government Specification** for its composition is: Copper, 88 per cent.; tin, 10 per cent.; zinc, 2 per cent. A variation of 1 per cent. is allowed above or below. Iron not to exceed 0.06 per cent.; lead not over 0.2 per cent.

Detail specifications are given in "Specifications for Inspection of Materials, Part II, Bureau of Steam Engineering, Navy Department, Revised July 1, 1910," which call for the following minimum **physical properties:** Tensile strength, 30,000 lb.; yield point, 15,000 lb.; elongation in 2 in., 15 per cent. This alloy is susceptible to quite wide variation in its physical properties, due to the heat treatment which it has had, the manner of attaching test bar, gating and provision for shrinkage. It can also be modified by slight additions of deoxidizing agents.

Table 14. Melting Points of Various Alloys

(From Technical Paper No. 60, U. S. Bureau of Mines)

Alloy	Composition by analysis				Melting point, deg. fahr.
	Cu	Zn	Sn	Pb	
	P. ct.	P. ct.	P. ct.	P. ct.	
Gun metal	......	......	......	......	1825
Leaded gun metal	85.4	1.9	9.7	3.0	1795
Red brass	......	......	......	......	1780
Low-grade red brass	81.5	10.4	3.1	5.0	1795
Leaded bronze	......	......	......	......	1735
Bronze with zinc	84.6	5.0	10.4	......	1795
Half yellow, half red	75.0	20.0	2.0	3.0	1690
Cast yellow brass	66.9	30.8	......	2.3	1645
Naval brass	61.7	36.9	1.4	......	1570
Manganese bronze	......	......	......	......	1600

COPPER-ZINC ALLOYS

Parts by weight		Melting point
Copper	Zinc	Deg. fahr.
95	5	1960
90	10	1930
85	15	1880
80	20	1830
75	25	1795
70	30	1725
65	35	1660
60	40	1635

COPPER-TIN ALLOYS

Parts by weight		Melting point
Copper	Tin	Deg. fahr.
95	5	1920
90	10	1840
85	15	1760
80	20	1635

COPPER-LEAD ALLOYS

Copper	Lead	Deg. fahr.
95	5	1950
90	10	1920
85	15	1895

Effect of Heat Treatment. As these alloys contain different constituents which are stable only at certain elevated temperatures, the effect of quenching alloys at those temperatures at which the various constituents are stable will result in the properties of the alloys being modified by reason of containing such constituents. This accounts for the fact that bronze acts in a manner reverse to steel when quenched above 930 deg. fahr. in water. If quenched at this temperature the alloy is more malleable and stronger, for the reason that the hard and brittle constituent Cu_4Sn has been prevented from forming. This constituent does not form unless the alloy is cooled in the air to below 930 deg. fahr. This important fact is taken advantage of commercially in modifying the properties of copper-tin alloys in a way analogous to the annealing treatments of steel.

Strength of Bronze at Various Temperatures in per cent. of the strength at 68 deg. fahr. averages about as follows:

Temperature, deg. fahr	212	392	572	752	932
Tensile strength (relative)	101	94	57	26	18

Bell Metal alloys are alloys of copper and tin containing from 15 to 25 per cent. of tin. The higher the tin content the more brittle the alloys become, and the higher the note which they will produce. The tones of a bell may also be greatly modified by its general design, the thickness of its walls, etc., by casting at different temperatures, by the method of casting, and by slight additions of other metals, such as lead and zinc. When slowly cooled the alloy is very hard and brittle. If bells are chilled above the temperature of 932 deg. fahr., they are more malleable and rather yellowish in color.

Statuary Bronze as now made is U. S. Government gun metal, to which has been added a small percentage of lead. In the cheaper alloys less tin and more zinc are used. It is a very fluid alloy, easily cast, and capable of being finished and filed. Under the influence of the atmosphere it assumes a pleasing oxidation tint or "patina."

Coinage Bronze. In the United States the alloy used for copper coins contains 95 per cent. copper and 2½ per cent. each of tin and zinc. This alloy is fairly hard (to resist wear), and yet sufficiently malleable and ductile when cold to take the impression of the die.

Speculum Metal, used for **reflectors**, is a hard, flint-like and extremely brittle alloy consisting of two parts copper and one part tin. It takes and retains a high polish. Slight additions of arsenic or nickel increase its whiteness.

Copper-Zinc Alloys (Brasses)

The term **brass** is properly applied to alloys of copper, the properties of which are modified chiefly by the addition of zinc up to approximately 42

Table 15. Properties of Various Copper-Zinc Alloys
(From Report of U. S. Test Board, 1879–1881)

Composition by analysis		Specific gravity	Color	Fracture	Tensile strength, lb. per sq. in.	Elastic limit, per cent. of breaking load	Modulus of rupture, lb. per sq. in.	Elongation in 5 in., per cent.	Deflection, in in. of 1-in. bar 22 in. long	Crushing strength, lb. per sq. in.	Max. torsion moment, ft.-lb.	Angle of torsion, deg.
Copper	Zinc											
97.83	1.88	8.791	Yellow-red..	Vesicular.	27,240	.....	......	....	.....	....	130	357
82.93	16.98	8.633	Red-yellow..	Earthy...	32,600	26.1	23,197	26.7	Bent	....	155	329
81.91	17.90	8.598	Yellow.....	Earthy...	32,670	30.6	21,193	31.4	Bent	....	166	345
77.39	22.45	8.574	Yellow.....	Earthy...	35,630	20.0	25,374	35.5	Bent	....	169	311
76.65	23.08	8.528	Yellow.....	Earthy...	30,520	24.6	22,325	35.8	Bent	42,000	165	267
73.20	26.47	8.465	Yellow.....	Earthy...	31,580	23.7	25,894	38.5	Bent	....	168	293
71.20	28.54	8.444	Yellow.....	Earthy...	30,510	29.5	24,468	29.2	Bent	....	164	269
69.74	30.06	8.384	Yellow.....	Earthy...	28,120	28.7	26,930	20.7	Bent	....	143	202
66.27	33.50	8.371	Red-yellow..	Earthy...	37,800	25.1	28,459	37.7	Bent	....	176	257
63.44	36.36	8.411	Red-yellow..	Earthy...	48,300	32.8	43,216	31.7	Bent	....	202	230
60.94	38.65	8.405	Red-yellow..	Earthy...	41,065	40.1	38,968	20.7	Bent	75,000	194	202
58.49	41.10	8.363	Red-yellow..	Earthy...	50,450	54.4	63,304	10.1	Bent	....	227	93
55.15	44.44	8.283	Red-yellow..	Earthy...	44,280	44.0	42,463	15.3	Bent	78,000	209	109
54.86	44.78	8.301	Red-yellow..	Coarsely granular.	46,400	53.9	47,955	8.0	Bent	....	223	72
49.66	50.14	8.291	Red-yellow..	Coarsely granular.	30,990	54.5	33,467	5.0	1.26	117,400	172	38
48.95	50.82	8.216	Pinkish-gray	Coarsely granular.	26,050	100.0	40,189	0.8	0.61	....	176	16
47.56	52.28	.....	Pinkish-gray	Coarsely granular.	24,150	100.0	48,471	0.8	1.17	121,000	155	13
43.36	56.22	8.035	Pinkish-gray	Finely granular.	9,170	100.0	17,691	....	0.10	....	88	2
41.30	58.12	8.061	Silver white.	Conchoidal	3,727	100.0	7,761	....	0.04	....	18	2
32.94	66.23	7.811	Silver white.	Conchoidal	1,774	100.0	8,296	....	0.04	....	29	1
29.20	70.80	7.766	Light gray...	Vitreous.	6,414	100.0	16,579	....	0.04	....	40	2
20.81	77.63	7.418	Bluish-gray.	Finely granular.	9,000	100.0	22,972	0.2	0.13	52,152	65	1
12.12	86.67	7.238	Bluish-gray.	Finely granular.	12,413	100.0	35,026	0.4	0.31	....	82	3
4.35	94.57	7.108	Bluish-gray.	Finely granular.	18,065	100.0	26,162	0.5	0.46	....	81	22
0.00	100.00	7.143	Bluish-white	Crystalline	5,400	75.0	7,539	0.7	0.12	22,000	37	142

Test pieces for compression tests were 2 in. long and ⅝ in. diam.
Test pieces for torsion tests were 1 in. long and ⅝ in. diam.

per cent. Many of the commercial brasses contain small percentages of tin and lead, and certain well-known alloys belonging under the general classification of brass have their properties modified by the addition of iron, manganese and aluminum. Alloys containing approximately 50 per cent. each of copper and zinc are known as **brazing solders.**

Industrial Brass may be divided into two classes, namely, cast brass and wrought brass.

Cast Brass varies in its zinc content from 30 to 40 per cent. The most desirable mixture contains 65 per cent. copper and 35 per cent. zinc. The alloy is given hardness by the addition of a small percentage of tin. The straight alloy is soft and ductile and drags severely under the tool, but the addition of 1 to 2 per cent. of lead allows it to be machined freely, producing short chips.

There are two classes of **yellow brass** ingots made from scrap on the market intended **for casting purposes:**

1. Ingots produced from rod chips, which when melted down make a very satisfactory casting brass; it contains little tin, has the necessary hardness and fluidity in casting, enough lead to insure free machining, and is practically free of iron. A typical analysis is 63 per cent. copper, 2 per cent. lead, no tin, balance zinc.

2. Ingots made from miscellaneous yellow brass scrap and containing from about ½ to 2 per cent. of iron, and often lead and tin in excess of the amounts desirable for good working; used only for cheap classes of work, such as plumbers' ferrules, etc.

Typical specifications for cast yellow brass are as follows:

1. Dept. of Steam Engineering, U. S. Navy Dept.

	Per cent.
Copper	59 to 63
Tin	0.5 to 1.5
Iron (maximum)	0.06
Lead (maximum)	0.60
Zinc	Remainder.

(Known as cast naval brass)

2. National Cash Register Co., Specification No. 39

	Per cent.
Copper, not less than	60
Lead, not to exceed	2
Iron	0.35
Zinc	Remainder.

Other impurities of a detrimental nature (except oxides) not to be present to exceed traces.

Wrought Brass. There is some confusion of nomenclature in relation to wrought brass. The names in the table on p. 538 are the usual commercial names in the U. S. In addition an alloy of 85 copper and 15 zinc is called **rich low brass.** The nomenclature is entirely different from that in use in England.

Hot-working Brass. Alloys containing approximately 56 to 62 per cent. copper and the balance zinc become plastic when heated to redness and are capable of hot working. The most widely used and known alloy within these specifications is **Muntz metal** (60 per cent. copper and 40 per cent. zinc), which was formerly largely used for ship sheathing, it being claimed that this alloy is corroded just sufficiently to prevent the attachment of barnacles. The alloy is also used for many purposes in which a hard sheet brass is desirable. According to Bengough and Hudson, it should not be rolled or extruded at a temperature below 1100 deg. fahr. It is hardened by quenching. Tensile strength, about 55,000 lb. per sq. in.

Modified Hot-working Brass is brass the properties of which have been somewhat altered by the addition of tin and iron. Two alloys of this nature which are largely used are Tobin bronze and Delta metal.

Tobin Bronze is a proprietary alloy of the American Brass Co., containing, according to published analyses, 58 to 60 per cent. of copper and about 40 per cent. of zinc, the remainder being iron, tin and lead. It is made in the form of sheets and plates ($\frac{1}{16}$ to 2 in. thick), rods (up to 7 in. diam.), rectangular bars and seamless tubes. At a cherry-red heat it can be **readily forged.** It is especially **resistant to corrosion,** and is used largely in naval work. **Tensile strength** of rods larger than 1 in. in diam., 60,000 lb. per sq. in. (62,000 for smaller rods); minimum **yield point,** $\frac{1}{2}$ × tensile strength; **elongation** in 2 in., rods larger than 1 in., 28 per cent. (25 per cent. for smaller rods); compressive strength, 170,000 to 180,000 lb. per sq. in. These properties make it adaptable for a wide variety of engineering purposes where a strong, reliable material is required. **Weights of sheets and rods** are $1\frac{1}{4}$ per cent. lower than the weights for brass rods and sheets in Tables 3 and 6.

Delta Metal is similar in composition and properties to Tobin bronze, except that it carries from 1 to 2 per cent. of iron. It is manufactured by the Phosphor Bronze Smelting Co., and is furnished in sheets and rods.

Cold-working Brass. For tubes, cartridge cases, wire drawing, etc., the standard alloy contains 70 per cent. copper and 30 per cent. zinc; this alloy possesses considerable strength and great ductility; higher-copper alloys have still greater ductility. This alloy **can be worked cold,** and when intended for rolling or drawing is cast into a mold of such size and shape that the alloy may be formed into its finished shape with as little working as possible. In wire drawing, ingots of suitable size are first rolled into rods, these being finally drawn into wire. **Spring brass** has the composition copper 72, zinc 28, or copper $66\frac{2}{3}$, zinc $33\frac{1}{3}$. This alloy **hardens under** the effect of **working** and must be frequently annealed in order to correct this condition and allow further reduction. After each **annealing** process the brass must be washed with acid to rid the surface of oxide. The **temperature of annealing** is an **important** matter and must be carefully observed. Annealing below 536 deg. fahr. has practically no effect. There is a particular temperature at which the best results are obtained for each alloy, depending on the amount of hardening the alloy has undergone. At 620–650 deg. fahr. there is a marked softening of 70–30 brass. The maximum effect of annealing is reached at 1400 deg. Above this temperature the brass is burnt. Alloys containing small percentages of tin and lead can be raised to higher temperatures than those not containing these metals without being burnt.

Lead and tin are frequently added to cold-working-brass alloys, in order to confer certain properties. **Lead is added to facilitate machining,** chips of a leaded brass leaving the tool easily, as they are short and brittle. Lead is mechanically mixed in brass and therefore breaks up the continuity of the structure. In brass containing an excess of lead, the lead is present in solid solution, rendering the alloy more brittle. Turnings from brass which does not contain lead come off in long, tenacious curls. Leaded brass must be rolled cold because it is "hot-short." The best alloy is one which contains 60 per cent. copper, 38 per cent. zinc and 2 per cent. lead. This has an average **tensile strength** in the rolled form of about 60,000 lb.; elongation, 15 to 20 per cent.; reduction of area, over 50 per cent. Tin is often added to brass to increase the hardness and to add to the resistance of the metal to the corrosive action of sea water. Antimony and bismuth are detrimental impurities. Arsenic is considered beneficial by some makers.

The following specifications for brass sheets, strips and rods are recommended by the S. A. E. (1912). In all cases the iron must not exceed 0.10 per cent. See also pp. 548 and 1197.

Specification number	Name	Copper, per cent.	Zinc, per cent.	Lead, per cent. not greater than	Tensile strength, lb. per sq. in.	Elongation in 2 in., per cent.
33*	Standard sheet brass, hard.	64 to 67	33 to 36	0.50	60,000	5
	Standard sheet brass, soft.	64 to 67	33 to 36	0.50	48,000	50
34†	Low brass, hard	78 to 81	19 to 22	0.20	75,000	5
	Low brass, soft	78 to 81	19 to 22	0.20	42,000	50
35‡	Brazing brass..	74 to 76	24 to 26	0.25	Same as low brass	
36§	Free-cutting brass, hard.	61 to 64	33 to 38	1.25 to 2.00	75,000	3
	Free-cutting brass, soft..	61 to 64	33 to 38	1.25 to 2.00	50,000	35
	Red metal or commercial bronze, hard.	88 to 91	9 to 12	0.20	55,000	5
37‖	Red metal or commercial bronze, soft.	88 to 91	9 to 12	0.20	37,000	40
38	Gilding metal, hard..	94 to 96	4 to 6	0.15	45 to 55,000	5
	Gilding metal, soft..	94 to 96	4 to 6	0.15	35,000	35
39	Brass rods.............	61.5 to 64.5	35.5 to 38.5	0.50	35 to 40,000	50
40	Free-cutting brass rods.	61.5 to 64.5	31.5 to 35.5	2.25 to 3.50	65,000	15

* Sheet brass is furnished annealed or hard-rolled. Annealed is either light annealed or soft. Hard-rolled is furnished in five tempers, quarter hard, half hard, hard, extra hard, spring. Used for ornamental work. † Resists corrosion well. ‡ Used for parts where brazing or silver soldering is required. § Does not form or bend well. ‖ Rich gold color; resists corrosion well.

For **weights of brass sheets and plates,** see p. 524. For **weights of round and square brass rods,** see p. 525.

Table 16. Approximate Weight of Brass Wire
(Lb. per 1000 Ft.)

B. & S. gage No.	Weight	B. & S. gage No.	Weight	B. & S. gage No.	Weight	B. & S. gage No.	Weight
0000	610.0	8	47.60	19	3.710	30	0.2900
000	483.0	9	37.70	20	2.940	31	0.2300
00	383.0	10	29.90	21	2.330	32	0.1820
0	304.0	11	23.70	22	1.850	33	0.1440
1	241.0	12	18.80	23	1.470	34	0.1150
2	191.0	13	14.90	24	1.160	35	0.0908
3	152.0	14	11.80	25	0.923	36	0.0720
4	120.0	15	9.38	26	0.732	37	0.0571
5	95.4	16	7.44	27	0.581	38	0.0453
6	75.6	17	5.90	28	0.460	39	0.0359
7	60.0	18	4.68	29	0.365	40	0.0285

Heat-treatment of Brass

The physical properties of annealed brass vary according to the temperature, and to some extent to the time in which the annealing has been done. When annealed at 650 to 750 deg. fahr. under ordinary annealing conditions, brass strips containing two parts copper and one part zinc may have a tensile strength of about 55,000 lb. per sq. in., with elongation in 2 in. of 40 per cent. When annealed at 1300 to 1400 deg. fahr., under similar conditions, they may have a tensile strength of about 40,000 lb. per sq. in., with elongation in 2 in. of 55 per cent. The process of annealing relieves those internal strains which result from cold-working the material. Annealing of cold-worked brass is accompanied by a re-distribution of the molecules resulting in the formation of a new crystalline structure. The average minimum temperature required to effect re-crystallization of the brasses in one-half hour is 750 deg. fahr. The size of the grain is roughly proportional to the temperature within the safe annealing range (750–1400 deg. fahr.), and for a given time, temperature and alloy the grain size is constant. Under these conditions the grain size has a definite relation to the physical properties; also, grain size has considerable influence on resistance to corrosive action. Impurities such as iron, nickel, etc., retard the growth of the grain size and render necessary the use of a higher temperature to secure a given grain size than with pure brass.

Annealing removes, to a very great extent, the liability of brass to crack or disintegrate on prolonged exposure under unfavorable conditions. For this reason the best practice requires that in structural work all material for tubes, rods, bolts, etc., shall be lightly annealed.

Special Bronzes and Brasses

Casting Manganese Bronze. Manganese bronze is in reality a brass, consisting largely of copper and zinc. The term bronze, however, has been generally applied to it for many years, probably because it has a fine red color similar to that of bronzes. It is very **much stronger than** the **ordinary brasses.** Manganese is simply used as a deoxidizing agent, and the finished alloy very often contains but mere traces of it.

Manganese bronze is now one of the standard alloys on the market and is highly **recommended for propeller wheels** which are used in salt water, both because of the resistance of the metal to the action of salt water, and its remarkable strength and ductility. It is also a favorite alloy for many castings used in automobile construction. In casting, large risers are necessary, due to the high shrinkage of the alloy.

Specifications for manganese bronze have been adopted by the United States Government, by the Society of Automobile Engineers and by the Society for Testing Materials. The Society for Testing Materials' specifications for manganese bronze ingots having notched flat bottoms approx. 3 × 2¾ in. wide by 12 in. long, call for the following percentage chemical composition: Copper, 53 to 62; zinc, 36 to 45; aluminum, 0.05 to 0.5; lead, not over 0.15. Ultimate tensile strength not to be less than 70,000 lb. per sq. in.; elongation in 2 in., not less than 20 per cent. Test specimen to be turned to 0.5 in. diam. and 2 in. gage length, from a piece cut from one corner near bottom of ingot. The compressive strength is about 120,000 lb. per sq. in.

Wrought Manganese Bronze. Manganese bronze is capable of being wrought similar to Muntz metal, Delta metal and Tobin bronze.

Tensilite, a high-strength bronze manufactured by the American Manganese Bronze Co., is stated to have the following physical properties: Tensile strength, rolled or forged, 120,000 lb. (cast, 105,000); elastic limit, 75,000 (60,000); elongation in 2 in., 18 per cent. (15); reduction of area, 22 per cent. (20); elastic limit in compression, 70,000 (60,000); permanent set under a load of 100,000 lb. per sq. in., 0.015 in. (0.020).

Aluminum Bronze is an alloy of copper and aluminum, containing up to 11 per cent. of the latter metal. Some years ago it was a strong competitor of manganese bronze, and in general it may be said that 11 per cent. aluminum bronze (the strongest mixture) has a somewhat higher tensile strength than manganese bronze, but a lower percentage of elongation. It was found that manganese bronze was more easily handled in the foundry and could be better depended upon; and as it could be produced more cheaply than aluminum bronze, it naturally became the more popular alloy for resistance to severe stresses. By the addition of titanium good solid castings of aluminum bronze can be obtained. Aluminum beyond 11 per cent. makes a brittle bronze which is hard to work. Aluminum bronze can be readily soldered.

For S. A. E. specifications for **aluminum alloys** containing 65 per cent. aluminum and over see p. 1197. See also p. 542.

Dagger's tests on copper-aluminum alloys yielded the following results:

Aluminum, per cent.	Specific gravity	Average tensile strength, lb. per sq. in.	Elongation, per cent.
11.00	7.23	95,000	8
10.00	7.69	82,000	14
7.50	8.00	61,000	40
5.00	8.37	37,000	40
2.50	8.69	31,500	50
1.25	....	27,000	55

Aluminum Brass is a copper-zinc alloy containing up to 3 per cent. of aluminum, and a very valuable series of alloys is thus produced with a wide variation of tensile strength and elongation. The alloys are quite analogous to the so-called manganese bronzes, and are often substituted for them. Aluminum brass, as made by the Aluminum Co. of America, has a tensile strength of 40,000 to 50,000 lb. per sq. in. (elastic limit, 30,000), and an elongation of 3 to 10 per cent. in 8 in.

Aluminum, when added in the form of aluminized zinc to brass in quantities up to 1 per cent., makes the brass flow freely, and the resulting castings have smooth surfaces and are free from blowholes. Added in larger quantities (up to 10 per cent.) it imparts increased strength to the castings.

Casting Red Brass. The term red brass is applied to all alloys containing a sufficient amount of copper to be red in color, with the exception of those reddish alloys of copper and zinc carrying approximately 45 per cent. of zinc and 55 per cent. of copper: manganese bronze, aluminum bronze and aluminum brass. Red brasses usually carry upward of 75 per cent. of copper and varying amounts of tin, zinc and lead. These alloys, because of their ease of machining, ease of casting and red color, are used for steam fittings, ornamental castings, miscellaneous brass trimmings, hardware, etc. Different alloys for various purposes have the following percentage compositions:

	Copper	Tin	Lead	Zinc
Valve metal*	85	5	5	5
Steam metal	86	8	3	3
Gas cocks	76	2	6	16
Polishing metal	87	....	1	12
Brazing metal	80	....	....	20

* S. A. E. Specification No. 27 for light castings.

Red Brass Ingots made from scrap, turnings, etc., are sold upon the market, subject to specifications with a variation of 1 per cent. either way on the constituents, at a price below the cost of producing the same composition from new metals. Such ingots are now produced of very satisfactory quality, and show a material saving as compared with making the alloys from virgin metals.

Phosphor Bronze in the **wrought form** (bars and plates) consists of upward of 96 per cent. of copper and up to 4 per cent. of tin which has been deoxidized by sufficient phosphorus. Its tensile strength is approximately 65,000 lb.; elastic limit, 60,000 lb.; elongation, 16–18 per cent. A small percentage of phosphorus can be left in the alloy, and adds to its hardness. Its qualities of toughness and great tensile strength, combined with resistance to corrosion and crystallization, make it superior to German silver for exposed work. It has superior elasticity and can be furnished in wire and sheets to various tempers for use in making **springs.**

Cast Phosphor Bronze (commercially known as "B" grade) contains from 88 to 92 per cent. copper and from 8 to 12 per cent. tin. To get the best results, just enough phosphorus is used to thoroughly cleanse the metal of oxides, none being retained in the finished product. The United States Government Specifications covering phosphor bronze call for a tensile strength of 40,000 lb. and an elongation of 20 per cent. in 2 in. The American Manganese Bronze Co. give for cast phosphor bronze, tensile strength, 25-40,000 lb. per sq. in.; elastic limit, 18-30,000 lb. per sq. in.; elongation in 2 in., 42 per cent.; reduction of area, 32 per cent. To obtain the best results this alloy should contain no lead or zinc. It is a particularly desirable metal for gears, being hard and tough and having a low rate of wear.

SPECIAL ALLOYS

Copper-Nickel Alloys

Cupro Nickel (85 per cent. copper and 15 per cent. nickel) is largely used for bullet jackets for military rifles in which smokeless powder is loaded. The addition of a small percentage of manganese or magnesium greatly facilitates the proper working of the alloy in ingots and under rolls. Outside of this use, which has now become quite extended, the alloy is used for coinage purposes, the American five-cent piece being an alloy of 75 per cent. copper and 25 per cent. nickel. It is also used for the driving bands of projectiles, and then contains from 3 to 5 per cent. nickel. This same alloy has also been tried with success for steam-boiler tubes. **Constantan** (60 per cent. copper, 40 per cent. nickel) has a very low temperature coefficient and is used for electrical resistance purposes.

German Silver is an alloy of copper, nickel and zinc. It is very **ductile** and can be rolled, hammered, stamped and drawn. At the same time it is **hard, tough** and not easily corroded, and above all possesses the valuable property of being **white.** It is softened by annealing.

Hiorns gives the following table showing the **composition** of various qualities of German silver made by the best makers in Birmingham, England. These different alloys are sold under the trade names indicated.

Name	Composition, per cent. Copper	Nickel	Zinc	Name	Composition, per cent. Copper	Nickel	Zinc
Extra white metal..	50	30	20	Seconds................	62	14	24
White metal.......	54	24	22	Thirds..................	56	12	32
Arguzoid..........	48½	20½	31	Special thirds...........	56½	11	32½
Best best.........	50	21	29	Fourths.................	55	10	35
Firsts or best......	56	16	28	Fifths, for plated goods..	57	7	36
Special firsts.......	57	17	27				

Hiorns states that alloys containing less than 16 per cent. of nickel should contain 30 per cent. of zinc in order to give the best results; while with alloys containing more than 16 per cent. of nickel, the quantity of zinc should be less than 30 per cent. The **impurities** found in German silver are iron, lead and tin. Iron increases the strength, hardness and elasticity, and makes the alloy whiter. Tin renders the alloy brittle and unfit for rolling. It also gives the alloy a decidedly yellow color. Lead does not enter the alloy but simply remains mechanically mixed, and therefore should be added if the alloy is intended for casting and subsequent working.

The **weight of German silver wire** made by the American Brass Co. and containing 18 per cent. nickel is 2.5 per cent. lower than that of copper wire of the same size and length; of 30 per cent. nickel it is 1.8 per cent. less.

Monel Metal is a natural alloy consisting of 68 to 70 per cent. nickel, 1½ per cent. iron, and the remainder copper. It is smelted and refined from ore mined in Canada without disturbing the proportions given. It is silver-white in color, takes a high polish and is **highly resistant to corrosion,** not being affected by atmospheric conditions, fresh or salt water, acid fumes or superheated steam. Its **specific gravity** (cast) is 8.87, and its **melting point** 2480 deg. fahr. **Tensile strength**: rods, 36,900 lb. per sq. in. (castings, 78,-240); **elongation** in 2 in.: rods, 40 per cent. (castings, 38.5 per cent.). It is used for ship propellers, in marine construction work, in sheets for roofing metal, and in the form of rods and castings in places where steel and bronze are unsuited because of their corrodibility. The Bureau of Steam Engineering, U. S. Navy Dept., uses the following **specification** for hot-rolled Monel metal rods 1 in. in diam. and smaller: Yield point, 47,000 lb. per sq. in. (45,000); tensile strength, 84,000 lb. per sq. in. (80,000); elongation in 2 in., 25 per cent. (28 per cent.). Figures in parentheses are for rods larger than 1 in.

Aluminum Alloys

Aluminum Alloys are extensively used in automobile construction. The S. A. E. Specification No. 23 for **aluminum-copper** alloy calls for 7 to 8 per cent. of copper. This is a tough alloy having a tensile strength of from 15,000 to 20,000 lb. per sq. in., and is especially adaptable for castings to withstand severe shocks and stresses. Zinc produces the strongest alloys with aluminum, the tensile strength running as high as 30,000 to 35,000 lb. per sq. in. The S. A. E. Specifications Nos. 24 and 25 are for **aluminum-zinc alloys**, the former containing 12.5 per cent. of zinc and 2.5 per cent. of copper, and the latter 35 per cent. of zinc. The copper-carrying alloy is tough, well adapted to forging, and easily machined. See p. 1197.

Magnalium is an alloy of aluminum and magnesium containing from 2 to 23 per cent. of the latter metal. **Specific gravity,** 2.40 to 2.57; **melting point,** 1110 to 1290 deg. fahr. The alloy is lighter than pure aluminum, of much greater strength, and does not tarnish. **Tensile strength**: Cast in chills, 42,000 to 64,000 lb. per sq. in.; sand castings, 17,000 to 30,000 lb.; rolled, 28,000 to 35,000 lb.; stamped, 52,000 to 70,000 lb.

Effect of High Temperatures on the Physical Properties of Metals and Alloys

For a list of investigations on the effects of high temperatures on the strengths of metals and alloys, see M. Rudeloff, 1909, International Ass'n. for Testing Materials. Most of the investigations have been limited in scope and have not dealt sufficiently with the cast and rolled metals at present in commercial use. An extensive series of tests by the Crane Co. on a wide range of metals and alloys is reported by I. M. Bregowsky and L. M. Spring, in *The Valve World*, Jan., 1913. The values given in Tables 17 and 18 are taken from this article, and are averages of from two to ten tests at each temperature.

ANTI-FRICTION ALLOYS (BEARING METALS)

Anti-friction Alloys are divided into **four** distinct **classes**: 1. Copper base, or those carrying over 50 per cent. copper, usually from 65 to 80 per cent. 2. Tin base, or those containing over 50 per cent. tin. 3. Lead base, or those containing over 50 per cent. lead. 4. Zinc base, or those containing over 50 per cent. zinc. Copper-base alloys are harder and stronger than white metals having tin, lead or zinc bases, and are used for bearings which are required to resist heavier pressures. Frequently, however, **compound metal bearings** are used, having a copper-base bearing metal for the backing and lined with an anti-friction metal usually of a tin or lead base.

The **copper-base metals** operate with a lower coefficient of friction because they are harder, but they are more liable to heat under abnormal conditions because they are lacking in the necessary plasticity to conform to irregularities

Table 17. Tensile Tests on Metals and Alloys Subjected to High Temperatures

[Tensile strength and elastic limit in lb. per sq. in.; elongation (in 2 in.) and reduction of area in per cent.]

Materials	Temperatures, deg. fahr.							
	70	300	450	500	600	750	950	1000
CAST MATERIALS								
[1] Hard Metal:								
Tensile strength	33,735	34,280	31,810	......	23,150	19,170	10,825	5,710
Elastic limit	25,035	19,630	20,845	......	17,160	15,925	8,775	5,500
Elongation	7.4	10.7	7.8	......	2.8	2.9	0.0	0.0
Reduction of area	9.4	12.8	9.2	......	4.1	3.5	0.5	0.0
[2] Aluminum Bronze:								
Tensile strength	35,205	32,330	35,815	......	24,265	6,425	......	6,230
Elastic limit	27,740	......	15,645	......	18,420	6,425	......	6,230
Elongation	56.3	78.1	65.7	......	45.3	0.0	......	0.0
Reduction of area	46.9	31.4	61.7	......	38.4	0.0	......	0.0
[3] Phosphor Bronze:								
Tensile strength	33,107	33,640	26,376	......	16,537	15,775	......	......
Elongation	12.5	12.5	10.9	......	4.7	0.0	......	......
Reduction of area	11.9	16.5	12.3	......	0.8	0.0	......	......
Steam Metal:								(550°)
Tensile strength	31,780	26,370	21,900	20,260	12,180	10,280	6,630	12,230
Elastic limit	16,900	12,770	13,130	12,410	11,370	10,280	6,630	11,230
Elongation	21.1	18.8	10.9	8.9	1.1	0.0	0.0	0.0
Reduction of area	23.8	18.2	14.5	10.9	1.2	0.4	0.0	0.7
[4] Aluminum Bronze:								
Tensile strength	41,360	......	42,308	......	38,425	26,800	25,960	......
Elongation	9.4	......	12.5	......	11.5	1.6	3.1	......
Reduction of area	13.1	......	4.8	......	11.6	2.9	6.7	......
[5] Cast Manganese Bronze:							(900°)	(925°)
Tensile strength	56,350	47,775	40,870	37,200	21,450	7,350	2,365	1,625
Elastic limit	31,875	26,900	25,580	22,200	13,525	5,230	1,500	1,625
Elongation	10.9	12.5	21.4	24.5	32.1	39.6	40.2	48.4
Reduction of area	15.8	17.1	25.3	33.7	41.0	54.0	61.3	63.0
Soft Cast Iron:							(860°)	
Tensile strength	22,060	23,260	20,730	......	21,240	21,925	21,590	19,820
[6] Strong Cast Iron:							(900°)	
Tensile strength	32,692	33,290	33,400	......	33,110	32,860	25,780	27,310
Malleable Iron:							(822°)	(950°)
Tensile strength	37,625	33,505	33,280	......	34,000	34,055	31,830	27,110
Elongation	63.0	......	......	......	6.3	6.3	......	......
Cast Steel:						(720°)	(750°)	
Tensile strength	73,325	76,570	81,167	......	67,366	58,713	41,388	17,568
Elastic limit	39,817	34,020	34,267	......	29,422	33,313	27,223	9,650
Elongation	27.0	15.6	19.8	......	23.1	22.7	2.5	31.3
Reduction of area	35.0	24.5	23.4	......	34.6	34.6	45.4	57.5
[7] Cast Nickel:							(900°)	
Tensile strength	38,029	40,850	36,667	......	35,896	36,536	27,800	16,775
Elastic limit	23,798	25,123	25,137	......	22,875	22,570	14,150	14,740
Elongation	5.7	9.6	6.5	......	6.2	7.2	6.3	3.9
Reduction of area	6.1	14.6	10.3	......	7.9	9.2	11.6	4.8
Monel Metal (cast):		(285°)	(435°)	(450°)				(1030°)
Tensile strength	52,870	53,130	54,100	47,200	39,450	41,787	......	26,400
Elastic limit	30,088	24,910	23,850	22,350	21,901	21,700	......	23,480
Elongation	16.6	20.3	25.8	15.6	18.2	14.1	......	6.3
Reduction of area	19.8	19.9	23.6	29.5	22.0	16.9	......	6.8
[8] U. S. Navy Brass:								
Tensile strength	28,930	27,000	25,600	20,900	13,130	9,025	......	......
Elastic limit	16,130	13,800	10,450	12,075	11,350	9,025	......	......
Elongation	19.3	18.8	19.5	17.2	1.0	0.0	......	......
Reduction of area	23.2	20.1	18.6	16.0	3.1	0.6	......	......
[9] U. S. N. Gun Bronze:								
Tensile strength	34,170	36,025	33,050	......	21,380	19,640	9,650	......
Elastic limit	25,650	21,900	19,650	......	18,780	18,140	9,150	......
Elongation	8.0	8.6	8.3	......	3.7	0.0	0.0	......
Reduction of area	7.3	7.1	10.0	......	2.9	0.7	0.0	......

Table 17. Tensile Tests on Metals and Alloys Subjected to High Temperatures—(*continued*)

Materials	Temperatures, deg. fahr.							
	70	300	450	500	600	750	950	1000
CAST MATERIALS								
[10] U. S. N. Valve Bronze:							(900°)	
Tensile strength............	35,345	34,260	27,630	28,160	16,100	13,000	9,530	6,400
Elastic limit...............	18,740	13,210	15,220	14,520	15,225	12,275	9,530	6,400
Elongation..................	23.2	26.2	17.2	12.8	3.8	0.6	0.0	0.0
Reduction of area...........	24.9	26.0	21.0	17.4	4.3	1.1	1.7	0.0
ROLLED MATERIALS								
[11] Rod Brass:							(900°)	
Tensile strength............	54,450	52,700	49,000	......	35,050	18,740	10,170	....
Elastic limit...............	45,000	43,200	39,100	......	23,735	15,050	8,060	....
Elongation..................	16.4	26.6	21.9	......	14.9	17.2	21.9	....
Reduction of area...........	18.0	34.2	26.0	......	17.8	21.3	26.0	....
[12] Nickel Steel:				(525°)				(1030°)
Tensile strength............	99,498	97,000	84,950	83,000	69,575	45,650	......	36,350
Elastic limit...............	39,850	31,700	32,250	26,200	25,650	21,100	......	15,500
Elongation..................	51.2	64.1	62.5	59.4	56.3	43.0	......	37.5
Reduction of area...........	59.8	65.0	65.0	66.8	72.6	59.4	......	55.7
Monel Metal:				(525°)				(1030°)
Tensile strength............	104,900	97,400	97,800	96,400	89,600	67,600	......	47,200
Elastic limit...............	78,350	58,500	58,600	58,400	57,950	42,550	......	26,800
Elongation..................	31.3	29.7	29.7	32.8	32.8	28.1	......	28.1
Reduction of area...........	61.7	57.8	51.0	51.5	59.5	58.1	......	60.7
[13] Cold-rolled Shafting:				(525°)		(795°)		
Tensile strength............	82,800	91,850	96,083	96,250	88,525	59,500	39,250	....
Elastic limit...............	76,800	77,100	72,850	75,300	54,275	53,200	30,400	....
Elongation..................	21.9	21.9	21.9	18.8	25.0	25.0	35.2	....
Reduction of area...........	49.5	39.1	38.7	37.5	44.2	58.5	78.0	....

[1] A strong copper-tin bronze. [2] Containing 5 per cent. of aluminum. [3] Highly resistant to acids; elastic limit not determined. [4] Containing 10 per cent. of aluminum; elastic limit not determined. [5] At 550 deg.: 32,050, 19,300, 23.4 and 31.6. [6] Ferro-steel, used in extra heavy valves over 7 in. [7] Rendered malleable by special process and used for rings, disks and trimmings for iron and steel fittings for high temperatures. [8] For screw pipe fittings (Cu, 77–80; Sn, 4; Pb, 3; Zn, 13–19). [9] (Cu, 88; Sn, 10; Zn, 2.) [10](Cu, 87; Sn, 7; Pb, 1; Zn, 5.) At 550 deg.: 17,200, 14,025, 4.4 and 6.2. [11] (Cu, 62.5; Pb, 2.5; Zn, 35.) [12] Rods, 30 per cent. nickel. [13] Bessemer steel, for valve stems.

of the journals or of foreign particles which may become lodged between the journal and the bearing surface. If bearings could be maintained in perfect adjustment, the harder alloys would give the most efficient service because of their low coefficient of friction and their lower operating temperature. The term anti-friction is generally applied to those alloys which have the highest coefficient of friction, namely, the soft white-metal alloys.

The rate of wear is greater in the hard alloys than in the soft alloys, because the particles of the hard alloys split off in service, whereas the particles of the soft alloys become compressed or flattened out before being torn off. The bearing should be just sufficiently hard and strong to carry its load. If the bearing is too soft it will roll out or be pounded out, and will probably hug the journal, squeeze out the oil film, become heated and be destroyed.

The physical properties of some of the bearing metals, as determined at the Penna. Ry. laboratories, are given in Table 19.

NOTES ON TABLE 18

[1] Bessemer steel, 0.093 per cent. carbon. [2] Cumberland cold-rolled shafting, 0.083 per cent carbon. [3] Open-hearth steel, 0.084 per cent. carbon. [4] (Ni, 3.25; V, 0.45; C, 0.365), oil tempered. [5] 25 per cent. nickel. [6] 30 per cent. nickel; turned from 1 in. to 0.8 in. diam. [7] Turned to 0.75 in. diam. from a ⅞-in. bar. [8] Cumberland shafting, 0.375 per cent carbon. [9] Annealed (Cr, 0.49; V, 0.145; C, 0.722). [10] Rolled rod (Cu, 62.5; Zn, 35; Pb, 2.5). [11] Elephant (phosphor) bronze (Cu, 95.5; Sn, 4; P, 0.31).

Table 18. Torsional Tests on Metals and Alloys Subjected to High Temperatures

(Torsional strength and elastic limit in in.-lb. per sq. in. Total twist in number of turns, with number of degrees excess. Test bars turned to 0.855 in. diam. from 1⅛-in. rods.) See notes at foot of opposite page.

Materials	Temperatures, deg. fahr.			
	70	385	600	800
[1] **Cold-rolled Shafting:**				
Torsional strength	72,400	82,350	41,175	12,350
Elastic limit	41,050	32,800	26,350	6,550
Twist	3 and 152°	1 and 265°	16	8 and 132°
[2] **Cold-rolled Shafting:**				
Torsional strength	68,990	77,210	33,680	17,250
Elastic limit	42,710	29,430	25,460	9,034
Twist	3 and 180°	1 and 345°	9 and 115°	12
[3] **Machinery Steel:**				
Torsional strength	59,590	49,260	33,061	26,530
Elastic limit	24,530	9,830	7,310	2,450
Twist	7 and 130°	3 and 190°	12 and 120°	61 and 260°
[4] **Nickel-vanadium Steel:**				
Torsional strength	101,200	82,500	19,590	14,340
Elastic limit	57,200	36,200	13,060	6,560
Twist	3 and 140°	3 and 10°	3 and 110°	8 and 35°
[5] **Nickel Steel:**				
Torsional strength	91,900	64,490	41,300	....
Elastic limit	17,250	8,150	6,560	6,560
Twist	9 and 30°	8 and 115°	8 and 40°	8 and 132°
[6] **Nickel Steel:**				
Torsional strength	104,800	78,550	53,170	25,350
Elastic limit	21,800	15,700	10,900	6,040
Twist	11 and 100°	6 and 330°	6 and 195°	9 and 210°
Rolled Monel Metal:				
Torsional strength	91,990	78,030	54,210	38,600
Elastic limit	37,780	36,140	19,800	10,680
Twist	11 and 150°	4 and 320°	4 and 90°	7 and 50°
[7] **Rolled Monel Metal:**				
Torsional strength	94,610	83,030	72,290	40,610
Elastic limit	45,510	33,940	31,300	10,910
Twist	12 and 150°	5	5 and 205°	6 and 240°
[8] **Cold-rolled Shafting:**				
Torsional strength	83,840	76,650	15,920	7,180
Elastic limit	42,540	36,800	2,040	1,630
Twist	2 and 280°	2 and 30°	8 and 230°	39 and 195°
[9] **Vanadium Tool Steel:**				
Torsional strength	137,295	124,080	67,710	....
Elastic limit	54,100	37,750	17,820	....
Twist	345°	1 and 250°	320°	....
[10] **Rod Brass:**				
Torsional strength	51,200	43,780	14,190	....
Elastic limit	32,600	22,180	4,100	....
Twist	2 and 225°	2 and 185°	5 and 40°	....
Tobin Bronze:				
Torsional strength	61,200	36,560	8,860	....
Elastic limit	26,850	9,800	1,630	....
Twist	2 and 155°	4	3 and 255°	....
[11] **Phosphor Bronze:**				
Torsional strength	70,000	51,020	19,920	....
Elastic limit	34,250	21,240	6,560	....
Twist	12 and 95°	1 and 215°	15 and 200°	....
Delta Metal:				
Torsional strength	61,630	42,860	3,265	....
Elastic limit	26,940	14,600	1,360	....
Twist	180°	3 and 235°	4 and 300°	....
[12] **Manganese Bronze:**				
Torsional strength	61,630	37,630	8,980	....
Elastic limit	22,040	13,060	3,260	....
Twist	2 and 5°	3 and 110°	2 and 10°	....

Table 19. Physical Properties of Bearing Metals

Kind of material	Composition in per cent.						Tensile test			Brinell test, 500-kg. load, 10-mm. ball, Hardness number		Compression test, Original section	
	Copper	Lead	Tin	Antimony	Nickel	Zinc	Tensile strength, lb. per sq. in.	Elongation, per cent. in 2 in.	Per cent. reduction of area	Natural	⅛" comp.	lb. per sq. in.	Per cent.
Phosphor bronze.	79.70	9.50	10.00	.....	...	.....	23,920	2.875 in 8"	.....	55.00	....	....	...
Cyprus bronze.	64.75	30.00	5.00	.....	...	.....	17,380	6.5 in 8"	.....	41.25	....	....	...
Plastic bronze.	64.00	30.00	5.00	.....	1.0	.....	21,340	10.10	8.20	45.00	53.0	64,490	44.2
Demo bronze.	60.67	32.97	4.60	.....	2.1	.....	18,170	3.00	0.35	52.00	59.0	56,220	30.4
Plumbic bronze.	50.00	50.00	.....	.....	...	.....	......	....	.....	21.80	....	....	...
Standard Babbitt.	3.70	.....	88.89	7.41	...	.....	11,780	5.00	3.14	27.70	29.0	29,150	59.0
German Babbitt.	5.55	.....	88.33	11.11	...	.....	8,726	....	.....	21.10	22.5	33,895	59.2
Souther Babbitt.	7.00	.....	84.00	9.00	...	.....	10,000	1.25	.....	34.80	48.0	28,410	44.1
Parsons white brass.	2.25	0.15	64.90	.....	...	32.93	21,416	11.25	18.72	18.00	19.0	75,460	73.8
Shonberg M. metal.	2.50	0.25	58.38	.....	...	38.93	11,710	4.00	5.76	25.10	31.5	36,755	54.8

Copper-base Bearing Metals

The copper-base alloys (usually known as bronzes) consist of the following classes: 1. Copper and tin. 2. Copper, tin and zinc. 3. Copper, tin and lead. 4. Copper, tin, lead and zinc. Occasionally there are added small proportions of other metals, which modify the general properties of the alloys.

Copper-Tin Bearing Alloys. About 30 years ago these alloys were in popular use, not only for machinery bearings but also for car and locomotive bearings, the alloy of 7 copper and 1 tin being the one most commonly used. This alloy has a high compressive strength, is quite hard and should be used only in cases where heavy pressures are encountered. Their use in other bearings is not justifiable, because the alloys have very little plasticity and therefore become heated upon little provocation, and have a high rate of wear.

Bell metal is a still harder metal than the 7-to-1 bronze, and carries from 16 to 25 per cent. of tin, the balance being copper. Such bearings have been tried because of the false idea that the harder metals will better resist wear. Their use has always met with failure.

Copper-Tin-Zinc Bearing Alloys. The addition of zinc to the copper-tin alloys improves their casting qualities. Such alloys are very rarely used to-day, except for backing a softer lining metal. The housings or backs on marine bearings are usually of this alloy. It is used by the United States Government and to some extent in the automobile trade, the bearings usually being lined with a high-tin-base Babbitt metal. This is a very expensive form of bearing and its use in many cases is not justified, as more satisfactory service could be rendered by cheaper alloys.

Copper-Tin-Lead Bearing Alloys. This class of alloys is the standard bronze alloy in use to-day for bearings. The standard phosphor-bronze bearing metal is an alloy of these three metals containing a small percentage of phosphorus. It was found about 25 years ago that the addition of lead to the then standard copper-tin alloy made the alloy more plastic and therefore better able to conform to irregularities of the service without heating, and diminished the rate of wear.

Dr. C. B. Dudley as a result of many investigations finds that the rate of wear in a bearing metal and its tendency to become heated in service diminish with the increase of lead and with the diminution of tin in the alloy.

The following series of alloys were experimented upon by the Pennsylvania Railroad under the direction of Dr. Dudley:

Metal Tested	Percentage Composition					Relative wear
	Copper	Tin	Lead	Phosphorus	Arsenic	
Phosphor bronze, standard	79.70	10.00	9.60	0.80	....	1.00
Ordinary bronze	87.50	12.50	....	....	....	1.49
Arsenic bronze, "A"	89.20	10.00	....	....	0.80	1.42
Arsenic bronze, "B"	82.20	10.00	7.00	....	0.80	1.15
Arsenic bronze, "C"	79.70	10.00	9.50	....	0.80	1.01
Bronze, "K"	77.00	10.50	12.50	....	....	0.92
	77.00	8.00	15.00	....	....	0.86

An **alloy of 85 per cent. copper, 10 per cent. tin, and 5 per cent. lead** will withstand compression of approximately 24,000 lb. on a 1-in. cube at the yield point; a 1-in. cube will compress 26 per cent. under a load of 100,000 lb. without fracture. The compressive strength is sufficiently high for it to be used in severe service; it is an admirable alloy for connecting-rod brasses.

An **alloy of 80 per cent. copper, 10 per cent. tin, and 10 per cent. lead,** as compared with the alloy carrying but 5 per cent. of lead, shows a diminished resistance to compression, namely, 23,000 lb. on a 1-in. cube at the yield point, and a compression under 100,000 lb. of 29 per cent. This alloy is very largely **used for locomotive, car, and general machinery bearings.** A small percentage of phosphorus is often added and the alloy is then known as standard phosphor-bronze bearing metal. (S. A. E. Specification No. 26—0.05 to 0.25 per cent. P.) The phosphorus adds somewhat to the resistance to compression and to the fluidity of the metal, but it has not been proved that it benefits in any way the quality of the alloy from the standpoint of service as a bearing. In fact, the limited data at hand tend to show that it acts adversely in service, because of the production of the brittle phosphide of copper in the alloy which frequently tends to cause heating. This alloy, nevertheless, is called for in the specifications of many prominent consumers.

The **alloy of 77 per cent. copper, 8 per cent. tin, and 15 per cent. lead** is extensively used, and together with a small percentage of phosphorus (0.2 per cent.) is known on the Pennsylvania Railroad as Ex. B. metal. This is the alloy of highest lead and lowest tin content which Dr. Dudley was able to produce, owing to foundry difficulties. Alloys of lower tin and higher lead segregated, the lead liquating to the bottom of the casting, due probably to the presence of phosphorus which lowers the solidifying point of the alloy and therefore maintains the alloy in a liquid condition for a longer period. This alloy will support a load of 21,000 lb. on a 1-in. cube without distortion. It has a slow rate of wear and is less liable to heat in service than either of the two preceding alloys.

Alloys containing from 4 to 7 per cent. of tin, and 20 to 30 per cent. of lead, are known in the trade as **Ajax plastic bronze** alloys, and are manufactured by the Ajax Metal Co. The patent controlling these alloys is based upon the fact that lead which is only mechanically held in the alloy can be prevented from liquating to the bottom of the casting if the copper and tin matrix which holds it can be made to solidify quickly. Quick solidification of the matrix will result if the copper and tin are in correct proportions, that is, in such proportions that only a solid solution of the two metals is formed. The following table gives the results of tests made in the author's laboratory.

Copper, per cent.	Tin, per cent.	Lead, per cent.	Wear, in grams	Copper, per cent.	Tin, per cent.	Lead, per cent.	Wear, in grams
85.76	14.90	....	0.2800	81.27	5.17	14.14	0.0327
90.67	9.45	....	0.1768	75?	5?	20?	0.0277
95.01	4.95	....	0.0776	68.71	5.24	26.67	0.0204
90.82	4.62	4.82	0.0542	64.34	4.70	31.32	0.0130
85.12	4.64	10.65	0.0380				

It was found that the addition of a small percentage of nickel adds to the compressive strength of the above series of alloys, and also elevates the temperature of solidification of the matrix. Ajax plastic bronzes of various grades, having compositions within the limits above set forth and also with the addition of 1 per cent. nickel, are in use very extensively for locomotive and car bearings, cold-roll neck mill bearings, etc.

Copper-Tin-Lead-Zinc Bearing Alloys are the cheapest form of bronze bearing-metal alloys on the market. Zinc is an adulterant which usually enters the alloy through the use of miscellaneous scrap. The following tests made in the author's laboratory show the effect on the rate of wear of increasing amounts of zinc.

Percentages				
Copper	Tin	Lead	Zinc	Wear in grams
85.12	4.64	10.64	....	0.0380
82.27	5.28	10.25	2.07	0.0415
79.84	4.71	10.30	5.44	0.0466
77.38	5.62	11.42	6.54	0.0672
74.28	4.68	10.61	11.04	0.0848

Zinc adds to the brittleness of the alloys, and this probably accounts for the more rapid rate of wear due to zinc content. The carrying of considerable zinc in the presence of fairly high percentages of lead has the further detrimental effect of causing the alloys to become somewhat weakened under the influence of elevated temperatures. The best bearing bronzes do not contain zinc, and its presence is restricted to below 1 per cent. in the specifications of the most prominent consumers. The Society of Automobile Engineers specification (No. 27) for **red brass** is copper, 85; tin, lead and zinc, each 5 per cent. This is said to machine well and to be an excellent bearing metal where speed and pressure are not excessive.

Tin-base Bearing Metals

Tin-base bearing metals (tin over 50 per cent.) come within the classification commonly known as **Babbitt metals**, and are divided into three classes: 1. Tin, antimony and copper alloys. 2. Tin, antimony, copper and lead alloys. 3. Tin, copper and zinc alloys. They possess in general a higher resistance to compression than the lead-base metals. The resistance to compression, however, is due in a very large measure to the relative percentages of copper and antimony which the alloys contain. Copper and antimony are the hardening constituents in such alloys, and, although they may also contain a certain percentages of lead and infrequently small percentages of other metals, the essential properties are derived from these two metal

The name "Babbitt" is derived from that of the inventor (Isaac Babbitt) of soft-metal-lined bearings. The term "babbitting" has been applied to the process of applying soft anti-friction metals inside of a harder shell for the purpose of producing bearings. The best-known soft-metal alloy at the time of this invention was the high-grade **pewter** alloy then used, the composition of which was copper, 3.70 per cent.; antimony, 7.40 per cent., and tin, 88.90 per cent. This particular alloy became known as "Genuine Babbitt," although the term is now more generally applied and includes all tin-antimony-copper alloys in which tin is the base metal, and which do not contain lead or zinc.

Soft Genuine Babbitt is usually made to the formula above given for Isaac Babbitt's original metal. Sometimes, however, this composition is slightly varied between 91 per cent. tin, 4½ per cent. lead, and 4½ per cent. antimony, and 88 per cent. tin, 4 per cent. copper and 8 per cent. antimony. The latter composition, as quoted by Charpy on a test piece 0.6 in. in height and 0.0155 sq. in. in sectional area, under a load of 1929 lb. showed a compression of 0.008 in., and under a load of 4960 lb. showed a compression corresponding to 0.3 in. This metal is free-flowing, tough, and gives excellent service in bearings where the loads are not heavy.

The specification of the Society of Automobile Engineers for Babbitt Metal No. 1 is: tin, 89 per cent.; antimony, 7 per cent., and copper, 4 per cent. A variation of 2 per cent. either way is permissible in the tin content, and a variation of 1 per cent. either way in the antimony and copper. Only traces of other metals are permissible.

Hard Genuine Babbitt. The composition of this alloy may range from 83 per cent. tin, 8½ per cent. antimony, 8½ per cent. copper, to 80 per cent. tin, 10 per cent. antimony and 10 per cent. copper. For the former Charpy gives the following figures: A test piece 0.6 in. in height and 0.0155 sq. in. in sectional area, under a load of 2646 lb., gave a compression of 0.008 in., and under a load of 5622 lb. a compression of 0.3 in. The metal of the latter formula would sustain a higher load with equal compression.

The method of casting and rate of cooling have a very important influence upon the structure and service which these metals will give. The rate of cooling affects the hardness of the matrix, and the size and number of the tin-antimony crystals. The proper temperature for pouring is approximately 810 deg. fahr., and the metal should preferably be cast against a chill heated to a temperature of approximately 212 deg. fahr., so that the rate of cooling is not too rapid.

The specification of the Society of Automobile Engineers for Babbitt Metal No. 2 is: tin, 84 per cent.; antimony, 9 per cent.; copper, 7 per cent. A variation of 2 per cent. either way is permissible in the tin, and a variation of 1 per cent. either way in the antimony and copper. Only traces of other metals. Specifications Nos. 1 and 2, above given, are also used in making **die-cast bearings.** An alloy of 80 tin, 10 copper and 10 antimony is used on the high-speed bearings of De Laval turbines.

Tin-base Babbitts Containing Lead. Lead has the property of increasing the fusibility of the tin-base alloys by forming a eutectic of low melting point, but the tin present should preferably be over 50 per cent., the antimony not less than 12 per cent., and the copper not less than 3 per cent. for the best results. The alloys may be made within a wide variation of composition, the hardness depending on the increasing percentages of copper and antimony.

Tin-base Babbitts Containing Zinc. The most important alloy of this class is (Parsons') white brass, which consists, approximately, of 62 per cent. tin, 35 per cent. zinc, and 3 per cent. copper. (S. A. E. Specification: Cu, 3 to 6; Sn > 65; Zn, 28 to 30.) It is largely **used for marine** bearings and to some extent for **automobile bearings.** Such bearings should be well lubricated. It is hard, tough, is rather sluggish to pour, and cannot for this reason be cast into thin sections after the manner of using the ordinary Babbitt metals. It must be poured at a fairly high temperature, approaching redness, and is preferably peined after casting to compress the metal and so add to its durability under the influence of wear.

Lead-base Bearing Metals (Babbitts)

Lead-base bearing metals (lead over 50 per cent.) are divided into three classes: (1) Lead-antimony alloys; (2) lead-antimony-tin alloys; (3) lead-antimony-tin-copper alloys. The useful alloys of lead and antimony for bearings contain from 7 to 20 per cent. of antimony. Below 7 per cent. the alloys are too soft, and above 20 per cent. too brittle, for practical uses. Lead and antimony are insoluble in each other in the cold, and the solidified alloys therefore consist of lead and eutectic alloy if antimony is present below 12.8 per cent., and of antimony and eutectic alloy if antimony is present in excess of 12.8 per cent. This is the **cheapest class of anti-friction metal** which can be produced, as lead sells at the lowest price of any of the metals, and antimony usually sells at only two or three times the price of lead. The alloy commercially known as No. 4 Babbitt consists of 88 to 90 per cent. of lead and 10 to 12 per cent. of antimony. **Magnolia metal** is a lead-tin-antimony alloy, together with traces of copper, zinc, iron and possibly bismuth. A recent analysis shows lead 79, tin 5, antimony 16.

Charpy has determined the compressive strength of several of these alloys and his results are given in the following table (test pieces as given under "Soft Genuine Babbitt"):

Composition of alloy, per cent. Lead	Antimony	Load corresponding to a permanent set of 0.008 in., lb.	Load corresponding to a permanent set of 0.3 in., lb.	Remarks
100	0.0	220	1100	
90	10	1433	2866	
82.5	17.5	1433	3197	Broke at
80	20	1676	....	2756 lb.

Lead-Tin-Antimony Alloys are very largely in use as Babbitt metals because of their low price, and because they perform very satisfactorily if used in the proper service and under the proper conditions. Many of the widely advertised brands of Babbitt metal come within this class.

Charpy states that the alloys of lead, tin and antimony are similar to those of lead and antimony; but the addition of tin diminishes the hardness and brittleness of the hard grains and also increases the compressive strength of the eutectic alloy. For this reason the alloys of lead, tin and antimony are superior to those of lead and antimony alone. He states that the tin must be present to the extent of more than 10 per cent., but not necessarily more than 20 per cent., and the antimony may vary between 10 and 18 per cent. The following table compiled by him shows the compressive strength of this series of alloys, test pieces as given under "Soft Genuine Babbitt," p. 549.

Composition		Load corresponding to a compression of		Composition			Load corresponding to a compression of	
Lead	Tin	0.008 in.	0.3 in.	Lead	Tin	Antimony	0.008 in.	0.3 in.
....	100	661	2337	10	80	10	2425	5952
20	80	1323	3858	20	60	20	2976	4850
40	60	1433	3252	40	40	20	2535	4023
60	40	1323	3086	60	20	20	2315	3748
80	20	1047	2535	80	10	10	1764	3913

Lead-Antimony-Tin-Copper Alloys. The addition of copper to the alloy of lead, tin and antimony increases the hardness and causes an increased sluggishness in pouring, depending on the amount of copper present in proportion to the tin.

In the high-tin-carrying alloys the percentage of copper can be increased without increasing the sluggishness. With 10 per cent. of tin, not over ½ per cent. of copper should be added; with 20 per cent. of tin, not over 1 per cent.; with 30 per cent. of tin, not over 1½ per cent.; with 40 per cent. of tin, not over 2 per cent., and with 50 per cent. of tin, not over 3 per cent. copper. A great many Babbitt metals are on the market having these four constituent metals in their composition, which are known in the trade as "copper-hardened Babbitt," and are designated by grades according to the amount of tin which they contain.

Zinc-base Bearing Metals

Lumen Metal consists of 85 per cent. zinc, 10 per cent. copper, and 5 per cent. aluminum. This alloy has a **low coefficient of friction,** a low specific gravity, and is capable of being cast in sand after the manner of brass and bronze castings. It is **easily machined,** and because of its low price and low specific gravity has found considerable favor in certain classes of service such as motor bearings, crane bearings, etc. It should be used, however, in bearings which can be maintained in fairly good adjustment, because, owing to the high zinc content of the alloy, it becomes brittle under the effects of heating, and in the absence of an oil film is quite liable to grip the journal. It is an **excellent** metal **for high-speed bearings carrying little weight. Fenton's alloy** contains 16 per cent. tin, 5 per cent. copper, and 79 per cent. zinc.

Alloys for Die Casting. Castings in which the molten metal is forced under pressure into accurately cut metal dies are now largely used in the place of small sand castings formerly made of brass and machined to size. It is possible in this way to produce parts of more or less intricate shape that are very accurate in size and smoothly finished. Babbitt metals are used in making die-cast bearings, while other machine parts are made from zinc-base alloys, the two most extensively used having the following compositions:

	I	II
Zinc	89.00	82.00
Aluminum	0.25	0.25
Tin	7.00	14.50
Copper	3.25	3.25

These and similar zinc alloys have a tensile strength not exceeding 18,000 lb. per sq. in., and an exceedingly low elongation and reduction of area. They compare favorably in strength with cast iron. They are corroded by aqueous solutions and should not be used for food containers or conveyors. Commercial gasoline when in direct and constant contact will also corrode them; copper plating, however, will aid in resisting its attack.

MISCELLANEOUS WHITE-METAL ALLOYS

Type Metals, or printer's metals, are divided into five classes: 1. Alloys for casting small individual type. 2. Linotype metal. 3. Monotype metal. 4. Stereotype metal. 5. Electrotype metal.

Alloys for Individual Type consist mostly of lead and antimony; sometimes small quantities of other metals are added. The essential requirements of a good type metal are that it shall give good, sharp castings and that it shall be sufficiently strong to withstand the necessary wear and pressure without losing its form.

The first of these requirements is fulfilled by alloys containing not less than 15 per cent. of antimony, which possess the property of expanding on cooling but are not strong enough to stand hard wear. In order to increase the strength of this alloy a certain quantity of tin is added. The composition of type metal varies considerably. An alloy containing lead 50, tin 25, antimony 25, is said to give the best results for high-class work; but the price of such an alloy is too high on account of the tin it contains, and a more usual composition is approximately lead 60, antimony 30, and tin 10. Frequently alloys are made carrying even more lead and less antimony and tin; for example, lead 75, antimony 20, tin 5.

In the **linotype** machine, where molten metal is forced under pressure into matrices to cast an integral line of type, the composition found to give the best results and which has been adopted as a standard, is lead, 79 per cent.; antimony, 16 per cent.; tin, 5 per cent. The **monotype** machine casts but one type at a time, using the same process as that employed in the linotype machine. The best metal for this machine consists of 76 per cent. of lead, 16 per cent. of antimony, and 8 per cent. of tin. **Stereotype metal,** used in the process of reproducing type in the form of an entire page or part of a page, must have the property of casting sharply and being sufficiently hard and durable to maintain a good face for the printing of many copies. The most desirable alloy for this purpose has the following composition: Lead, 83.75 per cent.; tin, 4.00 per cent.; antimony, 11.75 per cent.; and copper, 0.50 per cent. **Electrotype metal** is used as a backing for the thin, electrolytically deposited copper films of electrotypes, to give them the necessary rigidity. The alloy for this purpose does not require to be hard, as the wearing surface of the type or cut is of copper. Composition mostly used: Lead, 92 per cent.; tin, 4 per cent.; antimony, 4 per cent.

Alloys for Metallic Packing. For casting metallic packing rings the following inexpensive alloy has been found to give the best results: Lead, 83⅓ per cent.; tin, 8⅓ per cent.; antimony, 8⅓ per cent. Metallic packing for use with superheated steam should not fuse under the temperature produced by the combined effect of the temperature of the steam and the friction on the rod, should have sufficient plasticity to produce a tight joint, and should show but a slight rate of wear. A composition fulfilling these requirements consists of 80 per cent. lead and 20 per cent. antimony.

Fusible Alloys. By the addition of varying percentages of bismuth or cadmium to lead-tin alloys of different compositions, the melting points may be greatly lowered. Such alloys expand on cooling and are used for soldering lead, pewter and electrotypes, for taking casts from anatomical specimens and impressions from wood and other combustible materials, and for automatic fire sprinklers and alarms. Some of the compositions used (in parts by weight), together with their melting points, are as follows:

Alloy	Tin	Lead	Bismuth	Cadmium	Melting point, deg. fahr.
Lipowitz's	4	8	15	3	140
Wood's	4	8	15	4	158
Darcet's	25	25	50	....	203
Cliché metal	2	2	5	....	221
Rose's	24.6	28.1	50	....	230
Bismuth solder	24.8	22.1	53.1	....	250

Solders and Brazing Metals. Soft solders are used for joining tin plate and other metal sheets. A 90-tin-10-lead solder melts at about 410 deg. fahr., and a 70-tin-30-lead alloy at 384 deg. **Plumber's solder** consists of 2 parts lead and 1 part tin. **Brazing solder,** or brazing spelter, as it is commercially termed, consists of 50 to 55 per cent. of zinc, the remainder being copper. This is cast into ingots and granulated under a drop hammer into grades known as "long grain," "short grain," "fine grain," etc. The alloy mainly used as **brazing metal** consists of 80 per cent. copper and 20 per cent. zinc. Brazing solder and metal are used to unite brass, copper, iron and steel in strong joints. **Silver solder** is used by jewelers and for other fine metal work. The hardest consists of 4 parts silver to 1 of copper; the softest, 2 parts silver to 1 of copper. **Fluxes:** for soft solders, powdered rosin, hydrochloric acid "killed" by the addition of zinc scraps, and tallow (by plumbers); for brazing and with silver solder, powdered borax. Sal ammoniac is sometimes used in brazing copper.

Prices of Non-ferrous Metals, 1910–1915. The accompanying table gives the range of monthly average New York prices (cents per lb.) during 1910–1914 for purchases in large quantities, as well as the abnormal prices current Aug. 1, 1915, and brought about by war conditions in Europe.

	1910	1911	1912	1913	1914	Aug.1, 1915
Copper (Lake)..	12.7–14	12.3–13.8	14.4–17.8	14.8–17.0	11.7–15.0	21.0
Lead...........	4.4– 4.7	4.3– 4.5	4.0– 5.0	4.1– 4.7	3.5– 4.1	5.0
Spelter.........	5.2– 6.3	5.5– 6.6	6.5– 7.6	5.2– 7.2	5.0– 5.7	17.5
Tin.............	32.6–38.2	41.1–46.3	42.8–50.0	37.1–50.3	30.3–48.3	35.5
Antimony (Cookson's)...	7.6– 8.5	7.8– 9.6	7.3–10.4	7.5–10.0	7.2–17.8	48–50
Nickel.........	(40–45 during 1910–1914; electrolytic nickel, 45–50)					45–50
Aluminum (ingot).......		19.3–21.3	19.1–26.6	18.9–27.1	17.7–20.0	31–33

CORROSION

BY

MORGAN B. SMITH

REFERENCES: Cushman and Gardner, "Corrosion and Preservation of Iron and Steel," McGraw-Hill. Sang, "Corrosion of Iron and Steel," McGraw-Hill. "Corrosion of Iron and Steel," Speller, *Iron Age*, vol. 79, p. 478. "Corrosion of Iron," Whitney, *Jour. Am. Chem. Soc.*, vol. 25, p. 394. "Corrosion of Iron and Steel and Modern Methods of Prevention," Walker, *Jour. Iron and Steel Inst.*, 1909, vol. 90. "Boiler Corrosion an Electrochemical Action," Burgess, *Jour. West. Soc. Engrs.*, vol. 14, p. 375.

General Considerations

The atmospheric corrosion of a metal is a reaction between the metal and water. It may be represented by the equation, metal + water = hydroxide of metal + hydrogen. There are six factors to be considered.

1. THE METAL. Each metal has a tendency to dissolve in water; this tendency is known as the "solution pressure" of the metal and is measured by the potential of the metal when in a solution of one of its salts. The order of **solution pressures** of the more common elements is as follows: Potassium, sodium, barium, strontium, calcium, magnesium, aluminum, chromium, manganese, zinc, iron, cobalt, nickel, lead, cadmium, tin, bismuth, copper, hydrogen, mercury, silver, antimony, gold, iridium, rhodium, platinum. This series indicates the relative tendencies of the metals to corrode, but must be used with the caution mentioned later. Potassium has the maximum and platinum the minimum solution pressure of the metals given.

2. THE WATER. Water is dissociated into its ions hydrogen and hydroxyl. The greater the concentration of hydrogen ions in the water the more rapidly will the metal dissolve; hence, any acid or acid gas such as carbon dioxide or sulphur dioxide dissolved in the water will increase the corrosion of the metal. Increasing the hydroxyl ions by the addition of an alkali or alkaline salt, will retard or prevent corrosion; thus lime water or dense cement concrete will inhibit rusting.

3. THE HYDROXIDE OF THE METAL can influence directly the corrosion of the metal only when in solution in water. Most of the metal hydroxides are insoluble and have little direct bearing on corrosion, but they may have an indirect influence in the tendency to form sponge-like masses on the surface of the metal, thus holding moisture and oxygen in contact with the corroding parts.

4. THE HYDROGEN. When a metal dissolves in water, an equivalent quantity of hydrogen is set free. If the action is violent, the hydrogen appears as an escaping gas; in most cases, however, it separates from the solution and plates out as an insulating gaseous film on the surface of the metal, retarding or even preventing further action (polarization).

5. OXYGEN OF THE AIR. Oxygen and hydrogen unite to form water with extreme slowness under ordinary conditions, but when brought into contact with certain surfaces they unite easily. Upon platinum the reaction is very rapid; on iron much slower, and on zinc or aluminum it is negligible. It is in preventing the formation of this hydrogen film (depolarization) that oxygen dissolved in the water is so necessary to corrosion. The greater the oxygen pressure, or the more active the surface of the metal in accelerating the union of oxygen and hydrogen, the more rapid is the solution of the metal. Thus it happens that while zinc has a larger solution pressure than iron, it is also

more easily polarized by hydrogen than is iron; consequently, iron corrodes in a damp atmosphere, while zinc does not. When, however, zinc and iron are in contact, the hydrogen equivalent of the zinc which dissolves, can and does plate out on the iron; under these circumstances the zinc corrodes rapidly, while the iron does not corrode at all. If air is removed from the water in a boiler or heating system no corrosion will occur.

The depolarizing action takes place with extreme ease on forge or mill scale—the magnetic oxide of iron which covers a piece of iron or steel after it has been highly heated. If this scale forms a close coherent coat it is of value as a protecting surface; but if it is ruptured at any point, corrosion at this point takes place with great rapidity, and a pit is the result. For this reason mill scale should be removed as carefully as possible from ferrous structures exposed to corrosive conditions. An entirely secondary action of oxygen is the oxidation of the ferrous hydroxide formed by the action of the water, producing the red ferric hydroxide which is familiar as rust.

6. STRAY ELECTRIC CURRENTS. The solution of a metal in water and the consequent deposition of hydrogen is always accompanied by a flow of electricity. This action must not be confused with "electrolytic corrosion." The inherent electromotive force driving a metal into solution is, with the metals used in engineering structures, relatively small and the currents flowing very minute. When, however, an electric current due to some external electromotive force flows from a metal through its surroundings, the metal is carried into solution at a rate which may be calculated from Faraday's law.

Factors Stimulating Corrosion. Corrosive action is stimulated by. A very damp atmosphere, thus maintaining a film of water on the metal; atmospheric oxygen dissolved in this water film; acids or acid gases; salts which dissociate in water, producing an acid reaction; contact of dissimilar metals; the presence on the metal of a depolarizing surface, such as mill scale on iron; improperly annealed metals, where one part is under strain; soils containing cinders, coke, coal dust, etc., which carry acid sulphur compounds; an electric current due to an external source leaving a metal surface.

Factors Inhibiting Corrosion are: Reversed polarity-to counteract the effects of stray currents; the use of a self-sacrificing metal electropositive to the metal it is desired to protect, as zinc to prevent the corrosion of iron; protective coatings; the rendering of metals "passive" (or so treating them that they are insoluble in acids and do not precipitate metals from solutions); neutralization of corrosive fumes or liquors; alkaline solutions of sufficiently high alkalinity.

Corrosion Due to Electrolysis is generally caused by the leakage of current from electric circuits, and often takes place at a point far removed from where the leakage occurs. Stray currents of extremely feeble intensity and voltage tend to accelerate corrosion, even when they have not initiated it. Corrosion due to electrolysis may be minimized by providing thorough insulation; grounding all metallic conduits; avoiding combinations of dissimilar metals in a circuit; and by maintaining apparatus in an electronegative state with reference to possible sources of current, either by the so-called "drainage" system, or by "tying up" with some source of current of a higher potential.

Corrosion of Metals

Iron and Steel. Under similar conditions, iron and steel corrode at practically the same rate. Steel, however, corrodes more uniformly than iron.

Tests on steel skelp, wrought-iron skelp, charcoal iron, pipe steel and puddled iron exposed for long periods to the action of river water, sea water, aerated distilled water, aerated brine and the weather, showed on the average that steel lost 97.2 per cent. as much weight by corrosion as did wrought or puddled iron, and 96.1 per cent. as much as charcoal iron. (*Proc. A. S. T. M.*, 1908.)

The results of an investigation by J. H. Woolson of the hot-water systems of a number of bath houses in New York City, showed that there was no perceptible difference in the rate of corrosion of galvanized wrought-iron and galvanized steel pipes.

There is no appreciable difference in the rate of attack on hot-drawn and cold-drawn seamless iron and Bessemer steel tubes, when immersed in air-saturated distilled water. (J. D. Ford, *Jour. Am. Soc. Nav. Engrs.*, May, 1904. See also W. H. Walker, *Jour. Am. Water Works Assn.*, 1912, vol. 26.)

According to G. E. and M. C. Whipple, there is practically no difference in the tendency to form pits in the different types of ferrous metal (8th Int. Cong. Appl. Chem.).

Metals in Brine. M. B. Smith (*Ice and Refrigeration*, vol. xii, p. 159, and *Proc. A. S. R. E.*, 1911–1912–1913), from the results of tests of various commercial metals immersed for 150 days in calcium chloride brine (sp. gr., 1.2) at 73.4 deg. fahr., found that they resist corrosion in the following order: Bronze, brass, iron-copper alloy (3 per cent. Cu), copper, wrought iron, mild steel, cast iron, solder, lead, zinc, galvanized iron—bronze being the least attacked Aluminum showed a gain in weight, due to the accumulation of a coating of oxide. Combinations of these metals showed in each case greater losses, which were at the expense of the electropositive metal.

Couples of different metals similarly tested resisted corrosion in the following order: Brass-iron, copper-iron, wrought iron-galvanized iron, and copper-galvanized iron, the latter being least resistant.

When the brine was air-saturated, it was found the rate of corrosion was about three times as rapid as with normal brine, being especially rapid with metals which are oxidized readily and whose oxides do not adhere to the surface at which they are formed. For combinations of metals in air-saturated brine, the rate of corrosion is only about twice that when normal brine is used.

The **relative corrosiveness of brines** depends upon local conditions except when the brines are of inferior character. If the chlorides used—whether they are calcium chloride, calcium-magnesium chloride, calcium-magnesium-sodium chloride or sodium chloride—are neutral products or even slightly alkaline in reaction, there seems to be no appreciable difference in their attack upon metals, provided that all conditions are similar in each case.

Brines should be maintained in a slightly alkaline condition by the occasional addition of milk of lime, caustic soda or other alkali. When in proper condition they should give the characteristic pink color with the phenolphthalein indicator.

Corrosion of Pipes, Boilers and Structural Work

Pipe Materials. Tests by M. B. Smith of the relative corrodibility in artificial sea water, in fresh water, and when exposed to the weather, at 20–25 deg. cent. for 6 months, gave the following results:

Metals	Weather exposure	Artificial sea water	Fresh water
Soft steel	100	105	100
Nickel steel (3 per cent. Ni)	58	79	72
Nickel steel (18 per cent. Ni)	19	20	19
Wrought iron, best	100	100	100

Mill scale on pipes generally forms centers of corrosion, and should be removed in the process of manufacture.

Boiler Tubes. Ford's investigations of 3-in. lap-welded steel and wrought-iron tubes in air-saturated distilled water at 68–77 deg. fahr. show minimum corrosion for welded Bessemer steel and maximum for welded wrought iron; weldless Bessemer tubes corrode more rapidly than welded Bessemer, but less rapidly than weldless wrought-iron tubes.

Unger investigated the corrosion of boiler tubes in sea water, brine, ordinary well water and in 1 per cent. solution of sulphuric acid and ferrous sulphate. He found that the metals yielded to corrosion in the following order: Ordinary wrought iron, medium quality wrought iron, low-carbon Bessemer steel, best grades of wrought iron, open-hearth steel; the last being the most resistant. Open-hearth steels of high and low manganese contents offered about the same resistance to corrosion.

Steam Boilers. Corrosion in boilers takes place in one or all of three ways, namely, general, pitting, and grooving. **General corrosion** is the least dangerous, but the boiler must be watched closely lest it be gradually weakened to an unsafe degree. **Pitting** is readily detected as a rule and is frequently very difficult to stop without a thorough search after the causes. **Grooving** is often very hard to locate before leaks take place, since such corrosion takes place at points where the metal has been bent or strained, and is hidden from any but the most careful inspection. General corrosion of steam boilers may usually be traced to the water employed, but it is also caused by the action of certain boiler **feed-water compounds** containing tannic acid, sulphate of copper, etc. Copper sulphate reacts with the iron or steel in the well-known reaction wherein copper plates out on the iron, an equivalent amount of iron passing into solution. Great caution should be exercised in the use of such materials in boiler practice.

Where there is a corrosive action because of the **presence of acid** in the water, or of oil containing fatty acids which will decompose and cause pitting wherever the sludge can find a lodging place, it may be overcome by the neutralization of the water by carbonate of soda. This should be carried to a point where the water will just turn red litmus paper blue. As a preventive of such action, only the highest grades of hydrocarbon oils should be used.

Acidity may appear where salt water makes its way into a boiler, as may occur in marine practice from leaky condenser tubes, priming in the evaporators, etc. This acidity is caused by the dissociation of magnesium chloride into hydrochloric acid and magnesium under high pressure. The acid in contact with the metal forms an iron salt, which, as soon as formed, is neutralised by the free magnesia in the water, thereby precipitating iron oxide and re-forming magnesium chloride. Where it is unavoidable that some salt water should make its way into a boiler, the water should be neutralised by milk of lime, which will convert the magnesium chloride into magnesia and chloride of calcium, neither of which is corrosive, but both of which are scale-forming.

Decomposition of Boiler Waters. Constituents which may form **hydrochloric acid** are: Chloride of magnesium plus steam; sulphate of magnesium and the alkaline chlorides; silica and alkaline chlorides; ferric chloride; ferrous chloride; carbonate of magnesium plus chlorides; chloride of ammonium. Constituents which may form **sulphuric acid** are: Normal ferric sulphate; ferrous sulphate; sulphurous acid and ferric chloride; sulphites; hydrogen sulphide; sulphate of calcium plus organic matter; sulphate of ammonia; sulphate of copper. Constituents which may form **nitric acid** are: normal ferric nitrate; alkaline nitrates and acid sulphates or sulphuric acid; nitrate of ammonia.

Air sucked in by the feed pumps is a well-recognised cause of corrosion. Air bubbles form below the water line and the oxygen of the air attack the metal. The rust thus formed is washed away by circulation or dislodged by expansion, leaving a minute pit which serves as a lodging place for other air bubbles, and in this way continues the action. In marine practice it has been found advantageous to pump the water from the hot well to a filter tank above the feed-pump suction valves. The bubbles are liberated from the surface of the tank and a head is assured for the suction end of the pump. The corrosive action of air may also be reduced by introducing the feed water into the steam space above the water line.

Both general corrosion and pitting may be reduced or even eliminated by using an open feed-water heater, thus expelling the air, or better still, a feed-water heater connected to a vacuum pump.

Galvanic action takes place in certain instances. The remedy for it is usually the installation of zinc plates within the boiler, which must have positive metallic contact with the metal of the boiler. The zinc plates are corroded instead of the boiler, and the latter is protected at the expense of the former. The positive contact necessary is difficult to maintain, and the efficiency of such plates is questionable, except for a short period after their installation.

Some of the **other causes of corrosion** in steam boilers are: Polluted feed water containing manufacturing wastes—auto-electrolysis between the boiler metal and carbonaceous matter such as soot or ashes, especially around mud drums and blow-off pipes; rain water which, through contact with acid-containing soot on roofs, etc., becomes acid in nature; decomposition of sulphate scale, especially calcium sulphate, with the formation of sulphuric acid (some feed waters, apparently neutral at ordinary temperatures, become acid when heated strongly); coal-pit water—generally acid; boiler metal containing much impurity, especially if segregated; straining of the boiler metal, frequently caused by lack of proper staying, which leads to buckling of the plates (torsional strain is far worse than tensile and should be guarded against); certain alkaline waters, the addition of barium chloride to which would probably be advisable; water containing organic matter, as that from peat bogs, swamps, etc.

Copper ferrules should not be used on the ends of boiler tubes when expanding them into the tube sheets, as auto-electrolysis is almost certain to be set up when such dissimilar metals come in contact. Soft iron ferrules may be used. For the same reason, copper feed pipes should not be joined directly to the boiler metal.

Corrosion in **boilers out of service** may be prevented in several ways. In the French Navy the boiler is filled with water, after which milk of lime or soda is added. The exterior is then painted with red lead or tar, inaccessible places being coated by burning tar beneath them. Boilers in American naval vessels are first cleaned thoroughly and then coated inside and outside with a heavy mineral oil. (See also p. 937.)

Condenser Tubes. The corrosion of condenser tubes is often caused by the formation of oxides which are electronegative toward the metal of which the apparatus is constructed, resulting in auto-electrolysis. Other causes are: the combination of dissimilar metals; the decomposition of sulphate scale, liberating sulphuric acid; sudden changes in temperature. Air either in solution in the cooling waters or present as a gas, tends greatly to accelerate corrosion.

Corrosion takes place in one or more of three ways, namely, splits or cracks, pits, and dezincification, the latter conducing to brittleness. Resistance to corrosion apparently depends more on the microstructure of the metal than on its composition; hard-drawn tubes, annealed slowly at a temperature not exceeding 750 deg. fahr., last much longer than untreated tubes.

As a result of tests of several alloys used in making tubes, in sea water at 64 to 104 deg. fahr., Bengough and Jones (*Proc. Inst. of Metals*, 1913) recommend the use of either "Admiralty" metal (70 Cu, 29 Zn, 1 Sn) or Muntz special brass (70 Cu, 28 Zn, 2 Pb). Iron, if present in brass, greatly accelerates corrosion. Copper-aluminum alloy (8 per cent. Al) has given excellent service in salt water. Monel metal is not non-corrosive in salt water (J. J. Sparrow, *Power*, Oct. 14, 1913, p. 531).

The most common method of protecting condenser tubes against corrosion is to use a metal electropositive toward brass. Zinc has been superseded for this purpose by iron. Electrical contact is absolutely essential for protection.

Pumps may be corroded by the solvent action of the liquor passing through them, provided they are constructed from unsuitable material. When dissimilar metals are used, *e.g.*, a cast-iron body with brass glands, seats, etc., auto-electrolysis is almost certain to result when brines or other electrolytes are pumped. If it is not possible to confine the construction to one metal, the attacked metal should be used only for easily renewable parts.

Corrosion is often set up by the leakage of current from the driving motor. This may be prevented by using an insulating coupling between the pump and motor shafts.

Air in the pump, either free or in solution in the liquid pumped, accelerates corrosion. Pumps should be vented so that they may be completely filled with the liquid they are handling.

Underground Pipes. Pipes buried in the ground are often corroded by the action of stray electric currents from trolley-car tracks, the corrosion manifesting itself at the points where the current leaves the pipes. They should therefore be maintained in an electronegative relation to the tracks.

Soils containing organic or carbonaceous matter such as coke, coal, cinders, etc., or impregnated with acid wastes from manufacturing plants, are highly corrosive in their action.

Steel, wrought iron, cast iron and lead corrode in certain natural soils containing soluble electrolytic salts or polluted with corrosive wastes, etc., in localities where there are no stray electric currents. Scofield and Stenger (*El. Ry. Jour.*, Nov. 14, 1914) find that black peaty soils and clays as electrolytes in contact with metals give rise to the following voltages: Sheet iron, 0 to 0.5; steel pipe, 0 to 0.5; cast-iron pipe, 0 to 0.3; pitted cast iron, 0 to 0.7; clean cast iron, 0 to 0.15; the current leaving the metal at clean spots, returning to the impurities in the metal through the external circuit. Clean iron is therefore electropositive toward impurities. The auto-electrolysis of iron buried in soils is recurrent, since oxygen is always present in sufficient amount to act as a depolariser. Tests on 120 different soils with steel test strips showed comparative corrosive action as follows, beginning with the least corrosive soil: Clean dry sand; light sandy soils, loam and drier dark soils; heavy clays; wet black peaty soils. The minimum attack on steel in these tests was equal to a loss in weight of 0.04 gram, the maximum being 0.6 gram, on test pieces 1 × 2¼ in. × No. 16 gage thickness in 66 days. The average is 0.25 gram, or 1.39 grams per year, indicating a probable life for ¼-in. thick pipe of 7.6 years.

Pipes buried in mixtures of two dissimilar soils or in two unlike soils in contact (not mixed) generally corrode more rapidly than in either of the two soils separately. Lead in a mixture or simple contact of dissimilar soils will often corrode markedly. Pipes buried in trenches fulfill these conditions of soils in contact or mixed. Metals corrode at the junction line of dissimilar soils. Cast iron in soils takes on a hard coat of rust and soil, pits being filled with carbon and black iron oxide. Lead shows both the gray and brown oxides when corroded in soils. Potentials up to 1 volt may readily be generated by placing unlike metals in a given soil or by using one metal in two dissimilar soils.

The tendency to corrode, especially in the form of pitting, is said to be reduced by a process, known as "**Spellerizing,**" used by the National Tube Co. This process, applicable to welded pipes up to 5 in. in diam., consists in subjecting the bloom to the repeated alternate action of rolls provided with regularly shaped projections and of smooth rolls, resulting in the production of surface metal of a uniformly dense texture. The same company also furnishes pipes up to 18 in. in diam. which are first dipped in a hot bituminous compound and then wrapped helically on the outside with one or more layers of fabric impregnated with the hot compound. This "National coating" is said to effectually prevent moisture from reaching the metal, making corrosion or electrolysis impossible.

Pipes are often destroyed by the action of dissimilar metals to which they are connected, such as brass valves, etc., in which case short connections, readily replaced, should be used on either side of the brass fittings (see also Pipe Materials, p. 556).

Bridges, Roofs, Stacks, Etc. Iron and steel structures are corroded by the presence in the atmosphere of moisture and waste products from manufacturing and metallurgical plants such as carbon monoxide, carbon dioxide, sulphur dioxide, chlorine, ammonia, zinc and acid fumes, soot, etc. As a rule, they are not corroded in a dry atmosphere. To minimize the corrosive effects of gases, protective coatings (see p. 563) are resorted to. Methods of coating metals with other metals are given below. Metal work exposed to the action of sulphur fumes should be covered with brick or with a paint highly resistant to sulphur dioxide.

The presence of from 0.2 to 0.25 per cent. of copper in sheet steel has been shown to greatly increase its durability both in atmospheric corrosion and in the presence of strong acid fumes. Such steels are now available and much used in structural work. (Buck, *Jour. Ind. & Eng. Chem.*, 1913.)

Concrete Structures. The following notes are taken from Bulletin No. 18 of the U. S. Bureau of Standards:

When current passes from an iron anode into concrete, oxides of iron form on the anode, occupying about 2.2 times the volume of the equivalent iron, and giving rise to mechanical pressure (sometimes as high as 4700 lb. per sq. in.) which may result in cracking the concrete. At temperatures below 113 deg. fahr., and with a potential gradient less than 60 volts per foot, this action is very slight, even in wet concrete.

Concrete near the cathode, or metal through which the current leaves the concrete, becomes softened and remains brittle and friable after drying, destroying the bond between the iron and concrete. This effect is noted at all voltages, high or low, and is due to the concentration of sodium or potassium near the cathode. The content of these elements in the cement should therefore be kept low.

Salt (NaCl) or calcium chloride should never be added to concrete used in structures which will be subject to electrolytic action. Concrete in contact with salt water is very susceptible to electrolysis. In structures exposed to the action of salts, pickling solutions, etc., the potential gradient must be kept low.

In the absence of metallic electrodes the action of current is similar to slow seepage, the water-soluble elements in the concrete migrating toward the cathode. Grounding of electric conductors in such a structure is equivalent to installing electrodes.

Waterproofing compounds when mixed with concrete have but little effect in preventing electrolysis. Waterproofing membranes, properly applied, are fairly efficacious in preventing the entry of earth currents. Painting or otherwise coating the reinforcing metal may minimise the danger from electrolysis, but it prevents proper bonding between the metal and the concrete.

All direct-current circuits within a building should be kept free from grounds. All pipe lines entering a building should be installed with insulating joints outside the building. If passing through, they should have insulating joints on both sides of the building. If the potential drop around the isolated section is 8 volts or more, the isolated section should be shunted by a copper cable. Lead-covered cable should be kept out of contact with the building.

All metallic structures within a building should be interconnected, providing all lines entering the building are installed with insulating joints, but they should not be grounded to any ground plate lying outside of the insulating joints. Maintaining the reinforcing metal negative is worse than no protection at all.

Methods for Minimizing Corrosion

Corrosion may be minimized by (*a*) the use of a coating of protective metal such as zinc, tin, lead, nickel or copper; (*b*) by the production of magnetic oxides on iron and steel surfaces; (*c*) by the application of protective paints (p. 563), and (*d*) by rendering the surface of the metal passive.

Coating Metals with Zinc.—Galvanizing. Zinc is applied to metal surfaces either by the Sherardizing process, by dipping in a bath of molten zinc, or by electrodeposition.

In the **Sherardizing process** the articles, after being thoroughly cleaned by pickling and sand-blasting, are placed in a metal drum together with zinc dust, and heated to a temperature of from 500 to 600 deg. fahr., depending on their size and shape. The coating which results is not pure zinc, but an alloy of about 90 per cent. zinc and 10 per cent. iron (melting point, 1260 deg. fahr.), and is even more highly resistant to corrosion than pure zinc. It is covered in the final stages, however, by pure zinc. Articles of very complex shape may be readily coated by this process, since the zinc fumes reach all parts of the articles. The process is adapted for watch screws, wire, roofing sheets, bolts and nuts, chains, pipe fittings, nails, small castings, and such other articles as may conveniently be placed within a drum 30 in. in diam. and 22 ft. long. In coating screws it is only necessary to allow for the thickness of the zinc coating. The metal coated is often improved by the incidental heat treatment of the process, which is akin to annealing. The cost varies with the character of the articles coated. For a maximum drum charge of closely packed articles of similar shape, it is about $1 per 100 lb.; for thin, bulky material not packing to advantage, from $4 to $8 per 100 lb.

In the **hot process** the articles after being thoroughly cleaned are dipped in a bath of molten zinc. The bath must be maintained at a temperature somewhat higher than the melting point of zinc, making a large consumption of fuel essential and also resulting in a considerable loss in zinc through oxidation at the surface of the bath. The process is best suited to large articles such as wire, large sheets, etc., which may be readily drawn through the zinc bath. The cost is about double that of the Sherardizing process when all losses are taken into account. In order to cut down losses due to oxidation, the surface of the zinc in the bath is covered with a layer of ammonium chloride. From 2 to 3 per cent. of tin is frequently added to accelerate crystallization of the zinc on the metallic surfaces. In order to prevent the formation of hard zinc in coating sheet metal, the bath is often filled to within an inch or so of the required depth with lead, on top of which about 1 in. of zinc containing from 2 to 3 per cent. tin is placed. The lead serves as a reservoir for the heat, maintaining the zinc-tin alloy at an even temperature.

The **electrolytic or cold process** consists in setting up the articles to be coated as cathodes in an electrolytic bath of soluble zinc salts, the anode being metallic zinc. In the Cowper-Cowles process the solution is zinc sulphate, which is regenerated by passing it through tanks containing zinc dust. In the S. Wagners process the zinc anode is covered with flannel saturated with a solution of zinc and is moved over the surface of the object. Small articles are placed in metallic baskets in contact with the basket and with each other, the basket in turn being attached to the cathode of the system. The cost varies widely, but falls between that of the Sherardizing and the hot processes.

In the **Schoop spraying process** a stream of finely divided or molten metal is forced through a blow-pipe flame under air or gas pressure and played upon the surface to be treated, yielding a thin and firmly adherent coating. Lead, copper, brass or tin coatings may be applied by this process to any desired surface, whether of metal, wood, fabric, etc. It is being used commercially in France for coating iron sheets with tin and zinc, and has been proposed as a means for coppering the hulls of ships.

Coating with Tin, Nickel or Copper. Coatings of **tin** or of a **tin-lead alloy** are used principally on thin sheets of iron, being applied as a rule in a manner similar to that of the hot process of galvanising. When so-called **terne plate** is made, a mixture of tin (70 per cent.) and lead (30 per cent.) is generally used. Such coatings, if free from pin-holes, are highly resistant to corrosion.

In coating or electroplating with **nickel or copper,** the object to be coated is made the cathode, the anode consisting of a block of the metal to be deposited and the electrolyte a solution of the metal to be deposited. In nickel plating, a copper coating is generally applied before the nickel to render the latter more adherent.

Copper is sometimes applied in very thin coats by the Monnot and Willis processes, in which copper is cast around the object, which is afterward rolled down to the required thickness.

Magnetic Oxide on Iron Surfaces. In the **Bower-Barff process** the iron or steel articles to be coated are heated in a closed retort to a temperature of 871 deg. fahr., after which superheated steam is admitted. This results in the formation of red oxide (Fe_2O_3) and magnetic oxide (Fe_3O_4). Carbon monoxide (CO) is then admitted to the retort to reduce the red oxide to magnetic oxide, which is highly resistant to corrosion. Each operation takes about 20 min. The **Wells process** is similar to the Bower-Barff. The glossy black coating of magnetic oxide on **Russia iron** is produced by laying up sheets of iron with powdered charcoal between, the whole mass being then heated and hammered.

Rendering Iron Surfaces Passive may be accomplished in several ways, the most common being to immerse the metal in nitric acid (sp. gr., 1.4) after highly polishing it. Other methods consist in immersing the metal in fuming sulphuric acid, potassium ferrocyanide or potassium chromate solution or in chromic acid; coating with a manganese dioxide paint; pickling in a weak acid solution, the metal being connected as the cathode in a circuit of low voltage; treatment with arsenic, sodium nitrite, etc. Thus far, passive treatment has been of very doubtful value.

PAINTS AND PROTECTIVE COATINGS

BY

HENRY A. GARDNER

REFERENCES: Gardner, "Paint Technology and Tests," McGraw-Hill. Cushman and Gardner, "Corrosion and Preservation of Iron and Steel," McGraw-Hill. Hurst, "Painters' Colors, Oils and Varnishes," Lippincott. Wood, "Rustless Coatings," Wiley. Maire, "Modern Pigments and Their Vehicles," Wiley. Reports of Committee D-1 of the A. S. T. M. 1903–1915.

Preparation of Surfaces for Painting. If **new wood** is to be painted, it should be sandpapered lightly in rough spots. All knots should be brushed with turpentine or benzol just before applying the paint. This treatment will allow the paint to form a bond with the softened resin in the knots and thus give better results. At least 15 per cent. of turpentine should be used in the priming coat of paint, to secure penetration. Before applying a second coat, all nail holes and crevices should be stopped with putty. If the surface needs repainting, all "alligatoring," scaling or blistering appearing on the old paint should be leveled with sandpaper. When **new metal** is to be painted, it should be cleaned of rust spots with a wire brush, scraper or sand blast. Small articles are generally pickled in a hot 15 per cent. solution of sulphuric acid to remove the oxide scale from the surface, and then thoroughly washed and dried before painting. Three coats of paint should be used on all structural materials if good results are to be obtained.

Spreading Rates. Although the spreading rate of paints varies greatly, the average rate for a paint of normal consistency is as follows:

	Square Feet per Gal.		
	Wooden Surfaces	Metal Surfaces	Cement and Concrete
Priming coat	300 to 400	500 to 700	150 to 250
Second and third coats	400 to 600	700 to 800	300 to 400

Cost of Painting. The cost of labor for painting any structure is generally twice the cost of the paint, allowing on an average $2 per gal. for the paint. Where the preparation of surface requires chiseling and wire-brushing, or other extra preparation, the cost will sometimes exceed three times the price of the paint. The best paints from the standpoint of efficiency are those described below.

Pigments and Prepared Paints for Wooden Surfaces. The opaque white pigments used in paints for the preservation of wood are corroded **white lead** (basic carbonate white lead), sublimed white lead (basic sulphate white lead), **zinc white** (zinc oxide), and **leaded zincs** (lead sulphate combined with zinc oxide). Lead pigments used alone are apt to chalk and "alligator," while zinc pigments often scale and check. By combining the two types of pigments these defects are overcome. Equal parts of lead and zinc form a very good paint. Mixtures of lead and zinc pigments are often combined with a small percentage of the crystalline pigments such as **barytes** (barium sulphate), **china clay** (aluminum silicate), **silex** (silicon oxide), **asbestine** (magnesium silicate), etc., to increase the durability of the product. Such combinations when finely ground by machine with pure linseed oil form the best paints. They are tinted with **colored pigments** such as chrome yellow (lead chromate), Prussian blue (ferro-ferricyanide of iron), sienna, ochre, umber, lampblack, etc., and sold as prepared paints.

Paint Oils. Pure **linseed oil** (see p. 638), raw or boiled, on account of rapid and hard drying, is preferable for general painting work. Refined raw oil is generally used in paints for wooden surfaces. A mixture of raw and boiled oils is sometimes preferred in paints for metal surfaces. Oils are sold on the basis of 7½ lb. per gal., but are paid for by the gallon. The price of linseed oil varies widely but averages about 60 cents per gal. in 5-bbl. lots. **Menhaden oil,** which is extracted from the menhaden fish, is occasionally used in special paints for sea exposure. This oil is very resistant to moisture, but is apt to take dust and become darkened. **Soya bean oil,** extracted from the soya bean, a legume grown in the South for forage, has been used as a partial substitute for linseed oil. In some cases it has given fair results in combination with linseed oil in different types of paint. **China wood oil** (see p. 637) is heat-treated with driers and used in technical and water-proofing paints. Its rapid drying to a clear, glossy film makes it valuable for such purposes. **Corn oil** and **cottonseed oil** are very slow drying and are apt to make a paint "tacky." **Rosin oil** and **petroleum oil** are dangerous to use in paints for the protection of wooden or steel surfaces, on account of their slow drying and tendency to check.

Paste Paints. **White-lead paste** contains approximately 9 lb. of oil per 100 lb. For spreading this amount there should be added from 4 to 5 gal. of oil. **Zinc-oxide paste** contains 16 lb. of oil per 100 lb. of pigment. For spreading, there should be added 6 gal. of oil or an amount sufficient to produce an easy working paint. **Iron-oxide paste** contains approximately 30 per cent. of oil. **Red lead** is generally bought in the dry form and mixed with oil in the proportion of 20 to 30 lb. of lead to 1 gal. of oil. Red lead containing 15 per cent. of free litharge protects metal much better than red lead which contains only 4 per cent. of litharge. The litharge forms a hard, water-resistant film. Combination pigment paints contain on an average 30 per cent. oil and 70 per cent. pigment. Instructions for reduction and application are given on the label.

Paint Thinners. Paints of a very heavy body or thick consistency are difficult to apply and often show brush marks after application. Such paints should be thinned with **turpentine** or other neutral thinner. Because of the high price of turpentine, **petroleum spirits** of the same boiling point and gravity are being used in some paints, with very satisfactory results.

Driers. Salts or oxides of lead and manganese are boiled into oil to form driers. When these driers are added to paint, they act as catalytic agents and accelerate drying by attracting oxygen. **Litharge** (PbO) is most generally used. **Sugar of lead,** incorporated in oil, is used for light tints. **Sulphate of zinc** and **manganese dioxide** are used by the grinder for certain paints having zinc white as a base.

Paints for Structural Steel. The priming coat of paint should be made of linseed oil containing thoroughly inhibitive or inert pigments such as basic chromate of lead, red lead, sublimed white or blue lead, zinc oxide, iron oxide, zinc chromate, etc. A mixture of the lead, zinc and iron pigments is sometimes preferred to the use of any single pigment.

The **surface area of a steel structure** may be calculated by use of the following formulæ in connection with the material bills giving the tonnage of the various shapes.

Let S = sq. ft. of surface per ton of metal, and w = weight of shape per running foot, lb. Then, for

I-beams: $S = (56{,}800h + 54{,}800)/w$, where h = depth of beam, in.

Channels: $S = (434b + 500)/w$, where b = width of channel, in.

Angles (equal-leg): $S = (660b - 21)/w$, where b = width of each leg, in.

Angles (unequal-leg): $S = 3220b/w$, where b = sum of widths of both legs, in.
Z-bars: $S = (528h + 1090)/w$, where h = depth of web (out to out), in.
Plates (both sides): $S = 100/t$, where t = thickness, in.

One gallon of pure red-lead paint (20.3 lb. red lead in 5.62 lb. of linseed oil without turpentine or other thinner) when applied to new steel surfaces will cover from 500 to 700 sq. ft. for the first coat, 650 to 850 sq. ft. for the second coat, and from 800 to 1000 sq. ft. for the third coat; or, for two-coat work, 1 gal. will cover from 300 to 400 sq. ft. and for three-coat work from 215 to 285 sq. ft. (Cloyd M. Chapman, *Eng. Rec.*, Feb. 15, 1913.)

Carbon Paints. Paints containing graphite, carbon black or lampblack are unsafe to use as a priming coat for metal. Because of electrical conductivity which these pigments possess, they are apt to excite corrosion. Their use as second-coaters or top-coaters is to be recommended.

Ship-bottom Paints. Anti-corrosive priming paints made of the rust-inhibitive pigments noted above and topped with quick-drying anti-fouling paints made of shellac dissolved in alcohol and mixed with powdered zinc and the oxides of zinc, iron and mercury, are widely used in the Navy and merchant marine service for the protection of steel vessels.

Paints for Water Carriers. Cast-iron and riveted-steel water carriers are generally heated and dipped in hot preparations of mineral asphalt, coal tar and lime, or special baking enamels. This treatment is occasionally followed by baking at a high temperature to make the coating more resistant to abrasion.

Paints for Tinned and Galvanized Surfaces. After wiping off the greasy surface with benzine, a linseed-oil paint made of red lead, iron oxide and chromate pigments, thinned with a little varnish. gives the best results. Applying a 5 per cent. water solution of copper nitrate and sal ammoniac to galvanised iron previous to painting is resorted to when peeling is feared.

Rubber Paints. Paints made of caoutchouc, gutta percha, etc., dissolved in acetone, carbon bisulphide and benzol, form elastic films which give protection to metal for a short time. Upon weathering, the films soon show pinholing and brittleness.

Metal Lacquers. Amyl and ethyl acetate solutions of nitrated cotton are used as dipping lacquers for small metallic art objects. These lacquers dry rapidly and produce excellent and clear films. When brushed on, they are apt to show white brush marks. A new material called "**Bakelite,**" which is a condensation product of phenol and formaldehyde, is the base of some excellent clear lacquers and plastics that possess great preservative properties.

Varnishes. Gums melted into oils and thinned with turpentine form oleo-resinous varnishes. For exterior purposes a "long "oil varnish (one that contains a high percentage of oil) is superior. For interior use, a "short" oil varnish gives a highly lustrous surface that is easily rubbed or washed. High-grade varnishes should be judged comparatively by their body, working properties, flowing, clearness, gloss, and durability, Practical exposure **tests** are best suited to determine the latter point. Hardness of film, resistance to scraping, elasticity when rolled with a knife blade, and resistance to moisture, are some of the quicker tests used. Setting in from 6 to 8 hr. and hardening in 24 hr. are the limits sometimes specified. Gilsonite, elaterite, and other **asphalts** are melted into oils to form black, lustrous asphalt varnishes that are used for protection of water tanks, piping, etc., with great success. Clearness when flowed on glass, miscibility, rapid drying, and moderate flash point, are characteristics of the best asphalt varnishes. Hard, flexible, clear, and rapid-drying **insulating varnishes** are used extensively for dipping and brushing commutator rings, transformer coils and other electrical equipment. **Baking varnishes** for armature coils must be resistant to heat without blistering. **Shellac** (see p. 643) and other spirit varnishes, as well as Bakelite compounds, are used extensively for such purposes.

Enamel Coatings. Metal fittings that are to be permanently protected are often coated and baked with a mixture of feldspar, borax, tin and lead oxides, and other similar materials. The enamel coating produced by this method is very resistant to corrosive agencies. It is, however, subject to cracking from expansion or contraction of the metal upon which it is placed.

Concrete Coatings. Paints made of treated China wood oil mixed with hard, abrasion-resisting pigments, such as zinc oxide and barium sulphate, are finding a wide application as protective coatings for cement and concrete. They have given excellent results even when applied to relatively fresh surfaces. Better results are obtained however if the surfaces are first treated with zinc sulphate solution.

Macnichol Treatment for Cement Surfaces. This treatment consists of the application of a 25 per cent. solution of zinc sulphate to the surface of cement or concrete that is to be painted. The lime-neutralizing and void-filling properties of the zinc sulphate are sufficient to produce an excellent surface that will receive ordinary linseed-oil paints without showing signs of disintegration.

Paint-destroying Agencies. Atmospheres containing sulphurous, acid, or ammoniacal gases are very severe in their action upon paint coatings. Sea air is also very destructive in its effect upon paint. Saline drippings from underground tunnels, and fatty acids from machinery are also to be avoided. In dry climates and in communities of little industrial activity, paint coatings last for several years without decay.

CEMENT, MORTAR AND CONCRETE

BY

SANFORD E. THOMPSON

REFERENCES: Taylor and Thompson, "Concrete, Plain and Reinforced," Wiley. Sabin, "Cement and Concrete," McGraw-Hill. Taylor, "Practical Cement Testing," Myron C. Clark Pub. Co. Baker, "A Treatise on Masonry Construction," Wiley.

CEMENT, LIME, SAND, ETC.

Portland Cement is used for concrete, reinforced concrete, and, either with or without lime, for mortar and stucco. It is made from a mixture of about 80 per cent. of carbonate of lime (limestone, chalk, or marl), with about 20 per cent. of clay (in the form of clay, shale, or slag). After being intimately mixed, the materials are finely ground by a wet or dry process and then calcined in kilns to a clinker. When cool, this clinker is ground to a fine powder. A typical chemical analysis shows the following percentage composition: Silica (SiO_2), 21.31; alumina (Al_2O_3), 6.89; iron oxide (Fe_2O_3), 2.53; calcium oxide (CaO), 62.89; magnesium oxide (MgO), 2.64; sulphuric oxide (SO_3), 1.34; other constituents, 0.75; loss on ignition, 1.39.

White Cement is used for architectural and ornamental work because of its white color. It is of comparatively recent manufacture, is high in alumina, and low in iron. The best brands are true Portlands in composition.

Natural Cement is used, either with or without lime, for common mortar for brick or stone work. It is manufactured from limestone containing clay. The chemical constituents are similar to those for Portland cement, but it is made from the rock just as it comes from the quarry, so that its chemical composition varies with the composition of the rock in the ledge. The natural rock is fed directly to the kilns without being ground, is calcined at a temperature much lower than that required for Portland cement, and then crushed and ground.

Puzzolan or Slag Cement is used under certain conditions for concrete not exposed to the air. It is made by mixing and grinding together slaked lime with granulated blast-furnace slag or other puzzolanic material like natural lava, without burning. The ancient cements were mixtures of lime and volcanic material.

Portland Cement Tests. Cement should be tested for all but very small and unimportant work. Tests should be made in accordance with methods recommended by the Committee on Uniform Tests of Cement of the American Society of Civil Engineers, and the cement should be required to pass the standard specifications of the American Society for Testing Materials. Samples should be taken at random from sound packages, one from every 10 barrels or 40 bags, and mixed. The total samples should weigh about 10 lb. The **Standard requirements** for Portland cement in brief are as follows (the corresponding figures for natural cement are added in parentheses):

Chemical Limits: The following limits shall not be exceeded: Loss on ignition, 4 per cent.; insoluble residue, 0.85 per cent.; sulfuric anhydride (SO_3), 2 per cent.; magnesia (MgO), 5 per cent.

Specific Gravity: Not less than 3.1 (2.8); White Portland, 3.07.

Fineness: Residue by weight on a No. 200 sieve not more than 22 (30) per cent.

Soundness: A pat of neat cement about 3 in. in diameter and ½ in. in thickness, (1) shall remain firm and hard after 24 hr. storage in moist air, and (2) shall show no signs of distortion, cracking, checking or disintegration when exposed for 5 hr. to steam above boiling water in a loosely closed vessel.

Time of Setting: Initial set not less than 45 (10) min. when Vicat needle is used or 60 min. when Gilmore needle is used; final set not more than 10 (3) hr.

Tensile Strength: Minimum requirements for average tensile strength of not less than three briquettes, 1 sq. in. in cross-section, composed of 1 part cement to 3 parts standard sand, by weight:

Age of Test, days	Storage of Test Pieces	Tensile Strength, lb. per sq. in.
7	1 day in moist air, 6 days in water.	200 (25–75)
28	1 day in moist air, 27 days in water.	300 (75–150)

Common Lime, or Quicklime, is used for interior plastering and for lime mortar. Mixed with cement, it is used for lime and cement mortar and for stucco. It is not fit for mortar for thick walls because of slow setting qualities, and must never be used under water. It is made by burning pure limestone in kilns about 40 ft. high by 10 ft. in diam. at temperatures ranging from 1400 to 2000 deg. fahr. A typical percentage chemical composition is calcium oxide (CaO), 97; iron oxide (Fe_2O_3), 1.3; silica (SiO_2), 1.0; magnesium oxide (MgO), 0.7. It slakes with water, forming calcium hydrate (CaH_2O_2). With proper addition of water it becomes plastic, and the volume of putty obtained is 2 or 3 times the loose volume of the lime before slaking, while its weight is about 2¼ times the weight of the lime. Plastic lime sets by drying, by crystallization of calcic hydrate, and by absorbing carbonic acid from the air. The process of hardening is very slow.

Magnesium Lime is used for the same purposes as common lime. It contains magnesium oxide up to 40 per cent. In slaking, it evolves less heat, expands less, and sets more rapidly than pure lime.

Hydrated Lime is a finely divided white powder manufactured by slaking quicklime with the requisite amount of water. It is used because it does not contain unslaked particles of lime.

Sand to be used for mortar and concrete should be preferably of siliceous material, clean, with grains either coarse or else graded from fine to coarse, free from loam and other deleterious matter. The specific gravity of dry sand may be taken ordinarily as 1.65. The physical condition of sand is more important than the chemical composition, except that vegetable or organic impurities are harmful. A quantity of vegetable matter so small that it cannot be detected by the eye, may render a sand absolutely unfit for use with Portland cement.

Test of Sand. Sand should always be tested for use in important concrete structures. The strength of concrete depends to a great degree upon the quality of the sand and the coarseness and relative coarseness of the grains. Sand or other fine aggregate when made into a mortar of 1 part Portland cement to 3 parts fine aggregate by weight should show a tensile strength at least equal to the strength of 1 : 3 mortar of the same consistency made with the same cement and standard sand. If the aggregate is of poor quality, then the proportion of cement in the mortar should be increased to secure the desired strength. If the strength is less than 70 per cent. of that of Ottawa sand mortar, the aggregate should be rejected. The standard Ottawa sand will pass a No. 20 sieve and be retained on a No. 30 sieve. This sand is supplied by the Ottawa Silica Co., Ottawa, Ill.

Broken Stone and Gravel. Coarse aggregate for concrete may consist of broken stone or gravel. The particles should be clean, hard, durable, and free from deleterious matter and range in size from ¼ in. in diam. up to the coarsest size permissible for the structure. For reinforced concrete and small masses of unreinforced concrete, the maximum size is governed by that which will readily pass around the reinforcement and fill all parts of the forms. Either 1-in. or 1½-in. diam. is apt to be the maximum. For heavy mass work the maximum size may run up to 3 in. or larger.

MORTARS

Lime Mortar consists of lime paste and sand, and is made by slaking lime and mixing with sand or else by making up lime putty in advance to be mixed with sand when needed. Either the mortar or the putty should stand for 10 days or 2 weeks to thoroughly slake. Proportions customarily stated are 2.5 to 3 volumes of sand to 1 volume of lime before slaking. In practice, the proportion of sand is apt to be greater than this, as the mortar man determines the proportion by the way the mortar falls from his hoe in mixing. A large proportion of sand is advantageous. According to Taylor and Thompson, the **volume of lime mortar** made with different proportions of sand is as follows:

One 200-lb. cask of lime and 6 cu. ft. of sand will make 10.2 cu. ft. of 1 : 2 mortar.
One 200-lb. cask of lime and 9 cu. ft. of sand will make 12.2 cu. ft. of 1 : 3 mortar.
One 200-lb. cask of lime and 12 cu. ft. of sand will make 14.5 cu. ft. of 1 : 4 mortar.
One 200-lb. cask of lime and 15 cu. ft. of sand will make 17.1 cu. ft. of 1 : 5 mortar.

Lime and Cement Mortar is made by mixing 1 part of cement with 3 parts of lime mortar having a large proportion of sand. The lime mortar should be made up several days before cement is added. Only small quantities of cement should be mixed at a time, so that there will be no danger of the cement attaining a set before the mortar is used.

Cement Mortar is composed of cement and sand or screenings mixed with water. Proportions in practice are ordinarily 1 part cement to 2 parts sand (1:2), or 1 part cement to 3 parts sand (1:3). Table 1 gives the quantities of sand and cement required for 1 cu. yd. of cement mortar. The **strength** of Portland cement mortar (1) increases with the proportion of cement, (2) in general increases with the coarseness of the sand, and (3) increases with the density of the mortar. With the same aggregate, the strongest and most impermeable mortar is that containing the largest percentage of cement in a given volume of mortar. With the same percentage of cement, the strongest mortar (provided the sand is absolutely clean) is that which has the greatest density, that is, which in unit volume of mass has the largest percentage of solid materials. A small addition of lime, not exceeding 10 per cent., is apt to increase the strength of a lean, say a 1 : 3 or 1 : 4 mortar, and to slightly decrease the strength of a richer mortar. Unslaked lime or imperfectly slaked lime must never be used in mortar either alone or with cement. Even in as small an amount as 2 per cent. it may seriously reduce the strength of mortar because it is liable to produce expansion in the masonry by slaking after the mortar is in place. Table 2 shows the effect of different sizes of grains upon the strength of mortar.

Table 1. Volume of Compacted Plastic Portland Cement Mortar and Quantities of Materials per Cubic Yard*

(Taylor and Thompson, p. 229)

Relative proportions by volume		Volume		Materials for 1 cu. yd., based on barrel of 4 bags or 4 cu. ft.		Relative proportions by volume		Volume		Materials for 1 cu. yd., based on barrel of 4 bags or 4 cu. ft.	
Cement.†	Sand‡	Cubic feet from 1 bag of cement weighing 94 lb.	Cubic feet from 1 bbl. (4 bags) cement containing 4 cu. ft.	Packed cement, barrels	Loose sand, cubic yards	Cement†	Sand‡	Cubic feet from 1 bag of cement weighing 94 lb.	Cubic feet from 1 bbl. (4 bags) cement containing 4 cu. ft.	Packed cement, barrels	Loose sand, cubic yards
1	0	0.80	3.2	8.31	0.0	1	4½	3.91	15.6	1.72	1.15
1	½	1.02	4.1	6.61	0.49	1	5	4.28	17.1	1.58	1.17
1	1	1.38	5.5	4.88	0.72	1	5½	4.64	18.5	1.46	1.19
1	1½	1.74	7.0	3.87	0.86	1	6	5.00	20.0	1.35	1.20
1	2	2.11	8.4	3.21	0.95	1	6½	5.36	21.4	1.26	1.21
1	2½	2.47	9.9	2.74	1.01	1	7	5.72	22.9	1.18	1.22
1	3	2.83	11.3	2.39	1.06	1	7½	6.08	24.3	81.11	1.23
1	3½	3.19	12.8	2.12	1.10	1	8	6.44	25.8	1.05	1.24
1	4	3.55	14.2	1.90	1.13	......	......	......	......	......	......

* Variations in the fineness of the sand and cement and in the consistency of the mortar may affect the values by 10 per cent. in either direction.

† Cement as packed by manufacturer (4 bags = 1 bbl.). ‡ Coarse bank sand measured loose.

Table 2. Tests by New York Board of Water Supply of 1 : 3 Mortar Made with Sands of Different Mechanical Analyses

(Taylor and Thompson, p. 159.)

Percentages passing sieves				Tensile test, lb. per sq. in.		Compression test, lb. per sq. in.	
No. 4	No. 8	No. 50	No. 100	7 days	90 days	7 days	90 days
100	70	12	5	213	613	2690	5640
100	86	21	6	263	412	1915	4660
100	99	26	2	177	325	905	2170
100	97	28	6	178	282	1070	1500
100	94	44	12	139	228	905	1130
100	100	52	14	122	170	275	810
100	100	94	48	80	149	330	490

Mortar for Stone Work. Lime mortar should not be used for thick walls, as it hardens very slowly; for masonry under water, or in wet soil; nor in structures requiring great strength or which are subject to shock. Lime and cement mortar may be used for masonry above water. The amount of mortar required for a cubic yard of masonry is given in Table 3.

Table 3. Amount of Mortar Required for a Cubic Yard of Stone Masonry
(From Baker's "Treatise on Masonry Construction")

Description of masonry	Mortar, cu. yd.	
	Min.	Max.
Ashlar, 18-in. courses and ¼-in. joints	0.03	0.04
Ashlar, 12-in. courses and ¼-in. joints	0.06	0.08
Rubble, small, rough stones	0.33	0.40
Rubble, large stones, rough hammer-dressed	0.20	0.30
Squared stone masonry, 18-in. courses and ¾-in. joints	0.12	0.15
Squared stone masonry, 12-in. courses and ¾-in. joints	0.20	0.25

Mortar for Brickwork may be made with pure Portland cement and sand, natural cement and sand, lime and sand, or mixtures of lime, cement, and sand, according to the character of the work. For ordinary brickwork, mortars of lime and cement are used; for water-tight brickwork, Portland cement mortar without lime. The amount of mortar required in laying brickwork is given in Table 4.

Table 4. Amount of Mortar Required for Brickwork
(From investigations of Taylor and Thompson)

Mortar for common brick (20 brick per cu. ft.,* size 8¼ × 4 × 2¼ in., with ⅜-in. joints)			Mortar for selected face brick† (8.8 brick per sq. ft., size 7¾ × 3½ × 2¼-in., with 5/16-in. joints)			Mortar for pressed face brick‡ (7½ brick per sq. ft., size 8¼ × 4 × 2 5/16 in., with ¼-in. joints)		
Thickness of joints, in.	Cu. ft. per cu. ft. of masonry	Cu. ft. per 1000 brick	Thickness of joints, in.	Cu. ft. per sq. ft. of surface	Cu. ft. per 1000 brick	Thickness of joints, in.	Cu. ft. per sq. ft. of surface	Cu. ft. per 1000 brick
½	0.37	19.4	⅜	0.07	8.4	¼	0.05	6.8
⅜	0.28	14.0	5/16	0.06	6.8	3/16	0.035	4.7
¼	0.19	9.0	¼	0.05	5.6	⅛	0.02	2.6

* Actual brick laid in wall. For figuring quantities use 22½ brick per cu. ft.
† Laid with headers every 6th course. ‡ Laid running bond (all stretchers).

Mortars for Plastering. Mortar for **interior plastering** consists of lime, clean coarse sand, and cattle hair or fiber. To get best results slake the lime thoroughly, run it through a sieve, if not pure, and allow to stand for at least 24 hr., or, better yet, up to 7 days. Then mix the hair thoroughly with the lime, add sand, and throw in a pile. After at least 7 days, add water and apply immediately on the lathing. The mortar should be protected from freezing. Patented mortars also are largely used. One hundred pounds of lime will make about 3½ cu. ft. of lime putty or paste. The volume of a lime barrel is about 3 cu. ft. **Back plaster** is a coat of mortar placed usually on laths fastened to the boarding between the studs. **Scratch coat** consists of mortar of lime, sand and considerable hair—about 2 to 2½ bu. to one 200-lb. cask of dry lime. It is put directly on the laths. **Brown coat** consists of mortar with less hair, about 1 to 1½ bu. per 200-lb. cask of dry lime. When 3-coat work is used, this is troweled directly on to the scratch coat. **Skim coat** is a finish coat composed of lime putty and fine white sand. It is spread on to the scratch coat in 2-coat work or on to the brown coat in 3-coat work. It is placed in two layers and troweled to a hard finish. **Gaged skim coat** is skimming mixed with a certain amount of plaster of Paris, which makes it practically a hard finish. **Hard finish** consists of 1 part of lime putty to 1 or 2 parts of plaster of Paris. Certain patented plasters are also used for hard finish and are preferred because of uniform composition. **Two-coat work** is used for the cheaper class of houses for plastering on wood laths. For the first coat, the mortar, which contains considerable hair, is spread on in one layer usually on the walls and two layers on the ceiling, smoothed with a darby (a board with two handles) and, when nearly dry, floated with a wooden hand float. The finish coat may be of skim or hard finish. **Three-coat work** has a first or scratch coat of mortar spread on and scratched when nearly dry. The second coat or brown coat consists of mortar with less hair, smoothed off, darbied and floated with a wood float. This is then covered for the third coat with skim or hard finish. The first coat on metal or wire lathing for three-coat work requires a fatter mortar than laths and with less hair in it. Three-fourths-inch grounds or rails are generally used. The total thickness for three-coat work is therefore from ¾ in. to ⅞ in. **Stucco** is used for **exterior plastering**, and is applied to brick or stone or plastered on to wood or metal lath. The first coat may consist of 5 parts Portland cement, 3 parts slaked lime putty, a small amount of hair, and 12 parts clean coarse sand. The second or finish coat may consist of Portland cement mortar with a small quantity of lime paste. For covering wooden buildings, the stucco is plastered either on wood lath or on metal lath in two coats, using mortar similar to that for brick or stone. Concrete in northern climates exposed to frost should never be plastered, but should be finished by rubbing down with carborundum brick or similar tool when the surface is comparatively green; it may also be tooled in various ways.

CONCRETE

Concrete is made by mixing cement and an aggregate composed of hard, inert particles of varying size, such as a combination of sand or broken stone screenings, with gravel, broken stone, cinders, broken brick, or other material. Portland cement should always be used for reinforced concrete, for mass concrete subjected to stress and for all concrete laid under water.

Proportioning Concrete. The object in proportioning concrete is to produce a mix of the greatest possible density, since this with a given quantity of cement produces the highest strength. The proportions to be selected vary with the character of the aggregates and with the use to which the concrete is to be put. Proportions are determined for any special condition in various ways: (1) Arbitrary selection, such as 1 part cement, 2 parts sand, 4 parts stone (written 1:2:4). (2) Determining the voids in stone and selecting proper proportions for mortar by test or judgment; then using mortar slightly in excess of voids in stone. (3) Making trial mixtures of concrete with a given percentage of cement and different aggregates and selecting the mixture producing the smallest volume of concrete. (4) Mixing the cement and aggregate by methods of mechanical analysis curves. The method to select depends upon the conditions. For small structures the proportions may be arbitrarily selected, while methods (3) or (4) should be employed to obtain the most economical results on large constructions.

Proportions for Different Structures. The following four mixtures will serve as a rough guide to the selection of proper proportions for various classes of work (Taylor and Thompson).

(*a*) **Rich Mixture,** for columns and other structural parts subjected to high stresses or requiring exceptional water-tightness. Proportions, 1 : 1½ : 3.

(*b*) **Standard Mixture,** for reinforced floors, beams and columns, for arches, for reinforced engine or machine foundations subject to vibrations, for tanks, sewers, conduits, and other water-tight work. Proportions, 1 : 2 : 4.

(*c*) **Medium Mixture,** for ordinary machine foundations, retaining walls, abutments, piers, thin foundation walls, building walls, ordinary floors, sidewalks, and sewers with heavy walls. Proportions, 1 : 2½ : 5.

(*d*) **Lean Mixture,** for unimportant work in masses, for heavy walls, for large foundations supporting a stationary load, and for backing for stone masonry. Proportions, 1:3:6.

The above specifications give fair average practice. If the aggregate is carefully graded and the proportions are scientifically fixed, smaller proportions of cement may be used for each class of work.

Quantities of Materials. The volume of concrete obtained by a mixture of certain volumes of aggregates is much smaller than the total volume of the aggregates taken separately, since the finer materials (including the cement) enter more or less into the voids of the coarse aggregate. Table 5 gives for different proportions the volume of cement and aggregates necessary to produce 1 cu. yd. of concrete. The table is based on volumes of a barrel of cement equivalent to 4 cu. ft. The percentages of voids assumed in the broken stone or gravel are given at the head of the table.

Table 5. Quantities of Material for 1 Cu. Yd. of Rammed Concrete

(Taylor and Thompson. Based on a barrel of 4 cu. ft.)

Proportions by parts			Volume of mortar in terms of percentage of volume of stone	Broken stone screened to uniform size, voids 50 %			Average conditions, broken stone with dust screened out, voids 45 %			Gravel or mixed stone and gravel, voids 40 %			Scientifically graded mixtures, voids 30 %			Scientifically graded mixtures, voids 20 %		
Cement	Sand	Stone	%	Cement, bbl.	Sand, cu. yd.	Stone, cu. yd.	Cement, bbl.	Sand, cu. yd.	Stone, cu. yd.	Cement, bbl.	Sand, cu. yd.	Stone, cu. yd.	Cement, bbl.	Sand, cu. yd.	Stone, cu. yd.	Cement, bbl.	Sand, cu. yd.	Stone, cu. yd.
1	1	1½	96	3.08	0.46	0.68	2.97	0.44	0.66	2.87	0.42	0.64	2.69	0.40	0.60	2.53	0.38	0.56
1	1	2	73	2.74	0.41	0.81	2.63	0.39	0.78	2.52	0.37	0.75	2.33	0.34	0.69	2.17	0.32	0.64
1	1	2½	59	2.47	0.37	0.91	2.35	0.35	0.87	2.25	0.33	0.83	2.06	0.31	0.76	1.90	0.28	0.71
1	1	3	50	2.25	0.33	1.00	2.13	0.32	0.95	2.03	0.30	0.90	1.85	0.27	0.82	1.70	0.25	0.76
1	1½	2	92	2.39	0.53	0.71	2.30	0.51	0.68	2.22	0.49	0.66	2.07	0.46	0.61	1.94	0.43	0.58
1	1½	3	62	2.01	0.45	0.89	1.91	0.42	0.85	1.83	0.41	0.81	1.68	0.37	0.75	1.56	0.35	0.69
1	1½	4	48	1.73	0.38	1.03	1.64	0.36	0.97	1.56	0.35	0.92	1.42	0.32	0.84	1.30	0.29	0.77
1	1½	5	39	1.52	0.34	1.13	1.43	0.32	1.06	1.35	0.30	1.00	1.22	0.27	0.90	1.11	0.25	0.82
1	2	3	74	1.81	0.54	0.80	1.74	0.52	0.77	1.67	0.50	0.74	1.54	0.46	0.68	1.44	0.43	0.64
1	2	4	56	1.58	0.47	0.94	1.51	0.45	0.89	1.44	0.43	0.85	1.32	0.39	0.78	1.21	0.36	0.72
1	2	5	46	1.40	0.42	1.04	1.33	0.39	0.98	1.26	0.37	0.93	1.15	0.34	0.85	1.05	0.31	0.78
1	2	6	39	1.26	0.37	1.12	1.19	0.35	1.06	1.13	0.34	1.00	1.02	0.30	0.91	0.93	0.28	0.83
1	2½	3	86	1.65	0.61	0.73	1.59	0.59	0.71	1.53	0.57	0.68	1.42	0.52	0.63	1.33	0.49	0.59
1	2½	4	66	1.46	0.54	0.87	1.39	0.51	0.82	1.33	0.49	0.79	1.23	0.46	0.73	1.14	0.42	0.68
1	2½	5	54	1.31	0.48	0.97	1.24	0.46	0.92	1.18	0.44	0.87	1.08	0.40	0.80	0.99	0.37	0.73
1	2½	6	45	1.18	0.44	1.05	1.12	0.41	1.00	1.06	0.39	0.94	0.96	0.36	0.85	0.88	0.33	0.78
1	2½	7	39	1.08	0.40	1.12	1.02	0.38	1.06	0.96	0.36	1.00	0.87	0.32	0.90	0.79	0.29	0.82
1	3	4	75	1.35	0.60	0.80	1.30	0.58	0.77	1.25	0.56	0.74	1.15	0.51	0.68	1.08	0.48	0.64
1	3	5	60	1.22	0.54	0.90	1.16	0.52	0.86	1.11	0.49	0.82	1.02	0.45	0.76	0.94	0.42	0.70
1	3	6	50	1.11	0.49	0.99	1.06	0.47	0.94	1.01	0.45	0.90	0.92	0.41	0.82	0.84	0.37	0.75
1	3	7	44	1.02	0.45	1.06	0.97	0.43	1.01	0.92	0.41	0.95	0.83	0.37	0.86	0.76	0.34	0.79
1	3	8	39	0.94	0.42	1.11	0.89	0.40	1.05	0.84	0.37	1.00	0.76	0.34	0.90	0.69	0.31	0.82
1	5	10	47	0.70	0.52	1.04	0.66	0.49	0.98	0.63	0.47	0.93	0.57	0.42	0.84	0.52	0.38	0.77
1	6	12	46	0.59	0.52	1.05	0.56	0.50	1.00	0.53	0.47	0.94	0.48	0.43	0.85	0.44	0.39	0.78

Mixing. In order to get good concrete, the cement and aggregates must be thoroughly mixed so as to distribute evenly the cement and sand throughout the whole mass of concrete. Mixing may be done either by hand or machine. Concrete is prepared by **hand mixing** for small jobs or where only a small amount of concrete is deposited in one place. The mixing must be done thoroughly on a water-tight mixing platform. A good method of mixing concrete is as follows: The sand is measured and spread on the platform, then a proper amount of cement placed on the top, and the two materials mixed until the color is uniform. In the meantime, screened gravel or stone is placed in measuring box on the platform. After removing the box, the gravel is hollowed out slightly in the center and sand and cement are shoveled on top, covering the gravel with a layer of even thickness. A definite number of pails of water is poured on the top of these layers, and then the whole mass is mixed by shovels, each man lifting a shovelful of material, turning it, and spreading it on the platform about 2 ft. from its original position. The operation is repeated several times. As a rule, three turnings are sufficient. **Machine-mixed concrete** is employed almost universally on important work. A good concrete mixer not only stirs the mass, which undisturbed would tend to separate into light and heavy particles, but cuts it again and again, and repeatedly transfers the material from one part of the machine to another and in that way makes the product homogeneous.

Concrete Mixers are of two classes: (1) Continuous mixers, into which material is dumped continuously; and (2) batch mixers. Batch mixers are apt to give more uniform results because it is easier to supply the materials in the same proportions. Continuous and batch mixers may be classified in three general types: (a) **Rotating mixers**, consisting of a revolving drum or of a square box revolving about its diagonal axis. Some of these are provided with deflectors and blades which make the mixing more thorough; (b) **Paddle mixers**, consisting of a stationary box with movable paddles which perform the mixing; (c) **Gravity mixers**, which consist either of a steel trough of sufficient length with blades and deflectors, where materials are mixed by falling through the trough and striking the obstructions in their descent in the machine, or a number of boxes placed one above the other so that the materials become mixed in the passage from one box to another. The mixers are placed in many cases under the storage bin, so that the materials are directly discharged into them. In choosing between power and gravity mixers, it must be remembered that the only saving effected by using the gravity mixer is the cost of the power, and this saving may be overbalanced by the poorer quality of the concrete obtained. Rotating mixers are more generally used than those of the paddle type.

Consistency of Concrete. The consistency to be used depends upon the character of the structure. **Medium or quaking concrete** is adapted for ordinary mass concrete such as foundations, heavy walls, large arches, piers, and abutments. **Mushy concrete** is suitable for rubble concrete and for reinforced concrete, such as thin building walls, columns, floors, conduits, and tanks. **Dry concrete** may be employed in dry locations for mass foundations which must withstand severe compressive strain within one month after placing, providing it is carefully spread in layers not over 6 in. thick and is thoroughly rammed. The term "medium" or "quaking" mixture applies to a tenacious, jelly-like consistency which shakes on ramming. A "mushy" mixture will settle to a level surface when dumped in a pile and will flow very sluggishly into the forms or around the reinforcing bars; a "dry" mixture has the consistency of damp earth. A very wet mixture is much weaker than a quaking or a mushy mixture, the quality of the cement being frequently injured by an excess of water.

Weight of Concrete. The following are average weights per cubic foot of Portland cement concrete:

Cinder concrete	112 lb.	Limestone concrete	148 lb.
Conglomerate concrete	150 lb.	Sandstone concrete	143 lb.
Gravel concrete	150 lb.	Trap concrete	155 lb.

Loose unrammed concrete is from 5 per cent. to 25 per cent. lighter than concrete in place, varying with the consistency.

Strength of Concrete

The strength of concrete increases (1) with the quantity of cement in a unit volume; (2) with the density of the concrete; (3) with the size of the coarsest aggregate. Angular aggregates, like broken stone, produce slightly stronger concrete than rounded gravel. Specially graded mixtures of aggregates produce concrete of higher strength. Strength is decreased by an excess of sand over that required to fill the voids in the stone.

Compressive Strength of Concrete. Table 6 gives the results of tests with first-class materials and under first-class conditions. Growth in strength with age depends in a large measure upon the consistency. For medium consistency, the strength at 6 months is apt to be one-third greater than at 28 days. With wet consistency the strength is usually 50 per cent. greater at 6 months than at 30 days.

Table 6. Compressive Strengths of Different Mixtures of Concrete, Lb. per Sq. In.

(From Report of Joint Committee on Concrete and Reinforced Concrete)

Aggregate	Proportions by parts				
	1 : 1 : 2	1 : 1½ : 3	1 : 2 : 4	1 : 2½ : 5	1 : 3 : 6
Granite, trap rock	3,300	2,800	2,200	1,800	1,400
Gravel, hard limestone and hard sandstone	3,000	2,500	2,000	1,600	1,300
Soft limestone and sandstone	2,200	1,800	1,500	1,200	1,000
Cinders	800	700	600	500	400

Transverse Strength of Plain Concrete. The tensile strength of concrete is of less importance than the crushing strength, as the tensile strength of concrete is seldom relied upon and beams are built with steel reinforcement placed in the tensile part of the beam. There is no fixed relation between the tensile fiber stress of concrete beams and the crushing strength. The growth of strength is different in the two classes of strength. Experiments by the author comparing 8-in. cubes and 8-in. beams of 1 : 2½ : 5 concrete give a ratio of crushing strength to modulus of rupture at 1 and 2 months of 6 to 1.

Strength of Concrete in Direct Shear is from 60 to 80 per cent. of the compressive strength of concrete. The direct shear must not be confounded with shear in a beam involving diagonal tension where the concrete may break with a shearing stress equal to 5 to 10 per cent. of its crushing strength.

Water-tightness. Concrete can be made practically impervious to water by proper proportioning and mixing and placing. Leakage through concrete walls is usually due to poor workmanship or occurs at the joints between two days' work or through cracks formed by contraction. New concrete may be bonded to old by wetting the old surface, plastering it with neat cement, and then placing the concrete before the neat cement has set. Contraction cracks are almost impossible to prevent entirely, although a sufficient amount of reinforcement may reduce their width so as to permit only seepage of water. To get the best results, either a quaking or wet consistency should be used; the concrete must be placed carefully so as to leave no visible stone pockets; the entire structure should be made without joints and preferably in one continuous operation.

For maximum water-tightness, mortar and concrete may require more fine material than would be used for maximum strength. Gravel produces more water-tight concrete than broken stone under similar conditions. Pure **clay** finely powdered used in proportions up to 5 per cent. of the weight of sand has been found to appreciably increase the water-tightness of concrete. **Lime** or puzzolan cement added to concrete increases its water-tightness, but decreases its strength. **Patented compounds** are on the market for producing water-tight concrete, but under most conditions results as good may be obtained for less cost by increasing the percentage of cement in the mix. **Membrane waterproofing,** consisting of asphalt or tar with layers of felt or tarred paper with asphalt or tar, is advisable in certain cases. **Mortar** troweled on very hard by a plan similar to that followed in the hydrolithic process, may produce water-tight work.

According to Fuller and Thompson (*Trans. Am. Soc. C. E.*, vol. 59, p. 67), **water-tightness increases** (1) as the percentage of cement is increased and in a very much larger ratio; (2) as the maximum size of stone is increased, provided the mixture is homogeneous; (3) materially with age; and (4) with thickness of the concrete, but in a much larger ratio. It **decreases** uniformly with increase in pressure.

Protection of Metal from Rust and Fire. Reinforcement properly imbedded in dense concrete is protected against rust and against damage by fire. According to tests made by Prof. Charles Norton in America and Probst in Germany, dense concrete not only protects clean steel against rust, but also checks the progress of rusting in rusted steel imbedded in concrete. Small invisible cracks which form in the tensile part of a loaded reinforced beam or slab do not lower appreciably the protective qualities of concrete. On the other hand, with a porous concrete or one which contains serious structural cracks, moisture may penetrate and seriously rust the steel reinforcement. (See also p. 560.)

Freezing retards the setting and hardening of Portland cement concrete and lowers its strength at short periods, but the ultimate strength is but little affected. On exposed surfaces, such as walls and sidewalks, placed in freezing weather, a thin scale is apt to crack from the surface. The setting can be hastened and the action of frost retarded by heating the materials and also by adding salt in quantities up to 10 per cent. of the weight of water. Salt lowers the freezing point of water and its addition does not affect the strength of the concrete, although its use in reinforced concrete is somewhat questionable. Calcium chloride, using 2 per cent. of the weight of the cement, also retards freezing. Natural cement is completely ruined by freezing. Concreting in freezing weather should be avoided if possible; if unavoidable, precautions must be taken to prevent freezing of the concrete.

Sand vs. Broken-stone Screenings. Sand may often be replaced by broken-stone screenings, which will give satisfactory results provided the quantity of dust is not excessive. For most purposes not more than 10 per cent. should pass a No. 100 screen. For floors and sidewalks, the dust must be entirely removed.

Effect of Mica and Clay on Sand. Mica in sand, if over 2 per cent., reduces the density of mortar and consequently the strength. In crushed-stone screenings the effect of the same percentage of mica in the natural state is less marked. Black mica, which has a different crystalline form, is not injurious to mortar. Clay in sand may be injurious because of introducing too much fine material or forming balls in the concrete. When not excessive in quantity, it may increase the strength and water-tightness of a mortar of proportions 1 : 3 or leaner.

Effect of Regaging Mortar or Concrete. Although laboratory tests show that the ultimate strength of Portland cement mortars is not lowered by regaging, it is not permissible in practice to use mortar or concrete that has been regaged after beginning to set. The hardening is greatly retarded and in cases where moisture cannot reach it, it may never harden.

Effect of Oils, Acids, Manure and Electrolytic Action on Mortar and Concrete. Certain tests indicate increased water-tightness from the introduction of oil in cement or mortar. Mineral **oils** externally applied do not injure concrete. Animal fats and vegetable oils, however, tend to disintegrate concrete unless it has thoroughly hardened. Concrete after it has thoroughly hardened resists the attack of diluted **acids**, but is disintegrated by strong acids. Green concrete is injured by **manure**, but is not affected after it has thoroughly hardened. **Electrolysis** injures concrete under certain conditions (see p. 560), and electric currents should be prevented from reaching it.

Effect of Sea Water. Sea water attacks cement and disintegrates concrete unless it is made with the very best materials under the best possible conditions. Deleterious action is greatly accelerated by frost. To prevent serious damage, the concrete must be made with a rich mix (not leaner than 1 : 2 : 4), and with exceptionally good aggregates including a coarse sand, and must be allowed to thoroughly harden before it is touched by the sea water. Tests indicate that there is no essential difference in the strength of mortar gaged with fresh and with sea water, although the latter tends to retard the setting to a slight degree.

WOOD

BY

HERMANN VON SCHRENK AND WILLIAM KENDRICK HATT

GENERAL PROPERTIES OF WOOD

BY

HERMANN VON SCHRENK

REFERENCES: Bulletins and Circulars of the U. S. Forest Service. Weiss, "Preservation of Structural Timbers," McGraw-Hill. Record, "Identification of the Economic Woods of the United States," Wiley. Johnson, "Materials of Construction," Wiley.

Definitions and Classification. Timbers are classed commercially as hardwoods and softwoods. Typical **hardwoods** are oak, ash, chestnut, hickory, maple and poplar; typical **softwoods** are pines, hemlock, spruce, larch, fir and cedar. The terms "hard" and "soft" do not necessarily refer to actual hardness (see table below). In all timbers two forms are distinguished, heart and sapwood. **Heartwood** is the inner part of a tree, usually darker in color than **sapwood** (the outer part), frequently heavier, and more decay-resistant. Knots or branch inclusions (loose or solid) are defined as **pin knots** (not over ½-in. in diam)., **standard knots** (not over 1½ in.) **large knots** (more than 1½ in.), **sound knots** (solidly grown together with the surrounding wood), **loose knots** (not held firmly in place by growth or position), **pith knots** (sound knots with pith holes not more than ¼ in. in diam.), **encased knots** (*i.e.*, entirely surrounded by bark or pith), **rotten** (decayed) **knots, spike knots** (knots sawed lengthwise). Recognized **defects** are **wane,** bark or the lack of wood from any cause on the edge; **shakes,** splits or checks in timber, which usually cause a separation of the wood between annual rings; **sap stain,** a discoloration of the sapwood; **pitch pockets,** openings between the grain of the wood, containing more or less pitch; **dote** and **red heart,** various forms of decay.

Timbers are sold under various names. The following list of **standard names of timbers** has been adopted by the Master Car Builders' Association, and in part by the American Railway Engineering Association and the American Society for Testing Materials. The varieties included in each name are given in parentheses.

Ash (white, black, blue, green and red ash); **basswood** (linden, linn, lind or lime-tree); **beech** (red and white beech); **birch** (red, white, yellow and black birch); **buckeye** (wood from the horse-chestnut tree); **butternut** (butternut also known as white walnut); **cherry** (sweet, sour, red, black and wild cherry); **chestnut; cottonwood; cypress** [red, gulf, yellow and East Coast cypress (bald cypress)]; **elm, soft** (white, water, gray, red or slippery and winged elm); **elm, rock** (rock or cork elm); **Douglas fir** (yellow, red, Western, Washington, Oregon, Puget Sound fir or pine, West Coast fir); **gum** (red gum, sweet gum or satin walnut); **hemlock** (Eastern hemlock, *i.e.*, from all states east of and including Minnesota); **Western hemlock** (hemlock from the Pacific Coast); **hickory** (shellbark, kingnut, mockernut, pignut, black, shagbark and bitternut); **Western larch** (larch or tamarack from the Rocky Mountain and Pacific Coast regions); **maple, soft** (soft and white maple); **maple, hard** (hard, red, rock and sugar maple); **white oak** (white, burr or mossy cup, rock, post or iron, overcup, swamp, post, live, chestnut or tan bark, yellow and basket or cow oak); **red oak** (red, pin, black, water, willow, Spanish, scarlet, Turkey, black jack or barn, and shingle or laurel oak); **pecan; Southern yellow pine** (all pines of the Southern States manufactured into lumber, including longleaf, shortleaf, loblolly and Cuban pines. The lumber is divided according to quality into "dense southern yellow pine" and "sound southern yellow pine;" dense southern yellow pine should show on either end an average of at least six annual rings per inch and at least one-third summerwood, or else the greater number of rings should show at least one-third summerwood, all as measured over the third, fourth and fifth inches on a radial line from the pith; wide-ringed material excluded by this rule is acceptable, provided the amount of summerwood, as above measured, is at least one-half; the contrast in the color between summerwood and springwood should be sharp, and the summerwood should be dark in color, except in pieces having considerably above the minimum requirement for summerwood; sound southern yellow pine includes pieces of southern yellow pine without any ring or summerwood requirement); **white pine** (wood from tree of that name grown in Maine, Michigan, Wisconsin, Minnesota and Canada); **Norway pine** (Norway or red pine grown in Michigan, Minnesota, Wisconsin and Canada); **Idaho white pine** (varieties of white pine grown in Western Montana, Northern Idaho and Eastern Washington); **Western pine** (timber known as white pine grown in Arizona, California, New Mexico, Colorado, Oregon and Washington; sometimes known as Western yellow or ponderosa pine, or California white pine or Western white pine); **poplar** (wood from the tulip tree, otherwise known as whitewood, yellow poplar and canary wood); **redwood; spruce** (spruce timber from points east of and including Minnesota and Canada, covering white, red and black spruce); **Western spruce** (spruce timber from the Pacific Coast); **sycamore; tamarack** (tamarack or eastern tamarack, grown in states east of and including Minnesota); **tupelo** (tupelo gum and bay poplar); **walnut** (black walnut).

Physical Properties

Weight and Specific Gravity. The weight of wood will vary with the amount of water contained, and (within a given species) with the age, part of tree from which the wood is cut, geographical location, etc. In general, green wood will contain 50 to 75 per cent., air-dry wood 10 to 20 per cent. water. In general practice, a wood weighing less than 30 lb. per cu. ft. is called light; one between 30 and 40 lb., medium; and one more than 40 lb., heavy. The approximate air-dry weight of any wood is from 10 to 20 per cent. higher than its absolute dry weight. For the **specific gravity** of various woods, see p. 586; for weights per cu. ft., see pp. 454 and 608.

Hardness. According to Schenck ("Forest Utilisation"), dense woods are the harder. Wide rings in oak and narrow rings in pine indicate superior hardness. Heartwood is harder than sapwood, and dry wood generally harder than green wood. Frost increases hardness.

Relative Hardness of Woods

Hard: Hickory, dogwood, sugar maple, sycamore, locust, hornbeam, persimmon.
Medium: Ash, oak, elm, beech, cherry, mulberry, birch, sour gum, longleaf pine.
Soft: Chestnut, tulip tree, sweet gum, Douglas fir, yellow pine, larch, basswood, horse-chestnut, hemlock, cottonwood, spruce.
Very soft: White pine, sugar pine, redwood, willow.

A scale of hardness frequently used is as follows:

Scale of Hardness of Wood

Shellbark hickory	100	White hazel	72	Yellow oak	60	Chestnut	52
Pignut hickory	96	Apple tree	70	Hard maple	56	Yellow poplar	51
White oak	84	Red oak	69	White elm	58	Butternut	43
White ash	77	White beech	65	Red cedar	56	White birch	43
Dogwood	75	Black walnut	65	Wild cherry	55	White pine	30
Scrub oak	73	Black birch	62	Yellow pine	54		

Cleavability is inversely as the resistance to splitting in a lengthwise direction. According to Schenck ("Forest Utilisation"), cleavability is affected by: (*a*) the straightness, length and elasticity of the fiber; (*b*) the heaviness of the medullary rays; (*c*) straightness of growth; (*d*) branchiness; (*e*) moisture (the higher the moisture the easier the wood can be split); (*f*) frost (reduces cleavability); (*g*) hardness.

Relative Cleavability of Woods

Hard to split: Black gum, elm, sycamore, dogwood, beech, holly, maple, birch, hornbeam.
Medium: Oak, ash, larch, cottonwood, linden, yellow poplar, hickory.
Easy to split: Chestnut, pines, spruce, fir, cedar.

Calorific Value. The **specific heat** of practically all kinds of wood when oven-dry is 0.327 (Dunlap, *Forest Service Bulletin* No. 110, 1912). The calorific value of wood depends on its specific gravity (dry), heavier woods giving more heat than light woods. According to Schenck, 1 cord of green wood contains 250 gal. of water, and the heat required to evaporate this into steam is not available for other heating purposes. According to German experiments, wood with 45 per cent. moisture gives only 50 per cent. as much heat as dry wood. Rosin increases the heating power by about 12 per cent. According to Roth, 100 lb. of wood, as sold in wood yards, contains 25 lb. of water, 74 lb. of (dry) wood, and 1 lb. of ashes. Thus, 100 lb. of green wood (50 per cent. moisture) furnish about 270,000 B.t.u., 100 lb. of air-dry wood (10 per cent. moisture) about 580,000 B.t.u., and 100 lb. of kiln-dry wood (2 per cent. moisture) about 630,000 B.t.u.

Relative Values of Woods as Fuels

Best: Hickory, beech, hornbeam, locust, heart pine.
Good: Oak, ash, birch, maple.
Moderate: Spruce, fir, chestnut, hemlock, sap pine.
Poor: White pine, alder, linden, cottonwood.

Strength of Woods. See p. 585.

Analyses of Wood. See p. 609.

Decay and Destruction of Timber

Decay in timber is caused by fungi growing in the wood fiber and is favored by the presence of oxygen, water, heat and food supply. Timber suffering from the forms of decay usually distinguished as dry **rot**, moist rot, wet rot, brown rot, etc., has little practical value. Decayed wood is lighter than sound wood, and loses its strength rapidly as the decay progresses.

Absolutely dry wood will not decay, nor will wood decay when constantly submerged in water, nor when kept more than 4 ft. under ground. Poles and posts decay chiefly at the ground line. Construction timbers in buildings, bridges, etc., decay most rapidly at points where they come in contact either with other timbers, with the ground, or with stone or concrete walls. Any conditions favoring the retention of water in the timbers, particularly where the temperature is from 60 to 85 deg. fahr., will bring about decay. Sapwood of all timbers decays very rapidly, heartwood is usually more resistant. In structural timbers decay frequently appears in the inner sapwood, so as not to be observable from the outside, even by the most careful inspection. This form of decay is usually termed "internal sap rot." Where strength is the principal requirement, the most careful examination of timbers with large amounts of sapwood should be made, particularly if there is evidence that such timbers have been cut from the tree more than 2 or 3 months after felling. This applies particularly to all forms of piling, stringers, posts, caps, etc.

Blue Stain is a grayish-blue discoloration found in the sapwoods of pines and other coniferous woods, due to a minute fungus (*Ceratostomella pilifera*) growing in the wood fiber. It has no effect on the strength of the wood, and may be prevented by dipping the freshly sawed lumber in a 5-per cent. solution of sodium carbonate kept at about 140 deg. fahr.

Decay of Living Trees. Where decay, or dote, is found in the heartwood of timbers, it is usually due to disease of the living trees. Different forms of such decay are distinguished in conifers as red heart, dote, rot, and in hardwoods as piped rot, brown rot, speckled rot, etc. All of these forms of decay cease after trees are felled, and cannot be communicated to other pieces of sound structural timber.

Weathering is the wearing away of the surface of timbers caused by exposure to the elements, and is of importance only with comparatively soft woods. Different forms are distinguished according to the color, as white, gray or brown weathering.

Life of Woods. The natural length of life of wood and its resistance to decay vary with the kind of wood and the conditions under which it is used. In general, woods may be classed as long-lived, medium-lived and short-lived, as indicated below.

Long-lived: Cypress, redwood, red cedar, white cedar, osage orange, catalpa.

Medium-lived: White oak, slippery elm, black walnut, hickory, longleaf pine, tamarack, Douglas fir.

Short-lived: Red oak, red gum, beech, elm, spruce, shortleaf pine, hemlock.

Destruction of Wood by Marine Animals. Piling and other timbers exposed to salt water in warm climates are destroyed by various species of marine wood borers, of which the principal forms are the teredo (known as the ship worm) and the limnoria (sometimes, through resemblance, called a wood louse). The teredo thrives in waters with a saline density above 1.0054, and at temperatures of from 55 deg. fahr. to the highest found along our coasts. It lives in clear and turbid water, but seldom to a depth below 30 ft. It works most rapidly in warm water. Unprotected pine piles will be destroyed in approximately the following times at various points: Norfolk, Va., 1 to 5 years; Pensacola, Fla., 1 to 3 years; Galveston, Tex., 5 months to 1½ years; Colon, Panama, 9 months to 1 year; Puget Sound points, 1 year; Klawak, Alaska, 1½ to 3 years. The limnoria requires pure salt water and cannot live in dirty water. It occurs sparingly in Long Island Sound, is quite abundant along the coast of Massachusetts, and does great damage along the Gulf of Mexico and along the North Pacific Coast. All woods used for piling are subject to the attack of marine borers. (For preventive measures, see Creosoting, p. 581. For description, see United States Forest Service Circular No. 128, "Preservation of Piling against Marine Borers," and Chas. H. Snow, New York University, Lectures on Marine Borers, 1908.)

Timber Preservation

Seasoning of Timber. Timber can be seasoned either by air drying or by artificial means. In **air drying** excessive splitting may be prevented either by painting the exposed ends of logs, timbers or planks with common paint or preferably with coal-tar creosote, or by driving in sharp-edged irons of wedge section, shaped in the form of the letter S, and usually called S-irons. The base of the wedge section should be ⅛ in. thick. **Artificial drying** is usually done in kilns. Coniferous woods can be dried more rapidly than hardwoods and at higher temperatures, without affecting their strength. The latter should be dried very slowly. Small pieces of wood can be dried so as to prevent checking by soaking them from 1 to 7 days in a concentrated salt solution. The soaked pieces should then be piled in the air to dry. Another plan for drying small pieces is to pile them in bone charcoal.

Timber Preservation. The prevention of decay due to fungi and the protection of timber against wood borers are accomplished by the injection of various chemicals. The principal requirements for successful chemical preservation are: As thorough impregnation of the wood as can be obtained; the injection of sufficiently large quantities of the preservative; the injection of efficient chemicals; and the use of thoroughly sound and seasoned wood.

The **penetration** obtained will vary with the preservative. Water solutions, such as mercuric chloride and zinc chloride, will usually penetrate clear through a stick. Coal-tar creosote will penetrate sapwood only in coniferous woods (the heartwood of pines is slightly penetrable in large pieces, wholly so in small pieces like paving blocks), and in many hardwoods such as gum, white oak, hickory and ash. It will penetrate into the heartwood of red oak, elm and sycamore, and into the heartwood of small pieces of pine, spruce, etc., such as paving blocks. Individual pieces of timber absorb different quantities of preservative. In one run of a thousand ties treated with creosote, where all the ties were exactly the same size, the average absorption was 23 lb. of creosote, but some of the pieces absorbed as low as 2 lb. and some as high as 90 lb.

Wherever possible, only thoroughly air-seasoned timber should be treated, because the higher the percentage of moisture in a wood, the poorer the penetration obtained. Wherever possible, all injuries to wood after treatment should be avoided. All framing, boring and adzing should be done before treatment.

The **principal preservatives** used, mentioned in the order of their efficiency, are coal-tar creosote (known also as dead oil of coal tar, or creosote oil), mercuric chloride and zinc chloride.

Processes of Treatment

Zinc-chloride Treatment or Burnettizing Process. In this process a solution of zinc chloride at 140 deg. fahr., is injected into timber under pressure, with preliminary steaming in the case of green timber. According to standard practice ½ lb. of dry zinc chloride is injected per cu. ft. of timber. For pine and other coniferous woods, the strength of the solution will vary from 2½ to 4 per cent.; for hard-wood, such as red oak, it will usually be about 4 per cent.; under no circumstances should it exceed 5 per cent. The strength of the solution should be controlled either by hydrometer readings or, preferably, by chemical titration. Zinc chloride may be purchased either in crystalline form or in the form of a 50 per cent. solution. Care should be observed that the material does not contain free hydrochloric acid or basic chloride of zinc; the former reducing the strength of the timber and the latter reducing the antiseptic properties. The material is soluble in water and consequently tends to be washed and leached out of timber in damp locations or where subjected to rain. Combinations of zinc chloride with creosote have been used to offset this characteristic, using either the Card or the Allardyce process (see below).

Kyanizing or Mercuric-chloride Treatment. In this process the timber is soaked in a vat built entirely of wood and filled with a solution of corrosive sublimate, one part of sublimate to 150 parts of water. The strength of the solution decreases as it is used, and must therefore be renewed by adding more sublimate from time to time. The vat should be kept covered. The wood is left in the vat for from 5 to 10 days, according to the size of the timber, until a thorough absorption of the liquid has taken place. Corrosive sublimate is an excellent preservative, particularly for fence posts, sticks, and for use where creosote, because of its odor and physical nature, cannot be used. Care should be exercised to prevent unnecessary handling of this very poisonous salt. (In case of poisoning, drink large quantities of milk or water into which well-beaten fresh eggs have been stirred.)

The Factory Mutual Insurance Companies of Boston recommend (1915) that longleaf pine used in mill construction be soaked for a week in a 1 per cent. solution of corrosive sublimate, and state that the cost of treatment at the mills is about $3 per thousand. They require that portions of the wood exposed after treatment by cutting should be thoroughly swabbed with the solution.

Creosoting, with its several modifications, is the best preservative for practically all purposes. There are several processes in use, distinguished as the Bethel or plain creosoting process, the Lowry process, the Rüping process, and the boiling process.

Creosote or Dead Oil of Coal Tar is a by-product of coal tar in the manufacture of illuminating gas and by-product coke. Tar is distilled and the condensed vapors are separated into the light oils or naphtha, the dead oil or creosote, and pitch. Creosote is not a simple substance but contains a large number of chemical constituents. At 65 deg. fahr., it weighs about 8.7 lb. per gal.; at 100 deg. fahr. the specific gravity ranges from 1.03 to 1.08. It has high antiseptic properties and is insoluble in water. There are several grades of coal-tar creosote recognized in the trade, which are distinguished chiefly by their specific gravities and percentages of low and high boiling compounds. The heavier oils are considered the more valuable. Coal-tar creosote is usually injected into timber under pressure. In addition to creosote there are a number of trade-mark compounds (Carbolineum, Barol, etc.) which are usually used for brush applications.

Wood tar, when distilled, yields a **wood creosote** possessing weak antiseptic properties. Water-gas creosote is obtained as a by-product of water-gas tar, and is probably inferior to coal-tar creosote. Very little commercial use has been made of wood creosote for wood preservation, and the term "creosote" generally refers to the dead oil of coal tar.

Difference of opinion exists as to the desirable components of creosote. In view of this fact, most creosote specifications confine themselves to the requirement that the material be a straight product of coal tar distillation, free from adulteration, with limited material insoluble in benzol, and that certain definite percentages distill at standard temperatures.

For Standard Specifications for Creosote, see also Manual of the Am. Ry. Eng. Assn., 1912, p. 65; and *Proc. Am. Ry. Eng. Assn.*, 1915, p. 826.

In the **Bethel Process** a preliminary vacuum is maintained, after which creosote is injected under pressure varying from 40 to 200 lb. per sq. in., which is continued until the desired absorption has been obtained. The amount of creosote injected will vary from 10 to 25 lb. per cu. ft. The plain Bethel or full-cell creosoting process is used largely for the treatment of piling, bridge materials, paving blocks, telegraph poles, etc. It is applied chiefly to timber of which long service is expected, and where mechanical or destructive processes, aside from decay, are of minor importance. In general, bridge materials, piling, poles, and similar timbers are treated with 10 to 15 lb. of creosote per cu. ft. for regions north of the Ohio River and with 15 to 20 lb. for regions south of the Ohio River. Exceptions to this rule are made for piling, for which the following standards may be used: For marine exposure on the Atlantic Coast north of Delaware Bay, 12 to 15 lb. per cu. ft.; on the Atlantic Coast south of Delaware Bay and on the Gulf and Pacific Coast, 20 to 25 lb. per cu. ft. (see *Proc. Am. Ry. Eng. Assn.*, vol. 9, p. 352, 1908).

Lowry Process. This process aims to secure a good penetration with comparatively small quantities of creosote. Air-dry timber only is treated. Creosote oil is forced into the timber without a preliminary vacuum until a large quantity of creosote is absorbed, usually stated as "treatment to a refusal." A quick final vacuum is then applied and a considerable amount of the oil first injected is withdrawn. According to standard practice in railway tie treatment, about 2½ gal. of creosote remain in a 6 × 8 × 8 tie.

Rüping Process. This process likewise is intended to secure a good penetration with comparatively small quantities of oil. Compressed air is first forced into the wood up to a pressure of 75 lb. per sq. in., after which creosote oil is forced in at a higher pressure until treatment to a refusal is obtained. A final vacuum aids the compressed air in driving out a considerable quantity of the oil originally injected. According to standard practice, from 1.8 to 2 gal. are left in a railway tie at the end of the treatment.

Boiling Process. This process is used chiefly for the treatment of Douglas fir on the Pacific Coast. The green timber is placed in the creosoting cylinder, which is then filled with creosote and heated to a point slightly above the boiling point of water. This heating is maintained until practically no water comes out of the condenser. Pressure is then applied and the preservative is forced into the timber to the requisite amount.

Allardyce Process. In this process zinc chloride is first injected into the timber followed by a subsequent injection of creosote, usually 3 lb. of creosote per cu. ft.

Card Process. The preserving liquid used in this process is made up of 15 to 20 per cent. of creosote and the remainder of a 3 to 5 per cent. solution of zinc chloride. The creosote and zinc chloride are mixed in a centrifugal pump and the emulsion thus produced is forced into the timber under pressure.

Open-tank Treatment. Under this name a number of methods have been devised for treating timber, without pressure, with either zinc chloride or creosote. The timber to be treated is put into an open tank or vat and heated in a hot liquid for several hours. It is then quickly immersed in a cold liquid, either zinc chloride or creosote. During the heating process a partial vacuum is produced in the interstices of the wood, and when the hot wood is put in the cold bath, the atmospheric pressure is sufficient to bring about considerable absorption. This process is adapted particularly to the treatment of small quantities of timber in localities where larger treating plants are not available. (For details, see Forest Service Circulars Nos. 101, 104, 111 and 117.)

Cost of Treatment. The cost of treatment with zinc chloride varies from 4 to 6 cents per cu. ft. of timber treated, and with corrosive sublimate from 4 to 5 cents per cu. ft. In the full-cell creosote process the cost ranges from about \$15 to \$25 per 1000 ft. B.M., depending upon the amount of creosote used. In the treatment of 6 × 8 × 8 railroad cross-ties, with injections of ½ lb. of zinc chloride per cu. ft., the cost is from 11 to 15 cents per tie; with injections of creosote according to the Lowry process, about 30 cents per tie; with injections of creosote according to the Rüping process, from 20 to 25 cents per tie.

Length of Life Obtained from Treatment. Red oak and pine cross-ties treated with zinc chloride have given an average service of from 6 to 11 years. Pine timbers treated with creosote have lasted 15 to 20 years in the Southern States, with many of them still in service. Piling, when treated with an oil of high specific gravity, will last 15 years or more under ordinary conditions of service. (For a brief account of wood preservation, see Forest Service Bulletin No. 78.)

Standard Sizes and Shipping Weights of Lumber

Lumber is manufactured in various sizes. The standard of measurement is the board foot, equal to a piece of lumber 1 ft. square and 1 in. thick. Thus, a timber 12 in. × 12 in. × 20 ft. contains 240 feet B.M. (Board Measure). A distinction is made between rough lumber and finished lumber. Excepting only a few rarer woods, lumber and timbers are usually cut in even lengths, that is, 2 × 4 in., 2 × 6 in. 2 × 8 in. etc., in lengths of from 12 to 30 ft. Finished lumber is designated as S1S when surfaced on one side, S2S when surfaced on both sides, as D & M when dressed and matched, and as E when edged.

All lumber is classified into standard grades, these being determined by the size, absence or presence of standard defects. These grades have been standardized by associations of lumber manufacturers.

Shingles. Pine shingles are usually 16 in. long, 2¼ to 14 in. wide, $\frac{1}{16}$ in. thick at the small end and $\frac{5}{16}$ in. thick at the butt end. A standard shingle is 4 in. wide. Cypress shingles are 16 in. long and 5 butts measure 2 in. in thickness. A thousand shingles (4 to 5 bundles) will cover from 100 to 150 sq. ft. of roof, according as laid from 4 to 6 in. to the weather. Shingles are usually laid to show 4 in. of their length, forming with 16-in. shingles a quadruple layer on a roof. The higher the grade of the shingles the larger the weather face permissible. Cypress shingles are the most durable, followed by redwood, cedar, pine and spruce (poor). Redwood shingles are the least inflammable. The **weight** of shingles (lb. per 1000 shingles) is approximately as follows: Cypress, 300; western white pine, 250; Washington cedar, 160 to 220; redwood (6 × 16-in., per bundle of 147), 42.5.

Cordwood. Firewood, pulpwood and other small pieces of wood are usually bought and sold by the cord. A cord is equal to 128 cu. ft. of stacked wood. The **standard cord** is a stack of wood cut into 4-ft. lengths, that is, 4 ft. high and 8 ft. long. A cord foot is one-eighth of a cord. The expression "surface feet" means the number of square feet measured on the side of the stack. In countries using the metric system, wood is piled in cubic meters. (1 cu. m. = 0.274 cord.) The actual volume of solid wood in a cord will depend, among other factors, on the form and size of the sticks, the amount of bark, whether split, and on the method of stacking (see Table 3). Ordinary conifers produce a greater amount of wood per cord than hardwoods, because of the smoother, straighter sticks. One cord of first-class split wood obtained from sound pieces 12 in. in diam. contains 102.4 cu. ft. of solid wood. Composed of inferior pieces having a diam. of 6 in., a cord contains 97 cu. ft. of solid wood. While no absolute ratio can be given of the relation between cord measure and board measure, a fair average is 6 board-feet to each cubic foot.

Table 1. Contents in Feet (B.M.) of Joists, Scantlings and Timbers

Size in inches	Length in Feet									
	12	14	16	18	20	22	24	26	28	30
2× 4	8	9	11	12	13	15	16	17	19	20
2× 6	12	14	16	18	20	22	24	26	28	30
2× 8	16	19	21	24	27	29	32	35	37	40
2× 10	20	23	27	30	33	37	40	43	47	50
2× 12	24	28	32	36	40	44	48	52	56	60
2× 14	28	33	37	42	47	51	56	61	65	70
3× 8	24	28	32	36	40	44	48	52	56	60
3× 10	30	35	40	45	50	55	60	65	70	75
3× 12	36	42	48	54	60	66	72	78	84	90
3× 14	42	49	56	63	70	77	84	91	98	105
4× 4	16	19	21	24	27	29	32	35	37	40
4× 6	24	28	32	36	40	44	48	52	56	60
4× 8	32	37	43	48	53	59	64	69	75	80
4× 10	40	47	53	60	67	73	80	87	93	100
4× 12	48	56	64	72	80	88	96	104	112	120
4× 14	56	65	75	84	93	103	112	121	131	140
6× 6	36	42	48	54	60	66	72	78	84	90
6× 8	48	56	64	72	80	88	96	104	112	120
6× 10	60	70	80	90	100	110	120	130	140	150
6× 12	72	84	96	108	120	132	144	156	168	180
6× 14	84	98	112	126	140	154	168	182	196	210
8× 8	64	75	85	96	107	117	128	139	149	160
8× 10	80	93	107	120	133	147	160	173	187	200
8× 12	96	112	128	144	160	176	192	208	224	240
8× 14	112	131	149	168	187	205	224	243	261	280
10× 10	100	117	133	150	167	183	200	217	233	250
10× 12	120	140	160	180	200	220	240	260	280	300
10× 14	140	163	187	210	233	257	280	303	327	350
12× 12	144	168	192	216	240	264	288	312	336	360
12× 14	168	196	224	252	280	308	336	364	392	420
14× 14	196	229	261	294	327	359	392	425	457	490

Table 2. Shipping Weights of Lumber, Lb. per 1000 Ft. B.M.

	Green from saw	Shipping dry	Well seasoned	Kiln dried		Green from saw	Shipping dry	Well seasoned	Kiln dried
Ash, black....	4600		3200	3000	Gum, sap....	5000	3300	3000	2750
Ash, white...	4600		3800	3300	Hickory.....	6000		4500	
Basswood....	4200	2800	2500	2100	Mahogany...	4500		3500	
Beech........	5750		4000		Maple, hard..	5400	4150	3900	3400
Birch.........	5500		4000		Maple, soft	5000	3650	3300	3000
Butternut....	4000		2500		Oak, red.....	5500	4250	4000	3400
Chestnut.....	5000		2800	2450	Oak, white...	5700	4500	4100	3600
Cherry.......	5000				Poplar.......	3900	3000	2800	2400
Cottonwood..	4600	3100	2800	2400	Poplar bay				
Elm, rock ...	5400	4300	4000	3500	(Tupelo)....	4200		3000	
Elm, soft	4750	3300	3100	2900	Sycamore....	4750		3000	
Gum........	5400	3600	3300	3050	Walnut......	4900	4000	3800	

Table 3. Number of Cubic Feet of Solid Wood per Cord, with Corresponding Diameters of the Average 4-ft. Stick

(From U. S. Forest Service Bulletin No. 96)

Diam. of standing timber breast-high, inches	Chestnut		Black Oak		White Oak	
	Diam. of average stick, inches	Cubic feet per cord	Diam. of average stick, inches	Cubic feet per cord	Diam. of average stick, inches	Cubic feet per cord
1	0.9	....	....	....	....	....
2	1.8	63	1.8	63	1.8	63
3	2.6	70	2.5	69	2.5	69
4	3.3	75	3.1	74	3.1	74
5	4.0	79	3.6	77	3.5	76
6	4.7	83	4.1	80	3.9	79
7	5.2	85	4.5	82	4.2	81
8	5.8	88	4.8	84	4.5	82
9	6.2	89	5.0	85	4.7	83
10	6.7	91	5.3	86	4.9	84
11	7.0	92	5.4	86	5.0	85
12	7.4	93	5.6	87	5.1	85
13	7.7	94	5.7	88	5.2	85
14	7.9	94	5.7	88	5.2	85
15	8.2	95	5.8	88	5.3	86
16	8.4	95	5.9	88	5.4	86
17	8.5	95	5.9	88	....	....
18	8.7	95	6.0	89	....	....

Bark is usually sold and bought by the cord. The tanneries, however, apply the name to a weight of 2,240 lb., instead of to 128 cu. ft. Twelve cords of bark fill one common (old) freight car. A stack of bark contains from 30 to 40 per cent. solid bark. The specific gravity of fresh oak bark is 0.874; dried, it is 0.764. The bark of white oak varies from 55 per cent. of the wood in trees 20 years old to 21 per cent. in trees 140 years old. The average bark yield of chestnut oaks per tree (in cords) is as follows: Trees 6 in. in diam., 0.013; 12-in. trees, 0.073; 18-in. trees, 0.195; 24-in. trees, 0.375.

STRENGTH OF WOOD

By

WILLIAM KENDRICK HATT

(For references, see p. 577)

The strength of wood-substance, as arranged in wood structures peculiar to different species, must be distinguished from strength of large timbers containing defects such as knots and crooked grain Wood-substance exhibits an elastic limit under usual rates of test loading; under continuous loads it is markedly plastic and the modulus of elasticity may be only half as great as listed in the tables. Under impact, the elastic limit is frequently twice as great as listed in the tables. Strength is also affected by temperature, and very greatly affected by moisture. Table 1 represents results of a series of timber tests of the Forest Service on representative trees collected from the forest. Inasmuch as the strength of species is affected very greatly by the age of the tree, site conditions, and the inherent variability of individuals, strength values of wood cannot be quoted with any exactness.

Table 1. Strength of Green and Air-seasoned Timber

(Average physical and mechanical properties, based on tests of small clear specimens 2 × 2 in. in cross-section; bending span, 28 in. Mainly from Circular No. 213, U. S. Forest Service)

Species, common name	Condition, green or air dried	Moisture content, per cent.	Specific gravity based on volume when tested*	Shrinkage in volume from green to oven-dry condition, per cent.	Static bending: Fiber stress at elastic limit, lb. per sq. in.	Static bending: Modulus of rupture, lb. per sq. in.	Static bending: Modulus of elasticity, 1000 lb. per sq. in.	Static bending: Work to max. load, in.-lb. per cu. in., U_R	Compression parallel to grain: Crushing strength, lb. per sq. in.	Compression perpendicular to grain: Stress at elastic limit, lb. per sq. in.	Hardness—load required to imbed a 0.444-in. ball to one-half its diam.: End, lb.	Hardness—load required to imbed a 0.444-in. ball to one-half its diam.: Side, lb.	Shearing strength, parallel to grain, lb. per sq. in.
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Hardwoods													
Ash, black.........	Green	84.0	.456	....	2595	6000	1030	12.2	2300	430	588	548	860
	Dried	11.6	.497	....	6310	11,620	1395	13.1	5590	893	1101	792	1660
Ash, blue..........	Green	39.0	.533	11.7	5700	9650	1241	14.7	4180	994	1140	1028	1544
Ash, green.........	Green	48.0	.534	13.3	6110	10,040	1480	13.0	4360	1012	1073	1007	1318
Ash, pumpkin.....	Green	51.0	.485	12.0	4470	7600	1043	9.4	3360	989	885	752	1214
Ash, white.........	Green	42.0	.550	12.8	5260	9850	1460	15.6	4300	828	1036	941	1381
	Dried	10.8	.580	....	9860	15,665	1778	13.7	7600	1300	1870	1285	2023
Aspen, large-tooth.	Green	96.0	.354	11.6	3190	5850	1185	6.1	2720	269	443	366	813
Basswood..........	Green	104.0	.323	15.5	2660	4860	995	5.4	2135	209	282	243	607
	Dried	9.2	.345	....	5910	7310	1674	3.2	4770	557	445	364	1524
Beech.............	Green	63.0	.544	16.1	4495	8165	1242	12.5	3280	607	951	824	1210
	Dried	13.1	.620	....	8140	14,830	1830	16.4	6450	1185	1463	1217	1908
Birch, sweet........	Green	61.0	.588	15.0	4510	8590	1490	15.6	3560	525	1023	894	1220
Birch, yellow.......	Green	68.0	.55	16.8	4580	8600	1543	16.6	3460	454	818	745	1112
	Dried	10.3	.624	....	13,360	19,400	2396	17.9	9560	1340	1542	1280	1428
Buckeye, yellow....	Green	141.0	.326	12.0	2650	4820	981	5.4	2050	210	357	286	662
Butternut..........	Green	102.0	.354	9.4	3140	5870	1008	8.4	2580	258	414	394	762
Cherry, black......	Green	55.0	.471	11.5	4180	8030	1208	12.8	3540	444	754	664	1127
Cherry, wild red...	Green	46.0	.361	....	2880	5040	1042	6.2	2170	265	435	386	678
Cucumber-tree.....	Green	80.0	.440	....	4220	7420	1565	10.0	3140	408	595	515	991
Dogwood, flowering	Green	62.0	.638	19.9	4820	8790	1175	21.0	3640	1033	1413	1408	1516
Elm, rock..........	Green	46.0	.578	....	4290	9430	1222	19.4	3740	696	954	888	1270
	Dried	11.2	.630	....	8000	16,350	1755	20.4	7570	1603	1593	1257	2154
Elm, slippery.......	Green	73.0	.507	14.4	4650	8610	1264	13.9	3585	599	817	687	1138
	Dried	11.6	.591	....	7940	13,950	1622	14.4	7080	1145	1631	1214	2090
Elm, white.........	Green	79.0	.434	14.4	3330	6945	1040	11.3	2810	351	580	511	873
	Dried	10.8	.469	....	6790	12,140	1504	13.4	5840	727	892	679	1447
Gum, red..........	Green	71.0	.434	....	3460	6450	1138	....	2690	....	....	....	....
Hackberry.........	Green	59.0	.493	13.9	3080	7005	1040	16.6	2915	525	785	730	1093
	Dried	11.4	.547	....	7250	14,070	1426	18.6	6720	1320	1505	1159	1796
Haw, pear.........	Green	64.0	.623	....	3880	7650	964	22.7	3110	980	1218	1204	1356
Hickory, bitternut.	Green	65.0	.624	....	5470	10,280	1399	20.0	4570	986	....	....	1237
	Dried	9.7	.696	....	10,260	18,850	1880	17.9	10,600	2390	....	....	2048
Hickory, nutmeg..	Green	76.0	.558	....	4860	9060	1289	22.8	3980	938	....	....	1032
	Dried	9.0	.620	....	9150	19,270	1821	25.7	7960	2815	....	....	2105
Hickory, water.....	Green	74.0	.630	....	5980	10,740	1563	18.8	4660	1088	....	....	1440
	Dried	8.9	.652	....	11,750	20,200	2165	19.4	10,140	2208	....	....	1865
Laurel, mountain..	Green	62.0	.616	14.4	5800	8440	924	12.5	4310	1110	1401	1299	1669

* Both green and dried wood were oven-dry when their specific gravities were determined. The difference in specific gravity in the two cases is due mainly to shrinkage.

Table 1. Strength of Green and Air-seasoned Timber—*(continued)*

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Hardwoods													
Locust, honey	Green	53.0	.695	8.6	6020	12,360	1732	17.3	4970	1684	1862	1846	1990
Hickory, big shell-	Green	59.0	.633	19.3	5585	10,500	1330	30.2	3890	997	....	....	1187
bark.	Dried	9.3	.734	....	9800	20,550	2045	22.5	9780	2676	....	....	2430
Hickory, mockernut	Green	56.0	.645	17.7	6445	11,560	1672	24.8	4720	1011	....	....	1276
	Dried	9.3	.739	....	13,580	21,580	2352	21.5	10,650	2511	....	....	1899
Hickory, pignut	Green	55.0	.657	17.1	6310	11,850	1648	29.5	4820	1142	....	....	1348
	Dried	9.5	.763	....	12,620	22,570	2450	27.6	10,540	2776	....	....	2516
Hickory, shagbark	Green	62.0	.631	16.6	5895	10,870	1532	20.2	4597	1070	....	....	1298
	Dried	9.6	.730	....	12,400	22,300	2186	23.4	10,420	2385	....	....	2324
Maple, red	Green	70.0	.488	12.5	4110	7890	1420	10.5	3385	531	740	614	1157
	Dried	12.1	.548	....	8650	13,420	1761	12.4	6610	1291	1531	1024	1789
Maple, silver	Green	66.0	.439	12.0	3120	5820	943	11.0	2490	456	671	592	1053
Maple, sugar	Green	60.0	.559	14.5	4990	9060	1474	12.0	3850	742	997	880	1380
	Dried	11.4	.612	....	9660	15,260	1878	13.1	7840	1367	1925	1344	2250
Oak, burr	Green	70.0	.578	12.7	3640	7180	877	10.7	3280	836	1158	1108	1354
Oak, post	Green	84.0	.590	16.0	4720	7380	913	9.1	3330	1148	1139	1074	1299
	Dried	11.2	.677	....	7860	12,480	1321	10.0	6660	1970	1364	1286	1888
Oak, red	Green	85.0	.567	13.5	3930	8000	1330	11.3	3370	777	1055	982	1146
	Dried	11.2	.661	....	8790	14,080	1915	12.9	7270	1100	1528	1359	1796
Oak, swamp white	Green	74.0	.637	17.7	5380	9860	1593	14.5	4360	943	1205	1158	1296
	Dried	13.2	.711	....	9910	17,210	2020	18.9	8030	1408	1609	1556	1998
Oak, tanbark	Green	88.0	.562	....	6580	10,710	1678	....	4840	1355	....	....	1414
	Dried	14.2	.633	....	9080	15,512	2083	....	8170	1656	....	....	1960
Oak, white	Green	66.0	.599	15.4	4500	8160	1214	11.4	3510	850	1128	1063	1251
	Dried	11.6	.688	....	8090	15,190	1779	13.7	7750	1345	1577	1425	2141
Oak, yellow	Green	78.0	.561	....	4390	8110	1170	12.5	3390	857	970	926	1237
	Dried	11.6	.608	....	8035	14,180	1652	13.8	6760	1179	1437	1208	2004
Osage orange	Green	31.0	.761	8.9	7760	13,660	1329	37.9	5810	2260	1838	2037	....
Rhododendron, great	Green	99.0	.501	16.2	4630	6905	872	12.1	3470	886	1000	864	1240
Sumac, staghorn	Green	45.0	.449	....	3030	5845	809	10.8	2680	477	671	590	....
Sycamore	Green	81.0	.454	13.5	2820	6300	964	7.1	2790	433	664	580	1001
	Dried	11.5	.484	....	5680	9350	1365	6.4	5340	885	957	814	1554
Tupelo	Green	121.0	.475	12.4	4300	7380	1045	7.8	3550	451	814	700	1031
	Dried	12.8	.510	....	6310	8970	1286	5.4	5850	865	1244	847	1577
Willow, black	Green	148.0	.330	13.3	1360	3340	489	12.9	1320	193	305	334	562
Witch hazel	Green	70.0	.558	18.8	5000	8280	1112	19.5	3400	619	1013	977	1118
Softwoods													
Arborvitæ	Green	55.0	.293	7.0	2600	4250	643	5.7	1990	288	321	225	617
Cedar incense	Green	80.0	.363	....	3950	6040	754	....	3030	518	....	....	613
Cypress, bald	Green	79.0	.452	11.5	4430	7110	1378	5.1	3960	548	460	355	836
Fir: Alpine	Green	47.0	.306	9.0	2366	4450	861	4.4	2060	307	284	203	573
Amabilis	Green	117.0	.383	....	4060	6570	1323	....	3040	334	....	....	517
Douglas	Green	32.0	.418	10.9	3570	6340	1242	6.6	2920	427	415	399	853
White	Green	156.0	.350	10.2	3880	5970	1131	5.2	2800	445	381	322	742
Hemlock	Green	129.0	.340	9.2	3410	5770	917	6.6	2750	420	463	354	790
Pine: Lodgepole	Green	44.0	.370	11.3	3080	5130	1015	5.1	2530	364	288	307	672
Lodgepole	Green	58.0	.371	10.1	2750	5170	972	5.3	2400	332	316	318	709
Longleaf	Green	63.0	.528	12.8	5090	8630	1662	8.1	4280	491	574	502	1060
Red	Green	54.0	.440	11.5	3740	6430	1384	5.8	3080	358	355	345	812
Shortleaf	Green	52.0	.477	....	4360	7710	1395	....	3570	400	....	....	746
Sugar	Green	123.0	.360	8.4	3330	5270	966	5.0	2600	353	334	307	702
Western yellow	Green	98.0	.353	9.2	2660	4760	879	4.9	2220	342	310	311	644
Western yellow	Green	125.0	.377	11.5	3180	5180	1111	4.3	2420	326	316	306	686
Western yellow	Green	93.0	.391	9.9	3310	5460	1053	6.0	2600	410	315	330	694
White	Green	74.0	.363	7.8	3410	5310	1073	5.9	2720	314	304	294	649
Redwood	Green	81.0	.334	....	4530	6560	1024	....	3820	539	....	....	....
Redwood	Green	69.0	.366	....	5020	7460	1101	....	4160	618	....	....	....
Spruce: Engelmann	Green	45.0	.325	10.5	2740	4550	866	4.8	2170	302	272	253	607
Engelmann	Green	156.0	.299	10.3	2180	3850	798	5.0	1800	279	231	216	538
Red	Green	31.0	.396	....	3440	5820	1143	6.0	2920	322	....	....	754
White	Green	41.0	.318	....	3160	5200	968	6.6	1940	262	....	....	645
Tamarack	Green	52.0	.491	13.6	4200	7170	1236	7.2	3480	480	401	380	883

Table 2. Strength of Green Wood in Small Perfect Specimens*
(Summary of Table 1. Authority, U. S. Forest Service)

Group	Modulus of rupture, lb. per sq. in.	Compressive strength (parallel to grain), lb. per sq. in.	Woods included in group
I	10,000 to 13,000	4,200 to 6,000	Eucalyptus corynocalyx, osage orange, honey locust, mockernut hickory, pignut hickory, eucalyptus globulus and tereticornia, shagbark hickory, shellbark hickory, eucalyptus viminalis.
II	8,000 to 10,000	3,600 to 4,200	White, green and blue ash, slippery elm, eucalyptus rostrata, rock elm, sugar maple, longleaf pine, beech, yellow oak, yellow and sweet birch, red maple, Douglas fir, red oak, white oak, red wood in compression.
III	7,000 to 8,000	3,400 to 3,600	Slippery elm, loblolly pine, hackberry, poorer qualities of white and red oaks, shortleaf pine, tupelo gum, western hemlock, western larch, black cherry, white elm, cypress, tamarack.
IV	5,000 to 7,000	2,400 to 3,400	Redwood in bending, red gum, sycamore, black ash, white fir, red spruce, eastern hemlock, balsam fir, common catalpa, black spruce, sugar pine, white pine, white spruce, western yellow pine, Norway pine, lodgepole pine, chestnut, cedar, aspen, silver maple.
V	below 5,000	below 2,400	Basswood, hardy catalpa.

* Woods are listed in this table in each group according to strength, which, however, varies with locality and conditions of growth. Strength of wood from young, thrifty trees of hardwood is generally greater than that from old, overmature trees. For hardwoods rapid growth yields greatest strength and toughness. For conifers greatest strength is associated with slower growth. The rates of growth (annual rings per in.) shown in the following list are associated with maximum strength. A ring comprises the spring wood and summer wood of one year's growth: Loblolly pine, 6; shortleaf pine, 12; western hemlock, 14; western larch and Norway pine, 18; tamarack, 20; Douglas fir, 24; redwood, 30.

Relation of Physical Properties to Specific Gravity. As a rule, strength varies directly with the dry specific gravity. An approximate value, S, of various strength functions is given by the equation, $S = mg^n$, where g is the specific gravity of the dry wood and m and n have values which vary with the condition of the wood and the kind of stress to which it is subjected. Thus for small clear specimens, the modulus of rupture is obtained by putting $m = 25{,}530$, and $n = 1.20$; when the wood is green, $m = 19{,}140$ and $n = 1.27$; for green longleaf pine, $m = 20.800$ and $n = 1.5$. (Betts, International Engineering Congress, 1915.) See also Fig. 1. The value of n varies with the strength function under consideration and ranges from unity for compression parallel to the grain to 3 for ultimate rupture work. For shearing stress n is approximately unity.

The density and strength of *summer wood* of the pines are about twice as great as in the *spring wood*. The proportion of summer wood forms an excellent measure of strength.

Moisture Content, Shrinkage, Specific Gravity. The **percentage of moisture** in wood is based upon the oven-dry weight of the wood. Thus wood which weighs 35 lb. per cu. ft. when oven-dry and contains 50 per cent. moisture when green, will then weigh 35 plus 17½ or 52½ lb. per cu. ft. In large beams the cubic foot reckoned is not the shrunk cubic foot when oven-dry, but the cubic foot of volume occupied by the wood at the time of the test.

The **specific gravity** as based on oven-dry volume may be obtained from the specific gravity as based on green volume by allowing for shrinkage.

The **volumetric shrinkage** of wood from the green to the oven-dry state, expressed in percentage of volume, is 26.5 times the specific gravity as based on green volume. Thus, for white oak, shrinkage $= 0.46 \times 26.5 = 12.2$ per cent., and consists almost entirely of lateral contraction.

The specific gravity of **cellulose** is very nearly 1.5. A solid cubic foot of wood-substance would weigh nearly 93.6 lb.

Toughness and Strength. The order of **technological values** of various woods does not coincide with that indicated by the strength values. Toughness and strength combined indicate this value, which is shown by the work U_R required to break a beam. Values of U_R (in.-lb. per cu. in.) are given in Table 1; for hickories and rock elm U_R varies from 24 to 40; ashes and elms, 14 to 24; oaks, black spruce, and maple, 8 to 14; white spruce, firs, and other pines, below 8. This factor U_R does not vary directly with specific gravity. For instance, eucalyptus, a strong and heavy wood is comparatively brittle and ranks low in rupture work.

Tensile Strength. The tensile strength of wood parallel to the fibers (or grain) exceeds all other strength values. At 15 per cent. moisture, values of T.S. (lb. per sq. in.) according to Johnson, are: elm, 29,000; hickory, 32,000; larch, 19,000; longleaf pine, 17,300. Bovey gives 11,612 for Douglas fir, and Isaacs, 15,900. Tensile strength parallel to fiber is not important, for other stresses, as shear, or tension perpendicular to fiber, bending, or bearing govern design. Tensile strength **perpendicular to the fibers** may be taken at 900 lb. per sq. in. for hardwoods and 250 lb. per sq. in. for conifers.

Shearing Strength. The shearing strength **parallel to the fibers** of small, clear, green specimens may be taken as approximately 1100 lb. per sq. in. for hard woods and 600 lb. per sq. in. for conifers except the following: Cypress, Douglas fir, red pine, tamarack (800), and longleaf pine (1000). The values are increased from 50 to 100 per cent. by careful seasoning. For **shear in flexure,** see Tables 3 and 4.

The Strength of Large-sized Pieces of structural timber **containing knots** and other defects is usually not more than two-thirds of the values in Table 1. Tables 3 and 4 (abstracted from publications of Forest Service) give results of tests. The strength of small specimens cut from the beams after test is also included. The strength of timbers of the same apparent grade varies greatly. The average of the lowest 10-per cent. group of a series of values will be from 60 to 70 per cent. of the general average value for the group. The range of individual values low to high may be from 50 to 150 per cent. of the average. Bulletin 108, Forest Service, U. S. Dept. of Agriculture (Cline and Heim), is an authoritative discussion of tests of large sizes of timber and the effects of defects upon strength.

Table 3. Strength of Green Timber, Structural Sizes
(U. S. Forest Service)

Species	Weight of oven-dry wood, lb. per cu. ft.	Number of rings per in.	Bending				Compression				Shear
							Parallel to fibers			Perp. to fibers	
							Crushing strength				
			Fiber stress at elastic limit, lb. per sq. in.	Modulus of rupture, lb. per sq. in.	Modulus of elasticity, 1000 lb. per sq. in.	Horizontal shear;* lb. per sq. in.	At elastic limit, lb. per sq. in.	At maximum load, lb. per sq. in.	Modulus of elasticity, 1000 lb. per sq. in.	Crushing strength at elastic limit, lb. per sq. in.	Shearing strength, lb. per sq. in.
Longleaf pine:											
Structural sizes	35	13.8	3,700	6,099	1,463	353	3,480	4,800	......	568	......
Small specimens	....	....	4,950	9,070	1,540	......	......	4,400	......	......	973
Douglas fir:											
Structural sizes	28	11.0	3,968	5,983	1,517	166	2,770	3,495	1,414	570	......
Small specimens	....	....	5,227	8,280	1,597	......	3,500	4,030	1,925	......	765
Shortleaf pine:											
Structural sizes	30	12.1	3,237	5,548	1,473	332	2,460	3,435	1,548	351	......
Small specimens	....	....	4,350	7,710	1,395	......	......	3,570	......	400	704
Western larch:											
Structural sizes	28	24.3	3,324	4,948	1,301	288	2,675	3,510	1,575	456	......
Small specimens	....	....	4,274	7,251	1,310	......	3,026	3,696	1,545	......	700
Loblolly pine:											
Structural sizes	31	5.9	3,040	5,084	1,387	335	2,050	2,940	548	500	......
Small specimens	....	....	4,100	7,870	1,440	......	......	3,340	......	......	630
Tamarack:											
Structural sizes	30	14.0	2,813	4,556	1,220	261	2,400	3,230	1,373	......	......
Small specimens	....	....	3,875	6,820	1,141	......	......	3,190	......	......	668
West. hemlock:											
Structural sizes	27	15.6	3,516	5,296	1,445	288	2,905	3,355	1,617	434	......
Small specimens	....	....	4,406	7,294	1,428	......	2,938	3,392	1,737	......	630
Redwood:											
Structural sizes	22	18.8	3,760	4,472	1,042	302	3,194	3,882	1,240	434	......
Small specimens	....	....	4,750	6,980	1,061	......	3,490	3,980	1,222	569	742
Norway pine:											
Structural sizes	25	13.7	2,492	3,864	1,133	232	2,065	2,555	1,002	......	......
Small specimens	....	....	2,808	5,173	960	......	......	2,504	......	......	589

* Only those pieces which failed first by horizontal shear are included in this column.

Influence of Knots and Other Defects on Strength. Forest Service tests indicate the following relations: Knots which interrupt the grain in the tension face of a beam seriously influence its strength. Knots greater than 1½ in. in diam. diminish compression strength from 15 to 20 per cent. They do not influence stiffness or the elastic limit of beams. For beams the range of stress between the elastic limit and modulus of rupture is seriously influenced by knots, crooked grain and other defects. Checks and shakes both in green and seasoned timber bring about failure of beams by splitting at the ends under horizontal shear. When failure by shear occurs, however, the modulus of rupture is approximately equal to that of beams failing otherwise.

Tests of **large sizes in compression** show the following general ranges of ultimate strength (Lanza): White pine and spruce, 2000 to 3000; yellow pine, 3500 to 5500; white oak, 3000 to 5000.

REFERENCES: Lanza's "Applied Mechanics;" Talbot, *Bulletin* 41, Eng. Exp. Station, Univ. of Ill.; McFarland, *Bulletin Am. Ry. Eng. Assn.*, vol. 14, No. 149, Sept., 1912. For complete list of full-sized tests, see *Trans. Am. Ry. Eng. Assn.*, vol. 10, Part I, 1909.

Table 4. Strength of Air-seasoned Timber, Structural Sizes
(U. S. Forest Service)

Species	Weight of oven-dry wood, lb. per cu. ft.	Number of rings per in.	Bending: Fiber stress at elastic limit, lb. per sq. in.	Bending: Modulus of rupture, lb. per sq. in.	Bending: Modulus of elasticity, 1000 lb. per sq. in.	Bending: Horizontal shear,* lb. per sq. in.	Compression parallel to fibers: Crushing strength at elastic limit, lb. per sq. in.	Compression parallel to fibers: Crushing strength at maximum load, lb. per sq. in.	Compression parallel to fibers: Modulus of elasticity, 1000 lb. per sq. in.	Compression perp. to fibers: Crushing strength at elastic limit, lb. per sq. in.	Shear: Shearing strength, lb. per sq. in.
Longleaf pine:											
Structural sizes	39	15.4	3,691	5,749	1,705	272	3,480	4,800	......	572	......
Small specimens	....	....	6,750	11,520	1,740	......	......	......	......	......	934
Douglas fir:											
Structural sizes	28	13.1	4,563	6,372	1,549	221	3,271	4,258	1,038	639	......
Small specimens	....	....	6,686	10,378	1,695	......	3,842	5,002	1,064	......	822
Shortleaf pine:											
Structural sizes	31	12.4	4,675	6,573	1,726	364	4,070	6,030	1,951	796	......
Small specimens	....	....	7,780	12,120	1,792	......	......	6,380	......	926	1,135
Western larch:											
Structural sizes	29	22.7	3,503	5,856	1,487	340	......	5,746	......	597	......
Small specimens	....	....	5,880	10,254	1,564	......	......	5,934	......	......	905
Loblolly pine:											
Structural sizes	32	6.3	3,504	6,118	1,487	434	3,011	4,292	1,206	655	......
Small specimens	....	....	5,170	9,400	1,467	......	......	5,547	......	......	1,115
Tamarack:											
Structural sizes	31	12.7	3,730	5,498	1,341	299	3,349	4,320	1,351	......	......
Small specimens	....	....	7,630	13,080	1,620	......	......	4,790	......	697	879
West. hemlock:											
Structural sizes	28	17.7	4,398	6,420	1,737	307	4,840	5,814	2,140	473	......
Small specimens	....	....	6,333	10,369	1,666	......	4,560	5,403	1,923	......	924
Redwood:											
Structural sizes	22	20.1	3,442	3,891	890	......	......	4,276	......	525	......
Small specimens	....	....	4,777	7,798	1,146	......	......	5,119	......	564	671
Norway pine:											
Structural sizes	27	10.0	4,069	6,054	1,418	278	3,047	4,228	1,367	......	......
Small specimens	....	....	5,280	8,470	1,158	......	......	7,550	......	924	1,154

* Only those pieces which failed first by horizontal shear are included in this column.

Other **tests of large sizes** (beams) have given the following results:

	Authority	Modulus of rupture	Modulus of elasticity
Red pine	Bovey	5,370	1,520,000
White pine	Bovey	3,380	750,000
Douglas fir	Bovey	6,000	2,000,000
Spruce	Lanza	4,521	1,310,000
White oak	Lanza	5,863	1,131,000
Eastern hemlock	Lanza	3,825	1,922,000
White pine	Lanza	4,451	1,122,000
Yellow pine	Lanza	7,442	1,783,000
Maple	Lanza	7,146	1,517,000
Yellow pine	U. S. F.*	9,200–11,500	....
Longleaf pine	McFarland	4,955	....
Longleaf pine	Talbot	5,308	1,597,000
Loblolly pine	Talbot	4,090	1,833.000
Longleaf pine	Talbot	5,470	1,648,000
Douglas fir (old)	Talbot	4,284	1,780,000
Douglas fir (new)	Talbot	4,544	1,499,000

* U. S. Forestry, 1896.

Strength as Affected by Moisture Content. A comparison of the results of tests on air-seasoned material with those on green material shows that, in general, all of the **mechanical properties** are **improved by seasoning.** Increase in strength is especially marked on small pieces free from defects. Increase in strength of wood fiber due to drying is, in the case of large timbers, largely offset by a weakening of the timber due to the formation of checks. If the moisture content of a seasoned timber is increased it loses strength rapidly, and if thoroughly soaked with water will become slightly weaker than when green. On this account it is not safe in practice to depend upon an increase in strength in timbers due to seasoning. When, however, large beams are seasoned with ordinary care, it is safe to assume that they will not at any time be weaker than they were when green.

Seasoned wood in small pieces is much stronger than green wood. The relation of strength to moisture content depends upon species, whether softwood or hardwood, and upon the kind of loading.

Table 5 shows the **reduction in compressive strength** of several woods **due to increasing the moisture content** from 2 per cent. to the amount indicated. Thus, red spruce with 16 per cent. of moisture has but half (0.505) the strength it has with 2 per cent. of moisture. The table also serves to permit calculations of change in strength for other changes in moisture content.

Table 5. Relation of Moisture Content to Compressive Strength

(From Circular 108, U. S. Forest Service. Compression parallel to grain)

Percentage of moisture in the wood	Relative maximum crushing strength compared to that of wood containing 2 per cent. of moisture		
	Red spruce	Longleaf pine	Douglas fir
2	1.000	1.000	1.000
4	0.926	0.894	0.929
6	(c)0.841	0.790	0.850
8	0.756	0.702	0.774
10	0.681	0.623	0.714
12	0.617	0.552	0.643
14	(b)0.554	0.488	0.589
16	0.505	0.431	0.535
18	0.463	0.377	0.494
20	0.426	(a)0.328	0.458
22	0.394	0.278	0.428
24	0.362	....	(a)0.398
26	0.335	....	....
28	0.314	....	....
30	0.292	....	....
32	0.271	....	....
34	0.255	....	....

(a) Green. (b) Air-dry.
(c) Kiln-dry (approx.).

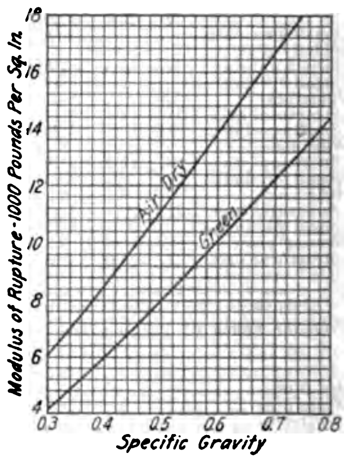

Fig. 1.—Modulus of Rupture of Wood.

The relation between the specific gravity (dry) and the modulus of rupture of both air-dry and green wood as determined by the U. S. Forest Service is shown in Fig. 1. (Newlin, *American Lumberman*, Jan. 16, 1915.)

Working Stresses for Timber for railway trestles and bridges are given in Table 6, and are to be used without increasing the live-load stress for impact. For highway bridges, unit stresses may be increased by 25 per cent. For buildings protected against weather and free from impact, the unit stresses may be increased 50 per cent. The stresses given are for green timbers, but no increase may be allowed for seasoned timber except as stated. **Factors of safety** used in designing timber structures must recognize the uncertainty of the material. Table 6 yields a factor of safety of approximately 5 in bending and 3 in compression.

Table 6. Unit Stresses for Structural Timber in Lb. per Sq. In.
Recommended by the Committee on Wooden Bridges and Trestles of the Am. Ry. Eng. Assn., 1909)

Kind of timber	Bending			Shearing				Compression				
	Extreme fiber stress		Modulus of elasticity	Parallel to grain		Longitudinal shear in beams		Perpendicular to grain		Parallel to grain		For columns under 15 diameters, safe stress‡
	Average ultimate	Safe stress	Average	Average ultimate	Safe stress	Average ultimate	Safe stress	Elastic limit	Safe stress	Average ultimate	Safe stress	
Douglas fir*	6,100	1,200	1,510,000	690	170	270	110	630	310	3,600	1,200	900
Longleaf pine*	6,500	1,300	1,610,000	720	180	300	120	520	260	3,800	1,300	980
Shortleaf pine*	5,600	1,100	1,480,000	710	170	330	130	340	170	3,400	1,100	830
White pine*	4,400	900	1,130,000	400	100	180	70	290	150	3,000	1,000	750
Spruce	4,800	1,000	1,310,000	600	150	170	70	370	180	3,200	1,100	830
Norway pine	4,200	800	1,190,000	590	130	250	100	....	150	2,600	800	600
Tamarack	4,600	900	1,220,000	670	170	260	100	....	220	3,200	1,000	750
Western hemlock	5,800	1,100	1,480,000	630	160	270	100	440	220	3,500	1,200	900
Redwood	5,000	900	800,000	300	80	....	....	400	150	3,300	900	680
Bald cypress	4,800	900	1,150,000	500	120	....	....	340	170	3,900	1,100	830
Red cedar	4,200	800	860,000	....	....	....	....	470	230	2,800	900	680
White oak†	5,700	1,100	1,150,000	840	210	270	110	920	450	3,500	1,300	980

* Ratio of length of stringer to depth = 10. † Ratio of length of stringer to depth = 12.
‡ For unit stresses in wooden columns of 15 diameters and over, see p. 1277.

The following table gives statistical information on the percentage amount and the average costs of the commoner woods used in New York State in the year 1912.

Kind of wood	Quantity used, per cent.	Average cost per 1000 ft.	Kind of wood	Quantity used, per cent.	Average cost per 1000 ft.
White pine	24.09	$27.70	Red oak	3.41	$ 38.49
Spruce	9.64	21.31	Yellow poplar	3.25	40.47
White oak	7.43	46.25	Basswood	3.25	27.36
Shortleaf pine	7.42	27.29	Birch	2.52	30.07
Sugar maple	5.14	27.07	Beech	2.43	20.54
Hemlock	4.73	19.82	Red gum	2.39	29.16
Chestnut	4.05	28.56	Cottonwood	1.30	21.00
Loblolly pine	4.02	20.77	Ash	1.00	38.49
Longleaf pine	3.67	32.41	Elm	0.99	28.37
Cypress (bald)	3.44	39.97	Red cedar	0.96	37.98

FUELS

BY

OZNI P. HOOD

REFERENCES: Publications of the U. S. Bureau of Mines and of the U. S. Geological Survey. Somermeier, "Coal," McGraw-Hill. Groves and Thorp, "Chemical Technology," Vol. I, Mills and Rowan. Lewes, "Liquid and Gaseous Fuels," Constable. "The Coal Resources of the World," McInnes, Dowling and Leach. Kent, "Steam Boiler Economy," Wiley.

SOLID FUELS

Coal

General. Coal consists of the more resistant parts of ancient vegetation. Different vegetable material has entered into the composition of the different coals, and the geologic experiences of the several beds have been sufficiently different to produce wide variations in composition, constitution and physical characteristics. Any division into groups must be arbitrary, with more or less overlapping of characteristics at the boundaries of each division.

The most evident constituents of any coal can be roughly observed when the coal burns. First moisture is driven off, then volatile matter, leaving coals of fixed carbon, and finally a residue of ash. An analysis which so divides the coal is called a "**proximate analysis.**" These several constituents cannot be sharply separated. The moisture is both free and combined with the coal substance and the amount driven off will depend upon the temperature. The volatile matter is so complex that the amount produced depends upon the temperature and the rate of heating. The ash is also both free and inherent. A proximate analysis depends upon more or less arbitrary standardized methods which, if not rigidly followed, give discordant results for the same coal sample. This analysis is, however, an acceptable indicator of the type of coal.

In the "**ultimate analysis**" of fuels the chemical elements are determined without regard to their combinations. Greater accuracy of determination is possible than with the proximate analysis. This analysis is expressed in terms of carbon, hydrogen, nitrogen, oxygen, sulphur and ash. Either of these analyses may be for fuel "as received," in which case the **moisture** is included in the weight, or may be computed to a "moisture-free" condition or a "moisture-and-ash-free" condition. The moisture in mine samples of fuel is about 2 to 4 per cent. for anthracite coals, 2 to 4 per cent. for bituminous coals of the Appalachian field, 4 to 10 per cent. for Ohio coals and 8 to 17 per cent. for Indiana, Illinois, Iowa and Missouri coals. Lignite fuels increase the amount to 35 per cent. The amount of moisture in commercial coal may be greater or less than that of the coal in the mine. It is usually reduced during shipment and handling, but may be increased by exposure to rain or snow. The free moisture in coal is largely a function of the surface exposed, each surface being capable of holding a film of moisture. Fine sizes of coal will therefore hold much more moisture than lump. Coal which in the mine does not contain more than 4 per cent. of moisture, may, in slack sizes, hold 15 to 18 per cent. moisture. Slack from stock piles in the open may contain as high as 20 per cent. moisture in winter and spring. Lump coal from subaqueous storage or from coal washers contains but little more moisture than the normal coal. The volatile matter in dry fuel covers an equally wide range, from anthracite having as little as 3 per cent. through all gradations to lignite with over 40 per cent.

The character of the **volatile matter** varies quite as much as its quantity, being composed of combustible gases as hydrogen, CO, methane and other hydrocarbons, as well as incombustible gases such as CO_2, water vapor, etc. From 2 per cent. to as high as 42 per cent. of the volatile matter may be incombustible. The ratio of the volatile matter to fixed carbon gives a general indication of the type of coal and its probable behavior in burning. Volatile matter escaping combustion furnishes carbon, tar, acids and ammonia, which, with some ash, form smoke.

The **ash** is an incombustible mineral residue consisting partly of mineral matter originally incorporated in the plant substance, but to a greater extent of clay, silt or sand, laid down with the vegetable matter, either intimately or in well-recognized layers. Parts of the roof or floor of the seam may also furnish some of the ash in commercial coal. To this is added water-borne mineral deposits in fissures or lenses, such as pyrites, gypsum, calcite, etc. Ash and moisture both reduce the net amount of combustible coal substance, but ash also greatly interferes with the combustion process by the formation of **clinkers**. Ash has no true melting point, but it has various degrees of plasticity depending on temperature and whether within an oxidizing or a reducing atmosphere. An arbitrary degree of plasticity is taken as a so-called melting point, ashes showing a low fusibility being most likely to give trouble from clinker, although this is by no means always true. The amount and mode of occurrence of the ash, together with the method of handling the fire, are equally important factors. Iron lowers the fusing temperature and is to be avoided in coals pulverized for burning.

Classification of Coals. Coals are classified according to the relative proportions of volatile matter to fixed carbon, together with their physical peculiarities (U. S. Geol. Survey, Bulletin 531). The content of fixed carbon divided by that of volatile matter is called the **"fuel ratio."** **Anthracite** is a hard coal, mostly obtained in Pennsylvania, having a fuel ratio not less than 10. **Semi-anthracite** has a fuel ratio of 6 to 10. **Semi-bituminous** coal has a fuel ratio of 3 to 6, and is of great commercial importance, embracing the Pocahontas, New River, Georges Creek and Clearfield fields in the Virginias, Maryland and Pennsylvania, which yield the highest-grade steam coals in the country. A small amount is found in the West. **Bituminous coal** is the most important class in the country, and includes most of the coals found east of the Rocky Mountains. Bituminous coals are either **caking** or **non-caking**, the former fusing and swelling in size when heated, the latter burning freely without fusing. **Cannel coals** are bituminous coals rich in hydrogen and hydrocarbons, burning with a bright flame without fusing, and are exceedingly valuable as gas coals. **Sub-bituminous coal** is a class lying between bituminous coal and lignite, the latter being brown and generally woody, with high moisture content.

Trade Names and Sizes of Coal. Bituminous coal as broken in the mine without any screening is called "run of mine." When run over screens to remove fine coal "Pittsburgh" coal is referred to as "lump," 1¼ or ¾, depending on the size of openings in the screens. **Sizing** is the separation of coal into groups of particles which will pass one size of screen and fail to pass another size. (For table of sizes of coal, see pp. 881 and 882.) The results are somewhat dependent on the way the coal breaks and the kind of screen used, whether bar, woven wire or perforated metal with round or square holes. There is no universal standard of sizes or kinds of screens nor of sizes of coal corresponding to certain names. Each coal district has certain sizes and names peculiar to itself and to the trade it supplies. Sizes and trade names of Illinois washed coals are given in University of Illinois Bulletin No. 9. All anthracite coal passing through the 3/32-in. openings when being screened is sometimes designated as buckwheat No. 4. In the Delaware, Lackawanna and Western region the term "birdseye" is used in place of buckwheat No. 3. In the Lehigh region the four sizes below 1/2 in. are designated as buckwheat, rice, barley and birdseye, respectively. In some localities, everything under stove size is sold as screenings. Coal which has been properly sized and graded may be greatly affected by breakage due to frequent handling and to weathering, so that inspection of size to be just should be at the place of sizing. No figures are available showing the degradation of size in bituminous coal due to handling.

Weight and Specific Gravity of Coals. According to Walter R. Johnson, in a report on American coals to the U. S. Navy Department, the specific gravity of anthracite (average of 7 samples) is 1.5, and its average weight (from 312 weighings made in a box holding 2 cu. ft.) is 53.4 lb. per cu. ft. The specific gravity of bituminous coal (average of 12 samples from Md. and Pa.) is 1.359, and its average weight (based on 328 weighings) is 52.8 lb. per cu. ft. The weight of coal as received (in box holding 8 cu. ft.) has been determined by the U. S. Bureau of Mines to be as follows:

Location of mine	Lb. per cu. ft.	Location of mine	Lb. per cu. ft.
Alabama (1 sample)	51.1	Northern China (avg. of 3 samples)	50.5
Illinois (1 sample)	49.3	Japan (avg. of 4 samples)	51.4
West Virginia (1 sample)	52.4	Australia (1 sample)	48.4

Solid coal weighs 85 to 94 lb. per cu. ft. Wilkesbarre anthracite lump weighs 69.4 lb. per cu. ft.; pea coal of same mine, 61 lb.

Heating Value of Coals. The calorific value of a fuel may be approximated by computing the various calorific values of its constituents as shown by the ultimate analysis. **Dulong's formula,** which follows, is used for this purpose, and gives results within 1 1/2 per cent. for most coals not high in moisture, such as anthracite, semi-bituminous and bituminous.

$$\text{B.t.u. per lb.} = 14{,}544C + 62{,}028\,[H - (O/8)] + 4050S$$

This formula assumes that the oxygen in the coal is combined with part of the hydrogen to form water, the remaining hydrogen being available for combustion together with the carbon and sulphur. The more direct determination of the calorific value is that made by using a **calorimeter.** See p. 503. With careful calorimetric work the errors can be kept within 0.3 per cent., or about 50 B.t.u. in high-grade coal. The greatest errors are due to the difficulty of obtaining a truly representative sample. With ordinary careful commercial sampling it is believed the errors are of the order of about 100 B.t.u. on high-grade coal. Where the sampling and calorimetric methods are not known to be of the highest order, the last 200 B.t.u. should be considered as in a region of more or less doubt. For the combustion of coal, see pp. 369 and 881 to 892.

Weathering. Coal suffers a **deterioration in calorific value** by exposure to air, but the amount is small. There is a slight oxidation at the surface of coal at ordinary temperatures, so that fine coal deteriorates more than lump. **West Virginia "smokeless" fine slack exposed to the weather in a temperate climate lost less than 1 per cent. in heating value in 1 year;** run of mine less than half as much; Pittsburgh run of mine less than half of 1 per cent.; Illinois run of mine about 2.5 per cent.; and Wyoming sub-bituminous about 3.5 per cent. The loss of gases or volatile matter from coal during storage at ordinary temperatures is so slight as to be inconsiderable in its effect on heating value (H. C. Porter, Bureau of Mines).

Table 1. Analyses of Mine Samples and Average Analyses of Delivery Samples of Anthracite Coal

Locality, bed, etc., or commercial name of coal	Proximate analysis "as received"				Ultimate analysis "as received"				B.t.u.	
	Moisture	Volatile matter	Fixed carbon	Ash	Sulphur	Hydrogen	Carbon	Oxygen	"Dry" coal	Ash- and moisture-free
Mine Samples										
ALASKA										
Carbon Mountain, 10½-ft. bed	13.89	5.01	73.87	7.23	0.82	.....	.....	.....	14,096	15,388
Second Berg Lake, 2⅔-ft. bed	3.74	5.41	85.92	4.93	1.10	.....	.....	.....	.....	.....
Matanuska, 38-ft. bed	2.55	7.08	84.32	6.05	0.57	.....	.....	.....	14,071	15,003
COLORADO										
Crested Butte, highest bed worked	2.7	3.32	88.15	5.83	0.80	3.28	85.38	3.59	14,490	15,413
Crested Butte, No. 1 bed	3.25	3.65	87.72	5.38	0.94	3.50	84.53	4.12	14,503	15,356
Floresta, Ruby anthracite mine	3.0	3.0	86.5	7.5	0.69	2.67	83.20	4.48	13,920	15,080
Somerset	4.22	3.98	76.02	15.78	0.88	.....	.....	.....	12,807	15,334
Cokedale, (natural coke)	0.57	0.39	78.24	20.80	0.54	.....	.....	.....	.....	.....
NEW MEXICO										
Madrid, White Ash bed	5.70	2.18	86.13	5.99	0.69	2.38	82.87	6.81	14,071	15,025
PENNSYLVANIA										
Minersville, Diamond bed	2.76	2.48	82.07	12.69	0.54	2.23	79.22	4.64	12,933	14,873
St. Nicholas, Mammoth (middle split) bed	2.30	1.54	82.77	13.39	1.05	2.13	78.96	3.81	12,818	14,855
Tower City, Lykens (No. 5) bed	3.33	3.27	84.28	9.12	0.60	3.08	81.35	5.06	13,810	15,248
Bernice, B(6 ft.) bed (semi-anthracite)	3.38	8.47	76.65	11.50	0.63	3.58	78.43	4.86	13,617	15,457
RHODE ISLAND										
Portsmouth, "Middle" 6-ft. bed	13.26	2.56	65.30	18.88	0.30	1.88	64.23	14.49	10,737	13,723
Delivery Samples of Pennsylvania Anthracite										
Morea and Lehigh Valley broken	3.38	4.86	82.45	9.31	0.60	.....	.....	.....	13,514	15,120
Lehigh egg	3.90	.....	.....	.....	.....	.....	.....	.....	13,270	14,988
Bernice, Sullivan County, (semi-anthracite) egg	2.12	8.80	73.43	15.65	0.58	.....	.....	.....	12,847	15,292
Deringer, Luzerne County, pea	5.11	4.99	74.87	15.03	0.62	.....	.....	.....	12,468	14,815
Philadelphia & Reading, buckwheat No. 1	4.58	4.56	72.99	17.87	0.59	.....	.....	.....	11,993	14,757
Girard Mammoth, Schuylkill County, buckwheat No. 2	6.21	5.64	70.70	17.45	0.56	.....	.....	.....	11,879	14,593
Philadelphia & Reading, rice	5.07	4.99	71.58	18.32	0.75	.....	.....	.....	11,989	14,856

Table 2. Analyses of Mine Samples and Average Analyses of Delivery Samples from the Principal Bituminous Coal Regions of the United States

[Analyses marked (*) represent delivery samples believed to be comparable to mine sample immediately preceding]

Locality, bed, etc., or commercial name of coal	Proximate analysis "as received"				Ultimate analysis "as received"				B.t.u.	
	Moisture	Volatile matter	Fixed carbon	Ash	Sulphur	Hydrogen	Carbon	Oxygen	"Dry coal"	Ash- and moisture-free
ALABAMA										
Belle Ellen, coke bed	3.16	31.05	59.56	6.23	1.20	5.33	78.28	7.59	14,602	15,604
Blocton, Thompson bed	3.21	32.05	60.79	3.95	0.60	5.52	77.91	10.82	14,490	15,106
* Blocton Cahaba red ash lump	1.90	32.84	60.68	4.58	1.05	....	....	....	14,658	15,376
Dolomite, Pratt bed	3.08	26.86	66.31	3.75	0.55	5.30	82.04	6.88	15,147	15,757
* Pratt lump	1.07	29.06	62.20	7.67	1.50	....	....	....	14,289	15,489
Warrior, Jefferson bed	2.18	31.71	63.32	2.79	1.07	4.98	80.86	8.59	15,145	15,590
Tillman, Harkness bed	3.39	30.69	57.08	8.84	2.34	5.18	73.81	8.30	13,832	15,224
Carbon Hill, Jagger bed	4.71	31.80	53.32	10.17	1.33	....	....	....	13,219	14,798
ALASKA (Bering River)										
Bering Lake	5.14	13.90	75.96	5.00	1.16	4.50	80.68	7.28	14,828	15,651
Carbon Mt. (elevation 950 ft.)	7.64	9.82	76.31	6.23	0.57	4.43	77.30	10.02	14,139	15,161
Carbon Mt. (elevation 900 ft.)	2.95	6.81	75.74	14.50	1.08	3.69	74.09	5.46	13,176	15,491
Kings Creek, 9-ft. 11-in. bed	2.93	21.85	63.09	12.13	0.59	....	....	....	13,757	15,723
Chicago Creek	38.31	24.41	30.20	7.08	0.90	6.78	37.86	46.72	....	....
ARKANSAS										
Clarksville, Spadra bed	1.72	10.46	79.50	8.32	2.49	....	....	....	14,110	15,415
Bonanza, Hartshorne bed	1.99	15.90	75.05	7.06	1.05	....	....	....	14,373	15,489
Midland, Hartshorne bed	3.97	16.86	73.26	5.91	1.53	....	....	....	14,824	15,797
COLORADO										
Lafayette	19.15	30.82	44.27	5.76	0.25	5.93	56.38	30.60	11,894	12,807
Canon City, lower, 3⅔-ft. bed	11.19	36.77	45.75	6.29	0.92	5.44	62.50	23.89	12,708	13,676
Crested Butte, 11-ft. bed	2.98	33.62	56.16	7.24	0.39	5.39	74.46	11.00	13,840	14,956
Walsenburg, Robinson bed	6.90	34.94	49.70	8.46	0.53	5.45	69.15	15.39	12,973	14,269
Berwind, Berwind bed	3.31	32.55	53.75	10.39	0.74	5.30	72.61	9.72	13,644	15,287
* Green Canon run of mine	3.87	35.48	47.98	12.67	0.72	....	....	....	12,816	14,762
ILLINOIS										
Benton, No. 6 bed	9.46	33.55	48.87	8.12	1.63	....	....	....	13,243	14,548
Ziegler, No. 6 bed	11.50	26.70	52.67	9.13	0.60	5.28	65.40	18.51	12,893	14,377
* Ziegler	8.11	....	....	....	....	....	....	....	12,433	14,347
LaSalle, No. 2 bed	13.87	37.26	38.56	10.31	3.44	....	....	....	12,755	14,486
Staunton, No. 6 bed	13.29	37.07	40.74	8.90	4.12	....	....	....	12,872	14,344
* Staunton lump	11.29	35.95	41.41	11.35	3.80	....	....	....	12,291	14,094
* Staunton lump	10.89	36.93	39.80	12.38	3.90	....	....	....	12,151	14,111
Collinsville, No. 6 bed	11.87	36.57	39.98	11.58	4.75	....	....	....	12,218	14,067
O'Fallon, (Belleville), No. 6 bed	11.17	39.31	39.20	10.32	4.22	....	....	....	12,634	14,296
Harrisburg, No. 5 bed	5.56	34.41	51.31	8.72	2.87	5.16	68.75	13.20	13,386	14,747
Springfield, No. 5 bed	13.89	33.96	40.89	11.26	3.83	....	....	....	12,352	14,209
* Sangamon County run of mine	13.86	....	....	....	....	....	....	....	12,209	14,162
Carterville, No. 6 bed	7.88	31.20	49.89	11.03	2.99	5.14	66.50	13.30	12,746	14,479
INDIANA										
Linton, No. 4 bed	13.53	33.54	45.38	7.55	0.95	....	....	....	13,573	14,873
Bicknell, No. 6 bed	10.60	38.06	43.04	8.30	3.69	....	....	....	13,145	14,492
Diamond, Brazil Block upper bed	13.70	35.94	44.45	5.91	2.66	....	....	....	13,824	14,841
Rosedale, No. 3 bed	11.54	39.49	49.35	9.62	4.41	....	....	....	13,176	14,783
Hymera, No. 5 bed	12.14	35.17	43.73	8.96	3.54	....	....	....	13,108	14,594
Terre Haute, No. 7 bed	13.73	35.54	42.08	8.65	3.00	....	....	....	13,167	14,636
Boonville, No. 5 bed	10.41	39.18	41.96	8.45	3.51	....	....	....	13,192	14,566
IOWA										
Centerville, lower Mystic bed	17.13	35.44	40.36	7.07	4.00	....	....	....	13,190	14,422
Chariton, lower bed	18.69	31.80	41.78	7.73	2.39	....	....	....	12,920	14,278
Hamilton, Big Vein bed	15.65	36.87	35.84	11.64	5.10	....	....	....	12,197	14,150
Altoona, third bed	14.42	37.81	36.78	10.99	5.89	....	....	....	12,433	14,265

Table 2—(continued)

Locality, bed, etc., or commercial name of coal	Proximate analysis "as received"				Ultimate analysis "as received"				B.t.u.	
	Moisture	Volatile matter	Fixed carbon	Ash	Sulphur	Hydrogen	Carbon	Oxygen	"Dry" coal	Ash- and moisture-free
IOWA—(Con'd)										
Laddsdale, third bed	11.35	38.65	39.49	10.51	4.72				12,798	14,519
KANSAS										
Scammon, lower Weir-Pittsburg bed	2.54	35.31	52.28	9.87	4.47				13,687	15,230
West Mineral, lower Weir-Pittsburg bed	5.11	32.60	53.39	8.90	4.34				13,622	15,032
* Deep shaft lump	3.28	34.01	52.05	10.66	4.28				13,379	15,036
Fleming, lower Weir-Pittsburg bed	2.91	35.81	51.73	9.55	3.79				12,336	14,791
* Fleming run of mine	4.21								13,268	15,062
Frontenac, lower Weir-Pittsburg bed	5.28	33.95	51.61	9.16	3.99				13,738	15,208
* Cherokee run of mine	4.74	30.63	48.09	16.54	5.37				12,246	14,817
KENTUCKY										
Straight Creek, Straight Creek bed	2.91	36.01	57.55	3.53	0.89				14,753	15,309
Barnsley, No. 9 bed	7.98	37.55	45.17	9.30	4.03				13,001	14,467
* St. Charles and Fox Run coal	7.97								12,450	14,447
Central City, No. 9 bed	8.76	35.02	46.80	9.42	4.07				13,113	14,623
Hellier, lower Elkhorn bed	3.41	32.08	58.78	5.73	0.53	5.22	77.01	10.33	14,420	15,328
Wheatcroft, No. 11 bed	4.61	38.17	49.82	7.40	3.33				13,482	14,616
Kensee, Jellico bed	5.02	36.08	54.47	4.43	0.92				14,328	15,028
MARYLAND										
Eckhart, Pittsburg bed	3.64	16.07	72.09	8.20	0.99				14,413	15,754
* Georges Creek run of mine	2.89								14,213	15,612
Eckhart, Tyson bed	2.99	15.73	73.76	7.52	0.95				14,591	15,817
* Georges Creek run of mine	2.99								14,361	15,622
Frostburg, Pittsburg bed	2.63	16.19	73.86	7.32	0.94				14,623	15,811
* Big Vein, Georges Creek	1.83								14,437	15,696
Lord, Big Vein or Pittsburg bed	1.86	15.70	74.82	7.62	0.90	4.52	80.87	4.33	14,467	15,683
Midland, Tyson bed	3.66	16.88	71.41	8.05	1.02				14,455	15,773
Westernport, 6-ft. bed	2.47	14.03	73.95	9.55	1.23				14,204	15,743
MICHIGAN										
Saginaw, Saginaw bed	11.91	31.50	49.75	6.84	1.24	5.84	66.56	18.33	13,374	14,499
St. Charles, Saginaw bed	12.12	30.10	48.67	9.11	1.40					
MISSOURI										
Novinger, Bevier bed	17.19	34.05	39.48	9.28	2.76				12,798	14,414
Windsor, Bowen bed	14.17	32.30	43.33	10.20	3.73					
Lexington, Lexington bed	14.45	33.12	39.73	12.70	3.69					
Bevier, Bevier bed	14.74	38.53	38.95	7.78	3.79				13,118	14,436
MONTANA										
Red Lodge, No. 1 bed	11.69	36.14	40.19	11.98	1.05	5.26	55.46	25.05	11,083	12,821
Belt, Belt Creek bed	7.05	25.47	49.34	18.14	1.67	4.36	58.10	17.09	10,888	13,529
Lewiston, 58-in. bed	15.35	28.27	48.08	8.30	4.53	5.42	61.15	19.89	12,539	13,903
Musselshell, Buckey bed	16.66	27.85	48.07	7.42	1.00	5.61	59.22	25.78	12,271	13,469
NEW MEXICO										
Raton, Sugarite bed	2.12	36.06	50.22	11.60	0.64	4.94	69.96	11.53	13,246	15,026
Gallup, Thatcher, 4-ft. bed	9.68	41.42	40.82	8.08	1.55				12,869	14,134
NORTH DAKOTA										
Scranton, upper bed	41.43	23.86	28.45	6.26	0.74				10,656	11,932
Wilton, 9½-ft. bed	40.53	27.05	27.37	5.05	0.76				11,171	12,209
Williston, middle bed	36.60	32.93	25.69	4.78	0.48				10,762	11,626
OHIO										
Bellaire, No. 8 bed	3.10	40.76	50.11	6.03	3.42				14,031	14,962
Flushing, Meigs Creek bed	4.13	39.22	48.69	7.96	4.15				13,651	14,888
Danford, No. 7 bed	6.28	35.81	50.61	7.30	3.55				13,552	14,695
Wellston, No. 5 bed	9.38	36.74	46.26	7.62	4.08				13,129	14,335

Table 2—(*continued*)

Locality, bed, etc., or commercial name of coal	Proximate analysis "as received"				Ultimate analysis "as received"				B.t.u.	
	Moisture	Volatile matter	Fixed carbon	Ash	Sulphur	Hydrogen	Carbon	Oxygen	"Dry coal"	Ash- and moisture-free
OHIO—(*Con'd*)										
* Milton Jackson lump	9.28	....	....	....	....	....	....	....	12,995	14,377
Bradley, No. 8 bed	4.06	38.49	49.70	7.75	3.67	....	....	....	13,703	14,908
Dixie, No. 6 bed	8.92	38.58	46.65	5.85	3.00	....	....	....	13,536	14,465
OKLAHOMA										
Lehigh, McAlester bed	7.07	36.41	45.68	10.84	3.64	5.13	64.38	14.57	12,341	13,970
Chant, McCurtain bed	2.37	19.26	69.54	8.83	1.03	4.57	79.39	4.63	14,177	15,586
Henryetta, Henryetta bed	8.87	34.82	47.68	8.63	1.62	....	....	....	13,273	14,661
Buck, lower Hartshorne bed	3.55	34.01	54.88	7.56	1.22	....	....	....	14,083	15,282
PENNSYLVANIA										
Creighton, upper Freeport bed	2.53	33.99	54.50	8.98	2.21	1.00	....	....	13,703	15,094
Scott Haven, Pittsburg bed	2.60	32.67	59.41	5.32	0.77	5.39	78.16	8.91	14,461	15,297
Barnesboro, lower Freeport bed	2.64	22.98	67.80	6.58	1.38	....	....	....	14,609	15,667
* Delta run of mine	1.98	21.43	67.75	8.84	2.23	....	....	....	14,185	15,591
Johnstown, upper Kittanning bed	5.60	14.04	70.86	9.50	2.33	....	....	....	14,074	15,649
Nanty Glo, lower Kittanning bed	1.85	20.14	72.0	6.01	1.84	....	....	....	14,835	15,802
* Cardiff run of mine	2.12	....	....	....	....	....	....	....	14,671	15,728
* Star run of mine	3.05	19.86	70.25	6.84	1.79	....	....	....	14,565	15,673
St. Bonifacius, lower Freeport bed	2.19	23.84	65.36	8.61	1.84	4.80	78.47	4.99	14,188	15,557
Vintondale, lower Kittanning or B bed	1.78	20.24	71.0	6.98	2.34	....	....	....	14,650	15,771
* Vintondale	2.18	....	....	....	....	....	....	....	14,445	15,674
Blue Ball Station, Brookville bed	1.9	22.0	66.3	9.8	1.95	4.66	78.05	4.40	14,030	15,590
Lower Kittanning or B bed	3.84	19.62	69.25	7.29	2.51	....	....	....	14,474	15,660
* Acme run of mine	2.56	21.00	68.56	7.88	2.19	....	....	....	14,423	15,693
Smoke Run, D "lower split" bed	3.2	21.0	69.3	6.5	0.69	4.85	79.90	6.76	14,530	15,570
Connellsville, Pittsburg bed	2.82	29.97	59.84	7.37	1.22	....	....	....	14,396	15,579
Glen Campbell, upper Kittanning bed	3.46	23.09	67.17	6.28	1.03	....	....	....	14,558	15,572
Punxsutawney, lower Freeport bed	2.53	27.82	63.95	5.70	1.19	....	....	....	14,670	15,581
* Punxsutawney	3.04	29.79	68.85	6.32	1.23	....	....	....	14,480	15,490
Boswell, upper Kittanning bed	4.68	15.64	73.55	6.13	0.75	....	....	....	14,610	15,615
* Orenda run of mine	2.26	....	....	....	....	....	....	....	14 060	15,614
Elk Lick, Pittsburg bed	3.04	19.59	70.33	7.04	0.74	....	....	....	14,618	15,763
* Elk Lick run of mine	2.66	....	....	....	....	....	....	....	14,131	15,665
Jenner, upper Kittanning bed	3.99	15.67	74.06	6.28	0.67	....	....	....	14,722	15,754
Jerome, upper Kittanning bed	2.78	16.14	73.63	7.45	0.77	....	....	....	14,413	15,610
Meyersdale, Pittsburg bed	2.71	19.34	71.29	6.66	0.72	....	....	....	14,692	15,772
* Pine Hill and Elk Lick	2.43	20.89	70.85	5.83	1.19	....	....	....	14,758	15,695
Marianna, Pittsburg bed	1.44	34.61	57.77	6.18	0.78	5.23	78.76	7.61	14,450	15,417
Greensburg, Pittsburg bed	2.73	30.34	57.80	9.13	1.33	....	....	....	13,997	15,446
Herminie, Pittsburg bed	2.01	33.56	58.11	6.32	1.39	....	....	....	14,441	15,439
* Youghiogheny gas lump	2.10	35.02	54.84	8.04	1.72	....	....	....	13,948	15,196
* Youghiogheny screenings	4.89	29.89	55.13	10.09	1.39	....	....	....	13,499	15,101
TENNESSEE										
Dean bed	3.25	35.63	54.51	6.61	0.85	....	....	....	13,968	14,992
Lafollette, Rex bed	3.03	34.01	58.05	4.91	1.77	5.19	75.78	10.73	14,290	15,052
Fork Ridge, Mingo or Ralston bed	3.71	35.61	55.94	4.74	1.28	....	....	....	14,335	15,079
Waldensia, Lower Sewanee bed	3.80	30.72	60.98	4.50	0.78	....	....	....	14,742	15,466
Orme, Battle Creek bed	3.31	31.71	51.87	13.11	1.30	....	....	....	12,611	14,589
TEXAS										
Medina County	32.92	27.42	27.08	12.58	1.46	....	....	....	10,197	12,550
Rockdale, "Big Vein"	35.30	26.22	29.58	8.90	0.76	....	....	....	10,661	12,362
Hoyt	28.86	35.96	27.26	7.92	0.50	....	....	....	11,239	12,647

Table 2—(continued)

Locality, bed, etc., or commercial name of coal	Proximate analysis "as received"				Ultimate analysis "as received"				B.t.u.	
	Moisture	Volatile matter	Fixed carbon	Ash	Sulphur	Hydrogen	Carbon	Oxygen	"Dry" coal	Ash- and moisture-free
UTAH										
Castlegate, Castlegate bed	6.13	40.07	45.45	8.35	0.56	....	.....	.....	13,014	14,285
Thompson, 69¼-in. bed	6.35	31.89	42.74	19.02	0.58	5.01	59.10	14.97	11,320	14,206
Coalville, Wasatch bed	14.20	36.00	44.80	5.00	1.41	5.79	61.40	25.31	12,390	13,160
VIRGINIA										
Darbyville, No. 5 bed	3.42	34.36	58.83	3.39	0.58	5.25	77.98	11.51	14,634	15,167
Pocahontas, No. 3 bed	3.65	17.27	76.27	2.81	0.43	4.95	84.65	5.95	15,437	15,901
* Pocahontas run of mine	3.03	20.30	71.43	5.24	0.66	....	.....	.....	14,931	15,785
Richards, No. 4 bed	2.60	24.47	68.45	4.48	1.35	....	.....	.....	15,026	15,752
Georgel, Upper Banner bed	2.48	31.71	60.30	5.51	0.52	5.59	79.69	7.13	14,614	15,489
Stonega, Imboden bed	2.16	33.10	58.27	6.47	0.68	5.29	77.85	8.24	14,303	15,319
WASHINGTON										
Black Diamond, McKay bed	7.40	39.50	49.00	4.10	1.28	5.56	68.25	18.92	13,500	14,110
Cle Elum, Roslyn bed	8.00	34.50	45.46	12.04	0.45	5.50	63.64	16.98	12,451	14,326
Carbonado, No. 2 coking bed	3.84	27.05	53.74	15.37	0.39	4.99	68.20	9.03	12,825	15,264
WEST VIRGINIA										
Ansted, Ansted or No. 2 gas bed	1.90	33.34	59.89	4.87	0.64	....	.....	.....	14,731	15,502
Fayette, Sewell bed	3.22	22.28	71.68	2.82	0.55	5.11	83.07	6.89	15,192	15,647
* New River run of mine	2.22	20.07	73.90	3.81	0.69	....	.....	.....	15,160	15,777
* New River run of mine, Loup Creek mines	2.22	21.94	70.92	4.92	0.84	....	.....	.....	14,877	15,667
* New River, Admiralty Smokeless run of mine	2.14	21.34	71.79	4.73	1.01	....	.....	.....	14,895	15,651
McDonald, Sewell bed	4.40	18.50	72.80	4.30	0.70	....	.....	.....	14,950	15,660
Page, Eagle or No. 1 gas bed	2.39	31.72	61.33	4.56	1.62	5.09	79.85	7.76	14,819	15,545
Powellton, Powellton bed	1.98	34.41	59.85	3.76	0.85	....	.....	.....	15,035	15,637
* Kanawha gas run of mine	3.18	30.57	59.40	6.85	0.92	....	.....	.....	14,304	15,392
Clarksburg, Pittsburg bed	2.80	38.51	53.14	5.55	2.40	....	.....	.....	14,512	15,390
Acme, No. 2 gas bed	2.66	33.30	59.60	4.44	1.14	....	.....	.....	14,760	15,466
* Kanawha run of mine	2.84	32.24	57.76	7.16	1.66	....	.....	.....	14,263	15,398
Charleston, Black Band bed	3.91	34.83	54.48	6.78	0.64	....	.....	.....	14,020	15,082
Monarch, Monarch mine, Cedar Grove bed	3.13	35.5[illegible]	57.82	3.54	0.59	....	.....	.....	14,414	14,960
Maybeury, Pocahontas No. 3 bed	3.35	14.72	78.14	3.79	0.45	4.79	84.38	5.46	15,179	15,799
Roderfield, Welch bed	2.70	18.50	75.50	5.30	....	....	.....	.....	14,930	15,782
West Vivian, Pocahontas No. 3 bed	1.90	14.00	79.20	4.90	0.55	....	.....	.....	15,000	15,780
* Pocahontas run of mine	2.63	18.25	73.87	5.25	0.64	....	.....	.....	14,921	15,771
* Pocahontas run of mine	2.07	16.84	74.14	6.95	0.68	....	.....	.....	14,653	15,773
Monongah, Pittsburg bed	3.13	32.96	56.41	7.50	0.90	5.35	74.99	9.77	13,941	15,111
Coopers, Pocahontas No. 3 bed	3.78	15.40	76.80	4.02	0.84	4.69	84.04	5.26	15,187	15,849
Elk Garden, Sewickley bed	2.82	17.70	71.15	8.33	1.04	....	.....	.....	14,365	15,711
Elk Garden, Big Vein (Georges Creek) bed	0.82	16.10	73.87	9.21	1.10	....	.....	.....	14,220	15,674
Price Hill, Sewell bed	4.20	18.00	74.30	3.50	1.40	....	.....	.....	15,190	15,760
* New River run of mine	2.06	22.06	71.60	4.28	0.87	....	.....	.....	15,052	15,740
Sophia, Beckley bed	3.30	14.00	77.60	5.14	0.63	4.60	82.94	5.28	14,990	15,830
* New River run of mine	2.50	20.58	71.70	5.22	0.74	....	.....	.....	14,905	15,747
Thomas, Upper Freeport bed	1.69	20.85	71.16	6.30	0.60	....	.....	.....	14,772	15,783
Thomas, lower Kittanning or Davis bed	3.90	20.22	70.06	5.82	0.71	....	.....	.....	14,768	15,721
WYOMING										
Rawlins, 4½-ft. bed	10.14	33.87	47.55	8.44	0.49	5.17	63.75	20.83	12,251	13,520
Diamondville, Main Kemmerer bed	5.13	40.51	49.75	4.61	0.49	5.63	72.95	15.14	13,664	14,362
Wiley	13.43	35.16	42.86	8.55	0.44	5.92	58.42	25.62	11,723	13,009
Dietz, Dietz No. 2 bed	22.38	31.85	39.42	6.35	1.16	6.32	52.25	32.73	11,912	12,974
Rock Springs, No. 1 bed	8.53	35.60	50.39	5.48	0.78	5.36	66.15	21.04	12,937	13,761

Spontaneous Combustion (H. C. Porter, Bureau of Mines) of coal results from accumulation of the heat of slow oxidation beginning at ordinary temperatures. If conditions are such that the oxygen of the air can be supplied so as to produce heat by oxidation faster than the heat can be dissipated, the temperature rises continuously until ignition results. Two main factors determine the liability of any coal to spontaneous ignition, (1), its friability or tendency to break up into fine coal and dust, and (2) the chemical character of its substance as manifested particularly in its rapidity of oxidation. The compactness of piling or the facility of penetration of air through the coal is one of the largest factors in promoting the progress of spontaneous heating. The oxidation rate of the sulphur compounds occurring in coal is not as great as that of the coal itself, and sulphur therefore is a lesser factor in spontaneous combustion. The relative amount of volatile matter is not in itself an important factor, although coals of high volatile matter frequently show a high rate of oxidation. The influence of moisture is uncertain, varied experience having led to contradictory opinions on the subject.

Spontaneous combustion in stored coal may be avoided by observance of the following precautions, and, of the first three, any one alone will in most cases serve to insure safety:

(1) Store in small piles, no interior part of which is more than 8 ft. from an air-cooled surface.

(2) Store only screened coal, larger than 1 in., eliminating fines and dust.

(3) Follow closely, by driving down iron rods for example, any rise of temperature, and rehandle those portions reaching 120 deg. fahr.

(4) In bunkers, bins or piles avoid draft of air *through* the coal, from open doors or chutes at the bottom or from the collection of lumps near the source of air supply. *Through* ventilation by pipes is dangerous.

(5) Avoid freshly crushed or freshly mined coal.

(6) Remove storage a reasonable distance from external sources of heat, such as boilers, furnaces, flues, steam pipes or hot cinder piles.

(7) Use the older portions of the storage first, avoiding accumulations of old coal in out-of-the-way corners and underneath new lots.

(8) For large stocks procure coal of the least possible friability and reduce to a minimum the breakage caused by drops and mechanical handling. Pile evenly, not in cones.

Coal Sampling. (G. S. Pope, Bureau of Mines Bulletin 63.) Portions are selected from all parts of a delivery of coal, combined into a gross sample, thoroughly mixed and reduced to quantities required by the chemist for his determination. A large gross sample and fine crushing reduce errors. Coal relatively easy to sample showed variations of 200 B.t.u. and 1.2 per cent. ash when 200-lb. gross samples were taken. When 1000-lb. gross samples were taken the variations were 37 B.t.u. and 0.22 per cent. ash. Further increase in the size of the gross sample gains but little in accuracy. It is practically impossible to get a representative sample of coal except while it is being handled. For approved method of obtaining laboratory sample (3 to 5 lb.), see "Specifications for the Purchase of Coal for Steam Power Plants," p. 604.

Methods Used in Proximate Analysis and Calorimetry. The following methods are those described by J. D. Davis in Bulletin 41, U. S. Bureau of Mines.

Preparation of Sample. The sample received at laboratory (2 to 4 lb.) is first reduced in a roll crusher so that it will pass a 20-mesh screen, and 3 oz. are immediately placed in a rubber-stoppered bottle for a later total moisture observation. The remainder is then thoroughly mixed and reduced by riffling to a quantity not more than 3 oz. For most anthracite samples and for any samples that are very wet or contain much foreign matter, the whole sample is thoroughly air-dried by placing it on a pan and in an oven through which air at 90 deg. fahr. is caused to circulate, until it loses no further moisture, as determined by successive weighings. It is then ground to pass a 20-mesh screen, riffled down to 8 oz. and then ground to a powder in a revolving porcelain jar about 2/3 full of flint pebbles. Three ounces of this powder are then placed in a rubber-stoppered bottle for the laboratory.

Determination of Moisture. One gram from the bottle sample (about 1/30 oz.) is weighed into a porcelain capsule (No. 2 size) of known weight, and placed in a drying oven and left there for about 1½ hr. This oven is of sheet copper and has four compartments each holding 12 samples, each compartment being surrounded by a bath of boiling glycerine (2 glycerine : 1 water), by which a uniform temperature of about 221 deg. fahr. is maintained. The capsule is then removed, covered with an aluminum lid and placed in a glass desiccator to cool without absorbing moisture. When cool it is again weighed, and the loss represents moisture.

Determination of Ash. The capsule containing the residue from the moisture determination is placed in an ordinary assayer's furnace or muffle and heated for 1½ hr. at about 1650 deg. fahr., with a slow draft of air circulating over it. This burns out all the combustible and drives off some volatile matter that is not combustible. The capsule is then allowed to cool and is weighed. To make sure that the combustible has been burned, the process is repeated.

Determination of Volatile Combustible Matter. One gram of the coal is weighed into a 30-c.c. platinum crucible of known weight, and the crucible is then covered, placed on supports and subjected for 7 min. to the heat of the 20-cm. carbon-free flame of a Fletcher burner, the bottom of the crucible being 7 cm. from the top of burner. It is then taken off, allowed to cool, and then weighed, the loss in weight representing volatile combustible matter.

Determination of Sulphur. See Bulletin 41, U. S. Bureau of Mines.

Determination of Calorific Value. The standard calorimetric outfit consists of a platinum-lined steel cup or bomb closed with a screw cap, and fitted with an oxygen valve, electrodes for electrical firing of the charge, a metal can for holding distilled water and the bomb, a mechanical device for stirring the water, a thermometer which can be read accurately to 0.001 deg. cent. by means of a cathetometer, and a double-walled felt-lagged metal jacket containing water, in which the can containing the bomb fits. (For calibration of calorimeters, see Circular No. 11, Bureau of Standards, 1911.) In determining the calorific value of a coal sample, 1 gram is weighed into a platinum tray and placed on a support inside of the bomb, a piece of platinum fuse wire being then connected to the electrodes and allowed to dip into the coal. The bomb cap is screwed into place and oxygen is forced in at a pressure of about 350 lb. per sq. in. The bomb is then placed in the weighed water. The temperature of the calorimeter is observed at minute intervals for 5 consecutive minutes, and at the end of the fifth minute the electric circuit is closed, firing the coal. The thermometer is now observed. The first two readings after firing are taken at half-minute intervals. Three more readings are taken at minute intervals. The maximum temperature will now have been reached and the thermometer is observed for five more consecutive minutes.

All the data for calculating the heating value of the coal have now been determined. Corrections must be made for the nitrogen content burned to nitric acid, and for the sulphur content which is burned to sulphuric acid. The **net heating value** is obtained by multiplying the rise of temperature caused by the combustion of the coal by the water value of the calorimeter. Calculations and corrections involved in this determination are given in detail in Technical Paper 8[a] of the Bureau of Mines.

Lignite contains so large an amount of water that the process of drying, either in storage or in the furnace fire, disintegrates the lumps into relatively fine stuff with attendant difficulties of handling and burning. It must be used shortly after being mined, requires a large grate with narrow air passages (see p. 888) and a large combustion space, and burns out quickly. The moisture in lignite as mined varies from 25 to 35 per cent. North Dakota dry lignite will average 50 per cent. fixed carbon, 42 per cent. volatile, and 8 per cent. ash. In a stove for house-heating twice as much lump lignite as mined was required as of Youghiogheny coal. With air-dry egg-size lignite 1½ times the amount was required. Some lignites are particularly liable to spontaneous combustion. Excellent producer gas can be made from lignite, see pp. 1057 to 1059.

Briquetting of Fuel. The advantages claimed for briquetted fuel are uniform size, smokelessness, uniform condition of fire bed, absence of clinkers, reduced attention to fires, increased heating value and efficiency, higher rates of combustion, reduced loss from breakage and weathering. Briquets should possess coherence to stand handling, be hard but not brittle, have a density equal to the lump coal from which they are made, should not absorb more than 3 per cent. of water, and should withstand exposure and retain their shape in the fire. The binder most commonly used is coal-tar pitch, but a great variety of substances singly or in combination has been proposed. The cost of briquets in eight plants in the United States ranges from $1.00 to $2.25 per ton plus the cost of the raw fuel. In Europe the cost is 25 to 50 cents, plus 50 to 80 cents for binder, plus the cost of the raw coal. The low price of good coal in the United States and the small difference in price between slack and lump confine the industry to relatively small fields at this time. Briquetting of North Dakota lignite residue after driving off moisture and a part of the volatile matter produces a product of value nearly equal to anthracite coal. The most favorable field for briquets is for locomotive and household use. Some fuels contain sufficient binding material to make good briquets under pressure and slight heat. Usually from 3 to 8 per cent. of binder is required. Too much pressure shatters the coal particles and weakens the briquets. The lack of waterproofing qualities is a serious defect in many binders. Mixing refuse coal dust as a mortar with clay, making it into balls by hand and air-drying them, is a household industry in some countries. The largest briquetting plant in the United States, at Superior, Wis., utilizes fine refuse coal from stock piles of Virginia coal and produces a product competing with anthracite for household use.

Pulverized Coal Fuel. Coal dust and low-grade coal may be successfully burned in the furnaces of specially designed vertical water-tube boilers (*Iron Age*, Oct. 2, 1913). The coal is fed into a hopper leading to a pulverizer, from which it is blown by air preheated by the waste gases through a vertical nozzle at the bottom of the furnace and then ignited. The flame is mushroom-shaped, returning upon itself on reaching the top of the furnace. The ash is converted by the intense heat into a liquid spray which strikes the fire-brick lining of the furnace and trickles down to its lower edge where it falls in the form of chilled globules into the ash pit. This spray protects the lining, rendering the use of fire clay unnecessary in laying the bricks. Preheating the air used by the blower makes it possible to burn coal containing 15 per cent. or more of moisture. Ignition is first effected by the use of a torch. The heat stored in the incandescent brick lining is sufficient to restart the fire after a temporary stop. The fuel consumption may be regulated in exact proportion to the demands made on the boiler, the air supply adjusted closely to the amount theoretically required, and the production of smoke reduced to a minimum. A very small stack will suffice, and steam can be raised very quickly—an advantage in taking care of peak loads.

Specifications for the Purchase of Coal for Use in Steam Power Plants. Following are extracts from the more important stipulations embodied in the form of specifications and contract devised by the U. S. Bureau of Mines.

Sampling. Samples to be regularly and systematically collected in increments of from 10 to 30 lb. (5 to 10 lb. may be taken for slack and small sizes of anthracite), so regulated that the gross sample shall not be less than 1000 lb. If coal contains an unusual amount of impurities, or slate, or the pieces of such impurities are very large, gross sample to be 1500 lb. or more. For slack and small sizes of anthracite without impurities in abnormal quantities or in pieces larger than ¾ in., a 500-lb. gross sample will be sufficient.

Gross sample to be crushed, mixed and reduced in quantity to convenient size for transmissal to laboratory. Crushing, by mechanical crusher or by hand tamping on smooth, tight floor (or on a heavy canvas). Mixing and reduction, by hand with a shovel, or by means of riffles and sampling machines. When prepared by hand, coal to be crushed to following approx. sizes before each reduction:

Weight of sample, lb.	1000 or more	500	250	125	60
Size, in.	1	¾	½	⅜	¼

The 60-lb. sample to be reduced to about 15 lb. by quartering or by use of riffling and sampling machines and divided into three 5-lb. parts, each placed in air-tight containers, one going to laboratory agreed upon, one to coal company and one to consumer, each part bearing an unbroken seal.

If moisture content is desired a separate special moisture sample must be taken. This sample, of approximately 100 lb., is to be accumulated by placing in a waterproof receptacle with a tight-fitting and waterproof lid small parts of the gross sample as collected, or it may be collected independently of the gross sample. The moisture sample is to be immediately crushed and reduced mechanically or by hand to about a 5-lb. quantity, which is to be immediately placed in a container and sealed air-tight. If prepared by hand, it should be rapidly crushed so that no pieces of coal or impurities are larger than ½ in., and it should be rapidly reduced by the "quartering" method on a canvas to the 5-lb. quantity.

Description of Coal Offered. The description of the coal should include the percentage content of moisture in the coal "as received," volatile matter in "dry coal," ash in "dry coal," sulphur in "dry coal," and the B.t.u. per lb. in "dry coal." The source of the coal should be indicated by giving the name of mine or mines and their location; the name or other designation of coal bed; the name of operator of mine or mines; the commercial name of coal; and the railroad on which mine or mines are located.

Award of Contract. Bids may be rejected from further consideration offering coals on which the consumer has information as possessing physical characteristics or volatile matter or sulphur or ash content, or because of clinkering or excessive refuse, or as having failed to meet the requirements of city smoke ordinances, or for other cause which would indicate them to be of a character or quality which the consumer considers unsuited for use in his storage space or in his power-plant furnace equipment.

In order to compare bids as to the quality of the coal offered, all proposals shall be adjusted to a common basis. The method used is to merge the four variables—ash content, moisture content, heating value, and price bid per ton—into one figure, the cost of 1,000,000 B.t.u. as follows:

(*a*) All bids are adjusted to the same ash percentage by selecting as the standard the proposal that offers coal containing the lowest percentage of ash. The difference in ash content between any given bid and this standard is divided by 2 and the price quoted in such bid multiplied by the quotient. The result is added to the price bid.

(*b*) The price as determined under (*a*) above is divided by the difference between 100 per cent. and the percentage of moisture guaranteed, the result being computed to the nearest tenth of a cent.

(*c*) The price as determined under (*b*) is multiplied by 1,000,000 and the result divided by the product of 2240 (or 2000) multiplied by the number of B.t.u. guaranteed.

After elimination of undesirable bids, the selection of the lowest bid of those remaining on the basis of the cost per 1,000,000 B.t.u. may be considered by the consumer as a tentative award, the consumer reserving the right to have practical service test or tests to determine the final award of contract.

Causes for Rejection. Coal containing 3 per cent. more moisture, or 4 per cent. more ash, or 3 per cent. more volatile matter, or 1 per cent. more sulphur, or 4 per cent. fewer B.t.u. than the specified guarantees or coal furnished from a mine or from mines other than those specified by the company, unless upon the written permission of the consumer, shall be considered subject to rejection, and the consumer may, at his option either accept or reject the same.

Price. The price named is to be adjusted as follows for variations in heating value, ash content, and moisture content from the standards guaranteed:

(a) When the coal is considered on a "dry-coal" basis, no adjustment in price shall be made for variations of 2 per cent. or less in the number of B.t.u. from the guaranteed standard. When the variation in heat units exceeds 2 per cent. of the guaranteed standard, the adjustment shall be proportional.

(b) No adjustment in price shall be made for variations of 2 per cent., or less, below or above the guaranteed percentage of ash on the "dry-coal" basis. When the variation exceeds 2 per cent. the adjustment in price shall be determined as follows: The difference between the ash content by analysis and the ash content guaranteed shall be divided by 2 and the quotient multiplied by the bid price and the result added to or deducted from the price adjusted for B.t.u. according to whether the ash content by analysis is below or above the percentage guaranteed. Adjustment to be figured to the nearest tenth of a cent. As an example, consider a coal delivered under a contract guaranteeing 10 per cent. ash at a bid price of $3 per ton. If the coal shows by analysis an ash content of 13.26 per cent. the adjustment in price would be $[(0.1326 - 0.10)/2] \times \$3 = 4.9$ cents per ton (deduction).

(c) The price shall be further adjusted for moisture content in excess of the amount guaranteed by the contractor, the adjustment being determined by multiplying the price bid by the percentage of moisture in excess of the amount guaranteed. *E.g.*, if a coal delivered under a contract guaranteeing a moisture content of 3 per cent., the price bid being $3 per ton, shows on analysis 4.58 per cent. moisture, the bid price should be multiplied by 1.58 (representing excess moisture), giving 4.7 cents as the deduction per ton.

Coke

Coke is the solid substance left after bituminous coal has been deprived of its volatile constituents by distillation. Distilled in a beehive oven the product has a long columnar structure, while from closed retorts (as in the by-product process) the structure is short and blocky, friable and large-celled. Both are used for metallurgical purposes. The theoretical **yield** of coke is the sum of the fixed carbon and ash of the coal, the actual yield averaging 64.7 per cent. for beehive and 75.1 per cent. for by-product coke. The average **weight** of coke is 28 lb. per cu. ft., *i.e.*, approximately 71 cu. ft. per short ton. About 58 per cent. of the sulphur in the coal is found in the coke. From 12 to 20 boiler h.p. can be generated from the heat from each beehive oven.

Cokes differ greatly in appearance and properties. They may be dull gray, light gray, silvery and occasionally iridescent. Coke is generally rough-surfaced, but that from near the walls of the oven is often smooth and glassy. Color is not a safe indication of quality. The structure may vary from porous to dense and heavy. For metallurgical purposes it should be hard and not easily crushed or broken, giving a clear, ringing sound when struck. Some cokes are soft, brittle, easily broken and give a soft, punky sound when struck. The ease of ignition varies widely. The cellular structure is what gives to coke its great advantage as a metallurgical fuel. Other things being equal, within certain limits, the more porous the coke the better the fuel. The limit to this advantage of porosity is reached when the strength of the cell walls is too greatly lessened due to the enlargement. Generally speaking, dense cokes are also soft, and porous cokes hard. The real **specific gravity** of coke is the specific gravity of the coke substance; the apparent specific gravity is the relation between the whole mass and an equal volume of water. Laboratory tests to determine the physical qualities of cokes are not very satisfactory. Shatter tests are probably of more value than crushing tests.

Coke made in beehive ovens from Connellsville coal is the **standard** with which all other cokes in the United States are compared. Average analyses of this coal and the coke produced from it are given in Table 3. Analyses of beehive and by-product cokes from various regions are given in Tables 4 and 5.

Table 3. Average Analyses of Connellsville Coal and Coke

	Volatile matter	Fixed carbon	Ash	Sulphur	Phosphorus	B.t.u.	Specific gravity	Apparent sp. gr.	Per cent. cells	Per cent. coke substance
	Per cent.									
Coal	30.00	62.00	8.00	1.25	0.012	15,000	......	......	......	......
Coke	0.93	87.43	11.64	0.90	0.017	12,600	1.90	1.03	46	54

Table 4. Percentage Analyses of Beehive Cokes from Various Regions

	Pa.	Va. and W. Va.	Ala., Ga., Tenn., Ky.	Colo., N. M., Utah	Mont.	Wash.
Moisture	0.23–0.91	0.07–0.60	0.75–1.34	0.28–1.46	0.27–1.18	0.92–1.20
Volatile matter	0.29–2.26	0.46–2.35	0.75–1.95	0.60–2.94	0.93–2.37	1.10–1.50
Fixed carbon	92.53–80.84	95.47–84.09	91.20–77.81	87.12–76.10	80.80–75.45	83.58–79.30
Ash	6.95–15.99	4.00–12.96	7.30–18.90	12.00–19.50	18.00–21.00	14.40–18.00
Sulphur	0.81–1.87	0.53–2.26	0.58–1.77	0.50– 0.75	2.00 and up	0.38–0.52

Table 5. Percentage Analyses of By-product Cokes

	Ala.	Ill. and Ind.	Md., Mass., N. J.	Mich., Minn., Wis.	New York	Ohio and W. Va.	Pa.
Moisture	1.92–10.64	0.73–7.00	8.00–11.00	1.80–2.64	0.58–5.50	1.35–3.00	0.20–11.00
Volatile matter	0.95–1.58	0.80–1.45	1.61–3.30	0.87–2.70	1.48–2.26	0.90–1.69	0.60– 1.95
Fixed carbon	89.02–84.59	89.84–87.00	86.99–85.73	91.93–87.98	89.49–79.95	88.10–87.35	89.68–84.65
Ash	9.40–14.16	8.63–11.55	9.75–12.12	7.20–9.70	8.48–16.26	8.75–10.92	9.52–13.40
Sulphur	0.71–1.08	0.60–0.83	1.18–2.37	0.68–0.97	0.69–1.58	0.79–0.95	0.93– 1.40

Peat

Peat is a mixture of water (85 to 95 per cent.) and partly decayed vegetable matter that is exceedingly friable and variable in quality. Color, from yellowish to brown and black; dry weight, from 7 to 60 lb. per cu. ft. Air-dried peat is from 8 to 18 times as bulky as coal, is easily kindled, burns freely, gives quick, intense heat, is clean—without soot or dust—and is liked for domestic use. Illuminating gas, producer gas and a variety of valuable by-products can be made from peat (for various uses, see Bulletin No. 16, U. S. Bureau of Mines). Table 6 gives percentage analyses and calorific values of peat from various localities.

Table 6. Analyses and Calorific Values of Air-dried Peat
(U S. Bureau of Mines)

Kind of peat	Locality	Water	Ash	Sulphur	Calorific value		
					Calories	B.t.u.	
						Air-dried	Water-free
Brown, fibrous	Fremont, N. H.	6.34	7.93	0.69	5161	9290	9920
Brown, fibrous	Hamburg, Mich.	7.50	6.55	0.28	5050	9090	10026
Light brown, fibrous	Rochester, N. H.	11.64	4.06	0.22	5042	9083	10280
Dark brown	Westport, Conn.	12.70	4.12	0.24	4772	8590	9839
Brown, structureless	New Durham, N. H.	6.06	17.92	0.88	4415	7947	8460
Brown	New Fairfield, Conn.	9.63	7.93	0.46	4367	7861	8698
Brown, fibrous	Westport, Conn.	19.69	3.23	0 19	4273	7691	9578
Brown	Kent, Conn.	12.10	7.22	0 83	4269	7684	8743
Brown, fibrous	Cicero, N. Y.	14.57	7.42	0 25	4209	7576	8869
Brown	Black Lake, N. Y.	8.68	16.61	0 99	4179	7522	8237
Brown, fibrous	La Martine, Wis.	9.95	16.77	0.79	4149	7468	8293
Salt marsh	Kittery, Me.	13.50	12.04	1.94	4066	7319	8462
Black	Greenland, N. H.	6.62	24.11	1.01	3992	7186	7695
Light brown, structureless	Waupaca, Wis.	6.62	24.44	0.65	3872	6970	7465
Brown, fibrous	Madison, Wis.	6.99	18.77	0.38	3857	6943	7628
Brown, sandy	Kent, Conn.	9.06	36.06	1.46	3291	5924	5924
Black	———, N. Y.	6.52	28.50	0.57	2867	5161	5521

Wood

The fuel values of different woods are nearly proportional to their dry weights, except that resinous woods give higher values. Ordinary dry cord wood contains about 25 per cent. of moisture. The weight and ash content of dry wood are given in Table 7 (see also p. 454). The proximate percentage analysis

Table 7. Weight and Ash Content of Dry Wood
(Wood dried to a constant weight at 212 deg. fahr.)

Name of wood	Ash, per cent.	Weight, lb. per cu. ft.	Name of wood	Ash, per cent.	Weight, lb. per cu. ft.
Ash, black	0.72	39	Hickory, pignut	0.99	56
Ash, white	0.42	39	Hickory, shagbark	0.73	51
Basswood	0.55	28	Locust, honey	0.80	42
Beech	0.51	45	Maple, red	0.37	38
Birch, sweet	0.26	47	Maple, sugar	0.54	43
Birch, yellow	0.31	41	Oak, red	0.26	45
Cedar, northern white	0.37	20	Oak, white	0.41	50
Cedar, red	0.13	31	Osage orange	0.68	48
Cedar, southern white	0.33	21	Pine, loblolly	0.26	34
Cypress, bald	0.42	29	Pine, lodgepole	0.32	25
Elm, rock	0.60	45	Pine, longleaf	0.25	44
Elm, slippery	1.69	44	Pine, shortleaf	0.29	32
Elm, white	0.80	34	Pine, sugar	0.22	22
Fir, Douglas	0.08	32	Pine, western yellow	0.35	29
Fir, white	0.85	23	Pine, white	0.19	24
Gum, red	0.61	37	Poplar	0.45	29
Gum, tupelo	0.52	39	Redwood	0.14	26
Hackberry	1.22	45	Spruce, Engelmann	0.32	21
Hemlock	0.46	26	Sycamore	0.46	35
Hickory, bitternut	1.03	47	Tamarack	0.09	46
Hickory, mockernut	1.06	53	Walnut, black	0.79	38

of oak wood made by the U. S. Bureau of Mines, the samples being obtained by sawing across the sticks, is as follows: Moisture, 10.3; volatile matter, 77.6; fixed carbon, 11.2; ash, 0.9; sulphur, 0.03. B.t.u. as received, 7589; in dry wood, 8460. Ultimate analysis shows that about 50 per cent. of the weight of wood is carbon, and about 6 per cent. hydrogen. Analyses, by Gottlieb, of different woods (dry) are given in Table 8.

Table 8. Analyses of Various Woods (Dry)

Name	C	H	N	O	Ash	Calories	B.t.u.
Oak	50.16	6.02	0.09	43.36	0.37	4620	8316
Ash	49.18	6.27	0.07	43.91	0.57	4711	8480
Elm	48.99	6.20	0.06	44.25	0.50	4728	8510
Beech	49.06	6.11	0.09	44.17	0.57	4774	8591
Birch	48.88	6.06	0.10	44.67	0.29	4771	8586
Fir	50.36	5.92	0.05	43.39	0.28	5035	9063
Pine	50.31	6.20	0.04	43.08	0.37	5085	9153

A small house-heating boiler in which a variety of fuels were burned evaporated 15,345 lb. of water from and at 212 deg. fahr. with amounts of fuel approximately as follows:

1 cord of dry oak wood,	1700 lb. of anthracite coal,
$1\frac{1}{8}$ cords of yellow pine,	1800 lb. of semi-bituminous coal,
$1\frac{1}{2}$ cords of spruce or birch,	2300 lb. of bituminous coal,
$1\frac{3}{4}$ cords of poplar, white pine, or cottonwood,	2800 lb. of sub-bituminous coal,
	3500 lb. of lignite.

Charcoal

Beech, birch and maple produce about 46 bushels of excellent charcoal per cord. Charcoal absorbs moisture rapidly up to 10 to 15 per cent., an average piece containing 84 per cent. carbon, 12 per cent. water, 3 per cent. ash and 1 per cent. hydrogen. Incomplete distillation leaves as much as 10 per cent. of volatile matter in some charcoal. The heating value is about 12,850 B.t.u. per lb.

Other Solid Fuels

Sawmill Refuse, consisting of saw dust, "hogged" or shredded wood, chips, etc., contains from 40 to 60 per cent. of moisture. The calorific value of redwood, pine, fir, hemlock, spruce and cedar refuse is practically 9000 B.t.u. per lb. of dry fuel.

Straw from which grain has been threshed has a calorific value of from 5000 to 6500 B.t.u. per lb., depending on its degree of dryness. **Corn** (shelled), sometimes used as an emergency fuel in the grain-growing states west of the Mississippi, has a calorific value of from 7800 to 8500 (dry) B.t.u. per lb.

Tan Bark, or the fibrous portion of oak bark remaining after it has been used in the tanning industry, contains about 66 per cent. moisture. The available heat per lb. is approximately 2700 B.t.u. The spent tan resulting from 1 lb. of dry ground bark used will produce an available heat of about 5700 B.t.u. In burning tan bark it is necessary to have a large combustion space, large areas of heated brickwork radiating to the fuel bed, and draft sufficient for high combustion rates. According to D. M. Meyers, from 1.5 to 2.08 h.p. may be developed from 1 sq. ft. of grate surface in horizontal return-tubular boilers.

Bagasse is sugar cane from which the juice has been extracted by pressure between the rolls of a mill. Its moisture content ranges from 42 to 53 per cent. Dry bagasse contains 43–47 per cent. C., 5.4–6.6 per cent. H, 45–49 per cent. O, and 1.5–3 per cent. ash, its calorific value ranging from 8000 to 8700 B.t.u. per lb. Porto Rico bagasse contains 0.4 per cent. N. The fuel value of green mill bagasse, according to "Steam," is approximately as follows, allowance being made for the heat required to evaporate its moisture and superheat the resulting steam to the stack-gas temperature:

Per cent. of extraction of weight of cane	Per cent. moisture in bagasse		B.t.u. available for steam generation		Lb. bagasse equivalent to 1 lb. of 14,000-B.t.u. coal	
	(a)	(b)	(a)	(b)	(a)	(b)
75	42.64	51.00	4139	3294	3.38	4.25
77	39.22	48.07	4475	3630	3.13	3.86
79	35.15	44.52	4874	3976	2.87	3.52
81	30.21	40.18	5359	4392	2.61	3.19
83	24.12	35.00	5958	5005	2.35	2.80
85	16.20	28.33	6716	5558	2.08	2.52

(a) Based upon tropical cane of 12 per cent. fiber and juice containing 18 per cent. of solid matter. (b) Based upon Louisiana cane of 10 per cent. fiber and juice containing 15 per cent. of solid matter.

LIQUID FUELS

Crude Oils and Their Distillates. Petroleum is a mineral oil composed of a series of hydrocarbons in various proportions. **Fuel oil** is either crude oil or the residue after the lighter oils have been removed. There is available for fuel 2 or 3 per cent. of the product from the Pennsylvania and Ohio fields; an amount from Ohio, Indiana and Oklahoma depending on a low price that will compete with coal, most of the California oil, and practically all from the Texas field. Crude oils from various fields show somewhat different compositions. As a rule, the specific gravity lies between 0.80 and 0.97, the flash point between 76 and 93 deg. fahr., and the composition by weight is 84 to 88 per cent. carbon and 11.5 to 14.5 per cent. hydrogen, with a small per cent. of impurities. Tables 9 and 10 give data on crude oils from various fields, and on their distillates. W. Inchley (*The Engineer*, vol. III, p. 155) finds that the higher heating value of liquid fuels is given very closely by the empirical expression

$$13{,}500\ C + 60{,}890\ H,$$

in which C and H denote respectively the parts by weight of carbon and hydrogen in the fuel. The last column of Table 10 (due to Mr. Inchley) gives values calculated from this formula. The heating value may also be roughly estimated by the formula: B.t.u. per lb. = 18,650 + 40 × (Baumé reading − 10).

Table 9. Analyses and Calorific Values of American Fuel Oils

Field	Gravity, Baumé	Specific gravity at 15 deg. cent.	B.t.u. per lb.	Wt. per gal., lb.	Ultimate analysis						Flash point deg. fahr.
					C	H	N	S	O	Undetermined	
Kern River, Cal.	14.78	0.9670	18,562	8.06	86.36	11.27	0.74	0.89	....	0.74	216
Colinga, Cal.....	17.29	0.9505	18,720	7.92	86.37	11.30	1.14	0.60	....	0.59	162
McKittrick, Cal.	15.83	0.9600	18,335	8.00	86.51	11.41	0.58	0.74	....	0.76	165
Midway, Cal....	16.14	0.9580	18,565	7.98	86.58	11.61	0.74	0.82	....	0.25	142
Sunset, Cal......	14.26	0.9705	18,419	8.09	85.64	11.37	0.84	1.06	....	1.09	160
Beaumont, crude	21.6	0.924	19,060	7.69	84.6	10.90	....	1.63	2.87	....	....
Beaumont, crude	21.3	0.926	19,481	7.71	83.2	12.41	....	0.50	3.83	....	....
Tampico, crude..	12 to 23	....	18,493	7.82	83.8	12.2	1.7	2.8	0.43	....	....

Table 10. Analyses and Calorific Values of Various Crude Oils and Their Distillates

	Specific gravity	Composition by weight				High heating value, B.t.u.	
		S	C	H	O + N	Observed	Calculated
Russolene	0.890	......	85.95	13.50	....	19,622	19,823
American kerosene	0.780	......	85.05	14.40	....	20,093	20,250
Russian crude	0.877	......	86.90	13.10	....	19,500	19,706
Java crude	0.867	......	87.10	12.70	....	19,177	19,490
Canadian crude	0.859	0.35	86.92	12.87	....	19,435	19,571
Texas crude	0.947	0.63	86.62	11.80	....	18,931	19,878
Solar oil	0.896	0.30	86.61	12.60	....	19,409	19,364
Coal oil	0.917	1.56	83.20	11.87	....	18,401	18,461
Russian crude, Caucasian	....	......	84.90	11.63	1.46	18,690	18,541
American Royal Daylight	0.797	......	85.70	14.20	....	20,101	20,214
American crude petroleum	....	......	86.894	13.107	....	19,642	19,712
Refined American petroleum.	....	......	80.585	15.10	4.32	19,955	20,074
Refined Russian petroleum	0.825	......	86.00	14.00	....	20,286	20,225

Oil is sold by the barrel of 42 gal. The A. T. & S. F. R. R. Co. found the evaporative value of coal and oil the same when the price of coal in tons was three and a half times the price of oil in barrels. Most experience falls within the limits of 3 to 4½ bbl. of oil as the equivalent of one long ton of coal.

Tests of 17 burners by the U. S. Naval Board showed the water evaporated from and at 212 deg. per lb. of oil to vary from 11.48 to 13.96 lb. Tests of stationary plants show equivalent evaporations of 14 to 16 lb. Fuel oil may be too viscous to flow in cold weather, and steam-heating devices are used to raise the temperature to about 150 deg. Reliable temperature control is necessary for successful handling as well as for economical use. Noxious and explosive gases that are heavier than air evolve from fuel oil, requiring the ventilation of containers and isolation from ignition sources.

Coal Tar, a by-product of the manufacture of coal gas and of coke, is a black, viscous liquid composed largely of aromatic hydrocarbons, 100 lb. of gas coal yielding from 4 to 5 lb. of tar weighing from 9.2 to 10.5 lb. per gal. (sp. gr., 1.10 to 1.26). It has a low flash point, and its lower calorific value is about 15,000 B.t.u. per lb. (higher, about 16,500). It can be burnt as a fuel in any furnace suitable for crude oil, by preheating to make it sufficiently fluid. It should pass through a strainer before reaching the burners.

Tar Oils (creosote oil, anthracene, etc.), resulting from the distillation of tar, have been used as Diesel-engine fuels in Germany. Specific gravity, 0.97 to 1.11; mean lower calorific value, about 16,000 B.t.u. per lb.

Gasoline. The average percentage composition of gasoline used in motors is carbon, 83.5 to 85; hydrogen, 15.5 to 15; nitrogen, oxygen, sulphur, etc., 1.0 to 0. The higher heating value lies between 19,000 and 21,000 B.t.u. per lb. From Inchley's formula it should be 20,710 B.t.u., but calorimetric determinations frequently show lower values. Specific gravity, 0.67 to 0.73. For the combustion of 1 lb. of gasoline the weight of air required is about 15 lb., or the volume of air at 62 deg. fahr. is approximately 200 cu. ft. When gasoline is used as a fuel for internal-combustion motors the liquid is vaporized and the vapor mixed with a suitable amount of air. Levin gives the following data regarding carbureted gasoline: One pound of gasoline vapor at 62 deg. fahr. and atmospheric presssure has a volume of

4.2 cu. ft. With 15 per cent. excess, 1 cu. ft. of vapor requires 54.2 cu. ft. of air, and taking 18,500 B.t.u. as the lower heating value per lb., the heating value per cu. ft. of the normal charge is 69.3 B.t.u. Approximate heating value, B.t.u per lb. = 18,320 + 40 (Bé. − 10).

Kerosene. The average composition is C = 84.5, H = 15.5, per cent. by weight; higher heating value, 20,000 to 23,000 B.t.u. per lb. Volume of 1 lb. of kerosene vapor at 62 deg. fahr., about 2.5 cu. ft.; and with 15 per cent. excess, 87.5 cu. ft. of air must be supplied per cu. ft. of kerosene vapor. The lower heating value per cu. ft. of normal charge (Levin) is 64.4 B.t.u. Approximate heating value, B.t.u. per lb. = 18,440 + 40 (Bé. − 10).

Benzol, obtained from by-product coke ovens, is used for denaturing and enriching alcohol. The specific gravity is 0.88 and the higher heating value about 18,000 B.t.u. per lb. (lower, 17,300 B.t.u.). Benzol has the chemical formula C_6H_6; hence 13.32 lb. of air are required for complete combustion of 1 lb. of benzol. 1 cu. ft. of benzol vapor at 62 deg. fahr. requires 36 cu. ft. of air for combustion and gives up 3700–3880 B.t.u.

Alcohol. See p. 621. The laws of the United States and France require that alcohol used as fuel must be **denatured.** It is required that: "To 100 volumes of ethyl or grain alcohol of strength of not less than 90 per cent. there must be added 10 volumes of methyl or wood alcohol and ½ volume of benzine; or 2 volumes of wood alcohol and ½ volume of pyridine bases." Commercial alcohol always contains water. Table 11, by Schöttler, gives the specific gravity and heating value of various mixtures of alcohol and water.

Table 11. Specific Gravity and Heating Value of Various Mixtures of Alcohol and Water

Per cent. of alcohol in mixture by volume	Per cent. of alcohol in mixture by weight	Specific gravity	Lower heating value per lb., B.t.u.
95	93.8	0.805	10,880
90	87.8	0.815	10,080
85	81.8	0.826	9,360
80	76.1	0.836	8,630
75	70.5	0.846	7,920
70	65.0	0.856	7,200

The equation of complete combustion of alcohol is:

$$C_2H_6O + 3O_2 = 2CO_2 + 3H_2O$$

1 cu. ft. of alcohol vapor at 62 deg. fahr. requires 14.5 cu. ft. of air for combustion. If the combustion is incomplete, according to the equation

$$C_2H_6O + O_2 = C_2H_4O_2 + H_2O,$$

the products are acetic acid and water. It is found that the presence of acetic acid causes rusting and corrosion, and that the addition of a considerable quantity of benzol to the alcohol serves as a safeguard against such corrosion. In France a mixture of equal parts of denatured alcohol and benzol has been used as an engine fuel. The analysis of the fuel was as follows: C, 0.690; H, 0.095; O, 0.146; H_2O, 0.069; higher heating value by test, 14,180 B.t.u. Table 12 gives combustion data for the various alcohol fuels. Approximate heats of combustion for American commercial alcohol in B.t.u. are as follows: Wood alcohol, 8140; 95 per cent. ethyl alcohol, low value 10,500, high value 11,900; denatured alcohol, 10,350.

Table 12. Combustion Data for Various Alcohol and Benzol Fuels

Fuel	O_2 required for combustion, lb.	Air required for combustion, lb.	Products of combustion, lb.			Approximate higher heating value, B.t.u. per lb.	Approximate lower heating value, B.t.u. per lb.
			CO_2	H_2O	N_2		
Ethyl alcohol (C_2H_6O)..	2.08	9.04	1.91	1.17	6.95	12,780	11,520
Methyl alcohol (CH_4O)..	1.5	6.50	1.38	1.12	5.0	9,550	8,400
Benzol (C_6H_6)..........	3.1	13.32	3.39	0.69	0.24	18,000	17,260
Denatured alcohol.......	1.81	7.83	1.66	1.15	6.02	11,600	10,500
50 per cent. mixture of alcohol and benzol....	2.45	10.60	2.53	0.92	8.16	14,200	13,300

Vapor Pressures of Liquid Fuels are shown in Fig. 1.

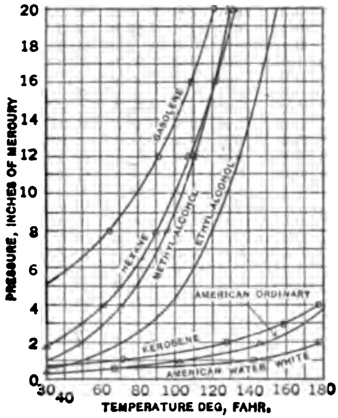

Fig. 1.

GASEOUS FUELS

The composition, heating value and other properties of gases employed as fuels are given in the following tables. See also pp. 316, 365 and 366. For **producer gas,** see pp. 372 and 1057 to 1059.

Table 13. Composition and Heating Value of Commercial Gases

Character of gas	Hydrogen, H_2	Methane, CH_4	Ethylene, C_2H_4	Nitrogen, N_2	Carbon-monoxide, CO	Oxygen, O_2	Carbon dioxide, CO_2	B.t.u. per cu. ft.	Authority
Oil gas.............	32.0	48.0	16.5	3.0	......	0.5	......	846	Wyer, S. S.
Coke-oven gas......	50.0	36.0	4.0	2.0	6.0	0.5	1.5	603	
Carbureted water gas	40.0	25.0	8.5	4.0	19.0	0.5	3.0	575	
Water gas..........	48.0	2.0	......	5.5	38.0	0.5	6.0	295	
Blast-furnace gas....	1.0	......	......	60.0	27.5	...	11.5	91	Wyer, S. S.
Pintsch gas.........	12.4	45.4*	35.7†	3.0	0.6	2.0	0.7	1500	

* Saturated hydrocarbons. † Unsaturated hydrocarbons.

Table 14. Properties of Natural Gas
(G. A. Burrell, Nat. Gas. Assn. of America, May, 1914)

Location of wells	Volumetric composition, per cent. Carbon dioxide, CO_2	Oxygen, O_2	Nitrogen, N_2	Methane, CH_4	Ethane, C_2H_6	Higher heating value, B.t.u. per cu. ft., 0 deg. cent. and 760 mm. pressure	Specific gravity, Air = 1
Armstrong Co., Pa.	0.05	0.0	1.45	81.6	16.9	1184	0.64
Osage Co., Okla.	1.10	0.0	4.6	94.3	0.0	1004	0.56
Kiefer, Okla.	2.40	0.0	1.8	64.1	31.7	1272	0.74
Barron Co., Ky*	2.5	0.0	1.3	23.6	69.7	1548	0.91
Barron Co., Ky.†	2.6	0.0	5.1	44.1	48.2	1367	0.84
Moab, Utah	3.6	0.0	5.6	90.8	0.0	967	0.61
Moab, Utah	3.5	0.0	6.5	90.0	0.0	959	0.62
Northwestern Oregon	3.0	0.0	0.9	96.1	0.0	1023	0.58
Crawford Co., Pa.	0.0	0.0	2.3	6.6	91.1	1766	1.01
Northwestern Oregon	0.5	0.0	12.5	87.0	0.0	927	0.60
Tillamook, Oreg.	0.1	0.0	97.9	2.0	0.0	21	0.96
Stillwater, Nev.	1.3	0.0	3.1	95.6	0.0	1018	0.58
Clarion Co., Pa.	0.0	0.0	1.1	96.4	2.5	1073	0.57
Forest Co., Pa.	0.0	0.0	1.0	70.8	28.2	1279	0.70
Clarion Co., Pa.	0.0	0.0	1.7	80.5	17.8	1189	0.65
Butler Co., Pa.	0.0	0.0	0.9	53.3	45.8	1420	0.78
Kings Co., Cal.	30.4	0.0	2.4	66.2	1.0	724	0.85
Greybull Field, Wyoming	0.2	0.0	0.8	81.7	17.3	1192	0.64
Casing head gas	0.0	0.0	1.3	51.5	47.2	1427	0.77
	0.5	0.0	3.1	64.1	32.3	1282	0.68
McKean Co., Pa.	0.0	0.0	1.0	86.0	13.0	1159	0.59
Caddo Parish Field, La.‡	0.9	0.0	1.5	97.6	0.0	1039	0.57
Park County, Okla.	0.0	0.0	1.8	94.4	3.8	1076	0.59
Bradford, Pa.	0.0	0.0	8.9	18.9	72.2	1534	1.00
Nortonville, N. D.	1.3	0.0	13.6	85.1	0.0	907	0.62
Schulto Field, Okla.	0.5	0.0	1.5	76.4	21.6	1215	0.67
Casing head gas used for production of gasoline§.	0.0	0.0	3.3	78.7	18.0	2424	1.38
From Pittsburg gas supply.	0.0	0.0	1.2	79.2	19.6	1208	0.65
From Columbus gas supply.	0.0	0.0	1.6	80.3	18.1	1193	0.64

* H_2S = 2.9 per cent.
† H_2S = 0.1 per cent.
‡ Little Rock, Ark., gas supply.
§ Propane = 78.7 per cent.

Blast-furnace Gas. Plottings of gas analyses from a group of 11 furnaces in operation during a continuous period of 2 years yield the following formulæ, in which R = lb. of dry coke per ton of pig iron:

Per cent. by volume of methane = constant = 0.2
Per cent. by volume of hydrogen = $0.000276\,R + 2.944$
Per cent. by volume of CO_2 = $24.67 - 0.0054R$
Per cent. by volume of CO = $16.15 + 0.00476R$

Using a coke containing approximately 88 per cent. carbon, and assuming that the carbon charged as coke reappears in the gas and that the carbon in the limestone is offset by the carbon in the pig iron and flue-dust produced, the following formulæ have been derived, in which T = tons of iron per 24 hr., the calorific values of CO, H and CH_4 in B.t.u. per cu. ft. being taken respectively as 324, 275 and 915:

B.t.u. per cu. ft. of gas = $0.016R + 62.5$
Cu. ft. gas per ton of iron = $71.4R$
B.t.u. in gas per ton of iron = $1.143R^2 + 4462.5R$
Cu. ft. of air at tuyères per ton of iron = $51.4R$
Cu. ft. of air per min. = $0.0357RT$.

Table 15. Properties of Coal Gas

(From U. S. Bureau of Mines Bulletin No. 6)

Source	Coke (per cent. of coal charged)	Gas: Per lb. of coal charged (cubic feet, corrected)	Gas: Average candle-power (approximate)	Gas: Candle-feet per lb. of coal	Gas: Average heating value per cu. ft., 60 deg. fahr. and 30 in. barometer (B.t.u.): Higher value	Gas: Average heating value per cu. ft., 60 deg. fahr. and 30 in. barometer (B.t.u.): Lower value	Gas: Heating value per lb. of coal (B.t.u.)	Average gas analysis (per cent.): Carbon dioxide	Illuminants	Oxygen	Carbon monoxide	Methane	Hydrogen	Nitrogen
Scott Haven, Pa.	66.9	5.11	15.5	79.50	641	582	3280	1.3	3.7	0.8	6.5	34.4	48.2	5.0
Blocton, Ala.	68.3	5.18	14.0	72.50	598	538	3100	2.7	3.7	1.0	8.9	31.2	49.1	3.4
Oak Creek, Colo.	59.9	4.81	14.0	67.20	626	566	3010	4.4	5.3	0.8	11.3	29.8	43.9	4.5
Sopris, Colo.	75.0	4.90	12.0	58.80	614	550	3008	1.3	4.2	0.9	6.0	29.5	55.8	2.3
Harrisburg, Ill.	62.3	4.30	15.2	65.30	632	568	2718	2.5	3.7	0.8	7.9	33.5	45.6	6.0
Hellier, Ky.	67.5	4.90	14.3	70.12	636	570	3118	1.8	4.2	0.9	6.7	31.3	50.8	4.4
Saginaw, Mich.	59.5	4.30	17.4	74.80	593	526	2550	2.9	5.3	0.7	9.2	34.1	43.1	4.7
Van Houten, N. M.	68.9	4.70	16.7	78.80	678	612	3037	2.1	6.0	0.9	8.2	35.2	45.8	2.1
La Follette, Tenn.	66.8	5.50	15.9	81.90	641	575	3525	2.0	5.0	0.9	8.2	31.2	49.1	3.6
Page, W. Va.	73.4	5.10	16.0	81.40	620	556	3159	1.3	3 6	0.8	5.0	32.8	54.5	2.2
Hanna, Wyo.	50.2	5.40	8.8	47.50	564	502	3046	7.6	4.9	0.2	14.4	29.0	40 2	3.7

Acetylene (92.3 C + 7.7 H by composition) see Table 48, p. 365, is a colorless gas having a specific gravity of 0.91. It liquefies under a pressure of 700 lb. per sq. in. at 70 deg. fahr. Acetylene as a fuel is confined to the oxy-acetylene blowpipe for cutting and welding metals, in which the combustion of a mixture consisting of 1 volume of acetylene and 1.7 volumes of oxygen produce a flame having a temperature of 6300 deg. fahr. Acetylene is obtained by bringing water into contact with calcium carbide, 1 lb. of chemically pure carbide yielding (at 70 deg. fahr. and 30 in. of mercury) 5.83 cu. ft. of the gas and precipitating slaked lime. The commercial carbide, containing impurities, yields around 5 cu. ft. per lb. Calcium carbide, CaC_2(62.5 Ca + 37.5 C by composition), is made by the reduction of an intimate mixture of quicklime (100 parts by weight) and powdered coke (68 parts) in an electric arc furnace, about 1.8 lb. of the mixture being required to produce 1 lb. of carbide. It is a crystalline, semi-metallic solid having a specific gravity of 2.22.

MISCELLANEOUS NON-METALLIC MATERIALS

BY

CARL F. WOODS

ABRASIVES

REFERENCES: Printed matter issued by the Norton Company and the Carborundum Company; "The Relative Efficiency of Different Abrasives in Common Use," *Sc. Am. Supp.*, Dec. 8, 1894.

Abrasives are of two general types, those used for removing metal or other stock and those used for burnishing or polishing. In leather or wood-working, for instance, the abrasive must be sufficiently hard to maintain its sharpness, while in cutting steel and other metals the cutting quality is maintained by the particles of the abrasive breaking away and constantly presenting new cutting surfaces.

Natural Abrasives

Corundum, the best of the natural abrasives, is a mineral composed of nearly pure alumina with a hardness of 9, in Mohs's scale, namely, next to that of the diamond. The artificial product, however, is much harder.

Emery, an impure form of corundum, has been extensively used for grinding wheels and abrasive paper on account of its cheapness. It is now used to some extent to mix with corundum and artificial abrasives, but has been largely superseded by the artificial products. Corundum and emery are graded according to the mesh of the screen through which they pass. These screens run 4, 6, 8, 10, 12, 14, 16, 20, 24, 30, 36, 40, 46, 54, 60, 70, 80, 90, 100, 120, 150, 180, and 200 to the inch. The flour, which is obtained by settling the overflow from the washers, is graded as 1F, 2F, 3F, 4F, 5F, and 6F.

The abrasive action of emery depends upon its corundum content; but chemical analysis is not a reliable guide as physical characteristics are more important. The amount of attrition produced by rubbing the powder between two weighed pieces of plate glass for a definite time as compared with that produced by a powder of known quality, is a good indication of the abrasive quality. Corundum is used mainly for the manufacture of vitrified wheels. For polishing, emery flour is better than corundum, as it does not scratch so much.

Garnet. Certain deposits of garnet having a hardness between quartz and corundum are used in the manufacture of abrasive paper. Quartz is also largely used for this purpose. In the electrical trades, quartz colored to imitate garnet is sometimes substituted. Garnet costs about twice as much as quartz and generally lasts proportionately longer. It is better for wood-working than the artificial abrasives, as the grains do not break down. Corundum is superior to garnet for leather working. A small quantity of feldspar is also used as an abrasive, but is much inferior.

Grindstones and Pulpstones are quarried directly and generally made from sandstone.

Buhrstones and Millstones are generally made from cellular quartz. **Chasers** (or stones running on edge) are also made from same mineral.

Oilstones. The great majority of oilstones are quarried in Arkansas and are known as "Arkansas" and "Washita" stones. The **Arkansas stone** is a true novaculite, homogeneous, gritty, and of a fine, siliceous composition. The hard variety is used for sharpening tools requiring a very fine edge, such as those of surgeons, engravers, dentists, etc. The soft variety is more porous and coarser and is used for less careful work.

Of same hardness as quartz, but very brittle. **Washita stone** is less dense, more porous and of the same general composition. The chief use is for whetstones and for coarser tools.

Pumice, of volcanic origin, is extensively used in leather, felt, and woolen industries, and in the manufacture of polish for wood, metal, and stone. An artificial pumice is made from sand and clay in five grades of hardness, grain, and fineness.

Infusorial Earth or Tripoli resembles chalk or clay in physical properties; it can be distinguished by absence of effervescence with acid; is generally white or gray in color, may be brown or even black. Owing to its porosity it is very absorptive. It is used extensively in polishing powders, scouring soaps, etc.; and on account of its porous structure, in the manufacture of dynamite as a holder of nitroglycerine; also as a non-conductor for steam pipes and as a filtering medium. Also known as **kieselguhr.**

Artificial Abrasives

Artificial abrasives, although more expensive than the natural, are rapidly superseding them owing to the absolute uniformity which it is possible to impart to the coarseness or fineness of the grade, thus enabling a perfect stone to be made with either a coarse, medium, or fine grain. It is also possible to produce a stone of unvarying hardness or mixture throughout, thus avoiding soft or hard spots, coarseness, and other defects in natural abrasives.

Aluminum Oxide. The artificial aluminum oxide products are best adapted as abrasives for high-tensile-strength metals such as steel. **Alundum** is made by the Norton Company from bauxite in the electric furnace. The physical formation of the grain of alundum is such that when it is broken or fractured it leaves sharp cutting corners or edges, but the material is very tough and stands up under the hardest service. It is also used in the manufacture of refractories and electric furnace parts. **Aloxite** and **corubin** are trade names of similar products. Melting point, around 3720 deg. fahr. Coefficient of expansion, 0.00000328 per deg. fahr.

Carbide of Silicon. Two products known as **carborundum** (Carborundum Co.) and **crystolon** (Norton Co.) are available. They are made by fusing together in an electric furnace, coke, sand, sawdust and salt. The abrasive so produced is characterized by great brittleness, the grains readily separating and breaking, thus presenting new cutting surfaces and preventing glazing. Carbide of silicon can be used for grinding most materials, but is generally employed for grinding metals of low tensile strength such as cast iron, brass, and aluminum, also marble, granite, etc. It is found uneconomical on high-tensile-strength metal, such as steel, on account of its rapid wear.

Grinding Wheels

Vitrified Wheels. In these the abrasive grains are bonded with clay and, after drying, are fired at a temperature sufficiently high to melt the bonding material (3000 deg. fahr. approx.). The majority of smaller abrasive wheels are made by this process, but wheels over 30 in. in diam. are not, as they are liable to warp in the firing. The hardness or grade of vitrified wheels is controlled by the composition of the bonding compound and by the firing temperature. The grade cannot always be exactly predetermined, so each wheel is tested and assigned to its proper grade. A common method for **testing for grade** is to dig into the wheel with a hard steel tool, the resistance of the grain to being dug out and removed determining the hardness of grain. In the larger works special instruments are used which are designed to make the grading more exact and to prevent the liability of error due to personal judgment. Vitrified wheels are most satisfactory for general grinding operations.

Silicate Wheels are made with a bond of sodium silicate, the grains and the liquid silicate of sodium being mixed and tamped while plastic in iron molds. They are then baked at a low temperature for a few days. The hardness or grade is determined largely by the amount of tamping the wheel received, but the tamping must be uniform to avoid hard and soft spots. These wheels are **used** largely **for tool and knife sharpening** and are usually **run wet.** They are made either with or without a wire web, opinions differing as to the added strength so obtained. Silicate wheels should be specified for large sizes; in cases of emergency where a wheel not in stock is required for immediate use; in places where low wheel wear and tool cutting are required; and for grinding tools requiring keen cutting edges. The same grades and grains are obtainable in both silicate and vitrified wheels.

Elastic Wheels. In making these the grains are mixed with shellac, heated, and allowed to cool; the mass is then broken up and poured into molds which are heated until the shellac melts, the surface being rolled with hot rollers by hand. Very large sizes are formed under heavy pressure. These wheels are very elastic and can be used as thin as 1/32 in. thick in sizes as large as 12 in. in diameter. Very fine wheels of small diameter can be used as thin as 1/64 in. They are **used for slotting** and cutting metals, and for **gumming saws** and sharpening shaping cutters, where, in backing off, much sidewise strain is applied to the wheels; and for certain classes of work, such as roll grinding, to give a special finish. They are supplied in eight to twelve grades, the grade being determined largely by the amount of bonding material.

The ordinary abrasive wheel is operated at **speeds** of from 4000 to 6000 surface ft. per min.; wheels below 6 in. in diam. are frequently operated at higher speeds. The retail **prices** of silicate and vitrified wheels are the same; elastic wheels are slightly higher in price. Carbide of silicon wheels, whether made by the silicate or elastic processes, cost more than aluminum oxide wheels. Abrasive wheels are **trued** up by a diamond set in the end of a soft metal rod and held against the face of the wheel while revolving. Diamond dust, rolled into a soft copper disk, is also used.

The following information is furnished by the Norton Company.

The **grain** of a wheel is determined by the size or number of the abrasive grain used. The numbers employed are 10, 12, 14, 16, 20, 24, 30, 36, 46, 50, 60, 70, 80, 90, 100, 120, 150, 180, and 200. By No. 10 grain is meant a size that will pass through a grading sieve having ten meshes to the linear inch. The finer grades of alundum and crystolon known as **flour** are designated as F, FF, FFF, and XF.

The **grade** is determined by the degree of hardness of the wheel, or the resistance of the cutting particles under grinding pressure. The wheel from which the particles are easily broken, causing it to wear away rapidly, is called soft, while one which is able to retain its particles longer is called hard.

The following table gives the grade designations and letters for abrasive wheels.

Grade designation	Very soft		Soft	Medium soft	Medium	Medium hard	Hard	Very hard
Norton Co.	A	BC	DEFG	HIJK	LMNO	PQRS	TUVWX	YZ
Carborundum Co.	V	UT	SRQ	PO	NML	KJIH	GFE	D

The intermediate letters between those designated as soft, medium-soft, etc., indicate so many degrees harder or softer; *e.g.*, L is one grade or degree softer than medium for a Norton wheel.

Elastic wheels are graded as follows: 1, 1½, 2, 2½, 3, 4, 5, and 6. Grade 1 is the softest and grade 6 the hardest.

See also pp. 1448 to 1450.

Abrasive Paper

Flint, Garnet, Emery, Corundum, and the artificial abrasives are all used on paper and cloth, fastened by glue or some special adhesive. The abrasive papers are used mostly for soft materials, such as wood, while the abrasive cloths are used for metal work. They are furnished in a great variety of grain **sizes,** as follows:

	Sand	Emery	Alundum	Carborundum	Aloxite
Paper, No.	4-0 to 4	3-0 to 3½	180 to 20	180 to 20	180 to 20
Cloth, No.	....	FF to 3½	180 to 40	FF to 3½	FF to 3½

Crushed Steel is made by heating high-grade crucible steel to nearly white heat and then quenching in a bath of cold water. The fragments thus produced are crushed to particles varying from fine powder up to 1/16 in. in diameter, and are classified as diamond crushed steel, diamond steel, emery, and steelite. The chief use of crushed steel is in the stone, brick, glass and metal trades, the size of the steel used depending on the character of the stone to be cut, rubbed, ground, or polished.

ADHESIVES

REFERENCES: J. A. Taggart, "The Glue Book." Fernback, "Glues and Gelatine." Sherer, "Casein."

Glue is made by boiling out hides, skins, bones, and sinews of various animals and drying the resulting liquor. Fish glue is produced from the heads, bones, and sinews of fish. It is obtainable commercially in sheets, strips and flakes, or ground. There is no uniform method of testing and grading glues. The grades originally used by Peter Cooper, known as "Cooper Standards," are widely employed and, in order of their strength, are as follows: A Extra, No. 1, 1X, 1¼, 1⅜, 1½, 1⅝, 1¾, 1⅞. These terms are commonly accepted as designating the various commercial grades.

In grading glues, chemical **tests** are of little value, viscosity and jelly strength being the most important tests. Generally, the greater the tenacity of the glue the greater its cohesiveness and the higher its viscosity. Viscosity is generally determined in a pipette as compared with water. Jelly strength is of great importance; by making up jellies of standard glues and the glue to be tested, a very close idea of strength can be determined by pressing each with the tip of the finger.

Ground glue is frequently **adulterated** and should be carefully examined for foreign substances. **Decay** is evidenced by white bubbles on the surface of the glue, also by sour odor. Bubbles inside the glue do not indicate putrefaction. Bone glues are usually darker than hide glues, but some cheap glues are clarified to give the appearance of a higher grade. The best glues should be neutral to litmus paper. Excess of acid should be avoided. A thin piece of good glue should bend without breaking, and when broken should show a splintery edge. A clean **fracture** indicates brittle, low-grade glue. Glues containing much grease or which foam on beating the solution, should be avoided.

In **dissolving,** glue should always be soaked in cold water first and then warmed. It is of great importance to employ no more heat than is necessary to reduce the glue to a proper consistency. It should never be heated higher than 150 deg. fahr. and heat should never be applied by live steam but always indirectly. The "Glue Book," by J. A. Taggart, Toledo, Ohio, gives the following valuable suggestions regarding **selection:**

Wood Joints: High-test hide glues. Veneers: A moderately high-test mixture of bone and sinew or bone and hide. Sizing: Glue free from grease and foam and that flows freely. Paper Boxes: Quick-setting hide glue for setting up; lower-test bone glue for covering. Belting and Other Leathers: High-test glues preferred. Bookbinding: For pasting covers, a low-grade bone glue; for rounding and backing, a high-grade hide glue. For Emery Purposes: Very high-grade and pure glue.

After soaking in water for 48 hours, there should be no discoloration of the water or disagreeable odor. In general, the high-test glues, although more expensive, go farther and do better work. For some purposes their quick setting is an objection.

To prepare **waterproof glue**, about 1 per cent. of ammonium or potassium bichromate may be added to the glue liquid, or a small amount of formaldehyde.

A very strong but expensive adhesive is **isinglass**, prepared chiefly from the swimming bladder of the sturgeon. Fish glues other than isinglass are always in the form of heavy solutions or pastes. **Imitation liquid glues** are made by treating animal glues, but are deficient in adhesiveness and binding properties. Certain "liquid glues" are made entirely of dextrines and flour paste. **Flexible glue** is a mixture of glue, glycerine, and water. Marine glues contain no glue but are used for calking vessels. **Jeffrey's marine glue** is a mixture of shellac and India rubber in benzine. Others are mixtures of asphaltum and rubber in petroleum solvents.

Starch Pastes are made from wheat, corn, tapioca, sago, and potato starches, each having different properties. Potato-starch paste is of heavy body with little adhesiveness; tapioca is fluid and sticky; sago yields a dark, sticky paste; corn is not so suitable for paste-making. All starch pastes must be preserved with chemicals such as bichloride of mercury, formaldehyde, or carbolic acid. They may be rendered more adhesive if treated with a hydrolizing agent, such as dilute acids or calcium chloride. In preparing starch and flour pastes, the starch or flour should always be emulsified in cold water and then added to the balance of boiling water.

Dextrines are produced by treating starches with acids. Potato dextrines are best, tapioca, corn, and sago following in order of serviceability. They are excellent substitutes for glue but do not have the same binding strength, are easily decomposed and must be preserved. Extensively used in the textile industries for filling, sizing, and as a cheap adhesive. Photo-library paste is generally made from white dextrine, preferably potato dextrine known in the market as "White E. S." or "H. White." These pastes are generally composed of dextrine and water with a small amount of glycerine and a proper preservative. Essential oils are generally added to cover the odor of the preservative.

Gum Arabic is a natural gum soluble in water with considerable adhesiveness. Although cheaper than glue, a large quantity must be used to equal the adhesiveness. Used extensively with dextrine in the manufacture of **mucilage**. Glycerine is generally added to increase flexibility and prevent cracking in the dried mucilages. Envelope gums are generally prepared from tapioca or potato dextrines.

Water-glass is a solution of sodium or potassium silicate made by fusing quartz sand with potash or sodium hydrate. It is a heavy, oily liquid containing from 33 to 60 per cent. of sodium silicate, and must be protected from the air to prevent decomposition. It is important to ascertain the actual content of silicate, as the commercial solutions vary widely in strength. It is used extensively as a cheap adhesive and as a protective coating, producing a hard surface, resistant to moisture and fireproof; also as a fireproofing agent on cloth and wood. An ingredient in many putties and in the preparation of artificial stones.

Putty. Ordinary putty is made from whiting and pure raw linseed oil. **Glazier's putty** generally contains some varnish in addition. Linseed-oil varnishes mixed with litharge or red lead and ochre are extensively used as a **hard-drying putty**. **Mastic cement** is composed of powdered limestone, sand, and litharge, mixed with linseed-oil varnish. Litharge and glycerine produce a very hard-drying cement, resistant to alkalies, acids, and petroleum solvents. Water-glass and chalk produce a hard waterproof mass, as do also water-glass and Portland cement. **Magnesia cement** consists of calcined magnesia in a 30-per cent. solution of chloride of magnesium, and hardens to a stone-like mass.

Cements for Special Uses

Pipe Joints. Use a mixture of red lead with linseed oil in a thick paste. **Diamond cement** may be prepared by slowly stirring into 40 parts of linseed oil the following ingredients in the order given: litharge, 30 parts; slaked lime, 10 parts; whiting, 20 parts; graphite, 100 parts. It should be applied hot to the slightly roughened surface of the metal. A strong cement may be made by mixing whiting in a 40-deg. Baumé water-glass solution to a plastic mass.

Porcelain and Crockery. Mix equal parts of gutta-percha and shellac at a gentle heat, and apply hot to edges previously warmed. An effective cement consists of 10 parts of casein dissolved in 100 parts of silicate of soda.

Glass. Water-glass is the best cement, provided glassware is not to be heated. For cementing glass upon glass, a mixture of Russia isinglass and acetic acid may be used.

Leather. Glue is the best adhesive for use with leather. A mixture of gutta percha, asphaltum, and turpentine gives very good results, or a solution of rubber in carbon tetrachloride, the latter being advantageous as a solvent and non-inflammable.

Wood. Glue is the best cement; shellac dissolved in alcohol is very satisfactory.

Bone. Use equal parts of paraffin wax and rosin dissolved in turpentine.

ALCOHOL

(See also pp. 298 and 612)

Grain Alcohol (Ethyl Alcohol) is made by the distillation of fermented sugar solutions derived either from grain, molasses, or fruit syrups, or by the treatment of wood with sulphurous acid. It is sold in the United States by "degree proof" or "proof," the figure representing twice the percentage of alcohol by volume; thus, 100 deg. proof spirit at 60 deg. fahr. contains 50 per cent. of absolute alcohol. Common custom in the United States expresses the percentage of alcohol "by volume." Grain alcohol is miscible in all proportions with most liquids except fatty oils, which it dissolves very slightly.

Wood Alcohol (Methyl Alcohol) is produced by the destructive distillation of wood. It is a colorless, mobile liquid with a spirituous odor, the odor of commercial wood spirit being due to impurities. Its specific gravity at 32 deg. fahr. is 0.8142, and its boiling point 151.7 deg. fahr. It burns with a non-luminous flame, is miscible in all proportions with water, ethyl alcohol, and ether, and dissolves certain resins, fats, and volatile oils. It is frequently adulterated by the addition of water, which raises the specific gravity.

Denatured Alcohol. The object of denaturing alcohol is to permit the use of ethyl alcohol for purposes other than drinking free from the heavy excise tax. The common denatured alcohol of commerce is made under Government regulation by adding a small quantity of wood alcohol and benzine to grain alcohol. Such a mixture is unfit for consumption as a beverage, but for most industrial purposes it serves equally as well as the pure product. For special industries where such a denaturing would be objectionable, the Government has permitted the use of special denaturing materials, such as pyridine, nicotine, and cadmium and ammonium iodides.

BELTING

(See also p. 743)

Leather Belts. Cut from the proper hide, belts cut along the spine have almost equal lateral stretching qualities, and consequently run the straightest and quietest, and are best for high speeds. They can only be obtained up to about 7/32 in. in thickness. Double belts can be made for greater thicknesses. For wide belts great care in selection is necessary. Very wide belts are also made as double belts, laterally.

For slow speeds and large pulleys belts may be made from side leather, as the tensions on the fibers have more time to equalize each other. Such belts should not exceed 8 in. in width. Belts cut transversely across the shoulder may be used for slow-speed machinery.

Permanent elasticity is the most important characteristic, tensile strength being less decisive in judging quality, and weight and thickness of even less importance. Thickness is often increased by artificial means, such as filling the leather. In making the belt, the ends are joined with a mixture of sinew glue and fish glue, after tapering by using a leather plane. Sewed belts are not as good as properly glued belts.

The United States Navy specifications require oak-tanned leather cut entirely from the center or back of the hide and free from any loading. Single belts must show an average tensile **strength** of 4000 lb. per sq in.; double belts a tensile strength of 3600 lb. When subjected to a stress of 2250 lb. per sq. in. for 1 hr., single belts should show not more than 13.5 per cent. elongation and double belts not more than 12.5 per cent. The modulus of elasticity in tension varies from 17,800 lb. per sq. in. for new belts to 32,000 lb. for old. The proportional elastic limit is about 2300 lb. per sq. in. The strength of chrome leather varies from 8500 to 12,900 lb. per sq. in.

Leather belts should be cleaned with warm water and dressings only applied when the belt becomes dry and husky. Melted beef tallow is an excellent dressing, also prepared castor-oil dressings. Mixtures of tallow and beeswax are stated to be of service on belts used in damp places. Resinous substances should never be used, as they cause the fibers to adhere and wear each other out by friction. A little neatsfoot oil can be used on very dry belts, but frequent application of penetrating oils makes the leather soft and flabby and causes it to stretch.

Commercial **sizes** of leather belting as adopted by the Leather Belting Association range from ½ in. to 72 in. in width, the width increasing by ⅛ in. up to 1 in., by ¼ in. up to 4 in., then by ½ in. to 7 in., and above that by inches.

Rubber Belts are made by saturating a strong woven duck belt with rubber and subsequently vulcanizing. Their advantages over leather belts are claimed to be uniformity in width and thickness, durability when exposed to steam and dampness, great tensile **strength** and grip on pulley. With proper weight of duck, a three- or four-ply rubber belt is claimed to equal a single, and a five- or six-ply a double, leather belt. Balata is extensively employed for this purpose as it is very little affected by dampness or change of temperature and does not stretch readily. Rubber belts are cheaper than leather or other belts suitable for the same kind of work and are of particular value in damp locations or under unusual temperature conditions.

Rubber belts should never be treated with animal oils or greases, as these substances rapidly destroy their life. Good results are said to be obtained by painting with a mixture of equal parts of red lead, black lead, French yellow and litharge mixed with boiled linseed oil and sufficient japan to cause quick drying. This produces a smooth, polished surface and if the belt should slip the side next the pulley can be slightly moistened with boiled linseed oil. Commercial **sizes** run from 1 in. to 60 in., increasing by ¼ in. up to 2 in., by ½ in. to 4 in., and then by inches. Special seamless and endless belts can be obtained. See also p. 744.

Cotton Belts, when woven, have a tensile strength of about 5000 lb. per sq. in.; when sewed, from 6500 to 7500 lb. per sq. in. See also p. 744.

BRICK

REFERENCES: E. Bourry, "Ceramic Industries." A. B. Searle, "Modern Brickmaking." Wahl-Henius, "American Handy-book."

Commercial brick may be divided into two main classes: building brick and refractory brick.

Building Brick includes ordinary clay brick, shale brick and sand-lime brick. The **ordinary clay brick** is most extensively used, particularly the common red brick, the color being dependent upon the composition of the clay and nature of burning. Yellow, blue, black and gray bricks are also made. Perfectly white bricks are only made by surface treatment as in glazed brick. Whitish bricks are produced from mixtures of clays and chalk or from certain marls. Good bricks should be regular in shape, texture and color, equally burned throughout and free from cracks or flaws. They should give out a clear, ringing sound when struck. Soft or unburned bricks are of no value. Bricks which have been too heavily pressed or highly burned may have too smooth a face to adhere readily to mortar. For many purposes it is unnecessary to insist upon all these qualities, a hard, well-burned brick sufficing. The characteristics of these bricks are dependent upon the clay used and are commonly classified as follows:

1. Method of Molding: Soft mud brick, stiff mud brick, pressed brick, re-pressed brick, slop brick, sanded brick, machine-made brick.

2. Position in Kiln: Arch or clinker brick form the top and sides of arches; they are very hard, brittle and weak. Body, cherry or hard brick are taken from interior, and are the best bricks in the kiln. Salmon, pale or soft brick are from the exterior of the mass; they are too soft for general purposes, and are usually employed for filling.

3. Form or Use: Compass, feather-edge brick, face brick, sewer brick, paving brick, vitrified brick. Vitrified clay blocks much larger than ordinary brick are sometimes called paving brick, but are more correctly brick paving blocks.

For average values of the strength of brick, see Table 4, p. 385.

The following values of the **compressive strength of building brick** are taken from "Tests of Metals, Etc.," 1894, pp. 456–468:

	Compressive strength, lb. per sq. in.		
	Min.	Max.	Mean
Face Brick:			
Stiff-mud	8,930	15,330	12,766
Dry-pressed	8,930	17,990	11,190
Re-pressed soft-mud	5,770	7,560	6,780
Common Brick:			
Hard-burned, soft-mud, Cambridge	9,140	14,750	11,340
Hard-burned, soft-mud, Brookfield	4,340	4,580	4,475
Hard-burned, soft-mud, Mechanicsville	5,110	6,730	5,808
Medium-burned, soft-mud, Cambridge	4,610	8,590	6,590
Medium-burned, soft-mud, Brookfield	4,200	6,850	5,248

Shale brick is similar to the ordinary clay brick, but is made from shale.

Sand-lime brick (see *Bulletin* No. 18, Ill. State Geol. Survey) consists of sand particles bound together by a network of calcium silicate, calcium-magnesium silicate, or calcium hydrosilicate formed by the action of steam under pressure upon a mixture of sand or granular silicate and lime; the lime may be either a high-calcium lime or a magnesium lime. The formation of this calcium-silicate bond results in the production of a mass in many ways similar to that produced in dry-pressed brick when they are burned.

Among the important claims made in favor of the sand-lime brick over the common clay brick are its increased strength, fire-resistant properties and the general uniformity in size and shape of each brick. Sand-lime brick will withstand a heat test of 3000 deg. fahr., the only apparent damage after 96 hr. in a kiln being a slight discoloration. Sand-lime brick are waterproof, frostproof, possess good bonding properties, and can be produced in any color desired.

A consular report from Germany states that in 255 tests on **sand-lime brick** the compressive strength varied greatly, the average, however, being 2175 lb. per sq. in., deviations from the average being less than in clay brick, a result of the greatest symmetry in shape and structure.

Refractory Brick may be divided into the following classes, based on the materials used. (For further details regarding fire brick see pp. 914 and 915):

1. FIRE-CLAY BRICKS are made by fusing certain clays to such a temperature that partial vitrification occurs, together with a plastic clay usually known as "grog" which serves as a skeleton for the formation of the brick. **Paving brick** is a lower grade of fire-clay brick which is hard but not brittle. The common **requirements** in this country for paving brick are as follows: Standard size, $8\frac{1}{2} \times 3\frac{1}{2} \times 4$ in.; not over 14 per cent. loss on standard rattler test; modulus of rupture, not less than 2700 lb.; absorption of water, not more than 2 per cent.; specific gravity, not less than 2.35; hardness, not less than 6.5 on Mohs's scale; crushing resistance, not less than 8500 lb. per sq. in.

Another class of fire-clay bricks are **acid-proof bricks.** The best bricks of this kind are made from ball or stoneware clay. The **standard test** for acid-proof bricks is to ascertain their crushing strength before and after they have been soaked for 7 days in concentrated sulphuric acid maintained at a temperature of 90 deg. fahr. The best bricks should be unaffected by this treatment. **Glazed bricks** are chiefly made of second-grade fire clay, the clay being composed of feldspar, flint, whiting, etc., fired at a high temperature.

2. SILICA BRICKS are made by heating to vitrification practically pure silica with about $\frac{1}{50}$ of its weight of lime, forming a true calcium silicate. Fire clays are commercially obtainable which fuse at a temperature corresponding to Seger cone 36. The chief disadvantages of silica bricks are brittleness and liability to "spall" when exposed to sudden changes of temperature. As silica bricks expand when heated, twice-burned bricks should be insisted upon.

3. GANISTER BRICKS are intermediate between fire-clay and silica bricks and are made in a manner similar to the silica bricks but without the addition of lime.

4. BASIC BRICKS—usually made of magnesite or bauxite—are weak in resistance to pressure but extremely resistant to heat. **Bauxite bricks** are used in metallurgical furnaces and rotary cement kilns. **Magnesia bricks** are made from calcined magnesite, a mineral containing from 90 to 96 per cent. of carbonate of magnesia and found in Styria and Greece. This material is plastic under heavy pressure and the Styrian material may be molded for brick or furnace bottoms without using a binder. Magnesia brick are remarkably resistant to slag and limestone and are particularly suitable for certain types of metallurgical furnaces. They are, however, very sensitive to the action of silica and, owing to their tendency to expand on repeated heating, cannot be used in arches. The mortar used in laying magnesia bricks should consist of powdered magnesia mixed with $\frac{1}{6}$ of its weight of tar.

5. NEUTRAL FIRE BRICKS OR CHROME BRICKS are usually made of chromite mixed with fire clay or bauxite. They are practically infusible, dense in structure and are used in basic open-hearth steel furnaces as a neutral course between fire-clay brick and magnesia brick; they are not affected by sudden changes in temperature, but are not very resistant to crushing when hot.

Melting Points of Fire Bricks and Clays. The U. S. Bureau of Standards has determined the melting points of fire bricks and other refractories. The results are given below.

	Number of samples tested	Melting point, deg. fahr.		Number of samples tested	Melting point, deg. fahr.
Fire-clay brick........	41	2831–3137	Bauxite clay........	1	3308
Bauxite brick.........	8	2849–3245	Bauxite............	1	3263
Silica brick..........	3	3092–3101	Chromite...........	1	3956
Chromite brick.......	1	3732	Pure alumina.......	1	3650
Magnesia brick.......	1	3929	Pure silica..........	1	3182
Kaolin...............	3	3155–3164	Carborundum.......	1	4892

The value of 3182 deg. given for silica is not the true melting point, but represents the temperature at which the silica flows distinctly. It was found that silicon carbide does not melt below 4892 deg. but becomes unstable at much lower temperatures.

Standard Sizes and Shapes of Brick

Common brick..........................	8¼ × 4 × 2¼ in.
Paving brick...........................	8½ × 4 × 2½ in.
Pressed brick..........................	8⅜ × 4 × 2⅜ in.
Roman brick...........................	12 × 4 × 1½ in.
Norman brick..........................	12 × 4 × 2⅜ in.
Fire and silica brick...................	9 × 4½ × 2½ in.
Paving block...........................	from 9 × 4 × 3 in. to 9 × 5 × 4 in.

The following **shapes** are also standard:

Small 9-in............................	9 × 3½ × 2½ in.
Large 9-in............................	9 × 6¾ × 2½ in.

	Bricks per circle	Circle Diameters Inside	Outside	Dimensions of Brick, in.
No. 1 Arch...........	76	4 ft. 3 in.	......	9 × 4½ × (2½ − 2⅛)
No. 2 Arch...........	38	1 ft. 9 in.	......	9 × 4½ × (2½ − 1¾)
No. 1 Wedge.........	91	4 ft. 6 in.	6 ft.	9 × 4½ × (2½ − 1⅞)
No. 2 Wedge.........	57	2 ft. 3 in.	3 ft. 9 in.	9 × 4½ × (2½ − 1½)
No. 3 Wedge.........	57	3 ft.	4 ft. 6 in.	9 × 4½ × (3 − 2)
No. 1 Key............	113	12 ft.	13 ft. 6 in.	9 × 4½ × (4 − 2½)
No. 2 Key............	57	5 ft. 3 in.	6 ft. 9 in.	9 × 4½ × (3½ − 2½)
No. 3 Key............	38	3 ft.	4 ft. 6 in.	9 × 4½ × (3 − 2½)
No. 4 Key............	25	1 ft. 6 in.	3 ft.	9 × 4½ × (2¾ − 2½)
No. 1 Circle..........	12	24 in.	33 in.	(9 − 6 9/16) × 4½ × 2½
No. 2 Circle..........	14	36 in.	45 in.	(9 − 7 3/16) × 4½ × 2½
No. 3 Circle..........	18	42 in.	51 in.	(9 − 7 7/16) × 4½ × 2½
No. 1 Cupola.........	15	30 in.	42 in.	(9 − 6 7/16) × 6 × 3 or 4
No. 2 Cupola.........	17	36 in.	48 in.	(9 − 6¾) × 6 × 3 or 4
No. 3 Cupola.........	21	48 in.	60 in.	(9 − 7 3/16) × 6 × 3 or 4
No. 4 Cupola.........	25	60 in.	72 in.	(9 − 7½) × 6 × 3 or 4

The general practice in **measuring brickwork** is to compute the superficial area of the walls and multiply by the number of brick in the wall in 1 sq. ft. of surface, depending of course on the thickness of the wall. A fair average of the practice in the eastern states for a 4-in. wall, or ½ brick in thickness, is 7½ bricks per superficial foot. In the western states the usual average is 7 bricks per superficial foot for a 4½-in. wall.

Hollow Tile. Safe loads per sq. in. of effective bearing parts are as follows: For hard fire-clay tiles, 80 lb.; for ordinary clay tiles 60 lb.; for porous terra cotta tiles, 40 lb.

CLEANSING MATERIALS

Soap is made by chemically combining fats with alkalies. Toilet and washing soaps should be entirely free from caustic alkali, and uncombined fat, and as free as possible from carbonated alkali. Bar soaps may contain excessive quantities of moisture, which should be avoided in purchasing. Soft soaps are made by combining potash with fat. Liquid soaps are potash soaps with large quantities of water and considerable glycerine. Excessive quantities of common salt should be avoided in ordinary soaps which are made by salting the soap out of solution. For cleaning paint and varnish, care should be taken that no free caustic alkali is contained in the soap and that the carbonated alkali be as low as possible; cleaning powders composed of soap and very fine infusorial earth or pulverized pumice, are excellent, provided the mineral substances do not contain hard particles which will scratch varnish. Floor-washing soaps generally contain large quantities of free sodium carbonate, which is very effective in cutting oil and dirt.

Soda Ash, which is practically pure sodium carbonate, is extensively used for cleaning purposes, and in solution as a lubricant for grinding steel dies and cutting steel and as a preventive of rust on steel tools. For the latter purpose, the strength of solution should be kept low. **Sal soda** is a crystalline form of sodium carbonate containing about 63 per cent. of water by weight. Although extensively used, it can generally be replaced to advantage by soda ash—which should be at least 97 per cent. pure. **Caustic soda** (sodium hydrate) is extensively used in textile mills for scouring, cleaning, etc., and for cleansing metals from oil and dirt. A good grade should contain not less than 94 per cent. of caustic soda. It is very hygroscopic.

Caustic Potash, used for cleansing, should contain not less than 75 per cent. of caustic potash; a crude variety, however, containing 65 per cent. together with considerable caustic soda, is frequently employed. **Carbonate of potash** should contain not less than 75 per cent. of potassium carbonate.

ELECTRICAL INSULATING MATERIALS

REFERENCE: Turner and Hobart, "The Insulation of Electric Machines." See also pp. 1596 and 1597.

The insulating properties of any material are dependent upon two factors: first, **dielectric strength,** or the ability to withstand high voltages, and, second, **ohmic resistance,** or the ability to prevent small currents from leaking through it. Materials may have one of these qualities to a far greater extent than the other, for example, air has a very high specific resistance but very little dielectric strength; glass has great dielectric strength but much lower resistance than air. The **ideal insulator** is one having the maximum dielectric strength and resistance and also mechanical strength and chemical stability. Moisture is by far the greatest enemy of insulation, consequently the absence of hygroscopic quality is desirable.

The common insulating materials are described below. For their electrical properties, see p. 1597.

Rubber. See "Rubber, Gutta Percha and Balata," p. 641.

Mica and Mica Compounds. Mica is a natural mineral varying widely in color and composition, and occurs in sheets which can be subdivided down to a thickness of 0.00025 in. White mica is best for electrical purposes, the green shades are the softest varieties, and the white amber from Canada the most flexible. Mica has high insulating qualities, the best grades having a disruptive strength of 12,000 R.M.S. volts per tenth of a millimeter. Its lack of flexibility and non-uniformity and its surface leakage are disadvantages. To offset these, several mica products have been developed, in which small pieces of mica are built up into finished shapes by means of binders such as shellac, gum, bakelite, etc.

Micanite consists of thin sheets of mica built into finished forms with insulating cement. It can be bent when hot and machined when cold, and is obtainable in thicknesses of from 0.01 in. to 0.12 in. Flexible micanite plates, cloth and paper are also obtainable in various thicknesses. **Megohmit** is similar to micanite except that it is claimed not to contain adhesive matter, which is objectionable in the latter product. It can be obtained in plates, paper, linen, and finished shapes. **Megotalc**, built up from mica and shellac, is similar to the above-named products and obtainable in similar forms.

Insulating Varnishes. Two general types of insulating varnish are used: those consisting of hydrocarbons such as asphalt, paraffin, etc., dissolved in benzine, and those consisting essentially of linseed oil with and without asphalt and other bitumens. The former are cheaper and if baked produce a fairly hard coating. They are apt to distill under working temperatures, however, with rapid crumbling and deterioration. Properly made linseed-oil varnishes are more satisfactory, as they do not deteriorate as rapidly.

Impregnating Compounds. Bitumens of various kinds are used to impregnate motor coils, the melted bitumens being forced into the coil in a vacuum tank, forming a solid insulation when cooled. Brittle compounds, which gradually pulverize due to vibration in service, and soft compounds, which melt and run out under service temperatures, should be avoided as far as possible.

Oil. Refined grades of petroleum oils are extensively used for the insulation of transformers, switches, and lightning arresters. The following specification covers the essential points:

Specific gravity, 0.860; flash test, not less than 335 deg. fahr.; cold test, not more than 14 deg. fahr.; viscosity (Saybolt) at 100 deg. fahr., not more than 120 seconds; loss on evaporation (8 hr. at at 200 deg. fahr.), not more than 0.5 per cent.; dielectric strength, not less than 35,000 volts; freedom from water, acids, alkalies, saponifiable matter, mineral matter, or free sulphur. Moisture is particularly dangerous in oil. Skinner claims that 0.06 per cent. H_2O will reduce the dielectric strength approximately 50 per cent. See p. 648.

Paraffin is used to impregnate paper and cloth. Specific inductive capacity, about 2; dielectric strength, 8,000 volts per mm.; specific resistance, 240,000,000 to 3,900,000,000 megohms per cu. cm.

Impregnated Fabrics. Fabrics serve as a framework to hold a film of insulating material, and must therefore be of proper thickness, texture, and mechanical strength, and free from nap or acidity. Cambric, muslin, lonsdale, and batiste are commonly used.

Empire Cloth is cambric treated with linseed oil by a special process. Similar products are oiled linen, oiled canvas, varnished cambric.

Insulating Tape, also known as **friction tape** or **binding tape**, consists of a flat, woven braid or tape impregnated with an adhesive and insulating compound adhering firmly to the braid. ¾-in. tape should have a breaking strength of not less than 33 lb. and run not less than 150 ft. per lb.; other widths in proportion.

Ambroin is an insulating material made from fossil copal and silicates by a special process, and is claimed to be strong, firm, uniform, resistant to high temperatures, and non-hygroscopic. It is used for commutator rings, spools, and other molded insulation, and is cheaper than ebonite. It is obtainable in various grades. Tensile strength, about 2140 lb. per sq. in.; compressive strength, 2680 lb. per sq. in.

Bakelite is a plastic substance produced by condensing phenols with formaldehyde. It is used in electrical insulation, in making imitation amber, and as a covering to resist the action of chemicals and solvents. It is also produced in liquid form for use as a wood-impregnating medium and lacquer.

Paper. The present tendency is to use paper only as a backing or framework for an insulating film or compound, owing to hygroscopic qualities. **Manila, express** and **bond** papers possess the best dielectric and mechanical strength, and when coated with good insulating varnish are excellent insulators. **Japanese paper** is extremely thin and strong, and is used in tubular insulation and also as a backing for mica paper.

Vulcanized Fiber, also known as **hard fiber,** is prepared by treating paper pulp or a textile material with a saturated solution of zinc chloride and afterward consolidating it under heavy pressure. It has a consistency like horn, but is generally found in commerce treated with glycerine or glucose, which makes it very pliable. In the hard form it can be planed, sawed and worked like wood, and in the soft form can be readily molded. It is claimed to be an efficient non-conductor of heat and an excellent insulator. The soft varieties are used in making artificial leather and rubber products. Its **specific gravity** is about 1.3; **specific resistance,** 53 megohms per cu. cm. Furnished in 42 × 66-in. **sheets** from 0.010 to 1⅛-in. thick, and in **tubes** from 3⁄16 in. to 2½ in. in diam. by 22 to 36 in. long. It must be thoroughly seasoned, tough and flexible, and must stand, when unbaked, bending around a circle having a radius of 10 times its thickness without cracking or splitting. After 12 hours' baking at 250 deg. fahr. it must bend around a circle of radius 50 times its thickness. **Tensile strength** of sheets, lb. per sq. in.: Less than 1⁄16 in. in thickness and over ⅝ in., 5000; 1⁄16 in. to ¼ in., 8000; ¼ in. to ⅝ in., 7000. **Electrical strength,** about 1200 volts per 0.01 in. of thickness.

Presspahn or Fuller Board is a specially prepared paper board which, when impregnated with oil, is an excellent insulator. Insulated presspahn is very hygroscopic. The material is mechanically strong but its dielectric strength is greatly injured by creasing. **Pergamyn** is cellulose beaten into fine fibrillæ and consolidated under pressure without gelatinization or chemical treament. Its uses are similar to those of vulcanized fiber. **Cellulith** is produced by grinding wood pulp to a homogeneous mass and then drying. It is very hard, can be worked like wood, and is claimed to be a substitute for horn or ebonite. **Isolit** is a form of papier maché impregnated and covered with special insulating compound. **Adit** is a variety of isolit claimed to have great tensile strength and toughness and to ignite with difficulty. **Litholite** is a paper product used for insulation. It has high disruptive strength, good insulation resistance, and is stated to be inflammable but tough. **Psychiloid** is a chemically treated paper pulp obtainable in sheets from 1⁄16 in. to 1½ in. in thickness. It is very strong and rigid, and stated to have high disruptive strength and insulation resistance.

Porcelain is made from the purest white clay agglutinized by some substance such as powdered feldspar which softens and fuses at the temperature at which the ware is fired, rendering the mass semi-transparent. The material possesses great solidity, strength and ability to resist sudden changes of temperature. It is used extensively for electrical purposes on account of its strength, durability and electrical resistance. External glaze is of importance in protecting porcelain. A simple test for quality consists in chipping off the glaze and noting whether ink produces a flowing stain, which it will do on poor grades. **Properties:** The Locke Insulator Mfg. Co. gives the following average values: Specific gravity, 2.427; specific inductive capacity, 4.4; tensile strength, lb. per sq. in., 1800; compressive strength lb. per sq. in., 15,000; dielectric strength, volts per mm., 16,350. J. E. Boyd (A.S.M.E., Dec., 1915) gives the compressive strength of porcelain and high-grade stoneware as 20,000 lb. per sq. in.; the tensile strength of porcelain >3000 lb. per sq. in., of stoneware 1100–2200 lb.; the modulus of elasticity in tension and compression of porcelain, 10,000,000 lb. per sq. in., of stoneware, 6,000,000 to 9,000,000.

FIBERS

REFERENCES: Matthews, "The Textile Fibers." Hannan, "The Textile Fibers of Commerce." Mitchell and Prideaux, "Fibers Used in Textile and Allied Industries." Griffin and Little, "The Chemistry of Paper-making." Worden, "Nitrocellulose Industry."

Fibers for industrial purposes are either mineral, animal, or vegetable. The principal mineral fiber of importance is asbestos (see p. 634). Finely spun glass, slag wool and metal threads are also used. The animal fibers of commercial importance are either animal hairs or the silk of the silkworm and the larvæ of other moths. Sheep, goats and camels supply the principal hair fibers. Cotton, wool, linen, jute and hemp are the principal vegetable fibers.

Fibers may readily be distinguished under the microscope. Animal and vegetable fibers may be distinguished by their odor on burning, and by the fact that vegetable fibers burn off sharply while animal fibers fuse to a rounded bead-like end.

Animal Fibers

Silk fiber consists of a continuous thread spun by the mulberry silkworm. Inferior silks are obtained from other varieties of caterpillars and are known as "wild" silk.

Silk fiber possesses great tensile strength, the average breaking strength being approximately 64,000 lb. per sq. in., and the elongation 15 to 20 per cent.

Wool. The wool of the sheep is that principally employed. The tensile strength and elasticity of wool fibers vary widely; the average strength being about half that of cotton. The length of the fiber varies between 1 and 8 in. and the diam. between 0.0018 and 0.004 in. Wool is graded according to the length of staple into "tops" and "noils," the former being used for worsted yarns and the latter for making woolen and carded yarns.

Vegetable Fibers

All vegetable fibers consist mainly of cellulose and may be classified as follows: Seed hairs, such as cotton; bast fibers, such as flax, hemp, jute, and ramie; and vascular fibers. Cotton and ramie are nearly white; linen, grayish brown; jute and hemp have a decided brown color. Those containing large proportions of cellulose are flexible and elastic, while those which are highly lignified or woody are stiff and brittle. The tensile strength varies considerably, but the relative strength of the fibers is in the following order: Hemp, jute, manila, linen, cotton, ramie. Vegetable fibers are much less hygroscopic than wool or silk, the latter averaging from 12 to 16 per cent. of moisture normally, while cotton and linen average but from 6 to 8 per cent.

Cotton fiber is obtained from the seed hair of the cotton plant, the seeds being separated from the fiber by a process known as "ginning." The small hairs left on the seed after the first ginning, known as "linters," are used in the manufacture of cotton batting, guncotton, etc. The best quality of cotton fiber is known as "Sea Island" cotton, as it is raised on islands off the coast of Carolina.

The natural spiral-like twist of the fiber makes it specially adapted to spinning. The spinning quality is also dependent upon the length and fineness of the staple. Sea Island cotton is spun into very fine yarns, up to 300, *i.e.*, 300 hanks of 840 yd. each weigh 1 lb. The **tensile strength** of cotton is between that of silk and wool. For cotton rope, see p. 748.

Mercerized Cotton. When cotton is treated with strong caustic alkali and simultaneously subjected to mechanical tension to prevent contraction, the fiber is greatly changed, taking on a high luster and an increase in tensile strength of from 30 to 50 per cent. The mercerized fiber also has a greater power of absorption for dyestuffs.

Linen is obtained from the flax plant by "retting." The linen fiber consists of nearly pure cellulose. Natural linen varies from pale yellowish-white to gray. It may be bleached with chloride of lime, but the fiber suffers considerable deterioration. It is of pronounced luster, silky in appearance, stronger than cotton and a better conductor of heat. Its hygroscopic moisture is about the same as cotton. It does not withstand the action of boiling alkalies, bleaching powders, and other oxidizing agents, as well as cotton, nor does it dye so readily. Linen can be mercerized in the same manner as cotton.

Jute is obtained from the stalks of the jute plant by steeping in water. The fiber so obtained is free from woody fiber and generally has a length of from 4 to 7 ft. It is of pale yellowish-brown color, the best qualities showing a yellowish-white or silver-gray, and having considerable luster and tensile strength. The short fibers are employed as a raw material for paper-making. The fiber is smooth and lustrous and has no jointed ridges or transverse markings, such as in linen or other bast fibers. It cannot be readily bleached owing to the disintegration of the fiber. Its valuable qualities are fineness, silk-like luster, and adaptability for spinning. Its strength is less than that of most other bast fibers. Its chief defect is lack of durability, dampness causing rapid deterioration, and under ordinary conditions the fiber becomes brittle and weak. Bleached fiber is especially unstable; its principal uses are in the manufacture of coarse-woven fabrics such as gunny sacks, bagging, for binding thread in carpets and rugs. As a substitute for hemp it has been extensively used in the manufacture of twine and small ropes, and, owing to its cheapness, is largely used to adulterate better fibers.

Ramie, and China Grass are obtained from the bast of the stingless nettle. They are two distinct fibers, but the names are generally used interchangeably. The fiber is the strongest and most durable of all vegetable fibers and the least affected by moisture. Ramie can be separated into fibers of great fineness and possesses three times the strength of hemp. It is white in color, resembling bleached cotton, and possesses a higher luster than linen. It lacks the elasticity of wool and silk and the flexibility of cotton, and consequently produces a harsher fabric in which the fine fibers do not adhere well to each other.

Hemp is a name applied to a number of bast fibers of similar appearance and properties, and is obtained from the plant by "retting." Its composition is a mixture of cellulose and lignocellulose. It contains more hygroscopic moisture than cotton or linen. It is principally used for the manufacture of twine and cordage on account of its great strength and durability. It is not readily affected by water as contrasted with jute. It is not much used for woven textiles on account of its harshness and stiffness and lack of pliability and elasticity. The fiber is of dark brown color and cannot be successfully bleached without injury.

Manila Hemp is obtained from the leaf-stalks of the *Musa textilis*. The fiber is white and lustrous, light and stiff, easily separated, very strong, and of great durability. The coarser fibers are used for the manufacture of cordage on account of their great strength. Manila hemp ropes are stronger than English hemp ropes. See p. 748.

Sisal Hemp is obtained from the leaves of a plant found in Central America, and to some extent from the West Indies and Florida. It is of light-yellowish color, straight and smooth. Its principal use is in cordage manufacture, its strength being second only to that of Manila hemp. It is also used for paper-making.

Waste. The best **wool waste** consists of all-wool carpet yarn in threads not less than 3 in. long and comparatively free from moisture or dirt. **Cotton waste** should consist of equal parts of white and colored new cotton threads properly mixed and free from water and dirt. White cotton waste should be made up entirely of new white cotton threads not less than 3 in. long and free from dirt or water. It is customary to purchase waste by sample. Inferior quality is indicated by the presence of fibers other than wool or cotton, by sweepings or dirt, or where the waste gives evidences of having been previously used.

FREEZING PREVENTIVES

Common Salt is sometimes used to prevent the freezing of water; it does not, however, lower the freezing point sufficiently to be of use in very cold weather, and in concentrated solution tends to "creep" and to crystallize all over the receptacle. It also tends to corrode metals. For freezing temperatures, see p. 299.

Calcium Chloride ($CaCl_2$) is a white, solid substance widely used for preventing freezing of solutions and (owing to its great hygroscopic power) for keeping sizing materials and other similar substances moist. It does not "creep" as in the case of salt. It does not rust metal but attacks solder. It can be purchased in solid form containing 75 per cent. calcium chloride at from 1 to 1½ cents per lb. For freezing temperatures see p. 299.

Glycerine is a colorless, viscid liquor without odor and miscible with water in all proportions. It should have a specific gravity of approximately 1.25. It has no effect on metals but disintegrates rubber. Costs from 17 to 20 cents per lb. (1 gal. = 10½ lb. approx.). A solution containing 3½ lb. of glycerine per gal. has a freezing temperature of + 10 deg. fahr. With 5½ lb. per gal. the freezing temperature is reduced to − 10 deg. fahr. See p. 298.

Denatured Alcohol is free from the disadvantages of calcium chloride, salt, and glycerine solutions. A solution containing 50 per cent. alcohol becomes inflammable, but it is rarely necessary to use more than 30 per cent. Cost, approximately 50 cents per gal. See p. 298.

Per cent. solution	20	30	40	50
Alcohol per gal. of water, qt.	1	1½	2½	4
Freezing point, deg. fahr.	10	−5	−20	−35

GLASS

References: Rosenhain, "Glass Manufacture." Hovestadt, "Jena Glass."

Glass is generally a mixture of several silicates, produced by melting together silica, an alkali and lime or lead. There are two general kinds of glass: **lime glass** and **lead glass.** The former is the more common, is cheaper, harder, more resistive and less fusible than lead glass. The latter has greater luster and brilliancy and is used chiefly for cut-ware and optical purposes. In general, the higher the percentage of silica the harder, less fusible, and more brittle the glass.

Fusibility is decreased and hardness increased by increasing the lime, but the glass does not become so brittle as in the case of high silica. The use of a mixture of soda and potash produces a very fusible glass. In colored glass a part of the lime and lead is replaced by oxides of iron, manganese, cobalt, etc. The addition of borates and phosphates improves glass for various optical and chemical purposes, as do also zinc and barium. German optical glass contains both zinc and barium. Practically all glass is decolorized in manufacture by the addition of manganese dioxide which oxidizes the iron present to a less troublesome form. The remedy is not permanent, as is shown by the development of a violet tint in old window glass and particularly in the glass globes of gas lamps.

Glass possesses the property on cooling from a molten condition of assuming first a pasty, tough condition, and finally a rigid state. This property of plasticity while hot permits its being blown. Blowing was originally entirely done by "hand blowers" blowing through a pipe into a ball of molten glass. Mechanical blowing has recently superseded this practice for all the common forms of glass.

The **compressive strength** of glass is considerably greater than its **tensile strength,** and this can be still more increased by cooling between metal plates, although the glass becomes very brittle. Rosenhain gives the following values:

Authority:	Strength, Tons per Sq. In: Tensile	Crushing
Trautwine	1 to 4	9 to 16
Winkelmann and Schott	2 to 5½	3 to 8
Kowalski	5 to 6	20 to 27
Henrivaux	½ to 1¼	....

The values given by Winkelmann and Schott are probably the most reliable, but they refer to a series of special Jena glasses, not resembling ordinary glass.

The **modulus of elasticity** of glass may be taken at from 10,000,000 to 11,000,000 lb. per sq. in. **Modulus of rupture,** 3000 to 4000 lb. per sq. in. **Coefficient of expansion** per deg. fahr., from 21×10^{-7} to 68×10^{-7} (Rosenhain). **Specific gravity,** 2.5 to 3.45. Average **weight per cu. ft.,** 186 lb.

Poor qualities of glass are injured by rain and air, and even good glass if stored in wet places or if soaked in seawater becomes dull or iridescent. The best qualities of glass are not attacked by bases or by acids with the exception of hydrofluoric. Superheated steam is very corrosive, resulting in the destruction of boiler gage glasses.

Window Glass is generally a soda-lime glass and is always blown. The sheets are cut to marketable size, and since the surface is fused and not polished, it is brilliant, hard and less easily scratched or etched than plate glass. It is graded by quality as A, AA and AAA, and appears on the market in two thicknesses known as single thickness (about $\frac{1}{16}$ in.) and double thickness (about ⅛ in.). Single-thick glass weighs about 82 lb. per square, and double, 164 lb., no allowance being made for lap. A box of ordinary window glass contains as nearly 50 sq. ft. as the size of panes admits. Any size of pane can be made to order, but standard sizes run from 6 × 8 in. to 36 × 50 in. for single-thick and 60 × 70 in. for double-thick.

Plate Glass is usually soda-lime glass cast on large iron plates and subsequently ground and polished. It varies in thickness and can be obtained in any size from 6 × 16 in. to 144 × 200 in. Ground plate glass is extensively used for flooring.

Skylight Glass. The weight of fluted or rough plate glass required for one square of roof (no allowance being made for lap) is approximately 350 lb. for ¼ in. glass and in proportion for other thicknesses. A square of roof is 10 ft. square or 100 sq. ft.

Pressed Glass is made by forming heat-softened glass to shape in dies or mold under pressure. It is fairly inexpensive.

Luxfer Prisms are white glass plates whose interior side is provided with parallel, prismatic, transverse ridges. They are mounted vertically in front of windows or as protecting roofs more or less obliquely on the outside, and serve to diffuse the light passing through them into the darker portion of the apartment. They possess great strength and considerable resistance to fire.

Quartz Glass is made by melting rock crystal and, lately, purest quartz sand in the electric furnace. It is unaffected by changes of temperature, is fireproof and acid-resistant, does not conduct electricity and has practically no expansion under heat. It is used considerably for high-temperature laboratory apparatus.

Wire Glass is a glass having an iron wire screen thoroughly embedded in it. It offers about 1½ times the resistance to bending that plain glass does, and very thin sheets may be walked on. If properly made, it does not fall apart when cracked by shocks or heat, and is consequently fireproof. It is used for flooring, skylights, fireproof doors, fire walls, etc. It must be ordered exactly to measure as it cannot be readily cut. Less light passes through it than through plain glass.

GRAPHITE

REFERENCES: "Graphite, Its Properties, Occurrence, Refining and Uses," Department of Mines, Canada. "Mineral Resources of the United States," 1909. E. G. cheson, "Seventeen Years of Experimental Research and Development," Am. cademy of Arts and Science, 1908. "Mineral Industry," 1893, 1898, 1900, 1902, 1904.

Graphite is an amorphous form of carbon, other forms being the diamond and lampblack. Three commercial varieties are obtainable—flake, amorphous and artificial. Flake and amorphous graphite exist in nature and are separated from their accompanying rock mechanically. The latter is much more abundant but of less value commercially. The best flake graphite comes from Ceylon, although certain valuable varieties occur in the United States, particularly those in New York State controlled by the Jos. Dixon Crucible Company. The **specific gravity** of commercial graphite ranges from 2.015 to 2.583; for refined graphite it is 1.802. Its **hardness** on Mohs's scale is between 1 and 2. The **specific heat** of natural graphite is 0.2019; of furnace graphite, 0.1970. The **heating value** of natural graphite is 14,033 B.t.u. (Favre and Silberman), and of artificial graphite, 14,222 B.t.u. (Berthelot and Petit) per lb. According to Fizeau, the **coefficient of linear expansion** per deg. fahr. is 0.00000437 at 104 deg. fahr., and the elongation of a unit length from 32 deg. to 212 deg. fahr. is 0.000796. Graphite conducts heat better than diamond and is also a good conductor of electricity.

Specific Electrical Resistance: Natural, 14.20 ohms (Streints), 12.20 ohms (Muraoka); graphitized electrode, 12 ohms; Ceylon, 2 to 8 ohms (Zellner); Acheson electrode, 12 ohms (Zellner).

Flake Graphite is extensively used in the manufacture of crucibles and other refractories, owing to its resistance to heat and its ability to form a stiff mix with clay. It is also used in stove polish, foundry facings, paint and pencil manufacture and as a lubricant.

Amorphous Graphite is extensively used in paints owing to its resistance to acids and alkalies and ability to form an elastic film with oil; also for packings, lubrication, electrodes and pencils. For lubrication it must be entirely free from grit; owing to its unctuous nature, it is an excellent lubricant either alone or with oil.

Artificial Graphite is made by the International Acheson Graphite Company, of Niagara Falls, by converting amorphous carbon to graphite by subjecting carbonaceous material to a temperature in the electric furnace above the volatilization point of impurities such as iron, silicon, etc. It is possible to produce absolutely pure graphite in this manner. Its principal use at present is in the manufacture of electrodes. Large amounts are sold as a lubricant after a special treatment called "deflocculation," which makes possible its colloidal suspension in water or oil. "Aquadag" and "oildag" are products of this kind containing deflocculated graphite in water and oil, respectively.

Graphite for foundry use is often called **plumbago**, as it was originally supposed that graphite was a form of black lead. The United States Navy specifies that plumbago shall be dry, free from coal dust or grit and shall contain not less than 55 per cent. of graphitic carbon.

Graphite refractory brick are used for furnace linings on account of their great heat-resisting qualities. The product known as **kryptol**—composed of graphite, carborundum, and clay—is used for electric resistance furnaces, as its resistance is sufficient to localize a high degree of heat without the material itself being destroyed.

HEAT INSULATORS

References: "Asbestos, Its Origin and Production," *Chemical World*, Jan., 1913. Printed matter issued by the H. W. Johns-Manville Co. and the Armstrong Cork Co.

Air is the most efficient insulator available, and the essential requirement of an insulating material is that it retain a large amount of air entrapped in minute spaces. The material should not absorb moisture and should be free from rot, mold, or offensive odors. The principal materials used for this purpose are cork, asbestos, and mixtures of asbestos and magnesia.

Cork is the outer bark of the cork oak. It is highly compressible, possesses great permanent elasticity, is an excellent non-conductor of heat, waterproof, is unaffected by moisture and very slow-burning. It possesses considerable strength and can be readily bonded to concrete. While comparatively expensive initially, its lasting qualities offset the increased price. Cork is obtainable in granulated form and in sheets or boards, the latter being generally made by building up granulated cork under pressure with a binding material such as glue, asphalt, pitch, or cement. One commercial variety is obtainable in which the natural resin of the cork is used as the binding material.

The United States Navy specifies compressed **cork in sheets** 36 × 12 in. and 1 to 3 in. in thickness, and requires that it stand boiling for 3 hr. without disintegration or natural linear expansion. **Granulated cork** is obtainable in the following sizes: Unscreened, screened, 3/32, 8/20, and 12/30. Unscreened cork is the standard grade for insulating purposes and will pass through a 1/2-in. mesh screen. 8/20 granulated will pass through an 8-to-the-inch mesh, but not through a 20-mesh screen.

Cork is used extensively aside from insulation for stoppers, artificial stone, and, notably, **linoleum.** This material is made of cork dust specially treated with linseed or other drying oils and is generally reinforced on one side by fabric and varnished; it is used for floor covering and for dressing walls. It can be obtained in various thicknesses. Excellent **substitutes** are obtainable, the best being compounds of stearin, pitch, and similar substances.

Asbestos is a fibrous mineral, the most important deposits of which are found in Canada. The amphibole or hornblende variety of asbestos does not possess the fineness of fiber, tensile strength, elasticity, or flexibility of the chrysotile variety, but both are equally heat-resisting. The former variety is of no value for spinning or weaving. The chrysotile fiber is white in color and is difficult to spin on account of its perfectly smooth surface, consequently cotton or linen is generally woven with it. Asbestos thread weighs about 1 oz. to the 100 yd.

The valuable **properties** of asbestos are its flexibility, fibrous structure, incombustibility, its low heat and electrical conductivity, and its resistance to the action of most chemical agents. **Uses.** It is spun and woven into thread, rope and cloth, made up into paste for insulation, and in finely divided wool in the form of felt or paper is used for building purposes and also for building board. Mixed with caustic lime it produces a fireproof wall plaster; mixed with Portland cement it is used to make fireproof shingles; with asphalt, it is used as a roofing; and with magnesia, as an insulation for steam and hot water pipes. It is also extensively used as packing in sheet form with and without metallic insertion, and for gaskets. It has an approximate **dielectric strength of** 4000 volts per mm. and a specific resistance of 16×10^4. Good asbestos should not burn or show any flame under the blowpipe nor should it be affected by concentrated hydrochloric acid. **Asbestos wood** is obtainable in standard sheets 36 × 48 in., and in thicknesses from 1/4 in. every 1/8 in. to 1 in., and then by 1/4-in. increases to 2 in.

The United States Navy specifications calls for **asbestos millboard** in standard heets 40 × 40 in., the various thicknesses to have the following weights: 1/2 in., 25 to 9 lb., 3/8 in. 21–24 lb.; 1/4 in., 14–16 lb.; 3/16 in., 11–12 lb.; 1/8 in., 7–8 lb.; 1/16 in., 3 1/2–4 1/2 b. Asbestos millboard is generally made hard, but, if desired, can be made medium r soft. The material should stand a dry heat of 400 deg. fahr. without injury and should not be affected by acids.

The United States Navy specifies for **asbestos plaster for pipe covering** a mixture of 5 per cent. of long asbestos fiber, 65 per cent. of infusorial earth (p. 617) and 30 per cent. of fireproof binding material. The material must mix with water to proper consistency, and, after drying, should not burn or show any flame. It should not bake hard like a brick but be capable of compression. One bag, 140 lb. net weight, should cover 40 sq. ft. of surface 1 in. thick.

Mineral Wool is a fiber made by sending a blast of steam through molten slag or rock. It should be of fine, fibrous structure and as free as possible from lumps. Rock wool should be used in contact with metal, as the sulphur in slag wool may corrode the metal.

LEATHER

Belts. See Belting, p. 621.

Rawhide is prepared by rubbing oil or fats into a strip of hide, twisting and stretching it until the moisture is removed and the skin thoroughly filled with oil. It is very strong, tough and waterproof, and is chiefly used for belt lacing and similar purposes.

Leather Substitutes generally consist of refuse leather reduced to a pulp, molded, pressed or wound into suitable shapes, and frequently waterproofed by a final treatment. The earlier of these preparations were composed of glue, starch, waxes and adhesives and also included water-repellent ingredients. Of the leather-free artificial leathers, of which there are many types, the best known are **leatherine,** made by coating calico with rubber and rubber substitute; **leather board,** prepared by pulping fibrous materials of various sorts rendered absorptive by the addition of powdered chalk or whiting and after being molded into form covered with a glazing solution composed of starch, gelatine and turpentine; and **leatheroid,** a combination of chemically treated paper with rubber and sandarac often used for trunks and suitcases.

NATURAL STONES

REFERENCE: Baker, "Masonry Construction."

The important **qualities** in stone for building construction are cheapness, durability, and strength. Stones are seriously affected by weather, chemicals, gases, and temperature. Resistance to weathering depends upon both the hardness and the absorbent properties of the stone. Porosity is an objectionable element. If the constituents differ greatly in characteristics, weathering is apt to be unequal, consequently starting disintegration.

In general, crystalline structure is more durable than amorphous. Examination of the condition of the stone in the quarry affords some idea as to its durability. Further information can be obtained by artificial tests of weight, crushing strength, absorption, and resistance to freezing, to acids and to heat.

The **structure** of the stone is of prime importance where heavy loads are to be carried. All stones are classified on the basis of the mineral forming the chief constituent, namely, **siliceous stones,** such as granite, syenite, gneiss, trap, quartz, etc.; **argillaceous** or clayey **stones,** in which alumina is the predominating mineral, such as slate; **calcareous stones** in which carbonate of lime predominates, such as limestone, marble, and dolomite. From a practical standpoint, stones are also divided into two classes—**stratified** and unstratified—the former being represented by such stones as quartz, marble, slate, etc., whereas the **unstratified** stones consist of an aggregate of crystalline grains such as granite, trap, and basalt-lava.

The following table gives the composition and some of the properties of the more common minerals.

Properties of Common Minerals*

	Composition	Hardness, Mohs's scale	Density
Amphibole	$Ca_5(ClF)(PO_4)_3$	5.0	3.2
Andalusite	Al_2SiO_5	7.5	3.1 to 3.2
Anhydride	$CaSO_4$	3.0 to 3.5	2.9 to 3.0
Apatite	$CO_5(ClF)(PO_4)_3$	5.0	3.2
Aragonite	$CaCo_3$	3.5 to 4.0	2.9
Barite	$BaSO_4$	2.5 to 3.5	4.3 to 4.6
Bauxite	$Al_2O(OH)_4$	1.0 to 3.0	2.4 to 2.5
Biotite	$(HK)_2(MgFe)_2Al_2(SiO_4)_3$	2.5 to 3.0	2.7 to 3.1
Borax	$Na_2B_4O_7.10H_2O$	2.0 to 2.5	1.7
Calcite	$CaCO_3$	3.0	2.7
Chrysolite	$(MgFe)_2SiO_4$	6.5 to 7.0	3.3 to 3.6
Corundum	Al_2O_3	8.5 to 9.0	3.95to 4.1
Cryolite	$AlNa_3Fe_6$	2.5	2.9 to 3.0
Cuprite	Cu_2O	3.5 to 4.0	5.8 to 6.1
Dolomite	$CaMg(CO_3)_2$	3.5 to 4.0	2.8 to 2.9
Epidote	$Ca_2(AlFe)_2(AlOH)(SiO_4)_3$	6.0 to 7.0	3.2 to 3.5
Garnet	$(SiO_4)_3$	6.5 to 7.5	3.1 to 4.3
Graphite	C	1.0 to 2.0	2.1 to 2.2
Gypsum	$CaSO_4.2H_2O$	1.5 to 2.0	2.3
Hematite	Fe_2O_3	5.5 to 6.5	4.9 to 5.3
Hypersthene	$(MgFe)SiO_3$	5.0 to 6.0	3.4 to 3.5
Kaolinite	$H_4Al_2Si_2O_9$	2.0 to 2.5	2.6
Lepidolite	$(KLi)_3Al(SiO_3)_3$	2.0 to 2.5	2.8 to 3.2
Leucite	$KAl(SiO_3)_2$	5.5 to 6.0	2.4 to 2.5
Magnesite	$MgCO_3$	3.5 to 4.5	3.0 to 3.12
Magnetite	Fe_3O_4	5.5 to 6.5	4.9 to 5.2
Malachite	$Cu_2(OH)_2CO_3$	3.5 to 4.0	3.9 to 4.0
Microcline	$KAlSi_3O_8$	6.0 to 6.5	2.5 to 2.6
Nephelite (Elaeolite)	$(NaK)_8Al_8Si_9O_{34}$	5.5 to 6.0	2.5 to 2.6
Orthoclase	$KAlSi_3O_8$	6.0 to 6.5	2.4 to 2.6
Plagioclase	$nNaAlSi_3O_8 + mCaAl_2Si_2O_8$	5.0 to 7.0	2.6 to 2.7
Pyrite	FeS_2	6.0 to 6.5	4.9 to 5.2
Pyrolusite	MnO_2	1.0 to 2.5	4.7 to 4.86
Pyroxene	$RSiO_3$	5.0 to 6.0	3.2 to 3.6
Pyrrhotite	FeS	3.5 to 4.5	4.5 to 4.6
Quartz	SiO_2	7.0	2.6
Serpentine	$H_4Mg_3Si_2O_9$	2.5 to 4.0	2.5 to 2.65
Siderite	$FeCO_3$	3.5 to 4.0	3.8 to 3.9
Talc	$H_2Mg_3(SiO_3)_4$	1.0 to 1.5	2.5 to 2.9

* From "A Guide to the Sight Recognition of 120 Common or Important Minerals," by A. J. Moses.

Granite, composed of feldspar, quartz and mica, is the strongest and most durable of stones in common use and is quarried into shape with facility. It is extremely hard and tough and can be wrought into elaborate shapes only with great difficulty. The feldspar content determines the coloring, while the quartz determines essentially the hardness.

Trap is a very strong and durable stone, but quarried and wrought with great difficulty. It is exceedingly tough and is widely used for roads and railroad ballast.

Syenite, composed of feldspar and hornblende, is crystalline, granular, and speckled black and white; it is hard, and takes a good polish. **Diorite** and **diabase** are similar stones and are frequently known as **greenstone.**

Serpentine is generally green in color and disintegrates in the open air. **Soapstones** and **asbestos** are related to it.

Trachytes are dense, frequently porous, feldspathic rocks with admixtures of crystals of hornblende, biotite and magnetite. They are gray in color. **Pumice stone** is a variety.

Augitites. Basalt, a solidified product of volcanic emission is a common variety, and the most durable and pressure-resisting structural stone; it is valuable for road-building and for bank and supporting walls. **Basalt-lava** is very porous rock varying in color and hardness, used for stairs, paving blocks and millstones.

Gneiss, a schistose structure of granite, is more subject to decomposition than granite, especially if rich in feldspar and mica.

Quartzite (pure or almost pure quartz) is characterized by its vitreous crystallization. When porous, it is used for millstones; it is also used in the manufacture of glass, and as broken stones or as gravel in road building, for preparing concrete, etc.

Mica-schist consists of gray quartz between layers of mica. It is used for roofing plates.

Clay-schist is composed of hardened clay and quartz, often with little lamellæ of mica. It cleaves readily and is therefore suitable for roofing. See also Slate, p. 640.

Limestone (carbonate of lime), if capable of taking high polish, is called **marble**. The color varies with amounts of iron oxide, copper oxide, etc. By ignition it produces **caustic lime**. **Calcite** is transparent and used as sculptor's marble. **Chalk**, another form of limestone, is white and earthy.

Marl is a dense, earthy or slaty rock, consisting of a mass of carbonate of lime mixed with clay and siliceous sand. Certain species are well suited for the manufacture of cement.

Dolomite, a magnesian carbonate of lime, is whitish-yellow or gray to brown in color, crystallized and dense. Certain deposits produce a hard, durable building stone, well suited for road building.

Gypsum, hydrated sulphate of lime, is generally white, but often yellowish to reddish, or gray to blackish; it is not durable when exposed to weather.

Sandstone consists of sand grains made up into a more or less compact rock by a binder consisting generally of silicon, lime, or clay. It often contains calcite, mica, iron ore, and admixtures of red and green clay. Iron pyrites has a disintegrating effect; an abundance of mica is also undesirable.

Conglomerates consist of rounded pebble stones formed by the aggregation of various rocks, held together by a binder such as clay or marl.

Gravel and Sand consist of small fragments of quartsy minerals, often mixed with lime, marl, and clay, from which they can be liberated by washing. Pit sand is ordinarily gritty but frequently more impure than river sand.

Clay is a hydrous oxide of aluminum; as it occurs in nature it is frequently a mixture of clay with sand, limestone and oxide of iron produced by the erosion of rocks containing feldspar minerals. The purest variety, **kaolin**, is used for the production of chinaware and as a paper-making material. Plastic varieties are used for pipe clays, fireproof clays and potter's clays. Clay shrinks in drying and subsequently in burning without losing its form as a whole, and becomes extremely hard. Certain varieties known as **fuller's earth** are very absorbent of oils or dyes. See also Brick, p. 622.

For strength and other physical properties of natural stones, see Table 4, p. 385.

PAINT OILS

China Wood Oil (tung oil), expressed from the tung nut, is being increasingly used as an adulterant and **substitute for linseed oil** in paints and varnishes. It has a peculiar rancid odor and yellow color, usually darker than that of linseed oil. It has the unique property of apparently drying throughout at a uniform rate when spread in a film instead of forming a skin by surface drying as does linseed oil. It dries "flat" instead of glossy, with an opaque, white film. When heated to about 400 deg. fahr. it is converted into a jelly-like mass insoluble in ordinary solvents. Specific gravity, from 0.936 to 0.944; its other constants are very similar to those of linseed oil. It can be mixed with lead and manganese driers. Its peculiar drying properties and resistance to water when dry have been successfully **utilized in varnish making.**

Linseed Oil is a brownish-yellow vegetable oil with a specific gravity between 0.931 and 0.937. It possesses the property of absorbing oxygen and consequently drying to a greater extent than most other oils; hence largely used for paints and varnishes. It is frequently adulterated with cottonseed, rosin, fish, China wood and soya bean oils. **Adulteration** can sometimes be detected by odor, color or gravity, but chemical analysis is usually necessary. Mixtures of China wood, soya bean and linseed oils can be made which are extremely difficult to detect. Adulteration with mineral or rosin oil can frequently be detected by a green or blue fluorescence which appears when the oil is viewed on a black background with the back to the light.

As raw oil dries very slowly, it is frequently used as "**boiled oil.**" The name was derived from the original practice of boiling the oil in open kettles at a temperature of 300 to 500 deg. fahr. and incorporating oxides of lead and manganese, this process greatly increasing the drying properties. Little if any boiled oil is now made in this way, as the original process was expensive and darkened the oil greatly. So-called "boiled oil" is now made by heating a small quantity of oil with lead and manganese oxides and subsequently adding this drier to the raw oil. A third class, known as "**bung hole" oil**, is made by adding driers dissolved in benzine or turpentine to the raw oil. If properly made the quicker processes produce satisfactory oil. Boiled oil should have a similar odor and taste to raw oil; specific gravity, between 0.931 and 0.950.

PAPER

REFERENCES: Griffin and Little, "The Chemistry of Paper-making." L. E. Andes, "The Treatment of Paper for Special Purposes."

Paper is made from cotton, linen and hemp rags and waste, from chemically prepared wood, from straws, and from wood not chemically prepared. The best papers are made from cotton, linen and hemp; rags and spinning wastes having longer and stronger fibers and consisting of purer cellulose are less affected by the chemical and mechanical operations of paper-making and do not wear and crack so easily as paper made from wood, straw, or mechanically prepared wood. Paper is weakened in manufacture by the addition of mineral fillers, by large amounts of sizing materials, or by rapid drying. **Fillers** are added to increase weight, improve printing quality and increase opacity. **Sizing** is added to prevent the spreading of ink.

The highest grades of **writing and book papers** are made from rags alone, cheaper grades from rags and wood pulp, or from wood pulp alone. The lower grades of paper, such as **newspaper**, are made largely from ground wood (*i.e.*, wood reduced to fiber by mechanical grinding) and readily disintegrate.

The highest-grade **wrapping and tag papers** are made from old manila rope, which produces a strong, tough paper. Extensive adulteration is practiced by the use of rope stock other than manila and by the substitution of wood pulp, principally sulphite, in whole or in part. **Cable paper** should be made from pure high-grade manila stock and should be free from all chemical residues, acids, etc. The use of "**Kraft" paper** is rapidly extending as a wrapper. This is made by cooking wood by the sulphate process and produces a strong, tough paper of a characteristic brownish color.

Box boards, container boards, etc., are made from ground wood alone, mixtures of ground wood and sulphite, and in the form of lined boards which have a layer of paper on the outside and are filled with a waste material such as old newspapers beaten to a pulp, ground wood, liquorice root, etc. **Blotting papers** are made from wood pulp with or without small additions of rag. Comparison is readily afforded by the speed with which the paper absorbs water.

In general, light, strong papers should be selected as additional weight and bulk beyond that necessary to secure sufficient strength, opacity, and resistance to wear, results in less durability and greater cost both of material and handling of transportation.

In **testing** papers, **tensile strength** (500 to 2500 lb. per sq. in.), stretch, and bursting are of importance; tests should be made on special machines. Comparison of different samples of the same grade of paper can be roughly made by tearing, crumpling and by visual examination for freedom of dirt and other indications of careless manufacture.

ROOFING MATERIALS

See Report of Committee on Buildings, *Bulletin Am. Ry. Eng. Assn.*, Feb., 1913; also p. 1280 and pp. 1327 to 1329.

Asphalt. Asphalts are bitumens occurring naturally in the solid state In crude form they are too hard and brittle for most purposes, and are therefore generally softened or fluxed with petroleum oil products. Tars do not mix well with asphalt. Mixtures of different asphalts give better results than single ones. Asphalts become hard and brittle with age, losing their elasticity and binding power. Properly refined, they are excellent for roofing purposes.

Trinidad Asphalt is widely used, due to its uniform quality. It is affected by sunlight and water, but is not volatile. It is not suitable for underground waterproofing or for places where it cannot dry out. Contains about one-third mineral matter.

Bermudez Asphalt is not so uniform as Trinidad, and is softer than the other commercial asphalts. It contains little mineral matter.

Gilsonite and Elaterite are nearly pure bitumens obtained from Colorado and Utah. Gilsonite is hard and brittle, requiring considerable fluxing. It is not always uniform in quality, nor a good saturant, although used in many roofings and in making varnish. Elaterite contains considerable sulphur, but is considered valuable for roofing.

Rock Asphalt or Mastic is sandstone naturally saturated with bitumen. It is little used for roofing or waterproofing, but is excellent for paving and floors.

Petroleum Residuals are obtained by the distillation of petroleum. Pennsylvania oils have a paraffin base, California oils an asphaltic base, while the Middle Continent oils are between the two. Most paraffin compounds are useless as binding materials; some of fluid paraffins are excellent fluxes for asphalt. The California oil residues are very similar to native asphalts, and are extensively used for roofing and paving. The Gulf, Middle Continent, and Texas residues are widely used, are cheaper than the California, but weaker and less ductile. These residual oils are treated by blowing air through them when hot and by vulcanizing with sulphur, which produces a material very stable and unaffected by the sun, but not a good saturant.

Tars. Coal Tar is produced by the distillation of coal either in manufacture of illuminating gas or in by-product coke ovens. It contains from 5 to 35 per cent. of carbon in a finely divided state due to cracking of hydrocarbons, which is claimed to be of value in roofing materials, as it makes tar less affected by temperature. Moderate amounts do not affect saturating power. The percentage of carbon content and the melting point are of importance. The U. S. Treasury Department specifications for **roofing pitch** require a "straight-run" pitch from coal tar, having the following properties: Melting point (m.p.), 135 to 155 deg. fahr.; insoluble in benzol 15 to 35 deg. Bé.; specific gravity, 1.25 to 1.35; evaporation loss (7 hours, at 325 deg. fahr.) not over 9 per cent. for pitch of 145 to 155 deg. m.p. and 11 per cent. for pitch 135 to 145 deg. m.p.; specific gravity of distillate to 670 deg. fahr., not less than 1.07 at 140 deg. fahr.

Water-gas Tar is a by-product of carbureted water gas obtained from the petroleum used, and is sometimes called **oil tar**. It contains generally less than 2 per cent. of free carbon, is affected by water and forms an emulsion with it, and is generally considered less stable than coal tar. As many gas plants do not separate their coal and oil tars, mixtures of the two are very common.

When properly compounded, asphalts and oil residuals produce a wide range of valuable roofing materials, whose durability can be foretold. This is even more true of tars. Laboratory tests are of negative value only, and specifications must include factory inspection and knowledge of materials used.

Felts for use with asphalts must have high absorbing capacity. With coal tar this is not so important, as it is more liquid when hot. Felts are generally made of cotton rags, woolen felts being expensive and too soft and tender. A small proportion of wool rags improves the absorbing capacity, but this rarely exceeds 25 per cent. Wood pulp and straw are often added to cheapen the price, but they reduce the value of the felt. Felts should be free from lumps. Dry felts are sold by weight, the standard being the amount required to lay 480 sq. ft. A felt weighing 28 lb. to 480 sq. ft. is known as No. 28. Mineral fillers are often added to make weight. Specifications should limit ash to 5 per cent.

Asbestos felts are much more expensive, are poor absorbents and act merely as protection to asphalt, as they neither burn nor decay. Tar has not been successfully used with **asbestos felts.** Jute burlap or canvas is of doubtful value, as jute is not absorbent and decays more readily than cotton. Average weight, 15 lb. per 100 sq. ft.; about 10 per cent. extra is required for laps.

Prepared Roofings. A large amount of roofing is made with a cheap, oily saturant composed of asphalt and a large percentage of oil. This material leaks out from the felt and continually distills under heat, the roofing rapidly crumbling and deteriorating. The best **saturant** is made from a mixture of asphalt and stearin pitch. Such a compound gives body to the felt and durability to the material and enables it to remain flexible. After the felt is properly saturated it should be coated to prevent the elements from attacking the felt and saturant. The **coating** used on the best qualities of roofing is a mixture of stearin and **Calabria pitch**, which is the most permanent and substantial waterproof coating developed, as it remains flexible, does not deteriorate readily and is very resistant to moisture. Many cheaper coatings are on the market composed of asphalt and gilsonite fluxed with petroleum oil, but these are not durable and allow roofing to rapidly become hard and brittle. Most prepared roofings are inferior to properly built-up roofing.

Tile. Hard-burned clay tiles with overlapping or interlocking edges cost about the same as slate. They should have a durable glaze and be well made. Unvitrified tiles with "slip" glaze are satisfactory in warm climates, but vitrified tiles only should be used in the North. Weight, from 750 to 1200 lb. per square (100 sq. ft.). Properly made tile does not deteriorate, is a poor conductor of heat and cold, and is not so brittle as slate.

Slate should be hard and tough and have a well-defined vein that is not too coarse. If too soft, they will absorb water; if too brittle, they are easily broken on cutting and installing. The surface when freshly split should have a bright metallic luster, free from loose flakes or dull surfaces and straight and true. Slate should give a clear metallic ring when struck. Color is not an indication of quality. Black ribbon slate is cheaper than all-black slate and equally good if appearance is unimportant.

Stock **sizes** range from 7 × 9 in. to 14 × 24 in.; thickness, from 1/8 to 3/8 in., 3/16 in. being the usual thickness. Approximate weight, 650 lb. per square. Slate roofs rank well in regard to fire hazard, but are not so good as tile on exposure to adjacent conflagrations.

Cement Roofings. Reinforced-concrete tiles are used with steel roof trusses, and are generally formed with projections enabling them to be placed directly on the purlins and held in place by their own weight. Wire glass can be inserted, avoiding expensive skylights. Cost, about the same as slate. **Small cement tiles** are less expensive than clay tile but more absorbent and brittle. **Asbestos shingles** made of Portland cement and asbestos are excellent. They cost more than slate but can be laid in the French or diagonal methods better than slate. They are sometimes strengthened by the insertion of wire mesh or perforated steel sheets.

Shingles. Clear **white pine** shingles are the best; **red cedar** shingles of good quality are obtainable from the Pacific Coast. In the South **red cypress** from the Gulf States is preferable. Redwood shingles come 5½ butts to 2 inches; lesser thicknesses are more liable to crack, and have shorter life. Shingles 8 in. wide or over should be split before laying. Dimension shingles of uniform width are obtainable. Dipping in oil or creosote adds to life. Creosote stains are both preservative and decorative. Shingles should be dipped before laying for best results. Wood shingles make a lasting roof, but materially increase the fire hazard. See also p. 583.

Metallic Roofings are laid in large sheets without sheathing (often strengthened by corrugating), sometimes cut into small sizes and laid as shingles, bent into interlocking shapes like tile, or soldered into a single structure as in tin roofing. The first cost is less than that for clay tile. Sheet steel covered with a coating of bitumen and asbestos is a recently developed roofing material. For data on corrugated sheets, see p. 1303.

RUBBER, GUTTA PERCHA AND BALATA

REFERENCES: C. O. Weber, "Chemistry of India Rubber." F. Clouth, "Rubber, Gutta Percha and Balata." H. L. Terry, "India Rubber and Its Manufacture." P. L. Schidrowitz, "Rubber."

These three substances which are frequently confused are all produced by the coagulation of the latex from certain shrubs and trees and are similar in many respects. The essential difference between gutta percha and rubber is that the former becomes soft and plastic on immersion in hot water, retaining any shape then given to it on cooling, whereupon it becomes hard but not brittle like other gums. Rubber or caoutchouc, on the other hand, does not soften in hot water and retains its original elasticity and strength almost unimpaired. Balata behaves on heating very much the same as gutta percha but possesses very little elasticity and can be vulcanized only with difficulty, whereas rubber and gutta percha are readily vulcanized, although the two latter substances possess different properties after vulcanization.

Rubber is obtained principally from South America, Africa and Ceylon, the the former country producing most of the better grades. **Para rubber** is the best rubber obtainable, but in recent years "plantation" rubber from the Malay Peninsula has become an important factor. Certain lower-grade rubbers, notably Guayule, Jelutong and Ceara, are rapidly coming into use for the cheaper materials.

Unvulcanized rubber is little used except for insulating tape, dental rubber, rubber cements and a few minor purposes. The bulk of the rubber used is **vulcanized** either by heating under pressure with sulphur or by exposing in the cold to the fumes of sulphur monochloride.

High-grade rubber will stretch to approximately ten times its length and at this point will bear a **load** of 10 tons per sq. in. It can be compressed to one-third its thickness thousands of times without injury. When vulcanized for elasticity it behaves like other engineering materials up to a load of 30 per cent. of the breaking load and at an extension of one-half the maximum. From this point on, however, the resistance increases in greater proportion than the extension, and finally assumes a comparatively high value. Even when stretched almost to the point of rupture it restores itself very nearly to its original dimensions on being released, and gradually recovers a part of the loss of form at that instant observable.

The **specific inductive capacity**, of rubber ranges from 2.22 to 3.70; its **dielectric strength** per cm. is 476 kilovolts. The **color** of vulcanized rubber containing only rubber and sulphur is light gray; mineral and other additions vary the color from a more-or-less dark gray to black.

Raw rubber possesses the power of **self-adhesion** which practically disappears in vulcanized rubber. Cold water indefinitely preserves rubber, but if exposed to the air, and particularly to the sun, all rubber goods tend to become hard and brittle. Dry heat up to 120 deg. fahr. should have little deteriorating effect, but at temperatures of 360 to 400 deg. rubber begins to melt, loses its elasticity and becomes sticky; at higher temperatures it becomes entirely carbonized. Unvulcanized rubber is soluble in carbon bisulphide, benzine, petroleum ether and turpentine. These **solvents** have little effect on vulcanized rubber except to cause it to swell. Vulcanized rubber is very resistant to alkalies and acid.

Rubber is seldom used alone industrially, but mixed with mineral matter, waxes, rubber substitutes or reclaimed rubber. The highest grades, such as insulation, fire hose, etc., seldom contain more than 40 per cent. of rubber. A small quantity of paraffin or ozokerite is beneficial in reducing the tendency to oxidize. Rubber substitutes are only suitable for very cheap material. While some excellent reclaimed rubber is now made, its use is undesirable where permanency is desired.

Specifications should state the percentage and kind of rubber desired, the specific gravity of compound and suitable physical tests. Tensile strength and stretch tests are of importance, but vary widely with different compounds. A good 30 per cent. Para compound should have a **tensile strength** of at least 1000 lb. per sq. in. and should stretch to at least five times its length before breaking. A piece stretched to three times its length should show a permanent elongation of not more than 20 per cent. Chemical analysis is of value in detecting rubber substitutes, reclaimed rubber and foreign substances, but, generally speaking, does not serve to identify the grade of rubber. Red lead is stated to be undesirable in high-grade compounds as it tends to increase oxidation, which is the primary cause of deterioration of rubber products. Zinc oxide, whiting, clay, barytes and similar substances are generally used and are harmless. Rubber for permanent use should be slightly under-vulcanised rather than over-vulcanised. Rubber compounds are extensively used for electrical insulation, hose, steam and water packing, buffers, tiling and numerous other purposes where electrical strength, resilience and waterproofing qualities are desired.

Hard Rubber, also known as **ebonite** and **vulcanite,** is made by vulcanising rubber with a large quantity of sulphur and for a greater length of time. It can be softened by heat and then be shaped or molded as desired. It is capable of being highly polished when free from mineral admixtures. It does not oxidise readily and is not affected by air and sunlight. It is very impervious to electricity but becomes charged with static electricity when rubbed. The substance is black, inodorous and resembles horn in appearance. The solvents which dissolve raw rubber and partly dissolve vulcanised rubber have no influence on hard rubber. It offers great resistance to all acids. If exposed for a long time to temperatures above 400 deg. fahr. it does not melt but carbonizes. It is extensively used for switch handles, for covering tools for electrical purposes, insulating tubes for electrical conduits, fountain-pen handles, linings for acid vats and numerous other purposes. Its use is being somewhat superseded by rubber substitutes.

The United States Navy **specifications** require that "hard rubber shall be tough and hard, show a shiny black fracture and shall be capable of being worked with machine tools and readily take a fine jet-black polish and must show high insulation and dielectric strength. The samples shall be exposed to live steam at a temperature of 212 deg. fahr. for two hours and must show no signs of disintegration or softening."

A **semi-hard rubber** is also made by varying the degree of vulcanization.

This is more flexible and elastic than hard rubber and more durable than ordinary vulcanized rubber.

Reclaimed Rubber. Large quantities of recovered or reclaimed rubber are used, generally mixed with new rubber. This is prepared either by merely cleansing and grinding up the waste rubber or by a chemical-mechanical process of reclaiming which removes as much as possible of the sulphur and foreign ingredients from the rubber and brings it as nearly as possible to its original crude condition. The quality of reclaimed rubber is widely variable, but a good quality can be obtained which is of considerable value in the production of cheap rubber articles.

Synthetic Rubber. Artificial rubber has been produced by chemical synthesis on a laboratory scale and it has been claimed recently (1912) that commercial synthetic rubber is being produced in both England and Germany, although the details of manufacture are as yet secret. Production on a large scale, however, is a possibility in the near future.

Rubber Substitutes. Although numerous so-called substitutes for rubber have been prepared, there is at the present time no real substitute, all of these compounds being merely cheapening ingredients and in no way the equivalent of real rubber. The great bulk of this material is used in admixture with new or reclaimed rubber. These substitutes are of two kinds: **Black substitutes**, which are made by oxidizing vegetable oils either by blowing or by treatment with nitric acid, producing a solid, elastic, brown substance which softens in water and is soluble in turpentine, carbon bisulphide and alkalies, and largely used in admixture with rubber and alone on cheap waterproof fabrics; and **white and brown substitutes**, the white substitute being made by cold vulcanization of vegetable oils with sulphur monochloride, producing a spongy, elastic substance soluble in alkalies and largely used as a filler in white rubber goods. The brown substitute is made by heating vegetable oils with flowers of sulphur and produces a substance of rubber-like appearance but possessing little elasticity or density. It is sometimes employed alone on waterproof cloth, but generally in combination with rubber.

Gutta Percha. Pure gutta percha is colorless, and, when finely cut, transparent. It is tasteless, inodorous, softens at 99 deg. fahr., and can be molded readily at 195 deg. fahr. If heated above 265 deg., it melts to a colorless oil; it is not affected by cold and is very resistant to water. When exposed to air and sunlight, it rapidly deteriorates. It possesses excellent heat resistance and the greatest electrical resistance of any plastic material.

Wunschendorff states that the **resistance** of gutta percha at a temperature of 75 deg. fahr., copper taken as a unit, is 6×10^{19}. Gutta percha is insoluble in most reagents, is slightly soluble in pure alcohol and partly dissolved by turpentine, olive oil and benzine. The best **solvents** are carbon bisulphide and chloroform. It is very little affected by acids or alkalies. On account of its great electrical resistance, waterproof qualities and permanency under water, it was largely used in the first transatlantic cables. Is little used to-day except for the manufacture of golf balls and minor uses.

Balata. Raw balata is gray, brown or whitish-red and possesses little elasticity. It is softer than gutta percha under ordinary temperatures, does not become so firm when cooled, is soluble in turpentine, benzine and carbon bisulphide, but resistant to acid and alkalies. It can be readily molded at 125 deg. fahr. On account of its great toughness and resistance to moisture and air, it is particularly well suited for the manufacture of rubber belts; also used to a small extent in rubber sheets, shoe soles, etc.

SHELLAC

The most commonly used shellac for pattern and varnish work is the T. N. grade, which usually contains about 5 per cent. of rosin. A grade called Garnet-lac, sells at about the same price as T. N., and is free from rosin or lac-wax.

Shellac is **soluble** in alcohol, hot aqueous solutions of sodium and potassium carbonates, caustic alkalies, borax and acetic acid. Its **specific resistance** is approximately 9×10^9 megohms per cu. cm.; **specific inductive capacity**, about 2.74. It is slightly hygroscopic but not sufficiently so to impair its value as an electrical insulator. Shellac softens at 181 deg. fahr., flows readily at 230 to 248 deg., hardens again at 356 to 384 deg. and begins to carbonize at 446 deg. fahr.

Shellac is **used** extensively as a sticker and insulator in the manufacture of electrical apparatus, for bonding abrasive wheels, and as a bond for a variety of molded goods. The most common **adulterant** is rosin, which makes the shellac brittle. A common test is to heat a piece of shellac with a lighted match until one corner is melted, then blow out the flame and attach match to the piece of shellac and let the latter drop to the floor. If the shellac is of good quality and ordinarily free from adulteration, it will string out in a thin flexible threaded form the height of a man's hand held as high as possible from the floor. It has been found that for certain purposes as a sticker the addition of 5 per cent. of rosin increases the adhesiveness. **Bleached shellac** is used very largely in lacquers and has less adhesive strength than **orange shellac**. It is not completely soluble in alcohol and nearly always contains some chlorine. The ordinary spirit **varnish** is made by dissolving from 4½ to 9 lb. of shellac to a gallon of alcohol. 4½ lb. per gal. makes 1½ gal. of varnish and 9 lb. per gal. makes 2 gal. of varnish.

LUBRICANTS

BY

AUGUSTUS H. GILL

REFERENCES: Archbutt and Deeley, "Lubrication and Lubricants;" Brannt, "Petroleum and Natural Gas;" Holde-Mueller, "The Examination of Hydrocarbon Oils, Fats and Waxes;" Lewkowitsch, "Analysis of Fats, Oils and Waxes;" Gill, "Short Handbook of Oil Analysis" and "Engine-room Chemistry."

Properties. A good lubricant should possess: 1. Minimum cohesion among its own particles. 2. Maximum adhesion to the surfaces to be lubricated. 3. Slight changeability as regards oxidation by the air or by changes in the temperature or pressure of the bearings. 4. Freedom from acid. 5. Purity.

A thin film of oil should interpose itself between the bearing metal and the shaft; as the rotation of the latter separates the particles of oil, low cohesion facilitates this. High adhesion prevents the oil from running off the shaft and out of the bearing. The oil should be sufficiently viscous to prevent it from being squeesed out of the bearing except by unusual pressures.

Exposure to the air may result in rancidity with the animal and vegetable oils or in resinification with the mineral oils; this is aided by heat and dust.

Heat produces an evaporation of hydrocarbon gases from petroleum oils—particularly the lighter or spindle oils—increasing the danger from fire.

Inability to withstand cold—high cold test—may freeze the oil in the bearings and stop lubrication.

Acid attacks and roughens the shaft and bearings. It can be formed by oxidation of the oil or come from the sulphuric acid used in refining.

Purity is freedom (*a*) from water which may emulsify the oil and diminish its lubricating power (water also gets into the lubricating wicks and diminishes their capillarity); (*b*) from solid matter which would stop up the oilers. This may be metal from the machinery, stearin, dirt, chips, etc., glue from the barrels, coke with mineral oils, cracklings with animal oils, and cellulose or gelatinous matter with vegetable oils.

Tests of Lubricants. Viscosity is the degree of fluidity of an oil—its internal friction or shearing modulus. The **viscosity test** consists in noting the time in seconds required for a certain quantity of oil to flow through an orifice at a standard temperature. In this country the Saybolt universal **viscosimeter** is the one commonly used, although the European Engler viscosimeter is increasingly employed. Another instrument for testing viscosity is the Doolittle torsion viscosimeter.

The **gumming test** is performed by treating the oil with acid, which brings about in a short time the changes that take place in an oil when used; it is also a measure of the extent to which an oil will "carbonize" in a gas-engine cylinder.

The **flash test** determines the lowest temperature at which the oil will give off vapors in quantity sufficient to produce an explosive mixture when mixed with air. The **fire test** is the lowest temperature at which the vapors given off will burn continuously when ignited.

The **evaporation test** shows the quantity of oily vapor an oil will give off at the average temperature of the bearing. It represents the amount of oil serving its purpose, and of oily vapor affecting the fire risk.

The **free-acid test** shows the amount of uncombined acid contained in the oil. This, in the case of mineral oils, is sulphuric acid coming from the refining process; with the organic oils it is an indication of age or rancidity and is usually oleic acid. The amount of sulphuric acid (calculated as SO_3) should not exceed 0.3 per cent. The permissible amount of oleic acid varies according to the purpose for which the oil is to be used. For tallow for cylinder oils, not more than 0.15 per cent.; for prime lard for signal oil, not more than 0.2; and for extra No. 1 lard, not more than 1.5 per cent. Free acid in oil attacks the metals of the bearings, journals, cylinders and containers, and in the case of signal oils chars the wick.

The "Oil Pulp" or **Soap Test.** To increase artificially and temporarily the viscosity of oils—in order to pass specifications—recourse is had to the addition of a small percentage of "dope," "oil thickener" or "white gelatin." This greatly increases the viscosity and causes the oil to chill more easily and to emulsify, thus increasing the friction. Furthermore, it is precipitated by contact with water or steam, causing clogging of the machinery.

The **saponification value** indicates the number of milligrams of potassium hydrate required to saponify 1 gram of oil. As this is about 193 for most of the oils, it is no criterion by which to distinguish one oil from another. Its chief value is to determine if an organic oil be adulterated or compounded with an unsaponifiable one, as mineral or rosin oil.

The **specific gravity** of organic oils is generally stated as a decimal indicating its relation to that of water. The specific gravity of the mineral oils is expressed in degrees of the Baumé hydrometer scale (see p. 85) for liquids lighter than water.

The **iodine number** or value represents the percentage of iodine absorbed by an organic oil under fixed conditions, and is used in detecting adulteration. Vegetable oils are subject to the influence of changing seasons and animal oils to that of changes in feed; as in the case of olive oil the iodine number varies from 77 to 88, it is impossible by this method to ascertain the adulteration of olive and some other oils more closely than about 5 per cent.

The **Maumené value** is the rise in temperature (deg. cent.) produced by mixing 10 cu. cm. of strong sulphuric acid with 50 grams of oil contained in a jacketed beaker. It is used for detecting adulteration.

The **cold test** gives the temperature at which the oil will just flow, and indicates the availability of the oil at this temperature.

The **friction test** measures the power consumed by the oil. This is carried out upon a bearing under ideal conditions—regularity of feed, temperature and pressure. The effects of the oil previously used upon the machine may persist for many hours, even though the shaft and bearing be apparently clean. Reliable readings cannot be obtained until after the machine has run at least 8 hours with the oil to be tested.

Choice of a Lubricant. In choosing a lubricant, the following points should be considered: (1) The pressure on the moving surfaces; (2) the velocity with which the surfaces are moving; and (3) their temperature. The cardinal principle underlying all lubrication is to **use the least viscous oil that will stay in place and do the work.** Besides this, the following will aid in the selection of a suitable lubricant:

1. The flash point should be above 300 deg. fahr.
2. The result of an evaporation test should be under 5 per cent.
3. For light pressures and high speeds, mineral oils (of sp. gr. 30.5 deg. Bé., flash point 360 deg. fahr.), sperm, olive and rape oils may be used.
4. For ordinary machinery, mineral oils (of sp. gr. 27–29 deg. Bé., flash point 400–450 deg. fahr.), lard, whale, neatsfoot, and tallow or heavy vegetable oils may be employed.
5. For cylinder oils, mineral oils (of sp. gr. 27 deg. Bé., flash point 550 deg. fahr.) alone or with small percentages of animal or vegetable oils (degras, tallow, linseed, cottonseed and blown rape) may be used.

6. For watches and fine machinery, clarified sperm, jaw and "melon" oils may be employed.

7. For very heavy pressure and slow speed, lard, tallow and other greases, either by themselves or mixed with graphite and soapstone.

Properties of Various Lubricating Fats and Oils

Animal and vegetable fats and oils are distinguished from mineral oils by being saponifiable with caustic alkalies, making hard or soft soaps according as soda or potash lye is used. Organic oils oxidize, becoming rancid and setting free fatty acids; they also gum or "dry" to a certain extent, this being most noticeable with cottonseed and corn oil. Subjected to high temperatures they are somewhat decomposed, evolving acid products. They are refined to remove albuminous and mucilaginous substances by treatment with a small amount of caustic soda, and bleached with fuller's earth and air or by sunning.

Animal Oils are commonly obtained by rendering the fatty parts of the different animals, usually with high-pressure steam. Extraction with solvents is seldom practiced. As a result of the rendering operation a more-or-less solid fat is obtained, as beef tallow, mutton tallow or lard. When this is chilled and pressed, the corresponding oil and stearin result. **Tallow** should be "acidless," that is, free from sulphuric acid and contain only the smallest possible quantity of free fatty acid. It is hard, does not turn rancid easily, and should melt at 113 deg. fahr. It forms a constituent of belt dressings, cylinder oils and lubricating greases. **Tallow** (or ox) **oil** is obtained by chilling and pressing tallow; it is a light-yellow bland **oil** used for mixing with other oils and as a lubricant. **Lard oil** is prepared similarly to tallow oil and appears in the market in five or six different varieties including Prime, Pure, No. 1 and No. 2, which are graded according to color, the first being very light straw-yellow and the last dark brown and ill-smelling. It is used to increase the viscosity of lubricants, as a burning oil, and for oiling textile stock. **Neatsfoot oil** is obtained by boiling the bones of the feet of horned cattle with water and running off the oil. It is often adulterated with sheep and horse foot and hide oil, as well as rape, cottonseed and mineral oils. It is light yellow and bland, with little tendency to turn rancid. It is used similarly to lard oil, and particularly for softening leather. **Whale oil** (train oil) is made by rendering the blubber of various species of whales, and consequently is of very variable composition. It is used as a leather dressing, as a burning oil, and to mix with other oils as a lubricant. **Sperm oil** comes from the great cavity in the head of the sperm whale and also from its blubber. It is a limpid, pale yellow oil particularly well adapted for fine machinery. **Degras** is the grease obtained in scouring wool. It usually contains sulphuric acid, except when made by the naphtha extraction process. It possesses the property of emulsifying with water to a marked degree and finds use in increasing the viscosity of lubricants, as a leather dressing, and in cylinder oils.

Vegetable Oils are usually prepared by pressing the crushed cooked seed or by extraction with benzine or carbon bisulphide. In America, the chief vegetable lubricating oils are castor (and to a lesser extent), corn and cottonseed; in Europe, rape, olive and almond oils are used. **Castor oil**—obtained from the castor bean—is a very viscous oil which gums on standing and does not naturally mix well with mineral oils. To render this possible it is heated to 212 deg. fahr. and a current of air forced through, whereby it becomes more viscous and is known as "blown oil." **Cottonseed oil**

is a pale or deep yellow oil of slight drying properties, chiefly used as an adulterant for other oils and, as well as rapeseed and castor, for the manufacture of blown oils. **Corn** (or maize) **oil** is a somewhat viscous oil with some drying properties. It is used as a lubricant, but more particularly with cottonseed oil for the preparation of soft soaps for lubricating greases. **Rapeseed oil** should be clear, *i.e.*, free from gelatinous matter. It was formerly one of the most commonly used and highly prized lubricants, particularly in Germany. **Olive oil** resembles rapeseed oil, except that it turns rancid more easily and is more fluid.

Table 1. Properties of the Organic or Animal and Vegetable Oils

Oil	Sp. Gr. at 60 deg. fahr.	Maumené test, deg. cent.	Cold test, deg. fahr.	Flash test, deg. fahr.†	Viscosity,‡ Seconds	Price (1915), cents per gal.
Castor	0.963	47	14	505	1485	75
Colza or rape	0.916	55	32	455	247	69
Corn	0.922	80	14	480	187	48
Cottonseed	0.922	100	40	580	180	50
Degras	0.902*	....	....	....	....	7§
Lard, prime	0.915	43	32–50	565	214	67
Neatsfoot	0.915	46	32	440	224	95
Porpoise (blubber)	0.925	50	0	495	156	50
Porpoise jaw	0.925	....	0	415	101	1000–2000
Rosin "4th run"	0.981	32	....	257	228	35
Seal	0.925	92	32	515	164	50
Sperm	0.880	46	45	455	115	65
Whale (blubber)	0.927	88	....	515	184	45
25 deg. Paraffin	0.900	10	....	410	163	13

* At 100 deg. cent. † With covered tester. ‡ Results with Saybolt universal viscosimeter at 100 deg. fahr. § Price per lb.

Mineral Oils are obtained by the distillation of crude petroleum. They are refined by agitation with sulphuric acid, and washed with water and caustic soda, or are filtered through fuller's earth or bone black. They do not change on exposure to the air (except upon very prolonged standing), neither thickening nor turning rancid. Their colors range from straw yellow in the filtered oils through reddish-brown to dark brown, and they all have a bluish or greenish fluorescence or "bloom" except when specially refined or treated. Some at 50 deg. fahr. even separate out paraffin or become pasty, but the Russian oils remain fluid at much lower temperatures.

Table 2. Properties of Some of the Mineral Oils

Oil	Specific gravity, deg. Baumé at 60 deg. fahr.	Flash point, deg. fahr.	Viscosity (Saybolt), at 70 deg. fahr.	Cold test, deg. fahr.
Black	29	325	100–120	5–15
Ice machine	26–27	325–360	60–100	0–4
Crank case	26–27	455	100	
Transformer	....	340–380	400	25
Turbine	30	420	160	....
Spindle	30–35	320–390	58–156	....
Loom	28	360	203	....
Engine	27–30	410	190–210	....
Cylinder	23–25	525	200–300*	....
Cylinder	26–28	400–575	....	....

* At 212 deg. fahr.

Miscellaneous Oils and Lubricants

Belt Dressings are (1) mixtures of fats, waxes, degras or tallow with castor or fatty oils; (2) vulcanized corn or cottonseed oil thinned with naptha; (3) preparations containing wood tar; or (4) preparations containing rosin which is undesirable (see also p. 622). **Black oils, car oils, well oil or reduced oils** are crude oils from which the napthas and burning oils have been separated by distillation. **Crank-case oils** are pure mineral oils which emulsify but little with water. **Graphite, plumbago or black lead** (see p. 632), when mixed with grease or by itself, is an excellent lubricant for very heavy service where a more fluid lubricant would be squeezed out by the pressure. It should be finely ground and carefully freed from the rock with which it occurs. Artificial graphite may replace it. **"Oildag"** and **"aquadag"** are colloidal suspensions of artificial deflocculated graphite in oil and water, respectively (see p. 633). **Milling-machine or soluble oils** are lard, sulphonated oils or mineral oils held in suspension in water by soaps or alkalies, as borax or soda; the soaps used are either ammonium, sodium or potassium with resin, oleic or sulphofatty acids. **Rosin oils** are obtained by distilling or "running" rosin, each distillate being called a "run" and numbered according to the times it has been distilled. They oxidise quite rapidly and should not be used as lubricants except as soaps in lubricating greases. **Screw-cutting oils** are often mixtures of 27 deg. Bé. paraffin and 25 per cent. fatty oil, preferably cottonseed, although lard oil was formerly used. **Stainless oils** are spindle or loom oils mixed with fatty oils—lard or neatsfoot. **Soapstone, talc or steatite** is a hydrated magnesium silicate. Like graphite, it must be freed from the harder minerals with which it occurs. It is sometimes used by itself in valve packing, but is usually compounded with oil and grease. **Transformer oils** should be either pure mineral or rosin oils and as free as possible from water, acid, alkali and sulphur. **Turbine oils** should be of excellent quality, free from acid and tendency to resinify, and low in sulphur. **Watch oil** is obtained from the porpoise, dolphin, or blackfish, where it exists in cavities in the jaw and in the brain or "melon" of the fish. **Lubricating greases** are mixtures of soaps of palm oil, tallow or rosin oil (with lime or soda as bases) with various oils or fats such as rosin, tallow or mineral oil. The best are those made from tallow by saponification with caustic soda. They may also contain finely powdered talc or graphite. **Non-fluid oils** are oils or their greases stiffened with "oil pulp" or "dope," *i.e.*, aluminum oleate or palmitate.

Spontaneous Combustion of Oils. Certain conditions are necessary to produce spontaneous combustion, namely, 1. A substance capable of absorbing oxygen with avidity, as a drying oil; 2. An open or porous organic substance upon which the oil can be spread out in thin layers; 3. Sufficient air to furnish oxygen, but not enough to cool the mass off appreciably; 4. Effective heat insulators.

The oils likely to produce spontaneous combustion are, in their order of danger: The drying—linseed, menhaden and "cod;" the semi-drying—corn, cottonseed, fish, lard, and neatsfoot; finally, red oil and oleic acid. Mineral oils are not susceptible of spontaneous combustion. When mixed with other oils—for example, neatsfoot or lard to the extent of 40 per cent. and cottonseed to 75 per cent.—the resulting compounds will not spontaneously ignite. The same holds true of emulsions of oils with water and soap or alkali, so long as water is present. With the animal oils the greater the percentage of free acid (oleic acid) the greater the liability to heating. Textile fibers and sawdust serve as media upon which the oils oxidise. As a rule, the larger the iodine number the more liable the oil to spontaneous combustion.

SECTION 7

MACHINE ELEMENTS

BY

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CONTENTS

* Staff Contribution.

MECHANISM

BY

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REFERENCES: Reuleaux, "Kinematics of Machinery," Macmillan. Unwin, "Elements of Machine Design," Longmans Green. Schwamb and Merrill, "Elements of Mechanism," Wiley.

Definition. A mechanism is that part of a machine which contains two or more pieces so arranged that the motion of one compels the motion of the others according to a definite law depending upon the nature of the combination.

Linkages

Links may be of any form so long as they do not interfere with the desired motion. The simplest form is that of four bars, *A*, *B*, *C*, *D*, fastened together at their ends by cylindrical pins, and which are all movable in parallel planes. If the links are of different lengths and each is fixed in turn, there will be four possible combinations; but as two of these are similar there will be produced three mechanisms having distinctly different motions. Thus, in Fig. 1, if *D* is fixed *A* can rotate and *C* oscillate, giving the **beam-and-crank** mechanism, as used on side-wheel steamers. If *B* is fixed the same motion will result; but if *A* is fixed (Fig. 2) links *B* and *D* can rotate, giving the **drag-link** mechanism used to feather the floats on paddle wheels. Fixing link *C*

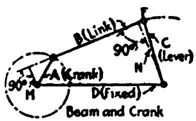


FIG. 1.

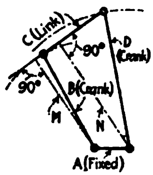


FIG. 2.

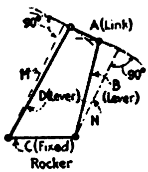


FIG. 3.

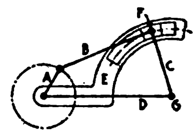


FIG. 4.

(Fig. 3), *D* and *B* can only oscillate, and a **rocker** mechanism sometimes used in straight-line motions is produced. It is customary to call a rotating link a **crank**; an oscillating link a **lever**, or beam; and the connecting link a **connecting rod.** The fixed link is often enlarged and used as the supporting frame.

If in the linkage (Fig. 1) the pin joint *F* is replaced by a slotted piece *E* (Fig. 4) no change will be produced in the resulting motion, and if the length of links *C* and *D* is made infinite the slotted piece *E* will become straight and the motion of the slide will be that of pure translation, thus obtaining the engine, or **sliding-block, linkage** (Fig. 5).

If in the sliding-block linkage (Fig. 5) the long link *B* is fixed (Fig. 6), *A* will rotate and *E* will oscillate and the infinite links *C* and *D* may be indicated as shown. This gives the **swinging-block linkage**. When used as a quick-return motion the slotted piece and slide are usually interchanged (Fig. 7) which in no way changes the resulting motion. If the short link *A* is fixed (Fig. 8) *B* and *E* can both rotate, and the mechanism known as the **turning-block linkage** is obtained. This is better known under the name of the

Whitworth quick-return motion, and is generally constructed as in Fig. 9. The **ratio of time of advance to time of return** H/K of the two quick-return motions (Figs. 7 and 9) may be found by locating, in the case of the swinging-block (Fig. 7), the two tangent points (t) and measuring the angles H and K made by the two positions of the crank A. If H and K are

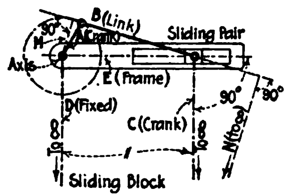


Fig. 5.

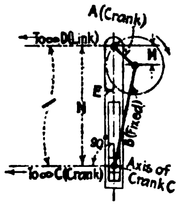


Fig. 6.

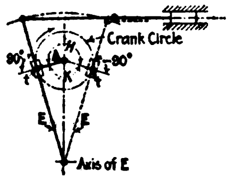
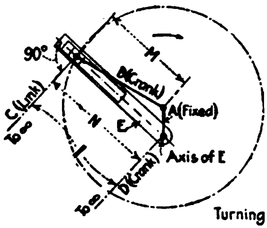
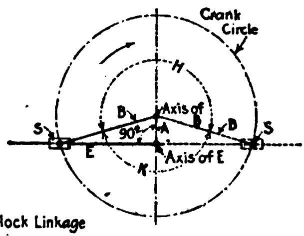


Fig. 7.

known the axis of E may be located by laying off the angles H and K on the crank circle and drawing the tangents E, their intersection giving the desired point. For the turning-block linkage (Fig. 9), determine the angles H and K made by the crank B when E is in the horizontal position; or, if the angles are known, the axis of E may be determined by drawing a





Turning Block Linkage

Fig. 8. Fig. 9.

horizontal line through the two crank-pin positions (S) for the given angle, and the point where a line through the axis of B cuts E perpendicularly will be the axis of E.

Velocities of any two or more points on a link must fulfill the following conditions (see p. 193): (1) Components along the link must be equal and

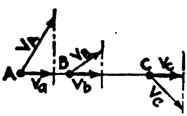


Fig. 10.

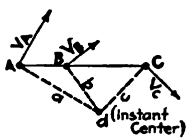


Fig. 11.

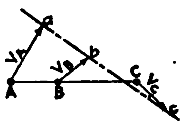


Fig. 12.

in the same direction (Fig. 10): $V_a = V_b = V_c$. (2) Perpendiculars to V_A, V_B, V_C from the points A, B, C, must intersect at a common point d, the **instant center** (or instantaneous axis). (3) The velocities of points A, B, and C are directly proportional to their distances from this center (Fig. 11): $V_A/a = V_B/b = V_C/c$. For a straight link the tips of the vectors representing the velocities of any number of points on the link will be on a straight line (Fig. 12): abc = a straight line. To find the velocity of any point when the velocity and direction of any two other points are known, condition (2) may be used, or a combination of (1) and (3). The **linear velocity ratio** of any two points on a linkage may be found by determining the distances e and f to the instant center (Fig. 13); then $V_e/V_b = e/f$. This may often be simplified by noting that a line drawn parallel to e and cutting B produced forms two similar triangles efB and sAy, which gives $V_e/V_b = e/f = s/A$. The **angular velocity ratio** for any position of two oscillating or rotating links A and C (Fig. 1), connected by a movable link B, may be determined by scaling the length of the perpendiculars M and N from the axes of rotation to the center line of the movable link. The angular velocity ratio is inversely proportional to these perpendiculars, or $O_C/O_A = M/N$. This method may be applied directly to a linkage having a sliding pair if the two infinite links are redrawn perpendicular to the sliding pair, as indicated in Fig. 14. M and N are also shown in Figs. 1, 2, 3, 5, 6, 8. In Fig. 5 one of the axes is at infinity, therefore N is infinite, or the slide has pure translation.

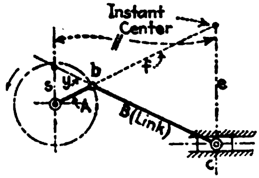


FIG. 13.

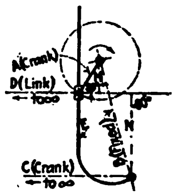


FIG. 14.

Forces. A mechanism must deliver as much work as it receives, neglecting friction, therefore the force at any point F multiplied by the velocity V_F in the direction of the force at that point must equal the force at some other point P multiplied by the velocity V_P at that point; or the forces are inversely as their velocities and $F/P = V_P/V_F$. It is at times more convenient to equate the moments of the forces acting around each axis of rotation (sometimes using the instant center) to determine the force acting at some other point. Applying this to Fig. 15, $(F \times a \times c)/(b \times d) = P$.

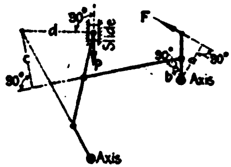


FIG. 15.

Cams

Cam Diagrams. A cam is usually a plate or cylinder which communicates motion to a follower by means of its edge or a groove cut in its surface. In the practical design of cams the angular velocity ratio is not directly involved, but the follower must assume a definite series of positions while the driver occupies a corresponding series of positions. The relationship of driver to follower may be represented by a diagram in which the ordinates represent the rise or fall of the follower and the abscissæ the angular motion of the cam.

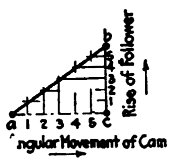


FIG. 16.

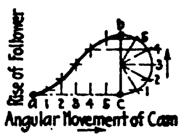


FIG. 17.

The three most common forms of motion used are uniform motion (Fig. 16), harmonic motion (Fig. 17), and uniformly accelerated and retarded motion

(Fig. 18). In plotting the diagrams (Fig. 18) for this last motion, divide *ac* into an even number of equal parts and *bc* into the same number of parts with lengths increasing by a constant increment to a maximum and then decreasing by the same decrement, as, for example, 1, 3, 5, 5, 3, 1, or 1, 3, 5, 7, 9, 9, 7, 5, 3, 1. In order to prevent shock when the direction of motion changes, as at *a* and *b* in the uniform motion, the harmonic motion may be used; if the cam is to be operated at high speed, the uniformly accelerated and retarded motion should preferably be employed; in either case there is a very gradual change of velocity.

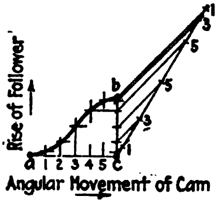


FIG. 18.

Pitch Line. The actual pitch line of a cam varies with the type of motion and with the position of the follower relative to the cam's axis. Most cams as ordinarily constructed are covered by the following four cases.

FOLLOWER ON LINE OF AXIS (Fig. 19). To draw the pitch line, subdivide the motion *bc* of the follower in the manner indicated in Figs. 16, 17, 18. Draw a circle with a radius equal to the smallest radius of the cam *a*0 and subdivide it into angles 0*a*1′, 0*a*2′, 0*a*3′, etc., corresponding with angular displacements of the cam for positions 1, 2, 3, etc., of the follower. With *a* as a center and radii *a*1, *a*2, *a*3, etc., strike arcs cutting radial lines at *d*, *e*, *f*, etc. Draw smooth curve through points *d*, *e*, *f*, etc.

OFFSET FOLLOWER (Fig. 20). Divide *bc* as indicated in Figs. 16, 17 and 18. Draw a circle of radius *ac* (highest point of rise of follower) and one tangent to *cb* produced. Divide the outer circle into parts 1′, 2′, 3′, etc., corresponding with

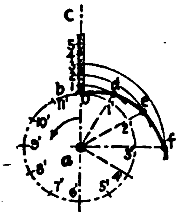


FIG. 19.

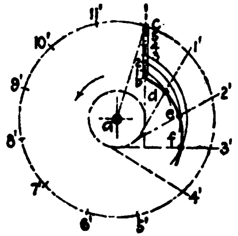


FIG. 20.

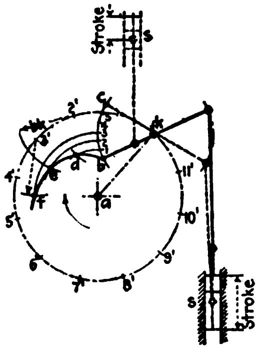


FIG. 21.

the angular displacement of the cam for positions 1, 2, 3, etc., of the follower, and draw tangents from points 1′, 2′, 3′, etc., to the small circle. With *a* as a center and radii *a*1, *a*2, *a*3, etc., strike arcs cutting tangents at *d*, *e*, *f*, etc. Draw a smooth curve through *d*, *e*, *f*, etc.

ROCKER FOLLOWER (Fig. 21). Divide the stroke of the slide *S* in the manner indicated in Figs. 16, 17, and 18, and transfer these points to the arc *bc* as points 1, 2, 3, etc. Draw a circle of radius *ak* and divide it into parts 1′, 2′, 3′, etc., corresponding with angular displacements of the cam for positions 1, 2, 3, etc., of the follower. With *k*, 1′, 2′, 3′ etc., as centers and radius *bk*, strike arcs *kb*, 1′*d*, 2′*e*, 3′*f*, etc., cutting at *bdef* arcs struck with *a* as a center and radii *ab*, *a*1, *a*2, *a*3, etc. Draw a smooth curve through *b*, *d*, *e*, *f*, etc.

CYLINDRICAL CAM (Fig. 22). In this type of cam more than one complete turn may be obtained, provided that in all cases the follower returns to its starting point. Draw rectangle *wxyz* (Fig. 22) representing the development of cylindrical surface of the cam. Subdivide the desired motion of the follower *bc* horizontally in the manner indicated in Figs. 16, 17 and 18, and plot the corresponding angular displacements 1′, 2′, 3′, etc., of the cam vertically; then through the intersection of lines from these points draw a smooth curve *bdef*, etc. This may best be shown by an example, assuming the following data for the diagram in Fig. 22: Total motion of follower = *bc*; circumference of cam = $2\pi r$. Follower to move with harmonic motion 4 units to the right in 0.6 turn, then rest (or "dwell") 0.4 turn, and finish with uniform motion 6 units to the right and 10 units to the left in two turns.

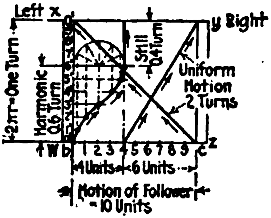


FIG. 22.

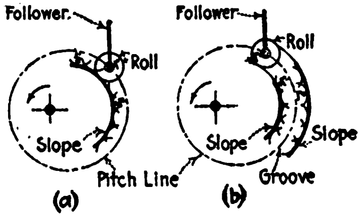


FIG. 23.

Cam Design. In the practical design of cams the following points must be noted. If only a small force is to be transmitted, sliding contact may be used, otherwise **rolling contact.** For the latter the pitch line must be corrected in order to get the true slope of the cam. An approximate construction (Fig. 23) may be employed by using the pitch line as the center of a series of arcs the radii of which are equal to that of the follower roll to be used; then a smooth curve drawn tangent to the arcs will give the slope

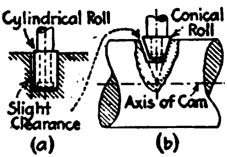


FIG. 24.

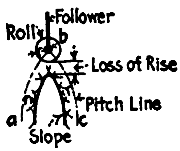


FIG. 25.

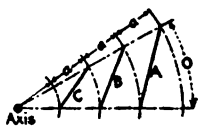


FIG. 26.

desired for a roll working on the periphery of the cam (Fig. 23*a*) or in a groove (Fig. 23*b*). For plate cams the roll should be a small cylinder, as in Fig. 24*a*. In cylindrical cams it is usually sufficiently accurate to make the roll conical, as in Fig. 24*b*, in which case the taper of the roll produced should intersect the axis of the cam. If the pitch line *abc* is made too sharp (Fig. 25) the follower will not rise the full amount. In order to prevent this **loss of rise** the pitch line should have a radius of curvature at all parts of not less than the roll's diameter plus ⅛ in. For the same rise of follower, *a*, and angular motion of the cam, *O*, the slope of the cam changes considerably, as indicated by the heavy lines *A*, *B*, and *C* (Fig. 26). Care should be taken to keep a moderate slope and thereby keep down the side thrust on the follower, but this should not be carried too far, as the cam would become too large and the friction increase.

Rolling Surfaces

In order to connect two shafts so that they shall have a definite angular velocity ratio, rolling surfaces are often used; and in order to have no slipping between the surfaces they must fulfill the following two conditions: the line of centers must pass through the point of contact, and the arcs of contact must be of equal length. The angular velocities, expressed usually in r.p.m., will be inversely proportional to the radii: $N/n = r/R$. The two surfaces most commonly used in practice, and the only ones having a constant angular velocity ratio, are cylinders where the shafts are parallel, and cones where the shafts (produced) intersect at an angle. In either case there are two possible directions of rotation, depending upon whether the surfaces roll in opposite directions (external contact) or in the same direction (internal contact). In Fig. 27, $R = nc/(N + n)$ and $r = Nc/(N + n)$; in Fig. 28, $R = nc/(N - n)$, and $r = Nc/(N - n)$. In Fig. 29, $\tan B = \sin A/[(n/N) + \cos A]$, and $\tan C = \sin A/[(N/n) + \cos A]$; in Fig. 30, $\tan B = \sin A/[(N/n) - \cos A]$, and $\tan C = \sin A/[(n/N) - \cos A]$. With the above values for the angles B and C, and the length d or e of one of the cones, R and r may be calculated.

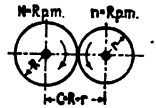


FIG. 27.

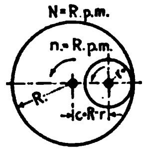


FIG. 28.

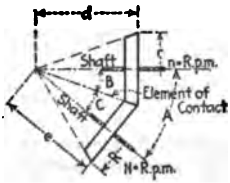


FIG. 29.

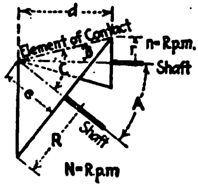


FIG. 30.

Epicyclic Trains

Epicyclic trains are combinations of gears in which some or all of the gears have a motion compounded of rotation about an axis and a translation or revolution of that axis. The gears are usually connected by a link called an arm, which often rotates about the axis of the first gear. Such trains may be calculated by first considering all gears locked and the arm turned; then the arm locked and the gears rotated. The algebraic sum of the separate motions will give the desired result. The following examples and method of tabulation will illustrate this. The figures on each gear refer to the number of teeth for that gear.

	A	*B*	*C*	*D*
Gear locked, Fig. 31..	+1	+1	+1	+1
Arm locked, Fig. 31..	0	−1	+1×59/20	−1×59/20×29/40
Value, Fig. 31........	+1	0	+3 1/5	−1/4
Gears locked, Fig. 32.	+1	+1	+1	+1
Arm locked, Fig. 32...	0	−1	+1×39/20	+1×39/20×29/40
Value, Fig. 32........	+1	0	+2 1/4	+1 3/4

For common epicyclic trains, see also p. 725.

In Figs. 31 and 32 lock the gears and turn the arm *A* right-handed through 1 revolution (+1), then lock the arm and turn the gear *B* back to where it started (−1); gears *C* and D will have rotated the amount indicated in the tabulation. Then the algebraic sum will give the relative turns of each gear. That is, in Fig. 31, for one turn of the arm, B does not move and C turns in the same direction $3\frac{1}{2}$ rev., and D in the opposite direction $\frac{1}{4}$ rev.; whereas in Fig. 32, for one turn of the arm, B does not turn, but C and D turn in the same direction as the arm respectively $2\frac{1}{2}$ and $1\frac{3}{70}$ rev. (Note: The arm in the above case was turned $+1$ for convenience, but any other value might be used.)

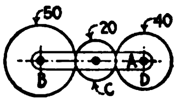


FIG. 31.

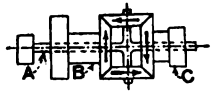


FIG. 33.

FIG. 35.

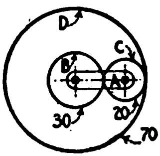


FIG. 32.

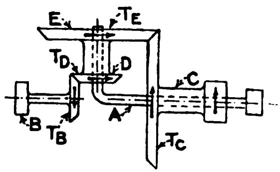


FIG. 34.

Bevel Epicyclic Trains are epicyclic trains containing bevel gears and may be calculated by the preceding method, but it is usually simpler to use the general formula which applies to all cases of epicyclic trains:

$$\frac{\text{Turns of } C \text{ relative to arm}}{\text{Turns of } B \text{ relative to arm}} = \frac{\text{Absolute turns of } C - \text{turns of arm}}{\text{Absolute turns of } B - \text{turns of arm}}$$

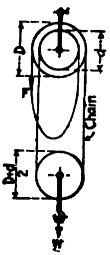


FIG. 36.

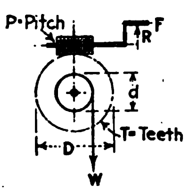


FIG. 37.

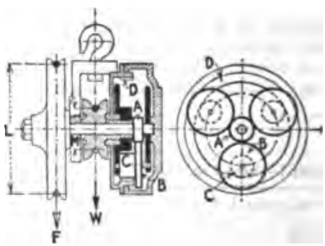


FIG. 38.

The left-hand term gives the value of the train and can always be expressed in terms of the number of teeth (T) on the gears; care must be used, however, to express it as either plus (+) or minus (−), depending upon whether the gears turn in the same or opposite directions.

$$\frac{\text{Relative turns of } C}{\text{Relative turns of } B} = \frac{C - A}{B - A} = -1, \text{in Fig. 33}; = +\frac{T_E}{T_C} \times \frac{T_B}{T_D} \text{ in Fig. 34.}$$

Hoisting Mechanisms

Pulley Block (Fig. 35). Given the weight W to be raised, the force F necessary is $F = V_W \times W/V_F = W/n$ = load/number of ropes; V_W and V_F being the respective velocities of W and F.

Differential Chain Block (Fig. 36). $F = V_W \times W/V_F = W(D - d)/2D$.

Worm and Wheel (Fig. 37). $F = \pi d(n/T)W/2\pi R = WP(d/D)/2\pi R$, where n = number of threads, single, double, triple, etc.

The **triplex chain block** (Fig. 38) is a geared hoist making use of the epicyclic train. $F = W \times M/L\{1 + [(T_D/T_C) \times (T_B/T_A)]\}$, where T = number of teeth on gears.

Toggle Joint (Fig. 39). $P = F \times s \cos A/t$.

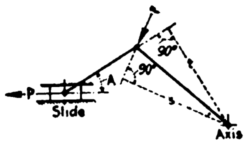


FIG. 39.

MACHINE ELEMENTS

BY

WALTER RAUTENSTRAUCH*

REFERENCES: Spooner, "Machine Design, Construction and Drawing," Longmans. Unwin, "Elements of Machine Design," Longmans. Kimball and Barr, "Elements of Machine Design," Wiley. Smith and Marx, "Machine Design," Wiley. Hess, "Machine Design," Lippincott. Benjamin and Hoffman, "Machine Design," Henry Holt. Reuleaux, "The Constructor." Bach, "Die Maschinen Elemente," Kröner.

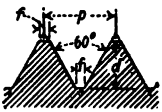
SCREW FASTENINGS

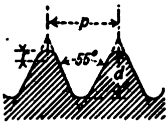
Machine Screw Threads: Forms and Proportions

The Sellers or U. S. Standard Thread for machine screws, shown in Fig. 1, is proportioned by the formulæ p = pitch = 1/No. of threads per inch, d = depth = $0.6495p$, f = flat = $p/8$. See Table 1.

The A.S.M.E. Standard Thread is of the same form as the Sellers thread, but has the following proportions:

$$d = 0.7037p;\ \text{flat (top)} = p/8;\ \text{flat (bottom)} = p/16$$





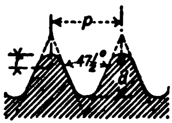


FIG. 1.—Sellers or U. S. FIG. 2.—Whitworth. FIG. 3.—British Association. Standard Screw Threads.

The Whitworth Standard Thread (Fig. 2) is used in Great Britain and is based on the formulæ $d = 0.6403p$, r = radius = $0.1373p$. See Table 2.

The British Association Screw Thread is shown in Fig. 3, and dimensions of bolts with such threads are given in Table 3.

The French (Metric) Screw Thread (Fig. 4) is based on the formulæ p = pitch in mm., $d = 0.6495p$, $f = p/8$. See Table 4.

Table 1. U. S. (Sellers) Standard Screw Threads

(See Fig. 1)

Diam. of screw, in.	Threads per in.	Diam. at root of thread, in.	Width of flat, in.	Area of section at root of thread, sq. in.	Diam. of screw, in.	Threads per in.	Diam. at root of thread, in.	Width of flat, in.	Area of section at root of thread, sq. in.
¼	20	0.1850	0.0063	0.027	2	4½	1.7113	0.0278	2.302
5/16	18	0.2403	0.0069	0.045	2¼	4½	1.9613	0.0278	3.023
⅜	16	0.2936	0.0078	0.068	2½	4	2.1752	0.0313	3.719
7/16	14	0.3447	0.0089	0.093	2¾	4	2.4252	0.0313	4.620
½	13	0.4001	0.0096	0.126	3	3½	2.6288	0.0357	5.428
9/16	12	0.4542	0.0104	0.162	3¼	3½	2.8788	0.0357	6.510
⅝	11	0.5069	0.0114	0.202	3½	3¼	3.1003	0.0385	7.548
¾	10	0 6201	0.0125	0.302	3¾	3	3.3170	0.0417	8.641
⅞	9	0.7307	0.0139	0.420	4	3	3.5670	0.0417	9.963
1	8	0.8376	0.0156	0.550	4¼	2⅞	3.7982	0.0435	11.329
1⅛	7	0.9394	0.0179	0.694	4½	2¾	4.0276	0.0455	12.753
1¼	7	1.0644	0.0179	0.893	4¾	2⅝	4.2551	0.0476	14.226
1⅜	6	1.1585	0.0208	1.057	5	2½	4.4804	0.0500	15.763
1½	6	1.2835	0.0208	1.295	5¼	2½	4.7304	0.0500	17.572
1⅝	5½	1.3888	0.0227	1.515	5½	2⅜	4.9530	0.0526	19.267
1¾	5	1.4902	0.0250	1.746	5¾	2⅜	5.2030	0.0526	21.262
1⅞	5	1.6152	0.0250	2.051	6	2¼	5.4226	0.0556	23.098

Table 2. Whitworth Standard Screw Threads (See Fig. 2)

Diam. of screw, in.	Threads per in.	Depth of thread, in.	Diam. at root of thread, in.	Diam. of screw, in.	Threads per in.	Depth of thread, in.	Diam. at root of thread, in.
1/4	20	0.0320	0.1860	1 5/8	5	0.1281	1.3689
5/16	18	0.0356	0.2414	1 3/4	5	0.1281	1.4939
3/8	16	0.0400	0.2950	2	4 1/2	0.1423	1.7154
7/16	14	0.0457	0.3460	2 1/4	4	0.1601	1.9298
1/2	12	0.0534	0.3933	2 1/2	4	0.1601	2.1798
9/16	12	0.0534	0.4558	2 3/4	3 1/2	0.1830	2.3841
5/8	11	0.0582	0.5086	3	3 1/2	0.1830	2.6341
11/16	11	0.0582	0.5711	3 1/4	3 1/4	0.1970	2.8560
3/4	10	0.0640	0.6219	3 1/2	3 1/4	0.1970	3.1060
13/16	10	0.0640	0.6844	3 3/4	3	0.2134	3.3231
7/8	9	0.0711	0.7327	4	3	0.2134	3.5731
1	8	0.0800	0.8399	4 1/2	2 7/8	0.2227	4.0546
1 1/8	7	0.0915	0.9420	5	2 3/4	0.2328	4.5343
1 1/4	7	0.0915	1.0670	5 1/2	2 5/8	0.2439	5.0121
1 3/8	6	0.1067	1.1616	6	2 1/2	0.2561	5.4877
1 1/2	6	0.1067	1.2866				

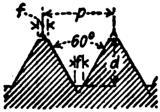
Table 3. British Association Screw Threads (See Fig. 3)

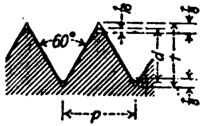
Number	Diam. of screw, mm.	Approx. diam., in.	Pitch, mm.	Approx. pitch, in.	Diam. at root of thread, mm.	Number	Diam. of screw, mm.	Approx. diam., in.	Pitch, mm.	Approx. pitch, in.	Diam. at root of thread, mm.
0	6.0	0.236	1.00	0.0394	4.8	13	1.20	0.047	0.25	0.0098	0.90
1	5.3	0.209	0.90	0.0354	4.22	14	1.00	0.039	0.23	0.0091	0.72
2	4.7	0.185	0.81	0.0319	3.73	15	0.90	0.035	0.21	0.0083	0.65
3	4.1	0.161	0.73	0.0287	3.22	16	0.79	0.031	0.19	0.0075	0.56
4	3.6	0.142	0.66	0.0260	2.81	17	0.70	0.028	0.17	0.0067	0.50
5	3.2	0.126	0.59	0.0232	2.49	18	0.62	0.024	0.15	0.0059	0.44
6	2.8	0.110	0.53	0.0209	2.16	19	0.54	0.021	0.14	0.0055	0.37
7	2.5	0.098	0.48	0.0189	1.92	20	0.48	0.019	0.12	0.0047	0.34
8	2.2	0.087	0.43	0.0169	1.68	21	0.42	0.017	0.11	0.0043	0.29
9	1.9	0.075	0.39	0.0154	1.43	22	0.37	0.015	0.10	0.0039	0.25
10	1.7	0.067	0.35	0.0138	1.28	23	0.33	0.013	0.09	0.0035	0.22
11	1.5	0.059	0.31	0.0122	1.13	24	0.29	0.011	0.08	0.0031	0.19
12	1.3	0.051	0.28	0.0110	0.96	25	0.25	0.010	0.07	0.0028	0.17

Table 4. French (Metric) Standard Screw Threads (See Fig. 4)

Diam. of screw, mm.	Pitch, mm.	Diam. at root of thread, mm.	Width of flat, mm.	Diam. of screw, mm.	Pitch, mm.	Diam. at root of thread, mm.	Width of flat, mm.	Diam. of screw, mm.	Pitch, mm.	Diam. at root of thread, mm.	Width of flat, mm.
3	0.5	2.35	0.06	18	2.5	14.75	0.31	40	4.0	34.80	0.50
4	0.75	3.03	0.09	20	2.5	16.75	0.31	42	4.5	36.15	0.56
5	0.75	4.03	0.09	22	2.5	18.75	0.31	44	4.5	38.15	0.56
6	1.0	4.70	0.13	22	3.0	18.10	0.38	45	4.5	39.15	0.56
7	1.0	5.70	0.13	24	3.0	20.10	0.38	46	4.5	40.15	0.56
8	1.0	6.70	0.13	26	3.0	22.10	0.38	48	5.0	41.51	0.63
8	1.25	6.38	0.16	27	3.0	23.10	0.38	50	5.0	43.51	0.63
9	1.0	7.70	0.13	28	3.0	24.10	0.38	52	5.0	45.51	0.63
9	1.25	7.38	0.16	30	3.5	25.45	0.44	56	5.5	48.86	0.69
10	1.5	8.05	0.19	32	3.5	27.45	0.44	60	5.5	52.86	0.69
11	1.5	9.05	0.19	33	3.5	28.45	0.44	64	6.0	56.21	0.75
12	1.5	10.05	0.19	34	3.5	29.45	0.44	68	6.0	60.21	0.75
12	1.75	9.73	0.22	36	4.0	30.80	0.5	72	6.5	63.56	0.81
14	2.0	11.40	0.25	38	4.0	32.80	0.5	76	6.5	67.56	0.81
16	2.0	13.40	0.25	39	4.0	33.80	0.5	80	7.0	70.91	0.88

The International Standard Screw Thread, as adopted by the "Congrès International pour l'Unification des Filetages," in Zurich, Oct 24, 1898, is shown in Fig. 5, in which $d = 0.7036\,p$ and $t = 0.866\,p$. See Table 5.





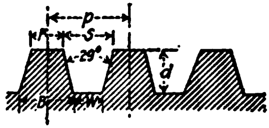


Fig. 4.—Metric. Fig. 5.—International. Fig. 6.—Acme.
Standard Screw Threads.

Table 5. International Standard Metric Screw Threads
(See Fig. 5)

Diam. of screw, mm.	Pitch, mm.	Diam. of screw, mm.	Pitch, mm.	Diam. of screw, mm.	Pitch, mm.	Diam of screw, mm.	Pitch, mm.	Diam. of screw, mm.	Pitch, mm.	Diam. of screw, mm.	Pitch, mm.
6	1.00	12	1.75	24	3.00	42	4.50	64	6.00	96	8.00
7	1.00	14	2.00	27	3.00	45	4.50	68	6.00	116	9.00
8	1.25	16	2.00	30	3.50	48	5.00	72	6.50	136	10.00
9	1.25	18	2.50	33	3.50	52	5.00	76	6.50	......	......
10	1.50	20	2.50	36	4.00	56	5.50	80	7.00	......	......
11	1.50	22	2.50	39	4.00	60	5.50	88	7.50	......	......

Power Transmission Screw Threads: Forms and Proportions

The Acme 29-deg. Screw Thread is shown in Fig. 6. The proportions of screws with Acme threads, given in Table 6, are obtained from the formulæ $d = 0.5p + 0.01$ in.; flat (top) $= 0.3707p$; flat (bottom) $= 0.3707p - 0.0052$ in.

Table 6. Acme 29-deg. Screw Threads
(Letters refer to Fig. 6)

N	*p*	*d*	*F*	*W*	*S*	*B*
Number of threads per in.	Pitch of single thread, in.	Depth of thread, in.	Width of top of thread, in.	Width of space at bottom of thread, in.	Width of space at top of thread, in.	Thickness at root of thread, in.
1	1.0	0.5100	0.3707	0.3655	0.6293	0.6345
1⅓	0.750	0.3850	0.2780	0.2728	0.4720	0.4772
2	0.500	0.2600	0.1853	0.1801	0.3147	0.3199
3	0.3333	0.1767	0.1235	0.1183	0.2098	0.2150
4	0.250	0.1350	0.0927	0.0875	0.1573	0.1625
5	0.200	0.1100	0.0741	0.0689	0.1259	0.1311
6	0.1667	0.0933	0.0618	0.0566	0.1049	0.1101
7	0.1428	0.0814	0.0530	0.0478	0.0899	0.0951
8	0.125	0.0725	0.0463	0.0411	0.0787	0.0839
9	0.1111	0.0655	0.0413	0.0361	0.0699	0.0751
10	0.10	0.0600	0.0371	0.0319	0.0629	0.0681

The Square Thread (Fig. 7) has the proportions given in Table 7 for the range of sizes tabulated.

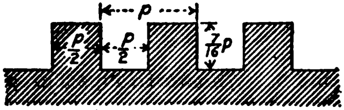


Fig. 7.—Sellers Square Thread.

Table 7. Sellers Standard Square Threads
(See Fig. 7)

Diam. of bolt, in.	Threads per in.	Diam. of root of thread, in.	Diam. of bolt, in.	Threads, per in.	Diam. at root of thread, in.	Diam. of bolt, in.	Threads, per in.	Diam. at root of thread, in.	Diam. of bolt, in.	Threads per in.	Diam. at root of thread, in.
1/4	10	0.1625	3/4	5	0.575	1 1/2	3	1.2084	3	1 3/4	2.5
5/16	9	0.2153	13/16	4 1/2	0.6181	1 5/8	2 3/4	1.307	3 1/4	1 3/4	2.75
3/8	8	0.2658	7/8	4 1/2	0.6806	1 3/4	2 1/2	1.4	3 1/2	1 5/8	2.962
7/16	7	0.3125	15/16	4	0.7188	1 7/8	2 1/2	1.525	3 3/4	1 1/2	3.168
1/2	6 1/2	0.3656	1	4	0.7813	2	2 1/4	1.612	4	1 1/2	3.418
9/16	6	0.4167	1 1/8	3 1/2	0.875	2 1/4	2 1/4	1.862	......	......	....
5/8	5 1/2	0.4666	1 1/4	3 1/2	1.00	2 1/2	2	2.0626	......	......	....
11/16	5	0.512	1 3/8	3	1.0834	2 3/4	2	2.3126	......	......	....

The Buttress Thread (Fig. 8) is a strong form, adapted for taking load in one direction only.

Screw Threads for Pipes

The Briggs Pipe Thread is the standard in use in America. Its form is shown in Fig. 9, and it is made to the following specifications: Taper of pipe end = 3/4 in. per ft. = 1/16 in. per in. Depth of thread (E) = 0.8/No. of threads per in. (n). See Table 8.

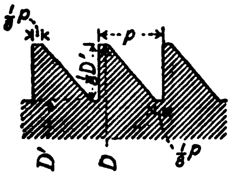


Fig. 8.—Buttress Thread.

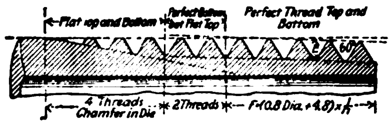


Fig. 9.—Briggs Pipe Thread.

Table 8. Briggs Pipe Threads

Nominal inside diam. of pipe, in.	No. of threads per in.	Diam. at end of pipe, in.	Diam. at bottom of thread, in.	Total distance pipe screws into fitting, in.	Nominal inside diam. of pipe, in.	No. of threads per in.	Diam. at end of pipe, in.	Diam. at bottom of thread, in.	Total distance pipe screws into fitting, in.
1/8	27	0.393	0.334	0.19	3	8	3.441	3.241	0.95
1/4	18	0.522	0.433	0.29	3 1/2	8	3.938	3.738	1.00
3/8	18	0.656	0.568	0.30	4	8	4.434	4.234	1.05
1/2	14	0.815	0.701	0.39	4 1/2	8	4.931	4.731	1.10
3/4	14	1.025	0.911	0.40	5	8	5.490	5.290	1.16
1	11 1/2	1.283	1.144	0.51	6	8	6.546	6.346	1.26
1 1/4	11 1/2	1.626	1.488	0.54	7	8	7.540	7.340	1.36
1 1/2	11 1/2	1.866	1.728	0.55	8	8	8.534	8.334	1.46
2	11 1/2	2.339	2.201	0.58	9	8	9.527	9.327	1.57
2 1/2	8	2.819	2.619	0.89	10	8	10.645	10.445	1.68

The Whitworth Pipe Thread is the standard used in Great Britain. It is cut either straight or with a taper of 3/4 in. per ft. as in the Briggs standard.

Table 9. Whitworth Pipe Threads

Pipe size, in.	Diam., in.	Diam. at bottom of thread, in.	Pipe size, in.	Diam., in.	Diam. at bottom of thread, in.	Pipe size, in.	Diam., in.	Diam. at bottom of thread, in.	Pipe size, in.	Diam., in.	Diam. at bottom of thread, in.
1/8	0.3825	0 3367	7/8	1.189	1.0975	1 5/8	2.021	1.905	2 3/4	3.247	3.1305
1/4	0 518	0.4506	1	1.309	1.1925	1 3/4	2.047	1.9305	3	3.485	3.3685
3/8	0.6563	0.5889	1 1/8	1.492	1.3755	1 7/8	2.245	2.1285	3 1/4	3.6985	3.582
1/2	0.8257	0.7342	1 1/4	1.65	1.5335	2	2.347	2.2305	3 1/2	3.912	3.7955
5/8	0.9022	0.8107	1 3/8	1.745	1.6285	2 1/4	2.5875	2.471	3 3/4	4.1255	4.009
3/4	1.041	0.9495	1 1/2	1.8825	1.765	2 1/2	3.0013	2.8848	4	4.339	4.223

Number of threads per inch: 1/8 in., 28; 1/4 and 3/8 in., 19; 1/2, 5/8, 3/4 and 7/8 in., 14; 1 in. and larger, 11.

Bolt and Machine Screw Heads and Nuts: Forms and Proportions

The tables and dimensions which follow relate to acceptable proportions; they are not, however, to be regarded as universally standard.

Table 10. U. S. Standard Bolts and Nuts

Diam. of bolt, in.	Rough dimensions				Finished dimensions of heads and nuts				
	Heads and nuts			Thickness of head,† in.	Hexagon heads and nuts			Exact size of hole in nut, in.	Size of tap drill used, in.
	Width across flats, hex. and square, in.	Width across corners* Hexagon, in.	Width across corners* Square, in.		Width across flats, in.	Width across corners, in.	Thickness, in.		
1/4	1/2	37/64	23/32	1/4	7/16	1/2	3/16	0.1850	0.191
5/16	19/32	11/16	27/32	19/64	17/32	39/64	1/4	0.2408	0.246
3/8	11/16	51/64	31/32	11/32	5/8	23/32	5/16	0.2938	19/64
7/16	25/32	29/32	1 7/64	25/64	23/32	53/64	3/8	0.3447	23/64
1/2	7/8	1 1/64	1 1/4	7/16	13/16	15/16	7/16	0.4001	13/32
9/16	31/32	1 1/8	1 3/8	31/64	29/32	1 1/64	1/2	0.4542	15/32
5/8	1 1/16	1 15/64	1 1/2	17/32	1	1 5/32	9/16	0.5069	33/64
3/4	1 1/4	1 29/64	1 25/32	5/8	1 3/16	1 23/64	11/16	0.6201	5/8
7/8	1 7/16	1 43/64	2 1/32	23/32	1 3/8	1 19/32	13/16	0.7307	47/64
1	1 5/8	1 7/8	2 19/64	13/16	1 9/16	1 13/16	15/16	0.8376	27/32
1 1/8	1 13/16	2 3/32	2 9/16	29/32	1 3/4	2 1/64	1 1/16	0.9394	61/64
1 1/4	2	2 5/16	2 53/64	1	1 15/16	2 15/64	1 3/16	1.0644	1 5/64
1 3/8	2 3/16	2 17/32	3 3/32	1 3/32	2 1/8	2 29/64	1 5/16	1.1585	1 11/64
1 1/2	2 3/8	2 3/4	3 23/64	1 3/16	2 5/16	2 43/64	1 7/16	1.2835	1 19/64
1 3/4	2 3/4	3 3/16	3 57/64	1 3/8	2 11/16	3 7/64	1 11/16	1.4902	1 35/64
2	3 1/8	3 39/64	4 27/64	1 9/16	3 1/16	3 17/32	1 15/16	1.7113	1 25/32
2 1/4	3 1/2	4 3/64	4 61/64	1 3/4	3 7/16	3 31/32	2 3/16	1.9613	1 31/32
2 1/2	3 7/8	4 31/64	5 31/64	1 15/16	3 13/16	4 13/32	2 7/16	2.1752	2 3/16
2 3/4	4 1/4	4 29/32	6 1/64	2 1/8	4 3/16	4 27/32	2 11/16	2.4252	2 7/16
3	4 5/8	5 11/32	6 17/32	2 5/16	4 9/16	5 9/32	2 15/16	2.6288	2 41/64

* Width across corners of hexagon heads or nuts = 1.155 × width across flats.
Width across corners of square heads or nuts = 1.414 × width across flats.
† Thickness of nut = diam. of bolt.

Weight of 100 Bolts and Nuts, Lb.

Length under head, inches	Bolts with square heads and nuts									Bolts with hexagon heads and nuts				
	Diameter of bolt, inches									Diameter of bolt, inches				
	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1/2	5/8	3/4	7/8	1
1	4	7	11	15	22	37	56	84	122	19	33	52	76	110
1 1/4	4	7	11	16	23	39	59	88	128	20	34	54	80	116
1 1/2	5	8	12	17	24	41	62	93	133	22	36	57	85	121
1 3/4	5	8	13	18	26	43	64	97	139	23	38	60	89	127
2	5	9	14	19	27	45	67	101	144	24	40	63	93	132
Lb. per inch additional	1.4	2.2	3.1	4.3	5.6	8.7	12.5	17.0	22.3	5.6	8.7	12.5	17.0	22.3

Weights of Nuts, Bolt Heads and Shanks

(For calculating the weight of large bolts)

Diameter of bolt, inches	1 1/4	1 1/2	1 3/4	2	2 1/2	3
Wt. of 1 hex. head and 1 hex. nut, lb.	1.7	2.9	4.6	6.8	13.0	22.0
Wt. of 1 sq. head and 1 sq. nut, lb...	2.0	3.5	5.5	8.1	15.5	26.2
Wt. of shank per inch, lb...........	0.35	0.5	0.68	0.89	1.40	2.00

Drills for Pipe Taps. The sizes of twist drills used in boring holes to be reamed with pipe reamers and threaded with pipe taps, for both the Whitworth and Briggs systems, are given in Table 11.

Table 11. Twist Drills for Use with Pipe Taps

Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.	Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.	Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.	Tap size, in.	Briggs: drill size, in.	Whitworth: drill size, in.
1/8	21/64	5/16	3/4	29/32	29/32	1 3/4	......	1 15/16	3	3 3/16	3 9/32
1/4	27/64	27/64	7/8	......	1 1/16	2	2 3/16	2 5/32	3 1/4	......	3 1/2
3/8	9/16	9/16	1	1 1/8	1 1/8	2 1/4	......	2 13/32	3 1/2	3 11/16	3 3/4
1/2	11/16	11/16	1 1/4	1 15/32	1 15/32	2 1/2	2 9/16	2 25/32	3 3/4	......	4
5/8	......	25/32	1 1/2	1 23/32	1 23/32	2 3/4	......	3 1/32	4	4 3/16	4 1/4

Table 12. Oval Fillister Head Machine Screws. A.S.M.E. Standard

A = Diam. of body. $B = 1.64A - 0.009$ = Diam. of head and radius for oval. $C = 0.66A - 0.002$ = height of side. $D = 0.173A + 0.015$. $E = \frac{1}{2}F$ = depth of slot. $F = 0.134B + C$ = height of head.

(Letters refer to Fig. 10. All dimensions in inches)

A	*B*	*C*	*D*	*E*	*F*	*A*	*B*	*C*	*D*	*E*	*F*
0.060	0.0894	0.0376	0.025	0.025	0.0496	0.216	0.3452	0.1405	0.052	0.093	0.1868
0.073	0.1107	0.0461	0.028	0.030	0.0609	0.242	0.3879	0.1577	0.057	0.105	0.2097
0.086	0.132	0.0548	0.030	0.036	0.0725	0.268	0.4305	0.1748	0.061	0.116	0.2325
0.099	0.153	0.0633	0.032	0.042	0.0838	0.294	0.4731	0.192	0.066	0.128	0.2554
0.112	0.1747	0.0719	0.034	0.048	0.0953	0.320	0.5158	0.2092	0.070	0.140	0.2783
0.125	0.196	0.0805	0.037	0.053	0.1068	0.346	0.5584	0.2263	0.075	0.150	0.3011
0.138	0.217	0.089	0.039	0.059	0.1180	0.372	0.601	0.2435	0.079	0.162	0.3240
0.151	0.2386	0.0976	0.041	0.065	0.1296	0.398	0.6437	0.2606	0.084	0.173	0.3469
0.164	0.2599	0.1062	0.043	0.071	0.1410	0.424	0.6863	0.2778	0.088	0.185	0.3698
0.177	0.2813	0.1148	0.046	0.076	0.1524	0.450	0.727	0.295	0.093	0.201	0.4024
0.190	0.3026	0.1234	0.048	0.082	0.1639	......	......	......	......	......	......

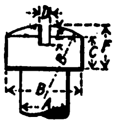


FIG. 10. Oval Fillister.

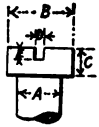


FIG. 11. Flat Fillister.

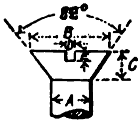


FIG. 12. Flat.

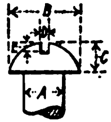


FIG. 13. Round.

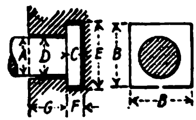


FIG. 14. Bolt Heads for Standard T-slots.

A. S. M. E. Standard Machine Screw Heads.

Table 13. Flat Fillister Head Machine Screws. A.S.M.E. Standard

A = Diam. of body. $B = 1.64A - 0.009$ = diam. of head. $C = 0.66A - 0.002$ = height of head. $D = 0.173A + 0.015$ = width of slot. $E = \frac{1}{2}C$ = depth of slot. (Letters refer to Fig. 11. All dimensions in inches)

A	B	C	D	E	A	B	C	D	E
0.060	0.0894	0.0376	0.025	0.019	0.216	0.3452	0.1405	0.052	0.070
0.073	0.1107	0.0461	0.028	0.023	0.242	0.3879	0.1577	0.057	0.079
0.086	0.132	0.0548	0.030	0.027	0.268	0.4305	0.1748	0.061	0.087
0.099	0.153	0.0633	0.032	0.032	0.294	0.4731	0.1920	0.066	0.096
0.112	0.1747	0.0719	0.034	0.036	0.320	0.5158	0.2092	0.070	0.104
0.125	0.196	0.0805	0.037	0.040	0.346	0.5584	0.2263	0.075	0.113
0.138	0.217	0.0890	0.039	0.044	0.372	0.601	0.2435	0.079	0.122
0.151	0.2386	0.0976	0.041	0.049	0.398	0.6437	0.2606	0.084	0.130
0.164	0.2599	0.1062	0.043	0.053	0.424	0.6863	0.2778	0.088	0.139
0.177	0.2813	0.1148	0.046	0.057	0.450	0.727	0.295	0.093	0.147
0.190	0.3026	0.1234	0.048	0.062	....	....	....	....	....

Table 14. Flat-head Machine Screws. A.S.M.E. Standard

A = Diam. of body. $B = 2A - 0.008$ = diam. of head. $C = (A - 0.008)/1.739$ = depth of head. $D = 0.173A + 0.015$ = width of slot. $E = \frac{1}{3}C$ = depth of slot. (Letters refer to Fig. 12. All dimensions in inches)

A	B	C	D	E	A	B	C	D	E
0.060	0.112	0.029	0.025	0.010	0.216	0.424	0.120	0.052	0.040
0.073	0.138	0.037	0.028	0.012	0.242	0.472	0.135	0.057	0.045
0.086	0.164	0.045	0.030	0.015	0.268	0.528	0.150	0.061	0.050
0.099	0.190	0.052	0.032	0.017	0.294	0.580	0.164	0.066	0.055
0.112	0.216	0.060	0.034	0.020	0.320	0.632	0.179	0.070	0.060
0.125	0.242	0.067	0.037	0.022	0.346	0.682	0.194	0.075	0.065
0.138	0.262	0.075	0.039	0.025	0.372	0.732	0.209	0.079	0.070
0.151	0.294	0.082	0.041	0.027	0.398	0.788	0.224	0.084	0.075
0.164	0.320	0.090	0.043	0.030	0.424	0.840	0.239	0.088	0.080
0.177	0.346	0.097	0.046	0.032	0.450	0.892	0.254	0.093	0.085
0.190	0.372	0.105	0.048	0.035	....	....	....	....	....

Table 15. Round-head Machine Screws. A.S.M.E. Standard

A = Diam. of body. $B = 1.85A - 0.005$ = diam. of head. $C = 0.7A$ = height of head. $D = 0.173A + 0.015$ = width of slot. $E = \frac{1}{2}C + 0.01$ = depth of slot. (Letters refer to Fig. 13. All dimensions in inches)

A	B	C	D	E	A	B	C	D	E
0.060	0.106	0.042	0.025	0.031	0.216	0.394	0.151	0.052	0.085
0.073	0.130	0.051	0.028	0.035	0.242	0.443	0.169	0.057	0.094
0.086	0.154	0.060	0.030	0.040	0.268	0.491	0.187	0.061	0.103
0.099	0.178	0.069	0.032	0.044	0.294	0.539	0.205	0.066	0.112
0.112	0.202	0.078	0.034	0.049	0.320	0.587	0.224	0.070	0.122
0.125	0.226	0.087	0.037	0.053	0.346	0.635	0.242	0.075	0.131
0.138	0.250	0.096	0.039	0.058	0.372	0.683	0.260	0.079	0.140
0.151	0.274	0.105	0.041	0.062	0.398	0.731	0.278	0.084	0.149
0.164	0.298	0.114	0.043	0.067	0.424	0.779	0.296	0.088	0.158
0.177	0.322	0.123	0.046	0.071	0.450	0.827	0.315	0.093	0.167
0.190	0.346	0.133	0.048	0.076	....	....	....	....	....

The American Screw Co.'s machine-screw standards are given below.

Machine Screw Standards
(American Screw Co., Providence, R. I.)

Trade number	Diam. of body, in.	Threads per in.	No. of drill	Size of drill, in.	Length, in.
2	0.0842	48, 56, 64	49	0.0730	1/8 - 7/8
3	0.0973	48, 56	45	0.0820	1/8 - 7/8
4	0.1105	32, 36, 40	42	0.0935	1/8 -2
5	0.1236	32, 36, 40	38	0.1015	1/8 -2 1/4
6	0.1368	30, 32, 36	35	0.1100	1/8 -2 1/4
7	0.1500	30, 32	30	0.1285	1/8 -3
8	0.1631	30, 32, 36	29	0.1360	1/8 -3
9	0.1763	24, 30, 32	27	0.1440	3/16-3 1/2
10	0.1894	24, 30, 32	25	0.1495	3/16-3 1/2
12	0.2158	20, 24	17	0.1730	1/4 -3 1/2
14	0.2421	18, 20, 24	13	0.1850	1/4 -4
16	0.2684	16, 18, 20	6	0.2040	5/16-4
18	0.2947	16, 18, 20	1	0.2280	3/8 -4
20	0.3210	16, 18	D	0.246	3/8 -4
22	0.3474	16, 18	J	0.277	3/8 -4
24	0.3737	14, 16, 18	N	0.302	3/8 -4
26	0.4000	14, 16	P	0.323	3/4 -4
28	0.4263	14, 16	R	0.339	3/4 -4
30	0.4526	14, 16	U	0.368	3/4 -4

Standard lengths vary by the following increments: 1/8 in. to 1 in. by 1/16 in.; 1 in. to 2 in. by 1/8 in.; 2 in. to 4 in. by 1/4 in.

Table 16. Dimensions of Bolt Heads for Standard T-slots
(See Fig. 14. All dimensions in inches)

A	*B*	*C*	*D*	*E*	*F*	*G*
1/4	9/16	1/8	5/16	5/8	5/32	3/8
5/16	5/8	3/16	3/8	11/16	7/32	7/16
3/8	3/4	3/16	7/16	13/16	7/32	7/16
7/16	7/8	1/4	1/2	15/16	9/32	9/16
1/2	1 1/8	3/8	5/8	1 3/16	13/32	3/4
5/8	1 1/4	1/2	3/4	1 5/16	17/32	1
3/4	1 9/16	5/8	7/8	1 5/8	11/16	1 1/16
7/8	1 13/16	3/4	1	1 7/8	13/16	1 3/16

Eyebolts used by the Union Iron Works are made to the proportions given in Fig. 18 and Table 17.

Table 17. Proportions of Eyebolts
(See Fig. 18)

Diam. of bolt, in.	Diam. of stock in eye, in.	*A*, in.	*B*, in.	*C*, in.	*D*, in.	*E*, in.	*F*, in.	Capacity, lb.
3/8	3/8	1 1/2	2 1/4	1 3/8	5/8	1	2	767
1/2	3/8	1 1/2	2 1/4	1 3/8	5/8	1	2	1,104
5/8	1/2	2	3	1 3/4	1	1 1/4	2 3/4	1,963
3/4	1/2	2	3	1 3/4	1	1 1/4	2 3/4	2,485
7/8	5/8	2 1/4	3 1/2	2 1/8	1 1/4	1 3/4	3 3/8	3,712
1	5/8	2 1/4	3 1/2	2 1/8	1 1/4	1 3/4	3 3/8	5,185
1 1/8	3/4	2 1/2	4	2 1/2	1 1/2	2 1/4	4	6,903
1 1/4	3/4	2 1/2	4	2 1/2	1 1/2	2 1/4	4	7,854
1 3/8	7/8	2 3/4	4 1/2	2 3/4	2	2 1/2	4 3/4	9,940
1 1/2	7/8	2 3/4	4 1/2	2 3/4	2	2 1/2	4 3/4	12,270
1 5/8	1 1/8	3 1/2	5 3/4	3 1/2	2 1/2	3	6	13,520
1 3/4	1 1/8	3 1/2	5 3/4	3 1/2	2 1/2	3	6	16,210
1 7/8	1 1/4	4	6 1/2	4	3	3 1/2	7	19,150
2	1 1/4	4	6 1/2	4	3	3 1/2	7	22,340

Strength of Eyebolts. The following formula by O. M. Sames (*Ind. Engg.*, Sept., 1911) closely represents the practice of the General Electric Co.: Load in lb. at which deformation begins = 33,000 $d^3/(D_1 - 1.2D_2)$, in which d = diam. of stock in eye, D_1 = mean diam. of eye (= internal diam. + d), and D_2 = diam. of bolt shank at root of thread—all in inches. In designing, for eyebolts forged from double refined iron, take safe load equal to ⅓ that given by formula.

Set Screws for fastening collars and the like to shafting may have forms of heads and points as shown in Figs. 15, 16 and 17.

The round point will be found to give better service, since it will not work loose under repeated load application.

The **safe holding power** (P) of cup or flat-point set screws (B. H. D. Pinckney, *Am. Mach.*, Oct. 15, 1914) may be calculated from P (lb.) = $63{,}025N/nr$, where N = h.p. transmitted, n = r.p.m., and r = radius of shaft, in. Values of P for various diameters (d) of set screws are as follows:

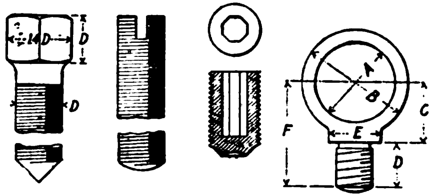
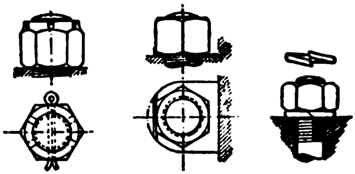


FIG. 15. FIG. 16. FIG. 17. Set Screws. FIG. 18. Eyebolt.

d (in.) =	¼	5/16	⅜	7/16	½	9/16	⅝	¾	⅞	1	1⅛	1¼
P (lb.) =	100	168	256	366	500	658	840	1280	1830	2500	3288	4198

Thus, for a pulley transmitting 6 h.p. at 300 r.p.m. on a 2-in. shaft, $P = 63{,}025 \times 6/(300 \times 1) = 1260$ lb., indicating the use of either one ¾-in. screw or two 9/16-in. screws.

Lock Nuts and special devices to prevent bolts and nuts from coming loose, as may result when vibrations are encountered, may be made to the forms shown in Fig. 19.



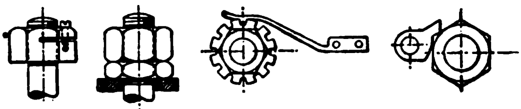


FIG. 19.—Lock Nuts.

Foundation Bolts may be made to the forms shown in Figs. 20 and 21. The difficulty in replacing such bolts as may become broken or injured in service, causes the form shown in Fig. 20 to be favored whenever it can be used. $l \geq 5d$.

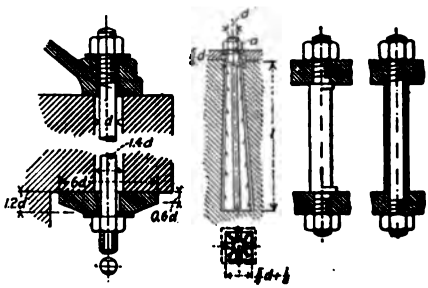


FIG. 20. FIG. 21. Foundation Bolts.

The Society of Automobile Engineers' (S.A.E.) Standard for Bolts and Nuts is shown in Fig. 22, proportions of bolts made to this standard being given in Table 18.

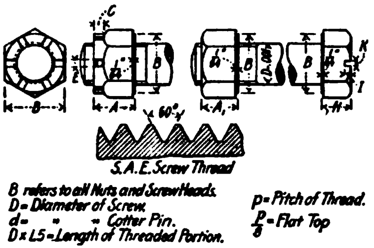


Fig. 22.—S.A.E. Standard for Bolts and Nuts.

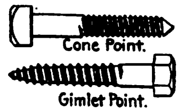


Fig. 23.—Coach and Lag Screws.

Fig. 24.—Wood Screw.

Table 18. S.A.E. Screw Standard
(See Fig. 22)

D	1/*p*	*A*	*A*₁	*B*	*C*	*E*	*H*	*I*	*K*	*d*
1/4	28	9/32	7/32	7/16	3/32	5/64	3/16	3/32	1/16	1/16
5/16	24	21/64	17/64	1/2	3/32	5/64	15/64	7/64	1/16	1/16
3/8	24	13/32	21/64	9/16	1/8	1/8	9/32	1/8	3/32	3/32
7/16	20	29/64	3/8	5/8	1/8	1/8	21/64	1/8	3/32	3/32
1/2	20	9/16	7/16	3/4	3/16	1/8	3/8	1/8	3/32	3/32
9/16	18	39/64	31/64	7/8	3/16	5/32	27/64	1/8	3/32	1/8
5/8	18	23/32	35/64	15/16	1/4	5/32	15/32	1/8	3/32	1/8
11/16	16	49/64	19/32	1	1/4	5/32	33/64	1/8	3/32	1/8
3/4	16	13/16	21/32	1 1/16	1/4	5/32	9/16	1/8	3/32	1/8
7/8	14	29/32	49/64	1 1/4	1/4	5/32	21/32	1/8	3/32	1/8
1	14	1	7/8	1 7/16	1/4	5/32	3/4	1/8	3/32	1/8
1 1/8	12	1 5/32	63/64	1 5/8	5/16	7/32	27/32	7/32	5/32	11/64
1 1/4	12	1 1/4	1 3/32	1 13/16	5/16	7/32	15/16	7/32	5/32	11/64
1 3/8	12	1 13/32	1 13/64	2	3/8	1/4	1 1/32	1/4	3/16	13/64
1 1/2	12	1 1/2	1 5/16	2 3/16	3/8	1/4	1 1/8	1/4	3/16	13/64

All dimensions in inches. 1/*p* = number of threads per inch. All heads and nuts to be semi-finish. Material—steel of not less than 100,000 lb. tensile strength and not less than 60,000 lb. elastic limit. These threads not to be used in cast iron, brass or aluminum.

Coach and Lag Screws as shown in Fig. 23 are made to the proportions given in Table 19.

Table 19. Coach and Lag Screws

Diam. of screw, in.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
No. of threads per in.	10	9 1/2	7	7	6	5	5	4 1/2	4 1/2	3
Across flats of hexagon and square heads, in.	3/8	15/32	9/16	21/32	3/4	27/32	15/16	1 1/8	1 5/16	1 1/2
Thickness of hexagon and square heads, in.	3/16	1/4	5/16	3/8	7/16	1/2	17/32	5/8	3/4	7/8

Length of Threads for Screws of all Diameters

Length of screw, in.	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2
Length of thread, in.	to head	1 1/2	2	2 1/4	2 1/2	3	3 1/2
Length of screw, in.	5	5 1/2	6	7	8	9	10-12
Length of thread, in.	4	4	4 1/2	5	6	6	7

Wood Screws are made in lengths from 1/4 in. to 6 in., increasing by 1/8 in. up to 1 in., by 1/4 in. up to 3 in., and by 1/2 in. up to 5 in. The standard of the American Screw Co. is shown in Fig. 24, and the proportions to which screws are made are given in Table 20.

Table 20. Dimensions of Wood Screws

(Included angle of flat head = 82 deg., see Fig. 24)

No. of Screw Gage	Diam. of screw, in.		Diam. of head, in.		Threads per in.	No. of Screw Gage	Diam. of screw, in.		Diam. of head, in.		Threads per in.
0	0.0578	1/16 −	0.110	7/64 +	32	16	0.268	17/64 +	0.526	17/32 −	9
1	0.0710	5/64 −	0.136	9/64 −	28	17	0.282	9/32	0.552	35/64 +	9
2	0.0842	5/64 +	0.162	5/32 +	26	18	0.295	19/64 −	0.578	37/64	8
3	0.0973	3/32 +	0.188	3/16	24	19	0.308	5/16 −	0.604	39/64 −	8
4	0.110	7/64 +	0.214	7/32 −	22	20	0.321	21/64 −	0.630	5/8 +	8
5	0.124	1/8 −	0.240	15/64 +	20	21	0.334	21/64 +	0.656	21/32	8
6	0.137	9/64 −	0.266	17/64 +	18	22	0.347	11/32 +	0.682	11/16 −	7
7	0.150	5/32 −	0.292	19/64 −	16	23	0.361	23/64	0.708	45/64 +	7
8	0.163	5/32 +	0.318	5/16 +	15	24	0.374	3/8 −	0.734	47/64	7
9	0.176	11/64 +	0.344	11/32 +	14	25	0.387	25/64 −	0.760	49/64 −	7
10	0.189	3/16 +	0.370	3/8 −	13	26	0.400	13/32 −	0.786	25/32 +	6
11	0.203	13/64 −	0.396	25/64 +	12	27	0.413	13/32 +	0.812	13/16	6
12	0.216	7/32 −	0.422	27/64	11	28	0.426	27/64 +	0.838	27/32 −	6
13	0.229	15/64 −	0.448	29/64 −	11	29	0.439	7/16 +	0.864	55/64 +	6
14	0.242	1/4 −	0.474	15/32 +	10	30	0.453	29/64	0.890	57/64	6
15	0.255	1/4 +	0.500	1/2	10	....	....	....	....	....	....

Washers for bolts and lag screws, either round or square, are made to the dimensions given in Table 21.

Table 21. Dimensions of Round and Square Washers

Size of bolt, in.	Size of hole, in.	U. S. Standard, round			Narrow gage, round		Standard sizes, square	
		Outside diam., in.	Thickness, in.	Approx. no. in 100 lb.	Outside diam., in.	Thickness, in.	Width, in.	Thickness in.
3/16	1/4	9/16	3/64	44,300	....	....	....	....
1/4	5/16	3/4	1/16	18,100	5/8	1/16	....	....
5/16	3/8	7/8	1/16	13,600	3/4	1/16	....	....
3/8	7/16	1	5/64	7,700	7/8	1/16	1 1/2	1/8
7/16	1/2	1 1/4	5/64	4,500	1 1/8	5/64	1 3/4	1/8
1/2	9/16	1 3/8	3/32	3,400	1 1/4	3/32	2	3/16
9/16	5/8	1 1/2	3/32	2,700	1 3/8	3/32	....	....
5/8	11/16*	1 3/4	1/8	1,400	1 1/2	1/8	2 1/4	1/4
3/4	13/16*	2	1/8	1,200	1 3/4	1/8	2 1/2	1/4
7/8	15/16*	2 1/4	5/32	760	2	5/32	3	1/4
1	1 1/16*	2 1/2	5/32	570	2 1/4	5/32	3 1/2	3/8
1 1/8	1 1/4	2 3/4	5/32	490	2 1/2	5/32	4	3/8
1 1/4	1 3/8	3	5/32	415	2 3/4	5/32	4 1/2	3/8
1 3/8	1 1/2	3 1/4	11/64	325	3	11/64	5	3/8
1 1/2	1 5/8	3 1/2	11/64	275	3 1/4	11/64	6	3/8
1 5/8	1 3/4	3 3/4	11/64	245	....	....	....	....
1 3/4	1 7/8	4	11/64	200	....	....	6	3/8
1 7/8	2	4 1/4	11/64	185	....	....	....	....
2	2 1/8	4 1/2	11/64	170	....	....	6	3/8
2 1/4	2 3/8	4 3/4	3/16	140	....	....	....	....
2 1/2	2 5/8	5	7/32	115	....	....	....	....

* Holes in square washers 1/32 in. larger for these four sizes.

Carriage Bolts (Fig. 25) may be obtained in the sizes given in Table 22 and to any lengths desired. Length of thread, 2 to 4 times thickness of nut, depending on length of bolt.

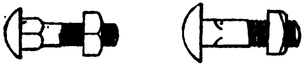


Fig. 25.—Carriage Bolts.

Fig. 26.—Stove Bolts.

Table 22. Dimensions of Carriage Bolts
(See Fig. 25)

Diameter of bolt, in.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
Diameter of head, in.	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 1/2	1 3/4	2
Thickness of head, in.	1/8	5/32	3/16	7/32	1/4	9/32	5/16	3/8	7/16	1/2

Stove Bolts (Fig. 26) are made in the sizes given in Table 23.

Table 23. Dimensions of Stove Bolts
(See Fig. 26)

Diam. of bolt, in.	1/8	5/32	3/16	7/32	1/4	5/16	3/8
No. of threads per inch	32	28	24	22	18	18	16

Materials, Strength and Service Adaptability of Machine Bolts and Screws

The Materials to be used for bolts, screws and nuts depend on service conditions and relative costs. The stresses to be allowed in determining the proportions in any case depend on the nature of the loading and the material.

As a guide to the selection of **bolts and nuts for fastenings** in stationary machinery, the specifications of the Bureau of Steam Engineering of the U. S. Navy are given in Table 24.

Table 24. Materials for Bolts and Nuts

Class	Material	Minimum tensile strength, lb. per sq. in.	Minimum elastic limit, lb. per sq. in.	Minimum elongation in 8 in., per cent.	Maximum percentage of	
					Phosphorus	Sulphur
A	Open-hearth nickel or carbon steel.	75,000	40,000	23	0.04	0.03
B	Open-hearth carbon steel	58,000	30,000	28	0.04	0.03

Bending Tests. Class A material must bend cold after quenching about an inner diameter equal to the thickness of the test piece in each case without cracking; quenching temperature, 80 to 90 deg. fahr. For Class B the same test applies, except that inner diameter = 1/2 thickness of test piece.

Hammer Test. Class A material must flatten out cold to 1/2 the original diameter without showing cracks. Class B must flatten out while heated to a cherry red in daylight to a thickness equal to 1/4 the original diameter without showing cracks.

Bolts requiring unusual strength are made from material specified under forgings in Table 25. Connecting-rod bolts, for example, are made from "high-grade forgings." For the S.A.E. specifications, see p. 464.

Set Screws may be made from the following material recommended by the Carnegie Steel Co.: Carbon, 0.15 to 0.20 per cent.; manganese, 0.6 to 0.8 per cent.; phosphorus, not over 0.06 per cent.; sulphur, 0.06 per cent.

Safe Loads in Tension for U. S. Standard Bolts, as determined by Harvey D. Williams, of the Bureau of Steam Engineering, U. S. N., are given in Table 26. Colvin and Stanley ("Am. Mach. Handbook") compute the tensile and shearing strengths as given in Table 27.

Table 25. Steel Forgings for Bolts

Class	Material	Treatment	Minimum tensile strength, lb. per sq. in.	Minimum elastic limit, lb. per sq. in	Minimum elongation in 2 in., per cent.	Maximum percentage of phosphorus	Maximum percentage of sulphur	Cold bend about an inner diam. of
High grade..	Open-hearth nickel steel	Anneal and oil-temper	95,000	65,000	21	0.06	0.04	1 in. through 180 deg.
A......	Open-hearth nickel or carbon steel	Anneal; oil tempering optional	80,000	50,000	25	0.06	0.04	1 in. through 180 deg.
B......	Open-hearth carbon steel	Anneal	60,000	30,000	30	0.06	0.04	½ in. through 180 deg.

Table 26. Safe Loads for U.S. Standard Bolts

Nominal diam., in.	No. of threads per in.	Ultimate strength, lb. per sq. in. 20,000	40,000	50,000	60,000	65,000	80,000	95,000
		Alloy Cu, 88%, Sn, 10%, Zn, 2%	Phosphor-bronze	Wrought iron and best rolled bronze	Class B bolt material	Class A bolt material	Class A Nos. 1 and 2 machinery forgings	High-grade machinery forgings
¼	20	57	115	143	172	186	229	272
5/16	18	99	198	247	297	322	396	470
⅜	16	150	301	376	451	488	601	714
7/16	14	207	415	519	623	675	830	986
½	13	282	564	704	845	915	1,125	1,340
9/16	12	365	730	912	1,095	1,186	1,460	1,730
⅝	11	456	913	1,140	1,370	1,480	1,820	2,170
¾	10	690	1,380	1,725	2,070	2,240	2,760	3,280
⅞	9	964	1,930	2,410	2,900	3,140	3,860	4,580
1	8	1,265	2,530	3,170	3,800	4,120	5,060	6,010
1⅛	7	1,595	3,190	3,990	4,790	5,180	6,380	7,570
1¼	7	2,070	4,140	5,180	6,210	6,730	8,280	9,830
1⅜	6	2,440	4,890	6,110	7,330	7,940	9,780	11,600
1½	6	3,020	6,040	7,540	9,060	9,800	12,050	14,300
1⅝	5½	3,530	7,060	8,820	10,600	11,500	14,100	16,750
1¾	5	4,060	8,120	10,150	12,200	13,200	16,200	19,250
1⅞	5	4,800	9,600	12,000	14,400	15,600	19,200	22,800
2	4½	5,360	10,750	13,400	16,100	17,400	21,500	25,500
2¼	4½	7,120	14,200	17,800	21,400	23,100	28,500	33,800
2½	4	8,750	17,500	21,900	26,300	28,400	35,000	41,500
2¾	4	11,000	22,000	27,500	33,000	35,700	44,000	52,200
3	4	13,400	26,800	33,500	40,200	43,600	53,600	63,600
3¼	4	16,100	32,200	40,200	48,400	52,400	64,400	76,400
3½	4	19,000	38,100	47,600	57,200	61,900	76,200	90,400
3¾	4	22,200	44,500	55,600	66,700	72,300	89,000	105,500
4	4	25,700	51,400	64,200	77,000	83,400	102,800	122,000
4¼	4	29,350	58,700	73,400	88,100	95,400	117,400	139,300
4½	4	33,300	66,600	83,200	100,000	108,000	133,000	158,000
4¾	4	37,400	75,000	93,700	112,000	122,000	150,000	178,000
5	4	41,900	83,800	105,000	126,000	136,000	167,500	199,000
5¼	4	46,600	93,200	116,500	140,000	151,000	186,000	221,000
5½	4	51,500	103,000	129,000	154,500	167,000	206,000	244,500
5¾	4	56,700	113,500	142,000	170,000	184,000	227,000	269,000
6	4	62,000	124,000	155,000	186,000	202,000	248,000	295,000

Table 27. Strength of U. S. Standard Bolts from ¼ to 3 in. in Diam.

Bolt		Areas		Tensile strength, lb.			Shearing strength, lb.			
							Full bolt		Bottom of thread	
Diam. of bolt, in.	No. of threads per in.	Full bolt, sq. in.	Bottom of thread, sq. in.	At 10,000 lb. per sq. in.	At 12,500 lb. per sq. in.	At 17,500 lb. per sq. in.	At 7,500 lb. per sq. in.	At 10,000 lb. per sq. in.	At 7,500 lb. per sq. in.	At 10,000 lb. per sq. in.
¼	20	0.049	0.027	270	340	470	380	490	200	270
5/16	18	0.077	0.045	450	570	790	580	770	340	450
⅜	16	0.110	0.068	680	850	1,190	830	1,100	510	680
7/16	14	0.150	0.093	930	1,170	1,630	1,130	1,500	700	930
½	13	0.196	0.126	1,260	1,570	2,200	1,470	1,960	940	1,260
9/16	12	0.248	0.162	1,620	2,030	2,840	1,860	2,480	1,220	1,620
⅝	11	0.307	0.202	2,020	2,520	3,530	2,300	3,070	1,510	2,020
¾	10	0.442	0.302	3,020	3,770	5,290	3,310	4,420	2,270	3,020
⅞	9	0.601	0.419	4,190	5,240	7,330	4,510	6,010	3,150	4,190
1	8	0.785	0.551	5,510	6,890	9,640	5,890	7,850	4,130	5,510
1⅛	7	0.994	0.693	6,930	8,660	12,130	7,450	9,940	5,200	6,930
1¼	7	1.227	0.890	8,890	11,120	15,570	9,200	12,270	6,670	8,900
1⅜	6	1.485	1.054	10,540	13,180	18,450	11,140	14,850	7,910	10,540
1½	6	1.767	1.294	12,940	16,170	22,640	13,250	17,670	9,700	12,940
1⅝	5½	2.074	1.515	15,150	18,940	26,510	15,550	20,740	11,360	15,150
1¾	5	2.405	1.745	17,450	21,800	30,520	18,040	24,050	13,080	17,440
1⅞	5	2.761	2.049	20,490	25,610	35,860	20,710	27,610	15,370	20,490
2	4½	3.142	2.300	23,000	28,750	40,250	23,560	31,420	17,250	23,000
2¼	4½	3.976	3.021	30,210	37,770	52,870	29,820	39,760	22,660	30,210
2½	4	4.909	3.716	37,160	46,450	65,040	36,820	49,090	27,870	37,160
2¾	4	5.940	4.620	46,200	57,750	80,840	44,580	59,400	34,650	46,200
3	3½	7.069	5.428	54,280	67,850	94,990	53,020	70,690	40,710	54,280

General Notes on the Design of Bolts

Bolts subjected to shock and sudden change in load are found to be more serviceable when the body of the bolt is turned down or drilled to the area of the root of the thread. The drilled bolt is stronger in torsion.

When a **number of bolts** are **employed in fastening** together two parts of a machine, such as a cylinder and cylinder head, the load carried by each bolt depends on its relative tightness, the tighter bolts carrying the greater loads. When the conditions of attendance are such as to occasion a great difference in tightness, lower working stresses must be used in designing the bolts than otherwise are necessary. On the other hand, it may be desirable to have the bolts the weakest part of the machine, since their breakage from overload in the machine will result in a minimum cost of replacement. In such cases the breaking load of the bolts may well be equal to the load which causes the weakest member of the machine connected to be stressed up to the elastic limit.

Bolts screwed up tight have an initial stress due to the tightening before any external load is applied to the machine member. According to Professor Barr, the initial tensile load due to screwing up for a tight joint varies about as the diam. of the bolt, and may be estimated at 16,000 lb. per in. of diam. There is consequently danger of excessive stresses for bolts of less than 1 in. diam. If the bolt is manifestly more yielding than the connected members, it should be designed simply to resist the initial tension or the external load, whichever is the greater. If the probable yielding of the bolt is from 50 to 100 per cent. of that of the connected members, take the resultant stress as the initial tension plus one-half the external load. If the yielding of the connected members is probably from four to five times that of the bolt (as when certain packings are used), take the resultant stress as the initial tension plus three-fourths the external load.

In any given machine the bolts and screws used should be of **as few sizes as possible** in order to reduce the number of spanners required to tighten them and the number of drill and tap sizes needed in manufacture.

In **drilling and tapping cast iron for studs,** it is necessary to tap to a depth equal to one and one-half times the stud diameter in order that the strength of the cast-iron threads in shear may equal the tensile strength of the bolt. Drill sizes and depths of hole and thread are given in Table 28.

Table 28. Depths to Drill and Tap Cast Iron for Studs

Diam. of stud, in	1/4	5/16	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
Diam. of drill, in	13/64	17/64	5/16	3/8	27/64	31/64	17/32	41/64	3/4	55/64
Depth of thread, in ...	3/8	15/32	9/16	21/32	3/4	27/32	15/16	1 1/8	1 5/16	1 1/2
Depth to drill, in	7/16	17/32	5/8	23/32	27/32	15/16	1 1/32	1 1/4	1 7/16	1 5/8

It is not good practice to drill holes to be tapped through the metal into pressure spaces, as the leakage resulting, even though the bolt fits tightly, will occasion much trouble.

Drill Sizes for Sellers or A. S. M. E. Standard Thread Taps. Diam. of drill = outside diam. of stud *minus* pitch of thread. Table 29 gives tap drill sizes for taps with U. S. (Sellers) Standard, "V" and Whitworth threads.

Table 29. Tap Drill Sizes

Size of tap, in.	No. of threads per in.	Size of drill, in.			Size of tap, in.	No. of threads per in.	Size of drill, in.		
		U. S. Standard	"V" thread	Whitworth			U. S. Standard	"V" thread	Whitworth
3/16	24	0.128	0.111	0.129	15/16	9	0.808	0.790	0.810
1/4	20	0.191	0.184	0.192	1	8	0.854	0.832	0.856
5/16	18	0.248	0.239	0.249	1 1/16	8	0.917	0.894	0.919
3/8	16	0.302	0.293	0.303	1 1/8	7	0.957	0.932	0.960
7/16	14	0.354	0.345	0.355	1 1/4	7	1.082	1.057	1.085
1/2	13	0.409	0.399	0.410	1 3/8	6	1.179	1.144	1.182
1/2	12	0.402	0.391	0.403	1 1/2	6	1.304	1.269	1.307
9/16	12	0.465	0.453	0.466	1 5/8	5 1/2	1.412	1.372	1.416
5/8	11	0.518	0.506	0.520	1 5/8	5	1.390	1.347	1.394
11/16	11	0.581	0.568	0.583	1 3/4	5	1.515	1.472	1.519
3/4	10	0.632	0.618	0.634	1 7/8	5	1.640	1.597	1.644
13/16	10	0.695	0.680	0.697	1 7/8	4 1/2	1.614	1.566	1.619
7/8	9	0.745	0.728	0.747	2	4 1/2	1.739	1.691	1.744

RIVET FASTENINGS

Forms and Proportion of Rivets. The forms of rivet heads for structural and boiler work, as well as proportions which represent good practice, are

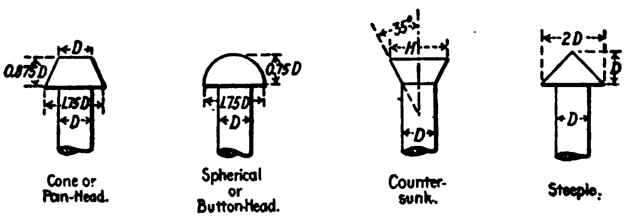


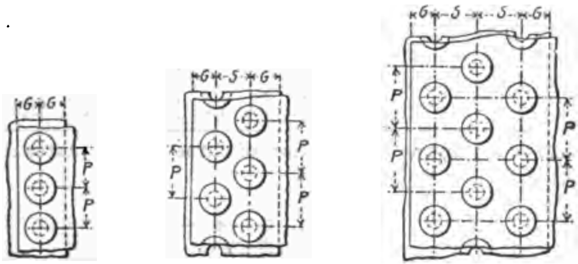
FIG. 27.—Rivet Heads.

shown in Fig. 27. The following values apply to rivets with countersunk heads:

For D (inches) = $\frac{1}{4}$ $\frac{5}{16}$ $\frac{3}{8}$ $\frac{7}{16}$ $\frac{1}{2}$ $\frac{9}{16}$ $\frac{5}{8}$ $\frac{11}{16}$ $\frac{3}{4}$ $\frac{13}{16}$ $\frac{7}{8}$ 1 $1\frac{1}{8}$ $1\frac{1}{4}$
H (inches) = $\frac{1}{2}$ $\frac{19}{32}$ $\frac{11}{16}$ $\frac{25}{32}$ $\frac{7}{8}$ $\frac{31}{32}$ $1\frac{1}{16}$ $1\frac{5}{32}$ $1\frac{1}{4}$ $1\frac{11}{32}$ $1\frac{7}{16}$ $1\frac{5}{8}$ $1\frac{13}{16}$ 2

Weights of 100 Structural Rivets with Button Heads, Lb.

Length under head, inches	Diameter of rivet, inches							
	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
1 1/2	7	13	23	35	50	68	91	130
1 5/8	7	14	24	36	52	71	95	134
1 3/4	8	15	25	37	54	74	98	139
1 7/8	8	15	26	39	56	77	102	143
2	9	16	27	41	58	80	105	148
2 1/8	9	17	28	43	60	82	109	152
2 1/4	9	18	29	44	62	85	112	156
2 3/8	10	18	30	46	64	88	116	161
Lb. per inch additional	3.0	5.6	8.7	12.5	17.0	22.2	28.2	34.7
	Weights of 100 button heads, lb.							
On rivets	2.4	5.0	9.7	16.0	24.0	35.0	49.0	78.0
As driven	1.9	4.0	7.5	12.5	18.5	27.0	37.5	51.0



Single Riveted. Double Riveted. Triple Riveted.

FIG. 28.—Riveted Joints.

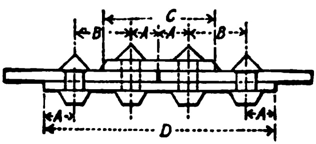


FIG. 29.

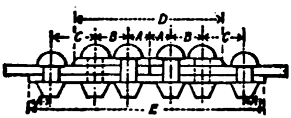


FIG. 30.

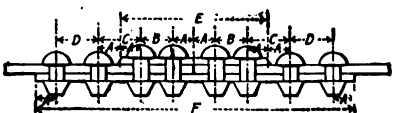


FIG. 31.

Forms and Proportions of Riveted Joints for Boilers and Tanks. Riveted joints for steam boilers and tanks may have the forms and proportions recommended by the Hartford Steam Boiler Inspection and Insurance Co., as given in Figs. 28 to 31, and Tables 30 to 33, inclusive.

Table 30. Dimensions of Riveted Lap Joints

(All dimensions in inches. Letters refer to Fig. 28. E = per cent. efficiency of joint. Dimensions based on a tensile strength of 60,000 lb. per sq. in. for the plate, and a shearing strength of 38,000 lb. per sq. in. for the rivets.)

Thickness of plate (steel)	Diam. of rivet (iron)	Single riveted			Double riveted				Triple riveted			
		P	G	E	P	G	S	E	P	G	S	E
1/4	1/2	1 1/4	1	50	1 13/16	1	1 3/4	69	2 1/2	1	1 7/8	76
	5/8	1 5/8	1 1/8	57	2 5/8	1 1/8	1 7/8	72	3 1/2	1 1/8	2 1/16	80
	11/16	1 7/8	1 3/16	60	2 7/8	1 3/16	1 15/16	74	4	1 3/16	2 1/8	81
5/16	5/8	1 3/8	1 1/8	50	2 3/16	1 1/8	1 5/8	68	2 15/16	1 1/8	2	76
	11/16	1 5/8	1 3/16	54	2 1/2	1 3/16	1 3/4	70	3 1/8	1 3/16	2 1/16	76
	3/4	1 7/8	1 1/4	56	2 7/8	1 1/4	1 15/16	72	4	1 1/4	2 3/16	79
3/8	3/4	1 11/16	1 1/4	52	2 9/16	1 1/4	1 7/8	68	3 3/8	1 1/4	2 3/16	76
	13/16	1 7/8	1 5/16	53	2 7/8	1 5/16	2	69	3 7/8	1 5/16	2 1/4	77
	7/8	2 1/8	1 7/16	55	3 1/4	1 7/16	2 3/16	71	4 7/16	1 7/16	2 1/2	79
7/16	3/4	1 9/16	1 1/4	47	2 5/16	1 1/4	1 7/8	65	3	1 1/4	2	73
	7/8	1 15/16	1 7/16	51	2 15/16	1 7/16	2 1/16	67	3 15/16	1 7/16	2 1/2	76
	15/16	2 1/8	1 1/2	53	3 5/16	1 1/2	2 3/16	70	4 3/8	1 1/2	2 5/8	77
1/2	7/8	1 13/16	1 7/16	48	1 11/16	1 7/16	2	65	3 1/2	1 7/16	2 1/4	73
	15/16	2	1 1/2	50	3	1 1/2	2 1/8	66	4	1 1/2	2 5/8	74
	1	2 3/16	1 5/8	51	3 5/16	1 5/8	2 1/4	68	4 7/16	1 5/8	2 11/16	76
9/16	15/16	1 7/8	1 1/2	46	2 3/4	1 1/2	2	63	3 5/8	1 1/2	2 5/16	72
	1	2 1/16	1 5/8	48	3 1/16	1 5/8	2 1/8	65	4 1/16	1 5/8	2 1/2	73

The form of the **joint at the junction of three or four plates** may be as shown in Figs. 32 to 35, inclusive.

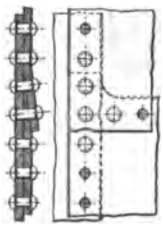


FIG. 32.

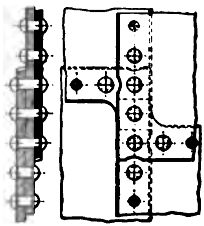


FIG. 33.

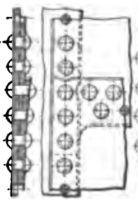


FIG. 34.

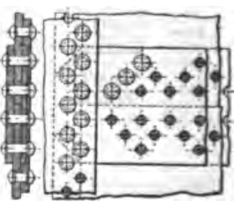


FIG. 35.

Forms of Riveted Joint at Junction of Three or Four Plates.

When **plates** are to be **joined at right angles** and riveted, the joint may have the form shown in Fig. 36. With such joints the outer radius of curvature of the plate should be at least four times the plate thickness. The overlap should be three times the rivet diameter. Frequently angle connections such as shown in Fig. 37 are used. In such construction it is good practice to have the rivet diameter $d = 2 \times$ plate thickness t, and the dimension $a = \frac{1}{2}(w - t_1)$, in which w = width of angle leg and t_1 = thickness of leg $= t + \frac{1}{16}$ in. The other dimensions in the figure are in terms of t.

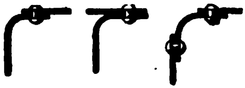


FIG. 36.

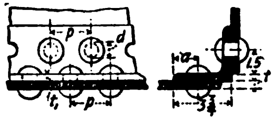


FIG. 37.

Joining Plates at Right Angles.

Table 31. Dimensions of Double-riveted Butt Joints (Steel Rivets in Steel Plates)

(All dimensions in inches. Letters refer to Fig. 29. Dimensions based on a tensile strength of 55,000 lb. per sq. in. for plates and straps, and a shearing strength of 42,000 lb. per sq. in. for rivets in single shear and 78,000 lb. for rivets in double shear.)

Thickness of plate	Diam. of rivet hole	Efficiency, per cent.	Long pitch	Short pitch	*A*	*B*	*C*	*D*	Thickness of straps
1/4	11/16	81.6	3 3/4	1 7/8	1 1/32	2 1/16	4 1/8	8 1/4	1/4
9/32	11/16	81.6	3 3/4	1 7/8	1 1/32	2 1/16	4 1/8	8 1/4	1/4
5/16	13/16	80.3	4 1/8	2 1/16	1 7/32	2 7/16	4 7/8	9 3/4	9/32
11/32	13/16	80.3	4 1/8	2 1/16	1 7/32	2 7/16	4 7/8	9 3/4	9/32
3/8	13/16	80.8	4 1/4	2 1/8	1 7/32	2 7/16	4 7/8	9 3/4	5/16
13/32	13/16	80.8	4 1/4	2 1/8	1 7/32	2 7/16	4 7/8	9 3/4	11/32
7/16	15/16	80.2	4 3/4	2 3/8	1 13/32	2 13/16	5 5/8	11 1/4	3/8
15/32	15/16	80.2	4 3/4	2 3/8	1 13/32	2 13/16	5 5/8	11 1/4	13/32
1/2	15/16	80.7	4 7/8	2 7/16	1 13/32	2 13/16	5 5/8	11 1/4	7/16

Table 32. Dimensions of Triple-riveted Butt Joints (Steel Rivets in Steel Plates)

(All dimensions in inches. Letters refer to Fig. 30. Dimensions based on the tensile and shearing strengths given for Table 31)

Thickness of plate	Diam. of of rivet hole	Efficiency, per cent.	Long pitch	Short pitch	*A*	*B*	*C*	*D*	*E*	Thickness of straps
1/4	9/16	87.5	4 1/2	2 1/4	27/32	1 3/8	1 11/16	6 1/8	9 1/2	3/16
9/32	5/8	86.3	4 9/16	2 9/32	15/16	1 3/8	1 7/8	6 1/2	10 1/4	3/16
5/16	3/4	88.0	6 1/4	3 1/8	1 1/8	1 7/8	2 1/4	8 1/4	12 3/4	1/4
11/32	3/4	88.0	6 1/4	3 1/8	1 1/8	1 7/8	2 1/4	8 1/4	12 3/4	1/4
3/8	13/16	87.5	6 1/2	3 1/4	1 7/32	2	2 7/16	8 7/8	13 3/4	5/16
13/32	13/16	87.7	6 5/8	3 5/16	1 7/32	2	2 7/16	8 7/8	13 3/4	5/16
7/16	15/16	86.1	6 3/4	3 3/8	1 13/32	2 1/32	2 13/16	9 11/16	15 5/16	3/8
15/32	15/16	86.6	7	3 1/2	1 13/32	2 1/8	2 13/16	9 7/8	15 1/2	3/8
1/2	1 1/16	85.84	7 1/2	3 3/4	1 19/32	2 1/4	3 3/16	10 7/8	17 1/4	3/8
17/32	1 1/16	85.83	7 1/2	3 3/4	1 19/32	2 1/4	3 3/16	10 7/8	17 1/4	13/32
9/16	1 1/16	85.95	7 9/16	3 25/32	1 19/32	2 9/32	3 3/16	10 15/16	17 5/16	7/16
19/32	1 1/16	86.0	7 5/8	3 13/16	1 19/32	2 9/32	3 3/16	10 15/16	17 5/16	7/16
5/8	1 1/16	86.3	7 3/4	3 7/8	1 19/32	2 5/16	3 3/16	11	17 3/8	15/32
21/32	1 1/16	85.8	7 3/4	3 7/8	1 19/32	2 5/16	3 3/16	11	17 3/8	1/2
11/16	1 3/16	84.7	7 3/4	3 7/8	1 25/32	2 5/16	3 9/16	11 3/4	18 7/8	17/32
23/32	1 3/16	84.5	7 3/4	3 7/8	1 25/32	2 5/16	3 9/16	11 3/4	18 7/8	17/32
3/4	1 3/16	84.1	7 7/8	3 15/16	1 25/32	2 3/8	3 9/16	11 7/8	19	9/16
25/32	1 3/16	83.6	7 7/8	3 15/16	1 25/32	2 3/8	3 9/16	11 7/8	19	19/32
13/16	1 5/16	82.8	7 7/8	3 15/16	1 31/32	2 3/8	3 15/16	12 5/8	20 1/2	5/8
27/32	1 5/16	82.2	7 7/8	3 15/16	1 31/32	2 3/8	3 15/16	12 5/8	20 1/2	5/8
7/8	1 5/16	82.5	8 1/4	4 1/8	1 31/32	2 1/2	3 15/16	12 7/8	20 3/4	21/32
29/32	1 5/16	82.0	8 1/4	4 1/8	1 31/32	2 1/2	3 15/16	12 7/8	20 3/4	11/16
15/16	1 7/16	81.7	8 1/2	4 1/4	2 5/32	2 9/16	4 5/16	13 3/4	22 3/8	23/32
31/32	1 7/16	81.5	8 5/8	4 5/16	2 5/32	2 5/8	4 5/16	13 7/8	22 1/2	3/4
[illegible]	1 7/16	81.0	8 5/8	4 5/16	2 5/32	2 5/8	4 5/16	13 7/8	22 1/2	3/4

Table 33. Dimensions of Quadruple-riveted Butt Joints (Steel Rivets in Steel Plates)

(All dimensions in inches. Letters refer to Fig. 31. Dimensions based on the tensile and shearing strengths given for Table 31)

Thickness of plate	Diam. of rivet hole	Efficiency, per cent.	Long pitch	Middle pitch	Short pitch	*A*	*B*	*C*	*D*	*E*	*F*	Thickness of straps
1/4	9/16	94.3	10	5	2 1/2	27/32	1 11/16	1 11/16	1 5/8	6 3/4	13 3/8	3/16
9/32	5/8	94.3	11	5 1/2	2 3/4	15/16	1 13/16	1 7/8	1 3/4	7 3/8	14 5/8	3/16
5/16	3/4	94.6	14	7	3 1/2	1 1/8	2 1/8	2 1/4	2	8 3/4	17 1/4	1/4
11/32	3/4	94.2	14	7	3 1/2	1 1/8	2 1/8	2 1/4	2	8 3/4	17 1/4	1/4
3/8	3/4	93.3	14	7	3 1/2	1 1/8	2 1/8	2 1/4	2	8 3/4	17 1/4	5/16
13/32	7/8	94.0	14 5/8	7 5/16	3 21/32	1 5/16	2 3/16	2 5/8	2 1/4	9 5/8	19 3/8	5/16
7/16	15/16	93.7	15	7 1/2	3 3/4	1 13/32	2 1/4	2 13/16	2 1/4	10 1/8	20 1/4	3/8
15/32	15/16	94.0	15 5/8	7 13/16	3 29/32	1 13/32	2 11/32	2 13/16	2 3/8	10 5/16	20 11/16	3/8
1/2	15/16	94.0	15 3/4	7 7/8	3 15/16	1 13/32	2 11/32	2 13/16	2 3/8	10 5/16	20 11/16	3/8
17/32	15/16	94.0	15 5/8	7 13/16	3 29/32	1 13/32	2 11/32	2 13/16	2 3/8	10 5/16	20 11/16	13/32
9/16	15/16	94.0	15 3/4	7 7/8	3 15/16	1 13/32	2 11/32	2 13/16	2 3/8	10 5/16	20 11/16	7/16
19/32	1 1/16	93.1	15 1/2	7 3/4	3 7/8	1 19/32	2 5/16	3 3/16	2 5/8	11	22 5/8	7/16
5/8	1 1/16	93.1	15 1/2	7 3/4	3 7/8	1 19/32	2 5/16	3 3/16	2 5/8	11	22 5/8	15/32
21/32	1 1/16	92.5	15 1/2	7 3/4	3 7/8	1 19/32	2 5/16	3 3/16	2 5/8	11	22 5/8	1/2
11/16	1 3/16	92.4	15 5/8	7 13/16	3 29/32	1 25/32	2 11/32	3 9/16	2 7/8	11 13/16	24 11/16	17/32
23/32	1 3/16	92.1	15 1/2	7 3/4	3 7/8	1 25/32	2 5/16	3 9/16	2 7/8	11 3/4	24 5/8	17/32
3/4	1 3/16	91.1	15 1/2	7 3/4	3 7/8	1 25/32	2 5/16	3 9/16	2 7/8	11 3/4	24 5/8	9/16
25/32	1 3/16	90.3	15 1/2	7 3/4	3 7/8	1 25/32	2 5/16	3 9/16	2 7/8	11 3/4	24 5/8	19/32
13/16	1 5/16	90.8	15 5/8	7 13/16	3 29/32	1 31/32	2 11/32	3 15/16	3 1/8	12 9/16	26 11/16	5/8
27/32	1 5/16	89.9	15 5/8	7 13/16	3 29/32	1 31/32	2 11/32	3 15/16	3 1/8	12 9/16	26 11/16	5/8
7/8	1 5/16	89.1	15 3/4	7 7/8	3 15/16	1 31/32	2 3/8	3 15/16	3 1/8	12 5/8	26 3/4	21/32
29/32	1 5/16	88.3	15 3/4	7 7/8	3 15/16	1 31/32	2 3/8	3 15/16	3 1/8	12 5/8	26 3/4	11/16
15/16	1 5/16	87.8	16	8	4	1 31/32	2 13/32	3 15/16	3 1/8	12 11/16	26 13/16	23/32
31/32	1 5/16	87.1	16	8	4	1 31/32	2 13/32	3 15/16	3 1/8	12 11/16	26 13/16	3/4
1	1 7/16	87.3	16	8	4	2 5/32	2 13/32	4 5/16	3 3/8	13 7/16	28 13/16	3/4
1 1/32	1 7/16	86.5	16	8	4	2 5/32	2 13/32	4 5/16	3 3/8	13 7/16	28 13/16	25/32
1 1/16	1 7/16	86.3	16 1/8	8 1/4	4 1/8	2 5/32	2 15/32	4 5/16	3 3/8	13 9/16	28 15/16	25/32
1 3/32	1 7/16	85.9	16 3/4	8 3/8	4 3/16	2 5/32	2 1/2	4 5/16	3 3/8	13 5/8	29	13/16
1 1/8	1 7/16	85.4	16 3/4	8 3/8	4 3/16	2 5/32	2 1/2	4 5/16	3 3/8	13 5/8	29	27/32
1 5/32	1 7/16	84.8	16 3/4	8 3/8	4 3/16	2 5/32	2 1/2	4 5/16	3 3/8	13 5/8	29	7/8
1 3/16	1 7/16	84.5	17	8 1/2	4 1/4	2 5/32	2 9/16	4 5/16	3 3/8	13 3/4	29 1/8	7/8
1 7/32	1 9/16	84.4	17	8 1/2	4 1/4	2 11/32	2 9/16	4 11/16	3 5/8	14 1/2	31 1/8	29/32
1 1/4	1 9/16	84.0	17 1/8	8 9/16	4 9/32	2 11/32	2 9/16	4 11/16	3 5/8	14 1/2	31 1/8	15/16

Straining Actions in Riveted Joints. The straining actions to which the several elements of a riveted joint are subjected are easily determined as to kind, but are most difficult to evaluate. More particularly is this true when the joint is one of several in a structure such as a boiler and receives its load according to the elasticity of the parts through which these loads are transmitted. Without regard to the friction between the plate and assuming all rivets in the joint to be equally well fitted and consequently sharing equally in taking up load, the accepted method of analyzing the stresses in the joint is as follows:

Let t = thickness of plate, in. d = diameter of rivet, in. p = pitch of rivets, in. n = number of rivets per pitch width of plate, in single shear. m = same, in double shear. f_t = intensity of stress in tension, lb. per sq. in. f_c = intensity of stress in crushing, lb. per sq. in., single shear. f_s = intensity of stress in shearing, lb. per sq. in., single shear (f'_c and f'_s for double shear). P = pressure existing in the tank or boiler, lb. per sq. in. D = internal diam. of tank, in. S = capacity of joint per pitch width to resist shearing with a given intensity of shearing stress, measured in lb. C = capacity of joint per pitch width to resist crushing with a given intensity of crushing stress, measured in lb. T = capacity of joint per pitch width to resist tearing of plate between the last or outside row of rivets with a given intensity of tensile stress, measured in lb. U = capacity of unpunched plate per pitch width to resist tearing with a given intensity of tensile stress, measured in lb.

$$\text{Then: } S = \pi d^2(nf_s + 2mf'_s)/4; \quad C = dt(nf_c + mf'_c); \quad T = (p - d)tf_t;$$
$$U = p \times t \times f_t;$$
$$\text{and } E_s = \text{efficiency in shearing} = S/U = \pi d^2(nf_s + 2mf'_s)/4ptf_t$$
$$E_c = \text{efficiency in crushing} = C/U = d(nf_c + mf'_c)/pf_t$$
$$E_t = \text{efficiency in tearing} = T/U = (p - d)/p$$

The above equations may be used to determine the **efficiency** of any form of riveted joint of known dimensions. When designing, the most economic relations between the pitch and diameter of rivet and the thickness of plate obtain when the three efficiencies are equal. If then these be equated and solved for p, d and t, it will be found that, for the **longitudinal seams** of boilers and tanks,

$$t = PD(nf_c + mf'_c + f_t)/2(nf_c + mf'_c)f_t$$
$$d = 4(nf_c + mf'_c)t/\pi(nf_s + 2mf'_s)$$
$$p = [\pi d^2(nf_s + 2mf'_s)/4tf_t] + d$$

For iron plates and rivets, $f_s = 38{,}000$, $f_c = 65{,}000$, $f'_s = 35{,}500$, $f'_c = 80{,}000$; for steel plates and rivets, $f_s = 44{,}000$, $f_c = 90{,}000$, $f'_s = 45{,}000$, $f'_c = 110{,}000$; $f_t = 40{,}000$ for iron and 55,000 for steel.

For the **girth seam** of single- or double-riveted lap joints, since the thickness of plate and diameter of rivets are established (all holes being of the same diameter for economy in fabrication), the pitch is determined such that a sufficient number of rivets may be placed to take up the total load on the joint with the proper limits of stress in shear and compression. The total load on the rivets in a girth seam is equal to $\pi D^2P/4$. The equations for t, d and p will be found to give proportions insuring steam-tightness up to pressures of 200 lb. per sq. in. when joints are properly calked.

The following **general practice rules** will also be found to be serviceable:

1. Butt joints are to be preferred to lap joints, since the latter occasion severe bending stresses in the rivet, which is a most frequent cause for failure.

2. The distance from the center line of the row of rivet holes nearest the edge of plate to edge of plate should be from 1½ to 2 times rivet diam.

3. The rivet diameter should be $d = 1.2\sqrt{t}$ to $1.4\sqrt{t}$.

4. In multiple-riveted joints the minimum distance between rows of rivets is 1.7d or 0.6 to 0.8 pitch for staggered riveting. For chain riveting this should be at least 2d and preferably 2½d.

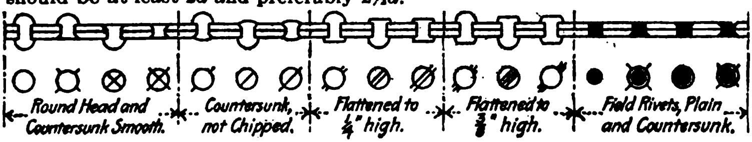


FIG. 38.—Conventional Signs for Rivets.

Materials Specifications for Rivets and Plates. See pp. 459–461, 866, 868. **A factor of safety** of from 4½ to 5 is usually employed for boiler joints.

Conventional Signs to indicate the form of the head to be used and whether the rivet is to be driven in the shop or the field at the time of erection, are given in Fig. 38. The **lengths of rivets for various grips** are given in Table 34 (see also Fig. 39).

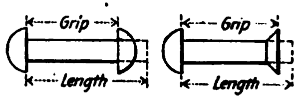


Fig. 39.

Table 34. Lengths of Rivets for Various Grips for Boilers

Grip in in. (See Fig. 39)	ROUND-HEAD RIVETS, Diam. in in. ½	⅝	¾	⅞	1	COUNTERSUNK-HEAD RIVETS, Diam. in in. ½	⅝	¾	⅞	1
	Length in in.					Length in in.				
1	2	2¼	2⅜	2½	2⅝	1⅝	1¾	1¾	1⅞	1⅞
1¼	2¼	2½	2⅝	2¾	2⅞	1⅞	2	2	2⅛	2⅛
½	2⅝	2⅞	3	3⅛	3¼	2⅛	2¼	2⅜	2⅜	2½
1¾	2⅞	3⅛	3¼	3⅜	3½	2⅜	2½	2⅝	2⅝	2¾
2	3⅛	3⅜	3½	3⅝	3¾	2⅝	2¾	2⅞	2⅞	3
2¼	3⅜	3⅝	3¾	3⅞	4	2⅞	3	3⅛	3⅛	3¼
2½	3⅝	3⅞	4	4⅛	4¼	3⅛	3¼	3⅜	3⅜	3½
2¾	3⅞	4⅛	4¼	4⅜	4½	3⅜	3½	3⅝	3⅝	3¾
3	4¼	4½	4⅝	4¾	4⅞	3¾	3⅞	3⅞	4	4⅛
3¼	4½	4¾	4⅞	5	5⅛	4	4⅛	4⅛	4¼	4⅜
3½	4¾	5	5⅛	5¼	5⅜	4¼	4⅜	4⅜	4½	4⅜
3¾	5	5¼	5⅜	5½	5⅝	4½	4⅝	4⅝	4¾	4⅞
4	5¼	5½	5⅝	5¾	5⅞	4¾	4⅞	4⅞	5	5⅛
4½	5¾	6	6¼	6⅜	6½	5¼	5⅜	5¼	5½	5⅝
5	6⅜	6⅝	6¾	6⅞	7	5⅞	6	6	6	6⅛

For **structural riveting,** see p. 1286.

Punched vs. Drilled Plates. Holes in plates forming parts of riveted structures are either punched, punched and reamed, or drilled. Punching, while cheaper, is objectionable. Further, the holes in different plates cannot be spaced with sufficient accuracy to register perfectly on being assembled. If the hole is punched out say 1/16 in. smaller than is required and then reamed to size, the metal injured by cold flow during punching will be removed. Annealing after punching also largely obviates the injury. Drilling, while more expensive, permits of greater accuracy and does not injure the metal. The Mass. Boiler Inspection Law states: "Rivet holes, excepting for attaching stays or angle bars to heads, shall be drilled full size with plates, butt straps, and heads bolted up in position; or they may be punched not to exceed ¼ in. less than full size for plates over 5/16 in. in thickness, and ⅛ in. less than full size for plates not exceeding 5/16 in. in thickness, and then drilled or reamed to full size with plates, butt straps and heads bolted up in position." The U. S. Navy Department specifies that all holes in boiler plates must be drilled with the plates in place. In structural work the holes are generally punched for shop riveting, while for field riveting it is usual to drill them to a template, or, if punched, to ream them with the parts to be connected bolted in place.

The **tensile strength** of the metal between holes drilled in a plate has been found to be greater than that of an undrilled plate. For the spacings usual in riveted work the increase in strength as shown by experiment is from 10 to 12 per cent. Punching, on the other hand, results in a loss of tensile strength, experiments showing the strength of the metal between the holes to be from 5 to 20 per cent. under that of unpunched plates in the case of iron, and from 8 to 35 per cent. in the case of steel plates. With the latter the loss increases with the thickness of the plate.

Hand-riveted joints are of practically the same strength as those machine-riveted, but the load at which visible slip occurs is much greater in the case of the latter. In hot riveting, pressures up to 150,000 lb. per sq. in. and even higher are used, and in cold riveting the pressure required is about 300,000 lb. per sq. in. of rivet section. Where the pressure exceeds 225,000 to 270,000 lb., however, there is danger that the lateral pressure of the rivet may crack the plate.

Frictional Resistance of Riveted Joints. Rivets in cooling contract longitudinally and draw the plates together with considerable force. They also contract laterally and therefore do not completely fill their holes when cold. Before shearing can take place it is consequently necessary that the plates shall slip on each other, such slipping, however, being resisted by the friction of the surfaces in contact. According to C. Bach, this frictional resistance when slipping begins ranges from 14,000 to 30,000 lb. per sq. in. of rivet section at each pair of surfaces in contact. As any appreciable slip of a boiler joint will result in leakage, it is the practice of European engineers to design such joints according to rules based by Bach on the resistance to slipping. The proportions specified in these rules, however, do not differ greatly from those based on a consideration of shearing strength.

KEYS, COTTERS AND PINS

Keys for fastening gears, pulleys, cranks and other pieces to round shafting may be made to the following forms and dimensions depending on the service conditions under which they are used.

Woodruff Keys (Fig. 40) are made by the Whitney Mfg. Co., of Hartford, Conn., in the sizes given in Table 35, and for use with shafts not larger than 2¼ in. in diameter. Cutters for milling out the key seats, as well as special machines for using the cutters, are to be had from the manufacturer. Where the hub of the gear or pulley is relatively long, two keys should be used. Slightly rounding the corners or ends of these keys will obviate any difficulty met with in removing pulleys from shafts. They are not to be used as sliding keys or feathers.

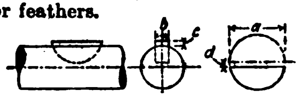


FIG. 40.—Woodruff Key.

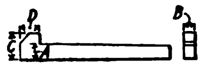


FIG. 41.—Gib-head Key.

Gib-head Keys (Fig. 41). This form of taper key is necessary when the smaller end is inaccessible for drifting out, and the larger end is accessible. It is successfully used with all sizes of shafts. Proportions are given in Table 36.

Sunk Keys are made by the Pratt & Whitney Co., of Hartford, Conn., to the form and dimensions given in Fig. 42 and Table 37. These keys are particularly adapted to the case of hubs fitting adjacent parts such that neither end of the key is accessible. The difficulties attending the sinking of keyways for these keys have militated against their frequent use. The Pratt & Whitney Co., however, has devised a successful spline miller which greatly facilitates this ordinarily troublesome operation.

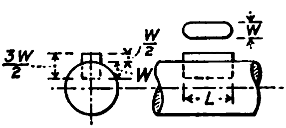


FIG. 42.—Sunk Key.

Feather Keys are used to prevent parts from turning on a shaft while allowing them to move in a lengthwise direction. They may be of the forms shown in Fig. 43, with dimensions as given in Table 37.

Table 35. Dimensions of Standard Woodruff Keys

(All dimensions in in. Letters refer to Fig. 40. d = distance from center of stock from which key is made to top of key)

No. of key	Diam. of key *a*	Thick-ness of key *b*	Depth of key-way *c*	*d*	No. of key	Diam. of key *a*	Thick-ness of key *b*	Depth of key-way *c*	*d*
1	1/2	1/16	1/32	3/64	19	1 1/4	3/16	3/32	5/64
2	1/2	3/32	3/64	3/64	20	1 1/4	7/32	7/64	5/64
3	1/2	1/8	1/16	3/64	21	1 1/4	1/4	1/8	5/64
4	5/8	3/32	3/64	1/16	*D*	1 1/4	5/16	5/32	5/64
5	5/8	1/8	1/16	1/16	*E*	1 1/4	3/8	3/16	5/64
6	5/8	5/32	5/64	1/16	22	1 3/8	1/4	1/8	3/32
7	3/4	1/8	1/16	1/16	23	1 3/8	5/16	5/32	3/32
8	3/4	5/32	5/64	1/16	*F*	1 3/8	3/8	3/16	3/32
9	3/4	3/16	3/32	1/16	24	1 1/2	1/4	1/8	7/64
10	7/8	5/32	5/64	1/16	25	1 1/2	5/16	5/32	7/64
11	7/8	3/16	3/32	1/16	*G*	1 1/2	3/8	3/16	7/64
12	7/8	7/32	7/64	1/16	26	2 1/8	3/16	3/32	17/32
A	7/8	1/4	1/8	1/16	27	2 1/8	1/4	1/8	17/32
13	1	3/16	3/32	1/16	28	2 1/8	5/16	5/32	17/32
14	1	7/32	7/64	1/16	29	2 1/8	3/8	3/16	17/32
15	1	1/4	1/8	1/16	30	3 1/2	3/8	3/16	13/16
B	1	5/16	5/32	1/16	31	3 1/2	7/16	7/32	13/16
16	1 1/8	3/16	3/32	5/64	32	3 1/2	1/2	1/4	13/16
17	1 1/8	7/32	7/64	5/64	33	3 1/2	9/16	9/32	13/16
18	1 1/8	1/4	1/8	5/64	34	3 1/2	5/8	5/16	13/16
C	1 1/8	5/16	5/32	5/64	....	....	....	....	....

STANDARD WOODRUFF KEYS TO USE WITH VARIOUS DIAMETERS OF SHAFTS

Diam. of shaft	Numbers of keys	Diam. of shaft	Numbers of keys	Diam. of shaft	Numbers of keys
5/16–3/8	1	7/8–15/16	6, 8, 10	1 3/8–1 7/16	14, 17, 20
7/16–1/2	2, 4	1	9, 11, 13	1 1/2–1 5/8	15, 18, 21, 24
9/16–5/8	3, 5	1 1/16–1 1/8	9, 11, 13, 16	1 11/16–1 3/4	18, 21, 24
11/16–3/4	3, 5, 7	1 3/16	11, 13, 16	1 13/16–2	23, 25
13/16	6, 8	1 1/4–1 5/16	12, 14, 17, 20	2 1/16–2 1/2	25

Table 36. Dimensions of Gib-head Keys

(All dimensions in in. Letters refer to Fig. 41)

A	*B*	*C*	*D*	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
1/8	1/8	1/4	7/32	1 1/8	1 1/8	1 7/8	1 5/16	2 1/8	2 1/8	4	2 1/2
3/16	3/16	5/16	9/32	1 3/16	1 3/16	1 15/16	1 3/8	2 3/16	2 3/16	4 1/8	2 9/16
1/4	1/4	15/32	11/32	1 1/4	1 1/4	2	1 7/16	2 1/4	2 1/4	4 1/4	2 5/8
5/16	5/16	9/16	13/32	1 5/16	1 5/16	2 1/8	1 1/2	2 5/16	2 5/16	4 3/8	2 11/16
3/8	3/8	11/16	15/32	1 3/8	1 3/8	2 1/4	1 9/16	2 3/8	2 3/8	4 1/2	2 3/4
7/16	7/16	3/4	17/32	1 7/16	1 7/16	2 3/8	1 5/8	2 7/16	2 7/16	4 5/8	2 13/16
1/2	1/2	7/8	19/32	1 1/2	1 1/2	2 1/2	1 3/4	2 1/2	2 1/2	4 3/4	2 7/8
9/16	9/16	1	21/32	1 9/16	1 9/16	2 5/8	1 13/16	2 9/16	2 9/16	4 7/8	2 15/16
5/8	5/8	1 1/8	23/32	1 5/8	1 5/8	2 3/4	1 7/8	2 5/8	2 5/8	5	3
11/16	11/16	1 3/16	25/32	1 11/16	1 11/16	2 7/8	1 15/16	2 11/16	2 11/16	5	3 1/16
3/4	3/4	1 1/4	7/8	1 3/4	1 3/4	3	2	2 3/4	2 3/4	5 1/8	3 1/8
13/16	13/16	1 5/16	15/16	1 13/16	1 13/16	3 1/8	2 1/16	2 13/16	2 13/16	5 1/8	3 3/16
7/8	7/8	1 1/2	1	1 7/8	1 7/8	3 3/8	2 1/8	2 7/8	2 7/8	5 1/4	3 1/4
15/16	15/16	1 5/8	1 1/16	1 15/16	1 15/16	3 5/8	2 3/16	2 15/16	2 15/16	5 1/4	3 5/16
1	1	1 3/4	1 1/8	2	2	3 3/4	2 1/4	3	3	5 3/8	3 1/2
1 1/16	1 1/16	1 13/16	1 3/16	2 1/16	2 1/16	3 7/8	2 7/16				

Table 37. Dimensions of Sunk Keys
(All dimensions in in. Letters refer to Fig. 42)

Key no.	L	W	Key no.	L	W	Key no.	L	W	Key no.	L	W
1	1/2	1/16	13	1	3/16	22	1 3/8	1/4	54	2 1/4	1/4
2	1/2	3/32	14	1	7/32	23	1 3/8	5/16	55	2 1/4	5/16
3	1/2	1/8	15	1	1/4	F	1 3/8	3/8	56	2 1/4	3/8
4	5/8	3/32	B	1	5/16	24	1 1/2	1/4	57	2 1/4	7/16
5	5/8	1/8	16	1 1/8	3/16	25	1 1/2	5/16	58	2 1/2	5/16
6	5/8	5/32	17	1 1/8	7/32	G	1 1/2	3/8	59	2 1/2	3/8
7	3/4	1/8	18	1 1/8	1/4	51	1 3/4	1/4	60	2 1/2	7/16
8	3/4	5/32	C	1 1/8	5/16	52	1 3/4	5/16	61	2 1/2	1/2
9	3/4	3/16	19	1 1/4	3/16	53	1 3/4	3/8	30	3	3/8
10	7/8	5/32	20	1 1/4	7/32	26	2	3/16	31	3	7/16
11	7/8	3/16	21	1 1/4	1/4	27	2	1/4	32	3	1/2
12	7/8	7/32	D	1 1/4	5/16	28	2	5/16	33	3	9/16
A	7/8	1/4	E	1 1/4	3/8	29	2	3/8	34	3	5/8

Saddle Keys and Flat Keys are used only for the transmission of small torques or turning efforts that are never liable to sudden changes in magnitude. Under excessive stresses they turn around the shaft and damage it. They may be made to the forms shown in Fig. 44.

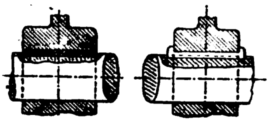


FIG. 43.—Feather Keys.

Saddle Key. Flat Key.

FIG. 44.

In **transmitting large torques** it is customary to use two or more keys as shown in Fig. 45. The arrangement shown at (*a*) permits more ready cutting of the keyway. If but one key is used with the arrangement shown at (*b*), torque can only be taken in one direction.

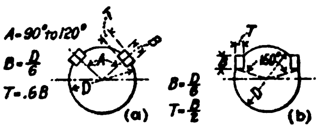


FIG. 45.—Double-keying of Shafts.

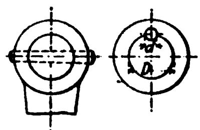


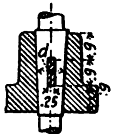
FIG. 46.—Taper Pins.

Taper Pins are sometimes used in transmitting very small torques, and may be placed in either of the ways shown in Fig. 46. They should be fitted so that the parts are drawn together when the pin is driven home to prevent their working loose. The Morse Twist Drill and Machine Co. furnishes taper pins in the sizes given in Table 38.

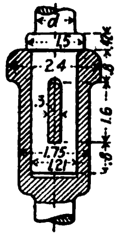
Table 38. Morse Standard Taper Pins
(Taper, 1/4 in. per ft. Lengths increase by 1/4 in.)

Size number	0	1	2	3	4	5	6	7	8	9	10
Diam. at large end, in.	0.156	0.172	0.193	0.219	0.250	0.289	0.341	0.409	0.492	0.591	0.70[illegible]
Do., approx.	5/32	11/64	3/16	7/32	1/4	19/64	11/32	13/32	1/2	19/32	23/32
Length	3/4-1 1/4	3/4-2	3/4-2 1/4	3/4-3	3/4-3	3/4-3	3/4-4	1-4	1 1/4-4 1/4	1 1/4-5 1/4	1 1/2-[illegible]

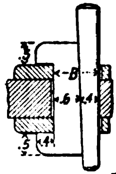
Cottered Joints may be employed for fastening rods to other rods, rods to pistons and cross heads, yokes to rods (as in the case of connecting rods) and for services of similar kinds. Some forms of such joints and proportions recommended are shown in Figs. 47–50, inclusive.

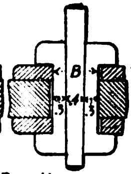


d = unit
FIG. 47.



d = unit
FIG. 48.





B = unit
FIG. 49. FIG. 50.

Cottered Joints.

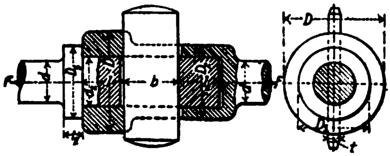


FIG. 51.—Cottered Joint.

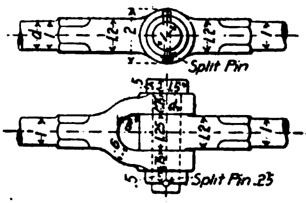


FIG. 52.—Knuckle Joint.

An analysis of the straining action in cotter joints which will serve as a guide in the determination of proportions for such joints in general, may be made with reference to the type of joint shown in Fig. 51. The joint may fail in any one of the following ways and the relation between load and stress is as specified. Let F = load on rod, lb.; d = diam. of rod, in.; f_t, f_s and f_c the tensile, shearing and crushing stresses in lb. per sq. in. Then (for wrought-iron rods),

1. Failure of rod by tension: $F = (\pi d^2/4)f_t$ (1)
2. Tearing of rod end across the cotter hole: $F = [(\pi d_2^2/4) - d_2t]f_t$ (2)
   Equating (1) and (2) and letting $t = d_2/4$, then $d_2 = 1.21d$ and $t = 0.3d$.
3. Tearing of socket across the hole: $F = [\pi/4(D_2^2 - d_2^2) - (D_2 - d_2)t]f_t$, whence (allowing for clearance), $D_2 = 1.75d$.
4. Double shearing of cotter: $F = 2btf_s$, whence
   $b = d_2(f_t/f_s) = 1.6d$ (approx.) for iron and steel.
5. Crushing the cotter bearing surface in rod end: $F = d_2tf_c$.
6. Crushing the cotter bearing surface in socket: $F = t(D - d_2)f_c$.
7. Shearing socket end: $F = 2[l(D - d_2)f_s]$. $(D = 2d_2.)$
8. Shearing rod end: $F = 2l_2d_2f_s$. (Let $f'_s = \frac{1}{2}f_s$ because of grain.)
   Letting $l = l_2$, then $l = 0.65d$, say, $l = 0.75d$.
9. Crushing collar on rod: $F = (\pi/4)(D_3^2 - d_2^2)f_c$, whence $D_3 = 1.4d$, say, $1.5d$.
10. Shearing collar off rod: $F = \pi d_2t_2f_s$, whence $t_2 = 0.42d$, say, $0.5d$.

When two rods are to be joined so as to permit movement at the joint, a round pin is used in place of a cotter. In such cases the proportions may be as shown in Fig. 52.

PRESS AND SHRINK FITS

Cranks, wheels, armature spiders and other parts are frequently secured to shafts and axles by pressing or shrinking them on. When shrink fits are employed the shaft and hole are usually cylindrical; with press fits the best results are obtained by slightly tapering one or both (from 1/16 to 3/16 in. per ft.), which permits the effective use of a lubricant. Experiments have shown that shrink fits resist torsion and axial load better than press fits.

Allowances for Shrink Fits. Practice differs widely in the allowances prescribed for both press and shrink fits. For shrink fits in iron or steel, the New York Shipbuilding Co. (Cathcart, "Machine Design," p. 24) allows 0.001 in. per inch of shaft diameter plus 0.001 in., and in large fits employs a taper of 1/16 in. per ft. for both shaft and hole. If the conditions are such that it is more convenient to ream the hole with a standard parallel reamer, the hole is tapered 0.0005 in. per inch of length unless the fit is so long that this taper will reduce the allowance at the small end of the hole to less than one-half that at the large end.

The Russell Engine Co. uses the following allowances in shrinking cast-iron cranks on to steel shafts:

Diam. of shaft, in.	4–5	5–7½	7½–9	10–12	12–16	16–18
Allowance, in.	0.0045	0.0030	0.0027	0.0025	0.0020	0.0015

A large railway company employs the following allowances in shrinking steel tires on to locomotive wheels:

Diam. of wheel center, in.	38	44	50	56	62	66
Allowance, in.	1/32	3/64	1/16	1/16	5/64	5/64

S. H. Moore (*Trans. A. S. M. E.*, vol. 24) gives the following formula for the allowance to be made for shrink fits:

Allowance in in. = $[(17D/16) + 0.5]/1000$, where D = shaft diam., in.

The Brown & Sharpe Mfg. Co. (Viall. *Trans. A. S. M. E.*, vol. 32, p. 1328) have the following grinding limits for shrink fits:

	Fits for shells, etc., having a thickness of more than 3/8 in.	Fits for hardened shells 3/8 in. thick and less
	Large	Large
To ½ in. diam., inc.	0.0005–0.001	0.00025–0.0005
To 1 in. diam., inc.	0.001 –0.0025	0.0005 –0.001
To 2 in. diam., inc.	0.0025–0.0035	0.001 –0.0015
To 3½ in. diam., inc.	0.0035–0.005	0.0015 –0.002
To 6 in. diam., inc.	0.005 –0.007	0.002 –0.003

The practice of the General Electric Co. for shrink fits (John Riddell, *Trans. A. S. M. E.*, vol. 24) is given in Table 38.

Table 38. Allowances for Shrink Fits
(General Electric Co.)

Diam., in.	Allowance, in.	Diam., in.	Allowance, in.	Diam., in.	Allowance, in.	Diam., in.	Allowance, in.
2	0.0015	18	0.0075	34	0.012	60	0.020
4	0.00275	20	0.008	36	0.01275	72	0.024
6	0.0035	22	0.00875	38	0.01325	84	0.027
8	0.0045	24	0.00925	40	0.01375	96	0.030
10	0.00525	26	0.00975	42	0.01425	108	0.033
12	0.00575	28	0.0125	44	0.015	120	0.0355
14	0.0065	30	0.011	46	0.0155	132	0.038
16	0.0070	32	0.0115	48	0.016	144	0.040

Allowances for Press Fits (Forced Fits). The New York Shipbuilding Co. uses the same allowances for press fits as for shrink fits (*q.v.*). For cast-iron cranks on steel shafts, the Russell Engine Co. employs the following allowances:

Diam. of shaft, in.	4–5	5–7½	7½–9	10–12	12–16	16–18
Allowance, in.	0.003	0.004	0.005	0.0055	0.006	0.009

S. H. Moore (*Trans. A. S. M. E.*, vol. 24) states that the allowance for press fits may be obtained from the formula $d = (2D + 0.5)/1000$ where d = allowance and D = shaft diam., both in inches.

For parallel shafts and holes the C. W. Hunt Co. finishes the shafts to from $0.001d$ to $0.001d + 0.001$ in. over size, and reams the holes with reamers kept between the following limits:

Diam. of shaft (D), exact size, in.	1 to 3	4 to 6	7 to 10
Reamers to be under size from	0.000 in. to 0.002 in.	0.003 in.	0.004 in.

For taper fits the shafts are kept to standard size (+ or −0.001 in.) and the holes are tapered 1/16 in. to the foot, the large end of the hole after reaming being small by from $(D + 4) \times 0.001$ in. to $(D + 5) \times 0.001$ in. That is, for a 3-in. fit the hole should be from 0.007 to 0.008 in. smaller than the nominal shaft diameter.

The Brown & Sharpe Mfg. Co. (Viall. *Trans. A. S. M. E.*, vol. 32, p. 1328) have the following grinding limits for press fits:

	Large
To ½ in. diam., inc.	0.00075–0.0015
To 1 in. diam., inc.	0.0015 –0.0025
To 2 in. diam., inc.	0.0025 –0.004
To 3½ in. diam., inc.	0.004 –0.006
To 6 in. diam., inc.	0.006 –0.009

The practice of the General Electric Co. for press fits is given in Table 39. The allowances for armature spiders are smaller than usual in work of this nature, as the limited facilities for assembling in the field are taken into consideration. For heavy parts, such as couplings, the values given in Table 39 are used.

Table 39. Allowances for Press Fits
(General Electric Co.)

A—Tight fits for parts with light hubs, as commutator shells. *B*—Press fits for solid steel armature spiders. *C*—Press fits for solid cast-iron armature spiders.

Shaft diam., in.	Allowances, in.			Shaft diam., in.	Allowances, in.			Shaft diam., in.	Allowances, in.		
	A	*B*	*C*		*A*	*B*	*C*		*A*	*B*	*C*
2	0.0005	0.0005	0.0005	18	0.0012	0.00225	0.00375	34	0.0017	0.003	0.006
4	0.00075	0.00075	0.00075	20	0.0012	0.00225	0.004	36	0.0017	0.00325	0.00625
6	0.001	0.00125	0.00125	22	0.0012	0.0025	0.00425	38	0.0017	0.00325	0.0065
8	0.001	0.0015	0.00175	24	0.0015	0.0025	0.0045	40	0.002	0.0035	0.00675
10	0.001	0.0015	0.002	26	0.0015	0.00275	0.00475	42	0.002	0.0035	0.007
12	0.001	0.00175	0.0025	28	0.0015	0.00275	0.005	44	0.002	0.00375	0.00725
14	0.001	0.002	0.003	30	0.0015	0.00275	0.0055	46	0.002	0.004	0.00775
16	0.0012	0.002	0.00325	32	0.0017	0.003	0.00575	48	0.00225	0.004	0.008

C. F. MacGill (*Jour. A. S. M. E.*, Nov., 1913) states as a result of 20 years experience that in making press fits it is not necessary to increase the allowance with the diameter of the shaft, as the increased surface area of the fit adds sufficient friction to bring the pressure up to the required tonnage; that an allowance of from 0.002 in. to 0.004 in. on steel shafts pressed into steel hubs, and an allowance of from 0.003 in. to 005 in. on steel shafts pressed into cast-iron hubs of ordinary hardness will give good results; and that allowances greater than 0.006 in. on steel shafts pressed into cast-iron hubs serve no useful purpose, but, on the contrary, tend to set up strains injurious to the castings.

Allowances for Drive Fits. Allowance in in. = $(\frac{1}{2}d + 0.5)/1000$ (S. H. Moore, *Trans. A. S. M. E.*, vol. 24), where d = shaft diam., in. For parallel shafts and holes the C. W. Hunt Co. finishes the shafts from $0.0005d$ to $0.0005d + 0.001$ in. over size, and the holes as for press fits (see above). For taper fits the holes are tapered as for press fits, the large end of the hole after reaming being smaller than the shaft diameter by from $0.5d \times 0.001$ in. to $(0.5d + 1) \times 0.001$ in.

The Brown & Sharpe Mfg. Co. (Viall. *Trans. A. S. M. E.*, vol. 32, p. 1328) have the following grinding limits for driving fits:

	For such pieces as are required to be readily taken apart	For other driving fits
	Large	Large
To ½ in. diam., inc.....	Standard-0.00025	0.0005-0.001
To 1 in. diam., inc.....	0.00025-0.0005	0.001 -0.002
To 2 in. diam., inc.....	0.0005 -0.00075	0.002 -0.003
To 3½ in. diam., inc.....	0.00075-0.001	0.003 -0.004
To 6 in. diam., inc.....	0.001 -0.0015	0.004 -0.005

Stresses in Hubs Due to Press and Shrink Fits on Steel Shafts. Maximum fiber stress in the hub (lb. per sq. in.) = 10,000 Kd, where d = allowance in in. per in. of diam. of hole, and K = constant having the following values derived by S. H. Weaver from a formula given by Professor Morley in *Engg.*, Aug. 11, 1911 (m = thickness of hub ÷ diam. of bore, usually from 0.3 to 0.5):

For m =	0.0	0.1	0.2	0.3	0.4	0.5	1.0	1.5	2	∞
K(steel hub) =	2790	2400	2110	1900	1820	1750	1550	1480	1440	1390
K(cast-iron hub) =	1400	1250	1150	1070	1030	1000	917	889	867	846

From a consideration of Lamé's formula for hoop stresses, A. L. Jenkins (*Eng. News*, Mar. 17, 1910) deduces the following expressions for radial unit pressure between surfaces (S) and unit tensile or hoop stress in the hub (S_1), both in lb. per sq. in.:

$S = 15{,}000{,}000d/(K + 0.6)$ for cast-iron hub on steel shaft
$S_1 = 15{,}000{,}000d/[1 + (0.6/K)]$ for cast-iron hub on steel shaft
$S = 30{,}000{,}000d/(1 + K)$ for steel hub on steel shaft
$S_1 = 30{,}000{,}000d/[1 + (1/K)]$ for steel hub on steel shaft

d = allowance in in. per in. of shaft diam., and K = constant depending on the ratio of hub thickness (t) to radius of shaft (r), as follows:

for t/r =	0.4	0.5	0.6	0.7	0.8	1.0	1.25	1.5	2.0	3.0
K =	3.08	2.60	2.28	2.06	1.89	1.67	1.49	1.38	1.25	1.13

Pressures Required in Making Press Fits. The pressure required for crank and crank-pin press fits may be computed by the following formulæ (*Am. Mach.*, Mar. 7, 1907): For crank fits with diam. D = 2 to 10 in., pressure in tons = $P = 9.9D - 14$; for D = 12 to 24 in., $P = 5D + 40$. For straight crank pins, $P = 13D$, and for tapered crank pins, $P = 14D - 7$.

According to Sanford A. Moss (*Jour. A. S. M. E.*, Sept., 1912), the average force in lb. required to press a hub on a shaft, the press fit being 0.001 in. per in. of bore diam., is $0.12dlS$, in which l and d are respectively the length and diam. of bore in in. and S is the radial stress at the bore in lb. per sq. in.

S has the following approximate values depending on the ratio d/D, in which D is the outer diam. of the hub, and on the ratio d_1/d, in which d_1 is the diam. of the hole (if any) in the shaft:

	Values of d/D					
	0.1	0.2	0.4	0.6	0.8	1.0
d_1/d	Values of S					
0.0	14,400	13,900	12,200	9,300	5,200	0
0.2	13,800	13,300	11,700	9,000	5,000	0
0.3	13,000	12,700	11,200	8,800	4,900	0
0.5	10,800	10,500	9,400	7,700	4,700	0
0.7	7,300	7,100	6,700	5,800	4,400	0
0.9	2,800	2,700	2,600	2,400	2,000	0
1.0	0	0	0	0	0	0

SHAFTS, AXLES, CRANKS

(For critical speeds of shafts see p. 783)

Strength and Stiffness of Shafting. (See Table 18, p. 424, for torsion of shafts of various cross-sections.) Shafts may be subjected to torsion, to torsion and bending in varying proportions, or to bending alone. To establish the relations between these actions and the resulting stresses and deflections, let T = torque on shaft, in.-lb. d = diam. of solid circular shaft, in. d_1 = inside diam. of hollow circular shaft, in. d_2 = outside diam. of hollow circular shaft, in. f_t = maximum intensity of tensile stress at outer surface of shaft, lb. per sq. in. f_s = maximum intensity of shearing stress at outer surface of shaft, lb. per sq. in. h.p. = horse power transmitted by shaft. N = rev. per min. of shaft. I_p = polar moment of inertia of shaft cross-section in inches to the fourth power (in.^{4}). I = rectangular moment of inertia of shaft cross-section, in.^{4}. M = bending moment on shaft at any section, in.-lb. r = radius of shaft to outer surface, in. a = angle passed through by any radius of shaft section under the influence of any torque and length of shaft, deg. E_s = modulus of elasticity in shear, lb. per sq. in. l = length of shaft, in.

Then for the **strength of a solid circular shaft** subjected to torque,

$$T = f_s I_p/r = 63{,}024 \text{ h.p.}/N;\ d = 68.5\sqrt[3]{\text{h.p.}/Nf_s} = \sqrt[3]{5.1T/f_s}$$

For the **torsional stiffness of a solid circular shaft,**

$$d = 4.9\sqrt[4]{Tl/aE_s} = 77.66\sqrt[4]{\text{h.p.}l/NaE_s}$$

The angle a is usually limited to 0.1 deg. per ft. of length.

The **strength of hollow circular shafting** subjected to torque may be determined from

$$d_2 = \sqrt[3]{5.1T/(1 - n^4)f_s} = 68.5\sqrt[3]{\text{h.p.}/(1 - n^4)Nf_s},\ \text{where } n = d_1/d_2.$$

For **non-circular shafts,** see Table 18, p. 424.

Strength of Solid Circular Shafts Subjected to Torque and Bending

Let M_e = that bending moment which when acting alone on the shaft will produce the same maximum intensity of tensile stress (f_t) as that produced by the combined action of a bending moment (M) and torque (T). = equivalent bending moment.

T_e = that twisting moment which when acting alone on the shaft will produce the same maximum intensity of shearing stress (f_s) as that produced by the combined action of a bending moment (M) and torque (T). = equivalent twisting moment.

Then $$M_e = \tfrac{1}{2}(M + \sqrt{M^2 + T^2}).$$

If $$M = kT,\ M_e = \tfrac{1}{2}(k + \sqrt{k^2 + 1})T$$

and $$T_e = \sqrt{M^2 + T^2} = T\sqrt{k^2 + 1}$$

Hancock's tests of metals under combined stresses (*Proc. Am. Soc. T. M.*, 1908, p. 376) show that shafts should be designed for their strength in shear. He recommends the following expression for the equivalent twisting moment (T_e), in which a = elastic limit in tension and b = elastic limit in shear: $T_e = \sqrt{M^2 + (4a^2T^2/b^2)}$.

Then for **solid circular shafts**

$$d = \sqrt[3]{5.1(M + \sqrt{M^2 + T^2})/f_t} = \sqrt[3]{5.1(k + \sqrt{k^2 + 1})T/f_t} \qquad (1)$$

or

$$d = \sqrt[3]{5.1\sqrt{M^2 + T^2}/f_s} = \sqrt[3]{5.1\,T\sqrt{k^2 + 1}/f_s} \qquad (2)$$

Example. A 2½-in. round shaft carries a 30-in. pulley weighing 500 lb. The total pull of the belt is 2000 lb. and the unbalanced pull is 1500 lb. horizontally. Pulley is 6 in. from hanger.

Resultant force acting on shaft $= \sqrt{500^2 + 2000^2} = 2060$ lb. $M = 2060 \times 6 =$ 12,360 in.-lb. at hanger, $T = 1500 \times 15 = 22{,}500$ in.-lb. From (1), $f_t = (5.1/d^3)(M + \sqrt{M^2 + T^2}) = (5.1/15.625)(12{,}360 + \sqrt{12{,}360^2 + 22{,}500^2}) = 12{,}400$ lb. per sq. in. Similarly, from (2) $f_s = (5.1/15.625)\sqrt{12.360^2 + 22{,}500^2} = 8{,}400$ lb. per sq. in.

For ductile materials the ratio of the allowable maximum intensity of shearing stress (f_s) to the allowable maximum intensity of tensile stress (f_t) is such as to result in the correct proportioning of shafts being accomplished by the use of equation (2).

See paper by James J. Guest in *Phil. Mag.*, July, 1900.

For **hollow circular shafts,**

$$d = \sqrt[3]{5.1\ (k + \sqrt{k^2 + 1})T/(1 - n^4)f_t} \text{ for tension,}$$

or

$$d = \sqrt[3]{5.1\ (\sqrt{k^2 + 1})T/(1 - n^4)f_s} \text{ for shear.}$$

The **relative torsional strength of solid and hollow shafts** may be determined by means of Table 40.

Examples. What is the diameter of a solid shaft equal in strength to a 10-in. shaft having a 4-in. hole? $d_1/d_2 = 4/10 = 0.40$; for this ratio $D/d_2 = D/10 = 0.991$, from which $D = 9.91$ in.

Required the outside diameter of a hollow shaft with hole = 0.6 × outside diameter equal in strength to an 8-in. solid shaft. d_2/D(for $d_1/d_2 = 0.60$) $= 1.047 = d_2/8$, whence $d_2 = 8.38$ in.

Required the dimensions of a hollow shaft but ⅔ the weight of a 12-in. solid shaft of the same strength. Here w = ⅔ = 66.7 per cent., for which (from table) $d_1/d_2 = 0.64$ and $d_2/D = 1.063$; hence $d_2 = 12 \times 1.063 = 12.76$ in., and d_1(from $d_1/12.76 = 0.64$) = 8.16 in.

Table 40. Relative Strengths of Solid and Hollow Shafts

D = diam. of solid shaft, d_2 = outside diam. of hollow shaft of equal torsional strength, d_1 = inside diam. of hollow shaft of equal torsional strength, w = weight of hollow shaft in percentage of the weight of a solid shaft of equal torsional strength, $D = \sqrt[3]{(d_2^4 - d_1^4)/d_2}$.

$\frac{d_1}{d_2}$	$\frac{d_2}{D}$	$\frac{D}{d_2}$	w	$\frac{d_1}{d_2}$	$\frac{d_2}{D}$	$\frac{D}{d_2}$	w	$\frac{d_1}{d_2}$	$\frac{d_2}{D}$	$\frac{D}{d_2}$	w
0.40	1.009	0.991	85.4	0.58	1.041	0.961	72.0	0.76	1.145	0.873	55.8
0.42	1.011	0.990	84.0	0.60	1.047	0.955	70.0	0.78	1.167	0.857	53.7
0.44	1.013	0.988	82.6	0.62	1.055	0.948	68.6	0.80	1.192	0.838	51.5
0.46	1.015	0.985	81.4	0.64	1.063	0.941	66.7	0.82	1.221	0.819	49.0
0.48	1.018	0.982	79.7	0.66	1.073	0.932	65.0	0.84	1.255	0.797	46.7
0.50	1.022	0.979	78.3	0.68	1.084	0.923	63.4	0.86	1.298	0.771	44.3
0.52	1.026	0.975	76.6	0.70	1.096	0.913	61.3	0.88	1.350	0.741	41.6
0.54	1.030	0.971	75.1	0.72	1.110	0.901	59.6	0.90	1.427	0.701	38.7
0.56	1.035	0.966	73.5	0.74	1.126	0.887	57.7	......	......	......	......

Forces in Different Planes Acting on Shaft. Frequently the forces acting on shafts causing bending do not act in the same plane. The following procedure may be followed in determining the resultant bending moment at any section.

In Fig. 53 (*a*), let *OQ* represent a shaft on which are acting forces normal to the axis at the points *A*, *B*, and *C*, and in the directions shown. In (*b*) draw *OQ* at *any* angle (conveniently 45 deg.) and to scale as in orthographic projection. Compute respectively the bending moments at *A*, *B*, and *C* occasioned by the force acting at each point. Lay off *AP*, *BN* and *CM* representing to scale the bending moments at *A*, *B*, *C* and in the direction of the force producing each. To determine the resultant bending moment at *B*, for example, there is required the combination by the parallelogram of forces of the bending moment *BN* due to the force acting at *B*, the bending moment *BT* due to the force acting at *C*, and the bending moment *BR* due to the force acting at *A*. Thus, the resultant of *BN* and *BR* is *BS* and the resultant of *BS* and *BT* is *BU*. The resultant bending moment at *B* due to all forces acting on the shaft is therefore *BU*, and it acts in the direction in which *BU* is shown. A similar procedure at *C* and at *A* will result in finding the resultant moments at these points. The following example will illustrate the application of the above methods to a particular case.

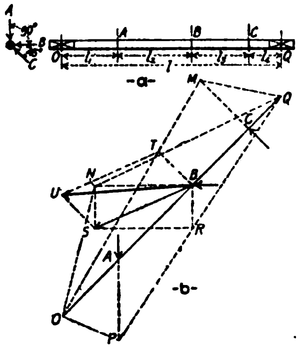


FIG. 53.

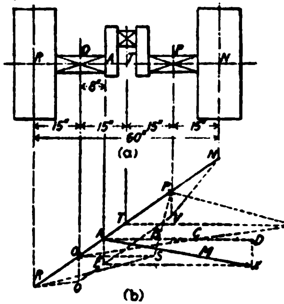


FIG. 54.

Let Fig. 54(*a*) represent the crank shaft of a 16 × 18-in. center-crank engine as made by the Atlas Engine Co., and carrying 100 lb. steam pressure. The flywheels *R* and *N* each weigh ¾ ton and *R* is assumed to carry the belt. The sum of the belt tensions for 50 per cent. overload is 2¼ tons. The worst straining action occurs when the belt leads off horizontally. Accordingly, the bending moments at *O* and *P* due to the flywheel weights will be *OQ* and *PV* (Fig. 54*b*), each equal to 11¼ in.-tons. The bending moment at *O* due to the belt pull will be *OS* and equal to 33¾ in.-tons. The bending moment due to the thrust of the connecting rod will be *TU* and equal to 75 in.-tons. Lay off these moments as shown in Fig. 54(*b*).

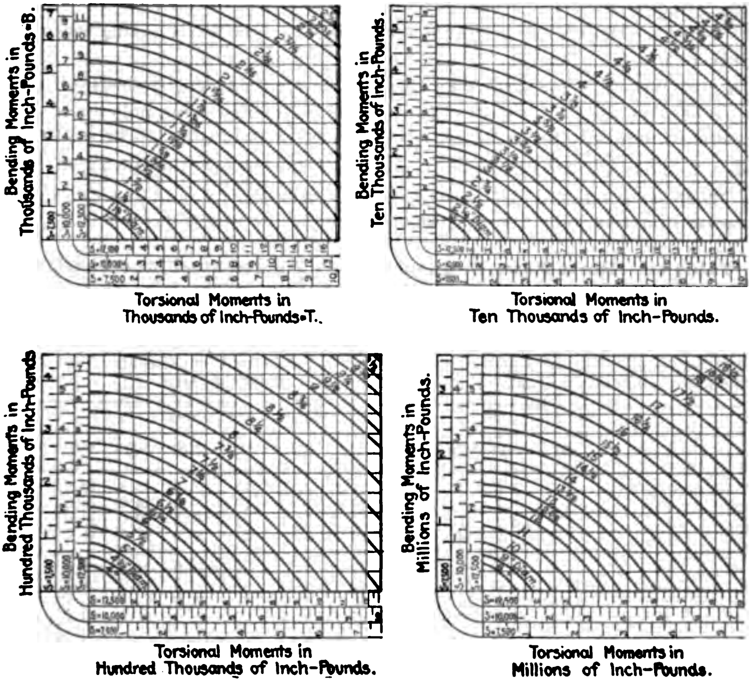
The torque in the shaft will exist in the length *RA*, and consequently it is necessary to determine the section in the length *RA* in which the greatest bending moment exists. This by inspection is seen to be at *A*. Therefore the resultant bending moment at *A* will be due to the bending moment *AB* caused by the belt pull, *AC* caused by the connecting-rod thrust and *AE* caused by the flywheel weight. Let *AD* equal *AB* plus *AC* and combine with *AE*, obtaining the resultant *AX* = *M*. It will be found that *M* scales 65 in.-tons. The torque on the shaft is 90 in.-tons, whence, for $f_t = 8000$,

$$d = \sqrt[3]{5.1\left(M + \sqrt{M^2 + T^2}\right)/8000} = 6\tfrac{1}{4}\text{ in.}$$

The diameter of the shaft is given in the catalog of the Atlas Engine Co. as 6½ in.

The crank pin is subjected to bending alone. The combined bending moments at *T* will give a resultant of 95 inch-tons, whence, $d = \sqrt[3]{10.2M/f_t} = 6\tfrac{1}{4}$ in.

Diameters of Shafts Subjected to Combined Torsional and Bending Stresses. Figs. 55–58 (*Machinery*, July, 1908) afford a convenient means of determining the diameters of shafts subjected both to bending and twisting. The diagrams are based on the formula $d = \sqrt[3]{(5.1/s)\,(M + \sqrt{M^2 + T^2})}$. For ductile materials it is well to check the value of d by means of the formula based on maximum intensity of shearing stress, namely, $d = \sqrt[3]{(5.1/s)\,\sqrt{M^2 + T^2}}$.; s = fiber stress in lb. per sq. in.



FIGS. 55–58.—Diagrams for Determining the Diameters of Shafts Subjected to Combined Torsional and Bending Stresses.

Practical Rules for Proportioning Mill Shafting. The impossibility of determining exactly the amount and position of the loads causing bending and twisting in shafting used for power transmission in mills and factories, as well as the probability that additional pulleys will be used at some time and changes required in the positions of pulleys due to the shifting of machines, make it necessary to depend upon empirical formulæ in determining sizes. Not only is **strength** to be considered but **stiffness** also, since through lack of proper stiffness much trouble may arise in the upkeep of bearings.

The **speed** of shafting in machine shops is about 150 r.p.m., with tendencies to higher values. In wood-working shops, speeds of 250 r.p.m. are usual. In textile mills speeds from 300 to 400 r.p.m. are common. The following rules are suggested by the Jones & Laughlin Steel Co. for determining suitable diameters of mill shafting.

For **shafts** subjected to torque alone, that is, **for power transmission over considerable distances between pulleys,** and for short countershafts having bearings not more than 8 ft. apart and pulleys close to bearings,

$d = \sqrt[3]{50 \text{ h.p.}/N}$ for turned shafting $= \sqrt[3]{40 \text{ h.p.}/N}$ for cold-rolled shafting, where h.p. = horse power to be transmitted, N = r.p.m., and d = shaft diam., in.

When the shaft is a first receiving shaft, that is, a line shaft having bearings, 8 ft. apart,

$d = \sqrt[3]{90 \text{ h.p.}/N}$ for turned shafting $= \sqrt[3]{70 \text{ h.p.}/N}$ for cold-rolled shafting.

When the shaft is a head shaft carrying the main driving pulley and distributing power to receiving or line shafts and is well supported by bearings,

$d = \sqrt[3]{125 \text{ h.p.}/N}$ for turned shafting $= \sqrt[3]{100 \text{ h.p.}/N}$ for cold-rolled shafting.

Table 41 is based on the formula $d = \sqrt[3]{90 \times \text{h.p.}/N}$, and affords a rapid means of determining the shaft diameter for any desired transmission.

Table 41. Horse Power Transmitted by Turned Steel Line Shafting

(Shafting well supported and with pulleys near to the bearings)

For cold-rolled line shafting (up to 5 in.) add 30 per cent. For head shafts of turned steel, subtract 30 per cent. For head shafts of cold-rolled steel, subtract 10 per cent. For transmission shafts (without pulleys) of turned steel, add 80 per cent. For transmission shafts (without pulleys) of cold-rolled steel, add 125 per cent.

Diam. of shaft, in.	Number of revolutions per minute										
	100	150	200	250	300	350	400	450	500	550	600
1½	3.7	5.6	7.5	9.4	11.2	13.1	15	16.9	18.8	20.5	22
1⅝	4.8	7.1	9.5	11.9	14.3	16.6	19	21.0	24.0	26.0	28
1¾	5.9	8.9	11.9	14.9	17.9	21.0	24	27.0	30.0	33.0	36
1⅞	7.3	11.0	14.7	18.3	22.0	26.0	29	33.0	37.0	41.0	44
2	8.9	13.3	17.8	22.0	27.0	31.0	35	40.0	44.0	48.0	53
2⅛	10.6	16.0	21.0	27.0	32.0	37.0	43	48.0	53.0	58.0	64
2¼	12.6	19.0	25.0	32.0	38.0	44.0	51	57.0	63.0	69.0	76
2⅜	14.9	22.0	30.0	37.0	45.0	52.0	60	67.0	74.0	81.0	89
2½	17.4	26.0	35.0	43.0	52.0	61.0	69	78.0	87.0	96.0	104
2⅝	20.0	30.0	40.0	50.0	60.0	71.0	80	90.0	100.0	110.0	120
2¾	23.0	35.0	46.0	58.0	69.0	81.0	92	104.0	115.0	125.0	138
2⅞	26.0	40.0	53.0	66.0	79.0	92.0	105	119.0	132.0	145.0	158
3	30.0	45.0	60.0	75.0	90.0	105.0	120	135.0	150.0	165.0	180
3⅛	34.0	51.0	68.0	85.0	102.0	119.0	136	152.0	170.0	187.0	203
3¼	38.0	57.0	76.0	95.0	114.0	134.0	153	172.0	191.0	210.0	229
3⅜	43.0	64.0	85.0	107.0	128.0	150.0	171	192.0	213.0	234.0	256
3½	48.0	72.0	95.0	119.0	143.0	167.0	190	214.0	238.0	262.0	286
3⅝	53.0	79.0	106.0	132.0	159.0	185.0	211	238.0	265.0	291.0	317
3¾	59.0	88.0	117.0	146.0	176.0	205.0	234	264.0	293.0	322.0	351
3⅞	65.0	97.0	129.0	161.0	194.0	226.0	258	291.0	322.0	354.0	387

Diam. of shaft, in.	Number of revolutions per minute										
	100	125	150	175	200	225	250	275	300	400	500
4	71.0	89.0	107.0	125.0	142.0	160.0	178	196.0	213.0	284.0	356
4¼	85.0	107.0	128.0	149.0	170.0	192.0	213	234.0	256.0	341.0	426
4½	102.0	127.0	152.0	178.0	203.0	228.0	253	278.0	305.0	405.0	507
4¾	119.0	149.0	179.0	209.0	238.0	268.0	298	328.0	357.0	476.0	595
5	139.0	174.0	208.0	244.0	278.0	313.0	347	382.0	417.0	557.0	695
5¼	161.0	201.0	242.0	281.0	322.0	362.0	403	443.0	483.0	644.0	805
5½	184.0	230.0	277.0	322.0	369.0	415.0	461	507.0	553.0	738.0	922
5¾	211.0	264.0	317.0	369.0	422.0	475.0	528	580.0	633.0	844.0	1055
6	240.0	300.0	360.0	419.0	480.0	540.0	600	660.0	720.0	960.0	1200
6¼	271.0	339.0	407.0	473.0	542.0	610.0	678	745.0	813.0	1084.0	1355
6½	305.0	382.0	459.0	535.0	611.0	687.0	764	840.0	917.0	1222.0	1528
6¾	341.0	427.0	513.0	598.0	682.0	767.0	853	938.0	1023.0	1364.0	1705
7	381.0	476.0	573.0	667.0	762.0	857.0	953	1048.0	1143.0	1524.0	1905
7¼	423.0	529.0	636.0	742.0	847.0	953.0	1059	1264.0	1270.0	1693.0	2116
7½	468.0	586.0	704.0	822.0	938.0	1055.0	1173	1290.0	1406.0	1875.0	2344
7¾	516.0	646.0	776.0	904.0	1033.0	1163.0	1293	1422.0	1550.0	2066.0	2583
8	568.0	712.0	855.0	998.0	1138.0	1280.0	1423	1565.0	1707.0	2275.0	2844
8½	681.0	853.0	1025.0	1197.0	1364.0	1535.0	1707	1878.0	2047.0	2728.0	3411
9	809.0	1013.0	1217.0	1421.0	1620.0	1823.0	2027	2230.0	2430.0	3240.0	......
9½	951.0	1191.0	1431.0	1671.0	1904.0	2143.0	2382	2620.0	2858.0	......	......
10	1111.0	1388.0	1666.0	1944.0	2222.0	2500.0	2778	3055.0	3333.0	......	......

Difficulty may be encountered in the use of **cold-rolled shafting** from the fact that its processes of manufacture being such that a state of stress exists in the shaft, any disturbance of this state by the cutting of keyways or wear in journals will tend to disalign the shafting and cause trouble in the operation of the bearings. The cost of upkeep to maintain satisfactory service may therefore more than offset the difference in installation cost.

Line shafting may be purchased from stock in lengths up to 24 ft., generally varying by 2 ft.

Shaft hangers should so be spaced that the deflection of the shaft will not exceed 0.01 in. per ft. of length to insure proper load distribution in the bearings. Accordingly, the Pencoyd Iron Works recommends the following rule for **spacing hangers:**

Let L = maximum distance in ft. between bearings for continuous shafting. Then $L = \sqrt[3]{873d^2}$ for bare shafting, $= \sqrt[3]{175d^2}$ for shafts carrying a fair proportion of pulleys placed near bearings.

In the use of any of the foregoing formulæ, the designer must have in mind the probability that more pulleys will be added and therefore more power transmitted as extensions are made to the mill.

Engine Shafts may be designed by the method given on p. 690. It is generally satisfactory, however, to employ certain empirical formulæ derived from data representing good practice. **Stationary steam-engine shafts** were investigated by Trooien, who found the average proportions to be as follows:

For slow-speed and Corliss engines, $d = 7.2\sqrt[3]{\text{h.p.}/N} - 0.3$ in.

For high-speed center-crank engines, $d = 6.6\sqrt[3]{\text{h.p.}/N}$.

For **marine-engine shafts,** see p. 1242. For **automobile crank shafts,** see p. 1194. **Gas-engine crank shafts** (see p. 1046) may be computed by the graphical method of p. 690, or by the methods given in Haeder and Hoskisson's "Handbook on the Gas Engine." In the latter, no account is taken of the belt pull, and hence for belted engines the results will be inaccurate.

Cranks of the Overhung Type may have the proportions given in Fig. 59, when made of wrought iron or steel, the stressing actions being as follows:

Bending at the section a-b: $Fl = tw^2f/6$; $t = 6Fl/w^2f$; $w = \sqrt{6Fl/tf}$;

Bending at the section c-e: $Fx = w_1t^2f/6$; $t = \sqrt{6Fx/w_1f}$; $w_1 = 6Fx/t^2f$;

where F = load on crank pin, lb.; l and x = lever arms, in. (see Fig. 59); w =

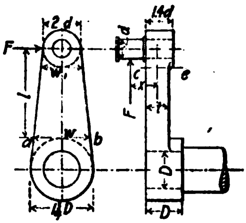


Fig. 59.—Overhung Crank.

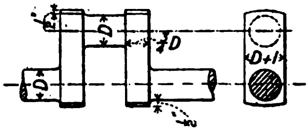


Fig. 60.—Center Crank.

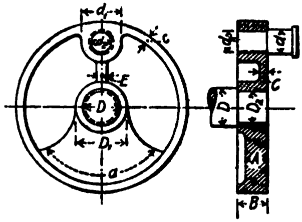


Fig. 61.—Crank Disk.

width of crank web near the shaft boss, in.; w_1 = width of crank web near pin boss, in.; t = web thickness, in.; f = allowable stress, lb. per sq. in. = 4500 to 6000 for wrought iron and steel. Both pin and shaft should be secured to the crank by press or shrink fits.

Center Cranks slotted from the solid may have the proportions shown in Fig. 60.

Crank Disks of cast iron (Fig. 61) may be designed with the following proportions:

$D_2 = 0.8D$	$B = \frac{3}{4}D$	$A = \frac{5}{8}D$	$C = \frac{1}{4}D$
$E = \frac{1}{2}D$	$d_1 = 2d$	$D_1 = 1\frac{3}{4}D$	$d_2 = 0.8d$

The arc dimension a varies with the amount of counterweight to be provided.

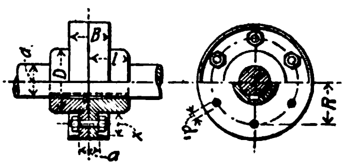


Fig. 62.—Flange Coupling.

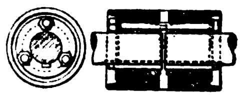


Fig. 63.—Sellers Coupling.

COUPLINGS AND CLUTCHES

Flange Couplings may be made as shown in Fig. 62. The proportions are in terms of $k = d + \frac{1}{2}$ in., where d = shaft diam. in in.

$D = d + 0.8k$ $\quad$ $B = 0.55k + 3d_1$
$a = 0.55k$ $\quad$ $l = 1.4k$
d_1 = bolt diam. $= 0.6d/\sqrt{N}$ $\quad$ $R = 1.5d$
N = number of bolts $\quad$ $X = 2.5d_1$

Flange couplings may be purchased in the market for shafts from 1 in. to 12 in. in diameter.

Sellers Coupling. This device (Fig. 63) is made by Wm. Sellers & Co., Inc., for shafts from 1 7/16 in. to 5 15/16 in. in diam. The principal dimensions are given in Table 42.

Table 42. Dimensions of Sellers Shaft Couplings
(See Fig. 63)

Diam. of shaft, in.	Approx. diam. of coupling, in.	Approx. length of coupling, in.	Approx. weight of coupling, each in lb.	Diam. of shaft, in.	Approx. diam. of coupling, in.	Approx. length of coupling, in:	Approx. weight of coupling, each in lb.
1 7/16	3 3/4	5 1/4	13	3 3/16	8 1/4	12	120
1 11/16	4 5/8	6 1/2	20	3 7/16	9 1/16	13	150
1 15/16	5 3/8	7 5/8	33	3 15/16	10	14 1/4	215
2 3/16	5 7/8	8 1/4	42	4 7/16	11 3/8	16	310
2 7/16	6 9/16	9 1/8	60	4 15/16	12 9/16	18	400
2 11/16	7	10	72	5 7/16	13 1/2	19 1/2	570
2 15/16	7 3/4	11	100	5 15/16	14 1/2	21	700

Clamp Couplings (Fig. 64) may be purchased in the market for shafts from 1 in. to 5 in. in diam. The proportions of the couplings made by the Jones & Laughlin Steel Co. are given in Table 43.

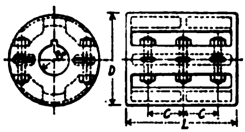


FIG. 64.—Clamp Coupling.

Table 43. Dimensions of Clamp or Split Compression Couplings
(Letters refer to Fig. 64)

B, in.	D, in.	L, in.	C, in.	No. of bolts	Diam. of bolts, in.	B, in.	D, in.	L, in.	C, in.	No. of bolts	Diam. of bolts, in.
15/16–1	3 1/4	3 1/2	1 5/8	4	3/8	2 9/16–2 5/8	7 3/4	9 3/16	2 29/32	6	5/8
1 1/16–1 1/8	3 5/8	3 15/16	1 7/8	4	3/8	2 11/16–2 3/4	8	9 5/8	3 1/16	6	5/8
1 3/16–1 1/4	4	4 3/8	2 1/16	4	3/8	2 13/16–2 7/8	8 1/4	10	3 7/32	6	5/8
1 5/16–1 3/8	4 1/4	4 13/16	2 1/4	4	3/8	2 15/16–3	8 1/2	10 1/2	3 3/8	6	5/8
1 7/16–1 1/2	....	......	......	4	....	3 1/16–3 1/8	.....	......	.....	6	....
1 9/16–1 5/8	4 7/8	5 11/16	2 11/16	4	1/2	3 3/16–3 1/4	8 7/8	11 3/8	3 5/8	6	3/4
1 11/16–1 3/4	5 1/4	6 1/8	2 7/8	4	1/2	3 5/16–3 3/8	.....	......	.....	6	....
1 13/16–1 7/8	....	......	......	4	....	3 7/16–3 5/8	9 1/2	12 1/4	3 15/16	6	7/8
1 15/16–2	6	7	3 1/4	4	1/2	3 11/16–3 7/8	10 1/4	13 1/8	4 3/16	6	7/8
2 1/16–2 1/8	6 3/8	7 7/16	3 1/2	4	5/8	3 15/16–4 1/8	11	14	4 1/2	6	1
2 3/16–2 1/4	6 3/4	7 7/8	3 3/4	4	5/8	4 3/16–4 3/8	.....	......	....	6	....
2 5/16–2 3/8	....	......	......	6	....	4 7/16–4 5/8	.....	......	.....	6	....
2 7/16–2 1/2	7 1/2	8 3/4	2 3/4	6	5/8	4 11/16–4 15/16	13 1/2	16 1/2	5	6	1

Friction Clip Couplings are frequently employed for shafts not over 1½ in. in diameter. The dimensions are as shown in Fig. 65. They are not easily removed and therefore should be used only when the shafting is permanent.

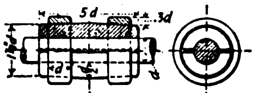


FIG. 65.—Friction Clip Coupling.

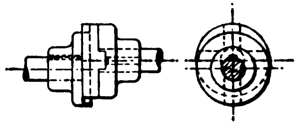


FIG. 66.—Oldham's Coupling.

Oldham's Coupling (Fig. 66) is frequently used aboard ship for connecting the engine and tail shafts. It permits a permanent displacement of the shaft center lines and maintains practically constant angular velocity of the connected shafts.

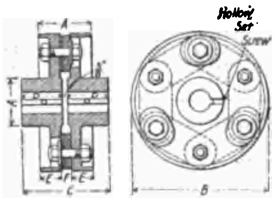


FIG. 67.

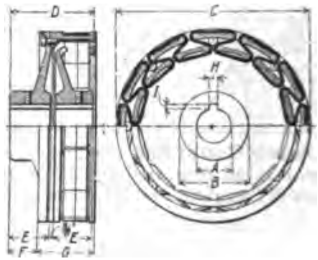


FIG. 68.

General Electric Co. Flexible Couplings.

Flexible Couplings are used for connecting two shafts not in exact alignment. Two types employed by the General Electric Co. for connecting engine and generator shafts are shown in Figs. 67 and 68, dimensions and power ratings being given in Tables 44 and 45. In one, the connection is through leather links; in the other, two endless belts are laced alternately over prongs on the spiders of each half of the coupling. The leather links and belts used serve also as insulation to prevent grounding of the generator through the engine frame. The De Laval coupling, shown in Fig. 69, consists of two forged steel disks, the driving half having a number of rigid studs which project into corresponding holes in the driven half. The torque is transmitted through steel-lined rubber bushings which allow for any defect in alignment and supply the flexibility necessary to prevent injurious stresses on the shafts and bearings.

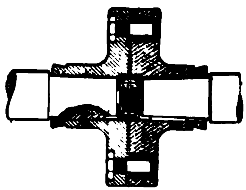


FIG. 69.—De Laval Flexible Coupling.

Hooke's Joint as shown in Fig. 70 is a form of coupling which is adapted to connecting shafts fixed at an angle which is either constant or variable. Suitable proportions are as follows:

$$d = \tfrac{1}{2}D;\quad L = 1\tfrac{1}{2}D;\quad t = \tfrac{1}{2}D;\quad x = d + \tfrac{1}{16}\text{ in.};\quad y = 0.05d;\quad T = 1.2D.$$

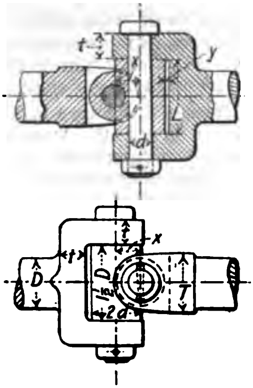


FIG. 70.—Hooke's Joint.

With this coupling a variable angular velocity ratio exists between the shafts during each quarter revolution.

The Jaw Coupling (or claw clutch) shown in Fig. 71 is used in cases where the shafts need frequently to be disengaged, as in punches and presses. Suitable proportions are as follows: Let d = shaft diam. in in. and $k = d + 1$ in. Then $L = 2.65k$; $D = 2.1k$; $D_1 = 1.6k$; $D_2 = 1.5k$; $l = 1.25k$; $l_2 = 0.6k$; $x = y = 0.3k$.

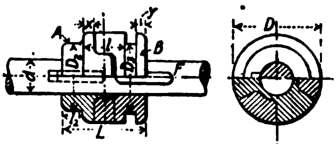


FIG. 71.—Jaw Coupling.

Table 44. Leather Link Flexible Couplings
(All dimensions in in. Letters refer to Fig. 67)

Bore	*A*	*B*	*C*	*D*	*E*	*F*	Rating per r.p.m.*		Max. r.p.m.	Weight, lb.
							kw.	h.p.		
3/4	1 3/8	3 3/8	2 7/8	1 1/4	11/16	3/8	0.0012	0.0016	1800	2 3/4
1	2	5	4	1 13/16	7/8	3/8	0.0032	0.0043	1800	7 1/4
1 1/2	2 3/4	6 3/4	6	2 13/16	1 1/16	3/8	0.0076	0.0102	1800	15
2	3 1/2	8 1/4	8	3 13/16	1 1/8	3/4	0.0149	0.0200	1800	27
2 1/2	4 1/8	9 1/2	10	4 13/16	1 7/16	3/4	0.0258	0.0346	1800	43
3	5	11 1/8	12	5 13/16	1 5/8	3/4	0.0410	0.0550	1800	75
3 1/2	6	12 1/2	12	5 13/16	1 15/16	1 1/8	0.0612	0.0821	1800	103

* With tensile stress of 400 lb. per sq. in. in the leather links.

Table 45. Laced Leather Belt Flexible Couplings
(All dimensions in in. Letters refer to Fig. 68)

Bore *A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	Rating per r.p.m.*		Max. r.p.m.	Weight, lb.
									kw.	h.p.		
1	2	5	3	1 5/16	11/16	2 7/16	1/4	1/8	0.0032	0.0043	1800	8
1 1/2	3	8 1/4	4	1 13/16	7/8	3 5/16	3/8	3/16	0.01492	0.02	1500	27
2	3 1/2	9 1/2	5	2 5/16	1 5/16	3 13/16	1/2	1/4	0.0258	0.0346	1500	39
3	5	15 1/2	6	2 13/16	1 5/8	5 1/16	3/4	3/8	0.1196	0.1604	1200	115
4	6 3/4	18 1/2	8	3 13/16	2 9/16	5 1/2	1	1/2	0.2066	0.277	900	189
5	8 3/8	24 1/2	10	4 13/16	3 7/16	6 13/16	1 1/4	5/8	0.4901	0.657	750	367
6	9 7/8	30 1/2	12	5 13/16	4 7/16	7 9/16	1 1/2	3/4	0.9573	1.2832	600	611
7	11 3/8	37	14	6 13/16	5 3/16	8 1/4	1 1/2	3/4	1.654	2.2176	450	1030
8	12 3/4	43	16	7 13/16	5 15/16	9 1/8	1 3/4	3/4	2.627	3.5215	350	1530
9	15 1/4	49	18	8 13/16	6 15/16	9 7/8	2	7/8	3.9214	5.2566	300	2200
10	16 3/4	49	20	9 13/16	7 15/16	9 7/8	2	7/8	3.9214	5.2566	300	2380
11	18 1/4	55	22	10 13/16	8 13/16	10 9/16	2 1/2	1	5.5833	7.4844	250	3170
12	19 3/4	55	24	11 13/16	9 13/16	10 9/16	2 1/2	1	5.5833	7.4844	250	3440
13	21 1/2	61	26	12 13/16	10 13/16	11 3/16	2 1/2	1	7.6591	10.2669	200	4490
14	23	61	28	13 13/16	11 13/16	11 3/16	2 1/2	1	7.6591	10.2669	200	4830

* With tensile stress of 400 lb. per sq. in. in the leather belts.

The teeth are frequently made in the shape of saw teeth, with one side normal to the direction of driving. This form of tooth permits more ready engagement of the coupling and limits the driving to one direction. This is frequently a decided advantage, as in the case of cranks for cranking gas engines.

Cone Friction Clutches (see also automobile clutches, p. 1197) are adaptable to connecting shafts whose loads are frequently thrown on and off. An automobile clutch is shown in Fig. 72. The force relations obtaining in the operation of this clutch are shown diagrammatically in Fig. 73 and may be formulated as follows:

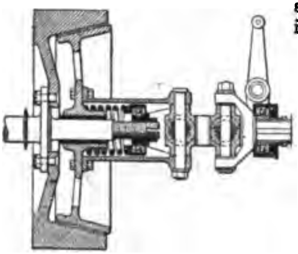


FIG. 72.—Cone Friction Clutch.

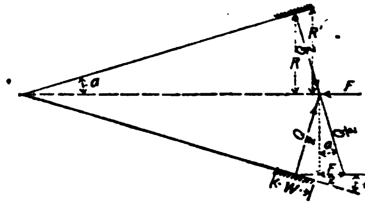


FIG. 73.

Let R = mean radius of clutch cone surface, in. T = torque transmitted through the clutch, in.-lb. Q = total force normal to conical surface, lb. P = tangential force at cone surface acting with leverage R, lb. F = axial force required to engage clutch (lb.) = force of the spring. a = angle between shaft axis and element of cone surface. f = coefficient of friction between the conical surfaces. N = r.p.m. of shaft. W = width of cone surface, in. p = pressure normal to cone surface, lb. per sq. in.

Then $T = QfR$; $F = Q(f \cos a + \sin a) = T(f \cos a + \sin a)/fR$.
Also $63{,}024 \times$ h.p.$/N = T = FfR/(f \cos a + \sin a)$
$\therefore F = 63{,}024 \times$ h.p. $(f \cos a + \sin a)/NfR$
and $Q = 63{,}024 \times$ h.p.$/NfR$. Also $Q = W \times 2\pi R \times p$.

Values of p are from 7 to 8 lb. per sq. in. for leather faces and from 40 to 50 lb. for metal faces. Values of f are given in Table 46. In order that the clutch may not disengage of its own accord, tan a must be less than the coefficient of friction f. Accordingly, for leather or greasy metal, a would be 12 deg. or less; or tan 12 deg. = 0.213, which is less than $f = 0.23$. For free disengagement, wood or metal surfaces, $a \overline{>} 20$ deg.; leather-faced cone clutches with a = 18 to 20 deg. need an operating device for disengagement (C. W. Hunt, *Trans. A. S. M. E.*, vol. 30).

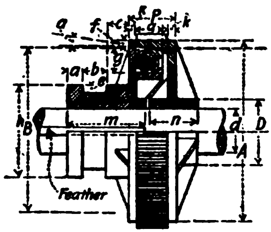
Table 46. Friction Coefficients for Cone Clutches

Cast iron on cast iron (dry)	0.15 to 0.20	Cork on metal	0.35
Cast iron on wood (dry)	0.20 to 0.25	Leather on metal (greasy)	0.23
Cast iron on brass (dry)	0.21	Cork on metal (greasy)	0.32
Leather on metal (dry)	0.56	Leather on metal (oily)	0.15

C. F. Blake (*Machy.*, Jan., 1907, p. 238) recommends the following proportions as suitable for cone clutches on hoisting devices (See Fig. 74):

$A = 4d$ to $8d$	$a = 0.3d + 0.3$ in.	$g = 0.8d$ to $2d$
$f = 0.2d + 0.1$ in.	$b = 0.4d + 0.4$ in.	$D = 1.8d + 0.5$ in.
$B = A - (2f + 0.257$ in.$)$	$e = 0.3d + 0.1$ in.	$m = 2d$
$h = 2d + 1$ in.	$k = 0.2d + 0.3$ in.	$n = d$ to $1.5d$
$c = 0.5d$		

Ring Friction Clutches of a great variety of designs may be purchased on the market. The type manufactured by The Hill Clutch Co. is illustrated in Fig. 75, dimensions (in.) and ratings being given in Table 47.



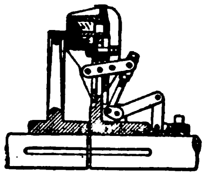
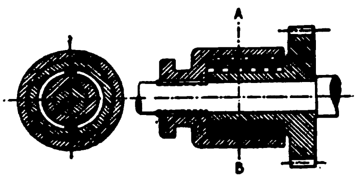


FIG. 74.—Cone Clutch for Hoists. FIG. 75.—Hill Friction Clutch.

Table 47. Dimensions and Ratings of Hill Friction Clutches
(See Fig. 75)

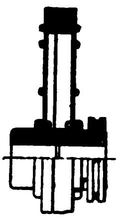
Diam. of coupling	No. of arms	H.p. at 100 r.p.m.	Equivalent shaft	Diam. of coupling	No. of arms	H.p. at 100 r.p.m.	Equivalent shaft	Diam. of coupling	No. of arms	H.p. at 100 r.p.m.	Equivalent shaft
12	3	5½	1 9/16	24	4	48	3 5/16	42	6	240	5⅝
14	3	8	1 13/16	27	4	65	3 11/16	48	6	325	6¼
16	3	11	2	30	4	85	4	54	6	450	7
18	3	15	2¼	34	4	110	4⅜	60	6	625	7¾
18	4	20	2 7/16	38	4	135	4 11/16	72	6	875	8½
20	4	27	2¾	42	4	165	4 15/16	84	6	1300	10
22	4	37	3	......	......	....	....	......	......	....	....

Plate Friction Clutches (see also automobile clutches, p. 1198) of the form known as the **Weston clutch** are as shown in Fig. 76. Alternate rings of wood and metal may be used, or thin metal disks only. The force relations are as follows: Let R = mean radius of rubbing surface of rings, in. T = twisting moment transmitted, in.-lb. F = axial force, lb. f = coefficient of friction. n = number of rubbing surfaces. N = r.p.m. of the shaft. Q = tangential force acting on ring surfaces at mean radius R, lb. h.p. = horse



Section A-B. Sectional Elevation.

FIG. 76.—Weston Plate Friction Clutch.



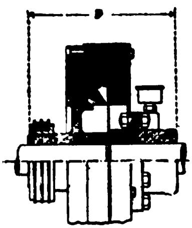


FIG. 77. FIG. 78.
Cutler-Hammer Magnetic Clutches.

power transmitted at N r.p.m. Then $Q = fFn$. $T = QR = fFnR = 63{,}024 \times \text{h.p.}/N$. H.p. = $NfFnR/63{,}024$. The values of f given in Table 46 may be used for the materials proposed for any design.

Magnetic Clutches are being successfully applied to the connecting of shafts even of large size. Clutches of this type manufactured by the Cutler-Hammer Mfg. Co. are shown in Figs. 77 and 78 and the principal dimensions of the standard sizes made are given in Tables 48 and 49.

Table 48. Dimensions and Ratings of Type "A" Cutler-Hammer Magnetic Clutches
(See Fig. 77)

No.	Outside* diam., in.	Torque, ft.-lb.	Approx. shipping weight, lb.	Current consumption, watts	No.	Outside* diam., in.	Torque, ft.-lb.	Approx. shipping weight, lb.	Current consumption, watts
12	12	184	45	55	38	38	5,790	830	165
14	14	263	60	66	40	40	6,700	950	220
16	16	368	85	55	42	42	8,050	1,120	220
18	18	552	110	66	44	44	8,630	1,330	209
20	20	815	160	88	46	46	10,000	1,490	297
22	22	1,185	210	110	48	48	12,000	1,850	286
24	24	1,471	270	132	50	50	12,900	2,000	275
26	26	2,060	340	132	54	54	18,200	2,600	352
28	28	2,260	380	121	60	60	21,800	3,000	308
30	30	3,160	490	143	64	64	30,000	3,950	418
32	32	3,550	550	154	68	68	32,900	4,400	396
34	34	4,470	670	187	72	72	43,600	5,500	495
36	36	5,000	730	175					

* Hub lengths vary with size of shaft. Horse power = 0.00019 × torque × r.p.m.

Table 49. Dimensions and Ratings of Type "C" Cutler-Hammer Magnetic Clutches
(See Fig. 78)

No.	Outside diam. in.*	B,* in.	Torque, ft.-lb.	Approx. shipping weight, lb.	Current consumption, watts	No.	Outside diam. in.*	B,* in.	Torque, ft.-lb.	Approx. shipping weight, lb.	Current consumption, watts
10	10¼	7⅜	26	100	49	22	22¼	16¼	499	575	173
11	11¼	8	39	125	58	24	24¼	......	605	690	193
12	12¼	8⅝	66	170	69	26	26¼	......	709	790	212
14	14¼	10⅛	131	230	90	28	28¼	......	840	900	234
16	16¼	12⅛	210	300	114	30	30¼	......	998	1000	244
18	18¼	13⅛	289	375	132	32	32¼	......	1155	1110	274
20	20¼	14⅛	367	475	153	......	......	......	....	....	....

* Hub dimensions vary with the size of shaft. H.p. = 0.00019 × torque × r.p.m.

BRAKES

(See also pp. 1108 and 1201)

Block Brakes are shown diagrammatically in Figs. 79, 80 and 81. The force relations obtaining in the operation of these brakes may be formulated as follows:

In Fig. 79 let F = load applied at end of lever arm, lb.; A = distance from point of application of P to block center, in.; B = distance from block center to center of fulcrum pin, in.; R = reaction between wheel and block, lb.; f = coefficient of friction; P = tangential frictional resistance, lb. Then, for rotation in either direction,

$$F(A + B) = RB, \quad R = P/f, \text{ and } F = PB/f(A + B)$$

In Fig. 80 let C = leverage distance from fulcrum pin to line of action of P, in. Then, for clockwise rotation, $F = PB[(1/f) - (C/B)]/(A + B)$. For counterclockwise rotation, $F = PB[(1/f) + (C/B)]/(A + B)$.

In the case of clockwise rotation it will be noted that C/B must be less than $1/f$, or the brake will be self-acting, that is, will bind.

In the arrangement shown in Fig. 81, for clockwise rotation, $F = PB[(1/f) + (C/B)]/(A+B)$, and for counterclockwise rotation, $F = PB[(1/f) - (C/B)]/(A+B)$.

In the latter case (for counterclockwise rotation) C/B must be less than $1/f$ or the brake will be self-acting, that is, will bind.

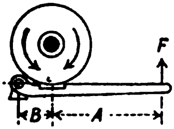


FIG. 79.

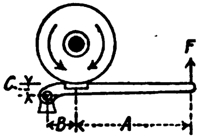


FIG. 80.

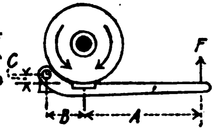


FIG. 81.

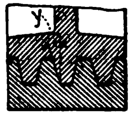


FIG. 82.

FIGS. 79–81.—Block Brakes.

Should the **face** of the brake wheel and blocks be **grooved,** as shown in Fig. 82, $f/(\sin y + f\cos y)$ must be substituted for f in the foregoing equations, y being equal to half the angle included by the faces of the grooves and not less than 23 deg., to prevent binding; y may have any value up to 30 deg.

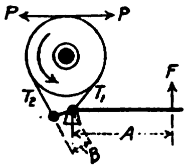


FIG. 83.

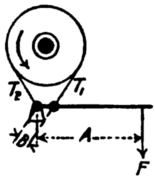


FIG. 84.

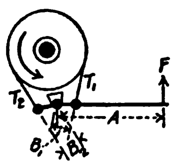


FIG. 85.

FIGS. 83–85.—Band Brakes.

Band Brakes are shown diagrammatically in Figs. 83, 84 and 85. The force relations obtaining in their operations are as follows:

In Fig. 83, let F = force at end of brake handle, lb.; P = tangential force at rim of wheel, lb.; f = coefficient of friction of materials in contact; a = angle of wrap of band, in deg.; T_1 = total tension in band on tight side, lb.; T_2 = total tension in band on slack side, lb. Then $T_1 - T_2 = P$ and $T_1/T_2 = 10^{0.0076fa} = 10^b$ where $b = 0.0076fa$. Also, $T_2 = P/(10^b - 1)$ and $T_1 = P \times 10^b/(10^b - 1)$.

For the arrangement shown in Fig. 83,

$$FA = T_2B = PB/(10^b - 1)$$

and

$$F = PB/A(10^b - 1).$$

For the construction illustrated in Fig. 84,

$$F = (PB/A)[10^b/(10^b - 1)].$$

For the differential brake shown in Fig. 85,

$$F = (P/A)[(B_2 - 10^bB_1)/(10^b - 1)].$$

In this arrangement the quantity $10^b \times B_1$ must always be less than B_2, or the band will grip the wheel and the brake, or part of the mechanism to which it is attached, will be ruptured.

It is usual in practice to have the leverage ratio A/B for block brakes about 5 : 1, and for band brakes about 10 : 1. The bands are faced with maple blocks.

If f for wood on iron be taken at 0.3 and the angle of wrap for the band be 270 deg., that is, subtends ¾ of the circumference, then, $10^b = 4$ approx. (see Table 11, p. 248, for values of $10^b = e^{fa}$), and C/B be taken equal to $1/0.5 = 2$, the loads required for a given torque will be as follows for the cases just considered:

Block brake, Fig. 79 .... $F = 0.55P$
Block brake, Fig. 80 (clockwise rotation) .... $F = 0.22P$

Block brake, Fig. 80 (counterclockwise rotation) .... $F = 0.90P$
Block brake, Fig. 81 (clockwise rotation).. $F = 0.90P$
Block brake, Fig. 81 (counterclockwise rotation) .... $F = 0.22P$
Band brake, Fig. 83 .... $F = 0.033P$
Band brake, Fig. 84 .... $F = 0.133P$
Band brake, Fig. 85 .... $F = 0.016P$

Clam-shell Brakes are block brakes adapted to absorbing great amounts of energy. Fig. 86 illustrates one way in which such brakes may be rigged. The force relations are as follows:

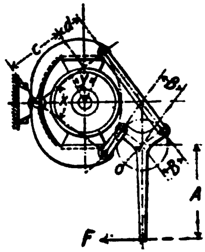


FIG. 86.

Let F = load applied at end of lever, lb.; R = reaction between wheel and *each* block, lb.; P = tangential frictional resistance on each block surface, lb.; f = coefficient of friction of materials in contact for a given condition of surface; r = drum radius, in., T = torque on drum shaft, in.-lb.; other notations as in the figure. Then,

$R = FA(c + d)/\cos(x/2)2Bc$; $P = Rf$; $T = 2Rfr$.

The point o should be a floating pivot to permit adjustment as the blocks wear. Values of f are given in Table 46.

In the case of Fig. 85 the dimension B_2 must be greater than $B_1 \times 10^b$. Accordingly, B_1 is taken at ¼, A at 10, and, since $10^b = 4$, B_2 is taken at 1¼.

The principal function of a brake is to absorb energy. This energy appears at the surface of the brake as heat, which must be carried away at a sufficiently rapid rate to prevent burning of the wooden blocks. Suitable proportions may be arrived at as follows:

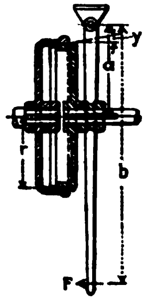


FIG. 87. Cone Brake.

Let p = unit pressure on brake surface, lb. per sq. in. = R (reaction against block)/area of block; v = velocity of brake rim surface, ft. per sec. = $2\pi rn/60$, where n = r.p.m. of brake wheel. Then pvf = work absorbed per sq. in. of brake surface per sec., and $pv \leqq 20$ for intermittent applications of load with comparatively long periods of rest and poor means for carrying away heat (wooden blocks); $pv \leqq 10$ for continuous application of load and poor means for carrying away heat (wooden blocks); $pv \leq 30$ for continuous application of load with effective means for carrying away heat (oil bath).

Cone Brakes may be made to the form shown in Fig. 87. The force relations are as follows:

Let F = load applied at end of lever, lb.; Q = normal pressure on cone surface, lb.; P = tangential force on the rim of the brake, lb.; r = mean radius of cone surface, in., and y = half angle of cone. Then $Q = F(b/a)/2(\sin y + f \cos y)$ and $P = fF(b/a)/(\sin y + f \cos y)$; $F = P(a/b)(\sin y + f \cos y)/f$. For $a = 1$, $b = 10$, $y = 15$ deg., and $f = 0.2$ for cast iron on cast iron, $F = 0.23P$, approx.

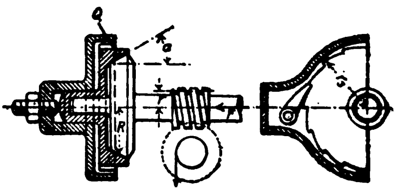


FIG. 88.—Cone Brake for Lowering Loads.

Such brakes are frequently used for lowering loads by the means shown in Fig. 88. The drum shaft is driven through the worm shaft by means of worm and wheel. In raising the load the ratchet runs free. As the load tends to lower the ratchet engages and the worm thrusts the cone surfaces together. Lowering of the load must be accomplished by the application of torque to the worm shaft. In the case of worms of small pitch no brake is required, for the wheel cannot turn the worm. This brake is therefore adapted to worms of large pitch, and the reason for employing a large pitch is that a more efficient drive is obtained (see p. 733). The force relations are approximately as follows:

Let R = mean radius of cone surface, in. Q = force normal to cone surface, lb. P = tangential force at cone surface, acting with the leverage R. F = axial force, lb. f = coefficient of friction between cone surfaces. a = angle between shaft axis and element of cone surface, deg. b = angle of pitch of worm on worm wheel, deg. f_1 = coefficient of friction of worm and wheel teeth. r = pitch radius of worm, in. r_3 = pitch radius of ratchet wheel, in. L = load on ratchet teeth, lb. Let $x = f \cos a + \sin a$, and $y = (\tan b - f_1)$; then Lr_3 = torque on worm shaft (in.-lb.) due to F, where F is the axial force due to load on drum = Fry approximately.

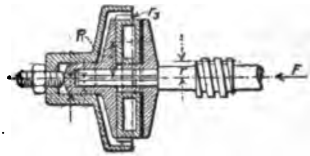


FIG. 89.—Disk Brake.

The resisting moment of the clutch is $PR = QfR = RFf/x$. To hold the load and prevent its lowering, the actuating torque, Fry, must be less than the reacting torque, RFf/x. Accordingly, $Rf/x \geqq ry$, or $R \geqq ryx/f$. The angles a and b may be assumed 22 deg., f at 0.08 and f_1 at 0.10, depending on the lubrication, whence x = 0.45 and y = 0.30. Then for the above assumptions $R \geqq 1.7r$. If a = 30 deg., b = 35 deg., then, for f and f_1 as before, x = 0.57, y = 0.60 and $R \geqq 4.3r$.

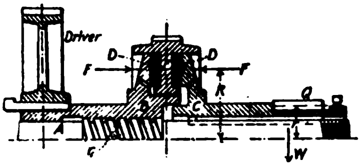


FIG. 90.—Disk Brake.

Disk Brakes of the same general type of construction but using a flat face instead of a cone, are as shown in Fig. 89. There are two faces to be moved against friction.

The force relations are $Lr_3 \leqq 2fFR = 2PR$, where $fF = P$. Also $R \geqq ry/2f$.

Frequently disk brakes are made as shown in Fig. 90. The motor pinion engages the gear in the drum (not shown). When the load is to be raised, power is applied through the gear and the connection between B and C is accomplished by the advancing of B along A and the clamping of the friction disks D and D and the ratchet wheel E. The reversal of the motor disconnects B and C. In lowering the load, only enough reversal of rotation of the gear is given as is needed to reduce the force in the friction disks so that the load may be lowered under control.

The force relations are as follows:

Let R = mean radius of friction plates, in.; f = coefficient of friction between plates; F = axial force along the screw = force in friction plates, lb.; P = tangential force on friction plates at mean radius R, lb., = fF; W = load on pinion teeth, lb.; r = radius of pinion pitch circle, in.; r_1 = radius of pitch circle of screw, in.; a = angle of screw thread, deg.; f_1 = coefficient of friction in thread.; $z = (\tan a + f_1)$.

Then the load in lowering causes a moment $Wr = fFR + Fr_1z$, approx. To sustain the load, $Wr \leqq 2fFR$ and $fR \geqq r_1z$.

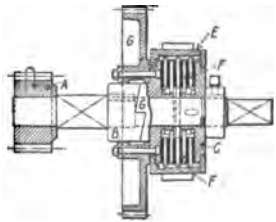


FIG. 91.—Multi-disk Brake.

Acceptable values of the several factors are: a = 10 deg.; f = 0.08; f_1 = 0.10; R = 9 in. Substituting these in the last equation, $r_1 \leqq 2.5$ in.+. Any radius of screw less than 2½ in. consistent with strength will be satisfactory for the above conditions.

A multi-disk brake is shown in Fig. 91. This type of construction results in an increase in the number of friction faces. The drum shaft is geared to the pinion A, while the motive power for driving comes through the gear G.

In raising the load, direct connection is had between G, B and A. In lowering, B moves relatively to G and forces the friction plates together, those plates fast to E being held stationary by the pawl on E. In the figure there are three plates fast to E, one fast to G and one fast to C.

In addition to the notation given in the previous case, let n = number of *faces* in sliding contact when the part C is moved relatively to part E which carries the ratchet. This condition obtains upon the load beginning to lower; and n_1 = number of *faces* in sliding contact when the parts G and C both move relatively to E. This obtains when full gripping of the plates takes place. The load in beginning to lower occasions the following force relation: $Wr = nfFR + Fr_1z$. To sustain the load it is necessary that $Wr \leq n_1fFR$. Hence, to prevent the load from dropping, $r_1z \leq fR(n_1 - n)$.

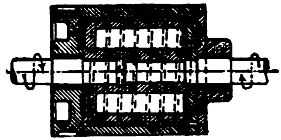


FIG. 92.—Coil Brake.

The Coil Brake shown in Fig. 92 is occasionally used for lowering loads under control. The coil is made by winding a Tobin bronze bar in a helix, or is cut from a solid steel cylinder. Wear occurs first and most severely at one end of the coil, which is an objectionable feature of this style of brake. The way to compensate for this is to bore the spring on a taper so that several turns of the coil will grip simultaneously.

BEARINGS

Plain Cylindrical Bearings

Dimensions of Plain Cylindrical Journal Bearings. The operation of a bearing depends upon the maintenance of an oil film between the sliding surfaces. The factors controlling the lubrication of the bearing are: 1. The intensity of pressure on the oil film. 2. The methods of introducing, distributing, carrying away and cooling the lubricant. 3. The characteristics of the lubricant under the operating conditions. 4. The characteristics of the sliding surface—oil grooves, etc. 5. The clearance between the journal and the bearing.

The intensity of pressure at any place on the oil film is dependent on the total load on the bearing, the projected area of the journal, the point or points of application of the load on the shaft with reference to the bearing, the stiffness of the bearing support and the journal, and the velocity of rubbing. According to the experiments of Prof. Herbert F. Moore (*Am. Mach.*, Sept. 10, 1903), the **pressure** on the projected area of a bearing **at which the oil film breaks down** is p(lb. per sq. in.) $= 7.47\sqrt{v}$, where v = velocity of rubbing, ft. per min. The **diameter of a bearing** in which a perfect oil film will be maintained at the final permissible temperature of the bearing, may be determined by the following formula derived by Axel K. Pedersen (*Am. Mach.*, Oct. 10, 1912) from a wide range of experimental data: $d^5 = 0.068453 \times (sW/x)^2(l/n)$ in which d = diam. of bearing, in.; s = factor of safety = p(in Moore's formula) divided by the *actual* pressure on projected area, lb. per sq. in.; W = total load on bearing, lb.; l = length of bearing, in.; x = l/d; and n = r.p.m. Values of l/d are given in Table 52. For ordinary light machinery, s may be taken as low as 2, and for heavy (especially high-speed) machinery, up to 8 or even 10. A fair average for preliminary design is 4 to 5.

The **allowable pressure** p (lb. per sq. in. of projected area) **with respect to the final temperature** (deg. fahr.) attained by the bearing may be determined from the following formulæ, in which v is the rubbing speed of journal, ft. per sec., and f the coefficient of friction:

For $v < 8.5$ ft. per sec., according to Pedersen, $p = 2.3\sqrt{60v}/f(t - 32)$.
For $v > 8.5$ ft. per sec., according to Lasche, $p = 51.2/f(t - 32)$.

The total load and the projected area (= length × diam.) of the bearing determine the bearing pressure. **Allowable bearing pressures,** according to Alford ("Bearings and Their Lubrication," p. 65), are given in Table 50. Representative German practice, according to "Hütte," is set forth in Table 51. For marine engines, see p. 1243; for gas engines, see also p. 1046.

Table 50. Allowable Bearing Pressures—Common Practice
(Pressures in lb. per sq. in. of projected area)

Gas engines		
Main bearings (total load)		500- 700
Crank-pin bearings		1500-1800
Cross-head-pin bearings		1500-2000
Air compressors (side-crank)		
Main bearings (total load)		160- 237
Crank-pin bearings		462- 850
Cross-head-pin bearings		628-1370
Air compressors (center-crank)		
Main bearings (total load)		122- 220
Crank-pin bearings		244- 402
Cross-head-pin bearings		400- 785
U. S. Navy		
Main engine bearings		275- 400
Main engine crank pins		400- 500
Steam turbine bearings		up to 85
Thrust bearings for torpedo boats		" " 50
Rolling mills	Rubbing vel., ft. per min.	
Pinion housing	350-600	30- 50
Roll housing	350-600	100-2000
Table roller	150	30- 50
Table line shafting	150	30- 50
Main bearing of shears	50-65	1800-2500
Railroads		
Cross-head-pin bearings		3000-4000
Crank-pin bearings		1500-1700
Driving wheel journal bearings		up to 556
Car-axle bearings		300- 325
Tender axle bearings		up to 425
Stationary engines (high-speed)		
Main bearings (dead load)		60- 120
Main bearings (total load)		150- 250
Crank-pin bearings		900-1500
Cross-head-pin bearings		1000-1800
Stationary engines (low-speed)		
Main bearings (dead load)		80- 140
Main bearings (total load)		200- 400
Crank-pin bearings		800-1300
Cross-head-pin bearings		1000-1500
Merchant marine		
Main bearings (total load)		400- 500
Crank-pin bearings		400- 500
Electrical machinery		
Horizontal steam turbine bearings		40- 60
Vertical steam turbine steps		200-1000
Generator and motor bearings		30- 80
Main bearings of engines driving generators		40- 80
Miscellaneous		
Bearings for slow speed and intermittent load, as for punch presses, shears, etc		3000-4000
Light line-shaft bearings, cast iron		15- 25
Heavy line shaft bearings, bronze or babbitt-lined		100- 150
Main bearings of slow-speed pumping engines		up to 600
Bearings for very low speeds and intermittent service, as for turn tables and bridges		7000-9000
Heavy slow-speed step bearings		up to 2000
Drill press thrust collars		" " 325
Angular thrust bearings for boring-mill tables		" " 75

Table 51. Allowable Bearing Pressures—German Practice

Materials in contact:	Pressure, lb. per sq. in. of projected area
Hardened crucible steel on hardened crucible steel	2130
Hardened crucible steel on bronze	1280
Unhardened crucible steel on bronze	850
Mild steel with smooth surface on bronze	570
Mild steel with slightly imperfect surface (or cast iron) on bronze	425
Mild steel with slightly imperfect surface on cast iron	355
Mild steel on lignum vitæ with water lubrication	355

If the total pressure maintains approximately the same magnitude and direction even when the journal is at rest (for example, in heavily loaded shafts, heavy gears, etc.), the unit pressures of Table 51 should be taken smaller than the tabular values. For journals (or bearings) which oscillate only, the unit pressure may be slightly higher. For the bearings of rope and chain sheaves, etc., which turn intermittently and on which the wear is either small or unimportant, the unit values—850 to 355 lb.—may be doubled or trebled. For crucible-steel crank and cross-head pins running on bronze in ordinary steam engines the unit pressure may be taken from 710 to 995 lb. and 1070 to 1130 lb., respectively; for similar purposes on locomotives, 1420 and 2130 lb., respectively; on high-speed steam engines, 570 to 710 lb., respectively. For flywheel bearings of steam engines use 213 to 217 lb. For crank-pin bearings of punches and shears the unit pressure may be as high as 2845 lb. per sq. in. of projected area.

A rule devised by Dr. Thurston to determine the allowable pressure in bearings having a steady load in one direction, is $pV < 50{,}000$, where p = load per sq. in. of projected area of journal, lb., and V = velocity of rubbing in ft. per min. This rule is followed quite closely in the practice of the General Electric Co. (H. G. Reist, *Trans. A. S. M. E.*, vol. 27, p. 476.)

The customary ratios of bearing length to bearing diameter for various classes of service are given in Table 52. These values, if observed in design, will insure journals of requisite stiffness.

Table 52. Length-Diameter Ratios for Journal Bearings

Type of Bearing	l/d	Type of Bearing	l/d
Marine Engines:		Stationary Engines:	
Main bearings	1 to 1.5	Main bearings	1.5 to 2.5
Crank-pin bearings	1 to 1.5	Crank-pin bearings	1
Ordinary Shafting:		Wrist-pin bearings	1 to 1.5
Heavy—fixed bearings	2 to 3	Generator and motor bearings	2 to 3
Light—self-adjusting bearings	3 to 4	Machine-tool bearings	2 to 4

Standard practice in regard to the **clearance between journal and bearing**, as followed by the General Electric Co., is given in Table 53. The practice of the Brown & Sharpe Mfg. Co. for running fits is given in Table 54.

Table 53. Clearances Between Journals and Bearings

Journal		Bore for bearings						Axle linings for ry. motors	
		Horizontal		Vertical		Step			
Nominal diam., in.	Allowable variation below maximum diam., in.	Allowable variation above the maximum diam., in.		Allowable variation above the maximum diam., in.		Allowable variation above the maximum diam., in.		Allowable variation above the maximum diam., in.	
		Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
3/8	0.0005	0.002	0.003	0.001	0.002	0.0005	0.001	0.005	0.009
1	0.0005	0.002	0.003	0.001	0.002	0.0005	0.001	0.005	0.009
2	0.0005	0.003	0.004	0.002	0.003	0.001	0.0015	0.005	0.009
3	0.0005	0.004	0.006	0.003	0.005	0.0015	0.0020	0.005	0.009
4	0.001	0.005	0.007	0.004	0.006	0.002	0.003	0.007	0.011
6	0.001	0.009	0.011	0.005	0.007	0.003	0.004	0.011	0.015
8	0.001	0.012	0.015	0.006	0.009	0.004	0.006	0.013	0.017
10	0.0015	0.014	0.019	0.007	0.012	0.005	0.007	....	....
21	0.002	0.018	0.023	0.013	0.018	0.008	0.010	....	....
25	0.003	0.020	0.028	0.013	0.018	....	....	....	....
32	0.003	0.024	0.034	....	....	....	....	....	....

Table 54. Grinding Limits for Various Classes of Fits

Size of work	Running fits, ordinary speed	Running fits: high speed, heavy pressure and rocker shafts	Sliding fits	Standard fits
	Small	Small	Small	Small
To ½ in. diam., inc...	0.00025–0.00075	0.0005–0.001	0.00025–0.0005	Standard to 0.00025
To 1 in. diam., inc...	0.00075–0.0015	0.001 –0.002	0.0005 –0.001	Standard to 0.0005
To 2 in. diam., inc...	0.0015 –0.0025	0.002 –0.003	0.001 –0.002	Standard to 0.001
To 3½ in. diam., inc...	0.0025 –0.0035	0.003 –0.0045	0.002 –0.0035	Standard to 0.0015
To 6 in. diam., inc...	0.0035 –0.005	0.0045–0.0065	0.003 –0.005	Standard to 0.002

Types of Plain Cylindrical Bearings and Details of Construction

Pillow Blocks for **small shafts and light service** may have the proportions shown in Fig. 93. Taking $d_1 = D + ½$ in. as a unit dimension (D = diam. of shaft, in.), the following dimensions are to be used:

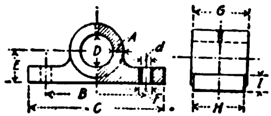


FIG. 93.—Light Pillow Block.

$A = 0.3d_1$		$G = 1.5d_1$	
$B = 2.5d_1$		$H = 1.25d_1$	
$C = 3.5d_1$		$I = 0.5d_1$	
$E = 0.8d_1$		d = bolt diam., in. $= 0.25d_1$	
$F = 0.3d_1$			

Pillow blocks for **heavy shafts** and greater loads may be made as shown in Fig. 94. The dimensions may be as follows, d_3 being the unit dimension $= D + ½$ in., where D = shaft diam., in.; $d_1(d_2)$ = diam. of bolts in base (cap), in.

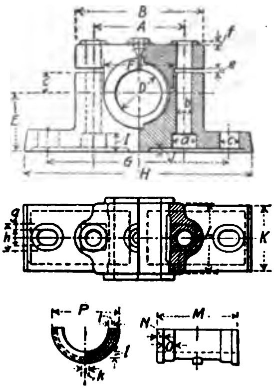


FIG. 94.—Pillow Block for Heavy Shafts.

$A = 1.75d_3$	$L = 1.25d_3$
$B = A + 2.75d_1$	$a = 1.5d_1 + ⅜$ in.
$C = 0.375d_3$	$b = 0.28d_3$
$E = d_3$	$c = 1.5d_2$
$F = 1.5d_3$	$d_1 = d_2 = 0.25d_3$
$G = 3.3d_3$	$e = D/16$
$H = G + 3.5d_2$	$f = 0.125d_1$
$I = 1.25d_2$	$g = 0.28d_2$
$J = {}^{11}/_{16}d_3$	$h = 1.5d_2$
$K = 1.125d_3$	

For the **brasses** the unit dimension is $D_1 = 0.09D + 0.15$ in., and

$i = 0.75D_1 \qquad N = l = D_1$

$k = 1.25D_1 \qquad O = 1.75D_1$

$M = L + 2N \qquad P = D + .36d_3$

Outboard Bearings or **pedestal bearings** with ring oiling may have the proportions shown in Fig. 95. Following are the principal dimensions in terms of the bearing diameter D:

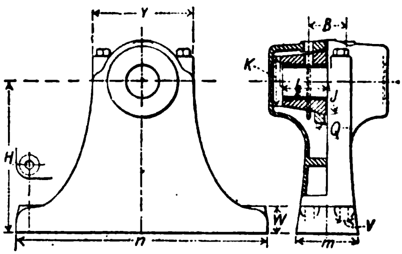


FIG. 95.—Pedestal Bearing.

L = length of bearing = $3D$ or $4D$	$K = 1.5D + \frac{1}{2}$ in.
$J = 2D + \frac{1}{2}$ in. for $L = 3D$ or $4D$	$m = 1.5D + (H - 2D)/5.55$ for $L = 3D$
$B = 1.25D$ for $L = 3D$	$= 1.8D + (H - 2D)/5.55$ for $L = 4D$
$B = 1.5D$ for $L = 4D$	$V = 0.25D$
$Q = 0.8D$ for $L = 3D$ } take nearest	$W = 0.625D + \frac{1}{4}$ in. } for $L = 3D$ or $4D$
$Q = 0.95D$ for $L = 4D$ } $\frac{1}{8}$ in. less	$n = 5D + 2H/3$ } for $L = 3D$ or $4D$
	$Y = 3D + 1$ in.

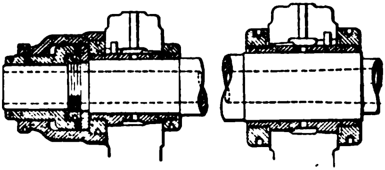
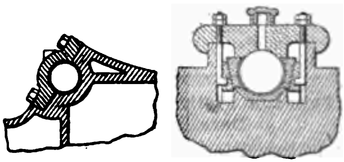


FIG. 96.—Lathe Headstock Bearings.



FIGS. 97 and 98.—Engine Bearings.

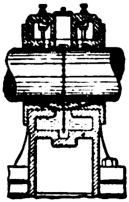


FIG. 99.—Chain-oiled Engine Bearing.

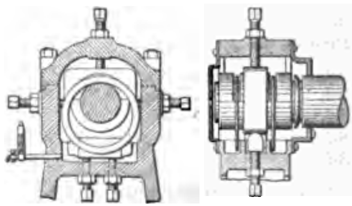
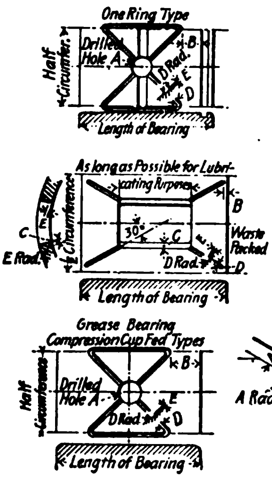
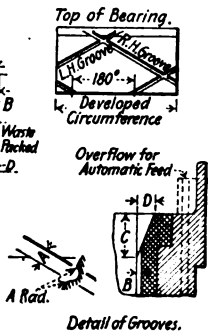


FIG. 100.—Ring-oiled Engine Bearing.





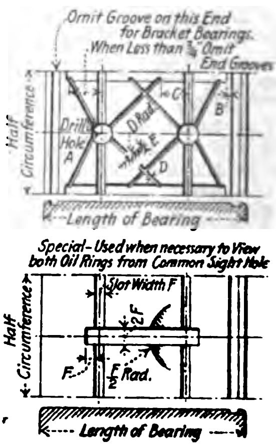


FIG. 101. FIG. 102. FIG. 103.

Grooving of Bearings for Oil Circulation.

Line-shaft Bearings are purchasable in standard sizes ranging from $1\frac{7}{16}$ in. to $3\frac{15}{16}$ in. in diam. by steps of $\frac{1}{4}$ in., and from $3\frac{15}{16}$ in. to $6\frac{15}{16}$ in. by steps of $\frac{1}{2}$ in. The lengths in all cases are either 3 or 4 diameters.

Hangers for line-shaft bearings may be purchased in types permitting of ceiling, floor or wall suspension. For ceiling and floor suspension the drop varies from 8 in. to 36 in., and for wall suspension from 6 in. to 9 in.

Main Shaft Machine Bearings. Average practice in proportioning the diameters and lengths of **headstock bearings in lathes** is as follows:

Front bearings: $d = s/5$; $l = s/4$. Back bearings: $d = s/8$; $l = s/6$. where d(diam.), l(length) and s(swing) are all in inches. Fig. 96 shows the usual type of design for small lathes—a construction which is also utilized in a large variety of other machine tools.

Engine Bearings for steam-, gas- and oil-engine crank shafts are constructed with regard to adjusting for wear and lubrication as shown in Figs. 97 to 100. The form shown in Fig. 97 is adapted to small engines. For larger engines, requiring more complete adjustment, the form shown in Fig. 98 is used. These bearings may be chain-oiled or ring-oiled, as shown in Figs. 99 and 100. Forced lubrication is used for important engines.

Bearing Metals. For the composition of bearing metals for use in different classes of machinery, see p. 542. The **anchoring and grooving** of bearings according to the practice of the Westinghouse Machine Co. is shown in Figs. 101 to 104 and Table 55. Fig. 101 relates to horizontal solid bearings, Fig. 102 to vertical bearings, and Fig. 103 to horizontal split bearings. Fig. 104 shows the standard method employed in anchoring babbitt metal in bearings.

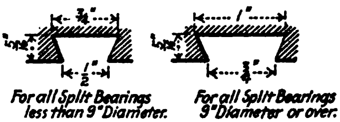


FIG. 104.—Method of Anchoring Babbitt Metal in Bearings.

Table 55. Oil Grooves in Bearings

(All dimensions in inches)

Horizontal solid bearings (see Fig. 101)					
Bearing diam.	Drilled hole A	B	C	D	E
3/4 to 1 inc.	3/8	1/8	3/16	3/32	3/32
Above 1 to 1 1/2 inc.	1/2	1/8	3/16	3/32	3/32
Above 1 1/2 to 1 3/4 inc.	5/8	3/16	1/4	3/16	1/8
Above 1 3/4 to 2 3/4 inc.	3/4	3/16	1/4	3/16	1/8
Above 2 3/4 to 3 1/2 inc.	1	1/4	3/8	1/4	1/8
Above 3 1/2 to 5 inc.	1 1/4	1/4	3/8	1/4	1/8

Horizontal split bearings (see Fig. 103)					
Bearing diam.	Drilled hole A	B	C	D	E
1 1/2 to 1 3/4 inc.	....	1/8	1/4	3/32	3/32
Above 1 3/4 to 2 1/4 inc.	....	1/8	1/4	1/8	1/8
Above 2 1/4 to 2 3/4 inc.	....	1/8	5/16	3/16	1/8

Horizontal split bearings (cont.)					
Bearing diam.	Drilled hole A	B	C	D	E
Above 2 3/4 to 3 inc.	....	1/8	3/8	3/16	1/8
Above 3 to 4 1/2 inc.	....	1/8	9/16	1/4	1/8
Above 4 1/2 to 7 1/8 inc.	....	1/4	13/16	1/4	3/16
Above 7 1/8 to 13 inc.	....	1/4	13/16	5/16	3/16

Vertical bearings (see Fig. 102)					
Bearing diam.	Drilled hole A	B	C	D	E
Up to 1 3/4 inc.	1/8	1/16	3/8	1/16	....
Above 1 3/4 to 2 1/2 inc.	3/16	3/32	3/8	3/32	....
Above 2 1/2 to 5 inc.	3/8	1/8	1/2	3/16	....
Above 5 to 10 inc.	3/8	1/8	1/2	1/4	....
....	....	....	....	....	....

Thrust and Step Bearings

Thrust and Step Bearings for Marine Service are sometimes made with adjustable horseshoe bearing rings as shown in Figs. 105 and 106, and with provision for water-cooling. The thrust collars are formed by grooving the shaft and have an outside diam. $D = 1.6d$ to $1.9d$, where d is the normal shaft diameter. The thickness of each collar $= w = 0.13d$ to $0.16d$, and the distance between collars $= s = 2w$ to $3w$. The number of collars required $= n = P/\pi d_1 bp$, where P = total thrust on bearing, lb.; d_1 = mean diam. of collar, in.; b = radial width of collar, in., and p = allowable bearing pressure, lb. per sq. in., as follows: For cargo steamers, $p = 40$ to 55; for passenger steamers, $p = 55$ to 80; for heavy warships, $p = 70$ to 85; for light warships, $p = 100$ to 130.

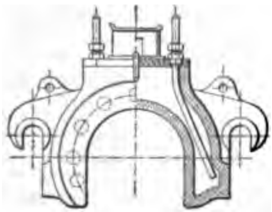
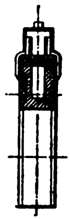



FIG. 105. FIG. 106.
Horseshoe Bearing Ring for Marine Thrust Bearing.

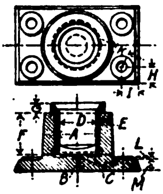


FIG. 107.—Step Bearing.

A simple **step bearing** may be made as shown in Fig. 107, acceptable proportions for which are as follows:

Shaft diam., in. $= D$	$C = 0.35D$	$H = I = K = 1.75d$
Bolt diam. $= d = 0.25D$	$E = 0.3D$	$L = 0.35d$
$A = 1.2D$	$F = D$	$M = 0.3D$
$B = 0.4D$	$G = 0.25D$	

The bearing pressures p depend on the mean velocity v of the rubbing surface and should be limited to the following values:

For v (ft. per min.) =	Slow and intermittent	50	50–100	100–150	150–200	Over 200
p (lb. per sq. in.) =	1500	200	100	75	60	50

Sliding Bearings

All sliding bearings, Fig. 108, to wear true must have the sliding parts of nearly equal lengths. Bearings made in this way will be found not to wear out of true. Oiling is accomplished in several ways, an acceptable method being that shown in Fig. 109. Short slides in many machine tools are lubricated by oil pads or direct oil application. The weight of the table and work and thrust of the tool cause wear on the bottom and sides of the guides. To compensate for the wear in both directions, bearings are sometimes made V-shaped, as shown in Fig. 110.

Simpler sliding bearings in machine tools are made with provision for adjustment, as shown in Figs. 111–113, of which there are many modifications.

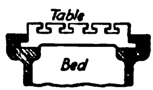


FIG. 108.

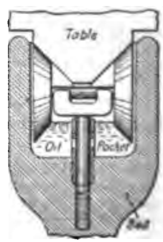


FIG. 109.

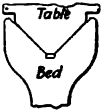


FIG. 110.

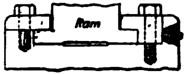


FIG. 111.

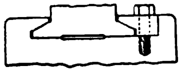


FIG. 112.

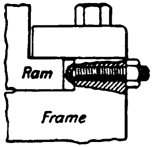


FIG. 113.

Ball Bearings

Ball bearings are used with great advantage in machinery, automobiles, line-shaft hangers, etc., under both thrust and radial loadings. **Typical mountings** for a variety of services are shown in Figs. 114, 115 and 116. Fig. 114 shows a typical mounting for a **pillow block.** The outer race should be made with a "sucking" or slip fit at *aa*, so that it may be easily inserted by hand. The inner race should be made with a light drive or force fit at *ee* and held securely against the shoulder *g* by firmly tightening the nut *f*, the nut being then pinned, as shown. The groove *c* has been found most effective in excluding dust and grit, which, if allowed to enter, will soon cause destruction of the ball and race surfaces. These general features of construction should be observed in mounting annular ball bearings of any sort.

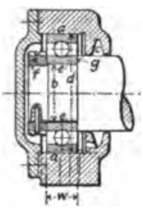


FIG. 114.—Hess-Bright Annular Ball Bearing.

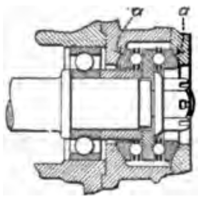


FIG. 115.—Combined Annular and Thrust Ball Bearing.

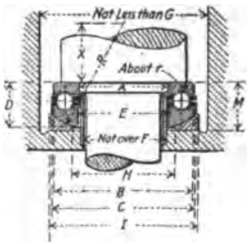


FIG. 116.—Hess-Bright Thrust Ball Bearing.

Fig. 115 shows a mounting of both **annular and thrust bearings.** In this case the thrust may occur in either direction. In thrust-bearing mountings the surfaces *aa* should be spherical in order to insure an even distribution of the load on the balls.

Annular and thrust ball bearings may be purchased in **standard sizes,** and are of different types according to the manufacturer. Knowing the speeds at which bearings are to operate and the nature of the service to which they are to be put, the manufacturers are able to specify the types best suited to the work and of the proper carrying capacities. Tables 56 and 57 give data on dimensions, loads and speeds of annular and thrust ball bearings made by the Hess-Bright Mfg. Co. It should be noted that the dimensions are given in millimeters, a practice due to the fact that German manufacturers were the first to take up their production on a large scale.

Table 56. Hess-Bright Annular Ball Bearings

(Dimension letters refer to Fig. 114)

	Light series					Medium series					Heavy series				
			Load in lb.					Load in lb.					Load in lb.		
			200	800	1500			200	900	1500			200	800	1500
b, mm.	d, mm.	w, mm.	r.p.m.	r.p.m.	r.p.m.	d, mm.	w, mm.	r.p.m.	r.p.m.	r.p.m.	d, mm.	w, mm.	r.p.m.	r.p.m.	r.p.m.
10	30	9	130	90	70	35	11	220	160	125	....	....	.....	.....	.....
12	32	10	145	100	75	37	12	265	175	130	....	....	.....	.....	.....
15	35	11	165	110	90	42	13	285	210	155	....	....	.....	.....	.....
17	40	12	240	175	130	47	14	395	285	220	62	17	770	550	440
20	47	14	350	240	200	52	15	440	330	240	72	19	1,145	815	640
25	52	15	395	285	220	62	17	660	485	350	80	21	1,385	990	750
30	62	16	550	395	310	72	19	880	625	485	90	23	1,650	1,210	880
35	72	17	660	460	350	80	21	1,100	770	615	100	25	1,980	1,430	1,080
40	80	18	860	605	440	90	23	1,430	990	770	110	27	2,310	1,650	1,255
45	85	19	945	660	495	100	25	1,760	1,265	970	120	29	2,640	1,870	1,385
50	90	20	990	705	550	110	27	2,090	1,485	1,100	130	31	3,080	2,155	1,595
55	100	21	1,255	880	660	120	29	2,530	1,760	1,320	140	33	3,465	2,465	1,825
60	110	22	1,630	1,145	860	130	31	2,970	2,090	1,540	150	35	4,400	3,080	2,200
65	120	23	1,760	1,210	925	140	33	3,410	2,420	1,760	160	37	4,950	3,475	2,530
70	125	24	1,870	1,320	990	150	35	3,895	2,750	1,980	180	42	6,095	4,290	3,030
75	130	25	2,200	1,540	1,155	160	37	4,400	3,080	2,245	190	45	6,710	4,620	3,300
80	140	26	2,750	1,870	1,375	170	39	4,995	3,520	2,530	200	48	7,370	5,105	3,630
85	150	28	3,080	2,145	1,540	180	41	5,500	3,850	2,750	210	52	8,030	5,590	3,960
90	160	30	3,630	2,530	1,760	190	43	6,160	4,290	3,025	225	54	9,460	6,600	4,685
95	170	32	3,850	2,750	1,980	200	45	6,820	4,730	3,300	250	55	10,890	7,525	5,170
100	180	34	4,180	2,970	2,090	215	47	7,435	5,170	3,520	265	60	12,650	8,710	5,940
105	190	36	4,840	3,410	2,420	225	49	8,140	5,610	3,850	290	65	13,500	9,045	6,345
110	200	38	5,280	3,630	2,640	240	50	9,680	6,710	4,620	320	70	15,400	10,320	7,240

Table 57. Hess-Bright Thrust Ball Bearings

(Dimension letters refer to Fig. 116)

LIGHT SERIES WITH BALL CAGE

A mm.	B mm.	D mm.	E mm.	F in.	G in.	Load in pounds									Crane hook load, lb.
						1500 r.p.m.	1000 r.p.m.	500 r.p.m.	300 r.p.m.	150 r.p.m.	100 r.p.m.	50 r.p.m.	25 r.p.m.	10 r.p.m.	
45	75	19	47	1 11/16	3 1/2	640	770	900	1155	1410	1630	2200	2970	4,180	5,280
50	80	20	52	1 7/8	3 5/8	705	860	990	1265	1540	1760	2420	3255	4,620	5,940
55	90	22	57	2 1/16	4 1/8	860	990	1210	1595	1980	2245	3035	4005	5,500	9,010
60	95	22	62	2 1/4	4 1/4	900	1045	1265	1670	2090	2355	3170	4180	5,940	9,460
65	100	23	67	2 7/16	4 5/8	1100	1320	1705	2090	2530	2970	4025	5280	7,480	12,320
70	105	24	72	2 5/8	4 11/16	1155	1385	1780	2200	2770	3125	4225	5500	7,920	13,200
75	110	24	77	2 13/16	4 15/16	1210	1450	1870	2310	2860	3255	4400	5785	8,140	13,640
180	120	25	83	3	5 3/8	1320	1585	2035	2530	3080	3520	4775	6270	9,020	14,740
85	125	25	88	3 3/16	5 15/16	1430	1715	2200	2750	3300	3830	5170	6765	9,680	16,000
90	130	25	93	3 3/8	6	1485	1780	2310	2860	3410	3960	5370	7040	10,120	16,500
95	135	26	98	3 5/8	6 3/8	1540	1850	2420	2970	3520	4115	5545	7260	10,340	17,160
100	140	26	103	3 13/16	6 3/4	1595	1915	2530	3080	3630	4270	5720	7525	10,780	17,820

Medium Series with Ball Cage (without case)

A	B	C	D	E	F	G	Load in pounds									Crane hook load, lb.
							1500	1000	500	300	150	100	50	25	10	
mm.	mm.	mm.	mm.	mm.	in.	in.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	
10	30	32	14	12	5/16	1 5/8	145	190	245	285	330	395	540	660	1,100	1,100
15	35	35	15	17	1/2	1 3/4	185	245	310	365	430	505	660	825	1,320	1,320
20	42	42	16	22	11/16	2 1/16	240	320	395	485	550	650	870	1,080	1,760	1,760
25	47	47	17	27	7/8	2 1/4	295	395	495	585	680	770	1,045	1,355	2,200	2,420
30	53	55	18	32	1 1/8	2 5/8	330	440	550	660	770	880	1,175	1,520	2,420	3,520
35	62	64	21	37	1 1/4	2 15/16	440	550	660	880	990	1,285	1,725	2,245	3,300	4,400
40	64	66	21	42	1 1/2	3 1/16	550	660	770	990	1,210	1,395	1,905	2,400	3,520	5,500
45	73	75	25	47	1 11/16	3 1/2	660	770	880	1210	1,540	1,890	2,585	3,255	4,620	7,700
50	78	80	25	52	1 7/8	3 5/8	770	880	1100	1430	1,760	2,110	2,880	3,650	5,060	8,800
55	88	90	28	57	2 1/16	4 1/8	880	1100	1320	1650	2,220	2,465	3,255	4,355	6,380	9,900
60	90	92	28	62	2 1/4	4 1/4	990	1210	1540	1870	2,420	2,605	3,430	4,620	6,820	11,000
65	100	102	32	67	2 7/16	4 5/8	1210	1430	1760	2200	2,640	3,235	4,235	5,720	8,360	13,200
70	103	105	32	72	2 5/8	4 11/16	1320	1540	1980	2420	3,080	3,300	4,335	6,070	8,800	15,400
75	110	110	32	77	2 13/16	4 15/16	1430	1650	2090	2530	3,300	3,485	4,555	6,380	9,240	16,280
80	115	118	35	82	3	5 3/8	1540	1760	2420	2640	3,740	4,180	5,455	8,790	11,000	17,600
85	125	132	38	88	3 3/16	5 15/16	1870	2090	2860	3300	4,400	4,950	6,600	9,295	13,200	22,000
90	135	135	38	93	3 3/8	6	1980	2200	2970	3520	4,620	5,225	6,950	9,790	13,860	23,100
95	140	145	41	98	3 5/8	6 3/8	2200	2530	3520	4180	5,280	5,995	7,965	11,255	15,400	26,400
100	150	150	41	103	3 13/16	6 3/4	2420	2640	3740	4400	5,500	6,510	8,745	11,440	16,280	29,040
105	155	157	46	108	4	7	2640	3080	3960	4840	5,940	7,370	9,845	12,640	17,600	33,000
115	165	167	49	118	4 3/8	7 1/2	2860	3520	4840	5500	7,040	8,635	11,605	14,830	22,000	39,600
125	175	180	52	128	4 3/4	8	3080	4180	5280	6380	8,140	10,340	13,970	17,600	24,200	46,200
140	200	206	58	143	5 3/8	9	3740	4840	6600	8140	10,560	13,510	18,215	23,010	28,600	61,600

Special Heavy Series with Ball Cage

A	B and C	D	E	F	G	H	M	Load in pounds						Crane hook load, lb.
								1500	1000	500	300	150	10	
mm.	mm.	mm.	mm.	in.	in.	mm.	mm.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	r.p.m.	
10	45	25	15	7/16	2	18	28	375	440	530	660	835	2,420	3,740
15	50	27	20	5/8	2 3/16	26	29	460	570	695	835	1,080	3,145	4,840
20	60	27	25	13/16	2 11/16	32	30	570	695	835	1,045	1,320	3,870	5,940
25	65	30	30	15/16	2 7/8	38	32	660	825	990	1,230	1,585	4,620	7,040
30	70	32	35	1 1/8	3 1/16	44	35	815	990	1,190	1,475	1,870	5,500	8,360
35	75	34	40	1 5/16	3 1/4	50	36	925	1100	1,365	1,650	2,090	6,380	9,680
40	80	36	45	1 1/2	3 9/16	54	38	1010	1190	1,485	1,815	2,355	6,820	10,780
45	90	38	50	1 3/4	3 15/16	61	40	1165	1320	1,780	2,135	2,770	8,250	13,200
50	95	38	55	1 7/8	4 1/8	67	42	1265	1430	1,925	2,310	2,990	8,800	14,300
55	105	42	60	2 1/8	4 1/2	74	44	1540	1740	2,330	2,795	3,630	10,780	17,160
60	110	43	65	2 5/16	4 3/4	75	46	1650	1870	2,530	3,015	3,915	11,660	18,700
65	115	45	70	2 9/16	4 15/16	78	48	1935	2200	2,925	3,520	4,400	13,200	22,000
70	125	48	75	2 3/4	5 3/8	86	50	2045	2375	3,125	3,805	4,730	14,300	23,760
75	130	50	80	2 15/16	5 5/8	94	52	2200	2595	3,430	4,180	5,170	15,400	25,960
80	140	52	85	3 1/8	6 1/16	98	55	2575	2860	3,850	4,510	5,940	17,160	30,140
85	150	56	90	3 3/8	6 7/16	105	58	2860	3300	4,400	5,060	6,820	19,800	35,200
90	155	57	95	3 9/16	6 11/16	114	60	3300	3740	5,060	5,940	7,810	20,460	39,600
95	165	62	100	3 3/4	7 1/16	119	65	....	3960	5,345	6,600	8,800	22,000	44,000
100	170	62	105	3 15/16	7 1/4	122	68	....	4290	5,720	8,250	9,570	23,760	47,740
110	190	67	115	4 5/16	8 1/16	139	70	....	4995	6,600	8,360	11,000	27,720	55,000
120	205	72	125	4 3/4	8 11/16	147	75	....	5940	7,920	9,570	12,870	33,000	66,000
130	220	75	135	5 1/16	9 1/4	165	79	....	6930	9,240	11,220	15,070	38,500	77,000
140	230	80	145	5 5/16	9 5/8	169	85	....	....	9,900	12,650	16,940	44,000	88,000
150	255	85	155	5 11/16	10 5/8	188	90	....	....	11,660	14,190	19,140	50,600	101,200
160	270	88	165	6 1/16	11 1/4	198	93	....	....	12,320	15,400	21,120	55,000	110,000
170	290	92	175	6 1/2	12 1/16	208	99	....	....	13,860	17,600	23,540	62,700	125,400

Table 58. Dimensions and Load Ratings of SKF Self-aligning Ball Bearings

(Letters refer to Fig. 117)

Bore d, mm.	Diam., D, mm.	Width, B or B_1, mm.	Revolutions per min. 150	500	1500	Weight, lb.	Diam., D, mm.	Width, B or B_1, mm.	Revolutions per min. 150	500	1500	Weight, lb.
			Maximum load, lb.						Maximum load, lb.			
	Light type						Medium type					
10	30	9	230	165	110	0.07	35	11	310	230	145	0.13
12	32	10	265	175	120	0.09	37	12	410	300	200	0.15
15	35	11	355	255	175	0.11	42	13	465	330	220	0.21
17	40	12	430	300	210	0.16	47	14	665	465	310	0.29
20	47	14	560	410	265	0.26	52	15	685	485	330	0.36
25	52	15	760	550	365	0.32	62	17	1,050	750	485	0.57
30	62	16	1,015	730	485	0.48	72	19	1,320	935	660	0.86
35	72	17	1,170	850	540	0.73	80	21	1,710	1,210	770	1.14
40	80	18	1,500	1,070	730	0.93	90	23	2,120	1,510	1,040	1.60
45	85	19	1,700	1,210	795	1.03	100	25	2,760	1,990	1,320	2.14
50	90	20	1,875	1,320	915	1.18	110	27	2,980	2,200	1,430	2.72
55	100	21	2,315	1,700	1,150	1.56	120	29	3,860	2,810	1,820	3.53
60	110	22	2,700	1,930	1,300	2.02	130	31	4,410	3,250	2,150	4.41
65	120	23	2,910	2,095	1,400	2.54	140	33	5,290	3,860	2,430	5.49
70	125	24	3,360	2,430	1,650	2.82	150	35	6,070	4,410	2,920	6.65
75	130	25	3,640	2,590	1,700	3.01	160	37	7,050	4,960	3,200	8.05
80	140	26	3,970	2,870	1,950	3.75	170	39	7,280	5,300	3,530	9.38
85	150	28	4,850	3,530	2,450	4.85	180	42	10,300	7,280	4,850	11.79
90	160	30	5,520	3,910	2,650	5.84	190	44	11,000	7,830	5,180	13.73
95	170	32	6,620	4,850	3,200	7.16	200	46	13,100	9,500	6,290	16.55
100	180	34	7,280	5,180	3,350	8.82	215	58	13,600	9,710	6,620	19.95
105	190	36	8,380	6,060	3,950	10.25	225	60	16,000	11,200	7,720	22.30
110	200	38	9,260	6,620	4,400	12.23	240	64	17,400	12,100	7,950	25.75
	Heavy type						Extra heavy type					
15	52	15.0	750	550	365	0.44	52	20.0	850	605	385	0.53
17	62	17.0	1,050	770	495	0.65	62	22.0	1,150	795	530	0.79
20	72	19.0	1,345	970	660	0.99	72	27.0	1,870	1,320	880	1.24
25	80	21.0	1,710	1,210	770	1.30	80	30.5	2,540	1,820	1,100	1.61
30	90	23.0	2,095	1,545	990	1.82	90	33.0	3,140	2,210	1,460	2.39
35	100	25.0	2,760	1,985	1,320	2.43	100	37.0	3,750	2,670	1,820	2.96
40	110	27.0	2,975	2,205	1,435	3.03	110	40.0	4,410	3,040	2,040	3.82
45	120	29.0	3,860	2,750	1,800	3.93	120	42.0	5,070	3,640	2,430	4.77
50	130	31.0	4,420	3,100	2,205	5.04	130	45.0	5,850	4,080	2,760	5.88
55	140	33.0	5,300	3,850	2,420	6.23	140	47.5	7,280	5,180	3,530	7.55
60	150	35.0	6,060	4,400	2,865	7.64	150	49.0	8,280	5,740	3,860	8.88
65	160	37.0	7,600	5,300	3,550	8.82	160	52.0	9,050	6,620	4,410	10.70
70	180	42.0	10,350	7,280	4,850	13.34	180	59.0	11,600	8,160	5,300	16.00
75	190	45.0	11,000	7,720	5,180	16.10	190	64.0	12,800	9,350	5,950	19.00
80	200	48.0	13,000	9,370	6,500	17.85	200	67.0	13,800	9,710	[illegible] 170	21.10
85	210	54.5	13,000	9,370	6,500	22.30	210	72.0	14,600	10,700	6,620	24.30
90	225	60.0	16,000	11,250	7,700	24.80	225	76.0	18,200	12,600	8,270	30.90
95	250	66.0	18,200	13,000	8,500	33.62	250	84.0	20,200	14,300	9,930	41.30
100	265	68.5	19,850	14,340	9,050	40.25	265	89.5	22,100	16,500	11,000	48.50

The S.K.F. self-aligning ball bearings have a double row of balls in a staggered arrangement. The outer ball race (Fig. 117) is ground spherical to a radius struck from the center of the bearing, thereby giving freedom for perfect adjustment to shaft flexure. The two grooves of the inner race are ground to a radius slightly larger than that of the balls. The inner cage, retainer and balls rotate as a unit within the spherical surface of the outer race. Tables 58 and 59 give data on bearings of this type. Fig. 117 relates to Table 58; Fig. 118 to Table 59.

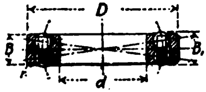


FIG. 117.—SKF Self-aligning Ball Bearing.

New Departure ball bearings for combined radial and axial loads are shown in Figs. 119 and 120. Tables 60 and 61 give the principal dimensions of these bearings and their capacities at various speeds.

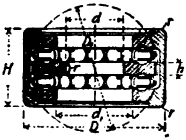


FIG. 118.
SKF Double Thrust Ball Bearing.

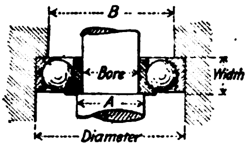


FIG. 119.

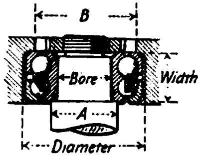


FIG. 120.
New Departure Ball Bearings for Combined Radial and Axial Loads.

Table 59. Dimensions and Load Ratings of SKF Self-contained Self-aligning Double Thrust Ball Bearings (1700 Series)
(Letters refer to Fig. 118)

d mm.	d_1 mm.	D mm.	H mm.	h mm.	Revolutions per min. 10	150	300	500	1,000	1,500	Weight, lb.
					Maximum load, in lb.						
20	27	52	32	5.5	2,920	905	730	640	520	410	0.63
25	32	62	32	5.5	3,400	1,060	880	770	605	485	0.97
30	42	72	40	6.5	4,190	1,265	1,100	925	730	550	1.45
35	47	80	46	7.0	5,730	1,710	1,500	1,265	1,025	770	2.02
40	52	90	46	7.0	6,620	1,985	1,765	1,435	1,215	880	2.49
45	57	100	54	8.5	7,280	2,205	1,875	1,545	1,320	990	3.97
50	62	110	54	8.5	7,280	2,205	1,875	1,545	1,320	990	4.41
55	72	120	60	10.0	9,700	2,870	2,540	2,205	1,765	1,325	6.06
60	77	125	60	10.0	10,140	3,530	2,810	2,330	1,820	1,455	6.25
65	82	130	66	11.0	11,020	3,640	3,030	2,425	1,930	1,575	7.50
70	88	140	72	12.0	12,560	4,350	3,530	2,755	2,205	1,820	10.15
75	93	150	72	12.0	15,210	5,300	4,190	3,310	2,650	2,205	11.02
80	98	160	80	12.5	17,410	6,060	4,850	3,640	2,870	2,425	13.25
85	103	170	80	12.5	18,180	6,280	5,070	3,860	3,140	2,650	15.87

Characteristics of Ball Bearings—Load Capacity. It is very necessary that the surfaces of the ball and of the race be not only accurate to dimensions but that the material of each should have a high elastic limit so that the loads encountered will not cause scaling and flaking. The ultimate or breaking load of the ball is in no way a measure of the capacity of the ball to take load safely.

When a hardened steel ball is placed between two other like balls and subjected to pressure, the **crushing load** is between $40{,}000d^2$ and $80{,}000d^2$ lb., where d is the ball diameter in inches. If placed between two plane surfaces the crushing load is greater. The **working load** must be much lower and may be determined from the formula $P = cd^2$, where P is the load on a single ball, lb., and c a coefficient having the following values determined by Stribeck and Bach:

Hard cast-iron balls between flat surfaces, 36; hard steel balls on flat, conical or cylindrical surfaces: intermittent running, 1400; continuous running, 700 to 1000; hard steel balls in races whose ball tracks have a radius of curvature = $\frac{2}{3}d$: intermittent running, 2800; continuous running, 1400 to 2000.

Table 60. New Departure Ball Bearings for Combined Radial and Axial Loads

(Single row of balls, Fig. 119)

Bore, mm.	Diam., mm.	Width, mm.	Balls		Inside shoulder, A, in.	Outside shoulder B, in.	Load, 100 r.p.m.	Load, 300 r.p.m.	Load, 600 r.p.m.	Load, 1500 r.p.m.	Load, 3000 r.p.m.
			Diam., in.	No.							
20	47	14	1/4	11	1.027	1.470	340	280	210	145	95
	52	15	5/16	10		1.595	485	435	345	220	125
	52	15	9/32	11	1.224	1.667	435	365	275	170	115
25	62	17	3/8	11		1.988	750	640	515	320	205
	80	21	5/8	8		2.541	1400	1070	775	450	295
	62	16	5/16	12	1.471	2.081	590	500	390	215	145
30	72	19	7/16	10		2.362	1000	840	690	435	265
	90	23	11/16	8		2.915	1990	1670	1270	610	380
	72	17	3/8	12	1.668	2.362	850	710	550	315	185
35	80	21	1/2	10		2.521	1320	1110	885	550	335
	100	25	3/4	8		3.309	2270	1970	1590	755	440
	80	18	3/8	14	1.915	2.657	1150	935	710	420	225
40	90	23	9/16	10		2.895	1620	1400	1140	690	425
	110	27	13/16	8		3.526	2640	2290	1920	980	530
	85	19	7/16	14	2.112	2.854	1250	1000	775	450	245
45	100	25	5/8	10		3.288	2050	1790	1500	890	540
	120	29	7/8	8		3.919	3130	2800	2370	1470	700
	90	20	3/8	16	2.339	3.031	1480	1230	910	520	290
50	110	27	11/16	10		3.662	2400	2050	1750	1100	650
	130	31	15/16	10		4.293	3360	3150	2630	1700	825
55	100	21	7/16	16	2.535	3.425	1800	1540	1180	640	375
	120	29	3/4	10		3.899	2800	2410	1990	1310	765
60	110	22	1/2	16	2.762	3.642	2000	1760	1370	750	430
	130	31	13/16	12		4.273	3350	2760	2325	1500	880
65	120	23	1/2	16	2.959	4.036	2340	2090	1680	925	520
	140	33	7/8	10		4.667	4250	3300	2625	1700	1030
70	125	24	1/2	18	3.182	4.213	2500	2220	1840	1020	580
	150	35	15/16	10		5.041	4900	4000	2960	1880	1200
75	130	25	9/16	16	3.383	4.409	2720	2440	2025	1200	650
80	140	26	5/8	16	3.610	4.783	3150	2820	2380	1500	825

Table 61. New Departure Ball Bearings for Combined Radial and Axial Loads

(Double row of balls, Fig. 120)

Bore, mm.	Diam., mm.	Width, in.	Balls per row Diam., in.	Balls per row No.	Inside shoulder A, in.	Outside shoulder B, in.	Load, 100 r.p.m.	Load, 300 r.p.m	Load, 600 r.p.m.	Load, 1500 r.p.m.	Load, 3000 r.p.m.
10	30	1/2	3/16	8	0.554	0.900	230	195	140	80	50
	35	3/4	1/4	8		1.038	375	315	255	150	85
12	32	1/2	3/16	10	0.632	0.960	295	250	200	95	65
	37	3/4	1/4	8		1.117	500	440	360	220	125
15	35	1/2	3/16	11	0.791	1.070	350	305	245	130	75
	42	3/4	1/4	10		1.294	625	540	440	270	160
17	40	1/2	3/16	12	0.869	1.255	440	380	320	195	95
	47	7/8	5/16	8		1.418	800	650	515	330	200
	62	1 3/16	7/16	8		2.008	1,700	1,500	1,220	620	370
20	47	3/4	1/4	11	1.027	1.470	590	490	415	270	150
	52	7/8	5/16	9		1.595	930	750	575	365	220
	72	1 3/8	1/2	8		2.226	2,020	1,800	1,500	.910	450
25	52	3/4	1/4	12	1.224	1.667	775	640	500	335	205
	62	1.00	3/8	9		1.988	1,550	1,310	1,000	550	340
	80	1 3/8	1/2	8		2.541	2,400	2,120	1,760	1,150	560
30	62	3/4	1/4	12	1.471	2.081	1,100	880	670	410	270
	72	1 3/16	7/16	9		2.362	2,000	1,730	1,385	750	445
	90	1 9/16	9/16	10		2.915	3,180	2,830	2,320	1,560	950
35	72	7/8	5/16	14	1.668	2.362	1,530	1,320	1,035	565	365
	80	1 3/8	1/2	10		2.521	2,530	2,220	1,815	1,070	570
	100	1 3/4	5/8	10		3.309	3,500	3,180	2,670	1,750	1,130
40	80	1.00	3/8	12	1.915	2.657	1,900	1,630	1,325	730	435
	90	1 7/16	17/32	11		2.895	3,100	2,700	2,245	1,390	750
	110	1 15/16	11/16	10		3.526	3,920	3,600	3,030	2,000	1,330
45	85	1.00	3/8	15	2.112	2.854	2,060	1,750	1,415	800	470
	100	1 9/16	9/16	11		3.288	3,750	3,390	2,850	1,850	1,100
	120	2 1/8	3/4	10		3.919	4,700	4,250	3,630	2,500	1,650
50	90	1.00	3/8	16	2.339	3.031	2,200	1,900	1,620	890	500
	110	1 3/4	5/8	11		3.662	4,400	3,800	3,275	2,180	1,330
	130	2 5/16	13/16	10		4.293	5,300	4,600	3,950	2,800	1,820
55	100	1 3/16	7/16	15	2.535	3.425	2,850	2,400	1,960	1,200	650
	120	1 15/16	11/16	11		3.899	5,250	4,500	3,690	2,510	1,600
	140	2 1/2	7/8	11		4.687	6,200	5,200	4,360	3,100	2,000
60	110	1 3/8	1/2	14	2.762	3.642	3,180	2,700	2,190	1,350	750
	130	2 1/8	3/4	11		4.273	6,200	5,200	4,200	2,850	1,850
	150	2 5/8	15/16	11		5.061	7,700	6,000	4,720	3,400	2,300
65	120	1 3/8	1/2	16	2.959	4.036	3,700	3,250	2,635	1,610	950
	140	2 5/16	13/16	11		4.667	7,000	6,000	4,825	3,200	2,100
	160	2 13/16	1	11		5.454	9,700	7,200	5,225	3,650	2,500
70	125	1 7/16	1/2	16	3.186	4.213	3,950	3,470	2,900	1,800	1,090
	150	2 1/2	7/8	11		5.041	8,100	6,900	5,550	3,500	2,400
	180	3 1/8	1 3/16	10		6.097	12,900	10,600	7,000	4,250	3,000
75	130	1 7/16	1/2	17	3.383	4.409	4,400	3,750	3,200	2,000	1,230
	160	2 11/16	15/16	11		5.434	9,500	8,100	6,400	3,900	2,700
	190	3 1/4	1 1/4	10		6.490	15,800	13,400	9,900	4,800	3,400

Table 61. New Departure Ball Bearings for Combined Radial and Axial Loads—(*continued*)

Bore, mm.	Diam., mm.	Width, in.	Balls per row: Diam., in.	Balls per row: No.	Inside shoulder A, in.	Outside shoulder B, in.	Load, 100 r. p. m.	Load, 300 r. p. m.	Load, 600 r. p. m.	Load, 1500 r. p. m.	Load, 3000 r. p. m.
	140	1⅝	9/16	18		4.783	5,300	4,500	3,700	2,400	1,470
80	170	2 11/16	1.00	12	3.610	5.808	11,000	9,100	7,200	4,400	2,950
	200	3 7/16	1 5/16	10		6.864	17,200	15,000	11,400	5,300	3,650
	150	1¾	9/16	18		5.177	5,900	4,850	4,000	2,640	1,620
85	180	2⅞	1 1/16	12	3.807	6.077	13,000	10,300	8,200	4,950	3,250
	210	3⅝	1⅜	10		7.258	18,700	16,300	12,750	5,800	3,850
	160	2.00	11/16	17		5.551	7,400	6,000	4,700	3,120	1,920
90	190	2⅞	1⅛	12	4.033	6.450	15,000	12,000	9,100	5,600	3,500
	225	3⅞	1½	10		7.828	21,600	18,700	15,000	7,500	4,300
	170	2 3/16	¾	16		5.788	8,600	7,200	5,400	3,500	2,200
95	200	3 1/16	1 3/16	12	4.230	6.844	16,800	14,000	10,000	6,200	3,800
	250	4 3/16	1⅝	10		8.813	26,000	23,000	18,600	11,000	5,000
	180	2⅜	13/16	16		6.162	10,000	8,400	6,500	3,900	2,500
100	215	3¼	1¼	12	4.457	7.415	18,700	16,000	12,000	6,800	4,200
	265	4½	1¾	10		9.383	29,000	26,000	21,000	13,000	5,900
105	190	2 9/16	⅞	16	4.654	6.555	11,700	9,800	7,600	4,300	2,900
	225	3 7/16	1 5/16	12		7.808	20,000	17,200	13,400	7,500	4,500
110	200	2¾	15/16	16	4.681	6.929	13,400	11,300	8,750	4,900	3,250
	240	3⅝	1⅜	12		8.379	22,000	19,000	15,000	8,200	4,900

For three-point or four-point contact but one-third of these latter values should be used. The above values are limits possible with good workmanship and materials. Lower values must be employed when the quality of either the materials or workmanship is uncertain and also when high speeds are called for. For a single ring of balls, from 10 to 20 in number, the greatest load on any one ball, according to Stribeck, is $P = 5Q/n$, where Q is the total load on the ring. Equating $P = cd^2$ with $P = 5Q/n$, it is found that $d = \sqrt{5Q/cn}$.

The **starting friction** of ball bearings is very low. For **running friction,** see p. 244.

The Hess-Bright Mfg. Co. use the following formulæ for determining the **load-carrying capacities of ball bearings:**

For radial or annular bearings, $P = knd^2$, where n is the number of balls required to fill the race, k is a coefficient depending on the type of bearing, the material and the speed, and d is the ball diameter in eighths of an inch (*i.e.*, for a 1-in. ball $d = 8$). For uninterrupted ball tracks, the inner race being grooved to a radius $= 0.52 \times$ ball diam. and the outer race to a radius $= 0.625 \times$ ball diam., separated balls, uniform load and steady speed up to 3000 r.p.m., $k = 9$; for full-type bearings, the filling opening being in one race at the unloaded side, otherwise as above, $k = 5$; for both ball tracks interrupted by filling openings, in elastic cage separators for the balls, or for the full ball type, speeds not above 2500 r.p.m., uniform load, $k = 2.5$; for thrust on a radial bearing of the first type, as above, $k = 0.9$. In general, the larger the balls the smaller the value of k. The type with filling openings in each race is not suitable for withstanding end thrust.

For thrust bearings consisting of one flat plate and one flat seat plate with grooved ball races, $P = k_1nd^2/\sqrt[3]{R}$, where R = r.p.m. (up to 3000) and $k_1 = 25$ to 40 for materials used by the Hess-Bright Co., with ball race grooved to a radius of $0.82d$; $k_1 = 0.5$ for unhardened steel occasionally used for very large races where there is no hammering or sharp blows. When both races are flat, $k_1 = 6$ to 10 and 0.125 respectively for hardened and unhardened races.

For ball thrust bearings with two-point contact having a groove in each washer, the Standard Roller Bearing Co. uses the formula $P = knd^2/a\sqrt{R}$, where d is the ball diam. in in., a the pitch diam. of the ball grooves and $k = 32{,}000$ for light-type bearings with 17 balls and 19,200 for medium and heavy bearings with 11 and 9 balls, respectively.

Professor John Goodman (*Engr*. Dec. 19, 1913) gives the following formula for determining the maximum working load P on a ball bearing:

$P(\text{lb.}) = knd^2/(RD + Cd)$, in which d = the ball diam., in.; D = inside diam. of outer ball race, being taken from point of contact of ball with race, or for a thrust bearing, D = diam. taken from the center of balls, both in in.; n and R as in previous paragraphs. C and K are coefficients having the following values:

	C	K
Thrust bearings, flat races	200	500,000
Thrust bearings, grooved races	200	1,000,000 to 1,250,000
Radial bearings, flat races	2000	1,000,000
Radial bearings, grooved races	2000	2,000,000 to 2,500,000

Roller Bearings

Roller bearings are used on automobiles, trucks, line shafts and in a variety of machines where heavy loads have to be carried. **Typical bearings** are shown in Figs. 121 and 122. Fig. 121 illustrates a step bearing with conical rollers. The end thrust in such construction is liable to be excessive, in which case a ball at the larger end of each roller is employed. Fig. 122 shows a step bearing with short cylindrical rollers which has been found to be very successful. To equalize the load on the rollers, the bottom plate is made to fit in a lubricated spherical seat.

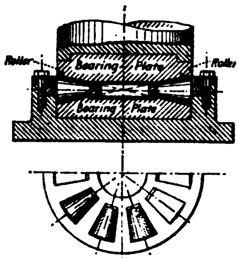


FIG. 121.—Roller Step Bearing.

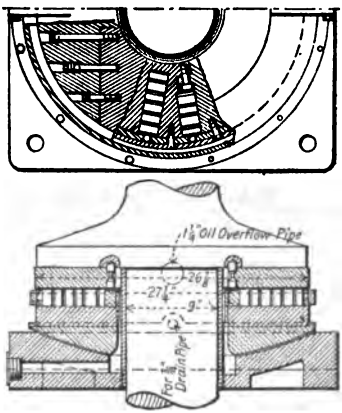


FIG. 122.—Roller Step Bearing.

In the **Hyatt roller bearing,** Fig. 123, the rollers are cylindrical and hollow, being wound helically from flat strip chrome-nickel steel and ground off on the ends to the proper length. The windings are alternately right- and left-hand. They are flexible and adjust themselves easily under ordinary pressures to small inaccuracies of the journal or box. In the "standard" type the rollers are in direct contact with the shaft; in the "heavy duty" type the specific pressures are greater and the shaft must be hardened by heat treatment at the place of contact with the rollers, or else a hard steel sleeve be interposed to form the bearing surface. For the standard type (*a*) the allowable pressure at speeds up to 500 r.p.m. is between 400 and 500 lb. per sq. in.; for line-shaft bearings up to $3\frac{15}{16}$ in. diam. running up to 600 r.p.m., 30 lb. per sq. in. For the high-duty type (*b*), allowable pressure at 1000 r.p.m. is 750 lb. per sq. in. Limiting maximum speeds are (*a*) 1500 r.p.m. and (*b*) 3000 r.p.m. The permissible loads fall off rapidly with increase of speed, for example, at 1500 r.p.m. the load on high-duty type should not exceed one-half that at 1000 r.p.m. Loads can be correspondingly increased at low speeds. Tables 62 and 63 give some dimensions of Hyatt bearings.


FIG. 123. Hyatt Roller Bearing.

In the **Norma roller bearing,** Fig. 124, the rollers are cylindrical and solid; the inner race is cylindrical while the outer race is slightly convex, giving some freedom for adjustment in case of flexure or imperfect alignment of the shaft. The capacities of these bearings at various speeds are shown in Table 64.

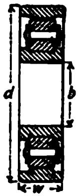


FIG. 124. Norma Roller Bearing.

Load Capacities of Roller Bearings. For rollers such as are used on bridge **turn tables** and in similar places where the speeds of rotation are slow, the safe load in pounds is $P = cnld$, in which n = number of rollers in the bearing, l and d the length and diameter of each roller, in., $c = 360$ for hard cast-iron rollers running on hard cast-iron tracks, and $c = 850$ for hard steel rollers on hard steel tracks. Where $l > 5d$, smaller values of c should be used. In case conical rollers are used, d is the mean diameter.

The Standard Roller Bearing Co. determines load capacities of roller bearings by the formula $P = 130{,}000d^2nl/3s$, in which P = load on bearing, lb.; d = diam. of roller, in.; n = number of rollers; l = length of each roller, in., and s = circumferential speed of each roller, ft. per min. In bearings with conical rollers d is the diameter of the roller at its mid-length. The safe load per inch of length of a solid roller is taken at 2000 lb., with the assumption that one-third the number of rollers take the whole load on the journal.

Table 62. Hyatt Roller Bearings. Standard Type

Diam. of shaft, in.	For medium and heavy loads and speeds up to 1500 r.p.m.		For light loads and speeds up to 500 r.p.m.		Diam of shaft, in.	For medium and heavy loads and speeds up to 1500 r.p.m.		For light loads and speeds up to 500 r.p.m.	
	Diam. of roller, in.	Bore for housing, in	Diam. of roller, in.	Bore for housing, in.		Diam. of roller, in.	Bore for housing, in.	Diam. of roller, in.	Bore for housing, in.
1	1/2	2 3/16	3/8	1 15/16	1 15/16	7/8	3 15/16	5/8	3 7/16
1 1/8	1/2	2 5/16	7/16	2 3/16	2	7/8	4	5/8	3 1/2
1 3/16	5/8	2 5/8	1/2	2 3/8	2 3/16	7/8	4 3/16	5/8	3 11/16
1 1/4	5/8	2 11/16	1/2	2 7/16	2 1/4	7/8	4 1/4	5/8	3 3/4
1 3/8	5/8	2 13/16	1/2	2 9/16	2 7/16	15/16	4 9/16	11/16	4 1/16
1 7/16	3/4	3 1/8	9/16	2 3/4	2 1/2	15/16	4 5/8	11/16	4 1/8
1 1/2	3/4	3 3/16	9/16	2 13/16	2 11/16	15/16	4 13/16	11/16	4 5/16
1 5/8	11/16	3 3/16	9/16	2 15/16	2 3/4	15/16	4 7/8	11/16	4 3/8
1 11/16	3/4	3 3/8	9/16	3	2 15/16	1	5 3/16	3/4	4 11/16
1 3/4	3/4	3 7/16	9/16	3 1/16	3	1	5 1/4	3/4	4 3/4
1 7/8	3/4	3 9/16	9/16	3 3/16	....	....	....	....	....

Length of bore from 2 in. to 7 in. by 1-in. intervals.

Table 63. Hyatt Roller Bearings. High-duty Type

Diam. of shaft with sleeve, in.	Diam. of shaft or axle without sleeve, in.	Outside diam. over all, in.	Short series: Length over all, in.	Short series: Safe load in lb.	Long series: Length over all, in.	Long series: Safe load in lb.	Diam. of shaft with sleeve, in.	Diam. of shaft or axle without sleeve, in.	Outside diam. over all, in.	Short series: Length over all, in.	Short series: Safe load in lb.	Long series: Length over all, in.	Long series: Safe load in lb.
0.750	1.000	2.249	1.000	460	2.000	1200	1.625	2.000	4.124	1.375	1470	2.750	3490
0.875	1.125	2.374	1.000	500	2.000	1340	1.750	2.125	4.249	1.375	1550	2.750	3700
.....	1.250	2.749	1.125	700	2.250	1700	1.875	2.250	4.374	1.375	1650	2.750	3925
1.000	1.375	2.874	1.125	750	2.250	1900	2.000	2.500	4.749	1.500	2060	3.000	4825
1.125	1.500	3.374	1.250	960	2.500	2340	2.250	2.750	4.999	1.500	2270	3.000	5300
1.250	1.625	3.499	1.250	1040	2.500	2530	2.500	3.000	5.374	1.750	3030	3.500	6890
1.375	1.750	3.624	1.250	1125	2.500	2730	2.750	3.250	5.624	1.750	3400	3.500	7600
1.500	1.875	3.749	1.250	1200	2.500	2925	......	......	......	......	......	......	....

The tabular safe loads may be doubled under favorable circumstances.

Table 64. Norma Roller Bearings

Inside diam., mm. (*b*, Fig. 124)	Load capacity in lb. at various r.p.m.											
	Light service				Medium service				Heavy service			
	10	300	500	1500	10	300	500	1500	10	300	500	1500
25	.....	.....	.....	.....	1,650	1,320	1,190	770	3,410	2,750	2,420	1,630
30	1,430	1,170	1,030	660	2,150	1,760	1,540	1,000	3,850	3,080	2,750	1,830
35	2,090	1,700	1,500	990	2,750	2,270	1,980	1,320	4,400	3,520	3,080	2,050
40	2,375	1,940	1,720	1,120	3,520	2,860	2,530	1,650	5,940	4,840	4,400	2,860
45	2,530	2,100	1,830	1,190	4,400	3,520	3,190	1,980	7,260	5,940	5,280	3,300
50	2,680	2,180	1,940	1,280	5,280	4,180	3,740	2,420	8,580	7,040	6,160	3,960
55	3,850	3,080	2,800	1,830	6,600	5,390	4,840	3,080	9,460	7,700	6,600	4,400
60	4,510	3,630	3,300	2,130	7,700	6,270	5,610	3,630	11,220	9,020	7,920	5,280
65	5,280	4,270	9,740	2,480	8,360	6,820	5,940	3,850	12,100	9,680	8,580	5,720
70	5,500	4,400	3,960	2,640	10,120	8,340	7,260	4,620	16,060	13,200	11,660	7,480
75	5,830	4,730	4,250	2,750	11,880	9,680	8,580	5,500	18,040	14,520	12,980	8,470
80	6,710	5,440	4,840	3,190	12,980	10,560	9,460	5,800	18,260	14,740	13,200	8,580
85	8,030	6,600	5,720	3,740	14,080	11,440	10,120	6,160	19,140	15,400	13,860	9,020
90	9,680	7,810	7,040	4,510	16,280	13,200	11,660	......	23,320	18,700	16,720	......
95	11,440	9,240	8,140	5,280	17,380	14,080	12,540	......	27,280	22,440	20,020	......
100	12,980	10,340	9,240	5,940	21,120	16,940	14,960	......	37,400	29,700	26,840	......

The above ratings are for steady loads and speeds; 50 per cent. temporary overload capacity can be added. External dimensions *d* and *w*, Fig. 124, are the same as for Hess-Bright bearings, Table 56. These bearings are also made with inside diameters (*b*) up to 215 mm. for the three types of service, with load capacities (at 300 r.p.m.) up to 34,300 lb. (light), 49,600 lb. (medium), and 67,500 lb. (heavy).

Another formula for the safe load on a roller bearing is $P = kld^2/(ND + 2000d)$, in which N = r.p.m. of the shaft; D = diam. of the sleeve or roller path, in.; and k = 1,200,000 to 2,000,000 for first-class workmanship, hardened steel rollers with $l = d$, running on hardened ground surfaces; k = 400,000 for ordinary workmanship and soft steel rollers running on a soft steel shaft.

GEARING

Spur Gears

Definitions. The various **parts of spur-gear teeth** are shown and named in Fig. 125. The **diametral pitch** (sometimes referred to as simply the pitch) is the ratio of the number of teeth in the gear to the diameter of the pitch circle measured in inches, or the number of teeth in the gear per inch of pitch circle diameter ($= \pi$/circular pitch in inches). The **line of action** of involute teeth is the tangent to the base circles of the two gears which passes through the point of tangency of their pitch circles; and the **angle of obliquity** (or **pressure angle**) is the angle included between the line of action and the common tangent to the pitch circles.

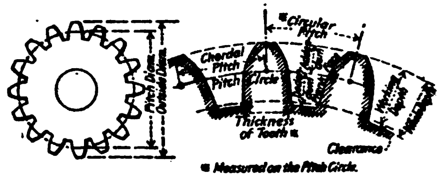


FIG. 125.—Spur Gear Teeth.

Outlines of Gear Teeth. The outlines of gear teeth are either involute curves or cycloidal curves. An **involute curve** is one generated by a point in a taut cord unwrapped from a **base circle**, or circle concentric with and inside the pitch circle. A cycloidal curve is one generated by a point on the circumference of a rolling circle. If the circle rolls on a straight line the curve is **cycloidal**; if on the outside of another circle, it is an **epicycloidal** curve, and if on the inside of another circle, a **hypocycloidal** curve.

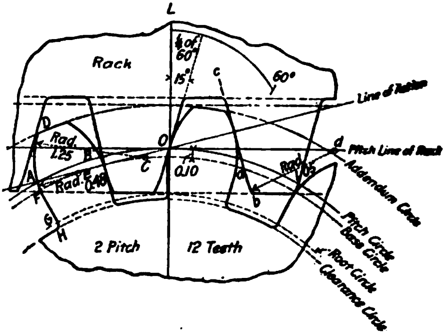


FIG. 126.—Grant's Odontograph for Involute Teeth.

The outlines of **involute teeth to be molded from patterns** may be laid out by means of Grant's **odontograph** for involute teeth, shown in Fig. 126. Referring to the figure, first draw the pitch, root, addendum and clearance circles. (For the relations of the latter three circles to the pitch circle, see p. 724.) Draw the base circle concentric with the pitch circle and with a radius of $^{29}/_{30}$ of the pitch circle radius. Lay off the circular pitch AO and thickness of tooth AB on the pitch circle. With a center C on the base circle and the face radius CA (obtained from Table 65) draw in the face outline AD of the tooth from the pitch circle to the addendum circle. With a center E on the base circle and the flank radius EA (obtained from Table 65) draw AF or that portion of the flank outline between the pitch circle and the base circle. From the base circle to the root circle the tooth outline FG is a radial line which rounds into the clearance circle at H. The outline of an **involute rack tooth** is drawn as follows: Draw a straight line ac making an angle of 15 deg. with the radius line OL. Draw ab from the center d on the rack pitch line with a radius equal to 0.67 in. multiplied by the circular pitch (or to 2.10 in.) divided by the diametral pitch. This arc establishes the necessary clearance in order to avoid interference.

The outlines of **epicycloidal teeth to be molded from patterns** may be similarly laid out by means of Grant's epicycloidal or three-point odontograph. See Fig. 127. Draw the pitch, addendum, root and clearance circles, and lay off the circular pitch and tooth thickness as before. Also lay off the face

Table 65. Grant's Odontograph for Involute Teeth

Pressure angle = 15 deg. Addendum = 0.3183 × circular pitch = 1/diametral pitch. Clearance = Addendum/8.

No. of teeth	Divide by the diametral pitch		Multiply by the circular pitch		No. of teeth	Divide by the diametral pitch		Multiply by the circular pitch	
	Face rad.	Flank rad.	Face rad.	Flank rad.		Face rad.	Flank rad.	Face rad.	Flank rad.
10	2.28	0.69	0.73	0.22	28	3.92	2.59	1.25	0.82
11	2.40	0.83	0.76	0.27	29	3.99	2.67	1.27	0.85
12	2.51	0.96	0.80	0.31	30	4.06	2.76	1.29	0.88
13	2.62	1.09	0.83	0.34	31	4.13	2.85	1.31	0.91
14	2.72	1.22	0.87	0.39	32	4.20	2.93	1.34	0.93
15	2.82	1.34	0.90	0.43	33	4.27	3.01	1.36	0.96
16	2.92	1.46	0.93	0.47	34	4.33	3.09	1.38	0.99
17	3.02	1.58	0.96	0.50	35	4.39	3.16	1.39	1.01
18	3.12	1.69	0.99	0.54	36	4.45	3.23	1.41	1.03
19	3.22	1.79	1.03	0.57	37–40	4.20	4.20	1.34	1.34
20	3.32	1.89	1.06	0.60	41–45	4.63	4.63	1.48	1.48
21	3.41	1.98	1.09	0.63	46–51	5.06	5.06	1.61	1.61
22	3.49	2.06	1.11	0.66	52–60	5.74	5.74	1.83	1.83
23	3.57	2.15	1.13	0.69	61–70	6.52	6.52	2.07	2.07
24	3.64	2.24	1.16	0.71	71–90	7.72	7.72	2.46	2.46
25	3.71	2.33	1.18	0.74	91–120	9.78	9.78	3.11	3.11
26	3.78	2.42	1.20	0.77	121–180	13.38	13.38	4.26	4.26
27	3.85	2.50	1.23	0.80	181–360	21.62	21.62	6.88	6.88

Table 66. Grant's Odontograph for Epicycloidal Teeth

Addendum = 0.3183 × circular pitch = 1/diametral pitch. Clearance = Addendum/8.

Number of teeth in the gear		For 1 diametral pitch; for any other pitch, divide by that pitch				For 1-in. circular pitch; for any other pitch, multiply by that pitch			
		Faces		Flanks		Faces		Flanks	
Exact	Intervals	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.	Rad.	Dis.
10	10	1.99	0.02	−8.00	4.00	0.62	0.01	−2.55	1.27
11	11	2.00	0.04	−11.05	6.50	0.63	0.01	−3.34	2.07
12	12	2.01	0.06	∞	∞	0.64	0.02	∞	∞
13½	13–14	2.04	0.07	15.10	9.43	0.65	0.02	4.80	3.00
15½	15–16	2.10	0.09	7.86	3.46	0.67	0.03	2.50	1.10
17½	17–18	2.14	0.11	6.13	2.20	0.68	0.04	1.95	0.70
20	19–21	2.20	0.13	5.12	1.57	0.70	0.04	1.63	0.50
23	22–24	2.26	0.15	4.50	1.13	0.72	0.05	1.43	0.36
27	25–29	2.33	0.16	4.10	0.96	0.74	0.05	1.30	0.29
33	30–36	2.40	0.19	3.80	0.72	0.76	0.06	1.20	0.23
42	37–48	2.48	0.22	3.52	0.63	0.79	0.07	1.12	0.20
58	49–72	2.60	0.25	3.33	0.54	0.83	0.08	1.06	0.17
97	73–144	2.83	0.28	3.14	0.44	0.90	0.09	1.00	0.14
290	145–300	2.92	0.31	3.00	0.38	0.93	0.10	0.95	0.12
∞	Rack	2.96	0.34	2.96	0.34	0.94	0.11	0.94	0.11

center and flank center circles with the distances from the pitch circle taken from columns 4 and 6 of Table 66. With radii for flanks and faces obtained from columns headed "Rad" in Table 66, draw the arcs forming the flanks and faces of the teeth. Fig. 127 shows the layout of a 20-tooth gear of 2 diametral pitch.

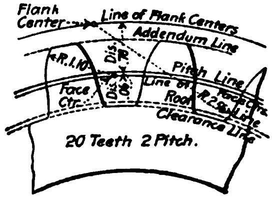


FIG. 127.—Grant's Odontograph for Epicycloidal Teeth.

Relative Proportions of Gear Teeth. In any set of interchangeable gears having cycloidal tooth outlines, the same rolling circle must be used for both faces and flanks of all the gears in the set. To avoid undercutting of the teeth, the rolling circle is made equal in diameter to one-half the pitch circle of the smallest gear in the set, which is usually limited to 12 or 15 teeth. Involute gears to be interchangeable must have the same angle of obliquity. Involute gears are usually preferred by manufacturers (one reason being that a slight change in the center distance does not impair the accuracy of their working), and the following relative proportions have been established (c.p. = circular pitch; d.p. = diametral pitch):

	Brown & Sharpe	C. W. Hunt Co.
Addendum	1/d.p. = 0.3183 c.p.	0.7854/d.p. = 0.25 c.p.
Dedendum	1.157/d.p. = 0.3683 c.p.	0.9424/d.p. = 0.30 c.p.
Working depth	2/d.p. = 0.6366 c.p.	1.5708/d.p. = 0.50 c.p.
Whole depth	2.157/d.p. = 0.6866 c.p.	1.7278/d.p. = 0.55 c.p.
Clearance	0.157/d.p. = 0.0500 c.p.	0.157/d.p. = 0.05 c.p.
Angle of obliquity	14½ deg.	14½ deg.

The proportions used by Wm. Sellers & Co., Inc. and the Grant Gear Works are the same as those given for Brown & Sharpe, excepting that the Sellers angle of obliquity is 20 deg. and the Grant 15 deg. The Fellows Gear Shaper Co. has established a stub-tooth standard having an angle of obliquity of 20 deg. Proportions are as given in Table 67.

Table 67. Fellows Stub-tooth System

Diametral pitch	Thickness of tooth	Addendum	Clearance	Whole depth of tooth	Diametral pitch	Thickness of tooth	Addendum	Clearance	Whole depth of tooth
4/5	0.3927	0.2000	0.0500	0.4500	8/10	0.1963	0.1000	0.0250	0.2250
5/7	0.3142	0.1429	0.0357	0.3214	9/11	0.1745	0.0909	0.0227	0.2045
6/8	0.2618	0.1250	0.0312	0.2812	10/12	0.1571	0.0833	0.0208	0.1875
7/9	0.2244	0.1111	0.0278	0.2500	12/14	0.1309	0.0714	0.0179	0.1607

Thickness of tooth same as for 14½-deg. gear of same pitch as numerator of stub tooth pitch fraction. Addendum equals reciprocal of denominator of stub tooth pitch fraction; clearance equals .25 divided by denominator of stub tooth pitch fraction.

Cutters for Gear Teeth. Standard cutters may be had for cutting either involute or cycloidal teeth of any system. For the stub-tooth system the Fellows gear shaper must be used. Gear teeth may also be hot-rolled, but for high-class work should be finished with a cutter.

The Brown & Sharpe Mfg. Co. has established **designations for rotating cutters** for gear teeth, which have become standard. These are given in Table 68.

Table 68. Standard Designations of Gear Cutters

Cutters for cycloidal teeth						Cutters for involute teeth			
Cutter	No. of teeth	Cutter	No. of teeth	Cutter	No. of teeth	Cutter	No. of teeth	Cutter	No. of teeth
A	12	*I*	20	*Q*	43–49	1	135–rack	5	21–22
B	13	*J*	21–22	*R*	50–59	1½	80–134	5½	19–20
C	14	*K*	23–24	*S*	60–74	2	55– 79	6	17–18
D	15	*L*	25–26	*T*	75–99	2½	42– 54	6½	15–16
E	16	*M*	27–29	*U*	100–149	3	35– 41	7	14
F	17	*N*	30–33	*V*	150–249	3½	30– 34	7½	13
G	18	*O*	34–37	*W*	250 & up	4	26– 29	8	12
H	19	*P*	38–42	*X*	Rack	4½	23– 25		

The Speed Ratios of Spur Gears are inversely proportional to their pitch diameters. The pitch diameters necessary for any given speed ratio and distance of shaft centers may be obtained in the following manner:

Let l = distance between shaft centers, in.; $r = \dfrac{\text{radius of smaller gear}}{\text{radius of larger gear}}$.

Then

Diam. of smaller gear (D_s), in. $= 2l/(1 + 1/r)$

Diam. of larger gear (D_l), in. $= 2l/(r + 1)$

Accordingly, if $l = 24$ in. and $r = 1/5$, $D_l = 48/(6/5) = 40$ in., and $D_s = 48/6 = 8$ in.

For **epicyclic gear trains,** see p. 657.

The speed ratios for the more common gear trains are as follows:

Fig. 128: r.p.m. of N = r.p.m. of $R \times \left(1 + \dfrac{\text{diam. of } F}{\text{diam. of } N}\right)$

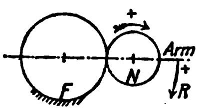


FIG. 128.

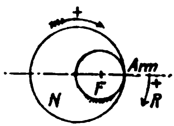


FIG. 129.

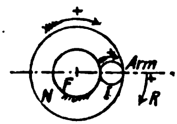


FIG. 130.

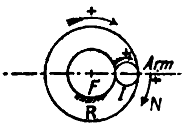


FIG. 131.

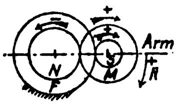


FIG. 132.

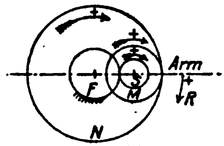


FIG. 133.

FIGS. 128–133.—Epicyclic Gear Trains.

Fig. 129: r.p.m. of N = r.p.m. of $R \times \left(1 - \dfrac{\text{diam. of } F}{\text{diam. of } N}\right)$

Fig. 130: r.p.m. of N = r.p.m. of $R \times \left(1 + \dfrac{\text{diam. of } F}{\text{diam. of } N}\right)$

Fig. 131: r.p.m. of N = r.p.m. of $R \times \left(\dfrac{\text{diam. of } R}{\text{diam. of } R + \text{diam. of } F}\right)$
(with internal gear driving)

Fig. 132: r.p.m. of N = r.p.m. of $R \times \left(1 - \dfrac{\text{diam. of } F \times \text{diam. of } M}{\text{diam. of } S \times \text{diam. of } N}\right)$

Fig. 133: r.p.m. of N = r.p.m. of $R \times \left(1 + \frac{\text{diam. of } F \times \text{diam. of } M}{\text{diam. of } S \times \text{diam. of } N}\right)$

The shading in Figs. 128–133 shows the gear whose shaft has no motion.

Elliptical Gears (having the pitch curve in the form of an ellipse instead of a circle as with spur gears) are employed to obtain variable velocity throughout the revolution.

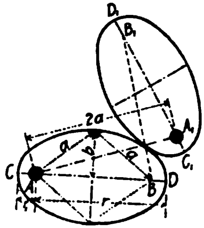


FIG. 134.—Elliptical Gears.

In Fig. 134, A = the axis of driving gear; A_1 = axis of driven gear; a = half major axis of ellipse; b = half minor axis of ellipse; A, B, A_1, B_1 are the focii of the ellipses; o = uniform angular velocity of the driving gear; o_1 = greatest angular velocity of the driven gear; o_2 = least angular velocity of the driven gear; r and r_1 as in figure.

Then $AA_1 = 2a$, and $b = 2a\sqrt[4]{V}/(\sqrt{V} + 1)$, where $V = o_1/o_2 = r^2/r_1^2$.

For strength of gear teeth, see p. 730; for efficiency of transmission by gears, pp. 247 and 733.

Bevel Gears

Bevel gears may be used to connect intersecting shafts in any given speed ratio. Referring to Fig. 135, let OA_1 and OA_2 represent the shafts to be connected and let a be the angle defined by their axes. Should it be required that OA_1 rotate at 100 r.p.m. while OA_2 rotates at 50 r.p.m., then the cones D_1OB and D_2OB defining the pitch surfaces of the bevel gears must have a common apex at O (the intersection of the shafts), and their common tangent must define the angles b_1 and b_2 whose ratio is 50 to 100 and whose sum is a. The tooth outlines will be generated by rolling cones having apexes at O. At the larger end of the larger gear the tooth outlines will be approximately those generated on a pitch circle of radius BA_2. BA_2 is drawn perpendicular to OB. The pitch circle for laying out the teeth at the smaller end is found in a similar manner.

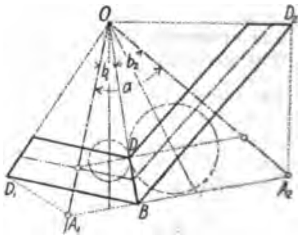


FIG. 135.—Bevel Gears.

Satisfactory relations for **proportioning the teeth of bevel gears** of the various inclinations shown in Figs. 136–139 are as follows:

Let P = diametral pitch
C = circular pitch
N = number of teeth in pinion
N_1 = number of teeth in gear
x = angle between shafts
$D(D_1)$ = pitch diam. of pinion (gear)
$O(O_1)$ = outside diam. of pinion (gear)
$J = 1/P = 0.3183C$
$K = 1.157/P = 0.3683C$

Then

$N = DP = \pi D/C$. $\quad N_1 = D_1P = \pi D_1/C$

$D = N/P = 0.3183CN$. $\quad D_1 = N_1/P = 0.3183CN_1$

$O = D + (2 \cos c/P) = D + 0.6366C \cos c$ [$= D + (1.4142/P) = D + 0.45C$ for the miter gear in Fig. 137]

$O_1 = D_1 + (2 \cos c_1/P) = D_1 + 0.6366C \cos c_1$
$= D_1 + (2 \sin c/P) = D_1 + 0.6366C \sin c$ for Fig. 136

$\tan c = \cos (x - 90°)/[(N_1/N) - \sin (x - 90°)]$ for Fig. 139
$= \sin x/[(N_1/N) + \cos x]$ for Fig. 138; $= N/N_1$ for Fig. 136

$c_1 = x - c$

$\tan s = 2 \sin c/N (= 1.4142/N$ for Fig. 137)

$2.314 \sin c/N (= 1.6362/N$ for Fig. 137)
$c + s\ (= 45° + s$ for Fig. 137); $a_1 = c_1 + s$
$c - f (= 45° - f$ for Fig. 137); $b_1 = c_1 - f$
$\frac{1}{2}O \cot a$; $M_1 = \frac{1}{2}O_1 \cot a_1$
$Y \cos a$; $H_1 = Y \cos a_1$.

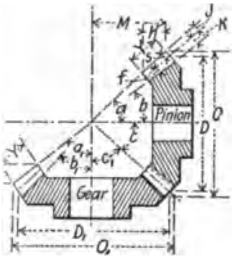


FIG. 136.

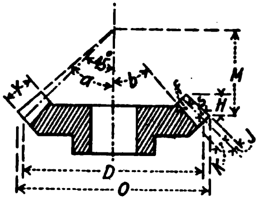


FIG. 137.

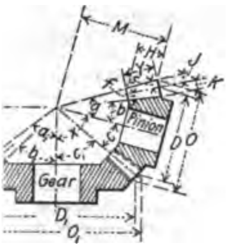


FIG. 138.

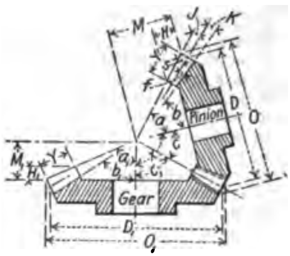


FIG. 139.

ır of teeth for which to select the cutter $= N/\cos c$ for pinions, and
ɡears. For Fig. 137 this equals $1.414N$.
h of bevel gears, see p. 732; for efficiency, pp. 247 and 733.

orm Gears

m-and-wheel gearing **y ratio** is the ratio number of teeth on the number of threads ı. Thus, a 30-tooth l meshing with a ed worm will have a o of 1 to 30, that is, ust make 30 revolu- to revolve the worm For a double- rm there will be 15 f the worm to one of ıeel, etc. High ve- are thus obtained y small wheels.

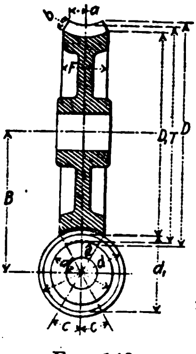


FIG. 140.

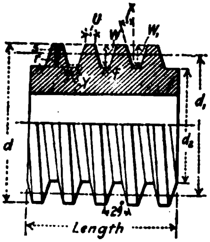


FIG. 141.

The following rules for **proportioning worm gears** are quoted from the "American Machinist Gear Book," by C. H. Logue, and refer to Figs. 140 and 141:

Let

N = number of teeth in worm wheel
n = number of threads in worm
p = circular pitch (dist. c to c. of teeth)
L = lead (advance of worm in 1 rev.)
D_1 = pitch diam. of worm wheel
T = throat diam. of worm wheel
D = outside diam. of worm wheel
F = face of worm wheel
a = distance from center line to point of tooth
b = length of side
d_1 = pitch diam. of worm
d = outside diam. of worm
d_2 = bottom diam. of worm
e = radius at throat of worm wheel
c = angle of sides of face, deg.
B = center distance
R = revs. of worm per rev. of wheel
x = angle of teeth in wheel with axis (used in gashing)
W = working depth
W_1 = whole depth
f = clearance
t = thickness of tooth at pitch line
t_n = normal thickness of tooth at pitch line
p_n = normal circular pitch
s = addendum
U = width of worm thread at top
Y = width of worm thread at bottom
P = diametral pitch

Then

$N = \pi D_1/p$
$T = 0.3183p(N + 2)$
$F = (0.5d + 0.17p) \sin c/0.5$
$a = F/2 - b \sin c$
$b = W_1 + 0.12p$
$d = d_1 + 2s$
$d_2 = d - 2W_1$
$e = \frac{1}{2}d_1 - s$
c = 30 to 35 deg.
$(\sin c = F/(d + 0.34p)$
$B = (D_1 + d_1)/2$
$D_1 = 0.3183Np$
$D = T + 2(e - e \cos c)$
$p = D_1/0.3183N$
$L = pn$
$n = N/R$
$\tan x = L/\pi d_1$
$t_n = t \cos x$, or
$t = t_n/\cos x$
$U = 0.335p = 1.0536/P$
$Y = 0.31p = 0.9744/P$
$p_n = p \cos x$, or $p = p_n/\cos x$

The formulæ for the tooth parts of spur gears apply also to worm gears. N, D, and p are calculated the same as for spur gears.

T has the same value as the outside diameter of a spur gear of the same pitch and number of teeth.

F will be sufficiently accurate when calculated from the formula in the parenthesis.

d_1. The efficiency of a worm gear drive depends greatly on the angle x of worm. When the lead is fixed, the angle is determined from $\tan x = L/\pi d_1$, and, in order to make it of a value that would indicate high efficiency, the diameter should be made as small as possible except when the angle approaches 45 deg., where either the worm or the wheel may drive. In such a case the conditions should be reversed and the diameter of the worm be made large enough to secure the desired angle, unless the wheel is to drive, which sometimes is the case.

c is generally from 30 to 35 deg., preferably 30 deg.

L = circular pitch × number of threads in the worm; *e.g.*, a double-threaded worm of 1½ in. circular pitch has a lead of 3 in.

n. The number of threads in a worm of given velocity ratio = number of teeth in the wheel divided by the ratio; *e.g.*, for a wheel with 60 teeth and a velocity ratio of 30 : 1, number of threads $= n = 60/30 = 2$.

W_1 should be proportioned from the normal pitch (p_n) rather than from the linear pitch (p), as is usually the case. It is wrong to take the depth from linear pitch even for single or double threads, and for quadruple and sextuple threads the error becomes so pronounced that it is likely to affect seriously the working of the gear.

t_n = thickness of tooth parallel with the axis of worm (= 0.5 × linear pitch) divided by the cosine of the helix angle x.

The **length of the worm** need be no greater than three times the circular pitch, as seldom are more than two teeth in contact at once. If the worm be made longer, how-

ɔan be shifted lengthwise when worn, as it always wears away more rapidly than d. It is common practice to make the length of worm = $6p$.
rength of worm gears, see p. 732; for efficiency, pp. 247 and 733.

Helical Gears

al gears (commonly misnamed "spiral" gears) may be used to connect llel shafts, (b) shafts at right angles and not intersecting, and (c) shafts at any angle to each other and not intersecting. Worm gears special case of (b). Fig. 142 represents two gears connecting non-intersecting shafts inclined other in plan at an angle of c degrees. Gear A, s R, has teeth of a pitch angle equal to x degrees, e pitch angle of the teeth of gear B is equal to x_1; the radius of gear B is R_1. Let n = r.p.m. of n_1 = r.p.m. of B; then the linear velocity of the urface of $A = 2\pi Rn$, and the component of the in the direction of the normal to the tooth pro- $\pi Rn \cos x$. Similarly, for gear B, the component velocity in the direction of the normal to the ofiles is $2\pi R_1n_1 \cos x_1$. When helical gears are in

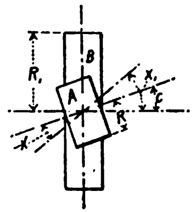


Fig. 142.

owever, the normals to the tooth profiles at the point of contact are nt and the velocities in the direction of the normals are identical in ence $2\pi Rn \cos x = 2\pi R_1n_1 \cos x_1$, or $n/n_1 = R_1 \cos x_1/R \cos x$.

oblem of connecting a pair of shafts by helical gears for any velocity ratio admits oer of solutions, since both the radii of the pitch surfaces and the angles of the tribute to establishing the velocity ratio. The formulæ given in Table 69 nd of assistance in the computation of helical gears. The notation used in this follows:

number of teeth in driver (follower)	a = tooth pitch angle of follower
pitch diam. of driver (follower)	b = tooth pitch angle of driver
circular pitch of driver (follower)	$L_1(L_2)$ = lead of driver (follower).
normal circular pitch for both gears	= lead of tooth helix
normal diametral pitch for both gears	$r_1(r_2)$ = r.p.m. of driver (follower)
outside diam. of driver (follower)	c = angle between shafts in plan
addendum of normal pitch	C = center distance.

Table 69. Formulæ for Helical Gear Calculations

Driver	Follower		Remarks
Formula	To find	Formula	
$\tan b = d_1r_1/d_2r_2$	a	90 deg. $- b$	Axes at right angles only.
$\tan b = p_1/p_2$	a	90 deg. $- b$	
$\cos b = p_n/p_1$	a	$c - b$	
$\tan b = \pi d_1/L_1$	a	$c - b$	
$\pi d_1 \cos b/N_1$	p_n	$\pi d_2 \cos a/N_2$	Same in both gears.
$\pi d_1/N_1$	p_2	$\pi d_2/N_2$	
$p_2N_1 = \pi d_1 \tan a$	L_2	$\pi d_2 \tan b = p_1N_2$	Axes at right angles only.
$d_1P \cos b$	N_2	$d_2P \cos a$	
$2C\Big/\left(\frac{r_1}{r_2}\tan a + 1\right)$	d_2	$2C\Big/\left(\frac{r_1}{r_2}\tan b + 1\right)$	Axes at right angles only.
$2C\Big/\left(\frac{r_1 \cos b}{r_2 \cos a} + 1\right)$	d_2	$2C - d_1$	
$0.3183N_1p_1$	d_2	$0.3183N_2p_2$	
$d_1 + 2S_n$	D_2	$d_2 + 2S_n$	
$d_1 + (2/P)$	D_2	$d_2 + (2/P)$	14½ deg. standard only.
$N_1/\cos^3 b$	Cutter*	$N_2/\cos^3 a$	

Center distance $C = (N_1/2P \cos b) + (N_2/2P \cos a)$

cutter to be used which is correct for the number of teeth given by the formulæ.

When helical gears are used to connect parallel shafts, the normal component of the tangential pressure on the teeth produces end thrust of the shafts. To remove this objection such gears are made with right-handed helical teeth on one side of the face and left-handed on the other, and are then known as **herringbone gears.** Correctly cut herringbone gears run much more smoothly than spur gears, and work silently and without vibration. They can also be run at higher speeds and higher velocity ratios. In the Wuest system, involute teeth with a 20-deg. angle of obliquity are employed, the pitch angle of the teeth being 23 deg. The face width is made equal to 6 × circular pitch p for gears with pinions of not less than 25 teeth, and from $6p$ to $12p$ for high ratios with small pinions. For convenience in manufacture the gears are grooved at the center of the face for a width equal to p and the two sets of teeth are stepped half the pitch apart. Pitch diam. $= n/p_1$ (for 20 teeth and over) $= (0.95n + 1)/p_1$ (for less than 20 teeth), where n = number of teeth and p_1 = diametral pitch. Addendum $= 0.8/p_1$; dedendum $= 1/p_1$; working depth $= 1.6/p_1$.

For strength of helical gears, see p. 732; for efficiency, p. 247.

Strength of Gear Teeth

Spur Gears. According to Wilfred Lewis (*Proc. Engr. Club of Phila.*, Jan., 1893), the load in pounds transmitted by spur gear teeth is

$$W = fbpy, \tag{1}$$

where f = intensity of stress of the material, lb. per sq. in.; b = width or face of gear, in.; p = circular pitch, in.; and y = a factor depending on the form of the tooth and the number of teeth in the gear. For involute teeth with an angle of obliquity of 20 deg., $y = 0.154 - (0.912/n)$, where n = number of teeth in the gear. For involute teeth with an angle of obliquity of 15 deg. and for cycloidal teeth, $y = 0.124 - (0.684/n)$. Values of y from these formulæ are given in Table 70.

Table 70. Values of Factor y in the Lewis Formula for Strength of Gears

No. of teeth	Value of factor y: Involute, 20 deg.	Involute 15 deg. and cycloidal	Radial flanks	No. of teeth	Value of factor y: Involute, 20 deg.	Involute 15 deg. and cycloidal	Radial flanks	No. of teeth	Value of factor y: Involute, 20 deg.	Involute 15 deg. and cycloidal	Radial flanks
12	0.078	0.067	0.052	20	0.102	0.090	0.060	43	0.126	0.110	0.068
13	0.083	0.070	0.053	21	0.104	0.092	0.061	50	0.130	0.112	0.069
14	0.088	0.072	0.054	23	0.106	0.094	0.062	60	0.134	0.114	0.070
15	0.092	0.075	0.055	25	0.108	0.097	0.063	75	0.138	0.116	0.071
16	0.094	0.077	0.056	27	0.111	0.100	0.064	100	0.142	0.118	0.072
17	0.096	0.080	0.057	30	0.114	0.102	0.065	150	0.146	0.120	0.073
18	0.098	0.083	0.058	34	0.118	0.104	0.066	300	0.150	0.122	0.074
19	0.100	0.087	0.059	38	0.122	0.107	0.067	rack	0.154	0.124	0.075

The 15-deg. involute system is the more universal in practice and further discussion of formula (1) will be confined to this particular case. If in (1) y be expressed as a function of the circular pitch, by writing $n = \pi D/p$, the formula becomes

$$p = D(0.28 - \sqrt{0.079 - (4.57W/bfD)}$$

Thus, for a gear of given diameter and given load, the circular pitch for any width of gear and limiting stress may be determined. In (1) the number of teeth and the circular pitch are both unknown. The application of this formula requires the selection of the width of gear and the working stress. It is usual to limit the width or face of gear to about 3 times the circular pitch, for under this assumption the strength of the tooth in case the load is applied at one corner will be equal to the strength when the load is uniformly distributed over the face. Table 71 gives values of the safe working stress f for different materials over a wide range of speeds. Metallic spur gears should not be run at a peripheral speed exceeding 2000 ft. per min.; above

Table 71. Values of the Factor f in the Lewis Formula for Strength of Gear Teeth

Material of gear	Speed of pitch circle, ft. per min.							
	100 or less	200	300	600	900	1200	1800	2400
Cast iron	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
Cast steel	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300
Forged steel	25,000	20,000	16,000	13,000	10,000	7,500	6,300	5,400
Rawhide	5,000	4,000	3,000	2,500	2,000	1,600	1,200	......
Bronze	12,000	9,000	7,000	6,000	4,500	3,600	3,000	2,500
Rawhide*	....	3,600	3,500	3,100	2,700	2,400	1,900	......

* General Electric Co. practice.

1200 ft. per min. ordinary cut gears are objectionable on account of the noise made. The General Electric Co. uses the formula $C = WV/nb$, for **rawhide pinions**, in which V = velocity at pitch diameter, ft. per min., the maximum allowable values for C being as follows:

For diametral pitch =	1	1½	2	2½	3	3½	4
C =	1600	1400	1200	1000	900	800	600

The New Process Rawhide Co. (*Am. Mach.*, Apr. 6, 1911) states that the working stress of high-grade rawhide gears may be taken as $150bp$, but in no case should the pressure per inch of face exceed 250 lb. Pinions with bronze flanges cut through and forming part of the working face may have pressures from 10 to 25 per cent. greater, depending on the quality of the metal and the thickness of the flanges. Rawhide gears should be slightly lubricated with a mixture of graphite and lard oil or tallow. **Cloth pinions** (layers of cotton cloth soaked in machine oil and clamped between end flanges) generally have the same strength as cast-iron gears of the same dimensions. The face width of the pinion should equal that of its mating gear plus the sum of the end play of both shafts. The flanges should never engage with the mating gear.

Should it be required to determine the circular pitch, p, of a pinion having a given number of teeth to take the twisting moment (T) of the shaft on which it is keyed, the following expression may be used: $p^3 = 2\pi T/[nfe(0.124 - 0.684/n)]$, where $e = b/p$, or diametral pitch$^3 = 5nfe \times (0.124 - 0.684/n)/T$. For the particular case of a 15-tooth pinion the last formula reduces to: diametral pitch $= 2.6\sqrt[3]{f/T}$, where the width of the gear is 3 times the circular pitch, that is, $e = b/p = 3$. T = in.-lb.

The **strength of stub teeth** cannot be computed by the Lewis formula. An 8–10 stub-tooth gear is one whose diametral pitch is 8 and whose addendum and dedendum circles are those of a standard 10-pitch gear. A stub-tooth gear will carry a greater load than a standard-tooth gear in the ratio of the two pitches dimensioning the stub tooth, that is, the 8–10 stub tooth will take 10/8 of the load of a standard 8-pitch tooth.

Bevel Gears. The strength of bevel-gear teeth is determined on the basis that a spur-gear tooth having the same face width and form and size as the bevel gear at the middle of its face, has the strength of the bevel-gear tooth.

Let Fig. 143 represent a bevel gear, in which p' = circular pitch at larger end; p = circular pitch at middle of gear face = $p'(A + a)/2A$; b = width of gear face; W_M = mean value of load on bevel-gear tooth per in. length of contact line. W_R = resultant of all unit pressures in bevel-gear tooth; W' = load which if applied at greatest pitch radius of gear (A) would produce the same torque in gear shaft as the actual load acting along face. Then $W_M = 3A(A^2 - a^2)W'/2(A^3 - a^3)b$. Assume that the pressure varies directly as the radius. After the bevel gear has been laid out so that A, a, R and c become known, the circular pitch at the middle of the gear face can be determined from the relation $p = D(0.28 - \sqrt{0.079 - 4.57\,W_M/bfD})$, where $D = 2R$ and W_M is found as above. Then $p' = 2Ap/(A + a)$.

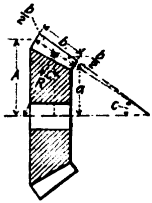


FIG. 143.

The strength of bevel-gear teeth may also be determined by using the following modification of the Lewis formula for spur-gear teeth: $W = fbpyd/D$, in which d and D are respectively the pitch diameters at the smaller and larger ends of the gear. This formula applies only when $d \geq 0.66D$.

Shrouding the teeth increases their strength very considerably, and it is safe to assume that full **shrouding** will give double the strength and half shrouding an increase in strength of nearly 50 per cent.

Herringbone Gears. Safe working pressure (lb.) on the teeth of Wuest gears = $W = 0.4pbk$, where p = circular pitch, b = width of face (both in in.) and k has the following values determined by experiment:

	Velocity of pitch circle, ft., per min.							
	200	400	800	1200	1600	2000	2400	2800
High-carbon-steel forgings	....	....	1550	1440	1320	1220	1120	1050
Steel castings	1300	1200	1100	1100	920	820	770	700
Phosphor bronze	1100	1040	920	830	750	690	630	590
Gun metal	960	880	750	650	580	530	500	460
Cast iron	760	690	590	500	450	410	390	370
Brass	600	520	440	400	360	330	310	300

Also W = h.p. × 33,000/V, where V = velocity of pitch circle, ft. per min.

Worm Gears. The allowable load (lb.) on worm-gear teeth = $W = cbp$, where b = width of tooth and p = circular pitch—both in in., and c = constant = 285 to 425 for cast-iron cut teeth = 455 to 711 for phosphor-bronze wheel and hardened steel worm such as are used for high speeds. These values are for the strength of the teeth only. For continuous running where heating and wear must be taken into consideration, c should have about the following values in the case of a phosphor-bronze wheel and a hardened and polished worm running in an oil bath:

For peripheral velocity of worm, ft. per sec. =	3.3	8.2	13.1	18	23
c =	426–570	356–426	285–340	213–256	142–170

Helical Gears. According to Unwin, $W = 0.0833bpf\cos^2 x$, where x = pitch angle of teeth, and f has the same values as for spur-gear teeth.

Proportions of Gear-wheel Parts

Gear Arms of **elliptical section** may be conveniently proportioned by means of the following equations:

Let Z = section modulus of arm at hub; p = circular pitch, in.; d.p. = diametral pitch; b = width of face; $R = b/p$; n = number of teeth in the gear; N = number of arms;

D = thickness of arm at base; $2D$ = width of arm at base. Then, upon equating the strength of the tooth to that of the arm considered as a cantilever and taking an equal share of the load, there results

$$Z = p^3R(n - 7)/50N = \pi^3R(n - 7)/[50N(\text{d.p.})^3]$$

$$\text{and } D = \sqrt[3]{p^3R(n - 7)/20N} = \sqrt[3]{1.57R(n-7)/N(\text{d.p})^3}.$$

In case **arms of cruciform or T section** are desired, find the dimensions of an arm of elliptical section as above and then determine a cruciform or T section of equal section modulus. The section moduli of elliptical, T and cruciform sections of the proportions shown respectively in Figs. 144, are $0.4E^3$, $0.04E^3$ and $0.04E^3$, respectively. The **number of arms** may be 6 for gears up to 60 in. in diam., 8 for 60 to 80 in., and 10 for gears over 80 in. in diam.

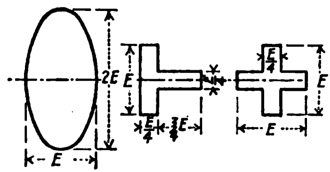


FIG. 144.—Gear-arm Sections.

Rims of gear wheels may be made of a thickness equal to half the circular pitch, tapering to an increased thickness where they join to the arms. Generally, **face width** = $2\frac{1}{2}\ p$ to $3p$.

Hubs may have an outside diam. equal to $1\frac{1}{2}$ to 2 times the shaft diameter, with a reinforcement at the keyway equal to one-half the key thickness.

Efficiency of Gearing in Power Transmission

(See p. 246)

Spur and Bevel Gears. The efficiency of power transmission by the use spur gears and bevel gears varies from 90 per cent. to 98 per cent. The conditions tending to a low efficiency are (1) high ratio of gear diameters; (2) non-lubrication of the tooth surfaces; (3) inaccurate tooth outlines; and (4) inaccuracy in alignment.

The efficiency of **worm gearing,** when the friction of the thrust bearing is neglected, is $E = (\tan a)(1 - f \tan a)/(\tan a + f)$ approx., where a = angle of thread, and f = coefficient of friction. With values of $f = 0.025$ and 0.05 the variation of efficiency with thread angle is as shown in Fig. 145.

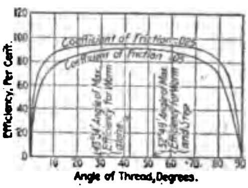


FIG. 145.—Efficiency of Worm Gearing.

Self-locking worms are obtained when $f = \tan a$. The efficiency resulting with such angles can never be as much as 50 per cent. In practice, $\tan a$ can be made greater than f and the gear will be self-locking because of the friction in other parts of the drive.

To find the **pitch** of worm-wheel teeth and the largest pitch diameter of worm **to transmit a given horse power** at a given speed at maximum efficiency the diagram shown in Fig. 146 may be employed (*Machy.*, May, 1912). To transmit 7 h.p. with a single thread worm at 360 r.p.m. there is required, as shown by the diagram, a worm pitch diam. of 3 in. and a circular pitch of worm-wheel teeth of $1\frac{1}{4}$ in. For cast-iron worm wheels and worms with **unfinished teeth,** the pitch should be $1\frac{1}{2}$ times that obtained from the diagram.

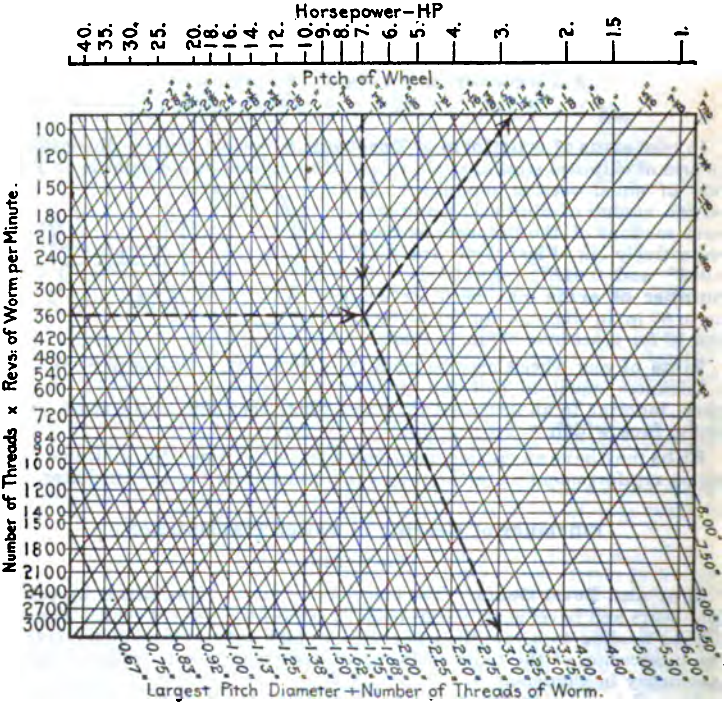


FIG. 146.—Diagram for Designing Worm Gearing of Maximum Efficiency.

Materials for Gears

(See also pp. 541 for phosphor bronze)

For cast-iron gears the compositions given in Table 72 are used. For phosphor-bronze worm wheel with steel worm, a satisfactory composition is: Cu, 89.5 per cent. Sn, 10 per cent., P, 0.5 per cent.

For **alloy steels for automobile transmission gears,** see S.A.E. specifications, p. 465.

Table 72. Compositions of Cast Iron for Gears

	Si	S	P	Mn	Total carbon
Large gears........................	1.00-1.50	0.08-0.10	0.30-0.50	0.80-1.00	Low
Medium gears.....................	1.50-2.00	under 0.09	0.40-0.60	0.70-0.90	....
Small gears........................	2.00-2.50	under 0.06	0.50-0.70	0.60-0.80	....

Friction Gearing

Friction gearing of either the spur or bevel gear type may be used for the transmission of power. In either case it is good practice for the driver to be of paper, leather, or pulp composition, and the driven wheel of metal, which may be iron, type metal, aluminum, etc. Goss (*Trans. A. S. M. E.*, vol. 29, 1907), reports as follows on the characteristics of friction drives:

The **safe working pressures** in lb. per sq. in. are as follows: Straw fiber, 150, leather fiber, 240; tarred fiber, 240; sulphite fiber, 140; leather, 150. They are taken as 20 per cent. of the ultimate resistance of the material to crushing.

The safe working values of the **coefficient of friction** may be 60 per cent. of the values found from laboratory experiments. The frictional and power coefficients in Table 73 are calculated upon this basis.

Table 73. Frictional and Power Coefficients for Friction Gearing

H.p. transmitted $= kwdN$

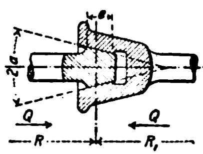
where $w =$ width of face of driver, in.; $d =$ diam. of driver, in.; $N =$ r.p.m.

Driver	Driven wheel	Coefficient of friction	k	Driver	Driven wheel	Coefficient of friction	k
Straw fiber.....	Iron........	0.25	0.00030	Tarred fiber....	Type metal.	0 16	0.00031
Straw fiber.....	Aluminum..	0.27	0.00033	Sulphite fiber...	Iron........	0.33	0.00037
Straw fiber.....	Type metal.	0.18	0.00022	Sulphite fiber...	Aluminum..	0.32	0.00035
Leather fiber...	Iron........	0.31	0.00059	Sulphite fiber...	Type metal..	0.31	0 00034
Leather fiber...	Aluminum..	0.30	0.00057	Leather........	Iron........	0.13	0 00016
Leather fiber...	Type metal.	0.18	0.00035	Leather........	Aluminum..	0.21	0.00026
Tarred fiber....	Iron........	0.15	0.00029	Leather........	Type metal.	0.25	0.00029
Tarred fiber....	Aluminum..	0.18	0.00035				

In the case of **bevel friction gears** the horse power may be had from the above values by taking for N a value which will correspond to the average velocity and radius of the pitch surface. In the case of **wedge-surface friction gears**, as shown in Figs. 147 and 148, the tangential force P for a given normal force Q pressing the wheels together is $P \leq Qf/(\sin a + f \cos a)$, where f is the coefficient of friction. Generally, $a = 10$ to 15 deg. The radial depth of contact of the surfaces e (Fig. 147) should not exceed ¼ in. The **efficiency** of friction drives, according to Ernst, may range from 88 to 90 per cent. In **disk drives** the distance between the face of the driver and the center of the driven wheel should not be less than 12 times the face width of the driver, otherwise the power capacity will be decreased due to slip.

PULLEYS, FLYWHEELS, SHEAVES, DRUMS

Pulleys for the transmission of power by belting may be purchased in the market in the following types and sizes.



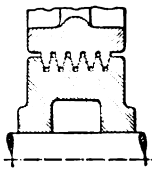


FIG. 147. FIG. 148.
Wedge-surface Friction Gears.

Cast-iron Pulleys, solid or split or with solid rims and split hubs, may have rims of the construction shown in Fig. 149. The **dimensions** to which the several forms of construction are made by the Hill Clutch Co., are as shown in Table 74. The **number of arms** is usually as follows: For pulleys less than 15 in. in diam., 4 arms; for 15-in. to 40-in. pulleys, 6 arms; for 40-in. to 150-in. pulleys, 8 arms. Pulleys with faces less than 20 in. wide, 1 set of arms; with faces from 20 to 50 in. wide, 2 sets; with faces over 50 in. wide, 3 sets. These proportions apply to thin-rim cast-iron belted pulleys and not to flywheels or heavy band wheels. The Medart Patent Pulley Co., of St. Louis, Mo., makes **pulleys with wrought-iron rims** and cast-iron spiders which are adapted to higher speeds than cast-iron pulleys.

Table 74. Face Widths of Standard Cast-iron Pulleys
(Letters refer to Fig. 149)

Diam., in.	A Max. face, in.	C Face, in.	Diam., in.	A Max. face, in.	C Face, in.	Diam., in.	A Max. face, in.	C Face, in.	Diam., in.	A Max. face, in.	C Face, in.
12	13	14	17	18	19	22	21	22–27	48– 59	24	25–36
13	14	15	18	19	20	23	21	22–28	60– 71	24	25–40
14	15	16	19	19	20	24–29	21	22–30	72– 83	24	28–40
15	16	17	20	20	21–25	30–35	21	22–30	84–144	24	......
16	17	18	21	20	21–26	36–47	22	23–36	....	......	......

Wood Split Pulleys may be obtained from a number of makers with diameters from 3 in. to 96 in. and with a large range of face widths, at prices approximately 20 per cent. less than for cast-iron pulleys. Wooden pulleys should be avoided in damp places and for high speeds.

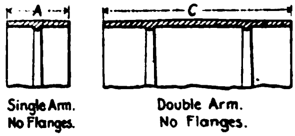


Fig. 149.—Pulley Rim Constructions.

Paper Pulleys transmit power at less loss than cast-iron-face pulleys.

Pressed-steel Pulleys are adapted to use at higher speeds than are cast-iron pulleys.

Stresses in Pulleys, Sheaves and Flywheels—Proportions

Arms of pulleys, sheaves and flywheels are subjected to stresses due to conditions of founding, to details of construction (such as split or solid), and to conditions of service, which do not readily admit of analysis. For this reason no accurate stress relations can be established, and the following formulæ must be understood to be only approximately correct. It has been established experimentally by Benjamin (*Am. Mach.*, Sept. 22, 1898) that thin-rim pulleys do not distribute equal loads to the several pulley arms. For these, it will be safe to assume the tangential force on the pulley rim as acting on half the number of arms. Pulleys with comparatively thick rims, such as engine band wheels, have all the arms taking the load. Furthermore, while the stress action in the arms is similar to that in a beam fixed at both ends, the amount of restraint at the rim depending on the rim's elasticity, it may, nevertheless, be assumed for purposes of design that cantilever action is predominant. The bending moment at the hub in arms of thin-rim pulleys will be $M = PL/\frac{1}{2}N$, where M = bending moment, in.-lb.; P = tangential load on the rim, lb.; L = length of the arm, in., and N = number of arms. For thick-rim pulleys and flywheels, $M = PL/N$.

For arms of elliptical section having a width of two times the thickness, where E = **width of arm section** at the rim, in., and f_t = intensity of tensile stress, lb. per sq. in.,

$$E = \sqrt[3]{40PL/f_tN} \text{ (thin rim)} = \sqrt[3]{20PL/f_tN} \text{ (thick rim)}$$

For single-thickness belts P may be taken as 50 B lb. and for double-thickness belts P = 75 B lb., where B is the width of pulley face, in. Then $E = k \times \sqrt[3]{BL/f_tN}$, where k has the following values: For thin rim, single belt, 13; thin rim, double belt, 15; thick rim, single belt, 10; thick rim, double belt, 12.

For cast iron of good quality f_t due to bending may be taken at 1500 to 2000. The arm section at the rim may be made from $\frac{2}{3}$ to $\frac{3}{4}$ the dimensions at the hub.

For high-speed pulleys and flywheels it becomes necessary to check the arm for tension due to rim expansion. It will be safe to assume that each arm is in tension due to one-half the centrifugal force of that portion of the rim which it supports. That is, $T = Af_t = Wv^2/2NgR$ (lb.) where T = tension in arm, lb.; N = number of arms; v = speed of rim, ft. per sec.; R = radius of pulley, ft.; A = area of arm section, sq. in.; W = weight of pulley rim in lb. and f_t = intensity of tensile stress in arm section, lb. per sq. in. Whence $f_t = WRn^2/5800NA$, where n = r.p.m. of pulley.

Arms of flywheels having heavy rims may be subjected to severe stress action due to the inertia of the rim at sudden load changes. There being no means of predicting the probable maximum to which the inertia may rise, it will be safe to make the arms equal in strength to $\frac{3}{4}$ of the shaft strength in torsion. Accordingly, for elliptical arm sections,

$$N \times 0.5E^3f_t = \tfrac{3}{4} \times 2f_sd^3, \text{ or } \quad E = 1.4d\sqrt[3]{f_s/f_tN}$$

For steel shafts with $f_s = 8000$ and cast-iron arms with $f_t = 1500$,

$$E = 2.4d/\sqrt[3]{N} = 1.3d \text{ (for 6 arms)} = 1.2d \text{ (for 8 arms)}$$

where E = width of elliptical arm section at hub, in. (thickness = $\frac{1}{2}E$) and d = shaft diameter, in.

Rims of belted pulleys cast whole may have the following proportions (see Fig. 150):

$$t_2 = \tfrac{3}{4}s + 0.005D; \quad t_1 = 2t_2 + C; \quad w = \tfrac{9}{8}B \text{ to } \tfrac{5}{4}B;$$

where s = belt thickness, $C = \frac{1}{24}w$, and B = belt width, all in inches.

Engine band wheels, flywheels and pulleys run at **high speeds** are subjected to the following stress actions in the rim:

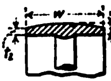


FIG. 150.

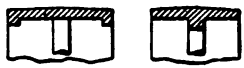


FIG. 151.

Rims for Belted Pulleys.

Considering the rim as a free ring (*i.e.*, without arm restraint) and made of cast iron or steel, $f_t = v^2/10$ (approx.), where f_t = intensity of tensile stress, lb. per sq. in. and v = rim speed, ft. per sec. For beam action between the arms of a solid rim, $M = Pl/12$ (approx.), where M = bending moment in rim, in.-lb.; P = centrifugal force of that portion of rim between arms, lb., and l = length of rim between arms, in.; from which $f_t = WR^2n^2/450N^2Z$, where W = weight of entire rim, lb.; R = radius of wheel, ft.; n = r.p.m. of wheel, and Z = modulus of rim section, in.^{3}. In case the rim section is of the forms shown in Fig. 151, care must be taken that the flanges do not reduce the section modulus from that of the rectangular section. For **split rims** fastened with bolts as shown in Fig. 152, the stress analysis is as follows:

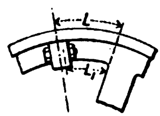


FIG. 152.

Let w = weight of rim portion L inches in length, lb.; w_1 = weight of lug, lb.; L_1 = lever arm of lug, in., and f_t = intensity of tensile stress in rim section joining arm. Then $f_t = 1.25n^2R(w_1L_1 + wL/2)/Z$, where n = r.p.m. of wheel; R = wheel radius, ft.; and Z = modulus of rim section. The above equation gives the value of f_t for bending when the bolts are loose, which is the worst possible condition that may arise. On this basis of analysis f_t should not be

47

greater than 8000. The stress due to bending in addition to the stress due to rim expansion as analyzed previously will be the probable maximum intensity of stress for which the rim should be checked for strength. The flange bolts, because of their position, do not materially relieve the bending action. In case a tie rod leads from the flange to the hub it will be *safe* to consider it as an additional factor of safety. When the tie rod is kept tight it very materially strengthens the rim.

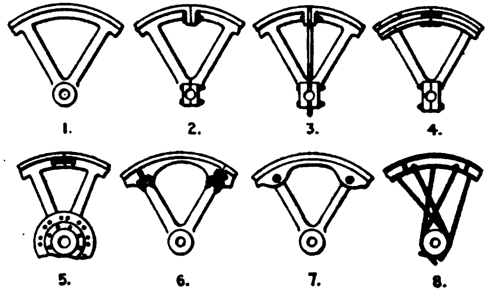


FIG. 153.—Types of Flywheel Construction (see Table 75).

The Relative Strengths of Different Types of Wheel Construction are shown by the results of Benjamin's experiments on the bursting of flywheels (*Trans. A. S. M. E.*, 1899, 1902). The types of wheels experimented with and the speeds at which they burst are shown in Fig. 153 and Table 75.

Table 75. Test Data on the Flywheels of Fig. 153

Number.*	1	2	3	4	5	6	7	8
Number of arms	6	6	6	6	8	6	6	24
Rims speed at failure, ft. per sec. = v	395	194	225	305	256	223	393	424
Comparative rim speeds at failure	100	49	57	77	65	56.5	100	107
Apparent rim tension at failure, lb. per sq. in. = $v^2/10$	15,625	3,764	5,062	9,302	6,502	4,973	15,445	17,978
Efficiency† of construction	0.85	0.19	0.265	0.49	0.34	0.26	0.84	0.94

Flywheels of the shrunk-link-joint type may be constructed as shown in Fig. 154 and to the dimensions given in Table 76. With cast-iron flywheels the bearing pressure on the link may be 20,000 lb. per sq. in. and the maximum shear on the head of the link 5600 lb. per sq. in. The length of the link may be made 0.999D, which gives an initial tension in the link of 30,000 lb. per sq. in. Proportions of **rims and arms of engine band wheels** are given in Tables 77 and 78.

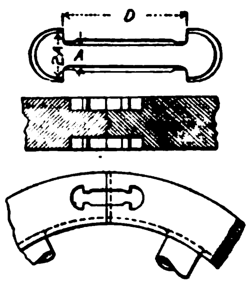


FIG. 154.—Shrunk-link Joint for Flywheels.

* Construction of flywheels: No. 1, solid wheel; No. 2, in halves, with flange joints; No. 3, in halves, with reinforced joints; No. 4, in halves, with link joints; No. 5, segmental, with link joints; No. 6, in halves, with pad joints; No. 7, solid rim, separate spider; No. 8, solid rim, with tangent spokes.

† Efficiency assuming tensile strength at 19,000 lb. per sq. in.

Table 76. Data on Flywheels with Shrunk Link Joints

Flywheel effect in lb. at 1 ft. radius — Weight of rim × (rad. gyr.²) or moment of inertia of rim	Max. safe speed, rev. per min.	Mean rim speed, ft. per sec. at max. safe r.p.m.	Dimensions					No. of arms	Weight in lb.	
			Rim			Hub				
			Diam., ft.	Face, in.	Thickness, in.	Std. bore, in.	Length, in.		Rim	Total
400,000	121	104	18	19	20	24	30	8	59,730	86,954
6,060,000	59	67	23	13½	18	20	28	10	52,466	94,725
7,200,000	74	84	23	16	18	20	28	10	62,466	104,008
9,850,000	85	71	19	27½	36	24½	30	8	154,000	172,500
11,900,000	140	193	28	18½	14	30	30	10	65,100	118,100
12,300,000	110	118	23	16½	27½	29½	34	10	116,000	144,760
15,000,000	83	84	22	24	30	33	38	10	143,300	190,188
17,600,000	54	64	25	22	28	32	38	10	137,000	180,790
20,900,000	67	79	25	25	30	37	40	10	165,000	242,387
33,400,000	105	140	28	26	29	39	40	10	205,000	299,450
24,200,000	100	133	28	19½	29	37	40	10	148,500	264,610

Table 77. Dimensions of Standard Engine-pulley Arms of Elliptical Cross-section (6 arms per pulley)

(The dimensions given are the long diameters in in. For very wide faces use two sets of arms. The hub diam. equals twice the shaft diam. Long diam. of elliptical cross-section = 2 × short diam. approximately)

Diam. of pulley, in.	Light pattern		Medium pattern		Heavy pulley flywheels		Diam of pulley, in.	Heavy pulley-flywheels	
	At rim	At hub	At rim	At hub	At rim	At hub		At rim	At hub
10	2	2¼	2⅛	2⅜	....	....	42	3¾	4⅛
12	2	2¼	2⅛	2⅜	....	....	44	3¾	4⅛
14	2	2¼	2⅛	2⅜	....	....	46	4	4½
16	2	2¼	2⅛	2⅜	....	....	48	4	4½
18	2¼	2½	2½	2¾	....	....	50	4½	5
20	2¼	2½	2½	2¾	....	....	54	4½	5
22	2¼	2½	2½	2¾	....	....	58	4½	5½
24	2¼	2½	2½	2¾	....	....	60	4½	5½
26	2⅜	2⅝	2¾	3	....	....	66	4¾	5⅞
28	2⅜	2⅝	2¾	3	....	....	68	4⅞	6
30	2⅜	2⅝	2¾	3	....	....	72	5	6⅛
32	2⅜	2⅜	2¾	3	....	....	78	5¼	6¼
34	2½	2¾	3⅛	3½	3½	3¾	84	6	7½
36	2½	2¾	3⅛	3½	3½	3¾	96	7	8½
38	2⅝	3	3⅜	3¾	3⅝	3⅞			
40	2⅝	3	3⅜	3¾	3⅝	3⅞			

Table 78. Average Thickness of Engine-pulley Rims

(Struthers, Wells & Co. practice)

Wide pulleys						Narrow pulleys					
Diam., in.	Face width, in.	Thickness of rim, in.	Diam., in.	Face width, in.	Thickness of rim, in.	Diam., in.	Face width, in.	Thickness of rim, in.	Diam., in.	Face width, in.	Thickness of rim, in.
12	3½	5/16	72	16½	1⅛	6	3½	¼	48	12½	1
18	4½	⅜	78	16½	1¼	8	4½	5/16	48	14½	1⅛
20	6½	7/16	78	18½	1⅜	10	5½	⅜	54	14½	1⅛
28	8½	½	78	20½	1½	14	7½	7/16	54	16½	1⅛
36	9½	9/16	78	22½	1½	16	10½	7/16	54	16½	1½
44	10½	½	84	24½	1⅝	20	10½	7/16	60	16½	1½
44	12½	9/16	84	26½	1¾	32	10½	½	72	18½	1¾
48	12½	9/16	96	31	2	32	10½	⅝	72	20½	1¾
54	12½	11/16	108	34	2¼	36	10½	⅝	78	22½	2
60	14½	¾	108	37	2½	36	12½	⅞	78	24½	2
60	14½	15/16	120	42	2¾	42	12½	⅞	......	....	....
72	16½	⅞	120	48	3	......	....	....	......	....	....
72	16½	15/16	....	....	....	......	....	....	......	....	....

Large flywheels for very high rim speeds (10,000 ft. and over per min.) are usually of "built-up" construction. The Mesta Machine Co. casts the wheel in two parts

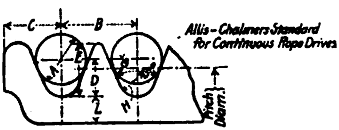


FIG. 155.

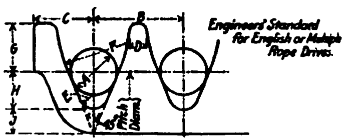


FIG. 156.

which are bolted together at the rim, the arms being bolted at their inner ends to a flange on the hub. This reduces the chance of the iron in the rim being spongy. Air-furnace iron of 30,000 lb. T. S. is used. A band saw mill flywheel designed by E. S. Newton has a cast-iron rim and hub with wrought-iron staggered spokes or arms. These are tinned on the ends before the hub and rim are cast around them. A wheel designed by the

Table 79. Dimensions of Pulley Rims for Rope Drives

(Standard grooves for cotton and Manila ropes. All dimensions in in.)

	A	B	C	D	E	F	G	H	J	L
Standard for continuous rope drives (see Fig. 155).	1	1½	1⅛	¾	1 1/16	......	3/16	11/32	......	½
	1¼	1¾	1⅜	15/16	1¼	......	¼	7/16	......	⅝
	1½	2	1½	1⅛	1 7/16	......	9/32	17/32	......	¾
	1¾	2½	1¾	1⅜	1 13/16	......	11/32	9/16	......	⅞
	2	2¾	2	1⅜	1¾	......	13/32	¾	......	1
Engineers' standard for English or multiple-rope drive (see Fig. 156).	¾	1⅜	15/16	¼	7/32	2 15/16	¾	⅝	⅜	.....
	⅞	1½	1 1/16	¼	¼	2 15/16	⅞	¾	7/16	.....
	1	1⅞	1¼	⅜	5/16	3⅞	1	13/16	½	....
	1⅛	2	1⅜	⅜	11/32	3⅞	1⅛	15/16	9/16	....
	1¼	2⅛	1½	⅜	⅜	4	1¼	1	⅝	....
	1⅜	2⅛	1 9/16	⅜	7/16	3⅛	1⅜	1⅛	11/16	.....
	1½	2¼	1 11/16	⅜	7/16	3 5/16	1½	1¼	¾	...
	1¾	2½	1 15/16	⅜	17/32	3⅜	1¾	1 7/16	⅞	.....
	2	2¾	2⅛	⅜	⅝	3 9/16	2	1⅝	1	......

Allis-Chalmers Co. has a two-part cast-iron hub, between which parts are bolted hollow cast-steel arms each cast integral with its proportion of a narrow, deep rim. The rim sections are connected by shrink links and the rim is reinforced on each side by several layers of steel plates riveted to it, the plates being arranged to break joints. A wheel made by the Westinghouse Elec. & Mfg. Co. for use in a rolling mill has a cast-steel spider with two sets of rectangular-section arms. The face of the spider is machined parallel to the shaft with dovetailed grooves, into which laminated steel sheets are fitted (the joints overlapping) and bound together by bolts passing through cast-steel end plates to form the rim.

Rims of Pulleys for Cotton and Manila Rope Drives may be made as shown in Fig. 155 and 156 and to the dimensions given in Table 79, which represent the practice of the Allis-Chalmers Co.

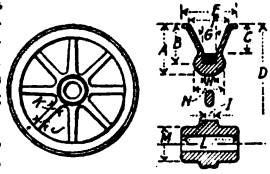


FIG. 157.—Sheaves for Wire Rope.

In the American (continuous) system of rope driving (see Rope Drives) of the Dodge Mfg. Co. the rim grooves have an angle of 60 deg., and for 1-in. rope, the dimensions *D*, *E*, *B* and *L* (see Fig. 155) are 11/16, 13/16, 1 1/8 and 5/8 in., respectively.

The practice of the Brown Hoisting Machinery Co. in the proportioning of **sheaves for iron and steel ropes** is given in Fig. 157 and Table 80.

Table 80. Proportions of Sheave Wheels for Iron and Steel Ropes
(Letters refer to Fig. 157. All dimensions in inches; weights in lb.)

Size of wheel	*D* ft.-in.		*A*	*B*	*C*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	Amt. of leather in groove, lb.	Amt. of rubber in groove, lb.	Weight of finished wheel	No. of arms
ft.																			
14	14	10	9 1/2	7	5	5 13/16	5	1 1/2	....	....	....	....	28	....	....	....	....	5120	8[1]
12	12	8	7 3/4	6 3/8	4 1/2	6 1/4	5	1 15/16	....	....	....	....	20	12	3 1/2	....	....	3442	8[2]
11	11	9 1/8	8 15/16	6 5/16	4 9/16	5 7/16	4 3/8	1 1/4	....	....	....	....	18	11	3 1/2	....	....	....	8[2]
10	10	8	6 1/4	5	3 1/2	5 1/8	4	1 5/16	1 1/4	1 3/4	3 3/4	5	15	12	3 1/2	16	42	2400	8[3]
9	9	8	6 1/4	5	3 1/2	5 1/8	4	1 5/16	1 1/4	1 5/8	3 1/2	4 1/2	12 1/2	9	3 1/2	15	36	1800	8[4]
8	8	8	7	5 3/4	4 1/8	5 7/16	4 3/16	1 3/8	2 1/2	3	2 1/2	3 1/2	13	8 1/2	3 1/2	13	31	1390	8[3]
7	7	8	6	5	3 1/2	4 7/8	4	1 5/16	1 1/8	1 7/16	2 1/2	3 1/2	12	8	3	11	27	975	8
6	6	6 3/4	5 1/2	4 1/2	3	4 5/16	3 7/16	1 5/16	1 3/8	1 1/2	2 1/2	3	10 1/2	7	3	8 1/2	21	800	8
5	5	5	4 1/2	3 3/4	2 1/2	3 15/16	3 1/8	1 1/8	3/4	7/8	2 1/4	2 3/4	6	6	2 11/16	5 1/2	12 1/2	450	8
4	4	5 1/4	3 7/8	3 1/16	2 5/8	3	2 3/16	13/16	3/4	7/8	1 5/8	2 1/4	6	4-6	1 15/16	3 1/2	7 1/2	275	8
3	3	3	3 1/2	2 3/4	1 1/2	2 7/8	2	13/16	3/4	7/8	1 3/4	2	5 1/2	4 1/2	2	2 1/2	5	101	6
in.																			
30	2	8 1/8	2 7/8	2 3/8	1 1/16	2 9/16	1 1/2	11/16	5/8	3/4	1 1/4	1 3/4	4 1/2	4	1 5/8	1 3/4	4 1/2	95	6
24	2	2	2 5/8	2 1/8	1	2 1/4	1 1/2	11/16	5/8	5/8	1 1/2	1 3/4	3 1/4	3	1 3/4	1	3 3/4	66	5
18	1	8	2 5/8	2 1/8	1	2 1/4	1 1/2	11/16	5/8	5/8	1 1/4	1 1/2	3	3 1/2	1 3/4	1/2	1 1/4	46	5
36	3	5	4 5/8	3 1/2	2	3 5/16	2 1/4	1 3/8	1 3/4	1 3/4	2 1/4	2 3/4	6 3/4	5 5/8	2 1/2	4	11 1/2	308	5
35	3	2	3 1/2	2 3/4	1 1/2	2 7/8	2	13/16	3/4	7/8	1 3/4	2	5 1/2	4 1/2	2	1 1/2	5	183	6
32	2	10 1/2	3 1/8	2 3/16	7/8	2	1 7/16	5/8	3/4	15/16	1 1/2	2	4 1/2	4 1/2	1 3/8	1 3/4	4 1/2	105	7[3]
24	2	4 1/4	3 7/16	2 7/8	1 1/2	3 3/16	2 7/16	7/8	5/8	3/4	1 9/16	1 3/4	4	3 7/8	2	1 1/2	4	108	6
21	2	0	3 1/2	2 3/4	1 1/2	2 7/8	2	13/16	5/8	3/4	1 1/2	1 3/4	4	3 1/2	2	1	3	92	5

R = distance center to center of ropes = $D - 2C$.

[1] Double wrought-iron ribs.
[2] Straight wrought-iron ribs.
[3] Curved elliptical arms.
[4] Straight ribbed arms.

For **hoisting sheaves,** see p. 1107.

Sheaves for ordinary **link chains** may be proportioned as shown in Figs. 158 and 159. The pitch diameter D is given by the equation

$$D^2 = (r/\sin a)^2 + (d/\cos a)^2$$

where a = 90 deg. divided by the number of teeth in the wheel. Values of sin a and cos a are given in Table 81.

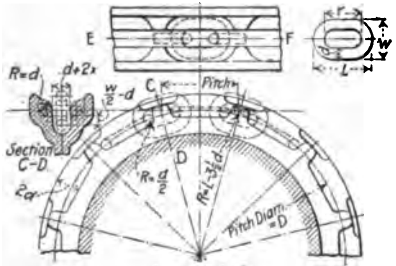


FIG. 158.—Sheave for Link Chain.

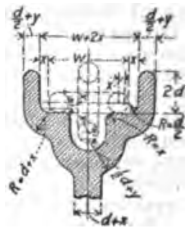


FIG. 159.

Table 81. Sprocket Wheels for Ordinary Link Chains

(Values of a, sin a and cos a for use in the formula:

Pitch diam. $D = \sqrt{(r/\sin a)^2 + (d/\cos a)^2}$. See Figs. 158–159)

No. of teeth	a	Sin a	Cos a	No. of teeth	a	Sin a	Cos a	No. of teeth	a	Sin a	Cos a
5	18° 0′	.30902	.95106	14	6° 25.7′	.11179	.99372	23	3° 54.78′	.06825	.99768
6	15° 0′	.25882	.96593	15	6° 0′	.10453	.99452	24	3° 45′	.06540	.99786
7	12° 51.4′	.22252	.97493	16	5° 37.5′	.09801	.99519	25	3° 36′	.06279	.99803
8	11° 15′	.19509	.98079	17	5° 17.64′	.09226	.99573	26	3° 27.69′	.06038	.99819
9	10° 0′	.17365	.98481	18	5° 0′	.08716	.99619	27	3° 20′	.05814	.99831
10	9° 0′	.15643	.98769	19	4° 44.22′	.08258	.99657	28	3° 12.85′	.05607	.99844
11	8° 10.9′	.14231	.98986	20	4° 30′	.07846	.99692	29	3° 6.18′	.05413	.98854
12	7° 30′	.13053	.99144	21	4° 17.14′	.07473	.99721	30	3° 0′	.05234	.99863
13	6° 55.4′	.12054	.99271	22	4° 5.45′	.07136	.99745	......	......	......	......

Table 82. Dimensions of Sprocket Wheel Chains

(Letters refer to Fig. 158. d = diam. of stock in link, in.; L and W = outside length and width of link, in.)

d	L	W	d	L	W	d	L	W	d	L	W
3/16	1 3/8	13/16	1/2	2 1/2	1 3/4	13/16	4	2 11/16	1 1/4	6	4 1/4
1/4	1 1/2	1	9/16	2 7/8	1 15/16	7/8	4 1/4	3	1 3/8	6 1/2	4 3/4
5/16	1 3/4	1 3/16	5/8	3 1/4	2 1/8	15/16	4 1/2	3 1/4	1 1/2	7 1/4	5 1/4
3/8	2	1 3/8	11/16	3 1/2	2 5/16	1	4 3/4	3 1/2	1 5/8	7 7/8	5 3/4
7/16	2 1/4	1 9/16	3/4	3 3/4	2 1/2	1 1/8	5 1/2	3 7/8	......	......	......

The value of r, the internal length of the link, is given by $r = L - 2d$, where L is the outside length of the link and d is the diameter of chain stock. These last quantities and the width of the link W are given in Table 82. In Figs. 158 and 159, x = 3/32 in. for d = 1/4 to 1/2 in.; = 1/8 in. for d = 1/2 to 1 1/8 in.; = 5/32 in. for d = 1 1/4 to 1 5/8 in. y is 3/32 in. for d = 3/16 to 7/16 in., and 1/16 in. for d = 1/2 to 15/16 in.

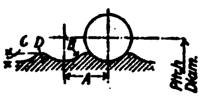
Drum Scores for Cables and Chains may be made as illustrated in Figs. 160–162, with the dimensions given in Table 83.

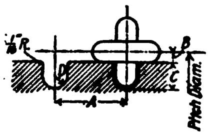
Table 83. Dimensions of Standard Drum Scores

(All dimensions in in. Letters refer to Figs. 160, 161 and 162)

See Fig. 160	Rope	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
	A	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/16
	B	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	17/32
	C	3/32	7/64	1/8	9/64	5/32	11/64	3/16	13/64	7/32	15/64	1/4
	D	3/32	3/32	3/32	1/8	1/8	1/8	5/32	5/32	5/32	3/16	3/16
See Fig. 161	Chain	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
	A	1 1/2	1 11/16	1 7/8	2 1/16	2 5/16	2 1/2	2 11/16	2 7/8	3 1/8	3 5/16	3 1/2
	B	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2
	C	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/16	1 1/8	1 3/16
	D	3/16	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	17/32	9/16
See Fig. 162	Chain	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
	A	1 1/4	1 7/16	1 9/16	1 3/4	1 7/8	2 1/16	2 3/16	2 3/8	2 1/2	2 11/16	2 13/16
	B	11/32	3/8	7/16	15/32	17/32	9/16	5/8	21/32	23/32	3/4	13/16
	C	3/16	7/32	1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2
	D	1	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	1 7/8	2	2 1/8	2 1/4

Strength of Hoisting Drums. Let W = total load to be hoisted, lb.; l = distance c. to c. of bearings, in.; d = diameter of drum at bottom of grooves, in.; d_1 = inside diameter of drum, in.; Z = section modulus = $0.0982\,[d^3 - (d_1^4/d)]$; M = bending moment on drum = $Wl/4$; P = tension in each





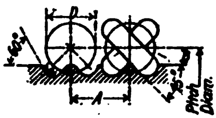


FIG. 160. FIG. 161. FIG. 162.

Drum Scores for Cables and Chains.

rope or chain supporting the load, lb.; p = pitch of grooves, in.; $t = \frac{1}{2}(d - d_1)$ = thickness of drum at bottom of grooves, in. Then, for cast-iron drums, to resist bending stresses, M/Z should not exceed 3000 lb. per sq. in.; to resist crushing stresses, $K = \sqrt{(M/Z)^2 + (P/pt)^2}$ should not exceed 6000 lb. per sq. in. For example, assume W = 10,000 lb., P = 5000 lb. (2 ropes), d = 15 in., l = 80 in., and $p = \frac{3}{4}$ in. Taking 12.5, 13 and 13.5 in. as trial values of d_1, the formula of K works out respectively as 5550, 6813 and 9060. The economical value (6000) therefore results when d_1 lies between 12.5 and 13 in. Taking d_1 = 12.75 in., K = 6060 and M/Z = 1263, both of which practically fulfill the conditions stated.

For **hoisting drums,** see p. 1107.

BELT DRIVES

(For the properties of belt materials, see p. 621)

Leather Belts of the best quality are those made from the "butt" of the hide. The ultimate tensile **strength** of leather used in belts varies from 3000 to 5000 lb. per sq. in. Average values of the breaking strength of good oak-tanned belting, as determined by Benjamin, are as follows:

	Breaking strength per inch of width, lb. Single belts	Double belts
Solid leather	900	1400
At riveted joint	600	1200
At laced joint	350	....

The **weight** of leather belting is about 0.035 lb. per cu. in. Single belts average from 0.22 in. to 0.25 in. in **thickness**, and double belts from 0.33 to 0.35 in. Rawhide, semi-rawhide, (*i.e.*, surface-tanned rawhide), and chrome-tanned (green) belts are not as serviceable in dry places as oak-tanned belts. Rawhide and chrome-tanned belts give good service in damp places, such as dye-houses.

Cotton and Canvas Belts weigh from 0.026 to 0.05 lb. per cu. in., the heavier qualities containing more sizing and frequently being waterproofed. Cotton belts are made in 2-ply from 1 to 24 in. wide; 3-ply, from 1½ to 24 in.; 4-ply, from 2 to 24 in.; 5-ply, from 4 to 24 in., and 6-ply, from 6 to 24 in. The widths increase by ½ in. from 1 in. up to 4 in., by 1 in. from 4 in. to 10 in., and by 2 in. from 10 in. to 24 in. Canvas belts are obtainable in 4-ply from 1½ to 18 in. wide; in 6-ply from 3 to 30 in.; in 8-ply from 4 to 48 in., and in 10-ply from 12 to 60 in. The widths increase by ½ in. from 1½ to 5 in.; by 1 in. from 5 to 14 in.; by 2 in. from 14 to 32 in.; widths above 32 in. are 36, 40, 48, 54 and 60 in. Canvas belting has about the same **strength** as leather belting.

Rubber Belts are specially adapted for use in damp places and out of doors. The rubber, however, when poor or softened by animal oil or grease, is liable to peel off, especially on belts which are frequently shifted. Rubber belting **weighs** about 0.045 lb. per cu. in. Four-ply belting, according to Benjamin, has an ultimate tensile **strength** of from 890 to 930 lb. per sq. in. Bach recommends a **working tension** of from 100 to 150 lb. per sq. in. Rubber belts are made in the following widths and thicknesses: 2-ply, 3-ply and 4-ply, from 1 to 60 in. wide; 5-ply, from 1½ to 60 in.; 6-ply from 2 to 60 in.; 7-ply, from 4 to 60 in.; 8-ply, from 6 to 60 in. The widths increase by ¼ in. from 1 in. to 2 in.; by ½ in. from 2 to 5 in.; by 1 in. from 5 to 16 in.; by 2 in. from 16 to 60 in.

Steel Belts have been used with success in Germany during the past few years, in thicknesses ranging from 0.008 to 0.044 in. and up to 10 in. in width. They are made from a special steel, tempered, and fitted with a joint fastened by screws. The belts are made at the factory to a length just enough less than the exact length around the pulleys where they are to be used to produce the proper initial tension when in working position. The pulleys on which they run are cylindrical—without crown—with cork-faced canvas cemented around them, which practically does away with slip. The effect of centrifugal force is negligible, and speeds up to 19,000 ft. per min. have been reported. The cross-section per h.p. transmitted at a speed around 3000 ft. per min. is about 0.00072 sq. in.

Belt Joints. Cemented joints when properly made have a strength equal to that of the solid leather. **Leather-laced and riveted joints** are about ½ and ⅔ as strong respectively as the solid leather. **Wire-laced joints** have from 85 to 90 per cent. of the strength of solid leather. Heavy metal belt joints should not be used on belts run at high speeds.

Arrangements for Belt Drives. In belt drives the center line of the belt advancing on the pulley should lie in a plane passing through the mid-section of the pulley at right angles to the shaft. Shafts inclined to each other require connection as shown in Fig. 163. In case **guide pulleys** are needed, their positions can be determined as shown in Figs. 164, 165 and 166. In Fig. 165 the center circles of the two pulleys to be connected are set in correct relative position in two planes, a being the angle between the planes (= supplement of angle between shafts). If any two ıs E and F be assumed on the line of intersection MN of the planes, and ts EG, EH, FJ and FK be drawn from them to the circles, the center of the guide pulleys must be so arranged that these tangents are also

63.

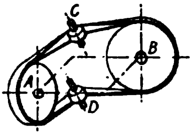


FIG. 164.

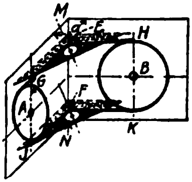


FIG. 165.

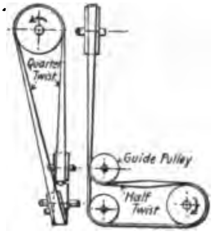


FIG. 166.

Arrangements for Belt Drives.

;s to them, as shown. In other words, the middle planes of the guide must lie in the planes GEH and JFK. When these conditions are ꜿ belts will run in either direction on the pulleys.

void the necessity of taking up the **slack** in belts which have become ꜿd and permanently lengthened, a **belt tightener** ı shown in Fig. 167 may be employed. It should ꜿed on the slack side of the belt and nearer the pulley than the driven pulley.

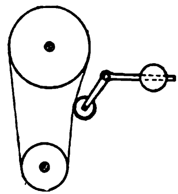


FIG. 167.—Belt Tightener.

;th of Belt for a Given Drive. The length of an **elt** for a given drive is equal to $L = \frac{1}{2}\pi D + \frac{1}{2}\pi d$ $c^2 + (R - r)^2$, where L = length of belt, in.; D = of large pulley, in.; d = diam. of small pulley, $= \frac{1}{2}D$; $r = \frac{1}{2}d$, and x = distance between pulley ı.

n a **crossed belt** is used, the length is

$$L = \tfrac{1}{2}\pi D + \tfrac{1}{2}\pi d + 2\sqrt{x^2 + (R + r)^2}.$$

Pulleys. In the case of belts operating on cone pulleys, the pulley ꜿrs must be such that the belt will fit over any pair with equal tightness. **·ossed belts,** it will be apparent from the equation for length of belt that ı of the pulley diameters need only be constant ıe belt may fit with equal tightness on each pulleys. With open belts the length is a function both the sum and the difference of the pulley ꜿrs, hence no direct solution of the problem is ꜿ.

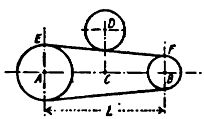


FIG. 168.

ıphical method devised by C. A. Smith (*Trans.* Γ. *E.*, vol. 10) is shown in Fig. 168. Let A and ıe centers of any pair of pulleys in the set, the diameters of which are or assumed. Bisect AB in C and draw CD at right angles to AB. $D = 0.314$ times the center distance L and draw a circle tangent to the e EF. The belt line of any other pair of pulleys in the set will then ʒent to this circle. If the angle EF makes with AB is greater than ., draw a tangent to the circle D making an angle of 18 deg. with AB ›m a center on CD distant $0.298L$ above C draw an arc tangent to this 18-deg. line. All belt lines with angles greater than 18 deg. will be tangent to this last-drawn arc.

A very slight error in a graphical solution drawn to any scale much under full size, will introduce an error seriously affecting the equality of belt tensions on the various pairs of pulleys in the set, and where much power is to be transmitted it is advisable to calculate the pulley diameters from the following **formulæ** derived from Burmester's graphical method ("Lehrbuch der Mechanik").

Let D_1 and D_2 be respectively the diameters of the smaller and larger pulleys of a pair, $n = D_2/D_1$, and l = distance between shaft centers, all in in. Also let $m = 1.58114l - D_0$, where D_0 = diam. of both pulleys for a speed ratio $n = 1$. Then $(D_1 + m)^2 + (nD_1 + m)^2 = 5l^2$. First settle on values of D_0, l and n and then substitute in the equation and solve for D_1. The diam. D_2 of the other pulley of the pair will then be nD_1. The values thus obtained are correct to the fourth decimal place.

The speeds given by cone pulleys should increase in a **geometrical ratio**, that is, each speed should be multiplied by a constant a in order to obtain the next higher speed. Let n_1 and n_2 be respectively the lowest and highest speeds (r.p.m.) desired and k the number of speed changes. Then $a = \sqrt[k-1]{n_2/n_1}$. In practice, a ranges from 1.25 up to 1.75 and even 2. The ideal value for a in machine-tool practice, according to Carl G. Barth, would be 1.189. In the example below this would mean the use of 18 speeds instead of 8.

Example. Let $n_1 = 16$, $n_2 = 400$, and $k = 8$, to be obtained with four pairs of pulleys and a back gear. From formula, $a = \sqrt[7]{25} = 1.584$, whence speeds will be 16, $(16 \times 1.584 =) 25.34$, $(25.34 \times 1.584 =) 40.14$, and similarly 63.57, 100.7, 159.5, 252.6, and 400. The first four speeds are with the back gear in, hence the back-gear ratio must be $100.7 \div 16 = 6.29$.

Transmission of Power by Belts. The turning force (tangential) on the rim of a pulley driven by a flat belt is equal to $T_1 - T_2$, where T_1 and T_2 are respectively the tensions in the driving (tight) side and following (slack) side of the belt. (For the relations of T_1 and T_2 at low peripheral speeds, see p. 248.) $\text{Log}\,(T_1/T_2) = 0.0076fa$ when the effect of centrifugal force is neglected. When the speeds are high, however, the relations of T_1 to T_2 are modified by centrifugal stresses in the belt, in which case $\log\,(T_1/T_2) = 0.0076f(1 - x)a$, where f = coefficient of friction between the belt and pulley surface, a = angle of wrap, in degrees, and $x = 12wv^2/gt$, in which w = weight of 1 cu. in. of belt material, lb.; v = belt speed, ft. per sec.; g = 32.2 ft. per sec.^{2} and t = allowable working tension, lb. per sq. in. Values of x for leather belting (with w = 0.035 and t = 300) are as follows:

For v =	30	40	50	60	70	80	90	100	110	120	130
x =	0.039	0.070	0.118	0.157	0.214	0.279	0.352	0.435	0.526	0.626	0.735

Researches by Barth (*Trans. A. S. M. E.*, 1909) seem to show that f is a function of the belt velocity, varying according to the formula $f = 0.54 - 140/(500 + V)$ for leather belts on iron pulleys, where V = belt velocity in ft. per min. For practical purposes of design, however, the following values of f may be used: For leather belts on cast-iron pulleys, f = 0.30; on wooden pulleys, f = 0.45; on paper pulleys, f = 0.55. The treatment of belts with belt dressings, pulleys with cork inserts, and dampness are all factors which greatly modify these values, tending to make them higher. Canvas belts operate satisfactorily with effective tensions on pulley rims varying from 20 lb. to 23 lb. per inch width per ply. Balata belting will take tensions 15 per cent. greater. These belts should not be laced.

horse power transmitted by a leather belt is

$$\text{h.p.} = (T_1 - T_2)V/33{,}000$$

$T_1 - T_2$ is the difference in belt tensions, lb., and V is the belt velocity, min. It is good practice to allow 35 lb. per in. of width for single id 50 lb. for double belts in roughly estimating the allowable values of ₂. Also, in estimating T_1, it is custo-) allow 70 lb. per in. of width for single id 110 lb. for double belts. When using leather belts it will be found advisable to ie pulley not less than 12 in. in diam.; le belts, not less than 20 in. in diam. 169 gives the **horse power transmit-· inch of belt width** at various speeds arc of contact of 180 deg., according to ctice of Wm. Sellers & Co., Inc. (based ie experiments of Wilfred Lewis, *Trans. I. E.*, vol. 7 and 20) and according to ommendation of C. G. Barth (*Trans. I. E.*, vol. 31). Barth's curves are in-to insure maximum durability of belts applicable to belts with any good joint. ("Handbook for Machine Designers") ends the use of the Sellers curves (*S*) ing belts and of Barth's (*B*) for ma-lts.

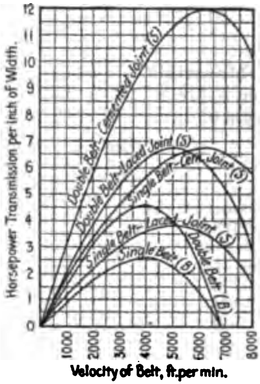


Fig. 169.—Power Transmitted by Belting.

an arc of contact differing from 180 values taken from Fig. 169 should be ed by a factor given in the following table:

con-g.) =	90	100	110	120	130	140	150	160	170	180	190	200	210
=	0.65	0.70	0.75	0.79	0.83	0.87	0.91	0.94	0.97	1.00	1.03	1.05	1.07

rc of contact on the smaller of two pulleys connected by an open belt, s, is approximately equal to $181 - [60(D - d)/l]$, where D and d are r and smaller pulley diameters and l the distance between their shaft all in in. This formula gives an error not exceeding 0.5 per cent.

ral Notes on Belting. The following conclusions are those arrived W. Taylor (*Trans. A. S. M. E.*, vol. 15) after 9 years of experimental machine-shop belting:

, thick belts give better satisfaction than wide, thin belts. Thickness should rith width.

st economical speed is from 4000 to 4500 ft. per min.

t distance between shaft centers is from 20 to 25 ft.

joints should be spliced and cemented. Double belts less than 10 in. wide ve a splice 10 in. long; belts wider than 10 in. should have a splice length he belt width. Rubber belts should have stepped splices, coated with rubber nised in place.

leather belts will last well when repeatedly tightened to an initial tension of 71 of width (= 240 lb. per sq. in.) but will not long maintain this tension.

ening belts, belt clamps with spring balances between them for weighing the ould be used. When their use is impracticable a double belt should be short-; in. for every 10 ft. of length for the ordinary rule of 110 lb. per in. of width 65 lb. per in. of width for the effective pull $T_1 - T_2$), and (*b*) 1 in. for every the more economical rule of 54 lb. for T_1(= 26 lb. for $T_1 - T_2$). Under (*a*) double belts should, with care, last 7 years; under (*b*), 18 years.

In figuring the total expense of belting, by far the largest item is the time lost on the machines while belts are being relaced and repaired.

Oak-tanned and fulled leather double belts should not have the effective pull in excess of 35 lb. per in. of width. At this value the number of linear feet of 1-in. double belt passing around a pulley per min. required to transmit 1 h.p. is 950. For other types of leather belts and 6- and 7-ply rubber belts the corresponding values are 30 lb. and 1100 ft.

ROPE DRIVES

Textile Rope Drives

Materials. Textile ropes are made from cotton, hemp, and Manila hemp fibers, see p. 629. The ultimate **strength,** working strength, and **weight** per ft. of such ropes are approximately as given in Table 84. The

Table 84. Weight and Strength of Textile Ropes

Diam. of rope, in.	Ultimate strength, lb.		Working strength, lb.		Weight, lb. per ft.	
	Cotton	Manila Hemp	Cotton	Manila Hemp	Cotton	Manila Hemp
½	1,150	1,900	50	50	0.06	0.08
⅝	1,800	2,900	78	78	0.10	0.13
¾	2,600	4,100	112	112	0.15	0.18
⅞	3,500	5,500	153	153	0.20	0.24
1	4,600	7,100	200	200	0.26	0.32
1¼	7,200	10,900	312	312	0.40	0.50
1½	10,400	15,000	450	450	0.58	0.72
1¾	14,000	19,800	612	612	0.79	0.98
2	18,400	25,100	800	800	1.04	1.28

values in the table are determined from the following formulæ, which agree with the results of tests and practice:

		Cotton	Hemp and Manila
Ultimate strength in lb.	=	$4600d^2$	$100d^2(81 - 9d)$
Working strength in lb.	=	$200d^2$	$200d^2$
Weight, lb. per ft.	=	$0.26d^2$	$0.32d^2$

Transmission Rope is made with three, four or six strands, the last two having an inner core. Three-strand rope is used for small drives where the rope diameter is less than ⅞ in. or where the rope is subject to much bending. Four and six strands are preferable for larger drives. Cotton rope is more flexible than Manila hemp and may be used for small installations where small pulleys are necessary. It is less desirable than Manila, less strong, more difficult to splice, more expensive, and less resistant to weather.

Transmission ropes **wear** out usually by internal friction, the wear being more rapid as the pulley sizes are diminished. For satisfactory wear the rope must be laid with a lubricant such as graphite. If run in a hot or dry place or exposed to the weather, a slight application of tallow to the rope when in motion will be found beneficial. The **pulley diameter** should never be less than 36 times the rope diameter. The use of a few large ropes, when consistent with the diameter of the smallest sheave, is the better practice, as there are fewer splices to be made, wear is less, and breakdowns are less likely to occur. In general, however, ropes larger than 1¾ in. in diameter should only be used on drives of great magnitude.

Capacity of Ropes for Power Transmission. Good three-strand cotton ropes with an arc of contact of 180 deg. will transmit power in accordance with the values given in Table 85 (Edward Kenyon, *Trans. S. Wales Inst.*

1909). These values correspond to those obtained from the formula: each rope = $0.003265Vd^2$, in which V = rope velocity, ft. per min., = diam. of rope, in. This formula may be used for intermediate ›s and rope sizes. Fig. 170 (from the catalog of the American Mfg.

le 85. Horse Power Transmitted by Three-strand Cotton Ropes

(For ropes running on pulleys having diameters ≧ 30 × rope diam.)

y,	Rope diameters, in.					Velocity, ft. per min.	Rope diameters, in.				
	1	1¼	1½	1¾	2		1	1¼	1½	1¾	2
	3.3	5.1	7.4	10	13.0	3500	11.3	17.7	26.0	35	45.7
	4.9	7.6	11.1	15	19.6	4000	13.0	20.2	29.7	40	52.2
	6.5	10.1	14.9	20	26.1	4500	14.6	22.7	33.4	45	58.7
	8.1	12.6	18.5	25	32.6	5000	16.2	25.3	37.1	50	65.3
	9.7	15.1	22.3	30	39.1	5500	17.8	27.8	40.9	55	71.8

›ws the relations between the speed, horse power and size of **Manila** .**ission rope.** The **most economical speed** is about 4500 ft. per

orce relations in power transmission by textile ropes (see also p. 746) approximately expressed by the equation

$$\log (T_1/T_2) = 0.0076fa(1-x)$$

ı and T_2 are respectively the tensions in the tight (driving) and loose ıg) sides of the rope, lb.; f = coefficient of friction × cosecant of half e of the rope groove; a = angle of wrap, in degrees, and x = a factor account of centrifugal stresses (see p. 746). Values of f are given in 6, and values of $(1 - x)$ in Table 87.

Table 86. Values of f in Rope Tension Ratios

f = 0.12 cosec ½ b, where b = angle of rope groove.

eg.) =	30	35	40	45	50	55	60
f =	0.46	0.40	0.35	0.31	0.28	0.26	0.24

Table 87. Values of $(1 - x)$ in Rope Tension Ratios

Values of $(1 - x)$ for a working stress equivalent to 200 d^2 lb.

ŕ,	$(1 - x)$	Velocity, ft. per min.	$(1 - x)$	Velocity, ft. per min.	$(1 - x)$	Velocity, ft. per. min.	$(1 - x)$
	0.93	3500	0.83	5500	0.58	7500	0.22
	0.94	4000	0.78	6000	0.50	8000	0.11
	0.91	4500	0.72	6500	0.41	8500	0.00
	0.87	5000	0.65	7000	0.32	....	....

ms of Rope Driving. There are two general systems of rope driving on use: the multiple, or English, system, and the continuous, or American tem. The **multiple system** consists of one or more independent ınning side by side in the grooves of the sheave wheels or pulleys and is adapted to the transmission of large powers. It is unlikely that ın one rope will fail at a time, and the system therefore is comparacure against a general breakdown. Power may be easily transmitted to the different floors of a mill and the amount of power transmitted may be easily increased by adding new ropes. The ropes always bend in the same direction and therefore wear less rapidly than in the continuous system. Disadvantages are the number of splices needed, unsuitability for outdoor use and for vertical drives. In the **continuous system** (Fig. 171), one rope is wound around the driving and driven sheaves several times, the rope from an outside groove of the driving sheave being guided by an independent sheave around a weighted tightener sheave, thence to the outside groove of the driven sheave. The tightener sheave or **tension carriage** produces a uniform tension in the rope and automatically regulates the slack due to stretch, inequalities of load, etc. The continuous system is especially adapted to vertical and quarter-turn drives (see Fig. 171), where shafts are inclined at an angle to each other, where rope is exposed to the weather, and in cases of complicated transmission. In general, it should also be used where the shafts to be connected are nearer together than 30 ft., as in the multiple system a small amount of stretch will decrease the initial tension to such an extent that the centrifugal force will lift rope from its groove, thereby lessening its driving capacity.

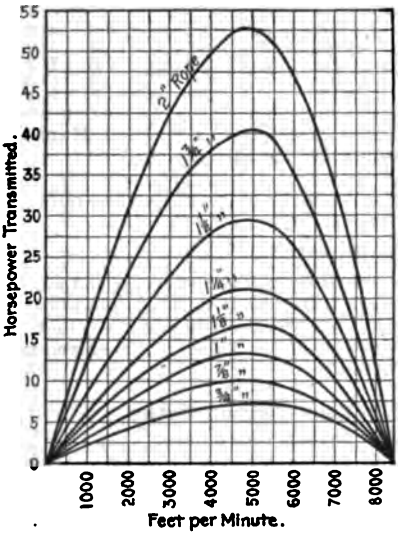


FIG. 170.—Power Transmitted by Cotton Rope.

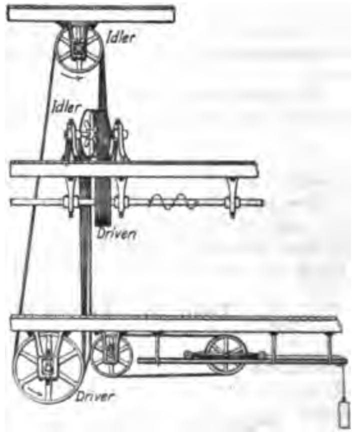


FIG. 171.—Continuous System of Rope Driving.

Rope Sheaves. Proportions for rope sheaves are shown in Figs. 155 and 156 and dimensions therefor are given in Table 79. The **groove angle** generally employed is 45 deg., larger angles permitting the rope to slip slightly under varying loads or overstress. In **idler or carrier sheaves** either 45-deg. or U-shaped grooves may be used. When the drive runs outdoors and the rope is exposed to the wind, the deep U-shaped groove should be used. This groove is $d + \frac{1}{4}$ in. wide at the periphery and slightly deeper than d, where d = rope diameter. Satisfactory driving may be done over distances up to 175 ft. without the use of idlers.

Tension Carriage. For drives over short center distances not more than laps of rope should be cared for by one tension sheave. For longer he number of laps should be reduced in proportion to the length and of the ropes between the driving and driven sheaves. Ordinarily, the m length of rope to be governed by a tension carriage should not 000 ft.

f Ropes. The sag on the driving side of both multiple and continu- ems, measured in feet, $= S = WL^2/8T$, where W = weight of 1 ft. , lb.; L = distance between shafts, ft.; T = maximum allowable in the rope, lb. per sq. in., taken as $200d^2$. The sag on the slack side nined by the same formula, using, however, the tension in the slack place of the maximum allowable tension. Table 88 gives values cal- from the formula. See pp. 147 to 151 for formulæ and tables for th of rope, etc., under various conditions.

Table 88. Sag in Power Transmission Ropes

Sag on driving side, all speeds, ft.	Sag on slack side, ft. Velocity, ft. per min.					Distance between sheaves, ft.	Sag on driving side, all speeds, ft.	Sag on slack side, ft. Velocity, ft. per min.				
	3000	4000	4500	5000	5500			3000	4000	4500	5000	5500
0.19	0.45	0.39	0.36	0.33	0.30	90	1.7	4.0	3.5	3.2	3.0	2.7
0.34	0.80	0.69	0.64	0.59	0.53	100	2.1	5.0	4.3	4.0	3.7	3.3
0.53	1.2	1.1	1.0	0.92	0.84	120	3.0	7.2	6.2	5.7	5.3	4.8
0.76	1.8	1.7	1.4	1.3	1.2	140	4.1	9.9	8.5	7.8	7.2	6.6
1.0	2.4	2.1	1.9	1.7	1.6	160	5.4	12.9	11.1	10.2	9.5	8.6
1.4	3.2	2.9	2.5	2.3	2.1	....	....	.....	......	......	....	....

ing Transmission Rope. Splices should have the same diameter ope itself, should be smooth and free from lumps, and should so be at the fastenings or "tucks" will not wear away and allow the rope to 1. The following instructions for making the English transmission r a 4-strand rope spliced on sheaves in the multiple system, are l from the American Mfg. Co.'s "Blue Book of Rope Transmission:"

t place the rope around the sheaves and stretch it taut with a tackle, the ing each other from 6 to 7 ft., and the passing point being marked with twine ope. Then slip the rope off the sheaves to give sufficient slack for making the . Unlay the strands in pairs as far back as twines M and M', crotch the four trands thus opened (Fig. 172), the cores c having been drawn out together on side. 3. Remove twine M and unlay strands 6 and 8 in pairs back 2 ft. to A rands 1 and 3 carefully in their place. Next unlay 5 and 7 to A' and replace nd 4 (see Fig. 173). 4. Separate strands 6 and 8, unlaying 8 four feet back to 6 at A (Fig. 174). 5. Separate strands 1 and 3, leaving 3 at A as a companion laying 1 in place of 8 until they meet at B. 6. Separate 2 from 4 and 5 from similarly, taking care that the original twist and lay of the strands is main- Trim all projecting strands to 2 ft. in length. 7. Tucking in the Strands. This for strands 2 and 7 (identical for the other three pairs) is performed as follows: nd 7 for 12 to 14 in. (Fig. 175), divide each in half by removing its cover yarns, ends 2′ and 7′ with twine; then, leaving cover 2, relay 2′ until near 7 and 7′, point join 2′ and 7′ with a simple knot (Fig. 176). Divide cover yarns 7 and hrough them, continuing on through the rope *under* the two adjacent strands, the core, thus locking 2′ (Fig. 177). In no event pass 2′ over these or any other At the right of knot made with 2′ and 7′, raise 2′ slightly with a marlinspike or tuck 7′ around it two or three times, these two thus forming a whole strand. lf-strand 7′ until cover is reached, the yarns of which are divided and 7′ rough them and drawn under the two adjacent strands, again forming the t off the strand ends at both locks, leaving about 2 in. so that the yarns may

draw slightly without unlocking (Fig. 178). After a few days service the projecting ends will wear away.

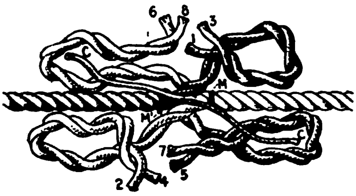


Fig. 172.

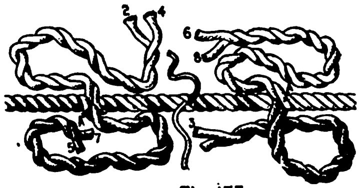


Fig. 173.

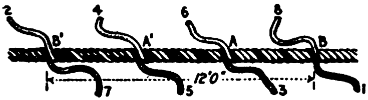


Fig. 174.

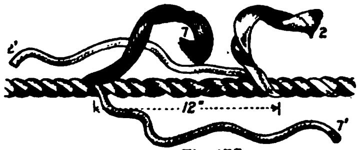


Fig. 175.

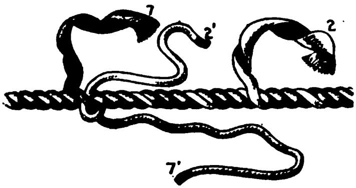


Fig. 176.

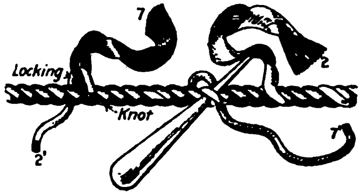


Fig. 177.

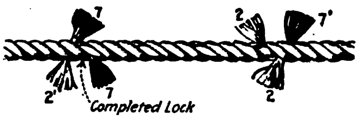


Fig. 178.

Splicing of Power Transmission Rope.

Efficiencies of Rope Drives. Results of extensive tests reported by E. H. Ahara (*Jour. A. S. M. E.*, Aug., 1913) show that the efficiency of both the multiple and continuous systems falls off slightly with an increase in speed, but is not affected by changes in center distances between 25 and 150 ft. Two types of drive were tested: the "open" (horizontally, over two sheaves) and the "up and over" (upward from driver, horizontally over two idlers, downward and then around driven wheel, upward a short distance, horizontally over two idlers, and thence downward around driver to point of beginning), with from one to eight 1-in. Manila ropes. The multiple system sheaves had 45-deg. grooves and the continuous system 60-deg. grooves in which the ropes lay freely and withdrew without effort. The following efficiencies at ¾ load were obtained on 100-ft. centers and at a rope speed of 4500 ft. per min.:

No. of ropes	1	2	3	4	6	8
Continuous system (American):						
Effy. of open drive, per cent	91	93	94	94	94	94
Effy. of up and over drive, per cent	58	72	79	80	80	81
Multiple system (English):						
Effy. of open drive, per cent	73	80	84	86	86	86
Effy. of up and over drive, per cent	56	67	71	72	75	77

Comparative Costs of Belt and Rope Drives. According to the Plymouth Cordage Co., belts cost 2½ times as much as Manila ropes of equal capacity, while rope sheaves cost about ⅓ more than belt pulleys.

Wire Rope Drives

The general **characteristics** and commercial **sizes** of wire ropes are given on p. 843.

Bending Stresses in Wire Ropes. Wire ropes when transmitting power are subjected to bending stresses in passing around pulleys or sheaves. The intensity of stress in any case may be determined by means of the formula

$$f = Ea/[2.06(R/d) + C]$$

where f = stress, lb.; E = modulus of elasticity of the material = 28,500,000 (average); a = aggregate cross-sectional area of the wires in the rope, sq. in.; R = radius of bend, in.; d = diam. of wire, in; C = 9.27 for ropes of six 7-wire strands (for which d = ⅑ rope diam.) = 15.45 for ropes of six 19-wire strands (for which d = 1/15 rope diam.). Ropes with six 7-wire strands are those generally used in power transmission, and their bending stresses are given in Table 89.

Table 89. Bending Stresses of Transmission Rope of Six 7-wire Strands, Lb.

(Trenton Iron Co.)

Diam. of bend, in.	10	12	14	16	18	20	22	24	26	28
Diam. of rope, in.										
¼	1,954	1,635	1,406	1,254	1,098	989	900	826	763	709
9/32	......	2,214	1,904	1,670	1,488	1,341	1,220	1,120	1,034	962
5/16	......	......	2,734	2,399	2,137	1,926	1,754	1,609	1,487	1,382
⅜	......	......	......	4,129	3,680	3,318	3,022	2,774	2,564	2,383
7/16	......	......	......	......	5,823	5,253	4,785	4,385	4,061	3,776
½	......	......	......	......	......	8,519	6,751	6,200	5,732	5,329
9/16	......	......	......	......	......	......	9,879	9,072	8,389	7,802
⅝	......	......	......	......	......	......	......	......	11,756	10,935
Diam. of bend, in.	**30**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**
Diam. of rope, in.										
¼	662	553	412	333	277	238	208	185	166	151
9/32	898	750	563	451	376	323	282	251	226	206
5/16	1,291	1,078	810	649	541	464	406	361	325	296
⅜	2,226	1,859	1,398	1,120	934	801	702	624	562	511
7/16	3,528	2,982	2,217	1,777	1,482	1,272	1,113	990	892	811
½	4,980	4,161	3,131	2,510	2,905	1,797	1,574	1,400	1,260	1,146
9/16	7,291	6,095	4,589	3,679	3,071	2,635	2,308	2,053	1,848	1,681
⅝	10,221	8,547	6,438	5,164	4,310	3,699	3,240	2,882	2,595	2,360
11/16	13,058	10,922	8,230	6,603	5,513	4,731	4,144	3,687	3,320	3,021
¾	......	14,202	10,706	8,591	7,174	6,158	5,394	4,799	4,322	3,931
⅞	......	22,592	17,045	13,685	11,431	9,815	8,599	7,651	6,892	6,260
1	......	......	25,476	20,464	17,100	14,686	12,869	11,452	10,317	9,369
1⅛	......	......	36,289	29,165	24,416	20,942	18,355	16,336	14,718	13,396
1¼	......	......	......	40,020	33,464	28,754	25,206	22,437	20,216	18,396
1⅜	......	......	......	......	44,551	38,290	33,571	29,888	26,933	24,510
1½	......	......	......	......	57,835	49,718	43,599	38,821	34,987	31,842

Table 90. Minimum Diameters of Sheaves for Power Transmission by Wire Ropes

(All dimensions in inches)

Rope diam.	Steel		Iron		Rope diam.	Steel		Iron	
	7-wire	19-wire	7-wire	19-wire		7-wire	19-wire	7-wire	19-wire
¼	20	12	40	24	¾	60	35	120	72
5/16	25	15	50	30	⅞	70	41	140	84
⅜	30	18	60	36	1	80	47	160	96
7/16	35	21	70	42	1⅛	90	53	180	108
½	40	24	80	48	1¼	100	58	200	120
9/16	45	27	90	54	1⅜	110	64	220	132
⅝	50	30	100	60	1½	120	70	240	144
11/16	55	32	110	66	......	......	......	......	......

Sheaves. The necessity for keeping bending stresses within safe limits has occasioned the establishment of minimum sheave diameters for the various sizes of rope. Table 90 is representative of good practice in this regard. The first cost of large sheaves is high but they conduce to long life for the rope and hence to low operating cost. For proportions of sheaves, see Fig. 157 and Table 80. Ordinarily, the best filling for the grooves consists of segments of leather and rubber soaked in tar and packed alternately in the groove. Where the pressure of the rope is very great, hardwood blocks are preferable. Sheaves should be as light as is consistent with strength, and well balanced.

Transmission of Power by Wire Rope. In wire-rope transmission the **working tension** on the rope should not exceed the difference between the maximum safe load and the bending load as obtained from Table 89 (for 7-wire rope) or from the formula given for f. Ordinarily, where the work is approximately uniform, the maximum safe load may be taken as 0.4 × ultimate load for cast steel and about 0.33 for plow steel. Thus, the ultimate load of a 1-in. cast-steel rope of six 7-wire strands is given in manufacturers' tables as 62,000 lb., whence maximum safe load = 24,800 lb. The bending load for this rope around a 96-in. sheave is (Table 89) 12,869 lb., or working tension = 24,800 − 12,869 = 11,931 lb. The usual **rope velocities** are from 50 to 100 ft. per sec., and centrifugal tension is generally neglected in power calculations. In the present case, centrifugal tension, lb., $= wv^2/32.2$, where w = weight of 1 ft. of rope, lb. For a 1-in. rope w = 1.58 lb., whence centrifugal tension = 490 lb. for v = 100 ft. per sec.

To avoid slipping, a certain ratio must exist between the tension T_1 in the tight side and T_2 in the slack side of the rope. This ratio is

$$T_1/T_2 = e^{fn\pi}$$

in which e = Napierian logarithmic base = 2.718, f = coefficient of friction between the rope and the material used in filling the sheave grooves, n = number of half laps of the rope about the sheaves at either end of the line. Following are values of f given by the Trenton Iron Co. for dry, wet, and greasy ropes running in various tracks:

	Grooved iron drum	Wood-filled sheaves	Sheaves with rubber and leather filling
For dry ropes f =	0.170	0.235	0.495
For wet ropes f =	0.085	0.170	0.400
For greasy ropes f =	0.070	0.140	0.205

The force available in transmitting power is the difference between the tensions T_1 and T_2, or $T_1 - T_2 = T_2\,(e^{fn\pi} - 1)$ and to obtain this the rope must be tightened when at rest so that the initial tension $= \frac{1}{2}(T_1 + T_2)$, or

$$\text{Initial tension} = \tfrac{1}{2}T_2(e^{fn\pi} + 1) = \tfrac{1}{2}(T_1 - T_2)\,(e^{fn\pi} + 1)/(e^{fn\pi} - 1)$$

Values of $e^{fn\pi}$ and of $(e^{fn\pi} + 1)/(e^{fn\pi} - 1)$ for various values of f and n are given in Table 91.

Table 91. Values of Coefficients in Wire Rope Power Transmission Formulæ

$f =$	$e^{fn\pi}$						$(e^{fn\pi} + 1)/(e^{fn\pi} - 1)$					
	n = number of half laps about sheaves or drums at either end of line						n = number of half laps about sheaves or drums at either end of line					
	1	2	3	4	5	6	1	2	3	4	5	6
0.070	1.246	1.552	1.934	2.410	3.003	3.741	9.130	4.623	3.141	2.418	1.999	1.729
0.085	1.306	1.706	2.228	2.910	3.801	4.964	7.536	3.833	2.629	2.047	1.714	1.505
0.140	1.552	2.410	3.741	5.808	9.017	13.998	4.623	2.418	1.729	1.416	1.249	1.154
0.170	1.706	2.910	4.964	8.467	14.445	24.641	3.833	2.047	1.505	1.268	1.149	1.085
0.205	1.904	3.626	6.904	13.146	25.031	47.663	3.212	1.762	1.338	1.165	1.083	1.043
0.235	2.092	4.378	9.160	19.166	40.100	83.902	2.831	1.592	1.245	1.110	1.051	1.024
0.400	3.514	12.346	43.376	152.405	535.488	1,849.140	1.795	1.176	1.047	1.013	1.004	1.001
0.495	4.716	22.425	106.194	502.881	2,381.400	....	1.538	1.093	1.019	1.004	1.001	1.000

Table 92. Horse Power Transmitted by Cast-steel Wire Ropes

(For ropes making a single lap on wood-filled sheaves. For iron ropes, take half the tabular values. For a given horse power the larger sheaves give the better results. For rubber- and leather-filled grooves, multiply values by 1.3.)

Diam. of sheave, in.	Diam. of rope, in.	Number of wires in each strand	Number of revolutions of sheaves per min.					
			75	100	125	150	175	200
18	3/8	19	5	7	9	11	13	15
24	3/8	19	7	10	12	15	17	20
30	3/8	7	9	12	15	18	21	25
36	7/16	7	15	20	25	30	35	40
36	1/2	19	20	26	33	39	46	53
42	1/2	7	23	31	38	46	54	61
48	1/2	7	26	35	44	53	61	70
36	9/16	19	25	33	42	50	58	67
48	9/16	7	33	44	55	67	8	89
36	5/8	19	31	41	51	62	72	82
48	5/8	7	41	55	68	82	96	110
60	5/8	7	51	68	86	103	120	137
36	11/16	19	37	50	62	75	87	99
48	11/16	19	50	66	83	99	116	133
60	11/16	7	62	83	104	124	145	166
36	3/4	19	44	59	74	89	104	118
48	3/4	19	59	79	99	118	138	158
60	3/4	7	74	99	123	148	173	197
48	7/8	19	81	108	134	161	188	....
60	7/8	19	101	134	168	201	....	....
72	7/8	7	121	161	201	....	....	....
48	1	19	105	140	175	....	....	....
60	1	19	132	175	....	....	....	....
72	1	*	158	210	....	....	....	....

* Specially flexible rope of 6 strands, each strand having 19 wires (9 on 9 on 1) the intermediate wires of which are of smaller cross-section.

Table 92 gives the horse power transmitted by cast-steel ropes making a single lap on wood-filled sheaves, and is calculated on the assumption that the sheaves are of equal diameters and of a size not less than the minimum diameters given in Table 90. It is impracticable to transmit more than 250 h.p. with filled sheaves. since the rope tension is so great that the filling material is rapidly cut out, and the frictional grip of an iron surface by the rope is insufficient with but a single lap at either end of the line. In this case the rope must be lapped several times about a pair of grooved drums at each end, or about sheaves so arranged in pairs that the rope makes a half lap about each and a final half lap at one end about the sheave of a tension carriage for taking up the slack.

Sag of Wire Ropes. The sag or deflection S of a wire rope at the center of the span may be calculated by the formula $S = WL^2/8T$, where W is the weight of 1 ft. of rope, lb.; L = span, ft. and T = tension in the rope, lb. Substituting for T the values of T_1 and T_2 in the tight and slack sides and expressing W in terms of the rope diameter, the following values are obtained:

		Steel rope	Iron rope
Deflection of still rope at center, ft.	=	$0.00004L^2$	$0.00008L^2$
Deflection of driving rope at center, ft.	=	$0.000025L^2$	$0.000025L^2$
Deflection of slack rope at center, ft.	=	$0.0000875L^2$	$0.000175L^2$

For sags, length of wire, etc., see pp. 147–151.

Span Lengths. For spans under 60 ft. in length, a heavier rope should be used than that actually required to transmit the power. This is made necessary by the difficulty of splicing short ropes so as to give the exact deflection required. The maximum length of span depends on the contour of the ground and the available height of the terminal sheaves. The lower side of the rope should always be the tight or driving side. The **maximum limit of span** is often determined by the sag of the upper side when running; for 10-ft. sheaves and towers of ordinary height it is about 600 ft., though transmissions have been built with spans of as much as 1700 ft. in length. In **long-distance transmissions,** when the distance exceeds the maximum for a single span, the rope is supported and its direction changed (if necessary) by intermediate sheaves having plain unfilled grooves. In very long transmissions slower speeds are necessary and the rope must be lapped about a grooved drum several times.

Splicing Wire Ropes. The general method for splicing wire ropes resembles closely that described for textile ropes. The length of splice used ranges from 16 ft. for ½ in. ropes up to 30 ft. for large sizes. Twelve strands have to be cared for instead of eight in the case of the textile rope. The ends for tucking in are left about 12 in. long. Tucking in is accomplished by untwisting the rope by means of long-armed wooden clamps, cutting out the hemp core for a sufficient distance and then forcing the ends into the space formerly occupied by the core. The rope is then smoothed and rounded with a wooden mallet.

CHAIN DRIVES

Chains may be used to advantage in the transmission of power when a positive, high-efficiency drive is desired and when the distances between the shaft centers are too short to use belts and too long for toothed gearing. They occupy but little space and will work satisfactorily in hot or damp places.

FIG. 179.—Roller Chain.

Roller Chains are made as shown in Fig. 179. The Diamond Chain & Mfg. Co. manufactures this type of chain in the following sizes and strengths:

Pitch, in.	2	1¾	1½	1¼	1	¾	⅝	½
Ultimate tensile strength, lb.	40,000 to 34,000	30,000 to 24,000	21,000 to 18,000	18,000 to 13,000	10,000 to 8000	5000	3500 to 1200	1200

For light and steady loads a **speed** of 2000 ft. per min. has been attained with this type of chain. As the loads become heavier and more irregular, however, the speed should be lower. About 1000 ft. per min. is a usual speed for a wide range of applications. The safe **working load** varies from ⅙ to ¹⁄₁₀ of the ultimate strength of the chain, according to the severity of the service and conditions of drive.

Block Chains (Fig. 180) are made by the Diamond Chain and Mfg. Co. with widths of ⅛, ³⁄₁₆, ¼, ⁵⁄₁₆, ½ and 1 in., and with pitches of ½, ⅝, 1, and 1½ in. The strength of the 1-in. pitch chain varies from 1200 to 2500 lb.; that of the 1½-in. pitch chain is 5000 lb. These chains are usually employed for speeds under 700 ft. per min.

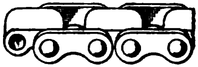


Fig. 180.—Block Chain.

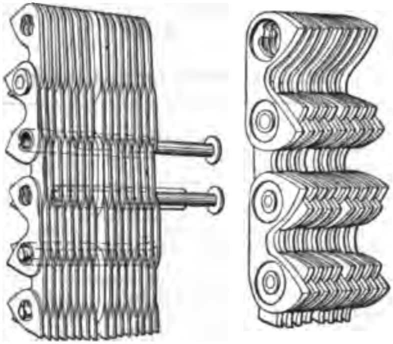


Fig. 181.—Renold Silent Chain.

Renold Silent Chain. The standard sizes, working loads and wheel widths of this chain (shown in Fig. 181) are given in Table 93. Speeds from 800 to 1200 ft. per min. are usual. One of the chief characteristics of this chain is that wear is automatically compensated for, and hence it operates with little noise and wear as compared with the roller and block types.

Table 93. Standard Sizes and Working Loads of Renold Silent Chain

Pitch, in.	Outside width, in.	Working load without shock, lb.	Wheel width, flanged or not, in.	Pitch, in.	Outside width, in.	Working load without shock, lb.	Wheel width, flanged or not, in.	Pitch, in.	Outside width, in.	Working load without shock, lb.	Wheel width, flanged or not, in.
½	¼	12	¾	¾	1¼	117	1¹⁵⁄₁₆	1¼	3	633	3¾
	½	24	1		1½	141	2³⁄₁₆		4	844	4¾
	¾	36	1¼		1¾	164	2½		5	1055	5¾
	1	48	1½		2	188	2¾		6	1266	6¾
	1¼	60	1¾		3	282	3¾	1½	2	·607	2¾
	1½	72	2		4	376	4¾		3	911	3¾
	2	96	2½	1	1	158	1⁹⁄₁₆		4	1214	4⅞
	3	144	3½		1½	237	2¹⁄₁₆		5	1518	5⅞
⅝	¾	57	1¼		2	316	2¹¹⁄₁₆		8	2428	9
	1	76	1½		2½	395	3³⁄₁₆	1¾	6	2250	7½
	1¼	95	1¾		3	474	3¹¹⁄₁₆		8	3000	9½
	1½	114	2³⁄₁₆		4	632	4¹¹⁄₁₆		10	3750	11½
	1¾	133	2⁷⁄₁₆		5	790	5¾		12	4500	13½
	2	152	2¹¹⁄₁₆	1¼	1½	316	2⅛	2	6	2500	7½
	3	228	3¹¹⁄₁₆		2	422	2⅝		8	3300	9½
¾	¾	70	1⁷⁄₁₆		2½	527	3⅛		10	4166	11½
	1	94	1¹¹⁄₁₆								

Morse Silent Chain. The principal feature of construction of this chain (Fig. 182) is the rocker joint, which requires but little oil for lubrication. These chains may therefore be run at speeds above which the oil is thrown off. Difficulty will be experienced if the wedges get out of alignment. When this occurs the destruction of the chain is very rapid. Table 94 gives the principal data on this type of chain.

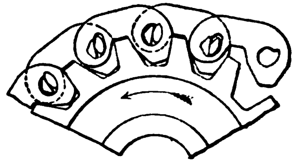


FIG. 182.—Morse Silent Chain.

Table 94. Data for the Design of Morse Silent Chain Drives

Pitch, in.	½	⅝	¾	9/10	1 2/10	1½	2	3
Minimum number of teeth:								
Small sprocket driver	13	13	13	15	15	17	17	17
Small sprocket driven	17	17	21	25	29	29	31	35
Desirable number of teeth in small sprockets	15–17	17–21	17–21	17–23	17–23	17–27	17–31	19–31
Maximum number of teeth in large sprockets. (See Note 2.)	99	109	115	125	129	129	129	131
Desirable number of teeth in large sprockets	55–75	55–75	55–85	55–95	55–105	55–115	55–115	55–115
To find pitch diam. of wheel multiply number of teeth by	0.159	0.199	0.239	0.2865	0.382	0.477	0.636	0.955
Addendum for outside diam. of sprockets 20 to 130 T. (See Note 1.) in.	0.05	0.06	0.075	0.09	0.12	0.15	0.20	0.30
Maximum r.p.m.	2400	1800	1200	1100	800	600	400	250
Tension per in. width chain, lb.:								
Small sprocket driver	80	100	120	150	200	270	450	750
Small sprocket driven	65	80	95	120	160	210	350	600
Radial clearance beyond tooth required for chain, in.	0.50	0.62	0.75	0.90	1.2	1.5	2.0	3.0
Approximate weight of chain per in. wide, 1 ft. long, lb.	1.00	1.20	1.50	1.80	2.50	3.00	4.00	6.00
C for solid pinions	0.0045	0.0063	0.009	0.013	0.023	0.035	0.058	0.145
C_1 for armed sprockets	0.16	0.25	0.35	0.45	0.7	1.0	2.0	4.0

Approximate Weights for Solid and Armed Sprockets. T = number of teeth. F = face in in. C and C_1 = constants in lb. per in. in face per tooth as per table. Weight of armed sprocket = $T \times F \times C_1$. Add 25 per cent. for split and 50 per cent. for spring and split sprockets. Weight of solid pinion = $T^2 \times (F + 1) \times C$.

Note 1.—Number of teeth = T. Exact outside diam. = D. For T less than 20 teeth, D = pitch diam. For T more than 20 teeth, D = pitch diam. + (2 × addendum).

Note 2.—Under special conditions a larger number of teeth than shown may be cut in large sprocket.

Thickness of sprocket rim, including teeth should be at least 1.2 times the chain pitch. The number of grooves in the sprocket, their width and distance apart, vary according to pitch and width of chain. The width of the sprocket should be ⅛ to ¼ in. greater on small drives, and ¼ to ½ in. greater on large drives than nominal width of the chain. An even number of links in the chain and an odd number of teeth in the wheels are desirable. Horizontal drives are preferred; tight chains on top are necessary for short drives without center adjustment, and desirable for long drives with or without center adjustment. Adjustable wheel centers are desirable for horizontal drives and necessary for vertical drives. Avoid vertical drives. Allow a side clearance for chain (parallel to axis of sprockets and measured from nominal width of chain) equal to the pitch. Maximum linear velocity for commercial service, 1200 to 1600 ft. per min. These data are for use in preliminary design. Engineering features should always be submitted to the company for approval before ordering.

Length of Chain. Referring to Fig. 183,

P = pitch of chain, in.

R and r = pitch radii of larger and smaller sprockets, respectively, in.

N and n = number of teeth on larger and smaller sprockets, respectively.

$(180° + 2a)$ and $(180° - 2a)$ = angles of contact on larger and smaller sprockets respectively.

$a = \sin^{-1}[(R - r)/D]$. $A = D \cos a$.

Then, **length of chain** in in. is equal to

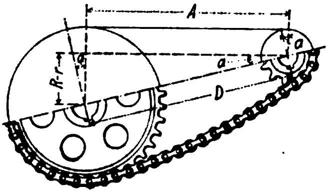


FIG. 183.

$$L = NP\left(\frac{180° + 2a}{360°}\right) + nP\left(\frac{180° - 2a}{360°}\right) + 2\,D\cos a$$

For a block chain the total length ordered should be a multiple of the pitch, while for a roller chain it should be a multiple of twice the pitch, since an outside and inside link are required for joining the ends.

Proportions of Sprockets. Sprocket wheels for **block chains** are designed as follows: In Fig. 184 let N = number of teeth; h = diam. of round part of chain block, in.; B = distance from center to center of holes in chain block, in.; A = distance from center to center of holes in side links, in. Then $a = 180°/N$, $\tan b = \sin a/[(B/A) + \cos a]$. Pitch diam. = $A/\sin b$; bottom diam. = $(A/\sin b) - h$; outside diam. = $(A/\sin b) + h$. Sprocket wheels (see p. 742) for **roller chains** are proportioned as follows: In Fig. 185, let N = number of teeth, P = pitch of chain, in., and D = diam. of roller, in. Then $a = \frac{1}{2} \times (360/N)$, pitch diam. = $P/\sin a$, bottom diam. = $(P/\sin a) - D$. For sprockets of 17 teeth and over, outside diam. = $(P/\sin a) + D$. For sprockets under 17 teeth, outside diam. = $(P/\sin a) + D - E$, in which E has the following values:

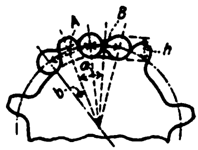


FIG. 184.

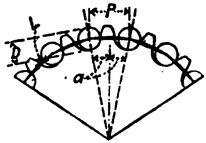


FIG. 185.

Pitch	E: 8 to 12 teeth	E: 13 to 16 teeth
½ in. to ¾ in.	0.062 in.	0.031 in.
1 in. to 2 in.	0.125 in.	0.062 in.

The **teeth for** both **roller and block chain sprockets** should have the following proportions: Referring to Fig. 186, let W = width of chain rollers or blocks, in. Then $A = W/2$; $B = W - \frac{1}{64}$ in. when $W = \frac{1}{4}$ in. or less, $= W - \frac{1}{32}$ in. when $W = \frac{5}{16}$ in. to ⅝ in., $= W - \frac{1}{16}$ in. when W = ¾ in. and over.

When the sprocket is flanged the following values of C obtain:

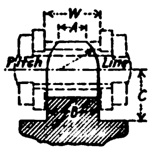


FIG. 186.

Roller, in.	A	B	1	1¼	1½	1¾	2
C, in., not less than	¼	11/32	½	9/16	⅝	13/16	15/16

A = 1-in. block and 1-in. twin roller, or ½-in. block and ⅝-in. roller.
B = 1½-in. block and ¾-in. roller.

The **pitch line clearance** for roller sprockets (see L, in Fig. 185) should be as follows:

Pitch, in.	1/2	5/8	3/4	1	1 1/4	1 1/2	1 3/4	2
Clearance, in.	1/32	1/32	1/16	3/32	3/16	7/32	1/8	5/32

Notes on Chain Drives. Short pitch is preferable to longer pitch for durability. The center-to-center distance between sprockets should not be less than 40 times the pitch of the chain. It is better to have the slack side of the chain at the bottom. Large sprockets and a large number of teeth give less wear. The ratio of sprockets should not be greater than 5 to 1 for roller chains, or 8 to 1 for block chains. Sprockets should not have less than 16 teeth. An odd or "hunting" tooth is advantageous on the pinion. Vertical drives, if impossible to avoid, should be short. A long vertical chain will tend to drop away from the lower wheel, resulting in bad chain action. Means should be provided for adjusting the distance between the shafts. Where the load is irregular or impulsive the use of a spring sprocket is advisable.

CRANE CHAINS, HOOKS, ETC.

Crane Chains. The researches of Goodenough and Moore (*Bulletin*, No. 18, Univ. of Ill. Exp. Station) on the strength of chain links results in the following formulæ:

$$P = 0.4d^2S \text{ for open links;} \qquad P = 0.5d^2S \text{ for stud links}$$

Table 95. Weights, Dimensions and Safe Loads* of Chains as Given by Standard Manufacturers

Size	Common coil				Crane				Stud link			
Thickness of link bar, in.	Length of link, *C*, in.	Width of link, *B*, in.	Approximate weight per ft., lb.	Safe load in 1000 lb.	Length of link, *C*, in.	Width of link, *B*, in.	Approximate weight per ft., lb.	Safe load in 1000 lb.	Length of link, *C*, in.	Width of link, *B*, in.	Approximate weight per ft., lb.	Safe load in 1000 lb.
3/16	1 3/8	7/8	0.46	0.5	......	......	......	......	......	......	......	......
1/4	1 1/2	1 1/16	0.75	0.8	......	......	......	......	......	......	......	......
5/16	1 3/4	1 1/4	1.10	1.3	......	......	......	......	......	......	......	......
3/8	2 1/8	1 1/2	1.55	1.8	......	......	......	......	......	......	......	......
7/16	2 1/4	1 11/16	2.00	2.3	......	......	......	......	......	......	......	......
1/2	2 1/2	1 7/8	2.60	3.3	......	......	......	......	3	1 3/4	2.3	4.8
9/16	2 7/8	2 1/8	3.25	4.0	......	......	......	......	3 3/8	2	3.0	5.9
5/8	3 3/8	2 1/4	4.00	4.8	3 1/8	2 1/8	4.0	6.9	3 3/4	2 1/4	4.0	6.3
11/16	......	......	......	......	......	......	......	......	4	2 1/2	4.8	8.5
3/4	3 7/8	2 11/16	5.90	6.8	3 5/8	2 1/2	6.3	9.6	4 3/8	2 3/4	5.7	10.1
13/16	......	......	......	......	......	......	......	......	4 3/4	3	6.7	11.9
7/8	4 3/8	3 1/8	8.0	9.3	4 1/8	2 7/8	8.0	13.5	5	3 1/4	7.3	14.0
15/16	......	......	......	......	......	......	......	......	5 3/8	3 1/2	8.5	15.8
1	5	3 5/8	10.0	12.0	4 3/4	3 1/4	10.0	17.0	5 7/8	3 3/4	9.8	18.0
1 1/8	5 1/2	4	13.0	14.5	5 1/4	3 3/4	13.0	21.5	6 1/2	4 1/8	12.5	22.8
1 1/4	6 1/8	4 3/8	15.0	19.5	5 7/8	4 1/8	16.0	27.0	7 1/8	4 1/2	15.2	28.1
1 3/8	......	......	......	......	6 9/16	4 9/16	19.0	31.0	7 3/4	4 7/8	18.8	34.0
1 1/2	......	......	......	......	7 1/8	5	23.0	36.0	8 1/2	5 3/8	22.0	40.5
1 5/8	......	......	......	......	7 7/8	5 1/2	28.0	41.5	9 1/4	5 7/8	26.0	47.5
1 3/4	......	......	......	......	8 5/8	5 7/8	31.0	44.8	10	6 1/4	29.2	55.1
1 7/8	......	......	......	......	9 3/8	6 3/8	35.0	51.3	10 1/2	6 3/4	34.2	63.3
2	......	......	......	......	10 1/8	6 3/4	40.0	58.3	11 1/8	7 1/4	40.0	72.0
2 1/8	......	......	......	......	10 7/8	7 1/8	47.0	65.8	12	7 3/4	44.2	81.3
2 1/4	......	......	......	......	11 5/8	7 5/8	53.0	73.7	13	8 1/4	50.0	91.1
2 3/8	......	......	......	......	12	8	58.5	82.0	13 1/2	8 3/4	54.2	101.5
2 1/2	......	......	......	......	12 5/8	8 3/8	65.0	90.9	14	9	60.0	112.5

* Safe loads based on one-half proof tests, or one-fourth of the approximate breaking load of chain.

where P = safe load on chain, lb.; d = diameter of bar from which chain is made, in.; and S = maximum permissible intensity of tensile stress, lb. per sq. in. Chains made of wrought iron may be designed for values of S from 15,000 to 20,000 according to the nature of the service. Standard **proportions for chains** are given in Table 95, the dimension letters of which refer to Fig. 187.

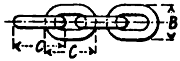


FIG. 187.

The load at which permanent deformation begins in wrought-iron **rings** used **on chain slings** is P(lb.) = $42{,}000d^3/D$, in which d = diam. of stock from which ring is forged, in., and D = mean diam. of ring, in. = inside diam. + d (W. Worsdell, *Engg.*, May 6, 1910). For safe load take $P = 14{,}000d^3/D$.

For proportions of **chain sheaves,** see p. 742. For **hoisting chains,** see p. 1109.

Crane Hooks may be designed to the proportions given in Fig. 188 (Axel K. Pedersen, *Am. Mach.*, Nov. 24, 1910). Letting P_1 = load on hook in tons (2000 lb.), $D_1 = \sqrt{P_1}$; $D_2 = \frac{7}{8}D_1$; $W = 1.5A$; $h = \frac{3}{4}H$; $L_1 = 2.3A$ to $2.6A$; $L_2 = 4.3A$ to $4.5A$. Values of A used by the Pawling & Harnischfeger Co. are as follows:

Capacity, lb.	6000	10,000	15,000	20,000	30,000	40,000
A, in.	1½	1⅞	2⅛	2½	3	3½
Capacity, lb.	50,000	60,000	80,000	100,000	120,000	
A, in.	3¾	4	4¾	5	5½	

At the section M–M, $b = 2A \tan 0.5\,T$, $B = b(x + 1)$, and $H = Ax$, where T = angle included between the side faces, deg., and x is a factor obtained from the curve in Fig. 188 and depending on the value of F. This latter value is determined from the equation

$$F = SA^2 \tan(0.5T)/P$$

where P = load on hook, lb., and S = maximum allowable intensity of tensile stress in the metal, lb. per sq. in. S may be taken at 17,000 lb. per sq. in. for a high grade of iron. Values of tan $(T/2)$ may range from 0.22 to 0.32. For the theory upon which the curve in Fig. 188 is based, together with experimental verifications, see p. 440 and the papers of Andrews and Pearson (Drapers' Company Research Memoirs, Technical Series, I), Goodman (*Proc. Inst. C. E.*, vol. clxvii), and Rautenstrauch (*Am. Mach.*, Oct. 7, 1907, and *Trans. A. S. M. E.*, vol. 31).

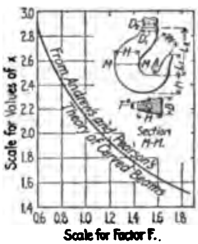


FIG. 188.

Open-side Machine Frames for punches, shears, presses, riveters, etc., are generally made of box or I-section, and should be designed as curved beams, using the analysis of Andrews and Pearson (Rautenstrauch, *Trans. A. S. M. E.*, vol. 31) instead of the ordinary formulæ for bending. The Andrews-Pearson formulæ, however, are difficult of application, and Mr. Andrews has accordingly amended the simple formulæ of the ordinary bending theory, as follows (*Am. Mach.*, Sept. 5, 1912): Corrected combined tensile stress at inner edge of section (lb. per sq. in.) $= t = P[1 + (Kld_1/k^2)]/A$; corrected combined compressive stress (lb. per sq. in.) at outer edge of section $= c = P[(Cld_2/k^2) - 1]/A$, where P = load or pressure (lb.) applied at the distance l (inches) from the neutral axis of the section; d_1 (d_2) = distance from neutral axis to inside (outside) edge, in.; k = radius of gyration of section about the neutral axis, in inch units; A = area of cross-section of frame, sq. in. K and C are correcting factors given by Mr. Andrews in the form of curves

(*The Engineer*, Apr. 19, 1912), from which the following formulæ have been derived (C. M. Sames, *Eng. Digest*, June, 1912, p. 477): $K = (7.84s + 0.42)/(8s - 3)$; $C = (s - 0.16)/(s + 0.2)$, in which $s = R/H$, where R = radius of curvature of the neutral axis and $h = d_1 + d_2$ = depth of section, both in inches. The working stresses employed should be based on the results of transverse breaking tests of cast iron rather than upon tensile tests.

ENGINE DETAILS

Cylinders

(See p. 1039 for the cylinders of gas engines; p. 1193 for automobile engines; and p. 1241 for marine engines)

Steam-engine Cylinders may have walls of the thickness

$$t(\text{in.}) = (PD/2000) + 0.3 \text{ in.}$$

where P = steam pressure, lb. per sq. in., and D = diam. of bore, in. The thickness thus obtained allows for reboring. For jacketed cylinders the **liner** when of cast iron may be of the thickness: $t_0 = [PD/(5125 + 10P)] + 0.4$ in.

The **thickness of** the **jacket** may be equal to that of the liner minus the 0.4 in. allowed for reboring. The space between the liner and jacket must not be less than ¾ in. and in large cylinders should be as much as 1¼ in. **Wrought-steel liners** need have but from 30 to 35 per cent. of the thickness of cast-iron liners. For compound engines the high-pressure cylinder is to be designed to withstand full boiler pressure, and the other cylinders as follows: Intermediate (triple) $P = 0.5 \times$ boiler pressure; low (triple), $0.3 \times$ boiler pressure; low (double), $0.4 \times$ boiler pressure.

Cylinder Flanges should not be thicker than $1.2t$ to $1.25t$. Heavier flanges cause severe cooling strains which materially weaken the casting.

Cylinder Heads (see also p. 1242) when flat may be proportioned as flat plates supported at the edges. $t_1 = 0.5D\sqrt{5P/6f}$, where t_1 = thickness of plate, in., and f = intensity of stress, lb. per sq. in. = 3000 for cast iron and 8000 for cast steel. When **heads** are **dished and ribbed** the same formula may be used for determining the maximum limit of dimension and the actual thickness made from 20 to 40 per cent. less, depending on the ribbing.

See also **Diaphragms, Casings** and **Heads**, p. 788.

Valve-chest Walls (see also Strength of Flat Plates, p. 420) when flat may have the following thickness (in in.): $t_2 = b\sqrt{a^2P/(a^2 + b^2)2f}$, where b = unsupported width of wall, in.; a = unsupported length of wall, in.; f = 6500 for cast iron.

Steam-chest Cover Plates for flat valves may have a thickness in inches given by $t_3 = b\sqrt{3a^2P/4(a^2 + b^2)f}$.

Large cover plates are usually dished and ribbed for added strength. A steam-chest cover such as shown in Fig. 189, when tested to destruction, cracks along a diagonal. The maximum intensity of stress will be $f = M/Z = Pb^2a^2/12Z\sqrt{b^2 + a^2}$, where M is the bending moment at the diagonal and Z is the modulus of the cross-section along a diagonal, or the moment of inertia of the section divided by the distance from the neutral axis to the outer fiber.

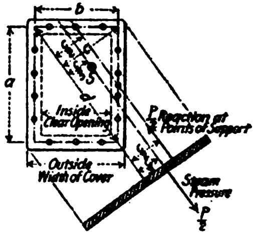


FIG. 189.

Bolts for holding down cylinder heads and steam-chest covers should never be less than ⅝ in. and preferably ¾ in. in diameter to prevent their being twisted off in tightening. The intensity of stress based on steam-pressure load should be limited to 6000 lb. per sq. in. and the spacing should not be greater than 6½ in. or less than 3½ times the diameter of the bolt. Stud holes should be tapped to a depth equal to 1½ times the diameter of stud, but never allowed to enter a steam space. The **strength** of the bolts should be much less than the strength of the frame or cylinder with load application along the center line of the engine. This will insure the safety of the more expensive castings in case of unusual loading such as may be caused by water in the cylinder. For **gas-engine cylinders**, see p. 1041.

Hydraulic Press Cylinders may be proportioned for strength by Lamé's formula (see p. 395). For cast-iron cylinders f is 6000; for cast steel, 15,000. For very high pressures cast iron is too porous and permits leakage through the cylinder walls. For pressures above 3000 lb. per sq. in. it is advisable to use steel castings, although a close-grained iron may serve up to 3500 lb. per sq. in. Pipe connections to press cylinders should be bossed. Press-cylinder bottoms should be hemispherical and not flat, in order to diminish shrinkage stresses.

Pistons and Piston Rods

Small Steam-engine Pistons may be made to the form and dimensions given in Fig. 190 and Table 96. A new piston, however, must be provided with each reboring of the cylinder. **Built-up pistons** may be made to the form and proportions given in Fig. 191 and Table 97. With the use of this type of piston, a reboring of the cylinder calls for a new bull ring and new piston rings, but permits the use of the same spider and follower plate.

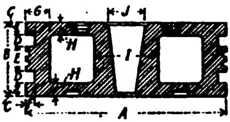


FIG. 190.—Small Steam-engine Piston.

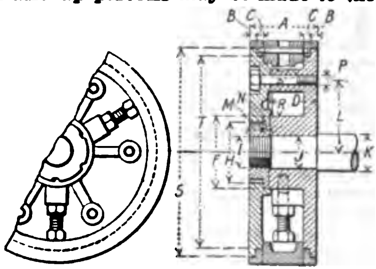


FIG. 191.—Built-up Engine Piston.

Conical Pistons for use in vertical engines may be proportioned for strength according to the formulæ given by Kraft (*Proc. Inst. C. E.*, vol. 127, p. 259):

$$T = p(R^2 - x^2)/2fx \sin a; \qquad t = px'/f \sin a$$

where p = steam pressure, lb. per sq. in.; f = maximum allowable stress, lb. per sq. in. (= 3000 for cast iron and 9000 for steel); T and t = minimum thickness of metal at sections near boss and rim, respectively, in.; x, x', a and R have the significance given in Fig. 192.

Bauer gives proportions for **cast-steel pistons** as found in Fig. 193

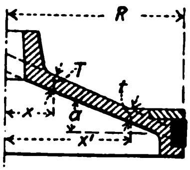


FIG. 192.

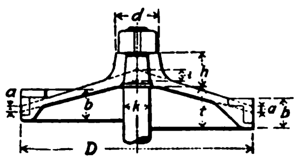


FIG. 193.—Conical Engine Piston.

and Table 98. Thickness of boss $d = 1.5k$ to $1.7k$. For small pistons and for heavily built engines use $1.7k$; for large pistons and light engines, $1.5k$. Height of the boss $= h = 1.1k$. The thickness of the steel at the middle or

Table 96. Dimensions of Steam-engine Pistons

(All dimensions in in. Letters refer to Fig. 190)

A	B	C	D	E	F	G	H	I	J	A	B	C	D	E	F	G	H	I	J
4*	2½	⅜	½	¾	⅜	....	....	....	1 3/16	11	4½	¾	⅝	1¾	½	1¼	⅞	3½	1⅝
5*	2½	⅜	½	¾	⅜	....	....	....	1 3/16	12	4½	¾	⅝	1¾	½	1¼	1	3¾	1⅞
6*	3	½	½	1	½	....	....	....	1⅛	13	5¼	¾	⅝	2½	9/16	1¼	1	3¾	2
7	3½	½	½	1½	7/16	1	⅝	2¾	1⅛	14	5¼	¾	⅝	2½	9/16	1¼	1	4	2
8	3½	½	½	1½	7/16	1	⅝	2¾	1¼	16	5½	¾	¾	2½	9/16	1¼	1	4¼	2⅜
9	4	⅝	⅝	1½	1½	1⅛	⅞	3	1⅜	18	6	⅞	¾	2¾	9/16	1½	1⅛	4½	2⅝
10	4	⅝	⅝	1½	½	1⅛	⅞	3¼	1⅜	20	6½	⅞	¾	3¼	9/16	1½	1¼	5	3

* Solid pistons.

Table 97. Dimensions of Built-up Steam-engine Pistons

(All dimensions in in. Letters refer to Fig. 191)

Cyl. diam.	A	B	C	D	F	H	I	J	K	L	M	N	P	R	S	T
10	3¼	⅜	⅜	2⅛	3½	2⅞	1 7/16	1 9/16	2 1/16	3	¾	⅜	⅝	1¼	9¼	8 7/16
12	3¾	7/16	7/16	2½	4⅛	3¼	1¾	1⅞	2⅜	3⅞	⅞	⅜	¾	1 13/16	11⅛	10⅛
14¼	4¼	½	½	2⅞	4¾	3⅝	2	2⅛	2 11/16	4¾	1	⅜	¾	2⅜	13⅜	12⅜
17½	5⅜	9/16	9/16	3¾	5⅜	4½	2 5/16	2 7/16	3⅛	5⅞	1⅛	½	¾	3 3/16	16½	15
20	6	11/16	11/16	4⅛	5¼	5	2¾	2 13/16	3⅝	6¾	1⅜	½	⅞	3⅝	18 13/16	17 3/16
24	7⅜	¾	¾	5	7¼	5¾	3 1/16	3¼	4	8¼	1½	⅞	1	4⅝	22 13/16	21⅛
28½	8¾	13/16	13/16	6½	8¾	6¾	3⅝	4	4⅝	10	1⅝	⅝	1⅛	5⅝	27 9/16	25 5/16

Number of bolts: 4 for cylinders up to 14¼-in.; 6 for 14¼-in. and larger cylinders.

Number of adjusting screws: 4 for 12- and 13-in. cylinders; 3 for larger cylinders; none for cylinders below 12 in. in diam.

Table 98. Values of k for Steel Pistons, In.

Cylinder, diam., in.	Steam pressures, lb. per sq. in., gage							
	0–15	15–45	45–75	75–105	105–135	135–165	165–195	195–225
16–24	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
24–32	1.4	1.6	1.8	2.0	2.6	2.8	3.0	3.2
32–40	1.6	1.8	2.0	2.6	3.2	3.6	3.8	4.0
40–48	1.8	2.0	2.4	3.0	3.6	4.0	4.2	....
48–56	2.0	2.2	2.8	3.2	3.8	4.2	....	....
56–64	2.4	2.6	3.2	3.6	4.0	4.4	....	....
64–72	2.6	3.0	3.4	4.0	4.4	....	....	....
72–80	2.8	3.2	3.6	4.2	....	....	....	....
80–88	3.0	3.4	3.8	4.4	....	....	....	....
88–96	3.0	3.6	4.0	....	....	....	....	....

Table 99. Dimensions of Equal-section and Eccentric Piston Rings

(All dimensions in in. Letters refer to Figs. 194 and 195)

Letters	Diameters, of cylinders, in in.														
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	6 5/32	7 5/32	8 3/16	9 3/16	10 7/32	11¼	12¼	13 9/32	14 9/32	15 5/32	16 11/32	17 11/32	18⅜	19 13/32	20 13/32
B	¼	9/32	5/16	⅜	13/32	7/16	15/32	½	9/16	19/32	⅝	11/16	¾	¾	13/16
C	½	½	½	⅝	⅝	⅝	⅝	⅝	⅝	¾	¾	¾	¾	¾	¾
D	35/64	39/64	21/32	23/32	25/32	27/32	⅞	15/16	1	1 1/16	1⅛	1 3/16	1¼	1 9/32	1 11/32
E	3/16	7/32	¼	9/32	9/32	5/16	11/32	⅜	13/32	7/16	15/32	15/32	½	17/32	9/16
F	9/32	5/16	11/32	⅜	13/32	7/16	15/32	½	9/16	⅝	11/16	11/16	¾	13/16	⅞

mean radius may be $i = k \times c$ where c may have the following values: $c = 1$ for flat pistons or those in which the inclination measured in the inside is very slight (angle 0 deg. to 6 deg.), $= 0.85$ to 0.95 when angle is 6 to 18 deg., $= 0.75$ to 0.85 when angle is 18 to 28 deg. $= 0.65$ to 0.75 when angle is 28 to 35 deg.; also $a = 0.45i$ to $0.55i$. For **marine-engine pistons,** see p. 1242; for **gas engines,** p. 1041; for **automobile engines,** p. 1242.

Piston Rings. Steam-engine piston rings up to 20 in. in diameter may be made to the forms and proportions given in Figs. 194 and 195 and Table 99. (For gas-engine piston rings, see p. 1043.) The eccentric rings give a more uniform pressure against the cylinder walls. This pressure should be from 3 to 4 lb. per sq. in. The ring is preferably in segments for large engines, each segment being pressed against the cylinder walls by a spring.

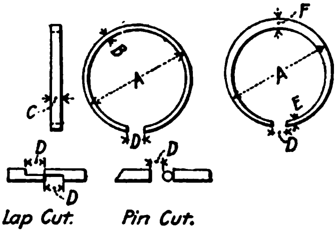


FIG. 194. FIG. 195.
Piston Rings.

Piston Rods are subjected mainly to compressive stresses and may be designed as round, free-ended columns, Euler's formula for which reduces to $P = 0.485Ed^4/sl^2$, in which P = total load, lb.; E = modulus of elasticity ($= 30{,}000{,}000$ for mild steel); d = diam. of rod, in.; l = length of rod, in.; [For hollow piston rods, substitute $(d_2^4 - d_1^4)$ for d^4 in formula, d_2 and d_1 being respectively the outer and inner diameters.] The factor of safety s has the following values: For vertical engines $s = 8$ to 11 for load fluctuating between P and 0; $s = 15$ to 22 for load fluctuating between $+P$ and $-P$. In horizontal engines the rods are also subjected to bending stress due to the combined weight of the piston and rod. In small engines with light pistons the formula for vertical engines may be used with $s = 11$ to 22. In large engines with heavy pistons and rods running through glands in both heads (as in tandem steam and gas engines), the rod must be of such stiffness that it will not deflect at mid-length between its points of support more than 0.08 in. In this case the piston diameter must be from 0.12 to 0.16 in. less than that of the bore. Deflection in in. $= l^3(W_1 + 0.625W_2)/48EI$, where l = length between supported points, in.; W_1 and W_2 = weights of piston and rod respectively, lb.; I = moment of inertia of rod section $= \pi d^4/64$. For **rods for marine engines,** see p. 1242; for **gas-engine piston rods,** see p. 1042. Most reliable values of d may be found by the use of Ritter's long-column formula. This reduces to the following form for mild-steel rods: $d^2 = P(1 + \sqrt{1 + fl^2/190P})/1.57f$, where f = allowable working stress, usually 5000 lb. per sq. in., l = rod length in inches, P = load on rod, lb.

Table 100. Dimensions for Ends of Locomotive Piston Rods
(All dimensions in in. Letters refer to Figs. 196 and 197)

A	B	C	D	E	F	G	H	R	K	T	M	N	O	P	S	L
2 3/4	3 1/4	3	2 1/2	4 1/2	3 3/4	3 1/8	2 3/8	4 3/4	2 5/8	1 15/16	5 3/4	5 1/4	3/4	1 1/2	2 3/8	9
3	3 1/2	3 1/4	2 3/4	4 1/2	3 3/4	3 1/8	2 3/8	5 1/4	2 7/8	1 15/16	5 3/4	5 1/4	3/4	1 1/2	2 3/8	9
3 1/4	3 3/4	3 1/2	3	5 1/2	4 3/4	4 1/8	2 3/8	5 3/4	3 1/8	2 7/16	6 1/2	6	3/4	1 3/4	2 5/8	9
3 1/2	4	3 3/4	3 1/4	5 1/2	4 1/2	3 7/8	2 7/8	6	3 3/8	2 7/16	6 1/2	6	3/4	1 3/4	2 5/8	9
3 3/4	4 1/4	4	3 1/2	6 1/2	5 1/2	4 7/8	2 7/8	6 1/2	3 5/8	2 15/16	7	6 1/2	3/4	1 3/4	3	10 1/2
4	4 1/2	4 1/4	3 3/4	6 1/2	5 1/2	4 7/8	2 7/8	6 7/8	3 7/8	2 15/16	7	6 1/2	3/4	1 3/4	3	10 1/2
4 1/4	4 3/4	4 1/2	4	7	6	5 3/8	3 3/8	7 1/4	4 1/8	3 7/16	7 1/2	7	3/4	2	3 1/4	11 1/2
4 1/2	5	4 3/4	4 1/4	7	6	5 3/8	3 3/8	7 3/4	4 3/8	3 7/16	7 1/2	7	3/4	2	3 1/4	11 1/2

To check for stress in an existing design the formula is more conveniently written as: $f = P[1 + (16SL^2/\pi^2Ed^2)]/A$, where A = area of rod section, sq. in.; E = modulus of elasticity in tension = 30,000,000 for mild steel; S = elastic limit of material = 30,000 lb. per sq. in. for mild steel.

Successful locomotive practice in the design of **piston-rod ends** is shown in Figs. 196 and 197, dimensions being given in Table 100.

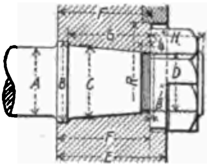


FIG. 196.

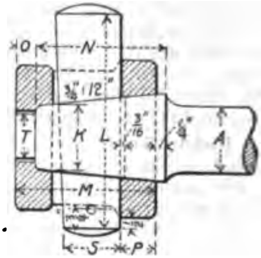


FIG. 197.

Piston-rod Ends.

Cross Heads

Engine Cross Heads of the **wing type** may have the features of construction shown in Fig. 198. The **box type** of cross head may be designed as shown in Fig. 199. The substitution of pin shoulders on the cross head and straight bores in the box for the taper construction reduces the cost of machining. The shoulders should bear on the face of the side opposite to which the nut holding the pin is seated in order that screwing up the nut will not tend to bind the connecting-rod end in the box and thus cause heating. It is well to flatten the top and bottom portions of the pin where wear is a minimum to prevent the pin wearing to an oval shape. It is sometimes the practice to give the pin a quarter turn occasionally to promote more uniform wear. In locomotive and marine service the type of cross head shown in Fig. 200 is frequently used.

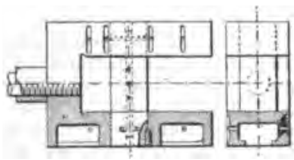


FIG. 198.—Wing Type of Cross Head.

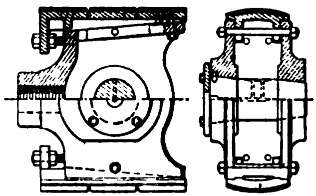


FIG. 199.—Box Type of Cross Head.

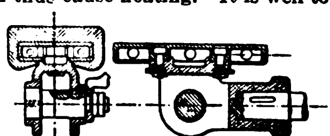


FIG. 200.—Marine-engine and Locomotive Type of Cross Head.

Cross-head Pins. In steam-engine practice the cross-head pin is designed for a bearing pressure of 1000 to 1500 lb. per sq. in. According to Barr, American practice in proportioning cross-head pins is as follows: $dl = CA$, $l = Kd$; where d = pin diam., in.; l = bearing length of pin, in.; A = piston area, sq. in., C = 0.08 (0.07) for high (low) speeds, and K = 1.25 (1.3) for high (low) speeds.

Shoes. The area of the shoes for stationary steam-engine cross heads should be such as to limit the maximum pressure to from 25 to 40 lb. per sq. in. The total maximum pressure of the cross-head shoes on the guide is $P \tan a$, where P = maximum steam-pressure load behind the piston, lb., and a = maximum angle of connecting rod with engine center line.

In **marine practice** (see also p. 1243) the area of the shoes, according to Bauer, is based in the following limiting pressures: 55 to 65 lb. per sq. in. for cargo and slow-running passenger vessels; 70 to 85 lb. per sq. in. for iron-clads and large cruisers; 65 to 80 lb. per sq. in. for mail steamers; 85 to 120 lb. per sq. in. for small cruisers and torpedo boats.

In **locomotive practice** the limiting pressure on the cross-head shoe is about 85 lb. per sq. in.

Connecting Rods

Connecting rods for **locomotive** service may be made to the form shown in Fig. 201. For **small stationary engines**, rods of the form shown in Fig. 202 are used. For **marine-engine rods**, see p. 1242; for **automobile engines**, p. 1193.

According to Haeder, **gas-engine connecting rods** may be proportioned as shown in Fig. 203 and Table 101.

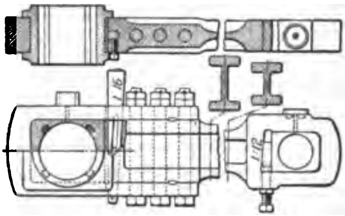


FIG. 201.—Locomotive Connecting Rod.

Shanks for connecting rods may be designed as long columns with round ends, using Ritter's formula (see p. 765), $d^2 = CP[1 + \sqrt{1 + (l^2kf/KCP)}]/kf$, where P = load on rod, lb.; f = maximum allowable stress, lb. per sq. in.; l = length of rod, in. For **rods of circular cross-section**, d = rod diam., in., C = 1.00, k = 1.57, and K = 300. For **rods of rectangular cross-section** having height of section equal to 2.7 times the thickness, d = height of rod section, in.; C = 1.35, k = 1, and K = 420. These constants are for rods of mild steel having an elastic limit of 30,000 lb. per sq. in. and a modulus of elasticity in tension of 30,000,000 lb. per sq. in. In high-speed steam-engine practice, f may be 4000. In slow-speed steam engines, gas engines and pumps, f may be 5000 to 5500. In general, the **working stress** for connecting rods should be from 1/6 to 1/8 the stress at the elastic limit of the material, when designing for long-column action. Rods for high-speed engines having important inertia stresses should be designed with a factor of 1/8. The sum of the stresses due to long-column action and to inertia (whipping action) should not exceed 1/3 the stress at the elastic limit of the material.

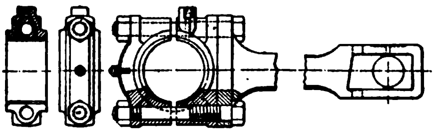


FIG. 202.—Connecting Rod for Small Engines.

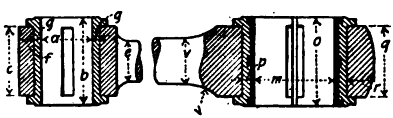


FIG. 203.—Gas-engine Connecting Rod.

The **maximum inertia stress** in a connecting-rod shank may be closely approximated by use of the following formulæ:

For round rods, $f' = l^2wv^2/(0.098d^3 \times 16gR)$, where f' = stress due to inertia, lb. per sq. in.; w = weight of 1 in. length of rod, lb; v = velocity of crank pin, ft. per sec.; g = 32.2 ft. per sec.^{2}; R = crank radius, ft.; d = mean diam. of rod, in. For rectangular rods, $f' = l^2wv^2/(0.167bh^2 \times 16gR)$, where b = thickness of rod section, in., and h = height of rod section, in.

Table 101. Dimensions of Connecting Rods
(All dimensions in in. Letters refer to Fig. 203)

Engine			Length of connecting rod	Piston end (small end)						Crank end (large end)					
H.p.	Stroke	Cyl. diam.		*a*	*b*	*c*	*e*	*f*	*g*	*m*	*o*	*p*	*q*	*r*	*v*
0.6	5	3 1/4	14 3/4	1 3/16	1 9/16	1 3/16	1	1/4	3/16	1 5/8	2	5/16	1 9/16	5/8	1 3/16
1.2	6 1/4	4	18 1/8	1 3/8	2	1 9/16	1 3/16	5/16	1/4	2	2 1/4	3/8	1 3/4	3/4	1 3/8
2.0	7 1/2	5	21 5/8	1 3/4	2 3/8	1 3/4	1 3/8	7/16	1/4	2 3/8	2 9/16	7/16	2	13/16	1 9/16
3.5	8 3/4	5 3/4	25 1/4	2	2 3/4	2 3/16	1 9/16	9/16	5/16	2 3/4	3 3/16	1/2	2 3/8	7/8	1 3/4
7.0	10 5/8	7 1/4	31 1/2	2 3/8	3 9/16	2 3/4	1 3/4	3/4	5/16	3 3/4	4 1/8	9/16	3 3/16	1	2 3/16
11.0	12 1/4	8 3/4	34 1/2	2 3/4	4 5/16	3 9/16	2 3/16	13/16	3/8	4 5/16	4 3/4	5/8	3 3/4	1 3/16	2 9/16
17.0	13 3/4	10 1/4	40 3/8	3 3/8	5 1/8	4 1/8	2 9/16	7/8	7/16	4 15/16	5	3/4	3 15/16	1 5/16	3
25.0	15 3/4	11 7/8	46 1/4	3 3/4	6	5	2 3/4	1	1/2	5 3/4	5 3/4	7/8	4 1/8	1 1/2	3 3/8
33.0	17 3/4	13	51 1/4	4 1/8	6 1/2	5 5/16	3 3/16	1 1/16	9/16	6 1/8	6 1/8	15/16	4 3/4	1 5/8	3 3/4
42.0	19 3/4	14 1/4	57 1/8	4 1/2	7 1/16	6	3 3/8	1 1/8	5/8	6 3/4	6 3/4	1	5 1/8	1 7/8	4 3/8
55.0	21 5/8	15 3/4	62	5 1/8	7 7/8	6 5/16	3 3/4	1 1/4	11/16	7 1/4	7 1/4	1 1/8	5 1/2	2 1/16	4 1/2
80.0	23 5/8	19	67	6	9 1/2	7 1/2	4 1/8	1 1/2	13/16	8 1/2	8 1/2	1 1/4	6 1/2	2 1/2	4 15/16
132.0	27 1/2	23 5/8	76 3/4	7 1/2	11 13/16	9 1/16	5 1/2	1 7/8	15/16	10 1/4	10 1/4	1 1/2	7 7/8	3 3/16	6 5/16

Connecting-rod Ends as in Fig. 202 should have their bolts designed such that each may take 5/8 the maximum load on the cap with a stress of 5000 lb. per sq. in. for wrought iron and 6000 lb. per sq. in. for steel. In steam engines the maximum load producing tension in the bolts may be occasioned by the steam pressure or the inertia of the reciprocating masses. In single-acting gas engines the load on the bolts is caused by the inertia forces. **Strap ends** as shown at the crank-pin end of the rod in Fig. 201, should have the side straps designed to carry 5/8 the load, with a limiting stress of 5000 to 6000 lb. per sq. in. The ends are made from 1 to 1.6 times the thickness of the sides for both milled and strap ends. Connecting-rod ends consisting of **strap and cotter fastenings** may be proportioned, according to Molesworth, as follows (see Fig. 204): $b = D + 1.4a$, $c = 0.3D + 0.06$ in., $e = 0.33D + 0.06$ in., $f = 0.37D + 0.12$ in., $g = 0.35D + 0.12$ in., $h = D$, $j = 0.5D + 0.4$ in., $k = 0.9D$, $l = 0.2D + 0.06$ in. Taper of cotter, 1 in 16.

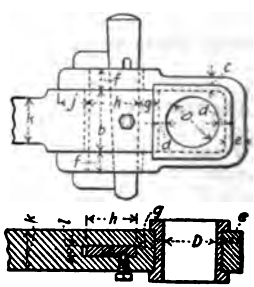


FIG. 204.—Connecting-rod End.

Stuffing Boxes

Stuffing boxes of the ordinary **bolted flange type** suitable for steam engines may be proportioned as shown in Fig. 205, where d = rod diam., in.; $d_1 = 1.22d + 0.6$ in.; $l_1 = 0.4d + 1$ in.; $l_2 = d + 1$ in.; $l_3 = 0.75l_2$; d_2 = bolt diam. = $0.12d + 1/2$ in. when 2 bolts are used, = $1.6\,(0.12d + 1/2 \text{ in.})/\sqrt{n}$ where the number of bolts n is greater than 2; $t = 0.1d + 0.6$ in.; $t_1 = 1.4t$; $t_2 = t$; $t_3 = 0.04d + 0.2$ in. (not to exceed 1/2 in.); $t_4 = 0.1d + 0.13$ in. (not to exceed 1 in.). All glands are to be of brass.

The **screw type of stuffing box** may be made as shown in Fig. 206, with

$d_1 = 1.3d + 0.6$ in. $\quad l_3 = 0.6d + 1$ in. $\quad h = 0.6d + 1$ in.
$d_2 = 0.15d + 0.5$ in. $\quad t = 0.1d + 0.3$ in. $\quad h_1 = 0.14d + 0.4$ in.
$l_1 = 0.4d + 1$ in. $\quad t_1 = 0.15d + 0.5$ in. $\quad h_2 = 0.67d + 1.2$ in.
$l_2 = d + 1.5$ in. $\quad t_2 = 0.13d + 0.4$ in. $\quad h_3 = 0.3d + 0.7$ in.

The gland is of brass.

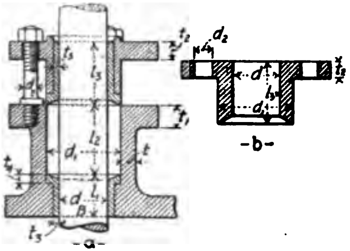


FIG. 205.—Bolted Flange Type.

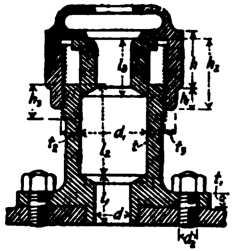
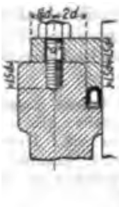
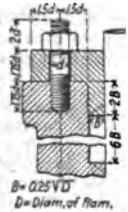


FIG. 206.—Screw Type.

Stuffing Boxes.

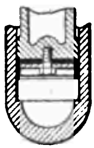
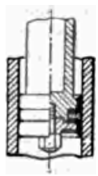
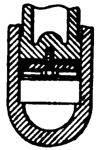
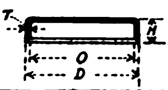





FIG. 207. FIG. 208. FIG. 209. FIG. 210. FIG. 211. FIG. 212.

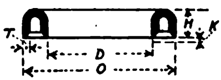
Packings for Hydraulic Cylinders.

Packings for Hydraulic Cylinders may be proportioned as shown in Figs. 207 and 208. The construction shown in Fig. 208 is adapted to pressures of 2500 to 3000 lb. per sq. in., and **hemp packing** is used.

Table 102. Proportions for Oak-tanned Leather Hydraulic Packing



(All dimensions in in.)
(K = Clearance in cavity)



End U-packing					U-wall or neck packing				Cup packing		Stock	
Diam. of ram	D	O	H	T	D	O	H	K	D	H	Diam.	Thickness
4	2½	4	¾	3/16	4	5½	1	1/16	4	1	6	3/16
6	4½	6	1	3/16	6	7½	1 1/16	1/16	6	1¼	8½	3/16
8	6½	8	1¼	3/16	8	9½	1¼	1/16	8	1½	11	3/16
10	8½	10	1¼	¼	10	11½	1⅜	⅛	10	1½	13	¼
12	10½	12	1½	¼	12	13½	1½	⅛	12	1½	15	¼
13	11½	13	1½	¼	13	14½	1½	⅛	13	1¾	16½	¼
14	12½	14	1½	¼	14	15½	1¾	⅛	14	1¾	17½	¼
18	16½	18	1⅞	¼	18	19½	1⅞	3/16	18	2	22	¼
21	19½	21	1⅞	¼	21	22½	1⅞	3/16	21	2	25	¼
23	21½	23	1⅞	¼	23	24½	1⅞	3/16	23	2	27	¼
24	22½	24	1⅞	¼	24	25½	1⅞	3/16	24	2	28	¼

Sometimes the ram of the press cylinder is **packed with leather,** as shown in Figs. 209–212. The proportions may be as given in Table 102. For high-pressure and superheated steam, stuffing boxes are usually provided with metallic packing.

For **gas-engine stuffing boxes,** see p. 1042.

Labyrinth Packing. Leakage past dummy or balance pistons of steam turbines of the Parsons type may be reduced to a small amount by a labyrinth packing (Fig. 213), where the steam pressure is throttled down by passing through a succession of very small clearances, c, formed by rings fitting alternately in grooves in the stator and rotor. These rings are usually composed of short brass strips with clearance between their ends in order to prevent them from being forced out by expansion when heated. The clearance edges of the rings are made from 0.01 to 0.015 in. thick. The number of rings n required to limit the leakage of steam to a given weight per second may be determined from the formula

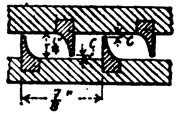


FIG. 213.

$$n = [40p - 2600(w/a)]/[540(w/a) - p]$$

where p = absolute pressure of steam, lb. per sq. in., w = permissible leakage of steam, lb. per sec., and a = cross-sectional area of the steam passage at the clearance rings, sq. in. This formula is derived from a chart given in Morrow's "Steam Turbine Design," p. 293.

Example. Let dummy piston circumference = 70 in.; pressure before passing through labyrinth = 180 lb. per sq. in. abs.; pressure after passing labyrinth = 85 lb. abs.; clearance c = 0.01 in.; leakage w not to exceed 0.7 lb. per sec. Here $a = 70 \times 0.01 = 0.7$ and $w/a = 0.7/0.7 = 1$. Substituting in formula, number of rings for 180 lb. = 13; also, number of rings for 85 lb. = 2; whence, number of rings for leakage of 0.7 lb. per sec. between the two pressures = 13 − 2 = 11.

CRANK GEARING

Analysis of the Motion of the Reciprocating Parts. Let r = crank radius, in.; R = crank radius, ft.; $s = 2r$ = stroke, in.; l = length of connecting rod, in.; x = displacement of piston from dead center, in.; a = crank angle corresponding to x; b = connecting-rod angle corresponding to x; v = crank-pin velocity, ft. per sec.; c = piston velocity, ft. per sec.; N = rev. per min. of the crank. Then, when the center lines of the piston rod and crank shaft lie in the same plane, as is customary,

$x = r(1 - \cos a) \pm l(1 - \cos b) = r(1 - \cos a \pm r \sin^2 a/2l)$ approximately, where the + sign is used for displacements from head-end dead center and the − sign for displacements from crank-end dead center. Table 103 gives **piston positions corresponding to different crank angles.** See also Table 18, p. 968, for crank angles corresponding to various piston positions.

When, however, the center line of motion of the reciprocating masses is located a distance h from the center of rotation of the crank, as shown in Fig. 214, the dead centers are unsymmetrical and the stroke is less than $2r$, having the value

$$s = \sqrt{(l + r)^2 - h^2} - \sqrt{(l - r)^2 - h^2}$$

Determination of the Velocity of the Reciprocating Parts. The velocity of the reciprocating parts is

$$c = v(\sin a + r \sin 2a/2l) \text{ approx.}$$

when the center lines of the piston rod and crank shaft lie in the same plane. When, however, the crank rotates about a center off the center line, as in Fig.

Table 103. Piston Positions for Various Crank Angles
(From beginning of stroke *toward* crank shaft. To find distance of piston from beginning of stroke, multiply tabular value by length of stroke)

Crank angles	Length of connecting rod ÷ length of crank							
	4	4.5	5	5.5	6	7	8	∞
5	0.0024	0.0023	0.0023	0.0022	0.0022	0.0022	0.0021	0.0019
10	0.0095	0.0092	0.0091	0.0089	0.0089	0.0087	0.0085	0.0076
15	0.0212	0.0207	0.0204	0.0201	0.0198	0.0194	0.0191	0.0170
20	0.0375	0.0367	0.0360	0.0355	0.0350	0.0344	0.0339	0.0302
25	0.0580	0.0568	0.0557	0.0549	0.0542	0.0532	0.0524	0.0468
30	0.0827	0.0809	0.0796	0.0784	0.0774	0.0759	0.0748	0.0670
35	0.1110	0.1087	0.1069	0.1054	0.1041	0.1022	0.1007	0.0904
40	0.1430	0.1400	0.1317	0.1358	0.1342	0.1318	0.1297	0.1170
45	0.1777	0.1742	0.1714	0.1692	0.1673	0.1643	0.1621	0.1468
50	0.2156	0.2115	0.2081	0.2054	0.2032	0.1996	0.1970	0.1786
55	0.2551	0.2505	0.2467	0.2436	0.2412	0.2371	0.2342	0.2132
60	0.2974	0.2920	0.2878	0.2843	0.2814	0.2769	0.2735	0.2500
65	0.3400	0.3343	0.3297	0.3260	0.3229	0.3180	0.3143	0.2886
70	0.3850	0.3786	0.3735	0.3694	0.3660	0.3607	0.3567	0.3290
75	0.4289	0.4224	0.4173	0.4130	0.4095	0.4039	0.3997	0.3705
80	0.4747	0.4677	0.4622	0.4576	0.4539	0.4480	0.4436	0.4132
85	0.5184	0.5115	0.5060	0.5015	0.4977	0.4918	0.4874	0.4564
90	0.5635	0.5563	0.5505	0.5453	0.5420	0.5359	0.5314	0.5000
95	0.6056	0.5987	0.5932	0.5886	0.5849	0.5790	0.5746	0.5436
100	0.6484	0.6414	0.6358	0.6313	0.6275	0.6216	0.6172	0.5868
105	0.6877	0.6812	0.6761	0.6718	0.6683	0.6627	0.6586	0.6294
110	0.7270	0.7206	0.7151	0.7110	0.7080	0.7027	0.6987	0.6710
115	0.7626	0.7569	0.7524	0.7486	0.7455	0.7406	0.7369	0.7113
120	0.7974	0.7921	0.7878	0.7843	0.7814	0.7769	0.7735	0.7500
125	0.8287	0.8240	0.8203	0.8173	0.8147	0.8107	0.8077	0.7868
130	0.8584	0.8542	0.8509	0.8481	0.8458	0.8424	0.8398	0.8214
135	0.8848	0.8813	0.8785	0.8763	0.8744	0.8714	0.8692	0.8535
140	0.9090	0.9061	0.9038	0.9019	0.9003	0.8978	0.8959	0.8830
145	0.9302	0.9279	0.9261	0.9245	0.9233	0.9213	0.9199	0.9096
150	0.9487	0.9469	0.9455	0.9444	0.9435	0.9419	0.9408	0.9330
155	0.9643	0.9631	0.9621	0.9612	0.9606	0.9595	0.9587	0.9531
160	0.9772	0.9763	0.9757	0.9751	0.9747	0.9740	0.9735	0.9698
165	0.9871	0.9867	0.9863	0.9850	0.9857	0.9853	0.9850	0.9829
170	0.9943	0.9941	0.9939	0.9938	0.9937	0.9935	0.9933	0.9924
175	0.9986	0.9985	0.9985	0.9984	0.9984	0.9984	0.9983	0.9981
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

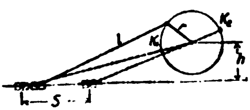


Fig. 214.

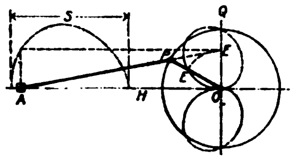


Fig. 215.

214, the above formula does not hold, and a graphical solution such as that shown in Fig. 215 may be resorted to. AH is the center line of the piston rod produced, AP is the connecting rod, OP the crank with center at O. Draw OQ

at right angles to AH. For any position AP of the connecting rod cutting OQ in E, the length OE represents the velocity of the reciprocating parts to the same scale to which OP represents the crank-pin velocity. The graphical construction of Fig. 215 shows how to construct velocity ellipses for the whole revolution.

Determination of the Acceleration of the Reciprocating Parts. The acceleration, p, of the reciprocating parts is given by the equation

$$p = v^2(\cos a \pm r \cos 2a/l)/R, \text{ approx.}$$

Table 104. Inertia Factors

(Values of $\cos a \pm (r/l) \cos 2a$. Algebraic signs relate to *forward* stroke; use opposite signs for return stroke)

Crank angles, forward (deg.)	Connecting rod length ÷ crank length							Crank angles, return (deg.)
	4	4.5	5	5.5	6	7	8	
0	1.250	1.222	1.200	1.182	1.167	1.143	1.125	360
5	1.242	1.216	1.193	1.175	1.160	1.137	1.119	355
10	1.219	1.194	1.173	1.156	1.141	1.119	1.102	350
15	1.183	1.158	1.139	1.124	1.110	1.089	1.074	345
20	1.131	1.110	1.093	1.079	1.067	1.049	1.035	340
25	1.067	1.050	1.035	1.023	1.013	0.998	0.987	335
30	0.991	0.977	0.966	0.957	0.949	0.937	0.928	330
35	0.905	0.895	0.888	0.881	0.876	0.868	0.862	325
40	0.809	0.804	0.801	0.797	0.795	0.791	0.788	320
45	0.707	0.707	0.707	0.707	0.707	0.707	0.707	315
50	0.599	0.604	0.608	0.611	0.614	0.618	0.621	310
55	0.488	0.498	0.505	0.511	0.517	0.525	0.531	305
60	0.375	0.389	0.400	0.409	0.417	0.428	0.437	300
65	0.262	0.279	0.294	0.306	0.315	0.331	0.342	295
70	0.150	0.172	0.189	0.203	0.214	0.233	0.246	290
75	0.032	0.067	0.086	0.102	0.115	0.135	0.151	285
80	−0.061	−0.035	−0.014	0.003	0.017	0.039	0.056	280
85	−0.159	−0.133	−0.110	−0.092	−0.077	−0.053	−0.036	275
90	−0.250	−0.222	−0.200	−0.182	−0.167	−0.143	−0.125	270
95	−0.333	−0.307	−0.284	−0.266	−0.251	−0.228	−0.210	265
100	−0.408	−0.382	−0.361	−0.344	−0.330	−0.308	−0.291	260
105	−0.476	−0.449	−0.430	−0.415	−0.401	−0.383	−0.357	255
110	−0.543	−0.512	−0.495	−0.481	−0.470	−0.451	−0.438	250
115	−0.583	−0.566	−0.551	−0.539	−0.530	−0.514	−0.503	245
120	−0.625	−0.611	−0.600	−0.591	−0.583	−0.571	−0.563	240
125	−0.659	−0.649	−0.642	−0.636	−0.631	−0.622	−0.616	235
130	−0.686	−0.681	−0.677	−0.674	−0.672	−0.668	−0.664	230
135	−0.707	−0.707	−0.707	−0.707	−0.707	−0.707	−0.707	225
140	−0.723	−0.727	−0.731	−0.735	−0.737	−0.741	−0.744	220
145	−0.734	−0.743	−0.751	−0.757	−0.762	−0.770	−0.776	215
150	−0.741	−0.755	−0.766	−0.775	−0.783	−0.795	−0.804	210
155	−0.746	−0.763	−0.778	−0.789	−0.799	−0.814	−0.826	205
160	−0.748	−0.769	−0.786	−0.801	−0.812	−0.830	−0.844	200
165	−0.750	−0.774	−0.793	−0.809	−0.822	−0.842	−0.858	195
170	−0.750	−0.776	−0.797	−0.814	−0.828	−0.851	−0.867	190
175	−0.750	−0.776	−0.799	−0.817	−0.832	−0.855	−0.873	185
180	−0.750	−0.778	−0.800	−0.818	−0.833	−0.857	−0.875	180

The **inertia force** in lb. per sq. in. of piston is $f = Mp/A = Wv^2(\cos a \pm r \cos 2a/l)/AgR = 0.00034WN^2R(\cos a \pm r \cos 2a/l)/A$, where W = total weight of reciprocating masses, lb.; A = piston area, sq. in. The value of W for any given engine is to be taken as the weight of the piston, piston rod, cross head and a portion of the connecting rod. It is usual to include from ½ to ⅔ the connecting-rod weight as part of the reciprocating masses. Values of $\cos a \pm (r/l) \cos 2a$ are given in Table 104.

Trooien shows that the **weight of reciprocating parts for steam engines** may be expressed by the formula $W = KD^2/SN^2$, where K = 1,370,000 to

Table 105. Tangential Factors

(Tangential pressure on crank = resultant horizontal pressure times tabular quantity. Forward stroke is *toward* crankshaft. Wrist-pin velocity = crank-pin velocity times tabular quantity.)

Crank angles, forward (deg.)	Connecting-rod length ÷ crank length						Crank angles, return (deg.)
	4	4.5	5	5.5	6	∞	
5	0.1089	0.1064	0.1045	0.1030	0.1016	0.0872	355
10	0.2164	0.2117	0.2079	0.2047	0.2022	0.1737	350
15	0.3215	0.3145	0.3089	0.3054	0.3005	0.2588	345
20	0.4227	0.4136	0.4065	0.4019	0.3957	0.3420	340
25	0.5189	0.5081	0.4995	0.4925	0.4866	0.4226	335
30	0.6091	0.5968	0.5870	0.5791	0.5724	0.5000	330
35	0.6923	0.6788	0.6682	0.6596	0.6523	0.5736	325
40	0.7675	0.7533	0.7421	0.7329	0.7253	0.6428	320
45	0.8341	0.8195	0.8081	0.7988	0.7910	0.7071	315
50	0.8914	0.8771	0.8657	0.8564	0.8488	0.7660	310
55	0.9392	0.9253	0.9144	0.9055	0.8982	0.8192	305
60	0.9769	0.9641	0.9540	0.9458	0.9390	0.8660	300
65	1.0046	0.9932	0.9842	0.9769	0.9709	0.9063	295
70	1.0224	1.0127	1.0052	0.9990	0.9939	0.9397	290
75	1.0304	1.0228	1.0169	1.0121	1.0082	0.9659	285
80	1.0289	1.0237	1.0199	1.0164	1.0137	0.9848	280
85	1.0186	1.0160	1.0139	1.0127	1.0109	0.9962	275
90	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	270
95	0.9738	0.9764	0.9785	0.9797	0.9816	0.9962	265
100	0.9407	0.9460	0.9500	0.9532	0.9559	0.9848	260
105	0.9016	0.9091	0.9150	0.9198	0.9237	0.9659	255
110	0.8571	0.8667	0.8743	0.8804	0.8855	0.9397	250
115	0.8080	0.8194	0.8285	0.8357	0.8418	0.9063	245
120	0.7552	0.7680	0.7781	0.7863	0.7931	0.8660	240
125	0.6992	0.7130	0.7239	0.7328	0.7401	0.8192	235
130	0.6407	0.6550	0.6664	0.6756	0.6833	0.7660	230
135	0.5801	0.5946	0.6061	0.6155	0.6232	0.7071	225
140	0.5181	0.5323	0.5435	0.5527	0.5603	0.6428	220
145	0.4549	0.4683	0.4790	0.4876	0.4949	0.5736	215
150	0.3909	0.4032	0.4130	0.4209	0.4276	0.5000	210
155	0.3264	0.3371	0.3458	0.3528	0.3586	0.4226	205
160	0.2614	0.2704	0.2776	0.2821	0.2884	0.3420	200
165	0.1962	0.2032	0.2088	0.2123	0.2171	0.2588	195
170	0.1309	0.1356	0.1394	0.1426	0.1451	0.1737	190
175	0.0655	0.0679	0.0698	0.0714	0.0727	0.0872	185
180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	180

3,400,000, with average value of 2,000,000; W = weight of parts, lb.; D = cylinder diam., in.; S = stroke, in., and N = r.p.m. For **gas engines**, see p. 1046.

Relation Between the Force in the Line of Piston Travel and the Tangential Reaction at the Crank Pin. In Fig. 216, if P is the force in the line of piston travel the force acting along the connecting rod is $C = P/\cos b$, and that on the guides $N = P \tan b$. The force tangent to the crank-pin circle at the crank pin is $T = C \sin (a + b) = P \sec b \sin (b + a) = P \sin a\,(1 + r \cos a/\sqrt{l^2 - r^2 \sin^2 a})$. Table 105 gives values of $\sec b \sin (b + a)$ for various values of l/r.

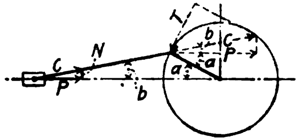


FIG. 216

DETERMINATION OF FLYWHEEL WEIGHT

Tangential Effort Curves. In Fig. 217 are shown the indicator cards $ABCD$ and $A'B'C'D'$ for the forward and return strokes of a single-cylinder double-acting steam engine. The total steam pressure on the piston at each instant is obtained by subtracting from the pressures on one face (*e.g.*, ABC) the corresponding pressures ($C'D'A'$) on the other face of the piston, giving such diagrams as abc and $a'b'c'$. The inertia effect of the reciprocating parts per sq. in. of piston area is represented by the ordinates to the curves xyz and $x'yz'$ for the forward and return strokes, respectively. The ordinates between the total steam pressure and the inertia curves (p_1, p_2, p' and p'') give the resultant pressures acting along the center line of the engine. The tangential reactions at the crank pin can be obtained as shown in the preceding paragraph. These tangential reactions are shown plotted against crank positions in Fig. 218.

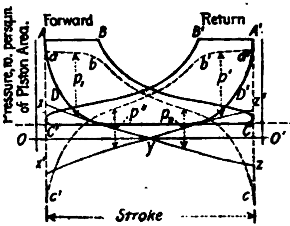


FIG. 217.

In engine design it is usual to construct the tangential effort curve from the cards obtained at normal or rated engine load and to assume that the engine is working against a constant and uniform torque. The uniform resisting torque is represented in Fig. 218 by XX, which is so located that the shaded areas above and below it are equal to one another. The areas BCD and FGH represent in ft.-lb. per sq. in. of piston area the amount of energy which is effective in accelerating the moving masses; and the areas DEF and HAB represent the energy per sq. in. of piston area liberated in the retardation of the moving masses.

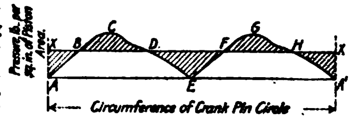


FIG. 218.

Determination of Flywheel Weight. Let M = mass of wheel rim. W = weight of wheel rim, lb. v_1 = maximum linear rim velocity, ft. per sec. v_2 = minimum rim velocity, ft. per sec. $v = \frac{1}{2}(v_1 + v_2)$ = average rim velocity, ft. per sec. $k = (v_1 - v_2)/v$ = coefficient of energy fluctuation. D = mean diam. of rim, ft. N = r.p.m. A = piston area, sq. in. E_m = energy represented by area BCD or FGH or DEF (Fig. 218) = maximum energy in ft.-lb. per sq. in. of piston area which must be absorbed or liberated by the wheel rim with velocity variation between v_1 and v_2. Then

$$E_m \times A = M\frac{v_1^2}{2} - M\frac{v_2^2}{2} = \frac{W}{g}kv^2 = \frac{kWD^2N^2}{11,744}; \text{ or } W = \frac{11,744\, E_m A}{kD^2N^2}$$

About 9/10 W may be placed in the rim to account for the flywheel effect of the arms and other rotating masses. Acceptable values of k are as follows: Pumps, 1/30 to 1/50; machine shops, 1/35 to 1/40; looms and paper mills, 1/40; spinning mills, 1/60; direct-current generators (lighting), 1/150; alternating-current generators, 1/300.

Approximate methods for determining **flywheel weights for steam engines** are as follows (for gas-engine flywheels, see p. 1047; for automobile engines, p. 1194): $W = \text{h.p.} \times C/kNv^2$, where h.p. = i.h.p. of the engine, other symbols as before. C has values as given in Table 106 (Theiss, *Am. Mach.*, Sept. 14, 1893).

Table 106. Weights of Steam-engine Flywheels
(Values of C in the formula: Weight in lb. = h.p. $\times\, C/kNv^2$)

Cut off	Compression*	Single-cylinder engines						Two-cylinder engines			Three-cylinder engines	
		Non-condensing				Condensing		Cranks at 90 deg.			Cranks at 120 deg.	
		Piston speeds in ft. per min.										
		200	400	600	800	200	400	200	400	800	200	800
1/6	p	272,690	240,810	194,670	158,200	234,160	174,380	71,980	70,160	70,040	33,810	30,190
1/6	$0.7p$	241,530	209,890	165,450	132,030	206,030	151,680	....	....	....	....	....
1/6	0	218,580	187,430	145,400	108,690	173,660	118,350	60,140†	60,140†	60,140†	32,240	31,570
1/4	p	242,010	208,200	168,590	162,070	204,210	164,720	59,420	57,000	60,140	33,810	35,140
1/4	$0.7p$	220,280	188,880	151,440	148,540	185,250	148,780	....	....	....	....	....
1/4	0	209,170	179,460	136,460	135,260	167,140	133,080	54,340†	54,340†	54,340†	35,500	33,810
1/3	p	220,760	188,510	165,210	....	189,600	174,630	49,272	49,150	....	34,540	36,470
1/3	$0.7p$	207,230	176,080	150,710	....	173,900	164,970	....	....	....	....	....
1/3	0	201,920	170,040	146,610	....	161,830	151,680	50,000†	50,000†	....	33,450	32,850
1/2	p	193,340	174,630	....	....	172,690	....	37,920	35,500	....	35,260	33,810
1/2	$0.7p$	187,670	167,860	....	....	165,930	....	....	....	....	....	....
1/2	0	182,840	167,860	....	....	159,990	....	36,950†	36,950†	....	32,370	32,370

* p = compression to initial steam pressure; $0.7p$ = compression to $0.7 \times$ initial pressure; 0 = no compression.
† Mean values.

In the operation of alternators in parallel, resonance must be avoided and the angular displacement must be within 2.5 deg. of the electrical circuit either side of the mean. To insure the time period of the alternator being 40 per cent. greater than that of the impulses of the prime mover (steam or gas engine), the following formula may be used:

$$WR^2 = T^2/P_0 \times 684{,}000/N^2,$$

where W = weight of flywheel, field and rotating parts, lb.; R = radius of gyration of combined rotating parts, ft.; T = time period of engine impulses, sec. = 60 ÷ ($N \times$ impulses per revolution); N = r.p.m. of engine; f = frequency of the system = No. of poles $\times$ r.p.m. ÷ 120; and P_0 = rated capacity of generator in kilowatts $\times$ shunt-circuit current ÷ full-load current. The above formula will ensure a lack of resonance but not necessarily a proper angular variation. The following formula insures an angular variation not more than 2½ electrical degrees either side of the mean: $WR^2 = \text{kw.} \times C \times 10^9/N^4$, where kw. = kilowatt rating of generator, and C has the following values:

	Type of Engine		
	Horizontal single crank	Vertical cross comp.	Horizontal cross comp.
25 cycles........	315	275	250
40 cycles........	505	440	400
50 cycles........	630	550	500
60 cycles........	755	660	600
125 cycles........	1575	1380	1250

Wittenbauer's Analysis for Flywheel Performance. Let the crank-pin velocity be represented by v_r and the velocity of any moving masses (m_1, m_2, m_3, etc.) at any instant or phase be represented by v_1, v_2, v_3, etc. The kinetic energy of the entire system of moving masses may then be expressed as

$$E = \tfrac{1}{2}(m_1v_1^2 + m_2v_2^2 + m_3v_3^2 \ldots.) = \tfrac{1}{2}M_rv_r^2$$

where the single reduced mass M_r at the crank pin which possesses the equivalent kinetic energy is

$$M_r = [m_1(v_1/v_r)^2 + m_2(v_2/v_r)^2 + m_3(v_3/v_r)^2 + \ldots]$$

In an engine mechanism sufficiently accurate values of M_r can be obtained if the weight of the connecting rod is put half at the crank pin and half at the cross head. M_r is a variable in engine mechanisms on account of the reciprocating parts, and should be found for a number of crank positions. It should include all moving masses, except the flywheel.

The total energy E used in accelerating reciprocating parts from the beginning of the forward stroke up to any crank position can be obtained by finding from the indicator cards the total work done in the cylinder (on both sides of the piston) up to that time and subtracting from it the work done in overcoming the resisting torque, which may usually be assumed constant. The mean energy of the moving masses is $E_0 = \tfrac{1}{2}M_rv_r^2$.

In Fig 219 the reduced weights of the moving masses $G_p + G_{rs}$ are plotted on the X axis corresponding to different crank positions. $G_p = gM_p$ is the reduced flywheel mass and $G_{rs} = gM_{rs}$ is the sum of the other reduced masses. Against each of these abscissæ is plotted the energy E available for acceleration measured from the beginning of the forward stroke. The curve 0123456 is the locus of these plotted points.

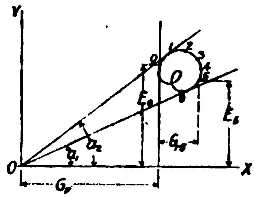


FIG. 219.

This diagram possesses the following property: Any straight line drawn from the origin O to any point in the curve is a measure of the velocity of the moving masses; tangents bounding the diagram measure the limits of velocity between which the crank pin will operate. The maximum linear velocity of the crank pin in ft. per sec. is $v_2 = \sqrt{2g \tan a_2}$, and the minimum velocity is $v_1 = \sqrt{2g \tan a_1}$. Any desired change in v_1 and v_2 may be accomplished by changing the value of G_p, which means a change in the flywheel weight or a change in the flywheel weight reduced to the crank pin. As G_p is very large compared with G_r, the tangents are best found by direct calculation.

GOVERNORS*

Governors are mechanisms designed to maintain the speeds of prime movers (r.p.m.) within reasonably constant limits, whatever the load may be. Nearly all governors depend for their action upon centrifugal force, and consist of a pair of masses rotating about a spindle driven by the prime mover and kept from flying outward by a controlling force. With an increase in speed this controlling force is overcome and the masses move outward until the controlling force is increased to balance their action, and this motion is transmitted to valves supplying the prime mover with its working fluid or fuel. (See also p. 215.)

Conical Pendulum Governor (Fig. 1). In this governor, invented by **Watt,** the revolving masses are balls attached to a vertical spindle by link arms, and the controlling force consists of the weights of the balls themselves. There is a definite ball position for any particular speed, and the governor is said to be "static." For any position of the balls the height of the cone of revolution of the balls and their arms is h (in.) $= 35{,}200/n^2$, where n = r.p.m. Energy (in.-lb.) $= 2wh$, where w = weight of one ball, lb. The sensitiveness of this governor, or the sleeve movement for a given change in speed, rapidly falls off as n increases.

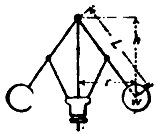


FIG. 1.

Loaded Governor. The **Porter governor** (Fig. 2) is a Watt governor with the addition of a heavy movable weight surrounding the spindle. For a given variation of speed the height h (in.) is greater than with the Watt governor, being, when the four arms are of equal length, $h = [(w + w_1)/w] \times (35{,}200/n^2)$, where w = weight of one ball and w_1 = weight surrounding spindle, both in lb. This type of governor can be run at much higher speeds than the Watt, and it is much more powerful with the same weights of balls. Energy (in.-lb.) $= 2wh + w_1h_1$, where h_1 = vertical rise of center weight, in.

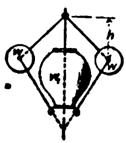


FIG. 2.

Crossed-arm Governor. This is a conical pendulum governor in which each arm is suspended from a pin placed at the opposite side of the spindle to that of its ball. By proper proportioning the balls may be made to move along an approximately parabolic path, and the height h made practically constant for the whole range of the governor, resulting in extreme sensitiveness.

Spring-loaded Governors are those in which the controlling force is wholly or partly produced by springs. This permits the use of a horizontal or inclined axis of rotation.

Shaft Governors consist essentially of a wheel keyed to the main shaft of the engine and carrying weights pivoted near its circumference, which are free to move outward under the influence of centrifugal force, this tendency being resisted by springs. In steam engines the movement of the weights is communicated by links to an eccentric, thus varying the travel of the valve.

In **inertia shaft governors** a weight arm is so pivoted and has its weight so distributed that its inertia assists in making the eccentric adjustment, and the more sudden the change in load the greater the assistance it renders. Thus, when the engine speeds up the weight tends to rotate at its old speed by reason of its inertia, and hence lags behind the wheel. Also,

* Staff contribution.

if the load is suddenly increased, the engine will slow down, but the weight arm will continue at its old speed, gaining on the wheel and again assisting the centrifugal force in altering the position of the eccentric with respect to the crank. In the Rites inertia governor the weight arm is a long bar with heavy ends which is pivoted off its center to a pin some distance from the center of the shaft. In the Armstrong governor a weight attached to the end of a laminated spring fixed to the hub of the governor wheel is acted on both by centrifugal force and inertia.

Helical **springs** for shaft governors may be calculated as follows: Let x = distance (ft.) from c. of g. of weight to center of pin around which the weight turns; x_1 = distance (ft.) from center of pin to point of attachment of spring. Then, pull P_1 on spring due to centrifugal force of weight W(lb.) $= 0.00034WR_1^2n_1^2\,x/x_1$, where R_1 = distance (ft.) from center of shaft to c. of g. of weight when running at n_1 r.p.m. Also, at n_2 r.p.m., $P_2 = 0.00034WR_2^2n_2^2x/x_1$. Then (see p. 429) P_2 = maximum tension (or compression) on spring, lb. $= f\pi d^3/8D$, where d = diam. of wire in spring, in.; D = mean diam. of coil, in.; and f (safe) = 50,000. After obtaining d from the equation for a given diameter D and pull P_2, the number of active coils z required to give the necessary extension or compression S (in.) of the spring may be obtained from the formula $z = Cd^4S/8(P_2 - P_1)D^3$, where P_1 is the initial tension or compression on spring, lb. and C = 12,000,000 for round steel wire. If with springs thus calculated the governing is too fine, reduce the number of active coils in order to stiffen spring and bring about a greater fluctuation in speed. If the governing is too coarse, a larger number of active coils is required. The **powerfulness** K of the two springs of a shaft governor depends upon the energy stored in the springs by extending them the amount S (from tension P_1 to tension P_2).

Stability, Isochronism, Hunting. The controlling force in a governor (whether gravity or that of springs) may be considered as a force F acting on each weight in the direction of the radius toward the axis of revolution. If w/g be the mass of the weight and r the radius of its c. of g., the governor will revolve in equilibrium when $F = 4\pi^2r(n/60)^2w/g = 0.00034rn^2w$. In order to insure stability F must increase more rapidly than r as the balls move outward. The frictional resistance to the action of a governor may be considered as a force f acting radially on each ball so that the whole controlling force is $F + f$ when the balls are moving out and $F - f$ when they are moving in. The corresponding speeds are given by the equation $F + f = 4\pi^2r[(n + \Delta n)/60]^2w/g$ and $F - f = 4\pi^2r[(n - \Delta' n)/60]^2w/g$. As a result of friction the speed may change from Δn above to $\Delta' n$ below the normal speed n while the position of the balls is unchanged. It is apparent that a stable governor cannot maintain an absolutely constant speed in the engine it controls. If a change takes place in the load on a steam engin, it is necessary for the weights or balls to be displaced a certain amount in order to alter the steam supply, and the weights can retain their new position only at a changed speed. The maximum range of speed depends on that amount of change of n which is required to alter the configuration of the governor between the limits of no steam supply and full steam supply. If this range is but a small percentage of n, the governor is said to be sensitive.

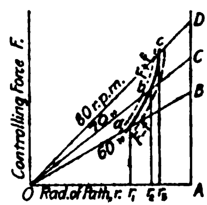


Fig. 3.

The **configuration** of a governor at any speed within its range may be determined as in Fig. 3, a numerical example of a Watt governor being taken, in which $w = 10$ lb. Assuming the upper end of ball arm (Fig. 1) to be pivoted at the axis of spindle, $r^2 + h^2 = L^2$ = constant, or, for any position, $r = \sqrt{L^2 - h^2}$. Now, $h = 35{,}200/n^2 = 9.78$ in. for 60 r.p.m., 7.19 in. for 70 r.p.m., and 5.5 in. for 80 r.p.m. Assuming radius $r = 10$ in. at 60 r.p.m., L^2 = constant = 195.6, whence $r = 12$ in. at 70 r.p.m. (for $h =$ 7.19 in.) = 12.8 in. at 80 r.p.m. In Fig. 3, at any radius greater than 12.8 in., say 16 in., erect an ordinate AD. Calculate the values of $F = 0.00034rn^2w$, taking $r = 16$ in., $w = 10$ lb. and $n = 60$, 70 and 80 respectively, and lay these values off to scale, as AB, AC, AD. Draw OB, OC and OD. Take $r_1 = 10$ in. by scale and erect an ordinate cutting OB or the 60 r.p.m. line at a; locate points b and c similarly, using radii r_2 and r_3 of 12 in. and 12.8 in. respectively, and connect a, b and c by a smooth curve. This is the **curve of controlling force** of the governor, and the ball path radius for any intermediate speed may be obtained by suitably scaling AD, drawing a line from the desired speed value on this scale to O and dropping an ordinate to the radius scale from its intersection with the curve abc. When a tangent to any point makes a greater angle with OA than does a line drawn from O through the same point, the governor is stable throughout its range. The influence of friction can be readily seen by plotting $F + f$ and $F - f$ curves.

A governor is said to be **isochronous** (or astatic) when the speed of rotation is the same for all positions within its limits of movement. On account of the frictional resistances of the joints, however, the speed must always increase slightly before any additional centrifugal force is available, and this will move the weights to their extreme outward position; conversely, when the speed falls sufficiently for gravity or spring force to overcome the frictional resistance, the weights will move to their extreme inward position. In this way the engine will fall into a state of speed oscillation or **hunting**, which is obviously objectionable and may be avoided by giving the governor a fair degree of stability, by reducing the frictional resistances as much as possible, and by the use of dash pots which offer great resistance to sudden movements but allow comparatively slow changes of position to take place freely.

ELEMENTS OF HIGH-SPEED MACHINES

BY

L. C. LOEWENSTEIN

DISK-WHEEL STRESSES

The wheels of steam turbines and the impellers of centrifugal pumps and centrifugal compressors are frequently subjected to high centrifugal forces. The correct mathematical determination of the stresses produced by these forces along the three principal axes of the given wheel is in general too laborious for practical purposes. As the axial width, however, of such wheels (usually spoken of as disk wheels) is generally small in comparison to their radial dimensions, the axial stresses are considered as negligible, and the computations, thus greatly simplified, are confined to the radial and tangential stresses.

The profiles of disk wheels are usually made up of straight lines and of circular arcs. For computation purposes, however, it is preferable to represent such profiles by one or more hyperbolas of the general form

$$t = br^{a} \tag{1}$$

in which t = thickness of disk at radius r, in.; a = shape constant, and may be either zero, positive, or negative; b = dimension constant, and does not enter into the stress computations unless the wheel is subjected to other centrifugal loads besides its own. For an hyperbola from radius r_1 to radius r_2, where the thicknesses are t_1 and t_2, respectively,

$$a = \log(t_1/t_2)/\log(r_1/r_2) \tag{2}$$

and

$$\log b = \log t_1 - a \log r_1 = \log t_2 - a \log r_2 \tag{3}$$

For the computations of the centrifugal radial and tangential stresses in a disk it is necessary to know besides its shape, as given by (1), either two stresses at the same radius (radial and tangential) or one stress of either kind at two different radii. Thus, if the radial stresses R_1 and R_2 at the radii r_1 and r_2, respectively, of the same hyperbolic section are given (or assumed), the corresponding tangential stresses T_1 and T_2 are found from the equations

$$T_1 = Ar_2^2 - BR_1 + CR_2 \tag{4}$$

$$T_2 = Dr_2^2 - ER_1 + FR_2 \tag{5}$$

The factors A, B, C, D, E, and F are functions only of the shape constant a and of the ratio of radii $K = r_1/r_2$, and are given by the following relations:

$$\left.\begin{aligned}
B &= -\frac{m_2K^{m_1-1} - m_1K^{m_2-1}}{K^{m_2-1} - K^{m_1-1}} & A &= -\frac{7.956\left(\frac{N}{1000}\right)^2}{8 + 3.3a}[1.9K^2 + 3.3(K^2B - C)] \\
E &= -\frac{m_2 - m_1}{K^{m_2-1} - K^{m_1-1}} & D &= -\frac{7.956\left(\frac{N}{1000}\right)^2}{8 + 3.3a}[1.9 + 3.3(K^2E - F)] \\
C &= \frac{E}{K^{a+2}} & m_1 &= -\frac{a}{2} - \sqrt{\frac{a^2}{4} - 0.3a + 1} \\
F &= B + a & & \\
N &= \text{r.p.m.} & m_2 &= -\frac{a}{2} + \sqrt{\frac{a^2}{4} - 0.3a + 1}
\end{aligned}\right\} \tag{6}$$

In formulæ (6) a density of 0.28 lb. per cu. in. and a ratio of lateral contraction to radial elongation (Poisson's ratio) of 0.3 have been assumed. When these functions have been computed in a given problem for a particular value of a and a particular ratio of radii r_1/r_2 their values may be preserved for future problems involving the same a and K, even though the actual dimensions may be entirely different. The values of A and D should be recorded for 1000 r.p.m.

Example. Fig. 1 shows the half-section of a turbine wheel designed to run at 3600 r.p.m. As at any given radius there is only one thickness, there is also only one set of values of T and R at any particular radius; so that T_{23} and R_{23} at the end of the third section are equal respectively to T_{14} and R_{14} at the beginning of the fourth section, and may be so equated by the use of formulæ (4) and (5). [In the notation used the first number of the subscript indicates the inner surface (1) or the outer surface (2) of a ring; the second number is the number of the ring. Thus, T_{23} indicates the tangential stress at the outer surface of the third ring.] At 3600 r.p.m. the shrink fit with which the wheel is forced on the shaft is supposed to have been almost entirely neutralized by the centrifugal expansion of the bore, so that the radial stress in the bore R_{11} may be taken as zero. By trial it is found that five hyperbolas give a sufficiently close approximation to the actual profile between the inner radius of 3¼ in. and the outer radius of 18¾ in. at which the buckets may be assumed as attached. The radial stress R_{25} produced by the pull of the buckets on the outer surface of the wheel (r = 18¾) is, say, 156.1 lb. per sq. in. at 1000 r.p.m. The available data may be summarized as follows:

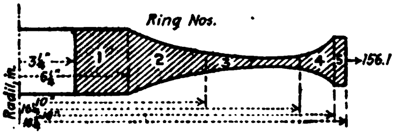


FIG. 1.

Ring No.	1	2	3	4	5
r_1 =	3.25	6.25	10.00	16.25	18.00
r_2 =	6.25	10.00	16.25	18.00	18.75
t_1 =	4.00	4.00	1.024	0.544	2.514
t_2 =	4.00	1.024	0.544	2.514	2.514
r_1/r_2 =	0.520	0.625	0.615	0.903	0.960
t_1/t_2 =	1.00	3.91	1.88	2.17	1.00
a =	0.0	−2.9	−1.3	15.0	0.0
R_1 =	0.0	......	......	......	......
R_2 =	......	......	......	......	156.1
A =	6.93	5.91	6.53	8.06	7.84
B =	1.74	4.08	2.94	3.90	24.5
C =	2.74	1.48	2.27	21.45	25.5
D =	3.17	3.31	3.57	7.04	7.44
E =	0.74	2.26	1.59	3.55	23.5
$F = B + a$ =	1.74	1.18	1.64	18.9	24.5

Equating the tangential stress at the end of each ring with that at the beginning of the adjoining ring, gives the following equations [from (4) and (5)]:

At $r = 6\frac{1}{4}$, $T_{21} = T_{12}$, or $3.17 \times (6\frac{1}{4})^2 - 0.74 \times 0 + 1.74R_{21} = 5.91 \times (10)^2 - 4.08R_{12} + 1.48R_{22}$.

At $r = 10$, $T_{22} = T_{13}$, or $3.31 \times (10)^2 - 2.26R_{12} + 1.18R_{22} = 6.53 \times (16\frac{1}{4})^2 - 2.94R_{13} + 2.27R_{23}$.

At $r = 16\frac{1}{4}$, $T_{23} = T_{14}$, or $3.57 \times (16\frac{1}{4})^2 - 1.59R_{13} + 1.64R_{23} = 8.06 \times (18)^2 - 3.90R_{14} + 21.45R_{24}$.

At $r = 18$, $T_{24} = T_{15}$, or $7.04 \times (18)^2 - 3.55R_{14} + 18.9R_{24} = 7.84 \times (18\frac{3}{4})^2 - 24.5R_{15} + 25.5 \times 156.1$.

The solution of these equations, considering that $R_{21} = R_{12}$, $R_{22} = R_{13}$, $R_{23} = R_{14}$ and $R_{24} = R_{15}$, gives the following stresses at 1000 r.p.m.:

For r =	3.25	6.25	10.00	16.25	18.00	18.75
R =	0.	462.	1500.	1652.	237.5	156.1
T =	1536.	927.	1060.	1265.	910.	860.

For 3600 r.p.m., all the stresses should be multiplied by $(3600/1000)^2 = 12.96$, giving

R =	0	5,990	19,440	21,410	3,080	2,020
T =	19,910	12,010	13,740	16,400	11,790	11,150

If the modulus of elasticity of the material is, say, 29,000,000, the radial stretch L at any radius is

$$L = (T - 0.3R)r/29{,}000{,}000 \tag{7}$$

making the radial elongation at the various radii as follows:

For r =	3.25	6.25	10.00	16.25	18.00	18.75
L =	0.00224	0.00220	0.00277	0.00560	0.00676	0.00682

For intermediate points in any ring, R_2 can be computed by (4) from the known values of T_1 and R_1, and then T_2 can be computed by (5) from the known values of R_1 and R_2.

Average Stress. Frequently it is desired to know only the average tangential stress for a given section of wheel disk without going through the computations for finding the local stresses. The formula for average tangential stress T_a in a wheel, due to its own weight alone, is

$$T_a = 28.42(N/1000)^2 SI/A_0 \tag{8}$$

in which N = r.p.m.; S = density of material, lb. per cu. in.; I = moment of inertia of the half-section about the axis of the shaft, in.^{4}; and A_0 = area of the half-section, sq. in. For hyperbolic cross-sections of the form $t = br^a$,

$$I = b\,(r_2^{a+3} - r_1^{a+3})/(a + 3) \tag{9}$$

and

$$A_0 = b\,(r_2^{a+1} - r_1^{a+1})/(a + 1) \tag{10}$$

For the preceding example the values in (9) and (10) are as follows:

Ring No.	1	2	3	4	5
b =	4.00	814.0	20.4	37.26×10^{-20}	2.514
A_0 =	12.00	7.8	4.6	2.3	1.9
I =	280.	471.	772.	686.	637.

Total $A_0 = 28.6$; total $I = 2846$.

The average tangential stress, due to the disk's own weight, at 3600 r.p.m. is $T_a = 28.42 \times (3.6)^2 \times 0.28 \times (2846/28.6) = 10{,}300$ lb. per sq. in.

If at the outer radius of 18¾ in. there is a radial stress of 2023 lb. per sq. in. at 3600 r.p.m. caused by the attached buckets and the non-continuous portions of the turbine wheel itself, the additional average stress caused by this dead load is $2023 \times 18.75 \times 2.514/28.6 = 3340$ lb. per sq. in. The total average tangential stress in the wheel disk section is therefore $10{,}300 + 3340 = 13{,}640$ lb. per sq. in. By plotting the local tangential stresses against total area of section counted from the inside of the bore (instead of against radii), and drawing on this curve of stresses a horizontal line to represent the average tangential stress as just found, the equality of areas above and below this horizontal line will serve as a very sensitive check on the correctness of the computations. In case there are bolt holes or other non-continuous portions in the section, the (projected) area of these holes should be subtracted from the gross area of the section, and all tangential stresses should be multiplied in the ratio of the gross area to the net area of the section.

Impellers. In impellers for centrifugal pumps such as shown in Fig. 2, the dead load caused by the blades cannot be regarded as acting at any particular radius. The usual procedure is to compute the local radial and tangential stresses by the method described above, on the assumption of zero radial stress both inside the bore and at the outer radius, thus leaving the dead load all out of account temporarily. Then the average tangential stress T_a in the section, and the total centrifugal force F in lb. exerted by all the blades are computed. If the weight of all the blades is W lb. and the radius of their center of gravity is R in., then, at N r.p.m., $F = 28.42(N/1000)^2WR$, and if the area of the impeller section on one side of the shaft line is A_0 sq. in., the additional average tangential stress T_e caused by the dead load is $T_e = F/2\pi A_0$. If there are any holes in the section, the sum of T_a and T_e should be multiplied in the ratio of the gross area to the net area of the section to give the corrected total average tangential stress in the impeller. The ratio of this total average stress to the original value of T_a is then applied as a correction factor to the values of the local tangential stresses as previously computed. The correction of the radial stresses is laborious and uncertain.

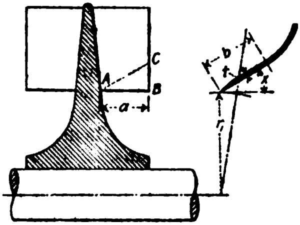


FIG. 2.

In connection with Fig. 2 it should be noted that if the width a of the impeller blade is excessive, the lower corner of the blade is subject to a crumpling stress on the section through AC. Referring to the lettering in the figure, if N = r.p.m. and S = density of the material (lb. per cu. in.), the crumpling stress in lb. per sq. in. is

$$C = 28.42\left(\frac{N}{1000}\right)^2 Sr_1 \cos x \frac{a^2b^2}{t(a^2+b^2)}$$

In case the value of C is too high, the blade must be reinforced by annular shrouds, or the radial inlet construction must be changed to that of the axial inlet type.

CRITICAL SPEEDS OF SHAFTS

The critical speed of a rotating shaft is the speed at which its elastic forces are completely neutralized so that it is incapable of offering any resistance to a deflecting force. This speed is equal numerically to the frequency of vibration of the shaft with the masses mounted on it, if deflected by an external force while the shaft is at a standstill. Its value depends on the length of the shaft, its various diameters, the manner in which it is supported, and on the magnitude and distribution of the loads it carries, but it is independent of whether the shaft is horizontal or vertical.

Shafts of Uniform Diameter. If a shaft carrying loads W_1, W_2, W_3, W_4. . . (lb.) has such dimensions and is so supported that the static deflections at those loads, if the shaft were horizontal (although actually it may be vertical), would be y_1, y_2, y_3, y_4 . . . (in.), respectively, the first or lowest critical speed N_c in r.p.m. is given by the formula

$$N_c = 187.7\sqrt{\frac{W_1y_1 + W_2y_2 + W_3y_3 + W_4y_4 + \ldots}{W_1y_1^2 + W_2y_2^2 + W_3y_3^2 + W_4y_4^2 + \ldots}} \quad (1)$$

The value obtained by using in this formula the static loads W and the corresponding deflections y is generally within 1 per cent. of the correct value. By repeating the calculation, using W_1y_1, W_2y_2, W_3y_3, W_4y_4 . . . as the concentrated loads acting on the shaft and finding the corresponding deflections y_{1e}, y_{2e}, y_{3e}, y_{4e}, and then writing for the correct value N_c

$$N_c^1 = N_c\sqrt{\frac{W_1y_1 + W_2y_2 + W_3y_3 + W_4y_4 + \ldots}{W_1y_{1e} + W_2y_{2e} + W_3y_{3e} + W_4y_{4e} + \ldots}} \quad (2)$$

a closer approximation to the true critical speed will be obtained, as the values y_{1a}, y_{2a}, . . . will correspond more nearly to the elastic curve of the shaft *at the critical speed.* Similarly, by repeating the calculation again with W_1y_{1a}, W_2y_{2a} . . . as loads and finding corresponding deflections y_{1b}, y_{2b} . . . ., a still closer approximation to the true critical speed will be obtained. Unless, however, extreme care is taken to avoid cumulative errors of computation, the increased accuracy obtained for successive trials is likely to be to a large extent illusory, and of no practical value. If the shaft carries any distributed loads, they should be replaced by approximately equivalent concentrated loads before using formula (1). In case the loads W_1, W_2, W_3, W_4 . . . are not all on the same span (*e.g.*, on a three-bearing shaft), the loads on one span must be assumed as acting vertically downward, and those on the other vertically upward, so as to preclude the possibility of a zero deflection at any point except at the bearings. The same rule applies when the shaft has three or more spans.

In practice, the loads are usually shrunk on the shaft so that its stiffness (or its moment of inertia) at the loads is greatly increased; the loads are not exactly concentrated at the assumed points; and, finally, the gyroscopic action of the loads on the bent shaft, especially toward the bearings, tends to straighten out the shaft. For these reasons, the value of N_c as computed by (1) will usually be found to be somewhat low.

If a single load W_1 alone is acting on the (weightless) shaft, (1) becomes

$$N_c = 187.7/\sqrt{y} \tag{3}$$

and in this form it is occasionally applied to single-span shafts carrying more than one load by taking for y the *maximum* deflection of the shaft, no matter where it occurs.

A shaft may have as many critical speeds as the number of loads it carries. A shaft carrying a distributed load may therefore have an infinite number of critical speeds. For engineering purposes, only the first critical speed is usually of importance, while the second critical speed is only occasionally reached. For shafts of uniform or average diameter d (in.), carrying one or two concentrated loads or a uniformly distributed load (lb.), formulæ for the critical speed can be written down directly. Tables 1, 2, and 3 are for steel shafts having a modulus of elasticity E = 29,000,000. Table 1 gives the single critical speed N_c. Table 2 gives the first critical speed N_{c1} and R the ratio of successive critical speeds to the first. Table 3 gives directly or indirectly the two critical speeds N_{c1} and N_{c2}. In all cases, the weight of the shaft itself is either neglected, or a part of it (one-half to two-thirds) is added to the concentrated loads. Shafts with very short or self-aligning bearings are considered as *supported* at the bearings, and those with long, rigid bearings as *fixed*.

As defined above, a shaft at its critical speed is in a state of maximum sensitiveness (or indifferent equilibrium), so that the smallest force may, if allowed sufficient time, deflect it to infinity and break it. The minutest deviation, however, from the mathematically exact critical speed is sufficient to restore to the shaft a considerable amount of its elastic resistance. In the neighborhood, therefore, of its critical speed a shaft merely undergoes more or less intense vibrations, which are generally transmitted also to the supporting frame. The deflecting force may be due to some external cause, but it is usually supplied by the almost unavoidable deviations of the centers of gravity of the various loads from the center line of the shaft. By reducing these deviations to a minimum by fine workmanship and very careful balancing, the vibration at the critical speed may be made scarcely noticeable.

Table 1. Critical Speeds of Shafts with a Single Concentrated Load

(Bearings R_1 and R_2; distance c. to c. = l)

Case	Bearings		Load W applied at distances from		Critical speed, N_c ($N_c = 187.7/\sqrt{y}$)	Static deflection, y
	R_1	R_2	R_1	R_2		
1	Supported	Supported	a	b	$387{,}500\frac{d^2}{ab}\sqrt{\frac{l}{W}}$	$\frac{Wa^2b^2}{3EIl}$
2	Supported	Supported	$l/2$	$l/2$	$1{,}550{,}000\frac{d^2}{l\sqrt{Wl}}$	$\frac{Wl^3}{48EI}$
3	Fixed.	Fixed.	a	b	$387{,}500\frac{d^2l}{ab}\sqrt{\frac{l}{Wab}}$	$\frac{Wa^3b^3}{3EIl^3}$
4	Fixed.	Fixed.	$l/2$	$l/2$	$3{,}100{,}000\frac{d^2}{l\sqrt{Wl}}$	$\frac{Wl^3}{192EI}$
5	Fixed.	Supported	a	b	$775{,}000\frac{d^2l}{ab}\sqrt{\frac{l}{Wa(3l+b)}}$	$\frac{Wa^3b^2}{12EIl^3}(3l+b)$
6	Fixed.	Supported	$l/2$	$l/2$	$2{,}377{,}000\frac{d^2}{l\sqrt{Wl}}$	$\frac{7Wl^3}{768EI}$
7	Fixed.	*	l	.....	$387{,}500\frac{d^2}{l\sqrt{Wl}}$	$\frac{Wl^3}{3EI}$

* One bearing, R_1; load W applied at free end of shaft, at distance l from bearing.

Table 2. Critical Speeds for Shafts Carrying Distributed Loads

(Bearings R_1 and R_2; distance c. to c. = l; total load = W)

Case	Bearings		First critical speed, N_{c1}	N_c for shaft alone	2d, 3d, 4th, etc., critical speeds = $N_{c1} \times$
	R_1	R_2			
1	Supported..	Supported..	$2{,}230{,}000\frac{d^2}{l\sqrt{Wl}}$	$4{,}760{,}000\frac{d}{l^2}$	4, 9, 16, . . .
2	Fixed.......	Fixed.......	$4{,}980{,}000\frac{d^2}{l\sqrt{Wl}}$	$10{,}620{,}000\frac{d}{l^2}$	$\frac{25}{9}, \frac{49}{9}, \frac{81}{9}$ . . .
3	Fixed.......	Free *.......	$795{,}000\frac{d^2}{l\sqrt{Wl}}$	$1{,}696{,}000\frac{d}{l^2}$	$\frac{9}{14.2}, \frac{25}{14.2}, \frac{49}{14.2}$.
4	Fixed.......	Supported..	$3{,}483{,}000\frac{d^2}{l\sqrt{Wl}}$	$7{,}020{,}000\frac{d}{l^2}$	$\frac{81}{25}, \frac{169}{25}, \frac{289}{25}$. .

* One bearing, R_1; load W distributed uniformly from R_1 to end of shaft, distant l from bearing.

Shafts of Variable Diameter. When the diameter of the shaft is too variable to assume a satisfactory average, or when the total number of loads is greater than two, no general critical speed formulæ are available, and each case must be worked out independently. For the deflections y_1, y_2, y_3 . . . in (1) the following general formula may be used:

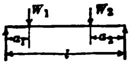
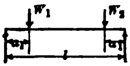
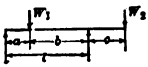
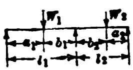
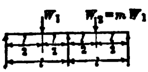
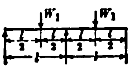
$$Ey_x = \int\int_0^x \frac{M}{I}\,dx + C_1x + C_2$$

in which E = modulus of elasticity (taken as 29,000,000 for steel); y_x = deflection at point x, in.; x = distance of point x from origin, in.;

Table 3. Critical Speeds of Shafts Carrying Two Concentrated Loads

General formula: N_{c1} and $N_{c2} = \sqrt{\frac{1}{2A}\left[U^2 + V^2 \mp \sqrt{(U^2+V^2)^2 - 4AU^2V^2}\right]}$

For N_{c1} use minus sign; for N_{c2} use plus sign. The values of A, U, and V are given in the table below, whenever N_{c1} and N_{c2} are not given directly. y_1 and y_2 = static deflections at W_1 and W_2 (shaft horizontal).

1		$A = 1 - \left(\frac{l^2 - a_1^2 - a_2^2}{2(l-a_1)(l-a_2)}\right)^2$ $U = 387{,}500\frac{d^2}{a_1(l-a_1)}\sqrt{\frac{l}{W_1}}$; $V = 387{,}500\frac{d^2}{a_2(l-a_2)}\sqrt{\frac{l}{W_2}}$
2		$N_{c1} = 548{,}000\frac{d^2}{a_1(l-2a_1)}\sqrt{\frac{l}{W_1}}$ $N_{c2} = 548{,}000\frac{d^2}{a_1}\sqrt{\frac{l}{W_1(3l-4a_1)}}$ $y_1 = \frac{W_1a_1^2}{6EI}(3l-4a_1)$
3		$A = 1 - \frac{(l+a)^2}{4l(l+c)}$ $U = 387{,}500d^2\sqrt{\frac{l}{W_1a^2b^2}}$; $V = 387{,}500\frac{d^2}{c\sqrt{W_2(l+c)}}$ $y_1 = \frac{W_1a^2b^2}{3EIl} - \frac{W_2ac}{6EIl}(l^2-a^2)$ $y_2 = \frac{W_2c^2}{3EI}(l+c) - \frac{W_1ac^2}{6EIl}(l^2-a^2)$
4		$A = 1 - \frac{9}{16}\,\frac{l}{l+c}$ $U = 1{,}550{,}000\frac{d^2}{l\sqrt{W_1l}}$; $V = 387{,}500\frac{d^2}{c\sqrt{W_2(l+c)}}$ $y_1 = \frac{W_1l^3}{48EI} - \frac{W_2cl^2}{16EI}$; $y_2 = \frac{W_2c(l+c)}{3EI} - \frac{W_1cl^2}{16EI}$
5		$C_1 = \frac{(l_1+a_1)^2}{4l_1(l_1+l_2)}$ $C_2 = \frac{(l_2+a_2)^2}{4l_2(l_1+l_2)}$ $A = 1 - \frac{C_1C_2}{(1-C_1)(1-C_2)}$ $U = 387{,}500\frac{d^2}{a_1b_1}\sqrt{\frac{l_1}{W_1(1-C_1)}}$; $V = 387{,}500\frac{d^2}{a_2b_2}\sqrt{\frac{l_2}{W_2(1-C_2)}}$
6		N_{c1} and $N_{c2} = 1{,}405{,}000\frac{d^2}{l\sqrt{W_1l}}\sqrt{1+m\mp\sqrt{1+m^2-1.388m}}$
7		$N_{c1} = 1{,}550{,}000\frac{d^2}{l\sqrt{W_1l}}$ $N_{c2} = 2{,}340{,}000\frac{d^2}{l\sqrt{W_1l}}$ $y_1 = \frac{7Wl^3}{768EI}$

M represents the moments at successive points on the shaft, lb.-in.; I represents the moments of inertia at the points where M is found, in.^{4}; C_1 = tangent at origin ($x = 0$) and C_2 = deflection at origin. In practice, the origin is taken at one of the bearings, where the deflection is zero; M and I are found at the points where the shaft diameter changes (generally designated as shoulders) and at the points where the loads are applied; values of dx are the finite (often large) distances between the successive points at which M and I are computed; and the integration indicated is in reality a mechanical summation.

If the total length of the span (between successive points where $y = 0$) is l, then

$$C_1 = -\frac{1}{l}\iint_0^l \frac{M}{I}\,dx$$

If M_1 and M_2 are the moments at the points x_1 and x_2, respectively, these points being separated by a piece of shaft of constant diameter d and moment of inertia I, then, for this piece of shaft,

$$\int_{x_1}^{x_2} \frac{M}{I}\,dx = \frac{M_1 + M_2}{2} \times \frac{x_2 - x_1}{I}$$

and proceeding step by step, the integral $\int_0^{x_1} \frac{M}{I}\,dx$ from the origin up to any point x_1 in. from the origin may thus be found. The double integral for the above piece of shaft of constant I is

$$\iint_{x_1}^{x_2} \frac{M}{I}\,dx = (x_2 - x_1)\int_0^{x_1} \frac{M}{I}\,dx + \frac{(2M_1 + M_2)(x_2 - x_1)^2}{6I}$$

so that the double integral $\iint_0^{x} \frac{M}{I}dx$ up to any point x in. from the origin may be found by a step-by-step process. If the distances $x_2 - x_1$ are very small, it is near enough to write for each individual step

$$\iint_{x_1}^{x_2} \frac{M}{I}\,dx = (x_2 - x_1)\int_0^{x_1} \frac{M}{I}dx$$

For critical-speed purposes, zero deflections are permissible only at the bearings, so that an overhanging load must be assumed as acting in an opposite direction from that of the loads between the bearings, and in a two-span shaft the loads on one span must be assumed as acting oppositely to (increasing the deflections of) those on the other span. The values of the reactions at the bearings of a single span, or two-bearing, shaft, are found in the usual way by taking moments. For a two-span, or three-bearing, shaft, one of the bearings is at first assumed as removed, and the deflections at the various loads, and also at the point of removal, are computed by the process outlined above. Then all the loads are supposed removed, a load of say 1000 lb. is assumed as applied at the point of bearing removal, and the deflections at all points for this condition are computed again. If the ratio of the deflection at the point of removal due to the actual loads to that due to the assumed 1000-lb. load is m, then $1000m$ is the desired third reaction, and the total deflections at all points can now be found with very little additional work. The same process may evidently be extended to the case of a four-bearing shaft by assuming two of the bearings removed and a 1000-lb. load applied successively at each of the points of removal.

For an illustration of the method of using the above principles and formulæ, see Loewenstein and Crissey, "Centrifugal Pumps," p. 203.

Long Drums. In addition to the deviation of the center of gravity of a load from that of the shaft on which it is carried, the axis of the load may be out of parallel with that of the shaft. When the shaft is put in rotation, a turning moment is developed which tends to increase the obliquity, but this is resisted by the elasticity of the shaft. Under certain conditions, however, a critical speed may be reached at which the elastic resistance of the shaft may be neutralized, and the increasing obliquity may strain the shaft to the breaking point. In Fig. 3, a continuous rigid drum is shown mounted on a steel shaft so that its principal axis AA deviates from the center line of the shaft by an angle t (shown greatly exaggerated). Let the diameter of the shaft be d, the span l, the distance of each end of the drum from the nearest bearing, a, (all in in.), the moment of inertia of drum about the principal axis AA, I_1, and that about the principal axis BB, I_2, both in in.^{4}. Then the critical speed N_{ct} due to the obliquity alone is $N_{ct} = 13{,}950 d^2 l/a\sqrt{a(I_2 - I_1)}$. The value of the angle of obliquity t does not enter into this formula, and the danger of this critical speed exists even when there is no initial obliquity. In impulse turbine wheels and in the impellers of centrifugal pumps and compressors, $I_2 - I_1$ is negative, and any initial obliquity to the center line of the bearings tends to decrease rapidly as the speed increases.

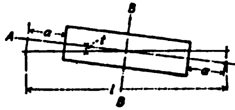


Fig. 3.

DIAPHRAGMS, CASINGS AND HEADS

The stationary parts of steam turbines, centrifugal pumps, and centrifugal compressors, such as diaphragms, casings and heads, are frequently subjected to high static pressures. Since a mathematical determination of the local stresses from point to point produced by these pressures is generally impracticable on account of the too irregular shape of such parts, the designer must depend on his judgment to avoid too sudden changes of contour or portions which are obviously too weak. If the general shape of the cross-section is satisfactory, the following method may be employed for determining the average stress in the extreme fiber.

The uniform static pressure of p lb. per sq. in. may be considered as resisted by the uniformly distributed reaction at the bolt circle (of diam. D in.) where the part in question is held to the adjoining parts. The total load W on the part, and also the total reaction, is $0.7854\ D^2p$ lb., and the distance L between the centers of gravity of the load and of the reaction is $0.1061\ D$ in. If the section modulus of the diametral cross-section is Z (in.^{3}), the average stress S at the extreme fiber is

$$S(\text{lb. per sq. in.}) = \tfrac{1}{2}W \times L/Z \qquad (1)$$

The following **graphical method for** finding the **section modulus of an irregular cross-section** will generally be found both convenient and sufficiently accurate. The heavy lines in Fig. 4 represent the half cross-section of a dished diaphragm on one side of the shaft line AB. The center line of gravity of the cross-section is assumed as nearly as possible, and two arbitrary lines at a distance K in. on each side of it are laid off outside of, but preferably close to, the cross-section. The cross-section is massed about AB (or any line parallel to it) by drawing a number of meridian lines perpendicular to AB and laying off their intercepts on the cross-section from AB; as replacing the intercept CD by the equal length $C'6$. The greater the number of these lines (especially furthest from the center of gravity line) the more accurate is the process; but it is always very important that such a line be taken wherever there is a sudden change of section. From the intersections $0'$ of the two arbitrary lines with AB, lay off the distances $0'1'$, $0'2'$, etc., equal respectively to the distances 01, 02, etc. From $5'$ draw a line to point a, which is the intersection of line 5 with the outline of the massed cross-section. Line $5'a$ intersects the arbitrary line at b. Connect b to 0 by a straight line, intersecting line 5 at c. Similarly, connect points $1'$ to $10'$ to the corresponding points of intersection of the meridian lines 1 to 10 with the boundary of the massed cross-section. Where these connecting lines intersect the arbitrary line are points to be connected to the origin 0. The intersection of each of these lines to the origin 0 with the original meridian lines will give intersection points on each meridian line similar to that of c. By connecting all these points the cross-hatched section as shown in the figure is found. If the center line of gravity was correctly chosen the area of the cross-hatched portion above the center line of gravity is equal to that below. If this is not the case, a new center line must be taken and the work repeated. If the diaphragm has been properly designed, the cross-hatched figure on each side of the center line of gravity will be very nearly a triangle. Therefore, any very marked deviations of the outer periphery of this figure from a straight line should give warning of a local excess or of a local deficiency of material. Also, the section selected for treatment in this way should be, as far as possible, a representative section, and must not be taken through a part where the body is specially strengthened by ribs or otherwise.

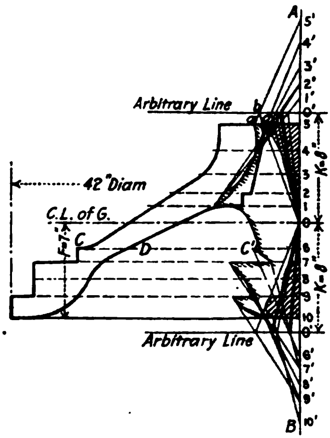


FIG. 4.

In Fig. 4, the total cross-hatched area on both sides of the center line of gravity of the half section is 12.69 sq. in. (after allowing properly for the scale of the drawing). As the diaphragm is symmetrical about the shaft line AB, the cross-hatched area A corresponding to the total diametral cross-section is 25.38 sq. in. If the distance F from the center line of gravity to the remotest fiber is 7 in., and the distance K on each side is 8 in., the modulus of the section $Z = AK^2/F = 232$ in.^{3} If the diameter of the bolt circle is 42 in., and the unbalanced pressure on the diaphragm is 100 lb. per sq. in., then $W = 0.7854 \times 42^2 \times 100 = 138{,}545$, and $L = 0.1061 \times 42 = 4.456$, so that from (1), $S = \frac{1}{2} \times 138{,}545 \times 4.456/232 = 1330$ lb. per sq. in. For safe design S should not exceed 3000 lb. per sq. in. for cast iron and 9000 lb. for cast steel.

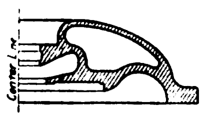


FIG. 5.

If the casing is hollow, as in the half cross-section of the centrifugal-pump casing shown in Fig. 5, the procedure is the same, the massing about the center line being the sum of the two intercepts for the hollow part of the casing.

PIPE AND PIPE FITTINGS

BY

C. W. HAM

CAST-IRON PIPE

Cast-iron Pipe is extensively used for water, gas, sewage, culverts, drains, etc., in a wide range of sizes and for varying pressures, and is particularly **adapted to underground and submerged service** because of its comparatively high corrosion-resisting qualities. It is more durable than wooden-stave, wrought-iron or steel pipe, but its first cost is greater for the ordinary sizes required in a pressure pipe line or in a distributing system. The tensile strength of commercial cast-iron pipe is uncertain, and, due to its low elasticity, it is not suitable for lines subject to the strains of expansion, contraction, and vibration, unless it is of very heavy weight. This pipe may be had in various thicknesses and weights with either **flanged ends** or **bell-and-spigot ends.** The latter are generally used for underground work, making a tight joint when properly put together, calked and leaded. For exposed piping, flanged ends are used, the joints being made up with gaskets. Flanged pipe has superior strength and tightness of the joint and is used where pipe

Table 1. Nominal Weight of Cast-iron Pipe Without Flanges, Lb. per Ft.*

Inside diam., in.	Thickness of metal in inches								
	1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
2	5.5	8.7	12.3	16.1	20.3	24.7	29.5	34.5	40.0
2 1/2	6.8	10.6	14.7	19.2	24.0	29.0	34.4	40.0	46.0
3	7.9	12.4	17.2	22.2	27.6	32.3	39.3	45.6	52.2
3 1/2	9.2	14.3	19.6	25.3	31.3	37.6	44.2	51.1	58.3
4	10.4	16.1	22.1	28.4	35.0	41.9	49.1	56.6	64.4
4 1/2	11.7	18.0	24.5	31.5	38.7	46.2	54.0	62.1	70.6
5	12.9	19.8	27.0	34.5	42.3	50.5	58.9	67.7	76.7
5 1/2	14.1	21.6	29.5	37.6	46.0	54.8	63.8	73.2	82.8
6	15.3	23.5	31.9	40.7	49.7	59.1	68.7	78.7	89.0
7	17.8	27.2	36.8	46.8	57.1	67.7	78.5	89.7	101.0
8	20.3	30.8	41.7	52.9	64.4	76.2	88.4	101.0	114.0
9	22.7	34.5	46.6	59.1	71.8	84.8	98.2	112.0	126.0
10	25.2	38.2	51.5	65.2	79.2	93.4	108.0	123.0	138.0
11	27.6	41.9	56.5	71.3	86.5	102.0	118.0	134.0	150.0
12	30.1	46.6	61.4	77.5	93.9	111.0	128.0	145.0	163.0
13	32.5	49.2	66.3	83.6	101.0	119.0	137.0	156.0	175.0
14	35.0	52.9	71.2	89.7	109.0	128.0	147.0	167.0	187.0
15	....	56.6	76.1	95.9	116.0	136.0	157.0	178.0	199.0
16	....	60.3	81.0	102.0	123.0	145.0	167.0	189.0	212.0
18	....	67.7	90.8	114.0	138.0	162.0	187.0	211.0	236.0
20	....	....	101.0	127.0	153.0	179.0	206.0	233.0	261.0
22	....	....	110.0	139.0	168.0	197.0	226.0	255.0	285.0
24	....	....	120.0	151.0	182.0	214.0	245.0	278.0	310.0
26	....	....	130.0	163.0	197.0	231.0	265.0	299.0	334.0
28	....	....	140.0	175.0	211.0	248.0	284.0	321.0	358.0
30	....	....	149.0	188.0	226.0	265.0	304.0	343.0	383.0

*Approximate weight of each flanged joint = weight of 1 ft. of pipe. Values in table are theoretical, and based on cast iron weighing 450 lb. per cu. ft. To these weights from 5 to 15 per cent. must be added, depending on the size.

Table 2. Standard Weights and Thicknesses of Cast-iron Bell-and-spigot Pipe (Fig. 1)*

(U. S. Cast Iron Pipe & Foundry Co.—Based on Specifications adopted by the American Society for Testing Materials, and by the American Water Works Association)

Nominal inside diam., in.	Approximate laying length, ft.	Class A 100 ft. head 43 lb. pressure			Class B 200 ft. head 86 lb. pressure			Class C 300 ft. head 130 lb. pressure			Class D 400 ft. head 173 lb. pressure			Approximate lb. lead per joint 2 in. thick	Approximate lb. hemp per joint
		Thickness, in.	Outside diam., in.	Wt. per ft., lb.	Thickness, in.	Outside diam., in.	Wt. per ft., lb.	Thickness, in.	Outside diam., in.	Wt. per ft., lb.	Thickness, in.	Outside diam., in.	Wt. per ft., lb.		
3	12	0.38	3.80	13.9	0.42	3.84	16.2	0.45	3.90	17.1	0.48	3.96	18.0	6.0	0.18
4	12	0.42	4.80	20.0	0.45	5.00	21.7	0.48	5.00	23.3	0.52	5.00	25.0	7.5	0.21
6	12	0.44	6.90	30.8	0.48	7.10	33.3	0.51	7.10	35.8	0.55	7.10	38.3	10.3	0.31
8	12	0.46	9.05	42.9	0.51	9.05	47.5	0.56	9.30	52.1	0.60	9.30	55.8	13.3	0.44
10	12	0.50	11.10	57.1	0.57	11.10	63.8	0.62	11.40	70.8	0.68	11.40	76.7	16.0	0.53
12	12	0.54	13.20	72.5	0.62	13.20	82.1	0.68	13.50	91.7	0.75	13.50	100.0	19.0	0.61
14	12	0.57	15.30	89.6	0.66	15.30	103.0	0.74	15.70	117.0	0.82	15.70	129.0	22.0	0.81
16	12	0.60	17.40	108.0	0.70	17.40	125.0	0.80	17.80	144.0	0.89	17.80	158.0	30.0	0.94
18	12	0.64	19.50	129.0	0.75	19.50	150.0	0.87	19.90	175.0	0.96	19.90	192.0	33.8	1.00
20	12	0.67	21.60	150.0	0.80	21.60	175.0	0.92	22.10	208.0	1.03	22.1[illegible]	229.0	37.0	1.25
24	12	0.76	25.80	204.0	0.89	25.80	233.0	1.04	26.30	279.0	1.16	26.30	307.0	44.0	1.50
30	12	0.88	31.70	292.0	1.03	32.00	333.0	1.20	32.40	400.0	1.37	32.70	450.0	54.3	2.06
36	12	0.99	38.00	392.0	1.15	38.30	454.0	1.36	38.70	546.0	1.58	39.20	625.0	64.8	3.00
42	12	1.10	44.20	513.0	1.28	44.50	592.0	1.54	45.10	717.0	1.78	45.60	825.0	75.3	3.62
48	12	1.26	50.50	667.0	1.42	50.80	750.0	1.71	51.40	908.0	1.96	52.00	1050.0	85.5	4.37
54	12	1.35	56.70	800.0	1.55	57.10	933.0	1.90	57.80	1140.0	2.23	58.40	1340.0	97.6	6.25
60	12	1.39	62.80	917.0	1.67	63.40	1100.0	2.00	64.20	1340.0	2.38	64.80	1580.0	108.0	8.25
72	12	1.62	75.30	1280.0	1.95	76.00	1550.0	2.39	76.90	1900.0	....	.....	......	131.3	12.50
84	12	1.72	87.50	1630.0	2.22	88.50	2100.0	....	.....	......	....	.....	......	152.0	15.00

* All weights are approximate and include allowance for bell; proportionate allowance to be made for any variation.

Table 3. Cast-iron Bell-and-spigot Pipe for Fire Lines and Other High-pressure Service

STANDARD WEIGHTS AND THICKNESSES

(U. S. Cast Iron Pipe & Foundry Co.)

Nominal inside diam., in.	Class E 500 ft. head 217 lb. pressure		Class F 600 ft. head 260 lb. pressure		Class G 700 ft. head 304 lb. pressure		Class H 800 ft. head 347 lb. pressure	
	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.
6	0.58	42.5	0.61	44.3	0.65	48.1	0.69	50.5
8	0.66	63.9	0.71	66.8	0.75	72.3	0.80	76.1
10	0.74	86.9	0.80	92.8	0.86	101.4	0.92	107.3
12	0.82	114.6	0.89	122.8	0.97	136.2	1.04	144.4
14	0.90	145.6	0.99	158.8	1.07	175.1	1.16	187.5
16	0.98	180.7	1.08	196.5	1.18	218.0	1.27	233.8
18	1.07	221.8	1.17	239.3	1.28	268.2	1.39	287.8
20	1.15	265.8	1.27	287.3	1.39	321.8	1.51	345.8
24	1.31	359.1	1.45	392.3	1.75	479.8	1.88	510.6
30	1.55	530.9	1.73	588.8	....	....	....	....
36	1.80	738.1	2.02	821.0	....	....	....	....

Laying lengths, 12 ft.; all weights are approximate and include allowance for bell; proportionate allowance to be made for any variation.

Table 3a. Weight of Lead Required for Cast-iron Bell-and-spigot Pipe

(U. S. Cast Iron Pipe & Foundry Co.)

Depth of joint, in.	Diameter of pipe, in.																
	3	4	6	8	10	12	14	16	18	20	24	30	36	42	48	54	60
	Approximate weight of lead per joint, lb.																
2	6	8	10	13	16	19	22	30	34	37	44	54	65	75	86	98	108
2½	7	9	12	16	19	23	26	36	40	44	53	65	77	86	102	117	130
Solid	10	13	18	23	31	37	39	65	72	80	95	118	140	155	202	239	256

lines can be well supported. The bell-and-spigot joint possesses greater flexibility, provides for expansion and contraction, and is therefore especially suitable for water pipe and almost exclusively used for that purpose. Fig. 1 shows the standard form of the joint for ordinary pressures. Other forms rarely used in the United States are the bell-and-plain-spigot and the turned-and-bored joints. Plain-end pipe with couplings for high-pressure gas mains and threaded pipe for corrosive liquids are also manufactured. Cast-iron pipe, fittings, and valves have not been found suitable for superheated steam service. German authorities state that cast iron should not be used for temperatures above 480 deg. fahr. Other authorities specify temperatures as high as 575 deg. fahr.

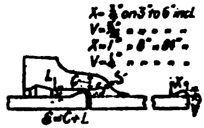


Fig. 1.

The **specifications** adopted by the American Water Works Association, May 12, 1908, are now used generally as the manufacturer's standard throughout the United States. The standards of the New England Water Works Association do not differ materially. Brackett's formula for the thickness of cast-iron pipe is $T = 0.25 + (P + P')r/3300$, in which T is the thickness in inches, P the maximum static pressure in lb. per sq. in. for which the pipe is designed, P' the allowance made for water ram, and r the radius in inches. This gives a large factor of safety to cover inequalities of the castings, strains brought on the pipe from other causes than the water pressure, and to give sufficient thickness to insure the pipe against breakage in shipping and laying. For ordinary water-works conditions for pipes, from 42 to 60 in. diameter, inclusive, 70 lb. per sq. in. is enough for P', but for smaller pipe Brackett allows the following values:

Diam. of pipe, in = 36	30	24	20	16	12	10 to 3
P', lb. per sq. in. = 75	80	85	90	100	110	120

For city work where great damage would be caused by breakage, and for single lines with no reserve where an interruption of the supply would be a very serious matter, the pipes may be made thicker than computed by this formula.

Cast-iron pipe should be made of a soft and tough quality of iron and should be subjected to a test pressure of at least twice the working pressure. A good grade of cast iron shows the following properties: Tensile strength, 20,000 lb. per sq. in.; a test bar 1 × 2 in. in section should support a central load of 2000 lb. when placed flatwise on supports 24 in. apart; if loaded to breaking, should show a deflection of not less than 0.30 in. before fracture. Cast-iron pipe is coated by dipping in hot coal-tar pitch varnish before being laid in the ground.

Flexible Joint Pipe. The necessity for crossing streams and other waterways and of laying pipe lines into them has developed various forms of flexible joint pipe adapted to laying under water, which, when calked with lead, are capable of motion through several degrees without leakage. Figs. 2 and 3 show two styles of such joints which have an adjustment of 10 to 12 deg. in standard sizes. The **lead flexible joint,** Fig. 2, is the one generally used. Fig. 3 shows a more expensive joint intended for the larger sizes of pipe, especially when used for conveying water under considerable pressure.

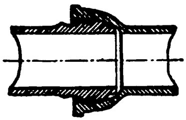


FIG. 2.

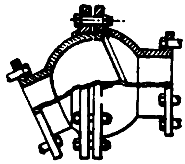


FIG. 3.

Flexible Pipe Joints.

In selecting the thickness of a pipe for a submerged line, the internal service pressure is seldom the determining factor, as ample allowance should be made to minimize the risk of breakage in laying and to withstand external shocks from floating ice or other objects. The thicknesses and weights given in Table 4 accord with good practice.

Table 4. Thicknesses and Weights of Flexible Joint Pipe*

(U. S. Cast Iron Pipe & Foundry Co.)

Nominal diam., in.	Class	Thickness, in.	Weight per 12-ft. length	Lead per joint, lb.	Nominal diam., in.	Class	Thickness, in.	Weight per 12-ft. length	Lead per joint, lb.
6	B†	0.48	500	12	16	D	0.89	2250	76
6	D†	0.55	550	12	18	B	0.75	2300	91
8	B	0.51	670	19	18	D	0.96	2760	91
8	D	0.60	780	19	20	B	0.80	2625	112
10	B	0.57	950	28	20	D	1.03	3220	112
10	D	0.68	1080	28	24	B	0.89	3530	136
12	B	0.62	1210	50	24	D	1.16	4300	136
12	D	0.75	1400	50	30	B	1.03	5100	181
14	B	0.66	1450	64	30	D	1.37	6400	181
14	D	0.82	1750	64	36	B	1.15	7000	225
16	B	0.70	1860	76	36	D	1.58	7900	225

* Joints furnished with screwed, flanged, or bell-and-spigot ends, as specified.
† Class B for 200-ft. head. Class D for 400-ft. head.

"Universal" Pipe, Fig. 4, is cast-iron pipe with hub-and-spigot ends, the contact surfaces of which are machined on a taper, giving an iron-to-iron joint. By making the tapers of slightly different pitch, the joint provides for flexibility while remaining tight. Two bolts to the joint are sufficient, except for pressures above 175 lb. per sq. in. "Universal" cast-iron pipe is largely used for carrying gas and water and is suitable for all pressures

Table 5. Standard Cast-iron Flanged Pipe

(U. S. Cast Iron Pipe & Foundry Co.)

Inside diam. of pipe, in.	Class A Head, 100 ft.; 43 lb. pressure			Class B Head, 200 ft.; 86 lb. pressure			Class C Head, 300 ft.; 130 lb. pressure			Class D Head, 400 ft.; 173 lb. pressure		
	Thickness of pipe, in.	Weights per		Thickness of pipe, in.	Weights per		Thickness of pipe, in.	Weights per		Thickness of pipe, in.	Weights per	
		Foot, lb.	12-ft. length with flange, lb.		Foot, lb.	12-ft. length with flange, lb.		Foot, lb.	12-ft. length with flange, lb.		Foot, lb.	12-ft. length with flange, lb.
3	0.39	13.0	168	0.42	14.6	188	0.45	15.5	199	0.48	16.4	211
4	0.42	18.2	235	0.45	19.6	254	0.48	21.1	272	0.52	23.0	297
6	0.44	27.8	357	0.48	30.5	391	0.51	32.6	417	0.55	35.3	451
8	0.46	38.1	492	0.51	42.5	547	0.56	47.0	603	0.60	50.6	648
10	0.50	51.5	666	0.57	59.1	762	0.62	64.6	831	0.68	71.2	914
12	0.54	66.4	869	0.62	76.7	1,000	0.68	84.5	1,100	0.75	93.7	1,220
14	0.57	81.4	1,060	0.66	94.9	1,230	0.74	107.0	1,390	0.82	119.0	1,540
16	0.60	97.6	1,280	0.70	115.0	1,500	0.80	132.0	1,710	0.89	147.0	1,910
18	0.64	117.0	1,520	0.75	138.0	1,780	0.87	161.0	2,070	0.96	178.0	2,290
20	0.67	136.0	1,770	0.80	163.0	2,120	0.92	189.0	2,440	1.03	212.0	2,730
24	0.76	185.0	2,410	0.89	217.0	2,820	1.04	255.0	3,300	1.16	286.0	3,690
30	0.88	266.0	3,490	1.03	313.0	4,080	1.20	367.0	4,760	1.37	421.0	5,440
36	0.99	359.0	4,740	1.15	419.0	5,500	1.36	498.0	6,500	1.58	582.0	7,560
40	1.06	427.2	5,684	1.23	497.0	6,590	1.48	602.0	7,921	1.72	703.0	9,280
42	1.10	465.0	6,180	1.28	543.0	7,200	1.54	658.0	8,640	1.78	764.0	10,000
48	1.26	608.0	8,120	1.42	688.0	9,130	1.71	833.0	11,000	1.96	960.0	12,600

Table 6. Cast-iron Flanged Pipe for High-pressure Service

(U. S. Cast Iron Pipe & Foundry Co.)

Standard Weights and Thicknesses*

Nominal inside diam., in.	Class E 500 ft. head, 217 lb. pressure		Class F 600 ft. head, 260 lb. pressure		Class G 700 ft. head, 304 lb. pressure		Class H 800 ft. head, 347 lb. pressure	
	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.
6	0.58	37.7	0.61	39.5	0.65	42.9	0.69	45.2
8	0.66	54.7	0.71	60.6	0.75	65.1	0.80	68.8
10	0.74	78.8	0.80	84.7	0.86	92.5	0.92	98.5
12	0.82	104.2	0.89	112.4	0.97	124.6	1.04	132.9
14	0.90	133.1	0.99	146.2	1.07	160.2	1.16	172.6
16	0.98	165.0	1.08	180.8	1.18	199.2	1.27	215.0
18	1.07	202.3	1.17	219.8	1.28	244.6	1.39	264.1
20	1.15	241.1	1.27	262.5	1.39	294.4	1.51	318.3
24	1.31	328.5	1.45	361.6	1.75	446.2	1.88	476.9
30	1.55	484.7	1.73	538.0	....	....	....	....
36	1.80	674.2	2.02	748.7	....	....	....	....

* Weights are approximate and do not include flanges. For each flanged joint add weight of 1 ft. of pipe; laying lengths, 12 ft.

and services. The pipe is tested with hydrostatic pressure of 300 lb. per sq. in. Pipe for gas is also tested with compressed air and soapsuds. All universal pipe and special castings of a given diameter and of any class are interchangeable with those of a different class. Standard laying lengths, 6 ft.

Table 7. Standard Weights and Thicknesses of Universal Cast-iron Pipe

(Central Foundry Co.)

Nominal inside diam., in.	Class No. 100, 100 lb. pressure: Approx. thickness, in.	Estimated weight, lb. per Ft.	Estimated weight, lb. per 6-ft. length	Class No. 130, 130 lb. pressure: Approx. thickness, in.	Estimated weight, lb. per Ft.	Estimated weight, lb. per 6-ft. length	Class No. 175, 175 lb. pressure: Approx. thickness, in.	Estimated weight, lb. per Ft.	Estimated weight, lb. per 6-ft. length	Class No. 250, 250 lb. pressure: Approx. thickness, in.	Estimated weight, lb. per Ft.	Estimated weight, lb. per 6-ft. length
2	......	......	......	......	......	......	0.35	8½	51	0.39	9½	57
3	......	......	......	......	......	......	0.37	13	78	0.42	14½	87
4	0.37	18	108	0.40	18¾	112½	0.43	20¼	121½	0.45	21¼	127½
5	0.40	24	144	0.425	25	150	0.45	26	156	0.49	29	174
6	0.43	30	180	0.45	31	186	0.47	32	192	0.51	35½	213
8	0.47	44¼	265½	0.49	46	276	0.525	49¼	295½	0.58	53¼	319½
10	0.50	60½	363	0.53	63½	381	0.58	67¾	406½	0.64	74	444
12	0.53	75½	453	0.57	80½	483	0.62	87	522	0.70	97½	585
14	0.565	94½	567	0.60	99½	597	0.66	107½	645	0.76	124	744
16	0.60	115½	693	0.65	123	738	0.72	134	804	0.83	156	936
20	0.67	166	996	0.73	178	1068	0.82	196	1176	0.94	223	1338

Inside diam., in....	2	3	4	5	6	8
Bolt sizes, in.......	½ × 3½	½ × 4	⅝ × 5	⅝ × 5½	¾ × 6	⅞ × 6½
Inside diam., in....	10	12	14	16	20	......
Bolt sizes, in.......	1 × 7½	1 × 8	1⅛ × 9	1¼ × 9½	1⅝ × 11	......

Fittings for Cast-iron Water Pipe. Flanged fittings such as those of the American 1914 Standard for steam are not used with cast-iron water pipe. The longer fittings of the American Water Works Association are generally preferred because of lower friction loss. The dimensions of the flanged fittings of this class conform very closely to the dimensions of the bell-and-spigot fittings of the American Water Works Association. The thickness of flanges equals approximately 1¼ times the thickness of the pipe plus ⅛ in., and they are drilled to American 1914 Standard templates. These fittings, both flange and bell-and-spigot type, are made in a great variety of forms known as "Standard Special Castings." For dimensions and weights, see catalog of U. S. Cast Iron Pipe & Foundry Co.

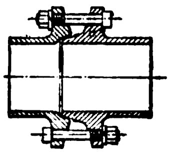


FIG. 4.—"Universal" C. I. Pipe.

WROUGHT-IRON AND STEEL PIPE

Grades. Wrought-iron and steel pipe are ordinarily adaptable for conveying steam, water, gas, oil, etc. The term wrought-iron pipe is commonly used indiscriminately to designate all butt- or lap-welded wrought-iron or steel pipe, with no special reference to either. As steel pipe has largely superseded wrought-iron pipe and is usually supplied unless otherwise specified, it must always be stated whether wrought iron or steel is required when either would not be acceptable.

Full-weight (or Standard) pipe (Table 9) is pipe of about card weight and can be obtained in random lengths of 18 to 20 ft. **Extra strong pipe** (Table 10) is heavy pipe used for high steam, gas, and hydraulic pressures when full-weight pipe will not stand the strain.

Table 8. Cast-iron Soil Pipe and Fittings

(Central Foundry Co.—In accordance with specifications prepared by the American Society of Inspectors of Plumbing and Sanitary Engineers and the National Committee of the Confederated Supply Associations.)

APPROXIMATE WEIGHTS IN LB.†

Fittings	Sizes of fittings, inches														
	2	3	4	5	6	3×2	4×2	4×3	5×2	5×3	5×4	6×2	6×3	6×4	6×5
Pipe, per ft.	5½	9½	13	17	20	....	....	....	....	....	....	....	....	....	....
¼ bends, regular	7¼	10¾	15	19	23½	....	....	....	....	....	....	....	....	....	....
¼ bends, short sweep	8½	13	17¾	22½	27½	....	....	....	....	....	....	....	....	....	....
¼ bends, long sweep	10¾	16	22	27½	33½	....	....	....	....	....	....	....	....	....	....
⅕ bends	6¾	10	13¾	17½	21½	....	....	....	....	....	....	....	....	....	....
⅙ bends	6½	9½	13	16½	20	....	....	....	....	....	....	....	....	....	....
⅛ bends	6	9	12¼	15¼	18¼	....	....	....	....	....	....	....	....	....	....
1/16 bends	5½	8	10¾	13¼	15¾	....	....	....	....	....	....	....	....	....	....
Return bends	9¼	14¼	20¼	26½	33½	....	....	....	....	....	....	....	....	....	....
Tees	10¼	15¼	21	26½	32½	13¼	16½	18¾	19¼	22	24½	22½	25	27½	30
Tapped tees*	8¾	11¾	15	17¾	20½	....	....	....	....	....	....	....	....	....	....
Sanitary tees	11	16¼	22½	28	34½	14	17¼	19¾	20	23	25½	23	26	29	31½
Tapped sanitary tees*	9	12	15¼	18	21¼	....	....	....	....	....	....	....	....	....	....
Y-branch	11	17	24	31½	39½	14	17¼	20½	20	23½	27½	23	27	31	35
½ Y-branch	10¼	15½	21½	27½	34	13½	16½	19	19½	22	25	22½	25½	28½	31
Tapped inverted Y-branch*	10¼	13¾	17½	21	24	....	....	....	....	....	....	....	....	....	....
Inverted Y-branch	11½	18	25½	33	41½	15	18¾	22	22	25½	29½	25½	29½	33	37
Combination Y and ⅛ bend	12	18¾	27	35	44½	15	18¼	22½	21	25½	30½	24	29	34	39
Upright Y-branch	12½	19½	28	36½	46	15¾	18¾	23	21½	26½	31½	24½	29½	35	40
Vent branch	11½	17¾	25	32½	41	14¾	17¾	21½	21	24½	28½	23½	27½	31½	36
Tapped vent branch*	11¼	14¼	17½	20¼	23	....	....	....	....	....	....	....	....	....	....
Double hubs	5¼	8¼	10¾	12¾	14¾	....	....	....	....	....	....	....	....	....	....
Single hubs	4¾	6¼	9¼	10¾	12½	....	....	....	....	....	....	....	....	....	....
Sleeves	6¼	8½	11¼	13¼	15¼	....	....	....	....	....	....	....	....	....	....
Reducers	....	....	....	....	....	6¼	7	8½	8	9¼	10¾	8¾	10¼	11¾	12¾
Increasers	....	....	....	....	....	8¾	10½	12	12	13½	14¾	13½	14¾	16¼	17½
Tapped increasers*	7½					9¼	11	....	12½	....	....	14	....	....	....

* Tapped up to 2 in.

† Weights of pipe include the hub. Laying lengths of pipe are 5 ft. From the data given for staple fittings, weights of other fittings may be calculated. For other data see "Specifications for Cast-iron Soil Pipe and Fittings," by Central Foundry Co.

Random lengths, from 12 to 20 ft. **Double extra strong pipe** (Table 11) has about twice the strength of extra strong pipe, and is suitable for very high hydraulic pressures. Random lengths, from 12 to 20 ft. **Outside diameter pipe** (Table 12): All regular sizes above 12 in., regardless of the thickness, are designated by outside diam. and thickness. Random lengths, 15 to 20 ft., depending on size. **Double-length pipe,** as made by the National Tube Co., has random lengths from 32 to 40 ft. for all kinds of pipe. The American Spiral Pipe Works furnish **large lap-welded steel pipe** in any inside or outside diameter, integral or fractional, in thicknesses from ⅛ to 1¼ in. and in lengths up to 40 ft. **Special hydraulic pipe,** for service on lines requiring the highest possible grade of material and workmanship, are bored from solid forgings and are made to order for pressures up to 10,000 lb. per sq. in.

For **physical properties** of wrought iron and steel for pipes and tubes, see p. 804.

Mill Tests on Pipe. There are no established standards for mill tests on wrought-iron and steel pipe, the various manufacturers having their own schedules of tests, of which the following (given by the National Tube Co.) is representative:

Wrought pipe	Sizes, in.	Testing pressure, lb. per sq. in.
Standard butt-welded	⅛ to 3	700 to 800
Standard lap-welded	1¼ to 15	600 to 1000
Extra strong butt-welded	⅛ to 3	700 to 1500
Extra strong lap-welded	1¼ to 15	1000 to 2500
Double extra strong butt-welded	½ to 2½	700 to 2200
Double extra strong lap-welded	1¼ to 8	2000 to 3000

Mill tests of this kind are made to ascertain possible defects in the welds or other portions of the pipe, and not the allowable working pressures.

The average ultimate strength of pipe steel is 57,000 lb. per sq. in. The relative strengths of steel pipes and tubes based on numerous tests are as follows: Butt-welded steel pipe, 73 per cent.; lap-welded steel pipe, 92 per cent.; and for seamless steel tubes approximately 100 per cent. Butt-welded wrought-iron pipe is 70 per cent. as strong as similar butt-welded steel pipe; lap-welded wrought-iron pipe is 60 per cent. as strong as similar lap-welded steel pipe. Tests on commercial tubes and pipes give the following average bursting pressures in lb. per sq. in.: Butt-welded steel, 41,000; lap-welded steel, 52,000; butt-welded wrought-iron, 29,000; lap-welded wrought-iron, 31,000; seamless steel, 62,000. The factor of safety for steam piping should not be less than six.

Pipe Threads. See p. 663.

Table 9. Standard Full-weight Wrought-iron and Steel Pipe*
(National Tube Co.)

Diam.			Nominal thickness, in.	Circumference		Transverse areas			Length of pipe per sq. ft. of		Length of pipe containing 1 cu. ft., ft.	Nominal weight per ft., lb.	No. of threads per in. of screw
Nominal internal, in.	Actual external, in.	Approx. internal diam., in.		External, in.	Internal, in.	External, sq. in.	Internal, sq. in.	Metal, sq. in.	External surface, ft.	Internal surface, ft.			
⅛	0.405	0.27	0.068	1.27	0.85	0.13	0.06	0.07	9.44	14.15	2513.00	0.24	27
¼	0.540	0.36	0.088	1.70	1.14	0.23	0.10	0.12	7.08	10.49	1383.30	0.42	18
⅜	0.675	0.49	0.091	2.12	1.55	0.36	0.19	0.17	5.66	7.76	751.20	0.57	18
½	0.840	0.62	0.109	2.63	1.95	0.55	0.30	0.25	4.55	6.15	472.40	0.85	14
¾	1.050	0.82	0.113	3.30	2.59	0.87	0.53	0.33	3.64	4.64	270.00	1.13	14
1	1.315	1.05	0.134	4.13	3.29	1.36	0.86	0.50	2.90	3.65	166.90	1.68	11½
1¼	1.660	1.38	0.140	5.22	4.34	2.16	1.50	0.67	2.30	2.77	96.25	2.27	11½
1½	1.900	1.61	0.145	5.97	5.06	2.84	2.04	0.80	2.01	2.37	70.66	2.72	11½
2	2.375	2.07	0.154	7.46	6.49	4.43	3.36	1.07	1.61	1.85	42.91	3.65	11½
2½	2.875	2.47	0.204	9.03	7.75	6.49	4.78	1.71	1.33	1.55	30.10	5.79	8
3	3.500	3.07	0.217	11.00	9.63	9.62	7.39	2.24	1.09	1.25	19.50	7.57	8
3½	4.000	3.55	0.226	12.57	11.15	12.57	9.89	2.68	0.96	1.08	14.57	9.11	8
4	4.500	4.03	0.237	14.14	12.65	15.90	12.73	3.18	0.85	0.95	11.31	10.79	8
4½	5.000	4.51	0.246	15.71	14.16	19.64	15.96	3.68	0.76	0.85	9.02	12.54	8
5	5.563	5.05	0.259	17.48	15.85	24.31	19.99	4.32	0.69	0.76	7.20	14.62	8
6	6.625	6.07	0.280	20.81	19.05	34.47	28.89	5.59	0.58	0.63	4.98	18.97	8
7	7.625	7.02	0.301	23.96	22.06	45.66	38.74	6.92	0.50	0.54	3.72	23.54	8
8	8.625	8.07	0.276	27.10	25.35	58.43	51.15	7.28	0.44	0.47	2.82	24.69	8
8	8.625	7.98	0.322	27.10	25.07	58.43	50.02	8.41	0.44	0.48	2.88	28.55	8
9	9.625	8.94	0.344	30.24	28.08	72.76	62.72	10.04	0.40	0.43	2.29	33.91	8
10	10.750	10.19	0.278	33.77	32.01	90.76	81.55	9.21	0.36	0.37	1.76	31.20	8
10	10.750	10.14	0.306	33.77	31.86	90.76	80.75	10.01	0.36	0.38	1.78	34.24	8
10	10.750	10.02	0.366	33.77	31.47	90.76	78.82	11.94	0.36	0.38	1.82	40.48	8
11	11.750	11.00	0.375	36.91	34.56	108.43	95.03	13.40	0.33	0.35	1.51	45.56	8
12	12.750	12.09	0.328	40.06	37.98	127.68	114.80	12.88	0.30	0.32	1.25	43.77	8
12	12.750	12.00	0.375	40.06	37.70	127.68	113.10	14.59	0.30	0.32	1.27	49.56	8
13	14.000	13.250	0.375	43.96	41.60	153.86	137.81	16.05	0.27	0.29	1.04	54.57	8
14	15.000	14.250	0.375	47.10	44.70	176.62	159.39	17.23	0.25	0.27	0.90	58.57	8
15	16.000	15.250	0.375	54.24	47.90	200.96	182.55	18.41	0.24	0.25	0.75	62.58	8

* Black steel pipe in random lengths with threads and couplings shipped unless otherwise specified.

Table 10. Extra Strong Wrought-iron and Steel Pipe*

(National Tube Co.)

Diam.			Nominal thickness, in.	Circumference		Transverse Areas			Length of pipe per sq. ft. of		Nominal weight per ft., lb.
Nominal internal, in.	Actual external, in.	Approx. internal diam., in.		External, in.	Internal, in.	External, sq. in.	Internal, sq. in	Metal, sq. in.	External surface, ft.	Internal surface, ft.	
1/8	0.405	0.21	0.100	1.27	0.64	0.13	0.03	0.10	9.43	18.63	0.31
1/4	0.540	0.29	0.123	1.70	0.92	0.23	0.07	0.16	7.08	12.99	0.54
3/8	0.675	0.42	0.127	2.12	1.32	0.36	0.14	0.22	5.66	9.07	0.74
1/2	0.840	0.54	0.149	2.64	1.70	0.55	0.23	0.32	4.55	7.05	1.09
3/4	1.050	0.74	0.157	3.30	2.31	0.87	0.43	0.44	3.64	5.11	1.47
1	1.315	0.95	0.182	4.13	2.99	1.36	0.71	0.65	2.90	4.02	2.17
1 1/4	1.660	1.27	0.194	5.22	4.00	2.16	1.27	0.89	2.30	3.00	2.99
1 1/2	1.900	1.49	0.203	5.97	4.69	2.84	1.75	1.08	2.01	2.56	3.63
2	2.375	1.93	0.221	7.46	6.07	4.43	2.94	1.50	1.61	1.98	5.02
2 1/2	2.875	2.32	0.280	9.03	7.27	6.49	4.21	2.28	1.33	1.65	7.66
3	3.500	2.89	0.304	11.00	9.09	9.62	6.57	3.05	1.09	1.33	10.25
3 1/2	4.000	3.36	0.321	12.57	10.55	12.57	8.86	3.71	0.96	1.14	12.50
4	4.500	3.82	0.341	14.14	12.00	15.90	11.45	4.46	0.85	1.00	14.98
4 1/2	5.000	4.28	0.360	15.71	13.45	19.64	14.39	5.25	0.76	0.89	17.61
5	5.563	4.81	0.375	17.48	15.12	24.31	18.19	6.11	0.69	0.79	20.78
6	6.625	5.75	0.437	20.81	18.07	34.47	25.98	8.50	0.58	0.66	28.57
7	7.625	6.63	0.500	23.96	20.81	45.66	34.47	11.19	0.50	0.60	38.05
8	8.625	7.63	0.500	27.10	23.96	58.43	45.66	12.76	0.44	0.50	43.34
9	9.625	8.63	0.500	30.24	27.10	72.76	58.43	14.33	0.40	0.44	48.73
10	10.750	9.75	0.500	33.77	30.63	90.76	74.66	16.10	0.36	0.40	54.73
11	11.750	10.75	0.500	36.91	33.77	108.43	90.76	17.67	0.33	0.35	60.07
12	12.750	11.75	0.500	40.06	36.91	127.68	108.43	19.25	0.30	0.33	65.41
13	14.000	13.00	0.500	43.96	40.82	153.86	132.59	21.21	0.27	0.29	72.09
14	15.000	14.00	0.500	47.10	43.96	176.62	153.84	22.78	0.25	0.27	77.43
15	16.000	15.00	0.500	54.24	47.10	200.96	176.61	24.35	0.24	0.25	82.77

Table 11. Double Extra Strong Wrought-iron and Steel Pipe*

(National Tube Co.)

Diam.			Nominal thickness, in.	Circumference		Transverse areas			Length of pipe per sq. ft. of		Nominal weight per ft., lb.
Nominal internal, in.	Actual external, in.	Approx. internal diam., in.		External, in.	Internal, in.	External, sq. in.	Internal, sq. in.	Metal, sq. in.	External surface, ft.	Internal surface, ft.	
1/2	0.840	0.25	0.294	2.64	0.77	0.55	0.05	0.51	4.55	15.67	1.70
3/4	1.050	0.43	0.308	3.30	1.33	0.87	0.14	0.73	3.64	9.05	2.44
1	1.315	0.60	0.358	4.13	1.84	1.36	0.27	1.09	2.90	6.51	3.65
1 1/4	1.660	0.89	0.382	5.22	2.78	2.16	0.62	1.55	2.30	4.32	5.20
1 1/2	1.900	1.10	0.400	5.97	3.42	2.84	0.93	1.91	2.01	3.51	6.40
2	2.375	1.50	0.436	7.46	4.68	4.43	1.74	2.69	1.61	2.56	9.02
2 1/2	2.875	1.77	0.552	9.03	5.51	6.49	2.42	4.07	1.33	2.18	13.70
3	3.500	2.30	0.600	11.00	7.18	9.62	4.10	5.52	1.09	1.67	18.60
3 1/2	4.000	2.73	0.636	12.57	8.53	12.57	5.79	6.77	0.96	1.41	22.80
4	4.500	3.15	0.674	14.14	9.85	15.90	7.72	8.18	0.85	1.22	27.50
4 1/2	5.000	3.58	0.710	15.71	11.20	19.64	9.98	9.66	0.76	1.07	32.50
5	5.563	4.06	0.750	17.48	12.76	24.31	12.97	11.34	0.69	0.94	38.10
6	6.625	4.89	0.864	20.81	15.32	34.47	18.67	15.81	0.58	0.78	53.10
7	7.625	5.87	0.875	23.96	18.46	45.66	27.11	18.56	0.50	0.65	63.08
8	8.625	6.87	0.875	27.10	21.60	58.43	37.12	21.31	0.44	0.55	72.42

* Black steel pipe with plain ends and without couplings shipped unless otherwise ordered.

Table 12. Outside Diameter (O. D.) Steel Pipe

(National Tube Co.—Nominal weight in lb. per running ft.)

Outside diam., in.	Thickness in inches											
	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1	1 1/8
14	36.8	45.7	54.6	63.4	72.2	80.8	89.4	97.8	106	123	139	155
15	39.4	49.1	58.6	68.1	77.5	86.8	96.0	105.0	114	132	150	167
16	42.1	52.4	62.6	72.8	82.9	92.8	103.0	113.0	122	141	160	178
17	44.8	55.7	66.6	77.5	88.2	98.8	109.0	120.0	130	151	171	191
18	47.4	59.1	70.7	82.1	93.5	105.0	116.0	127.0	138	160	182	203
20	......	65.8	78.7	91.5	104.0	117.0	129.0	142.0	154	179	203	227
21	......	69.1	82.7	96.2	110.0	123.0	136.0	149.0	162	......	......	...
22	......	72.4	86.7	101.0	115.0	129.0	143.0	157.0	170	......	......	......
24	......	......	94.7	110 0	126.0	141.0	156.0	171.0	186	......	......	......
26	......	......	103.0	120.0	136.0	153.0	170.0	186.0	202	......	......	......
28	......	......	....	129.0	147.0	165.0	183 0	201.0	218	......	......	......
30	......	......	....	138.0	158.0	177.0	196.0	215.0	234	......	......	......

Large Lap-welded Steel Pipe can be furnished in any inside or outside diam., integral or fractional, and in single lengths up to 40 ft. It can also be made in any thickness from 1/8 to 1 1/4 in. Forged-steel flanges and other connections are furnished in all sizes. Weights and bursting pressures are given in Table 13.

Table 13. Weights and Bursting Pressures of Large Lap-welded Steel Pipe

(American Spiral Pipe Co.)

Inside diam., in.	Thickness, in.	Weight per ft., lb.	Approx. bursting press., lb. per sq. in.	Inside diam., in.	Thickness, in.	Weight per ft., lb.	Approx. bursting press., lb. per sq. in.
12	3/16	25.8	1562	36	1/4	102	694
14	3/16	30.0	1339	40	5/16	142	781
16	3/16	34.2	1172	42	3/8	179	892
18	3/16	38.4	1041	48	3/8	204	781
20	3/16	42.6	937	54	1/2	306	926
22	3/16	46.8	852	60	5/8	426	1040
24	3/16	51.0	781	66	3/4	563	1132
30	3/16	64.0	625	72	1	822	1388

Table 14. Capacity of Pipes and Cylindrical Tanks of Various Diameters in Gallons per Foot of Length

(For capacities of standard-size pipe, see p. 797. For capacities in cu. ft., use table of areas, p. 33)

Feet	Inches											
	0	1	2	3	4	5	6	7	8	9	10	11
0	......	.0408	.1632	.3672	.6528	1.020	1.469	1.999	2.611	3.305	4.080	4.937
1	5.875	6.895	8.00	9.18	10.44	11.79	13.22	14.73	16.32	17.99	19.75	21.58
2	23.50	25.50	27.58	29.74	31.99	34.31	36.72	39.21	41.78	44.43	47.16	49.98
3	52.88	55.86	58.92	62.06	65.28	68.58	71.97	75.44	78.99	82.62	86.33	90.13
4	94.00	97.96	102.0	106.1	110.3	114.6	119.0	123.4	128.0	132.6	137.3	142.0
5	146.9	151.8	156.8	161.9	167.1	172.4	177.7	183.2	188.7	194.3	199.9	205.7
6	211.5	217.6	223.4	229.5	235.7	242.0	248.2	254.7	261.1	267.7	274.3	281.1

Table 14. Capacity of Pipes and Cylindrical Tanks of Various Diameters in Gallons per Foot of Length—(*continued*)

Feet	Inches 0	3	6	9	Feet	Inches 0	3	6	9	Feet	Inches 0	3	6	9
7	287.9	308.8	330.5	352.9	16	1504	1551	1600	1648	25	3672	3746	3820	3896
8	376.0	399.9	424.5	449.8	17	1698	1748	1799	1851	26	3972	4048	4126	4204
9	475.9	502.7	530.2	558.5	18	1904	1957	2011	2066	27	4283	4363	4443	4524
10	587.5	617.7	647.7	679.0	19	2121	2177	2234	2292	28	4606	4689	4772	4856
11	710.9	743.6	777.0	811.1	20	2350	2409	2469	2530	29	4941	5027	5113	5200
12	846.0	881.7	918.0	955.1	21	2591	2653	2716	2779	30	5288	5376	5465	5555
13	992.9	1032	1071	1111	22	2844	2909	2974	3041	31	5646	5738	5830	5923
14	1152	1193	1235	1278	23	3108	3176	3245	3314	32	6016	6111	6206	6302
15	1322	1366	1412	1457	24	3384	3455	3527	3599	33	6398	6496	6594	6692

Pipe Bends of any size and shape can be obtained to suit almost any condition arising in practice. Open-hearth steel is used in sizes over 6 in., while Bessemer is employed under 6 in. The main weakness with open-hearth

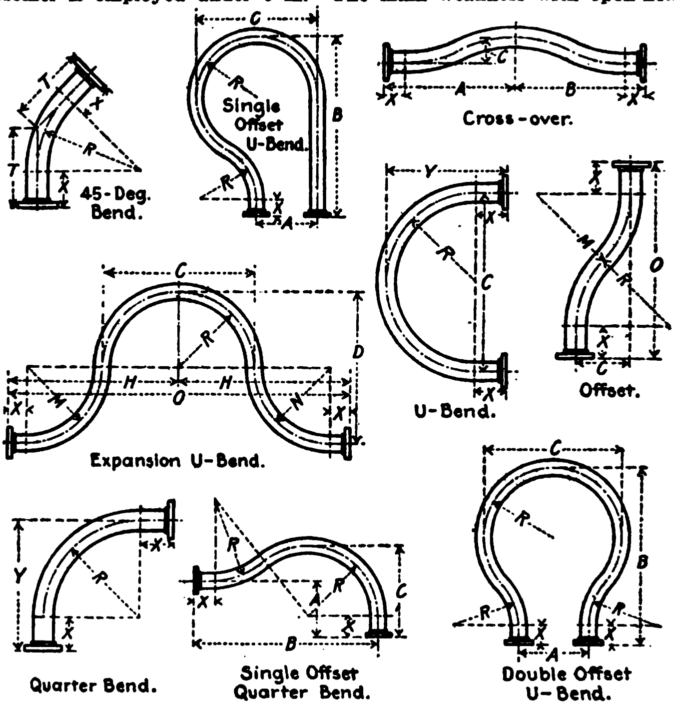


Fig. 5.—Pipe Bends.

is that the weld is not so secure. Open-hearth steel is not used on smaller-sized pipe mainly on account of difficulty in threading. Various shapes of

Table 15. Safe Expansion Values of 90-deg. or Quarters Wrought-steel Pipe Bends, in Inches

(Crane Co.)

(The values are for bends placed in line without springing. If bends are sprung to the amount of the safe expansion value, in a direction opposite to the expansion, the amount of expansion that can be taken care of will be double that shown in the table.)

Pipe diam., in.	Mean radius of bend, inches												
	12	15	20	30	40	50	60	70	80	90	100	110	120
1	1/4	3/8	3/4	1 3/4	3 1/8	......	......	......	......	......	......	......	......
2	1/8	3/4	1/2	1	1 3/4	2 3/4	3 7/8	5 3/8	......	......	......	......	......
2 1/2	....	1/4	3/8	7/8	1 1/2	2 1/4	3 1/4	4 1/2	5 3/4	......	......	......	......
3	....	1/8	3/8	5/8	1 1/8	1 7/8	2 5/8	3 5/8	4 3/4	6	......	......	......
3 1/2	....	....	1/4	5/8	1	1 5/8	2 3/8	3 1/8	4 1/8	5 1/4	......	......	......
4	....	....	1/4	1/2	1	1 1/2	2	2 7/8	3 3/4	4 3/4	5 3/4	......	......
4 1/2	....	....	....	1/2	7/8	1 3/8	1 7/8	2 1/2	3 3/8	4 1/4	5 1/4	......	......
5	....	....	....	3/8	3/4	1 1/8	1 5/8	2 1/4	3	3 3/4	4 5/8	5 5/8	......
6	....	....	....	3/8	5/8	1	1 3/8	1 7/8	2 1/2	3 1/8	3 7/8	4 3/4	5 5/8
8	....	....	....	......	1/2	3/4	1	1 1/2	1 7/8	2 1/2	3	3 5/8	4 3/8
10	....	....	....	......	......	5/8	7/8	1 1/8	1 1/2	2	2 3/8	2 7/8	3 1/2
12	....	....	....	......	......	......	3/4	1	1 3/8	1 5/8	2	2 1/2	2 7/8
14	....	....	....	......	......	......	......	7/8	1 1/8	1 3/8	1 3/4	2 1/8	2 1/2
15	....	....	....	......	......	......	......	......	1	1 3/8	1 5/8	2	2 3/8
16	....	....	....	......	......	......	......	......	7/8	1 1/4	1 1/2	1 7/8	2 1/4
18	....	....	....	......	......	......	......	......	......	......	......	1 5/8	1 7/8
20	....	....	....	......	......	......	......	......	......	......	......	......	1 3/4

Table 16. Thickness of Pipe for Various Bends

Pipe diam., in. (*D*)	Working pressures					
	Up to 125 lb.		125 to 250 lb.		250 to 350 lb.	
	Radius of bend	Thickness of pipe	Radius of bend	Thickness of pipe	Radius of bend	Thickness of pipe
7 and smaller	4*D*–5*D*	Extra strong 1/2 in.	4*D* to 5 and 6*D*	Extra strong 1/2 in.	4*D* and over	Extra strong 1/2 in.
8 and larger.						
7 and smaller	over 5*D*	Full weight	over 6*D*	Full weight	....	......
8..........		28.55 lb. per ft.		28.55 lb. per ft.	....	......
10..........		40.48 lb. per ft.		40.48 lb. per ft.	....	......
12..........		49.56 lb. per ft.		49.56 lb. per ft.	....	......
14–16 incl...		5/16 in.		3/8 in.	....	......
18–22 incl...		3/8 in.		7/16 in.	....	......
24–30 incl...		7/16 in.		1/2 in.	....	......

pipe bends are shown in Fig. 5, the dimensions of which are given in Table 17. Their uses may be broadly classed as follows: (1) For steam lines to provide flexibility and compensate for expansion and contraction; (2) to reduce the number of joints in a pipe line; (3) to avoid obstructions such as columns, pipe foundations, which otherwise would require several pieces of pipe and fittings; (4) to reduce friction in piping; (5) for coils in heaters, refrigerating systems, etc.

Crane Co. has recently made extensive tests with various types of bends in several sizes and weights of pipe to determine their expansion values (*i.e.*, the maximum ex-

Table 17. Pipe Bends Made from Lap-welded Steel Pipe

(Crane Co.—Letters at heads of columns 2–5 refer to Fig. 5)

Size of pipe, in.	R-M-N Advisable radius of bends, in.	T Center to end or face of flanges, ft.	in.	X Length of tangents or straight pipe on each bend, in.	Y Center of bends to face of flanges or ends of pipe, ft.	in.	Lin. ft. of pipe in each quarter bend, ft.	in.	Lin. ft. of pipe in each U-bend, ft.	in.	Lin. ft. of pipe in each 45-deg. bend, ft.	in.	Minimum radius to which bends can be made from extra strong pipe only, in.
2½	12½	0	9 3/16	4	1	4½	2	3¾	3	11¼	1	5⅞	7
3	15	0	10¼	4	1	7	2	7¾	4	7⅛	1	7⅞	8
3½	17½	1	0¼	5	1	10½	3	1½	5	5	1	11¾	10
4	20	1	1¼	5	2	1	3	5½	6	1	2	1¾	12
4½	22½	1	3 5/16	6	2	4½	3	11½	6	10¾	2	5¾	14
5	25	1	4⅜	6	2	7	4	3¼	7	6⅝	2	7½	15
6	30	1	7 7/16	7	3	1	5	1⅛	9	0¼	3	1½	20
7	35	1	10½	8	3	7	5	11	10	6	3	7⅜	24
8	40	2	1 9/16	9	4	1	6	9	11	11¾	4	1½	28
9	45	2	5⅝	11	4	8	7	8¾	13	7⅜	4	9¼	35
10	50	2	8¾	12	5	2	8	6½	15	1⅛	5	3¼	40
12	60	3	2⅞	14	6	2	10	0¼	18	0¼	6	3¼	50
14	70	3	9	16	7	2	11	10	21	0	7	3	65
15	75	3	11 1/16	16	7	7	12	6	....	....	7	7	70
16	80	4	3⅛	18	8	2	13	5¾	....	....	8	2¾	78
18	108	5	2¾	18	10	6	17	1¾	....	....	10	0⅞	88
20	120	5	7¾	18	11	6	18	8½	....	....	10	10¼	104
22	132	6	0⅝	18	12	6	20	3	....	....	11	7¾	132
24	144	6	5⅝	18	13	6	21	10	....	....	12	0⅝	144

Drawings submitted should include dimensions *A*, *B*, *C*, *D*, *H* and *O* (Fig. 5) where necessary, and any other variations from dimensions as given in the above table.

Lin. ft. of pipe used in bends will vary according to dimensions differing from those in the above table.

pansion which they may be expected to take up) for steam-pipe lines. (For complete data, see *The Valve World*, Oct., 1915.) From these tests the following facts were established:

(1) Extra strong pipe bends have practically the same expansion value as the corresponding shape full-weight pipe bends.

(2) The tangents ordinarily furnished on pipe bends do not add materially to the expansion value.

(3) Bends made to a shorter radius than 5 or 6 diameters of the pipe have practically no expansion value, as they will buckle when bending.

(4) Bends are superior to the ordinary pipe-and-fitting structure because they are more flexible and do not throw all the load on the joints. Comparative tests showed that the built-up structure invariably leaked at the flanged joints, and in many cases before the fitting had been stressed near to the maximum allowable fiber stress.

(5) A U-bend has twice the expansion value of a 90-deg. or quarter bend of the same size and radius, and an expansion U-bend four times the expansion value of a quarter bend or twice that of a U-bend. A double offset expansion U-bend has two- and one-half times the expansion value of a U-bend, and one and one-fourth times that of an expansion U-bend. Safe expansion values of these bends for various radii and sizes of pipe were determined in these tests and are given in Table 15.

(6) For a given size, radius of bend and working pressure, the thicknesses of pipe for various bends as recommended by Crane Co. are shown in Table 16.

Coils and Bends of every description are made from iron, steel (preferably open-hearth), brass and copper pipe and tubing. Iron pipes, however, cannot be bent to a radius smaller than 12 in. (See catalogs of Whitlock Coil Pipe Co., Hartford Conn., and National Pipe Bending Co., New Haven, Conn.) Limiting dimensions are given in Table 14.

Table 18. Dimensions of Pipe Bends and Coils

(National Pipe Bending Co.)

		Cold bent											Hot bent						
Nominal pipe size, in.		⅛	¼	⅜	½	¾	1	1¼	1½	2	2½	3	3½	4	4½	5	6	7	8
Least ordinary	R	1¼	1½	1¾	2	2½	3	4	6	8	10	18	20	22	24	27	30	36	42
	D	2½	3	3½	4	5	6	8	12	16	20	36	40	44	48	54	60	72	84
	D_1	5	6	7	8	10	12	15	18	20	24	42	44	49	54	60	72	84	96
Difficult	R	⅞	1	1¼	1½	2	2½	3	4	6	8	15	14	16	18	20	25	30	36
	D	1¾	2	2½	3	4	5	6	8	12	16	30	28	32	36	40	50	60	72
	D_1	3	4	5	6	8	10	12	14	16	20	36	32	36	42	48	60	72	84
Approx. limit*	R	¾	⅞	1	1¼	1½	1¾	2¼	3	4	5	6	10	12	14	16	20	25	30
	D	1½	1¾	2	2½	3	3½	4½	6	8	10	12	20	24	28	32	40	50	60
	D_1	2	2½	3	4	6	7	8	10	12	14	16	24	30	36	42	48	60	72

R = Center radius of 90-deg. bends. D = Center diam. of U-bends. D_1 = Outside diam. of coils. Ends on bends should be straight for a length equal to the diam. of the pipe, in order to permit perfect joints at the threads.

* Varying with circumstances.

Table 19. Standard Dimensions of Lap-welded Steel or Charcoal-iron Boiler Tubes

(National Tube Co.)

External diam., in.	Internal diam., in.	Nominal thickness, in.	Nearest Birm. wire gage No.	Circumference: External, in.	Circumference: Internal, in.	Transverse areas: External, sq. in.	Transverse areas: Internal, sq. in.	Transverse areas: Metal, sq. in.	Length of tube per sq. ft. of: External surface, ft.	Length of tube per sq. ft. of: Internal surface, ft.	Nominal weight per ft., lb.
1¾	1.560	0.095	13	5.498	4.901	2.405	1.911	0.494	2.183	2.448	1.68
2	1.810	0.095	13	6.283	5.686	3.142	2.573	0.569	1.909	2.110	1.93
2¼	2.060	0.095	13	7.069	6.472	3.976	3.333	0.643	1.698	1.854	2.19
2½	2.282	0.109	12	7.854	7.169	4.909	4.090	0.819	1.528	1.674	2.78
2¾	2.532	0.109	12	8.639	7.954	5.940	5.035	0.905	1.389	1.509	3.07
3	2.782	0.109	12	9.425	8.740	7.069	6.079	0.990	1.273	1.373	3.37
3¼	3.010	0.120	11	10.210	9.456	8.296	7.116	1.180	1.175	1.269	4.01
3½	3.260	0.120	11	10.996	10.242	9.621	8.347	1.274	1.091	1.172	4.33
3¾	3.510	0.120	11	11.781	11.027	11.045	9.676	1.369	1.018	1.088	4.65
4	3.732	0.134	10	12.566	11.724	12.566	10.939	1.627	0.955	1.024	5.53
4½	4.232	0.134	10	14.137	13.295	15.904	14.066	1.838	0.849	0.902	6.25
5	4.704	0.148	9	15.708	14.778	19.635	17.379	2.256	0.764	0.812	7.67
6	5.670	0.165	8	18.850	17.813	28.274	25.249	3.025	0.637	0.673	10.28
7	6.670	0.165	8	21.991	20.954	38.485	34.942	3.543	0.546	0.573	12.04
8	7.670	0.165	8	25.133	24.096	50.266	46.204	4.062	0.477	0.498	13.81
9	8.640	0.180	7	28.274	27.143	63.617	58.629	4.988	0.424	0.442	16.96
10	9.594	0.203	6	31.416	30.140	78.540	72.292	6.248	0.382	0.398	21.24
11	10.560	0.220	5	34.558	33.175	95.033	87.583	7.451	0.347	0.362	25.33
12	11.542	0.229	4½	37.699	36.260	113.098	104.629	8.469	0.319	0.330	28.79
13	12.524	0.238	4	40.841	39.345	132.733	123.190	9.543	0.294	0.305	32.44
14	13.504	0.248	3½	43.982	42.424	153.938	143.224	10.714	0.273	0.283	36.42
15	14.482	0.259	3	47.100	45.515	177.000	175.000	12.000	0.254	0.263	40.77
16	15.468	0.270	2½	50.265	48.569	201.060	187.710	13.350	0.239	0.247	45.36

NOTE.—In estimating effective steam-heating or evaporating surface of tubes, the surface in contact with air or gases of combustion, according to manner of application, as whether internal or external, is to be thus taken. For heating liquids by steam, superheating steam, or transferring heat from one liquid or one gas to another, mean surface of tubes to be computed.

Tubes. The most general applications of tubes are for oil, gas and water mains, boiler tubes, poles, conduits, ball-bearing cages and sleeves, bushings, spindles, gun barrels, axles, dies, cylinders, shrapnel cases, etc. Tubing is especially applicable to case-hardened bushings for wearing surfaces. The manufacture of tubes can be divided into two general classifications—welded and seamless. **Welded steel tubes** can be divided into two sub-classes—butt and lap. Both are made from strips of rolled iron or steel, commercially called skelp. Butt-welded tubes are made in sizes from ¼ in. to 3½ in. in diam.; lap-welded from 2 in. to 30 in.

Table 20. Physical Properties of Wrought-iron and Steel for Pipes and Tubes

	Grades	Chemical analysis					Elastic limit (lb. per sq. in.)	Ultimate strength (lb. per sq. in.)	Per cent. elongation	Per cent. contraction	Heat treatment
			S	P	Mn	C			In 2 in. (In 8 in.)		
Welded	Bessemer	Average	0.045	0.100	0.30	0.07	37,000	57,000	....(22)	50	F.h.
	Open hearth		0.035	0.030	0.38	0.10	32,000	52,000	....(25)	55	F.h.
	Puddled iron		0.030	0.200	Tr.	Tr.	27,000	45,000	...(12)	30	F.h.
	0.17 carbon		0.035	0.030	0.50	0.17	40,000	57,000	....(30)	50	F.h.
Seamless	0.17 carbon, cold-drawn open-hearth steel.	Max.	0.040	0.030	0.60	0.19	70,000	80,000	18(7)	30	N.a.
		Min.	0.015	0.010	0.40	0.14	55,000	65,000	12(3)	20	N.a.
		....	....	....	....	....	65,000	75,000	25(16)	45	F.a.
		....	....	....	....	....	50,000	60,000	18(10)	35	F.a.
		....	....	....	....	....	48,000	65,000	60(28)	60	M.a.
		....	....	....	....	....	35,000	52,000	50(22)	50	M.a.
		....	....	....	....	....	35,000	55,000	65(33)	62	S.a.
		....	....	....	....	....	27,000	47,000	55(28)	52	S.a.
	0.35 carbon, cold-drawn open-hearth steel.	Max.	0.040	0.030	0.60	0.40	90,000	100,000	15	18	N.a.
		Min.	0.015	0.010	0.40	0.30	75,000	85,000	10	12	N.a.
		....	....	....	....	....	85,000	95,000	30(18)	32	F.a.
		....	....	....	....	....	70,000	80,000	20(12)	25	F.a.
		....	....	....	....	....	65,000	80,000	45(30)	42	M.a.
		....	....	....	....	....	50,000	65,000	35(20)	35	M.a.
	3½ per cent. nickel, cold-drawn.	Max.	0.040	0.030	0.60	0.30	100,000	110,000	18	32	N.a.
		Min.	0.015	0.010	0.40	0.20	85,000	95,000	10	22	N.a.
		....	(Ni = 4.00 for max.)				90,000	105,000	25	35	F.a.
		....	(Ni = 3.00 for min.)				75,000	85,000	15	25	F.a.
		....	....	....	....	....	60,000	85,000	50(28)	50	M.a.
		....	....	....	....	....	45,000	70,000	40(20)	45	M.a.

F.h. = Finished hot; N.a. = Not annealed; F.a. = Finish annealed; M.a. = Medium annealed; S.a. = Soft annealed.

Seamless Steel Tubing, both hot-drawn and cold-drawn, is extensively used for various mechanical and engineering purposes. Owing to their smooth finish and slight variation in diameter and thickness, cold-drawn tubes can often be used to advantage and with economy in place of articles ordinarily machined from solid stock.

Seamless tubes are made by four different processes: (1) The **punching** of solid billets by hydraulic or power presses, and the subsequent rolling or hot-drawing of the hollow billet. (2) The **plate-cupping process,** which consists in taking a circular sheet or plate and cupping it while hot in a series of operations until it has been formed into a shallow cup of approximately required diam., then elongating it by hot-drawing operations on a solid mandrel through a succession of dies; this process is generally used for tubes of large diam. (3) The **cast-hollow-billet process,** where a cast-steel hollow tubular blank is used as a basis of manufacture, which is elongated by rolling over a plug or mandrel, or by hot-drawing operations. (4) The **piercing process,** which starts with a round, solid billet, which is passed through angularly-disposed rolls giving a high rotating speed and a slow advancing movement to the billet, forcing it over a pointed mandrel, increasing its length and changing it from a solid to a tubular form; this is commonly called the "Mannesmann Process."

Seamless tubes after being cold-drawn are very hard and inclined to be brittle. They are therefore furnished in three different anneals: (*a*) Hard, where great rigidity and stiffness are required and where tubes will not be bent or otherwise deformed; (*b*) medium, where strength and toughness are desired, and only slight changes of form are necessary; (*c*) soft, where a ductile and pliable material is wanted that will stand much manipulation and deformation. Outside and inside diameters vary from

Table 21. Round Seamless Steel Tubing

(National Tube Co.)

SHELBY COLD-DRAWN TUBING, APPROX. WEIGHT IN LB. PER FT.*

Thickness†	Outside diam. in in.													
	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	2 3/4	3	3 1/2	4	4 1/2	5	5 1/2
20	0.17	0.27	0.36	0.45	0.55	......	......	......	......	......	......	......	......	......
18	0.24	0.37	0.50	0.63	0.76	......	......	......	......	......	......	......	......	......
16	0.30	0.47	0.65	0.82	1.00	1.34	1.69	1.86	......	......	......	......	......	......
14	0.37	0.59	0.81	1.03	1.25	1.70	2.14	2.36	......	......	......	......	......	......
13	0.41	0.66	0.92	1.17	1.42	1.93	2.44	2.69	2.95	3.45	......	......	......	......
12	0.45	0.75	1.04	1.33	1.62	2.20	2.78	3.07	3.37	3.94	......	......	......	......
11	0.49	0.81	1.13	1.45	1.77	2.41	3.05	3.37	3.69	4.33	4.97	......	......	......
10	....	0.88	1.24	1.60	1.95	2.67	3.39	3.74	4.10	4.82	5.53	6.25	6.96	7.67
5/32	....	0.99	1.41	1.82	2.24	3.07	3.91	4.32	4.74	5.57	6.41	7.24	8.07	8.91
3/16	....	1.13	1.63	2.13	2.63	3.63	4.63	5.13	5.63	6.63	7.63	8.63	9.63	10.6
7/32	....	....	1.82	2.41	2.99	4.16	5.32	5.91	6.49	7.66	8.82	10.0	11.2	12.3
1/4	....	....	2.00	2.67	3.33	4.67	6.00	6.67	7.33	8.67	10.0	11.3	12.7	14.0
5/16	....	....	....	3.13	3.96	5.63	7.29	8.13	8.96	10.6	12.3	14.0	15.6	17.3
3/8	....	....	....	3.50	4.50	6.50	8.50	9.51	10.5	12.5	14.5	16.5	18.5	20.5
1/2	....	....	....	....	5.33	8.00	10.7	12.0	13.3	16.0	18.7	21.3	24.0	26.7
5/8	....	....	....	....	....	9.17	12.5	14.2	15.8	19.2	22.5	25.8	29.2	32.5
3/4	....	....	....	....	....	......	......	......	18.0	22.0	26.0	30.0	34.0	38.0
7/8	....	....	....	....	....	......	......	......	19.8	24.5	29.2	33.8	38.5	43.2
1	....	....	....	....	....	......	......	......	21.3	26.7	32.0	37.3	42.7	48.0

HOT-DRAWN TUBING, APPROX. WEIGHT IN LB. PER FT.‡

Thickness, in.	Outside diam. in inches										
	6	7	8	9	10	11	12	14	16	18	20
1/4	15.4	18.0	20.7	23.4	26.0	28.7	31.7	36.7	42.1	47.4	52.7
5/16	19.0	22.3	25.7	29.0	32.3	35.7	39.0	45.7	52.4	59.0	65.7
3/8	22.5	26.5	30.5	34.5	38.6	42.6	46.6	54.6	62.6	70.6	78.6
1/2	29.4	34.7	40.1	45.4	50.7	56.1	61.4	72.1	82.8	93.5	104.0
5/8	35.9	42.6	49.2	55.9	62.6	69.3	75.9	89.3	103.0	116.0	129.0
3/4	42.0	50.1	58.1	66.1	74.1	82.1	90.1	106.0	122.0	138.0	154.0
7/8	47.9	57.2	66.6	75.9	85.3	94.6	104.0	123.0	141.0	160.0	179.0
1	53.4	64.1	74.8	85.4	96.1	107.0	117.0	139.0	160.0	182.0	203.0
1 1/8	58.6	70.6	82.6	94.6	107.0	117.0	131.0	155.0	179.0	203.0	227.0
1 1/4	63.4	76.8	90.1	103.0	117.0	130.0	144.0	170.0	197.0	224.0	250.0
1 3/8	67.9	82.6	97.3	112.0	127.0	141.0	156.0	185.0	215.0	244.0	274.0
1 1/2	72.1	88.1	104.0	120.0	136.0	152.0	168.0	200.0	232.0	264.0	296.0

* For weights of intermediate sizes (5/8, 7/8, 1 1/8, 1 3/8, 1 3/4, 2 1/4, 3 1/4, 3 3/4, 4 1/4, 4 3/4 and 5 1/4 in. diam.), take one-half of sum of weights of next larger and smaller tabulated sizes.

† Nos. 20 to 10, B. W. G. numbers; remaining sizes in fractions of an inch.

‡ For weights of intermediate sizes (6 1/2, 7 1/2, 8 1/2, 9 1/2, 10 1/2, 11 1/2, 13, 15, 17 and 19 in. diam.), take one-half of sum of weights of next larger and smaller tabulated sizes.

Square Tubing made in sizes included between heavy horizontal rules. Weight per ft. = tabular weight × 1.27.

0.005 to 0.010 in. from true diam., thicknesses from 5 to 10 per cent. Furnished in random lengths of 5 ft. and over.

Riveted Steel Pipe. Straight riveted pipe and spiral riveted pipe are furnished in lengths up to 40 ft. with pressed- or forged-steel flanges riveted to the ends. The latter are preferable, being heavier and stronger. Such pipe may be galvanized to prevent corrosion and to make the riveted joints tighter after calking. It is used extensively in non-condensing power-plant work for exhaust-steam mains, large drains, etc., but should never be used for live steam, feed water, or similar high-pressure work. **Spiral riveted pipe** (Fig. 6), because of its construction, is **stronger than straight riveted pipe** (Fig. 7) of equal size, thickness, and quality. The single seam in this pipe, being continuous and helical from end to end of pipe, stiffens it throughout its length. Weights and dimensions are given in Tables 22 and 23.

Fig. 6.—Spiral Riveted Steel Pipe.

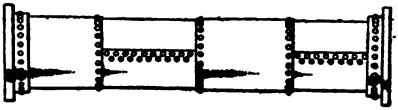


Fig. 7.—Straight Riveted Steel Pipe.

A **factor of safety** of not less than 4 should be used, and then only when the pressure is practically uniform. For pumping mains and similar piping where the pressure is apt to increase suddenly, or where shocks from water hammer are probable, a large factor should be employed; not less than 6 or 8, depending upon the conditions. The **tensile strength of the steel** plate varies from 50,000 to 70,000 lb. per sq. in., and the **efficiency of the riveted joint** in straight riveted pipe varies from 50 to 85 per cent. The friction losses in riveted pipe are greater than in smooth pipe.

Spiral riveted pipe can be furnished in lengths up to 40 ft. if asphalted or 20 ft. if galvanized. Coatings are applied by hot dipping after the pipe is made. Extra heavy and double extra heavy pipe can also be furnished for working pressures up to 400 lb. The flanges on this pipe are forged steel and of special dimensions adopted by riveted pipe manufacturers.

All diameters of pipe can be furnished with forged-steel flanges conforming to the dimensions and drilling of the A. S. M. E. Flanges are usually attached to the pipe at the factory and cannot be put on readily by the user.

Table 22. Standard Weight Spiral Riveted Pressure Pipe

(American Spiral Pipe Works)

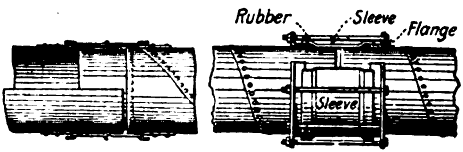
Inside diam., in.	Thickness, U. S. gage	Approx. weight per ft., lb.	Diam. of flanges, in.	Approx. bursting pressure, lb. per sq. in.	Inside diam., in.	Thickness, U. S. gage	Approx. weight per ft., lb.	Diam. of flanges, in.	Approx. bursting pressure, lb. per sq. in.
3	18	2.3	6	2000	15	14	17.0	19	625
4	16	3.7	7	1875	16	14	18.1	21¼	585
5	16	4.5	8	1500	18	14	19.9	23¼	525
6	16	5.3	9	1250	20	14	22.1	25¼	470
7	16	6.2	10	1070	22	12	33.7	28¼	595
8	16	7.1	11	935	24	12	36.5	30	540
9	16	8.0	13	1045	26	12	39.5	32	505
10	16	8.8	14	750	28	10	51.7	34	605
11	16	9.7	15	680	30	10	56.8	36	560
12	16	10.6	16	625	32	10	61.6	38	525
13	16	11.4	17	575	36	10	69.1	42	470
14	14	15.9	18	670	40	10	76.7	46	420

Table 23. Straight-seam Riveted Steel Pipe for Exhaust-steam and Water Pipe Lines

Inside diam. of pipe, in.	Thickness of material, U. S. standard gage	Equivalent thickness, in.	Theoretical safe working head, ft.	Approximate weight per lin. ft., lb.	Inside diam. of pipe, in.	Thickness of material, U. S. standard gage	Equivalent thickness in in.	Theoretical safe working head, ft.	Approximate weight per lin. ft., lb.
16	16	0.062	190	13.0	24	6	0.200	405	59.0
16	14	0.078	237	16.0	26	14	0.078	145	25.5
16	12	0.109	332	22.3	26	12	0.109	203	35.5
16	11	0.125	379	24.5	26	11	0.125	233	39.5
16	10	0.140	425	28.5	26	10	0.140	261	44.3
18	16	0.062	168	14.8	26	8	0.171	319	54.0
18	14	0.078	210	18.5	26	6	0.200	373	64.0
18	12	0.109	295	25.3	28	14	0.078	135	27.3
18	11	0.125	337	29.0	28	12	0.109	188	38.0
18	10	0.140	378	32.5	28	11	0.125	216	42.3
18	8	0.171	460	40.0	28	10	0.140	242	47.5
20	16	0.062	151	16.0	28	8	0.171	295	58.0
20	14	0.078	189	19.8	28	6	0.200	346	69.0
20	12	0.109	265	27.5	30	12	0.109	176	39.5
20	11	0.125	304	31.5	30	11	0.125	202	45.0
20	10	0.140	340	35.0	30	10	0.140	226	50.5
20	8	0.171	415	45.5	30	8	0.171	276	61.8
22	16	0.062	138	17.8	30	6	0.200	323	73.0
22	14	0.078	172	22.0	30	¼	0.250	404	90.0
22	12	0.109	240	30.5	36	11	0.125	168	54.0
22	11	0.125	276	34.5	36	10	0.140	189	60.5
22	10	0.140	309	39.0	36	3/16	0.187	252	81.0
22	8	0.171	376	50.0	36	¼	0.250	337	109.0
24	14	0.078	158	23.8	36	5/16	0.312	420	135.0
24	12	0.109	220	32.0	40	3/16	0.187	226	90.0
24	11	0.125	253	37.5	40	¼	0.250	303	120.0
24	10	0.140	283	42.0	40	5/16	0.312	378	150.0
24	8	0.171	346	50.0	40	3/8	0.375	455	180.0

The safe working heads given above are theoretical and based on ordinary working conditions. Judgment should be used in arriving at the practical safe working heads, due allowance being made for possible water hammer, settling, and the expansion and contraction of pipe, and causes which would tend to collapse same. The working heads are based on the longitudinal seams being double-riveted and the circumferential seams single-riveted.

Pipe can be furnished, if desired, with a special forged-steel coupling which is shipped separately and which allows 5 deg. deflection at each coupling. The slip joint (Fig. 8*a*) is used largely for medium- and low-pressure work. The sleeve which is riveted in one end of the pipe may be wrapped with burlap or canvas soaked in red lead or tar, then driven into the adjoining pipe. Lugs are then connected by wire in order to hold the pipe securely. The bolted joint is shown in Fig. 8*b*.



Slip Joint (*a*). Bolted Joint (*b*).
FIG. 8.—Joints for Spiral Riveted Pipe.

A complete line of cast-iron flanged fittings, gate valves, etc., is manufactured with the flange diameters shown in Table 22.

Table 24. Weights and Dimensions of Line Pipe

Size, in.	Diameters, in.		Thickness, in.	Weight per ft., lb.		Threads per in.	Couplings		
	External	Internal		Plain ends	Threads and couplings		Diam., in.	Length, in.	Weight, lb.
1/8	0.405	0.269	0.068	0.244	0.246	27	0.582	1 1/8	0.043
1/4	0.540	0.364	0.088	0.424	0.426	18	0.724	1 3/8	0.069
3/8	0.675	0.493	0.091	0.567	0.571	18	0.898	1 5/8	0.126
1/2	0.840	0.622	0.109	0.850	0.856	14	1.085	1 7/8	0.205
3/4	1.050	0.824	0.113	1.130	1.140	14	1.316	2 1/8	0.316
1	1.315	1.049	0.133	1.680	1.690	11 1/2	1.575	2 3/8	0.445
1 1/4	1.660	1.380	0.140	2.270	2.300	11 1/2	2.054	2 7/8	0.974
1 1/2	1.900	1.610	0.145	2.720	2.750	11 1/2	2.294	2 7/8	1.100
2	2.375	2.067	0.154	3.650	3.720	11 1/2	2.841	3 5/8	2.150
2 1/2	2.875	2.469	0.203	5.790	5.880	8	3.389	4 1/8	3.390
3	3.500	3.068	0.216	7.580	7.680	8	4.014	4 1/8	4.080
3 1/2	4.000	3.548	0.226	9.110	9.260	8	4.628	4 1/8	5.510
4	4.500	4.026	0.237	10.800	11.000	8	5.233	4 1/8	6.670
4 1/2	5.000	4.506	0.247	12.500	12.700	8	5.733	4 1/8	7.380
5	5.563	5.047	0.258	14.600	15.000	8	6.420	5 1/8	11.700
6	6.625	6.065	0.280	19.000	19.400	8	7.482	5 1/8	13.900
7	7.625	7.023	0.301	23.500	24.000	8	8.482	5 1/8	15.900
8	8.625	8.071	0.277	24.700	25.400	8	9.596	6 1/8	24.100
8	8.625	7.981	0.322	28.600	29.200	8	9.596	6 1/8	24.100
9	9.625	8.941	0.342	33.900	34.600	8	10.596	6 1/8	26.800
10	10.750	10.192	0.279	31.200	32.500	8	11.958	6 5/8	39.800
10	10.750	10.136	0.307	34.200	35.500	8	11.958	6 5/8	39.800
10	10.750	10.020	0.365	40.500	41.600	8	11.958	6 5/8	39.800
11	11.750	11.000	0.375	45.600	46.800	8	12.958	6 5/8	43.300
12	12.750	12.090	0.330	43.800	45.200	8	13.958	6 5/8	46.900
12	12.750	12.000	0.375	49.600	50.900	8	13.958	6 5/8	46.900
13	14.000	13.250	0.375	54.600	56.600	8	15.446	7 1/8	65.500
14	15.000	14.250	0.375	58.600	60.800	8	16.446	7 1/8	70.000
15	16.000	15.250	0.375	62.600	65.000	8	17.446	7 1/8	74.600

The permissible variation in weight is ±5 per cent. Furnished with threads and couplings and in random lengths unless otherwise ordered. Taper of threads is 3/4 in. diam. per ft. of length for all sizes. The weight per ft. of pipe with threads and couplings is based on a length of 20 ft., including the coupling, but shipping lengths of small sizes will usually average less than 20 ft.

Pipe for Special Purposes. For a complete treatment of the following pipe and other pipe of special make, including dimensions, weights, test pressures, etc., see the "Book of Standards" published by the National Tube Co.

Line Pipe is a special pipe that employs recessed and taper thread couplings and usually has a greater length of thread than that given by the Briggs standard. The pipe is also subjected to a higher test. (See Table 24.)

Drive Pipe is a pipe which is driven or forced into a bored hole, to shut off water courses or to prevent caving. It has a threaded joint in which the pipe butts in the center of the coupling.

Drill Pipe is pipe fitted with special couplings, used in well boring.

Matheson Joint Pipe is wrought-iron pipe with leaded joint of the bell-and-spigot type, very similar in appearance to bell-and-spigot cast-iron pipe. This pipe is often used for water and gas under low pressures. Sizes range from 2 in. to 30 in. outside diameter; the pipe is made of different thicknesses. (See Fig. 21, p. 824.)

Converse Lock-joint Pipe is wrought-iron pipe with leaded joint used for low-pressure water or gas. Sizes range from 2 in. to 30 in. outside diameter; the pipe is made of different thicknesses. (See p. 824.)

Signal Pipe is a special pipe used on interlocking switches and the signals on railroads. It has a peculiar joint that is both threaded and connected by a plug riveted to the pipe.

Tubular Electric Line Poles made of steel pipe are used for carrying the wires for the overhead construction on electric railways, telephone, telegraph and transmission lines. Poles are made up of standard and extra strong pipe where practicable, otherwise standard tubular material is used.

Rifled Pipe is used for conveying heavy oils. The pipe is rifled with helical grooves which make a complete turn through 360 deg. in about 10 ft. of length.

Casing is a pipe used for lining oil or gas wells. It is usually characterized by light weight and fine threads. Various kinds of casing are known to the trade, differing in the range of sizes and thicknesses and the kind of joint or coupling used.

Tubing is a special grade of high-test pipe fitted with threads and couplings of special design, and is made to the same outside diameters as standard pipe. (See p. 805.) It is similar to what is known in Europe as "hydraulic pressure pipe."

Weights and Dimensions of Air Line Pipe

Size, in.	Diameters, in.		Thickness, in.	Weight per ft., lb.		Threads per in.	Couplings		
	External	Internal		Plain ends	Threads and couplings		Diam., in.	Length, in.	Weight, lb.
1½	1.900	1.582	0.159	2.96	3.0	11½	2.387	2 15/16	1.36
2	2.375	2.043	0.166	3.92	4.0	11½	2.976	3½	2.42
2½	2.875	2.423	0.226	6.39	6.5	8	3.544	4	3.77
3	3.500	2.990	0.255	8.84	9.0	8	4.272	4½	5.90
4	4.500	3.996	0.252	11.40	11.8	8	5.500	4½	9.12
5	5.563	4.977	0.293	16.50	17.0	8	6.652	6	16.70
6	6.625	6.025	0.300	20.30	21.0	8	7.833	6	21.80

Weights and Dimensions of Oil Well Tubing*

Size, in.	Diameters, in.		Thickness, in.	Weight per ft., lb.		Threads per in.	Couplings		
	External	Internal		Plain ends	Threads and couplings		Diam., in.	Length, in.	Weight, lb.
1¼	1.660	1.380	0.140	2.27	2.30	11½	2.054	2⅞	0.974
1½	1.900	1.610	0.145	2.72	2.75	11½	2.294	2⅞	1.100
2	2.375	2.041	0.167	3.94	4.00	11½	2.841	3⅝	2.150
2	2.375	1.995	0.190	4.43	4.50	11½	2.841	3⅝	2.150
2½	2.875	2.469	0.206	5.79	5.90	11½	3.449	4⅛	3.640
2½	2.875	2.441	0.217	6.16	6.25	11½	3.449	4⅛	3.640
3	3.500	3.068	0.216	7.58	7.69	11½	4.074	4⅛	4.370
3	3.500	3.018	0.241	8.39	8.50	11½	4.074	4⅛	4.370
3	3.500	2.922	0.289	9.91	10.00	11½	4.074	4⅛	4.370
3½	4.000	3.548	0.226	9.11	9.26	8	4.628	4⅜	5.510
4	4.500	4.026	0.237	10.80	11.00	8	5.233	4⅜	6.670
4	4.500	3.990	0.255	11.60	11.80	8	5.233	4⅜	6.670

* See notes at foot of Table 24.

PIPES AND TUBES OF COPPER, BRASS, LEAD, TIN AND ALUMINUM

Seamless Brass and Copper Tubes. **Brass Pipe** is made up to the same dimensions as wrought-iron and steel pipe, and threaded to the wrought-iron standard. When ordering brass pipe, iron-pipe sizes should always be specified. **Thin brass tubing** commonly used for ornamental work, brass hand railings, etc., is not of the proper size to take the standard pipe thread and is too thin for pressure work. Brass pipe is not liable to corrosion. The average ultimate strength is about 18,000 lb. per sq. in.

Copper Pipe deteriorates rapidly under high temperatures and repeated stresses. At a temperature of 360 deg. fahr. its strength is reduced 15 per cent. and on this account it should never be used for high steam pressures and temperatures.

Table 25. Seamless Drawn Brass and Copper Tubing

(American Brass Co.)

Size, in.	Standard Iron Pipe Sizes				Extra Heavy Iron Pipe Sizes			
	Dimensions		Approx. weights		Dimensions		Approx. weights	
	Inside diam., in.	Outside diam., in.	Brass, Per lin. ft., lb.	Copper, Per lin. ft., lb.	Inside diam., in.	Outside diam., in.	Brass, Per lin. ft., lb.	Copper, Per lin. ft., lb.
1/8	0.281	0.405	0.246	0.259	0.205	0.405	0.353	0.371
1/4	0.375	0.540	0.437	0.459	0.294	0.540	0.593	0.624
3/8	0.494	0.675	0.612	0.644	0.421	0.675	0.805	0.847
1/2	0.625	0.840	0.911	0.958	0.542	0.840	1.191	1.253
3/4	0.822	1.050	1.235	1.298	0.736	1.050	1.622	1.706
1	1.062	1.315	1.740	1.829	0.951	1.315	2.386	2.509
1 1/4	1.368	1.660	2.557	2.689	1.272	1.660	3.291	3.460
1 1/2	1.600	1.900	3.037	3.193	1.494	1.900	3.986	4.191
2	2.062	2.375	4.017	4.224	1.933	2.375	5.508	5.791
2 1/2	2.500	2.875	5.830	6.130	2.315	2.875	8.407	8.839
3	3.062	3.500	8.314	8.741	2.892	3.500	11.24	11.82
3 1/2	3.500	4.000	10.85	11.41	3.358	4.000	13.66	14.37
4	4.000	4.500	12.29	12.93	3.818	4.500	16.41	17.25
4 1/2	4.500	5.000	13.74	14.44	4.250	5.000	20.07	21.10
5	5.062	5.563	15.40	16.19	4.813	5.563	22.51	23.67
6	6.125	6.625	18.44	19.39	5.750	6.625	31.32	32.93
7	7.062	7.625	23.92	25.15	6.625	7.625	41.22	43.34
8	8.000	8.625	30.05	31.60	7.625	8.625	47.00	49.42
9	8.937	9.625	36.94	38.84	....	....	....	....
10	10.019	10.750	43.91	46.17	....	....	....	....

Plumber's Sizes of Seamless Drawn Brass and Copper Tubing

Size, in.	Diam., in.		Lb. per ft.		Size, in.	Diam., in.		Lb. per ft.	
	Outside	Inside	Brass	Copper		Outside	Inside	Brass	Copper
5/8	0.654	0.521	0.452	0.475	1 1/4	1.245	1.060	1.233	1.297
3/4	0.768	0.631	0.554	0.583	1 1/2	1.508	1.311	1.606	1.689
7/8	0.875	0.728	0.682	0.717	1 3/4	1.756	1.564	1.844	1.939
1	1.000	0.836	0.871	0.916	2	2.007	1.815	2.123	2.232

Table 26. Weight per Ft. of Brazed Brass Tubing, Lb.
(American Brass Co.)

B. & S. gage No.	Approx. thickness, in.	Outside diam. in inches								
		1/8	3/16	1/4	5/16	3/8	1/2	5/8	3/4	1
12	0.0808	....	....	....	....	....	....	0.515	0.633	0.870
13	0.0720	....	....	....	....	....	....	0.466	0.571	0.782
14	0.0641	....	....	....	....	0.233	0.327	0.421	0.515	0.702
15	0.0571	....	....	....	0.171	0.212	0.296	0.380	0.463	0.630
16	0.0508	....	....	0.119	0.156	0.193	0.267	0.342	0.416	0.565
17	0.0453	....	....	0.109	0.142	0.175	0.241	0.307	0.373	0.506
18	0.0403	....	0.069	0.099	0.128	0.158	0.217	0.276	0.335	0.453
19	0.0359	....	0.064	0.090	0.116	0.143	0.195	0.248	0.300	0.405
20	0.0320	0.035	0.058	0.082	0.105	0.128	0.175	0.222	0.269	0.362
21	0.0285	0.032	0.053	0.074	0.095	0.115	0.157	0.199	0.240	0.324
22	0.0254	0.030	0.048	0.067	0.085	0.104	0.141	0.178	0.215	0.289
23	0.0226	0.027	0.044	0.060	0.077	0.093	0.126	0.159	0.192	0.258
24	0.0201	0.025	0.039	0.054	0.069	0.084	0.113	0.142	0.172	0.231

B. & S. gage No.	Approx. thickness, in.	Outside diam. in inches							
		1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3
12	....	1.106	1.343	1.579	1.816	2.052	2.289	2.525	2.762
13	....	0.993	1.203	1.414	1.625	1.835	2.046	2.257	2.467
14	....	0.890	1.077	1.265	1.453	1.640	1.828	2.015	2.203
15	....	0.797	0.964	1.131	1.298	1.465	1.632	1.799	1.966
16	....	0.714	0.862	1.011	1.160	1.309	1.457	1.606	1.755
17	....	0.638	0.771	0.903	1.036	1.168	1.301	1.433	1.566
18	....	0.571	0.689	0.807	0.925	1.043	1.161	1.279	1.397
19	....	0.510	0.615	0.720	0.825	0.930	1.036	1.141	1.246
20	....	0.456	0.549	0.643	0.737	0.830	....	....	....
21	....	0.407	0.490	0.574	0.657	....	....	....	....
22	....	0.363	0.438	0.512	0.586	....	....	....	....
23	....	0.324	0.390	0.457	....	....	....	....	....
24	....	0.289	0.348	0.407	....	....	....	....	....

Table 27. Weight of Lead Pipe, Lb. per Ft.
(National Lead Co.)

Thickness, in.	Caliber in inches						
	2 1/2	3	3 1/2	4	4 1/2	5	6
3/16	8	9	9.5	12.5	14	....	....
1/4	11	12	15.0	16.0	18	20	24.5
5/16	14	16	18.0	21.0	....	....	....
3/8	17	20	22.0	25.0	....	31	37.0

LEAD WASTE PIPE

Caliber, in....................	1 1/2	2	3	3 1/2	4	4 1/2	5	6
Lb. per ft.....................	2 & 3	3 & 4	3 1/2 & 5	4	5, 6 & 8	6 & 8	8, 10 & 12	12

Table 28. Weight of Lead Tubing
(American Lead Co.)

Caliber, in.	Outside diam., in.	Oz. per ft.	Caliber, in.	Outside diam., in.	Oz. per ft.
1/16	1/8	0.75	3/16	5/16	3.50
3/32	1/8	0 67	3/16	11/32	5.00
3/32	3/16	1.50	3/16	3/8	7.00
3/32	1/4	4.00	1/4	11/32	3.50
1/8	7/32	2.25	1/4	25/64	5.50
1/8	1/4	3.50	1/4	29/64	9.00
1/8	5/16	4.50	1/4	1/2	11.50
3/16	1/4	2.25	....	....	....

Table 29. Weights and Dimensions of Lead Pipe
(American Lead Co.)

Western standard	Caliber, in.	Outside diam., in.	Weight per ft., lb.	Avg. ft. in coil	Western standard	Caliber, in.	Outside diam., in.	Weight per ft., lb.	Avg. ft. in coil
A		0.49	0.50	140	A		1.17	1.50	112
XL		0.52	0.56	124	XL		1.23	2.00	84
L		0.57	0.75	94	L		1.27	2.50	67
M	3/8	0.61	1.00	70	M	1	1.35	3.25	52
S		0.73	1.50	93	S		1.43	4.00	42
XS		0.75	2.00	70	XS		1.49	4.75	35
XXS		0.79	2.50	56	XXS		1.56	5.50	58
					7 lb.		1.66	7.00	45
A		0.64	0.63	280	A		1.43	2.00	84
XL		0.66	0.75	230	XL		1.49	2.50	67
L		0.70	1.00	175	L		1.53	3.00	56
M		0.75	1.25	140	M		1.58	3.75	85
S	1/2	0.85	1.75	100	S	1 1/4	1.66	4.75	67
AA		0.89	2.00	87	XS		1.77	6.00	53
XS		0.94	2.50	70	XXS		1.83	6.75	47
XXS		1.01	3.00	58	9 lb.		1.89	9.00	35
4 lb.		1.12	4.00	43					
A		0.77	0.75	230	A		1.74	3.00	56
XL		0.84	1.25	138	XL		1.77	3.50	48
L		0.92	1.75	98	L		1.82	4.00	42
M		0.95	2.00	86	M		1.89	5.00	64
S	5/8	1.02	2.50	69	S	1 1/2	1.93	6.00	53
AA		1.05	2.75	62	XS		2.05	7.50	42
XS		1.08	3.00	57	XXS		2.13	9.00	35
XXS		1.15	3.50	49	11 lb.		2.19	11.00	29
4 lb.		1.19	4.00	43					
A		0.91	1.00	171	A		2.19	3.00	12′ length
XL		0.97	1.50	114	XL		2.23	4.00	12′ length
L		1.04	2.00	85	L		2.28	5.00	12′ length
M	3/4	1.07	2.25	76	M	2	2.41	7.00	43′ coil
S		1.16	3.00	57	S		2.46	8.00	38′ coil
XS		1.23	3.50	49	XS		2.52	9.00	34′ coil
XXS		1.26	4.00	43	XXS		2.61	10.50	29′ coil
5 lb.		1.35	5.00	34	15 lb.		2.83	15.00	20′ coil

Lead pipe of all sizes furnished on reels any length required.

Table 30. Weight of Block Tin Pipe and Tubing

Inside diam., in.	Outside diam., in.	Weight per ft., oz.	Inside diam., in.	Outside diam., in.	Weight per ft., oz.	Inside diam., in.	Outside diam., in.	Weight per ft., oz.
1/8	3/16	1	5/16	7/16	3 2/3	1/2	4[illegible]/64	8
1/8	1/4	1 5/6	5/16	1/2	5 2/3	1/2	4[illegible]/64	10
1/8	9/32	2 2/3	5/16	9/16	9	1/2	3/4	12 1/2
1/8	5/16	3 1/3	3/8	7/16	2	5/8	2[illegible]/32	9
3/16	1/4	1	3/8	1/2	4	5/8	1[illegible]/16	12
3/16	9/32	1 5/6	3/8	9/16	7 1/3	5/8	5[illegible]/64	16
3/16	5/16	2 1/2	3/8	5/8	9 2/3	3/4	5[illegible]/64	9
3/16	3/8	4 1/3	3/8	11/16	12 1/2	3/4	1[illegible]/16	12
3/16	7/16	6 1/6	7/16	1/2	2	3/4	[illegible]/64	16
1/4	9/32	3/4	7/16	9/16	5 1/3	1	1 1/8	12
1/4	5/16	1 1/3	7/16	5/8	7 2/3	1	1 [illegible]/16	16
1/4	3/8	3 1/6	1/2	19/32	4	1 1/4	1 7/16	20
1/4	7/16	5 1/6	1/2	5/8 scant	5	1 1/4	1 1/2 full	28
1/4	1/2	7 1/2	1/2	5/8 full	5 1/2	1 1/2	1 1[illegible]/16	24
5/16	3/8	1 2/3	1/2	41/64	6	2	2 [illegible]/16 full	32

Tin tubing of 1/8-in. caliber runs about 63 ft. per lb.

Table 31. Weights and Dimensions of Seamless Drawn Aluminum Tubes
(Aluminum Co. of America)

Nominal size, in.	Outside diam., in.	Inside diam., in.	Weight per ft., lb.	Nominal size, in.	Outside diam., in.	Inside diam., in.	Weight per ft., lb.
⅛	0.405	0.270	0.083	1½	1.90	1.61	0.928
¼	0.540	0.364	0.145	2	2.38	2.07	1 240
⅜	0.675	0.494	0.193	2½	2.88	2.47	1.980
½	0.840	0.623	0.290	3	3.50	3.07	2.590
¾	1.050	0.824	0.387	3½	4.00	3.55	3.110
1	1.320	1.050	0 577	4	4.50	4.03	3.690
1¼	1.660	1.380	0.777	....	....	....	....

Pipes with Special Linings. For use in lines through which are passed solutions containing more or less free acid or other corrosive agents, standard pipe, valves, and fittings may be **lead-lined** or **tin-lined** to resist corrosive action. This lining prolongs the life of the pipe and also gives it additional strength. For mine service in coal districts where the drainage water is more or less impregnated with sulphur or free sulphuric acid, **wood-lined pipe** and fittings are sometimes used. For special service, **seamless-copper-lined** pipe is also used.

VITRIFIED, WOODEN-STAVE AND CONCRETE PIPE

Vitrified Salt-glazed Pipe is used extensively for drains and sewerage systems. Burnt clay tile being rendered impervious to water by glazing, is by far the best material for sewage purposes as it is not attacked by acids.

Table 32. Approximate Weights, Dimensions, Etc., of Vitrified Pipe
(National Fire-proofing Co., New York)

Standard Sewer Pipe					Double-strength Pipe				
Caliber, in.	Weight per ft., lb.	Depth of socket, in.	Annular space, in.	Thickness, in.	Caliber, in.	Weight per ft., lb.	Depth of socket, in.	Annular space, in.	Thickness, in.
3	7	1½	¼	½	15	75	2½	½	1¼
4	9	1⅝	⅜	½	18	118	2¾	½	1½
5	12	1¾	⅜	⅝	20	138	3	½	1⅜
6	15	1⅞	⅜	⅝	21	148	3	½	1¾
8	23	2	⅜	¾	22	157	3	½	1⅝
9	28	2	⅜	13/16	24	190	3¼	½	2
10	35	2⅛	⅜	⅞	27	265	4	¾	2¼
12	43	2¼	½	1	30	290	4	¾	2½
15	60	2½	½	1⅛	33	335	5	1¼	2⅝
18	85	2¾	½	1¼	36	375	5	1¼	2¾
20	100	3	½	1⅜					
21	120	3	½	1½					
22	130	3	½	1⅝					
24	140	3¼	½	1⅝					
27	224	4	¾	2					
30	252	4	¾	2⅛					
33	310	5	1¼	2¼					
36	350	5	1¼	2½					

Vitrified Drain Tile
(Round and hexagon)

Caliber, in......	3	4	5	6	8	10	12
Lb. per ft........	5	7½	10	13	20	30	40
Y's, T's, L's, each in lb.....	10	10	15	20	30	50	75
Ft. per tile......	1	1	1	1	2	2	2

Wooden-stave Pipe (Figs. 9 and 10) is used to a considerable extent for water supply, sewers, mining, sluicing, irrigation lines, and chemical fluids. It is durable when completely buried and kept constantly saturated. When it is used for pressures exceeding 40 lb. it should be reinforced by a banding process and the bands covered with a protective coating to prevent corrosion. **Machine-made wooden-stave pipe** is made in sizes up to 48 in. in diam. and assembled in lengths of 12 ft. for shipping. Machine-made pipe possesses the advantage of uniformity.

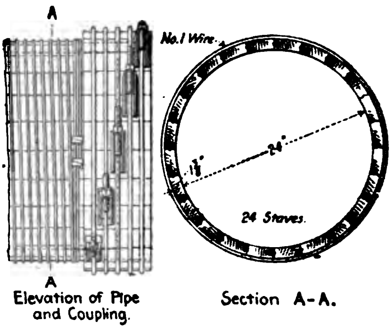


FIG. 9.—Wooden-stave Pipe.

For **continuous wooden-stave pipe,** the staves are dressed on the flat sides to circles corresponding to the diam. of the pipe to be laid, and on the edges to radial planes; and in machine-banded pipe the edges are tongued and grooved (Fig. 9). They are built into a continuous pipe by banding the staves together on the outside, so placing them as to break joints. The end of each stave is then cut off squarely and slotted for the insertion of a tight-fitting metallic tongue. The bands are spaced to conform to the pressure which the pipe is expected to withstand. Wooden-stave pipe can be made of any desired diam. and for any required pressure. Manufacturers claim that it is being used successfully for pressures as high as 200 lb. per sq. in. and for heads of over 350 ft. in extreme cases. Standard cast-iron fittings can be used for wood pipe, but special fittings may be obtained which are cheaper and better adapted to it.

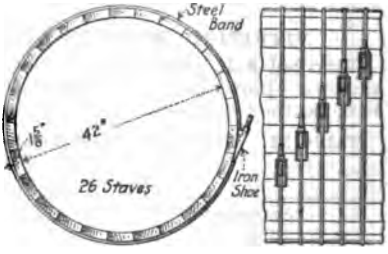


FIG. 10.—Wooden-stave Pipe.

Table 33. Approximate Weights of Machine-banded Wooden-stave Pipe

(Washington Pipe & Foundry Co., Tacoma, Wash.)

Inside diam., in.	Weight per ft., lb.	No. of ft. per carload	Inside diam., in.	Weight per ft., lb.	No. of ft. per carload
2	2½	9000	12	14½	1000
3	3⅞	6500	14	17	850
4	4¼	5500	16	22	700
6	7½	3500	18	26	500
8	9⅞	2500	20	33	500
10	12½	1500	24	50	400

Concrete Pipe has recently become an important factor in sewer, conduit, railroad, culvert, and water-pipe construction. The pipe, as usually made, is constructed of concrete reinforced longitudinally with bars and transversely with wire mesh or steel bands. It is made in sections of definite length, with the longitudinal reinforcement so disposed as to provide for the interlocking of one section with another, and so formed that when these are locked together and cemented they form a continuous line of pipe free from leakage or seepage. This pipe is made in diameters of 2 to 10 ft., and can be reinforced to withstand any strain. Fig. 11 shows the construction employed by the Reinforced Concrete Pipe Co., of Jackson, Mich. Pipes under 5 ft. in diam. are made in 3-ft. lengths; larger sizes in 5-ft. lengths. The wall thickness ranges from 2½ in. for a 2-ft. diam. pipe to 10 in. for a 10-ft. pipe. The flange lengths run from 3 in. for a 2-ft. pipe up to 5¼ in. for a 6-ft. pipe; above 6 ft. the flanges are all made 6 in. long.

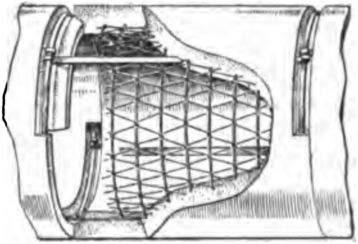


Fig. 11.—Reinforced-concrete Pipe.

FITTINGS FOR WROUGHT-IRON AND STEEL PIPE

Flanges and Flanged Fittings. Much confusion has resulted in the past, due to the various standards for flange dimensions and bolting adopted by manufacturers and engineering societies. In 1912, the American Society of Mechanical Engineers and the Master Steam and Hot Water Fitters' Association adopted what is known as "The 1912 U. S. Standard," and in the same year, at a meeting of manufacturers in New York City, the "Manufacturer's Standard" was promulgated. The disadvantages of having two standards in existence were immediately recognized, and committees of the A. S. M. E. and the manufacturers united in a compromise known as the "American Standard," to be effective after Jan. 1, 1914. The same dimensions are published by the Master Steam and Hot Water Fitters' Association under the title of "The 1915 U. S. Standard."

Notes on the American Standard. The following notes apply to the American Standard for flanges and flanged fittings:

(a) Standard and extra heavy reducing elbows carry the same dimensions center to face as regular elbows of largest straight size.

Standard and extra heavy tees, crosses and laterals, reducing on run *only*, carry same dimensions face to face as largest straight size.

Flanged fittings for lower working pressures than 125 lb. conform to this standard in all dimensions except thickness of shell.

Where long-radius fittings are specified, reference is had only to elbows made in two center-to-face dimensions and known as elbows and long-radius elbows, the latter being used only when so specified.

Standard weight fittings are guaranteed for 125 lb. working pressure and extra heavy fittings for 250 lb.

Extra heavy fittings and flanges have a raised surface 1/16 in. high inside of bolt holes for gaskets. Standard weight fittings and flanges are plain-faced. Bolt holes are ⅛ in. larger in diam. than bolts, and straddle the center line.

The size of all fittings scheduled indicates the inside diam. of ports.

The face-to-face dimension of reducers, either straight or eccentric, for all pressures, is the same as that given in table of dimensions.

Square-head bolts with hexagonal nuts are recommended. For 1⅝-in. and larger bolts, studs with a nut on each end are satisfactory. Hexagonal nuts for pipe sizes up to 46 in. on the 125-lb. standard, and up to 16 in. on the 250-lb. standard can be conveniently pulled up with open wrenches of minimum design of heads. For larger pipe sizes (up to 100 in. on 125-lb., and to 48 in. on 250-lb. standard) use box wrenches.

Twin elbows, whether straight or reducing, carry same dimensions center to face and face to face as regular straight-size ells and tees.

Side outlet elbows and side outlet tees, whether straight or reducing sizes, carry same dimensions center to face and face to face as regular tees having same reductions.

(*b*) Bull-head tees, or tees increasing on outlet, have same center-to-face and face-to-face dimensions as a straight fitting of the size of the outlet.

Tees, crosses and laterals 16 in. and smaller, reducing on the outlet, use the same dimensions as straight sizes of the larger port. Sizes 18 in. and larger, reducing on the outlet or branch, are made in two lengths, depending on size of outlet or branch as given in dimension table.

(*c*) The dimensions of reducing flanged fittings are always regulated by the reductions of the outlet or branch.

(*d*) For fittings reducing on the run only, always use the long-body pattern.

Y's are special and are made to suit conditions.

(*e*) Double-sweep tees are not made reducing on the run.

Steel flanges, fittings and valves are recommended for superheated steam.

Table 34. Templates for Drilling Standard and Low-pressure Flanged Valves and Fittings*

AMERICAN STANDARD

Size, in.	Diam. of flanges, in.	Thickness of flanges, in.	Bolt circle diam., in.	Number of bolts	Size of bolts, in.	Size, in.	Diam. of flanges, in.	Thickness of flanges, in.	Bolt circle diam., in.	Number of bolts	Size of bolts, in.
1	4	7/16	3	4	7/16	40	50¾	2½	47¼	36	1⅝
1¼	4½	½	3⅜	4	7/16	42	53	2⅝	49½	36	1⅝
1½	5	9/16	3⅞	4	½	44	55¼	2⅝	51¾	40	1⅝
2	6	⅝	4¾	4	⅝	46	57¼	2 11/16	53¾	40	1⅝
2½	7	11/16	5½	4	⅝	48	59½	2¾	56	44	1⅝
3	7½	¾	6	4	⅝	50	61¾	2¾	58¼	44	1¾
3½	8½	13/16	7	4	⅝	52	64	2⅞	60½	44	1¾
4	9	15/16	7½	8	⅝	54	66¼	3	62¾	44	1¾
4½	9¼	15/16	7¾	8	¾	56	68¾	3	65	48	1¾
5	10	15/16	8½	8	¾	58	71	3⅛	67¼	48	1¾
6	11	1	9½	8	¾	60	73	3⅛	69¾	52	1¾
7	12½	1 1/16	10¾	8	¾	62	75¾	3¼	71¾	52	1⅞
8	13½	1⅛	11¾	8	¾	64	78	3¼	74	52	1⅞
9	15	1⅛	13¼	12	¾	66	80	3⅜	76	52	1⅞
10	16	1 3/16	14¼	12	⅞	68	82¼	3⅜	78¼	56	1⅞
12	19	1¼	17	12	⅞	70	84½	3½	80½	56	1⅞
14	21	1⅜	18¾	12	1	72	86½	3½	82½	60	1⅞
15	22¼	1⅜	20	16	1	74	88½	3⅝	84½	60	1⅞
16	23½	1 7/16	21¼	16	1	76	90¾	3⅝	86½	60	1⅞
18	25	1 9/16	22¾	16	1⅛	78	93	3¾	88¾	60	2
20	27½	1 11/16	25	20	1⅛	80	95¼	3¾	91	60	2
22	29½	1 13/16	27¼	20	1¼	82	97½	3⅞	93¼	60	2
24	32	1⅞	29½	20	1¼	84	99¾	3⅞	95½	64	2
26	34¼	2	31¾	24	1¼	86	102	4	97¾	64	2
28	36½	2 1/16	34	28	1¼	88	104¼	4	100	68	2
30	38¾	2⅛	36	28	1⅜	90	106½	4⅛	102¼	68	2⅛
32	41¾	2¼	38½	28	1½	92	108¾	4⅛	104½	68	2⅛
34	43¾	2 5/16	40½	32	1½	94	111	4¼	106¼	68	2⅛
36	46	2⅜	42¾	32	1½	96	113¼	4¼	108½	68	2¼
38	48¾	2⅜	45¼	32	1⅝	98	115½	4⅜	110¾	68	2¼
						100	117¾	4⅜	113	68	2¼

* These templates are in multiples of four, so that fittings may be made to face in any quarter and bolt holes straddle the center line. Bolt holes are drilled ⅛ in. larger than nominal diam. of bolts.

Table 35. Templates for Drilling Extra Heavy Flanged Valves and Fittings*

AMERICAN STANDARD

Size, in.	Diam. of flanges, in.	Thickness of flanges, in.	Bolt circle diam., in.	Number of bolts	Size of bolts, in.	Size, in.	Diam. of flanges, in.	Thickness of flanges, in.	Bolt circle diam., in.	Number of bolts	Size of bolts, in.
1	4½	11/16	3¼	4	½	16	25½	2¼	22½	20	1¼
1¼	5	¾	3¾	4	½	18	28	2⅜	24¾	24	1¼
1½	6	13/16	4½	4	⅝						
2	6½	⅞	5	4	⅝	20	30½	2½	27	24	1⅜
2½	7½	1	5⅞	4	¾	22	33	2⅝	29¼	24	1½
						24	36	2¾	32	24	1⅝
3	8¼	1⅛	6⅝	8	¾	26	38¼	2 13/16	34½	28	1⅝
3½	9	1 3/16	7¼	8	¾	28	40¾	2 15/16	37	28	1⅝
4	10	1¼	7⅞	8	¾						
4½	10½	1 5/16	8½	8	¾	30	43	3	39¼	28	1¾
5	11	1⅜	9¼	8	¾	32	45¼	3⅛	41½	28	1⅞
						34	47½	3¼	43½	28	1⅞
6	12½	1 7/16	10⅝	12	¾	36	50	3⅜	46	32	1⅞
7	14	1½	11⅞	12	⅞	38	52¼	3 7/16	48	32	1⅞
8	15	1⅝	13	12	⅞						
9	16¼	1¾	14	12	1	40	54½	3 9/16	50¼	36	1⅞
10	17½	1⅞	15¼	16	1	42	57	3 11/16	52¾	36	1⅞
						44	59¼	3¾	55	36	2
12	20½	2	17¾	16	1⅛	46	61½	3⅞	57¼	40	2
14	23	2⅛	20¼	20	1⅛	48	65	4	60¾	40	2
15	24½	2 3/16	21½	20	1¼						

* These templates are in multiples of four, so that fittings may be made to face in any quarter and bolt holes straddle the center line. Bolt holes are drilled ⅛ in. larger than nominal diam. of bolts.

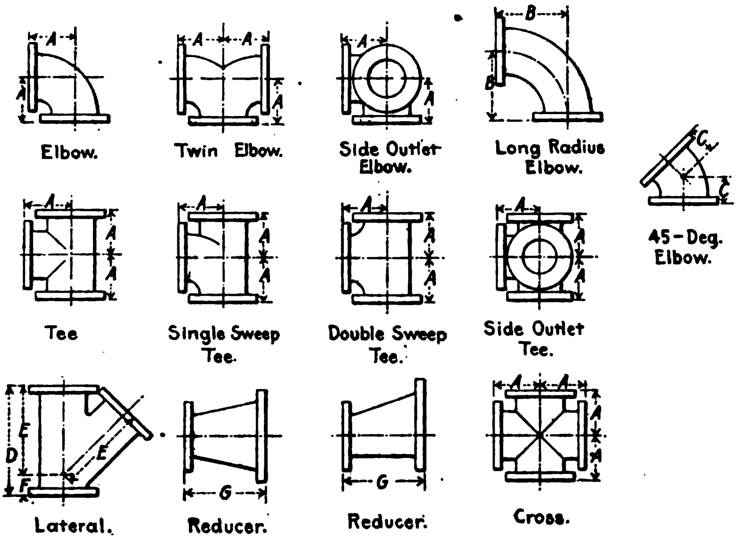


FIG. 12.—Standard Flanged Fittings for W. I. and Steel Pipe (see Table 36).

Table 36. General Dimensions of Standard Flanged Fittings—Straight Sizes

AMERICAN STANDARD

(All dimensions in inches. Letters refer to sketches in Fig. 12)

For sizes from 32 in. to 100 in., dimension G = size of pipe (S) and A-A = $2A$; also, in lower half of table, d and t = diameter and thickness of flanges, and t_1 = minimum metal thickness of body.

Size S	Face to face, tees and crosses A-A	Center to face, ells, tees and crosses A	Center to face, long-radius ells B	Center to face, 45-deg. ells C	Face to face, laterals D	Center to face, laterals E	Center to face, laterals F	Face to face, reducers G	Diam. of flanges	Thickness of flanges	Minimum metal thickness of body
1	7	3½	5	1¾	7½	5¾	1¾	......	4	7/16	7/16
1¼	7½	3¾	5½	2	8	6¼	1¾	......	4½	½	5/16
1½	8	4	6	2¼	9	7	2	......	5	9/16	5/16
2	9	4½	6½	2½	10½	8	2½	......	6	⅝	5/16
2½	10	5	7	3	12	9½	2½	......	7	11/16	5/16
3	11	5½	7¾	3	13	10	3	6	7½	¾	5/16
3½	12	6	8½	3½	14½	11½	3	6½	8½	13/16	5/16
4	13	6½	9	4	15	12	3	7	9	15/16	½
4½	14	7	9½	4	15½	12½	3	7½	9¼	15/16	½
5	15	7½	10¼	4½	17	13½	3½	8	10	15/16	½
6	16	8	11½	5	18	14½	3½	9	11	1	9/16
7	17	8½	12¾	5½	20½	16½	4	10	12½	1 1/16	⅝
8	18	9	14	5½	22	17½	4½	11	13½	1⅛	⅝
9	20	10	15¼	6	24	19½	4½	11½	15	1⅛	11/16
10	22	11	16½	6½	25½	20½	5	12	16	1 3/16	¾
12	24	12	19	7½	30	24½	5½	14	19	1¼	13/16
14	28	14	21½	7½	33	27	6	16	21	1⅜	⅞
15	29	14½	22¾	8	34½	28½	6	17	22¼	1⅜	⅞
16	30	15	24	8	36½	30	6½	18	23½	1 7/16	1
18	33	16½	26½	8½	39	32	7	19	25	1 9/16	1 1/16
20	36	18	29	9½	43	35	8	20	27½	1 11/16	1⅛
22	40	20	31½	10	46	37½	8½	22	29½	1 13/16	1 3/16
24	44	22	34	11	49½	40½	9	24	32	1⅞	1¼
26	46	23	36½	13	53	44	9	26	34¼	2	1 5/16
28	48	24	39	14	56	46½	9½	28	36½	2 1/16	1⅜
30	50	25	41½	15	59	49	10	30	38¾	2⅛	1 7/16

S	A	B	C	d	t	t_1	S	A	B	C	d	t	t_1
32	26	44	16	41¾	2¼	1½	68	50	89	34	82¼	3⅜	2 11/16
34	27	46½	17	43¾	2 5/16	1 9/16	70	51	91½	35	84½	3½	2¾
36	28	49	18	46	2⅜	1⅝	72	53	94	36	86½	3½	2 13/16
38	29	51½	19	48¾	2⅜	1 11/16	74	54	96½	37	88½	3⅝	2⅞
40	30	54	20	50¾	2½	1¾	76	56	99	38	90¾	3⅝	2 15/16
42	31	56½	21	53	2⅝	1 13/16	78	58	101½	39	93	3¾	3
44	32	59	22	55¼	2⅝	1⅞	80	59	104	40	95¼	3¾	3 1/16
46	33	61½	23	57¼	2 11/16	1 15/16	82	60	106½	41	97½	3⅞	3⅛
48	34	64	24	59½	2¾	2	84	62	109	42	99¾	3⅞	3 3/16
50	35	66½	25	61¾	2¾	2 1/16	86	63	111½	43	102	4	3¼
52	37	69	26	64	2⅞	2⅛	88	65	114	44	104¼	4	3 5/16
54	39	71½	27	66¼	3	2 3/16	90	67	116½	45	106½	4⅛	3⅜
56	41	74	28	68¾	3	2¼	92	68	119	46	108¾	4⅛	3½
58	42	76½	29	71	3⅛	2 5/16	94	69	121½	47	111	4¼	3 9/16
60	44	79	30	73	3⅛	2 7/16	96	71	124	48	113¼	4¼	3⅝
62	45	81½	31	75¾	3¼	2½	98	73	126½	49	115½	4⅜	3 11/16
64	47	84	32	78	3¼	2 9/16	100	74	129	50	117¾	4⅜	3¾
66	48	86½	33	80	3⅜	2⅝							

(For Weights, see Table 49)

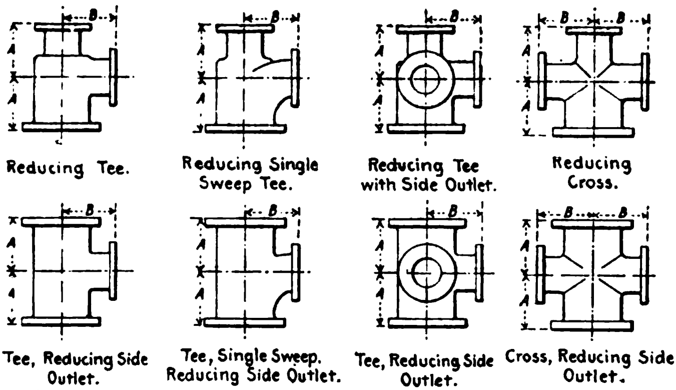


Fig. 13.—Standard Reducing Tees and Crosses (See Table 37).

Table 37. General Dimensions of Standard Reducing Tees and Crosses (Short-body Pattern)

American Standard

(All dimensions in inches. Letters refer to sketches in Fig. 13. $A\text{-}A = 2A$)

Size	Size of outlets and smaller*	Center to face, run *A*	Center to face, outlet *B*	Size	Size of outlets and smaller*	Center to face, run *A*	Center to face, outlet *B*	Size	Size of outlets and smaller*	Center to face, run *A*	Center to face, outlet *B*
1 to 16	All reducing fittings from 1 to 16 in. inclusive have the same center-to-face dimensions as straight-size fittings.			40	26	22	29	70	46	37	47
				42	28	23	30	72	48	40	48
				44	28	23	31	74	48	40	49
				46	30	24	33	76	50	42	50
				48	32	26	34	78	52	43	52
				50	32	26	35	80	52	43	53
18	12	13	15½	52	34	27	36	82	54	44	54
20	14	14	17	54	36	29	37	84	56	47	56
22	15	14	18	56	36	29	39	86	56	47	57
24	16	15	19	58	38	31	40	88	58	48	58
26	18	16	20								
28	18	16	21	60	40	33	41	90	60	50	61
				62	40	33	42	92	60	50	62
30	20	18	23	64	42	34	44	94	62	52	63
32	20	18	24	66	44	35	45	96	64	53	64
34	22	19	25	66	44	35	45	96	64	53	64
36	24	20	26	68	44	35	46	98	64	53	65
38	24	20	28					100	66	55	67

* Long-body patterns are used when outlets are larger than given above, and therefore have the same dimensions as straight-size fittings.

See also Notes (*a*), (*b*), (*c*), (*d*), and (*e*), p. 815.

Example of Note (*b*): A 12 × 12 × 18-in. tee will be governed by dimensions of the 18-in. long-body tee; namely, 16½ in. center to face of all openings and 33 in. face to face (Table 36).

Sizes of Wrenches for Bolting Up Flanges. Tests made by Crane Co. (*Valve World*, Sept., 1913) show that a compression of about 12,000 lb. can be obtained by one man using a 16-in. wrench on ¾- to 1-in. bolts, a 36-in. wrench on 1⅛- to 1⅜-in. bolts, a 60-in. wrench on 1½- to 1¾-in. bolts, and a 72-

in. wrench on 1¾- and 2-in. bolts. Square nuts give from 20 to 25 per cent. less compression than hexagon nuts with a given effort on the wrench. Rough-cut or uneven threads reduce compression 10 to 15 per cent. Lubrication of threads and bearing surfaces increases compression about 50 per cent. over that obtained with dry threads and surfaces. It is not necessary to over-strain bolts specified in the American Standard if proper wrenches are used, and almost impossible to overstrain bolts larger than 1 in. in diam. The following overall lengths of box wrenches are recommended (interpolate for intermediate sizes):

Size of bolt, in.	½	⅝	¾	⅞	1	1¼	1½	1¾	2	2¼
Length of wrench, in.	7¾	9⅞	11½	13¼	15	19	23	27	30½	34

FIG. 14.—Standard Reducing Laterals (see Table 38).

Table 38. General Dimensions of Standard Reducing Laterals (Short-body Pattern)

AMERICAN STANDARD

(Letters refer to sketches in Fig. 14)

Size, in.*	18	20	22	24	26	28	30
Size of branches and smaller, in.†	9	10	10	12	12	14	15
Face to face, run, *C*, in.	26	28	29	32	35	37	39
Center to face, run, *D*, in.	25	27	28½	31½	35	37	39
Center to face, run, *E*, in.	1	1	½	½	0	0	0
Center to face, branch, *F*, in.	27½	29½	31½	34½	38	40	42

* All reducing fittings from 1 to 16 in. inclusive have the same center-to-face dimensions as straight-size fittings.

† Long-body patterns are used when branches are larger than given in table, and have the same dimensions as straight-size fittings.

See also Notes (*c*) and (*d*), p. 816.

Table 39. Stresses in American Standard Flange Bolts

[Fiber stress in lb. per sq. in. of bolt metal (approx.)]

Pipe diam., in.	Standard*	Extra heavy†	Pipe diam., in.	Standard	Extra heavy	Pipe diam., in.	Standard	Extra heavy
1	250	400	8	2600	2500	28	3100	3650
1¼	400	650	9	2200	2400	30	2950	3600
1½	450	550	10	1950	2250	32	2800	3500
2	500	950	12	2850	2550	34	2750	3950
2½	750	1000	14	2900	2750	36	3100	3850
3	1100	750	15	2500	2500	38	2900	4300
3½	1500	1000	16	2850	2800	40	2900	4250
4	950	1300	18	2850	2950	42	3200	4700
4½	850	1650	20	2800	3100	44	3150	4600
5	1050	2100	22	2650	3050	46	3450	4550
6	1450	1950	24	3150	3100	48	3400	4900
7	2000	1900	26	3100	3150	50	3200	....

* For working pressure of 125 lb. per sq. in.

† For working pressure of 250 lb. per sq. in.

Table 40. General Dimensions of Extra Heavy Flanged Fittings—Straight Sizes

(All dimensions in inches. Letters refer to sketches in Fig. 12)

Size	Face to face, tees and crosses AA	Center to face, ells, tees and crosses A	Center to face, long-radius ells B	Center to face, 45-deg. ells C	Face to face, laterals D	Center to face, laterals E	Center to face, laterals F	Face to face, reducers G	Diam. of flanges	Thickness of flanges	Minimum metal thickness of body
1	8	4	5	2	8½	6½	2	......	4½	1 1/16	½
1¼	8½	4¼	5½	2½	9½	7¼	2¼	......	5	¾	½
1½	9	4½	6	2¾	11	8½	2½	......	6	13/16	½
2	10	5	6½	3	11½	9	2½	......	6½	⅞	½
2½	11	5½	7	3½	13	10½	2½	......	7½	1	9/16
3	12	6	7¾	3½	14	11	3	6	8¼	1⅛	9/16
3½	13	6½	8½	4	15½	12½	3	6½	9	1 3/16	9/16
4	14	7	9	4½	16½	13½	3	7	10	1¼	⅝
4½	15	7½	9½	4½	18	14½	3½	7½	10½	1 5/16	⅝
5	16	8	10¼	5	18½	15	3½	8	11	1⅜	11/16
6	17	8½	11½	5½	21½	17½	4	9	12½	1 7/16	¾
7	18	9	12¾	6	23½	19	4½	10	14	1½	13/16
8	20	10	14	6	25½	20½	5	11	15	1⅝	13/16
9	21	10½	15¼	6½	27½	22½	5	11½	16¼	1¾	⅞
10	23	11½	16½	7	29½	24	5½	12	17½	1⅞	15/16
12	26	13	19	8	33½	27½	6	14	20½	2	1
14	30	15	21½	8½	37½	31	6½	16	23	2⅛	1⅛
15	31	15½	22¾	9	39½	33	6½	17	24½	2 3/16	1 3/16
16	33	16½	24	9½	42	34½	7½	18	25½	2¼	1¼
18	36	18	26½	10	45½	37½	8	19	28	2⅜	1⅜
20	39	19½	29	10½	49	40½	8½	20	30½	2½	1½
22	41	20½	31½	11	53	43½	9½	22	33	2⅝	1 9/16
24	45	22½	34	12	57½	47½	10	24	36	2¾	1⅝
26	48	24	36½	13	......	......	......	26	38¼	2 13/16	1 13/16
28	52	26	39	14	......	......	......	28	40¾	2 15/16	1⅞
30	55	27½	41½	15	......	......	......	30	43	3	2
32	58	29	44	16	......	......	......	32	45¼	3⅛	2⅛
34	61	30½	46½	17	......	......	......	34	47½	3¼	2¼
36	65	32½	49	18	......	......	......	36	50	3⅜	2⅜
38	68	34	51½	19	......	......	......	38	52¼	3 7/16	2 7/16
40	71	35½	54	20	......	......	......	40	54½	3 9/16	2 9/16
42	74	37	56½	21	......	......	......	42	57	3 11/16	2 11/16
44	78	39	59	22	......	......	......	44	59¼	3¾	2 13/16
46	81	40½	61½	23	......	......	......	46	61½	3⅞	2⅞
48	84	42	64	24	......	......	......	48	65	4	3

(For Weights, see Table 49)

The Bursting Pressure of Flanged Fittings. Results of hydraulic-pressure tests on flanged tees and ells made by Crane Co. (*Valve World*, Sept., 1913) are approximately expressed by the formula: Bursting point, lb. per sq. in. $= tS/D$, where t = thickness of metal in the body of the fitting in in., S = 65 per cent. of the tensile strength (T.S.) of the metal for fittings up to 12 in. diam. (60 per cent. for larger diameters), and D = inside diam. of fitting, in in. A factor of safety of from 4 to 8 should be used, depending on the size of the fitting. The formula is based on averages of tests of 52 fittings made from cast iron (T.S. = 22,000) and "ferro-steel" (T.S. = 33,000).

Table 41. General Dimensions of Extra Heavy Reducing Tees and Crosses (Short-body Pattern)*

AMERICAN STANDARD

(All dimensions in inches. Letters refer to sketches in Fig. 13)

Size	Size of outlets and smaller*	Face to face, run *AA*	Center to face, run *A*	Center to face, outlet *B*	Size	Size of outlets and smaller*	Face to face, run *AA*	Center to face, run *A*	Center to face, outlet *B*
1 to 16	All reducing fittings 1 to 16 in. inclusive have the same center-to-face dimensions as straight-size fittings.				32	20	41	20½	26½
					34	22	44	22	28
18	12	28	14	17	36	24	47	23½	29½
20	14	31	15½	18½	38	24	47	23½	30½
22	15	33	16½	20	40	26	50	25	31½
24	16	34	17	21½	42	28	53	26½	33½
26	18	38	19	23	44	28	53	26½	34½
28	18	38	19	24	46	30	55	27½	35½
30	20	41	20½	25½	48	32	58	29	37½

* See notes at foot of Table 37.

Table 42. General Dimensions of Extra Heavy Reducing Laterals (Short-body Pattern) *

AMERICAN STANDARD

(Letters refer to sketches in Fig. 14)

Size, in.†	18	20	22	24
Size of branches and smaller, in.	9	10	10	12
Face to face, run, *C*, in.	34	37	40	44
Center to face, run, *D*, in.	31	34	37	41
Center to face, run, *E*, in.	3	3	3	3
Center to face, branch, *F*, in.	32½	36	39	43

* See notes at foot of Table 38.

† All reducing fittings 1 to 16 in. inclusive have the same center-to-face dimensions as straight-size fittings.

Flanged Pipe Joints. The flange is a most vital part of a piping system, and it is upon this that the principal cost of pipe maintenance depends. Flanges are made of cast iron, cast brass, semi-steel, and rolled steel. The important types are: the screwed, screwed and peened, shrunk and rolled, Van Stone, and welded flanges.

The **screwed joint** (Fig. 15), if properly made, compares favorably with other types of flanged joints for medium steam pressure and high water pressure in sizes up to 12 in.; above 12 in., however, it is unsatisfactory and unsafe. As an additional safeguard against leakage the inside of the pipe may be hammered or peened into the threads on the flange, making what is known as the **screwed-and-peened joint.** On account of the difficulty of properly threading sizes above 12 in., the only practical method of attaching flanges has been by riveting, or shrinking and peening. The **riveted-flange joint** (Fig. 16) and the **shrunk-and-peened joint** (Fig. 17) are mostly used for exhaust, water, and condenser service.

In the **shrunk-and-rolled joint** the flange is bored out to a shrink fit, then heated and placed on the pipe, after which the pipe is expanded by a power roller expander until it not only fits the barrel of the flange, but the metal of the pipe flows into the corrugations in the hub of the flange. Both pipe and flange are then faced off. This type of joint has been used successfully for high-pressure steam, but, having been superseded by the Van Stone joint, is used chiefly on exhaust, condenser and water lines.

The **Van Stone joint** (Fig. 18), also known by various other names, is extensively used for very-high-class work. The flange is first bored to fit over the outside of the pipe loosely. The pipe is then expanded or turned over on the end until it is at right angles to the axis of the pipe. It is then faced on the front and outer circumference. For purposes of erection this type of joint excels all others, as the flanges are loose and may be swiveled on the pipe. Rolled-steel flanges should be used, as they are practically unbreakable.

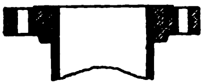


FIG. 15.—Screwed Joint.

FIG. 16.—Riveted-flange Joint.

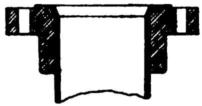


FIG. 17.—Shrunk-and-peened Joint.

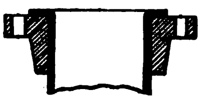


FIG. 18.—Van Stone Joint.

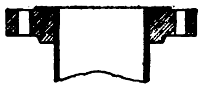


FIG. 19.—Improved Van Stone Joint.

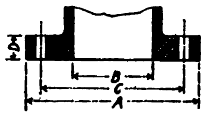


FIG. 20.—Welded Joint.

Flanged Pipe Joints.

For the **improved Van Stone joint** (Fig. 19) a distinct advantage over the ordinary joint of that name is claimed, in that the pipe is reinforced or strengthened at the turned-over portion by a band welded to the end of the pipe. It is desirable that the pipe be faced on the back surface as well as the front, as otherwise the scale formed when the pipe is heated prevents the flange from having a full and intimate bearing against the back of the pipe. After the turned-over portion is faced on the front, back, and circumference, the pipe has a thickness equal to the original thickness of the pipe and a thickness through the fillet equal to 1¼ times the original thickness of the pipe.

The **welded joint** (Fig. 20) is made by welding a rolled-steel flange to the end of a pipe, forming an integral piece of one homogeneous material. For lines subject to very high temperatures and pressures and heavy expansion strains, this joint is used very successfully. The cost of welded flanges is considerably higher than that of cast flanges, but their use makes it possible to reduce the number of joints in a run by 50 to 75 per cent., thus effecting a great saving in weight and minimizing the opportunities for leakage.

The diameters and thicknesses of the flanges shown in Figs. 15–19, as well as the diameters of bolt circles, are given in Tables 34 and 35. **Shrunk-and-peened flanges** are made in sizes 4 to 30 in. inclusive; **Van Stone flanges** in sizes 4 to 30 in. inclusive; **screwed flanges** in sizes 1 to 24 in. inclusive; and **riveted flanges** 4 to 32 in. inclusive. All but riveted flanges (cast iron) are made of either cast iron or forged steel; in the latter case somewhat smaller flange thicknesses are used. Dimensions of **welded-steel flanges** for pressures up to 250 lb. are given in Table 43.

Table 43. Dimensions of Forged- and Rolled-steel Flanges (Fig. 20)*

(American 1914 Standard for extra heavy pressure.—American Spiral Pipe Works)

Size of pipe (nominal inside diam.), in. *B*	Outside diam. of flange, in. *A*	Thickness of flange, in. *D*	Diam. of bolt circle, in. *C*	Number of bolts	Diam. of bolts, in.	Size of pipe (nominal inside diam.), in. *B*	Outside diam. of flange, in. *A*	Thickness of flange, in. *D*	Diam. of bolt circle, in. *C*	Number of bolts	Diam. of bolts, in.
1½	6	¾	4½	4	⅝	9	16¼	1 7/16	14	12	1
2	6½	⅞	5	4	⅝	10	17½	1½	15¼	16	1
2½	7½	1	5⅞	4	¾	12	20½	1⅝	17¾	16	1⅛
3	8¼	1	6⅝	8	¾	14†	23	1¾	20¼	20	1⅛
3½	9	1⅛	7¼	8	¾	15†	24½	1 13/16	21½	20	1¼
4	10	1⅛	7⅞	8	¾	16†	25½	1⅞	22½	20	1¼
4½	10½	1¼	8½	8	¾	18†	28	2	24¾	24	1¼
5	11	1¼	9¼	8	¾	20†	30½	2⅛	27	24	1⅜
6	12½	1¼	10⅝	12	¾	22†	33	2½	29¼	24	1½
7	14	1 5/16	11⅞	12	⅞	24†	36	2¾	32	24	1⅝
8	15	1⅜	13	12	⅞	......	......	......	......	......	......

* Dimensions given are for flanges to withstand 250 lb. pressure with superheated steam. Additional thicknesses can be furnished for working pressures up to 1000 lb.

† Outside diameter.

Pipe Joints for Special Service. Various styles of pipe joints made for special service lines are shown in Fig. 21, namely, the **inserted screwed joint; flush screwed joint; Matheson joint; and Converse lock joint** for plain-end pipe. Sizes and weights of these and other joints are given in the National Tube Co.'s "Book of Standards."

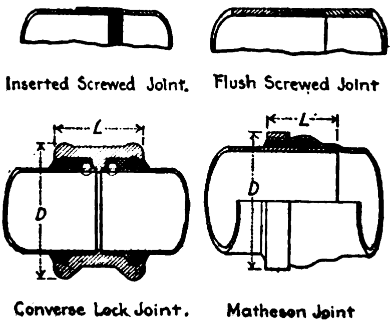


Fig. 21.—Pipe Joints for Special Service.

For water pipe which does not have to stand very high pressures, **leaded joints** are often used. The most common leaded joints are the Converse lock joint and the Matheson joint. The Converse joint is made by means of a special cast-iron coupling or hub which has a groove on each end extending around just inside of the end of the coupling, and two tee-shaped grooves on each end a short distance in from the circular groove. The pipe has two holes punched a short distance from the end on opposite sides into which rivets are driven. In making up this joint, the heads of the rivets slip into the tee-shaped slots of the hub, and the pipe is turned slightly, thus holding the pipe from pulling out of the hub endwise. This joint is then made tight by pouring lead into the circular slot and calking. The Matheson joint is another type of lead joint used for water or gas.

Gaskets. For steam and water lines where the pressure does not exceed 160 lb., wire-insertion rubber gaskets 1/16 in. thick give good service. For low-pressure lines, canvas-insertion black rubber gaskets are ordinarily used. For pressures above 160 lb. in lines carrying superheated steam, corrugated steel gaskets covering the entire annular area inside the bolt holes are highly satisfactory. Wire-inserted rubber gaskets are used for high-pressure water lines, and canvas-inserted rubber gaskets for low-pressure flanged joints.

Flange Faces for Retaining Gaskets. Flange faces are machined in various ways in order to retain the gaskets used to make fluid-tight joints. The **plain straight face** is commonly employed for pressures less than 125 lb. on steam and water lines. The best results are obtained by using a fairly thick gasket, so that the gasket will have sufficient pressure exerted on it by the bolts to make a tight joint before the outside edges of the flanges meet. The full-faced gasket is preferred by some, because it may be installed more readily and is more likely to be concentric with the bore of the flange than a ring gasket, but it has no further advantages.

In the **raised-face type** the face of the flange between the bore and inside of the bolt holes is from 3/32 to 1/16 in. above that of the remainder of the flange. It is most satisfactory on high-pressure steam lines and is very generally used. Ring gaskets are employed. The raised face prevents the touching of the outside edges of the flanges, and the entire pressure exerted by the bolts is transmitted to the gasket, which gives a maximum efficiency and resistance against leakage.

The **raised-face** type **with ground joints** has been extensively used in the past on superheated steam lines. As gaskets are now obtainable that will work successfully under temperatures as high as 800 deg. fahr., this type is becoming less frequently employed.

In the **tongue-and-groove** joint the gasket is prevented from blowing outward or squeezing inward, and its area may be considerably smaller than when unprotected. The pressure obtainable on the gasket is therefore greater than that in any other form of joint. It is especially adapted for high-pressure water, gas and air lines, and for ammonia piping.

In the **male-and-female** joint, used on high-pressure steam lines and on hydraulic lines with cup-shaped gaskets, the gasket width is greater than that in the tongue-and-groove joint. Both tongue-and-groove and male-and-female joints are expensive to install and difficult to disassemble.

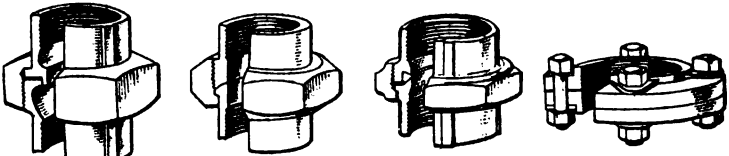
The **plain-face corrugated** joint has coarse concentric grooves upon each flange face, cut with a round-nose tool. It is used where an exceptionally thick gasket is called for, the corrugations tending to prevent the gasket from blowing out. In the **plain-face scored** joint the concentric grooves are finer and are made with a diamond-point tool. This type is used on oil or acid lines, where lead gaskets must be employed, the lead squeezing into the scores and effecting a tight joint without undue strain on the bolts and flanges.

The increasing use of **ball-shaped flanges** having inserted parts and non-corrosive rings is due to the fact that screwed unions of this type are being made to this construction.

Unions are usually classified as **nut unions** and **flange unions** (see Fig. 22). Nut unions are commonly used in sizes 2 in. and smaller, and flange unions in sizes larger than 2 in. However, many manufacturers make nut unions as large as 4 in. and flange unions smaller than 2 in.

Nut unions are made in malleable iron, brass and malleable iron, and all brass. The all-malleable-iron union (lip union) is the standard malleable union of the trade and requires a gasket. The brass and malleable-iron union (known as the "Kewanee" union) is a much better union, because no gasket is required and there is no possibility of the parts rusting together. The pipe end of the "Kewanee" union which carries an external thread, called the thread end, upon which the nut or ring screws, is made of brass, and the other pipe end (called the bottom) and nut or ring are made of malleable iron. The seat formed by the brass and iron pipe ends when brought together is truly spherical, and the harder iron is sure to make a perfect joint with the softer brass.

The brass and malleable-iron union with the inserted brass ring, known as the brass-to-iron seat, also eliminates the use of a gasket and is extensively used. The brass rings are forced into the groove under pressure and then



Lip Union. A. Kewanee Union. B Brass to Iron Seat Union. C. Flange Union. D.

FIG. 22.—Types of Pipe Unions.

ground to insure a tight joint, so that there is no possible chance for the ring to work loose. This union is also made with a brass union ring in order to provide a non-corrosive brass-and-iron thread joint that may be readily broken.

All-brass unions are made with a spherical or conical seat, no gaskets being required. Finished all-brass unions are often used where showy work is desired, such as oil piping for engines, etc.

Flange unions are made of both cast iron and malleable iron in three weights, standard, extra heavy, and hydraulic.

Expansion Joints for Steam Pipe Lines. The linear expansion of a long pipe must be taken care of by the use of some form of expansion joint or bend. The coefficients of expansion a of three metals used in pipe manufacture, within the range of temperatures common to boiler practice (32 to 392 deg. fahr.), were recently determined at the Bureau of Standards, Washington, D. C., as follows: Charcoal iron, 0.000006861; Bessemer steel, 0.000006989; seamless open-hearth steel (hot-finished), 0.000006883. The length of a tube at t deg. fahr. $= L_t = L\ (1 + at)$ in which L is the length of the tube at 0 deg. fahr. The theoretical expansion in 100 ft. of iron or steel pipe, carrying steam of various pressures and temperatures, based on an outside temperature of 32 deg. fahr., is as follows:

Gage pressure, lb. per sq. in.	30	100	125	150	175	200	Superheated steam				
Temperature of steam, deg. fahr.	274	338	353	366	377	388	420	500	550	600	650
Expansion of 100 ft. of pipe, in.	2.03	2.72	2.89	3.05	3.19	3.31	3.73	4.77	5.48	6.23	7.03

Table 44 gives the linear expansion in 100 ft. of cast-iron, wrought-iron, steel, brass or copper pipe. The expansion for a pipe of any length between any two given temperatures is found by taking the difference in length at these temperatures, dividing by 100 and multiplying by the length of the pipe in feet.

Table 44. Linear Expansion of Pipes
(Increase of length of 100 ft. of pipe, in inches)

Temp., deg. fahr.	Cast iron	Wrought iron	Steel	Brass and copper	Temp., deg. fahr.	Cast iron	Wrought iron	Steel	Brass and copper
0	0.00	0.00	0.00	0.00	450	3.89	4.28	4.08	6.18
50	0.36	0.40	0.38	0.57	475	4.20	4.62	4.41	6.68
100	0.72	0.79	0.76	1.14	500	4.45	4.90	4.67	7.06
125	0.88	0.97	0.92	1.40	525	4.75	5.22	4.99	7.55
150	1.10	1.21	1.15	1.75	550	5.05	5.55	5.30	8.03
175	1.28	1.41	1.34	2.04	575	5.36	5.90	5.63	8.52
200	1.50	1.65	1.57	2.38	600	5.70	6.26	5.98	9.06
225	1.70	1.87	1.78	2.70	625	6.05	6.65	6.35	9.62
250	1.90	2.09	1.99	3.02	650	6.40	7.05	6.71	10.18
275	2.15	2.36	2.26	3.42	675	6.78	7.46	7.12	10.78
300	2.35	2.58	2.47	3.74	700	7.15	7.86	7.50	11.37
325	2.60	2.86	2.73	4.13	725	7.58	8.33	7.96	12.06
350	2.80	3.08	2.94	4.45	750	7.96	8.75	8.36	12.66
375	3.15	3.46	3.31	5.01	775	8.42	9.26	8.84	13.38
400	3.30	3.63	3.46	5.24	800	8.87	9.76	9.31	14.10
425	3.68	4.05	3.86	5.85					

Figs. 23 and 24 show **corrugated copper expansion joints** for high and low pressures, respectively. Fig. 25 shows a **globe-type joint** and Fig. 26 a **non-collapsible joint,** both of copper, and for low-pressure work. Face-to-face dimensions of these joints are given in Table 45.

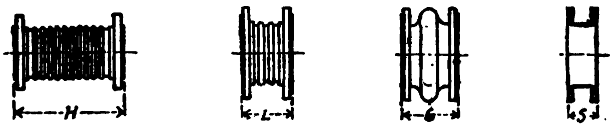


FIG. 23. FIG. 24. FIG. 25. FIG. 26.
Copper Expansion Joints for Steam Lines.

Table 45. Face-to-face Dimensions of Copper Expansion Joints
(All dimensions in inches. Letters refer to Figs. 23–26. Dimensions *H* and *L* are given by Alberger Pump & Condenser Co.; *G* and *S*, by Crane Co.)

Size, in.	*H*	*L*	*G*	*S*	Size, in.	*H*	*L*	*G*	*S*	Size, in.	*L*	*G*	*S*	Size, in.	*G*
3		......	....	....	14	30	10½	12	6	34	....	17	8½	56	22
3½	16	......	....	....	15	....	10½	12	6	36	17	17	8½	58	23
4	16	......	....	....	16	30	10½	12	6	38	....	18	9	60	23
4½	16	......	....	....	18	30	12¾	13	6½	40	....	18	9	62	24
5	16	......	....	....	20	....	12¾	13	6½	42	17	19	9	64	24
6	18	8⅝	....	....	22	....	12¾	14	7	44	....	19	9	66	25
7	18	......	....	....	24	....	12¾	14	7	46	....	20	9	68	25
8	18	8⅝	....	....	26	....	17	15	8	48	17	20	9	70	26
9	18	8⅝	....	....	28	....	17	15	8	50	....	21	......	72	26
10	24	8⅝	11	6	30	....	17	16	8½	52	....	21	......	......	....
12	24	8⅝	11	6	32	....	......	16	8½	54	....	22	......	......	....

Cast-iron standard and double expansion joints are shown in Figs. 27 and 28, and the dimensions of various sizes are given in Table 46. Cast-iron double expansion joints are of the same general proportions as standard expansion joints and are more particularly designed for underground work.

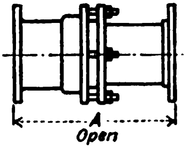


Fig. 27.

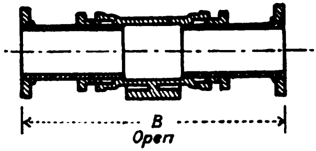


Fig. 28.

Cast-iron Expansion Joints for Steam Lines.

Table 46. Dimensions of Expansion Joints for 125 and 250 Lb. Steam Working Pressure

(Crane Co.)

(All dimensions in inches. Letters refer to Figs. 27 and 28)

Size, in.	125 lb. A	250 lb. A	125 lb. B	Size, in.	125 lb. A	250 lb. A	125 lb. B	Size, in.	125 lb. A	250 lb. A	125 lb. B
2	11 3/8	15 1/2	28	6	19 1/2	25 3/4	34	15	40	43 1/8	46 1/2
2 1/2	11 11/16	16	28 1/2	7	22 9/16	28 1/2	36 1/4	16	41	45	48 1/2
3	12 11/16	17 5/8	29 1/2	8	25 5/16	31 1/2	37 3/4	18	42 1/4	46 1/8	....
3 1/2	13 13/16	18 11/16	30 3/4	9	26 1/4	31 5/8	39 3/4	20	47 1/2	48	....
4	14 3/4	19 1/2	31 3/4	10	27	33 5/8	39 1/4	22	49 1/4	56	....
4 1/2	15 1/8	21	32	12	30 5/8	37 11/16	43	24	51	63	....
5	16 5/8	22 3/8	32 1/2	14	39 1/4	43	45	....	....	....	....

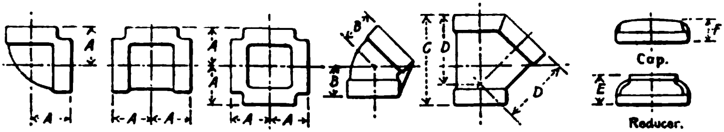
Balanced expansion joints are used where the expansion of the steam line is so great as to make it impracticable to use any design of joint or bend which depends for its operation upon the elasticity of the metal. They are similar in design to the ordinary unbalanced expansion joint, with the addition of a balancing cylinder on each side. These two cylinders are of practically the same combined area as the main sleeve, and eliminate the tendency of the unbalanced expansion joint to pull itself apart.

Screwed Fittings. Malleable-iron screwed fittings are made for standard, extra heavy, and hydraulic sizes of pipe. Plain standard fittings are generally used for low-pressure gas and water, as in house plumbing and railing work, while the beaded fitting is the standard steam, air, gas, or oil fitting. **Cast-iron** screwed fittings are made in standard and extra heavy sizes 1/4 in. to 12 in. inclusive. It is not considered good practice, however, to use screwed fittings of any kind in sizes larger than 6 in. The **length of thread** on pipe that is screwed into valves or fittings, as recommended by Crane Co., is as follows:

Size of pipe, in......	1/8	1/4	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	
Length of thread, in.	1/4	3/8	3/8	1/2	1/2	9/16	5/8	5/8	11/16	15/16	
Size of pipe, in......	3	3 1/2	4	4 1/2	5	6	7	8	9	10	12
Length of thread, in.	1	1 1/16	1 1/16	1 1/8	1 3/16	1 1/4	1 1/4	1 5/16	1 3/8	1 1/2	1 3/4

Table 47. Approximate Weights and Dimensions of Standard Screwed Fittings

(Crane Co.)



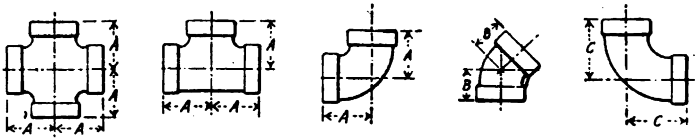
(Letters refer to figures above)

Size, in.	Malleable iron									
	Dimensions in inches						Weight in lb. per 100 pieces			
	A	B	C	D	E	F	Ells	45° Ells	Tees	Crosses
1/4	13/16	3/4	......	......	1	5/8	13	11	14	......
3/8	15/16	13/16	......	......	1 1/8	3/4	17	14	23	21
1/2	1 1/8	7/8	2 1/2	1 11/16	1 1/4	7/8	27	21	35	42
3/4	1 5/16	1	2 7/8	2	1 7/16	1 1/16	39	32	55	54
1	1 7/16	1 1/8	3 3/16	2 7/16	1 11/16	1 3/16	60	50	80	96
1 1/4	1 3/4	1 5/16	4 1/16	2 15/16	2 1/16	1 1/4	105	80	136	152
1 1/2	1 15/16	1 7/16	4 1/2	3 5/16	2 5/16	1 5/16	131	111	183	197
2	2 1/4	1 11/16	5 7/16	4 1/16	2 13/16	1 7/16	232	197	285	340
2 1/2	2 11/16	1 15/16	6 1/4	4 11/16	3 1/4	1 5/8	420	350	428	575
3	3 1/8	2 3/16	7 1/4	5 9/16	3 11/16	1 3/4	637	483	742	960
3 1/2	3 7/16	2 3/8	......	......	4	1 15/16	940	665	1,000	1,040
4	3 3/4	2 5/8	8 7/8	6 15/16	4 3/8	2	1,100	775	1,200	1,550

Size, in.	Cast iron									
	A	B	C	D	E	F	Ells	45° Ells	Tees	Crosses
1/4	13/16	3/4	......	......	......	......	14	......	20	......
3/8	15/16	13/16	......	......	......	......	24	24	32	......
1/2	1 1/8	7/8	2 1/2	1 7/8	......	......	40	37	53	70
3/4	1 5/16	1	3	2 1/4	......	......	55	55	81	100
1	1 7/16	1 1/8	3 1/2	2 3/4	......	......	93	84	122	150
1 1/4	1 3/4	1 5/16	4 1/4	3 1/4	2 1/8	......	152	138	200	238
1 1/2	1 15/16	1 7/16	4 7/8	3 13/16	2 1/4	......	192	196	268	350
2	2 1/4	1 11/16	5 3/4	4 1/2	2 7/16	......	318	284	430	530
2 1/2	2 11/16	1 15/16	6 1/4	5 3/16	2 11/16	......	500	440	650	785
3	3 1/8	2 3/16	7 7/8	6 1/8	2 15/16	......	700	660	1,000	1,100
3 1/2	3 7/16	2 3/8	8 7/8	6 7/8	3 1/8	......	920	850	1,325	1,550
4	3 3/4	2 5/8	9 3/4	7 5/8	3 3/8	2 1/16	1,250	1,125	1,780	2,150
4 1/2	4 1/16	2 13/16	11 5/8	9 1/4	3 5/8	2 3/16	1,600	1,450	2,330	2,700
5	4 7/16	3 1/16	11 5/8	9 1/4	3 7/8	2 3/8	2,100	1,650	2,620	3,000
6	5 1/8	3 7/16	13 7/16	10 3/4	4 3/8	2 5/8	3,000	2,500	4,000	4,300
7	5 13/16	3 7/8	15 1/4	12 1/4	4 13/16	2 7/8	4,400	3,500	5,500	6,600
8	6 1/2	4 1/4	16 15/16	13 5/8	5 1/4	3 1/8	5,500	4,600	7,900	8,300
9	7 3/16	4 11/16	20 11/16	16 3/4	5 11/16	3 3/8	7,800	6,900	10,200	13,600
10	7 5/8	5 5/16	20 11/16	16 3/4	6 3/16	3 5/8	11,100	8,600	14,900	15,400
12	9 1/4	6	24 1/8	19 5/8	7 1/8	4 1/4	16,800	12,500	21,500	25,500

Table 48. Approximate Weights and Dimensions of Extra Heavy Screwed Fittings

(Crane Co.)



(Letters refer to figures above)

Size, in.	Malleable iron						
	Dimensions in inches			Weight in lb. per 100 pieces			
	A	B	C	Ells	45° Ells	Tees	Crosses
¼	1 1/16	¾	....	20	20	34	42½
⅜	1¼	⅞	....	38	25	64	81
½	1½	1	....	62	49	92	106
¾	1¾	1⅛	....	97	69	133	163
1	2	1 5/16	2½	134	105	200	236
1¼	2¼	1½	3	223	175	320	378
1½	2½	1 11/16	3½	316	232	420	503
2	3	2	4	460	370	660	800
2½	3½	2¼	4¾	720	538	1,000	1,200
3	4⅛	2½	5½	1,065	763	1,600	2,000
3½	4⅝	2⅝	6¼	1,500	920	2,200	2,600
4	5⅛	2 13/16	7	2,000	1,250	2,950	3,240
	Cast iron						
1	2	1⅜	....	196	155	285	305
1¼	2¼	1½	....	292	248	400	510
1½	2 9/16	1⅝	....	408	335	525	680
2	3	1 15/16	....	650	548	925	1,080
2½	3½	2¼	....	900	950	1,400	1,750
3	4⅛	2½	....	1,350	1,400	2,000	2,980
3½	4 11/16	2 9/16	....	1,900	1,750	2,600	3,300
4	5⅛	2¾	....	2,500	2,300	3,800	4,900
4½	5½	3	....	3,000	2,800	4,400	6,300
5	6⅛	3 5/16	....	3,900	3,600	6,000	7,200
6	7¼	3¾	....	6,200	5,500	9,000	11,300
7	8⅛	4	....	8,800	7,500	12,700	16,300
8	9⅛	4¾	....	12,500	9,800	17,500	22,000
10	11⅜	4⅞	....	28,000	15,000	39,000	40,000
12	13⅜	5½	....	40,000	20,300	60,600	70,400

Railing Fittings. Fittings of special construction and of lighter weight than standard steam, gas, and water pipe fittings are widely used for hand railings around area ways, on stairs, for office enclosures with gates, and for permanent ladders. Railing fittings are made in various styles, generally globe-shaped in body, with ends reduced to take thread and recessed to cover all threads. They are furnished in malleable iron, black and galvanized, and in brass, in sizes up to 2 in. Larger sizes are made to order.

Special railing-fitting joints are available, such as the slip-and-screwed joint, where the post connection is screwed and the rim of the fitting is so made that the rail will slip into the fitting and allow for an angular variation of

Table 49. Weights of Cast-iron Flanged Fittings for Steam
(American Standard Dimensions)

Size, in.	Approximate weight per piece, lb.							
	Standard (125 lb.)				Extra heavy (250 lb.)			
	Ell	45° Ell	Tee	Cross	Ell	45° Ell	Tee	Cross
2	18	15	26	34	23	20	38	80
2½	22	20	34	43	34	29	50	85
3	30	27	45	58	46	38	70	90
3½	37	33	55	74	57	44	75	115
4	45	38	67	89	67	61	100	140
4½	46	43	75	100	85	70	120	170
5	63	53	90	121	95	85	130	190
6	75	68	115	152	125	105	190	250
7	100	90	150	200	160	145	235	325
8	120	100	170	236	190	175	280	370
9	150	130	220	305	240	195	330	480
10	205	160	285	400	320	250	450	580
12	285	230	430	570	450	380	680	900
14	390	300	550	750	640	520	970	1300
15	440	330	660	800	750	570	1050	1400
16	525	400	760	1000	840	675	1255	1675

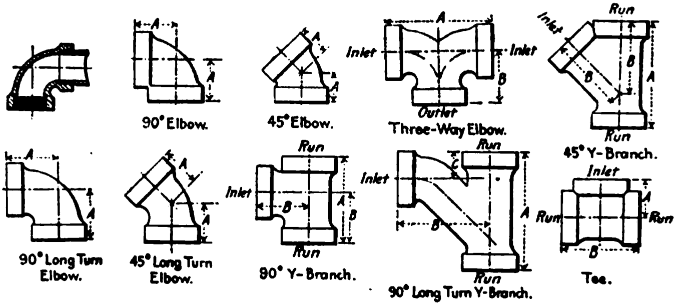
several degrees, being fastened by pins which are riveted over and filed smooth. The flush-joint stair-rail fitting is another special style of fitting which provides a hand rail with even surfaces at the joints. Approximate weights are as follows:

Approximate Weights of Standard Malleable Railing Fittings (Lb.)

Type of fitting	Size of fitting, inches					
	½	¾	1	1¼	1½	2
Ells, regular	0.32	0.55	0.67	1.08	1.77	2.49
Ells, side outlet	0.34	0.56	0.76	1.10	2.03	2.75
Tees, regular	0.36	0.57	0.77	1.17	1.95	2.52
Tees, side outlet	0.38	0.59	0.75	1.20	2.13	2.98
Crosses, regular	0.38	0.48	0.86	1.27	1.75	2.60
Crosses, side outlet	0.43	0.67	0.83	1.56	2.04	3.00
Railing flanges	0.47	0.40	0.52	0.79	1.14	1.67
Do., globe pattern	0.66	0.60	0.80	1.36	1.65	3.00
Railing acorns	0.35	0.36	0.47	0.83	1.10	1.90

Drainage Fittings, as shown in the figures accompanying Table 50, have no pockets for the lodgment of solids, and the length of the thread chamber is such that when the pipe is threaded to the Briggs standard dimensions, the end of the pipe will practically touch the shoulder when screwed in. They are especially adapted to plumbing work and vacuum cleaning pipe installations.

Table 50. Dimensions of Drainage Fittings*



(Crane Co.—All dimensions in inches)

Size, in.	90-deg. elbows	45-deg. elbows	90-deg. long-turn elbows	45-deg. long-turn elbows	Three-way elbows		Tees		90-deg. Y-branches		90-deg. long-turn Y-branches			45-deg Y-branches	
	A	A	A	A	A	B	A	B	A	B	A	B	C	A	B
1 1/4	1 3/4	1 5/8	2 1/4	......	......	......	......	....	3 3/4	2 1/4	4 3/4	3 5/8	1 3/16	5	3 1/4
1 1/2	2 3/16	1 7/16	2 1/2	1 7/8	5 1/4	2 5/8	2 3/16	4 3/8	4 1/4	2 1/2	5 3/8	4 1/8	1 1/4	5 1/2	3 5/8
2	2 3/8	1 3/4	3 1/16	2 1/4	6 1/4	3 1/8	2 5/16	4 5/8	5 3/16	3 1/16	7 1/16	5 7/16	1 5/8	6 3/16	4 5/16
2 1/2	2 13/16	2 1/16	3 11/16	2 5/8	7 3/8	3 11/16	......	....	6 5/16	3 11/16	8 1/4	6 1/4	2	7 7/8	5 3/8
3	3 3/16	2 3/8	4 1/4	2 7/8	8 5/8	4 5/16	3 3/16	6 3/8	7 1/4	4 1/4	9 13/16	7 1/2	2 5/16	9	6 3/16
4	3 13/16	2 3/4	5 3/16	3 7/16	10 3/8	5 3/16	4	8	8 3/4	5 3/16	13 3/4	9 7/8	2 7/8	10 7/8	7 11/16
5	4 1/2	3 3/16	6 1/8	4 1/8	......	......	4 5/8	9 1/4	10 5/16	6 1/8	15 3/4	12 1/4	3 1/2	12 15/16	9 3/16
6	5 3/16	3 1/2	7 1/8	4 7/8	......	......	5 3/16	10 3/8	11 15/16	7 1/8	18 11/16	14 9/16	4 1/8	14 3/4	10 3/4
7	5 13/16	3 7/8	8 1/8	5 1/2	......	......	5 13/16	11 5/8	13 5/8	8 1/8	21 5/8	16 7/8	4 3/4	16 11/16	12 1/4
8	6 1/2	4 3/16	9	6	......	......	6 1/2	13	15 5/16	9	24 9/16	19 5/16	5 1/4	18 13/16	13 3/16

* Other fittings which are available are as follows: 5 5/8-deg., 11 1/4-deg., 22 1/2-deg. and 60-deg. elbows; basin crosses; double 90-deg. Y-branches; double-90 deg. long-turn Y-branches; 45-deg. double Y-branches; S-traps; half S-traps; running traps; offsets.

Ammonia Fittings are made of especially homogeneous material and usually have the mouth countersunk and both the mouth and thread tinned.

Ammonia pipe is lap-welded except under 2 in., where it is butt-welded and redrawn, as leakage occurs when butt-welded pipe is used. It should be galvanised on the outside.

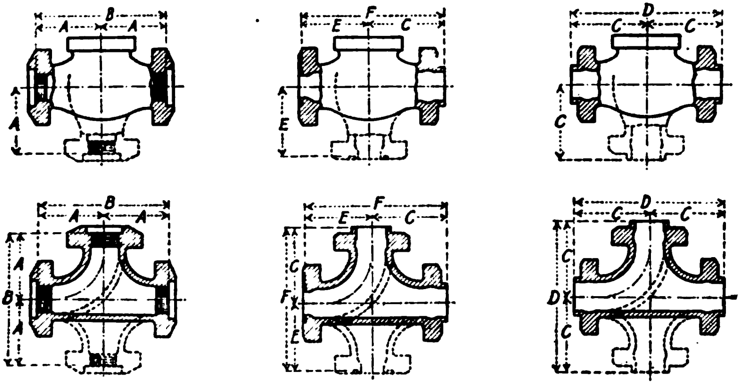
Ammonia joints should be made of wrought iron or steel, as ammonia attacks and eats away copper and its alloys, brass and gun metal. On account of the penetrating nature of ammonia, all flanges should be screwed on and then soldered to the pipes. Lead washers should be used for gaskets on all flange joints. Lead or white-metal packing should be used for all valves. See also p. 1728.

The following information on ammonia fittings, valves, etc., is taken from Crane Co.'s special catalog of "Valves and Fittings for Ammonia:"

Valves and fittings for ammonia are suitable for working pressures up to 250 lb. per sq. in., and are subjected to an air-under-water test of 300 lb. Different sizes of ammonia valves have been tested to 4000 lb. per sq. in., without breaking. The material used is ferro-steel, which has an average tensile strength of 34,500 lb. Malleable iron is used for screwed ammonia fittings and has an average tensile strength of 37,000 lb. These fittings have stood tests from 3500 to 8000 lb. per sq. in. without breaking.

Valves, elbows, tees and crosses of the same type and size have the same center-to-end dimensions. A screw gland-end globe valve, for example, may therefore be substituted for a screw gland-end cross or tee, a tongue-end angle valve for a tongue-end elbow, etc.

The trimmings of valves are standardized so that the bonnet of a globe valve may be placed on an angle or a cross-valve body, and *vice versa*. The trimmings of any size valve are the same, whatever the end connections of the valve may be. Bolted bonnets for ammonia are in every way superior to screwed bonnets. Ammonia valves have the same proportions, etc., as extra heavy steam valves.



Gland End. Tongue and Groove. Boyle End.

Fig. 29.—Ammonia Fittings.

Table 51. Dimensions of Ammonia Valves and Fittings

(Crane Co. All dimensions in inches. Letters refer to Fig. 29)

Style of flange	Size of valve or fitting, in.	Center to center of screw-gland-end valves, ells, tees and crosses A	Face to face of screw-gland-end valves, tees or crosses B	Center to face of screw-gland-end valves, ells, tees or crosses C	Face to face of Boyle-end valves, tees or crosses D*	Center to face of groove-end valves, ells, tees or crosses E	Face to face of tongue-and-groove valves, tees or crosses F	Center to end of plain screw valves	End to end of plain screw valves
Oval	¼	2¼	4½	2⅞	5¾	2 7/16	5 5/16	2¼	4½
	⅜	2⅜	4¾	3	6	2 9/16	5 9/16	2⅜	4¾
	½	2½	5	3⅛	6¼	2 11/16	5 13/16	2½	5
	¾	2¾	5½	3⅜	6¾	2 15/16	6 5/16	2¾	5½
Square	1	3⅛	6¼	3 9/16	7⅛	3⅛	6 11/16	3⅛	6¼
	1¼	3⅜	6¾	4	8	3⅜	7⅜	3⅜	6¾
	1½	3¾	7½	4⅜	8¾	3¾	8⅛	3¾	7½
	2	4¼	8½	4⅞	9¾	4¼	9⅛	4¼	8½
	2½	4¾	9½	5 9/16	11⅛	4¾	10 5/16	....	....
Round	3	6	12	6 13/16	13⅝	6	12 13/16	....	....
	3½	6½	13	7 5/16	14⅝	6½	13 13/16	....	....
	4	6⅞	13¾	7 11/16	15⅜	6⅞	14 9/16	....	....
	5	7¾	15½	8 11/16	17⅜	7¾	16 7/16	....	....
	6	8¾	17½	9 11/16	19⅜	8¾	18 7/16	....	....

Note.—On tongue-and-groove valves the groove end is on the end next to the under side of disk, and the tongue end or ends on the end or ends next to the upper side of disk.

* **Made to order only.**

Table 52. Dimensions of Ammonia Glands, Flanges and Templates for Drilling (Fig. 30)

(Crane Co.)

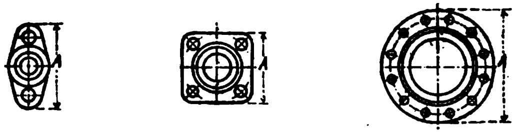


FIG. 30.

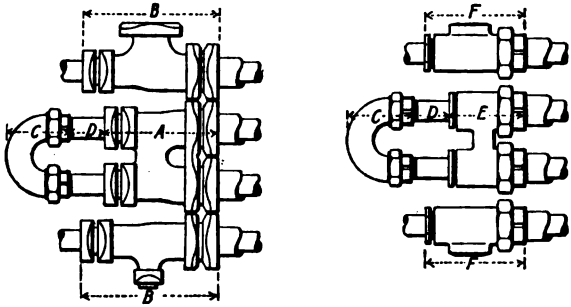
Style of flange	Oval				Square					Round				
Size, in.	1/4	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	4	5	6
Flange diam. *A*, in.	3 5/8	3 13/16	4	4 3/4	3 9/16	3 15/16	4 11/16	5 1/16	5 15/16	8 1/4	9	10	11	12 1/2
Bolt circle diam., in.	2 3/8	2 9/16	2 3/4	3 1/4	3 1/4	3 3/4	4 1/2	5	5 7/8	6 5/8	7 1/4	7 7/8	9 1/4	10 5/8
Number of bolts	2	2	2	2	4	4	4	4	4	8	8	8	8	12
Bolt diam., in.	1/2	1/2	1/2	5/8	1/2	1/2	5/8	5/8	3/4	5/8	5/8	3/4	3/4	3/4
Length of bolts, in.*	2 3/4	2 3/4	2 3/4	3 1/4	2 3/4	3	3 1/4	3 1/2	4 1/4	4 1/4	4 1/2	4 3/4	5 1/4	5 1/4
Length of bolts, in.†	2 3/4	2 3/4	2 3/4	3 1/4	2 1/2	2 3/4	3	3	3 1/2	3 3/4	3 3/4	4	4 1/2	4 1/2

* For Boyle joints. † For tongue-and-groove joints.

Table 53. Outside Dimensions of Double-pipe Ammonia Fittings in Inches (Fig. 31)

(Crane Co.)

Ammonia Condenser				Ammonia Brine Cooler				Water Cooler			
A	*B*	*C*	*D*	*A*	*B*	*C*	*D*	*C*	*D*	*E*	*F*
9 5/16	11 1/4	5 3/16	3	10 7/16	11 13/16	6 11/16	3	5 3/16	3	6 1/8	8



Ammonia Condenser and Brine Cooler.

Water Cooler.

FIG. 31.—Double-pipe Ammonia Fittings.

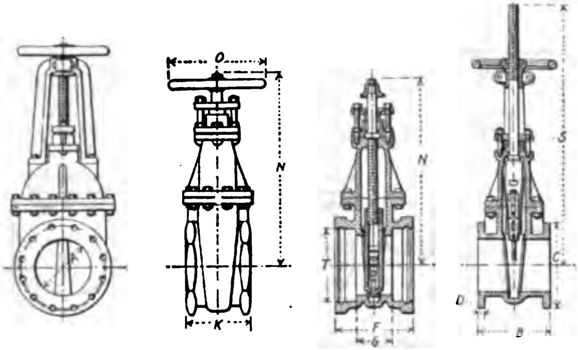
Standard thread lengths are used in making up all ammonia connections except Boyle joints (see p. 833). Lengths of threads for Boyle joints are as follows:

Size of pipe, in.	1/4	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	3 1/2	5	6
Length of thread, in.	1 1/4	1 1/4	1 1/4	1 3/8	1 3/8	1 1/2	1 5/8	1 5/8	2	2 1/8	2 1/8	2 1/2	2 5/8
No. of threads per in.	18	18	14	14	11 1/2	11 1/2	11 1/2	11 1/2	8	8	8	8	8

VALVES

Table 54. Dimensions of Standard Straight-way Valves

(Crane Co.)



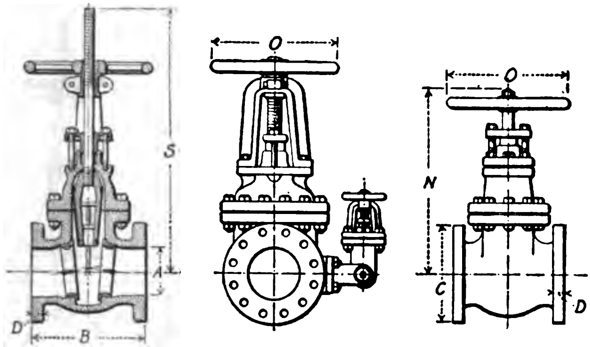
(Iron body with brass trimmings: sizes 16 in. and smaller for steam working pressures up to 125 lb.; sizes 18 in. and larger for steam working pressures up to 100 lb. Tested to 150 lb. hydraulic pressure per sq. in. All dimensions in inches. Letters refer to dimension sketches above. For drilling templates see Table 34.)

Size, A	B	C	D	K	N	O	S	F	T	G*	No. of turns to open
2	7	6	5/8	5 7/16	11 3/4	6 1/2	14 1/2	8 1/2	3 1/4	3	7
2 1/2	7 1/2	7	11/16	5 7/8	12 3/4	6 1/2	16	......	......	3 1/4	8
3	8	7 1/2	3/4	6 1/8	14 1/4	7 1/2	19	9	4 3/4	3 1/2	10 1/4
3 1/2	8 1/2	8 1/2	13/16	6 5/8	15 1/4	7 1/2	21 1/4	......	......	......	10 1/8
4	9	9	15/16	6 7/8	16 1/4	9	24	10 1/4	5 5/8	4 1/4	8 3/4
4 1/2	9 1/2	9 1/4	15/16	7 1/8	17 5/8	9	25 1/2	......	......	......	9
5	10	10	15/16	7 3/8	19	10	28 1/2	10 1/4	6 3/4	4 1/4	11
6	10 1/2	11	1	7 3/4	20 3/4	12	31 3/4	10 3/4	7 7/8	4 3/4	12 5/8
7	11	12 1/2	1 1/16	8 1/4	23	12	37 1/4	10 3/4	9	4 3/4	15 1/8
8	11 1/2	13 1/2	1 1/8	8 3/4	26	14	41	12	10	5	16
9	12	15	1 1/8	9 1/4	28	14	44 3/4	......	......	......	18 3/4
10	13	16	1 3/16	9 7/8	30 1/4	16	50	12 3/4	12 1/4	5 3/4	20 1/8
12	14	19	1 1/4	11 5/8	35 1/4	18	57 1/4	13 1/2	14 3/8	6 1/2	24 1/8
14	15	21	1 3/8	......	39 1/4	20	66 3/4	13 3/4	16 1/2	6 3/4	28 1/4
15	15	22 1/4	1 3/8	......	41 1/8	20	69 3/4	......	......	......	31 1/8
16	16	23 1/2	1 7/16	......	44 1/4	22	75 1/4	16	18 3/4	8	33 1/4
18	17	25	1 9/16	......	48 3/4	24	86	17	20 7/8	9	35 1/8
20	18	27 1/2	1 11/16	......	52 1/2	24	91	17	23	9	42 1/4
22	19	29 1/2	1 13/16	......	55 1/2	27	100	......	......	......	46
24	20	32	1 7/8	......	62	30	109	18	27 1/4	10	50
26	23	34 1/4	2	......	65 7/8	30	117 1/2	22	29 3/8	14	65
28	26	36 1/2	2 1/16	......	70	36	125	26	31 1/2	17	80
30	30	38 3/4	2 1/8	......	75 1/2	36	133	30	34	21	92 1/2
36	36	45 3/4	2 3/8	......	83	......	158 1/2	32	40 1/2	23	108

* End to end of pipes in hub, without by-pass.

Table 55. Dimensions of Medium and Extra Heavy Straight-way Valves

(Crane Co.)



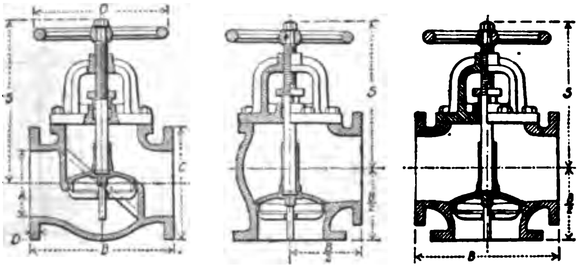
(Medium valves: ferro-steel body with special brass seats, for steam working pressures, sizes 16 in. and smaller, up to 175 lb.; sizes 18 in. and larger, up to 150 lb. Tested to 500 lb. hydraulic pressure per sq. in. Extra heavy valves: ferro-steel body with hard metal seats; for steam working pressure up to 250 lb. Tested to 800 lb. hydraulic pressure per sq. in. All dimensions in inches. Letters refer to dimension sketches above. For drilling templates see Table 35.)

Size, A	Medium								Extra heavy							
	B	B*	C	D	N	S	T†	W‡	B	B*	C	D	N	S	O	W‡
1¼	....	....	....	......	....	......	....	......	6½	5½	5	¾	8¾	10⅝	5	12
1½	....	....	....	......	....	......	....	......	7½	6¼	6	13/16	9⅝	12¼	5½	11
2	7½	5½	6½	⅞	11¼	14	....	6⅜	8½	7	6½	⅞	10½	13¾	6½	14
2½	8	6	7½	1	12¼	15½	....	7⅞	9½	8	7½	1	12⅞	16	7½	15
3	9½	7¼	8¼	1⅛	14	18⅜	....	9¾	11⅛	9	8¼	1⅛	14⅝	19½	9	14
3½	10	7½	9	1 3/16	14¾	20½	....	11¼	11⅞	10	9	1 3/16	15½	22	10	16
4	10½	7¾	10	1¼	16¼	23¾	....	9	12	11	10	1¼	17¾	24½	12	18
4½	11	8¼	10½	1 5/16	17	25	....	10	13¼	12¼	10½	1 5/16	18¾	27	12	21
5	11½	8½	11	1⅜	19	28⅛	....	11	15	13½	11	1⅜	20¼	29¾	14	23
6	12	8¾	12½	1 7/16	21	31⅞	1¼	25	15⅞	15⅞	12½	1 7/16	23	34⅛	16	28
7	12½	9¼	14	1½	22⅞	35¾	1¼	30	16¼	16¼	14	1½	24¾	38	18	30
8	13½	10	15	1⅝	25⅞	40¾	1½	34	16½	16½	15	1⅝	28¾	42¾	20	34
9	14	10¾	16	1¾	27½	44¼	1½	38	17	17	16	1¾	30½	47	20	40
10	15	11½	17½	1⅞	30⅛	49¾	1½	42	18	18	17½	1⅞	33¾	52¾	22	39
12	16	12½	20	2	33¾	56⅞	2	50	19¾	....	20	2	37¼	60	24	46
14	18	....	22½	2⅛	38½	64½	2	59	22½	....	22½	2⅛	42¾	67¾	24	52
15	18¾	....	23½	2 3/16	41	68¾	2	64	22½	....	23½	2 3/16	42¾	67¾	24	52
16	19½	....	25	2¼	44¾	74½	3	67	24	....	25	2¼	....	75¼	27	60
18	21	....	27	2⅜	47⅛	82¼	3	76	26	....	27	2⅜	....	82¼	30	67
20	22½	....	29½	2½	52	92	4	84	28	....	29½	2½	....	91½	30	74
22	....	....	....	......	....	......	....	......	29½	....	31½	2⅝	....	101	36	82
24	25½	....	34	2¾	61¾	108	4	87½	31	....	34	2¾	....	109	36	88

* End to end, screwed, inches.
† T = size of by-pass, inches.
‡ W = number of turns to open.

Table 56. Dimensions of Standard Globe, Angle and Cross Valves

(Crane Co.)



(Iron-body valves with yokes and brass trimmings, for steam working pressures up to 125 lb. Tested to 150 lb. hydraulic pressure per sq. in. All dimensions in inches. Letters refer to sketches above. For drilling templates, see Table 34.)

Size, A	B	B/2	C	D	S	O	Size, A	B	B/2	C	D	S	O
2	8	4	6	5/8	10 3/4	6 1/2	7	16	8	12 1/2	1 1/16	20 1/2	14
2 1/2	8 1/2	4 1/4	7	11/16	11 1/4	6 1/2	8	17	8 1/2	13 1/2	1 1/8	23 3/4	16
3	9 1/2	4 3/4	7 1/2	3/4	12 3/4	7 1/2	10	20	10	16	1 3/16	28	18
3 1/2	10 1/2	5 1/4	8 1/2	13/16	13	7 1/2	12	24	12	19	1 1/4	34	20
4	11	5 1/2	9	15/16	15 1/4	9	14	28	14	21	1 3/8	38 1/2	24
4 1/2	12	6	9 1/4	15/16	15 1/4	9	15	30	15	22 1/4	1 3/8	38 1/2	24
5	13	6 1/2	10	15/16	17 1/4	10	16	32	16	23 1/2	1 7/16	41 1/2	27
6	14	7	11	1	19	12	....	......	......	......	......	......	....

Crane Co. gives the following data relative to the quality of the **materials used for valves**: Tensile strength of brass, 30,000 lb. per sq. in.; cast iron, 22,500 lb.; ferro-steel, 33,500 lb.; hard metal (hard brass), 34,500 lb.; cast steel, 60,000 lb. Comparative destructive hydraulic tests on cast-iron and ferro-steel extra heavy gate valves showed the following results:

	Bursting pressures, lb. per sq. in.	
	Cast iron	Cast steel
Sizes 4 to 8 in.	1600 to 1900	2450 to 2600
Sizes 10 to 12 in.	1350 to 1550	1750 to 1900
Sizes 14 to 16 in.	1100	1200 to 1350

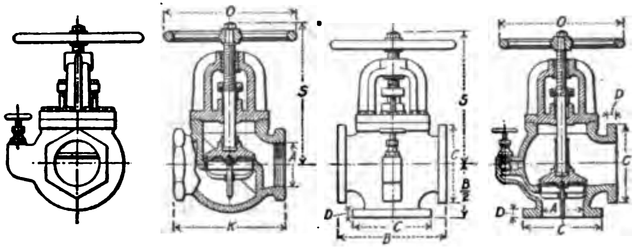
It is observed from these tests that the factor of safety is apparently very high, but in a steam line pressure is not the only consideration. Allowance must be made for the strains of expansion and contraction, settling, weight of piping, water hammer and the cutting effect of steam on the valve disk and seat.

The use of screwed valves larger than 6 in. is not recommended. On medium and extra heavy valves it is desirable to have a by-pass on sizes larger than 8 in.

For determining the water working pressures for valves and fittings based on the steam working pressures, the following rule is recommended as conservative. For sizes 12 in. and smaller add 40 per cent. to the steam working pressure; for sizes 14 in. and larger, add 20 per cent. A much greater range may be safely used for very small sizes.

Table 57. Dimensions of Medium and Extra Heavy Globe, Angle and Cross Valves

(Crane Co.)



(Medium valves: ferro-steel body with special brass seats, for steam working pressures up to 175 lb. Tested to 500 lb. hydraulic pressure per sq. in. Extra heavy valves: ferro-steel body with hard metal seats, for steam working pressures up to 250 lb. Tested to 800 lb. hydraulic pressure per sq. in. All dimensions in inches. Letters refer to dimension sketches above. For drilling templates, see Table 35.)

Size, A	Medium						Extra heavy					
	B	C	D	K	S	O	B	C	D	K	S	O
2	9	6½	⅞	7¾	11¾	6½	10½	6½	⅞	9½	13¾	7½
2½	10	7½	1	8	12⅝	7½	11½	7½	1	10¾	14½	9
3	11	8¼	1⅛	8¼	14¼	9	12½	8¼	1⅛	11¾	17½	10
3½	12	9	1 3/16	9½	15⅜	10	13¼	9	1 3/16	12¼	17½	10
4	13	10	1¼	10½	16⅜	10	14	10	1¼	13	19½	14
4½	13½	10½	1 5/16	11¼	17⅜	12	15	10½	1 5/16	14	19½	14
5	14½	11	1⅜	12¼	18¼	12	15¾	11	1⅜	15	21½	16
6	16	12½	1 7/16	14	20¼	14	17½	12½	1 7/16	16½	25	18
7	17½	14	1½	17	21¼	14	19¼	14	1½	18¼	26¼	20
8	20	15	1⅝	18½	24⅛	16	21	15	1⅝	20	29½	24
10	22½	17½	1⅞	22½	28½	20	24½	17½	1⅞	23¼	33½	27
12	25½	20	2	25½	31	20	28	20	2	......	39	30
14	......	......	......	......	......	......	33	22½	2⅛	......	42	36
15	......	......	......	......	......	......	33	23½	2 3/16	......	42	36

Cocks are generally designated under two headings, steam and gas, and are made with both brass and iron bodies. Brass-body cocks are regularly made in sizes from ¼ to 3 in., inclusive, and iron-body cocks in sizes from ¼ to 6 in., inclusive.

PIPE SUPPORTS

Pipes may be supported on rolls and chairs, anchored to brackets in various ways, and suspended from beams by hangers, or from below by floor stands. Some of the methods used are shown in Fig. 32.

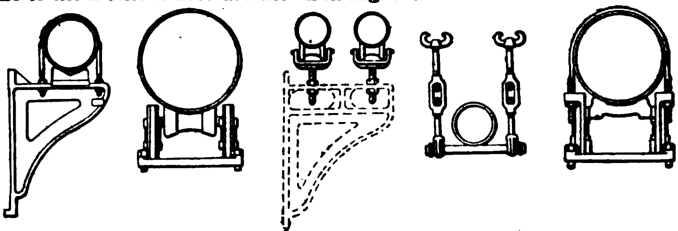


Fig. 32.—Methods of Supporting Pipes.

Tables 58 and 59 give dimensions of cast-iron rolls and chairs for pipe supports. Fig. 33 shows a heavy wall bracket adapted for any size of pipe from 5 to 20 in. in diam. Dimensions: *A*, 15 to 19 in.; *B*, 8 in.; *C*, 18¾ in.; *D*, 27 in. The universal adjustable bracket shown in Fig. 34 is made for pipe from 5 to 14 in. in diam. The inside dimension *A* ranges in six sizes from

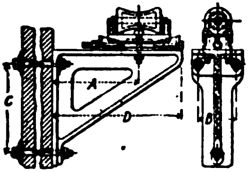


FIG. 33.

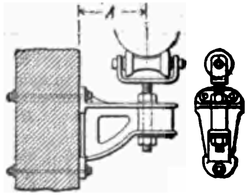


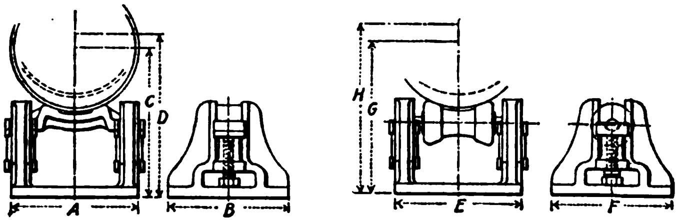
FIG. 34.

Wall Brackets for Supporting Pipes.

6 in. to 21 in.; the range of adjustment outward in each size is 3 in., and any adjustment *A* between 6 and 24 in. can be obtained from some one of the sizes made.

Table 58. Dimensions of Adjustable Cast-iron Rolls and Chairs for Supporting Pipe

(Crane Co.)

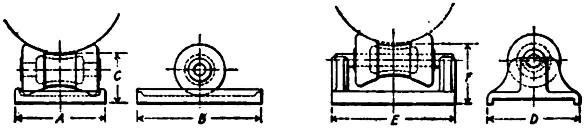


(Letters refer to figures above. All dimensions in inches)

Size	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
4	7½	7	7 9/16	9 1/16	7½	7	7⅝	9⅛
4½	7½	7	7 13/16	9 7/16	7½	7	7⅞	9⅜
5	7½	7	8 5/16	9 13/16	7½	7	8¼	9¾
6	7½	7	8 15/16	10 7/16	7½	7	8¾	10¼
7	10½	10	11 9/16	13 9/16	10½	10	11	13
8	10½	10	12¼	14¼	10½	10	11½	13½
9	10½	10	12 13/16	14 13/16	10½	10	12	14
10	10½	10	13½	15½	10½	10	12⅝	14⅝
12	12½	11	14¼	16¼	12½	11	14¼	16¼
14	12½	11	15⅛	17⅛	12½	11	14⅞	16⅞
15	12½	11	15¾	17¾	12½	11	15⅝	17⅝
16	14	12	16 1/16	18 1/16	14	12	16	18
18	14	12	17⅜	19⅜	14	12	17	19
20	14	12	18 9/16	20 9/16	14	12	18	20
22	15	12	19½	21½	15	12	19½	21½
24	15	12	20⅞	22⅞	15	12	20½	22½

Table 59. Dimensions of Cast-iron Rolls with Plates and Chairs

(Crane Co.)



(Letters refer to figures above. All dimensions in inches)

Size	*A*	*B*	*C*	*D*	*E*	*F*	Size	*A*	*B*	*C*	*D*	*E*	*F*
4	4½	8	2½	7⅛	6½	2¾	12	8½	10	4⅜	7⅛	11½	4¾
4½	4½	8	2½	7⅛	6½	2¾	14	8½	10	4⅜	7⅛	11½	4¾
5	4½	8	2½	7⅛	6½	2¾	15	8½	10	4⅜	7⅛	11½	4¾
6	4½	8	2½	7⅛	6½	2¾	16	9½	11	5	7⅛	13	5¼
7	6½	9	3¾	7⅛	9¼	4	18	9½	11	5	7⅛	13	5¼
8	6½	9	3¾	7⅛	9¼	4	20	9½	11	5	7⅛	13	5¼
9	6½	9	3¾	7⅛	9¼	4	22	10½	12	5¼	7⅛	14½	5⅝
10	6½	9	3¾	7⅛	9¼	4	24	10½	12	5¼	7⅛	14½	5⅝

PIPE COVERING

(See p. 305 for heat-transmission data, and p. 633 for properties of insulating materials.)

The value of a pipe covering is measured by the percentage of saving in condensation over that which occurs in bare pipe. This percentage ranges from 65 to 85 per cent., and even higher. The **standard covering** consists of 85 per cent. of carbonate of magnesia mixed with 15 per cent. of asbestos. For first-class work the thickness for high-pressure pipes is 2 in. up to 2 in. diam., 2½ in. up to 8 in. diam., and 3 in. on larger sizes. For low-pressure lines the thickness may be less. This covering is usually applied in molded sections about 3 ft. long, the seams of which are staggered and filled with magnesia plaster. For irregular flanges or fittings magnesia (or asbestos) cement is used. The sections are bound to the pipe by means of galvanized iron wire or netting, over which is wrapped a coat of rosin-sized paper, followed by 8-oz. canvas securely sewed on.

Asbestos air-cell covering consists of several layers of corrugated asbestos sheets, making a laminated covering with air spaces between the corrugations. Another covering of the **air-space type** consists of perforated iron furred out from the pipe and wrapped with raw silk, a second jacket in a similar way being placed upon the first.

A **wood casing** is sometimes used for underground steam or hot-water pipes, for exposed steam lines in mine shafts, etc., and for cold storage and brine pipes. This covering is lined with asbestos for high-pressure steam lines, and is waterproofed for underground service. (See catalog of Wyckoff Manufacturing Co., Elmira, N. Y.)

L. B. McMillan (*Trans. A. S. M. E.*, 1915) finds, for coverings with white canvas surfaces, that the total heat flow in B.t.u. per sq. ft. of pipe surface per hour through a covering $= W_1 = at_d$, where a is taken from the accompanying table, and t_d = temperature difference (deg. fahr.) between pipe surface and air. The total B.t.u. loss per hour per sq. ft. of outside surface of covering $= W_2 = W_1r_1/(r_1 + s)$, where r_1 = outside radius of pipe, in., and s = thickness of covering, in. The temperature difference between the outer covering surface and the air corresponding to the loss W_2 is

$T_d = (328W_2 - 220)/(W_2 + 390)$, and the conductivity for a given value of T_d is $k = W_1 r_2(\log_e r_1 - \log_e r_2)/T_d$, where $r_2 = r_1 + s$. Conductivities for $t_d = 300$ deg. fahr. are given in the accompanying table.

The heat loss through any thickness of any material of which k is known for $t_d \leq 500$ deg. fahr., is $W_2 = k(t_1 - t_2 - T_d)/[r_2(\log_e r_2 - \log_e r_1)]$, where t_1 and t_2 are the temperatures of the pipe surface and room, respectively, deg. fahr. The value of W_2 for any pipe covering surfaced with white canvas may be closely approximated by use of the equation: Reading of thermometer under the canvas minus $t_2 = (328W_2 - 220)/(W_2 + 390)$.

The heat loss from *bare* pipe per deg. temperature difference per hr. per sq. ft. is as follows:

Temp. diff., deg. fahr.......	50	100	200	300	400	500
Loss in B.t.u. (= b)........	1.95	2.152	2.665	3.26	4.035	5.18

The B.t.u. *saving* due to covering per deg. temperature difference per sq. ft. per hour = $b - a$ (values of a in table), and the *efficiency* of the covering in per cent. = $100(b - a)/b$. Thus, for the first covering in the table, the saving per year per sq. ft. of covering for a temperature difference of 300 deg. fahr. (for continuous operation, or 8760 hr.) = $(3.26 - 0.455) \times 300 \times 8760$ = 7,370,000 B.t.u., and the efficiency = $100(3.26 - 0.455)/3.26 = 86$ per cent. With steam costing $0.30 per 1,000,000 B.t.u. (about 1000 lb. of steam), the gross money saving is $2.212. The first cost of the coverings mentioned ranges from 16 to 38 cents per sq. ft., installed, an average price being 24 cents. Allowing 14 per cent. of first cost for interest, depreciation, repairs, etc., makes the annual cost of covering per sq. ft. = $0.24 × 0.14 = $0.034, whence the net saving would be $2.212 − $0.034 = $2.178.

Data on Commercial Pipe Coverings

Kind of covering (Thickness of single covering, inches)	k (for temp. diff. of 300 deg. fahr.)	B.t.u. loss per sq. ft. per deg. temp. diff. per hour for single-thickness covered pipes (= a)					
		Temperature difference (pipe and room), deg. fahr.					
		50	100	200	300	400	500
J-M 85 per cent. Magnesia (1.08)....	0.551	0.435	0.438	0.446	0.455	0.469	0.488
J-M Indented (1.12)...............	0.686	0.472	0.483	0.309	0.549	0.603	0.666
J-M Vitribestos (0.96)............	1.087	0.626	0.654	0.715	0.781	0.856	0.967
J-M Eureka (1.04).................	0.549	0.440	0.451	0.464	0.478	0.487*	......
J-M Molded Asbestos (1.25)........	0.778	0.517	0.522	0.539	0.561	0.596	......
J-M Wool Felt (1.10)..............	0.521	0.386	0.400	0.421	0.442	0.453*	......
Sall-Mo Expanded Asbestos (1.07)..	0.598	0.409	0.427	0.464	0.503	0.541	0.581
Carey Carocel (0.99)..............	0.540	0.358	0.378	0.421	0.466	0.510	0.562
Carey Serrated (1.00).............	0.682	0.454	0.468	0.506	0.546	0.587	0.634
Carey Duplex (0.96)...............	0.636	0.423	0.447	0.498	0.548	0.574*	......
Carey 85 per cent. Magnesia (1.10)..	0.546	0.413	0.418	0.424	0.436	0.454	0.472
Sall-Mo Wool Felt (1.01)..........	0.510	0.395	0.401	0.433	0.459	0.421†	0.455‡
Nonpareil High Pressure (1.16).....	0.543	0.399	0.402	0.412	0.426	0.444	0.465
J-M Asbestos Fire Felt (0.99).......	1.093	0.694	0.711	0.749	0.795	0.845	0.901
J-M Asbestos Sponge Felted (1.16)..	0.468	0.336	0.347	0.369	0.391	0.414	0.439
J-M Asbestocel (1.10).............	0.596	0.418	0.429	0.454	0.493	0.544	0.609
J-M Air Cell (1.00)...............	0.718	0.459	0.475	0.515	0.571	0.643	0.733
J-M Plastic 85 per cent. Magnesia (0.51–3.24).....................	0.587	......	......	......	......	......	......
Sall-Mo Air cell (0.95)............	0.802	......	......	......	......	......	......

* For a temperature difference of 350 deg. fahr. † 150 deg. fahr. ‡ 250 deg. fahr.

Identification of Piping by Colors. Pipe covering after being canvas-jacketed is frequently painted. The paint should be fireproof and preferably of a light color, to reduce radiation. It is well to paint pipes for different purposes of various colors. There is at present no generally accepted standard color scheme for piping. The following scheme, however, has been proposed by the A. S. M. E. Committee on Identification of Power House Piping (*Trans. A. S. M. E.*, 1911, vol. 33, p. 17).

(*a*) In the main engine room of plants which are well lighted, and where the functions of the exposed pipes are obvious, all pipes are to be painted to conform to the color scheme of the room; and if it is desirable to distinguish pipe systems, colors are to be used only on flanges and on valve-fitting flanges.

(*b*) In all other parts of the plant, such as the boiler house, basements, etc., all pipes (exclusive of valves, flanges and fittings), except the fire system, are to be painted black, or some other single, plain, durable, inexpensive color.

(*c*) All fire lines (suction and discharge), including pipe lines, valve flanges and fittings, are to be painted red throughout.

(*d*) The edges of all flanges, fittings or valve flanges on pipe lines larger than 4 in. inside diam., and the entire fittings, valves and flanges on lines 4 in. inside diam. and smaller, are to be painted the following distinguishing colors, numbered 1 to 12 inclusive:

Distinguishing Colors to be Used on Valves, Flanges and Fittings Only

Steam division	1. High pressure—white.
	2. Exhaust system—buff.
Water division	3. Fresh water, low pressure—blue.
	4. Fresh water, high-pressure boiler-feed lines—blue and white.
	5. Salt-water piping—green.
Oil division	6. Delivery and discharge—brass or bronze yellow.
Pneumatic division	7. All pipes—gray.
Gas division	8. City lighting service—aluminum.
	9. Gas-engine service—black; red flanges.
Fuel oil division	10. All piping—black.
Refrigerating system	11. White and green stripes alternately on flanges and fittings, body of pipe being black.
Electric lines and feeders	12. Black and red stripes alternately on flanges and fittings, body of pipe being black.

PRESSURE HOSE

Hose with durable rubber lining may be obtained to withstand any needed pressure. If the rubber compound is properly made the life of a hose will be from 7 to 10 years, while a cheaper hose, lined with inferior material, will probably not last more than 3 or 4 years. For fire hose, see p. 276.

National Standard Hose Couplings. There is a marked diversity in the character and dimensions of the hose couplings now in general use. Those specified below, however, have been adopted by the leading organizations supervising or controlling public utilities.

SPECIFICATIONS FOR NATIONAL STANDARD HOSE COUPLINGS

Inside diam. of hose, in.	2½	3	3½	4½
Number of threads per in.	7½	6	6	4
Male Couplings:				
Outside diam. of thread, finished, in.	3 1/16	3⅝	4¼	5¾
Diam. at root of thread, in.	2.8715	3.3763	4.0013	5.3970
Clearance between male and female thread, in.	0.03	0.03	0.03	0.05
Total length of threaded male end, in.	1	1⅛	1⅛	1⅜

Threads to be of the 60-deg. V pattern with 0.01 in. cut off the top of thread and 0.01 in. left in the bottom of the 2½-in., 3-in., and 3½-in. couplings, (0.02 in. top and bottom for 4½-in. couplings); ¼-in. blank end to be left on the male part of couplings in each case. Female ends to be cut ⅛ in. shorter for endwise clearance, and bored out 0.03 in. larger in the 2½-in., 3-in., and 3½-in. sizes, and 0.05 in. larger in the 4½-in. size in order to make up easily, without jamming or sticking. (See discussion by F. M. Griswold, *Trans. A. S. M. E.*, 1913, vol. 35, p. 423.)

Flexible Metal Hose or Tubing is made from a continuous metal ribbon or strip wound spirally. The edges are turned in during the winding so as to make an interlocked joint as shown in Fig. 35. A cord fed into a separate groove during the winding acts as a packing to keep the hose tight. The metal used for steam is bronze or copper with asbestos cord packing; for heavy pressures a braided wire jacket is added and an external spiral wire; sizes from 3/16 to 3 in. For oil, ammonia, and other non-rusting liquids steel wire is used with asbestos cord. For compressed air, cold water or suction, rubber packing is used in place of asbestos (see catalog of The American Metal Hose Co., Waterbury, Conn.).

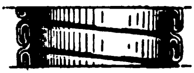


FIG 35.—Flexible Metal Hose..

WIRE ROPE, NAILS, ETC.*

Wire Rope

Wire ropes are built up of **strands** made of wires twisted together, the numbers of wires commonly used being four, seven, twelve, nineteen and thirty-seven. Ordinarily the wires are twisted into strands in the opposite direction to the twist of the strands into rope. When wires and strands are twisted in the same direction, the rope is known as **Lang lay rope. Standard wire rope** is made of six wire strands and a hemp core. Wire strands are twisted around the core, either to the right or left, and the resulting rope is thereby designated as right lay or left lay. The twist may be long or short, the shorter twist forming the more flexible rope. The **core** of a wire rope is, as a rule, hemp saturated with a lubricant. It provides little additional strength, but acts as a cushion to preserve the shape of the rope and helps to lubricate the wires. A wire strand or rope core adds from 7 to 10 per cent. to the strength of the rope, but will wear from the friction between it and the outer strands as rapidly as the outside of the rope. This does not apply to stationary ropes.

For great flexibility, the strands of a wire rope sometimes consist of wire ropes, which in turn are made of strands composed of wires, as in tiller rope. Running ropes and one construction of ship's hawsers are made with strands composed of 12 or 18 wires each, twisted about a hemp center. Ropes so made are very pliable and present good resistance to outside friction. Indi-

* Staff contribution.

vidual strands of wires are employed as smoke-stack guys, span wires for trolley roads, and wherever only moderate flexibility is needed.

Strength and Working Loads. The test **strength of wire ropes** seldom exceeds 90 per cent. of the aggregate strength of all of the wires, the average being about 82.5 per cent. The **working loads** in the following tables are calculated at one-fifth the breaking strengths, but these values are not recommended for all cases: for instance, elevator ropes seldom have a load of more than one-sixth or one-eighth of their breaking strength.

Sizes of Drums or Sheaves. (See also p. 754.) The sizes given in the tables are calculated on a basis of a working load of one-fifth the breaking strength, and provide that ropes shall not have their wires stressed beyond the elastic limit when passing around the drums or sheaves indicated. While the theoretical sizes for plow-steel ropes are smaller than for the corresponding cast-steel ropes, because the elastic limit of the former is much higher than that of the latter and the modulus of elasticity is nearly the same for both, still it is better to make drums and sheaves for the former at least as large as for the latter under ordinary conditions, since plow steel, due to its greater hardness, cracks more easily when worn. There are cases as, for example, electric cranes, where the wear is not severe, and here plow steel may be bent to more nearly its theoretical limit. Iron rope requires larger sheaves and drums than cast steel, due to its low tensile strength. When high speeds are used, much larger drums and sheaves must be employed. When working loads less than those given in the tables are used, or when the bending around sheaves and drums is very occasional, slightly smaller dimensions may be adopted. In general, the larger the drum or sheave, the longer the rope will last, while the use of diameters less than those stated in the tables will often result in a rapid deterioration of the rope. Wear increases with speed; it is therefore better to increase the load, within certain limits, than the speed.

Handling. Wire rope must not be coiled or uncoiled like hemp rope. When it is received upon a reel, the latter should be mounted upon a spindle or turn table and the rope then run off. When shipped in a coil, it should be rolled along the ground like a wheel. All untwisting and kinking must be avoided. When a wire rope is to be cut, soft iron wire should be served on each side of the place where the division is to be made to prevent the rope from untwisting.

Materials. Rope made from **Swedish iron** wire is soft, tough and pliable and has but little tendency to wear grooves in sheaves. This is of considerable advantage when wire rope is run over multiple-grooved sheaves. It is especially adapted for passenger elevators and similar service where the tendency to abrasion is comparatively slight, the speed is high, loads moderate. **Cast-steel rope** is recommended for general hoisting, such as mine hoists, derricks, coal hoists, cableways, conveyors, freight elevators, etc. It is tough and pliable, will bend over comparatively small sheaves and resist the abrasion which accompanies rapid hoisting. The wire in cast-steel rope has nearly twice the strength of that in Swedish iron rope and also has a much higher elastic limit. **Extra strong cast-steel rope** occupies an intermediate place between cast-steel and plow-steel rope. It is used in place of cast steel when it is desirable to increase the factor of safety for a given diameter. It may be used to advantage for general hoisting. **Plow-steel rope** is made of wire of great strength and toughness, capable of resisting severe abrasion. It can be used in place of cast-steel rope where it is desirable to reduce the dead weight of the rope itself, or where, by reason of increased loads, it is necessary to use a much stronger rope without increasing its diameter. Plow-steel rope is recommended especially for logging lines, scraper, dredge and wrecking ropes, heavy cranes, ballast unloader ropes, and for all rough uses requiring maximum strength and toughness. **Improved plow steel rope** is a special rope for extreme conditions. It has a maximum strength, toughness and uniformity.

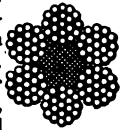


FIG. 1.

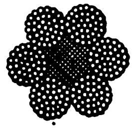


FIG. 2.

Standard Hoisting Rope (Fig. 1) is made of 6 strands, each of 19 wires, the strands being twisted around a hemp center.

Table 1. Standard Hoisting Rope

Composed of 6 strands and a hemp center, 19 wires to the strand

(John A. Roebling's Sons Co.)

Trade number	Diam., in.	Approx. wt. per ft., lb.	Diameter of drum or sheave advised, ft.		Proper working load, tons of 2000 lb.				
			For steel	For Swedish iron	Swedish iron	Cast steel	Extra strong cast steel	Plow steel	Improved plow steel
00	2¾	11.95	11	17	22.2	42.2	48.6	55.0	63.0
0	2½	9.85	10	15	18.4	34.0	40.0	46.0	42.0
1	2¼	8.0	9	14	14.4	26.6	32.0	37.0	42.0
2	2	6.3	8	12	11.0	21.2	24.6	28.0	33.0
2½	1⅞	5.55	8	12	10.0	19.0	22.4	25.0	30.0
3	1¾	4.85	7	11	8.8	17.0	19.8	22.0	27.0
4	1⅝	4.15	6½	10	7.6	14.4	16.6	19.0	22.0
5	1½	3.55	6	9	6.6	12.8	14.6	16.0	20.0
5½	1⅜	3.0	5½	8½	5.6	11.2	12.8	14.0	17.0
6	1¼	2.45	5	7½	4.56	9.4	10.6	12.0	14.0
7	1⅛	2.0	4½	7	3.72	7.6	8.6	9.4	11.0
8	1	1.58	4	6	2.90	6.0	6.80	7.6	[illegible]
9	⅞	1.20	3½	5½	2.36	4.6	5.20	5.8	7.0
10	¾	0.89	3	4½	1.70	3.5	4.04	4.6	5.3
10¼	⅝	0.62	2½	4	1.20	2.5	2.80	3.1	3.8
10½	9/16	0.50	2¼	3½	0.94	2.0	2.24	2.4	2.9
10¾	½	0.39	2	3	0.78	1.68	1.84	2.0	2.4
10a	7/16	0.30	1¾	2¾	0.58	1.30	1.45	1.6	1.9
10b	⅜	0.22	1½	2¼	0.48	0.96	1.06	1.15	1.35
10c	5/16	0.15	1¼	2	0.30	0.62	0.70	0.76	0.9
10d	¼	0.10	1	1½	0.22	0.44	0.49	0.53	0.53

Extra Pliable Hoisting Rope. Made of 6 strands of 37 wires each and a hemp center (Fig. 2). The wires in this rope are much finer than those used in the standard hoisting rope and consequently not as suitable to withstand abrasion. These ropes are used on electric cranes, dredges and for similar service requiring a strong, tough rope that will operate successfully over small sheaves.

Table 2. Extra Pliable Hoisting Rope

(Six 37-wire strands and hemp center)

Diam., in.	Approx. wt. per ft., lb.	Diam. of drum or sheave advised, ft.	Proper working load in tons of 2000 lb.			
			Cast steel	Extra strong cast steel	Plow steel	Improved plow steel
2¾	11.95	......	40.0	47.0	53.0	55.0
2½	9.85	......	32.0	37.0	43.0	45.0
2¼	8.0	......	25.0	30.0	35.0	37.0
2	6.30	......	21.0	23.0	26.0	27.0
1¾	4.85	......	17.0	19.0	22.0	23.0
1⅝	4.15	......	14.0	16.0	18.0	19.0
1½	3.55	3.75	12.0	14.0	16.0	17.0
1⅜	3.0	3.5	11.0	12.0	14.0	14.0
1¼	2.45	3.2	9.0	10.0	11.0	11.0
1⅛	2.0	2.83	7.0	8.0	9.0	9.2
1	1.58	2.5	6.0	6.4	7.0	7.4
⅞	1.20	2.16	5.0	5.0	5.0	5.8
¾	0.89	1.83	3.5	3.8	4.0	4.6
⅝	0.62	1.75	2.2	2.5	3.0	3.2
9/16	0.50	1.5	1.9	2.1	2.3	2.5
½	0.39	1.33	1.45	1.65	1.85	1.9
7/16	0.30	1.16	1.1	1.27	1.4	1.5
⅜	0.22	1.0	0.84	0.93	1.0	1.06

Table 3. Extra Pliable Hoisting Rope

(Eight 19-wire strands and hemp center)

Diam., in.	Approx. wt. per ft., lb.	Diam. of drum or sheave advised, ft.	Proper working load in tons of 2000 lb.			
			Cast steel	Extra strong cast steel	Plow steel	Improved plow steel
1½	3.19	3.75	11.6	13.0	14.8	16.0
1⅜	2.70	3.5	10.2	11.0	12.8	13.0
1¼	2.20	3.2	8.4	9.4	10.4	11.0
1⅛	1.80	2.83	6.8	7.6	8.6	9.2
1	1.42	2.5	5.2	5.9	6.6	7.2
⅞	1.08	2.16	4.0	4.6	5.2	5.6
¾	0.80	1.83	3.06	3.5	4.0	4.4
⅝	0.56	1.75	2.18	2.5	2.8	3.0
9/16	0.45	1.50	1.74	2.0	2.32	2.4
½	0.35	1.33	1.46	1.6	1.74	1.9
7/16	0.27	1.16	1.14	1.26	1.38	......
⅜	0.20	1.0	0.84	0.93	1.02	......
5/16	0.13	0.83	0.55	0.61	0.67	......
¼	0.09	0.75	0.36	0.40	0.45	......
....	......	......	......	......	......	......
....	......	......	......	......	......	......
....	......	......	......	......	......	......

Extra Pliable Hoisting Rope of 8 strands of 19 wires and a hemp center (Fig. 3), is much more pliable than the standard construction of 6 strands of 19 wires. The metallic area of an 8-strand rope is not so great as that of a 6-strand rope, and the wires are smaller, but under severe bending stresses the decrease in strength is largely offset by the great pliability. It can be used over comparatively small sheaves and drums such as are frequently found on derricks. It is not good practice to use it where there is much overwinding, because it would flatten or lose shape more quickly than 6 × 19 rope.

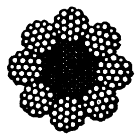


Fig. 3.

Galvanized extra pliable cast-steel hoisting rope is much more flexible than the six-strand hoisting rope, and is often used in preference to galvanized cast-steel running rope.

Standard Coarse-laid Rope (Fig. 4) is made of 6 strands and a hemp center, 7 wires to the strand. It is much stiffer than standard hoisting rope and requires larger sheaves. On account of the smaller number of wires, this rope should also be used with a higher factor of safety, as the breaking of one or two wires materially reduces the strength of the rope. The wires used are considerably larger in diameter than in hoisting rope, and consequently will stand greater wear. **Swedish iron rope** of this con-

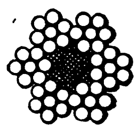


Fig. 4.

struction is recommended for power transmissions equipped with large sheaves. **Cast-steel and extra strong cast-steel rope** are recommended for mine haulages, tramways, sand lines and similar service where conditions tend to severe abrasion. **Plow-steel and improved plow-steel ropes** are recommended in place of cast steel when it is desirable to reduce the dead weight of the rope itself, or where, by reason of increased loads, it is necessary to use a stronger rope without increasing its diameter. This rope is particularly adapted for very long mine haulages.

Table 4. Extra Special Flexible Hoisting Rope
(Composed of six 61-wire strands and a hemp core)

Diam., in.	Approx. wt. per ft., lb.	Proper working loads, tons of 2000 lb.				Diam. of sheave or drum advised, ft.
		Cast steel	Extra strong cast steel	Plow steel	Monitor plow steel	
3¼	16.60	56	63	70	74	11
3	14.20	48	55	62	65	10
2¾	11.95	40	47	53	56	9
2½	9.85	32	37	43	45	8
2¼	8.00	25	30	35	37	7
2	6.30	21	23	26	27	6

Table 5. Standard Coarse-laid Rope for Haulages and Transmissions
(Composed of six 7-wire strands and a hemp center)

Trade number	Diam., in.	Approximate weight per ft., lb.	Diam. of drum or sheave advised for steel, ft.	Diam. of drum or sheave advised for Swedish iron, ft.	Proper working load, tons of 2000 lb.				
					Swedish iron	Cast steel	Extra strong cast steel	Plow steel	Improved plow steel
11	1½	3.55	11	16	6.4	12.6	14.6	16.4	18.0
12	1⅜	3.00	10	15	5.6	10.6	12.6	14.4	16.0
13	1¼	2.45	9	13	4.6	9.2	10.8	12.0	13.0
14	1⅛	2.00	8	12	3.8	7.4	8.6	9.4	10.0
15	1	1.58	7	10½	3.0	6.2	7.0	7.6	8.4
16	⅞	1.20	6	9	2.4	4.8	5.6	6.2	6.6
17	¾	0.89	5	7½	1.7	3.7	4.2	4.6	5.0
18	11/16	0.75	4¾	7¼	1.5	3.1	3.3	3.6	4.0
19	⅝	0.62	4½	7	1.2	2.6	2.9	3.2	3.5
20	9/16	0.50	4	6	0.96	2.0	2.2	2.4	2.6
21	½	0.39	3½	5½	0.74	1.5	1.8	2.0	2.2
22	7/16	0.30	3	4½	0.52	1.1	1.25	1.4	1.5
23	⅜	0.22	2¾	4	0.44	0.92	1.05	1.2	1.3
24	5/16	0.15	2¼	3½	0.34	0.70	0.79	0.88	....
25	9/32	0.125	1¾	3	0.24	0.50	0.59	0.68	....

Flat Rope is composed of a number of wire ropes called "flat rope strands," of alternate right and left lay, placed side by side, then secured or sewed together with soft Swedish iron or steel wire (Fig. 5). The sewing or filling wire is much softer than the steel wires composing the strands of the rope, acts as a cushion or soft bed for the strands, and wears out much faster than the harder wires com-

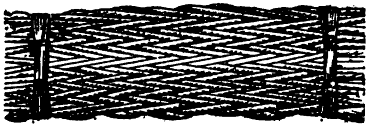


Fig. 5.—Flat Wire Hoisting Rope.

posing the latter. When the sewing wires are worn out, the flat rope can be resewed with new wire, and if any of the rope strands are also worn or damaged, these can be replaced by new portions. Flat rope is used principally for hoisting purposes. When large and long rope is used in hoisting heavy loads out of deep shafts, round rope requires large and heavy drums on which to wind, while flat rope, winding on itself, needs a reel but little wider than the width of the rope. Furthermore, flat rope does not spin or twist in the shaft. Flat rope is obtainable from 1¾ in. to 8 in. in width, and from ¼ in. to ⅞ in. in thickness, the length varying from 20 to 3000 ft. Flat rope is particularly applicable to the operating of spouts on coal and ore docks, also for raising and lowering emergency gates on canals and similar machinery. It combines flexibility and great strength, thus making possible the use of simple and compact hoisting machinery.

Table 6. Flat Rope—Crucible Steel and Plow Steel

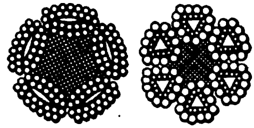
Width and thickness, in.	Approx. weight per ft., lb.	Proper working load, tons of 2000 lb.		Width and thickness, in.	Approx. weight, per ft., lb.	Proper working load, tons of 2000 lb.	
		Crucible steel	Plow steel			Crucible steel	Plow steel
¼ × 1½	0.65	2.6	3.10	½ × 4	3.15	13.8	16.4
¼ × 2	0.82	3.4	4.00	½ × 4½	3.85	16.6	19.8
¼ × 2½	1.06	4.4	5.30	½ × 5	4.20	18.0	21.6
¼ × 3	1.23	5.2	6.20	½ × 5½	4.55	19.6	23.6
5/16 × 1½	0.79	3.6	4.4	½ × 6	4.90	21.0	25.2
5/16 × 2	1.10	4.6	5.6	½ × 7	5.90	25.6	30.6
5/16 × 2½	1.35	6.0	7.0	⅝ × 3½	3.50	13.6	15.8
5/16 × 3	1.60	7.2	8.6	⅝ × 4	4.00	15.8	18.4
5/16 × 3½	1.88	8.2	10.0	⅝ × 4½	4.55	18.2	21.0
5/16 × 4	2.15	9.6	11.4	⅝ × 5	5.10	20.4	23.8
⅜ × 2	1.30	5.4	6.6	⅝ × 5½	5.65	22.8	26.4
⅜ × 2½	1.70	7.2	8.6	⅝ × 6	6.15	25.0	29.0
⅜ × 3	1.89	8.2	9.8	⅝ × 7	7.30	29.6	34.2
⅜ × 3½	2.30	10.0	12.0	⅝ × 8	8.40	34.0	39.4
⅜ × 4	2.43	10.8	13.0	¾ × 5	6.85	27.0	31.4
⅜ × 4½	2.85	12.6	15.2	¾ × 6	7.50	30.2	35.0
⅜ × 5	3.10	13.6	16.2	¾ × 7	8.25	33.6	38.8
⅜ × 5½	3.50	15.4	18.4	¾ × 8	9.75	40.4	46.8
⅜ × 6	3.73	16.2	19.4	⅞ × 5	7.50	31.0	34.4
½ × 2½	2.20	9.0	10.8	⅞ × 6	8.53	36.0	41.8
½ × 3	2.50	10.4	12.6	⅞ × 7	9.56	40.6	46.6
½ × 3½	2.80	12.0	14.4	⅞ × 8	10.60	45.0	51.6

The diameters of **drums and sheaves** for flat rope should be as large as possible, particularly for mine hoisting work. The desirable diameter of the drum at the bottom may be obtained from the formula $D = ct$, where D = diam. of drum at bottom, ft., t = thickness of flat rope, in., and c = 100 for drums = 160 for sheaves. For short flat ropes, drums are usually made smaller, as follows:

Thickness of flat rope, in.	¼	5/16	⅜	½	⅝	¾	⅞
Diam. at bottom of drum, in.	6	7½	9	12	15	18	21
Diam. of sheave, in.	12	15	18	24	30	36	42

Sheaves should be slightly crowned in the center and have deep flanges to guide the rope.

Flattened-strand Wire Rope is designed to give increased contact or wearing surface. The wear is consequently lessened upon any one individual wire and the necessity of the use of heavier wire diminished, which results in greater flexibility. The wearing surface is approximately 150 per cent. greater than that of a round-strand rope. Another feature of this type of rope is that the interstices between the strands being lessened, a greater number of wires are used for the same diameter. It is always made Lang lay. Flattened-strand rope has little tendency to kink, and, owing to its smooth wearing surface, saves wear on pulleys, sheaves and drums. It is made in the forms shown in Fig. 6. The working loads for types A and B are given in Tables 7 and 8, and are calculated for a factor of safety of five. They are used for elevators, hoisting, dredging, etc.



A. B.

FIG. 6.

Table 7. Flattened-strand Hoisting Rope, Type A
(Five 28-wire strands and a hemp core)

Diam., in.	Approx. weight per ft., lb.	Diam. of drum or sheave advised, ft. Iron	Steel	Proper working load, tons of 2000 lb. Iron	Cast steel	Extra strong cast steel	Plow steel
2¼	8.00	11¼	8½	14.4	26.6	32.0	42.0
2	6.30	10¾	8	11.0	21.2	24.6	33.2
1¾	4.85	9	7¼	8.8	17.0	19.8	26.6
1⅝	4.15	7½	6¼	7.6	14.4	16.6	22.0
1½	3.55	6¾	5¾	6.6	12.8	14.6	19.6
1⅜	3.00	6¼	5½	5.6	11.2	12.8	16.8
1¼	2.45	5¾	5	4.56	9.4	10.6	13.8
1⅛	2.00	5¼	4½	3.72	7.6	8.6	11.2
1	1.58	4¾	4	2.90	6.0	6.8	9.0
⅞	1.20	4	3½	2.36	4.6	5.2	7.0
¾	0.89	3½	3	1.70	3.5	4.04	5.26
⅝	0.62	3	2¼	1.20	2.5	2.80	3.8
9/16	0.50	2½	1¾	0.94	2.0	2.24	2.9
½	0.39	2	1½	0.78	1.68	1.84	2.42
⅜	0.22	1	......	0.48	......	......	......

Table 8. Flattened-strand Hoisting Rope, Type B
Six 25-wire strands and a hemp core

Diam., in.	Approx. weight per ft., lb.	Diam. of drum or sheave advised, ft.	Proper working load, tons of 2000 lb. Cast steel	Extra strong cast steel	Plow steel
2¼	9.20	12	29.2	35.2	46.2
2	7.25	11	23.4	27.0	36.6
1¾	5.60	9	18.8	21.8	29.2
1⅝	4.75	8½	15.8	18.2	24.2
1½	4.00	8	14.0	16.0	21.6
1⅜	3.45	7½	12.4	14.0	18.4
1¼	2.80	7	10.4	11.6	15.2
1⅛	2.30	6	8.4	9.4	12.4
1	1.80	5	6.6	7.4	10.0
⅞	1.38	4½	5.0	5.8	7.8
¾	1.00	4	3.86	4.44	5.8
⅝	0.72	3½	2.76	3.08	4.2
9/16	0.58	3	2.2	2.46	3.2
½	0.45	2¾	1.86	2.02	2.7

Non-spinning Hoisting Rope (Fig. 7) is constructed of 6 strands of 7 wires each, Lang lay (wires in the strands and strands themselves twisted to the left) laid around a hemp core and covered with an outer layer composed of 12 strands, 7 wires, regular lay (wires in the strands twisted to the left and strands themselves twisted to the right). The object of this combination of lays is to prevent a free load suspended on the end of a single line from rotating. This type of rope is recommended for "back-haul" or single-line derricks; also for shaft sinking and mine hoisting where the bucket or cage swings free without guides. It works best where it does not overwind on drum.

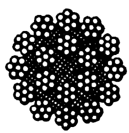


FIG. 7.

54

Either a closed or an open socket makes the best fastening on the end of non-spinning rope. These may be fastened in the same manner as any rope socket, but great care must be taken in attaching the socket to the rope to see that the strands do not untwist or allow any slack to work back into the rope. It is best to seize the end of the rope tightly for a distance of 4 or 5 in. just outside of the socket until the socketing is completed, when it may be taken off.

Table 9. Non-spinning Hoisting Rope

(Composed of 18 strands and a hemp core, 7 wires to the strand)

Diam., in.	Weight per ft., lb.	Diam. of drum or sheave advised, ft.	Proper working load, tons of 2000 lb.			
			Cast steel	Extra strong cast steel	Plow steel	Monitor plow steel
1¾	5.50	7.00	17.1	20.2	22.2	24.04
1⅝	4.90	6.50	14.8	17.5	19.2	....
1½	4.32	6.00	12.7	15.0	16.5	18.1
1⅜	3.60	5.50	10.4	12.4	13.7	15.1
1¼	2.80	5.00	8.7	10.3	11.3	12.5
1⅛	2.34	4.50	7.3	8.6	9.5	10.4
1	1.73	4.00	5.6	6.6	7.2	7.8
⅞	1.44	3.50	4.5	5.3	6.3	7.0
¾	1.02	3.00	3.3	3.9	4.9	5.4
⅝	0.70	2.50	2.2	2.6	3.1	3.4
9/16	0.57	2.25	1.8	2.1	2.5	....
½	0.42	2.00	1.3	1.6	1.9	2.1
7/16	0.31	1.75	0.98	1.1	1.3	....
⅜	0.25	1.50	0.78	0.92	1.1	1.2

Steel-clad Ropes (Fig. 8) are made in three constructions for the purpose of securing different degrees of flexibility: the 6 × 19, 6 × 37 and 6 × 61 types, respectively. Flat strips of steel wound spirally around each of the six strands composing the rope, give additional wearing surface without sacrificing flexibility. When the outer flat-steel winding is worn through, a complete hoisting rope remains, with unimpaired strength. These ropes are designed to meet very severe conditions of service. The increased life obtained by the use of steel-clad rope is, in places where conditions are suitable, from 50 to 100 per cent. It is recommended particularly for such service as dredging.

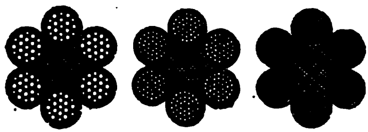


Fig. 8.—Steel-clad Ropes.

Galvanized Wire Rope has almost entirely superseded Manila rope for shrouds and stays aboard ship. It is cheaper in first cost; is not affected by the weather, and does not stretch and contract with changes in atmospheric conditions, and thus saves a great deal of labor in setting up; it is as elastic as Manila rope. There is great reduction in bulk and weight by its use, as it is only one-fifth or one-sixth as large as a Manila rope of equal strength. Consequently, it only offers half as much surface to the wind. It is much less liable to accidents by being cut or chafed, and does not rot and give way suddenly without warning. Galvanized rope is better suited for guys for derricks than hemp rope or rods linked together.

Table 10. Steel-clad Hoisting Rope
(Six 19-wire strands and a hemp core)

Finished diam. over serving, in.	Diam. of bare rope, in.	Approx. weight per ft., lb.	Diam. of drum or sheave advised, ft.	Proper working load, tons of 2000 lb.			
				Cast steel	Extra strong cast steel	Plow steel	Monitor plow steel
2¼	2	8.45	8.0	21.2	24.6	28.0	33.0
2	1⅞	6.70	7.5	19.2	22.4	25.0	30.0
1⅞	1¾	6.02	7.0	17.0	19.8	22.0	27.0
1¾	1⅝	5.25	6.5	14.4	16.6	19.0	22.0
1⅝	1½	4.62	6.0	12.8	14.6	16.0	20.0
1½	1⅜	3.95	5.5	11.2	12.8	14.0	17.0
1⅜	1¼	3.30	5.0	9.4	10.6	12.0	14.0
1¼	1⅛	2.80	4.5	7.6	8.6	9.4	11.0
1⅛	1	2.12	4.0	6.0	6.80	7.6	9.0
1	⅞	1.72	3.5	4.6	5.20	5.8	7.0
⅞	¾	1.30	3.0	3.5	4.04	4.6	5.3
¾	⅝	1.00	2.5	2.5	2.80	3.1	3.8
⅝	½	0.70	2.0	1.68	1.84	2.0	2.4

Table 11. Steel-clad Special Flexible Hoisting Rope
(Six 37-wire strands and a hemp core)

Finished diam. over serving, in.	Diam. of bare rope, in.	Approx. weight per ft., lb.	Diam. of drum or sheave advised, ft.	Proper working load, tons of 2000 lb.			
				Cast steel	Extra strong cast steel	Plow steel	Monitor plow steel
2¾	2½	12.05	8.0	32.0	37.0	43.0	45.0
2½	2¼	9.90	7.0	25.0	30.0	35.0	37.0
2¼	2	8.00	6.0	21.0	23.0	26.0	27.0
2	1⅞	6.60	5.25	18.8	21.2	23.8	25.0
1⅞	1¾	5.90	4.75	17.0	19.0	22.0	23.0
1¾	1⅝	4.90	4.25	14.0	16.0	18.0	19.0
1⅝	1½	4.30	3.75	12.0	14.0	16.0	17.0
1½	1⅜	3.75	3.5	11.0	12.0	14.0	14.0
1⅜	1¼	3.05	3.2	9.0	10.0	11.0	11.0
1¼	1⅛	2.40	2.83	7.0	8.0	9.0	9.2
1⅛	1	2.00	2.5	6.0	6.4	7.0	7.4
1	⅞	1.75	2.16	5.0	5.0	5.0	5.8

Table 12. Steel-clad Extra Special Flexible Hoisting Rope
(Composed of 6 strands and a hemp core, 61 wires to the strand)

Finished diam. over serving, in.	Diam. of bare rope, in.	Approx. weight per ft., lb.	Diam. of drum or sheave advised, ft.	Proper working load, tons of 2000 lb.			
				Cast steel	Extra strong cast steel	Plow steel	Monitor plow steel
3¼	3	16.80	10	48	55	62	65
3	2¾	14.35	9	40	47	53	55
2¾	2½	12.05	8	32	37	43	45
2½	2¼	9.90	7	25	30	35	37
2¼	2	8.45	6	21	23	26	27

Galvanized Steel Wire Strand (Fig. 9) is used chiefly for guying poles and smokestacks, for supporting trolley wire, and for operating railroad signals. For overhead catenary construction of suspending trolley wire, the special grades of strand are preferable because they possess greater strength and toughness. The last five sizes listed in Table 13 (sometimes called "galvanized seizing strand") are used for seizing or binding the ends of wire rope and thimble splices, and for tying rope into coils.

Fig. 9.

Table 13. Galvanized Steel Wire Strand

(Composed of 7 wires twisted together)

Diam., in.	Approx. weight per 1000 ft., lb.	Approx. strength, lb.	Diam., in.	Approx. weight, per 1000 ft., lb.	Approx. strength, lb.	Diam., in.	Seizing strand trade number	Approx. weight per 1000 ft., lb.	Approx. strength, lb.
5/8	800	14,000	5/16	210	3,800	9/64	18	40	700
9/16	650	11,000	1/4	125	2,300	1/8	19	32	500
1/2	510	8,500	7/32	95	1,800	7/64	20	25	450
7/16	415	6,500	3/16	75	1,400	3/32	21	20	400
3/8	295	5,000	5/32	55	900	5/64	22	13	300

Table 14. Extra Galvanized Special Strand

(Seven steel wires twisted into a single strand)

Diam., in.	5/8	1/2	7/16	3/8	5/16	9/32	1/4	3/16	1/8
	Tensile strength, lb.								
Siemens-Martin (*a*)	19,000	11,000	9,000	6,800	4,860	4,380	3,060	2,000	900
High Strength (*b*)	25,000	18,000	15,000	11,500	8,100	7,300	5,100	3,300	1,500
Ex. High Strength (*c*)	42,500	27,000	22,500	17,250	12,100	10,900	7,600	4,900	2,250

Minimum elongation, per cent. in 10 in.: (*a*), 10; (*b*), 6; (*c*), 4.

Table 15. Properties of Special Grades of Extra Galvanized Special Strands

Diam. of strand, in.	Number of wires in strand	Strength of strand, tons			Aprox. weight per ft., lb.	Diam. of strand, in.	Number of wires in strand	Strength of strand, tons			Approx. weight per ft., lb.
		Siemens-Martin steel	Crucible steel	Plow steel				Siemens-Martin steel	Crucible steel	Plow steel	
1 1/2	61	55.0	91.5	121	4.75	1	37	25.5	43.7	60.0	2.25
1 3/8	61	45.3	76.9	100	3.95	7/8	19	19.0	32.0	45.0	1.70
1 1/4	37	38.0	63.5	65	3.30	3/4	19	14.2	23.7	35.0	1.25
1 1/8	37	32.5	54.0	72	2.62	5/8	19	10.0	16.5	23.5	0.81

Table 16. Galvanized Steel Hawsers

(Composed of 6 strands and a hemp center, each strand consisting of 12 wires and a hemp core)

Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.
2 1/16	6 1/2	4.43	83	1 11/16	5 1/4	2.94	57	1 1/4	4	1.63	31
2	6 1/4	4.20	77	1 5/8	5	2.76	53	1 3/16	3 3/4	1.47	28
1 15/16	6	3.89	71	1 1/2	4 3/4	2.36	45	1 1/8	3 1/2	1.33	26
1 13/16	5 3/4	3.42	66	1 7/16	4 1/2	2.16	41	......	......	......	......
1 3/4	5 1/2	3.23	61	1 3/8	4 1/4	2.00	38	......	......	......	......

Table 17. Galvanized Cast-steel Yacht-rigging and Guy Ropes

(Composed of 6 strands and a hemp center, either 7 or 19 wires to the strand)

Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. Strength, tons of 2000 lb.
1¼	4	2.45	42	⅞	2¾	1.20	22.0	½	1½	0.39	7.0
1³⁄₁₆	3¾	2.21	38	¹³⁄₁₆	2½	1.03	19.0	¹⁵⁄₃₂	1⅜	0.34	6.0
1⅛	3½	2.00	34	¾	2¼	0.89	16.8	⁷⁄₁₆	1¼	0.30	5.0
1¹⁄₁₆	3¼	1.77	31	⅝	2	0.62	11.7	⅜	1⅛	0.22	4.2
1	3	1.58	28	⁹⁄₁₆	1¾	0.50	9.0	⁵⁄₁₆	1	0.15	3.2

Galvanized Mast-arm Rope is used for arc lights, mast-arms or other purposes where exposed to moisture. This rope is more durable than Manila rope and does not shrink.

Table 18. Galvanized Mast-arm Rope

(Composed of nine 4-wire strands and a cotton center)

Diam. in.	½	⁷⁄₁₆	⅜	⁵⁄₁₆	¼
Weight, lb. per ft.	0.335	0.245	0.163	0.107	0.077
Approx. breaking stress, lb.	4700	3400	2200	1530	1125

Table 19. Galvanized Steel Mooring Lines

(Fig. 10.—Composed of 6 strands and a hemp center, each strand composed of 24 wires around a hemp core)

Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.
2¹⁄₁₆	6½	5.81	113	1⅝	5	3.63	74	1⅛	3½	1.75	34
2	6¼	5.51	106	1½	4¾	3.10	63	1¹⁄₁₆	3¼	1.54	27
1¹⁵⁄₁₆	6	5.09	98	1⁷⁄₁₆	4½	2.92	55	1	3	1.38	25
1¹³⁄₁₆	5¾	4.48	88	1⅜	4¼	2.62	50	⅞	2¾	1.05	20
1¾	5½	4.24	82	1¼	4	2.15	42	¹³⁄₁₆	2½	0.90	17
1¹¹⁄₁₆	5¼	3.86	76	1³⁄₁₆	3¾	1.93	38	¾	2¼	0.78	14

Table 20. Galvanized-iron and Cast-steel Running Rope

(Fig. 11.—Composed of 6 strands and a hemp center, each strand consisting of 12 wires and a hemp center)

Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.		Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	
			Iron	Cast steel				Iron	Cast steel
1¹⁄₁₆	3¼	1.18	10.1	22.5	⁹⁄₁₆	1¾	0.33	2.8	6.5
1	3	1.05	8.7	19.5	½	1½	0.26	2.2	5.0
⅞	2¾	0.80	6.9	15.5	⁷⁄₁₆	1¼	0.20	1.7	3.9
¹³⁄₁₆	2½	0.68	6.0	13.5	⅜	1⅛	0.14	1.3	2.85
¾	2¼	0.59	5.1	11.5	⁵⁄₁₆	1	0.10	0.82	1.98
⅝	2	0.42	3.6	8.0	....	....	....	....	....

Table 21. Galvanized Ships' Rigging and Guy Ropes

(Composed of 6 strands and a hemp center, 7 or 12 wires to the strand)

Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 bl.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.
1¾	5½	4.85	42	1⅛	3½	2.00	18.0	½	1½	0.39	3.39
111/16	5¼	4.42	38	1 1/16	3¼	1.77	16.1	7/16	1¼	0.30	2.35
1⅝	5	4.15	35	1	3	1.58	14.1	⅜	1⅛	0.22	1.95
1½	4¾	3.55	30	⅞	2¾	1.20	11.1	5/16	1	0.15	1.42
1 7/16	4½	3.24	28	13/16	2½	1.03	9.4	9/32	⅞	0.125	1.20
1⅜	4¼	3.00	26	¾	2¼	0.89	7.8	¼	¾	0.09	0.99
1¼	4	2.45	23	⅝	2	0.62	5.7	7/32	⅝	0.063	0.79
1 3/16	3¾	2.21	19	9/16	1¾	0.50	4.46	3/16	½	0.04	0.61

Table 22. Galvanized Steel Hawsers

(Composed of 6 strands and a hemp center, 37 wires to the strand)

Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. circumference, in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.
2⅜	7½	8.82	188	1¾	5½	4.85	104	1 3/16	3¾	2.21	47.0
2 5/16	7¼	8.36	182	1 11/16	5¼	4.42	97	1⅛	3½	2.00	42.0
2¼	7⅛	8.00	171	1⅝	5	4.15	87	1 1/16	3¼	1.77	38.0
2⅛	6¾	7.06	155	1½	4¾	3.55	76	1	3	1.58	31.5
2 1/16	6½	6.65	140	1 7/16	4½	3.24	72	⅞	2¾	1.20	26.0
2	6¼	6.30	132	1⅜	4¼	3.00	66	13/16	2½	1.03	22.0
1 15/16	6	5.84	125	1¼	4	2.45	54	¾	2¼	0.89	20.0
1 13/16	5¾	5.13	112	......	......	......	......	......	......	......	......

Table 23. Galvanized Steel Cables for Suspension Bridges

(Fig. 12.—Composed of 6 strands with wire center)

Diam., in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.	Diam., in.	Approx. weight per ft., lb.	Approx. strength, tons of 2000 lb.
2¾	12.7	310	2⅛	7.60	185	1⅝	4.34	106
2⅝	11.6	283	2	6.73	164	1½	3.70	90
2½	10.5	256	1⅞	5.90	144	1⅜	3.10	75
2⅜	9.50	232	1¾	5.10	124	1¼	2.57	62
2¼	8.52	208	......	......	......	......	......	......

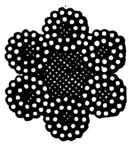


Fig. 10.

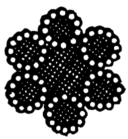


Fig. 11.

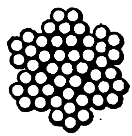


Fig. 12.

Tramway Strand (Table 24) has usually 19, 32, 37, 61 or 91 wires, according to the diameter of the strand and the conditions of its use. It is recommended for the track or trolley cable on an aerial tramway on account of its compact construction, and is found, by reason of its comparatively smooth surface, to greatly reduce the friction on the carriage wheels. In long spans where the weight of the rope itself is a disadvantage, plow-steel quality is generally used.

Table 24. Tramway Strand

Diam., in.	Weight per 100 ft., lb.	Approx. strength, tons of 2000 lb.		Diam., in.	Weight per 100 ft., lb.	Approx. strength, tons of 2000 lb.		Diam., in.	Weight per 100 ft., lb.	Approx. strength, tons of 2000 lb.	
		Cast steel	Plow steel			Cast steel	Plow steel			Cast steel	Plow steel
2½	1310	285.00	335.00	1¾	659	145.80	171.00	1⅛	270	60.00	70.70
2¼	1036	233.00	266.00	1⅝	563	124.00	146.00	1	220	49.20	58.00
2⅛	935	204.00	240.00	1½	488	108.40	127.50	⅞	169	37.60	44.40
2	840	185.00	218.00	1⅜	401	88.80	105.00	¾	124	27.60	32.50
1⅞	728	161.00	189.00	1¼	323	71.80	84.60	⅝	86	19.20	22.30

Table 25. Wire Sash Cord

(Six 7-wire strands and cotton core)

Diam., in.		¼	7/32	3/16	⅛	3/32	1/16
Weight per ft., lb.	Iron	.101	.077	.056	.025	.014	.006
	Copper	.115	.087	.064	.029	.016	.007
Approx. breaking stress, lb.	Bright iron	2200	1800	1400	550	320	140
	Annealed iron	1650	1411	1100	425	250	110
	Bright copper	1320	1080	840	350	200	90

Table 26. Tiller Rope or Hand Rope

(6 strands of 42 wires each, 252 wires in all—7 hemp cores)

Diam., in.	Approx. weight per ft., lb.	Diam. of drum or sheave advised, in.	Approx. breaking strength, lb.		Diam., in.	Approx. weight per ft., lb.	Diam. of drum or sheave advised, in.	Approx. breaking strength, lb.	
			Iron	Crucible cast steel				Iron	Crucible cast steel
1	1.10	24	22,000	35,000	½	0.28	12	5,800	9,000
⅞	0.84	21	15,500	26,000	7/16	0.21	10½	4,000	6,500
¾	0.62	18	11,000	18,000	⅜	0.16	9	3,000	4,800
⅝	0.43	15	7,000	13,500	5/16	0.11	7½	1,900	3,600
9/16	0.35	13½	6,300	11,000	¼	0.07	6	1,300	2,500

Nails and Spikes

Nails are either **wire nails** of circular cross-section and constant diameter, or **cut nails** of rectangular cross-section with taper from head to point. The larger sizes are called **spikes.** The length of the nail is expressed in the "**penny**" system, the equivalents in inches being given in the following tables; the letter "d" is the accepted symbol for penny. A keg of nails weighs 100 lb. **Heavy hinge nails** or **track nails** with countersunk heads have chisel points unless diamond points are specified. **Plaster-board nails** are smooth with diamond points and ½-in. flat heads for lengths, 1, 1¼ and 1½ in.; 7/16-in. head for sizes Nos. 9, 10 and 11. **American felt roofing nails** have an extra large head and thin shank. **Spikes** are made either with flat heads and diamond points or with oval heads and chisel points.

Wire Nails and Spikes

Size of nail	Length, inches	Casing nails, smooth and barbed box: Gage	No. to lb.	Finishing nails: Gage	No. to lb.	Clinch nails: Gage	No. to lb.	Fence nails: Gage	No. to lb.	Shingle nails: Gage	No. to lb.
2d	1	15½	1010	16½	1351	14	710	......	....	13	429
3d	1¼	14½	635	15½	807	13	429	......	....	12	274
4d	1½	14	473	15	584	12	274	......	....	12	235
5d	1¾	14	406	15	500	12	235	10	142	12	204
6d	2	12½	236	13	309	11	157	10	124	11	139
7d	2¼	12½	210	13	238	11	139	9	92	11	125
8d	2½	11½	145	12½	189	10	99	9	82	11	114
9d	2¾	11½	132	12½	172	10	90	8	62	10	83
10d	3	10½	94	11½	121	9	69	7	50	......	....
12d	3¼	10½	87	11½	113	9	62	6	40	......	....
16d	3½	10	71	11	90	8	49	5	30	......	....
20d	4	9	52	10	62	7	37	4	23	......	....
30d	4½	9	46	......	....	......	....	......	....	......	....
40d	5	8	35	......	....	......	....	......	....	......	....

Size of nail	Length, inches	Boat nails, Heavy: In.	No. to lb.	Boat nails, Light: In.	No. to lb.	Hinge nails, Heavy: In.	No. to lb.	Hinge nails, Light: In.	No. to lb.	Flooring brads: Gage	No. to lb.
4d	1½	¼	44	3/16	82	¼	50	3/16	82	......	....
6d	2	¼	32	3/16	62	¼	38	3/16	62	11	157
8d	2½	¼	26	3/16	50	¼	30	3/16	50	10	99
10d	3	⅜	14	⅜	22	⅜	12	¼	25	9	69
12d	3¼	⅜	13	¼	20	⅜	11	¼	23	8	54
16d	3½	⅜	12	¼	18	⅜	10	¼	22	7	43
20d	4	⅜	10	¼	16	⅜	9	¼	19	6	31

Size of nail	Length, inches	Common wire nails and brads: Gage	No. to lb.	Barbed car nails, Heavy: Gage	No. to lb.	Barbed car nails, Light: Gage	No. to lb.	Spikes: Size	Length in.	Gage	Approx. No. to lb.
2d	1	15	876	......	....	......	....	10d	3	6	41
3d	1¼	14	568	......	....	......	....	12d	3¼	6	38
4d	1½	12½	316	10	165	12	274	16d	3½	5	30
5d	1¾	12½	271	9	118	10	142	20d	4	4	23
6d	2	11½	181	9	103	10	124	30d	4½	3	17
7d	2¼	11½	161	8	76	9	92	40d	5	2	13
8d	2½	10¼	106	8	69	9	82	50d	5½	1	10
9d	2¾	10¼	96	7	54	8	62	60d	6	1	9
10d	3	9	69	7	50	8	57	......	7	5/16 in.	7
12d	3¼	9	64	6	42	7	50	......	8	⅜	4
16d	3½	8	49	6	35	7	43	......	9	⅜	3½
20d	4	6	31	5	26	6	31	......	10	⅜	3
30d	4½	5	24	5	24	6	28	......	12	⅜	2½
40d	5	4	18	4	18	5	21	......	....	......	....
50d	5½	3	14	3	15	4	17	......	....	......	....
60d	6	2	11	3	13	4	15	......	....	......	....

Wire Nails for Special Purposes

Length, in.	Gage No.	No. to lb.	Length, in.	Gage No.	No to lb.	Length, in.	Gage No.	No. to lb.
Barrel nails			**Barbed roofing nails**			**Slating nails**		
⅝	15½	1615	¾	13	714	1	12	411
¾	15½	1346	⅞	12	469	1¼	10½	225
⅞	14½	906	1	12	411	1½	10½	187
1	14½	775	1⅛	12	365	1¾	10	142
1⅛	14½	700	1¼	11	251	2	9	103
1¼	14	568	1⅜	11	230	**Fine nails**		
1⅜	13	400	1½	10	176	1	16½	1351
1½	13	367	1¾	10	151	1⅛	15	718
Barbed dowel nails			2	9	103	1½	14	473
⅝	8	290	**Clout nails**			1	17	1560
¾	8	250	¾	15	1160	1⅛	16	1015
⅞	8	210	⅞	14	808	**American felt roofing nails, ⅜-in. heads**		
1	8	190	1	14	705	⅞	12	195
1⅛	8	165	1⅛	14	628	1	12	180
1¼	8	150	1¼	13	423			
1⅜	8	130	1⅜	13	390			
1½	8	120	1½	13	358			

Approximate Number of Boat Spikes to a Keg of 200 Lb.

Size of spike	Length, in.											
	3	4	5	6	7	8	9	10	11	12	14	16
⅝ in. sq.	......	......	......	......	......	260	240	220	205	190	175	160
½ in. sq.	......	......	......	450	375	335	300	275	260	240	......	......
7/16 in. sq.	......	......	......	600	590	510	400	360	320	280	......	......
⅜ in. sq.	1320	1140	940	800	650	600	525	475	......	......	......	......
5/16 in. sq.	1660	1360	1230	1175	990	880	......	......	......	......	......	......
¼ in. sq.	3000	2375	2050	1825	......	......	......	......	......	......	......	......

Approximate Number of Railroad Spikes to a Keg of 200 Lb.

Size of spike	Length, in.						
	2½	3	3½	4	4½	5	5½
⅜ in. sq.	1342	1240	1190	1000	......	......	......
7/16 in. sq.	......	......	900	720	680	......	......
½ in. sq.	......	......	......	600	530	450	......
9/16 in. sq.	......	......	......	......	......	400	360

Cut Steel Nails and Spikes

(Sizes, lengths, and approximate number per lb.)

Sizes	Length, inches	Common	Clinch	Finishing	Casing and box	Fencing	Spikes	Barrel	Slating	Tobacco	Brads	Shingle
2d	1	740	400	1,100	......	......	......	450	340	......	......	......
3d	1¼	460	260	880	......	......	......	280	280	......	......	......
4d	1½	280	180	530	420	......	......	190	220	......	......	......
5d	1¾	210	125	350	300	100	......	......	180	130	......	......
6d	2	160	100	300	210	80	......	......	......	97	120	......
7d	2¼	120	80	210	180	60	......	......	......	85	94	......
8d	2½	88	68	168	130	52	......	......	......	68	74	90
9d	2¾	73	52	130	107	38	......	......	......	58	62	72
10d	3	60	48	104	88	26	......	......	......	48	50	60
12d	3¼	46	40	96	70	20	......	......	......	......	40	......
16d	3½	33	34	86	52	18	17	......	......	......	27	......
20d	4	23	24	76	38	16	14	......	......	......	......	......
25d	4¼	20	......	......	......	......		......	......	......	......	......
30d	4½	16½	......	......	30	......	11	......	......	......	......	......
40d	5	12	......	......	26	......	9	......	......	......	......	......
50d	5½	10	......	......	20	......	7½	......	......	......	......	......
60d	6	8	......	......	16	......	6	......	......	......	......	......
......	6½	......	......	......	......	......	5½	......	......	......	......	......
......	7	......	......	......	......	......	5	......	......	......	......	......

Sizes of American Wire Tacks

Oz.	Length, in.	Size of wire, steel wire gage			Oz.	Length, in.	Size of wire, steel wire gage		
		Upholsterers'	Carpet	Bill posters'			Upholsterers'	Carpet	Bill posters'
1	7/32	18	18	....	10	5/8	14½	15¼	12
1½	¼	18	18	....	12	11/16	14½	15	12
2	9/32	17	17	....	14	¾	14	14½	11½
2½	5/16	17	17	15	16	13/16	14	14½	11½
3	⅜	16½	16½	14	18	⅞	13½	13½	11
4	7/16	16	16	13½	20	15/16	13½	13½	11
6	½	15	15¼	13	22	1	13½	13½	....
8	9/16	15	15¼	12½	24	1⅛	13	13	....

Knots, Hitches, and Bends

No two parts of a knot which would move in the same direction if the rope were to slip, should lie alongside of and touching each other. The knots shown in Fig. 13 are known by the following names:

A, bight of a rope; B, simple or overhand knot; C, figure 8 knot; D, double knot; E, boat knot; F, bowline, first step; G, bowline, second step; H, bowline, completed; I, square or reef knot; J, sheet bend or weaver's knot; K, sheet bend with a toggle; L, carrick bend; M, "stevedore" knot completed; N, "stevedore" knot commenced; O, slip knot; P, Flemish loop; Q, chain knot with toggle; R, half hitch; S, timber hitch; T, clove hitch; U, rolling hitch; V, timber hitch and half hitch; W, blackwall hitch; X, fisherman's bend; Y, round turn and half hitch; Z, wall knot commenced; AA, wall knot completed; BB, wall knot crown commenced; CC, wall knot crown completed.

The bowline G, one of the most useful knots, will not slip, and after being strained is easily untied. Knots H, K and M are easily untied after being under strain. The knot M is useful when the rope passes through an eye and is held by the knot, as it will not slip, and is easily untied after being strained. The wall knot is made as follows: Form a bight with strand 1 and pass the strand 2 around the end of it, and the strand 3 around the end of 2, and then through the bight of 1, as shown at Z in the figure. Haul the ends taut, when the appearance is as shown at AA. The end of the strand 1 is now laid over the center of the knot, strand 2 laid over 1, and 3 over 2, when the end of 3 is passed through the bight of 1, as shown at BB. Haul all the

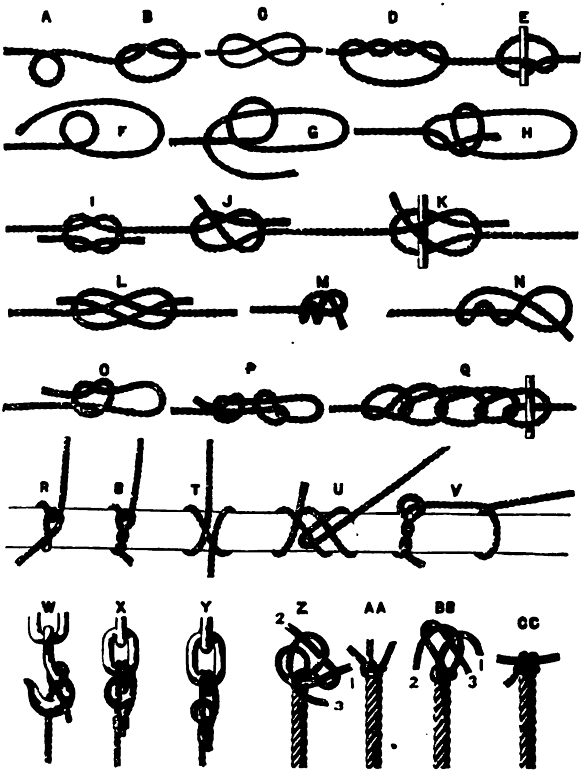


Fig. 13.—Rope Knots, Hitches and Bends.
(From C. W. Hunt Co.'s Catalog)

strands taut, as shown at CC. The "stevedore" knot (M, N) is used to hold the end of a rope from passing through a hole. When the rope is strained the knot draws up tight, but it can be easily untied when the strain is removed. If a knot or hitch of any kind is tied in a rope its failure under stress is sure to occur at that place. The shorter the bend in the standing rope the weaker is the knot. The approximate strength of knots compared with the full strength of (dry) rope (= 100), based on Miller's experiments (*Machy*, p. 198, 1900), is as follows: Eye splice over iron thimble, 90; short splice in rope, 80; S and Y (see Fig. 13), 65; H, O and T, 60; I and J, 50; B and P, 45.

Recommended Standard Cross-sections

The following cross-sections are recommended by the Cross-sections Committee of the A. S. M. E. Subdivisions of any of the materials may be

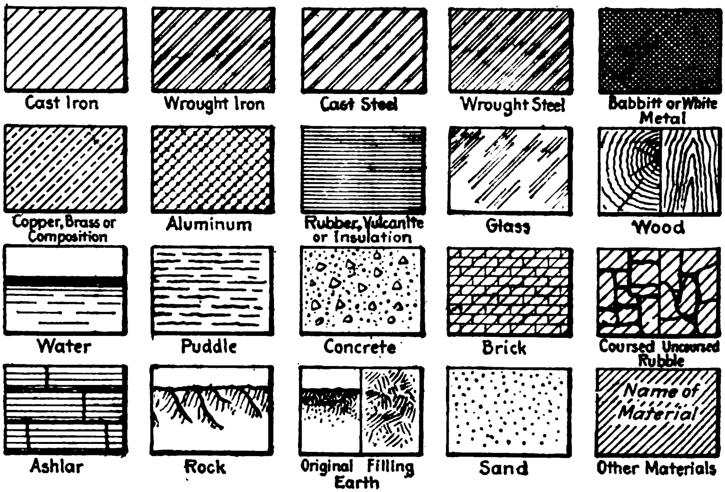


FIG. 14.—Recommended Standard Cross-sections.

made by taking one of these standard cross-sections as a basis and making minor changes or by writing on the standard cross-section the name of the material.

SECTION 8

POWER GENERATION

BY

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CONTENTS

* Staff Contribution.

* Staff contribution.

POWER GENERATION

MUSCULAR ENERGY OF MEN AND ANIMALS

The accompanying table gives results obtained by Poncelet, Morin, Rankine, and others. The work of Dr. F. W. Taylor shows that a maximum amount of shoveling may be accomplished by the use of a shovel taking up a load of 22 lb. Also that by the introduction of rest periods at stated intervals (determined from a study of the particular task), the amount of work done in a day by a laborer may be greatly increased above the figures given.

Nature of work	Weight moved or resistance overcome, lb. w	Velocity of movement, ft. per sec. v	Work done per sec., ft.-lb.	Time of working, hours per day	Work done per day, ft.-lb.
Raising Weights					
Man raising his own weight up a stair or ladder	143	0.5	71.5	8	2,059,200
Man hoisting weight with rope and pulley, and lowering unloaded rope	40	0.66	26.4	6	570,240
Lifting weights by hand	44	0.56	24.6	6	531,360
Carrying weights on the back up stairs or a ladder, returning unloaded*	143	0.13	18.6	6	401,760
Pushing loaded wheelbarrow up a 1:12 incline, returning unloaded	132	0.065	8.6	10	309,600
Shoveling up earth: lift, 5 ft. 3 in.	6	1.3	7.8	10	280,800
Operating Machines and Tools					
Pushing or pulling horizontally and continuously (capstan or oar)	26.4	2.0	52.8	8	1,520,640
Pushing and pulling alternately in a vertical direction (pump)	13.2	2.5	33	10	1,188,000
Turning a crank	17.6	2.5	44	8	1,267,200
Horse† operating a horse gin, walking	99	3.0	297	8	8,553,600
Horse operating a horse gin, trotting	66	6.6	436	4½	7,063,200

* Dr. Taylor ("Principles of Scientific Management," p. 60) cites the instance of a laborer lifting and carrying 1156 pigs of iron (each weighing 92 lb.) up an incline into a car during a 10-hour day. Average distance of travel, 36 ft.; total lift (probably not less than) 8 ft. Ft.-lb. of work (lifting) $= 8 \times 1156 \times 92 = 850{,}816$. Prior to a study of the task and the introduction of proper rest periods, the best day's accomplishment was the transporting of 305 pigs.

† Ox: $w = 132$, $v = 2$; mule: $w = 66$, $v = 3$; ass: $w = 31$, $v = 2.6$.

Nature of work	Weight moved or resistance overcome, lb. w	Velocity of movement, ft. per sec. v	Work done per sec., ft.-lb.	Time of working, hours per day	Work done per day, ft.-lb.
TRANSPORTING LOADS HORIZONTALLY*					
Man walking unloaded	143	5.0	715†	10	25,740,000†
Man wheeling load w in a 2-wheeled barrow, returning unloaded	220	1.7	374†	10	13,464,000†
Man wheeling load w in a 1-wheeled barrow, returning unloaded	132	1.7	224†	10	8,064,000†
Man traveling with load w on back	88	2.5	220†	7	5,544,000†
Man carrying load w on back, returning unloaded	143	1.7	243†	6	5,248,800†
Throwing earth with a shovel a distance of 13 ft.	6	2.2	13.2†	10	475,200†
Horse drawing cart, loaded with w, walking	1,540	3.6	5,544†	10	199,584,000†
Horse drawing cart loaded with w, trotting	770	7.2	5,544†	4½	89,812,800†
Horse walking with loaded cart, returning empty	1,540	2.0	3,080†	10	110,880,000†
Horse carrying burden w, walking	264	3.6	950†	10	34,200,000†
Horse carrying burden w, trotting	176	7.2	1,267†	7	31,928,400†

* Level roads, ordinary condition.
† Weight (lb.) × distance (ft.) transported.

According to D. K. Clark a laborer can exert 0.1 h.p. for 8 hours a day on a windlass or pump, and for periods of a few minutes as much as 0.5 h.p. The maximum force which a man can exert in pushing or pulling is from 110 to 130 lb.; the greatest weight he can ordinarily carry is about 330 lb.; the weight he is capable of sustaining ranges from 450 to 650 lb.

The draft of horses is reduced by working them in teams. The draft of a horse in a 2- (4-) [8-] horse team is but 98 (80) [49] per cent. of that exerted by the horse when worked alone. A 1 : 100 grade reduces the draft 10 per cent.; a 1 : 50 grade, 20 per cent.; a 1 : 30 grade, 35 per cent.; a 1 : 20 grade, 60 per cent., and a 1 : 10 grade, 75 per cent.

WINDMILLS

REFERENCES: R. M. Dyer, *Iowa Engineer*, July, 1906, and *Machy*, Aug., 1907; Bulletin No. 82, Agl. Expt. Station, Univ. of Wis.; Water Supply and Irrigation Papers Nos. 1, 8, 14, 20, 41 and 42, U. S. Geological Survey; *Eng. Mag.* (electric lighting), Dec., 1894, p. 475.

Windmills are reversed disk or propeller fans and are built in diameters up to 30 ft., for operation with wind velocities of from 8 to 30 miles per hour. The **theoretical horse power** of the wind passing through a windmill is h.p. $= 0.075(\pi D^2/4)V^3/(2g \times 550) = 0.000001663D^2V^3 = 0.000005247D^2W^3$, where D is the maximum diam. of the wheel, ft.; V the wind velocity, ft. per sec.; and W the wind velocity, miles per hour. The rear edge of a (curved) **vane** should be nearly parallel to the plane of motion, while the forward edge should be parallel to the direction of the impinging current of air. The efficiency is materially reduced by the presence of obstacles at the back of the vanes, such as arms. The weights of wheels increase twice as rapidly as the powers developed. Smaller wheels are the more efficient.

According to U. S. Government tests of a 12 ft. Aermotor, (1) the speed of the wheel for maximum load increases slightly faster than the first power of the wind velocity; (2) the power of the mill for a constant load varies as the square root of the wind velocity; (3) the maximum power of the mill varies as the square of the wind velocity; (4) the load for maximum power does not increase quite as fast as the wind velocity; and (5) the ratio of speed for maximum load to the speed for no load increases somewhat with the wind velocity.

Brake tests of two Aermotors show that the power increases approximately as the $\frac{3}{2}$ power of the diameter (for maximum h.p.); and that the number of revolutions per min. is inversely as the diameter. Taking mean annual wind conditions as found at Dodge, Kan., a 12-ft. Aermotor will furnish on an average 1.3 h.p. 10 hours a day for 26 days a month, and a 16-ft. mill 1.9 h.p. In Table 1, the maximum power developed by a 16-ft. mill at a wind velocity of 20 miles per hour is 1.55 h.p., and the theoretical h.p. of the wind passing through the mill, from formula, is 10.75 h.p., making the efficiency = 1.55/10.75 = 14.4 per cent.

In addition to their use for driving pumps, windmills are often employed as the motive power in small electric lighting installations, where they drive direct-current dynamos for charging storage batteries. Table 2 gives data on the electrical energy generated by windmills of various sizes.

Table 1. Results of Tests of Aermotors

	Revolutions per min.					Maximum horse power		
Wind velocity, miles per hour	8	12	16	20	25	10	15	20
12-ft. Aermotor....	30	49	63	75	87	0.21	0.58	1.05
16-ft. Aermotor	23	38	48	56	64.5	0.29	0.82	1.55
Ratio of peripheral velocities ($^{16}/_{12}$)...	1.02	1.03	1.02	0.99	0.99	......	......	......
Ratio of max. horse powers ($^{16}/_{12}$).....	......	......	......	......	......	1.38	1.41	1.48

Table 2. Kilowatts Generated by Windmill-driven Electric Plants

Wheel diam., ft.	Velocity of wind, miles per hour						
	8	10	12	16	20	25	30
12	....	1/16	1/8	1/4	1/2	3/4	1
14	....	1/8	1/4	1/2	7/8	1 1/4	1 1/2
16	1/16	3/16	3/8	3/4	1 1/8	1 5/8	2
18	1/4	3/8	5/8	1	1 1/2	2	2 1/2
20	3/8	5/8	1	1 1/2	2	2 3/4	3 1/2
25	5/8	7/8	1 1/2	2 1/4	3	4	5
30	1	1 1/2	2	2 3/4	3 1/2	4 1/2	6
35	1 1/2	2	2 5/8	3 3/8	4 1/8	5 1/4	7

STEAM BOILERS

BY

ARTHUR D. PRATT

MATERIALS AND CONSTRUCTION

REFERENCES: Dept. of Commerce and Labor: General Rules and Regulations Prescribed by the Board of Supervising Inspectors, as Amended Jan., 1912. Commonwealth of Massachusetts: Rules Formulated by the Board of Boiler Rules, 1911. Report of the Boiler Code Committee, A. S. M. E., Feb., 1915.

For marine boilers, see p. 1238; for heating boilers, p. 1347.

Rules for Use in the Manufacture of Steam Boilers

The rules governing the construction of steam boilers vary with different controlling boards. There appears, however, to be a tendency toward the adoption of uniform rules in the United States. The state of Massachusetts was the first to adopt a uniform law, and this is being used as the basis for such laws in other communities. The state of Ohio, the cities of Detroit, Memphis and Manila, P. I., have all adopted laws or ordinances almost exactly similar to those adopted by Massachusetts. Boilers built for merchant vessels in the United States must be constructed in accordance with the rules and regulations prescribed by the Board of Supervising Inspectors, Department of Commerce and Labor.

The most comprehensive and probably the most rigid rules for the construction of stationary boilers are those prepared by the Boiler Code Committee of the A. S. M. E. and accepted by the Council of that Society, Feb. 13, 1915. These rules were prepared as a suggested uniform standard for adoption by all states and communities. The states of Ohio, Indiana and Wisconsin have already passed legislation under which boilers built in accordance with the A. S. M. E. Standard Code are accepted as fulfilling the requirements of their state laws. These rules, together with the Massachusetts Law and the Rules and Regulations of the U. S. Board, are quoted below as typical and, in most instances, satisfactory. For the rational design of parts of steam boilers, see pp. 392, 421, 676.

Factor of Safety (MASS. LAW). For boilers with shells or drums exposed to the products of combustion and having lap-riveted longitudinal joints, lowest factor to be five (5).

(U. S. BOARD.) No definite factor of safety (F.S.) named. Under allowable pressure for shells or drums, F.S. = 6 for single riveting, which covers actual nominal F.S. as well as loss of strength at joint. Based on reduced section at rivets, F.S. < 5.

(A. S. M. E. CODE). For new construction, 5.

Rivet Steel (MASS. LAW). Tensile strength (T.S.) of rivet steel, 45,000 to 50,000 lb. per sq. in.; yield point not less than ½ T.S.; elongation not less than 28 per cent. in an 8-in. test piece. Rivet stock of full size as rolled to endure bending 180 deg. flat upon itself, both before and after being heated to a light cherry red and quenched, without fracture on outside of bent portion.

(A. S. M. E. CODE). T.S. = 45,000 to 55,000 lb. per sq. in.; yield point, min. lb. per sq. in. = ½ T.S.; elongation in 8 in., min. per cent. = 1,500,000/T.S., but need not exceed 30 per cent. Bending tests same as Mass. Law. Certain requirements of workmanship, finish, marking and inspection.

Rivet Formulæ (Mass. Law). Maximum shearing strength of steel rivets per sq. in. of cross-sectional area: single shear, 42,000 lb.; double shear, 78,000 lb.

On longitudinal joints distance from center of rivet to edge of plate not to be less than 1½ × diam. of rivet hole. Rivet holes, except for attaching angle-bars or stay to heads, to be either drilled full size with butt straps and heads bolted up in position, or punched ¼ in. less than full size for plates over 5/16 in. thick, and ⅛ in. smaller for plates not exceeding 5/16 in. thick, and drilled or reamed to full size, with plates, butt straps and heads bolted in position. Rivets to be of sufficient length to fill rivet holes and form a head equal in strength to the body of rivet. Rivets to be machine-driven wherever possible with sufficient pressure to fill rivet holes, and allowed to cool and shrink under pressure.

(A. S. M. E. Code). In computing the ultimate strength of steel rivets in shear, the following values in lb. per sq. in. of cross-sectional area of the rivet shank shall be used: single shear, 44,000; double shear, 88,000. The cross-sectional area used in the computation shall be that of the rivet shank after driving.

On longitudinal joints distance from center of rivet to edge of plate same as Mass. Law. The distance between center lines of any two adjacent rows of rivets, or the "back pitch" measured at right angles to direction of joint, shall be at least twice diameter of the rivets and shall meet the following requirements:

(a) Where a single rivet in the inner row comes midway between two rivets in the outer row, the sum of the two diagonal sections of the plate between the inner rivet and the two outer rivets shall be at least 20 per cent. greater than the section of the plate between the two rivets in the outer row.

(b) Where two rivets in the inner row come between two rivets in the outer row, the sum of the two diagonal sections of the plate between the two inner rivets and the two rivets in the outer row shall be at least 20 per cent. greater than the difference in the section of the plate between the two rivets in the outer row and the two rivets in the inner row.

Drilling, punching and riveting as in Mass. Law.

(U. S. Board.) Appendix of rules quotes the formulæ (equivalent to those of the British Board of Trade) for determination of pitch, distance between rows, diagonal pitch and maximum pitch of steel rivets, in which p = greatest pitch of rivets, in.; n = number of rivets in one pitch; p_d = diagonal pitch, in.; d = diam. of rivets, in.; T = thickness of plate, in.

$$p = \frac{23 \times d^2 \times 0.7854 \times n}{28 \times T} + d$$

Distance between rows for double chain-riveted joints not to be less than $2d$, and preferably not less than $(4d+1)/2$.

Diagonal pitch for double-zigzag-riveted lap joints, $p_d = (6p+4d)/10$.

Maximum pitch $= 1.31T + 1\frac{5}{8}$ in. for single-riveted lap joint, and $2.62T + 1\frac{5}{8}$ in. for double-riveted lap joint.

For data on riveted joints see also p. 674.

Drum Joints (Mass. Law). Longitudinal joints of boilers with shells or drums over 36 in. in diam. to be of butt-and-double-strap construction; and for diameters ≤ 36 in., may be of lap-riveted construction. Maximum pressure allowed on such shells or drums, not over 100 lb. per sq. in. Longitudinal joints of horizontal-return tubular boilers to be located above the fire line of setting. Horizontal-return tubular, vertical tubular, or locomotive-type boilers not to have continuous longitudinal joints over 12 ft. in length. Thickness of plates in a shell or drum to be of the same gage.

(A. S. M. E. CODE). Add to Mass. Law. With butt and double-strap construction, longitudinal joints of any length may be used provided the plates are tested transversely to the direction of rolling. Tests to meet standards prescribed under specifications for boiler-plate steel.

Butt straps and ends of shell plates forming longitudinal joints shall be rolled or formed by pressure, not blows, to the proper curvature.

Material for Cylindrical Shells or Drums (MASS. LAW). All open-hearth steel classified under flange or boiler steel, fire-box steel, and extra soft steel, the physical properties of which are given in Table 1.

Table 1. Physical Properties of Steel

	Flange or boiler steel	Fire-box steel	Extra soft steel
Tensile strength (T.S.), lb. per sq. in.	55,000 to 65,000	52,000 to 62,000	45,000 to 55,000
Yield point, lb. per sq. in., not less than	½ T.S.	½ T.S.	½ T.S.
Elongation, per cent. in 8 in., not less than	25	26	28

Shells, drums and butt straps to be of fire-box steel as above.

Heads, combustion chambers, furnaces, and plates that require flanging or staying to be of open-hearth flanged, fire-box or extra soft steel.

Test specimen to be subjected to both cold and quenched bending tests, and endure bending 180 deg. flat upon itself without fracture on the outside of bent portion.

(A. S. M. E. CODE). Specifications cover two grades of steel, flange and fire-box. Steel shall be made by the open-hearth process. T.S., lb. sq. in.: flange, 55,000–65,000; fire-box, 55,000–63,000; yield point, min. lb. per sq. in., 0.5 T.S.; elongation in 8 in. min., per cent. = 1,500,000/T.S. For material over ¾ in. thick, a deduction of 0.5 from above percentages of elongation shall be made for each increase of ⅛ in. in thickness above ¾ in. to a minimum of 20 per cent. Steel plates for any part of a boiler, when exposed to the fire or products of combustion and under pressure, shall be of fire-box quality; for any part under pressure, where fire-box quality is not specified, they shall be of fire-box or flange quality. Test specimen same as Mass. Law.

(U. S. BOARD.) Test specimen (see Fig. 19, p. 386) to show an elongation of at least 25 per cent. in a length of 4 in. up to a thickness of ¼ in. inclusive, in a length of 6 in. for all thicknesses above ¼ in., to show an average reduction of sectional area of at least 50 per cent. for thicknesses up to and including ½ in., 45 per cent. for thicknesses over ½ in. and up to ¾ in. inclusive, and 40 per cent. for thicknesses over ¾ in.

Quenching and bending test pieces to be at least 12 in. long, and from 1 to 3½ in. wide. Such pieces, after being heated to a cherry red, to be quenched and endure bending to a curve whose inner radius $\leq$ 1½ $\times$ thickness of sample, without cracks or flaws.

Maximum Allowable Pressure for Shells or Drums (MASS. LAW). Maximum allowable pressure (lb. per sq. in.) on steel or wrought-iron shells or drums to be determined from the minimum thickness (t) of shell plates, the lowest T.S. (lb. per sq. in.) stamped on plate by manufacturer, the efficiency (e) of longitudinal joint or ligament between tube holes (whichever is the least), the inside diam. ($2R$) of outside course, and a factor of safety (F.S.)

of not less than five (5), the formula being $p = \text{T.S.} \times t \times e/(R \times \text{F.S.})$. R and t in in., e in per cent.

When a shell or drum is drilled for tube holes in a line parallel to the axis of the shell or drum, the **efficiency (e) of ligament between the tube holes** is to be determined as follows:

(*a*) When pitch of tube holes on every row is equal, $e = (p-d)/p$; (p = pitch of tube holes, in.; d = diam. of tube holes, in.).

(*b*) When pitch of tube holes in any row is unequal, $e = (P-nd)/P$; (P = unit length of ligament, in.; n = No. of tube holes in length P; d = diam. of tube holes, in.).

(A. S. M. E. CODE). Maximum allowable pressure on shells and drums same as Mass. Law. No boiler shall be operated at a higher pressure than the maximum allowable pressure except when the safety valve or valves are blowing, at which time the maximum allowable working pressure shall not be exceeded by more than 6 per cent.

Efficiency of ligaments between tube holes to be determined as in Mass. Law for cases (*a*) and (*b*). When a shell or drum is drilled for tube holes in a line diagonal with the axis of the shell or drum, the efficiency of the ligament between tube holes shall be determined by the following methods and the lowest value used. (*a*) $e = 0.95\,(p_1 - d)/p_1$, (*b*) $e = (p - d)/p$, where p_1 = diagonal pitch and p = longitudinal pitch.

(U. S. BOARD.) Maximum working pressure in pound per sq. in. = $(\text{T.S.} \times t)/(R \times 6)$ for single riveting, in which R = radius of shell, in. For double riveting add 20 per cent. The pressure allowed to be based on the plate whose T.S. × thickness gives a minimum product.

The U. S. rule is unsatisfactory in not taking into account the efficiency of joint or class of seam. The F.S. of 6 simply covers the actual nominal F.S., as well as the loss of strength at joint.

Efficiency of Riveted Joints. See p. 679.

(A. S. M. E. CODE). The efficiency of a riveted joint is determined by calculating the breaking strength of a unit section of the joint, considering each possible mode of failure separately, and dividing the lowest result by the breaking strength of the solid plate of a length equal to that of the section considered. The strength of circumferential joints of boilers the heads of which are not stayed by tubes or through braces shall be at least 50 per cent. that of the longitudinal joints of the same structure. When 50 per cent. or more of the load which would act on an unstayed solid head of the same diameter as the shell is relieved by the effect of tubes or through stays, in consequence of the reduction of the area acted on by pressure and the holding power of the tubes and stays, the strength of the circumferential joints in the shell shall be at least 35 per cent. that of the longitudinal joints.

Plate Thickness (MASS. LAW). Minimum thickness of plates used in boiler construction, ¼ in. Minimum thickness (t) of shell plates as follows:

For shell diam. =	36 in. or under	37 to 54 in. incl.	55 to 72 in. incl.	Over 72 in.
t =	¼ in.	5/16 in.	⅜ in.	½ in.

Butt straps to be rolled or formed to proper curvature on forms made for that purpose and to have the following minimum thicknesses:

Thickness of shell plate, in.:	¼ to 11/32	⅜ and 13/32	7/16 and 15/32	½ to 9/16	⅝ and ¾	⅞	1 and 1⅛	1¼
Minimum thickness of butt straps, in.	¼	5/16	⅜	7/16	½	⅝	¾	⅞

Minimum thickness of tube sheets (t): When diam. (d) of tube sheet $\leq$ 42 in., $t = 3/8$ in.; for d = over 42 in. to 54 in. incl., $t = 7/16$ in.; for d = over 54 in. to 72 in. incl., $t = 1/2$ in.; for d = over 72 in., $t = 9/16$ in.

Bumped Heads (Mass. Law). Minimum thickness of convex head, in. $= t = R \times$ F.S. $\times P$/T.S.; minimum thickness of concave head, in. $= t = R \times$ F.S. $\times P/(0.6 \times$ T.S.), in which $R = 1/2 \times$ radius to which head is bumped, in., F.S. = 5, P = working pressure, lb. per sq. in. for which boiler is designed, and T.S. = tensile strength (lb. per sq. in.) stamped on heads by manufacturer.

(A. S. M. E. Code). Convex Heads: The thickness required in an unstayed dished head with the pressure on the concave side, when it is a segment of a sphere, shall be calculated by the formula: $t = 1/8 + [5.5 \times P \times L/(2 \times \text{T.S.})]$, where L = radius to which the head is dished, in. Where the radius is less than 80 per cent. of the diameter of the shell or drum to which the head is attached, the thickness shall be at least that found by the formula by making L equal to 80 per cent. of the diameter of the shell or drum.

Concave Heads: Dished heads with the pressure on the convex side shall have a maximum allowable working pressure equal to 60 per cent. of that for heads of the same dimensions with the pressure on the concave side. When a dished head has a manhole opening, the thickness shall be increased by not less than 1/8 in. When dished heads are of a less thickness than called for by the formula, they shall be stayed as flat surfaces, no allowance being made in such staying for the holding power due to the spherical form. The corner radius of an unstayed dished head measured on the concave side of the head shall not be less than 1½ in. or more than 4 in. and within these limits shall be not less than 3 per cent. of L. A manhole opening in a dished head shall be flanged to a depth of not less than three times the thickness of the head measured from the outside.

Flat Stayed Surfaces (Mass. Law). Minimum thickness of plate in stayed flat-surface construction, 5/16 in.

Stay-bolt ends to be riveted over or upset by equivalent process. Pitch allowed for stay bolts on flat surfaces and on furnace sheets of internally fired boilers in which the external diameter of the furnace is over 38 in. (except a corrugated furnace or one strengthened by an Adamson ring or the equivalent), not to exceed values given in Table 2.

Table 2. Maximum Allowable Pitch of Stay Bolts

Pressure, lb. per sq. in.	Thickness of plate, in.							Pressure, lb. per sq. in.	Thickness of plate, in.						
	5/16	3/8	7/16	1/2	9/16	5/8	11/16		5/16	3/8	7/16	1/2	9/16	5/8	11/16
	Maximum pitch of stay bolts in in.								Maximum pitch of stay bolts in in.						
100	5½	6¼	7	7¾	8½	....	....	170	4½	5	5⅝	6⅛	6¾	7¼	7⅜
110	5¼	6	6¾	7⅜	8⅛	....	....	180	4⅜	4⅞	5½	6	6½	7⅛	7¾
120	5⅛	5¾	6½	7⅛	7⅞	8½	....	190	4⅜	4⅞	5⅜	5⅞	6⅜	7	7½
125	5	5⅝	6⅜	7	7¾	8⅜	....	200	4¼	4¾	5¼	5¾	6¼	6¾	7⅜
130	5	5⅝	6¼	6⅞	7⅝	8¼	....	225	4⅛	4½	5	5½	6	6½	7
140	4¾	5½	6	6⅝	7⅜	8	....	250	4	4⅜	4¾	5¼	5¾	6¼	6⅝
150	4¾	5¼	5⅞	6½	7⅛	7¾	8⅜	300	3¾	4⅛	4½	4⅞	5⅜	5¾	6⅛
160	4⅝	5⅛	5¾	6¼	6⅞	7½	8..	....	....	....	....	....	....	....	..

For a pitch not exceeding 8½ in. that is not given in Table 2, use formula $P = C(t+1)^2/(S^2-6)$, in which S = max. pitch of stay bolts, in.; C = 66; t = thickness of plate in sixteenths of an inch, and P = working pressure, lb. per sq. in.

For hollow stay bolts with holes ¼ in. in diam. or over, maximum allowable pitch may be increased by the mean diam. of stay bolt. Mean diam. = (least outside diam. of stay bolt + diam. of hole)/2.

(A. S. M. E. Code). The ends of screwed stay bolts shall be riveted over or upset by equivalent process. The outside ends of such stay bolts shall be drilled with a hole at least 3/16 in. diameter to a depth extending ½ in. beyond the inside of the plates, except on boilers having a grate area not exceeding 15 sq. ft. where the drilling of the stay bolts is optional. The maximum allowable pitch in inches of screwed stay bolts, ends riveted over, shall not exceed values given in Table 2*a*.

Table 2a. Maximum Allowable Pitch of Screwed Stay Bolts, Ends Riveted Over

Pressure, lb. per sq. in.	Thickness of plate, in.					Pressure, lb. per sq. in.	Thickness of plate, in.					
	5/16	3/8	7/16	1/2	9/16		3/8	7/16	1/2	9/16	5/8	11/16
	Maximum pitch of staybolts, in.						Maximum pitch of staybolts, in.					
100	5¼	6⅜	7⅜	......	....	170	4⅞	5⅝	6¾	7½	8⅜	....
110	5	6	7	8⅜	....	180	4¾	5½	6½	7⅜	8⅛	....
120	4¾	5¾	6¾	8	....	190	4⅝	5⅜	6⅜	7⅛	7⅞	....
125	4¾	5⅝	6⅝	7¾	....	200	4½	5¼	6⅛	7	7¾	8½
130	4⅝	5½	6½	7⅝	....	225	4¼	4⅞	5⅞	6½	7¼	8
140	4½	5⅜	6¼	7⅜	8⅜	250	4	4⅝	5½	6¼	6⅞	7⅝
150	4¼	5⅛	6	7⅛	8	300	....	4¼	5	5⅝	6¼	7
160	4⅛	5	5⅞	6⅞	7¾	....	....	....	.....	.....	......	....

For a pitch not exceeding 8½ in. not given in the table, the maximum allowable working pressure may be calculated by the formula of the U. S. Board given below. The values of C are as follows: $C = 112$ for stays screwed through plates not over 7/16 in. thick, with ends riveted over; = 120 for stays screwed through plates over 7/16 in. thick, with ends riveted over; = 135 for stays screwed through plates and fitted with single nuts outside of plate; = 175 for stays fitted with inside and outside nuts and outside washers where the diameter of washers is not less than $0.4p$ and thickness not less than t.

In water-leg boilers the stay bolts may be spaced at greater distances between the rows than indicated in Table 2*a*, provided the portions of the sheet which come between the rows of stay bolts have the proper transverse strength to give a factor of safety of at least 5 at the maximum allowable working pressure. The diameter of a screw stay shall be taken at the bottom of the thread, provided this is the least diameter. Holes for screw stays shall be drilled full size or punched not to exceed ¼ in. less than full diameter of the hole for plates over 5/16 in. in thickness. and ⅛ in. less than the full diameter of the hole for plates not exceeding 5/16 in. in thickness, and then drilled or reamed to the full diameter. The holes shall be tapped fair and true, with a full thread. An internal cylindrical furnace which requires staying shall be stayed as a flat surface as in Table 2*a*.

(U. S. Board.) The maximum stress allowable on flat plates supported or stayed, to be determined by the following formulæ (all stayed surfaces formed to a curve, the radius of which is over 21 in., excepting surfaces otherwise provided for, shall be deemed flat surfaces):

$$\text{Working pressure (lb. per sq. in.)} = C \times T^2/P^2$$

in which T = thickness of plate in sixteenths of an inch, P = greatest pitch of stays, in., and C = a constant, varying for various types of stay bolts as follows:

C = 112 for screw stays with riveted heads, T = 7/16 in. and under.
C = 120 for screw stays with riveted heads, T = above 7/16 in.
C = 120 for screw stays with nuts, T = 7/16 in. and under.
C = 125 for screw stays with nuts, T = over 7/16 in. and under 9/16 in.
C = 135 for screw stays with nuts, T = 9/16 in. and over.
C = 175 for stays with double nuts, having one nut on the inside and one nut on the outside of plate, without washers or doubling plates.
C = 160 for stays fitted with washers or doubling strips having a thickness of at least ½P, riveted to outside of plates, and stays having one nut inside of plate and one nut outside of washer or doubling strip. For T take 72 per cent. of the combined thickness of plate and washer or plate and doubling strip.
C = 200 for stays fitted with doubling plates having a thickness equal to at least ½ the thickness of the plate reinforced, and covering the full area braced (up to curvature of flange, if any), riveted to either the inside or outside of plate, and stays having one nut outside and one inside of plate. Washers or doubling plates to be substantially riveted. For T take 72 per cent. of combined thickness of the two plates.
C = 200 for stays with plates stiffened with tees or angle bars having a thickness of at least ⅔T and depth of webs at least ¼P, and substantially riveted on the inside of plates, and stays having one nut inside bearing on washers fitted to the edges of webs, that are at right angles to plate. For T take 72 per cent. of combined thickness of web and plate.

No such flat plates or surfaces to be unsupported at a greater distance than 18 in.

British Board of Trade Rule (see notation for Mass. Law):

$$P = C(t+1)^2/(S-6)$$

In which

C = 125 for plates not exposed to heat or flame, the stays fitted with nuts and washers, the latter at least 3 × stay diam. and ⅔ × thickness of plate.
C = 187.5 for the same condition, but with washers = ⅔S in diam. and thickness not less than plate.
C = 200 for the same condition, but with doubling plates in place of washers, the width of which is ⅔S and thickness the same as the plate.
C = 112.5 for the same condition, but the stays with nuts only.
C = 75 when exposed to impact of heat or flame and steam in contact with plates, and stays fitted with nuts and washers 3 × stay diam. and ⅔ the plate's thickness.
C = 67.5 for the same condition, but stays fitted with nuts only.
C = 100 when exposed to heat or flame, and water in contact with the plates, and stays screwed into plates and fitted with nuts.
C = 66 for the same condition, but stays with riveted heads.

The Board of Trade rules are based on actual experiments and are satisfactory.

Tube Sheets—Internally Fired Boilers (U. S. Board). Maximum working pressure of a tube sheet supporting a crown sheet braced by crown bars to be

$$P = (D-d)T \times 27{,}000/(W \times D)$$

in which P = working pressure, lb. per sq. in.; D = least horizontal distance between tube centers, in.; d = inside diam. of tubes, in.; T = thickness of tube plates, in.; and W = extreme width of combustion chamber, in.

The compressive stress on tube plates, as determined by the following formula, shall not exceed 13,500 lb. per sq. in., when pressure on top of combustion chamber is supported by vertical plates of such chamber.

$$C = P \times D \times W/[2T \times (D - d)]$$

Where C = stress on tube sheet, lb. per sq. in.; P = working pressure, lb. per sq. in.; D = least hor. dis. between tube centers, in.; d = inside diam. of tubes, in.; W = extreme width of combustion chamber, in., and T = thickness of tube sheet, in.

(A. S. M. E. Code). Maximum allowable working pressure on a tube sheet of a combustion chamber where the crown sheet is *not* suspended from the shell of the boiler, same as in rule of U. S. Board, above.

Furnace Formulæ (Mass. Law):

$$L = \left(\frac{Ct^2}{Pd}\right)^2, \quad t = \sqrt{\frac{Pd\sqrt{L}}{C}}, \quad P = \frac{Ct^2}{d\sqrt{L}}, \quad d = \frac{Ct^2}{P\sqrt{L}}$$

in which P = working pressure, lb. per sq. in.; C = a constant = 110; t = thickness of furnace sheet in thirty-seconds of an inch, d = external diam. of furnaces, in., and L = longitudinal pitch of stay bolts, in in., or ½ × height of furnace if only one row of circumferential stay bolts is required.

(A. S. M. E. Code). Plain Circular Furnaces. The maximum allowable working pressure for unstayed, riveted, seamless or lap-welded furnaces, where the length does not exceed 6 times the diameter and where the thickness is at least 5/16 in., shall be determined by one or the other of the following formulæ:

(*a*) Where the length does not exceed 120 times the thickness of the plate, $P = 51.5\ (18.75T - 1.03L)/D$.

(*b*) Where the length exceeds 120 times the thickness of the plate, $P = 4250T^2/LD$, where P = maximum allowable working pressure, lb. per sq. in.; D = outside diameter of furnace, in.; L = length of furnace, in.; T = thickness of furnace walls, in sixteenths of an inch.

Where the furnaces have riveted longitudinal joints, no deduction need be made for the joint provided the per cent. efficiency of the joint is greater than $PD/1250T$.

A plain cylindrical furnace exceeding 38 in. in diameter shall be stayed in accordance with the rules governing flat surfaces.

(U. S. Board.) Tensile strength of steel used in construction of corrugated or ribbed furnaces not to exceed 67,000 lb. and not to be less than 54,000 lb.; for all other furnaces minimum T.S. not less than 58,000 lb. and maximum not more than 67,000 lb. Minimum elongation in 8 in. to be 20 per cent.

All corrugated furnaces having plain parts at the ends not exceeding 9 in. in length and made to practically true circles, shall be allowed a steam pressure in accordance with the formula $P = C \times T/D$, in which P = pressure lb. per sq. in.; T = thickness, in.; D = mean diam. in., and C = a constant, with the following values with various types of furnaces: Leeds, C = 17,300, $T \geq$ 5/16 in.; Morison, C = 15,600, $T \geq$ 5/16 in.; Fox, C = 14,000, $T \geq$ 5/16 in.; Purves, C = 14,000, $T \geq$ 7/16 in.; Brown, C = 14,000, T = 5/16 in.; type having sections 18 in. long, C = 10,000, T = not less than 7/16 in.

Material for Stays (Mass. Law). For maximum allowable stress per sq. in. on net cross-sectional area of stays and stay bolts for various materials.

see Table 3. When a stress greater than given in Table 3 is required, material should have a T.S. not over 62,000 lb. per sq. in., a yield point not less than ½ × T.S., and an elongation not less than 28 per cent. in an 8-in. test piece. Maximum allowable stress to be based on a factor of safety of not less than 6.5.

Table 3. Maximum Allowable Stress on Net Cross-sectional Area of Stays and Stay Bolts (lb. per sq. in.)

Material and type	Size up to and including 1¼ in. diam. or equivalent area	Size over 1¼ in. diam. or equivalent area
Weldless mild steel head to head or through stays....	8000	9000
Weldless mild steel diagonal or crowfoot stays.......	7500	8000
Weldless wrought-iron head to head or through stays..	7000	7500
Weldless wrought-iron diagonal or crowfoot stays....	6500	7000
Welded mild steel or wrought-iron stays............	6000	6000
Mild steel or wrought-iron stay bolts...............	6500	7000

(A. S. M. E. Code). The maximum allowable stress in lb. per sq. in. net cross-sectional area of stays and stay bolts shall be as follows:

(*a*) Unwelded stays less than twenty diameters long screwed through plates with ends riveted over, 7500.

(*b*) Unwelded stays and unwelded portions of welded stays, except as specified in (*a*), 9500 for lengths between supports not exceeding 120 diameters; 8500 for lengths exceeding 120 diameters.

(*c*) Welded portions of stays, 6000 for lengths between supports not exceeding 120 diameters; 6000 for lengths exceeding 120 diameters.

The length of the stays between supports shall be measured from the inner faces of the stayed plates. The stresses are based on tension only.

(U. S. Board.) Use of welded steel stay bolts prohibited. The maximum allowable working stress in lb. per sq. in. of cross-sectional area for stays accurately fitted and properly secured, is 9000 for tested steel stays 1¼ in. in diam. and above (stays thoroughly annealed); 8000 for tested Huston or similar type of brace, the cross-sectional area of which exceeds 5 sq. in.; 7000 for such tested braces when the cross-sectional area is not less than 1.227 and not more than 5 sq. in.; 7500 for wrought-iron through stays 1¼ in. and upward; 6800 for welded crowfoot stays of best quality refined wrought iron, and for all stays not provided for above when made of the best quality refined iron and steel without weld.

Tubes and Tube Tests (U. S. Board). All lap-welded tubes to be of charcoal iron, or mild steel, made by any process, and to undergo the following tests before shipment by the manufacturer:

(*a*) Test piece 2 in. in length cut from a tube to stand being flattened by hammering until sides are brought parallel with the curves on inside of ends not greater than 3 × thickness of metal without showing cracks or flaws, with bend at one side being in the weld.

(*b*) A second tube to have a flange turned over at right angles to body of tube and have a width of ⅜ in. Tests (*a*) and (*b*) to be done cold.

Each tube to be subjected to an internal hydrostatic pressure of 500 lb. per sq. in. without showing signs of weakness or defects. All steel tubes to have ends properly annealed by manufacturer before shipment, and stand expanding, flanging over on tube plate and beading without flaw, crack, or opening at weld.

Seamless Steel Boiler Tubes

Material. Tube steel, to be made by the open-hearth process.

Surface Inspection. Tubes to be free from all surface defects, particularly tears, snakes, checks, slivers, scratches, laps, pits, rings, and sinks. All seamless steel cold-drawn tubes to be annealed as a final process. One or more tubes to be selected at random from each charge of annealing furnace, and coupons cut from same for testing.

(a) A piece 3 in. long cut from the first tube to stand being flattened by hammering until its sides are brought parallel with a curve on inside of ends not greater than 3 × thickness of metal, without showing cracks or flaws.

(b) A flange to be turned all around the end of tube to a width equal to ⅜ in. beyond outside body of tube. Tests (a) and (b) to be done cold.

Hot-finished tubes to pass the same manipulating tests as cold-drawn tubes and be subject to the same conditions as to gage; annealing not required.

Each tube to be subject to an internal hydrostatic pressure of 1000 lb. per sq. in. without showing signs of weakness or defects.

All tubes to stand expanding, flanging over on the tube plate and beading without flaw or crack.

All individual tubes to be carefully gaged with a Birmingham wire gage, and come within the limits of one gage under or one gage over the specified thickness.

(A. S. M. E. Code). Lap-welded tubes shall be made of open-hearth steel or knobbled hammered charcoal iron. Seamless tubes shall be made of open-hearth steel.

A test specimen not less than 4 in. in length shall have a flange turned over at right angles to the body of the tube without showing cracks or flaws. This flange as measured from the outside of the tube shall be ⅜ in. wide.

A test specimen 3 in. in length shall stand test (a) of the U. S. Board.

Tubes under 5 in. in diameter shall stand an internal hydrostatic pressure of 1000 lb. per sq. in., and tubes 5 in. in diameter or over an internal hydrostatic pressure of 800 lb. per sq. in. Lap-welded tubes shall be struck near both ends, while under pressure, with a 2-lb. hand hammer or the equivalent.

All test specimens shall be taken from tubes before being cut to finished lengths and shall be smooth on the ends and free from burrs. All tests shall be made cold.

The finished tubes shall be circular within 0.02 in., and the mean outside diameter shall not vary more than 0.015 in. from the size ordered. All tubes shall be carefully gaged with a B. W. G. gage and shall not be less than the gage specified; but tubes on which the standard slot gage (specified) will go on tightly at the thinnest point, will be accepted. The length shall not be less, but may be 0.125 in. more, than that ordered.

The finished tubes shall be free from injurious defects and shall have a workmanlike finish and shall be practically free from kinks, bends and buckles.

Tube Holes and Ends. Tube holes shall be drilled full size from the solid plate, or they may be punched at least ⅛ in. smaller in diameter than full size and then drilled, reamed or finished full size with a rotating cutter. A fire-tube boiler shall have the ends of the tubes substantially rolled and beaded, or welded at the fire-box or combustion-chamber end. The ends of all tubes, suspension tubes and nipples shall be flared not less than ⅛ in. over the diameter of the tube hole on all water-tube boilers and superheaters, or they may be beaded. The ends of all tubes, suspension tubes and nipples of water-tube boilers and superheaters shall project through the tube sheets or headers not less than ¼ in. nor more than ½ in. before flaring.

Tubes for Water-tube Boilers. The minimum thicknesses of tubes used in water-tube boilers measured by Birmingham wire gage, for maximum allowable working pressures not exceeding 165 lb. per sq. in., shall be as follows: Diameters less than 3 in., No. 12 B. W. G.; 3 in. or over, but less than 4 in., No. 11 B. W. G.; 4 in. or over, but less than 5 in., No. 10 B. W. G.; 5 in., No. 9 B. W. G.

The above gages shall be increased for maximum allowable working pressures higher than 165 lb. per sq. in. as follows: Over 165 lb. but not exceeding 235 lb., 1 gage; over 235 lb. but not exceeding 285 lb., 2 gages; over

285 lb. but not exceeding 400 lb., 3 gages. Tubes over 4 in. diameter shall not be used for maximum allowable working pressures above 285 lb. per sq. in.

Tubes for Fire-tube Boilers. The minimum thicknesses of tubes used in fire-tube boilers measured by Birmingham wire gage, for maximum allowable working pressures not exceeding 175 lb. per sq. in., shall be as follows: Diameters less than 2½ in., No. 13 B. W. G.; 2½ in. or over, but less than 3¼ in., No. 12 B. W. G.; 3¼ in. or over, but less than 4 in., No. 11 B. W. G.; 4 in. or over, but less than 5 in., No. 11 B. W. G.; 5 in., No. 9 B. W. G.

For higher maximum allowable working pressures than given above the thicknesses shall be increased one gage.

Hydrostatic Pressure (Mass. Law). Maximum hydrostatic pressure applied to boiler not to exceed 1½ × maximum allowable working pressure. Boiler to be examined both internally and externally after test.

(A. S. M. E. Code). Same pressure as U. S. Board. The pressure shall be under proper control, so that in no case shall the required test pressure be exceeded by more than 6 per cent. During the test the safety valve or valves shall be removed, or each valve disk shall be held to its seat by means of a testing clamp and not by screwing down the compression screw upon the spring.

(U. S. Board.) Hydrostatic pressure applied = 1½ × steam pressure allowed in lb. per sq. in.; after applying hydrostatic test, inspector to thoroughly examine every part of boiler. All coil and pipe boilers when complete and ready for inspection to be subjected at the first inspection to a hydrostatic pressure double that of the steam pressure allowed in the certificate of inspection.

Recommendations Made by the Massachusetts Board of Boiler Rules

1. The installation of more than one safety valve on a boiler permitted to carry over 25 lb. pressure per sq. in.
2. Externally fired boilers over 84 in. in diam. not to be used.
3. Use of steam domes not recommended. Advise use of a reflecting plate, or a dry pipe located at the highest point of the steam space, and having closed ends and small slotted or drilled openings on its upper part, a total area of openings not less than twice the area of the outlet, and an ample drain opening at the lowest point.
4. Diagonal stays not to be attached to shell plates directly over the fire.
5. Cast-iron or copper steam pipe not to be used.
6. Elliptical handholes of the following sizes to be used: 2¼ × 3¼ in.; 2¾ × 3¾ in.; 3 × 4½ in.; 3½ × 5 in.; 4 × 6 in.
7. Boiler on which a longitudinal lap crack is discovered to be discontinued from service and not repaired.
8. O. G. form of construction at the lower end of furnace sheets not recommended.
9. Lead manhole gaskets below the water line are not to be used.

Miscellaneous (A. S. M. E. Code). Cross pipes connecting the steam and water drums of water-tube boilers, headers and cross boxes, and all pressure parts of the boiler proper over 2-in. pipe size or equivalent cross-sectional area for boilers in which the maximum allowable working pressure exceeds 160 lb. per sq. in.; mud drums of boilers used for other than heating purposes, and pressure parts of superheaters, separately fired or attached to stationary boilers, unless of the locomotive type, shall be of wrought steel or cast steel of Class B grade, see p. 463.

Cast iron shall not be used for boiler and superheater mountings such as nozzles, connecting pipes, fittings, valves and their bonnets, for steam temperatures of over 450 deg. fahr.

WELDED JOINTS. The ultimate tensile strength of a longitudinal joint which has been properly welded by the forging process, shall be taken as 28,500 lb. per sq. in. with steel plates having a range in tensile strength of 47,000 to 55,000 lb. per sq. in.

CALKING. The calking edges of plates, butt straps and heads shall be beveled. Every portion of the calking edges of plates, butt straps and heads shall be planed, milled or chipped to a depth of not less than 1/8 in. Calking shall be done with a round-nosed tool.

STOP VALVES. Each steam discharge outlet over 2 in. in diameter, except safety-valve and superheater connections, shall be fitted with a stop valve or valves of the outside screw and yoke type, located as near the boiler as practicable. The main stop valves of boilers shall be extra heavy when the maximum allowable working pressure exceeds 125 lb. per sq. in. When two or more boilers are connected to a common steam main, two stop valves, with an ample free blow drain between them, shall be placed in the steam connection between each boiler and the steam main. The discharge of this drain valve must be visible to the operator while manipulating the valve. The stop valves shall consist preferably of one automatic non-return valve (set next the boiler) and a second valve of the outside screw and yoke type; or, two valves of the outside screw and yoke type may be used.

When the maximum allowable working pressure exceeds 125 lb. per sq. in., the bottom blow-off pipe shall have two valves, or a valve and a cock, and such valves, or valve and cock, shall be extra heavy, except that on a boiler having multiple blow-off pipes, a single master valve may be placed on the common blow-off pipe from the boiler, in which case only one valve on each individual blow-off is required.

Every superheater shall have one or more safety valves near the outlet. The discharge capacity of the safety valve or valves on an attached superheater may be included in determining the number and sizes of the safety valves for the boiler, provided there are no intervening valves between the superheater safety valve and the boiler. Every safety valve used on a superheater, discharging superheated steam, shall have a steel body with a flanged inlet connection, and shall have the seat and disk of nickel composition or equivalent material, and the spring fully exposed outside of the valve casing so that it shall be protected from contact with the escaping steam.

Boiler Tubes

Holding Power of Expanded Tubes. C. B. Richards has investigated the holding power of expanded tubes (*The Locomotive*, 1881). The tubes tested were 3 in. diam. by 0.109 in. thick, and were expanded into plates 5/8 and 15/32 in. thick. The stresses at which the tubes yielded ranged from 5000 to 7500 lb. when the tubes were expanded, but neither flared nor beaded. When expanded and the ends were flared, the holding power was much greater, ranging from 19,000 to 20,500 lb.

Slipping Point of Expanded Joints. The slipping point is defined as that at which a leak will show. The working load, therefore, must be kept below the slipping load. O. P. Hood and G. L. Christensen (*Trans. A. S. M. E.*, vol. 30, p. 1193), after experiments with 3-in. No. 12 gage cold-drawn tubes expanded into 1-in. sheets, arrived at the following conclusions: (1) The slipping point of such tubes rolled into straight, smooth, machined holes in a 1-in. plate occurs with a pull of about 7000 lb.; (2) various degrees of rolling above the practice ordinarily followed do not affect particularly the point of initial slip; (3) the frictional resistance of such holes is about 750 lb. per sq. in. of tube-bearing area in sheets 5/8 and 1 in. thick; (4) for a higher resistance to initial slip, resistance other than friction must be depended upon; (5) the slipping point may be raised to three or four times that where a smooth hole is used, by serrating the tube seat by cutting or rolling grooves 0.01 in. deep into the machined hole; (6) a rolled joint may be made that will offer a resistance beyond the elastic limit of tubes and remain tight. As tubes increase in diameter and thickness, the normal pressure developed when rolled will be higher than that in the case just cited; the area of the tube seat will also be greater, and the resistance to slipping will, therefore, be increased.

Bursting Pressure of Boiler Tubes. Table 4 gives data on the bursting pressure of boiler tubes, derived from experiments conducted by I. Harter, Jr., in 1906, agreeing with other published data. The test pieces used were 2 ft. long. They were held at the ends from expanding, but were free to shorten, and this actually occurred before bursting. The figures are the results of six breaks in each instance.

Table 4. Physical Tests and Bursting Pressures of Boiler Tubes

Tube (4-in. No. 10 gage)	Lap-welded steel	Cold-drawn seamless	Charcoal iron	Charcoal iron
Original area of piece tested, sq. in.	0.1153	0.1045	0.1072	0.1132
Area after fracture, sq. in.	0.0628	0.0509	0.0718	0.0524
Elastic limit as tested, lb.	4,007	3,140	2,982	3,643
Maximum load, lb.	6,627	5,372	5,062	5,690
Elastic limit, lb. per sq. in.	34,850	29,928	27,800	32,180
Elongation, per cent. in 8 in.	16.7	21.5	15.65	22.21
Reduction of area, per cent.	45.6	51.3	33.1	53.6
Tensile strength, lb. per sq. in.	57,495	51,385	47,163	50,357
Bursting pressure, lb. per sq. in.	4,733	4,183	3,333	3,970

Collapsing Pressures of Tubes. See "Strength of Materials," p. 393, for results of tests by Stewart and Carman.

Working Pressures for Steel Flues (U. S. BOARD). The working pressure and corresponding minimum thickness of wall for long, plain, lap-welded and seamless steel flues, 7 to 18 in. in diam. and subjected to external pressure only, may be determined from the formula $t = [(F.S. \times P) + 1386] \times D/86{,}670$, in which t = thickness of wall, in.; P = collapsing pressure, lb. per sq. in.; D = outside diam. of tube, in.; and the factor of safety (F.S.) is not less than 5.

Standard Tubular Boiler Specifications

The following dimensions and specifications for fire-tube boilers, issued by the National Association of Tubular Boiler Manufacturers, have been generally adopted. For specifications of the American Boiler Manufacturers' Association, see p. 881.

Materials. The shell, heads and covering strips of the standard boiler shall be of flange steel as described in the standard specifications of the Association of American Steel Manufacturers. All such plates shall be marked 60,000 lb. tensile strength and in building the boiler the plates shall be so placed that these stamps are plainly visible on the outside. These plates shall have the full specified thickness at the edges and shall meet the tensile, quenching and bend tests described.

The tubes shall be standard quality lap-welded mild steel of standard manufacture. The thickness of the metal shall be 12 B. W. G. for 3-in. tubes, 11 B. W. G. for 3½-in. tubes and 10 B. W. G. for 4-in. tubes.

The rivets shall be of boiler-rivet steel as described in the standard specifications of the Association of American Steel Manufacturers and of proper size to suit the size of the hole and the thickness of the plates and to form up heads equal in strength to the pressed heads of same.

The diagonal braces shall be weldless, of the same quality as the flange steel previously described and pressed from the solid plate, or else forged from steel bars. The number of braces to be used shall be computed on an allowance of not more than 7500 lb. of load per sq. in. of section of brace neglecting, in this, the inherent strength of the heads and the slight angularity of the braces.

Suitable through rods and braces shall be installed below the tubes when necessary to sustain the pressure. These shall be of steel and shall be computed with the same allowance as diagonal braces.

Table 5. Dimensions for Standard Return Tubular Boilers

Working pressure	Seams	Horse-power of boiler as rated	Diam. of boiler, in.	Length of tubes, ft.	Number of 3½-in. tubes	Number of 4-in. tubes	Tube heating surface with 3½-in. tubes, sq. ft.	Tube heating surface with 4-in. tubes, sq. ft.	Shell heating surface, sq. ft.	Thickness of shell, in.	Thickness of heads, in.	Width of grates, in.	Length of grates, in.	Area of grates, sq. ft.	Diam. of stack, in.	Length of stack, ft.	Gage of stack	Length of guys, ft.	Diam. of guys, in.	Size of Trimmings: Size of steam opening, in.	Size of Trimmings: Size of pop safety valve, in.
100 lb. working pressure	Longitudinal seams all butt-jointed—, double-riveted	70	54	16	44	36	645.1	603.2	150.8	5/16	7/16	54	54	20.25	26	60	16	600	1/4	3½	2½
		80	60	16	54	44	791.7	737.2	167.6	5/16	7/16	60	54	22.5	28	60	14	600	5/16	4	2½
		90	60	18	54	44	890.6	829.4	188.5	5/16	7/16	60	60	25.0	28	60	14	600	5/16	5	3
		100	66	16	66	54	967.6	904.8	184.3	11/32	7/16	66	54	24.75	30	60	14	600	5/16	5	3
		110	66	18	66	54	1088.6	1017.9	207.4	11/32	7/16	66	60	27.5	30	60	14	600	5/16	5	3
		125	72	16	86	70	1260.8	1172.9	201.1	3/8	1/2	72	54	27.0	34	60	14	600	5/16	5	4
		150	72	18	86	70	1418.4	1319.5	226.2	3/8	1/2	72	60	30.0	34	60	14	600	5/16	6	4
		165	72	20	86	70	1576.0	1466.0	251.3	3/8	1/2	72	66	33.0	34	70	14	700	5/16	6	4
		180	78	18	110	88	1814.0	1658.8	245.2	13/32	1/2	78	60	32.5	38	60	12	600	3/8	6	4
		200	78	20	110	88	2015.9	1843.1	272.4	13/32	1/2	78	66	35.75	38	70	12	700	3/8	6	4
125 lb. working pressure	Longitudinal seams all butt-jointed—, triple-riveted	70	54	16	44	36	645.1	603.2	150.8	11/32	7/16	54	54	20.25	26	60	16	600	1/4	3½	2½
		80	60	16	54	44	791.7	737.2	167.6	3/8	7/16	60	54	22.5	28	60	14	600	5/16	4	2½
		90	60	18	54	44	890.6	829.4	188.5	3/8	7/16	60	60	25.0	28	60	14	600	5/16	5	3
		100	66	16	66	54	967.6	904.8	184.3	13/32	7/16	66	54	24.75	30	60	14	600	5/16	5	3
		110	66	18	66	54	1088.6	1017.9	207.4	13/32	7/16	66	60	27.5	30	60	14	600	5/16	5	3
		125	72	16	86	70	1260.8	1172.9	201.1	7/16	1/2	72	54	27.0	34	60	14	600	5/16	5	4
		150	72	18	86	70	1418.4	1319.5	226.2	7/16	1/2	72	60	30.0	34	60	14	600	5/16	6	4
		165	72	20	86	70	1576.0	1466.0	251.3	7/16	1/2	72	66	33.0	34	70	14	700	5/16	6	4
		180	78	18	110	88	1814.0	1658.8	245.2	1/2	9/16	78	60	32.5	38	60	12	600	3/8	6	4
		200	78	20	110	88	2015.9	1843.1	272.4	1/2	9/16	78	66	35.75	38	70	12	700	3/8	6	4
150 lb. working pressure	Longitudinal seams all butt-jointed—, quadruple-riveted	70	54	16	44	36	645.1	603.2	150.8	3/8	7/16	54	54	20.25	26	60	16	600	1/4	3½	2½
		80	60	16	54	44	791.7	737.2	167.6	13/32	1/2	60	54	22.5	28	60	14	600	5/16	4	2½
		90	60	18	54	44	890.6	829.4	188.5	13/32	1/2	60	60	25.0	28	60	14	600	5/16	5	3
		100	66	16	66	54	967.6	904.8	184.3	15/32	1/2	66	54	24.75	30	60	14	600	5/16	5	3
		110	66	18	66	54	1088.6	1017.9	207.4	15/32	1/2	66	60	27.5	30	60	14	600	5/16	5	3
		125	72	16	86	70	1260.8	1172.9	201.1	1/2	9/16	72	54	27.0	34	60	14	600	5/16	5	4
		150	72	18	86	70	1418.4	1319.5	226.2	1/2	9/16	72	60	30.0	34	60	14	600	5/16	6	4
		165	72	20	86	70	1576.0	1466.0	251.3	1/2	9/16	72	66	33.0	34	70	14	700	5/16	6	4
		180	78	18	110	88	1814.0	1658.8	245.2	17/32	9/16	78	60	32.5	38	60	12	600	3/8	6	4
		200	78	20	110	88	2015.9	1843.1	272.4	17/32	9/16	78	66	35.75	38	70	12	700	3/8	6	4

Other standardised dimensions of fittings are as follows: Size of water gage glass, ¾ in. Size of water column connections, 1¼ in. Size of gage cocks, ¾ in. Size of blow-off, 2 in. up to 125 boiler h.p.; 2½ in. from 125 to 200 b.h.p. Size of feed and check valves, 1½ in. up to 180 b.h.p., and 2 in. from 180 to 200 b.h.p. Size of steam gage dial, 5 in.

Braces as above described shall be carefully placed so that the pressure on the flat surfaces, both above and below the tubes, shall be as nearly equally distributed as possible.

For any boiler head the area to be supported by braces shall be the surface included within lines drawn 2 in. from the outside of the tubes and 3 in. from the inside of the shell.

Design. The longitudinal seam of the shell shall be of the butt-joint type having inside and outside covering strips and either double, triple or quadruple riveted as indicated in Table 5.

The circumferential seams shall be of the lap type of joint and single-riveted.

In all joints the size, number and spacing of the rivets shall be such as to provide the strength necessary to maintain the factor of safety of 5.

The tubes shall be placed in vertical and horizontal rows with ample space between the adjacent tubes and also between tubes and shell for the circulation of water and also with a large steam space above the water line.

Boilers 44 in. or less in diam. shall have a manhole above the tubes and a handhole below the tubes. These may both be in the front head, or the handhole may be in the front head and the manhole in the back head. Boilers 48 in. in diam. shall have two manholes, both in the heads, one above the tubes and one below the tubes. They may both be in the front head or one manhole may be in the front head below the tubes and the other in the back head above the tubes. Boilers larger than 48 in. shall have two manholes, one of which shall be in the front end below the tubes, the other shall be above the tubes in either head or preferably may be placed in the shell.

The opening in the manhole shall not be less than 10×15 in. Each manhole shall be equipped with a heavy plate, bale, bolt, nut and gasket.

Manholes placed in the shell shall be properly reinforced with a suitable steel saddle.

All openings 2 in. in diam. or larger shall have suitable steel reinforcing flanges riveted on. Cast-iron flanges or nozzles shall not be used.

Boilers for 125 to 150 lb. working pressure shall have the feed opening in the front head over the tubes and same shall be provided with a brass bushing and an internal feed pipe extending from the front end of the boiler to within about 3 ft. of the rear head, thence across to the side of the shell terminating in an elbow for discharge below the top row of tubes.

Boilers for 100 lb. working pressure shall be arranged to feed through the blow-off connection and the internal feed pipe be omitted.

Each boiler shall be provided with four steel lugs or brackets, two on each side for supporting the boiler, except that boilers 78 in. in diam. or boilers 20 ft. long shall be provided with eight such lugs, four on each side of the boiler and all of them arranged in pairs.

Workmanship. All rivet holes and all tube holes shall be either drilled from the solid or punched small and reamed to size. After drilling or reaming the plates shall be taken apart and all burrs removed and the tube holes chamfered with a rose reamer.

The edges of the plates shall be beveled to an accurate calking edge and after being riveted shall be calked tight with a round-nose tool.

The tubes shall be carefully placed, expanded tight in the heads with a roller expander and the ends carefully beaded over against the head at both ends.

Water Column. The water column shall be provided with three gage cocks and shall be so placed that the lowest gage cock shall be 2 in. above the tops of the uppermost row of tubes. The middle gage cock shall be 3 in. and the uppermost gage cock 6 in. above the lowest gage cock, measured vertically.

Tests. The completed boiler shall be subjected to a steady water pressure 50 per cent. above its proposed working pressure and made tight.

Equipment. A complete standard boiler equipment includes: The bare boiler with lugs; cast-iron front with anchors; grate bars with bearers; rear arch bars; rear ash-pit door and frame; safety valve with nipple to connect it to boiler; steam gage with siphon; water column with gage glass, three gage cocks and pipe connections to boiler; blow-off valve; check valve; stop valve for water feed line; smoke stack and guys; stacks less than 60 ft. long shall have four guy eyes, and guys six times as long as the stacks. Stacks 60 ft. long or longer shall have six guy eyes, two sets of three, and guys ten times as long as the stack. A stack plate or nozzle shall be included with each full-front boiler.

The boiler specifications adopted by the American Boiler Manufacturer's Association (1898) differ from those preceding in the following particulars:

Steel for shells heads and covering strips to be as specified in Table 1 *ante* (max. sulphur, 0.03 per cent.; max. phosphorus, 0.04 per cent.). Thickness of charcoal iron or mild steel tubes, B. W. G.: 1 to 1¾-in. tubes, No. 13; 2 to 2½-in., No. 12; 2¾ to 3½-in., No. 11; 3¾ and 4-in., No. 10; 4½ and 5-in., No. 9. Rivets to be of charcoal iron or of mild steel having the properties of fire-box steel in Table 1. Rivets to have height of head at periphery of shank = ½ shank diam.; shank to join head with a slight fillet. Pressures used in machine riveting: 1⅛ and 1¼-in. rivets, 80 tons; 1-in., 65 tons; 15/16-in., 57 tons; ¾-in., 35 tons. Make resistance to shear of aggregate rivet section at least 10 per cent. greater than tensile strength of the net or standing plate metal. Copper ferrules (Nos. 18 to 14 B. W. G.) to be used on ends of tubes of fire-tube boilers subject to direct heat. Iron stay bolts: T.S., $>$ 46,000 lb.; E.L., $>$ 26,000 lb.; elongation $>$ 22(20) per cent. for bolts $<$ ($>$) 1 sq. in. in cross-section. Steel stay bolts: T.S. $>$ 55,000 lb.; E.L., $>$ 33,000 lb.; elongation $>$ 25 (22) per cent. for bolts $<$ ($>$) 1 sq. in. in cross-section. Braces and stays: Allowable actual pull per sq. in. of net section of good stock, 6500 lb. for wrought iron and 8000 lb. for mild steel. Hydrostatic test pressure not to exceed 1½ $\times$ working pressure; water temperature $\geqq$ 125 deg. fahr. In flanging, bending and forming no work is to be done on any point which does not show red by daylight and at least 4 in. around it.

THE BURNING OF FUEL

Hand-firing with Coal

Anthracite. See p. 595. This fuel is marketed under the names given in Table 6 to designate sizes. The larger sizes are not generally employed for commercial steam generation, the grades now used in power-plant work being Nos. 1, 2 and 3 buckwheat.

Table 6. Anthracite Coal Sizes

Trade name	Round mesh, in.		Testing segments, standard square mesh, in.	
	Through	Over	Through	Over
Broken	4½	3¼	4	2¾
Egg	3¼	2⅜	2¾	2
Stove	2⅜	1⅝	2	1⅜
Chestnut	1⅝	⅞	1⅜	¾
Pea	⅞	⅝	¾	½
No. 1 buckwheat	⅝	⅜	½	¼
No. 2 buckwheat or rice	⅜	3/16	¼	⅛
No. 3 buckwheat or barley	3/16	3/32	⅛	1/16

This class of fuel is usually hand-fired. It should be spread evenly in small charges at frequent intervals and left alone after it strikes the fire bed, the fire tools being used as little as possible. The thickness of the fires is not over 2 or 3 in., but the fuel bed builds up between cleaning intervals and the total thickness may be from 14 to 16 in. just before such an interval. Cleaning periods should depend upon the combustion rate and the class of fuel. The fuel ignites with difficulty, and in cleaning fires this fact must always be taken into consideration. Combustion arches sprung over the grates assist in the ignition of freshly fired fuel and in keeping the furnace temperature high. A forced blast is usually employed.

The rated capacity of a boiler can be obtained with the ratio of grate to heating surface of 1 to from 35 to 40. With the finer sizes of anthracite and where overloads are desired, however, this ratio should preferably be 1 to 25. Grates 10 ft. deep, with a slope of 1½ in. to the foot, may be conveniently handled, and grates 12 ft. deep with the same slope, successfully handled. Wherever grates over 8 ft. deep are installed, the rear portion at least should

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be of the overlapping dumping type. Air openings in the grate bars should be from 3/16 in. for No. 3 buckwheat to 5/16 in. for No. 1 buckwheat. These openings should be equally distributed over the whole surface to prevent blowing holes in the fire.

For burning the smaller sizes of anthracite a blast up to 3 in. should be available. Stacks should be of such sizes as to insure a suction at all times within all parts of the boiler setting. Ashes may be economically disposed of when circumstances permit, by flushing with water. A forced blast with anthracite fuel causes rapid fouling of the heating surfaces, the dust carried over amounting often to as much as 10 per cent. of the total coal fired.

Bituminous Coal. See p. 595. The sizes and grades of bituminous and semi-bituminous coals vary so much according to kind and locality, that there are no standards of size for these coals which are generally recognized. The following classification is sometimes used:

EASTERN BITUMINOUS COALS

(*a*) Run of mine coal; the unscreened coal taken from the mine.

(*b*) Lump coal; that which passes over a bar screen with openings 1¼ in. wide.

(*c*) Nut coal; that which passes through a bar screen with 1¼-in. openings and over one with ¾-in. openings.

(*d*) Slack coal; that which passes through a bar screen with ¾-in. openings.

WESTERN BITUMINOUS COALS

(*e*) Run of mine coal; the unscreened coal taken from the mine.

(*f*) Lump coal; divided into 6-in., 3-in., and 1¼-in. "lump," according to the diam. of the circular openings over which the respective grades pass; also 6×3-in. "lump" and 3×1¼-in. "lump" according as the coal passes through a circular opening having the diam. of the larger figures and over that of the smaller diam.

(*g*) Nut coal; divided into 3-in. "steam nut," which passes through an opening 3 in. in diam.; 1¼-in. "nut," which passes through a 1¼-in. diam. opening and over a ¾-in. diam. opening; ¾-in. "nut," which passes through a ¾-in. diam. opening and over a ⅝-in. diam. opening.

(*h*) Slack coal; that which passes through a ⅝-in. diam. opening.

(*i*) Washed sizes; those passing through or over circular openings of the following diameters, in inches:

Number	1	2	3	4	5
Through	3	1¾	1⅛	¾	¼
Over	1¾	1⅛	¾	¼	0

In burning bituminous coals the volatile gases distilled from the coal should be brought into intimate contact with air sufficiently heated to cause them to ignite; sufficient space should be allowed for their thorough mixture with the air; and sufficient time should be allowed for their combustion before the gases strike the comparatively cool heating surfaces.

As the volatile content of the coal increases, the difficulty of burning it successfully will increase. Highly volatile coals need an added combustion space, which can only be obtained by the use of an extension furnace. The percentage of volatile matter will govern the thickness of fuel bed to be carried for the best results. The following suggestions for general conditions may be of service: Semi-bituminous coals, such as Pocahontas, New River, Clearfield, etc., require fires from 12 to 14 in. thick; bituminous coals from the Pittsburg District, from 4 to 6 in. thick; Kentucky, Tennessee, Ohio, and Illinois bituminous coals, from 4 to 6 in.; free-burning Wyoming coal from the Rock Spring District, from 6 to 8 in.; poorer grades of western coals, from Montana, Utah and Washington, about 4 in.

Firing Methods. (*a*) In the spreading method but little fuel is fired at one time, and is spread evenly from front to rear of the fuel bed. Where there is more than one firing door, the doors should be fired alternately, so that the whole surface of the fire will not be blanketed with green coal.

(b) In the coking method, fresh coal is fired to a considerable depth at the front of the grate and, after it is partly coked, is pushed back into the furnace. This preserves a bed of burning carbon at the rear of the grate, which burns the volatile matter driven off from the green coal. This method is not applicable where the gases pass directly upward from the fire. Modern practice for hand-fired boilers favors the spread-firing method. As a rule, the less a bituminous coal fire is worked, the better the results.

Table 7 gives the **ratios of grate surface to heating surface** for conditions of economy and capacity and the width of grate-bar openings which, under ordinary conditions, will give satisfactory results with various bituminous coals. In general, the ratio for capacity should be selected, as grates can always be cut down.

Table 7. Grate Dimensions and Ratios of Grate Surface to Heating Surface with Various Bituminous Coals

Coal	Grate-bar openings, in.		Ratio for economy		Ratio for capacity	
	Mine run	Slack	Mine run	Slack	Mine run	Slack
Virginia, West Virginia, Maryland and Pennsylvania	1/2	3/8	1:60	1:55	1:55	1:50
Ohio, Kentucky, Tennessee and Alabama	3/8 to 1/2	1/4	1:55	1:50	1:50	1:45
Illinois, Indiana, Kansas and Oklahoma	3/8 to 1/2	1/4	1:50	1:45	1:45	1:40
Colorado and Wyoming*	3/8	1/4	1:50	1:45	1:45	1:40

* Some of the latter classes can be best handled with ratios as given for Ohio coals, but in general the larger ratios should be used.

Lignites. See p. 603. The volatile content of this class of fuel is higher than in bituminous coal and the difficulties of burning it greater. A large combustion space is necessary and the best results are obtained with furnaces of the reverberatory type. A fuel bed of from 4 to 6 in. should be maintained, and firing should be in small quantities at frequent intervals by the spreading method. Clinker forms rapidly on exceeding certain combustion rates, and a steam jet in the ash pit for softening such clinker is often desirable. Shaking grates aid in cleaning fires, but must be handled carefully to prevent good fuel being lost in the ash pit. Lignites usually produce much smoke when hand-fired. For western lignites, air spaces in grate bars should be 1.2 in.; for Texas lignites, 7/16 in.

For **briquetted fuel** and **pulverized coal**, see p. 604.

Automatic Stokers

Stoker-fired furnaces have the advantage over hand-fired furnaces of minimizing smoke. In general, a stoker installation is not practicable unless both the coal and ashes are automatically handled. In large plants the labor saving made possible by the installation of stokers in connection with modern methods of coal storage and coal and ash handling, is considerable. In small plants, however, the labor saving may be negligible. Stokers are therefore advisable in small plants only where the fuel saving will be large, or where the smoke question is important.

The upkeep cost of stoker-fired furnaces will in general be higher than that of hand-fired furnaces. The use of stokers makes it possible to burn cheaper fuels with as high or higher efficiencies than those obtainable with better grades of fuel in hand-fired furnaces. This better efficiency is due to even and continuous firing as against intermittent firing, and a constant air supply as against a variation in such supply with varying furnace conditions in hand-fired practice. When properly proportioned and designed, the boiler capacity can be increased over hand-firing without loss of efficiency.

There is a tendency in all stokers to cause loss of unburned fuel in the ash pit. Other conditions being equal, there will be no appreciable difference in the efficiency or capacity of various types of stokers, provided care is taken to select the proper type for the fuel to be burned and the operating conditions to be fulfilled. No stoker will satisfactorily handle all classes of fuel.

Types of Stokers. Stokers can be classified under three general types: (1) traveling-grate, (2) overfeed, and (3) underfeed.

Traveling-grate Stokers. This type is illustrated by the Babcock & Wilcox Co.'s chain-grate stoker which consists of an endless grate composed of small bars passing over sprockets at the front and rear of the furnace. Coal is fed from a hopper by gravity on to the forward end of the grate, is ignited as it passes under an ignition arch and is carried by the grate to the rear as the combustion progresses. When properly operated, combustion is completed when the fire reaches the end of the grate and the refuse is carried over the rear end. Generally a bridge-wall water box or similar device at the rear serves as a seal to prevent the admission of excess air at that point, and in some cases closes up the rear portion of the fire by partly obstructing its passage over the rear of the grate. Some types are equipped with automatic dumping grates at the rear.

Advantages: Especial adaptability for burning low grades of coal running high in volatile matter—burned only with difficulty on other stokers; comparatively low upkeep cost; ease of withdrawal from furnace for inspection and repairs without in any way disturbing the boiler setting; continuous and automatic cleaning.

Overfeed Stokers may be divided into two general classes, the distinction being in the direction in which the coal is fed relatively to the furnace. In one class (illustrated by the Roney stoker) the coal is fed into hoppers at the front end of the furnace to the upper part of grates inclined downward toward the rear at about 45 deg. The grates are reciprocated, taking alternately straight and inclined positions, and this motion gradually carries the fuel as it is burned toward the rear and bottom of the furnace. At the bottom of the inclined grates flat dumping grates are supplied for completing the combustion and for cleaning. The fuel is coked on the upper portion of the grates, and the volatile gases driven off in this process are ignited and burned in their passage over the bed of burning carbon lower on the grates.

In the other class (represented by the Murphy stoker) the fuel is fed from the sides of the furnace for its full length on to the upper part of grates inclined downward toward the center. The inclined grates are moved by rocking bars and the fuel is gradually carried on to the bottom and center of the furnace as combustion proceeds, where a clinker breaker grinds out and removes the refuse.

Overfeed stokers are advantageous in the burning of coals of the caking class. The fires ordinarily carried are comparatively thin, and the movement of the grate bars keeps them broken up and open, thus preventing caking.

Underfeed Stokers. These are of two classes, either horizontal or inclined. In the Taylor inclined underfeed stoker the fuel is fed from beneath by plungers. The green fuel is fed under the coked and burning coal, the volatile gases from this fresh fuel being heated and partially burned in their passage through the hottest fire on the top. As in the overfeed stokers, the refuse is carried downward to the rear where there are dumping grates.

The American underfeed stoker is an example of the horizontal class of underfeed stoker. The principle of operation of this stoker is the same as that of the Taylor stoker. The fuel is fed continuously from beneath by a screw, which carries it as it is consumed upward and backward, depositing the ash on dead plates on either side of the retorts, from which it is removed either by dumping or by hand through doors at the front of the furnace.

Advantages: Underfeed stokers, as a class, have the ability to carry heavier fires. As they are practically all operated with a forced blast (driven in some cases by the same mechanism as the stoker drive), but a small amount of draft suction is necessary in the furnace, and, due to the thickness of the fuel bed, the air supply is kept at a minimum, with a resulting high CO_2 reading. As the plunger and screw feeds both tend to break up the fire, caking coals can be burned with satisfactory results.

Very high temperatures are ordinarily developed in the furnaces of underfeed stokers, which may result in brickwork troubles if the draft is not properly regulated.

Smoke

Many plants burning a wide variety of fuels in hand-fired furnaces operate under ordinary conditions without smoke. When, however, a load is suddenly thrown on and the fires have to be brought up quickly, the steam pressure can only be maintained by the free use of the slice bar. This working of the fire will cause smoke.

In hand-fired furnaces the spreading method with alternate firing should be used, the coal being fired evenly, quickly, lightly and often, and the fire worked as little as possible. Smoke can be diminished by giving the gases a long travel under the action of heated brickwork (in an extension furnace or otherwise) before striking the boiler heating surfaces. Air introduced over the fires and the use of heated piers, arches, etc., for mingling the air with the gases distilled from the coal, will diminish smoke.

Under ordinary operating conditions, stoker-fired furnaces are much more nearly smokeless than hand-fired, and may be to all intents and purposes smokeless. In stoker installations, the volatile gases as they are distilled should be acted upon by ignition or other arches before they strike the heating surfaces.

So-called "smoke consumers" are for the most part, impracticable and diminish smoke only at a loss of efficiency and capacity through the addition of excess air, or through using a considerable amount of steam.

Smoke Determinations. No wholly satisfactory methods for either quantitative or qualitative smoke determinations have come into use, or any reliable method for fixing even the relative density of smoke issuing from a chimney at different times. The character of the sky or the background has a bearing on the color and density, and the personal equation of the observer enters very largely. Density charts and smoked-glass apparatus in general have proven unsatisfactory. A method sometimes employed is that of making 1-min. observations and recording these according to the degree of blackness, giving the various degrees an arbitrary percentage value, rated about as follows: Dense black, 100; medium black, 80; dense gray, 60; medium gray, 40 light gray, 20; very light, 5; trace, 1; clear chimney, 0.

The most generally used basis of qualitative smoke determination is the chart known as **Ringlemann's chart.** This consists of four cards ruled with vertical and horizontal lines forming squares. No. 1 is ruled with lines 1 mm. thick, the spaces being 9 mm. wide. No. 2 is ruled with 2.3-mm. lines, the spaces being 7.7 mm. No. 3 is ruled with 3.7-mm. lines, the spaces being 6.3 mm. No. 4 is ruled with 5.5-mm. lines, the spaces being 4.5 mm. Cards thus ruled are placed about 50 ft. from the observer, in line with the chimney emitting smoke, together with a white card and one that is solid black. The observer glances rapidly from the chimney to the cards and judges which one corresponds with the color and density of the smoke.

Smoke Ordinances. Numerous city and town smoke ordinances have been passed recently, but many of these are defective or ineffective. Some of the features included in the Chicago ordinance, which is one of the most comprehensive in use, are:

(*a*) A department of smoke inspection, at the head of which there is a mechanical engineer, with necessary assistants, deputies, etc.

(*b*) No new power plant to be constructed or old plant reconstructed until plans and specifications are approved by inspector and a permit issued.

(*c*) No new or reconstructed plant to be put in operation until approved by board and certificate issued.

(*d*) The emission of dense smoke from any stack, except for a period of 6 min. in any one hour during which a fire box is being cleaned or a new fire is being built therein, is a nuisance and may be summarily abated. (Penalty for violation of this provision.)

(*e*) The smoke inspector shall keep a complete record of all plans submitted, of all plants examined and of all certificates issued.

The principal objection to this ordinance is the lack of a statement as to what constitutes dense smoke or the fixing of a standard to be used as a basis of comparison.

The Bureau of Mines, Department of Interior, in its Bulletin 49, 1912, offers a suggested form of smoke ordinance for large and medium-sized cities. This ordinance differs from that of Chicago, above quoted, mainly in that a standard of grading smoke is given. The paragraphs relative to the allowable emission of smoke read as follows:

The emission of dense smoke within the city from the smoke stack of any locomotive, steamboat, or steam tug for a period of more than 1 min., except for a period or periods aggregating not to exceed 10 min. in any one hour during which period or periods the fire box is being cleaned or a new fire is being built therein, is hereby declared a nuisance: *Provided*, that the fire engines or the fire boats of the city fire department, or both of them, shall be exempt from these restrictions.

The emission of dense smoke within the city from the smoke stack of any steam roller, steam derrick, steam pile-driver, tar kettle, or other similar machine or contrivance, or from the smoke stack or chimney of any building or premise, except for a period or periods aggregating not to exceed 6 min. in any hour during which period or periods the fire box is being cleaned or a new fire is being built therein, is hereby declared a nuisance.

For the purpose of grading the density of smoke, the Ringlemann smoke chart shall be the standard of comparison. Smoke shall be considered "dense" when it is of greater density than No. 3 of the chart.

Wood Burning

Wood, as used commercially for steam-generating purposes, is usually a waste product from some industrial process, refuse from lumber mills forming the greater portion of this class of fuel. In some refuse the moisture content may run as high as 60 per cent. of the total weight, and the composition may vary widely during different portions of the mills' operations. Such fuel consists of sawdust, hogged wood and slabs, the percentage of each varying widely.

Wet Wood. It is essential that a large combustion space be provided and, where there is a high moisture content, heated brickwork to radiate heat to the fuel bed and evaporate the moisture is necessary. Extension furnaces are usually necessary to give good results. Where this fuel alone is burned, added combustion space can be obtained by dropping the grates to the floor line. A comparatively thick fuel bed is necessary to reduce the excess air to a minimum. Where hogged fuel and sawdust alone are used, they are best fed to the furnace through chutes on the top, the fuel from such chutes being allowed to pile up to a height of 3 ft. or 3 ft. 6 in. In properly designed furnaces, one chute can take care of approximately 30 sq. ft. of grate surface with this fuel. With proper draft conditions, 150 lb. of wet fuel can be burned per sq. ft. of grate surface per hour, and in well-designed wood-burning furnaces 1 sq. ft. of grate surface can readily develop from 5 to 6 boiler h.p. A strong natural draft is to be preferred to a forced blast with this fuel.

Unconsumed particles of wood fuel may be carried over into the heating surfaces and lodged in the setting out of the path of the gases. Such particles may ignite if air leaks to them, with harmful results. This is especially true where particles are carried into the base of a stack and settle at a point below the flue entrance. Collecting chambers should be provided in the setting and should be kept clean where wood fuel is used.

Dry Wood. Dry sawdust, chips and blocks from wood-working industries are sometimes used as fuel. Here, too, ample combustion space is required. The action of heated brickwork is not as necessary as where the moisture content is high. Where the fuel is largely sawdust, and an extension furnace is not used, the sawdust can be fed through chutes in the boiler front over the grates. The fire doors should be large for the introduction of such fuel as cannot be fed through the chutes.

Bagasse. See p. 609. In practically all cases where bagasse is burned, some auxiliary fuel will be required, the amount depending largely upon the proportion of fiber in the cane. Much better results are secured where large quantities are burned; that is, a given quantity burned in one furnace between two boiler units will give better results than the same quantity burned in two separate furnaces with separate boilers. (See Geerlig's "Cane Sugar and Its Manufacture.")

The best combustion is secured by burning the fuel on a hearth rather than on grates, air for combustion being admitted through several rows of tuyères placed above and symmetrically around the hearth. High combustion rates give the best results. With a natural draft in the furnace of, say, 0.3 in. water column, a combustion rate of from 250 to 300 lb. per hour per sq. ft. of grate surface may be secured. With a blast of 0.5 in. available, this rate may be increased to 450 lb. per hour per sq. ft. These rates are for bagasse containing approximately 50 per cent. of moisture. The most economical combustion rate is approximately 300 lb.

Considerable dirt is deposited from this class of fuel, and the gas ducts and furnace should be thoroughly cleaned out once in 24 hr. If this cleaning is not properly taken care of, the dirt will fuse into a glass-like clinker which is particularly difficult to remove.

Oil Fuel

Petroleum (see p. 610) is the only liquid fuel sufficiently abundant and cheap to be used for the generation of steam. It possesses many advantages over coal and is extensively used in some localities where its cost as compared with coal makes it the cheaper fuel. The petroleum most generally used for fuel purposes in the United States is that which upon distillation yields asphalt, the other classes yielding upon distillation such a variety of lighter oils that their use as fuel is prohibitive because of price. To the asphalt petroleums belong the oils found in Texas and California.

Oil vs. Coal. Advantages of the use of oil over coal may be summarised as follows:

1. The handling cost is low, oil being fed by simple mechanical means.
2. A general labor saving throughout the plant in the elimination of stokers, coal passers, ash handlers, etc.
3. For equal heat value, oil occupies very much less space than coal, and the storage space can be at a distance from the boiler without detriment.
4. Combustion with oil is more perfect than with coal, the excess air being more readily reduced to a minimum, and there being no ash which in the burning of coal

usually contains considerable combustible matter. The furnace temperature may be kept practically constant and smoke may be eliminated with the resulting increased cleanliness of heating surfaces. The more perfect combustion results in higher efficiencies than are attainable with coal.

5. The intensity of the fire can be almost instantaneously regulated to meet fluctuations in load.
6. There is no loss in calorific value or difficulties arising from disintegration and slow oxidation where oil is stored such as are possible in coal storage.
7. Oil is cleanly and free from dust, with a consequent saving in wear on machinery.

The disadvantages of oil are as follows:

1. The necessity that the oil have a high flash point to minimise the danger of explosions.
2. Ordinances relative to location and isolation of oil storage tanks may in certain cases make the use of oil fuel prohibitive.
3. Unless boilers and furnaces are especially adapted for oil fuel, the upkeep cost will be higher than if coal were used.

Table 8, comparing oil and coal, calculated by C. C. Moore & Co., Engineers, of San Francisco, takes into consideration the variation in boiler efficiency, and is a good basis for rough approximations.

Table 8. Relative Values of Coal and Oil Fuel

Gross boiler efficiency with oil fuel	Net boiler efficiency* with oil fuel	Net evaporation, lb. from and at 212 deg. fahr. per lb. of oil	Water Evaporated from and at 212 Deg. Fahr. per Pound of Coal							
			5	6	7	8	9	10	11	12
			Pounds of Oil Equal to 1 Lb. of Coal							
73	71	13.54	0.3693	0.4431	0.5170	0.5909	0.6647	0.7386	0.8124	0.8863
74	72	13.73	0.3642	0.4370	0.5099	0.5827	0.6556	0.7283	0.8011	0.8740
75	73	13.92	0.3592	0.4310	0.5029	0.5747	0.6466	0 7184	0.7903	0.8621
76	74	14.11	0.3544	0.4253	0.4961	0.5670	0.6378	0.7087	0.7796	0.8505
77	75	14.30	0.3497	0.4196	0.4895	0.5594	0.6294	0.6993	0.7692	0.8392
78	76	14.49	0.3451	0.4141	0.4831	0.5521	0.6211	0.6901	0.7591	0.8281
79	77	14.68	0.3406	0.4087	0.4768	0.5450	0.6131	0.6812	0.7493	0 8174
80	78	14.87	0.3363	0.4035	0.4708	0.5380	0.6053	0.6725	0.7398	0.8070
81	79	15 06	0.3320	0.3984	0.4648	0.5312	0.5976	0.6640	0.7304	0.7968
82	80	15.25	0.3279	0.3934	0.4590	0.5246	0.5902	0.6557	0.7213	0.7869
83	81	15.44	0.3238	0.3886	0.4534	0.5181	0.5829	0.6447	0.7125	0.7772

* Net efficiency = gross efficiency less 2 per cent. for steam used in atomizing oil. Heat value of oil = 18,500 B.t.u.

Burning Oil Fuel. The requirements for burning petroleum with the highest economy are as follows:

1. Its atomization must be thorough.
2. When atomised it must be brought into contact with the requisite quantity of air for its combustion, and this quantity kept at a minimum.
3. The mixture must be burned in the presence of refractory material, which radiates heat to assist in the combustion. A high grade of fire brick must be used for the furnace to stand up under the high temperatures developed.
4. Combustion must be complete before the gases are cooled by coming into contact with the boiler heating surfaces.
5. There must be no localisation of the heat on certain portions of the heating surfaces such as may be produced by a blow-pipe action in the burners, or trouble will result from overheating and blistering of the boiler tubes.

See "Developments in Oil Burning," E. H. Peabody, *Trans. Soc. Nav. Arch. and Mar. Eng.*, 1912. B. R. T. Collins, *Trans. A. S. M. E.*, vol. 22, 1911; *Jour. Am. Soc. Nav. Eng.*, vol. 23, 1911.

Oil Furnaces. The best results are secured where a furnace is used in which the oil is introduced in the direction in which it increases in volume, insuring free expansion and a thorough mixture of the oil with the air, and complete combustion of the gases before they come into contact with the heating surfaces. In such a furnace a flat-flame burner gives the best results. Burners should be so located that the flames from individual burners will not interfere or impinge to any extent on the side walls or lanes of smoke and an uneven distribution of heat will result.

Burners

Spray Burners are almost universally used and the simplicity and excellent economy of the steam atomizing burners make them the most satisfactory where the loss of steam and the consequent requirement of an equal added amount of fresh make-up water is not of great consequence. Air atomizing burners have practically gone out of use in this country.

Steam atomizing burners are of the inside or outside mixing type. In the former, steam and oil come into contact inside of the burner and the mixture is atomized in passing through the burner nozzle. In the latter type the steam passes through a narrow slot or row of small holes below a similar slot through which the oil flows. The oil is picked up by the steam outside of the burner and is atomized by it.

Heating the oil is advantageous in causing it to be atomized more readily and in securing economical combustion. Such heating should be done as close to the burners as is practicable. If the temperature of the oil is raised too near its flash point and any material vaporization occurs, the oil will flow irregularly and the flame will splutter. By-passes should be provided between the oil and steam ducts of a burner for blowing out the oil piping and burner to clean out any deposits. Strainers should be installed in duplicate in the oil lines for removing the silt carried by the oil, and should be so located as to allow for rapid removal, cleaning and replacing without interfering with the operation of the boilers.

Mechanical Atomizing Burners are usually of the round-flame type. The oil is fed to the burner under pressure varying with the rates at which it is to burn. It is given a rotary motion before issuing from the burner nozzle. Heating the oil is an aid to atomization. Heating and the pressure affect the capacity of individual burners, and provide a means of regulation of individual burners. Large variation in demand for steam is taken care of by changing the number of burners in service.

Steam Consumption of Burners. With a good type of steam atomizing burner, the steam required for atomization is less than 2 per cent. of the total steam generated with the boiler running at its rated capacity. This figure will decrease as the boiler rating increases.

The steam required by mechanical burners in heating and pumping is in the neighborhood of 0.25 per cent. of the total generated, and this steam can be returned to the system. It is this fact that makes a mechanical atomizing burner more satisfactory for marine work than a spray burner, or for any work where the question of make-up feed water is important.

Gaseous Fuel

Natural Gas (see p. 614) is best burned by the use of a large number of small burners, each being capable of handling approximately 30 rated h.p. This obviates the danger of any laning or blow-pipe action of the gases which might be present where large burners are used. Where natural gas is the only fuel, the burners should be evenly distributed over the lower portion of the boiler front. If this fuel is used as an auxiliary to coal, these burners may be placed through the fire doors. A large combustion space gives the best results, say, 2 cu. ft. per rated h.p. A checkerwork wall is sometimes placed in the furnace about 3 ft. from the burners to break up the flame, but with a good type of burner and furnace this wall is not necessary. Good results are secured with the burners inclined slightly downward toward the rear of the furnace where the gas is burned alone and there are no coal-firing grates.

Blast-furnace Gas. See p. 614. For good efficiency a long flame travel and ample furnace volume are necessary. The combustion space should be from 1½ to 2 cu. ft. per rated boiler h.p. The most satisfactory type of burner is that in which both the amount of gas and air can be regulated. The question of handling dust when burning this fuel is important. If auxiliary grates are used for igniting the fires, these may be placed immediately under the burners; much of the dust is then precipitated on the fire and will be cleaned out when the fires are cleaned. Dust precipitated in the furnace, if allowed to accumulate, becomes fused and is difficult to remove.

With gas passages of 0.8 sq. in. per rated boiler h.p. a boiler can be run at its rated capacity with a gas pressure of 2 in. of water. The pressure in the main is usually from 6 to 8 in., which allows for overloads and throttling of the gases to equalise the flow to the various boilers in a plant. The air passage should be from 0.75 to 0.85 sq. in. per rated h.p.

Stack Sizes with Blast-furnace Gas. Stacks 130 ft. high are sufficient for this class of work. Such stacks will give sufficient draft to obtain 175 per cent. of the boiler's rated capacity. The capacity beyond a certain point will not increase proportionately with the draft. With too much draft the mixture of gas and air is not good, sometimes resulting in a pulsating action of the flame—if not actual explosion, danger of burning out dampers, and increase in wear and tear on settings. Pulsations within the setting may also be occasioned through a lack of air for combustion.

The brickwork must be well laid. The pulsating action of the flame shakes the setting, and the possibility of explosions makes it necessary that the whole should be properly tied and buckstayed. Extension furnaces, where used, should be particularly well buckstayed.

Flue-gas Analysis

The quantities determined in the ordinary gas analysis are the relative proportions by volume of carbon dioxide, oxygen and carbon monoxide, the determinations being made in the order named. In boiler practice the nitrogen content of the flue gases is obtained by deducting the sum of the other constituents from 100. See also pp. 369 and 1695.

Incomplete combustion is indicated by the presence of carbon monoxide in the analysis. It is the result either of a deficient air supply or an imperfect mixture of the combustible gases with the air supplied. The loss due to incomplete combustion of the carbon in the fuel, in B.t.u. per lb. of fuel, is

$$L = 10{,}150\,C \times CO/(CO + CO_2),$$

where 10,150 = difference in heat evolved in burning 1 lb. of carbon to CO_2 and to CO; CO and CO_2 = percentages by volume of carbon monoxide and carbon dioxide as found by analysis; and C = fraction by weight of carbon in the fuel which is burned and passes up the stack. The presence of 1 per cent. of CO in the flue gases will represent a decrease in the boiler efficiency of 4.5 per cent.

C is to be distinguished from the per cent. of carbon by ultimate analysis of the fuel, and is that quantity, less the per cent. of unconsumed carbon in the ash. Where an ultimate analysis of the ash is not available, the percentage of carbon in the ash is determined by the difference in the ash percentage by proximate analysis of the fuel and as found at the time of the boiler test by actual weight.

It is generally assumed that high CO_2 readings are indicative of good combustion, and hence of high efficiencies. Such readings are not satisfactory when considered apart from the CO determination. The best percentage of CO_2 to maintain varies with different fuels, and is lower for those with a high hydrogen component than for fuel mainly composed of carbon.

For the air supply, combustion products, etc., with complete combustion, both with and without excess of air, see p. 362. If part of the carbon is burned to CO (**incomplete combustion**), the equations are as follows:

The air supply per pound of fuel is: $W^1 = 3.036(C \times N/(CO_2 + CO))$, where W^1 = pounds of air supplied per lb. of fuel and N = percentage by volume of nitrogen in the flue gases. The error in using this formula does not exceed 0.2 per cent.

Hydrogen in the volatile content of fuel increases the nitrogen content of the flue gases. This is due to the fact that the water vapor formed by the combustion of hydrogen will condense at the temperature at which the analysis is made, while the nitrogen which accompanied the oxygen maintains its gaseous form and passes in such form into the sampling apparatus. For this reason, where highly volatile coals, producing a large quantity of water vapor, are used, the apparent nitrogen content is increased.

The weight of flue gases per pound of fuel,* including moisture formed by the hydrogen component $= 3.036\,[N/(CO_2 + CO)]C + (1 - A)$, where A = per cent. of ash as found in test.

The weight of dry flue gases per pound of fuel* may be determined from the formula: $W^1 = C[11\ CO_2 + 8O + 7\ (CO + N)]/3\ (CO_2 + CO)$.

The ratio of air supplied per pound of fuel to the air theoretically required is: $W^1/W = 3.036C \left(\frac{N}{CO_2 + CO}\right) / 34.56 \left(\frac{C}{3} + H - \frac{O}{8}\right)$.

The ratio of air supplied per pound of combustible to that theoretically required is: $N/[N - 3.782\ (O - \frac{1}{2}\ CO)]$, on the assumption that all the nitrogen in the flue gas comes from the air supplied. Fig. 1 gives the value of this ratio for varying flue-gas analyses where there is no CO present.

Effects of Excess Air. The heat loss in the dry stack gases is: $L = 0.24 \times (T - t) \times W$, where L = B.t.u. lost per lb. of fuel; W = weight of flue gases in lb.; T = temperature of flue gases, deg. fahr.; t = temperature of atmosphere, deg. fahr.; and 0.24 = specific heat of flue gases.

The heat lost in dry chimney gases is shown in Fig. 2, which is calculated on the assumption that the gas contains (in addition to the percentage of carbon dioxide marked on the curves) 80 per cent. of nitrogen, and that the rest of the gas is oxygen. In those cases where some CO is present, the correction is quite negligible if the CO is reckoned as CO_2. In using Fig. 2 the heat contained in the gases at boiler-room temperature must be subtracted from the heat at chimney temperature in order to find the heat carried away by the dry gases. For loss per lb. of coal, multiply by per cent. of carbon in coal as shown by ultimate analysis. The heat going up the chimney with the superheated steam resulting from evaporation of moisture in the coal and from the combustion of H should be added to that in the dry gases when finding the total heat escaping up the chimney. See p. 896.

* In this formula, the weight of gas is per lb. of dry or moist fuel as the percentage of C is referred to a dry or moist basis.

The **minimum quantity of air** varies widely with the fuel. In a boiler test where the highest authentic efficiency ever secured with oil was obtained, the ratio of the air supplied to that theoretically required was 1.18 : 1. In tests with low volatile bituminous coals where similar efficiencies (the highest ever secured with such fuels) were obtained, these ratios were respectively

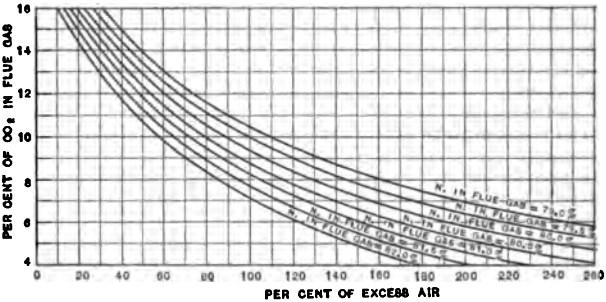


FIG. 1.—Ratio of Air Supplied per Pound of Combustible to That Theoretically Required.

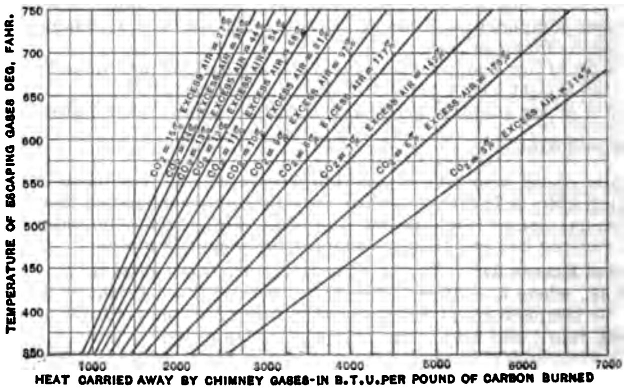


FIG. 2.

1.20 and 1.26 to 1. In a test with a highly volatile coal in which an efficiency was obtained at least as high as any that has been authentically secured for such coal, the ratio of air supplied to that required was approximately 2 : 1. The difference in efficiencies between the last case cited and the first two was from 5 to 7 per cent. The fact that the efficiencies taken are the highest on record indicates that a considerable excess of air is necessary even where combustion is as nearly perfect as practicable. The ratios of air supplied to that theoretically required, as given above, are under test conditions and in ordinary practice will be much larger. Table 9 gives the average losses as indicated by a heat balance for test conditions and for ordinary operating conditions.

THE CAPACITY AND EFFICIENCY OF STEAM BOILERS

The capacity of a boiler should be stated in terms of its evaporative power. As temperatures of feed water and steam pressures are different in various boilers, it is necessary, in order to secure a means for comparing the operation of different boilers, to reduce all results to some common basis. The method adopted is to transform the evaporation to an equivalent evaporation under a set of standard assumed conditions.

The Committee of Judges in charge of the boiler trials at the Centennial Exposition at Philadelphia in 1876, recommended that an evaporation of 30 lb. of water from an initial temperature of 100 deg. fahr. to steam at 70 lb. gage pressure be considered **1 boiler h.p.** A boiler h.p. so defined is really a measure of the amount of heat transmitted. The standard conditions now accepted are a feed temperature of 212 deg. fahr. and evaporation at the same temperature. These conditions are described by the term "evaporation from and at 212 deg." The weight of water which would be evaporated (under the assumed standard conditions), by the amount of heat actually absorbed by the boiler, is called the **equivalent evaporation from and at 212 deg.** The committee on boiler tests of the A. S. M. E. defines the **boiler h.p. as the equivalent evaporation of 34.5 lb. of water from and at 212 deg. per hour.** This is the same as 33,479 B.t.u. per hour.

The **factor of evaporation** is the ratio of the heat required to generate 1 lb. of steam under actual conditions to that required to generate 1 lb. of steam "from and at 212 deg." This ratio may be expressed for either saturated or superheated steam, as follows:

$$\text{Factor of evaporation} = (H - h)/970.4,$$

where H = total heat in 1 lb. of steam at boiler pressure and steam temperature; h = heat of the liquid at feed temperature, and 970.4 = latent heat of evaporation of 1 lb. of steam at 212 deg.

A method of determining the factor of evaporation for any set of conditions from a chart is indicated in Fig. 3. This chart, prepared by Professor Marks and based on the Marks and Davis Steam Tables, give factors for the usual range of steam pressures, steam qualities, superheat and feed-water temperatures. The main vertical scales give the factor of evaporation for a feed-water temperature of 32 deg. fahr. To correct for the actual feed-water temperature, go vertically from the feed-water temperature as given on the top horizontal scale to the diagonal line; the correction is shown on the extreme vertical scales. This quantity must in all cases be subtracted.

Unit of Boiler Capacity. In view of the very arbitrary unit adopted as a basis for boiler work, many attempts have been made to devise a unit of greater utility. H. G. Stott has suggested the **myriawatt** as a means of connecting mechanical units directly with electrical units. It has its basis in the fact that the present boiler h.p. (33,479 B.t.u.) and the kilowatt-hour (3412 B.t.u.) bear such a close relation. The "myriawatt," or 10 kw.-hr., would be equivalent to 34,120 B.t.u., or within 1.91 per cent. of the value of a boiler h.p.

Rating. As sold to-day, boilers are rated at a stated or nominal h.p. In stationary practice it is customary to rate a boiler on the basis of 10 sq. ft. of heating surface per h.p., *i.e.*, 10 sq. ft. of heating surface under normal operating conditions are considered capable of evaporating 34.5 lb. of water per hour from and at 212 deg. This method of rating a boiler is arbitrary, and has been adopted simply as a matter of convenience.

The capacity of a boiler usually designates its maximum capacity for

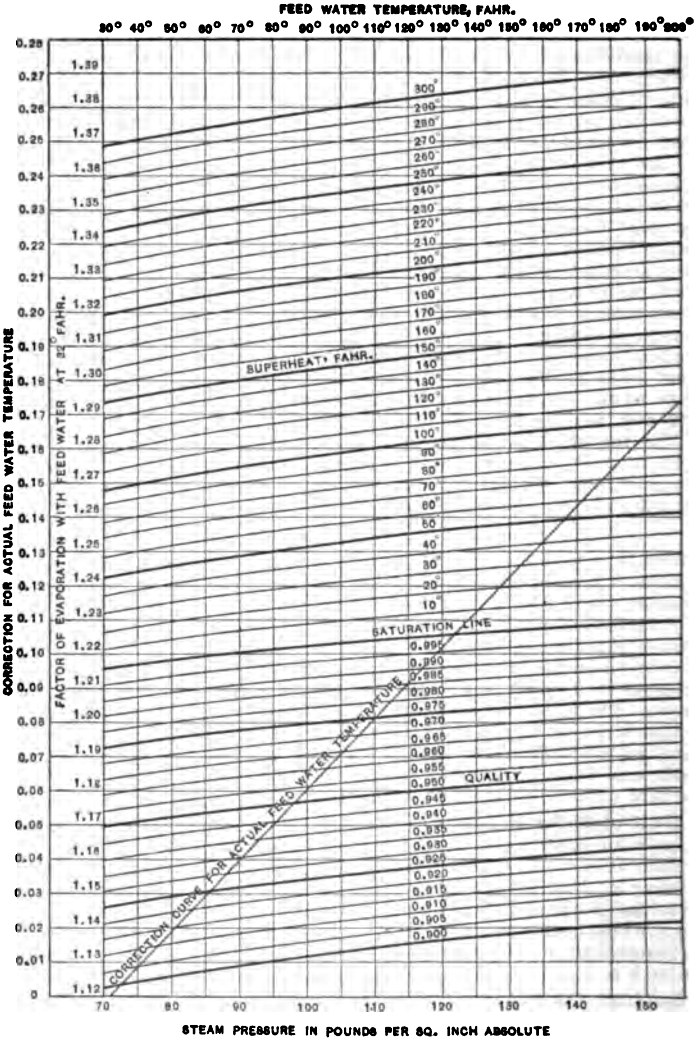
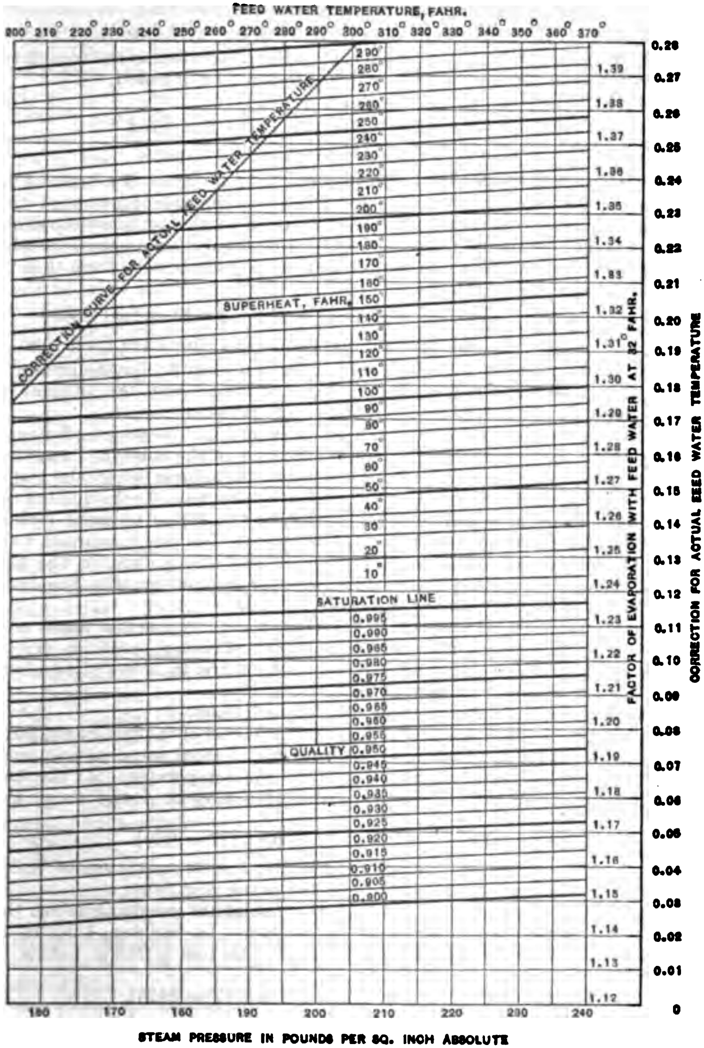


FIG. 3.—Chart for Determining



Factors of Evaporation.

evaporation. This maximum capacity, expressed in boiler h.p., is ordinarily given in terms of per cent. of the builder's nominal rated h.p.

Efficiency. The efficiency of a boiler, as ordinarily stated, is the combined efficiency of boiler, furnace and grate. Expressed as a formula,

Efficiency of boiler, furnace and grate =

$$\frac{\text{Heat absorbed by boiler per lb. of fuel fired}}{\text{Heat of perfect combustion per lb. of fuel}}$$

This efficiency will be the same whether based on moist or dry fuel. The heat absorbed by the boiler comes from the combustible portion of the fuel actually burned, irrespective of what fraction of the total combustible fired this may be. This has led to a second efficiency, based on combustible, which is called the **efficiency of boiler alone**, as distinguished from that of boiler and grate. As a formula,

$$\text{Efficiency of boiler} = \frac{\text{Heat absorbed by boiler per lb. of fuel fired}}{\text{Heat actually developed in furnace per lb. of fuel}}$$

The furnace is not entirely eliminated from the last formula, for the presence of excess air in the gases lowers the efficiency obtained, and the amount of excess air depends upon the furnace design and operation.

It is extremely difficult, if not impossible, to determine the actual efficiency of a boiler alone, as distinguished from the efficiency of the combined apparatus. When guarantees are to be met by a boiler manufacturer, where the boiler and stoker or grate are different installations, the only way of establishing an efficiency that has a practical value is to require the stoker or grate manufacturer to make certain guarantees as to minimum CO_2 and maximum CO, and that the amount of combustible in the ash and blown away in the flue gases shall not exceed certain limits. In usual commercial practice the combined efficiency of boiler, furnace and grate is used.

For approximate results a chart such as Fig. 4 may be used in place of a computation of efficiency. This may be used for efficiency based on dry or moist coal, and, where evaporation and heat values are in terms of combustible, for efficiency based on combustible.

The **heat losses** in a boiler may be divided as follows:

(1) Loss due to the utilization of a portion of the heat of the fuel in **evaporating** the **moisture** contained in it, after raising it from atmospheric temperature to 212 deg. and superheating it to the temperature of the flue gases. This loss in B.t.u. per lb. of combustible may be found from the formula:

$$\text{Loss} = W[(212 - t) + 970.4 + 0.47\ (T - 212)]$$

where W = moisture per lb. of combustible, lb.; t = temperature of boiler room, deg. fahr., and T = temperature of flue gases, deg. fahr.

(2) Loss due to heat carried away by the **moisture** resulting **from** the burning of the **hydrogen** component of the fuel. In burning, 1 lb. of H unites with 8 lb. of O to form 9 lb. of steam. This loss in B.t.u. per lb. of combustible is:

$$\text{Loss} = 9H[(212 - t) + 970.4 + 0.47(T - 212)],$$

where H = weight of hydrogen in the fuel per lb. of combustible.

Where an ultimate analysis of the fuel is not available, the hydrogen content may be determined approximately, for American fuels, from the proximate analysis of the fuel, according to a method devised by Professor L. S. Marks from a study of proximate and ultimate analyses carried out in 1904–1908 by the U. S. Government on about 240 different coals, from 28 states and territories, representing every kind of fuel from anthracite to peat, and a range of volatile matter in the combustible of from 6 to 70 per cent. Prof. Marks has shown (*Power*, Dec. 1, 1908) that the hydrogen in coal, not contained in moisture, can be found from the results of the proximate analysis, and further, that the computation of the chemical analysis of a coal can be made from its proximate analysis with sufficient accuracy for all usual heat-engine-investigation purposes.

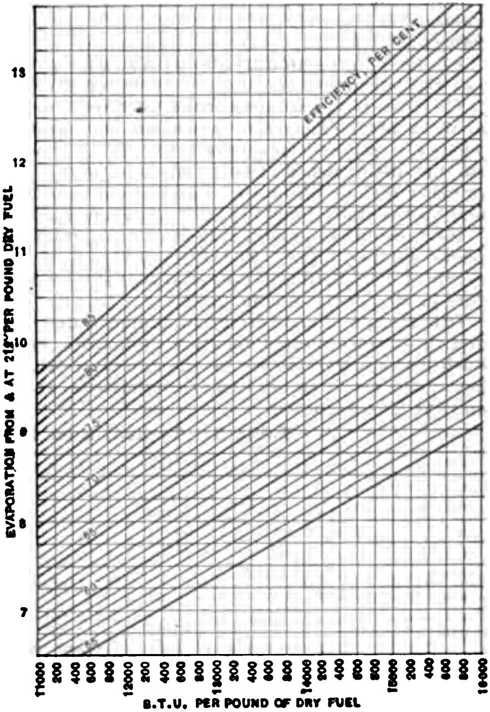


Fig. 4.—Chart for Determining Efficiency of Steam Boilers.

In proximate analysis the combustible = volatile matter + fixed carbon = coal as received − moisture and ash. Let V, H, C and N be respectively the percentage by weight of volatile matter, hydrogen, volatile carbon and nitrogen in the combustible. Then $H = V[7.35/(V + 10) - 0.013]$, which is accurate for American coals to about ±0.2 per cent.

The volatile carbon may be calculated from the content of volatile matter with an accuracy of ± 2 per cent., approx., by the following formulæ:

$C = 0.9 (V - 10)$ for anthracite and semi-anthracite.
$C = 0.9 (V - 14)$ for bituminous and semi-bituminous.
$C = 0.9 (V - 18)$ for lignites.

Sulphur in coal directly increases the value of V; values of C calculated as above will be too high practically by the sulphur content of the combustible.

The nitrogen (coming off in the volatile matter) may be calculated with an accuracy of ± 0.5 per cent. by the formulæ,

$N = 0.07 V$, for anthracite and semi-anthracite.
$N = 2.10 - 0.012 V$, for bituminous and lignite.

The formulæ given above are taken from Carpenter and Diederichs' "Experimental Engineering" (p. 507, 1911 edition).

(3) Loss due to heat carried away by **dry chimney gases.** This loss has been discussed on p. 891. In cases where the total carbon content of the fuel has not been determined by an ultimate analysis, it is possible to get its value from the proximate analysis, adding the fixed carbon content to the value of C obtained by the method given above.

(4) Loss due to **incomplete combustion** of the carbon content of the fuel, *i.e.*, the burning of carbon to CO instead of CO_2. See p. 890.

(5) Loss due to **unconsumed carbon** in the ash. This item requires an analysis of the ash and is, per lb. of combustible, Loss $= C \times A \times Q$, where C = per cent. of carbon in the ash; A = per cent. of ash as found in test (to be distinguished from that determined by an analysis, and should properly include unconsumed particles of fuel carried into the boiler setting and up the stack); and Q = heat value per lb. of combustible in the ash.

(6) Loss due to superheating **moisture in** the **air** supplied, from the atmospheric temperature, t, to the temperature of the flue gases, T. This loss per lb. of combustible is, Loss $= M \times 0.47(T - t)$, where M = weight of moisture in air per lb. of combustible.

Other losses—due to radiation, to unconsumed hydrogen and hydrocarbons, and those unaccounted for—are ordinarily grouped together in one item. Such losses will obviously be the difference between 100 per cent., the total heat value per lb. of dry fuel or combustible, and the sum of the above items and the heat absorbed by the boiler.

Table 9. Typical Heat Balances

TEST DATA AND ANALYSES

		Test conditions	Operating conditions
Temperature of boiler room, deg. fahr		60	60
Temperature of exit gases, deg. fahr		500	530
Ultimate analysis, dry coal	Moisture %	4.00	4.00
	C %	82.90	82.90
	H %	4.13	4.13
	O %	2.61	2.61
	N %	1.31	1.31
	S %	0.95	0.95
	Ash %	8.10	8.10

		Test conditions	Operating conditions
B.t.u. per lb. dry coal		14460	14460
B.t.u. per lb. combustible		15717	15717
Per cent. ash found by test		10.00	12.50
Per cent. combustible in ash		25.0	35.0
Flue-gas analysis.	CO_2 %	12.5	9.5
	O %	6.8	9.2
	CO %	0.3	0.3
	N %	80.4	81.0
Evaporation from and at 212 deg. per lb. dry coal		11.32	10.22

HEAT BALANCE

	Test conditions		Operating conditions	
	B.t.u.	%	B.t.u.	%
Heat absorbed by boiler	10,984	75.96	9,917	68.58
Loss due to moisture in coal	50	0.35	51	0.35
Loss due to moisture formed in burning H in coal	467	3.23	472	3.26
Loss due to heat carried away in dry chimney gases	1,721	11.90	2,367	16.37
Loss due to incomplete combustion of carbon	197	1.36	257	1.78
Loss due to unconsumed carbon in the ash	388	2.68	687	4.75
Loss due to radiation and unaccounted losses	653	4.52	709	4.91
Total	14,460	100.00	14,460	100.00

Relation Between Boiler Efficiency and Capacity

Efficiency. Thermal efficiencies of boilers, under various test conditions of operation are given in Table 10.

Capacity. The evaporation from a definite amount of heating surface in a well-designed boiler is limited only by the amount of fuel that can be burned under that heating surface. The factors limiting the production of high power from small heating surfaces are:

1. **Efficiency.** As capacity increases there will be in general a decrease in efficiency (see below).

2. **Possible or Practicable Grate Ratio.** Since all fuels have a combustion rate above which satisfactory results cannot be obtained, regardless of draft available, the only method of increasing capacity beyond such rates will be to increase grate surface. There is a point beyond which the grate surface for a given boiler cannot be increased, due to impracticability of handling or of features of construction.

3. **Gas Friction.** The frictional resistances to the passage of the gases increase very rapidly at high capacities, and may require excessive draft to overcome them.

4. **Feed Water.** The quality of feed water used is frequently the limiting factor in the capacity to be obtained from a definite amount of heating surface. With increased capacities comes an added concentration of such ingredients in feed as will cause priming, foaming or rapid scale formation. Certain waters, giving no difficulty with boilers operating at ordinary ratings, will cause difficulties at higher ratings out of all proportion to any advantage secured by an increase in capacity from the heating surface available.

Relation of Efficiency and Capacity. The increase in furnace efficiency at moderate capacities is usually at a greater rate than is the decrease in boiler efficiency, with the result that the combined efficiency of a boiler and furnace increases at first with an increase in capacity. This makes the ordinary point of maximum combined efficiency somewhat above the boiler's rated capacity, and in many instances the combined efficiency is approximately the same for a considerable range of ratings.

The usual variation of efficiency with capacity in modern boilers and furnaces is as follows:

Per cent. of boiler's rated capacity developed	80–100	120	140	160	180	200	220	240	260	280
Combined efficiency of boiler and furnace, per cent.	75	74.9	74.8	74.5	73.9	73.2	72.2	70.8	69.1	66.9

These values are derived from a number of tests, the results of which have, as far as possible, been modified to place them on a comparable basis. They are based on a combination of boiler and furnace which will give a maximum efficiency at or about the boiler's nominal rated capacity. If the arrangement is such as to give the maximum efficiency at a point considerably above the rated capacity of the boiler, the values for combined efficiency will drop off at the lower ratings. While the efficiencies represent those secured under test conditions, they nevertheless may be assumed to be an approximate representation of the variation in efficiency with capacity under any conditions of operation.

Performance of Steam Boilers

The highest authentic efficiencies that have been obtained with coal fuel are those secured by Dr. D. S. Jacobus in a series of tests at the Delray plant of the Detroit (Mich.) Edison Co. These tests were run on Stirling boilers containing approximately 23,650 sq. ft. of heating surface, the principal results of some of the tests being given in Table 11. The boilers tested were large-sized units, and the furnace was specially designed for the securing of the highest possible efficiency.

Table 10. Performance of Various Boilers and Furnaces with Different Fuels Under Test Conditions

Number	Plant and location	Boiler	Rated h. p.	Furnace	Grate surface	Coal	Duration of test	Draft in furnace or blast in ash pit	Temperature exit gases	Dry coal per sq. ft. grate surface per hr.	Evaporation from and at 212 deg. per sq. ft. heating surface per hr.	Rated capacity developed	B.t.u. per lb. dry coal	Evaporation from and at 212 deg. per lb. dry coal	Combined efficiency, boiler and furnace
			h.p.		sq. ft.		hr.	in.	deg. fahr.	lb.	lb.	%	B.t.u.	lb.	%
1	Susquehanna Coal Co., Shenandoah, Pa.	Babcock and Wilcox	300	Hand-fired	84	No. 1 Anth. buckwheat	8	+0.41	490	16.6	4.10	118.7	11,912	8.81	71.8
2	Consolidated Gas & Elec. Co., Baltimore Md.	Babcock and Wilcox	640	Hand-fired	118	Penn. bituminous	8	−0.34	487	21.0	4.19	121.5	14,602	10.83	72.0
3	Public Service Corp. of N. J., Hoboken, N. J.	Babcock and Wilcox	520	Roney stoker	103.2	Penn. bituminous	9	−0.18	609	24.6	5.06	146.7	14,022	10.40	72.0
4	Edison Electric Illuminating Co., Boston, Mass.	Babcock and Wilcox	508	Murphy stoker	90	New River	16.25	−0.23	560	26.0	5.30	153.5	14,910	11.51	74.9
5	Union Electric Lt. & Pr. Co., St. Louis, Mo.	Babcock and Wilcox	508	B. & W. chain grate	103.5	Illinois slack	8	−0.60	523	33.5	5.67	161.5	10,576	8.16	74.9
6	Boston Elevated Ry. Co., Boston, Mass.	Stirling	350	Hand-fired	61	Pocahontas	10	−0.22	625	20.5	4.18	121.3	14,637	10.75	71.3
7	General Electric Co., Lynn, Mass.	Stirling	1000*	Roney stoker	180	New River	24	−0.21	667	20.3	3.96	114.8	14,833	10.84	70.9
8	Babcock & Wilcox Co., Barberton, O.	Stirling	1128	B. & W. chain grate	187	Pittsburgh slack	8	+1.09	624	43.4	6.84	198.3	12,130	9.51	76.1
9	Poughkeepsie Lt. Ht. & Pr. Co., Poughkeepsie, N. Y.	Stirling	306	Murphy stoker	.....	Penn. bituminous	8	−0.23	560	......	5.56	161.0	14,797	11.25	73.8
10	National Tube Co., Republic Works.	Rust	335	Hand-fired	68	Pittsburgh	10	−0.17	503	17.01	3.63	105.1	13,428	10.51	75.6
11	National Tube Co., Republic works.	Rust	335	Hand-fired	68	Pittsburgh	8	−0.34	718	38.0	7.26	210.5	13,202	9.42	68.9
12	Service Sta., Grand Cent. Terminal, N. Y.	Heine	635	Hand-fired	125	No. 2 Anth. buckwheat	8.16	......	505	24.3	4.09	118.9	11,448	8.58	72.67
13	Monarch Leather Co., Chicago.	Heine	400	Green chain grate	83.5	Illinois slack	8	−0.19	661	23.3	4.17	123.8	11,608	8.79	73.5

* Two boilers. Tests Nos. 1, 2 and 5 by N. E. Lewis, No. 3 by E. H. Peabody, No. 4 by Lionel S. Marks, No. 6 by C. H. Johnson, No. 7 by C. F. Dallas, No. 8 by I. Harter, No. 9 by W. W. Clark, Nos. 10 and 11 by Wm. Kent, No. 12 by Z. de Nemeth, No. 13 by Chas. L. Atkins.

Table 11. Tests of the 2365-h.p. Stirling Boilers at the Detroit Edison Co.'s Delray Plant

(By D. S. Jacobus. See *Trans. A. S. M. E.*, vol. 33, 1911)

Stoker	Duration	Superheat	Draft: Blast in ash-pit	Draft: Draft in furnace	Draft: Draft at damper	Temperature of exit gases	Dry coal per hour	Dry coal per sq. ft. grate surface per hour	Evaporation from and at 212 deg. per hour
	hr.	deg. fahr.	in.	in.	in.	deg. fahr.	lb.	lb.	lb.
Roney	24.0	115	0	0.16	0.05	480	5,609	12.58	65,671
	24.0	108	0	0.24	0.16	483	6,606	14.81	76,768
	25.0	135	0	0.32	0.42	576	7,463	16.73	85,948
	30.0	136	0	0.22	0.55	670	11,582	25.97	124,410
	16.5	133	0	0.34	0.95	636	13,761	30.85	157,722
Taylor	48.0	134	0.67	0.22	0.20	487	6,656	16.43	75,808
	24.0	128	0.76	0.26	0.20	493	7,622	18.82	88,061
	24.0	155	1.30	0.37	0.58	575	11,094	27.39	123,407
	24.0	168	1.56	0.33	0.84	647	13,469	33.25	151,447
	26.5	165	2.53	0.26	0.84	651	15,696	38.75	172,456

Stoker	Evap. from and at 212° per sq. ft. heating surface per hour	Flue-gas Analysis: CO_2	Flue-gas Analysis: O	Flue-gas Analysis: CO	B.t.u. per lb. dry coal	Evap. from and at 212 deg. per lb. dry coal	Rated capacity developed	Combined efficiency, boiler and furnace
	lb.	%	%	%		lb.	%	%
Roney	2.78	14.33	4.54	0.11	14,225	11.71	80.0	79.88
	3.24	14.40	4.54	0.35	13,896	11.62	94.0	81.15
	3.63	11.95	7.55	0.05	14,362	11.52	105.0	77.84
	5.26	14.74	3.96	0.54	13,756	10.74	152.4	75.78
	6.67	14.69	4.55	0.20	14,493	11.46	193.3	76.73
Taylor	3.22	11.86	7.96	0.06	14,188	11.39	92.9	77.90
	3.72	13.69	5.82	0.10	13,965	11.55	107.9	80.28
	5.22	14.00	5.50	0.42	14,000	11.12	151.2	77.07
	6.40	14.20	5.08	0.08	14,272	11.24	185.5	76.42
	7.29	15.45	3.86	0.17	14,061	10.99	211.3	75.84

With oil fuel a gross efficiency of somewhat over 83 per cent. has been obtained as a maximum, and this figure is frequently approached.

The highest authentic capacity obtained with a standard stationary type of steam boiler, over a period of time of any duration, is 294 per cent. of the boiler's rated capacity, or at a rate of evaporation of 10.14 lb. of water from and at 212 deg. per sq. ft. of heating surface per hour. The test in which this capacity was secured was of 5 hours' duration, and was run at the plant of the Edison Electric Illuminating Co. of Boston. There are undoubtedly plants in which boilers are operated during short intervals of peak load at ratings considerably in excess of this figure, but the intervals are short and the exact capacity hard to determine.

In a series of tests on a Babcock & Wilcox marine boiler at the company's shops, Bayonne, N. J., with coal fuel, an equivalent evaporation of 14.76 lb. per sq. ft. of heating surface per hour was obtained. This rate is equivalent to approximately 428 per cent. of the boiler's rated capacity. In a test on the same boiler with oil fuel, an equivalent evaporation of 15:83 lb. per sq. ft. of heating surface per hour was obtained, or, on the ordinary stationary boiler basis of rating, 458 per cent. of the boiler's rated capacity. In these two cases a boiler h.p. was developed with 2.34 and 2.18 sq. ft. of heating surface, respectively.

Combustion Rates. Table 12 gives the draft required in the furnace for various combustion rates of different coals, and covers the ordinary range of such rates as found in modern practice.

Table 12. Pounds of Coal Burned per Sq. Ft. of Grate per Hour

Draft, inches of water	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2
Pea	11.3	15.1	18.6	21.5	24.1	26.3	28.3	30.3	32.0	....
No. 1. Buckwheat	9.0	11.8	14.3	16.6	18.6	20.5	22.1	23.6	24.9	27.3
No. 3. Buckwheat	....	8.3	10.1	11.7	13.2	14.4	15.7	16.8	17.8	19.6
Bituminous (*A*)	16.3	22.2	27.2	31.7	35.8	39.6	43.1	....	....	....
Bituminous (*B*)	17.9	24.3	30.1	35.3	40.0	44.1	....	....	....	....
Bituminous (*C*)	19.8	27.8	34.9	41.1						

(*A*) Md., Pa., Va., and W. Va. semi-bituminous coals.
(*B*) Ill., Ind., and Kan. bituminous coals.
(*C*) Ala., Ky., Pa., and Tenn. bituminous coals.

It may be of interest to note the following high combustion rates actually obtained in boiler-testing work:

(1) Burning low-grade Illinois slack coal on Babcock & Wilcox chain-grate stokers with a natural draft of 0.69 in. in the furnace, a rate of 45.2 lb. of dry coal per sq. ft. of grate surface per hour was secured. With the same class of fuel on a similar stoker with a draft of 0.75 in. in the furnace, 53.2 lb. of dry coal per sq. ft. of grate were burned per hour. This coal averaged approximately 15 per cent. of moisture, which would make the combustion rates, based on coal as fired, respectively 53.2 and 62.6 lb.

(2) Burning New River coal in a Murphy furnace, a rate of 39.2 lb. per sq. ft. of projected grate area per hour was obtained with a natural draft of 0.44 in. in the furnace. With a blast of 1.03 in. and a draft in the furnace of 0.03 in., with the same stoker and coal, 58.3 lb. were burned per sq. ft. of projected grate area per hour.

(3) In a test of a Babcock & Wilcox marine-type boiler burning Pocahontas hand-picked coal, with an ash-pit blast of 3 in., a combustion rate of 69.72 lb. of dry coal, or 70.24 lb. of coal as fired, per sq. ft. of grate surface per hour was obtained.

GENERAL DESIGN

Heat Transfer in Steam Boilers

Experimental determinations have been made in recent years of the heat transfer rate in cylindrical tubes at comparatively low temperatures and small temperature differences, and Nusselt (*Z. V. D. I.*, 1908, p. 1750), from a study of his own work as well as that of others, has derived the empirical formula:

$$a = 0.0255 \frac{L_w}{d^{0.214}}\left(\frac{Wc_p}{AL}\right)^{0.786}$$

in which a = transfer rate in B.t.u. per hour per sq. ft. of surface per deg. fahr. difference in temperature; W = lb. of gas flowing through tube per hour; A = cross-sectional area of tube, sq. ft.; d = diam. of tube, ft.; c_p = specific heat of the gas at constant pressure; L = conductivity of the gas at the mean temperature and pressure in B.t.u. per hour per sq. ft. of surface per deg. fahr. drop in temperature per ft., and L_w = conductivity of the steam at the temperature of the wall of the tube. Values of L are: for air, 0.0122 (1 + 0.00132 T); CO_2, 0.0076 (1 + 0.00229 T); superheated steam, 0.0119 (1 + 0.00261 T), where T = temperature in deg. fahr. See also p. 301.

When a fluid is flowing through a pipe at low velocities the loss of head is directly proportional to the velocity and the fluid flows in straight stream lines. When, however, the velocity exceeds a value which is determinable for any size of tube, the direct or stream-line motion is replaced by an eddy or mixing flow, in which the loss of head by friction is approximately proportional to the square of the velocity. Croker and Clement (*Proc. Roy. Soc.*, vol. 71) have shown that the heat transferred during the mixing flow is at a much higher rate than with direct or stream-line flow. Nusselt's formula applies only to mixing flow, *i.e.*, above the critical velocity. Below the critical velocity the transfer rate seems to be little affected by change in velocity, and he concludes that, while it is approximately constant as regards the velocity in a straight cylindrical tube, it will vary from point to point of the tube, growing less as the surface passed over increases.

A large proportion of the heat absorbed by a boiler is received direct as radiation from the furnace. The lower row of tubes of a Babcock & Wilcox boiler absorb heat at an average rate per sq. ft. of surface between the first baffle and the front headers equivalent to the evaporation of from 50 to 75 lb. of water from and at 212 deg. fahr. per hour. Part of this heat is absorbed through actual contact between the hot gases and the tubes, but a large portion of it must have been received through radiation.

The average heat transfer rate, R (B.t.u. per sq. ft. per hour), in Babcock & Wilcox boilers with ordinary clean surfaces can be determined to a fairly close approximation from the formula $R = 2 + 0.0014\ (W/A)$, in which A is the average area in sq. ft. between all the passes of the boiler, and W is the total weight of gases passing per hour.

The relation between the boiler heating surface, the capacity being developed and the flue-gas temperatures for normal conditions of boiler operation is shown in the following table, which represents the average result of a large number of tests:

Boiler surface per boiler horse power developed, sq. ft....	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0
Flue gas temp., deg. fahr.....	660	570	510	470	440	420	405	395	390

Proportioning of Gas Passages in Steam Boilers

Experience has shown that an area of 10 sq. in. per nominal rated h.p. at the point where the gases leave the boiler furnace to enter the heating surface will give satisfactory results. In water-tube boilers this applies not only to the throat of the furnace, but also to the net area between tubes at the point where the gases enter the heating surfaces. This area has been found sufficient to allow for present-day overloads, without undue throttling action at this point. Assuming that for any given set of furnace conditions the best results are to be secured with a constant resistance to the gases throughout the heating surface, the cross-sectional area of passage per h.p. for varying amounts of heating surface passed over will be approximately as follows:

Heating surface passed over, in per cent	0	20	40	60	80	100
Gas-passage area per h.p., in sq. in	10	8.4	6.8	6.2	5.8	5.7

From the nature of their design, it is impossible to get with cylindrical or return-tubular boilers a variation in gas-passage areas that will at all correspond to these figures. Water-tube boilers lend themselves more readily to such a variation in gas-passage areas, and in the modern designs will be found to approximate these figures closely.

Superheaters

Superheaters are of two general classes, those integral with the boiler and those separately fired. The first type is the more common and is more efficient. The efficiency of a boiler and furnace containing a properly designed integral superheater will be slightly higher than of a boiler in which there is no superheater installed.

In the design of integral superheaters, the following features are to be observed:

The superheater should be so located that it will be in the direct path of the products of combustion and that no by-passing of any portion of these gases is necessary to divert them from their regular path over the boiler heating surface in order to give the required superheat. At the same time it should be so placed that the frictional resistance of the gases through the boiler is not unduly increased. Superheaters should be so designed that they are free to expand without affecting the boiler parts or the setting. The location should be such that there is no possibility of the superheater tubes burning when the boiler is started up, and that there will be no excessive or sudden fluctuation of temperature with fluctuating loads. All superheaters should be equipped with independent safety valves of the outside spring type, set slightly lower than the boiler safety valves. This is essential to provide a flow of steam through the superheater and thus prevent burning in case the load should suddenly be thrown off the boiler or plant. The superheater should be equipped with a drain for drawing off, before starting up, any water that may have collected in the superheater headers or manifolds. The area for steam passage and steam connections should be such that there will be no possibility of the steam by-passing any of the tubes; that is, the cross-sectional area of the tubes must be such, in relation to the area of the manifolds or headers, and the steam connection must be such, that approximately the same quantity of steam will pass through all the tubes. Any tubes that may be by-passed are in danger of burning. The velocity of steam through superheaters may be figured slightly higher than is good practice in designing boiler-room piping, provided this does not result in too great a drop of pressure through friction.

In properly designed and located integral superheaters, while the heat transfer rate will vary widely with the ratings at which a boiler is run and operating conditions in general, under ordinarily good conditions, with a boiler delivering approximately its rated capacity, a transfer rate of about 1500 B.t.u. per sq. ft. of heating surface per hour may be expected.

Separately Fired Superheaters. In some instances where it is desired to use superheated steam in a plant already in operation, or where the prime movers are a long distance from the steam generators, the best solution of the problem may be the installation of a separately fired superheater on the main steam line. In certain special processes temperatures are required that it is not feasible to attempt with the integral type, and here only separately fired superheaters can fill the requirements.

There is danger in the separately fired type that too high superheat may be obtained by reason of the temporary checking of the demand for steam, and the fires must be watched carefully to avoid such danger. These superheaters should also be equipped with safety valves set below the boiler safety valves. The best economy is obtained where the counterflow principle of steam and gas flow is used, having the gas make three or four passes over the superheating surfaces. In some designs the steam is first brought to the hottest section of the setting, as a measure to guard against burning of tubes, and from this section is passed to the coldest part of the setting and thence forward toward the hottest part. In a properly designed, separately fired superheater an efficiency of from 55 to 60 per cent. may be secured, and in figuring surface a heat transfer rate of 2500 B.t.u. per sq. ft. of superheating surface per hour may be safely assumed.

The velocity of the steam in passing through a superheater should be from 30 to 50 ft. per sec. The temperature loss of superheated steam in steam mains varies from 30 to 60 deg. fahr. per 100-ft. length, according to the kind of lagging used.

Waste Heat

The power that can be obtained by utilizing the waste heat from certain commercial processes depends upon the temperature, specific heat and weight of the gases. The temperatures vary over a wide range in the different commercial operations. Table 13 gives a list of certain of these operations yielding waste heat available for the generation of steam and the approximate temperatures of the waste gases.

Table 13. Temperatures of Waste Gases from Various Industries

Waste heat from	Temperatures, deg. fahr.*
Brick kilns	2000–2300
Zinc furnaces	2000–2300
Copper matte reverberatory furnaces	2000–2200
Bee-hive coke ovens	1800–2000
Cement kilns	1200–1600†
Nickel refining furnaces	1500–1750
Open-hearth steel furnaces	1100–1400

* Temperatures are average over one cycle or operation, and may vary widely as to maximum and minimum. † Dependent upon length of kiln.

Where the temperatures of the waste gases are high, approaching those found in direct-firing boiler practice, boilers modified but little from standard may be used. As the temperatures are lower the velocity of the gases must be increased in order to secure a heat-transfer rate at all comparable with those found in direct-firing practice. Sufficient draft must be provided to overcome the frictional loss in drawing the waste gases through the boiler.

Table 14, of tests on waste-heat boilers, gives the relations between the temperatures, draft resistance, and horse power developed.

Table 14. Results Obtained with Waste Heat from Commercial Processes

	Bee-hive coke ovens	Reverberatory copper furnaces	Nickel refining furnaces
Type of boiler	Stirling	Stirling	Rust
Heating surface, sq. ft	8416*	3560	3927
Temperature of entering gases, deg. fahr	1804	1728	1700
Temperature of exit gases, deg. fahr	490	733	612
Draft at boiler entrance, in	0.36	0.35	0.33
Draft at boiler exit, in	0.56	0.60	0.50
H.p. developed	824	241.3	263
Sq. ft. of heating surface per h.p	10.2	14.8	14.9

* Two boilers.

The following table shows the relation between gas temperatures, heating surface passed over, and the amount of work done by such surfaces in generating steam for a boiler in which the velocity of gases is that usual in direct-fired boilers and the entering gas temperature is assumed to be 2500 deg., the exit gas temperature 500 deg., and the unit of heating surface per h.p. assumed to be 10 sq. ft.

Evaporation in Waste-heat Boilers

Temperature of hot gases, deg. fahr....	2000	1500	1200	1000	900	800	700	650	600	550
Water heating surface passed over by the gases, per cent......	8	18.5	27.5	37	43	50.5	60	66	73.2	82.5
Per cent. of the total steam generated in the boiler..........	25	49	64	75	80	83	90	92	94	96

This table has been shown by experiment to be very nearly correct for boilers not absorbing heat by direct radiation from a fire, and for this reason is applicable in waste-heat work. From this table it is possible to determine the approximate heating surface required per h.p. for any other entering and exit gas temperatures, provided the boilers are baffled about as they would be for direct-fired practice. Decreasing or increasing the velocity of the gases over the heating surfaces, *i.e.*, decreasing or increasing the frictional resistance through the boiler, will increase or decrease the heating surface necessary. This method is best illustrated by example.

Assume the entering gas temperatures to be 1470 deg. and that the gases are cooled to 570 deg. From the table, under what are assumed to be standard conditions, the gases have passed over 19 per cent. of the heating surface by the time they have been cooled 1470 deg. When cooled to 570 deg., 78 per cent. of the heating surface has been passed over. The work done in relation to the standard of the table is represented by $(1470-570)/(2500-500) = 45$ per cent. (These figures may also be read from the table in terms of the per cent. of the work done by different parts of the heating surfaces.) That is, 78 per cent. $-$ 19 per cent. $=$ 59 per cent. of the standard heating surface has done 45 per cent. of the standard amount of work. $59/45 = 1.31$, which is the ratio of surface of the assumed case to the standard case of the table. Expressed differently, there will be required 13.1 sq. ft. of heating surface in the assumed case to develop 1 h.p., as against 10 sq. ft. in the standard case.

Practically all waste gases contain much dust, and provision should be made for removing deposits from the heating surfaces. In some instances, as in the case of waste-heat boilers set in connection with cement kilns, dust-precipitating chambers should be provided between the kilns and the boiler proper.

When the operation of the furnace is intermittent, or the gas supply varies in quantity and temperature at different stages of the process, auxiliary coal grates may be supplied to insure a constant steam output from the boiler. No deleterious action on the heating surfaces of the boilers installed has been observed to result from sulphur fumes or other constituents of certain waste gases.

BOILER FEED WATERS AND ECONOMIZERS

REFERENCES: W. H. Booth, "Water Softening and Treatment." W. W. Christie, "Water, Its Purification and Use."

Where water of sufficient purity is not available for the purposes of feed, it is usually found necessary to render suitable the supply which must be used by some method of treatment. Against the cost of any such treatment there is to be placed the interest and depreciation on the increased number of boilers necessary where there is a tendency for the boilers to become scaled; the losses due to the cost of taking off boilers for cleaning; and the decrease in efficiency and capacity accompanying an increased incrustation of the boilers in use.

Troubles arising from the use of poor feed water may be classified as foaming, priming, formation of scale and corrosion.

Impurities in Feed Water. The following table gives an approximate classification of the impurities found in boiler-feed water, the difficulties arising from their presence, and the means ordinarily adopted for the treatment of the water to overcome such effects.

Approximate Classification of Impurities Found in Feed Waters, Their Effect and Ordinary Methods of Relief

Impurity	Nature of Difficulty	Ordinary Method of Overcoming or Relieving
Sediment, mud, etc.	Incrustation	Settling tanks, filtration, blowing down.
Readily soluble salts	Incrustation	Blowing down.
Bicarbonates of lime, magnesia, etc.	Incrustation	Heating feed. Treatment by addition of lime or of lime and soda. Barium carbonate.
Sulphate of lime	Incrustation	Treatment by addition of soda. Barium carbonate.
Chloride and sulphate of magnesium.	Corrosion	Treatment by addition of carbonate of soda.
Acid	Corrosion	Alkali.
Dissolved carbonic acid and oxygen.	Corrosion	Heating feed. Keeping air from feed. Addition of caustic soda or slacked lime.
Grease	Corrosion	Filter. Iron alum as coagulant. Neutralization by carbonate of soda. Use of best hydrocarbon oils.
Organic matter	Corrosion	Filter. Use of coagulant.
Organic matter (sewage)	Priming	Settling tanks. Filter in connection with coagulant.
Carbonate of soda in large quantities.	Priming	Barium carbonate. New feed supply. If from treatment, change.

Foaming is likely to occur in waters contaminated with sewage or organic growths, due to the fact that the suspended particles collect on the surface of the water in the boiler and render difficult the liberation of the steam bubbles arising to that surface. It is sometimes due to the light, flocculent precipitates from carbonates in solution, and in certain cases where any animal or vegetable oil is present to the fact that there is an excess of sodium carbonate added to the water during treatment.

Priming is the passing off of steam in belches which carry over water; it is caused by the concentration of sodium carbonate, sodium sulphate, or sodium chloride in solution. Sodium sulphate occurs in many southern waters and is also found where calcium or magnesium sulphate is precipitated with soda ash. The degree of concentration that may occur before priming will commence varies widely with conditions of operation and may be definitely determined only by experience with individual cases.

Scale is formed on boiler heating surfaces by the depositing of impurities in the feed water in the form of a more or less hard, adherent crust. Such deposits are due to the fact that the water loses its soluble power at high temperatures, or, through evaporation, the concentration becomes so high that the impurities crystallize and adhere to the surfaces. The salts usually responsible for this incrustation are the carbonates and sulphates of lime and magnesia.

The most widely known evidence of the presence of scale-forming ingredients in feed water is what is known as hardness. **Temporary hardness** is due to the presence of the carbonates of lime and magnesium which may be precipitated by boiling at 212 deg. fahr., and water containing no other scale-forming elements than these becomes "soft" under such treatment. **Permanent hardness** is due mainly to the presence of sulphate of lime, which is precipitated only at temperatures above 300 deg. fahr. Boiler-feed treatment consists in the main of getting rid of these salts more or less completely.

Scale is also formed by the settling of sediment and mud carried in suspension, which may bake or be cemented to a hard scale when mixed with other scale-forming ingredients.

Scale may have various characteristics. The sulphates of lime form the hardest, most flint-like scale. The scale formed from the carbonates of lime is somewhat softer. Sulphate of magnesia, when present alone, does not form a hard scale, but when carbonate of lime is present a chemical reaction takes place, forming hydrate of magnesia and calcium sulphate, which forms a particularly hard and stony scale. Carbonate of magnesia when present often separates out as a floury precipitate which floats for a time on the surface of the water or may be carried off in priming water. Its presence is particularly dangerous if the water contains grease, as it will combine with such grease to form a peculiar spongy scale that is less conductive of heat than a solid, stone-like scale. Such a spongy scale forming on heating surface directly exposed to the fire may lead to disastrous results.

Treatment of Water

Foaming. Where this is caused by organic matter in suspension, it may be largely overcome by filtration. Such matter as is not left in the filter may usually be taken care of by surface blowing. In some instances settling tanks are used to overcome this difficulty, but where large quantities of water must be provided, filtration is used as a substitute for the time and area necessary in settling tanks. When caused by overtreatment of feed water, such treatment should, of course, be changed.

Priming. Where this is caused by excessive concentration, it may be overcome largely by frequent blowing down of the boilers. Where the salts in the boiler feed exist in such quantities that to blow down sufficiently to keep the concentration below the priming point is not feasible because of the resultant losses, it may be overcome by some process of feed treatment.

Scale Prevention. The treatment of waters carrying scale-forming ingredients is along two main lines: (1) By chemical means, and (2) by means of heat.

Chemical Methods. (1) Lime Process. This process is used where waters contain bicarbonates of lime and magnesia. Slaked lime in solution (as lime water) is used, combining with the carbonic acid which is present either free or as carbonates, to form an insoluble monocarbonate of lime, the soluble bicarbonates of lime and magnesia losing their carbonic acid but thereby becoming insoluble and precipitating.

(2) Soda Process. This process is used for waters containing sulphate of lime and magnesia. Carbonate of soda and hydrate of soda (caustic soda) are used either alone or together as reagents. Carbonate of soda added to water containing a little (or no) carbonic acid or bicarbonates, decomposes the sulphates to form insoluble carbonates of lime and magnesia, which precipitate, the neutral soda remaining in solution. If free carbonic acid or

bicarbonates are present, bicarbonate of lime is formed and remains in solution, though under the action of heat the carbon dioxide will be driven off and insoluble monocarbonates will be formed. Caustic soda used in this process causes a more energetic action.

(3) Lime and Soda Process. This method, which is a combination of (1) and (2), is by far the most general in water purification, and is used where water contains sulphates of lime and magnesia and carbonic acid or bicarbonates in such quantities as to impair the action of soda. Sufficient soda is added to decompose the sulphates of lime and magnesia, with as much lime as is required to absorb all the carbonic acid not taken up by the soda reaction.

All apparatus for effecting this treatment is approximately the same in its chemical action, the different systems varying in the mechanical methods of introducing and handling the reagents. In an apparatus of this description water is ordinarily tested for hardness, alkalinity and causticity. The **scale of hardness** usually accepted (grains of dissolved salts per gal.) is as follows: Soft water, 1 to 10; moderately hard water, 10 to 20; very hard water, above 25. (One grain per U. S. gal. = 1.714 parts per 100,000.)

Alkalinity is a general term used for the presence of compounds having the power to neutralize acids. **Causticity**, as used in water treatment, is a term to indicate the presence of an excess of lime added during treatment. Though such presence would also indicate alkalinity, the term is arbitrarily used to apply to those hydrates whose presence is indicated by phenolphthalein.

When properly treated, the alkalinity, hardness and causticity should be in the approximate relation of 6, 5 and 4. When too much lime is used in the treatment, the causticity will be high in the purified water, nearly equaling the alkalinity. If too little lime is used, the causticity falls to approximately half the alkalinity. The hardness should not be in excess of two points less than the alkalinity. Where too much soda is used, the hardness is lowered and the alkalinity raised. If too little, the hardness is raised and the alkalinity lowered.

Other Reagents. Barium carbonate is sometimes used in removing calcium sulphate, the products of the reaction being barium sulphate and lime carbonate, both of which are insoluble. It is in itself an insoluble salt and cannot be added as a solution. It is a most satisfactory reagent for use in settling tanks, though until recently its price has made its use prohibitive. Barium hydrate is also used to reduce permanent hardness or the calcium sulphate component. It sets free any lime hydrate, which will reduce any lime carbonate present to insolubility. Where both carbonate and sulphate of lime are present, both barium hydrate and carbonate must be used. Silicate of soda will precipitate lime carbonate with the formation of a gelatinous silicate of lime and carbonate of soda. If sulphate of lime is present, carbonate of soda is formed in the above reaction, which will decompose the sulphate as well. Oxalate of soda is an expensive, but efficient, reagent which forms a precipitate of calcium oxalate of a particularly insoluble nature. Alum and iron alum will act as coagulants in waters where there is organic matter present. Iron alum has not only this property, but also that of reducing oil from water discharged from surface condensers to a condition in which it can be removed by filtration.

Testing Water as Treated by the Lime and Soda Process

A standard solution of sulphuric acid is used in testing for alkalinity and causticity. A soap solution is used in testing for hardness. A silver nitrate solution may also be used to determine whether an excess of lime has been used in the treatment.

Alkalinity. To 50 cu. cm. of treated water, to which sufficient methylorange has been added to color it, add the acid solution drop by drop until the mixture is just on the point of showing red. This point can be determined by shaking the mixture. As the acid solution is first added the red color quickly disappears, and more slowly as the critical point is reached. One-tenth cu. cm. of the standard acid solution corresponds to 1 deg. of alkalinity.

Causticity. To 50 cu. cm. of treated water to which one drop of phenol, dissolved in alcohol, has been added (resulting in a pinkish color), add the acid solution drop by drop, shaking until all color disappears. One-tenth cu. cm. of acid solution corresponds to one unit of causticity.

The alkalinity may be determined from the same sample after the causticity by coloring with methylorange and adding the acid to the point where the solution turns red. The *total* acid added (in determining both causticity and alkalinity) is the indication of the alkalinity.

Hardness. 100 cu. cm. of water are used for this test 1 cu. cm. of soap solution corresponds to 1 deg. of hardness. The soap solution is added a small quantity at a time to the sample, and the whole violently shaken. Enough must be added to make a permanent lather or foam, that is, when the shaking is stopped the soap bubbles should not disappear.

Nitrate of Silver Test for Excess of Lime. If there is an excess of lime in the water, the addition of silver nitrate will cause the sample to become dark brown. Otherwise a milky white solution will be formed.

Effects of Scale on Efficiency. Many tests have been made to indicate the loss in boiler efficiency due to the presence of scale. Some such tests would indicate that a scale ⅛ in. in thickness will decrease efficiency as much as 20 per cent., but, unfortunately, the figures that have been published are not to be implicitly relied upon. This is due largely to difference in conditions between tests with clean boilers and those in which scale has been allowed to accumulate, but to a greater extent to the impossibility of knowing the thickness of scale that has formed over *all* the heating surfaces.

The rate of such a falling off of efficiency as the thickness of scale increases will depend upon the nature of the scale formed. The effect of a very thin coat of scale on the heating surfaces of a boiler would probably be imperceptible in the efficiency of a boiler.

Allowable Degree of Concentration. The allowable concentration of salts in the water in a boiler depends on the operating conditions; in some instances trouble will be experienced with a concentration as low as 30 gr. per gal., while in others 300 gr. can be handled without priming. A trace of oil or the presence of a small amount of organic matter will cause priming with waters which would otherwise give no trouble.

Where the soluble salts are mainly sodium chloride, if the degree of concentration is kept below 150 to 175 gr. per gal., no priming difficulty should be experienced. In a test for the amount of sodium chloride concentration possible before priming would start, 330 gr. per gal. were handled with no priming, the steam showing 0.5 per cent. of moisture. With the concentration increased to 400 gr. per gal., priming was practically continuous.

Where the soluble salts are mainly sodium carbonate, the permissible degree of concentration may be increased. With a boiler properly operated at moderate loads, this concentration, if kept below 300 gr. per gal., should cause no difficulties from priming. In a test with a particularly bad water, containing salts which were mainly sodium carbonate, there was no priming with a concentration of 440 gr. per gal., the steam showing 1 25 per cent. of moisture. With a concentration of 560 gr. per gal., there was no priming, but the moisture content was increased to 2 per cent. With the idea of indicating the maximum concentration possible, special arrangements were made in the manner of drawing off steam and the concentration increased to over 1500 gr. per gal. without any priming action. Any such degree of concentration, however, is decidedly inadvisable.

The limiting factor in deciding whether a water carrying a large amount of soluble salts may be used for boiler feed purposes, is the amount of blowing down necessary to keep the degree of concentration within the limits shown by experience to be necessary.

Allowable Degree of Concentration in Treated Water. Where boiler feed water is treated with soda ash, experience has shown that there are certain limits of concentration of this chemical, above or below which difficulties will be encountered. Ordinarily, these limits are between 25 and 70 gr. of soda per gal., with the best results secured where they are between 30 and 40 gr.

The usual method employed in keeping between such limits of concentration, when the treated water is fed to all boilers of a plant, is to titrate the water from each boiler once a week to determine the amount of soda contained. Where this falls below the lower limit an additional amount of the saturated solution of soda is introduced by a hand pump. If the amount of soda is above the upper limit, the boiler is blown down sufficiently to reduce the soda to the proper amount.

Purification Plants. The approximate cost of a water purification plant will be between $500 and $750 per 1000 gal. treated per hour, the smaller plants costing more per unit of capacity than the larger. The space occupied by some typical apparatus of this nature is given in Table 15.

Table 15. Floor Space Required by Water Purification Apparatus

System	Capacity, Gal. per Hour			
	2,000	5,000	10,000	20,000
L. M. Booth Co.	7′ 9″ diam.	12′ 3″×19′10′	17′ 6″×26′ 2″	24′ 6″×35′ 7″
The Kennicott Co.	9′ diam.	13′ 6″ diam.	18′ diam.	24′ diam.
Wm. B. Scaife Co. (Intermittent.)	24′×15′	42′×18′	57′×23′	80′×30′
Wm. B. Scaife Co. (Continuous.)	25′×15′ 6″	25′×15′ 6″	30′ 6″×20′	48′× 25′

Heights of Apparatus, Ft.: 2000 gal.—Booth, 26½; Kennicott, 16½. 5,000 gal.—Booth, 26⅙; Kennicott, 20. 10,000–20,000 gal.—Booth, 27⅙; Kennicott, 23.

Time for reaction is approximate and variable, depending upon the water treated. With the Booth system it is claimed that the time (3½ to 4 hr.) is shortened through mechanical agitation. The Kennicott Co. states that the time will vary with the temperature of water treated, from 4 hr. at 50 deg. fahr. to as low as 1 hr. at 200 deg. fahr.

Table 16 gives a means of computing the cost of treatment of feed water by a typical "lime and soda" apparatus. It is based on using lime containing 90 per cent. of calcium oxide and soda ash containing 58 per cent. of sodium oxide, the amounts of these reagents required for each grain per gal. of various impurities found being given in lb. per 1000 gal. From local prices of the reagents the costs of purification may be readily computed. Lime ordinarily costs about $0.50 per 100 lb., while soda ash costs about $1 per 100 lb.

When treated by the lime and soda process, water containing less than 20 gr. of scale-forming matter will ordinarily, after treatment, contain less than 3 gr. Water containing 20 to 30 gr. will be reduced to 4 gr.; 30 to 40 gr. reduced to 5 gr., and 40 to 50 gr. reduced to approximately 6 gr. In general, only waters containing over 5½ gr. per gal. will deposit scale.

Boiler Compounds are almost invariably based on soda, with certain tannic substances, and in some instances a gelatinous substance which is presumed to encircle the scale particles and prevent their adhering to the boiler surfaces. The action of such compounds is usually to reduce the lime sulphate by means of carbonate of soda and to precipitate it as a muddy form of carbonate of lime, which may be blown off. The object of using the tannic compounds with soda is to introduce organic matter into scale already formed, so that when it has penetrated to the surface of the metal, below the scale, decomposition will set up, causing a disruptive effect which uplifts scale, sometimes in large slabs. The latter effect should be guarded against in the use of compounds, or trouble will arise due to the presence of loose scale and the consequent danger of burning the boiler tubes. See p. 557 for effects of compounds or corrosion.

Table 16. Reagents Required in Lime and Soda Process for Treating 1000 U. S. Gal. of Water per Grain of Contained Impurities per Gal.*

	Lime†	Soda‡		Lime	Soda
Calcium carbonate	0.098 lb.	....	Ferrous carbonate	0.169 lb.	....
Calcium sulphate	....	0.124 lb.	Ferrous sulphate	0.070	0.101 lb.
Calcium chloride	....	0.151	Ferric sulphate	0.074	0.126
Calcium nitrate	....	0.104	Aluminum sulphate	0.087	0.147
Magnesium carbonate	0.234	....	Free sulphuric acid	0.100	0.171
Magnesium sulphate	0.079	0.141	Sodium carbonate	0.093	....
Magnesium chloride	0.103	0.177	Free carbon dioxide	0.223	....
Magnesium nitrate	0.067	0.115	Hydrogen sulphide	0.288	....

* L. M. Booth Co. † Based on lime containing 90 per cent. calcium oxide.
‡ Based on soda containing 58 per cent. sodium oxide.

When proper care is taken to suit the compound to the water used, it may be fairly effective. Compounds should be used, however, to prevent the formation of scale rather than to remove scale already formed. The cost of compounds is usually high as compared with the actual amount of active material they contain.

Oil as a Scale Preventive. The introduction of crude oil or kerosene into a boiler has from time to time been used as a means of preventing scale formation on the heating surfaces, but this use of kerosene or of crude oil is not to be recommended. While cases may arise in which boiler waters can be effectively treated within the boiler itself, oil is not the reagent to be used. The distilling off of the lighter oils finally leaves a heavy, gum-like carbonaceous deposit on the heating surfaces, which will tend to cause a burning out of the affected parts. Further, such oils may contain materials which will saponify where the feed is sufficiently alkaline, and severe foaming will result.

Economizer Practice

The **efficiency due to an economizer installation** will increase as the temperature of the gases entering the economizer increases, as the velocity of the gases through the economizer for a given weight of gas handled per hour increases, as the temperature of the water entering the economizer decreases, and as more heating surface is supplied in the economizer. The heating surface that can best be installed is limited by the expense warranted and the added draft resistance of the larger economizer.

The **heat transfer rate** to be expected **through economizers** will vary somewhat with the difference between the mean temperatures of the gases and the water. Under ordinary conditions, with a gas velocity through the economizer of approximately 1000 ft. per min., a heat transfer of from 2.25 to 3 B.t.u. per hour per sq. ft. of surface per deg. fahr. of difference in temperature may be expected. With the velocity of gas increased to about 1800 ft., this transfer rate will be approximately 3 B.t.u.

Where boilers are run at slightly above their rated capacity, the efficiency with an economizer having approximately 60 per cent. of the boiler heating surface will be increased approximately 6 per cent. With boilers run at about their rated capacity, with exit gas temperatures of approximately 500 deg. and gas velocities such as the present design of economizer ordinarily gives, the added efficiency will be in the neighborhood of 5 per cent. With economical boilers run at 200 per cent. of their rated capacity, with exit gas temperatures somewhat above 600 deg. and a gas velocity of about 1800 to 2000 ft. per min., with economizers containing approximately 70 per cent. of the boiler heating surface this increase will approach 10 per cent. and with uneconomical boilers may be still greater.

The saving in B.t.u. per year of continuous operation for various amounts of economizer surface per boiler h.p. and for each 100 deg. difference in temperature between the entering flue-gas temperatures and the entering boiler-feed temperature is shown in the following table which represents the average of results obtained in 200 plants. For shorter periods of operation the saving is decreased proportionately; the saving varies also as the temperature difference.

Economizer surface, sq. ft. per 100 deg. fahr. difference per boiler h.p.	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Millions of B.t.u. saved per sq. ft. per year	11.1	6.7	4.6	3.6	2.9	2.5	2.2	1.9

The installation of economizers generally necessitates the use of an induced-draft fan. For the ordinary arrangement of sections, the draft loss may be considered as about 0.15 to 0.2 in. of water with boilers operated at 200 per cent. of their rated capacity.

Materials of Economizer Construction. Cast iron is ordinarily used in economizer construction to obviate danger of corrosion. Wrought-iron and steel tubes have been used and are being used in European practice, and while some economizers with wrought-iron tubes appear to have stood up well, others have failed through corrosion in a comparatively short time. Much depends upon the nature of the feed water, and, in certain instances, economizers with wrought-iron tubes which have stood up for a time have afterward failed rapidly when the amount of make-up water at the plant was so reduced that the water fed through the economizer was purer than before. Air in the water appears to have important influence. Corrosion occurring in such apparatus may be external or internal, though the former, which comes through condensing vapor from the gases on the colder economizer surfaces, is unlikely where the water fed to the economizer is at a higher temperature than 120 deg. fahr.

Practice in this country in removing dust from the outside of economizer tubes has been largely limited to the use of scrapers. European practice has shown that such removal can be done in a satisfactory manner through the use of compressed air.

The average cost of economizers installed is about $1.20 per sq. ft. of surface.

BOILER-SETTING BRICKWORK, GASKETS, ETC.

Red Brick should be used in boiler settings only where it is protected from heat, and should be of the best grade, sound, hard, well-burned and uniform. For properties of bricks, see p. 622.

Fire Brick. The quality of fire brick is ordinarily judged by the following features:

Plasticity: The tendency of a brick to become plastic, at a temperature much below the melting point, and to a degree that may cause the brick to become deformed under the stress to which it is subjected. This is now considered the determining factor in considering the suitability of fire brick. To enable a brick to be used with any degree of satisfaction, the plastic point should be above 2400 deg. fahr. under a unit stress of 100 lb. per sq. in.

Fusing Point: The temperature at which the brick will fuse. Important only in that a high fusing point ordinarily indicates a high critical temperature of plasticity.

Expansion. The lineal expansion will vary from 0.009 in. to 0.08 in. in 9 in. Within these limits, the expansion of the brick is not an important factor in a properly designed furnace.

Compression. The strength necessary to cause crushing at the center of the 4½-in. face by a steel block 1 in. square.

Hardness: Relative, usually figured on an arbitrary scale of 10.

Size of Nodules: The average size of flint grains when a brick is carefully crushed. The scale of nodules is: Small, or the size of anthracite rice; large, or the size of anthracite pea.

Ratio of Nodules: Percentage of a given volume occupied by flint grains. This scale is ordinarily: High, 90 to 100 per cent.; medium, 50 to 90 per cent.; low, 10 to 50 per cent.

See Technologic Papers Nos. 7 and 10 of the U. S. Bureau of Standards, entitled "The Testing of Fire-Clay Refractories," and "Melting Points of Fire Bricks."

The values of fire brick are necessarily relative, but, generally **speaking**, what are known as **first-grade fire brick** may be divided into **three classes** suitable for various operating conditions as follows:

Class A: For stoker-fired furnaces where high overloads are to be expected, or where other extreme conditions of service are liable to occur.

Class B: For ordinary stoker settings where there will be no excessive overloading of the boiler, or in hand-fired settings where the rates of driving will be high.

Class C: For ordinary hand-fired settings where the presumption is that the boilers will not be overloaded, except at rare intervals and for short periods only.

The characteristics of these three classes are given in Table 17.

Table 17. Characteristics of First-grade Fire Brick

Characteristics	Class A	Class B	Class C
Fuse point, deg. fahr.	Safe at 3,200 to 3,300 deg.	Safe at 2,900 to 3,200 deg.	Safe at 2,900 to 3,000 deg.
Compression, lb.....	6,500 to 7,500	7,500 to 11,000	8,500 to 15,000
Hardness, relative...	1 to 2	2 to 4	4 to 6
Size of nodules......	Medium	Medium to medium-large	Medium to large
Ratio of nodules.....	High	Medium to high	Medium low to medium

Many fire brick are sold for No. 1 grade that are in reality No. 2. The characteristics of such fire brick are: Fuse point, 2400 to 2700 deg. fahr.; hardness, 6 to 10; compression, 14,200 to 32,000 lb.; size of nodules, small to very small: ratio of nodules, low to very low. It will be seen that as the brick is poorer it is harder, but it is recognized that if a brick be of sufficient hardness to withstand its load, any additional hardness is a detriment rather than an advantage.

A rough indication of the **quality of a fire brick** is afforded by the appearance of a **fracture**. A fine, uniform texture, like bread, shows a poor brick. If the fracture is open, clean. white and flinty, the brick is probably good. The best fire brick are manufactured from Pennsylvania fire clays. South and west from Pennsylvania the quality of fire clay in general becomes poorer as the distance increases. Some of the southern fire clays contain a considerable amount of metallic iron.

Red brick should be laid in a thoroughly mixed mortar, composed of 1 volume of Portland cement, 3 volumes of unslaked lime, and 16 volumes of clean sharp sand. Not less than 2¼ bushels of lime should be used in laying 1000 brick. Every brick should be thoroughly imbedded and each joint filled.

All fire brick should be protected from moisture, and should be dry when used. Each brick should be dipped in a thin clay wash, "rubbed and shoved" into place, and tapped with a wooden mallet until it touches the brick next below it. Fire clay is not a cement, and has little or no holding power. Its action is not that of a binder, but of a filler, and a consistency in fire-clay wash sufficient to permit the use of a trowel should not be allowed.

Walls of red and fire brick should be carried up together. Furnace linings should be laid with four courses of headers and one stretcher. Furnace center walls should be entirely of fire brick.

Fire-brick arches should be constructed of selected brick that are smooth, straight and uniform. Arch centers or frames should be built of batten strips not more than 2 in. in width. Brick should be dipped on one side and two edges only, and tapped into place with a mallet. They should be laid in courses along the arch center and not in rings, and each joint should be broken with a bond equal to half a brick. Each course should be tried in place dry, and checked with a straight edge to insure a uniform thickness of joint between courses. A course of wedge brick should be used only where necessary to keep the bottom of the straight brick in even contact with the arch center, and where it is impossible to obtain such contact by the use of wedges, the straight brick should lean away from the center line of the furnace, rather than toward it. When an arch is about two-thirds completed, a trial ring should be laid to determine if the key course will fit. Where cutting is necessary to fit the keying course, it should be done on the two adjacent courses on the side of the brick away from the key. The keying course should be a true fit from top to bottom, and after being dipped and driven it should not extend below the surface of the arch; preferably, it should extend about ¼ in. above the surface. After fitting, the keys should be dipped, replaced loosely, and then driven uniformly into place by a heavy hammer and a piece of wood extended along the full width of the keying course. Arch centers should be so constructed that they can be dropped down free of the arch after it is keyed, and removed from the setting without being burned out.

Care of Boiler Brickwork. To avoid cracking it is essential that the setting be dried out thoroughly before a boiler is placed in service. As a preliminary drying process, the boiler damper and ash pit doors should be blocked open and a circulation of air maintained through the setting for two or three days. When it is desired to place the boiler in service, a light wood fire should be started and this fire built up very gradually until the walls are thoroughly warm, at which time coal may be fired and steam raised.

Cracks in the setting cause deterioration in addition to a decrease in efficiency. Soot will fill up such crevices, become packed tight and will have a tendency to loosen the brickwork. Cracks about cleaning-door frames should be at once filled and the whole setting kept pointed up. Any leakage of steam or water within a setting will cause the latter to disintegrate rapidly. The draft suction available has also a direct bearing upon the life of a setting. In installations where furnace temperatures are high, if there is not sufficient draft, the hot gases will tend to percolate outward through the brickwork, and the temperature of the furnace walls may rise above the fusing point of the brick. Such temperatures are often found in stoker-fired and oil installations, and care should be taken to see that there is no back pressure at any point within the boiler setting, or what is often referred to as a "bottling action" of the gases in the boiler furnace.

Steel Casing. Radiation losses from the surface of brick boiler settings, combined with losses due to air leakage, have been estimated as varying from 2 to 10 per cent., depending upon the amount of radiating surface and condition of the setting, which in turn depends upon the size of the boiler unit. In modern efforts to secure the highest possible plant economies, much has been done in reducing these losses by the use of an insulated steel casing. In a reasonably large-sized unit, such a casing, properly installed, will reduce the losses approximately 1 or 2 per cent. over what can be accomplished with a brick setting without a casing. Steel plate, or steel plate backed by asbestos millboard, is not as effective as a casing properly insulated from the brick portion of the boiler setting by magnesia block.

Table 18 gives the approximate brick quantities required in the setting of various-sized units of different types of boilers, and Table 19 the approximate floor space occupied by different types of boilers ranging from 100 to 500 h.p. in capacity.

Table 18. Comparative Approximate Brick Quantities Required in Settings of Different Types of Boilers

Type of boiler	Size of unit							
	100 h.p.		200 h.p.		400 h.p.		600 h.p.	
	Red	Fire	Red	Fire	Red	Fire	Red	Fire
Horizontal water-tube	14,500	3,500	16,900	4,150	21,400	5,450	22,600	6,500
Semi-vertical water-tube	14,900	4,800	17,000	5,400	20,100	6,800	30,250	8,850
Vertical water-tube	10,700	7,900	15,600	10,000	20,750	14,500	25,800	19,700
Horizontal return-tubular	20,500	1,750	25,600	3,025	36,600	5,900	....	....

Table 19. Comparative Approximate Floor Space Occupied by Different Types of Boilers

Type of boiler	Size of unit			
	100 h.p.	200 h.p.	300 h.p.	500 h.p.
Horizontal water-tube	7′ 3″ × 19′ 9″	9′ 0″ × 19′ 9″	10′ 2″ × 19′ 9″	13′ 8″ × 19′ 9″
Semi-vertical water-tube	8′ 6″ × 14′ 0″	9′ 6″ × 16′ 9″	11′ 0″ × 17′ 10″	13′ 0″ × 19′ 7″
Vertical water-tube	9′ 5″ × 20′ 10″	9′ 5″ × 22′ 10″	10′ 10″ × 23′ 10″	17′ 6″ × 23′ 10″
Horizontal-return tubular	10′ 0″ × 21′ 7″	11′ 0″ × 23′ 10″	12′ 0″ × 25′ 0″	....
Vertical fire-tube*	68″ × 23′ 1″	89″ × 24′ 10″	104″ × 26′ 11″	....

* Diam. and height.

Boiler Gaskets and Coverings*

Gaskets may be classified in four groups:

(1) Rubber, or partial rubber composition, gaskets. Tight joints can be made with these, though their use is limited to those places where they will only come into contact with water or saturated steam.

(2) Asbestos, or asbestos composition, gaskets. These, when properly "followed up" after being placed under steam, will neither leak nor blow out. Possibly, for most cases, a gasket with an asbestos fiber and litharge base will be found to give as satisfactory results as may be obtained.

* See Paulding's "Steam in Covered and Bare Pipes."

(3) All-metal gaskets. It is difficult to make tight joints between plane surfaces with soft metal interposed unless bolt holes be on 2½- to 3-in. centers. Some gaskets made entirely of metal have a tendency to break down mechanically under compression by bolts, and to be attacked by moisture resulting from incipient leaks.

(4) Metal and fiber, or metal and composition, gaskets. Such gaskets are ordinarily of copper and asbestos, copper and rubber, copper and lead. Copper gaskets with lead insertion are ordinarily used for water, and with asbestos insertion for steam.

A good gasket must not blow out under strains arising from temperature changes; must not harden; should be reversible and interchangeable; should not spread or stick to plates; should be such as to enable it to be used more than once.

Coverings. Exposed parts of boilers, if not properly covered, will radiate heat to the surrounding atmosphere at the approximate rate of 3 B.t.u. per sq. ft. of exposed surface per hour per deg. difference in temperature between the contained steam or water and the external air. With sufficient thickness of insulating material, this radiation may be reduced to approximately 0.5 B.t.u. (see pp. 305, 308 and 840).

Ordinarily, the tops of the drums of both water-tube and fire-tube boilers are covered with one row of brick on edge and a thin layer of cement. No figures are available on losses through such a covering, but the radiation is probably decreased to from 1.5 to 2 B.t.u. per sq. ft. In certain plants a thin layer of magnesia is placed on the drums, then a layer of cement, and an ash insulation built up.

Drumheads are ordinarily covered with magnesia block or plastic magnesia held in place by netting and covered with painted canvas. Connections from boilers to superheaters should be covered with 2-in. magnesia block, canvas-covered and painted.

Where steel casings are furnished over boiler brickwork, they should be insulated with 2-in. 85 per cent. magnesia block, between which and the brickwork should be inserted ¼-in. asbestos millboard.

SAFETY VALVES FOR STEAM BOILERS

The safety valve (or valves) should have a relieving capacity at least as great as the evaporative capacity of the boiler when operating at maximum capacity.

Safety valves are of two general types, lever and spring-loaded. The first, though still manufactured, is seldom used except in low-pressure work. The objections to this type are that there is no definite "pop" point, the valve lifting slowly in opening, and settling gradually in closing. A comparatively long range of blowing is necessary for the valve to effectively open, and a considerable overpressure is necessary in specifications for such valves. Spring-loaded valves, on the other hand, have a clean, positive opening to practically the full amount. At the popping point a properly designed spring valve will lift its maximum, say 0.15 in., and this lift will decrease approximately 0.01 in. per lb. that the pressure in the boiler falls below the setting point, until the closing point, which is at from 4 to 5 lb. below the popping point. Other pressures may force the lift slightly higher with such a valve, but not sufficiently to make these pressures necessary to obtain the full valve efficiency. In specifying spring valves, therefore, an overpressure should not be allowed, at least not over 1 or 2 lb.

Rules for Design. The following rules for the design and use of safety valves are given by the Massachusetts State Board of Boiler Rules, by the A. S. M. E. code, and by the United States Board of Supervising Inspectors.

(Mass. Board.) Table 20 gives areas of grate surfaces in sq. ft. for other than direct spring-loaded safety valves.

Table 20. Areas of Grate Surfaces for Other than Direct Spring-loaded Safety Valves

Diam. of valve, in.	Maximum allowable pressure on boilers, lb. per sq. in. — 0 to 25	Over 25 to 50	Over 50 to 100	Diam. of valve, in.	Maximum allowable pressure on boilers, lb. per sq. in. — 0 to 25	Over 25 to 50	Over 50 to 100
	Area of grate in sq. ft.				Area of grate in sq. ft.		
1	1.50	1.75	2.00	3	11.75	14.25	16.00
1¼	2.25	2.50	3.00	3½	16.00	19.50	21.75
1½	3.00	3.75	4.00	4	21.00	25.50	28.25
2	5.50	6.50	7.25	4½	26.75	32.50	36.00
2½	8.25	10.00	11.00	5	32.75	40.00	44.00

Each boiler shall have one or more safety valves.

Table 21 gives areas of grate surfaces in sq. ft. for direct spring-loaded safety valves.

Table 21. Areas of Grate Surfaces for Direct Spring-loaded Safety Valves

	$W=\frac{75}{3600}$ $P=40$ $A=0.401$	$W=\frac{100}{3600}$ $P=65$ $A=0.329$	$W=\frac{160}{3600}$ $P=115$ $A=0.297$	$W=\frac{160}{3600}$ $P=140$ $A=0.244$	$W=\frac{200}{3600}$ $P=190$ $A=0.224$	$W=\frac{240}{3600}$ $P=240$ $A=0.213$
Max. allowable pressure on boiler, lb. per sq. in.	0 to 25 lb.	Over 25 to 50 lb.	Over 50 to 100 lb.	Over 100 to 150 lb.	Over 150 to 200 lb.	Over 200 lb.
Diam. of valve, in.	Area of grate in sq. ft.					
1	2.00	2.50	2.75	3.25	3.5	3.75
1¼	3.25	4.00	4.25	5.00	5.5	5.75
1½	4.50	5.50	6.00	7.25	8.0	8.50
2	8.00	9.75	10.75	13.00	14.0	15.00
2½	12.50	15.00	16.50	20.00	22.0	23.00
3	17.75	21.50	24.00	29.00	31.5	33.25
3½	24.00	29.50	32.50	39.50	43.0	45.25
4	31 50	38.25	42.50	51.50	56.0	59.00
4½	40 00	48.50	53.50	65.00	71.0	74.25
5	49.00	60.00	66.00	80.00	88.0	92.25

When the conditions exceed those on which Table 20 is based, use formula $A = 11(70\ W/P)$, in which A = area of direct spring-loaded valve in sq. in. per sq. ft. of grate surface; W = lb. of water evaporated per sq. ft. of grate surface per sec., and P = pressure (absolute) at which the safety valve is set to blow.

If more than one safety valve is used, minimum combined area to be in accordance with Table 20. More than one safety valve to be installed on a boiler carrying over 25 lb. pressure per sq. in.

Safety valves hereafter (1908) installed on boilers not to exceed 5 in. in diam., and to be of the direct spring-loaded type, with seat and bearing surface of the disk inclined at an angle of about 45 deg. to center line of spindle; designed with a substantial lifting device so that disk can be lifted from its seat with spindle, not less than ⅛ diam. of valve, when pressure on boiler is 75 per cent. of that at which the safety valve is set to blow.

(A. S. M. E. CODE). Each boiler shall have two or more safety valves, except a boiler for which one safety valve 3-in. size or smaller is required by these Rules.

The safety-valve capacity for each boiler shall be such that the safety valve or valves will discharge all the steam that can be generated by the boiler without allowing the pressure to rise more than 6 per cent. above the maximum allowable working pressure, or more than 6 per cent. above the highest pressure to which any valve is set.

One or more safety valves on every boiler shall be set at or below the maximum allowable working pressure. The remaining valves may be set within a range of 3 per cent. above the maximum allowable working pressure, but the range of setting of all of the valves on a boiler shall not exceed 10 per cent. of the highest pressure to which any valve is set.

Safety valves shall be of the direct spring-loaded pop type with seat and bearing surface of the disk either inclined at an angle of about 45 deg. or flat at an angle of about 90 deg. to the center line of the spindle. The vertical lift of the valve disk measured immediately after the sudden lift due to the pop may be made any amount desired up to a maximum of 0.15 in. irrespective of the size of the valve. The nominal diameter measured at the inner edge of the valve seat shall be not less than 1 in. or more than 4½ in.

Each safety valve shall have plainly stamped or cast on the body: (*a*) The name or identifying trade-mark of the manufacturer, (*b*) the nominal diameter with the words "Bevel Seat" or "Flat Seat," (*c*) the steam pressure at which it is set to blow, (*d*) the lift of the valve disk from its seat, measured immediately after the sudden lift due to the pop, (*e*) the weight of steam discharged in pounds per hour at the pressure for which it is set to blow.

The minimum capacity of a safety valve or valves to be placed on a boiler shall be determined on the basis of 6 lb. of steam per hour per sq. ft. of boiler heating surface for water-tube boilers, and 5 lb. for all other types of power boilers, and upon the relieving capacity marked on the valves by the manufacturer, provided such marked relieving capacity does not exceed that given in Table 22. In case the relieving capacity marked on the valve or valves exceeds that given in Table 22, the minimum safety valve capacity shall be determined on the basis of the maximum relieving capacity given in Table 22 for the particular size of valve and working pressure for which it was constructed. The heating surface shall be computed for that side of the boiler surface exposed to the products of combustion, exclusive of the superheating surface. In computing the heating surface for this purpose, only the tubes, shells, tube sheets and the projected area of headers need be considered.

Safety-valve capacity may be checked in any one of three different ways, and if found sufficient, additional capacity need not be provided: (*a*) By making an accumulation test by shutting off all other steam discharge outlets from the boiler and forcing the fires to the maximum. The safety-valve equipment shall be sufficient to prevent an excess pressure beyond 6 per cent.; (*b*) by measuring the maximum amount of fuel that can be burned and computing the corresponding evaporative capacity upon the basis of heating value of the fuel; (*c*) by determining the maximum evaporative capacity by measuring the feed water. The sum of the safety-valve capacities marked on the valves shall be equal to or greater than the maximum evaporative capacity of the boiler.

Table 22. Discharge Capacities for Direct Spring-loaded Pop Safety Valves with 45-deg. Bevel Seats*

Gage pressure, lb. per sq. in.	Diameter, in.															
	1	1¼	1½		2		2½		3		3½		4		4½	
	Lift, in.															
	Max.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
	0.05	0.05	0.03	0.06	0.04	0.07	0.04	0.08	0.05	0.10	0.06	0.11	0.07	0.12	0.08	0.13
	Discharge capacity, lb. per hr.															
15	163	203	146	293	261	456	326	651	489	977	684	1254	912	1564	1173	1906
25	218	272	196	392	349	610	435	871	653	1307	914	1676	1219	2090	1568	2547
50	354	444	320	639	568	994	710	1419	1064	2129	1490	2732	1987	3406	2555	4151
75	492	615	443	886	787	1377	984	1968	1475	2951	2066	3788	2754	4722	3542	5756
100	629	786	566	1133	1007	1761	1258	2516	1887	3774	2642	4843	3522	6038	4529	7358
125	767	957	689	1379	1224	2145	1532	3064	2299	4596	3218	5899	4290	7354	5516	8963
150	904	1129	813	1625	1438	2529	1806	3613	2710	5419	3794	6954	5058	8670	6503	10566
175	1040	1301	936	1872	1664	2913	2081	4161	3121	6242	4369	8010	5824	9984	7490	12173
200	1178	1472	1060	2119	1884	3296	2354	4709	3532	7064	4946	9068	6593	11305	8475	13773
225	1315	1643	1183	2366	2104	3680	2629	5258	3944	7890	5521	10120	7361	12616	9465	15383
250	1451	1814	1307	2613	2322	4064	2903	5807	4355	8708	6097	11175	8130	13938	10448	16980
275	1589	1986	1430	2860	2542	4448	3177	6355	4766	9533	6672	12233	8895	15248	11438	18585
300	1746	2157	1553	3107	2762	4832	3452	6903	5177	10358	7248	13290	9668	16568	12428	20195

* The discharge capacity of a flat-seat valve of a given diameter with a given lift may be obtained by multiplying the discharge capacity given in the table for a 45-deg. bevel-seat valve of same diameter and same lift, by 1.4. Discharge capacity is proportional to the lift.

When two or more safety valves are used on a boiler, they may be either separate or twin valves made by mounting individual valves on Y-bases, or duplex, triplex or multiplex valves having two or more valves in the same body casing. The safety valve or valves shall be connected to the boiler independent of any other steam connection, and attached as close as possible to the boiler, without any unnecessary intervening pipe or fitting. Every safety valve shall be connected so as to stand in an upright position, with spindle vertical, when possible. Each safety valve shall have full-sized direct connection to the boiler. No valve of any description shall be placed between the safety valve and the boiler, nor on the discharge pipe between the safety valve and the atmosphere. When a discharge pipe is used, it shall be not less than the full size of the valve, and shall be fitted with an open drain to prevent water from lodging in the upper part of the safety valve or in the pipe.

Safety valves shall operate without chattering and shall be set and adjusted as follows: To close after blowing down not more than 4 lb. on boilers carrying an allowed pressure less than 100 lb. per sq. in. gage; after blowing down not more than 6 lb. on boilers carrying pressures between 100 and 200 lb. per sq. in. gage, inclusive; and after blowing down not more than 8 lb. on boilers carrying over 200 lb. per sq. in. gage.

Each safety valve used on a boiler shall have a substantial lifting device, and shall have the spindle so attached that the valve disk can be lifted from its seat a distance not less than one-tenth of the nominal diameter of the valve, when there is no pressure on the boiler. The seats and disks of safety valves shall be of non-ferrous material.

Springs used in safety valves shall not show a permanent set exceeding $\frac{1}{32}$ in. 10 min. after being released from a cold compression test closing the spring solid. The spring in a safety valve shall not be used for any pressure more than 10 per cent. above or below that for which it was designed.

(U. S. BOARD.) Area of all safety valves in boilers to be determined by the following formula: $A = 0.2074\ W/P$, where A = area of safety valve in sq. in. per sq. ft. of grate surface; W = lb. of water evaporated per sq. ft. of grate surface per hour, and P = absolute pressure, lb. per sq. in. The value of A so found multiplied by the sq. ft. of grate surface gives the area of safety valve, or valves, required. The seats of all safety valves to have an angle of inclination of 45 deg. to the center line of their axes. After 1911, no safety valves having a set-screw arrangement at top of the valve casing, designed to hold the valve down while the hydrostatic pressure is being applied, to be allowed.

Darling's Formulæ for Design. The factors entering into the discharge area or relieving capacity of a safety valve are the diameter of the inlet opening at the seat and the vertical valve lift. In figuring the valve capacity by either of the rules given above, it is assumed that the valve lift has a definite ratio to the diameter at the inlet opening. Philip G. Darling (*Trans. A. S. M. E.*, vol. 31, 1909, p. 109) has pointed out that such an assumption would make the relieving capacity of all valves of the same nominal diameter the same. Special experiments show that this is not the case; that the lift does not vary with the diameter, but is nearly a constant for any make of valve regardless of the diameter; and that the actual lifts are in some cases only approximately one-half the lifts as assumed above. Table 23 gives a comparison of lifts, discharge areas, and relieving capacities of several different makes of stationary boiler valves, all of the same nominal diameter.

Table 23. Comparison of Lifts, Discharge Areas and Relieving Capacities of Seven Different Makes of Boiler Safety Valves

(Diam. of all valves, 4 in.; popping pressure in all cases, 200 lb. per sq. in.)

Lifts		Effective area of discharge with opening lift, sq. in.	Per cent. of largest opening	Relieving capacities	
Opening, in.	Closing, in.			Lb. of steam per hour, from $E = 105\ LDP$	H.p. on basis of 30 lb. evap. per hour = 1 h.p.
0.064	0.024	0.568	46.6	5,780	193
0.031*	0.017	0.398	31.4	3,960	132
0.056	0.032	0.496	40.8	5,060	169
0.094	0.039	0.834	68.5	8,500	283
0.094	0.055	0.834	68.5	8,500	283
0.082	0.054	0.727	59.7	7,400	247
0.137	0.088	1.22	100.0	12,400	413

* Flat seat; all other valves with 45-deg. seats.

The conclusions reached by Mr. Darling were to the effect that the rules in use are not safe and should not be followed. A more satisfactory rule for determining valve capacity is obtained from Napier's formula for the flow of steam, combined with the actual discharge area of a valve as determined by its lift. The constant, or coefficient of flow, when applied to the irregular steam discharge passages of safety valves, was found by a series of actual tests to be 47.5, and Napier's formula modified becomes $E = 47.5aP$, where E = lb. of steam discharged per hour; a = effective discharge opening in sq. in., and P = absolute pressure in lb. per sq. in. Expressed in terms of valve diameter and lift, to make it generally serviceable, and assuming the almost universal 45-deg. valve seat, this formula becomes,

$$E = 105LDP \tag{1}$$

$$\text{or } D = 0.0095E/LP \tag{2}$$

where D = diam. in in., and L = lift in in.

It will be noted that the nominal area of discharge does not enter into this formula. If D is found to have a value of, say, 12, three 4-in. or four 3-in. valves will be necessary. Formula (2) is probably the best now available for use in designing safety valves.

To meet the demand for a formula expressed in terms of heating surface, modified values of the constant of flow have been advanced for various classes of work as follows, the nominal evaporation per sq. ft. of heating surface (H = heating surface in sq. ft.) and the maximum overloads being assumed as given:

For cylindrical multi-tubular and vertical water-tube stationary boilers,

$$D = 0.0068H/LP \tag{3}$$

where a nominal evaporation of 3½ lb. of water per sq. ft. of heating surface and a maximum overload of 100 per cent. (giving 7 lb. per sq. ft. of heating surface) are assumed.

For water-tube, marine and Scotch marine boilers,

$$D = 0.0095H/LP \tag{4}$$

where a maximum evaporation of 10 lb. of water per sq. ft. of heating surface is assumed.

For locomotive boilers (the constant being determined by actual test),

$$D = 0.0055H/LP \tag{5}$$

The constants given in the above formulæ are for 45-deg. angle seats. For flat seats these values become: (1) 149; (2) 0.0067; (3) 0.0065; (4) 0.090; (5) 0.052. Formulæ (1) and (2) are the most accurate, in that the total maximum evaporation is taken as a definite quantity, without any assumptions.

Installation and Care of Safety Valves. All safety valves should be connected directly to the boiler with a close nipple or a short steam nozzle of the full valve size, or larger. In attaching the flange, the bolts should be drawn up evenly, as distortion of the valve seat may otherwise occur. In making a hydraulic test on a boiler the valve should be properly gagged, and not made to blow at a higher pressure by screwing down the spring. Safety-valve springs are designed for a definite pressure, which is usually stamped on a tag fastened to the valve. If a valve is to be set at a pressure differing from the designed pressure by more than 5 or 10 lb. (depending upon the make of the valve), either above or below, a new spring should be furnished. Where safety valves are used with superheated steam, those of the outside-spring type should be used in order to protect the metal of the spring from the high temperatures. In operation, all valves should be made to blow periodically to avoid the danger of sticking.

CHIMNEYS AND DRAFT

The factors involved in designing a stack are so numerous that as yet no formulæ have been advanced that will take all of them satisfactorily into account. The existing formulæ are purely empirical.

The intensity of the draft, usually measured in inches of water, is equal to the difference in weight between the column of heated gas within a chimney and the weight of a similar column of external air. This difference in weight for a given temperature of gas and air will vary directly with the stack height, so that the height of a stack is the governing factor in the intensity or amount of draft to be expected. The draft must be sufficient to overcome the friction of the air in passing through the fuel bed, of the gases in passing through the boiler proper, and through the flues connecting the boiler to the stack.

The theoretical force or **intensity of the draft** obtainable from any stack is given by the formula:

$$D = 0.52\,PH[(1/T) - (1/T_1)],$$

in which D = draft produced, in in. of water; H = height of stack above grates, ft.; P = atmospheric pressure, lb. per sq. in.; T = atmospheric temperature, absolute, deg. fahr.; and T_1 = temperature of stack gases, absolute, deg. fahr.

For various stack temperatures, assuming atmospheric pressure and temperature as 14.7 lb. and 60 deg., respectively, the formula reduces to $D = KH$, in which the values of K for various stack temperatures are as follows:

Temp. of stack gases, deg. fahr.	750	700	650	600	550	500	450	400	350
Value of K in formula.....	0.0084	0.0081	0.0078	0.0075	0.0071	0.0067	0.0063	0.0058	0.0053

This draft is the theoretical draft and cannot be obtained in practice. The theoretical draft which a stack will give, less the loss in overcoming frictional and inertia resistance, will be the draft available at the stack base or at the point where a flue enters, provided the stack height be measured above that point. The **draft loss due to friction and inertia** is

$$d = fW^2CH/A^3,$$

where d = draft loss, in. of water; W = weight of gases passing per sec., lb.; C = circumference of stack, ft.; A = area of passage, sq. ft.; H = height of stack, ft.; and f is a constant having the following values: 0.0015 for steel stacks with gases at 600 deg. fahr. (0.0011 at 350 deg.), and 0.0020 for brick or brick-lined stacks with gases at 600 deg. (0.0015 at 350 deg.).

Available Draft. The available draft, d_1, for any stack is therefore $d_1 = KH - fW^2CH/A^3$.

Table 24, calculated from this formula, gives the available draft at the base of stacks of various sizes to which different horse powers of boilers are connected.

Of the empirical formulæ advanced for proper height and diameter of stack for various horse powers of boilers, one largely used by engineers is that of Wm. Kent (see *Trans. A. S. M. E.*, vol. 6), namely,

$$\text{H.P.} = 3.33\,(A - 0.6\,A^{1/2})H^{1/2},$$

where H.P. = horse power based on a coal consumption of 5 lb. per rated h.p. per hour; A = actual area of chimney, sq. ft.; and H = height of chimney, ft.

$(A - 0.6A^{1/2})$ = effective area of stack, based on the assumption that the retarding effect by friction of the ascending gases may be considered as equivalent to the diminution of the area of the stack equal to a layer of gas 2 in. thick about the perimeter.

The height must first be determined from the known or assumed resis ances. Table 25 gives the sizes of stacks for various horse powers of boilers based on this formula.

For approximate work Table 25 gives satisfactory results, but no stack table will satisfactorily serve all cases, there being too many varying factors which enter into each individual case.

Table 24. Available Draft for 100-ft. Steel Stacks of Different Diameters

(Based on a stack temperature of 500 deg. fahr.* and 100 lb. of gas per h.p. For other heights of stack, multiply draft by height ÷ 100)

Horse power	Diam. of stack in in.														
	36	42	48	54	60	66	72	78	84	96	108	114	120	132	144
200	.55	.62	....	....	....	....	....	....	....	....	....	....	....	....	....
400	.21	.46	.56	.61	....	....	....	....	....	....	....	....	....	....	....
600	....	.19	.42	.53	.59	....	....	....	....	....	....	....	....	....	....
800	....	....	.23	.43	.52	.58	.61	.63	....	....	....	....	....	....	....
1000	....	....	....	.29	.45	.53	.58	.61	.63	....	....	....	....	....	....
1200	....	....	....	....	.35	.47	.54	.58	.61	.64	....	....	....	....	....
1600	....	....	....	....	....	.31	.43	.52	.56	.62	.64	.65	.65	....	....
2000	....	....	....	....	....	....	....	.43	.50	.59	.62	.63	.64	....	....
2500	....	....	....	....	....	....	....	....	.41	.54	.60	.61	.63	....	....
3000	....	....	....	....	....	....	....	....	....	.48	.56	.59	.61	.63	.64
3500	....	....	....	....	....	....	....	....	....	.40	.52	.56	.58	.62	.64
4000	....	....	....	....	....	....	....	....	....	....	.48	.52	.56	.60	.62
4500	....	....	....	....	....	....	....	....	....	....	.43	.49	.53	.58	.61
5000	....	....	....	....	....	....	....	....	....	....	....	.44	.49	.56	.60

* For other stack temperatures add or deduct before multiplying by height ÷ 100, as follows:

For 750 deg. fahr., add 0.17 in.	For 550 deg. fahr., add 0.04 in.
For 700 deg. fahr., add 0.14 in.	For 450 deg. fahr., deduct 0.04 in.
For 650 deg. fahr., add 0.11 in.	For 400 deg. fahr., deduct 0.09 in.
For 600 deg. fahr., add 0.08 in.	For 350 deg. fahr., deduct 0.14 in.

Table 25. Stack Sizes by Kent's Formula

(Assuming 5 lb. of coal per h.p. per hour)

Diam., in.	Area, sq. ft.	Height of stack in ft. / Commercial h.p.										Side of equivalent square stack, in.	Diam., in.
		50	60	80	100	125	150	175	200	225	250		
33	5.94	106	115	133	149	....	....	....	....	....	....	30	33
36	7.07	129	141	163	182	....	....	....	....	....	....	32	36
39	8.30	155	169	196	219	245	....	....	....	....	....	35	39
42	9.62	183	200	231	258	289	316	....	....	....	....	38	42
48	12.57	246	269	311	348	389	426	460	....	....	....	43	48
54	15.90	318	348	402	449	503	551	595	....	....	....	48	54
60	19.64	400	437	505	565	632	692	748	....	....	....	54	60
66	23.76	490	537	620	694	776	849	918	....	....	....	59	66
72	28.27	591	646	747	835	934	1023	1105	....	....	....	64	72
78	33.18	700	766	885	990	1107	1212	1310	....	....	....	70	78
84	38.48	818	896	1035	1157	1294	1418	1531	....	....	....	75	84
90	44.18	....	....	....	1338	1496	1639	1770	1893	2008	2116	80	90
96	50.27	....	....	....	1532	1713	1876	2027	2167	2298	2423	86	96
102	56.75	....	....	....	1739	1944	2130	2300	2459	2609	2750	91	102
108	63.62	....	....	....	1959	2190	2392	2592	2770	2939	3098	98	108
114	70.88	....	....	....	2192	2451	2685	2900	3100	3288	3466	101	114
120	78.54	....	....	....	2438	2726	2986	3226	3448	3657	3855	107	120
126	86.59	....	....	....	2697	3016	3303	3568	3814	4046	4265	112	126
132	95.03	....	....	....	2970	3321	3637	3929	4200	4455	4696	117	132
144	113.10	....	....	....	3554	3973	4352	4701	5026	5331	5618	128	144
156	132.73	....	....	....	4190	4684	5131	5542	5925	6285	6624	138	156
168	153.94	....	....	....	4878	5454	5974	6454	6899	7318	7713	158	168

A simpler formula, sufficiently accurate for approximate purposes in considering stacks for above 1000 h.p., as given by Wm. Kent, is, $H.P. = 2.5D^2H^{1/2}$.

A convenient and satisfactory method of determining the proper diameter of large stacks, 200 ft. and over in height, is to allow 30 sq. ft. of cross-sectional area for each 1000 nominal rated h.p. of boilers which the stack is to serve.

From the first formula and others of like nature, it will be seen that, by varying the height and diameter, different sizes of stacks will give the requisite theoretical draft for a given h.p. Among such various sizes there must be one cheaper than the others to build, and it appears from a study of stack costs that the minimum-cost stack has a diameter directly dependent upon the h.p. of the boilers it is to serve and a height proportional to the available draft required. Expressed as a formula, this diameter for unlined steel stacks is, diam. in in. $= 4.68\ (\mathrm{H.P.})^{2/5}$. This is based on the assumption of 120 lb. of flue gas per hour per boiler h.p., which figure is sufficiently high to take care of reasonable overloads and the use of poor coal. The diameter of stacks at sea level and the rated h.p. which they will serve by this formula are as follows:

Horse power of stack........	100	200	300	400	600	800	1000	1500	2000	3000	4000
Diameter of stack, in........	29	38.5	45	50.5	59	67	73.5	87.2	98	115	129

Draft Loss in Furnace. The draft loss in the furnace or through the fuel bed will vary widely with various grades of coal and different combustion rates. No fixed rules can be given for the draft required to burn certain quantities of different coals. Table 12 gives the furnace draft required at various combustion rates, allowing a fairly safe margin.

Draft Loss through Boiler. The draft resistance offered by the boiler heating surface will vary largely with the design of the boiler and the percentage of its capacity at which it is being run. This loss in a well-designed water-tube boiler will be about 0.25 in. at rating; 0.4 in. at 150 per cent. of rating; and 0.65 in. at 200 per cent. of rating.

Draft Loss in Flues. The draft loss in straight flues may be determined by the formula given for draft loss in stacks, in which case C is the circumference of flue in ft. when the flues are round, (if square or rectangular flues are used, the retarding effect will be approximately 12 per cent. greater than that of a circular flue of the same area), and H is the length of flue in ft. The draft loss due to right-angle turns in flues may be taken care of by an allowance of 0.05 in. loss for each such turn; the turns from a boiler into a flue and from a flue upward into the stack should be included in the calculation. Cross-sectional areas of flues should be ample, and a design allowing 35 sq. ft. per 1000 rated boiler h.p. is to be recommended.

Correction of Stack Sizes for Altitude. To develop a given boiler h.p. requires a constant weight of air for combustion. As the altitude is increased, the density is decreased, and the gas velocity must be increased. The mean velocity for a given boiler h.p. and constant weight of gas will be inversely proportional to the barometric pressure; and the velocity head, measured in column of external air, will be inversely proportional to the square of the barometric pressure. The added frictional loss due to added gas velocity and added height is compensated by an increase in diameter, such increase being proportional to the two-fifths power of the ratio of barometric pressures. Table 26 indicates the necessary changes in stack dimensions for varying altitudes above sea level.

Stack Sizes for Oil Fuel. The requirements for stacks for oil fuels are entirely different from those connected to coal-fired boilers, as there are certain losses through the boiler proper that are eliminated. Gas tempera-

Table 26. Stack Capacities. Correction Factors for Altitudes

Altitude, height in ft. above sea level	Normal barometer, in.	R Ratio barometer reading sea level to altitude	R^2 Ratio increase in stack height	$R^{2/5}$ Ratio increase in stack diam.
0	30.00	1.000	1.000	1.000
1,000	28.88	1.039	1.079	1.015
2,000	27.80	1.079	1.164	1.030
3,000	26.76	1.121	1.257	1.047
4,000	25.76	1.165	1.356	1.063
5,000	24.79	1.210	1.464	1.079
6,000	23.87	1.257	1.580	1.096
7,000	22.97	1.306	1.706	1.113
8,000	22.11	1.357	1.841	1.130
9,000	21.28	1.410	1.988	1.147
10,000	20.49	1.464	2.144	1.165

tures entering the stacks are lower for a given capacity, and the volume of gas is less than with coal. The cross-sectional area of the stack may therefore be less. In most instances this may safely be taken as 60 per cent. of the area that would be furnished for coal-fired boilers. In determining the height, care must be taken to design a stack that will give a sufficient draft for the maximum requirement of the boiler, but not more. In this way the efficiency is safeguarded, and the danger of a draft such as to pull through too large an excess of air is eliminated.

Table 27, adapted from one calculated by C. R. Weymouth (*Trans. A. S. M. E.*, vol. 34, 1912) and based on actual test data, will be found to give satisfactory results under the assumed conditions as given for stacks with oil-burning boilers. For stack sizes with blast-furnace gas, see p. 890.

Table 27. Stack Sizes for Oil Fuel

Stack diam., in.	Height in ft. above boiler-room floor					
	80	90	100	120	140	160
33	161	206	233	270	306	315
36	208	253	295	331	363	387
39	251	303	343	399	488	467
42	295	359	403	474	521	557
48	399	486	551	645	713	760
54	519	634	720	847	933	1000
60	657	800	913	1073	1193	1280
66	813	993	1133	1333	1480	1593
72	980	1206	1373	1620	1807	1940
84	1373	1587	1933	2293	2560	2767
96	1833	2260	2587	3087	3453	3740
108	2367	2920	3347	4000	4483	4867
120	3060	3660	4207	5040	5660	6160

Figures represent nominal rated h.p.; sizes as given are good for 50 per cent. overloads. Based on centrally located stacks, short direct flues and ordinary operating efficiencies.

Stack Design and Construction

There are four methods of building stacks: (1) Masonry construction, in which the stability of the stack is dependent upon the dead weight; (2) reinforced-concrete construction, in which the tension of the stack is taken care of by the steel reinforcing members which also assist in securing stability;

(3) self-supporting steel construction, anchored to a foundation; (4) guyed steel stacks, which are carried on foundations or structural supports and depend largely for their stability upon guy wires. There are certain instances in which any of these methods may be the best construction to follow, though the reinforced-concrete construction may almost be considered in an experimental state. Steel stacks are not as long-lived as masonry stacks, but no definite figure can be given on their life, as it depends upon local conditions, the thickness of the material used, and upon the care given in the way of painting and cleaning. When properly constructed and cared for, a steel stack should last at least 15 to 20 years.

Masonry Stacks are ordinarily built of round, octagonal or square section. Square stacks are ordinarily limited to short heights. Octagonal stacks require special brick for the angles in alternating courses to produce the best work. Circular stacks are the most efficient for their area and take the least material.

Circular stacks are usually built of specially formed brick, known as radia brick. While common brick are sometimes used, it is not advisable when the diameter becomes less than 5 ft.

In designing stacks the wind pressure to be allowed for is frequently established by building ordinances. The ordinance of the city of Chicago reads: "All chimneys shall be so designed as to safely resist a wind pressure of 30 lb. per sq. ft. of surface exposed to the action of the wind." Such a value includes whatever suction there may be on the leeward side of the chimney. In ordinary calculations, 100 miles per hour is used as the maximum velocity of the wind. This is equivalent to 50 lb. pressure per sq. ft. on a flat surface, or, as is ordinarily assumed in the case of a circular stack, 25 lb. per sq. ft. of projected area.

Experiments made by Professor Kernot on the Forth Bridge showed that the average pressure on large surfaces does not exceed about two-thirds that upon small surfaces of 1 or 2 sq. ft. A pressure of 50 lb. per sq. ft. represents the maximum obtainable on a small surface with a wind velocity of 100 miles per hour, which would be classed as a severe hurricane. German practice is to ordinarily allow 33⅓ lb. per sq. ft. of projected area.

In the computation of stresses in a masonry stack, the stack is ordinarily considered as a hollow cantilever beam. This beam is acted upon by the force of wind and by gravity, the single or combined effect of which tends to (1) overthrow a part of the chimney, (2) crush the masonry at the leeward side of any section of the chimney, and (3) produce too great pressure on the soil under the leeward side of the foundation.

Stack Foundations. Table 28, for self-supporting stacks, gives foundation sizes based on 160 lb. per sq. ft. on the earth due to weight and 160 lb. due to wind, or 320 lb. per sq. ft. per ft. of depth of foundation.

In computing wind pressures (*Eng. Rec.*, Nov. 28, 1908), wind moments, section moduli and bearing pressures, a stack is divided into sections from the top downward, each section considered separately at its lower edge and computations made above such edge. The wind pressure on each section is determined by multiplying the projected area of the section by the unit wind pressure. In octagonal stacks, the wind-resisting area is ordinarily assumed as 70 per cent. of actual area.

Stack specifications ordinarily set the maximum wind pressure to be figured upon and the allowable compression or tension of the masonry in any portion of the stack. A safe figure to be used as a maximum bearing pressure due to both static load and wind pressure is 20 tons per sq. ft. of any section.

Table 28. Data for the Construction of Self-supporting Steel Stacks

Diam.	Height	Horse power	Concrete foundation (square)			Foundation bolts		Bottom section, including flare		2nd section		3rd section		4th section		5th section		6th section		Flare, straight conical	
			Side	Height	Earth pressure	Number	Diam.	Height	Material	Height	Material	Height	Material	Height	Material	Height	Material	Height	Material	Diam. base	Height
in.	ft.		ft.	ft.	lb.		in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft.	in.	ft. in.	ft.
48	80	311	15.0	5.0	1600	12	1⅜	25	5/16	15	¼	40	3/16	....	....	....	....	....	....	4–7	10
54	100	449	17.2	5.7	1840	16	1⅜	40	5/16	15	¼	45	5/16	....	....	....	....	....	....	6–0	25
60	125	632	19.8	6.6	2110	14	1⅞	45	⅜	15	5/16	20	¼	45	3/16	....	....	....	....	6–7	30
66	125	776	20.3	6.8	2180	22	1⅜	55	5/16	20	¼	50	3/16	....	....	....	....	....	....	8–1	40
72	150	1023	22.8	7.6	2430	18	1⅞	65	⅜	15	5/16	20	¼	50	3/16	....	....	....	....	9–0	50
78	150	1212	23.3	7.8	2500	18	1⅞	60	⅜	15	5/16	20	¼	55	3/16	....	....	....	....	9–4	45
78	175	1310	25.1	8.4	2690	20	2⅛	70	7/16	15	⅜	15	5/16	20	¼	55	3/16	....	....	9–6	55
84	175	1531	25.6	8.5	2720	20	2⅛	65	7/16	15	⅜	20	5/16	20	¼	55	3/16	....	....	9–10	50
90	175	1770	26.0	8.7	2780	20	2⅛	60	7/16	15	⅜	20	5/16	20	¼	60	3/16	....	....	10–1	45
90	200	1893	27.8	9.3	2960	20	2½	70	½	15	7/16	15	⅜	20	5/16	20	¼	60	3/16	10–4	55
96	200	2167	28.3	9.4	3010	20	2½	65	½	15	7/16	20	⅜	20	5/16	20	¼	60	3/16	10–8	50
96	225	2298	30.0	10.0	3200	20	2¾	75	9/16	15	½	25	7/16	20	⅜	20	5/16	20	¼	10–11	60
108	225	2939	30.9	10.3	3240	24	2½	80	½	20	7/16	10	⅜	20	5/16	25	¼	60	3/16	12–3	60
120	225	3657	31.8	10.6	3300	24	2⅛	75	½	20	7/16	20	⅜	20	5/16	25	¼	65	3/16	13–3	55
120	250	3855	33.4	11.1	3550	26	2¾	80	9/16	20	½	20	7/16	20	⅜	20	5/16	25	¼	13–2	60
132	225	4455	32.5	10.8	3460	32	2⅛	85	7/16	20	⅜	25	5/16	95	¼	....	....	....	....	15–5	65
132	250	4696	34.2	11.4	3640	28	2½	90	½	20	7/16	20	⅜	25	5/16	95	¼	....	....	15–3	70
144	250	5618	34.9	11.6	3710	30	2⅛	85	½	20	7/16	20	⅜	25	5/16	100	¼	....	....	16–3	65
144	275	5890	35.8	11.9	3800	28	2¾	90	9/16	20	½	20	7/16	20	⅜	25	5/16	100	¼	16–1	70

Self-supporting Steel Stacks. The following method of calculating stresses in self-supporting steel stacks has given successful results.

The pressure exerted by the wind is taken at 25 lb. per sq. ft. of projected area. The strains produced in the stack are such that there would be no danger of failure even should there be considerable corrosion so as to reduce the plate thickness, say, 1/16 in.

The stacks have a straight conical flare at the base, the apex of the cone being at the top of the stack and the height of the flared part made approximately one-fourth the height of the stack. This makes the diameter at the base about one-third larger than that of the straight part of the stack. The flared part is built of plates of a uniform thickness. This construction provides a larger diameter for admitting the flue opening to the side of the stack and leads to a more easy flow of the gases than in a cylindrical stack. Again, less reinforcement is necessary where the breeching joins the stack with the flared-base construction than would be necessary with a parallel stack.

The maximum stress S per lineal inch of circumference due to the wind pressure is determined by considering the stack to act as a beam, and represents the stress which acts on the part of the circumference most remote from the neutral axis; this stress will be $S = M/A$, where M = wind moment in in.-lb., and A = cross-sectional area of stack in sq. in.

For the parallel part of the stack, with a wind pressure of 25 lb. per sq. ft. of projected area, this reduces to $S = 150\ H^2D/A$, where H = height in ft. from section considered to top of stack; D = diam. of stack in ft. and A = cross-sectional area of stack in sq. in.

The maximum stress due to the wind pressure per lineal inch is determined at each of the joints.

Table 29 gives the basis of calculation used by The Babcock & Wilcox Co. in designing self-supporting steel stacks, and indicates the maximum allowable stress in plates of varying thicknesses and rivet data. The figures represent stresses due to a wind pressure of 25 lb. per sq. ft. of projected area without regard to the weight of stack, which will add to compression side and deduct from tension side from 5 to 12 per cent.

Table 28 gives foundation data, plate thicknesses, height of sections, etc., for a number of self-supporting steel stacks calculated on the basis of Table 29.

Table 29. Basis of Calculation of Self-supporting Steel Stacks

(The Babcock & Wilcox Co.)

Pull per in. of circumference, lb.	Plate thickness, in.	Diam. of rivets, in.	Pitch of rivets, in.	Stress at rivets, lb. per sq. in.			
				Bearing	Shear	Tension	Tension in body of plate
600	3/16	3/8	2	15,880	9,270	4,020	3,200
1,100	1/4	1/2	2	16,600	9,950	5,980	4,400
1,700	5/16	5/8	2	16,600	10,000	8,080	5,440
2,360	3/8	3/4	(2 rows) 4	16,100	9,860	7,830	6,295
3,100	7/16	3/4	(2 rows) 3	13,600	9,750	9,600	7,090
3,930	1/2	7/8	(2 rows) 3	13,000	9,150	11,400	7,690
4,800	9/16	1	(2 rows) 3½	14,480	10,050	12,100	8,545

The stacks of Table 28 are unlined. If a lining is to be installed, special calculations should be made for bearing and for shear of rivets, which should be kept within the limits of Table 29.

Guyed Steel Stacks. Guyed steel stacks are placed on a foundation on the ground, or are carried on a structural framework that may be a part of the boiler support. The material used in their construction is ordinarily lighter than that for self-supporting stacks. It need only be sufficiently heavy to allow a margin for corrosion, to support its own weight, and to give strength sufficient to prevent buckling under a wind pressure of 25 lb. per sq. ft. of projected area. The tendency of wind and weight to overthrow the stack is taken care of by the guy wires. Such guys are ordinarily furnished in two sets of three each, attached to guy bands at two points in the height of the stack, the guys being at an angle of approximately 60 deg. with the vertical.

A satisfactory basis of calculation for loads on base, stress in guys due to wind pressure and initial stress, for stacks guyed as described, is as follows:

$$\text{Total wind pressure} = 25DL;$$
$$\text{Overturning moment} = 12.5DL^2.$$

where D = diam. of stack, ft., and L = height of stack, ft.

Horizontal pull on each guy = overturning moment divided by the sum of the heights of the guy bands = $12.5DL^2/(h_1+h_2)$, where h_1 and h_2 are heights in ft. of first and second guy bands from ground.

Direct pull on each guy due to wind = horizontal pull × cosecant 60 deg.

Initial stress on each guy = ½ × direct pull due to wind.

Therefore,

$$\left.\begin{array}{l}\text{Maximum stress in}\\ \text{guys due to wind}\\ \text{and initial stress}\end{array}\right\} = \frac{21.65\,DL^2}{h_1+h_2} \qquad \left.\begin{array}{l}\text{Vertical load due}\\ \text{to maximum stress}\\ \text{in guys}\end{array}\right\} = \frac{50.53\,L^2D}{h_1+h_2}$$

$$\left.\begin{array}{l}\text{Vertical load due}\\ \text{to weight of stack}\end{array}\right\} = W \qquad \left.\begin{array}{l}\text{Total vertical load}\\ \text{on base}\end{array}\right\} = \frac{50.53\,L^2D}{h_1+h_2} + W$$

Where a number of guyed stacks are placed in one continuous row, instead of two sets of three guy wires being furnished, a lattice bracing is frequently used between stacks, each stack having in addition two sets of two guy wires each, and the end stacks having two sets of three each.

Table 30 gives the approximate weight per foot of guyed steel stacks of varying thicknesses. To these figures there should be added 10 per cent. for guy ropes, bands, clips, etc.

Table 30. Approximate Weight of Guyed Stacks per Foot of Height

(Including laps, rivets, and manufacturer's maximum allowance for overweight)

Stack diam., in.	Material				
	No. 12 B. W. G.	No. 10 B. W. G.	No. 8 B. W. G.	3/16 in.	¼ in.
30	41.2 lb.	50.6 lb.	....	....	....
33	45.2	55.5	....	....	....
36	49.3	60.6	74.5 lb.	....	....
39	53.2	65.4	80.5	91.4 lb.	....
42	57.1	70.4	85.4	97.0	129.3 lb.
48	65.2	80.2	97.2	111.1	150.0
54	....	91.1	110.9	124.6	168.2
60	....	101.0	122.7	139.9	183.8
66	....	....	134.8	153.8	202.3
72	....	....	146.7	167.2	219.7

All steel stacks should be supplied with a top band, painter's trolley and suitable clean-out doors. A stack ladder is frequently advisable.

Table 31 gives the approximate comparative costs of guyed steel stacks, self-supporting steel stacks, brick stacks and concrete stacks.

Table 31. Comparative Approximate Costs of Chimneys

Height, ft.	Diam., in.	Horse power	Brick, cost	Concrete, cost	Self-supporting steel		Guyed steel	
					Weight, lb.	Cost	Weight, lb.	Cost
100	42	258	$1,400	$1,300	....	....	8,250	$420
150	54	551	2,700	2,150	....	....	21,080	850
150	72	1,023	3,500	2,800	51,750	$2,200	31,450	1,230
175	84	1,531	4,300	3,500	76,250	3,250	53,230	2,075
200	96	2,167	5,600	4,500	108,100	4,600	....	....
200	120	3,448	7,200	5,800	117,000	4,975	....	....
225	132	4,455	8,700	7,000	155,980	6,650	....	....
250	144	5,618	10,000	8,100	206,800	8,800	....	....

All costs include foundation material and erection. Freight not included in steel stack costs. Costs of brick and concrete stacks will vary from 5 to 15 per cent. with local prices. Total cost of steel stacks will vary with freight charges.

Mechanical Draft

Mechanical draft may be divided into three classes: (1) That produced by a steam jet or jets; (2) that produced by suction on the stack side of a boiler, or induced draft; and (3) that produced by a pressure under the grates or forced draft.

Mechanical draft produced by steam jets is either forced, as when jets are introduced into the ash pit, or induced, as when jets are installed in the flue or stack. The advantage of draft produced by this means lies almost entirely in its cheapness of installation and low cost of repair. Its use in general is confined to small plants where the cost of a fan is not warranted, or in larger plants to help carry an occasional peak load. Its disadvantages are large steam consumption, the limitations of draft that may be produced by this means, and noise.

The steam consumption of simple steam jets, either under grates or in the stack, may run as high as 30 per cent. of the total amount of steam generated, a fair figure being from 5 to 10 per cent. For some systems of ash-pit steam blast the steam consumption may be as low as 2½ per cent. under test conditions. A good system, under ordinary operating conditions, will probably require from 4 to 6 per cent. of the total steam generated.

The maximum ash-pit blast that can be obtained with steam jets with a reasonable economy, is approximately 1 in. of water. Where steam jets are placed in the flue or stack, the draft suction ordinarily cannot be increased more than 0.75 in. over what the stack alone would give. If but a single jet is supplied, the increase in draft may be inappreciable in a stack of a moderate diameter, regardless of the amount of steam used. A ring of jets or two such rings should be used in a stack of medium or large diameter.

A jet or jets of steam improperly introduced into the ash pit may lead to an uneven distribution of the draft pressure beneath the grates and cause the blowing of holes in the fire. The combustion of many coals that have a tendency to fuse or mat on the grates may be materially aided by the introduction of steam beneath the grates, and a steam-jet blast so used serves a double purpose. In certain cases steam is so used without any intention of producing blast pressure.

Induced-draft Fans are introduced in the stack side of a boiler. The fan handles gases of a high temperature which are frequently dust-laden.

This severe service often results in excessive maintenance cost. In early constructions the bearings were exposed to the heated gases and frequently required cooling. In modern designs the bearings are usually out of the path of the gases.

The nature of the service of an induced-draft fan is necessarily more severe than that of a forced-draft fan, and as the temperature of the gases handled by an induced-draft fan is higher and the weight of the gases passing through the fan is greater than that of the air that would be handled by a forced-draft fan, it might seem that a forced-draft apparatus could be used to advantage in all cases. In most constructions; however, it is important that a draft suction be maintained within the boiler setting, and this may necessitate the use of an induced-draft fan. There is no danger of an uneven distribution of draft suction over the surface of the fire with an induced-draft fan, whereas, with forced draft, care must be taken that the air is introduced properly.

The steam consumption of an induced-draft fan, when in good order, will be approximately 2 to 4 per cent. of the total steam generated. In waste-heat work, induced-draft fans are the only type practicable.

Forced-draft Fans. With this type of apparatus the fan forces air drawn from the boiler room into a closed ash pit. The steam consumption of a forced-draft fan, when new, will be from 2 to 4 per cent. of the total steam generated.

In the installation of a forced-blast fan it is necessary to ensure that the stack is capable of maintaining a suction throughout all parts of the boiler setting under all conditions of operation. If a suction is not maintained, trouble will result with the brickwork of the setting, as the high-temperature gases will tend to pass out through cracks in the setting and will cause the brickwork to "soak up" heat, whereas with a draft suction the cold air from the outside percolates inward through the brickwork or through any cracks in the setting and reduces the temperature.

In installing a forced-blast apparatus the air should be introduced in such a manner as to give an even pressure under all portions of the grate. Such an even distribution is best accomplished by introducing the air through a blast box in the bridge wall extending approximately the full width of the furnace. It is necessary that the ash pit be air-tight.

It is generally desirable that an induced- or forced-draft apparatus be controlled by some automatic means. The automatic control may be (1) by the steam pressure, or (2), where it is desired to keep a constant suction in the furnace, by the draft suction.

Where the steam pressure is utilized to control the mechanical draft, a damper regulator is ordinarily employed to give the initial movement to an actuating lever with but a slight variation in steam pressure. In most instances the fan engine is not placed wholly under such automatic control, but a by-pass connection is provided in the steam-supply line to allow a partial hand regulation. The fan engine should never stop, as if it does it ceases to be automatic, and if the balanced throttling valve is retained in a position where a small volume of steam is passing through at a high velocity it will cut the valve and valve seat. The adjustment on the balanced throttling valve should be such that when the fan speed is minimum it will be in a closed position and be tight, and to start the fan at its minimum speed the steam required will go through the by-pass valve and pipe.

In any plant where a number of boilers are in operation, the main flue damper is ordinarily under some method of automatic control. Where this exists in a plant utilizing forced draft, there is danger that as the steam pressure increases and the main damper closes, the blast will cause the gases to come out into the fire room through the firing or inspection doors. Aside from the inconvenience to the operatives, a back pressure within the boiler setting is detrimental to the setting. Devices actuated by the pressure of the gases within the furnace may so regulate the damper that any suction desired may be obtained, and in this way eliminate the difficulties above outlined.

Where a stack of sufficient size may be used to give the required draft and combustion rate, natural draft is preferable to mechanical draft. Mechanical draft has advantage over natural draft in the ease with which it may be controlled, and in the ability to start the fire.

MANAGEMENT AND INSURANCE

Economical Boiler-plant Loads

The rating at which a boiler plant should be run depends solely upon the nature of the load it carries. The controlling factor in the cost of a plant, regardless of the nature of the load, is the capacity to carry the maximum peak load that may be thrown on the plant under any condition.

Loads may be grouped in three classes: (1) The approximately constant 24-hr. load; (2) the steady 10- or 12-hr. load, usually with a noon-day period of no load; (3) the 24-hr. variable load found in central-station practice. The economical load at which a boiler plant may be run will vary with these groups.

(1) **For a constant load, 24 hr. a day,** the economical load, or that at which, all things considered, a given amount of steam can be produced most cheaply, is ordinarily considerably over the boiler's rated capacity. What this overload will be is dependent very largely upon the cost of coal at the plant. The capital investment must be weighed against the coal saving through added thermal efficiency, and the labor account, which increases with the number of units installed, must be given proper consideration. The economical rating will ordinarily be between 25 and 50 per cent. above the rated capacity of the boilers.

Where this question arises in the case of a plant already installed the conditions may be different from those that will exist in a new plant. If there are enough boilers to operate at lower ratings the capital investment leads only to a fixed charge, and the gain in thermal efficiency due to low ratings will make it advisable to run at lower capacities than in a contemplated plant.

(2) **For a 10- or 12-hr. load,** either approximately constant or one in which a peak occurs, the economical load will ordinarily be found to be somewhat higher than in (1). This is due to the fact that the return on the invested capital will increase, with the capacity secured, to a point where the actual boiler efficiency decreases sufficiently to offset such added return. Here again the determining factors are the fuel and labor lost as against thermal efficiency.

Due to operating difficulties that may be encountered at high overloads, the rating to show the best return on invested capital will for a load of this nature be ordinarily found to be between 150 and 175 per cent. of the rated capacity of the boilers.

The necessity for "spares" (see below) may change the aspect from which the economical rating for a load of this nature is to be considered. Their presence leads to a fixed charge and it may be advisable to use such spare boilers as part of the regular equipment, except at such times as certain boilers are off for cleaning and repairs, and in this way secure an added thermal efficiency through decreased capacity of the plant as a whole. Under such conditions these boilers could only be considered as spares during such time as actual cleaning or repairs are being done. The feed water available is the governing factor in such a consideration.

(3) **For the 24-hr. variable load** the point of maximum plant economy is, to an extent greater than in the other groups, dependent upon individual plant conditions, and the methods of handling a load of this nature will vary widely with fuel, labor, flexibility of apparatus, type of boiler and stoker, etc. Under ordinary conditions in city central-station plants, where the maximum peaks occur but a few times a year, the installation should be such as to enable these peaks to be carried with the boilers operating at their maximum capacity, a margin being allowed for insurance and continuity of service. Provided these maximum peaks can be so carried, a large sacrifice in thermal efficiency is permissible.

Some methods of handling a load of this nature are:

(*a*) The carrying of whatever load is on the plant at any time on only such boilers as will furnish the requisite power when operated at from 150 to 200 per cent. of rated capacity. All boilers in operation are run at such ratings at all times, increasing loads being handled by cutting in banked boilers. This method of handling central-station loads is probably the most generally used in practice to-day and under many conditions the ratings described are the economical loads.

(*b*) Other conditions of operation make it advisable to carry the load on a definite number of boiler units, operating such units slightly below their rated capacity at light plant loads, and carrying the peaks on the same number of boilers operated at high capacities. In such a method there are no boilers on bank and the spares are such in all senses of the word.

(*c*) A third method is that of considering the plant as divided into two parts, one of which handles the constant load, and the other the fluctuating load. That portion of the plant carrying the constant load is designed to operate at a maximum thermal efficiency, this point being raised to such a maximum by the installation of any apparatus by which such result may be obtained. The portion carrying the variable load is operated by either method (*a*) or (*b*), with the efficiency as high as the nature of load and ability to carry peaks allow.

The foregoing paragraphs have dealt largely with central-station practice; they are, however, generally applicable to smaller plants.

Spare Boilers. Where the load is to be reasonably constant it is advisable to install at least two spare boilers if continuity of service is essential. This permits the taking off of one boiler for cleaning and repair, and still allows a spare boiler in the event of the blowing out of a tube or some other unforeseen occurrence. In small plants two spares are not warranted. A large plant is ordinarily laid out in sections or panels, and each section should have its spare boiler, or boilers, even though the sections be cross-connected.

Stand-by Boiler Losses. The amount of fuel required to carry a boiler on banked fires is approximately the same as that required to maintain full steam pressure. Some approximate figures of tests for this requirement are as follows:

A hand-fired boiler of 640 (rated) h.p., using W. Va. bituminous coal, required (on a 72-hr. test) 1 lb. of coal per sq. ft. of grate surface per hour, or approximately 0.2 lb. of coal per rated h.p. This was the best figure obtained in a number of trials, the average being from 1.25 to 1.50 lb. per sq. ft. of grate per hour, or 0.25 to 0.30 lb. per (rated) h.p. per hour. These figures would indicate that a 600-h.p. unit would require for banking over a 24-hr. period between 3000 and 4000 lb. of coal.

A hand-fired boiler, rated at 604 h.p., using No. 3 anthracite buckwheat, required approximately 0.33 lb. of coal per rated h.p. per hour, or 4800 lb. for 24 hr. A stoker-fired boiler rated at 600 h.p. required 8000 lb. of coal for a period of 24 hr. or 0.55 lb. per h.p.-hour. This figure is probably higher than will ordinarily be found necessary.

The lay-over losses as above noted are with reasonably tight ash pits and proper damper regulation.

The exact quantity of oil fuel required for maintaining full pressure is difficult to determine. Two per cent. of the fuel ordinarily used in generating a boiler's rated capacity will maintain the full steam pressure for an indefinite period. This 2 per cent., however, does not represent the total stand-by loss, for the reason that during lay-over periods the setting is cooled, and on taking a load again, the rate at which oil is burned will be, for a period after starting, considerably in excess of that rate when the setting has been brought up to its normal operating temperature.

Boiler Insurance and Inspection

(Charles S. Lake, in *The Locomotive*, Oct., 1910)

Insurance policies agree within certain limits to indemnify absolutely the assured's property, boilers, buildings, machinery or stock, and personal injuries for which the assured may be liable, provided such injuries and losses are caused directly by the explosion of the insured boiler. Boiler insurance policies do not cover fire, whether caused by an explosion or not.

Ordinarily, the insurance policy states what maximum working pressure the insured boiler may carry, though in some policies this is based upon inspectors' reports or certificates, this latter system allowing a change in working pressure during the life of a policy. An increase in the working pressure over that allowed makes a policy void in all instances.

Table 32. Summary of Defects Found in 352,674 Boiler Inspections Made in 1911

(Hartford Steam Boiler Inspection and Insurance Co.)

Nature of defect	Total	Dangerous	Nature of defect	Total	Dangerous
Cases of deposit of sediment	19,710	1,400	Cases of leakage around tubes	11,188	1,627
Cases of incrustation and scale	42,879	1,699	Tubes too light	1,901	521
Cases of internal grooving	2,756	305	Leakage at joints	5,417	373
Cases of internal corrosion	14,083	649	Water gages defective	3,447	773
Cases of external corrosion	9,755	898	Blow-offs defective	4,509	1,373
Defective braces and stays	2,485	545	Cases of deficiency of water	313	90
Settings defective	5,686	741	Safety valves overloaded	1,124	319
Furnace out of shape	7,191	397	Safety valves defective	1,225	329
Fracture of plates	3,479	440	Pressure gages defective	7,836	525
Burned plates	4,837	477	Boilers without pressure gages	532	71
Laminated plates	509	44	Unclassified defects	29	19
Cases of defective riveting	3,026	636			
Defective heads	1,349	234			
Cases of defective tubes	9,447	2,935	Total	164,713	17,410

The definition of an explosion, as covered by practically all policies, is "a violent separation of the metal or the component parts of the boiler," and the contract is usually extended to all water or steam pipes up to and including the first shut-off valve on each pipe. As a necessary provision of practically all policies, a statement is made to the effect that the explosion must be caused by steam pressure.

No company agrees in its policy contract to make inspection, though all policies provide that a company's inspectors be given reasonable opportunity to inspect insured boilers and appliances upon which their safety depends. All companies reserve the right to suspend insurance at any time.

The Hartford Steam Boiler Inspection & Insurance Co. in 1911 inspected 352,674 boilers in 180,842 visits of inspection, inspected 140,896 boilers both internally and externally, subjected 12,724 to hydrostatic pressure, and found 653 unsafe for continued insurance. The total number of defects discovered was 164,713, of which 17,410 were considered dangerous. Table 32 indicates the nature and proportion of the number of defects thus found.

Boiler-room Management

For any plant, a standard of operating efficiency should be set, and a knowledge of operating results should be secured through a continuous record of plant conditions.

Some of these records are: The amount of fuel burned; the calorific value; and the ash content of the coal used. The ash content compared with the actual weight of ash found acts as a check on the grate efficiency. The weight of water evaporated. A record of flue-gas analysis, which is important as a check on the furnace efficiency. The flue temperature, which serves as a check on stack losses, and in general indicates the state of cleanliness of the boiler heating surfaces. The feed temperature, which is an indication of the degree of utilisation of exhaust steam. Steam temperature, where superheated steam is used. Draft records, as an indication that proper draft conditions exist for the particular combustion rate necessary to correspond to the evaporation.

Recommendations Leading to Economy, Safety and Continuity of Service. When placing a new boiler in service, a quantity of soda ash should be placed within it, the boiler filled with water to its normal level and a slow fire started. After 12 hr. of slow simmering, the fire should be allowed to die out, the boiler cooled slowly and then opened and washed out thoroughly. Such a proceeding will **remove all oil and grease from the interior** and prevent the possibility of foaming and tube difficulties when the boiler is placed in service.

A **boiler** should not be **cut into** the **line** with other boilers until the pressure within it is approximately that in the steam main. The boiler stop valve should be opened very slowly until it is fully opened. The arrangement of piping should be such that there can be no possibility of water collecting in a pocket between the boiler and the main, from which it can be carried over into the steam line when a boiler is cut in.

The **water column** should be **blown down** thoroughly at least once on every shift and the height of water indicated by the glass checked by the gage-cocks. The bottom blow-offs should be kept tight. These should be opened at least once daily to blow from the mud drum any sediment that may have collected and to reduce the concentration.

In case of **low water,** resulting either from carelessness or from some unforeseen condition of operation, the essential object to be obtained is the **extinguishing of the fire** in the quickest possible manner. A method generally recommended is to cover the fire with wet ashes or fresh fuel. A boiler so treated should be cut out of line after such an occurrence and a thorough inspection made to ascertain what damage, if any, has been done before it is again placed in service.

The **efficiency and capacity depend** very largely **upon** the **cleanliness of the heating surfaces,** both externally and internally. The most efficient way of **blowing soot from** the **tubes** is by means of a steam lance, with which all parts of the surfaces are reached and swept clean. If permanent soot-blowing devices are fixed within the boiler setting, certain features must be watched to avoid trouble. If there is any leakage of water of condensation within the setting coming into contact with the boiler tubes, it will tend toward corrosion, or, if in contact with the heated brickwork, will cause rapid disintegration of the setting. If the steam jets are so placed that they impinge directly against the tubes, erosion may take place.

A large percentage of **tube losses is due** directly **to** the presence of **scale.** The **internal cleaning** can best be accomplished by means of an **air- or water-driven turbine,** the cutter heads of which may be changed to handle various thicknesses of scale. Where a water-driven turbine is used, it should be connected to a pump which will deliver at least 120 gal. per min. per cleaner at 150 lb. pressure. This pressure should never be less than 90 lb. if satisfactory results are desired. Where an air-driven turbine is used, the pressure should be at least 100 lb., though 150 lb. is preferable, and sufficient water should be introduced into the tube to keep the cutting head cool and assist in washing down the scale as it is chipped off.

Where scale has been allowed to accumulate to an excessive thickness, the work of removal is difficult and tedious. Where such a **heavy scale** is of sulphate formation, its removal may be assisted by filling the boiler with water to which there has been added a quantity of soda ash, a bucketful to each drum, starting a low fire and allowing the water to boil for 24 hr. with no pressure on the boiler. It should be cooled slowly, drained, and the turbine cleaner used immediately, as the scale will tend to harden rapidly under the action of the air.

Where **pitting** or **corrosion** is noted, the parts affected should be carefully cleaned and the interior of the drums should be painted with white zinc if the boiler is to remain idle. The cause of such action should be immediately ascertained and steps taken to apply the proper remedy. See p. 558.

Boilers should be taken out of service at regular intervals for **cleaning** and **repairs.** When this is done, the boiler should be cooled slowly, and, when possible, be allowed to stand for 24 hr. after the fire is drawn before opening. The cooling process should not be hurried by allowing cold air to rush through the setting, as this will invariably cause trouble with the brickwork. When a boiler is off for cleaning, a careful examination should be made of its condition, both external and internal, and all leaks of steam, water and air through the setting stopped. If water is allowed to come into contact with brickwork that is heated, rapid disintegration will take place. If water is allowed to come into contact with the metal of the boiler when out of service, there is a likelihood of corrosion.

If a boiler is to remain idle for some time, its deterioration may be much more rapid than when in service. If the period for which it is to be laid off is not to exceed 3 months, it may be filled with water while out of service. The boiler should first be cleaned thoroughly, internally and externally, all soot and ashes being removed from the exterior of the pressure parts and any accumulation of scale removed from the interior surfaces. It should then be filled with water, to which five or six pails of soda ash have been added, a slow fire started to drive the air from the boiler, the fire drawn and the boiler pumped full. In this condition it may be kept for some time without bad effects.

If the boiler is to be out of service for more than three months, it should be emptied, drained and thoroughly dried after being cleaned. A tray of quicklime should be placed in each drum, the boiler closed, the grates covered and a quantity of quicklime placed on top of the covering. Special care should be taken to prevent air, steam or water leaks into the boiler or on to the pressure parts, to obviate danger of corrosion. See also p. 558.

Cost of Boilers and Equipment. The cost of boilers and their equipment, in dollars per boiler h.p., for medium- and large-size plants varies roughly as follows: Boilers without setting, 8.00 to 11.00; superheaters about 3.00; stokers, 3.00 to 5.50; masonry setting, 2.00 to 3.50; flues, 0.75 to 1.50; stacks, 2.00 to 4.00; economisers, about 4.00; mechanical draft, about 3.00; feed pumps, 0.50 to 1.50; feed heaters, 0.40 to 1.00; piping and pipe covering, 6.00 to 10.00; coal chutes and ash hoppers, up to 1.25; miscellaneous, 0.50 to 1.00. The total costs average from \$30 to \$40 per boiler h.p.

THE STEAM ENGINE

BY

WILLIAM D. ENNIS

REFERENCES: Ewing, "The Steam Engine and Other Heat Engines," 1910; Ripper, "Steam Engine Theory and Practice;" Heck, "The Steam Engine and Turbine," 1911.

WORK AND DIMENSIONS OF THE STEAM ENGINE

Simple Engines Using Saturated Steam

Ideal Hyperbolic Diagram. Fig. 1 shows the **typical indicator diagram** *abcdef*. Nomenclature as to valve action is shown at the points *a*, *c*, *d* and *f*. Nomenclature as to behavior of steam is designated on the intervening lines. The **idealized diagram** on which design is based is *ghjkl*, which differs from the actual diagram in showing no clearance or compression, in extending between the throttle pressure *P* and atmospheric or condenser pressure *p*, in showing sharp corners at *h*, *j* and *k*, and in following the law, pv = constant, between *h* and *j*.

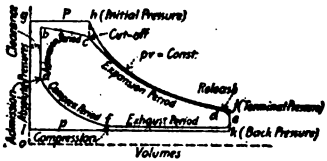


FIG. 1.—Typical Steam-engine Indicator Diagram.

Note the nomenclature adopted for pressures at *h*, *j* and *k*. The volume ratio $v_j \div v_k = R$ is called the **ratio of expansion.**

The average ordinate or **mean effective pressure** of the diagram *ghjkl* is, in lb. per sq. in.,

$$p_m = P\frac{(1 + \log_e R)}{R} - p, \tag{1}$$

in which *P* and *p* are taken in lb. per sq. in., absolute. Table 1 gives the ratio p_m/P for zero back pressure ($p = 0$) for various values of *R*.

Table 1. Ratio of Mean Effective Pressure to Initial Pressure for Various Ratios of Expansion (Zero Back Pressure)

Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)	Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)	Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)	Ratio of expansion, R	Ratio of mean to initial pressure (p_m/P)
1.67	0.907	3.33	0.662	6.67	0.435	18	0.216
1.82	0.874	3.64	0.631	7.00	0.421	20	0.200
2.00	0.847	4.00	0.597	8.00	0.385	22	0.186
2.22	0.810	4.44	0.561	10.00	0.350	24	0.174
2.50	0.766	5.00	0.522	12.00	0.290	26	0.164
2.67	0.744	5.71	0.481	14.00	0.260	28	0.155
2.86	0.717	6.00	0.465	16.00	0.236	30	0.147

Pressures and Expansion Ratios [for Equation (1)]. In ordinary practice, *P* ranges from 75 to 250. Common values for simple engines are from 95 to 115. For non-condensing engines exhausting to the atmosphere at sea level, *p* is between 15 and 17. For condensing engines, *p* is between 1 and 2½. The value 2 should be used in design (Ennis, "Vapors for Heat Engines," 1912, p. 38). If the exhaust steam from an engine is used for heating buildings or in manufacturing processes, *p* may have any value from 15 upward.

The value of *R* is commonly around 4 in simple engines. It should increase as *P* increases and as *p* decreases, being usually between 3 and 5. It should be higher in jacketed than in unjacketed engines. The efficiency of the engine depends largely on the value chosen for R (see pp. 954, 955; also see under "real" and "apparent" ratios of expansion, p. 941). High values of R will be adopted for an engine to be used where fuel is costly or the load steady. The overload capacity is similarly influenced: low values of R lead to high mean effective pressures—and hence to large output from a cylinder of given size—but also to low overload capacities. In general European practice cut-off is fixed so as to give $p_m = 1.2 + 0.2\,P$.

The **maximum overload capacity** (ratio of maximum excess load to the load for which the engine is designed) with unvarying pressure limits is

$$\frac{P - p}{\frac{P}{R} + \frac{P}{R}\log_e R - p} - 1 = \frac{P-p}{p_m} - 1 \qquad (2)$$

Upper Limits of Cut-off for Maximum Overall Efficiency and Least Steam Consumption (Hrabak)

Throttle pressure, lb. per sq. in. abs.	Non-condensing		Condensing		
	Simple		Simple		Compound
	Slide valve	Expansion valve	Unjacketed	Jacketed	
60	0.53–0.42	0.39–0.31	0.20–0.14	0.15–0.10	....
75	0.46–0.32	0.33–0.27	0.17–0.13	0.13–0.09	0.10–0.08
90	0.40–0.28	0.28–0.23	0.15–0.125	0.11–0.08	0.09–0.07
120	0.34–0.25	0.22–0.19	0.14–0.12	0.09–0.07	0.08–0.06
150	0.29–0.20	0.19–0.17	....	....	0.07–0.05

The higher values are for operation with maximum commercial efficiency; the lower values for minimum steam consumption. The higher values given are the upper limits (latest desirable cut-off) and apply with such conditions as small engines, cheap fuel and intermittent operation; with large engines, high-priced fuel and continuous operation, the cut-off for maximum commercial efficiency should be made earlier.

Mean Effective Pressures Realized. The values of p_m, obtained from (1), are multiplied by a **diagram factor**, f, always less than 1.0, to obtain the mean effective pressure likely to be realized in the actual engine. The value of f is always between 0.45 and 0.95 and usually between 0.80 and 0.95. Values given by Seaton are

Type of Engine	f
Independent cut-off, cylinder jacketed....................	0.90
Single valve, automatic cut-off, cylinder jacketed............	0.86 to 0.88
Single valve, automatic cut-off, without jacket...............	0.77 to 0.82
Unjacketed throttling engines of small size and high speed....	0.58 to 0.77

The following summarizes the principal influences which determine the value of f:

Influence	Effect on the Value of f
Governing by throttling as compared with cut-off regulation....................................	decrease 0.10 to 0.25
Jackets..	increase 0.05 to 0.15
Very early cut-off (prior to ⅛ stroke)............	decrease 0.025 to 0.125
High speed (above 225 r.p.m.).................	decrease 0.025 to 0.10
Excessive clearance (over 5 per cent.)...........	decrease up to 0.08
Abnormally small ports and passages..............	decrease 0.025 to 0.10
Interrelated valve movements (as with a single valve)....................................	decrease 0.025 to 0.175

Cylinder Dimensions.

$$\text{h.p.} = \frac{fp_mAS}{33{,}000} = \frac{fp_mALN}{16{,}500} = fp_mK \qquad (3)$$

$$A = \frac{33{,}000 \times \text{h.p.}}{fp_mS} = \frac{16{,}500 \times \text{h.p.}}{fp_mLN} \qquad (4)$$

in which

h.p. = indicated horse-power to be expected from the engine,
A = average effective area of the piston, sq. in.,
L = stroke of piston, ft.,
N = revolutions per minute (r.p.m.),
S = $2LN$ = piston speed, ft. per min.,
K = $ALN/16{,}500 = AS/33{,}000$, a constant for a given engine.

In an ordinary double-acting engine, A is the cross-sectional area of the cylinder *minus* half the cross-sectional area of the piston rod. When a tail rod is used, the deduction is the whole cross-sectional area of the rod.

Piston Speed, Revolutions per Minute [for Equation (4)]. Values of S range from 500 upward, with 800 as the usual limit excepting in very large engines. The higher the value of S the greater is the power realised from an engine of given size; or the less is the weight, size and cost of an engine of given power. With very high values of S the question of port size and arrangement becomes especially important. Maximum values of S usually accompany maximum values of L.

The value of N is chiefly limited by the type of valve and gear. It may be fixed by the mode of application of the power. Releasing-gear engines seldom run over 100 r.p.m. Rotary-valve engines without releasing gear may work up to N = 240. Ordinary slide- and piston-valve engines may run as fast as 350 r.p.m.; the Porter-Allen engine (see p. 976) runs up to 600 r.p.m.; poppet-valve engines to 225 r.p.m. for horizontal and 350 r.p.m. for vertical engines. Large engines with releasing gear almost always run between 80 and 100 r.p.m. Small engines generally employ maximum values of N for the type of valve gear used. At 24-inch stroke, the speed with any type of valve gear seldom exceeds 175 r.p.m.

Strokes of engines running around 100 r.p.m. vary from 1½ to 3 times the piston diameter, the ratio being less with larger-size engines. In engines running above 150 r.p.m., the stroke is short, usually about equal to the diameter. Pumping engines, which must run at low piston speeds, have long strokes, and even then the value of N is abnormally low. Long strokes favor low clearances.

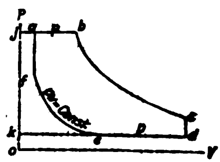


FIG. 2.—Modified Hyperbolic Diagram.

Modified Hyperbolic Diagram. In Fig. 2, the ideal diagram *abcdef* includes the influences of clearance and compression, and thus more closely approximates the indicator card than the diagram of Fig. 1. The mean effective pressure is

$$p_m = \frac{P}{R}(1 + c - cR) + \frac{P}{R}(1 + c)\log_e R - p[1 + c(1 - R_c)] - pR_c c \log_e R_c \qquad (5)$$

in which c = proportion of clearance = $v_a/(v_e - v_a)$, R_c = ratio of compression = v_a/v_f. When Equation (5) is used for p_m, corresponding values of f are usually between 0.90 and 1.0.

Clearance depends upon the size of the engine, its ratio of diameter to stroke, and the type and location of valves. The **clearance volume** includes all spaces between the piston and the valve faces, when the former is at the (adjacent) end of its stroke. The **linear clearance** (distance from piston to cylinder head) may account for only a small part of the total clearance volume. The clearance volumes at the two ends of the cylinder are usually unequal. Designers aim to **keep clearance low.** This is more important in simple engines than in compounds, and is most important with high ratios of expansion. In the low-pressure cylinders of compounds, clearances are relatively high, on account of the large ports.

Clearance data	Percentage of piston displacement
Extreme range	1 to 15
Range with	
Flat slide valves at the side of the cylinder	5 to 10
Piston valves " " " " " "	7 to 15
Corliss valves	2 to 8
Poppet valves, mean for 1000 h.p. (Lens)	4
Flat or piston valves in the cylinder heads	2 to 7
High-speed engine } comparative values	8 to 12
Slow-speed engine } comparative values	4 to 6

"Real" and "Apparent" Ratios of Expansion. The ratio $(v_e - v_a)/(v_b - v_a)$, Fig. 2, is the "apparent" ratio of expansion. Following the notation of Equation (5), its value is

$$R_a = R/(1 + c - Rc) \tag{6}$$

and

$$R = (R_a c + R_a)/(R_a c + 1) \tag{7}$$

Engines are usually designed by assuming R_a and c, and then finding R from Equation (7).

Table 2. Real Ratio of Expansion
[Values of R calculated from Equation (7)]

Apparent ratio, R_a	Fraction of stroke at which cut-off occurs	Clearance, per cent. (= 100 c)							
		1	2	3	4	5	6	8	10
10	0.10	9.18	8.50	7.92	7.43	7.00	6.63	6.00	5.50
8⅓	0.12	7.77	7.30	6.86	6.50	6.18	5.89	5.40	5.00
7 1/7	0.14	6.73	6.37	6.06	5.78	5.53	5.30	4.91	4.58
6¼	0.16	5.94	5.67	5.42	5.20	5.00	4.82	4.50	4.23
5	0.20	4.81	4.64	4.48	4.33	4.20	4.08	3.86	3.67
4	0.25	3.88	3.77	3.68	3.58	3.50	3.42	3.27	3.14
3⅓	0.30	3.26	3.19	3.12	3.06	3.00	2.94	2.84	2.75
2½	0.40	2.46	2.43	2.40	2.36	2.33	2.30	2.25	2.20
2	0.50	1.98	1.96	1.94	1.92	1.90	1.89	1.86	1.83
1⅔	0.60	1.66	1.65	1.64	1.63	1.62	1.61	1.59	1.57

Compression has for an **extreme limit** the condition $p_f = p_a$, Fig. 2. Values of R_c, Equation (5), reach a maximum at normal values of R, falling off at extreme values. The "apparent" compression ratio $(v_e - v_f)/(v_c - v_a)$ varies from 0.08 to 0.65 at 7 per cent. clearance. Where Equation (4) is used in design, compression is regarded only as influencing the value of f. These values are low for high-speed engines partly because such engines use excessive compression. They are low for single-valve engines because such engines have high compression at light loads (see p. 970). Practical limits of value of p_f, Fig. 2, may be taken at $0.7(P - p) + p$ for high-speed and $0.1\ (P - p) + p$ for low-speed engines.

Illustrative Design. Assume a simple condensing engine to develop 500 i.h.p. at 115 lb. initial absolute pressure, with an overload capacity of about 0.80, the engine to have Corliss valves and to be unjacketed.

First, assume a piston speed of 800 ft. per min. and 2 lb. back pressure, the engine being double-acting. Also 100 r.p.m. as an allowable rotative speed (p. 940).

From Equation (2), $0.80 = [(115 - 2)/p_m] - 1$, or $p_m = 62.9$. Also, $p_m/P = 62.9/115 = 0.547$. Table 1 shows that this ratio is obtained when R is slightly over 4.5. To be on the safe side, make $R = 4.5$. Then, from Equation (1) $p_m = [(115/4.5) \times (1 + \log_e 4.5)] - 2 = 62$.

From p. 939, the value of f for an unjacketed Corliss engine may be taken at 0.85. Applying Equation (4), $A = (33{,}000 \times 500)/(0.85 \times 62 \times 800) = 390$ sq. in. If the piston-rod diameter be assumed to be 1/10 the diameter of the piston, the latter is $\sqrt{(4 \times 390 \times 200)/199\pi} = 22.4$ in.

The stroke is $L = S/2N$ [nomenclature of Equation (4)] or $800/200 = 4$ ft. The engine is 22.4 by 48 in. Its piston-rod diameter must be not far from 2.24 in. Its apparent ratio of expansion with, say, 4 per cent. clearance (p. 941) is, from Equation (6), $4.5/[1 + 0.04 - (4.5 \times 0.04)] = 5.22$: or, cut-off will occur at $1/5.22 = 0.191$ of the stroke. The ratio of stroke to piston diameter is $48/22.4 = 2.15$ (see p. 940). The probable mean effective pressure in the actual engine is $0.85 \times 62 = 52.8$ lb. per sq. in. With p_f, Fig. 2, $= 15$, Equation (5) would have given an ideal mean effective pressure of 58.3 lb. The diagram factor which would just reduce this to the value here employed would be $52.8/58.3 = 0.905$ (see p. 939).

Simple Engines Using Superheated Steam

The ideal diagram of Fig. 2 represents the action of a simple engine using highly superheated steam, excepting that the expansion curve bc no longer approximates the hyperbolic. Its equation is, pv^n = constant, where n has a value depending on the amount of superheat and the ratio of expansion, as follows:

Ratio of Expansion	Superheat, deg. fahr. 200	250	300	350	400	450	500
	Values of exponent n						
8.0	1.05	1.07	1.09	1.11	1.13	1.15	1.18
7.0	1.06	1.08	1.10	1.12	1.14	1.16	1.19
6.0	1.07	1.09	1.11	1.14	1.16	1.18	1.21
5.0	1.08	1.10	1.12	1.15	1.17	1.19	1.22
4.0	1.09	1.11	1.13	1.16	1.18	1.20	1.23
3.0	1.10	1.12	1.15	1.17	1.20	1.22	1.25

With nomenclature as in Equations (5) and (6), the **mean effective pressure** is (8)

$$p_m = \frac{P}{R_a} + \frac{P}{n-1}\left(\frac{1 + cR_a}{R_a} - \frac{1+c}{R^n}\right) - p\left\{1 + c(1 - R_c)\right\} - cpR_c \log_e R_c$$

compression being assumed to be still hyperbolic. With $n > 1.0$ this equation gives values always less than those by Equation (5), as the latter gives values less than those by Equation (1). **Diagram factors** will be usually between 0.95 and 0.98, though throttling, excessive speed, large clearance, restricted ports and passages or interrelated valve movements may cause fluctuations as listed on p. 939. Jackets are not used. Ratios of expansion may be 40 per cent. higher than with saturated steam, without reduction of diagram factor. Piston speeds may be higher without causing undue port friction. Corliss valves are rarely if ever used with high superheat: the gridiron slide, poppet or piston valve may be employed.

Having determined fp_m, the **cylinder dimensions** are obtained from Equation (4).

With slight superheat, Equation (1) is used for determining p_m, and the value of f will be governed by the considerations of p. 939, superheating being here regarded as equivalent to jacketing.

Compound Engines

Preliminary Diagram. In Fig. 3 let the ideal diagram $habg$ correspond with $ghjkl$ of Fig. 1. The diagrams differ in that with the former the terminal pressure equals the back pressure. There is no "terminal drop" (jk, Fig. 1). In Fig. 3, the line bg represents the flow of steam, without fall of pressure, from the "high-pressure" cylinder to the "receiver." The latter in turn delivers steam to a "low-pressure" cylinder, the action of which is represented by the diagram $gbcef$. The two cylinders constitute a compound engine.

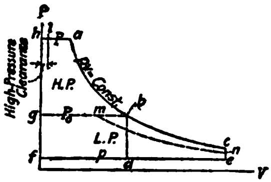


FIG. 3.—Ideal Diagram for Compound Engine.

To avoid noticeable fluctuations of pressure along gb, the receiver must be large, from ½ to 1½ times the size of the high-pressure cylinder; the former ratio being used for tandem engines, the latter for cross-compounds, in which the cranks are usually 90 deg. apart.

Maximum stresses are greater in tandem engines than in cross-compounds, and close regulation of speed is more difficult. On the other hand, the tandem costs less and occupies less space.

If this ideal diagram correctly represented the action, then a single simple cylinder of the same size as the low-pressure cylinder of the compound, working between the same extreme pressure limits and with the same ratio of expansion $R = v_c/v_a$, would give the diagram $hacef$. The simple cylinder would give the same power, then, as the compound engine.

Basis for Design. The diagram $hacef$ of Fig. 3 and the output of the whole engine are determined when P, p and R ($= v_c/v_a$) are given. The proportion of work done by each cylinder depends upon the receiver pressure, P_o. The designer may proceed in any one of five ways:

(1) The **receiver pressure** may be **assumed.**

(2) The **temperature ranges** may be made **the same** in both cylinders. This is probably the best basis with modern high initial pressures. Then $T_b = (T_a + T_e)/2$, where T_b, T_a and T_e are (for saturated steam) the temperatures corresponding respectively with the pressures P_o, P and p, from which T_b and P_o may be found.

(3) The **cylinders** may develop the **same power,** in which case

$$\log_e P_o = \tfrac{1}{2}\left(\log_e R + \frac{Rp}{P} - 1\right) - \log_e \frac{R}{P} \tag{9}$$

(4) The total maximum **piston pressures** may be **equalized,** when

$$P_o = P^2/(P + P_oR - pR) \tag{10}$$

(5) The **ratio of volumes** of the **cylinders** may be assumed, as $C = v_c/v_b$;

$$P_o = CP/R \tag{11}$$

Mean Effective Pressures. Having determined P_o by any one of these five methods, and having noted the corresponding value of $C = RP_o/P$, the **approximate mean effective pressures** are, based on Fig. 3,

high-pressure, $p_{mh} = [P/R_h] \log_e R_h = [PC/R] \log_e R_h$ (12)

low-pressure, $p_{ml} = [(P_o/R_l)(1 + \log_e R_l)] - p = [(P/R)(1 + \log_e C)] - p$ (13)

where R_h = ratio of expansion in high-pressure cylinder $= v_b/v_a$;

R_l = ratio of expansion in low-pressure cylinder $= v_c/v_b = C$;

other symbols being as in Equation (1).

Discussion of Factors. The value of C is usually from 3 to 4 in non-condensing and from 4 to 5 in condensing engines. It should vary directly with R. If it is made too great, the engine will, though probably economical of steam (p. 956), be costly to build and deficient in overload capacity; for the maximum power of the engine, working as a compound, is obtained when $R = C$, the low-pressure cylinder receiving steam at full boiler pressure, and the high-pressure doing no work. In usual practice, the low-pressure cylinder is not built to withstand full boiler pressure, but only a lower pressure P_1, which is realised either by raising the receiver pressure to P_1 or (in unusual emergencies) by running the two cylinders as two simple engines between the pressure limits P and p and P_1 and p, respectively. The maximum overload capacities for the two cases [defined as in Equation (2)] with the high-pressure cylinder receiving steam throughout the stroke are respectively

$$\frac{\dfrac{P_1}{C}\left(\dfrac{P}{P_1} + \log_e C\right) - p}{\dfrac{P}{R}(1 + \log_e R) - p} - 1, \qquad (14) \qquad\qquad \frac{\dfrac{P - p}{C} + P_1 - p}{\dfrac{P}{R}(1 + \log_e R) - p} - 1 \qquad (15)$$

In the latter case there is no cut-off in either cylinder.

At a fixed ratio of expansion R, the output of the engine is independent of the size of the high-pressure cylinder. Exceptionally high efficiencies have been obtained with values of C around 6 or 7 (see p. 956). When on account of having reached the limit of size (about 100 in. diameter), more than one low-pressure cylinder is used, the value of C is the ratio of the combined volumes of the low-pressure cylinders to that of the high-pressure cylinder. Following are proposed values of R for maximum thermal efficiency in compound condensing engines with initial pressure P, lb. per sq. in. abs. See also p. 945, under "Size of Cylinder."

P....	90	120	150	180	200	220	250	275
R....	10 to 13	11 to 14	13 to 17	16 to 21	19 to 23	22 to 27	25 to 32	28 to 36

P will range from 115 to 265, preferably not under 165 for condensing engines. Values of p, non-condensing, are from 15 to 17; condensing, 1 or 2, but preferably 1. Half the tabulated values of R may be used for compound non-condensing engines. Generally, in such engines, p_c is between 18 and 25 lb., increasing as P increases. The use of jackets warrants high values for R. High values are indicated when fuel is costly or the load is steady.

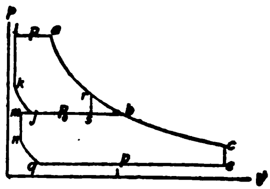


FIG. 4.—Diagram for Compound Engine with Clearance and Compression.

Definitive Diagram. Equations (12) and (13) may be employed in connection with Equation (4) for the immediate determination of **cylinder dimensions.** Some refinement is added by considering the diagrams *labjk*, *mbceqn*, Fig. 4, in which the effects of clearance and compression are included. With these diagrams,

$$p_{mh} = (Pc_h/R_h)[\log_e R_h - R_h + R_{hc}(1 - \log_e R_{hc})] + (P/R_h)\log_e R_h \qquad (16)$$

$$p_{ml} = (P/R)(1 + c_l - c_lR_l) + (P/R)(1 + c_l)\log_e R_l - p[1 + c_l(1 - R_{lc})] - pR_{lc}c_l\log_e R_{lc} \qquad (17)$$

in which $c_h = v_l/(v_b - v_l)$ = clearance of high-pressure cylinder;

$R_h = v_b/v_a = P/P_e$ = real ratio of expansion, high-pressure cylinder;

$R_{hc} = v_j/v_k = p_k/P_o$ = real ratio of compression, high-pressure cylinder;

$c_l = v_m/(v_c - v_m)$ = clearance of low-pressure cylinder;

$R_l = v_c/v_b = C = P_oR/P$ = real ratio of expansion, low-pressure cylinder;

$R_{lc} = v_q/v_n = p_n/p$ = real ratio of compression, low-pressure cylinder;

nomenclature being otherwise as in Equations (12) and (13). Also,

$p_h = 0.1(P - P_o) + P_o$ for slow speeds up to $0.4(P - P_o) + P_o$ for high speeds;

$p_n = 0.1(P_o - p) + p$ for slow speeds up to $0.3\ (P_o - p) + p$ for high speeds.

Clearances in compound engines are about the same, proportionately, as in simple engines. Usually $c_l > c_h$.

Diagram Factors for compound engines expanding R times are about the same as those for simple engines with the $\sqrt{R}$ ratio of expansion (see p. 939). High compression and excessive clearance reduce the value of f less than they do in simple engines. The value of f for Equations (16) and (17) may be taken at 0.92 to 1.0. Superheat less than 150 deg. fahr. may be regarded as equivalent to jackets. Excessive terminal drop in the high-pressure cylinder (p. 946) reduces the value of f. The use of a reheater between the cylinders is assumed; its omission may reduce the value of f by 0.05.

Size of Cylinder. Given f, p_{mh}, p_{ml},

$$\text{h.p.} = (LNA_lf/16{,}500)[(p_{mh}/C) + p_{ml}] \tag{18}$$

$$A_l = (\text{h.p.} \times 16{,}500)/LNf[(p_{mh}/C) + p_{ml}] \tag{19}$$

$$A_h = A_l/C$$

where A_h and A_l denote the net piston areas, other symbols being as in Equations (3) and (17). (The strokes of the pistons are usually equal, although other obvious methods of arrangement are possible.) The quantity $[(p_{mh}/C) + p_{ml}]$ is **the mean effective pressure referred to the low-pressure cylinder** ("equivalent mean effective pressure"). This important design constant is usually between 23 and 31 lb. in present power-station practice. In Europe, it is about $1.2 + 0.05\ P$ for best thermal efficiency, increasing to $1.2 + 0.12\ P$ for best commercial efficiency, in condensing engines. The outputs of the two cylinders will have the ratio

$$(\text{h.p.}_h/\text{h.p.}_l) = (p_{mh}/Cp_{ml}),\ \text{where h.p.}_h + \text{h.p.}_l = \text{h.p.} \tag{20}$$

Note also that $2LN = S$. Values for L, N and S are as given on p. 940. Common values of p_{ml} in stationary practice are between 8 and 15; with the usual division of work and cylinder ratio the corresponding values of $(p_{mh}/C + p_{ml})$ are from 24 to 32.

Illustrative Design. To outline a **500-i.h.p. compound engine,** condensing, with reheater, without jackets, piston speed 800 ft., r.p.m. 100, releasing gear, $P = 120$, $p = 2$, $R = 25$, the maximum pressure allowable in the low-pressure cylinder being 60 lb.

From Table 1 (interpolating) for $R = 25$, $p_m/P = 0.169$, or $p_m = 0.169 \times 120 = 20.2$ which, less back pressure p ($= 2$) gives the equivalent mean effective pressure as 18.2. Using $f = 0.9$, this gives in Equation (19),

$$A_l = (16{,}500 \times 500)/(4 \times 100 \times 0.9 \times 18.2) = 1260 \text{ sq. in.},$$

while A_h may have any value whatever. If temperature ranges are to be equalized, the steam table and p. 943 give $T_b = (T_a + T_c)/2 = (341 + 126)/2 = 233.5$ and $P_o = 22$, which would make $C = RP_o/P = (22 \times 25)/120 = 4.6$. If maximum piston pressures are equalized, $P_o = 120 \times 120/(120 + 25P_o - 50)$ and $P_o = 22.7$ [Equation (10)]. Again, if the two cylinders are to develop equal power, Equation (9) gives $\log_e P_o = [(2.3 \log 25 + 0.417 - 1)/2] - 2.3 \log 0.208$, whence $P_o = 17.8$, which would make $C = 3.7$. This is a reasonable value for the low initial pressure prescribed, and will be adopted. Following Equations (14) and (15), the overload capacities are (a) working compound, with increased receiver pressure and no high-pressure cut-off,

$$\frac{\frac{60}{3.7}\left(\frac{120}{60} + 2.3 \log 3.7\right) - 2}{\frac{120}{25}\left(1 + 2.3 \log 25\right) - 2} - 1 = 1.85;$$

or (b) with both cylinders running without expansion and exhausting to the atmosphere,

$$\left[\left(\frac{120-2}{3.7}+60-2\right)\Big/\left\{\frac{120}{25}\left(1+2.3\log 25\right)-2\right\}\right]-1=3.92.$$

Using Equations (16) and (17), and assuming $c_h = 0.025$, $c_l = 0.03$, p_b (Fig. 4) $= P_o$, $p_n = 0.1(P_o - p) + p = 3.6$,

$$p_{mh} = 32.3,\quad p_{ml} = 8.8,\quad R_{ho} = 1.0,\quad R_{lo} = 1.79,\quad (p_{mh}/C) + p_{ml} = 17.6,$$

the last result being of course somewhat below 18.2, found from Table 1. Using 18.2, with $f = 0.90$ [the corresponding value of f for Equations (16) and (17) would be $(18.2/17.6) \times 0.90 = 0.93$], the low-pressure piston area is 1260 sq. in., that of the high-pressure piston is $1260/3.7 = 340$ sq. in., the stroke is 48 in., and Equation (20) gives the check result that the cylinders develop equal amounts of power.

Receiver Steam Used for Heating. When the receiver delivers part of its steam for heating or process work, the action is as suggested by the diagrams *habg*, *gmnef*, Fig. 3, and the mean effective pressure of the low-pressure cylinder becomes

$$p_{ml} = \frac{P_o}{C}(1-x)\left(1+\log_e\frac{C}{1-x}\right)-p \qquad (21)$$

where x is the proportion of steam supplied to the high-pressure cylinder which is drawn off for heating. Equations (18) and (19) should then be used in design. A simple engine should be used if a large amount of low-pressure waste steam can be utilized.

Superheated Steam in Compound Engines. With superheat exceeding 150 deg. fahr., Equations (12), (13), (16) and (17) are invalid. The curve *abc*, Fig. 3, may be represented by pv^n = const., the value of n for the R ratio of expansion being that given on p. 942 for a simple engine in which the ratio of expansion is $\sqrt{R}$. **For equal division of work between the cylinders,**

$$C = \left[\frac{1}{2n}\left\{\frac{p}{P}R^n(n-1)+nR^{n-1}+1\right\}\right]^{\frac{1}{n-1}},\ \text{and}\ P_o = P\left(\frac{C}{R}\right)^n \qquad (22)$$

Applying this to Fig. 3,

$$p_{mh} = \frac{n}{n-1}\left(\frac{P}{R_h}-P_o\right),\quad p_{ml} = \frac{P_o n}{C(n-1)}-\frac{P}{R^n(n-1)}-p \qquad (23)$$

With these mean effective pressures, the value of f to be employed is that for a jacketed simple engine expanding $\sqrt{R}$ times (see p. 939).

Governing Compound Engines: High-pressure Terminal Drop. In early (Woolf) compound engines, without a receiver, the arrangement was necessarily either tandem or opposed and the low-pressure cylinder admitted steam throughout the full stroke. Present-day engines may have the pistons out of phase, the receiver and pipes storing steam between the cylinders; and the low-pressure cylinder works expansively while the division of work between the cylinders is controlled by variation of receiver pressure.

In Fig. 4, *larsjk* and *mbceqn* represent the action with "terminal drop" (*rs*) in the high-pressure cylinder. Such drop occurs whenever the high-pressure terminal pressure (at r) exceeds the receiver pressure. Its effect is to reduce the value of f, say, by about 0.05.

If an engine with any definite amount of drop at normal load has a fixed point of low-pressure cut-off, increase of load will increase the receiver pressure and the proportion of work done by the low-pressure cylinder without changing the drop. German engines are thus designed, with rather low values of C.

With variable low-pressure cut-off, as in American practice, delay in such cut-off lowers the receiver pressure and increases the drop, without any appreciable influence on the output of the whole engine. The output of the low-pressure cylinder is decreased because of the reduced receiver pressure, in spite of its later point of cut-off. The output of the whole engine is increased by delaying high-pressure cut-off. The division of work between the cylinders may then be kept equal by also delaying low-pressure cut-off; but this will cause increased drop.

Whether any **drop** should be permitted **at normal load** is a controverted subject (see B. C. Ball, *Trans. A. S. M. E.*, vol. xxi, p. 1002). There should certainly be none in an engine intended for frequent overload conditions. To eliminate drop at normal load, the normal point of low-pressure cut-off must be fixed to suit the type of engine, size of receiver and cylinder ratio. The graphical determination is described by Ennis in "Applied Thermodynamics for Engineers," chap. xiii. For **tandem compounds,** $R_{la} = (QC + 1)/(Q + 1)$, and for **cross compounds** with normal low-pressure cut-off not later than half stroke, $Q[(C/R_{la}) - 1] = 0.5 - (\sqrt{R_{la}} - 1/R_{la})$, where R_{la} = apparent ratio of expansion in low-pressure cylinder $= (v_c - v_m)/(v_b - v_m)$, Fig. 4, Q = ratio of receiver volume to volume displaced by high-pressure piston, and C = cylinder ratio.

Triple and Quadruple Engines

In Fig. 5, the **ideal diagrams** for a triple engine are 1234, 4378 and 87*a*09. For a quadruple, let the middle diagram be divided into 4365 and 5678, the pressures noted being absolute pressures. For equalization of work,

$$\left.\begin{array}{l} \log_e P_1 = \log_e P - \dfrac{\log_e R}{3} + \dfrac{pR}{3P} - \dfrac{1}{3}, \\ P_2 = \dfrac{P_1^2}{P}, \qquad C_2 = \dfrac{v_7}{v_3} = \dfrac{P_1}{P_2}, \qquad C_3 = \dfrac{v_a}{v_7} = \dfrac{P_2}{P_a} = \dfrac{P_2R}{P}, \end{array}\right\} \text{in triples;}$$

$$\left.\begin{array}{l} \log_e P_1 = \log_e P + \dfrac{pR}{4P} - \dfrac{\log_e R}{4} - 0.25, \\ P_1 = \sqrt{PP_2}, \quad P_2 = \sqrt{P_1P_3}, \quad C_2 = \dfrac{P_1}{P_2}, \quad C_3 = \dfrac{P_2}{P_3}, \quad C_4 = \dfrac{P_3R}{P}, \end{array}\right\} \text{in quadruples;}$$

in which C_2, C_3, C_4 denote the successive cylinder ratios. (Equalization of maximum total piston pressures is a more common requirement in marine design.) The **equivalent mean effective pressures referred to the low-pressure cylinder** are,

$$\text{for triples, } p_e = \frac{P}{R}\left(\log_e \frac{P}{P_1} + \log_e \frac{P_1}{P_2} + \log_e \frac{P_2R}{P} + 1\right) - p;$$

$$\text{for quadruples, } p_e = \frac{P}{R}\left(\log_e \frac{P}{P_1} + \log_e \frac{P_1}{P_2} + \log_e \frac{P_2}{P_3} + \log_e \frac{P_3R}{P} + 1\right) - p.$$

European designers keep this value around $1.2 + 0.05\,P$ for ordinary engines, or $1.5 + 0.07\,P$ for marine triples with large drop. The **diagram factor,** f, is that for a simple engine of $\sqrt{R}$ expansions, and the **low-pressure piston areas** are $A_3 = 33{,}000 \times \text{h.p.}/fSp_e$ for triples and $A_4 = 33{,}000 \times \text{h.p.}/fSp_e$ for quadruples. Values of R range from 20 to 36. **The terminal pressure** P_a, Fig. 5, should be between 7 and 10 lb. (condensing). **Cylinder ratios are about** $\sqrt[m]{R}$, where m denotes

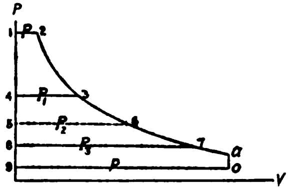


FIG. 5.—Ideal Diagrams for Triple and Quadruple Engines.

the number of expansive stages; they may successively somewhat increase, *e.g.*, $C_3 > C_2$. The overall cylinder ratio varies as the initial pressure.

Factors Beyond the Cylinder

Mechanical Efficiency. The power lost in friction, f.h.p., may be regarded as constant (see p. 960), so that the brake horse power, b.h.p., is i.h.p. − f.h.p. at all loads. The mechanical efficiency at full load is

$$M = \text{b.h.p./i.h.p.} = (\text{i.h.p.} - \text{f.h.p.})/\text{i.h.p.}$$

i.h.p. being taken at full load. **As** the **load increases,** the **mechanical efficiency** steadily **increases.** At the Q proportion of full rated load, it is

$$M_1 = (Q + M - 1)/Q$$

The **commercial efficiency** of the engine is determined jointly by its thermal and mechanical efficiencies, and the most economical ratio of expansion is commercially somewhat less than that which gives highest thermal efficiency. From a standpoint of capacity alone, moreover, the maximum practicable value of R is determined by the expression

$$\frac{P}{R} - p = (1 - M)\left[\frac{P(1 + \log_e R)}{R} - p\right] \qquad (24)$$

Values of M range from 0.82 to 0.97, tending to vary inversely with the initial pressure, the number of cylinders, number of journals and size of journals, and directly with the size of engine and the ratio of mean effective to maximum pressure. These variations are obscured by the factors of workmanship and lubrication, so that definite values for standard types of engine cannot be given.

Non-rotative Engines (direct-acting pumps) must give a fluid pressure p_p (lb. per sq. in.) in the pump at the lowest steam pressure, p_s (lb. per sq. in.). If A_p and A_s are the respective pump and steam cylinder areas, $p_pA_p = Mp_sA_s$. In duplex pumps or where inertia may be depended on, p_s may be taken as the fp_m of Equation (3), providing P is the lowest pressure likely to be reached under plant operating conditions. With a single cylinder and no inertia, p_s may be as low as fP/R, the value of R for ordinary small pumps, running almost without expansion, being taken at 1.5.

Traction and Hoisting Engines. In a double-acting single-cylinder engine

$$F = p_m ASM/S_1 \qquad (25)$$

where F = tractive or hauling force exerted, in lb.; p_m = average net piston pressure (the mean effective pressure, in expansive engines), lb. per sq. in., A = piston area, sq. in., S = piston speed, ft. per min., M = mechanical efficiency, and S_1 = speed of part at which the tractive force is applied—the peripheral speed of locomotive drivers or of hoisting engine drum—ft. per min.

Cost of Engines

(See also pp. 1102 to 1104)

Prices per horse power range from $4 to $30, increasing with the number of expansive stages and the ratio of expansion. Prices per lb. vary from 6 to 10 cents, or higher, inversely with the size. The more economical engines are generally higher priced. At present (1916), engines are relatively cheap. Comparative costs per i.h.p. are as follows (Gebhardt):

Simple high speed, $8 to $11
Installation of same, $0.80 to $1.35
Compound high speed, $16 to $25
Compound low speed, $13.40 to $23
Simple low speed, $11 to $20
Installation of same, $1.80 to $6.30

Depreciation on low-speed engines is 3 to 5 per cent. annually; on ordinary high-speed engines, 5 to 10 per cent.

STEAM-ENGINE ECONOMY

Variables Affecting Economy

The influences which determine steam-engine efficiency may be studied partly by thermodynamic analysis and partly by experiment. Both methods are here employed.

Initial Pressure. The efficiency varies directly with the initial pressure. With slight expansion the variation is unimportant. At high ratios of expansion, high initial pressures have marked influence on efficiency. The **ideal economies** of Tables 3 to 8, computed for **adiabatic expansion** and **no compression,** illustrate these points. The results of varying initial pressure with superheated steam are shown in Tables 5 to 8. In condensing engines with superheat, high pressure is less important than in non-condensing engines.

Table 3

Steam Initially Dry. Back Pressure, 2 Lb. Abs.

Ratio of expansion	Absolute steam pressure, lb. per sq. in.														
	70	100	130	160	215	70	100	130	160	215	70	100	130	160	215
	B.t.u. per i.h.p. per min.					Thermodynamic efficiency, per cent.					Steam consumption, lb. per i.h.p. per hour				
5	249	241	234	230	....	17.0	17.6	18.0	18.2	18.3	13.8	13.2	12.8	12.5	......
7	229	221	216	213	208	18.5	19.2	19.7	19.9	20.1	12.6	12.1	11.8	11.6	11.3
10	213	205	200	197	192	19.9	20.8	21.4	21.6	22.1	11.8	11.2	10.9	10.7	10.4
15	202	191	186	181	176	21.0	22.2	23.0	23.4	24.0	11.2	10.5	10.1	9.9	9.6
20	199	185	178	173	168	21.5	22.9	23.8	24.2	25.2	10.9	10.2	9.7	9.5	9.1
25	198	182	174	169	163	21.8	23.3	24.3	24.8	26.0	10.8	10.0	9.55	9.3	8.9
30	197	180	173	167	160	21.9	23.5	24.7	25.3	26.5	10.5	9.8	9.4	9.1	8.7
40	...	....	172	165	156	...	....	25.0	25.8	27.2	....	......	9.3	9.0	8.5
50	....	....	....	164	155	....	....	....	26.0	27.6	....	......	......	8.9	8.4

Table 4

Steam Initially Dry. Back Pressure, 16 Lb. Abs.

Ratio of expansion	Absolute steam pressure, lb. per sq. in.														
	70	100	130	160	215	70	100	130	160	215	70	100	130	160	215
	B.t.u. per i.h.p. per min.					Thermodynamic efficiency, per cent.					Steam consumption, lb. per i.h.p. per hour				
2	423	390	354	329	304	9.8	10.9	11.9	12.4	13.4	26.4	23.2	21.2	20.0	18.5
3	395	338	317	297	279	10.8	12.5	13.4	14.2	15.2	23.8	20.4	19.0	17.7	16.6
4	390	321	294	277	260	10.9	13.2	14.4	15.4	16.3	23.0	19.4	17.5	16.4	15.3
5	....	316	281	265	246	....	13.4	15.1	16.1	17.2	....	19.0	16.8	15.7	14.5
6	....	....	278	258	237	....	....	15.2	16.4	17.8	....	......	16.6	15.3	14.0
7	....	....	....	256	232	....	....	....	16.6	18.3	....	......	......	15.2	13.7
10	....	....	....	....	228	....	....	....	....	18.6	....	......	......	......	13.5

Table 5

Steam Superheated 150 Deg. Fahr. Back Pressure, 2 Lb. Abs.

Ratio of expansion	Absolute steam pressure, lb. per sq. in.														
	70	100	130	160	215	70	100	130	160	215	70	100	130	160	215
	B.t.u. per i.h.p. per min.					Thermodynamic efficiency, per cent.					Steam consumption, lb. per i.h.p. per hour				
5	234	226	220	217	....	18.3	18.8	19.2	19.5	....	12.0	11.5	11.3	11.0	.....
7	216	208	203	200	196	19.6	20.4	20.9	21.2	21.6	11.2	10.6	10.4	10.1	9.8
10	203	198	188	185	182	20.9	21.9	22.5	22.9	23.4	10.5	10.0	9.6	9.4	9.1
15	195	194	177	173	169	21.7	23.1	24.0	24.5	25.2	10.1	9.4	9.0	8.8	8.5
20	191	183	171	167	162	22.2	24.7	24.8	25.5	26.2	10.0	9.1	8.7	8.4	8.1
25	190	179	168	162	158	22.5	25.0	25.3	26.1	27.0	9.9	9.1	8.6	8.3	7.9
30	188	178	167	161	157	22.7	25.2	25.7	26.4	27.0	9.8	9.0	8.5	8.2	7.9
40	....	....	166	160	156	....	....	25.9	26.6	27.2	....	.....	8.5	8.1	7.8

Table 6

Steam Superheated 150 Deg. Fahr. Back Pressure, 16 Lb. Abs.

Ratio of expansion	Absolute steam pressure, lb. per sq. in.														
	70	100	130	160	215	70	100	130	160	215	70	100	130	160	215
	B.t.u. per i.h.p. per min.					Thermodynamic efficiency, per cent.					Steam consumption, lb. per i.h.p. per hour				
2	359	316	296	284	269	10.7	12.0	12.9	13.3	14.9	22.9	19.7	18.1	17.1	16.5
3	338	289	265	251	236	11.4	13.4	14.6	15.4	16.1	20.8	17.6	16.1	15.1	14.6
4	....	281	252	239	222	....	13.8	15.4	16.4	17.4	....	17.0	15.1	14.2	13.4
5	....	....	250	234	215	....	....	15.7	16.9	18.3	....	......	15.0	13.8	12.6
6	....	....	....	232	212	....	....	....	17.1	18.8	....	......	......	13.7	12.3
7	....	....	....	....	210	....	....	....	....	19.1	....	......	......	......	12.1

Table 7

Steam Superheated 300 Deg. Fahr. Back Pressure, 2 Lb. Abs.

Ratio of expansion	Absolute steam pressure, lb. per sq. in.														
	70	100	130	160	215	70	100	130	160	215	70	100	130	160	215
	B.t.u. per i.h.p. per min.					Thermodynamic efficiency, per cent.					Steam consumption, lb. per i.h.p. per hour				
5	214	208	202	199	....	19.8	20.4	20.9	21.2	....	10.4	10.0	9.7	9.6	......
7	202	194	189	186	181	21.0	21.9	22.5	22.9	23.3	9.8	9.3	9.0	8.8	8.6
10	192	182	177	174	169	22.1	23.2	24.0	24.5	25.1	9.3	8.8	8.5	8.3	8.0
15	186	174	167	163	158	23.1	24.4	25.3	25.9	26.7	9.0	8.4	8.1	7.8	7.5
20	182	171	163	159	154	23.9	25.0	26.0	26.7	27.6	9.0	8.3	7.8	7.6	7.3
25	180	169	161	156	152	24.6	25.5	26.6	27.2	28.3	8.9	8.2	7.7	7.5	7.1
30	178	168	160	155	151	25.0	25.9	27.0	27.5	28.7	8.8	8.1	7.6	7.3	7.0
40	....	....	159	154	150	....	....	27.4	28.2	29.3	....	......	7.5	7.2	6.9

Table 8

Steam Superheated 300 Deg. Fahr. Back Pressure, 16 Lb. Abs.

Ratio of expansion	Absolute steam pressure, lb. per sq. in.														
	70	100	130	160	215	70	100	130	160	215	70	100	130	160	215
	B.t.u. per i.h.p. per min.					Thermodynamic efficiency, per cent.					Steam consumption, lb. per i.h.p. per hour				
2	360	311	295	281	264	11.8	13.4	14.4	14.9	15.5	19.0	16.6	15.3	15.0	13.8
3	337	289	265	251	237	12.5	14.7	16.1	16.9	17.9	17.7	15.0	13.7	12.9	12.1
4	....	282	253	239	222	....	15.1	16.8	17.8	19.1	....	14.6	13.1	12.3	11.4
5	....	....	250	233	215	....	.. .	17.0	18.1	19.7	....	......	12.9	12.0	11.0
6	....	....	....	231	212	....	....	....	18.3	20.0	....	......	......	11.9	10.8
7	....	....	....	....	211	....	....	....	....	20.2	....	......	......	......	10.7

Fig. 6 gives typical experimental results. The "total steam" lines illustrate the **Willans Law**, that at fixed cut-off and variable initial pressure the steam consumed per hour is $a + bh$, where h = i.h.p. and a and b are constants. Simple non-condensing engines, as built in this country, with initial pressures seldom much over 100 lb., use 24 to 28 lb. of steam per i.h.p-hr. The corresponding steam rates in German practice with 150 to 180 lb. steam pressure, are 19 to 23 lb.

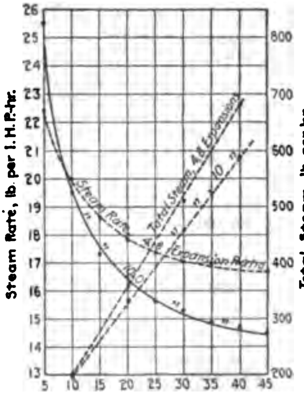


FIG. 6.—Diagram Illustrating Willans' Law of Steam Consumption.

Initial Dryness. Thermodynamic analysis shows that no variation of moisture content at the throttle to be reasonably expected can have appreciable influence on economy. The experimental evidence leads to an even bolder statement. Carpenter and Marks (*Trans. A. S. M. E.*, vol. xv) have shown that the **introduction of water** in proportions from 1 to 42 per cent. is practically **without influence on** the **dry steam consumption.** The water remains inert, neither helping nor hindering.

Superheat. Tables 5 to 8 show that **superheat increases efficiency,** and that ideally the efficiency is very nearly in direct proportion to the amount of superheat. In practice, superheat is justified by its **influence on cylinder condensation** rather than on thermodynamic grounds. Cylinder condensation is practically eliminated when the steam is kept dry at the point of cut-off. This requires ordinarily about 150 deg. (fahr.) of superheat (Ripper, "Steam Engine Theory and Practice," 1905, p. 150). The gain by superheating is progressive, probably up to that point at which the exhaust becomes superheated. **Factors favoring superheat** are slow speed, high ratios of expansion, high fuel cost and steady load.

With constant initial pressure and constant superheat, the total hourly steam consumption varies directly with the output (Ripper, *op. cit.*, p. 151; see Fig. 7). Experiments on H.M.S. "Brittania" show, moreover (Fig. 8), that the variation with load in overall efficiency of plant is about the same with saturated as with superheated steam.

The tests quoted in Table 9 show, in general, an increase in economy with

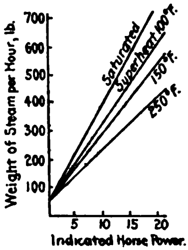


FIG. 7.

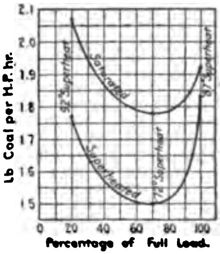


FIG. 8.

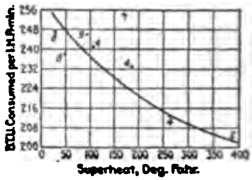


FIG. 9.—Increase in Economy of Compound Condensing Engines with Increasing Superheat.

Table 9. Performance of Engines Using Superheated Steam

Number of records	Size of engine, i.h.p.	Initial absolute pressure, lb. per sq. in.		Superheat, deg., fahr.		Steam consumption, lb. per i.h.p-hr.			B.t.u. per i.h.p-min.			References
		Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Average	Minimum	Maximum	Average	
					SIMPLE NON-CONDENSING							
2	32–300	80	91	4	41	25.6	40.0	32.8	442	668	555	1
					SIMPLE CONDENSING							
5	300–600	68	94	16	59	23.6	29.8	26.2	361	408	374	2
					COMPOUND NON-CONDENSING							
1	1,000	151	151	168	168	......	......	15.4	......	......	283	3
					COMPOUND CONDENSING							
8	650–1,100	115	165	12	45	11.9	15.1	13.2	220	283	247	4
8	400–10,000	135	190	9	86	9.0	14.0	12.7	214	259	238	5
9	130–1,500	103	191	50	132	11.3	15.1	13.0	219	291	246	6
4	1,000–1,800	123	147	71	139	11.7	13.1	12.3	230	255	241	7
1	200	154	154	167	167	......	......	12.6	......	......	254	6
4	400–1,000	147	150	176	195	10.4	11.9	11.3	208	260	233	8
4	400–1,200	142	150	216	310	9.7	10.8	10.3	205	219	213	8
2	400	132	157	375	398	9.0	9.6	9.3	198	208	203	9
					TRIPLE CONDENSING							
1	1,000	166	166	39	39	......	......	12.7	......	......	240	10
3	3,000	203	211	13	223	9.0	11.3	9.9	......	......	......	8
2	900, 3,000	170	213	110	213	9.6	9.7	9.6	187	196	192	11

REFERENCES: [1] Barrus, "Engine Tests," 1901 (Nos. 10, 21). [2] Barrus, Do. (Nos. 4, 8, 10, 15). [3] Sulzer power plant engine. [4] Barrus, *op. cit.* (Nos. 43, 48, 50, 51). [5] Marks, *Trans. A. S. M. E.*, vol. xxv, p. 443; Denton, *Trans. A. S. M. E.*, vol. xxv, p. 882; Stott, *Jour. A. S. M. E.*, Mar., 1910; Barrus, *Eng. Rec.*, 1902, vol. ii, p. 436. [6] Josse, "Neuere Kraftanlagen," 1911. [7] Sulzer engines tested by Schröter and Weber. [8] Sulzer engines. [9] Jacobus, *Trans. A. S. M. E.*, vol. xxv, p. 264; Longridge, *The Engineer*, 1905, vol. i, p. 546. [10] Barrus, *op. cit.* (No. 59). [11] *Zeit. Ver. d. Ing.*, 1900, p. 606.

increasing superheat. This is illustrated (for compound condensing engines) in Fig. 9. High superheat (to a temperature of 600 deg. fahr.) causes a saving of about 20 per cent. in simple engines, 16 per cent. in compounds and 8 per cent. in triples (see Table 10). The simple engine with superheated steam is as good as the compound engine with saturated steam, and **superheating** may be regarded **as a substitute for compounding.** The compound engine with superheated steam excels the triple using saturated steam.

Table 10. Comparative Results of Steam-engine Tests: Saturated vs. Superheated Steam

Type of engine	Initial pressure, (absolute) lb. per sq. in.	Superheat, deg. fahr.	Steam rate, lb. per i.h.p-hr.	B.t.u. consumed per i.h.p-min.	Remarks and references
Simple non-condensing	150–180	0	19.0–22.3	380–446	Figures represent best current German practice; results not reached in American plants
	150–180	260	13.4–16.1	300–353	
Simple condensing	120–150	0	16.3–17.5	306–334	
	120–150	260	10.0–11.6	226–254	
Compound condensing	145	0	11.98	225	32 expansions, Denton, *Stevens Inst. Ind.* Jan., '05
	145	307	8.99	192	
	165	0	....	213–246	Schröter engine
	165	250	....	155–223	" "
	149–151	0	13.0–14.1	(800-h.p. engine)	Sulzer power-station engines tested by Schröter and Weber
	150	181	11.3		
	144–145	0	14.1–14.5	(1000-h.p. engine)	
	147	138	11.7		
	160	0	13.8	250	Rice and Sargent engine; Jacobus, *Trans. A. S. M. E.*, vol. xxv., p. 264
	157	375	9.5	208	
	120–180	0	12.3–16.8	246–333	Figures represent current German practice
	120–180	160	10.7–13.4	226–294	
	120–180	260	9.4–11.2	213–246	
Triple condensing	218	0	11.8	217	*Zeit. Ver. Deutsch Ing.*, 1900, p. 606
	213	213	9.6	196	
	174	0	10.8	195	Holly pumping engine, Louisville, 1909.
	170	110	9.7	187	
	180–225	0	11.4–13.4	226–253	Figures represent current German practice
	180–225	160	10.0–11.2	213–239	
	180–225	260	8.9–10.0	200–220	

Back Pressure. Unnecessary back pressure causes absolute loss. Comparison of experimental results from condensing and non-condensing engines illustrates the effect of changing the back pressure from 1 or 2 to 15 or 16 lb. Such a comparison is made in Table 11. It shows that the condensing engine saves 26 per cent. over the non-condensing engine in simple four-valve types and 35 per cent. in releasing-gear compounds (both with saturated steam). The automatic engines profit less when running condensing, partly on account of their high compression. The simple condensing engine is more efficient than the ordinary compound non-condensing engine. With superheat, the condensing engine is more efficient than the non-condensing by 20 to 30 per cent. for simple types and by 23 per cent. for compounds.

Table 11. Performance of Condensing and Non-condensing Steam Engines

(Mostly from Barrus, "Engine Tests," 1901)

Type of engine	Non-condensing			Condensing			Difference in steam rate, per cent.
	Size, i.h.p.	Initial absolute pressure, lb. per sq. in.	Lb. steam per i.h.p-hr.[1]	Size, i.h.p.	Initial absolute pressure, lb. per sq. in.	Lb. steam per i.h.p.-hr.[1]	
Simple, four-valve....	50–500	80–117	29.0(13)	200–600	67–144	21.5(9)	26
Compound, automatic cut-off............	50–350	125–182	23.6(7)	90–350	120–145	19.6(7)	17
Compound, four-valve	100	144–150	21.9(2)	300–900	110–166	14.2(18)	35
Simple, slight superheat.[3]............	32–300	80–91	32.8	300–600	68–94	26.2	20
Simple, 260° superheat.[3]............	......	150–180	13.4–16.1	....	120–150	10.0–11.6	27
Compound, superheated steam.[2].....	1000	151	15.4	130–10,000	103–191	9.3–13.2	23

[1] Figures in parentheses in this column denote number of records. [2] See Table 9. [3] See Table 10.

Altitude correspondingly **affects** engine **efficiency and capacity,** since atmospheric pressure varies about 1 lb. per 2000 ft. of elevation. In non-condensing engines, the mean effective pressure varies about ¾ lb. for simples and nearly 1 lb. for compounds, for this elevation. The efficiencies of such engines vary directly with the altitude. With condensing engines, the variation is in the other direction, but is so slight as to be negligible.

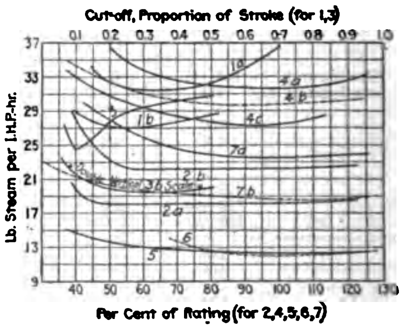


FIG. 10.—Relation Between Ratio of Expansion and Steam Consumption.

1. Meyer Valve: (*a*) 60 lb. initial gage pressure; (*b*) 90 lb. initial gage pressure. *Trans. A. S. M. E.*, vol. x, p. 722.
2. Westinghouse: (*a*) condensing; (*b*) non-condensing. *Trans. A. S. M. E.*, vol. xiii, p. 537.
3. Jacketed; 3*b*, unjacketed. Ripper, "Steam Engine Theory and Practice," 1905, p. 168.
4. Single-valve. Gebhardt, "Steam Power Plant Engineering," 1908, p. 262.
5. 5500-h.p. Westinghouse Engine, N. Y. Edison Co.
6. 700-h.p. Compound, *Trans. A. S. M. E.*, vol. xxiv, p. 1274.
7. (*a*) Non-condensing; (*b*) condensing. *Proc. Inst. C. E.*, vol. cxiv.

Ratio of Expansion. Tables 3 to 8 show that ideally the **efficiency varies directly with the ratio of expansion,** and that a given amount of increase in cylinder volume pays better when the ratio is previously low than when it is high. From this standpoint, some expansion is certainly, and complete expansion probably, desirable. (By "complete" expansion is to be understood expansion continuing until the terminal pressure is that in the exhaust pipe). Complete expansion leads to excessive cylinder condensation, so that the ratio is standardized between 3 and 5 for simple engines and between 6 and 36 for multiple-expansion engines—multiple expansion itself serving to mitigate the influence of cylinder condensation. **Maximum economy** is obtained with relatively high ratios of expansion where jackets, reheaters or superheat are used, and where the initial pressure is high or the engine runs condensing. Low ratios may be employed, regardless of thermal efficiency, to secure low first cost (*Trans. A. S. M. E.*, vol. ii, pp. 147-281).

Most stationary engines are governed by varying the ratio of expansion, so that the change in economy with load illustrates the effect of ratios of expansion on efficiency. The curves of Fig. 10 show this. The flatter curves represent the type of engine best adapted for running economically over a wide range of loads. (For the shape of such curves with superheated steam, see p. 952.) The effect of engine friction on desirable ratio of expansion is discussed on p. 948.

Cylinder Condensation. The walls of a steam cylinder are alternately heated and cooled during each revolution. The cold walls cause condensation of steam during admission and possibly during the early part of expansion, the heat transfer being at the rate of 20 to 25 B.t.u. per sq. ft. per deg. of temperature difference per min. This is followed by a drying of the steam during late expansion or exhaust; but even if the reverse transfer covered the same amount of heat as was first lost, a reduction of efficiency would ensue. This action would go on even in a cylinder perfectly insulated.

See Callendar and Nicolson, *Proc. Inst. C. E.*, vol. cxxxi, pp. 147–268; Duchesne, *Révue de Mécanique*, vol. xix, pp. 1–40; Mallanby, *Proc. Inst. Engrs. and Shipbuilders of Scotland*, 1911; Barraclough and Marks, *Min. Proc. Inst. C. E.*, vol. cxx, ii.

Since most of the condensation occurs during admission, the dryness at cut-off is a measure of economy. This may be ascertained by comparison of the indicator diagram with the experimentally determined steam consumption. The dryness at cut-off, and the economy, are directly influenced by the size of the engine, its speed and, chiefly, by its ratio of expansion. Fig. 11 shows the direct relation between cut-off dryness and ratio of expansion. It must be remembered, however, that up to a certain point there is a thermodynamic gain associated with high expansive ratios which more than offsets initial condensation (p. 954).

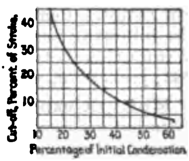


FIG. 11.

J. P. Clayton (*Bulletin* No. 58, Engineer. Exper. Station, Univ. of Illinois; also *Trans. A. S. M. E.*, vol. xxxiv) found that for non-condensing, unjacketed engines, unless excessive leakage exists, the expansion curve follows the law pv^n = constant, and the value of n depends on the quality, x, of the steam at cut-off. With superheated steam, the the value of n is higher than 1.0 and as high as 1.34; for small engines using saturated steam n is lower than 1.0 and may be as low as 0.70. For unjacketed engines exhausting at or near atmospheric pressure, the relation between n and x is given by $x = 1.258n - 0.614$. By means of this relation the value of x, and consequently the steam consumption of an engine, may be determined very closely from an accurate indicator diagram. This method applied to a large number of tests of engines of all sizes, with different pressures and speeds, yields a maximum difference of 4.6 per cent. between the calculated steam consumption and that determined from the test. Further experiments are required to establish relations between n and x for jacketed engines and for condensing engines.

Jackets. The steam jacket, by keeping the walls at constant maximum temperature, may eliminate heat transfer. The jacket should be supplied with high-pressure steam. The circulation must be free, and air pockets must be avoided. Excellence of circulation involves large condensation in the jacket. A separator should be used on the steam supply to the cylinder. Conditions favoring jackets are those already listed for superheat (p. 951). In large power stations they are mostly used to keep engines ready for service. Superheat is a better device than jacketing if high thermal efficiency is sought. Jackets should not be needed when superheat is used. Heck ("The Steam Engine and Turbine," 1911) reports for three compound condensing engines with slight superheat 235 B.t.u. consumed per i.h.p-min. without jackets or reheaters, 233 B.t.u. with both jackets and reheaters (average results). See also *Trans. A. S. M. E.*, vol. xiv, xvi; *Stevens Institute Indicator*, 1905.

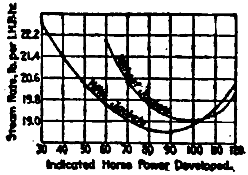


FIG. 12.—Saving Due to Jacketing.

In nearly all tests reported the jacket shows some net gain; but unfavorable results are apt not to be published. From 0 to 30 per cent. (usually 0 to 10 per cent.) saving in total steam consumption may be effected, the jacket meanwhile consuming 7 to 16 per cent. (usually 7 to 10 per cent.) of the total steam. (Heck, *op. cit.*: Average in 10 compounds, 10.2 per cent.; in 5 triples, 5.6 per cent. The lower percentages occur when superheated steam is used in the cylinders. See p. 957 as to the omission of low-pressure jackets when reheaters are used. See also *Trans. A. S. M. E.*, vol. xiv, p. 1412; *Proc. Inst. Mech. Eng.*, 1886; Isherwood, "Experimental Researches;" Barrus, *Eng. Rec.*, 1902, vol. ii, p. 436.) Fig. 12 shows results of a test by Carpenter, the saving by jacketing varying directly with the ratio of expansion and becoming negative at very low ratios. Peabody ("Thermodynamics of the Steam Engine," 1907) gives the following heat rates—B.t.u. per i.h.p-min.—on a triple expansion engine: Without jackets, 274, jackets on heads, 261, jackets on whole of cylinders, 233; jackets on whole of cylinders and receivers, 239.

Multiple expansion permits of a high ratio of expansion without excessive cylinder condensation, and therefore **increases efficiency**. Roughly, the compound uses 20 to 30 per cent. less steam than the simple engine. The triple may use 10 to 15 per cent. less than the compound. These savings (particularly those of the compound over the simple engine) are due partly to the higher initial pressure and lower leakage and clearance losses usually associated with multiple expansion. Willans (*Proc. Inst. C. E.*, vol. xciii) obtained the following results on a triple expansion engine:

Running normally	18.7 lb. steam per i.h.p-hr.
Running compound (high-pressure cylinder eliminated)	19.2 lb. steam per i.h.p-hr.
Running simple (low-pressure cylinder only)	26.0 lb. steam per i.h.p-hr.

Table 12. Average Steam Rates (Lb. per I.h.p-hr.) with Saturated Steam

(From Stanwood, Barrus, *Proc. Inst. C. E.*, vol. xciii, Willans, Clark, *Trans. A. S. M. E.*, vol. xxi, etc.) Compare with Table 9

Type of engine	Non-condensing			Condensing		
	Single valve	Double valve	Four valve	Single valve	Double valve	Four valve
Simple	33	30	28	27	24	21
Compound	24	23	22	20	16	14
Triple	..	..	18½	..	..	12

High-ratio Compounds. The best triples have only slightly surpassed the best compounds. With high cylinder ratio, 6 or 7 to 1 (p. 943), about

30 expansions, and 150 to 200 lb. initial pressure, jacketed compound engines have repeatedly consumed about 12 lb. of steam per i.h.p-hr. (*Trans. A. S. M. E.*, vol. xiii, p. 647; xix, pp. 155, 167, 189; *Power*, July, 1904, p. 424; Barrus, "Engine Tests," No. 47). Very few triple engines are used in power-station service in this country.

Non-condensing Compound Engines are generally regarded as commercially undesirable (Church, *Am. Mach.*, Nov. 19, 1891). A large unit has, however, reached a steam rate of 16.13 lb. per i.h.p-hr. (*Jour. A. S. M. E.*, March, 1910).

Reheaters. The reheater does for the low-pressure cylinders of a multiple-expansion engine just what superheat does for a single cylinder (*Trans. A. S. M. E.*, vol. xxv, 482–492). Adequate reheat involves superheating the steam in the receivers by from 30 to 100 deg. fahr. With the latter amount, Marks has shown (*op. cit.*) that the efficiency of the (compound) engine may be increased 6 to 8 per cent. Good reheating makes low-pressure jackets unnecessary. As a rule, not nearly enough reheating surface is installed. With moderately superheated steam in both cylinders, the steam consumption per i.h.p-hr. is about constant for the range from ½ load to 1¼ load.

Speed and Size. Cylinder condensation is reduced by high rotative speed, but the efficiency is not usually increased thereby. Along with high speed comes a limitation of choice of valve gear and (usually) imperfect distribution of steam. Best efficiencies are mostly obtained with speeds at or under 100 r.p.m. (With poppet valves, somewhat higher.) Slight variations in speed are practically without effect on the economy.

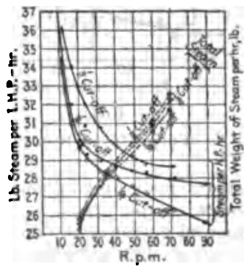


FIG. 13.

Fig. 13 shows results of a test by Denton and Jacobus, Fig. 14 those of a test by Willans. In both cases the dotted lines are based on the assumption (only approximately true) that the output is directly proportional to the r.p.m. These dotted lines show perceptible curvature; yet Marks has found (*Min. Proc. Inst. C. E.*, vol. cxx, ii) in a 9 by 36-in. condensing engine with constant initial pressure and practically constant cut-off, a straight-line relation between total steam consumed per hour and indicated horse power (compare the Willans Law, p. 951). In this as in the Denton and Jacobus tests the range of speeds was within that at which impairment of steam distribution was unlikely, so that the variation in efficiency was due to changes in cylinder condensation alone. The tests agree in showing that for slow speeds, the efficiency varies directly with the speed.

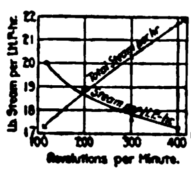


FIG. 14.

As to size, small engines are usually wasteful; more wasteful, in fact, than they need be. (Still, *St. Ry. Jl.*, Dec. 1, 1906, p. 1058: Carpenter, *Trans. A. S. M. E.*, 1907, p. 579.) In sizes from 2 to 5 h.p., non-condensing, a compound engine used 42 lb. of steam per i.h.p-hr., simple engines, 78 to 89 lb. (*Engg.*, June 27, 1890). So far as indicated thermal efficiency is concerned, there is no argument in favor of engines of particularly large size; some of the best records have been made by units of not much over 100 h.p. capacity.

Clearance and Compression. Clearance is a necessary evil and should be kept as small as possible. The heat losses to the walls increase as the clearance increases. This is one of the reasons why small engines (with relatively large clearances) are wasteful and compound engines (which usually have small clearance) are economical. Klemperer gives (*Z. V. D. I.*, 1905, vol. i, p. 797), for a 7 by 18-in. Corliss engine, non-condensing, with 4½ per cent. clearance, 25.6 lb. steam per i.h.p-hr.; with 15.2 per cent., 28.5 lb.

Compression improves the mechanical action of the engine, but advocacy thereof as a means for warming the cylinder walls prior to the supply of steam rests on a fallacious basis. A moderate amount of compression (varying with the speed) is justified on mechanical grounds, and from the experimental evidence of Figs. 15 and 16. These warrant the conclusions that

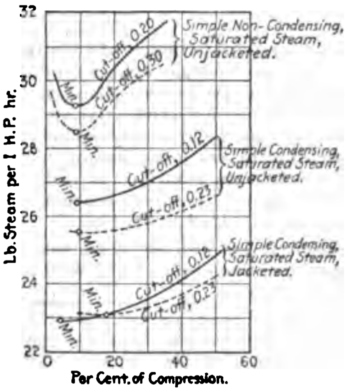


FIG. 15.

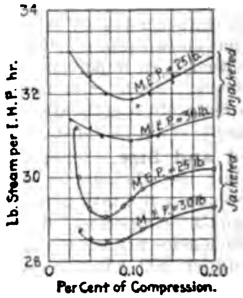


FIG. 16.

Variation of Steam Consumption with Percentage of Compression.

(*a*) Jacketing reduces the desirable amount of compression and makes low compression particularly advisable at early cut-off.

(*b*) Changes of 20 per cent. in mean effective pressure or of 50 per cent. in cut-off are practically without effect on the desirable amount of compression.

(*c*) Under only one condition did the desirable compression exceed about 10 per cent. Multiple-expansion engines should have relatively small compression. Superheat warrants exceedingly small compressions.

Valve and Port Details. With respect to these points the efficiency may be modified by: (*a*) throttling by inadequate ports, which by reducing the initial pressure or raising the back pressure may increase the steam consumption. **Governing** by throttling the supply of steam gives close regulation and makes an engine easy to handle and start under load. Moderate throttling with steady loads, slow speeds and high ratios of expansion may even be thermally desirable, but with ordinary variable loads an excellent steam rate for a throttling engine—simple non-condensing—would be 33 lb. per i.h.p-hr., and cases have been known—small direct-acting pumps—where it reached 300 lb. The great majority of constant-speed stationary engines are governed by adjustment of cut-off.

(*b*) Speed of valve movement. Quick opening and closure are necessary to avoid wire-drawing, the effect of which is shown by a rounding of the corners of the indicator diagram. To reduce friction, the movements of the valves at other parts of their travel should preferably be of small amplitude and slow speed.

(c) Interrelation of valve movements. For each end of the cylinder, four events are to be separately controlled, if highest efficiency is to be reached. This involves the use of two valves for each end. The common slide valve, with its correlation of functions, cannot control admission independently of compression, or cut-off independently of release (see p. 970).

Valves may be considered as having evolved in the following order: (1) The single valve, with interrelation and speed dependent on travel: the lowest type. (2) The double valve, which makes cut-off independent and in some cases rapid. (3) Four valves, all of the events of the stroke independently controlled; and (4) Four-valve releasing, giving especially rapid cut-off: the highest type. The last type is costly, is not applicable to very small engines, and is for the most part limited to speeds not over 100 r.p.m. (The question of leakage in relation to type of valve is considered below.) Table 13 is perhaps unduly favorable to the four-valve (type 3) engine, in that most of the rotative speeds were low, so that cut-off was fairly sharp even without releasing gear.

Table 13. Typical Steam Rates (Lb. per I.h.p-hr.) with Various Valves

Type of engine	Single valve	Double valve	Four valve	Releasing
Simple non-condensing	33	30	29	26*
Simple condensing	27	23	21½	21½*
Compound non-condensing	25½	23	22	22
Compound condensing	20	17	15	15
Average	26⅜	23⅝	21⅞	21⅛
Progressive saving, per cent	..	12	7	3

* See Table 10 as to exceptional records in German practice.

Leakage may occur past either steam valve, exhaust valve or piston. In single-valve engines, steam may escape directly from the chest to the exhaust port. The amount of leakage varies directly with the pressure difference and (in a vague way) inversely with the amount of seal of the valve. It is somewhat reduced by jacketing. The Callendar and Nicolson experiments cited on p. 955 showed that the unaccounted-for cylinder loss was largely leakage. A valve may be tight when cold or stationary, and leak when heated or moving. The steam-valve leakage in a small engine was 300 lb. per hour. Through an unbalanced slide valve, known to be tight when stationary, 29 to 38 lb. of steam escaped per hour. In a small automatic engine leakage of 10 lb. per hour was reduced to 2 lb. by refitting.

In the tests of Table 14 the valves were adjusted but not refitted. The differences between the last two columns indicate that after from 1 to 5 yr. of 24-hr. service steam rates with either flat- or piston-valve engines may be 1 to 45 per cent. (average 16 per cent.) higher than those known to be attainable with the same types of engine when new.

The figures in Table 15, though less extreme, show loss due to leakage ranging up to 12 per cent. (In this tabulation, engines have been classed as "tight" which are described as "fairly tight," "tight" or "practically tight." Those called "leaking" were originally described by such expressions as "some leakage," "considerable leak" or "excessive leakage." Presumably all of the engines were in fair shape.)

Table 14. Tests of Engines with Leaky Valves
(Dean and Wood, *Trans. A. S. M. E.*, vol. xxx, p. 6)

Type of engine	Valves	Load, per cent. of rating	Previous length of service, hours	Lb. steam per i.h.p-hr.—By test	Lb. steam per i.h.p-hr.—Computed, new engine*
(*a*) Simple non-condensing...	Single, flat	5/8	15,200	43.5	36.3
Simple non-condensing	Single, flat	5/8	20,000	38.5	36.3
Simple non-condensing	Single, flat	5/8	28,644	37.4	36.3
(*a*) Simple non-condensing...	Four, flat	3/4	719	43.9	39.2
Compound condensing	Single, piston	3/4	15,000	22.6	20.8
Simple non-condensing	Single, piston	3/4	32,000±	34.8	34.3
Simple non-condensing	Single, piston	5/8	5,600	42.4	36.3
Simple non-condensing	Four	7/8	10,800	38.3	29.3

Cases marked (*a*) were with slightly superheated steam; others with saturated steam.

* These figures are based on Table 13, modified to suit the load in accordance with curve 7*a*, Fig. 10.

Table 15. Steam Consumption of Engines with Tight and Leaky Valves
(Barrus, in "Engine Tests")

Class of engine	No. of tests Tight	No. of tests Leaking	Lb. steam per i.h.p-hr. Tight	Lb. steam per i.h.p-hr. Leaking	Per cent. difference in steam rate
Simple non-condensing, single-valve.....	4	6	31.6	34.4	+ 8.8
Simple non-condensing, four-valve	10	4	28.8	32.3	+ 12.2
Simple condensing, four-valve	4	5	21.0	21.2	+ 1.0
Compound non-condensing, single-valve..	2	4	22.7	23.2	+ 2.2
Compound condensing, single-valve......	2	4	18.6	20.5	+ 10.2
Compound condensing, four-valve.......	7	11	14.2	13.9	+ 2.1
Triple condensing.....................	1	1	12.6	12.7*	− 1.0

* Steam slightly superheated.

(See Clayton's paper, *Trans. A. S. M. E.*, vol. xxxiv, for a study of the indicator diagram for the detection of leakage.)

Friction. The known laws of friction indicate that the power lost in friction should increase somewhat with the load in either a variable-cut-off or a variable-speed engine. Conditions of workmanship and lubrication make it difficult to confirm this by actual tests of engines (see p. 948), and the friction loss is usually regarded as constant for all loads—an assumption which underlies the determination of mechanical efficiency by taking a "no-load" or "friction" indicator diagram.

The mechanical efficiency in important engines is usually between 0.9 and 0.96 at full load. The highest record seems to be nearly 0.98. Values below 0.85 are exceptional. The steam rate and thermal efficiency may be referred to brake output by introducing the mechanical efficiency as a factor.

The greater part of engine friction occurs at main bearings. Friction of piston rings and stuffing boxes comes next in importance, and the latter item may be considerably varied in operation. Unbalanced slide valves add greatly to friction losses.

Records by Special Types of Engine

Rotary Engines, since the advent of the turbine, have ceased to be even desirable. Of the thousands which have been invented and forgotten, practically all manifested excessive leakage due to the substitution of line contact of wearing parts for the surface contact of an ordinary piston. Even those which were tight at first soon developed leakage.

Fireless Engines (using stored hot water, vaporized by expansion to a lower pressure) are in occasional use. (Zeuner, "Technical Thermodynamics," Klein Ed., 1907, vol. ii, p. 449; Riedler, *Z. V. D. I.*, vol. xxvii, p. 729; Gutermuth, *op. cit.*, vol. xxviii, pp. 69, 533; xxix, p. 101.) The same principle underlies the Halpin plan for equalizing the load on power plants.

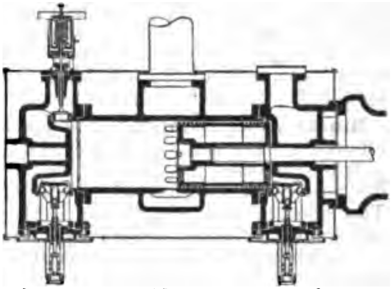
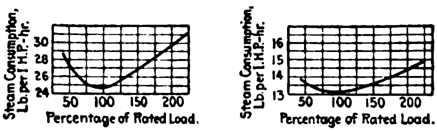


FIG. 17.—Uniflow Engine (built by Nordberg).

The Stumpf (Uniflow) Engine. In this (Fig. 17), the long piston acts as an exhaust valve, the ports being slots in the middle of the cylinder barrel, uncovered at 9/10 stroke. Piston leakage is reduced, exhaust valve leakage eliminated, back-pressure is slight and the uni-directional flow of steam decreases cylinder condensation. On the other hand, compression is (in non-condensing engines) excessive, a difficulty which is overcome by a special piston construction at the cost of some disadvantages. Clearances are 1½ to 2 per cent. The mechanical efficiency (simple) is 0.88 to 0.89. Fig. 18 shows results of tests with saturated steam. Other tests (on condensing engines) follow (*Power*, Oct. 31, Dec. 26, 1911).



Non-condensing Condensing

FIG. 18.—Test Results on Nordberg Uniflow Engines with Saturated Steam.

Observer	Engine	Steam	Lb. steam per i.h.p-hr.
Burmeister and Wain....	184 h.p.	Saturated, 140 lb.	13.64
Stumpf................	24½ by 39 in.	140 lb., 376 deg. fahr.	12.25
Stumpf................		120 lb., 490 deg. fahr.	10.85
Stumpf................		162 lb., 518 deg. fahr.	9.9
Burmeister and Wain....	115 to 220 h.p	140 lb., 667 deg. fahr.	9.06 to 9.68

These unjacketed simple engines are thus about equal in economy to our best compounds. Tests on 100-h.p. condensing engines by Lentz (*Zeit. Oest. Ing. u. Arch. Ver.*, Nos. 45, 46, 1911) gave the following results:

Initial steam pressure, lb. absolute..................	235.0	461.0
Steam temperature, deg. fahr	923.0	1018.0
Lb. steam per i.h.p-hr..................	6.52	5.67
B.t.u. per i.h.p-min..................	162.0	144.0

The Locomobile is a compact engine and internally-fired boiler outfit using high superheat and multiple expansion, with flue-gas jackets, economisers and (sometimes) flue-gas reheating. It originated in Germany under the name "lokomobile." It is reported to have developed, with 220 lb. boiler pressure and 660 deg. fahr. steam temperature, an indicated horse power on 8.25 lb. of steam or 0.97 lb. of fuel, with mechanical efficiencies between 0.70

and 0.80 (compound condensing). Tests of both Wolf and Lanz machines are given by Josse, "Neuere Kraftanlagen," Berlin, 1911. (See papers by Miller, *Eng. Mag.*, Oct., 1910, p. 64; and by Junge, *Power*, Dec. 31, 1912, p. 966.) A Buckeyemobile of 169 h.p., with steam at 209 lb. per sq. in. superheated 218 deg. fahr. and with 25.7 in. vacuum, has developed an i.h.p-hr. on 9.2 lb. of steam and on 1.08 lb. of coal of calorific value 14,209 B.t.u. per lb. The locomobile uses particularly high ratios of expansion.

Binary Vapor Engines. The principle of the steam-ether engine of 1850 (Rankine, "The Steam Engine," 1897, p. 444) has been revived by Josse, in a triple-expansion machine for steam (initially superheated) which finally discharges to a condenser in the coils of which sulphur dioxide is circulated. This substance is vaporized and works in a fourth cylinder at a pressure of 120–180 lb. The best record made was 167 B.t.u. per i.h.p-min.

Condensing engines have frequently been built to use various vapors other than steam, alone. The thermal properties of a vapor influence its efficiency in a heat engine, and under certain conditions it appears possible that a substitute vapor might lead to a heat saving of 10 to 20 per cent. over steam, along with some increase in cylinder capacity. (Ennis, "Vapors for Heat Engines," 1912.)

Regenerative Cycle: Quadruple Engines. By using the receivers as successive feed-water heaters, remarkably high thermal efficiencies have been reached in certain quadruple engines. The cycle of such engines is approximately that of Carnot (p. 319). The action is described in *Trans. A. S. M. E.*, vol. xxi, p. 181; xxviii, No. 2, p. 221; and by Ennis, "Applied Thermodynamics," 1913, Art. 550. The Wildwood pumping engine, 712 h.p., 200 lb. pressure, partly jacketed, used 186 B.t.u. per i.h.p-min. A 1000-h.p. Nordberg compressor with 257 lb. steam pressure used 169 B.t.u. (The steam rate does not measure the efficiency in this type of cycle.) These results were reached with saturated steam.

Summary of Recorded Engine Performances

The data on pp. 951–960 are typical for saturated and superheated steam in usual engines. Those in Table 16 may be regarded as best results attainable.

Table 16. Best Performances of Reciprocating Steam Engines

Type of engine	Saturated steam, lb. per i.h.p-hr.	Superheated steam,* B.t.u. per i.h.p-min.
Simple non-condensing.............	21–26 (*a*)	300 (*b*)
Simple condensing..................	18½ (*c*)	226 (*d*)
Compound non-condensing.........	16	245 (*e*)
Compound condensing.............	12	200
Triple condensing..................	10½ (*f*)	190
Quadruple condensing (regenerative)	(169 B.t.u. per i.h.p-min.)	...

* High superheat (exceeding 150 deg. fahr.) presupposed.

(*a*) German practice—American practice. 23.4 lb. has been realized in an American locomotive. With slight superheat (50 deg. fahr.), a Lentz engine gave 19.2 lb. (*Power*, Jan. 28, 1913, p. 104.)

(*b*) German practice. There seems to be no American record below 500 B.t.u.

(*c*) Barrus, "Engine Tests," No. 22. In German engines, 16.3 lb. has been realized.

(*d*) Barrus, *loc. cit.*, reaches 336 B.t.u. Figure given (corresponding with 10 lb. steam rate) is not reached in American practice.

(*e*) Carpenter, *Trans. A. S. M. E.*, 1907, p. 579. Motor-car engine.

(*f*) *Eng. News*, Aug. 23, 1900, p. 125.

Ratio of Actual and Ideal Efficiencies. Where E is the efficiency of the actual engine and E_1 is the efficiency of the ideal cycle (Tables 3 to 8), the quantity E/E_1 is called the efficiency ratio, E_R. The B.t.u. consumed per i.h.p-min. by the actual engine will be $42.42/E_R$. Average values of E_R are as follows:

	Saturated steam Condensing	Saturated steam Non-condensing
Simple	0.4	0.6
Compound	0.5	0.65
Triple	0.6	....

With high superheat, E_R lies between 0.6 and 0.7. Jackets on an engine using saturated steam increase E_R by 0.03 to 0.05.

Table 17. Historical Development of the Reciprocating Steam Engine

Engine	Ratio of expansion	Boiler pressure, lb. per sq. in.	Type	Lb. steam per i.h.p-hr.
Cornish pumps, 1840	1½ to 3½	45	Simple condensing	16½ to 24
Marine, 1850–1890	6	60 to 80	Simple or compound, condensing	19½
Marine, 1850–1890	14	150	Triple condensing	15
Marine, 1850–1890	15	210	Quadruple condensing	13½
Leavitt at Lawrence, 1872	16	120	Compound (Woolf) condensing, 12 r.p.m.	16½*
Corliss at Pawtucket, 1878	16	120	Compound condensing	13¾*
Various power-plant engines	...	...	Triple condensing	12½
Rockwood and Greene	26	150	Compound condensing, 7:1 cyl. ratio	12½*
Rice and Sargent	33	150	Compound condensing	12⅛*
Van den Kerchove	32	130	Compound condensing	12*
Westinghouse	29	185	Compound condensing	12
Leavitt, Snow, Allis, pumps	25 to 33	175	Triple condensing	11.05 to 11.26*
Allis pump	...	85	Triple condensing	10.33†
Nordberg pump	...	257	Quadruple condensing, regenerative	(A)
Stumpf, uniflow	...	140	Simple condensing	13.64
Van den Kerchove	...	130	Compound condensing	8.99‡
Stumpf	...	190	Simple condensing	9.06§
Josse (binary vapor)	...	...	Triple binary	(B)
Locomobile (p. 961)	.	220	Compound condensing	8.25‖
Stumpf uniflow (p. 961)	...	461	Condensing	5.67(C)

Mostly quoted by Denton, *Stevens Institute Indicator*, Jan., 1905; all engines used saturated steam except the last five. * Engines jacketed. † 196 B.t.u. per i.h.p-min. (A) 169 B.t.u. per i.h.p-min. ‡ 192 B.t.u. per i.h.p-min. (steam superheated 307°). § Steam superheated to 667°. (B) 167 B.t.u. per i.h.p-min. (steam superheated). ‖ Steam superheated to 660°. (C) 144 B.t.u. per i.h.p-min. (steam superheated to 1018°).

Efficiencies of Practice. The records given above are, except as specified, test records, that is, exceptionally good records. The tests giving high economies were made in most cases at full steady load, with good vacuum, on engines of fair size and good design, with leakage eliminated and valves properly adjusted. The figures reported may be duplicated under duplicate conditions in these respects, and with the same features of valve gearing, jackets, reheaters, speed and clearance. They will not be guaranteed by manufacturers without a margin of about 10 per cent.

In ordinary operation, with allowance made for the proportion of load carried, in accordance with Fig. 10, the steam consumptions should not exceed those here given by more than 15 per cent., assuming good design, no leakage and proper speed.

Efficiency may be increased by: A change of cycle, as in the regenerative quadruples; increase of temperature range, by special vapors or superheating; increase of efficiency ratio by better design. Development is going on in all of these directions. The last decade has seen marked progress in the development of the reciprocating engine, particularly in increasing the efficiency attainable in the average plant.

VALVES AND VALVE GEARING

REFERENCES: Peabody, "Valve Gears for Steam Engines"; Zeuner, "Treatise on Valve Gears (Klein tr.)"; Spangler, "Valve Gears"; Furman, "Valves, Valve Gears and Valve Diagrams"; Dalby, "Valves and Valve Gear Mechanisms"; Ewing, "The Steam Engine" (1906), chap. viii, for a brief treatment.

Valves and Valve Diagrams

Action of the Valve. In Fig. 1, the **slide valve,** shown in central position, admits steam from the steam chest, *A*, through the steam ports *a*, *a'*, and allows exhaust steam to flow from the cylinder through *a*, *a'*, to the exhaust port *b*. **Steam laps** (or "laps") are represented by *de* and *nm*; **exhaust laps** are represented by *fg* and *jk*. (Either or both may be negative. A negative exhaust lap must be less than the adjoining steam lap.) **Admission** occurs through *a* when the valve is displaced to the right by the lap *de*; or through *a'* when it is displaced to the left by the lap *mn*. **Maximum port opening,** usually less than the **port width,** *ef*, which is designed for the passage of exhaust steam, is attained (on the left side of the piston) when the valve is displaced its maximum distance to the right; or (on the right side of the piston) when the valve is displaced its maximum distance to the left. **Cut-off** occurs when the valve is displaced as for admission, but is traveling in the opposite direction to that which produces admission. **Release** occurs through *a* when the valve is displaced by the amount *fg* of exhaust lap to the left; or through *a'* when it is displaced by the exhaust lap *jk* to the right. **Compression** occurs at the same displacements, the directions of valve movement being opposite to those for release. Admission, cut-off, release and compression are described as **critical events.** For B-valves see p. 1494.

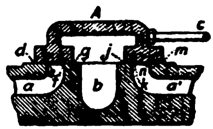


FIG. 1.—Slide Valve.

Valve and Piston Positions. Usually, the valve will have moved a greater distance than *de* when the piston has reached the left-hand end of its stroke. The excess of movement over *de* is called the **lead.** It is the amount of opening of the steam port when the piston is on the (adjacent) dead center. There may also be (and usually is) **exhaust lead,** defined as the amount of opening of the port on the exhaust side when the piston is at the (more remote) dead center.

The valve is moved by an eccentric, equivalent to a crank, the eccentric rod being of great length in proportion to the eccentric throw. Displacements of the valve are therefore regarded as equal to horizontal displacements of the eccentric center, and the **travel** of the valve is equal to **twice the eccentric radius.** If a rocker is used, it may be necessary to write "proportional" for "equal" in these two statements.

With neither laps nor lead, the valve would be central (*i.e.*, in mid-travel) when the piston was at dead center: the two would be 90 deg. out of phase. Steam would flow into the cylinder throughout one stroke, and out of it throughout the succeeding stroke. Lap permits of cut-off and expansive working. With lap, a 90-deg. phase difference between piston and valve would delay admission, and to avoid this disadvantage the phase angle is increased, the valve being displaced from its central position by an amount equal to (**lap + lead**) when the piston is on dead center. The increase of phase angle is called the **angle of advance,** $= j$. The total phase angle is 90 deg. $+ j$.

Choice of a Valve Diagram. For most purposes the Reuleaux diagram offers the most convenient method of solving problems connected with a simple eccentric valve gear. If the travel of the valve is not given, the Bilgram diagram should be used. The Zeuner diagram is best for Meyer gear, or other double eccentric gears.

The Reuleaux Valve Diagram. In Fig. 2, describe about the center *o* a circle of horizontal diameter *aok*, representing the valve travel. Through *o* draw *bo*, making the angle *boa* = *j*; and *oz*, perpendicular to *bo*. Lay off *oc* = lap, *op* = exhaust lap. Draw *af*, *dcm*, *qpr*, parallel to *bo*. Then the lead is *cf*, and the four critical events occur at piston-crank positions denoted by the points *d*, *m*, *r*, *q*. In interpreting these, the large circle is to be regarded as that described by the piston crank, rotating clockwise. The laps are those for the left-hand side of the valve (*de* and *fg*, Fig. 1) and the crank positions found are those for the critical events referred to that side of the piston which is controlled by the left-hand side of the valve.

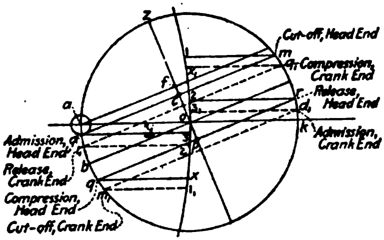


FIG. 2.—Reuleaux Valve Diagram.

On *ka* produced, find the center for an arc *ox*, described through *o*, with radius representing the length of connecting rod to the same scale as *ak* now represents the stroke of the piston. For an engine running "over," this center will be to the left of the diagram. For one running "under," it will be to the right. Then horizontal lines *qx*, *d*3, *m*1, *r*2, represent by their lengths the displacements of the piston from its central position at the critical points, again to the same scale as *ak* represents the piston stroke.

The dotted lines show the construction for the other side of the valve and the other end of the piston. Angularity of the eccentric rod may be allowed for by replacing *af*, *dcm* and *qpr* by circular arcs with centers on *of* produced, and radii representing the length of eccentric rod to the same scale as *ak* represents the travel of the valve. On account of angularity of the connecting rod the steam distribution is different on the two sides of the piston. Cut-off may be equalised by using different steam laps on the two sides of the valve, but this would lead to inequality of lead. For the effect of a reversing rocker in correcting distribution, see Furman, *op. cit.*, p. 23. The distance cs, Fig. 2, represents the maximum port-opening (scale oc = lap). Note that **(lap + lead) > exhaust lap,** if release is to occur prior to the end of the stroke.

Examples. (1) **Given travel, admission, release and cut-off.** Draw the circle (Fig. 2). Fix points d and m. Draw dm, determining **angular advance.** Fix r and draw rq parallel to dm, determining q. Draw ps perpendicular to dm and qr. Then oc = **lap,** op = **exhaust lap.** Through a draw af parallel to dm. Then cf = **lead,** cs = **maximum port opening.**

(2) **Given travel, lead, angle of advance and release.** Draw the circle. About a draw a circle of radius = lead. Tangent to this circle, draw dcm making the angle j with the horizontal, determining d, m. Fix r and draw rq parallel to dm, determining q. **Laps** and **maximum port opening** are found as in Case (1).

(3) **Given port opening, cut-off, angular advance and release.** Draw the circle ask of any convenient radius. Fix m. Draw md making the angle j with the horizontal, determining d. Similarly, fix r and draw rq parallel to md to determine q. Through a draw af parallel to md. Through o draw os perpendicular to md. Let p = required port opening. Then

$$\text{travel} = p.\frac{ak}{cs};\quad \text{lap} = p.\frac{oc}{cs};\quad \text{exhaust lap} = p.\frac{op}{cs};\quad \text{lead} = p.\frac{cf}{cs}.$$

The action of the valve depends on the four variables, lap, exhaust lap, angle of advance and travel. These determine port opening, lead, and the timing of the four events of opening and closure. If the timing of a related pair of these four events is specified, the angle of advance is fixed, and only one of the other two events may be independently assumed. The specification of an *unrelated* pair (admission and compression or cut-off and release) leaves the designer free to assume independently, either: one of the laps, the angle of advance, or (within certain limits) the lead; but only one of these three assumptions is possible.

The Zeuner Diagram. In Fig. 3, draw jl horizontally, AB vertically, intersecting at o; aok making the angle j with the vertical, and the large circle about o of diameter representing the valve travel. On the diameters ok, oa, describe the small circles. For any crank position 2 the valve displacement is $o3$, determined by drawing the vector $o2$. About o describe arcs bc, de, of radii equal to lap and exhaust lap, respectively. Through the intersecting points b, c, e, d, draw radii from o, determining g, f, m and n as the critical piston-crank positions. The distance ih is the

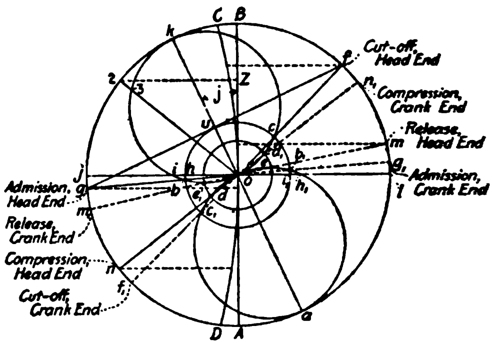


FIG. 3.—Zeuner Valve Diagram.

lead, and uk is the maximum **port opening.** Piston positions are found as in Fig. 2, by drawing the circular arc CD through o, and drawing horizontal lines from g, f, m and n to this arc. This construction gives the piston positions for the left-hand (head end) side of the piston in an engine running "over," rotation in Fig. 3 being clockwise. The positions for the crank end are worked out by dotted lines.

The Zeuner diagram gives valve displacements and port openings for all crank positions in a satisfactory way, but is **less accurate than** the **Reuleaux diagram** for determining the crank positions at which the critical events occur.

Example. Given cut-off, angle of advance and port opening. Draw (in Fig. 3) *AB*, *jl* and *aok*, and the circle on *jl*, of any convenient radius. Fix *f*, and draw *of*. Draw *fg* perpendicular to *aok*, determining *g*. On *ok* as diameter describe the small circle, fixing *c*. About *o* describe the arc *cb* through *c*, determining *h* and *u*. Let *p* be the required port opening. Then **lead** $= p.\frac{ih}{uk}$; **travel** $= p.\frac{jl}{uk}$; **lap** $= p.\frac{oc}{uk}$.

The Bilgram Diagram. (Hugo Bilgram, "Slide Valve Gears," 1878.) In Fig. 4, draw *ab* horizontally and 34 vertically, and about their intersection *o*, describe the large circle of diameter = valve travel. From *o* draw *oe* making the angle *eoa* = angle of advance, *j*. To find the valve displacement for any position *d* of the piston crank, draw the crank radius *od* and from *e* draw *ef* perpendicular to *od*. The length of *ef* represents the displacement of the valve from mid-travel. About *e* describe circular arcs *hi*, *jk*, of radii representing the lap and exhaust lap, respectively. From *o*, draw radii tangent to these arcs, cutting the large circle at the critical crank positions *A*, *B*, *C*, *D*. Draw *et* perpendicular to *oa*. Then *st* is the **lead.** *Eo* is the **maximum port opening.** The locus of the point *f* is the circle whose diameter is *eo*. Piston positions are found as in Figs. 2 and 3. Dotted lines show the construction for the crank end. With negative exhaust lap = *er*, release would be determined by DoD_1 (produced) and compression by *ok*.

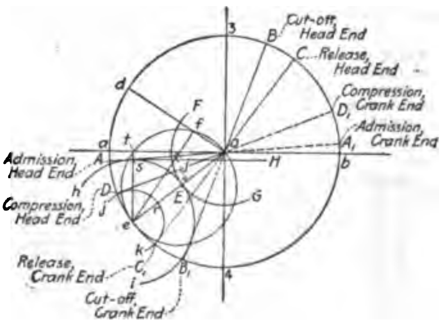


Fig. 4.—Bilgram Valve Diagram.

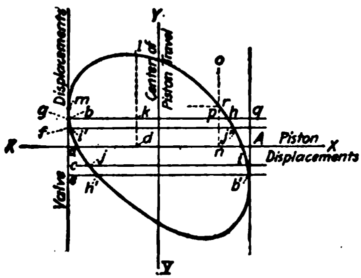


Fig. 5.—Valve Ellipse.

Example. Given port opening, cut-off, release and lead. In Fig. 4, through *o*, draw *aob* horizontally. Fix the angular position of cut-off by the line BoB_1. About *o* describe the circular arc *FEG*, of radius = port opening. Draw *sH* parallel to *aob* and distant from it by the amount of lead. Find a center, *e*, for a circle *hi*, which shall be tangent to *sH*, oB_1 and *FEG*. The angle *aoe* is the **angle of advance.** A radius from *o* tangent to the upper limit of the circle *hi* determines the angular position of the crank at **admission.** Fix the crank angle at release by the line CoC_1. About the center *e*, describe the circle *jk* tangent to this line. From *o*, draw *oD* also tangent to *jk*, determining the crank angle at **compression.** Then **travel** = 2*oe*, **lap** = *eB*, exhaust lap = *eK*.

Valve and Piston Positions and Velocities—The Valve Ellipse. In Fig. 5, ordinates are vectors of Fig. 3; abscissæ are corresponding piston displacements, *i.e.*, the co-ordinates of the point 2 (Fig. 3) are $o3$ and $2Z$. In the lower portion of this diagram, the valve displacements are taken from the lower of the two small circles of Fig. 3. The departure of the diagram from the form of an ellipse is due to angularity of the connecting rod.

Lay off $ag = ae$ = **lap**, $af = ac$ = **exhaust lap.** Draw gq, eb', fj', ci, horizontally. Then the four critical events occur at b, h, i, j. At the piston position d, the port opening is kl. This is the **maximum port opening** if the vertical line dl strikes the highest point of the curve at l. If the curve is tangent to eg at m, the distance gm is the **lead.**

Table 18. Crank Angles and Piston Positions for Connecting Rods of Different Lengths

(F = forward stroke; R = return stroke. Angles in degrees)

Fraction of stroke from commencement	Ratio of length of connecting rod to length of stroke: 2, *F*	2, *R*	2½, *F*	2½, *R*	3, *F*	3, *R*	3½, *F*	3½, *R*	4, *F*	4, *R*	Infinite *F* or *R*
	Angle through which crank has advanced from dead center										
0.005	7.3	9.3	7.4	9.1	7.5	9.0	7.6	8.8	7.7	8.7	8.1
0.01	10.3	13.2	10.5	12.8	10.6	12.6	10.7	12.4	10.8	12.3	11.5
0.02	14.6	18.7	14.9	18.1	15.1	17.8	15.2	17.5	15.3	17.4	16.3
0.03	17.9	22.9	18.2	22.2	18.5	21.8	18.7	21.5	18.8	21.3	19.9
0.04	20.7	26.5	21.1	25.7	21.4	25.2	21.6	24.9	21.8	24.6	23.1
0.05	23.2	29.6	23.6	28.7	24.0	28.2	24.2	27.8	24.4	27.5	25.8
0.06	25.4	32.5	25.9	31.5	26.3	30.9	26.6	30.5	26.8	30.2	28.4
0.07	27.5	35.1	28.1	34.1	28.5	33.5	28.8	33.0	29.0	32.7	30.7
0.08	29.5	37.5	30.1	36.4	30.5	35.8	30.8	35.3	31.0	35.0	32.8
0.09	31.3	39.8	31.9	38.7	32.4	38.0	32.7	37.5	33.0	37.2	34.9
0.10	33.1	41.9	33.8	40.8	34.3	40.1	34.6	39.6	34.9	39.2	36.9
0.15	41.0	51.5	41.9	50.2	42.4	49.3	42.9	48.7	43.2	48.3	45.6
0.20	48.0	59.6	48.9	58.2	49.6	57.3	50.1	56.6	50.4	56.2	53.1
0.25	54.3	66.9	55.4	65.4	56.1	64.4	56.6	63.7	57.0	63.3	60.0
0.30	60.3	73.5	61.5	72.0	62.2	71.0	62.8	70.3	63.3	69.8	66.4
0.35	66.1	79.8	67.3	78.3	68.1	77.3	68.8	76.6	69.2	76.1	72.5
0.40	71.7	85.8	73.0	84.3	73.9	83.3	74.5	82.6	75.0	82.0	78.5
0.45	77.2	91.5	78.6	90.1	79.6	89.1	80.2	88.4	80.7	87.9	84.3
0.50	82.8	97.2	84.3	95.7	85.2	94.8	85.9	94.1	86.4	93.6	90.0
0.55	88.5	102.8	89.9	101.4	90.9	100.4	91.6	99.8	92.1	99.3	95.7
0.60	94.2	108.3	95.7	107.0	96.7	106.1	97.4	105.5	98.0	105.0	101.5
0.65	100.2	113.9	101.7	112.7	102.7	111.9	103.4	111.2	103.9	110.8	107.5
0.70	106.5	119.7	108.0	118.5	109.0	117.8	109.7	117.2	110.2	116.7	113.6
0.75	113.1	125.7	114.6	124.6	115.6	123.9	116.3	123.4	116.7	123.0	120.0
0.80	120.4	132.0	121.8	131.1	122.7	130.4	123.4	129.9	123.8	129.6	126.9
0.85	128.5	139.0	129.8	138.1	130.7	137.6	131.3	137.1	131.7	136.8	134.4
0.90	138.1	146.9	139.2	146.2	139.9	145.7	140.4	145.4	140.8	145.1	143.1
0.91	140.2	148.7	141.3	148.1	142.0	147.6	142.5	147.3	142.8	147.0	145.1
0.92	142.5	150.5	143.6	149.9	144.2	149.5	144.7	149.2	145.0	149.0	147.2
0.93	144.9	152.5	145.9	151.9	146.5	151.5	147.0	151.2	147.3	151.0	149.3
0.94	147.5	154.6	148.5	154.1	149.1	153.7	149.5	153.4	149.8	153.2	151.6
0.95	150.4	156.8	151.3	156.4	151.8	156.0	152.2	155.8	152.5	155.6	154.2
0.96	153.5	159.3	154.3	158.9	154.8	158.6	155.1	158.4	155.4	158.2	156.9
0.97	157.1	162.1	157.8	161.8	158.2	161.5	158.5	161.3	158.7	161.2	160.1
0.98	161.3	165.4	161.9	165.1	162.2	164.9	162.5	164.8	162.6	164.7	163.7
0.99	166.8	169.7	167.2	169.5	167.4	169.4	167.6	169.3	167.7	169.2	168.5
0.995	170.7	172.7	170.9	172.6	171.0	172.5	171.2	172.4	171.3	172.3	171.9
1.00	180.0	180.0	180.0	180.0	180.0	180.0	180.0	180.0	180.0	180.0	180.0

For the other end of the cylinder, critical events occur at b', h', i', j', cut-off being earlier on this side. Cut-off might be equalized thus: Lay off $An = eh'$, draw no vertically, and make the lap nr instead of np on that side of the piston the forward action of which is described by the upper half of the diagram. But this would practically eliminate lead and would decrease the maximum port opening.

The "steepness" of the curve measures the relative rapidity of valve movement. Thus the valve is moving at nearly maximum speed during admission.

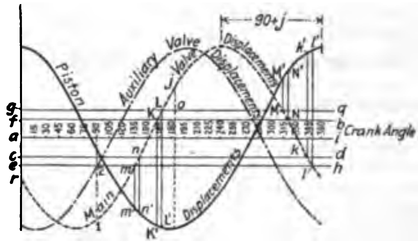


FIG. 6.—Sinusoidal Valve Diagram.

Crank Angles and Piston Positions for various ratios of connecting-rod length to length of stroke are given in Table 18. See also p. 771 for piston positions corresponding to different crank angles.

Sinusoidal Diagram. In Fig. 6 the dotted "main valve" curve represents valve displacements on a crank-angle base, the angle 0 deg. = 360 deg. corresponding with crank position j, Fig. 3. Similarly, the solid line curve represents piston displacements (to a different scale). The valve curve is truly sinusoidal. The shape of the piston curve depends on the ratio of lengths of connecting rod and crank. Draw qg and he distant from the horizontal axis by the amount of lap, and bf and dc distant therefrom by the amount of exhaust lap. The critical events occur (on one side) at valve positions L, M, N, K and (on the other side) at l, m, n, k. Vertical lines from these points give the corresponding piston positions, L', M', N', K' and l', m', n', k'. Vertical lines gr, oj, through crank angles 0 deg. and 180 deg., show **lead** = er or oj. Changes in angular advance may be studied by shifting one of the curves horizontally. The "steepness" of the valve curve again measures the speed of valve movement. This diagram illustrates lead more clearly than Fig. 5.

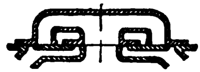


FIG. 7.—Multiported Valve.

Piston Valves may be analyzed as in Figs. 2, 3, and 4, but with "inside edge" distribution the displacement of the valve is (Fig. 1) to the left for admission to the left-hand side of the piston.

Multiported Valves reduce friction and clearance. Fig. 7 shows one form. With a large number of ports, this becomes the **gridiron valve.**

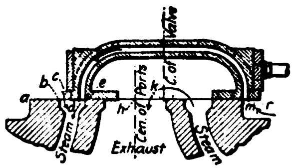


FIG. 8.—Allen Valve.

The Allen Valve is shown in Fig. 8. The passage in the valve gives double the opening for admission, with travel equal to that of a plain valve. In the figure, the valve is in its extreme right-hand position. The steam lap = $kc - hb$, should exceed ce. Also, $cb = mr = de$. The distance from a to d, when the valve is at mid-travel, must equal the steam lap at the *other end of the valve.* (Furman, *op. cit.*, p. 28.) Also called "trick" valve.

For descriptions of various "divided port" valves—Straight Line, Woodbury, Armstrong, Rice, Armington and Sims, Ide and Giddings—both with and without balance plates, see Halsey, "Slide Valve Gears," 1890, pp. 70–76.

The Shifting Eccentric—Limits of the Plain Valve. Early cut-off is impracticable with the plain slide valve. The point of admission being fixed, hastening of cut-off involves larger lap, earlier release and compression, and either increased travel or reduced maximum port opening. Release can then be delayed by increase of exhaust lap, but this makes compression still earlier. The plain valve and gear, moreover, must work always with the fixed point of cut-off for which they were designed.

Change in Advance and Travel. In Fig. 9, let *ab*, *cod*, *ef* and *iA* be the elements of a Zeuner diagram fixing the four critical events at *h*, *g*, *k* and *j*. Let it be required to fix cut-off at *l*, without change in admission. Draw *ol*, and *om* bisecting the angle *hol*. Find a center, on *om*, for a circle *epn*, which shall pass through *o* and *n*, the point of intersection of *ol* and *ef*. The required conditions are met when *epn* becomes the valve circle; *i.e.*, when the angle of advance is increased by *com* and the travel is reduced from *cd* to 2 × *op*. The symmetrically placed circle on *oB* now fixes release at k_2 and compression at j_2, the latter event being considerably hastened. Maximum port opening is decreased, but the distribution is superior to that in a plain slide-valve gear.

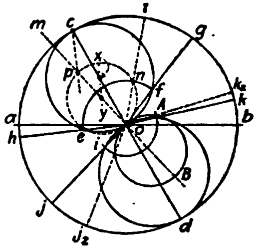


FIG. 9.—Zeuner Diagram for Slide Valve Operated by Shifting Eccentric.

The **change in throw and angle of eccentric** is accomplished in various ways: by a combination of two eccentrics (Armington and Sims); by the use of a shaft guide in which wings attached to the eccentric move back and forth (Russell, Giddings); or, as in recent practice, by using a pivoted arm on the flywheel, under governor control. The action may be described by plotting the "path of the eccentric center," *cp*, Fig. 9. If this path is *straight* and *vertical*, the lead will be constant. (Equality of lead does not imply an unchanged crank position at admission.) If it is *convex* toward *o*, the travel will be relatively great at late cut-off, and port openings small at early cut-off. Equalization of steam distribution is facilitated when the path is *concave* toward *o*.

Separate Cut-off Valves. Some of the disadvantages of the plain valve are overcome by employing a second, auxiliary mechanism, driven from an independent eccentric, for control of cut-off only, the point of cut-off being varied by hand or by the governor. In the **Gonzenbach valve** this was accomplished by control of the supply to the steam chest. (Bilgram, "Slide Valve Gears," 1878, pp. 91–96; Halsey, "Slide Valve Gears," 1890, pp. 102–108.) In the **Meyer valve** (Fig. 10) cut-off occurs (on the left-hand side of the piston) when the auxiliary valve has moved *to the left* a *relative* distance equal to its lap, *l*. The *absolute* movement of the auxiliary necessary to produce this relative displacement depends on the amount of movement of the main valve meanwhile taking place. The lap (and point of cut-off) may be varied by hand, by turning the valve rod; or the latter may be under the control of a governor which shifts the eccentric so as to change the angle of advance of the auxiliary.

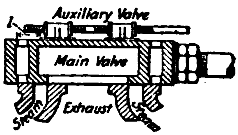


FIG. 10.—Meyer Cut-off Valve.

Fig. 11 gives the Zeuner diagram of a Meyer valve, *cod*, *aob*, *vep* and *tsf* being the elements of the main valve. Lay off *og* determining the angle $K = goc$, usually 45 deg., the **relative angle of advance**. The length of *og* represents half the travel of the auxiliary valve. On the diameter *og* describe the auxiliary valve circle *gjo*. Draw *gc*, and *oh* parallel to *gc*. On *oh* lay off $oi = gc$ and on the diameter *oi* describe the circle *olim*, vectors to which represent **relative displacements** of main and auxiliary valve. Then for a crank position 9, the main valve displacement is *ok*, that of the auxiliary is *oj*, and the relative displacement is $ok - oj = kj = ol$.

Suppose cut-off to be desired at 9. Draw *o*9, intersecting the circle *olim* at *l*. Through *l*, with *o* as a center, describe the arc 3*lm*1. The radius of this circle is the auxiliary lap, *l*, Fig. 9. Readmission by the auxiliary valve occurs at *n*, determined by the intersection *m*; but before this, the main valve will have closed at *q*. The four events occur (for one side of the piston) at *w*, 9, *r*, *u*; for the other side (with the same laps), at *y*, 2, 6, 8, the circle on *ox* being symmetrical with that on *oi*. To equalise cut-off, $(j + K) = Aog$ may equal 90 deg.; but this may lead to sluggish action at cut-off. Inequality of auxiliary laps is preferable. By increasing the travel of the auxiliary valve, greater port openings could be obtained. At very late cut-off, the point *n* might precede *q*, which would be inadmissible.

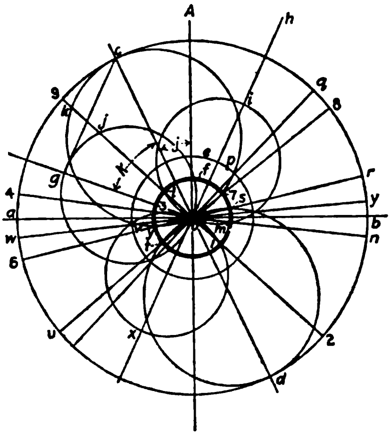


Fig. 11.—Zeuner Diagram of Meyer Valve.

In Fig. 6, the auxiliary valve displacements are plotted for a case in which both main and auxiliary valves have the same travel, the ordinates of the curve being the vectors of the circles *gjo*, Fig. 11. Vertical distances between main and auxiliary curves represent relative displacements. Cut-off occurs when this relative displacement equals the auxiliary lap. Thus if it be desired to cut off at 90 deg. crank angle, the necessary auxiliary lap is 12. With very early cut-off (about 20 deg. crank angle) the auxiliary valve is unfortunately moving at minimum speed.

The **Rider valve,** like the Meyer, incorporates an auxiliary cut-off member, but the lap is changed either by rotating or transversely moving the auxiliary. In the former case, the expansion valve is of the piston type, working in a hole bored lengthwise in the main valve. From this passage the steam ports are cut obliquely, as are also the lap ends of the auxiliary. When transverse displacement is practiced, the auxiliary is flat and of trapezoidal outline.

In the **Polonceau valve,** one long block is substituted for the two short ones of Fig. 10. Cut-off is controllable from the eccentric alone. A multiported ("gridiron") Polonceau valve has been used on recent important engines. The objections to it are its high first cost, limited range of cut-off and exceedingly small linear lead of the main valve.

The **Buckeye valve** has a pair of auxiliary blocks, like the Meyer, but fixed on the valve rod. The auxiliary moves in a steam-tight box formed in the main valve, its stem being within the main valve stem and the two being operated from a special compound rocker. Both valves have the same travel. Clearance is small and cut-off sharp.

Poppet Valves have vertical spindles, and usually lift to open. There are commonly four independent valves per cylinder. They may be located like Corliss valves, or as in gas engines, or may connect with the cylinder heads. In vertical engines, they may be on meridian planes at the sides of the cylinder. The exhaust valves are always below the steam valves. The seats are usually double (occasionally single, and sometimes, to decrease travel, quadruple), and are nearly balanced, a sufficient unbalanced pressure being provided to insure tightness when closed. The maximum angle of seat with a plane perpendicular to the spindle is 65 deg. It is usually attempted to equalize the two steam-passage areas provided. Fig. 12 shows some forms.

FIG. 12.—Poppet Valves.

In some types of engine the valves are operated direct from eccentrics, the minimum number of the latter being two, one each for steam and exhaust valves. More commonly, a pair of **miter gears** connects the engine shaft with a valve shaft, from which the valves are driven by eccentrics and cams. **Variation of cut-off** is accomplished by (*a*) a shifting eccentric (Widnmann); (*b*) **change** in angular advance and travel (old Collmann gear); or by (*c*) disengagement, as in the Corliss gear. (The later Collmann gear has disengagement control.)

In the design of poppet valves, the desired distribution of steam is assumed as in Figs. 5 and 6, and the operating cams are proportioned to give the movement necessary by ordinary kinematic methods.

Valve Gears

Rotary Gear. The **Corliss valve** is skeleton-cylindrical, located transversely. It rocks back and forth through a small arc. Separate steam and exhaust valves are provided at each end of the cylinder. Some forms are shown in cross-section in Fig. 13. The **Wheelock valve** (Fig. 14), though of Corliss form, belongs in principle with the Gonzenbach class (p. 970). The **typical Corliss valve gear** is shown in Fig. 15. In more elaborate design separate eccentrics are used for steam and exhaust valves. With releasing gear, this is usually necessary if releasing cut-off is to be effective at cut-off later than half-stroke. The **diameters of** such **cylindrical valves** range from 4 to 5 times the maximum port opening (less, if "trick" ports are used). The actuating crank attached to the end of the valve is ½ in. to 1 in. longer. The **length of ports** is about equal to the bore of the cylinder.

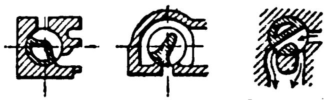


FIG. 13.—Corliss Valves.

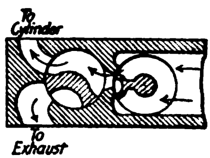


FIG. 14.—Wheelock Valve.

Valves on horizontal engines are usually located at the ends of the cylinder, the steam valve above and the exhaust valve below. The Corliss gear has small travel and slight friction. It gives particularly small clearances, though designs differ somewhat in this respect. Live steam and exhaust steam

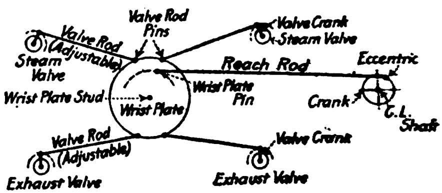


FIG. 15.—Typical Corliss Valve Gear.

never pass through the same port. The valve is **not suitable for use with high steam temperatures.**

A fundamental feature of a Corliss engine is the **releasing gear.** In Fig. 16, the crank A is loosely mounted on the valve stem, and the grab hook H is loose on a pin fixed in A. The spring S presses the grab hook against the knock-off cam C, the position of which is controlled by the governor. C rocks freely on the valve stem, but the lever B is keyed to the stem.

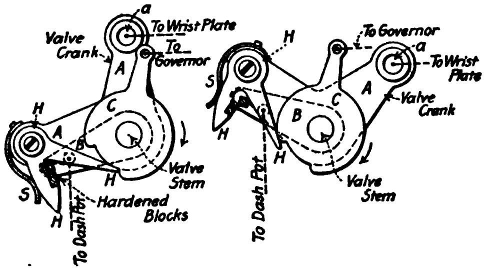


FIG. 16.—Releasing Gear of Corliss Engine.

In the left half of Fig. 16 the valve is about to rotate clockwise. This rotation is produced by the train A, H, B. Simultaneously, the rod running from H to the dash pot rises. As the motion proceeds, the upper of the hardened-steel blocks slides to the right, disengaging from the lower block—as indicated in the right half of Fig. 16. A vacuum dash pot then pulls down on H by means of the vertical rod, closing the valve by a quick counter-clockwise rotation. The point of disengagement is determined by the position of the cam C.

Ordinary **releasing gears do not work well** at speeds **above 100 r.p.m.** Corliss valves without disengagement, operated by one fixed exhaust eccentric and a steam eccentric under control of the shaft governor, are a feature of the so-called "four-valve" engine.

Design of Corliss Valves. In Fig. 17, the circle 79 is described about the center of the wrist-plate stud with a radius equal to half the horizontal travel of the point, H, midway between the valve-rod pins. Assuming harmonic motion, divide this circle into equal arcs, corresponding with equal angular movements of the eccentric. From the points of division, 1, 2, 3, . . ., draw vertical lines intersecting the line of travel of the valve-rod pins at 1′, 2′, 3′, etc. These determine successive positions of the point H. From these, by laying off the distance HF to the left, we find 1″, 2″, 3″ . . . as the positions of the left-hand valve-rod pin corresponding with eccentric positions represented by 1, 2, 3 . . .

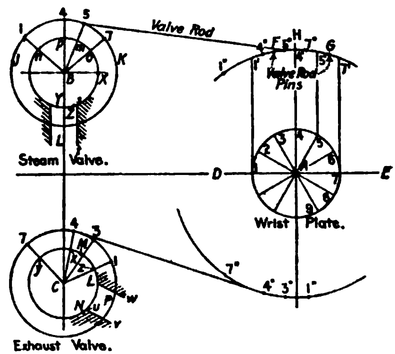


FIG. 17.—Layout of Corliss Valve Gear.

About B draw the circle JK of radius equal to length of valve crank. From 1″, 2″, 3″, . . . with radii equal to the length of the valve rod, strike arcs intersecting JK in 1, 2, 3 . . . which points thus determine valve-crank positions. Also, about B, describe the circle no of **radius representing the valve diameter** (this doubles the scale for this part of the construction). To the same scale, indicate the steam port, L. From s, lay off sZ = lead. Then the right-hand (active) edge of the valve, moving clockwise to open, is at Z when the piston is on dead center. The eccentric will than be ahead of the dead center; at 5, if the angle of advance is made 30 deg., the valve crank will be at the point 5 on JK.

Draw the radii $B1$, $B2$, $B3$ . . From Z lay off $ZY = mo$, $ZX = mn$, determining Y and X as the two extreme positions of the right-hand edge of the valve. The left-hand edge must fully cover the left-hand edge of the port when the active right-hand edge is at X. The valve face may then be described as **width of port** $+ sX + a$, where a is the amount of cover required. The **lap** is $mp - Zs$, where $Bp4$ represents the midway phase of the valve crank.

For the exhaust valve, valve-crank positions on the valve-crank circle are found in the same way. The valve outline $4s$ and exhaust port P being indicated (again preferably to double scale), note that the valve rotates counter-clockwise to close. Then the right-hand edge of the valve must be at L when release or compression occurs. Suppose these events to be prescribed at 30 deg. before the (piston) dead center is reached. The eccentric will then be at mid-throw, and the valve crank at 4. From L lay off $LM = xy$, $LN = xz$, determining M and N as the extreme points of valve travel, referred to the right-hand edge. The maximum port opening slightly exceeds the port width. The valve face must cover the arc $uL + LM +$ **desired amount of seal**. The central position of the valve is at L, so that it has no lap. Care should be taken to provide **ample seal** or cover for the valves for all possible adjustments of valve-rod length.

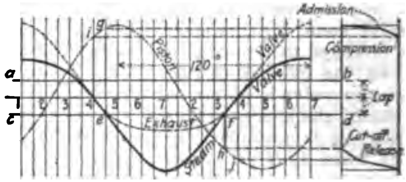


FIG. 18.—Harmonic Valve Diagram, Corliss Gear.

The **harmonic or sinusoidal diagram** is plotted in Fig. 18, with eccentric positions as a base and the rectified travel arcs xy, xz, np, pm_r etc., as ordinates of the valve curves (see Fig. 6 and explanation). The piston positions g, h, j, i, being found for one end of the cylinder, the indicator card may be sketched by projecting these points horizontally to the right. Cut-off may be independently changed by the releasing gear. The four critical events occur at maximum valve speeds.

Typical Laps for Corliss engines are as follows (exhaust laps being negative):

Cylinder diam.. in.	12	14–16	18–22	24–28	30–36	38–42
Lap, in.	1/4	5/16	3/8	7/16	1/2	9/16
Exhaust lap. in.	−1/32	−1/32	−1/16	−3/32	−1/8	−1/16

Reversing Gears. An engine may be arranged to run in either direction at will, by (a) interchanging steam and exhaust ports; (b) using two alternative eccentrics, with optional engaging grabs (as in the Stevens gear, p. 977); (c) shifting the eccentric back by an angle exceeding 180 deg. $+ j$; (d) using a reversing gear.

The **reversing gear is virtually equivalent to a shifting eccentric,** but with greater flexibility of control, since it changes both position and travel of the valve. In addition, the linkage may correct inequality of distribution due to angularity of connecting rod. It does not actually move the eccentric, but its effect is the same as that due to a movement of the extremity of the valve circle from c along some such path as cp, Fig. 9. This path being determined, the steam distribution may be examined as in Fig. 9. Where the path cannot be predicted, the action may be studied by models (Bilgram, *op. cit.*, pp. 59–71), or by more or less approximate graphical methods (*e.g.*, for the Stephenson link, see Furman, *op. cit.*, p. 85). A reversing gear may be used for gradual variation of cut-off to give constant speed at variable loads or (as usually) to give constant power at various speeds.

In the **Stephenson link** (Fig. 19), the eccentrics are treated as equivalent to two short cranks, driving the eccentric rods, which are hinged to the link.

The radius of curvature of the link is equal to the length of eccentric rods. The valve is moved by the link by means of the slide block, rocker and valve rod. The link may be raised or lowered by the reversing lever, reversing rod, bell crank and link hanger. When the link is in its lowest position, the valve derives its motion from the upper eccentric rod entirely, the lower rod serving merely to rock the link about the slide-block pin as a fulcrum. In its uppermost position, the link and valve are effectively moved by the lower eccentric rod alone. At these positions, the valve travel is a maximum. At mid-height of the link, the travel is a minimum, and the motion is determined by both eccentric rods. The movement of the link from one extreme position to the other thus gradually changes the angle of advance and the travel; and both may be fixed temporarily at any desired value between the extreme limits. To minimize slip at the slide-block bearing, the slide block should be immediately under the saddle block at the usual running adjustment.

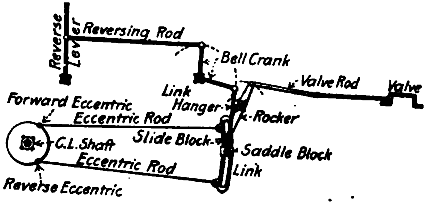


FIG. 19.—Stephenson Link Reversing Gear.

Fig. 19 shows a link motion with "open" rods. Sometimes the rods are "crossed;" *i.e.*, when the crank is at left dead-center and the eccentrics are between it and the link, the projections of the rods cross each other. (In either construction, the rods appear crossed at certain crank positions.) With crossed rods, the supply of steam may be cut off at mid-position of the link. With open rods, there is positive lead at all positions.

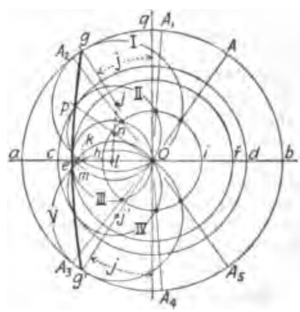


FIG. 20.—Diagram of Stephenson Link Motion.

Fig. 20, following the construction of Fig. 9, shows the successive equivalent valve circles I, II, III, IV, V, as the link is shifted from "forward" to "reverse" position. The "equivalent eccentric path" is *gpeg'*. In progressing from "full" to "mid" gear, travel and port opening decrease and lead increases. The crank positions at cut-off are A, A_1 and A_2. Proceeding from mid gear to full reverse, cut-off occurs successively at A_3, A_4, A_5 (rotation now counter-clockwise).

With constant lead, the path *gpeg'* would be a straight vertical line. In the Stephenson link, it is a parabola, closely coinciding with a circular arc drawn through g, g' and e, such that $oe = og\,(\cos goa \pm m \sin goa)$, where m is the ratio of link length to rod length. (The + sign is for open rods, the − sign for crossed rods.) Lead at full gear, with open rods, is usually from 1/16 to 3/16 in.

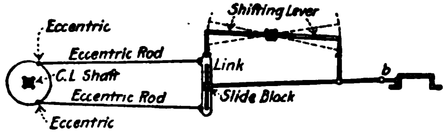


FIG. 21.—Allen Valve Gear.

Fig. 21 shows the **Allen gear**, which differs from the Stephenson in providing a simultaneous and opposite shifting of link and slide block, so that a straight slot may be employed. It is so proportioned as to cause the slide block to move nearly in a circular arc about b, as the position is varied by the shifting lever. If this were accomplished, the lead would be constant, and the path *gpeg'*, Fig. 20, would be a straight vertical line. In usual designs, the path resembles that produced by the Stephenson link with open rods, but its curvature is less.

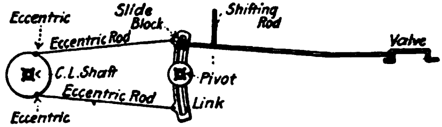


FIG. 22.—Gooch Valve Gear.

Stationary Link Gears. The **Gooch gear** (Fig. 22) suspends the link at constant height. The slide block is moved at the end of a rod of length equal to the link radius. The lead is constant and the path *gpeg'*, Fig. 20, is a straight vertical line.

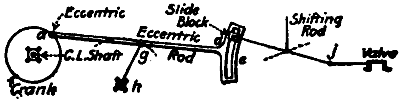


FIG. 23.—Fink Valve Gear.

In the **Fink gear** (Fig. 23) a single eccentric is placed opposite the crank, and the eccentric rod is guided by the arm *hg* so as to move nearly in a straight line. The construction from link to valve may be as in the Gooch gear, and the action is the same. The **Porter-Allen gear** is of the Fink type, the distance *ag* being greatly reduced so that the arm *ag* is a part of the eccentric strap; point *d* coincides with *g*, and (since reversing is not contemplated) the lower half of the link is omitted (Fig. 24). As used in high-speed automatic engines, this gear includes a separate radius bar, pivoted at *p* to operate the exhaust valve with constant release and compression.

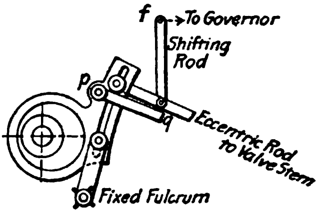


FIG. 24.—Porter-Allen Valve Gear.

Radial Gears employ a guiding arm in place of a link. In most of them the lead is constant, and the path *gpeg'*, Fig. 20, is a straight vertical line.

The **Hackworth gear** (Fig. 25) has the eccentric rod, *EQ*, terminating at *Q* in a slide block. This block moves in a straight slot, the position of which is adjustable. The pin *P* moves in an ellipse, the proportions of which are determined by the position of the slide-block slot. The gear gives good steam distribution with sharp cut-off, and is compact. In the alternative **Marshall** (or **Bremme-Marshall**) **gear**, the slot is replaced by the end of lever which rotates about an adjustable center, constraining the pin *Q* to move in some one of a series of circular arcs. This form is largely used in marine engines, particularly with "inside edge" steam distribution. The **Brown gear** is like the Hackworth, but the slotted guide (Fig. 25) is placed *between E* and *P*.

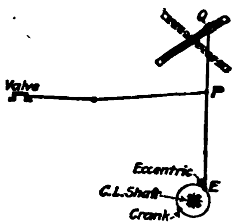


FIG. 25.—Hackworth Valve Gear.

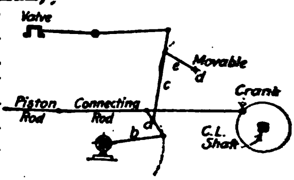


FIG. 26.—Joy Valve Gear.

In the **Joy gear** (Fig. 26) the valve is moved partly from the connecting rod by the linkage *a*, *b*, *c*, but the movement is modified by the position at which the movable fulcrum *d* of the radius bar *e* is set. There is no eccentric. A slotted guide (as in the Hackworth gear) may replace *e*. The Joy gear gives quick cut-off, well balanced, and moderate compression.

The **Walschaerts gear**, Fig. 27, also combines two methods of driving: one from the cross-head, through *e, f, g, h*, and one from the eccentric (90 deg. out of phase with the crank) through the eccentric rod, link and radius rod attached at *g*. The slide block by which the radius rod is moved from the link is vertically adjustable. When the slide block is in line with the link fulcrum, the valve is moved by the cross head alone. The link radius is equal to the length of the radius rod. If when the piston is on dead center the center of the link arc coincides with *g*, the lead will be constant. Without the cross-head linkage, there would be no lap or lead or expansion. The effect of this linkage is to increase the displacements of the valve. Minimum travel is determined by analyzing this linkage as it operates when the slide block is under the link fulcrum. Maximum travel occurs when the slide block is at one of the ends of the link. Unless the prescribed proportions are followed, the lead will vary about as it does in the Stephenson gear. The Walschaerts gear as applied to **locomotives** uses a supplementary outside crank in place of an eccentric. The mechanism is all outside, and occupies little width. Cut-off and lead may be equalized for the two ends.

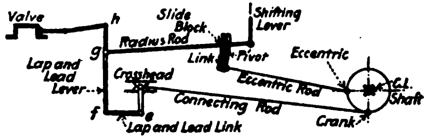


FIG. 27.—Walschaerts Valve Gear.

The **Stevens gear**, used on paddle-wheel steamers, is applied to poppet valves. The valves are operated by separate steam and exhaust eccentrics. Cut-off is fixed and the speed is varied by throttling. Reversing is accomplished either by throwing out the gear and moving the valves by hand, or by shifting to a secondary set of reversing eccentrics.

The **Lentz gear** is **used** not for reversing but **for regulation.**

Floating Gears are used on steam hammers, steering engines, etc. Fig. 28 shows one used to shift a Stephenson link. The cylinder, etc., are parts of the gear; the engine cylinder, not shown, is vertical. The link is shown in mid-position at *rs*. Its position is controlled by the linkage *ompq*, the lever *po* being pinned to the shaft *o*.

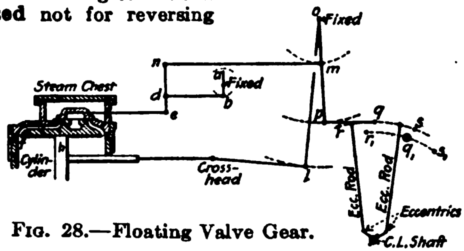


FIG. 28.—Floating Valve Gear.

The lever *ab* is thrown counter-clockwise. This gives the lever *nde* a counter-clockwise rotation about *n*, moving the valve to the right and admitting steam through the port *f*. The lever *ab* is then locked, fixing the position of *d*. The piston moves to the right, throwing the lever *lo*, pinned at *o*, and moving the link to the position r_1s_1 by means of the linkage *ompq*. The pin *m* thus pulls the pin *n* to the right, rocking the lever *edn* about the new center *d* and returning the valve to its original position. Small steam lap and about ¼ in. exhaust lap are used. Should the piston move the link further than is desired, the valve would be displaced to the left, admitting steam to stop the piston. (Furman, *op. cit.*, p. 115.)

STEAM TURBINES

BY

L. C. LOEWENSTEIN

REFERENCES: Stodola, "Die Dampfturbinen," Springer. Stodola, "Steam Turbines" (translated by L. C. Loewenstein), Van Nostrand. Bauer and Lasche, "Marine Steam Turbines," Henley Publishing Co. Martin, "Steam Turbines," Longmans, Green & Co. Guido Zerkowitz, "Thermodynamik der Turbomaschinen," Oldenbourg. Thomas, "Steam Turbines," Wiley. Moyer, "Steam Turbines," Wiley.

A steam turbine consists of a rotor (one or more wheels or drums) carrying blades or buckets against which one or more jets of steam impinge.

Classification. Steam turbines are classified as follows:

1. According to the method of steam expansion: (*a*) **Impulse turbines,** with steam expansion only in the stationary blades or nozzles; (*b*) **reaction turbines,** with steam expansion both in the stationary and in the moving blades.

2. According to the method of subdividing the available energy: (*a*) **Pressure-stage turbines,** where the available steam pressure drop is subdivided among two or more successive sets of rotating blades, each set taking care of a part of the drop of pressure; (*b*) **velocity-stage turbines,** where the steam at constant pressure passes through two or more successive sets of rotating blades, starting with a high steam velocity and decreasing the same from set to set.

3. According to the direction of the principal part of the steam flow with relation to the shaft: (*a*) **Axial-flow turbines** (flow parallel to the shaft); (*b*) **radial-flow turbines** (flow radial or at right angles to the shaft); (*c*) **tangential-flow turbines** (flow tangential to the rotor or shaft).

In a **pure impulse turbine** steam passes through stationary nozzles where it suffers a drop of pressure, the energy of which is converted into kinetic energy. The steam then impinges on the blades or buckets of the rotor, producing rotation and thereby mechanical work. There is no drop of pressure in the steam as it passes through the rotating blades. In a **pure reaction turbine** the steam passes through a set of stationary blades or guide vanes which direct the steam against the rotating blades. As the steam passes through these rotating blades, there is a drop of pressure from the entrance side to the exit side, which increases the velocity of the steam and produces rotation by the reaction of the steam on the buckets. The energy of the pressure drop is converted into kinetic energy in the moving blades, producing rotation and thereby mechanical work. In practice, impulse turbines work with some reaction, and reaction turbines work with some impulse; but each type is designated according to whether its predominant feature is impulse or reaction.

The energy of the total available pressure drop cannot usually be efficiently converted into mechanical work with one set of moving blades or buckets, and **pressure staging** is resorted to. In a reaction turbine this is accomplished by alternating rows of stationary and moving blades. Each set of moving blades takes care of a small drop of pressure. In an impulse turbine each pressure stage has a set of stationary blades or nozzles in which a drop of pressure occurs, the steam being directed against the rotating blades or buckets on the wheel of that stage. By sufficient subdivision of the total available pressure drop the steam velocities produced are such as can be efficiently utilized in the moving blades of a turbine. Velocity staging is also resorted to in impulse turbines. If the velocity of the steam from a nozzle is high, several sets or rows of moving blades can be used alternating with sets of stationary blades or reversing vanes. The kinetic energy is abstracted progressively in the succeeding rows of moving blades.

Applications. Steam turbines are necessarily high-speed prime movers and are most efficiently used direct-connected to electrical generators, centrifugal pumps, centrifugal compressors and ship propellers. They can be built to operate with **high-pressure** steam, running condensing or non-condensing; or with **low-pressure** steam only, running condensing; or as **mixed-pressure turbines**, utilizing any available low-pressure steam, but automatically resorting to the high-pressure steam supply if the low-pressure supply is insufficient; or as **steam-extraction turbines**, in which low-pressure steam can be extracted from the turbine for heating or other purposes; or as a combination high-pressure condensing or non-condensing unit, low-pressure unit, mixed-pressure unit, or steam-extraction unit controlled automatically to suit the user's requirements.

Advantages. Compared with other prime movers, steam turbines require less floor space, lighter foundations and less attendance, have better utilization of high vacuum and lower oil consumption, eliminate reciprocating masses and their resulting vibrations, have no rubbing parts outside of the bearings; they have extreme overload capacity, and freedom of the exhaust steam from oil, increased reliability due to simplicity of construction, excellent regulation and with favorable conditions as regards to steam pressure, superheat and vacuum, economies as good as steam engines for moderate powers and much better economies for units of larger sizes and greater powers.

Utilization of Vacuum. High vacuum can be better utilized in a steam turbine than in a reciprocating engine. The steam turbine can use advantageously a vacuum of more than 98 per cent. High vacuum reduces the windage or rotation losses of the rotor. Condensing turbines usually have from 27 to 29 in. vacuum, based on a 30-in. mercury barometer. The power required for producing this vacuum when the cooling-water conditions are favorable is, for large units, from 1½ to 3 per cent. of the turbine output. Low temperature of the cooling water is highly important, and separate condensing plant for each large turbine is desirable.

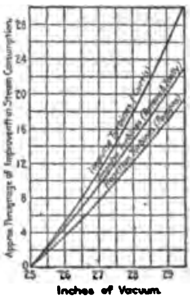


Fig. 1.—Influence of Vacuum on Steam Consumption of Turbines.

The **effect of vacuum on the steam consumption** varies greatly with the type and size of the turbine, so that no general statement is of value. For high-pressure units there is a decrease in steam consumption of about 5 per cent. for each inch of vacuum secured between 24 and 27 in. vacuum, a saving of 6 to 7 per cent. between 27 and 28 in., and of 7 to 8 per cent. between 28 and 29 in. For low-pressure units the gain due to increase of vacuum is greater. If the velocity of the steam as it leaves the last rotating buckets reaches the acoustic velocity (about 1400 ft. per sec.), further increase of vacuum has no effect on the steam economy. Fig. 1 shows the saving in steam consumption with improved vacuum for several types of steam turbines.

Effect of Superheat. Owing to the absence of rubbing parts in contact with the steam, steam turbines can work without difficulty with steam temperatures up to 650 deg. fahr., and have been designed for temperatures up to 800 deg. fahr. It is important, however, to keep the high-temperature steam away from the wheel casing and the turbine blading on account of its destructive effect on cast iron and other metals, and also because the great expansion interferes with small clearances. High steam temperatures are most easily applied with turbines having a large drop of pressure in the first stage, in which case the very-high-temperature steam comes only in contact with the steam passages to the first-stage nozzles. Single-stage impulse turbines are most suitable, while multi-stage reaction turbines are least suitable for use with steam of high initial superheat. Multi-stage units which have velocity stages in the first pressure stage, are more suitable than those without velocity stages. Superheat, besides improving the steam consumption, eliminates the erosion of the turbine blades by moisture and reduces the rotation losses of the turbine rotor. In high-pressure turbines, the steam consumption is improved approximately 1 per cent. for every 7 to 13 deg. fahr. superheat; the better value holding good for about 50 deg. fahr. superheat, while the other value holds for about 200 deg. fahr. superheat. The advantage of superheat is greater with non-condensing than with condensing units, and is even more marked with low-pressure units. A change of 35 deg. fahr. superheat causes a change of 1 per cent. of thermodynamic efficiency in the average high-pressure condensing unit of moderate power. The saving in steam consumption due to superheat is not as marked as that due to increase of vacuum. Fig. 2 shows the influence of superheat on steam consumption for two types of steam turbines.

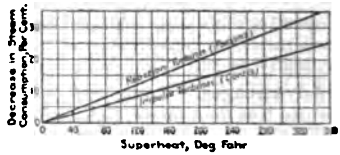


Fig. 2.—Influence of Superheat on Steam Consumption of Turbines.

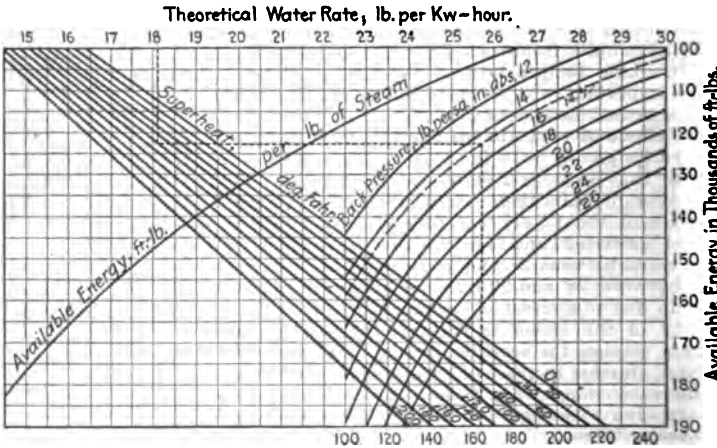


Fig. 3.—Theoretical Water Rates of High-pressure Non-condensing Steam Turbines.

Steam Consumption. The water rate of the steam turbine varies considerably. Other conditions being equal, the larger the power of the unit and the higher the speed the better the water rate. The theoretical water rates of steam turbines in lb. per kw-hour are given in Figs. 3, 4 and 5. To

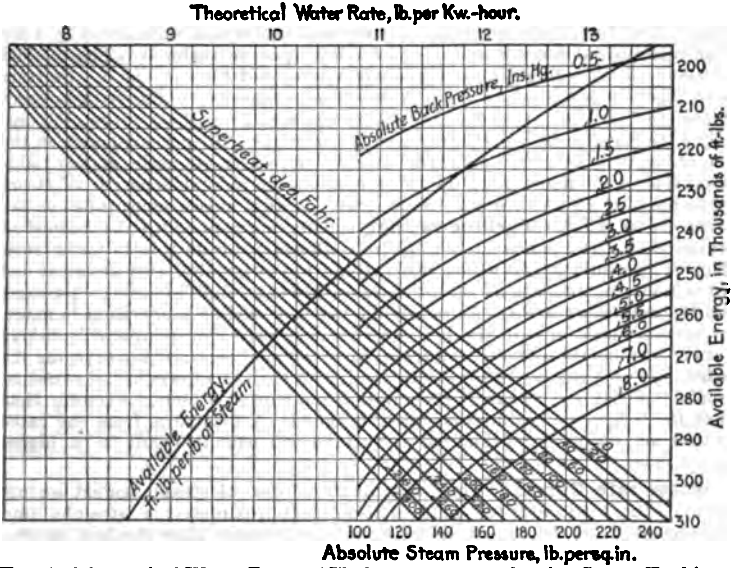


FIG. 4.—Theoretical Water Rates of High-pressure Condensing Steam Turbines.

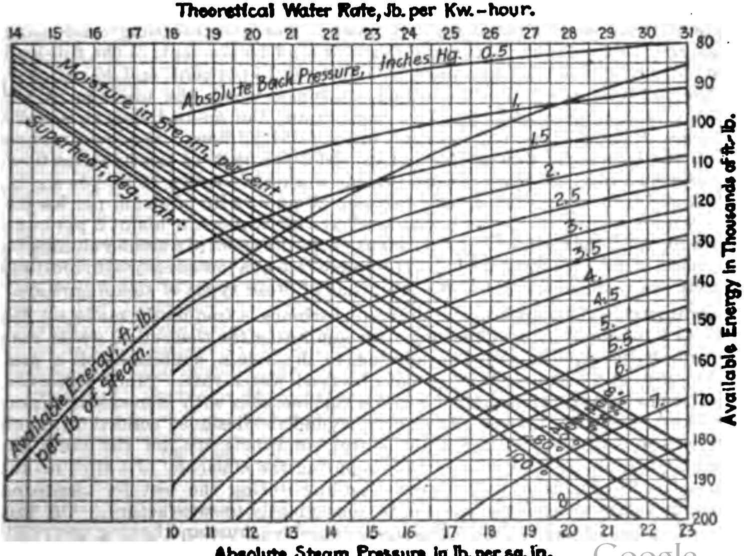


FIG. 5.—Theoretical Water Rates of Low-pressure Steam Turbines.

convert to lb. per h.p.-hour, take three-fourths of the water rate per kw.-hour. Fig. 3 gives the values for high-pressure non-condensing turbines; Fig. 4 for high-pressure condensing, and Fig. 5 for low-pressure turbines. The method of use of all three figures is indicated by the dotted lines in Fig. 3. Starting with the steam pressure (165 lb. abs.), go vertically to the back-pressure curve (14.7 lb. abs.), then horizontally to the initial superheat curve (100 deg. fahr.), and vertically to the water-rate scale (18.1 lb. per kw-hour) at the top of the diagram. The available-energy curve with its scale at the right-hand side of the figure gives the theoretical energy available per lb. of steam corresponding to the various water rates. Actual steam consumptions are given on p. 1006. **Turbine ratings** usually correspond to the output at which each turbine has its best overall efficiency, or its lowest water rate. This is the output for which the turbine has originally been designed and for which the dimensions of the nozzles, buckets, and interstage passages are theoretically correct. In nearly all types of turbines, provision is made, by excess nozzle area in the first stage or otherwise, for temporarily taking care of large overloads ranging from 25 to 50 per cent., or for carrying the rated load under unfavorable steam conditions such as a lower initial pressure, smaller initial superheat, higher initial moisture, or poorer vacuum than that for which the turbine has been designed. A departure from the rated output or steam conditions of a turbine will usually result in a higher water rate.

Water-rate Curves. The curve of total flow of steam against output between no load and full load is usually approximately a straight line. Beyond full load the total flow increases more rapidly than the load so that the flow line curves somewhat. If the water rates for two outputs not above rated load are given, an approximate flow line and water-rate curve of the turbine can be drawn. If only the water rate at full load is given, an estimate of the flow at no load must be made. For non-throttling condensing machines, this will vary between 5 per cent. and 15 per cent. of the full-load flow; for non-condensing machines, it is between 15 per cent. and 25 per cent. of the full-load flow. For throttling machines, where the pressure at the turbine nozzles rises and falls with the output, the no-load flow is usually between 25 per cent. and 35 per cent. of the full-load flow. For low-pressure turbines, the no-load flow should not exceed 5 per cent. of the full-load flow.

The water rate of a low-pressure turbine working between 16 lb. per sq. in. abs. and 28-in. vacuum is usually between 27 and 32 lb. of steam per kw-hr. Every pound change in initial pressure affects the water rate about 2 per cent. The effect of a change in vacuum is about twice the effect of a similar change on a high-pressure turbine (Fig. 1). The water rate of a low-pressure turbine will usually rise with overloads faster than that of other types of steam turbine.

Economy. A comparison of different turbines is frequently made on the basis of "thermodynamic efficiency," by computing the theoretical water rate (for adiabatic expansion) corresponding to the steam conditions of a particular test, and dividing this by the observed water rate. Since, however, the thermodynamic efficiency of a given turbine rises with increase of superheat and falls with increase of vacuum beyond the designed conditions, such comparison is misleading when the steam conditions are not the same for different turbines. The practice of computing the efficiency of a turbine for steam conditions other than those obtaining during the test by assuming constant thermodynamic efficiency, is consequently unsatisfactory. On this basis, turbines of large output, designed for high vacuum and having unavoidably high steam exit velocities from the last buckets on account of the enormous volume of steam handled, show, in general, a poorer thermodynamic efficiency than turbines designed for a poorer vacuum.

STEAM FLOW THROUGH NOZZLES AND BUCKETS IN IMPULSE TURBINES

(For general treatment of flow of steam, see p. 352)

Nozzles. The flow of **saturated steam**, dry or moist, through a nozzle when the back pressure is less than the critical pressure, can be computed approximately by Grashof's formula

$$W = 60 A_t p_1^{0.97} / \sqrt{x_1} \qquad (1)$$

in which W = flow of steam, lb. per hour; A_t = cross-sectional area of nozzle throat, sq. in.; p_1 = initial pressure, lb. per sq. in. absolute; x_1 = quality of steam corresponding to p_1. The results given by this formula are accurate to 1 to 2 per cent. within the range of the usual steam pressures encountered. Values of $p_1^{0.97}$ are given on p. 355. Within the range of $p_1 = 40$ to $p_1 = 300$ the above equation may be put in the form

$$\frac{W}{A_t} = \frac{50.5 p_1 + 150}{\sqrt{x_1}} \qquad (2)$$

with an error not exceeding 1 per cent.

For **superheated steam** the steam flow W as given by equation (1) is diminished, and when the discharge pressure p_2 is less than $0.58 p_1$,

$$W_s = \frac{W}{1 + 0.00065 T_s} = \frac{60 A_t p_1^{0.97}}{1 + 0.00065 T_s} \qquad (3)$$

in which W_s = flow of superheated steam, lb. per hour; and T_s = amount of superheat in deg. fahr.

When $p_2 > 0.58 p_1$, the flow of steam as given by (1) and (3) must be multiplied by a factor K.

$$K = 2.182 \sqrt{n(1 - 1.19n)} \qquad (4)$$

in which $n = 1 - (p_2/p_1)$. Table 1 gives values of K for pressure ratios p_2/p_1 down to 0.60.

Table 1

p_2/p_1	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84	0.82	0.80
K	0.321	0.428	0.512	0.585	0.646	0.698	0.744	0.784	0.818	0.850
p_2/p_1	0.78	0.76	0.74	0.72	0.70	0.68	0.66	0.64	0.62	0.60
K	0.877	0.901	0.922	0.940	0.956	0.970	0.981	0.988	0.995	0.998

Nozzles designed to convert the pressure energy into velocity energy when the final pressure is less than 0.58 of the initial pressure (unaffected region), must have a diverging nozzle beyond the throat or orifice (section of minimum area). The divergence should not exceed 20 deg. included angle and should preferably be made about 15 deg. The ratio R of the mouth area to the throat area for various values of p_1/p_2, both for steam initially dry and saturated and for superheated steam, is given in Table 2.

Table 2. Ratio of Mouth Area to Throat Area of Nozzles for Various Pressure Ratios

p_1/p_2	100	80	70	60	50	20	10	8	6	4	2
Saturated steam: R =	13.802	11.555	10.395	9.163	7.980	3.966	2.436	2.069	1.716	1.349	1.015
Superheated steam: R =	9.681	8.271	7.529	6.761	5.959	3.214	2.075	1.818	1.545	1.258	1.005

When a nozzle having an actual expansion ratio r is used for steam pressures having a ratio p_2/p_1 which calls for an expansion ratio R, a percentage nozzle mouth error is introduced of a value $d = 100\ (r - R)/r$, which may be positive or negative, and the nozzle efficiency is reduced.

Table 3 gives values of the velocity coefficient C_v, by which the theoretical spouting velocity must be multiplied to obtain the actual spouting velocity for various values of the nozzle mouth error d. When the actual expansion ratio of the nozzle is greater than required, the nozzle is said to be overexpanded; when smaller, underexpanded. From Table 3 it appears that it is preferable to have a nozzle underexpanded than overexpanded. Thus, when the nozzle is overexpanded by 20 per cent. (a positive nozzle mouth error of 20 per cent.) the loss in nozzle efficiency is $1 - (0.953/0.970)^2$, or 3.5 per cent., while when the nozzle is underexpanded 20 per cent. the loss in nozzle efficiency is only $1 - (0.959/0.970)^2$, or 2.3 per cent.

Table 3

Nozzle mouth error, per cent.	−40	−30	−20	−10	0	10	15	20	25	30
Velocity coefficient, C_v	0.935	0.948	0.959	0.967	0.97	0.967	0.963	0.953	0.936	0.906

Bucket Velocity Coefficients. Theoretically, the velocity of the steam through the bucket in an impulse turbine wheel, *relatively to the bucket*, remains constant. Actually, however, as explained below, the relative exit velocity from the bucket is less than the relative inlet velocity by an amount depending on the magnitude of the inlet relative velocity. The ratio of exit velocity to inlet velocity is called the bucket velocity coefficient. Table 4 gives values of bucket velocity coefficients for the usual shapes of impulse turbine buckets. These coefficients are derived from tests and include not only the friction loss in the moving buckets but also the total losses from the nozzle mouth to the entrance of the succeeding intermediates or stationary reversing buckets, if any. These losses are the loss in the clearance space between the nozzles and the moving buckets, the loss due to the bluntness of the buckets at inlet, steam friction loss in passing through the moving buckets, and finally the loss in the clearance space between the moving buckets and the succeeding intermediates.

Table 4

Velocity relative to buckets, ft. per sec.	200	400	600	800	1000	1500	2000	2500	3000	4000
Bucket velocity coefficient	0.953	0.918	0.888	0.863	0.841	0.801	0.774	0.754	0.737	0.716

Bucket Velocity Diagram. Fig. 13 shows diagrammatically the usual arrangement of a nozzle with impulse turbine buckets having two velocity stages. Fig. 6 shows the corresponding bucket velocity diagram, giving the relative magnitudes and directions of the steam velocities in the nozzles and in the buckets. V_0 represents in magnitude and direction the theoretical spouting velocity of the steam from the nozzle, corresponding to the pressure drop in the nozzle. V_1 is the actual spouting velocity ($= 0.97V_0$, approx.) when there is no nozzle mouth error. The wheel speed at the pitch diameter of the buckets is U. The relative inlet velocity to the first row of buckets V_2 is obtained from the triangle of veloc-

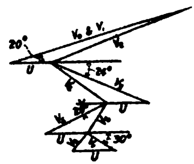


FIG. 6.—Bucket Velocity Diagram (Impulse Turbine).

ties. Multiplying V_2 by the proper velocity coefficient from Table 4 gives V_3, the relative exit velocity from the first row of buckets. Combining V_3 with U gives V_4, the *absolute* exit velocity from the first row of moving buckets, and the inlet velocity to the succeeding intermediates or stationary buckets. The exit velocity from the intermediates V_5 is equal to $0.95V_4$ regardless of the numerical value of V_4 (the difference between V_4 and V_5 representing only a friction loss, all other losses being taken care of by the velocity coefficients of the moving buckets). The relative inlet velocity to the second row of moving buckets V_6 is found by combining V_5 and U. Multiplying V_6 by the proper velocity coefficient from Table 4 gives V_7, the relative exit velocity from the second row of moving buckets. Finally, combining V_7 with U gives V_8, or the absolute velocity with which the steam is discarded from the turbine wheel.

Nozzle and Bucket Efficiency is the ratio of the energy utilized by the buckets to the energy represented by the pressure drop in the nozzle on the basis of adiabatic expansion.

Example 1. Let p_1 the initial pressure of the steam be 115 lb. per sq. in. absolute, and p_2 the final pressure be 15 lb. per sq. in. abs. An adiabatic expansion from p_1 to p_2, as read from a Mollier diagram, gives 148 B.t.u. per lb. of steam. Let the exit angle of the nozzle be 20 deg., and the exit angles of the first row of buckets, intermediates, and second row of buckets, be 24 deg., 26 deg., and 30 deg., respectively. (The best values for these exit angles are determined by experience). Let the wheel speed at the pitch diameter of the buckets be 620 ft. per sec.

From p. 355, $V_0 = 223.7\sqrt{148} = 2720$ ft. per sec., and $V_1 = 0.97V_0 = 2638$ ft. per sec. The wheel speed $U = 620$ ft. per sec. From the velocity diagram V_2 equals 2068 ft. per sec. (Fig. 6). From Table 4, the bucket velocity coefficient for 2068 ft. per sec. is 0.773. Then $V_3 = 0.773V_2 = 1597$ ft. per sec. Drawing V_3 at an angle of 24 deg. with U gives V_4 as 1061 ft. per sec. Then $V_5 = 0.95V_4 = 1007$ ft. per sec. and is drawn at an angle of 26 deg. with U. From the velocity diagram V_6 is 526 ft. per sec. Table 4 gives 0.900 for the velocity coefficient corresponding to 526 ft. per sec., so that $V_7 = 0.900V_6 = 473$ ft. per sec. Finally V_7 drawn at an angle of 30 deg. with U gives V_8 as 317 ft. per sec. Tabulating the above results, and converting velocity into equivalent B.t.u. [B.t.u. = $(\text{velocity}/223.7)^2$] the following is obtained:

	Ft. per sec.	Equivalent B.t.u.
Theoretical spouting velocity from nozzle	2720	148.0
Actual spouting velocity from nozzle	2638	139.0
First row buckets, relative inlet velocity	2068	85.5
First row buckets, relative exit velocity	1597	51.0
First row buckets, absolute exit velocity	1061	22.5
Intermediates, absolute exit velocity	1007	20.2
Second row buckets, relative inlet velocity	526	5.5
Second row buckets, relative exit velocity	473	4.5
Second row buckets, absolute exit velocity	317	2.0

The nozzle and bucket losses in B.t.u. are then as follows: Nozzle loss, $148.0 - 139.0 = 9.0$; first row buckets loss, $85.5 - 51.0 = 34.5$; intermediates loss, $22.5 - 20.2 = 2.3$; second row buckets loss, $5.5 - 4.5 = 1.0$; discarded velocity loss $= 2.0$. The total losses $= 48.8$ B.t.u. and the energy utilized is $148.0 - 48.8 = 99.2$ B.t.u. The nozzle and bucket efficiency is then $99.2/148 = 0.671$. If there had been only one row of moving buckets, the losses would have been $9.0 + 34.5 + 22.5$, a total of 66.0 B.t.u., and the nozzle and bucket efficiency only $82/148 = 0.554$. The addition of the second row of buckets in this particular example increases the nozzle and bucket efficiency by over 21 per cent.; although, if the wheel were originally designed for a single row of buckets, the efficiency could have been improved by the use of a smaller exit angle for the first row of buckets.

Rotation Loss. Of the many different formulæ proposed by various experimenters for the rotation loss of a bucket wheel in steam, the following have been found to give consistent results for the usual design and construction of turbine rotors:

$$L_d = 0.042D^2w \times (U/100)^{2.9} \quad (5)$$

$$L_b = 0.187Dwh^{1.25} \times (U/100)^{2.9} \quad (6)$$

in which L_d = rotation loss of the disk carrying the buckets, kilowatts; L_b = rotation loss of one row of buckets, kilowatts; U = wheel speed at pitch diam., ft. per sec.; D = pitch diam. (at center line of nozzle), ft. w = specific weight of steam, lb. per cu. ft.; h = mean bucket height, in. L_b must be figured for each row of buckets. L_d plus the sum of the values of L_b gives the total rotation loss in kilowatts for dry saturated steam. For wheels of pitch diameter greater than 36 in., the rotation loss computed from the above formulæ should be increased by 4 per cent. If the wheel rotates in moist or in superheated steam, the rotation loss computed from the above formulæ must be multiplied by an experimentally found factor such as given in Table 5.

Table 5

Superheat deg. fahr., or moisture, per cent.	100	50	0	2	4	6	8	10	12
Rotation loss factor	0.82	0.91	1.0	1.04	1.09	1.19	1.33	1.55	1.81

According to Buckingham (*Bull. Bureau of Standards*, vol. 10), a safe maximum value of the windage loss of wheels of ordinary shapes with one row of shrouded blades, running in the open or in casings which leave large clearances, in air or dry steam, may probably be computed from the following equation deduced from Stodola's results: Loss in horse power = $wn^3D^5[1 + 590(h/D)^2]/10^{16}$, where D = diam. of wheel at root of buckets, in., and n = r.p.m., other notation as above. Reducing the clearance around the buckets reduces the windage loss. With small clearance the resistance running backward is but little greater than when running forward. Wet steam offers more resistance than dry steam or air of the same density. The foregoing equation is for a wheel when driven through its shaft; if working in the usual manner the coefficient 590 may be reduced in the ratio of the idle buckets to the whole number. No formula, however, based on data now available can be trusted to give accurate results. For conditions similar to the above, Stodola gives the following formulæ for wheel friction, the second being derived from results of experiments carried out by the Allgemeine Elektricitäts-Gesellschaft: h.p. = $(0.061D^2 + 0.454Dh^{1.5})U^3w/10^6$; h.p. = $aD^4hn^3w/10^{10}$, where a = 0.78, 0.93, 1.25 and 1.89 respectively for 1, 2, 3 and 4 rows of buckets.

The friction loss in the buckets L_b may be approximately calculated from the formula $L_b = (1/3087)kL(C/4A)(v^2/2g)$ in B.t.u. per lb. of steam, where C = circumference of passage through which the steam flows, ft., A = sectional area of passage, sq. ft., v = relative entrance velocity of steam into buckets, ft. per sec., L = length of passage, ft., g = 32.2, and k = 0.06 to 0.10.

Bearing Loss. The friction losses of the bearings together with the usual friction losses of the shaft packings amount to not over 1 per cent. of the rating of the turbine.

Water Rate. The water rate of a turbine driving a generator is usually expressed in pounds of steam per hour per kilowatt of output at the terminals of the generator. If the turbine is connected to a centrifugal compressor the water rate is usually expressed in pounds of steam per 100 cu. ft. of air or gas delivered against a certain pressure, or if connected to a centrifugal pump or other mechanical drive the water rate is usually expressed in pounds of steam per shaft horse power delivered. In turbine design the water rate per bucket kilowatt or horse power is also used.

In the example given under nozzle and bucket efficiency, the available energy in the steam is 148 B.t.u. per lb. of steam. The nozzle and bucket efficiency is 0.671, hence the water rate per bucket kw. is 3412/(148 × 0.671) = 34.35 lb. of steam per hour. (1 kw-hr. is equivalent to 3412 B.t.u.). If 20,100 lb. of steam per hour are actually used in the turbine, the actual bucket kilowatts generated = 20,100/34.35 = 584. Assuming the rotation loss plus bearing losses to be 40 kw. the shaft output would be 584 − 40 = 544 kw.; and the water rate per shaft kw. would be 20,100/544 = 37 lb. per hour. If the turbine is direct-connected to a generator having an overall efficiency of 0.92 the power delivered to the switchboard is 544 × 0.92 = 500 kw. The water rate per kw. at the switchboard (terminals of the generator) is then 20,100/500 = 40.2 lb. of steam per hour.

IMPULSE TURBINES

Single-Stage (De Laval)

The total available pressure drop takes place in a single nozzle or set of nozzles, and the resulting kinetic energy is converted into mechanical work by a single turbine wheel carrying one or more rows of rotating buckets.

The **details of design** of a single-stage turbine are best explained by employing a numerical example. Let the initial pressure of the dry saturated steam used be 115 lb. per sq. in. absolute, and the exhaust pressure 15 lb. per sq. in. abs. It is desired to drive a 100-kw. generator at 3600 r.p.m. A water rate of 45 lb. per electrical kw-hour delivered is considered satisfactory. (Within reasonable limits a low water rate means a turbine with a rotor of large pitch diameter, hence large dimensions and high cost; a high water rate means a correspondingly cheaper turbine; therefore, either cost of unit or reasonable water rate must be determined upon.)

Nozzle. The necessary steam flow is 100 × 45 = 4500 lb. per hour. The theoretically required throat area [from (1)] is 4500/5980 = 0.752 sq. in. This area should be increased to allow the turbine to deliver full output when the initial steam pressure decreases or the exhaust pressure increases. Let the nozzle throat area be increased 25 per cent. or be 0.940 sq. in. The corresponding expansion ratio of the nozzle from Table 2 (by interpolation) is 1.99; hence the area of the nozzle at the exit is 0.940 × 1.99 = 1.870 sq. in. If the axis of the nozzle makes an angle of 20 deg. with the face of the nozzle plate or turbine rotor, the nozzle area on this plate is 1.870/sin 20 deg. = 5.47 sq. in. If the nozzles extended around the entire circumference of the rotating wheel, the nozzle height corresponding to this area would be very small. In an impulse turbine it is not necessary for the nozzles to extend around the entire circumference as is the case with reaction turbines. A reasonable nozzle height can therefore be assumed, in this case, 0.50 in. The necessary length of nozzle arc is 5.47/0.50 = 10.94 in. It is preferable to divide the necessary nozzle area into a number of small nozzles of the same total equivalent area, and to space them as close together as possible in order to deliver a practically continuous steam jet.

Rotor. Assuming that a peripheral speed of 620 ft. per sec. is satisfactory, the pitch diameter of the rotating buckets when running at 3600 r.p.m. must be $(620 \times 12 \times 60)/(\pi \times 3600) = 39.5$ in. The bucket angles and heights (bucket diagram) are usually determined by assuming certain bucket exit angles based on the results of experience, and calculating the corresponding correct inlet angles and bucket heights. Small values of exit angles give greater extraction of energy than larger exit angles but require correspondingly higher buckets at exit, which increases the rotation losses and mechanical difficulties of construction. Increase of bucket exit height (due to smaller exit angles) is practical only within certain limits because the steam jet in passing through the buckets will not diverge sufficiently to fill them. The angle of divergence should not exceed 20 deg.

Bucket Diagram. The steam velocities, bucket angles, and the nozzle and bucket efficiency are calculated in the examples under Nozzle and Bucket

Efficiency. The volume of steam remains constant as it passes through the turbine buckets and intermediates, but the steam velocity is decreasing; therefore, the heights of the buckets and intermediates must increase. The **exit heights** are as follows (for a nozzle height equal to 0.50 in.):

Buckets, 1st row: (2638 sin 20 deg./1597 sin 24 deg.) × 0.50 = 0.69 in.
Intermediates: (2638 sin 20 deg./1007 sin 26 deg.) × 0.50 = 1.02 in.
Buckets, 2d row: (2638 sin 20 deg./473 sin 30 deg.) × 0.50 = 1.91 in.

The **inlet height** of any bucket or intermediate is theoretically the same as the exit height of the nozzle or preceding bucket, but for mechanical reasons and also to allow for the slight spreading of the steam in the axial clearance between the nozzles and the buckets, and between the successive rows of buckets, the inlet height of each row of buckets is made somewhat larger than the exit height of the nozzle or of the preceding bucket. In the present example, the inlet height of the first row of buckets may be 0.56 in.; of the intermediates, 0.76 in.; and of the second row of buckets, 1.09 in.

The **rotation loss** may be computed by the use of equations (5) and (6) *ante*. In the present example, $U = 620$; $D = 39.5/12 = 3.291$; h, for first row of buckets $= (0.56 + 0.69)/2 = 0.625$; h, for second row of buckets $= (1.09 + 1.91)/2 = 1.50$; and w, for steam of 15 lb. per sq. in. abs., is 0.0381. The rotation loss uncorrected for moisture is then

$$L_d = 0.042 \times (620/100)^{2.9} \times (3.291)^2 \times 0.0381 = 3.44 \text{ kw.}$$
$$L_b = 0.187 \times (620/100)^{2.9} \times 3.291 \times 0.0381 \times [(0.625)^{1.25} + (1.50)^{1.25}] = 10.32 \text{ kw.}$$

As the pitch diameter of the buckets in the present case is greater than 36 in., the rotation loss as just computed must be increased by 4 per cent., giving a total uncorrected rotation loss of (3.44 + 10.32) × 1.04 = 14.30 kw.

The steam in expanding adiabatically from 115 lb. per sq. in. abs. to 15 lb. per sq. in. liberates 148 B.t.u., and has a final total heat of 1041 B.t.u. and a final moisture of 11.3 per cent. Assuming a shaft efficiency of 57 per cent. (to be checked presently), the 43 per cent. losses of energy in the steam due to steam friction, rotation loss, etc., appear in the exhaust steam in the form of heat, or 0.43 × 148 = 64 B.t.u. go back into the steam by reheating it. The total heat of the steam after it has passed through both rows of buckets is therefore 1041 + 64 = 1105 B.t.u. At a pressure of 15 lb. per sq. in. this corresponds to a moisture of about 4.7 per cent. The average moisture of the steam is then (11.3 + 4.7)/2, or 8 per cent. From Table 5 the rotation loss correction factor is 1.33, so that the corrected rotation loss is 14.30 × 1.33 = 19 kw.

Water Rate. Referring to Fig. 3, the theoretical water rate for the steam conditions of the present problem is 3412/148 = 23.0 lb. of steam per kw.-hour. With a nozzle and bucket efficiency of 0.671, the water rate per bucket-kw. is 23.0/0.671 = 34.4 lb. per hour. The bucket-kw. therefore from the assumed flow of 4500 lb. of steam per hour is 4500/34.4 = 131. The rotation loss, as found above, is 19 kw. Allowing about 1 kw. for bearing losses, the total loss is about 20 kw. The net shaft output of the turbine is therefore 131 − 20 = 111 kw.; and the water rate per shaft kw. is 4500/111 = 40.6 lb. per hour. The shaft efficiency is then 23.0/40.6 = 0.568. If the overall efficiency of the generator is 90 per cent., the generator output is 111 × 0.90 = 100 kw.; and the water rate per electrical kw. is 4500/100 = 45 lb. per hour.

Multi-stage (Rateau, Curtis, Zoelly)

The total available pressure drop is subdivided between two or more stages, each stage consisting of a set of nozzles and of a turbine wheel or rotor carrying one or more bucket rows. As the energy to be utilized in each stage is comparatively small, a good efficiency may be attained with a moderate wheel speed.

Details. The number of stages in an impulse turbine varies considerably, but it is always small in comparison with the number of stages in a reaction turbine for the same steam conditions and output. The pressure drop can be subdivided between the various stages so that each stage has the same available B.t.u. All wheels can be then of the same pitch diameter and have the same number of velocity stages.

Frequently, the first stage is allowed to take more than its equal share of the available pressure drop, in order to reduce the steam pressure in the first stage. This lowers the maximum pressure to which the casing is subjected, reduces the mechanical difficulties in the design of a packing to prevent leakage of steam from the first stage to the atmosphere, and, by lowering the density of the steam in the first stage, reduces the rotation loss of the first-stage wheel which is generally the greatest. In this case, the first-stage wheel usually has two or more velocity stages; while the following wheels, with a comparatively smaller B.t.u. per stage, have only one velocity stage each. The pitch diameters of the last wheels are frequently made larger so as to increase their nozzle and bucket efficiency, while their rotation loss remains fairly low on account of the low density of the steam in which these last wheels revolve. The nozzles of the first stage usually take up only a small part of the pitch circumference. As the pressure drops and the volume of the steam increases, the arc of admission increases from stage to stage till it reaches 360 deg. in the last stages, giving full peripheral admission.

Labyrinth Packings are designed to reduce the leakage between stages along the shaft to a minimum. This leakage may be computed by the following formulæ (see also p. 770).

$$W = 5.67A\sqrt{\frac{p_1^2 - p_2^2}{zp_1v_1}} \quad \text{when } p_2 > p_1 \times \frac{0.85}{\sqrt{z + 1.5}} \tag{7}$$

and

$$W = 5.67A\sqrt{\frac{1}{z + 1.5} \times \frac{p_1}{v_1}} \quad \text{when } p_2 \leq p_1 \times \frac{0.85}{\sqrt{z + 1.5}} \tag{8}$$

in which W = leakage, lb. per sec.; A = leakage area, sq. ft. (= circumference of shaft times the radial clearance, in ft.); p_1 = pressure before packing, lb. per sq. in. abs.; p_2 = pressure after packing, lb. per sq. in. abs.; v_1 = specific volume corresponding to p_1, cu. ft. per lb.; z = number of steps or subdivisions in packing. In practice, the loss due to leakage through the packings is usually from 2 to 4 per cent. of the rated output of the turbine.

Example. It is desired to design a turbine to drive a 1000-kw. generator at 3600 r.p.m.; initial pressure of the saturated steam to be used, 165 lb. per sq. in. abs., exhaust, 1 lb. per sq. in. abs. A water rate of 15.7 lb. per electrical kw. is assumed as satisfactory, and experience indicates a five-stage turbine should give this water rate. The first stage is assigned a pitch diameter of 36 in. and the other four stages one of 48 in. each. The first-stage pressure is chosen as 50 lb. per sq. in. abs. in order to keep down the maximum pressure existing in the turbine wheel, thereby reducing also the difficulty of designing the high-pressure packing to withstand the steam pressure of the first stage, and lowering the rotation losses of that stage. The available energy in the last four stages is usually equally divided, but the amount available in any stage depends upon the amount of reheating occurring in the previous stage. The reheating in any stage is equal to unity minus its shaft efficiency. A shaft efficiency is assumed and the output and amount of reheating of each stage are calculated and by trial and error the final distribution of energy and shaft efficiency determined. The following results are those obtained after several trial calculations.

Nozzle and Bucket Diagram, First Stage. The necessary steam flow from the assumed water rate is 1000 × 15.7 = 15,700 lb. per hour. The theoretical throat area, from equation (2) is therefore 1.85 sq. in., and should be increased 50 per cent. to allow the turbine to deliver full output with lower steam pressures and also full output when running non-condensing. This increase of area will not affect the efficiency or the design of the machine, because multi-stage impulse turbines are designed with governing mechanisms which admit the steam through a considerable number of valves opened or closed successively by the governor, each valve admitting steam only to a few nozzles of the first stage.

Pressure ratio for nozzles = 165/50 = 3.3; expansion ratio, from Table 2, = 1.23. Taking nozzle height = 0.5 in. and nozzle axis at angle of 20° with nozzle plate, area of nozzle mouth = 1.85 × 1.50 × 1.23/sin 20° = 10 sq. in., or net length = 10/0.5 = 20 in. Assuming exit angles of 24° and 30° for first and second rows of buckets and of 26° for intermediates, and drawing velocity diagram, Fig. 6, the steam velocities and equivalent B.t.u. are

	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8
Ft. per sec.	2157	2092	1574	1253	774	735	336	313	332
B.t.u.	93	87.5	49.5	31.4	12	10.8	2.3	2.0	2.2

and the exit heights (as per p. 988) are 0.7 in., 2.28 in., and 1.11 in., respectively, for the first and second bucket rows and the intermediates. Nozzle and bucket efficiency (as per Example 1) = 0.706. With assumed shaft efficiency of 0.60, reheating = 0.40 × 93 = 37.2 B.t.u. Moisture after adiabatic expansion through nozzle = 7.7 per cent. (from Mollier diagram, Fig. 7) and total heat = 1102 B.t.u. ∴ total heat after reheating in first stage = 1102 + 37.2 = 1139.2 B.t.u.; corresponding moisture = 3.6 per cent., hence average moisture in stage = 5.7 per cent. The corrected rotation loss, taking inlet heights of first and second rows of buckets as 0.56 and 1.78 in., respectively, is 44 kw. (using method on p. 988). Bucket output per lb. steam = 93 × 0.706 = 65.7 B.t.u.; total bucket output = 15,700 × 65.7/3412 = 302 kw. Shaft output for first stage = 302 − 44 = 258 kw., and theoretical water rate = 3412/93 = 36.7 lb. steam per kw.-hr. Actual water rate = 15,700/258 = 60.8, making shaft effy. = 36.7/60.8 = 0.603.

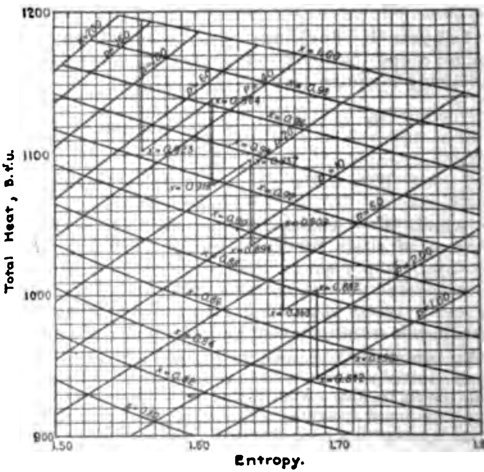


Fig. 7.—Mollier Diagram for Steam.

Nozzle and Bucket Diagrams, Second to Fifth Stages. Assume shaft efficiencies of 0.70, 0.75, 0.77 and 0.77, respectively, for these stages (higher for latter stages because of decreased rotation losses), and an available energy of 61.5 B.t.u. per stage. The second-stage nozzles expand from a pressure of 50 lb. and total heat of 1139.2 B.t.u. to a total heat of (1139.2 − 61.5 =) 1077.7 B.t.u. and a pressure (from Mollier diagram) of 21.3 lb. Reheating = entire loss of stage = (1 − 0.70) × 61.5 = 18.4 B.t.u., so that total heat of steam entering third stage = 1096.1 B.t.u. The third-stage nozzles expand the steam from 21.3 lb. through a B.t.u. drop of 61.5 to 8.5 lb. and 1034.6 B.t.u. Reheating = (1 − 0.75) × 61.5 = 15.4 B.t.u., making total heat of steam entering fourth stage = 1050 B.t.u. Similarly for fourth and fifth stages, the fifth-stage nozzles expanding from 3.1 lb. to 1 lb. (abs.).

Fig. 7 shows on a Mollier entropy-total heat diagram the expansion of the steam in this turbine from its initial to its final pressure (165 lb. abs. to 1 lb. abs.). It first expands to a pressure of 50 lb. and at the end of expansion has $(1 - x = 1 - 0.923 =)$ 7.7 per cent. of moisture. In passing through the buckets of the first stage the steam was reheated at constant pressure and finally had a moisture content of only $(1 - 0.964 =)$ 3.6 per cent. Similarly for the remaining stages.

Taking minimum nozzle height = 0.50 in., the nozzles of the first, second and third stages will extend only partially around the circumference. In the fourth and fifth stages they must extend entirely around and have respective heights of 0.76 in. and 2.10 in., allowing 10 per cent. for spaces between nozzles. As the available energy in each of the last four stages is 61.5 B.t.u., the nozzle and bucket angles are the same. Taking nozzle and bucket-exit angles each as 20°, the inlet angle (Fig. 8) is found to be 35°, and the steam velocities and equivalent B.t.u. are:

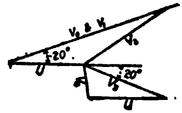


FIG. 8.

	V_0	V_1	V_2	V_3	V_4
Ft. per sec....................	1754	1701	1027	860	299
B.t.u..........................	61.5	57.8	21.1	14.8	1.8

By methods previously described, the nozzle and bucket efficiency of each stage is found to be 0.808, and the exit heights from the second-, third-, fourth- and fifth-stage buckets 0.99 in., 0.99 in., 1.5 in. and 4.16 in., respectively. The mean bucket heights are 0.80 in., 0.80 in., 1.22 in. and 3.58 in., taking 0.61 in., 0.61 in., 0.95 in., and 3 in. as the respective inlet heights. The corrected rotation losses are approximately 30 kw., 15 kw., 10 kw. and 10 kw., respectively, for the second, third, fourth and fifth stages. The bucket output per lb. of steam in each stage is $61.5 \times 0.808 = 49.7$ B.t.u., and the total bucket output per stage $= 15{,}700 \times 49.7/3412 = 229$ kw. The shaft outputs for the last four stages are then 199, 214, 219 and 219 kw., or a total for all five stages of 1109 kw. Allowing 4 per cent. for bearing losses and leakage, net shaft output = 1064 kw. Output at generator terminals (overall efficiency, 94 per cent.) $= 1064 \times 0.94 = 1000$ kw. Theoretical water rate for last four stages $= 3412/61.5 = 55.5$ lb. per kw.-hr. Actual shaft water rates for the four stages are $(15{,}700/199 =)$ 79, 73.4, 71.6 and 71.6 lb., respectively, and the shaft efficiencies $(55.5/79 =)$ 0.703, 0.755, 0.775 and 0.775.

The energy corresponding to a single adiabatic expansion from 165 lb. per sq. in. abs. to 1 lb. abs. is 322 B.t.u., and the theoretical water rate therefore $(3412/322 =)$ 10.6 lb. of steam per kw. per hour. In the turbine considered 15,700 lb. of steam develop 1064 kw. at the shaft, giving a shaft water rate of $15{,}700/1064 = 14.75$ lb. per kw. per hour, and making the overall shaft efficiency of the turbine $10.6/14.75 = 0.719$.

REACTION TURBINES (PARSONS)

The reaction turbine is always a multi-pressure-stage turbine with small pressure drop per stage. Each stage consists of a stationary set of blades or nozzles and a row of rotating buckets. As there is a continuous drop of pressure throughout each stage, partial peripheral admission of the steam (as in the impulse turbine) is impossible. Steam must be admitted around the entire circumference of the wheels, and therefore the stationary blades must extend around the entire circumference. The total available pressure drop for each stage is divided into two parts, one being converted into kinetic energy in the stationary blades, and the other in the moving blades. The residual velocity from one stage is utilized in the next stage.

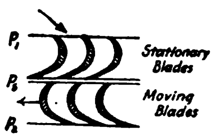


FIG. 9.—Reaction Turbine.

Details. The steam approaches each set of stationary blades (from the exit of the preceding stage) with a velocity of V_4 ft. per sec. and at an absolute pressure of p_1 lb. per sq. in. The steam expands adiabatically while passing through the stationary blades from pressure p_1 to p_s (see Fig. 9). The theoretical spouting velocity V_1 from the stationary blades is found from the available energy corresponding to the velocity of steam approach, V_4, plus the energy in the available pressure drop p_1 to p_s.

$$V_1 = 223.7\sqrt{(V_4/223.7)^2 + H_s} \qquad (9)$$

in which H_s is the available energy in B.t.u. per lb. of steam corresponding to adiabatic expansion from p_1 to p_s. This spouting velocity V_1 combined with the wheel speed U (see Fig. 10) gives the relative entrance velocity V_2 to the moving blades. The relative exit velocity from the moving blades is obtained from the available energy corresponding to the velocity V_2 plus the energy in the available pressure drop p_s to p_2. Or it can be expressed by the equation

$$V_3 = 223.7\sqrt{(V_2/223.7)^2 + H_2} \qquad (10)$$

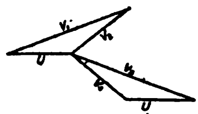


FIG. 10.—Bucket Velocity Diagram (Reaction Turbine).

in which H_2 is the available energy in B.t.u. per lb. of steam corresponding to adiabatic expansion from the absolute pressure p_s to the absolute pressure p_2. The absolute exit velocity V_4 from these moving buckets is found (Fig. 10) by combining V_3 with the wheel speed U. This absolute velocity V_4 is again available as the entrance velocity to the next stage.

Degree of Reaction. If H represents the B.t.u. corresponding to the total pressure drop p_1 to p_2 of any stage and H_2 the B.t.u. corresponding to the pressure drop p_s to p_2 in the moving blades alone, then H_2/H is the degree of reaction. Usually H_2/H is taken as 0.5; or the B.t.u. drop in the moving blades is equal to the B.t.u. drop in the stationary blades. This division of energy between the stationary and the moving blades is best obtained by having the entrance and exit angles of the stationary buckets equal respectively to those of the moving blades.

Grouping of Stages. As the pressure drop per stage is small, a reaction turbine has a large number of stages. It is usual to subdivide these stages into a number of main divisions, each with a different pitch diameter and usually called first, second, third, etc., drums. As the steam in passing through the turbine drops in pressure, its volume increases; and therefore the pitch diam. of the various drums increases from the first to the last. Experience has shown that three drums give a convenient arrangement for most reaction turbines; and for the usual ranges of steam pressures, the pitch diameters of the drums have usually a ratio of $1:\sqrt{2}:2$. The distribution of energy among the three drums in practice is approximately in the ratio of $1:1:1.5$.

Theoretically, each set of stationary and moving buckets on any one drum should either continually increase in height from one end of the drum to the other to accommodate the increasing volume of steam, or, with equal heights of blades, the various exit angles of the blades should gradually increase to accommodate the increase of steam volume. Practically, for constructive reasons, it is preferable to divide each drum into a number of groups, each group consisting of stationary and moving buckets of the same height and shape but so assembled that the exit angles of the buckets gradually grow larger as the steam volume increases. This is accomplished by "gaging" the buckets, that is, by turning the buckets in the successive rows after they are assembled on their respective drums, so that they have gradually increasing exit angles.

A convenient empirical formula for initial calculations of reaction turbines leading to the proper choice of the **number of stages per drum with three-drum construction**, is

$$z_1U_1^2 + z_2U_2^2 + z_3U_3^2 = \frac{25{,}000 \times H_0}{(k/K_a^2) - 1 + (2\cos a/K_a)} \quad (11)$$

in which z_1, z_2, and z_3 are the number of stages on the drums having respectively the wheel speeds U_1, U_2 and U_3 at their pitch diameters, H_0 is the total available B.t.u. for adiabatic expansion from the initial to the final steam pressures; k is a loss coefficient varying from 0.18 to 0.35, and generally assumed as 0.25 for the usual types of bucket construction; $K_a = U/V_1$ = wheel speed divided by steam velocity, and is usually assumed as 0.5; a is the mean exit angle of any group and usually assumed as 25 deg. Substituting the usual values in the above equation it can be expressed as

$$z_1U_1^2 + z_2U_2^2 + z_3U_3^2 = \frac{25{,}000H_0}{[0.25/(0.50)^2] - 1 + (2 \times 0.906/0.50)} = 6900H_0 \quad (12)$$

As the drum diameters are usually chosen in the ratio of $1:\sqrt{2}:2$, $U_2^2 = 2U_1^2$ and $U_3^2 = 4U_1^2$, so that equation (12) becomes

$$z_1 + 2z_2 + 4z_3 = 6900H_0/U_1^2 \quad (13)$$

The **wheel speed** U_1 can readily be obtained from the pitch diameter of the first drum (D_1) and the prescribed r.p.m. The pitch diameter of the first drum can be expressed by the equation

$$D_1 = \sqrt[3]{60G_1v_1KK_a/\pi^2e_1N \sin a} \quad (14)$$

in which G_1 = flow of steam through the buckets of the first drum, lb. per sec.; v_1 = initial specific volume of the steam, K = ratio of pitch diam. to bucket height; K_a = ratio of wheel speed to steam speed; e_1 = ratio of net or clear exit area from the buckets to the exit area if the buckets were infinitely thin; N = r.p.m.; and a = exit angle from the buckets of the first stage on the drum.

Leakage Losses. As there is a continuous drop of pressure from one end of a reaction turbine to the other, the radial mechanical clearances between the rotating buckets and the casing, and between the stationary buckets and the drums, are of considerable importance, because these clearance areas form a means for considerable leakage losses. The radial clearances are made as small as good mechanical operation permits, small clearances being of more importance in the first drum than in the succeeding drums on account of the small bucket heights and greater steam density. The mechanical clearance must be sufficient to allow for the stretch of the buckets and drum when subjected to centrifugal forces at running speeds, and also for expansions due to temperature changes of the revolving and the stationary parts. The leakage losses in the first drum are usually from 5 to 8 per cent.; in the second drum, from 3 to 6 per cent.; and in the third drum from 1 to 2½ per cent.; the lower values calling for very close clearances.

As all reaction turbines are subject to an axial **end thrust** of the rotating parts due to the difference of steam pressures at the ends of each drum, this thrust is usually balanced by **balancing pistons** mounted on the rotor running with close clearance to the casing and of the same diameter as the overall diameter of each drum. Each balancing piston is then subjected to the same difference of pressure as the rotating drums by means of steam-pipe connections, but so that the thrust is in the opposite direction from that of the rotating drums. In large turbines the drums are sometimes divided into two groups with steam flow in opposite directions to balance axial thrust without the use of a balancing piston. The leakage between the balancing piston and the casing is usually from 5 to 8 per cent. for the first piston; from 4 to 7 per cent. for the second piston; and from 2 to 3½ per cent. for the third piston.

Energy and Efficiency. The **B.t.u. drop** in the first stage of each drum can be obtained from the equation

$$H_1^{*} = [2(1 + k)V_1^2 - V_4^2]/50{,}040 \tag{15}$$

The B.t.u. drop for any other stage on a drum includes the energy in the steam velocity approaching the stage (exit velocity from preceding stage) and can be expressed by the equation

$$H_1 = 2[(1 + k)V_1^2 - V_4^2]/50{,}040 \tag{16}$$

The B.t.u. drop for the first group of $z_1{}^1$ stages on any drum is

$$H_s = [2z_1{}^1(1 + k)V_1^2 - (2z_1{}^1 - 1)V_4^2]/50{,}040 \tag{17}$$

The B.t.u. drop for any other group of (z^1) stages on any drum is

$$H_s = 2z^1[(1 + k)V_1^2 - V_4^2]/50{,}040 \tag{18}$$

The **efficiency** E **of any stage** is obtained from the equation

$$E = \frac{2(U/V_{1s})\cos a_s - (U/V_{1s})^2}{k + 2(U/V_{1s})\cos a_s - (U/V_{1s})^2} \tag{19}$$

The angle a_s being the exit angle of the rotating bucket of the stage in question, it is necessary to know this angle for any certain stage. As the B.t.u. drop per stage is known, the absolute pressure, quality and specific volume of the steam for any stage can be determined. Therefore the exit angle for the z stage can be found from the following relations:

$$V_{1s} \sin a_s = V_1 \sin a_1(v_s/v_1) \tag{20}$$

and

$$V_{1s} \cos a_s = V_1 \cos a_1 \tag{21}$$

in which V_{1s} is the absolute exit velocity from the stationary bucket of the z stage in question, which is also equal to the relative exit velocity from the moving bucket of the z stage (only in case the degree of reaction equals 0.5); a_s is exit angle of the rotating bucket of the z stage; v_s is the specific volume of the steam after the z stage; and V_1, a_1 and v_1 are the respective values for the first stage.

Equation (20) is a strictly rational formula, whereas (21) is an approximation based on general experience and used for all stages except where the steam volumes (and therefore the bucket heights) increase very rapidly, as in the last few stages of a reaction turbine. Combining equations (20) and (21), the **exit angle** can be solved by the relation

$$\tan a_s = \tan a_1(v_s/v_1) \tag{22}$$

and the value for V_{1s} can then be solved from equations (20) and (21).

Having angle a_s and the value of V_{1s}, the **efficiency** E in equation (19) can be solved. As equation (21) is only approximate, the final exit angle is sometimes revised, as will be explained in the numerical example following. The mean of the efficiencies of the first and the last stage in a group gives the mean efficiency E_{mean} of the group. Finally, the **losses** in the group are given by the equation

$$L = (1 - E_{\text{mean}}) \times \frac{2z_1{}^1(1 + k)V_1^2 - V_4^2}{50{,}040} \tag{23}$$

where L is the loss in B.t.u. in the group, this loss being returned to the steam as heat, thus increasing both the total heat and the quality of the steam at the end of the group.

Example. It is desired to design a reaction turbine to drive a 2000-kw. generator at 3600 r.p.m., using dry saturated steam with an initial pressure of 165 lb. per sq. in. abs., and exhausting at 1 lb. per sq. in. abs. A water rate of 16.5 lb. per electrical kw. per hour is considered satisfactory.

Three drums will be used with their pitch diameters in the ratio of $1 : \sqrt{2} : 2$. The total flow of steam $G = 16.5 \times 2000 = 33{,}000$ lb. per hour or 9.17 lb. per sec. In the first drum, the leakage in the clearance between the blades and opposing surfaces is from experience about 6 per cent., as is also that past the balancing pistons, so that steam flow through blades of first drum $= G_1 = 0.88G$. For second drum (blade leakage, 4 per cent.; piston leakage, 5 per cent.), $G_2 = 0.91G$, and for third drum (blade leakage, 1.5 per cent.; piston leakage when double steam-flow connection is not used, 2.5 per cent.), $G_3 = 0.96G$. From equation (14) the pitch diam. $D_1 = 1.125$ ft. $= 13.5$ in., assuming $K = 20$, $e_1 = 0.77$ and $a = 20$ deg., and the wheel speed $U_1 = 212$ ft. per sec., so that from equation (13), $z_1 + 2z_2 + 4z_3 = 49$, H_0 being taken as 322. If the total output is to be distributed among the three drums approximately in the ratio 1 : 1 : 1.5, $z_1 = 15$, $z_2 = 7$ and $z_3 = 5$ will be a satisfactory solution.

Design of First Drum. The fifteen stages on the first drum may best be divided into four groups, all the buckets in any one group being of equal height. In the first stage in each group the exit angle $= 20$ deg., $U = 212$, and $V_4^2 = U^2 + V_1^2 - 2UV_1 \cos a$. The B.t.u. developed in one stage (except in first stage of first group), from equation (17) $= 6.76$. The efficiency of the first stage in each group, from equation (19) $= 0.734$.

First Group (5 stages). Initial pressure $p_1 = 165$ lb. abs.; quality, 1.00; sp. vol., 2.753 cu. ft. per lb.; total heat, $H = 1195$ B.t.u. per lb. B.t.u. drop from equation (17) $= 34.9$; final total heat per lb. $= 1195 - 34.9 = 1160.1$ B.t.u., with corresponding pressure after adiabatic expansion of 108 lb., quality $= 0.969$, and sp. vol. $= 4.00$. Exit angle a_5 from fifth stage [equation (22)] $= 27° 55'$; this, however, gives too large a drop. Taking $a_5 = 28° 30'$, equation (20) gives $(V_1)_5 = 442$, whence $(V_4)_5^2 = 75{,}700$ and H_1, from (16) $= 6.74$ B.t.u. which is close enough to 6.76 B.t.u. before determined Angles of intermediate stages may be similarly calculated or interpolated between first and last stages. Velocity ratio of fifth stage $= 212/442 = 0.48$ and efficiency of stage, from (19) $= 0.71$. Mean efficiency of group $= (0.734 + 0.71)/2 = 0.722$. Losses, from (23) $= 9.4$ B.t.u. Neglecting losses due to radiation, these revert to the steam, making total heat per lb. at exit from first group $= 1160.1 + 9.4 = 1169.5$ B.t.u.

Steam flowing through blades of first drum $= 0.88 \times 9.17 = 8.07$ lb. per sec., sp. vol. $= 2.753$ amd $V_1 = V_3 = 424$, whence bucket exit area $= 8.07 \times 2.753/424 = 0.0524$ sq. ft. $= 7.54$ sq. in. With a 20° exit angle and $e_1 = 0.77$, annulus area between drum and outer ends of buckets $= 7.54/(\sin 20° \times 0.77) = 28.6$ sq. in.; height of all buckets in first group $= 28.6/13.5\pi = 0.675$ in. Before entering the second group of buckets the steam from the first group may be considered as mixing with the 6 per cent. of steam leaking through the clearance spaces. This leakage steam, having done no work, has a total heat of 1195 B.t.u. per lb., and the final total heat of mixture x as obtained from the equation $0.88G(x - 1169.5) = 0.06G(1195 - x)$, is 1171.1 B.t.u. per lb.

Second Group (4 stages). $p_1 = 108$ lb. abs.; $H = 1171.1$; quality, 0.981; sp. vol., 4.05. B.t.u. drop, from (18) $= 27$; final total heat $= 1171.1 - 27 = 1144.1$; corresponding pressure after adiabatic expansion $= 77$; quality, 0.959, sp. vol., 5.45. Exit angle a_4 from fourth stage, from (22) $= 26° 6'$, which is too small. Taking value of $26° 30'$, $(V_1)_4$ from (20) $= 437$ and $(U/V_1)_4 = 0.485$. Efficiency of fourth stage from (19) $= 0.717$. Efficiency of first stage of group is same as for first stage of first group, 0.734, hence mean effy., of group $= (0.717 + 0.734)/2 = 0.725$. Losses, from (23), $= 7.4$ B.t.u., which added to steam make total heat at exit $= 1151.5$ B.t.u. per lb. Bucket exit area $= 8.07 \times 4.05/424 = 0.0771$ sq. ft. $= 11.1$ sq. in., from which annulus area $= 42.1$ sq. in. and bucket height 0.994 in. Final total heat x after mixing with leakage steam, from $0.88G(x - 1151.5) = 0.06G(1171.1 - x)$, $= 1152.8$ B.t.u. per lb.

Third Group (3 stages). $p_1 = 77$ lb. abs.; $H = 1152.8$; quality, 0.969; sp. vol., 5.50. B.t.u. drop, from (18) $= 20.3$; final total heat $= 1152.8 - 20.3 = 1132.5$ B.t.u.; corresponding pressure after expansion $= 59$; quality, 0.952; sp. vol., 6.95. Exit angle a_3 from third stage, from (22) $= 24° 42'$. Taking a value of 25°, $(V_1)_3$ from (20) $= 433$, $(U/V_1)_3 = 0.489$, and efficiency of third stage, from (19) $= 0.722$. With efficiency of first stage of 0.734 as in previous groups, mean effy. $= (0.734 + 0.722)/2 =$

0.728, and losses by (24) = 5.5 B.t.u., which added to steam give total heat at exit of 1138 B.t.u. Bucket exit area of third group = 8.07 × 5.50/424 = 0.1047 sq. ft. = 15.08 sq. in., annulus area = 57.3 sq. in. and bucket height = 57.3/13.5π = 1.352 in. Final total heat x after main steam mixes with blade leakage, from $0.88G(x - 1138) = 0.06G \times (1152.8 - x)$, = 1138.9 B.t.u.

FOURTH GROUP (3 stages). p_1 = 59 lb. abs.; H = 1138.9; quality, 0.959; sp. vol., 7. The first stage of this group is similar to that in second and third groups, and B.t.u. drop = that in third group = 20.3. Final total heat per lb. = 1138.9 − 20.3 = 1118.6 B.t.u.; corresponding pressure after adiabatic expansion, 45; quality, 0.943; sp. vol., 8.8. Exit angle a_3 from third stage, from (22) = 24° 34′. Taking value of 24° 55′ $(V_1)_3$ from (20) = 433, and $(V_4)_3{}^2$ = 65,700. From (16), H_1 = 6.73 B.t.u., which agrees with value first determined (6.76 B.t.u.). Velocity ratio for stage = 212/433 = 0.49, and efficiency from (20) = 0.722. Mean efficiency = (0.734 + 0.722)/2 = 0.728. The losses in the fourth group are equal to those in the third, namely, 5.5 B.t.u. There is a residual velocity loss due to change in pitch diam. from first to second drum = $(V_4)_3{}^2$/50,040 = 1.3 B.t.u., making total loss = 6.8 B.t.u., and total heat per lb. of steam at exit = 1118.6 + 6.8 = 1125.4 B.t.u. Bucket exit area = 19.21 sq. in., annulus area, 73 sq. in. and bucket height, 1.72 in. Total heat after mixing with leakage = 1126.2 B.t.u. per lb. Before entering second drum 1 per cent. of the total steam that has traveled over balancing piston mixes with main steam entering second drum (leakage past balancing piston of first drum, 6 per cent., minus that past piston of second drum, 5 per cent.). As this (at 1195 B.t.u.) has done no work, the new total heat = x = 1126.9 B.t.u.

Design of Second Drum. Pitch diam. = 13.5 × $\sqrt{2}$ = 19.09 in., and wheel speed U = 300 ft. per sec. Because of larger B.t.u. drop per stage and the more rapid increase in sp. vol. and in required exit angle, the smaller number of stages allotted (7) is divided into three groups. In first stage of each group a = 20°, V_1 = 300/0.50 = 600, $V_4{}^2$ = 111,600. B.t.u. developed in one stage (except first stage of first group) from (17) = 13.54. Efficiency of first stage in each group, from (20), = 0.734. The method of design of the three groups follows the same processes as for the first drum.

Design of Third Drum. Pitch diam. = 13.5 × 2 = 27 in. Five stages will be used, each computed separately on account of the very rapid increase in sp. vol. In the first four stages exit angle a will be taken as 20 deg. and V_1 as 848 ft. per sec. Max. bucket height for mechanical reasons is taken as 5.4 in. As this is too small to accommodate the flow, a double-flow construction will be used, where the steam at exit from second drum will divide in parallel between two 27-in. drums at opposite ends of the turbine. The heights of buckets on each drum will then be only one-half of the values found in the computation of total bucket height.

For method of design of the five groups, follow the procedure as given for the first drum. The complete calculation yields the following results:

	First drum	Second drum	Third drum
Pitch diameter, in.	13.5	19.09	27.0
Initial pressure, lb. per sq. in. abs.	165.0	45.0	11.0
Quality of steam	1.0	0.952	0.919
Flow through buckets, lb. per hr., G*	29,040.	30,030.	32,550.
Total bucket B.t.u. developed per lb. of steam, P	74.4	69.8	102.1
Total bucket output (GP/3412), kw.	633.0	614.0	975.0

	Group No.	Number of stages	Available B.t.u.	Mean efficiency	Bucket B.t.u.	Total bucket height, in.
First drum	1	5	34.9	0.722	25.2	0.675
	2	4	27.0	0.725	19.6	0.994
	3	3	20.3	0.728	14.8	1.352
	4	3	20.3	0.728	14.8	1.720
Second drum	1	3	42.8	0.714	30.6	1.022
	2	2	27.1	0.724	19.6	1.767
	3	2	27.1	0.723	19.6	2.510

* 33,000 × 0.88, 0.91, and 0.985, respectively.

	Group No.	Number of stages	Available B.t.u.	Mean efficiency	Bucket B.t.u.	Total bucket height, in.
Third drum....	1	1	31.5	0.734	23.1	2.910
	2	1	27.0	0.734	19.8	4.310
	3	1	27.0	0.734	19.8	6.580
	4	1	27.0	0.734	19.8	10.400
	5	1	28.1	0.708	19.7	10.800

The total bucket output = 633 + 614 + 975 = 2222 kw. Allowing 5 per cent. of this output for radiation and for mechanical loss, shaft output = 2222 × 0.95 = 2111 kw. With a generator efficiency of 94.7 per cent. the electrical output is 2000 kw. and the water rate per electrical kw. is 33,000/2000 = 16.5 lb. per hour.

LOW-PRESSURE, MIXED-PRESSURE, AND EXTRACTION TURBINES

Low-pressure Turbines are those designed to utilize low-pressure steam, usually exhaust steam from other apparatus (at pressures ranging from atmospheric to 25 lb. gage), and should operate exhausting into high vacua. Installations having large reciprocating engines can greatly increase the overall steam economy and output by the addition of low-pressure turbines taking the steam from the engines at pressures slightly above atmosphere. The expansion of the steam in the cylinders of reciprocating engines is limited by mechanical difficulties, because cylinder and piston diameters become mechanically impracticable if expansion to a high vacuum is attempted, while on the other hand steam turbines can easily accommodate almost any volumes of steam by providing sufficient steam-flow area. Generally speaking, a plant having first-grade reciprocating condensing engines can be remodeled by the installation of low-pressure turbines so that the reciprocating units run non-condensing and exhaust into the turbines. The additional capacity thus secured varies with local conditions, but is usually about 100 per cent. That is, for each 1000 h.p. output of the condensing engines the remodeled plant will have approximately 2000 h.p. output, with a moderate increase in the steam output of the boilers. By resetting their valve motions the engines are made to give their former output running non-condensing, and the low-pressure turbines are designed to produce equal output with the available low-pressure steam from the reciprocating units. The methods used in the design of low-pressure turbines are exactly similar to those described for high-pressure turbines.

Governing of Low-pressure Turbines. If both the engine supplying the exhaust steam and the low-pressure turbine are driving electric generators, the two generators, after being brought up independently to their correct speeds, should be interlocked electrically by being connected to the same busbars; as the load varies, the engine automatically supplies the turbine with the necessary amount of steam to enable the latter to take its proper share of the load.

If the engine is under a mechanical load while the turbine has an electrical load, a synchronous motor connected to the busbars of the turbine generator and also belted to the countershaft driven by the engine serves as a convenient means for transferring power from the turbine to the engine and *vice versa*.

If the load on the engine is of an intermittent character, a regenerator or heat accumulator is frequently used, in which the excess steam from the engine is condensed in water, and from which steam is evaporated and sent to the turbine when the steam supply from the engine is cut off. Or, one or two stages may be added to the turbine to take high-pressure steam when the low-pressure steam fails, this stage or stages being automatically cut out when the low-pressure steam is again available. (See Mixed-pressure Turbines.)

Mixed-pressure Turbines are designed to operate with two sources of steam supply, a high-pressure source and a low-pressure source. They are usually low-pressure turbines which automatically utilize high-pressure steam when the low-pressure steam supply fails or is insufficient in amount to carry the load. They can be designed as a very highly efficient low-pressure turbine with a high-pressure end of poorer efficiency and merely considered as a reserve or protection against loss of power when the low-pressure steam is not available; or they can be designed as a highly efficient high-pressure turbine utilizing incidentally any available low-pressure steam. An ordinary straight high-pressure turbine operating at various loads does not have constant stage pressures, and in designing a mixed-pressure turbine this fact must be recognized. If an efficient low-pressure turbine is desired, some means of holding a constant pressure at the stage into which the low-pressure steam is introduced is desirable. If, however, high-pressure steam economy is more important, the stage pressure should be permitted to vary and the low-pressure steam introduced at a pressure equal to or greater than the pressure existing in that stage when the turbine is carrying full load with high-pressure steam only.

Extraction Turbines. High-pressure turbines which permit the extraction of some of the low-pressure steam from any of their stages are known as extraction turbines. These turbines are used where quantities of low-pressure steam are needed constantly or intermittently. If used constantly, the areas for steam flow in the latter stages of the turbine are designed for less steam than passes through the high-pressure stages. Should the low-pressure extraction cease and all the steam be required to pass through the final exhaust of the turbine, the stage pressures would increase and the turbine would operate less efficiently. If, however, the steam extraction is intermittent and the steam flow to the turbine exhaust is usually the full amount, the latter stages of the turbine are designed for full steam flow, which results in a somewhat lower efficiency when steam is being extracted.

It is possible to design a mixed-pressure turbine so that it can be an extraction machine when desired. Some installations require turbines which will utilize low-pressure steam from various sources when there is an excess of this low-pressure steam, and will automatically supply low-pressure steam when there is a deficiency. To accomplish this the turbine governor controls both the supply of low- and of high-pressure steam by means of suitable valves. An auxiliary governor responsive to the stage pressure in which the low-pressure steam is introduced influences the main governing so that constant stage pressure is held. If the stage pressure tends to drop due to a diminution of the low-pressure steam supply, the governor, in order to hold constant stage pressure, admits more high-pressure steam; and when more low-pressure steam again becomes available, the stage pressure tends to rise and the governor cuts off some of the high-pressure steam, and the turbine utilizes more low-pressure steam.

TURBINE TYPES

During the life of the fundamental turbine patents, turbine types were more distinctive than to-day. At present there is a gradual tendency toward a more uniform type, especially in the case of large units.

CURTIS TURBINE OF THE GENERAL ELECTRIC CO.

The Curtis turbine is the most extensively used type of turbine in the United States. It is built in sizes of from less than 1 kw. up to 40,000 kw. capacity. Although some of the large units have been built with a vertical shaft, the horizontal-shaft machine is now available in all sizes. The smaller units up to about 100 kw. are designed with a single pressure stage having two or three velocity stages and throttling governors. From about 100 kw. to 3000 kw. capacity the turbines are built with two to five pressure stages, the first stage having usually two bucket rows (two velocity stages) and the succeeding stages either one or two bucket rows. Fig. 11 gives a sectional view of a 2000-kw. four-stage turbine. The governor controls a series of double-seated valves which admit steam to successive groups of nozzles in the first stage. The turbine is primarily arranged for condensing operation, but is provided with an exhaust opening from the second stage if it is necessary to operate at times non-condensing. From 3000-kw. upward the number of pressure stages varies from five to twelve, the first stages usually having two bucket rows and the succeeding stages having one bucket row each. In the very large units, the last one or two stages are arranged for double steam flow; that is, the steam passes in parallel through two wheels instead of one. This is necessary on account of the large volumes of steam and the otherwise excessive heights of buckets.

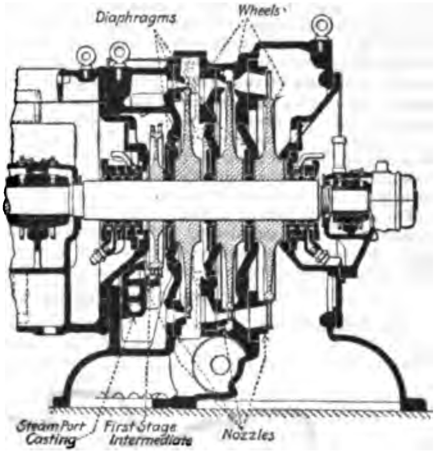


FIG. 11.—2000-kw. Four-stage Curtis Turbine.

The peripheral wheel speeds are generally between 400 and 850 ft. per sec. The buckets are inserted in the wheel rim by a dovetail construction as shown in Fig. 12, and are surrounded by a thin shroud mounted in sections and secured to each bucket. The buckets are made of a special non-corrosive copper alloy, nickel steel or chrome-vanadium steel according to the height of bucket and the peripheral speed employed. The buckets are equally spaced around the circumference of the wheel either by space blocks inserted in the dovetail of the wheel between the buckets, or by having these spacing blocks forged integral with the bucket. The nozzles are either expanding or non-expanding. In the first and earlier

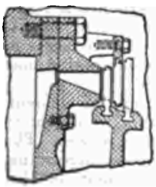


FIG. 12.—Mounting of Buckets in Curtis Turbine.

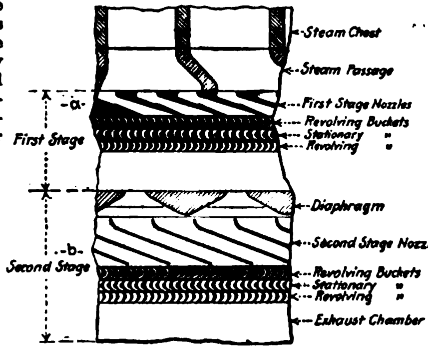


FIG. 13.—Arrangement of Nozzles and Buckets in a Two-stage Curtis Turbine.

stages where the steam temperatures are high the nozzles are machined out of bronze alloy, as shown at *a* in Fig. 13, while in the latter stages, the nozzles are made of plates of nickel steel cast into a nozzle ring of cast iron, as shown at *b* in Fig. 13, or cast directly into the diaphragms between stages.

To prevent steam leakage from stage to stage or to the atmosphere, either carbon packings or metallic labyrinth packings are employed. Carbon packings consist of one or more rings of pure carbon made in sections of 90 or 120 deg. and held toward the shaft with small clearances by means of springs. Metallic labyrinth packing is made of a light special alloy and is used similarly to the carbon packing. The labyrinth packing has the usual sawtooth arrangement where it comes in contact with the shaft, so that it will readily wear its own clearance when the shaft rotates. To prevent steam leakage from the high-pressure end to the atmosphere and air from the atmosphere to the vacuum end of the turbine, the packing is arranged in the form of three to five rings. On the high-pressure end the leakage of steam past the first few rings is carried to some of the low-pressure stages of the turbine, so that the outer rings need only prevent the leakage of this low-pressure steam to atmosphere. On the vacuum end, low-pressure steam is introduced between the packing rings in just sufficient quantity to supply the leakage toward the vacuum end of the turbine, and again the outer rings are employed to prevent the leakage of this low-pressure steam to atmosphere.

The turbine governor is connected by levers directly to a pilot valve which operates a steam- or oil-actuated piston. This piston controls directly or by means of a shaft and cams the successive opening or closing of a series of valves. Each valve controls the admission of high-pressure steam to a certain number of nozzles in the first stage.

WESTINGHOUSE TURBINES

The Westinghouse Machine Co. builds turbines ranging in size from 1 to 30,000 kw. The smaller machines up to 200 kw. capacity have a re-entry impulse wheel furnished with one row of buckets. The steam being expanded in a nozzle, passes the

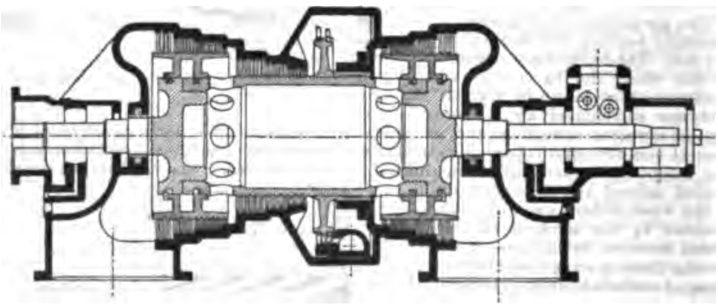


Fig. 14.—Westinghouse Turbine with Semi-double-flow Rotor.

wheel, and by means of suitably formed reversing chambers is re-directed into the same row of buckets. Depending upon the operating conditions, there may be two or more expansions, all operating on the same row of blades. In the larger machines a combination of impulse and reaction blading is used. The combination machines are built (1) single-flow, (2) semi-double-flow, and (3) double-flow.

In the single-flow unit, the steam after acting on the impulse element flows in one direction only, and the end thrust is compensated either wholly or in part by balance pistons provided with labyrinth packings on their outer edges. The reaction blading may be carried on drums of one or two different diameters, depending upon the size of the machine. This type is used for 60-cycle units up to about 1000 kw., and for 25-cycle up to about 1500 kw. capacity. Fig. 14 shows a semi-double-flow rotor such as is used for 1800-r.p.m., 60-cycle machines up to 6000 kw. capacity, and for 25-cycle 1500-r.p.m. machines up to about 7500 kw. normal rating. The intermediate reaction blading is made in a single section large enough to pass the entire quantity of steam. A balance piston similar to that used on the single-flow type (shown at the right of the impulse wheel) compels all of the steam to pass through the single intermediate section of the reaction blading and balance the end thrust due to this section. When the steam issues from the intermediate section the current is divided, one-half passing directly to the adjacent low-pressure section, while the other half passes through holes in the periphery of the hollow rotor and through the rotor itself, beyond the dummy ring, into the other low-pressure section at the right-hand end of the turbine.

In the double-flow rotor, the steam after acting on the impulse element divides, one-half going through the reaction blading at the left, the remainder passing over the top of the impulse wheel and through the reaction blading at the right. The double-flow construction is used for 60-cycle, 1800-r.p.m. units up to 15,000 kw., 60-cycle, 3600-r.p.m. units up to 5000 kw. and for 25-cycle, 1500-r.p.m. units up to 20,000 kw. capacity. The larger turbines may have their expansion carried out in two separate elements, and it is sometimes convenient to operate these at two different synchronous speeds, permitting the use of reaction blading in both elements.

Leakage along the turbine shaft at the ends of the casing is prevented by a water packing. This is virtually a single-stage centrifugal-pump impeller mounted on the shaft and supplied with water. Centrifugal force confines the water at the periphery of the impeller and between it and the surrounding packing casing. The impeller is made of sufficient diameter to seal against the pressure differences between casing and atmosphere. In the smaller turbines the governor acts directly on the steam admission valves, opening first the primary and then the secondary valve. Governors for the larger turbines employ an oil-relay mechanism for operating the steam valves. Non-condensing and low-pressure turbines are both built of reaction elements only, the latter generally being double-flow. Bleeder turbines are generally of the combined type.

DE LAVAL TURBINE

The original De Laval turbine was a single-pressure-stage impulse turbine with one revolving row of buckets, rotating at high speeds, usually from 10,000 to 30,000 r.p.m., and connected to generators or other apparatus by means of double helical reduction gears. The multi-stage turbine built by the De Laval Steam Turbine Company, Fig. 15, is of the impulse type and is a pure pressure-stage turbine, each stage consisting of a De Laval single-stage turbine; in some designs, however, the turbine is divided into a number of pressure stages, each stage contains several velocity stages. It is built in capacities from 50 h.p. to 15,000 h.p., with the turbine wheels mounted directly on a rigid shaft. The maximum peripheral bucket speed is not greater than 650 ft. per sec.; maximum rotative speed, 6000 r.p.m.

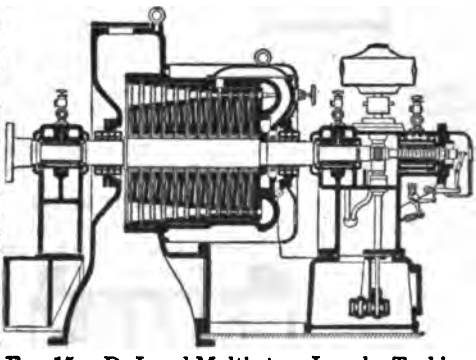


FIG. 15.—De Laval Multi-stage Impulse Turbine.

The De Laval single-stage turbine is built in capacities up to 600 b.h.p., peripheral speeds up to 1300 ft. per sec. being employed. The turbine wheel is made of chrome-nickel steel without any hole in the center, the shaft being joined to the disk by flange connections. On account of the high rotative speed and the mechanical impossibility of perfect rotative balance the shaft is made small in diameter, thus permitting the turbine rotor to revolve more nearly about its own true center of gravity. The buckets are of forged steel, with bulb shanks which fit into axial cylindrical grooves in the bucket wheel; on the outer end of the bucket the contributing section of the ring is forged as an integral part.

INGERSOLL-RAND TURBINES

The Ingersoll-Rand Co. turbines are primarily designed for driving turbo-compressors and turbo-blowers, and are generally of the mixed-pressure type. They are built in sizes up to 5000 kw., and designed for high-speed operation. High-pressure turbines are also built for driving electric generators through reduction gears.

Ingersoll-Rand turbines are of the impulse type and the high-pressure steam turbine has five to six wheels with single rows of buckets, the first wheel being of larger diameter than the following wheels. The first wheel works with partial, and the following low-pressure stages with nearly full, peripheral admission; peripheral velocity of wheels, 500 to 750 ft. per sec. Fig. 16 shows a typical high-pressure steam turbine used for turbo-compressors, having a capacity of 450 kw. at 6200 r.p.m.

The I.-R. mixed-pressure turbines are mostly designed on the double-flow principle and the steam admission to the low-pressure turbines is controlled by two separate inlet gears which open one after the other as the load increases. The discharge steam of the high-pressure stage passes to one of the low-pressure turbines only. The efficiency is practically constant from 40 per cent. load up to full load, when exhaust steam alone is used. Two kinds of steam control are used—steam throttling and nozzle regulation.

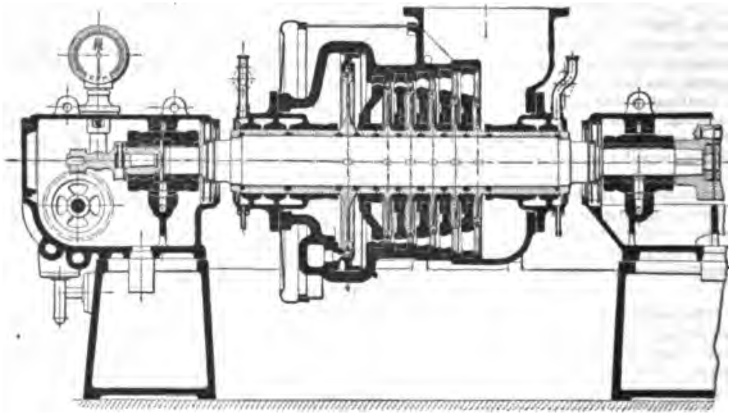


Fig. 16.—Ingersoll-Rand High-pressure Impulse Turbine.

KERR TURBINE

The Kerr turbine (Fig. 17) is of the multi-pressure-stage type with one bucket row per stage, and is built in sizes from 1 h.p. to 1500 h.p. The turbine is divided into stages

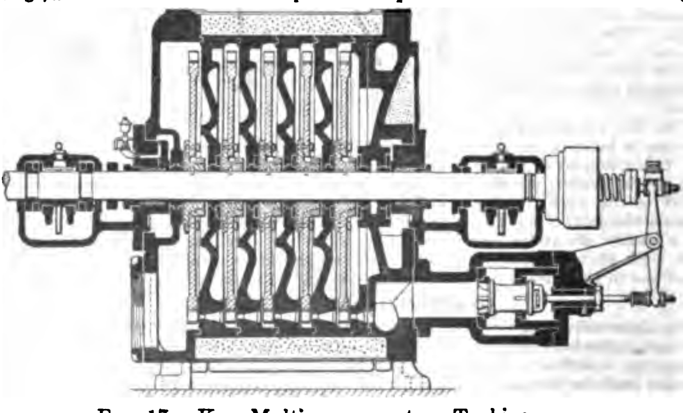


Fig. 17.—Kerr Multi-pressure-stage Turbine.

vertically, the wheels being mounted on the shaft by means of detachable hubs. In sizes from 300 to 1000 kw., the turbine casings are split horizontally, so that the upper half of the casing, frequently carrying the upper halves of the diaphragms with it, can be removed and the rotor taken out without breaking the steam and exhaust connections. The smaller turbines are built with from two to eight stages, and the larger machines up to thirteen stages, depending upon the power developed, steam pressure and vacuum. Governing is accomplished by throttling.

The buckets, made of 3.5 per cent. nickel steel, are set into dovetailed slots in the rim of the wheel and stiffened by a shroud ring. Peripheral speeds, from 300 to 600 ft. per sec. The nozzles are of the non-expanding type, except in the case of two- and four-stage turbines, or in mixed-pressure turbines. They are formed by casting in the diaphragms nickel-steel blades about 1/16 in. in thickness.

TERRY TURBINE

The Terry turbine is usually built as a single-pressure-stage turbine (with multi-velocity staging) having tangential admission of steam. The wheel has buckets or semi-circular recesses milled into its periphery, mounted around which are groups of nozzles with contiguous reversing chambers. The steam is expanded down to the exhaust pressure in a single nozzle, and, after striking one side of the wheel bucket, is reversed in direction, leaving the opposite side of the same wheel bucket. It then enters the first stationary bucket or reversing chamber, Fig. 18, which directs the steam back again into another bucket of the same wheel. This operation is repeated as many times as is necessary to reduce the kinetic energy in the discarded steam to a relatively low value. The turbine casing is split horizontally, the steam and exhaust connections being in the lower half, and is subjected to exhaust pressure only. Steam leakage between the shaft and the casing is prevented by bronze glands with an annular ball seat, no soft packing being used. Governing is by throttling. The single-wheel Terry turbine is used for non-condensing service and for condensing service in the smaller sizes. In the larger sizes, the condensing turbines are a combination of the above-described wheel for the high-pressure end with a series of single-velocity multi-pressure stages for the low-pressure end.

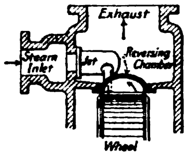


FIG. 18.—Terry Turbine.

THE MARINE TURBINE

Its smaller dimensions and weight for a given output, the rapidity with which it may be reversed, and its comparative freedom from vibration, have among other important reasons caused the steam turbine in recent years to be more and more frequently preferred for marine work, especially for torpedo boats and cruisers, and for fast passenger steamers. The main objection against its universal adoption on shipboard has been the fact that the steam turbine is essentially a high-speed machine, while ship propellers designed for very large powers must run at low relative speeds. Some of the methods by which this difficulty is overcome, are described in the following paragraphs.

Series Arrangement. In the Parsons marine turbine the expedient has been adopted of having the total steam expansion from boiler pressure to condenser pressure take place successively in several separate turbines, each turbine driving an independent propeller. Thus, in a 16,500-h.p., 290-r.p.m. turbine for an 18-knot steamer, the total power is distributed between three turbines, the high-pressure turbine driving the central shaft and exhausting into two low-pressure turbines located in the wings and taking the exhaust steam in parallel. The pitch diameter of the high-pressure turbine is over 6 ft., and that of the low-pressure turbines is nearly 9 ft. The reversing turbines are mounted on extensions of the low-pressure drums, their casings forming a continuation of the low-pressure casings.

Velocity Stages. In the Curtis marine turbine both the number of velocity stages per wheel and the number of pressure stages are much larger than in stationary turbines. On the U. S. Destroyer "Perkins," the two Curtis turbines (mounted on independent propeller shafts) developed a total of 12,000 h.p. at 600 r.p.m., attaining a speed of 30 knots. At 300 r.p.m. the combined output was 1550 h.p., and the speed 16.6 knots. Each turbine comprises six Curtis wheels, of which the first has four bucket rows and the others three rows each, and eight low-pressure stages arranged on a single drum. The common pitch diameter is 6 ft. The reversing turbine located in the same casing with the main turbine consists of two wheels, each having four bucket rows.

Gear Reduction. By the interposition of suitable gearing, the r.p.m. of the turbine and of the propeller become independent of each other, and each may therefore be designed for the r.p.m. at which it works most advantageously. On the U. S. S. "Neptune" a 4000-h.p. turbine running at 1250 r.p.m. drives a propeller shaft at 130 r.p.m. through a gear and pinion of the double-helical or herringbone type. The gearing in this case has been designed by Messrs. Melville and McAlpine, and is characterised by the elastic support of the pinion shaft in a floating frame carried on hydraulic rams. This support renders the gearing practically noiseless and insures automatically more nearly perfect alignment between gear and pinion under all conditions. With turbines of smaller output satisfactory operation with reduction gearing has been obtained without the use of a floating frame. The transmission efficiency of reduction gearing may exceed 98 per cent.

Hydraulic Transmission. In this method the shaft of the steam turbine carries two centrifugal pump impellers, each impeller being enclosed by a casing which contains suitable stationary passages and a water-turbine wheel. The two water-turbine wheels are mounted directly on the propeller shaft, which is entirely independent of the steam-turbine shaft. In operation, one of the casings is filled with water which circulates through the pump, stationary passages, and water wheel, driving the latter at the r.p.m. that the propeller was designed for. To reverse the direction of rotation, the first casing is emptied out and the water quickly transferred to the second casing, the water wheel of which is designed to drive the propeller in the opposite direction. In an hydraulic transmission gear designed by Prof. Föttinger for a 10,000-h.p., 850-r.p.m., steam turbine driving a propeller shaft at 170 r.p.m., a transmission efficiency of nearly 90 per cent. was attained.

Electrical Transmission. In this method the propeller may be driven by an electric motor mounted directly on its shaft, and supplied with current from a high-speed turbo-generator set located in any convenient part of the ship. On the U. S. Collier "Jupiter," a 5000-kw., nine-stage steam turbine driving a bipolar 2200-volt, 2000-r.p.m., 3-phase alternator, supplies current to two induction motors, each of which drives a propeller shaft at 110 r.p.m. The facility of starting, stopping, speed variation, and reversal, and the ability to tell at any time the r.p.m. and the power consumed by the propellers, are important advantages.

Reversing Turbines. Where the driving turbine is mounted directly on the propeller shaft, an additional reversing turbine must be supplied for use during maneuvering. As the usual output of the reversing or astern turbine is only from 25 to 50 per cent. of that of the ahead turbine, and as steam consumption during maneuvering is only of secondary importance, the astern turbine comprises comparatively few stages and the wheels are of a smaller diameter. This turbine is generally located close to the low-pressure part of the ahead turbine, and, when not in active operation, it runs in a vacuum with a low rotation loss. The reversing process requires very little skill and consists merely in closing the valve of the ahead turbine and opening that of the astern turbine.

Combination Drive. Steam turbines may be used to drive some of the propellers, the others being driven by steam or oil engines. On the French liner "Rochambeau," two reciprocating steam engines drive the two central shafts and each engine exhausts into a nearby low-pressure steam turbine driving a propeller in the wings. During maneuvering, the steam turbines are by-passed and the engines exhaust directly into the condensers, thus obviating the necessity for a reversing turbine. Similarly, English destroyers have been designed with oil engines driving the center shaft during slow-speed cruising, and with steam turbines driving the wing shaft during fast travel.

GENERAL TURBINE DATA

Foundations. Turbine foundations need not be as heavy as those used for reciprocating apparatus. On the other hand, high-speed apparatus produces vibrations which are sometimes not perceptible at the unit itself, but which may be transmitted to other parts of a building which vibrate in synchronism and become more marked than at the source. For this reason it is desirable not to tie turbine foundations to the structural ironwork of a building. Where this is necessary, the ironwork must be reinforced and the flooring or floor plates of the station must not be connected to the foundation.

Expansion Joints. If the foundations are satisfactory the only external means by which a turbine can be pulled out of alignment is by the steam and exhaust piping. The steam pipe should have large bends to allow for heat expansion. The steam-exhaust piping is of larger dimensions and should be installed with expansion joints, preferably of corrugated sheet copper of about one pipe diam. length between flanges. It is preferable to install the expansion joint directly to the exhaust flange of the turbine and so to anchor the succeeding piping that no movement or strains from the exhaust piping beyond the expansion joint can be transmitted to the turbine.

Efficiencies. An approximate idea of the water rate of the steam turbine may be obtained from the knowledge of the number of wheels, the number of bucket rows per wheel, the pitch diameters of the various wheels, and the r.p.m. For most commercial machines within the usual range of sizes, the shaft efficiency of the steam turbine is a function only of the summation of the squares of its wheel speeds, each bucket row counting as a distinctive wheel. Consequently, if a few sizes of a given type of turbine have been tested out and their shaft efficiencies have been plotted against the summation of the squares of the wheel speeds, the shaft efficiencies of any intermediate sizes operating under approximately the same steam conditions may safely be predicted. In Table 6 the efficiencies of a number of impulse and reaction turbines are given against a function $K = N(d/10)^2(\text{r.p.m.}/100)^2$, where N is the total number of rows and d the pitch diameter in in. of the various wheels. These efficiencies hold good only for the particular load for which the turbine is best suited.

Table 6

$K/1000$	40	60	80	100	140	180	220
Overall efficiency of turbine, per cent.	54	60	64	67.5	72	75	78

Water Rates and Prices. On account of the decided differences in actual turbine design, and more so on account of the various uses to which turbines are applied, no general statement as to water rates for any given size can apply. Industrial turbines for driving centrifugal pumps or other plant auxiliaries usually run non-condensing and the water rate is of no moment on account of the availability of the exhaust steam for heating purposes. These machines are designed with a view of obtaining low costs, and the water rate is poor. Larger turbines for driving centrifugal pumps or centrifugal compressors are designed for high economy, and are more costly per horse power than the former class. They have not as yet been as fully standardized as the turbines for generator drive. Table 7 gives approximately the limitations of water rates that can be expected for turbo-generators. The better figure is probably the best that can be obtained in general practice.

Table 7. Water Rates and Prices of Turbo-generators*

(60-cycle, 2300-volt a-c. generators)

Condensing						Non-condensing		
Steam at 150 lb. gage, 28-in. vacuum			Steam at 200 lb. gage, 150 deg. superheat, 29-in. vacuum			Steam at 150-lb. gage, exhaust to atmosphere		
Rating, kw.	Water rate, lb. per hr. per kw. incl. excitation	Price per kw. of rating	Rating, kw.	Water rate, lb. per hr. per kw. incl. excitation	Price per kw. of rating	Rating, kw.	Water rate, lb. per hr. per kw.	Price per kw. of rating
50	32.0 to 42.0	....	4,000	12.0 to 13.3	$11.2	50	52.0 to 60	....
100	21.2 to 27.5	$45.0	5,000	11.9 to 13.0	10.9	100	43.5 to 50	$42.0
200	19.5 to 25.0	33.0	7,500	11.6 to 12.6	10.5	200	38.4 to 44	30.0
300	18.7 to 22.5	26.0	10,000	11.2 to 12.3	....	300	36.5 to 42	24.0
400	18.2 to 21.0	21.7	12,500	11.1 to 12.0	....	400	35.2 to 40	20.3
500	17.7 to 19.3	19.0	15,000	11.1 to 11.8	....	500	34.1 to 38	18.5
600	17.4 to 19.0	17.5	17,500	11.0 to 11.6	....	600	33.4 to 37	17.1
750	17.0 to 18.8	15.8	20,000	11.0 to 11.4	....	750	32.7 to 36	15.5
1,000	16.5 to 18.6	14.3	25,000	10.9 to 11.3	....	1,000	31.7 to 35	14.1
1,250	16.2 to 18.3	13.7	30,000	10.8 to 11.2	....	1,250	31.0 to 34	13.5
1,500	16.0 to 18.0	13.3	35,000	10.7 to 11.1	....	1,500	30.7 to 33	13.2
2,000	15.7 to 17.7	12.6	40,000	10.6 to 11.1	....	2,000	30.1 to 32	12.5
2,500	15.5 to 17.4	12.1	50,000	10.5 to 11.0	....	2,500	29.4 to 31	12.0
3,000	15.4 to 17.1	11.7	60,000	10.5 to 10.9	....	3,000	28.9 to 30	11.6
3,500	15.3 to 16.9	11.4	75,000	10.4 to 10.8	....	3,500	28.6 to 29	11.3

* Speeds: 50 to 3500 kw., inc., 3600 r.p.m.; 4000 to 20,000 kw., inc., 1800 r.p.m.; 25,000 kw. and over, 1200 r.p.m.

The **prices** given in the table can be used for general estimating purposes, but are subject to large fluctuations; higher-class turbo-generators cost more than the prices quoted, and turbo-generators of poorer water rate can probably be obtained at lower prices.

CONDENSATION

BY

G. A. ORROK

REFERENCES: Gebhardt, "Steam Power Plant Engineering," Wiley. Morrow, "Steam Turbine Design," Longmans Green. Hausbrand, "Evaporating, Condensing and Cooling Apparatus," Scott, Greenwood & Co. Dalby, "Heat Transmission," *Proc. Inst. M. E.*, 1909. Orrok, *Trans. A. S. M. E.*, vols. 32 and 34.

General. Condensation is either that by direct contact between steam and water, as in the case of the jet condenser; or surface condensation, where a film of metal prevents the mixing of the steam and the cooling medium, as in the case of the ordinary surface condenser type. The cooling medium is generally water. Direct-contact condensers may be divided into three classes: (*a*) jet (*b*) barometric, and (*c*) ejector condensers. Surface condensers may be classed as (*d*) water-cooled, (*e*) air-cooled, and (*f*) evaporative.

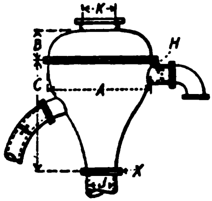
In a **jet condenser** the exhaust steam and the cooling water enter at or near the top of the condenser bell and the steam is condensed by the water falling in a fine spray, the resulting mixture of condensed steam, cooling water and air being removed by a tail pump; or an independent pump may be used to remove the air. For high vacuum, the independent air pump is necessary. The tail pump is replaced by the tail pipe in the **barometric condenser,** and the cooling water and condensed steam flow from the bottom of the tail pipe without the aid of a pump. An independent air pump is generally used, but is not necessary with some types. The **ejector condenser** acts on the same principle as a steam ejector; the exhaust steam enters the ejector through a series of orifices and is condensed by the cooling water, resulting in the water being given a velocity high enough to discharge it against atmospheric pressure. This type is not suitable for intermittent service or for a load which varies widely.

In a **water-cooled surface condenser** the water flows through tubes and the steam is condensed by being brought into contact with the outside of the tubes. The steam generally enters at the top of the condenser and the cold water first passes through the lower tubes and then through the upper tubes. This is known as the counter-flow type. If the condensing water enters at the top near the steam inlet, the condenser is of the parallel-flow type. In **air-cooled condensers** the heat is removed from the steam by the passage of cool air through the tubes or over plates. In **evaporative condensers,** steam is brought into contact with one side of a plate or tube and water is allowed to flow over the other side. The heat which is absorbed by the water is carried away by a stream of air passing over the surface of the water, part of the water being evaporated.

DIRECT-CONTACT CONDENSERS

Condensing Water Requirements. Let t_s = temperature of the steam to be condensed, t_0 that of the injection water, and t_1 that of the outlet or hot-well water, all in deg. fahr.; H = total heat in the steam at t_s, and q = heat in the liquid at t_1, both in B.t.u. per lb.; W = weight of steam per hour to be condensed, and Q = weight of water per hour needed for condensing, both in lb. Then $Q/W = R = (H - q)/(t_1 - t_0)$, and R is the ratio of water to steam required for condensation. $H - q$ may usually be taken as 1000 B.t.u. Theoretically, t_1 is equal to t_s, but in practice it is from 5 to 15 deg. lower than t_s, owing to the presence of air and to imperfect mixing.

In proportioning ordinary jet or barometric condensers, W is the normal amount of steam to be condensed; a 50 per cent. overload is common at some reduction of vacuum.



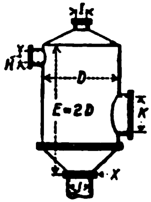


FIG. 1. FIG. 2.
Jet Condenser Bells.

Design of Condenser Bells. Good proportions are as follows (see Figs. 1 and 2): Cubical contents of cone $= G = 0.00143W + 8.25$ cu. ft. With an allowable velocity of 5 ft. per sec. in the tail pipe, $J = 0.0765\sqrt{W}$ in., $A = 15.7\sqrt[3]{G}$ in., $B = 0.3A$, and $C = 1.2A$. Diam. of injection pipe $H = J - 1$ in. ($J - 2$ in.) for small (large) sizes. Diam. of air pipe $I = J/3$ for 26- to 27-in. vacuum, to be increased slightly for 28-in. vacuum. The steam velocity in K is about 600 ft. per sec. K is equal approximately to $\sqrt{W/50}$. In the barometric type the height of the flange X above the level of the hot well should never be less than 35 ft., and may be greater with advantage. In the counter-current type, Fig. 2, D (inches) $= 20 + 0.125\sqrt{W}$.

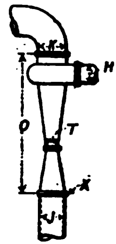
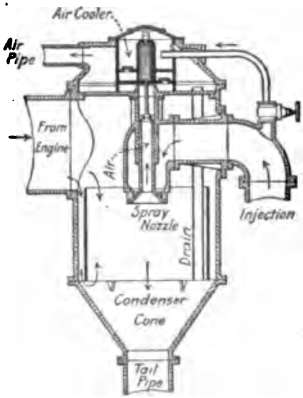


FIG. 3. Ejector Condenser.

The condenser bell should be as near to the exhaust flange as possible, as friction and velocity head increase rapidly with increasing vacuum. In the ordinary jet condenser the bell is generally placed over the suction chamber of the pump. In all arrangements of this type, with a reciprocating pump, there must always be a sufficient head of water over the suction valves to insure their rising. Condenser bells of practically any shape may be used for the jet and barometric types, and are equally efficient if the water and steam are brought into contact and the air is collected and carried away. The latter may be done by a separate dry-air pump, or the air pipe may be lead into the throat of the tail pipe.

In ejector condensers with tail pipes (Fig. 3) the throat is usually figured for a velocity of 15 to 20 ft. per sec. Dimensions in inches are as follows: $K = \sqrt{W/50}$; $H = 0.0765\sqrt{W} - 1$; $T = 0.6H$; $J = 0.0765\sqrt{W}$; $O = 5K$, approx. The flange X should be about 40 ft. above the level of the tail water. If it must be placed near the level of the water in the discharge well, the velocity through throat T must be increased in order to discharge the water



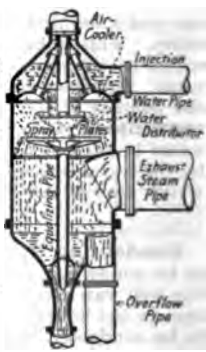


FIG. 4a.—Alberger. FIG. 4b.—Tomlinson.
Barometric Condensers.

and air against atmospheric pressure. Ejector condensers without the tail pipe are common, and when properly installed and operated, work very well.

Pumps Required by Direct-contact Condensers. Barometric condenser: circulating pump and sometimes a dry air pump; jet condenser: tail pump; ejector condenser: circulating pump; ejector condenser without tail pipe: circulating pump.

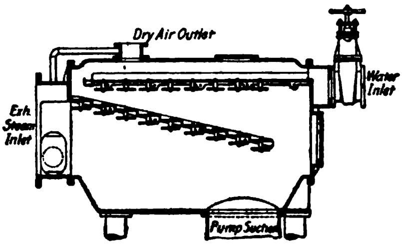


FIG. 4c.—Wheeler Jet Condenser.

SURFACE CONDENSERS

Surface Condensation. Let N = total heat to be transmitted per hour, B.t.u.; S = total outside surface of tubes, sq. ft.; W = steam condensed per hour, lb.; Q = condensing water required per hour, lb.; $R = Q/W$; k_0 = coefficient of heat transmission; t_m = mean temperature difference of water and steam, t_s = vacuum temperature, t_w = hot-well temperature, t_0 and t_1 = temperature of circulating water at inlet and outlet, respectively (all temperatures in deg. fahr.); H = total heat at t_s and q = heat in liquid at t_w. Then $N = k_0t_mS = W(H - q)$, $Q = W(H - q)/(t_1 - t_0)$, and $S = Q(t_1 - t_0)/k_0t_m = W(H - q)/k_0t_m$. For practical work k_0 may be taken as constant for any one condition; although it has been shown by experiment to be subject to small variations with t_m. The mean temperature difference for rough calculation with small rise in temperature of the circulating water may be taken as the arithmetical mean without serious error, but for most calculations the logarithmic mean should be used. See p. 306. The quantity of circulating water Q is a function of the number and size of the tubes, the number of water passes in the condenser and the velocity of the water in the tubes. The values of k_0 depend on the velocity of the water, as well as on the material of the tube, its cleanliness and the richness of the steam and air mixture in the condenser, see p. 307. The water velocity w in the tubes should be around 8 ft. per sec. The material coefficient may usually be taken at 0.95, and the cleanliness coefficient at about 0.9 for such waters as are found in New York or Chicago. The air richness ratio (p_s/p_t) is exceedingly difficult to measure experimentally, but for tight condensers with efficient air pumps it may be taken at from 0.95 to 0.97. Under these conditions k_0 = 782.

Design of Surface Condensers. In condenser design the given quantities usually are W, t_0, and the required vacuum. It is important that the place of measurement of the vacuum should be stated, and this is usually in the nozzle connecting the prime mover to the condenser. The highest vacuum will always be found at the air-pump suction, less in the body of the condenser, and the lowest vacuum in the nozzle. The vacuum inside the prime mover will be less by the velocity head necessary to give motion to the exhaust and by friction in the nozzle. The allowable velocity in a turbine nozzle is about 600 ft. per sec. The **loss** (or "drop") in a well-designed condenser should not exceed 0.2 in. of mercury, and t_s should be taken as the temperature corresponding to this reduced vacuum. For good practice, t_1 should be from 8 to 10 deg. fahr. lower than t_s. Q/W, or the **ratio of cooling water to condensed steam,** usually ranges from 50 to 100 for turbines and from 20 to 40 for engines; a large ratio requires more power in the circulating pumps.

Small **tubes** are best for the transmission of heat, but cannot be used with dirty water, so that the usual sizes are ⅝ in., ¾ in. and 1 in., and in some cases with very bad water 1¼ in., or even larger. Let a = internal cross-sectional area of tube, sq. in.; l = length of tube (sum of all passes), ft.; d = outside diam. of tube, in.; n = number of tubes in one pass; A = area of one pass, sq. ft.; f = number of passes. Then $n = 144A/a = Q/1560aw$, and $l = 3.82S/nd$. The length of a single tube is l/f, and the tube ratio l/d should be between 25 and 50. The best value of this ratio, however, has not been established by experiment.

Example. Design a condenser for turbine service, 30,000 lb. of steam to be condensed per hour, circulating water at 70 deg. fahr., with a vacuum in the nozzle of 28.4 in. referred to a 30-in. barometer. Clean salt water to be used for condensing and Admiralty tubes of 1 in. diam., No. 18 B. W. G. (internal diam. = 0.902 in.).

SOLUTION. $k_0 = 350 \times 0.9 \times 0.98 \times 0.92 \times \sqrt{8} = 804$ B.t.u. Vacuum (28.4 in. + 0.2 in. for drop) = 28.6 in. From the steam tables, t_s = 89.6 deg. fahr., whence $t_1 = 89.6 - 10 = 79.6$ deg. fahr. Also q = 47.6 B.t.u. and H = 1099 B.t.u., making $R = (1099 - 47.6)/(79.6 - 70) = 109.5$, from which $Q = 30{,}000 \times 109.5 = 3{,}285{,}000$ lb. per hour. $t_m = 9.6/\log_e(19.6/10) = 9.60/0.6729 = 14.3$ deg., and $S = 30{,}000 \times 1051.4/(804 \times 14.3) = 2743$ sq. ft. Tube area $= \pi \times 0.902^2/4 = 0.64$ sq. in.; number of tubes $n = 3{,}285{,}000/(1560 \times 0.64 \times 8) = 411$; total length of tube $l = 3.82 \times 2743/411 \times 1 = 25.5$ ft., or say, a two-pass condenser with tubes 13 ft. long.

Condensing Surface Requirements. Table 1 gives the **amount of cooling surface** for various values of temperature rise in the water and ratios of cooling water to condensed steam. This table is based on a value of $k_0 = 350$, which is an average value taken from a large number of installations. For other values of k_0 multiply the figure taken from the table by 350 and divide by the new value of k_0 to obtain the surface necessary. Surface condensers for large turbine installations have generally from 1¼ to 2 sq. ft. of **condensing surface per kw.**, or approximately 1/12 to ⅛ sq. ft. per lb. of steam condensed. Small turbo-generators require from 2 to 4 sq. ft. per kw., or from 1/10 to ⅙ sq. ft. per lb. of steam.

For **dry-air condensers** the coefficient of conductivity in B.t.u. per hour per sq ft. of surface per deg. fahr. difference of temperature, varies from 3 in still air, to 14 with air moving at a velocity of 1500 ft. per min.

Materials of Construction. The materials generally used for **tubes** are brass, copper, aluminum bronze, and Muntz metal. Any of these will give satisfactory service with fresh water, but with salt water considerable trouble may be experienced. Admiralty mixture (70Cu, 29Zn, 1Sn) is generally used for salt water. Tubes are made of No. 20, No. 18 and sometimes No. 16 B. W. G. The variation in heat transfer due to the thickness may be neglected. **Tube spacing** is important, as there must be room for the glands and sufficient metal between them for strength. The minimum allowable spacing or pitch of tubes is as follows (number of tubes per sq. ft. $= 166/s^2$, approx., s being the pitch in inches):

Diam. of tube, in.	⅝	¾	⅞	1	1¼
Pitch of tubes, in.	15/16	1 1/16	1¼	1⅜	1⅝
Number of tubes per sq. ft.	189	147	106	88	63

The **tube sheets** should be of Muntz metal or brass. A rolled sheet will give the best service, although cast sheets are used. Thickness should be at least ⅛-in. greater than the tube diameter. **Glands** should be of the same metal as the tubes and be provided with an inside lip to prevent creeping of tubes. The entrance of the gland should be rounded. **Rubber rings** are

Table 1. Condensing Surface Necessary for 1000 Lb. of Steam per Hour*

Vacuum, in. 30in. bar.	Temperature rise in water, deg. fahr.	Ratio $R = Q/W$	Inlet water temperature, t_0 (deg. fahr.)						
			50	55	60	65	70	75	80
29	5	200.0	107.1	132.3	172.0	248.0			
	10	100.0	119.8	152.3					
	15	66.7	137.4	184.2					
	20	50.0	165.0						
28.5	5	200.0	72.9	83.3	98.0	117.5	149.0	202.0	
	10	100.0	78.1	90.5	107.9	133.3	175.3		
	15	66.7	84.5	99.7	121.5	156.0			
	20	50.0	93.2	112.0	141.5	196.0			
	25	40.0	104.2	130.0	176.4				
	30	33.3	120.5	161.0					
28	5	200.0	58.6	65.2	73.7	84.9	99.5	121.0	153.7
	10	100.0	61.8	69.7	79.5	92.3	110.3	137.0	182.3
	15	66.7	66.1	74.6	86.4	102.0	124.8	161.8	
	20	50.0	70.7	81.2	95.2	113.2	146.0		
	25	40.0	76.6	89.0	107.0	134.0	184.7		
	30	33.3	84.0	99.9	124.0	168.0			
	40	25.0	108.8	143.6					
27.5	5	200.0	50.3	55.5	61.6	69.4	79.0	91.5	108.2
	10	100.0	53.3	58.4	65.8	74.0	85.2	100.1	121.5
	15	66.7	56.1	62.3	69.9	80.0	93.2	111.8	140.1
	20	50.0	59.5	66.5	75.4	87.3	103.6	128.2	169.4
	25	40.0	63.4	71.4	82.0	96.6	118.2	153.7	
	30	33.3	68.0	77.8	91.0	110.0	141.3		
	40	25.0	81.6	97.3	122.2	174.6			
27	5	200.0	45.7	49.4	54.0	59.8	67.2	76.4	86.5
	10	100.0	47.4	51.8	57.2	63.6	71.5	82.2	96.1
	15	66.7	50.0	54.5	60.6	67.8	77.0	89.6	106.2
	20	50.0	52.7	57.8	64.3	72.8	83.6	98.7	120.4
	25	40.0	55.6	61.5	69.0	79.0	92.4	111.6	142.3
	30	33.3	58.8	65.8	75.0	87.0	104.3	131.3	184.0
	40	25.0	68.2	78.2	92.5	114.3	155.7		
26.5	5	200.0	41.8	45.4	49.3	53.6	59.2	66.7	74.6
	10	100.0	43.5	47.4	51.4	56.6	62.8	70.7	80.6
	15	66.7	45.5	49.5	54.2	59.8	67.1	75.8	87.8
	20	50.0	47.6	52.0	57.2	63.6	71.7	82.5	97.0
	25	40.0	50.0	54.8	61.1	68.2	77.9	90.8	109.4
	30	33.3	52.7	58.2	65.2	73.9	85.6	102.2	128.0
	40	25.0	59.7	67.3	75.6	90.9	111.7	150.0	
26	5	200.0	39.6	42.1	45.4	49.4	54.0	59.2	66.8
	10	100.0	40.8	43.4	47.4	51.3	56.9	63.2	71.1
	15	66.7	42.1	45.5	49.5	54.2	60.0	67.1	76.3
	20	50.0	43.9	47.7	52.1	57.4	63.8	72.1	82.7
	25	40.0	45.9	50.0	55.0	60.9	68.5	78.1	91.1
	30	33.3	48.3	52.9	58.4	65.2	73.9	86.0	102.7
	40	25.0	54.0	59.8	67.5	77.2	91.2	112.5	151.0
25	5	200.0	34.8	37.3	40.2	43.1	46.5	50.9	55.3
	10	100.0	36.0	38.7	41.5	44.8	48.6	53.3	58.6
	15	66.7	37.4	40.2	43.2	46.8	50.8	56.2	62.2
	20	50.0	38.95	41.9	45.1	48.9	53.6	59.4	66.5
	25	40.0	40.55	43.6	47.3	51.5	56.7	63.3	71.3
	30	33.3	42.2	45.5	49.6	54.5	60.5	68.0	77.6
	40	25.0	46.3	50.6	55.7	62.2	70.5	81.5	97.2

* $H - q$ assumed to be 1000 in calculating this table.

much used for **packing** on European condensers with fresh condensing water, but the screw gland with corset-lace packing put in with an automatic gun is probably best. Tube packings may be of fiber, woven hose or corset lacing. No animal or vegetable fats should be used on the packing, as they form soluble compounds with copper; paraffin is the best wax to use with woven packings.

Condenser **shells** are usually cast iron, ribbed outside against collapsing pressure, but may be of steel plate or sheet brass (navy practice) stiffened with angles. Tubes should be supported at distances of 60 to 70 diameters by supporting plates (usually of cast iron) drilled with 1/16-in. clearance around tube. **Water boxes** should be large and designed to offer little friction to the passing of the water. A hole 1/8-in. in diameter in the partition will allow the upper box to drain when not in use, and the condenser is usually set on a slope of 1 in. in 15 ft., so that the tubes may drain. Where possible, the steam should enter from the top and water at the bottom (counter-current principle), but this is not essential, as parallel-flow condensers give good results. The bottom of the circulating-water outlet should be above the highest point of the tube bank. If this cannot be done at the water box the discharge pipe should be carried up to the same height away from the condenser.

The steam passage should be direct to the tube bank, and, if possible, the nozzle should be spread so that no dead pockets may be left away from the path of the steam. Baffle plates and guide plates are also used for the same purpose but are not as efficient. The upper bank of tubes may have a wider spacing than the lower, or channels may be made in the tube bank to afford a free passage for the steam. The **steam flow** should be directed to the coldest part of the condenser and here the dry-air suction should be taken out. The suction should be baffled to prevent water being carried into it. **Water connections** should be figured for a velocity of 8 ft. per sec. and the air-pump connection should be at least twice that of the hot-well pump suction, which should be figured for about 4 ft. per sec.

AIR PUMPS

The amount of air in feed water varies from 0.75 to 5 per cent. by volume at atmospheric temperature and pressure. If closed heaters are used, all of the air passes through the boilers and prime movers into the condenser. If open heaters are used, the air in the feed water (at 200 deg. fahr.) should not be over 1 per cent. The **leakage in surface condensers** is an extremely variable quantity, ranging from 1 per cent. in tight condensers to 25 per cent. or more in condensers in poor condition. The average leakage is from 5 to 10 per cent. of the volume of the feed water. The **volume of air passing into the air pump** may be calculated as follows, if the volume of free air is known: $V = V_aP_a(t_s + 460)/(P_c - P_s)(t_a + 460)$, where V = volume of the air in vacuum, cu. ft.; V_a = volume of the air at atmospheric pressure and temperature entering the condenser, cu. ft.; P_a = atmospheric pressure, P_c = total pressure at pump suction, and P_s = pressure of vapor at t_s, all in in. of mercury, absolute; t_s = temperature at pump suction, and t_a = atmospheric temperature, both in deg. fahr. As there is always leakage through glands, pipes or joints, the air pump must have a displacement greater than that given by the above formula. In practice the dry-air pump has a displacement of from 10 to 60 times the volume of the condensed steam. The average ratio for engines is about 20, and for steam turbines 25 at low vacua and 35 to 45 at high vacua. If the air pump must remove the condensed steam, allowance is made therefor. Fig. 5 shows the relation between the temperature in the air-pump suction pipe and the volume of saturated air in cu. ft. per lb. of dry air.

The circulating water may contain as much as 5 per cent. air (by volume), but an extra allowance of from 3 to 5 per cent. should be made for leakage. The combined circulating and air pump generally has a displacement approximately three times the volume of the cooling water. If an independent air pump is used, its displacement should be only twice the volume of the cooling water. The **effect of air leakage on vacuum** in surface condensers is shown by Fig. 6, from a paper by G. A. Orrok, in *Jour. A. S. M. E.*, Nov., 1912, which describes the method of determining the amount of air handled by the air pump and also the method of determining the quantity of air in the feed water.

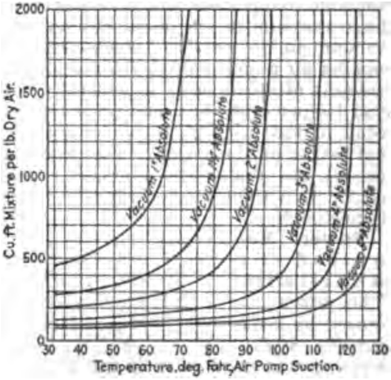


FIG. 5.—Relation Between Temperature in Air-pump Suction Pipe and Volume of Saturated Air.

Fig. 7 shows three of the centrifugal hydraulic or **rotary air pumps** now on the market. In Fig. 7*a* the centrifugal force imparted to the water forces it out through the discharge, in the meantime creating a vacuum which draws in more water which is hurled in thin sheets at a high speed into the diffuser, each sheet of water carrying with it a layer of air drawn in through the air inlet. In Fig. 7*b* the action is similar but the arrangement is different. Fig. 7*c* shows a design of pump in which the impeller is entirely removed from the air-pump casing. An ordinary centrifugal pump may be used to supply the hurling water to the revolving jet wheel shown. The jets of water form a large number of pistons which discharge the air through the tail pipe. This type of pump is now in common use and is rapidly eliminating the reciprocating pump in the large central stations. It is driven by a motor or turbine, is very compact, and, owing to the absence of valves and reciprocating parts, requires very little attention. The efficiency of

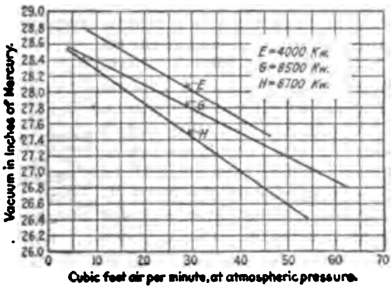


FIG. 6.—Effect of Air Leakage on Vacuum in Surface Condensers.

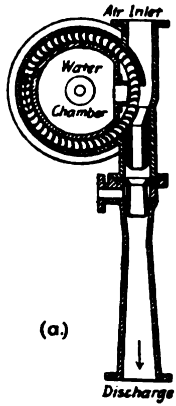


FIG. 7*a*.—Westinghouse-Leblanc Rotary Air Pump.

the hydraulic pump is low, the power required being from three to eight times that for a reciprocating pump.

In the **Parsons vacuum augmenter** (Fig. 8) the air is exhausted from the condenser by means of a steam jet into an auxiliary condenser. The dry vacuum pump does not have to maintain the high vacuum in the main condenser, as the vacuum in the auxiliary condenser is lower; a smaller air pump may therefore be used. The amount of steam used is generally between 1 and 1.5 per cent., but may be as low as 0.6 per cent. of that used by the main turbine. The net saving on the average condenser due to the use of the augmenter averages about 5 per cent.; with tight condensers the saving made is negligible.

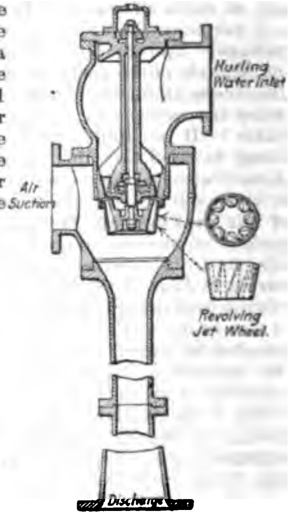


FIG. 7c.—Worthington Air Pump.

Power Required by Auxiliaries. The power required by large turbine auxiliaries varies from 1 to 3 per cent., generally between 1 and 2 per cent. of the power of the main unit. In small installations the power consumed varies from 2 to 10 per cent. Centrifugal hot-well pumps for surface condensers generally require from 0.05 to 0.10 h.p. for each 1000 lb. steam condensed per hour. Reciprocating air pumps require from 0.10 to 0.3 h.p. per 1000 lb. steam per hour. Centrifugal air pumps require 0.25 to 0.5 h.p. per 1000 lb. steam per hour. The power required by circulating pumps varies with the design of the condensers, quantity of water pumped, and the efficiency of the pump. The pumping head may be as low as 10 ft. in some condensers and as high as 35 ft. or more in others, the average being about 20 ft. Pump efficiencies range from 60 to 80 per cent. for this service. Assuming a ratio of cooling water to steam of 50, a pumping head of 20 and an efficiency of 70 per cent., the brake horse power per 1000 lb. steam per hour would be approximately 0.75 h.p. The **steam required** for auxiliaries in lb. **per brake horse power per hour** is as follows:

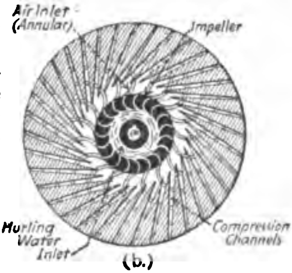


FIG. 7b.—A. E. G. Air Pump.

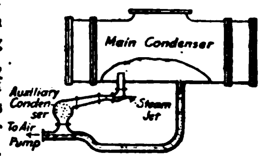


FIG. 8.—Parsons Vacuum Augmenter.

Circulating pumps, slow-speed, engine-driven	25 to 75
Circulating pumps, high-speed, engine-driven	40 to 100
Circulating pumps, turbine-driven, geared	25 to 30
Circulating pumps, turbine-driven, direct-connected	30 to 60
Air pumps, reciprocating	40 to 100

Hydraulic air pumps	40 to 55
Hot-well pumps, reciprocating	50 to 100
Hot-well pumps, centrifugal	40 to 60

The **power required by jet condensers** varies from 1 h.p. per 1000 lb. steam used per hour by the main turbine engine in the case of large units, to 3 h.p. per 1000 lb. steam for small units. Barometric condensers require from 0.5 h.p. to 1.5 h.p. per 1000 lb. steam per hour.

Most Economical Vacuum. No definite rule can be laid down for the most economical vacuum to be used. Each case must be considered separately on account of such factors as the cost of condensers, auxiliaries and piping (all of which vary with temperature of inlet water), foundations, space, cost of coal, load factor, and fixed charges on main unit. For steam turbines a vacuum of 28 in. is generally considered the most economical with inlet water at 70 deg. fahr. For engines a vacuum of 26 in. is the most economical with water of ordinary temperatures. A higher vacuum will not increase engine economy.

Salt-water Leakage. Where salt water must be used for circulating water in surface condensers it is important that there should not be an excessive leakage into the condensed steam which is to be used for boiler feed. If the hot-well water contains more than 1 per cent. of salt, it should not be supplied to the boilers. The method of determining this percentage is as follows:

Ten cu. cm. of the condenser water are measured out by a pipette, placed in a beaker, and two or three drops of potassium chromate solution added. A standard solution of silver nitrate is then added drop by drop from a burette, but only until the liquid in the beaker maintains a faint pink color on agitating. The volume of silver nitrate solution used is then recorded. Ten cu. cm. of the sample of circulating sea water are now measured out, 90 cu. cm. distilled water added, mixed, and 10 cu. cm. of the mixture taken. A few drops of the chromate solution are now added, and the silver nitrate added as before, the amount of the latter used being recorded. The amount of silver nitrate used for the circulating water multiplied by ten and divided into the amount used for the condenser water and the quotient multiplied by 100, gives the percentage of sea-water leakage in the condenser water. The silver nitrate solution usually used is made up by dissolving 9.6 gm. silver nitrate in 2 liters of water. This is of such strength that 1 cu. cm. is equivalent to one milligram of chlorine.

COOLING EQUIPMENT

Cooling Ponds. Where an inexpensive and sufficient supply of circulating water is not available, a **cooling pond** may be provided for reducing the temperature of the discharge water so that it may be recirculated again and again. The weight of water evaporated per sq. ft. of water surface per hour in still air (Box, p. 152) $= E(\text{lb.}) = (243 + 3.7t)(V - v)/7000$, where t = temperature of water, deg. fahr., V = maximum vapor tension at temperature t, and v = actual vapor tension of air, both in in. of mercury. The evaporation is increased considerably by wind and may be as much as ten times that given by the above formula. Approximately 35 lb. of water will be cooled 30 deg. by the evaporation of 1 lb. from the surface of a pond. Under average conditions 1 sq. ft. will be sufficient to cool from 4 to 6 lb. of water per hour from 100 deg. fahr. to 70 deg. fahr. Ruggles (*Jour. A. S. M. E.*, April, 1912) states that under ordinary conditions in the northern part of the United States with engines using 15 lb. of water per h.p.-hour and a vacuum of 26 in., a reservoir having a surface of 120 sq. ft. per h.p. is ample for cooling and condensing water.

As cooling ponds require considerable space, their use is often impracticable, and **cooling sprays** are used to accomplish the same object. The nozzle used must be of such design that the water jet emerging will be broken up into fine spray, under pressures ranging from 3 to 15 lb. Approximately 3 per cent. of the water is lost by evaporation. The power consumed ranges from 0.75 per cent. for the low heads to 2 per cent. at the high heads. The area required for the pond is approximately 1 sq. ft. for every 150 lb. water per hour for plants of 500 h.p. and 1 sq. ft. for every 250 lb. for plants of 5000 h.p.; the depth need not be more than 3 ft. A spray system for cooling 4800 gal. per min. could be laid out with three rows of eight nozzles (or sets of nozzles), each nozzle (or set) having a capacity of 200 gal. per min. The rows should be spaced 20 ft. apart in this case and the nozzles 13 ft. c. to c. in each row. In order to prevent spray from being carried away from the pond, there should be 24 ft. between the outer nozzles and the borders of the pond, but if all of this space is not available, a fence should be erected with sufficient openings for the passage of air. A spray system of this capacity would require a pond 139 ft. long by 88 ft. wide. Test made on nozzles have shown the following results:

Temp. of air, deg. fahr.	80	87	78	72	30	74
Humidity, per cent.	65	48	65	84	50	84
Temp. of water before spraying, deg. fahr.	120	122	117	112	79	114
Temp. of water after spraying, deg. fahr.	85	88	82	88	58	89
Drop in temp., deg. fahr.	35	34	35	24	21	25

Cooling Towers. A cooling tower consists of an iron or wooden casing surrounding a network of wooden partitions, sheet-iron trays or iron-wire mats, or layers of short, staggered cylindrical earthen or metallic tiles. The water to be cooled is distributed at the top of the tower and falls down over the boards or mats in such a manner that a large surface of the water is exposed to the current of air passing upward. The water is cooled partly by the evaporation of a portion of its volume, and partly by conduction of some of the heat to the air. If the tower is of sufficient height, the air can be circulated without the use of a fan (natural-draft tower). If the air must be forced or induced through the tower, it is known as a forced-draft tower. The **volume of air necessary per pound of steam** condensed may be determined with sufficient accuracy for practical purposes from the formula $W(q_t - q_b) = 0.238(T_t - T_b)V_bW_a[(p - p_bh_b)/29.92] + r(V_tW_th_t - V_bW_bh_b)$, where W = weight of cooling water per lb. steam condensed per hour, W_a = weight of 1 cu. ft. dry air at temperature T_b and 29.92-in. barometer, $W_t(W_b)$ = weight of 1 cu. ft. of water vapor at temperature $T_t(T_b)$, all in lb.; $h_b(h_t)$ = relative humidity of air entering (leaving) tower; p = atmospheric pressure, and $p_t(p_b)$ = pressure of vapor corresponding to $T_t(T_b)$, all in in. of mercury; $q_t(q_b)$ = heat of liquid in water entering (leaving) tower, B.t.u.; $T_b(T_t)$ = temperature of air entering (leaving) tower, deg. fahr. abs.; V_b = volume of air entering tower at atmospheric temperature and pressure, cu. ft.; V_t = volume of air leaving tower at T_t, cu. ft., = $[V_b(p - p_bh_b)/(p - p_th_t)](T_t/T_b)$; r = heat of vaporization at T_t.

Example. An engine consumes 20,000 lb. steam per hour; temperature of the injection water, 90 deg. fahr., of the discharge water, 120 deg., of the outside air, 70 deg.; barometer, 30 in.; relative humidity of air entering tower, 70 per cent., of discharge air, 95 per cent. Determine the amount of air per hour necessary to cool the water.

Solution. $W = (H - q)/(t_1 - t_0) = 1000$ (approx.)$/(120 - 90) = 33.3$ lb. $W_a = 0.0749$; $W_t = 0.00377$; $W_b = 0.00115$; $h_t = 95$; $h_b = 70$; $p = 30$; $p_t = 2.59$; $p_b = 0.74$; $q_t = 88$; $q_b = 58$; $T_t = 110 + 460 = 570$; $T_b = 70 + 460 = 530$; $r = 1030$. T_t is generally from 5 to 10 deg. below the temperature of the water entering the tower; h_t generally ranges from 90 to 100. Substituting the foregoing values in formula and solving for V_b, $V_b = 243$ cu. ft. of air per lb. steam condensed = 4,860,000 cu. ft. per hour for 20,000 lb. of steam condensed.

For cooling towers 60 to 80 ft. high, filled with layers of staggered 4- to 5-in. sheet-metal cylinders (8 to 12 in. long) to a depth of 15 to 20 ft., over the surface of which water trickles from a revolving cross-arm spray, 1 sq. ft. of ground area is required for each 80 lb. of steam condensed per hour; cooling surface (of cylinders), about 3.5 sq. ft. per lb. steam condensed per hour. For towers enclosing lattice-work systems of cooling surface, ground area required ranges from 1 sq. ft. for each 16 lb. steam condensed per hour (in a size to handle the circulating water needed to condense 16,000 lb. steam per hour) to 1 sq. ft. for 26 lb. steam in the largest sizes.

POWER FROM SOLAR HEAT*

The heat received per minute from the sun on 1 sq. ft. of surface normal to the sun's rays above the atmosphere of the earth amounts to 7.12 B.t.u. (0.168 h.p. per sq. ft.), but, before reaching the earth where it can be utilized, part of this heat is absorbed by water vapor and dust suspended in the atmosphere. In vast areas in the tropics and in certain semi-tropical and arid sections (*e.g.*, Arizona, Egypt), however, the air is dry and clear, this loss is small, and sun power can be profitably produced provided coal is very expensive and the cost of the considerable land area required for a plant is low.

A. S. E. Ackermann (*Jour. Roy. Soc. Arts*, Apr. 30, 1915) states the theoretical thermal efficiency of a solar heat absorber to be $e = [Dsa - pk \times (T^4 - \frac{2}{3}A^4) - (1 - r)Dsa]/Dsa$, where D = width of reflector, ft.; s = solar constant = 7.12 B.t.u. per sq. ft. per min.; a = coefficient of atmospheric transmission = 0.7; p = perimeter of boiler, ft.; k = boiler radiation constant = $10^{-10} \times 0.36$ B.t.u. per sq. ft. per min.; T = boiler temperature, deg. fahr. abs.; A = temperature of reflectors, deg. fahr. abs. (= atmos. temp. + 9 deg.); r = efficiency of silvered glass as a reflector of heat—taken as 0.6. Also, theoretical overall thermal efficiency of a sun-power plant $= e_1 = e \times (T - 568)/T$, where 568 is the absolute temperature of the condenser, deg. fahr. (assumed constant). In the Cairo plant, D = 12.67 ft. and p = 2.92 ft. Assuming A = 561 deg. and solving equation for e_1 for various values of T and plotting the results, it is found that the maximum value for e_1 (5.9 per cent.) is obtained when T = 231 deg. (= 692 deg. abs.) or the temperature for a steam pressure of 21 lb. abs. The actual maximum overall efficiency of the plant was found to be 4.32 per cent., showing that about 75 per cent. of the boiler h.p. theoretically possible was obtained.

In a 50-b.h.p. plant installed in 1913 at Cairo, Egypt, by Frank Shumann (Manchester Assn. of Engrs., March 14, 1914), the sun's rays are concentrated on the flat bottom of a cast-iron boiler by silvered panes of ordinary window glass arranged in frames so as to form approximate parabolic reflectors, which frames are so geared that the engine (working through a friction clutch governed by a thermostatic device) intermittently turns them and keeps them facing the sun throughout its course during the entire day. This boiler generates steam at atmospheric pressure (14.7 lb. per sq. in.) which is utilised in a specially designed condensing engine that yields 1 b.h.p. on a consumption of 22 lb. of steam. (For larger plants the use of low-pressure turbines is proposed.) When the sun is obscured by clouds the engine will continue to generate power economically until the pressure drops to about 4 lb. abs. To provide power over rainy spells of 2 or 3 days, additional plant must be provided to heat water to 212 deg. for storage in insulated tanks. The latitude of Cairo is 30 deg. N.; at a location 1000 miles nearer the equator the Cairo plant would yield 65 b.h.p.

The steam-producing part of the Cairo plant cost $7600. With interest and depreciation at 10 per cent., the annual charge would be $760. An equivalent coal-burning plant with stack, boiler and buildings would cost $3750, and the annual charge at the same rate would be $375, or $385 less than for the sun-power plant. Assuming a coal consumption of 2 lb. per b.h.p.-hour, the fuel annually required for 365 ten-hour days would amount to 163 long tons. That is, the sun-power plant will compete on an even basis with a coal-burning plant using coal costing but ($385/163 =) $2.36 per long ton. For particulars regarding sun-power plants using less efficient heat absorbers consisting of

* Staff contribution.

shallow, glass-covered, water-filled wooden basins of large area, and also the use of ether or sulphur dioxide as the working fluid instead of steam, see articles by Messrs. Shumann and H. E. Willsie, *Eng. News*, May 13, 1909. A brief sketch of former efforts to utilise solar energy, and a bibliography of the subject, are included in the paper by Mr. Ackermann cited above.

HOT-AIR ENGINES*

Hot-air engines are heat engines in which air is employed as the working substance, operating on the Stirling or Ericsson cycles (see p. 319) or modifications of them. Their bulk per h.p. of capacity is great as compared to steam or gas engines and their efficiency low. They find use, however, in small sizes for domestic pumping work. Bryan Donkin ("Gas, Oil and Air Engines") gives the following data on such motors:

Engine	Cyl. diam., in.	Stroke, in.	R.p.m.	I.h.p.	B.h.p.	Lb. fuel per hr. per I.h.p.	Lb. fuel per hr. per B.h.p.
Buckett..............	24.0	16.0	61	20.20	14.40	1.8	2.5
Beniér................	13.4	13.8	117	5.80	4.00	3.1	3.6
Bailey................	14.6	6.9	106	2.40	1.30	4.2	7.6
Rider.................	6.7	9.5	138	0.81	0.23	.....	.....

The actual thermal efficiency of the Buckett engine, assuming the fuel (coke) to have a calorific value of 12,000 B.t.u. per lb., is $2546/(2.5 \times 12{,}000) = 8.48$ per cent. Similarly, that of the Bailey engine is 2.8 per cent. A steam engine of the size and speed of the Buckett engine, with a mean effective pressure of but 30 lb. per sq. in., would develop over three times the indicated horse power.

* Staff contribution.

INTERNAL-COMBUSTION ENGINES

BY

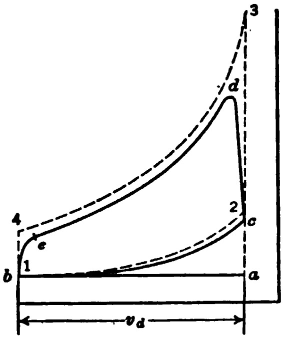
LIONEL S. MARKS

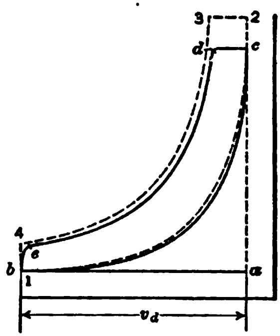
REFERENCES: Güldner, "Verbrennungskraftmaschinen,"3rd edition, Springer, also translation of 2nd edition by Diederichs, Van Nostrand. Clerk, "The Gas, Petrol and Oil Engine," Wiley. Schöttler, "Die Gasmaschine," Springer. Carpenter and Diederichs, "Internal Combustion Engines," Van Nostrand. Haeder and Huskisson, "Handbook on the Gas Engine," McGraw-Hill.

GENERAL FEATURES

Methods of Operation

The term "internal-combustion engine" is applied indiscriminately to gas and oil engines, whether combustion takes place instantaneously (explosively), at constant volume (as in Fig. 1), or gradually, at constant pressure (as in Fig. 2). Both types of operation are used in four- and two-cycle, single- and double-acting, horizontal and vertical engines.





Internal-combustion-engine Cycles.

FIG. 1.—Otto Cycle, Explosive Combustion.

FIG. 2.—Diesel Cycle, Constant-pressure Combustion.

Processes of the cycle.

a–b	Suction stroke; admission of the charge.
b–c	Compression.
at *c*	Ignition of the compressed mixture.
c–d	Combustion (explosion).
d–e	Expansion of the products of combustion.
at *e*	Opening of the exhaust valve.
e–b–a	Exhaust.

Processes of the cycle.

a–b	Suction stroke, admission of air.
b–c	Compression of the air.
c–d	Injection and combustion of the fuel.
d–e	Expansion of the products of combustion.
at *e*	Opening of the exhaust valve.
e–b–a	Exhaust.

The Four-stroke Cycle (four-cycle) is most commonly used at the present time for stationary gas engines. All the processes occur in a single cylinder, so that each cycle requires for completion four strokes, of which one only produces useful work. The work of the admission and exhaust strokes amounts to from 5 to 10 per cent. of the power of the engine. This type of engine up to 150 h.p. is ordinarily built single-acting and without a cross head. For larger powers the engines are built double-acting, and have an external cross head.

Advantages of the Four-cycle Single-acting Engine: Simplicity; trunk piston operating as a cross head; water-cooling of the piston not needed except in the largest sizes; can be built for very high rotative speeds; cost of construction for small powers is low. **Disadvantages:** A very variable crank effort, which necessitates for a small cyclic variation of angular velocity a very heavy flywheel or the use of a number of cylinders; power per unit volume of piston displacement is low; imperfect scavenging.

The Two-stroke Cycle (two-cycle) has been used economically only in large, slow-running engines, in which the suction and pre-compression of the mixture take place in separate gas and air pumps. Many small liquid-fuel engines utilise the crank case for the precompression of the charge. In most cases the mixture is not admitted to the combustion cylinder until a blast of scavenging air has been passed through. The exhaust takes place through ports in the cylinder bore, which are uncovered by the working piston; the mixture and scavenging air may be admitted through valves in the cylinder head. Many small motors with crank-case compression are valveless. An explosion takes place at each revolution of the crank. With the present construction, the indicated work of suction and admission of the charge amounts to from 7 to 12 per cent. of the total power. By using scavenging air at a low pressure—from 1.5 to 3.0 lb. per sq. in.—and supplying larger passages for the admission and exhaust, the pump work can be diminished.

Advantages as compared with the four-stroke cycle: The power per unit volume of piston displacement is greater by from 70 to 90 per cent.; the crank effort is more even; with a proper design there is a more perfect scavenging of the exhaust gases; in small engines the construction is cheaper, due to the elimination of valves. **Disadvantages:** Greater difficulty in cooling and oiling the piston in small engines because of its being completely enclosed; with present construction the negative work is larger; a loss of unburnt gases into the exhaust, amounting in some cases to 20 per cent., due to the impossibility of having the exhaust valve open long enough to get the exhaust gases out of the cylinder and still have it close in time to prevent the charge blowing through.

Two- and Four-cycle Double-acting Engines. Double-acting engines are being increasingly used for larger powers—from 75 to 800 i.h.p. per cylinder end.

Advantages as large engines: More uniform running with light flywheels; greater power generated in a single cylinder; a quicker response to change in load, since charges are drawn in twice as often as in the case of the single-acting engine. **Disadvantages:** Complicated and costly construction; great length; heavy reciprocating masses little adapted to high speed and requiring unusually good foundations; the necessity of cooling the piston and piston rod in order to avoid preignitions and distortion of the piston and rod, to insure tightness between the piston and cylinder walls and for adequate oiling; the great difficulties encountered in the design and construction of the piston, cylinder, and cylinder heads; larger consumption of oil and cooling water as compared with single-acting engines.

The **Diesel engine** is usually built 4-cycle and single-acting in sizes up to about 1000 h.p. per cylinder. The 2-cycle single-acting type is made by a number of European builders in sizes up to 1250 h.p. per cylinder; this type is more easily made reversible and consequently has been adopted for marine use. A 2-cycle cylinder develops from 170 to 180 per cent. of the work of a 4-cycle cylinder of the same piston displacement, but its fuel consumption per b.h.p. is about 10 per cent. greater. The best 2-cycle engines have scavenging pumps of from 1.2 to 1.8 times the cylinder volume, which compress air to 4 to 8 lb. per sq. in. and use up about 4 per cent. of the rated power of the engine. The mechanical efficiency of the 4-cycle (2-cycle) type is about 75 (70) per cent.

The air for spraying the oil is supplied to Diesel engines by 2- or 3-stage compressors at a pressure of from 800 to 1100 lb. per sq. in. The air required varies from 16 to 34 cu. ft. of free air per b.h.p. per hour; the power required by the compressor is from 4 to 7 per cent. of the rated power of the engine. Diesel engines are usually rated with an overload capacity of 10 to 15 per cent. The heat carried away by the jackets is 2500 to 3000 B.t.u. per b.h.p. per hour, corresponding to a heat transfer of about 4000 (9000) B.t.u. per sq. ft. of cylinder surface per hour in 4-cycle (2-cycle) engines. Regulation usually within 3 per cent. Fuel consumption as low as 0.4 lb. per b.h.p. per hour, with good fuel (19,500 B.t.u. per lb.). Lubricating oil, about 0.01 lb. per b.h.p. per hour. Attendance, one engineer for 1000 to 1500 b.h.p. Cost of engine per b.h.p., from $63 for 200 b.h.p. to $48 for 1000 b.h.p. units. Weight per b.h.p. from 250 to 500 lb. in American practice; reported as low as 62 lb. in European practice. Fuels containing more than 2 to 4 per cent. of sulphur are liable to cause corrosion.

Heavy liquid fuels can also be used in engines of the **semi-Diesel or hot-surface** type. This type (exemplified by the De La Vergne engine, type FH) has a hot bulb (uncooled) in the clearance space into which the fuel is injected at the end of compression. The compression pressure is from 250 to 300 lb. per sq. in. and the injection air pressure about 600 lb. per sq. in. as compared with 550 lb. and 1000 lb. per sq. in. respectively for the Diesel cycle. The air in the hot bulb is always hot enough to burn the injected fuel. The combustion, at first, is at nearly constant volume, resulting in a combustion pressure at full load of about 500 lb. per sq. in.; it then continues at approximately constant pressure. The lower compression pressures of the semi-Diesel type make the engine less expensive. The efficiency is only slightly lower than that of the Diesel engine; thermal brake efficiencies up to 30.5 per cent. have been recorded. A preliminary heating of the bulb is necessary before the engine will start.

Fuels

The thermal efficiency of an internal-combustion engine increases with an increase in its compression pressure (see pp. 320 and 1031). Furthermore, the compression of the charge results in an increase of the power per unit volume of piston displacement and permits the use of lean gases which otherwise would be difficult to ignite. It is therefore advantageous to compress the charge to as high a pressure as is possible. The upper limit of compression is set by the ignition temperature of the explosive mixture used. The ignition temperature varies with the composition of the fuel and the strength of the mixture. Generally, the lower the heat value of the charge the higher is its point of ignition. Rich fuels can be highly compressed only as dilute mixtures, while lean gases can withstand a higher compression in the presence of a slight excess of air. Pure mixtures rich in hydrogen ignite in the neighborhood of 950 deg. fahr. (the temperature of ignition of one part of hydrogen and one part of oxygen is 957 deg. fahr.), while charges rich in carbon monoxide ignite at about 1110 deg. fahr. The compression pressure can be raised by suitable cooling of the mixture or by separate compression of the fuel and air. The ignition temperature of ordinary liquid fuels lies between 950 and 1020 deg. fahr. Mixtures in which they are present cannot therefore be highly compressed. See p. 1024.

Table 1. Usual Compression Pressures for Internal-combustion Engines

Fuel	Type of engine	Range of compression, lb. per sq. in. gage	Average compression in lb. per sq. in. gage
Gasoline	Automobile	45–95	65
Gasoline	Stationary	60–105	70
Kerosene	Hot bulb, 250–500 r.p.m.	30–75	60
Kerosene	Vaporized before entering cylinder	45*–85†	65
Alcohol	Vaporized before entering cylinder	120–210	150
Fuel oil	Injected into hot bulb before compression — Hornsby-Akroyd	45	45
Fuel oil	Injected after compression	255	255
Fuel oil	Diesel cycle	510	510
Natural gas	Medium and large engines.‡	75–160	120
Coke-oven gas	Large engines (in Germany)	105–135	120
Coal gas	Mostly small, very few large engines.	75–120	100
Carburetted water gas	Mostly small, very few large engines.	75–105	90
Producer gas	Both large and small engines	100–160	130
Blast-furnace gas	Largest engines built	120–190	155

* With hot mixture without water injection. † With water injection.
‡ Wide variation due to variation in composition of natural gas from various localities.

Explosion Limits. In order that a mixture of combustible gas or vapor with air shall be explosive, the ratio of air to gas must lie within certain limits depending on the character of the fuel. In general, the mixture that has just sufficient oxygen for complete combustion of the fuel gives the highest pressures and temperatures and very nearly the highest speed of ignition. If the proportion of oxygen (or air) is increased beyond or decreased from the

Table 2. Limits of Proportion for Explosive Air-Gas Mixtures
(Lucke, "Thermodynamics," p. 869)
Percentage of gas in the mixture by volume

Gas	Combining proportion	When air is in excess	When gas is in excess	Gas	Combining proportion	When air is in excess	When gas is in excess
Carbon monoxide	29.6	16.5	74.95	Benzene	2.7	2.65	6.5
Hydrogen	29.6	9.45	66.4	Pentane	2.6	2.4	4.9
Water gas	....	10.5	52.3	Gasoline, 86° Bé	....	1.54	4.76
Coal gas	....	6.7	18.4	Gasoline, 71° Bé	....	1.54	4.76
Illuminating gas	....	7.9	19.1	Gasoline, 65° Bé	....	1.31	4.76
Acetylene	7.9	3.35	52.3	Alcohol	6.5	3.95	13.65
Ethylene	6.5	4.1	14.6	Blue oil gas	....	4.0	8.0
Methane	9.5	6.1	12.8	Ethane	....	4.0	22.0
Ether	3.4	2.75	7.7				

theoretical proportion, the maximum pressures and temperatures are lowered and the speed of ignition decreases until at certain upper and lower limits the mixture ceases to be explosive and only slow combustion can occur.

Tables 2 and 3 give the explosion limits and the theoretical combining proportions for various fuels at atmospheric pressure. Table 2 is for usual room temperatures. The explosion is assumed to be caused by an electric spark.

Table 3. Limits of Percentage of Gas in Explosive Air-Gas Mixtures at Different Temperatures

(Roszkowski)

Gas	Temperatures, deg. fahr.							
	59		212		392		572	
	Limits							
	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower
Hydrogen	64.7	9.5	68.2	9.5	72.1	9.6	79.3	9.6
Carbon monoxide	74.6	14.3	77.2	13.2	80.4	12.5	57.4	21.0
Methane	13.0	6.8	12.6	5.8	12.8	5.8	13.0	5.7
Illuminating gas	22.6	7.0	24.7	7.0	26.7	6.5	28.6	6.5

Ignition Temperatures in air at atmospheric pressure are as follows (deg. fahr.): Hydrogen, 1075–1100; acetylene, 760–820; illuminating gas, 1100; ethylene, 1005–1015; alcohol, 950; methane, 1200–1240; ether, 750; benzol, 970; benzene, 780.

Speed of Ignition. The speed with which flame spreads through a mixture of fuel gas and air depends upon the composition of the mixture, the pressure and temperature, the shape of the combustion chamber, and the location of the point of initial ignition. Experiments on the speed of flame propagation at the present give little definite information as to what may be expected in the gas-engine cylinder. In the transmission of the explosion two entirely different processes may be noted. 1. In the process of **slow combustion** the layers of gas adjacent to the ignited layer are brought to the temperature of ignition by conduction. 2. An explosion may be transmitted by a **compression wave,** which is transmitted simultaneously with the ignition at a speed of several thousand feet per second. The character-

Table 4. Speed of Ignition of Combustible Gas-Air Mixtures

(Nägel, *Mitteil. über Forsch-Arb.*, Heft 54)

Mixture of hydrogen and air, initial temp., 59 deg. fahr.			Illuminating gas and air, initial temp., 167 deg. fahr.			Producer gas and air, initial temp., 167 deg. fahr		
Per cent. of fuel in mixture by volume	Absolute initial pressure, atmospheres	Speed of ignition, ft. per sec.	Per cent. of fuel in mixture by volume	Absolute initial pressure, atmospheres	Speed of ignition, ft. per sec.	Per cent. of fuel in mixture by volume	Absolute initial pressure, atmospheres	Speed of ignition, ft. per sec.
10.30	0.5	1.96	8.0	0.5	1.22	30.0	0.5	1.75
10.43	1.5	2.20	8.0	1.0	0.63	30.0	3.5	1.19
10.13	2.5	1.67	8.0	2.0	0.35	30.0	7.5	0.49
18.13	0.5	14.38	11.0	0.5	5.12	46.5	0.5	7.16
18.23	1.5	18.74	11.0	3.5	4.69	46.5	3.5	7.24
18.23	2.5	20.02	11.0	5.5	4.71	46.5	5.5	6.27
23.97	0.5	27.0	16.0	0.5	13.20	....	...	....
24.37	1.5	38.8	16.0	3.0	12.35	....	...	....
24.40	2.5	45.6	16.0	5.5	11.88	....	...	....

istics of compression waves have been studied by Berthelot, Mallard and Le Chatelier, and by Dixon (*Proc. Royal Soc.*, vol. 200 A). As an explosion wave is accompanied by the generation of an enormous pressure almost instantaneously, the production of such waves in gas engines is likely to cause damage. Nägel has investigated the speed of ignition (slow combustion) in a spherical bomb with a capacity of 1.06 cu. ft. Table 4 gives the results.

Gases. The available gases are natural gas, illuminating gas (coal or carburetted water gas), oil gas, coke-oven gas, producer gas, blast-furnace gas and other industrial by-product gases. For average compositions and heating values, see pp. 365, 613 and 1057.

Liquid Fuels. Any combustible liquid can be used as a fuel in an engine adapted to it. Liquid fuels are vaporized either before or during their mixture with the air for combustion. Gasoline and similar light hydrocarbons can be vaporized either by passing air over the surface or bubbling it through the liquid (carburetting by evaporation), or by atomizing them in the air current (carburetting by injection). Petroleum, crude oil, coal-tar oil, alcohol, and other liquid fuels whose boiling points are high, are vaporized by heating the liquid. The lighter of these liquids—petroleum, light crude oil, and alcohol—are used in engines of the explosive type, in which the liquid may be vaporized (*a*) in a vaporizer outside of the engine, which is kept hot by the exhaust gases; (*b*) in a vaporizer—hot bulb—connected to the combustion chamber of the engine and kept hot by the explosions; or (*c*) against a hot part of the combustion chamber—a plate projecting from the cylinder head. In the last case the engine must be started and run on gasoline until the plate becomes sufficiently hot to vaporize the fuel—kerosene. The heavier liquid fuels can be used only in engines of the constant-pressure-combustion type. The heavier the fuel used the more finely must it be divided when injected into the cylinder, in order to insure complete vaporization and ignition. If the heavier oils are injected in large particles only the surface of the particles will be burnt, while the center will be converted by the heat into a pitchy substance, which will be deposited on the cylinder walls and valves. For properties of liquid fuels, see p. 610.

Fuel Consumption. The fuel consumption per horse-power-hour increases with decrease of load below the normal, as is shown in Table 5.

Table 5. Relative Increase of Fuel Consumption per B.h.p. per Hour at Partial Loads

	Full-load	¾ load	½ load	¼ load
Natural-gas engine	1.00	1.05–1.20	1.20–1.50	1.75–2.00
Illuminating-gas engine	1.00	1.05–1.20	1.20–1.50	1.75–2.00
Producer-gas plant (engine and producer)	1.00	1.15–1.25	1.45–1.60	2.30–2.70
Gas producer	1.00	1.04–1.06	1.10–1.15	1.30–1.40
Blast-furnace gas engine	1.00	1.05–1.20	1.20–1.50	1.75–2.00
Low-pressure oil engine	1.00	1.15–1.35	1.45–1.70	1.90–2.30
Diesel oil engine	1.00	1.02–1.15	1.07–1.25	1.40–1.90

In explosion engines the total no-load consumption goes as high as 30–45 per cent. of the total consumption at normal load; in constant-pressure-combustion engines this figure is only 20–25 per cent. In producer-gas plants the coal consumption increases faster at partial loads, more because of the loss of efficiency of the producer than because of the heat consumption of the engine itself. For fuel consumptions under various conditions, see test results, Table 6.

Table 6. Results of Gas and Oil Engine Tests

Reference letter	Kind of fuel	Make of engine	Engine dimensions: Diam. of cylinder	Stroke in inches	No. of cylinders	Horizontal or vertical	Rated: B.h.p.	R.p.m.	Conditions†	Test data and results: Load factor, %	I.h.p.	Lower heat value of fuel. B.t.u. (per cu. ft. or per lb.)	Cu. ft. or lb. of fuel per b.h.p.-hr.	Heat, B.t.u. per b.h.p.-hr.	Indicated thermal efficiency, %	Mechanical efficiency, %	Brake thermal efficiency, %
A	N.G.	Otto	11.25	18.00	1	H.	36	220.0	T	80.0	36.3	1,086.0	14.58	15,820	20.2	79.5	16.1
B	N.G.	Walrath	13.00	14.00	3	V.	75	250.0	T	105.0	86.7	1,041.0	11.50	11,960	27.1	78.7	21.3
C	N.G.	Westinghouse	25.00	30.00	3	V.	550	150.0	T	100.0	618.0	1,000.0	10.57	10,570	27.1	89.0	24.1
D	N.G.	Snow, *D*	25.00	48.00	2	H. Tan.	....	....	T	......	736.7	1,175.0	9.15	10,740	29.4	80.7	23.7
E	N.G.	Westinghouse, *D*	23.50	33.00	4	H.T.T.	1500	150.0	T	96.1	1,826.0	1,041.0	10.00	10,430	31.0	78.8	24.4
									T	73.4	1,460.0	1,054.0	10.50	11,060	30.6	75.4	23.1
									T	48.0	1,053.0	1,054.0	12.00	12,680	29.4	68.4	20.1
									T	0.0	273.0	(per i.	h.p.)	15,830	16.1	0.0	0.0
F	I.G.	Koerting	......	......	..	....	....	160.0	T	......	......	500.0	18.07	9,030	....	....	28.2
G	I.G.	Westinghouse	......	......	3	V.	....	235.0	T	......	196.5	597.4	15.96	9,540	35.2	76.0	26.7
H	P.G.	Westinghouse	......	......	3	V.	....	235.0	T	......	173.6	123.8	86.80	10,740	33.0	71.8	23.7
I	P.G.	Westinghouse*	19.00	22.00	3	V.	235	200.0	T	83.5	677.0	100.0	112.20	11,230	26.0	87.0	22.6
									10d	52.9	452.0	100.0	149.60	14,960	20.6	82.5	17.0
J	P.G.	Westinghouse	19.00	22.00	3	V.	235	200.0	T	95.0	271.0	137.3	83.20	11,420	27.1	82.3	22.3
K	P.G.	R. D. Wood	25.00	30.00	2	H. Tan.	300	150.0	T	56.3	242.0	145.0	102.8	14,910	24.4	70.0	17.1
L	P.G.	Westinghouse, *D*	23.50	33.00	2	H. Tan.	500	150.0	T	95.7	579.0	114.6	89.0	10,200	29.9	83.5	25.0
									T	100.0	598.0	114.6	88.2	10,100	30.1	83.8	25.2
M	B.G.	Cockerill, *D*	51.20	55.20	1	H.	....	....	......	......	786.0	99.5	125.6	12,480	27.9	73.2	20.4
N	B.G.	Nürnberg, *D*	33.50	43.40	2	H. Tan.	....	....	......	......	1,427.0	88.0	102.7	9,030	33.9	83.1	28.2
O	B.G.	Westinghouse	......	......	..	....	....	....	1 yr.	......	......	87.5	143.8	12,580	25.3	80.0	20.2
P	B.G.	Snow,* *D*	42.00	60.00	4	H.T.T.	3000	83.3	1 yr.	85.2	......	......	......	11,545	29.0	76.0	22.0
Q	B.G.	Allis-Chalmers,* *D*	42.00	54.00	4	H.T.T.	3000	83.3	1 yr.	93.5	14,000.0	98.3	125.8	12,370	25.8	80.0	20.6
R	G.	Losier (2-cycle)	5.00	6.00	1	....	5	....	T	98.3	6.28	18,520.0	0.877	16,210	20.1	78.3	15.7
S	G.	Springfield	6.50	12.00	1	....	6	230.0	T	102.8	9.63	20,078.0	1.240	24,950	16.0	64.0	10.2
T	G.	Fairbanks	6.50	9.00	1	....	7	300.0	T	103.3	11.28	18,200.0	1.060	19,290	20.6	64.1	13.2
U	G.	Westinghouse	5.75	8.00	2	V.	10	....	T	55.8	9.19	......	......	14,710	28.5	60.7	17.3
V	G.	Daimler	3.56	5.11	4	V.	16–20	....	T	400	r.p.m.	17,500.0	0.875	15,300	19.3	86.1	16.6
									T	600	r.p.m.	17,500.0	0.774	13,530	22.0	85.4	18.8
									T	1000	r.p.m.	17,500.0	0.753	13,170	24.2	79.8	19.3
W	K.	Diesel	6.65	10.60	1	V.	8	270.0	T	107.5	11.19	18,610.0	0.496	9,220	36.0	76.8	27.6
X	K.	Hornsby	8.20	14.00	1	H.	....	....	T	......	6.07	18,600.0	1.050	19,580	15.9	81.6	13.0
Y	K.	Diesel	10.23	16.16	1	V.	....	....	T	......	30.46	18,600.0	0.530	9,870	37.8	68.4	25.8

Reference letter	Kind of fuel	Make of engine	Engine dimensions: Diam. of cylinder	Stroke in inches	No. of cylinders	Horizontal or vertical	Rated: B.h.p.	R.p.m.	Conditions†	Test data and results: Load factor, %	I.h.p.	Lower heat value of fuel, B.t.u. (per cu. ft. or per lb.)	Cu. ft. or lb. of fuel per b.h.p.-hr.	Heat, B.t.u. per b.h.p.-hr.	Indicated thermal efficiency, %	Mechanical efficiency, %	Brake thermal efficiency, %
									T	107.0	31.00	18,870	0.730	13,770	21.4	86.3	18.5
									T	71.8	22.40	18,870	0.920	17,390	18.2	80.2	14.6
Z	K.	Hornsby	14.50	17.00	1	H.	25	200.0	T	36.0	13.10	18,870	1.320	24,930	14.8	68.8	10.2
									T	0.0	4.28	18,870	...	...	10.1	0.0	0.0
AA	K.	Diesel	15.75	23.65	1	V.	70	160.0	T	99.5	88.00	18,870	0.430	7,980	40.3	79.1	31.9
BB	T.O.	Diesel	...	...	1	V.	(50	per cyl.)	Avg.	...	...	17,000	0.460	7,790	...	...	32.6
									T	116.0	...	18,350	0.510	9,370	...	...	27.2
									T	105.0	...	18,350	0.480	8,820	...	...	28.9
CC	C.O.	Haselwander	...	...	1	H.	10	250.0	T	92.0	...	18,350	0.510	9,370	...	...	27.2
									T	72.0	...	18,350	0.570	10,460	...	...	24.4
									T	52.0	...	18,350	0.600	11,000	...	...	23.2
										100.0	...	18,000	0.45	8,110	...	...	31.4
DD	C.O.	Diesel	Average German practice.—				300	...		75.0	...	18,000	0.48	8,650	...	...	29.4
							to	...		50.0	...	18,000	0.545	9,810	...	...	26.1
							450	...		25.0	...	18,000	0.815	14,665	...	...	17.4
EE	C.O.	Diesel	...	...	...	...	225	170.0	T	111.0	...	19,460	0.470	9,070	...	...	28.1
FF	C.O.	De La Vergne (FH)	17.00	27.50	1	H.	85	180.0	T	101.0	123.1	19,090	0.498	9,510	38.4	69.7	26.8
									T	76.1	93.4	19,090	0.485	9,270	40.2	68.6	27.5
GG	C.O.	De La Vergne (FH)	...	...	...	H.	125	155.0	T	102.0	...	18,500	0.484	8,950	...	...	28.4
HH	F.O.	De La Vergne (FH)	17.00	25.50	2	H. Twin.	200	200.0	1 yr.	54.0	...	17,750	0.506	8,990	...	...	28.3
II	F.O.	Trinkler-Koerting	...	...	1	...	12	...	T	100.0	...	18,500	0.480	8,890	...	...	28.6
									T	50.0	...	18,500	0.520	9,620	...	...	26.5
JJ	A.	Koerting	6.16	9.95	1		6	300.0	T	123.0	...	9,900	1.160	11,670	...	...	21.8
KK	A.	Deuts	8.35	11.80	1	...	12	275.0	T	140.0	...	9,900	0.800	8,050	...	...	31.6
LL	A.	Marienfeld	9.95	15.75	1	...	14	200.0	T	141.0	...	9,900	0.770	7,780	...	...	32.7
MM	A.	Bánki	...	...	1	...	20	225.0	T	160.0	...	9,700	0.820	8,430	...	...	30.2
NN	G.O.	Nürnberg-Diesel	...	...	4	V.	800	...	T	100.0	...	18,090	0.366	6,630	...	...	37.8

D means double-acting; all engines not so marked are single-acting.

Abbreviations used for fuels are: N.G., natural gas; I.G., illuminating gas; P.G., producer gas; B.G., blast-furnace gas; G., gasoline; K., kerosene; T.O., coal-tar oil; C.O., crude oil; F.O., fuel oil; A., 86 per cent. alcohol; G. O., gas oil.

* Test results of plant I for 3 units; P for 2 units; Q for 4 units.

† This column gives the conditions under which the results were obtained: T, special test; 10d., ten days normal operation; 1 yr., one year normal operation. For plant J is given the average of the U. S. Geol. Survey test on low-grade fuels; for plants BB and DD the average of German practice.

Table 7. Usual Absolute Compression, Explosion, and Mean Effective Pressures for an Average Fuel for Each Group

Fuel	Usual compression pressure, lb. per sq. in.	Usual explosion pressure, lb. per sq. in.	Mean effective pressure, lb. per sq. in.
Rich gas:			
Rich mixture: 1 part of gas to 6–7 parts of air by vol.	60– 75	225–300	70–85
Lean mixture: 1 part of gas to 10–15 parts of air by vol.	75–120	300–375	65–80
Lean gas	120–180	270–375	65–80
Liquid fuel with low boiling point	45– 75	180–300	55–60
Liquid fuel with high boiling point	60–120	150–300	50–70

Regulation

The methods of regulation are: (*a*) The hit-and-miss system; (*b*) quantitative governing—variation of the quantity of charge, the ratio of fuel to air remaining constant; (*c*) qualitative governing—variation of the ratio of fuel to air with change of load; (*d*) variation of the time of ignition; (*e*) combination systems.

The **hit-and-miss system** is not used in modern practice, because of the irregular crank effort it produces; but it is the most economical in fuel. Few of the systems in use at present are purely of one type—they may be mainly of one type, but they have characteristics of the others to a greater or less extent. **Quantitative governing** is open to the objection that the compression decreases with decrease of load, and therefore the efficiency decreases. Cut-off regulation is the only purely quantitative governing system, but is not much used because of the complexity of valve gear required. **Throttle governing** is never purely of the quantitative type, since the mixture proportions are upset by the increased suction on the air, particularly where the gas is supplied under pressure. In such a case, the mixture becomes richer as the load diminishes.

With **qualitative governing,** as the load decreases the amount of fuel should be reduced and the air increased so as to give constant cylinder filling. In an engine using gas fuel, this system actually partakes of the character of quantitative governing, since, as the gas is throttled, the air suction increases and the pressure at the end of admission is thereby lowered. Consequently, with this system the compression is also variable, though less so than with quantitative governing. At low loads this system is inferior to quantitative regulation; the fuel consumption per h.p. increases very rapidly as the load decreases, because as the fuel ratio is decreased the mixture rapidly becomes difficult to ignite and slow in burning. This method of regulation is unsatisfactory at light loads on account of the misfires. Liquid-fuel engines using the heavier fuels, both explosively and at constant pressure, govern qualitatively by forcing more or less oil into the engine, and frequently show maximum thermal efficiency at about 80 per cent. of rated load. Automobile

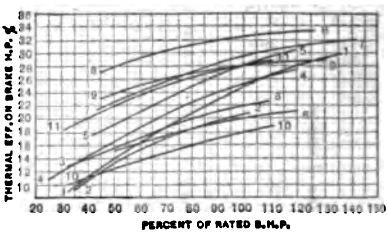


Fig. 3.—Efficiency Curves of Internal-combustion Engines (Figures refer to Table 8).

and motor-boat engines using gasoline and kerosene explosively, regulate by combined throttling and variation of the ignition point.

A few typical **efficiency curves of engines** fitted **with** the **various systems of regulation** for various percentages of the rated load, are shown in Fig. 3, taken from Carpenter and Diederichs' "Internal-combustion Engines." Table 8 gives the explanatory data corresponding to these curves.

Table 8. Data on Engines Referred to in Fig. 3

Curve No.	Type of engine	Rated b.h.p.	Rated r.p.m.	Fuel	Governing system
1	Deuts, single cylinder..	50	200	Illuminating gas.....	Throttling gas
2	Westinghouse, 3-cylinder vertical..........	100	270	Natural gas..........	Throttling mixture
3	Deuts	450	.....	Producer gas.........	
4	Güldner, single cylinder.	35	220	Producer gas.........	Throttling
5	Nürnberg	1200	106	Blast-furnace gas....	Throttling gas
6	Swiderski, single cylinder.	15	235	Alcohol..............	Throttling
7	Deuts, single cylinder..	12	285	Alcohol..............	Throttling
8	Diesel	70	158	Russian kerosene.....	Cut-off
9	Diesel	8	275	Kerosene.............	Cut-off
10	Hornsby-Akroyd	25	202	Kerosene.............	Regulating oil
11	Bánki	25	210	Gasoline with water injection	Hit-and-miss

Table 9. Size, Weight, and Fuel and Oil Consumption of 5 Air-ship Motors Entered for the Kaiser's Prize, 1913

Builder of engine	Cylinders: No.	Cylinders: Bore, in.	Cylinders: Stroke, in.	R.p.m.	M.e.p., lb. per sq. in.	B.h.p.	Consumption for 1 h.p.-hr. of gasoline (72°), lb.	Consumption for 1 h.p.-hr. of lubricating oil, lb.	Weight of motor*: total, lb.	Weight of motor*: per h.p., lb.	Weight of installation per h.p., lb.†
Benz & Co................	4	5.12	7.08	1288	107.0	101.2	0.465	0.042	373.5	3.64	7.83
Daimler Motor Co......	6	4.127	5.51	1387	114.2	88.9	0.503	0.038	343.4	3.81	8.44
Neue Auto. Co. (NAG)	4	5.32	6.30	1344	100.9	95.8	0.478	0.038	420.2	4.34	8.80
Daimler Motor Co.......	4	4.725	5.51	1412	103.7	71.4	0.498	0.046	306.2	4.23	8.95
Argus Motor Co..........	4	5.51	5.51	1368	106.5	96.8	0.528	0.038	366.6	3.75	8.99

* Does not include weight of the initial supply of cooling water, gasoline, and lubricating oil. † Includes weight of water, gasoline and oil necessary for 7 hr. operation.

Table 10. Heat Balances of Gas and Oil Engines

(Per cent. of heat of combustion.—Lucke, "Thermodynamics")

Engine	I.h.p.	B.h.p.	Friction (*a*)	Jacket	Exhaust	Radiation and unaccounted for
General (Mathot)...............	33.0	28.0	5.0	36.0	31.0*b*	....
300-h.p. engine at 197 h.p........	43.5	33.5	10.0	34.3	24.1	1.9*c*
300-h.p. engine at 294 h.p........	45.8	32.2	13.6	31.8	23.9	1.5*c*
300-h.p. engine at 335 h.p........	41.5	30.9	10.6	33.8	24.8	0.1*c*

a. Including pump work. *b*. Including radiation, etc. *c*. Excess.

Table 11. Mechanical Efficiencies
(Lucke, "Thermodynamics")

Type of engine	Mech. efficiency 4-cycle	2-cycle
Small high-speed auto, multi-cylinder, single-acting	0.75	....
Small single-cylinder, boat engine, single-acting	0.85	0.68
Small or medium single-cylinder, stationary, single-acting	0.87	0.70
Small or medium 2-cylinder, stationary, single-acting	0.84	....
Small or medium 3-cylinder, stationary, single-acting	0.82	....
Small or medium 4-cylinder, stationary, single-acting	0.80	....
Large single-cylinder, stationary, single-acting	0.90	0.70
Large 2-cylinder, stationary, single-acting	0.86	to
Large 4-cylinder, stationary, single-acting	0.84	0.80
Large single-cylinder, stationary, double-acting	0.83	0.75
Tandem 2-cylinder, stationary, double-acting	0.81	0.73
Tandem twin 4-cylinder, stationary, double-acting	0.77	0.69

GENERAL DESIGN

Notation

Let t and T = temperature, deg. fahr. and absolute, respectively
p = absolute pressure, lb. per sq. in.
v = volume of gas or of mixture of gases, cu. ft.
c_p and c_v = specific heats at constant pressure and constant volume
$x = c_p/c_v$ = ratio of specific heats
$J = 1/A = 778$ = mechanical equivalent of heat
$R = 778\ (c_p - c_v)$ = gas constant. For air, $R = 53.2$
$M = 144\ pv/RT$ = weight of volume v of gas
Q = B.t.u. added or abstracted during a given operation
W = work in ft.-lb. done during this operation
n = exponent of the polytropic curve pv^n = const.
$r_c = v_1/v_2$ = volumetric compression ratio (see Figs. 1 and 2)
$r_d = v_3/v_2$ = constant pressure ratio, ratio of cut-off volume to clearance volume, in the case of constant-pressure combustion
$r_e = v_1/v_3$ = ratio of expansion
b.h.p.$_n$ = rated brake horse power of an engine, usually 0.85 of the maximum power
C = fuel consumption in cu. ft. or lb. per b.h.p.$_n$ per hour
C_h = total fuel consumption per hour at normal load in cu. ft. or lb.
L = volume of air actually in the mixture per cu. ft. of gas or per lb. of fuel, in cu. ft.
H = lower heat value of 1 cu. ft. or 1 lb. of the fuel under standard conditions. (29.9 in. of mercury and 62 deg. fahr.)
d = piston diam., ft.
l = stroke, ft.
v_d = piston displacement per stroke, cu. ft.
N = revolutions of the crank shaft per minute
p_m = indicated mean effective pressure, lb. per sq. in.
E_t = theoretical indicated thermal efficiency
E_i = actual indicated thermal efficiency
E_m = b.h.p./i.h.p. = mechanical efficiency
E_v = volumetric efficiency of the suction stroke (taken at 29.9 in. of mercury and 62 deg. fahr.)
$E_c = E_i/E_t$ = ratio between the indicated and theoretical thermal efficiencies = diagram factor
$E_e = E_cE_tE_m$ = economic efficiency = thermal efficiency at the brake
Unexplained suffixes refer to points on Figs. 4 and 5.

Cycles

The Explosion Method. For equations of work done and thermal efficiency, see p. 320. From the equation for thermal efficiency it is seen (assuming that x, the ratio of specific heats, is constant) that the efficiency depends on the ratio of the compression. The compression pressure should be made as high as possible.

The effect of the variability of the specific heats (see p. 366) on the efficiency as compared to the efficiency with constant specific heats, is shown in the following table based on an average of the results of Clerk and Wimperis:

Ratio of compression, r_c	3	4	5	6	7
Theoretical thermal efficiency (E_t) when the specific heats are assumed					
Constant	35.50	42.75	47.75	51.5	54.25
Variable	28.25	34.90	39.50	43.0	45.50

Constant-pressure Combustion. For general equations in terms of temperature for work done and thermal efficiency, see p. 321. The equation for efficiency in terms of volume ratios, is

$$E_t = 1 - (r_d^x - 1)/[xr_c^{x-1}(r_d - 1)]$$

The efficiency of the constant-pressure cycle is the same as for the explosion cycle with the exception of the factor $(r_d^x - 1)/x(r_d - 1)$. The efficiency depends, therefore, not only on the compression but also upon r_d, that is, the volume at the end of combustion. The smaller the value of r_d—the earlier in the stroke the fuel supply is cut off—the greater is the thermal efficiency. A large number of tests of Diesel engines have shown a greater thermal efficiency at three-quarters than at full load. That this does not hold for still lower loads is due to the influence of other factors.

The Indicated Work. The actual indicated work, W_i, is always less than the theoretical, in consequence of incomplete combustion, losses due to cooling, radiation, etc. (See full lines, Figs. 1 and 2, p. 1020.)

$W_i = p_m(v_1 - v_2) = p_m v_d$ ft.-lb. per combustion stroke.

The volume of piston displacement per stroke in cu. ft. is

$v_d = 66{,}000 \times$ i.h.p.$/Np_m$ for 4-cycle engines

$= 33{,}000 \times$ i.h.p.$/Np_m$ for 2-cycle engines.

p_m depends upon the heat value of the gas and air mixture, $H_m = H/(1 + L)$:

$$p_m = 778\ H_m E_t E_e/144 \text{ (lb. per sq. in.)}$$

With producer gas where $H = 125$ B.t.u. per cu. ft., $L = 1.2$, $E_t = 30$ per cent, and $E_e = 85$ per cent., $H_m = 125/(1 + 1.2) = 56.8$ B.t.u. per cu. ft. Then $p_m = 7.8 \times 56.8 \times 0.30 \times 0.85/144 = 78.25$ lb. per sq. in.

If the total hourly consumption of an engine is C_h lb. or cu. ft. of fuel, of a heat value of H, the actual heat added for each combustion stroke (in B.t.u.) is $Q_a = C_h H/30N$ for 4-cycle $= C_h H/60N$ for 2-cycle.

The **indicated thermal efficiency** $E_i = 2545 \times$ i.h.p.$/C_h H$. The average limiting values of $E_e = 0.5$ to 0.8. The **economic efficiency** $E_e = 2545 \times$ b.h.p.$/C_h H$. For average values of E_e, see Table 15, p. 1036; for values for special engines, see Table 6, p. 1026.

The Working Processes in Four-Cycle Engines

The Suction Stroke. At the innermost position of the piston (Fig. 4) the compression space is filled with v_c cu. ft. of unexpelled exhaust gases whose weight in lb. is $M_r = 144 p_r v_c/R_r T_r$.

If the temperature of the charge before admission is T_m, the weight of the mixture drawn in is $M_m = (E_s v_d / T_m)\ (144 \times 14.7/R_m)$ lb. at the end of the suction stroke, and the weight of the total charge in lb. is

$$M_s = M_r + M_m = 144 p_s v_T / T_s R_s,$$

Then $\quad p_s v_T / T_s = p_r v_o / T_r + 14.7 E_s v_d / T_m.,$

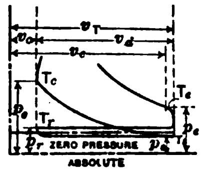


Fig. 4.

provided it is assumed that $R_s = R_m = R_r$, which can be done without sensible error. In order to obtain a maximum amount of work per stroke E_s must be made as large as possible, by diminishing the suction and exhaust resistances and the suction temperature. The influence of atmospheric pressure on E_s and consequently on i.h.p. must be especially reckoned with in plants located at high altitudes.

Table 12. Influence of the Height above Sea Level on the Volumetric Suction Efficiency

(Atmospheric pressure as given by average barometer reading b.

Height above sea level, ft.	Atmos. pressure, in. of mercury (b)	Relative volumetric efficiency (E_s), b : 29.92	Height above sea level, ft.	Atmos. pressure, in. of mercury (b)	Relative volumetric efficiency (E_s), b: 29.92
0	29.92	1.000	4,000	25.85	0.865
500	29.41	0.984	4,500	25.37	0.848
1,000	28.85	0.965	5,000	24.92	0.833
1,500	28.33	0.948	5,500	24.46	0.818
2,000	27.82	0.931	6,000	24.00	0.803
2,500	27.31	0.914	8,000	22.17	0.742
3,000	26.82	0.892	10,000	20.34	0.681
3,500	26.35	0.882			

Practical Values: p_r = 15.4 to 16.4 lb. per sq. in. abs.; t_r = 800 to 980 deg. fahr.; t_s = 170 to 260 deg. fahr. Too early closing of the exhaust valve and a very long (or too small) exhaust pipe may increase these values considerably.

Engine	Inlet valve	p_s (lb. per sq. in.)	E_s
Slow speed.........	Mechanically operated..	12.5 to 14.2	0.87 to 0.90
High speed.........	Mechanically operated..	11.4 to 12.1	0.78 to 0.83
Slow speed.........	Automatic.............	12.1 to 12.8	0.80 to 0.85
High speed.........	Automatic.............	11.1 to 11.8	0.65 to 0.75
Very high speed (automobile)	Automatic (with air cooling)	8.5 to 10.7	0.50 to 0.65

Suction producers and carburetors increase the suction resistance, and in unfavorable cases may decrease the above values of E_s by from 2 to 5 per cent.

The Compression Stroke. The compression line follows the equation pv^n = constant. Values of compression pressures (p_c), temperatures (T_c) and theoretical thermal efficiencies (E_t) for different values of r_c and n are given in Table 13. If, in any actual case, the real suction pressure should be some other value than p_s = 12.5, as p'_s, the values of p_c in Table 13 can be corrected in the ratio $p'_s/12.5$.

Values of n from Practice. The mean value of the exponent n of the compression line varies generally between 1.25 and 1.35. High temperature of the walls may raise it to 1.4 or higher, while leakages of gas diminish its apparent value.

Table 13. Values of Compression Pressures, Temperatures and Theoretical Thermal Efficiencies

[p_s taken at 12.5 lb. per sq. in. abs., and T_s at 700 deg. fahr. (t_s = 240 deg. fahr.)]

	Ratio of compression	r_c =	3.5	4.0	4.5	5.0	6.0	7.0	8.0	9.0	10.0	12.0	15.0
n = 1.30		p_c =	63.7	75.8	88.3	101.3	128.4	156.9	186.6	217.5	249.4	316.1	422.5
	$1 - \frac{1}{r_c^{n-1}} =$	E_t =	0 313	0.340	0.363	0.383	0.416	0.442	0.464	0.483	0.499	.....	.....
		T_c =	1019	1061	1099	1134	1198	1255	1306	1353	1397	1475	1577
n = 1.35		p_c =	67.8	81.2	95.2	109.8	140.4	172.9	207.1	242.7	279.8	357.9	483.8
		E_t =	0.355	0.384	0.409	0.431	0.466	0.494	0.517	0.5 7	0.553	.....	.....
		T_c =	1085	1137	1185	1230	1311	1383	1449	1510	1567	1670	1806
n = 1.41		p_c =	73.1	88.3	104.2	120.9	156.4	194.3	234.6	276.9	321.3	415.5	569.1
		E_t =	0.402	0.434	0.460	0.483	0.520	0.550	0.574	0.594	0.611	.....	.....
		T_c =	1170	1236	1297	1354	1459	1554	1642	1723	1799	1939	2125

The Clearance Volume for 4-cycle engines averages as follows (assuming p_s = 12.8 lb. per sq. in.)

Fuel	p_c, lb. per sq. in.	Clearance, per cent.	t_c, deg. fahr.
Gasoline	42.5	40	530
Illuminating gas	85	25	620
Producer gas	130	15	765
Blast-furnace gas	155	12	850
Fuel oil (constant-pressure engine)	500	7 to 8	1070

According to Illmer (*A. S. M. E.*, Dec., 1915) the volume of the vaporiser in oil engines should vary with the compression pressure. The following values are given as minimum ratios of vaporiser volume, V_v, to total clearance volume, V_c.

Compression, lb. per sq. in. gage	135	160	200	250	300	350	400
V_v/V_c	1	0.8	0.55	0.34	0.18	0.07	0

With compression above 400 lb. per sq. in. the vaporiser could be dispensed with except to allow for cold starting and other contingencies.

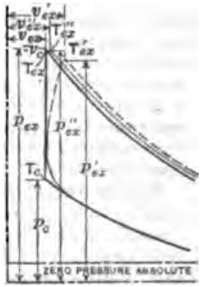


Fig. 5.

The Combustion and Expansion Stroke. For causes of differences between the ideal and the actual explosion temperatures, see section on Combustion, p. 368. The variation of the explosion from the theoretical constant-volume combustion is indicated by the inclination of the actual combustion line and by the varying expansion lines in Fig. 5.

When the explosion is retarded so that $v'_{ex} > v_c$,

$$T'_{ex} = T_c p'_{ex} v'_{ex} / p_c v_c = p'_{ex} v'_{ex} / MR_e$$

If the value of p'_{ex} can be taken directly from an indicator diagram, the above equation will give the actual explosion temperature T'_{ex}; if, however, p_{ex} or T_{ex} is taken from a hypothetical indicator card, the values of T'_{ex} or p'_{ex} can be found only by multiplying by a reduction factor which takes into account the decrease of temperature or pressure due to heat losses, cooling, etc. The value of this factor is not far from that of the card factor E_c of the cycle.

Values of n from Practice: The mean value of the exponent n of the expansion line varies, generally, between 1.30 and 1.50; infrequently an indicator card shows as high a value of n as 1.70. Loss of charge through leaks increases appreciably the apparent value of the exponent, while an indicator with too much frictional resistance gives an apparent error in the opposite direction.

Herberg (Doctor's Thesis, 1903) gives the following values, derived from investigation of an 8-h.p. Koerting illuminating gas engine:

Compression	p_c = 213.0	128.0	100.0	71.0	42.5	28.5	lb. per sq. in.
Mean exponent	n = 1.285	1.295	1.305	1.335	1.395	1.430	

Münsinger found the following results from an investigation of a 15-h.p. Diesel engine:

p_m =	35.3	56.3	68.4	83.0	83.5	88.7	98.3	105.0	107.2	116.7
n_1 =	1.29	1.36	1.33	1.13	1.37	1.33	1.30	1.25	0.85	0.85
n_2 =	1.48	1.46	1.44	1.41	1.45	1.44	1.39	1.40	1.35	1.38

where p_m = mean effective pressure, lb. per sq. in., n_1 is the smallest exponent—at the beginning of the expansion—and n_2 the largest exponent—occurring toward the end of the expansion line. As in the case of the explosion engine, a diminution of the gas temperature also resulted in an increase of n.

Values of explosion and terminal pressures and temperatures from practice are as follows:

t_{ex} = 2240 to 3140 deg. fahr.; p_{ex} = 200 to 375 lb. per sq. in. abs.;
t_e = 1190 to 1940 deg. fahr.; p_e = 37.5 to 75 lb. per sq. in. abs.

The Exhaust Stroke. (See Fig. 4.) The exhaust valve usually begins to open at between 80 per cent. and 90 per cent. of the expansion stroke. During the establishment of pressure equilibrium, the velocity of the gases is very great, in the neighborhood of 2600 to 3000 ft. per sec.

If the opening of the exhaust valve is retarded the negative work is increased and the cylinder temperature is increased, which in turn decreases the volumetric efficiency, E_s, and the allowable compression pressure. With correctly timed valve opening, sufficient valve lift, few changes of direction of the gases, and sufficiently large exhaust pipe, the resistance p_r will be a minimum, and together with properly designed inlet valves and passages, will insure a maximum value of the volumetric efficiency, E_s. Under these conditions and with an exhaust pipe fitted with a long vertical riser, to act as a chimney, it is possible through the kinetic energy of the exhaust-gas column to bring the resistance p_r down to atmospheric or even a little below. It is, however, impossible to predict with certainty the occurrence of this vacuum—several manufacturers having tried without success to utilize this phenomenon.

Values Obtained in Practice. The pressure along the exhaust line is usually from 15 to 17 lb. abs., sometimes more; the exhaust temperature (outside of the cylinder and close to the exhaust valve) varies from 575 to 1000 deg. fahr.; this may be raised considerably by the use of rich mixtures, and by after-burning due to late ignition or improper mixtures. The average percentage composition of the exhaust gases by volume is, CO_2, 10 (about); O_2, 5 to 10; CO, none or only very slight trace; H_2 or CH_4, none; N, 80 to 85. For economical running there should be no trace of combustible gases in the exhaust gas.

The variation of the specific heats of an average exhaust gas with change in temperature, is as follows:

Temperature, deg. fahr.	t = 32	t = 1000	t = 1750	t = 2500	t = 3500
	T = 492	T = 1460	T = 2210	T = 2960	T = 3960
c_p	0.2488	0.2873	0.3173	0.3472	0.3872
c_v	0.1795	0.2183	0.2482	0.2780	0.3180

Determination of the Principal Dimensions of an Engine

1. Dimensions Based on Volume of Air Used

(B.h.p. taken throughout as b.h.p. per cylinder end. For symbols see p. 1030)

A simple basis for determining the principal dimensions is the amount of air necessary for combustion. This method permits of taking into consideration the properties of the various fuels, and requires but few assumptions and established empirical factors.

Let C_s = fuel consumption per suction stroke at the rated brake load, b.h.p.$_n$; for gases in cu. ft., for liquids in lb.

L_s = actual amount of air required per suction stroke at the normal load, determined from C_s and L under standard conditions (29.9 in. Hg and 62 deg. fahr.), in cu. ft. Then

for **four-cycle engines** (per cylinder end),

$C_h = 2545 \times \text{b.h.p.}_n/HE_e$ (cu. ft. or lb. per hour)

$C_s = 2C_h/60N = 84.8 \times \text{b.h.p.}_n/E_eHN$ (cu. ft. or lb.)

$L_s = C_sL = 84.8L \times \text{b.h.p.}_n/E_eHN$ (cu. ft.)

For **two-cycle engines** C_s and L_s must be divided by 2, since there is a charging stroke every revolution.

General Equations for Engines Using Gaseous Fuels. The actual charge drawn into the cylinder during one suction stroke is $= C_s + L_s$ cu. ft. and for **single-acting engines** requires an actual piston displacement of

$$V_d = 84.8 \times \text{b.h.p.}_n \times (1 + L)/E_vE_eHN \text{ cu. ft.}$$

$d = \sqrt{216 \text{ b.h.p.}_n \times (1 + L)/E_vE_eHS}$ ft., where S is the allowable piston speed, ft. per min.;

$l = 108 \times \text{b.h.p.}_n \times (1 + L)/E_vE_eHNd^2$ ft.

$N = 108 \times \text{b.h.p.}_n \times (1 + L)/E_vE_eHd^2l$ r.p.m.

In the case of a **double-acting engine** with a piston rod (of diameter d_r) through each cylinder head, the equation becomes

$$d = \sqrt{216 \times \text{b.h.p.}_n \times (1 + L)/E_vE_eHS\,(1 - y^2)} \text{ ft. for gas fuel,}$$

where $y = d_r/d$. Writing $K = 216(1 + L)/E_vH$, the last equation reduces to

$$d = \sqrt{K \times \text{b.h.p.}_n/E_eS(1 - y^2)}$$

For **engines using liquid fuels**, substitute L for $(1 + L)$ in the preceding formulæ. For values of y, see p. 1042; for E_v, see p. 1032; for E_e, see Table 15; for S, see below; for L, H and K, see Table 14.

Table 14. Values of K in Formula for Cylinder Diameter
(Table based on a value of E_v = 0.85)

Fuel	Lower heat value (H) B.t.u. per		Air required—cu. ft.				K
			Theoretically (L_t) per		Actually, to give best results (L) per		
	cu. ft.	lb.	cu. ft.	lb.	cu. ft.	lb.	
Natural gas	950	....	9.4	....	14	....	4.01
Illuminating gas	600	....	5.5	....	8	....	3.81
Coke-oven gas	580	....	5.4	....	7.8	....	3.86
Producer gas	135	....	0.99	....	1.3	....	4.33
Blast-furnace gas	100	....	0.73	....	1.1	....	5.34
Gasoline	....	20,500	....	189	....	300	3.72
Kerosene	....	20,300	....	187	....	300	3.76
Alcohol, 90% by vol.	....	10,900	....	101	....	165	3.85
Crude oil	....	18,000	....	176	....	305	4.31

2. Dimensions Obtained by Use of Practical Coefficients

The following coefficients are based on experience with actual machines operating under certain conditions, and their use presupposes similar conditions. For special cases, or those which require exact treatment, it is better to use the previous method.

The **ratio of stroke to diameter** (l/d) lies between 1.0 and 2.0. For automobile engines the range is from 1.0 to 1.3. Small stationary engines usually have a little larger ratio, 1.2 to 2.0. In average American practice for the very large double-acting tandem or twin-tandem stationary engines, l/d = 1.37. The **allowable piston speed** S for stationary engines varies between 500 and 1000 ft. per min., as follows:

Horse power	Over 1000	1000	700	500	150	50	Small
Limits, ft. per min.	700–1000	700–1000	700–900	650–850	600–800	500–700	450–700
Average practice, ft. per min.	850	800	750	700	650	600	550

For automobile engines the usual limits are 600–1400 ft. per min., and average practice 750 ft.

Values of the **thermal brake efficiency** for various types of engines are given in Table 15, and **diagram factors** for four-cycle explosion engines using different fuels in Table 16.

Table 15. Thermal Brake Efficiencies of Different Types of Engines

Type of Engine	E_e, per cent.
Small high-speed auto, multi-cylinder, single-acting	15.0 to 27.5
Small or medium, single-cylinder, stationary, single-acting	10.0 to 17.5
Small or medium, two-cylinder, stationary, single-acting	15.0 to 20.0
Small or medium three-cylinder, stationary, single-acting	20.0 to 27.5
Large single-cylinder, stationary, single-acting	17.5 to 25.0
Large two-cylinder, stationary, single-acting	17.5 to 25.0
Large single-cylinder, stationary, double-acting	20.0 to 25.0
Double-acting tandem, two-cylinder	20.0 to 30.0
Double-acting twin tandem, four-cylinder	20.0 to 27.5

Table 16. Diagram Factors for Four-cycle Explosion Engines

Fuel	Compression, lb. per sq. in. gage	Diagram factor, E_e, per cent.	Fuel	Compression, lb. per sq. in. gage	Diagram factor, E_e, per cent.
Natural gas	90 to 140	40 to 52	Gasoline	80 to 105	40 to 65(*b*)
Illuminating gas	80	45	Kerosene, vaporized before injection	45 to 75	30 to 40
Coke-oven gas	100 to 135	45(*a*)	Kerosene, injected. May run as low as 20		
Producer gas	100 to 160	40 to 56	Alcohol	75 to 210	72 to 74(*b*)
Blast-furnace gas	130 to 180	30 to 48			

(*a*) Probable. (*b*) Strong, U. S. Geol. Survey.

In Diesel engines operating at rated load with fuel admission for about 10 per cent. of the stroke, the diagram factor varies between 50 and 70 per cent.; in well-designed engines of good workmanship the diagram factor averages 70 per cent. The diagram factor for a two-cycle engine may be taken as 0.8 of that for a four-cycle engine.

When the cylinder diameter is based on the mean effective pressure of the theoretical air card (p'_m), the diagram factor (E_c), the pressure ratio of compression, and the heat supplied per cu. ft. of best mixture, then, for a four-cycle engine

$$d\ (\text{inches}) = \sqrt{168{,}100 \times \text{b.h.p.}_n \times (1 + a)/E_m p_m S(1 - y^2)}$$

where a = overload factor = (total b.h.p. − b.h.p.$_n$)/b.h.p.$_n$, and $p_m = E_c p'_m$

$$= 5.4\left[1 - (p_s/p_c)^{\frac{s-1}{s}}\right]E_cH/(1 + L) = KE_cH/(1 + L), \text{ lb. per sq. in.}$$

Values of K for various values of p_c/p_s are as follows:

p_c/p_s =	3	4	5	6	7	8	9	10	12	15	18
K =	1.46	1.77	1.99	2.17	2.30	2.44	2.52	2.60	2.75	2.91	3.04

For oil fuels, substitute L for $(1 + L)$.

Capacity Coefficients. In Table 17 the values are referred to normal brake horse power (b.h.p.$_n$) of single-acting four-cycle engines. For maximum load they should be increased in proportion. The table includes:

1. Indicated mean effective pressure p_m, from which b.h.p.$_n$ = 0.066 $p_mE_mV_s$, where V_s = piston displacement, cu. ft. per sec.
2. Specific capacity W_o = power per cu. ft. of piston displacement per sec., expressed in ft.-lb. per cu. ft. per sec.; b.h.p.$_n$ = $W_oV_s/550$.
3. Specific piston displacement V_o, in cu. ft. per sec. for 1 b.h.p.

$$\text{b.h.p.}_n = \frac{d^2S(1-y^2)}{11{,}000\ V_o},\ \text{or } d(\text{inches}) = \sqrt{\frac{11{,}000 \times \text{b.h.p.}_n \times V_o}{S(1-y^2)}}$$

4. Capacity constant K_c. b.h.p.$_n$ = $K_c\, d^2S(1 - y^2)$.

Table 17. Capacity Coefficients for Single-acting Four-cycle Engines

Fuel, assuming E_m = 80 per cent.	M.e.p. p_m lb. per sq. in.	Specific capacity $W_o = \frac{144 p_mE_m}{4}$ ft.-lb. per cu. ft. per sec.	Specific displacement $V_o = \frac{550}{W_o}$ cu. ft. per sec. per b.h.p.	Capacity constant $K_c = \frac{p_mE_m}{168{,}000}$	Relative capacity referred to illuminating gas engine = 1.00
Natural gas	85.0	2450	0.224	0.000405	1.10
Illuminating gas	77.5	2230	0.247	0.000369	1.00
Coke-oven gas	77.5	2230	0.247	0.000369	1.00
Producer gas	62.5	1800	0.306	0.000298	0.81
Blast-furnace gas	57.5	1660	0.331	0.000274	0.74
Gasoline	75.0	2160	0.255	0.000357	0.97
Kerosene	55.0	1580	0.348	0.000262	0.71
Alcohol	55.0	1580	0.348	0.000262	0.71
Crude oil (Diesel)	100.0	2880	0.191	0.000477	1.29

The specific capacity W_o for double-acting four-cycle engines is twice as large as the values given above for single-acting engines. For two-cycle single- and double-acting engines with separate gas and air pumps W_o can be assumed to be respectively from 1.75 to 1.90 and from 3.5 to 3.8 times larger than the values in Table 17. The specific displacement is decreased in the inverse ratio. In small high-speed two-cycle engines with crank-case compression the specific capacity is no larger than for four-cycle engines of the same cylinder dimensions, due to low volumetric and mechanical efficiency.

CONSTRUCTION AND DESIGN

Vertical Engines are used for small and medium powers because of their simplicity and low cost of construction; for medium powers the capacity is obtained by using multi-cylinders, keeping the cylinder dimensions small and retaining the simplicity. Double-acting vertical engines are not built at the present time because of the trouble in locating the valves in the lower head, and because of the fact that the piston must then be water-jacketed.

Advantages of the vertical over the horizontal type: The foundations need not be as large or heavy, since, without counterbalancing, the free force is in the vertical plane. The floor space occupied is about two-thirds of that required for a horizontal single-acting engine of the same power. It is easier to lubricate the cylinder uniformly, and the wear in the cylinder is less.

Disadvantages: In large sizes less accessibility; dust and grit tend to remain in a vertical cylinder and score the cylinder bore and piston packing rings.

Horizontal Engines are built single-acting, using a trunk piston for small and medium powers, and are advantageous because of the simplicity of construction, decreased length of the machine, and consequent lower manufacturing cost. Some medium and all large engines are built double-acting. In the larger sizes of single-acting engines a cross head has to be used to keep down the cylinder wear.

Multi-cylinder Engines. For higher capacity and high coefficients of regulation, multi-cylinder engines can be built at less cost than single-cylinder engines. The arrangements of horizontal multi-cylinder engines in use at the present time are single-cylinder twin, two-cylinder tandem, and twin-tandem engines made up of two two-cylinder tandems. Two-cylinder opposed and twin-two-cylinder opposed engines were formerly built but have now become obsolete, due principally to the difficulty of building a forked connecting rod—to run on the same crank pin as the opposed cylinder—which would run even reasonably cool.

Relative Advantages. The single-cylinder twin engine has the advantage over the two-cylinder tandem that the weight of reciprocating parts is less; the cost of construction, however, is greater. Two-cylinder tandem engines are almost universally constructed with main, intermediate, and tail-guide cross heads. With this construction special provision must be made to allow the engine to expand. In a 600-b.h.p. double-acting tandem producer-gas engine, 45 ft. in overall length, the **expansion from cold to full-load condition** amounts to $^{3}/_{16}$ in. The expansion is generally taken care of by anchoring the main frame rigidly to the foundation and by supporting the tie piece and tail guide on foundation plates. The contact surfaces of the tie piece, tail guide and foundation plates are planed and lubricated. In order that the engine may expand freely in the direction of the center line but be restrained from distorting laterally, the tie piece and tail guide are gibbed to the foundation plates with only enough clearance (0.015 in. total sideways and 0.015 in. up and down) to allow them to slide freely on the foundation plates. The engine is suitably prevented from lifting by bolting the tie piece and tail guide to the foundation plates with bolts which are screwed down on to bushings and machined washers, the bushings being of such a length that there is only a sufficient clearance between the lower side of the washers and the machined pads on the tie piece and tail guide to allow them to slide freely (about 0.015 in.). Care must be taken that there is sufficient clearance between the bushing and the sides of the holes through the pads to allow of the full movement due to expansion.

A larger factor of safety must be employed than in the case of steam engines, because accidental occurrences such as preignitions, the force and time of occurrence of which are not subject to calculation. In few of the smaller engines does the maximum pressure exceed 360 lb. per sq. in., while many of the modern large engines have a maximum limit of 450 to 500 lb.

Foundations. The reciprocating weights of gas engines are greater and the speeds of rotation generally higher than in the case of steam engines, necessitating more mass to the foundation, which should always be carried down to a firm footing. Concrete floors and the walls of the engine house should not be tied rigidly to the foundations, because of the vibration. The average **volume of material in foundations** for the different types of engines may be taken as follows:

For horizontal engines without outboard bearing, 14 to 18 × b.h.p.$_n$ cu. ft.
For horizontal engines with outboard bearing, 21 to 25 × b.h.p.$_n$ cu. ft.
For vertical engines without outboard bearing, 7.7 to 8.8 × b.h.p.$_n$ cu. ft.
For vertical engines with outboard bearing, 9.8 to 10.5 × b.h.p.$_n$ cu. ft.

Frames. The allowable fiber stress for cast iron of the best grade should not exceed 3500 lb. per sq. in. in bending, as computed from the total explosion pressure. It is especially important to take up the maximum total explosion stress as nearly as possible in an axial direction. Frames for vertical engines lend themselves easily to this, but those for horizontal engines do not. Types of frames for horizontal engines in general use are all subject to considerable bending stress; the moment arms of the bending stress, where unavoidable, must be made as short as possible. The foundations should not be figured as taking any of the stress. The unit pressure or load on the foundations due to the weight of the machine and the inertia forces should not exceed ½ to ⅓ of the usual values for the various materials under the main bearings of horizontal engines.

Box and "A" Frames for Vertical Engines. Determinations of the thickness of walls or webs in frames, on the basis of strength, give webs so thin as to give rise to molding and casting difficulties and to bad casting strains. The walls are therefore made thicker than is required for strength. The **box frame** is used more commonly in the United States for vertical engines than the "A" frame. The facility with which it can be completely enclosed makes it especially suitable for dusty locations, and the oiling is simplified as the splash system can be utilized. A special application of this type is its use for automobile engines, where lightness as well as dust-proof construction are prime requisites; lightness is secured by using aluminum alloys and strength by ribbing the crank case on the outside. The usual thicknesses, s, for various cylinder diameters, d, are as follows:

d(inches) =	5	7	9	11	13	15
s(inches) =	½	9/16	⅝	11/16	¾	13/16

Check computations should be made to determine the tensile stress in the smallest cross-section of the frame (and of the jacket wall, if the cylinder and frame are cast in one piece) and the bending stress in the cross-section below the main bearing, taking the distance between the foundation bolts as the moment arm. This latter is especially necessary in case the main bearings are located in a separate base plate, on top of which the frame proper is bolted (the distance between the holding-down bolts in this case being the moment arm).

The **"A" frame**, adapted from steam-engine practice, is used in Germany for medium-sized vertical engines. It is considerably lighter than the box frame, due to the better disposition of the metal. A check computation for this type—in addition to the two mentioned above—should be made for the legs of the "A," which are subject to tension and bending (the forces being the maximum total explosion pressure resolved into components along and normal to the axis of the "A" leg, respectively). The **allowable stresses** should not exceed the following values: Tensile and bending stress, each 2150 lb. per sq. in.; combined tensile and bending stress, 2500 lb. per sq. in.

Frames or Beds for Horizontal Engines. The dangerous section is usually in a plane perpendicular to the center line of the engine, at a place somewhere between the main bearing and the cylinder connection, where the bending moment arm, *i.e.*, the distance between the center line of the engine and the neutral axis of the section, is a maximum. To make this bending moment a minimum the neutral axis of the dangerous section must be as close as possible to the center line of the engine. The side walls should be carried as high as possible—in small engines at least to a level with the center line of the engine, and in large engines still higher. The allowable combined stress is the same as for "A" frames.

The other part of the frame under severe stress is the vertical section through the center line of the main bearing. This section is subject to bending. The best form for the main bearing is the one in which the explosion load is taken up directly by the frame. The bottom shell of the bearing should be fitted with a ball and the frame with a socket. If the bottom of the bearing jaws conforms closely to the curvature of the ball socket, the jaws are stronger than if the bottom of the bearing is horizontal and filleted at the jaws.

Cylinders. The **material** used is generally a hard, close-grained cast iron, and cylinders are designed with an allowable stress of 4250 lb. per sq. in. when the stress conditions are fairly well defined; in other cases the allowable stress is taken at a lower figure.

In medium-sized engines of 8 in. diameter and up, the cylinder wall or barrel is cast separate from the jacket, provided the inlet and exhaust ports do not compel the use of a one-piece casting. The advantages of this construction are that suitable hard iron can be used for the barrel, to withstand the wear, and the jacket can be made of a soft iron to give ease of casting; the cylinder barrel can expand axially independently of the jacket, provided it is fastened rigidly only at the head end.

In large single-acting engines the **one-piece cylinder** has its advantages on account of being a stiffer construction, but the greater difficulty in manufacture and the unequal expansion partially offset this. The total of the stresses due to expansion and to the explosion pressure may far exceed the elastic limit of the material. It is not easy to relieve the jacket walls of double-acting cylinders with valve ports cast integral with the cylinder, of longitudinal temperature stresses, and it is difficult to get good castings. Additional temperature stresses are caused by bad distribution of metal (particularly at the valve ports), therefore a cylinder of this type should be designed with symmetry. Unequal distribution of metal also causes irregular radial expansion of the cylinder bore, which gives rise to cutting and leaking between the cylinder and piston. When this type of cylinder is employed, a hard, close-grained iron liner is almost always shrunk into the cylinder to take the wear. This liner must be securely locked at the middle of the cylinder and left free to expand at the ends; otherwise it may be pulled out of place should a piston ring break and become jammed between the piston and liner. The liner is not figured to take the radial stress at all, but its thickness is determined by the reboring allowance.

A method of obtaining **cylinder castings** which are **free from longitudinal temperature stresses** is to cast the barrel and jacket walls integral, with the exception of a ring of the jacket wall around the center of the cylinder, which is cored out. This ring is closed by split jacket bands or saddles of sheet metal or cast iron, bolted together at opposite ends of a diameter and packed with flexible packing to permit of expansion. When a cast-iron saddle is used it may be cast in such a shape as to form gas and air pipe belts leading to the mixing chambers. A further extension of this method is to cast the cylinder in halves with the joint end up (the poor metal can then be machined off); the faces of the joint are then ground, the two halves held together by means of shrink links, and the opening in the jacket wall covered as in the previous case. A very few manufacturers in this country and Germany adopted cast steel because of the difficulty in obtaining good iron castings, but their use has not been entirely satisfactory in practice. The greater expansion and the much higher modulus of elasticity of cast steel as compared with cast iron, cause increased stresses, while the strength of cast steel is not proportionately greater.

The **water jacket** should cover the entire length of the stroke, to avoid unequal expansion in the cylinder bore and burning of the lubricating oil. The water space should be wide, and, in large cylinders, cleaning holes should be provided. The life of the cylinder can be materially increased if cleaning is done at regular intervals of from 4 to 8 weeks, depending on the water.

The maximum explosion pressure causes tensile stresses in the cylinder walls, both longitudinal and radial. To these must be added, under certain circumstances, secondary stresses due to drawing up on studs or bolts, thrust of the connecting rod (with trunk piston), unequal temperature stresses, etc. If the cylinder and jacket walls are cast integral the longitudinal stress may be neglected since the radial stress is always more than twice the former. If the jacket wall and cylinder are cast separate, the jacket wall alone must sustain the longitudinal tensile stress.

Gas-engine cylinders may have the following wall thickness (in inches) according to Bach:

$$t = \tfrac{1}{2}d[\sqrt{(k_t + 0.4p)/(k_t - 1.3p)} - 1]$$

where d = bore of cylinder, in.; k_t = safe stress in material, lb. per sq. in. and p = maximum explosion pressure, lb. per sq. in. Taking $p = 356$ and $k_t = 3500$, $t = 0.047d$. Where a cylinder liner is not used, a **reboring allowance** X, values of which are given in the table below for various diameters (all dimensions in inches), should be added to this thickness. Where a **liner** is used, X gives its necessary thickness.

d =	4	6	8	10	12	14	16	18	20	24	28	32	40	48
X =	3/8	3/8	13/32	7/16	15/32	15/32	15/32	1/2	17/32	9/16	19/32	5/8	23/32	13/16

The inner fibers of the cylinder wall are under a greater stress than the outer, and this inequality increases rapidly with an increase in the thickness of the wall. Excessive thickness also causes the jacketing to be less effective, consequently **flanging or ribbing the cylinder** is more effective than thickening the wall. The cylinder should be checked for tensile, bending and shearing stresses, due to drawing up the cylinder head. This pressure must at all times exceed the total internal head pressure due to the explosion in order to maintain **tight joints.** Experience has shown that an addition of from 10 to 20 per cent. to the internal head pressure is sufficient to maintain the joint, but by excessive tightening of the bolts this may be increased by 50 per cent. The **allowable stress** for both tension and shear should be 2800 lb. per sq. in. The **jacket wall,** from foundry considerations, should not be less than 7/16 in. thick for small engines.

The **allowable stress in bolts** at the base of the threads may be taken for soft steel as 5500 to 6500 lb. per sq. in. All **joints** should have as little packing material between the surfaces as possible. Wherever possible, the joints should be made metal-to-metal, with possibly the interposition of a very little white lead.

Pistons are usually made from a good grade of close-grained cast iron with an allowable stress of from 5000 to 6000 lb. per sq. in. In the larger engines cast steel is sometimes used.

For single-acting engines the trunk piston is the type generally used (see Table 18 and Fig. 6 for dimensions from Haeder). The length is determined by considering the piston as a cross head, using an allowable unit pressure of 20 lb. per sq. in. without deducting the ring grooves.

Table 18. Dimensions of Gas-engine Pistons
(All dimensions in inches. Letters refer to Fig. 6)

Engine		Piston body					Piston pin (cross head)		Piston rings	
Stroke	*D*	*a*	*b*	*c*	*h*	*k*	*d*	*e*	*f*	*g*
5	3¼	3 11/16	2½	⅜	5/16	3/16	1 3/16	1 9/16	3/32	3/16
6¼	4	4 11/16	3¼	½	⅜	3/16	1⅜	2	⅛	3/16
7½	5	5⅞	4	9/16	7/16	¼	1¾	2⅜	3/16	¼
8¾	5¾	6⅞	4½	⅝	½	¼	2	2¾	3/16	¼
10⅝	7¼	8¼	5½	¾	⅝	¼	2⅜	3 9/16	¼	5/16
12¼	8¾	9 13/16	6⅝	⅞	11/16	5/16	2¾	4 5/16	¼	⅜
13¾	10¼	11 13/16	7⅞	1	¾	⅜	3⅜	5⅛	5/16	7/16
15¾	11⅞	13 3/16	8⅞	1⅛	1	7/16	3¾	6	⅜	9/16
17¾	13	14 9/16	9⅞	1 3/16	1 1/16	½	4⅛	6½	⅜	9/16
19¾	14¼	15 5/16	10¼	1 5/16	1 3/16	9/16	4½	7 1/16	7/16	⅝
21⅝	15¾	17 5/16	11⅜	1½	1¼	⅝	5⅛	7⅞	½	¾
23⅝	19	20 7/16	13⅜	1¾	1½	11/16	6	9½	9/16	⅞
27½	23⅝	24¾	16½	2 3/16	1¾	¾	7½	11 13/16	11/16	1 1/16

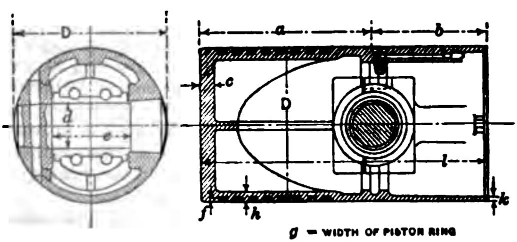


FIG. 6.—Gas-engine Piston.

The maximum thickness of the piston barrel (at the ring grooves) may be made $t = 0.02d +$ thickness of the rings + 0.2 in. If the ribbing is sufficient, the thickness at the open end of a trunk piston may be one-half to one-third of the maximum—the taper to start at about the first ring groove. To allow for expansion, the inner end of the piston barrel should be tapered down by from 0.2 to 0.5 per cent., starting at the last ring groove; when water-cooled, relieve only as far as thickness of the piston face. The **width of the bridge between the ring grooves** should be at least equal to the width of the groove. The **depth of the groove** should be but about 0.02 in. greater than the thickness of the rings. In order to distribute the wear evenly over the cylinder length, a few of the rings are sometimes put near the open end of the piston; this also serves to retain the oil on the piston surface in better shape. In vertical engines with splash lubrication, this arrangement is essential in order to prevent too much oil being sucked up into the combustion chamber during the suction stroke. In order to obtain a uniform distribution of the connecting-rod thrust, the wrist pin should be located at the center of that part of the piston which carries the cross-head load.

If **large pistons** are properly **ribbed**, their stiffness and strength are materially increased. For pistons up to 20 in. in diam. from 6 to 8 radial ribs, sometimes only on the bottom, each from 3/8 in. to 5/8 in. thick, and a couple of strong ribs each side of the wrist-pin bosses, are sufficient. For larger pistons, the lower radial ribs at least should be extended to the open end, and the wrist-pin bosses should be stiffened by two circular ribs. Thin ribs, properly placed and in sufficient numbers, give a lighter construction for equal strength than a few thick ribs, and assist in the cooling of the piston by increasing the radiating surface.

Cast iron is generally used in making **disk pistons** for double-acting engines. Such pistons have been found by American builders to give trouble by cracking across between the cleaning plugs, although they are rendering satisfactory service in Germany. **Cast-steel pistons** have occasionally given trouble either by cutting the cylinder walls or by the slots widening, or by beading over and binding the rings. The trouble due to slot widening was taken care of by making the bridges between the slots much wider than the slots, so that when the slots had become too wide they could be re-turned and wider or double rings fitted. This increased the maintenance and caused most builders to go back to cast iron for all except the largest engines. Modern cast-iron pistons are usually without cleaning holes other than the jacket-water inlet and outlet openings. A double-acting piston is a cantilever beam subjected to a shock load which is applied alternately in opposite directions; the connection between the face of the piston and the rod boss must therefore have enough metal to prevent fatigue of the metal due to this alternating force, otherwise the piston will crack here.

Piston Rods for large engines are either forged hollow, or solid and then bored. Open-hearth steel of the following percentage composition has been found to be very satisfactory for piston rods: C, 0.45 to 0.60; Mn, 0.45 to 0.60; P, under 0.04; S, under 0.04; Si, 0.10 to 0.20. Such steel has an elastic limit of 50,000 lb. per sq. in. and an ultimate strength of 95,000 lb.; elongation in 2 in., 12 per cent. These rods were formerly cambered, but this refinement is not practised at the present time as the deflection is not serious (about 0.035 in. for a 34-in. piston). The outside diameter of the rod, from average American practice, is 31 per cent. of the cylinder diameter (German, 26 per cent.). The diameter of the bore averages 38 per cent. of the outside diameter of the rod. The ends of the rod for the cross-head, intermediate and tail-guide connections should be from 1/4 in. to 5/8 in. smaller in diameter than the body of the rod.

The **strength** of the rods as checked by using the long-column formula of Euler (see p. 435) should not give a factor of safety of less than 20. The ends of the rods should also be checked for tension and shear at the base of the threads if this method of connection is used. The Acme thread is the most satisfactory.

Piston-rod Packing. Metallic packing is universally used, the material generally employed being a good grade of close-grained soft cast iron. Softer alloys have been tried, but the wear and the consequent maintenance are too great.

The packing generally consists of a single or double sectional or segmental ring held to the rod by garter springs. The packing effect is obtained by closely fitting the ring to the rod and by the use of a number of rings or groups of rings in series, separated by solid one-piece rings or the bridges of the packing case. The garter spring should only be tight enough to keep the parts of the ring from displacement by the movement of the rod. The principal trouble with these packings is that the explosion pressure leaks into the space on top of the rings, forces the segments on to the rod and shoulders the rod and wears out the ring in a short time. To reduce the refitting of segmental rings to a minimum, they should be made of as many segments as possible (not less than three, preferably five).

In German practice the packing is housed in the cylinder head itself without a separate packing case, but in American practice a separate packing case is used, which fits into the cylinder head and is sometimes itself water-cooled. This packing case should be uncooled only when the cylinder head is free of valve and igniter openings. Forced lubrication distributing the oil around at least one ring must be used for successful operation.

Piston Rings. Cast-iron snap rings of uniform cross-section give the best service; narrow rings, in a sufficient number, make a tighter packing and with less friction than a smaller number of wider rings; two narrow rings in a groove give better results than one wide ring.

The stress caused by spreading the ring to slip it on the piston should be the same as the stress during operation. With an allowable stress, K_b, of 12,000–17,000 lb. per sq. in., where r_m is the mean radius of the ring $= \frac{1}{2}(d - s)$, the **thickness** in inches $= s = r_m\sqrt{K_b}/2145$. The **width** of the ring $= b = 1.25s$ to $2s$ for one wide ring to the slot. If two rings to the slot are used, width of each $= \frac{1}{2}b$. Dimensions of concentric snap piston rings for the working cylinders of constant-pressure engines, as made by the Davy Robertson Piston Ring Co., Berlin, are as follows (to the nearest 0.001 in.):

Cylinder bore, in........	3.94 to 11.83	11.83 to 15.77	15.77 to 27.57
Width of ring, in........	0.315	0.355	0.394
Thickness of ring, in.....	0.158 to 0.335	0.374 to 0.453	0.473 to 0.788

(The thickness advances by 0.02 in. for an increment of 0.787 in. in the cylinder bore.) The **number of rings** to be used $= n = d/5b$, approx. The gap distance (in.) to be cut from the ring after preliminary turning $= a = 9.5r_m dK_b/2sE$, where E = the modulus of elasticity. The external diam. (in.) for preliminary turning is $d_1 = d + (a/\pi) + 0.12$. The gap is cut either straight across, as a step joint (not used below 8 in. diam.), or diagonally across (the simplest and the best joint for all sizes).

A clearance allowance for expansion a_1 (in.) $= 0.000006\pi d\ (t_r - t_s)$ should be filed out at the joint. t_r is the mean temperature of the piston rings and t_s that of the cylinder barrel. The following estimates are based upon practical experience: Mean temperature of the body of a trunk piston without water-cooling, about 525 deg. fahr.; of the first ring, $t_r = 435$ deg. fahr.; and of the cylinder wall, $t_s = 165$ deg. fahr., approx. Therefore $t_r - t_s = 270$ deg. and $a_1 = 0.0051d$. For trunk pistons without water-cooling, provided the workmanship is good, the clearance for expansion may be taken as $a_1 = 0.0060d$ to $0.0075d$. The temperature of the piston rings decreases very quickly toward the outer end—at the last ring the difference $(t_r - t_s)$ will scarcely amount to more than 90 to 110 deg. fahr. Nevertheless, all the rings of a set or of a size should be given the same clearance in order to make them interchangeable. For large water-cooled pistons the temperature difference is very small and a much smaller clearance (a_1) suffices—from 0.040 in. to 0.2 in., depending upon the diameter.

Ignition is generally by electric spark in modern engines using gas or gasoline as fuel. See p. 1608. Explosion engines working on the lighter of the liquid fuels (kerosene, light crude oil, etc.) use either an electric spark or the hot walls of the gasifier. In large gas engines high-tension electric ignition is never used because of the difficulty of insulation. The most common system is a mechanical make-and-break using a low-tension direct-current make-and-break, with choking coils in the circuit to give sufficient surge voltage to cause the current to arc across the igniter electrodes (see p. 1611). The use of magnetic make-and-break ignition is being discontinued because of the difficulty in accurately timing the ignition, due to the magnetic lag. A high-tension jump-spark ignition is the most commonly used system for gasoline, and in some cases for the other light liquid fuels. If the charge is thoroughly mixed the best results are obtained with the ignition starting at the center of the combustion space. For maximum efficiency in an explosion engine the combustion should take place as nearly at constant volume as possible, but when the explosion takes place in that manner there is too much shock to the engine. The igniter should be set with the ignition advanced so as to give indicator cards with a slightly rounded top. This occurs on the average with the igniter advanced about 35 deg.

For details of ignition systems, see p. 1608. Two arrangements of igniters for double-acting cylinders are used in American practice. When the gas valve closes before the inlet valve—thus filling the mixing chamber and at least part of the inlet port with a stratum of pure air—the usual arrangement is to place two igniter plugs in each combustion chamber, one on either side of the cylinder, 45 deg. down from the vertical, where the mixture is probably purest. When the gas valve closes at about the same time as the inlet valve, the mixture is purest in the inlet port next to the inlet valve. In this case one igniter is located in the onion-shaped inlet port and the other on the opposite side of the combustion chamber, 45 deg. down from the vertical. In the largest engines built, using blast-furnace gas, a third igniter, placed 45 deg. up from the bottom and set to trip 2½ deg. earlier than the other two, has increased the capacity 20 per cent.

Make-and-break igniters are now generally of the hammer-and-anvil type. Steel points are very generally used. An improvement is in the use of hammers and anvils whose surfaces come into contact without the use of points, for which purpose cast iron is the best material. For any type of make-and-break ignition, a known resistance—such as a lamp—should be in each plug circuit to govern the amount of current flowing, otherwise the electrodes will be rapidly burned out. The best results are obtained from 110-volt ignition with from 0.5 to 1.5 amp. per plug, depending upon atmospheric, gas, and operating conditions. With cast-iron hammer and anvil and continuous 24-hr. operation, a surface oxide forms on the electrodes, so that at least 1-amp. current must be used for the igniter. The best results are obtained by using a separate choking coil (so designed as to give a surge voltage of 5000 at the instant of break) for each igniter, or at the most, each two igniters.

Valves. Valve cages or valve seats are made of hard, close-grained cast iron. Valves for small cylinders are made of carbon or alloy steel. Valve disks for large engines are made of cast iron with stems of carbon steel fastened to the disks. The allowable stress in valve disks should be low on account of the high temperature, to avoid warping, and to give enough metal for regrinding.

Gas-engine valves at the present time are of the poppet type, with conical seats, with the exception of some in automobile engines. Automatic inlet valves are used only on small, slow-speed, cheaply constructed engines because of the noise of operation, rapid wear, liability to fracture, and low volumetric efficiency. All valves should be placed in a vertical position to avoid, as far as possible, leakage, sticking, and the extra length of guide necessary for a horizontal valve. The inlet valve should be placed directly over the exhaust valve, in order to cool the exhaust valve by the incoming charge. The exhaust-valve chamber must be cooled. In engines above 100 h.p. the exhaust-valve disk and stem are often water-cooled to keep down the cutting effect of the hot gases upon the valve seat, and to aid lubrication. In Germany the present tendency is to make the exhaust valves for largest engines of forged steel or Durana metal and largely of the non-cooled, mushroom type.

The **dimensions of the ports and passages** in four-cycle engines may be determined from the formula

$$A_g = C_s N/2v = C_s N/9000, \text{ and } A_a = L_s N/2v = L_s N/9000$$

where A_g and A_a are respectively the cross-sectional areas of the gas and air ports and passages in sq. ft., C_s and L_s the volumes of gas and air respectively in cu. ft. per suction stroke (see p. 1035) and v the allowable mean gas, air or mixture velocity = 4500 ft. per min. The smallest **diameter of the valve seat** is d_1 (in.) $= \sqrt{v_d N/49.1}$. For N and v_d see p. 1030. Sometimes it is impossible to realize mean velocities as low as 4500 ft., in large engines the velocity going as high as 6000 to 8500 ft. per min. The **diameter of the exhaust-valve seat** should be larger than that of the inlet valve.

The **valve cages** should be seated in the cylinder or cylinder-head castings on a flat surface with a metal-to-metal joint. The **thickness of the disks** should be determined by considering them as flat plates supported at the circumference, and, because of the high temperature and necessity for regrinding, the allowable stress should be taken at 5700 lb. per sq. in. for mechanically operated valves and up to 11,000 lb. for automatic valves. The **width of** the **valve seat** may be made approximately $b = 0.01d_1 + 3/16$ **in., the angle of the seat being generally 45 deg. The diameter of the valve stem** is $d_s = 1/8 d_1 + 1/4$ to $3/8$ in. The stem of the exhaust valve should be larger than this in order to decrease the wear and to increase the water-cooling surface. The **diameter of** the **valve cage** should be not less than $1.6d_1$.

With mechanically actuated valves the valve spring should exert a tension of not less than 7 lb. per sq. in. of valve surface in engines with throttle governing; with automatic valves from 0.7 to 1.0 lb. per sq. in. according to the engine speed. The strength of the spring must be increased considerably if it has to move part of the valve gearing. In large engines the deflection of springs in operation is considerable, in addition to the load at no deflection; at the maximum deflection of the spring, *i.e.*, with the valve wide open, there should be from 0.05 to 0.10 in. clearance between each two consecutive coils.

Valve Gear. Cam or lay shafts are generally made from machine steel; cams and rollers from hardened steel; eccentrics and eccentric straps from cast iron; valve levers from soft or cast steel (sometimes case-hardened); driving gears generally from steel upon cast iron or upon bronze, or if well lubricated, cast iron on cast iron for screw gears; and cast iron on cast iron for spur or bevel gears.

In small engines the valves are generally actuated by cams, while in medium-sized and large engines the wear on a cam motion is too great and eccentrics are generally employed. In the larger engines the main lay shaft is driven from the crank shaft through an intermediate shaft, so that the half time is obtained through two sets of gears. In such a case the wear on the gears is reduced by introducing a "hunting tooth." Common tooth angles for the screw gears used are 63 deg. 25 min. for the driving gear and 26 deg. 35 min. for the driven.

Large cam diameters permit of accurate adjustment of the valve motion, but the peripheral velocity of the cam should not exceed 3 ft. per sec., as knocking against the cam rollers is likely to result and the torque in the shaft is increased. The **width of cam rollers** for inlet valves can be 0.3 of the diameter of the roller, and for the exhaust valve 0.4. These dimensions should be checked to see that the maximum pressure does not exceed 3000 lb. per in. of width. Computations for strength of **valve levers** should be based on a pressure at the moment of opening of at least 30 lb. per sq. in. for the inlet, and 75 lb. per sq. in. for the exhaust, valve.

Valve Setting. In a single-cylinder, single-acting engine a typical timing of the valve events would be as follows:

Inlet valve starts to open when crank is 20 deg. from head-end dead center (approaching)
Gas valve starts to open when crank is 10 deg. from head-end dead center (approaching)
Exhaust valve closes when crank is 15 deg. from head-end dead center (leaving)
Max. lift of inlet and gas valves occurs when crank has moved through 95 deg. from head-end dead center.
Gas valve closes when crank has moved through 200 deg. from head-end dead center.
Inlet valve closes when crank has moved through 210 deg. from head-end dead center.
Exhaust valve opens when crank has moved through 485 deg. from head-end dead center.
Max. lift of exhaust-valve occurs when crank has moved through 610 deg. from head-end dead center.

Regulation. The regulating device should be as close to the inlet valve as possible, particularly in a multi-cylinder engine. In the case of double-acting cylinders, especially with large mixing chambers, control remote from the inlet valves gives rise to especially poor results, in that one of the ends will take the greater part of the gas in the mixing chamber, and when the suction stroke of the other end starts, the gas has not had time to flow into and fill the mixing chamber. As a result, the first end may take from 60 to 75 per cent. of the load. This is not serious while the load is light, but at and above 0.75 normal load, the first end will be considerably overloaded, often above its designed capacity, and will heat up, resulting in preignitions and back-firing.

Table 19. Weights of Reciprocating Parts for Various Types of Engines

Kind of engine	Weight per sq. in. of piston surface, lb.: Explosion engine	Constant-pressure engine
Single-acting engines:		
Trunk piston, short stroke ($l < 1.5d$)	5.5– 8.5	7.0–10.0
Trunk piston, long stroke ($l \geq 1.5d$)	8.5–10.5	10.0–11.5
With cross head ($l = 1.5d$ to $1.33d$)	12.5–17.0	14.0–18.5
Two-cylinder tandem	17.5–21.5	19.0–23.0
High-speed automobile	0.35–0.60	....
Double-acting engines:		
Single-cylinder, without tail rod	14.0–18.0	....
Single-cylinder, with tail rod	17.0–20.0	18.5–21.5
Two-cylinder tandem	21.0–25.5	23.0–27.0
Three-cylinder tandem, one of which is a blowing tub.	28.5	....

German builders use the center crank exclusively, objecting to the side or overhung crank because of the greater strains due to its being supported only on one side. American builders, however, generally employ the side crank, because of the fact that the center crank requires three bearings, which are difficult to keep in line.

Crank Shafts, especially for large engines, are generally made from open-hearth steel with a tensile strength of at least 64,000 lb. per sq. in. and from 18 to 20 per cent. elongation. The **allowable stress**—on account of the impact effect of the explosion pressure—should be taken at about 12,000 lb. per sq. in.

Crank-shaft Bearings. The allowable **bearing pressure** should be taken at 400 lb. per sq. in. The maximum unit pressure, due to the explosion pressure, should not be so great that the oil film can be squeezed out of the bearing, but say 1400 to 1600 lb. per sq. in. See also pp. 704–708.

Main Bearings. The material generally used for journal diameters (d_j) up to 5 in. is bronze, and the thickness may be $t = (d_j/15) + 0.2$ in. For larger journals a cast-iron shell lined with white metal is used. The thickness of the shell (without the lining) may be $t = 0.1d_j + 0.4$ in. The white-metal alloy used must be hard, but not so hard as to crack under pounding. A mixture found to give satisfaction contains 8 parts by weight of copper, 12 of antimony, and 80 of tin. G. W. Lewis and A. G. Kessler (*Am. Mach.*, 1911) give the data in Table 20 as representing average American practice in allowable unit pressures for a maximum explosion pressure of 350 lb. per sq. in.

Table 20. Allowable Unit Bearing Pressures for Gas Engines

(All dimensions in inches. Allowable unit bearing pressures in lb. per sq. in.)

Item	Horizontal stationary engines	Vertical stationary engines	Automobile engines	Item	Automobile engines
Cylinder diam.	4–20	4–20	4–$5\frac{1}{2}$	Cylinder diam.	4–$5\frac{1}{2}$
Crank-pin diam.	$1\frac{1}{2}$–$8\frac{1}{8}$	$1\frac{5}{8}$–$8\frac{3}{16}$	$1\frac{9}{16}$–$2\frac{1}{16}$	Front-bearing diam.	$1\frac{9}{16}$–$2\frac{1}{16}$
Crank-pin length	$1\frac{5}{8}$–$8\frac{3}{8}$	$1\frac{5}{8}$–$9\frac{5}{8}$	$2\frac{1}{8}$–$2\frac{3}{4}$	Front-bearing length	$2\frac{11}{16}$–$3\frac{9}{16}$
Allowable pressure	1800–1620	1660–1400	1320–1470	Allowable pressure	780 –930
Wrist-pin diam.	$1\frac{5}{16}$–$6\frac{7}{16}$	$1\frac{5}{8}$–$4\frac{1}{2}$	$\frac{7}{8}$–$1\frac{3}{8}$	Center-bearing diam.	$1\frac{9}{16}$–$2\frac{1}{16}$
Wrist-pin length	$1\frac{5}{8}$–$11\frac{5}{32}$	$2\frac{7}{8}$–$8\frac{1}{8}$	2–$3\frac{1}{16}$	Center-bearing length.	$2\frac{3}{16}$–$3\frac{5}{8}$
Allowable pressure	2950–1540	2120–3010	2510–1980	Allowable pressure	970–810
Main-bearing diam.	$1\frac{5}{16}$–$8\frac{13}{32}$	$1\frac{1}{2}$–$9\frac{1}{2}$	....	Rear-bearing diam.	$1\frac{9}{16}$–$2\frac{1}{16}$
Main-bearing length	$2\frac{5}{8}$–$19\frac{1}{8}$	$3\frac{1}{2}$–16	....	Rear-bearing length	3–$5\frac{5}{8}$
Allowable pressure	647–342	415–360	....	Allowable pressure	760–605

Flywheels (see also pp. 736 and 774). In stationary engines the flywheels are generally made of cast iron, though if the regulation must be close they are sometimes cast steel or occasionally a cast-iron rim with steel or cast-steel arms and hub. For high-speed automobile and motor-boat engines, a steel disk is generally used. The **allowable stress** should not exceed 3000 lb. per sq. in. for cast iron, or 7000 lb. for steel. The **coefficients of fluctuation** may have the following values (for other values see p. 775):

For ordinary power purposes	$1/30$ to $1/40$
For direct-current generators, direct-connected	$1/100$ to $1/120$
For direct-current generators, belt-driven	$1/70$ to $1/80$
For alternating-current generators, direct-connected	$1/175$ to $1/200$
For alternating-current generators, belt-driven	$1/125$ to $1/150$

For the parallel operation of several alternators still smaller values are generally chosen.

In four-cycle engines the inertia effects of the reciprocating weights may be neglected in flywheel computations. The mean pressure of suction and exhaust is only about 1 per cent. of the mean pressure of the positive work in modern large engines with mechanically operated valves of large area, and therefore this negative work has practically no effect on the turning moment; it has less effect upon the coefficient of fluctuation than has the frictional resistance of the reciprocating parts, of whose magnitude and distribution over the four strokes of the cycle nothing is known. Sufficient accuracy is attained by considering only the negative work of the compression stroke and the positive work of the combustion and expansion stroke, neglecting the suction and exhaust strokes.

The following **method of determining the necessary flywheel weight**, devised by Güldner, gives results more quickly than the graphical method and is of sufficient accuracy.

Let M = necessary flywheel weight for a single-cylinder single-acting engine, lb.
i.h.p. = maximum indicated horse power.
p_{mc} = mean indicated pressure of the compression stroke, lb. per sq. in.
p_m = mean effective pressure of the cycle, lb. per sq. in.
k = coefficient of fluctuation.
V = velocity of the center of gravity of the rim cross-section, ft. per sec.
R_w = $\frac{1}{2}D_w$ = radius of the equivalent wheel = distance from center of shaft to center of gravity of rim cross-section, ft.

Then

$$M = \frac{2{,}125{,}000 \times \text{i.h.p.} \times [0.75 + (p_{mc}/p_m)]}{kV^2N} = \frac{77.5 \times 10^7 \times \text{i.h.p.} \times [0.75 + (p_{mc}/p_m)]}{kD_w^2N^3}$$

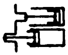
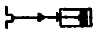
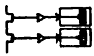
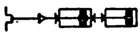
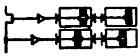
About one-tenth of the total weight of M is usually assigned to the arms of the wheel, so that only $0.9M$ goes into the rim. This is in error in gas-engine practice, however, since the arms are usually designed much heavier than in steam-engine practice; the added weight in the arms, however, improves the regulation. In the above equation it should be noted that the weight varies inversely as the cube of the speed of rotation. Therefore, the weight of the flywheel may be cut down or the regulation be improved materially by a slight increase in the speed of rotation. The value of the ratio (p_{mc}/p_m) does not depend upon the differences of construction of the various machines as much as it does upon the heating value of the fuel employed and the quality of the mixture. The following results have been obtained experimentally by Güldner:

Illuminating gas.. p_{mc}/p_m = 0.25 to 0.35	Gasoline......... p_{mc}/p_m = 0.10 to 0.2[illegible]
Producer gas..... p_{mc}/p_m = 0.35 to 0.45	Alcohol.......... p_{mc}/p_m = 0.25 to 0.32
Kerosene......... p_{mc}/p_m = 0.30 to 0.40	Diesel oil engine... p_{mc}/p_m = 0.48 to 0.52

For a single cylinder, single-acting four-cycle engine to give a regulation of $k = 1/80$, having a ratio $p_{mc}/p_m = 0.35$, a rim speed V of 100 ft. per sec. and $N = 200$ r.p.m., a flywheel weight of 128.5 lb. per i.h.p. (including the weight of the arms) is required. (For different values of p_{mc}/p_m, and V the required weight can be found by proportion.) Taking the above weight as unity, the relative flywheel weights, under the same conditions, for various types of engines and cylinder combinations, are given in Tables 21 and 22.

Table 21. Comparison of Horizontal Engines Operating on Lean Gases

(Relative flywheel weight obtained from turning moment diagram, derived from actual indicator cards—Güldner)

	Number of cylinders	Cylinder and crank arrangement	Angle between the cranks, deg.	4-cycle engines			2-cycle engines		
				Crank travel between explosions, deg.	Relative flywheel weight for		Crank travel between explosions, deg.	Relative flywheel weight for	
					Equal d, l, & N	Equal i.h.p. max.		Equal d, l, & N	Equal i.h.p. max.
I	1		....	720	1.00	1.00	360	0.80	0.40
II	2		0	360	0.85	0.425	......	......	......
III	2		180	540&180	1.20	0.60	180	0.25	0.063
IV	1		....	540&180	1.20	0.60	180	0.24	0.06
V	2		0	180	0.62	0.155	90	0.25	0.03
VI	2		....	180	0.325	0.08	180	0.56	0.07
VII	4		90	90	0.28	0.035	90	0.40	0.025

Accessories

Lubrication. The hourly bearing oil circulation should be about 0.05 gal. per b.h.p. to give satisfactory lubrication. The amount of oil actually used up and not recoverable will vary between 0.00035 and 0.00075 gal. per b.h.p. per hour with well-designed lubricating systems; in a poorly designed system it will reach as high as 0.001 gal. The consumption of cylinder oil, none of which can be recovered, varies between 0.00024 and 0.00042 gal. per b.h.p. per hour, depending upon the care and attention of the operators.

Cooling System. While some small automobile engines are air-cooled, most engines are water-jacketed. About one-third of the heat supplied per b.h.p-hr. is lost to the jacket water, and consequently the cooling water must carry off about 4,500 B.t.u. per b.h.p-hr. With a temperature range of 90 deg. fahr. (from 60 to 150 deg.) this gives a jacket-water consumption of 50 lb. (6.0 gal.) per b.h.p. per hour. In large engines the heat given to the jackets will be much less than this, but the jacket water cannot be heated above about 120 deg. to give reliable operation when the hydrogen content of the gas is high, as there may be danger of heating the engine to the point where preignitions are liable to occur. In such a case, the amount of cooling water required may reach 9 gal. per b.h.p. per hour. In designing the cooling system a maximum figure of 10 gal. per b.h.p. per hour should be taken, as the initial temperature may be above 60 deg. fahr. The cooling system for small engines consists of but one inlet pipe and one discharge pipe into and out of each cylinder jacket; the head jacket is in communication with the cylinder jacket, and the exhaust valve chamber a part of the cylinder casting and hence

Table 22. Comparison of Vertical Four-cycle Engines Operating on Various Fuels

	No. of cylinders	Crank angles, deg.	Crank travel between explosions, deg.	Relative flywheel weights					
				Explosion engines operating on lean gas (producer, blast-furnace gas, etc.)		Explosion engines operating on rich gas (illuminating, coke-oven gas, gasoline, etc.)		Constant-pressure oil engines	
				Equal d, l, and N	Equal i.h.p. max.	Equal d, l, and N	Equal i.h.p. max.	Equal d, l, and N	Equal i.h.p. max.
I	1		720	1.0	1.0	1.0	1.0	1.0	1.0
II	2		360	0.86	0.43	0.85	0.425	0.89	0.445
III	2		540&180	1.44	0.72	1.20	0.60	1.17	0.585
IV	3		240	0.72	0.24	0.65	0.22	0.75	0.25
V	4		180	0.304	0.076	0.265	0.066	0.25	0.06

cooled by the cylinder jacket. Large horizontal engines have separate inlet and discharge pipes for the jackets of important parts, cylinder, cylinder head, pistons, exhaust valves and separate valve chambers. It is undesirable to lead the cold jacket water into parts most exposed to the heat of the explosions (cylinder and piston jackets), and consequently it is well to divide the inlet jacket water between the exhaust valves, exhaust valve chambers and cylinder heads and to send the discharge from each exhaust-valve chamber to the corresponding end of the cylinder jacket. It is often the practice to run the discharge from both the exhaust-valve chamber and exhaust valve (a parallel arrangement) through the cylinder jacket, but this is objectionable because the flow through the exhaust valve may be stopped and not be observed. It is better to run the discharge from the valve jacket directly into the drain pipe through a visible discharge. The usual method of connecting the moving exhaust valve with the stationary supply and drain pipes, is by means of a rubber hose.

In a tandem double-acting engine the discharges from the cylinder heads of one cylinder are usually run through the jacket of the other piston—thus equalising the final temperatures of the two discharges (a parallel-series arrangement)—and are discharged into the common drain through a visible discharge. The water is generally led into and out of the piston by means of swing joints. A telescopic water connection does not give as good results. The discharge from the piston jacket may be through an open pipe which travels with the cross head and leads down through a slot into the base of the frame or tail guide.

To keep the piston jackets of large engines filled, the cooling water must be under a pressure of from 60 to 80 lb. per sq. in. For the other jackets 10 to 15 lb. per sq. in. is sufficient, and therefore two cooling systems at the different pressures may be used or the water may be throttled down at the entrance to the other jackets. Valves on the discharge sides of the jackets should not be used, since by so doing the jacket is put under pressure, and will be more apt to leak. There is one exception to this statement, namely, in the case of the piston jackets. Even with the high pressure of the jacket water there will be a water hammer in the piston jackets unless the area of the discharge is restricted to about 21 per cent. of the area of the inlet opening. The average percentage distribution of the water to the various jackets is approximately as follows: 60 to the cylinder jacket and 40 to the cylinder-head jacket with uncooled piston; 50 to the cylinder jacket, 25 to the cylinder-head jacket, and 25 to the piston and piston rod with water-cooled piston.

The **size of piping** required **for** the **inlet to the jacket** may be determined from the formula d_i (in.) $= \sqrt{0.023 \times \text{b.h.p.}_n \times X/\text{No. of cylinder ends}}$; and the size of the discharge pipe from $d_d = 1.25\ d_i$ to $1.75 d_i$, depending on the length and drop, where X is the percentage of the total jacket water used in the jacket under consideration. This is based on the assumption of 10 gal. per b.h.p. per hour and an allowable velocity in the pipe of 3 ft. per sec.

Air and Gas Suction Pipe. The **mean air velocity** V is taken as 3600 ft. per min. for lines up to 35 ft. long, but should not be greater than 1800 ft. per min. for long lines. The free **cross-section of** the **air suction pipe** may be taken approximately as ⅓ of the effective area of the piston. For abnormally long lines, the diameter should be computed by means of the general formulæ on p. 357, taking into account the pipe friction, pressure drop, etc.

The same velocities may be allowed for gas as for air. For lines of considerable length the pressure drop may be taken at from 0.07 in.–0.12 in. of water for every 100 ft. of pipe. Close figuring on the **size of gas pipe** for industrial gas installations is undesirable, because of the reduction of the normal cross-section of the pipe by tar and dust deposits. In some cases of multi-cylinder engines where the gas for each cylinder is led off from the gas main opposite each cylinder, and where the pipe is not enlarged, there may be robbing of the charge of some cylinders by the others, due to the momentum of the gas column. If a storage chamber of large diameter is formed in the main opposite the engine, the difficulty is overcome. The same troubles may occur in plants where two or more engines are fed from the same gas line, the engine farthest from the source of gas supply refusing to carry as much load as the engine nearest the supply, and both engines backfiring heavily. The same cure must be applied. Formerly gas-engine plants were equipped with holders or pressure regulators, in which case this trouble did not arise. Most of the recent producer-gas power plants have been built without holders or pressure regulators—depending upon a regulating valve at the fan or exhauster to maintain the pressure. This is apt to cause the troubles stated above. The gas line should be free from traps and as free as possible from changes in direction of flow, and should be pitched slightly (¼ in. to the foot) from both ends toward a low point in the center, where a sealed drain should be located. All water seals in the gas line should be as cool as possible, to prevent the gas from picking up moisture.

The **cross-sectional area of the exhaust pipe** from the engine to the muffler should be made from 1.1 to 1.3 times the area of the free cross-section of the exhaust valve, depending upon the length of the connection and the changes in direction of flow. In a multi-cylinder engine the exhaust pipe should be firmly anchored at the center of the engine, so that the bending moment on the connections to the exhaust valves, due to the expansion, may be equalized between the connections and reduced to a minimum. Careful provision must be made for the expansion of the line, especially in the connection to the muffler—if one is used—and therefore the line must be supported by expansion hangers. A drain should be located in the lowest point of the line.

Exhaust **mufflers**, to be efficient as sound deadeners, must have a volume of from fifteen to twenty times the total piston displacement per stroke. Generally the volume is made only from 6 to 8 times the piston displacement. Better muffling and cheaper construction are obtained by connecting two or three small mufflers in series. The use of any device to increase gradually the volume or to change gradually the direction of flow by baffles, etc., permits of a smaller volume in the muffler, but increases the back pressure in the engines. The discharge opening from the muffler should have the same cross-sectional area as the exhaust pipe. When a perforated plate or its equivalent (for instance a grating covered with a bed of cobblestones) is used between the inlet and outlet of the muffler, the free area through such plate or equivalent should be from 5 to 6 times the cross-sectional area of the exhaust pipe. For large engines cast-iron or sheet-steel mufflers are not satisfactory because of the size, radiation of heat and corrosion, and **concrete muffle pits** are more generally used. The following proportions on engines up to 300 b.h.p. give good results: The volume of the pit should be 20 times the total piston displacement per stroke, with a horizontal cross-sectional area of 20 times the area of cross-section of the exhaust pipe. The pit should be divided into three equal horizontal layers by iron gratings and the space between these gratings filled with cobblestones or rubble—care being taken to see that the free area through the grating is at least 5 or 6 times the cross-sectional area of the exhaust pipe. The exhaust pipe is led into and across the bottom division of the pit, with the end flange extending just through the further wall. The end of this pipe should be covered with an explosion door, the best and safest form of which is a wooden cover about ¼-in. thick, scarfed across to a depth of half the thickness, and held in place by only two bolts placed at the ends of the scarf mark. The length of pipe between the pit walls should be drilled full of small holes, whose combined area should equal from 5 to 6 times the area of the pipe. The top of the pit should be covered with a removable iron cover, and from the top a cast-iron pipe of the same diameter as the exhaust pipe should rise high enough to carry off the products of combustion. The pit must be provided with an automatic drain.

The **recoverable waste heat contained in the jacket water and** the **exhaust gases** (amounting to about two-thirds of the total heat consumption) in the case of large installations is approximately distributed as follows:

	Large gas engines	Constant-pressure oil engines
Hourly heat consumption, B.t.u. per h.p.	11,100 to 9,150	7,950 to 7,350
Hourly heat wasted, B.t.u. per h.p.	7,150 to 5,950	4,550 to 4,000
Distribution of the waste heat:		
Hourly heat in jacket water, B.t.u. per h.p.	2,800 to 2,650	2,000 to 1,800
Hourly heat in exhaust gases, B.t.u. per h.p.	4,350 to 3,300	2,550 to 2,200

Where there is need for warm water of from 100 to 120 deg. fahr. the jacket water can be used directly; the utilisation of the heat of the exhaust is much more difficult, however, because of the high temperature existing, the great velocity of flow, the small heat content, and the reluctance with which the exhaust gas parts with its heat (according to Kutzbach about 200 times worse than is the case with steam).

The exhaust gases themselves are not adapted to the development of power; therefore their heat must be absorbed, as close to the engine as possible, by the hot jacket water. In this transfer much of the heat is lost; in large gas engines about 2000 B.t.u., and in Diesel engines about 1400 B.t.u. per h.p-hour of the engine rating is recovered. The back pressure resulting from the use of an exhaust heater may be serious. A crude-oil engine with an exhaust heater shows a back pressure of 2.55 to 2.85 lb. per sq. in. without apparent harmful effect on the action of the engine. A gas engine will give trouble at a much lower back pressure. Cast iron is the best material for the heater, since it is capable of withstanding the action of heat and acids, but wrought iron is also sometimes used. The area of the heating surface varies between 1.05 and 2.15 sq. ft. per h.p., and the heat transfer is about 1100 B.t.u. per sq. ft. According to Cochand and Hottinger, the heat transfer coefficient k, referred to a 1-deg. fahr. temperature difference, for favorable circumstances (high exhaust temperature, clean heating surface, etc.) ranges from 2.5 to 1.2 B.t.u. per sq. ft. per hour. Ordinarily the exhaust-gas temperature at entrance to the heater is 575 to 750 deg. fahr., though with heavily loaded engines or delayed combustion it is much higher, and therefore the heat return is increased.

The following test results of a 1350-h.p. Cockerill blast-furnace gas engine show what is obtainable in the **generation of high-pressure steam by the use of the waste heat from large gas engines:**

Load during test, b.h.p.	1,290.0
Hourly heat consumption, per h.p., B.t.u.	10,080.0
Thermal brake efficiency, per cent	29.0
Water evaporated in the exhaust heater, lb. per hr.	2,405.0
Pressure of the steam generated, lb. per sq. in.	101.7
Percentage of waste heat utilised	30.0

If the steam generated were used in a turbine requiring 14 lb. per h.p-hour., 172 h.p. or 13.3 per cent. of the gas-engine power would be obtained from the waste heat.

RECENT DEVELOPMENTS IN GAS POWER

Precompression of the Mixture. A recent method of operating large four-cycle gas engines consists in scavenging and forcing in the fresh mixture under a pressure somewhat higher than atmospheric. This is accomplished by the use of turbo-blowers compressing to a few inches of mercury above atmospheric pressure. The exhaust heat of the engines can be utilized to generate steam in waste-heat boilers for driving the turbo-blowers. In one plant in Germany to which this method has been applied, an overload of 35 per cent. is obtained without raising the maximum pressure, and the thermal efficiency and the mechanical efficiency are both increased. In another plant two tandem engines of 1360 kw. normal rating (at 65 lb. m.e.p.) carried loads of 1800 to 2000 kw., corresponding to an average increase in capacity of approximately 40 per cent. H. J. Freyn states that in a blast-furnace-gas plant a mean effective pressure of from 70 to 71 lb. per sq. in. was obtained before the adoption of the new method of operation; with a precompression of 3 lb. per sq. in., mean effective pressures of from 90 to 100 lb. per sq. in. were obtained without raising the maximum pressure. At the present state of development it is not known how the life of the cylinders will be affected by the increased average cylinder temperature.

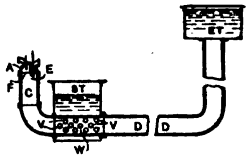


FIG. 7.—Humphrey Gas Pump.

The Humphrey Gas Pump. (Abstract of paper read by H. A. Humphrey, M. I. C. E., before the Manchester Association of Engineers, Nov. 13, 1910.) All types of the Humphrey pump, with the exception of the two-cycle, are fitted with a simple interlocking gear arranged between the inlet and exhaust valves so that when either one closes it locks itself closed and releases the other one. In the **simple type** of pump, referring to Fig. 7, water has to be raised from a suction tank *ST* to an elevated tank *ET*, and the cycle of operations is as follows:

1. All valves are closed when an explosion of the compressed charge occurs, driving water out of the combustion chamber *C* and setting the whole water column in motion. The expanding gases do work on the water column, and, when expansion reaches atmospheric pressure, the column is moving with considerable velocity and cannot be suddenly arrested, and therefore a vacuum is established in the combustion chamber, opening the exhaust valves *E* and water-suction valves *V*. The water sucked in follows the moving column toward the elevated tank, and also rises in the combustion chamber.
2. When the kinetic energy of the column has expended itself by forcing water into the elevated tank, the column comes to rest and starts back toward the pump, gaining velocity until the water reaches the level of the exhaust valve, which it shuts by impact. The exhaust valve projects into the combustion chamber far enough to form the clearance volume desired. The energy of the moving column is expended in compressing the products of combustion remaining in the cushion space *F*.
3. The column is brought to rest by this cushion and a second outward movement of the column results. When the water reaches the level of the exhaust valve, the pressure in the chamber is again atmospheric, and further movement of the column opens the spring-loaded inlet valve *A* and draws in a fresh combustible charge. If there were no friction the water would fall to the same level as that before the last upward movement, but in the actual machine the volume of charge drawn in is slightly less than this movement would represent.
4. The column of water again returns, actuated by the head in the elevated tank, and compresses the charge, which is then ignited and starts a fresh cycle.

In this simple form the degree of compression of the charge depends on the height to which the water is raised and exceeds the static equivalent of the head, due to the kinetic energy of the moving water column. The amount of cushion pressure is also governed by the same considerations, except that in this case the initial volume of the residual gases is the clearance volume, and the cushion pressure rises rapidly as the discharge head increases. At the maximum lift of 40 ft.—the limit of the simple pump—the cushion pressure may exceed the explosion pressure when using producer gas, or may approximately equal the explosion pressure when using illuminating gas or gasoline.

As an example of the pressures obtaining in a pump of this type, the following values—taken from a 45-water-h.p. four-stroke pump, delivering 306 gal. per working stroke against a head of 39.4 ft. and working on illuminating gas—are given: Compression pressure at ignition, 56.5 lb. per sq. in.; explosion pressure, 163 lb. per sq. in.; water suction at start, 2 to 3 ft. of water, decreasing to atmospheric pressure; cushion pressure, 185 lb. per sq. in.; vacuum at start of suction of fresh charge, 4 lb. per sq. in., decreasing to atmospheric pressure.

The action of the pump is not altered if, instead of delivering into an elevated tank, it discharges into an air dome, or into an open-top standpipe—both devices serving to make the flow from the outlet pipe continuous. The reciprocating column of water is analogous to the pendulum of a clock, the length of the column governing the period of the swing. If other things are equal and the column is of uniform cross-section, the period is almost proportional to the square root of the length. The four unequal strokes which result from the free oscillations of the water column give a cycle which is superior to the Otto cycle.

The following table compares the assumed practical efficiencies (0.71 of air standard card) of the Humphrey pump cycle with that for the Otto cycle for various compressions, taking the specific heats as constant:

Compression pressure, lb. per sq. in. gage......	0	20	40	60	80	100	120
Thermal efficiency of pump cycle, per cent....	16.2	26.5	31.5	34.6	37.0	39.0	40.5
Thermal efficiency of Otto cycle, per cent.....	0.0	15.5	22.2	26.5	29.4	31.6	33.4

With a compression of 150 lb. per sq. in. gage, the theoretical thermal efficiency of the Humphrey cycle is 52.5 per cent., and that of the Otto cycle 40 per cent. when all corrections for varying specific heats are taken into account. Using compression pressures under 50 lb. per sq. in., an actual thermal efficiency of over 23 per cent. was obtained on a four-cycle pump working against a head of 35 ft.

Certain **modifications of the simple type** consist in (*a*) the addition of a scavenging valve so that the residual gases are diluted with air; (*b*) the addition of an internal bell in which the cushioning on the residual gases takes place, so that the exhaust gases outside of the bell are all driven out of the combustion chamber and thus the incoming charge is not diluted with the exhaust gases; (*c*) the addition of a second combustion chamber so that the explosions occur alternately in the two chambers and require only two strokes of the main water column to complete a cycle (the other two strokes occurring between the chambers). A two-cycle pump without valve locking gear has also been built.

GAS PRODUCERS

(For general **theory**, see p. 370.)

General Features and Types

The general arrangement of the parts of a gas-producer installation and the type and design of producer depend mainly upon the properties of the fuel to be gasified. The main factors are the moisture content, the quantity and properties of the ash, the proportion of tar-forming constituents, tendency to coke and clinker, and size and heat value of the fuel. A high moisture content reduces the temperature of the producer seriously and causes a lack of uniformity of quality of the gas generated, so that immediately after charging the gas quality falls off rapidly to a minimum and then increases slowly until the next charging period. A large percentage of **ash** quickly clogs up both the fuel and grate, necessitating frequent poking and cleaning. If, with a high ash content, the fuel bed is not frequently cleaned, holes will form in the fire, allowing free passage of air through the fuel bed and resulting in the more-or-less complete combustion of the gas above the bed. An ash which clinkers badly, so that it forms a barrier across the fuel bed, increases the resistance to the air blast, while if it clinkers only partially across the bed, it tends to burn a hole through the area not so covered, by increasing the volume of air and consequently the rate of combustion through that area. In any case, a **clinkering** ash adds greatly to the difficulties of cleaning the fire. Tar-forming coals cause serious trouble because of the extra gas cleaning necessary, and because the **tar** which is not cleaned out of the gas condenses in the pipe lines and valves, soots up the cylinder during combustion, and sticks to the piston-rod packing, necessitating frequent cleaning. Fuels that show a strong tendency to coke interfere with the operation of the producer in much the same way as an excessive ash content.

Small-sized **fuels** have a tendency to pack and thus increase resistance to the air blast, and when the blast pressure is increased to meet this difficulty, the whole fuel bed will lift and dance because of the lightness of the particles. Large-sized or coarse fuel does not burn uniformly, offers too little surface for gasification, and allows steam and carbon dioxide to reach the upper part of the producer without being decomposed. Those fuels which are most free from the above disadvantages, and especially those low in tar-forming constituents, are most commonly used in producer gas plants. The fuels that come nearest to falling within these limitations are anthracite and coke with low ash content and a high fusing point. The sizes of **anthracite** most commonly used are No. 1 pea or No. 1 buckwheat.

The use of **bituminous coal**, lignite and peat has until recently been attended with many difficulties. In the early stages of development the same type of producer was used for the tar-forming coals as for anthracite, but without success, because of the difficulty in cleaning the gas. The next step was so to conduct the gasification process that the tar vapor distilled from the freshly charged fuel should be led through the incandescent zone of the fuel bed, broken up, and fixed as a gas, or partly burnt. With these coals the temperature of the incandescent zone necessary to fix the tar is so great that there is serious clinkering. Part of the tar vapor is also burnt to lampblack, so that the gas washing must be as thorough for clean gas as if the tar were not fixed; the only difference being that the lampblack deposits are not as serious a detriment to operation as is tar. The most recent development is a reversion to the standard type of anthracite producer with its attendant formation of tar, made possible by the invention of a simple process of tar extraction (see p. 1063).

Producer **types** are usually classified by the manner in which the air or air-steam mixture is caused to travel through the producer and washer.

In **suction producers** the air or air-steam mixture is drawn through the fuel bed either by the suction of the engine or by an exhauster, in which latter case the gas is delivered to the engine under pressure. The former type is used only for small installations where the disadvantage of the increased suction resistance and diminished capacity of the engine is offset by the simplicity of the installation. The latter type is employed for large installations. Suction producers are rapidly displacing other types because any accidental leaks are inward and the producer can be cleaned in operation without the loss of gas through the cleaning openings. This type is consequently well adapted to continuous operation.

In **pressure producers** the air or air-steam mixture is forced through the fuel bed by means of blowers. In this type poking of the fuel bed in order to fill up holes that have burned through the fire, to push down clinker, etc., is possible, but it results in a great deal of discomfort to the operator because of the escaping gas. A thorough cleaning of the fuel bed can be accomplished only when the producer is out of operation.

In balanced-draft or combination producers the air or air-steam mixture is supplied underneath the fuel bed under pressure and the gas is drawn off from the top of the fuel bed and forced over to the engine under such a suction that the pressure on top of the fire is exactly atmospheric. In a properly designed system of this sort, cleaning can be accomplished during operation, and there will be no leakage at the poke holes, either of gas into the producer room or of air into the top of the producer.

A further classification is made according to the direction of flow of the air-steam mixture, into (1) **up-draft,** (2) **down-draft** and (3) **double-zone producers.** In the last type the gas is taken off at the mid-height of the producer, the blast is introduced at the bottom, and the fresh coal is fired at the top, where the hydrocarbons are distilled off and partly burned in the presence of a little air admitted at the top—the products of this distillation being drawn down through an incandescent fuel bed and drawn off with the rest of the gas at the gas outlet.

Prevention of Clinkering. The use of steam in the blast is necessary with most coals to prevent clinkering. In some producers the steam is blown in intermittently; clinker is allowed to form during the period in which the bed is not steamed, and then the bed is thoroughly steamed, its temperature rapidly reduced, and the clinker broken into small pieces. During this period water gas is formed, which must be thoroughly mixed with a large volume of producer gas, otherwise preignition troubles will result in the engine. The hydrogen from constant steaming can be kept within allowable limits and improves the quality of the gas. Another method of reducing the fuel-bed temperature below the clinkering point, is to use the exhaust gases from the engine instead of steam—the heat absorbed by the reduction of the carbon dioxide in the exhaust to carbon monoxide being assumed to be sufficient to keep the temperature of the fuel bed below the clinkering point. In practice, except with a few coals low in ash, this method has not proved as successful as the use of steam, there being a great tendency to clinker and a wide variation in the quality of the gas. The gas generated by this method will stand a high compression, since it consists principally of carbon monoxide and nitrogen, but requires larger cylinder capacity than ordinary producer gas on account of its low heat value.

Control. The proportion of air and steam in the blast must remain constant for all loads to produce a gas uniform in quality. To obtain this uniformity some form of automatic control must be used. Up to within a few years it has been attempted to obtain this control by hand regulation, but, while the average quality of the gas could be controlled as desired, there were fluctuations amounting to as much as 40 per cent. in the heat value of the gas corresponding to sudden changes in load, charging, or cleaning the fire. With automatic regulation, charging and cleaning the fire may produce a variation of 5 per cent. as a maximum, and a sudden change of load may cause a variation of 20 per cent. as a maximum. In a plant that is properly designed and operated, these variations will not be over 1 per cent. and 5 per cent., respectively.

Gas from By-product Plants. In England and on the Continent, by-product recovery plants (generally according to the Mond process) are successfully operated. In this process a very deep fuel bed, run at a low temperature (in order not to break up the ammonia), is maintained; the gas generated is consequently of a very uniform quality (about 150 B.t.u.) and because of the by-product recovery is very clean and is therefore well adapted for use in gas engines. Low-grade coals—peat, lignite and brown coal— as well as the ordinary coals, can be successfully gasified. In some plants the sulphate of ammonia recovered is worth $2 per ton of coal fired, which in many cases is as much or more than the cost of the coal. This process has not yet been successfully applied in this country.

A. H. Lymn (*A. S. M. E.*, Dec, 1915) reports the following results from a by-product plant. Coal consumption per day, 64.6 tons; per kw-hr., 1.58 lb.; yield of ammonium sulphate, 60 lb. per ton; of tar, 230 lb. per ton; avg. heating value of gases, 155 B.t.u. per cu. ft.; nitrogen efficiency, 70 per cent. American coals average 1.32 per cent. of nitrogen on dry coal.

Design

Rating. It is customary to rate the capacity of producers in horse power producible in the engine. To do this a definite weight of fuel must be gasified per sq. ft. of grate area per hour. This weight and the quality of gas generated vary with the kind of fuel used.

The following **rates of fuel consumption** are allowable for the various American fuels as fired per sq. ft. of grate surface per hour: Anthracite, 8 to 10 lb.; bituminous, 5 to 11 lb.; sub-bituminous, 7½ to 12 lb.; lignite, 7 to 10 lb.; peat, 12 lb. Consumptions of lignite of more than 40 lb. per sq. ft. have been reported, but it is unwise to choose so high a figure as this unless the characteristics and actions of the particular lignite to be used are very well known. A **moderate rate of driving** for all fuels is absolutely **essential for continuous operation,** since thereby clinkering is reduced to a minimum, and consequently the producer can be easily cleaned in operation.

Dimensions. Producer **shells** are generally built of ¼-in. to ⅜-in. steel boiler plate, the joints being calked gas-tight as carefully as in boiler construction. The **height of the ash pit** from the bottom (which is usually in the form of a truncated cone formed by plates riveted to the shell, or formed in the foundation for the shell) to the top of the grates is generally from 2½ to 5 ft., the distance from the bottom of the shell to the top of the grate varying between 2 and 3 ft. The grate, or the blast hood, if this is used, must be protected from the hot coal by an ash bed from 6 to 12 in. thick. The **depth** of the **fuel bed** varies with the character of the fuel, but will in general be from 2 to 5 ft. The distance from the top of the fuel bed to the bottom of the gas outlet pipe should not be less than 3 ft., otherwise the draft will be pulled toward this opening and the fuel bed will not burn uniformly. The diameter of this outlet pipe should give a gas velocity not to exceed 30 ft. per sec. The **top of the shell must be cooled.** This is generally done by water-cooling, either by having a flanged steel plate riveted to the shell with the flange upward to form an open water basin, which serves as a seal for the cleaning holes as well as cooling the top, or by using for the top a water-jacketed iron casting, the jacket serving either as a vaporizer or a heater for the vaporizer water. Another method of keeping the top cool is to have the part of the producer above the height necessary for the fuel bed serve as a fuel magazine, *i.e.*, fill the producer to the top. This method has the advantage that much of the sensible heat of the outgoing gases is withdrawn and returned to the fuel in the magazine, thus rendering it immediately ready to enter into combustion as soon as the demand is made by a sudden increase in load. The bottom of the shell, except for the blast opening, must be sealed either by means of a **water seal,** the top of which must be high enough above the bottom of the shell to prevent ordinary operating pressures from blowing gas out into the room, or by tightly closing the bottom of the shell by means of a riveted plate, or by grouting it tightly to the foundation. If a water seal is not used, cleaning doors must be provided for the removal of ash. If a water seal is used, the smallest distance between the seal basin and the bottom of the shell must be large enough to enable the removal of the largest clinker that is likely to be formed (at least 5 in.). At the level of the grate, **cleaning doors** of the same size as those in the ash pit must be provided. The top of the producer must be provided with small cleaning holes so located that all parts of the fuel bed can be reached for cleaning.

Distribution of the blast over the whole area of the fuel bed is of the greatest importance in producer design. There is always a tendency for the blast to be strongest next to the lining. This tendency can be minimised only by proper design of grate and by locating the gas outlet at the center of the cross-section of the producer. The grate should be designed to give as nearly uniform a blast distribution as possible, and to permit of easy cleaning and removal of ash. Above the grate, inclined **bosh plates,** projecting inward from the shell, are generally used to prevent the fuel bed from running out over the edges of the grate. The space between the top of the grate and the bottom of the bosh should be between 5 and 6 in.

The **producer lining** should be of a good quality of fire brick set in fire clay. The bricks should be so molded as to give a smooth interior and require a minimum amount of fire-clay binder between the bricks. A space of from ½ in. to 1 in. should be left between the back of the bricks and the shell and filled with a pliable refractory material (such as dry fire clay, or a mixture of fire clay and asbestos fiber) to reduce the radiation of heat, permit of the expansion and contraction of the lining independent of the shell, and prevent the leakage of air between the shell and lining.

Test Results

(For methods of conducting and reporting tests of gas producers, see p. (1767.)

Tables 23–27 summarise results obtained in government tests made at St. Louis, Norfolk, and Pittsburgh, as averaged and reported by R. H. Fernald. In Table 23 all results affected by the load factor are given for loads ranging only from 90 to 100 per cent. of full load, and no test of less than 30 hr. is included in the average. All coal results are referred to weight of coal as fired. All heat values are higher values.

Table 23. Results of Tests of Gas-producer Fuels

Item	Bituminous coal	Sub-bituminous coal	Lignite	Peat
Number of tests averaged	112	7	7	1
B.t.u. per lb	12,370	9,910	7,110	8,130
Yield, cu. ft. of gas per lb	61.1	39.3	27.7	30.3
Rate of gasification, lb. per sq. ft. of grate surface per hr.	7.64	11.02	13.28	16.2
B.t.u. per cu. ft. of gas, standard	151	159	161	175
Producer (cold) efficiency	74.6	63.1	62.8	65.2
Pounds of tar, soot, etc., per ton of fuel:				
Water not extracted	354	259	157	240
Water extracted	287	224	....	157
Composition of fuel:				
Moisture, per cent	6.6	15.0	35.7	21.0
Volatile, per cent	32.8	34.3	29.2	51.7
Fixed carbon, per cent	50.6	39.4	27.2	22.1
Ash, per cent	10.0	11.3	7.9	5.2
Sulphur,* per cent	2.32	0.90	1.12	0.45
Volumetric analysis of the gas:				
CO_2, per cent	9.71	11.16	9.90	12.40
O_2, per cent	0.02	0.12	0.13	0.00
C_2H_4 (ethylene), per cent	0.19	0.20	0.10	0.40
CO, per cent	19.03	17.52	20.86	21.00
H_2, per cent	13.48	14.41	14.30	18.50
CH_4, per cent	2.78	3.64	2.88	2.20
N_2, per cent	54.79	52.95	51.83	45.50

* Separately determined.

Table 24 gives data on 103 coals, classified into 10 groups according to calorific value, each group embracing a range of 500 B.t.u., the mean figure of which is assigned as the approximate calorific value (on a dry-coal basis) of the group. The duration of the tests from which the results were obtained ranged from 29 to 74½ hr., with an average of 48 hr.

Table 24. Gas Yield of Coals of Different Calorific Values

Approximate calorific value, B.t.u. per lb. of dry coal	Coal per sq. ft. of grate surface per hour, lb.		Gas under standard conditions (62 deg. fahr. and 14.7 lb.)			Producer efficiency, per cent.
			Per lb. of dry fuel		B.t.u. per cu. ft., high value	
	As fired	Dry	Calorific value, B.t.u.	Yield, cu. ft.		
15,000	5.64	5.50	13,350	91.1	153	74.7
14,500	6.10	5.92	12,460	80.5	160	74.2
14,000	6.45	6.22	10,890	72.1	152	71.9
13,500	6.97	6.54	10,070	67.8	150	69.3
13,000	7.88	7.30	9,360	61.8	152	67.5
12,500	8.76	7.84	8,770	58.6	149	65.0
12,000	8.54	7.69	8,780	59.4	148	67.6
11,500	10.11	8.88	8,010	54.8	146	64.3
11,000	11.61	10.53	6,110	45.9	133	52.5
10,500	13.26	11.53	5,790	40.9	135	50.5

Table 25. Typical Analyses of Producer Gases from Various Fuels
(Values in percentage by volume)

Constituents	Up-draft pressure-producer gas				Down-draft producer gas		
	Anthracite	Bituminous coal	Lignite	Peat	Bituminous coal	Lignite	Peat
Carbon dioxide..... CO_2	5.2	9.84	10.55	12.40	6.22	11.87	10.94
Oxygen............ O_2	0.4	0.04	0.16	0.00	0.13	0.01	0.41
Ethylene........... C_2H_4	0.0	0.18	0.17	0.40	0.01	0.00	0.06
Carbon monoxide... CO	22.9	18.28	18.72	21.00	21.05	16.01	16.91
Hydrogen.......... H_2	15.3	12.90	13.74	18.50	12.01	14.76	10.19
Methane........... CH_4	1.0	3.12	3.44	2.20	0.49	0.98	0.66
Nitrogen........... N_2	55.2	55.64	53.22	45.50	60.09	56.37	60.83
	100.0	100.00	100.00	100.00	100.00	100.00	100.00

The quantity of gas obtained varies with the fuel used, the type of producer plant, and the method of operation. Table 26 gives the average yield per pound of fuel (in cu. ft. and B.t.u.) as obtained at the government testing plant at St. Louis (with the addition of values for coke or charcoal, and anthracite).

Table 26. Average Yield of Gas per Pound of Fuel

Character of fuel	Yield of gas, cu. ft. per lb. of fuel		Higher heat value of the gas B.t.u. per cu. ft.	Higher heat value of the gas, B.t.u. per lb. of fuel	
	As fired	Dry		As fired	Dry
Coke or charcoal.....	85	90	140	11,900	12,600
Anthracite coal......	70	75	135	9,450	10,100
Bituminous coal.....	60	65	152	9,120	9,880
Lignite..............	36	46	158	5,690	7,270
Peat................	30	38	175	5,250	6,650

The results obtained from some of the low-grade fuels are summarized in Table 27.

Table 27. Typical Analyses of Producer Gas from Low-grade Fuels
(From Bulletin No. 13, U. S. Bureau of Mines)

Fuel	H_2	CH_4	C_2H_4	N_2	CO	O_2	CO_2	B.t.u. per cu. ft. at 62 deg. fahr. and 14.7 lb. pressure
Coke	11.10	0.20	0.10	57.50	21.90	0.00	9.20	120.6
Coke breeze	12.81	0.37	0.00	54.80	23.28	0.00	8.74	136.9
Illinois bituminous coal	15.60	1.90	0.40	52.00	20.90	0.00	9.20	156.1
Pennsylvania bituminous coal	12.50	2.50	0.00	55.60	20.70	0.00	8.70	140.6
California lignite	12.67	4.16	0.00	57.82	13.19	0.25	11.91	126.7
Montana lignite	16.00	2.90	0.60	52.90	14.20	0.20	13.20	147.5
North Dakota lignite	14.33	4.85	0.00	51.00	20.90	0.23	8.69	188.5
Texas lignite	9.63	4.81	0.00	57.54	18.22	0.20	9.60	156.2
Florida peat	18.50	2.20	0.40	45.50	21.00	0.00	12.40	175.2
North Carolina peat	10.19	0.66	0.06	60.83	16.91	0.41	10.94	109.7

Producers Using Crude Oil have been developed to such an extent that fair results are obtainable. The heat value of the gas generated varies from 105 to 450 B.t.u. per cu. ft., depending upon the producer and method of operation. In these producers, the tar-forming constituents of the oil are cracked and fixed as gases and lampblack. The quantity of lampblack exceeds that made in down-draft producers fired with bituminous coal so that gas cleaning in an oil-producer installation is a serious problem. Owing to the price of oil, the oil producer has only a limited application.

GAS CLEANING

(See "The Cleaning of Blast-furnace Gas," W. A. Forbes, *Bul. A. I. M. E.*, Oct., 1913)

Cleanliness Requirements. Most engine builders specify that the dust and other solid impurities contained in the gas shall not be in excess of 0.02 gr. per cu. ft. at 62 deg. fahr. and 30 in. of mercury, and that the moisture shall not be in excess of 10 gr. per cu. ft. The German standard is 0.009 gr. per cu. ft., which can easily be attained by the use of Theisen or similar washers. Such a cleanness, however, is out of proportion in a blast-furnace plant unless the air also is washed, as the air often contains much more dust than this. Most producer manufacturers guarantee their scrubber plants to clean the gas to a dust content not to exceed 0.01 gr. per cu. ft. at 62 deg. fahr. and 30 in. of mercury.

Determining the Dust or Tar Content of Gases. The simplest method consists in filling the vertical legs and bulbs of a so-called "**Peligot tube**" (a glass U-tube with an enlarged bulb in each vertical leg and in the bottom connection of the U) with clean, dry, absorbent cotton, weighing the tube and cotton, allowing a metered quantity of the gas to be tested to pass through, and weighing again. The increase in weight gives the weight of impurities removed from the gas by the cotton. This apparatus is accurate only within about 16 per cent., as in operation the cotton in the discharge leg becomes nearly as dirty as that in the inlet leg, showing that some of the impurities pass out with the gas. The **Leo Martius dust determinator** consists of two flanges, held together by wing screws, between which a piece of chemical filter paper is placed. The filter paper is sometimes backed on the discharge side by a perforated plate of thin sheet metal or a wire screen to prevent the paper breaking if there is much moisture in the gas. The filter paper is weighed before and after passing through it a metered quantity of gas, and from this the dust content per cu. ft. is determined. This method is very accurate. The apparatus as used generally consists of two determinators in parallel (so that alternate determinations can be carried on continuously) and a wet meter. The **Brady filter** consists of a brass cylinder, in which is supported the filter itself—an ordinary Soxhlet extraction shell. When the gas contains moisture the cylinder must be heated. This method has the advantage that the cylinder filter resists the gas pressure much better than the sheet of filter paper used in the other; the filter shell maintains a porous condition even though much fine dust has been deposited, owing to the formation of concentric layers of dust and their subsequent cracking by action of gravity and slight jarring, whereas in the filter-paper method the pores of the paper fill up, clog, and finally prevent the passage of gas, if the gas is very dirty. On account of maintaining a porous condition, gas samples of twice the size permissible in other instruments can be passed through this filter. As is the case in the other methods, the shell is weighed in an analytical balance before and after passing the metered gas sample through it.

With any of these methods, in case the gas contains moisture, the filter must be kept at a temperature a little above the boiling point. The moisture can be determined by passing the sample through a tube filled with a dehydrating agent, such as calcium chloride, and weighing the tube before and after. If the gas contains much heavy dust, a sample pipe bent in a 90-deg. curve with a radius of not less than 6 in. should be used. The end of this pipe should be reduced to a sharp edge and it should be inserted in the gas main, if possible, on a horizontal diameter at least 15 ft. away from any bend or obstruction, to a distance of one-quarter to one-third of the diameter of the main.

Gas Cleaning. The general methods for the removal of both moisture and solid impurities from gas are: (*a*) A reduction of temperature; (*b*) a sudden change of direction of flow; (*c*) a sudden reduction of velocity; (*d*) utilization of the weight of the impurities to effect separation by centrifugal force; (*e*) by increasing the weight of each particle by enveloping it in a particle of water. For use in gas engines the temperature of gas must be reduced to as low a point as is conveniently possible, to decrease the amount of saturation vapor and to increase the weight of gas per cu. ft. To wash gas in the most efficient manner the gas must be brought into intimate contact with the smallest particles of water possible. If the drops of water are as small or smaller than the size of the dust particle, the dust particle will penetrate the water film, be enveloped by the water particle, weighted down, and carried to the bottom of the washer by the increased density of the combined mass. Combined dust-and-water particles too small to have sufficient weight to resist the upward force of the gas current, tend to unite with similar particles, and soon become heavy enough to fall to the bottom of the washer.

Types of Gas-cleaning Apparatus. There are in general two types of apparatus for gas cleaning, static and mechanical. **Static cleaners** are either (1) **dry cleaners** which remove the impurities either by an abrupt change in the direction of flow, a sudden reduction of velocity of flow, giving the gas a whirling motion and thus utilizing centrifugal force, or a combination of some or all of these methods; (2) **filters** which are mechanically cleaned; or (3) **tower washers** of the hurdle, baffle, rain, or impinging type. **Mechanical washers** are those in which centrifugal force is used by giving the gas a whirling motion, and in which the water is broken up into a very fine mist by means of (1) **fans** or (2) **disintegrators.**

Dry Cleaning is used only in blast-furnace gas plants, as it is effective only in removing the heavier impurities. Blast-furnace gas in its raw state contains from 3 to 10 gr. per cu. ft., while at the time of slips of the charge in the furnaces or other sudden changes in furnace conditions, the dust content will be much more. The gas is usually conducted from the furnace to some form of dry-dust catcher through a self-cleaning zigzag flue.

The velocity in the catcher should be about 1.5 ft. per sec. An improved form has conical top and bottom and introduces the gas through a tangential, inclined inlet near the top of the shell and takes it off through a vertical riser projecting down through the center of the shell to a point near the bottom of the shell; the gas travels from the top to bottom in long spirals. Two of these catchers are usually used in series. This type of apparatus will clean the gas on an average to a fineness of 1.8 to 3 gr. A very recent dry-dust catcher is the Dyblie. Three of these catchers (which can be much smaller than the older type) are generally used in series and clean the gas to an average fineness of a little less than 0.5 gr. per cu. ft.

Some plants use a horizontal centrifugal separator after the dry-dust catchers, consisting of a helical gas passage with a dust trap at the bottom of each turn. Five or six turns of the helix are usually as many as can be effectively used. Efficiency, about 70 per cent. Mains or risers in the dry gas system must be either vertical or inclined so as to be self-cleaning. The mains and dust catchers are generally unlined so as to cool by radiation. The average radiation is about 1⅛ B.t.u. per sq. ft. of the radiating surface per deg. difference in temperature per hour. The average temperature of the raw gas as it leaves the furnace is 400 deg. fahr., but it may reach 700 or 800 deg.

In Germany, a **dry filtration process** (the Halberger-Beth), which takes the place of the primary and secondary wet cleaning plants, is coming into use. The gas is cooled by radiation to about the dew point (approximately 130 deg. fahr.), to condense as much vapor as possible, and then is superheated 20 or 30 deg. by the sensible heat of the raw gas, to avoid clogging the filter bags with moisture. The gas is then passed through a number of filter bags in parallel. Once every 4 min. each group of bags is shaken for a period of from 15 to 20 sec., and the gas current reversed through it long enough to remove the accumulated dust by causing a partial collapse of the filter bags, and deposit it in dust traps. With this apparatus the dust content is reduced below 0.0004 gr. per cu. ft.—which is 15 to 30 times the fineness obtained with Theisen washers. The power consumed is ⅗ to ⅓ of that used for hydraulic fans or Theisen washers and the water consumption is only ⅙ to ⅛.

Wet Cleaning is accomplished in **tower washers.** In these washers it is important to secure a uniform distribution of both gas and water over the cross-sectional area.

Hurdle scrubbers are towers filled with staggered rows of slats, set on edge, the alternate rows running at right angles so as to form a checkerwork. In blast-furnace plants, the slats are spaced 3 in. on centers, with about 2 ft. left between the hurdles to allow of cleaning and repairs without removing all the hurdles from each compartment. Hurdle scrubbers will deliver gas containing from 0.15 to 0.3 gr. per cu. ft.

At the South Chicago plant of the Illinois Steel Co., on the average the gas reaches the wet scrubbers containing 1.533 gr. of dust per cu. ft. and after passing through two of these towers in series contains 0.3183 gr. of dust and 6.62 gr. of moisture per cu. ft., or 2¼ times the moisture content of the atmosphere (efficiency, 79.3 per cent.). Most of the cooling is effected in the first tower as the gas is here brought to within 1½ deg. of the water temperature—with a water temperature rise of 20 deg. fahr. The water consumption varies between 103.2 and 68.6 gal., with an average of 82.8 gal. per 1000 cu. ft. of gas washed. The power consumption for pumps is about 0.33 per cent. of the total output of the engines. When the amount of gas cleaned was increased 25 per cent. the dust content delivered to wet scrubbers averaged 1.806 gr. per cu. ft., the wet scrubbers cleaned to an average fineness of 0.1488 (efficiency, 91.5 per cent.), the water consumption averaged 63.1 gal. per 1000 cu. ft. gas cleaned, and the moisture content 4.735 gr. per cu. ft. or 2.38 times the moisture content of the atmosphere.

In the **rain-type scrubber** the water is broken up and distributed by means of a revolving screen, or by spraying or atomizing. The water particles are a little finer than with splash plates, and there is a consequent increase in efficiency. The water consumption and power to drive the pumps is increased, since the water must be delivered to the spray heads at a considerable pressure in order to be atomized. **Baffle scrubbers,** in which the gas is caused to pass many times through sheets of water flowing from one baffle to the next, are very efficient but require a great deal of water. In the **impinging type of washer** the gas comes into the top of the washer, passes down through tubes, impinges upon a water surface, and then is taken off in a reverse direction. The efficiency of this type is very high but it does not cool the gas. All of these types of static wet scrubbers are applied to producer-gas plants with about the same efficiency and water consumption as with blast-furnace plants.

Mechanical Washers. The **Bian gas washer,** which is a primary washer, consists of a stationary horizontal steel cylinder with a slowly revolving shaft carrying a number of vertical disks of fine-meshed wire netting. The clearance between the edges of the disks and the inside of the cylinder is very small. The disks dip for nearly half their diameter in water at the bottom of the cylinder and are consequently kept covered with thin films of water, compelling thorough contact with the gas as it passes through the perforations.

It is necessary to use **tar extractors** in bituminous-coal producer plants, since static washers will not remove the tar. The most common type is the **fan washer.** The gas enters around the shaft on one side of the fan and is taken off on the other side. The water is injected at the top of the casing and the water and tar are drawn off at the bottom through a water-sealed tar leg. The fan is built with a central partition plate which causes the gas to pass through the water film which seals the edges and end of the fan. The impurities are separated by centrifugal force. Efficiency rarely exceeds 30 per cent. When the tar is sticky, the extractor may be shut down by it, necessitating boiling out the tar with steam or even removing the casing and scraping it off. The **inertia extractor** is another variation in which centrifugal force is utilized only in an incidental manner, the real cleaning effect being produced by an extremely rapid change in the direction and velocity of flow of the gas current. The gas is passed through alternate sets of steel blades revolving in opposite directions, and the particles of impurities are projected violently from the blades running rapidly in one direction against the next series running with equal speed in the opposite direction. The impurities are separated by inertia and are thrown out against the housing and washed away by the scrubber water. This type is much more efficient than the previous one, but still leaves much to be desired. In both types the speed of rotation is of vital importance; any deviation from the proper speed seriously affects the successful working, especially if the variation is a reduction. Both types require between 3 and 4 per cent. of the total output of the engines to drive them.

The **Theisen washer** is very efficient and requires relatively little power and water. The apparatus consists of a closed cylindrical drum fitted on its outer surface with longitudinal blades arranged in spirals, which decrease in depth from inlet to outlet, rotating at high speed in a stationary conical housing. The drum is equipped at the inlet end with suction vanes, while on the discharge end an exhaust fan is attached. The suction vanes draw the gas from the inlet pipe and deliver it to the longitudinal vanes, which have an inclination to the axis of the drum so as to oppose the flow of the gas through the washer. The discharge fan at the outlet end of the drum overcomes this tendency and discharges the gas under a positive pressure of 8 to 10 in. of water higher than the pressure or suction on the inlet side. The clearance between the outer edge of the blades and the inner surface of the housing is not more than 1 in., and the gas passing through this narrow space under high pressure imparts to the water introduced at several points into the casing a movement in long spirals in an opposite direction to its own travel—this flow of water, in the form of a film covering the inner surface of the housing, being assisted by the conical shape of the latter. The surface of contact between the gas and water is increased by wire netting closely fitting the inside of the housing. The dust particles weighted down by the water drops are thrown by centrifugal force into the rotating water film. The moisture in the gas leaving the washer is removed in a separator consisting of a removable box filled with iron shavings held in place by wire netting and located close to the discharge of the washer. This washer is self-cleaning except for a slight mud deposit at the inlet ends of the blades. The wire netting is entirely self-cleaning. The efficiency of the Theisen washer—taking gas from wet scrubbers—is from 80 to 85 per cent., the dust content in the discharge gas is about 0.005 gr. per cu. ft., the water consumption is 17 to 20 gal. per 1000 cu. ft. of gas, and the power consumption 3 per cent. of the total power of the engine.

The Theisen system of final wet cleaning is now being superseded by systems whose first cost and operating expense are less. Most of these systems can be used for primary cleaning as well as for final cleaning, by installing in two stages. The most important of the wet-cleaning systems which clean as efficiently with the consumption of less power and water than the Theisen and Schiele systems are the **disintegrator systems** of Theisen and of Schwarz-Bayer, the Fowler and Medley rotary washer, and the Feld rotary washer, while the Halberger-Beth dry cleaning system of filtration through canvas (see p. 1061) is remarkably efficient in cleaning and is cheap to operate. The **Feld washer** is built in seven sections; the upper four acting as cooling chambers and the lower three as cleaning chambers. Its action is based on the fact that fine powders will mix readily with hot water, Each section contains a series of perforated truncated cones, the lower ends of which dip into water contained in a tray provided with gas ports. By revolving these cones (with a peripheral speed of 1600 ft. per min.) the water is carried up inside the cones and hurled out through the perforations in a finely divided spray. The gas is cooled to about 85 deg. fahr. in the upper four sections by the use of 31 gal. per 1000 cu. ft. of gas. Then 7½ gal. of this hot water is taken into the three lower sections and used for cleaning. The washer is self-cleaning and the water and dust leave it in the form of mud which is easily handled in centrifugal pumps. The first cost of installation, as well as the ground space occupied, is less than with static towers, and the power required is less than for Theisen washers. The Feld washer will clean to a fineness of at least 0.01 grain per cu. ft.

A New Type of Tar Extractor (Smith). With this type of apparatus, the standard anthracite producer is **used with bituminous coal** and no attempt is made to fix the resulting tar. The raw gas on leaving the producer is first cooled to a point where the tar vapors are condensed, by being passed through a primary cooler or condenser, from which the gas is carried into a rotary gas pump or exhauster, which delivers it under pressure into the main. It is then delivered through a porous diaphragm of spun glass into the engine main, where a sump or separator is provided in which the tar accumulates. The diaphragm must be sufficiently porous to permit the gas and tar to pass freely, and is in the form of a uniform layer of glass wool retained between two metal screens. Ordinarily, the thickness is about ¼ in. and the diameter must be adjusted in accordance with the quantity of gas to be treated. About 400 cu. ft. per hour can be handled per sq. in. of diaphragm area. No tar is retained in the diaphragm, both tar and gas being discharged together, but in passing through, change in the physical state of the tar occurs. On the entering side the tar exists in a large number of minute particles, known as tar fog, while in passing the diaphragm these particles are caused to coalesce, so that on the discharge side the tar particles are of relatively large dimensions and separate out by gravity and drain into the sump. It appears to be possible to secure any desired degree of gas cleanness by regulating the velocity of flow through the diaphragm. In ordinary commercial operation, it is found that a difference in pressure of from 2½ to 4 lb. per sq. in. will clean the gas so that no discoloration will be produced on a white filter paper through which 30 cu. ft. of gas has been passed. No water is used except that required to cool the gas, and therefore the tar separated is practically water-free (less than 1 per cent.). This process is not well adapted for gas containing large quantities of lampblack or from coals yielding a very heavy, viscous tar. It has, however, been used with success with Ohio, Illinois and Indiana high-volatile coals and with lignite.

Removal of Sulphur from Gas is accomplished chemically by the use of iron oxide sponge, which must be frequently renewed; the efficiency of the process is from 80 to 85 per cent. It has been found that sulphur as high as 2 gr. per cu. ft. can be successfully handled in an engine if water can be kept out of the gas in the cylinder and exhaust so that acid is not formed, and if the piston rods are kept at a sufficiently high temperature to prevent condensation and the consequent formation of acid. The piston rods must also be thoroughly flushed with a continuous stream of oil, as with normal lubrication the rods tend to become dry in places.

INSTALLATION AND OPERATING COSTS OF GAS-POWER PLANTS

Cost of Gas Engines and Gas Producers. Table 28 gives average costs taken from those published by R. H. Fernald in Bulletin No. 55, Bureau of Mines, 1913. **Producer costs** are made up approximately in the following percentages: Producer, 35; gas-cleaning plant, 21.5; piping and auxiliaries 43.5. Blast-furnace-gas-cleaning plants cost approximately $5.75 per b.h.p.

Cost of engine accessories, auxiliaries and erection, in percentages of the cost of engine F.O.B. factory, are given in Table 29.

Cost of producer accessories, auxiliaries and erection, in percentages of the cost of producer F.O.B. factory: Foundations, 3.5; erection, 25.5; coal-handling and storage plant, 34.0.

Costs of installation of large plants using blast-furnace gas (H. J. Freyn, Am. Iron and Steel Inst., May, 1913) are given in Table 30.

Cost of buildings includes cost of parts of buildings occupied by engines and part of buildings housing final gas cleaning apparatus which serves to clean the gas for the engines.

Table 28. Average Costs of Gas Engines and Gas Producers

(Costs F.O.B. factory)

Brake horse power	Cost of engine per b.h.p.	Cost of producers, piping, auxiliaries, and gas cleaning plant, per b.h.p.	Brake horse power	Cost of engine, per b.h.p.	Cost of producers, piping, auxiliaries, and gas cleaning plant, per b.h.p.	Brake horse power	Cost of engine, per b.h.p.	Cost of producers, piping, auxiliaries, and gas cleaning plant, per b.h.p.
15	....	$28.20	100	$39.50	$12.00	200	$38.83	$12.35
20	$55.00	....	125	36.10	....	250	37.73	10.93
25	....	20.30	150	38.70	9.83	300	38.33	10.33
35	....	16.00	165	41.20	....	500	35.33	14.50
45	43.30	....	180	36.10	....	1000	33.33	13.25
50	42.00	18.65	190	44.75	....	3000	30.00	13.25(a)
75	44.00	12.33						

(a) Three 1000-h.p. producer plants.

Table 29. Cost of Engine Accessories, Auxiliaries and Erection

(Costs in percentages of cost of engine F.O.B. factory)

Item	Cost, per cent.	Item	Cost, per cent.
Foundations, engines under 1000 h.p.	5.0	Station wiring, installed (average)	20
Foundations, engines over 1000 h.p.	3.5	Engine auxiliaries, air compressor, storage tank, motor-generator set, etc., installed (average)	20
Erection, engines under 1000 h.p.	10.0	Station piping from ends of foundation to producer piping or street mains, exhaust stacks, etc., installed (average)	5
Erection, engines over 1000 h.p.	8.5		
Piping and mains to end of foundations, installed (average)	10 0		
Switchboard, erected (average)	8.0		

Table 30. Actual Installation Costs of Gas Electric Stations*

(In plants of subsidiary companies of the United States Steel Corporation)

Power plant No.	1		2		3		4		5	
No. of units	17		2		4		4		5	
Capacity: kw.	40,000		4,500		9,000		9,000		11,400	
Capacity: b.h.p.	56,400		6,400		12,800		12,800		16,300	
Cost of installation per kw. max. con. rating:		%		%		%		%		%
Buildings	$9.87	11.3			$10.17	10.6	$10.90	10.8	$10.52	10.3
Engine equipment	71.78	82.0	$75 50	81 8	72.75	74.6	77.78	76.4	80.32	77.3
Gas-cleaning plant	5.85	6.7	16.80	18.2	14.40	14.8	13.00	12.8	12.76	12.4

* All cost figures based on maximum continuous rating of generator at 40 deg. cent. temperature rise. Engines have a greater b.h.p. capacity than corresponds to this generator rating and the figures in table are computed assuming a mean effective pressure of 70 lb. per sq. in. (a value universally adopted by gas-engine builders) and a mechanical efficiency of 80 per cent.

Engine equipment includes engine, generator, switchboard, station wiring (but no transmission lines) and all auxiliaries, installed. Does not include rotary converters storage batteries and switchboards for these appliances.

Cost of gas-cleaning plant includes cost of final gas-cleaning plant for the engines only, as the gas must be cleaned in the primary plant for use in stoves and boilers.

Operating Costs

Depreciation. The average life of cylinders in large steel plants is from 5 to 6 years, consequently the expense of a new cylinder must be distributed over this period. In general, an allowance of from 7 to 10 per cent. of the capital outlay for the power plant should cover depreciation, the lower figure for a producer power plant, and the higher for engines alone.

Attendance. From data by H. J. Freyn on the cost of attendance in three large blast-furnace gas-electric stations containing 26 units of an average capacity of 2300 kw., 3 men were required per unit per 24 hr. at an average attendance cost of about $8 per unit per 24 hr.

Maintenance and Repairs. The annual charge for these items should not exceed 3 per cent. of the purchase price of the engines and producers in a well-designed and built plant where the operation is normal. Freyn gives the following as average annual percentages of the cost of the separate items for a blast-furnace gas-power plant: Engines and electric generators, 2.5; cleaning plant, 7.0; air compressor used for starting, etc., 5; piping, etc., 2. To these may be added from 1 to 2 per cent. of the cost of the building for its maintenance.

Freyn (*op. cit.*) gives the following figures on the average repair and maintenance costs for 3 years operation, in eight American blast-furnace gas-engine plants. The use factors vary between 22 and 71.5 per cent. The repair and maintenance costs are expressed as a percentage of the net operating expenses (*i.e.*, total cost of power production less the fixed charges and cost of fuel):

Plant capacity, kw.	40,000 to 50,000	11,400	9,000	9,000	5,000	4,500	2,500	1,500	Avg. 11,600
Costs, 1910, per cent.	28.0	50.5	31.0	50.5	34.5	38.0	55.0	52.5	42.5
Costs, 1911, per cent.	29.5	51.0	35.5	64.5	21.5	31.5	50.5	48.0	41.5
Costs, 1912, per cent.	33.0	61.0	41.0	60.0	17.0	43.5	57.5	51.0	45.5

Average for 3 years' operation, 43 per cent.; average use factor, per cent.: 1910, 46.3; 1911, 46.8; 1912, 53.9. Average for 3 years' operation, 49.0 per cent. See also Tables 32 and 33.

Power Costs. The costs given in Table 31 are reported by J. R. Bibbins (*Trans. A.S.M.E.*, vol. 28, p. 559) as approximate power costs. This plant consists of three 150-kw. 3-cylinder vertical Westinghouse single-acting four-cycle engines running on producer gas. The coal has a heat value of 13,500 B.t.u. per lb. and costs $2.30 per short ton delivered. The plant is run 24 hrs. a day for 6¼ days a week at a labor cost of $14 a day for the two shifts. Supplies and repairs are estimated as 15 per cent. of the total other operating costs at 50 per cent. load factor. The cost of the plant, including engines, generators, switchboard, gas holder, producers, auxiliary pumps, and motors, coal-handling apparatus erected, is given as $125 per b.h.p. ($180 per kw.), or a total of $81,000.

Tables 32–34 (Freyn, Am. Iron and Steel Inst., 1913) give operating cost data for large blast-furnace gas plants.

Table 31. Power Costs in a 450-kw. Producer-gas Power Plant
(Running on bituminous coal)

	Minimum load factor	Normal load factor	Maximum load factor
Load factor, per cent	50	80	100
Output. kw-hr. per day	5,000	8,000	10,000
Coal consumption, lb. per kw-hr	2.175	1.940	1.880
Fuel cost cents per kw-hr	0.250	0.223	0.216
Labor cost, cents per kw-hr	0.280	0.175	0.140
Supplies and repairs, cents per kw-hr	0.147	0.092	0.074
Total operating cost, cents per kw-hr	0.677	0.490	0.430
Fixed charges. 15 per cent., cents per kw-hr	0.718	0.449	0.359
Total power cost, cents per kw-hr	1.395	0.939	0.789

Table 32. Cost of Producing Electric Power at Gary, Ind.
(Cents per kilowatt-hour)

	1910		1911		1912	
Capacity in kw	40,000		40,000		50,000	
Kw-hr. produced	116,535,000		157,742,510		286,575,000	
Use factor, per cent	33.3		45.0		64.5	
Cost of installation per kw	$88.00		$88.00		$88.00	
Labor	0.0678		0.0421		0.0302	
Repairs and maintenance	0.0366		0.0305		0.0273	
Lubricants	0.0116		0.0100		0.0085	
Water	0.0074		0.0057		0.0036	
Miscellaneous	0.0064	%	0.0153	%	0.0128	%
Total net operating expenses	0.1298	17	0.1036	17	0.0824	17.5
Value of gas	......	...	0.1508	...	0.1464	.....
Cost of final cleaning	......	...	0.0219	...	0.0144	.....
Total cost of fine gas	0.1951	25	0.1727	28	0.1608	34.0
Operating cost without fixed charges	0.3249	42	0.2763	45	0.2432	51.5
Fixed charges at 15 per cent	0.4520	58	0.3360	55	0.2310	48.5
Grand total at switchboard	0.7769	100	0.6123	100	0.4742	100.

Table 33. Average Cost of Producing Electric Power in Eight Blast-furnace Gas-power Plants
(Plants of the U. S. Steel Corp. Subsidiaries. All costs in cents per kw-hr.)

	Maximum	Average			Minimum	Average for 1912		
Plant capacity, kw	50,000		11,600		1,500		11,600	
Use factor, per cent	22.0		49.0	%	71.5		54	%
Labor	0.0881		0.0550	33.0	0.0302		0.0463	30.6
Repairs and maintenance	0.1282		0.0733	44.0	0.0273		0.0726	47.9
Lubricants	0.0237		0.0125	7.5	0.0054		0.0104	6.9
Water	0.0162		0.0120	7.2	0.0036		0.0068	4.5
Miscellaneous	....	%	0.0137	8.3	....	%	0.0153	10.1
Total net operating expense	0.2438	52.2	0.1665	100.0	0.0824	50.6	0.1514	100.0
Cost of 1 million B.t.u. in cents	10.37		8.11		5.89		7.89	
Cost of fuel	0.2441	47.8	0.1530		0.0963	49.4	0.1480	
Total cost of power production without fixed charges	....	100.0	0.3195		....	100.0	0.2994	
Heat consumption. B.t.u. per kw-hr	26,000		18,400		16,200		18,450	
Thermal efficiency at switchboard, per cent	13.12		18.54		21.0		18.5	

Table 34. Average Costs of Cleaning Blast-furnace Gas
(Cost in dollars per 100,000 cu. ft. of gas cleaned at the Duquesne furnaces of the Carnegie Steel Co., in 1912)

	Primary cleaning in Duquesne towers		Final cleaning in Theisen washers	
		per cent.		per cent.
Producing labor	0.01052	16.62	0.01965	16.57
Labor in repairs and maintenance	0.00630	9.95	0.00822	6.94
Materials in repairs and maintenance	0.00481	7.60	0.01701	14.35
Electric current	0.02660	42.03	0.06710	56.59
Water	0.01506	23.80	0.00658	5.55
	$0.06329	100.00	$0.11856	100.00

Total cost of cleaning 100,000 cu. ft. = $0.06329 + $0.11856 = $0.18185.
Cost of cleaning gas for blowing engines per ton of iron, $0.022082
Cost of cleaning gas for power engines per 1000 kw-hr., $0.32379.

GAS TURBINES

By L. C. LOEWENSTEIN

The fundamental difference between a gas turbine and a steam turbine is that in the gas turbine the products of combustion have, with the exception of a little water vapor usually present, no latent heat of vaporization, and also that their specific heat is comparatively low. For these reasons, the utilization of large quantities of heat in the gas turbine generally involves the employment of very high temperatures.

Classification. The rotor of a gas turbine may be of the impulse or of the reaction type, and may be classified in the same manner as the steam turbine. The classification of gas turbines is usually according to the manner in which the fuel is burned in the combustion chamber, as follows: (*a*) **Continuous combustion**, in which the combustion takes place continuously and at constant pressure, the fuel and the air entering and leaving the combustion chamber in a uniform stream; (*b*) **explosion**, in which the combustion takes place intermittently, and at constant volume, the inlet and the exit of the combustion chamber being closed while the actual combustion is in progress. The isothermal generation of heat in the combustion chamber according to the Carnot cycle and its various derivatives (like the Diesel, the Stirling, and the Ericsson) has not found practical application in the gas turbine.

The Air Compressor. The effective utilization of the heat generated in the combustion chamber requires a large pressure range between the combustion chamber and the exit of the rotor. For large powers, the pressure in the combustion chamber may reach 8 to 10 atmospheres, the turbine exhaust being atmospheric. For smaller powers, to avoid too short bucket heights on the rotor and to reduce rotation losses, the pressure in the combustion chamber is lower, the exhaust gases being cooled and their pressure reduced to a fraction of an atmosphere by means of an exhauster. Except in the very smallest sizes of the explosion type, an air compressor or an exhauster, therefore, forms a part of every gas-turbine installation.

The Regenerator. Only a small part of the chemical energy of the combustible is recoverable directly as mechanical work; another part must be recovered as sensible heat. The heat contained in the exhaust gases may be transferred from them by means of a regenerator to the compressed air on its way from the compressor to the combustion chamber. With gaseous fuel, a regenerator may also be used for preheating the gas. The regenerator may consist of a series of iron tubes like those of a surface condenser, or of stoves like those used with blast furnaces, or of a combination of the two, and may have a transmission efficiency as high as 75 per cent. Another type of regenerator consists of a tubular boiler for generating steam from the heat of the exhaust gases, this steam being used in an independent steam turbine. When working with exhaust at a fraction of an atmosphere, regeneration through steam will in general be impracticable.

Injection of Water. The injection of water or preferably of steam into the combustion chamber makes possible the use of fuels of high calorific value without excessive air dilution. Water injection may thus reduce the size of both the compressor and the turbine for a given output.

Overall Shaft Efficiency. The overall shaft efficiency of a gas-turbine plant is the ratio of the power given out by the plant for external uses to the chemical energy in the fuel; it depends on the shaft efficiency of the gas turbine (or gas turbine and steam turbine combined) and the efficiency of the air compressor. If L_t represents the theoretical power developed by the gas turbine (including that of the steam turbine, if any), L_c that required for compression, and L that available for external purposes; then $L = L_t - L_c$. If the efficiency of the gas turbine (and steam turbine) is e_t, and that of the compressor is e_c, the **actual external power available** $L_a = L_te_t - (L_c/e_c)$.

To illustrate the sensitiveness of a gas-turbine plant to the efficiencies of its component parts, assume the two extremes of efficiencies which are likely to be met in practice. Let the heat developed per lb. of gas mixture be 500 B.t.u., and let 80 per cent. of this, or 400 B.t.u. per lb., be available for mechanical work. The theoretical work of air compression is assumed as 80 B.t.u. per lb. of gas mixture. With a turbine efficiency of only 0.40, and a compressor efficiency of 0.50, the actual external power available is $L_a = 400 \times 0.40 - (80/0.50) = 0$, so that the overall shaft efficiency of the plant is zero. Again, assuming a turbine efficiency of 0.65, and a compressor efficiency of 0.75, the external power is $L_a = 400 \times 0.65 - (80/0.75) = 153$ B.t.u. per lb. of gas mixture, giving an overall efficiency of $153/500 = 0.306$.

The **power** available for external purposes may be about **doubled by** the use of a **regenerator**, whether it is for gas to gas or for gas to water. The efficiency of the regenerating apparatus is therefore of great importance. Thus, in the case of gas-to-gas regeneration, an increase in the number of stoves used may increase the regenerative efficiency from 0.50 to 0.75, and thereby increase the net output of the turbine plant by over 30 per cent. The **efficiency** of a gas turbine **increases with** the **pressure ratio and** with the **temperature** in the combustion chamber, but increasing the pressure ratio beyond 10, or the combustion chamber temperature beyond 3000 deg. fahr., results in very little gain.

Advantages and Applications. The gas turbine should present the same advantages as the steam turbine, namely, smaller dimensions and weight for a given output, lower first cost, less lubrication and attendance, less wear and repairs, comparative freedom from vibrations, and perfectly uniform torque. Not all of these advantages have as yet been demonstrated. A number of gas turbines have been installed on torpedo boats. The best prospects, however, for the gas turbine are for large units of, say, 2000 kw. and over, where the dimensions of the gas engine become unwieldy.

In a **continuous-combustion turbine** the compressor, in the case of liquid fuel or of powdered coal, forces air alone or gas and air into a chamber where combustion takes place continuously at constant pressure. If the temperature of combustion is T_3 and the temperature within the turbine to which the gas mixture expands adiabatically is T_4, there becomes available for mechanical purposes c_p $(T_3 - T_4)$ B.t.u. per lb. If the compressor takes in air and fuel at a temperature T_1, there is available for regenerative purposes, as explained before, c_p $(T_4 - T_1)$ B.t.u. per lb. of gas mixture, the turbine exhaust pressure being the same as the compressor inlet pressure. The overall shaft efficiency of a continuous-combustion gas-turbine plant will, without regeneration, rarely exceed 15 per cent. With regeneration, 25 per cent. and even 30 per cent. (with steam-turbine regeneration) may be reached.

In the **explosion turbine** air and fuel are introduced into a comparatively cold combustion chamber at a pressure p_2, temperature T_2, and specific volume v_2. The inlet of the chamber is then closed and the gas mixture is ignited, when the pressure and the temperature rise to p_3 and T_3, respectively, at constant volume. With the inlet still closed, the exit from the combustion chamber opens, and the contents are expelled adiabatically into a region of pressure p_4 and temperature T_4. The energy liberated for mechanical work is c_p $(T_3 - T_4) - (144/778)v_2(p_3 - p_4)$ B.t.u. per lb. of gas mixture. When the pressure p_4 has been fully established in the combustion chamber, the inlet is opened mechanically and the remaining gases are expelled by the new charge or by scavenging air. If the inlet temperature of the compressor is T_1, there is available for regenerative purposes $c_p(T_4 - T_1)$ B.t.u. per lb. of gas mixture. The compressor may be replaced by an exhauster so that p_2 is atmospheric pressure. To eliminate wheel-speed irregularities while the pressure in the combustion chamber is gradually falling from p_3 to p_4, a number of combustion chambers may be arranged around the periphery of the turbine wheel and the various chambers opened and closed successively one after another by a special mechanism. This mechanism, by varying the lapse of time between the opening of the successive chambers, may also regulate the power developed by the turbine. Steam-turbine regeneration only has been used with explosion gas turbines. With a high-pressure air compressor isothermally cooled and with efficient regeneration, the overall efficiency of an explosion gas-turbine plant may exceed 30 per cent.

WATER POWER

WATER WHEELS

By C. M. SAMES

Water wheels—the earliest forms of hydraulic motors—have been almost entirely displaced by turbines. They are in general of three **classes**: Overshot wheels, which receive the water at the top and are of a diameter less than the total head (commonly adapted to high heads); breast wheels, which receive the water on their upstream side and pass it underneath (used for medium heads); and undershot wheels, which are rotated by water passing below them and are adapted to the lowest heads.

Overshot Wheels are suitable for water falls ranging between 10 and 70 ft. where the water supply is from 2 to 30 cu. ft. per sec., corresponding to a maximum development of 75 h.p. when the economical range of head (10 to 40 ft.) is considered.* In these the water enters along a sluice at the summit of the wheel and falls in a parabolic path into the buckets. The diameter D of the wheel (ft.) may be from $H - 1.33$ ft. to $H - 2.25$ ft., where H = available fall, ft., and the circumferential velocity v from 5 to 7 ft. per sec. For maximum efficiency of operation the velocity of the entering water should be approximately $1.1v$.* The radial depth d of the (steel) buckets may range from $0.16\sqrt[3]{H}$ to $0.25\sqrt[3]{H}$ (and up to $0.5\sqrt[3]{H}$ where narrow wheels are desired), and the number of buckets about $\pi D/d$. The width of the buckets b (ft.) between shrouds should be from $2Q/vd$ to $4Q/vd$, where Q = flow, cu. ft. per sec. When b is greater than 5.5 ft. the buckets should be braced at their mid-width.

The **horse power of a water wheel** $= 0.1134QHe$, where e = total efficiency (hydraulic efficiency × mechanical efficiency) expressed as a decimal. The efficiency of a properly designed overshot wheel, operating under favorable conditions, computed on the basis of output at the wheel shaft, may be as high as 90 per cent. (a 10-ft. wheel tested at the University of Wisconsin gave e = 89 per cent.).* For maximum efficiency the point of impact of the water should lie as high in the wheel as possible. Submergence of the wheel in the tail water seriously decreases the efficiency.

Breast Wheels. In these the water enters the wheel at the upstream side, either (1) flooding the buckets or (2) being directed against them in a stream making an angle of about 27 deg. with a tangent to the circumference at the point of entry, and is prevented from leaving them by a breast wall just clearing the lower quadrant of the wheel on the entry side. Type (1) is available for heads of from 1.3 to 12 ft., and type (2) for heads of from 5 to 17 ft. For (1), $D = 3H$ to $4H$; for (2), $D = H + 11.5$ ft.; $v = 4.6$ to 5.6 ft. per sec. for (1), and 5.3 to 7.2 ft. per sec. for (2). For both types $d = (0.4 \text{ to } 0.5) \times \sqrt[3]{D/H}$; $b = (2 \text{ to } 3) \times Q/vd$; circumferential pitch of bucket $= 0.5d$ to $0.7d$; efficiency, up to 85 per cent.

Undershot Wheels. Where the head does not exceed 6 ft., the Poncelet undershot wheel is probably the best type to employ. In this (impulse) wheel the water jet (thickness t = 8 to 10 in.) flows down a 1 : 10 slope,

* From Bulletin of the University of Wisconsin No. 529, by Carl R. Weidner, entitled "Theory and Test of an Overshot Water Wheel."

enters the wheel without shock, follows up the curved vanes or buckets (never, however, entirely filling them), falls back and leaves the wheel practically without velocity. A close-fitting breast at the bottom should cover 15 deg. each side of the vertical wheel diameter. $D = 2H$ to $4H$ (generally not less than 14 ft.); $d \geq 0.5H$; $b = Q/tv$, where t is taken in ft.; $v = 0.55\sqrt{2gH}$. Circumferential bucket pitch, about 1 ft.; efficiency, ≥ 70 per cent.

Curvature of Buckets. Overshot Wheels (Fig. 1): Take $AB = \frac{1}{3} \times$ radial distance AC, and $CE = 1.2 \times$ circumferential pitch GC. Draw FE at an angle of from 10 to 15 deg. with radius EH. From a point on FE as a

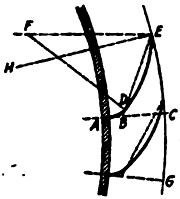


Fig. 1.—Overshot Wheel.

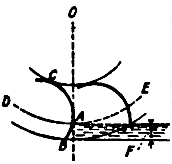


Fig. 2.—Breast Wheel.

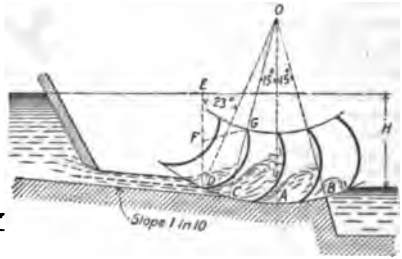


Fig. 3.—Undershot Wheel.

Curvature of Water-wheel Buckets.

center, describe an arc passing through E and D (near B), and round this arc into the radial portion AB of the bucket. Breast Wheel (Fig. 2): From the wheel center O draw the dotted circle DE with a radius equal to the radius of the wheel minus the depth F of tail water. The outer part (AB) of the bucket should be an involute unwrapped from DE; the inner part(AC) should curve sharply after leaving A. Poncelet Undershot Wheel (Fig. 3): Draw vertical radius OA of wheel and lay off AC and AB (each 15 deg. of circumference), making breast BC. At mid-depth D of jet as it strikes the wheel, draw radius OD; also draw DE at an angle of 23 deg. to OD. Take $DF = 0.5H$ to $0.7\ H$ and draw arc DG, which is the bucket curve desired.

HYDRAULIC TURBINES

BY

W. M. WHITE

References: Daniel W. Mead, "Water Power Engineering;" Gelpke and Van Cleve, "Modern Turbine Design;" Pfarr, "Hydraulic Turbines." German Texts: Brauer, "Theory of Turbines;" Budau, "Computation of Hydraulic Governors," and "The Speed Regulation of Hydraulic Motors;" Escher, "The Theory of the Water Turbine;" Gelpke, "Turbines and Turbine Plants;" Honold and Albrecht, "Francis Turbines;" Mueller, "Francis Turbines;" Pfarr, "Turbines for Hydraulic Power Plants;" Quantz, "Water Power Machines;" Thomann, "Water Turbines;" Wagenbach, "Modern Hydraulic Turbine Plants."

Water may do useful work in four general ways: (1) By its pressure; (2) by its velocity; (3) by its weight, and (4) by combinations of (1), (2) and (3). Of these, (2) and (4) are employed most generally in impulse and reaction turbines, respectively. Other types of hydraulic prime movers such as the overshot wheel and breast wheel of the third class, the undershot and current wheels of the second class, and all forms of pressure motors of the first class except in small sizes, as well as modifications of reaction and impulse turbines, including Girard wheels, Fourneyron wheels, Jonval wheels, etc., are not in general use because of inherent disadvantages as to size,

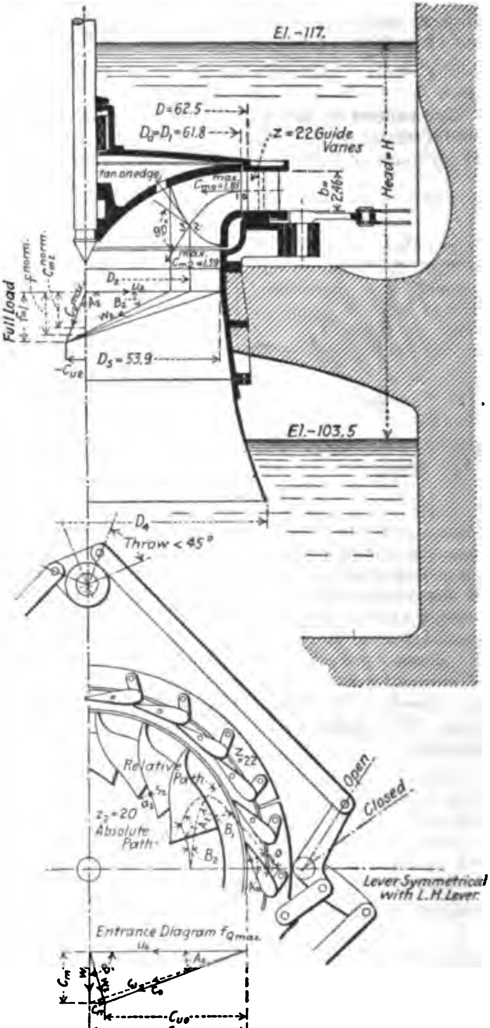
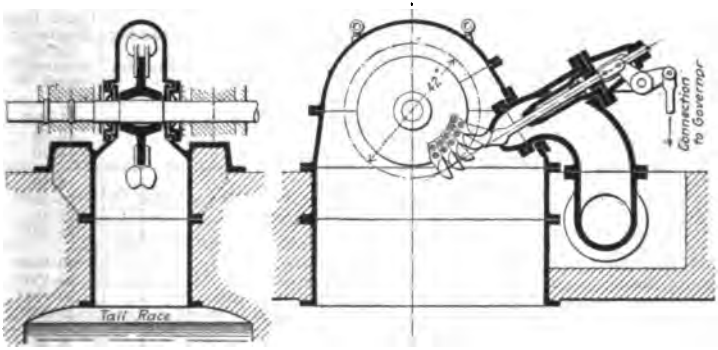


Fig. 1.—Francis Reaction Turbine for Low Head.

cost, efficiency, durability or regulation which make them uneconomical or impractical.

CHARACTERISTICS AND CLASSIFICATION OF TURBINES

Types of Turbines. Of the many possible designs, two types only are of special interest. These are the **reaction** (or pressure) **type,** where the head is only partly transformed into velocity in a stationary guide case, and the **impulse** (or action) **type,** where the head is completely transformed into velocity in a stationary nozzle before acting on the runner. In either case, the direction of the water flowing through the runner is forcibly changed. The impulse turbine revolves at a lower speed than the reaction turbine for a given head, and because it offers advantages of simplicity in design it is preferred for high heads and small discharges. Its lower speed makes it more adaptable to direct connection to electrical generators of economical speeds, and its simplicity of construction permits of a design less subject to deterioration under severe pressure and velocity conditions.



$Q = 14$ cu ft/sec., $H = 650$, $n = 500$, $Q_1 = 0.55$ cu ft/sec., $n_1 = 19.62$, $n_s = 4.4$, $D_0 = 42$, $u_0 = 3.6$ ft/sec., Jet Diam $3\frac{5}{8}$ in.

FIG. 2.—Impulse Turbine (Pelton Wheel).

Reaction (Francis) turbines have normally a radial inlet and discharge the water in a direction more or less parallel to the shaft. Regulation is usually accomplished by wicket-type gates or guide vanes. A plurality of smaller runners may be placed on a single shaft for the purpose of obtaining higher speeds (twin turbine, quadruplex turbine, etc.). An example of a reaction turbine for low head is given in Fig. 1.

Impulse (action) turbines (Fig. 2) are designed with a runner in the form of a disk, upon the rim of which buckets are mounted, these buckets usually being of the double-lobe form, the two parts coming together in a common splitter. Tangent to the runner one or more jets impinge upon the splitters of the buckets and are diverted approximately 90 deg. from their initial absolute direction. The jet is usually circular in section and is adjusted by moving a needle which is concentric with the axis of the nozzle. The water is brought to the wheel through a pipe line. For the purpose of obtaining higher speeds in connection with low or moderate heads or for extremely large units, a plurality of wheels or of jets per wheel may be made use of, though simplicity and (usually) efficiency are sacrificed by so doing.

General Arrangements of Turbine Installations. Turbines are either set in an open flume or in an enclosed casing of concrete or metal. Impulse turbines have the latter arrangement, while reaction turbines have either. The limits of head to which the various types and forms of both reaction and impulse turbines are adapted are dependent upon the comparative economy (*i.e.*, first cost, maintenance charges, efficiency and value of power) effected by the use of the various arrangements. These limits are fairly well defined in modern engineering practice and are outlined roughly in Table 1. The arrangements marked* are those most generally adopted, no attempt having been made to list the various modifications of construction sometimes used. The limits given should not be taken as representing absolute points beyond which the various types are not suitable, but rather as indications of general practice.

Table 1. General Arrangements of Turbine Installations and Usual Head Limits Employed

Type	Setting	Construction	Number of runners	Usual head limits for direct-connected units, ft.
Reaction turbines, 5 to 600 ft. head	Open flume, 5 to 50 ft. head.	Vertical	1	15-30*
			2	10-30
			3	12-30
		Horizontal	1	18-50
			2	16-50*
			3	15-35
			4	14-30*
	Encased, 20 to 600 ft. head	Plate Steel: Horizontal	1	40-150
		Plate Steel: Vertical	1	30-150*
		Plate Steel: Hor., top inlet	2	30-150*
		Plate Steel: Hor., end inlet	2	20-50*
		Concrete spiral, vert.	1	20-80*
		Cast-iron spiral: Hor. or vert.	1	50-600*
		Cast-iron spiral: Horizontal	2	80-200*
Impulse wheels, 300 to 3000 ft. head.	....	(Stationary or deflecting nozzles).	1	300-3000*
			2	400-2000*

Selection of Turbine Types. After the available head, quantity of water and power available are determined, the probable turbine arrangement may be decided upon, though additional factors such as extent of variation of head- or tail-water levels, penstock or pipe-line length and closeness of speed regulation required, have considerable influence upon the selection of type. The following general rules will serve as an additional guide in selection of type.

1. If the head is under 30 ft. the open-flume type should nearly always be used, except for small single-unit plants of less than 100 h.p. in capacity where an encased turbine might be preferable.

2. If the head is between 30 and 50 ft. either the open-flume or plate-steel-encased type will be used, the latter usually working out the most economically for units less than 500 h.p. in capacity. The final decision in any case depends on the local conditions, being based on the relative cost of concrete retaining walls as compared with plate-steel casing.

3. If the head is between 50 and 100 ft., the plate-steel-encased construction is ordinarily the most economical, except in units of small capacity where the cast-iron casing is often less costly.

4. If the head is above 100 ft. and not over 600 ft., the cast-iron casing is usually used on reaction turbines of all moderate or large sizes, cast steel being more economical for large turbines under high heads.

5. For heads between 300 and 600 ft., the impulse type is used for all wheels under about 500 h.p.

6. For heads between 600 ft. and 3000 ft., the impulse wheel is used, though the reaction type is suitable for heads above 600 ft. under special conditions where little or no regulation is required.

Having determined the general forms of construction, the following more specific points may be given consideration.

Vertical Shaft Layouts are ordinarily the more suitable (1) where the most efficient setting of runner is desired; in this case the single-runner vertical-shaft type of reaction wheel is the best and permits of the highest efficiency; (2) in connection with direct-connected hydroelectric units where a large fluctuation of water levels exists and it is of importance to place the electric generator above the extreme high water level and not run into excessive draft-tube length or place the turbine too near the head water level; (3) in connection with direct-connected hydroelectric units when a limited space is available for the power house and forebay. In this case, the width of the power house, measuring in the direction of flow, may be greatly reduced, as the generators are located directly above the turbines.

Horizontal Shaft Arrangements, particularly when used with direct-connected hydroelectric units, ordinarily possess advantages over the vertical-shaft types when considered from standpoints of first cost of machinery, accessibility of parts, ease of erection and inspection, and by reason of having only journal bearings in the place of a special thrust bearing.

Characteristic Speed. Turbine runners of different types may possess characteristics of power, speed, dimension and operating head which vary widely, and it therefore is difficult to draw direct comparisons. In order that there may be available a common basis for comparison of all turbine runners, the value designated hereinafter as "characteristic speed" is used. The *characteristic speed of a runner* (sometimes designated "specific speed" or "type characteristic") *is the speed in r.p.m. which a model of that runner would have if operated under a head of* 1 *ft., this model to be reduced proportionally in all dimensions from the original until it will develop* 1 *h.p. under* 1 *ft. head.*

Since the angles of the buckets of a runner are fixed, the linear speed of the bucket should at all times bear a constant relation to the linear velocity of the entering water in order to secure operation without disturbance. The linear velocity or r.p.m. (n) of a runner is proportional to $\sqrt{H}$, where H in the head in ft., and the speed at 1 ft. head $(= n/\sqrt{H})$ = is called **unit speed,** and designated n_1.

The power (N) developed by a runner is a function both of the pressure and the amount of the operating water. The pressure is proportional to the head and the amount proportional to the velocity, which in turn is proportional to the square root of the head. Therefore the power of the runner is proportional to $H \times \sqrt{H}$, and the power of a runner at 1 ft. head is equal to $N/H\sqrt{H}$, and is called **unit power,** being designated N_1.

The **characteristic speed** $n_s = n_1\sqrt{N_1} = \sqrt{N/H\sqrt{H}} \times n/\sqrt{H} = n\sqrt{N}/H^{5/4}$. In general, impulse wheels have a characteristic speed of about 3 (and not over 6), and reaction wheels as ordinarily used one ranging between 12 and 120. In European practice the characteristic speed for impulse wheels goes occasionally as high as 9 for a single nozzle, and may be increased proportionately to the square root of the number of nozzles. The characteristic speed of reaction wheels may similarly be increased by using two or more runners. The practical **limits** of characteristic speed are determined by the following considerations: With low characteristic speed the narrowness of the passages may result in clogging, their increased length tends to high frictional loss, and the increase in overall diameter of the unit runs up initial cost. In securing low speed, the peripheral coefficient (or ratio of the peripheral velocity of the runner rim to the spouting velocity of the water) is first reduced as far as is possible without causing pressureless flow at the runner entrance (as in impulse turbines). The practical limit in this direction is given in Fig. 3. With this as a basis, speed might be reduced indefinitely by increasing the diameter of the runner, but in so doing the net area of the runner inlet must be kept constant in order to conform to the above velocity requirements, consequently the runner entrance narrows directly as the diameter is increased. A practical limit is reached in this direction very quickly by reason of the likelihood of clogging in the runner passages.

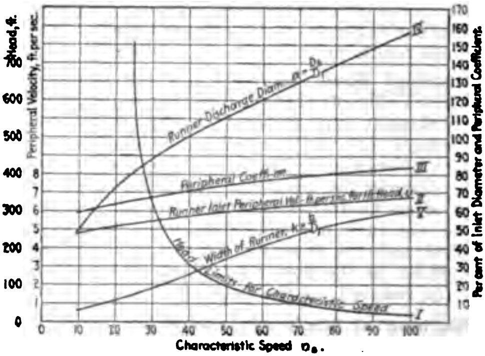


Fig. 3.—Practical Limits of Characteristic Speed of Turbine Runners.

In obtaining high characteristic speed, the reverse of the above holds. The peripheral coefficient is made as high as possible with limits as per Fig. 3 and the runner diameter reduced. This reduction of diameter requires a corresponding increase in the width of the runner inlet in order to conform to velocity requirements. Reducing the diameter of the runner causes the discharge diameter to be relatively small and the discharge velocity correspondingly high. This latter feature is undesirable because the energy of the discharging water can be but partially regained in a flaring draft tube.

TURBINE DESIGN

Notation:

Q = Quantity of water flowing, cu. ft. per sec.
N = brake horse power.
n = revolutions per minute (r.p.m.).
H = head, ft.
Q_1 = unit discharge, or discharge for 1 ft. head (cu. ft. per sec.) $= Q/\sqrt{H}$.

N_1 = unit horse power, or h.p. for 1 ft. head = $N/H\sqrt{H}$.
n_1 = unit speed, or r.p.m. for 1 ft. head = $n/\sqrt{H}$.
n_s = characteristic speed = $n_1\sqrt{N_1}$, r.p.m.
z = number of buckets or vanes in the runner.
t = thickness of blades, ft.
s = pitch of blades, ft.
e_m = mechanical efficiency.
e_h = hydraulic efficiency.
e = $e_m \times e_h$ = total efficiency.
j = percentage of gate opening.
u = peripheral velocity of runner rim at unit speed n_1 ft. per sec. (The corresponding velocities at any given head H are obtained by multiplying by $\sqrt{H}$.)
m = peripheral coefficient, or ratio of the linear velocity of the runner rim to the spouting velocity of the water = $u/\sqrt{2gH}$ where H = 1 ft., = $0.000544D_1n_1$. (D_1 = runner inlet diameter in inches.)
c = absolute velocity of the water, ft. per sec.
w = relative velocity of the water with reference to the runner, ft. per sec.

Velocities at various points are indicated by subscripts (see Fig. 1):
No subscript at the outlet of the guide vanes.
0, immediately in front of the runner inlet.
1, immediately back of the runner inlet.
2, immediately in front of the runner outlet.
3, immediately back of the runner outlet.

Further, c_u indicates the component of c in the direction of u, w_u the component of w in the direction opposite to u, and c_m is the component of c at right angles to u (in the meridian plane).

The angles between the absolute runner and water paths are designated A_0, A_1, A_2, etc., and the relative angles (with the negative direction of movement of the runner), B_0, B_1, B_2, etc.

For a given turbine the values of discharge and speed are proportional to the square root of the head. $Q_1 = Q/\sqrt{H}$ and $n_1 = n/\sqrt{H}$ are therefore constant for any one turbine or runner. It is sufficient to know the values Q_1, n_1 and the operating head H when computing the proportions of a turbine. For different sizes of turbines homologously built, any dimension, such as runner diameter, D_1, may be expressed as follows, k being a constant: $D_1 = k\sqrt{Q_1}$ (areas are proportional to discharge). Hence $n_1 = 60u/\pi k\sqrt{Q_1} = K\sqrt{Q_1}$. K is constant for turbines of the same characteristic speed. It is now customary to introduce the value N_1 in place of Q_1, as turbines are rated more often in h.p. than in discharge, whence $N_1 = Q_1e_m/8.82$, and $n_1 = 60u/\pi k\sqrt{8.82N_1/e_m} = n_s/\sqrt{N_1}$. N_1 and Q_1 are maximum values at wide-open gate.

Francis Turbines (n_s = 12 to 90 for one runner). It has been found in actual practice that there are certain practical limits of characteristic speed beyond which it is not desirable to go for any given head. These limits may be imposed by reason of runner strength, tendency toward erosion or pitting of runner vanes, or may be fixed by limits in generator construction. In Fig. 4 there are plotted a number of runner characteristics taken from actual American installations which have worked out satisfactorily, the examples being selected by reason of their being rather extreme. The arbitrary curve is drawn in as nearly as possible to include all the points representing runners that are known to be commercially successful, and to exclude those which are questionable or that are known to show signs of pitting or other defects. A limit is thus obtained giving the highest characteristic speed which is suited to any particular head. The curve thus derived is drawn in on Fig. 3 (Curve I). The dimensions A, B, C, D, E, F, G and H show the ranges of heads appropriate for the corresponding turbine types as listed in Table 2.

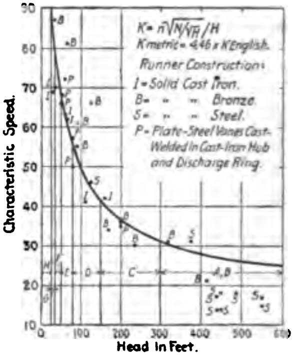


FIG. 4.—Relation of Characteristic Speed to Head (American Practice).

Fig. 3 serves as a guide for the selection of the allowable characteristic speeds and the main runner dimensions. The several curves are based largely on experimental data, and represent modern practice as followed by the leading turbine designers. Knowing the head, the limiting characteristic speed may be taken from Curve I. The power (N) per runner being decided upon, N_1 can be obtained from $N_1 = N/H\sqrt{H}$. Knowing N_1 and n_s, n_1 is obtained from $n_1 = n_s/\sqrt{N_1}$, and the r.p.m. n computed from $n = n_1\sqrt{H}$. As stated above, Curve I is an approximate upper limit and is not to be adhered to rigidly. If the r.p.m. as thus computed is higher than desirable, say, for an electric drive, a lower value of n_s is obtained by computing n_1 = desired r.p.m./$\sqrt{H}$, whence $n_s = n_1\sqrt{N_1}$.

If the speed as first computed is lower than desirable and a higher characteristic speed is not advisable, a remedy is found by dividing the desired h.p. between two or more runners and computing n_1 and n from the smaller values of N_1 and n_s, as before. Knowing n_s, the peripheral speed u may be taken directly from Curve II, whence the entrance diameter of the runner D_1 is obtained from $D_1 = 60u/\pi n_1$. Expressed in inches as is usually customary, D_1 (in.) $= 229u/n_1 = 1835m/n_1$, where m, the peripheral coefficient, is obtained from Curve III or by dividing u by $\sqrt{2gH}$, the value H being unity. The diameter D_1 is measured at the center line of the inlet of the runner (see Fig. 1). The diameter of the runner D_2 at its discharge side fixes the diameter of the draft-tube top and equals aD_1, the constant a being taken from Curve IV, Fig. 3. The width b of the guide case or speed gate $= kD_1$, the constant k being taken from Curve V, Fig. 3.

Examples. LOW HEAD: Given $H = 25$ ft., $N = 4000$ h.p., four units of 1000 h.p. each being desired. From Curve I, Fig. 3, $n_s = 90$ (permissible). For a single-runner unit $N_1 = 1000/25\sqrt{25} = 8$; $n_1 = 90/\sqrt{8} = 31.8$; $n = 31.8\sqrt{25} = 159$ r.p.m. For a 60-cycle installation a synchronous speed ($2 \times 60 \times 60$/No. of poles) must be used, and 150 r.p.m. would probably be adopted. Recomputing on this basis, $n_1 = 150/\sqrt{25} = 30$; and $n_s = \sqrt{8} \times 30 = 85$. From Curve III, Fig. 3, and $n_s = 85$, $m = 0.81$, whence runner diam. $D_1 = 1835 \times 0.81/30 = 50$ in., approx. Similarly, width of runner inlet (guide-case height) $b = 0.54 \times 50 = 27$ in., and runner discharge diam. $D_2 = 1.44 \times 50 = 72$ in. These values would be typical of a single-runner vertical-shaft layout, but considerations of higher speed (lower cost), balanced arrangement and more symmetrical setting would probably warrant the use of a twin turbine if a horizontal unit were desired. In this case N per runner $= 500$ h.p.; $N_1 = 4.0$; $n_s = 90$;

Table 2. Characteristics of Standard Turbine Runners*

(Allis-Chalmers Co.—See Fig. 4*a* for types of blade profiles)

Diam., in. D_1	TYPE A $n_s = 13.55$; $m = 0.585$			TYPE B $n_s = 20.3$; $m = 0.625$			TYPE C $n = 29.4$; $m = 0.665$			TYPE D $n_s = 40.7$ $m = 0.70$		
	n_1	N_1	Q_1	n_1	N_1	Q_1	n_1	N_1	Q_1	n_1	N_1	Q_1
15	71.7	0 0358	0.394	76.6	0.0705	0.776	81.4	0.150	1.43	85.7	0.226	2.49
18	59.8	0.0514	0.565	63.8	0.105	1.155	67.8	0.187	2.06	71.4	0.324	3.56
21	51.2	0.0705	0.776	54.7	0.138	1.523	58.2	0.225	2.48	61.3	0.442	4.86
24	44.8	0.0915	1.007	47.8	0.182	2.00	51.0	0.333	3.66	53.6	0.577	6.35
27	39.8	0.116	1.276	42.5	0.229	2.52	45.2	0.423	4.65	47.6	0.731	8.04
30	35.8	0.142	1.562	38.3	0.284	3.12	40.7	0.520	5.72	42.8	0.902	9.92
34	31.6	0.184	2.024	33.8	0.363	3.99	35.9	0.668	7.35	37.8	1.158	12.74
38	28.3	0.230	2.53	30.2	0.453	4.98	32.2	0.835	9.19	33.9	1.444	15.88
42	25.6	0.280	3.08	27.4	0.551	6.06	29.1	1.016	11.18	30.6	1.765	19.4
46	23.4	0.336	3.69	25.0	0.665	7.32	26.6	1.225	13.48	28.0	2.12	23.3
50	21.5	0.398	4.38	23.0	0.79	8.69	24.4	1.450	15.95	25.7	2.50	27.5
55	19.5	0.480	5.28	20.9	0.95	10.45	22.2	1.745	19.20	23.4	3.04	33.4
60	17.9	0.573	6.30	19.1	1.13	12.43	20.4	2.08	22.88	21.4	3.61	39.7
65	16.5	0.672	7.39	17.7	1.33	14.63	18.8	2.44	26.84	19.8	4.22	46.4
70	15.4	0.785	8.64	16.4	1.53	16.83	17.5	2.82	31.00	18.4	4.90	53.9

Diam., in. D_1	TYPE E $n_s = 51.7$ to 60.5 $m = 0.75$			TYPE F $n_s = 72$; $m = 0.80$			TYPE G $n_s = 82$; $m = 0.77$			TYPE H $n_s = 92.5$; $m = 0.815$		
	n_1	N_1	Q_1	n_1	N_1	Q_1	n_1	N_1	Q_1	n_1	N_1	Q_1
14	98.4	0.277	3.05	105.0	0.47	5.17	101.0	0.66	7.26	107.0	0.74	8.15
16	86.1	0.367	4.04	92.0	0.61	6.71	88.5	0.86	9.46	94.0	0.96	10.55
18	76.5	0.471	5.18	82.0	0.775	8.52	78.5	1.09	12.0	83.5	1.22	13.4
20	69.0	0.597	6.57	73.5	0.96	10.55	70.5	1.35	14.85	75.0	1.51	16.6
22	62.6	0.731	8.04	67.0	1.15	12.65	64.0	1.63	17.9	68.5	1.82	20.0
24	57.4	0.883	9.70	61.5	1.37	15.1	59.0	1.94	21.3	62.5	2.17	23.9
26	53.0	1.055	11.60	56.5	1.62	17.8	54.5	2.27	25.0	58.0	2.55	28.1
28	49.2	1.243	13.67	52.5	1.87	20.6	50.5	2.64	29.0	53.5	2.95	32.5
30	46.0	1.436	15.80	49.0	2.15	23.6	47.0	3.02	33.2	50.0	3.4	37.4
32	43.0	1.65	18.15	46.0	2.45	27.0	44.0	3.45	38.0	47.0	3.85	42.3
34	40.5	1.89	20.80	43.5	2.76	30.4	41.5	3.88	42.7	44.2	4.36	48.0
36	38.3	2.15	23.65	41.0	3.1	34.1	39.0	4.35	47.9	41.7	4.88	53.7
38	36.3	2.42	26.60	39.0	3.44	37.8	37.0	4.85	53.3	39.5	5.45	60.0
40	34.4	2.75	30.25	37.0	3.82	42.0	35.3	5.37	59.0	37.5	6.02	66.2
42½	32.4	3.09	34.0	34.5	4.32	47.5	33.3	6 06	66.6	36.3	6.8	74.8
45	30.6	3.53	38.8	33.0	4.85	53.3	31.4	6.8	74.8	33.4	7.64	84.0
47½	29.0	4.01	44.1	31.0	5.4	59.4	29.7	7.6	83.6	31.6	8.5	93.5
50	27.6	4.45	49.0	29.5	5.95	65.5	28.3	8.4	92.5	30.0	9.4	103.4
52½	26.3	4.95	54.5	28.0	6.6	72.5	27.0	9.28	102.0	28.3	10.6	116.5†
55	25.1	5.52	60.7	26.7	7.25	79.8	25.7	10.2	112.0			
57½	24.0	6.10	67.1	25.6	7.9	87.0	24.5	11.1	122.0	26.8	11.8	130.0‡
60	23.0	6.80	74.8	24.5	8.6	94.6	23.5	12.1	133.0	25.0	13.5	148.5
64	21.6	7.63	83.9	23.0	9.75	107.0	22.0	13.75	151.5	23.5	15.4	169.5
68	20.3	8.57	94.3	21.7	11.0	121.0	20.7	15.5	170.5	22.0	17.4	191.5
72	19.2	9.58	105.4	20.4	12.3	135.0	19.6	17.4	191.5	20.8	19.5	214.5
76	18.2	10.92	120.1	19.4	13.8	152.0	18.6	19.4	213.0	19.8	21.7	239.0
80	17.3	12.30	135.3	18.4	15.3	168.0	17.6	21.5	237.0	18.8	24.1	265.0
85							16.6	24.3	267.0	17.7	27.2	299.0
90							15.7	27.2	299.0	16.7	30.5	336.0
95							14.8	30.3	333.0	15.8	34.0	374.0
100							14.1	33.6	370.0	15.0	37.7	415.0

* All values of discharge Q_1 are calculated from unit horse power N_1, using an efficiency of 80 per cent.

† $D_1 = 53$ in. ‡ $D_1 = 56$ in.

$n_s = 45$, and $n = 225$ r.p.m. Whence $D_1 = 33$ in.; $b = 18.5$ in.; $D_2 = 1.48 \times 33 = 49$ in.

HIGH HEAD: Given $H = 420$ ft. and $N = 4000$ h.p. in one unit. From Curve I, Fig. 3, n_s (permissible) $= 27$; $N_1 = 4000/420\sqrt{420} = 0.465$; $n_1 = 27/\sqrt{0.465} = 38.6$; $n = 38.6\sqrt{420} = 810$ r.p.m. This would ordinarily be too high a speed for economical generator construction. On the basis that a 450-r.p.m. machine would be best, the hydraulic end would work out as follows: $n_1 = 450/\sqrt{420} = 21.9$; $n_s = 21.9\sqrt{0.465} = 15$; $D_1 = 1835 \times 0.62/21.9 = 52$ in.; $D_2 = 0.85 \times 52 = 43$ in.; $b = 0.09 \times 52 = 4.7$ in.

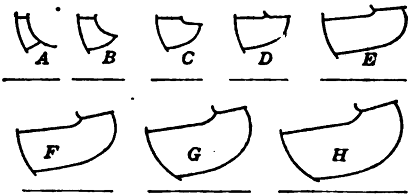


FIG. 4a.—Typical Profiles of Runner Blades.

In general practice it is advisable to design the runner for each given set of conditions. This procedure is adhered to for all large or important installations by the leading designers. It is possible, however, to design a line of wheels which will cover more or less closely all ordinary requirements. A typical line of this kind is shown in Table 2, the field of the various types and the heads to which they are adapted being shown diagrammatically in Fig. 4. The approximate outline or profile of the runner blade of each type is shown in Fig. 4a, the proportions being in accordance with the ratios shown in Fig. 3.

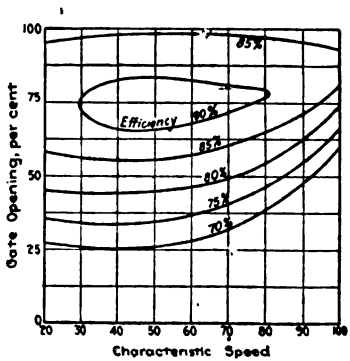


FIG. 5.—Relations Between Characteristic Speed, Efficiency and Gateage of Reaction Turbines.

Fig. 5 shows diagrammatically the general relations between characteristic speed, efficiency and gateage of reaction turbines, it being assumed that the proportions and main dimensions of the turbine are determined from Fig. 3. The vertical scale as shown is based on the percentage of gate opening, but for approximate purposes may be considered as the percentage of discharge, which is roughly proportional to gate opening. Above 50 per cent. the vertical scale may also be taken as percentage of h.p. output for approximate work.

These values of efficiency are relative rather than absolute and represent the efficiency of the guide case, runner and draft tube, rather than that of the complete unit as installed, the efficiency of which may depend as much on the setting as on the turbine design. Besides the proper selection of main dimensions and the proper design and construction of buckets, the other points of considerable influence on the efficiency may be cited as size, draft tubes and settings.

Influence of Size on Efficiency. Turbines which have a large output either by reason of their size or the head under which they operate, are more favorable to high efficiency than those of smaller capacity. Fig. 5 may be said to hold roughly down to $D_1 = 30$ in., $H = 10$ ft. With the larger sizes or higher heads the frictional losses become relatively smaller, consequently the efficiency—particularly at the smaller gate openings—becomes relatively higher than shown.

Draft Tubes. The draft tube is an essential part of the Francis turbine, as it serves the double purpose of permitting the placing of the machine high above tail water, thereby making it more accessible, and of reducing the loss incident to the velocity of discharge to a negligible amount. If a Francis turbine is so placed above the tail water, part of the total head acts on the wheel by suction.

The velocity at the discharge of the draft tube may be determined as follows: For low heads up to 20 ft., the velocity at the discharge end of the draft tube should correspond to a loss of not more than 1 per cent. of the head. For medium heads from 20 ft. up to 60 ft., the velocity at the discharge end of the draft tube should be limited to 0.08 to $0.1\sqrt{2gH}$ (or less for the upper limit of heads), on account of the resulting high absolute velocity. For heads from 60 ft. up to 600 ft., the velocity should be still lower, on account of the great disturbance in the tail race. For instance, for a head of 450 ft., $0.1\sqrt{2gH} = 17$ ft. per sec. Good practice would limit the discharge velocity in this case to about 7 ft. per sec.

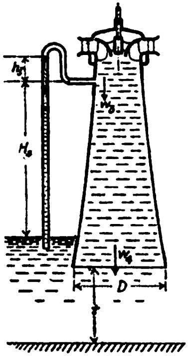


FIG. 6.—Conical Draft Tube.

Shape of Draft Tube. In a conical draft tube (Fig. 6) part of the loss due to the discharge velocity from the runner can be utilized; theoretically, to the amount $h_s = (w_3^2 - w_4^2)/2g$ in ft. of head, where w_3 and w_4 = velocity in ft. at entrance to and exit from the draft tube; or, more correctly, $h_s = \frac{w_3^2 - w_4^2}{2g} - \frac{(w_3 + w_4)^2}{2g} \sin a - h_f$, wherein the second member expresses the loss due to the widening of the passage, with the angle of flare, a, and h_f the friction loss due to the flow through the draft tube. Not more than from one-half to two-thirds of the theoretical gain can be expected in actual operation.

Assuming a straight conical draft tube, the space t between the draft-tube discharge and the bottom of the flume should not be less than $0.6D$ if the water can flow away on all sides, and not less than D if it can flow off on one side only. The angle of flare a should be small to avoid the danger of the flow leaving the walls. For straight conical draft tubes, the side angle should not exceed 7 deg. In curved concrete draft tubes a larger flare than 7 deg. can be used below tail-water level. The **correct shape** (see Fig. 7) can be designed, according to Prof. Prášil (Zürich), by the rule $\pi D^2 Z/4$ = constant. In many cases a shape like this can be approximated in steel-plate construction. Where sufficient depth is not available, it is of advantage to bend the draft tube around to discharge in the direction of the flow in the tail race; this is best accomplished in concrete draft tubes. The shape of the draft tube should in any

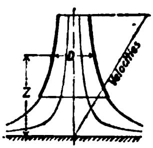


FIG. 7.

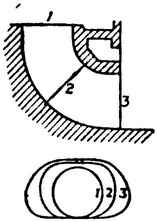


FIG. 8.

case be such as to join correctly to the discharge ring of the runner, and the cross-section be approximately circular in form down to the bend. An oval discharge opening (Fig. 8), possibly unsymmetrical, with a flat bottom and arched top, is the best design because it combines the circular cross-section with a shape which allows the water to spread in going through the bend and permits of minimum excavation. The increase of sectional area should take place as much as possible before the bend, that is, in the straight part of the draft tube, because a gain in head in the bend and beyond that point is doubtful.

With the horizontal twin-runner arrangement of turbines a center discharge casing is required to conduct the water to the draft tube proper. This arrangement necessitates a 90 deg. bend in the flow practically at the point of highest discharge velocity. At best it introduces considerable losses, particularly in connection with high-speed runners (characteristic speeds above 65). The length of the casing between runners should not be less than 1.6 times the discharge diameter of the runner. The discharge opening at the bottom of the casing (top of draft tube) is preferably circular in form and should have an area of approximately 1.25 times the sum of the areas of the runner discharges.

Limits for Draft Height. The theoretical maximum is 33.8 ft. for a barometric pressure of 30 in. This will naturally be reduced in the case of a conical draft tube by the amount $(w_3^2 - w_4^2)/2g$; for example, if $w_3 = 13$ ft. and $w_4 = 4$ ft., this is equal to 2.38 ft., leaving the net theoretical maximum $H_s = 31.42$ ft. (see Fig. 6). In actual practice a draft-tube height of 20 ft. is to be considered as a safe limit, and 22 ft. to 24 ft. possible but risky, especially for part-load operation or when the action of a governor is apt to impose sudden and severe velocity changes. In many cases the water contains a large quantity of air which will impair the action of the draft tube, and this must be considered in case the draft-tube head is more than 20 ft. For the same reason also the flare of the draft tube should be small in cases where large amounts of air can be anticipated. The draft head is to be measured from the tailwater level to the highest point of the turbine on the discharge side of the runner where the vacuum will occur.

For small sizes the **material** used in the construction of draft tubes is steel plate or cast iron; for large sizes, generally concrete. The surface of the concrete should be smooth.

Setting. The efficiency of a turbine is influenced by its setting. The waterways should be so formed as to properly guide the water in its flow to and from the turbine. Smooth and unobstructed passages are desirable and low velocities should be adopted as much as possible. Velocities through trash racks are commonly kept below 2½ ft. per sec. for all heads. In low head (up to 30 ft.) open-flume plants the maximum velocities in the turbine chamber seldom reach 3 ft. per sec., and from the draft tube about 4 ft. maximum. The velocity in the turbine chamber or penstock should preferably be under $0.08\sqrt{2gH}$ for all heads, an absolute velocity limit of 10 ft. per sec. being common for large high-head plants. The exit velocity from the draft tube should be kept under $0.1\sqrt{2gH}$, with an additional limit of 6 ft. per sec. for all loads. The spiral form of turbine chamber is possible with vertical-shaft single-runner units of the open-flume and concrete-cased types, and represents best practice for low- and medium-head (up to 100 ft.) plants.

Impulse Wheels (n_s not over 6 for one jet). Fig. 9 shows the approximate relations between characteristic speed, efficiency and gateage. These values are given with the same general reservations mentioned in connection with Fig. 5. General computations as to speeds are obtained as for reaction wheels, using a constant peripheral coefficient of about 0.47. Detailed methods of calculation of overall dimensions are given later.

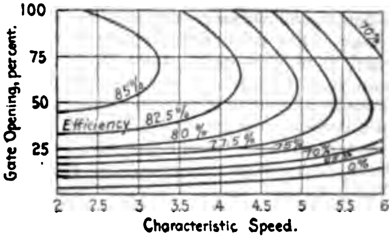


FIG. 9.—Relations Between Characteristic Speed, Efficiency and Gateage of Impulse Wheels.

COMPUTATION AND CONSTRUCTION OF TURBINES

Francis Turbines

The Runner. After determining the type of turbine and the characteristic speed of the runner, the main dimensions are determined by the use of the curves of Fig. 3 and the outlines and diagrams of Figs. 1 and 10. The blades should not be too long measured along the flow lines, as this produces too much friction. On the other hand, blades that are too short cause excessive eddy currents due to the abrupt curvature.

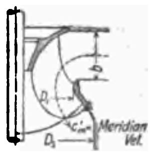


FIG. 10.

In determining the size of the blades in connection with the number to be used and the power to be developed it is often convenient to figure with the working pressure per sq. ft. of blade area, which should not be more than $h = 0.15$ to $0.25H$. From this the product of blade area, projected on the meridian plane (blade profile), and the number of blades (z) $= Az = Q_1 e_h H / h u_s$, where u_s is the peripheral speed of the center of gravity of the profile of the blade for $H = 1$ ft.

Prof. Zowski gives the following rules for determining the number of vanes: "High-capacity runners which have a high guide case should have a larger number of vanes than low-capacity or high-head runners. In all cases an uneven number (z_1) of vanes should be used, determined approximately by $z_1 = C\sqrt{D_1}$ (in.), where C may be taken as about 3.7 for low characteristic speed runners, 3 for medium characteristic speeds and 2.2 for high characteristic speeds." In order to prevent clogging, the minimum width between blades should never be less than 1¼ in. to 2 in. Abrupt turns in the runner profile should be avoided, particularly at the outer rim. The discharge loss $c_3^2/2g$ varies from $0.02H$ to $0.16H$, for characteristic speeds of from 10 to 100, respectively. Theoretically, all of this energy can be regained in the draft tube down to the small amount contained in the necessary velocity at the discharge end of the tube. Actually, it can be only partially regained by reason of frictional and eddy-current losses in even the best-designed tubes. A safe assumption is that one-half of the theoretical energy may be regained.

The materials used for runners are cast iron, bronze, plate steel cast into hubs of cast iron and bronze, and cast steel. The choice is governed by local conditions of head, water and characteristic speed, as it is a generally accepted fact that the chief cause of pitting is improper design, or conditions of loading for which the wheel was not intended, rather than the material of which the runner is made. Erosion should be distinguished from pitting, as the former is due to the abrasive action of foreign material in the water, rather than to design. The principal advantage of bronze is its strength and the ease with which it may be cast. Cast steel also gives strength and toughness, but the complicated shapes of buckets, excepting in high-head Francis-type runners, make it difficult to obtain good castings with this metal. Where the conditions do not require great strength or stiffness, as in runners of high characteristic speed, both cast iron and composite runners are satisfactory and much cheaper. [From Report of Committee on Prime Movers, N. E. L. A., Chicago, June, 1913.] Except where the water carries a heavy content of acid, alkali or air, the material used for any particular condition is usually fixed by requirements of strength, rigidity or ease in manufacture.

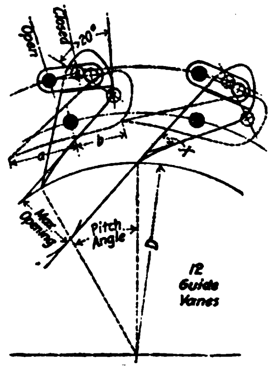


FIG. 11.—Guide Vanes for Francis Turbines.

The Guide Case is commonly designed for the maximum discharge rather than for the normal. The **number of guide vanes** is selected arbitrarily and should preferably be a multiple of four, from manufacturing reasons. The number used varies from about 12 for small runners to 28 for large wheels. For equal diameters, high-speed wheels have fewer guide vanes than low-speed wheels. The gate opening varies from 2 in. for small runners to 10 in. for large runners. The **form of guide vanes** generally used is shown in Fig. 11. The inner end of the vane is wedge-shaped for the purpose of guiding the water between parallel or converging walls at all gate openings. The angle X should never exceed the pitch angle, and is usually somewhat less. For a small number of vanes the angle X becomes large, and the vane is extended at the tip so as to reduce the disturbances between the water flowing in adjacent openings. The form shown in Fig. 11 is adapted to the open-flume type of setting where the water approaches the guide case in practically a radial direction.

In laying out the guide case, the **pivot** should, when possible, be located slightly nearer the large end so as to divide the vane into parts (a, b) proportioned approximately in the ratio 6 to 4 for the average form of vane. This permits the gates to be balanced when about one-third or one-fourth open, and makes the maximum tendency to open approximately equal to the maximum tendency to close. It is undesirable to locate the pivot close to the point of the vane in order to secure a tendency to close at all positions, as so doing places the pivot in a smaller vane width and either necessitates a smaller and weaker pivot or a reduction of vane strength. At the same time, the unbalanced moment of the vane at the wide-open position is greatly increased, being roughly doubled if the gates are laid out so as to be balanced when closed.

The guide vanes are controlled by the regulating cylinder of the governor through a shifting ring to which all the vanes in a guide case are connected by means of links. Two general styles of connections are used and are designated as the "inside" and "outside" types of **gate gear.** The former is operated inside the wheel pit and is submerged. The vanes are usually made of cast iron, but in some cases this material is not strong enough and steel is used. All working surfaces exposed to the action of the water should be bronze-bushed or lined. The outside type of gear works outside the turbine chamber and permits of oil or grease lubrication. The guide vane is practically always cast solid of steel and is provided with a stem which projects through a stuffing box to a lever-and-link connection to the shifting ring. The inside type of mechanism is commonly used on all open-flume turbines, while the outside type of gear is used with single-runner concrete-spiral-flume turbines and all high-head units.

Pressures Due to Thrust. Exclusive of the weight of the runner (which has to be considered when the setting is vertical) there is a certain resultant hydraulic thrust to be taken care of in computing the load to be carried by the thrust bearing. The load imposed by the runner may be divided into three parts; the weight of the runner, P_1; the static pressures upon the exterior areas of the runner, P_2; and the dynamic pressure or reaction resulting from acceleration which the water attains within the runner in an axial direction, P_3. For characteristic speeds above 30 the weight P_1 (lb.) may be determined with a sufficient degree of accuracy from $P_1 = CD^2b$, where C varies from 0.1 to 0.12 for plate-steel-vane-runners and from 0.9 to 0.11 for solid cast runners, the runner diameter D and width b, being in inches. This weight P_1 should be reduced by the weight of the water displaced by the runner, which introduces a reduction of about 13 per cent., but usually this reduction is neglected.

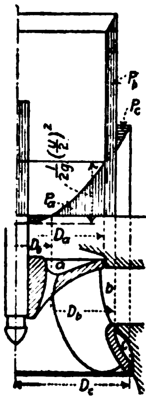


FIG. 12.

In calculating P_2, assume the runner to be adjacent to the tail-water level and consider the static pressures on the external surfaces of the runner as equal to or a function of the working head. This is practically correct regardless of the position of the runner with respect to the water levels. The water rotates in the spaces a and c (Fig. 12), and consequently the pressure increases with the radius. Assuming that the water rotates with one-half the velocity of the runner, or $u/2$, the pressure difference in ft. between any two points x and y will be $(1/2g)[(u_x/2)^2 - (u_y/2)^2]$. In addition to the pressure difference between the two points, the actual pressure at one of them must be determined. For very accurate work the pressure at the diameters D_a and D_c (Fig. 12) may be determined from the known areas of the clearance and drain openings. A relationship between total pressure available, velocities and coefficients of contraction may be drawn, keeping in mind the fact that the same quantity passes through both openings. The drain openings should ordinarily have a net area about five times that of the clearance space, in which event the pressure above them may be assumed to be 0. Similarly, the pressure is 0 at the diameter D_c on account of the large lower clearance.

Considering that the volume of a paraboloid of revolution equals that of a cylinder of equal base and half its height, $P_a = (62.4/144)A_a(1/2g) \times [\frac{1}{2}(u_a/2)^2 - (u_c/2)^2]$, where A_a is the annular area between the diameters D_a and D_c. If large drain openings connected to the draft tube are provided on the cover at the diameter D_a and the inner clearance is closed off, zero pressure may be assumed at the outer periphery D_a and the total due to pressure in the chambers will be $P_a = -A_a(0.434/4g)(u_a/2)^2$. If no drain holes are provided, the maximum obtainable pressure h_{max} at D_a reaches the value of the pressure in the guide case adjacent to the clearance, or $h_{max} = H - (c_0^2/2g)$, and the total load due to this pressure will be $R = 0.434A_a[h_{max} - (1/4g)(u_a/2)^2]$. Assuming that the guide-case pressure is constant from D_a to D_b, the load on the ring area between D_a and $D_b = P_b = 0.434A_b h_{max}$. In Fig. 12, P_b is shown as increasing with the radius. Actually this is more correct, though only a slight error is introduced in determining P_b as above.

Considering the pressure as zero at the lower clearance, the pressure in the chamber c is negative and the load will be

$$P_c = \frac{0.434}{2g}\left\{A_b\left[\left(\frac{u_c}{2}\right)^2 - ½\left(\frac{u_b}{2}\right)^2\right] - \frac{A_c}{2}\left(\frac{u_c}{2}\right)^2\right\},$$

where A_c is the annular area between D_b and D_c. For runners of higher characteristic speeds (over 40) P_c is negative and tends to reduce the load on the thrust bearing (when the lower clearance is large). The same result is obtained for runners with characteristic speeds below 40 by making the lower clearance small so that positive pressure is built up underneath the discharge ring which for these runners slopes inward. In extreme cases this effect may be increased by supplying the chamber c with pressure directly from the penstocks.

The **total load resulting from static pressure** is $P_2 = P_a + P_b + P_c$. To this should be added the pull exerted by the suction action on the end of the shaft. This is shown in Fig. 12, but it is practically negligible. The reaction force P_3 produced by the axial acceleration is $P_3 = (62.4Q/g)(c_{z0} - c_{z2})$, where c_{z0} and c_{z2} are the axial components of the water velocity at the runner inlet and outlet, respectively. For radial inlet c_{z0} will be zero and c_{z2} will be the velocity figured from the discharge area of the runner. P_3 has a balancing effect which increases with the gate opening. In determining the load on the thrust bearing P_3 is commonly neglected, as the unit should be capable of operating at low gate openings for an indefinite length of time.

An arrangement shown in Fig. 13 (Allis-Chalmers patent) is frequently used to balance automatically the end thrust of single-runner horizontal turbines. Annular spaces (a, c) are provided on opposite sides of the runner and so arranged that a slight axial movement of the runner causes a throttling and consequently a pressure increase in the flow through one side and a corresponding relief on the other. The result is that the runner is returned automatically to its original position. The clearances are kept small to prevent loss, so that the rotor has practically a fixed position and permanent oscillations are prevented. In connection with this device it is usual to provide at least a small thrust bearing for starting service, but it is given considerable clearance so as to allow the automatic balancing.

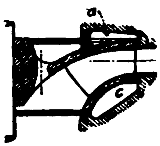


FIG. 13.

Thrust Bearings. Plain collar thrust bearings are commonly used for horizontal-shaft units, steel shoulders or collars on the shaft turning against the babbitt of the bearing. For vertical-shaft units thrust bearings of the collar, ball, roller or oil-pressure type are used. Close-grained cast iron has given very satisfactory results in connection with the former, though great care is required in fitting the disks to the shaft and base and securing perfect finish of the bearing surfaces. Pressures (p) as high as 850 lb. per sq. in. are allowable. For bearings not supplied with water cooling, $pv \leq 1000$; for cooled bearings, $pv \leq 3500$, where v = mean rotating velocity of rings, ft. per sec. Cooling is usually effected by bringing the lubricating oil in contact with water-cooling coils, either within the housing or outside of it. The coefficient of friction f varies from 0.01 to 0.005 (see p. 243). Ball bearings are frequently used for small sizes of bearings and roller bearings for the larger sizes, their small coefficients of friction (see p. 244) resulting in a saving of power and a very low starting torque.

The **Kingsbury thrust bearing**, now in general use, consists of a rigid collar, usually of cast iron, secured to the shaft and having a carefully finished bearing surface. This collar rests on several babbitted shoes or bearing blocks, which are so supported that they may tip at a very slight angle. The entire bearing is submerged in an oil bath and the coefficient of friction is stated to be from 0.001 to 0.003 for ordinary conditions.

For extreme cases an **oil-pressure type of bearing** is often used, though it usually necessitates a complicated and expensive auxiliary oil-pressure system. Sometimes it is possible to combine the regular pressure system of a plant with the thrust-bearing supply or utilize the penstock pressure for emergency purposes and reduce the cost and complications. This type of bearing has the lowest coefficient of friction of any here considered, approximately 0.0005. Roughly, it consists of two disks, one rotating and one stationary, between which oil is pumped under pressure. When sufficient pressure is maintained the disks separate slightly so that the runner practically floats on an oil film of greater or less thickness depending on the pressure used and the amount of oil pumped.

For a discussion on thrust and guide bearings, alignment, starting, shaft deflection, etc., see N. E. L. A. Reports, 1913 and 1914.

The Holyoke Testing Flume

The testing flume of the Holyoke Water Power Co. at Holyoke, Mass., was reconstructed to its present form about 1883 under the direction of Clemens Herschel. Between 2000 and 3000 tests have been made to date. Much experimental work has been carried on at this flume, and standard and special runners developed.

The tests furnish an absolute comparison of runners, tested under uniform conditions. Their application to actual installations requires good judgment in estimating the influence of difference between the test conditions and the actual power-house setting. A high-speed runner tested in Holyoke with a long flaring draft tube will not show equally good performance in a power-house setting with practically no draft tube.

The capacity of the Holyoke Testing Flume is desirably limited by the discharge conditions to 150 sec.-ft., although tests are frequently made with about 200 sec.-ft. The maximum head is 17 ft., but falls to as low as 9 ft. with a discharge of 300 sec.-ft. Test runners 30 in. in diameter are commonly used for the high-speed types, larger sizes up to 40 in. or 50 in. being permissible for medium- or low-speed forms. The higher the head and the larger the runner, the better will be the performance of a hydraulic turbine of a given type. Consequently, the majority of actual installations should show efficiencies exceeding these obtained at Holyoke.

Impulse Wheels

The Nozzle (Fig. 14). Jets of circular section are used exclusively in modern design, needle regulation being employed for controlling the diameter of the jet. The velocity in the upstream end of the nozzle pipe proper should be from 0.06 to $0.10\sqrt{2gH}$, and increase toward the nozzle tip. Smooth surfaces with carefully designed curves and passages are of great importance in reducing frictional and eddy-current losses resulting from the high velocities used. The velocity should be increased quickly at the nozzle tip and kept practically constant along the nozzle body. Also the diameter of the needle should be kept small further to reduce frictional losses. Directly at the nozzle tip f_0 the transformation of pressure into velocity is completed only at the outer surface of the jet, while in the inner portions there is still slightly more than atmospheric pressure due to the curvature of the flow, and a slightly lower velocity. On an average the velocity at the nozzle tip is 0.75 to $0.85\sqrt{2gH}$, depending on the shape of the nozzle, and the maximum velocity in the free jet is about $0.985\sqrt{2gH}$, corresponding to an efficiency of about 97 per cent. The angle of the cone of the nozzle tip at exit should be from 70 to 90 deg. With the velocities given above, the

diameter of the nozzle pipe in inches will be $d_1 = 20$ to $15\sqrt{Q_1}$, and the **jet diameter** in inches $d = 4.8\sqrt{Q_1}$. At the largest opening the needle ordinarily occupies 0.10 to 0.16 of the area of the nozzle opening. The largest diameter of the needle is fixed by the requirement that the point of inflection W (Fig. 14) of the profile of the needle should be within the nozzle when the needle is closed. This condition is necessary to prevent the jet from leaving the needle for the smaller openings, which would result in rapid corrosion. Buckets are usually made of bronze or cast steel for larger sizes of turbines and the higher heads, and frequently of cast iron for low heads (under 400 ft.). The nozzle body is of cast iron or cast steel, depending upon its diameter and the working pressure. The nozzle tip is ordinarily of the same material as the nozzle body and is frequently equipped with a renewable wearing ring of bronze. The needle is made of steel and should be provided with a removable tip to permit replacement in case of wear. All surfaces adjacent to water flowing at high velocity should be carefully finished and polished smooth to avoid eddies. Jet diameters up to about 8 in. are common, but for commercial reasons larger diameters are seldom used.

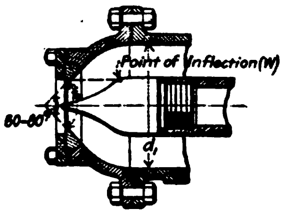


FIG. 14.—Impulse Wheel Nozzle.

Wheel Disk and Buckets. The horse power of an impulse wheel is fixed by the working head and size of the jet. The **wheel diameter** D may be chosen to suit any desired speed, except that D should never be less than about $9d$, the most favorable ratio being between 15 and 20. If a sufficiently high speed is not obtained using one nozzle, two jets may be used in connection with two wheels. Sometimes two or more jets are directed against one wheel, but this frequently results in a reduction of efficiency owing to interference of the jets. Theoretically, the most favorable **peripheral speed** of the buckets should be one-half the jet velocity, but in practice a lower value is taken, as the water must leave the wheel with some velocity and at an angle to avoid interference with the following bucket. On this account the peripheral velocity should be from 0.42 to $0.48\sqrt{2gH}$, the smaller coefficient applying to high characteristic speeds (6) and the higher value to lower speeds (up to 2).

In **laying out the buckets** care must be taken that no water is allowed to pass through the wheel without reacting fully on the buckets. To determine this the path of the water relative to the wheel should be drawn, as in Fig. 15. Relatively to the buckets every point of the jet describes an extended involute which may be constructed by rolling a straight line LL upon the circle K, assuming that the points of the jet are rigidly connected to the straight line LL. The circle K has a radius r_0c_0/u_0, so that its peripheral

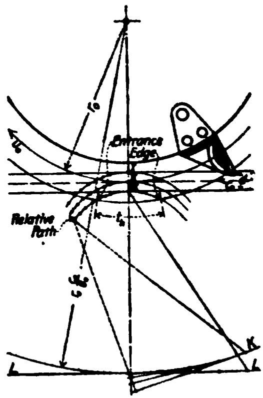


FIG. 15.—Layout of Impulse Wheel Buckets.

velocity is equal to c_0. As all of the involutes traced by the points on one flow line are congruent, it is most convenient to draw them but once as shown for the jet lines *I*, *II*, and *III*, and to turn the buckets into their relative positions. A section of the jet is cut off between adjacent buckets, the outline of this intercepted portion being shown in its relative form in Fig. 16. The ends of the various intercepted threads of the jet lie along a cycloid generated by a point which moves with the circle *K* (Fig. 15) as it rolls on the line *L*. The curve is actually the path of the entering edge of the bucket relative to the jet. Bucket No. 2 is shown in Fig. 16 in its position relative to the jet at the instant of entering the jet. The preceding bucket 1 is shown at the instant when the last thread *III* of the jet is intercepted. Position 1′ shows the bucket at the instant when the last end of jet line *I* reaches the bucket, and position 2′ when the last end of thread *III* reaches it. It should be clearly understood that the bucket positions 1, 2, 1′ and 2′ are not the actual positions corresponding to the events just mentioned, but show only the relations between bucket and jet at those times. (See also article by S. J. Zowski, *Eng. News*, Feb. 10, 1910.)

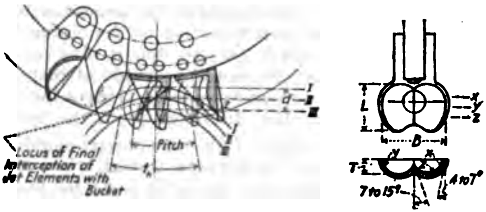


Fig. 16.—Layout of Impulse Wheel Buckets.

Where there is doubt as to whether complete reaction will be obtained, the length of the intercepted portion of the jet may be decreased by increasing the number of buckets. The limit in this direction is fixed by the difficulty in properly fastening the buckets to the disk. The alternative remedy consists in lengthening the arc t_k (necessitating increasing the wheel diameter, which in turn reduces the r.p.m.) or in increasing the radial height of the bucket. The latter is undesirable as it results in excessive friction and increases the difficulties in securing a good mechanical connection between the buckets and disk. Experiments have shown that a large number of buckets is desirable.

The connections between the buckets and disk must receive both the intermittent impact of the jet and the centrifugal force of the bucket itself. The latter increases to about 3.25 times its normal value in case of runaway, and if the wheel is blocked the strain due to impact is about 1.8 times the normal. As far as possible the splitter should be at right angles to the relative path of the jet. The main proportions of the bucket are shown in Fig. 16. The width *B* in inches should be about 15 to 19 times $\sqrt{Q_1}$, which corresponds to about 3 to 4 times the jet diameter. The length should be $L = 11$ to $14 \times \sqrt{Q_1}$, and the depth $T = 4.5$ to $5\sqrt{Q_1}$. The angle of the splitter should be between 14 and 30 deg. The exit angle is ordinarily made between 4 and 7 deg., and should be kept as small as possible, as so doing increases the efficiency even though the discharging water interferes slightly with the adjacent bucket at full gate opening. In securing the extended outer lips of the discharge edge of the bucket, the edge receiving the jet takes the form of a recess. This recess is shown in Fig. 16 and has a width only slightly greater than the jet diameter, as otherwise flow line *III* would not fully react toward the end of its flow in the bucket. The back surface of the bucket adjacent to this recess is finished at such an angle that the jet does not strike the back of the bucket, or at most at a very small angle.

The Casing (Fig. 2) consists of a lower housing into which the nozzle extends. This housing is usually of cast iron and either rests on a base flange or is secured directly to the foundations. For very small units it may support the bearings, though usually the wheel disk is attached on an extension of the generator shaft and carried by the generator bearing when used for hydro-electric units. The upper portion of the casing, called the cover, is usually separate from the lower part so that it may be removed for inspecting the wheel or nozzle tip. It should be made of plate steel so that it will not be destroyed if a bucket comes loose and leaves the disk. The width (in.) of the casing adjacent to the discharge from the buckets should be at least 30 to $40\sqrt{Q_1}$. The width of the upper part should be of sufficient radius and width to safely clear the buckets, though when its dimensions are reduced to a minimum it is advisable to put in deflector plates to prevent water which may splash on the disk and buckets from being carried around by the wheel. At high heads the surfaces exposed to the discharging jet should be carefully protected by plates of steel or cast iron, as the jet passes through the wheel freely and with almost no reduction of velocity when the wheel runs away. Efforts have been made to utilize a draft head by making the casing and housing tight and controlling the amount of air in the casing by means of floats. The complication involved can hardly be compensated for, as the maximum additional head gained is so small a percentage of the heads to which impulse wheels are usually adapted.

Regulation. Impulse wheels are regulated either by varying the size of the jet (needle regulation) or by varying the amount of the jet that impinges on the buckets (using a deflecting nozzle). The former is employed when economy in the use of water is of greatest importance. The latter finds its greatest application when a constant amount of water is to be discharged (as for irrigation purposes) or where the length of the penstock is so excessive as to preclude the possibility of quickly changing the velocity of flow by the governor. Special arrangements of either type may be adapted to this condition of extreme pressure surges.

Efficiency. A rough indication of the variation of efficiency of impulse wheels is given in Fig. 9, which is based on wheels designed in accordance with the proportions given above. As contrasted to reaction wheels the efficiency remains high over a wide range of characteristic speed and for a large variation in percentage output. Power-plant efficiencies between 85 and 90 per cent. at the maximum point are reached with careful design.

REGULATION OF HYDRAULIC TURBINES

General. Direct regulation of hydraulic turbines is prevented by reason of the considerable amount of power required to operate the gates. It is therefore necessary to make use of indirect regulation, a flyball governor (see p. 777) being employed to control the distribution of power developed in the servomotor or **regulating cylinder.** The parts which transmit the gate motion back to the flyballs in order to bring about "dead beat" regulation are usually designated as the **relay.** Means for adjusting this relay are necessary for starting, stopping or changing the speed of the unit. The earlier forms of governor embodied a design wherein the power was developed mechanically through gears from the turbine itself, but at present the hydraulic- or oil-pressure form is used almost exclusively. In this type oil or other working fluid under pressure is distributed to the regulating cylinder through a control or regulating valve actuated by the flyball governor.

The Regulating Cylinder. The design of the regulating cylinder is dependent upon the stroke used to give full travel to the gates, the maximum resistance occurring along this stroke, and the operating pressure used in the cylinder. The product of the maximum resistance (in lb.) and the stroke (in ft.) may be considered as the maximum energy required for regulation, though it is larger than the actual energy required for moving the gates from one extreme position to the other. The maximum energy required for regulating the gates of Francis turbines with guide vanes of normal design may be determined approximately from $E_r = KBDH$ or $E_r = kN_{max}/\sqrt{H}$, where E_r = energy required in the regulating cylinder, ft.-lb., D = runner diam., B = width of guide case—both in ft., H = maximum head (ft.) under which the turbine is to be regulated, and N_{max} = b.h.p. of turbine at full gate under head H. For large turbines with horizontal shafts, $K = 12.5$ and $k = 30$; for medium-sized and small turbines with vertical shafts, $K = 16.5$ and $k = 40$; for small turbines and those with slot-and-sliding-block connections, $K = 20.5$ and $k = 56$.

Determination of Speed Regulation and Flywheel Effect. In general practice a certain percentage of speed variation (usually 3 per cent.) is allowed between normal operation at full load and at no load. Within this range of speed, which corresponds to a certain definite travel of the governor collar, any one position of the collar corresponds to a certain gate opening of the turbine and consequently to a fixed number of revolutions. Let n and n_2 be the r.p.m. at full load (considered normal) and no load, respectively. Then the ratio $(n_2 - n)/\frac{1}{2}(n_2 + n) = J_r$ is termed the "**degree of speed variation of the regulation,**" and is commonly expressed as so many per cent. no load to full load. Let n_0 be the r.p.m. of the governor corresponding to the highest position of the collar travel and n_3 that at its lowest position. Then the ratio $(n_0 - n_3)/\frac{1}{2}(n_0 + n_3) = J_g$ and is termed the "**degree of speed variation of the governor**" and is commonly expressed (for example) as "a 6 per cent. flyball." J_g should always be equal to or greater than J_r.

The ratio of the change of load from N_1 to N_2 to the maximum load N_{max} is $(N_1 - N_2)/N_{max} = L$ and is termed the "**relative change of load.**" By "**speed change**" is meant the temporary percentage change in speed resulting from change of load. If n_1 is the initial speed and n_{max} is the extreme speed (n_{min} for load thrown on) the speed change is expressed by

$$(n_{max} - n_1)/\tfrac{1}{2}(n + n_2) = S \qquad (1)$$

"**Flywheel effect**" is the polar moment of inertia of the rotating masses, wr^2, in which w is the total weight of the rotating masses (lb.) and r the radius of gyration (ft.). (See also p. 218.) The maximum turning moment (ft.-lb.), corresponding to the maximum capacity of the unit at normal speed $= N \times 550 \times 60/2\pi n = 5250\ N/n$. For any load change from N_1 to N_2 the average excess or deficiency of turning moment will be proportional to $(N_1 - N_2)/2$ and the total kinetic energy (ft.-lb.) stored in or absorbed from the rotating masses will be $E = 550\ T(N_1 - N_2)/2$, where T is the regulating time in seconds. (See Fig. 17; area $AOC = \frac{1}{2}NT$.)

The corresponding energy absorbed or given out by the rotating masses is expressed by $[2wr^2\pi^2/(g \times 60^2)](n^2_{max} - n_1^2)$. Equating these two quantities and designating the speed change by S, approximately

$$S = 800{,}000\,(N_1 - N_2)\,T/wr^2n^2 \qquad (2)$$

It is evident that the speed changes are inversely proportional to the constant

$$K = wr^2n^2/N$$

where N is the maximum turbine rating. For satisfactory regulation K should ordinarily be 5,000,000 or greater, the higher the value the better being the regulation that is possible. (For derivation of formula for S, see Mead "Water Power Engineering," p. 462.) The constant K may be expressed in terms of the "starting time," T_a, in sec., which would be required for the turning moment M to start the rotating masses from rest and bring them up to normal speed n_1. $T_a = wr^2n_1^2/417N_{max}$.

The regulating time T is the time which elapses between the start of the load change and the time at which the power produced is equal to the load. (For full-load condition see Fig. 17.) It is the time required for the regulating cylinder to make its initial stroke and must not be confused with the total duration of the regulating period, which is usually considerably greater than T, as it extends up to the time when conditions have become steady. The regulating time T is composed of three parts (see Fig. 17): $T = T_1 + T_2 + T_3$. T_1 is the time which elapses until the speed is changed sufficiently to overcome the frictional resistance and lost motion in the flyballs and connections to the control valve and to open the ports of the valve enough to cause the start of the motion. As the gates are not moved during this period the speed change that takes place during the time T_1 is termed the total "unsensitiveness" S_i, and is practically uniform for all load changes. S_i = 0.5 to 1.0 per cent. T_2 is the time which elapses between the beginning of the motion of the regulating cylinder and the instant when the control valve has attained its maximum opening. During this time the gates are moved with increasing velocity. T_3 is the time from the termination of T_2 up to the instant when the output of the turbine first equals the load.

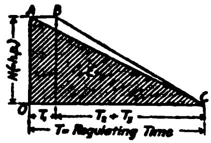


FIG. 17.

In actual practice T is nearly constant for all load changes, this resulting from the fact that the control valve does not open its full amount for the smaller loads. For very small changes (under one-fourth load) T often shows considerable reduction, but in the general determination of speed changes this reduction is neglected. The relative values of T_2 and T_3 depend upon the details of the design and on the amount of load change. On the basis of the foregoing and considering that T_1 is practically a fixed quantity, it follows that: (1) If T_1 is always large in proportion to T_3, it must be practically constant for all load changes and the speed variations occurring during the time T_2 are roughly proportional to the load changes. (2) If T_3 is large in proportion to T_2 for all load changes, the resulting regulation approaches that secured with mechanical governors where energy is supplied at a constant rate. T_3 is then proportional to the load change and the speed variations are roughly proportional to the square of the load change.

The relative values of T_2 and T_3 in proportion to the total time T will determine the relationship between the speed variations due to partial load changes and those due to a full-load change. With modern governors this relationship may be approximated from the equation S(effective) $= S_i + aS$, where a has the following values:

Load change L, per cent..........	100	75	50	25	10
a........................	1	0.6 to 0.7	0.3 to 0.4	0.15 to 0.2	0.07 to 0.1

S is ordinarily determined directly from (2), but may be closely approximated from T_2 and T_3, assuming that T_2 is small compared with T_3, that for T_3 the closing time T_s may be inserted, and that the turning moment varies directly with the time, as follows:

$$T_aS = C(T_s + T_2) \tag{3}$$

T_s is the "closing time," or the time required by the regulating cylinder for closing or opening the turbine gates, with maximum opening of the regulating cylinder valve. The usual value of T_2 is betweeen 0.1 and 0.2 sec. For short closing time (T_s = 1 to 2 sec.) $C = 40$; for long closing time (T_s = 5 sec. or more) $C = 45$.

So long as L does not exceed J_rT_a/T_s, the regulation is "dead beat" (Fig. 18*a*). If $L > J_rT_a/T_s$, the "dead beat" condition is reached only after a series of decreasing oscillations of the speed (Fig. 18*b*). The more nearly the regulation approaches the condition shown in Fig. 18*c*, the better it becomes for any given value of S (maximum). In order to obtain this condition, the governor motion is so influenced that its sensitiveness is temporarily decreased. This is accomplished either by arranging the connections in such a manner that the motion of the regulating cylinder has the effect of a brake on the governor or by means of a double or flexible relay. Both means are theoretically of equal value and in actual practice have given good results, but the latter is more commonly adopted as it is slightly more sensitive.

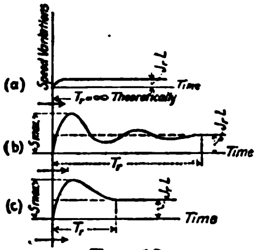


FIG. 18.

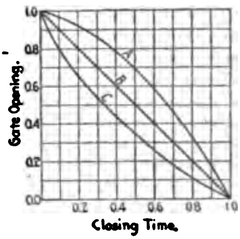


FIG. 19.

Influence of the Pipe Line. When the turbine is set in a closed flume or casing and supplied with water through a pipe line of length L (ft.) and of such dimensions that the velocity in the pipe is c_{max} for the maximum load N_{max}, the characteristics of the pipe line influencing the regulation are given by $T_l = Lc_{max}/gH$, which represents a period of time. The inertia of the mass of water causes a pressure change in the penstock close to the turbine whenever the velocity of the water is changed. This change of pressure depends on the rate of change of the gate opening. For a total closing of the gates, Fig. 19 shows three typical examples of the relationship between closing time and gate opening. Curve A indicates a slow initial motion which is accelerated toward the end of the closing time. C shows a rapid initial motion, while B indicates a fair average (used in Fig. 17).

On the basis of the average condition B and taking the pressure rise in per cent. of the head as dH,

$$dH/H = (1.3 \text{ to } 1.5)T_l/T_s \tag{4}$$

The elasticity of the water and material of the pipe line should be taken into account if $Ac_{max}/gH < 1$, where A is the average velocity of a pressure wave in the pipe. For steel-plate penstocks $A = 4700/\sqrt{1 + 0.008(d/s)}$, where d is the inside diameter of the pipe (in.) and s the thickness of the walls (in.). This expression also takes into account the effect of rivets, flanges and supports, and is based on data taken from actual installations.

For a total unloading of the turbine the effect of the penstock on speed regulation may be approximated very closely by extending the regulating time T the amount $1.35T_l$. From this, in place of (3), $T_aS = C(T_e + T_2 + 1.35T_l)$. For partial loads the speed changes will lie slightly above the values originally given in the table preceding (3), but the proportional increase will be less than that for full load.

For Francis turbines in closed casings it becomes necessary in extreme cases to take into consideration the effect of the moving water in the casing and guide case, and with open-flume turbines the draft tube sometimes has considerable effect. For casing inlet velocities of 0.25 to $0.3\sqrt{2gH}$ the pipe-line length and that of the casing may be reduced to equivalent lengths for a single velocity. The length of path in a spiral-cased turbine will be approximately $6(D + 3.3)$ where D is the runner diameter, ft.

No permanent or accumulative pressure surges will be set up by the regulation provided that $J_rT_a > 1.5T_l$, which represents the condition of stability. Pressure variations of more or less severity are permissible in connection with pipe lines, the following values of dH/H representing extreme upper limits: For heads up to 150 ft., 40 per cent.; up to 400 ft., 20 per cent.; up to 600 ft., 15 per cent.

In some instances, particularly in connection with high head installations (200–600 ft.) with long penstocks (above $3H$ in length), the closing time T_e obtained from (4) gives values of T_a larger than necessary for certain speed variations which may be desirable. Supplying additional wr^2 in the form of a flywheel gives a direct solution, and pressure regulators or by-passes are frequently used. The method adopted to take care of the surges depends upon the comparative cost of the regulators and of additional flywheel effect. If the water-saving by-pass arrangement be used care must be taken to make the time for opening the gates sufficiently long to prevent injurious pressure decreases which would be succeeded by corresponding surges on account of the elasticity of the penstock. Lengthening the opening time is not ordinarily detrimental to good speed regulation, as load increases usually occur gradually or at least very slowly in comparison to the extreme load decreases.

Often an **equalizing reservoir or standpipe** works out economically and may obviate entirely all necessity of regulators or flywheel. The maximum rise or **surge** of water level h_{max} when the total flow is suddenly shut off can be computed by equating the change in kinetic energy (ft.-lb.) in the penstock to the work done (ft.-lb.) in raising water in the standpipe or reservoir. In this determination that portion of the penstock upstream from the reservoir is all that need be considered. Let L_p = length of penstock (ft.); A_p = area of penstock (sq. ft.); v_1 and v_2 = initial and final velocities in penstock (ft. per sec.); A_s = area of standpipe or reservoir (sq. ft.); h_0 = theoretical surge neglecting friction; h_w = friction loss in ft. in penstock length L_p due to velocity v_1. Then $h_0 = \sqrt{(A_pL_p/A_sg)(v_1^2 - v_2^2)}$. For full load off, *i.e.*, $v_2 = 0$, this becomes $h_0 = v_1\sqrt{A_pL_p/A_sg}$. $h_{max} = h_0 - 0.6h_w$, and is measured above the final static or running water level. It is assumed that h_0 is greater than h_w.

The time required for the water level to reach its maximum is very nearly one-quarter of the oscillating period T of the system, which is determined from $T = 2\pi\sqrt{A_sL_p/A_pg}$. On account of the automatic action of the governor, accumulative oscillations or surges may occur if h_{max} is too large by reason of choosing too small a value of A_s. This can be obviated by making

$h_0 < \sqrt{2Hh_w}$, where H is the net working head on the turbines under steady conditions and with the velocity v_1 in the penstock.

The foregoing approximations are based on conditions of surge resulting from closing of the turbine gates. This corresponds to sudden decrease of load and is the most extreme condition ordinarily experienced in power plants. Conditions during increasing load are not usually of like intensity, as loads are picked up more or less gradually and an abrupt drop in pressure or level is avoided. For approximate purposes it is ordinarily assumed that the maximum drop below running level for any given increment of velocity does not differ greatly from the corresponding surge figured as above for a like decrease of velocity, though in actual practice when there is a large friction head to be considered it will usually exceed it. In special instances where it is desirable to determine the extent of water-level variation very closely, the method described by R. D. Johnson (*Proc. A. S. M. E.*, June, 1908), by D. W. Mead ("Water-power Engineering"), or by Prášil (*Schweiz. Bauzeitung*, 1908), may be used.

Pressure regulation by means of a standpipe or reservoir is frequently uneconomical on account of the local limitations, and pressure regulators in the form of automatic by-passes or compressed air tanks are made use of (see Johnson, *op. cit.*). By-passes or relief valves may be either governor-operated or pressure-operated, or a combination of the two. They regulate the pressure variations by causing the desired velocity change to take place gradually, the time of this change being fixed by the by-pass adjustment, which is independent of the governor time. The usual form of by-pass is so connected directly to the turbine gates that upon closure of the latter the regulator opens sufficiently to by-pass the amount of water rejected by the turbine. If it is being used as a synchronous by-pass (water-wasting) it continues discharging this water until there is an opening motion of the gates calling for additional flow. Ordinarily, however, the adjustment is such that the by-pass will close slowly so as to permit of greater water economy. The time of this closing is regulated by an adjustable dash pot and is fixed so that no injurious surge is produced. The synchronous by-pass gives ideal results so far as regulation alone is concerned, as it keeps the penstock velocity constant for both increase or decrease of load on the turbine, but is employed infrequently on account of its wastefulness in the use of water.

Working Fluid. Oil is ordinarily used as the working fluid in hydraulic governors. Water pressure is economical only for higher head installations and even then presents such operating disadvantages that it is not made use of frequently. Oil pressure is used to operate the turbine gates in practically all modern plants. The oil pumps may deliver into pressure tanks or accumulators or they may be of sufficient size to supply continuously the oil required for the regulation; the former method is followed in the majority of cases, the oil being pumped against air pressure. The usual operating pressures are between 100 and 300 lb. per sq. in., but preferably not over 200 lb.

For a plant consisting of several units a separate pump and tank may be used for each turbine or there may be a central pumping system for the station supplying oil to all the governors. The former is designated as a unit system. The pump should have a capacity per min. of three to five times the displacement of the regulating cylinder; the volume of the air space of the pressure tank should be approximately equal to the oil space (each being about five to nine times the volume of the regulating cylinder) for the unit system, and six to ten times the cylinder displacement of all units simultaneously operated for central systems. Central systems are usually in duplicate. The rotary type of oil pump is preferable except possibly in extremely large plants, as it is very simple and reliable, and the high speeds permissible allow direct connection to motor drive without special gearing. The closed arrangement of oil system wherein the oil is prevented from coming in contact with air by maintaining a vacuum in the discharge lines and receiving tanks, is not found to give satisfactory results. It greatly shortens the life of the oil and requires greater attention than does the open system. The latter is more commonly used in important installations on account of its simplicity and because it requires no special brand of oil.

Usual Governor Guarantees. Large direct-current or alternating-current generators usually furnish sufficient flywheel effect for obtaining commercial speed regulation. Values of the constant K (p. 1092) varying from 4,000,000 to 10,000,000 are found, the larger values occurring with larger sizes and higher speeds where best regulation is obtained. The closing time for open-flume units varies from 1 to 3 sec., 2 sec. being the usual rate. The difference between no-load speed and full-load speed is chosen between 2 per cent. and 6 per cent., but can be reduced to zero (constant speed) by special relay construction. For paralleling of generators some inherent drop is necessary. The higher grades of governors are so constructed as to make this difference adjustable to suit any desired operating condition. Conservative speed-regulation guarantees for open-flume settings are ordinarily made as in Table 3, these values to be corrected to allow for the influence of a pipe line.

Table 3. Approximate Speed Regulation of Open-flume Turbines

(Speed variation in percentage of normal speed)

Value of K	Regulating time of governor								
	1 sec.			1½ sec.			2 sec.		
	Percentage of load change								
	100	50	25	100	50	25	100	50	25
5,000,000	16	7.5	4.5	24.0	11.0	7.0	32	15	9.0
6,000,000	14	6.5	4.0	21.0	10.0	6.0	28	13	8.0
7,000,000	12	5.5	3.5	18.0	8.0	5.0	24	11	7.0
8,000,000	10	4.5	3.0	15.0	7.0	4.3	20	9	6.0
9,000,000	9	4.0	2.5	13.5	6.0	4.0	18	8	5.0
10,000,000	8	3.5	2.3	12.0	5.5	3.5	16	7	4.5

Overgating. A wheel is said to be "overgated" at the guide-vane opening where any increase in opening results in decrease in horse power. It is the point where the efficiency begins to drop faster than the discharge increases. The gate stops should be set so that this point cannot be reached when the unit is controlled by a governor, as it is impossible to secure regulation at or beyond this critical opening.

Hydroelectric Units. In proportioning hydroelectric units it has been general practice to choose a turbine of such size as to be able to drive the generator at 25 per cent. overload when operating at full gate. The average turbine shows best efficiency at about 0.8 load, a condition that permits of best hydraulic operation when the generator delivers its rated output and still permits of overload capacity. A more recent practice, and one which is rapidly gaining favor, consists of rating the turbine to give only a very slight percentage increase (for regulating purposes) on a generator rating known as "maximum continuous capacity." This practice reduces the probability of operating the unit continuously at some overload which, while being entirely within the range of output for which the turbine was designed, might be injurious to the generator. Furthermore, recent developments in turbine work have been in the direction of higher-speed types which show their highest efficiency usually above 90 per cent. load. It is better to place the generator and turbine ratings on a common basis and discard the separate "normal" generator rating and "full-gate" turbine rating. In alternating-current work it is necessary to consider the probable power factor in order to place the generator rating on a power basis (kw.) rather than on a current basis (kva.) which offers no indication on which to fix turbine capacity unless the power factor is known.

Outside of speed and capacity, two other features of generator design may depend on or be limited by the hydraulic end of the installation. They are the moment of inertia, upon which speed regulation depends (see p. 1091), and the rotor construction, which depends on the maximum speed attainable. The overspeed of a turbine never exceeds certain definite limits (Mead, *Proc. A. I. E. E.*, July, 1912, p. 1391), and it is customary to design the generator rotor to safely withstand this speed. Failure to do this has caused several very serious accidents in hydroelectric plants. The speed of reaction turbines at wide-open gate and no load varies from 68 to 80 per cent. above normal, while that of impulse wheels is theoretically between 90 and 100 per cent. These limits are based on normal head only, that is, the head at which the turbine gives best efficiency at normal speed. If the operating head should exceed this at times, the runaway speed limit should be raised in proportion to the square root of the head.

For example, assume 25 ft. head and a speed of 100 r.p.m. with a medium- or high-speed runner correctly designed for these conditions. The maximum overspeed would not exceed about 170 r.p.m. If, however, the head at times reached 36 ft., the maximum overspeed would not exceed about $170\sqrt{36}/\sqrt{25} = 204$ r.p.m.

This varying head feature is of importance principally in low-head plants where the ordinary head variation is a large percentage of the total. In higher-head plants this variation is not so noticeable, though the higher overspeed characteristic of low-speed reaction runners used for such heads requires a usual overspeed of about 75 to 80 per cent.

Costs of Hydroelectric Machinery. Hydraulic machinery must generally be adapted to each particular set of natural conditions. The variation of costs is so wide that a special study is required for each installation. The limits given in Table 4 represent in a rough way the usual limits of cost for the more common forms of machinery. These approximate costs are f.o.b. manufacturers' works per h.p. of maximum turbine capacity, and cover complete generating equipment from the penstock up to but not including the switchboard. Turbines, governors, generators, exciting equipment and any auxiliary gate valves or pressure regulators are included. Considerable variation from these values may be met with, though such variation will not exceed 25 to 30 per cent. on the average except where individual units, developed machinery or extremely large installations are the influencing factors. A single machine may be high in price by reason of requiring a relatively large outlay for patterns and drawings, or the equipment may be installed at an exceptionally low rate on account of some manufacturer having on hand suitable patterns. As a rule, however, the larger the installation the lower the cost per h.p. for a given head, though extremely large developments are not readily approximated on account of the special features of each case.

Table 4. Approximate Costs of Hydroelectric Machinery
(For complete generating equipment from the penstock to switchboard, but not including the latter)

H.p. of unit	Head, ft.	Cost per h.p., dollars	H.p. of unit	Head, ft.	Cost per h.p., dollars
Plants with single- or multiple-runner, horizontal or vertical shaft turbines for open flume setting			Plants with single-vertical or twin-horizontal turbines for concrete flume setting		
8,000	45	7.25	12,000	80-30	5.50-10.00
6,000	45-35	7.50- 9.00	8,000	70-25	6.00-11.00
4,000	45-25	8.00-11.50	4,000	60-20	8.00-14.00
2,000	45-25	9.50-15.00	2,000	50-20	12.00-18.00
1,000	45-15	12.00-23.00	....	....	....
500	45-15	14.00-27.00	....	....	....
Plants with single- or twin-runner turbines of the plate-steel cased type, horizontal or vertical			Plants with single- or twin-runner turbines of the cast-iron or cast-steel cased type, horizontal or vertical		
12,000	100	5.00	12,000	500-100	4.50- 8.50
8,000	100-65	5.50- 7.50	8,000	500-100	5.00- 9.00
4,000	100-65	7.00- 8.50	4,000	500-100	6.00-11.00
2,000	100-30	8.00-14.00	2,000	500-100	8.50-13.50
1,000	100-30	10.00-20.00	1,000	300-100	13.00-17.00
500	65-30	17.00-25.00	500	300-100	17.00-24.00
Plants with single- or twin-runner impulse wheels					
12,000	2,000-1,000	4.00-6.00	2,000	1,800-600	6.00- 9.00
8,000	2,000- 700	4.25-7.00	1,000	1,500-500	7.00-11.00
4,000	2,000- 700	5.00-8.00	500	1,200-400	9.00-10.00

HYDRAULIC POWER TRANSMISSION *

The use of water under high pressure as a medium for transmitting energy offers decided advantages (1) where it is required to operate slow-acting but powerful machinery whose movements must be regulated with great precision; and (2) where the motion is continuous in one direction and the demand for power is very intermittent, a large force being required at intervals for but a very short time. Water under high pressure is therefore well adapted to the operation of presses, flanging and riveting machinery, elevators, hoists, cranes, and testing machines. A number of cities in Great Britain have central stations for supplying high-pressure water to docks and factories through underground mains of cast iron or steel (when over 10 in. in diam.) and at pressures varying from 700 to 1600 lb. per sq. in. The mains have flanged joints which are grooved for the insertion of a triangular-section gutta-percha packing ring, and are bolted together. The water is received by **accumulators**, or vertical cylinders having heavily weighted plungers working through stuffing boxes at the tops, by means of which a quantity of water is stored and made available at the pressure which the weighted plungers will yield. In the **differential accumulator** the plunger is fixed and made with two diameters, the pressure acting upon the difference of area of the two diameters and the cylinder moving upon the plunger. The energy (ft.-lb.) stored in an accumulator = weight of plunger and load (lb.) × height of lift, ft. Capacity in h.-p. hours = $pAL/(33{,}000 \times 60)$, where p = pressure, lb. per sq. in., A = sectional area of plunger, sq. in. and L = length of plunger

* Staff contribution.

travel, ft. Mechanical efficiency, from 95 to 98 per cent. Where it is desired to use a pressure higher than that afforded by the accumulator, or where a moderately high pressure is wanted and only the ordinary city water pressure is available, an **intensifier** is employed. In this the low-pressure water is conducted into a large cylinder having a piston connected to a plunger of smaller diameter which works in another cylinder. Thus, if city water at 40 lb. pressure works in a 20-in. cylinder to drive a 5-in. plunger in the second cylinder, the pressure of the water in the latter, neglecting friction losses, will be $40 \times 20^2/5^2 = 640$ lb. per sq. in.

The essential apparatus of a **hydraulic crane** consists of a cylinder, a ram or plunger, and pulleys. For a direct lift the efficiency is 87 to 90 per cent. and even higher. Where pulleys are used to multiply the effective pull, the efficiency in per cent. is roughly equal to $84 - 2m$, where m = number of pulleys used. In direct-acting **hydraulic elevators** the principal resistance to be overcome is the friction of the cup leathers. According to Hick, total friction, lb. $= F = kpd$, where d = plunger diam., ft., p = water pressure, lb. per sq. ft., and $k = 0.00393$ for new or badly lubricated leathers $= 0.00262$ for leathers in good condition and well lubricated (see also p. 236). Since the total pressure on the plunger is $P = \pi d^2p/4$, the fraction of P required to overcome the friction of the leathers $= F/P = 0.005/d$ to $0.0033/d$. The velocity of the water in the mains should, in general, not exceed from 3 to 5 ft. per sec., nor the velocity through the valves of hydraulic machines exceed 100 ft. per sec.

The percentage **loss of head in hydraulic transmission** $= 63.5flH^2/p^3d^5$, where f = coefficient of friction of pipe, say 0.006; l = length of pipe, ft.; H = h.p. transmitted; p = pressure in main, lb. per sq. in.; d = pipe diam., ft. For economy, a size of pipe should be chosen that will allow a pressure drop p_d of about 10 lb. per sq. in. per mile.

Example. $H = 200$ h.p.; $p = 800$ lb. per sq. in.; $f = 0.01$; $p_d = 10$; $l = 1$ mile. The percentage loss of head is $\frac{10}{800} \times 100 = 63.5flH^2/p^3d^5$, from which $d^5 = 0.20955$, or $d = 0.732$ ft. = 8.78 in., say 9 in. Velocity of water through main, ft. per sec. $= 550H/(p \times 9^2 \times \pi/4) = 2.16$.

TIDAL POWER

Where a tide range H of 6 ft. or more is available, water may be impounded in natural or artificial basins and power developed by its flow back into the ocean through low-head turbines or undershot water wheels. According to J. Royden Peirce (*Eng. Rec.*, Jan. 27, 1912), the maximum number of horse-power hours obtainable from a tide basin is $N = [HQ - (Q^2/2A)]/31{,}800$, where Q is the available drop in the level of the basin (ft.), multiplied by the area A of the basin (sq. ft.). At a basin in Mamaroneck, N. Y., $H = 7$ ft., $A = 1{,}200{,}000$ sq. ft., and the drop = 1.4 ft., making $N = 330$ h.p.-hr. The power is used over a period of 6⅔ hr., developing on an average about 50 h.p. The author estimates that the power derivable from natural basins along the N. Y. and Conn. coasts of Long Island Sound would amount to less than 5000 h.p. Decoeur, a French engineer, proposed in 1890 to construct two large basins in the estuary of the Seine, the basins being separated by a bank or wall rising above high water, in which were to be placed low-head turbines of about 300 h.p. The upper basin (A) was to be open to the sea during the higher one-third of the tidal range (H) when rising, and the lower basin (B) during the lower one-third of the range when falling. The level in A was never to fall below ⅔H (ft.) measured from low water, nor that in B to rise above ⅓H. The available head would then vary between

0.53H and 0.8H, with a mean value of $\frac{2}{3}H$. Letting S (sq. ft.) be the area of B, $\frac{1}{3}SH$ cu. ft. of water would flow from A to B during 9¼ hr., or a mean flow of SH/99,900 cu. ft. per sec.; or, taking mean fall = $\frac{2}{3}H$, available gross horse power = $0.033S_1H^2$, where S_1 = area of basin B in acres. (See *Proc. Inst. C. E.*, 1890.)

WAVE POWER

According to Albert W. Stahl, U. S. N. (*Trans. A. S. M. E.*, vol. 13 p. 438), the total energy of an indefinite series of trochoidal deep-sea waves may be expressed as follows: H.p. per ft. of breadth of wave = $0.0329 \times H^2\sqrt{L}[1 - 4.935(H^2/L^2)]$, where H = height of wave, ft., and L = length of wave between successive crests, ft. For example, with L = 25 ft., and L/H = 50, h.p. = 0.04; with L = 100 ft. and L/H = 10, h.p. = 31.3. Not much more than a quarter of the total energy of such waves would probably be available after reaching shallow water, and apparatus rugged enough for this purpose would doubtless be unable to utilize more than a third of this amount. **Wave motors** brought out from time to time have depended for their operation largely on the lifting power of the waves. One installed at Atlantic City, N. J. (*Power*, Jan. 17, 1911), consisted of six 4-ft. cylindrical floats 4 ft. high. These, each weighing about 3100 lb., were lifted 2 ft. by the waves about eleven times per minute, and drove a horizontal shaft by means of chains and ratchets, developing but 12 h.p., steadiness being obtained by the use of heavy flywheels. The fixed charges on the excess cost of wave motors thus far proposed over steam-power plants of equal capacity, have been more than sufficient to care for the fuel and other additional costs necessitated by the use of the latter.

COST OF POWER

By L. S. MARKS

The cost of producing power is made up of two classes of costs—**fixed charges** and **operating costs.** The former are those dealing with the plant itself, practically regardless of its use, and consist of interest on the capital invested, rental of premises or an equivalent allowance, depreciation of the machinery, apparatus and buildings, taxes, insurance, etc.; also, in the case of companies engaged in the sale of power, office rental, salaries of officers and assistants, and all expenses not directly chargeable to the power plant. In addition to interest allowance, central-station engineers maintain that a *profit* charge should be made when costs of central-station power and isolated-plant power are compared; and that this profit charge should be the amount that could have been earned (over and above interest) by the capital invested in the plant had it been employed in extending the most profitable part of the business which was capable of further development. The rate of **interest** varies, but is usually taken as 5 per cent. in calculations. **Taxes** range from 0.1 per cent. to 2.0 per cent. of the investment, depending on location; for estimating purposes 1 per cent. of actual value may be assumed. **Insurance** against loss by fire or explosion may be roughly covered by an allowance of 0.5 per cent. There is little agreement among engineers on the subject of **depreciation** charges, or the amounts which should be set aside annually in an interest-bearing **amortization fund** to provide for the purchase of new apparatus, etc., to replace that worn out. Estimates of the **useful life** of the different parts of a plant vary within wide limits, one published list being as follows:

	Years		Years
Buildings, brick or concrete	50	Motors	20
Buildings, wooden or sheet-iron	15	Pumps	25
Chimneys, brick	50	Condensers, surface	20
Steel stacks, self-supporting	25	Condensers, jet	35
Sheet-iron stacks, guyed	10	Feed-water heaters	20–30
Boilers, water-tube	25	Economisers	20
Boilers, fire-tube	15	Wiring	20
Engines, slow-speed	25	Belts	7
Engines, high-speed	15	Piping	12–20
Steam turbines	25	Coal-handling machinery	10–15
Generators	25–30	Transformers	25–30

Due to obsolescence or inadequacy of apparatus, however, these periods are in practice very much shorter; the **average life** of a plant is probably but from 12 to 18 years. The average life of central power plants is still less as a consequence of the rapid developments in generating units. The **percentage to allow for depreciation,** according to eleven published estimates, varies as follows (a rough approximation would be 2 per cent. for buildings and 5 to 6 per cent. for other items):

	Min.	Max.	Avg.		Min.	Max.	Avg.
Buildings	1.0	2.5	1.9	Wires and cables	2.0	6.6	4.4
Boilers	2.5	10.0	6.2	Switchboards	2.0	10.0	5.1
Steam piping	2.5	10.0	5.5	Rotary converters	4.0	10.0	5.4
Auxiliaries	3.0	10.0	6.1	Motors	4.0	10.0	4.6
Steam engines	2.5	10.0	5.1	Transformers	3.0	6.6	5.0
Generators, belted	3.3	10.0	6.1	Shop equipment	3.0	15.0	7.4
Generators, direct-connected	3.3	8.0	5.3	Supplies and misc.	1.5	7.5	4.9

Inclusive allowances for fixed charges vary from 10 to 14 per cent.

Scrap Values in per cent. of the original cost may be assumed to average as follows: Buildings, 5; boilers, stokers and furnaces, 5; conveyors, elevators and hoists, 1; turbines, complete, 10; engines and condensers, 10; piping, valves and traps, 3; pumps, 5; synchronous converters, transformers and exciters, 10; switching apparatus and instruments, 5; alternators and motors, 10; storage batteries, 10; tools and sundries, 3.

Operating Costs include labor or attendance; supervision; fuel and water; oil, waste and other supplies; and repairs. It is difficult to give adequate information regarding **labor** costs. Generally, it may be assumed that plants of under 100 h.p. capacity can be attended to by one man for each shift of 8 or 10 hr. When the fuel consumed in one shift exceeds 2 tons, a fireman will be needed; where it exceeds 3 tons, a fireman should be employed for every 3 tons burnt or fraction thereof. Published data on the labor costs in street-railway power plants (continuous operation, 8760 hr. per annum) are as follows:

Capacity, kw.	100	300	500	1000	3000	5000	10,000	20,000
Labor cost per kw-year	$27	9.20	8.52	6.96	4.98	4.10	3.76	3.56

Estimates of labor cost are also given in Tables 3 and 4. Oil, waste and supplies form from 2 to 10 per cent. of the total operating costs, inversely as the capacity of the plant.

Load Factor. The load factor of a power plant is the ratio of the average power to the maximum power during a certain period of time. The average power is taken over a certain period of time, such as a day, a month or a year, and the maximum is taken over a short interval of the maximum load within that period. In each case both should be specified. The **plant or capacity factor** is the ratio of the average load to the rated capacity of the plant. The **demand** of an installation or system is the load which it puts on the source of supply, as measured at the receiving terminals. The **maximum demand** is the greatest demand, as measured over a suitable and specified interval (as a "5-min. maximum demand"). The **demand factor** is the ratio of the maximum demand to the total connected load. The **diversity factor** is the ratio of the sum of the maximum power demands of the subdivisions of any system to the maximum demand of the whole system, measured at the point of supply. The **connected load** is the combined continuous rating of all the receiving apparatus on consumers' premises connected to the system. [From A. I. E. E. Standardization Rules, 1914.] Few industries have a load factor as high as 80 per cent. In machine shops it is stated to range from 40 to 50 per cent.

Costs of Steam-power Plants. Tables 1 and 2 give approximate costs of the various items entering into the construction of steam-power plants. According to the authorities of Table 2, power-plant buildings contain from 50 to 100 cu. ft. per kw. of capacity and require a ground area of from 0.8 to 2 sq. ft. per kw.; buildings cost from 8 to 12 cents per cu. ft., and foundations from $1.50 to $4.00 per sq. ft. of building area. The cost of turbo-generators is given on p. 1006. Costs of completed plants, exclusive of land, are also given in Tables 3 and 4.

Table 1. Cost of Equipment of Isolated Power Plants

(P. R. Moses, *A. I. E. E.*, Jan. 12, 1912)

	Cost per kw. of capacity		Cost per kw. of capacity
Boilers (erected and set in masonry):		Gas producers	15–20
Horizontal-tube	$14–18	Electric generators:	
Water-tube	16–20	Direct-connected to high-speed engines	13–16
Steam engines:		Belt-connected to engines	12–15
High-speed, simple, direct-connected	20–25	Direct-connected to Corliss engines	16–20
Medium-speed, compound, non-condensing, direct-connected	28–35	Switchboards	5–10
Low-speed, compound, condensing, belted	20–25	Foundations	5–10
Low-speed, simple, belted	25–30	Steam fitting, including feed heater, separator, exhaust heads, tanks, pipe covering, etc.	20–30
Gas engines	50–60		
Oil engines (Diesel)	75–85		

Cost of Steam Power. Plant, fuel and labor costs, as well as estimates of fixed charges, vary so widely that it is impossible to predict power costs with any degree of accuracy. The estimates in Table 3 are for mill or factory power plants, the first five items being taken from tables devised by W. O. Webber (*Eng. Mag.*, July, 1908). They are for plants running 10 hr. a day, 308 days in the year, and at from 80 to 100 per cent. of their rated capacity. The figures for fuel consumption are based on steam consumptions ranging from 34 lb. to 14 lb. per h.p-hour, according to size and type of plant, and on evaporations of from 6 to 9 lb. of water per lb. of coal burned. With high-grade equipment, good fuel and efficient attendance, as good, or even better, results can be obtained. Very low fuel consumptions are reported for **locomobiles** (p. 961), or semi-portable engines mounted on internally fired tubular boilers with short steam connections and built-in superheaters. Such units, built in sizes from 100 to 1000 h.p., develop under test conditions a brake horse power on less than 1 lb. of coal, and should operate regularly on a consumption of 1.5 lb.

Table 2. Cost of Steam-Electric Power Stations, 2000–20,000 Kw. Capacity

(O. S. Lyford, Jr. and R. W. Stoval, *Elec. Jour.*, 1912, vol. 9, p. 322. Based on maximum continuous capacity of generators at 50 deg. cent. temperature rise)

	Dollars per kw. High	Dollars per kw. Low
Preparing site (dismantling and removing structures, making construction roads, tracks, etc.)	$0.25	0.00
Yard work (intake and discharge flumes for condensing water, railway siding, grading, fencing, sidewalks)	2.50	1.00
Foundations (including foundations for buildings, stacks and machinery, together with excavation, piling, waterproofing, etc)	6.00	1.00
Boiler-room equipment (including boilers, stokers, flues, stacks, feed pumps, feed heater, economizers, mechanical draft and all piping and pipe covering except for condenser water)	24.00	12.00
Turbine-room equipment (including steam turbines and generators, condensers, condenser auxiliaries, water piping, oiling system, etc.)	22.00	12.00
Electric switching equipment (including exciters, masonry switch structure with all switchboards, switches, instruments, etc., and all wiring except for building lighting)	5.00	2.00
Service equipment (including cranes, lighting, heating, plumbing, fire protection, compressed air, furniture, permanent tools, coal and ash-handling machinery, etc.)	5.00	2.50
Building (including frame, walls, floors, roofs, windows and doors, coal bunkers, but exclusive of foundations, heating, plumbing, etc.)	12.00	4.00
Starting up (comprising labor, fuel and supplies for getting plant ready to carry useful load)	1.00	0.50
General charges (such as engineering, purchasing, supervision, clerical work, construction plant and supplies, watchman, cleaning up, etc.)	6.00	3.00
Total, except for land and interest during construction	$83.75	38.00

Table 3. Annual Cost of One Horse Power in Factory Steam-Power Plants*

	Simple non-condensing engines			Compound condensing engines				
Rated capacity of plant, h.p.	20	40	80	100	300	500	1000	2000
Cost of plant per h.p.†	$200	190	175	170	126	96	60	56
Fixed charges at 14 per cent	$28.00	26.60	24.50	23.80	17.65	13.45	8.40	7.85
Attendance	$30.00	20.00	13.00	12.00	8.60	6.20	3.50	3.00
Oil and other supplies	$6.00	4.00	2.60	2.40	1.72	1.24	0.70	0.60
(a) Cost per horse power less fuel	$64.00	50.60	40.10	38.20	27.97	20.89	12.60	11.45
(b) Lb. steam per h.p-hour	34	32	30	20	18	16	15	14
(c) Lb. steam per lb. coal burnt	6	7	7	8	8	9	9	9
(d) Lb. coal per h.p-hour = b/c	5.67	4.57	4.29	2.5	2.25	1.78	1.67	1.56
(e) Coal per h.p-year at $1 per ton	$7.79	6.28	5.90	3.44	3.09	2.45	2.30	2.15
Cost of 1 h.p-year $(a + e)$‡	$71.79	56.88	46.00	41.64	31.06	23.34	14.90	13.60

* Plants operating 10 hours a day, 308 days in the year, at 80 to 100 per cent. of rated capacity; boilers hand-fired.

† Includes buildings, entire equipment, and erection.

‡ Cost with coal at $1 per ton. For any rate x (dollars) of fuel cost per ton, cost of 1 h.p-year = $a + ex$.

Where the load factor is low, the cost of a horse power actually delivered is higher than given in Table 3. For example, if it is necessary to have a capacity of 500 h.p., but the average load is only 250 h.p., the total cost will be made up of the cost less fuel, $20.89 × 500 = $10,445, plus the cost of the coal burned [= 250 × 1.78 × 3080 (hours) × $1/2240 (lb.)], or $612, making $11,057 per annum for 250 h.p., or $44.23 per h.p-year as against $23.34 when the plant is working at 100 per cent. capacity.

One fireman could be dispensed with, but the saving due to this would probably be counterbalanced by the increased fuel consumption at half load. In industries requiring a supply of heat as well as power, and where this heat may be obtained from the exhaust steam of a non-condensing engine, it is proper to deduct the cost of independently developing this heat from the gross power cost.

Table 4. Costs and Operating Expenses of Steam-Electric Power Plants*

	Capacity of plant, kilowatts			
	4000	8000	15,000	30,000
Cost of Plant:				
Boiler-room equipment and piping	$21.50	19.50	16.75	15.00
Generating equipment	22.50	20.50	18.25	15.50
Electric control equipment	5.00	5.00	4.50	4.00
Service equipment	5.00	5.00	4.50	4.00
Buildings, foundations and yard work	17.50	14.00	11.75	9.25
Engineering supervision	9.50	8.00	6.25	5.25
Total cost per kw. (except for land and interest during construction)	$81.00	72.00	62.00	53.00
Operating Cost:				
Labor	0.22	0.15	0.123	0.095
Water, supplies and repairs	0.14	0.09	0.073	0.050
Coal at $1.00 per 10,000,000 B.t.u. (= $3 per ton, approx.)	0.42	0.36	0.329	0.300
Total, cents per kw-hr	0.78	0.60	0.525	0.445

* From curves in the "American Handbook for Electrical Engineers," embodying average results from a large number of plants operating with load factors ranging from 25 to 33 per cent.

The **total cost of a kilowatt-hour** of electric energy at the switchboard terminals may be calculated approximately from Table 4. Assume a plant of 15,000 kw. rated capacity. According to a preceding paragraph (taking average values), such a plant would occupy a ground area of 15,000 × 1.5 = 22,500 sq. ft. which, at $2 per sq. ft. (value assumed only for purpose of calculation) would cost $45,000. The cost of the plant (Table 4) = 15,000 × $62 = $930,000. Interest on average investment during construction would probably be covered by $25,000, making a total investment of $1,000,000. Fixed charges on this amount at 12 per cent. = $120,000. The rated output of such a plant would be 15,000 × 8760 = 131,400,000 kw-hr. per annum. Assuming the load factor to be 30 per cent., the actual output would be 39,420,000 kw-hr., whence the fixed charges per kw-hr. would be $120,000/39,420,000 = 0.304 cent. Adding this to the operating cost from Table 4 (0.525 cent), gives the total cost of 1 kw-hr., with coal at $3 per ton, as 0.829 cent. The actual power costs in large plants are frequently much lower than this figure.

Cost of Water Power. Water-power plants cost from $20 to $200 per h.p., depending on the capacity, the head employed, whether or not a dam is required, difficulties of construction, etc. Ordinary low-head plants cost from $60 to $100 per h.p. The cost of power per h.p-year ranges from $10 to $30, being usually under $15 for ordinary low-head plants. For costs of hydro-electric machinery, see p. 1097.

Cost of Gas Power. See p. 1065.

SECTION 9

HOISTING AND CONVEYING

BY

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CONTENTS

HOISTING AND CONVEYING

BY

C. KEMBLE BALDWIN

HOISTING MACHINERY

TYPES OF DRIVES

Hand Power is applied by crank, lever, chain or rope, and is used where an apparatus is required intermittently for short periods. Hand-power drives are limited to short-lift, slow-speed hoisting. They are of low efficiency, owing to the large speed reduction required.

Transmission Drives are applied by means of belts, ropes or friction wheels, and are used only where power is already installed for other purposes. The capacity of a transmission drive is limited by the connecting member. The maximum power for belt drives is about 15 h.p.; for friction drives, about 5 h.p.

Compressed-air Drives are applied in two ways: (1) By direct-lift apparatus in which the air acts on a piston to which the load is attached directly or through intermediate pulley blocks. (2) The air is used in engines of either the reciprocating or rotary type, to which the load is connected by gearing, drum and cable. Compressed-air hoists are used where air supply has been installed for other purposes. Their radius of action is relatively small, being limited by the flexible connection to the source of air supply.

Combustion-motor Drives are used for operating small or medium-capacity hoists in locations where steam or electric power is not available. Combustion motors must be started without load, therefore clutches are required to connect the load after the engines are up to speed. They are not satisfactory under fluctuating load.

Hydraulic Drives are applied by water under pressure acting on a piston, to which the load is attached directly or through intermediate pulley blocks. They were formerly used universally for heavy lifting in steel mills, for elevators, cranes, etc. Electric drives have practically replaced hydraulic drives in all hoisting machinery except in the case of elevators. The radius of action is limited by the flexible connection where hoist is movable, and where stationary by the cost of installation and maintenance of pumps, piping, etc.

Steam Drives are used principally where a central steam supply is available, as in the case of mine hoists, and on portable hoists where the engine and boiler are mounted on the same base. By the simple manipulation of throttle and brakes, heavy loads may be hoisted and lowered very rapidly and smoothly, the steam cushion absorbing the shocks of sudden starting and stopping. Stationary hoists are limited only by their distance from the source of steam supply. Portable hoists are limited by the weight of boiler and engine to about 50 h.p.

Electric Drives are now used where transmission, compressed-air, hydraulic and steam drives were formerly employed. The economy, ease of transmission and flexibility of electricity and the great improvement of the control apparatus have resulted in the general adoption of the electric drive for all types of hoisting apparatus.

PARTS OF HOISTING APPARATUS

Drums are made with smooth surfaces on hand-power hoists and on power hoists where the rope speed is low; and with grooved surfaces when the rope speed is over 1000 ft. per min. Up to about 4 ft. diameter they are usually made of cast iron in one piece. Larger drums have cast-iron or cast-steel spiders with either smooth wooden lagging or grooves attached as cast-iron or hard-wood segments. To reduce weight, large high-speed drums are built with cast-iron hubs, channel spokes and steel-plate outer surface reinforced with angles. Conical grooved drums are used on mine hoists to equalize the work on the engine. The diameter of the drums is fixed by the diameter and construction of the hoisting ropes (see p. 844). The face of a drum is made wide enough to hold the required amount of rope plus three or four holding turns. The hole in the drum face through which the end of the rope passes to be secured should be as shown in Fig. 1, in order to prevent excessive bending.

Fig. 1.

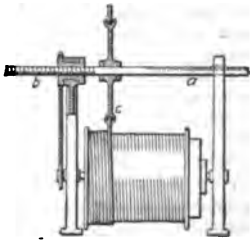


Fig. 2.—Hoisting Drum.

Where there is side draft on the rope, movable **idlers** are provided to align the rope with groove. The idlers may be moved parallel to the face of the drum by the side pressure of the rope or driven positively sideways, thus eliminating friction and increasing the life of rope. In Fig. 2, the idler sheave *c* revolves between fixed collars on shaft *a*, which is connected to the drum shaft by sprocket and chain. The shaft is prevented from rotating by a feather key. The sprocket *b* being a nut held from moving sideways by flanges, the shaft *a* with the sheave *c* is moved in the direction of its axis. In an alternative construction the idler shaft is threaded but held stationary, and the sheave hub is a nut. The sheave is turned by the friction of the rope, which causes it to travel back and forth. Fig. 3 shows construction used when the side draft is excessive. The upright rollers *a*, *a* are moved sideways by screw *b* driven by sprocket and chain *c* from the drum shaft. Where necessary to wind the rope on the drum in several layers, a clutch is provided to reverse the direction at the end of the travel.

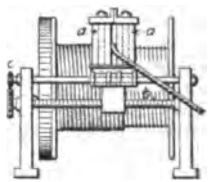


Fig. 3.—Hoisting Drum.

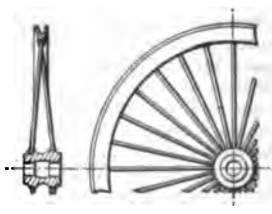


Fig. 4.—Rope Sheave.

Sheaves (see p. 741) should be grooved to fit the rope as closely as possible, to prevent the rope from assuming an oval or elliptical shape under heavy stress. They should be balanced and properly aligned to prevent swaying of the rope and its consequent abrasion against the sheave flanges. When the tension is great and speed high, the grooves are filled with renewable hardwood blocks. Sheaves up to 5 ft. pitch diameter are cast in one piece in iron or steel, Fig. 157, p. 741. Sheaves up to about 10 ft. diameter, Fig. 4, are made with cast-iron hubs and rims with round steel spokes cast in. Sheaves larger than 10 ft. are built as shown in Fig. 5, with cast-iron or steel segmental rims, cast hubs and steel spokes bolted with turned and fitted bolts.

Counterweights are used with elevators, skip hoists, mine hoists and the like, to equalize the engine load. The engine or motor is thereby smaller, as it must only accelerate the total mass and overcome friction. The brakes of a counterbalanced system must be more powerful than for the same hoist without counterweights, as they must retard greater masses. Skip and mine hoists are counterbalanced by means of duplicate cars, one traveling up while the other goes down. The effective load is the difference between the weights of a full and empty car, corrected for the weight of the rope. Elevators are usually overbalanced, *i.e.*, the counterweight exceeds the light weight of the car by the average expectation of load. With high lifts, the weight of rope is neutralized by compensating chains hung from the top of the shaft and attached to the bottom of car, producing no pull when the car is down and the maximum when at the top.

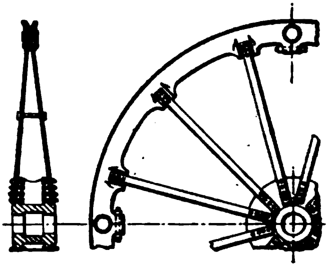


Fig. 5.—Segmental Rope Sheave.

Brakes (see p. 700). The smaller hoists are provided with band brakes, usually hand-operated, and the larger hoists with post brakes, mechanically operated. On electric hoists, it is usual to apply the brake with a weight or spring and remove it by a solenoid. Should the current fail, the brakes are automatically applied. A dash pot is sometimes used to avoid shock. On steam hoists, the brake is taken off by a steam piston and cylinder instead of a solenoid. On overhead cranes, load brakes are used to sustain the load automatically at any point and to regulate the speed when lowering.

In one type of load brake (Fig. 6), the motor *A* drives drum *B* through the load brake on the intermediate shaft *D*. This consists of a spider *C* keyed to shaft *D*, its inner end supporting one end of a coiled bronze spring of square section *E*. The opposite end of spring is fixed in the flange *F* loosely fitted to shaft *D* and directly attached to pinion *G*. Any relative angular motion of flange *F* and spider *C* alters the closeness of the coiling of spring *E*, consequently also altering its outside diameter considered as a drum. This outer surface is one of the friction surfaces of the brake, the other being provided by the internal face of drum *H*, which revolves loosely on the shaft *D* at one end and on flange *F* at the other. It is restrained from moving in one direction by ratchet *I* and pawls *J*. The exterior of drum *H* is grooved for heat dissipation.

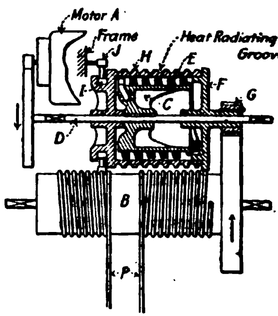


Fig. 6.—Load Brake.

The action of the load brake in Fig. 6 is as follows: When hoisting, the brake revolves as shown by the arrow. Pawls *J* permit drum *H* to revolve, consequently the whole mechanism is locked and revolved as one piece. When stopping the load, the downward pull of the load reacts so as to drive drum *H* against pawl *J*. Flange *F* therefore moves slightly in an angular direction relative to spider *C*, and spring *E* consequently untwists until it grips interior of drum *H*, thus locking the load. The action is such that the grip is slightly more than necessary to hold the load. Reversing the motor for lowering the load drives the interior of brake surface against drum *H*, so that the power consumed is the amount necessary to overcome the excess holding power of the brake over the load reaction.

Load brakes of the disk type and cone type (see pp. 702, 703) are also used, which embody the same principle of pawl locks and differential action. The type should be chosen with a view to smoothness of working and lack of chatter, as well as to the power required for lowering at different values of load within the range of the crane.

LOAD SUSPENSION

Wire Rope (see p. 843). For **hoisting** devices, where flexibility is the chief consideration, the rope generally used is made of 6 strands of 19 wires each on a hemp core (Fig. 1, p. 845). Extra pliable ropes are made with 6 strands, 37 wires each, or 8 strands, 19 wires each, on a hemp core. These ropes, while more flexible and permitting of the use of smaller drums and sheaves, are made of smaller wire, and hence are less durable. For **haulage**, where resistance to abrasion is the determining consideration, the rope is made of 6 strands of 7 wires each on hemp core (Fig. 4, p. 846). This construction presents heavier wire to resist the wear of cable grips, etc.

Fig. 7.—Haulage Rope.

Fig. 8.—Lang Lay Rope.

Hoisting ropes are constructed with the relative twist of the wire in the strands the reverse of the twist of the strands about the core, Fig. 7, and should not be spliced. Haulage ropes are of the same construction or of the Lang lay type, Fig. 8 (twist of wire and strand in the same direction), which construction increases the distribution of the surface wear and also the life. Lang lay ropes have a tendency to untwist, so should not be used when the load is in free suspension. They are difficult to splice.

Reverse bends should be avoided in hoisting ropes, and all drums and sheaves should be of the same diameter to obtain the maximum service from the ropes. Hoisting and haulage ropes should be frequently greased to prevent corrosion; boiled linseed oil is used on ropes subjected to atmospheric action, and crude petroleum and graphite on haulage and hoisting ropes in wet places. The greatest wear on ropes is at the points of attachment to the load, which should be frequently inspected and new connections made at the first sign of wear. Sufficient extra rope is usually coiled up inside the drum to be paid out to make these connections. For **strength** and working loads of wire ropes, **sizes of drums** and sheaves, etc., see pp. 843–851.

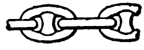


Fig. 9.

Hoisting Chains are used for handling very heavy loads through short distances and also in cases where abrasion would destroy ropes. Chains require greater drum faces than ropes. For light loads, plain welded link chain is used (Fig. 187, p. 761); for heavy loads, stud link chain (Fig. 9), the stud preventing the link from distorting. Chains should be frequently inspected and annealed. For weights, dimensions and safe loads, see Table 95, p. 760.

Hooks for cranes, etc., are usually made with swivel, on the larger sizes with ball, bearings, as in Fig. 10. Fig. 11 shows a safety hook, the sliding collar *a* on the shank locking the safety lever *b*. Fig. 12 shows a hook with a safety handle to protect the workman when guiding the hook. For design of hooks, see p. 761.

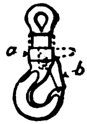
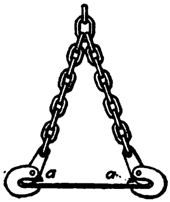



FIG. 10. FIG. 11. FIG. 12.
Crane Hooks.

Lifting Tongs. Fig. 13 shows the type used for lifting plates. The cams *a* grip the plate when the chains tighten, preventing slipping. They are safer than plain hooks. Self-closing tongs, Fig. 14, are used for handling logs, manure, straw, etc. The rope *a* is attached to and makes several turns around drum *b*; chains *c* are attached to the bucket head *e* and to drum *b*. When power is applied to rope *a*, drum *b* is revolved, winding itself upon chains *c* and closing the tongs. To open, slack off on rope *a*, holding tongs on *d* attached to head *e*.



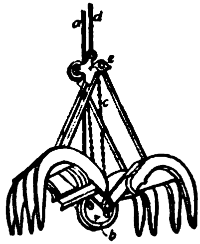


FIG. 13. FIG. 14.
Lifting Tongs.

Lifting Magnets (see also p. 1600) are used for handling pig iron, scrap iron, castings, billets, tubes, rails, plates and other finished materials; also used to handle skull-cracker balls in scrap yards, and in some localities for handling magnetic iron ore. They are of two **types**: circular magnets, for handling pig iron, scrap and similar detached material; and bi-polar, for handling rails, beams and similar products of fairly heavy section and considerable length. Rectangular magnets are used to some extent in handling plates which are flexible and tend to pull away from the magnet face by their deflection. Circular magnets are the most rugged and rectangular the least rugged.

Lifting magnets should not be used for carrying material over the heads of workmen, as failure of current will drop the load. Safety tongs are sometimes used in connection with magnets, but they are cumbersome and limit output. If 10 tons or more of material is to be handled per day and a crane is available for service, the installation of a magnet will be profitable.

Circular magnets, Fig. 15, consist of a steel casing *a* having an annular slot in its lower face in which the magnetizing winding *b* is placed. Strap copper windings are usually employed, permitting a greater number of ampere-turns in the same space than circular wire. The slot is covered by a non-magnetic plate *c* of manganese steel held in position by removable pole shoes *d*. Circular magnets are usually suspended by three chains. The headroom required is given in Table 18, p. 1601. Bi-polar magnets, Fig. 16, have a horizontal core *a* enclosed by a magnetizing winding *b*. The core has poles *c* which project below the clearance line of the winding.

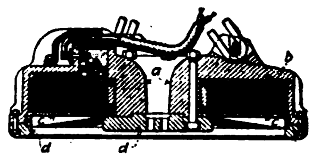


FIG. 15.—Lifting Magnet.

Lifting magnets are operated on 220-volt direct-current circuits. The magnet controllers automatically shunt a discharge resistance across the magnet terminals just prior to the opening of the magnet circuit, and automatically disconnect it just prior to the re-establishment of the circuit. This renders harmless the strong inductive reaction when the circuit is suddenly opened and protects the magnetizing winding. To release the load instantly, the current in the coil is reversed, overcoming the residual magnetism that causes the load to cling after the circuit is opened.

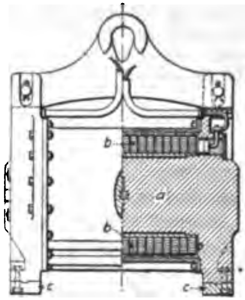


Fig. 16.—Bi-polar Lifting Magnet.

Cable take-ups, Fig. 17, are usually used to keep cable taut. Special types are used on locomotive cranes. The diameter of sheaves should be at least 30 times the diameter of cable. **Capacity** depends on the nature of the material handled (see p. 1601). When handling heavy steel scrap, the capacity will be about 10 per cent. greater than with pig iron. In figuring the kw.-hours required per ton of material handled, assume the magnet energised but 50 per cent. of the time when in continuous service.

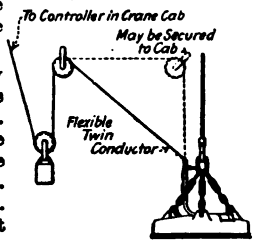


Fig. 17.—Cable Take-up for Lifting Magnet.

Tubs. Self-dumping tubs, Fig. 18, are rapidly being superseded by self-filling buckets. Tubs are mounted on steel rollers and filled by hand. When empty, the center of gravity is back of the trunnions; when filled, it is in front, so that the tub will dump itself when trip bar *a* is raised. They are also made with trip levers on the bale. By raising the tub until the levers come in contact with a stop on the trolley, the latch is released. To prevent breakage of material, the latch is arranged so that tub may be lowered on to pile, automatically releasing the latch. When the the tub is raised, it dumps. Tubs are made in sizes of from 10 to 20 cu. ft. for hand filling, and up to 140 cu. ft. for spout filling. Bottom-dump tubs are used when the load must be accurately placed in such work as charging furnaces, placing concrete, etc. Various types of bottom doors are used, Fig. 19 showing a single door with hand-operated latch.

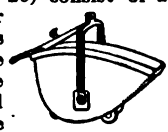


Fig. 18.

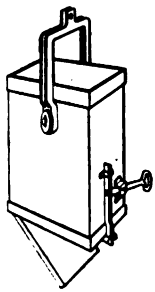


Fig. 19.

Self-filling Drag-line Buckets (Fig. 20) consist of a scraper bowl which is drawn by rope *b* over the material to be handled. The shackles of *b* are so attached that the cutting edge will penetrate the material. When the bucket is filled, it is raised by rope *a* and carried to the dumping position, when rope *b* is released, allowing the load to dump. These buckets are built with capacities from 1½ to 3½ cu. yd. Their design is determined by operating conditions.

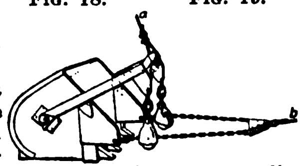


Fig. 20.—Self-filling Drag-line Bucket.

Grab Buckets. The more important **types** of grab buckets described below have their approximate **weights, capacities** and **dimensions** given in Table 1. The **Hayward** type, Fig. 21, is used for handling coal, sand, gravel, etc., and for other materials which do not pack tightly and which may be easily dug. The holding rope *a* is made fast to the head of the bucket. The closing rope *b* makes several wraps around and is made fast to drum *d* mounted on the shaft to which the scoops are pivoted. The chains *c* are made fast to the head of the bucket

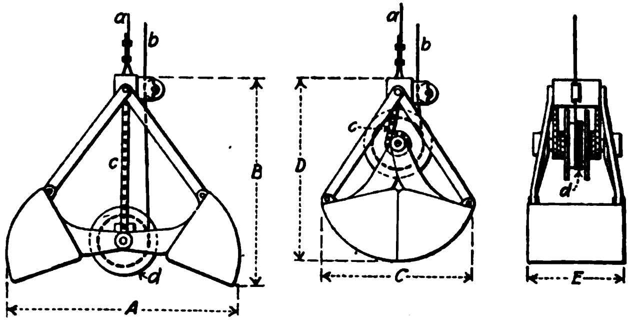


Fig. 21.—Hayward Grab Bucket.

and to the small diameter of drum *d*. When power is applied to rope *b*, it causes the drum to wind itself up on chain *c*, raising the drums and closing the bucket. To dump, hold rope *a* and slack rope *b*. The digging power of the bucket is determined by the weight and the ratio of the diameters of the large and small parts of the drum. In the **Williams** type, Fig. 22, used for the same service as the Hayward, the holding rope *a* is attached to the head

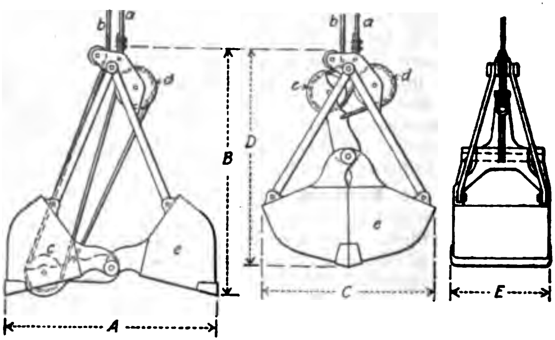


Fig. 22.—Williams Grab Bucket.

of the bucket. The closing rope *b* passes around a bend sheave in the head of bucket, over sheaves *c* and *d*, and is made fast to the arm carrying sheave *c*, which is attached to shell *e*. To close the bucket, rope *a* is slacked and power applied to *b*, drawing sheaves *c* and *d* together. To open the bucket, the weight is carried on rope *a*, rope *b* is slacked and the bucket then opens automatically. The **Andresen-Evans** type, Fig. 23, is used for handling coal, stone and (in the heavier types) ore. The hold rope *a* is attached to the bucket arms by chain shackles to open the bucket. The closing line *b* is made fast to and makes several wraps around the large-diameter drum *c*, which is mounted on a shaft carried by the frame of the bucket to which the scoops are pivoted. The four chains *f* are made fast to the small diameter of the drum *e*, the other ends being made fast to the bent strut *d* above the drum *e*. When power is applied to rope *b* it re-

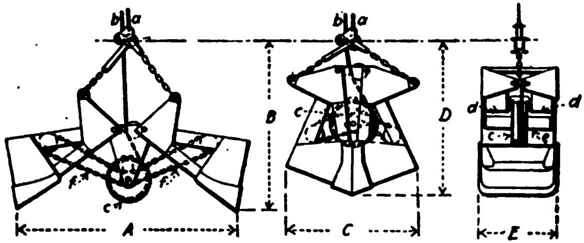


FIG. 23.—Andreesen-Evans Grab Bucket.

volves drums *c* and *e*, causing chains *f* to wind upon the small-diameter drum *e*, closing the bucket. To dump bucket, the weight is held by rope *a*, and rope *b* is slacked, allowing the bucket to dump automatically.

In the **Brownhoist** type, Fig. 24, used for handling coal, ore and similar materials, the path of the bucket gives a true scraping motion and the bucket head does not lower when opening, as is the case with other types. The hold rope *a* is made fast to the trolley above and passes over sheave *c* in the head of the bucket, the other end of the rope going to the hoisting drum. The

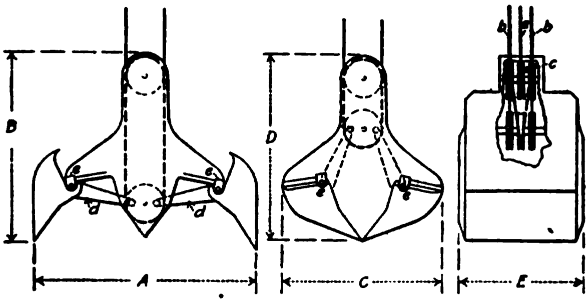


FIG. 24.—Brownhoist Grab Bucket.

speed of hoisting can be doubled by leading both ends of rope *a* to the drum. Rope *b* passes over the two sheaves in the head of the bucket and the three sheaves in the sliding cross head. When power is applied to rope *b*, the sliding cross head is raised. The arms *d* of the scoops are pivoted to the sliding cross head and a sliding connection *e* pivoted to the scoop causes the scoops to be drawn in as the cross head is raised. To dump the bucket, the weight is carried by ropes *a* and rope *b* is slacked, allowing the bucket to dump automatically.

The **Hulett** type, Fig. 25, is used for handling coal, ore and **similar materials**. The scoops are pivoted at points *c*, from quadrants on shaft *d*, and on links

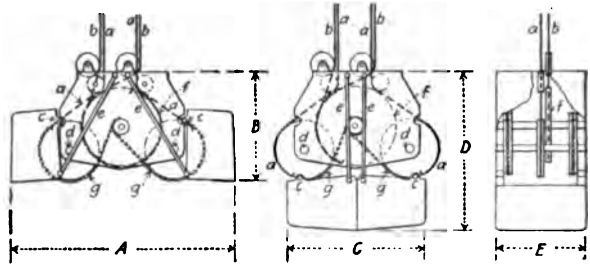


Fig. 25.—Hulett Grab Bucket.

e, so as to give a scraping motion to the bucket. These buckets are operated by four ropes, except in the smaller sizes. The holding ropes *a* pass around sheave on shaft *d* to open the bucket. The closing lines *b* pass around sheave *f*, and the chains *g* fastened to quadrants on shaft *d* are made fast to small diameter of drum *f*. To open the bucket, slack cables *b* and hold cables *a*. To close the bucket, slack cables *a* and pull in on cables *b*.

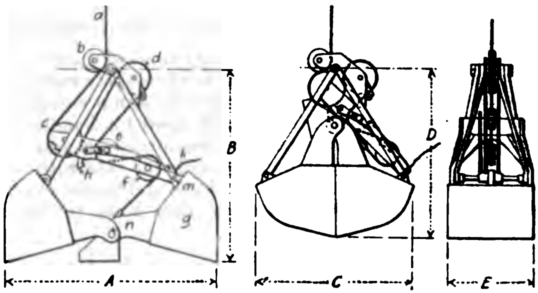


Fig. 26.—Williams Single-rope Grab Bucket.

The **Williams** single-rope type, Fig. 26, is used for handling coal, sand in foundries, and in other places where the digging is not difficult and where but a single-drum engine is available, or where it is desirable to hook the bucket on a derrick or crane hook. Rope *a* passes around sheave *b*, around sheaves *c* (carried by the latching arm), *d* (carried by the head of the bucket), *e* and *f* (carried by the latching arm), and is made fast to the arm of scoop *g*. The open view of Fig. 26 shows the latching arm when the bucket is resting on the material and rope *a* is being slackened. The latching arm, pivoted at point *m*, is lowered until latch *h* engages a projection on extended arm of scoop *g*. When power is applied to rope *a*, the latch *h* raises the shaft *n*, closing the bucket. To discharge, the trip rope *k* releases latch *h*, allowing the bucket to

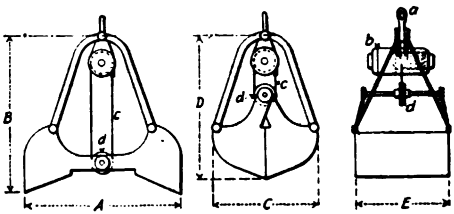


Fig. 27.—Hayward Electric Single-rope Grab Bucket.

open automatically. The **Hayward electric** single-rope type, Fig. 27, is used for handling coal, sand in foundries, and in other places where it is desirable to hook the bucket on a derrick or crane hook. The bucket is hung from the crane or derrick hook by the eye *a*. The four bucket arms are pivoted on this head, and suspended therefrom is a self-contained electric hoist *b*, consisting of motor, gear-reduction box and drum. One end of the rope *c* is made fast to the casing of the hoist, and the other to the hoist drum. When current is turned on, the drum revolves, winding up the rope, thus

Table 1. Weights, Dimensions and Capacities of Self-filling Grab Buckets

Capacity in cu. yd.	Hayward, Fig. 21						Williams, Fig. 22					
	Weight in lb.	Dimensions in inches					Weight, in lb.	Dimensions in inches				
		A	*B*	*C*	*D*	*E*		*A*	*B*	*C*	*D*	*E*
½	2,100	71	69	50	62	40	1,900	72	88	53	77	33
¾	2,500	84	82	60	74	40	2,350	79	89	60	80	33
1	2,700	93	87	66	77	40	2,800	83	98	67	83	38½
1¼	3,000	93	87	66	77	46	3,200	88	99	72	86	38½
1½	3,800	105	98	74	87	47	3,800	100	114	80	99	45
1⅝	4,000	105	98	74	87	50	3,900	105	117	80	103	45
2	4,800	105	99	74	89	60	4,200	100	114	80	99	51
2½	5,800	120	113	84	101	60	5,200	103	119	80	105	57½
3	6,500	120	113	84	101	70	6,000	111	121	82	109	57½
4	9,000	136	134	108	111	70	....	....	......	....	......	....
5	11,000	144	140	114	117	74	....	....	......	....	......	....
7½	17,500	171	154	125	134	84	....	....	......	....	......	....
	Andresen-Evans, Fig. 23						Brownhoist,* Fig. 24					
1	3,200	114	92½	69	85	43	2,900	93	80	66	78	45
1¼	....	....	......	....	......	....	3,830	115	91	81	88	42
1½	4,000	129	104½	77	96½	46	....	....	......	....	......	....
2	5,400	144	115½	86	106½	51	4,350	115	91	81	88	61
2½	....	....	......	....	......	....	5,200	125	100	90	93	61
3	7,500	165	130½	98	121½	58½	6,750	142	107	102	102	61
4	10,000	183	144½	107½	133	63½	7,800	153	100	116	102	73
5	12,500	195	155	115	142	70½	....	....	......	....	......	....
	Williams, single-rope, Fig. 26						Hayward electric, Fig. 27					
½	2,400	81	81½	54	74	33	....	....	......	....	......	....
¾	2,650	88	82½	60	77	33	2,300	65	77	49	69	46
1	3,250	93	93	67	84	39	3,900	76	88	56	79	48
1¼	3,600	100	95	72	87	39	4,000	88	97	61	89	51
1½	4,750	109	111½	80	111	45	4,500	88	97	61	89	59
2	5,300	109	111½	80	111	51	....	....	......	....	......	....
	Hulett,* Fig 25						Orange-peel, Fig. 28					
¾	4,350	101	48	77	70	39	3,800	75	100	61	90	....
1	....	....	......	....	......	....	4,200	82	105	68	94	....
1¼	4,950	108	49	84	74	39	4,600	87	108	72	96	....
1½	5,200	116	51	88	75	41	7,750	94	124	76	112	....
1⅝	....	....	......	....	......	....	....	....	......	....	......	....
2	7,200	120	60	102	84	44	8,500	102	132	84	118	....
2½	8,000	144	72	110	100	53	10,500	111	138	92	122	....
3	9,700	148	80	120	108	56	11,500	115	142	96	124	....
4	11,000	156	84	126	114	58	18,000	126	150	106	130	....

* Dimensions and weights given are for coal buckets.

raising sheave d and closing the bucket. To dump the bucket, the hoist is reversed. The hoist is provided with a disk clutch, which is adjusted to slip should the bucket be prevented from closing by a large lump. This clutch also slips if the motor is not stopped when bucket is entirely open or closed. This prevents overloading the motor. The ¾-cu. yd. bucket is provided with a 4-h.p. motor. The 1-, 1¼- and 1½-cu. yd. buckets have motors of from 6 to 10 h.p., according to the duty. The **orange-peel bucket,** Fig. 28, used for digging material which cannot be dug with grab buckets, consists of four blades pivoted to a moving cross head. The operation of the bucket is the same as that of the Hayward type, Fig. 21.

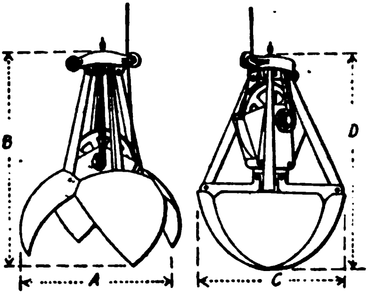


FIG. 28.—Orange-peel Bucket.

HAND-POWER HOISTING APPARATUS

Winches and Crabs are hand-power hoisting devices used with ropes and sheaves. (Called crabs when mounted on skids as independent hoists, and winches when attached to cranes, derricks, etc., forming their hoisting mechanism.) They are limited to short lifts, slow speeds, and intermittent hoisting. They consist of a rope drum geared by one or more reductions to hand cranks, and are usually provided with a band brake for lowering and ratchet and pawl for holding the load. Fig. 29 shows a double-purchase crab; Fig. 30 a double-purchase winch. Cranks may be used on intermediate or drum shafts to handle lighter loads quickly. Winches and crabs are built in capacities up to 5,000 lb. on a single rope. By the use of block and tackle heavier loads may be handled at corresponding slower speeds.

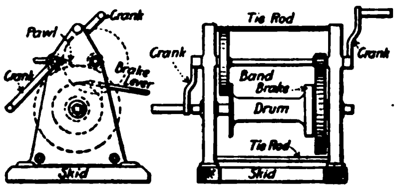


FIG. 29.—Double-purchase Crab.

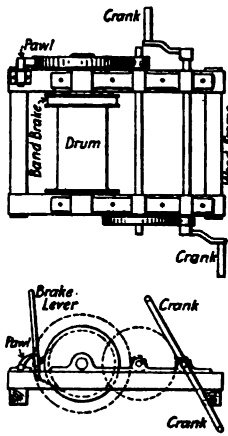


FIG. 30.—Double-purchase Winch.

Capstans, Fig. 31, are used mainly on shipboard for applying power to Manila rope. The base a is bolted to the deck and carries shaft b. Base a has recesses to receive pawls c and d. The gear plate e carried the pawls d and three pinions, f, meshing with

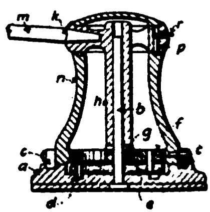


FIG. 31.—Capstan.

with the center gear *g* which is attached to the head *h* by a jaw clutch. The head *h* has holes *k* to receive the capstan bars *m*. The barrel *n* carries the pawls *c*, also the annular gear at the lower part, meshing with pinions *f*. When the head *h* and barrel *n* are attached by pin *p*, light loads are lifted by giving the rope several turns about the barrel *n*, the pawls *c* keeping the barrel from reversing. When hoisting heavier loads at a slower speed, the release pin *r* disengages the head *h* from the barrel *n*, and the motion from the head is transmitted through the center gear *g*, the pinions *f*, to the annular gear of *n*, thus increasing the power, due to the gearing. The pawls *d* and *c* hold the barrel from reversing. A capstan may be operated by a steam engine or electric motor placed below deck and geared to shaft *b*.

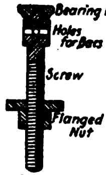


FIG. 32. House-raising Screw Jack.

Jacks are portable tools used for lifting heavy loads through short distances. There are three types in common use: Screw jacks, used in house raising, shoring, etc,; rack-and-lever jacks, for light work; and hydraulic jacks, for very heavy work. Owing to the large reduction, the lifting speed is slow. Their range is limited by weight of the jacks, which must be portable. **House-raising screw jacks**, Fig. 32, consist of a heavy screw threaded into a flanged nut. The screw head is fitted with a ball with socket in the bearing plate. The screw is turned by a bar passing through holes in the screw head. Fig. 33 shows a bell-base screw jack sometimes fitted with a ratchet head in place of the bar, thus increasing the speed. **Rack-and-lever jacks**, Fig. 34, consist of a cast-steel or malleable-iron base and post to which is pivoted the lever and pawl. The rack-toothed bar passes through the hollow post and is raised by the pawls actuated by the lever. The load is both raised and lowered by the lever. **Hydraulic jacks** are made in a great variety of forms according to service. Fig. 35 shows the common type, consisting of a base or cylinder and a head or piston. The head serves as a reservoir for the fluid and contains also the pump rods. At the lower end is the pump piston which draws the fluid from the reservoir and forces it through the valve in the bottom of the head into the base, thus forcing the piston or head up. The operating lever is so constructed that during pumping the travel is limited, but by reversing the lever the pump piston travels beyond the pumping position, touching the check-valve stem and allowing the fluid to pass back through a by-pass into the reservoir in the head, thus lowering the load. House-raising jacks are made with lifts up to 50 in. and are used in multiple, thus raising any desired load. Bell-base screw jacks have a maximum lift of about 30 in. Lever-and-rack jacks will lift 15 tons through 19 to 20 in. Hydraulic jacks of the type illustrated are made to lift 50 tons 18 in. Special types will lift 250 tons 18 in. Bridge jacks are built to lift 600 tons 12 in.

FIG. 33. Screw Jack.

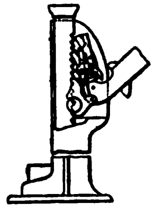


FIG. 34. Rack-and-lever Jack.

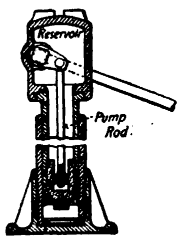


FIG. 35. Hydraulic Jack.

Chain Hoists are portable lifting devices suspended from a hook and operated by hand chain. They are used for short lifts and for intermittent service on loads up to 20 tons. The limit of weight of hoist is about 1400 lb. and that of the maximum practical downward pull on the operating chains is about 500 lb. Three types are in common use, each being provided with a hook to suspend the hoist from the overhead member, hook and chains for supporting the load, and hand chain for imparting power. The **differential hoist** (see Fig. 36, p. 658) consists of three chain sheaves with an endless chain for lifting and operating. One of the upper sheaves is constructed with an extra link pocket. The difference in diameter being too small to overbalance the friction of the parts, the load is automatically sustained at any point, making the hoist self-locking. The **worm-gear hoist**, Fig. 36, consists of an operating chain *a* passing over pocket sheave *b* mounted on worm shaft *c*. The worm imparts motion to the two load sheaves *d*. The load chain is attached to the swivel hook and passes over the load sheaves, the slack hanging in a loop. The worm gear makes the hoist self-locking. Separate chains are used for hoisting and operating. The **spur-gear hoist** (see Fig. 38, p. 658) has a higher efficiency than the other types, as it does not depend on friction of the mechanism to sustain the load. A mechanical brake, consisting of a ratchet and pawl and friction plate, sustains the load. To lower, the hand chain is pulled, overcoming the friction of the friction plates and revolving the ratchet. Table 2 gives **capacities**, weights and general data of the three types.

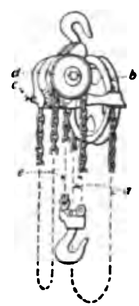


FIG. 36. Worm-gear Hoist.

Table 2. Data on Chain Hoists

Capacity, tons	Differential hoists: Lift, in ft.	Approx. weight, lb.	Pull on chain to lift full load, lb.	Ft. of chain overhauled to lift full load 1 ft.	Minimum distance between hooks, in.	Worm-gear hoists: Lift, in ft.	Approx. net weight, lb.	Pull on chain to lift full load, lb.	Ft. of chain overhauled to lift full load 1 ft.	Minimum distance between hooks, in.	Spur-gear hoists*: Lift, in ft.	Approx. weight, lb.	Pull on chain to lift full load, lb.	Ft. of chain overhauled to lift full load 1 ft.	Minimum distance between hooks, in.
⅛	5	11	....	....	16	....	....	....	....	....	....	....	....	....	....
¼	6	22	72	18	17	....	....	....	....	....	....	....	....	....	....
½	7	30	122	24	21	8	43	68	40	13	8	53	62	21	15
1	8	51	216	30	26	8	57	87	59	16	8	80	82	31	17
1½	8½	81	246	36	32	8	76	94	80	19	8	124	110	35	19⅛
2	9	122	308	42	39	9	104	115	93	21	9	188	120	42	24
3	10	180	557	38	44	10	180	132	126	25	10	200	114	69	32
4	....	....	....	....	....	10	215	142	155	29	10	283	124	84	37
5	....	....	....	....	....	12	330	145	195	31	12	380	110	126	45
6	....	....	....	....	....	12	340	145	252	33	12	390	130	126	46
8	....	....	....	....	....	12	380	160	310	36	12	455	135	168	51
10	....	....	....	....	....	12	560	160	390	45	12	570	140	210	57
12	....	....	....	....	....	....	....	....	....	....	12	900	130	126	57
16	....	....	....	....	....	....	....	....	....	....	12	967	270	336	61
20	....	....	....	....	....	....	....	....	....	....	12	1375	280	240	77

* Spur-gear hoists—12-, 16- and 20-ton—have 2 operating chains. The pull on chains and the amount of chain overhauled is the total for the 2 chains.

POWER HOISTS

Pneumatic Hoists are used for short lifts in places where compressed air is already installed. The movable hoist is limited by the flexible air connection to a radius of about 25 ft. The direct-acting hoist is made with stroke from 4 to 10 ft. or more, but with rope and sheaves may be geared up to 6 : 1 or more. Fig. 37(*a*) shows the direct-acting hoist, consisting of a cylinder *d*, piston *e* and piston rod *f*, on the end of which is a hook for the attachment of the load. This hoist is made both single- and double-acting. The single-acting hoist is hung vertically, the upper end of the cylinder being open to the atmosphere, and air under pressure admitted at the lower end, lifts the piston and load. The double-acting or balanced-pressure hoist has the motive end of the cylinder under constant air pressure and the balancing end subject at will to a variation of from full to no pressure. The force exerted by the hoist is due to the difference in pressure on the two sides of the piston. As the area of the piston rod makes the effective area on the two sides unequal, the piston is forced toward the stuffing box when the air pressure is equal on the two sides. The double-acting hoists may be used vertically or horizontally. Fig. 37(*b*) shows the reeving for a horizontal cylinder lifting its load vertically. Fig. 37(*c*) shows a rope-geared hoist which may be reeved 2, 4 or 6 to 1. The hoist is operated by chains *h*, controlling valve *g*. Special automatic attachments are supplied when the load is to be suspended for a period of time, and speed boxes are used to govern the speed of stroke independent of the operator. Check valves are placed in the supply pipe to maintain pressure in case of failure of air supply. Lubrication is admitted through the air supply. Table 3 gives clearance dimensions, capacities and air consumption,

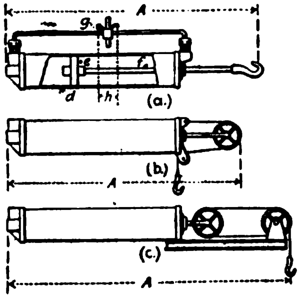


FIG. 37.—Direct-acting Hoist.

Table 3. Direct-acting Pneumatic Hoists

(Curtis Type)

(4-ft. stroke. Air consumption based on 80 lb. pressure with no allowance for leaks or slip)

Nominal diam. of hoist, in.	Vertical and horizontal hoists, Fig. 37 (*a*)				Double-acting rope-geared hoist, Fig. 37(*b*)			Double-acting rope-geared hoists, Fig. 37(*c*)								
								Rope-geared 2 : 1			Rope-geared 4 : 1			Rope-geared 6 : 1		
	Capacity, lb. (10 per cent. friction)	Cu. ft. free air to lift hook 1 ft.	A, in., for vert. hoists	A, in., for hor. hoists	Capacity, lb. (20 per cent. friction)	Cu. ft. free air to lift hook 1 ft.	Approx. length A, in.	Capacity, lb. (25 per cent. friction)	Cu. ft. free air to lift hook 1 ft.	Approx. length A, in.	Capacity, lb. (25 per cent. friction)	Cu. ft. free air to lift hook 1 ft.	Approx. length A, in.	Capacity, lb. (32 per cent. friction)	Cu. ft. free air to lift hook 1 ft.	Approx. length A, in.
4	861	0.54	61	58	......	......	....	......	....	....	......	....	....	......	....	....
5	1,356	0.85	63	58	......	......	....	......	....	....	......	....	....	......	....	....
6	2,050	1.22	67	59	900	0.61	72	800	0.61	139	400	0.30	139	250	0.21	134
7	2,791	1.73	67	59	1,200	0.87	75	1,050	0.87	139	525	0.43	139	325	0.29	134
8	3,616	2.24	68	61	1,600	1.12	75	1,450	1.12	145	700	0.56	145	450	0.38	137
9	4,592	2.85	68	61	2,000	1.43	78	1,900	1.43	146	950	0.71	146	575	0.47	142
10	5,636	3.29	72	62	2,500	1.70	81	2,400	1.70	147	1,150	0.82	147	750	0.55	142
12	8,154	5.06	72	62	3,600	2.55	82	3,500	2.53	152	1,400	1.25	148	1,100	0.84	148
14	11,270	7.13	76	64	5,000	3.57	88	4,800	3.57	155	2,400	1.78	153	1,500	1.17	153
17	16,500	10.10	77	65	7,000	5.05	91	7,000	5.05	161	3,500	2.50	154	2,250	1.67	154
19	20,900	12.50	78	65	9,000	6.25	93	9,000	6.25	161	4,500	3.10	154	2,900	2.10	154

The **motor hoist** consists of a small-cylinder air engine driving a hoisting drum through a spur-gear or worm reduction. Where spur gears are used, an automatic brake must be employed to hold the load. Fig. 38 shows the worm-gear type. The air engine *a* drives drum *b* through worm-gear and spur-gear reductions *c* and *d*. The hoist rope is made fast to the drum and carries a hook at the lower end for attachment of the load. Lever *e* attached to the motor coming in contact with the top of the hook, stops the motor, acting as an automatic limit stop. The hoist is operated by chains from the floor. Table 4 gives clearance dimensions.

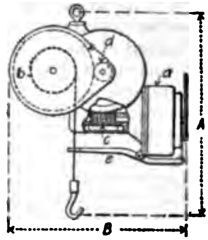


FIG. 38.—Air-motor Hoist.

Electric Hoists are used where rapid and frequent hoisting is required. When attached to a trolley, they are used for moving material in shops, yards, etc., and largely on small cranes as the power unit. They are very flexible and well adapted

Table 4. Pneumatic Motor Hoists

(Ingersoll-Rand Type)

(Air consumption based on 80 lb. pressure, with no allowance for leaks or slip. Dimension letters refer to Fig. 38)

Capacity, lb.	Max. lift, ft.	Feet lifted per min. at 80 lb. pressure	Cu. ft. free air per min.	Shortest distance bet. hooks, *A*	Length in in., *B*	Width in in.	Weight, lb.
1,000	20	32	45	33	25	18	270
2,000	20	16	45	33	25	18	270
4,000	20	8	45	40	33	21	395
7,000	20	8	80	47	40	30	785
10,000	20	7	80	47	40	30	785

Table 5. Data on Electric Hoists

(Pawling & Harnischfeger type. Dimension letters refer to Fig. 39)

Type of current	Capacity, tons	Lift, ft.	Hoist		Trolley		Net weight			Add weight for 30-ft. lift	Dist. between hooks. Hook suspension, in.	*A*, trolley type, in.	*B*, in.	*C*, in.
			Speed, ft. per min.	Motor h.p.	Speed, ft. per min.	Motor h.p.	For hook suspension	With hand-power trolley	With motor-driven trolley					
D. C.	½	13	30-50	2¼	250-300	1	700	900	1100	50	40	39	39	29
D. C.	1	15	23-50	2¼	250-300	1	700	900	1100	50	40	39	39	29
D. C.	2	15	18-40	4	175-225	1	1300	1600	1850	100	51	48	51	36
D. C.	3	15	15-40	4	150-200	1	1300	1600	1850	100	51	48	51	36
D. C.	4	15	14-30	6½	175-225	2½	2000	2600	3000	200	55	52	65	38
D. C.	5	15	13-30	6½	175-225	2½	2000	2600	3000	200	55	52	65	38
D. C.	6	15	12-30	6½	150-200	2½	2000	2600	3000	200	55	52	65	38
A. C., 60 cycle	½-1	15	33-35	3	275	1	700	900	1100	50	40	39	39	29
	2-3	15	18-20	5	150	1½	1400	1700	1950	100	51	48	51	36
	4-6	15	16-18	7½	175	2½	2200	2800	3200	200	55	52	65	38
A. C., 25 cycle	½-1	15	14-15	1¼	175	¾	750	950	1150	50	40	39	39	29
	2-3	15	12-13	3	150	1¼	1400	1700	2000	100	51	48	51	36
	4-6	15	11-12	5	135	2	2250	2850	3300	200	55	52	65	38

to expansion, as each hoist being an independent unit, several may be used on the same trackage. They are limited in capacity to about 6 tons, and can be operated by chains from the floor. If the hoist is to be moved frequently over a considerable distance, it is built in the form of a telpher, the man riding with the hoist. (See p. 1155.)

Fig. 39 shows one of the many types of electric hoists. The motor *a* drives the drum *b* through the gear reductions *c* and load brake *e*. The hoisting cable is made fast to the hoist, passes around the sheave *f*, and is made fast to the drum. The solenoid brake *g* on the motor stops the motor when the current is shut off, thus enabling the operator to place the hook accurately. The type shown in Fig. 39 is carried by the four wheels *h*, moved by the hand wheel *k* and chains *m* through a spur-gear reduction. Current is brought to the hoist by conductors on either side of the rail through the sliding shoes or trolleys *n*. The controller is operated from the floor by chains *p*. These hoists are arranged for hook suspension where used in a stationary position. Other types of electric hoists employ a worm-gear reduction between motor and drum, this type not requiring the load brake, but depending on the worm gear to hold the load. All types are provided with an arm attached to the hoist, so that when the sheave *f* approaches the hoist it will come in contact with arm, throwing out the circuit breaker. Capacities are given in Table 5, together with speeds, sizes of motors, weights and clearance dimensions.

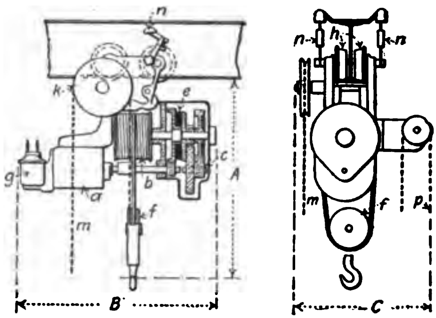


FIG. 39.—Electric Hoist.

PLATFORM ELEVATORS

Platform elevators are used for raising and lowering freight and passengers between the various floor levels in buildings. Five types are in general use: Hydraulic plunger, low-pressure hydraulic cylinder rope-geared, high-pressure cylinder rope-geared, electric drum, and electric traction. The hydraulic plunger type is limited to speeds of 400 to 600 ft. per min., due to difficulty in making accurate stops at higher speeds, and also by the cost of sinking deep wells for plunger cylinders. The low-pressure cylinder type is limited by the space required for cylinders. The efficiency is less and the cost of installation greater than for the high-pressure type. The high-pressure cylinder type is used for high speeds and lifts. The electric drum type is limited to medium lifts, owing to the width and diameter of drums on which cable must be coiled. The traction geared type is used for speeds up to 350 ft. per min. and long lifts. The traction gearless type is used universally for high speeds and long lifts.

Hydraulic Plunger Type, used with water pressure from 140 to 200 lb. per sq. in. In this type the car is lifted by a steel tube plunger, working in a pipe cylinder set in the ground in an outside steel casing. The plunger must be several feet longer than the travel of the car. The cylinder is supplied with water through an operating valve of the balanced-piston type, this valve being controlled from the car by a rope. Limit valves in the supply line and in the exhaust line are operated automatically by ropes when the car reaches the upper and lower limits. Buffers of the spring or oil type attached to the head of the cylinder protect it from shock due to excess speed. A counterweight attached by ropes to the top of the car counterbalances the weight of the car and part of the plunger. The car is raised by the water pressure in the cylinder, assisted by the counterweight, and the descending car must exert sufficient pressure to force the water out of the cylinder. To maintain the lifting pressure of the plunger constant, the weight of the counterweight rope is made to counterbalance the water displaced by the plunger. The plungers are from 5 to 7 in. in diameter, depending on the length. Water pressure may be obtained from city mains or from closed steel pressure tanks six-tenths full of water and four-tenths full of air. The capacity of tank is from ten to twenty times the volume of water used per trip. The pumps are either steam- or electrically driven and provided with automatic regulators to stop or start the pumps when the pressure in the tank reaches its maximum or minimum.

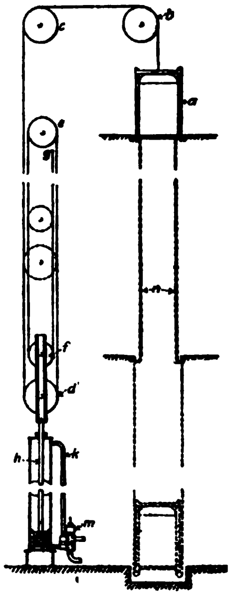


Fig. 40.—Hydraulic Rope-geared Elevator.

Vertical Hydraulic Cylinder Rope-geared Type (Fig. 40). This consists of car *a* suspended by ropes passing over sheaves *b*, *c*, *d*, *e* and *f* and made fast at *g*, near the fixed sheave *e*. Sheaves *d* and *f* are carried by a traveling frame attached to the end of piston rod *h*. To raise the car, water is admitted to the cylinder at the top through pipe *k*, forcing the piston down. To lower, water is allowed to escape from the bottom of the cylinder, the flow being controlled by valve *m*, operated by a running or standing rope from the car. Uniform working stroke is secured by circulating the water from the upper to the lower end of the cylinder and discharging from the latter into an overhead tank or through a suitable valve. This maintains a constant back pressure under the piston and gives better control. Automatic top and bottom limit stops are provided to shut off the water when the limits are approached. Relief valves are placed in the exhaust to allow water to pass to the upper part of the cylinder from the lower part, if the stop is too sudden. The cage is counterbalanced by the heavy piston and traveling sheave, and, if necessary, by a separate weight. The chains *n* attached to the bottom of the cage or counterweight and midway in the elevator shaft, balance the weight of the lifting cables.

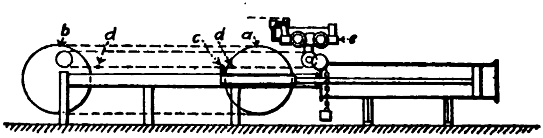


Fig. 41.—Hydraulic Rope-geared Elevator.

Fig. 41 shows the **horizontal cylinder pulling type**, which is located in the basement. The operation is similar to the vertical type, except that a separate counterweight is required to take the place of the traveling sheaves, piston and rod. To raise the car, sheaves *a* are pulled away from sheaves *b*, the reeving being sufficient to give the desired lift. The fork *c* attached to the cross head carrying sheaves *a* automatically limits the length of travel by coming in contact with the stops *d* attached to rope which closes the valve *e*. The **horizontal cylinder pushing type** is similar to the pulling type, except that the fixed sheaves are supported on the back head of the cylinder and the piston connected to the traveling sheave so as to push them apart. Limit stops similar to the pulling machine are used, and rubber-buffers on the piston and on the cross head coming in contact with a projection of the cylinder head reduce shock in case of overtravel. Either of these types may be mounted in double or single deck to economize floor space. The usual reeving is 8 to 1, although this may be made as low as 4 to 1 or up to 12 to 1.

High-pressure Elevators, Fig. 42, are similar in operation to the low-pressure type, but designed for water pressures of from 700 to 800 lb. per sq. in. This reduces the size of piping and cylinders and makes them suitable for high-speed equipment. The elevator is shown with a 6 to 1 reduction, but this may be from 4 to 1 to 8 to 1. Rope *a*, attached to the car, passes around the traveling sheaves *b*, *c* and *d* and the fixed sheaves, *e* and *f*, and is made fast at *g*. The cylinder *h* is inverted, the piston *k* being forced down by the pressure. The required counterweight is attached to the frame of the moving sheaves. The pump *m* forces the water into the accumulator *n*, from which it passes to the cylinder *h* through high-pressure valve *p*. The discharge water passes from the cylinder to the main discharge reservoir *r* through valve *p*. Valve *p* is operated by the low-pressure valve *s*, which in turn is controlled by the pilot valve under pressure from the water from reservoir *r*. The plunger operates under a pressure of from 700 to 800 lb., but the valve *s*, which is controlled by rope from the car, operates under a pressure of from 80 to 100 lb. The discharge from the pilot valve is piped to open reservoir *t* and returned by pump *v* to the main discharge system.

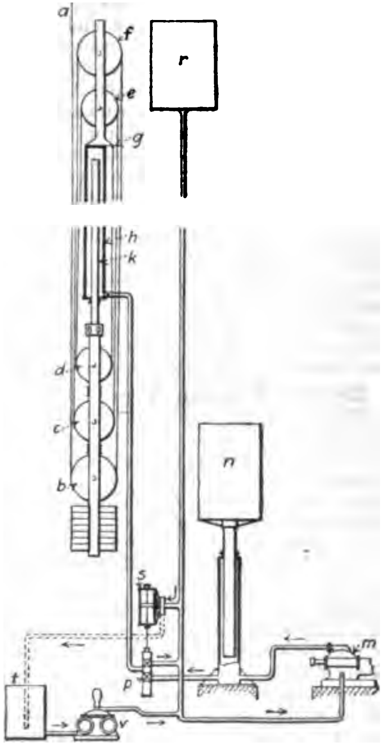


FIG. 42.—High-pressure Hydraulic Elevator.

Electric Drum Type. This type may have hoists mounted at the top of the shaft or in the basement. They may be single- or double-geared. In the double-worm-gear machine the motor drives a shaft carrying a right- and a left-hand worm; these worms mesh with worm gears, which also mesh with each other. This arrangement eliminates the end thrust due to the worms. The drum is driven from one of the gears by arms extending from the gear engaging similar arms on the drum, thus applying power to the drum near its face without torsional strains in the shaft. A solenoid brake on the worm shaft is applied by a spring and held off by a magnet operated by the line current. A magnet control system is used automatically to accelerate and retard the car, so that the brake is not used until the car has come nearly or quite to a stop. Compound-wound direct-current or slip-ring alternating-current motors may be used. The cars are overbalanced by a counterweight attached to cables made fast to the drum on the opposite side from the hoisting rope. For very heavy service, where the tooth pressure becomes excessive, the hoists are driven by two motors. Limit stops of a mechanical type attached to the winding drum limit the travel of the car. Additional limit switches placed in the hatchway are operated by the car itself or the counterweight, should they run beyond their limits. These switches cut off the current and apply the brake. Potential switches cut off the current in case of failure of the supply. Slack cable switches cut off the current and apply the brake, should the car or counterweight be arrested in their travel by the regular safety devices or by obstructions, thus preventing the cables from unwinding on the drum. The hand-operated emergency switch in the elevator cuts off the current to the motor independent of all the automatic devices.

Electric Traction Type. This latest development in elevator construction, shown in Fig. 43, is used for very high lifts and high speeds. It consists of a slow-speed motor *a*, connected to traction sheave *b* through a rigid form of connection described under the drum type. The ropes from car *d* pass over sheave *b*, over sheave *c*, back over *b*, and are made fast to counterweight *e*. The solenoid brake *f* is attached to the shaft of the motor and sheave. The motor is of the special slow-speed shunt-wound direct-current type, with magnet controller for the variable speed and electric braking so that the brake *f* acts only after car stops. An inherent safety feature is due to the reduction in the tractive effort of the sheave when car or counterweight reach the end of travel and the cables become slack. On very-high-lift elevators, such as used in the Woolworth Building, New York City, elevators of this type may have the bottom of the hatchways enclosed in air-tight chambers to act as a cushion to prevent a rapid drop. Traction-type elevators for car speeds approximately 450 ft. per min. are also used rope-geared 2 to 1 for both car and counterweight. Traction types are built with worm-gear drive for car speeds of 350 to 400 ft. per min. This type uses a high-speed motor with drive similar to the drum machine.

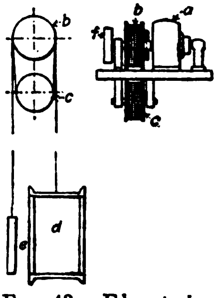


FIG. 43.—Electric Traction Elevator.

Efficiencies. The approximate efficiency of hydraulic apparatus is as follows: E_1, hydraulic cylinders, 75 to 90 per cent.; E_2, friction loss in piping and valves, 5 to 6 per cent. for high pressure and 10 to 30 per cent. for low pressure; E_3, the load efficiency of elevators, which depends upon the unbalanced weight of the car, etc. $= eL/(L + U)$, where L = live load, U = unbalanced weight, e = mechanical efficiency of elevators, sheaves, etc. = 90 to 95 per cent. The total efficiency from pump to work done $= E_1 \times E_2 \times E_3$. Some comparative total percentage efficiencies are as follows: Vertical high-pressure, 55 per cent.; vertical low-pressure, 52 per cent.; horizontal low-pressure, 44 per cent.; plunger low-pressure, 40 to 45 per cent.

For electric elevators of the drum or geared traction type, the mechanical efficiency will range from 60 to 72 per cent.: for the direct-connected gearless type, about 75 to 85 per cent. The total efficiency of drum geared electric elevators is from 50 to 55 per cent., and of the gearless traction type, 55 to 65 per cent. Average motor efficiencies given by T. E. Barnum, *Proc. A. I. E. E.*, vol. 30, are as follows (per cent.): ½ load and maximum speed, geared traction, 90 per cent.; gearless traction, 76 per cent. ½ load and normal speed, geared traction, 78 per cent.; gearless, 58 per cent. ¾ load and maximum speed, geared traction, 91 per cent.; gearless, 80 per cent. ¾ load and normal speed, geared traction, 81 per cent.; gearless, 67 per cent. Normal speed = ½ maximum speed.

Speeds. Hydraulic plunger elevators have a range of speed of from 300 to 600 ft. per min., but are not well suited to high speeds. Hydraulic cylinder elevators operate up to 600 ft. per min., for both the high- and low-pressure types. Electric elevators of the drum type travel as high as 400 ft. per min. but usually from 250 to 350 ft.; worm-geared traction type up to 400 ft. per min. and rope-geared up to 450 ft. per min.; gearless traction type, from 400 to 700 ft. per min. Elevators running at from 600 to 700 ft. per min. are express elevators, while those making frequent stops run from 400 to 600 ft. For ordinary freight service the speed is from 50 to 250 ft. per min.

Loads and Weights. The live load for passenger elevators is usually from 75 to 80 lb. per sq. ft., and the **weight of** the **car** itself from 100 to 125 lb. per sq. ft. of platform. For hydraulic elevators of the plunger type, the **counterweight** is usually taken equal to 75 or 80 per cent. of the weight of the car and plunger. For the cylinder type machines, the unbalanced weight is from 500 to 1500 lb., depending on the speed of operation required and ease of handling; for freight service, 500 lb. up, and for passenger service, 800 to 1500 lb. For electric elevators, the counterweight is usually equal to the weight of the car plus 40 to 50 per cent. of the live load.

Capacity of freight elevators, from 1000 to 10,000 lb.; of passenger elevators, from 2000 to 4000 lb. **Power** requirements depend upon the speed, capacity, etc. Power is usually required for motion in one direction only for hydraulic elevators, but for electric it may be required in both directions. Let L = weight of live load, lb., U = unbalanced weight of car, lb., S = speed of car, ft. per min., and e = efficiency of complete elevator, per cent. Then h.p. $= 100(L + U)S/33{,}000\,e$. In determining the horse power required to operate hydraulic elevators, allow for power on upward trip only, and for an approximate result compute ⅔ of the amount required to operate all elevators at one time under full load. For electric elevators, power may be required in both directions, depending on direction and amount of travel. Compute power for 70 per cent. of amount required to raise all elevators at the same time, assuming 70 per cent., as they will not all be running all the time. Operation may be such that they will all lift full loads and return empty, depending on the time of day. For hydraulic elevators, the steam required per car-mile, using a flywheel type pump, ranges from 150 to 250 lb., based on 25 lb. of steam per i.h.p. of the pumps. Other types of pumps in proportion. For electric elevators, the car-mile energy consumption ranges from 3.5 to 8 kw.-hr., depending on the frequency of stops.

TRAVELING CRANES

A traveling crane consists of a steel bridge carried by wheels on the ends, traveling on elevated track. The bridge carries the hoisting unit which may be hand power, pneumatic or electric. The hand-power crane is limited to a capacity of 50 tons and a span of 40 ft., where the duty required is slow and infrequent. The pneumatic crane is used for short lifts where electricity is not available and where air is already installed for other purposes. The electric crane is built in capacities up to 200 tons and with spans up to 125 ft. Larger capacities and longer spans may be built, but are seldom required in present engineering practice.

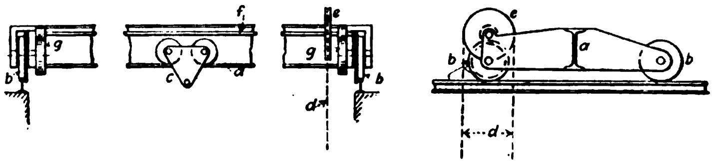


Fig. 44.—Hand-power Crane.

Hand-power Crane (Fig. 44). In its simplest form this consists of an I-beam *a* supported by four wheels *b*. The trolley *c* traveling on the lower flanges carries the chain hoist forming the lifting unit. The crane is moved by hand chain *d* turning pocket wheel *e* keyed to shaft *f*. The pinions on shaft *f* mesh with the gears *g* keyed to the axles of the two wheels. For heavier duty and longer span, the bridge is constructed of two I-beams side by side, a four-wheel trolley running on rails secured to the upper flanges. Fig. 45 shows a crane of this type operated by a winch supported by a structural frame hung from the bridge. One end of the hoisting chain is made fast to the end of the bridge, passes over sheave *a* in the trolley, around the pendant sheave *b* carrying the swivel hook, over sheave *c* in the trolley, around *d* on the end of bridge, and is then made fast to the drum of the winch. The trolley is moved by a separate winch.

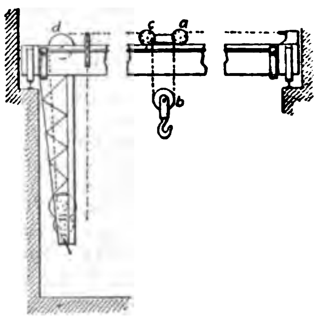


Fig. 45.—Heavy-duty Hand-power Crane.

Electric Traveling Crane (Fig. 46). This consists of two bridge girders *a*, on top of which are rails on which travels the self-contained hoisting unit *b*. The girders are supported at the ends by trucks with two or four wheels, according to the size of the crane. The crane is moved along the track by motor *c*, through shaft *d* and gearing to the truck wheels. Suspended from the girders on one side is the operator's cab *e*, containing the controller and operating levers. The resistance boxes are usually suspended under the floor of the cage. The bridge girders for small cranes are of the I-beam type, but on the longer spans box girders are used to give lateral stiffness. The girders are rigidly attached to the end truck framing, which carries the double-flanged wheels for supporting the bridge. The end frames project over the rails so that in case of a broken wheel or axle, the frame will rest on the rail, preventing the crane from dropping. One wheel axle on each truck is fitted with gears for driving the crane. A powerful foot brake on shaft *d* enables the operator in the cab to stop the crane.

The self-contained hoisting unit consists of a motor-driven truck carried by double-flanged wheels geared direct to the moving motor. This truck carries the hoisting drums which are geared direct to their motor, being provided with solenoid brake, together with a load brake to hold the load, should the current fail. The larger sizes of cranes have auxiliary hoists of smaller lifting capacity carried by the truck of the main hoist. The drums are spiral-grooved, the rope being made fast at each end of the drum and passing around sheaves in the pendant block and on the trolley truck. As the load is lifted, the ropes wind on the drum toward the center so that the load on each of the bridge girders is the same. Limit switches are provided to stop the motors when the limits of travel are reached. Current is brought to the crane by sliding contacts from trolley wires strung along the runway girders on the cab side. Current from the cab to the hoisting truck is carried by wires attached to the girders with sliding contacts carried by the truck.

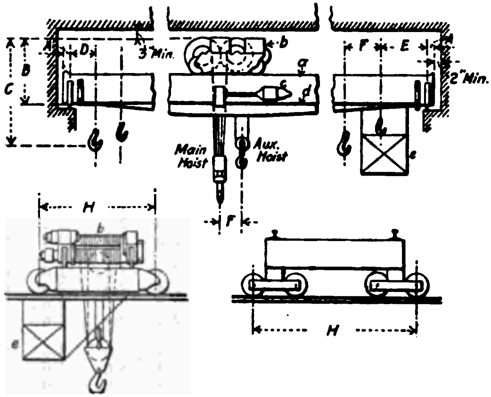


Fig. 46.—Electric Traveling Crane.

Table 6. Typical Capacities and Speeds of Electric Traveling Cranes for General Service

Capacity, tons (2000 lb.)	Hoisting speed, ft. per min.	Bridge travel speed, ft. per min.	Trolley travel speed, ft. per min.	Capacities of auxiliary hoists, tons	Speed of aux. hoists, ft. per min.
5	25-100	300-450	100-150	....	....
10	20-75	300-450	100-150	3	30-75
15	17-60	300-400	100-150	3 or 5	40-125
20	12-50	250-350	100-150	3 or 5	40-125
25	10-40	250-350	100-150	3-5 or 10	25-125
30	10-35	250-350	100-150	5 or 10	25-100
40	9-30	250-350	100-150	5 or 10	25-100
50	8-30	200-300	100-150	5 or 10	25-100
60	8-30	200-300	100-150	10 or 15	20-60
75	6-25	200-250	100-150	15	20-50
100	5-18	200-250	100-150	20	20-50
125	5-15	200-250	100-150	25	20-50
150	5-15	200-250	100-150	25	20-50

Electric cranes are built for either alternating current or direct current. The motors for both kinds of current are of special slow-speed types designed particularly for crane service. Direct-current motors are series-wound and alternating-current are of the slip-ring type. The usual voltage is 220.

Table 7. Typical Clearances and Loadings for Electric Traveling Cranes*

Capacity, tons	Span, ft.	Letters refer to Fig. 46							Maximum load for each wheel, lb.	Runway rail, lb. per yd.
		A, in.	*B*, ft. in.	*C*, ft. in.	*D*, ft. in.	*E*, ft. in.	*F*, ft. in.	*H*, ft. in.		
5	40	8	5-5	5-3	2-3	2-5	....	9-10	13,300	50
	60	8	5-1	5-3	2-3	2-5	....	10-0	15,300	50
	80	8	5-1	5-3	2-3	2-5	....	11-4	18,500	50
	100	9	5-5	5-3	2-3	2-5	....	14-3	23,300	50
10	40	9	5-10	6-0	2-7	2-6	....	10-4	18,000	50
	60	9	5-9	6-0	2-7	2-6	....	10-9	21,500	50
	80	9	5-11	6-0	2-7	2-6	....	11-7	26,500	60
	100	9	5-11	6-0	2-7	2-8	....	14-4	32,000	60
15	40	9	6-0	7-1	2-0	2-2	3-0	11-5	30,000	60
	60	9	6-0	7-1	2-0	2-2	3-0	11-9	32,500	60
	80	9	6-1	7-1	2-0	2-2	3-0	12-1	35,500	60
	100	9	6-6	7-1	2-0	2-2	3-0	14-5	42,000	70
20	40	9	6-7	8-1	2-3	2-6	2-6	11-5	35,300	60
	60	9	7-0	8-1	2-3	2-6	2-6	11-10	39,500	70
	80	9	7-1	8-1	2-3	2-6	2-6	12-2	43,700	70
	100	10	7-1	8-1	2-3	2-9	2-6	14-4	50,800	80
30	40	10	7-4	8-11	2-9	3-0	3-0	12-5	50,800	80
	60	10	7-4	8-11	2-9	3-0	3-0	12-7	54,500	80
	80	11	7-10	8-11	2-9	3-0	3-0	12-11	59,300	80
	100	11	7-10	8-11	2-9	3-0	3-0	14-6	67,500	80
40	40	11	7-11	9-11	3-0	3-3	3-5	12-10	67,000	80
	60	11	7-11	9-11	3-0	3-3	3-5	13-2	71,500	80
	80	13	8-7	9-11	3-3	3-3	3-5	14-0	78,500	100
	100	13	8-7	9-11	3-3	3-3	3-5	14-4	86,000	100
50	40	13	8-6	10-5	3-3	3-3	3-5	13-7	80,000	100
	60	13	8-7	10-5	3-3	3-3	3-5	13-9	84,500	100
	80	13	8-7	10-5	3-3	3-3	3-5	14-0	90,000	100
	100	13	8-7	10-5	3-3	3-3	3-5	14-6	97,500	100
60	40	13	9-9	12-2	3-9	3-8	4-0	16-0	97,300	100
	60	13	9-9	12-2	3-9	3-8	4-0	16-3	102,000	100
	80	13	10-6	12-11	4-0	4-0	4-5	12-4	52,000	150
	100	13	10-6	12-11	4-0	4-0	4-5	12-4	54,000	150
75	40	13	11-6	15-0	4-6	4-6	4-1	16-0	55,000	100
	60	13	11-6	15-0	4-6	4-6	4-1	16-0	60,000	150
	80	13	11-6	15-0	4-6	4-6	4-1	16-0	64,000	150
100	40	17	13-6	15-6	4-1	4-1	Special	16-0	83,000	150
	60	17	13-6	15-6	4-1	4-1	Special	16-0	86,000	150
	80	17	13-6	15-6	4-1	4-1	Special	16-0	89,000	150
150	40	17	15-6	18-8	6-0	6-0	Special	18-0	130,000	150
	60	17	15-6	18-8	6-0	6-0	Special	18-0	134,000	150
	80	17	15-6	18-8	6-0	6-0	Special	18-0	139,000	150

* 60-ton cranes (over 60-ft. span) and larger have 4-wheeled trucks; smaller sizes have 2-wheeled trucks.

The **efficiency** of electric cranes ranges for **the motor** from 65 to 90 per cent., and for the mechanical part from 70 to 80 per cent., giving a combined efficiency of from 45 to 72 per cent. between the crane hook and the switchboard. The **lifting speed** for hand-power cranes is from 2 to 25 ft. per min.; for pneumatic cranes, from 5 to 35 ft. The speeds of electric cranes, for general service, are given in Table 6. The speed of bridge travel in ft. per min. should not exceed the length of the runway plus 100 ft. The **capacities**, clearance dimensions, wheel loadings, etc., for standard electric cranes are given in Table 7. These figures should be used for preliminary work only, as the data vary with different manufacturers.

BRIDGE CRANES

Gantry Cranes are modifications of traveling cranes used out of doors where it is not convenient to erect an overhead runway. The bridge, Fig. 47, is carried at the ends by the legs *a*, which may be bolted to foundations or mounted on wheels so that the crane may travel. The bridge carries a hoisting unit, the same as the traveling crane, but covered to protect the machinery from the weather. The crane is driven by motor *b* through a gear reduction to shaft *c*, which drives the vertical shafts *d* through bevel gears. Bevel-gear and spur-gear reductions connect the axles of the wheels with shafts *d*. Gantry cranes are made in the same sizes as standard traveling cranes. They are, however, usually built to suit local conditions.

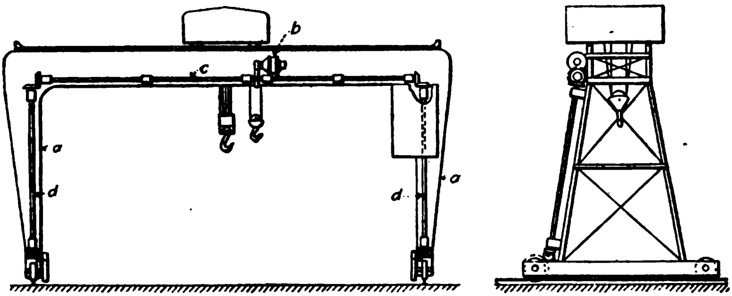


FIG. 47.—Gantry Crane.

Bridge Storage Cranes are used to unload material in bulk, such as coal, ore, stone, gravel, sand, etc., from open-top cars and vessels and to place it in open storage piles; also to reclaim material from storage piles and deposit it in cars or vessels. Their capacity is limited by the size and speed of the self-filling grab bucket carried by the bridges. These buckets are from 2 to 10 tons capacity, the average being from 2 to 5 tons. The span of the bridges varies with local conditions, one in use being 500 ft. from c. to c. of supporting towers.

Storage bridges consist of a trussed steel span with a cantilever at one or both ends, as required by local conditions. When used for unloading material from vessels, the cantilever end is hinged so that it may be raised to clear the masts of vessels and be housed inside of the dock line. The bridge span is flexibly mounted on the supporting towers to prevent distortion, should one end run ahead of the other. Fig. 48 shows a bridge with an inverted tower. A king pin is provided at *a*, and sliding contacts, well greased, between the span and the tower, to carry the load. The other end is carried by the A-frame, the span being hung by the links *b* from the lower chord, a pin being provided to take the wind load. The hoisting unit is carried by wheels resting on tracks *c*, and is usually of the man-trolley type consisting of a carriage containing the hoisting drum, motors, controlling apparatus and the operator for the grab bucket. The trolley is propelled by motors geared to the truck wheels. The hoisting unit may also be of the rope-driven type, in which the hoisting drums with their motors and other apparatus are fixed in one of the towers, the bucket and trolley being moved by wire ropes.

Fig. 49 shows a bridge in which the tower spans one or more railroad tracks, giving it a wide base. It is connected to the bridge span through a king pin, the load being carried by well-greased bearing plates. The other end is carried by an A-frame, the bridge being provided with a girder and socket resting on the ball on top of the A-frame. The bridges are mounted on wheels, the trucks being pin-connected so as to take up inequalities in the

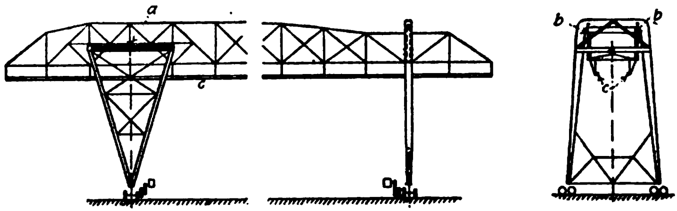


FIG. 48.—Bridge Crane with Inverted Tower.

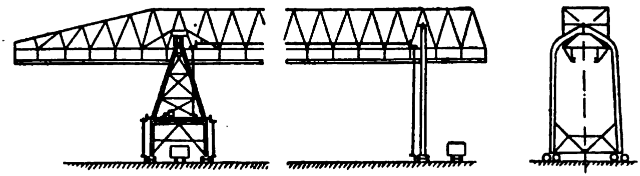


FIG. 49.—Bridge Crane with Shaft-driven Trucks.

tracks. They are propelled in various ways, the most usual drive consisting of motors geared to the truck-wheel axles. Two drum-type controllers are used, each controlling the motors at one end of the bridge. Indicators are provided so that the operator at all times knows the relative position of the two ends, and the two controllers enable him to move the ends of the bridge at the same speed. One manufacturer gears the motors to the axles of the truck wheels by worm-gear and spur-gear reductions, the worm gears acting as an automatic lock when the bridge is stationary.

Fig. 49 shows the shaft-driven bridge, in which the truck-wheel axles are driven through spur-gear and bevel-gear reductions by a vertical shaft from a motor at the tower end. The shear-leg end of the bridge is propelled by a shaft (carried by the bridge span) through bevel gears. Universal joints are used in the shaft at the tower and shear-leg ends to compensate for the movement at these points. A jaw clutch is provided so that the bridge may be lined up by moving one end while the other end is stationary. One manufacturer employs cables anchored at the ends of the bridge travel, passing over motor-driven drums on the bridge trucks. This method pulls the bridge along independent of the friction of wheels on rails, solenoid motor brakes locking the bridge when not in motion. The two ends are handled by separate controllers.

Capacity of storage bridges, from 100 to 500 tons per hour, depending on the size of the buckets, motor equipment, etc. **Speed** of bridge traverse, from 50 ft. to 200 ft. per min., depending upon the service; speed of the man trolley, from 500 to 1500 ft. per min.

ROTARY CRANES

Rotary cranes are used for lifting material and moving it to points covered by a boom pivoted to a fixed or movable structure. Derricks are used out of doors, in quarries, for construction work, etc., being built so that they may be easily moved. Pillar cranes always are fixed and used for light, infrequent service. Jib cranes are used in manufacturing plants. Locomotive cranes mounted on car wheels are used to handle loads by hook, or bulk material in tubs or grab buckets. Wrecking cranes are of the same general type as locomotive cranes and are used for handling heavy loads on railroads.

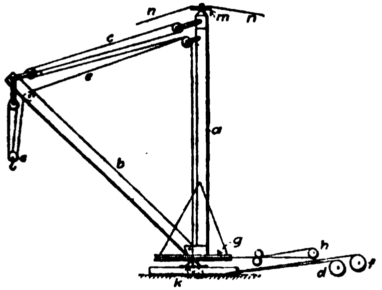


Fig. 50.—Guyed Wooden Derrick.

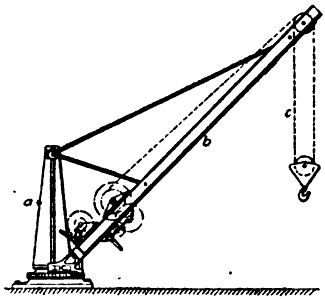


Fig. 51.—Pillar Crane.

Derricks are made with either wood or steel masts and booms of the guyed or stiff-leg type, and are either self-slewing or swung with a bull wheel. Fig. 50 shows a guyed wooden derrick of the bull-wheel type. The mast *a* is carried at the foot by pivot *k*, and at the top by pivot *m*, held by rope guys *n*. The boom *b* is pivoted at the lower end of the mast. The rope *c*, passing over sheaves at the top of the mast and at the end of the boom and through the pivot *k*, is made fast to drum *d*, and varies the angle of the boom. The hoisting rope *e*, from which the load is suspended, is made fast to drum *f*. The bull wheel *g* is attached to the mast and swings same by a rope made fast to the bull wheel and passing around the reversible drum *h*. In derricks of the self-slewing type the engine is mounted on a platform attached to the mast and the derrick is swung by a pinion meshing with a gear attached to the foundation. Either the bull-wheel or self-slewing type may be made of steel or wood construction and of the guyed or stiff-leg type.

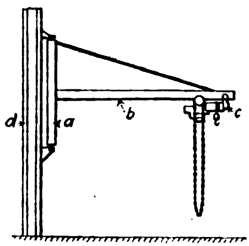


Fig. 52.—Column Jib Crane.

Fig. 51 shows a fixed-radius **pillar crane**, largely used for unloading cars in railroad yards, and usually of the hand-power type. It consists of a pillar *a*, bolted to a foundation, carrying the boom *b*, which may be swung around a full circle. The load is lifted by ropes *c* operated by a hand winch. Fig. 52 shows a **column jib crane** consisting of pivoted post *a*, carrying boom *b*, on which travels the hoist *c*. The post *a* is attached to column *d* so that it may swing through approximately 270 deg. The **yard jib crane**, illustrated in Fig. 53, consists of mast *a*, carrying fixed boom *b*, on which travels the hoist *c*. The mast is carried by a pivot bearing *d* at the bottom, and bearing *e* at the top, held by guys *f*. The crane is swung by gear *g* attached to the foundation through pinion *h* from motor carried by the mast.

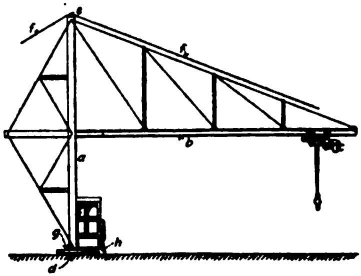


FIG. 53.—Yard Jib Crane.

Fig. 54 shows a **locomotive crane** traveling on standard-gage track and self-propelled by bevel gears from the crane engine. These cranes require but one operator and are usually steam-driven, the boiler, engine and boom being carried by a turntable so that they may turn through a full circle. A two-drum engine is used when lifting loads with a hook, one drum raising and lowering the boom, the other lifting the load. When a two-rope grab bucket is used, an extra drum is provided on the engine. These cranes propel themselves at about 7 or 8 miles per hour, and in addition to handling the material are used for shifting cars. Table 8 gives general data on locomotive cranes. Fig. 55 shows a **wrecking crane** which travels on standard-gage track, the engine, boiler and boom being mounted on a turntable so that they may swing through a full circle. Two hoists are

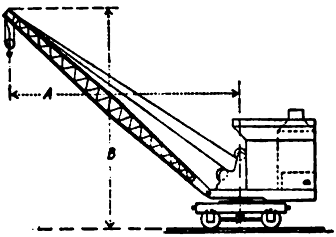


FIG. 54.—Locomotive Crane.

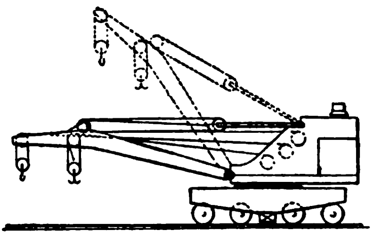


FIG. 55.—Wrecking Crane.

usually provided, one in the end of the boom and one midway between the pivot and end. These cranes are provided with outriggers so that very heavy loads may be handled without overturning the cranes. Table 9 gives general data on this type of crane.

Capacities of Rotary Cranes. Wooden derricks are made with booms from 24 to 80 ft. in length and from 3 to 20 tons capacity. Steel derricks are made with booms from 50 to 100 ft. long and from 5 to 50 tons capacity. The booms operate at an angle of 15 deg. to 45 deg. from the horizontal. Fixed-radius pillar cranes are made with 15-ft. radius to handle from 1 to 30 tons; 20-ft. radius, 2 to 20 tons; 25-ft. radius, 2 to 15 tons. Column jib cranes are made up to 20-ft. radius for 3-ton loads. Yard jib cranes are designed especially for the work to be done. Capacities of locomotive and wrecking cranes are given in Tables 8 and 9. Wrecking cranes are built with booms up to 40 ft. radius.

Table 8. Data on Locomotive Cranes
(Brownhoist type. Letters refer to Fig. 54)

Size	No. of wheels	Minimum working radius of boom			Usual working radius of boom			Maximum working radius of boom			Approx. weight, lb.	Wheel base, ft. in.	Maximum load on any wheel, lb.
		Radius, ft., A	Height, ft., B	Capacity, lb. at min. rad.	Radius, ft., A	Height, ft., B	Capacity, lb. at usual rad.	Radius, ft., A	Height, ft., B	Capacity, lb. at max. rad.			
3	4	10	24	10,000	20	17	4,900	25	9	4,000	30,000	5–6	31,000
10	4	12	40	26,300	30	24	7,900	39	11	5,400	88,000	8–0	61,000
15	8	13	41	31,500	30	25	11,400	40	11	7,800	125,000	19–2	35,800
20	8	17	65	31,600	35	46	12,600	50	28	7,600	156,000	19–2	39,000

Table 9. Data on Wrecking Cranes
(Bucyrus Type)

Size	No. of wheels	Lifting capacity, in tons									Approx. weight in pounds, self-propelling	Size of engine, double-cylinder, in.
		with all outriggers			with end outriggers only			without outriggers				
		17-ft. rad.	20-ft. rad.	25-ft. rad.	17-ft. rad.	20-ft. rad.	25-ft. rad.	17-ft. rad.	20-ft. rad.	33-ft. rad.		
115	8	115	75	54	56	43	31	18	15	7	195,000	10 × 12
150	8	150	110	75	85	64	45	24	18	9	231,000	12 × 12

EXCAVATING MACHINES

Steam Shovels are used for excavating on dry land such loose material as sand, earth, ore, blasted rock, etc., and placing it in cars or wagons so that it may be moved to the dumping point. Shovels are also used to deposit excavated material directly on dikes, waste piles, etc. They are made in two types: the revolving-shovel, in which the dipper and machinery revolve 360 deg. on a turntable, and the railroad type, where the dipper and boom alone revolve through approximately 180 deg. The smaller machines may be mounted on traction wheels for use in cellar excavation, road grading, trench work, etc.. Larger sizes travel on standard-gage track or are equipped with caterpillar traction, and are used in railroad construction, in stripping overburden from coal or ore mines, loading material into cars, canal construction, etc. Shovels discharging to spoil banks are provided with booms 60 to 75 ft. long. Steam shovels cannot excavate under water, as they stand below the level of the material they are digging. Except in the case of long-boom shovels, they are dependent upon some method of transportation to move the excavated material. Steam shovels consist of a structural-steel frame, similar to that of a car, which is supported on traction wheels, caterpillar traction, or railroad trucks, as the size and use determines. This frame supports the power plant and the boom for carrying the dipper and dipper beam. Fig. 56 shows a typical shovel of the railroad type. The cutting edge of the dipper is usually provided with teeth, and is pulled through the material by a rope or chain passing over end of the boom and operated by a drum, driven by double-cylinder engine *a*. The double-cylinder thrusting

engine *b*, mounted on the boom and operated independently, forces the beam, to which the dipper is attached, in or out. In this way the dipper is held into the work while the cutting edge is being pulled through the material by engine *a*. When the dipper is filled, the swinging engine, attached by chains to the turntable *c*, swings the dipper over the wagon or car to be loaded. Two operators are required, one to control the hoisting and swinging engine, the other located on a platform attached to the boom to operate the thrusting engine and to trip the latch of the door on the bottom of the dipper, which

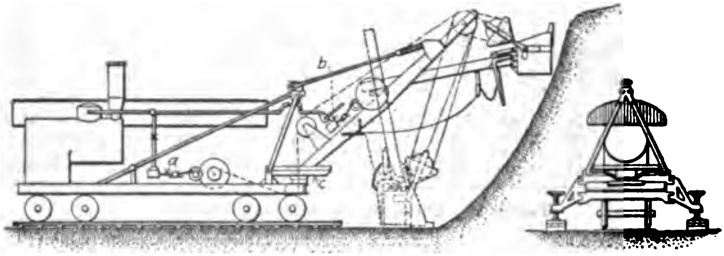


FIG. 56.—Steam Shovel.

allows the material to be discharged by gravity. The revolving shovel consists of the upper frame carrying the boom, boiler, engines, etc., mounted on a turntable so that it may turn through a full circle. The hoisting of the bucket is usually accomplished by chain, but wire rope is used where the digging is comparatively easy and where high speed is desired. Table 10 gives capacity and general data for shovels of different sizes of the chain-hoist type. These shovels make from 3 to 4 cycles per min. when operating at full speed. The wire-rope type will make from 3 to 5 cycles per min. The hoisting and thrusting engines are usually under steam from one-quarter to one-third of the time, and the swinging engine about one-half the time, when operating steadily. The engines are of the high-speed type and the determining factor in their size is not the horse power they develop, but the amount of pull they can exert on the dipper when moving it about 100 ft. per min. The working boiler pressure is about 125 lb. Electric shovels are used to some extent.

Table 10. Data on Steam Shovels
[Chain-hoist (Bucyrus) type]

Type of shovel	Weight, tons	Capacity of dipper, cu. yd.	Size of engines in in.			Boiler		Effective pull on dipper, lb.	Capacity, cu. yd. per hour
			Main	Swing-ing	Thrust	Type	Diam. (in.) × length (ft.-in.)		
Revolving.	18	5/8	5 ×6	4 ×5	4 ×5	Vertical.	40×7-6	12,250	20-50
	23	7/8	6 ×7	4½×5	4½×5		44×7-6	18,250	30-60
	27	1¼	7 ×8	5 ×6	5 ×6		54×8-11	25,500	40-100
Railroad....	42	1½	8 ×8	5½×6	5½×6	Locomotive.	42×13-6	33,000	75-175
	65	2½	10 ×12	7½×7	7½×7		44×17-0	56,000	100-300
	75	2½-3	10 ×14	7½×7	7½×7		44×18-0	64,000	150-350
	89	3 -4	12 ×15	8 ×8	8 ×8		50×18-0	70,000	150-400
	101	3½-5	12½×16	8 ×8	8 ×8		54×18-0	91,000	150-500
	116	3½-6	13 ×16	9 ×9	9 ×9		58×18-3	98,000	150-550

Dipper Dredges are used for excavating material from the beds of harbors, rivers, canals, etc. The excavated material is discharged either on to the bank or on to scows which carry it to the dumping point. The water must be deep enough to float the dredge. Depth of digging limited to approximately 55 ft. The operation of dipper dredges is similar to that of railway-type steam shovels, except the apparatus is mounted on a scow instead of a car, and held in its operating position by spuds. Dredges of this type are built with dippers from 1 to 15 cu. yd. capacity, and are designed especially for the work to be done.

Ladder Dredges are used for excavating sand, gravel, etc., especially gold-bearing gravel in placer mining. They are built to dig to a depth of 75 ft. below water line, and will dig through soft rock. The digging unit consists of an endless chain of buckets passing over tumblers at each end of a ladder pivoted at its upper end, and driven by the upper tumbler; speed, from 12 to 18 buckets per min. Dredges are made with buckets having a capacity of from 3 to 16 cu. ft. each. Table 11 gives the average monthly capacities of placer dredges in the California gold fields.

Table 11. Capacity and Operating Costs of Ladder Dredges

Size of buckets, cu. ft.	3	4	5	7	7½	8	13	13½
Capacity, 1000 cu. yd. per month	34	40	74	81	101	149	230	255
Cost of handling, cents per cu. yd.	8.0	6.5	5.3	4.85	4.34	3.58	2.91	2.41

Drag-line Excavators are used for digging open cuts, drainage ditches, canals, sand and gravel pits, etc., where the material is to be moved from 20

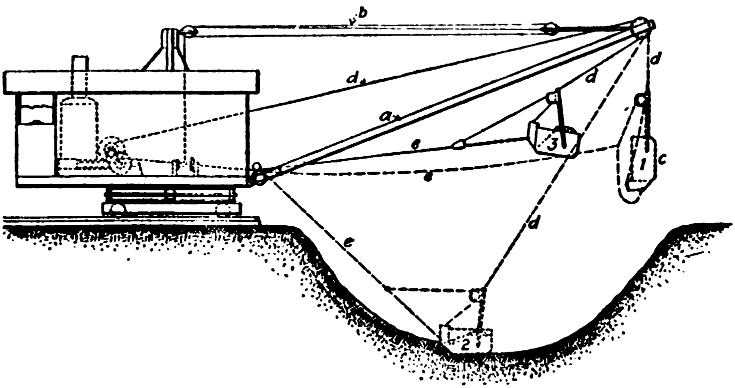


FIG. 57.—Drag-line Excavator.

to 200 ft. before dumping. They cannot handle rock unless blasted. Being provided with long booms and mounted on turntables, permitting them to swing through a full circle, these excavators may deposit material directly on the spoil bank farther from the point of excavation than any other type of machine. While a steam shovel stands below the level of material it is digging, a drag-line excavator stands above, so it may be used to excavate material under water. The bank on which it operates must have sufficient bearing value to support the weight of the machine. Fig. 57 shows a drag-line excavator consisting of a turntable mounted on skids, trucks or caterpillar traction. The turntable supports the boiler and engine. The boom *a* is pivoted at its lower end to the turntable, the outer end being supported by cable *b* so that it may be raised and lowered to the desired angle. The scraper bucket *c* is supported by cable *d* attached to a bale on the bucket, passing over a sheave at the head of the boom and made fast to the engine. A second cable, *e*, is attached to the front of the bucket and made fast to the second drum of the engine. The bucket is shown in three positions. Position 1 is the dumping position, the bucket being suspended from cable *d*, *e* being slack. The bucket is dropped and dragged along the surface of the material by cable *e* until filled (position 2). It is then hoisted by rope *d*, drawn back to its dumping position, *e* being kept tight until the dumping point is reached, when *e* is slacked, allowing the bucket to dump by gravity. After the bucket is filled, the boom is swung to the dumping position, while the bucket is being hauled out. **Speed,** about 1 bucket per min.; under favorable conditions, 2 or even 3 trips per min. are possible. Table 12 gives general data and **capacities** of the various sizes.

Table 12. Data on Drag-line Excavators
(Lidgerwood Type)

Weight, tons	Capacity of bucket, cu. yd.	Working radius of boom elevated 25 deg., ft.	Size of engines		Boiler		Capacity, cu. yd. per hour
			Bucket engine, in.	Swinging engine, in.	Type	Size, in.	
35	1½	54	7 ×10	5 ×6	Vertical.	48×102	75-90
40	1½	64	9 ×10	6¼×8		48×114	75-90
55	2	64	8¼×10	6¼×8		60×114	100-120
90	2	90	8¼×10	8¼×10		60×114	100-120
90	2½	86	9 ×10	8¼×10		60×114	120-150
145	2½	104	9 ×10	7 ×10	Horizontal.	60×114	120-150
90	3	76	10 ×12	8¼×10		66×144	150-180
145	3	104	10 ×12	7 ×10		66×144	150-180
90	3½	67	10 ×12	8¼×10		66×144	180-210
145	3½	104	10 ×12	7 ×10		66×144	180-210

VESSEL-UNLOADING MACHINERY

Bucket-operating machines used to unload bulk material such as coal, ore, stone, etc., from the holds of vessels, have their bucket capacity fixed by the dimensions of the vessel hatches. Mine-run material may be handled, provided the lumps do not exceed about 24 in. in size. The type of unloading device is determined by the nature of the material to be handled and the capacity desired.

Mast and Gaff Unloader (Fig. 58). This is the simplest type of unloader, consisting of a mast *a* set in the ground and supported by stiff legs *b*, and rope guys, where possible. The gaff *c* is pivoted at its lower end to the mast above the crosstrees *d*. The outer end of the gaff is supported by rope *e*, and the angle may be adjusted. The clos-

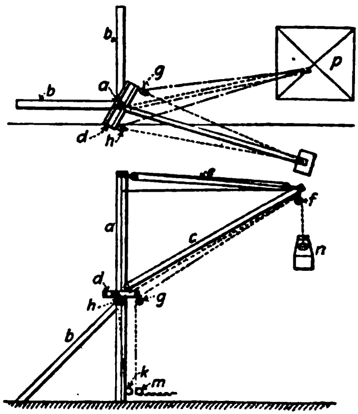


FIG. 58.—Mast and Gaff Unloader.

ing rope from one of the drums of the engine passes over sheave *m*, sheave *g* attached to the crosstrees, and sheave *f* suspended from the end of the boom, and is made fast to the closing drum of the bucket. The hold rope from the other drum of the engine passes over sheaves *k*, *h* and *f*, and is made fast to the head of the bucket *n*. After the bucket is closed, the operator hoists on the closing line, slacks on the hold line, raising the bucket and swinging the gaff over the hopper *p*. After the bucket is empty, the gaff is swung out by putting greater tension on the hold line than on the closing line. These unloaders are used mainly on coal, sand, gravel, crushed stone, etc., and handle from 50 to 60 tons per hour. They cannot unload from boats wider than from 25 to 30 ft.

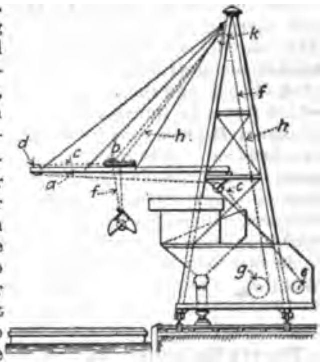


FIG. 59.—Boston Tower Unloader.

Boston Type Tower. The Boston or two-man tower (Fig. 59) consists of a tower of steel or wood, carrying the boom *a*, on the upper flanges of which travels the trolley *b*. The closing line *f* attached to the closing drum of the bucket passes over one sheave of the trolley, over sheave *k* at the head of the tower, and is attached to one of the engine drums *g*. The hold line *h*, secured to the head of the bucket, passes over a sheave of the trolley, over a sheave at the head of the tower and is attached to a drum on the same shaft as *g*. The trolley rope *c* is secured to the front of the trolley, passes over sheave *d* on the end of the boom and is attached to drum *e* of the trolley engine. Two operators are required, one controlling the raising, lowering, opening and closing of the bucket, the other the movement in and out on the boom. The drums of the hoisting and closing engine are provided with frictions. After the bucket is closed, it is raised on the closing and hold lines, the trolley operator slacking on rope *c*, allowing the bucket to run in over the hopper. After the bucket is dumped, the trolley is hauled out by rope *c* and the bucket is lowered into the vessel. Capacity, 75 to 300 tons per hour, depending on size of bucket (rarely over 2 cu. yd.).

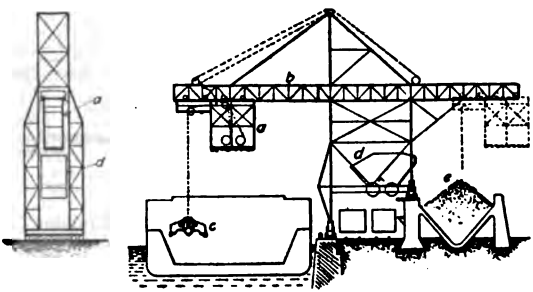


FIG. 60.—One-man Unloader.

One-man Type Unloaders (Fig. 60) are used for the rapid unloading of coal, ore, etc. The steel-frame tower carries a self-contained hoisting unit or man trolley, *a*, which may be moved along the boom *b*. The trolley carries the drums, motors, controllers and operator required for the grab bucket *c*. After the bucket has been filled and hoisted, the trolley is moved along the boom, discharging the material into the hoppers *d*, from which it may be fed to railroad cars; or to the trough *e*, back of the unloader, from which it will be moved to the storage pile by the storage bridges. This constitutes the Hoover & Mason method of handling ore. The boom *b* is hinged so that it may be raised to clear the masts of vessels. Towers of this same general form are used with rope-operated buckets, the drums, motors and controlling apparatus being located in the tower. Fig. 61 shows one method of reeving the ropes for such a tower. The reeving of the closing line is shown at *A*, the bucket being attached to *a*, sheave *b* being on the end of the boom, and sheave *c* carried by an auxiliary trolley which may be moved by rope winding on drum *d*. Diagram *B* shows the holding line which is attached to the head of the bucket, these ropes being reeved the same as the closing line. To close the bucket, drum *d* is revolved, pulling sheave *c* back, closing the bucket without movement of the trolley, as it is in balance. When the bucket is closed, drums *d* and *f* are revolved at the same speed, raising the bucket. To move the trolley, the endless rope, Diagram *C*, is attached to the two ends of the trolley, making several wraps around drum *g*. The trolley is therefore hauled in either direction. Capacity, 100 to 1000 tons per hour, depending on size of bucket (2 to 10 tons for coal, up to 10 tons for ore).

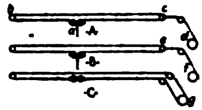


FIG. 61.

Hulett Unloader (Fig. 62). This consists of a bucket *a* attached to arm *b* which is pivoted with a parallel motion from beams *c* and *d*. The bucket is opened and closed by drum *e* on arm *b*, and the arm is raised and lowered

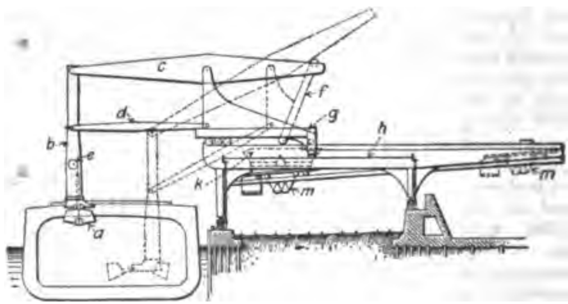


FIG. 62.—Hulett Unloader.

by cable *f* attached to the arm and drawing down the extended end of lever *c*. Levers *c* and *d* are pivoted on traveler *g* so that the whole device may be moved on the framework *h*, permitting the bucket to discharge into hopper *k*. The material from hopper *k* is discharged to the motor-driven car *m*, which travels on the inclined track, carrying the material to the storage back of the machine, where it may be reached by the storage bridges. The car *m* may also discharge into railroad cars running under the unloader. Capacity, 500 to 1000 tons per hour with buckets of from 10 to 15 tons capacity.

Self-unloading Vessels are made having a hopper bottom with a series of gates discharging to two pan or belt conveyors which elevate the material, feeding an elevator which lifts the material above the deck line and discharges to a belt conveyor carried by a swinging boom. The boom carrying the conveyor may be elevated to an angle of 18 deg., and is made from 100 to 150 ft. long. From 500 to 2000 tons may be unloaded per hour.

CONVEYING MACHINERY

General. Conveyors are devices for moving material from one point to another at the same or a different elevation. In **intermittent conveying** the material is moved in a succession of separate loads; in **continuous conveying** the material is delivered in a practically steady stream.

The **angle of repose** of any material is the angle with the horizontal at which material will stand when piled. For anthracite coal it is about 27 deg.; for coke of the same size, about 40 deg. The percentage of fine material in the mass has a decided influence on the angle, as the fines carry the bulk of the moisture. Screened material has an angle of repose of from 35 to 40 deg., depending on the shape, smoothness and method of storing. Thirty-seven degrees has been found to be the average angle for crushed and screened limestone, iron and copper ore, and similar materials. Mine-run soft coal will stand at from 35 to 37 deg.

The **angle of slide** is the angle at which material will flow on an inclined surface. Anthracite coal will flow on steel plate inclined at about 20 deg., coke at about 25 to 30 deg. Ore, stone, etc., will slide at about 30 deg. where the fine material is removed, and at 35 to 40 deg. for mine-run material. It is customary to build chutes on an angle of 45 deg. for such material as coal, stone, ore, etc., this angle being increased when there is a large percentage of fine, damp material. Where gentle handling is required to prevent breakage, special tests should be made to determine the minimum angle of slide.

The **weights of loose materials** commonly handled by conveying apparatus range approximately as follows:

	Lb. per cu. ft.		Lb. per cu. ft.		Lb. per cu. ft.
Ashes	45–50	Coke	26–30	Ore	105–215
Barley	37–40	Earth	75–115	Rye	44–50
Cement	90–118	Gravel	90–135	Sand	75–120
Charcoal	17–27	Lignite	31–47	Slag, blast-furnace	37–63
Clay	95–169	Lime	50–80	Stone, broken	90–120
Coal, lump	50–54	Limestone	90–110	Wheat	44–50
Nut and screenings	53–60	Oats	28–31		

TRANSPORTATION BY DIRECT-IMPELLED VEHICLES

Loading, Hauling and Dumping Material with Carts and Wheelbarrows (Condensed from Taylor and Thompson's "Concrete Costs"). Capacity of double carts, 33⅓ cu. ft.; single carts, 16 cu. ft. Let g = time of shoveling per cu. yd. in minutes per one man (for gravel, 29.5 min.; sand, 19.7 min.); c = time of changing carts at face per cu. yd. in minutes per team (for double carts, 2 min.; single carts, 3.5 min.); d = time of dumping per cu. yd. in minutes per team (for double carts 1.4 min.; single carts, 1.5 min.); n = number of men shoveling; m = ratio of cost of team and teamster to cost of labor of one man (for double carts, 3; single carts, 1.5). Then, **time per cu. yd. loading** material when teamster does not help load =

$$T = (g + nc) + m[(g/n) + c + d],$$

where $(g + nc)$ = time of loading gang reduced to time of one man, and $m[(g/n) + c + d]$ = time of team and teamster waiting during loading, changing carts and dumping.

When teamster helps loading, the time per cu. yd. loading and dumping

$$T = (g + nc) + [(m - 1)\,g/(n + 1)] + m(c + d)$$

On short hauls, the amount of work done depends on the time of loading, as the horses have ample resting time.

Let l = distance hauled one way in hundreds of feet or miles; t_s = actual time of hauling 100 ft. or per mile and return per cu. yd. (= 0.9 min. for 100 ft. and 47.5 min. per mile). Then, **time per cu. yd. loading, hauling and dumping** (for short hauls) when teamster does not load =

$$T_1 = (g + nc) + m[(g/n) + c + d] + m(l \times t_s)$$

When teamster helps load, $T_1 = (g + nc) + [(m - 1)\,g/(n + 1)] + m(c + d) + m(l \times t_s)$.

When teamster loads alone, $T_1 = m(g + c + d) + m(l \times t_s)$

On long hauls, the work a team can do is limited by the distance a horse can travel. The average horse can travel per day 17 miles, 8½ miles hauling a load and 8½ miles with empty cart.

The loading time is not enough to give the horses rest, so that the time of changing carts, loading and dumping does not enter the formula. Let l = distance hauled one way in miles; t_l = time of team per cu. yd. per mile of actual hauling and return plus required time to rest (=56.9 min. based on 17 miles in 10 hr.; 48.35 min. based on 20 miles in 10 hr.; 45.5 min. based on 17 miles in 8 hr.) Then, **time per cu. yd. loading and hauling** where teamster does not help load = $T_2 = (g + nc) + m(l \times t_l)$

When teamster helps load, $T_2 = \left(\dfrac{ng}{n + 1} + nc\right) + m(l \times t_l)$

When teamster loads alone, $T_2 = m(l \times t_l)$

In **transporting by wheelbarrows**, gangs should be arranged to give room for the wheeler to load his own barrow. Should two men load with the wheeler idle, add 35 per cent. to the time and cost of loading; when one man loads with the wheeler, add 25 per cent. In carrying loads up or down a slope, add 5 per cent. for each 4 deg. of slope. Hauling by wheelbarrows is

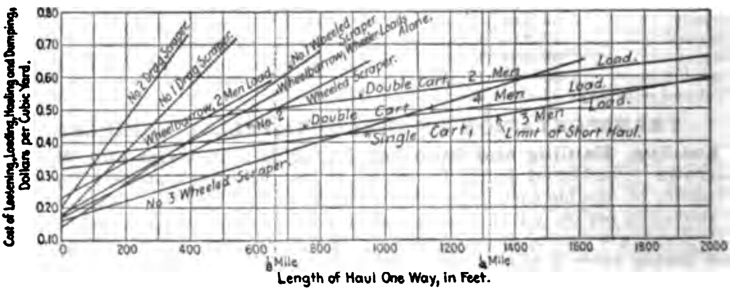


Fig. 1.—Cost of Loading and Hauling Gravel.

more economical than by carts up to a distance of about 250 ft. The capacity of a wheelbarrow is 3 cu. ft. Fig. 1 gives costs of loading and hauling gravel by double carts and wheelbarrows, based on wages of 20 cents per hour.

Scrapers. Drag scrapers are used for excavating and hauling plowed earth or sand when the haul is short. Wheeled scrapers are used for longer hauls where the quantity is greater and in cases where it would be impossible to use dump carts. Fresno or Buck scrapers are used for leveling land and roads, for thin cutting and spreading, and also for dragging dirt down hill, pushing material ahead of the pan, filling ditches, etc. Scrapers should be used in gangs, the number depending on the length of haul and local conditions. Either one or two men are required to load, depending upon the capacity of scraper and material handled. The driver usually dumps without help, but for larger capacities, a helper is frequently used. Drag scrapers are limited to hauls of 400 to 500 ft.; wheeled scrapers to hauls of 800 to 900 ft., but may be used up to 1600 ft. where impossible to use dump carts. Fresno scrapers are limited to short hauls and leveling. All scrapers are limited to easy-cutting material, unless it is plowed first.

The **drag scraper** (Fig. 2) consists of a steel pan *a*, bent from one piece, the bottom being reinforced by a hard-steel wearing plate. The hardwood handles *b* are attached to the pan and the bale *c* is pivoted at the sides and carries a swivel eye, to which the team is attached. To load the scraper, the handles are raised until the cutting edge gathers the load, when they are lowered and the scraper is dragged along the ground. To dump, the handles are raised until the cutting edge catches the ground and the pull of the team throws the pan over.

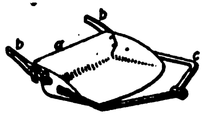


Fig. 2.—Drag Scraper.

The **wheeled scraper**, Fig. 3, consists of an arched axle *a* carrying two wheels of steel or wood with wide tires. The lever *c* is attached to the axle and the scraper pan *d* is hung from lever *c* by straps *e*. The tongue is attached to the pan and carries a pair of hooks *f* connected to lever *c* by straps *g*. To load the pan, lever *c* is raised until hooks *f* engage the lugs at the front of the pan so that the pull is applied to the cutting edge. After the pan is filled, lever *c* is lowered, which lifts the front end of the pan, releasing the hook *f*. The latch *h* engages the back end of the pan, holding it several inches above the ground, in which position it is moved to the dumping point. To dump, the back end of the pan is raised until the cutting edge catches the ground, when the pull of the team turns the pan over. The hook *k* pivoted to the draft gear holds the pan in its dumped position during the return trip. When handling sand, gravel or other loose material, automatic end gates are used to prevent leaking of material from the pans.

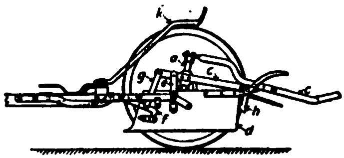


Fig. 3.—Wheeled Scraper.

The **Fresno or Buck scraper**, Fig. 4, consists of pan *a* carrying the runners *b* and lever *c*. The draft attachment *d* is attached to the pan by links. To load, the lever *c* is raised until the cutting edge gathers the desired load. The scraper is then dragged along the ground until the dumping point is reached, when lever *c* is raised until the cutting edge catches the ground, throwing the pan over on the runners. When empty, it travels on the runners.

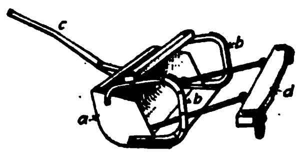


Fig. 4.—Fresno or Buck Scraper.

The speed of scrapers depends on the nature of the ground. For continuous operation, a team of horses can travel about 17 miles per day or about 2¼ miles per hour. Table 1 gives capacities and weights of the various types of scrapers. Fig. 1 gives costs of handling material by drag and wheeled scrapers. Two horses are required for drag and wheeled scrapers, with a snap team to help load large wheeled scrapers. Four horses are required for sizes Nos. 1 and 2 Fresno scrapers and two horses for No. 3.

Table 1. Weights and Capacities of Scrapers

Wheeled scrapers			Drag scrapers (western type)			Fresno scrapers		
Size No.	Capacity, cu. ft.	Weight, lb.	Size No.	Capacity cu. ft.	Weight, lb.	Size No.	Cutting width, ft.	Weight, lb.
1	9	500	1	5½	110	1	5	340
2	12	650	2	4½	105	2	4	310
2½	15	700	3	3½	100	3	3½	270
3	16	800	....	....	....	....	....	....

Industrial Cars. Various types of narrow-gage cars are used for handling bulk and package material inside and outside of buildings. Those used for bulk material are usually of the dumping type, the form of the car being determined by the duty. They are either pushed by men or drawn by mules, locomotives or cable. The **rocker side-dump car,** Fig. 5, consists of a truck, on which is mounted a V-shaped steel body supported on rockers so that it may be tipped to either side, discharging material. Largely used on con-

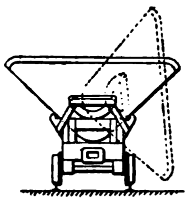


Fig. 5.—Rocker Side-dump Car.

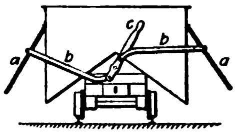


Fig. 6.—Gable-bottom Car.

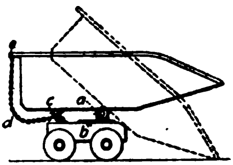


Fig. 7.—Scoop Dump Car.

struction work. Table 2 gives capacities, weights and general dimensions of standard sizes. In the **gable-bottom car,** Fig. 6, the side doors *a* are hinged at the top and controlled by levers *b* and *c* which lock the doors when closed. As this type of car discharges material close to the ground on both sides of the track simultaneously, it is used mainly on trestles, as described under "Endless Rope System," p. 1152. Table 3 gives capacities and general dimensions of various sizes. The **scoop dump car,** Fig. 7, consists of a scoop-shaped steel body pivoted at *a* on turntable *b* carried by the truck. The latch *c* holds the body in a horizontal position, being released by chain *d* attached to handle *e*. The body being mounted on a turntable, the car is used for service where it is desirable to discharge material at any point in the circle. Made with capacities from 12 to 27 cu. ft., to suit local requirements. The **hopper-bottom car,** Fig. 8, consists of a hopper on wheels, the bottom

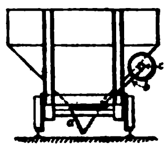


Fig. 8.—Hopper-bottom Car.

opening being controlled by door *a* operated by chain *b* winding on shaft *c*. The shaft is provided with hand wheel and ratchet and pawl. The type of door or gate controlling the bottom opening varies with different materials.

Table 2. Cradle and Rocker Double-side Dump Cars*

(Cars made for any gage)

Capacity, cu. yd.	Standard track gage, in.	Overall dimensions			Wheel diam., in.	Weight, lb.
		Length, ft. in.	Width, ft. in.	Height, ft. in.		
¾	24	5-8	4-7	3-7	12	785
1	24	6-2	5-1	4-0	12	880
1⅛	24	6-10	5-3	4-3	14	1045
1⅓	30	7-2	5-3	4-5	14	1080
1½*	36	7-4	5-5	4-5	16	1300
2	36	7-5	6-8	4-8	16	2160
3	36	8-7	7-4	5-6	18	3150
4	36	10-9	7-4	5-9	20	5540
5	4 ft. 8½ in.	12-8	8-4	7-4	26	8500

* Cars of 1½ cu. yd. capacity and larger are of the rocker type.

Table 3. Typical Gable-bottom Cars

Capacity, cu. ft.	Track gage, in.	Overall dimensions			Wheel diam., in.	Capacity, cu. ft.	Track gage, in.	Overall dimensions			Wheel diam., in.
		Length, ft. in.	Width, ft. in.	Height, ft. in.				Length, ft. in.	Width, ft. in.	Height, ft. in.	
29	24	5-2	3-11	3-11	14	100	40	10-0	5-11	4-4	16
55	24	9-0	4-8	4-4	14	135	4 ft. 8½ in.	10-6	7-7	5-1	18
73	24	8-11	5-1	3-7	14	175	36	12-9	8-2	6-3	24

Table 4. Box-body Dump Cars

(Dimensions given are for wooden cars, except 2-cu. yd car)

Capacity, cu. yd.	Track gage, ft. in.	Inside dimensions of body, in. × in. × ft. in.	Overall dimensions			Wheel base, ft. in.	Weight (wood), lb.	Weight (steel), lb.
			Height, ft. in.	Width, ft. in.	Length, ft. in.			
1¼	2-0	14 × 54 × 5-9	3-9	5-0	8-4	2-8	1,500	....
1½	2-0	18 × 54 × 6-3	4-2	5-0	8-10	2-10	1,800	2,700
2	2-6	14 × 72 × 7-6	4-5	6-8	10-2	3-6	....	2,800
3	3-0	20 × 75 × 8-0	5-4	7-6	11-3	....	....	....
4	3-0	21 × 81 × 9-0	5-7	8-0	12-0	4-2	6,000	....
5	3-0	22 × 84 × 10-6	6-0	8-0	14-4	5-0	10,500	....
6	4-8½	21 × 100 × 11-0	6-7	9-8	15-6	5-6	11,000	....
7	4-8½	22 × 104 × 12-0	6-11	9-8	16-0	6-6	14,000	....
8*	3-0	20 × 84 × 19-0	6-0	8-0	23-0	4-2	17,000	....
10	4-8½	31 × 104 × 12-0	7-3	9-3	16-0	6-6	14,500	....
12*	4-8½	23 × 108 × 19-0	7-6	10-5	23-0	5-2	25,000	28,000
12*	4-8½	17 × 108 × 26-0	7-0	10-5	30-0	5-2	29,000	....
16*	4-8½	22 × 108 × 26-0	7-6	10-0	30-0	5-4	35,000	33,200
20*	4-8½	28 × 108 × 26-0	8-0	10-0	30-4	5-4	44,300	42,500
30*	4-8½	32 × 108 × 34-0	8-8	10-0	38-8	5-6	52,500	....

* Cars with two trucks; all others have one truck.

Table 5. Maximum Permissible Wheel Loadings, Lb.
(Wheel at mid-distance between tie centers)

Ties, c. to c., in.	Weight of rail per yard, lb.						
	12	16	20	25	30	40	45
20	2,200	3,550	4,970	6,390	8,340	12,780	14,920
24	1,830	2,950	4,140	5,320	6,950	10,650	12,420
30	1,460	2,360	3,310	4,260	5,560	8,520	9,940
36	1,220	1,970	2,760	3,550	4,630	7,100	8,280
42	1,050	1,690	2,360	3,040	3,970	6,080	7,100

The **box-body dump car**, Fig. 9, consists of a rectangular body pivoted on the trucks at *a*, and held in horizontal position by chains *b*. The side doors of the car are attached to levers so that the door is automatically raised when the body of the car is tilted to its dumping position. The cars may be dumped to either side, and on the large sizes, where rapid dumping is required, this is accomplished by compressed air. This type of car is largely used in excavation and quarry work, being loaded by steam shovels. The greater load is placed on the side on which the car will dump, so that dumping is automatic when operator releases the chain or latch. The car bodies may be steel or wood, steel-lined. Table 4 gives capacities, weights and general dimensions of standard sizes. **Mine cars** are usually of the four-wheel type with low bodies, the doors being at one end, pivoted at the top, with latch at the bottom. Industrial **tracks** are made with rails from 12 to 45 lb. per yd., gage from 24 in. to 4 ft. 8½ in. Either steel or wooden ties are used. The steel tie is preferred where tracks are frequently moved, owing to lighter weight, the track being made up in sections. Table 5 gives wheel loadings for different weights of rail with various tie spacings. It is not economical to move material over 200 ft. in industrial cars pushed by men. Industrial cars are frequently built with one wheel attached to the axle, the other wheel loose to enable the car to turn on short-radius tracks. The frictional resistance per ton (2000 lb.) for different types of mine-car bearings is as follows:

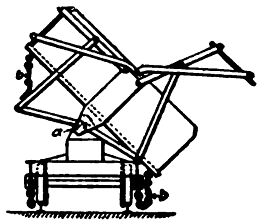


FIG. 9.—Box-body Dump Car.

Type of bearing	Drawbar pull, lb. per ton: Level track	2 per cent. grade	4 per cent. grade
Spiral roller	13	15	46
Solid roller	14	18	53
Self-oiling	22	31	..
Babbitted, old style	24	40	..

Motor Haulage

Electric Locomotives are used for hauling industrial cars in mines, quarries, yards, etc., taking the place of mules, steam locomotives and rope haulage. On standard-gage railroads, electric locomotives of 130 tons or more are used on passenger and freight service and for yard shifting. They are also largely used in mines, the locomotives being provided with cable and reel for hauling cars from side entries not equipped with trolley wire. Electric locomotives consume power only when in use. They have large momentary overload capacity, easy control, low maintenance cost and produce no smoke or fire.

Their use is limited to grades of approximately 10 per cent. for traction, and 16 per cent. for rack, locomotives. The size of track limits the weight of the locomotive, but for heavy service two locomotives are frequently used in tandem. The voltage is limited in many states by law to 275 volts for mine work; the usual gathering locomotive weighs from 5 to 6 tons, for haulage from 10 to 30 tons. Fig. 10 shows an electric locomotive of the mine type. The motors *a* are pivoted on the axles, the back end supported by rods *b*. The axles are driven through pinions and gears running in dust-tight cases. The motors are usually direct-current 250-volt. They are of the rugged enclosed type, designed for sparkless operation under any load, with capacity usually sufficient to drive the wheels to the slipping point. One motor is usually provided for each axle. Controllers are usually of the drum type with magnetic blow-out for series-parallel operation, and may be located at both ends or one end of locomotive, as required. Trolley poles are used to take current from the overhead wire, double poles being used where it is not possible to bond the rails. **Gathering locomotives,** used in mines and yards for collecting cars from tracks not equipped with trolley wire, are supplied with reel *c*, carrying conductor cable *d*. This reel is positively driven from the axle of the locomotive through sprocket chain and gears so that the cable will be paid out or drawn in as the locomotive moves. The carriage *e*, carrying horizontal and vertical rollers, guides the cable on the drum, being driven by screw *f* through sprocket chain *g* from the reel shaft. When the locomotive leaves the main track, the conductor *d* is attached to the trolley wire and to the track if the siding is not bonded. The cable is automatically paid out and taken up as the locomotive moves. The reel may be motor-driven, predetermined tension being maintained in the cable. This arrangement is used where steep grades are encountered. Locomotives may be equipped with a motor-driven winch on which is coiled a wire cable. In this case the locomotive stays on the main track, the cable being carried out and attached to the cars by an attendant. The winch pulls the cars out to the main track. This type requires an extra man; it cannot return cars up grade on entries, but can haul cars out on tracks not strong enough to support the locomotive. Locomotives may be equipped with either or both devices for gathering cars. Special locomotives equipped with powerful winches are frequently used for gathering only.

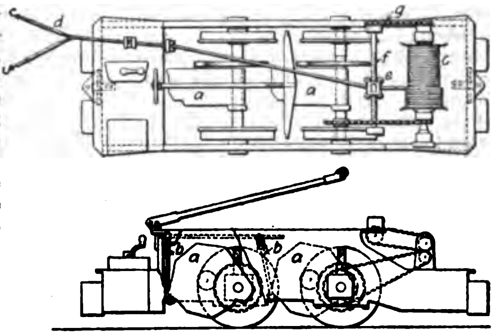


Fig. 10.—Electric Mine Locomotive.

The **Goodman rack-rail locomotives** are used on grades up to 16 per cent. A rack rail is mounted between the T-rails so the teeth of the sprocket wheel on the locomotive axle will engage therewith. With this system, locomotives may be operated by traction on level track, using the rack only on grades, or may operate on rack at all times. The rack may be used as a conductor if properly insulated and protected. Locomotives may be equipped with hand- or motor-driven winches for loading and unloading material.

Table 6 gives the minimum and recommended weights of rail for different locomotives, the size depending on the character of roadbed, spacing of ties, etc. The rail gages are from 18 in. to 4 ft. 8½ in. The electrical efficiency is from 70 to 85 per cent., depending on the size of motors and load factors. Total efficiency ranges from 60 to 75 per cent. The **weight** and **drawbar pull** for standard-size locomotives are given in Table 6 and are based on a train resistance of 30 lb. per ton for friction and 20 lb. per ton for each per cent. of grade. For cast-iron wheels and clean, dry rails, the drawbar pull is taken at 20 per cent. of the locomotive weight, and 25 per cent. with steel wheels. For sanded rails, the starting drawbar pull is 25 to 30 per cent. with cast-iron, and 30 to 33 per cent. with steel, wheels. Table 6 is based on locomotives with steel wheels. If cast-iron wheels are used, take 80 per cent. of the values given in the table. If roller-bearing cars are used, larger gross train loads may be handled.

Table 6. Standard Electric Locomotive Performances

Loco. weight, tons	Drawbar pull, lb.	Speed, miles per hour	Gross train load in tons for a resistance of 30 lb. per ton								Minimum weight of rail per yd., lb.	Recommended weight of rail per yd., lb.
			Per cent. of grade									
			level	1	2	3	4	5	6	7		
4	2,000	6.1	67	38	26	20	15	12	10	8	16	25
6	3,000	6.3	100	58	39	29	23	18	15	13	16	30
8	4,000	6.7	133	77	53	39	30	25	20	17	20	40
10	5,000	7.0	167	96	66	49	38	31	25	21	25	45
12	6,000	6.2	200	115	79	59	46	37	30	25	30	45
15	7,500	6.6	250	144	99	73	57	46	38	32	40	50
20	10,000	7.2	333	192	131	98	76	61	51	42	50	60
25	12,500	7.1	417	240	164	122	95	77	63	53	60	80

Storage-battery Locomotives are used for hauling industrial cars where it is impossible or not economical to install trolley wires and bond the rails. They are largely used in storage yards and in buildings of manufacturing plants; also to some extent in mines for gathering cars. Their first cost is frequently less than that for trolley installation. They possess many of the advantages of the trolley locomotive but eliminate the danger and obstruction of the trolley wire. Being charged at night, they make no demand on the day load, as do trolley locomotives. Storage-battery locomotives are limited by the energy which may be stored in the battery. They should not be used on steep grades or where large continuous overloads are required. They give best results where light and medium loads are to be handled intermittently over short distances with a grade of not over 3 per cent. against the haul. The general construction and mechanical features are similar to those of the trolley type, Fig. 10, with battery boxes located either on top of the truck or between the frames, according to the type of service. Motors are of the special low-voltage vehicle type with drive to the axle of spur gears, or special high-efficiency worm gears. For light work the batteries may run several days on a single charge, but are usually charged every night. The charging may be done from existing generators in the power house where the voltage is sufficiently constant, but a separate generator set is usually employed.

The **efficiency** of batteries varies from 50 to 75 per cent., under average working conditions. The motor efficiency is from 65 to 80 per cent. The **speed** varies from 3 to 7 miles per hour, the average being from 3½ to 4½ miles. Table 7 gives the **weight and drawbar pull** of standard locomotives.

Table 7. Performances of Typical Battery Locomotives

Weight, tons	Drawbar pull, lb.	Miles per hour at rated draw-bar pull	Min. gage, in.	Wheel base, in.	Approx. total ton-miles on one charge, assumed friction = 30 lb. per ton.	Battery capacity, amp-hr.
2½	400	3	18	30	66	126
3	500	4	24	34	200	220
4	700	5	30	36	280	270
4	1000	3½	18	34	230	225
4	1000	3½	24	44	300	300
5	2000	3½	30	36	265	300
7	2400	3½	36	66	320	450

In determining the capacity of the **battery**, both the current and the energy factors must be considered. The speed at which the nominal drawbar pull is developed forms one basis for estimating the size of the battery. Assuming the overall efficiency of locomotives to be 60 per cent., I = the current discharge of the battery, amperes; E = voltage of battery; D = drawbar pull, lb.; M = speed, miles per hour, then $I = 3.32DM/E$. If the current is required almost continuously, the battery selected should have a normal discharge rate of $0.5I$ to $0.67I$; for intermittent service, from $0.25I$ to $0.2I$. Generally, $0.33I$ is used. If the current discharge for a required drawbar pull is not in excess of the normal discharge rate, then the size of the battery will be determined by the energy which must be stored to perform the desired work between charging periods. Assuming the efficiency of the locomotive at 60 per cent., rolling friction of locomotive 20 lb. per ton and of cars 30 lb. per ton, each per cent. of grade adding an additional 20 lb. per ton for both locomotive and cars, the kilowatt-hours required for handling locomotives and cars during the day is determined. This result is multiplied by 1.35 to allow for loss of greater than normal discharge, switching, excessive starting and stopping, etc. The result is the kilowatt capacity at a normal discharge rate required of the battery.

Gasoline Locomotives, being self-contained units, are used where it would not be economical to use electric or steam locomotives and in localities where the cost of oil is less than that of coal. They require no external charging station or licensed engineer. The first cost and maintenance are low. They are limited to hauls up grade of from 10 to 12 per cent., and used to some extent in mines (which must be well ventilated, due to exhaust gases and danger from fire). Mine-type locomotives are of the same general form as the trolley and storage-battery locomotives, the power unit being a four-cylinder four-cycle water-cooled engine. Power is transmitted to the axles by bevel and spur gears or by sprocket chains. The transmission provides two or three speeds both forward and reverse. The low speed is used for starting and pulling heavy loads and the high speed for long-distance runs. The engine runs in one direction and is provided with friction clutches for the forward and reverse motion and jaw-type clutches for shifting from high to low speed. The clutches are interlocking so that the jaw clutches cannot be thrown until the friction clutches are thrown out in the neutral position. For mine work, the gasoline tanks are filled outside the mine and attached to the locomotive through connections which prevent leakage during the change. The intake and exhaust of the engine and the magneto are screened with safety gause to prevent sparks igniting gas in the mine. For surface haulage, the same type of motor is used, but the gasoline tanks are attached to the frame and filled while in place. The method of transmitting power from engine to axles varies with different makers, but the general principles are as described under mine types. Gasoline locomotives operate at speeds from 3 to 15 miles per hour, and require from ¼ to ⅒ of a gallon of gasoline per h.p.-hour. Table 8 gives approximate performances of Milwaukee type locomotives.

Table 8. Performance of Gasoline Locomotives
(Train loads in tons of 2000 lb.)

Brake horse power	Weight, tons	Drawbar pull, lb.	Gear	Speed, miles per hour	Gross train load in tons (exclusive of locomotive) that may be hauled on level track and different grades with train resistance of 20 and 30 lb. per ton.					
					Level		2%		4%	
					20	30	20	30	20	30
17	2½	675	....	6	33.7	22.5	11.0	9.4	6.5	5.9
17	3½	1000	Low High	4 8	50.0 36.5	33.3 23.2	16.3 9.8	14.0 7.9	9.6 4.5	8.7 3.8
21¼	4	1140	Low High	4 8	57.0 45.8	38.0 29.2	18.6 12.6	15.9 10.2	11.0 6.0	10.0 5.1
21¼	5	1540	Low High	4 8	77.0 44.8	51.3 28.2	25.1 11.6	21.6 9.2	14.8 5.0	13.1 4.0
29¾	6	2000	Low High	4 8	100.0 63.6	66.6 40.4	32.7 17.2	28.0 14.0	19.2 7.9	17.5 6.7
29¾	7	2333	Low High	4 8	116.5 62.6	77.7 39.4	38.1 16.2	32.7 12.9	20.8 6.9	18.3 5.6
42½	8	2900	Low High	4 8	145.0 92.0	97.0 58.0	47.3 25.2	40.6 20.5	27.8 12.0	25.3 10.1
42½	9½	3800	Low High	4 8	190.0 90.2	126.6 57.0	62.1 23.7	53.2 19.0	36.5 10.5	33.1 8.6
63¾	12	4800	Low High	4 8	240.0 137.0	160.0 87.7	78.6 37.8	67.2 30.7	46.1 18.0	41.9 15.2

Motor-driven Cars and Trucks are used for moving material in bulk or packages, where it is not possible or convenient to lay tracks. **Electrically driven cars** used in industrial establishments are generally designed for special conditions. Self-discharging cars may be provided with scale beams mounted on the truck to weigh the charge. They are driven by railway-type motors, the operator riding with the car. **Electric automobiles** for freight and package handling are built with chassis of pressed steel and provided with solid-rubber-tired wheels. The motor is of the low-voltage vehicle type, driving the rear axle through gears or chain. The controller is of the drum type. The batteries are arranged in trays so they may be removed when discharged and charged batteries substituted, giving continuous operation of the truck; or batteries may be charged at night without being removed. Capacities range from 500 lb. (delivery wagons) up to 10 tons (trucks). Speeds range from 8 to 15 miles per hour for wagons up to 1000 lb. capacity and 4 to 8 miles for larger sizes; average speed, from 5 to 6 miles.

The average mileage is from 15 to 20 miles per charge on city streets where hills are encountered, and 20 to 30 miles on level streets. Cost of operation, from 25 to 40 cents per ton-mile.

In the transportation of package material in industrial plants, freight houses, stations, etc., **electric baggage and freight trucks** are extensively used. Fig. 11 shows the drop-frame truck, consisting of a steel frame carried by four wheels, provided with solid rubber tires. One pair of wheels is driven from the motor through a gear reduction, the storage battery being located at the other end of the truck. A hinged driver's platform is provided at each end, so arranged that when the operator steps on the platform, he releases the brakes and closes the main circuit between the battery and controller. He may then insert the controller handle. The steering bars are connected to all four wheels so the truck may turn in a radius of 15 ft. As controllers and steering bars are provided at each end, the truck may run in either direction without turning. The drop-frame type of truck is about 12 ft. in length and 45 in. wide, has a capacity of about 4000 lb. and a speed of 4 to 6 miles an hour, loaded; weight, about 3000 lb. Freight trucks are made with large driving wheels at one end, small wheels at the other end. They are about 7 ft. long and 42 in. wide, the platform being 10½ in. above the floor. Weight, about 1900 lb.; capacity, 4000 lb. A 30-day record of an Elwell-Parker drop-frame truck handling baggage showed an average daily service of 11 hours, mileage of 13½ miles, handling 225 tons. The truck was moving one-third of the time and loading and unloading the balance. A gang of eight men was employed and the cost of handling was 10.4 cents per ton.

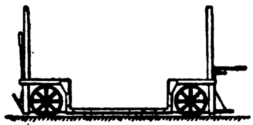


Fig. 11.—Electric Freight Truck.

For heavy haulage both on country roads and city streets **gasoline trucks** are rapidly supplanting horses. They are made with capacities from 1000 lb. up to 10 tons, and run at speeds varying from 16 miles per hour for the 1000-lb. truck to 5 miles for the 10-ton truck. In the tractor type the motor unit, either of the three- or four-wheel type, is not provided with a body for receiving the load. The load is carried in a separate body, one end of which is attached to the tractor. The tractor pulls the load instead of pushing it, and time is saved, as the tractor does not have to remain idle during the loading and unloading period. Fig. 12 gives the cost per ton-mile of hauling material with two-horse teams and trucks and tractors of various sizes, as compiled by the New England Audit Company.

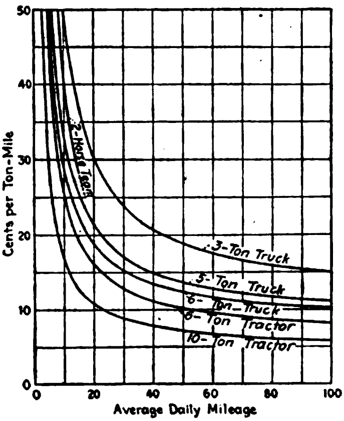


Fig. 12.—Costs of Hauling by Horse Teams, Trucks and Tractors.

Automatic Electric Haulage. The Woodford system of central control (patented) is used in quarries, surface mines, brickyards, excavation work, etc., where cars are to be moved within sight of operator in a central tower. As operators may be placed at intervals, the length of haul is without limit. The minimum gage is 36 in., owing to the motor equipment of cars. Maximum grade, about 15 per cent. It is not economical on capacities of less than 10 tons per hour. The system consists of a series of motor-driven cars, each provided with a selector switch. Between the T-rails there is located a third rail insulated by wooden blocks. It is made in sections that may be energized through feeder wires at two different voltages, under control of operator in central tower. The lower voltage operates the brakes on the car and the higher voltage the motor. The operator may thus either apply the brakes or move the car or cars in any of the sections at will. 220-volt direct current is generally used. Where the feeders are longer than 1½ miles, 550 volts is used, necessitating standard insulators on the third rail. When running down grade, the cars are retarded by the motors which run in the opposite direction, generating energy which is dissipated by resistance coils in the circuit. Should the circuit breaker be thrown out, a loaded car would back down the grade at a predetermined speed depending on the resistance and capacity of coils. The system may be applied to any size or type of car and the cars may be dumped at any desired point by the central-station operator. For spotting cars at the loading point, if at a long distance from the central station, a sub-controlling lever is installed to enable a local operator to spot the cars and when loaded to move them on to the main line. The operating speed varies from 4 to 12 miles per hour, depending upon local conditions.

CARS DRAWN BY CABLE

The haulage of cars by cable is divided into five general systems: Gravity inclined planes, engine inclined planes, tail-rope systems, endless-rope systems and the Hunt automatic railway. Their use is limited to mine and industrial cars.

Gravity Inclined Planes are used for lowering material in cars down an inclined plane. The grade must be sufficient for the loaded car to haul the empty car up the incline, overcoming the frictional resistance of cars, rollers supporting the rope, etc. If too steep the cars will attain too much velocity, causing excessive work on the drum brake. The system includes a drum located at the top of incline under which the cars may pass, the drum being provided with a brake which puts the speed of the cars under the control of operator. Fig. 13 shows four arrangements of track. Arrangement *A* consists of two tracks, one for empty and one for loaded cars, joined by switches at top and bottom. In *B*, three rails are used with turnouts in the middle of the plane. *C* requires still less rail, but involves a self-acting switch at the lower end of the parting. *D* has but a single track with a counterweight running on a separate track of narrower gage than the car track and under same for hauling empty cars to the top of the incline. The capacity of arrangement *D* is one-half that of *A*, *B* and *C*, as loaded and empty cars cannot be handled simultaneously.

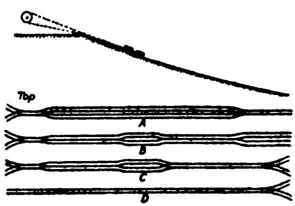


FIG. 13.

In all of these arrangements, the cable may wind on a drum at the top of the incline, or pass over a grip wheel at the same place. Where a single drum is used, one rope leads off the top and one off the bottom of the drum. Two drums are frequently used geared together and rotating in opposite directions, both ropes leading from the under side of the drums, preventing the tendency to lift the cars when at the top of the plane. The grip wheel is used only for light service. The ropes are supported by rollers between the track, and derailing switches and safety devices are provided in case the rope breaks. The proper grade for the incline depends on the length, the number and weight of the cars handled each trip. For ordinary inclines up to 2000 ft. in length, it varies from 4.8 to 10 per cent. To give the car an initial velocity, the top of the incline is steeper for a short distance, called the "knuckle." Operating speeds, 7 to 15 miles per hour.

Engine Inclined Planes are used for raising material in cars up an incline where the empty cars will return by gravity. The plane must have sufficient grade to allow the empty cars to descend the full length by gravity, hauling the rope after them. Long, straight grades will operate on $2\frac{1}{4}$ per cent. or even less, under the best conditions. Curved inclines should have steeper grades. The construction consists of a single or double track with an engine-driven drum at the top; or the engine may be at the bottom with a return sheave at the top. This requires twice the length of rope, therefore greater friction must be overcome by the descending car. The road may be straight or curved and the grade uniform or varying, if all one way, and sufficient for the empty cars to descend hauling the rope. An example of a plane with engine at the top is a 4600-ft. incline, $72\frac{1}{2}$ ft. total fall, with 3 per cent. grade for 200 ft., 2 per cent. for 100 ft. and $1\frac{1}{2}$ per cent. for the remainder. The train, consisting of 25 to 30 cars, is hauled by $\frac{9}{16}$-in. steel rope, a round trip occupying 9 min. The empty cars, weighing 15 to 18 tons, descend at a velocity of about 15 miles per hour. The cars are usually provided with brakes, and a derailing bar pivoted and dragged behind the last car sticks into the ground, derailing the train in case it starts backward, due to a broken rope. Operating speeds, 7 to 15 miles per hour.

The Tail-rope System is used principally for collecting and moving cars in mines. As the cars are pulled in both directions, this system does not depend on grades for its operation, so the road may be straight, curved, level or undulating. It is frequently used in connection with gravity and engine planes in large mines but is being replaced by electric-locomotive haulage, except on steep grades. Fig. 14 shows a two-drum engine, one for the main rope, the other for tail rope, both drums being provided with brakes and clutches.

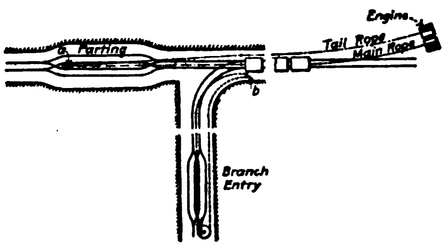


Fig. 14.

The main rope hauls the loaded cars in one direction, the tail rope pulling the empty cars back. The main rope is attached to the front of the train; the tail rope is carried over sheave *a* at the parting and attached to the rear. As the train is hauled out, the tail rope is dragged after it. To return the empty cars, the operation is reversed, the main rope being dragged in by the train. Either rope may be used for braking the speed of the train through brakes and clutches on the engine drum. The tail ropes are usually smaller than the main rope, but local grade may be such as to reverse this condition. For handling branch entries by power, a rope twice the length of the branch is laid in it with ends *b* in the main entry. Shackle connections are placed in the main and tail ropes, which are stopped opposite the branch. When the ends of the branch rope are connected to the main and tail ropes, the cars may be drawn in and out of the branch entry. As many connections of this kind as necessary may be provided. Operating speeds, 7 to 15 miles per hour.

The Endless-rope System is used for moving material in cars on level

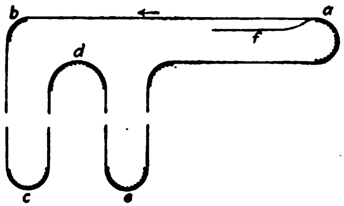


Fig. 15.

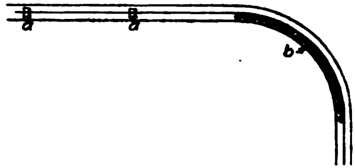


Fig. 16.

or inclined tracks. It is employed to some extent in mines, but its most frequent use is on elevated structures, forming storage piles where cars run continuously from the loading hopper over the circuit of track back to the loading point. The length of the system depends upon the grade, number of curves, weight, capacity and number of cars used, all of which determine the stress in the rope. Systems of this type have been installed with the circuit over a mile in length. The system consists of a constantly running endless steel rope traveling in the middle of a narrow-gage track. Fig. 15 shows a typical plan of road, the cars being loaded on the straight section between *a* and *b* and discharged on the sections, *bc*, *cd* and *de*. The section of track *f* is provided for spare cars. The driving mechanism is located at point *a*. The rope is supported on the straight section, Fig. 16, by cast-iron rollers, *a*, spaced 18 to 20 ft. apart, and guided around the curves by a series of rollers, *b*, about 7 in. in diam., set on vertical spindles and having concave faces. Fig. 17 shows one type of driver located beneath the track. The rope passes over sheave *a*, makes several wraps over the grooved drums *b* and *c*, passes over tension sheave *d*, guide sheave *e*, and

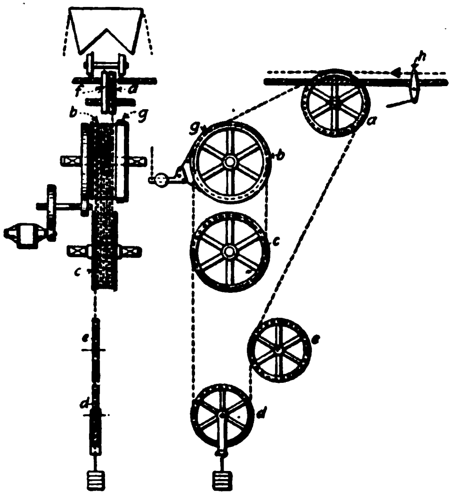


Fig. 17.—Driver for Cable with Endless-rope System.

over sheave *f*, which is on the same shaft as sheave *a*. Drum *b* is driven from the engine or motor by gears, and drum *c* has its shaft tilted so as to lead the rope to the grooves of drum *b*. The tension sheave *d* is provided with a weight to keep the rope at the proper driving tension. It is necessary to ungrip the cars where the rope passes over sheave *a* to the driver, which is done by hand. Should the operator fail to do this, the car will strike the trip bar *h*, which is connected by levers to the band brake *g* by a latch. When the car strikes the trip *h* it stops the motor and applies the brake. The rope cannot be started until the operator resets the trip, releasing the brake.

The cars are usually of the gable-bottom type with side doors hinged at the top and opened automatically by movable stops at the desired dumping point. The cars are attached to the rope by grips as shown in Fig. 18. These grips consist of a casting *b*, forming the upper jaw of the grip, with a sliding piece *c* guided by *b* and forming the lower part. The casting *b* is pivoted around point *e* by the bar *d*, and slides on the curved supports *h*. This enables the car and grip to make the turns. The moving section of the grip *c* is threaded at the top and raised and lowered by the shaft *f* and hand wheel *g*. To attach the grip to the rope, the operator swings the grip against the rope and turns the hand wheel *g* until the lower half of the grip raises the rope against the upper half. The cars are from 1 to 5 tons in capacity. The ropes are from ⅝ to ⅞ in. in diameter. The curves have a minimum radius of 15 ft. The speed of the endless-rope system is from 1 to 3 miles per hour. The **capacity** of gravity and engine planes depends on the length of haul, number of cars per train and the amount of time consumed in shifting, making fast and releasing the cars from the cable. Endless-rope systems will handle up to 250 tons per hour.

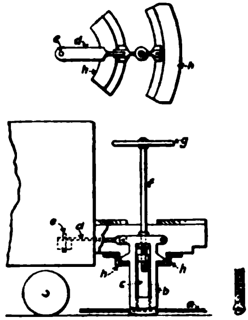


FIG. 18.

The Hunt Automatic Railway is used for moving bulk material on elevated track which may be placed on a down grade from the loading point. Its use is limited to a length of 600 ft. and curves may be placed only at the loading end. Fig. 19 shows the general form. The cars are loaded at *a*

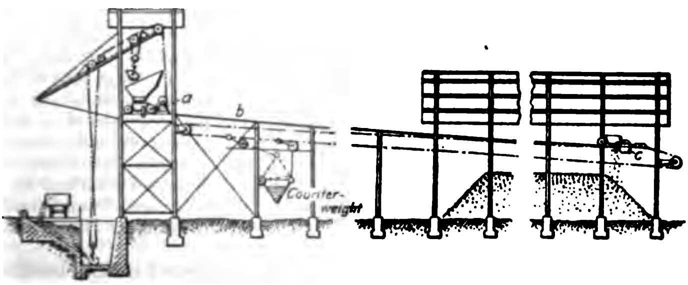


FIG. 19.—Hunt Automatic Railway.

and started by the operator down the first section of track inclined on about a 10 per cent. grade. The balance of the track is on from 2½ to 3 per cent. grade so that the car will travel the length of the track by gravity. The car is of the gable-bottom type (see Fig. 6) with the back inclined so that the discharging material will assist in starting the car backward. The side doors are hinged at the top and are controlled by a wire rope made fast to one door passing through ties on the end of the car, the other end being made fast to a toggle lever. Attached to this lever is an extension carrying a roller so arranged that when it strikes the dumping board on the stringers of the runway it will break the toggle, allowing the doors to open. The car is stopped by contact with a block attached through a spring and a cable to a counterweight which is lifted by its momentum; the falling of this counterweight imparts sufficient impetus to the car to return it up the grade to the loading point. The dumping block on the cable must be moved when the dumping point is changed; its movement is about 40 ft. The gage of the track is 21 in. outside and the curves at the loading end must have a radius of 30 ft. or over. The counterweight may be located at any point under the track. The cars are made in one- and two-ton capacities and a round trip on a 300-ft. track can be made in about 1 min. Only one man is required to operate the device and no power is required.

OVERHEAD TRACKAGE

Light Rigid Trackage consists of tracks of various forms suspended from overhead structures, for carrying trolleys to which loads are attached by hooks or chain blocks. Fig. 20 shows the **bar type** of track, consisting of a steel bar *a* with rounded edges carried by hangers *b* from the structure above. The trolley consists of four wheels *c* provided with roller bearings, each pair of wheels being carried by frame *d*, pivoted at points *e*, enabling the trol-

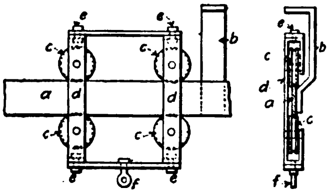


Fig. 20.—Bar Track.

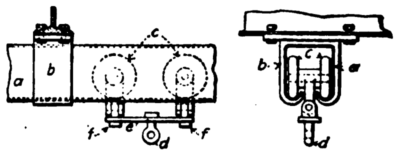


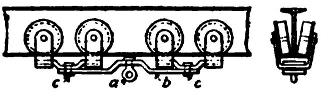
Fig. 21.—Coburn Track.

Overhead Trackage.

ley to turn on track of small radius. The load is attached to eye *f*. This construction is used for loads up to 4000 lb., the hangers being spaced from 2 to 5 ft. apart. In its simpler form, it is largely used for handling meat in packing houses. The **Coburn type** track, Fig. 21, consists of a rolled-steel track *a* supported by cast brackets *b* bolted to the overhead structure. The trolleys consist of four wheels *c* with rounded treads, provided with roller bearings. The load is suspended from the swivel eye *d* attached to bar *e*, which is pivoted at *f* so that the trolley may turn on track of small radius. This type is used for loads from 300 to 4000 lb., the hangers being spaced from 20 in. to 5 ft. apart.

The **single I-beam track** and trolley (Fig. 22) consists of an I-beam suspended from the overhead structure, with trolley wheels running on the lower flange. The type shown has eight wheels, the load being suspended from the swivel eye *a* attached to bar *b*, which is pivoted at *c* so that the trol-

ley may turn on track of small radius. This type of trolley is used in capacities of from 1 to 10 tons. Fig. 23 shows a double I-beam track used for the heavier work, the trolley wheels running on the lower flanges. The trolley is driven by a hand chain *a* through chain wheel *b*, pinion *c*, and gear *d*, the latter being keyed to the axle of one pair of wheels. Capacity, from 10 to 20 tons. The types shown in Figs. 20 and 21 are also made with a greater number of wheels, spreading the load over a greater length of track by the use of equalizing bars, such as shown in Fig. 22. They are usually moved along the track by hand, while those



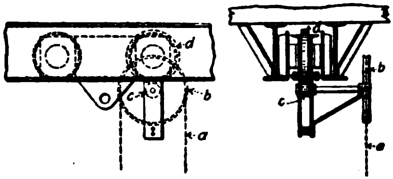


Fig. 22.—Single I-beam. Fig. 23.—Double I-beam.
Overhead Trackage.

shown in Figs. 22 and 23 may be moved either by hand or by chain wheel.

Telphers. **The telpher is a form of electric hoist which not only lifts the load but transports it on overhead track from one portion of an industrial plant to another, the operator riding on the telpher and controlling all operations. It is used to lift and transport material which may be suspended from a hook, such as pieces of machinery, trucks or boxes containing merchandise, lumber on skids, and bulk material such as coal, ore, etc., in tubs or grab buckets; also for great variety of service in industrial plants both in-**

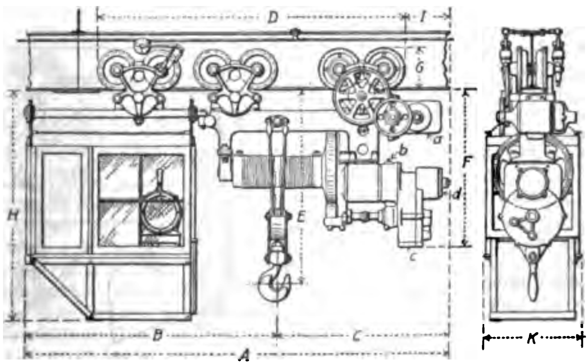


Fig. 24.—Hook Type Telpher.

side and outside of buildings, handling freight, etc., in freight houses and warehouses, unloading open-top cars and carrying the material to storage piles, bins, etc., as well as handling lumber, pipe, structural steel, etc., on skids. The length of haul is limited only by the length of the runways, the apparatus being used principally within the limits of a manufacturing plant. The minimum radius of track is 15 ft. when telphers are provided with swivel trucks. The load is limited to about 5 tons on standard machines.

A typical arrangement of track consists of an I-beam suspended from the overhead structure, the telpher wheels running on the lower flanges. The

switches consist of a pivoted section of beam supported by rollers at the moving end from a curved track above, and provided with a system of levers to enable the operator to throw the switch from the cab and to lock it in position. Turntables also operated from the cab enable the telpher to be turned at right angles. Fig. 24 shows the general form of a telpher of the **hook type.** The trucks are pivoted to the telpher frame to enable them to turn on short radius. The telpher is propelled by motor *a* geared to four wheels of the forward truck. The hoisting drum is driven by gears from motor *b*, provided with load brake *c* and solenoid motor brake *d*. Multi-speed controllers are used on both hoist and travel, limit switches being provided to prevent overhoisting. Table 9 gives general data on this type of hoist. When used to handle skids of lumber the same type is used, except that the drum is made longer and two hoisting ropes are used—winding on the same drum. The **lumber grapple hook,** Fig. 25, carried by turntable *a*, is suspended from the drum by ropes *b*. The hooks *c* take hold of the sticks under the pile of lumber, being controlled through lever *d* by the operator in the cab. The timbers *e*, to which are attached the pipes *f*, form a fork to hold the pile together. This system handles lumber in units of about 1500 ft. B. M. each, and is largely used to transport material between the storage yard, kilns, cars and boats.

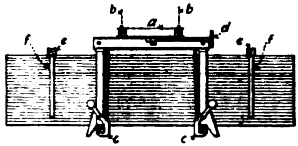


FIG. 25.—Lumber Grapple Hook.

Fig. 26 shows a telpher of the **grab-bucket type.** The closing rope of the bucket is attached to the closing drum *a*, and the hold rope to the hoist drum *b*. Each of the drums is driven by its own motor through spur-gear reductions, each being provided with solenoid brakes. The two motors are controlled by one drum controller. The telpher is moved by one or more motors geared to the truck wheels, operated by a drum-type controller. Table 10 gives general data on this type of hoist. Telphers may be operated by either direct or alternating current. The hoisting **speeds** vary from 25 to 200 ft. per min., and the travel speeds from 250 to 700 ft. per min., depending upon local conditions.

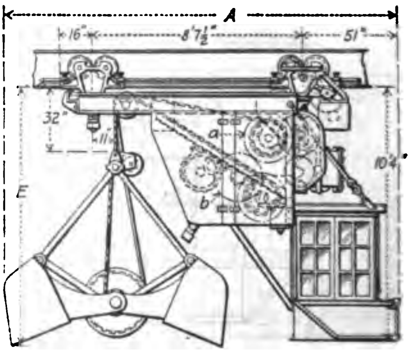


FIG. 26.—Grab-bucket Telpher.

Cableways are aerial hoisting and conveying devices using suspended steel cable for track, the loads being suspended from carriages and moved by gravity or power. The most common uses consist in transporting material from open pits and quarries to the surface, handling construction material in the building of dams, docks, etc., where the construction of tracks across rivers or valleys would be uneconomical, loading logs on to cars, coaling vessels at sea, etc. The maximum clear **span** is from 2000 to 3000 ft.; usual spans, 300 to 1500 ft. The gravity type is limited to conditions where at least a 20 per cent. grade is obtainable on the track cable. **Transporting**

Table 9. Data on Telphers of the Hook Type

(Pawling & Harnischfeger Co.)

Capacity, tons	Standard lift, ft. in.	Hoist: Speed, ft. per min.	Hoist: H.p. of motor	Trolley: Speed, ft. per min.	Trolley: H.p. of motor	Weight, lb.	Clearance dimensions* (Letters refer to Fig. 24): *A* ft. in.	*C* ft. in.	*F* ft. in.	*I* ft. in.
1	19 3	32–65	3½	250–300	3	3600	10 4	3 10	2 10	1 0
1	19 3	35–75	4½	350–400	4½	3600	10 4	3 10	2 10	1 0
2	19 3	25–60	4½	250–300	3	3600	10 4	3 10	2 10	1 0
2	21 6	32–70	6½	250–300	3½	4600	10 7	4 1	3 1	1 4
2	19 3	25–60	4½	350–400	4½	3600	10 4	3 10	2 10	1 0
2	21 6	35–75	8	350–400	6½	4600	10 7	4 1	3 1	1 4
3	21 6	25–60	6½	250–300	3½	4600	10 7	4 1	3 1	1 4
3	21 6	30–65	8	350–400	6½	4600	10 7	4 1	3 1	1 4
5	16 7	25–60	10	250–300	6½	7000	12 2	4 8	4 2	1 3
5	16 7	30–65	13	350–400	8	7000	12 2	4 8	4 2	1 3

* *B* = 6 ft. 6 in. (7 ft. 7 in.); *D* = 7 ft. (8 ft. 11 in.); *E* = 4 ft. 3 in. (5 ft. 5 in.), *G* = 10 in. (14 in.); *H* = 5 ft. 10 in. (5 ft. 11 in.); *K* = 3 ft. 8 in.; dimensions in parentheses refer to telphers of 5 tons capacity.

Table 10. Data on Telphers of the Grab-bucket Type

(Sprague type. Travel speed, 350 to 700 ft. per min.)

Capacity, cu. yd.	Weight, lb.: Empty bucket (Hayward type)	Weight, lb.: Bucket loaded with coal	Weight, lb.: Telpher with empty bucket	Standard lift, ft.	Hoisting speed, ft. per min.	Clearance dimensions (Letters refer to Fig. 26): *A* ft. in.	*E* ft. in.
½	2,100	2,775	11,400	50	150	15 6	9 4
¾	2,500	3,515	11,800	50	150	15 6	9 4
1	2,700	4,050	12,600	50	150	16 1	9 10
1¼	3,000	4,690	13,800	50	150	16 1	9 10
1½	3,800	5,825	14,600	50	150	16 6	10 8
1¾	4,000	6,200	15,000	50	150	16 6	10 8

cableways move the load from one point to another. Hoisting-transporting cableways hoist the load as well as transport it. A **transporting cableway** may have one or two fixed track cables, inclined or horizontal, on which the carriage operates by gravity or power. The gravity transporting type, Fig. 27(*I*), will either raise or lower material. It consists of one track cable *a*, on which travels the wheeled carriage *b* carrying the bucket. The traction rope *c* attached to the carriage is made fast to the power drum *d*. The inclination must be sufficient for the carriage to coast down and pull the traction rope after it. The carriage is hauled up by the traction rope *c*. Drum *d* is provided with brake to control the lowering speed, and material may be either raised or lowered. When it is not possible to obtain sufficient fall to operate the load by gravity, the traction rope *c*, Fig. 27(*II*) is made endless so that the carriage *b* is drawn in either direction by the power drum *d*. Another type of inclined cableway, shown in Fig. 27(*III*), consists of two track cables *a*, *a*, with an endless traction rope *c* driven and controlled by drum *d*. When material is being lowered, the loaded bucket *b* raises the empty carriage *bb*, the speed being controlled by the brake on the drum. When material is being raised, the drum is driven by power, the descending

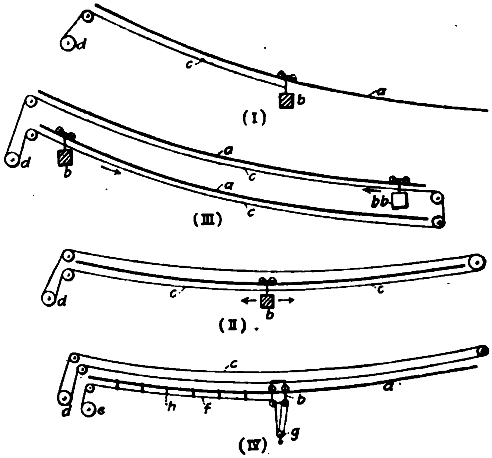


Fig. 27.—Cableways.

empty carriage assisting the engine in raising the loaded carriage. This type has twice the capacity of that shown in Fig. 27(*I*).

A **hoisting and conveying cableway,** Fig. 27(*IV*), hoists the material at any point under the track cable and transports it to any other point. It consists of a track cable *a* and carriage *b* moved by the endless traction rope *c* and by the power drum *d*. The hoisting of the load is accomplished by the power drum *e* through fall rope *f* raising the fall block *g* suspended from the carriage. The fall-rope carriers *h* support the fall rope, otherwise the weight of this sagging rope would prevent the fall block *g* from lowering when without load. Where it is possible to obtain a minimum inclination of 20 deg. on the track cable, the traction-rope drum *d* is provided with a brake and is not power-driven. The carriage then descends by gravity, pulling the fall and traction ropes to the desired point. Brakes are applied to drum *d*, stopping the carrier. The fall block is lowered, loaded and raised. If the load is to be carried up the incline, the carriage is hauled up by the fall rope. With this type, the friction of the carriage must be greater than that of the fall block, or the load will run down. The most recent development is the use of self-filling grab buckets, operated from the carriages of cableways, which are lowered, automatically fill themselves, are hoisted, carried to dumping position and discharged.

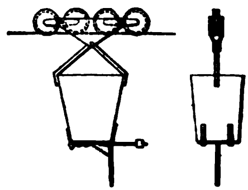


Fig. 28.—Bucket Carriage.

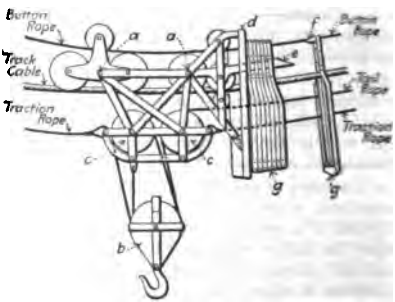


Fig. 29.—Lidgerwood Carriage.

Fig. 28 shows one type of bucket **carriages** used on transporting cableways. It consists of a four-wheeled carriage, from which is suspended an automatic bottom-dumping bucket. These carriages may also be provided with hooks or slings to which loads of various types may be attached. Fig. 29 shows the Lidgerwood type of carriage used on hoisting-transporting cableways. The trucks are pivoted at *a*, so as to distribute the load equally on the wheels. The fall block *b* is suspended from the carriage by sheaves *c*. At the back of the carriage is the pivoted frame *d* carrying the horn *e* for picking up the fall-rope carriers *g*. As the carriage runs out, the buttons *f* on the button rope distribute the fall-rope carriers at intervals, and as the carriage returns these carriers are picked up by the horn *e*. The button rope is made fast to one tower and kept tight by the counterweight at the other tower. Fig. 30 shows the Roebling type of carriage, in which the fall hook *a* is carried by a flat steel rope winding like a watch spring on drum *b*. The fall rope *c* is endless and wound around the two multiple-grooved sheaves *d* in the carriage, driving the drum *b* through pinions *e* and gear *f*, thus raising or lowering the load. This type does away with fall-rope carriers.

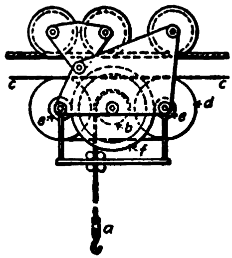


FIG. 30.—Roebling Carriage.

The **fall-rope carriers** are light steel frames for carrying the traction and fall ropes. In the Lidgerwood system these are distributed by the button rope above the track cable. The buttons are about 100 ft. apart and of different diameters, the eyes of the carriers being made to correspond to the buttons, so that as the carriage moves out the carriers are distributed by their respective buttons. Other types of carriers are carried by wheels on the track cable and attached to the carriage, to each other and to the tower by chains or rods which space them along the track cable as the carriage moves out. **Supporting towers** may be of steel or wood, fixed or movable. For short spans, simple A-frames are used, while on the long spans they are made with four posts. The movable towers are made with four or more posts securely braced and mounted on wheels. Both towers may move longitudinally or one may be fixed, the other moving on curved tracks. Fixed towers have been built as high as 235 ft. and traveling towers 125 ft. The towers are sometimes provided with propelling mechanisms where the cableways are frequently moved. The movable towers must be provided with counterweights to keep them from overturning. The engines and boilers serve this purpose at one end, the other end being weighted down with the required quantity of rock.

Table 11. Typical Installations of Gravity Transporting Cableways for Handling Coal (Roebling)

Type	Span, ft.	Diam. of track cable, in.	Type of carriage	Time of round trip, min.	Capacity per day of 10 hours, tons
Gravity incline, Fig. 27(*I*)	500	2	1½-ton bottom-dump bucket	2	500
Gravity 2-rope, Fig. 27(*III*)	1190	2¼	3½-ton bottom-dump bucket	4	1000
Gravity 2-rope, Fig. 27(*III*)	2077	2	2-ton bottom-dump bucket	2	1200

The carriage **speed** is from 300 to 1000 ft. per min. (in special cases up to 1800 ft.); average hoisting speed, from 100 to 300 ft. per min. The **average loads** for coal, earth, etc., are from 1 to 5 tons, for rock from quarries, 5 to 20 tons. Cableways have been built to handle loads of 50 tons on short spans. Tables 11 and 12 give data of typical installations. Steam-driven cableways require one fireman and one engineer; electric cableways, one engineer. Traveling and radial cableways require, in addition, one rigger, and all types require a signalman if it is not possible for the engineer to see the carriage and load at all times.

Table 12. Typical Installations of Hoisting and Conveying Cableways (Lidgerwood)

Service	Type	Material handled	Span, ft.	Load, tons	Diam. of track cable, in.	Diam. of conveying rope, in.	Diam. of hoisting rope, in.	Ht. head tower, ft.	Ht. tail tower, ft.
Building concrete bridge	Stationary	Concrete materials, forms, etc.	2038	6–10	2½, plow steel	¾, plow steel	¾, plow steel	165	165
Excavating and building dam	Stationary	Grab bucket and concrete materials	1436	10	2¼, patent locked steel	¾, Hercules steel	¾, Hercules steel	80	125
Building power house	Stationary	Building materials and machinery	1100	10–30	2¼, special patent locked steel	¾, special plow steel	¾, special plow steel	45	60
Quarry	Stationary	Stone	525	5	1¾, patent locked steel	⅝	⅝	50	50
Logging	Traveling	Logs	800	3–4	1½, patent locked steel	⅝	⅝	74	74
Clay pits	Radial traveling	Clay in skips	975	10	2¼, patent locked steel	¾	¾	74	80

The **deflection of** the **track cable** with its maximum gross load at mid-span is usually taken as one-twentieth of the span. Let S = span between supports, ft., l = one-half the span, ft.; w = weight of rope, lb. per ft., P = total concentrated load on rope, lb.; h = deflection, ft.; H = horizontal tension in rope, in lb. at one-fourth breaking strength. Then $h = S/20 = (wl + P)l/2H$, $P = (2hH - wl^2)/l = (8hH - wS^2)/2S$.

For track cables, a factor of safety at least of 4 is advised. The traction and fall ropes should have the sum of the load and bending stress well within the elastic limit of the rope, or for general hoisting about two-thirds of the elastic limit (which is taken at 65 per cent. of the breaking strength). Let P = load on the rope, lb.; A = area of metal in rope section, sq. in.; E = 29,500,000; R = radius of curvature, in.; d = diam. of individual wires in rope, in. (For 6-strand 19-wire rope, $d = 1/15$ rope diam.; for 6-strand 7-wire rope, $d = 1/9$ rope diam.) Then, load stress per sq. in. = $T_l = P/A$, and bending stress per sq. in. = $T_b = Ed/2R$. In determining the horse power required, the load on the traction or on the fall ropes will govern, depending upon the degree of inclination.

Cable Tramways are aerial conveying devices using suspended cables, carriages and buckets for transporting material over level and rough, mountainous country, across rivers, valleys, hills, etc. They are used for handling small quantities over long distances, and their construction cost is insignificant as compared with railroads and bridges. Two types are in use, single-rope, which both supports and propels the carriages; and double-rope, where one rope supports and the other propels the carriages. Table 13 gives data on typical installations of cable tramways. Single-rope tramways must run in a straight line but may have vertical curves. Owing to the high ratio of the weight of the moving parts to the weight of material carried, they are adapted only to short lengths ($\leq$ 1000 ft.), short spans and small capacities. They are not extensively used. The double-rope tramways are made in two types. The underhung type handles large capacities at high speeds and over long lengths, but must be in a straight line, although vertical curves are allowable. The overhead type is adapted to almost any requirement, with lateral as well as vertical curves. It is the best type for large capacity, but the spans must be shorter. The stress in the traction rope limits the length of tramways. For small capacity on level ground, 7 to 10 tons per hour may be carried 6 miles in one section. For 60 tons per hour the limit is about 3 miles. Larger capacities and steep inclines require shorter sections. Tramways are in operation 22 miles long, made in eight sections, the carriages being transferred from one section to the other.

Table 13. Typical Installations of Cable Tramways

Total length of tramway system, miles	Number of sections	Tramway elevates or lowers	Vertical distance between terminals, ft.	Material handled	Capacity of each carriage, lb.	Capacity, tons per hour	Speed of carriages, ft. per min.
1	1	Lowers	400	Ore	750	10	300
2	1	Lowers	812	Ore	1,200	50	333
2¼	1	Lowers	600	Rock	1,000	60	300
2½	1	Lowers	1,150	Ore	1,000	40	300
4	1	Lowers	200	Coal	1,250	50	500
12	3	Lowers	650	Log wood	300	10	300
22	8	Lowers	11,500	Ore	1,000	40	500

The **single-rope type**, Fig. 31, consists of an endless cable *a* passing over horizontal sheaves *d* and *e* at the ends and supported at intervals by towers. This cable is moved continuously and both supports and propels the carriages *b* and *c*. The carriages are either attached permanently to the cable, in which case they must be loaded and dumped while in motion, or they are attached by friction grips so that they may be connected automatically or by hand at the loading and dumping points. When the tramway is lowering material from a higher to a lower level, the grade is frequently sufficient for the loaded buckets *b* to raise the empty buckets *c*, operating the tramway by gravity, the speed being controlled by brake on grip wheel *d*.

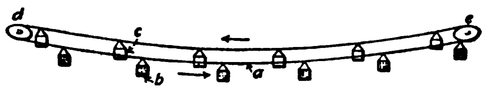


Fig. 31.—Single-rope Cable Tramway.

The **double-rope type**, Fig. 32, consists of two stationary track cables *a*, on which the wheeled carriages *c* and *d* travel. The endless traction rope *b* propels the carriages, being attached by friction grips. Fig. 33 shows the arrangement of the **overhead type**. The track cable *a* is supported at intervals by towers *b*, carrying the saddles *c*, in which the track cable rests. Each tower also carries the sheave *d* for supporting the traction rope *e*. The self-dumping bucket f is suspended from the carriage g. The grip h which attaches the carriage to the traction rope e is controlled by lever k. In the **underhung type,** shown in Fig. 34, the track cable a is carried above the traction rope e. The saddle c on top of the tower supports the track cable, and sheave d the traction cable. The sheave is provided with a rope guard m.

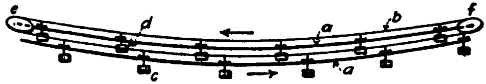


FIG. 32.—Double-rope Cable Tramway.

The lever h with roller on the end automatically attaches and detaches the grip by coming in contact with guides at the loading and dumping points. The carriages move in one direction only on each track. On steep downgrades, special hydraulic speed controllers are used to fix the speed of the carriages.

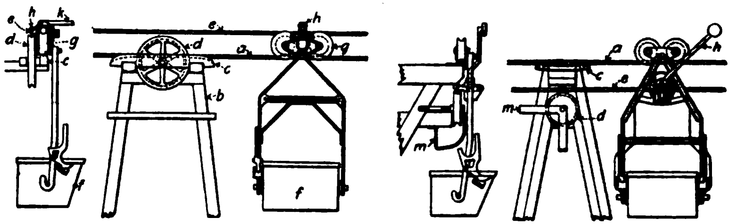
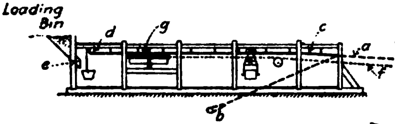


FIG. 33.—Overhead Type. FIG. 34.—Underhung Type.
Double-rope Cable Tramways.

The **track cables** are of the special locked-joint smooth-coil or tramway types. Cast-steel ropes are used for short spans and plow-steel for long spans. The usual spans for level ground are from 200 to 300 ft., and in rough country as required. One end of the track cable is anchored, the other end counterweighted to one-quarter the breaking strength of the rope so that the horizontal tension is a known quantity. Special couplers of machine steel are used to splice the rope, designed to develop the full strength of the rope and to have as small a diameter as possible to allow the carriage wheels to roll over them. The **traction ropes** are made 6-strand 7-wire, or 6-strand 19-wire, of cast or plow steel on hemp core. The maximum diameter is 1 in., which limits the length of the sections. The traction rope is endless and is driven by a drum or grip wheel at one end, passing over a counterweighted sheave at the other end.

Fig. 35 shows a **loading terminal.** The track cables a are anchored at b. The carriage runs off the cable to the fixed track c, which makes a 180-deg. bend at d. The empty buckets are loaded by the chute e from the loading bin, continue around the track c, are automatically gripped to the traction cable f, and pass on to the track cable a. The traction cable f passes around and is driven by the grip wheel or drum g. When the carriages are permanently attached to the traction cable, they are loaded by a moving hopper which is automatically picked up by the carriage and carried with it a short distance while the bucket is being filled. Fig. 36 shows a **discharge terminal.** The carriage rolls off from the track cable a to the fixed track c, being automatically ungripped. It is pushed around the 180-deg. bend of track c, discharging into the bin underneath, and continues on track c until it is automatically gripped to the traction cable *f*. The counterweights *h* are attached to the track cables *a*, and the counterweight *k* is attached to the carriage of the traction-rope sheave *m*. The **supporting towers** are A-frames of steel or wood. At abrupt vertical angles the supports are placed



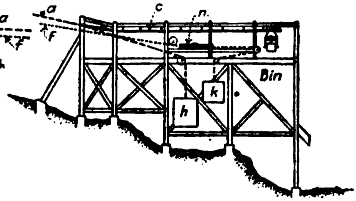


FIG. 35.—Loading Terminal. FIG. 36.—Discharge Terminal

Cable Tramway.

close together and steel tracks installed in place of the cable. The best practice spaces the towers, where possible, approximately the same as the carriages, so that but one carriage will be on a span at a time.

The operating **speed** for single-rope tramways is from 100 to 200 ft. per min., for double-rope tramways, from 200 to 550 ft. Single-rope tramways have a **capacity** of 5 to 15 tons per hour. Double-rope systems, up to 200 tons per hour, and under extremely favorable conditions, 300 tons. The standard buckets are made with capacities up to 1 ton.

Fig. 37 shows the **Lawson type tramway,** in which the car *b* is carried by the two track cables *a*. These cables rest at the towers on the pivoted saddles *c*. The cars are attached to traction rope *d*. The cars are loaded by a special traveling hopper filled from bin above and traveling with the car a short distance while it is being filled. They are discharged as they pass around the drum at the dumping end.

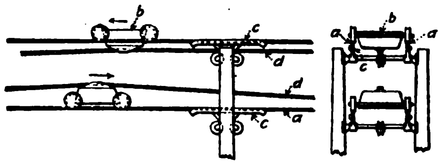


FIG. 37.—Lawson Cable Tramway.

STRESS IN ROPES (Roebling). The **deflection** for track cables of tramways is taken as 1/40 to 1/50 of the span to reduce the grade at the towers. Let S = span between supports, ft.; h = deflection, ft.; P = gross weight of buckets and carriages, lb.; Z = distance between buckets, ft.; W_1 = total load per foot of rope, lb.; H = horizontal tension of rope, lb. The best condition is when $S = Z$, with only one bucket in the span. The formulæ given under Cableways (p. 1160) then apply. When several buckets come in the span at the same time, special treatment is required for each span. For large capacities the buckets are spaced close together, and the load may be assumed as uniformly distributed and the live load per lineal foot of span $= P/Z$. Then $H = W_1S^2/8h$, where W_1 = weight of rope per ft. $+ (P/Z)$. When the buckets are not spaced closely, the equilibrium curve can be plotted

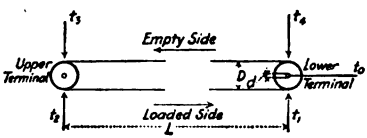


FIG. 38.

with known horizontal tension and vertical reactions at points of support.

For figuring the traction rope, t_0 = tension on counterweight rope, lb., t_1, t_2, t_3, t_4 = tensions (lb.) at points shown in Fig. 38; n = number of carriers in motion; a = angle subtended between the line connecting the tower supports and the horizontal; W_1 = weight of each loaded carrier, lb.; W_2 = weight of each empty carrier, lb.; w = weight of traction rope, lb. per ft.; L = length of tramway of each grade a, ft.; D = diam. of end sheave, ft.; d = diam. of shaft of sheave, ft.; f_1 = 0.015 = coefficient of friction of shaft; f_2 = 0.025 = rolling friction of carriage wheels. Then, if the loads descend, the maximum stress on the loaded side of traction rope is

$$t_2 = t_1 + \Sigma\,[L\,w \sin a + \tfrac{1}{2}nW_1 \sin a] - f_2\,\Sigma\,[Lw \cos a + \tfrac{1}{2}nW_1 \cos a]$$

where $t_1 = \tfrac{1}{2}t_0[1 - f_1(d/D)]$. If the load ascends, there are two cases: (1) driving power located at the lower terminal; (2) driving power at the upper terminal. If the line has no reverse grades, it will operate by gravity at a 10 per cent. incline for 10 tons per hour capacity and at 4 per cent. grade for 80 tons per hour. The above formula will determine whether it will operate by gravity.

The **power required** or developed by tramways is as follows: Let V = velocity of traction rope, ft. per min.; P = gross weight of loaded carriage, lb.; p = weight of empty carriage, lb.; N = number of carriages on one track cable; $P/50$ = friction of loaded carriage; $p/50$ = friction of empty carriage; W = weight of moving parts, lb.; E = length of tramway divided by difference in levels between terminals in feet. Then, power required =

$$\text{h.p.} = \frac{NV}{33{,}000}\left(\frac{P-p}{E} \pm \frac{P+p}{50}\right) \pm 0.0000001WV$$

Where power is *developed* by tramways, use 80 instead of 50 under $P + p$.

CAR-UNLOADING MACHINERY

Five types of devices are in common use for unloading material from all types of open top cars: Cross-over and horn dumps, used to unload mine cars with swinging end doors; rotary car dumps, for mine cars without doors; tipping car dumps, for unloading standard-gage cars where large unloading capacity is required; and the plow type, used for unloading flat cars.

Cross-over Dump. Fig. 39 shows car in the act of dumping; Fig. 40 shows loaded car pushing empty car off the dump. A section of track is

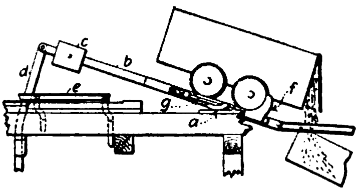


FIG. 39.

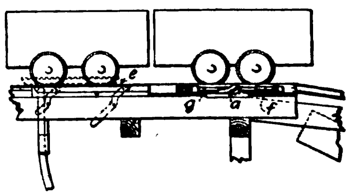


FIG. 40.

Cross-over Dump.

carried by a platform supported on rockers a. An extension bar b carries weight c and brake friction bar d. A hand lever controls brake, acting on friction bar and placing dumping under control of operator. A section of track e in front of the dump is pivoted on a parallel motion and counterbalanced so that it is normally raised. The loaded car depresses rails e, and through levers pivots horns *f* around shafts *g*, releasing the empty car. The loaded car strikes the empty car, starting it down the inclined track. After the loaded car has passed rails *e*, the springs return horns *f* so that they stop the loaded car in the position to dump. Buffer springs on shaft *g* absorb the shock of stopping the car. The center of gravity of the loaded car being forward of the rockers, the car will dump automatically under control of the brake. No power is required for this dump and one operator can dump from three to four cars per minute.

Horn Dump (Fig. 41). This consists of a curved track inclined so that the bottom of the car will be at the angle of slide of the material. A shaft *a* carries two horns *b* which engage the front wheels of the car as it starts down the incline. A band brake *d* controlled by lever *e* enables the operator to regulate the speed of car. The extension on one of the horns engaging the end of lever *c* holds the car in the dumping position. No power is required; one operator can dump from three to four cars per minute.

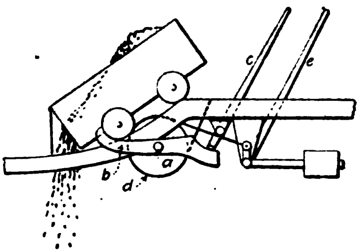


Fig. 41.—Jeffrey Horn Dump.

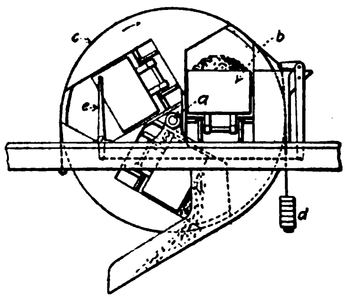


Fig. 42.—Rotary Gravity Dump.

Rotary Gravity Dump (Fig. 42). This consists of a steel cylinder supported by shaft *a*, its three compartments carrying three tracks. The loaded car *b* being to one side of the center, causes the cylinder to rotate, the material rolling to the chute beneath. The band brake *c*, with counterweight *d*, operated by lever *e*, places dumping under control of the operator. No power is required; one operator can dump from two to three cars per minute.

Rotary Power Dumps are made to handle from one to six cars at a time. They consist of a framework carrying track for cars and supported by steel rings, some of which consist of gears meshing with pinions on a shaft which is driven through gear reduction from motor. A band brake is provided so the operator may control dumping. Longitudinal beams in the framework engage the top of car to prevent it from leaving the track. A stop engaging the axle of the car wheels holds the car in proper longitudinal position. When built for one car, one operator can handle about three cars per minute; when long enough for six cars, about eighteen cars per minute. They require about 15 h.p. when handling three 5-ton cars.

Tipping Car Dump. Tipping dumps are made both electrically and steam-driven, and are built to handle cars up to 100 tons capacity. They are capable of handling 30 cars per hour, and under extremely favorable conditions, 40 to 45 cars. For 100-ton cars, 450 to 500 boiler h.p. is provided, where steam-driven, and about 400 motor h.p. where electrically driven.

Four to five men are required on the dump itself, this not including the men required to bring cars to the dump.

Plow-type Car Unloaders are used for unloading flat cars. They consist of a flat car carrying a single-drum reversible engine taking steam from the boiler of the locomotive. A flat car at the other end of the train carries a steel plow. Plows are made to discharge on one side or both sides at the same time. The spaces between the cars are covered with hinged steel plates, to bridge the openings. The trains consist of 17 to 20 cars each. Two posts, located at a convenient point along the track, support a chain spanning the track. The end of the hauling rope coiled on the drum is made fast to the chain and the train moved forward. The rope is drawn out along the top of the load until it reaches the plow, when it is released and hooked to the plow. The train is then moved to the dumping point and the plow drawn through the train, sweeping the load off to one side or the other. The cars may have side doors hinged at the top when used for fine material, but where large rock is handled the cars are provided with stakes about 2 ft. high to guide the plow. Two sizes of engines are used: 25-ton pull, with double steam cylinders 10 by 12 in., used with $1\frac{1}{4}$ in. rope; and 60-ton pull, with cylinders 12 by 12 in., used with $1\frac{1}{2}$ in. rope. A 17-car train can be unloaded in about 5 min., and any material which can be loaded on to the cars with steam shovels may be unloaded with plows. The hoisting engine is operated by the locomotive engineer or fireman, and, by moving the train at the same time the plow is in operation, it is possible to unload the entire train at one point or to distribute the train load over as great a length as desired.

CONTINUOUS CONVEYORS

Screw or Spiral Conveyors are used for conveying horizontally dry non-abrasive materials, such as grain, flour, seeds, cement and fine coal; also for sand, gravel, fine ashes, etc., although the wear from these abrasive materials is so rapid that the maintenance cost is excessive. The maximum length when

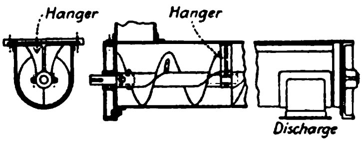


FIG. 43.—Spiral Conveyor.

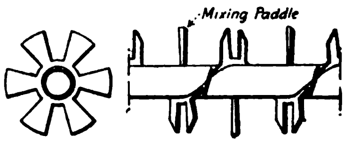


FIG. 44.—Cut-flight Conveyor.

handling grain or light pulverised materials is 400 to 500 ft.; for more abrasive materials, 100 to 150 ft. They may be used as feeders in single sections on inclines up to 15 deg. They should not be used for handling material containing lumps. The **spiral conveyor** (Fig. 43) consists of a stamped or rolled steel spiral secured by lugs to a shaft formed of pipe. The sections (6 to 12 ft. long) are joined by a piece of shafting which serves as coupling and bearing. The screw revolves in a box of steel, wood, cast iron or concrete which carries the hanger bearings. Wooden boxes are usually lined with light steel plate. The power is applied at one end by pulley, gear or sprocket. When used for mixing materials or breaking up lumps, a portion of the spiral is cut away, as shown in Fig. 44, forming a **cut-flight conveyor.** Mixing paddles are sometimes inserted both in cut-flight and standard conveyors to mix the material or retard the flow. When handling abrasive materials, cast-iron screws and boxes should be used.

The **ribbon conveyor,** Fig. 45, is used when handling damp or sticky material which would build up around the shaft of the standard type. It consists of a ribbon flight secured to the shaft by lugs, but with an open space between the ribbon and shaft.

Table 14 gives the maximum advisable speeds for various materials, and capacities based on conveyors with standard-width hangers, continuously fed. The capacity is limited by the quantity of material passing the hangers (1¼ to 3 in. wide). The end of one flight delivers material into the hanger space, and it must flow by gravity (seeking its natural angle of repose) assisted by the crowding action of the flight across this space to the next flight. The **power required** is given by h.p. $= TLC/1000$, in which T = short tons per hour, L = length of conveyor, ft., and C = 0.8 for grain, = 1.6 for coal and cement, = 2.4 for sand, gravel and ashes.

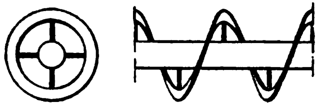


Fig. 45.—Ribbon Conveyor.

Table 14. Speeds and Capacities of Screw or Spiral Conveyors

(For ribbon conveyors take one-third of the capacities given)

Diam. of screw, in.		3	4	5	6	7	8	9	10	12	14	16
Grain	Max. r.p.m.	200	200	190	180	175	175	170	165	165	160	160
	Cu. ft. per hour	34	73	175	244	353	732	910	1206	2181	2937	5125
Cement (90 lb. per cu. ft.)	Max. r.p.m.	....	....	....	125	115	110	100	100	95	90	85
	Bbl. per hour	....	....	....	40	56	112	130	174	290	390	655
Coal, ¾-in. and under	Max. r.p.m.	....	....	....	......	110	105	100	95	90	85	80
	Tons per hour	....	....	....	......	6½	13½	16	21	36½	47	80
Sand, gravel, fine ashes	Max. r.p.m.	....	....	....	115	110	105	100	95	90	85	80
	Cu. ft. per hour	....	....	....	126	180	360	420	540	930	1200	2000

Conveyor and Elevator Chains. Fig. 46 shows a **detachable-link chain** used for the lighter service in handling non-abrasive material, and consisting of a series of malleable cast-iron links hooked together. When straightened out, the links cannot come apart, but when folded up, as shown in Fig. 46, the links may be separated by slipping the bar sideways out of the hook through the recess *a*. When this chain is overstressed, the hooks straighten out, throwing the chain out of pitch. The articulating joint between the bar and hook being exposed, grit may enter, causing wear and elongated pitch preventing chain from properly fitting the sprockets. Fig. 47 shows in part cross-section a **closed-link riveted chain.** The riveted pins are kept from turning by the projection *a*, so that articulation takes place between link and pin, giving a large, well-protected bearing area. This chain will hold its pitch better than the detachable chain. Fig. 48 shows in part cross-section a **closed-link mal-**

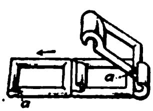


Fig. 46.—Detachable Link.

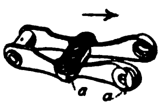


Fig. 47.—Closed Link.

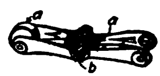


Fig. 48.—Closed Link with Roller.

leable chain with roller *a* to decrease wear on sprockets and support the chain in its horizontal travel. The pin is kept from turning by the projection *b*, and is usually made in the form of a bolt so that rollers may be replaced when worn. This type of chain is also made with bushings between the pins and links. These three types of chain are also made with projections of various forms on the links for the attachment of buckets, flights, etc.

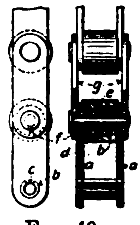


FIG. 49. Steel Bushed Roller Chain.

Fig. 49 shows a **steel bushed roller chain** used for heaviest service. It consists of steel side-bar links, the inner links *a* carrying the bushing *b*, which does not form a complete circle and is kept from turning by the projection *c* of the link. The cast-iron roller *d* turns on the steel bushing *b*. The pin *e* has a projection *f* under the head which fits into the recess of the outer bar *g* and keeps the pin from turning. The pin is riveted on the other end and, being attached to the outer bars and the bushing to the inner bars, the turning takes place between pin and bushing, both of which are replaceable. Attachments are riveted to the side bars for carrying buckets, flights, etc. In some cases, the side bars are made of malleable iron in place of steel. Chains of the type shown in Figs. 46 and 47, not being provided with rollers, will produce unnecessary wear on the sprockets if not properly installed. Fig. 50 shows a sprocket acting as a driver with the conveyor or elevator chain properly installed with the bar running first so that the articulation takes place between bar and hook, producing no wear on the sprocket tooth. If the hook ran first, the turning would take place between the hook and the sprocket tooth. The arrows on Figs. 46 and 47 show the proper direction in which these chains should run on conveyors and elevators.

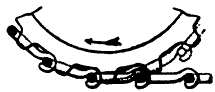


FIG. 50.

Scraper or Flight Conveyors are used for conveying non-abrasive materials such as coal horizontally and on inclines. Their length is limited by the strength of the chains. They may be built 200 to 300 ft. long, if horizontal, and operated on inclines up to 45 deg. Lumps are limited to sizes which may be fed readily between flights and between chains. Single-strand conveyors, Fig. 51, consist of a single chain *a*, to which are bolted flights *b*, of steel or malleable iron. Double-strand conveyors, Fig. 52, have two chains *a*, with the flights *b* suspended from them. Either type may be equipped with sliding blocks *c*, as Fig. 51, or with rollers *c*, Fig. 52, preventing the flights from touching the trough. For heavy duty, double-strand conveyors with rollers should be used. Troughs *d* are made of steel plate or cast iron. Chains are driven by sprockets at one end, take-ups being provided at the other end. The flights scrape the material along the trough, discharging at the end or through openings in the trough bottom controlled by gates. Scraper conveyors always pull the material toward the driving end. Single-strand conveyors should only be used in short lengths and for light duty. The working **speed** of the chain should not exceed 100 ft. per min. when large material is being handled, but it may be increased to 125 to 150 ft.

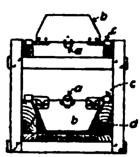


FIG. 51. Single Chain.

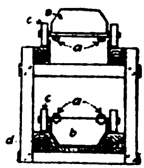


FIG. 52. Double Chain.

Scraper or Flight Conveyors.

when the material is small. The best results are obtained when the speed is kept low. Table 15 gives **capacities** and weights, based on an even and continuous feed. When the angle exceeds 30 deg. from the horizontal, the capacity figures given should be reduced 1 per cent. for each degree. For **power required**, see p. 1172.

Table 15. Capacities and Approximate Weights of Scraper Conveyors

(All double-strand conveyors except those with 4×10-in. and 5×12-in. flights; capacity based on handling coal at a speed of 100 ft. per min.)

Size of flights, in.	Spacing of flights, in.	Capacity with sliding blocks, tons per hour	Weight of chain and flights with sliding blocks per ft. of chain, lb.	Capacity with roller flights, tons per hour	Weight of chain and flights with rollers per ft. of chain, lb.	Size of flights, in	Spacing of flights, in.	Capacity with sliding blocks, tons per hour	Weight of chain and flights with sliding blocks per ft. of chain, lb.	Capacity with roller flights, tons per hour	Weight of chain and flights with rollers per ft. of chain, lb.
4×10	12	25	10	......	......	10×24	24	150	26	120	42
5×12	12	40	12	......	......	10×30	36	185	22	150	36
6×18	18	70	18	......	......	10×36	36	230	30	180	45
8×18	18	90	21	74	32	12×30	36	230	30	175	45
8×20	24	100	18	81	30	12×36	36	275	32	200	48
8×24	24	125	20	97	34	12×42	48	325	31	235	47
8×30	36	150	21	122	35	12×48	48	350	33	275	49
10×20	24	120	25	100	41	......	....	......	......	......	......

Apron Conveyors may be used for practically any material which will not adhere to the carrying surface, except pulverized material which will leak through the joints between the apron plates. As the load is carried and not dragged, less power and smaller maintenance are required than by scraper conveyors. They are used with stationary skirt or side plates as feeders for taking material from hoppers or bins. The length is limited by the strength of the chains. They may be built 300 to 400 ft. long, if horizontal, and operated on inclines up to 30 deg. Sizes of lumps are limited by the width of the pans and the ability of conveyor to withstand the impact of loading. End discharge only is possible. The apron conveyor, Fig. 53, consists of two strands of roller chain separated by overlapping apron plates, forming the carrying surface, with sides 2 to 6 in. high. The chains are driven by sprockets at one end, take-ups being provided at the other end. The conveyors always pull the material toward the driving end. For light duty, flangeless rollers on flat rails are used; for heavy duty, single-flanged rollers and T-rails. Apron conveyors may be run without feeders, provided the opening of the feeding hopper is made sufficiently narrow to prevent material from spilling over the sides of the conveyor after passing from the opening. When used as a conveyor, the **speed** should not exceed 60 ft. per min.; when used as a feeder, 30 ft. per min. Table 16 gives weights and **capacities** based on an even and continuous feed. For **power required**, see p. 1172.

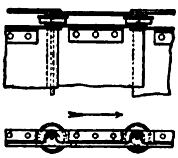


Fig. 53. Apron Conveyor.

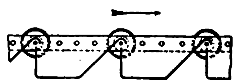


Fig. 54. Open-top Carrier.

Table 16. Capacities and Weights of Apron Conveyors

Width of conveyor apron, in.	Capacity handling coal in tons per hour at speed of 50 ft. per min.				Approximate weight of chains and apron per ft., lb.			Operating as feeder at 10 ft. per min.	
	Sides 2 in. high	Sides 4 in. high	Sides 6 in. high	Sides 8 in. high	9-in. pitch ⅜ × 2½-in. links 3½-in. rollers	12-in. pitch ⅜ × 3½-in. links 6-in. rollers	12-in. pitch ½ × 4-in. links 6-in. rollers	Depth of material on pan, in.	Capacity, tons of coal per hour
18	15	34	52	71	50	....	....	12	20
24	28	53	78	103	56	....	....	18	41
30	45	77	108	139	62	....	....	24	70
36	66	103	141	178	67	115	....	24	85
42	90	134	178	222	73	122	....	24	100
48	109	159	209	259	78	127	145	30	144
54	141	197	254	310	....	133	155	30	162
60	180	243	305	368	....	137	165	30	181
66	217	286	355	424	....	143	175	36	240
72	261	336	411	486	....	150	185	36	262

Open-top Carriers, Fig. 54, are similar to apron conveyors, except that dished or bucket-shaped receptacles take the place of the flat or corrugated apron plates used on the apron conveyor. They will operate on steeper inclines than apron conveyors (up to 70 deg.), as the buckets prevent material from sliding back. Neither sides extending above top of buckets nor skirt boards are necessary. **Speed** when loaded by a feeder, 60 ft. per min. (max.); when dragging the load from a hopper or bin, ≦ 30 ft. per min. The **capacity** should be figured on a basis of the buckets being three-fourths full, the angle of inclination of the conveyor determining the loading condition of the bucket. For **power required**, see p. 1172.

V-bucket Carriers are used for elevating and conveying non-abrasive materials, principally for handling coal when it is desired to elevate and convey with one piece of apparatus. The length and height lifted are limited by the strength of the chains, neither of which should exceed 75 ft. They may operate on any incline and may discharge at any point on the horizontal run. The size of lumps is limited by the size and spacing of the buckets. The carrier consists of two strands of roller chain separated by V-shaped steel buckets. Fig. 55 shows the most common form, where material is received on the lower horizontal run, elevated and discharged through openings in the bottom of the trough of the upper horizontal run.

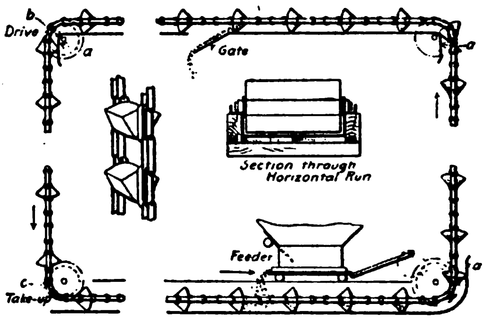


Fig. 55.—V-bucket Carrier.

The material is scraped along the horizontal portions of the conveyor, as in a scraper conveyor. The steel guard plate *a* at the lower bend, guides the material into the bucket. Fig. 56 shows a different form, where material is dug by the elevator from a boot, elevated vertically, scraped along the horizontal run and discharged through gates in the bottom of the trough. Fig. 57 shows a variation of Fig. 56, requiring one less bend in the conveyor. The troughs are of steel or wood steel-lined. When feeding material to the horizontal run, it is advisable to use an automatic feeder driven by power from one of the bend shafts to prevent overloading. Should the buckets of this type of conveyor be overloaded, they will spill on the vertical section. The drive is located at *b*, with take-up at *c*. The **speed** should not exceed 100 ft. per min. when large material is being handled, but when material is small, it may be increased to 125 ft. The best results are obtained when speeds are kept low. Table 17 gives the **capacities** and weights based on an even and continuous feed. For **power required,** see p. 1173.

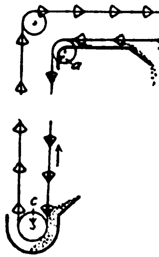


FIG. 56.

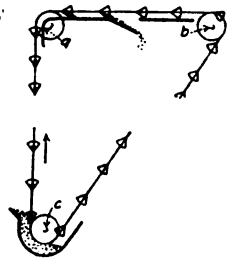


FIG. 57.

V-bucket Carriers.

Table 17. Capacities and Weights of V-bucket Carriers

Buckets				Capacity, tons of coal per hour at 100 ft. per min.	Weight per ft. of chains and buckets, lb.	Buckets				Capacity, tons of coal per hour at 100 ft. per min.	Weight per ft. of chains and buckets, lb.
Length, in.	Width, in.	Depth, in.	Spacing, in.			Length, in.	Width, in.	Depth, in.	Spacing, in.		
12	12	6	18	19	36	30	20	10	24	98	70
16	12	6	18	25	40	36	24	12	30	135	94
20	15	8	24	39	55	42	24	12	30	158	105
24	20	10	24	78	65	48	24	12	36	150	150

Pivoted-bucket Carriers are used for elevating and conveying practically any material which will not adhere to the buckets, although at a disadvantage when handling finely ground abrasive material which can get into the bearings of the moving parts. They require less power than V-bucket carriers, as the material is carried and not dragged on the horizontal run. The length

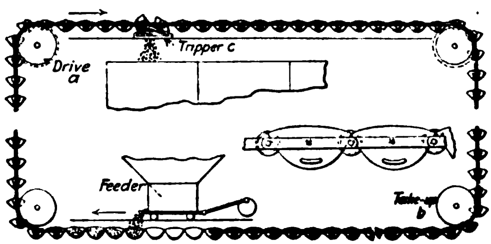


FIG. 58.—Pivoted-bucket Carrier.

and height lifted are limited by the strength of the chains. The length should not exceed 500 ft., and the height lifted 100 ft. They may be operated on any incline, and may discharge at any point on the horizontal run. The size of lumps is limited by the size of buckets. The pivoted-bucket carrier consists of two strands of roller chain with flanged rollers, between which are pivoted buckets of steel or malleable iron. Fig. 58 shows the usual form in which these conveyors are installed. The drive is located at *a*, the take-up at *b*. The material is fed to the buckets on the lower horizontal run (usually by an automatic feeder), elevated and discharged on the upper horizontal run. The tripper *c*, mounted on wheels so that it may be moved to the desired dumping position, engages the cams on the buckets, tipping the buckets until the material is discharged. The buckets, being pivoted above the center of gravity, remain in the carrying position on both horizontal and vertical runs, except when passing the tripper. The rollers of the chains run on T-rails on the horizontal section and between guides on the vertical runs. The conveyor operates best at from 40 to 50 ft. per min., the **speed** being limited by the impact of the buckets on the dumping tripper. Table 18 gives the **capacities** and **weights** based on an even and continuous feed. For **power required**, see p. 1173.

Table 18. Capacities and Weights of Pivoted-bucket Carriers

(Capacity based on a speed of 50 ft. per min.)

Size of buckets, in.	18×16	24×18	24×24	24×30	24×36	30×36	36×36
Pitch of chain, in.	18	24	24	24	24	30	36
Capacity, tons coal per hr.	30	50	67	102	122	160	208
Wt. per ft. of empty carrier, lb.	65	75	80	85	95	115	130

Power Required for Chain Conveyors. The following formulæ give the power required to be applied to the conveyor driveshaft. Allowance in engine or motor size must be made for the speed reduction through gears, driving chains and belts. The weights and capacities of each type will be found in the tables. For scraper conveyors, apron conveyors and open-top carriers,

$$\text{h.p.} = (AWLS/1000) + (BLT/1000) + X$$

where A and B are constants from Table 19; W = weight of conveyor per ft. of one run, lb.; L = length from center of head sprocket to center of tail sprocket, ft.; S = speed of conveyor, ft. per min.; T = capacity of conveyor per hour in tons (2000 lb.); X = 1 for conveyors up to 100-ft. centers and 2 for longer conveyors.

If the conveyor is composed of portions on different inclines, the various portions should be considered independently and the results added. Where such changes in the inclination occur, 10 per cent. should be added for each change in direction.

Example. Determine the power required to drive a scraper conveyor having 10 × 24-in. flights spaced 24 in. apart, and carrying 140 tons of bituminous coal per hour; sliding blocks to be used, and the conveyor to run as shown in Fig. 59. As the conveyor is inclined at 36 deg., 1 per cent. must be deducted from the rated capacity of 150 tons per hour for each degree over 30 deg. so that the conveyor will carry 141 tons per hour at a speed of 100 ft. per min. The values for the 30-ft. inclined section are A = 0.025, W = 26 lb., L = 30 ft., S = 100 ft., B = 1.08, T = 141 tons. The values for the 80-ft. horizontal section are A = 0.03, L = 80 ft., B = 0.60, X = 2, and 10 per cent. must be added for power consumed at the bend between the inclined and horizontal sections. Therefore

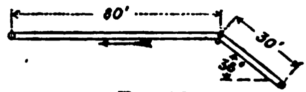


Fig. 59.

$$\text{h.p.} = \left[\left(\frac{0.025 \times 26 \times 30 \times 100}{1000} + \frac{1.08 \times 30 \times 141}{1000}\right) + \left(\frac{0.03 \times 26 \times 80 \times 100}{1000} + \frac{0.60 \times 80 \times 141}{1000}\right) + 2\right] \times 1.10 = 23.68 \text{ h.p.}$$

For V-bucket and pivoted-bucket carriers,

$$\text{h.p.} = (AWL_hS/1000) + (BL_cT/1000) + (TH/1000) + \tfrac{1}{2}X$$

where L_h = horizontal length of conveyor, ft.; L_c = total horizontal length traversed by loaded bucket, ft.; H = total vertical height traversed by loaded bucket, ft.; X = total number of 90-deg. turns in the conveyor; and A, B, W, S and T are the same as for the first formula.

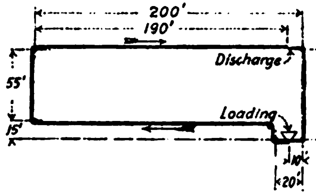


Fig. 60.

Example. Determine the power required to drive a pivoted-bucket carrier having 24 × 24-in. buckets carrying 67 tons of coal per hour; rollers 6 in. in diameter with 1⅜-in. pins; the path of the conveyor to be as shown in Fig. 60. The double lines show the distance traversed by the loaded buckets. $A = 0.0046$, $W = 80$ lb., $L_h = 200$ ft., $S = 50$ ft., $B = 0.076$, $L_c = 380$ ft., $T = 67$ tons, $H = 70$ ft., $X = 6$ bends. Therefore

$$\text{h.p.} = \frac{0.0046 \times 80 \times 200 \times 50}{1000} + \frac{0.076 \times 380 \times 67}{1000} + \frac{67 \times 70}{1000} + \tfrac{1}{2} \times 6 = 13.3 \text{ h.p.}$$

Table 19. Values of Constants in Chain Conveyor Power Formulæ

Angle of conveyor with horizontal, deg.	A				B (for scraper conveyors)					B (for carrier conveyors)		
	Sliding block	3½-in. roller, ¾-in. pin	6-in. roller, 1⅜-in. pin	6-in. roller, 1½-in. pin	Anthracite coal	Bituminous coal	Ashes	Limestone	Corn	3½-in. roller, ¾-in. pin	6-in. roller, 1⅜-in. pin	6-in. roller, 1½-in. pin
0	0.030	0.0043	0.0046	0.0050	0.33	0.60	0.54	0.59	0.33	0.071	0.076	0.083
6	0.030	0.0043	0.0046	0.0050	0.43	0.69	0.63	0.69	0.43	0.17	0.18	0.19
12	0.030	0.0042	0.0045	0.0049	0.54	0.79	0.73	0.79	0.53	0.28	0.28	0.29
18	0.029	0.0041	0.0044	0.0048	0.63	0.88	0.82	0.87	0.62	0.38	0.38	0.39
24	0.028	0.0039	0.0042	0.0046	0.72	0.95	0.90	0.95	0.71	0.48	0.48	0.49
30	0.026	0.0037	0.0040	0.0043	0.79	1.02	0.97	1.02	0.79	0.57	0.57	0.58
36	0.025	0.0035	0.0037	0.0040	0.86	1.08	1.03	1.07	0.86	0.65	0.66	0.66
42	0.023	0.0032	0.0034	0.0037	0.92	1.12	1.07	1.12	0.92	0.73	0.73	0.74
48	0.020	0.0029	0.0031	0.0033	0.97	1.15	1.11	1.15	0.97	0.80	0.80	0.81

Bucket Elevators are of two types, chain-and-bucket, where the buckets are attached to one or two chains; and belt-and-bucket, where the buckets are attached to canvas or rubber belts. Either type may be vertical or inclined, and have continuous or non-continuous buckets. They are used to elevate any bulk material that will not adhere to the bucket. Belt-and-bucket elevators are particularly well adapted to handling abrasive materials which would produce excessive wear on chains. Chain-and-bucket elevators are frequently used with perforated buckets when handling wet material, to drain off surplus water. The length of elevators is limited by the strength of the chains or belts. They may be built up to 100 ft. long, but average from 25 to 75 ft. Inclined belt elevators operate best on an angle of about 15 deg. to the vertical. At greater angles the sag of the return belt is excessive, as it cannot be supported by rollers between the head and foot pulleys. This applies also to single-strand chain elevators. Double-strand chain elevators, however, if provided with roller chain, may run on any angle, as both the upper and return chains are supported by rails. The size of lumps is limited by the size and spacing of the buckets and the speed of the elevator.

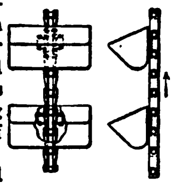


FIG. 61.—Single-strand.

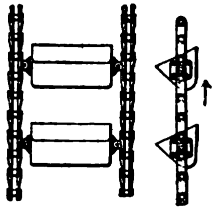


FIG. 62.—Double-strand.

Fig. 61 shows a **single-strand elevator** with back-hung buckets of the **Salem** type bolted to the ears of the **attachment links.** This type should be used only for short elevators handling free-flowing material without lumps, otherwise the buckets will be twisted off the chain. Fig. 62 shows a **double-strand elevator** with end-hung buckets and pivoted attachments, which is used for the heavier service. Fig. 63 shows the discharge end of a double-strand elevator with choke sprockets forming a perfect discharge. This type of discharge allows the non-continuous type of bucket to discharge the material into the chute, placed well under the buckets. Single-strand elevators cannot be built with perfect discharge, and must therefore be operated at sufficient speed to throw the material clear of the buckets into the chute. Fig. 64 shows **continuous overlapping end-hung buckets.** The back of one bucket forms the chute for the discharge of material from the bucket back of it, thus giving a clean discharge of material. When these buckets are made non-overlapping, they may be used with perfect discharge, as illustrated in Fig. 63. Overlapping buckets may run at slower speeds than the Salem type, owing to the nature of the discharge, and therefore handle larger and heavier material. The general operation of the **V-bucket elevator** is described under "V-bucket Carriers," p. 1170. Fig. 65 illustrates the **cast-iron boot** used in connection with Salem-bucket and V-bucket chain elevators and with Salem-bucket belt elevators. The shaft is provided with takeups and carries the tail sprockets or pulley. Fig. 66 illustrates the general form of a vertical belt-and-bucket or back-hung chain-and-bucket elevator with **structural-steel boot and casing.** The casing is made in sec-

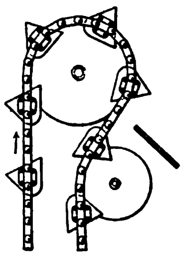


FIG. 63.—Perfect Discharge End.

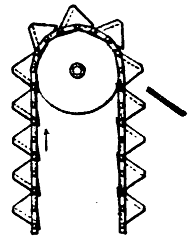


FIG. 64.—Continuous Overlapping Buckets.

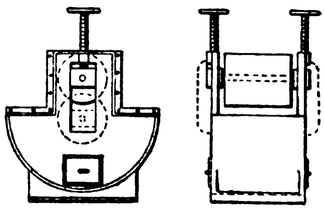


FIG. 65.—Cast-iron Boot..

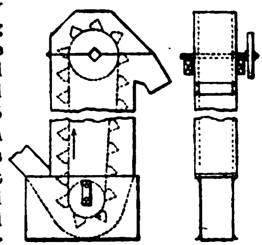
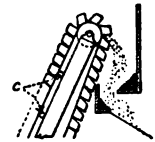


FIG. 66.—Structural Steel Boot and Casing.

tions of light steel plate fastened together with angles. Elevators of this type must be run at sufficient speed to throw the material from one bucket clear of the bucket ahead of it into the chute. Fig. 67 shows a continuous belt-and-bucket elevator of the inclined type. These elevators give the best results when the material is fed directly into the buckets, because when they dig from boots the material becomes wedged between the bucket and belt above the bolts which attach the bucket to belt. The stone box *a* (Fig. 67), used for **feeding stone and ore** to elevators, allows the material to flow at its angle of repose to the buckets. The extension *b* of the box prevents the material from spilling when the bottom of one bucket passes the bottom of the stone box, directing the material into the bucket below. Stone boxes at the discharge end receive the impact of the material, thus reducing the wear on the chutes. The loaded side of the belt is supported by the rollers *c*, the empty side hanging unsupported between the head and foot pulleys.



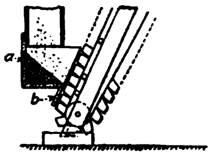


Fig. 67.—Bucket Elevator for Stone or Ore.

Single-strand non-continuous elevators should have a bucket **speed** of from 175 to 200 ft. per min. to insure good discharge. Perfect-discharge and gravity-discharge elevators should run at 150 ft. per min. maximum. Vertical belt-and-bucket elevators require 200 ft. per min. to insure good discharge. Inclined continuous-bucket elevators should not run over 150 ft. per min., and if handling heavy material in large lumps, these speeds should not exceed 100 ft. Table 20 gives the **capacity** of Salem-type bucket elevators and continuous bucket elevators at a speed of 100 ft. per min., handling coal weighing 50 lb. per cu. ft.; also the approximate weights of the elevators per foot of run. The capacities are based on an even and continuous feed, the buckets being figured three-quarters full. **Horse power** required = $TH/500$, where T = load in tons (2000 lb.) per hour, and H = vertical height of lift, ft. If the horizontal run of the V-bucket elevator is over 8 ft., use the power formula for V-bucket carriers.

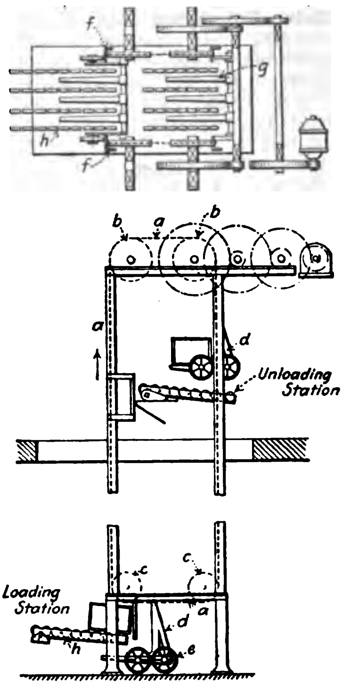


Fig. 68.—Tray Elevator.

Fig. 68 illustrates one type of **tray elevator** used for raising material in packages or boxes. The two chains *a* run over sprockets *b* at the head and *c* at the foot. Suspended from these chains are the trays *d*, carrying the guide wheels *e* which engage the columns *f* of the elevator, preventing the trays from tipping. The bottom of the tray consists of three fingers *g*, which pass between the pivoted rollers *h*. In Fig. 68, the tray is about to pick up the box in the basement and another tray is about to deliver a box on the floor above. As the tray descends, the box rests on the rollers and rolls off automatically. The pivoted rollers forming the loading and discharging stations are controlled by levers so that they may be moved out of the way when desired. Tray elevators operate at a maximum speed of 60 ft. per min.

Belt Conveyors will handle practically any material in bulk which may be properly fed to the conveyor and will not adhere to the belt. Light package material is also handled by belt conveyors.

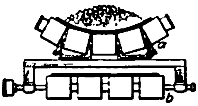


Fig. 69.—Robins Belt Conveyor.

Their length is limited by the strength of the belt. They may be built up to 1000 ft. long, if level, and be operated on inclines up to 20 deg. for material of uniform size and 18 deg. for unsized material. Under favorable conditions these angles may be increased 1 or 2 deg. The size of lumps is limited by the width of the feeding chute, and the temperature of material handled to 180 deg. fahr. Fig. 69 shows the cross-section of **conveyor** (Robins type, patented), which consists of an endless belt passing over pulleys at the conveyor ends. The loaded side is supported by troughed idlers *a*, and the empty belt by the straight idlers *b*. The troughing idlers are of the three- or five-pulley type. The five-pulley

Table 20. Capacities and Weights of Bucket Elevators

Salem-type bucket elevators							Continuous bucket elevators						
Buckets			Weight in lb. per ft.—one run			Capacity, tons coal per hour at 100 ft. per min.	Buckets			Weight in lb. per ft.—one run			Capacity, tons coal per hour at 100 ft. per min.
			Chain and buckets		Belt and buckets					Chain and buckets		Belt and buckets	
Length, in.	Width, in.	Spacing, in.	Single-strand	Double-strand			Length, in.	Width, in.	Depth, in.	Single-strand	Double-strand		
8	5	12	8	12	6	7	8	5	8	17	....	13	14
10	6	12	11	15	9	15	10	6	9	22	....	17	22
12	6	12	13	19	10	18	12	8	12	33	....	24	36
14	6	12	15	21	12	20	14	8	12	36	42	28	41
16	6	12	17	23	15	23	16	8	12	38	44	30	47
18	7	16	17	24	16	25	18	8	12	....	47	34	53
20	7	16	18	25	17	28	20	8	12	....	56	37	60
20	8	18	19	26	18	31	24	8	12	....	70	52	70
24	8	18	21	28	19	38	24	12	18	....	80	53	110
30	8	18	26	36	24	46	30	12	18	....	90	67	140
36	8	18	29	40	27	56	36	12	18	....	100	77	165

type is best for heavy service, as it supports the belt at five points instead of three, giving an easier curve to the belt. The pulleys are 5 in. in diameter on 12- to 18-in. idlers, and 6 in. on 20- to 48-in. idlers. They turn on hollow-steel shafts supported by cast-iron brackets and are lubricated from the interior by large grease cups on the shaft ends through holes in the shaft walls. The return idlers consist of a series of pulleys turning on hollow shafts with grease cups at one end, or pulleys attached to solid shafts turning in adjustable hangers. They are spaced about 10 ft. apart. For troughing idler spacing, see Table 21.

The **belts** generally used are of the rubber-covered type consisting of plies of cotton duck cemented together with rubber friction and protected from moisture and abrasion by a rubber cover. The duck gives the belt its tensile strength. The rubber cover on the carrying side is from 1/16 to 3/16 in. thick, according to the service. When the cover runs straight across the belt, as shown in Fig. 70, it seldom exceeds 1/8 in. in thickness. In the Robins patent belt, Fig. 71, several of the upper plies are stopped off at varying distances from the edges, forming a flexible middle and stiff edges. The rubber cover is 3/16 in. thick at the middle, where the wear is greatest, tapering to 1/16 in. at the edges, where the wear is least. A layer of loosely woven fabric is embedded in the cover over the duck, passing around the edges, increasing the adhesion of the cover. Stitched canvas and woven cotton belts are also used. The fabric of these belts is not protected from abrasion, hence they wear more rapidly. They also shrink and contract, due to atmospheric changes, requiring frequent adjustment of the take-ups to keep them at proper driving tension. The ends of the belts are joined by metallic **lacing** of the Bristol, Turtle, or Crescent type. The holes for the prongs should be punched with an awl to prevent cutting of the fabric.

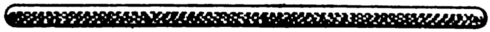


Fig. 70.

Fig. 71.

Rubber-covered Conveyor Belts.

Belt conveyors are driven by power applied to one or more of the pulleys over which the belt bends. The **drive** may be located at either end or at any point between the ends on the return belt, as required by local operating conditions. On long, heavy conveyors elevating material from below ground to a bin or building, the drive is usually located at the ground level and is of the tandem type, in which the belt makes a part wrap around two pulleys geared together and turning at the same speed. This results in an effective arc of driving contact of 360 deg. or more, while the single-pulley drive has a maximum arc of contact of about 250 deg. The tandem drive will therefore operate with less initial tension on the belt. Driving pulleys for the heavy service are rubber-covered to increase the coefficient of friction. The pulleys over which the conveyor belt bends under driving tension should have a diameter of from 4 to 5 in. per ply of belt. Take-up bearings are provided, usually at the loading end, to take up slack in the belt. The adjustment varies from 12 to 36 in. When the power required to raise the load, given by the second part of the formula for inclined conveyors (p. 1179), exceeds that to drive a level conveyor of the same length, given by the first part of the formula, a hold-back device should be attached to the drive to prevent the conveyor from running backward, should it be stopped loaded.

Trippers of the fixed or movable type are used for discharging material between the two ends of conveyors. Fig. 72 shows a self-propelling tripper which consists of two pulleys *a* and *b*, over which the belt passes, the material being discharged into the chute *c* as the belt bends around the upper pulley *a*. The pulleys are mounted on a frame carried by four wheels driven by worm

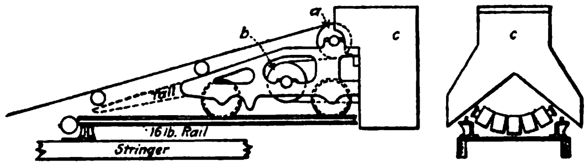


Fig. 72.—Self-propelling Tripper for Belt Conveyor.

gears. A lever on the frame and stops on the rails enable the tripper, taking power from the conveyor belt, to move automatically between the stops, thus distributing the material. Rail clamps are provided to hold the tripper in a fixed position when discharging. The smaller sizes may be moved by hand cranks. Fixed trippers have their pulleys mounted on the conveyor framework instead of on a movable carriage.

When the material handled is damp and sticks to the belt, **rotary brushes**, driven by chain and gears, are used to clean the belt. When handling material of uniform size that will flow readily through the chute, a simple feeding gate may be used. But when handling unsized material, a **feeder** of the shaking type or reciprocating plate type is advised, to insure an even and continuous feed to the conveyor.

Fig. 73 shows **typical arrangements of belt conveyors.** *A* is a level conveyor receiving material at one end and discharging at the other. *B* shows a level conveyor with traveling tripper. The receiving end of the conveyor is depressed so that the belt will not be lifted against the chute when the tripper is at its extreme loading end. *C* is a level conveyor with fixed trippers. *D* shows an inclined end combined with a level section. In passing from the inclined to the level section a crown-faced pulley is used, the belt flattening out at the bend; but as belt and material are moving at the same speed, there is no spill. *E* is a combination of level conveyor, vertical curve and horizontal section. The radius of the vertical curve depends upon the weight of the belt and the tension in the belt at the point of tangency. This must be figured in each case, the radius for average conditions being about 200 ft. *F* is a combination of level conveyor receiving material from a bin, a fixed dump, an inclined section and a series of fixed trippers.

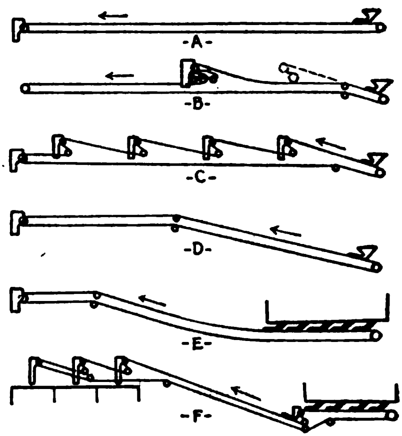


Fig. 73.—Typical Arrangements of Belt Conveyor.

When the quantity of material to be handled is small and the pieces are large, the size of the material fixes the **width of the belt;** when the quantity is large, the capacity fixes the width. In all cases the speed should be as slow as possible. When handling large material, the **belt speed** should not exceed 400 to 500 ft. per min. When feeding abrasive material to inclined conveyors, it should not exceed 400 ft. per min. owing to the wear produced in giving the material the same speed as the belt. Table 21 gives the maximum advisable speed for various widths of belt, but these values should not be ussd when handling the maximum-size pieces given in the table. Belt conveyore may be built to handle practically any quantity of material which can be fed to them. 1500 tons per hour of mine-run coal, 1500 tons per hour of limestone, ranging in size from 18 in. to sand, and 2000 tons per hour of iron ore, crushed to 4 in., are being successfully handled on single conveyors. Table 21 gives the **capacities** and maximum **size of lumps** for belts of different widths, based on an even and continuous feed to the conveyor.

The **power required** to drive belt conveyors depends upon the spacing of idlers, type of drive, etc. The belt should run no faster than required to safely carry the desired load, and if, for any reason, it is necessary to increase the speed, the power should be figured for the full capacity at the chosen speed. Let C = power constant from Table 21; T = load in tons of 2000 lb. per hour; H = vertical height that material is lifted, ft.; S = belt speed, ft. per min.; B = width of belt, in.; L = length of conveyor between centers, ft., then

$$\text{for level conveyors, h.p.} = CTL/1000$$

$$\text{for inclined conveyors, h.p.} = (CTL/1000) + (TH/1000)$$

For each movable or fixed tripper, add the horse power given in Table 21.

Table 21. Data on Belt Conveyors

Size		Speed	Capacity		Power			Belts		Idlers	
					Constant C for material weighing					Spacing of troughing idlers (ft.) for material weighing	
Width of belt, in	Max. size of pieces, in.	Max. advisable speed, ft. per min.	Capacity in cu. ft. per hour at belt speed of 100 ft. per min.	Capacity in tons per hour at belt speed of 100 ft. per min. material weighing 100 lb. per cu. ft.	50 lb. per cu. ft.	100 lb. per cu. ft.	H.p. required for each movable or fixed tripper	Min. plies of belt	Max. plies of belt	50 lb. per cu. ft.	100 lb. per cu. ft.
12	2	300	460	23	0.177	0.127	1/2	3	4	5	4 1/2
14	2 1/2	300	630	31	0.175	0.124	1/2	3	4	5	4 1/2
16	3	300	820	41	0.172	0.121	3/4	4	5	5	4 1/2
18	4	350	1,040	52	0.160	0.116	1	4	5	4 1/2	4 1/2
20	5	350	1,280	64	0.189	0.133	1 1/4	4	6	4 1/2	4 1/2
22	6	400	1,550	73	0.185	0.131	1 1/2	4	6	4 1/2	4
24	8	400	1,850	93	0.181	0.130	1 3/4	4	7	4 1/2	4
26	9	450	2,180	110	0.169	0.124	2	5	7	4 1/2	4
28	10	450	2,500	125	0.161	0.121	2 1/4	5	7	4 1/2	4
30	12	450	2,900	145	0.161	0.119	2 1/2	5	7	4 1/2	4
32	15	500	3,300	165	0.156	0.117	2 3/4	5	8	4 1/2	3 1/2
34	16	500	3,700	185	0.151	0.116	3	5	8	4	3 1/2
36	18	500	4,200	210	0.150	0.115	3 1/4	5	8	4	3 1/2
38	19	550	4,600	230	0.149	0.115	3 1/2	6	9	4	3 1/2
40	20	550	5,100	255	0.148	0.114	4	7	10	4	3
42	20	550	5,600	280	0.147	0.112	4 1/4	7	10	4	3
44	22	600	6,200	310	0.144	0.110	4 1/2	7	10	3 1/2	3
46	22	600	6,800	340	0.141	0.108	4 3/4	7	11	3 1/2	3
48	24	600	7,400	370	0.138	0.105	5	7	11	3 1/2	3

For friction of conveyor ends and drive, add 80 per cent. for conveyors 25 ft. in length, 50 per cent. for conveyors 50 ft. in length, 30 per cent. for conveyors 75 ft. in length, 20 per cent. for 100-ft. conveyors, 10 per cent. for 200-ft., 4 per cent. for 500-ft. To determine the number of plies P of belt, let G = constant determined by the type of drive (= 3600 for single bare pulley drive, 2700 for single rubber-lagged pulley drive, 2250 for tandem bare pulley drive and 2025 for tandem rubber-lagged pulley drive, based on 18.4 lb. per in. per ply working strength of duck) and h.p. = power required to drive the conveyor. Then $P = (\text{h.p.} \times G)/(S \times B)$.

Gravity Conveyors. Boxed, wrapped or bundled material may be lowered from one level to another by **spiral package chutes,** which are largely used in warehouses, stores, etc., to lower packages, boxes, bundles, baskets, etc., from upper floors to basement or intermediate floors. The size of packages is limited by the width of the spirals. Table 22 gives the maximum sizes of packages for spirals of different diameters. The height of package should not exceed its length, otherwise it would overturn. Packages or bundles containing abrasive materials which may leak, should not be handled, owing to destructive wear on the plates. Packages containing sticky material should not be handled, as it will affect the angle of slide. Packages from a few ounces to 500 or 600 lb. may be handled by the same spiral and they will descend at approximately the same speed. Where the spirals are built to handle only heavy material, the drop is less than where light packages only are to be handled.

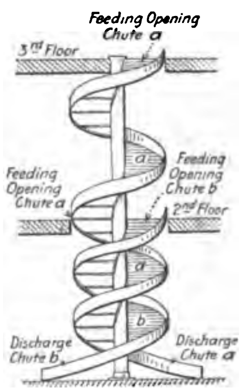


Fig. 74. Standpipe Chute.

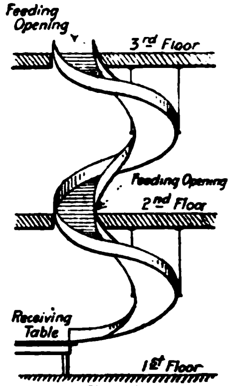


Fig. 75. Open-center Chute.

Spiral Package Chutes.

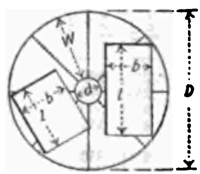


Fig. 76.

Two types of chutes are in use. The **standpipe chute,** Fig. 74, consists of a central column or pipe, supported by a foundation at the bottom or hung from the top. The wing plates forming the bottom of the chute are bolted to the column on the inside, and to a guard plate on the outside. The bottom plates are made of from No. 12 to No. 20 gage steel. When handling very heavy packages, the bottoms are made of cast iron. This type of spiral may be loaded only from the outside. The **open-center type,** Fig. 75, is suspended from the various floors by rods and may be loaded from either the inside or the outside. Where several classes of packages are to be handled and kept separate, the chutes may be made double- or triple-thread. Fig. 74 shows a double-thread. Chute *a* may be fed on either the third or second floor, while chute *b* is fed on the second floor. The open-center type may be constructed in the same manner. The type shown in Fig. 74 is frequently enclosed in a cylinder, in which case the opening in the floor is circular. Fire doors with fusible links may be installed in all of the chutes to close the openings in the floors. These spirals operate on a pitch of from 17 deg. to 29 deg. on the outside of the runway, depending on the material handled.

Table 22. Dimensions of Spiral Package Chutes

(Letters refer to Fig. 76)

Width of runway, in.	Standpipe type			Open-center type		
	Diam. *d* of post, in.	Outside diam. *D* of chute, in.	Maximum size of package, $b \times l$, in.	Inside diam. *d*, in.	Outside diam. *D* of chute, in.	Maximum size of package, $b \times l$, in.
18	4	42	12 × 24, 13 × 18	12	48*	9 × 15
24	4	54	18 × 24, 20 × 20	24	72	14 × 22
30	6	68	20 × 36, 24 × 30	30	90	18 × 30
36	12	86	24 × 48, 28 × 36	36	108	22 × 39
42	12	98	30 × 52, 33 × 42	42	126	27 × 45
48	12	110	36 × 54, 39 × 48	48	144	36 × 66

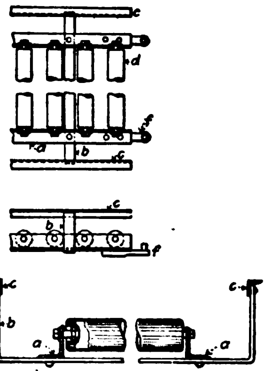


Fig. 77.—Gravity Roller Conveyor.

The **gravity roller conveyor** is used for conveying material on rollers down grade by gravity. It is adapted to handle all classes of merchandise packages and materials that will move by their own weight on the bed of rollers and have one flat, hard surface. Roller conveyors are used in warehouses, brickyards, sawmills, etc., for moving material in the process of manufacture and to cars for shipment. They cannot be used for material so rough that it will catch on the rollers. The length of a conveyor is limited only by the drop, although motor-driven inclines are used to raise the packages by endless chain to higher levels, delivering to another section of roller conveyor. Fig. 77 shows the most common form, consisting of angles *a* connected by bars *b* and carrying a series of rollers *d* provided with ball bearings. These rollers are made of steel tubing about 2½ in. in diam., spaced on 5-in. centers. This type is used for handling boxes, barrels, etc., and is usually provided with guard rail *c*. The conveyors are made in sections 10 ft. long, joined at the ends by the stud *f* entering a hole of the adjoining section. The sections are supported by light steel frames having adjustable legs so that the height may be varied to give the conveyor the proper slope. The legs may be provided with castors. Curved sections are similar in construction to straight sections, except that the rollers are tapered so as to keep the package in the center of the conveyor. The usual radius of curve is 4½ ft. at the outside.

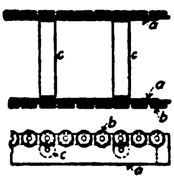
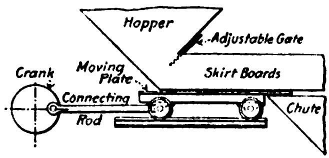


Fig. 78.

The conveyor for handling lumber, shingles in bundles, large, light cases, etc., is shown in Fig. 78. The shingles and cases travel on small wheels *b* carried by the side plates *a*. The lumber travels on the rollers *c*, **spaced 12 in.** apart. **Lumber conveyors** are made in straight sections only, but curved sections are provided for the shingle and case conveyors. The rollers may be made any desired length, depending upon the size of material to be handled. Filled boxes with smooth bottom surfaces, weighing from 25 to 100 lb. require an inclination of about 3/8 in. to the foot, empty boxes, 1/2 in. For the curved section, filled boxes require 3/4 in. and empty boxes 1 in. to the foot. Bricks require 3/8 in. to 1 in. per foot, depending on their smoothness. Lumber and shingles require about 1/2 in. to the foot. The **capacity** is usually determined by the quantity of material that can be placed on, and will move away from, the loading point.

Feeders. When drawing material from a hopper or bin to a conveyor, an **automatic feeder** should be used, unless the material is dry and free-running, such as grain. The satisfactory operation of any conveyor depends on the material being fed to it in an even and continuous stream. The automatic feeder not only insures a constant and regular feed, irrespective of the size of material, but saves the expense of a man who would otherwise be required at the feeding point. Fig. 79 shows a **reciprocating plate feeder**, consisting of a plate mounted on four wheels and forming the bottom



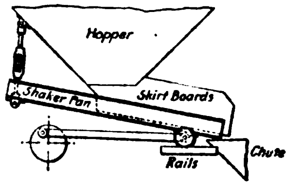


FIG. 79.—Reciprocating Plate Feeder. FIG. 80.—Shaking Feeder.
Automatic Feeders.

of the hopper. When the plate is moved forward, it carries the material with it; when moved back, the plate is withdrawn from under the material, allowing it to fall into the chute. The plate is moved by connecting rods from cranks or eccentrics. The capacity of this feeder is determined by the length and number of strokes, width of plate and the location of the adjustable gate. The number of strokes should not exceed 60 to 70 per min. When used under a track hopper, the material remaining on the plate will freeze in winter, as this type of feeder is not self-clearing.

Fig. 80 shows a **shaking feeder**, consisting of a shaker pan located under the opening in the bottom of hopper at such an angle that the material will not flow when the pan is stationary. When given a reciprocating motion by the cranks and connecting rods, the material is moved forward on the pan. The front end of the pan is carried by a pair of flanged wheels, the back end by two hanger rods provided with turnbuckles so that the angle of the pan may be varied. The crank having an adjustable length of stroke, there are three variables, *viz.*, number of strokes, length of stroke, and inclination of the pan. The number of strokes should not exceed 75 per min., the angle of the pan varying from 8 to 15 deg. This type of feeder, having a wide range of adjustment and being self-clearing so that material cannot freeze to the pan in winter, is adapted to the feeding of practically any material. Automatic feeders are usually driven by the conveyor which they feed, but a clutch should be provided so that the feeder may be stopped, should it become necessary to clear the conveyor for inspection or repairs.

Automatic Scales are often used for weighing and recording the weight of material being carried by belt conveyors, apron conveyors, open-top carriers or pivoted-bucket carriers. They may also be adapted to recording the gross weight of loaded cars such as are used on industrial cable railways or overhead transporters, without stopping the car on the weighing track. By returning the empty cars, the mechanism may be made automatically to deduct the tare weight from the gross weight, leaving the net weight of the material transported. Fig. 81 shows the Merrick Patent Weightometer attached to belt conveyor. Three of the supporting idlers *a* are carried by the steel framework hung by the four rods *b* from the scale beams. These scale beams are attached by levers to the weigh beam *c*. The weight of the load on the suspended portion of the conveyor at any instant is automatically balanced by the buoyancy of a cylindrical iron float suspended from the long end of the weighing beam and partially immersed in a bath of mercury. An increase or decrease of the load on the levers will raise or lower the float in the mercury until the loss or gain in buoyancy compensates for the variation in load. The function of the float is to insure that the movement of the beam from its zero position, when the conveyor is empty, be proportional to the weight of the material at any instant on the suspended portion of the conveyor. The extreme end of the weigh beam is connected by rod *d* to a totalising mechanical integrator. The Messiter Patent Electric Weigher is similar to the weightometer in the method of suspension of the loaded portion of the conveyor. The deflection of the weigh beam, due to load on the conveyor, is measured electrically. Either of these scales will weigh with an error of not over 1 per cent.

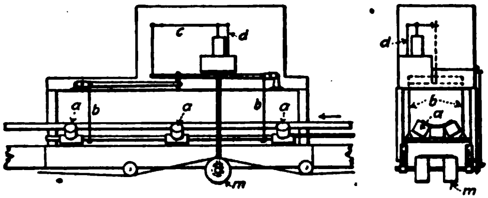


FIG. 81.—Automatic Scales for Conveyors.

Pneumatic Conveyors are used for handling material in bulk, such as ashes, coal, grain, wood waste and particles thrown off by grinding wheels and woodworking machinery. They are usually of the suction type, which makes the apparatus dustless in operation. Incandescent inflammable material should not be handled unless thoroughly quenched before it comes to rest in the separator. Compressed-air conveyors are used to some extent for feeding pulverised coal to kilns and boilers under pressure and handling mixed concrete or grout where it is desired to deliver the material under pressure. Their length is limited by the power required to move material by a current of air, which is not an economical method of conveying. The height material can be lifted is limited to about 125 ft., and the size of lumps by the length of pipe and frequency and location of bends. The latter is a most important consideration, because the velocity of the material is decreased at the bends and, if they are to close together, the air current will be unable to give the required velocity to carry it past the next bend.

The suction ash conveyor system, Fig. 82, consists of a conveyor pipe *a* with an inlet at one end and feeding openings *b* in front of the boilers. The conveyor pipe *a* discharges to the air-tight separator storage tank *d* near the top. Just before entering the tank, the ashes are sprayed by small streams of water *c* to thoroughly quench them and settle the dust. The suction pipe f leads from the top of the tank to the exhauster g, preferably of the positive type, and produces a suction of air from the feeding openings so that material is drawn through the conveyor pipe to the separator tank, where the material is stored ready to be drawn off by gravity through gate e. The air passes through the suction pipe, exhauster and discharge pipe.

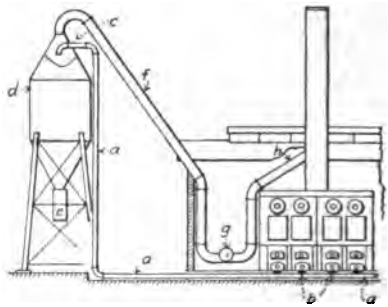


FIG. 82.—Suction Ash Conveyor System.

The method of handling coal, grain, wood waste and other materials is similar to that described above, except that where light material, such as wood waste, is handled, the separator is made circular with the inlet at the periphery, the material entering tangentially. The air outlet is in the center at the top and the material outlet at the bottom. The mixed air and material enter the separator with a whirling motion, but, as the velocity is reduced by the increase of area, the material settles and is thus separated from the air. When dusty material is handled, the water spray is used to prevent the dust and abrasive material from being drawn into the exhauster. The elbows are usually made with removable wearing pieces of manganese steel. The wear on the pipes is slight, as the air current draws the material away from the sides toward the center. Table 23 gives the approximate capacity and power required for different sizes of suction conveyors handling ashes, based on even and continuous feed.

Table 23. Capacities and Power Requirements of Suction Ash Conveyors

Diam. of conveyor pipe, in.	6	8	10
Capacity, tons of ashes per hour	3–4½	6–10½	9–18
Greatest advisable length, ft	200	500	800
Horse power required	20–30	25–50	50–150

STORAGE OF MATERIAL

Bulk material is stored in open piles and reclaimed in a variety of manners. For temporary storage piles, material is dumped from cars on trestles to the ground below and reclaimed by locomotive cranes, steam shovels or other types of excavating machinery. The type of permanent installation depends on the nature of the material, the quantity to be stored and the rate at which it must be reclaimed. Bituminous coal cannot be piled higher than 30 to 40 ft., owing to the danger of fire, some grades of coal being limited to 20 ft. Anthracite coal, stone, ore, sand, gravel and similar materials may be stored in high piles.

Fig. 83 shows a **storage pile spanned by a traveling bridge,** provided with a power-driven self-filling grab bucket. The material to be stored is received in hopper-bottom cars a, discharged into trench b, from which it is removed by the grab bucket c and placed in the storage pile. The material is removed from the storage pile by the grab bucket, discharged to hopper d carried by the bridge, from which it is fed to cars e. As the bridge is carried by wheels, it may travel the length of the pile. The capacity of a storage of this type is determined by the span of the bridge, the length of pile and the height to which material is stored. The capacity of bridges of this type is from 100 to 300 tons per hour from trench to storage and from storage to cars. The **Robins system** (patented) employs a bridge of the type shown in Fig. 83, on the truss of which is carried a belt conveyor with automatic tripper. The loading end of the conveyor is on an incline, receiving material

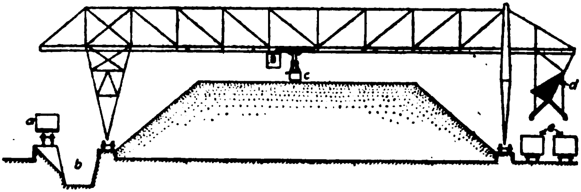


Fig. 83.—Storage System with Traveling Bridge.

from a belt conveyor running parallel to the bridge travel and supported by a trestle 15 to 20 ft. high. The tripper of this elevated conveyor discharges the material to the conveyor carried by the bridge, the tripper of which discharges to the storage pile. With this system, material may be put into storage without rehandling at a rate of from 500 to 1200 tons per hour, depending on the capacity of the conveyors. The **Mead-Morrison system** (patented) employs an endless-cable railroad on an elevated structure running parallel to the bridge travel, the cable being looped over the bridge so that the cars leave the trestle, pass over the bridge, dump and return to the trestle without being detached from the cable. Bridge-type storage is frequently made with one end of the bridge pivoted, forming a circular pile where the shape of the ground makes such a pile more economical than a rectangular one.

Fig. 84 shows **storage pile formed with a locomotive crane** (Link Belt Co. system, patented). The coal is received in hopper-bottom cars *a*, discharged into the track hopper *b*. The crane operates on a track of 80 ft. radius, digging the material from the hopper, discharging to the storage pile; or it may remove the material from the storage pile, discharging to cars. Plants of this type have a capacity of from 75 to 150 tons per hour, depending upon the size of the grab bucket. The height of storage is limited to from 25 to 30 ft.

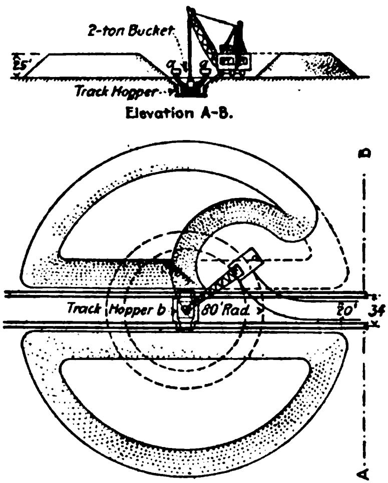


Fig. 84.—Locomotive Crane Storage System.

Fig. 85 shows a type of open **storage with overhead and tunnel conveyors**, used where the height of the pile is not limited and where it is desirable to have a large capacity both into and out of storage. It consists of a gallery supported by A-frames, carrying a conveyor which will discharge the material to the storage pile below. The material is withdrawn by conveyor in a concrete tunnel underneath the pile through gates in the top of the tunnel. The material forms its own angle, and the material which will not flow by gravity is used as reserve storage to be reclaimed in emergencies by locomotive crane or steam shovel. This type of storage is used for storing anthracite coal, crushed stone, sand, gravel, ore and other material which will flow by gravity. The capacity into and out of storage is determined by the size and type of conveyors. Where belt conveyors are used, capacities of 1000 to 1500 tons per hour are obtained both in and out.

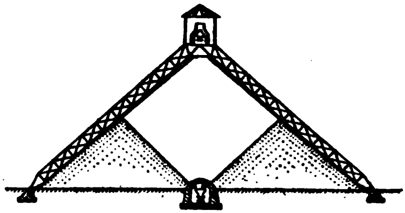


Fig. 85.—Storage System with Overhead and Tunnel Conveyors.

Fig 86 shows the **Dodge system** (patented) **of storing anthracite** coal. This consists of two conical piles, each spanned by the trusses *a* and *b*, which are pivoted at the lower ends to foundations and at the upper ends to each other. They are guyed by the wire-rope cable *c*. A scraper conveyor is carried by truss *b* and receives its material from track hopper *d*. The conveyor drags the material up the truss, discharging through openings in the trough, the crest of the pile increasing in height as the quantity increases. The material thus rolls on itself, eliminating breakage. Material is reclaimed by the pivoted scraper conveyor *e*, made reversible and operating in a horizontal plane. The frame of the conveyor is pivoted at *f* and is supported on a series of circular tracks *g*, of which *f* is the center. As conveyor *e* is swung into the pile, it drags the material to the pivot point *f*, from which

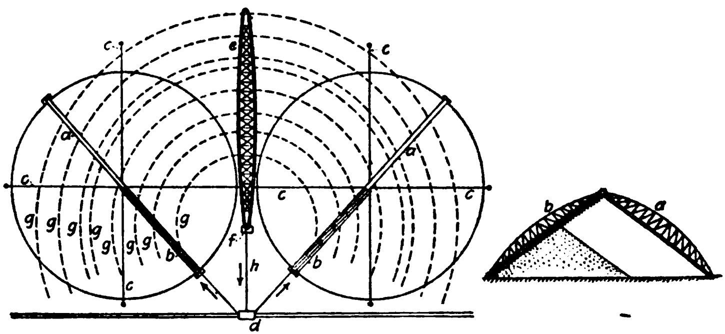


Fig. 86.—Dodge System of Storing Anthracite.

it is elevated on conveyor *h* to cars. A pile 300 ft. in diameter on the base will store approximately 50,000 tons of coal. The capacity in and out of storage is from 100 to 150 tons per hour, determined by the size of the conveyors.

Fig. 87 shows the **Robins-Messiter system** (patented) **of storing and averaging ore**, used to prepare charges of ore, flux, etc., for metallurgical furnaces. The material is crushed, automatically sampled, weighed and discharged to the storage pile by the tripper of conveyor, *a*, carried by gallery above the pile. The tripper discharges material only when moving in one direction, and distributes each carload of material over as many feet of the bed as possible. Analysis of the contents made from the samples enables the operator to keep accurate record of the chemical contents of material stored, so that when the bed is completed it forms a proper furnace charge. Material is removed from the bed by the reclaiming machine *b*, carried by four wheels and moved into the bed at a predetermined rate. A chain conveyor *c* consisting of a series of plows is carried

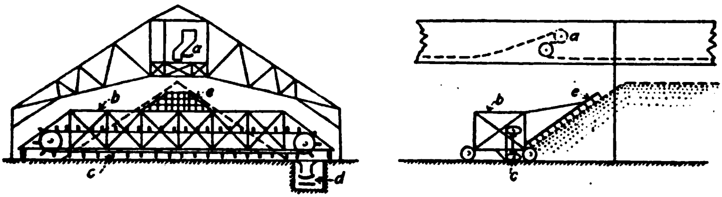


Fig. 87.—Robins-Messiter System of Storing and Averaging Ore.

by the reclaimer, dragging the material from the toe of the pile to the belt conveyor *d* in the trench, which carries the material to the furnace bins. On the front of the reclaimer there is mounted a mechanical harrow *e*, inclined at the angle of repose of the material. This harrow is given a reciprocating motion across the pile, loosening the material and causing it to flow at its natural angle of repose so that when the reclaimer has moved into the bed a predetermined distance it will remove a fixed quantity which is an exact average of the cross-section of the pile. The capacity of the system, both storing and reclaiming, is determined by the size of the reclaimers and conveyors, the average capacity being from 100 to 150 tons per hour reclaiming, and 200 to 300 tons per hour storing. The minimum number of beds is three, one being in the process of forming, one being reclaimed and one a surplus bed to store any excess of material received.

SECTION 10

TRANSPORTATION

BY

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CONTENTS

TRANSPORTATION

AUTOMOBILES

BY

P. M. HELDT

REFERENCES: "Cyclopedia of Automobile Engineering," American Technical Society. A. Graham Clark, "Text Book of Motor Car Engineering," Van Nostrand. Heldt, "The Gasoline Automobile," The Horseless Age Co. Cushing and Smith, "The Electric Vehicle Hand Book," H. C. Cushing.

General. Motor road vehicles are usually propelled by multi-cylinder four-cycle gasoline motors. Practically all commercial-car engines have four cylinders; most pleasure-car engines have either four or six cylinders, a few eight and twelve. All parts of the mechanism are supported by the vehicle frame, which is generally made of channel-section members pressed from sheet steel. The **typical pleasure car** "chassis" (vehicle without body), as shown in plan view in Fig. 1, comprises the following main divisions or assemblies: The motor *A*, with its carburetor, ignition device, cooling and lubrication systems; a friction clutch, *B*; a change-speed gear or transmission *C*; a rear propelling axle *D* with wheel-hub brakes *EE*; a front steering axle *F*; a steering gear *G*; a control lever assembly *H*; frame *I*; springs

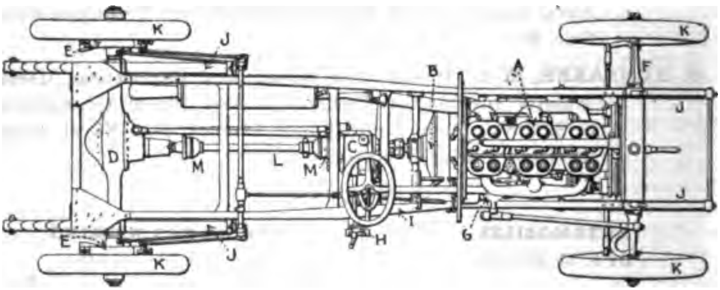

FIG. 1.—Plan of Chassis of Typical Pleasure Car.

J J J J; wheels *K K K K*. The change-speed gear, which is carried on the spring-supported frame, is connected to the driving rear axle through a jointed propeller shaft *L* having either one or two universal joints *MM* inserted in it. The rear axle is driven through a pair of bevel gears and embodies a differential gear through which the driving torque is under all conditions divided equally between the two rear wheels, even when the two wheels run at different speeds, as in turning corners.

In **heavy commercial vehicles** the power is transmitted from the change-speed gear either through a pair of bevels to a differential countershaft and thence by a pair of side chains to the rear wheels, or by a pair of bevels and a pair of internal gears to the rear wheels, or by a worm and worm gear to a live rear axle.

Resistances to Vehicular Motion. The total resistance to the motion of a road vehicle is made up of three items, (1) traction resistance, (2) air resistance and (3) resistance due to gradients. The **traction resistance** on most ordinary pavements can be figured at 25 lb. per 1000. The **air resistance** is about $0.0042\ V^2A$ lb., where V is the vehicle speed in miles per hour and A the forwardly projected area in sq. ft. The resistance due to an up grade of n per cent. in lb. $= nW/100$, where W is the weight of the vehicle. On a down grade there is an equal negative resistance, and on a down grade of 2½ per cent. or more, with smooth, hard road surface, a vehicle will coast.

According to M. Forestier (*Proc. Sec. Int. Aut. Congress*, Paris, 1902), the **traction resistance** f per 1000 lb. weight of vehicle and load is as follows on different kinds of road surfaces:

	f in lb.
Smooth dry macadam (or good dry dirt road)	15
Slightly defective macadam, reasonably dry	18
Good macadam softened by rain	20
Slightly defective macadam softened by rain	22
Irregular Belgian block pavement	22
Macadam in bad state of repair	25
In loose gravel	75–100
In sand	150–200

For pneumatic tires running on asphalt pavement at 15 miles per hour, f ranges from 21 to 31 lb. See also p. 236.

The Motor Horse Power Required for different types of vehicles is determined by empirical rules. Touring cars and roadsters are fitted with motors of 1 rated horse power per 60–100 lb. of vehicle weight. Motor trucks of t tons load capacity are fitted with motors of $15 + 5t$ rated horse power.

Tread and Wheelbase. The standard tread (center to center of tire-ground contact) for pleasure and light commercial vehicles is 56 in. For the Southern States' trade the tread is often made 60 in. For motor trucks there is no standard, the treads varying from 62 to 70 in. Wheelbases range as follows:

Four-cylinder cars:	H.p.	Wheelbase, in.
Runabouts and roadsters	30 and under	90 to 105
Runabouts and roadsters	>30<40	105 to 115
Taxicabs (4 to 5 passengers)		96 to 100
Touring cars	30 and under	100 to 115
Touring cars	>30<40	110 to 120
Touring cars	Over 40	120 to 130

Six-cylinder cars: About 10 in. longer than for four-cylinder cars of the same class.

AUTOMOBILE MOTORS

Motor Output. According to the A. L. A. M. **horse-power rating formula,** an automobile gasoline motor should develop $nb^2/2.5$ brake horse power at 1000 ft. piston speed per min., where n is the number of cylinders and b the cylinder bore in in. This formula is based on a "brake mean effective pressure" (product of the mean effective pressure by the mechanical efficiency) of 67.2 lb. per sq. in. This is a fair average figure, although many designers have improved upon it in recent years. The assumption involved in the formula that the horse-power output is independent of the length of stroke, is incorrect. Long strokes, large valves and light reciprocating parts have raised the piston speed corresponding to maximum engine output to 1400 ft. per min. and higher. The following formula, adopted in 1911 by the Society of Motor Manufacturers and Traders (Great Britain), takes account of the length of stroke:

$$\text{b.h.p.} = 0.45(b - 1.18)(l + b),$$

where l is the length of stroke and b the cylinder bore, both in in.

The Technical Committee of the Automobile Club of France has developed some motor-rating formulæ for the French Ministry of Public Works, to serve as a basis for taxation. These formulæ are based on a brake mean effective pressure of 75.2 lb. per sq. in. for touring-car motors whose piston speed does not exceed 1200 ft. per min.; 78.1 lb. per sq. in. for touring-car motors whose piston speed exceeds 1200 ft. per min.; 82.3 lb. per sq. in. for commercial-vehicle motors, and 92.3 lb. per sq. in. for aeronautic motors (*Bull. Off. d. l. Comm. Tech.*, June, 1912).

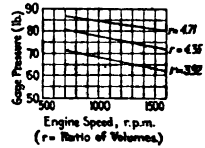

FIG. 2.—Compression Pressures in Motor Cylinders.

Compression. The compression space is made equal to from 0.25 to 0.35 the piston displacement. The **compression** obtained may be calculated by means of the equation $C = (V/v)^{1.3}p$, where C is the compression pressure in lb. per sq. in., absolute; V is the initial volume of the gas before compression begins; v the final volume and p the initial pressure. The value of p corresponding to operation at full load will not exceed 13 lb. per sq. in., owing to the resistance of the valve passages. Fig. 2 shows the results of some compression tests made by Prof. W. Watson on a four-cylinder Clement-Talbot engine of 3.35-in. bore and 4.72-in. stroke. The change in the compression ratio was obtained by inserting packing pieces of different thicknesses below the cylinder.

The **normal explosion pressure** may be taken as four times the gage compression pressure. This will be exceeded when one or more explosions are missed and the cylinder is completely cleared of dead gas.

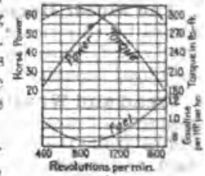

FIG. 3.—Variation of B. H. P., Motor Torque, and Fuel Consumption with Speed.

Characteristic Curves. The average four-cylinder automobile motor can be operated under full throttle over a wide range of speed, down to about one-third the speed of maximum output (depending upon flywheel capacity and carburetor characteristics), and a six-cylinder motor is even more flexible. The variation of the torque, brake horse power and specific fuel consumption with variations in the speed is shown by the diagram, Fig. 3. The torque is a maximum at a speed slightly above the lowest speed of stable running, and decreases at first slowly and then more rapidly. The fuel economy is highest slightly below the speed of maximum output, the **specific fuel consumption** then being about 0.76 lb. per h.p.-hr. These curves were obtained from an Alco six-cylinder motor of 4¾-in. bore by 5½-in. stroke. A good figure for the average fuel consumption of automobile motors under full load is 0.8 lb. per h.p.-hr. For maximum fuel economy the amount of air should exceed that theoretically required by from 15 to 50 per cent., according to speed and throttle position. Fig. 4 shows the variation of the **thermal efficiency** for maximum output at different throttle positions, and the variation of the speed of maximum output with the throttle position. These curves are based on the tests (Taylor, *Horseless Age*, March 4, 1908) of a four-cylinder Mercédès Motor of 4.72-in. bore and 5.9-in. stroke.

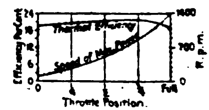

FIG. 4.—Variation of Thermal Efficiency and Speed of Maximum Power with Throttle Position.

Engine Parts

For the theory of the gasoline motor, see p. 320. See also Internal-combustion Motors, pp. 1020–1050. A part sectional elevation of a four-cylinder automobile motor is shown in Fig. 5 and a cross-section through another motor in Fig. 6.

Cylinders and pistons are cast of gray iron. Cylinders of four-cylinder motors are cast either in pairs or all in one block, rarely singly; cylinders of six-cylinder motors either in pairs, in threes, or all in one block. Cylinder-wall thickness can be made $(b/30) + 1/8$ in. Water spaces are made comparatively deep, from 1/2 in. for a bore of 3 to 4 in. to 3/4 in. for a bore of 6 in. The jacket wall is made of a thickness varying from 3/16 in. for 3- to 4-in. bore to 1/4 in. for 6-in. bore. There are three common arrangements of valves, viz., all valves on one side of cylinders, with inlet between exhaust valves; inlet valves on one side and exhaust on the other; and all valves in the cylinder heads. The first arrangement is gaining, for one reason because it lends itself most readily to enclosure of the valve mechanism.

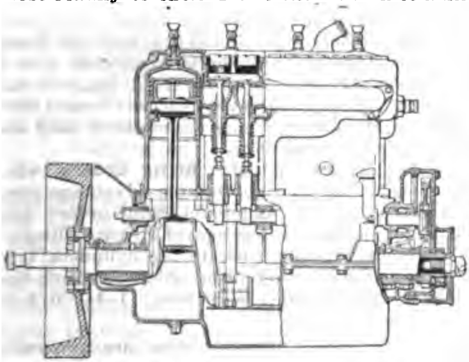

FIG. 5.

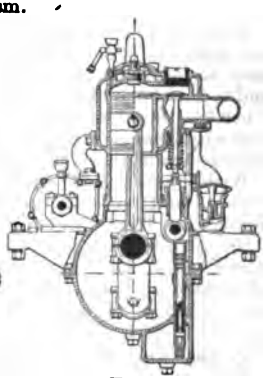

FIG. 6.

FIGS. 5 AND 6.—Sectional Elevations of Automobile Motors.

Pistons are of the trunk type with internal bosses for piston-pin bearings. They are generally provided with three or four packing rings above the piston pin. Rings are turned from cast iron to an outside diam. $= 1.035b$ and an inside diam. $= 0.973b$, have a slot cut in them to form either a 45-deg. diagonal or a lap joint and so as to remove $0.085b$ in the direction of the circumference; the rings are then compressed and clamped on an arbor and ground to an outside diameter b.

The piston head and side wall above the piston pin (underneath rings) can be made of the same thickness as the cylinder wall. Below the piston pin the wall is made thinner. The length of the piston is usually made about $1.25b$; the piston pin is placed substantially at the middle. The length of the piston-pin bearing in connecting rod $= 0.5b$. The pin diameter is figured on the basis of 2000 lb. explosion pressure per sq. in. of projected bearing area. Pins are usually made hollow and case-hardened.

Connecting Rods. Length $= 2l$ to $2.25l$, where $l =$ length of stroke. They are generally drop-forged, of I-section so proportioned that the cross-sectional area is about one-half of the full section of same overall dimensions and height = 1.5 × width of section. If forged of carbon steel, the cross-sections can have 1 sq. in. of area per 7500 lb. explosion pressure on the piston for a length of rod of 14 in., and 10,000 lb. for a length of rod of 8 in. For other lengths and other materials, in proportion.

Crank Shafts are made of 0.45 per cent. carbon steel or nickel steel, and are generally drop-forged. For four-cylinder motors they are supported mostly in three bearings, occasionally in two (small motors) and in five. The crank-pin bearing area is figured on the basis of 900 lb. per sq. in. of projected area with three-bearing support and 700 lb. per sq. in. for two-bearing support, the ratio of length to diameter being made 1.35 in the first case and 1 in the latter. Main journals are made of the same diameter as crank pins; provided the flywheel is at the rear, the rear bearing can be made 1.75 times the length of the crank pins and the other two bearings 1.25 times the crank-pin length. All crank arms are made the same width, about 1.3 times the crank-pin diameter. The thickness or small dimension of the long arms is made equal to $\sqrt{d^3/2w}$, and the thickness of the short arms to $\sqrt{d^3/3.5w}$, where d = shaft diameter and w = width of arm.

Four-cylinder crank shafts for four-cycle motors are made with their two inner crank pins in line and the two outer ones at 180 deg. from the inner ones, to insure proper balance. Six-cylinder crank shafts have crank pins 1 and 6, 2 and 5, 3 and 4 in line, respectively, and each pair at 120 deg. with the other two pairs, for the same reason. Crank shafts are generally forged with a flywheel flange and a clutch pilot at their rear end. Sometimes they are drilled through the arms and pins for oil circulation.

With four-cylinder motors there are two possible **firing orders**, viz., 1–2–4–3 and 1–3–4–2, neither of which has any appreciable advantage over the other. With six-cylinder motors there are six possible firing orders, and if the cylinders are cast in pairs and the exhaust passages of the two cylinders in each pair are "siamesed" or come together close to the cylinders, it is advantageous to choose one of the following three firing orders in which the two cylinders of any pair never fire in direct succession: 1–4–2–6–3–5, 1–5–3–6–2–4, 1–3–2–6–4–5.

Valves and Valve Gear. Conical-head poppet valves are generally employed. With all valves on the same side the clear diameter of the valve port is limited to about $0.4b$; with other valve arrangements it is made $0.5b$ and even greater. The lift is made small, for noiseless operation at high speed, usually about $\frac{1}{8}d + \frac{1}{8}$ in., where d = clear diam. of valve in in. The valve is held to its seat by a coiled compression spring of such strength as to equal the inertia force on the valve and intermediate reciprocating members at the highest engine speed. Following is a good average **timing** of the valves: Inlet opens 15 deg. past top dead center; inlet closes 30 deg. past bottom dead center; exhaust opens 45 deg. ahead of bottom dead center; exhaust closes 10 deg. past top dead center. Early exhaust opening and late inlet closing go together with high speed and great maximum output, but interfere with operation at low speeds. Inlet and exhaust pipes can be calculated for a mean gas speed of 7200–8000 ft. per min.

Weights of Flywheels and Engines. Flywheels are generally made with a solid web which is bolted to an integral flange on the crank shaft. The outside diameter varies between 15 and 20 in., the ground clearance required setting the latter limit for pleasure cars. The weight in the rim for either four- or six-cylinder motors can be made $8.5(bl/r)^2$ lb., r being the mean radius of the wheel rim in in. The weight of a complete motor with aluminum crank case is about $5b^2l$ lb. for four cylinders, and $7b^2l$ lb. for six cylinders. This is equivalent to about 15 lb. per h.p. for a stroke of average length.

Lubrication. Engines are generally lubricated by combined circulating and splash systems.

Carburetors. Nearly all automobile carburetors work on the spray jet principle (Fig. 7). The gasoline is kept at a constant level just below the top of the spray nozzle A by means of a float B and float valve C in a communicating float chamber, and is drawn from the nozzle by the suction created in the air passage D around it by the downward stroke of the engine piston. The fuel delivery increases faster than the air delivery with an increase in suction (due to increased engine speed or greater throttle opening), and to compensate for this effect a supplementary air valve E is placed in the wall of the passage beyond the spray nozzle A, which opens automatically with

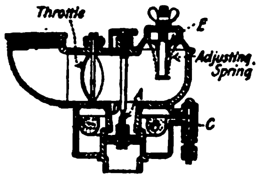

Fig. 7.—Spray Jet Carburetor.

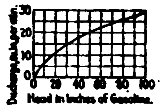

Fig. 8.—Gasoline Flow Through Carburetor Nozzles.

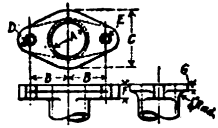

Fig. 9.—S. A. E. Standard Carburetor Flange.

increasing suction and dilutes the charge to keep its proportions constant. The rate of gasoline flow Q/t is connected with the suction head h by an equation of the form $C_1(Q/t)^2 + C_2(Q/t) = h$ where C_1 and C_2 are constants and Q is the volume or weight flowing in a time t. The curve Fig. 8 gives the rates of gasoline flow from an 0.83 mm. (0.0327 in.) carburetor nozzle under different suctions, as determined by Lauret. The standard carburetor flange recommended by the Society of Automobile Engineers is shown in Fig. 9. Following are the dimensions in in. for different sizes, the letters referring to Fig. 9:

Carburetor, size, in.	*A*	*B*	*C*	*D*	*E* *	*F*	*G*
3/4	15/16	1 1/16	1 5/8	11/32	5/16 × 18	3/8	1/8
7/8	1 1/16	1 1/8	1 3/4	11/32	5/16 × 18	3/8	1/8
1	1 3/16	1 3/16	1 7/8	11/32	5/16 × 18	13/32	5/32
1 1/4	1 7/16	1 11/32	2 3/16	13/32	3/8 × 16	15/32	3/16
1 1/2	1 11/16	1 15/32	2 1/2	13/32	3/8 × 16	15/32	3/16
1 3/4	1 15/16	1 21/32	2 13/16	15/32	7/16 × 14	9/16	7/32
2	2 3/16	1 25/32	3 1/8	15/32	7/16 × 14	9/16	7/32

* U. S. Standard threads.

Ignition. For ignition systems, see p. 1608. High-tension **magnetos** usually furnish the ignition current in regular operation and a storage or dry-cell battery in starting the motor. Magnetos must be positively driven at crank-shaft speed for four-cylinder motors and at 1½ times the crank-shaft speed for six cylinder. The conventional **battery equipment** consists of three storage cells of about 60 amp-hr. capacity or six 6-in. dry cells. Magnetos are usually clamped down to their bases (which are cast integral with the crank case) by means of screw-clamp straps, and are located in position by four dowel pins at the corners of a 50-mm. square. The shaft center is 45 mm. above the bottom plane of the base. With the **high-tension magneto** ignition system the only parts besides the magneto that have to be accommodated on the engine are the **spark plugs**, which are usually screwed into holes in the caps over the inlet valves. The part screwing into the cylinder or valve cap, known as the spark-plug shell, has been standardized. The S. A. E. standard spark-plug shell is illustrated in Fig. 10. It has 18 threads to the inch, of the U. S. S. type, cut perfect into the recess. Besides the S. A. E. standard shell, a shell with a ½-in. pipe thread is used, and a metric standard shell with a thread 18 mm. in diam. and of 1.5 mm. pitch.

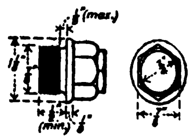

FIG. 10.—S. A. E. Standard Spark-plug Shell.

Engine Cooling. Cooling water is generally circulated through a **radiator** at the front of the engine by means of a pump (generally of the centrifugal type) positively driven from the engine crank shaft, and a **fan** driven from the engine is placed back of the radiator to increase the air circulation through it. The radiator should be provided with from 2 to 2¼ sq. ft. of **radiating surface** per motor horse power, counting only one side of gills or flanges which are not in direct contact with the water. With natural or thermo-siphon circulation, and on slow-running motor trucks, more radiating surface is required.

Two-cycle Motors (for principle of operation, see p. 1021) are comparatively little used on automobiles, but much for marine work. The conventional type employs a crank-case precompression of about 8 lb. per sq. in. To get this compression the connecting rod must be made as short as possible (length = $1.8\,l$) and the crank-chamber walls brought close to the crank sweep. The inlet port to the cylinder should be of such height as to open and close 44 deg. of crank motion on either side of the dead center; exhaust port, 58 deg. Both ports should be flush with the bottom end of piston stroke. In case the charge is admitted to the crank case through a piston-controlled port, the latter can be made of the same dimensions as the cylinder inlet port. The piston must be made longer than the stroke. Ports in cylinder walls must have at least one "bridge" to prevent piston rings getting caught. According to Roberts ("Gas Engine Handbook"), the output of a two-cycle motor is given by the equation h.p. $= nb^2lr/9000$, where n = number of cylinders; b = bore in in.; l = stroke in in., and r = r.p.m. Engines with crank-case precompression cannot be run economically above 750 ft. per min. piston speed. Very large ports make higher speeds possible but entail an appreciable loss of fuel through the exhaust port. For automobile work so-called concentric-piston engines are generally used, in which the charge is precompressed in an annular space at the bottom of the working cylinder.

Metals and Alloys Used in Automobile Construction

Standard Specifications have been adopted by the Society of Automobile Engineers. For **Iron and Steel Specifications**, see p. 464.

Specifications Nos. 24 to 29 for brass, bronze and bearing metals are given in the following table:

Brass, Bronze and Bearing Metals

(S. A. E. Standard Specifications)

Spec. No.	Alloy	Cu	Sn	Pb	Zn	Sb	P
24*	Babbitt metal	7	84			9	
25†	White brass	3–6	≥65		28–30		
26‡	Phosphor bronze bearing metal	80	10	10			0.05–0.25
27§	Brass casting metal	84–86	5	5	5		
28‖	Yellow brass	62–65	2–4		36–31		
29**	Cast manganese bronze	60			40		

Values in percentages. * Permissible variations: In tin, 2 per cent. either way; in antimony and copper, 0.5 per cent. either way. No impurities permitted other than lead, and that not in excess of 0.10 per cent. † Shipments containing more than 0.10 per cent. of other metals may be rejected. ‡ Tolerances of 1 per cent. plus or minus permitted, except in the phosphorus. Iron, zinc and antimony in excess of 0.1 per cent. not permitted. § Tolerance of 1 per cent. plus or minus permitted in tin, lead and zinc. ‖ Iron in excess of 0.5 per cent. not permitted. ** Main dependence placed upon physical properties, which should be as follows: Tensile strength, 60,000 lb. per sq. in.; elastic limit, 30,000 lb.; elongation in 2 in., 20 per cent.

Specifications Nos. 30 to 32 for aluminum alloys are given below.

Aluminum Alloys

(S. A. E. Standard Specifications)

Spec. No.	Al	Zn	Cu	Mn
30	≥90	....	8.50–7.00	....
31	≥80	≤15	2.00–3.00	....
32	65	35	....	≤0.40

The alloy according to Specification No. 30 is not to contain more than 1.5 per cent. impurities and no other elements than carbon, silicon, iron and manganese. No. 31 is not to contain more than 1.65 per cent. impurities, of which not more than 0.50 per cent. should be silicon, not more than 1.00 per cent. iron and not more than 0.15 per cent. lead. No. 32 is not to contain more than 1.65 per cent. impurities. Nos. 30 and 31 can be used for machinery casings and other parts requiring considerable strength; No. 32 should only be used where no great strength is required, as in foot boards, running boards, etc.

Specifications Nos. 33 to 40 for brasses are given on p. 538. Specifications 1 and 2 for Babbitts are given on p. 549.

Standard Steel Tube Sizes

(Society of Automobile Engineers)

Outside diameters, in.	1/2, 5/8, 3/4, 7/8, 1, 1 1/8, 1 1/4, 1 3/8, 1 1/2, 1 3/4, 2, 2 1/4, 2 1/2, 2 3/4, 3, 3 1/4, 3 1/2
Wall thicknesses, in.	.035, .049, .065, .095, .120, .134, .156, .188, .250, .313, .375, .500, .625

Steel tubes of 2 in. outside diam. (o.d.) or less vary between 0.005 in. plus and minus on both the inside and outside diameters; tubes above 2 in. o.d., between 0.010 in. plus and minus on both inside and outside diameters.

Transmission Mechanism

Friction Clutches. (See also Machine Elements, p. 698.) Either leather-faced cone clutches or multiple-disk clutches are generally used. The latter have either metal-to-metal contact surfaces and operate in oil, or metal-to-asbestos-fabric contact surfaces and operate dry. The driven part of the clutch should have a low moment of inertia to facilitate gear changing and minimize injury to the change-gear teeth by "clashing." In cone clutches (see

Fig. 72, p. 698), the female cone is usually formed by the flywheel rim and the male cone is an aluminum casting. The angle of the cone varies between 10 and 13 deg., 12½ deg. being most frequently used. The male cone is guided on an extension of the engine crank shaft (clutch pilot) and connects with the change-speed gear through a sliding and flexible coupling. The clutch diameter is determined by the flywheel diameter. The width of the leather facing is based on a unit friction force of 12 lb. per sq. in. A friction coefficient of 0.2 can be figured on for either leather or asbestos fabric facing. The clutch is normally held in engagement by a coiled spring whose pressure may be determined from the inequality $P > Tf \sin a$, where T = tangential force at the mean circumference of the cone; f = coefficient of friction, and a = angle of cone. The clutch is disengaged by means of a foot lever whose leverage should be such that a pressure of less than 40 lb. on it will overcome the clutch spring.

For metal-to-metal disk clutches a friction coefficient of 0.04 can be figured on, and for dry disk clutches (asbestos fabric), 0.2. The unit friction in the case of the former should be restricted to 1 lb. per sq. in. and in the case of the latter to 3 lb. The "disks" are generally made in the form of comparatively narrow rings, to minimize the difference in wear at their inner and outer circumferences. Let h.p. = horse power to be transmitted at N r.p.m. and let the clutch have n disks of r in. mean radius. Then, with a friction coefficient of 0.04 the spring pressure required, $P > (\text{h.p.} \times 33{,}000 \times 12)/(Nnr \times 2\pi \times 0.04)$. For dry plate clutches substitute 0.2 for 0.04 in this inequality. The disks are made of saw steel and ground. From 20 to 50 are used in metal-to-metal clutches and from 8 to 14-in. dry plate clutches.

Change-speed Gears. Most change gears are of the sliding type with either three or four forward speeds and one reverse. The highest forward speed is a direct drive through the change-gear box. The lowest is made ⅓ to ¼ the high, depending upon the proportion of motor power to vehicle weight and the number of gear changes. Intermediate gear ratios are so proportioned as to approximately form a geometric series with the extreme ratios. The reduction ratio of the reverse gear is made equal to or lower than the lowest forward gear. The sliding pinions slide on a fluted shaft. There are generally either two or three sliding members, which are operated selectively by means of a hand lever. In order to insure noiseless operation, short, heavy shafts are used. The shaft center distances are usually about as follows: 20 h.p., 3.5 in.; 30 h.p., 4¼ in.; 40 h.p., 4¾ in. Gears are generally cut with stub teeth with a 20-deg. pressure angle, 6-8 pitch being very common, and are chamfered at their engaging sides. All gears in one set are usually made of the same width of face. This width can be determined for the driving pinion by means of the Lewis method (see p. 730) or a modification of same for stub teeth, on the basis of a stress of 15,000 lb. per sq. in. for case-hardened gears and 30,000 lb. per sq. in. for alloy-steel gears hardened all through.

An efficiency test of a three-speed-and-reverse sliding-pinion change gear made at the works of the Franklin Mfg. Co., showed maximum efficiencies of 98 per cent. on the direct drive; 95.5 per cent. on the intermediate gear; 94 per cent. on the low gear and 87 per cent. on the reverse. At full load and maximum engine speeds the efficiencies were as follows: Direct drive, 97 per cent.; intermediate speed, 95 per cent.; low speed, 93 per cent.; reverse, 86 per cent.

Final Drive. The drive from the change-speed gear to the rear axle is effected in pleasure cars by means of a propeller shaft with one or two universal joints and a pair of bevel gears, or spiral bevel gears giving a reduction ratio of between 3 : 1 and 5 : 1. In motor trucks the transmission is effected either by means of a pair of bevel gears to a countershaft and thence by side chains to the rear wheels, or by a propeller shaft and worm and worm wheel to the rear axle. In case only a single universal joint is used, the propeller shaft should be made relatively long—by placing the change-gear box close to the engine—and the engine and change gear should be placed at such a level or at such an angle that the shafts connected by the universal joint are substantially in a straight line when the car carries its normal load—in order to prevent periodic fluctuations in the drive.

In a chain-driven car the rear axle must be connected to the frame by a pair of radius rods which, in heavy vehicles at least, should have a universal connection at each end. These rods take up the chain pull and transmit the driving thrust from the rear axle to the frame. They should be of adjustable length with a range of adjustment equal to at least one pitch of the chain. In shaft-driven cars the rear axle must be provided with a torque arm or a torque tube concentric with the propeller shaft which takes up the reaction due to the pressure of the bevel pinion or worm. Means must also be provided for transmitting the driving thrust to the frame, and the rear-axle housing must either be made inherently strong enough, or else must be braced to the propeller-shaft housing, to enable it to withstand any shocks due to one road wheel striking an obstruction. Tests made by the Franklin Mfg. Co. show that the efficiency of a rear-axle bevel-gear drive may attain a value of 97 per cent.

Running Gear and Control

Rear Axle. A live rear axle consists of the axle housing, axle shafts, and differential and driving gear. The axle housing is often made of pressed steel, in halves which are welded together by the autogenous process; occasionally it is made of a single drop-forging, but most frequently it is assembled from a malleable iron or aluminum driving-gear housing and two axle tubes. Brake-supporting brackets and spring seats are riveted to the tubes near their outer ends. Nearly all live axles are stiffened in a vertical plane by an underrunning truss anchored to the brake supports, the driving-gear housing forming the strut.

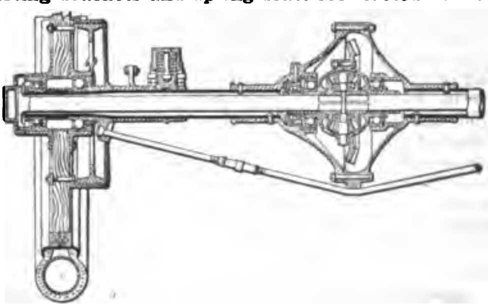

Fig. 11.—Full-floating Live Axle.

Live Axles are divided into full-floating, semi-floating and three-quarter-floating axles. In the first (Fig. 11) the shafts are subjected to torsional loads only, the wheel bearings being mounted on the outside of the axle tubes and the inner bearings of the axle on the hubs of the differential casing, the axle shafts being connected to the driving-wheel hubs by floating driving dogs. Three-quarter-floating differ from full-floating axles in that the floating driving dog is replaced by a driving flange rigidly fastened to shaft and wheel hub. In a semi-floating axle the axle tube extends only to the inner edge of the wheel hub and the load due to the vehicle weight there is transferred to the shaft. Shafts for full-floating axles can be calculated for torsion only, allowing a maximum stress of 20,000 lb. per sq. in. for heat-treated carbon steel (taking account of any reduction in the section at the joints) and 30,000 lb. per sq. in. for nickel-chrome steel. In the case of semi-floating axles the combined bending and torsional stress must be figured with, the same maximum stresses being allowed. Axle tubes are generally made of an outside diameter equal to about twice the shaft diam., and in full- and three-quarter-floating axles are reduced at the wheel hub.

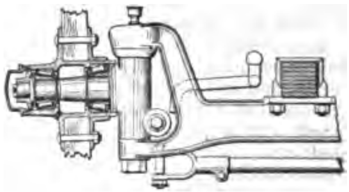

Fig. 12.—End Construction of Pleasure-car Front Axle.

Differential Gear. This serves to divide the torque equally between the two driving wheels, and permits one wheel to run faster than the other in turning curves. It consists of a frame or housing receiving the power and carrying three or four bevel pinions with their axes radial to the axis of the frame. Each pinion meshes with two master gears secured to the two rear axle shafts (or countershafts) respectively. A spur type of differential is also used, in which the master spur gears each mesh with three or four spur pinions. The faces of the pinions are of about twice the width of those of the gears, and each pinion meshing with one spur gear also meshes with a pinion meshing with the other gear.

Rear Dead Axles of motor trucks are made of solid rectangular section, about 1.5 times as high as wide, and are calculated on the basis of 15,000 lb. per sq. in. stress under full-load.

Front Axles for pleasure cars are drop-forged with a relatively light I-section so proportioned as to give from 2½ to 3 times as much bending strength in the vertical as in the horizontal plane, and can be calculated on the basis of 10,000 lb. per sq. in. stress under full load. Truck front axles are made with either a solid or a heavy I-section. Fig. 12 shows one end of a pleasure-car front axle.

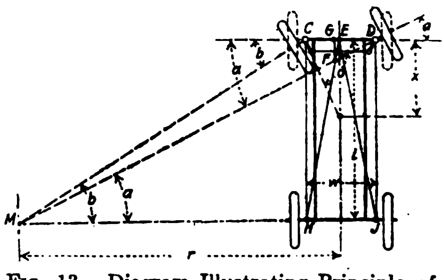

Fig. 13.—Diagram Illustrating Principle of Ackermann Steering Gear.

Steering Gear. Fig. 13 illustrates the principle of the Ackermann steering gear generally employed on automobiles. In order that there may be no slippage of any of the wheels when describing a curve of radius r, the point of intersection of the two front wheel axes produced must always fall in a vertical plane through the rear axle center. When this condition is fulfilled, $\cot a - \cot b = w/l$.

The value of b corresponding to any given angle, a, may be found without laying off the point M, which falls generally beyond the limits of the drawing board, since the angle $FCE = b$. The above-mentioned condition of perfect steering is not fully attainable by means of the customary steering linkage consisting of two steering arms of equal length secured to the steering knuckles which by means of a steering tie rod are connected up into a trapezium.

According to an investigation of Lutz (*Der Motorwagen*, 1908, p. 699), for values of l/w between 1.5 and 2.5 and with maximum steering deflections

$a_{max} = b_{max} =$	35°	40°	45°
o (for tie rods located in front of the axle) may be chosen between	58° and 69.5°	59° and 71°	60° and 72°
or x/w between	0.8 and 1.3	0.84 and 1.4	0.9 and 1.5

Also,

o (for tie rods located back of the axle) may be chosen between	66° and 74.5°	67° and 75°	68° and 76.5°
or x/w between	1.12 and 1.79	1.17 and 1.86	1.26 and 2.0

without in any case exceeding an error of 1 per cent. in the steering.

These values are calculated for the ratio: (Length of steering arms)/(Distance between knuckle pivots) = 0.14, but are sufficiently accurate for values of the ratio between 0.11 and 0.17. The tie rod is preferably placed back of the axle, where it is protected against injury.

Pleasure cars are steered by means of a hand wheel 15 to 18 in. in diam., usually with a cast aluminum center and a hardwood or hard rubber rim. The motion is reduced through a worm and sector (or worm wheel) or a screw and nut mechanism, the pitch being so chosen that the gear is completely or very nearly self-locking or irreversible. The worm or screw is usually cut with 4 or 5 threads, and the leverages are so arranged that from 1 to 1½ turns of the hand wheel will move the steering wheels from hard over one way to hard over the other way.

Brakes. American designers of pleasure cars usually place both the service and emergency brakes on the rear wheel hubs, either making both sets expanding or one expanding and the other contracting. In Europe, it is customary to place one brake on the transmission shaft directly back of the change-gear box. In motor trucks with side-chain drive, one set of brakes is placed on the rear hubs and the other on the outer ends of the countershaft. **Service brakes** are operated by a pedal, and **emergency brakes,** as a rule, by a hand lever. Brake shoes or bands are generally faced with asbestos fabric secured by copper rivets (in Europe with cast-iron blocks), and brake drums are pressed from sheet steel. Wheel brake drums are made of a diameter equal to about 0.4 the wheel diam., except in the very lightest cars, and each set of wheel brakes is provided with 1 sq. in. of **braking surface** per 15 lb. of car weight (unloaded). Half this amount of surface is sufficient in the transmission brake. The operating **effort applied to** the two **brakes** of each set, respectively, must be equalized in order to prevent **skidding.** A maximum pressure of about 100 lb. can be exerted on the brake pedal, from which the operating linkage can be calculated for strength.

Frames of pleasure cars are almost universally made of sheet steel, all members being of channel section, the height of the section of the side members decreasing from the center to both ends. Cross members are riveted to side members with substantial gusset plates, generally formed integral with the former. Golden (*Horseless Age*, Aug. 26, 1908) gives the following formula for the strength of pressed-steel side members: $Z = 39W/L_e$, where Z is the section modulus; W the weight of car with load, in lb., and L_e the elastic limit of the frame material. Truck frames are made of pressed steel or of rolled I-section or channel steel.

Automobile frames weave and distort more or less under road shocks.

Hence bearings in different parts supported on the frame cannot be kept in alignment, and shafts supported by them must be flexibly joined together. To prevent injury to housings from distortion of the frame, they should preferably be supported at three points. Engine and change-gear bearings can be kept in alignment by rigidly bolting their housings together into a unit power plant.

Springs. Chassis frames are usually supported by half-elliptic springs in front and three-quarter or full-elliptic or cantilever springs in the rear. Cross springs are employed in a few instances both in front and rear. The front springs are pivoted to the frame at their forward ends and shackled to it at their rear ends. The connection of the rear springs to the frame depends upon the axle linkage employed; three-quarter elliptic and cantilever springs are generally shackled at their forward ends. Spring bolts must be provided with grease cups. Spring seats usually turn on the rear axle housing and sometimes have a spherical seat thereon.

Tires. The load ratings of pneumatic tires adopted as standard by the Society of Automobile Engineers, are given in the following table. The oversize tires, which fit on the rims of the regular tires on the same line of the table, are not intended to be fitted by car manufacturers.

Load Ratings of Pneumatic Tires

Regular sizes			Over sizes		
Size, inches	Rear, pounds	Front, pounds	Size, inches	Rear, pounds	Front, pounds
30 × 3	375	450	31 × 3½	475	575
30 × 3½	450	550	31 × 4	635	775
32 × 3½	500	625	33 × 4	675	850
32 × 4	650	800	33 × 4½	825	1050
34 × 4	700	875	35 × 4½	935	1175
34 × 4½	900	1125	35 × 5	1000	1250
36 × 4½	1000	1250	37 × 5	1100	1350
36 × 5	1000	1375	37 × 5½	1150	1400
38 × 5½	1350	1600	39 × 6	1800	2100

The load ratings of solid rubber tires are given in the following table. The Society of Automobile Engineers recommends the use of three wheel diameters only for commercial vehicles, 32, 36, and 40 in.

Load Ratings of Solid Rubber Tires, Lb.

	Diameters						Miles per hour
Cross-section	32 in.	34 in.	36 in.	38 in.	40 in.	42 in.	
2-in. singles	450	475	500	525	550	575	20
2.5-in. singles	670	710	750	790	830	870	20
3-in. singles	900	950	1000	1050	1000	1150	20
3.5-in. singles	1130	1190	1250	1310	1370	1430	18
4-in. singles	1350	1425	1500	1575	1650	1725	16
5-in. singles	1800	1900	2000	2100	2200	2300	14
6-in. singles	2250	2375	2500	2625	2750	2875	12
7-in. singles	2700	2850	3000	3150	3300	3450	10
2-in. dual	1125	1188	1250	1312	1375	1438	18
2.5-in. dual	1675	1775	1875	1975	2075	2175	18
3-in. dual	2250	2375	2500	2625	2750	2875	16
3.5-in. dual	2825	2975	3125	3275	3425	3575	14
4-in. dual	3375	3560	3750	3940	4125	4310	13
5-in. dual	4500	4750	5000	5250	5500	5750	12
6-in. dual	5625	5940	6250	6565	6875	7190	10
7-in. dual	6750	7125	7500	7875	8250	8625	10

ELECTRIC VEHICLES

Electric vehicles carrying storage batteries as the source of current, are used almost exclusively as city vehicles, on account of their limited mileage capacity on a single charge. Aside from the running gear, the essential parts of an electric vehicle are the battery, the motor or motors, the controller and the driving gear.

Batteries. Either a special light type of lead storage battery or an Edison nickel-iron type of battery is used. Lead battery cells for vehicles are made with specially thin plates and give about 10 watt-hr. per lb. of cell weight. These cells are relatively frail and require constant care. They must not be overcharged nor be discharged below 1.8 volts per cell. The average discharge voltage is 2 volts per cell. Most lead batteries increase in capacity during the first period of service and then gradually lose capacity until it becomes necessary to renew the plates. Thin-plate lead batteries used the year around will cost for washing and renewal of plates from two-thirds to four-fifths of their initial cost. In the so-called "iron clad" type of lead battery the annual maintenance cost is about one-third the initial cost. Formerly, batteries of 40 or 44 cells were generally used, because of the convenience in charging from 110-volt circuits, but since most electric distribution lines now carry alternating current and a transformer has to be used in any case, a smaller number of cells is now frequently used.

The Edison nickel-iron battery occupies about one-third more space than the thin-plate lead battery and costs about three times as much for the same capacity, but is guaranteed to retain its full capacity for 4 years. This battery has an average discharge voltage of 1.2 volts and a specific capacity of 13.5 watt-hr. per lb. of cell weight. For charging from 110-volt circuits, 60-cell batteries are used.

The weight of the battery usually constitutes from 30 to 40 per cent. of the weight of the car complete without load. Since too rapid a rate of discharge is injurious to the battery, electric cars must be geared to run at moderate speeds. For pleasure vehicles 20 miles per hour is usually the maximum speed and for commercial vehicles on solid or cushion tires, from 6 to 12 miles per hour according to load capacity. As the rate of discharge increases, lead batteries lose in ampere-hour capacity, Edison batteries mainly in discharge voltage.

Motors. Vehicle motors are generally of the multipolar (4 to 6 poles) iron-clad type with forged or laminated poles. Formerly, two motors were frequently used on the heavier vehicles in order to dispense with the differential, but single-motor equipment is now well-nigh universal. Motors are generally hung from the spring-suspended frame. Series-wound motors are used, on account of their high starting torque. Motors of from 2 to 2.5 kw. rating are generally used for pleasure cars and the following sizes for trucks: One ton load capacity, 2 kw.; 2 tons, 2½ kw.; 3½ tons, 3½ kw.; 5 tons 4¼ kw. These motors generally have a momentary overload capacity of 250–300 per cent. A double reduction is frequently employed between the motors and rear axle or rear wheels, such combinations as a silent chain and bevel gear set, a silent chain and herringbone gear set, and a spur gear set and side chains being used. For electric pleasure vehicles the worm drive is being extensively adopted. Correctly cut worm gears may operate at efficiencies as high as 98 per cent. (Lanchester, *Proc. Inst. Auto Eng.*, London, 1913.)

Control by paralleling the battery is no longer used. From five to six forward speed combinations are generally provided. The field coils are generally arranged so that they can all be connected in series or in two parallel groups. One control system is as follows: 1st speed, fields in series, resistance in series; 2nd speed, fields in series, resistance out; 3rd speed, fields in series, resistance shunted across fields; 4th speed, fields in parallel, resistance out; 5th speed, fields in parallel, resistance shunted across fields.

A two-passenger electric runabout weighs about 2000 lb.; a four-passenger open car, 2500 lb. Pleasure cars have a radius on one charge of from 60 to 100 miles on hard, level road; motor trucks, from 30 to 40 miles.

Electric Vehicle Data

Rated load capacity, lb.	Two passenger.	1,000	2,000	4,000	7,000	10,000
Body allowance, lb(*a*)	....	500	600	800	1,100	1,400
Total weight, lb.	2,000	3,000	4,200	6,500	8,500	10,000
Max. speed (miles per hour).	20	14	12	10	8	6
Energy consumption (watt-hours per mile)(*b*)	300	550	650	830	1,100	1,400

(*a*) Recommended by the Commercial Vehicle Committee of the National Automobile Chamber of Commerce.

(*b*) From a report made by the Vehicle Research Committee of the Massachusetts Institute of Technology.

The weights and speeds given are approximate averages of figures obtained from manufacturers.

Electric Vehicle Cost Estimates

(Based on figures given in Reports of the Vehicle Research Committee of the Massachusetts Institute of Technology)

Rated load capacity, lb.	1,000	2,000	4,000	7,000	10,000
Estimate based on miles per year	10,500	10,000	9,100	8,850	8,000
Cents per mile for tires, repairs and battery (pasted lead type)	6.8	7.3	8.3	11.2	14.3
Electricity, cents per mile, at 3 cents per kw.-hr.	1.5	1.8	2.2	3	3.6
Garage and lubricant (dollars per yr.).	$215	215	235	255	285
Driver and helper	$1,000	1,140	1,140	1,210	1,210
Depreciation, interest and insurance	$380	440	464	532	685
Total annual expense	$2,455	2,705	2,794	3,252	3,610

RAILWAY ENGINEERING

BY

EDWARD C. SCHMIDT

REFERENCES: Statements in this section, unless qualified, apply to American equipment built within, say, the last 10 years, and to American practice of about the same period. More detailed information may be found in the following books: *Proceedings* of the American Railway Master Mechanics' Association, Annual Reports. *Proceedings* of the Master Car Builders' Association, Annual Reports. "Statistics of Railways in the United States," Annual Reports of the Interstate Commerce Commission. "Railway Traffic Statistics" (and other publications), Bureau of Railway Economics, Washington, D. C. "Locomotive Tests and Exhibits," Pennsylvania Railroad Company. *Bulletins* of the Pennsylvania Railroad Test Department. "Locomotive Performance," W. F. M. Goss. "Locomotive Operation" (2d Ed.), G. R. Henderson. "Railway Track and Track Work" (3d Ed.), E. E. R. Tratman.

LOCOMOTIVE DESIGN

Types. The types shown in Fig. 1 comprise all but about 1/10 of 1 per cent. of the locomotives in service in the United States. The numerals of the type symbol indicate successively the number of leading wheels, drivers and trailing wheels, beginning at the front end. For example, the Atlantic locomotive is designated as the 4-4-2 type.

Eight of these types—the American, Atlantic, Mogul, Prairie, Ten-wheel, Pacific, Consolidation, and 6-wheel Switcher—comprise 95 per cent. of the locomotives in service. Those of the Consolidation type alone constitute 31 per cent. of the total number. The trailing-wheel types, which permit the use of larger grate areas, are being built in ever-increasing numbers, and will probably eventually displace the other types in road service.

Service. The usual service of the ordinary types of road locomotive is shown in Table 1. Service requirements and character of road occasionally necessitate variations from the uses there indicated.

Table 1. Service of Locomotives of Ordinary Types

Type	Service	Type	Service
American (4-4-0)	Ordinary passenger	Consolidation (2-8-0)	Freight
Atlantic (4-4-2)	Fast passenger	Mikado (2-8-2)	Freight
Mogul (2-6-0)	Light freight	Twelve-wheel (4-8-0)	Freight
Prairie (2-6-2)	Heavy passenger and fast freight	Decapod (2-10-0)	Freight, on heavy grades
Ten-wheel (4-6-0)	Heavy passenger and fast freight	Articulated	Freight, on heavy grades
Pacific (4-6-2)	Fast and heavy passenger		

Costs. The cost of recently built locomotives varies from about 8.3 to about 10.8 cents per pound. These costs include the cost of the tender, but

Table 2. Weights and Costs of Various Types of Locomotives

Type	Weight, lb.	Cost	Type	Weight, lb.	Cost
6-Wheel switcher	138,500	$12,500	Consolidation	203,500	$17,000
Atlantic	187,000	17,500	Consolidation	223,000	18,000
Atlantic	191,000	20,500	Consolidation	229,000	22,500
Pacific	224,000	21,000	Mikado	284,000	24,000
Pacific	245,000	22,000	Mikado	285,000	24,500
Pacific	258,000	23,500	Mallet	465,000	40,000
Ten-wheel	209,000	21,000			

they are based on the weight of the locomotive only, in working condition (with water in the boiler). In general, the cost per lb. is likely to decrease

Type Symbol	Wheel Arrangement	Type Name
0-4-0		4-Wheel Switcher
0-6-0		6-Wheel Switcher
0-8-0		8-Wheel Switcher
0-10-0		10-Wheel Switcher
0-4-2		4-Coupled & Trailing
0-4-4		Forney 4-Coupled
2-4-0		4-Coupled
2-4-2		Columbia
2-4-4		4-Coupled
4-4-0		American
4-4-2		Atlantic
4-4-4		4-Coupled Double-Ender
0-6-2		6-Coupled & Trailing
2-6-0		Mogul
2-6-2		Prairie
2-6-4		6-Coupled
4-6-0		10 Wheel
4-6-2		Pacific
4-6-4		6-Coupled Double-Ender
2-8-0		Consolidation
2-8-2		Mikado
4-8-0		12 Wheel
4-8-2		Mountain
2-10-0		Decapod
2-10-2		10 Coupled or Santa Fé
0-4-4-0		Articulated (Mallet)
2-4-4-0		" "
0-6-6-0		" "
2-6-6-0		" "
2-6-6-2		" "
0-8-8-0		" "
2-8-8-0		" "
2-8-8-2		" "
2-10-10-2		" "

FIG. 1.—Types of Locomotives.

as the weight increases. Table 2 presents total costs of locomotives and tenders of various types of locomotives, all built within the last 5 years. The weights are for the locomotive only, in working order,

Locomotive Dimensions. Table 3 presents the principal dimensions of the ordinary types of road locomotives. Where two locomotives of a type are included, the first is one whose main dimensions are approximately the average of the dimensions of all those of its type in service, whereas the second is the largest or almost the largest of its type. Where only one of a type is given, it is the largest or nearly the largest of its type recently built.

Clearance Limits. The maximum width of existing locomotives is about 11 ft. 6 in., and the maximum height about 16 ft. The maximum width occurs usually at the cylinders. On curves there is some **overhang** sidewise, which, in articulated locomotives, is a considerable amount. In particular localities and on individual roads, the clearance dimensions are considerably less than those above stated.

Wheel Loads. The allowable **maximum wheel load** varies on different roads and depends upon the strength of track and bridges. The weight on each driver varies from 25,000 to 31,000 lb. in those types which demand the heaviest wheel loads. It rarely exceeds the latter amount. The maximum is at present 36,500 lb., which is, however, quite exceptional. The load on each trailing wheel is occasionally almost equal to the load per driver; but generally less. The load on each front truck wheel is generally about one-half the load per driver. Wheel loads on locomotives over 10 years old are generally less than those here cited.

Boiler Pressure, in locomotives using saturated steam built within the last 12 or 15 years, is in the majority of cases 200 lb. per sq. in. Higher pressures—up to 225 lb.—are occasionally used. Pressures have been carried to this point in the desire to obtain maximum capacity within the available space limits. General overall economy is better promoted by pressures in the neighborhood of 180 lb. ("High Pressure Steam in Locomotive Service," W. F. M. Goss, Publication 66 of the Carnegie Institution, Washington); but the demand for capacity has overshadowed that for maximum economy. The economy resulting from the use of superheated steam has greatly enhanced the capacity of the locomotive as a whole and has therefore rendered it feasible to return to a lower boiler pressure, which in superheated-steam locomotives is generally from 170 to 200 lb.

Valve Gears. The great majority of American locomotives are equipped with the Stephenson link motion (p. 974). In recent years the larger locomotives have been quite generally equipped with the Walschaerts gear (p. 977), which is rapidly becoming their characteristic valve motion. This gear has been adopted chiefly because of its greater accessibility to inspection and repair, and because of the difficulty of finding adequate space for the Stephenson gear on large locomotives.

Master Mechanics' Association Standards. The annual volumes of the *Proceedings* of the American Railway Master Mechanics' Association (Am. Ry. M. M. Assn.) contain the association's standards concerning such features of design as sizes of wheels, journals, axles, tires, etc.; specifications for boiler and fire-box steel, iron and steel boiler tubes, iron and steel axles; and rules for boiler inspection, locomotive efficiency tests, etc.

Counterbalance. The weights of the revolving parts, such as the crank pins and side (parallel) rods, if unbalanced, would produce equal radial forces for all positions of the crank. These forces can be and are perfectly balanced for all crank positions. The inertia of the reciprocating parts produces at the main crank pin radial forces which vary in magnitude throughout each revolution, and which attain their maximum values when the crank is at either the front or back dead center. Since the cranks on opposite sides are set 90 deg. apart, these maximum forces occur four times during each revolution and they amount, at high speed, in a modern passenger locomotive, to as much as 90,000 lb., which is exerted first forward on one side, then forward on the other; next backward on the first side, then backward on the other. These forces, if unbalanced, would throw the front end of the loco-

Table 3. Locomotive Dimensions

Table item No.	Locomotive type	General dimensions: When built	Tractive effort, lb.	Weights (in working order): Total locomotive, lb.	Weights: On drivers, lb.	Weights: On leading truck, lb.	Weights: On trailing truck, lb.	Weights: Tender loaded, lb.	Wheel base: Driving wheels, ft.-in.	Wheel base: Total locomotive, ft.-in.	Wheel base: Locomotive and tender, ft.-in.	Diam. of drivers, in.	Capacity of tender: Fuel, tons	Capacity of tender: Water, gal.	Engines: Simple or compound	Cylinders: Number	Cylinders: Diam. in.	Cylinders: Stroke, in.	Cylinders: Volume (total) cu. ft.
1	4-4-0	1906	15,700	126,600	85,100	41,500	......	105,200	9-0	25-1	48-5	63	10	4,300	S	4	12½	20	5.70
2	4-4-0	1914	27,850	173,500	120,540	52,960	......	146,500	9-0	24-9	56-11	68	10½	7,000	S	2	21	24	9.64
3	4-4-2	1909	29,600	202,000	116,000	49,000	37,000	149,900	7-0	30-10	6-28	73	13	7,500	S	4	17½	26	14.50
4	4-4-2	1913	29,400	240,000	133,100	55,000	51,900	158,000	7-5	29-7	63-10	80	13	7,000	S	2	23½	26	13.10
5	2-6-0	......	33,300	187,000	159,000	28,000	......	146,000	14-9	23-10	56-10	63	13	7,500	S	2	21	28	11.20
6	2-6-2	1906	33,300	209,500	152,000	......	......	139,500	11-0	28-11	57-3	63	12	7,000	S	2	21	28	11.20
7	2-6-2	1906	37,800	248,200	174,700	31,300	42,200	175,000	13-8	33-9	65-0	69	14	9,000	C	4	17½ & 29	28	12.10†
8	4-6-0	......	24,200	153,500	114,000	39,500	......	97,000	14-10	25-10	52-2	73	11	4,350	S	2	20	26	9.46
9	4-6-0	1908	31,000	208,000	158,000	50,000	......	155,800	15-10	26-11	59-3	69	12	8,000	S	2	22	26	11.40
10	4-6-2	1907	31,560	229,500	142,500	46,500	41,000	134,000	13-1	33-5	61-2	73	14	6,000	S	2	22	28	12.70
11	4-6-2	1914	46,600	312,600	191,460	56,670	64,470	184,500	13-0	34-9	71-11	69	14	9,500	S	2	27	28	18.55
12	2-8-0	......	41,140	197,300	176,200	......	......	......	17-0	25-3	56-6	56	10	7,000	S	2	22	28	12.70
13	2-8-0	1913	55,900	266,500	236,000	30,500	......	177,300	17-0	27-0	62-3	57	15	9,000	S	2	26	30	18.42
14	2-8-2	1912	57,000	312,000	236,000	......	......	159,700	17-0	35-5	67-7	63	14	8,000	S	2	28	30	21.36
15	4-8-0	1910	52,450	262,000	221,700	40,220	......	146,700	16-0	27-1	62-0	56	14	9,000	S	2	24	30	15.80
16	4-8-2	1911	58,000	330,000	239,000	44,000	47,000	173,400	16-6	37-5	70-6	62	15	9,000	S	2	29	28	21.40
17	2-10-0	1902	62,900	267,800	237,800	30,000	......	......	20-4	29-10	59-6	57	12	7,000	C	4	19 & 32	32	17.60†
18	2-10-2	1912	71,500	378,700	301,800	26,600	50,300	183,300	20-9	39-8	74-4	60	15	10,000	S	2	30	32	26.18
19	2-10-2	1914	84,500	406,000	336,800	22,700	46,500	178,000	21-0	40-3	76-6	58	16	10,000	S	2	30	32	26.18
20	2-6-6-2	1912	82,000	400,000	337,500	......	......	163,000	17-0	25-3	56-6	56	10	7,000	C	4	22 & 35	32	21.20†
21	0-8-8-0	1911	105,000	461,000	461,000	......	......	181,500	15-0*	40-8	77-3	56	16	9,500	C	4	26 & 41	32	29.70†
22	2-10-10-2	1911	111,600	616,000	550,000	......	......	234,000	19-9*	66-5	108-2	57	4,000‡	12,000	C	4	28 & 38	32	30.20†

* For each group. † Volume of equivalent simple cylinders. ‡ Gallons.

Table 3. Locomotive Dimensions.—*Continued*

Table item No.	Locomotive type	Boiler: Steam pressure, lb.	Boiler: Fuel	Firebox: Grate area, sq. ft.	Firebox: Length, in.	Firebox: Width, in.	Heating surface, Water heating surface: Tubes, sq. ft.	Heating surface, Water heating surface: Firebox, sq. ft.	Heating surface, Water heating surface: Total, sq. ft.	Heating surface: Superheater, sq. ft.	Tubes, Fire tubes: No.	Tubes, Fire tubes: Diameter, in.	Tubes, Fire tubes: Length, ft.-in.	Tubes, Superheater: No.	Tubes, Superheater: Diameter, in.	Ratios: Weight on drivers ÷ tractive effort	Ratios: Weight on drivers ÷ total heating surface ‖	Ratios: Total heating surface ‖ ÷ grate area	Ratios: Total heating surface ‖ ÷ cylinder volume	Ratios: Grate area ÷ cylinder volume
1	4-4-0	180	Bit.	21.0	96	31	1,326	140	1,466	......	187	2	13-7	....	....	5.40	58.0	69.8	257	3.68
2	4-4-0	210	Anth.	86.0	114	108	1,297	220†	1,517	257	208	1¾	10-5	....	....	4.32	63.4	22.1	197	8.94
3	4-4-2	170	Bit.	42.8	102	60	2,521	195	2,716	479	206	2	18-0	24	5¼	3.92	33.8	80.3	237	2.95
4	4-4-2	205	Bit.	55.1	110	72	2,660	196	2,856	721	242	2	15-0	....	....	4.52	33.8	71.3	300	4.21
5	2-6-0	200	Bit.	52.0	108	69	2,755	180	2,935	......	390	2	13-7	....	....	4.78	54.2	56.4	262	4.64
6	2-6-2	200	Bit.	43.5	96	65	2,105	235	2,340	......	306	2	13-3	....	....	4.57	65.0	53.8	209	3.88
7	2-6-2	225	Bit.	53.8	107	71	3,803	217	4,020	......	342	2¼	18-10	....	....	4.63	45.5	74.7	332	4.44
8	4-6-0	200	Bit.	30.8	108	41	2,273	200	2,473	......	312	2	14-0	....	....	4.71	46.1	80.3	262	3.25
9	4-6-0	280	Bit.	54.9	105	75	3,104	203	3,307	......	400	2	14-11	....	....	5.10	47.8	60.2	290	4.81
10	4-6-2	200	Bit.	53.5	108	71	3,743	204	3,947	......	310	2¼	20-6	....	....	4.51	36.1	73.8	311	4.21
11	4-6-2	185	Bit.	80.3	120	96	4,196	283	4,479	991	244	2¼	20-6	....	....	4.12	32.3	74.3	322	4.33
12	2-8-0	200	Bit.	46.9	90	75	2,527	157	2,710	......	329	2	14-9	....	....	4.28	65.0	57.8	213	3.69
13	2-8-0	185	Bit.	66.8	114	84	3,293	224	3,517	774	293	2	15-6	....	....	4.23	55.0	64.3	232	3.60
14	2-8-2	180	Bit.	63.1	103	84	4,593	261	4,854	1,065	304	2	21-0	....	....	4.14	36.6	102.2	302	2.95
15	4-8-0	200	Bit.	45.0	100	64	4,281	179	4,460	......	386	2¼	18-10	....	....	4.22	49.7	99.2	282	2.85
16	4-8-2	180	Bit.	66.7	114	84	3,897	334	4,231	845	243	2¼	19-0	40	5½	4.12	43.5	82.4	257	3.13
17	2-10-0	225	Bit.	58.5	108	78	5,156	244	5,390	......	463	2¼	19-0	....	....	3.78	44.1	92.2	306	3.32
18	2-10-2	175	Bit.	88.0	132	96	4,841	320	5,161	970	285	2¼	22-7	30	6	4.22	45.6	75.2	253	3.36
19	2-10-2	200	Bit.	88.0	132	96	5,215	293†	5,638*	1,332	269	2¼	23-0	....	....	3.99	48.8	78.4	264	3.36
20	2-6-6-2	225	Bit.	72.2	108	96	4,674	367	5,041	911	244	2¼	24-0	36	5½	4.11	52.7	88.8	302	3.40
21	0-8-8-0	210	Bit.	99.9	123	114	5,206	321	5,527	1,002	277	2¼	24-0	....	....	4.50	64.6	71.4	240	3.36
22	2-10-10-2	225	Oil	81.9	150	78	3,625	295	6,579§	2,328	377	2¼	16-5	....	....	4.93	....	....	....	2.71

§ Includes surface of a feed water heater. ‖ For superheaters this is total equivalent heating surface, = total water heating surface + 1.5 × superheating surface. † Includes arch tube heating surface. * Includes 65 sq. ft. of combustion chamber heating surface.

motive from side to side, causing it to "nose," and they would also cause great fluctuations in the pull exerted on the tender drawbar. **The purpose of counterbalancing** is to reduce these disturbing forces. They could be almost entirely eliminated by adding, opposite the cranks, counterbalance equal in weight to the main rod and reciprocating parts, but so great a weight of counterbalance would produce, when it was on the bottom center, undue rail pressure, and when on the top center, would lift the driving wheels from the rails at high speed. In the face of these difficulties counterbalancing becomes a compromise, and these horizontal forces are only partially balanced.

The following **system** of counterbalancing is **in general use.** (See "Locomotive Operation," by G. R. Henderson, 2d Ed., p. 41, and *Proc. Am. Ry. M. M. Assn.*, 1896, vol. 29, p. 148, and 1897, vol. 30, p. 117.) It consists essentially of three steps:

1. A weight (R_1, R_2, etc.—see below) is put in each driver to counterbalance the weight of the purely revolving parts which pertain to that wheel.

2. A weight, M, is put in the main driver (to which connecting rod is attached) to counterbalance the vertical forces due to the weight of main rod and reciprocating parts when the crank is on either the top or bottom center.

3. Additional weight, P, is distributed among the drivers to counterbalance, on the horizontal centers, that portion of the weight of the main rod and reciprocating parts which is to be balanced and which remains after M in item 2 is provided for.

These weights and their distribution may be determined by means of the following formulæ, in which all weights are expressed in lb. and all dimensions in ft.:

B = Weight of total combined counterbalance in all drivers on one side, placed opposite the cranks and assumed to be placed with its center of gravity at radius r from the wheel center. When, as is usual, it is placed nearer the wheel rim it must be so reduced that its centrifugal force equals that of the weight B at radius r. This weight is all carried in the driving wheels.

R = Total weight of balance for revolving parts.

M = Weight of balance for *vertical thrust* on main pin due to main rod and reciprocating parts, when crank is on top or bottom centers.

P = Total weight to partially balance, on the front and back centers, the horizontal forces due to main rod and reciprocating parts.

W = Total weight of the locomotive, excluding the weight of the tender.

W_1 = Total weight of the main rod.

W_2 = Total combined weight of all those parts which have a purely reciprocating motion, namely, cross head, piston rod, piston, and all parts pertaining thereto. For locomotives with Walschaerts valve gear, W_2 includes also the weight of the valve-gear arm, link, and the lower half of the combining lever.

$R_1, R_2 \ldots\ldots R_n$ = The respective total weights of the purely revolving parts attached to each driver. (Subscripts refer to the position of the drivers, No. 1 being the front driver.) R_1, for example, is the combined weight of the revolving parts on the first driver, namely, the weight of the forward end of the front side rod, the weight of the crank pin with its washer and nut, and the equivalent weight of the crank-pin hub. (The hub weight must be reduced to a weight which, with its center of gravity placed at crank radius, would have a centrifugal force equal to that of the actual hub weight.) In dealing with locomotives of more than two drivers on a side, it must be borne in mind that one or more crank pins carry portions of the weight of two side rods. In locomotives equipped with Walschaerts valve gear, the weight of the eccentric arm and approximately one-half the weight of the eccentric rod are revolving weights pertaining to the *main* driver.

n = Number of drivers on each side.

r = Crank radius.

l = Length of the main rod, center to center of pins.

d = Distance of the center of gravity of the main rod from crank-pin center.
k = The radius of gyration of the main rod about cross-head-pin center.

Under this notation:

$$R = R_1 + R_2 \ . \ . \ . \ . + R_n \tag{1}$$

$$M = W_1\left[\frac{k^2}{l^2} + \frac{d}{l} \times \frac{r^2}{l^2 - r^2}\right] + W_2\frac{r^2}{l^2 - r^2} \tag{2}$$

or, for rods of usual design

$$M = W_1\left[\frac{1}{2} + \frac{3}{8} \times \frac{r^2}{l^2 - r^2}\right] + W_2\frac{r^2}{l^2 - r^2} \tag{3}$$

$$M = W_1/2 \text{ approximately,} \tag{4}$$

$$P = W_1 + W_2 - M - (W/400) \tag{5}$$

$$B = R + M + P \tag{6}$$

The value of M is given exactly by equation (2); almost exactly, for rods of usual design, by equation (3); and less exactly by equation (4). Equation (4) is frequently used, and its use results in a deficiency of the main wheel balance which is generally not more than 20 lb. The last term of equation (5) is the amount of main-rod and reciprocating weight which is allowed to remain unbalanced. Variations in practice from the system here set forth consist chiefly in differences in the denominator of this term. Other variations lie in the determination of M, which is occasionally—though less logically—assumed equal to the weight of the crank-pin end of the main rod. P is sometimes determined by means of the formula $P = (W_1 + W_2) \times A$, in which the value of A varies from 0.50 to 0.67.

The **total weight of counterbalance on one side** of the locomotive is given by equation (6). Its **distribution** is effected as follows: R is divided into its components R_1, R_2, etc., each of which is placed on the wheel to which it pertains. M is placed in the main wheel. P is usually divided equally among the drivers, each of which receives an amount equal to P/n. When, however, the static weights on drivers are unequal, the weight P may be unequally divided among them, in such a way as to result in ecual maximum rail pressures under all drivers. The components of the counterbalance weight thus determined for each driver are added and the total placed in the wheel opposite the crank. If placed at crank radius this weight would give correct balance. The actual balance cast in the wheel weighs less, in the ratio $r \div$ radius to center of gravity of the actual balance.

The balance for a 4–6–2-type locomotive, whose main rod is attached to the middle driver, would be distributed thus:

Driver No.	1	2(Main)	3
Balance for revolving parts	R_1	R_2	R_3
Balance for main rod	......	M	......
Balance for main rod and reciprocating parts	$P/3$	$P/3$	$P/3$
Balance per wheel =	$R_1 + (P/3)$	$R_2 + (P/3) + M$	$R_3 + (P/3)$
Total balance =	$R_1 + R_2 + R_3 + M + P = B.$		

All of the foregoing applies to one side only of the locomotive. Balance for the opposite side is determined in like manner. The system applies only to locomotives having two main cranks, set 90 deg. apart.

When the cranks are on either the top or bottom center, the counterbalance weight P/n ("excess balance") produces in each wheel a centrifugal force which is not neutralized by the action of the engine parts. At maximum speed, the sum of this force and the static weight on the driver must not exceed the allowable rail pressure, nor should this force exceed 75 per cent. of the static weight on each driver.

Electric Locomotives. A few of the principal dimensions of characteristic electric locomotives are given in Table 4.

Railway Motor Cars. During the last ten years self-propelled "motor cars" have made their way into service on steam roads. Their use is thus far infrequent, is limited to passenger and express service, and has been warranted only on branch lines with very light traffic. With respect to their **general design** they group themselves as follows:

1. Cars with an internal-combustion engine, using gasoline or similar fuel, usually located within the car and connected by chain or friction drive to the driving axle. In some designs the engine is carried in the truck and is geared to the driving axle. Examples of this type are the McKeen and the Daimler cars.

2. Cars with an internal-combustion engine located within the car and driving an electric generator which furnishes current for motors carried in the trucks, as in the ordinary electric car. Cars of this type are those built by the General Electric Co., those of the North-Eastern Railway (England) and the Dra-Car. Others, now rarely used, like the Strang and the Hicks cars, have a storage battery added to this equipment.

3. Cars equipped with simple or compound steam engines, supplied from boilers which use oil —and occasionally coal—for fuel and which generally supply superheated steam. In most cases the engine is carried in the truck and is either directly connected to the driving axles or is geared to them. In one design the engine is carried on the car frame and is chain-connected to the driving axle. Examples of this type are the Ganz and the Purrey cars, and those of the Canadian Pacific Railway.

4. Cars carrying storage batteries as the sole source of power, from which current is supplied to electric motors carried in the trucks as in the ordinary electric car. Cars of this type are thus far rarely used on steam roads.

Table 4. Data on Modern Electric Locomotives

Road	Date	Service		Type	Maximum tractive effort, lb.	Weights		Wheel base		No. motors	Current				H.P. rating	
		Character	Location			Total locomotive, lb.	On drivers, lb.	Total, ft. in.	Drivers, ft. in.		D.C. or A.C.	Voltage	Phase	Cycles	1 hour	Continuous
Pennsylvania	1910	Pass.	N.Y. terminal	4-4-4-4	69,300	314,000	200,000	55–11	7–2	2	D.C.	660	..	..	2,500	1,600
N. Y. Central	1909	Pass.	N.Y. terminal	4-8-4	33,500	230,000	142,000	36–0	13–0	4	D.C.	600	..	..	2,200	1,000
N. Y. Central	1913	Pass.	N.Y. terminal	4-4-4-4	......	220,000	220,000	46–5	46–5	8	D.C.	600	..	..	2,600	2,000
N.Y., N.H. & H.	1908	Pass.	N.Y. division	2-4-4-2	19,200	204,000	154,000	30–10	8–0	4	A.C.	11,000	1	25	960	800
N.Y., N.H. & H.	1911	Frt.	N.Y. division	4-8-4	40,000	232,000	232,000	39–0	11–0	8	A.C.	11,000	1	25	1,396	......
Baltimore & Ohio	1910	P. & F.	Belt line tunnel	0-4-4-0	50,000	184,000	184,000	27–6	9–6	4	D.C.	600	..	..	1,100	460
Grand Trunk	1908	P. & F.	Sarnia tunnel	0-6-0	50,000	132,000	132,000	16–0	16–0	3	A.C.	3,000	1	25	720	570
Great Northern	1909	P. & F.	Cascade tunnel	0-4-4-0	52,000	230,000	230,000	31–9	11–0	4	A.C.	6,600	3	25	1,700	1,500
Norfolk & Western	1915	Frt.	Bluefield, W. Va.	2-8-4-8-2	133,000	540,000	440,000	105–8	83–10	8	A.C.	11,000	1	25	......	......
Chi., Mil. & St. Paul	1915	Frt.	Montana	4-8-8-4	120,000	520,000	400,000	102–8	75–0	8	D.C.	3,000	..	..	3,430	3,000
Italian State	1909	Frt.	Genoa-Giovi	0-10-0	......	134,000	134,000	20–2	20–2	2	A.C.	3,000	3	15	1,980	1,440
Italian State	1915	Pass.	Monza-Lecco	2-6-2	......	161,000	112,000	......	......	2	A.C.	3,300	1	17	2,600	......
Swedish State	1911	Frt.	Kiruna	0-6-6-0	68,760	220,000	270,000	......	......	2	A.C.	15,000	1	15	2,000	......
Swiss Federal	1913	Pass.	Lötschberg	2-10-2	39,600	236,000	172,000	37–2	24–9	2	A.C.	15,000	1	15	2,500	......

The **cost of operation** of these cars in American service, including the cost of labor, fuel and supplies, is from 11 to 20 cents per mile, under ordinary branch-line conditions. The cars weigh from 30 to 60 tons. More detailed information may be found in *Proc. Am. Ry. M. M. Assn.*, 1906, vol. 39, p. 416; 1907, vol. 40, p. 194; 1909, vol. 42, p. 86; *Proc. International Ry. Fuel Assn.*, vol. 5, 1913.

LOCOMOTIVE PERFORMANCE

The locomotive is a power plant which is required to work under loads that vary widely and suddenly, and which is designed to come within space limits nowhere approached in other practice. Under ordinary conditions of service, the load on boiler and engines frequently decreases from the maximum to one-fifth or one-sixth of that amount within a period of a few minutes. Under such conditions, the efficiency of the boiler performance depends in a peculiar degree upon the judgment of the fireman, and of the engine performance upon the skill of the engineer. These facts account for great variations in general or average performance. Throughout this article, unless otherwise specifically stated, coal and steam, when mentioned, refer respectively to bituminous coal and dry saturated steam.

Firebox Performance. The rate of combustion in locomotive boilers is much greater than in boilers of other types. The usual range of **relations between draft and the coal burned** per sq. ft. of grate per hour is shown by the shaded area in Fig. 2. Variations in draft for the same rate of combustion are due to differences in the thickness and in the condition of the fire, and also to differences in the construction of the ash pan and front end. In the diagram the draft is the difference between the pressure of the atmosphere and the pressure existing in front of the diaphragm in the front end. A rate of 150 lb. per sq. ft. of grate per hour is rarely exceeded—indeed, rarely attained—in modern locomotives, and this rate is reached only under unusual demands and for short periods. Before the advent of the wide firebox, the combustion rate on small grate areas was occasionally carried up to 200 lb.

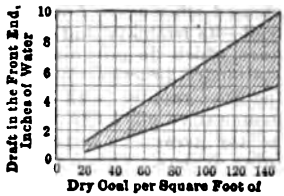

FIG. 2.

In freight service, with large locomotives of the usual types, and in runs of from 8 to 10 hr. duration, the average **hourly coal consumption** is frequently about 4000 lb., occasionally about 5000 lb. In locomotives hauling heavy and fast passenger trains, this hourly consumption occasionally—though rarely—averages 6500–7500 lb. in runs of about 3 or 4 hr. duration. For periods of an hour or less, an unusually skillful fireman may fire at the rate of 9000 lb. per hour; but such a performance is rare. In general, the average hourly coal consumption is much less than these maximum rates. The rates stated may be exceeded by the use of mechanical stokers.

Draft. In locomotives of ordinary design, the draft necessary to draw the air from the ash pan through the fire is only from 0.2 to 0.3 of the total draft shown in Fig. 2. The remainder is required to draw the gases through the tubes and under the diaphragm. Total draft varies directly with the weight of steam discharged through the front end per min. and maximum draft is therefore attained when the boiler is working at maximum power. Ten inches of water is about the **maximum draft** ordinarily obtained. Under the action of maximum draft, from 12 to 15 per cent. of the coal fired may be ejected as sparks into the front end and through the stack.

Mechanical Stokers. The ultimate capacity of the grates and heating surface of the largest locomotives cannot be realised by hand firing, and, under the demand for increased capacity, mechanical stokers have been applied in considerable number to such locomotives during the last 5 or 6 years. Their design is, however, just emerging from the experimental state and their use is not yet general. About 950 are now in service. Descriptions of the various designs and data concerning their performance are contained in *Proc. Am. Ry. M. M. Assn.*, 1909–1915.

Boiler Performance. Under ordinary conditions, the **maximum equivalent evaporation** from and at 212 deg. per sq. ft. of heating surface per hour is about 12 lb. Under the variable rates of power consumption which prevail in service, the evaporation averages much less, from 6 to 8 lb. per hour. Under unusually severe demand, this rate is occasionally raised, for short periods, to 15 or even 16 lb. per hour, but the modern locomotive with ample heating surface is rarely driven beyond a rate of 12 lb. per hour. Each sq. ft. of heating surface therefore delivers, on the average, about 0.2 boiler h.p., and, when driven, will supply from 0.35 to 0.45 h.p.

When driven at very low rate, the equivalent evaporation per lb. of coal in locomotive boilers is about equal to that obtained in good stationary practice. This evaporative efficiency falls off rapidly as the rate of evaporation is increased. The heavy draft employed causes large quantities of the coal fired to be ejected into the front end and through the stack, resulting in decreased efficiency. For this reason differences in the physical character of coal—its friability and size—produce variations in locomotive boiler performance which do not make themselves felt in stationary practice. Since these stack losses increase rapidly with increased power, the evaporative efficiency falls off more rapidly as the power is increased than it does in other boilers. Other losses also are greater in locomotive boilers when driven at high rates.

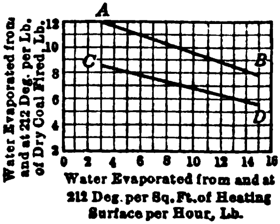

FIG. 3.

The range in values of **equivalent evaporation per lb. of dry coal** and of **evaporation per sq. ft. of heating surface per hour** is indicated in Fig. 3, which shows also the ordinary relation existing between these two quantities. The line *AB* represents about the best performance obtainable with the highest grade of coal, ample grate area, and excellent firing. Line *CD* represents poor performance, such as may be expected with either poor coal or poor firing. Performance in individual cases may be expected to be represented by a line lying between these two. **Results with** good **anthracite** coal will, with skillful firing, equal those indicated by the line *AB*.

In locomotives of modern design, from 35 to 50 per cent. of the total evaporation takes place on the heating surface of the firebox, although the latter constitutes generally only from 4 to 8 per cent. of the total heating surface.

Cylinder Performance. In a locomotive, cylinder performance is affected by variations in both speed and cut-off. Run at constant speed, the steam consumed per i.h.p. per hour in simple locomotives is a minimum at from 30 to 40 per cent. cut-off, and increases at either earlier or later cut-offs. When run at constant cut-off, the steam consumption decreases rapidly with speed until it reaches its minimum at a speed which lies generally between 120 and 180 r.p.m., and then increases rapidly as the speed is increased beyond this point. Within approximately these limits, the speed of best performance varies, depending on the cut-off and the steam pressure. (It is generally lower in compounds.) There is therefore in any locomotive a combination of a particular cut-off and a particular speed which produces **maximum economy.** These facts are exemplified in Figs. 4 and 5, which are fairly typical high-grade performance curves for a simple freight locomotive with 22 × 28-in. cylinders, 56-in. drivers, and 200 lb. boiler pressure. It is difficult, under the circumstances, to formulate any general statement for cylinder performance, but it may be approximately represented as in Fig. 6, in which the range in values of steam consumption is shown, for various speeds, by the shaded belts there drawn. Any of the larger modern simple locomotives, using steam at from 190 to 210 lb. may, when well handled, be expected to give a performance within the range shown in the upper part of Fig. 6. In using this diagram to predict the probable steam rate for a particular locomotive, the influence of cut-off should be remembered.

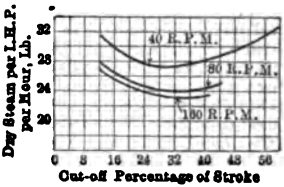

FIG. 4.

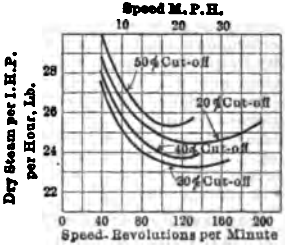

FIG. 5.

Compound Locomotives, under the most favorable conditions, give a cylinder performance similar to that shown by the lower part of Fig. 6. Under the fluctuations in load which prevail in ordinary operating conditions, their economy is reduced and is variously claimed at from 10 to 20 per cent. less than that of simple engines. Their performance is sensitive to changes in operating conditions, and their first cost and cost of maintenance are somewhat higher than for simple locomotives. Save on a few roads, they have never established themselves very firmly in American practice. Of the locomotives in service in the United States, only 3 per cent. are compounds. Of this number, about one-fourth are articulated freight locomotives in which, since 4 cylinders are required, compounding follows as a matter of course, and without being chargeable with complicating the design.

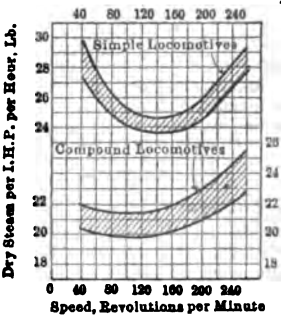

FIG. 6.

Use of Superheated Steam. What has preceded relates to saturated steam. The most important recent development in locomotive practice consists in the use of superheated steam. It was first introduced in Germany in 1898, and 10 years later was firmly established in German practice. It was in the meantime experimented with on American railways, being here first extensively used on the Canadian Pacific Railway, which in 1906 had about 200 superheater locomotives in service. Within the last 6 years superheaters have been rapidly applied to locomotives in the United States and Canada, where at present more than 14,000 are in service; most of them being of the fire-tube type. For descriptions of the various types and data concerning their performance in service, see *Proc. Am. Ry. M. M. Assn.*, 1910–1915.

The degree of superheat used in locomotive service varies up to a maximum of about 270 deg. fahr. Most superheaters now in use are designed to produce a superheat varying from 150 to 250 deg. The use of low superheat (up to 100 deg. fahr.) results in a relatively slight decrease in steam and coal consumption. The use of medium and high superheats (150 to 250 deg.) results, on the other hand, in very significant **reductions in the amount of steam and coal consumed** per i.h.p. per hour, as compared with the performance of locomotives using saturated steam. The consumption of steam and coal not only decreases as the superheat is increased, but the rate of this decrease is greatest in the highest ranges of superheat, *e.g.*, a greater gain in economy results from increasing the superheat from 200 to 250 deg. fahr. than in increasing it from 150 to 200 deg. Comparative tests of simple locomotives using saturated and superheated steam (150 to 250 deg.), made on testing plants, show the steam and coal consumptions of the latter to be from 15 to 35 per cent. less than those of the former, for continuous running under constant loads. Not all of this saving is realized on the road, for under the conditions of service much coal and some steam are used in ways upon which the superheater can have no beneficial effect. Numerous road tests and long-term comparisons of road performance make it clear, however, that, even under the conditions of service, simple locomotives using moderately or highly superheated steam may be relied upon to consume from 10 to 20 per cent. less coal and from 15 to 25 per cent. less steam than simple saturated-steam locomotives performing like service. The greater gains are apt to be obtained in passenger service. The cylinder performance of superheater locomotives is also less sensitive to changes in speed and cut-off, and their water rate at speeds beyond the point of minimum consumption increases but slightly. The best test-plant performance yet recorded is that of a simple Pacific type locomotive using steam at 204 lb. pressure and 223 deg. superheat, in which 1.93 lb. of dry coal and 14.6 lb. of steam were required per i.h.p. per hour.

On account of this economy in steam consumption, superheater locomotives may be designed—within given limits of space and weight—to have a considerably greater capacity than saturated-steam locomotives; and this increase in capacity is perhaps an even more important factor in bringing about their general adoption than is their economy of operation. Nearly all the locomotives most recently built have been equipped with superheaters.

General Performance. Among locomotives of other types than the articulated, the **maximum tractive effort** thus far attained is 84,500 lb., reached with a 2–10–2 type weighing 406,000 lb. The most powerful articulated locomotive thus far built, a 2–8–8–8–2 type weighing 853,000 lb., has a tractive effort of 160,000 lb. working as a compound. Among passenger locomotives the current maximum tractive effort is about 47,000 lb. These tractive forces are attained only at very late cut-offs and can be maintained only at very low speeds. The average maximum tractive effort of all locomotives in service is about 30,400 lb.

Maximum horse power is usually attained at speeds of from 25–35 miles per hour in freight locomotives, and at speeds of from 50–60 miles per hour in passenger locomotives. At higher speeds the power decreases in saturated-steam locomotives; but is maintained nearly constant in those using superheated steam. The i.h.p. developed under service conditions, even with the largest locomotives, rarely exceeds 1800; and the maximum thus far attained and measured in road or laboratory tests is about 3200 i.h.p.

Efficiency. The losses between the cylinder and the tender drawbar include (1) friction of all parts of the engine mechanism; (2) net resistance on level track at uniform speed (journal resistance, resistances at the wheel rim, and air resistance); (3) grade resistance, and (4) acceleration resistance. Items 3 and 4 may be absent. Items 1 and 2 are always present and are at their minimum at low speeds. Consequently, the losses between the cylinders and the tender drawbar are least at low speeds, on level track, and at uniform speed.

Defining locomotive efficiency as the ratio of the horse power developed at the tender drawbar to the i.h.p. developed in the cylinders, it is obvious that the efficiency is 0 whenever the locomotive moves only itself and its tender. It reaches its maximum when the locomotive hauls the heaviest train it is capable of handling, on level track at very low and uniform speed. This maximum is about 92 per cent. Between these limits efficiency may have any value, depending on the speed and on the relative weights of the locomotive and the train. Table 5 presents values of efficiency obtained in tests of four freight locomotives hauling their maximum tonnage on a 0.58 per cent. grade 4.5 miles long, at practically uniform speed. These values are all averages for two trips of each locomotive over the whole 4.5 miles.

Table 5. Efficiencies of Freight Locomotives

Type of loco-motive	Speed, miles per hour	Ratio of train weight to weight of train, locomotive and tender, per cent.	Efficiency, per cent.	Equivalent values on level track	
				Ratio of train weight to weight of train, locomotive and tender, per cent.	Efficiency, per cent.
2-6-0	11.8	89.3	84.2	95.8	90.1
4-6-0	14.2	88.9	83.6	96.0	90.7
2-8-0	11.7	92.3	87.8	97.0	92.5
4-8-0	10.1	92.0	86.0	97.1	90.9

The values in columns 2 to 4 are the actual results of tests and they represent performance which may be attained in ordinary service where the ruling grades do not exceed about 0.5 per cent. The efficiency values of column 4 are not likely to be exceeded except on lines of unusually low grade or in starting trains on level track. Column 6 shows what the efficiency of these locomotives would have been, had they been operated on level track at the same power output. These values indicate what the locomotive may do; but they are not likely to be realized in service, because they imply ratios of train weight to total weight which are generally impracticable.

Tractive Effort. While the tractive effort of a locomotive is determined primarily by the action within the cylinders, it is limited on the one hand by the adhesion and on the other by the amount of steam which the boiler can supply. Cylinders are so designed that, at the latest cut-off used in their normal operation, the tractive effort does not exceed the weight on drivers $\times$ A, in which A is the coefficient of adhesion between the drivers and the rails.

The coefficient of adhesion, A, varies with the weight on the drivers and with the condition of the driver and rail surfaces. It probably does not vary much with speed. When the rail and drivers are very clean, A reaches its maximum of 0.35, which, though frequently attained, may never be relied upon. When the surfaces are greasy, or muddy, or moist, or frost-covered, the value of A decreases to from 0.15 to 0.20; but even under such conditions it can be restored to almost its maximum value by the use of sand. In design, it is not safe to assume that A will be greater than 0.25. The moment the drivers slip, this coefficient of *static* friction ceases to act, and its place is taken by a very much smaller coefficient of *sliding* friction. A committee of the American Railway Master Mechanics' Association recommends that for purposes of cylinder design the coefficient of adhesion be assumed as follows: For passenger locomotives, 0.250; for freight locomotives, 0.235; for switching locomotives, 0.220. A later committee, in view of the general use of effective sanders, recommends that the value 0.25 be assumed for all locomotives. In late designs of all classes of locomotives the ratio of tractive effort to weight on drivers varies from 0.18 to 0.25.

Maximum Tractive Force. The maximum tractive force of locomotives is realized only under full boiler pressure, wide-open throttle, and latest cut-off. Latest cut-off can be maintained without exhausting the steam supply, only below a speed which, with ordinary ratios of heating surface to cylinder volume, varies from about 6 to 10 miles per hour. This maximum tractive force may be determined by means of the following formulæ, in which T = maximum tractive force as herein qualified, lb.; P = steam pressure in the boiler, lb. per sq. in., gage; S = stroke of the piston, in.; D = diam. of the driving wheels, in.; d = diam. of the cylinders, in.; d_H = diam. of the high-pressure cylinder, in., and d_L = diam. of the low-pressure cylinder, in.:

Type	Working Simple	Working Compound
Simple, 2-cylinder.....	$T = 0.85\,d^2(PS/D)$....(1)	
Compound, 2-cylinder.	$T = 0.80\,d_H^2(PS/D)$..(2)	$T = (2\,d_H^2/3)(PS/D)$.............(3)
Compound, 4-cylinder.	$T = 1.6\,d_H^2(PS/D)$...(4)	$T = [(2\,d_H^2/3) + (d_L^2/4)](PS/D)$....(5)
Mallet compound, 4-cylinder................		$T = 0.52\,d_L^2(PS/D)$..............(6)

Tractive force determined by the formulæ is the maximum force available at the tender drawbar when the locomotive is moving on level track at uniform speeds up to 6 or 8 miles per hour. A more conservative view assumes the values thus obtained to be the maximum force available at the rim of the drivers; but if the maximum possible mean effective pressure and the maximum possible efficiency between cylinders and tender drawbar are both actually attained, the forces defined by the formulæ may be safely assumed to be available at the tender drawbar. If the locomotive is moving up grade and if its speed is accelerating, the value of T must be reduced by the sum of the forces required to lift and accelerate its mass, in order to find the force available for work upon the train. If the value of T exceeds the adhesion as above defined, it cannot, of course, be made available.

Formula (1) is almost universally used in American practice, although a few authorities use a constant 0.80 instead of 0.85. Formulæ (2) to (6) are in general use for compound locomotives of current design, although occasionally divergence from common practice as regards ratio of cylinder diameters and steam distribution, makes desirable slight variations in the constants of these formulæ. Compound locomotives at very low speeds (in starting trains) are occasionally worked "simple," *i.e.*, high-pressure steam is used in all cylinders. Under such conditions, formulæ (2) and (4) are applicable. In formula (6) the value of the constant generally lies between 0.51 and 0.53, although it occasionally is as great as 0.56. When the ratio $d_L^2/d_H^2 = 2.47$, formulæ (5) and (6) give identical values of T.

As the speed increases above 6 to 10 miles per hour, the steam supply becomes inadequate to maintain maximum cut-off and the cut-off must be continually decreased, resulting in a continuous decrease in tractive force as the speed increases.

COSTS OF OPERATION

The information here presented relates to the steam railways of the United States. It is compiled from the latest available statistics published by the Interstate Commerce Commission.

Summary of Operating Expenses. The annual operating expenses for railways which have operating revenues above $100,000 per year, are summarised in Table 6.

In Table 6, item 1 includes expenses for superintendence, advertising, traffic associations, industrial and immigration bureaus, etc. Item 2 includes expenses for general officers and clerks, office supplies and expense, legal service, insurance, relief department, pensions, etc. Item 3 covers the expenses for all features of the permanent way, such as track material, roadway tools and supplies, bridges, tunnels, station and other buildings, telegraph and telephone, signals, docks, wharves, etc. The content of item 4 is indicated in Table 8, and that of item 5 in Table 7.

Transportation Expenses. The expenses for conducting transportation amount to more than half of the total operating expense. They are composed of the expenditures shown in Table 7, which presents an analysis of item 5 in the first column of Table 6.

Table 6. Annual Operating Expenses for Railways Which Have Operating Revenues Above $100,000 per Year

Items	Year Ending June 30, 1913		Year Ending June 30, 1914	
	Amount	Per cent. of total	Amount	Per cent. of total
1. Traffic Expenses....	$62,826,186	2.89	$63,769,677	2.90
2. General Expenses...	78,028,425	3.60	83,529,665	3.80
3. Maintenance of Way and Structures..	421,030,360	19.41	419,277,779	19.06
4. Maintenance of Equipment..........	511,467,852	23.58	532,138,606	24.20
5. Transportation Expenses............	1,095,909,503	50.52	1,101,597,432	50.04
Total*..............	2,169,282,326	100.00	2,200,313,159	100.00

* Total for *all* railways, year ending June 30, 1913, $2,182,769,000.

Table 7. Transportation Expenses for the Year Ended June 30, 1913, for Railways Which Have Operating Revenues Above $100,000 per Year

Item	Amount	Per cent. of total operating expense
1. Fuel for locomotives..................	$249,507,624	11.503
2. Water for locomotives................	15,416,010	0.710
3. Lubricants and other supplies...........	11,311,205	0.521
4. Enginehouse expense..................	46,195,703	2.129
5. Enginemen..........................	170,216,533	7.846
6. Train supplies and expenses............	39,305,182	1.812
7. Conductors and brakemen..............	202,573,960	9.339
8. Yard employees and expense...........	36,053,328	1.661
9. Station employees and expense..........	159,621,463	7.359
10. Superintendence and dispatching trains.	44,848,036	2.067
11. Flagmen, gatemen, signalmen..........	25,302,813	1.166
12. Loss and damage to property, injury to persons............................	71,139,555	3.278
13. Miscellaneous expense................	24,418,091	1.125
	1,095,909,503	50.516

Maintenance of Equipment. The expenses for maintaining equipment, which constitute a little more than one-fifth of the total operating costs, are composed as indicated in Table 8 which presents the maintenance of equipment costs for four representative roads which differ in the character of their territory and traffic.

Operating costs vary greatly with the character and volume of traffic, with the character of the roadway and equipment, and with the conditions of service. No specific conclusions should therefore be drawn from them, nor comparisons made between them, without definite knowledge of the underlying conditions. Operating costs and statistics are fully presented in "Statistics of Railways in the United States," Interstate Commerce Commission, Washington; and well analyzed in "Railroad Operating Costs," Suffern and Son, New York.

Table 8. Expense for Maintenance of Equipment of Four Representative Roads—Year Ending June 30, 1911

	Baltimore and Ohio R.R.	Illinois Central R.R.	Atchison, Topeka, and Santa Fé R.R.	Denver and Rio Grande R.R.
Steam Locomotives:		Cents per locomotive-mile		
Repairs	9.0807[1]	10.5390	13.4520	12.4423
Renewals	.0177	....	.0081	....
Depreciation	.9621	.7751	1.4989	.8153
Freight Cars:		Cents per ton-mile		
Repairs	0.0448[2]	0.0724	0.0567	00.916
Renewals	.0025	.0022	.0024	.0019
Depreciation	.0129	.0173	.0177	.0218
Passenger-train Cars:		Cents per passenger-mile		
Repairs	0.0998[3]	0.1129	0.1143	0.0780
Renewals	.0018	.0000	.0006	.0013
Depreciation	.0116	.0117	.0185	.0190
Superintendence of equipment		Cents per train-mile		
	1.1810[4]	0.6656	1.3766	1.2897
Shop machinery and tools	1.1976	1.0122	.7613	1.5544
Injuries to persons	.0408	.1325	.0681	.0911
Other expenses[5]	.7772	.5611	.6547	.3598

[1] Cents per locomotive-mile. Includes both revenue and non-revenue service. [2] Cents per ton-mile. Includes revenue service only. [3] Cents per passenger-mile. Includes revenue service only. [4] Cents per train-mile. Includes both revenue and non-revenue service. [5] Includes expense for electrical equipment; floating, work, and power-plant equipment; joint terminal equipment, etc.

CARS

Dimensions. Tables 9 and 10 present the weights and the principal dimensions (to the nearest full inch) of a few representative freight cars and passenger cars, built during the last six years.

Table 9. Dimensions of Freight Cars

Kind	Description	Capacity, lb.	Weight empty, lb.	Maximum width, ft. in.	Length over end-sills, ft. in.	Height, rail to floor, ft. in.	Maximum height from rail, ft. in.
Box	....	60,000	....	9-8	36-7	3-7	12-11
Box	....	60,000	....	9-11	36-6	3-10	13-1
Box	All-steel	100,000	37,400	9-4	40-0	3-6	12-7
Box	All-steel	100 000	39,000	9-6	41-3	3-7	12-2
Box	All-steel	100 000	43,160	10-0	41-10	...	14-2
Furniture	....	60,000	44,000	9-10	51-10	3-6	14-2
Gondola.	Hopper bottom	80,000	....	9-9	28-6	4-0	9-3
Gondola.	Twin hopper	80,000	....	9-0	36-0	3-4	5-10
Gondola.	....	80,000	....	10-3	36-0	4-3	7-10
Gondola.	....	100 000	41,500	10-1	42-9	4-0	8-3
Gondola.	Drop bottom	100 000	37,700	10-1	40-10	3-10	8-0
Gondola.	Drop bottom	100 000	43,300	10-2	41-0	4-5	8-9
Flat	....	80,000	29,400	10-1	42-0	3-11	....
Flat	....	80,000	33,100	9-8	42-2	4-3	....
Flat	....	100,000	....	9-11	40-0	4-0	....
Flat	....	100,000	....	9-5	41-2	3-11	....

Dimensions of Wheels and Axles. Practically all freight-car wheels are made of cast iron and are 33 in. in diam. They vary in weight as follows:

For cars of 95,000 lb. max. gross weight, from 615 to 625 lb. per wheel.

For cars of 132,000 lb. max. gross weight, from 665 to 675 lb. per wheel.

For cars of 161,000 lb. max. gross weight, from 715 to 725 lb. per wheel.

Passenger-car wheels are generally of cast iron with steel tires, or of solid steel. They vary in diam. from 33 to 36 in.

Table 10. Dimensions of Passenger Cars

Kind	Description	Road	Weight, lb.	Maximum width, ft. in.	Length over end-sills, ft. in.	Length over buffer-beams, ft. in.	Maximum height from rail, ft. in.
Coach...	....	C. of N. J.	90,000	10-5	60-0	64-8	13-11
Coach...	....	D. L. & W.	100,250	9-11	60-8	67-0	14-0
Coach...	....	Southern	120,000	9-10	73-5	....	14-0
Coach...	Steel.....	B. & O.	....	9-10	....	70-0	14-0
Coach...	....	Atl. Coast	....	9-10	....	74-9	14-4
Coach...	Steel.....	L. S. & M. S.	142,000	9-10	....	77-10	....
Chair car	....	C. B. & Q.	119,300	10-0	70-0	....	14-0
Pullman.	....	....	137,200	....	73-6	80-0	....
Pullman.	....	C. M. & St. P.	152,300	10-4	72-6	79-8	....
Pullman.	Observation Buffet	Nor. Pacific	131,300	10-0	72-0	80-0	....

Table. 11. Dimensions of Standard Car Axles

Capacity of cars, lb.	Size of journals (diam. × length), in.	Diam. at wheel-seat, in.	Smallest diam. at center of axle, in.	Carrying capacity, lb.
40,000	3¾ × 7	5⅛	4⅛	15,000
60,000	4¼ × 8	5¾	4¾	22,000
80,000	5 × 9	6⅛	5⅜	31,000
100,000	5½ × 10	7	5⅞	38,000
....	6 × 11	7⅝	6 7/16	50,000

Costs. The cost of cars varies greatly, depending upon their size and capacity and type of construction. The allowed values of freight-car parts and of entire freight cars are given in detail in the Interchange Rules of the Master Car Builders' Association (*Proc. M. C. B. Assn.*, vol. 48, 1914, pp. 490 to 547). Costs of a few typical freight cars and passenger cars are presented in Table 12.

Table 12. Costs of Typical Cars

Kind of car	Description	Capacity, lb.	Year built	Cost
Freight Cars:				
Box..............	Wood....................	80,000	1906	$853
Box..............	Steel center sills............	80,000	1907	825
Box..............	Steel underframe...........	80,000	1908	1,013
Box..............	Steel underframe...........	100,000	1908	1,224
Stock............	Wood....................	60,000	1902	664
Refrigerator.........	Wood....................	60,000	1906	929
Refrigerator.........	Steel underframe...........	60,000	1912	1,375
Flat.............	Wood....................	80,000	1912	610
Flat.............	Steel underframe...........	100,000	1907	980
Coal.............	Wood....................	80,000	1903	721
Coal.............	Steel underframe...........	100,000	1906	941
Coal.............	All-steel..................	100,000	1912	825
Coal.............	Steel underframe, hoppers...	100,000	1910	1,404
Passenger Cars:		Length, ft.		
Coach............	Wood....................	60	1907	9,307
Coach............	Steel....................	60	1911	12,492
Coach............	Steel....................	60	1911	13,271
Pullman..........	Steel....................	70	....	20,000
Diner............	Wood....................	70	1907	18,011
Diner............	Steel....................	70	1911	22,555

TRAIN RESISTANCE

Elements of Train Resistance. The term **gross train resistance**, as here used, means the total force exerted by the locomotive on the train, that is, the force exerted at the rear tender drawbar. The various elements which make up gross resistance are: 1. Resistance due to grade. 2. Resistance due to track curvature. 3. Resistance due to acceleration. 4. Resistance due to wind (as distinguished from still-air resistance). 5. Net resistance on straight and level track, at uniform speed, in still air. Item 5 is always in operation to retard a moving train. One or more (or none) of the others may also be acting with item 5 to form gross resistance.

The resistances due to grade, curvature, and acceleration (items 1, 2, 3) need no comment in addition to what is given below. Wind resistance (item 4) is herein considered as the resistance due to wind only, and is distinguished from still-air resistance which is treated as one of the elements of net resistance.

Net resistance (item 5), as here defined, is what is ordinarily meant by the unqualified term "train resistance." It comprises all those resistances which are always encountered when a train moves in still air, at uniform speed, over straight and level track, namely: (*a*) Air resistance, due to friction and impact of the air. (*b*) Rolling resistances, including all resistances which develop at the wheel rim, *viz.*, those due to the rolling of the wheel on the rail and those due to the yielding of the track under the wheel load. (*c*) Journal friction. (*d*) Flange friction, due to the rubbing of the wheel flanges on the rail head. (*e*) Resistances due to the energy-absorbing oscillations and impacts of the car bodies and trucks.

Of these elements of net resistance, air resistance is probably the greatest at very high speeds. At very low speed the journal friction probably exceeds the sum of all the others. Various attempts have been made to differentiate these components of net resistance; but there is great diversity in the results. Net resistance, therefore, is here treated as a unit, and no attempt is made to distinguish its various elements.

Determination of Train Resistance

The data needed for the determination of train resistance are ordinarily best obtained by the use of a **dynamometer car**, in which there is produced a curve of drawbar pull, a curve of speed, a record of the location of the train on the road, and a record of time. These records, supplemented by an accurate track profile and train data, will provide the information required for the application of the following formulæ.

Momentary values of resistance may be determined at the instant the train passes a point on the road (method 1). Average values of resistance may be determined for the period during which the head of the train passes over a definite section of track (method 2). The significance of the notation as applied in method 1 is illustrated in Fig. 7; and its significance in method 2 is illustrated in Fig. 8. The notation used in the formulæ is as follows:

P = Total gross resistance = drawbar pull, lb.
R = Net resistance on tangent, level track, at uniform speed, in still air, lb. per ton.
R_g = Resistance due to grade, lb. per ton.
R_c = Resistance due to curvature, lb. per ton.
R_a = Resistance due to acceleration, lb. per ton.
R_w = Resistance due to wind, lb. per ton.
W = Total train weight, tons.
w = Average gross weight of cars, tons.
V, V_1, etc. = Train speed, miles per hour.
G = Grade, ft. per mile.
A = Acceleration of the train speed, miles per hour per second.

E_1 and E_2 = Elevations of the center of mass of the train, ft.
S = Length of track section used in method 2, ft.
N = Number of cars in the train.

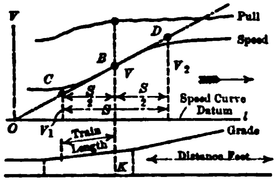

FIG. 7.

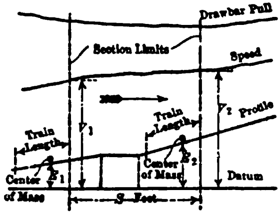

FIG. 8.

Wind Resistance. Resistance due to wind (R_w, item 4) varies greatly with the wind velocity and with the angle at which the wind strikes the train. Its value is generally indeterminate. It is frequently great enough to necessitate reductions in train tonnage and, in rare instances, great enough to stall trains. It may be eliminated from resistance calculations by running tests in quiet weather. When not so eliminated it will appear as part of net resistance, from which it cannot be separated.

Curve Resistance. The resistance due to track curvature (R_c, item 2) varies with the curvature, and by some authorities is considered to vary also with speed. Ordinarily, the influence of speed is ignored and the value of curve resistance is assumed at from 0.5 to 0.8 lb. per ton per degree of curve. In experimental determinations of resistance, curve resistance is ordinarily eliminated by making experiments on straight track only.

Grade Resistance. The resistance due to grade (R_g, item 1) varies only with the grade and may be exactly determined by the following formulæ. Momentary resistance at a point =

$$R_g = 0.379\,G \tag{1}$$

Average resistance over a section =

$$R_g = 2000(E_2 - E_1)/S \tag{2}$$

Acceleration Resistance. The resistance due to acceleration (R_a, item 3) is made up of two parts. The first is the force needed to produce the acceleration in the motion of translation of the train as a whole; the second is the force needed to produce the acceleration in the rotation of the wheels and axles. The sum of these two forces is the total acceleration resistance R_a. Its value, both for momentary resistance at a point and for average resistance over a section, is given by the formula

$$R_a = \left[91.05 + 145.5\left(\frac{N}{W}\right)\right] \times A \tag{3}$$

In this equation the second constant (145.5) applies to cars with 4 axles and is approximately correct for all standard weights of wheels and axles. The maximum error in R_a, resulting from changes in this constant due to differences in the weight of wheels and axles, is about 1 per cent. For cars with 6 axles this constant is 218.3. When the acceleration resistance due to translation only is desired, the second term within the brackets, or 145.5 (N/W), should be dropped.

The value of A in (3) may be determined by means of the formula

$$A = 0.733(V_2^2 - V_1^2)/S \tag{4}$$

in which, for the determination of resistance at a point, V_1 and V_2 are found by drawing at B (the point at which resistance is to be determined—see Fig. 7) a tangent to the speed curve, prolonging it to C and D equidistant from B, and dropping from C and D ordinates which are proportional to V_1 and V_2, respectively. For the determination of average resistance over a section, V_1 and V_2 are the speeds at entrance to and exit from the section respectively (see Fig. 8).

Net Resistance. If, as above assumed, the resistances due to wind and curvature are eliminated, the value of net resistance (R, item 5) may be found by the use of the following formulæ.

Momentary resistance at a point =

$$R = \frac{P}{W} - 0.379\,G - \left[91.05 + 145.5\left(\frac{N}{W}\right)\right] \times A \qquad (5)$$

Average resistance over a section =

$$R = \frac{P}{W} - \frac{2000\,(E_2 - E_1)}{S} - \left[91.05 + 145.5\left(\frac{N}{W}\right)\right] \times A \qquad (6)$$

If the train is on a down grade, the sign of the second term in equation (5) must be changed to plus.

In applying formulæ 1 to 6, the following **precautions** must be taken in selecting the point or section at which resistance is to be determined:

METHOD 1:

The entire train must be on tangent track and on a uniform grade.

The speed curve must be nearly straight for a short distance both sides of the point chosen, in order to permit the tangent to be accurately drawn.

The point should preferably be chosen where the acceleration is low, so that acceleration resistance is not a large share of the gross resistance.

METHOD 2:

The track must be straight over the section, and also before the entrance to the section for a distance equal to the train length.

To facilitate the determination of E, the entire train should be on a uniform grade at the moment its head end enters the section, and again when it leaves the section. These grades need not, however, be alike.

There should not be much variation in speed over the section.

Values of Net Resistance

The resistance in still air, at uniform speed, on straight and level track (R) increases with the speed and decreases as the weight of the cars increases. This decrease with increase of car weight is due chiefly to the increase in journal load. Net resistance increases also as the air temperature decreases. For ordinary weather conditions net resistance cannot, therefore, be predicted, unless both the speed and the average car weight be known.

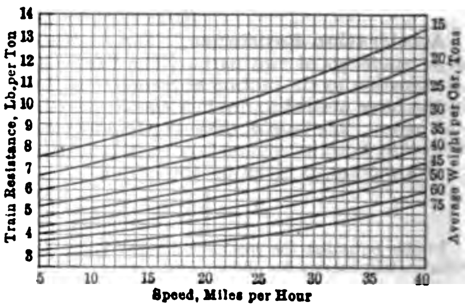

FIG. 9.—Freight-train Resistance.

Freight-train Resistance. Fig. 9 gives the values of net resistance for freight trains running on well-constructed track in weather when the air temperature is above 30 deg. fahr. The use of the figure requires a knowledge of speed and of the average gross-weight of the cars of which the train is composed. Values of R are more conveniently and accurately given in Fig. 9, but they may be approximately derived from the empirical formula

$$R = (V + 39.6 - 0.031w)/(4.08 + 0.152w) \qquad (7)$$

Passenger-train Resistance. The net resistance of passenger trains increases with speed. It presumably varies also with the load per axle, as in freight trains; but experimental evidence on this point is deficient. Since, however, the load per axle of passenger cars does not vary through a wide range, there is justification for ignoring the influence of axle load, as is usually done in considering passenger-train resistance.

The formula proposed by Mr. F. J. Cole, of the American Locomotive Company, gives values for net passenger-train resistance which are reliable for the heaviest cars, the best track, and good weather conditions. This formula is

$$R = 5.4 + 0.002(V - 15)^2 + 100/(V + 2)^3 \qquad (8)$$

Formula (8) gives the following values of resistance R for various values of V:

For V (mi. per hr.) =	5	10	15	20	25	30	35	40	45	50	60	70	80	90
R (lb. per ton) =	5.9	5.5	5.4	5.5	5.6	5.9	6.2	6.7	7.2	7.9	9.5	11.5	13.9	16.7

For ordinary cars and track and to make allowance for adverse weather, it is preferable to use formula (9), which is based on the Berlin-Zossen tests, and which gives results considerably greater than those derived by means of formula (8).

$$R = 4 + 0.1V + 0.278(V^2/W) \qquad (9)$$

References. Reviews of train-resistance experiments and formulæ are presented in Bulletin 84 of the American Railway Engineering and Maintenance of Way Association, Feb., 1907, and in a series of articles by F. J. Cole, in the *Railroad Age-Gazette*, vol. 47, Nos. 9 to 14, Aug. 27 to Oct. 1, 1909. Data on freight-train resistance are given in Bulletins 43 and 59 of the Engineering Experiment Station of the University of Illinois.

TRACK

Mileage. For the year ended June 30, 1914, the total number of miles of single track in operation in the United States was 252,000. Including second, third, and fourth track, and the track in yards and sidings, the total mileage in operation was 387,000.

Gage. The gage of railway track is the distance between the inner sides of the rail heads. The gage to which practically all American railways are laid is 4 ft. 8½ in., which is known as **standard gage.** Rail wear causes an increase in this distance. Owing to the wear of rails and to the practice of widening the gage of curved track at the rate of about ¼ in. for each 5 deg. of curvature, it is not uncommon to find the gage on sharp curves as great as 4 ft. 9½ in.

The distance from center to center of rails is variable, depending upon the width of the rail head. This width varies from 2⅜ in. in rail weighing 60 lb. per yd. to 2¾ in. in rail weighing 100 lb., so that the distance from center to center of rails varies usually from 4 ft. 10⅞ in. to 4 ft. 11¼ in.

In America, the gage of **narrow-gage** track varies usually from 2 ft. 6 in. to 3 ft. 6 in., although 3 ft. 0 in. is most common. One meter is the prevailing narrow gage on European railways.

Track Spacing. The distance between centers of main-line tracks is usually 13 ft., although it is occasionally only 12 ft. In yards it has been customary to space tracks 12 ft., or even as little as 11 ft. 6 in. apart; but 13 ft. is coming to be the standard distance. This is none too great, since the largest cars are occasionally as wide as 10 ft. 4 in.

Clearances. Obstructions adjacent to track should be kept far enough away to clear a man riding on the side of a car. This, with large cars, requires that the minimum distance from the center of track to the nearest point of the obstruction should be 8 ft.

Any overhead obstruction should be high enough to clear a man standing on the highest car. This implies a minimum height of at least 21 ft. On main lines 22 ft. is commonly specified wherever practicable. In some states 21 ft. or 22 ft. is a minimum clearance height fixed by law. Where it is certain that trainmen will be kept off the cars and where conditions require the minimum clearance, as in tunnels, these clearances are much reduced. For tracks entering buildings, a door opening 12 ft. wide and 16 ft. high will ordinarily suffice to pass the largest locomotive or car.

Curvature. The curvature of track is designated in terms of "degree of curve." **Degree of curve** is the number of degrees of central angle subtended by a chord of 100 ft. length (measured on the track center line). Table 13 gives the radii for ordinary steam railway curves.

Table 13. Radii of Steam Railway Curves

Degree of curve	Radius to track center line, ft.	Degree of curve	Radius to track center line, ft.	Degree of curve	Radius to track center line, ft.
1	5730	9	637.3	17	338.3
2	2865	10	573.7	18	319.6
3	1910	11	521.7	19	302.9
4	1433	12	478.3	20	287.9
5	1146	13	441.7	21	274.4
6	955.4	14	410.3	22	262.0
7	819.0	15	383.1		
8	716.8	16	359.3		

On important main lines where trains are operated at high speed the curves are ordinarily not sharper than 6 or 8 deg. In mountainous territory in very rare instances main-line curves as sharp as 22 deg. occur. Even in restricted yard spaces the curvature is very seldom as great as 22 deg. A 22-deg. curve, if operated over with great care and at very low speed, will pass even the largest locomotive. A car alone will pass considerably sharper curves.

Rails. Until within a few years ago the **standard length** of rails was 30 ft.; but it is now 33 ft. The weight of rail varies between different railways and between the different parts of the same railway. Table 14 shows the proportions of the various weights of rail used in main tracks of the railways of the United States in January, 1912.

Table 14. Rail Used in Main Tracks of U. S. Railways

Weight of rail per yard, lb.	Percentage of total in use
100 lb. and upward	5.845
90 lb. and less than 100 lb.	8.324
80 lb. and less than 90 lb.	32.941
75 lb. and less than 80 lb.	12.809
70 lb. and less than 75 lb.	8.564
60 lb. and less than 70 lb.	18.158
Less than 60 lb., mixed, and unknown	13.359

Material Required per Mile of Single Track. Details of rails, splice-bars, bolts and spikes are given in Tables 15 and 16.

Table 15. Rails and Accessories, Weights and Dimensions

Rail section (Numbers indicate lb. per yd.)	Length of splice bar, in.	Size of bolt, in.	Size of spike, in.	For one joint: Weight of one pair of splice bars, lb.	For one joint: Number and weight of bolts and nuts, lb.		For one mile, single track: Number: Pairs of splice bars	Number: Bolts and nuts	Number: Spikes	Weight in gross tons: Splice bars	Weight in gross tons: Bolts and nuts	Weight in gross tons: Spikes	Weight in gross tons: Total accessories	Weight in gross tons: Rail
100A	34	1 × 4½	5½ × 9/16	87.00	6 BOLTS	10.53	326	1956	10,640	12.66	1.54	2.80	17.00	157.14
90A	34	⅞ × 4¼	5½ × 9/16	74.00		7.41	326	1956	10,640	10.77	1.08	2.80	14.65	141.43
85A	34	¾ × 4¼	5½ × 9/16	68.13		5.20	326	1956	10,640	9.92	0.76	2.80	13.48	133.57
80A	34	¾ × 4⅛	5½ × 9/16	63.13		5.05	326	1956	10,640	9.07	0.75	2.80	12.62	125.71
75A	34	¾ × 4	5½ × 9/16	58.50		4.96	326	1956	10,640	8.52	0.74	2.80	12.06	117.86
70A	34	¾ × 3¾	5½ × 9/16	54.64		4.76	326	1956	10,640	7.96	0.71	2.80	11.47	110.00
65A	24	¾ × 3¾	5½ × 9/16	35.55	4 BOLTS	3.18	326	1304	10,640	5.17	0.48	2.80	8.45	102.12
60A	24	¾ × 3½	5½ × 9/16	32.40		3.12	326	1304	10,640	4.71	0.46	2.80	7.97	94.29
55A	24	¾ × 3½	5½ × 9/16	28.90		3.12	326	1304	10,640	4.20	0.46	2.80	7.46	86.43
50A	24	¾ × 3¼	5½ × 9/16	25.50		3.00	326	1304	10,640	3.71	0.44	2.80	6.95	78.57
45A	20	¾ × 3	5½ × 9/16	18.75		2.90	364	1456	10,640	3.04	0.47	2.79	6.31	70.71
40A	20	¾ × 3	5½ × ½	16.10		2.90	364	1456	10,640	2.62	0.47	1.94	5.03	62.86
35A	16½	⅝ × 2½	4½ × ½	12.10		1.74	364	1456	10,640	1.97	0.30	1.78	4.05	55.00
30A	16½	⅝ × 2½	4 × ½	10.45		1.74	364	1456	10,640	1.70	0.30	1.59	3.59	47.14
25A	16½	½ × 2¼	4 × ½	5.70		0.97	364	1456	10,640	0.93	0.16	1.59	2.68	39.29
20A	16½	½ × 2	3½ × ½	4.86		0.91	364	1456	10,640	0.79	0.16	1.53	2.48	31.43
16A	16½	½ × 1¾	3½ × ⅜	4.36		0.865	364	1456	10,640	0.71	0.15	0.81	1.67	25.14
14A	16½	½ × 1¾	3 × ⅜	3.44		0.865	364	1456	10,640	0.56	0.15	0.77	1.48	22.00
12A	16½	½ × 1¾	3 × ⅜	3.44		0.865	364	1456	10,640	0.56	0.15	0.77	1.48	18.86
10A	16½	⅜ × 1½	2½ × 5/16	2.60		0.45	364	1456	10,640	0.42	0.07	0.43	0.92	15.71
8A	16½	⅜ × 1½	2½ × 5/16	2.00		0.45	364	1456	10,640	0.33	0.07	0.43	0.83	12.57

Above table is based on 90 per cent. of rails to be 33 ft. long, and 10 per cent. not less than 24 ft. long, varying by even feet. Ties 2 ft. to centers, or 2640 ties per mile, Rails below 50 lb. per yd. are furnished 30 ft. long, and 10 per cent. not less than 20 ft. long. No excess has been allowed.

The amount of **ballast** required depends upon the depth to be used under the ties, the spacing and dimensions of the ties, the width of ballast on top, and the angle of side slope. It must be separately computed for each set of conditions. Ordinarily, it will vary from ½ to ¾ cu. yd. per lineal foot of track.

Costs. Ordinarily, a single-track railway built through favorable country will cost—including equipment and all other charges—not less than $25,000 to $30,000 per mile. Where land is very valuable, as on the outskirts of large cities, and where frequent overhead highway or stream crossings are necessary the cost may be as high as $150,000 per mile, and in rare cases even higher.

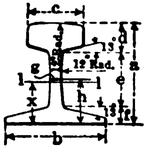

FIG. 10.

Table 16. A. S. C. E. and Light Rails*
(Letters refer to Fig. 10)

Section index	Weight per* yd., lb.	Area of section, sq. in.	Dimensions in inches								Axis 1-1			
			a	b	c	d	e	f	g	h	I	r	S	x
											In.⁴	In.	In.³	In.
110A	110	10.80	6 1/8	6 1/8	2 7/8	1 25/32	3 11/32	1	37/64	2 43/64	55.2	2.26	17.2	2.92
100A	100	9.84	5 3/4	5 3/4	2 3/4	1 45/64	3 5/64	31/32	9/16	2 65/128	44.0	2.11	14.6	2.73
95A	95	9.28	5 9/16	5 9/16	2 11/16	1 41/64	2 63/64	15/16	9/16	2 55/128	38.8	2.05	13.3	2.65
90A	90	8.83	5 3/8	5 3/8	2 5/8	1 19/32	2 55/64	59/64	9/16	2 45/128	34.4	1.97	12.2	2.55
85A	85	8.33	5 3/16	5 3/16	2 9/16	1 35/64	2 3/4	57/64	9/16	2 17/64	30.1	1.90	11.1	2.47
80A	80	7.86	5	5	2 1/2	1 1/2	2 5/8	7/8	35/64	2 3/16	26.4	1.83	10.1	2.38
75A	75	7.33	4 13/16	4 13/16	2 15/32	1 27/64	2 35/64	27/32	17/32	2 15/128	22.9	1.77	9.1	2.30
70A	70	6.81	4 5/8	4 5/8	2 7/16	1 11/32	2 15/32	13/16	33/64	2 3/64	19.7	1.70	8.2	2.22
65A	65	6.33	4 7/16	4 7/16	2 13/32	1 9/32	2 3/8	25/32	1/2	1 31/32	16.9	1.63	7.4	2.14
60A	60	5.93	4 1/4	4 1/4	2 3/8	1 7/32	2 17/64	49/64	31/64	1 115/128	14.6	1.57	6.6	2.05
55A	55	5.38	4 1/16	4 1/16	2 1/4	1 11/64	2 11/64	23/32	15/32	1 103/128	12.0	1.50	5.7	1.97
50A	50	4.87	3 7/8	3 7/8	2 1/8	1 1/8	2 1/16	11/16	7/16	1 23/32	9.9	1.43	5.0	1.88
45A	45	4.40	3 11/16	3 11/16	2	1 1/16	1 31/32	21/32	27/64	1 41/64	8.1	1.36	4.3	1.78
40A	40	3.94	3 1/2	3 1/2	1 7/8	1 1/64	1 55/64	5/8	25/64	1 71/128	6.6	1.29	3.6	1.68
35A	35	3.44	3 5/16	3 5/16	1 3/4	61/64	1 25/32	37/64	23/64	1 15/32	5.2	1.23	3.0	1.60
30A	30	3.00	3 1/8	3 1/8	1 11/16	7/8	1 23/32	17/32	21/64	1 25/64	4.1	1.16	2.5	1.52
25A	25	2.39	2 3/4	2 3/4	1 1/2	25/32	1 31/64	31/64	19/64	1 29/128	2.5	1.02	1.8	1.33
20A	20	2.00	2 5/8	2 5/8	1 13/32	23/32	1 15/32	7/16	1/4	1 11/64	1.9	0.99	1.4	1.27
16A	16	1.55	2 3/8	2 3/8	1 11/64	41/64	1 23/64	3/8	7/32	1 7/128	1.2	0.89	1.0	1.15
14A	14	1.34	2 1/16	2 1/16	1 1/16	5/8	1 3/32	11/32	1/4	57/64	0.76	0.75	0.73	1.02
12A	12	1.18	2	2	1	9/16	1 1/32	11/32	3/16	57/64	0.66	0.75	0.63	0.96
10A	10	0.96	1 3/4	1 3/4	15/16	33/64	15/16	19/64	3/16	49/64	0.40	0.65	0.46	0.87
8A	8	0.77	1 9/16	1 9/16	13/16	15/32	13/16	9/32	5/32	11/16	0.26	0.58	0.32	0.75

* American Railway Association Rails range from 100 to 60 lb. by 10-lb. intervals, and differ slightly in dimensions from the A. S. C. E. standard.

MARINE ENGINEERING

BY

W. F. DURAND

REFERENCES: "Speed and Power of Ships," Taylor (J. Wiley & Sons). "Naval Architecture," Peabody (J. Wiley & Sons). "Resistance and Propulsion of Ships," Durand (J. Wiley & Sons). "Design and Construction of Ships," Biles (J. B. Lippincott Co.). "Marine Engines and Boilers," Bauer and Robertson (N. W. Henley Pub. Co.). "Manual of Marine Engineering," Seaton (D. Van Nostrand Co.). *Transactions* of the Institute of Naval Architects (London). *Transactions* of the American Society of Naval Architects and Marine Engineers (New York). *Journal* of the American Society of Naval Engineers (Washington, D. C.).

PRINCIPAL DIMENSIONS OF SHIPS

Definitions. The **dimensions of a ship** may refer to the molded body or form defined by the outside of the frames; to general outside or overall dimensions; and to dimensions on which the determination of tonnage or of classification is based. There are thus (1) molded dimensions, (2) overall dimensions, (3) tonnage dimensions and (4) classification dimensions.

The **molded length** is the distance between two vertical lines. The forward vertical is located at the forward edge of the stem. With sailing and twin-screw ships the after vertical is located at the after face of the stern or body post. With single- or triple-screw ships there is some difference in practice by different authorities and in different countries. In some cases the after vertical is located at the after face of the body post, and in others at the after face of the rudder post. This difference depends primarily on whether the after boundary of the geometrical form of the ship is the rudder or stern post. The **length between perpendiculars** is again the distance between two verticals. Often, though not necessarily, it coincides with the molded length. The forward perpendicular is usually located at the forward edge of the stem. In the United States the after perpendicular is usually taken at the after edge of the body post on the load water line. In Great Britain it has the same location except for single- and triple-screw ships, where it is usually taken at the after edge of the rudder post at the load water line. The **length on the load water line** is the length between two verticals touching the bow and the stern at the load water line. The **length overall** is the extreme length between two verticals touching the ship at the forward tip of the stem and the overhang at the stern. In the case of a war ship with ram bow and war-ship stern, it may be defined as the extreme length between two verticals touching the ship at bow and stern. The **tonnage length** is especially defined in various countries according to the rules for computing tonnage. The **classification length** is likewise specially defined by the various societies (Lloyd's, Bureau Veritas, Am. Bureau of Shipping, etc.) for purposes of classification and rating.

The **molded beam** is the extreme breadth of the molded form. With steel ships this is the only value ordinarily considered. The extreme or overall breadth is occasionally used, referring to the extreme transverse dimension on the outside of the plating.

The **depth** is the vertical distance from the inner surface of the plating at the keel to the inner surface of the upper continuous deck. There is, however, some difference of meaning of the term depth in ships of different types of construction.

The **draft** is the vertical distance from the bottom of the keel to the water line.

The **displacement** is the weight (in tons of 2240 lb.) of the water displaced by the immersed part of the ship, and is equal to the weight of the ship and everything on board. The density of sea water averages about 64 lb. per cu. ft. (1 ton = 35 cu. ft.), hence the displacement in sea water is measured by the immersed volume divided by 35. In fresh water the corresponding divisor is usually taken as 36.

The **tonnage** is an arbitrary measure of capacity used in estimating navigation charges of various kinds, and is intended to be proportional to the earning capacity of the vessel. In general, tonnage is based on the **cubic capacity** of the ship below the main or upper continuous deck, with an allowance for such part of this volume as is occupied by machinery, fuel, crew quarters, light and air shafts and certain like uses. According to U. S. law, the tonnage is equal to the residual volume in cu. ft. divided by 100. (For directions in detail, see U. S. Revised Statutes, Sec. 4153, with subsequent amendments.)

Coefficients of Form. Assume the following notation: L = length on water line, ft., B = beam, ft., H = draft, ft., D = displacement in tons of sea water, V = displaced volume in sea water, cu. ft. (= $35D$), A = area of water plane at surface of water, sq. ft., M = area of midship section below water plane, sq. ft.

Then b = block coefficient of fineness = $35D/LBH$
w = water plane coefficient = A/BL
m = midship section coefficient = M/BH
p = mean length or prismatic coefficient = $35D/ML$
q = mean depth coefficient = $35D/AH$

and the following relations may be noted:

$$p = b/m;\ q = b/w;\ p/q = w/m;\ b = \sqrt{pqmw}$$

The typical and usual **values of** these **coefficients** range as follows:

	b	w	m	p	q
Freight Steamers	.65–.80	.75–.85	.85–.95	.75–.85	.80–.95
Passenger Liners	.55–.65	.65–.75	.80–.90	.60–.70	.80–.90
Battle Ships	.60–.65	.70–.75	.85–.92	.65–.70	.80–.90
Fast Cruisers	.50–.55	.60–.65	.80–.90	.55–.60	.75–.85
Torpedo Craft, Fast Yachts, etc.	.35–.45	.55–.60	.75–.85	.50–.60	.65–.80

STABILITY

The Problem of Stability. (See p. 253 for Buoyancy and Flotation.) A ship or other floating body is in vertical equilibrium under the action of two systems of forces: (*a*) gravity, acting down, and (*b*) buoyancy, acting up. The center or point of application of the gravity system considered as a whole is at the center of gravity of the ship and its contents. The corresponding center for the buoyancy system is at the center of volume of the immersed body of the ship, called the center of buoyancy. These two centers lie in the same vertical line, when the body is in vertical equilibrium.

If a couple with a longitudinal axis be brought to bear on a ship in such equilibrium, a rotation about such axis will result. Notwithstanding this rotation, the center of the gravity system will remain fixed at the center of gravity of the ship. On the other hand, the buoyancy system will change its center due to the changing form of the immersed body. Furthermore, these two points will no longer lie in the same vertical, and the two systems will constitute a couple with equal and opposite vertical forces and with a moment arm measured by the distance between their present lines of action. This **gravity-buoyancy moment** developed as a result of such an inclination of the ship is a measure of the so-called "**stability**" of the ship.

For small inclinations, as, for example, up to 5 or 10 deg., the lines of action of the buoyancy system will all pass very near a single point M in the ship known as the **metacenter.** If this be taken as a fixed point for small inclinations, the expression for stability becomes very simple. For positive stability the metacenter must lie above the center of gravity, G. Let $\overline{GM}$ denote the distance between these points. Then it is readily seen that $\overline{GM}$ sin c will be the moment arm for a small inclination of c degrees, and $\overline{GM} \sin c \times D$ will be the **stability moment,** D being the displacement or weight of the ship. For inclinations of greater amount and for which the line of action of the buoyancy system no longer passes through the metacenter, the problem becomes that of finding the center of figure of the immersed body in the inclined position and the resulting value of the arm drawn from the center of gravity to a vertical passing through this buoyancy center. To carry out this program requires the location of the water plane for the inclined position and the center of volume of the under-water body thus determined. The entire process involves only the elementary principles of geometry and mechanics applied to the immersed body of the ship.

Dynamical Stability. Starting with a ship in equilibrium in the upright position, suppose it to be heeled over continuously to any ultimate angle c. The moment opposed by the statical stability must be overcome, and work will therefore be performed. The amount of such work is a measure of the so-called **dynamic stability** of the ship for any angle of inclination c. Let M_s denote the moment of static stability, and R the dynamic stability. Then $R = \int_0^c M_s dc$. The determination of dynamic stability therefore requires the previous determination of a series of values of the static stability, which may then be integrated either by numerical means or plotted in the form of a curve and integrated by a planimeter or integraph.

The dynamical stability may also be directly computed for any given inclination as the work represented by the expression $D(\overline{G_0B_0} - \overline{G_1B_1})$, where D = displacement and $(\overline{G_0B_0} - \overline{G_1B_1})$ is the difference in the vertical distances between the centers of gravity and buoyancy in the upright and in the inclined positions.

Trim. The problem of trim or change of trim is simply that of rotation about a transverse axis, and hence of longitudinal stability. For all ordinary changes of trim the vertical through the center of buoyancy passes through a point practically fixed relative to the ship—the longitudinal metacenter. Let $\overline{GM_1}$ denote the distance of this point above the center of gravity. Then moment of longitudinal stability $= \overline{GM_1} \sin c \times D$. Also, if L = length of ship in ft. and h = change of trim in in., then for small angles, approximately, $\sin c = \tan c = h/12L$. Hence, moment $= \overline{GM_1}hD/12L$ (ft.-tons). In general, the moving of a weight of W tons a distance l ft. in a fore-and-aft direction will produce a disturbing moment which must be balanced by the development of an equal trim moment. Hence, $Wl = \overline{GM_1}hD/12L$. This equation serves to connect together the moving of weights and the resulting change of trim.

SHIP RESISTANCE AND POWERING

Ship Resistance. The resistance to the movement of a ship through the water may be classified under the four following heads: (1) **Stream-line resistance,** due to the energy required for the maintenance of a system of stream lines in a liquid not perfectly free from viscosity; (2) **eddy or head resistance,** due to the energy required for the maintenance of systems of large eddies of water in vortex motion (such eddies are of special importance in ships of full form, in all bodies of an irregular form and in particular in approximately plane surfaces moving at an angle with their own plane); (3) **skin resistance,** due to the energy required for the maintenance of a belt or blanket of eddying water close about the surface of the ship resulting from the action of minute irregularities of form and of adhesive forces between the water and the ship's surface; and (4) **wave resistance,** due to the energy required for the maintenance of systems of waves which surround and accompany the ship. These waves are a surface manifestation of differences of liquid pressure in the water immediately surrounding the ship. In all cases (1) is relatively small. In ships of normal form (2) is relatively small and most of the resistance arises from (3) and (4). The sum of (1), (2) and (4) is often called **residual resistance.**

The proportion between skin and residual resistance decreases with increase in speed relative to the length. This is due to the rapidly increasing importance of the wave-making resistance (4) at relatively high speeds. Let v = speed in knots and L = length in ft. Then the ratio v^2/L is called the **speed-length ratio.** With ships of fine form and moving at speeds such that the speed-length ratio is less than 1, the residual resistance is likely to comprise only some 10 to 20 or 25 per cent. of the total resistance, and the skin resistance will comprise the remainder. With increase in the speed-length ratio the relative proportion of residual resistance (composed mostly of the wave-making component) will rapidly increase to 40 or 50 per cent., and at high values of the ratio to 60 or 70 per cent. of the total.

Skin resistance is expressed by the formula $R_s = fAv^{1.83}$, where R_s = skin resistance, in lb., f = coefficient of skin resistance, A = wetted surface, in sq. ft. and v = speed in knots. With these units the values of f for clean, painted steel plates vary approximately and gradually from 0.0085 to 0.0100 for lengths from 600 ft. to 20 ft., respectively. With rough bottoms due to barnacles and marine growths the coefficient may increase, varying with the degree of roughness, to values 50 to 100 per cent. greater, and in extreme cases even to three or four times the values for smooth plates.

To find **wetted surface** A: For approximate purposes Taylor proposes the formula $A = 15.6\sqrt{DL}$, where D = displacement in tons and L = length in ft. A method requiring more labor but giving more accurate results consists of the rectification of the wetted girth lines and their integration throughout the length of the ship. This gives an area known as the **reduced wetted surface,** A_r. This value is then used in the formula $A = A_r[1 + (p - 0.2)(B/L)^2]$. Residual resistance [composed largely of (4)] varies with the form and dimensions of the ship as well as with the speed. Resistance as a whole will vary with the size and form of the ship, with the character of wetted surface and, for moderate speeds, closely as the square of the speed, while as the speed increases the index of resistance variation with speed fluctuates between 2 or less to 3.5 or more. These fluctuating terms arise from varying relations between the ship and combinations of the various trains of waves resulting from the movement of the ship through the water. For relatively moderate speeds, which may be defined as speeds for which the speed-length ratio (see above) is not greater than 1, Taylor suggests the formula $R = fAv^{1.83} + 12.5\,bDv^4/L^2$, where R = total resistance, lb. The first term gives the skin resistance as above, and the second the residual resistance.

In shallow water or in restricted channels, as in the case of canals, the **resistance is greatly increased.** With a depth equal to or greater than half the length of the wave corresponding to the speed of the ship the influence of the bottom should, on the simple theory of wave formation, be negligible. This is equivalent to a depth in ft. of $v^2/3.6$, where v = speed in knots. Experience indicates that for small high-speed ships such as torpedo boats, destroyers, fast launches, etc., this depth is somewhat excessive. For large ships it is presumably none too much. At the **critical depth,** $v^2/11.3$, the resistance undergoes a very marked increase. The desirable depth should apparently be not less than about $v^2/5$ and preferably nearer $v^2/4$, or $v^2/3$, especially for large, full-formed ships. From a consideration of stream-line phenomena, Taylor has suggested a depth of 6 to 10 times the draft, or, for boats with a high speed-length ratio, a depth not much less than the length.

For restricted channels, as in canal navigation, at moderate speeds, if the transverse dimensions of the canal section are as much as 6 to 10 times those of the ship, the added resistance will be negligibly small, while with a ratio of 2.5 or 3 the increase in resistance is excessive and with further restriction compels a speed so low as to be prohibitive from the standpoint of operation.

Power Required for Propulsion. The relation between the size of ship, speed and power required is conveniently represented by the Admiralty formula $H = D^{2/3}v^3/K$, where H = the indicated horse power required; D = displacement of ship in tons of 2240 lb.; v = speed of ship in knots of 6080 ft. per hour; and K = factor or coefficient intended to include the influence of all factors in the problem other than those above noted.

The more important of the factors included in K are those representing the influence due to the characteristics of form and that due to the overall dimensions (and in particular the length) relative to the speed. The factor K must also include recognition of any special variation from the normal in the efficiency of the propeller or in the mechanical efficiency of the engines. Assuming normal values for the latter, the speed-length ratio (see Resistance) serves in some degree, in combination with the characteristics of form, as a guide in the selection of a suitable value of K. The following brief indications may be given.

Values of K in Admiralty Formula. For ships of large size and with fine to medium characteristics of form (in particular prismatic coefficient) at speeds such that $v^2/L \leq$ 0.8 to 0.9, values of K = 250 to 350 are found.

For similar ships at higher speeds when $v^2/L \geq 1$, the values of K range from 220 to 250 or 275.

With medium to full forms at speeds where $v^2/L < 1$, values of 220 to 250 are common.

With medium to small ships, other things being equal, the values range somewhat smaller than for large ships.

With medium to small ships of average form characteristics and at speeds where v^2/L may considerably exceed 1, values of 150 to 200 for K are common.

With small boats of moderately full forms and urged to speeds where v^2/L may rise to values of 5 to 10 or more, the value of K may fall to 100 to 150, or in extreme cases to less than 100.

For small craft of fine lines driven at excessively high speed and of such under-water form as to rise partly out of the water, the resistance is decreased with corresponding reduction in power and increase in the value of K. In such cases values of 200 to 250 and more are met with.

No exact rules can be given for selecting the value of K in any particular case. Judgment must enter largely in the use of such a formula. In case the speed v is measured in miles per hour instead of knots, the values of K will be increased in the ratio of 1.526 to 1, or approximately by 50 per cent.

By the application of **Froude's law of comparison** extended to cover approximately all components of ship resistance, much more reliable results may be determined. This law may be stated in many ways. For the present purpose it may be put as follows: For ships of similar geometrical form and proportion, *corresponding speeds* are in the ratio of the square roots of the lengths. For such ships *at corresponding speeds*, the values of K in the Admiralty formula are the same.

Still otherwise the law may be put in the form: For similar ships *at corresponding speeds* the powers will be in the ratio of the products of the displacements by the speeds, or the power ratio will equal the product of the displacement ratio by the speed ratio.

Thus, given a type ship having a length of 400 ft., a displacement of 7500 tons, a speed of 16 knots, and requiring 6300 i.h.p., by substitution in the formula the value of K

is readily found to be 250. Then, assuming a similar ship 480 ft. in length, length ratio = 480/400 = 1.2; corresponding speed ratio $= \sqrt{1.2} = 1.095$; corresponding speed = 16 × 1.095 = 17.53; displacement ratio = $(1.2)^3 = 1.728$; displacement = 7500 × 1.728 = 12,960. Then, using the same value of $K(= 250)$ and substituting in the Admiralty formula, H for the larger ship at the higher speed = 11,900 i.h.p. Or, again, by the second form in which the law is stated, $H = 6300 \times 1.728 \times 1.095 =$ 11,900 as before.

Where a sufficient number of test results are available, the method by comparison gives a means of determining closely the power which any given ship will require at any proposed speed. For satisfactory application, however, the ships employed in such a comparison must not differ widely in form or proportion. The precise application of the rule assumes exact geometrical similarity. Closely approximate results, however, may be found in case the departure from exact geometrical similarity is not extreme.

SCREW PROPELLERS

The **performance** of a screw propeller may be conveniently related to the following characteristics and conditions of operation:

Diam. (in ft.) = d.
Pitch (in ft.) = p.
Projected blade area (in sq. ft.) = A.
Revolutions per minute = N.
Speed in knots = v.
Speed of advance of propeller relative to water at stern in which it is working, knots = u.
Shaft horse power = S.

Pitch is often replaced by its **ratio** to the diameter, thus: $p = dl$, or $l = p/d$.

The relation between projected **blade area** and actual or helicoidal area is given by the following approximate empirical equation due to Taylor:

$$\text{Actual Area} = \text{Projected Area}/[1.067 - 0.229(p/d)].$$

This formula is intended for blades of elliptical form, but will hold less accurately for any blade with a generally oval contour.

Speed is often represented by its relation to N and p through the **slip ratios**. Thus,

$$\text{True slip ratio } s = (pN - 101.3u)/pN \quad (1)$$
$$\text{Apparent slip ratio } s' = (pN - 101.3v)/pN \quad (2)$$
$$\text{whence } u = pN(1 - s)/101.3 \text{ (knots)} \quad (3)$$
$$v = pN(1 - s')/101.3 \text{ (knots)}. \quad (4)$$

The **true slip** s is measured relative to the body of water at the stern in which the propeller is working, and which has had impressed upon it by the movement of the ship a certain forward motion relative to the surrounding and outlying body of still water. The **apparent slip** s' is measured relative to the surrounding or outlying body of still water through which the ship is moving. Since the propeller should be judged and determined relative to the water in which it is actually working, the true slip should be used in all formulæ directly relating to its design.

The relation between true and apparent slip is indicated by means of a so-called "**wake factor**" w, through the equation

$$1 - w = (1 - s)/(1 - s') \quad (5)$$

Values of w vary from small or almost vanishing amounts for high-speed yachts, torpedo boats and similar craft, up to 0.10 for vessels of average size and moderate speed, and to values of 0.15 and more for vessels of very full form or large dimensions. In the lack of special information, the value 0.10 is often taken as an approximate mean value.

The speeds u and v have the following relation:

$$u = v(1 - w) \quad (6)$$

The mechanical efficiency of marine engines is usually taken, in the absence of more direct information, at 88 or 90 per cent. Hence, approximately, $S = 0.9 \times$ i.h.p.

For purposes of **propeller design** or the analysis of propeller performance, Taylor's coefficients, which are defined by equations (7) and (8), may be employed:

$$R = N^{1/2}S^{1/4}/u^{5/4} \tag{7}$$

$$D = dN^{3/4}/(Su)^{1/4} \tag{8}$$

Numerical values of R and D for **three-bladed propellers** based on Taylor's researches at the Government Experimental Tank at Washington and adapted for use by Peabody will be found in Tables 1–3. These tables likewise include values of the efficiency e which may be anticipated in any given case. These three quantities, R, D and e, are given for various values of the pitch ratio, true slip, area ratio and thickness ratio. The **area ratio** is defined as the area of the blades projected on a plane perpendicular to the shaft, divided by the disk area of the propeller or by the area of the circle described by the tips of the blades. The **thickness ratio** is defined as the ratio of OE in Fig. 1 to the diameter.

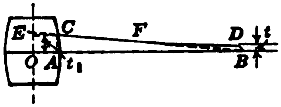

Fig. 1.

It should be noted that the values of R, D and e presuppose *projected* blade contours elliptical in form with smooth, true pitch surfaces and clean, sharp edges. No serious error will be involved in their application to a generally oval form of projected area so long as the departure from the ellipse is moderate. Any marked departure from these general characteristics or in the thickness ratio as noted, will seriously affect the value of these quantities.

Table 1. Three-bladed Propellers

(Projected area ratio, 0.27; thickness ratio, 0.06)

True slip ratio		Pitch ratio									
		0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
0.12	R	4.56	3.89	3.40	3.01	2.71	2.24	1.93	1.73	1.56	1.40
	D	70.2	66.7	63.7	62.4	59.2	55.6	52.8	50.3	48.2	46.1
	e	0.540	0.592	0.626	0.649	0.676	0.710	0.727	0.739	0.736	0.721
0.16	R	4.86	4.16	3.64	3.23	2.92	2.43	2.09	1.86	1.69	1.54
	D	70.6	66.8	63.7	62.3	59.1	55.4	52.4	49.9	47.6	45.5
	e	0.531	0.589	0.625	0.649	0.676	0.712	0.729	0.741	0.740	0.729
0.20	R	5.16	4.47	3.90	3.47	3.14	2.63	2.27	2.01	1.84	1.68
	D	71.0	67.1	63.8	62.3	59.0	55.3	52.1	49.5	47.2	45.1
	e	0.519	0.583	0.620	0.646	0.671	0.705	0.724	0.735	0.736	0.726
0.24	R	5.48	4.80	4.18	3.73	3.38	2.85	2.47	2.18	2.01	1.84
	D	71.6	67.5	64.0	62.3	59.0	55.2	51.9	49.2	46.8	44.7
	e	0.506	0.575	0.613	0.639	0.662	0.693	0.712	0.721	0.720	0.715
0.28	R	....	5.17	4.50	4.02	3.64	3.09	2.69	2.39	2.20	2.01
	D	....	68.1	64.4	62.5	59.1	55.2	51.8	49.0	46.6	44.4
	e	....	0.565	0.602	0.628	0.649	0.676	0.693	0.702	0.699	0.698
0.32	R	....	....	4.90	4.35	3.93	3.35	2.93	2.62	2.40	2.20
	D	....	....	64.9	62.9	59.3	55.4	52.0	49.0	46.5	44.2
	e	....	....	0.587	0.613	0.632	0.657	0.673	0.679	0.677	0.677

Table 2. Three-bladed Propellers
(Projected area ratio, 0.36; thickness ratio, 0.05)

True slip ratio		Pitch ratio 0.60	0.70	0.80	0.90	1.00	1.20	1.40	1.60	1.80	2.00
0.12	*R*	4.12	3.62	3.15	2.82	2.57	2.16	1.80	1.65	1.49	1.35
	D	74.3	70.0	66.3	63.5	61.2	57.3	54.1	51.5	49.3	47.4
	e	0.558	0.594	0.626	0.658	0.676	0.704	0.716	0.709	0.689	0.673
0.16	*R*	4.43	3.91	3.42	3.06	2.78	2.36	2.05	1.82	1.65	1.50
	D	73.9	69.8	66.0	63.0	60.6	56.6	53.3	50.6	48.3	46.4
	e	0.561	0.599	0.630	0.660	0.680	0.710	0.721	0.719	0.704	0.687
0.20	*R*	4.78	4.23	3.75	3.36	3.03	2.58	2.24	1.99	1.81	1.66
	D	73.6	69.5	65.8	62.7	60.1	56.1	52.7	49.9	47.6	45.6
	e	0.560	0.595	0.627	0.654	0.678	0.704	0.716	0.716	0.705	0.690
0.24	*R*	5.15	4.58	4.09	3.66	3.31	2.80	2.44	2.16	1.97	1.82
	D	73.3	69.3	65.8	62.5	59.8	58.8	52.3	49.4	47.1	45.1
	e	0.557	0.588	0.618	0.643	0.668	0.693	0.704	0.706	0.697	0.684
0.28	*R*	....	4.97	4.44	3.97	3.61	3.00	2.66	2.35	2.15	2.00
	D	....	69.2	65.7	62.5	59.7	55.5	52.1	49.0	46.7	44.7
	e	....	0.577	0.604	0.628	0.649	0.676	0.688	0.689	0.681	0.670
0.32	*R*	....	5.38	4.83	4.34	3.95	3.36	2.93	2.01	2.38	2.20
	D	....	69.0	65.8	62.7	59.8	55.4	51.9	48.7	46.4	44.5
	e	....	0.562	0.585	0.608	0.628	0.655	0.664	0.666	0.660	0.640

Table 3. Three-bladed Propellers
(Projected area ratio, 0.45; thickness ratio, 0.04)

True slip ratio		Pitch ratio 0.60	0.70	0.80	0.90	1.00	1.20	1.40	1.60	1.80	2.00
0.12	*R*	3.97	3.43	3.04	2.75	2.51	2.10	1.83	1.62	1.46	1.34
	D	77.2	72.1	68.2	65.3	62.6	58.2	54.8	52.1	50.0	48.1
	e	0.539	0.573	0.603	0.629	0.647	0.663	0.671	0.659	0.642	0.630
0.16	*R*	4.29	3.71	3.31	3.00	2.75	2.32	2.02	1.80	1.62	1.49
	D	76.5	71.4	67.5	64.5	61.8	57.3	53.8	51.0	48.7	46.8
	e	0.537	0.576	0.609	0.638	0.657	0.678	0.684	0.675	0.657	0.642
0.20	*R*	4.61	4.02	3.61	3.27	2.99	2.55	2.21	1.98	1.79	1.65
	D	76.0	71.0	67.1	63.8	61.2	56.7	53.0	50.2	48.0	45.9
	e	0.530	0.573	0.609	0.637	0.658	0.681	0.682	0.676	0.659	0.643
0.24	*R*	4.96	4.37	3.93	3.55	3.27	2.79	2.42	2.16	1.97	1.82
	D	75.6	70.7	66.8	63.4	60.8	56.2	52.4	49.5	47.3	45.3
	e	0.517	0.562	0.600	0.628	0.651	0.673	0.675	0.669	0.654	0.637
0.28	*R*	5.38	4.76	4.27	3.86	3.55	3.05	2.64	2.37	2.18	2.01
	D	75.4	70.5	66.6	63.1	60.4	55.8	51.9	49.0	46.8	44.8
	e	0.508	0.543	0.584	0.613	0.635	0.660	0.664	0.656	0.642	0.626
0.32	*R*	....	5.19	4.65	4.21	3.86	3.34	2.90	2.61	2.40	2.23
	D	....	70.5	66.5	63.0	60.2	55.6	51.6	48.7	46.5	44.6
	e	....	0.515	0.561	0.592	0.614	0.639	0.647	0.639	0.626	0.610

Four-bladed Propellers. The characteristics and proportions of propellers with four blades may be derived from those with three blades by means of the following relations:

Assuming the blades to be of the same shape and proportions, and the pitch ratio, speed and true slip to be alike in both cases, then for the diameter and

pitch of the four-bladed propeller, divide those dimensions of the three-bladed propeller by 1.145; and for the **revolutions** per minute of the four-bladed propeller, multiply the value of N for the three-bladed propeller by 1.145. The **efficiencies** may be expected to range from 90 to 95 per cent. of the values for three-bladed propellers, or, under usual conditions, from 0.04 to 0.07 less on the actual values. These reductions in efficiency range from the smallest values (0.04) for low area ratio and low pitch ratio to the higher values (0.07) for the highest area ratios and highest pitch ratios.

Example showing method of using Tables 1–3. Let i.h.p. = 3600 and speed v = 18 knots. Assume the power absorbed in friction to be 12 per cent. = 432 h. p. Then S = 3168 h.p. Assume wake factor = 0.10. Then $u = 0.90 \times 18 = 16.2$. Assume also that N is fixed at 120. Then, by substitution in (7), the value of R = 2.53. Referring next to Table 1 for three-bladed propellers with area ratio 0.27, it is possible to find by interpolation the following combinations of pitch ratio and slip with the corresponding values of the efficiency, all for a value of R = 2.53:

Pitch ratio	True slip	Efficiency	Pitch ratio	True slip	Efficiency
1.08	0.12	0.689	1.68	0.32	0.678
1.16	0.16	0.705	1.20	0.18	0.708
1.25	0.20	0.710	1.40	0.25	0.707
1.37	0.24	0.710	1.60	0.305	0.689
1.51	0.28	0.698			

An indefinite number of combinations may be selected. The above are simply those for the tabular values of the slip, 0.12, 0.16, etc., and for the pitch ratios 1.2, 1.4, etc.

So far as practicable, the values finally chosen should be such as to indicate the highest values of the efficiency. In the present case the true slip may be taken equal to 0.20 and the pitch ratio to 1.25. This gives for D the value 54.5. Then, substituting in (8) and solving for diam., d = 13.65, and $p = 13.65 \times 1.25 = 17.06$. The apparent slip results from equation (5), substitution in which gives s' = 0.11.

For a **four-bladed propeller** working at the same slip and having the same pitch ratio and with blades of the same proportions and hence with area ratio 0.36, from the values just calculated:

Diameter = 13.65/1.145 = 11.92 ft.; pitch = 17.06/1.145 = 14.9 ft.; rev. per min. = 120×1.145 = 137.4. The speed and slip are the same for both propellers.

In the **selection of propeller characteristics** the following points may be noted. The action of the propeller in producing an augmentation of the resistance of the ship as a result of the defect of pressure produced at and about the stern increases with the diameter and with the amount of rotation impressed on the water and hence is more pronounced with high than with low pitch ratios. This argues for the use of relatively low pitch ratios and high revolutions for ships of full form for which such augmentation represents a more serious item, while for ships of fine form, high pitch ratios with the resulting larger diameters may be employed. The use of low pitch ratios, high revolutions and resulting small diameters must not, however, be carried too far with full forms, as in such case the propeller will work almost wholly in the dead or eddy water close to the stern of such forms, with resulting irregularity and loss of efficiency.

The number of blades is usually three or four, depending partly on the desired total area, partly on questions of vibration. In the absence of special considerations, three blades is standard practice.

The immersion of the tips of the blades when nearest the surface of the water should be not less than 0.2 to 0.3 the diameter. If this distance is less there may be indraft of air, with resulting irregularity of working and probable loss of efficiency.

At excessive speeds of the propeller blade through the water, as may result with high-powered turbine-driven craft, **cavitation** may occur. This is due to the movement of the blade, especially at the tip, at a speed faster than the water under its hydraulic head can close in around it. Cavities are formed in the wake of or about the moving blade, resulting in unsteadiness of motion and in great loss of thrust in proportion to revolutions. It has been found empirically that this condition can be controlled by a suitable value of the thrust per unit of projected blade area. With an immersion of the blade tip of about 12 in., a safe value of the average thrust per sq. in. of projected blade area was found by Barnaby and Parsons to be about 11 lb., while, according to the former, every additional foot of immersion will permit an increase in this value of about ¾ lb. For approximate purposes the actual thrust T in lb. may be estimated by the formula $T = 220H/v$, where H = i.h.p. and v = speed in knots. Above a limiting value of the peripheral speed of about 12,000 ft. per min., cavitation is very likely to develop. Peripheral speeds should therefore be kept below this value and the intensity of thrust should not exceed 11 or 12 lb. per sq. in. of projected blade area.

Thickness of Propeller Blades. The thickness of blades at the tip is determined by the possibility of making smooth, reliable castings. Such thickness, t, Fig. 1, is usually from ½ to ¾ in. The thickness AC at the root is to be determined by the formula given below and a straight line BC drawn. This line is then joined by a smooth curve FD to the thickness value at the tip, thus giving a continuous line CFD representing relative to AB the maximum or mid-blade thickness from root to tip. For the thickness AC, employ the formula

$$t_1 = A\sqrt{Hf/bNn} \tag{9}$$

where t_1 = thickness, in., H = i.h.p.; b = length of section at root, in.; N = rev. per min.; n = number of blades; A = 9 to 12 for bronze or steel and 14 to 17 for cast iron; and f = a factor having the following values for various ratios of p to d:

p/d	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3
f	1.10	1.02	0.95	0.89	0.85	0.81	0.77	0.74	0.72	0.70	0.68	0.66	0.64	0.63

PADDLE WHEELS

Paddle Wheels are of two types: (1) **Radial,** with fixed radial floats, and (2) **feathering,** with movable or oscillating floats so connected as to enter and leave the water as nearly as may be possible in the direction of their own plane.

The **effective diameter** of a paddle wheel is usually taken as the distance between the centers of opposite floats. This is not exact, but sufficiently so for practical purposes. The **pitch circle** is then defined as the circle on this diameter. Let D = diam. of pitch circle, ft.; N = r.p.m.; U = speed in miles per hour, and s = apparent slip ratio. Then $s = (\pi DN - 88U)/\pi DN$. For **feathering floats,** Seaton gives the following formulæ: Number = $n = 60/\sqrt{N}$; area of one float = (i.h.p. $\times K)/n(DN)^3$, where $K = 40 \times 10^6$ for vessels plying in smooth water $= 45 \times 10^6$ for sea-going steamers $= 50 \times 10^6$ for tugs and craft requiring special control.

When working in smooth water the immersion of the top edge need not exceed ⅛ × breadth. For general service at sea the immersion should be about ½ × breadth. The length of the float should be from 2.5 to 3 times its breadth.

Radial Wheels. The area of each float is usually made from 70 to 80 per cent. that for feathering floats. The number may be taken as one for each foot of diameter. The length should be from 3 to 3.5 times the breadth. Under these conditions the slip for radial wheels will be some 5 to 10 per cent. greater than for feathering wheels.

BOILERS

Performance of Scotch Boilers. With coal fuel and natural draft, from 15 to 20 or 25 lb. of coal may be burned per sq. ft. of grate surface per hour. With mechanical draft the combustion may be raised to 30 or 40 lb. or more. The usual ratio of heating to grate surface is 30–40 to 1, and with such ratio and under usual operative conditions, an evaporation of 4 to 8 lb. of steam per sq. ft. of heating surface may be realised.

With an overall boiler efficiency of 70 to 75 per cent. and the usual type of triple-expansion marine engine, an indicated horse power will be developed in the main engines for 2½ to 3 sq. ft. of heating surface, or 10 to 15 h.p. per sq. ft. of grate surface. The weight of Scotch boilers with water and fire-room auxiliaries, all in steaming condition, will usually range from 80 or 90 to 150 lb. per h.p. at the main engines.

Structural Design. The various marine registration societies, as well as the British Board of Trade and the U. S. Treasury Department have laid down extended sets of rules for the structural proportions of Scotch boilers. Below are given excerpts from the rules of the American Bureau of Shipping, which cover the more important items. In any individual case care must be taken to insure the fulfillment of all rules which may be significant for the case in hand for purposes of registry, classification or governmental inspection, and as laid down in the particular codes to which due reference must be made.

Rules for the Construction of Scotch Boilers

Riveted Joints. For circumferential seams, single- or double-riveted lap joints with efficiencies from 0.55 to 0.70 are employed. For longitudinal seams, butt-strap joints are employed with double cover plates and double or multiple riveting in various patterns, and with efficiencies rising to 0.85. For discussion of riveted joints see p. 674.

Pressure, Diameter and Thickness may be connected by the formula: $p = tTe/Df$, where p = pressure to be allowed, lb. per sq. in.; t = thickness of shell, in.; T = lowest tensile strength of any plate in the shell, lb. per sq. in.; e = efficiency of riveted joint; D = mean diam. of shell, in.; and f = 2.05 (2.10), when longitudinal seams are made with double butt straps of equal (unequal) width and the circumferential seams are at least double-riveted.

Bumped Heads. For shells or drums with bumped heads the design may be determined as follows: For heads convex on the outside, $p = 0.4tT/R$; and for heads concave on the outside, $p = 0.2tT/R$, where R = radius in in. to which head is bumped—other notation as above.

Stays. For steel screw stays whose smallest diam. is 1½ in. or less, a maximum tensile stress of 8000 lb. per sq. in. may be allowed. For steel screw stays whose smallest diam. is above 1½ in. and for all other stays, a maximum tensile stress of 9000 lb. per sq. in. may be allowed.

Girders Supporting Combustion Chambers. Let d = depth of girder, W = width of combustion chamber, P = pitch of supporting bolts, D = pitch of girders, L = length of girders, and t = thickness of girders (both plates), all in in., p = working pressure, lb. per sq. in., and C = a factor depending on the number of bolts per girder = 6600 for 1 bolt, 10,000 for 2 or 3 bolts, 11,000 for 4 or 5, 11,500 for 6 or 7, and 12,000 for 8 or more. Then $d^2 = p(W - P)DL/Ct$, or $p = d^2Ct/(W - P)DL$.

Flat Plates. The allowable pressure on flat surfaces $= p = C(t - 1)^2/A$, where p = working pressure, lb. per sq. in., t = thickness in sixteenths of an inch, A = area supported, sq. in., and C = constant having the following values:

1. Plate reinforced with a doubling plate over whole area to be supported, and of thickness not less than one-half that of main plate, and stays fitted with double nuts: for t take 0.8 combined thickness of both plates and C = 160.

2. Stays fitted with double nuts and riveted doubling strips or double nuts and riveted washers, the width of strip or diam. of washer not less than 0.6 pitch of stays and thickness *equal* to that of plate to be supported: C = 277 for doubling strips and 259 for washers.

3. Stays fitted with double nuts and riveted doubling strips or double nuts and riveted washers, the width of strip or diam. of washer not less than 0.6 pitch of stays and thickness *not less than* 0.8 thickness of plate to be supported: C = 238 for doubling strips and 226 for washers.

4. Stays fitted with double nuts and riveted doubling strips or double nuts and riveted washers, the width of strip or diam. of washer not less than 0.6 pitch of stays and

thickness not less than 0.6 thickness of plate to be supported: $C = 199$ for doubling strips and 193 for washers.

5. Stays fitted with double nuts without doubling strips or washers: $C = 160$.
6. Stays screwed through plates and fitted with nuts: $C = 160$.
7. Stays screwed through plates and riveted over: $C = 140$.

For other proportions of riveted doubling strips or riveted washers, the values of the constant may, if desired, be found from the following formulæ. It is required, however, that the diameter of washer or width of doubling strip shall be at least 0.4 the pitch of the stays, and thickness at least 0.5 that of the plate to be supported:

For doubling strips, $$C = 160 + 156\left(\frac{d - 0.3p}{0.4p}\right)\left(\frac{t_1 - 0.4t}{0.6t}\right)$$

For washers, $$C = 160 + 132\left(\frac{d - 0.3p}{0.4p}\right)\left(\frac{t_1 - 0.4t}{0.6t}\right)$$

where d = diam. of washer or width of strip, p = pitch of stays, t = thickness of plate to be supported, and t_1 = thickness of washer or strip, all in in. For boiler fronts in the steam space unprotected from flame in the uptakes, the above values of C must be reduced by 20 per cent.

Thickness of Tube Sheets. The thickness of tube sheets should not be less than that given by the formula $t = 1 + \sqrt{pA/200}$, where t = thickness in sixteenths of an inch, p = pressure, lb. per sq. in., and A = area supported by each stay tube, sq. in.

Angle-iron Stiffeners for Flat Surfaces. Flat surfaces of boilers stiffened with double angle irons riveted on, with thickness of leaf at least two-thirds that of plate and depth at least one-fourth the pitch, may be allowed pressure as given by the formula $p = 200(t - 1)^2/h^2$, where h = pitch of angle irons in in., p and t as before. The spacing of such stiffeners must in no case exceed 16 in.

Angle-iron Stiffeners for Rounded Convex Surfaces. Where single angle-iron stiffeners are used to stiffen the rounded bottoms of combustion chambers, such angles must be efficiently riveted to the plate to be supported, and must have a thickness of leaf $= 0.8t$ and a depth at least equal to $\frac{1}{2}h$. Where such angles are used to stiffen the rounded tops of combustion chambers, they must be of thickness and depth of leaf not less than as specified above for rounded bottoms, and such angles must be carried on thimbles and riveted through with rivets not less than 1 in. in diam. and spaced not more than 6 in. between centers. Rounded surfaces supported in this manner may be allowed a pressure: $p = 700t^2/hD$, where h = pitch of angle stiffeners, in., and D = diam. of circle to which plates are curved; p and t as before.

Compressive Stress on Tube Plates. Where the pressure on top of combustion chambers is borne by the vertical plates of such chambers, the compressive stress on tube plates as found by the following formula or rule must not exceed 12,500 lb. per sq. in.: $S = pWP/2(P - d)t$, where S = stress on tube sheet and p = pressure to be carried, both in lb. per sq. in.; W = width of combustion chamber, P = pitch of tubes, d = inside diam. of tubes, and t = thickness of tube sheet, all in in.

Reinforce for Holes Cut in Boiler Shell. All holes exceeding 6 in. in diam. cut in either the flat heads or circumferential shell of steel boilers, should be reinforced with wrought- or cast-steel rings to compensate for the material removed. In lieu of such a reinforce ring, holes in flat heads may, if preferred, be reinforced by flanging the metal about the hole inward to a depth of not less than ¾ in., measured from the inner surface. Reinforce rings on flat heads must be efficiently riveted to the head, and must have a sectional area not less than 0.8 the section of metal removed, the latter being measured across the shorter axis of the opening.

Reinforce rings on the circumferential shell must be efficiently riveted to the shell, and must have a sectional area not less than 0.7 the section of metal removed, the latter being measured across the hole in a direction parallel to the length of the boiler. Reinforce rings should be of thickness not less than that of plate to which attached.

Corrugated Furnaces circular in section and with corrugations 8 in. from center to center, the radius of outer corrugation being not more than one-half that of the reverse or suspension curve, and with plain parts at the ends not exceeding 9 in. in length, may be allowed steam pressure in accordance with the formula $p = 15{,}000t/D$, where p = pressure, lb. per sq. in., t = thickness of metal, in., and D = mean diam., in. = inside diam. + thickness + 1½ in. For the thickness, the formula becomes $t = pD/15{,}000$.

Water-tube Boilers. Of the many types of water-tube boilers, a certain number have been developed with special reference to the requirements of marine service. Among these may be mentioned the Thornycroft, Yarrow, Mosher, Marine Babcock and Wilcox, and Normand boilers.

In general, such boilers fall into two classes: (1) Those with relatively large and heavy straight tubes expanded into headers or junction boxes and intended for use where the saving of weight is relatively less imperative, and furnishing thus a larger margin for safety and for durability under the normal conditions of use; and (2) those with relatively small and light tubes, either straight or curved, expanded usually into upper and lower drums and intended for use where the saving of weight is imperative in character, as on torpedo boats, destroyers, fancy racing craft, etc. The latter are often termed the **"express" type** of water-tube boiler.

Performance of Water-tube Boilers. With coal fuel and forced mechanical draft, a combustion from 20 lb. per sq. ft. of grate surface to 50 lb. and above may be realized. The ratio of heating surface to grate surface is usually between 40 to 1 and 60 to 1, increasing with the proposed rate of combustion per sq. ft. of grate. This ratio, with an actual evaporation of from 7 to 11 lb. of steam per lb. of coal (decreasing with increase of draft), will give an evaporation of 6 to 10 lb. of steam per sq. ft. of heating surface. This, with good triple-expansion engines or turbines, will give an indicated horse power for 1.5 to 2.5 sq. ft. of heating surface. Such values by no means indicate the limit, and tests have shown an evaporation of 13 to 16 lb. per sq. ft. of heating surface, or practically 1 h.p. per sq. ft.

The weight of water-tube boilers of the heavy straight-tube type is but slightly less than of Scotch boilers of like capacity. Boilers with water and fire-room auxiliaries all in steaming condition will weigh usually from 50 to 90 lb. per i.h.p. with good triple-expansion engines or turbines. With the lighter express types, the weights range from 16 to 30 lb.

MARINE ENGINES

The typical marine steam engine is of the vertical inverted triple- or quadruple-expansion form, and acts on three, four or five cranks spaced at equal or nearly equal angular intervals about the circumference. Compound engines acting on three cranks, or more rarely on two, are also met with.

The **steam pressure** employed for such engines ranges about 200 lb. per sq. in. and lower for those of the compound type, or higher for torpedo boats, destroyers and fast yacht designs. The **number of expansions** ranges from 10 to 15 or higher in accordance with special conditions.

Design of Structural Features

Cylinders. Large marine-engine cylinders are very commonly made with liners, thus securing a working barrel and at the same time a jacket space between outer and inner barrels. For thickness, t, of liner and other cylinder dimensions see p. 762.

The **cylinder end** may be made with either a single or double shell, in either case with supporting ribs. For a single cylinder end, take the thickness $= 0.9t$ to t, and for a double cylinder end $0.8t$ to $0.9t$. The ribs are made of about the same thickness as the end, and of depth 5 to 6 times the thickness. They must be so spaced that the intermediate flat surfaces will fulfill the conditions for strength as given by the formula $t = b\sqrt{p}/100$, where t = thickness of flat plate and b = distance between ribs, both in in., and p = gage pressure, lb. per sq. in.

Cylinder Covers. (See also p. 762.) For flat single-shell cover of cast iron the thickness $= 0.9t$; for conical single cover of cast iron, $0.8t$; for double-shell cover of cast iron $= 0.75t$; for steel covers the form is usually single and the thickness $= 0.5t$, where t is the thickness of cylinder barrel. In any case the surface must be supported by ribs of dimensions and spacing similar to those for the cylinder end.

Pistons (see also p. 763) are usually of cast steel, dished or conical in form and made of a single thickness of metal. The various pistons of a compound or triple-expansion engine should, as a rule, be made of the same total depth. This gives a steep angle to the high-pressure and a flat angle to the low-pressure piston. The latter should not be less than about 1 : 5. The **thickness near boss** may be obtained from the formula $t = 0.004\ (d + 10\sqrt{p})$, where d = diam. in in., and p = maximum effective difference of pressure on two sides, lb. per sq. in. The **thickness near the edge** may be taken about 0.6 that near the boss. The **thickness of** the **wall of boss** for the piston rod should be about the same as that of the piston near boss, as given by formula above. The **height of boss** should be made 1.10 to 1.15 times the diam. of the piston rod (see below).

Piston Rods. The average piston load with suitable empirical factors may be used in the determination of many of the moving parts of the engine. This is defined as follows: If H = i.h.p. developed in one cylinder of engine, L = length of stroke, ft., N = r.p.m., and $2LN$ = piston speed, ft. per min., $= S$, then **mean piston load** is proportional to H/S, and the **diameter** d **of** the **piston rod** (in in.) may be obtained from $d = 5\sqrt{H/S}$.

When the rod is secured by threaded fastenings at the ends, the diameter at the root of the thread should be such as to insure a stress not exceeding 3000 lb. per sq. in. based on *maximum* piston load as defined by pA, where p = maximum difference of pressure between the two sides (lb. per sq. in.) and A = area (sq. in.)

Connecting Rods. The length of the connecting rod is usually from 4½ to 5 times the length of the crank, and between these limits the relation of the connecting rod to the piston rod may be determined on an empirical basis in accordance with the following proportions: Diam. of connecting rod at upper end = diam. of piston rod; and area of section of connecting rod at lower end = 1.2 to 1.3 times that of piston rod.

If the usual form of section is employed with the fore-and-aft dimension nearly constant from top to bottom of rod and transverse dimension increasing as above, then the section modulus for cross breaking at the lower end will be some 40 to 60 per cent. in excess of that at the upper end, and sufficient stiffness and strength will be secured.

Crank Shafts. The design of the crank shaft may be conveniently related to the mean turning moment (M) given by the equation: $M = 33{,}000 \times H \times 12/2\pi N = 63{,}000H/N$, where H = total indicated horse power, and N = rev. per min.

For a **solid shaft,** $D = 1.72\sqrt[3]{eM/T}$, where D = diam., in., M = moment as above, T = shearing stress allowed, usually from 8000 to 10,000 lb. per sq. in., and e = factor intended to relate empirically the mean to the maximum turning moments, and usually with values of from 1.2 to 1.5.

For a **hollow shaft,** $D = 1.72\sqrt[3]{(eM/T)/[1 - (d^4/D^4)]}$ where d = diam. of hole, in., other notation as above.

Only that section of the shafting which is aft of the after crank is required to transmit the entire turning moment. Usually, however, in order to secure uniformity of size in the bearings, all parts of the crank shaft are made of the same diameter.

The diameters of shafting may also be found by the following empirical rules:

Crank Shafts for Compound and Multiple-expansion Engines.

$$C = \sqrt[3]{pD^2L/K[(D^2/d^2) + 2.4]}$$

where C = diam. of shaft, in.; p = gage pressure at boilers, lb. per sq. in.; D = diam. of l.-p. cylinder, in. (if there is more than one l.-p. cylinder, then for D^2 use the sum of the squares of the diameters); d = diam. of h.-p. cylinder, in. (if there is more than one h.-p. cylinder, then for d^2 use the sum of the squares of the diameters); L = length of stroke, in., and K = factor having the following values:

Type of engine	Number of cranks	Angle between cranks, deg.	Value of factor K
Compound	2	90	1450
Compound	2	180 or 0	1200
Compound (3-cyl.)	3	120	1500
Triple	3	120	1700
Triple (4-cyl.)	4	90	1600
Quadruple	4	90	1800
Quadruple (5-cyl.)	5	72	2100

In four-crank engines with unequal spacing of cranks, K must be decreased as follows: If the smallest angle lies between 75 deg. and 85 deg., decrease K by 100; if between 65 deg. and 75 deg., decrease K by 200; and if between 55 deg. and 65 deg., decrease K by 300.

Crank Shafts for Single-expansion Condensing Engines.

$$C = \sqrt[3]{pD^2L/3K},$$

where C, p and L are the same as in the preceding paragraph, D = diam. of cylinder, in. (if there is more than one cylinder, then for D^2 use the sum of the squares of the diameters), and K is a factor having the following values:

Number of cranks	Angle between cranks, deg.	Value of factor K
1	...	1000
2	180	1000
2	90	1225
3	120	1320
4	90	1225

The diameter of the **propeller shaft** should be made equal to 1.06C; the diameter of the **tunnel shafting** may be taken as 0.95C. The diameters of **paddle-wheel crank shafts** may be found by the preceding formulæ and rules according to the type of engine employed. For simple beam engines and smooth-water navigation, however, the value of K may be multiplied by 2.5, while for other types of engine and smooth-water navigation the values of K may be multiplied by 1.7.

Bearing Pressures. Let W = i.h.p. developed by any given cylinder; S = piston speed, ft. per min.; a = length of crank and b = length of connecting rod, both in in. Then, **area of cross-head slipper** in sq. in. = 33,000 Wa/B_1Sb, where B_1 may vary from 15 to 20. In multi-stage engines all **guides** are made equal in area. The area for taking the load when backing may be made from 0.50 to 0.70 that for going ahead. See also p. 767.

The **projected area of engine bearings** in sq. in. = 33,000 W/B_2S, where

B_2 may range from 240 to 300 for the **cross-head pin**, from 100 to 120 for the **crank pin**, and from 65 to 80 for the **main bearings**.

The **area of the thrust bearing** in sq. in. = 220 H/B_3U, where H = total i.h.p., U = speed in knots, and B_3 ranges from 50 to 70.

The Problem of Balance

Unbalanced Forces in Ships. A ship is an elastic structure with natural principal and secondary periods of vibration and will respond in varying degrees to unbalanced forces in the engine. In case the periodicity of the applied disturbing forces agrees or nearly agrees with the natural period of the ship, vibrations of a serious character may develop. Two **types of disturbing action** are noted:

1. The resultant of the forces due to reciprocating and rotating parts, which with vertical engines have maximum values in the vertical direction and usually produce maximum disturbance in the same direction. These forces tend to throw the ship into **lateral** (usually vertical) **vibration,** either in a primary or some higher period.

2. A rocking moment tending to rock the engine bed plate in a vertical fore-and-aft plane, due to the periodic application of the forces (1) at any one instant with components in opposite directions and thus developing periodic **rocking moments.** The maximum values of such moments are usually with the forces vertical, and hence the induced vibrations are usually vertical in direction.

The forces (1) will be most effective when they are applied near the middle of a loop of the vibrating system which the ship may be considered as constituting. The rocking moment (2) will be most effective when applied near a node of the same vibrating system.

Means Available for Balancing. The following means are available for reducing the effects due to the action of these unbalanced forces:

1. The application to each **crank** of **counterbalance weights** to counteract the vertical first-order forces.

2. The **variation of** the fore-and-aft **distances between crank centers** for the purpose of controlling or reducing the resultant rocking moment, the individual forces of which are assumed to be applied at the crank centers. Usually, the best results are realized by such adjustment of weights as shall place the smaller forces at the ends and the greater forces between. Also, with four cranks, the two inner cylinders should be placed as near together as possible, and the outer ones in such position as to secure the best balance of moments.

3. The **variation of crank angles between** the **successive cranks.** This factor is of special significance in four-crank engines, and in particular with reference to the entire system of forces, including those due to the valves and valve gears. It is found in such case that a marked reduction in the final resultant forces and moments may be realized by suitable spacing of the cranks, departing in some cases 20 or 25 deg. from the conventional 90-deg. arrangement.

In general, it must be recognized that no combination of factors or arrangements as noted in (1), (2) and (3) will give a complete balance. The primary forces alone may be thus balanced, but not those of higher orders. The so-called Yarrow-Schlick-Tweedy system of balance takes full cognizance of these various forces and corrective means (see *Trans. Inst. Naval Architects*, 1893, 1894).

Auxiliaries

Condenser Surface. Surface condensers alone are used in salt-water navigation and are the general rule in all cases. The amount of surface is usually 1 to 1.5 sq. ft. per i.h.p. of the main engines, corresponding to a heat transfer of 200 to 250 B.t.u. per min. per sq. ft. of surface. The **amount of condensing water required** may be taken from 30 to 60 times the amount of feed water or from 60 to 120 gal. per h.p.-hr., (= 1 to 2 gal. per min. per i.h.p.). For cold water and moderate vacuum, the amount will range toward the lower figure; for high vacuum, especially with turbine drive, or with warm water, or both, the amount will range upward toward the higher value.

The **volume displaced by an air-pump plunger** may be taken as follows:

Displacement volume in cu. in. $= C \times (\text{i.h.p.})/N$,

where N = number of displacing strokes per min.

C = 50 to 70 for a triple- or quadruple-expansion engine with independent air pump.

C = 100 to 120 for a triple- or quadruple-expansion engine with air pump driven from the main engine.

C = 75 to 110 for a compound engine with independent air pump.

C = 150 to 180 for a compound engine with air pump driven by the main engine.

Steam Piping. The size of steam pipes is given by $a = vA/V$, where v and V are respectively the piston-speed and steam velocity in ft. per min., a is the cross-sectional area of the pipe and A the piston area, both in sq. in. V may be taken from 5000 to 8000, smaller values being suited to small pipes and high-pressure steam, and larger values to large pipes and low-pressure steam. The size of low-pressure exhaust pipes may be based on a value of V = 4,000 to 6,000.

U. S. Rules for Thickness of Steel and Copper Steam Pipes. Let p = working steam pressure in lb. per sq. in., t = thickness of pipe in in., and d = diam. of pipe in. in. Then for steel pipe, $t = (pd/10{,}000) + 0.125$; for copper pipe, $t = (pd/6000) + 0.065$.

AERONAUTICS

BY

J. C. HUNSAKER

REFERENCES: G. Eiffel, "La Résistance de l'Air et l'Aviation," Dunod et Pinat, Paris (1912 and 1914). L. Marchis, "Cours d'Aeronautique," Dunod et Pinat. G. H. Bryan, "Stability in Aviation," Macmillan. "Technical Report of the Advisory Committee for Aeronautics," 1909–10, 10–11, 11–12, 12–13, H. M. Stationery Office, London. A. Fage, "The Aeroplane," Griffin, London. S. Huppert, "Leitfaden der Flugtechnik," Springer, Berlin. Reber, "An Outline of the Theory of Ballooning," *Jour. Franklin Inst.*, Oct., 1912. Eberhardt, "Theorie und Berechnung von Motorluftschiffen," Berlin, 1912.

Aeronautics embraces all that pertains to the navigation of the air, and includes the use of aeroplanes or heavier-than-air machines (**aviation**) and air ships and balloons or lighter-than-air machines (**aerostation**). In aviation, sustentation is obtained from the dynamic lift of wings, while in aerostation, sustentation is obtained from the buoyancy of a volume of light gas. In either case, propulsion is by means of propellers producing thrust sufficient to overcome the resistance of the air. **Aerodynamics** is the study of the motion of air and its dynamic effects.

Lifting Power of Balloons. Let V = cubic contents of balloon, cu. ft.; $w'(w)$ = density of balloon gas (atmosphere), lb. per cu. ft.; $a = w - w'$ = lifting power per cu. ft., lb. Then gross lifting power A (lb.) = aV. If W = weight of balloon and equipment (lb.), net lifting power available = $A - W$. Let $s = w'/w$; then for temperature T_0 and pressure P_0 of gas and air, $a_0 = w_0 - w'_0 = w_0(1 - s)$, and for any given gas $(w,' P', T')$ and air (w, P, T) conditions, $a = w\,[1 - s(P'T/PT')] = a_0[PT_0/P_0(1 - s)] \times [(1/T) - (sP'/T'P)]$. Taking P_0 = 760 mm., T_0 = 492 deg. abs. (or 32 deg. fahr.), and w_0 for dry air = 0.0807 lb. per cu. ft., the following values hold:

	w'_0	s	a_0
Coal gas	0.0418–0.0281	0.52–0.35	0.0387–0.0524
Hydrogen, pure	0.00559	0.0692	0.0751
Hydrogen, commercial	0.0936	0.12	0.0687

P' is practically always equal to P; T' is often 30 or more degrees higher than T, due to heat from the sun's rays.

Taking gas and air temperatures at 32 deg. fahr., the **height** to which a **balloon will rise** = h (ft.) = 60,350 log (W/a_0V). If V is constant, a decrease of W by an amount W' (*i.e.*, throwing out ballast) will result in the additional ascent h_1 (ft.) = 63,050 log $[1 - (W'/W)]$. For $W' \leq 0.1W$, $h_1 = 26{,}240\ W'/W$, or h_1 = 262 ft. for each per cent. W is lightened. For any values of T' and T, the height to which the balloon will ascend $= h' = h - 53.4(t - 32) + 26{,}240\ [s/(1 - s)] \times [(T' - T)/T]$, where $t = T - 460$. An increase of 1 deg. fahr. in the air temperature will increase the altitude about 55 ft. and the lifting power 0.2 per cent., and *vice versa*. If the gas temperature T' be assumed as 572 deg., the altitude will be approximately increased as follows for each deg. fahr. difference between T' and T: Coal gas (s = 0.435), 37.4 ft.; hydrogen, pure, 3.64 ft.; commercial hydrogen, 6 ft.

Air ships or dirigible balloons are provided with air bags (**ballonets**) within their hulls for regulating the height of ascent. These are pumped full before ascent, emptied while ascending, and refilled when it is desired to descend. The total cubic contents of these air bags when filled = $V' = mV$. Values of m for various heights to which it is desired the air ship should rise, are as follows:

Height, ft.	1,000	2,000	4,000	6,000	8,000	10,000
$m = V'/V$	0.04	0.075	0.141	0.20	0.26	0.318

RESISTANCE OF BODIES OF EASY FORM

Stream-line Bodies. A body of "easy" form (*e.g.*, resembling a fish or torpedo) when moved through air or water at ordinary speeds makes little disturbance, and no turbulent wake is left behind. Air for ordinary transportation speeds is practically incompressible. The resistance to motion through air for such bodies is almost entirely frictional. A body of such easy form that the only resistance to its motion is frictional, is called a "**stream-line body**;" and the motion of the fluid caused by its passage is a steady **stream-line motion.** In practice, the term stream-line body is applied to bodies which give rise to an approximate stream-line motion. The nose of the body may be fairly blunt, with the maximum section well forward of the middle. The after body is very fine. The ratio of length to maximum diameter should be from 4 to 6.

Viscosity Resistance. The viscosity of a fluid is a measure of its strength in shear. Air is a very viscous fluid and, further, tends to adhere to a solid body immersed in it. If this body be in motion a fluid shear or viscous drag is set up. The drag so produced is felt as a resistance to motion R_v, which is a function of the coefficient of kinematic viscosity of the fluid μ/d, the area in shear A_s, and the relative velocity V. $R_v = a(\mu/d)^{0.5}A_s^{0.75}V^{1.5}$, where a is a constant for a given geometrical form.

Density Resistance. In a real fluid, except for very low velocities, some momentum is always imparted to particles of fluid near the moving body. This may be manifest as a wake left behind or as small friction rollers next the skin, and represents work done by the body and hence the overcoming of a resistance to motion which is called density resistance. Its magnitude is given by $R_d = bdAV^2$, where d is the density of the fluid, A some area of the body (usually its projected area normal to the wind), V the relative velocity, and b a constant for a given geometrical form.

Skin Friction (or total resistance of a thin plane surface moving edgewise). In air, a thin plane surface (or board) moving edgewise experiences a viscous drag and also considerable density resistance. For smooth surfaces not longer than 16 ft., Zahm's experiments on air friction lead to the following empirical formula:

$$R = R_v + R_d = 0.0000077 8L^{0.93}V^{1.86}b,$$

where R = resistance for one side of board, lb.; L = length in direction of wind, ft.; b = width, ft.; V = velocity, ft. per sec. The effect of small changes in temperature and barometer are here neglected.

Total Resistance. For solids of revolution of "stream-line form" there may be more or less turbulence in the wake, indicating an augmentation of density resistance beyond that of a plane surface. The degree of turbulence set up is inversely as the excellence of the shape from an aerodynamic standpoint. The total resistance is given by

$$R = R_v + R_d = K_1 d\,(\mu/d)^{2-n}\,A^{n/2}V^n$$

where K_1 is a constant, n an exponent to be found by experiment for every geometrical form, and A is some area on the body. The ratio μ/d is called the coefficient of **kinematic viscosity,** and may be considered nearly constant for air at ordinary temperatures and pressures. The above expression then reduces to $R = KA^{n/2}V^n$. The value of n for six air-ship hulls tested both in air and in water at the National Physical Laboratory, England, varies between 1.84 and 1.96 in an irregular manner, with a mean value of 1.86. The tests are summarised in Table 1, in which K is computed from $K = R/A^{0.93}V^{1.86}$, where R is resistance of the hull (lb.), A the surface area (sq. ft.), and V the velocity (ft. per sec.). The values of K for a round-end and blunt-end cylinder are also given to show the large resistance coefficient for the latter shape.

Table 1. Air-ship Shapes*

Shape

Model	Beta	Gamma	B.F. 36
L/D =	4.18	5.15	5.83
K =	0.0000164	0.0000165	0.0000142

Shape

Model	B.F. 32	Lebaudy	Cylinders	
L/D =	6.0	8.5	7.0	8.0
K =	0.000014	0.0000124	0.0000693	0.0000138

* All values of K are from the British Report (1912–13) except for cylinders (Eiffel 1914). L is the length of the hull.

NOTE. In this and subsequent tables of this section, constants involving density refer to standard conditions, *i.e.*, dry air at 15 deg. cent. (59 deg. fahr.) and 760 mm. (29.92 in.) barometer.

Resistance of Wires and Cylinders. The resistance of long wires or cylinders stretched across the wind increases with the velocity, but with a power less than the square. Consequently, in the formula, $R = KLDV^2$, the coefficient K is not strictly a constant, but increases for high speeds and large cylinders. L is length of wire (ft.), D diameter (ft.), V speed (ft. per sec.), and R resistance (lb.). The variation of K with the product DV is given in Table 2. The coefficient for stranded rope and cable may be taken 20 per cent. greater than the corresponding value for smooth wires and cylinders. The variation of resistance of a wire with varying inclination to the wind is given below.

Inclination of wire to wind, deg.	30	40	50	60	70	80
Per cent. of normal resistance	20	37	54	72	87	97

Table 2. Wire Resistance Coefficients, K

(K = Resistance in lb. per ft. run $/DV^2$)

DV in ft.-sec. units	0.5	1.0	1.5	2.0	2.5	3.0
K	0.00120	0.00130	0.00137	0.00141	0.00143	0.00145

Resistance of Struts and Bars. To keep head resistance a minimum, structural members of an aeroplane are given a stream-line section. A good

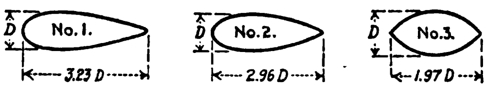

FIG. 1.—Aeroplane Strut Sections.

section is found to have less than one-eighth the resistance of a cylinder with long axis across the wind. The coefficient K for such struts as are used in biplanes decreases with increase in the product DV, where D represents the maximum thickness of the strut section in feet. Values of K for the three struts shown in section in Fig. 1 are given in Table 3. A strut inclined 30 deg. to the wind is there shown to have its resistance materially reduced if it be of poor shape, while the resistance of a good shape is not affected.

Table 3. Strut Resistance Coefficients, K

(K = Resistance in lb. per ft. run/DV^2)

Strut	DV in ft.-sec. units									
	1	2	3	4	5	6	7	8	9	10
No. 1. Normal.........	.00047	.00030	.00022	.00020	.00018	.00017	.00016	.00015	.00015	.00015
No. 2. Normal.........	.00055	.00034	.00022	.00017	.00016	.00014	.00014	.00014	.00013	.00013
No. 2 at 30 deg.........	......	......	......	.00015	.00014	.00013	.00013	.00012	......	......
No. 3. Normal.........	.00084	.00083	.00081	.00080	.00078	.00076	.00073	.00069	.00066	.00062
No. 3 at 30 deg.........	.00062	.00061	.00060	.00058	.00056	.00053	.00050	.00046	.00041	.00036

Struts give less resistance when more elongated, but a thin strut is structurally weak. In aeroplanes a strut of depth-to-thickness ratio between 3 and 4 is used. Fig. 2 shows the effect of elongating a strut. The coefficients K_x refer to a value of $VD = 2.5$. For larger values of VD, the coefficients will be reduced as in Table 3. The "fineness ratio" is the ratio of depth to maximum thickness of the cross-section. A good cross-section is shown in Fig. 2. The relative thicknesses at each tenth of the depth, starting from the front, are 0.75, 0.94, 1.00, 0.99, 0.96, 0.91, 0.81, 0.69, 0.54, and 0.

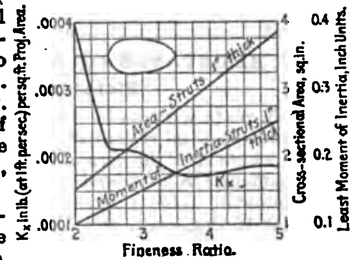

Fig. 2.—Effect of Lengthening an Aeroplane Strut.

Resistance of Miscellaneous Objects. The objects in Table 4 are generally supposed to give rise to such turbulence that density resistance preponderates, and the resistance in pounds may be represented by $R = KAV^2$, where A is the area (sq. ft.) of the projection of the object on a plane normal to the wind. An aeroplane **radiator** of the honeycomb type may be taken as having a resistance of one-half that of a flat plate normal to the wind and of the same projected area.

Table 4. Resistance of Miscellaneous Objects (Eiffel)

Object	K	Limits	Attitude	Object	K	Attitude
Sphere............	0.000209	$VD > 32$		Farman wheel* (covered in)..	0.00061	
Hemispherical shell.	0.0018	$VD > 11$		Cone, closed base........	0.00061	60°
	0.00038	$VD > 22$			0.00040	30°
Circular disk......	0.00123			Cone, hemispherical end.	0.000192	30°
Farman wheel* (uncovered)	0.00124	....			0.000104	30°

* Ratio D/t = 7.9.

Resistance of Aeroplane Bodies. The resistance of such an unsymmetrical form as an aeroplane body is estimated from wind-tunnel tests on small models, on the assumption that $R = KAV^2$, where A is the area (sq. ft.)

projected on a plane normal to the wind. Table 5 gives the values of K for three of the best bodies in present use. No. I is a monoplane body, No. II a "tractor" biplane and No. III a "pusher" biplane body. It is to be noted that the resistance of bodies I and III includes that of a propeller not turning. Nos. II and III include the resistance of the heads of the personnel. The value of K given should be less for full size and high speed, but due to the blast from the propeller this saving in resistance should not be counted on.

Table 5. Resistance of Aeroplane Bodies

Body............			
	Deperdussin	B.E. 3	Farman
Speed, ft. per sec..	92	30	32.8
Length/max. depth	5.6	7.35	3.2
$K = R/AV^2 =$....	0.000541	0.000336	0.000397
Authority........	Eiffel, '14	Brit. Rep., '13	Eiffel, '14

RESISTANCE OF SHAPES PRESENTING AN EDGE

Plates in Normal Presentation

A thin plate with its face normal to the wind has a turbulent wake in its rear, and the total resistance (almost wholly density resistance) is R(lb.) $= KAV^2$, where A is the area of plate (sq. ft.) and V the velocity (ft. per sec.). The coefficient K is found to increase slightly with the size of the plate. The following table gives values of K for square (or circular) plates (air taken at 15 deg. cent. and 760 mm.).

Side (or diam.) of plate, ft.	0.5	1.0	2.0	3.0	4.0	5.0	10.0
K.....................	0.00125	0.00133	0.00146	0.00148	0.00150	0.00152	0.00152

Aspect Ratio is the ratio of the span of a rectangular plate or wing measured across the wind to the depth measured in the direction of the wind. For high aspect ratios the value of K increases as shown below. The coefficient for a square plate of the same area is taken as unity.

Aspect ratio............	1.0	1.5	3.0	6.0	10.0	14.6	20.0	30.0	50.0
K....................	1.0	1.04	1.07	1.10	1.15	1.25	1.34	1.40	1.47

Inclined Surfaces

When any thin plane or cambered surface is moved through the air so that the direction of motion makes a small angle with the lower side, the fluid motion resulting is unsymmetrical and the resultant pressure of the air is very nearly normal to the plane of the surface. On top of a wing, the flow of air is turbulent and the entire upper surface is under suction. This suction produces a lift which in good wings amounts to more than ⅔ of the total lift. Skin friction on an inclined surface is present only on the face or lower side where there is rubbing, and may generally be neglected. The total resultant force R in pounds for a velocity V ft. per sec. of a wing of S sq. ft. area is given by the formula: $R = K_iSV^2$, where K_i is a coefficient which varies in an irregular manner for different shapes of wing and for different attitudes of the same shape. Since R is a density resistance the coefficient K varies with the density of the air, which is here assumed to be at 760 mm. and 15 deg. cent. The vertical component of R is the **lift** L in pounds, or $L = K_ySV^2$, and the horizontal component of R (along the direction of the wind) is the resistance to motion called **drift**, or $D = K_xSV^2$. Hence $K_i^2 = K_x^2 + K_y^2$, and the direction of the resultant force is defined by the angle between its line of action and the wind, or $\theta = \tan^{-1}(K_y/K_x)$. The coefficients K_y and K_x are called lift and drift coefficients and are characteristic of various surfaces. The angle of incidence is defined arbitrarily as the acute angle between the wind direction and the lower chord of a section of the wing cut by a vertical plane parallel to the wind direction. K_y and K_x vary with the angle of incidence.

Flat Plates. For a smooth, thin flat plate Table 6 gives K_y and K_x for various values of the angle of incidence i and for different aspect ratios. Note in this table that the lift for each plate increases with the angle until a critical angle is reached, called the "**burble point,**" where the lift begins to decrease. For aspect ratio 9 the burble point is about 20 deg., while for aspect ratio 1 it is 35 deg. Rudders should therefore be given a low aspect ratio if large angles are to be used. The lift-drift ratio is not much improved for flat plates by high aspect ratio.

Table 6. Lift and Drift Coefficients for Various Angles of Incidence and Aspect Ratios

i (deg.)	Aspect ratio									
	1		2		3		6		9	
	K_y	K_x	K_y	K_x	K_y	K_x	K_y	K_x	K_y	K_x
3	......	....	......	....	......	....	.00026	.00006	......	......
5	.00021	.000062	.00023	.000025	......	....	......	......	......	......
6	.00045	.000087	.00034	.000046	.00042	.000055	.00051	.00008	.00063	.00012
9	......	....	......	....	......	....	.00077	.00014	......	......
10	......	....	.00057	.00010	.00065	.00013	......	......	.00081	.00016
15	......	....	......	....	......	....	.00095	.00025	......	......
20	.00097	.00034	.00115	.00042	.00098	.00036	.00092	.00034	.00098	.00033
25	......	....	.00083	.000415	.00090	.00042	......	......	......	......
30	.00137	.00081	.00082	.00052	.00090	.00052	.00095	.00056	.00098	.00059
40	.00113	.00096	.00078	.00068	......	....	......	......	......	......
60	.00065	.00102	.00058	.00101	.00065	.00101	.00065	.00102	.00065	.00102

Curved Wings. Aeroplanes obtain a maximum K_y/K_x ratio by the use of wings of curved or arched section. The lift of a good wing is more than double that of a plane, and the K_y/K_x ratio is 18 or 20 as against 6 or 7 for a plane. The K_y/K_x ratio of curved wings is much improved by high aspect ratio. The following table shows the effect of aspect ratio for wings of usual form. An aspect ratio 6 is taken as the standard of comparison. The gain in lift-drift ratio is obtained without change in lift by a decrease in drift at high aspect ratios.

Aspect ratio..................	3	4	5	6	7	8
Drift........................	1.28	1.18	1.08	1.00	0.92	0.89

Camber of the upper surface of a wing is expressed as a fraction of the chord of that surface. In practice, a decrease of camber below 0.05 or increase of camber much above 0.08 is disadvantageous. For the incidence giving maximum lift-drift ratio, the lift for camber 0.08 may be twice as great as for camber 0.05, but the lift-drift ratio is diminished 27 per cent. Also, the burble point comes at 6 deg. instead of about 16 deg. Camber of lower surface made by hollowing out is found to have only a slight effect on burble point and lift-drift ratio. The lift at any angle is increased about 17 per cent. when a flat surface is hollowed out to a camber of 0.06. Camber of lower surface is not very important, and if excessive makes a wing structurally weak. The maximum ordinate of the upper surface should be placed about ⅓ of the chord from the leading edge.

The wing sections whose camber and thickness are given in Table 7 are shown in Fig. 3. Eiffel No. 32 is suitable for a racing machine, having a lift-drift ratio of 20 for small angles. R. A. F. 3 gives a very large lift at large angles and is suitable for a weight-carrying aeroplane or sea plane. R. A. F. 6 is the wing used on British army biplanes and has the great structural advantage of being thick enough to permit the use of large wing spars. Coefficients for these wings are shown in Fig. 3 and refer to an aspect ratio 6. The corresponding angles of incidence are marked along the curves.

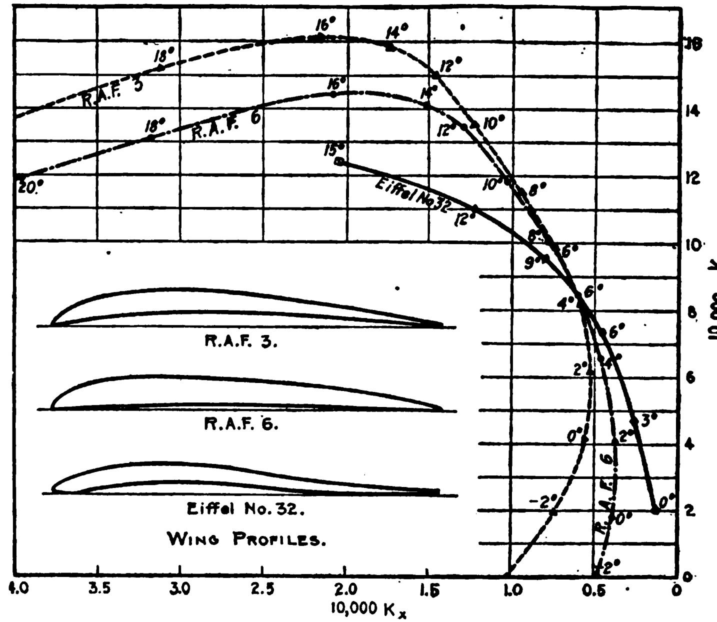

Fig. 3.

Table 7. Cambers and Thicknesses of Wing Sections in Fig. 3
(The camber A is that of the lower surface of the wing. t = thickness. All quantities are expressed as decimal fractions of the chord)

		Distance from leading edge of wing												
		0	0.025	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Eiffel No. 32	A....	0.01	.....	.....	0.007	0.023	0.029	0.027	0.020	0.011	0.005	0.003	0.0	0.003
	t.....	0.0	.....	.....	0.047	0.051	0.050	0.045	0.039	0.033	0.028	0.021	0.015	0.007
R.A.F. 3	A....	0.0	.....	0.009	0.016	0.024	0.029	0.032	0.031	0.026	0.021	0.014	0.006	0.0
	t.....	0.008	0.030	0.035	0.048	0.060	0.059	0.053	0.047	0.043	0.035	0.027	0.016	0.007
R.A.F. 6	A....	0.0	.....	0.002	0.004	0.007	0.008	0.007	0.005	0.004	0.003	0.002	0.001	0.0
	t.....	0.005	0.032	0.042	0.056	0.067	0.068	0.068	0.066	0.061	0.054	0.042	0.026	0.005

Center of Pressure. The intersection of the resultant force with the plane of the chord of the lower surface is called the center of pressure (c.p.).

This center varies with the angle of incidence in a characteristic manner for every wing shape. The center of pressure is located for a given angle of incidence by giving its distance from the leading edge in per cent. of the chord length. Fig. 4 shows the c.p. motion for a flat plate and for the Eiffel No. 32 and R. A. F. 6 wings. The curve for the R. A. F. 3 wing is very close to the R. A. F. 6 wing. The motion of the flat plate is such that for small angles the wind lifts the front edge and at larger angles the rear edge. This gives longitudinal stability if the center of gravity is at an intermediate point. On the contrary, the c.p. motion of the R. A. F. 3 and R. A. F. 6 is unstable, which is the case for most cambered surfaces. The artifice of a reversed curvature near the trailing edge, as in Eiffel No. 32, is seen to hold the c.p. motion nearly constant. This is an advantage, but is obtained at the expense of lift at large angles.

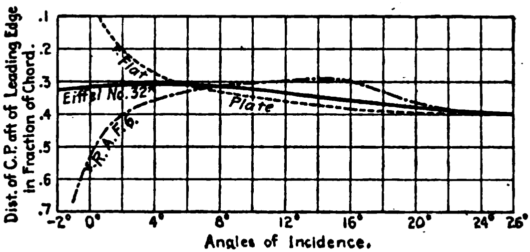

FIG. 4.

Speed and Scale. The coefficients of wings are obtained from wind-tunnel tests on small models. The drift coefficients are lower for the full-scale machines, but the lift coefficients are practically unaffected. By using model coefficients without correction the designer is on the safe side by about 10 per cent.

Interference

Biplane Coefficients. When a pair of superposed wings is used, as in a biplane, the suction on the back of the lower wing is reduced by the interference of the upper wing, and the lift of the lower wing may be reduced 30 per cent. The lift of the upper wing is not affected. The drift of a biplane is somewhat less than that of a monoplane, but the reduction is not important compared with the loss in lift. The **gap** (Fig. 5) is the perpendicular distance between the wings of a biplane. It is usually given in terms of the chord length. To obtain biplane coefficients from the known coefficients for a monoplane wing, use the following multiplying factors.

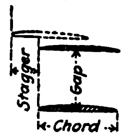

FIG. 5.

Ratio of gap to chord	0.40	0.80	1.00	1.20	1.60
Factor for K_y	0.61	0.76	0.81	0.86	0.89
Factor for K_y/K_x	0.75	0.79	0.81	0.84	0.88

The usual case in practice is for biplane wings to be equal in area, parallel, and with gap equal to the chord. A **staggered biplane** (Fig. 5) is a biplane in which the upper wing is not directly over the lower wing. A slight improvement may be made by making the lower wing, which is less effective, of smaller area than the upper. In this case the coefficients are to be obtained by considering the overhanging portions of the upper wing as monoplane surface.

The center-of-pressure motion for ordinary biplanes is not sensibly different from that for a monoplane of the same section. Efforts to make center-of-pressure motion zero by inclining the upper and forward wing of a staggered biplane 3 or 4 deg. away from the lower wing involve a loss of lift-drift ratio of 5 per cent.

A horizontal fixed **tail** surface is usually flat, of area about $\frac{1}{10}$ the wing area, and placed to the rear of the after edge of the wing a distance equal to the half-span of the wings. Such a tail, if placed parallel to the axis of the body (or propeller shaft), will be inclined to the wing chord by 3 or 4 deg., making what is known as the "longitudinal vee," which insures longitudinal stability. The tail surface is in the wash of the wings, which deflect the air downward by an angle $\alpha = 1 + \frac{1}{2}i$, where i is the incidence of the wings. This air, when the body is horizontal, strikes the tail on the top with an incidence α, and from a knowledge of the coefficients of the tail surface the moment of the air forces about the center of gravity of the aeroplane can be computed. Combine this moment with moment of the wings. The resultant moment should provide a righting moment tending to turn the machine back to the horizontal if it tends to pitch.

THEORY OF AEROPLANE DESIGN

Lift and Power Equations. The aeroplane in horizontal flight is sustained in the air by the lift of the wings. The lift $W = K_ySV^2$, where W is the total weight (lb.), K_y the wing coefficient for a particular incidence, S the effective area of the wings (sq. ft.), and V the relative velocity of the wings through the air (ft. per sec.). In order to maintain the speed necessary for sustentation, the propeller must overcome the resistance to forward motion at this speed. The **horizontal force on the wings** is $R_w = K_xSV^2$. The resistance of the wing bracing, body, radiator, landing carriage, etc., is called "parasite" resistance, since no lift is obtained from these parts. Their resistance is $R_b = K_bV^2$, where K_b is a constant characteristic of the machine and is independent of the incidence.

The **propeller thrust** T (lb.) is then $T = R_b + R_w = (K_xS + K_b)V^2$. If the efficiency of the propeller is e, the **brake horse power required** for flight at speed V is given by: $P = TV/550e = (K_xS + K_b)\ V^3/550e$.

In this equation K_x is the drift coefficient corresponding to K_y of the lift equation, and both are functions of the incidence. The lift and power equations must be satisfied simultaneously by a concordant set of values of W, S, V, P, e, K_b, K_x and K_y. In a design W, S, V, e, K_b are known by assumption or calculation, and it is required to determine P, K_x and K_y for a series of values of V. Since K_x and K_y are functions of i, there are only two independent variables, i and P, to be found.

Weight. The final weight of the machine is estimated from the plans, but a preliminary estimate must be made from similar machines. The useful load the aeroplane is required to carry is the weight of fuel, oil, passengers, etc. This useful weight may be taken as 45 per cent. of the total weight in the air for light monoplanes, and 30 per cent. for heavy military biplanes. In practice, racing machines show 15 lb. of total weight per horse power, military machines 20, and flying boats over 25. The former are said to be "overpowered" and the latter "underpowered."

The **motor weight** is estimated by taking t = duration of flight in hours; P = b.h.p. of motor; a = gross weight per b.h.p. of motor and accessories, lb; c = fuel and oil per b.h.p.-hour, lb.; and tanks and oil pipes = $\frac{1}{6}$ weight of liquid carried. Whence, total supply weight = $\frac{7}{6}tPc$; total motor weight = Pa; and total power-plant weight = $P(a + \frac{7}{6}tc)$.

For rotary air-cooled motors, $a = 3$ and $c = 0.9$; for water-cooled four-cycle fixed-cylinder motors, $a = 5$ and $c = 0.6$, including water, radiator, pump, piping, magneto and all accessories. With these values for a flight of more than 5 hours, the fixed-cylinder motor is the lighter. Figures for a and c are for the best motors of their type in the year 1917.

Wing Weight. The weight of wings may be estimated at about ¾ lb. per sq. ft., not including external struts and wires. Where the latter are included, allow 1 lb. per sq. ft.

The area of wings is roughly such that the air load is from 4 lb. per sq. ft. on large biplanes to 8 on small monoplanes. Above 300 sq. ft. wing area the biplane arrangement is structurally preferable to the monoplane.

The following table shows the upper limits of present practice. The loads there given should be approached with caution.

Area of wings, sq. ft.	150	200	250	300	400	500	600	700
Load per sq. ft. of wing, lb.	9.4	8.1	7.4	6.8	6.2	5.7	5.4	5.1

The following table gives the weights of a typical military aeroplane (125 b.h.p.) in per cent. of the total weight (2438 lb.).

Useful load		Structural weight	
Personnel and equipment	13.1	Fuselage or body	8.2
Gasoline and oil, 6 hr.	19.8	Landing carriage	8.2
Motor weight		Directive surfaces	4.1
Tanks and pipes	3.3	Wings	16.5
Motor and accessories	17.9	Wing bracing	4.0
Radiator (empty)	2.2	Useful load	32.9
Cooling water	1.7	Motor weight	26.1
Propeller and hub	1.0	Structural weight	41.0

Principal Dimensions. The **wings** of a monoplane have an aspect ratio from 4 to 6; for a biplane, 6 to 10 for each wing. From the assumed area S the span of the wings can be estimated. Place the tail surfaces in rear of the rear edge of the wings a distance about equal to half the span. The overall dimensions of the motor fix the **diameter of the body**, and if the motor and propeller are mounted in front, the body will extend to the tail surfaces, giving a length somewhat over ¾ span. The body is placed between the wings of a biplane so that a propeller of 8 or 9 ft. diameter may be swung. The **chassis** or landing-carriage height is fixed by the propeller clearance desired. The wheels of the chassis should be 24 in. or 26 in. for rough ground, with 4-in. tires for machines weighing over 2000 lb. The center of gravity (c.g.) of the whole machine should be near the propeller axis and about 1 ft. to the rear of the wheels of the landing carriage. The gasoline tank should be near the c.g., so that as fuel is consumed balance is not altered.

The **horizontal rudder** is about equal in area to the fixed **horizontal tail** surface. Let the sum of these areas be Q, l the distance of their center from the center of gravity of the aeroplane, S the total wing area, and b the chord length of the wings. Then a coefficient $c = Ql/Sb$ can be used to estimate Q or l from present practice, which indicates that machines which are easily controlled have c between 0.45 and 0.65. The former figure is for fast monoplanes and the latter for slow biplanes. Flying boats have such great wing area that c is often as low as 0.3.

In a similar manner let s be the span of the wings and V the total area of **vertical fin and rudder**. Then $f = Vl/Ss$. The coefficient f has a value of about 0.02. The vertical fin is not in general necessary on long-body machines, but in pusher-type machines when the body extends far in advance of the center of gravity, a fixed fin aft is often fitted to give better steering.

It is rarely more than 25 per cent. of the rudder area. **Ailerons** or **wing flaps** are hinged to the rear edge of the wings so that by pulling down on one side and up on the other, the air forces on the ailerons produce a rolling moment on the machine. Let a be total aileron area, S the wing area and s the wing span. In practice, $a/S = x = 10$ to 20 per cent. The product sx is 650 for hydroplanes and 500 for military aeroplanes. **Warping** the rear outer portion of the wings gives a more powerful control than ailerons, but is structurally undesirable. Six degrees is normal warping.

Characteristic Aeroplane Curves are used to predict the probable performance of the designed aeroplane. From the given corrections to be applied to the K_x, K_y coefficients to take account of biplane gap, aspect ratio, etc., plot curves of K_x and K_y for the selected wing section against incidences i as abscissæ. From the lift equation $W = K_ySV^2$ compute K_y for a series of speeds. For these values of K_y take from the wing-coefficient curves corresponding values of i and K_x. Compute R_w, R_b and T at each speed. The **effective horse power** required at each speed is then e.h.p. $= TV/550$. The efficiency of the propeller is a maximum at full speed and falls off rapidly at low speeds. If P is the b.h.p. of the motor selected, then eP at each speed is the **propeller horse power** (p.h.p.) available. Flight is possible over the range where the p.h.p. exceeds the e.h.p.

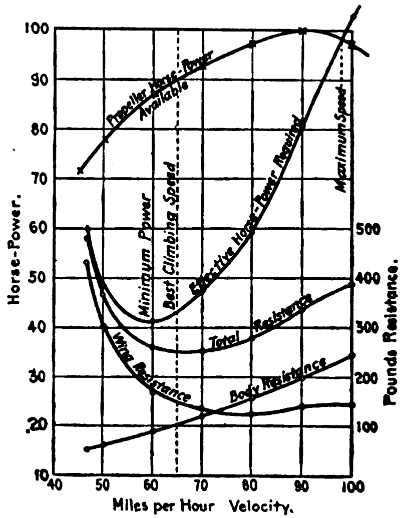

FIG. 6.—Characteristic Aeroplane Curves.

The characteristic curves of Fig. 6 are for the 125-h.p. machine whose weights were given above. The maximum speed is 97 miles per hour, the speed of minimum power required is 60 miles, which is also the minimum safe speed. Speeds between 47 and 60 miles are in the region of reversed controls and not considered safe (see Alex. Sée; "Les Lois Expérimentales de l'Aviation," Paris, 1912, p. 200). The greatest margin of power available over power required is 47 h.p. at 65 miles. This is the **best climbing speed,** for if the throttle be opened out, the machine will climb at a rate $47 \times 33{,}000/W = 650$ ft. per min. A machine whose curves showed no margin of power available for climbing could fly at only one speed. However, it never could get off level ground because it could not climb. Variable speed is obtained by over-powering and by the use of large wings of light loading per sq. ft. and a wing section giving a high value of K_y at the burble point.

Stability

Aeroplane Control. An aeroplane is controlled in flight by impressing on it yawing, pitching or rolling moments by manipulation of the vertical or horizontal rudders, or by use of wing warping or ailerons. This is the **three-rudder system** in almost universal use. Controllability is quite independent of stability. The aeroplane may be made automatically stable by gyroscopic or other devices which actuate the controls mechanically; it is inherently stable if, on deflection by any accidental cause, the aerodynamic forces introduced are such as to produce restoring moments tending to return the aeroplane to its original state of motion.

Stability may be either **static** or **dynamic.** The aeroplane is statically stable if the restoring moments tend to turn the machine back to its original equilibrium position. It is dynamically stable in addition if the oscillation produced by these restoring moments is rapidly damped out. The aeroplane is dynamically unstable if an oscillation tends to increase in amplitude with time. Static stability or **stable equilibrium** is necessary to obtain dynamic stability in oscillation, but is no guarantee of obtaining it. For small oscillations, **symmetric or longitudinal stability** may be considered independently of **asymmetric or lateral stability.** The former deals with **pitching,** and involves change of forward and upward components of velocity and angular motion of pitching. Lateral stability involves changes of lateral component of velocity or side-slipping (**"skidding"**) and angular velocities of roll and yaw. The term **"directional" stability** as used for ships to designate a tendency to persist on a course without yawing, is intimately bound up with the rolling and side-slipping and cannot be considered separately.

The righting moments due to the combination of wings and tail (see p. 1254) may be plotted on angle of pitch. If this curve of statical righting moments is too steep, the machine is stiff and difficult to control; it is too stable. The **righting moment** at any angle of pitch must always be less than the moment which the pilot may produce with his horizontal rudder. Dynamic stability in pitching is practically always secured when the machine is statically stable longitudinally. A large tail makes pitching slow and easy to control.

Rolling does not in general produce a marked center-of-pressure motion on the wings, and hence there is no restoring moment. However, if an aeroplane tilts to one side, it starts to skid or side-slip toward the low side. If a **fin** or its equivalent be placed above the center of gravity, a **lateral restoring moment** will be produced as soon as side-slipping starts. This will correct the roll and stop the side-slip. Such a fin is made by raising the wing tips or by broadening the tops of the struts. A strong moment is produced in an ordinary machine by inclining the wings upward and outward 2 or 3 deg. A greater dihedral angle than this (174 or 176 deg.) may make the aileron control too weak, and may accentuate the rolling oscillation.

When the aeroplane side-slips, if there be more vertical surface forward of the center of gravity than aft, a yawing couple is produced which tends to throw the nose of the machine up and away from relative wind; if the fin surface aft preponderates, the aeroplane may dive on a side-slip. The latter is preferable to the former condition, but as near a balance as possible should be attempted in the longitudinal distribution of fin surface. In some machines it is necessary to fit a small fin on the tail to balance a forward projecting body, skids, covered wheels and propeller. Curves of rolling and yawing moment are best obtained from wind-tunnel tests on a complete model.

When the side-slip resulting from a roll creates a righting moment without introducing an appreciable yawing moment, **static lateral stability** is obtained. If the fin surface, or the equivalent, be excessive, the aeroplane rolls badly in side gusts and may be difficult to control when landing. Too much fin aft causes the machine to turn from her course to head into side gusts. **Dynamic lateral stability** is not assured by the existence of static stability. An oscillation in roll alone is heavily damped by the wide-spreading wings, and the majority of modern aeroplanes if given proper fin disposition are unlikely to permit an oscillation in roll to increase. If the vertical rudder be large it acts as a fin and causes the machine to yaw toward the low side on a side-slip, or to dive. The first effect of this angular velocity in yaw is to cause a greater lift on the outer and more swiftly moving wing than on the inner one. The difference in lift on the two wings is an upsetting moment, tending to augment the side-slip. The result may be a **"spiral dive"**—the most common form of lateral instability.

A tendency toward spiral instability may be removed by the following alterations in a machine: (1) Provide a slight preponderance of fin surface above the center of gravity. (2) Reduce the rolling moment due to angular velocity of yaw by shortening the span of the wings. (3) Correct the undesirable preponderance of fin surface aft by fitting a smaller rudder or by fitting fin surface forward. (4) Damp any oscillation in yaw by balanced fin surface forward and aft. Thus a deep flat-sided body with a fin aft and another forward will damp an oscillation in yaw. A panel between the wings of a biplane, small upward dihedral of wings, or wings swept back in plan like an arrowhead are equivalent means of obtaining a righting moment. A sweep back of about 15 deg. for each wing to right and to left is equivalent to the small dihedral recommended. Neither expedient involves any appreciable loss in efficiency of wing.

For the mathematical theory of the oscillations of an aeroplane and the criterion for dynamic stability based on analysis of the differential equations of motion, see "Stability in Aviation," by G. H. Bryan, MacMillan, London, 1911, and for extension of the theory, determination of aerodynamic constants, and application to a Blériot monoplane, see Report No. 77, "Technical Report of the Advisory Committee for Aeronautics," 1912–1913, H. M. Stationery Office, London.

The above discussion of stability assumes constant power from the motor to maintain speed. In case of sudden stoppage of the motor a sudden pitching moment is produced if the center of gravity be higher or lower than the propeller axis. In **hydro-aeroplanes** with heavy floats the c.g. is low but the propeller must be high to clear the water. Such machines tend to nose up if the motor stops. Headed up the machine cannot climb without power, and speed would be quickly lost if the pilot did not head down into a glide at once. A stable machine may be "stalled" by an inexperienced pilot, but after a fall of sufficient height it will take its natural gliding path.

Air Propellers

Theory. Air propellers are commonly designed to fit a given motor and aeroplane by application of the theory of similitude to a propeller whose characteristics are known. The characteristics of a given propeller are the following quantities: T, thrust (lb.); Q, torque (ft.-lb.); b.h.p., brake horse power; t.h.p. thrust horse power; e, efficiency = t.h.p./b.h.p. Each of these five quantities is given by an expression involving density of air d, diam. of propeller D, rev. per sec. N, and speed of advance of aeroplane V. For a family of **geometrically similar propellers**, in which size, speed, and revolutions vary,

$$T = dN^2D^4f_1(V/ND) = K_tN^2D^4, \quad Q = dN^2D^5f_2(V/ND)$$
$$\text{b.h.p.} = \pi QN/550 = dN^3D^5f_3(V/ND)$$
$$\text{t.h.p.} = TV/550 = dN^3D^5f_4(V/ND), \quad e = f_5(V/ND).$$

Here f_1, f_2, f_3, f_4 and f_5 are unknown functions of the **slip function** V/ND, and $K_t = df_1(V/ND)$ is the **thrust coefficient**. All the characteristics of a family of propellers vary with the slip function, and from any two of them the rest may be calculated. Knowing the thrust and efficiency for one propeller for a series of values of V/ND, all the characteristics may be computed for a larger or smaller propeller geometrically similar to it, for the same values of V/ND.

The thrust coefficient and efficiency calculated from tests on a particular propeller may be applied to similar propellers for corresponding values of slip function by use of the above equation for thrust. The error should be within 3 per cent.

Construction. Air propellers are usually made of about six black-walnut or mahogany planks glued together. Two blades are commonly employed, but where great power on small diameter is required, four blades may be desirable. The thrust of a four-bladed propeller may be taken as nearly twice that of a two-bladed propeller of the same diameter and blade shape. If the blades are not broad, the efficiency is the same for two and four blades. As a general rule, the blades of a two-bladed propeller may be given a rectangular plan form of width ⅙th of radius. For four-bladed propellers the width should be about ⅛th of the radius. Experiments show that the plan form of the blade has little effect on efficiency.

The sections of the blades should be similar to the aeroplane wing section

R. A. F. 6, Fig. 3, near the tip. As the hub is approached, the blades must be thickened up for strength. The useful part of the blade is the outer three-quarters, hence the size of hub is not important and the blades may be thickened up in an arbitrary manner near the hub.

Strength. Any section of a blade is subject to centrifugal tension and to a bending moment due to thrust. A factor of safety of six should be used. The bending moment due to thrust may be computed by considering that the thrust of the blade is a single force acting at a point ¾ of the radius from the hub.

Efficiency. The efficiency of good aeroplane propellers at best speed V is about 80 per cent. The efficiency should usually be a maximum for the normal speed of the aeroplane. At 10 per cent. excess speed, for the same r.p.m. the efficiency of an 80-per cent. propeller may be 70 per cent. or less. At 10 per cent. below normal speed the efficiency may be 78 per cent. At 50 per cent. of normal speed, the efficiency may be about 50 per cent.

Thrust. A propeller is designed to give a thrust equal to the resistance of the aeroplane at normal speed, and to turn at the r.p.m. corresponding to full power of the motor. Gasoline motors develop practically constant torque and hence the b.h.p. varies directly as the r.p.m. For reduced power, the thrust delivered by the propeller at the reduced revolutions may be estimated by selecting values of K_t corresponding to the new values of slip

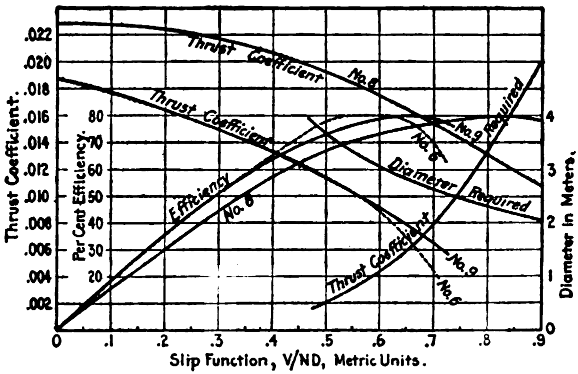

FIG. 7.—Thrust-coefficient and Efficiency Curves of Aeroplane Propellers.

function V/ND computed from assumed values of N and V. Thus curves of thrust plotted on V may be constructed for several revolution speeds of the motor. Such curves will intersect the curve of aeroplane resistance on speed, and these intersection points will indicate the motor revolutions necessary to make the corresponding speeds.

Full-size tests on three aeroplane propellers at the University of Paris ("Bulletin de l' Institut Aerotechnique de l' Université de Paris," Paris, 1913) give curves of thrust coefficient and efficiency as shown in Fig. 7. (Metric units are here used: thrust in kilograms, diameter in meters, velocity in meters per sec.)

Example. An aeroplane requires a thrust of 380 lb. (or 172 kg.) at 90 miles per hour (40.2 meters per sec.), and has a motor which delivers 125 b.h.p. at 1300 r.p.m.

($N = 21.66$ rev. per sec.). From $S = V/ND$ and $D = V/21.66\,S$, compute for various values of S the diameters required to preserve similitude. A curve of D required is plotted on Fig. 7. The thrust must be 172 kg., or $T = K_t N^2 D^4$ and $K_t = 172/(21.66)^2 D^4$. For every value of D, K_t is determined, so also is D determined by S. Hence K_t varies with S and is so plotted on Fig. 7. The intersection of the curve of K_t required with the curve of K_t for any propeller is a possible solution. For propeller No. 6, the intersection is at $V/ND = 0.66$. The corresponding efficiency is 77 per cent.; the diameter required is 2.75 meters (9 ft.), probably too large. Propeller No. 8 gives the best solution. The intersection of the K_t required curve takes place at $V/ND = 0.795$, efficiency being 80 per cent. and diam. 2.3 meters (7.54 ft.).

The propeller (No. 8) for which the curves in Fig. 7 were obtained had a diameter of 2.596 meters and was run at 700 r.p.m. The propeller just selected, if made of the same material as the model and geometrically similar, will be relatively weaker in the proportion of $N_1^2D_1^2/N_2^2D_2^2 = 1300^2 \times 2.3^2/700^2 \times 2.596^2 = 2.7$, or roughly one to three.

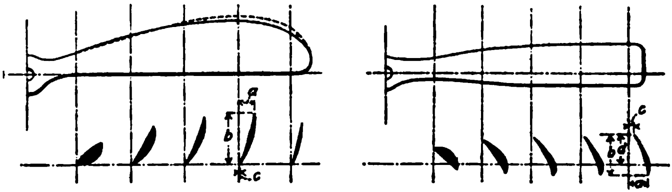

FIG. 8.—Aeroplane Propeller Blades.

The factor of safety in the original design is thus cut down to one-third. Since the centrifugal force and bending moment are important only for sections near the root of the blade, it is necessary to thicken up only those sections materially. Such thickening of the root will have very little effect on the functioning of the propeller, as the major part of the work is done by the outer half of a blade.

In very light propeller blades the distortion at high speeds is considerable and may have an important effect on efficiency. To preserve the same efficiency, a geometrically similar propeller must be run with N^2D^2 constant, so that the distortions, or the stresses which cause them, shall be the same.

The propellers used in Fig. 7 as prototypes are all two-bladed wood propellers. Their principal features are shown in Fig. 8 and Table 8. The pitch of the blade face increases toward the tip in each case and the propeller with greatest pitch (No. 8) gives the greatest thrust coefficient. It is possible to make anything that looks like a standard propeller of pitch equal to diameter with assurance that the efficiency will be at least 60 per cent. However, it may not allow the motor to turn at its required revolutions and may otherwise not fit the aeroplane.

Table 8. Aeroplane Propeller Dimensions

(Dimension letters refer to Fig. 8. D = distance from center of shaft to the section whose dimensions are given. p = pitch. All dimensions in millimeters)

	Propeller No. 6 Diam., 2.142 m. Tested at 1300 r.p.m.					Propeller No. 8 Diam., 2.596 m. Tested at 780 r.p.m.					Propeller No. 9 Diam., 2.90 m. Tested at 750 r.p.m.				
D	220	411	602	793	984	246	492	738	984	1230	250	525	800	1075	1350
p	1434	1495	1480	1460	1545	1850	2120	2260	2440	2520	1840	1890	2025	2050	2075
a	91.5	84.0	75.0	69.0	42.5	126	118	107	102	96	133	118	103	84.5	58
b	84.0	145.0	189.0	208.0	170.0	105	170.5	195	201.5	206.5	113.5	201.5	248	265	212.5
c	0.0	0.0	0.0	0.0	0.0	0.0	2.5	12	20.5	28.5	0.0	2.0	3	4	4.0
d	....	.....	.....	.....	.....	75	121	140	146.5	151.5	.....	.....	....	.....	.....

Table 9. Data on Existing Aeroplanes

Name	Nationality	Type	Seats	Velocity, max.-min., miles per hr.	Weight loaded, lb.	Useful load, lb.	Useful load in % of weight loaded	Weight per sq. ft. wing area, lb.	Weight per h.p., lb.	Span: Upper, ft.	Span: Lower, ft.	Length, ft.	Chord, ft.	Wing area, sq. ft.	Motor	B. h.p.	Number of cylinders	Hours fuel
Biplanes																		
Avro, 1	E	T	1	100–40	1165	490	42	5.0	14.5	26.0	26.0	22.3	4.8	235	Gn	80	7	3.0
Avro, 2	E	P	2	65–35	1800	800	44	3.8	22.5	44.0	44.0	....	5.5	468	Gn	80	7	4.5
Bristol, 1	E	T	2	62–33	1665	695	42	4.0	20.8	37.7	37.7	....	6.1	420	Gn	80	7	..
Bristol, 2	E	T	1	95–47	957	340	36	6.0	12.0	22.0	22.0	....	4.3	162	Gn	80	7	...
Sopwith	E	T	2	70–40	3200	1000	31	5.3	32.0	38.0	38.0	....	...	600	Gn	100	9	...
G. White, 1	E	P	2	80–42	1550	550	35	4.3	15.5	37.0	35.0	....	5.0	358	Gn	100	9	...
G. White, 2	E	P	5	51–30	3100	1100	35	3.7	31.0	62.5	43.6	....	7.1	849	Gr	100	6	...
Vickers	E	P	2	100–45	1700	850	50	4.3	17.0	25.0	25.0	21.0	...	400	Gn	100	9	2.5
R.E. 1	E	T	2	82–43	1580	580	37	5.0	22.0	34.0	34.0	....	4.8	316	R	70	8	...
B.E. 2	E	T	2	73–40	1530	530	35	4.1	22.0	38.5	38.5	29.6	5.6	374	R	70	8	3.0
Bréguet	F	T	4	70	3090	1545	50	5.7	24.0	51.0	41.0	31.0	...	538	S	130	....	...
Farman	F	P	2	70	1980	660	33	3.5	28.0	51.0	33.0	....	...	560	R	70	8	...
Albatross	G	T	2	66	....	....	....	....	....	47.0	47.0	34.4	5.6	420	M	100	6	...
D.F.W	G	T	2	80	....	....	....	....	....	56.0	39.0	29.5	5.2	495	M	100	6	...
Monoplanes																		
Blériot	F	T	2	72	1388	618	45	5.6	20.0	34.2	....	....	...	248	Gn	70	7	...
Blériot	F	T	1	80	927	309	33	4.8	11.6	29.3	....	....	...	194	Gn	80	7	...
Nieuport	F	T	1	84	947	352	37	6.1	12.0	....	....	....	6.0	156	Gn	80	7	...
Borel	F	T	2	80	1234	617	50	6.3	15.3	39.4	....	26.4	...	194	Gn	80	7	...
Morane	F	T	2	85	1355	661	49	7.9	17.0	33.5	....	21.5	...	172	Gn	80	7	...
Ponnier	F	T	1	125	1299	529	40	15.4	8.1	23.0	....	17.8	...	84	Gn	160	....	1.0
Deperdussin	F	T	1	125	1411	419	30	14.5	8.8	21.7	....	19.7	...	97	Gn	160	....	1.0

E = English. F = French. G = German. T = Tractor. P = Pusher. Gn = Gnôme. Gr = Green. R = Renault. S = Salmson. M = Mercedes.

Table 10. Additional Data on Biplanes of Table 9

	Gap, ft.	Area, sq. ft.: Elevator	Area, sq. ft.: Fixed tail	Area, sq. ft.: Rudder		Gap, ft.	Area, sq. ft.: Elevator	Area, sq. ft.: Fixed tail	Area, sq. ft.: Rudder
Avro, 1	5.5	13	17	8.0	**Vickers**	......	30	35	13
Avro, 2	6.0	26	32	13.0	**R.E. 1**	5.5	....	....	....
Bristol, 1	5.9	14	15	8.6	**B.E. 2**	6.0	25	52	12
Bristol, 2	4.3	....	....	....	**Farman**	......	22	43	....
Sopwith	......	30	40	10.0	**Albatross**	6.6	32	....	10
G. White, 1	6.0	23	26	15.0	**D.F.W**	7.2	....	....	....
G. White, 2	6.5	38	138	11.0					

SECTION 11

BUILDING CONSTRUCTION AND EQUIPMENT

BY

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CONTENTS

BUILDING CONSTRUCTION

BY

LIONEL S. MARKS

FOUNDATIONS

Bearing Pressure of Soils. The bearing pressure which may be allowed on soil may vary over a large range. Where borings have not been made, tests should be carried out in the open trenches or in test pits by applying a concentrated load of known weight upon a limited area of the bearing soil and making daily level readings of the supported load. Table 1 (from Baker's "Treatise on Masonry Construction") gives a general classification of soils and the safe pressures which they may support.

Table 1. Safe Bearing Power of Soils

Nature of Soil	Safe Bearing Power, Tons per Sq. Ft. Min.	Max.
Rock, the hardest, in thick layers in native bed	200	..
Rock, equal to best ashlar masonry	25	30
Rock, equal to best brick masonry	15	20
Rock, equal to poor brick masonry	5	10
Clay, in thick beds, always dry	4	6
Clay, in thick beds, moderately dry	2	4
Clay, soft	1	2
Gravel and coarse sand, well cemented	8	10
Sand, compact and well cemented	4	6
Sand, clean, dry	2	4
Quicksand, alluvial soils, etc	0.5	1

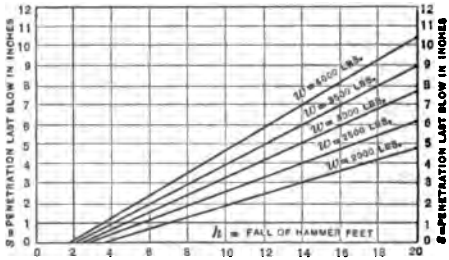
These values are somewhat higher than are customarily used in foundation work for low buildings, that is, for five stories or less, where preliminary exploratory work has not been made, and are higher than most building laws allow. The foundation for a building housing heavy vibrating machinery such as steam hammers, heavy punches, shears, etc., should receive some allowance for possible compression and rearrangement of soil due to the vibrations transmitted through it. The foundation for a tall chimney should be designed with a comparatively low pressure upon the soil, because of the disastrous results which might occur from local settlements.

Footings. The purpose of footings is to spread the concentrated loads of building walls and columns over an area of soil so that the unit pressure will come within the allowable limits. Footings are usually constructed of either concrete or stone. Brick is occasionally used, especially in the South, but is generally more expensive than concrete. Stone, where available in quantity and of the proper quality, can often be used economically, but concrete is the customary footing material (see p. 1314). This material is either used plain in mass, or reinforced with steel rods. The offset on each side of the footing for each successive course is determined by figuring the overhanging portion as a cantilever. For plain footings of 1 : 2½ : 5 (or better) concrete and soil pressures of 2 tons per sq. ft. (or less) a good all-around rule is to make the offset 0.6 of the thickness. The center of pressure should always pass through the center of the footing.

Pile Foundations. Where satisfactory bearing soil is not to be had at a reasonable depth, piles become necessary. Piles are either of wood or of concrete. **Wood piles,** barring the presence of marine borers or other animal destroyers, are as permanent as stone when continually submerged in water. Marine borers will not attack them below the mud line. A subsequent lowering of the ground-water level, however, may be disastrous, because when alternately wet and dry they will rot. **Concrete piles** are indestructible, and hence are adaptable to many conditions. Wood piles are usually designed to carry from 8 to 12 tons per pile. They should be straight and not less than 6 in. in diameter under the bark at the tip. With a concrete pile, from 25 to 35 tons are carried, 30 tons being a fair average.

Methods of Driving Wood Piles. The drop hammer and the steam hammer are usually employed in driving wood piles. The steam hammer, with its comparatively light blows delivered in rapid succession, is of advantage in case of an elastic soil, the speed with which the blows are delivered preventing the readjustment of the soil. It is also of advantage in soft soils where the driving is easy. The water jet is used in sandy soils and in mud. Water supplied under pressure at the point of the pile through a pipe or hose run alongside it erodes the soil, allowing the pile to settle into place.

Determination of Safe Loads for Piles. Piles may obtain their supporting power from friction of the surrounding soil or from columnar action. In the latter case, the bearing power is limited only by the strength of the pile, considered as a column, and since it is supported against deflection sideways, the load that can be carried is very nearly equal to the crushing strength of the pile. In the former case no precise determination of the bearing power can be made. Many formulæ have been developed for determining the safe bearing power in terms of weight of hammer, the fall, and the penetration of the pile per blow, the most generally accepted of which is that known as the *Engineering News* formula, namely, $L = 2wh/(s + 1)$, where L = safe load, lb.; w = weight of hammer, lb.; h = fall of hammer, ft.; s = penetration of last blow, in. A factor of safety of six is assumed. This formula and similar ones are based on the determination of the energy in the falling hammer, and from this the pressure which it must exert on the top of the pile. It is a wise practice to drive test piles and determine their bearing value before proceeding with the final designs of important structures to be supported on piles. The designs are then based upon the safe bearing power so determined, and the piles are driven to the penetration which the above formula shows necessary to give the bearing power for which they were designed. Fig. 1 shows the relation between penetration and fall for weights of different sizes driving a pile equal to supporting a load L of 14,000 lb. Diagrams are easily constructed for other loads.



NOTE: THE LAST FIVE BLOWS SHOULD BE MADE WITH A FALL OF HAMMER OF BETWEEN 8 AND 12 FT.
THE FORMULA IS BASED ON A FREE FALL OF THE HAMMER; A SUITABLE REDUCTION SHOULD BE MADE IN THE ACTUAL WEIGHT OF THE HAMMER TO ALLOW FOR FRICTION IN THE FALL ROPE.

FIG. 1.—Relation Between Penetration and Fall of Hammer in Pile Driving.

Concrete Piles may be broadly divided into two classes: (1) Those molded in place, and (2) those molded, cured and driven. The first of these classes (Raymond) is made by driving a mandrel into the ground and filling the hole so formed with concrete. In one well-known pile of this type, a thin steel sheet is fitted over a tapered mandrel before driving. This shell, which is left in the ground when the mandrel is withdrawn, is filled with concrete and left to rust away. Another well-known pile (Simplex) of the molded-in-place type uses a hollow cylindrical mandrel which is filled with concrete after having been driven to the desired depth and raised a few feet at a time, the concrete flowing out at the bottom and filling the hole in the earth. Most separately molded and many molded-in-place piles are reinforced with steel. Jetting is used extensively in placing separately molded concrete piles, but most are driven into place. Jetting is not often practical or safe in city work on account of danger to the foundations of adjoining buildings. Driving by hammer necessitates a cushioned driving head. The choice of type of concrete pile should be governed by considerations such as whether the pile derives its supporting power from skin friction or column action, the character of the soil, presence of stones, springs, etc.

Spacing of Piles. Wood piles are preferably spaced not closer than 2½ ft. or 3 ft. on centers. If driven closer than this, one pile is liable to force another up; piles relying on skin friction lose some of their bearing power when driven too close together. Occasions arise, however, when closer spacing is inevitable. Twenty-one inches is probably a minimum center distance.

Capping of Piles. Wood piles are capped with timber or concrete. The former is used where stonework is to be supported, and sometimes with concrete. Where concrete is used it is customary to omit the timber platform and place the concrete directly around the pile heads. Concrete placed in this way necessarily gets a bearing on all piles. The piles should be embedded 6 in. in the concrete.

Cost of Piles. Concrete piles cost from $1.00 to $1.10 per lin. ft.; wood piles from $0.25 to $0.30 per ft.

MASONRY CONSTRUCTION

Brickwork. Building walls are built of hard, well-burned brick. Hard-burned or arch brick may be laid in the core of the walls only; soft-burned or salmon brick should never be used on important work. Strong brick masonry is laid up in mortar composed of one part Portland cement to three parts of clean, coarse, sharp sand, tempered with one part of slaked-lime paste. For work of less importance the mortar is made cheaper by decreasing the amount of cement and increasing the amount of sand. An average mortar consists of one part cement, one part lime paste and six parts sand (see p. 569). The poorest mixture that should be used is one part lime to six parts sand. Brick should be laid in a full bed of mortar and shoved into place in order to secure a solid bearing and a joint of even thickness.

Laying and Bonding Brickwork. Brick laid with the long dimension parallel with the wall are called stretchers, and when laid with the long dimension across the wall are called headers. In order to tie or bond brickwork together, the various bonds shown in Fig. 2 are used. Arches over windows are laid in consecutive rings one-half brick high with small joints, and each ring is plastered over before the next ring is placed. Adjacent rings should break joints. Radius of arch should be from one to one and a quarter times the width of the opening. Before laying, all brick should be thoroughly wetted if the weather is warm, but should be kept perfectly dry in freezing weather. The **cost of labor in laying brickwork** in ordinary manufacturing buildings is about $8.00 per 1000 brick.

Strength of Brickwork. The safe working stress for brickwork, including walls and piers, with heights less than six times the least dimension, is 8 tons per sq. ft. if laid in lime mortar (1 lime, 6 sand). If laid in 1 : 3 cement mortar, 20 tons is allowed by the New York and Boston building laws. In isolated piers in which the height is from 6 to 12 times the least dimension, the stresses should be taken as 12 and 7 tons per sq. ft., respectively.

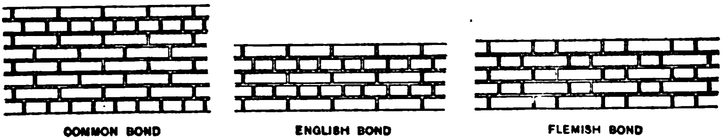


Fig. 2.—Bonds Used in Bricklaying.

Stone Work. Stone masonry is divided into two general classes, viz., rubble and ashlar (see Fig. 3). **Rubble masonry** work is built up of rough stone of irregular shape and size, coursed and uncoursed. Rubble work is used principally for retaining walls, and is sometimes laid up without mortar. Mortar, when used, occupies from 1/5 to 1/3 of the mass, the amount increasing as the stones are smaller. **Ashlar masonry** is built of up stones of rectangular shape on the exterior face of the wall, and is further classified into coursed and broken ashlar, depending on whether the stones of the same course are of the same height or of different heights.

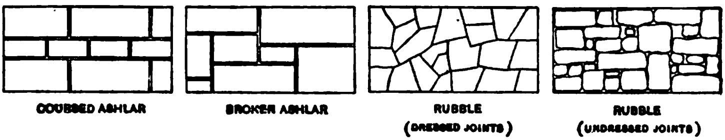


Fig. 3.—Ashlar and Rubble Masonry.

Allowable Working Compressive Stresses on Masonry in tons per sq. ft. according to the building laws of the principal American cities, range as follows:

Granite ashlar	60–170	Rubble in lime	4–6
Limestone ashlar	40–166	Brickwork in Portland cement	15–18
Marble ashlar	40–86	Brickwork in natural cement	12–15
Sandstone ashlar	30–115	Brickwork in lime and cement	10–12
Rubble in Portland cement	12–20	Brickwork in lime	8
Rubble in natural cement	7–12	Concrete, Portland cement, 1 : 2 : 4	15–29
Rubble in lime and cement	8–12	Concrete, natural cement, 1 : 2 : 4	8–9

Retaining Walls. A wall used to sustain the pressure of earth behind it is called a retaining wall. The proportion generally used for walls with fill level at the top is to make the thickness four-tenths of the height, the faces of the wall being plumb. An additional factor of safety is obtained by building the face on a batter. Care should be taken in the design of a wall that the allowable soil pressure is not exceeded, and that drainage is provided for the back of the wall. The foundation should be at least 4 ft. below the ground level where frost is expected.

It is impossible to derive formulæ for the earth pressure on the back of the wall which will take account of all the actual conditions. Assuming the earth to be a loose, homogeneous, granular mass, and the coefficient of friction to be independent of the pressure, Rankine deduced the following formula:

$$P = (\tfrac{1}{2}wh^2 + vh)\tan^2(45° - \tfrac{1}{2}a)$$

the center of pressure being at a height $\tfrac{1}{3}h(wh + 3v)/(wh + 2v)$ above the base, where P = earth pressure per lin. ft. of the wall, lb.; h = height of the wall, ft.; w = weight of earth per cu. ft., lb.; v = weight of superposed load per sq. ft. of surface, lb., and a = angle of repose of the earth, deg. (For values of a see p. 232.) The retaining wall should have sufficient thickness at the base so that the resultant of the earth pressure, P, combined with the weight of the wall, will fall within the middle third of the base. If this resultant falls at the outside edge of the middle third, the maximum vertical pressure on the foundation (at the outer edge of the base) will be equal to $2W/T$ lb. per sq. ft., where W is the weight of 1 lin. ft. of the wall, and T the thickness of the wall at the base, ft.

Building Walls. The following table gives the thickness in inches of brick, stone or solid concrete walls as specified in the Chicago Building Ordinances as amended to September, 1912: Walls less than 50 ft. long from end to end or between cross walls may be 4 in. thinner, but not less than 12 in. thick, and walls of dwelling houses may be 8 in. for a one-story house, and 8 in. in the second story of a two-story house with a story height of less than 14 ft.

Stories	Basement	1	2	3	4	5	6	7	8	9	10	11	12
One story	12	12											
Two stories	16	12	12										
Three stories	16	16	12	12									
Four stories	20	20	16	16	12								
Five stories	24	20	20	16	16	16							
Six stories	24	20	20	20	16	16	16						
Seven stories	24	20	20	20	20	16	16	16					
Eight stories	24	24	24	20	20	20	16	16	16				
Nine stories	28	24	24	24	20	20	20	16	16	16			
Ten stories	28	28	28	24	24	24	20	20	20	16	16		
Eleven stories	28	28	28	24	24	24	20	20	20	16	16	16	
Twelve stories	32	28	28	28	24	24	24	20	20	20	16	16	16

Curtain Walls. When the entire weight of a building is sustained on a steel or concrete framework, the walls may be thin, since they have to support only their own weight in each story. Such walls are called curtain walls, and should be at least 12 in. thick for an 18-ft. height, 16 in. for a 24-ft. height, and 20 in. for a 30-ft. height.

Partitions or thin walls used to subdivide a building into rooms, are of various types, the principal ones being wooden or stud partitions (see Table 14, p. 1279), terra cotta tile, concrete block, and metal lath and plaster. **Terra cotta partitions** are formed of hollow cellular tile, usually 12 in. square and 3, 4 or 6 in. thick, and are laid up with lime mortar like ordinary brickwork. The outside of the tile is then plastered in the usual way for finish. Partitions constructed in this way are both fire- and sound-proof. A safe height for 3-in. block partitions is 12 ft.; for 4-in., 16 ft; and for 6-in., 20 ft. The unsupported length should not exceed the safe height. The crushing strength of solid terra cotta is from 6000 to 7000 lb. per sq. in. **Concrete block partitions** are made up in the same manner as terra cotta, and serve the same purpose. They are much heavier, and for this reason not so generally used. Common practice, shown by the building laws of Philadelphia and Buffalo, allows hollow spaces in the blocks equal to

¼ the cross-section, and working stresses of from 100 to 150 lb. per sq. in. of solid concrete, excluding the area of the hollow spaces. **Metal-lath partitions** are much used, on account of their lightness and economy of space. They are usually formed of ⅞-in. or 1-in. steel channels fastened to the floors and ceilings and spaced 12 in. on centers for spans of over 10 ft., and up to 16 in. apart for smaller spans. These channels are then covered with wire fabric or expanded metal wired on, and plastered over on both sides, usually with two coats of mortar and a finish coat on each side of hard-setting mortar, with a total thickness of 2 in.

Table 2. Terra Cotta Partitions: Dimensions and Weights of Stock Sizes

Thickness of partition, in.		2	3	4	5	6
Dimensions of blocks, in.		2 × 6 × 12 2 × 8 × 12 2 × 12 × 12	3 × 6 × 12 3 × 8 × 12 3 × 12 × 12	4 × 6 × 12 4 × 8 × 12 4 × 12 × 12	5 × 8 × 12 5 × 12 × 12	6 × 8 × 12 6 × 12 × 12 ....
Weight, lb. per sq. ft.	Semi-porous	12	15	16	18	24
	Porous.....	14	17	18	20	26

Terra Cotta Floor Construction. The floors and roofs of steel-frame buildings are often constructed with terra cotta hollow-tile arches, spanning between and protecting the steel beams. These arches are of two classes,

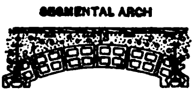


FIG. 4.

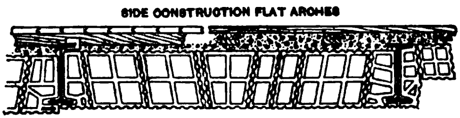


FIG. 5.

Terra Cotta Floor Construction.

segmental (Fig. 4) and flat (Figs. 5 and 6), the latter being of end construction or side construction. The segmental arch is the cheapest and strongest and is well adapted to factories, warehouses and locations where a flat ceiling is not required. The end-construction type of flat arch is nearly 50 per cent. stronger, for the same weight, than the side construction. Proper protection of the bottom flange of the steel beams should be provided for all

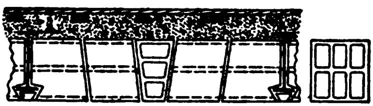


FIG. 6.—Terra Cotta Tile Floor Arch, End Construction.

types of this construction and the beams securely tied together with rods to resist thrust. All blocks should be set in Portland cement mortar. Cinder concrete filling is placed on top of the arches and leveled off to receive flooring. If wood flooring is used, nailing strips are bedded in the concrete and the flooring nailed to these strips, the strips being anchored to the tops of steel beams. Table 3 gives the spacing (in ft.) of ¾-in. tie rods to provide a sufficient cross-section to withstand the thrust of the arch with a total load on the arch of 100 lb. per sq. ft. For other loadings, multiply the tabular values by 100 and divide by the loading to be used. Tie rods should be placed in the line of thrust if possible, usually 3 in. above the bottom of the beam. In segmental arches the effective rise is equal to the vertical distance between the highest point of the concave surface and the springing line or chord; the effective rise of a flat arch may be taken at 2.4 in. less than the arch depth. Tables 4 and 5 give safe loads for end-construction flat arches and segmental arches. The weights of the arches themselves are included in the safe loads in Table 4, and may be computed at 0.06 lb. per cu. in. of solid terra cotta.

Table 3. Spacing of 3/4-in. Tie Rods for Terra Cotta Tile Arches

(Spacing in feet from center to center of rods for total load on arch of 100 lb. per sq. ft. For any other loading W, multiply tabular values by 100 and divide by W)

Span, ft.	Effective Rise of Arch, Inches											
	4	5	6	7	8	9	10	11	12	13	14	15
3	14.3	......	......	......	......	......	......	......	......	......	......	......
4	8.1	10.1	12.1	14.1	......	......	......	......	......	......	......	......
5	5.2	6.4	7.7	9.0	10.3	11.6	12.9	14.2	......	......	......	......
6	3.6	4.5	5.4	6.3	7.2	8.1	8.9	9.8	10.7	11.6	12.5	13.4
7	......	3.3	3.9	4.6	5.3	5.9	6.6	7.2	7.9	8.5	9.2	9.9
8	......	......	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.6
9	......	......	......	......	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
10	......	......	......	......	......	......	3.2	3.5	3.9	4.2	4.5	4.8

Table 4. Safe Loads for End-construction Flat Arches of Standard Terra Cotta Blocks, Lb. per Sq. Ft.

(National Fireproofing Co. Sectional areas given are per lin. ft. of arch parallel to the beam. Factor of safety, 7)

Depth of arch, in.	6	7	8	9	10	12	15
Sectional area, sq. in.	31	34	37	40	43	49	58
Spans, ft. and in.	Safe loads, lb. per sq. ft.						
3-0	482	617	767	933	1114	1524	2255
3-3	410	525	654	795	950	1299	1922
3-6	354	453	563	685	819	1120	1657
3-9	308	394	491	597	713	975	1443
4-0	271	347	431	525	627	857	1268
4-3	240	307	382	465	555	759	1124
4-6	214	274	341	414	495	677	1002
4-9	192	246	306	372	444	608	900
5-0	173	222	276	336	401	548	812
5-3	....	201	250	304	364	497	736
5-6	....	183	228	277	331	453	671
5-9	....	168	208	254	303	415	614
6-0	....	....	191	233	278	381	563
6-3	....	....	176	215	256	351	519
6-6	....	....	163	198	237	324	480
6-9	....	....	....	184	220	301	445
7-0	....	....	....	171	204	280	414
7-6	....	....	....	....	178	243	360
8-0	....	....	....	....	....	214	317
8-6	....	....	....	....	....	190	281
9-0	....	....	....	....	....	169	250
9-6	....	....	....	....	....	....	225
10-0	....	....	....	....	....	....	203

Table 5. Safe Loads for Terra Cotta Segmental Arches, Lb. per Sq. Ft.

(Weight of arch blocks not included. Sectional areas are per lin. ft. of arch parallel to the beams. Factor of safety, 7)

Depth of arch, in.		4	6	8	10
Sectional area, sq. in.		28	36	43	47
Spans, ft.-in.	Rise, in.	Lb. per sq. ft.			
4–0	1	920	1148	1414	1545
	1½	1353	1740	2079	2272
	2	1736	2233	2667	2915
4–6	1	812	1044	1247	1363
	1½	1196	1539	1838	2009
	2	1536	1975	2359	2578
5–0	1	744	951	1143	1249
	1½	1072	1379	1647	1800
	2	1379	1773	2118	2315
5–6	1	672	864	1032	1128
	1½	984	1266	1512	1652
	2	1258	1619	1933	2113
6–0	1	612	788	941	1028
	1½	898	1154	1379	1507
	2	1148	1476	1763	1927
6–6	1	562	724	864	944
	1½	823	1058	1264	1382
	2	1055	1358	1622	1772
7–0	1	520	669	799	873

Depth of arch, in.		4	6	8	10
Sectional area, sq. in.		28	36	43	47
Spans, ft.-in.	Rise, in.	Lb. per sq. ft.			
7–0	1½	762	981	1171	1280
	2	983	1264	1510	1650
7–6	1	482	621	741	810
	1½	715	920	1099	1201
	2	915	1176	1405	1536
8–0	1	457	588	703	768
	1½	668	859	1026	1122
	2	854	1099	1312	1434
8–6	1	428	551	658	719
	1½	626	806	963	1052
	2	807	1037	1239	1354
9–0	1	403	518	619	677
	1½	590	758	906	990
	2	759	977	1167	1275
9–6	1	380	489	584	638
	1½	561	721	862	942
	2	717	923	1102	1204
10–0	1	359	462	552	603
	1½	531	683	816	892
	2	683	879	1050	1147

TIMBER CONSTRUCTION

Holding Power of Nails, Spikes and Screws in Wood. The **holding power of nails and spikes** is given in Table 6 and that of **lag screws and wood screws** in Table 7. The resistance of a lag screw increases about as the square root of the diameter; the size of the hole bored apparently has but little influence on the resistance.

Table 6. Holding Power of Nails and Spikes, in Lb. per In. of Penetration

(Tests at Watertown Arsenal, 1884)

Cut nails	Parallel to the grain			Perpendicular to the grain			Wire nails and spikes	Perpendicular to the grain				Parallel to grain
	Yellow pine	White pine	White oak	Yellow pine	White pine	White oak		Cedar (dry)	White oak	Yellow pine	White pine	White pine
6 d	89(*a*)	....	....	154(*b*)	77(*b*)	317(*b*)	6 d	......	129(*b*)	108(*b*)	60(*b*)	30(*a*)
8 d	206(*a*)	89	520	327	211	630	10 d	......	390	132	70	50
10 d	222	108	580	324	181	650	60 d	......	731	465	.....	....
20 d	320(*a*)	148	692	407	298	800	⅜ in.	283(*c*)	1188(*e*)	590(*d*)	450	370
50 d	439(*a*)	170	820	570	316	991	7/16 in.	......	....	....	436	344
60 d	445	200	950	639	324	1040	½ in.	338(*c*)	744	700	364	113

(*a*) R. C. Carpenter, *Trans. A. S. M. E.*, 1895. (*b*) F. W. Clay, *Eng. News*, Jan. 11 1894. (*c*) A. W. Wright, West. Soc. Engrs., 1881. (*d*) Prof. W. R. Johnson. (*e*) Ship-spikes (square), Report of Chief of Engineers, U. S. A., 1884. Chestnut offers about the same resistance as yellow pine.

Table 7. Holding Power of Wood Screws and Lag Screws

(Wood-screw tests by P. J. Haler and H. T. Wright, *Wood Craft*, Oct., 1912. Lag-screw tests (*a*) by A. J. Cox, Univ. of Iowa, 1891, and (*b*) by Watertown Arsenal, 1884)

	Kind of wood	Diam. of screw, in.	Length of screw, in.	Holding power per in. of penetration, lb.	Method of applying screw
Wood screws	Oak	0.194	2½	850	Small hole bored, then screwed in.
	Oak	0.19	1½	620	Small hole bored, then screwed in.
	Oak	....	2½	820	Brass screw, broke at top.
	White pine	....	2	600(*a*)	....
	White deal	0.19	1½	400	Screwed into wood direct.
	White deal	0.19	1½	400	Half driven in and then screwed.
	White deal	0.19	1½	100	Driven in.
Lag screws	White oak	5/8	4½	1780(*b*)	In ½-in. hole.
	White oak	9/16	3	2160(*b*)	In 7/16-in. hole.
	White oak	½	4½	1950(*b*)	In 3/8-in. hole.
	Yellow pine	5/8	4	950(*b*)	In ½-in. hole.
	White cedar (unseasoned)	5/8	4	860(*b*)	In ½-in. hole.
	White pine	5/8	2½	1060(*a*)	No hole.
	White pine	½	4¼	1270(*a*)	No hole.
	White pine	1	6½	1580(*a*)	No hole.
	Douglas fir	....	....	....	(½ values given for white oak.)

Lateral Resistance of Nails in Wood. According to W. K. Hatt (*Ind. Eng. Soc.*, 1900) the load in pounds supported by a nail when it fastens two boards together and is subjected to a transverse load, is $P = cd$, where d is the pennyweight designation of the nail and c has the following values:

	Common Nails		Finish Nails		Fence Nails	
	Cut	Wire	Cut	Wire	Cut	Wire
Oak	82	70	49	52	100	..
Yellow pine	50	48	42	32	90	84

The joints will slip at about 60 per cent. of these loads. The **resistance of common railway spikes** (W. K. Hatt, *Proc. Am. Ry. Eng. Assn.*, 1910) is as follows:

Resistance against	Red oak	Short-leaf pine	Long-leaf pine	Lob-lolly pine	Red gum	Screw spikes are stronger by
Direct pull, lb.	7639	4359	3955	3930	3883	1.7 to 3.8 times
Lateral pull, lb.	2026	1704	1650	1633	1619	1.2 to 2.4 times

The ultimate **pulling resistance of drift bolts** perpendicular to the grain of the wood, in lb. per lin. in., is as follows (Watertown Arsenal, 1902): 1-in. bolt driven in a 15/16-in. hole: yellow pine, 375; white oak, 3250; Douglas fir, 2250. 1-in. bolt in 13/16-in. hole: yellow pine, 740; white oak, 1300; Douglas fir, 886. Douglas fir: ¾-in. bolt in 11/16 hole, 800; in 9/16-in. hole, 1000; ½-in. bolt in 7/16-in. hole, 374; do., in 5/16-in. hole, 1000. The resistance is only about half these values when driven parallel to the grain. The holding power varies directly with the area of contact and with the compressive strength of the wood. Square drift bolts have the same resistance as round drift bolts of the same diameter.

Floors (See also p. 1329 for factory floors). The framing of wooden floors may be divided into two general types: joist construction, and solid or mill construction. The first consists of **joists** 2 or 3 in. wide, of the necessary depth, and spaced about 12 in. to 16 in. on centers. The wall ends should be built solidly into walls, and the interior ends carried by a line of girders on columns. These joists should be securely cross-bridged in each span to prevent twisting and to assist in distributing concentrated or suddenly applied loads. The floor is formed of a thickness of rough boarding on which the finish flooring is laid. **Solid or mill-construction floors** are designed to do away with the small pockets which exist in joist construction and thus reduce the fire hazard. They are generally framed with beams spaced from 8 to 11 ft. on centers and spanning from 20 to 25 ft. The wall ends of beams are built into and anchored to the wall, and the interior ends carried on columns and tied together to form a continuous tie across the building. In all cases care should be taken to provide sufficient bearing at the points of support so that the allowable intensity of compression across the grain, as given in the tables on p. 593, is not exceeded. In case it is desirable to omit columns, or the floor load requires a closer spacing of beams, girders are run lengthwise of the building over the columns to take the beams, the ends of which are hung in hangers or stirrup irons and tied together over or through the girders. This is called **intermediate framing.** Steel beams are sometimes used in place of wooden beams in this type of construction, in which case a wooden strip is secured to the top flange of the beam to take the nailing of the plank, or the plank is laid directly on top of the beam and secured by spikes driven from below and clinched over the flange. The floor is formed of 3- or 4-in. plank, grooved in each edge and put together with splines and securely spiked to beams. On top of the plank is laid an intermediate or top flooring having a layer of sheathing paper between the plank and flooring. In case the floor loads require an excessive thickness of plank, or in localities where heavy plank is not easily obtainable, the flooring is built up of 3 × 4-in., 4 × 6-in., or other sized pieces, placed on edge and securely nailed together.

Floor Loads are regulated by the building laws in all municipalities, and vary slightly in different localities. For districts not thus regulated, the following values will serve as a guide for live loads (lb. per sq. ft.): Cotton mills, 75 to 100; machine shops, 125 to 250; foundries, 200 to 300; roof loads (flat), 40 to 50, depending on locality and snowfall.

Table 8 gives the properties of mill floors made of matched and dressed plank, and of laminated floors made of planks on edge, laid close. For the meanings of "coefficient of strength" and "coefficient of deflection" see the following two pages.

Table 8. Properties of Yellow Pine Floors per Foot Breadth

b = Breadth = 12 in.

	1	2	3	4	5	6	7	8
	Nominal thickness, in.	Actual thickness, in., d	Area of of section bd. (A) sq. in.	Weight per sq. ft., lb.	Moment of inertia $bd^3/12$ (I) in.^{4}	Section modulus, $bd^2/6$ (S) in.^{3}	Coefficient of strength, fiber stress 1000 lb. per sq. in.	Coefficient of deflection, uniform load, in. $E = 1,620,000$ (N)
Mill floors	2	1⅝	19.5	5.4	4.29	5.28	3500	0.00323
	2½	2⅛	25.5	6.9	9.59	9.03	6000	0.00144
	3	2⅝	31.5	8.7	18.09	13.78	9700	0.000768
	4	3⅝	43.5	12.0	47.63	26.28	17500	0.000291
	5	4⅝	55.5	15.4	98.93	42.78	28500	0.000144
	6	5⅝	67.5	18.7	177.97	63.28	42200	0.000078
Laminated floors	6*	5½*	66.0	18.3	166.37	60.50	40300	0.0000835
	8	7½	90.0	25.0	421.87	112.50	75000	0.0000329
	10	9½	114.0	31.6	857.37	180.50	120300	0.0000166
	12	11½	138.0	38.3	1520.87	264.50	176300	0.00000913

* Use for 2½ × 6, 3 × 6, and 6 × 6 pieces; for 2 × 6 and 6 × 6, use values for mill floors.

Beams

Properties of Wooden Beams. Table 9 is adapted from the "Yellow Pine Handbook" for 1913. The "nominal size" of a timber is indicated by the breadth and depth of the section in inches. The "actual size" indicates the size of the dressed timber, and these sizes are dressed as indicated. "S1S1E" indicates that the piece is "dressed on one side and one edge," "S4S" indicates that the piece is "dressed on four sides." The moment of inertia and section modulus are with the neutral axis perpendicular to the depth at the center. The **coefficients of strength** are given for the fiber stress of 1000 lb. per sq. in. and are the safe loads in pounds uniformly distributed, including its own weight, for a beam 1 ft. long. The safe load

Table 9. Properties of Wooden Beams (Actual Size)

Nominal size, in.	Actual size, in. $b \times d$	Dressed	Area of section, bd. (A). sq. in.	Weight per ft. at 40 lb. per cu. ft., lb.	Moment of inertia, $bd^3/12$ (I), in.^{4}	Section modulus, $bd^2/6$ (S), in.^{3}	Coefficient of strength, fiber stress 1000 lb. per sq. in.	Coefficient of deflection, uniform load. E = 1,620,000
2 × 4	1⅝ × 3⅝	S1S1E	5.89	1.63	6.45	3.56	2400	0.00215
4 × 4	3½ × 3½	S4S	12.25	3.40	12.50	7.15	4700	0.00111
2 × 6	1⅝ × 5⅝	S1S1E	9.14	2.53	24.10	8.57	5700	0.000576
4 × 6	3⅝ × 5⅝	S1S1E	20.39	5.65	53.76	19.12	12700	0.000258
6 × 6	5½ × 5½	S4S	30.25	8.38	76.25	27.73	18500	0.000182
2 × 8	1⅝ × 7½	S1S1E	12.19	3.38	57.13	15.23	10100	0.000243
4 × 8	3¾ × 7½	S1S1E	28.12	7.80	131.83	35.16	23400	0.000105
6 × 8	5½ × 7½	S1S1E	41.25	11.43	193.36	51.56	34300	0.0000718
8 × 8	7½ × 7½	S4S	56.25	15.58	263.67	70.31	46800	0.0000527
2 × 10	1⅝ × 9½	S1S1E	15.44	4.28	116.10	24.44	16300	0.000120
6 × 10	5½ × 9½	S4S	52.25	14.47	392.96	82.73	55100	0.0000353
8 × 10	7½ × 9½	S4S	71.25	19.74	535.86	112.81	75200	0.0000259
10 × 10	9½ × 9½	S4S	90.25	25.00	678.75	142.89	95300	0.0000205
2 × 12	1⅝ × 11½	S1S1E	18.69	5.18	205.95	35.82	23800	0.0000674
6 × 12	5½ × 11½	S4S	63.25	17.52	697.07	121.23	80800	0.0000199
8 × 12	7½ × 11½	S4S	86.25	23.89	950.55	165.31	110200	0.0000146
10 × 12	9½ × 11½	S4S	109.25	30.26	1204.03	209.39	139600	0.0000115
12 × 12	11½ × 11½	S4S	132.25	36.63	1457.51	253.48	168800	0.00000953
6 × 14	5½ × 13½	S4S	74.25	20.57	1155.17	167.10	111400	0.0000123
8 × 14	7½ × 13½	S4S	101.25	28.05	1537.73	227.81	151800	0.00000903
10 × 14	9½ × 13½	S4S	128.25	35.53	1947.80	288.56	192200	0.00000713
12 × 14	11½ × 13½	S4S	155.25	43.00	2357.86	349.31	232800	0.00000589
14 × 14	13½ × 13½	S4S	182.25	50.48	2767.92	410.06	273300	0.00000502
8 × 16	7½ × 15½	S4S	116.25	32.20	2327.42	300.31	200200	0.00000597
10 × 16	9½ × 15½	S4S	147.25	40.79	2948.07	380.39	253600	0.00000471
12 × 16	11½ × 15½	S4S	178.25	49.37	3568.71	460.48	307000	0.00000389
14 × 16	13½ × 15½	S4S	209.25	57.96	4189.36	540.56	360200	0.00000332
16 × 16	15½ × 15½	S4S	240.25	66.55	4810.00	620.65	413600	0.00000289
10 × 18	9½ × 17½	S4S	166.25	46.05	4242.84	484.89	323100	0.00000327
12 × 18	11½ × 17½	S4S	201.25	55.75	5136.07	586.98	382900	0.00000270
14 × 18	13½ × 17½	S4S	236.25	65.44	6029.29	689.06	459000	0.00000230
16 × 18	15½ × 17½	S4S	271.25	75.14	6922.53	791.14	527200	0.00000201
18 × 18	17½ × 17½	S4S	306.25	84.83	7815.76	893.23	595300	0.00000178
20 × 20	19½ × 19½	S4S	380.25	105.33	12049.17	1235.81	823900	0.0000011

for any span is obtained by dividing the proper coefficient by the length of the span in feet, and multiplying by the ratio of the fiber stress it is desired to employ to 1000 lb. per sq. in.

To select a **beam to support a given uniformly distributed load** on a given span, find the coefficient of strength required as follows: Multiply the given uniformly distributed load in pounds by the span in feet, and by the ratio of 1000 lb. per sq. in. to the fiber stress it is desired to employ, and refer to the table for a beam having that coefficient. To select a beam for supporting a **load concentrated at the center of** the **span**, multiply the concentrated load by 2 and consider the result an equivalent uniformly distributed load. If the loads are not applied as above, the necessary size must be found from the section modulus, as explained on p. 404. A beam whose span is less than from 10 to 15 times its depth is likely to have its safe load limited by the longitudinal shearing stress, and the safety of such beams should always be examined.

The **coefficients of deflection** given are the deflections in inches of beams 1 ft. in span with a uniformly distributed load of 1000 lb., the modulus of elasticity being taken at 1,620,000 lb. per sq. in., which is a reasonable value for yellow pine under loads not long continued. For continuous dead loads, the tabular coefficient of deflection should be multiplied by 2. For other woods than longleaf yellow pine, multiply the tabular coefficient of deflection by the ratio of 1,620,000 to the modulus of elasticity of the wood used. (See the table of the properties of different varieties of structural timber, p. 591.) The deflection of a beam of a given span under uniformly distributed load is obtained by multiplying the coefficient of deflection of the beam by the cube of the span in feet and by the number of 1000-lb. units in the given load. Coefficients of deflection under concentrated loads applied at the middle of the span may be obtained by multiplying the values in the table by 1.6.

Safe Loads for Wooden Beams. Table 10 is based upon the working unit stresses adopted by the American Railway Engineering Association (see p. 593), and gives the uniformly distributed safe loads for rectangular sections 1 in. thick; the safe load for a beam of any thickness is found by multiplying the tabular value by the thickness of the beam in inches. The safe loads include the weight of the beams and are computed on the assumption that the beams are braced against lateral deflection. The table also gives minimum and maximum spans and coefficients of deflection.

The maximum safe loads as limited by the allowable shearing stresses along horizontal axes of beams have been calculated from the formula: Maximum safe load = $\frac{4}{3}$ × area of section × safe unit stress for longitudinal shear. These limits should not be exceeded to avoid failure of the beam by horizontal shear. The deflection in the center of the span for uniformly distributed and permanently applied loads is obtained from the coefficients of deflection by dividing the depth of the beam, in inches, into the corresponding coefficients; the results are only approximate, as the modulus of elasticity varies with the moisture content of the wood.

The deflection of beams intended to carry plastered ceilings should not exceed $\frac{1}{360}$ of the span; the table gives the maximum spans for this limit and for uniform and permanently applied loads.

For loads concentrated in the center of the span, use one-half the values for the tabular loads and four-fifths of the coefficients of deflection. For special cases of loading, see Table 12*b*, p. 403.

Table 10. Rectangular Wooden Beams—1 In. Thick

Maximum Safe Loads and Limiting Spans

Depth, in.	White oak		Longleaf pine		Shortleaf pine		White pine		Douglas fir		Western hemlock		Spruce	
	Max. load, lb.	Min. span, ft.	Max. load, lb.	Min. span, ft.	Max. load, lb.	Min. span, ft.	Max. load, lb.	Min. span, ft.	Max. load, lb.	Min. span, ft.	Max. load, lb.	Min. span, ft.	Max. load, lb.	Min. span, ft.
2	293	1.7	320	1.8	347	1.5	187	2.1	293	1.8	267	1.8	187	2.4
4	587	3.3	640	3.6	693	3.1	373	4.3	587	3.6	533	3.7	373	4.8
6	880	5.0	960	5.4	1040	4.6	560	6.4	880	5.5	800	5.5	560	7.1
8	1173	6.7	1280	7.2	1387	6.2	747	8.6	1173	7.3	1067	7.3	747	9.5
10	1467	8.3	1600	9.0	1733	7.7	933	10.7	1467	9.1	1333	9.2	933	11.9
12	1760	10.0	1920	10.8	2080	9.2	1120	12.9	1760	10.9	1600	11.0	1120	14.3
14	2053	11.7	2240	12.6	2427	10.8	1307	15.0	2053	12.7	1867	12.8	1307	16.7
16	2347	13.3	2560	14.4	2773	12.3	1493	17.1	2347	14.5	2133	14.7	1493	19.0
18	2640	15.0	2880	16.3	3120	13.8	1680	19.3	2640	16.4	2400	16.5	1680	21.4
20	2933	16.7	3200	18.1	3467	15.4	1867	21.4	2933	18.2	2667	18.3	1867	23.8
22	3227	18.3	3520	19.9	3813	16.9	2053	23.6	3227	20.0	2933	20.2	2053	26.2
24	3520	20.0	3840	21.7	4160	18.5	2240	25.7	3520	21.8	3200	22.0	2240	28.6

Coefficients of Deflection for Permanent Loads

Span in feet	White oak	Long-leaf pine	Short-leaf pine, western hemlock	White pine, Douglas fir	Spruce	Span, feet	White oak	Long-leaf pine	Short-leaf pine, western hemlock	White pine, Douglas fir	Spruce
1	0.06	0.05	0.05	0.05	0.05	21	25.31	21.37	19.67	21.05	20.20
2	0.23	0.19	0.18	0.19	0.18	22	27.78	23.44	21.59	23.10	22.17
3	0.52	0.44	0.40	0.43	0.41	23	30.37	25.63	23.59	25.25	24.23
4	0.92	0.78	0.71	0.76	0.73	24	33.06	27.91	25.69	27.49	26.38
5	1.44	1.21	1.12	1.19	1.15	25	35.88	30.28	27.88	29.83	28.63
6	2.07	1.74	1.61	1.72	1.65	26	38.80	32.75	30.15	32.27	30.96
7	2.81	2.37	2.19	2.34	2.24	27	41.85	35.32	32.51	34.80	33.39
8	3.67	3.10	2.85	3.06	2.93	28	45.00	37.99	34.97	37.42	35.91
9	4.65	3.92	3.61	3.87	3.71	29	48.27	40.75	37.51	40.14	38.52
10	5.74	4.85	4.46	4.77	4.58	30	51.66	43.61	40.14	42.96	41.22
11	6.95	5.86	5.40	5.78	5.54	31	55.16	46.56	42.86	45.87	44.01
12	8.27	6.98	6 42	6.87	6.60	32	58.78	49.61	45.67	48.88	46.90
13	9.70	8.19	7.54	8.07	7.74	33	62.51	52.76	48.57	51.98	49.88
14	11.25	9.50	8.74	9.36	8.98	34	66.35	56.01	51.56	55.18	52.95
15	12.92	10.90	10.04	10.74	10.31	35	70.32	59.35	54.64	58.47	56.11
16	14.69	12.40	11.42	12.22	11.73	36	74.39	62.79	57.80	61.86	59.36
17	16.59	14.00	12.89	13.79	13.24	37	78.58	66.33	61.06	65.34	62.70
18	18.60	15.70	14.45	15.47	14.84	38	82.89	69.96	64.40	68.92	66.14
19	20.72	17.49	16.10	17.23	16.53	39	87.31	73.69	67.84	72.60	69.66
20	22.96	19.38	17.84	19.09	18.32	40	91.84	77.52	71.36	76.37	73.28

Maximum Spans in Feet for Permanent Loads

Species of timber	Depth of beam in inches											
	2	4	6	8	10	12	14	16	18	20	22	24
White oak	2.3	4.7	7.0	9.3	11.6	13.9	16.3	18.6	20.9	23.2	25.6	27.9
Longleaf pine	2.8	5.5	8.3	11.0	13.8	16.5	19.3	22.0	24.8	27.6	30.3	33.1
Shortleaf pine	3.0	6.0	9.0	12.0	15.0	17.9	20.9	23.9	26.9	29.9	32.9	35.9
Western hemlock	3.0	6.0	9.0	12.0	15.0	17.9	20.9	23.9	26.9	29.9	32.9	35.9
White pine, Douglas fir	2.8	5.6	8.4	11.2	14.0	16.7	19.5	22.3	25.1	27.9	30.7	34.5
Spruce	2.9	5.8	8.7	11.6	14.6	17.5	20.4	23.3	26.2	29.1	32.0	37.9

Columns

Wooden Columns are usually of hard pine, either square or round, should have cast-iron caps for supporting the beams and rest on cast-iron pintles which carry the load down to the iron column cap or base plate below. In place of pintles, the columns are often run down and rest directly on the cap or plate. For hard-pine columns whose length is not more than twelve times the diameter or least side, the safe load may be found by multiplying the sectional area in square inches by 1000; when the length exceeds the above ratio, the column is liable to bend. For safe loads for square columns with a unit stress of 1000 lb. per sq. in., see Table 28, p. 438. The **working unit stresses for wooden columns** adopted by the American Railway Engineering Association are given in Table 11 for ratios of l/d ranging between limits of 15 and 30.

Table 11. Unit Working Stresses for Wooden Columns, Lb. per Sq. In.

l = effective length of column, in inches; d = least side or diam., in inches; unit working stress = $K[1 - (l/60d)]$

$\frac{l}{d}$	Longleaf pine, white oak, $K = 1300$	Douglas fir, western hemlock, $K = 1200$	Shortleaf pine, spruce, bald cypress, $K = 1100$	White pine, tamarack, $K = 1000$	Red cedar, redwood, $K = 900$	Norway pine, $K = 800$	$\frac{l}{d}$	Longleaf pine, white oak, $K = 1300$	Douglas fir, western hemlock, $K = 1200$	Shortleaf pine, spruce, bald cypress, $K = 1100$	White pine, tamarack, $K = 1000$	Red cedar, redwood, $K = 900$	Norway pine, $K = 800$
15	975	900	825	750	675	600	23	802	740	678	617	555	493
16	953	880	807	733	660	587	24	780	720	660	600	540	480
17	931	860	788	717	645	573	25	758	700	643	583	525	467
18	910	840	770	700	630	560	26	737	670	623	567	510	553
19	888	820	752	683	615	547	27	715	660	605	550	495	440
20	867	800	733	667	600	533	28	693	640	587	533	480	427
21	845	780	715	650	585	520	29	672	620	568	517	465	413
22	823	760	697	633	570	507	30	650	600	550	500	450	400

Cast-iron Columns are often used in place of wooden columns on account of the saving in size. Building laws generally limit the stresses and ratio of length to diameter of this type of column. Table 12 gives allowable loads according to the New York Building Law formula, $p = 9000 - (40\,l/r)$, r being the least radius of gyration and l the length—both in the same units—and p the allowable load in lb. per sq. in. of cross-sectional area.

Table 12. Round and Square Cast-iron Columns

(Allowable Loads in Thousands of Pounds)

ROUND CAST-IRON COLUMNS

Outer diam. or width, in.	Thickness, in.	Area, sq. in.	Weight per ft, lb.	Least radius of gyration, in.	Effective length of column in feet: 8	10	12	14	16	18	20	22	24	26	28
6	½	8.64	27.0	1.95	61	56									
	¾	12.37	38.7	1.88	86	80									
7	¾	14.73	46.0	2.23	107	101	95								
	1	18.85	58.9	2.15	136	128	119								
8	¾	17.08	53.4	2.58	128	122	116	109							
	1	21.99	68.7	2.50	164	156	147	139							
9	1	25.13	78.5	2.85	192	184	175	167	158						
	1¼	30.43	95.1	2.78	232	221	211	200	190						
10	1	28.28	88.4	3.20	221	212	204	195	187	178					
	1¼	34.36	107.4	3.13	267	257	246	235	225	214					
11	1¼	38.29	119.7	3.48	302	292	281	271	260	250	239				
	1½	44.77	139.9	3.40	352	340	327	314	302	289	277				
12	1¼	42.22	131.9	3.83	338	327	316	306	295	285	274	264			
	1½	49.48	154.6	3.75	395	382	369	357	344	331	319	306			
13	1½	54.19	169.4	4.10	437	424	412	399	386	374	361	348	335		
	1¾	61.85	193.3	4.03	498	483	468	454	439	424	409	395	380		
14	1½	58.91	184.1	4.45	479	467	454	441	429	416	403	390	378		
	1¾	67.35	210.5	4.38	547	532	518	503	488	473	459	444	429		
15	1¾	72.85	227.6	4.73	597	582	567	552	537	523	508	493	478	463	
	2	81.68	255.3	4.65	668	651	634	617	600	583	566	550	533	516	
16	1¾	78.34	244.8	5.08	646	631	616	601	587	572	557	542	527	513	498
	2	87.97	274.9	5.00	724	707	690	673	657	640	623	606	589	572	555

SQUARE CAST-IRON COLUMNS

Outer diam. or width, in.	Thickness, in.	Area, sq. in.	Weight per ft, lb.	Least radius of gyration, in.	8	10	12	14	16	18	20	22	24	26	28
6	½	11.00	34.4	2.26	80	76	71								
	¾	15.75	49.2	2.17	114	107	100								
7	¾	18.75	58.6	2.57	141	134	127	120							
	1	24.00	75.0	2.48	179	170	160	151							
8	¾	21.75	68.0	2.98	168	161	154	147	140						
	1	28.00	87.5	2.89	215	205	196	187	178						
9	1	32.00	100.0	3.29	251	241	232	223	213	204					
	1¼	38.75	121.1	3.21	302	291	279	268	256	244					
10	1	36.00	112.5	3.70	287	277	268	259	249	240	231				
	1¼	43.75	136.7	3.61	347	336	324	312	301	289	277				
11	1¼	48.75	152.3	4.01	392	380	369	357	345	334	322	310			
	1½	57.00	178.1	3.93	457	443	429	416	402	388	374	360			
12	1¼	53.78	168.1	4.42	437	426	414	402	391	379	367	356	344		
	1½	63.00	196.9	4.33	511	497	483	469	455	441	427	413	399		
13	1½	69.00	215.6	4.74	565	551	537	523	509	495	481	467	453	439	
	1¾	78.75	246.1	4.65	644	627	611	595	579	562	546	530	514	497	
14	1½	75.00	234.4	5.14	619	605	591	577	563	549	535	521	507	493	479
	1¾	85.75	267.9	5.05	707	690	674	658	641	625	609	593	576	544	544
15	1¾	92.75	289.8	5.46	769	753	737	721	704	688	672	655	639	623	606
	2	104.00	325.0	5.37	862	843	824	806	787	769	750	731	713	694	676
16	1¾	99.75	311.7	5.86	832	816	800	783	767	751	734	718	702	685	669
	2	112.00	350.0	5.77	934	915	896	878	859	840	822	803	785	766	747

Wrought-iron Pipe is often used for columns under light loads. Table 13 gives the safe loads on Standard size pipes used as columns, according to the New York building laws. For Extra Strong and Double Extra Strong pipe used as columns, the safe loads will increase in the same proportion as the weight per foot.

Table 13. Standard Pipe Columns

(Loads in tons of 2000 lb., based on New York Building Code.)

$S = 15{,}200 - (58\,l/r)$

S = allowable compressive stress for steel, lb. per sq. in.;
l = length of column, in.;
r = least radius of gyration, in.

Length, ft.	Size of pipe, in.												
	3	4	5	6	7	8	9	10	11	12	13	14	15
	Thickness, in.												
	.216	.237	.258	.280	.301	.322	.342	.365	.375	.375	.375	.375	.375
24	......	.....	13.55	21.66	30.32	39.96	50.44	63.43	74.03	82.98	94.17	103.12	112.08
22	......	8.02	15.15	23.39	32.18	41.95	52.55	65.69	76.35	85.30	96.49	105.44	114.40
20	......	9.49	16.74	25.12	34.04	43.94	54.66	67.94	78.67	87.62	98.81	107.76	116.71
18	......	10.95	18.34	26.85	35.90	45.93	56.78	70.20	80.99	89.94	101.13	110.08	119.03
16	6.27	12.42	19.93	28.58	37.76	47.92	58.89	72.46	83.30	92.26	103.45	112.40	121.35
14	7.61	13.88	21.52	30.31	39.62	49.90	61.01	74.71	85.62	94.57	105.76	114.72	123.67
13	8.27	14.61	22.32	31.17	40.55	50.90	62.06	75.84	86.78	95.73	106.92	115.88	124.83
12	8.94	15.34	23.12	32.04	41.48	51.89	63.12	76.97	87.94	96.89	108.08	117.03	125.99
11	9.61	16.07	23.91	32.90	42.41	52.89	64.18	78.10	89.10	98.05	109.24	118.19	127.15
10	10.27	16.81	24.71	33.77	43.34	53.88	65.23	79.22	90.26	99.21	110.40	119.35	128.31
9	10.94	17.54	25.51	34.63	44.27	54.88	66.29	80.35	91.42	100.37	111.56	120.51	129.47
8	11.60	18.27	26.30	35.50	45.20	55.87	67.35	81.48	92.57	101.53	112.72	121.67	130.62
7	12.27	19.00	27.10	36.36	46.13	56.87	68.40	82.61	93.73	102.69	113.88	122.83	131.78
6	12.94	19.73	27.90	37.23	47.06	57.86	69.46	83.74	94.89	103.85	115.04	123.99	132.94
5	13.60	20.46	28.69	38.09	47.99	58.86	70.52	84.86	96.05	105.00	116.20	125.15	134.10

NOTE.—Loads above or to the left of the zigzag line correspond to values of l/r greater than 120.

Stud Partitions as Columns. Table 14, taken from the "Yellow Pine Handbook," gives the safe load in pounds per lin. ft. of partition, based on the formula: allowable stress $= 1000[1 - (l/80\,d)]$; it is assumed that the partition is bridged midway in height.

Table 14. Safe Loads on Stud Partitions

(Based on studs being bridged at center)

Nominal size of studs, in.	Distance on centers, in.	Height, ft.	Per lineal foot of partition			Nominal size of studs, in.	Distance on centers, in.	Height, ft.	Per lineal foot of partition		
			Safe load, lb.	Weight, lb.	Board feet				Safe load, lb.	Weight, lb.	Board feet
2×4	12	8	3,723	16.30	6.66	2×6	16	8	4,326	20.24	8.00
2×4	12	10	3,180	19.56	8.00	2×6	16	10	3,699	24.03	9.50
2×4	12	12	2,631	22.82	9.33	2×6	16	12	3,057	27.83	11.00
2×4	16	8	2,793	13.04	5.33	3×6	12	8	11,823	42.00	15.00
2×4	16	10	2,385	15.50	6.33	3×6	12	10	10,992	50.40	18.00
2×4	16	12	1,974	18.75	7.66	3×6	12	12	10,175	59.80	21.00
2×6	12	8	5,767	25.30	10.00	3×6	16	8	8,868	33.60	12.00
2×6	12	10	4,926	30.56	12.00	3×6	16	10	8,244	39.90	14.25
2×6	12	12	4,076	35.42	14.00	3×6	16	12	7,630	46.20	16.50

Stairs. The sum of the rise and run should be equal to from 17 in. to 17½ in. In case of greater rise the tread should narrow accordingly, or *vice versa.*

Roofs

The roofs of buildings of joist and mill construction are framed in a manner similar to the floors of each type, and should be securely anchored to the walls and columns. In case columns are not desired in the top story, trusses of either steel or wood of any of the regulation types are used. For spans up to 35 ft. trussed beams can often be used to advantage.

Loads on Roofs. Where **snow** is likely to occur, the minimum load should be 25 lb. per horizontal sq. ft. of roof for slopes up to 20 deg., and 1 lb. less for each additional degree up to 45 deg. In severe climates this load must be increased. Table 15 gives the **wind pressure** in lb. per sq. ft. normal to the roof, according to Hutton's formula $P_n = P \sin a^{1.84 \cos a - 1}$ where P is the normal pressure (assumed to be 30 lb.) per sq. ft. on a vertical surface, and a is the inclination of the roof surface to the horizontal, in degrees. For other pressures than 30 lb. per sq. ft., the values change in proportion. For slopes over 60 deg. the values given in the table for 60 deg. are applied.

Table 15. Normal Wind Pressure, Lb. per Sq. Ft.

Slope a, deg.	Pressure, P_n, per sq. ft., lb.	Slope a, deg.	Pressure, P_n, per sq. ft., lb.	Slope a, deg.	Pressure, P_n, per sq. ft., lb.	Slope a, deg.	Pressure, P_n, per sq. ft., lb.
5	3.9	20	13.8	35	22.6	50	28.6
10	7.2	25	17.0	40	25.0	55	29.6
15	10.7	30	19.9	45	27.0	60	30.0

Dead Loads. Table 16 gives the approximate weight per sq. ft. of the roofing materials commonly used.

Table 16. Approximate Weights of Roofing Material

Roofing Material	Weight per sq. ft., lb.
Copper, No 22 B. W. G.	1¼
Corrugated galvanized iron, No. 20 B. W. G. (No. 26 B. W. G)	2¼ (1¼)
Felt, 2 layers	½
Felt and asphalt or coal-tar	2
Glass, ⅛ in. thick	1¾
Lath and plaster ceiling	6–8
Lead, ⅛-in. thick	7½
Mackite, 1 in. thick, with plaster	10
Sheathing, hemlock, 1 in. thick (white pine, spruce) [yellow pine]	2 (2¼–2½) [3½]
Shingles, 6 × 18 in. 6 in. to weather	2
Skylight, glass 3/16 to ½ in. including frame	4–10
Slag roof, 4-ply, with cement and sand	4
Slate, ⅛ in. thick, 3-in. double lap (3/16 in. thick)	4½ (6¾)
Terneplate, IC (IX)	½ (⅝)
Tiles (plain), 10½ × 6¼ × ⅝ in., 5¼ in. to weather	18
Tiles (Spanish), 14½ × 10½ in., 7¼ in. to weather	8½
Zinc, No. 20 B. W. G	1½

Combined Roof Loads. In climates corresponding to that of Pittsburgh, ordinary roofs up to 80 ft. span should carry the minimum loads given in Table 17 per sq. ft. of exposed surface, applied vertically, to provide for dead, wind and snow loads combined. For roofs in climates where no snow is likely to occur, reduce these loads by 10 lb. per sq. ft., but no roof or any part thereof should be designed for a total live and dead load less than 40 lb. per sq. ft. The weight of the jack rafters, purlins, and supporting trusses must be added (see p. 1283 for empirical formulæ giving the weights of roof trusses).

Table 17. Combined Roof Loads

Roof Covering		Roof load per sq. ft., lb.
Gravel or Composition Roofing	on boards, flat slope, 1 to 6 or less	50
	on boards, steep slope, more than 1 to 6	45
	on 3-in. flat tile or cinder concrete	60
Corrugated sheeting on boards or purlins		40
Slate	on boards or purlins	50
	on 3-in. flat tile or cinder concrete	65
Tile on steel purlins		55
Glass		45

Timber Trusses

Stresses in Trussed Beams. Referring to Fig. 7, for a uniformly distributed load of W lb., assuming that the load is carried by the members as a simple truss (without allowance for the continuity of the beam) and writing $h = \sqrt{d^2 + l^2/4}$.

	In Fig. 7a	In Fig. 7b
Tension in rod	$= Wh/4d$	$= Wh/3d$
Compression in strut	$= W/2$	$= W/3$
Compression in beam	$= Wl/8d$	$= Wl/9d$

For a single concentrated load, W, in the middle of the truss in Fig. 7a, the above stresses will all be doubled. For a concentrated load $W/2$ on each of the vertical struts of Fig. 7b, the above stresses should be multiplied by 1.5

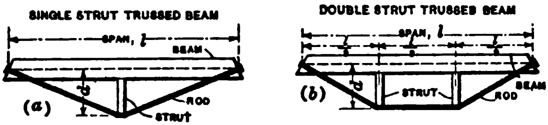


Fig. 7.—Trussed Wooden Beams.

The beam should be large enough so that the unit stress found by adding the unit stress due to the direct compression to the compressive stress caused by bending (considering the length between the strut and the support as a simple beam) shall not exceed the allowable compressive stress for the kind of wood used.

Stresses in Timber Trusses. The most common types of trusses involving wooden compression members are shown in Figs. 8–17 (from Godfrey on "Buildings"). For the **roof trusses** with sloping top chords, Figs. 8–11, the stress in any member may be found by multiplying the coefficient from Tables 18–21 by the total load on the truss (lb.), if the load is vertical and uniformly distributed. The − signs indicate tensions; the + signs, compressions. For other loadings, it will be necessary to draw a stress diagram. For members under both axial and flexural stress, see preceding paragraph.

For trusses with parallel chords shown in Figs. 12–17, the stress in any member is written next to the member, in the figure, in terms of the load on each panel point, for equal vertical loads on each panel point. For other loadings, a stress diagram should be drawn.

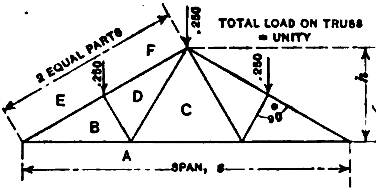


FIG. 8.

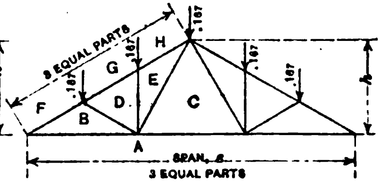


FIG. 9.

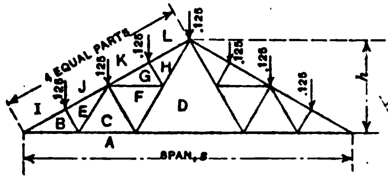


FIG. 10.

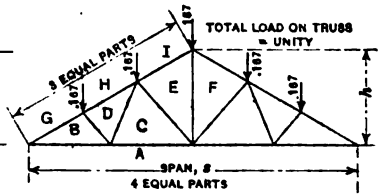


FIG. 11.

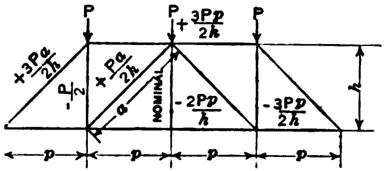


FIG. 12.

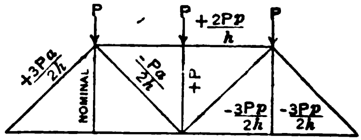


FIG. 13.

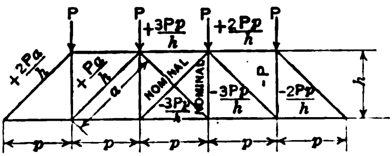


FIG. 14.

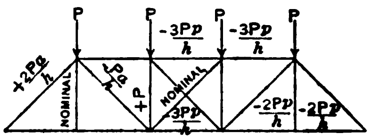


FIG. 15.

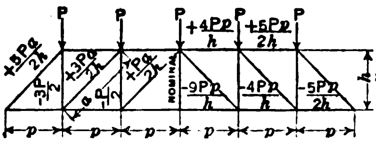


FIG. 16.

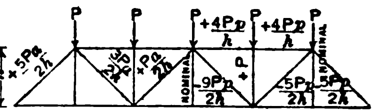


FIG. 17.

FIGS. 8–17.—Types of Timber Trusses.

Table 18. Coefficients for Truss Shown in Fig. 8

Pitch, h/s	AB	AC	BD	CD	BE	DF
⅓	−.563	−.375	+.208	−.188	+.676	+.537
.288*	−.650	−.433	+.217	−.217	+.750	+.625
¼	−.750	−.500	+.224	−.250	+.838	+.727
⅕	−.938	−.625	+.232	−.313	+1.010	+.917

Table 19. Coefficients for Truss Shown in Fig. 9

Pitch, h/s	AB	AC	BD	DE	CE	BF	DG	HE
⅓	−.625	−.375	+.150	+.167	−.250	+.751	+.600	+.600
.288*	−.722	−.433	+.167	+.167	−.289	+.833	+.667	+.667
¼	−.833	−.500	+.187	+.167	−.333	+.932	+.748	+.748
⅕	−1.042	−.625	+.224	+.167	−.417	+1.122	+.895	+.895

Table 20. Coefficients for Truss Shown in Fig. 10

Pitch, h/s	AB	AC	AD	BE	CE	CF	DF
⅓	−.656	−.562	−.375	+.104	−.093	+.208	−.187
.288*	−.758	−.650	−.433	+.108	−.103	+.216	−.217
¼	−.875	−.750	−.500	+.112	−.125	+.224	−.250
⅕	−1.094	−.938	−.625	+.116	−.158	+.232	−.312

Pitch, h/s	DH	FG	GH	BI	EJ	GK	HL
⅓	−.280	−.093	+.104	+.789	+.719	+.650	+.581
.288*	−.325	−.108	+.108	+.875	+.813	+.750	+.688
¼	−.375	−.125	+.112	+.978	+.922	+.866	+.810
⅕	−.469	−.156	+.116	+1.178	+1.132	+1.085	+1.039

Table 21. Coefficients for Truss Shown in Fig. 11

Pitch, h/s	AB	AC	BD	DC	CE	EF	BG	DH	EI
⅓	−.625	−.500	+.139	−.119	+.208	−.333	+.751	+.651	+.451
.288*	−.722	−.577	+.147	−.121	+.220	−.333	+.833	+.722	+.500
¼	−.833	−.667	+.157	−.124	+.236	−.333	+.932	+.807	+.559
⅕	−1.042	−.833	+.178	−.131	+.267	−.333	+1.122	+.972	+.673

* 30 deg. pitch.

For a tabulation of coefficients for **Stresses in Steel Roof Trusses**, see the Carnegie Steel Co.'s "Pocket Companion," 16th edition, pp. 308–310.

Weights of Trusses. For **trusses with wooden compression members and steel tension rods**, the **approximate weight** of the truss, in lb. per sq. ft. of the area of the horizontal projection of that portion of the roof supported by one truss, may be taken as w = (span/25) + (span^2/6200), where the span is measured in feet. For **steel trusses**, for a roof load of about 40 lb. per sq. ft. and span L (ft.), the approximate additional load due to the weight of the truss itself will be $\frac{1}{3}(\sqrt{L} + \frac{1}{3}L)$ lb. per horizontal sq. ft. For greater loads, multiply by the ratio: load per sq. ft./40.

Joints for Timber Trusses. A plain butt joint (Figs. 18 and 19) with steel splice plates is probably the best way to join two members whose axes lie in the same straight line. By extending the splice plates, three or more members may be securely joined. The splice plates are often pieces of plank, in which case h should equal ½b. Special cast-iron pieces (Fig. 20) are often used to join web members and chords. For timber roof trusses, a satisfactory end joint is that shown in Fig. 21.

Choice of Roof Trusses. Wooden Trusses. For pitched roofs with span $L \leq 20$ ft., the simple king-post truss (Fig. 4, p. 225) may be used, and for $L \leq 40$ ft., the type shown in Fig. 10, p. 228. When $L = 40$ to 80 ft., a triangular truss with six vertical struts and a center tie rod, equally spaced, with a tie rod running from the bottom of each strut to the top of the next inner one and from the bottoms of the two inner struts to the peak, is an approved construction. For flat roofs the Howe truss (Fig. 14) may be used up to $L = 130$ ft. The top chord is then made to extend at each end to the full length of the

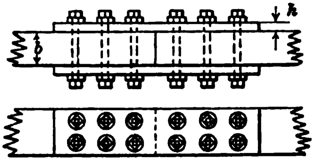


Fig. 18.

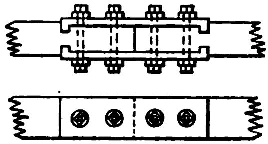


Fig. 19.

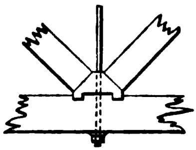


Fig. 20.

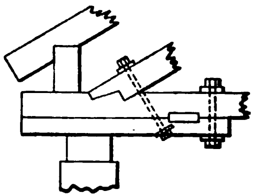


Fig. 21.

Joints for Timber Trusses.

truss, the ends being supported by struts or built into the walls, and may be slightly inclined to the bottom chord for drainage (see Fig. 10, p. 1327). Depth of truss, $\leq 0.1L$; length of each panel, $\leq 0.1L$; spacing of trusses, 10 to 15 ft. apart. Steel Trusses. For pitched roofs, trusses of the types shown in Figs. 8, 9 and 10 are the most satisfactory, Fig. 8 being suitable for $L < 40$ ft., Fig. 9 for $L \leq 50$ ft., and Fig. 10 for $L \leq 90$ ft. For flat roofs, latticed girders (*i.e.*, two superposed and staggered Warren trusses, Fig. 7, p. 227) and Howe trusses may be employed. Spacing of steel trusses, 20 to 25 ft. For various types of trusses suitable for spans up to 200 ft., see *The Brickbuilder*, Mar., 1915.

STEEL FRAMED STRUCTURES

(For loads allowed by various city ordinances, see p. 388)

Specifications. The following excerpts are taken from the American Bridge Co. specifications.

Design

Loads. For structures carrying traveling machinery, such as cranes, conveyors, etc., add 25 per cent. to the stresses resulting from the live load, to provide for the effect of impact and vibrations.

Wind Pressure (assumed to act horizontally in any direction). For finished structures, 20 lb. per sq. ft. on the sides and ends of buildings and on the vertical projection of roof surfaces. For buildings in process of construction, 30 lb. per sq. ft. on vertical surfaces and the vertical projection of inclined surfaces of all exposed metal or other framework.

Unit Stresses. For allowable unit stresses according to the building laws of various cities, see Table 7, p. 388. All parts of structures to be proportioned so that the sum of the dead and live loads, together with the impact, if any, shall not cause the stresses to exceed the following amounts in lb. per sq. in.

Tension, net section, rolled steel	16,000
Direct compression, rolled steel and steel castings	16,000
Bending, on extreme fibers of rolled shapes, built sections, girders, and steel castings	16,000
Bending on extreme fibers of pins	24,000
Shear on shop rivets and pins	12,000
Shear on bolts and field rivets	10,000
Shear—average—on webs of plate girders and rolled beams, gross section	10,000
Bearing pressure on shop rivets and pins	24,000
Bearing on bolts and field rivets	20,000

The pressure per linear inch on expansion rollers shall not exceed 600 times the diam. of rollers in inches.

Axial compression of gross sections of columns, for ratio of l/r up to 120, 19,000 − (100 l/r), with a maximum of 13,000, where l = effective length of members, and r = corresponding radius of gyration of section—both in inches. For ratios of l/r up to 120, and for greater ratios up to 200, use the amounts given in the following table. For intermediate ratios, use proportional amounts.

Ratio	60	70	80	90	100	110	120
Amount	13,000	12,000	11,000	10,000	9,000	8,000	7,000
Ratio	130	140	150	160	170	180	190
Amount	6,500	6,000	5,500	5,000	4,500	4,000	3,500

For bracing and combined stresses due to wind and other loading, the permissible working stresses may be increased 25 per cent.—provided the section thus found is not less than that required by the dead and live loads alone.

Proportion of Parts

General. The effective or unsupported length of main compression members not to exceed 120 ×, and for secondary members 200 ×, the least radius of gyration.

In proportioning columns, provision to be made for eccentric loading. In proportioning tension members, net section to be used. Rivet holes deducted to be taken ⅛ in. larger than the nominal size of rivets.

Members subject to the action of both axial and bending stresses to be proportioned so that the greatest fiber stress will not exceed the allowed limits in that member. Members subject to alternate stresses of tension and compression to be proportioned for the stress giving the largest section, but their connections to be proportioned for the sum of the stresses.

Girders. Rolled I-beams and channels and built-up members used as beams and girders to be proportioned by their moments of inertia. Plate-girder webs to have a thickness not less than 1/160 of the unsupported distance between flange angles. Webs to have stiffeners, generally in pairs, over bearings, at points of concentrated loading, and at other points where the thickness of the web is less than 1/60 of the unsupported distance between flange angles, generally not farther apart than the depth of the web plate, with a maximum limit of 6 ft.

Lateral unsupported length of beams and girders not to exceed 40 × the width of compression flange. When the unsupported length (l) exceeds 10 × the width (b) of compression flange, the stress per sq. in. in the compression flange shall not exceed 19,000 − (300 l/b).

Details of Steel Construction

General. Adjustable members in any part of structures preferably to be avoided. Sections preferably to be made symmetrical. No connection, except lattice bars, to have less than two rivets. Trusses preferably to be riveted structures. Heavy trusses of long span, where the riveted field connections would become unwieldy, or for other good reasons, may be designed as pin-connected structures.

Abutting joints in compression members faced for bearing to be spliced sufficiently to hold the connecting members accurately in place. All other joints in riveted work whether in tension or compression, to be fully spliced.

Lateral, longitudinal and transverse bracing in all structures preferably to be composed of rigid members, and designed to be sufficient to withstand wind and other lateral forces when building is in process of erection as well as after completion.

Girders. Two or more rolled beams used to form a girder are to be connected by bolts and separators at intervals of not more than 5 ft. All beams having a depth of 12 in. and more to have at least two bolts to each separator. Flange plates of all girders to be limited in width, so as not to extend more than 6 in. beyond the outer line of rivets connecting them to the angles, or 8 × thickness of thinnest plate.

Web stiffeners to be in pairs, and have a close bearing against the flange angles. Those over the end bearing or forming the connection between girder and column to be on fillers. Intermediate stiffeners may be on fillers or crimped over the flange angles. Web plates of girders to be spliced at all points by a plate on each side of the web, capable of transmitting the full stress through splice rivets.

Riveting. Minimum distance between centers of rivet holes to be three diameters of the rivet; but preferably not less than 3 in. for ⅞-in., 2 ½ in. for ¾-in., 2 in. for ⅝-in., and 1¾ in. for ½-in. rivets. Maximum pitch in line of stress for members composed of plates and shapes to be 6 in. for ⅞-in. rivets, 6 in. for ¾in., 4½ in. for ⅝-in., and 4 in. for ½-in. rivets.

For angles in built sections with two gage lines, with rivets staggered, maximum pitch in each line to be twice as great as given above. Where two or more plates are in contact, rivets not more than 12 in. apart in either direction to be used to hold plates together.

Minimum distance from center of any rivet hole to a sheared edge to be 1½ in. for ⅞-in. rivets, 1¼ in. for ¾-in., 1⅛ in. for ⅝-in., and 1 in. for ½-in. rivets; and to a rolled edge, 1¼, 1⅛, 1, and ⅞ in., respectively. Maximum distance from any edge to be 8 × thickness of plate.

Pitch of rivets at ends of built compression members not to exceed four diameters of the rivets for a length equal to 1½ × maximum width of the member.

In transmitting stresses between riveted pieces it is customary to disregard friction and to proportion rivets to the entire stress to be transmitted. They must be of sufficient size and number to resist shear and to afford such bearing area as not to cause distortion of the metal at the rivet holes. In the case of beams which frame opposite and of single-web girders, this latter condition often necessitates a greater thickness of web than required by the shearing stresses.

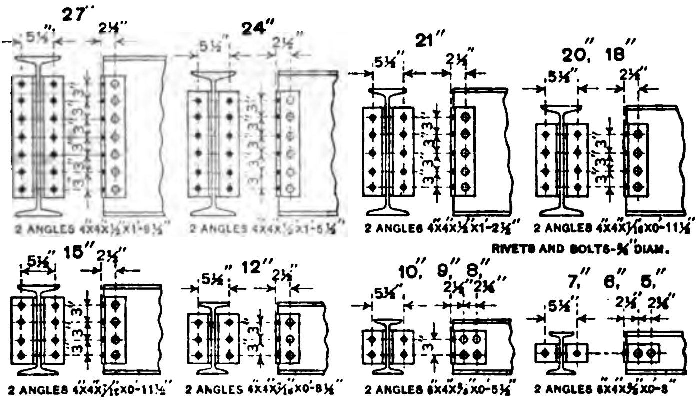
Latticing. Open sides of compression members to be provided with lattice bars, having tie plates at each end and at intermediate points where the lattice is interrupted. Tie plates to be as near the ends as practicable. In main members carrying calculated stresses, end tie plates to have a length not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones not less than half this distance. Their thickness not to be less than 1/50 of same distance.

Latticing of compression members to be proportioned to resist a shearing stress equal to 2 per cent. of the direct stress. Minimum thickness of lattice bars to be for single lattice, 1/40, and for double lattice, 1/60 of the distance between the end rivets. Minimum widths as follows. For 15-in. channels, or built sections with 3½- and 4-in. angles, 2½ in. (⅞-in. rivets); for 12-, 10- and 9-in. channels, or built sections with 3-in. angles, 2¼ in. (¾-in. rivets); for 8- and 7-in. channels or built sections with 2½-in. angles, 2 in. (⅝-in. rivets); for 6- and 5-in. channels, or built sections with 2-in. angles, 1¾ in. (½-in. rivets).

Inclination of lattice bars with axis of member generally to be not less than 45 deg. When distance between rivet lines in flanges is more than 15 in., if a single rivet bar is used, lattice to be double. The pitch of lattice connections along flange, divided by least radius of gyration of member between connections, to be less than the corresponding ratio of member as a whole.

Pins. Pin holes to be reinforced by plates where necessary. At least one plate to be as wide as the projecting flanges will allow; where angles are used, this plate to be on the same side as the angles. Plates to contain sufficient rivets to distribute their portion of the pin pressure to the full cross-section of the member. Pins to be long enough to insure a full bearing of all parts connected upon the turned-down body of the pin. Members packed on pins to be held against lateral movement.

Table 22. Limiting Values of Beam Connections



I-beams		Value of web connection	Values of outstanding legs of connection angles					
			Field rivets			Field bolts		
Depth, in.	Weight, lb. per ft.	Shop rivets in enclosed bearing, lb.	¾ in. rivets or turned bolts, single shear, lb.	Min. allowable span in ft., uniform load	t, in.	¾ in. rough bolts, single shear, lb.	Min. allowable span in ft., uniform load	t, in.
27	83	66,800	61,900	18.4	⅝	49,500	23.1	⅝
24	80	67,500	53,000	17.5	⅝	42,400	21.9	⅝
24	69½	52,700	53,000	16.3	⅝	42,400	20.2	⅝
21	57½	40,200	44,200	15.5	9/16	35,300	17.6	⅝
20	65	45,000	35,300	17.6	⅝	28,300	22.1	⅝
18	55	41,400	35,300	13.3	⅝	28,300	16.7	⅝
18	46	29,000	35,300	15.0	½	28,300	15.4	⅝
15	42	36,900	35,300	8.9	⅜	28,300	11.1	⅝
15	36	26,000	35,300	11.1	7/16	28,300	11.1	9/16
12	31½	23,600	26,500	8.1	9/16	21,200	9.0	⅝
12	27½	17,200	26,500	10.3	7/16	21,200	10.3	½
10	25	27,900	17,700	7.4	⅜	14,100	9.2	⅝
10	22	20,900	17,700	6.9	⅜	14,100	8.6	⅝
9	21	26,100	17,700	5.7	⅜	14,100	7.1	⅝
8	18	24,300	17,700	4.3	⅜	14,100	5.4	⅝
8	17½	18,900	17,700	4.4	⅜	14,100	5.5	⅝
7	15	11,300	8,800	6.2	⅜	7,100	7.8	⅝
6	12¼	10,400	8,800	4.4	⅜	7,100	5.5	⅝
5	9¾	9,500	8,800	2.9	⅜	7,100	3.6	⅝

t = web thickness, in bearing, to develop max. allowable reactions, when beams frame opposite.

Connections are figured for bearing and shear (no moment considered).

The above values agree with tests made on beams under ordinary conditions of use.

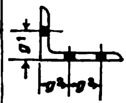
Where web is enclosed between connection angles (enclosed bearing), values are greater because of the increased efficiency due to friction and grip.

Special connections shall be used when any of the limiting conditions given above are exceeded—such as end reaction from loaded beam being greater than value of connection, shorter span with beam fully loaded; or a less thickness of web when maximum allowable reactions are used.

Standard Details and Connections are shown in the figures accompanying Tables 22 and 23. Connections in riveted steel framing (built-up members) are usually made with **gusset plates** (Figs. 22 and 23) and the standard connection angles of Table 22.

Table 23. Gages for Angles, Inches

Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1	¾
g1	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	⅞	⅞	¾	⅝	½
g2	3	2½	2½	2											
g3	3	3	2¼	1¾											
Max. rivet	1⅛	1	⅞	⅞	⅞	⅞	⅞	¾	⅝	½	⅜	⅜	⅜	¼	¼



For column details, 6 in. leg (½ in. thick or less) against column shaft, g2 = 1¾ in., g3 = 3 in. For diagonal angles, etc., gage in middle, where riveted leg equals or exceeds 3 in. for ¾ in. rivets, 3½ in. for ⅞ in. rivets. Use special gages to adapt work to multiple punch, or to secure desirable details.

Properties of Structural Steel

Safe Loads for I-beams. The coefficient of strength, fiber stress 16,000 lb. per sq. in., given in Tables 25 to 29, is the safe load in pounds uniformly distributed, including the weight of the beam itself, for a beam 1 ft. long. The safe uniformly distributed load for any span, so far as flexural stresses are concerned, is obtained by dividing the proper coefficient by the span in feet. If it is desired to use some stress other than 16,000 lb. per sq. in., multiply the tabular coefficient by the ratio $f/16,000$, where f is the fiber stress to be employed.

To select a beam to support a given uniformly distributed load, multiply the given total load by the span in feet, and by the ratio $f/16,000$, and refer to the table for a beam having that coefficient.

For a given load concentrated at the center of the span, multiply the given concentrated load by 2 and consider the result as an equivalent uniformly distributed load.

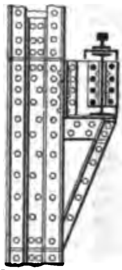
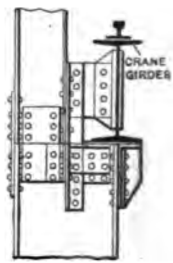


PLATE AND ANGLE COLUMN

FIG. 22.



BETHLEHEM COLUMN

FIG. 23.

Short beams should be investigated for shear, by dividing the maximum shear, in pounds, by the area of the web, excluding the flanges.

Formulæ for the safe loads and deflections of beams with various methods of support and of loading are given in Tables 12*a* and 12*b*, p. 402.

Deflection of I-beams and Other Structural Shapes. Table 24 gives **coefficients of deflection** for steel shapes under uniformly distributed loads, and is based on the formula: Deflection in inches = $30fL^2/Ed$, the table giving the values of $30fL^2/E$. (f = fiber stress, lb. per sq. in., L = span, ft.; d = depth of section, in.).

To find the deflection in inches of a section symmetrical about the neutral axis, such as a beam, channel, zee, etc., divide the coefficient in the table corresponding to given span and fiber stress by the depth of the section in inches.

To find the deflection in inches of a section not symmetrical about the neutral axis, such as an angle, tee, etc., divide the coefficient corresponding to given span and fiber stress by twice the distance of extreme fiber from neutral axis obtained from table of elements of sections.

To find the deflection in inches of a section for any other fiber stress than those given, multiply this fiber stress by any of the coefficients in the table for the given span and divide by the fiber stress corresponding to the coefficient used.

Table 24. Coefficients of Deflection for Steel Shapes Under Uniformly Distributed Loads

(Am. Bridge Co. For a load concentrated at the center of the span, producing a stress given in the table, use ⅘ of the coefficient given)

Span, ft.	Fiber stress, lb. per sq. in.			Span, ft.	Fiber stress, lb. per sq. in.			Span, ft.	Fiber stress, lb. per sq. in.		
	16,000	14,000	12,500		16,000	14,000	12,500		16,000	14,000	12,500
1	0 017	0.014	0.013	18	5.363	4.692	4.190	35	20.276	17.741	15.841
2	0 066	0.058	0.052	19	5.975	5.228	4.668	36	21.451	18.770	16.759
3	0.149	0.130	0.116	20	6.621	5.793	5.172	37	22.659	19.827	17.703
4	0.265	0.232	0.207	21	7.299	6.387	5.703	38	23.901	20.913	18.672
5	0.414	0.362	0.323	22	8.011	7.010	6.259	39	25.175	22.028	19.668
6	0.596	0.521	0.466	23	8.756	7.661	6.841	40	26.483	23.172	20.690
7	0.811	0.710	0.634	24	9.534	8.342	7.448	41	27.824	24.346	21.737
8	1.059	0.927	0.828	25	10.345	9.052	8.082	42	29.197	25.548	22.810
9	1.341	1.173	1.047	26	11.189	9.790	8.741	43	30.603	26.779	23.909
10	1.655	1.448	1.293	27	12.066	10.558	9.427	44	31.954	28.039	25.034
11	2.003	1.752	1.565	28	12.977	11.354	10.138	45	33.517	29.328	26.185
12	2.383	2.086	1.862	29	13.920	12.180	10.875	46	35.023	30.646	27.362
13	2.797	2.448	2.185	30	14.897	13.034	11.638	47	36.562	31.992	28.565
14	3.244	2.839	2.534	31	15.906	13.918	12.427	48	37.135	33.368	29.793
15	3.724	3.259	2.909	32	16.949	14.830	13.241	49	39.741	34.773	31.047
16	4.237	3.708	3.310	33	18.025	15.772	14.082	50	41.379	36.207	32.328
17	4.783	4.186	3.737	34	19.134	16.742	14.948				

Properties of Standard Structural Shapes. Tables 25–30 give the properties of Carnegie standard I-beams, channels, angles, tees and zees. In these tables I = moment of inertia, r = radius of gyration, S = section modulus, and x = distance from neutral axis to face.

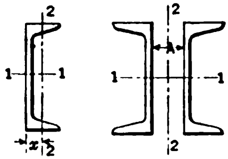
Table 25. Standard I-beams

Section index	Depth of beam	Weight per ft.	Area of section	Width of flange	Thickness of web	Neutral axis perpendicular to web at center			Neutral axis coincident with center line of web			Coefficient of strength in thousands of pounds for fiber stress of 16,000 lb. per sq. in.
						I	r	S	I	r	S	
	In.	Lb.	Sq. in.	In.	In.	In.⁴	In.	In.³	In.⁴	In.	In.³	
B 31	27	83.0	24.41	7.500	0.424	2888.6	10.88	214.0	53.1	1.47	14.1	2282
B 24	24	115.0	33.98	8.000	0.750	2955.5	9.33	246.3	83.2	1.57	20.8	2627
		110.0	32.48	7.938	0.688	2883.5	9.42	240.3	81.0	1.58	20.4	2563
		105.0	30.98	7.875	0.625	2811.5	9.53	234.3	78.9	1.60	20.0	2499
B 1	24	100.0	29.41	7.254	0.754	2379.6	9.00	198.3	48.6	1.28	13.4	2115
		95.0	27.94	7.193	0.693	2309.0	9.09	192.4	47.1	1.30	13.1	2052
		90.0	26.47	7.131	0.631	2238.4	9.20	186.5	45.7	1.31	12.8	1990
		85.0	25.00	7.070	0.570	2167.8	9.31	180.7	44.4	1.33	12.6	1927
		80.0	23.32	7.000	0.500	2087.2	9.46	173.9	42.9	1.36	12.3	1855
B 32	24	69.5	20.44	7.000	0.390	1928.0	9.71	160.7	39.3	1.39	11.2	1714
B 33	21	57.5	16.85	6.500	0.357	1227.5	8.54	116.9	28.4	1.30	8.8	1247
B 2	20	100.0	29.41	7.284	0.884	1655.6	7.50	165.6	52.7	1.34	14.5	1766
		95.0	27.94	7.210	0.810	1606.6	7.58	160.7	50.8	1.35	14.1	1714
		90.0	26.47	7.137	0.737	1557.6	7.67	155.8	49.0	1.36	13.7	1661
		85.0	25.00	7.063	0.663	1508.5	7.77	150.9	47.3	1.37	13.4	1609
		80.0	23.73	7.000	0.600	1466.3	7.86	146.6	45.8	1.39	13.1	1564

Table 25. Standard I-beams—(*continued*)

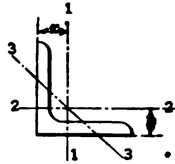
Section index	Depth of beam	Weight per ft.	Area of section	Width of flange	Thickness of web	Neutral axis perpendicular to web at center			Neutral axis coincident with center line of web			Coefficient of strength in thousands of pounds for fiber stress of 16,000 lb. per sq. in.
						I	r	S	I	r	S	
	In.	Lb.	Sq. in.	In.	In.	In.[4]	In.	In.[3]	In.[4]	In.	In.	
		75.0	22.06	6.399	0.649	1268.8	7.58	126.9	30.3	1.17	9.5	1353
B 3	20	70.0	20.59	6.325	0.575	1219.8	7.70	122.0	29.0	1.19	9.2	1301
		65.0	19.08	6.250	0.500	1169.5	7.83	117.0	27.9	1.21	8.9	1248
		90.0	26.47	7.245	0.807	1260.4	6.90	140.0	52.0	1.40	14.4	1494
		85.0	25.00	7.163	0.725	1220.7	6.99	135.6	50.0	1.42	14.0	1447
B 81	18	80.0	23.53	7.082	0.644	1181.0	7.09	131.2	48.1	1.43	13.6	1400
		75.0	22.05	7.000	0.562	1141.3	7.19	126.8	46.2	1.45	13.2	1353
		70.0	20.59	6.259	0.719	921.2	6.69	102.4	24.6	1.09	7.9	1092
B 80	18	65.0	19.12	6.177	0.637	881.5	6.79	97.9	23.5	1.11	7.6	1045
		60.0	17.65	6.095	0.555	841.8	6.91	93.5	22.4	1.13	7.3	998
		55.0	15.93	6.000	0.460	795.6	7.07	88.4	21.2	1.15	7.1	943
B 34	18	46.0	13.53	6.000	0.322	733.2	7.36	81.5	19.9	1.21	6.6	869
		75.0	22.06	6.292	0.882	691.2	5.60	92.2	30.7	1.18	9.8	983
B 5	15	70.0	20.59	6.194	0.784	663.7	5.68	88.5	29.0	1.19	9.4	944
		65.0	19.12	6.096	0.686	636.1	5.77	84.8	27.4	1.20	9.0	905
		60.0	17.67	6.000	0.590	609.0	5.87	81.2	26.0	1.21	8.7	866
		55.0	16.18	5.746	0.656	511.0	5.62	68.1	17.1	1.02	5.9	727
B 7	15	50.0	14.71	5.648	0.558	483.4	5.73	64.5	16.0	1.04	5.7	688
		45.0	13.24	5.550	0.460	455.9	5.87	60.8	15.1	1.07	5.4	648
		42.0	12.48	5.500	0.410	441.8	5.95	58.9	14.6	1.08	5.3	628
B 35	15	36.0	10.63	5.500	0.289	405.1	6.17	54.0	13.5	1.13	4.9	576
		55.0	16.18	5.611	0.821	321.0	4.45	53.5	17.5	1.04	6.2	571
B 8	12	50.0	14.71	5.489	0.699	303.4	4.54	50.6	16.1	1.05	5.9	539
		45.0	13.24	5.366	0.576	285.7	4.65	47.6	14.9	1.06	5.6	508
		40.0	11.84	5.250	0.460	269.0	4.77	44.8	13.8	1.08	5.3	478
B 9	12	35.0	10.29	5.086	0.436	228.3	4.71	38.0	10.1	0.99	4.0	406
		31.5	9.26	5.000	0.350	215.8	4.83	36.0	9.5	1.01	3.8	384
B 36	12	27.5	8.04	5.000	0.255	199.6	4.98	33.3	8.7	1.04	3.5	355
		40.0	11.76	5.099	0.749	158.7	3.67	31.7	9.5	0.90	3.7	339
B 11	10	35.0	10.29	4.952	0.602	146.4	3.77	29.3	8.5	0.91	3.4	312
		30.0	8.82	4.805	0.455	134.2	3.90	26.8	7.7	0.93	3.2	286
		25.0	7.37	4.660	0.310	122.1	4.07	24.4	6.9	0.97	3.0	261
B 37	10	22.0	6.52	4.670	0.232	113.9	4.18	22.8	6.4	0.99	2.7	243
		35.0	10.29	4.772	0.732	111.8	3.29	24.8	7.3	0.84	3.1	265
B 13	9	30.0	8.82	4.609	0.569	101.9	3.40	22.6	6.4	0.85	2.8	242
		25.0	7.35	4.446	0.406	91.9	3.54	20.4	5.7	0.88	2.5	218
		21.0	6.31	4.330	0.290	84.9	3.67	18.9	5.2	0.90	2.4	201
		25.5	7.50	4.271	0.541	68.4	3.02	17.1	4.8	0.80	2.2	183
B 15	8	23.0	6.76	4.179	0.449	64.5	3.09	16.1	4.4	0.81	2.1	172
		20.5	6.03	4.087	0.357	60.6	3.17	15.2	4.1	0.82	2.0	162
		18.0	5.33	4.000	0.270	56.9	3.27	14.2	3.8	0.84	1.9	152
B 38	8	17.5	5.15	4.330	0.210	58.3	3.37	14.6	4.5	0.93	2.1	156
		20.0	5.88	3.868	0.458	42.2	2.68	12.1	3.2	0.74	1.7	129
B 17	7	17.5	5.15	3.763	0.353	39.2	2.76	11.2	2.9	0.76	1.6	119
		15.0	4.42	3.660	0.250	36.2	2.86	10.4	2.7	0.78	1.5	110
		17.25	5.07	3.575	0.475	26.2	2.27	8.7	2.4	0.68	1.3	93
B 19	6	14.75	4.34	3.452	0.352	24.0	2.35	8.0	2.1	0.69	1.2	85
		12.25	3.61	3.330	0.230	21.8	2.46	7.3	1.9	0.72	1.1	78
		14.75	4.34	3.294	0.504	15.2	1.87	6.1	1.7	0.63	1.0	65
B 21	5	12.25	3.60	3.147	0.357	13.6	1.94	5.5	1.5	0.63	0.92	58
		9.75	2.87	3.000	0.210	12.1	2.05	4.8	1.2	0.65	0.82	52
		10.5	3.09	2.880	0.410	7.1	1.52	3.6	1.0	0.57	0.70	38
B 23	4	9.5	2.79	2.807	0.337	6.8	1.55	3.4	0.93	0.58	0.66	36
		8.5	2.50	2.733	0.263	6.4	1.59	3.2	0.85	0.58	0.62	34
		7.5	2.21	2.660	0.190	6.0	1.64	3.0	0.77	0.59	0.58	32
		7.5	2.21	2.521	0.361	2.9	1.15	1.9	0.60	0.52	0.48	21
B 77	3	6.5	1.91	2.423	0.263	2.7	1.19	1.8	0.53	0.52	0.44	19
		5.5	1.63	2.330	0.170	2.5	1.23	1.7	0.46	0.53	0.40	18

Table 26. Standard Channels



Depth of channel	Weight per ft.	Area of section	Width of flange	Thickness of web	Axis 1-1 I	Axis 1-1 r	Axis 1-1 S	Axis 2-2 r	x	Distance, A, back to back required to make radii of gyration equal, in.	Coefficient of strength in thousands of pounds for fiber stress of 16,000 lb. per sq. in.
In.	Lb.	Sq. in.	In.	In.	In.⁴	In.	In.³	In.	In.		
15	55.0	16.18	3.818	0.818	430.2	5.16	57.4	0.87	0.82	8.53	612
	50.0	14.71	3.720	0.720	402.7	5.23	53.7	0.87	0.80	8.72	573
	45.0	13.24	3.622	0.622	375.1	5.32	50.0	0.88	0.79	8.92	533
	40.0	11.76	3.524	0.524	347.5	5.43	46.3	0.89	0.78	9.16	494
	35.0	10.29	3.426	0.426	319.9	5.58	42.7	0.91	0.79	9.42	455
	33.0	9.90	3.400	0.400	312.6	5.62	41.7	0.91	0.79	9.51	445
12	40.0	11.76	3.418	0.758	196.9	4.09	32.8	0.75	0.72	6.60	350
	35.0	10.29	3.296	0.636	179.3	4.17	29.9	0.76	0.69	6.83	319
	30.0	8.82	3.173	0.513	161.7	4.28	26.9	0.77	0.68	7.06	287
	25.0	7.35	3.050	0.390	144.0	4.43	24.0	0.79	0.68	7.35	256
	20.5	6.03	2.940	0.280	128.1	4.61	21.4	0.81	0.70	7.68	228
10	35.0	10.29	3.183	0.823	115.5	3.35	23.1	0.67	0.70	5.18	246
	30.0	8.82	3.036	0.676	103.2	3.42	20.7	0.67	0.65	5.41	220
	25.0	7.35	2.889	0.529	91.0	3.52	18.2	0.68	0.62	5.66	194
	20.0	5.88	2.742	0.382	78.7	3.66	15.7	0.70	0.61	5.96	168
	15.0	4.46	2.600	0.240	66.9	3.87	13.4	0.72	0.64	6.33	143
9	25.0	7.35	2.815	0.615	70.7	3.10	15.7	0.64	0.62	4.83	168
	20.0	5.88	2.652	0.452	60.8	3.21	13.5	0.65	0.59	5.14	144
	15.0	4.41	2.488	0.288	50.9	3.40	11.3	0.67	0.59	5.48	121
	13.25	3.89	2.430	0.230	47.3	3.49	10.5	0.67	0.61	5.62	112
8	21.25	6.25	2.622	0.582	47.8	2.77	11.9	0.60	0.59	4.22	127
	18.75	5.51	2.530	0.490	43.8	2.82	11.0	0.60	0.57	4.37	117
	16.25	4.78	2.439	0.399	39.9	2.89	10.0	0.61	0.56	4.53	106
	13.75	4.04	2.347	0.307	36.0	2.98	9.0	0.62	0.56	4.72	96
	11.25	3.35	2.260	0.220	32.3	3.11	8.1	0.63	0.58	4.92	86
7	19.75	5.81	2.513	0.633	33.2	2.39	9.5	0.56	0.58	3.49	101
	17.25	5.07	2.408	0.528	30.2	2.44	8.6	0.57	0.56	3.65	92
	14.75	4.34	2.303	0.423	27.2	2.50	7.8	0.57	0.54	3.82	83
	12.25	3.60	2.198	0.318	24.2	2.59	6.9	0.58	0.53	4.00	74
	9.75	2.85	2.090	0.210	21.1	2.72	6.0	0.59	0.55	4.21	64
6	15.5	4.56	2.283	0.563	19.5	2.07	6.5	0.53	0.55	2.90	69
	13.0	3.82	2.160	0.440	17.3	2.13	5.8	0.53	0.52	3.08	62
	10.5	3.09	2.038	0.318	15.1	2.21	5.0	0.53	0.50	3.29	54
	8.0	2.38	1.920	0.200	13.0	2.34	4.3	0.54	0.52	3.51	46
5	11.5	3.38	2.037	0.477	10.4	1.75	4.2	0.49	0.51	2.35	44
	9.0	2.65	1.890	0.330	8.9	1.83	3.6	0.49	0.48	2.57	38
	6.5	1.95	1.750	0.190	7.4	1.95	3.0	0.50	0.49	2.79	32
4	7.25	2.13	1.725	0.325	4.6	1.46	2.3	0.46	0.46	1.88	24
	6.25	1.84	1.652	0.252	4.2	1.51	2.1	0.45	0.46	1.96	22
	5.25	1.55	1.580	0.180	3.8	1.56	1.9	0.45	0.46	2.08	20
3	6.0	1.76	1.602	0.362	2.1	1.08	1.4	0.42	0.46	1.10	14
	5.0	1.47	1.504	0.264	1.8	1.12	1.2	0.42	0.44	1.17	13
	4.0	1.19	1.410	0.170	1.6	1.17	1.1	0.41	0.44	1.29	12

Table 27. Selected Standard Angles, Equal Legs

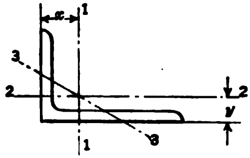


A single angle should never be used as a beam. Two angles, riveted at frequent intervals, should always be used.

Size	Weight per ft.	Area of section	Axis 1-1 and Axis 2-2				Axis 3-3	Coefficient of strength in thousands of pounds for fiber stress of 16,000 lb. per sq. in.
			I	r	S	x	r min.	
In.	Lb.	Sq. in.	In.^{4}	In.	In.^{3}	In.	In.	
8 × 8 × 1⅛	56.9	16.73	98.0	2.42	17.5	2.41	1.55	186.99
8 × 8 × 1	51.0	15.00	89.0	2.44	15.8	2.37	1.56	168.53
8 × 8 × ⅞	45.0	13.23	79.6	2.45	14.0	2.32	1.56	149.55
8 × 8 × ¾	38.9	11.44	69.7	2.47	12.2	2.28	1.57	130 03
8 × 8 × ⅝	32.7	9.61	59.4	2.49	10.3	2.23	1.58	109.87
8 × 8 × ½	26.4	7.75	48.6	2.51	8.4	2.19	1.58	89.28
6 × 6 × 1	37.4	11.00	35.5	1.80	8.6	1.86	1.16	91.41
6 × 6 × ⅞	33.1	9.73	31.9	1.81	7.6	1.82	1.17	81.39
6 × 6 × ¾	28.7	8.44	28.2	1.83	6.7	1.78	1.17	71.04
6 × 6 × ⅝	24.2	7.11	24.2	1.84	5.7	1.73	1.17	60.37
6 × 6 × ½	19.6	5.75	19.9	1.86	4.6	1.68	1.18	49.17
6 × 6 × ⅜	14.9	4.36	15.4	1.88	3.5	1.64	1.19	37.65
5 × 5 × 1	30.6	9.00	19.6	1.48	5.8	1.61	0.96	61.87
5 × 5 × ⅞	27.2	7.98	17.8	1.49	5.2	1.57	0.96	55.15
5 × 5 × ¾	23.6	6.94	15.7	1.50	4.5	1.52	0.97	48.32
5 × 5 × ⅝	20.0	5.86	13.6	1.52	3.9	1.48	0.97	41.17
5 × 5 × ½	16.2	4.75	11.3	1.54	3.2	1.43	0.98	33.60
5 × 5 × ⅜	12.3	3.61	8.7	1.56	2.4	1.39	0.99	25.81
4 × 4 × ¾	18.5	5.44	7.7	1.19	2.8	1.27	0.77	29.97
4 × 4 × ⅝	15.7	4.61	6.7	1.20	2.4	1.23	0.77	25.60
4 × 4 × ½	12.8	3.75	5.6	1.22	2.0	1.18	0.78	21.01
4 × 4 × ⅜	9.8	2.86	4.4	1.23	1.5	1.14	0.79	16.21
3½ × 3½ × ¾	16.0	4.69	5.0	1.03	2.1	1.15	0.67	22.51
3½ × 3½ × ⅝	13.6	3.98	4.3	1.04	1.8	1.10	0.68	19.31
3½ × 3½ × ½	11.1	3.25	3.6	1.06	1.5	1.06	0.68	15.89
3½ × 3½ × ⅜	8.5	2.48	2.9	1.07	1.2	1.01	0.69	12.27
3½ × 3½ × ¼	5.8	1.69	2.0	1.09	0.79	0.97	0.69	8.43
3 × 3 × ⅝	11.5	3.36	2.6	0.88	1.3	0.98	0.57	13.87
3 × 3 × 9/16	10.4	3.06	2.4	0.89	1.2	0.95	0.58	12.69
3 × 3 × 7/16	8.3	2.43	2.0	0.91	0.95	0.91	0.58	10.13
3 × 3 × 5/16	6.1	1.78	1.5	0.92	0.71	0.87	0.59	7.57
3 × 3 × ¼	4.9	1.44	1.2	0.93	0.58	0.84	0.59	6.19
2½ × 2½ × ½	7.7	2.25	1.2	0.74	0.73	0.81	0.47	7.79
2½ × 2½ × 7/16	6.8	2.00	1.1	0.75	0.65	0.78	0.48	6.93
2½ × 2½ × 5/16	5.0	1.47	0.85	0.76	0.48	0.74	0.49	5.12
2½ × 2½ × 3/16	3.07	0.90	0.55	0.78	0.30	0.69	0.49	3.20
2½ × 2½ × ⅛	2.08	0.61	0.38	0.79	0.20	0.67	0.50	2.13
2 × 2 × ⅜	4.7	1.36	0.48	0.59	0.35	0.64	0.39	3.73
2 × 2 × ¼	3.19	0.94	0.35	0.61	0.25	0.59	0.39	2.67
2 × 2 × ⅛	1.65	0.48	0.19	0.63	0.13	0.55	0.40	1.39
1¾ × 1¾ × ⅜	3.99	1.17	0.31	0.51	0.26	0.57	0.34	2.77
1¾ × 1¾ × ¼	2.77	0.81	0.23	0.53	0.19	0.53	0.34	2.03
1¾ × 1¾ × ⅛	1.44	0.42	0.13	0.55	0.10	0.48	0.35	1.07
1½ × 1½ × ⅜	3.35	0.98	0.19	0.44	0.19	0.51	0.29	2.03
1½ × 1½ × ¼	2.34	0.69	0.14	0.45	0.13	0.47	0.29	1.39
1½ × 1½ × ⅛	1.23	0.36	0.08	0.46	0.07	0.42	0.30	0.77
1¼ × 1¼ × ¼	1.92	0.56	0.08	0.37	0.09	0.40	0.24	0.97
1¼ × 1¼ × ⅛	1.01	0.30	0.04	0.38	0.05	0.35	0.25	0.52
1 × 1 × ¼	1.49	0.44	0.04	0.29	0.06	0.34	0.19	0.60
1 × 1 × ⅛	0.80	0.23	0.02	0.31	0.03	0.30	0.19	0.33

Table 28. Selected Standard Angles, Unequal Legs

(Intermediate sizes vary in thickness only, and by intervals of 1/16 in.)



Section index	Size	Weight per ft.	Area of section	Axis 1-1				Axis 2-2				Axis 3-3	Coefficient of strength,* Axis 1-1
				I	r	S	x	I	r	S	y	r min.	
	In.	Lb.	Sq. in.	In.⁴	In.	In.³	In.	In.⁴	In.	In.³	In.	In.	
A 138	8 × 6 × 1	44.2	13.00	80.8	2.49	15.1	2.65	38.8	1.73	8.9	1.65	1.28	161.2
A 135	8 × 6 × 13/16	36.5	10.72	67.9	2.52	12.5	2.59	32.8	1.75	7.4	1.59	1.29	133.9
A 132	8 × 6 × 5/8	28.5	8.36	54.1	2.54	9.9	2.52	26.3	1.77	5.9	1.52	1.30	105.3
A 139	8 × 6 × 7/16	20.2	5.93	39.2	2.57	7.1	2.45	19.3	1.80	4.2	1.45	1.30	75.4
A 320	8 × 3½ × 1	35.7	10.50	66.2	2.51	13.7	3.17	7.8	0.86	3.0	0.92	0.73	146.0
A 323	8 × 3½ × 13/16	29.6	8.68	55.9	2.54	11.4	3.10	6.7	0.88	2.5	0.85	0.73	121.6
A 326	8 × 3½ × 5/8	23.2	6.80	44.7	2.57	9.0	3.03	5.4	0.90	2.0	0.78	0.74	95.8
A 329	8 × 3½ × 7/16	16.5	4.84	32.5	2.59	6.4	2.95	4.1	0.92	1.5	0.70	0.74	68.8
A 150	7 × 3½ × 1	32.3	9.50	45.4	2.19	10.6	2.70	7.5	0.89	3.0	0.96	0.74	112.9
A 153	7 × 3½ × 13/16	26.8	7.87	38.4	2.21	8.8	2.64	6.5	0.91	2.5	0.89	0.74	94.1
A 156	7 × 3½ × 5/8	21.0	6.17	30.9	2.24	7.0	2.57	5.3	0.93	2.0	0.82	0.75	74.4
A 310	7 × 3½ × 3/8	13.0	3.80	19.6	2.27	4.3	2.48	3.5	0.96	1.3	0.73	0.76	46.2
A 89	6 × 4 × 1	30.6	9.00	30.8	1.85	8.0	2.17	10.8	1.09	3.8	1.17	0.85	85.6
A 161	6 × 4 × 13/16	25.4	7.47	26.1	1.87	6.7	2.10	9.2	1.11	3.2	1.10	0.86	71.5
A 164	6 × 4 × 5/8	20.0	5.86	21.1	1.90	5.3	2.03	7.5	1.13	2.5	1.03	0.86	56.6
A 168	6 × 4 × 3/8	12.3	3.61	13.5	1.93	3.3	1.94	4.9	1.17	1.6	0.94	0.88	35.4
A 92	6 × 3½ × 1	28.9	8.50	29.2	1.85	7.8	2.26	7.2	0.92	2.9	1.01	0.74	83.5
A 170	6 × 3½ × 13/16	24.0	7.06	24.9	1.88	6.6	2.20	6.2	0.94	2.4	0.95	0.75	69.9
A 173	6 × 3½ × 5/8	18.9	5.55	20.1	1.90	5.2	2.13	5.1	0.96	1.9	0.88	0.75	55.4
A 177	6 × 3½ × 3/8	11.7	3.42	12.9	1.94	3.3	2.04	3.3	0.99	1.2	0.78	0.77	34.7
A 178	5 × 4 × 7/8	24.2	7.11	16.4	1.52	5.0	1.71	9.2	1.14	3.3	1.21	0.84	53.2
A 181	5 × 4 × 11/16	19.5	5.72	13.6	1.54	4.1	1.64	7.7	1.16	2.7	1.14	0.84	43.2
A 185	5 × 4 × 7/16	12.8	3.75	9.3	1.58	2.7	1.55	5.3	1.19	1.8	1.05	0.85	28.8
A 187	5 × 3½ × 7/8	22.7	6.67	15.7	1.53	4.9	1.79	6.2	0.96	2.5	1.04	0.75	52.1
A 190	5 × 3½ × 11/16	18.3	5.37	13.0	1.56	4.0	1.72	5.2	0.98	2.1	0.97	0.75	42.4
A 193	5 × 3½ × ½	13.6	4.00	10.0	1.58	3.0	1.66	4.0	1.01	1.6	0.91	0.75	31.9
A 96	5 × 3½ × 5/16	8.7	2.56	6.6	1.61	1.9	1.59	2.7	1.03	1.0	0.84	0.76	20.7
A 196	5 × 3 × 13/16	19.9	5.84	14.0	1.55	4.5	1.86	3.7	0.80	1.7	0.86	0.64	47.5
A 199	5 × 3 × 5/8	15.7	4.61	11.4	1.57	3.5	1.80	3.1	0.81	1.4	0.80	0.64	37.9
A 203	5 × 3 × 3/8	9.8	2.86	7.4	1.61	2.2	1.70	2.0	0.84	0.89	0.70	0.65	23.9
A 204	4½ × 3 × 13/16	18.5	5.43	10.3	1.38	3.6	1.65	3.6	0.81	1.7	0.90	0.64	38.6
A 207	4½ × 3 × 5/8	14.7	4.30	8.4	1.40	2.9	1.58	3.0	0.83	1.4	0.83	0.64	30.8
A 211	4½ × 3 × 3/8	9.1	2.67	5.5	1.44	1.8	1.49	2.0	0.86	0.88	0.74	0.66	19.5
A 212	4 × 3½ × 13/16	18.5	5.43	7.8	1.19	2.9	1.36	5.5	1.01	2.3	1.11	0.72	31.2
A 215	4 × 3½ × 5/8	14.7	4.30	6.4	1.22	2.4	1.29	4.5	1.03	1.8	1.04	0.72	25.1
A 219	4 × 3½ × 3/8	9.1	2.67	4.2	1.25	1.5	1.21	3.0	1.06	1.2	0.96	0.73	16.0
A 220	4 × 3 × 13/16	17.1	5.03	7.3	1.21	2.9	1.44	3.5	0.83	1.7	0.94	0.64	30.6
A 223	4 × 3 × 5/8	13.6	3.98	6.0	1.23	2.3	1.37	2.9	0.85	1.4	0.87	0.64	24.5
A 226	4 × 3 × 7/16	9.8	2.87	4.5	1.25	1.7	1.30	2.2	0.87	1.0	0.80	0.64	17.9
A 283	4 × 3 × ¼	5.8	1.69	2.8	1.28	1.0	1.24	1.4	0.89	0.60	0.74	0.65	10.7
A 229	3½ × 3 × 13/16	15.8	4.62	5.0	1.04	2.2	1.23	3.3	0.85	1.7	0.98	0.62	23.5
A 232	3½ × 3 × 5/8	12.5	3.67	4.1	1.06	1.8	1.17	2.8	0.87	1.3	0.92	0.62	18.8
A 235	3½ × 3 × 7/16	9.1	2.65	3.1	1.08	1.3	1.10	2.1	0.89	0.98	0.85	0.62	13.8
A 286	3½ × 3 × ¼	5.4	1.56	1.9	1.11	0.78	1.04	1.3	0.91	0.58	0.79	0.63	8.32
A 238	3½ × 2½ × 11/16	12.5	3.65	4.1	1.06	1.9	1.27	1.7	0.69	0.99	0.77	0.53	19.7
A 241	3½ × 2½ × ½	9.4	2.75	3.2	1.09	1.4	1.20	1.4	0.70	0.76	0.70	0.53	15.0
A 245	3½ × 2½ × ¼	4.9	1.44	1.8	1.12	0.75	1.11	0.78	0.74	0.41	0.61	0.54	8.00
A 252	3 × 2½ × 9/16	9.5	2.78	2.3	0.91	1.2	1.02	1.4	0.72	0.82	0.77	0.52	12.27
A 254	3 × 2½ × 7/16	7.6	2.21	1.9	0.92	0.93	0.98	1.2	0.73	0.66	0.73	0.52	9.92
A 255	3 × 2½ × 3/8	6.6	1.92	1.7	0.93	0.81	0.96	1.0	0.74	0.58	0.71	0.52	8.64
A 257	3 × 2½ × ¼	4.5	1.31	1.2	0.95	0.56	0.91	0.74	0.75	0.40	0.66	0.53	5.97

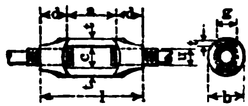
* Coefficient of strength in thousands of pounds. Fiber stress 16,000 lb. per sq. in.

Table 28. Selected Standard Angles, Unequal Legs—(*continued*)

Section index	Size	Weight per ft.	Area of section	Axis 1-1				Axis 2-2				Axis 3-3	Coefficient of strength.* Axis 1-1
				I	*r*	*S*	*x*	*I*	*r*	*S*	*y*	*r* min.	
	In.	Lb.	Sq. in.	In.⁴	In.	In.³	In.	In.⁴	In.	In.³	In.	In.	
A 258	3 × 2 × 1/2	7.7	2.25	1.9	0.92	1.0	1.08	0.67	0.55	0.47	0.58	0.43	10.67
A 260	3 × 2 × 3/8	5.9	1.73	1.5	0.94	0.78	1.04	0.54	0.56	0.37	0.54	0.43	8.32
A 262	3 × 2 × 1/4	4.1	1.19	1.1	0.95	0.54	0.99	0.39	0.57	0.25	0.49	0.43	5.76
A 264	2 1/2 × 2 × 1/2	6.8	2.00	1.1	0.75	0.70	0.88	0.64	0.56	0.46	0.63	0.42	7.47
A 267	2 1/2 × 2 × 5/16	4.5	1.31	0.79	0.78	0.47	0.81	0.45	0.58	0.31	0.56	0.42	5.01
A 268	2 1/2 × 2 × 1/4	3.62	1.06	0.65	0.78	0.38	0.79	0.37	0.59	0.25	0.54	0.42	4.05
A 523	2 1/2 × 2 × 1/8	1.86	0.55	0.35	0.89	0.20	0.74	0.20	0.61	0.13	0.49	0.43	2.13
A 610	2 1/2 × 1 1/2 × 5/16	3.92	1.15	0.71	0.79	0.44	0.90	0.19	0.41	0.17	0.40	0.32	4.69
A 612	2 1/2 × 1 1/2 × 3/16	2.44	0.72	0.46	0.80	0.28	0.85	0.13	0.42	0.11	0.35	0.33	2.99
A 270	2 1/4 × 1 1/2 × 1/2	5.6	1.63	0.75	0.68	0.54	0.86	0.26	0.40	0.26	0.48	0.32	5.76
A 272	2 1/4 × 1 1/2 × 3/8	4.4	1.27	0.61	0.69	0.42	0.81	0.21	0.41	0.20	0.44	0.32	4.48
A 273	2 1/4 × 1 1/2 × 5/16	3.66	1.07	0.53	0.70	0.36	0.79	0.19	0.42	0.17	0.42	0.32	3.84
A 275	2 1/4 × 1 1/2 × 3/16	2.28	0.67	0.34	0.72	0.23	0.75	0.12	0.43	0.11	0.37	0.33	2.45
A 631	2 × 1 1/2 × 3/8	3.99	1.17	0.43	0.61	0.34	0.71	0.21	0.42	0.20	0.46	0.32	3.63
A 615	2 × 1 1/2 × 1/4	2.77	0.81	0.32	0.62	0.24	0.66	0.15	0.43	0.14	0.41	0.32	2.56
A 525	2 × 1 1/2 × 1/8	1.44	0.42	0.17	0.64	0.13	0.62	0.09	0.45	0.08	0.37	0.33	1.39
A 646	2 × 1 1/4 × 1/4	2.55	0.75	0.30	0.63	0.23	0.71	0.09	0.34	0.10	0.33	0.27	2.45
A 645	2 × 1 1/4 × 3/16	1.96	0.57	0.23	0.64	0.18	0.69	0.07	0.35	0.08	0.31	0.27	1.92
A 618	1 3/4 × 1 1/4 × 1/4	2.34	0.69	0.20	0.54	0.18	0.60	0.09	0.35	0.10	0.35	0.27	1.92
A 620	1 3/4 × 1 1/4 × 1/8	1.23	0.36	0.11	0.56	0.09	0.56	0.05	0.37	0.05	0.31	0.27	1.00
A 670	1 1/2 × 1 1/4 × 5/16	2.59	0.76	0.16	0.45	0.16	0.52	0.10	0.35	0.11	0.40	0.26	1.71
A 623	1 1/2 × 1 1/4 × 1/4	2.13	0.63	0.13	0.46	0.13	0.50	0.08	0.36	0.09	0.38	0.26	1.39
A 624	1 1/2 × 1 1/4 × 3/16	1.64	0.48	0.10	0.46	0.10	0.48	0.07	0.37	0.07	0.35	0.26	1.07

* Coefficient of strength in thousands of pounds. Fiber stress 16,000 lb. per sq. in.

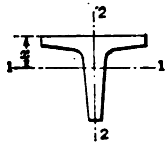
Turnbuckles (American Bridge Co. Standard)



(All dimensions in inches. For turnbuckles marked,* *a* = 9 in.; for all others, *a* = 6 in. Pitch and shape of thread, A. B. Co. Standard.)

Diam. of screw, *u*	Standard dimensions *d*	*l*	*c*	*t*	*g*	*b*	Weight, lb.	Diam. of screw, *u*	Standard dimensions *d*	*l*	*c*	*t*	*g*	*b*	Weight, lb.
3/8	9/16	7 1/8	9/16	3/16	1/2	1 1/16	1	2	3	12	2 3/8	1 1/16	2 1/4	4 1/4	14
7/16	21/32	7 5/16	5/8	1/4	5/8	1 3/8	1	2 1/8	3 3/16	12 3/8	2 1/2	23/32	2 1/2	4 1/2	17
1/2	3/4	7 1/2	5/8	1/4	5/8	1 3/8	1	2 1/4	3 3/8	12 3/4	2 11/16	13/16	2 1/2	4 3/4	20
9/16	27/32	7 11/16	13/16	5/16	3/4	1 9/16	1 1/2	2 3/8	3 9/16	13 1/8	2 3/4	13/16	2 3/4	4 7/8	22
5/8	15/16	7 7/8	13/16	5/16	3/4	1 9/16	1 1/2	2 1/2	3 3/4	13 1/2	3 1/16	27/32	3	5 3/8	25
3/4	1 1/8	8 1/4	1 1/16	11/32	7/8	2	2	2 3/4	4 1/8	14 1/4	3 1/4	15/16	3 1/4	5 3/4	33
7/8	1 5/16	8 5/8	1 1/4	3/8	1	2 1/4	3	2 7/8	4 5/16	14 5/8	3 7/16	1 1/32	3 1/4	6 1/16	36
1	1 1/2	9	1 5/16	7/16	1 1/4	2 7/16	4	3	4 1/2	15	3 5/8	1 1/32	3 1/2	6 3/8	40
1 1/8	1 11/16	9 3/8	1 7/16	1/2	1 1/4	2 9/16	5	3 1/4	4 7/8	15 3/4	3 7/8	1 1/16	4	6 3/4	50
1 1/4	1 7/8	9 3/4	1 9/16	1/2	1 1/2	2 3/4	6	3 1/2	5 1/4	16 1/2	4 1/4	1 7/32	4	7 1/4	65
1 3/8	2 1/16	10 1/8	1 11/16	1/2	1 5/8	3 1/16	7	3 3/4	5 5/8	17 1/4	4 7/16	1 5/16	5	8 1/4	95
1 1/2	2 1/4	10 1/2	1 3/4	5/8	1 3/4	3 3/16	8								
1 5/8	2 7/16	10 7/8	2	5/8	1 7/8	3 1/2	10	4	6	18	4 5/8	1 7/16	5	8 3/4	108
1 3/4	2 5/8	11 1/4	2 1/8	5/8	2	3 3/4	11	*4 1/4	6 1/4	21 1/2	4 5/8	1 5/8	5 5/32	9 1/4	140
1 7/8	2 13/16	11 5/8	2 3/16	11/16	2 1/8	3 7/8	12	*4 1/2	6 3/4	22 1/2	5 1/2	1 3/4	6 1/2	10 3/4	195
....	......	......	......	.....	.....	.....	....	*4 3/4	7 1/4	23 1/2	5 5/8	2	6 1/2	11 1/4	205
....	......	......	......	.....	.....	.....	....	*5	7 1/2	24	6	2 1/4	6 1/2	11 7/8	250

Table 29. Selected Carnegie Tees



EQUAL LEGS

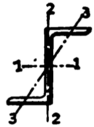
Size: Flange	Size: Stem	Min. thickness: Flange	Min. thickness: Stem	Weight per ft.	Area of section	Axis 1-1: I	Axis 1-1: r	Axis 1-1: S	Axis 1-1: x	Axis 2-2: I	Axis 2-2: r	Axis 2-2: S	Coefficients of strength,* Neutral axis 1-1
In.	In.	In.	In.	Lb.	Sq. in.	In.⁴	In.	In.³	In.	In.⁴	In.	In.³	
4	4	½	½	13.5	3.97	5.7	1.20	2.0	1.18	2.8	0.84	1.4	21.55
4	4	⅜	⅜	10.5	3.09	4.5	1.21	1.6	1.13	2.1	0.83	1.1	16.85
3½	3½	½	½	11.7	3.44	3.7	1.04	1.5	1.05	1.9	0.74	1.1	16.32
3½	3½	⅜	⅜	9.2	2.68	3.0	1.05	1.2	1.01	1.4	0.73	0.81	12.69
3	3	½	½	9.9	2.91	2.3	0.88	1.1	0.93	1.2	0.64	0.80	11.73
3	3	⅜	⅜	7.8	2.27	1.8	0.90	0.86	0.88	0.90	0.63	0.60	9.17
3	3	⁵⁄₁₆	⁵⁄₁₆	6.7	1.95	1.6	0.90	0.74	0.86	0.75	0.62	0.50	7.89
2½	2½	⅜	⅜	6.4	1.87	1.0	0.74	0.59	0.76	0.52	0.53	0.42	6.29
2½	2½	⁵⁄₁₆	⁵⁄₁₆	5.5	1.60	0.88	0.74	0.50	0.74	0.44	0.52	0.35	5.33
2¼	2¼	⁵⁄₁₆	⁵⁄₁₆	4.9	1.43	0.65	0.67	0.41	0.68	0.33	0.48	0.29	4.37
2¼	2¼	¼	¼	4.1	1.19	0.52	0.66	0.32	0.65	0.25	0.46	0.22	3.41
2	2	¼	¼	3.56	1.05	0.37	0.59	0.26	0.59	0.18	0.42	0.18	2.77
1¾	1¾	¼	¼	3.09	0.91	0.23	0.51	0.19	0.54	0.12	0.37	0.14	2.03
1½	1½	¼	¼	2.47	0.73	0.15	0.45	0.14	0.47	0.08	0.32	0.10	1.49
1½	1½	³⁄₁₆	³⁄₁₆	1.94	0.57	0.11	0.45	0.11	0.44	0.06	0.32	0.08	1.17
1¼	1¼	¼	¼	2.02	0.59	0.08	0.37	0.10	0.40	0.05	0.28	0.07	1.01
1¼	1¼	³⁄₁₆	³⁄₁₆	1.59	0 47	0.06	0.37	0.07	0.38	0.03	0.27	0.05	0.78
1	1	⅛	⅛	0.89	0.26	0.02	0.30	0.03	0.29	0.01	0.21	0.02	0.35

UNEQUAL LEGS

Size: Flange	Size: Stem	Min. thickness: Flange	Min. thickness: Stem	Weight per ft.	Area of section	Axis 1-1: I	Axis 1-1: r	Axis 1-1: S	Axis 1-1: x	Axis 2-2: I	Axis 2-2: r	Axis 2-2: S	Coefficients of strength,* Neutral axis 1-1
5	3	½	¹³⁄₃₂	13.40	3.93	2.40	0.78	1.10	0.73	5.40	1.17	2.20	11.41
5	2½	⅜	⁷⁄₁₆	10.90	3.18	1.50	0.68	0.78	0.63	4.10	1.14	1.60	8.96
4½	3½	⁷⁄₁₆	¹¹⁄₁₆	15.70	4.60	5.10	1.05	2.10	1.11	3.70	0.90	1.70	22.72
4½	3	⅜	⅜	9.80	2.88	2.10	0.84	0.91	0.74	3.00	1.02	1.30	9.72
4½	2½	⅜	⅜	9.20	2.68	1.20	0.67	0.63	0.59	3.00	1.05	1.30	6.71
4	5	½	½	15.30	4.50	10.80	1.55	3.10	1.56	2.80	0.79	1.40	33.39
4	5	⅜	⅜	11.90	3.49	8.50	1.56	2.40	1.51	2.10	0.78	1.10	25.92
4	4½	½	½	14.40	4.23	7.90	1.37	2.50	1.37	2.80	0.81	1.40	27.09
4	4½	⅜	⅜	11.20	3.29	6.30	1.39	2.00	1.31	2.10	0.80	1.10	21.12
4	3	⅜	⅜	9.20	2.68	2.00	0.86	0.90	0.78	2.10	0.89	1.10	9.60
4	2½	⅜	⅜	8.50	2.48	1.20	0.69	0.62	0.62	1.10	0.92	1.00	6.61
4	2	⅜	⅜	7.80	2.27	0.60	0.52	0.40	0.48	2.10	0.96	1.10	4.27
3½	4	½	½	12.60	3.70	5.50	1.21	2.00	1.24	1.90	0.72	1.10	21.12
3½	4	⅜	⅜	9.80	2.88	4.30	1.23	1.50	1.19	1.40	0.70	0.81	16.53
3½	3	½	½	10.80	3.17	2.40	0.87	1.10	0.88	1.90	0.77	1.10	12.05
3½	3	⅜	⅜	8.50	2.48	1.90	0.88	0.89	0.83	1.40	0.75	0.81	9.49
3	4	½	½	11.70	3.44	5.20	1.23	1.90	1.32	1.20	0.59	0.81	20.69
3	4	⅜	⅜	9.20	2.68	4.10	1.24	1.50	1.27	0.90	0.58	0.60	16.11
3	3½	½	½	10.80	3.17	3.50	1.06	1.50	1.12	1.20	0.62	0.80	15.89
3	3½	⅜	⅜	8.50	2.48	2.80	1.07	1.20	1.07	0.93	0.61	0.62	12.37
3	2½	⅜	⅜	7.10	2.07	1.10	0.72	0.60	0.71	0.89	0.66	0.59	6.40
3	2½	¼	¼	5.00	1.47	0.78	0.73	0.43	0.66	0.61	0.64	0.40	4.59
2½	3	⅜	⅜	7.10	2.07	1.70	0.91	0.84	0.95	0.53	0.51	0.42	8.96
2½	1¼	³⁄₁₆	³⁄₁₆	2.87	0.84	0.08	0.31	0.09	0.32	0.29	0.58	0.23	0.93
2	1½	¼	¼	3.09	0.91	0.16	0.42	0.15	0.42	0.18	0.45	0.18	1.60
1½	2	³⁄₁₆	³⁄₁₆	2.45	0.72	0.27	0.61	0.19	0.63	0.06	0.92	0.08	2.03
1½	1¼	⅛	⅛	1.25	0.37	0.05	0.37	0.05	0.33	0.04	0.32	0.05	0.57

* Coefficient of strength in thousands of pounds. Fiber stress 16,000 lb. per sq. in.

Table 30. Carnegie Zees



Section index	Size: Depth	Size: Flanges	Size: Thickness	Weight per ft.	Area of section	Axis 1-1: I	Axis 1-1: r	Axis 1-1: S	Axis 2-2: I	Axis 2-2: r	Axis 2-2: S	Axis 3-3: r min.	Coefficient of strength,* neutral axis 1-1
	In.	In.	In.	Lb.	Sq. in.	In.⁴	In.	In.³	In.⁴	In.	In.³	In.	
Z 3	6 1/8	3 5/8	7/8	34.6	10.17	50.2	2.22	16.4	19.2	1.37	6.0	0.83	174.93
	6 1/16	3 9/16	13/16	32.0	9.40	46.1	2.22	15.2	17.3	1.36	5.5	0.82	162.35
	6	3 1/2	3/4	29.4	8.63	42.1	2.21	14.0	15.4	1.34	4.9	0.81	149.76
Z 2	6 1/8	3 5/8	11/16	28.1	8.25	43.2	2.29	14.1	16.3	1.41	5.0	0.84	150.40
	6 1/16	3 9/16	5/8	25.4	7.46	38.9	2.28	12.8	14.4	1.39	4.4	0.82	136.75
	6	3 1/2	9/16	22.8	6.68	34.6	2.28	11.5	12.6	1.37	3.9	0.81	123.20
Z 1	6 1/8	3 5/8	1/2	21.1	6.19	34.4	2.36	11.2	12.9	1.44	3.8	0.84	119.68
	6 1/16	3 9/16	7/16	18.4	5.39	29.8	2.35	9.8	11.0	1.43	3.3	0.83	104.85
	6	3 1/2	3/8	15.7	4.59	25.3	2.35	8.4	9.1	1.41	2.8	0.83	90.03
Z 6	5 1/8	3 3/8	13/16	28.4	8.33	28.7	1.86	11.2	14.4	1.31	4.8	0.76	119.47
	5 1/16	3 5/16	3/4	26.0	7.64	26.2	1.85	10.3	12.8	1.30	4.4	0.74	110.29
	5	3 1/4	11/16	23.7	6.96	23.7	1.84	9.5	11.4	1.28	3.9	0.73	101.01
Z 5	5 1/8	3 3/8	5/8	22.6	6.64	24.5	1.92	9.6	12.1	1.35	3.9	0.76	102.08
	5 1/16	3 5/16	9/16	20.2	5.94	21.8	1.91	8.6	10.5	1.33	3.5	0.75	91.95
	5	3 1/4	1/2	17.9	5.25	19.2	1.91	7.7	9.1	1.31	3.0	0.74	81.92
Z 4	5 1/8	3 3/8	7/16	16.4	4.81	19.1	1.99	7.4	9.2	1.38	2.9	0.77	79.36
	5 1/16	3 5/16	3/8	14.0	4.10	16.2	1.99	6.4	7.7	1.37	2.5	0.76	68.16
	5	3 1/4	5/16	11.6	3.40	13.4	1.98	5.3	6.2	1.35	2.0	0.75	56.96
Z 9	4 1/8	3 3/16	3/4	23.0	6.75	15.0	1.49	7.3	11.2	1.29	4.0	0.68	77.44
	4 1/16	3 1/8	11/16	20.9	6.14	13.5	1.48	6.7	10.0	1.27	3.6	0.67	70.93
	4	3 1/16	5/8	18.9	5.55	12.1	1.48	6.1	8.7	1.25	3.2	0.66	64.53
Z 8	4 1/8	3 3/16	9/16	18.0	5.27	12.7	1.55	6.2	9.3	1.33	3.2	0.68	65.92
	4 1/16	3 1/8	1/2	15.9	4.66	11.2	1.55	5.5	8.0	1.31	2.8	0.67	58.67
	4	3 1/16	7/16	13.8	4.05	9.7	1.55	4.8	6.7	1.29	2.4	0.66	51.52
Z 7	4 1/8	3 3/16	3/8	12.5	3.66	9.6	1.62	4.7	6.8	1.36	2.3	0.69	49.81
	4 1/16	3 1/8	5/16	10.3	3.03	7.9	1.62	3.9	5.5	1.34	1.8	0.68	41.71
	4	3 1/16	1/4	8.2	2.41	6.3	1.62	3.1	4.2	1.33	1.4	0.67	33.49
Z 12	3 1/16	2 3/4	9/16	14.3	4.18	5.3	1.12	3.4	5.7	1.17	2.3	0.54	36.59
	3	2 11/16	1/2	12.6	3.69	4.6	1.12	3.1	4.9	1.15	2.0	0.53	32.64
Z 11	3 1/16	2 3/4	7/16	11.5	3.36	4.6	1.17	3.0	4.8	1.19	1.9	0.55	31.79
	3	2 11/16	3/8	9.8	2.86	3.9	1.16	2.6	3.9	1.17	1.6	0.54	27.41
Z 10	3 1/16	2 3/4	5/16	8.5	2.48	3.6	1.21	2.4	3.6	1.21	1.4	0.56	25.39
	3	2 11/16	1/4	6.7	1.97	2.9	1.21	1.9	2.8	1.19	1.1	0.55	20.48

* Coefficient of strength in thousands of pounds. Fiber stress 16,000 lb. per sq. in.

Bethlehem Girder Beams from 8 in. to 24 in. in depth, inclusive, have a strength, or section modulus and coefficient of strength, equal to that of two minimum-weight Standard I-beams of the same depth. The girder beam, however, weighs generally 12½ per cent. less than the combined weight of the two Standard beams, not considering the saving in weight of separators needed for assembling the Standard beams into a girder. For example, a Bethlehem 15-in. girder beam, weighing 73 lb. per ft., has a coefficient of strength of 1,256,000. Two Standard 15-in. I-beams, each weighing 42 lb. per ft., have a total coefficient of strength of 1,256,000. Thus, for equal depth and coefficient of strength, the girder beam weighs 11 lb. per ft. less than the two Standard beams. **Bethlehem I-beams** from 8 in. to 24 in. in depth, inclusive, have the same strength, or section modulus and coefficient of strength, as Standard beams of the same depth. Bethlehem beams weigh generally 10 per cent. less than Standard beams of the same depth and strength. For example, a Bethlehem 15-in. I-beam, weighing 54 lb. per ft., has a coefficient of strength of 867,000. The corresponding Standard section is a 15-in. I-beam weighing 60 lb. per ft., having a coefficient of strength of 866,100.

Bethlehem H-Columns are rolled I-sections with heavy, wide flanges. The greater width of flange makes the difference between the maximum and minimum radii of gyration much less than with standard I-beams; these sections are therefore especially adapted for use as columns. The values in Table 31 are selected from those published by the Bethlehem Steel Co.

Table 31. Selected Bethlehem H-Columns

(See Fig. 24)

For D = 16⅞ to 13¾ in., L = 11.06 in. For D = 11½ to 10 in., L = 7.67 in.
For D = 13½ to 11¾ in., L = 9.21 in. For D = 9½ to 8 in., L = 6.14 in.

Weight of section, lb. per ft.	Dimensions, in in.				Area of section, sq. in.	Axis XX			Axis YY			"Square" ends, safe loads, tons of 2000 lb.		
	D	B	Thickness of web	M		Moment of inertia I	Section modulus S	Radius of gyration, in. r	Moment of inertia I'	Section modulus S'	Radius of gyration, in. r'	$l < 55r'$	$l = 90r'$	$l = 125r'$
287.5	16⅞	14.90	1.41	2.183	84.50	3836.1	454.7	6.74	1226.7	164.7	3.81	549.3	467.0	386.3
261.5	16½	14.78	1.29	1.995	76.93	3402.1	412.4	6.65	1095.6	148.3	3.77	500.0	425.4	351.0
227.5	16	14.62	1.13	1.745	66.98	2859.6	357.5	6.53	929.4	127.1	3.73	435.4	370.5	305.6
195.0	15½	14.47	0.98	1.495	57.35	2359.7	304.5	6.41	774.2	107.0	3.67	372.8	317.9	263.0
162.0	15	14.31	0.82	1.245	47.71	1894.0	252.5	6.30	626.1	87.5	3.62	310.1	263.7	217.7
130.5	14½	14.16	0.67	0.995	38.38	1466.7	202.3	6.18	486.9	68.8	3.56	249.5	212.1	175.4
99.0	14	14.00	0.51	0.745	29.06	1070.6	153.0	6.07	356.9	51.0	3.50	188.9	160.8	132.6
83.5	13¾	13.92	0.43	0.620	24.46	884.9	128.7	6.01	294.5	42.3	3.47	159.0	135.3	111.8
161.0	13½	12.47	0.94	1.442	47.28	1444.3	214.0	5.53	477.0	76.5	3.18	307.3	261.3	216.2
132.5	13	12.31	0.78	1.192	38.97	1141.3	175.6	5.41	380.7	61.9	3.13	253.3	215.4	177.8
105.0	12½	12.16	0.63	0.942	30.94	866.8	138.6	5.30	291.7	48.0	3.07	201.1	171.1	141.1
78.0	12	12.00	0.47	0.692	22.94	615.6	102.6	5.18	208.1	34.7	3.01	149.1	126.8	105.4
64.5	11¾	11.92	0.39	0.567	19.00	499.0	84.9	5.13	168.6	28.3	2.98	123.5	105.0	86.6
123.5	11½	10.47	0.86	1.327	36.32	790.4	137.5	4.67	259.3	49.5	2.67	236.1	200.8	165.8
99.5	11	10.31	0.70	1.077	29.32	607.0	110.4	4.55	201.7	39.1	2.62	190.6	162.1	133.4
77.0	10½	10.16	0.55	0.827	22.59	443.6	84.5	4.43	149.1	29.4	2.57	146.8	124.8	103.3
54.0	10	10.00	0.39	0.577	15.91	296.8	59.4	4.32	100.4	20.1	2.51	103.4	88.0	72.8
90.5	9½	8.47	0.78	1.212	26.64	385.3	81.1	3.80	125.1	29.6	2.17	173.2	147.3	121.6
71.5	9	8.32	0.63	0.962	21.05	285.6	63.5	3.68	94.4	22.7	2.12	136.8	116.3	96.0
53.0	8½	8.16	0.47	0.712	15.53	197.8	46.5	3.57	66.3	16.3	2.07	101.0	86.0	71.0
34.5	8	8.00	0.31	0.462	10.17	121.5	30.4	3.46	41.1	10.3	2.01	66.1	56.2	46.4

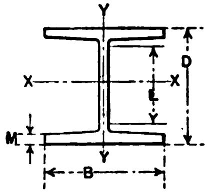


Fig. 24.

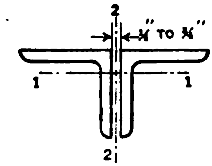


Fig. 25.

Table 32. Radii of Gyration for Two Angles, Unequal Legs

(See Fig. 25)

Single angle: Size, in.	Single angle: Weight, lb. per ft.	Two angles: Area, sq. in.	Long legs vertical: Axis 1-1	Long legs vertical, Axis 2-2: In contact	Long legs vertical, Axis 2-2: ⅜ in. apart	Long legs vertical, Axis 2-2: ¾ in. apart	Short legs vertical: Axis 1-1	Short legs vertical, Axis 2-2: In contact	Short legs vertical, Axis 2-2: ⅜ in. apart	Short legs vertical, Axis 2-2: ¾ in. apart
8 × 6 × 1	44.2	26.00	2.49	2.39	2.52	2.66	1.73	3.64	3.78	3.92
7/16	20.2	11.86	2.57	2.31	2.43	2.56	1.80	3.55	3.68	3.82
8 × 3½ × 1	35.7	21.00	2.51	1.26	1.40	1.55	0.86	4.04	4.19	4.34
7/16	16.5	9.68	2.59	1.15	1.28	1.41	0.92	3.93	4.07	4.22
7 × 3½ × 1	32.3	19.00	2.19	1.31	1.45	1.60	0.89	3.48	3.63	3.78
3/8	13.0	7.60	2.27	1.20	1.33	1.46	0.96	3.36	3.50	3.65
6 × 4 × 1	30.6	18.00	1.85	1.60	1.74	1.89	1.09	2.85	2.99	3.14
3/8	12.3	7.22	1.93	1.50	1.62	1.76	1.17	2.74	2.87	3.02
6 × 3½ × 1	28.9	17.00	1.85	1.37	1.51	1.66	0.92	2.92	3.07	3.22
5/16	9.8	5.74	1.95	1.25	1.37	1.50	1.00	2.81	2.95	3.09
5 × 4 × 7/8	24.2	14.22	1.52	1.66	1.80	1.95	1.14	2.29	2.43	2.58
3/8	11.0	6.46	1.59	1.58	1.70	1.85	1.20	2.20	2.34	2.48
5 × 3½ × 7/8	22.7	13.34	1.53	1.42	1.56	1.71	0.96	2.36	2.50	2.65
5/16	8.7	5.12	1.61	1.33	1.45	1.59	1.03	2.26	2.39	2.54
5 × 3 × 13/16	19.9	11.68	1.55	1.18	1.32	1.47	0.80	2.42	2.57	2.72
5/16	8.2	4.80	1.61	1.09	1.22	1.35	0.85	2.33	2.47	2.61
4½ × 3 × 13/16	18.5	10.86	1.38	1.21	1.36	1.51	0.81	2.15	2.30	2.45
5/16	7.7	4.50	1.44	1.13	1.26	1.40	0.87	2.06	2.20	2.34
4 × 3½ × 13/16	18.5	10.86	1.19	1.50	1.64	1.79	1.01	1.81	1.96	2.11
5/16	7.7	4.50	1.26	1.42	1.55	1.69	1.07	1.73	1.86	2.00
4 × 3 × 13/16	17.1	10.06	1.21	1.25	1.40	1.55	0.83	1.88	2.03	2.18
1/4	5.8	3.38	1.28	1.16	1.28	1.43	0.89	1.78	1.92	2.06
3½ × 3 × 13/16	15.8	9.24	1.04	1.30	1.45	1.60	0.85	1.61	1.76	1.91
1/4	5.4	3.12	1.11	1.20	1.34	1.48	0.91	1.52	1.65	1.80
3½ × 2½ × 11/16	12.5	7.30	1.06	1.03	1.18	1.33	0.69	1.66	1.80	1.96
1/4	4.9	2.88	1.12	0.95	1.09	1.23	0.74	1.58	1.71	1.86
3 × 2½ × 9/16	9.5	5.56	0.91	1.05	1.20	1.35	0.72	1.37	1.51	1.66
1/4	4.5	2.64	0.95	1.00	1.13	1.28	0.75	1.31	1.45	1.59
3 × 2 × ½	7.7	4.50	0.92	0.80	0.94	1.10	0.55	1.42	1.57	1.72
1/4	4.1	2.38	0.95	0.74	0.88	1.03	0.57	1.38	1.52	1.67
2½ × 2 × ½	6.8	4.00	0.75	0.84	0.99	1.15	0.56	1.15	1.30	1.46
1/4	3.62	2.12	0.78	0.80	0.93	1.08	0.59	1.11	1.25	1.40

Table 33. Radii of Gyration for Two Angles, Equal Legs
(See Fig. 25)

Single angle		Two angles	Radii of gyration, in.			
				Axis 2-2		
Size, in.	Weight, lb. per ft.	Area, sq. in.	Axis 1-1	In contact	3/8 in. apart	3/4 in. apart
8 × 8 × 1 1/8	56.9	33.46	2.42	3.42	3.55	3.69
1/2	26.4	15.50	2.50	3.33	3.45	3.59
6 × 6 × 1	37.4	22.00	1.80	2.59	2.72	2.87
3/8	14.9	8.72	1.88	2.49	2.62	2.75
5 × 5 × 1	30.6	18.00	1.48	2.19	2.33	2.47
3/8	12.3	7.22	1.56	2.09	2.21	2.35
4 × 4 × 13/16	19.9	11.68	1.18	1.75	1.89	2.04
1/4	6.6	3.88	1.25	1.66	1.79	1.93
3 1/2 × 3 1/2 × 13/16	17.1	10.06	1.02	1.55	1.70	1.85
1/4	5.8	3.38	1.09	1.46	1.59	1.73
3 × 3 × 5/8	11.5	6.72	0.88	1.32	1.46	1.61
1/4	4.9	2.88	0.93	1.25	1.38	1.53
2 1/2 × 2 1/2 × 1/2	7.7	4.50	0.74	1.09	1.24	1.39
1/4	4.1	2.38	0.77	1.05	1.19	1.34
2 × 2 × 7/16	5.3	3.12	0.59	0.88	1.03	1.19
1/4	3.19	1.88	0.61	0.85	0.99	1.14

Plate and Angle Columns. Table 34 gives the dimensions in inches and the properties of columns built up either of a web and four equal angles, or of a web, four equal angles and two plates assembled as in Fig. 26. The safe loads are calculated for an allowable fiber stress of 13,000 lb. per sq. in. for lengths of 60 radii or under; reduced for lengths over 60 radii. Weights do not include rivet heads or other details.

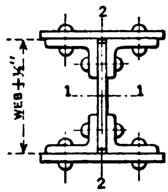


Fig. 26.

Table 34. Plate and Angle Columns, Safe Loads in Thousands of Pounds

(American Bridge Co.)

Effective length, ft.	Web pl. 6×¼			Web pl. 8×¼				Web pl. 8×5/16				Web pl. 8×⅜		
	4 Angles													
	2½×2×¼	3×2×¼	3×2½×¼	3×2½×¼	3×2½×5/16	3½×2½×¼	3½×2½×5/16	3½×2½×5/16	3½×2½×⅜	4×3×5/16	4×3×⅜	4×3×⅜	4×3×7/16	4×3×½
6	69	81	88	94	110	101	119	125	142	141	161	168	188	208
10	43	60	63	65	78	83	100	104	121	128	149	154	175	196
14	28	40	42	43	52	57	70	73	86	97	114	118	135	152
18	....	29	29	28	36	43	52	54	64	71	83	86	98	110
22	....	....	....	....	....	30	38	39	47	55	66	68	78	89
26	....	....	....	....	....	....	....	....	......	40	48	49	58	67
Area, sq.in.	5.74	6.26	6.74	7.24	8.48	7.76	9.12	9.62	10.94	10.86	12.42	12.92	14.48	16.00
I_{1-1}, in.^{4}	34.3	39.1	42.6	81.2	96.9	90.1	107	110	127	122	141	143	161	178
r_{1-1}, in	2.45	2.50	2.51	3.35	3.38	3.41	3.43	3.38	3.40	3.35	3.36	3.33	3.34	3.33
I_{2-2}, in.^{4}	6.2	10.3	10.3	10.3	12.9	16.0	20.2	20.7	24.9	30.3	36.3	37.2	43.5	50.2
r_{2-2}, in	1.04	1.28	1.24	1.19	1.23	1.44	1.49	1.47	1.51	1.67	1.71	1.70	1.73	1.77
Weight, lb. per ft.	19.6	21.5	23.1	24.8	29.2	26.4	31.2	32.9	37.3	37.3	42.5	44.2	49.4	54.6

Effective length, ft.	Web pl. 10 × ¼			Web plate 10 × 5/16			Web plate 10 × ⅜							Web plate 10 × ½			Web pl. 10 × ⅝
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	4 Angles																
	3×2½×¼	3½×2½×¼	3½×2½×5/16	3½×2½×5/16	4 × 3 × 5/16	4×3×⅜	4×3×⅜	4×3×7/16	5×3½×⅜	5×3½×7/16	6×4×⅜	6×4×7/16	6×4×½	6×4×½	6×4×9/16	6×4×⅝	6×4×⅝
8	82	100	119	125	149	170	178	198	207	232	236	266	296	312	341	370	386
12	52	71	87	91	116	135	140	160	194	220	236	266	296	312	341	370	386
16	36	50	61	64	82	98	101	116	157	140	209	238	267	280	309	337	351
20	....	36	45	47	64	76	78	90	121	180	175	201	226	236	262	287	298
24	....	....	30	....	47	57	58	68	98	113	141	163	185	192	214	236	245
28	....	....	....	....	....	....	....	47	80	93	117	134	152	158	175	192	200

Column	1	2	3	4	5	6	7	8	9
Area, sq. in.	7.74	8.26	9.62	10.25	11.49	13.05	13.67	15.23	15.95
I_{1-1}, in.^{4}	134	148	176	181	201	232	237	267	279
r_{1-1}, in	4.16	4.23	4.28	4.20	4.18	4.22	4.17	4.19	4.18
I_{2-2}, in.^{4}	10.3	16.0	20.2	20.7	30.3	36.3	37.2	43.5	70.6
r_{2-2}, in	1.15	1.39	1.45	1.42	1.62	1.67	1.65	1.69	2.10
Weight, lb. per ft.	26.5	28.1	32.9	35.0	39.4	44.6	46.8	52.0	54.4

Column	10	11	12	13	14	15	16	17
Area, sq. in.[2]	17.87	18.19	20.47	22.75	24.00	26.24	28.44	29.69
I_{1-1}, in.^{4}	315	319	361	401	412	451	489	500
r_{1-1}, in	4.20	4.19	4.20	4.20	4.14	4.15	4.15	4.10
I_{2-2}, in.^{4}	82.3	119	139	160	165	186	206	213
r_{2-2}, in	2.15	2.56	2.61	2.65	2.62	2.66	2.69	2.68
Weight, lb. per ft.	60.8	62.0	70.0	77.6	80.8	89.4	97.0	101.3

Table 34. Plate and Angle Columns, Safe Loads in Thousands of Pounds—(*continued*)

Effective length in ft.	Web pl. 12 × ¼			Web pl. 12 × 5/16		Web plate 12 × ⅜						Web plate 12 × ½					Web pl. 12 × ⅝	Web pl. 12 × ¾
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	4 Angles																	
	3½ × 2½ × ¼	3½ × 2½ × 5/16	4 × 3 × 5/16	4 × 3 × 5/16	4 × 3 × ⅜	4 × 3 × ⅜	5 × 3½ × ⅜	5 × 3½ × 7/16	5 × 3½ × ½	6 × 4 × 7/16	6 × 4 × ½	6 × 4 × ½	6 × 4 × 9/16	6 × 4 × ⅝	6 × 4 × 11/16	6 × 4 × ¾	6 × 4 × ¾	6 × 4 × ¾
10	89	106	131	138	159	167	217	242	266	276	305	325	354	383	411	439	458	478
14	59	72	97	101	119	124	181	205	229	264	295	312	342	373	403	433	451	469
18	44	54	71	75	87	91	142	162	184	224	252	265	292	319	347	373	388	403
22	28	37	55	56	67	69	110	125	141	184	209	218	242	266	290	314	325	338
26	....	....	38	38	47	48	91	104	118	147	166	173	192	213	234	254	262	272
30	....	....	....	....	....	...	71	82	95	127	143	150	166	183	199	215	223	232

Column	1	2	3	4	5	6	7	8	9
Area, sq. in.	8.76	10.12	11.36	12.11	13.67	14.42	16.70	18.62	20.50
I_{1-1}, in.⁴	222	264	295	304	350	359	421	476	526
r_{1-1}, in.	5.04	5.11	5.09	5.01	5.06	4.99	5.02	5.05	5.07
I_{2-2}, in.⁴	16.0	20.2	29.6	30.3	36.3	37.3	70.6	82.3	94.6
r_{2-2}, in.	1.35	1.41	1.61	1.58	1.63	1.61	2.06	2.10	2.15
Weight, lb. per ft.	29.8	34.6	39.0	41.6	46.8	49.3	56.9	63.3	69.7
Column	10	11	12	13	14	15	16	17	18
Area, sq. in.	21.22	23.50	25.00	27.24	29.44	31.60	33.76	35.26	36.76
I_{1-1}, in.⁴	544	605	623	683	741	794	849	867	885
r_{1-1}, in.	5.06	5.07	4.99	5.01	5.02	5.01	5.01	4.96	4.91
I_{2-2}, in.⁴	139	160	165	186	206	228	249	257	266
r_{2-2}, in.	2.56	2.61	2.57	2.61	2.65	2.69	2.72	2.70	2.69
Weight, lb. per ft.	72.5	80.1	85.2	92.8	100.4	107.6	114.8	119.9	125.0

Effective length, ft.	Web plate 12 × ⅜			Web plate 12 × ½				Web plate 12 × ⅝				
	4 Angles, 2 Plates											
	6 × 4 × ⅜ 14 × ⅜	6 × 4 × ⅜ 14 × ½	6 × 4 × 7/16 14 × ½	6 × 4 × ½ 14 × ½	6 × 4 × ½ 14 × ½	6 × 4 × ½ 14 × ⅝	6 × 4 × 9/16 14 × ⅝	6 × 4 × ⅝ 14 × ⅝	6 × 4 × ⅝ 14 × ⅝	6 × 4 × ⅝ 14 × ¾	6 × 4 × ⅝ 14 × ⅞	6 × 4 × ⅝ 14 × 1
16	379	428	458	487	506	553	582	610	630	675	721	766
20	334	383	407	433	447	495	520	544	558	606	654	700
24	289	334	355	377	388	432	454	475	486	529	574	616
28	244	285	303	321	329	369	388	405	413	453	494	532
32	203	237	250	265	272	307	321	336	341	377	414	448
35	186	215	229	243	249	279	293	306	313	342	371	399
Area, sq. in.	29.44	32.94	35.22	37.50	39.00	42.50	44.74	46.94	48.44	51.94	55.44	58.94
I_{1-1}, in.⁴	916	1073	1136	1197	1215	1377	1437	1495	1513	1682	1856	2037
r_{1-1}, in.	5.58	5.71	5.68	5.65	5.58	5.69	5.67	5.64	5.59	5.69	5.79	5.88
I_{2-2}, in.⁴	291	348	368	388	394	451	472	492	499	556	613	671
r_{2-2}, in.	3.14	3.25	3.23	3.22	3.18	3.26	3.25	3.24	3.21	3.27	3.33	3.37
Weight, lb. per ft.	100.2	112.1	120.1	127.7	132.8	144.7	152.3	159.9	165.0	176.9	188.8	200.

Table 34. Plate and Angle Columns, Safe Loads in Thousands of Pounds—(*continued*)

Effective length, ft.	Web plate 12 × 5/8								Web plate 14 × 3/8				
	4 Angles, 2 Plates												
	6×4×5/8 14×1 1/8	6×4×5/8 14×1 1/4	6×4×5/8 14×1 3/8	6×4×5/8 14×1 1/2	6×4×5/8 14×1 5/8	6×4×5/8 14×1 3/4	6×4×5/8 14×1 7/8	6×4×5/8 14×2	6×4×3/8 14×3/8	6×4×7/16 14×3/8	6×4×1/2 14×3/8	6×4×1/2 14×7/16	6×4×1/2 14×1/2
18	791	840	888	937	986	1034	1082	1130	363	390	417	442	468
22	703	748	793	837	882	926	970	1014	317	340	363	387	410
26	615	657	697	738	779	818	858	898	270	289	309	331	353
30	527	565	601	638	675	710	746	782	223	239	255	275	295
34	439	473	505	538	571	603	634	667	194	208	222	237	253
Area, sq. in.	62.44	65.94	69.44	72.94	76.44	79.94	83.44	86.94	30.19	32.47	34.75	36.50	38.25
I_{1-1}, in.^{4}	2224	2418	2618	2825	3038	3259	3486	3721	1261	1351	1436	1539	1643
r_{1-1}, in	5.97	6.06	6.14	6.22	6.30	6.38	6.46	6.54	6.46	6.45	6.43	6.49	6.55
I_{2-2}, in.^{4}	728	785	842	899	956	1014	1071	1128	291	311	331	360	388
r_{2-2}, in	3.41	3.45	3.48	3.51	3.54	3.56	3.58	3.60	3.10	3.09	3.09	3.14	3.19
Weight, lb. per ft.	212.6	224.5	236.4	248.3	260.2	272.1	284.0	295.9	102.8	110.8	118.4	124.3	130.3

Effective length, ft.	Web plate 14 × 3/8		Web plate 14 × 1/2		Web plate 14 × 5/8							
	4 Angles, 2 Plates											
	6 × 4 × 1/2 14 × 9/16	6 × 4 × 1/2 14 × 5/8	6 × 4 × 1/2 14 × 5/8	6 × 4 × 9/16 14 × 5/8	6 × 4 × 5/8 14 × 5/8	6 × 4 × 5/8 14 × 5/8	6 × 4 × 5/8 14 × 3/4	6 × 4 × 5/8 14 × 7/8	6 × 4 × 5/8 14 × 1	6 × 4 × 5/8 14 × 1 1/8	6 × 4 × 5/8 14 × 1 1/4	6 × 4 × 5/8 14 × 1 3/8
16	520	543	566	595	623	643	691	737	782	828	873	919
20	463	487	502	527	551	568	615	664	711	758	805	851
24	403	426	437	459	479	493	536	581	625	667	711	753
28	344	364	373	390	407	417	457	498	538	577	617	655
32	284	303	308	322	336	345	378	415	452	487	522	556
35	260	275	282	295	309	317	346	375	404	432	461	489
Area, sq. in.	40 00	41.75	43.50	45.74	47.94	49.69	53.19	56.69	60.19	63.69	67.19	70.69
I_{1-1}, in.^{4}	1749	1857	1885	1970	2053	2081	2302	2529	2764	3006	3255	3512
r_{1-1}, in.	6.61	6.67	6.58	6.56	6.54	6.47	6.58	6.68	6.78	6.87	6.96	7.05
I_{2-2}, in.^{4}	417	446	451	472	492	499	556	613	671	728	785	842
r_{2-2}, in	3.23	3.27	3.22	3.21	3.20	3.17	3.23	3.29	3.34	3.38	3.42	3.45
Weight, lb. per ft.	136.2	142.2	148.1	155.7	163.3	169.3	181.2	193.1	205.0	216.9	228.8	240.7

Corrugated Sheets are used for roofs and sides of buildings. They are usually laid directly upon the roof purlins and held in place by means of clips of steel hoops which encircle the purlin and are placed about 12 in. apart. The projecting edges of the corrugated sheets at the eaves and gable ends of the roof should be well secured, otherwise the wind will loosen the sheets.

Corrugated sheets are made in the sizes given in Table 35. The size most generally used has nominally 2½-in. corrugations, (actual width 2⅔ in.) about ½ in. in depth. One corrugation is the double curve between corresponding points; depth of corrugation is the greatest deviation of the curved surfaces from the straight line. The gages frequently used for roofing are Nos. 20 and 22, U. S. Standard Gage.

One and one-half corrugations are allowed for lap in the width of the sheet and 6 in. in the length for the usual quarter-pitch roof; one corrugation in width and 4 in. in the length of the sheet are usually allowed for sidings. Corrugated sheets are furnished in **standard lengths** of 5, 6, 7, 8, 9 and 10 ft. and with a covering width of 24 in., when laid with a lap of either one or one and one-half corrugations.

Table 35. Corrugated Sheets

(American Sheet and Tin Plate Co. Standard)

Description of Corrugated Sheets						Areas of Corrugated Sheets						
Corrugations				Width, in.		Length of sheet, in.	Sq. ft. in 1 sheet			Sheets in 100 sq. ft.		
Width, in.		Depth, approx., in.	Number per sheet	Full sheet	Covers approx.		Corrugations			Corrugations		
Nominal	Actual						5 in.	3 in., $2\frac{1}{2}$ in., 2 in.	$1\frac{1}{4}$ in., $\frac{5}{8}$ in.	5 in.	3 in., $2\frac{1}{2}$ in., 2 in.	$1\frac{1}{4}$ in., $\frac{5}{8}$ in.
5	$4\frac{4}{5}$	$\frac{7}{8}$	6	28	24	60	11.67	10.83	10.42	8.57	9.23	9.60
3	$2\frac{6}{7}$	$\frac{5}{8}$	9	26	24	72	14.00	13.00	12.50	7.14	7.69	8.00
$2\frac{1}{2}$	$2\frac{2}{3}$	$\frac{1}{2}$	10	26	24	84	16.33	15.17	14.58	6.12	6.59	6.86
2	$2\frac{4}{11}$	$\frac{1}{2}$	11	26	24	96	18.67	17.33	16.67	5.36	5.77	6.00
$1\frac{1}{4}$	$1\frac{1}{4}$	$\frac{3}{8}$	20	25	24	108	21.00	19.50	18.75	4.76	5.13	5.33
$\frac{5}{8}$	$\frac{25}{26}$	$\frac{3}{16}$	26	25	24	120	23.33	21.67	20.83	4.29	4.62	4.80
Standard lengths 5, 6, 7, 8, 9 and 10 ft. Maximum length, 12 ft. for 5-in. to $1\frac{1}{4}$-in. corrug'n.						144	28.00	26.00	25.00	3.57	3.85	4.00

Weight of Galvanized Corrugated Sheets, Lb. per 100 Sq. Ft.

Nominal corrugation, in.	Thickness, U. S. Standard Gage and Decimals of an Inch												
	12	14	16	18	20	21	22	23	24	25	26	27	28
	.109	.078	.063	.050	.038	.034	.031	.028	.025	.022	.019	.017	.016
5	....	354	286	232	178	165	151	138	124	111	98	91	85
3	....	....	286	232	178	165	151	138	124	111	98	91	85
$2\frac{1}{2}$	488	354	286	232	178	165	151	138	124	111	98	91	85
2	....	....	286	232	178	165	151	138	124	111	98	91	85
$1\frac{1}{4}$	....	....	....	....	185	....	157	....	129	....	101	94	87
$\frac{5}{8}$	....	....	....	....	....	....	....	....	129	....	101	94	87

The weights per 100 sq. ft. given above do not include allowances for end or side laps. The following table gives the approximate number of square feet of sheeting necessary to cover an area of 100 sq. ft., and is based on sheets of standard width, 96 in. long. If longer or shorter sheets are used, the number of square feet required will vary accordingly.

Square Feet of Corrugated Sheets to Cover 100 Sq. Ft.

Side lap	End lap, in.					
	1	2	3	4	5	6
1 Corrugation	110	111	112	113	114	115
$1\frac{1}{2}$ Corrugations	116	117	118	119	120	121
2 Corrugations	123	124	125	126	127	128

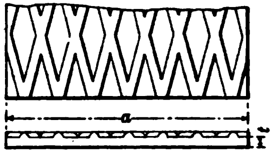
By experiment it has been determined that corrugated sheet steel, 5/8 in. deep and 0.035 in. thick, spanning 6 ft., will begin to give a permanent deflection with a load of 30 lb. per sq. ft., and that it collapses with a load of 60 lb. per sq. ft. The distance between centers of purlins should, therefore, not exceed 6 ft. and should preferably be less than this.

Approximately the uniformly distributed **safe load** of corrugated sheets may be obtained from the formulæ

$$W = 25{,}000tdb/l; \quad w = 25{,}000td/L^2$$

where W = total allowable uniform load, lb.; w = allowable uniform load, lb. per sq. ft.; b = width of sheet, in.; l = unsupported length of sheet, in.; L = unsupported length of sheet, ft.; t = thickness of sheet, in., and d = depth of corrugations, in.

Table 36. Carnegie Steel Checkered Plates



Section index	Width a, Min., in.	Width a, Max., in.	Thickness t, in.	Lb. per sq. ft.	Section modulus for 1 ft. width, in.^{3}	Span 1	2	3	4	5	6	7	8	9
						Allowable uniform load, lb. per sq. ft. for fiber stress of 16,000 lb. per sq. in.								
M 54	12	60	1/2	21.4	0.500	5333	1333	593	333	213	148	109	83	66
M 53	12	60	7/16	18.9	0.383	4083	1021	454	255	163	113	83	64	....
M 52	12	60	3/8	16.3	0.281	3000	750	333	188	120	83	61	....	....
M 51	12	60	5/16	13.8	0.195	2083	520	232	130	83	58	....	....	....
M 50	12	60	1/4	11.2	0.125	1333	333	148	83	53	....	....	....	....
M 49	12	48	3/16	8.7	0.070	746	187	83	47	....	....	....	....	....

Prices of Structural Steel. Standard structural shapes are sold (Aug. 1, 1915) at a **base price** (F.O.B. Pittsburgh) of 1.30 cents per lb. in carload lots (Jan. 1, 1915, 1.05–1.10 cents). This base price applies to beams and channels 3 to 15 in. inclusive, angles 3 to 6 in. on one or both legs and 1/4 in. thick or over, and to zees 3 in. and over. **Extras** to be added to the base price (per lb.) are as follows: For angles over 6 in. on one or both legs, 0.10 cent.; tees, 0.05 cent.; bulb beams and angles, 0.30 cent. For cutting to lengths < 1 ft., 1.55 cents.; 1 to 2 ft., 0.50 cent.; 2 to 3 ft., 0.25 cent; over 3 ft., no charge. Structural rivets in quantity, 1.60–1.75 cents per lb. (Jan. 1,1915, 1.35–1.50 cents). **Freight rates** from Pittsburgh in cents per 100 lb. (carload lots, 36,000 lb. minimum) for structural shapes and fabricated material are as follows: Atlanta, 45; Birmingham, 45; Boston, 19; Buffalo, 12; Chicago, 19; Cincinnati, 16; Cleveland, 11; Denver, 69; Indianapolis, 18; Kansas City, 44; New Orleans, 30; New York, 17; Omaha, 44; Pacific Coast, 80; Philadelphia, 16; St. Louis, 24; St. Paul, 33.

REINFORCED-CONCRETE CONSTRUCTION

BY

SANFORD E. THOMPSON

REFERENCES: Taylor and Thompson, "Concrete, Plain and Reinforced," Wiley. Turneaure and Maurer, "Principles of Reinforced Concrete Construction," Wiley. Marsh, "Reinforced Concrete," Van Nostrand." Taylor and Thompson, "Concrete Costs," Wiley.

Reinforced Concrete is a combination of concrete and steel acting as a unit in consequence of the adhesion between the two materials. Concrete has very low tensile and high compressive strength. Beams of plain concrete fail by tension under very low stresses, but if reinforced with steel in their tensile portion they may be stressed to the compressive working limit of concrete. Reinforced-concrete structures are practically monolithic, more rigid than steel and substantially fireproof. Reinforced concrete is made **use** of in parts of the structure in which tensile and compressive stresses exist, as in beams and slabs, and also in members subject to secondary bending stresses, such as columns and struts. Reinforcement for concrete is also used to prevent cracks due to changes of temperature and shrinkage, as in walls.

Concrete. For reinforced work, only first-class Portland cement concrete must be used, and the aggregates must be carefully selected (see p. 569). The **proportions** are governed by the kind and importance of the construction and by the quality of aggregates (see p. 572). The most common proportions for building constructions are 1 : 2 : 4. For more massive structures proportions of 1 : 2½ : 5 are common. Lean mixtures must be avoided because the adhesion to the steel is small and there is danger of slipping of the rods. A wet consistency, so soft that it will flow sluggishly, but not so wet as to produce a separation of the material in transferring to the work, must be used for all reinforced-concrete work.

Steel. Reinforcing steel is generally in the shape of bars, plain round bars being most extensively used. A number of **deformed bars,** that is, bars with irregular surfaces, have been designed to produce mechanical bond and greater adhesion between steel and concrete. For ordinary beams and slabs, plain bars may be used with safety. Deformed bars are of advantage for short, heavily loaded beams where large bending moments are developed in a short distance, or where the construction is subject to unusual vibration, or to resist temperature stresses. **Fabric reinforcement,** such as triangular mesh, expanded metal, welded wire, etc., is adapted in certain cases for slabs, walls, and partitions. Usually, however, bars are more economical. Angles and structural shapes, either riveted together or loose, are sometimes used as reinforcing material, but are unsatisfactory because of their poor adhesion to concrete. **Mild steel is preferred** by the majority of engineers because, if bought in open market, it is more reliable than high-carbon steel. The latter, however, if properly tested, may be safely used. The steel should comply with the following specifications of the American Society for Testing Materials.

Specifications for Steel for Reinforced Concrete

1. **Manufacture.** Steel may be made by either the open-hearth or Bessemer process. Bars shall be rolled from billets.

2. **Chemical and Physical Properties of Reinforcing Steel.** See p. 460 for the structural steel grade and cold-twisted bars; p. 461 for the hard grade (rail steel). The hard grade will be used only when specified.

3. Number of Tests. At least one tensile and one bending test shall be made from each melt of open-hearth steel rolled, and from each blow or lot of 10 tons of Bessemer steel rolled. In case bars differing ⅜ in. and more in diam. or thickness are rolled from one melt or blow, a test shall be made from the thickest and thinnest material rolled. Should either of these test specimens develop flaws, or should the tensile test specimen break outside of the middle third of its gaged length, it may be discarded and another test specimen substituted therefor. In case a tensile test specimen does not meet the specifications, an additional test may be made.

4. Number of Twists. Cold-twisted bars shall be twisted cold with one complete twist in a length equal to not more than twelve (12) times the thickness of the bar.

Adhesion or Bond Strength is the resistance which the steel bars embedded in concrete offer to longitudinal motion. Tests show that the bond strength is proportional to the area of the surface of steel in contact with the concrete, and varies with the character of the surface and the nature of concrete. The ultimate bond strength for 1:2:4 concrete varies from 350 to 460 lb. per sq. in. of contact surface for plain bars. A length of 50 diameters of embedment is a normal requirement for developing the tensile strength of the bars. Deformed bars give a higher bond resistance, depending upon the nature of deformity.

Table 1. Working Unit Stresses

Kind of stress		No. of formula (see pp. 442 to 445)	Allowable working stress: Percentage of crushing strength at 28 days	Allowable working stress: For 2000-lb. concrete, lb. per sq. in.	Remarks
Bearing		....	32.5	650	
Axial compression		....	22.5	$f_c = 450$	(a)
Columns (as described on p. 1313).	Vertical steel* (1 to 6 %)	(1)	22.5	$f_c = 450$	(b)
	Vertical steel* (1 to 6 %) and spirals (1 %)	(1)	35.0	$f_c = 700$	(c)
Compression in extreme fiber.	Ordinary	(5) to (22)	32.5	$f_c = 650$	(d)
	In continuous beams adjacent to support.	(5) to (22)	32.5	$f_c = 750$	
Shear (punching shear)			6	120	
Shear (as measure of diagonal tension).	Beams without web reinforcement.	(27)	2	$v = 40$	
	Beams with web reinforcement.	(27)	6	$v = 120$	(e)
Bond	Plain bars	(25), (26)	4	$u = 80$	
	Deformed bars	(25), (26)	5 to 6	100 to 120	
Steel in tension.	Structural grade	(4) to (23)	$f_s = 16{,}000$ lb. per sq. in.		
	First-class high-carbon steel.	(4) to (23)	$f_s = 18{,}000$ lb. per sq. in.		

The notation in the fourth column is explained on p. 442.

* Longitudinal steel is to be considered as taking a unit stress equal to nf_c, where n = ratio of moduli of elasticity of steel to concrete.

(a) Height of pier not to exceed 6 diameters; (b) height not to exceed 15 diameters; (c) height not to exceed 10 diameters of core; (d) use in beam formulæ; (e) two-thirds of this stress must be provided for with web reinforcement.

Moduli of Elasticity. For 1 : 2 : 4 concrete the value of the modulus when applied to beams may be taken at the average value of 2,000,000. The modulus of elasticity of steel is practically constant and may be taken as 30,000,000. The ratio of moduli of steel and concrete is of great importance in designing reinforced-concrete structures, and its value may be accepted as 15 for concrete having a strength of 2200 lb. per sq. in. or less. For concrete between 2200 and 2900 lb., use a ratio of 12, and for a strength over 2900 lb., use 10.

Working Unit Stresses. The working unit stresses given in Table 1 have been recommended by the Joint Committee on Concrete and Reinforced Concrete. The third column gives the unit stresses in per cent. of the crushing strength of concrete tested in cylinders 8 in. in diam. and 16 in. long. When a test of the concrete to be used is not available, the expected crushing strength for the concrete of the proportions used may be taken from Table 6, p. 575, at the same time adopting the corresponding ratio of moduli of elasticity of steel to concrete given above. The fourth column in the table gives allowable stresses for 1 : 2 : 4 concrete the strength of which at 28 days is 2000 lb.

UNITS OF CONSTRUCTION

Structures may be divided into the following parts: slabs, beams, columns and footings.

Beams and Slabs

Loads and Spans. The dead and live loads are added together in figuring the allowable stresses. Span lengths of slabs and beams should be taken as the distance between centers of supports, but not to exceed the clear span plus the depth of beam or slab.

Moments for Dead Load and Uniformly Distributed Live Load for Continuous Beams. (*a*) For interior spans of beams and slabs continuous over more than three panels, the maximum positive bending moment at the center and the maximum negative bending moment at the supports should be taken as $M = wL^2/12$, and for end spans at the center and adjoining the support as $M = wL^2/10$. (*b*) For beams and slabs continuous for two spans only, the bending moment at the central support should be taken as $wL^2/8$, and near the middle of the span as $wL^2/10$. (*c*) At the extreme ends of continuous beams the amount of negative moment which will be developed will depend on the conditions of fixedness, but must not be taken at less than $M = wL^2/20$.

Slabs

Slabs supported by steel and by concrete beams respectively are shown in Figs. 1 and 2. In Fig. 3 a part of the tensile portion is replaced by tiles, thus reducing the weight of the slab. Flat slabs supported on columns with flaring heads are also used to a great extent. Slabs are designed by the formulæ on p. 446. The theoretical depth is the distance from center of steel to top of slab. Thickness of concrete below steel should be ¾ in. Shear and bond stresses do not need to be considered for ordinary slabs.

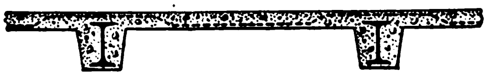


FIG. 1.—Slab Supported by Steel Beams.

Reinforcement of Slabs. Slabs **simply supported** at the ends need to be reinforced at the bottom of the slab only, the principal reinforcement being placed at right angles to the supporting beams. Simply supported slabs, however, seldom occur in construction. **Continuous slabs,** *i.e.*, slabs continuous over several supports, require at each support the same amount of steel placed near the upper surface of the beam as is used at the bottom of slab in the middle of the span. Part of the steel, usually one-half or two-thirds is bent up at a point about one-fifth of the span distant from the support, and carried over the support into the adjoining span. By proper arrangement of bent bars, as regards the lapping of the ends, the required amount of steel at the support may be obtained without introducing extra short bars. To prevent shrinkage cracks, bars of small diameter are placed (preferably in the top of the slab, from 2 to 3 ft. apart) transversely to the principal reinforcement. The **usual reinforcement** of slabs consists of bars, preferably of small diameter, spaced from 4 to 10 in. apart, depending upon the diameter and upon the thickness of slab. **Fabric reinforcement** also may be used for reinforcing slabs.

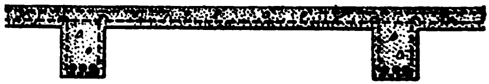


FIG. 2.—Slab Supported by Concrete Beams.

Table 2. Dimensions of Slabs for Different Spans and Loads*

(Based on $M = wL^2/12$; $f_c = 650$ lb. per sq. in.; $f_s = 16{,}000$ lb. per sq. in.; $n = 15$)

h = total thickness of structural slab; A_s = area of cross-section of steel per foot of width of slab

Span, ft.	Live load per square foot†																	
	40		60		80		100		125		150		175		200		250	
	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.	h, in.	A_s, sq. in.
4	3	0.04	3	0.05	3	0.06	3	0.07	3	0.08	3	0.10	3	0.11	3	0.12	3	0.14
5	3	0.06	3	0.08	3	0.09	3	0.11	3	0.13	3	0.15	3	0.17	3½	0.16	3½	0.19
6	3	0.09	3	0.11	3	0.13	3	0.16	3	0.19	3½	0.18	3½	0.20	4	0.19	4	0.23
7	3	0.12	3	0.15	3	0.18	3½	0.18	3½	0.21	4	0.21	4	0.24	4	0.27	4½	0.30
8	3	0.16	3	0.21	3½	0.21	4	0.21	4	0.24	4½	0.27	4½	0.30	4½	0.33	5	0.35
9	3	0.20	3½	0.22	4	0.23	4	0.26	4½	0.30	5	0.31	5	0.35	5	0.38	5½	0.41
10	3½	0.22	4	0.24	4	0.28	4½	0.32	5	0.33	5	0.38	5½	0.38	5½	0.43	6	0.46
11	4	0.24	4	0.29	4½	0.33	5	0.35	5½	0.37	5½	0.42	6	0.43	6	0.47	6½	0.52
12	4	0.28	4½	0.34	5	0.37	5½	0.38	6	0.41	6	0.46	6½	0.48	6½	0.52	7½	0.54
13	4½	0.33	5	0.37	5½	0.40	6	0.42	6½	0.45	6½	0.50	7	0.53	7½	0.55	8	0.60
14	5	0.36	5½	0.40	6	0.43	6½	0.46	7	0.50	7	0.55	7½	0.58	8	0.60	8½	0.66
15	5	0.41	6	0.43	6½	0.47	6½	0.53	7½	0.54	7½	0.60	8	0.63	8½	0.65	9	0.73
16	5½	0.44	6½	0.47	7	0.51	7	0.57	7½	0.61	8	0.65	8½	0.68	9	0.71	9½	0.79
17	6½	0.46	7	0.51	7½	0.55	7½	0.62	8	0.66	8½	0.70	9	0.74	9½	0.77	10½	0.83
18	6½	0.51	7	0.57	7½	0.62	8	0.66	9	0.69	9	0.76	9½	0.80	10	0.83	11	0.90
19	7	0.55	7½	0.61	8	0.66	8½	0.71	9½	0.74	10	0.78	10½	0.83	11	0.87	11½	0.96
20	7½	0.59	8	0.65	8½	0.71	9	0.76	10	0.79	10½	0.84	11	0.89	11½	0.93	12	1.03

† Live load in this table is total load exclusive of weight of slab but inclusive of weight of flooring or granolithic finish.

The cross-sectional area of the strands must be computed to resist the stresses.

Rectangular Slabs. Slabs supported on four sides (when the length does not exceed 1½ times the width) must be reinforced in two directions, and the reinforcement in each direction designed to carry its proportion of the load. For transverse reinforcement, the proportion r for conservative design may be found from the equation $r = (l/b) - 0.5$, where l equals length and b width of slab.

Concrete Floor with Hollow Tile (see Fig. 3) is adapted to certain cases where long spans with flat plastered ceilings between beams are required. The widths of the concrete beams or ribs between the lines of tiles must be determined by shear requirements. The compression portion consists of concrete of sufficient depth above the tiles. The bottoms of the ribs are flush with the bottoms of the tiles. Care must be taken to see that the bond stress does not exceed the allowable working limits. The spacing of the ribs is fixed by the size of the tiles, which are generally 12 in. wide.

Fig. 3.—Concrete Floor with Hollow Tile.

Beams

Reinforced-concrete beams are seldom of rectangular shape. Usually, the rectangular beams are built monolithic with the slab, so that a portion of the latter may be considered as an integral part of the beam, forming the so-called **T-beam.** In practice, the depth of the beam is governed by the rigidity of construction, available head room, and economy. The ratio of length of span to depth is usually restricted to a value of 12. The width of beam, which must be large enough to enable convenient placing of steel, is determined by the shearing stresses. The ratio of depth to width must not exceed four.

For the **design of rectangular beams,** use formulæ (2) to (8), p. 443; for T-beams, formulæ (9) to (18) p. 444. For rectangular beams with steel in top and bottom and for T-beams at the supports, use formulæ (19) to (24) p. 445. The theoretical depth d is the distance from center of tension steel to top of beam. For one layer of bars the center of steel coincides with the center of the bars. For two layers of bars the center of steel is at the center of gravity of the group of bars. To obtain total depth, h, add to the theoretical depth, d, 2 in. or the distance from center of steel to the lowest surface of steel plus 1½ in. Concrete below steel is necessary for fireproofing.

Reinforcement of Beams usually consists of bars placed at the bottom of the beam at the center of the span. Some of the bars are bent in two or more places and carried at the top of the beams, the remaining ones being carried straight. The bent bars help to resist the diagonal tension.

Continuous Beams, *i.e.*, beams continuous over several supports. In these, the bent bars are carried over the support and extend a sufficient length into the beam of the adjoining span to provide anchorage. For 2000-lb. concrete and a working stress of 16,000 lb. in the steel, this requires, by Joint Committee recommendations, a length of embedment of 50 diameters beyond the point of maximum stress. The amount of steel required at the top of the support is substantially the same as at the center of the span. One-half of the bottom steel is generally bent up and carried at the top of the beam and the remainder of the steel required for the negative bending moment is carried over from the adjoining span. If the sum of the cross-sectional areas of the bent bars and the bars carried over from the adjacent span is not large enough, short bars must be used to make up the required area. Fig. 4 illustrates good practice in reinforcing continuous beams. The bars may be either bent and assembled in place in the forms, or unit frames built at the shop may be used.

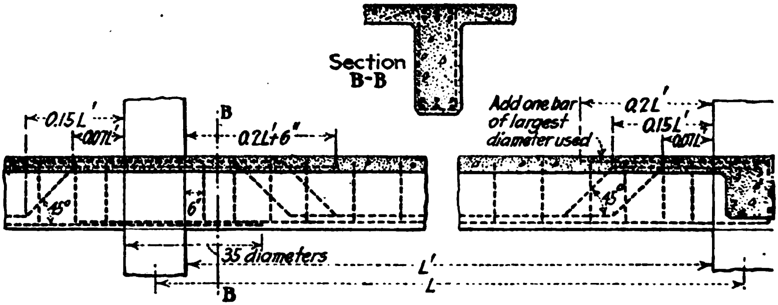


FIG. 4.—Reinforcement of Continuous Beams.

Continuous T-beam at the Support. The compressive area of a T-beam in the middle of the span includes not only the upper part of the web but also a portion of the slab which acts as a flange. At the support, however, the beam is subject to a negative bending moment, so that the flange of the T-beam, which is in tension, is not effective. This reduces the T-beam at the support to a rectangular beam with a compressive concrete area considerably smaller than at the center of the span. To keep the compressive stress in concrete at the support within working limits, the depth of beam must be increased or a sufficient amount of compression steel introduced. In computing, the beam is considered as a rectangular beam with steel in top and bottom. The straight bars in the bottom of the beam at the support are figured to take a part of the compression, and, if necessary, additional short bars may be placed. [See formulæ (19) to (24), p. 445.]

Cantilever Beams must be reinforced with bars placed at the top. The maximum stress in steel is at the support, and the bars must therefore be embedded for a sufficient length to develop this stress or be specially anchored. For steel stressed up to 16,000 lb. per sq. in. and an allowable bond stress of 80 lb. per sq. in., the required length of embedment is equal to 50 diameters.

Diagonal Tension Reinforcement consists of stirrups of U-shaped form. The spacing of stirrups and their sectional area may be found by formulæ (28) and (30), p. 445. Stirrups must be firmly attached to the horizontal bars which are in tension. At the support of continuous beams, since the tension bars are at the top, the attachment must be made to these bars. The ends in the compression part of the beam must be provided with hooks or other means of increasing the bond. Wherever stirrups are necessary, they should not be spaced farther apart than three-fourths the depth of the beam.

Spacing of Bars in Beams. Parallel bars should not be spaced nearer together than 3 times their diameter measured from center to center, nor should the distance from side of the beam to center of the nearest bar be less than 2 diameters. When necessary, two layers of bars may be placed one above the other, with 1 in. clear spacing in between. More than two layers of bars is not advisable. The clear spacing between the lowest layer of bars and the bottom of the beam should be 1½ in.

Table 3. T-beams for Uniform Loading

[Dimensions and reinforcement of beams for 1 : 2 : 4 concrete. $n = 15$, $v = 120$, $f_c = 650$ at center and 750 at support, $f_s = 16{,}000$. (For notation, see p. 442)]

Load, lb. per linear ft.	Width of beam, in.	Depth (top of slab to bottom of beam), in.	Minimum thickness of slab, in. †	Simple beam $M = WL/8$: A_s	Simple beam $M = WL/8$: No. and diam. (in.) of round bars	Continuous beam $M = WL/12$: A_s	Continuous beam $M = WL/12$: No. and diam. (in.) of round bars	End beam $M = WL/10$: A_s	End beam $M = WL/10$: No. and diam. (in.) of round bars	Round U-stirrups: Diam., in.	Round U-stirrups: No.	Round U-stirrups: Spacing in inches at each end
SPAN 10 FT.												
800	6	12	3	0.86	2-¾	0.57	2-⅝	0.68	1-¾+1-⅝	⅜	8	6,8,8,8
1000	6	12	3	1.07	1-⅞+1-¾	0.71	1-¾+1-⅝	0.86	2-¾	⅜	8	6,8¾,8¾,8¾
1200	8	12	3	1.29	3-¾	0.86	2-¾	1.03	2-⅝+1-¾	⅜	8	6,8,8,8
1400	8	12	3	1.50	2-⅞+1-⅝	1.00	2-⅝+1-¾	1.20	2-⅝+1-⅞	⅜	10	6,6,8,8,8
1600	8	14	3	1.43	2-¾+1-⅞	0.95	2-⅝+1-¾	1.14	2-¾+1-⅝	⅜	12	6,6,8,8,8,8
1800	8	15	3	1.48	2-¾+1-⅞	0.99	2-⅝+1-¾	1.19	2-¾+1-⅝	⅜	10	6,6,9,9,9
2000	8	15	3	1.79	*2-⅞+2-⅝	1.10	2-¾+1-⅝	1.32	3-¾	⅜	12	6,6,7,7,9,9,
2200	8	16	3	1.68	2-⅞+1-¾	1.12	2-¾+1-⅝	1.34	3-¾	⅜	8	6,8,11½,11½
2400	9	16	3	1.83	3-⅞	1.22	3-¾	1.46	2-¾+1-⅞	⅜	12	6,6,7,7,9,9,
2600	9	18	3	1.74	3-⅞	1.16	2-¾+1-⅝	1.39	2-¾+1-⅞	⅜	10	6,6,6,9,9
3000	9	18	3	2.01	2-1 +1-¾	1.34	3-¾	1.61	2-⅞+1-¾	⅜	10	6,6,8,8,12
SPAN 12 FT.												
800	8	12	3	1.23	3-¾	0.82	2-½+1-¾	0.99	2-⅝+1-¾	⅜	8	8,8,8,8
1000	8	12	3	1.55	2-⅞+1-¾	1.03	2-⅝+1-¾	1.24	3-¾	⅜	10	6,8,8,8,8
1200	8	14	3	1.55	2-⅞+1-¾	1.03	2-⅝+1-¾	1 24	3-¾	⅜	10	6,8,8,8,8
1400	8	15	3	1.80	*2-⅞+2-⅝	1.01	2-⅝+1-¾	1.33	3-¾	⅜	12	6,6,6,10,10,10
1600	8	15	3	2.06	*2-¾+2-⅞	1.27	3-¾	1.52	2-⅞+1-⅝	⅜	12	6,6,8,8,10,10
1800	8	17	3	1.98	*2-¾+2-⅞	1.23	3-¾	1.48	2-¾+1-⅞	⅜	10	6,6,10,10,10
2000	8	17	3	2.21	*3-⅞+1-¾	1.37	2-¾+1-⅞	1.64	2-⅞+1-¾	⅜	14	6,6,6,6,10,10,10
2200	8	21	3	1.89	*3-¾+2-⅝	1.19	2-¾+1-⅝	1.43	2-¾+1-⅞	⅜	12	6,6,6,10,10,10
2400	8	21	3	2.06	*2-¾+2-⅞	1.30	3-¾	1.56	2-⅞+1-¾	½	8	6,12,12,12
2600	8	21	3	2.23	*3-⅞+1-¾	1.41	2-¾+1-⅞	1.79	*2-⅞+2-⅝	½	8	6,12,12,12
3000	10	21	3	2.57	*6-¾	1.63	2-⅞+1-¾	2.05	*2-⅞+2-¾	½	8	6,12,12,12
SPAN 14 FT.												
800	8	14	3	1.40	2-¾+1-⅞	0.93	3-⅝	1.12	2-¾+1-⅝	⅜	8	6,9,9,9
1000	8	15	3	1.76	*4-¾	1.08	2-¾+1-½	1.29	3-¾	⅜	10	6,10,10,10,10
1200	8	15	3	2.10	*2-¾+2-⅞	1.29	3-¾	1.55	2-⅞+1-¾	⅜	12	6,8,8,10,10,10
1400	8	17	3	2.10	*2-¾+2-⅞	1.31	3-¾	1.57	2-⅞+1-¾	⅜	12	6,8,8,10,10,10
1600	8	17	3	2.40	*4-⅞	1.49	2-⅞+1-⅝	1.92	3-¾+1-⅞	⅜	12	6,6,8,8,12,12
1800	10	17	3	2.70	*2-1 +2-⅞	1.68	2-⅞+1-¾	2.16	*3-⅞+1-¾	½	10	6,10,12,12,12
2000	10	17	3	3.00	*5-⅞	2.00	*2-⅞+2-¾	2.40	*4-⅞	½	10	6,10,12,12,12
2200	10	19	3	2.88	*4-⅞+1-¾	1.81	3-⅞	2.30	*4-⅞	½	10	6,10,12,12,12
2400	10	19	3	3.15	*3-⅞+3-¾	2.10	*2-⅞+2-¾	2.52	*5-¾+1-⅝	½	12	6,8,8,12,12,12
2600	0	21	3	3.03	*5-⅞	2.02	*2-⅞+2-¾	2.42	*4-⅞	½	10	6,10,12,12,12
3000	10	23	3	3.15	*3-⅞+3-¾	2.10	*2-⅞+2-¾	2.52	*6-¾	½	10	6,9,12,12,16

* Reinforcing bars are arranged in two layers.

† The thickness in this column is the minimum required for compression.

Table 3. T-beams for Uniform Loading—(*continued*)

[Dimensions and reinforcement of beams for 1 : 2 : 4 concrete. $n = 15$, $v = 120$, $f_c = 650$ at center and 750 at support, $f_s = 16{,}000$. (For notation, see p. 442)]

Load, lb. per linear ft.	Width of beam, in.	Depth (top of slab to bottom of beam), in.	Minimum thickness of slab, in. (t)†	Simple beam $M = WL/8$		Continuous beam $M = WL/12$		End beam $M = WL/10$		Round U-stirrups		
				A_s	No. and diam. (in.) of round bars	A_s	No. and diam. (in.) of round bars	A_s	No. and diam. (in.) of round bars	Diam., in.	No.	Spacing in inches at each end
						SPAN 16 FT.						
800	8	15	3	1.83	*2-7/8+2-5/8	1.13	2-3/4+1-5/8	1.35	2-3/4+1-7/8	3/8	10	6,10,10,10,10
1000	8	17	3	1.95	*2-3/4+2-7/8	1.22	3-3/4	1.46	2-3/4+1-7/8	3/8	10	6,10,10,12,12
1200	8	17	3	2.35	*4-7/8	1.46	2-3/4+1-7/8	1.75	3-7/8	3/8	12	6,6,9,9,12,12
1400	10	17	3	2.74	*2-1 +2-7/8	1.71	3-7/8	2.20	*3-7/8+1-3/4	3/8	14	6,7,7,7,9,9,9,
1600	10	17	3½	3.14	*3-7/8+3-3/4	2.09	*2-3/4+2-7/8	2.51	*5-3/4+1-5/8	3/8	16	6,6,6,6,8,8,10,10
1800	10	19	3½	3.09	*3-7/8+3-3/4	2.06	*2-3/4+2-7/8	2.47	*2-1 +2-3/4	3/8	16	6,6,6,6,8,8,10,10
2000	10	19	3½	3.42	*5-7/8+1-3/4	2.28	*3-7/8+1-3/4	2.74	*2-1 +2-7/8	1/2	12	6,9,12,12,12,12
2200	10	23	3½	3.02	*5-7/8	2.01	*2-3/4+2-7/8	2.41	*4-7/8	3/8	16	6,6,6,6,8,8,10,10
2400	10	23	3½	3.30	*4-7/8+2-3/4	2.20	*3-7/8+1-3/4	2.63	*6-3/4	1/2	10	6,10,14,14,14
2600	10	23	3½	3.57	*6-7/8	2.38	*4-7/8	2.85	*4-7/8+1-3/4	1/2	12	6,9,9,9,14,14
3000	12	23	3½	4.11	*3-1 +3-7/8	2.61	1-1 +3-7/8	3.29	*5-7/8+1-5/8	1/2	12	6,8,8,12,12,16
						SPAN 18 FT.						
800	8	17	3	1.98	*2-3/4+2-7/8	1.23	3-3/4	1.48	2-3/4+1-7/8	3/8	10	6,10,10,12,12
1000	10	17	3	2.48	*2-1 +2-3/4	1.54	2-7/8+1-3/4	1.85	3-7/8	3/8	12	6,6,9,9,12,12
1200	10	17	3	2.97	*5-7/8	1.85	3-7/8	2.38	*4-7/8	3/8	16	6,6,7,7,8,8,10,10
1400	10	20	3	2.85	*4-7/8+1-3/4	1.80	3-7/8	2.28	*3-7/8+1-3/4	3/8	16	6,6,7,7,8,8,10,10
1600	10	20	3½	3.27	*4-7/8+2-3/4	2.18	*3-7/8+1-3/4	2.61	*6-3/4	1/2	10	6,10,12,16,16
1800	10	20	3½	3.68	*6-7/8	2.45	*4-7/8	2.94	*5-7/8	1/2	12	6,9,10,10,16,16
2000	10	23	3½	3.46	*5-7/8+1-3/4	2.31	*4-7/8	2.77	*2-1 +2-7/8	1/2	12	6,10,12,12,12,12
2200	10	23	3½	3.82	*5-1	2.55	*6-3/4	3.05	*3-7/8+3-3/4	1/2	12	6,9,10,10,16,16
2400	12	23	3½	4.17	*3-1 +3-7/8	2.78	*2-1 +2-7/8	3.33	*5-7/8+1-5/8	1/2	12	6,9,9,12,12,16
2600	12	23	4	4.52	*3-1 1/8+2-1	3.01	*5-7/8	3.61	*6-7/8	1/2	14	6,6,9,9,12,12,16
3000	12	27	4	4.34	*4-1 +2-7/8	2.89	*4-7/8+1-3/4	3.47	*5-7/8+1-3/4	1/2	12	6,9,9,12,12,17
						SPAN 20 FT.						
800	8	19	3	2.14	*3-7/8+1-3/4	1.35	3-3/4	1.61	2-7/8+1-3/4	3/8	8	6,14,14,14
1000	8	21	3	2.38	*4-7/8	1.50	2-7/8+1-5/8	1.91	*3-3/4+1-7/8	3/8	12	6,6,10,10,12,12
1200	8	21	3	2.85	*4-7/8+1-3/4	1.90	*3-3/4+1-7/8	2.28	*4-7/8	3/8	14	6,8,8,10,10,13,13
1400	9	22	3½	3.16	*3-7/8+3-3/4	2.11	*2-3/4+2-7/8	2.53	*6-3/4	3/8	16	6,7,7,9,9,9,12,12
1600	9	22	3½	3.60	*6-7/8	2.41	*4-7/8	2.89	*5-7/8	1/2	12	6,10,12,12,16,16
1800	9	24	3½	3.68	*6-7/8	2.45	*2-1 +2-3/4	2.94	*5-7/8	1/2	14	6,10,12,12,16,16,16
2000	10	24	3½	4.08	*3-1 +3-7/8	2.72	*2-1 +2-7/8	3.27	*4-7/8+2-3/4	1/2	12	6,10,12,12,16,16
2200	10	24	4	4.49	*5-1 +1-7/8	3.00	*5-7/8	3.60	*6-7/8	1/2	14	6,10,12,12,16,16,16
2400	12	24	4	4.90	*5-1 1/8	3.27	*4-7/8+2-3/4	3.92	*5-1	1/2	16	6,8,8,10,10,12,12,12
2600	12	24	4	5.31	*3-1 1/8+3-1	3.54	*6-7/8	4.25	*4-1 +2-7/8	1/2	15	6,6,9,9,9,12,12,16
3000	12	27	4	5.36	3-1 1/8+3-1	3.57	*6-7/8	4.28	*4-1 +2-7/8	1/2	16	6,8,8,10,10,12,12,12

* Reinforcing bars are arranged in two layers.

† The thickness in this column is the minimum required for compression.

Explanations for T-beam Table (Table 3)

How to Use.

(1) Take span of beam as distance center to center of supports, but not to exceed clear span plus depth of beam.

(2) Find from Table 2 required thickness of slab to support the load.

(3) Compute total live and dead load per linear foot of beam. Dead load to include weight of slab, beam, and granolithic finish or other flooring used.

(4) Note whether beam is an interior span, an end span, or a simply supported beam, and from the table take off the required dimensions and reinforcement.

(5) If there is no table for the span required, use table for the next higher span, but reduce the load in the ratio of required span and span used. For example, when designing a beam for 18 ft. 6 in., use the table for 20 ft. span and multiply the load by 18.5/20.

(6) See that thickness of slab determined for the load is not smaller than minimum thickness given in column 4.

Bending of Steel. Bend at least half of the bars in each beam. For intermediate support, carry the bent bars over support into adjoining span the length required. For end support, hook bent bars in column for a length of at least 9 diameters, and add one short hooked bar of largest diameter used. For an even number of bars of same diameter, bend half the bars. For an odd number of bars, bend four out of seven, three out of five, and two out of three. With reinforcement in two rows the bending should start at $0.07L'$ and $0.15\ L'$ respectively from the support, as shown in Fig. 4. With reinforcement in one row this distance should be $0.1\ L'$.

When a combination of bars of various diameters is used, see that the cross-sectional area of the bent bars is at least equal to half the total cross-section of the bars.

When more than two bars are to be bent, bends should be made in two places at each end of beam, as shown in Fig. 4. For an even number of bars, bend half of them in each place. For an odd number of bars, the number of bars bent near the support should be one greater than the number at the other bend.

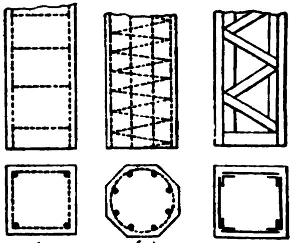
Stirrups. Beams are provided with web reinforcement of stirrups where shearing unit stress, taken as the measure of diagonal tension, exceeds 40 lb. per sq. in. Bent-up bars were not counted as web reinforcement; therefore, if desired, the spacing of stirrups may be increased where they overlap with bent portions of the horizontal bars, or altogether omitted.

Bond Stresses for Tension Bars in Beams must be calculated by formula (27), p. 445. If the bond stress is too large, the diameter of bar must be decreased or the depth of beam increased to prevent slip, Where high bond resistance is required, deformed bars, *i.e.*, bars with uneven surfaces, may be used to advantage.

Columns

Piers of short length up to 6 times the least diameter and loaded centrally may be built of plain concrete. Columns longer than this and also piers and columns subject to cross-bending must be reinforced. Formulæ for reinforced-concrete columns are given on p. 442. An outside shell of a thickness equal to 1½ in. should be considered as fireproofing. The usual mixtures for concrete columns are 1 : 2 : 4 and 1 : 1½ : 3. For columns carrying heavy loads the cross-section may be reduced by using a rich mixture of concrete. As the steel is stressed only to 15 times the allowable stress in concrete, the use of a large amount of steel is not economical; better economy is obtained by using rich mixtures. Reinforcement of columns usually consists of (1) longitudinal bars; (2) circular hoops or spirals; (3) longitudinal bars with circular hoops or spirals; (4) rigid structural shapes.

Columns with Longitudinal Reinforcement. [Fig. 5(*a*).] When the allowable unit stresses given on p. 1306 are used, the length of the column must not exceed 15 times its least diameter. The percentage of steel in longitudinal reinforcement should not be less than 1 per cent. nor more than 6 per cent. of the concrete area. Longitudinal reinforcement usually consists of bars, but small angles may also be used. The steel is placed 2 in. to 2½ in. from the surface. To prevent the steel from buckling and to keep it in place during erection, horizontal ties of small diameter are used, spaced not farther apart than the diameter of the column nor more than 15 diameters of the bar. Joints in vertical bars may be provided by lapping them for a length of about 30 diameters. For bars of large diameter, the ends should be milled and butted, with a sleeve around the joint. Steel in the basement column either must be carried 30 diameters into the footing, or dowels of sufficient length provided; or else the bars may rest on steel plates of large enough area to transfer the stress to the footing by bearing on the concrete.



(a) (b) (c)

Fig. 5.—Types of Reinforcement for Concrete Columns.

Columns with Spiral Reinforcement. When spirals are counted upon as increasing the strength of the column, the amount of such reinforcement must not be less than 1 per cent. of the volume of the concrete enclosed and the length of column not more than 10 diameters of the core. The pitch of the spiral should not be greater than one-sixth of the diameter of the column, and preferably not greater than one-tenth and in no case more than 2½ in. The ends of the spirals must be lapped or held together in such a way as to develop their full strength in tension. It is not advisable to use concrete columns with spirals only.

Columns with Longitudinal Bars and Spirals. [Fig. 5(*b*)]. When the stresses given on p. 1306 are used, the length of column must not exceed ten diameters of the core. Suggestions as to the amount of longitudinal steel and hoops and also as to their arrangement are given in the two preceding paragraphs. A column of this type is very reliable.

Concrete Columns with Rigid Structural Shapes. When small shapes are used of an amount not exceeding 4 per cent. of the concrete cross-sectional area, the column may be considered as reinforced concrete and treated as suggested under "Columns with Longitudinal Reinforcement." When a larger amount of steel is used, 10 per cent. or 15 per cent. of the concrete area, the column may be considered as a structural-steel column strengthened by the concrete. Concrete must not be relied upon as a binding material, and the shapes composing the structural-steel column must be riveted together and tied by tie plates or lattice bars, and in other respects designed in conformity with standard practice for structural steel. A satisfactory arrangement of steel shapes is shown at (*c*), Fig. 5.

Footings

Footings for separate columns are usually square or rectangular. The area of the footing is determined by the allowable unit pressure on the ground, or, if the footing is carried on piles, by the number and spacing of the piles. In figuring the moments, the projecting footing may be taken as cut along each diagonal, and each part acting as a cantilever supported at the edge of the column and loaded uniformly by the reaction of the ground, or, for pile foundations, by the reaction of the piles. The depth of the footing is determined by shear and bond stresses, formulæ (25), (26) and (27) p. 445, and is generally larger than would be required by the moment formula (4) (p. 445). The steel determined from the bending moment and the depth consists of bars, preferably of small diameter, placed at the bottom and usually in two transverse directions. Four-way reinforcement is sometimes used, but is of no special advantage. Deformed bars may be used to advantage because of their greater bond resistance. Use slab formulæ (9) to (18), p. 444, for determining depth and amount of reinforcement, and formulæ (25) to (27) to determine shear and bond. The sizes of bars to select are usually governed by the bond stresses.

Table 4. Round and Square Reinforced-concrete Columns

Table for determining cross-section of concrete and steel for required load, or safe load P for given dimensions of column.

(Based on $f_c = 450$; $n = 15$; for 1 : 2 : 4 concrete, and formula $P = Af_c[1 + (n - 1)p]$. For notation, see p. 442)

Square column, side, in.	Equivalent diam. of round column, in.	Effective area, sq. in.*	Ratio of area of steel to effective area of concrete $= p$													
			0.01		0.015		0.02		0.025		0.03		0.035		0.04	
			P/1000, lb.	As, sq. in.	P/1000, lb.	As, sq. in.	P/1000, lb.	As, sq. in.	P/1000, lb.	As, sq. in.	P/1000, lb.	As, sq. in.	P/1000, lb.	As, sq. in.	P/1000, lb.	As, sq. in.
10	10.9	49	25	0.5	27	0.7	28	1.0	30	1.2	31	1.5	33	1.7	34	2.0
11	12.0	64	33	0.6	35	1.0	37	1.3	39	1.6	41	1.9	43	2.2	45	2.6
12	13.1	81	42	0.8	44	1.2	47	1.6	49	2.0	52	2.4	54	2.8	57	3.2
13	14.3	100	51	1.0	54	1.5	58	2.0	61	2.5	64	3.0	67	3.5	70	4.0
14	15.4	121	62	1.2	66	1.8	70	2.4	74	3.0	77	3.6	81	4.2	85	4.8
15	16.5	144	74	1.4	78	2.2	83	2.9	87	3.6	92	4.3	97	5.0	101	5.8
16	17.7	169	87	1.7	92	2.5	97	3.4	103	4.2	108	5.1	113	5.9	119	6.8
17	18.8	196	101	2.0	107	2.9	113	3.9	119	4.9	125	5.9	131	6.9	138	7.8
18	19.9	225	115	2.3	123	3.4	130	4.5	137	5.6	144	6.8	151	7.9	158	9.0
19	21.0	256	131	2.6	139	3.8	147	5.1	156	6.4	164	7.7	172	9.0	180	10.2
20	22.2	289	148	2.9	157	4.3	166	5.8	176	7.2	185	8.7	194	10.1	203	11.6
22	24.4	361	185	3.6	197	5.4	208	7.2	219	9.0	231	10.8	242	12.6	253	14.4
24	26.6	441	226	4.4	240	6.6	254	8.8	268	11.0	282	13.2	296	15.4	310	17.6
26	28.9	529	271	5.3	288	7.9	305	10.6	321	13.2	338	15.9	355	18.5	371	21.2
28	31.2	625	321	6.3	340	9.4	360	12.5	380	15.6	399	18.8	419	21.9	439	25.0
30	33.4	729	374	7.3	397	10.9	420	14.6	443	18.2	466	21.9	489	25.5	512	29.2
32	35.6	841	431	8.4	458	12.6	484	16.8	511	21.0	537	25.2	564	29.4	590	33.6
34	37.9	961	493	9.6	523	14.4	554	19.2	584	24.0	614	28.8	644	33.6	675	38.4
36	40.2	1089	559	10.9	593	16.3	627	21.8	662	27.2	696	32.7	730	38.1	765	43.6
38	42.4	1225	628	12.3	667	18.4	706	24.5	744	30.6	783	36.8	821	42.9	860	49.0

* 1½ in. of concrete on each side considered as fireproofing is not included in the effective area of column.

Combined Footings for two or more columns must be considered as a slab supported at the columns and loaded uniformly by the reaction of the ground. The area of the footing is determined by the allowable pressure on the ground, and its shape must be such that its center of gravity coincides with the center of gravity of superimposed loads. This requirement is very important, as otherwise unequal settlement may result. The depth and amount of reinforcement are determined from formulæ (2) and (8) p. 443, and bond and shear from formulæ (25) to (27) p. 445. Bars placed near the top surface between columns are bent down and carried near to the bottom surface under the columns and in the projections. Transversely to this reinforcement, bars must be placed at the bottom of the footing of sufficient strength to distribute the load in transverse directions. Shear reinforcement may be required.

Spread Foundation, consisting of a slab over the entire area under the columns, or of a slab supported by beams, may be considered as loaded by a uniform upward reaction of the ground. The principle of design is exactly the same as that applied to a floor system, except that the load acts upward instead of downward.

Walls and Partitions

Reinforced concrete is well adapted to the construction of walls, especially where they have to withstand heavy pressures, such as the retaining walls of a cellar or basement, walls for coal pockets, silos, reservoirs and grain elevators. For buildings, the cost of forms may make concrete walls more expensive than brick walls, but concrete walls may be made one-half to two-thirds the thickness of brick for the same conditions. Partitions may be built of solid concrete from 4 to 6 in. thick, reinforced to prevent temperature and shrinkage cracks. The amount of steel necessary for temperature reinforcement varies from 0.3 to 0.5 per cent., the last figure to be used for thin walls in exposed locations. Bars of small diameter (1/4 to 5/8 in.) or fabric reinforcement may be used.

Forms

Forms may be built of wood or metal. Wooden forms are usually most economical unless the construction allows for the repeated use of the same forms. For wooden forms, spruce, Norway pine, and shortleaf southern pine are generally used. For slab forms and sides of beams, 1-in., 1 1/4-in. or 1 1/2-in., stock is generally used; for bottoms of beams, 2-in. stock; and for columns 1 1/2-in. stock Forms must be designed so that they can be easily erected, removed, and re-erected. The usual order of removing of forms is: (1) column sides; (2) joists; (3) girder and beam sides; (4) slab bottoms; (5) girder and beam bottoms. **Column forms** are held together by clamps made of wood or steel, the spacing of which is smallest at the bottom and increases with the decrease in pressure. **Beam forms** consist of the bottom and two sides held together by clamps or cleats and supported by posts. **Slab forms** consist of boards supported by joists spaced 2 or 3 ft. apart. The joists either rest on a horizontal joist bearer nailed to the clamps of the beam or girder, or are supported by stringers and posts or both.

Removal of Forms. The time that forms should remain in place depends upon the character of the members and upon weather conditions. The hardness of concrete must be ascertained before removing the forms. As an approximate guide for the minimum time for form removal, the following rules (from Taylor and Thompson's "Concrete Costs") may be observed:

WALLS IN MASS WORK. One to three days, or until concrete will bear pressure of the thumb without indentation.

THIN WALLS. In summer, 2 days; in cold weather, 5 days.

COLUMNS. In summer, 2 days; in cold weather, 4 days, provided girders are shored to prevent appreciable weight reaching the columns.

SLABS UP TO 7-FT. SPAN. In summer, 6 days; in cold weather, 2 weeks.

BEAMS AND GIRDER SIDES. In summer, 6 days; in cold weather, 2 weeks.

BEAMS AND GIRDERS AND LONG-SPAN SLABS. In summer, 10 days or 2 weeks; in cold weather, 3 weeks to 1 month. If shores are left without disturbing them, the sheeting in summer may be removed after one week.

CONDUITS. Two or three days, provided there is not a heavy fill upon them.

ARCHES. If of small size, 1 week; large arches with heavy dead load, 1 month.

INDUSTRIAL BUILDINGS

BY

CHARLES DAY

REFERENCES: "Principles of Industrial Engineering," Charles B. Going; "Industrial Plants," Charles Day; "The Principles of Scientific Management," F. W. Taylor; "The Principles of Industrial Management," John C. Duncan; "Factory Organization and Administration," Hugo Diemer.

THE PLANNING OF INDUSTRIAL PLANTS

The development and extension of existing manufacturing plants and the building and equipping of entirely new plants should be based upon a thorough preliminary study of the factors involved. After the influences of the product, the physical equipment, the processes employed and the methods of administration upon the proposed plant have been investigated, it is possible to determine intelligently the areas and arrangement of departments, the number and character of buildings required to house them, and the size and location of the site.

Character of Data Required. Complete data should be obtained concerning the nature and amount of the products of manufacture. In the case of the metal industries, for example, the calculations should show the **annual consumption** of gray-iron castings, forgings, bar stock steel, lumber for all purposes, molding sand, steel castings, structural material, brass or other alloys, coal and coke, core sand, etc.

The **rate of consumption** of these materials and the **supply** that trade conditions make it desirable to carry from standpoints of **price** and **delivery,** should be ascertained and recorded. Data of the foregoing character should also be secured with regard to the **finished and partially finished products.**

The following classification covers the principal items of information required: (*a*) Character and amount of raw materials that should be carried as stores; (*b*) character and amount of materials to be carried in stock after work has been done on them; (*c*) character and amount of finished product carried as stores or pending shipment.

With this information as a basis, the following data should be compiled: (*d*) The **character and extent of** the various kinds of hand, machine or process **work required to be done upon the product**; (*e*) the **weight and bulk** of the units in which the various materials are required to be **handled**; (*f*) The **cubic contents and weight of** the various **raw materials, worked materials and finished products** which will have to be **stored**; (*g*) special **notes of** any **exceptional conditions** that products may impose, such as fire, chemical and explosive hazards.

Manufacturing Equipment Needed. The types of **machinery or equipment required** should be decided upon and a list prepared comprising each specific machine that should be installed. The **probable equipment requirements for the future** should be approximated, as these govern the amount of floor space that it should be possible to secure by further extensions. All factors limiting complete freedom of choice in the selection of machinery and processes must be taken into account.

Safety Devices. Provision should be made for safety devices for machines, belting, etc., and for conditions which will increase the health and efficiency of the workers. For example, the supply of slightly heated water in the winter for tool-grinding operations has been found to increase efficiency.

Power Generation and Distribution. In the great majority of cases presented to the designer of manufacturing plants, power requirements are met most satisfactorily by the use of electric motors operating the machines either individually or in groups. The advantages of this practice as opposed to belting and line shafting connected directly with a central motive power unit are stated on p. 1467. The relative advantages of building an isolated plant or purchasing power when the latter is a possibility should be carefully ascertained, taking into account the heating and industrial requirements in connection with which exhaust or live steam may be needed.

If power from a central generating point is to be transmitted throughout the plant by means of belt, rope or gear drives and line shafting, the buildings must be designed to allow the ready installation of the apparatus in a manner that will interfere the least with the work in each department. The additional dead load and the probable effects of vibration must be taken into account. In the case of heavy machinery, such as rolling-mill equipment, the transmission apparatus may present unusual requirements necessitating special design of the building structure.

Necessary Equipment Data. The following statistics with regard to each item of machinery and equipment to be installed should be determined: Total weight; power required; supplementary handling facilities; total floor space occupied; method of applying power; head room required; special conditions. The small equipment and hand operations entering into erecting and other work performed for the most part without the aid of large machinery, should be covered with equal thoroughness.

Administration Methods. The mistake is frequently made of assuming that matters involving questions of administration can be deferred until the completion of the plant. The management of an industrial plant will never be confronted with a more opportune time to effect improvements in its methods of management than when building a new plant. The limitations imposed by the layout and character of many existing plants are often such as to prohibit the most efficient introduction of modern methods of management. Owing to the great diversity of requirements presented by industrial plants generally, it is not practicable to do more than to suggest the importance of this phase of the subject. The following specific illustrations should serve this purpose.

(*a*) The system of storekeeping requiring the use of two bins or spaces for each kind of material (to be used alternately for issuing and receiving) requires about 50 per cent. more room for the stores department than does the ordinary system in connection with which but one bin is used.

(*b*) The amount of floor space required for shop offices, as well as their location, is dependent upon the manner in which the routine of work of manufacture will be planned and directed.

(*c*) In certain cases it is desirable to group together machinery of a like character, such as lathes, boring mills, drill presses, etc., owing to the administrative advantages gained thereby, even though such a course necessitates considerable extra handling.

(*d*) Under one system of management it may be found necessary to inspect for quality and quantity in an isolated department; in another the inspection may be accomplished by "traveling inspectors," thus eliminating the necessity of moving the work for this purpose.

The ideal **arrangement of equipment** and processes is that which minimises the travel of materials of manufacture and at the same time lends itself readily to the particular plan of industrial administration that is to be used. In general, the procedure which should be followed is outlined below.

Industrial plants can be classified into two main divisions from the standpoint of **sequence of operations**, as follows:

1. Those employing **continuous operations**, as in the case of refining sugar, milling flour, manufacturing paper, etc.
2. Those employing **intermittent operations** involving the manufacture and assembly of parts in which the sequence of operations is not fixed rigidly.

The determination of the correct sequence of operations and the preparation of graphical routing diagrams as a basis for the arrangement of equipment and departments is a relatively simple matter for industries of the first class. This problem, however, becomes much more complex when dealing with industries of the second class, as it is possible frequently to attain the same final result by performing certain of the operations in different sequences. In addition, the advantage gained through arranging equipment strictly to minimize handling must be compared with that resulting from an arrangement made purely to facilitate administration and minimize investment. In one case it might be required that each department comprise the various types of equipment needed for the performance of all necessary operations on a given unit; in the other that each department contain equipment of a like character, thus requiring the movement of the product from one department to another. Then, again, in certain industries the importance of centralized inspection between operations is so great as to justify the excessive handling of all parts necessitated through such procedure.

In general, in the metal-working industries it is found best to group together equipment of a like character, such as automatic lathes, drill presses, gear cutters, etc. This permits of the smallest investment, the minimum number of operatives and satisfactory local inspection within the department. As most of this work is relatively light in weight, the handling problem is not a serious one. On the other hand, the equipment required to perform work on heavy and bulky materials, such as engine beds, flywheels, etc., is usually arranged solely with a view to minimizing the handling costs incident to a given sequence or performance. The determination of the proper sequence is governed by such considerations as the following: Planing usually precedes boring and drilling operations, the purpose being to secure finished surfaces from which subsequent operations can be laid out accurately.

Floor Space Needed. Templates should be made to a scale consistent with the magnitude of the work (¼ in. to the foot is satisfactory usually), and should indicate the overall dimensions and outlines of foundations for machinery. The floor space required by each machine and process and for the temporary storage of the materials upon which work is to be done or has been done, should be determined.

In certain instances where the work is of the continuous class, as in the case of a bleach house where the work passes continuously between certain processes in the rope form, little or no space is needed for storage. The opposite extreme is exemplified by the conditions presented in the machine shop which handles a wide diversity of work. In such a plant it is desirable usually to have several jobs laid out in advance for each machine and at a point directly accessible to it. It may also be desirable to allow the work to remain in proximity to the machine last engaged upon it until it has been inspected for quality and quantity.

Routing Diagrams. Typical forms of routing diagrams are illustrated by Figs. 1, 2 and 3.

Fig. 1 is a partial routing diagram of a sugar refinery, illustrating industries of the first class where the manufacturing **work is continuous in character.** As the diagram shows the routing in elevation only, it gives no indication of the amount of floor space required on the respective floors nor of the total amount of equipment of various classes that enter into the work. It is an example of those industries in which the material is carried to the upper floors and the processes succeed each other as it is carried down through the plant by gravity. In such a layout the handling costs are minimized. The continuous class of manufacture is exemplified in part by steel and brass rolling mills. These industries, however, require the location of the heavy equipment on a single floor and the replacement of gravity by mechanical handling, owing to the weight and bulk of the product. Cotton and paper mills, on the other hand, represent to a certain extent a combination of the principles exemplified by each of the illustrations just referred to.

Fig. 2 illustrates the use of the routing diagram as a means of **determining the correct location of machines** in each department. It exemplifies the layout of a small machine shop or machine department and shows the exact

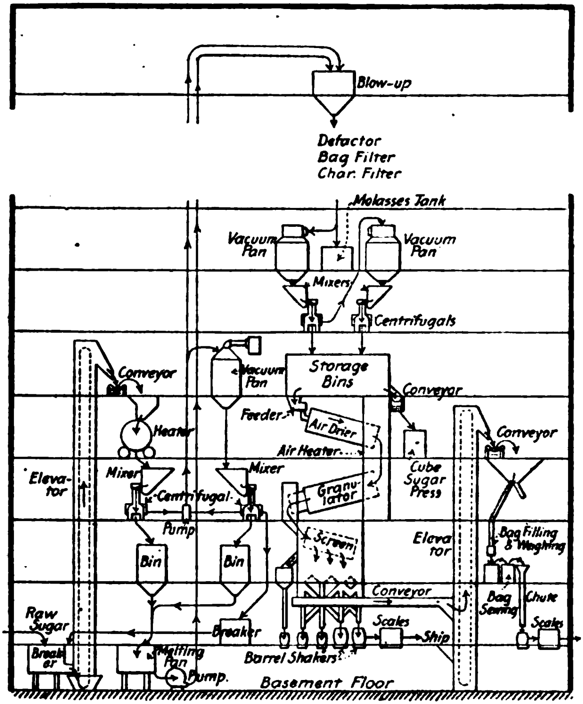


FIG. 1.—Routing Diagram of a Sugar Refinery.

paths followed by three units of the product. The diagram exemplifies the best possible condition from the standpoint of the routing of material. To avoid confusion, all of the equipment has not been shown nor the routing of the entire product indicated.

Fig. 3 illustrates the routing in a machine shop where it is necessary to inspect the work after each operation. It will be noted that equipment of like character is grouped into departments and these departments are located around the central inspection department in order to reduce travel to the minimum under the conditions imposed. The shop in question provides for the independent manufacture of small parts made from bar stock and forgings.

The **designs of the buildings** must be such that the work to be performed can go forward with practically as much freedom as though the buildings did not exist; that is, the workers, whether employed at individual machines or engaged in moving material from point to point, should be unimpeded by the housing structure. Certain of the more important factors are considered below.

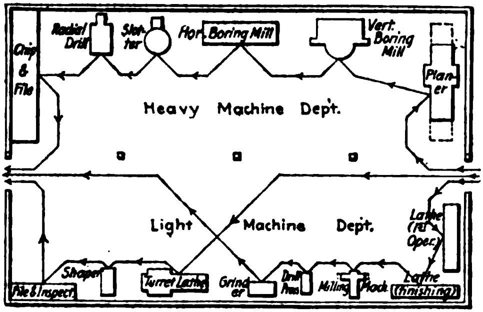


FIG. 2.—Routing Diagram of Small Machine Shop.

Certain operations and departments must be housed in single-story buildings; others may be located on the first floor of a multiple-story building, and others may be located advantageously on upper floors. If the plant is located

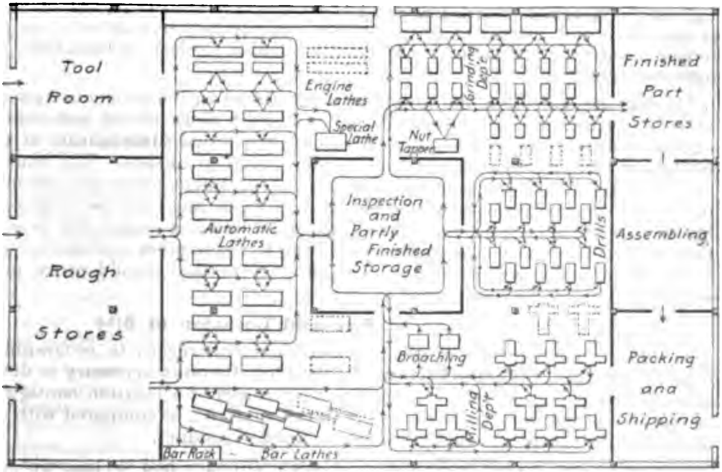


FIG. 3.—Routing Diagram of Machine Shop, Providing for Inspection of Work After Each Operation.

in a place where the cost of land is high, it may be necessary to resort to multiple-story structures wherever this is practicable; on the other hand, if the cost of land is low it may be desirable to locate the departments in single-story buildings, owing to the handling and supervising advantages that would result.

The **number of buildings** into which the requisite area should be divided, is dependent largely upon the extent of the plant and character of work to be done. The interrelationship of certain processes frequently requires that they should all be housed in the same structure; again, certain groups or processes may present certain governing requirements that are common to all the processes, as, for example, the necessity of overhead traveling cranes, or the need of exceptional conditions in regard to heating and ventilation (exemplified by certain of the textile and chemical industries), or by the exceptional fire hazard where alcohol, gasoline or other combustible materials are used extensively. Materials and parts must be made to **travel in one general direction through** the **plant,** just as through a given department, and their routing through departments where no work is done upon them should be avoided. Care should also be exercised to **prevent congestion** at any point **in the path of travel.** Departments for the receipt of raw materials and shipment of finished product and, in some instances, erecting departments, should be located adjacent to railroad sidings, docks or roads on which shipments are received or made.

In the case of a plant laid out for the manufacture of stiff derby hats, it was found that the operations entering into the work could be broadly classified into three main groups. The first group, consisting of operations performed in a dry state, was housed in a multiple-story building. The second group, consisting of operations practically all of which must be performed in the presence of water or steam, was housed in a single-story structure designed with special reference to ventilation and floor drainage. A sub-group was isolated in a fireproof structure owing to the hazard incident to the use of alcohol in large quantities. The third group, comprising operations certain of which are performed through the medium of dry heat and the balance of which are hand work, was housed in a multiple-story building.

Having decided upon the grouping of machines in departments and the grouping of departments into buildings or at least structures of well-defined character, it becomes necessary to determine the actual **dimensions of the building structures** required to meet the specified output. The routing diagram and a knowledge of the necessary department areas are the most important factors in this connection, although there must not be overlooked the requirements of good natural lighting, good building design, fire prevention, etc. With these data and those relating to the number and character of buildings required, the combination of dimensions that should prove most advantageous can be ascertained.

Determination of Size and Location of Site

In many industries the governing condition with regard to geographical location is that of **transportation.** It therefore becomes necessary to determine by actual calculation the relative advantages of a location contiguous to one or more of the sources of supply of raw material, as compared with one at or near the center of distribution for the finished product.

Where a plant is dependent upon raw materials such as clay, iron ore, limestone, etc., and particularly where the materials or product resulting from the utilisation of such materials represent but a small part of the original bulk or weight, the location of such materials determines unqualifiedly the location of a plant.

In many cases several different kinds of raw material coming from widely separated localities enter into a single product, so that it becomes necessary to decide upon the relative transportation costs. Brickyards, for example, are located at the clay deposits, as the tonnage ratio between the clay, brick and coal used is about 5:3:0.2. Blast furnaces working 33⅓ per cent. iron ore, require three tons of ore and about one ton of coke to produce one ton of pig iron. Consequently, from the standpoint of transportation, such blast furnaces should be located at the ore mines. When the quality of the ore is considerably better, and especially for puddling furnaces, Bessemer and open-hearth steel works and rolling mills, the quantity of coal is the first desideratum, for which reason the most profitable location from a transportation standpoint is at the coal mines.

In the majority of industries, however, the materials upon which work is done are purchased in a partially manufactured state, and it is frequently impossible to decide definitely in advance as to the source from which they can be secured most advantageously. In such instances, other factors should govern in the selection of the site from a geographical standpoint.

Occasionally, the **distribution of** the **finished product is the more important factor,** although this relates to those industries which depend principally upon local demand, such as bakeries, breweries, printing establishments, etc.

Labor Market. In many industries it is important to have a supply of skilled workmen in the immediate vicinity of the plant. In other cases, the necessity of being able to command employment of a large number of unskilled men is so important as to necessitate the selection of a location in or adjoining a large city. Many communities have become the centers of certain classes of manufacture, and skilled workers are attracted thereto by the prospect of more steady employment. The desirability of taking advantage of such a labor market is frequently so great as to outweigh all other considerations. The possibility of securing **cheap power** is sometimes a determining factor. The segregation of plants in the vicinity of Niagara Falls exemplifies this.

Urban vs. Suburban Sites. Having decided upon the geographical location, the next step is to determine the relative advantage of a site directly in or adjacent to a large city as compared with one at some distance from such a community. The question of labor supply also enters, as in certain industries it is possible to get skilled men in large numbers only in relatively big cities. Workmen in cities are usually more intelligent and able than those in the country; on the other hand, they are successful in demanding higher wages.

Selection of Property. Before selecting an actual site, specific information should have been compiled in regard to the following factors:

1. Amount of property required for immediate and future use.
2. Desirable shipping facilities—water, rail, highways, etc.
3. Public conveniences, such as fire-protection facilities, water supply, gas, electricity, police protection, street railways, etc.
4. Desirable topography for drainage and foundation conditions.
5. The sum of money for acquisition of property that is consistent with the estimated income of the business, and which bears a logical proportion to the total justifiable disbursement.

Other factors which must be taken into account in laying out the plant are as follows: Future growth must always be provided for. Large, unobstructed areas are generally preferable to small, enclosed areas. Large window areas should be provided and direct sunlight avoided as far as possible. Processes hazardous from the standpoints of fire or explosion, or areas too large to merit insurance at the lowest rate, should be isolated by are walls. Large plants should have separate sidings for shipping and receiving. The plant should be laid out so that an interruption in one department will not make it necessary to shut down the entire plant. That arrangement of equipment and departments which results in a minimum handling of product should not be departed from without strong justification. The pressure in the water mains must be sufficient for effective fire service. Legislation dealing with sanitary and safety equipment must not be overlooked. The requirements of underwriters in regard to fire protection must be taken into account. The minimum radius fixed by the principal steam railroads for track over which their equipment will operate should be considered, as well as the height and width of doorways through which cars are to be shifted and the platform clearances for loading and unloading. Legal requirements in these respects should also be ascertained. The possibility of purchasing electric current for light and power more cheaply than it can be generated in a private plant, should not be overlooked. The possibility of arranging with electric traction companies to transfer freight cars or to transport freight should be considered. The floors of storehouses and manufacturing departments should be designed to withstand the excessive loads imposed at times by congestion of work or the carrying of an abnormally large stock. Safety devices of all kinds should be provided, as well as adequate fire exits.

Preparation of Detail Plans. A brief statement or specification accompanied by the necessary block plans, statistical data, etc., should be prepared, defining the specific conditions imposed from a manufacturing standpoint and by the site finally selected. The information in question should include a complete survey of the property, a block plan of the proposed plant, a schedule of minimum inside dimensions, floor, crane and elevator loads, etc. All passageways, doors, and special building features should be clearly defined, and, where certain latitude is permissible, limits of possible variation should be stated.

It is of the utmost importance that data of the foregoing character should be complete in every important particular before authorizing the preparation of detail structural specifications. It is only in this way that cost can be minimized through eliminating many changes in the plans subsequent to their initial completion. This preliminary work is in no sense a function of the designing engineer or architect, but rather of the manufacturing expert.

Contracts for Building. The principal forms of contract and the onditions for which they are particularly suited, are as follows:

General Lump-sum Contract. This form of contract requires that idders shall make lump-sum bids on the entire work covered by the plans nd specifications. Until recently it has been the accepted form of contract or practically all building operations. It is now being superseded rapidly by the minor lump-sum contract.

Minor Lump-sum Contract. This form of contract relates to but a single feature of the work, the entire structure being provided for by entering into as many minor contracts as there are distinct classes of construction work. All structural steel work would be covered by one contract, brickwork by another, millwork by still another, and so on. It has the great advantage of restricting the competition to units of a like character, so permitting of a more intelligent consideration and comparison of the bids. It also results in lower prices if care is exercised in soliciting bids only from those who are thoroughly experienced in the respective functions. This form of contract therefore presents many advantages to those about to build industrial plants, unless the work involves exceptional hazards or uncertainty of construction conditions. In such cases, the percentage, or fixed-fee, contract should be adopted.

Percentage or Fixed-fee Contract. This form of contract provides that the contractor shall be reimbursed for all expenses, and, in addition, paid either a percentage on the cost or a fixed sum of money as a profit. It is adopted principally where difficulty is encountered in defining in advance certain essential conditions affecting the work, such as the character and amount of excavation that may be necessary, or the necessity of performing the work in such a manner that it will not interfere with certain other activities, as in the case of extensions to industrial plants where the manufacturing work must not be interfered with by the construction force.

DETAILS OF BUILDING CONSTRUCTION

Types of Industrial Buildings

The determination of the proper type of buildings that will house most efficiently the departments comprising a given business, involves a consideration of the following characteristics.

Single-story Buildings allow of the greatest flexibility in the arrangement of departments; provide for unlimited and uninterrupted growth in any direction to the boundary of property; allow (through the adoption of overhead skylights) of the enclosure under a single roof of an unlimited area—frequently a great industrial advantage; afford the best natural lighting, secured by sawtooth or other type of roof sash; facilitate supervision of manufacturing work through a minimum amount of isolation of departments; and in general, present less fire hazard than multiple-story structures.

Multiple-story Buildings are desirable when cost of real estate is high or a limited amount procurable; have a greater utility than single-story structures, and therefore usually bring a larger return in the event of sale or rental; are readily heated in cold weather with minimum cost of equipment; cost less per sq. ft. of floor area for most classes of occupancy, and are desirable for continuous manufacture where gravity travel can be used or for light manufacturing purposes.

Construction Materials. The decision as to whether buildings should be of reinforced concrete, mill construction, steel-frame or other construction depends upon the specific requirements dictated by the manufacturing work to be performed within them; climatic conditions; fire hazard; the type of building; the character of the locality in which the buildings are to be erected; and the limitations imposed by available capital.

For multiple-story buildings there is a decided tendency toward **reinforced concrete**, as it is the least expensive type of fully fireproof construction. It is especially adapted to plants using high-speed machinery, owing to its resistance to vibration. On the other hand, as the structure is virtually a monolith, changes are more costly than in other types of construction. Fireproof multiple-story buildings can be constructed of steel frame with tile or reinforced-concrete floors and brick or concrete walls, but in such cases all columns and exposed steel work must be thoroughly fireproofed, which, as a rule, makes the total cost greater than that of a reinforced-concrete building. Multiple-story buildings of **mill-type construction** (brick walls, wood columns, heavy timber girders and floors), if not sprinkled, generally cost from 10 to 15 per cent. less than those of reinforced concrete. If a sprinkler system is installed, such structures present a low fire hazard. Single-story structures, such as those required for heavy machine work, are usually constructed with steel frame and plank (or some form of fireproof) roof. While such structures are not combustible, intense heat may weaken them to the point of collapse, but they nevertheless meet satisfactorily the requirements of businesses that deal principally in non-combustible products. **Steel-frame construction** has been used chiefly in the past in connection with single-story buildings with sawtooth roofs, but reinforced-concrete construction is now being adopted for this purpose.

Roof Trusses

The types of roof construction which have been frequently adopted (see also pp. 1281–1284) are as follows:

The Fink truss (Fig. 4) is suitable for buildings from 40 to 80 ft. in width. It does not permit of good natural lighting nor ventilation in the greater widths.

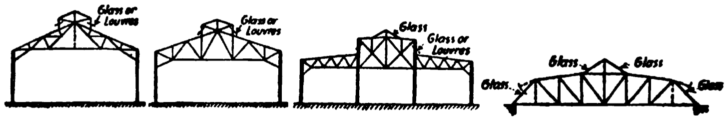


FIG. 4. FIG. 5. FIG. 6. FIG. 7.

FIGS. 4–7.—Roof Trusses for Industrial Buildings.

Fig. 5 represents a more expensive type of roof truss with the same objectionable features as the Fink truss. It gives, however, a stiffer building and overhead machinery can be hung from it readily and economically. In general, the span should not be greater than 80 ft. if effective natural lighting is to be obtained.

Fig. 6 represents an economical construction for spanning excessive widths, which provides good natural lighting and ventilation.

Fig. 7 shows a method of securing fairly satisfactory natural lighting and ventilation for buildings of large spans.

The customary **spacing** of the trusses illustrated by Figs. 4 to 7 varies from 15 to 25 ft., depending upon local conditions.

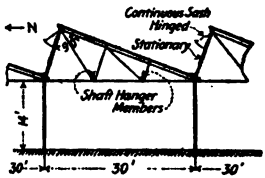


FIG. 8.—Steel Sawtooth Roof Truss.

Fig. 8 represents a steel roof truss of the **sawtooth type,** which is particularly adapted to single-story buildings covering a large area. Its use should result in relatively low cost, good natural lighting and ventilation, and ease with which line shafting can be attached. It is possible to obtain these conditions with a minimum story height and irrespective of area covered. The sawtooth should face due north to exclude the direct rays of the sun. If it were not for the inclination of the earth's axis the angle between the horizontal and the sawtooth should be 90 deg. minus the latitude in which the building is located. On account of the inclination of the earth's axis this angle must be increased by 23½ deg. In northern latitudes ranging from 35 to 45 deg. it is customary to make the inclination of the sawtooth to the horizontal about 70 deg. Where spans range from 25 to 30 ft. in width, the trusses are spaced from 15 ft. to 20 ft. center to center. **Sawtooth trusses with vertical glass faces** are not recommended, as they do not admit light as well as the type shown in Fig. 8.

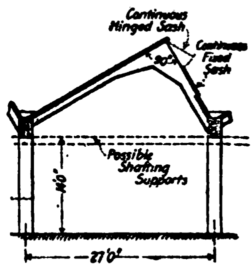


FIG. 9.—Reinforced-concrete Sawtooth Roof Truss.

Fig. 9 shows a **reinforced-concrete sawtooth truss.** It costs from 15 to 25 per cent. more than one of structural steel, but it is fully fireproof. The truss shown without a lower chord is used where overhead transmission shafting is not required and gives full sweep to the rays of light coming through the continuous sawtooth skylight. If shafting is required in the building, the truss can be designed with a lower chord of reinforced concrete, or a steel member can be inserted between the building columns made up of two channels, which makes an ideal support for shafting.

Fig. 10 illustrates a type of **building** that has been used extensively **for heavy machine and erection work**, the high bay being used for work requiring heavy crane service. In many cases the side galleries are omitted. The ratio of high-bay area to low-bay area being fixed, such a shop is capable of extension longitudinally only. Good natural lighting and ventilation can be obtained with this type of shop.

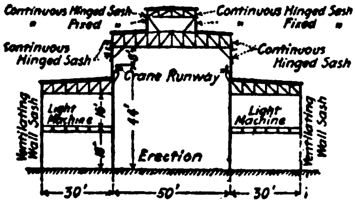


FIG. 10.

Figs. 11 and 12 illustrate types of construction that combine high center crane bays with single-story side bays of sawtooth roof construction. Such buildings permit of extension in any direction, so that any desired ratio of high-bay area to low-bay area can be secured. They are replacing to a considerable extent those illustrated by Fig. 10 on account of the ease with which they can be adapted to various manufacturing conditions.

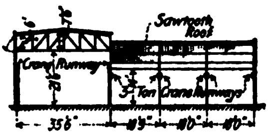


FIG. 11.

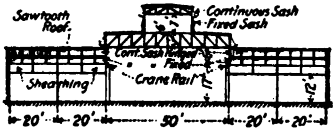


FIG. 12.

Fig. 13 represents a type of building adapted particularly to classes of industrial work which require good natural lighting combined with the best possible ventilation, such as **foundries, smith shops, drop-forge plants,** etc. Hanging gutters are dispensed with, and roof drainage is efficiently provided for, even in the coldest climates, by providing rain-water drainage inside the building where it is not subjected to outside temperatures. This design also provides excellent natural ventilation owing to the pitch of the roof toward the center and the large openings that can be provided on either side of the building directly under the eaves. As this form of truss (the Pond) is more expensive than the one illustrated in Fig. 10, its use is largely confined to those industries in which it is necessary to dispose of hot gases or fumes rapidly.

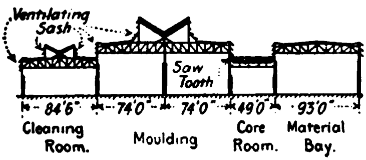


FIG. 13.

Roofs

For weights of roofing material, see p. 1280.

Wood Roofs. The wood roof is the cheapest form of substantial construction, but its life is relatively short as compared with all other types except metal, and it is combustible. If, however, the building is sprinkled and the roof covering is reasonably fireproof, the fire hazard is low. Generally, the wood roof comprises a layer of 2- or 3-in. tongued-and-grooved yellow pine fastened to wood nailing strips which are in turn bolted to purlins. Such a roof costs between 12 and 15 cents per sq. ft.

Metal Roofs. The simplest form of this construction comprises bare black or galvanized corrugated iron or steel (see p. 641) laid directly on the purlins. This is advocated only for buildings of a more-or-less temporary character. A more permanent form of metal roof is secured through the use of asbestos-covered corrugated iron, but even this material is not particularly durable if not otherwise protected. Metal roofs should have a very steep pitch in order to insure positive roof drainage. In general, they cost from 8 to 16 cents per sq. ft., depending upon the gage and grade of metal used and the type of protection afforded.

Reinforced-concrete Roofs. Various types of reinforced-concrete roofs are in general use, the principal differences being in the kind of reinforcement. Several of the commercial types of reinforcing material such as Ferro-inclave, Hy-Rib and asbestos-protected metal eliminate in large measure the use of temporary forms. In the case of steel buildings the concrete slabs may rest either directly upon the steel purlins, which procedure entails the minimum construction costs, or the concrete may be made to embrace all of the purlins for the purpose of stiffening the building and fireproofing the steel. More frequently the concrete roof is used in connection with reinforced-concrete buildings, in which case it is an integral part of the concrete construction. A concrete roof will cost from 18 to 30 cents per sq. ft.

The **Ferro-inclave type of roof** is a combination of corrugated sheet steel, concrete and cement plaster. The corrugations in the steel are dovetail in shape so as to hold the plaster coat on the under side and secure the concrete on the upper side. For spans from 5 to 6 ft. between purlins, the plaster coat on the under side is usually about 3/8 in. thick below the corrugations and the concrete on the upper side about 2½ in. thick.

The **Hy-Rib type of roof** is similar in construction to the Ferro-inclave except that the steel sheet is perforated and formed with parallel ribs an inch or two in width at regular intervals. Concrete is applied on the upper side and plaster on the under side in much the same manner as in the case of a Ferro-inclave roof.

Asbestos-protected Metal is corrugated iron coated on both sides with asbestos. When used in roof construction the concrete is applied on the upper side, usually being reinforced with steel fabric or rods depending upon the span. Plastering on the under side is usually dispensed with.

Composition Roofs. Several kinds of composition roof are in common use, some of which are built up in the field, whereas others are manufactured in units and shipped to the building site for use in much the same manner as ordinary tile. The composition in most cases consists of a mixture of gypsum and wool or straw with a steel reinforcing element. The desirable features of this type of roof are its fire-resisting qualities and its light weight (from 30 to 60 per cent. lighter than concrete). Its cost varies from 12 to 18 cents per sq. ft., depending on purlin spacing, thickness of slab, etc.

Tile Roofs. The tiles used for manufacturing buildings are usually made of reinforced cement concrete in units from 4 to 6 ft. in length by about 2 ft. in width. The mixture used in their manufacture is both fireproof and waterproof and a roof of this character does not require any additional covering for protection against the weather. The units are usually supported on purlins spaced to suit the size of tile used and are laid up somewhat like slate without cementing at the joints. Where skylights are required, tile of this character is often used with a pane of glass imbedded in the tile itself. The **cost** of such a roof varies from 15 to 35 cents per sq. ft., depending upon size of the units and the quality of material used.

Roof Coverings

For properties of roofing materials, see pp. 639 and 1280.

Slag is more commonly used than any other roof covering. First-class work requires four or preferably five layers of high-grade felt, each coated with pitch and surfaced with slag, gravel, pebbles or crushed stone. When the slope of the roof exceeds about 10 deg. a specially prepared pitch must be used to obviate the tendency to melt and run where high temperatures may prevail. A good slag covering **costs** approximately 5 cents per sq. ft. in place. It is considered practically fireproof and is guaranteed by reputable contractors to last 10 years.

Composition. Many excellent patented combinations of felt, pitch, asphalt, asbestos or other materials may be purchased in rolls. This covering is applied by nailing to the sheathing a single thickness and sealing at the seams with a waterproof compound. Practically all manufacturers guarantee their product to be fireproof and durable. Composition roof covering is ideal for roofs of steep pitch; it is also used extensively on flat roofs. The **cost** varies from 5 to 7 cents per sq. ft.

Shingles. Wood shingles (see p. 583) have been largely superseded by various patented shingles composed of asbestos and cement and furnished in units about 16 in. square. They are nailed to the roof structure like wood shingles and **cost** from 15 to 25 cents per sq. ft., depending upon the size, thickness, color and quality. Their use is restricted usually to roofs having a decided pitch.

Metal. The use of tin and copper roof covering for industrial plants is comparatively limited owing to the excessive cost, although the use of the proper materials results in the most permanent form of roof covering. The wide variation in quality of tinplate causes a range in **cost** from 8 to 18 cents per sq. ft., while copper covering costs from 30 to 40 cents per sq. ft., depending upon the gage of the metal used.

Slate. The use of slate for roof covering is limited to roofs of decided pitch and buildings where the roof forms a part of the architectural effect desired. The grades of slate in common use vary in **cost** from 8 to 15 cents per sq. ft. or even more when slate of an unusual quality, thickness or color is used.

Floors

Floors for industrial plants (see also p. 1272) may be classified broadly as (*a*) concrete, cement or asphalt, (*b*) wood plank and (*c*) wood block, the choice being governed by the nature of service to which the floor is to be subjected, the cost, and whether it is to rest upon the soil or to be supported on beams or columns.

Concrete, Cement or Asphalt Floors. A common form of construction consists of a wearing surface of ¾ in. of cement and sand laid on 4 in. of concrete over 10 in. of cinders. Such a floor is desirable where considerable moisture results from manufacturing processes or where it is necessary to flush the floor frequently. It tends, however, to create a dust where considerable trucking is done and it is difficult to make satisfactory repairs to the wearing surface. Moreover, its hardness and coldness make it uncomfortable for the operatives. A floor with a base of 1 : 3 : 4 concrete and a wearing surface composed of 1 part cement, 2 parts sand and 1 part granite screenings, will cost about $2 per sq. yd.

In another type of concrete floor the wearing surface consists of a coat of mastic asphalt compound. Such a surface is often desirable in shops where unusual traffic conditions prevail or where the floor is subjected to the action of gases or acids injurious to ordinary concrete. Cost, about $3 per sq. yd.

Wood-plank Floors. A wood-plank floor usually consists of a hardwood wearing surface supported either directly on sleepers or on an underfloor composed of plank laid in the opposite direction. Maple and birch are generally used for the wearing surface, varying from 13/16 in. to 1 3/16 in. in thickness. The underfloor is usually made of 1½- to 3-in. yellow pine or spruce planking. It is customary to use tongued-and-grooved material throughout.

For heavy machine and erecting shops tar-rock floor is frequently used. The sub-floor consists of a mixture of broken stone and tar tamped or rolled in place and covered with a 1-in. coating of sand mixed with hot pitch. A 3-in. yellow-pine underfloor is laid on this sand cushion while still hot and tamped securely in place without spiking, after which a 13/16-in. maple wearing surface is laid. The tar-rock sub-base acts as a preservative for the timber and prevents dry rot. Cost, about $3 per sq. yd.

A cheaper floor which is not recommended for heavy service and is less durable than tar-rock floor, has a wearing surface of 13/16-in. maple laid on a ⅞-in. underfloor resting on 3 × 4-in. sleepers, spaced 16 in. on centers. The sleepers are imbedded in a 4-in. thickness of cinder concrete supported on a 6-in. concrete sub-base. Cost, about $2.50 per sq. yd. A floor costing about 25 cents less per sq. yd. can be made by laying 13/16-in. yellow-pine flooring directly on the sleepers.

In another construction an underfloor of 3-in. planking is substituted for the sleepers in the floors previously described. The cost is about $2.75 per sq. yd.

Fig. 14 shows how provision may be made for the accommodation of piping or conduit work by supporting the underfloor on 3 × 6-in. sleepers resting on the concrete sub-base. Such construction has been found desirable in machine shops where the use of automatic machinery makes it necessary to install a complete system of piping for lubrication, or where any other complicated system of piping must be provided for. Cost, about $2.75 per sq. yd.

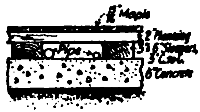


FIG. 14.

Wood-block Floors give better service than any other type where the conditions are unusually severe, such as in heavy machine and erecting shops, smith shops, drop-forge plants, warehouses, etc. They can be repaired more readily than wood-plank floors, and repairs do not have to be made so often as they will stand more abuse than any other floor. The blocks are usually rectangular, and are made from long-leaf yellow pine, maple, tamarack or other hard woods. They are generally impregnated with creosote or some other preservative, so that the liability of dry rot is practically eliminated.

Blocks for shop use vary in thickness from 2½ in. to 4 in. and generally are laid on a 1-in. cushion of sand resting on a 6-in. concrete sub-base. This floor costs about $2.25 per sq. yd. Where the loads imposed are comparatively light, and where service conditions are less severe, the concrete sub-base is sometimes omitted, the blocks merely being laid on a 1-in. cushion of sand resting directly on the soil. Such a floor costs about $1.75 per sq. yd.

Windows and Skylights

Windows. The development of metal sash has resulted in a marked improvement in the natural lighting of industrial plants. The use of metal in place of wood for the structural members of window frames and sash increases the glass area for a given window opening by over 25 per cent. Fig. 15 indicates several methods in which both steel and wood sash may be used.

Bay 1 (Fig. 15) illustrates an old type of wood window frame; the glass area amounts to only about 70 per cent. of the total window opening. With counterbalanced or double-hung sash, it is possible to secure 50 per cent. ventilation. Windows of this character can be furnished in place at from 30 to 45 cents per sq. ft.

Bay 2 shows the same size window openings as Bay 1 but with steel sash substituted for wood with a resulting increase in effective glass area from 70 per cent. to 90 per cent. of the total opening. It is possible to secure from 25 per cent. to 100 per cent. ventilation by means of ventilating panels indicated by dotted diagonal lines in the cut.

Bay 3 shows wood window frames arranged in groups of three so that the relative area of window opening to wall area is materially increased, but the net effective glass area as compared to wall opening remains the same as in the sash shown in Bay 1 (about 70 per cent.). In this case the greater size of the wood window frames increases the cost to from 40 to 60 cents per sq. ft.

Bay 4 shows the same size window openings as Bay 3 with metal sash instead of wood. This results in an increase in effective glass area from 70 per cent. to 90 per cent. with the same size opening in the wall.

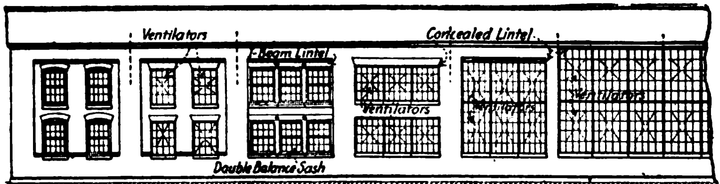


FIG. 15.—Types of Window Frames for Industrial Buildings.

Bay 5 shows a method that permits of a still greater glass area by extending the sash from pilaster to pilaster and from sill to lintel. This results in an increase of effective glass area of about 25 per cent. over that shown in Bay 4. It applies, however, only in single-story buildings of generous ceiling heights, whereas the arrangements shown for Bays 1 to 4 inclusive are equally satisfactory for single- or multiple-story buildings.

In Bay 6 no pilasters are used at the panel points, the result being that the entire side of the building is of glass. In this case, steel sash is located outside of the building columns which support the roof, giving maximum natural light through the side walls and making it possible to secure almost any degree of ventilation.

Steel sash varies in cost from 45 to 55 cents per sq. ft., depending largely on the size of units, number of ventilating panels, character of glass and quantity of sash required.

Multiple-story buildings up to 80 ft. in width can be supplied with adequate natural light by the liberal use of steel sash, whereas 60 ft. is about the limit when equipped with wood sash unless the ceiling heights are excessive.

Occasional applications of an inexpensive, white cold-water paint to walls and ceilings increase the light in all buildings.

Skylights. Formerly skylights of comparatively small area were used on buildings too wide to be sufficiently lighted through windows in the side walls. The introduction of monitors and sawtooth construction created the necessity for a continuous type of skylight with metal frames and ample provision for ventilation. Metal skylight construction must be waterproof, must permit of the automatic adjustment of the glass to unusual conditions of temperature, vibration, etc., and must provide positive drainage for moisture resulting from condensation.

Figs. 16–19 illustrate designs of skylight bars commonly used, most of which are equally suitable for flat, double-pitch, four-hip or continuous skylights.

Fig. 16 shows a common type of bar for small, non-continuous sheet-metal skylights. Cost, from 35 to 40 cents. per sq. ft., depending upon area.

In Fig. 17 the bar consists of a specially rolled section combined with a zinc or copper cap. Cost, 45–50 cents per sq. ft. Modifications of this type provided with lead liners and copper flashings, cost from 70 to 80 cents per sq. ft.

Figs. 18 and 19 illustrate bars that are easier to clean and repair than the other sections described. In these the glass rests on oakum or felt and the metal caps are held in place by large stud bolts which facilitate removal and replacement. The bar shown in Fig. 19 possesses an advantage in that the channel section or bridge connecting the bolts braces the whole construction and provides a substantial resting place for planks used in cleaning. Cost. from 50 to 65 cents per sq. ft.

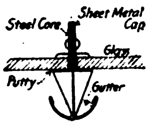


Fig. 16.

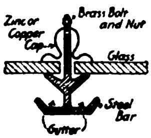


Fig. 17.

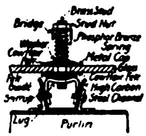


Fig. 18.

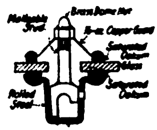


Fig. 19.

Figs. 16–19.—Types of Skylight Bars.

Heating

(See Heating and Ventilation, p. 1334)

The governing factors in the selection of a **steam-heating system** for an industrial plant are: (*a*) The degree and kind of heat required (moist or dry). (*b*) The amount of heat required for process work. (*c*) The size and type of buildings. (*d*) The amount of exhaust steam available.

The **direct system of radiation** is more desirable in multiple-story buildings and is usually more economical because of the ease with which coils or radiators can be located along side walls under the window sills. It is also more desirable in buildings of the sawtooth type because the coils located on the roof trusses will prove to be less of an obstruction to light than the large heating ducts necessary in connection with an indirect system.

The **indirect system** is usually more desirable in buildings of extreme width where large, unobstructed areas must be maintained and where the ceiling height is very much greater than in most multiple-story or sawtooth buildings. The high-bay type of erecting shop, for example, affords very little opportunity for the effective location of the heating coils necessary in a direct system, but the ducts required by the indirect system usually can be provided for satisfactorily, positive circulation of heated air being insured by fan pressure. It is particularly effective in plants where ventilation is of much importance.

Hot water is sometimes used in place of steam in direct heating systems. The greater amount of radiating surface required makes an excessive cost of installation which has been offset by combining sprinkler piping with a **hot-water heating system,** such a combination usually costing about as much as independent sprinkler and direct steam-heating systems.

Cost of Buildings

The only way to determine accurately the cost of a proposed building is to prepare detail estimates from complete plans and specifications. It is often desirable, however, to secure **preliminary estimates** before the completion of plans and specifications. For this purpose, certain **unit figures** are available, representing the results of past experience in erecting buildings of various kinds. These figures in most cases can be modified to suit conditions involved in proposed structures so that a fairly accurate estimate can be made. A number of unit figures relating to buildings of various kinds are given below, all referring to the **cost per cubic foot of space** and including all expense involved in construction work together with a reasonable contractor's profit. They do not include, however, architect's fees, cost of real estate, cost of preparing the building site or the cost of overcoming any unusual conditions such as treacherous soil, close proximity to other construction resulting in a lack of adequate working space, or unusual carriage charges owing to inaccessibility of the building site. All of these factors must be taken into consideration when using unit figures of this character and proper **allowances** made.

It should also be understood that these figures do not include anything in the nature of manufacturing, transportation or handling equipment. They represent only the **cost of a building** equipped **with the necessary service fixtures** to fit it for occupancy. The figures given cover a wide range of conditions for each type of building. Allowance must be made for all unusual conditions.

Approximate Cost of Industrial Buildings, Cents per Cu. Ft.

	Bare building, including plumbing	Complete building, including lighting and heating
Sawtooth buildings (first-class construction in structural steel and brick walls, using steel sash, a high-grade floor and slag roof)	7½ to 9	9 to 12
High monitor-type shops (foundries and heavy machine or erecting shops of structural steel, with brick walls, steel sash, first-class floor and slag roof)	5 to 6½	6 to 7½
Multiple-story buildings (slow-burning type of mill construction, with brick walls, wood sash and slag roof)	8 to 9½	10 to 12
Reinforced-concrete buildings (with maple floors, brick spandrel walls, steel sash and slag roof)	10 to 12	12 to 15
Multiple-story reinforced-concrete buildings (6 to 11 stories in height, with brick spandrel walls, steel sash and slag roof)	10½ to 13	12 to 15

HEATING AND VENTILATION

BY

KONRAD MEIER

REFERENCES: Allen (J. R.), "Notes on Heating and Ventilation," Chicago, 1911. Domestic Engineering Co. Carpenter (R. C.), "Heating and Ventilating Buildings," New York, 1914, J. Wiley and Sons. Dietz (Ludwig), "Ventilations- und Heizungs-Anlagen," Munich, 1909, R. Oldenbourg. Gramberg (A.), "Heizung und Lüftung von Gebäuden," Berlin, 1909, J. Springer. Hoffman (J. D.), "Handbook for Heating and Ventilating Engineers," New York, 1913, McGraw-Hill Book Co. Meier (Konrad), "Mechanics of Heating and Ventilating," treatise on the flow of water, steam and air in heating practice; New York, 1912, McGraw-Hill Book Co. Rietschel (H.), "Leitfaden zum Berechnen und Entwerfen von Lüftungs- und Heizungs-Anlagen," general treatise, with theory and tables, 2 vol.; Berlin, 1913, J. Springer. *Proceedings* of the American Society of Heating and Ventilating Engineers. *Heating and Ventilating Magazine.*

REQUIREMENTS

Heating Capacity. A heating plant for a building should be capable of delivering a quantity of heat (E_t, in B.t.u. per hour) sufficient to replace that which is transmitted through the structure to the atmosphere and the surroundings under a stated extreme temperature difference, with allowances for exposure to winds, leakage and heat losses in piping and ducts. When the service is to be interrupted at night or periodically for one or more days, extra capacity must be provided for the heat that will be absorbed by the building until the normal temperature and rate of transmission are reached. The heat-storage capacity of the structure (E_r, in B.t.u.) and the time available for reaching full temperature will determine the allowance (see Reheating).

The temperatures required for various purposes and the lowest outside temperatures to be met are given in Table 1.

Table 1. Temperatures Used in Heating Determinations

INSIDE TEMPERATURES RECOMMENDED, DEG. FAHR.

Living-rooms	68°	Auditoriums	66°	Massage and operating	78°
Bed-rooms	64°	Churches	60°	Steam bath	114°
Bath-rooms	74°	Prisons	64°	Warm-air bath	122°
Sick-rooms	72°	Offices	68°	Hot-air bath	140°
School-rooms	68°	Workshops	60° to 64°	Greenhouses	60° to 80°
		Gymnasiums	60°		

OUTSIDE TEMPERATURES FOR DIFFERENT LOCALITIES, DEG. FAHR.

Alabama	0° to 10°	Louisiana	10° to 20°	Ohio	−10° to 0°
Alaska	−30° " −40°	Maine	−25° " −10°	Oklahoma	−10° " 0°
Arizona	−15° " 0°	Maryland	0° " 10°	Oregon	0° " 20°
Arkansas	−10° " 5°	Massachusetts	−20° " 0°	Pennsylvania	−15° " 0°
California	20° " 40°	Michigan	−20° " −10°	Rhode Island	−5° " 0°
Colorado	−20° " 0°	Minnesota	−40° " −20°	S. Carolina	10° " 20°
Connecticut	−10° " 0°	Mississippi	5° " 15°	S. Dakota	−30° " −20°
Delaware	0° " 10°	Missouri	−15° " 5°	Tennessee	0° " 10°
Dist. Col.	0° " 5°	Montana	−40° " −20°	Texas	0° " 20°
Florida	25° " 35°	Nebraska	−25° " −15°	Utah	−20° " −10°
Georgia	0° " 20°	Nevada	−20° " 0°	Vermont	−25° " −15°
Idaho	−25° " −10°	N. Carolina	5° " 15°	Virginia	0° " 10°
Illinois	−20° " −10°	N. Dakota	−40° " −20°	Washington	−10° " 10°
Indiana	−20° " −10°	New Hampshire	−20° " −10°	W. Virginia	−20° " 0°
Iowa	−25° " −15°	New Jersey	−10° " 0°	Wisconsin	−20° " −10°
Kansas	−20° " 0°	New Mexico	−10° " 0°	Wyoming	−30° " −20°
Kentucky	−10° " 0°	New York	−20° " 0°		

When rooms are to be **ventilated** by any method introducing air from outdoors, additional heat (E_v, in B.t.u.) must be supplied to bring it up to the room temperature. If ventilation is combined with heating, as with the indirect system, this air volume (Q, in cu. ft. per hour), must be heated through a greater range. In this case the total heat is $E_t + E_v = Q(t_f - t_o)sw$, where $E_t = Q(t_f - t_r)sw$ and $E_v = Q(t_r - t_o)sw$; t_o, t_r and t_f being the outside, room and flue temperatures respectively, w the density of the air and s its specific heat ($= 0.2375$) at constant pressure. $sw = 0.018$ for 70 deg. fahr., at which temperature the air volumes are generally based. If ventilation is independent of heating, E_v is the heat required for tempering the air supply.

In view of the increased tendency to natural ventilation in cold weather, it is proper, when calculating E_v, to figure on reduced volumes for lowest outside temperatures. A reduction to three-fourths of the volume for temperatures below 10 or 15 deg. fahr. and to one-half or two-thirds for temperatures below zero, is permissible, unless the volumes are absolutely prescribed.

Hygienic Requirements in Heating. The effect of heating apparatus on health and comfort depends on three factors, namely, the wholesomeness of the room air, the sensible temperature and the humidity. To obtain the **best results,** the following requirements should be met.

1. Even temperature within the rooms.

2. Selection of heating system. Continuous service and massive, impermeable structure are more favorable for heating by air, while the necessity for frequent airing by windows calls for direct heating. Light and permeable structure make heat reserve in apparatus (hot water) desirable.

3. Overheating of rooms depresses the vitality of occupants. The heat should always be under control, either centrally, as by variation of water temperature, individually by graduating valves on radiators, or automatically when regulation cannot be left to personal attendance.

4. The properties of fresh, cool, outdoor air should be preserved as far as possible. This means lowest room *air* temperature at which comfort can be secured and consequently the least drop in humidity. For this purpose it is best to utilize radiant heat in mild form, such as may be obtained from direct radiating surfaces at moderate temperature, spread out and spaced liberally. When air must be used to carry the heat it is better to use large volumes at low temperatures, not exceeding 125 deg. fahr. Rietschel (5th ed., p. 45) recommends a limit of 95 to 104 deg. fahr. for indirect, and 122 deg. fahr. for furnace heating.

5. Contamination of air by dust should be avoided. Clean heating surfaces of moderate temperature prevent decomposition, drying and stirring of dust. Concealed or screened heating surfaces gather dust and contaminate the air. Hot-air flues should be protected from falling dirt. Floor registers and dirt pockets generally should be avoided.

Ventilating Capacity. The **volume of fresh air** that must be introduced artificially, in order **to maintain wholesome indoor conditions,** depends on the amounts of heat, moisture, organic matter and other contamination produced in a room, less the amounts taken care of by natural absorption and leakage. The air volume varies also according to the standard of temperature, humidity and vitiation permissible in a given case. In schools, hospitals and wherever physical weakness and predisposition to disease are expected, the highest standard should prevail, and provision should be made in addition for periodical contingencies such as occur in class-rooms during breathing spells and in hospital wards at certain hours. See "Ventilation Laws," Heating & Ventilating Magazine Co., 1914.

Table 2. Heat Developed by Persons and by Various Methods of Lighting

	B.t.u. per hour		B.t.u. per c.p. per hour
Adult at hard work.....	550		
Adult at medium work.	470		
Adult at rest..........	13.2 (98.6 − t)	Incandescent alcohol light.	35 to 45
Children..............	6.6 (98.6 − t)	Petroleum................	110 to 250
	B.t.u. per c.p. per hour	Candles..................	320
		Electric, carbon filament..	10 to 16
Ordinary gas jet........	220 to 320	Electric, metal filament...	3.3 to 5.2
Incandescent gas light..	22 to 40	Electric, arc.............	4 to 6

The heat developed by persons and by various methods of lighting can be estimated from Table 2. The difference between the heat generated and the heat transmitted or absorbed represents the cooling to be effected by ventilation, E_c, in B.t.u. per hour. The volume per hour figures approximately $Q = E_c/0.018(t_m - t_s)$, t_m and t_s being the mean room and air-supply temperatures. If the volume is predetermined, the difference $t_m - t_s$ is to be figured from the same formula. Q is limited to the amount of air that may be introduced at the stated temperature without causing drafts, depending on height of room, location, style and number of inlets. From 6 to 12 renewals per hour may be permissible, depending again on the difference between t_m and t_s. If well diffused and inlets are placed away from occupants, the entering air may be kept from 5 to 10 deg. fahr. below room temperature.

The average hourly production of vapor per person, according to Pettenkofer, is 0.09 lb. for adults at rest, and 0.18 lb. for adults at work. Children will produce about one-half these amounts. The production varies with the air conditions, clothing, etc. It may be absorbed by walls or condensed by windows. The above figures therefore give no basis for calculation of the resulting relative humidity.

Air is rendered unwholesome by perspiration, by respiration, excessive heat, humidity, effluvia from the human body and other impurities directly or indirectly imparted by the occupants of a room. The percentage of carbonic acid may be regarded as a measure of the vitiation from respiration and from combustion, but not from the heat and moisture resulting from the same source. Air may be polluted with dust and other harmful matter of which CO_2 gives no indication. CO_2 tests should be used only for checking the renewal of air and its distribution within the room. The production of this gas can only be assumed as a basis for calculating the air supply where respiration and combustion (gas lights) are the preponderating factors of vitiation; in such cases the CO_2 should not exceed 8 or 10 parts in 10,000.

Table 3. Air Volume per Person in Cu. Ft. at 68 Deg. Fahr. Required if CO_2 Shall Not Exceed the Given Percentages of CO_2

(CO_2 in outside air is 0.04 per cent.)

Per cent. of CO_2	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15
	Cu. ft. per person per hour								
Adult at work................	4650	3500	2800	2330	2000	1750	1560	1400	1270
Adult at rest................	2330	1750	1400	1170	1000	880	780	700	640
Children.....................	1500	1130	900	750	640	560	500	450	410
Gas burned, per cu. ft......	2100	1560	1250	1050	890	780	700	630	570

Table 4. Carbonic Acid Produced per Person and by Illuminants

	Cu. ft. at 68 deg. fahr.
Adult at work	1.2 to 1.6 per person per hour
Adult at rest	0.6 to 0.8 per person per hour
Children	0.3 to 0.6 per person per hour
Gas	0.5 to 0.75 per cu. ft.
Petroleum	27 per lb.
Candles	24.5 per lb.

The quantities of air based on dilution of carbonic acid may be calculated from the formula $Q = nc/(p - o)$ where Q is the number of cu. ft. per hour, p the permissible ratio of CO_2, o the original ratio in the outer air, n the number of sources producing CO_2, and c the volume produced by each source per hour. Table 3 gives the volume of air to be supplied per capita for certain limits of CO_2 under various conditions, and Table 4 the average rate of production by persons and by various illuminants.

Where the conditions affecting vitiation are uncertain factors, the air supply may be determined from general experience, according to Table 5.

Table 5. Air Supply for Ventilation

	Air supply per occupant, cu. ft. per hour	Number of renewals of air contents per hour	Remarks
Hospital wards	2700 to 3600*		*Billings
Hospital, epidemic	5000*		*Carpenter
Schools	1800* to 2400		*Legal requirements
Prison cells	500 to 750		Carpenter 1700
Prison wards	350		
Barracks	3000*		*Billings
Water closets	2400*	10 to 20**	*Per fixture, Billings **Wolff's practice
Living rooms	700 to 1200	1 to 2	
Kitchens		8 to 12*	*Wolff's practice
Auditoriums	1000 to 2000*		*Carpenter
Theaters	1500 to 1800*		*Snow-Nolan
Restaurants		5 to 6*	*Allen

The volumes given above may be reduced for rooms with very high ceilings when occupied for short periods only, and in general when leaky or not easily overheated. They should be increased where these conditions are unfavorable. Greater volumes or provision for increased air movement are required for summer.

Quality of the Air Supply. The **temperature** of the entering air should be that of the room, or lower when required for cooling. It is not advisable to combine the air supply with heating, which tends to raise the room air temperature. The latter can be kept lower by applying mild radiation independently. The air supply should help to maintain equable room temperature. The **relative humidity** may range between 30 and 70 per cent. of saturation, when the air is relatively cool and pure. Extremes are felt and become harmful when the air is overheated and dusty. The **purity** of the air supply is of prime importance. Where metallic, mineral and organic dust is present, it should be removed by washing or filtering and the air ways designed to be self-cleaning. **Movement** of air in a room is desirable, but it must not cause a sensible draft. Natural, gentle movement is favored by low air temperature. Drafts are felt quicker in hot air. For the velocity of the incoming air, see p. 1359.

SYSTEMS OF HEATING AND VENTILATING

Heating Systems

Open Fireplaces and grates are adapted only to mild climates or to rooms well protected from the weather. As auxiliaries to other apparatus or for ventilating they are desirable, but uneconomical as to fuel. Franklin stoves, which retain the advantages of the open fire, are somewhat better in this respect.

Closed Stoves, for wood or coal, can be fairly economical when of ample size, and are satisfactory for single-room heating. Iron stoves are too hot for wholesome effect. Tile ovens are better in this respect, especially for intermittent wood firing.

Individual Gas Stoves are cheap in first cost, but not economical in operation if connected to a flue, unless the heating surfaces are ample. Their surface temperature is generally too high for wholesome heating. When the products of combustion are allowed to escape into the room, the air is rapidly vitiated. This practice ought not to be tolerated for permanent installation.

Gas-fired Boilers for water and steam differ from coal-fired steam or hot-water systems only in that grates are replaced by burners. A greater number of small heaters may be used, by which long mains are avoided and the apparatus made more adaptable to control. This last feature reduces the gas consumption to a minimum and may often bring the cost of operation down to a reasonable figure. Whether it will prove advantageous depends on the price of gas, the saving in attendance and the value of the convenience.

Electric Heating is limited to where current is very cheap, where the cost or inconvenience of any other form of heat outweighs the operating expense, as for railways and where heat must be conveyed quickly to distant or inaccessible points, as in emergencies. When resistance coils are used as heating surface, their temperature is likely to be higher than desirable for hygienic reasons. This sanitary objection is avoided by heaters in which the surface temperature is reduced by some intermediate substance.

Hot-water Heating can be used to advantage wherever several rooms or buildings are to be heated from one service point. Water is best adapted to absorb heat from fuels, waste gases and exhaust steam. It can carry it economically for short or long distances. Through variation of water temperature it offers considerable range for general regulation, which is an important factor in reducing losses from pipe lines. The absence of rust and sediment resulting from the continuous use of the same water insures durability and reliability of apparatus. The danger of freezing is negligible with proper design and installation. High water pressure in tall buildings can be overcome by division of the system.

The **open system** should be used for gravity and forced circulation, with an expansion tank connected to the flow and return mains, so that air and steam may escape freely. The **cost of hot-water heating** depends upon the limit set to the water temperature. It is not necessarily higher than with steam heat and generally offers a better investment.

The choice between **gravity circulation and pumping** is decided in the main by the yearly expenses for carrying the heat. The **method of distribution** is determined generally by structural conditions. A small ratio of height to length of mains favors the overhead system. Tall risers generally make underfeed more desirable. Where the bulky lines are not objected to, single mains may prove the cheapest. Good circulation can be obtained by any of these methods where sufficient head is available. **Artificial acceleration** is recommended only to overcome special conditions.

Steam Heat is indicated where exhaust can be utilized with the least complication of apparatus, where general heat control and moderate temperature of surface are of secondary importance, and where heat is to be carried to distant points under a considerable pressure drop. In its simplest form, steam heat is generally cheaper in first cost than hot water, but less economical in operation and less desirable from the hygienic point of view. Steam heat is applied in **closed systems** under 1 to 10 lb. pressure, with two- or one-pipe distribution, the latter being the cheapest form. **Vapor systems** carry steam at practically atmospheric pressure. They are generally open through the returns. The flow of steam is restricted to the condensing capacity of the radiators, which permits graduated hand control through special valves. **Vacuum** is applied in different ways, through air lines or returns, according to the problem of drainage or removal of air. This auxiliary is used extensively in connection with exhaust-steam heating with the idea of reducing back pressure. When considering it with this in view, the additional apparatus involved will not always justify itself, since good circulation can be effected without suction in a correctly designed pipe system at not more than 0.2 lb. per sq. in. pressure drop, and the vacuum, being in practice maintained on return and air lines only, cannot increase the power output of the engine to any extent. Such apparatus is indicated mainly for special cases and for correcting difficulties of circulation on old plants.

The choice of the **method of distribution,** which may be overhead or from below or both, with wet or dry returns, is partly dictated by structural conditions, but it may also be influenced by the problem of equalization.

Warm-air Heating by furnace depends entirely on air as the heat carrier. This system should be used only where the first cost must be kept down and where the conditions are favorable. Sometimes it may be indicated for churches and halls to be heated only at intervals during which a steam or hot-water apparatus might freeze up and where it will not pay to keep the fires going in cold weather for better service and for the preservation of the building. When applied and designed with judgment, warm-air heating can give fair service, but its efficiency in utilizing the fuel is lower than that obtained with other heat carriers.

Indirect Steam and Hot-water Heating with the air circulating under gravity, is likely to be inefficient. This system is expensive in first cost as well as in operation and not desirable from the sanitary point of view, because the heating surfaces are never kept clean.

The **hot-blast system,** which is also a form of air heating, reduces the bulk of the ducts, the uncertainties of delivery and also the sanitary objections, in so far as the heating surfaces and air passages can be made self-cleaning. The heat is absorbed more readily under forced circulation, which greatly reduces the surface and often pays for the necessary machinery. Hot blast will pay generally for delivery of warm air at considerable distance, and is indicated where direct radiation is not practicable or not desirable, as in certain types of factory buildings, see p. 1332. **Hot blast with double ducts** for tempered and heated air is often used for schools. Unless distribution is made by a plenum chamber and well-proportioned individual ducts to each room, distribution is uncertain, because of the changing pressures in the ducts. The varying temperatures of the air supply are also liable to be felt in the rooms. From the heating point of view the system shares the uncertainties of hot air.

Ventilating Systems

Natural Ventilation by leakage through walls and around windows gives about one renewal of the air contents of a room per hour for average conditions during the winter season. This amount may vary considerably according to building structure and exposure. When supplemented by occasional airing, this natural exchange may suffice for rooms with light occupancy. Windows and transoms are also natural (and in some ways the best) means for ventilation, but they should be relied upon only where an occasional quick airing will suffice during the winter season. For crowded rooms or to meet some particular source of vitiation calling for a steady renewal of air, window ventilation is too uncertain, and if attempted to any extent, makes proper heating almost impossible.

Sash Ventilators depending on wind pressure or temperature difference will inevitably be closed during cold and windy weather and give little effect in still, mild weather when active renewal is most needed. Even where vent flues are provided to draw in the air, their action is most uncertain and liable to be reversed by outside influences. These devices should be used only for light duty, with the idea of increasing natural leakage when desired. **Air inlets**, behind or in connection with radiators (the **direct-indirect system**), are unsanitary when designed with box bases and screens that are bound to fill with dirt from the street. With large inlets the effect is even more uncertain than that of the sash ventilators. Considerable extra heating surface is required to cover the losses, which adds to the difficulty of the heat control.

Gravity Vents in the form of flues leading above the roof with ventilating or protecting hoods depend on temperature difference and wind. They should be considered as an aid to natural or window ventilation. When used in connection with a forced air-supply system, they should be calculated liberally, for a small temperature difference, so as to reduce the back pressure. Vents can be made to create a positive movement under all conditions when accelerated by heat. Since the greater part of this heat is lost, this method will not pay unless waste heat can be utilized for such purpose.

Mechanical Exhaust by fans is indicated for the removal of any special sources of vitiation and excess heat. It is most effective when applied locally, so as to prevent the spreading of heat or impure matter. Exhaust is also indicated when it is desired to put rooms under vacuum, to confine odors. Provision should be made in such cases for free access of air to these rooms from a locality not affected by wind.

Mechanical Air Supply by fan without any positive means for the removal, is sometimes called the **plenum system**, the object being to keep a room under pressure to avoid indraft. The results of this system depend largely upon the natural leakage of the rooms or the capacity of any vent flues, which are both influenced by the weather. The plenum is also used in distribution as a means for equalizing pressure by a large duct or a plenum space.

Mechanical Air Supply Combined with Exhaust, both by fans, is the proper equipment where positive, continuous ventilation is desired and prescribed volumes of air, tempered or otherwise conditioned, are to be introduced, and must be under complete control and independent of the heating apparatus. The needs of heating and ventilation do not coincide. The best results are obtained if each equipment is designed for itself and to best advantage for its own duties.

Open Disk Fans for circulation within a room do good service when air movement is desirable to relieve heat stagnation which cannot always be taken care of by apparatus designed principally for use in winter.

THE TRANSFER OF HEAT

Transmission of Heat in Buildings. The safe way to determine the hourly heat loss of rooms, etc., is by calculation on the basis of the surfaces exposed to cooling, and to estimate the effects of exposure and leakage under wind action. This loss is given by the formula, $E_t = [ak(t_i - t_o) + lQ(t_i - t_o)c](1 + e)$, where a is the wall, window or other area in sq. ft. exposed to cooling, k the heat transmission per deg. fahr. per hour per sq. ft., $(t_i - t_o)$ the temperature difference between inside and outside, l the number of hourly renewals of air, Q the cubical contents of the room, c the specific heat of the air per cu. ft. (0.018) and e the allowance for exposure and wind action according to the points of compass, as a fraction of the heat transmission and leakage. Table 6 gives the rate of heat transmission k, and is based on German and American practice. The values for corrugated iron are from a test by A. H. Blackburn. They apply to quiet air only. Extra large allowance must be made for wind action and height of walls. The figures for frame walls are those used by the Supervising Architect of the U. S. Treasury Department, Washington, D. C. Only one coefficient is given for single glass, to be used for windows and skylights with allowances for height, which induces air currents and increases the transmission. The factor l, giving the number of hourly renewals of air by leakage, is estimated from the exposure and material. With one outside wall, l varies from 0.5 to 0.8; for corners, from 0.8 to 1.2, and for rooms with through or updraft, 1 to 1.4.

Table 6. Coefficients of Heat Transmission in Buildings

(k = B.t.u. per hour per sq. ft. per 1 deg. fahr. difference in temperature)

WINDOWS AND DOORS

	k		k
Single windows	1.03	1-in. wood doors	0.50
Double windows	0.48	2-in. wood doors	0.46
Double glass on single sash	0.72		

(For single glass over 10 ft. high, add 10 per cent. for every 5 ft. above 10 ft.)

WALLS AND PARTITIONS

Solid brick walls, thickness in in.

=	4	8	12	16	20	24	28	32	36	40
k =	0.51	0.39	0.31	0.26	0.22	0.19	0.17	0.15	0.135	0.125

Solid concrete walls, thickness

in in. =	4	6	8	10	12	16	20	24
k =	0.62	0.55	0.50	0.455	0.42	0.365	0.32	0.28
Solid thickness in in. =	12	16	20	24	28	32	36	40
For sandstone k =	0.46	0.41	0.37	0.34	0.31	0.29	0.26	0.24
For limestone k =	0.51	0.455	0.41	0.375	0.34	0.32	0.29	0.27
For granite or marble k =	0.42	0.38	0.35	0.32	0.29	0.27	0.26	0.24
Hollow terra cotta, thickness, in. =	4	6	8	10	12			
k =	0.42	0.365	0.32	0.27	0.23			

	k
Corrugated iron (on total surface)	1.18
Corrugated iron (on projected surface)	1.50
Wire lath and plaster	0.64
Studding, wood lath and plaster on one side	0.53
Studding, wood lath and plaster on both sides	0.375
Frame wall with clapboard only	0.44
Frame wall with sheathing and paper	0.28
Frame wall with sheathing and asbestos	0.23

FLOORS

Cement floor on concrete arch	0.35
Single wood floor on beams	0.33
Double wood floor on beams	0.30
Double wood floor, air space and plaster	0.10
Double wood floor, sleepers on brick arch	0.08
Cement floor on concrete arch, air space and plaster	0.06
Double wood floor on concrete arch, air space and plaster	0.05

CEILINGS

Plaster, concrete arch, cement floor above	0.36
Plaster, air space, concrete arch and cement	0.24
Plaster, air space, single wood floor above	0.18
Plaster, air space, double wood floor above	0.16
Plaster, brick arch, wood floor on sleepers	0.14

For exposure and wind action add the following percentages, e, to outside transmission and natural ventilation: North, 20 per cent.; Northwest, 25 per cent.; West, 20 per cent.; Southwest, 10 per cent.; South, 0 per cent.; Southeast, 5 per cent.; East, 10 per cent.; Northeast, 15 per cent. These percentages are to be modified according to situation. They are to be increased where glass areas are large and decreased for small windows, solid construction and a preponderance of cooler inside surfaces not exposed to wind. The area a should be measured from c. to c. of intersecting walls, floors and ceilings. Window areas are understood to be the clear openings in walls.

For very tall buildings it is necessary to take into account the pressure created by the column of warm air within, as the several connecting stories act in the manner of chimneys, drawing air from below and discharging it above the "neutral zone." Where halls, elevator shafts and rooms interconnect freely, also in tall rooms, the resulting tendency for indraft of cold air through windows and doors at the lower levels must be taken care of by additional heat. Such allowances may run up to 50 per cent. of all other losses. It is not safe to deduct correspondingly from the heat for upper floors, since the warm air from below will not always and should not travel through the rooms above.

This method of estimating the heat requirements may be applied to all classes of buildings, excepting only very massive structures (see p. 1343), but in general it should be borne in mind that the figures given are necessarily approximate, since the heat transfer varies with quality of materials, dryness of surface and interior, finish and other minor factors. Considerable judgment should be used in estimating coefficients.

Carpenter's Rule for Ordinary Buildings, which should be used for approximation only or for estimating, is as follows: B.t.u. per hour $= (Cn/55 + W/4 + G)d$, where C = volume of room in cu. ft., W = sq. ft. of exposed wall surface, G = sq. ft. of windows, d = difference between inside and outside temperatures in deg. fahr., and n = the number of times per hour that the air is changed in the room ($n = 1$ for ordinary rooms = 2 to 3 for halls and vestibules).

Reheating Buildings. Allowances for reheating are not necessary where heat will be needed for the greater part of every day. In such cases the service should be continuous in coldest weather in order to maintain a desired temperature. This applies in general to residences, to most business buildings and to greenhouses. When heat is needed only for the smaller part of the day, as in many schools, some allowance should be made for reheating. Substantial buildings with considerable heat reserve will cool off more slowly, but will also take much time for reheating. Leaky, exposed buildings cool off quickly and are slower to heat up under extreme conditions, as the rapid transmission leaves little margin for raising the temperature. The allowance should be increased according to the ratio of the heat transmission to the heat reserve E_t/E_r, and according to the time stipulated for reheating. From 10 to 25 per cent. should be added to the boiler capacity and radiating surface for daily reheating. The same principle applies to buildings heated periodically and allowed to cool to any extent, such as churches. The maximum heat required per hour will then figure approximately $E = (E_r/H) + E_l + E_t[½(t_l + t) - t_o]/(t - t_o)$, where H = number of hours allowed for reheating, t_l and t = lowest and normal inside temperatures, t_o = outside temperature, E_l = heat losses in transit from generator to heating surfaces, and E_r = thermal absorption of building and apparatus during the period of reheating.

Assuming the mean temperature of the structure to be halfway between the indoor and outdoor temperatures, the total heat absorbed is $E_r = 0.5\,(t - t_l) \times cW + (t_m - t_l)cW$, where cW represents the sum of products of specific heat and weight of the materials warmed in building and apparatus, as calculated from Tables 7 and 8, and t_m the maximum temperature of the water to be reheated. Where a banked fire is kept up, t_l (the lowest temperature of the apparatus) can be assumed higher than the lowest room temperature.

Table 7. Thermal Capacity of Building Materials. (Approx.)

(B.t.u. per deg. fahr.)

	Per lb.	Per cu. ft.		Per lb.
Masonry	0.20	25	Wrought iron	0.114
Hardwood	0.57	35	Cast iron	0.130
Softwood	0.47	20	Tin	0.056
Glass	0.19	30	Brass	0.094

Table 8. Approximate Weights and Thermal Capacities of Heating Apparatus

	Approx. wt. per sq. ft. rating, lb.	B.t.u. per 1° F. (sp. ht.) per sq. ft. rating	Water contents per sq. ft. in lb. at 60° F. or B.t.u per 1° F.
Cast-iron sectional boilers	(S) 2 to 4; (W) 1.2 to 2.4	(S) 0.26 to 0.52 (W) 0.16 to 0.32	(S) 0.35* (W) 0.25*
Cast-iron sectional radiators	(S) 6.5; (W) 7	(S)0.85;(W)0.9	(W) 1 to 1.5

(S) for steam, (W) for water. * Obtained for "Mills" boilers only.

For weight and capacities of piping, see p. 797.

Rietschel gives two formulæ for reheating. For rooms which are heated daily, but with interruptions during the night, the heat per hour A to be added to the other heat losses is $A = 0.0625(n - 1)W_1/z$ and for rooms which are not heated daily, $A = 0.1(8 + z)W/z$, where W_1 = the heat lost per hour in normal operation through the outside wall and windows and other exposed surfaces which have to be rewarmed, W = the total heat loss of the room per hour, from all causes when normally heated, n = the number of hours from the ending of the daily heating in the evening to the beginning in the morning, z = the number of hours from the beginning of the heating of the morning to the time when the building is thoroughly heated. A should never be taken greater than $\frac{1}{2}W$.

In exceptional cases, when a very **massive structure** is used occasionally for a few hours, the heat transmission will not be normal and the walls keep on absorbing heat until some time after the room temperature is reached. Rietschel gives the following approximations for these problems:

For direct heating $E = \frac{A_g k(t - t_o)}{2} + A_w\left[8.5 + \frac{(t - t_i)}{H}\right]$;

For indirect, or hot-air heating $E = \frac{A_g k(t - t_o)}{2} + A_w\left[14.8 + \frac{2(t - t_i)}{H}\right]$

where A_g and A_w are the glass and total inside and outside wall areas, and k the coefficient of transmission for glass (1.02). For rooms over 40 ft. high, add 15 per cent. for every 10 ft. of additional height.

It will be found in some cases that the capacity of apparatus figured from these formulæ will be several times greater than that based on continuous heating. The question will then arise, whether in mid-winter a steady service, reduced between periods of occupancy, will not pay better. As a general rule, little or no fuel is saved by interruption of service. Continuous heating reduces the capacity of apparatus required, the time of reheating, gives a steadier load on boilers, and results in warmer walls and more equable, agreeable conditions in rooms.

Calculation of Heating Surface for Rooms. The heating surface required to make up the losses by transmission and for reheating $E_t + E_r = E$ is figured by the formula $S = E/(t_r - t_a)k$, where E is the total heat to be emitted in B.t.u. per hour, t_r and t_a the mean temperatures of the heat carrier in the radiator and of the surrounding air, and k the coefficient of transmission in B.t.u. per hour per deg. fahr. difference.

For **direct surfaces** the average values of $(t_r - t_a)k$ used in **steam and hot-water heating practice** are given in Table 9. They apply closely to the typical sectional radiators and pipe coils standing not more than 3 in. and not less than 2 in. from a wall.

Table 9. Emission from Direct Heating Surfaces, in B.t.u. per Hour per Sq. Ft. of Surface

(Surfaces exposed, at room temperature of 70 deg. fahr., under natural air circulation)

Height, in.	Gravity hot water, 180 deg. fahr.		Forced hot water, 200 deg. fahr.		Low-pressure steam, 1 lb. (215 deg. fahr.)		Low-pressure steam, 5 lb. (227 deg. fahr.)	
	Single column	Two column	Single column	Two column	Single column	Two column	Single column	Two column
20	191	180	233	217	294	274	326	303
26	187	174	227	211	286	266	317	295
32	183	170	223	205	279	258	308	285
38	181	165	220	200	273	250	302	277
44	178	162	216	196	268	244	298	271
	Three column	4 and 5 column	Three column	4 and 5 column	Three column	4 and 5 column	Three column	4 and 5 column
14	....	165	....	200	....	250	....	277
20	168	156	202	189	255	238	282	263
26	161	150	195	180	245	225	272	250
32	155	142	188	172	236	216	261	240
38	150	137	181	166	227	207	251	230
44	146	....	176	....	219	....	242	....
Wall radiators, vertical loops..........	200		245		310		340	
1 horizontal pipe (1½ in.).........	272		331		402		445	
1 vertical pipe (1¼ in.)............	250		305		370		410	
1 × 4 − 1 ½-in. coil..	250		305		370		410	
1 × 8 − 1 ¼-in. coil..	237		287		349		386	
1 × 12 − 1 ¼-in. coil.	225		273		333		370	

Corrections.—For lower room temperature than 70 deg. fahr., increase the heat emission by 1 per cent. for every degree lower; for a higher room temperature, reduce by 1 per cent. for every degree higher.

For surfaces under windows add 5 per cent. to the emission.
For surfaces on inside walls subtract 5 per cent. from the emission.
For radiators of less than 6 sections add 5 per cent.
For radiators of less than 4 sections add 10 per cent.
For 1-in. pipe add 5 per cent.; for 2-in. pipe subtract 5 per cent.

When sections are widely spaced apart the coefficient is likely to be somewhat higher. When spaced close together k is generally lower, except when the design favors air currents. Corrections should be made for higher or lower room temperatures, for location, length of radiators and pipe diameter of coil surfaces, as stated. The heat emission is further affected by the finish of the surfaces, which bears on radiating qualities. This applies particularly to styles of radiators giving a greater proportion of radiant heat. According to Prof. J. R. Allen (see *Trans. A. S. H. & V. E.*, 1909) the customary bronzing materially reduces efficiency, while lead paints and enamel finish are as good or better than bare surfaces.

Table 10. Relative Values of Radiator Paints

(John R. Allen, in "Notes on Heating and Ventilation," Domestic Engg. Co.)

Surface	Relative transmission	Surface	Relative transmission
Bare iron............	1.000	Terra cotta enamel.......	1.038
Copper bronze.......	0.760	Maroon gloss japan.......	0.997
Aluminum bronze....	0.752	White lead paint..........	0.987
Snow-white enamel..	1.010	White zinc paint.....	1.010
No-luster green enamel	0.956		

(The heat transmission depends upon the last coat put on.)

Table 11. Effect of Humidity on the Transmission of Heat through Cast-iron Radiators

Percentage of moisture saturation	20	30	40	50	60	80	100
Coefficient of transmission	1.79	1.77	1.75	1.72	1.69	1.63	1.59

Table 12. Heat Transmission through Cast-iron Radiators at Varying Temperature Differences

(John R. Allen, Nat. Dist. Htg. Assn., June, 1911)

Temperature difference, deg. fahr	80	100	120	150	170	190
Coefficient of transmission	1.56	1.58	1.62	1.65	1.69	1.72

Marble or iron tops, shelves or any feature that will obstruct circulation or radiant heat will also keep down the heat emission. For screened heating surfaces, according to the experiments by Brabbée (*Gesundheits-Ingenieur*, No. 44, 1911), the efficiency is reduced most for low radiators and where the air circulation is through the front only. The reduction may reach 40 per cent. for metal screens. For insulated wood boxing with low solid top the emission is reduced by at least one-half, and still more when surfaces are dusty. Screens practically stop the radiant heat and turn the apparatus into an unsanitary, inefficient and expensive hot-air heater. Direct heating surface should not be concealed.

For **indirect heating surface** in stacks the calculation should cover the losses by transmission and ventilation $E_t + E_v$, including the losses in flues. The rate of heat emission depends upon the obtainable velocity of the air, the character of surface, which is usually extended by ribs or pins, and upon the depth. For steam heat, with air at about 3.5 ft. velocity per sec., $k = 3.0$ for plain, and 2.4 for extended, surface. For hot-water heat with air at about 3 ft. vel. per sec. $k = 2.75$ for plain, and 2.2 for extended, surface. For deep sections (20 sq. ft.) the coefficients should be taken about 10 per cent. smaller. For higher and lower velocities the efficiency, according to Carpenter (5th ed., p. 103), varies as the square root of the velocity. The velocity obtainable in the stack should be verified (see p. 1358).

For **direct indirect radiators**, where fresh air enters a room through a direct radiator, the heat emission is extremely variable and uncertain, depending on the arrangement of the air inlets. It is not safe to assume the efficiency of the surface higher than that of ordinary direct radiation. The calculation should be based on $E_t + E_v$, according to the renewal of air assumed.

Hoffman ("Handbook for Heating and Ventilating Engineers'" p. 116), gives the following rules for indirect heating, which may be used for approximation: Duct area in sq. in. = sq. ft. of radiation installed $\times\ C$; and cu. ft. of air supplied = sq. ft. of radiation $\times\ K$, values of C and K being as follows:

	C		K	
	Steam	Hot water	Steam	Hot water
First floor	1.5 to 2	1.00 to 1.33	200	150
Second floor	1.0 to 1.25	0.66 to 0.83	170	130
Other floors	0.9 to 1.0	0.6 to 0.66	150	115

Allen gives the data of Table 13 on the heat emission from the standard types of indirect radiators.

For **hot-blast surface**, box coils or cast-iron sectional radiation may be used. For cast-iron surface specially designed for blast, the manufacturers give figures for various depths, velocities and temperatures of steam and air. The efficiency of the type of coils in general use, with 1-in. pipes, 2⅛ in. on centers in staggered rows, giving about 40 per cent. free area, the rate of emission for different speeds and depths of heater, may be obtained from Fig. 1, which represents the average results of several series of tests by differ-

Table 13. Heat Emission from Indirect Radiators

Cu. ft. of air per sq. ft. of radiation	Increase in temperature of the air passing the radiator, deg.		Steam condensed per sq. ft. of radiation, lb.		B.t.u. transmitted per sq. ft. of radiation per deg. diff. in temp. of the steam and the air	
	Standard pin	Long pin	Standard pin	Long pin	Standard pin	Long pin
50	147	140	0.125	0.15	0.80	0.95
75	143	137	0.17	0.21	1.17	1.27
100	140	135	0.24	0.26	1.51	1.60
125	138	132	0.295	0.31	1.85	1.90
150	135	129	0.355	0.36	2.22	2.20
175	132	126	0.41	0.405	2.57	2.47
200	130	123	0.47	0.45	2.90	2.72
225	127	120	0.53	0.49	3.25	3.00
250	123	118	0.585	0.53	3.60	3.20
275	121	115	0.645	0.57	3.90	3.40
300	119	112	0.700	0.61	4.22	3.60

ent experimenters. For other air and steam temperatures the efficiencies will be roughly proportional to the mean differences, but increasing for colder and decreasing for warmer air. The rate of emission is not affected to any extent by the condition or finish of the surfaces, but is liable to be influenced by interior circulation, or lack of it.

EFFICIENCIES OF HOT-BLAST COILS

40% FREE AREA

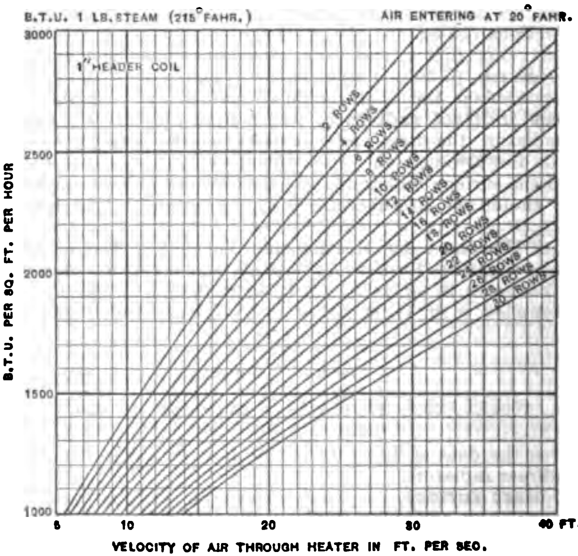


FIG. 1.

Laws of Convection, Steam to Air (From tests of 1-in. wrought-iron pipe, by Carrier and Busey, *Htg. and Vent. Mag.*, Jan., 1912):

$$\log [(T_s - T_1)/(T_s - T_2)] = f/(0.1119V + 127); \text{ also,}$$
$$\log [(T_s - T_2)/(T_s - T_1)] = 0.3994\, fK/V,$$

where T_s is the steam temperature, T_1 the entering air temperature, T_2 the final air temperature, $f = S/A$ = ratio of total surface to clear area, V = velocity of air through clear area (at 70 deg.) in ft. per min., and K = B.t.u. per sq. ft. per hour per deg. temperature difference between steam and air.

For ordinary calculations where it is desired to solve for the initial and final temperatures when f and V are known, first determine approximate values of T_1 and T_2 from the first-given equation. Then solve for the mean temperature difference between steam and air, using the above-determined values of T_1 and T_2 in the formula $(T_s - T)_m = 0.434\ (T_2 - T_1)/\log [(T_s - T_1)/(T_s - T_2)]$ where $(T_s - T)_m$ represents the mean effective temperature difference between the steam and air.

Boilers. See also pp. 866 to 937. For low-pressure ($\leq$ 20 lb.) steam heating, **cast-iron sectional boilers** are usually best adapted, especially for the smaller units. They are rated by the makers in terms of the radiation surface, S, in sq. ft., which they will supply; in B.t.u. per hour the capacities are $250S$ for steam and $160S$ for hot water. Ratings are given for anthracite; for bituminous coal boilers should be from 10 to 30 per cent. larger. In small boilers allow 1 sq. ft. of **grate surface** to every 15 to 30 sq. ft. of heating surface. Over half the heating surface should be exposed to direct radiation from the fire. With average draft they burn from 2 to 6 lb. of anthracite per sq. ft. of grate surface per hour, and with bituminous coal from 3 to 8 lb. Efficiencies run from 50 to 70 per cent.

Hot-air furnaces burn from 2½ to 6 lb. of coal per sq. ft. of grate surface per hour. Their efficiencies vary from 50 to 70 per cent. **Grate surface** in sq. ft. = $H/20{,}000$ to $H/45{,}000$, where H = B.t.u. required per hour. The **furnace capacity** should be increased 25 per cent. for bituminous coal and 33⅓ per cent. for Rocky Mountain bituminous or lignite.

The heat transmission from hot gases to air under the conditions of a hot-air furnace is from 550 to 750 B.t.u. per sq. ft. per hour for smooth surfaces and from 450 to 550 B.t.u. for extended or ribbed surfaces. Recknagel gives 1.0 to 1.4 B.t.u. per 1 deg. fahr. difference of temperature per sq. ft. for air velocities of about 3 ft. per sec. Allen allows 50 to 70 sq. ft. of heating surface per sq. ft. of grate surface.

MECHANICS OF HEATING

The movement of the heat carrier is effected either under its own power, as in gravity circulation and by the pressure of steam, or the necessary power is applied by machinery, as in forced circulation. The principles of distribution remain the same. The energy is used in part to overcome pipe friction and local resistances, the balance being expended in the desired movement. The problem of distribution is the adjustment of the resistances to the pressure available for each point of delivery. Tables based on total pressure alone can only give average sizes and may serve for estimates. The correct **sizes** of pipes and ducts to assure good distribution should be obtained by calculating and equalizing the resistances of the individual circuits.

Gravity Hot-water Circulation. The distribution of heat to a number of radiating units is calculated for a uniform temperature difference between all the rising and falling columns of water, making the difference in weight of the two columns proportional to the height available. The actual height h of the columns required to produce the flow, or the difference in mean level between the heat-emitting and receiving surfaces, usually represented by the radiator and boiler, is given by the equation $(w_1 - w)h = H(w + w_1)/2$, or $h = 0.5H(w + w_1)/(w_1 - w)$, where w and w_1 are the densities of flow and return water, H the sum of resistance heads for the circuit in question. The weight (W) in lb. per hour and the velocity (v) in ft. per sec. of the water quantity for which H must be figured, are $W =$ B.t.u. per hour$/(t - t_1)$ and $v = W/[3600a(w + w_1)/2]$, where t and t_1 are the temperatures, w and w_1 the densities of flow and return water in lb. per cu. ft. and a the pipe area in sq. ft.

A distributing pipe system is to be proportioned so that the sum of friction and local resistances, as read from Fig. 2, expressed in differential heads for the chosen temperature drop, in this case 30 deg. fahr., equals the actual height available between the radiator and boiler for each of the circuits in question. As the water expands 1 ft. in 100.3 ft., the true head is multiplied on this chart by 100.3. The slope of the lines is based on Tutton's general formula, as used by Schoder (see *Eng. Rec.*, Sept. 3, 1904, and *Trans. A. S. C. E.*, vol. 51, p. 308) and applied by Meier to heating with allowances for conditions in practice. The friction and resistance heads for runs of even diameter and velocity between junctions will figure $h_f = 0.0257 \times (v^{1.86}/2g)(l/d^{1.25})$ and $h_r = 1.38rv^{1.86}/2g$. In the above equations v is the velocity of flow in ft. per sec., r is a resistance factor whose values for different types of obstruction are given in Fig. 2, l and d are respectively the length and diameter of the pipe in ft. For a circuit of several runs the total resistance H equals the sum of h_f and h_r, which is $H_f + H_r = H$.

The readings from the chart in Fig. 2 agree substantially with the results of formulæ with variable coefficients of friction in so far as they apply to conditions in actual heating practice. The local resistances are read along the same velocity line, where it intersects with that for the features to be calculated. The resistance heads h_r are not strictly proportional to the velocity head, but taken to vary in this respect as the friction, the results being close enough to established facts for all practical purposes.

In using Fig. 2, start with the amount of heat to be given up by the water (as shown on the lower horizontal scale) and follow vertically till the assumed pipe size (lines inclining down to the left) is reached. The velocity in the pipe is given by the lines inclining down to the right. The vertical scale gives the height required to overcome the frictional head in 10-ft. length of the pipe, or the resistance offered by various obstructions. These heights must be read at the level of the intersection of the inclined velocity line with the pipe size and the obstruction lines. For example, for a transmission of heat of 240,000 B.t.u. per hour, with 4-in. water main, the velocity is seen to be 0.4 ft. per sec. The height required to overcome frictional resistance is 0.29 ft. per 10-ft. length of pipe; to overcome the resistance of a gate valve on the same run it is 0.1 ft.; of an angle valve 0.35 ft.; of a globe valve 0.77 ft.

The **calculation** is the same in principle for single or multiple circuits and for any method of distribution and means of acceleration. The choice between single-main distribution with secondary circulation, the underfeed or the overhead distributing system, is sometimes decided by the structural and service conditions. The feasibility of increasing or decreasing the effective head by the heat losses in transit, as may be needed to equalize, will decide in favor of one or the other method. In any event, it is necessary to follow the individual circuits and to calculate the mains so that the radiating surfaces in least favorable locations will have sufficient head available with connections of reasonable size.

For **rational calculation** it is necessary to establish all factors systematically by the use of a pipe schedule in the shape of an isometric drawing, showing all lengths and elevations to scale, also all the features causing local resistances. The B.t.u. carried may then be entered for all runs of even

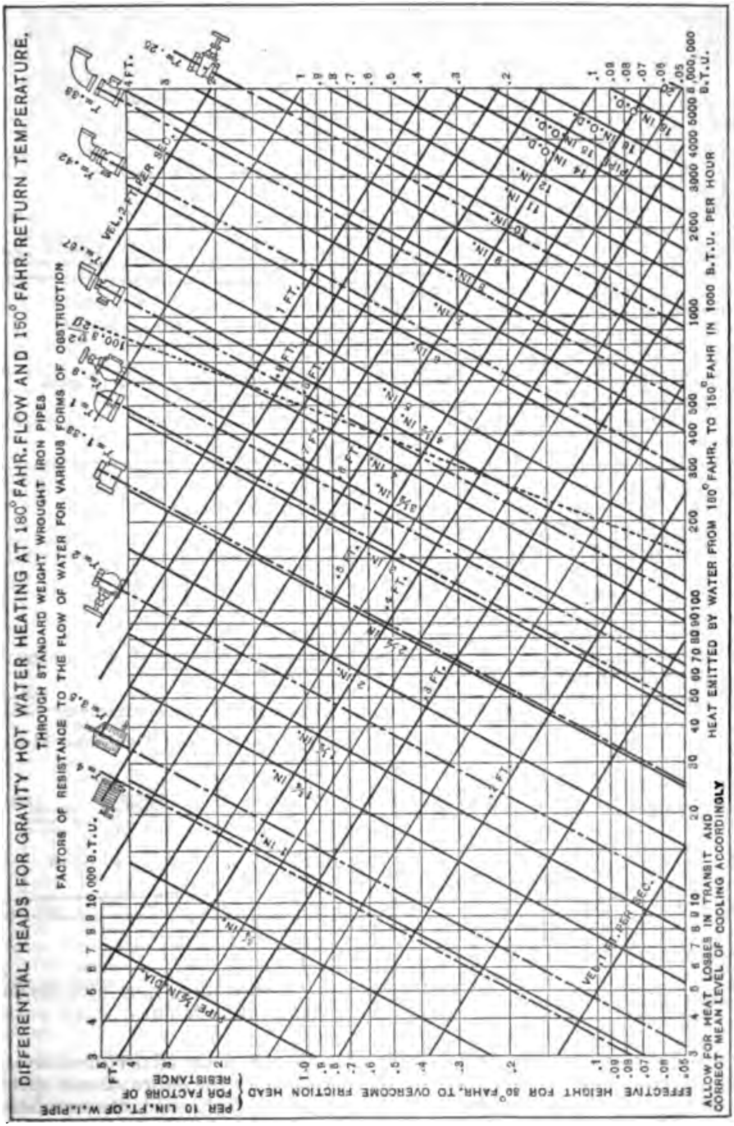


Fig. 2.—Chart for Gravity Hot-water Circulation.
(Condensed from Meier's "Mechanics of Heating and Ventilation.")

diameter on branches and added up for the mains. The piping is then to be sized tentatively by means of the table of approximation. On the basis of these preliminary sizes the resistances can be figured and modified where necessary to balance the effective available heights. This calculation is facilitated by the chart in Fig. 2, which permits direct readings of the losses of head by friction and local resistances for any quantity of water, or heat carried under the same range of temperatures. For smaller or larger range the volumes as well as the difference in density will vary. The B.t.u. and friction head in Fig. 2 should be multiplied by 1.15 for 26 deg. fahr. drop, 1.07 for 28 deg., 0.94 for 32 deg., and 0.88 for 34 deg.

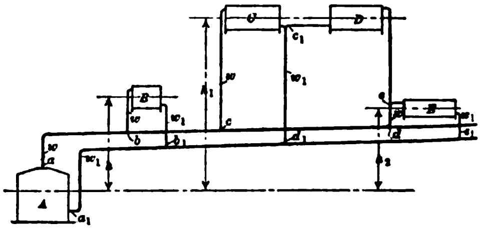


FIG. 3.—Hot-water Heating System with Underfeed Mains.

Greater drop of temperature is preferable for long mains. Smaller drop may be indicated for great heights; in other cases it results in excessive water bulk, slow heating and high cost.

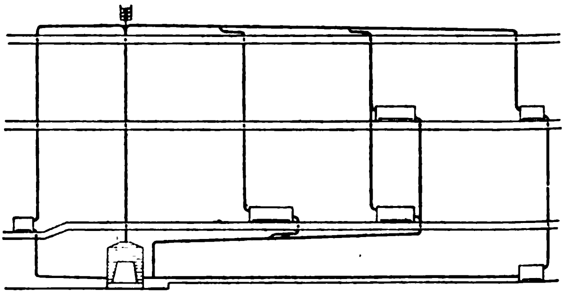


FIG. 4.—Hot-water Heating System with Overhead Mains.

The **heat losses along** the **uncovered flow mains** and bare risers should be estimated in B.t.u. and added to the tax on the lines. If these losses occur mainly at higher or lower levels, the mean height of cooling should be established and taken as the effective head.

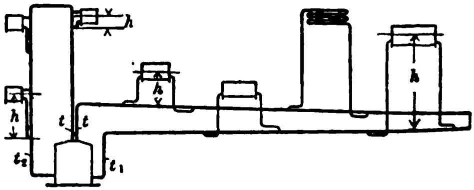


FIG. 5.—Hot-water Heating System with Single Distributing Main.

When using Fig. 2 the resistances are obtained for runs of even diameter, from junction to junction, usually together for flow and return. Starting at the boiler the items h_f and h_r are added successively, up to the radiator closing the circuit. The sum for the entire circuit should then approximate the head available. Where properly sized, the circulating system will need no adjustments.

For an **underfeed system** (Fig. 3), the resistances are figured first for the runs $a - b - c - d - e$ plus $e_1 - d_1 - b_1 - a_1$ plus $e - E - e_1$, for which the sum of differential heads, as read from the chart, should approximate the actual height h_2 in ft. The circuits nearer to the boiler are figured in the same way, the connections being modified to make up the desired totals h_1 and h.

With an **overhead system,** the circuits are followed in the same manner, but flow and returns must be figured separately from the first to the last junction. It pays in such cases to take advantage of the increased effective head due to the cooling of the mains at a higher level. Fig. 4 will suggest the method for calculating the mean effective head available for a branch carrying a stated total heat, also for radiators placed in sequence.

For **single-main distribution,** as shown by Fig. 5, the main should be calculated as a circuit by itself for a temperature drop $t - t_1$ or $t - t_2$ that will assure sufficient heating effect for the branches farthest from the boiler. The branch circuits are to be calculated independently for any range and height h above main, and need not be balanced against each other.

Table 14 gives sizes of mains and risers for various lengths of circuit and height h for gravity hot-water systems. It may be used for approximation and estimating.

Table 14. Gravity Hot-water Heating. Approximate Capacities of Mains and Risers for Range from 180 to 160 Deg. Fahr.

Capacities (including losses in transit) are in 1000 B.t.u. per hour and allow for average resistance of boilers, radiators and piping. For sq. ft. of radiating surface supplied (160 B.t.u. per sq. ft.), multiply the tabular figures by 6.25.

MAINS

Length,* ft.	Height,* ft.	Diameter of main, in. 1¼	1½	2	2½	3	3½	4	4½	5	6	7	8	9	10	12
		Capacity in 1000 B.t.u.														
100	7	15	22	40	60	98	133	188	240	315	480	675	900	1180	1550	2350
200	8	12.5	18	32	50	82	114	157	206	270	415	590	800	1040	1380	2100
300	9	11	16	29	45	75	106	144	190	250	385	550	740	970	1300	1980
400	10	10	15	27	42	70	100	135	180	238	367	520	700	920	1240	1900

RISERS

Height,† ft.	Diameter of riser, in. ¾	1	1¼	1½	2	2½	Height,† ft.	Diameter of riser, in. ½	¾	1	1¼	1½	2
	Capacity in 1000 B.t.u.							Capacity in 1000 B.t.u.					
10	4.0	7.5	15.0	22.0	42	67	30	3.4	7.1	13.1	26	38	74
15	5.0	9.2	18.7	27.4	52	82	40	4.0	8.2	15.2	31	45	86
20	5.8	10.6	21.7	31.8	60	95	50	4.5	9.2	17.0	35	51	97
25	6.4	11.8	24.2	35.5	67	106	60	4.9	10.1	18.5	38	56	107

* The length and mean height above boiler are those of the circuit for the most distant radiator in lowest location.

† The mean height above boiler is that of the circuit in question. This table is for a circuit 200 ft. long. For other lengths allow about in proportion as given above for mains.

Accelerated Gravity Circulation. Where difficult conditions are to be overcome, the circulation may be accelerated by the injection of steam into the flow main. The increased differential weight gained thereby is the motive power for most of the artificial systems of quick circulation on the market.

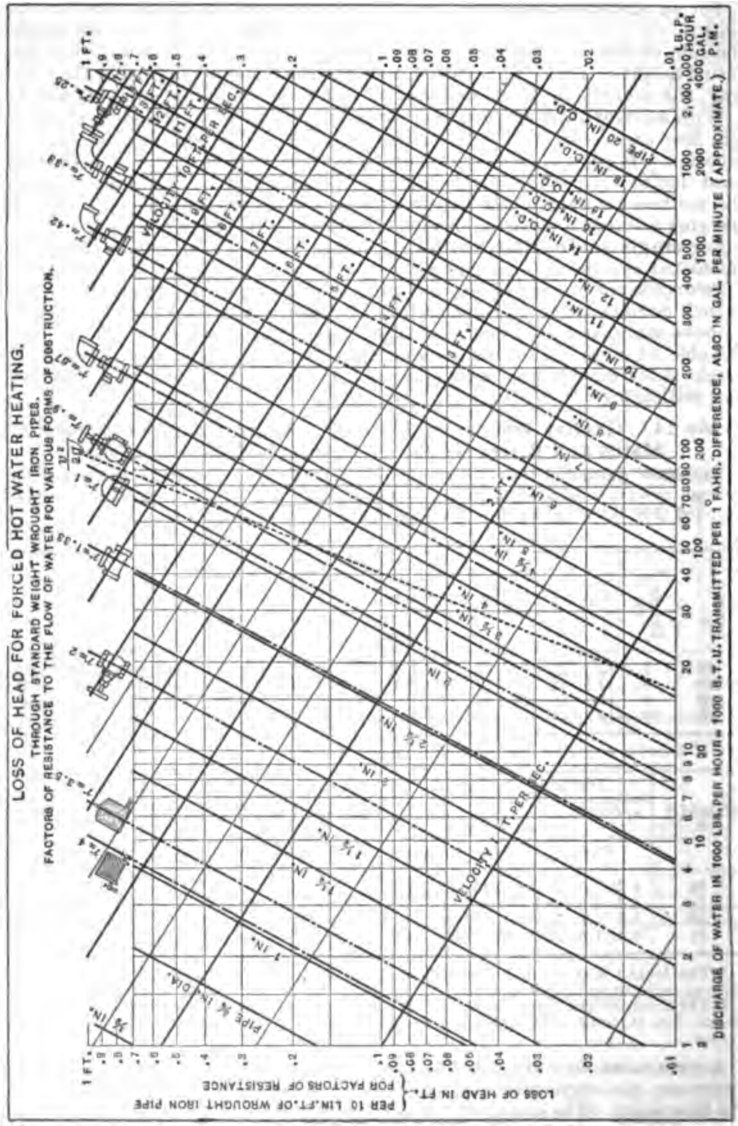


Fig. 6.—Chart for Forced Hot-water Circulation.
(Condensed from Meier's "Mechanics of Heating and Ventilation.")

With closed systems, permitting water temperatures above the boiling point, the circulation can be forced by greater ranges between flow and return. Either method depends on high flow temperatures and should not be recommended, except to meet special conditions.

Forced Hot-water Circulation is used where the saving due to the use of smaller pipe sizes will pay for the cost of pumping, which is often the case in connection with power plants. Centrifugal or rotary pumps are preferred for this work.

The **range of temperature between flow and return** with forced circulation, or the volume and speed of the water, are determined by the temperature available in the generator and that desired at the radiators. Ranges of 10 to 20 deg. fahr. difference are within the limits of good practice.

To approximate, the system is first sized on the basis of an even rate of pressure losses, along one horizontal line of the chart in Fig. 6, selected according to power and length, or the initial velocity desired. The losses of head throughout the system and the resulting differential pressures are then obtained conveniently, and the sizes of branches can be modified to make up the same total. The losses of head in this case are expressed in actual ft. of water column. It is advisable to eliminate any gravity head by using sufficient pump head.

In using Fig. 6 proceed as in Fig. 2, noting however that the abscissæ are weights of water circulated and that the ordinates are losses of head. For 240,000 B.t.u. per hour, and a temperature range of 12 deg. fahr., the weight of water circulated is 20,000 lb. per hour. With 2½-in. pipe the velocity is seen by the chart to be 2.75 ft. per sec. The losses in head due to this velocity are 0.19 ft. per 10-ft. length of pipe; 0.037 ft. per gate valve; 0.13 ft. per angle valve and 0.27 ft. per globe valve. The pressure loss for instance from a to a_1 on Fig. 7 must be made to equal that for $a - b - b_1 - a_1$ and $a - b - c - c_1 - b_1 - a_1$.

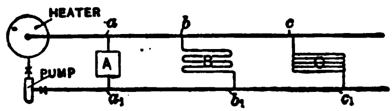


Fig. 7.

The **sizes** may be **varied** for flow and return or for portions of runs **to effect close equalization,** which will obviate the need of adjustments. The lines to and from the expansion tank should also be equalized to prevent short circuit, as well as the possibility of freezing. The suction at the pump inlet should be overbalanced by the static head of water to prevent boiling at the highest return temperature. The calculation of pressure losses will give the proper head, speed and motive power for centrifugal pumps.

For **central-station heating** the distribution to several buildings can be figured on the same principle, the service connections being the points of delivery. Allowance should be made in arrangement and sizing for future additions and flexibility in general.

Steam Heating. Although a part of the sensible heat is utilized, it is safer to count only on the latent heat of condensation as the energy delivered. The volumes carried by any run of pipe should be understood to include the losses beyond that run as well as half of its own losses. In **high-pressure distribution,** the friction, local resistances and head of discharge can be calculated from the following formulæ, which give substantially the same results as those of Babcock and Darcy having variable coefficients: $p_f = 0.0257wv^{1.95}l/(144 \times 2g \times d^{1.2})$; $p_r = 1.12wv^{1.95}r/(144 \times 2g)$; $p_v = wv^2/(144 \times 2g)$, where p_f, p_r and p_v are the pressure losses in lb. per sq. in. due to friction, obstructions and velocity respectively, d and l the length and diam. in ft., v the mean velocity in ft. per sec., and w the weight in lb. per cu. ft. of the steam at mean pressure. $v = W$ per hour$/3600wa =$ B.t.u. per hour$/3600waC$, C being the latent heat per lb. of steam, W the weight in lb. and a the area of the pipe in sq. ft. The pressure drop should be

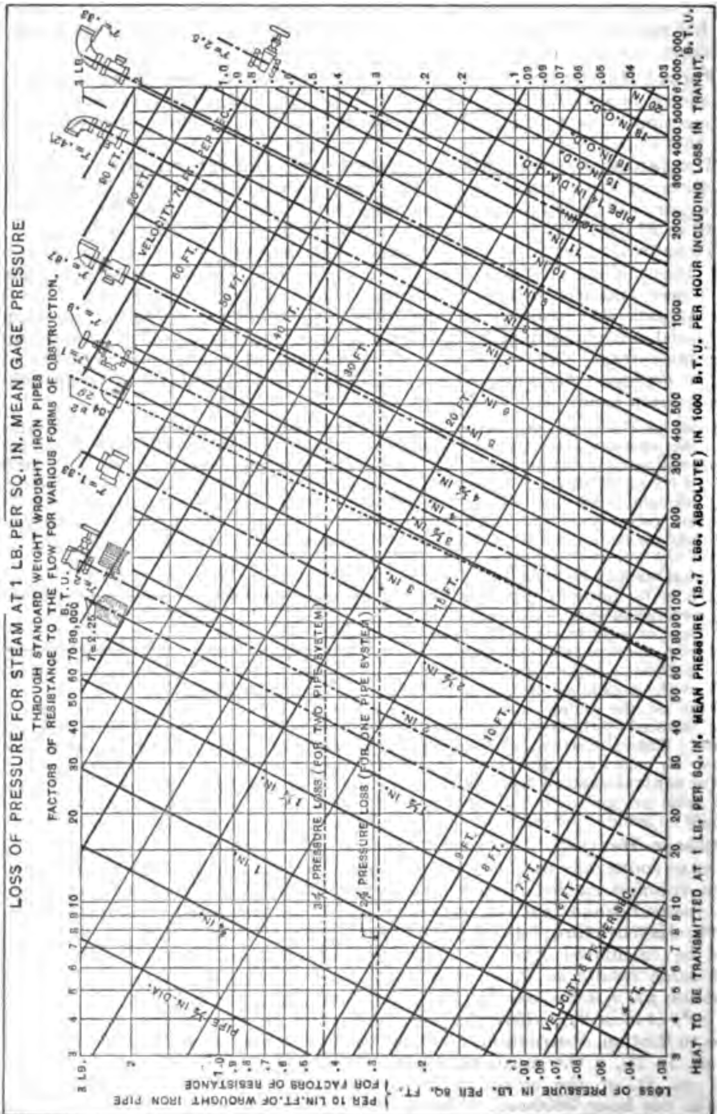


Fig. 8.—Chart for Steam Heating.
(Condensed from Meier's "Mechanics of Heating and Ventilation.")

equalized for all points of delivery by the resistance of piping. The returns may be sized for one-half the diameter of the steam pipes. When carrying steam, they should be made larger up to the trap.

Low-pressure Steam Distribution. The flow of steam in low-pressure systems is expressed by the same general formulæ given above for high pressure, except that different values are taken for the coefficients of friction and local resistances, or for the exponents of v and d, owing to the lesser density of the fluid. For pressures near the atmospheric, the friction and resistance losses in lb. per sq. ft. are $p_f = 0.0257\ wv^{1.97}l/(2g \times d^{1.16})$ and $p_r = 1.07\ wv^{1.97}r/2g$, where v = B.t.u. per hour/$3600waC$ and should represent the mean velocity of the steam.

The chart in Fig. 8 gives the **pressure losses for 1 lb. steam pressure.** For other pressures the velocities as well as the friction and resistance heads must be corrected according to the latent heat and the density. The resistances decrease about as the volume and velocity, and the heat carried for the same pressure drop increases as the square root of the density. For approximation the pipe sizes may be read from the lines of equal pressure loss, taking 0.288 lb. per sq. ft. per 10 lin. ft. for a rate of 2 per cent. and 0.432 lb. for a rate of 3 per cent. loss in 10 lin. ft. The former will serve for one-pipe, the latter for two-pipe systems.

In using Fig. 8 note that the ordinates are pressure losses measured in *lb. per sq. ft.* For a heat transmission of 240,000 B.t.u. per hour, follow vertically from the corresponding abscissa to the selected pipe size, which in this case may be assumed to be 4 in. The intersection is at a velocity of 19.5 ft. per sec. Corresponding to this velocity the pressure loss for a 10-ft. length of pipe is 0.2 lb. per sq. ft.; for a gate valve 0.06 lb. per sq. ft.; for an angle valve 0.21 lb. per sq. ft.; and for a globe valve 0.47 lb. per sq. ft.

In any system, equalize roughly the pressure losses from the boiler up to all points of delivery. In a **closed or sealed system** with wet returns, when the working pressure is generally higher than that needed for distribution, the equalization need not be close, but in dry-return systems the pressure losses should be equalized more closely in order to avoid short circuits and the flow of steam against condensation, which causes water hammer.

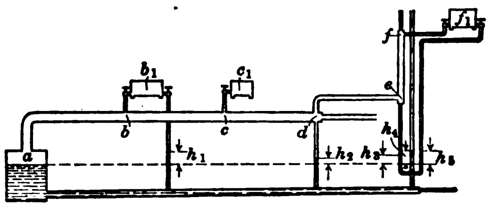


Fig. 9.—Steam Distribution with Wet Returns; with or without Vacuum on Air Lines; with Single or Double Piping to Individual Radiators.

With the **open-return system** the pressure drop in distribution must equal the working pressure, and the delivery is to be limited to the condensing capacity of the radiation so that no steam escapes to the atmosphere. The main advantage of an open system is the graduated control, and if that is to be realized, the equalization must be accurate. For this purpose liberal sizes of mains and relatively small branches closely calculated are desirable, even when final adjustment is made by valves.

For **rational calculation** all factors should be established by the use of a pipe schedule. The principle of equalization of pressure loss to the point of delivery remains the same for all systems, but the procedure varies. For a wet-return apparatus, as represented by Fig. 9, the pressure loss for the run $a - b - b_1$ should be made equal to that of $a - b - c - c_1$ and $a - b - c$

$- d - e - f - f_1$. The heights of the water columns correspond to the losses of head between the boiler and the various radiators and junctions. Those for radiators, h_1 and h_2, should be equal. The total pressure drop should be assumed no larger than necessary to equalize with reasonable pipe sizes for nearest and farthest radiator. It will range from 6 to 20 lb. per sq. ft., according to situation, for two-pipe systems, and from 4 to 12 lb. for one-pipe systems. For the latter, which generally do not require close equalization, allowance should be made for extra resistance on the lines carrying the whole condensation against the steam, by doubling the values of p_f and p_r as obtained from the charts.

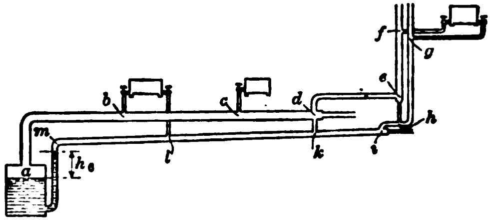


FIG. 10.—Steam Distribution with Dry Returns; with or without Vacuum on Returns or Air Lines; with Single or Double Piping to Risers and Radiators.

The resulting difference in the water line is $H = 0.192P$ in in. of water column. Thus, for a total pressure drop of 20 lb. per sq. ft. it would figure 3.84 in. To allow for increased steam volumes while reheating, the radiators or mains should be kept about four times higher above the water line in the boiler.

The losses of steam in transit are often a considerable factor. For a dry-turn apparatus the volumes carried are further increased by the condensation in the returns. The points of delivery for which to equalize, for instance, Fig. 10, would be l, k, i and h. If the pressure loss for $a - b - l - m$ is made to approximate $a - b - c - k - l - m$, no backing of steam against water, and no disturbance, will take place. Dry-return systems may be sized for the same total pressure drop as wet-return systems. The mains will figure somewhat larger when all factors are considered.

With the open-return system (Fig. 11), the working pressure or total loss is expressed by the water column H. It may be assumed to be from 5 to 10 lb. per sq. ft. The pressure drop of run $a - b - b_1 - b_2 - i - k$ should be made to equal that for $a - b - c - d - e - e_1 - e_2 - f - g - h - i - k$. In practice, the losses beyond radiators are negligible.

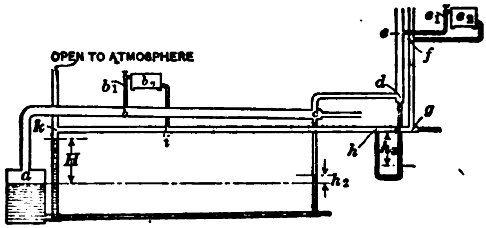
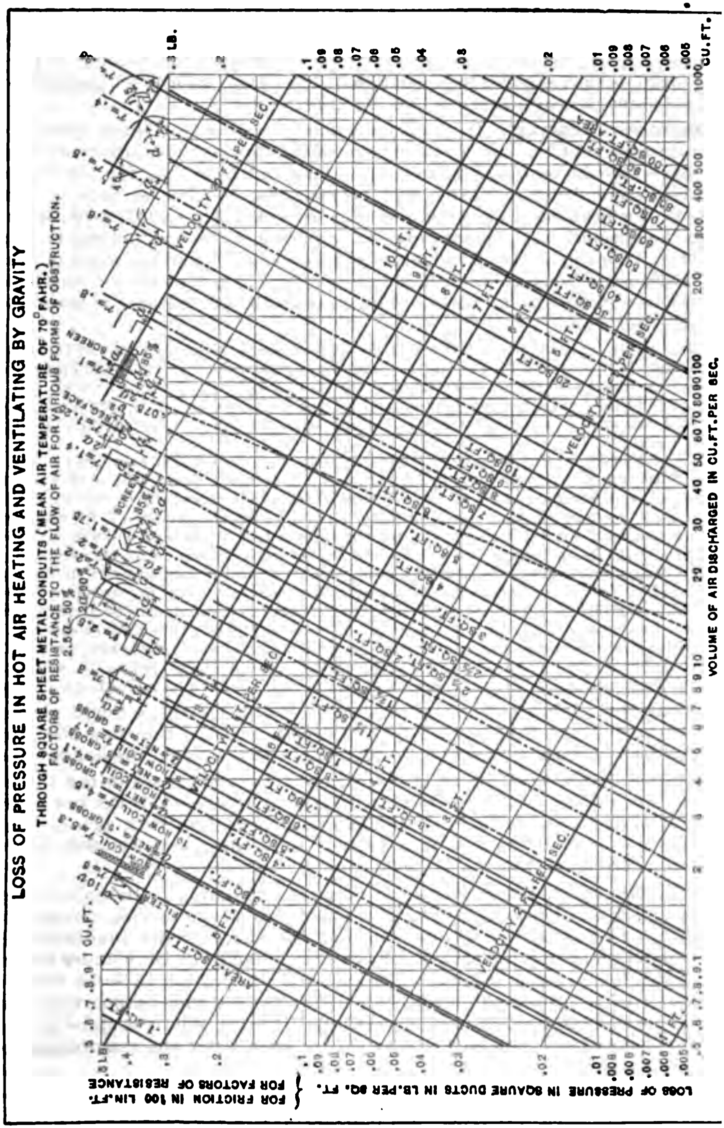


FIG. 11.—Steam Distribution with Open Returns, with Adjustable Valves Only.

The return pipes should be taken about one-half the diameter of the steam pipe, rounded up for smaller sizes and dry returns and rounded down for larger sizes and wet returns. Air lines are taken about one-fourth of the diameter of the steam pipe.

The application of **vacuum** to air lines through automatic valves will


LOSS OF PRESSURE IN HOT AIR HEATING AND VENTILATING BY GRAVITY
THROUGH SQUARE SHEET METAL CONDUITS (MEAN AIR TEMPERATURE OF 70° FAHR.)
FACTORS OF RESISTANCE TO THE FLOW OF AIR FOR VARIOUS FORMS OF OBSTRUCTION.
VOLUME OF AIR DISCHARGED IN CU.FT.PER SEC.
LOSS OF PRESSURE IN SQUARE DUCTS IN LB.PER SQ. FT.
FOR FRICTION IN 100 LIN.FT.
FOR FACTORS OF RESISTANCE

accelerate the removal of air during reheating, but will not affect distribution when the valves close under the action of steam. The pipe size should be determined, irrespective of vacuum devices, for greatest volume and at pressure drop permitting fair equalization. The same applies to vacuum on returns through thermostatic valves or traps.

Warm-air Heating. In gravity systems the ducts and flues should be calculated so that the sum $P_f + P_r + p_v = P$, the total pressure created by the difference in temperature between room and heat flue, for the height between mean level of heater and transmitting surfaces in rooms. The pressure for heat flues above 70 deg. fahr. $P = 0.075[1 - (530/T_f)]h$, where T_f is the absolute flue temperature. The friction heads for uniform runs of ducts and flues $p_f = 0.075 \times 0.00624(v^{1.9}/2g)l(c/a)^{1.18}$, and the losses of heat by local obstructions $p_r = 0.075 \times 1.25(v^{1.9}/2g)r$, as well as the velocity head p_v may be obtained from Fig. 12. The sums for the whole length of main and branch are P_f and P_r.

The pressure for a flue temperature of 135 deg. fahr. and a room of 70 deg., 20 ft. above the heater, is $P = 0.165$ lb. per sq. ft. The theoretical velocity corresponding to this pressure is $V = 12$ ft. per sec.; the approximate actual velocity resulting $= 0.4V = 4.8$ ft. per sec. This velocity may be assumed for the estimates and as a preliminary for the final calculation, rounding it off for systems with considerable obstruction, as with indirect stacks.

To find the size of ducts that will approximately use up the total pressure available while delivering the desired volume, Fig. 12 is used as follows: For a volume of 5 cu. ft. per sec. at, say, a velocity of 4 ft. per sec., the intersection shows an area of 1.25 sq. ft. The nearest commercial flue size is 12 × 16 in., or 1.33 sq. ft., giving 3.7 ft. velocity. The friction head for this size is given by the intersecting abscissa as 0.038 lb. per 100 ι. ft., or 0.019 lb. for 50 ft. The resistance p_r for the two registers, for the same velocity that of the flue, happens to be read at the same point and makes up 0.038 lb. per sq. for each. The elbow, at the intersection of the diagonal for $r = 0.4$, on the same veloc- r line, uses 0.007 lb. The resistance of a typical stack is estimated to be about equal that of an 8-row coil, in this case for 2 sq. ft. area = 0.03 lb. The casings should be .ken separately, entrance and exit being figured like flanged flue ends: $r = 1$, for which $p_r = 0.017$ lb. each. The above items make up the sum of 0.166 lb. per sq. ft., which is the total head available, the velocity head p_v being included in the losses for the registers.

Chimneys. For steam plants the gases should leave the boiler at not more than 400 deg. fahr., and for water boilers at not more than 350 deg. On this basis, which assumes ample fire surface in the boiler, the flue area in sq. in. $= A = E/C\sqrt{H}$, where H is the height from the grate to the top of the flue, ft., E is the total heat to be generated by the boiler, B.t.u. per hour, and C a coefficient varying from 400 for retarding factors, such as long connections, exposure to cooling, flat shape of flue, offsets, etc., to 600 where conditions are favorable.

Conduits. Split-tile conduits laid on good foundation, with underdrains below and pipe rollers inside, filled with mineral wool, or asbestos sponge, are durable and generally the best investment. Wooden sectional conduits with sheet-metal lining and pipe insulation are cheaper and will serve under favorable conditions, in dry ground, when installed with care. In some cases a concrete trench, properly drained, with slanting cover, pipes supported clear of the floor and insulated, is used successfully. A layer of broken stone should be provided under all conduits to take care of snow water and assist drainage in general.

Heating Guarantees. In heating installations the customary guarantee is for the maintenance of stated inside temperatures at stated lowest outside temperature when all rooms are heated continuously.

MECHANICS OF VENTILATING

The static pressure in rooms may differ from the atmospheric on account of wind pressure or suction from the outside, temperature conditions, and the ventilating equipment itself. Temperature differences between inside and outside create a suction and indraft at bottom and pressure causing outward leakage near the top, with a zone of atmospheric pressure, called the **neutral zone,** between. This zone is maintained at its natural height, the mean level of leakage, when air supply and exhaust are of equal capacity. An excess of supply will depress the neutral zone, increase the area of outward leakage and put the room under a pressure, which reduces the head available for supply. An excess of exhaust will raise the zone and increase the head for the air supply.

When it is desired to maintain a plenum or excess pressure in a room, for the purpose of checking indraft through doors, allowance must be made for delivery against back pressure. This can be calculated approximately from the height of the room and the temperature difference, as for gravity vents. (See p. 1361.) Taking for example a hall of 30 ft. in clear height, in which the zone of inward leakage should be depressed below the doors, at floor level, the natural height of the zone, being at half height, must be brought down by 15 ft. For a temperature difference of $t_r - t_o = 70$ deg. fahr. this will equal a pressure of 0.17 lb. per sq. ft., which the supply fan is to meet. The capacity of the exhaust system is to be reduced correspondingly by the assumption of greater height of draft or greater pressure difference available.

An air-supply system depending on outward leakage and exhaust depending on inward leakage cannot be relied upon to give the desired volumes unless the excess pressure is made sufficient to overcome the resistance of the structure to leakage, which is most variable and beyond calculation.

When a room is to be kept under suction to localize odors, free ingress must be assured from a neutral source not affected by weather.

In tall buildings, to assure proper action of ventilating equipment, whether gravity or mechanical, it is essential to stop intercommunication between different stories, since the pressure disturbances affecting the delivery may be considerable and change constantly with atmospheric conditions.

Forced Ventilation. A system of conduits is calculated for even pressure loss for all individual outlets while delivering the desired volumes. The run and sizes of certain parts of conduits being determined by various considerations, the pressure losses of a system can be equalized by calculating friction and local resistances in these fixed portions, such as main ducts, flues and outlets, and proportioning the variable sections, usually the connections between ducts and vertical flues, to make up the differences of pressure to the individual outlets.

For air-supply registers, when air is being delivered at about room temperature, the velocity should not exceed 2 ft. per sec. when the outlet is below or near the headline and sensible drafts must be avoided. When the registers are at least 8 ft. above the floor, velocities from 4 to 10 ft. may be allowed, according to the distance from the opposite wall or the space to be covered. The velocities through ceiling registers should be from 2 to 4 ft., according to height. These velocities are based on the gross area as determining the current of air in front of the register. As a general rule, colder air should be introduced more slowly, while warmer air may be brought in at higher speed. For exhaust, the velocities given above for bottom and top registers may be doubled, if resistances allow, without causing objectionable draft.

The velocities in vertical flues should average about twice the velocity given for the registers, except for extremes. The ducts and other parts of the system, such as intakes, discharge flues and air passages in heaters, are determined largely to suit the total pressure or power, while the branch ducts and connections between ducts and flues are graded to effect equalization of resistances.

For **calculation of friction** and local resistances in the customary square **sheet-metal conduits,** the chart in Fig. 13 may be used. This chart is

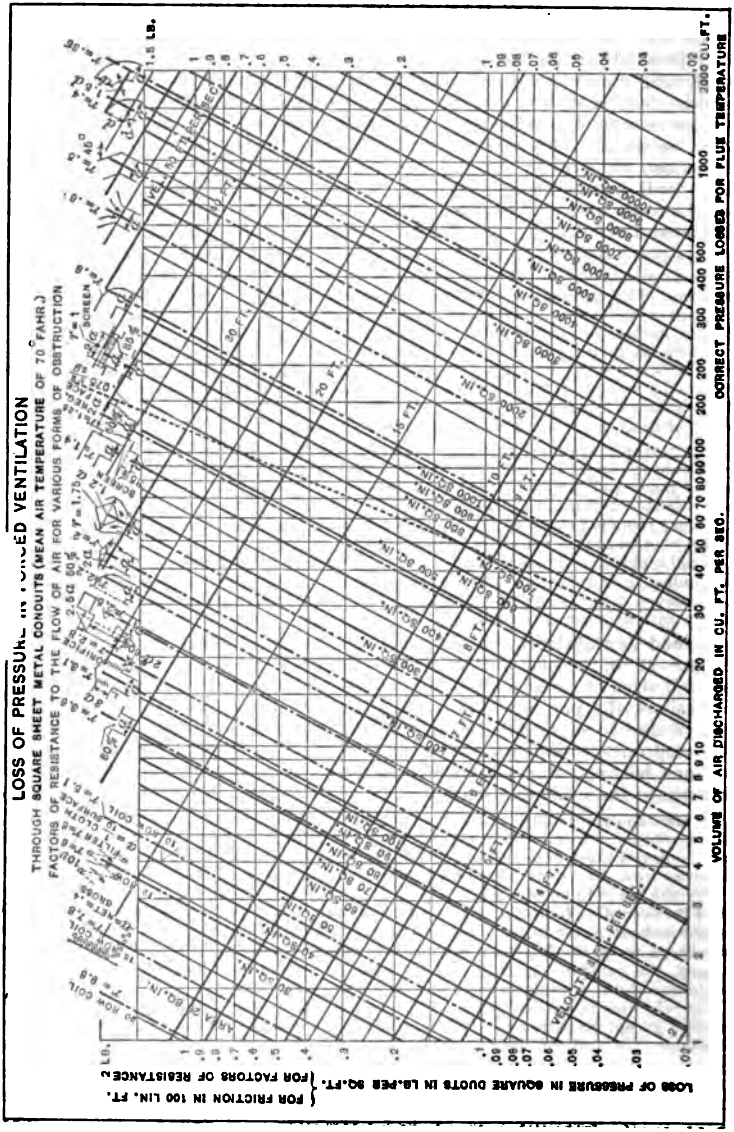


Fig. 13.—Chart for Forced Ventilation.
(Condensed from Meier's "Mechanics of Heating and Ventilation.")

based on the formula for friction loss $p_f = wfv^{1.9}l(c/a)^{1.18}/2g$, and that for local resistances $p_r = 1.25\ wrv^{1.9}/2g$ (approximation), where $w = 0.075$ for air at 70 deg. fahr., coefficient of friction $f = 0.00624$, c, a and l the circumference, area and length in ft., and r the factor of resistance, expressed in velocity heads, v = velocity in ft. per sec. and p_f and p_r are expressed in lb. per sq. ft.

The true velocity head of discharge $p_v = 0.075\ v^2/2g$, as an item in the sum for the whole length of main and branch, $P_f + P_r + p_v = P$, is included in the factors for typical outlets given, but may be read separately for any velocity by means of the dotted line. For round pipes of equal area, multiply values of p_f by 0.87. For flat ducts of equal area, multiply p_f by 1.07, 1.18, 1.30 or 1.43, respectively, according as the duct sectional dimensions are in the proportions 1:2, 1:3, 1:4 or 1:5.

The total resistance of the distributing system, heating coils, intakes, filters and other apparatus gives the total **pressure to be produced by the fan,** which determines its speed and blast area. The theoretical velocity $V = \sqrt{2gP/0.075}$, the blast area $A = Q/V$, Q being the cu. ft. per sec., and the theoretical power $= QP/550$. The actual power depends on the efficiency of the fan. It averages twice the theoretical power.

As an example in the use of the chart, Fig. 13, if 100 cu.ft. of air are to be discharged per sec. through a square duct of 1000 sq. in. area, the velocity will be 14.4 ft. per sec., the loss of pressure 0.2 lb. per sq. ft. per 100 linear feet of duct, and the resistance of an elbow $1.5d$ = 0.085 lb. The velocity head (given by the diagonal dotted line) p_v = 0.25 lb.

To calculate systematically, it is advisable to work out a duct schedule from the scale plans, showing all runs, volumes to be carried with allowances for leakage and flue sizes as predetermined by brick dimensions, speed at outlet desired and space. The main ducts may then be sized for an even rate of pressure loss of say, 0.4 lb. per sq. ft. per 100 linear ft. of duct, or 0.5 lb. for higher speeds and shorter runs, as experience will show. The resistances by friction and obstructions may then be figured from the outlets or registers backward to the first junction and from there with increasing volumes toward the fan. If the total for two branches is made equal for the proper volume by variation of sizes on connections, the equalized total at junctions will give the actual pressures that will obtain at these points. The resistances along the mains are then figured likewise, from junction to junction, up to the fan, giving the pressure for which to equalize other branches. When equalization becomes difficult, owing to excessive speeds in certain parts, it is advisable to ease the total losses by enlargement of mains, using a lower rate of resistance.

For **exhaust systems** the procedure is the same, except that the suction is to be equalized for all branches, and the resistance of discharge represents the back pressure to be added to the suction in order to obtain the total to be created.

Gravity Vents. The total pressure available for a gravity vent is $P = 0.075[(530/T_o) - 1]h$, for the density of the air at 70 deg. fahr. in the room and the flue, T_o being the absolute outside temperature and h the height of the flue, measured from the neutral zone in the room to the top of the flue. When the flue temperature is higher than 70 deg., the pressures should be corrected by multiplying P by $T_f/530$.

When several flues are joined into a main vent duct, the pressure losses in the several branches should be equalized for the desired volumes. If the heights of draft vary, the resistances must be made to conform to the respective heads, but any such differences in head must be made up in the branches, so that the least pressure given by any flue still leaves a margin for the resistances in the main duct. In other respects the calculation is carried through on the same principles as for mechanical ventilation and gravity warm-air heating, using the chart, Fig. 12, for the latter.

AIR CONDITIONING

BY

W. H. CARRIER

REFERENCES: Carrier, "Rational Psychrometric Formulæ" and "Air Conditioning Apparatus," *Trans. A. S. M. E.*, vol. 33. Lyle, "Atmospheric Cooling and Dehumidifying," *Trans. Am. Soc. Refrig. Engrs.*, 1912.

(For properties of moist air, dewpoint, wet-bulb readings, etc., see pp. 338–341.)

Air conditioning may be divided into three different operations, air washing, humidifying and dehumidifying.

Air Washing. Air washers contain sprays and eliminators. The air is drawn through the mist in the spray chamber, and then through the eliminator where an additional washing effect is obtained. The latter may or may not be independently flooded. The efficiency of an air washer will vary from 60 to 70 per cent. with fine dirt up to 98 per cent. with coarse dirt. Some **cooling** may be obtained with an air washer, even when the spray water is recirculated. In a well-designed apparatus this will vary from 70 to 80 per cent. of the wet-bulb depression. **Air velocities** of from 450 to 550 ft. per min. through the washer are used. The resistance will vary from 0.20 to 0.50 in. water gage, according to the velocity of the air and the type of washer. The water pressure on the sprays should be from 15 to 25 lb. per sq. in., depending on the nature of the impurities to be removed and on the design of the nozzles. From 3 to 3.5 gal. of water per 1000 cu. ft. of air will ordinarily be required.

Humidifying. Humidifiers may be divided into the following general types according to their method of operation: Direct (spraying into the room); indirect (introduction of moistened air); and combined (direct and indirect). **Indirect humidifiers** are similar in operation to the spray-type air washer, except that the water is sprayed directly against the incoming air. During cool weather the dewpoint or saturation temperature at the apparatus is secured and controlled artificially. During warm weather, it is often impossible to obtain as low a dewpoint as desired without refrigeration, which in the majority of cases is impracticable. The lowest saturation temperature that can be obtained is the same as the outside wet-bulb temperature. Therefore, the dewpoint in the room will always be the same as the outside wet-bulb temperature. The room temperatures corresponding to certain percentages of relative humidity are given in Table 1.

Table 1. Room Temperatures (Deg. Fahr.) at Various Outside Wet-bulb Temperatures and Various Percentages of Humidity in the Room

Outside wet-bulb temperature, deg. fahr.	Percentage of humidity in room					
	55	60	65	70	75	80
50	67.0	64.3	62.0	59.8	57.7	56.0
55	72.0	69.6	67.5	65.2	63.1	61.1
60	77.5	74.9	72.5	70.4	68.3	66.3
65	82.8	80.2	77.6	75.6	73.5	71.6
70	87.2	85.6	83.1	80.0	78.7	76.7
75	93.7	90.8	87.3	85.9	83.9	81.9
80	99.1	96.2	93.4	91.2	89.0	87.0

Table 2. Dewpoint Temperatures (Deg. Fahr.) for Various Percentages of Humidity and Room Temperatures

Room temperature, deg. fahr.	Percentage of relative humidity										
	85	80	75	70	65	60	55	50	45	40	35
65	60.5	58.8	56.75	54.75	52.8	50.7	48.3	45.8	43.0	40.0	36.75
70	65.4	63.5	61.6	59.6	57.5	55.3	53.0	50.5	47.5	44.5	41.0
75	70.1	68.25	66.3	64.3	62.25	60.0	57.75	55.25	52.75	49.0	45.5
80	74.8	73.2	71.2	69.25	67.2	64.8	62.3	59.75	56.75	53.5	49.75
85	79.75	78.0	76.2	74.1	71.75	69.4	66.9	64.25	61.25	58.0	54.2
90	84.75	83.0	80.9	78.8	76.7	74.2	71.6	68.75	65.75	68.55	59.0
95	89.7	87.8	85.75	83.6	81.3	78.8	76.25	73.35	70.3	67.0	63.2

In the majority of industrial applications the problem during warm weather is as much a question of cooling as of humidifying, and in the moist-air system one is dependent on the other. The air required per B.t.u. of cooling effect is given in Table 3. The **degree of saturation of the air** leaving the humidifier depends upon the intimacy of contact of the air and water, upon the relation of the water temperature to the wet-bulb temperature of the entering

Table 3. Cooling Capacity of Carrier Humidifying System

Per cent. humidity in room.....	50	55	60	65	70	75	80
Diff. bet. dewpoint and room temp., deg. fahr..............	20.3	17.7	15.2	12.8	10.8	8.8	6.8
Cu. ft. air (at 70 deg. fahr.) required per B.t.u. cooling effect..	2.71	3.11	3.63	4.31	5.10	6.27	8.11

air and upon the length of the spray chamber and the air velocity. The smaller the nozzle orifice the greater will be the air and water contact, and the greater the degree of saturation attained. It is customary to use smaller nozzles more closely spaced than in an air washer, with a water pressure of about 35 lb. Higher air velocities are also used than through the air washer, 600 to 700 ft. per min. being the ordinary range. The resistance at these velocities will vary from 0.35 to 0.45 in. water gage.

Methods of Humidity Control. The various methods of controlling the humidity in a room or building may be classed as follows: (1) **Humidistat**—operated by some hygroscopic material. (2) **Wet-and-dry-bulb hygrostat,** either of the **differential or vapor pressure type,** consisting of two members, one exposed to the wet-bulb and one to the dry-bulb temperature of the air, and maintaining a constant relative humidity in the room. (3) **Dewpoint or Differential Thermostat.** The dewpoint thermostat gives a constant dewpoint at the apparatus, and consequently a constant moisture content in the air of the room. A second thermostat may be arranged to maintain a constant temperature in the room. The differential thermostat is used when it is impracticable to maintain either a constant dewpoint or a constant room temperature, controlling either one with respect to the other. Table 4 gives the heat required for tempering and humidifying air at various wet-bulb and dewpoint temperatures.

Dehumidifying. In many industrial establishments a comparatively low moisture content is required in the room, or at least lower than exists in the outdoor air. This reduction in moisture content may be obtained by reducing the dewpoint of the incoming air and condensing the excess moisture in either one of three ways: (1) By means of some chemical process, by bringing the air into contact with calcium chloride or with concentrated sulphuric acid —only practical where small quantities of air are to be treated. (2) By

Table 4. Heat (in B.T.U.) Required to Condition 1000 Cu. Ft. of Air (Measured at 70 Deg. Fahr.) from Various Entering Wet-bulb Temperatures to Various Dewpoint Temperatures

Wet-bulb temperature of entering air, deg. fahr.	Relative humidity, per cent. at 70 deg. fahr. (and dewpoint, deg. fahr.)					
	30% (37.25)	40% (44.5)	50% (50.5)	60% (55.3)	70% (59.6)	80% (63.5)
−10	1194	1452	1653	1860	2044	2245
0	984	1246	1447	1663	1840	2039
10	750	1025	1228	1445	1621	1822
20	510	779	983	1200	1377	1581
30	300	496	700	920	1097	1300
40	....	178	384	603	783	987
50	....	....	....	220	394	619
60	....	....	....	....	....	181

employing surface condensation, the air being passed over brine or direct-expansion ammonia pipe coils or other means of refrigeration. (3) By condensation with a cold spray and the elimination of any free or entrained moisture. Either artificially cooled water or brine or cold well water is used.

Surface Required for Surface Condensation. The rate of heat transfer in cooling air where condensation occurs is variable and depends upon the proportion of latent to sensible heat removed from the air. Providing the mean temperature of the water in the cooling coils is maintained constant, the ratio of initial to final temperature difference between the cooling water and the air is substantially the same whether there is moisture condensed from the air or not, so that the following approximate formula (for transverse flow of air) may be used:

$$\log_{10}[(t_w - t_1)/(t_w - t_2)] = S/(0.1119Q + 127A)$$

where t_1 = temperature of the entering air; t_2 = temperature of the leaving air; t_w = mean temperature of the water (all in deg. fahr.); S = sq. ft. of condensing surface; A = clear area in sq. ft. through coils; Q = cu. ft. of air per min. The amount of heat removed is equal to the difference between the total latent and sensible heat in the air at temperatures t_1 and t_2, respectively. It should be noted that the air leaving a surface dehumidifier is not necessarily saturated. The weight of water per lb. of air leaving the dehumidifier may be determined approximately from the equation

$$W_2 = W_1 - [(W_1 - W_m)(t_1 - t_2)/(t_1 - t_w)]$$

where W_1 = weight of water per lb. of air entering dehumidifier; W_2 = weight of water per lb. of air leaving dehumidifier; W_m = weight of water per lb. of air contained in air saturated at a temperature corresponding to the mean water temperature in the cooling coils.

The **spray-type dehumidifier** is similar in design to the spray-type humidifier, except that two banks of nozzles are ordinarily used to each stage, spraying against the incoming air, with a water pressure of from 25 to 30 lb. In a properly designed apparatus of this type the temperature of the leaving air will be not more than 1 deg. above the final water temperature. The air will always become saturated, and thorough elimination is required to remove entrained moisture. The **water required** for a one-stage-type dehumidifier may be determined by means of the formula

$$W = N[(H_1 - H_2)/(t_w - t_2)] = N[(t'_1 - t_2)/(t_w - t_2)]h,$$

where W = weight of water, lb.; N = weight of air, lb.; t_w = initial water temperature; t'_1 = initial wet-bulb temperature of air; t_2 = final dewpoint temperature of air (all in deg. fahr.); H_1 = initial total heat in 1 lb. of air at wet-bulb temperature, t'; H_2 = total heat in 1 lb. of air at final dewpoint t_2; h = approximately the average total change in latent and sensible heat per degree change in temperature in saturated air as determined from psychrometric charts or tables. The **refrigeration required** for spray-type dehumidifiers is given in Table 5.

Table 5. . B.T.U. Refrigeration Required to Cool 1000 Cu. Ft. of Air (Measured at 70 Deg. Fahr.) from a Given Wet-bulb Temperature to a Given Dewpoint

Leaving dewpoint, deg. fahr.	Entering wet-bulb temperature, deg. fahr.							
	50	55	60	65	70	75	80	85
65	....	....	....	....	296	606	961	1350
60	....	....	....	259.0	553	865	1220	1609
55	....	....	221.5	480.5	777	1086	1440	1840
50	....	203	425.0	683.0	980	1290	1570	2030
45	185	388	611.0	869.0	1165	1474	1830	2220
40	359	569	791.0	1050.0	1345	1656	2010	2400

ILLUMINATION

BY

LOUIS BELL

REFERENCES: Barrows, "Light, Photometry and Illumination," McGraw-Hill. Bell, "The Art of Illumination," McGraw-Hill. Bloch, "Beleuchtungstechnik," Springer. Cravath and Lansingh, "Practical Illumination," McGraw-Hill. Gaster and Dow, "Modern Illuminants and Illuminating Engineering," Macmillan. Högner, "Lichtstrahlung und Beleuchtung," Vieweg u. Sohn. "Ill. Eng. Soc. Lectures on Illuminating Engineering," Johns Hopkins Press. Trotter, "Illumination, Its Distribution and Measurement," Macmillan. Wickenden, "Illumination and Photometry," McGraw-Hill.

Intensity of Light

The Physical Basis of Light. The sensation of light is a function of the quantity of luminous energy which produces it, and of the coefficient of sensibility of the eye for the particular kind or kinds of luminous energy concerned. The limits of the electromagnetic wave lengths sensible to the eye lie practically between 800 $\mu\mu$ and 400 $\mu\mu$,* differing slightly for different eyes and different conditions of the same eye.

The common sources of light are hot bodies, whether minutely divided as in particles which glow in a flame, or continuous solids as in filaments of incandescent lamps or the mantle of a gas burner. Certain established laws define, to a close approximation at least, the relation between the temperature of a hot body and the radiant energy sent out by it. The first is Stefan's law (see p. 309). This law, however, does not tell anything about the relation of the luminous energy to the total energy.

A second relation connects the temperature of a hot body with the amount of radiant energy of each particular wave length delivered by it. This relation, known as Planck's law, is: $E_l = Cl^{-5}/(e^{k/lT} - 1)$, where E_l is the intensity of radiation at the wave length l, e is the Napierian logarithmic base, and C and k are experimental constants. It is also found that the product of a given temperature by the wave length of maximum energy is a constant; and that this constant is approximately 2940 when the wave length is taken in thousandths of a millimeter. The practical significance of this fact is that a hot body must attain a temperature of at least 5000 deg. cent. absolute before the maximum energy radiated falls in the middle of the region of luminous energy already referred to. For gaseous substances in which luminous radiation may be excited by electric discharges, Planck's law does not in general hold.

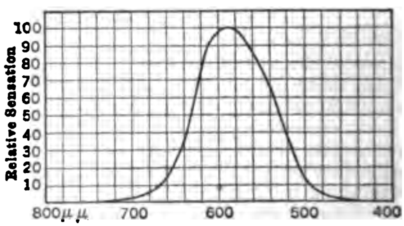


FIG. 1.—Typical Luminosity Curve of the Human Eye.

Physiological Relations of Luminous Energy. The sensibility of the eye for stimuli of various wave lengths varies with the intensity of the luminous radiations. In daylight or brilliant artificial light the eye is most sensitive to the wave lengths near 560 $\mu\mu$, that is, to those which correspond to a yellowish-green in color; in twilight or in faint artificial light the maximum sensibility of the eye lies in the full green near

* 1 $\mu\mu$ = 1 micromillimeter = 1/1 000 000 000 of a meter.

wave length 510$\mu\mu$, and with varying intensities of light shifts between these limits. Fig. 1 shows the typical "luminosity curve" of the human eye. The ordinates show the relative amount of luminous sensation derived in daylight from a given amount of luminous energy at each of the wave lengths concerned. The curve indicates that in the full red (650 $\mu\mu$) and in the blue (490 $\mu\mu$) a given amount of energy produces less than one-fifth the sensation that it would produce if it were in the yellowish-green. This shows how difficult it is to get bright lights of strong color. The eye sees best by nearly monochromatic light of a wave length corresponding approximately to the maximum of the sensibility curve.

Two characteristics of the eye are important in considering illumination. The first of these is its power of shade perception. Objects are perceived chiefly by their difference in luminosity, although nearby help is obtained from binocular vision, and, to a less extent than is generally supposed, from color differences; but the main thing in vision is difference of luminosity. The visual picture, according to Helmholtz, is about equally distinct, whatever the absolute brightness may be, provided the relative brightness of the parts remains the same. The eye sees almost equally well in the bright sunshine of noon and at sunset, although the absolute intensity of the light observed is changed in the ratio of more than 100 to 1. For ordinary eyes, and illumination as bright as diffused daylight or bright artificial light, the proportional difference for shade perception is about 0.01; that is, two adjacent surfaces can be distinguished as separate if one reflects to the eye about 1 per cent. more light than the other. In very dull light this fraction, generally known as Fechner's, increases, and may be several tenths, as, for example, in moonlight. One of the determining facts in artificial illumination is the quantity necessary to bring the shade perception of the eye to its normal value.

The second matter regarding vision that is important in practical illumination, is visual acuity; that is, the power of perceiving as distinct, closely placed objects which present no difficulty from mere lack of contrast. Acuity varies very widely in different eyes, but, like shade perception, it remains nearly uniform over a very wide range of illumination, beginning to fail only in the same dim light that causes shade perception to fail. Fig. 2 gives practically the relation between visual acuity, shade perception, and illumination for the human eye, as determined for the former factor by Dr. Uhthoff, and for the latter by Drs. Koenig and Brodhun. The lower shade-perception curve, *a*, is for white light; *b* is for crimson light. The abscissæ are expressed in meter-candles, the meter-candle being merely the light given by a standard candle at a distance of 1 meter. Both the shade-perception curves and the acuity curves reach practically steady values at a light intensity of from 20 to 30 meter-candles.

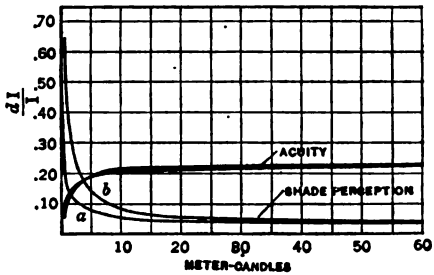


FIG. 2.—Relations Between Visual Acuity, Shade Perception and Illumination.

These curves show the approximate amount of light which must be supplied and need not be greatly exceeded. These values of the illumination, however, refer to the observation of white or very light colored surfaces. If the surfaces are dark so that they reflect comparatively little light to the eye, the illumination required for good seeing must be proportionately increased.

Photometric Units

Primary Standards of Light. Some of the more important standards which have been used in the past are as follows: The **British candle**, which loses in burning 120 grains per hour; the **French Carcel lamp**, an Argand burner of specified dimensions which gives the standard brightness when consuming 42 grams of colza oil per hour; the **amyl-acetate lamp**

designed by **von Hefner-Alteneck,** giving the standard brightness when burned at its normal flame height of 40 mm.; the **Vernon-Harcourt pentane lamp,** an Argand gas burner fed by air saturated with pentane vapor and used extensively in England; and the **bougie decimale,** or 1/20 of the light given off normally by 1 sq. cm. of surface of platinum at its point of solidification. In 1909 the national laboratories of the United States, England and France agreed upon a common unit called the **international candle,** which has since been adopted by several other countries. This unit has the value of the bougie decimale. The relations between these primary standards are shown in Table 1.

Table 1. Relations Between the Primary Standards of Light

Standard	International candle	Carcel	Hefner	British candle
International candle....	1.0	0.104	1.11	0.98
Carcel..................	9.6	1.0	10.7	9.4
Hefner..................	0.9	0.0935	1.0	0.88
British candle..........	1.02	0.106	1.14	1.0

In actual practice incandescent electric lamps properly seasoned and compared are the usual custodians of this standard for laboratory use. In gas lighting, however, a flame standard is often desirable as tending to eliminate the variations due to moisture, CO_2 and barometric pressure, and for this purpose both gas lamps and kerosene lamps—of which a definite area of flame is exposed—are in common and successful use.

Photometric Units. Table 2 gives the practical photometric units in fairly well-established use. The unit of luminous flux, or the **lumen,** is the flux from a source of unit intensity through one unit solid angle. In the sphere there are 4 π such angles, so that 1 mean spherical international candle is a source of 4 π lumens. Any surface subtending one unit solid angle from such a source receives a total flux of one lumen. The practical unit of illumination is the **lux,** which is that illumination received from unit source at 1 meter distance. Hence the total luminous flux in lumens received over a given area, divided by that area in square meters, gives the illumination in lux.

Table 2. Practical Photometric Units

Magnitude	Unit	Symbol*
Intensity....................................	Int. candle.........	I (J)
Luminous flux................................	Lumen..............	F (Φ)
Illumination.................................	Lux (foot-candle)..	E (E)
Intrinsic brightness.........................	Candles/sq. cm. (candles/sq. in.)	b (e)

* Symbols in parentheses are those used in connection with the Hefner unit by German writers.

The defining equation for illumination is $E = F/S = I/R^2$, where S is surface in square meters and R is distance in meters. In English-speaking countries many measurements of illumination are expressed in the **foot-candle** unit, this being the illumination received from an international candle at 1 ft. distance. The flux in lumens is reduced to illumination in foot-candles by taking the area concerned in square feet. The intensity of a source is known as intrinsic brightness, and is measured in candles per square centimeter, or per square inch. The **intrinsic brightness** customarily denotes the total intensity of a source in any direction divided by its apparent area when viewed from that direction. The lux and the foot-candle are connected by the following equations: 1 lux = 0.0929 foot-candle; 1 foot-candle = 10.76 lux

COMPUTATION OF ILLUMINATION

In computing illumination, two procedures are open: first, the determination by the flux-of-light method; second, the determination of the illumination at a point or points from the intensity of the source and from the distance. The first is extremely convenient in dealing with conditions by general average, and also in determining the effect of secondary sources of light, like brilliantly lighted surfaces. The **flux-of-light method**, for example, gives very readily the average illumination over any enclosing surface about the source considered. It does not, however, readily yield directly the illumination on a particular working plane. To facilitate the computation of this case, it is necessary either to determine the solid angle or angles involved, or to proceed by a process of averaging. The latter plan is usually adopted in dealing with ordinary illumination problems, the former being reserved for the illumination produced by secondary sources. In working out the illumination by the flux-of-light method, dependence is usually placed on experimental coefficients determined for different kinds of lighting installations. For example, with a working plane 10 m. square and light sources giving 1000 lumens, it is expedient in determining the illumination to fall back on the **coefficient of utilization** of the particular sort of installation adopted. This coefficient is the proportion of the total lumens falling on the plane in question. For example, it is found that the coefficient of utilization in using incandescent lamps with certain diffusing glass reflectors is 0.5. This means that of 1000 lumens delivered by the sources assumed, 500 reach the working plane. This being 10 m. square, the mean illumination on it is therefore $1000 \times 0.5/10^2 = 5$ lux. The formal equation is $E = FK/S$, wherein K is the coefficient of utilization of the installation and S the surface of the plane with reference to which K is assumed. The method is a very quick and easy one for approximate average results. Transformed to the form $F = ES/K$, it serves to give the necessary flux for a required illumination at the given working plane. Table 6 gives values of K found in practice.

The second method, of **computing illumination from the distance and intensity of the source**, is readily applied to points or small areas, but is less easy to use for quick approximations over considerable areas than is the flux-of-light method. The simple radiation from a luminous point follows the law of inverse squares, $E = I/l^2$, the element of surface at which E is reckoned being normal to the ray. The criteria for the applicability of the law of inverse squares are (1) that the angle subtended by the dimensions of the luminous area as seen from the point under consideration shall be small, and (2) that the luminous radiation shall be free and not complicated by reflections or refractions, as, for example, by mirror surfaces or lenses. When the light is received on a surface inclined to the ray, the illumination is proportional to the cosine of the angle of incidence. (**Lambert's Law.**)

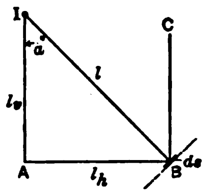


FIG. 3.

In Fig. 3 let a source of light I be located at a distance l_v above the plane of reference AB, and consider, at a distance l_h along this plane, an element of surface ds normal to a ray received from I over a distance l. On ds, $E = I/l^2$, but if ds lies in the plane AB, the illumination is reduced by the obliquity to $E = (I/l^2) \cos a$. If ds is projected on a vertical plane BC, the illumination becomes $E = (I/l^2) \sin a$. Sometimes it is convenient to compute the illumination by starting with the height of the source l_v and obtaining the illumination for various values of a. Noting that $l = l_v/\cos a$, the formula for the normal illumination E_n becomes

$$E_n = (I/l_v^2) \cos^2 a$$

For illumination on the horizontal plane AB,

$$E_h = (I/l_v^2) \cos^3 a$$

and for the vertical projection on the plane BC the expression becomes

$$E_v = (I/l_v^2) \sin a \cos^2 a$$

When artificial light is derived from a number of sources at varying distances and at the same height, the general summation for the normal illumination received at a point is

$$E_n = I_1(1/l_1^2) + I_2(1/l_2^2) + \ldots\ldots I_n(1/l_n^2)$$

or, for equal sources,

$$E_n = I[(1/l_1^2) + (1/l_2^2) + \ldots\ldots (1/l_n^2)]$$

and for equal lights carried at a uniform height,

$$E_n = (I/l_v^2)(\cos^2 a_1 + \cos^2 a_2 + \ldots\ldots \cos^2 a_n)$$

where a_1, a_2, etc., are measured from the various sources.

Inasmuch as an element of surface cannot be illuminated normally by a number of light sources, Lambert's law must be taken into consideration in a general case, which, for lights of uniform height and intensity, becomes

$$E = (I/l_v^2)(\cos^3 a_1 + \cos^3 a_2 + \ldots\ldots \cos^3 a_n)$$

Usually $\cos^3 a$ is negligible for a part of the sources or several nearly equally distant sources can be lumped together; or some sources are cut off by obstacles so that the summation need seldom include more than three or four terms. The average value of I in any of these formulæ must be taken from the polar distribution curve of the light as installed.

In lighting indoor spaces, the light received at any given point will be a certain amount, E_0, derived directly from the radiant object according to the formulæ already given, plus a certain other amount, E_r, reaching it after one or more reflections from the walls. The effect of these reflections cannot be calculated, since they come from useless as well as useful directions, and the law of inverse squares does not hold for large and near surfaces like those of walls. It is necessary to evaluate E_r by an indirect method.

Every surface has a **coefficient of reflection**, k, giving the proportion of the incident light reflected. This reflection may be specular, as from a mirror, diffuse, as from a piece of white blotting paper, or mixed, as from a glossy painted surface, in which the reflected image of a light source can be plainly seen, while yet there is a considerable amount of diffuse reflection. For the purposes of computation one need not discriminate between these kinds of reflections, and the first approximation to the effective illumination as reinforced by reflection, is $E = E_0 [1/(1 - k)]$, where k is the average coefficient of reflection of the surfaces concerned. Large values of k indicate considerable assistance from reflection. Table 3 gives the reflection coefficients from a number of typical surfaces.

The variation in k is very great, dark-colored papers and paints reflecting very little light, and very few wall surfaces of any kind giving k above 0.4 to 0.5. The majority of light painted or papered walls have k within these limits. Light creams, buffs and greens give the highest coefficients. A very slight admixture of black or red in a painted surface, resulting in an apparently light and warm tone, will reduce the reflecting power very materially.

Table 3. Reflection Coefficients

Material	k	Material	k
Polished silver	0.92–0.95	Yellow cardboard	0.30
Mirror silvered on back	0.82–0.88	Light blue cardboard	0.25
White blotting paper	0.82	Brown cardboard	0.20
White cartridge paper	0.80	Yellow painted wall (dirty)	0.20
Polished brass	0.70–0.75	Emerald green paper	0.18
Mirror backed with amalgam.	0.70	Dark brown paper	0.13
Ordinary foolscap paper	0.70	Vermilion paper	0.12
Chrome yellow paper	0.62	Bluish-green paper	0.12
Orange paper	0.50	Cobalt blue paper	0.12
Yellow wall paper	0.40	Black paper	0.05
Yellow painted wall	0.40	Ultramarine blue paper	0.035
Light pink paper	0.36	Black velvet	0.004

The floors and window spaces in rooms are practically negligible as aids to illumination. Unless special effort has been made to get light surfaces, the quantity $1/(1 - k)$ rarely exceeds 1.5. With dark walls, the aid received is negligible, so that the amount of light required to illuminate a room with dark walls may easily exceed that required for the same room with light walls by 50 per cent. or more.

PRACTICAL SOURCES OF LIGHT

Flames. The flame sources commonly used are candles, kerosene, and gas flames from ordinary illuminating gas, air-gas and acetylene. Ordinary commercial **candles** are made of stearin, paraffin or wax, or mixtures of these substances, give a little more than 1 international candle power (c.p.), and usually burn from 110 to 130 gr. per hour. The cost per candle-hour is from ¼ to ½ of a cent; specific consumption of energy, about 90 watts per c.p.

Ordinary **kerosene lamps** give from 10 to 30 c.p., according to size, on a consumption of from 50 to 60 gr. per hour per c.p., or about 800 candle-hours per gal.; cost per candle-hour, ordinarily not over 0.02 cent. The light given by a well-designed kerosene lamp properly shaded is very steady and soft, and not surpassed in quality by any light ordinarily used.

Gas lamps burning illuminating gas are inefficient, often unsteady and rather rapidly passing out of use. Ordinary slit **gas burners** consume about 5 cu. ft. of gas per hour at the usual pressure and give 14 to 17 c.p., or about 3 c.p. per cu. ft. Argand burners with proper chimneys give a somewhat higher efficiency—up to 3.5 c.p. per cu. ft.—and a light admirable in quality and steadiness.

Air-gas, commonly known as gasoline gas, has been widely used for isolated plants in villages and country houses, and consists of air charged with gasoline vapor to a point in excess of the maximum percentage which can form an explosive mixture. More rarely a mixture too lean to explode is used with fair results in Welsbach burners. The mixture commonly used in this country is about 6 gal. of gasoline per 1000 cu. ft. of air. The illuminating power of such **air-gas** is not widely different from that of ordinary commercial gas.

A flame illuminant occasionally very useful is **acetylene**, see p. 615. Acetylene is so rich in carbon that it must be burned with a very liberal supply of air, but when so burned gives a very brilliant whitish flame and produces something like 40 c.p. per cu. ft. It is very easily generated, and furnishes a most useful local source of light. The ordinary burners are of about 20 c.p., consuming about 0.5 cu. ft. per hour. Commercially it is frequently supplied dissolved in acetone and under pressure in metallic cylinders containing about 100 times their volume of free acetylene at atmospheric pressure. In this form it may be very safely and easily transported. Acetylene is exceedingly valuable for the temporary lighting of interior or exterior spaces and is a reasonably cheap illuminant. One ton of carbide, costing $60 to $80, will produce between 9000 and 10,000 cu. ft. of acetylene.

It should be noted that, despite appearances, flames are very nearly transparent and give about the same illumination in all directions in which the light is not obstructed by some part of the burner.

Mantle Burners. The standard types of mantle, consisting of about 99 per cent. thoria and 1 per cent. ceria, give, when the burner which heats the mantle is suitably regulated, from 15 to 18 spherical c.p. per cu. ft. of gas as the initial efficiency. The inverted mantles perform in this respect slightly better than the older erect mantles. They are therefore 5 or 6 times as efficient as light producers as the gas flame. The efficiency of the mantle gradually falls off with use, the older mantles dropping fully half their c.p. in the first 500 to 800 hr. The later mantles, made on a basis of ramie fiber or of artificial silk, *i.e.*, collodion cotton dissolved and squirted into fibers, hold up much better than the cotton basis formerly used. The illuminating value just given is based on ordinary pressures of 2.5 in. of water. By using gas at high pressure, which takes in a correspondingly larger portion of air, the combustion of the burner can be forced as in a blast lamp so as to carry the mantle to a very high temperature, with a corresponding increase in efficiency. Pressures of from 40 to 90 in. of water are in use in these intensive burners. So burned the gas gives 30 to 50 c.p. per cu. ft. according to the quality of the gas and pressure. The mantles used with this "press gas" are several inches in length and give from 1000 to 2000 c.p. each at the higher pressures. So great is the luminous efficiency of the Welsbach mantle that its use has practically wiped out the objections to gas on the score of pollution of the air. Mantle burners are frequently used with gasoline gas and more rarely with acetylene.

Electric Illuminants

Open Arc Lamps. The earliest electric illuminant was the arc struck between two carbons burned in the open air. The passage of the current carries the two carbon electrodes to a very high incandescence, and in the upper and positive one is formed a crater in which the carbon is heated practically to its boiling point. This point of highest temperature gives the major part of the light, which is therefore turned somewhat downward, with a maximum at about 40 deg. below the horizontal. The light from an open arc in good order and supplied with high-grade carbons, is steady, powerful, and gives a low specific consumption, somewhat better than 1 watt per mean spherical candle power (m.s.c.p.). The difficulty of getting proper carbons and the considerable cost of the frequent renewal of the carbons necessary in view of the rather rapid consumption per hour (at least ½ in.) in ordinary lamps, caused the open arc to come into disfavor, and it was rapidly displaced about 1900 by the **enclosed arc lamp.** In this type the consumption of the carbon electrodes is reduced by keeping the arc in an enclosure of hard glass, closed by caps with holes for snugly fitting carbons. In this arrangement the air in the enclosure is rapidly burned to CO and nitrogen, and the wasting away of the carbon is reduced to ⅙ in. or so an hour in ordinary lamps. No crater is formed with the enclosed arc, although the carbon points are highly heated. A considerable proportion of the light comes from the arc itself, which gives a strongly bluish cast to the illumination. The efficiency is much less than that of the open arc, the specific consumption ranging between 2 and 3 watts per m.s.c.p.

Flame Arc Lamps. Carbon is the ideal substance for the electrodes of an arc lamp, inasmuch as it burns to a gaseous, odorless, and non-corrosive oxide, but its light-giving power is strictly due to the high temperature to which it can be forced by the electric current. Hope of higher efficiency than that thus attained has rested in the choice of electrodes capable of giving selective radiation in the visible spectrum. From work along this line has originated the whole group of flame and luminous arcs. The first-named of these burn carbon electrodes like ordinary arcs, but the electrodes carry an admixture of from 10 to 20 per cent. of metallic salts, the vaporisation of which gives a powerful arc stream radiating selectively according to the character of the material with which the carbon is mineralised. The commonest substance thus used is calcium fluoride, which gives a band spectrum as fluoride, intensively brilliant in the orange and green and giving light of a strong yellow cast. The specific consumption of such lamps, which usually operate at 10 or 12 amperes and a voltage at the arc of 40 to 45, ranges from 0.35 to 0.45 watt per m.s.c.p. Such lamps are made in two forms, one with converging carbons meeting at an acute angle from above, and another with carbons vertical, as in the ordinary lamp, the negative carbon being uppermost. Such lamps are made both for alternating current and direct current. Of late, mineralized carbons have been introduced, giving a white light at a slightly reduced efficiency. These carbons are mineralised chiefly with the by-products of the Welsbach gas-mantle industry, and occasionally with some admixture of other substances. Fig. 4 gives the polar distribution curve of an ordinary converging-carbon yellow-flame lamp.

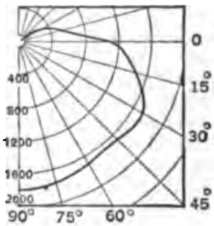


FIG. 4.—Candle Power of Yellow-flame Arc Lamps.

Lamps of the flaming class give a white fume of oxide, which somewhat militates against their use in contracted spaces. The flame arcs can be enclosed to prolong the life of the carbons, which ordinarily burn away at the rate of something like ¾ in. per hour. The arc is maintained in a tight-fitting globe of refractory glass, and provided with merely enough circulation to help carry the fumes into a cooler part of the lamp structure where they may condense and settle so as not to obstruct the light. Such long-burning flame arcs are rather less efficient than the flame arcs burned more intensively, but show a life of the carbons of 75 to 100 hr. Fig. 5 shows the distribution curve from one of the well-known types of long-burning flame arc which gives a specific consumption at the lamp terminals of 0.58 watt per m.s.c.p. Enclosed flame arc lamps are made both for direct and alternating currents.

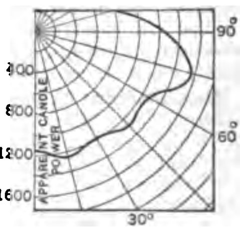


FIG. 5.—Candle Power of Long-burning Flame Arc Lamp.

Luminous Arc Lamps. Still more important in American practice is the so-called luminous arc, of which the well-known **magnetite arc** is the chief type. In this lamp the arc is struck between a positive electrode of thick copper and a negative lower electrode consisting of an iron tube packed with a mixture of metallic oxides consisting chiefly of magnetite with an addition of 20 to 25 per cent. of titanium oxide and occasionally small quantities of other substances. The arc stream is chiefly from the negative electrode, and is itself the chief source of light, giving highly selective radiation of a good, whitish color. The life of the electrodes is, according to size and current, from 75 to 150 or more hours. The positive copper electrode contributes very little vapor and has to be renewed only at intervals of months. As in case of all arcs, the efficiency of such lamps rises with the current. Two forms are commonly used, one taking 4 amp., and the other 6.6 amp. The former gives with a clear globe about 240 m.s.c.p. at a specific consumption of about 1.25 watts. The latter gives a little over 800 m.s.c.p., and its specific consumption is a little over 0.6 watt per candle. Fig. 6 shows the distribution curve of the 6.6-amp. lamp in the clear globe, and Fig. 7 the same lamp with a light opal globe, highly desirable with so powerful an illuminant to reduce the intense brilliancy of the arc. The m.s.c.p. is reduced by such a globe by a little over 0 per cent. Magnetite cs in burning give fumes of brown oxides, which have to be disposed of to ep them from settling on the globe and clogging the mechanism. This is rried out in practice by a flue through which the fumes are carried away, n the whole, pretty thoroughly. This lamp cannot be used on alternating currents. A similar lamp with a carbon upper electrode and a lower tube packed with a mixture chiefly titanium carbide has been found applicable to alternating currents, but has not yet come into considerable use, although highly efficient, on account chiefly of the difficulty of getting rid of the fumes.

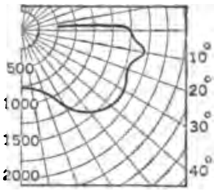


Fig. 6.
With Clear Globe

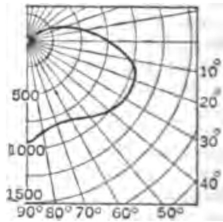


Fig. 7.
With Light Opal Globe

Candle Power of Magnetite Arc Lamps.

Incandescent Lamps. The **carbon incandescent lamp** has a specific consumption of from 3.5 to 4 watts per m.s.c.p. and survives only in lamps of 16 c.p. and less, and in miniature lamps. For most practical illumination with units of 16 c.p. and above, the only form of carbon filament in extensive use is the so-called "metallized filament," produced by firing the carbonized filament material in the electric furnace at so high a temperature as to alter its structure. Lamps of ordinary or metallized carbon filaments are available in all ordinary voltages from 2 c.p. up to about 50. Their efficiency is very low but they are suitable, on account of their low cost, for use in places where the lamps are infrequently used, and where a smaller light is desired than is now available with **metallic filament lamps,** as in some decorative work, and occasionally in domestic use.

The **tantalum lamp,** on account of the comparatively low resistance of the metal, requires a very long and slender filament that in the usual commercial size, giving 20 to 25 m.h.c.p. (mean horizontal candle power), is about 0.002 in. in diameter and about 2 ft. long. The specific consumption of the tantalum lamp is slightly below 2 watts per m.h.c.p., and the effective life down to 80 per cent. of the initial c.p. is 750 to 1000 hr. It has been nearly driven out in the last few years by the tungsten lamp, which is the chief reliance of efficient illumination at the present time. The life of the tantalum lamp on a.c. circuits is scarcely two-thirds of that on d.c. circuits.

The use of **tungsten** for **filaments** marked a great advance in efficiency, tungsten being the metal with the highest known melting point, about 3200 deg. cent. Tungsten is found in liberal amounts in the minerals wolframite and scheelite. The metal was at first obtained only as a black powder, and filaments were made by mixing the finely divided metal with a binder, squirting into filaments and then sintering. Such filaments were extremely fragile except when hot. Tungsten is now obtained as a pure, solid metal, bright steel-gray in color (very hard), yet sufficiently ductile to enable it to be drawn into wire having the extraordinary tensile strength of nearly 500,000 lb. per sq. in., although when in service in filaments it tends to decrease in tensile strength. The drawn-wire filaments are very much stronger than those previously used and it has been possible by their use to increase the efficiency of the lamp materially, in the larger sizes up to a specific consumption of not more than 1 watt per candle and very commonly less, without undue sacrifice of life. The working temperature of the filament at 1 watt per candle is approximately 2300 deg. cent. The **tungsten lamp** has its drawn-wire filament carried on a spider because the specific resistance of the material is so low as to require a very thin and long filament. Tungsten lamps are known under a wide variety of trade names (that best known in this country being "Mazda") and are now available in sizes taking 10, 15, 20, 25, 40, 60, 100, 150, 250, 400 and 500 watts at voltages between 100 and 130, and with a specific consumption varying from about 1.3 watts per candle in the smallest sizes to 0.95 watt per candle in the largest. Lamps are made for series circuits at the ordinary amperages in most of the sizes mentioned except the two smallest. Some odd and intermediate sizes not mentioned here are occasionally found. All these sizes except the smallest ones are now to be had for the higher voltages of 200–260. The general average life of the lamps in all these sizes runs above 1000 hr. The specific consumption quoted above is in watts per m.h.c.p. For spherical c.p. the reduction factor is 0.78 to 0.80.

Gas-filled Lamps are lamps with tungsten filaments enclosed in bulbs filled with nitrogen at nearly atmospheric pressure. The effect of the nitrogen is two-fold: (1) It diminishes surface evaporation from the filament; (2) the products of evaporation are carried to the upper part of the bulb. By this use of nitrogen it becomes possible to push the filament to a temperature of 2600 to 2700 deg. cent. and to reduce the specific consumption to from 0.5 to 0.7 watt per m.h.c.p. The loss of heat by convection is nevertheless severe, and this convection loss falls most heavily upon the slender filaments. Therefore high efficiency can be best reached by lamps carrying large current. With currents of 2 amp. or less the gain in efficiency by the use of the nitrogen-filled bulb becomes small, but for multiple distribution this type of lamp works well in sizes taking from 200 watts upward at 110 volts.

Lamps for series circuits, in which the filament is short and thick, work well and are made for currents of 5 or 6 amp. and upward in sizes from about 50 watts up to 500 or more. The larger sizes are preferably made for currents of 10 or even 20 amp. derived locally from series transformers. Such lamps give specific consumption down to 0.5 watt. Within proper limits of current the nitrogen lamp at its high efficiency bids fair to give about the same useful life as other tungsten lamps and to be of large use in replacing the inefficient enclosed arcs. The high filament temperature gives a much whiter light than ordinary tungsten lamps, although distinctly less white than the intensified or the white flame arcs. The nitrogen lamp has not been fully standardized. Series lamps of the ordinary candle powers up to 1000 are available, and multiple 110-volt lamps from 600 c.p. to above 2000. Some very recent lamps are filled with argon instead of nitrogen, which gives a still higher working efficiency.

Nernst Lamps. The Nernst lamp employs a thin pencil of material which conducts tolerably well when hot, although a non-conductor when cold. The pencil forms the glower and is about 1 in. in length and about 0.02 in. in diameter. For starting, a heating resistance close above the glower heats the latter until it begins to conduct, when the heater is automatically cut out of circuit. A certain amount of resistance has to be placed in series with the glower to steady its action. Including the ballast resistance, the specific consumption of these lamps is between 1.5 and 2 watts per candle and the light is of good color and steady. They are manufactured in sizes of from 50 to 500 m.s.c.p., the larger sizes having several glowers in multiple. They are rapidly being driven out of use by tungsten lamps.

Vapor Lamps. In these lamps the source of light is a gaseous mass rendered luminous by the passage of electric current. The efficiency of such sources is not a mere matter of temperature as with incandescent illuminants. The spectra given are discontinuous and the efficiency depends on the relation of the intensities of the various spectral lines. The most important source of this kind is the **Cooper Hewitt mercury vapor lamp,** in which a column of mercury vapor is rendered luminous by the passage of the current. The ordinary type of lamp consists of a glass tube about 1 in. in diameter and 2 ft. in length, terminated by bulbs containing the negative) mercury and (positive) iron electrodes. The hood above the tube ontains inductance coils, a steadying resistance, an automatic tilting magnet nd the necessary contacts. When the current is hrown on the lamp is tilted, and as it drops back he current follows the mercury back through the tube and establishes a luminous body of vapor. The light is of a strong greenish cast, but steady, of relatively low intrinsic brilliancy, 10 to 11 c.p. per sq. in., and of high efficiency. The ordinary lamp gives about 300 m.l.h.c.p. (mean lower hemispherical c.p.) when worked at 3.5 amp. with about 192 watts at the terminals; in other words, it works at a specific consumption of about 0.67 watt per candle.

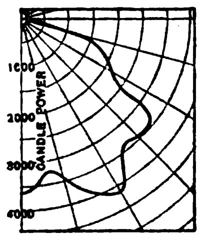


FIG. 8.—Candle Power of Quartz Mercury Vapor Lamp.

An important development from this principle is the **quartz mercury lamp,** in which the same plan is used, but the containing tube is of fused quartz which is so refractory as to allow the lamp to be worked intensively with a great concentration of energy in the vapor column. In the quartz lamp the tube is less than 0.5 in. in diameter and but 4 or 5 in. long in the usual lamp, taking 3.5 amp. at something over 200 volts. Lamps of the same amperage are available as well, for use on 110-volt circuits. The life of the tubes is 2000–4000 hours. This intensive working results in a very great increase of efficiency. Fig. 8 shows the distribution from such a quartz lamp which gave a little over 2300 m.l.h.c.p. at a specific consumption of 0.3 watt per candle. The quartz lamp is one of the steadiest and one of the most powerful known illuminants and has come into considerable use. Both the ordinary form of Cooper Hewitt lamp and the quartz lamp are essentially d.c. lamps, and when only alternating sources are available, it is necessary to employ a current rectifier.

The only other gaseous illuminant is the **Moore tube,** which is practically a gigantic Geissler tube excited by high-tension alternating currents delivered directly to the tube by an attached step-up transformer. The tube is generally about 1.5 in. in diameter and may be of any length up to several hundred feet. It is installed in the form of an enclosed loop or loops around the area to be illuminated, and serves itself as the secondary of the transformer system. The tube is commonly filled with nitrogen at a pressure of about 0.1 mm. of mercury, and under normal conditions gives a specific consumption of about 2.4 watts per m.s.c.p. The light is pleasant, steady, and of very low intrinsic brilliancy, not exceeding 0.5 c.p. per sq. in. Similar tubes coiled up into a small space and filled with CO_2 have been introduced for color-matching purposes, the light giving a close approximation to daylight; the efficiency, however, is very greatly reduced by the use of CO_2 instead of nitrogen.

Table 4, due to Dr. Lux, summarizes some of the essential properties of sources of light.

Table 4. Efficiency, Specific Power Consumption and Specific Output of Various Illuminants

Illuminant	Watts consumed	Per cent. power radiated as light	Mean spherical candle power	Watts per m.s.c.p.	Lumens per watt
Hefner lamp	86.3	0.103	0.726	128.8	0.106
Kerosene lamp	508.0	0.25	10.56	48.2	0.261
Acetylene flame	96.0	0.65	5.31	18.1	0.695
Gas mantle with chimney	716.7	0.46	78.9	8.97	1.385
Inverted mantle with glass	571.0	0.51	72.4	7.88	1.593
Carbon filament lamp	98.23	2.07	21.6	4.55	2.76
Nernst lamp with ballast	181.4	3.85	83.5	2.18	5.76
Nernst lamp without ballast	165.0	4.21	83.5	1.98	6.35
Tantalum lamp	44.0	4.87	23.5	1.87	6.71
Osram lamp	38.3	5.36	24.1	1.60	7.91
D. c. open carbon arc	435.0	5.6	461.0	0.95	13.3
D. c. enclosed carbon arc	541.0	5.15	260.0	2.08	6.04
A. c. carbon arc	180.6	1.84	96.0	1.88	6.67
Yellow flame arc	349.7	15.0	1010.0	0.343	36.3
White flame arc	348.0	7.56	668.0	0.521	22.2
Quartz tube mercury arc	691.0	6.0	2640.0	0.261	48.0

METHODS OF LIGHTING

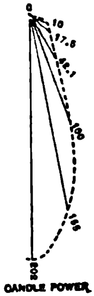
Shades and Reflectors. The intrinsic brilliancy of a light source, as the eye sees it, should not exceed 2 or 3 c.p. per sq. in. Most sources run to hundreds of c.p. per sq. in., and consequently require shading. The surface of a properly designed shade will act as the apparent source of light, and will keep down the intrinsic brilliancy. The effect of the shade on the source is (1) to cut off some of the light and (2) to redirect the light. Usually it rounds out the distribution curve, but if this is very irregular in shape it may produce a noticeable redistribution of the light.

Table 5. Loss of Light in Passing through Closed Shades

	Loss, per cent.		Loss, per cent.
Alabaster glass	15 to 20	Opal glass	25 to 60
Opaline glass	20 to 40	Milk glass	30 to 70
Alba glass	20 to 30	Paper	40 to 80
Ground glass	25 to 30	Textiles	40 to 80

The loss of light in closed shades of medium density through which the whole light flux has to pass, is approximately as given in Table 5.

Reflectors are made of polished or enameled metal, of porcelain, opal glass, of silvered metal or mirror glass, or of prismatic glass which sends back the light from its prismatic elements by total reflection. Prismatic glass shades may act both by transmission (the light being directed by annular prisms) and total reflection. In any case, the purpose is to reflect such portion of the light as may be radiated in useless directions so that it will fall upon the working plane.



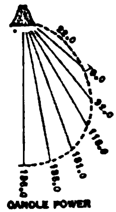


FIG. 9. FIG. 10.

FIG. 9.—10-in. Cone Reflector Lined with Mirror.

FIG. 10.—Reflector Lined with Corrugated Mirror Surface.

If the reflector covers, as is common, something like half of the total solid angle about the source, the total light below the lamp will be that directly radiated in the lower hemisphere plus that radiated above in the upper hemisphere multiplied by the coefficient of reflection of the surface used. This coefficient may vary from 30 to 40 per cent. in the case of some forms of glass and enamel, up to 80 or 85 per cent. in the case of silvered surfaces. The light delivered in the lower hemisphere is consequently reinforced by about 50 per cent. of its own value in the form of light turned down by the reflector. Reflectors may be divided into three classes according as the effect of the reflector is designed (1) to concentrate the light in a narrow area, (2) to spread it over a moderate area, or (3) to scatter it widely. Reflectors of the first class are usually deep, so that comparatively little light escapes from them unreflected and undirected, and find their most perfect development in the parabolic reflectors such as are used in automobile headlights. **Commercial reflectors** of this type are usually made of polished metal or mirror surfaces, or of prismatic glass. Fig. 9 shows the distribution of light from a 16-c.p. electric lamp placed in a 10-in. cone reflector lined with mirror. The intensity in the axis is very high, but the cone of rays is narrow and covers only a small space.

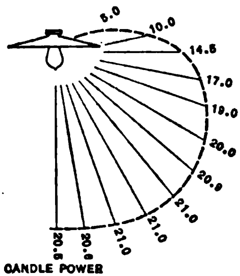


FIG. 11.—Flat Reflector.

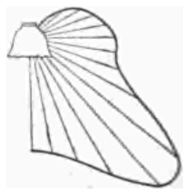


FIG. 12.—Prismatic Glass Shade.

The second type of reflector is the one most commonly used for general illumination, inasmuch as the light is considerably intensified in a large space underneath the lamp. Fig. 10 shows the distribution from a reflector of this character lined with corrugated mirror surface.

The third type of reflector is useful when for any reason a large area has to be lighted from a single source. It takes two general forms, one (Fig. 11) being a comparatively flat reflector, used slightly above the source of light.

This turns down much useful light while allowing lateral rays to escape unreflected. This arrangement is objectionable in that it allows the light of the source itself to be really visible, but is occasionally useful when the lights are not of very high intensity, or are located where they are somewhat out of the field of view. The typical distribution is a rounded one, as shown. Fig. 12 shows a more exaggerated type of wide distribution obtained from a prismatic glass shade which more completely shields the lamp itself, the light in this case being reflected backward from the wall of the shade and out of the shade in the other direction.

Indirect Lighting. In indirect lighting the reflectors are inverted so as to throw the light against the ceiling, from which it is diffused and reaches the working plane. This tends toward uniformity of illumination and completely conceals the source of light, but causes loss of light flux from the secondary diffusion by the ceiling. This loss, commonly 40 to 50 per cent., may be somewhat reduced by giving the ceiling an exceptionally good diffusing surface and keeping it clean. For a given intensity of illumination the indirect lighting usually requires in the neighborhood of 1.75 times the intensity of source necessary in case of direct lighting from open reflectors.

Semi-indirect Lighting. In semi-indirect lighting, which is frequently used to good advantage, the reflecting screen is placed as in indirect lighting, but is of translucent material so as to let through considerable light besides turning a large fraction backward to the ceiling. It lies in efficiency and general characteristics about midway between the systems already described.

The Planning of Illumination

Table 6 shows about the range of efficiencies of utilization which may be expected in practice, using modern shades and reflectors. It will be seen that the most efficient reflectors are the mirror or prismatic glass ones of a fairly concentrating character. Next in order come the usual types of open diffusing reflectors of prismatic or opal glass or metal, and after these follow diffusing shades which completely enclose the lamp, semi-indirect, and completely indirect lighting.

Table 6. Efficiencies of Utilization

	Coeff. K		Coeff. K
Concentrating reflectors	0.7 to 0.8	Half-globes, prismatic	0.45 to 0.50
Concentrating prismatic reflectors	0.7 to 0.8	Coves, indirect	0.15 to 0.36
Concentrating mirror reflectors	0.8 to 0.85	Indirect lighting	0.25 to 0.35
Diffusing reflectors	0.44 to 0.63	Semi-indirect lighting	0.35 to 0.45
Diffusing balls	0.34 to 0.36	Semi-indirect (enclosed arcs and diffusers)	0.45 to 0.50
		Arc with opal globe	0.45 to 0.50

To utilize Table 6, see p. 1369. At any required intensity in lumens per sq. ft. the total flux which must be obtained is this intensity multiplied by the sq. ft. of area, and this flux is that derived from a total flux in lumens, F, multiplied by the coefficient of utilization, K. The illuminants may be subdivided as convenience dictates, bearing in mind that good utilization of the reflectors generally requires the lights to be spaced apart from 1½ to 2 times their height above the working plane. As the space becomes bigger in area and height, it becomes permissible to use larger and larger units for obtaining the same effective illumination.

To pass from the rating of illuminants in the customary horizontal c.p. to the spherical c.p., it is necessary to know the **spherical reduction factor** of the lamp, or the quantity which multiplied by the horizontal c.p. gives the mean spherical c.p. This is usually about 0.8, but in case of an unusual type of illuminant it should be ascertained and not taken by average. For example, with tungsten lamps the illumination is about 0.8 horizontal c.p. per watt, or about 0.64 m.s.c.p. per watt, or, multiplying by 4π, practically 8 lumens per watt. With suitable reflectors it is easy to reach a utilization factor of 50 per cent., which means that about 4 lumens per watt can be obtained on the working plane. An allowance of 1 watt per sq. ft. should give approximately an illumination of 4 foot-candles. Lamps should preferably be rated in lumens.

Sometimes it is necessary to compute **special illumination** at certain points in addition to a general illumination, and in this case the flux method as here indicated may be used in combination with a point-by-point computation for the special places to be considered.

Table 7. Illumination Requirements

Locality	Illumination				
	Monasch (lux)	Bloch (lux)	Toone (ft.-candles)	Various sources (ft.-candles)	Average in ft.-candles (very approx.)
Spinning mills	15	15–20	1.5–2.0	....	1.5
Weaving (light colors)	25–30	25–35	1.5–2.0	....	2.5
Weaving (dark colors)	30–40		2.5–4.0	....	3.5
Machine shops	30	25–35	3.0–4.5	1.0	3.0
Foundries	30	....	....	3.0	3.0
Fine work (mechanical)	40	35–50	5.0–10.0	....	5.5
Living rooms:					
Drawing room	....	20–30	....	1.5–2.0	2.0
Dining room	....	....	1.0–2.0	1.5	1.5
Bedroom	....	....	....	1.5	1.5
Commercial offices	30–40	35–50	2.0–5.0	4.0	4.0
Drafting rooms	50–60	60–80	5.0–6.0	8.0	6.0
Concert and entertainment halls	40–50	35–50	2.0–4.0	2.0	3.5
Theaters	....	....	1.0–3.0	2.0	2.0
Churches	....	....	2.0–4.0	....	3.0
Printing offices	40–50	....	4.0–5.0	....	4.5
Compositors	40–50	60–80	....	8.0	6.0
Shops	30–50	35–50	4.0–4.5	2.5–5.0	4.0
Shop windows	....	....	....	8.0–20.0	14.0
Restaurants	....	35–50	....	2.5	4.0
Hotels:					
Small rooms	....	10–15	....	1.5	1.5
Large rooms	....	15–20	....	2.0	2.0
Corridors	....	....	0.5–1.5	0.6	1.[illegible]
Railway cars	....	....	1.0–2.0	2.0	2.0
Post Office (sorting dept.)	....	....	2.0–4.0	7.0	4.5
School rooms	....	35–50	....	2.0–2.5	3.0
Lecture theaters	30–40	35–50	2.0–4.0	....	3.5
Ball rooms	....	....	2.0–3.0	2.0	2.5
Libraries:					
General	....	....	1.0–2.0	....	1.5
Local	....	....	3.0–4 0	....	3.5
Reading tables	....	....	2.5–10 0	3.5	5.0
Book shelves	....	....	4.0	1.5	2.5
Reading (ordinary print)	....	....	3.0–4.0	2.0	3.0
Streets:					
Main	....	3–6	0.1	0.5	0.5
Side	....	1.5–3	....	0.1	0.15

Amount of Light Required for Various Kinds of Work. The estimates made of the light required for various classes of work differ very much, being based mostly on specific examinations of practical examples which vary widely in the actual requirements represented. Table 7, published in the *Illuminating Engineer* (London), gives a consensus of opinion on the subject, and the average values indicated are probably as near the requirements of sound practice as the data allow.

PREVENTION OF ACCIDENTS

BY

D. S. BEYER

REFERENCES: "Unfallverhütung und Betriebssicherheit," by Schlesinger (Carl Heymanns, Berlin), which includes several hundred illustrations of safety devices used in different industries; "Prevention of Railroad Accidents," Bradshaw (Norman Henley, New York); "Work-accidents and the Law," Charities Pub. Comm., No. 105 East 22nd St., New York City; "Accident Prevention and Relief," Schwedtman and Emery, Nat. Assn. of Mfrs., No. 30 Church St., New York City; "Industrial Accident Prevention," D. S. Beyer (Houghton Mifflin Co.).

Legislation, Etc. The following States have recently passed workmen's compensation laws: Arizona, California, Connecticut, Colorado, Illinois, Indiana, Iowa, Kansas, Louisiana, Maine, Maryland, Massachusetts, Michigan, Minnesota, Montana, Nebraska, Nevada, New Hampshire, New Jersey, New York, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, Texas, Vermont, Washington, West Virginia, Wisconsin and Wyoming. Several of the other states now have commissions at work drafting additional legislation of the same nature (Nov., 1915).

The common tendency of this legislation is to eliminate the defenses which were formerly open to the employer (notably the "assumption of risk," "fault of fellow servant" and "contributory negligence" rules), and in most cases a fixed schedule of payments for injuries has been established, to apply in ll cases except those where the "serious and wilful misconduct" of the injured an is directly responsible for the accident. In addition to these compensa- ›n laws, more stringent factory inspection legislation is being adopted by ırious states and municipalities.

The increased responsibility of industry for its accidents has multiplied :eatly the cost of insurance against industrial accidents. This increased cost, and the certainty with which it is applied, has put a premium on accident prevention work; this cost can be materially reduced by the installation of safety devices. Experience has shown that approximately 50 per cent. of all industrial accidents are preventable.

The logical time to install safety devices is while general construction work is being done, or when alterations or repairs are being made; results can be accomplished with a minimum of expense and delay at the time plans and specifications are being prepared.

Checking for Safety. In order to make sure that the question of safety will not be overlooked, it is well to have all plans, specifications and drawings "checked for safety," making special provision for this in each set of specifications, and in the title plate of each drawing.

Buildings

Accidents involving the greatest loss of life in relation to the construction and arrangement of buildings are chiefly attributable to explosions, fires or conflagrations; and collapse or failure of building structures.

Less important from the standpoint of individual catastrophes, but involving a wider range of minor injuries, are insufficient lighting, inadequate ventilation, general features such as defective stairways, platforms, etc.

Explosions. The most serious accidents in this class are caused by (*a*) boilers and other high-pressure steam apparatus; (*b*) flywheels (principally those on engines); (*c*) miscellaneous explosives such as dynamite, benzine, naphtha, etc.; (*d*) hot metals.

Classes (*a*), (*b*) and (*c*) should be considered with reference to their location relative to buildings commonly occupied. Engines, boilers and explosives of any sort should be located in separate rooms or buildings sufficiently isolated from other parts of the establishment where workers are employed, to reduce to the lowest point danger of personal injuries in case of an explosion. The planes of rotation of engine flywheels should not be in line with workrooms. High-pressure steam piping should not pass through rooms or buildings in locations which would endanger human life if the piping or fittings should burst. (*d*) This class of accidents is due principally to molten metal being caught in a confined space where moisture is present, causing instantaneous generation of steam. All furnaces and receptacles of molten metal should be so located in respect to sewers, water piping, etc., as to minimize this danger.

Fires or Conflagrations. Means for preventing accidents from this source are (*a*) adequate exits and escapes; (*b*) non-combustible construction; (*c*) apparatus and facilities for detecting and controlling fires. See also section on Fire Protection, p. 1390.

(*a*) **Exits and Escapes.** So far as practicable, all **doors** should open outward, or *with* the natural direction of egress; they must not block passageways from other floors or parts of the building. This rule should be observed particularly in all cases where a sufficient number of persons are involved to make hasty exit important.

For buildings of two stories or higher, two or more "fireproof" **stairways** or exits should be provided for each of the upper floors (including the roof); these exits should be separated in such a manner that they are not liable to be cut off by a single local fire. (The location of stairways around or adjacent to passenger elevators is undesirable, unless they are adequately isolated by fire walls, or unless other means of egress are provided.)

Where stairs are used they should be at least 22 in. in width, and any increase in this width should be in increments of 22 in.; the bottom of each flight should be in the immediate vicinity of and in line of travel with the top of the next lower flight. All stairways should have handrails on both sides; those 8 ft. or more in width should have an intermediate handrail, centrally located. Every story below the street grade should have at least two means of egress to the ground level. Stairways and fire exits should be of such capacity and arrangement that persons coming from upper floors will not find their passage blocked by those escaping from the floors below. Outside fire escapes are inferior to stairways as means of egress; where used they should be located on dead walls, or arranged so that persons on them will be protected from flames issuing from windows or openings underneath by the use of wired glass in standard metal frames, fire doors, etc. The best arrangement is that known as the **"Philadelphia" fire escape, or "smoke-proof tower."** This consists of a stairway contained in an outside tower entirely isolated by fire walls from, and without any direct openings to, the building proper; communication with this tower is by means of vestibules or outside balconies on each floor, with doors opening to the balconies from both building and tower.

MUNICIPAL REQUIREMENTS REGARDING STAIRWAYS, EXITS, FIRE ESCAPES, ETC. are extremely varied, and notably lacking in definition and standardization at the present time. The following paragraphs from the "Specifications for Construction of a Standard Building" prepared by the committee on Fire-resistive Construction of the National Fire Protection Association, are recommended as indicating good practice from the standpoint of safety to human life. See also N. F. P. A. reports of "Committee on Safety to Life."

Height from Grade to Roof Line, irrespective of its occupancy, not to exceed 70 ft., unless the building be fully equipped with automatic sprinklers, in which case the height shall not exceed 125 ft.; nor shall any standard building exceed a height of two and one-half times the width of the widest street upon which it is located.

Area Within Inclosing or Fire Walls not to exceed 7,500 sq. ft., with no dimension greater than 125 ft. for a building without automatic sprinklers; or 20,000 sq. ft., unrestricted as to linear dimensions, for a building fully equipped with automatic sprinklers.

Exits to be one of the two following forms: (1) Stair exit, consisting of the direct connection of any floor area to an approved stairway built either as (*a*) an enclosed interior staircase, or (*b*) a smoke-proof tower. (2) Horizontal exit, or the connection of any floor area through a fire-exit partition, fire wall, or an open-air balcony or vestibule to another floor area in the same or an adjoining building having its own independent stair exits; the other area to be of sufficient size to contain temporarily the joint occupancy of the two areas thus joined, allowing not less than 5 sq. ft. of unobstructed floor space per person, and providing at least ¼ normal exit capacity for joint occupancy.

Number of Exits Required. Every floor area to have at least two separate exits, and whenever any floor area exceeds 10,000 sq. ft. at least one additional exit to be provided.

The occupants of every floor above the first to be provided with exits computed on the basis of at least 22 in. in width for every 14 persons for stair exits, or 22 in. in width for every 50 persons for horizontal exits. At least one of the exits provided for every such floor area to be a stair exit.

No width of exit stairway or passageway required by these rules to be reduced at any subsequent point in the direction of exit travel.

Exits to be remote from each other, and no point of any floor area to be more than 100 ft. distant from an exit. Whenever any building is more than four stories high, and has an occupancy greater than twenty-five people above the fourth floor, then each floor area of such building to be connected either directly or indirectly through horizontal exit to a smoke-proof tower.

Automatic sprinklers (see p. 1390) are one of the best safeguards for human life in high buildings; the record of some 13,000 fires in sprinkled buildings, covering a period of 16 years, shows an efficiency of 95.1 per cent. in fires extinguished or held in check.

(*b*) **Non-combustible Construction.** See p. 1390.

(*c*) **Fire Apparatus.** See pp. 276, 1393.

Lighting. Adequate light has an important bearing on the prevention of accidents. Walls, ceilings, columns, etc. should be light in color in order to ›flect and diffuse the light. Widely separated light units of high intensity ıould be avoided, since they give a dazzling effect in their immediate vicinity, ith heavy shadows and outlying zones of intense darkness. From the andpoint of safety, small lighting units of frequent spacing are preferable.

Ventilation. The lack of adequate ventilation helps to bring on fatigue and reduces the alertness of the workmen.

Accessibility, Stairways, Platforms, Openings, Etc. All **valves, operating mechanism, etc.**, which require attention at intervals, should be so located as **to be readily accessible.** Where they cannot be reached from the floor, suitable platforms should be erected, with convenient means of access. For this purpose stairways are preferable to ladders, see p. 1389. Falls from, or with, ladders constitute a material percentage of all industrial accidents.

All **stairways** should be provided with hand rails at both sides. Stair treads should be of such material and construction as will prevent their wearing smooth and becoming slippery with use. Iron or steel plates with "checkered" or roughened surfaces may be used; a good non-slipping tread with large capacity for wear consists of plates surfaced with an abrasive material such as alundum or carborundum. Crystals of these materials, about the size of buckwheat, may be imbedded in the surface of concrete while it is still soft, giving a similar but somewhat less efficient form of "safety tread."

All **overhead platforms** should be equipped with railings or barriers of a construction suited to the materials handled on them.. Such railings should not be less than 42 in. in height, and unless they are solidly enclosed, should have an intermediate member. Coamings or "toeboards" not less than 6 in. high should be placed along the edges of the platform at foot of railing, to prevent loose objects from falling off. **Railings** should preferably be of riveted structural steel shapes; if pipe is used it should be "extra heavy," with special hand-rail fittings.

As far as possible, **openings through floors or platforms** should be avoided; where they necessarily occur, they should be protected with railings, as described above, or with **covers**. Such covers should be hinged or permanently attached so as to prevent their being entirely removed. It is often possible to equip covers or trap doors with rest rods which support the door at such an angle as will permit easy egress from the open end, but still form a natural barrier at the other three sides.

Power Equipment

Power Generation and Distribution. **Electric power is to be preferred** to steam from the standpoint of safety, on account of its flexibility and ready adaptability to various methods of control. Individual or group motor drives naturally bring the controlling switch near the point where power is used, and can readily be controlled from a distance. With motor drives the momentum of moving parts is relatively small, so the machinery comes to a standstill quickly after power is shut off.

There are certain features common to all forms of power transmission, namely, power-disconnecting devices; shafting, pulleys and couplings; belting; and gearing.

Power-disconnecting Devices. There should be provided means for quickly disconnecting the power in each room or floor where machinery is used. This may take the form of an "automatic engine stop" operated with electric push buttons, or a quick-closing valve with mechanical tripping device where steam power is employed, or clutches (preferably of friction or magnetic type) may be applied to the various lines of shafting. Push buttons located near the machinery driven may also be used for controlling water wheels, gas engines, etc.

Shafting, Pulleys, and Couplings. The **elimination** from revolving shafting, couplings and pulleys of all **projecting parts** which might catch in the clothing, such as set screws, bolt heads, keys, etc., is of primary importance; these can generally be countersunk or covered so as to render them safe. Where transmission **shafting** is near the floor or ordinary working positions of operators, it should be enclosed or **protected by** suitable **barriers.** Pulleys should be spaced far enough from hangers to allow adequate clearance if a belt should drop off on the hanger side. Where lines of shafting are placed high above the floor, it is desirable to provide suitable means for oiling them from the floor, such as long-spout oil cans.

Belting. Guards should be placed under main transmission belts, to protect workmen underneath in case the belt should break. Such **guards** should be substantially constructed and should be continuous to the ceiling or adjacent supports so that they will not leave an end over which a breaking belt might catch. Belts on individual machines should be guarded as specified later.

Gearing. All gearing should be **completely enclosed**, the covers being made preferably of cast iron or sheet steel, of substantial construction, and shaped to conform to the gear outlines. It is often possible to design these guards so that the gears can be run in an oil bath, thus increasing their life and efficiency.

Boiler Plant. The nature of this equipment and the danger of explosions or other accidents which may fill the room with hot steam, make it particularly important to provide adequate means for quickly and safely reaching all working positions. A system of walks or **runways** should be erected to give convenient access to overhead valves, water columns, etc., also passages from boiler to boiler and suitable platforms at individual valves from which they can be safely operated or repaired. Walks and platforms should be well lighted and free from breaks or obstructions which might interfere with their use; they should be equipped with standard rails and coamings (see "Buildings," *ante*) and wherever possible stairways should be used in preference to ladders for giving access to these walks. Means of egress should be provided for **overhead walks** or **underground tunnels** at both ends of each line of boilers, and at one or two intermediate points in large plants. From the standpoint of **safety**, water-tube **boilers** are preferable to those of return tubular type. The use of "**non-return valves**" of the triple-acting type is recommended in plants where two or more boilers are connected in one battery. Gage glasses should be equipped with suitable guards to prevent eye injury when the glass bursts. Where these **gage glasses** are located near the floor, or close to ordinary operating positions, so that constant protection is desired, a guard of wired glass, or other suitable construction, which can be in place between the eyes of the operator and the gage glass at all times, is desirable; where the glass is high above the floor, a swiveling guard consisting of a semicircular shield of sheet metal gives adequate protection.

Escape pipes from safety valves should be arranged so as to discharge steam outside the building when a valve blows off; wherever practicable these pipes should extend vertically through the roof, and open drain connections should be provided for each pipe, close to the valve.

Distributing Piping. Systems of steam piping should be laid out with great care to avoid danger from expansion and contraction. It is particularly necessary to see that the **expansion** does not throw undue strain on any part of the piping system, or on the steam chests of engine cylinders, etc. to which the pipes are connected. Where main lines branch, or a distributing pipe leads off from the main line, a valve should be placed close to the connection so that the branch line can be shut off independently. Care should be taken to avoid "**water pockets,**" and where these necessarily occur suitable drain pipes should be provided for the removal of condensed steam. Separators should be placed in the pipe connections to individual engines, with traps for discharging water from same located where their operation may be constantly under observation.

Engines. Governors, if not of the shaft type, should be driven with ropes (not less than three in number). Corliss **valve gear** should be equipped with safety cams which unhook the valves when the governor weight drops to low position; a bracket should be provided for supporting this weight when shutting down the engine, so arranged that it will drop out automatically after the engine is started up. Where a fly-ball governor is used, the revolving parts should be adequately guarded or enclosed. Where **main bearings** cannot readily be reached from the floor, they should be equipped with steps having standard handrails. Some type of safety tread, with non-slipping surface of lead or abrasive material, is particularly desirable for engine-room steps on account of their liability to become slippery with oil. Cross heads, connecting rods and cranks, and tail rods should be guarded or enclosed. A **relief valve** should be placed in each end of each cylinder; these valves should have hand levers or other means for lifting the valve from seat for testing.

Care should be taken in the design of valve mechanism and other moving parts to avoid places where a hand or foot might be caught and crushed; where such places necessarily occur, they should be properly safeguarded. **Flywheels** should be suitably railed or guarded; standard coamings should be placed around the edges of flywheel pits, engine platforms, etc. Railings around rope sheaves should be sheeted solidly from top to floor to prevent breaking ropes from catching in the railing.

Electricity. In considering electrical equipment for industrial establishments, it should be borne in mind that the **hazard** to human life **increases with** increase in **voltage**, and economy in transmission wiring and copper parts which is achieved through the use of high-tension motors, may be offset by increased danger of accident. For small motors, lights, and general service inside industrial plants, installations of 110 or 220 volts are recommended.

All **switches, fuse boxes,** terminals, starting **rheostats, motors, etc.,** should be **enclosed** or guarded in such a manner as will prevent accidental contact with live parts, irrespective of voltage. **Switches** should be arranged so that they can be **locked** in the open position, to guard against a switch being thrown in accidentally while men are at work on the lines or equipment which the switch controls. High-tension **equipment of over 550 volts** should be isolated from other operating equipment, in separate rooms or enclosures, with provision for locking up these enclosures. All **metallic cases, frames,** and **supports** of such equipment should be permanently grounded. Foundation bolts should not be depended upon for this purpose, but substantial ground conductors should be used. It is preferable to have these ground wires accessible for inspection. All **low-tension secondary circuits** of 550 volts or less (such as light circuits, motor circuits, and meter circuits) should be permanently grounded whenever a neutral point is available for the purpose. Secondary circuits of 250 volts or less should be permanently grounded, even though a neutral point is not available. Whenever grounding has to be omitted, the frame or casing of the apparatus should be permanently grounded, and all live parts of the secondary circuit should be shielded to prevent accidental contact therewith. It is also desirable to have floors or platforms adjacent to switchboards built of non-conducting material, or suitably insulated. In the absence of such construction, rubber mats may be used.

General Machinery

Machine Design. All gearing should be completely enclosed with substantial cast-iron or sheet-steel covers, so designed as to be readily detachable; all set screws, keys, bolts, etc. in moving parts should be countersunk or covered in such a way as to eliminate danger of accident; unused portions of keyways should be filled so as to present a smooth surface.

Where machines are belt-driven, shifters should be provided. All shifters should be so arranged that they will lock automatically in both positions; this may be accomplished by notching the sliding parts or by locking crank motion.

Belts to individual machines should be guarded so as to prevent danger of anyone being caught between the belt and pulley. A height of 5 ft. above the floor line is recommended for such guards, if within 6 in. of the belt.

Care could be taken to avoid shearing or crushing motions between the moving parts of machines, or between such parts and the adjacent framework. Counterweights should be placed in such positions as will minimize danger of injury in case the support gives way and allows the weight to drop; where they cannot be guarded by natural location, they should be suitably enclosed.

Construction Details

Beds of **metal-working planers** should be covered or sheeted over, to avoid shearing action with the reciprocating table. **Emery wheels** should have safety hoods (preferably of steel, not less than $\frac{3}{16}$ in. in thickness), and all dry grinders should be provided with suitable exhaust connections; emery-wheel flanges should be not less than half the diameter of a new wheel. **Wood jointers** or **planers** should have standard safety cylinder or circular heads, and should preferably be equipped with exhaust connections; an additional guard which automatically covers the unused portion of the revolving knives is desirable in addition to the safety head. **Band saws** should have upper and lower wheels enclosed, and all portions of the saw except that over the table subject to immediate use should be guarded against accidental contact. **Circular saws** should be guarded and where possible should be equipped with a "splitter," except in the case of self-feed saws.

Elevator Specifications and Installation Rules. The following requirements are prepared with freight elevators particularly in mind, although similar requirements also apply to passenger elevators.

There should be at least two lifting and two counterweight cables each, and both lifting and counterweight cables of a drum-machine elevator should have not less than two full turns of cable on the drum when the car is at the lowest and the highest points of travel. Sheaves of operating ropes should be thoroughly guarded, so that the ropes cannot be misplaced. An approved device should be provided for locking or securing the starting rope in the cage so that the elevator cannot be started from any other point. Unused sides in the car should be enclosed to a height of at least 6 ft. from the floor; oven steel fabric of about No. 9 wire, 1-in. mesh, makes a good material for is enclosure. Unless otherwise specified, the elevator car should be provided ith a substantial cover to prevent danger of tools or material falling from erhead and striking persons in the car; this cover should preferably be ranged in two or more sections, hinged to the supports so they will swing pward if they strike any obstruction as the car descends. Where an elevator serves two stories or more it should be equipped with an approved speed governor.

All elevators except those of the hydraulic plunger type should have an approved type of safety clamp which will engage the guides and stop the car in case the hoisting cables should break or become detached. This clamp should be thrown into engagement by a spring or mechanical device, and not by gravity; it should be arranged, however, so that the gripping power of the clamp is not obtained from a spring. The elevator well should be completely enclosed on the unused sides and from the head clearance (7 ft.) up to the ceiling, where entrances occur. Doors or gates should be placed at all entrances, and, wherever practicable, should be arranged so that they can be opened only from within. If gates are used they should be preferably of the semi-automatic type; but they may be manipulated entirely by the operator, if necessary, and full automatic gates may be used at upper and lower floors. Wherever possible gates should be at least 6 ft. in height, to prevent any one looking over the top of them; if this height is impracticable, suitable means should be provided to prevent accidents from the descending cage (such as tell-tale chains, suspended from bottom of cage). Where the elevator car passes close by beams, floors, etc., the latter should be beveled off and sheeted over to prevent possibility of a foot being crushed between them and the floor of the cage.

Platforms with railing and toeboards should be placed at the top sheaves, for use of oilers, repairmen, etc., with permanent ladders to these platforms where necessary. A pit at least 3 ft. 6 in. in depth should be provided at the lower floor, and an equal clearance allowed between overhead platform, or beams, and the top of cage when it is at the upper floor level. Bumpers should be placed in the bottom pit, arranged so as to secure a minimum clearance of at least 30 in. between bottom of cage and pit floor. For further details see Massachusetts "Elevator and Escalator Regulations."

Ladders. The following precautions should be taken:

Fixed ladders are safer than portable ones; they should be made of steel to secure the highest degree of permanence and stability. At least 8 in. clearance should be allowed back of each ladder, and care should be taken to avoid any obstruction which would decrease this clearance and interfere with a proper foothold on the rungs of the ladder.

Where a high ladder is necessary (say 30 ft. or over), it is desirable to provide it with a "safety cage" consisting of a series of parallel bars arranged so as to enclose any one using the ladder in a cage or tube. A good form of construction is to make this cage approximately 27 in. inside diam., with seven parallel bars (about $\frac{3}{8} \times 1\frac{1}{2}$) supported at intervals of 6 to 8 ft. by a circular band rigidly attached to the ladder proper. It is also well to divide such ladders into sections of 15 or 20 ft., offsetting each section and constructing a railed landing between sections.

Where portable ladders are used for oiling shafting, suitable hooks should be placed at the top of the ladder for engaging with the shaft or hanger.

"Safety feet" should be attached to the bottom of each ladder; for wood floors a radial steel plate having a series of sharp spurs is desirable; for concrete floors it is well to use a foot which is pivoted, so as to give a large bearing surface on the floor; this foot may be advantageously made of metal surfaced with an abrasive material such as carborundum or alundum.

Safety Organizations, Societies, Etc.

The National Safety Council serves as a general clearing house of safety information. It sends out weekly distributions of safety literature, including bulletins, statistical information, descriptions of safety practices, rules, etc. It has branch Councils in many of the principal cities of the United States and holds an annual safety congress. It has a Standardization Committee at work on the standardization of safety devices and practices for various industries. It is a co-operative organization conducted without profit. Secretary, Mr. W. H. Cameron, Continental and Commercial Bank Bldg., Chicago.

The National Affiliated Safety Organizations represent the National Association of Manufacturers (30 Church Street, N. Y. City), the National Founders Association (29 South La Salle Street, Chicago, Ill.), the National Metal Trades Association (Peoples Gas Building, Chicago, Ill.) and the National Electric Light Association (29 West 39th Street, N. Y. City). A joint "Conference Board on Safety and Sanitation" representing these associations is working on the development of safety devices and practices, and the dissemination of information on them through various publications, bulletins and magazine articles. Such devices at present (1915) include first-aid jars (see p. 1747), stretchers, respirators, chip guards, ladder feet, foundrymen's shoes and leggings, knuckle guards for trucks, etc. Mr. M. W. Alexander, West Lynn, Mass., is secretary of the Conference Board.

The American Museum of Safety is a museum of industrial safety and hygiene. It issues publications, assists in public safety campaigns and other lines of safety work and holds an annual exposition of safety and sanitation in New York. The museum is located at 14–18 West 24th St. New York City. Dr. W. H. Tolman is the director.

The Underwriters' Laboratories (207 E. Ohio Street, Chicago, Illinois) which, for a number of years, has examined fire and electrical appliances for approval, under the direction of the National Board of Fire Underwriters, has arranged to test and approve safety devices also. It maintains a card service on approved devices.

FIRE PROTECTION

BY

H. O. LACOUNT

Construction

Types of Construction. The use of reinforced concrete affords a **non-combustible construction** that greatly reduces the conflagration hazard. **Slow-burning mill construction** (see p. 1325) can be easily protected by automatic sprinklers, and fires can generally be extinguished before the building is seriously damaged. In any case, roofs or roof coverings should be of non-combustible construction in order to prevent ignition through sparks brought from a distant fire.

Areas. (See also p. 1383.) Small areas tend toward limiting the damage from any given fire. Usually, they should not exceed 20,000 to 30,000 sq. ft., but with safe occupancies, such as weaving, machine shops, etc., larger areas would not be unreasonable. The more hazardous processes should be placed in separate buildings or separate rooms. Processes of widely different hazards should not occupy the same room. Processes in which fires are liable to start should not be located on floors above valuable occupancies specially susceptible to water damage.

Cut-offs. Subdivision should be obtained by substantial fire walls parapeted through the roof, and necessary openings protected by self-closing fire doors. Fire curtains should be provided in very large areas. Good cut-offs floors are especially important to prevent fire, heat, and smoke in a lower floor from quickly spreading throughout the upper floors. Stairs, elevators and main belts should be enclosed, preferably with non-combustible construction, though for many of the smaller stairways a good plank or double matched sheathing partition would be satisfactory. All openings in these enclosures, except those of plank, should have fire doors, and good wood doors should be provided at either top or bottom of stairs enclosed with plank. Belts, except the smaller ones, especially need enclosing, for they create drafts which readily carry fire from one floor to another. Brick or concrete towers are usually provided for main drives, but expanded metal and cement plaster on iron framework is satisfactory for belts in general.

For **Fire Hose** and **Fire-extinguishing Streams** see p. 276.

Automatic Sprinklers

Where Needed. Sprinklers should be installed throughout the premises, including basements and lofts, under stairs, platforms, and large shelves, in towers, elevator wells, dry boxes, etc.; in small enclosures such as belt boxes, chutes, conveyor trunks, etc.; also in cupboards and closets unless they have tops entirely open or are covered with light, inflammable material, and are so located that sprinklers can effectively spray therein. Sprinklers should not be omitted in any room merely because it is wet or of non-combustible construction.

Sprinklers **may** generally **be omitted** under non-combustible construction when the occupancy does not involve the introduction of combustible material.

Sprinklers **should not be omitted** over electrical apparatus except in dynamo or transformer rooms where voltages exceed 600. With these higher voltages the apparatus is liable to be damaged from a wet-down, and it is important to guard against possible loss in case of accidental opening of a head. If the sprinklers are omitted, the room or building should be of non-combustible construction or all exposed woodwork should be adequately fireproofed. In some cases suitable non-combustible shields can be erected over the machines and thus permit the use of sprinklers without danger of wetting the electrical apparatus.

In **power houses** having non-combustible walls the need of sprinklers may be avoided by making the roof also non-combustible, or, if the roof is of wood, by fireproofing the underside with metal laths and cement plaster.

Table 1. Spacing of Automatic Sprinklers

Water pressure at highest sprinkler	Kind of hazard*	Spacing of sprinklers, ft.						
		Standard mill construction					Open-joisted ceilings	
		Width of bay, ft.					Sprinklers at right angles to joists	Sprinklers parallel with joists
		12	11	10	9	8 and under		
Exceeding 20 lb. per sq. in.	Medium..	8	9	10	11	12	8	10
	Special....	7	8	9	10	11	7½	9
Less than 20 lb. per sq. in., or supplied by tank.	Medium..	7	8	9	10	11	7½	9
	Special...	6	7	8	9	10	6½	8

* The terms "medium hazard" and "special hazard" relate to the contents or occupancy of each room. Specially hazardous places are such as picker rooms, planing or sawing departments of wood-working establishments, painting or varnishing rooms, etc.

Arrangement. In general, at least one sprinkler should be allowed for every 100 sq. ft. of floor area, the sprinklers never to be more than 12 ft. apart in either direction and preferably not more than 10 or 11 ft. No branch line should contain more than 7 sprinklers.

Spacing. For standard mill construction (6- to 12-ft. bays), one row of automatic sprinklers is placed midway between beams in each bay, and the sprinklers should be spaced according to Table 1. Spacing for open-joisted ceilings is given in the same table.

Under joisted construction sprinklers should be staggered at right angles to, or across, the joists, so that no two adjacent heads will distribute water into the same channel between joists, but in no case should the end or intermediate sprinklers in each line violate the rules for spacing.

Sizes of Pipes. The number of automatic sprinklers in any one room supplied through a given size of pipe should not exceed that given in the following table:

Size of pipe, in.	¾	1	1¼	1½	2	2½	3	3½	4	5	6
No. of sprinklers	1	2	3	5	10	20	36	55	80	140	200

These sizes apply to the ordinary runs of pipe in the average equipment. Longer runs, such as feeders across wide buildings, or feeders to sections remote from a riser, must usually be larger, and should be figured to carry the amount of water without excessive friction loss.

Discharge of Sprinklers at Different Pressures. All of the approved sprinklers give practically the same discharge with the same pressure at the sprinkler. For diaphragm or ring nozzles (½-in. orifice), the discharge is as follows:

Pressure at sprinkler, lb. per sq. in.	5	10	20	30	40	50	60	70	80	90	100
Discharge, gal. per min.	12	18	25	31	36	41	45	49	52	55	58

Cost of Sprinkler Equipment. An average figure for a sprinkler equipment, including the risers and all inside work but no outside work, is $3 per sprinkler. This might be reduced to $2.50 for plain, straightway work under favorable conditions, and run as high as $4 to $5 when the cost of labor is high or roofs are difficult to reach.

Open Sprinklers

Where Needed. Where window openings must be guarded against exposure, if the exposure is moderate and such as not to warrant much expense, considerable protection may be obtained from sprinklers located just above the windows so as to discharge water directly upon the glass and frames. This protection will be greatly increased if the windows are made of wired glass in metal frames.

Size of Discharge Orifices. Where there is but one horizontal line of sprinklers the orifices should be 3/8 in. in diameter. Where there is more than one horizontal line the size of the orifices should be as follows: top line, 3/8 in.; next line below, 5/16 in.; all other lines, 1/4 in. When there are over six horizontal lines of sprinklers they should be divided into groups, each group to be supplied through an independent riser. Even with a system of six horizontal lines subdivision may sometimes be advisable in order to obtain better control of the equipment.

Sizes of Pipe. The number of sprinklers on a branch line should not exceed six. The pipe sizes should be as given in Table 2.

Table 2. Sizes of Pipe for Open-sprinkler Branch Lines

Size of pipe, in.	Maximum number of heads allowed				
	Size of orifice				
	3/8 in.	5/16 in.	1/4 in.		
	Top line	2d line from top	3d line from top	4th line	5th and 6th lines
3/4	....	1	1	2	4
1	1	3	3	5	6
1 1/4	3	6	6	6	....

The pipe sizes on riser and feed mains, should be as follows:

Number of heads	6	10	18	30	48	72
Pipe size, in.	1 1/2	2	2 1/2	3	3 1/2	4

Where the heads are over 12 ft. apart, larger pipe than given in the table should be used. Where the feed main (including risers to the first branch line) is over 25 ft. in length, it should be at least a size larger than the table requires.

The **discharge** from an open-sprinkler equipment with such pressures as are usually available, will vary from about 8 gal. to 12 gal. per min. per head, depending on the pressure and the arrangement of the pipes. For efficient service the pressure should not be less than 5 lb. at any head while water is running.

Pipes of the best quality, galvanized inside and outside, should be used for the equipment as far back as the controlling valve, and the piping should be run inside the building as far as practicable.

Location of Sprinklers. For windows not exceeding 6 ft. in width, one sprinkler should be placed in the center with the deflector about on a line with the top of the upper sash, and 7, 8, 9, or 10 in. in front of the glass, according as the windows are 3, 4, 5, or 6 ft. in width, respectively. Where windows are over 6 ft. in width or where mullions interfere, two or more sprinklers should be used.

Discharge of Open Sprinklers at Different Pressures. The discharge of open sprinklers having smooth-bore conical nozzles and outlets of various diameters, is as given in the following table:

Pressure at sprinkler, lb. per sq. in.	5	10	15	20	25	30	40	50
Discharge, gal. per min.:								
¼-in. outlet	4.3	5.6	6.7	7.7	8.5	9.2	10.4	11.5
5/16-in. outlet	5.8	8.0	9.7	11.1	12.3	13.4	15.3	17.0
⅜-in. outlet	8.3	11.5	13.9	15.9	17.6	19.2	21.9	24.3

Standpipes

Stair towers often furnish excellent opportunity for use of hose streams at fairly close range, especially in case of fire in upper floors, and for this purpose standpipes should be run up through most main stair towers. These standpipes should not be smaller than 4 in. in diam., and when two connections are taken off at each floor, 6-in. pipe would be advisable. At each connection should be provided a 2½-in. hose valve and attached to this a length of 2⅝-in. unlined linen hose not less than 50 ft. long and equipped with a standard playpipe with a ⅞-in. or 1-in. nozzle. Depending on the width of building, a 75-ft. length of hose or two 50-ft lengths coupled together should be provided, stored on suitable racks and arranged to be easily pulled out for immediate use. The hose valve should be absolutely tight, as leakage will wet the hose, causing mildew, which seriously weakens the fabric.

Underground Supply Pipes

Size of Pipes. This will depend upon the size of system, the amount of water per min. liable to be needed, and the allowable friction loss. For the supply of a 2-way hydrant a 6-in. pipe should be used, and for a 3- or 4-way hydrant an 8-in. pipe.

Cost of Cast-iron Pipe Laid. The approximate cost per linear foot of cast-iron pipe laid in earth, including an average number of tees, elbows, etc., as ordinarily required for mill-yard systems, is as follows.

Inside diam. of pipe, in.	4	6	8	10	12	14	16	20	24	30	36
Cost per lin. ft.	\$0.84	1.08	1.42	1.80	2.13	2.57	2.95	4.20	5.37	7.46	9.50

The above figures are based upon iron pipe at \$30 per ton, and include cost of teaming at 30 cents per ton-mile for an average distance of 2½ miles. For rock excavation and back-filling with earth, add \$4 per cu. yd.

Hand Apparatus for Extinguishing Small Fires

Fire Pails. Ordinary pails kept filled with water are very efficacious in extinguishing small fires in factories. Generally, one pail should be provided for every 750 to 1000 sq. ft. of area, with additional pails where there are large amounts of light combustible materials, as in packing and shipping rooms, woodworking shops, etc.

Small Hose (1¼-in. unlined linen hose with a ⅜-in. nozzle) should be provided, and may be connected to the sprinkler system if the pipes are not less than 2½ in. in diam. An independent supply, however, is preferable.

Chemical Extinguishers. For general use the standard 2½-gal. soda-and-acid extinguisher is to be preferred. Extinguishers using carbon tetrachloride are especially useful in connection with electrical and gasoline fires, where water should not be employed.

Sawdust. Fires in inflammable liquids in moderate-sized open tanks such as used for paint dipping, lacquering, etc., can be easily extinguished by spreading sawdust over the burning liquid with a long-handled shovel. The best results are obtained when 10 lb. of bicarbonate of soda are mixed with each bushel of sawdust.

SECTION 12

MACHINE-SHOP PRACTICE

BY

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CONTENTS

MACHINE TOOLS AND MACHINE-SHOP PRACTICE

BY

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REFERENCES: *Transactions* of the American Society of Mechanical Engineers. Convention Reports, National Machine Tool Builders' Association. Files of *American Machinist, Machinery, Foundry,* and *Wood-Craft.* "Engine Lathe Work," Colvin. "Treatise on Planers," Cincinnati Planer Co. "Modern Milling Machines," Horner. "Automatic Screw Machine," Goodrich and Stanley. "Accurate Tool Work," Goodrich and Stanley. "American Machinists' Grinder Book," Colvin and Stanley. "High Speed Steel," Becker. "American Machinist Gear Book," Logue. "Mechanical Engineering and Machine-shop Practice," Moore. "American Foundry Practice," West. "American Machinists' Handbook," Colvin and Stanley. "Woodworking Safeguards," Van Schaack.

Machines for shaping the materials of machinery construction are classified into the seven following groups:

1. Molding machines
2. Core-making machines
3. Forging machines
4. Bending, forming and shearing machines
5. Welding machines and apparatus
6. Metal-cutting machines
7. Wood-working machines

MOLDING MACHINES

Small Hand Squeezer. This (the simplest) form of molding machine is used to save the greater part of the work of hand ramming. It consists of a metal frame supporting a table upon which the pattern and flask rest during the molding operation. Above the table is a yoke or presser top connected to the frame by strain bars which can be swung back out of the way, or brought over the flask.

To make a mold, the pattern in its match plate is placed on the table and over it the drag part of the flask. The correct amount of sand is then shoveled in, a presser board is put on, and by means of a long lever the sand is squeezed into the flask and around the pattern. This produces a half mold—the drag. The cope is made in the same manner, except that the drag is turned over, the match removed and the cope half of the flask put over it. **Used only on light work**—usually for snap flasks.

Power Squeezer. The power squeezer is next in point of simplicity to the hand squeezer. The platen rests on the top of a plunger fitting in a cylinder beneath and actuated (usually) by compressed air. The yoke at the top that resists the pressure is hinged at the bottom and can be swung back out of the way. It is adjustable on the strain bars at the sides to accommodate flasks of different depths. A **vibrator,** or small air-operated hammer, is provided to jar the mold while the pattern is being drawn and overcome the friction of the sand. It does not appreciably enlarge the mold.

The **operation** is the same as with a hand squeezer, except that the **ramming** is done **by power** and the vibrator is used in drawing the pattern. The air pressure is from 80 to 100 lb. per sq. in.

In **commercial sizes,** cylinder diameters range from 10 to 18 in., spans between strain bars from 32 to 62 in., and platens from 18 × 26 to 24 × 42 in.

The **air consumption** may be figured at 4 cu. ft. of free air per min. This includes squeezing the mold, vibrating when drawing the pattern and blowing off the pattern when drawn. In the larger machines, more air is used per operation, but fewer molds are made in a given time.

Vibrators are made from $\frac{7}{16}$ to 2 in. inside cyl. diam. in size. They are not proportioned to the size of molding machines except incidentally, but to the patterns and pattern carriers that they agitate. The material of the carrier is another factor. It is often advisable to use a large vibrator with a wooden plate of a given size than with a corresponding metal one. The wood transmits the vibrations (5,000 to 30,000 per min.) more slowly. For metal plates and patterns the practice of the Mumford Molding Machine Co. is to make the **mass of** the **vibrator plunger** $\frac{1}{50}$ of that of the plates and patterns. The **production** from these machines exceeds that of hand squeezers by 15 to 30 per cent., depending upon the character of the work. Expert operators can put down from 270 to 325 molds in 6 hr., although the average is probably not greater than 200 molds. On some lines of work it is possible to increase production over hand molding by 100 per cent.; on other lines, as stove work, 20 per cent. is a satisfactory margin.

Split-pattern Machine. The split-pattern machine is a **power squeezer** particularly adapted to make molds from symmetrical patterns that can be split through the center, or for flat-back work. **Patterns** adapted to this machine are not limited to these two classes, however, as any pattern that can be split on a true plane may be molded.

This type of machine is **used for** a **wide variety of work** and for castings ranging from a few ounces in weight to others weighing several hundred pounds. They are made for both hand and power draft, are provided with vibrators, and may be provided with stripping plates.

In **operation** a half flask is put in position on the flask frame, parting sand is shaken over the pattern and the flask shoveled full of sand. The yoke is drawn forward and pressure applied. When the proper sand density is reached, as indicated by the pressure gage, the pressure is released and the platen and connected parts sink to normal position. The yoke is then thrown back, the vibrator started, and the flask lifted from the pattern by hand or power-operated mechanical means. The half mold is now finished and ready to be lifted from the flask frame.

The hand-draft, power-squeezing, split-pattern machine is made in two sizes, for flasks 14 × 16 in. and 13 × 18 in. **Air consumption,** 4 cu. ft. free air per min. per machine; pressures used, 60 to 80 lb. per sq. in.

Commercial **sizes** of power-draft machines range from 13 × 18 to 36 × 36 in. in flask size, 4 to 12 in. in draft and 13 to 36 in. in cylinder diameter.

In using **symmetrical patterns,** but a single plate of patterns is necessary, as the cope and drag are molded from a double set of half patterns. The impressions of the right-hand set in the drag match those of the left-hand set in the cope, and *vice versa.*

Flat-back patterns are mounted on the pattern plate for the drag half of the mold and a blank plate used for the cope half. It is customary to mold a "floor" of drags, then change the plate and mold the copes to complete. Split-pattern machines must be built for flasks of fixed dimensions, or at least fixed in length or width, to fit the flask pins of the machines. For this reason there has recently been a tendency to supersede them with other forms.

In the use of these machines it is not unusual to damage the finished mold when the pattern is drawn. This difficulty is obviated in part by drawing the pattern through a closely fitting hole in a fixed plate known as the **stripping plate.** As the pattern is carefully guided there is little danger of damaging the mold.

Jarring or Jolt Ramming Molding Machine. In this type of machine, developed in recent years and **adapted** especially **for large work,** the flask is struck a forceful blow to compact the sand. It is essentially a **sand-packing machine.**

In some forms the jarring table is struck from beneath by a heavy plunger actuated by compressed air. The blow raises the table a little distance from its support, by which it is struck a second blow when it falls back. Other types are mounted upon a heavy concrete foundation in such a manner that the force of the blow is kept within the machine itself and is not transmitted to the surrounding ground—to the damage of finished molds that may be in the vicinity. However, it is not safe to set up finished molds with hanging sand in the neighborhood of a machine of this type.

They are made with fixed and variable length of stroke, and with and without means to use the air expansively. Wilfred Lewis (*Trans. A. S. M. E.*, 1910, vol. 32, p. 794) gives the **strokes** used in practice as from 3/8 to 4 in., with an average of perhaps 2½ in.

Commercial sizes range from 3 to 36 in. in cylinder diam., 15 × 20 to 96 × 144 in. in platen dimensions, and from 350 to 50,000 lb. rated capacity with 80 lb. air pressure.

Shockless Jarring Machine. The Tabor Mfg. Co., Philadelphia, Pa., has recently developed a jar ramming machine, known as the "shockless," in which the table is mounted on a plunger fitted into a cylindrical base and supported upon vertical springs. When the table is lifted by air pressure the springs are compressed. When the air is released the dropping table is met by the rising anvil. Both come to rest with a ramming effect on the sand, but with little or no shock to the foundations and surroundings. (Wilfred Lewis, *Trans. A. S. M. E.*, 1910, vol. 32, p. 789.)

Commercial sizes range from 6 to 36 in. in cylinder diam., 24 × 32 to 96 × 144 in. in platen dimensions, and 1000 to 50,000 lb. rated capacity with air at 80 lb. pressure.

Combination Jarring and Roll-over Machine. Combination machines are built having a jarring machine for ramming, a roll-over attachment, and a pattern-drawing mechanism. They are **adapted for large "side-floor" work**, or for molds where the ramming, finishing and handling time is worth saving. Commercial **sizes** of machines made by the Tabor Mfg. Co. are given in Table 1.

Table 1. Molding Machine Weights and Prices

(From information supplied by the Tabor Mfg. Co.)

Machine	Weight, lb.	Approx. price per lb.	Machine	Weight, lb.	Approx. price per lb.
Small power squeezer..	800	$0.20	24-in. roll-over.......	800	$0.34
14 × 16-in. split-pattern................	1,600	0.25	Shockless jarring machines:		
Plain jarring machines:			6-in. cylinder......	3,000	0.27
6-in. cylinder........	1,000	0.40	20-in. cylinder.....	30,000	0.08
20-in. cylinder......	10,000	0.12	36-in. cylinder.....	100,000	0.075

Special Molding Machines. In many foundries special molding machines have been developed for individual pieces. Among these are **machines for** making cast-iron **pipe fittings** and **gears.** Of the latter, those for small gears use a complete pattern, while those for large gears often use a block representing a single tooth space.

These machines are of **two general types**, one having resemblance to a vertical boring mill in that it has two housings extending upward from a base and a cross rail along which the head carrying the pattern block can be moved to get the proper setting for the various sizes of gears within the capacity of the machine. The second type resembles a radial drilling machine in that it has a single arm carrying the head for the tooth block. In both of these types the table can be rotated by an indexing mechanism to space the mold for any number of teeth desired.

Automatic Molding Machines. A limited use has been made of automatic molding machines in which the **operations are performed in sequence by the machine itself.** The sand is conveyed to the top of the machine by bucket conveyor, is fed into a hopper, from which it is discharged in proper quantities into an empty flask on a feeding table. After the flask is filled with loose sand, a back board is slipped on, the mold is squeezed, the pattern drawn and the molded half flask is ready to be taken away by helper.

These machines are only adapted for special lines of work, but are capable of producing a very large number of flasks per day under favorable conditions. They are used for both iron and brass molding. The Berkshire Mfg. Co., builders of the Berkshire automatic molding machine, report **outputs** varying from 60 to 100 molds per hour.

Molding Sand

The individual grains of molding sand should constitute approximately 90 per cent. of the mass, and be completely covered with a bond of alumina or clay. The more uniform the size and shape of the grains, the better is the porosity in relation to the average size of the grains.

A. E. Outerbridge, Jr. (*Trans. A. S. M. E.*, vol. 29, p. 868), gives the following **formulæ for molding sands**, known as strong, special strong, and fine sand; proportions by volumes:

	Lumberton sand (new) Strong	Lumberton sand (new) Weak	Gravel (new)	Floor sand (old)	Fine floor sand (old)	Coal dust
Strong sand	14	..	7	6	..	2
Special strong sand	9	..	14	..	..	2
Fine sand	..	14	..	..	4	2

The **strong sand is used for** the majority of **large molds**, such as planer beds, housings and the like, particularly the principal parts of large machine tools. Most of these **molds** are **skin-dried**, that is, baked on the surface by means of portable drying ovens, after having been wet-blacked.

The **special strong sand** is **used** only **for molds for** the very **heaviest castings**, such as large anvil blocks and the like. These **molds** are **wet-blacked, and** when **baked** are as hard as a baked loam mold.

The **fine sand** is **used for** all **light castings** and for much medium work. These molds are all **green-sand molds**, *i.e.*, are not dried before use. It is essential that molding sand be carefully mixed before put into use.

In Mr. Outerbridge's **tests with strong sand**, bars 1 × 1 × 6 in. were used, (*a*) with sand that had been dampened and turned over several times with a spade, and with sand that had been run through a centrifugal mixer from once (*b*) in the case of the first bar, to 10 times (*c*) in the case of the last bar. These bars of sand were slid over the edge of a plate until they broke. The length of the overhang was taken as indicative of the strength of the sand. These overhangs varied in length from 2½ in. for case (*a*), to 3 in. for (*b*), and to 3¾ in. for (*c*). The corresponding increases in strength were 75 and 225 per cent., respectively, over the spaded mixture.

CORE-MAKING MACHINES

Core-making Machines. Two general types of core-making machines are in use. The first produces cores of a uniform section which may be circular, square, or of other shape. The second produces cores of more complex form, and in principle is identical with some of the molding machines previously described. **Screw-feed machines** are used largely **for making plain cylindrical cores.** The mixed core sand is fed into a hopper communicating with an Archimedean screw driven either by hand or power, which forces the sand out in a compact core of any length. A second kind of machine of this type uses a **plunger** for forcing out the core instead of a screw.

It is not unusual to mount these machines to feed the core down a slight incline. The core as it is fed is received upon a metal plate which acts as a guide to keep it straight. After the plate is filled, it is taken to the oven for baking. From the long cores thus produced, pieces are cut off to any length desired.

The second type of machine is employed in making large cores, and includes **simple squeezers and jolt ramming machines.**

Core Ovens. **Permanent ovens** are usually of brick or concrete, fitted with iron shelves on which the cores are placed to be baked, or provided with a track in the floor to receive cars carrying large cores.

Portable ovens are of sheet iron or steel and are placed over very large cores at the points where these are made. There are no standard sizes, except a few small-chamber shelved ovens for light work.

The essential feature in **core-oven construction** is to maintain a uniform temperature throughout the entire drying chamber. There should not be a difference in the various parts of the oven exceeding 60 deg. fahr. Furthermore, there should be a proper circulation of hot air through the oven to insure carrying off steam generated during the first part of the period of baking, and supplying an ample amount of oxygen to oxidize and dry oil binders. A common error in core-oven design is to have the firebox too near the drying chamber. This renders some of the volume unusable because of its being so hot as to burn the cores instead of baking them.

The usual **core-oven fuels** are anthracite coal, coke, fuel oil and gas. For some classes of binders, **steam-heated ovens** are excellent, as their construction nsures easy control of the temperature and precludes the possibility of its rising above a desired maximum. The fuel economy of steam-heated ovens is very low. For this reason direct firing of some other fuel is usually resorted to.

Core Sands. (See paper by Henry M. Lane, *Trans. A. S. M. E.*, 1911, vol. 33.) Cores are composed of sand and a binder which hardens under the action of heat and binds the sand grains together into a hard mass. A core **must be** sufficiently **porous to vent freely.** The individual grains should be bound one to another at the contact points only. **Rounded grains** give a maximum area of the vent passages. For different metals and mixtures, different-sized sand grains must be used.

Core Binders. The demand for large output in this country has brought about the extensive use of green-sand molds requiring an extensive use of **baked cores.** In Europe, skin drying is common practice and **dextrine** or "core gum" is the common core binder. Pea meal, rye flour, and linseed oil are also used.

In America an extensive line of **prepared binders** is on the market. These are divided into **two** general **classes,** depending upon their action on the sand: the true pastes which do not flow to the contact points of the grains of sand, and the binders that do flow.

Molasses has an extensive use but is open to the objection of a lack of uniformity in quality from deterioration through fermentation and of the danger of burning unless care is used in regulating the baking temperature. If underbaked, the molasses is not hardened and does not properly cement the sand grains; if overbaked, it is burnt or charred and its strength is lost.

Another water-soluble bond extensively used is **glutrine.** It forms an excellent emulsion with clay and tends to carry to the contact points of the sand grains.

The flour and dextrine mixtures are pastes which form in small masses on the faces of the sand grains and dry thereon. They do not flow to the contact points. Thus, much of the binder is inactive as binding material and is located in such a way as to tend to reduce the area of the vent passages.

Resin and pitch are also used to bind by melting and flowing over and between the grains, collecting to a certain extent at the contact points as the core cools. But in this respect they are not as efficient as oil.

Oil of a proper quality is undoubtedly the strongest, weight for weight, and the most **efficient binder** when used with clean silica grains. Linseed oil is the best for cores. When thoroughly mixed with the sand each grain is uniformly coated with oil or an emulsion of oil and water. In baking, the moisture is driven off and the oil tends to accumulate at the contact points of the grains only.

Table 2 gives the **relative strength of** a number of core **binders** and the results of a determination of their solid contents.

Table 2. Percentages of Solids in Liquid Core Binders

Name	Dried 24 hr. at 212 deg. fahr.	Dried 1 hr. at 400 deg. fahr.	Burned to ash	Remarks	Relative strength of binders
Glutrine	51.47	41.98	8.03	Skin	....
Raw linseed oil	100.22	96.98	0.22	No skin	....
Boiled linseed oil	93.49	90.23	0.54	Skin	32
Soya bean oil	100.91	96.45	0.65	No skin	....
Fish oil	100.52	92.70	1.20	No skin	14
Paraffin oil	72.05	33.36	0.11	No skin	Negligible
Corn oil	100.72	95.35	0.42	No skin; crawled	20
Cottonseed oil	100.68	96.08	0.07	No skin	16
Light tar oil	31.18	11.65	0.09	Slick; crawled	4
Heavy tar oil	67.21	38.27	0.06	No skin; crawled	26
Resin oil	62.89	19.59	0.06	No skin; crawled	1
Crude tar oil	51.83	34.32	0.06	No skin; crawled	Negligible
Resin oil	63.64	10.12	0.03	No skin; crawled	2
China wood oil	102.20	98.75	0.03	Skin	54

Oil-Sand Cores for the most exacting requirements have a tensile strength of at least 75 lb. per sq. in. In view of the high cost of linseed oil, foundrymen have turned to other varnish oils, including China wood, soya bean, corn, and cottonseed oils. If made from green sand and oil they have no tendency to absorb moisture and will **retain** their **strength for an indefinite time.** Cores with this composition have been used when four years old and seem to give as good results as when new. Core mixtures in foundry use vary in **tensile strength** from 5 to 150 lb. per sq. in.

Baking Temperatures for Cores made with flour, starch or dextrine binders should be from 350 to 375 deg. fahr.; under no circumstances should they exceed 400 to 410 deg. fahr. For pitch and resin binders a temperature of 350 to 400 deg. fahr. is suitable. A temperature of 400 deg. fahr. will destroy the dextrine which is a component in many such compounds. Resin oil does not distill much below 550 deg. fahr.; at 640 deg. fahr. 90 per cent. will be driven off. Cores bonded with oil will not as a rule be injured up to 500 deg. fahr., though a temperature from 400 to 410 deg. fahr. is sufficient. All binders containing carbon begin to char and to lose strength at 600 deg. fahr.

FORGING MACHINES

General Data on the Hot-forging of Metal. The tensile strength of a metal determines to a certain extent its resistance when being forged. The strength decreases with an increase of temperature but the decrease is not appreciable for wrought iron and steel below 375 deg. fahr. For the **relation between temperature and tensile strength**, see pp. 486 and 543.

The **forces required to produce deformation** are greater than the resistances indicated by strength tests: first, because the deformation occurs more quickly than the application of the load during test; second, because the density of the structure of wrought iron and steel increases during the working process; third, because the dies and tools abstract a considerable amount of heat from the piece being worked.

Let S = tensile strength at forging temperature, lb. per sq. in.; K = resistance to be overcome in forging, lb. per sq. in.; P = total forging pressure, lb.,

A = area of surface under pressure, sq. in. (projected area if surface is inclined to direction of pressure); and C = factor from experience (always greater than unity). Then, $K = SC$, and $P = AK = ASC$. If the piece is to be flattened out to a considerable extent, $P = 2AK = 2ASC$. The resistance is still further increased if there are deep impressions in the die to be filled.

If the pressure P is due to a blow, as from a hammer, the value of C is high, up to 10. If the pressure is slow-acting, as from the ram of a press, C = 1.4 to 2. If d represents the distance (in.) traveled by the die in the operation of forging, the work W expressed in ft-lb. is

$$W = dP/12 = dASC/12 \tag{1}$$

The **work required to bend a** sheet or **plate** t in. thick, with a volume of V cu. in. **into a curved shape** with a radius of r in. is $W = K(t/r)V/48$ ft-lb.

Hot-forging Machines, hammers and presses are used for a wide variety of work, mainly on steel, wrought iron and copper where final shape and size are obtained by causing the metal to flow under the action of anvil or die faces.

Forging Hammers divide in the following classes: Helve, strap, belt and rope drop, board drop, steam drop, single-frame steam, double-frame steam, and pneumatic.

Helve Hammers (Bradley Type) are usually belt-driven and carry the hammer face or swage on the end of a beam. The belt is provided with a tightening device, treadle-controlled, permitting the operator to regulate the number and speed of the blows. They are **used for general** and duplicate **forging,** welding, plating, drawing, swaging, collaring, spindle-making, etc.

Commercial sizes are 15, 25, 40, 60, 80, 100, 200, 300 and 500 lb. The 200-lb. size requires approximately 2 h.p. to operate.

Strap Hammers carry the hammer slung from a strap, usually of leather. The control and operation are the same as for the helve type. They are **adapted for general work,** tool dressing and the like.

Commercial sizes are 15, 30, 50 75, 100, 125, 150 and 200 lb.

The Board Drop-hammer is one of the most important types of forging hammer and is used principally to produce **drop-forgings** of wrought iron, steel, and copper.

Let W = work of blow, in ft-lb.; H = weight of hammer and die, lb.; g = acceleration due to gravity (= 32.2 ft. per sec. per sec.); h = actual hammer stroke, ft.; and v = terminal velocity of hammer, ft. per sec. Then, if the hammer and die fall of their own weight, $W = (H/g)v^2/2 = Hh = 0.015Hv^2$ (approx.). Combining with formula (1) by making the work done in forging equal to that of the hammer blow, $Hv^2 = 5.5dASC$, and $Hh = 0.084dASC$.

The hammer head is fastened to a board (usually of hard maple) mounted between two pulleys or rollers, one or both of which are continually revolving. The hammer is lifted by pressing the rollers into contact with the board by means of a treadle-operated mechanism. When the treadle is released, the rolls spread and the hammer falls. Or, the hammer can be operated automatically, the height to which it is lifted being determined by an adjustable dog. Usually, drop-hammer boards are uniform in width for the entire length; occasionally, however, they are slightly tapering toward the upper end. It is essential that they shall be made from clear, straight-grained wood, otherwise they are liable to split. The **lifting power** $L = fF$ when the board is lifted by one roller (when both rollers are driven $L = 2fF$), where f = coefficient of friction of roll and board, and F = force with which rolls are pressed together. Since $f = 0.25$ (approx.), $L = 0.25F$ and $0.5F$ (approx.), respectively.

Part of the lifting power must at first accelerate the motion of the hammer head. Consequently, $L = bH$, where b is a constant which is always > 1.

The motion of the hammer is accelerated with the effort $(b - 1)H$ until the speed v has been attained. The friction driving wheels or belt must consequently slip at first. Disregarding frictional resistances, **time** (in seconds) **required to lift hammer** $= t_1 = [0.016v/(b - 1)] + (h/v)$, and the **duration of drop** $= t_2 = 0.124\sqrt{h}$. Generally, $v = 2.5$ to 3.5 ft. per sec.; $b = 1.2$ to 2; $h = 3$ to 6 ft., and $H = 100$ to 3000 lb.

Weight of hammer, lb.	250	400	600	800	1000	1500
Approximate horse power required	2	2.5	3	3.5	4	5

Steam Hammers may be divided into **three classes.** In the first, the hammer is lifted by steam and drops of its own weight; in the second, steam is admitted above the piston and through its expansion increases the force of the blow; in the third, live steam is admitted above the piston throughout the stroke and the force of the blow is from combined weight of the falling hammer and the pressure of the steam.

In the **first class,** that of the single-acting steam hammer, the lifting power $L = bH$. That part of the force of the steam equal to $(b - 1)H$ is used for accelerating the lifting motion continuing after the steam has been cut off as long as the pressure under the piston is $> H$. These hammers are **used** only **for very large work.** The **weight of** the **hammer** ranges from 25 to 125 tons, and the value of b ranges from 1.5 to 1.2. The disadvantage of this type lies in the fact that the height of the clearance under the piston is directly dependent upon the thickness of the piece of work.

In the **second class,** in which steam is admitted above the piston and allowed to expand, there is an economy in steam consumption, a greater acceleration of the hammer head, and a larger number of blows per unit of time. The reliability of operation is not of the best.

To overcome this defect in operation the **third class** was developed, in which live steam is admitted above the piston. Here the **weight of** the **hammer** head varies from 1 to 25 tons. The control is such that the weight of the hammer itself is available for light blows, and for heavier blows steam is admitted above the piston.

In comparatively small hammers where the head weighs from 150 to 2000 lb., $b = 2$ to 3.5, and the diameter of the piston rod equals 0.5 to 0.65 of the diameter of the piston. The area of the upper piston face is consequently from 1.3 to 1.7 times that of the lower face. Up to 350 **blows per min.** can be obtained, depending on the length of the stroke and the tightness of the stuffing box. These hammers are provided with an automatic reversing gear. The **number and force of** the **blows** can be regulated by throttling the steam, changing the center position of the operating valve and changing the backlash in the operating gear. The frame of these hammers is usually C-shaped.

The **anvil** in small hammers is usually a single casting. In large hammers it is usually divided into upper and lower anvil blocks. In smaller hammers the anvil is connected with the shears and upper part of the machine. In the larger hammers, however, this is not the case, as the concussions tend to injure the hammer mechanism. For good practice the **weight of** the **anvil** ($= Q$) for hammers used in forging iron is at least eight times the weight of the hammer head; for forging steel, at least 12 times.

The pressure Q_1 exerted by the anvil block on the surface which it supports is assumed to be as follows: For blooming hammers, $Q_1 = (30 \text{ to } 60)hH + Q$; for billet-forging hammers, $Q_1 = (60 \text{ to } 95)hH + Q$; for hammers for steel forging, $Q_1 = (95 \text{ to } 125)hH + Q$.

Commercial Sizes. From mechanical arrangement steam hammers are called steam drops, single-frame and double-frame steam hammers. Sizes of steam drops range from 400 to 7000 lb. Sizes of single- and double-frame hammers range from 250 to 4000 lb. and 1500 to 30,000 lb. respectively. A series of small double-frame steam hammers, known as "tilting hammers," is also made. They are used chiefly by forges producing the smaller sizes of steel bars, as squares and octagons of tool steel. Uniformity in force of blow and length of stroke are the impertant features. Commercial sizes range from 500 to 2500 lb.

Pneumatic Hammers. A self-contained type of pneumatic forge hammer (the Bêché) has an air-operated ram with an air-compressing cylinder integral with the frame. The ram is raised by admitting compressed air beneath the ram piston; at the same time a partial vacuum is caused above it. The ram is forced down by a reversal of this action.

This type is made in a number of different models by the Nazel Engineering and Machine Works, Philadelphia. Tests by the maker give the force of the blows for the various sizes, and capacity in maximum sizes of stock, as follows:

Size, lb	66	165	250	350	500	770
Blow, in ft-lb	268	948	1338	2351	4116	6452
Maximum diameter of stock forged, in	2	4	5	7	8	9

In exceptional cases, where very pure water or oil was used, the pistons have been accurately ground in place and used without packing. Many ingenious methods of control have been developed, in some of which all of the motions of the hammer are controlled by a single lever.

Energy of Hammer Blows. Where the striking velocity of a hammer ram is greater than 10 ft. per sec., the energy E of the blow (in in-lb.) may be determined from the compression of lead cylinders. From experiments made by W. T. Sears, of the Niles-Bement-Pond Co. (*Am. Mach.*, Mar. 10, 1910), using $1\frac{1}{2} \times 1\frac{1}{2}$-in. cylinders, the following results were obtained (c = compression or shortening of cylinder, in.):

c (in.)	0.2	0.4	0.6	0.8	1.0	1.2	1.3
E (in.-lb.)	900	3000	6000	10,500	18,000	34,000	54,000

For any value of c close results may be obtained from the formula $E = (10{,}800c - 870)/(1.55 - c)$. Where the speed of compression is slow, as in **presses**, it is necessary to know the speed to estimate the energy. Mr. Sears gives the following values at two rates of speed, and also quotes static-pressure results obtained at Purdue University:

c (in.)	0.2	0.4	0.6	0.8	1.0	1.2	1.25
E (0.0005 ft. per sec.)	600	2100	4000	7000	11,100	19,200	23,500
E (0.00007 ft. per sec.)	400	1600	3000	5000	8000	14,800	18,000
E (Purdue)	400	1400	2600	4100	6600	12,000	15,000

Riveting Machines divide themselves into **two classes**, one resembling steam hammers in that a series of blows is struck to form the rivet head, and the other resembling forging presses in that the snap is slowly guided against the rivet, the other end being held in the rivet die.

These machines are **operated** either **by compressed air or** by **hydraulic pressure.** The frames are usually C-shaped, and may have a **length** up to 15 ft. They may either be set in a vertical or horizontal direction, depending upon the nature of the work, or the riveting machine may be suspended or moved about to any desired position. The usual **air pressure** is from 80 to 100 lb. per sq. in. **Riveting pressures** are commonly taken as 150,000 lb. per sq. in. of the rivet section for hot-working, and 300,000 lb. per sq. in. for cold-working. The plates should be pressed together by a **pressure** of from 0.3 to 0.4 that used in riveting.

Forging Dies. Drop-forge dies are of steel, steel castings and cast iron. A good all-around grade of **steel** is a 0.60 per cent. carbon open-hearth. Dies of this will forge mild steel, copper and tool steel satisfactorily if the number of forgings required is not too large. For a large number of tool-steel forgings tool-steel dies of 0.80 to 0.90 per cent. carbon may be used, and for extreme conditions, 3½ per cent. nickel steel.

For very large pieces with deep inpressions, **cast-steel** dies are sometimes used. For large dies liable to spring in hardening, 0.85 **carbon steel** high in manganese is sometimes used unhardened.

A very recent development is the use of **cast-iron dies.** Several large shops are experimenting with them and report encouraging results. In some cases they have outlasted steel dies on the same kind of forging at a die cost of only ⅙ to ⅟₁₆ that of steel dies.

Good **die-block proportions** for width and depth are

Width, in.	8	10	12	14
Depth, in.	6	7	7	7 or 8

For ordinary work, 1¼ in. of metal between impression and edge of block is sufficient.

Dimensions of **dovetailed die shanks:** For hammers up to 1200 lb., 4 in. wide and 1¼ in. deep, with sides dovetailed at angles of 6 deg. with the vertical; for hammers from 1200 to 3000 lb. in size, 6 in. wide and 1½ deep, with 6 deg. angles.

The **minimum draft for** the **impressions** is 7 deg., although for parts difficult to draw this may be increased up to 15 deg. It is not uncommon to have several drafts in the same impression.

Open-hearth and tool-steel **dies** are **hardened** by heating in a carbonizing box packed in charcoal and dipping face downward over a jet of brine. The jet is allowed to strike into the impression, thus freeing the face of steam and producing uniform hardness. After hardening, they are drawn in an oil bath to a temperature of 500 to 550 deg. fahr.

The forging production per pair of dies is largely affected by the size and shape of the impression, the material forged, the material in the dies, the quality of heating of stock to be forged, and the care exercised in use. It may vary from a few hundred pieces to 50,000 or more. A normal **life** for a pair **of dies** under average conditions may be 20,000 pieces.

BENDING, FORMING AND SHEARING MACHINES

Plate-straightening machines. In these, the plate or sheet is passed through a system of **rolls,** the lower row generally consisting of three rolls in stationary bearings with a common drive, and the upper row consisting of four adjustable rolls. These can usually all be adjusted in one operation, but in some cases the two outside ones must be independently adjusted. Work-supporting rolls are also usually provided. For **rolled sections** such as beams and angles, suitably grooved rolls are used.

The horse power required for plate-straightening machines is as follows:

Thickness of plate, in.	0.25	0.4	0.6	0.8	1.0	1.2	1.4	1.6
Width of plate, in.	48.0	52.0	60.0	72.0	88.0	102.0	120.0	140.0
Diameter of rolls, in.	5.0	8.0	10.0	12.0	13.0	14.0	15.0	16.0
Horse power (approx.)	6.0	8.0	12.0	20.0	30.0	55.0	90.0	130.0

Power required for angle-iron-straightening machines: For 4-in. angles, 12 h.p.; for 6-in., 18 h.p.; for 8-in., 25 h.p.

Power or hydraulic presses are used to straighten **large rolled sections.** The presses make from 20 to 30 strokes per min., and the amount of flexure is regulated by inserting wedges or pieces of flat iron. The beams are supported on rolls so they can be easily handled. The **power required for presses** of this kind is as follows:

Depth of girder, in.	4	6	8	10	12	16	20	24
Horse power (approx.)	3	5	7	11	13	19	23	35

Horizontal Plate-bending Machines consist of two stationary rolls and a third vertical adjustable upper roll which can be fitted obliquely for taper bending, and is held in bearings with spherical seats. The diameter of the rolls can be approximately determined from the equation $r^2 = bt$, in which r is the radius of the roll, b the width of the plate or sheet, and t its thickness, all in in.

The **power requirements** of horizontal plate- and sheet-bending machines are as follows:

Thickness of plate, in.	0.5	0.6	0.8	1	1.2
Horse power for plate 120 in. wide	10.0	12.0	18.0	27.0	40.0
Horse power for plate 240 in. wide	30.0	30.0	40.0	55.0	75.0

Vertical Plate-bending Machines have a hydraulically operated piston which moves an upper and a lower pair of rolls between inclined surfaces of the stationary upright and the cross head. The bending is done piece by piece against a second stationary upright. Heavy ship plates are rigidly clamped down and bent by a roll operated by two hydraulic pistons. For angular bends or for the production of warped surfaces, the pistons can be operated independently or together. In vertical machines, angles and other rolled shapes are bent between suitably shaped rolls. Pipes are filled with sand to prevent flattening when being bent. For some work, pipes are bent hot between suitable forms operated by hydraulic pressure.

Drawing Presses. Eccentric or crank **presses for rough punching and rough drawing** of comparatively short pieces are provided with tools to cut out or form in single or successive operations. Chain drawing machines **(draw benches)** are generally used to produce drawn pieces of considerable length. The **number of draws and** the graded **size of** the **tools** depend largely on the quality of the material. If d is the original diameter, and d_1 that after drawing, then, according to Musiol, $d_1/d = m$, in which m is a factor depending on the diameter, thickness and nature of the work. The average value of m for the first draw is 0.5 to 0.63, and for subsequent draws 0.65 to 0.85.

In making metal molding and parts for metal building trim and fireproof features, **draw frames** are largely used. Two **types of dies** are employed, the first being solid with an opening or openings, the final one giving the finished shape of the desired section. In the second form, hardened steel rolls are used, the number of sets depending upon the shape and size of the final section. These rolls are not driven by power but are free to turn in suitable bearings. The raw material is strip. It is customary to weld the ends of strips together, either electrically or by the oxy-acetylene process. In this way a strip of any length desired can be produced.

Otto S. Beyer (*Am. Mach.*, vol. 34, p. 447) gives the following formula for **determining the dimensions of drawn shells:** $h = D[1 + (d/D)^2]/4(d/D)$, in which D = diameter of the blank, d = shell diameter, and h = shell height—all in in.

Experience shows that the ratio (h/d) must not exceed 0.5 for the first operation with shells of steel, and when h approaches ¾ D, annealing becomes necessary for further operations. R. R. Pratt (*Am Mach.*, vol. 31, Part 2, p. 858) gives the smallest diam. of cup that can be drawn from a given soft-steel blank as equal to $D/1.9$, and for successive reductions, $D_1/1.28$.

Screw Presses. In the screw press the ram is forced downward by a steep-pitch-threaded spindle having on its upper end a heavy flywheel. The **effective power** is that due to the energy in the revolving parts (flywheel and spindle). The energy produced by the parts in downward motion (flywheel, spindle and ram) is relatively small, and can be disregarded. If I_f is the moment of inertia of the flywheel rim, I_s the moment of inertia of the spindle, v the angular velocity of both, and P the work done in stamping, then $P = \frac{1}{2}I_fv^2 + \frac{1}{2}I_sv^2$. The friction between the ram and the spindle and between the ram and its guides may be disregarded.

Hand-operated screw presses are used for stamping, embossing, lettering and forming on work requiring more power than can be easily developed in foot presses. The larger sizes are used for "hubbing" dies and punches. Some forms are provided with a lever and balls instead of a flywheel. In Europe, this type of press is frequently **power-driven** through a friction device acting on the flywheel.

The hand-operated machines are made in two types, the "arch," and "overhanging pattern." Commercial **sizes** are from 5 × 5 in. to 10 × 10 in. for the opening in the bed, and from 65 to 700 lb. for flywheel weight for the arch pattern. For the overhanging pattern, the bed opening ranges from 5 × 5 in. to 15 × 16 in., and the flywheel weight from 65 to 500 lb.

Hydraulic Presses using high pressures are **used to shape hollow objects,** as silverware, long tubular pieces, etc., **and for extruding metals.**

For hollow ware the work is placed in a die (or mold) and protected by packing (rubber, cement, putty, etc.) against the infiltration of water between it and the die. Both work and die are then introduced into the press cylinder and subjected to pressure, which forms the metal to the mold and makes it take the desired shape. The pressure required depends upon the elastic limit and yield point of the material, and may be as high as 120,000 lb. per sq. in.

Drop Presses are extensively **used in drawing and forming sheet-metal** articles, hardware, cutlery, silverware, etc., and for stamping and embossing thin sheet metal used in building construction. They are made **in sizes** having the following hammer weights: 50, 80, 120, 175, 225, 400, 600, 800, 1,000, 1,200 lb.

The **efficiency of drop presses** is given as 90 per cent. by R. H. Thurston (*Trans. A. S. M. E.*, vol. 5, p. 53), and of **drop-hammers** as 70 per cent. by A. W. Moseley and J. L. Bacon (*Trans. A. S. M. E.*, vol. 27, p. 605).

Spinning Lathes perform operations similar to drawing, that is, the forming of sheet metal cold, starting either from a flat blank or a shell that has been drawn in a press. They are used for seamless-drawn tin and sheet-iron ware, and hollow objects of brass and aluminum, as automobile horns, phonograph horns, and for some silversmithing work.

The **work** is formed into a mold or metal or wooden chuck in inside spinning, or around shaping rolls in outside spinning. Hand **tools** and burnishing rolls are used to press and draw the metal into shape. Turning, beading and wiring are also done on these machines in the finishing of hollow ware. They are made commercially in the following **sizes:**

Swing over bed, in.	16	20	24	30	38
Length of bed, in.	42	60	70	90	132

Punch Presses and Shears. In subsequent paragraphs the following notation is used: t = thickness of sheet, in.; d = diam. of punched hole, in.;

d_1 = diam. of punch, in.; d_2 = diam. of hole in die, in.; P = resistance at cutting edge, lb.; s = shearing stress, lb. per sq. in.; W = approximately $1.7s$.; h = stroke of punch or shear blade, in.; u = cutting speed, ft. per min.

The following tabulation gives the values of s and $1.7s$ for common sheet metals:

Metal	s Lb. per sq. in.	$1.7s(= W)$ Lb. per sq. in.
Sheet steel, soft	57,000	85,000 to 100,000
Wrought iron	34,000 to 50,000	57,000 to 85,000
Wrought iron heated to a dark red	11,000 to 14,000	17,000 to 28,500
Sheet copper	28,500 to 35,500	35,500 to 57,000
Sheet zinc	8,500 to 12,800	12,800 to 21,500
Tin	1,850 to 2,850	2,850 to 4,250
Lead	1,140 to 2,560	2,130 to 3,400

To obtain **smoothly punched holes**, make $d_1 = d - 0.005t$, and $d_2 = d + 0.005t$; or $d_1 = d$, and $d_2 = d + 0.01t$. To avoid **burrs**, the clearance between punch and die must be kept down to a small amount. Both punch and die are slightly tapered away from the cutting edge to reduce friction. The **maximum resistance** in lb. at the cutting edge of the punch is $P = Wd\pi t$. The **stroke** h is generally taken between $2t$ and $3t$, and the **cutting speed** u from 3 to 4 ft. per min.

The **horse power required to punch** is $N = N_0 + N_1$, where N = total h.p., N_0 = friction h.p. of machine, and N_1 = h.p. used in punching the metal. For thickness of sheet t = 0.16 to 2.2 in., Hartig gives for wrought iron $N_1 = 0.513fF$, $f = 0.167 + 0.245t$, where F = area of sheared surface produced per hour, in sq. ft. (= $1.31\ ndt$; d = diam. of punch in in. and n = number of holes punched per min.).

Thickness of plate, in.	0.40	0.80	1.20	1.60
No. of holes punched per min.	10.0	9.2	8.3	7.5
Friction h.p. of machine = N_0	0.16	0.32	0.55	0.82
H.p. per sq. ft. of surface sheared per hr. (= f)	0.265	0.363	0.460	0.558

Plate Shears have the angle of the front cutting edge B about 75 deg. (see Fig. 1), the clearance angle Y about 2 deg., and the angle x between the edges of the shear blades from 5 to 10 deg. For shears with parallel blade edges, the **maximum pressure required** is $P = Wbt$. For shear blades inclined to each other, $P = 0.225Wt^2/\tan x$. For x = 9 deg., $P = 1.4Wt^2$. The **cutting speed** varies from 3 to 6 ft. per min.

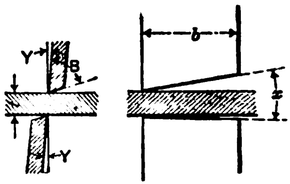


FIG. 1.—Angles of Shear Blades.

Power Required for Punch Presses, straight side, eccentric type:

Thickness of plate, in.	0.32	0.4	0.6	0.8	1.0	1.2	1.6
Diam. of hole, in.	0.65	0.8	0.88	1.04	1.2	1.4	1.6
Approximate horse power	3.0	4.0	7.0	10.0	14.0	25.0	40.0

Circular Shears are used for cutting sheets and plates up to ¼ in. in thickness. The diam. of the blades should $\geq$ 70 times the thickness of the sheet. Circumferential **cutting speed** is from 90 to 180 ft. per min. **Power requirements** are as follows:

Thickness of plate, in.	0.1	0.15	0.3
Gap or opening of throat, in.	12.0	20.0	28.0
Approximate horse power	3.0	5.0	8.0

These machines are made in a wide variety of styles and sizes for all classes of work to be punched and sheared. The tools are also made in great variety of form, and the output is dependent upon so many factors that it is impossible to generalise in regard to it.

WELDING MACHINES AND APPARATUS

Welding processes may be divided into pressure welding and casting welding. In the first category is electric resistance welding; in the second are electric arc welding and all of the gas or flame welding processes.

Electric Welding

Resistance Electric Welding Machines. Resistance welding (Thomson process) involves the passage of an electric current through the abutting parts of the pieces of metal to be welded. Heat is generated at the points of contact, where the greatest resistance to the flow of the current is encountered. Meanwhile, pressure is applied to force the pieces together and form the weld.

Various forms of machines are built, the main element in all being a transformer provided with a pair of clamps (each insulated from the other) for holding the pieces to be welded, and a mechanical, hydraulic or other device to force the abutting ends together. In some forms there are additional devices (striking dies) to shape the weld after it has been formed and to remove the fin. The pieces of work begin to heat at the interior, thereby tending to expel impurities from the weld. No current is used except when heating is being done.

In range these machines have a **capacity** from small wire to sections 3 in. square. A reactive coil is used to control the current for varying sections of stock. In the smaller types pressure is applied and the current shut off automatically. In all but the smaller types a circulation of water is maintained in the secondary coil of the transformer. Only **alternating electric current is used**, taken from a single phase of constant potential and usually from 40 to 60 cycles and 100 to 500 volts.

The process and machines are particularly **adapted to butt welding** of small and similar sections, although by no means confined to these. Common applications are the welding of metal tires, hoops, bales, parts of the running gear of wagons, single-pointed cutting tools, automobile parts, parts used in street railway construction, and a wide variety of products made from wire, such as fencing, screens, chain, and the like. Machines have been made for welding pipe and tubular sections such as house hot-water heaters.

The horse power and time required to make a given weld vary nearly directly as the cross-sectional area. Within certain limits, the greater the power the less the time, and *vice versa*. A ¾-in. round steel bar can be welded in 15 sec. using 15 kw. of current. The same bar can be welded in 6 sec. using 23 kw.

A variety of metals can be welded either to themselves or in many cases to other metals. In practice, iron, steel and copper are the most commonly worked upon. Copper requires more power and less time to weld a given section than iron or steel.

Table 3 gives data for power consumption and seconds per weld for iron and steel rods, while the following values, from German sources, are for copper:

Area of section, sq. in.	0.031	0.062	0.124	0.233	0.388	0.775
Power required, kw	1.5	3.0	7.5	15.0	40.0	60.0

Some of the latest applications of the process are in welding platinum points on steel or brass pins, and welding halves of casters, handles, etc., in semi-automatic machines to make the whole article. Many manufacturers of hollow ware attach handles or bail ears by "spot" welding.

Another method of incandescent welding is that of La Grange and Hoho, in which the bar to be heated is attached to the negative pole of an electric circuit and then plunged into a tank containing acidulated water and a metal plate connected to the positive pole of the circuit. Strictly speaking, it is a method of heating preparatory to welding, for the welding proper must be done by mechanical means similar to those employed for welding pieces that are fire-heated.

Table 3. Power and Time Required for Electric Butt-welding Iron or Steel Rods

(Thomson Electric Welding Co.)

Diam. of rod, in.	Kilo-watts required	Seconds per weld	Approximate kw-hr. for 1000 welds	Diam. of rod, in.	Kilo-watts required	Seconds per weld	Approximate kw-hr. for 1000 welds
1/4	4	2	2	1	20	20	113
3/8	6	3	5	1 1/8	25	24	167
1/2	9	6	15	1 1/4	30	33	275
5/8	12	10	30	1 1/2	38	40	422
3/4	15	15	65	1 3/4	48	48	640
7/8	18	18	90	2	60	60	1000

Electric Arc Welding. There are **three** different **processes** of electric arc welding, namely, the Zerener, in which an arc is drawn between two carbon electrodes; the Bernardos, where an arc is drawn between the work (metal) and one carbon electrode; and the Slavianoff, in which an arc is drawn between the work (metal) and one metal electrode.

These processes are made **use** of in steel foundries to fill blowholes and to weld on projections which were improperly formed by failure of the metal to fill the mold. Also to cut holes in metal, to cut bars, plates, or castings in two, to cut sink heads from steel castings, and for repair work on machine parts, particularly locomotives.

In the **Zerener process** an arc is first drawn between two carbons by bringing them into contact with each other and then separating them a given distance. The arc is then blown upon the metal by means of an electromagnet. As a result, the metal in the vicinity of the point of the arc is raised to a welding temperature. This apparatus has been called the **electric blowpipe**.

The metal to be welded is placed so that the arc can be readily directed upon it, the blowpipe being suspended from a chain or rope so that it may be moved as desired. Because of the difficulty in obtaining a close regulation of the arc—as well as of the construction of the apparatus, which does not lend itself conveniently to handling large currents—the Zerener process is **limited in its application** to a narrow range of small, rough work such as the welding of small steel and brass castings and wrought-iron plates, tubes and tanks.

In the **Bernardos process** the metal to be welded is made one terminal of a direct-current circuit, the other being formed by a carbon or sometimes a metallic electrode. The carbon or metallic electrode is brought into contact with the metal, thus closing the electric circuit. It is then instantly pulled away and an arc is thus drawn between the terminals. The temperature reached by the arc is between 5500 and 7000 deg. fahr. and the metal may be either entirely melted away, molded into a different shape, or fused to another piece of metal.

In spite of the best of care, hard welds are at times produced by the Bernardos and Zerener processes, due to the entrance of the electrodes. The **Slavianoff process** was developed in an endeavor to overcome this difficulty. It is a modification of the Bernardos process, in which an electrode of the same metal as the one being welded is substituted for the carbon electrode. In operating, the metal bar melts and drops on the place to be welded. This process is particularly **suitable for repairing broken** pieces of **cast and wrought iron.** For welding iron and steel, the electrode should be of the best grade of soft iron wire, not larger than 3/16 to 1/4 in. in diam. and about 1 ft. long. Moreover, it should be covered with a flux, applied in paste form and allowed to dry on before use. A **flux** originated by the Westinghouse Elec. & Mfg. Co. and used with good results particularly on cast and malleable irons, has the following percentage composition: Oxide of copper (CuO), 5; oxide of manganese (MnO_2), 15; oxide of iron (Fe_2O_3), 30; borax ($Na_2B_4O_7 + 5H_2O$), 50. The ingredients should be mixed, ground to pass through an 80-mesh sieve, and during welding applied dry. Direct current of about 130 amp., 24 to 26 volts across the arc, is satisfactory. The arc itself is short, not over 1/8 to 3/16 in. long.

In all of these processes **protective coverings** must be provided **for the operator,** as the light from the arc produces severe burns on exposed skin. This light is peculiarly injurious to the eyes.

Data on variations in voltage and current for arcs of different lengths, comparison of fire-welded and electric-welded bars and labor costs of blacksmith-welded versus arc-welded rings are given in Tables 4–6 (from article by C. B. Auel, *Am. Mach.*, Mar. 16, 1911, p. 479).

Table 4. Variations of Voltage and Current for Arcs of Different Lengths

Length of arc, in.	6	5	4	3	2
Volts across arc, including carbon	101	98	93	86	80
Current, in amperes	300	350	400	605	750
Size of carbon, in.	1×6½	....	....	....	....

Table 5. Results of Tests of Fire-welded and Electric-welded Bars

Brand and size F. = fire-welded E. = electric-welded Brand	Inches		Ultimate tensile strength per sq. in., T, tons	Contraction of area at fracture, C, per cent.	Extension in 10 in., per cent.	Ratio of weld to solid, per cent.	Figure of merit ($T \times C$) F.	E.
Lowmoor iron	2 × 3/16	F.	20.3	15.2	7.3	77.9	308	
		E.	21.1	17.3	7.3	81.1		365
Lowmoor iron	2 × 5/16	F.	21.5	22.3	11.3	90.7	479	
		E.	21.8	20.7	9.7	91.8		451
Netherton best iron	2 × 1/4	F.	18.4	10.1	3.4	84.4	185	
		E.	20.1	10.8	4.5	92.0		217
Parkgate steel	2 × 1/8	F.	20.9	9.3	1.9	69.1	194	
		E.	22.3	18.4	3.8	73.6		410
Parkgate steel	2 × 1/2	F.	20.4	15.9	8.1	82.3	324	
		E.	21.0	15.4	7.3	86.4		323
							1490	1766

Ratio of electric-welded to fire-welded bars = 1766/1490 = 118.5 per cent.

Table 6. Labor Costs of Blacksmith-welded vs. Arc-welded Rings

Section of ring, in.	1 × 1½	1¼ × 1½	1½ × 2	1¾ × 2½	2 × 6
Cost, smith-welded	\$0.59	0.66	1.13	1.25	3.05
Cost, arc-welded	\$0.51	0.30	0.45	0.45	0.85

With the **Bernardos process** a drilled hole 1¼ in. in diam., 2 in. deep in an axle cap was filled in 56 sec. A sink head was cut from an axle cap in 4 min. 45 sec.; cross-section cut, 2¼ × 6 in. A hole 1¾ in. in diam. by 1½ in. deep was burned in a wrought-iron plate in 3 min. 30 sec. (including 45 sec. used in reversing plate). Size of carbon in all three cases 1½ × 6 in.

The peculiar **field of arc welding,** according to C. B. Auel, is work of the rougher kind where appearance, finish and strength are not of paramount importance. Material of the same kind as the metal to be filled is usually used for **filling.** For wrought iron or steel this may be soft iron rod, punched iron scrap or broken parts of steel castings. For cast and malleable iron it may be soft iron rod, punched iron scrap, copper or special cast iron high in silicon. For brass it may be lead and zinc rods or castings of the same material.

Precautions to be Observed in Making Electric Arc Welds. The positive terminal should always be fastened to the metal to be welded; the negative terminal to the carbon electrode. The arc should be as long as possible. The weld should be made with the fewest possible applications of the arc. The arc should be given a rotary motion and should have as large an area as possible instead of being concentrated on one spot. The metal to be welded should be free from dirt and slag before commencing to weld. A suitable flux should be used. Vigorous hammering should accompany the making of each weld.

Gas Welding

Oxy-hydrogen Process. This is the older of the gas-welding processes. It makes use of a blowpipe flame from the combustion of oxygen and hydrogen gases, a temperature of about 4000 deg. fahr. being reached. Hydrogen must be in excess to avoid oxidizing the metal. The usual ratio is 4 volumes of hydrogen to 1 of oxygen. The flame has a great power of penetration and for this reason is capable of cutting considerable thicknesses of steel and wrought iron. It is especially **useful in welding thin sections** and metals of low fusibility. It has the advantage over processes giving a higher flame temperature that less skill is required in handling.

The essentials of the **apparatus** are a torch for producing the flame with suitable safeguards against back-firing. In some types the speed of flow of the gases at the orifice is greater than the velocity of flame propagation; in others, screens are introduced similar to the screens of miners' lamps. Torches with different-sized openings and kinds of tips are used for different work. Gas is supplied either from metal bottles or from generating apparatus, usually the former. Connection from the bottles to the torches is made by means of flexible hose so that the torches can be easily moved to the work. In addition, clamping devices are provided to hold the work.

Oxy-acetylene Process. In oxy-acetylene welding apparatus the flame, which has an intensely high temperature (6300 deg. fahr. maximum), is obtained by the combustion of acetylene and oxygen. There are two **types of torches** in use, usually spoken of as low-pressure and high-pressure, or better, as injector and pressure. In the first the oxygen is admitted under pressure and draws in the acetylene after the general principle of the steam injector. An objection to this form is the difficulty of maintaining the proper relative proportions of the two gases. In the pressure-type torch both gases are introduced under pressure, and the proportion of the mixture can be varied by varying the pressure of either.

As a rule, the tips of torches are inclined at an angle of from 45 to 60 deg. with the frame. In the injector type the **proportion of oxygen used to acetylene** is 1.5 or 1.7 to 1. The pressure type only requires 1.28 oxygen to 1 of acetylene.

Gas welding has found its most general **application** in working thin metal, as parts of steel railway cars, repairs on railway rolling stock, and the cutting up of large masses of steel, as the wrecked frames of buildings, bridges, and the like.

Metal thicker than ⅛ in. to be welded should be joined by **scarfing** or chamfering to give a V-groove in which to work. This permits the flame to penetrate to the bottom of the joint. It is necessary to add metal to fill, except in the case of very thin sheets which are not scarfed. This metal is melted in from a wire or strip usually of the same material as those being joined. In **making the weld,** after the metal contiguous to the joint is in a running condition, molten metal is added drop by drop until the groove is filled, and, where allowable, more is added to make the joint a little thicker and stronger than the normal section. If dissimilar metals are joined, it is usual to use a welding metal of the same material as the one which melts at the lower temperature.

Long seams are usually spot-welded or tacked at intervals of 6 or 12 in. Care must be taken to permit the work to adjust itself for expansion and contraction during the process of heating and welding. Iron castings are pre-heated in a forge or other fire before welding. The temperature of heating should not exceed 500 deg. fahr.

The flame processes of welding are especially valuable for metal of from No. 20 gage up to 5/8 in. thick. Plates up to 1/8 in. are butt-welded.

The process is also of value in **cutting**. Hydrogen gas torches have been used to cut plates up to 24 in. thick. Oxy-acetylene torches reach their limit at 12 to 15 in. **Acetylene** develops more heat, does the work more quietly, is cheaper and **more suitable for thicker articles than hydrogen.** It tends, however, to increase the amount of carbon in the weld.

Thus far but little has been done commercially in developing gas-flame-welding machines. A few have been made for cutting circular sections, consisting of mechanism to guide the torch in a circular path. Others have been developed to cut pipe, and still others resembling a radial drilling machine in arrangement for making long cuts or welding long seams.

The following **labor and material costs** for common locomotive repairs are given by H. W. Jacobs (*Am. Mach.*, Nov. 15, 1911, p. 913): Building up piston rods at cross-head fits, \$3.00; building up worn places on links, \$4.02; building up reverse lever, \$2.25; putting two teeth in reverse lever latch, \$0.71; putting two teeth in quadrant; \$0.71; repairing cracked cylinder heads, \$2.55.

Herbert L. Whittemore, in *Bulletin* No. 45, Engineering Experiment Station, University of Illinois, gives results of an extensive series of **tests on oxy-acetylene welds.** The following information is derived from that source.

Gas Consumption. Table 7 gives sizes, capacities and approximate gas consumption for Fouché or injector-type torches or blowpipes for the welding of steel plates. For copper plates, larger blowpipes are required for corresponding thicknesses.

Table 7. Sizes, Capacities and Approximate Gas Consumption of Fouché Blowpipes

Blowpipe No.	2	3	4	5	6	7	8	10	12	15
Thickness of plate, in.	1/32	3/64	3/32	1/8	3/16	1/4	3/8	1/2	5/8	3/4
Oxygen, cu. ft. per hr.	2	4	6	10	16	25	36	45	65	100
Acetylene, cu. ft. per hr.	1½	2½	3¾	6	10	15	22	28	40	60

Creep. The satisfactory allowance for "creep," or the drawing together of the edges as the metal expands, was found to be 2.2 per cent. Some blowpipe manufacturers recommend 2.5 per cent. If the allowance is too small the thin edges of the plate crowd together more or less with no bad results, but if the edges fail to meet it is almost impossible to weld at a satisfactory rate.

Strength of Welds. Strength tests on welds show that **85 per cent. efficiency** is about as high as can be expected from a weld of the same thickness as the plate. **Forging after welding increases** the **strength** apparently about 10 per cent. The **welding rate** is nearly constant at 17.5 sq. in. of welded section per hr.

Cost of Welding. With acetylene at 1 cent per cu. ft., oxygen at 3 cents per cu. ft., and labor at 30 cents per hour, the cost for beveled plates is about 25 cents per cu. in. of filler. The cost of operation rises very rapidly as the thickness of the plate increases, reaching possibly \$4 per hour for gas and labor on plates 3/4 in. thick.

Welding with Water Gas. This method is used especially for **welding thick plates** which cannot be easily heated in an open fire, where the heating is from only one side. If the sections are narrow, flame can be applied from both sides. Water gas for welding has a composition of 3 per cent. CO_2, 44 per cent. CO, 50 per cent. H and 3 per cent. N. The temperature of the flame is 3300 deg. fahr. The mixture takes place in a proportion of about 5 volumes of air to 2 of gas. For producing a thin flame, both gas and air are compressed.

Aluminothermic Welding

Thermit Welding. Very large sections are successfully welded by the thermit process, which employs a compound consisting of finely powdered aluminum and oxide of iron mixed in such proportions and manner as to give the reaction $Fe_2O_3 + 2Al = Al_2O_3 + 2Fe$. The temperature generated by this reaction is estimated at 5400 deg. fahr.

There are two general **methods** by which thermit is used. The first is designated **butt-welding** and is **used** chiefly **on pipes, tubes,** and **small rods.** The ends of the pieces to be united are closely fitted and fastened in contact by means of suitable clamps. The joint is then surrounded with a casting mold designed to hold just enough of the molten mass to bring the parts to be united to a welding heat. The necessary amount of thermit is then ignited in a small, flat-bottomed crucible and as soon as the reaction is over the contents of the crucible are poured into the mold. The aluminum oxide, on account of its low specific gravity, flows first, and coming in contact with the cold iron adheres to it. This forms a refractory coating which prevents the flowing molten steel from adhering to the mold or parts to be welded. As soon as the molten mass has been in the mold long enough to allow heat to penetrate to the parts to be welded (a period determined by experience), the pieces are forced together by the clamps and the weld completed. Rods up to 2 in. and pipe up to 6 in. in diam. are welded by this method.

In the second method, called **intermediate welding,** the parts to be united are not brought close together nor are they fitted in any way. Instead, a space varying from 1 to 2 in. is provided to permit a free flow of the molten thermit between the ends. After the parts have been thoroughly cleaned of grease, dust, etc., they are surrounded by a refractory mold similar to that used in making steel castings. The mold is provided with a pouring gate and riser, and also an opening at the bottom used for preheating.

If the parts to be welded are of a uniform section, the molds are made from wooden patterns as in standard foundry practice. If the parts are irregular, the wax method is used. In this the space between the parts to be united is filled with wax and a reinforcing collar is formed of the same material. As soon as the wax is hard, the parts are surrounded with a flask or mold box and the mold rammed in place, wooden patterns being used for the risers and gates. Heat is then applied through the preheating gate at the lowest part of the mold. This melts the wax, which flows out and leaves a cavity of the dimension desired. The mold is thoroughly dried and the ends to be united are brought to a bright red heat by the continuation of the preheating. The amount of thermit necessary to fill such a mold is found by weighing the wax and multiplying this weight by 32. A so-called automatic crucible is placed over the mold and into it are poured the thermit and filling metal. The latter consists of carbon-steel punchings, nickel, chromium, manganese, etc., to give a resulting steel of approximately the same analysis as the parts to be welded. This mixture after being melted is tipped into the mold. In making welds in this manner, precautions have to be taken to provide for the contraction of the metal.

This method has been **used for welding engine frames, crank shafts,** rudder posts of vessels, **and large castings** of every description. It has been extensively used for filling steel rails and for repairs which have to be done rapidly or in place. Table 9 gives the results of physical tests of thermit welds.

Table 9. Bending Tests of Thermit-welded Bars

Description	Diam. of bar, in.	Span, ft.	Total load, tons	Angle bent through, deg.	Effects
Solid bar	2	15	10.18	180	Uncracked.
Thermit-welded bar, bulb turned off	2	15	10.09	46	Broken at weld.
Thermit-welded bar, bulb left on	2	15	17.30	125	Bent as far as practicable. Uncracked.

Soldering and Brazing

Soldering is the operation of uniting two pieces of metal with a third soft metal that is applied in a molten state. Solders are alloys of lead and tin. W. F. Brannt (Metal Workers' Handbook) gives the following list of solders with their melting points (see also p. 552):

Composition	Tin	1	1	1	1	1	1	1½	2	3	4	5	6
	Lead	25	10	5	3	2	1	1	1	1	1	1	1
Melting point, deg. fahr.		558	541	511	482	411	370	334	340	356	365	378	380

The parts to be soldered are brought in contact and the melted solder is flowed over and into the joint. On small pieces the solder is melted by an "iron" which is copper-tipped. Heat may be from a small forge or furnace or from electric current. Large pieces are frequently dipped in a bath of melted solder. This is true of some parts of electrical machinery, tin cans, etc.

The common **fluxes** used to prevent the oxidisation of the metals are borax, cream of tartar, sal ammoniac, resin, chloride of zinc and hydrochloric acid. There are many soldering pastes and solutions on the market.

Brazing is the uniting of two pieces of metal by a thin film of soft brass. **Brazing alloys** have the following composition (see also p. 552):

	Tin	Copper	Zinc
Hard	0	3	1
Medium	0	1	1
Soft	1	4	3

There are numerous brazing alloys or spelters on the market. The common **fluxes** are borax and boracic acid. The parts to be brazed are brought into contact, dusted with borax or coated with some other flux and the spelter is melted into the joint. On large quantities of work **immersion brazing** is used. This consists in dipping the parts to be brazed into a tank of molten spelter after fitting the parts together, brightening them, clamping or wiring them securely and coating with flux.

METAL-CUTTING MACHINES

The most important machines used in making machine parts are included in the chip-removing group. The following classification is that adopted by the National Machine Tool Builders' Association:

Boring machines	Grinding machines	Screw machines
Boring and turning mills	Milling machines	Shaping machines
Broaching machines	Hobbing machines	Slotting machines
Cutting-off machines	Planing machines	Tapping machines
Drilling machines	Key-seating machines	Turning machines (Lathes)
Gear-cutting machines	Rack-cutting machines	

A number of these names are different from those in common use in technical literature and in shops. Throughout this section the nomenclature used is that of the *American Machinist*.

Standardization of Machine Parts

A movement is now under way in the National Machine Tool Builders' Association to standardize machine parts. The requirements as stated by the author before the Convention of the Association in October, 1911, are as follows:

1. Standardize corresponding designations and capacities and establish a method of power rating. 2. Standardize devices for holding cutting tools.

3. Standardize devices for holding work and fixtures. 4. Standardize operating movements. 5. Standardize parts concerned in the setting up of machines with reference to the permanent shop equipment. 6. Accept the geometric progression as a fundamental requisite in designing feeds and speeds.

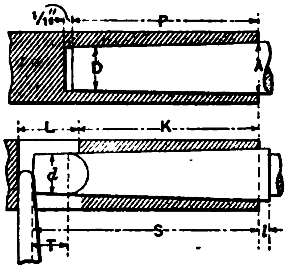


FIG. 2.—Machine Taper.

A few **standards** are **already in use** to a greater or less degree. There are six spindle and shank **tapers**: Morse, Brown & Sharpe, Jarno, Sellers, Reed and the metric. The designating numbers and corresponding dimensions of these tapers are given in Tables 10–15.

The **Morse taper** is largely used for drill shanks, taper holes in drilling-machine spindles, collets, and other machine parts arranged to take drills, and for the shanks of lathe centers. The **Brown & Sharpe** taper is commonly used in milling-machine spindles. The **Jarno** taper is used by a few firms in lathe and small drilling-machine spindles. The **Sellers** taper is used in lathes and drilling and boring machines made by the firm of Wm. Sellers & Co., Inc., and some others. A spline is provided in connection with each taper, as shown in Fig. 3. The **Reed** tapers are used in lathe spindles. The taper per foot is the same as that of the Jarno, but the diameters are different and in most cases the lengths are somewhat less. The **Metric** tapers are used in Europe.

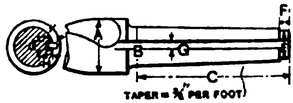


FIG. 3.—Sellers Taper.

Table 10. Proportions of Morse Tapers

(All dimensions in in. Letters refer to Fig. 2)

No. of taper	Diam. of plug at small end, *D*	Diam. at end of socket, *A*	Standard plug depth, *P*	Taper per in.	No. of taper	Diam. of plug at small end, *D*	Diam. at end of socket, *A*	Standard plug depth, *P*	Taper per in.
0	0.252	0.356	2	0.05208	4	1.02	1.231	4 1/16	0.05191
1	0.369	0.475	2 1/8	0.05000	5	1.475	1.748	5 3/16	0.05250
2	0.572	0.700	2 9/16	0.05016	6	2.116	2.494	7 1/4	0.05216
3	0.778	0.938	3 3/16	0.05016	7	2.75	3.270	10	0.05208

Table 11. Proportions of Brown & Sharpe Tapers

(All dimensions in in. Letters refer to Fig. 2)

No. of taper	Diam. of plug at small end, *D*	Diam. at end of socket for length *P*, *A*	Standard plug depth, *P*	No. of taper	Diam. of plug at small end, *D*	Diam. at end of socket for length *P*, *A*	Standard plug depth, *P*
1	0.20	0.2391	15/16	9	0.90	1.0770	4 1/4
2	0.25	0.2995	1 3/16	10*	1.0446	1.2596	5
3	0.312	0.3952	2	10*	1.0446	1.2891	5 11/16
4	0.35	0.4020	1 1/4	11	1.25	1.5312	6 3/4
5	0.45	0.5229	1 3/4	12	1.50	1.7968	7 1/8
6	0.50	0.5989	2 3/8	13	1.75	2.0729	7 3/4
7	0.60	0.7250	3	14	2.00	2.3437	8 1/4
8	0.75	0.8985	3 9/16	15	2.25	2.6145	8 3/4
9	0.90	1.0667	4	16	2.50	2.8855	9 1/4

* Taper per in. = 0.043 in. (For all other sizes = 0.0416 in.)

Table 12. Proportions of Jarno Tapers

(All dimensions in in. Letters refer to Fig. 2)

Diam. at large end = No. of taper ÷ 8. Diam. at small end = No. of taper ÷ 10. Length of taper = No. of taper ÷ 2.

No. of taper	Diam. at large end, *A*	Diam. at small end, *D*	Length of taper, *P*	No. of taper	Diam. at large end, *A*	Diam. at small end, *D*	Length of taper, *P*
1	0.125	0.10	0.5	11	1.375	1.10	5.5
2	0.250	0.20	1.0	12	1.500	1.20	6.0
3	0.375	0.30	1.5	13	1.625	1.30	6.5
4	0.500	0.40	2.0	14	1.750	1.40	7.0
5	0.625	0.50	2.5	15	1.875	1.50	7.5
6	0.750	0.60	3.0	16	2.000	1.60	8.0
7	0.875	0.70	3.5	17	2.125	1.70	8.5
8	1.000	0.80	4.0	18	2.250	1.80	9.0
9	1.125	0.90	4.5	19	2.375	1.90	9.5
10	1.250	1.00	5.0	20	2.500	2.00	10.0

Table 13. Proportions of Sellers Tapers

(All dimensions in in. Letters refer to Fig. 3)

Diam. of drill	Diam. of shank at gage point *B*	Length of shank from point *B*	Diam. at reduced portion of shank	Length of reduced portion of shank	Width of spline in shank	Depth of spline in shank	Height of key
A	*B*	*C*	*E*	*F*	*G*	*H*	*K*
1/4 – 1/2	1/2	2 1/4	11/32	3/16	3/32	5/64	9/64
9/16 – 13/16	11/16	2 3/4	1/2	1/4	1/8	3/32	11/64
7/8 – 1 1/16	7/8	3 1/2	5/8	5/16	1/8	3/32	11/64
1 1/8 – 1 9/16	1 1/8	4 1/2	25/32	3/8	3/16	1/8	15/64
1 5/8 – 2	1 5/8	6 1/2	1 1/8	7/16	1/4	5/32	21/64

The length *L* of the drill body from point *B* is given below

A	*L*	*A*	*L*	*A*	*L*	*A*	*L*	*A*	*L*	*A*	*L*
1/4	4 1/4	9/16	6 1/4	7/8	8	1 3/16	9	1 1/2	10	1 13/16	10 1/2
5/16	4 1/4	5/8	6 3/4	15/16	8	1 1/4	9	1 9/16	10	1 7/8	11
3/8	4 3/4	11/16	6 3/4	1	8 1/2	1 5/16	9	1 5/8	10	1 15/16	11
7/16	5 1/4	3/4	7 1/4	1 1/16	8 1/2	1 3/8	9 1/2	1 11/16	10	2	11 1/2
1/2	5 3/4	13/16	7 1/4	1 1/8	9	1 7/16	9 1/2	1 3/4	10 1/2		

Table 14. Proportions of Reed Lathe-center Tapers

(Taper per foot = 0.6 in., taper per in. = 0.05 in.)

Size of lathe, in.	Diam. of small end of taper, in.	Length of taper, in.	Size of lathe, in.	Diam. of small end of taper, in.	Length of taper, in.
12	9/16	3 5/8	20	1 1/2	5 5/16
14	15/16	4 1/8	22	1 1/2	5 5/16
16	1 1/4	4 1/8	24	1 3/4	5 1/2
16*	1 1/4	4 5/8	27	1 3/4	5 1/2
18	1 1/4	5 1/16	30	2	5 3/4

* Special.

Table 15. Proportions of Metric Tapers
(Letters refer to Fig. 2; t = thickness at d)

Dimension letters	Number of taper										
	1	2	3	4	5	6	7	8	9	10	11
	Dimensions in millimeters										
A	12	18.0	24	32	40	50	60	70	80	90	100
d	9	14.0	19	26	33	42	51	60	69	78	87
T	8	10.0	12	14	16	18	20	22	24	26	28
t	5	6.5	8	11	14	17	20	23	26	29	32
S	60	80.0	100	120	140	160	180	200	220	240	260
l	4	4.0	4	4	4	5	6	7	8	9	10

A **Standard for T-slots and T-headed bolts,** devised by Carl G. Barth (*Am. Mach.*, vol 36, p. 516) is shown in Table 16. The Sellers standard for T-slots is given in Table 17.

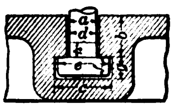
Table 16. Proportions of T-slots and T-headed Bolts (Barth)

d = diam. of bolt shank in inches.

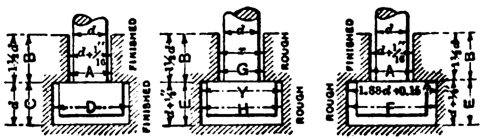
$a = d + \frac{1}{16}$ in. $\quad f = \frac{5}{8}d + \frac{1}{16}$ in. $\quad b = 1\frac{1}{2}d$.

$c = 1\frac{7}{8}d + \frac{3}{16}$ in. $\quad h = \frac{7}{16}d + \frac{1}{16}$ in. $\quad e = 1\frac{3}{4}d + \frac{1}{16}$ in.

$g = \frac{11}{16}d + \frac{1}{8}$ in.



Barth



Sellers

Proportions of T-slots.

Table 17. Proportions of T-slots, Wm. Sellers & Co.
(All dimensions in inches)

d	A	B	C	D	E	F	G	H
1/2	9/16	3/4	1/2	1	5/8	1 1/16	9/16	1
5/8	11/16	15/16	5/8	1 3/16	3/4	1 5/16	11/16	1 3/16
3/4	13/16	1 1/8	3/4	1 3/8	7/8	1 9/16	7/8	1 7/16
7/8	15/16	1 5/16	7/8	1 9/16	1	1 13/16	1	1 5/8
1	1 1/16	1 1/2	1	1 3/4	1 1/8	2	1 1/8	1 13/16
1 1/8	1 3/16	1 11/16	1 1/8	1 15/16	1 1/4	2 1/4	1 1/4	2
1 1/4	1 5/16	1 7/8	1 1/4	2 1/8	1 3/8	2 1/2	1 3/8	2 3/16
1 3/8	1 7/16	2 1/16	1 3/8	2 5/16	1 1/2	2 3/4	1 1/2	2 3/8

For $d = \frac{1}{2}$ and $\frac{5}{8}$ in., $X = d + \frac{1}{16}$ in. and $Y = 1.5d + \frac{1}{4}$ in.
For other values of d, $X = d + \frac{1}{8}$ in. and $Y = 1.5d + \frac{5}{16}$ in.

During the last few years **geometrical progression** has been largely used **in designing** machine-tool **feeds and speeds.** No ratios have been used to a sufficient extent to consider them as standard. Carl G. Barth recommends a leading ratio of 2 and a secondary ratio of 1.189, equal to $\sqrt[4]{2}$. (See *Am. Mach.*, vol. 36, p. 52.)

Electric Motors for Machine Tools

The National Machine Tool Builders' Association has adopted (1910) the following recommendations for the **standardization of motor drives** for machine tools:

1. Horse Power. The following ten sizes of motors are considered sufficient for practically all requirements of machine tools: 1, 1½ (for direct current only), 2, 3, 5, 7½, 10, 15, 20, and 25 h. p.

2. Voltage: For direct-current motors, 115 and 230 volts; for alternating-current motors, 110 and 220 volts.

3. Horse-power Ratings:

(a) Where approximately standard load conditions exist, motors are to be given a continuous constant horse-power rating.

(b) For variable-speed motors used for intermittent service the standard two-hour continuous-duty rating is to be used for ordinary shop conditions.

4. Direct-current Motors. Variable-speed motors are to be standardized for two ranges, namely, 2 to 1 and 3 to 1.

5. Speeds. The following table includes the recommended speeds for variable-speed direct-current motors:

Horse Power	Speed Ratio 2 : 1	Speed Ratio 3 : 1	Horse Power	Speed Ratio 2 : 1	Speed Ratio 3 : 1
1	1500–750	1500–500	7½	1200–600	1200–400
1½	1500–750	1500–500	10	1200–600	1200–400
2	1500–750	1500–500	15	1200–600	1200–400
3	1500–750	1500–500	20	900–450	900–300
5	1200–600	1200–400	25	900–450	900–300

6. Alternating-current Motors. The following table gives the recommended speeds for polyphase 60-cycle alternating-current motors:

Horse power	Speeds	Horse power	Speeds	Horse power	Speeds
1	1800 and 1200	5	1200	15	900 and 600
2	1200	7½	1200 and 900	20	900 and 600
3	1200	10	1200 and 600	25	900 and 600

7. Alternating-current Motors. In considering constant-speed alternating-current motors, 60 cycles is to be used as the basis.

Methods of Applying Motors to Machine Tools. A. L. DeLeeuw (*Trans. A. S. M. E.*, vol. 32, p. 137) outlines the method of applying motors to machine tools as follows:

Bench Lathes to be driven from a countershaft attached to the bench and driven in its turn by motor. Any type of motor may be used except one series-wound or heavily compounded.

Speed Lathes to be driven from a countershaft located upon each by a direct-connected motor. In the latter case a variable-speed machine is to be preferred.

Engine Lathes. Both variable-speed and constant-speed motors are used, although in general engine lathes are group-driven. Where individual motors are used, each is placed on a bracket attached to the lathe and connected with a geared or chain drive.

Heavy Engine Lathes, Forging Lathes, Etc., to be driven by direct-connected motors. The type should be direct-current. The position of the motor should be low to reduce vibrations in its support. The output of these lathes may be increased from 20 to 25 per cent. by motor drive.

Axle Lathes, Wheel Lathes and Driving-wheel Lathes, to be driven by direct-current motors.

Chucking Lathes, in general, not to be motor-driven. If motors are used they should be variable-speed.

Automatic Screw Machines. Small machines to be group-driven; the larger preferably driven by variable-speed motors.

Sensitive Drilling Machines, in general, should not be motor-driven. However, if machine is placed in an isolated location, a motor may be directly applied to the machine itself or the machine may be driven from a motor on the floor through a countershaft.

Vertical and Radial Drilling Machines. In general, these machines are group-driven unless set up in an isolated location. If motor-driven, the variable-speed type should be selected. The motor may be direct-connected to the machine itself or set up to drive the machine countershaft.

Boring Machines. If used for specialized work a belt drive is preferred. If used for a variety of operations, motor drive from a variable-speed motor is of value.

Grinders to be driven by constant-speed motor belted to the regular machine countershaft.

Planers. Small planers, particularly if set up under a crane, to be driven by variable-speed motors. A recent development is the reversing-motor planer drive which is now being extensively tested and promises success.

Shapers, Slotters, Etc. In general, these machines are not to be motor-driven.

Milling Machines, Knee-and-Column Type. In the larger sizes to be driven by variable-speed motors, especially if used on gang work.

Milling Machines, Planer Type. To be motor-driven. Either the constant-speed or variable-speed type is successful.

Table 18, prepared by the Westinghouse Electric & Mfg. Company gives the horse power recommended for machine-tool motors in average practice.

Table 18. Sizes of Motors for Driving Machine Tools

CLASSES OF MOTORS REFERRED TO IN TABLE

(*A*) Adjustable-speed shunt wound direct-current motor, wherever a number of speeds are essential.

(*B*) Constant-speed shunt-wound direct-current motor, where the speeds are obtainable by a gear box or cone-pulley arrangement or where only one speed is required.

(*C*) Squirrel-cage induction motor, where direct current is not available. A gear box or cone-pulley arrangement must be used to obtain different speeds.

Bolt and Nut Machinery

BOLT CUTTER (*A*, *B* OR *C*)

	Size, in.	H.p.
Single	1, 1¼, 1½	1-2
	1¾, 2	2-3
	2¼, 3½	3-5
	4, 6	5-7½
Double	1, 1½	2-3
	2, 2½	3-5
Triple	1, 1½, 2	3-7½

BOLT POINTERS (*B* OR *C*)

	1¼, 2½	1-2

NUT TAPPERS (*A*, *B* OR *C*)

Four-spindle	1, 2	3
Six-spindle	2	3
Ten-spindle	2	5

NUT FACING (*B* OR *C*)

	1, 2	2-3

BOLT HEADING, UPSETTING AND FORGING (*A**, *B*† OR *C*‡)

Size, in.	H.p.	Size, in.	H.p.
¾-1½	5-7½	2½-3	20-25
1½-2	10-15	4 -6	30-40

* Speed variation is sometimes desired when different sizes of bolts are headed on the same machine. † Compound-wound direct-current motor. ‡ Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

Bulldozers or Forming or Bending Machines (*B** or *C*†)

Width, in.	Head movement, in.	H.p.
29	14	5
34	16	7½
39	16	10
45	18	15
63	20	20

* Compound-wound motor. † Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

Drilling and Boring Machines (*A*, *B* or *C*)

	H.p.
Sensitive drills up to ½ in	¼-¾
Upright drills, 12-20 in.	1
Upright drills, 24-28 in.	2
Upright drills, 30-32 in.	3
Upright drills, 36-40 in.	5
Upright drills, 50-60 in.	5 -7½

Radial drills	HORSE POWER Heavy	HORSE POWER Avg.
3-ft. arm	3	1-2
4-ft. arm	5-7½	2-3
5, 6 and 7-ft. arm	5-7½	3-5
8, 9 and 10-ft. arm	7½-10	5-7½

Multiple Spindle Drills (*A*, *B* or *C*)

Size of drills, in.	Number of spindles, up to	H.p.
1/32-1/4	6-10	3
1/16-3/8	10	5
3/16-1/2	10	7½
1/4 -3/4	10	10
3/8 -1	10	10-15
2	4	7½
2	6	10
2	8	15

Boring and Turning Mills (*A*, *B*, or *C*)

Size	Horse power	
	Average	Heavy
37-42 in.	5 - 7½	7½-10
50 in.	7½	7½-10
60-84 in.	7½-10	10 -15
7- 9 ft.	10 -15	....
10-12 ft.	10 -15	30 -40
14-16 ft.	15 -20	....
16-25 ft.	20 -25	....

Cylinder Boring Machines (*A*, *B* or *C*)

Diam. of spindle, in.	Max. boring diam, in.	H.p.
4	20	7½
6	30	10
8	40	15

Pipe-threading and Cutting-off Machines (*A*, *B* or *C*)

Size pipe, in.	H.p.	Size pipe, in.	H.p.
1/4-2	2	3-10	5
1/2-3	3	4-12	5
1 -4	3	8-18	7½
1¼-6	3-5	24	10
2 -8	3-5	....	....

Planers (*A**, *B** or *C*)

Width, in.†	H.p.	Width, in.†	H.p.
22	3	48	15-20
24	3-5	54	20-25
27	3-5	60	20-25
30	5-7½	72	25-30
36	10-15	84	30
42	15-20	100	40

Normal length of bed in ft. is about 1/4 the width in inches.

*Compound-wound motor. † Distance under rail = width.

Buffing Lathes (*B* or *C*)

Diam. of wheels (2), in.	H.p.	Notes
6	1/4-1/2	For brass tubing and other special work use about double these values.
10	1-2	
12	2-3	
14	3-5	

Hydrostatic Wheel Presses (*B* or *C*)

Size, tons	H.p.	Size, tons	H.p.
100	5	400	10
200	7½	600	15
300	7½	...	..

Gear Cutters (*A*, *B* or *C*)

Size, in.	H.p.	Size, in.	H.p.
36×9	2-3	60×12	5 -7½
48×10	3-5	72×14	7½-10
30×12	5-7½	64×20	10 -15

Punching and Shearing Machines

Presses for Notching Sheet Iron (*A*, *B*, or *C*), ½ to 3 h.p.

PUNCHES (*B** or *C*†)

Diam., in.	Thickness, in.	H.p.
3/8	1/4	1
1/2	1/2	2 -3
5/8	5/8	2 -3
3/4	3/4	3 -5
7/8	3/4	5
1	1/2	5
1	1	7½
1¼	1	7½-10
1¾	1	10 -15
2	1	10 -15
2½	1½	15 -25

* Compound-wound motor. † Wound secondary or squirrel-cage motor with approximately 10 per cent. slip on the larger sizes.

SHEARS (*B** or *C*†)

Width, in.	Horse power	
	Cut 1/8-in. iron	Cut 1/4-in. iron
30-42	3	5
50-60	4	7½
72-96	5	10

Bolt shears, 7½ h.p. Double-angle shears, 10 h.p.

* Compound-wound motor. † Wound secondary or squirrel-cage induction motor with 10 per cent. slip.

LEVER SHEARS (*B** or *C*†)

Size, in.	H.p.	Size, in.	H.p.
1 × 1	5	1 × 7	.....
1½ × 1½	7½	2¾ × 2¾	20
2 × 2	10	1½ × 8	.....
6 × 1	.....	3½ × 3½	30
2½ × 2½	15	4½ round	.....

* Compound-wound motor. † Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

PLATE SHEARS (*B** or *C*†)

Size of metal cut, in.	Cuts per min.	Length of stroke, in.	H.p.
⅜ × 24	35	3	10
1 × 24	20	3	15
2 × 14	15	4½	30
1 × 42	20	4	20
1½ × 42	15	4½	60
1¼ × 54	18	6	75
1½ × 72	20	5½	10
1¼ × 100	10-12	7½	75

* Compound-wound motor. † Wound secondary or squirrel-cage motor with approximately 10 per cent. slip.

Shapers (*A*, *B* or *C*)

Stroke, in.	H.p.*	Transverse head shaper: Stroke, in.	Transverse head shaper: H.p.
12-16	2		
18	2-3		
20-24	3-5	20	7½
30	5-7½	24	10

* Horse power for single head.

Rolls—Bending and Straightening (*B** OR *C*†)

Width, ft.	Thickness, in.	H.p.
4	⅜	5
6	5/16	5
6	7/16	7½
6	¾	15
8	⅞	25
10	1⅛	35
10	1½	50
24	1	50

* Standard bending roll motor.
† Wound secondary induction motor.

Saws—Cold and Cut-off (*A*, *B*, or *C*)

Saw diam., in.	H.p.	Saw diam., in.	H.p.
20	3	36	10-15
26	5	42	20
32	7½	48	25

Slotters and Key Seaters (*A*, *B* or *C*)

Stroke, in.	H.p.	Stroke, in.	H.p.
6	3	16	7½
8	3-5	18	7½-10
10	5	20	10 -15
12	5	24	10 -15
14	5-7½	30	10 -15

Horizontal Boring, Drilling and Milling Machines (*A*, *B* or *C*)

Size of spindle, in.	H.p.
3½-4½	5 -7½
4½-5½	7½-10
5½-6½	10 -15

For machines with double spindles use motors of double the horse power given.

Rotary Planers (*A*, *B* or *C*)

Cutter diam, in.	H.p.	Cutter diam., in.	H.p.
24	5	60	20
30	7½	72	25
36-42	10	84	30
48-54	15	96-100	40

Emery Wheels, Grinders, Etc. (*B* OR *C*) (With two wheels)

Wheel size, in.	H.p.	Wheel size, in.	H.p.
6	¼-1	18	5-7½
10	2	24	7½-10
12	3	26	7½-10

Miscellaneous Grinders (*B* or *C*)

	H.p.
Wet tool grinder................	2-3
Flexible swinging, grinding, and polishing machine.............	3
Angle-cock grinder...............	3
Piston-rod grinder...............	3
Twist-drill grinder...............	2
Automatic tool grinder...........	3-5

Grinding Machines (GRINDING SHAFTS, ETC.) (*A*, *B* OR *C*)

Wheel diam., in.	Length of work, in.	H.p.
10	50-120	5-7½
14	72	10-15
18	120-168	10-15

For heavy work use larger h.p. values.

Hammers (*B** or *C*†)

Size, lb.	H.p.
15-75	½-5
100-200	5 -7½

* Compound-wound motor. † Wound secondary squirrel-cage motor with approximately 10 per cent. slip.

Bliss drop-hammers require approximately 1 h.p. for every 100 lb. weight of hammer head.

Lathes (*A*, *B* or *C*)

ENGINE LATHES

Swing, in.	Horse power	
	Average	Heavy
12	½	2
14	¾-1	2-3
16	1-2	2-3
18	2-3	3-5
20-22	3	7½-10
24-27	5	7½-10
30	5-7½	7½-10
32-36	7½-10	10-15
38-42	10-15	15-20
48-54	15-20	20-25
60-84	20-25	25-30

AXLE LATHES

	H.p.
Single	5, 7½, 10
Double	10, 15, 20

WHEEL LATHES

Size, in.	H.p.	Tail Stock motor,* h.p.
48	15-20	5
51-60	15-20	5
79-84	25-30	5
90	30-40	5-7½
100	40-50	5-7½

* Standard machine-tool traverse motor.

Milling Machines (*A*, *B*, or *C*)

VERTICAL SLABBING MACHINES

Width of work, in.	H.p.
24	7½
32-36	10
42	15

VERTICAL MILLING MACHINES

Height under work, in.	H.p.
12	5
14	7½
18	10
20	15
24	20

PLAIN MILLING MACHINES

Table feed, in.	Cross feed, in.	Vert. feed, in.	H.p.
34	10	20	7½
42	12	20	10
50	12	21	15

UNIVERSAL MILLING MACHINES

Machine No.	H.p.	Machine No.	H.p.
1	1-2	3	5 -7½
1½	1-2	4	7½-10
2	3-5	5	10 -15

HORIZONTAL SLAB MILLERS

Width between housings, in.	Horse power	
	Average	Heavy
24	7½-10	10-15
30	7½-10	10-15
36	10 -15	20-25
60	25	50-60
72	25	75

Line-shaft and Individual Motor Drives for Machine Tools. For discussion of line-shaft, individual motor and group drives, see p. 1467.

Depreciation of Machine Tools and Shop Buildings. A report on cost accounting adopted by the National Machine Tool Builders' Association in 1908 recommended charging off 10 per cent. of the original cost per year on all machinery, 5 per cent. on frame factory buildings and 3 per cent. on brick buildings. The cost of fixtures, special tools and patterns should either be charged off at once, or in 24 equal parts during a period of as many months.

Lathes

Classification of the National Machine Tool Builders' **Association:**

Cone drive	Axle	Precision
Small	Gap for car axle	Brass finishers'
Speed	Pulley turning	Automatic threading
Single pulley drive	Bench	Gun boring
Chucking	Projectile turning and boring	Extension gap
Turret	Special bevel-gear turning	Driving wheel
Heavy turret	Automatic	Steel-tire turning
Vertical turret		Shafting turning
Turret on carriage shears		

Most of these terms are descriptive either of the tool itself or of the work for which it is intended.

There is no fixed method of stating sizes and capacities of lathes, but it is common practice to give the **swing** (diameter) and length of bed, that is, a lathe might be 16-in. × 6-ft. bed. The actual swing is often as much as 2 in. greater than the nominal swing. The capacity in length is determined by the maximum distance between centers when the footstock is in its extreme position, and not by length of bed.

The common **sizes of engine lathes** are as follows: 11, 12 and 13 in. × 5 ft.; 14 in. × 6 ft.; 16, 18 and 20 in. × 8 ft.; 22. in. × 10 ft.; 24, 26, 28, 30, 36, 42, 48, 60 and 72 in. × any length of bed desired.

The following **details** of lathe design should be **standardized** so far as possible in any given shop: The diameter, thread and length of spindle nose; taper of hole in spindle; diam. of hole through spindle; taper of centers; hole, keyway, face, pitch and kind of change gears for the ordinary screw-cutting type; number of threads per in. of the lead screw; size of T-slots in wings of carriage; position and direction of motion of operating handles controlling movements of the footstock spindle, carriage, tool block and compound rest.

Cone-driven lathes are commonly back-geared, with the gearing either single or double. The first multiplies the number of cone speeds by two and the second by three.

All-geared lathe heads have lately been developed with the purpose of increasing the power of the drive and to provide easy means for obtaining a large number of finely graduated spindle speeds. Similarly, all-geared devices have been produced for the feed.

In Germany, lathes are divided into roughing and general-purpose lathes. The mean value for the **cutting resistance** in lb. of the roughing lathes is fixed as 725 times the heights of the centers in inches. For general-purpose lathes half this value should be taken.

The maximum **power required to move** the **carriage** may be taken as 2½ to 3 times the cutting resistance, the maximum axial pressure on the headstock center about three times and on the back center about twice this same amount.

Feeds for light work have a minimum of about 0.002; for average work, 0.008; and for heavy work, 0.02 in. Maximum feeds are about 0.02, 0.07 and 0.12 in. per revolution of the spindle. The thickness of the chip may vary from almost nothing to 2½ in.

Power Required by Lathes. According to German practice, the power required for small and medium-sized lathes is approximately as follows:

Swing, in.	12	16	24	32	48	60	80	100*	120*	140*	160*
Horse power	1.5	2	3	4	5	7	10	12	15	20	25–30

* With double beds.

Table 19 (Charles Robbins, *Trans. A. S. M. E.*, vol. 32, p. 211) gives the power required for lathes according to American practice.

Table 19. Sizes and Speeds of Motors on Lathes

Swing, in.	Engine lathes							Driving-wheel lathes	
	Light duty		Medium duty			Heavy duty			
	H.p.	Constant speed, r.p.m.	H.p.	Adjust. speed	Constant speed, r.p.m.	H.p.	Constant speed, r.p.m.	Size, in.	H.p.
14	2.0	1800	3.0		1600	5.0	1200	51	15
16	3.0	1800	5.0		1200	5.0	1200	60–69	20
18–20	3.0	1800	5.0	Ratio 1:3	1200	7.5	1200	79 & 84	25
22–24	5.0	1200	7.5		1200	10.0	1200	90	30
27–30	7.5	1200	10.0		1200	15.0	1200	100	50
36–48	7.5	1200	10.0		1200	20.0	900	....	......

Special Lathes: Car wheel, 48-in., 20 h.p.; double axle, moderate duty, 15 h.p.; heavy duty, 25 h.p.

Lathe Gearing for Thread Cutting. To select the change gears for thread cutting if the lathe is simple-geared and the stud runs at the same speed as the spindle, select some convenient gear for the screw and multiply its number of teeth by the number of threads per inch of the lead screw. Divide this product by the number of threads per inch to be cut. The quotient will be the number of teeth in the gear for the stud.

If the lathe is compound-geared, select at random a set of driving gears and multiply together all of the numbers of teeth and multiply this product by the number of threads per inch to be cut. Then select at random all of the driven gears except one. Multiply together the numbers of teeth of these gears and this product by the number of threads per inch on the lead screw. Divide the first result by the second. The quotient is the number of teeth in the remaining driven gear.

Metric Screw Threads may be cut on lathes having lead screws cut to a number of threads per inch by using change gears having 50 and 127 teeth, respectively. This is because 127 centimeters = 50 in. (127 × 0.3937 in. = 49.9999 in.).

Cutting Tools. Three factors must be determined by every machinist operating a metal-cutting machine on every metal-cutting operation. These are: (*a*) The tool to be used; (*b*) the cutting speed; and (*c*) the feed.

F. W. Taylor, in a paper entitled "The Art of Cutting Metals" (*Trans. A. S. M. E.*, vol. 28, p. 32), points out that there are twelve elements or variables which affect the choice of cutting speed and feed for the lathe. The influence of certain of these elements on the amount of metal cut is set forth in the following recapitulation:

(*a*) Quality of metal to be cut: 1 in the case of semi-hardened steel, to 100 in the case of very soft carbon steel.

(*b*) Depth of cut: In the proportion of 1 for ½-in. depth to 1.36 for ⅛-in. depth.

(*c*) Thickness of chip: In the proportion of 1 with a thickness of 3⁄16-in., to 3½ with a thickness of 1⁄64 in.

(*d*) Elasticity of work and tool: In the proportion of 1 for a chattering tool to 1.15 for a smooth-cutting tool.

(*e*) Shape or contour of cutting edge: In the proportion of 1 for a thread tool to 6 for a broad-nosed tool.

The **angles of tools** for materials of three different degrees of hardness are given in the following tabulation:

Material Cut	Clearance angle, deg.	Back slope, deg.	Side slope, deg.	Lip angle, deg.
Medium and soft steel	6	8	22	61
Cast iron and hard steel	6	8	14	68
Very hard steel and tire steel	6	5	9	74

Fig. 4 shows these angles with reference to the point of a tool.

The hard steel classed with cast iron begins with a low limit of carbon content of about 0.45 per cent., tensile strength of 100,000 lb. per sq. in. and an elongation of 18 per cent. The medium and soft steels listed have their upper limit with these same properties.

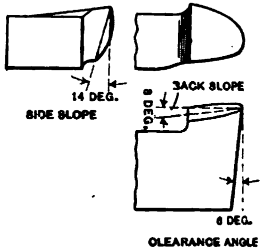


FIG. 4.—Clearance and Slope Angles of Cutting Tools.

For chilled iron a lip angle of from 86 to 90 deg. is to be used. For very soft steels with a carbon content from 0.1 to 0.15 per cent., lip angles keener than 61 deg. may be used. The most important consideration in choosing the lip angle is to make it sufficiently blunt to avoid crumbling and spalling. In choosing between side slope and back slope to produce a sufficiently acute lip angle, the following considerations given in order of importance call for a steep side slope and are therefore opposed to a steep back slope: (*a*) With side slope the tool can be ground many times more without weakening it. (*b*) The chip runs off sidewise and does not strike the tool posts or clamps. (*c*) As the pressure of the chip tends to deflect the tool in one direction, a steep side slope tends to correct this by bringing the resultant line of pressure within the base of the tool. (*d*) The tool is easier to feed.

Mr. Taylor gives the following equations for the radius r of the point of a roughing tool in terms of the width a of the tool, both in inches. For cutting hard steel and cast iron, $r = 0.5a - 5/32$. For cutting medium and soft steel, $r = 0.5a - 3/16$.

Pressure on the Tool. The pressure per sq. in. of cross-sectional area of the chip increases slightly as the thickness of the chip decreases. There is but little difference in pressure between slow and fast cutting speeds, but the total power required to remove a given amount of metal is less at the faster speeds. A slower cutting speed is necessary for hard metals than for soft, even when the total pressure of the chip on the tool is the same, for the following reasons: (*a*) In cutting a hard metal the intensity of the pressure per sq. in. of the lip surface of the tool which comes in contact with the chip is much greater than in cutting soft metals. (*b*) The center of pressure is closer to the cutting edge. (*c*) The section of metal directly below the center pressure is smaller for carrying away the heat.

Taylor's equation for the **pressure on a tool when cutting** cast iron is

$$P = CD^{14/15}F^{3/4},$$

and for steel,

$$P = CDF^{14/15},$$

in which P = pressure, lb.; D = depth of cut, in.; F = feed, in., and C = a constant = 230,000 for steel, 45,000 for soft cast iron, and 69,000 for hard cast iron. Table 21 gives the values of numbers n when raised to the fractional powers $3/4$ and $14/15$. The **specific cutting pressures,** in lb. per sq. in., have been determined as follows:

Authority	Cast Iron	Wrought Iron	Steel
Nicolson	106,000 to 188,000	....	242,000 to 336,000
Fischer	99,500 to 170,500	156,000 to 240,000	226,000 to 340,000
Taylor	70,000 to 197,500	....	240,000 to 296,000

Table 20. Fractional Powers of Numbers for Use in Taylor's Equation for the Pressure on a Tool

n	$n^{3/4}$	$n^{15/16}$	n	$n^{3/4}$	$n^{15/16}$	n	$n^{3/4}$	$n^{15/16}$	n	$n^{3/4}$	$n^{15/16}$
1/32	0.0746	0.0392	7/32	0.320	0.241	13/32	0.508	0.430	19/32	0.677	0.614
1/16	0.126	0.0750	1/4	0.353	0.274	7/16	0.539	0.461	5/8	0.703	0.644
3/32	0.170	0.110	9/32	0.388	0.308	15/32	0.568	0.491	21/32	0.729	0.672
1/8	0.210	0.143	5/16	0.418	0.337	1/2	0.595	0.522	11/16	0.754	0.703
5/32	0.249	0.176	11/32	0.450	0.369	17/32	0.622	0.552	23/32	0.780	0.732
3/16	0.285	0.210	3/8	0.480	0.400	9/16	0.650	0.583	3/4	0.807	0.762

Cutting Speeds for Cast Iron and Steel. Tables 21 and 22 give practical cutting speeds for Taylor high-speed steel tools of the shape shown in Fig. 4. These speeds (**for roughing cuts**) are given for various combinations of depth of cut and feed, and are those at which the tools will cut properly for 90 min. before regrinding is necessary.

Effect of Cooling Water. In cutting steel the gain through the use of a heavy stream of water thrown directly upon the chip at the point where it is removed for modern high-speed steel tools is 40 per cent.; for old-style self-hardening steel tools 33 per cent.; and for carbon-steel tools 25 per cent. Not less than 3 gal. of water per min. should be used for a 2 × 2½-in. tool. The gain is practically the same for all qualities of steel regardless of hardness.

Table 21. Practical Cutting Speeds for Cast Iron. Taylor

(Speeds for tools to last 1½ hr. before regrinding)

Depth of cut in inches	Feed in inches	½-in. Tool: cutting speeds, ft. per min.			⅝-in. Tool: cutting speeds, ft. per min.			¾-in. Tool: cutting speeds, ft. per min.		
		Soft	Medium	Hard	Soft	Medium	Hard	Soft	Medium	Hard
3/32	1/64	209.0	104.0	60.9	214.0	107.0	62.3	218.0	100.0	63.6
	1/32	148.0	77.8	43.0	158.0	78.9	46.0	165.0	82.5	48.1
	1/16	99.2	49.6	28.9	110.0	54.9	32.0	118.0	58.9	34.4
	3/32	77.1	38.5	21.9	87.0	43.7	25.5	95.0	47.5	27.1
	1/8	64.4	32.2	18.8	74.0	37.0	21.6	81.3	40.7	23.7
	3/16	......	......	......	......	......	......	65.0	32.5	19.0
1/8	1/64	197.0	98.4	57.4	198.0	99.0	57.7	200.0	99.8	58.2
	1/32	139.0	69.4	40.5	146.0	73.0	42.6	151.0	75.5	44.0
	1/16	93.4	46.7	27.3	102.0	50.8	29.7	108.0	53.9	31.4
	3/32	72.5	36.3	21.2	81.8	40.4	23.6	87.0	43.5	25.4
	1/8	61.2	30.6	17.7	68.4	34.2	20.0	74.4	37.2	21.7
	3/16	......	......	......	......	......	......	59.4	29.8	17.4
3/16	1/64	181.0	90.7	52.9	179.0	89.4	52.1	177.0	88.7	51.7
	1/32	128.0	64.1	37.4	132.0	65.9	38.4	134.0	67.1	39.1
	1/16	86.0	43.0	25.1	91.8	45.9	26.8	95.7	47.9	27.9
	3/32	66.9	33.5	19.5	73.0	36.5	21.3	77.2	38.6	22.5
	1/8	......	......	......	61.8	30.9	18.0	66.1	33.0	19.3
	3/16	......	......	......	......	......	......	52.9	26.4	15.4
1/4	1/64	172.0	86.0	50.2	167.0	83.4	48.6	164.0	81.9	47.8
	1/32	121.0	60.5	35.3	123.0	61.5	35.9	124.0	61.9	36.1
	1/16	81.6	40.8	23.8	85.6	42.8	25.0	88.4	44.2	25.8
	3/32	......	......	......	68.1	34.1	19.9	71.3	35.7	20.8
	1/8	......	......	......	......	......	......	61.1	30.5	17.8
3/8	1/64	......	......	......	152.0	76.0	44.3	147.0	73.5	42.9
	1/32	......	......	......	112.0	56.1	32.7	111.0	55.6	32.5
	1/16	......	......	......	78.0	39.0	22.7	79.3	39.6	23.1
	3/32	......	......	......	......	......	......	64.0	32.0	18.7

Table 21. Practical Cutting Speeds for Cast Iron. Taylor—(*continued*)
(Speeds for tools to last 1½ hr. before regrinding)

Depth of cut in inches	Feed in inches	7/8-in. Tool: cutting speeds, ft. per min.			1-in. Tool: cutting speeds, ft. per min.			1¼-in. Tool: cutting speeds, ft. per min.		
		Soft	Medium	Hard	Soft	Medium	Hard	Soft	Medium	Hard
3/32	1/64	223.0	112.0	65.2	231.0	116.0	67.2	252.0	126.0	73.5
	1/32	172.0	86.0	50.2	180.0	90.2	52.7	200.0	100.0	58.4
	1/16	122.0	61.0	35.6	133.0	66.4	38.7	150.0	74.9	43.7
	3/32	102.0	50.9	29.7	109.0	54.6	31.8	124.0	62.2	36.3
	1/8	87.7	43.9	25.6	94.5	47.2	27.6	109.0	54.4	31.7
	3/16	70.9	35.4	20.7	76.9	38.5	22.5	89.5	44.7	26.1
1/8	1/64	204.0	102.0	59.6	207.0	103.0	60.4	224.0	112.0	65.4
	1/32	157.0	78.6	45.8	162.0	81.2	47.4	178.0	88.9	51.9
	1/16	112.0	55.8	32.5	120.0	59.8	34.9	133.0	66.6	38.8
	3/32	93.0	46.5	27.1	98.2	49.1	28.6	111.0	55.4	32.3
	1/8	80.2	40.1	23.4	85.1	42.5	24.8	96.6	48.3	28.2
	3/16	64.8	32.4	18.9	69.3	34.6	20.2	79.6	39.8	23.2
3/16	1/64	178.0	88.9	51.9	181.0	90.3	52.7	191.0	95.5	55.7
	1/32	137.0	68.4	39.9	141.0	70.5	41.1	152.0	75.9	44.3
	1/16	100.0	50.1	29.2	104.0	51.8	30.2	114.0	56.8	33.1
	3/32	91.0	40.5	23.6	85.2	42.6	24.9	94.4	47.2	27.5
	1/8	69.8	34.9	20.4	73.8	36.9	21.5	82.4	41.2	24.0
	3/16	56.4	28.2	16.5	60.1	30.0	17.5	67.8	33.9	19.8
1/4	1/64	163.0	81.3	47.4	164.0	82.1	47.9	171.0	85.6	50.0
	1/32	125.0	62.6	36.5	128.0	64.0	37.3	136.0	68.0	39.7
	1/16	90.9	45.5	26.5	94.2	47.1	27.5	102.0	51.0	29.7
	3/32	74.1	37.1	21.6	77.4	38.7	22.6	84.6	42.3	24.7
	1/8	63.9	31.9	18.6	67.1	33.5	19.6	73.9	37.0	21.6
	3/16	51.6	25.8	15.1	54.6	27.8	15.9	60.8	30.4	17.8
3/8	1/64	144.0	72.2	42.1	144.0	72.1	42.1	147.0	73.9	43.1
	1/32	111.0	55.6	32.4	112.0	56.2	32.8	116.0	58.2	33.9
	1/16	80.6	40.3	23.5	82.7	41.4	24.1	87.9	44.0	25.7
	3/32	65.7	32.9	19.2	68.0	34.0	19.8	73.0	36.5	21.3
	1/8	56.7	28.3	16.5	58.9	29.4	17.2	63.8	31.9	18.6
	3/16	45.8	22.9	13.4	47.9	24.0	14.0	52.5	26.2	15.3
1/2	1/64	133.0	66.4	38.7	132.0	65.9	38.4	133.0	66.7	38.9
	1/32	102.0	51.1	29.8	103.0	51.4	30.0	106.0	53.0	30.9
	1/16	74.2	37.1	21.7	75.6	37.8	22.1	79.4	39.7	23.2
	3/32	60.5	30.3	17.7	62.1	31.1	18.1	66.0	33.0	19.3
	1/8	52.2	26.1	14.9	53.8	26.9	15.7	57.6	28.8	16.8
	3/16	......	....	......	43.8	21.9	12.8	47.4	23.7	13.8

Duration of Cut. The formula expressing the relation between cutting speed and duration of cut between grindings for a given material, feed and depth of cut is $V = C/T^{1/8}$, in which V = the cutting speed in ft. per min., T = duration of cut between grindings in min., C = a constant. Consequently there is a material gain in cutting speed if the tools are ground frequently. Table 23 gives the most economical duration of cut and the number of regrindings for various sizes of tool.

Table 22. Practical Cutting Speeds for Steel. Taylor

(Speeds for tools to last 1½ hours before regrinding)

Depth of cut in inches	Feed in inches	½-in. Tool: cutting speeds, ft. per min.			⅝-in. Tool: cutting speeds, ft. per min.			¾-in. Tool: cutting speeds, ft. per min.		
		Soft	Medium	Hard	Soft	Medium	Hard	Soft	Medium	Hard
1/16	1/64	500	250.0	114.0	536	268.0	122.0	......	....	......
	1/32	316	158.0	72.2	350	175.0	79.6	......	....	......
	1/16	199	100.0	45.3	229	115.0	52.1	......	....	......
3/32	1/64	425	218.0	99.0	456	228.0	104.0	465	233.0	106.0
	1/32	275	137.0	62.4	299	149.0	67.9	302	156.0	70.9
	1/16	173	86.6	39.4	195	97.7	44.4	209	105.0	47.6
	3/32	132	66.1	30.0	152	76.2	34.6	165	82.8	37.2
1/8	1/64	396	198.0	90.0	410	205.0	93.1	413	207.0	93.9
	1/32	250	125.0	56.8	268	134.0	60.9	277	139.0	62.9
	1/16	158	78.8	35.8	175	87.6	39.8	186	92.9	42.2
	3/32	......	....	......	137	68.4	31.1	147	73.5	33.4
	1/8	......	....	......	......	....	......	123	61.6	28.0
3/16	1/64	350	175.0	79.6	351	176.0	79.9	350	175.0	79.6
	1/32	221	110.0	50.2	230	115.0	52.2	235	118.0	53.4
	1/16	......	....	......	151	75.7	34.4	157	78.8	35.8
	3/32	......	....	......	......	....	......	125	62.4	28.3
1/4	1/64	322	161.0	73.3	319	160.0	72.6	313	157.0	71.2
	1/32	......	....	......	209	105.0	47.5	210	105.0	47.8
	1/16	......	....	......	......	....	......	141	70.5	32.0
3/8	1/64	......	....	......	280	140.0	63.6	269	135.0	61.3
	1/32	......	....	......	......	....	......	181	90.4	41.1

Depth of cut in inches	Feed in inches	⅞-in. Tool: cutting speeds, ft. per min.			1-in. Tool: cutting speeds, ft. per min.			1¼-in. Tool: cutting speeds, ft. per min.		
		Soft	Medium	Hard	Soft	Medium	Hard	Soft	Medium	Hard
3/32	1/64	477	238.0	108.0	490.0	245.0	111.0	533.0	266.0	121.0
	1/32	325	163.0	73.9	340.0	170.0	77.2	375.0	188.0	85.2
	1/16	222	111.0	50.4	235.0	118.0	53.5	264.0	132.0	60.0
	3/32	177	88.3	40.2	189.0	94.6	43.0	215.0	108.0	48.9
1/8	1/64	420	210.0	95.5	427.0	214.0	97.2	461.0	231.0	105.0
	1/32	286	143.0	65.1	296.0	148.0	67.3	325.0	163.0	73.9
	1/16	195	97.8	44.4	205.0	102.0	46.6	229.0	115.0	52.1
	3/32	156	77.8	35.4	165.0	82.5	37.5	186.0	93 2	42.4
	1/8	133	66.7	30.3	142.0	71.0	32.3	161.0	80.6	36.7
3/16	1/64	352	176 0	80.1	357.0	179.0	81.2	377.0	189.0	85.8
	1/32	240	120.0	54.6	247.0	124.0	56.2	265.0	133.0	60.4
	1/16	164	82.0	37.3	171.0	85.6	38.9	187.0	93.6	42.6
	3/32	130	65.2	29.7	138.0	68.9	31.3	152.0	76.2	34.7
	1/8	112	55.9	25.4	119.0	59.3	26.9	132.0	65.9	30.0
	3/16	......	....	......	95.4	47.7	21.7	107.0	53.7	24.4
1/4	1/64	312	156.0	70.9	314.0	157.0	71.4	328.0	164.0	74.6
	1/32	213	106.0	48.4	218.0	109.0	49.4	231.0	116.0	52.5
	1/16	145	72.6	33.0	151.0	75.3	34.2	163.0	81.3	37.0
	3/32	116	57.8	26.3	121.0	60.6	27.5	132.0	66.2	30.1
	1/8	......	....	......	104.0	52.1	23.7	114.0	57.2	26.0
	3/16	......	....	......	......	....	......	93.2	46.6	21.2
3/8	1/64	265	132.0	60.1	265.0	133.0	60.3	270.0	135.0	61.3
	1/32	181	90.3	41.0	183.0	91.9	41.8	190.0	95.1	42.2
	1/16	123	61.6	28.0	127.0	63.6	28.9	134.0	66.9	30.4
	3/32	......	....	......	102.0	51.2	23.3	109.0	54.5	24.8
	1/8	......	....	......	......	....	......	94.2	47.1	21.4
1/2	1/64	237	118.0	53.8	234.0	117.0	53.2	236.0	118.0	53.6
	1/32	162	80.8	36.7	162.0	80.9	36.8	166.0	83.0	37.7
	1/16	......	....	......	112.0	55.9	25.4	117.0	58.5	26.6
	3/32	......	....	......	......	....	......	95.2	47.6	21.6

Table 23. Duration of Cut Between Grindings. Taylor

S = Size of tool shank in inches; N = Number of regrindings for each dressing; I = Interval between grindings for most economical cutting speed, min.

S........	½×¾	⅝×1	¾×1⅛	⅞×1⅜	1×1½	1¼×1⅞	1½×2¼	1¾×2¾	2×3
N.......	13	15	17	18	19	20	20	20	20
I........	75	75	75	90	90	105	120	150	165

Let t = feed, a = area of cut, S = cutting speed; then, according to E. G. Herbert (*Am. Mach.*, vol. 32, p. 1063), for the same durability of the cutting tool under two sets of working conditions, represented by subscripts 1 and 2, respectively, $S_2{}^3t_2a_2 = S_1{}^3t_1a_1$.

Speeds for parting and thread tools are respectively ½.7 and ¼ the values given in Tables 21 and 22.

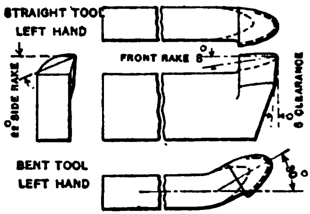


FIG. 5.—Sellers Angles for Round-nosed Roughing Tools for Wrought-iron and Steel (Lathe and Planer Tools).

During the past 4 or 5 years numerous high-speed steels have been put on the market, known generally as **"superior" steels.** Results of careful tests are wanting, though isolated trials have indicated that cutting speeds on soft materials 50 to 60 per cent. higher, and on hard materials 25 per cent. higher, are possible than with the Taylor high-speed steels. Another group of steels, known as **"intra" steels,** has been developed; these have many of the qualities of high-speed steel, but can be successfully hardened in water.

Experiments are also reported with alloys of cobalt, tungsten and chromium used for cutting tools. These have the trade name **"stellite."** Tools ground from a cast bar containing 25 per cent. tungsten, 15 per cent. chromium and 60 per cent. cobalt, have cut annealed nickel-chrome steel at a speed more than double that possible with high-speed steel tools.

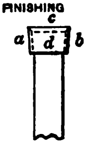
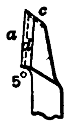
KIND OF TOOL	FACE	CLEARANCE	KIND OF TOOL	FACE	CLEARANCE	KIND OF TOOL	FACE	CLEARANCE
FINISHING 	SIDE a	4°	SIDE (RIGHT) 	SIDE a	6°	SQUARE THREAD (RIGHT) 	SIDE a	10°
	SIDE b	4°		END c	6°		SIDE b	0°
	END c	6°		TOP d	12°		END c	6°
	TOP d	15°					TOP d	0°
NICKING 	SIDE a	3°	60-DEG. V THREAD (RIGHT) 	SIDE a	12°	BRASS 	SIDE a	10°
	SIDE b	3°		SIDE b	7°		SIDE b	6°
	END c	6°		END c	15°		END c	10°
	TOP d	1°		TOP d	0°		TOP d	0°

FIG. 6.—Sellers Shapes for Straight-face Lathe Tools.

Lathe Tools. Fig. 5 gives angles for both straight and bent roughing tools, and Fig. 6 angles for straight-face tools. All of these show the practice

Table 24. Dimensions of Round-nose High-speed-steel Roughing Tools

(Letters refer to Fig. 7)

W	*S*	*A*	*B*	*C*	*D*	*E*	*H*	*R*	*L** Length of bar stock	*X** 2nd opr.
1/2	3/4	3/4	1/2	3/16	13/16	1 7/16	1 5/8	1/8	11	3 1/2
5/8	1	7/8	21/32	13/64	1	1 11/16	2 1/32	3/16	12	4 1/4
3/4	1 1/8	1	13/16	7/32	1 5/32	1 15/16	2 5/16	1/4	15	5
7/8	1 3/8	1 1/8	31/32	15/64	1 5/16	2 3/16	2 23/32	5/16	16	5 1/2
1	1 1/2	1 1/4	1 1/8	1/4	1 1/2	2 1/2	3	3/8	18	6 1/4
1 1/4	1 7/8	1 1/2	1 7/16	9/32	1 27/32	3	3 11/16	1/2	21	7 1/2
1 1/2	2 1/4	1 3/4	1 3/4	5/16	2 1/8	3 9/16	4 3/8	5/8	24	9
1 3/4	2 5/8	2	2 1/16	11/32	2 7/16	4 1/8	5 1/16	3/4	27	10 1/4
2	3	2 1/4	2 3/8	3/8	2 3/4	4 5/8	5 3/4	7/8	30	11 1/2

* See directions for forging tools, p. 1458.

of William Sellers & Co., Inc. Tools used for roughing in cast iron and the harder grades of steel (lathe and planer tools) are similar to the tools of Fig. 5, except that the angle of side rake is 14 deg. Fig. 7 and Table 24 show common dimensions for round-nose high-speed roughing tools.

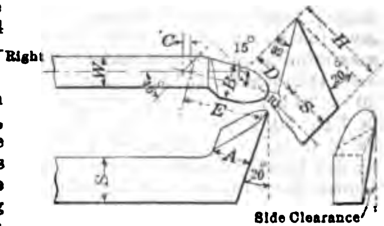


Fig. 7.—Standard Forged Straight Round-nose Roughing Tools.

Boring Tools. Dempster Smith (Manchester Assoc. of Engineers, October, 1911), from trials with single and two-lipped boring tools, deduces equations for the cutting pressure and thrust for two-lipped boring cutters in soft steel. The cutting pressure

$$F = 362{,}000wt + 20{,}000t + 80w,$$

in which F is the cutting pressure, lb., w the width of each cutting lip, in in., and t the feed in in. per rev. An approximation of this is: $F = 400{,}000wt$. The end thrust in lb. is: $P = 38{,}000wt^{0.7}$.

Approximate equations for medium cast iron for two-lipped boring cutters are

$$F = 57{,}000wt^{0.7} \qquad P = 27{,}500wt^{0.7}$$

Trials with single-lipped cutters both for trepanning and facing show that if an initial groove is cut, into which the chips are free to spread, the cutting pressure is about 80 per cent. of that necessary when trepanning from the solid. The formula for cutting pressure for a single cutter for a cutting angle of 75 deg. used in trepanning from the solid, is

$$F = 10{,}000t(30w + 5) + 600w,$$

The thrust when cutting in the solid is: $P = 1000(40w + 1)t^{0.7}$

For facing with a two-lipped cutter the equations are:

$$F = 300{,}000wt \qquad P = 32{,}500wt^{0.7}$$

The corresponding equations for trepanning medium cast iron are:

$$F = (62{,}000w + 2000)t^{0.7} \qquad P = 28{,}800wt^{0.7}$$

For facing, the equations are:

$$F = 55{,}000wt^{0.7} \qquad P = 31{,}000wt^{0.7}$$

Horse Power Required to Remove Metal by Lathe Tools. Charles Robbins (*Trans. A. S. M. E.*, vol. 32, p. 202) gives the following values of the power required to remove 1 cu. in. of metal per min., using ordinary lathe tools:

Metal	Horse power	Metal	Horse power
Brass and similar alloys	0.2 to 0.3	Mild steel (0.30 to 0.40 per cent. carbon)	0.6
Cast iron	0.3 to 0.5	Hard steel (0.50 per cent. carbon)	1 to 1.25
Wrought iron	0.6		
Very hard tire steel	1.5		

Planers, Shapers and Slotters

In using these machines the work is fastened to a platen or table which is traversed in the case of planers and fed in shapers and slotters. As a rule, **cutting is only** done **on one stroke,** although some large planers, especially plate planers, are arranged for double-cutting. Some large shapers cut on the return stroke only, or with a draw cut. On planers, the feed motion acts upon the saddle carrying the cutting tool.

Planers. The **classification** of planers of the National Machine Tool Builders' Association is as follows: Geared, rotary, open side, crank, frog, crossing and switch point, second belt drive, locomotive connecting rod, duplex, textile feed roll fluting. The common **method of designation** is to give the width of the table or platen, or distance between housings, the maximum height cleared under the rail (these dimensions usually being equal), and the length of the platen between pockets. **Common sizes** are 20 × 20 in. × 4 ft.; 22 × 22 in. × 5 ft.; 24 × 24 in. × 6 ft.; 26 × 26 in. × 7 ft.; 30 × 30 in. × 8 ft.; 36 × 36 in. × 8 ft.; 42 × 42 in. × 10 ft.; and 48 × 48 in., 54 × 54 in., 60 × 60 in., 72 × 72 in., 84 × 84 in., 96 × 96 in. by any length of platen desired. The largest and heaviest planer built in the United States is 14 × 20 × 30 ft., weighs 422 tons and has a 100-h.p. motor for the main drive.

In general, planers are **used to produce flat surfaces,** although arcs and special forms can be made with proper tools and attachments. Surfaces to be finished by scraping, particularly parts of machine tools, are with few exceptions planed.

Planers are divided into **two types,** depending upon the way in which they are driven. The **spur-gear drive** is the more common, consisting of a rack running the length of the table on its under side, usually along its center line, into which meshes a large toothed wheel known as the "bull" gear. Power is transmitted to the bull gear from the driving belt through intermediate shafts. In the second, or spiral, type, the **drive** is **through a worm** set at an angle to mesh with the rack.

Large planers may have an additional gear reduction using a second intermediate shaft. This is known as a **four-shaft drive.** Small and medium-sized planers usually have but one **tool head.** Larger machines usually have two on the cross rail and frequently one on each housing below the rail, known as **side heads.** The latter are used to plane down the sides of work and for undercutting.

The smaller machines, up to about 60 × 60-in. gap, usually have but one return speed, this being 3 or 4 times greater than the cutting speed. Larger machines are usually driven by double racks and gears with a return speed 2 or 3 times greater than the forward.

At the moment of reversing the platen the accumulated energy in the driving pulley and train of gears must be overcome, and the reversing belt must supply power to accelerate the table to its normal return speed. This factor limits the **return speed.** To aid this reversal, machines have been built with springs that were compressed by the platen on the cutting stroke. These are commonly referred to as **regenerative planers.**

To smooth out the **reversal,** flywheels have sometimes been mounted on the shaft with the intermediate gear. This arrangement is effective in reducing shocks on the driving motor. Another method employs driving pulleys made from aluminum composition, that is, pulleys that are light in weight and have but little momentum to be overcome at reversal.

To bring about **easy reversal,** other devices used are pneumatic clutches, magnetic clutches, and reversing motors. The latter is the newest and most promising. Table 25 gives a comparison of cutting results on belt-driven and reversing-motor-driven planers.

Table 25. Comparison of Cutting Results on Belt-driven and Reversing-motor-driven Planers

Castings planed, size in in.	Depth of cut	Feed of tools per cut	Length of stroke	Speeds		Stock Removed		Total time of operation		Load at Motor	
				Cutting	Return	Volume of metal per minute	Weight of metal per minute			Power developed on cutting stroke	Power expended per cu. in. of metal removed per min.
	In.	In.	In.	Ft. per min.	Ft. per min.	Cu. in.	Lb.	Min.	Sec.	H.p.	H.p.
Steel 24 × 72....	0.375	0.143	80	32	98	23.2	6.6	27	25	19.3*	0.84
Cast iron 24 × 36	0.625	0.125	48	28	94	25.8	6.74	21	......	25.8*	1.00
Cast iron 24 × 36	0.625	0.111	48	24	87	21.4	5.58	24	58	24.3†	1.13
Cast iron 24 × 36	0.625	0.125	48	19	44	17.4	4.55	31	4	18.2‡	1.04
Cast iron 24 × 36	0.625	0.125	48	17	40	16.9	4.41	32	......	18.3§	1.08

* 48-in. planer, motor drive. † 48-in. planer, motor drive; tools not reground. ‡ 60-in. planer, belt drive. § 60-in. planer, belt drive; tools not reground.

For squaring and finishing the edges of boiler, tank, bridge and ship plates and the like, **plate-edge planers** are used. The work is usually clamped by hydraulic pressure and the machine cuts on both the forward and return strokes. Sizes and power requirements of plate-edge planers are as follows:

Length of stroke, inches........................	160	200	280	to 400
Approximate horse power......................	7	8	10	to 20

Rotary Planers. In these, the cutting tools, of which there are a number, are held in a circular disk or "cathead." The work is clamped to a table provided with automatic feed. There are frequently two heads, so that both sides of the work can be finished at a single setting. They are used largely in structural steel shops to trim and finish the ends of rolled sections.

Motors for Planers. Table 26, for motor sizes and speeds, is adapted from one by Charles Robbins (*Trans. A. S. M. E.*, vol. 32, p. 211).

Table 26. Sizes and Speeds of Motors for Planers

Width and height planed, in....	24×24	30×30	36×36	42×42	48×48	56×56	144×120
Horse power, medium duty.....	5	7.5	10	......	15	15	....
Horse power, heavy duty......	7.5	......	......	25	....	25	60

14 × 12-Ft. Heavy-duty Planer, motor horse-power required: For main drive, 100; for slotting and cross planing, 50; for clutches and feed, 30; for vertical travel of rail, 20; for horizontal travel of heads, 7½.

Cutting Speeds for planers range from 18 to 75 ft. per min.; **feeds** from 0.005 to 3/32 in. for roughing and from ⅛ to 2 in. for finishing. The **depth of chip** ranges from ⅛ in. to 1¼ in., and may be even greater.

The following values for cutting speeds are from the Cincinnati Planer Co.'s "Treatise on Planers." Table 27, from the same source, gives the number of feet of **platen travel** per hour on cut for various cutting and return speeds.

Material	Cutting speed, ft. per min.	Material	Cutting speed, ft. per min.
Cast iron, roughing cut	40 to 50	Wrought iron, roughing cut	30 to 45
Cast iron, finishing cut	20 to 25	Wrought iron, finishing cut	20
Steel castings, roughing cut	30 to 35	Bronze and brass	50 to 60
Steel castings, finishing cut	20	Machinery steel	30 to 35

Table 27. Number of Feet Planer Platen Travels per Hour on Cut

Speed of cut, feet per minute	Speed of return, feet per minute							
	50	60	70	80	90	100	120	150
20	857	900	933	960	981	1000	1028	1058
25	1000	1058	1105	1142	1173	1200	1241	1285
30	1125	1200	1260	1309	1350	1384	1440	1500
35	1235	1321	1400	1460	1512	1555	1625	1702
40	1333	1440	1527	1600	1661	1714	1800	1894
45	1421	1542	1643	1728	1800	1862	1962	2076
50	1500	1636	1750	1846	1928	2000	2117	2250

Shapers are classified by the National Machine Tool Builders' Association, as follows: Crank, and back-geared crank, triple-geared rack, traverse, open-side, friction, draw-cut.

The cutting tool is mounted on the free end of a horizontal ram which is operated usually by a crank and link, or, in the larger sizes, by a rack and pinion. The slotted-link device produces a **return speed** equal at the most to three times the cutting speed. As each revolution of the driving mechanism produces one stroke of the ram, changes in speed are obtained by means of a cone pulley, variable-speed motor, or other device. The **length of stroke** is changed by changing the position of the crank pin.

The shaper is used for miscellaneous facing, surfacing, notching, key-seating and the production of flat surfaces on all kinds of small machine parts. It is more especially a tool-room, repair-shop and job-shop machine, although in some places it is used on manufacturing operations. It is well adapted to handle special and unusual jobs within its capacity and requiring the operations mentioned above.

The **cutting speeds** employed are usually up to 50 ft. per min., with **return speeds** up to 160 ft. per min. **Feeds** range from 0.005 to 3/32 in. **Commercial sizes** are 14, 16, 20, 24, 28 and 36 in., the size being stated in terms of the length of the stroke. The **horse power** required for shapers is as follows: 14 to 20 in., 3 h.p.; 24 to 28 in., 5 h.p.; 36 in., 7.5 h.p.

Slotters are classified by the National Machine Tool Builders' Association respectively as vertical, frame, and traveling head. This classification does not include the small machines used to saw slots in screw heads, which are also known as slotters.

The machines covered by the above classification are similar in type to shapers, except that the ram carrying the cutting tool is operated vertically.

They are used on general work for keyseating, notching, facing and surfacing, and in some cases, for manufacturing, such as finishing brush holders for generators and motors. **Cutting speeds,** up to 50 ft. per min.; **return speeds,** up to 160 ft. per min.; feed, 1/100 in. to 1/4 in. per cut.

Motors for Slotters. Commercial sizes determined by length of stroke and motor sizes for both light and medium work are as follows: 10 in. stroke, 3 to 5 h.p.; 12 to 16 in., 5 to 7.5 h.p.; 20 in., 7.5 to 10 h.p.; 26 to 30 in., 15 h.p.; and for geared slotters, 24 to 60 in. stroke, 20 h.p.

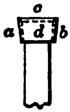
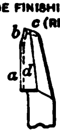
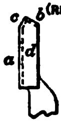
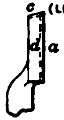
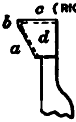
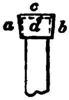
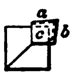
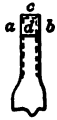
PLANER TOOLS								
KIND OF TOOL	FACE	CLEARANCE	KIND OF TOOL	FACE	CLEARANCE	KIND OF TOOL	FACE	CLEARANCE
FINISHING 	SIDE a	3°	SIDE FINISHING (RIGHT) 	SIDE a	6°	BENT FINISHING (RIGHT) 	SIDE a	3°
	SIDE b	3°		SIDE b	4°		END c	4°
	END c	4°		END c	6°		TOP d	7°
	TOP d	7°		TOP d	18° 5°			
30-DEG. ANGLE (RIGHT) 	SIDE a	6°	CHAMFERING 	SIDE a	4°	CUTTING DOWN (LEFT) 	SIDE a	6°
	SIDE b	4°		SIDE b	4°		END c	4°
	END c	4°		TOP c	0°		TOP d	0°
	TOP d	0°						
30-DEG. ANGLE SLOT (RIGHT) 	SIDE a	4°	SPLINING 	SIDE a	2°	BENT SIDE FINISHING (RIGHT) 	SIDE a	6°
	SIDE b	4°		SIDE b	2°		SIDE b	4°
	END c	4°		END c	4°		END c	6°
	TOP d	0°		TOP d	3°		TOP d	18° 5°
SLOTTER TOOLS								
CORNER 	SIDE a	4°	SQUARE 	SIDE a	4°	SPLINING 	SIDE a	0.3°
	SIDE b	4°		SIDE b	4°		SIDE b	0.3°
	END c	7°		SIDE c	4°		END c	3°
				SIDE d	4°		TOP d	4°
				END e	0°			

FIG. 8.—Angles of Sellers Straight-face Planer and Slotter Tools.

Planer and Slotter Tools. The practice of William Sellers & Co., Inc., with respect to tool shapes and angles, is given by Fig. 8. The shapes and angles for round-nose roughing tools are the same for planers as for lathes and are given in Fig. 5, p. 1430.

Boring Machines

Boring Machines are of two general types, horizontal and vertical, and are frequently referred to as horizontal boring machines and vertical boring and turning mills. The classification of the National Machine Tool Builders' Association is: Horizontal boring machines, vertical boring and turning mills, vertical multi-spindle cylinder boring mills, vertical cylinder boring mills, vertical turret boring mills (vertical turret lathes), and car-wheel boring mills.

The **horizontal type** is made both for precision work and general manufacturing. The first type of machine is particularly adapted for work not conveniently revolved, for long holes, and for work having more than one hole to be bored, particularly if the holes are in different planes. The "Precision" type of machine, first built by the Lucas Machine Tool Co., is adapted for both boring and milling on tool-room, experimental, repair and special work. Also for interchangeable work that must be produced without jigs and fixtures. The second type is used for general manufacturing with fixtures that determine the accuracy of the operation. In automobile building this type is used to bore differential housings, crank cases, etc.

Motor Sizes for Horizontal Boring Machines. Table 28, due to Charles Robbins, gives sizes and speeds of motors for horizontal boring, drilling and milling machines.

Table 28. Sizes and Speeds of Motors for Horizontal Boring, Drilling and Milling Machines

Spindle diameter, in.	3½	4	5	6	7
Horse power of motor	3	5	7½	10	15
R.p.m. of motor (constant-speed)	1800	1200	1200	1200	1200

Adjustable speed ratio, 1 : 3

The precision type of machine is made in the following sizes:

Spindle diam., in.	3	3¾	4½
Horse power of motor (constant-speed)	5	7½	10

Table 29. General Dimensions of Standard Boring Mills, Rapid-production Mills and Vertical Turret Lathes

Size of machine, in.	Table diam., in.	Capacity: Diam. of work, in.	Capacity: Under rail, in.	Capacity: Under slide, in.	Capacity: Vertical feed, in.	Table speeds, ft. per min.	H.p. of motor
Standard Boring Mills							
30	28	34	16	25	18	114.4 -4.16	3½
42	37½	42	32½	40½	24	54.6 -2.2	5
51	50	51½	40	47½	26	54.9 -1.58	7½
60	58	61	40	47½	26	47.1 -1.38	7½
Rapid-production Mills							
42	37½	43	33½	44½	30	54.6 -2.2	7½
54	52	56	42	53	36	23.33-1.72	12
Vertical Turret Lathes							
24	24½	26	18	26½	18	62.5-4.67	5
36	34	36½	24	35	26	38.8-2.9	7½
42	40½	44	32	43	27	60-3	10-12

Vertical Boring Mills are adapted to a wide range of face-plate work—work that can be revolved. The advantage lies in the ease with which work is fastened to the horizontal table, and in the lessened effect of centrifugal forces arising from unsymmetrically balanced work. Pulleys, flywheels, gear blanks, piston heads, bearing shells, centrifugal-pump casings, steam-turbine disks, steam-turbine casings, electric-motor and -generator frames and spiders are commonly finished on these machines. Table 29 gives **commercial sizes,** capacities and sizes of motor required—from the practice of the Bullard Machine Tool Co.

Table 30 gives sizes and speeds of **motors for vertical boring mills** (Charles Robbins, *Trans. A. S. M. E.*, vol. 32, p. 212), and following is the practice of the Colburn Machine Tool Co:

Size of mill, in.	42	48	54	60	72
H.p. of motor (Speed, 1200 r.p.m. for all sizes)	7½	10	10	15	15

Table 30. Sizes and Speeds of Motors for Vertical Boring Mills

Size,	24–30 in.	36–42 in.	60–90 in.	100 in.	10 ft.	12 ft.	14 ft.	16 ft.
Horse power	5	7.5	10	15	20	20	25	30
Horse power for rail	.....	.....	5	5	7½	7½	7½	10
Motor speed, r.p.m.	1200	1200	1200	1200	900	900	900	900

Tools for Boring Mills. The forged tools used are either the same as those for lathes, see Figs. 5 and 6, or small bits in holders. In addition, many special tools are used made for particular jobs, and these are frequently furnished by the machine builder.

Drilling Machines

Drilling machines are made in a wide variety of kinds and sizes and are intended for drilling holes with flat, twist or other drills, for tapping, counterboring, reaming, and general boring operations. In some shops they are employed on manufacturing work commonly sent to the lathe, including facing, threading, and outside turning.

The classification of the National Machine Tool Builders' Association is as follows:

Vertical, light cone drive
Vertical, heavy cone drive
Vertical, all-geared
Vertical, light electric drive
Vertical, triple-geared
Radial
Radial, electric drive
Sensitive, plain bearing
Sensitive, ball bearing
Sensitive, electric
Horizontal
Multiple spindle, gang type
Multiple spindle, adjustable gang type
Multiple spindle, adjustable with independently varying spindle speeds.
Multiple spindle, adjustable cluster type
Horizontal multiple spindle
Automatic, revolving table
Boiler makers' radial countersinking
Bench
Bench, electric drive

Sizes of Drilling Machines. Vertical drilling machines are usually designated by a dimension, in inches, which roughly indicates the diameter of the largest circle which can be drilled at its center under the machine. This dimensioning, however, does not hold universally true for all makes of machines. The sizes begin with about 20-in. and continue to 50-in.—the largest size commercially built. The common sizes are: 20, 22, 24, 26, 28, 30, 32, 36, 42 and 50-in.

The size of a **radial drill** is designated by the length of the arm, this dimension being expressed both in feet, as 2, 3, 4, 5 and 6-ft. machines, and in inches.

Sensitive and multiple-spindle machines are not regularly designated by trade sizes.

Motors for Drilling Machines. Sizes and speeds of motors are given by Charles Robbins, as follows:.

Radial drilling machines:

Size, ft.	4	5	6	10
Horse power	3	5	5	7.5

Vertical drilling machines:

Size, in.	15	20–26	28–34	42–50
Horse power	0.5	1	2	3

The **power required for drilling** can be expressed in terms of the number of cubic inches of metal removed per minute.

Material	Horse power per cu. in. of metal removed per minute
Brass and similar alloys	0.4 to 0.6
Cast iron	0.6 to 1.0
Wrought iron	1.2
Mild steel (0.3 to 0.4 per cent. carbon)	1.2
Hard steel (0.5 per cent. carbon)	2 to 2.5

Twist Drills are the most common tools used in drilling, and are made in many sizes and lengths. The smaller are designated by number, as shown in Table 31. Smaller drills than No. 80 have been made, one firm regularly

Table 31. Numbered Twist Drill Sizes

No. of drill	Diam. in inches	No. of drill	Diam. in inches	No. of drill	Diam. in inches	No. of drill	Diam. in inches	No. of drill	Diam. in inches
1	0.2280	17	0.1730	33	0.1130	49	0.0730	65	0.0350
2	0.2210	18	0.1695	34	0.1110	50	0.0700	66	0.0330
3	0.2130	19	0.1660	35	0.1100	51	0.0670	67	0.0320
4	0.2090	20	0.1610	36	0.1065	52	0.0635	68	0.0310
5	0.2055	21	0.1590	37	0.1040	53	0.0595	69	0.02925
6	0.2040	22	0.1570	38	0.1015	54	0.0550	70	0.0280
7	0.2010	23	0.1540	39	0.0995	55	0.0520	71	0.0260
8	0.1990	24	0.1520	40	0.0980	56	0.0465	72	0.0250
9	0.1960	25	0.1495	41	0.0960	57	0.0430	73	0.0240
10	0.1935	26	0.1470	42	0.0935	58	0.0420	74	0.0225
11	0.1910	27	0.1440	43	0.0890	59	0.0410	75	0.0210
12	0.1890	28	0.1405	44	0.0860	60	0.0400	76	0.0200
13	0.1850	29	0.1360	45	0.0820	61	0.0390	77	0.0180
14	0.1820	30	0.1285	46	0.0810	62	0.0380	78	0.0160
15	0.1800	31	0.1200	47	0.0785	63	0.0370	79	0.0145
16	0.1770	32	0.1160	48	0.0760	64	0.0360	80	0.0135

producing them with a diameter of 0.007 in. and experimentally having produced them as small as 0.005 in. Continuing the series beyond the **numbered sizes** are the **lettered sizes,** running from A to Z, inclusive, as shown in Table 32.

Table 32. Lettered Twist Drill Sizes

Letter	Size	Diam., in.	Letter	Size	Diam., in.	Letter	Size	Diam., in.	Letter	Size	Diam., in.
A	15/64	0.234	H	17/64	0.266	N	......	0.302	U	......	0.368
B	......	0.238	I	......	0.272	O	5/16	0.316	V	3/8	0.377
C	......	0.242	J	......	0.277	P	21/64	0.323	W	25/64	0.386
D	......	0.246	K	9/32	0.281	Q	......	0.332	X	......	0.397
E	1/4	0.250	L	......	0.290	R	11/32	0.339	Y	13/32	0.404
F	......	0.257	M	19/64	0.295	S	......	0.348	Z	......	0.413
G	......	0.261	......	......	......	T	23/64	0.358	......	......	......

Regular twist drills listed in **fractional sizes** begin at 1/16 in. and advance by 1/64 in. to 3 in. for both straight and taper shanks. **Taper-shank drills** from 3 1/16 to 5 in. advance in size by sixteenths, and from 5 1/8 to 6 in. by eighths. Some makers recommend that three- and four-flute drills be used in sizes larger than 2 in.

Twist drills are backed off from the cutting edge (radial relief) and also are decreased in diameter from point to shank to prevent binding. This longitudinal **clearance** varies from 0.00025 to 0.0015 per in. of length.

The web is increased gradually in thickness from point to shank to increase the strength. It is customary to increase the pitch of the spiral as it approaches the shank. The shape of the groove is important, that one which allows a full curl to the chip being the best. The **spiral angles** of the flutes vary from 18 to 35 deg. The usual **lip-edge angle** is 59 deg. The best all-round **clearance angle** is 12 deg., though 15 deg. is sometimes used for soft metals. There are a number of automatic and hand **twist-drill grinders** on the market designed to give the proper angles.

Among the common types of drills are the **center drill**—a short drill used to center shafts before squaring and turning; the **core drill**, used to cut a core from a piece of metal instead of reducing all of the metal removed to chips; the **gun-barrel drill**, run at a high speed under a light feed and used to drill small, long holes; the **hog-nosed drill**, used to bore out cored holes; the **oil-tube drill**, having holes or tubes in its body through which oil is forced to the cutting lips; **three- and four-grooved drills**, used to enlarge large holes after a leader hole has been drilled with a two-fluted drill; **twisted drills** made from flat high-speed steel.

Speeds and Feeds for Drills. Table 33, giving speeds and feeds for drilling, is copyrighted, 1911, by the Henry & Wright Manufacturing Co., and reproduced here by permission. It gives speeds and feeds for various classes of materials and sizes of drills, which are the results of extensive experiments and probably can be considered as maximum practice.

Table 33. Speeds and Feeds for Drilling with Carbon-steel Drills

For high-speed-steel drills double the tabulated speeds
(Figures in parentheses are speeds in ft. per min.)

Size of drill, in.	Feed per rev., in.	Bronze, brass (150 ft.), r.p.m.	Cast iron ann'ld (85 ft.), r.p.m.	Hard cast iron, (40 ft.), r.p.m.	Mild steel (60 ft.), r.p.m.	Drop forgings (30 ft.), r.p.m.	Malleable iron (45 ft.), r.p.m.	Tool steel (30 ft.), r.p.m.	Cast steel (20 ft.) r.p.m.
1/16	0.003	....	5185	2440	3660	1830	2745	1830	1220
1/8	0.004	4575	2593	1220	1830	915	1375	915	610
3/16	0.005	3050	1728	813	1220	610	915	610	407
1/4	0.006	2287	1296	610	915	458	636	458	305
5/16	0.007	1830	1037	488	732	366	569	366	245
3/8	0.008	1525	864	407	610	305	458	305	203
7/16	0.009	1307	741	349	523	261	392	261	174
1/2	0.010	1143	648	305	458	229	343	229	153
5/8	0.011	915	519	244	366	183	275	183	122
3/4	0.012	762	432	204	305	153	212	153	102
7/8	0.013	654	371	175	262	131	196	131	87
1	0.014	571	323	153	229	115	172	115	77
1 1/4	0.016	458	260	122	183	92	138	92	61
1 1/2	0.016	381	216	102	152	77	106	77	51
1 3/4	0.016	327	186	88	131	66	98	66	44
2	0.016	286	162	87	115	58	86	58	39

Table 34 gives results of experiments made by the American Tool Works Co. with flat twisted high-speed-steel drills.

Table 34. Power Required to Drill Cast Iron and Steel with Flat Twisted High-speed-steel Drills

Drilling cast iron*					Drilling steel				
Size of drill, in.	Rev. per min.	Cutting speed, ft. per min.	Feed, in. per min.	H.p.	Size of drill, in	Rev. per min.	Cutting speed, ft. per min.	Feed, in. per min.	H.p.
1 1/32	313	84.5	14.4	13.2	9/16	356	52.3	4.27	4.2
1 7/32	313	99.8	14.4	15.3	3/4	313	61.5	3.75	10.8
1 15/32	216	83.1	7.1	12.6	1 1/32	188	50.9	4.51	9.0
1 23/32	216	97.0	7.1	16.8	1 5/32	188	56.9	4.55	9.3
1 31/32	128	66.0	4.22	15.6	1 23/32	128	57.6	3.07	8.4
3 1/2	60	55.0	1.44	10.2	1 31/32	167	86.2	2.00	7.8

* Length of hole, 2 in.

The Colburn Machine Tool Co. report the following **drilling records in cast iron** with high-speed twisted drills under a vertical machine:

Size of drill, in.	1	1 1/4	1 3/8	1 1/2	1 3/4	3	3	3
Speed of drill, r.p.m.	520	520	520	520	370	140	140	188
Feed per rev., in.	0.060	0.060	0.060	0.060	0.060	0.041	0.060	0.060
Cutting speed, ft. per min.	130	162	178	190	160	105	105	141
Inches drilled per min.	31.2	31.2	31.2	31.2	22.2	5.75	8.40	11.25

A similar report for **drilling steel** is:

Size of drill, in.	1 1/4	1 1/4	1 1/2	1 1/2	1 1/2	1 3/4	1 3/4	2 1/2	3	3
R.p.m. of drill	370	370	262	262	262	188	188	140	100	100
Feed per rev., in.	0.019	0.028	0.019	0.028	0.041	0.028	0.041	0.041	0.028	0.041
Cutting speed, ft. per min.	115	115	98	98	98	81	81	87	75	75
Inches drilled per min.	7	10.4	5	7.3	10.75	5.25	7.7	5.75	2.8	4.1

Thickness of block drilled in all tests, 4 in.

In all these tests the drills were in good condition after the holes were finished.

What may be considered as **maximum results in high-speed drilling** under radial drills, are given in Table 35 (F. E. Bocorselski, in *Am. Mach.*, vol. 33, part 1, p. 479).

Table 35. Power Required to Drill 0.70 Carbon Steel

Size of drill, in.	Rev. per min.	Cutting speed, ft. per min.	Feed, in. per min.	H.p.	H.p. per cu. in. of metal removed per min.	H.p. per lb. per min.
1 1/2	290	113.0	6.0	25.0	2.43	8.74
1 1/2	312	123.0	10.08	56.6	3.19	11.4
1 5/16	312	107.0	10.08	57.6	4.23	15.1
1 1/4	330	107.0	6.83	24.8	4.10	10.6
1 1/8	208	61.3	4.58	17.3	3.57	13.6
1 1/16	330	91.0	6.83	23.2	3.86	13.8

From these tests Mr. Bocorselski concluded that it is best to run a 1-in. drill at about 300 r.p.m., with a feed of 0.015 in. per rev., and a 1 1/2-in. drill at 225 r.p.m., with a feed of 0.02 in. per rev.

Dempster Smith and P. Poliakoff (*Am. Mach.*, vol. 32, part 1, pp. 739 and 830) give the results of an extensive series of experiments with twist drills of high-speed steel, cutting both steel and cast iron. The cast iron was of medium hardness with the skin removed. The steel was Whitworth's fluid-pressed, of medium hardness, 0.29 per cent. carbon and 0.625 per cent. manganese.

Tables 36 and 37 give the results of these experiments. The revolutions in column 2 of Table 36 for cast iron are based on a cutting speed of 48 ft. per min., and those in the corresponding column in Table 37 (for steel) on a speed of 60 ft. per min.

Table 36. Results of Drilling Tests in Cast Iron

Diam. of drill, in.	Speed, rev. per min.	Feed, in. per rev.	Cu. in. of metal removed per min.	Cutting h.p.	Feeding h.p.	Total h.p.	H.p. per cu. in. of metal removed per min.
¼	735.0	0.0075	0.27	0.29	0.005	0.295	1.092
⅜	490.0	0.0086	0.462	0.435	0.0055	0.4405	0.954
½	368.0	0.0094	0.682	0.58	0.0059	0.586	0.862
¾	245.0	0.0109	1.17	0.87	0.0066	0.8766	0.748
1	184.0	0.0119	1.715	1.16	0.007	1.167	0.681
1¼	147.0	0.0129	2.32	1.45	0.0073	1.457	0.628
1½	122.0	0.0136	2.92	1.74	0.0078	1.748	0.598
1¾	105.0	0.0144	3.63	2.03	0.0081	2.038	0.563
2	92.0	0.015	4.32	2.32	0.0084	2.328	0.539
2¼	81.7	0.0156	5.05	2.61	0.0086	2.619	0.519
2½	73.5	0.0162	5.82	2.9	0.0089	2.909	0.500
2¾	66.75	0.0167	6.6	3.19	0.0091	3.199	0.486
3	61.3	0.0172	7.4	3.48	0.0093	3.489	0.472
3¼	57.5	0.0176	8.22	3.77	0.0095	3.78	0.46
3½	52.5	0.0181	9.05	4.06	0.0096	4.07	0.45
3¾	49.0	0.0185	10.0	4.35	0.0098	4.36	0.436
4	46.0	0.019	10.8	4.64	0.00995	4.65	0.431

Table 37. Results of Drilling Tests in Steel

Diam. of drill, in.	Speed, rev. per min.	Feed, in. per rev.	Cu. in. of metal removed per min.	Cutting h.p.	Feeding h.p.	Total h.p.	H.p. per cu. in. of metal removed per min.
¼	920.0	0.006	0.284	0.712	0.0092	0.721	2.54
⅜	614.0	0.007	0.485	1.068	0.0102	1.078	2.22
½	460.0	0.008	0.716	1.425	0.0109	1.426	1.99
¾	306.0	0.009	1.23	2.14	0.0121	2.152	1.75
1	230.0	0.01	1.8	2.85	0.012	2.863	1.59
1¼	184.0	0.011	2.44	3.56	0.0138	3.574	1.47
1½	153.0	0.012	3.08	4.27	0.0145	4.285	1.39
1¾	131.0	0.012	3.81	4.99	0.015	5.005	1.31
2	115.0	0.013	4.54	5.7	0.0155	5.715	1.26
2¼	102.0	0.013	5.3	6.42	0.0159	6.436	1.21
2½	92.0	0.014	6.12	7.12	0.0163	7.136	1.165
2¾	83.5	0.014	6.92	7.84	0.0167	7.857	1.135
3	76.5	0.014	7.76	8.55	0.0171	8.567	1.105
3¼	70.5	0.015	8.66	9.25	0.0175	9.267	1.07
3½	65.6	0.015	9.5	9.98	0.0178	9.998	1.05
3¾	61.25	0.015	10.8	10.7	0.0181	10.718	1.024
4	57.5	0.016	11.4	11.4	0.0184	11.42	1.0

End Thrust and Torque of Twist Drills. Approximate equations for **the end thrust** P, in pounds, **and** the **torque** T, in lb.-ft., for ordinary twist **drills of any diameter** cutting soft cast iron without lubricants, are:

$$P = 35{,}500d^{0.7}t^{0.75}$$

$$T = 740d^{1.8}t^{0.7} = 10d^2 + 100t(14d^2 + 3),$$

where d = the drill diam., in., **and** t = the feed in in. per rev. of the drill.

The corresponding equations for mild steel are:

$$P = 35{,}500d^{0.7}t^{0.6};\quad T = 1640d^{1.8}t^{0.7} = 28d^2(1 + 100t)$$

High-speed Drilling. Speeds up to 10,000 r.p.m. in cast iron and soft steel are in use for drills No. 37 and smaller. Experimentally, speeds up to 20,000 r.p.m. have been used. The advantages of high speed in drilling small holes (in addition to the increased production) are: drill breakage is reduced; heating effect is lessened; practicable rates of feed are increased; burring is practically eliminated. Similar speeds have been used for tapping small holes. Taps Nos. 10–24 and smaller have been run at 10,000 r.p.m. in aluminum and soft brass.

Milling Machines

Milling machines use cutters with multiple teeth, in contrast to the single-pointed tools of the lathe and planer. The cut is not continuous. Though originally developed as a tool-room machine, the miller is to-day largely used as a manufacturing machine. It is **used for** an exceedingly wide variety of work, both of a job-shop and repetitive nature. Irregular sections, including spur and bevel **gears**, are easily produced by using formed cutters. **Flat surfaces** are economically produced by the modern wide-spaced toothed cutters. The universal machines are used to cut **spirals**, including threads, worms, helical gears, etc. In the tool room the vertical type is frequently used to bore jigs. Most milling machine work is held in a vise or fixture.

The **classification** of the National Machine Tool Builders' Association is: Column-and-knee type, Lincoln, hand, planer type (slab millers) and thread millers.

The Association has fixed the **maximum limits for** the vertical, cross and longitudinal **feeds** (in inches) of the common sizes of column-and-knee-type machines as follows:

Number of machine	Plain machines			Universal machines		
	Longitudinal	Cross	Vertical	Longitudinal	Cross	Vertical
0	18	6	15	..	..	..
1	24	7	19	20	7	18
1½	24	7	19	20	7	18
2	28	8	19	25	8	18
3	34	10	20	30	10	19
4	42	12	20	35	12	20
5	50	12	21	..	..	..

The vertical millers of one prominent builder have the following capacities:

Number of machine	Longitudinal traverse, in.	Cross traverse, in.	Vertical traverse, in.	Head traverse, in.
2	28	12	14	6
3	34	13	14	8
4	42	15	14	10

Column-and-knee-type Machines for manufacturing are **plain**, that is, they are provided with three traverses with power feed: vertical, longitudinal and cross. For milling spirals and similar work, the **universal machine** is necessary, in which the table can be swiveled at an angle with the face of the column.

Vertical Milling Machines are frequently provided with a rotary table, permitting them to be used for making cylindrical surfaces. It is not uncommon to provide them with fixtures of such a nature that finished work can be removed and unfinished work inserted while the machine is in operation.

Within a short time, a few American firms have placed vertical-type **circular milling machines** upon the market. These are intended for continuous milling similar to that of vertical machines with rotary tables. In Germany, a horizontal type is extensively used on parts having formed cross-sections, such as sheaves, belt pulleys, rollers, wheels, gear blanks, flywheels, and the like. The work is mounted on the end of a slowly revolving spindle, supported by a foot center if necessary. The cutter is brought in contact with the rim, or edge, which is entirely finished in one or two revolutions, the quality determining whether or not a finishing cut is required. These machines are adapted for manufacturing on a large scale, as one operator can attend to as many as six. The **power required** for a machine for a capacity of work up to 24 in. in diam, is about 2½ h.p.

Lincoln-type Millers are rigid machines for their size and weight, and are particularly adapted for repetitive manufacturing.

Hand Millers have the feeding movements hand-controlled. They are sensitive, as the cuts can be felt. They are used only on light work.

Planer-type Machines are used only on the heaviest work. Many are specially built to machine a number of surfaces on a particular part or small group of parts, and have a number of cutter spindles.

Thread Millers are used to cut threads and worms.

The **tapers used in** the **spindles** of horizontal machines are generally as follows:

Machine number	0	1½	2	3	4	5
Brown & Sharpe taper number	9	10	10	11	12	12

Motors for Millers. Motor sizes and speeds for various types of milling machines, as determined by Charles Robbins, are given in Table 38.

Table 38. Sizes and Speeds of Motors for Milling Machines

Horizontal machines, plain or universal				Vertical machines			Slab milling machines	
Table feed, in.	Cross feed, in.	Vertical feed, in.	H.p.	Table diam., in.	Spindle diam., in.	H.p.	Width of table, in.	H.p.
24	8	18	3	28	4	5	24–30	10
30	10	18	5–7½	32	4	7½	36	15
36	12	20	7½–10	40	4½	10	60	25
50	12	20	10–15	54	5	15	36 heavy	25
....	....	....	....	70	6	20	42 heavy	50

Power Required to Remove Metal with Milling Cutters. The power requirements of different forms of cutters vary with their construction. Table 39 gives values obtained from tests made by A. L. DeLeeuw with Cincinnati plain and high-power millers.

Milling Cutters are made from both carbon and high-speed steel. Common kinds are enumerated below.

Angular, for cutting tooth spaces in other cutters, ratchets and the like; **convex and concave,** used to cut half-round grooves and projections, the convex often used to flute taps; **corner rounding; cotter mills,** for cutting keyseats, slots and grooves; **dovetail; end mills,** for milling edges of work, cutting slots, cam outlines and grooves; **formed cutters,** made solid in small sizes—with inserted teeth in large, used to produce irregular and other contours; **fish-tail cutters,** used at high speed and with light feed for slots, grooves and keyways; **helical cutter,** consisting of a cylindrical body with teeth in the form of a screw thread wound around it at an angle of about 60 deg. with the axis—used on all classes of work, free cutting and remarkably efficient; **shell end cutters,** made to be screwed to a shank or keyed to end of an arbor, used for same work and heavier cuts than end mills; **side mills or straddle mills,** comparatively thin cutters used to finish sides of work or to cut slots, frequently used in pairs to straddle the work; **slabbing cutters,** having a comparatively long face for surfacing; **saws, T-slot cutters.**

The tendency in milling cutter design is toward **wide-spaced teeth.** Chip space is necessary for free cutting. A. L. DeLeeuw (*Trans. A. S. M. E.*, vol. 33) gives propor-

tions for end mills, shell cutters, side mills, face mills, and helical cutters. The number of teeth recommended are respectively as follows:

Diam. of cutter, in.	1	1¼	1½	2	2½	3	3½	4	5	6
Number of teeth:										
End mills	4	5	6	8	...	..	...	..	..	..
Side mills and spiral shell cutters	..	...	...	..	8	8	9	10	11	12

The metal-removing capacity of these is given in Table 39.

The **helical cutters** are made with a helix angle of the teeth with the axis of the body of 60 deg., rake of 15 and clearance of 5 deg. for cutting steel, and rake of 8 and clearance of 7 deg. for cutting cast iron. They push off the chip in the direction of the cutter axis and at right angles to the feed. Roughing cuts in steel take only about one-third the power required by old-style spiral mills.

Table 39. Power Requirements of Various Types of High-speed Steel Milling Cutters

Material cut	Size and type of cutter	H.p. per cu. in. of metal removed	Material cut	Size and type of cutter	H.p. per cu. in. of metal removed
Mild steel.*	8-in., 12-bladed, high-power face mill.	1.4 to 1.9	Steel.**	4-in. spiral mill with nicked teeth.	1.5 to 2.2
Mild steel.†	Do.	1.0 to 1.6	Cast iron	3½-in., 14-toothed spiral nicked cutter	0.8 to 1.0
Mild steel.†	Do.	1.0 to 1.2	Cast iron	3-in., 14-toothed end mill	0.9 to 1.2
Mild steel.‡	Do.	0.8 to 1.0	Cast iron	9½-in., 22-toothed regular face mill.	0.7 to 1.0
Mild steel.§	10-in., 16-bladed, high-power face mill.	0.8 to 1.2	Cast iron	5-in., 20-toothed end mill.	1.0 to 1.6
Mild steel.§	4½-in., 10-toothed spiral nicked cutter.	1.0 to 1.3	Cast iron.	8-in, high-power face mill.	0.5 to 0.7
Mild steel.§	9½-in., 22-toothed regular face mill.	1.6 to 1.7	Cast iron††	10-in. high-power face mill.	0.9 to 1.2
Mild steel.§	3½-in., 4¼-in. lead, 2⅝-in. pitch helical cutter.	0.7 to 1.0	Cast steel††	4-in. spiral nicked cutter.	1.6
Mild steel.§	3⁷⁄₁₆-in., 4¼-in. lead, 1⁵⁄₁₆-in. pitch helical cutter.	0.7 to 1.0	Spiegeleisen.††	4-in. spiral nicked cutter.‡‡	1.0
Steel.‖	8-in. face mill.	1.8 to 2.7	Cast iron§§	8-in., 18-toothed face mill.	0.5 to 0.7
Steel.‖	4½-in. spiral nicked cutter.	1.7 to 1.8	Cast iron§§	10-in., 16-bladed, high-power face mill.	0.4 to 0.6
Steel.**	10-in. high-power face mill.	1.3 to 1.7	Cast iron§§	4½-in. nicked spiral mill.	0.4 to 0.9

* Elastic limit (E. L.), in lb. per sq. in., 36,200. † E. L. = 36,400. ‡ E. L. = 37,400. § Tensile strength (T. S.) in lb. per sq. in., 55,000; carbon, 0.2 per cent.; manganese, 0.5 per cent. ‖ German Siemens-Martin steel; T. S. = 85,000 to 90,000. ** Chrome-nickel steel; T. S. = 122,000 to 141,000. †† German metal. ‡‡ Cutter made from carbon steel. §§ German gray iron.

Performance of Large High-speed-steel Milling Cutters with Inserted Helical Blades. Results of numerous experiments made with large milling cutters with inserted helical blades, known as Taylor-Newbold cutters, are given in Table 40 (Wilfred Lewis and W. H. Taylor, *Trans. A. S. M. E.*, vol. 30, p. 867). The angle of the helix of these blades is 19 deg. 15 min., given by the formula: Diam. × 9 = pitch. The blades are set in grooves in the steel body and held by pouring around them a molten, fusible, non-shrinking alloy. A gain of 33 per cent. in cutting speed in milling steel and wrought iron is made by throwing a heavy stream of lubricant upon the cutter along its entire face. A similar gain of 15 per cent. is made in milling cast iron. For a large milling cutter, the nozzles in the lubricating pipe should be spaced about 4 in. apart and in sufficient number to cover the face of the cutter.

Table 40. Results of Tests of Taylor-Newbold High-speed Steel Milling Cutters with Inserted Helical Blades

Feed		Depth of cut, in.	Width of cut, in.	Metal removed per min., lb.	Duration of test, min. sec.	Speed of cutter		H.p.	H.p. per cu. in. of metal removed per min.	Cu. in. removed per in. width of cut
Table advance per min., in.	Advance per blade, in.					Rev. per min.	Feet per min.			
Slab milling cast iron—Cutter, 8 in. diam. × 18 in. face.										
3½	0.00636	½	15	6.83	10 2	25	53½	40.21	0.53	1.75
5⅞	0.01332	½	15	11.47	6 8	24½	51⅛	44.23	1.00	2.94
7½	0.01736	½	15	14.74	4 48	24	50¼	44.70	0.794	3.75
6⅛	0.01361	½	15	11.96	5 5	25	53½	48.79	1.000	3.06
7¾	0.01722	½	15	15.13	4 39	25	53½	60.32	1.030	3.87
7⅜	0.01781	11/16	15	13.20	4 53	23	48	70.96	1.390	3.38
8	0.01851	11/16	15	21.48	1 15	24	50¼	68.52	0.830	5.50
7	0.01760	1	15	27.34	3 17	22	47	89.14	0.849	7.00
Slab milling 0.30 per cent. carbon steel—Cutter, 8 in. diam. × 18 in. face										
11¼	0.01785	3/16	18	8.96	1 47	35	73⅓	56.03	1.77	1.76
11¼	0.01785	5/16	18	17.92	0 11	35	73⅓	......	....	3.51
6⅝	0.01041	⅜	18	12.19	1 30	34	71	78.41	1.82	2.99
7	0.01080	⅜	18	13.38	1 26	36	75½	96.51	2.04	2.62
7	0.00925	⅜	18	13.38	2 51	42	88	92.74	1.96	2.62
Channel milling 0.35 per cent. carbon-steel forging—Cutter, 8½ in. diam. × 4⅜ in. face										
6⅞	0.01320	5/16	4⅜	2.66	3 25	37	78.69	22.74	2.42	2.14
1 15/16	0.00307	1	4⅜	2.40	7 12	35	74.44	23.56	2.72	1.93
1½	0.00231	1	4⅜	1.86	4	36	76.57	21.85	3.33	1.49
2⅝	0.00394	1 5/16	4⅜	4.27	2 40	37	78.69	30.29	2.00	3.44
3⅝	0.00544	1 5/16	4⅜	5.89	1 56	37	78.69	35.28	1.69	4.75
5	0.00750	1 5/16	4⅜	8.13	1 24	37	78.69	50.93	1.77	6.56

Speeds and Feeds of Milling Cutters. Good general practice with carbon-steel cutters is, for wrought iron and mild steel, a surface velocity of 30 ft. per min., with a feed of 0.01 to 0.02 in. per rev.; for cast iron, 40 to 50 ft. per min. and 0.02 to 0.03 in. per rev.; for brass and bronze, 60 to 75 ft. per min. and 0.03 to 0.04 in. per rev. With high-speed steel cutters these speeds may be increased from 50 to 75 per cent.

High-speed Milling. Under a system worked out by the Cincinnati Milling Machine Co. with very rigid machines, and with the employment of about 10 times as much lubricant as usual, peripheral cutting speeds up to 800 ft. per min. in soft steel and feeds up to 112 in. per min. were successfully reached in tests. In daily practice cutting speeds up to 450 ft. per min. and feeds up to 30 in. per min. are now common.

Gear-cutting Processes

Teeth in gears are formed by either a milling, shaping or hobbing process. **The Four Principles of Action** of gear-cutting machines are:

(*a*) The formed-tool principle, using a tool or cutter shaped to the tooth space.

(*b*) The template principle, in which the motion of the cutting tool is controlled by a template corresponding to the tooth curve.

(*c*) The odontographic principle, in which the tool is guided by suitable mechanism so that its path closely approximates the tooth curve.

(*d*) The generating principle, in which a tool or series of tools representing a rack are moved with reference to the gear blank being cut in the same manner as a rack meshing with its gear; or a cutter representing a mating gear is rolled with the gear blank while cutting. As a finishing operation, an abrasive wheel sometimes takes the place of the cutting tools.

Machines using method (*a*) produce spur, spiral, helical and worm gears; (*b*) and (*c*) spur and bevel gears; and (*d*) spur, helical, bevel and worm gears.

Processes for Cutting Spur Gears. The most common rocess is **milling,** which uses a circular formed cutter corresponding with the shape of the tooth space. The gear blank is carried upon an arbor in the main spindle of the machine, which has a dividing wheel on its other end. This wheel is usually larger than the largest gear to be cut, to diminish inaccuracies in indexing.

The template principle is rarely used, except for very large gears, as in rolling-mill work. A pair of such gears are designed to run together without being interchangeable with others. The machines operate on the **planing or shaping process.**

The **generating** principle has numerous applications. The shaping **process** is employed, using a tool shaped like a rack tooth. The operation is continuous, succeeding cuts being taken in successive tooth spaces. The rolling action takes place after the gear has been cut around, thus setting for the second cut in each tooth space, and so on. The blank is rotated and the tool ram carrying the tool is fed sidewise, the motion corresponding with the motion of the gear meshing with an imaginary rack.

The **shaping process** is employed in another way, where multiple tools are used representing a rack. There are two sets, one on each side of the blank. The blank is rotated and the tools fed sidewise simultaneously. For large gears, the tools reciprocate in a vertical direction and the blank only rotates. For small gears, this arrangement is reversed. The most widely used machine employing a shaping process and the generative principle is the Fellows gear shaper. The cutter is a pinion which reciprocates and also rotates as if in mesh with the gear to be cut from the blank. In cutting, it is usually fed in to its full depth at the start and then rotated, finishing the gear with one cut around.

During the past few years **hobbing machines** have grown rapidly in favor. They employ the generative principle and the milling process. A section through a hob along a plane passing through the center of figure and making an angle with the axis equal to its helix angle at the pitch surface, is the section of a rack. Thus the hob is in effect a generating rack. Both hob and blank are rotated to give the effect of a meshing gear and rack. The hob starts the cut at one edge of the blank and is fed across its face.

Processes for Cutting Bevel Gears. The **milling process and formed circular cutter principle** are employed most widely in cutting bevel gears, except for those that are required to be especially accurate; at least two cuts are necessary through each tooth space, one on each side. Bevel gears cut in this way have only approximately correct tooth curves. As a rule, the cutter is designed to give the correct shape at the large end of the tooth, and necessarily leaves the smaller end too thick. This is corrected by filing.

The **template principle and planing process** are used in the Gleason bevel-gear planing machine. Three templates are used, a straight-faced one for gashing, and two formed ones, one for each side of the tooth space. Simple planing tools are used. After all of the spaces are gashed, one side of each tooth is completely finished, and then the other side to complete the gear.

The **generating principle and shaping process** have been used in a number of different machines. The general arrangement consists in providing a crown gear or its equivalent with the cutting tools representing its sides in successive positions. Meshing with the crown gear is a master bevel gear carrying on its shaft or arbor the blank to be cut. The master and crown gears rotate in mesh, thus presenting the blank to the cutting tool in successive positions to generate the teeth. Details of this general arrangement have been worked out in a number of different ways.

The **generating principle and milling process** are used in an ingenious machine built by the Brown & Sharpe Mfg. Co. The sides of the teeth of the imaginary crown gear are represented by the plane faces of milling cutters. In operation, the imaginary crown gear is stationary and the master bevel gear is rolled over it, presenting the work to the cutting tool. Large milling cutters are used to minimize the deepening of the tooth space at the center. They interlock and cut on opposite sides of the same tooth space. The Warren machine is similar in its action, except that the two cutters operate on facing sides of alternate teeth.

Processes for Cutting Internal Gears employ the following principles and processes: (1) Formed circular cutter principle and milling process; (2) generative principle and shaping process (Fellows machine); (3) formed cutter principle and shaping process. This is the one most commonly used.

Processes for Cutting Racks. The following principles and processes are used: (1) Formed circular cutter principle and milling process, this being by far the most common; (2) generative principle and shaping process (Fellows machine).

Processes for Cutting Helical Gears employ the following principles and processes: (1) Formed cutter principle and shaping process; (2) formed circular cutter principle and milling process, this being the most common; (3) generative principle and milling process (hobbing); (4) generative principle and shaping process (Fellows machine).

Processes for Cutting Worms use the (1) formed circular cutter principle and milling process; and the (2) formed cutter principle and turning process.

Processes for Cutting Worm Wheels employ the generating principle and milling process (hobbing). Three **forms of cutters** are used, the straight hob, tapered hob and fly cutter. The first requires the simplest mechanism; an ordinary miller is sufficient, but the hobs are expensive. The fly cutter is easily and accurately made, but requires a complicated machine to use it. The taper hob is adapted to large work and a large number of pieces.

Motors for Gear Cutters. Most gear-cutting machines are of comparatively small capacity, and for this reason are driven in groups. For power requirements, see Table 18.

Gear Cutters. Circular formed cutters for involute toothed gears in an interchangeable standard system are made in sets of eight, to cut gears ranging from a 12-toothed pinion to a rack.

Number of cutter.........	1	2	3	4	5	6	7	8
Number of teeth..........	135–∞	55–134	35–54	26–34	21–26	17–20	14–16	12 and 13

For more accurate gears, a 15-cutter set is made by introducing a half number between each two in the foregoing list, each cutter therefore having a smaller range of teeth, as follows:

Number of cutter...	1	1½	2	2½	3	3½	4	4½
Number of teeth....	135–∞	80–134	55–79	42–54	35–41	30–34	26–29	23–25

Number of cutter.........	5	5½	6	6½	7	7½	8
Number of teeth..........	21 and 22	19 and 20	17 and 18	15 and 16	14	13	12

For still more accurate gears, cutters are made for the exact number of teeth to be cut.

The **duplex gang cutters** made by Gould & Eberhardt are to be used in gangs of two or more, the number depending upon the number of teeth in the gear to be cut. The following tabulation gives the number that may be used in cutting different numbers of teeth:

Number of cutter..............	1	2	3	4	5
Number of teeth..............	Under 30	30–49	50–69	70–94	95–119
Number of cutter..............	6	7	8	10	12
Number of teeth..............	120–149	150–179	180–229	230–259	Over 260

To rough out the tooth spaces, **stocking cutters** are used. These leave only a small amount of metal to be removed by the regular cutter, increase the accuracy of the gears, and save wear on the finishing cutters. One stocking cutter serves for all gears of the same pitch. The term stocking cutter is also applied to a concave cutter ganged beside a regular gear cutter and used to finish the periphery of a gear blank by milling ahead of the gear cutter. **Hobs** are cutters with helical teeth arranged like the thread of a screw and fluted to give cutting edges. They are backed off and can be ground on the faces of the teeth without changing the shape.

Grinding Machines

The **classification** of grinding machines of the National Machine Tool Builders' Association is as follows: Grinding wheel stand (for bench); electric drive (for bench); cock; cup and cone; cutter; cylinder; cylindrical; center; drill; grinding wheel stand (for floor); frog and switch; hob; internal; multiple spindle; polishing stand; roll; semi-universal; surface (both vertical and horizontal); surface, rotary table type; disk; tool; universal.

All of these machines except the disk type use **grinding wheels** made by fixing an abrasive in a binder which is usually hardened by baking. The disk type uses an abrasive or abrasive cloth pasted to the surface of a metal disk. **Cutter grinders** are designed to grind all kinds of milling and similar toothed cutters. **Cylinder grinders** are a special type for grinding the cylinders of automobile engines. One form has a planetary motion for the grinding spindle. The **cylindrical grinder** is a companion machine to the engine lathe. Shafts, cylinders, rods, rolls, studs, and a wide variety of other cylindrical parts are first roughed out on the lathe, then finished accurately to size by the cylindrical grinder. **Drill grinders** are provided with rests so mounted that by a simple swinging motion correct cutting angles are automatically produced on the lips of drills. A cupped wheel is usually employed. **Internal grinders** are used for finishing the holes in bushings, rolls, sleeves, cutters and the like. High speed is a feature of these machines. Speeds from 15,000 to 30,000 r.p.m. are common and speeds of 100,000 r.p.m. are claimed for the Rivett machines. **Horizontal surface grinders** are in general of small capacity and are used mainly in tool making. **Vertical surface grinders** are used for producing flat surfaces on manufacturing work. **Vertical and horizontal disk grinders** are used for surfacing on both cast iron and steel. **Universal grinders** are cylindrical machines arranged with a swiveling table so that both straight and taper internal and external work can be ground. They are used on tool work and in refined manufacturing.

Motors for Grinders. Charles Robbins (*Trans. A. S. M. E.*, vol. 32, p. 215) gives motor sizes for cylindrical grinders for medium (heavy) work as follows: 10 in. diameter, 5 (7.5) h.p.; 14 and 18 in. diameter, 10(15) h.p. For a 44-in. wheel grinder 30 h.p. is required.

Grinding Wheels. Grinding wheels are made from the commercial abrasives, emery, corundum, carborundum, alundum, and others marketed under special trade names. For **grades** and for **abrasives in grain form**, see p. 616. The marks left by these various grits have been compared with file marks, as follows:

Table 41. Grit Marks and Corresponding File Marks

Size of grains	File which leaves corresponding marks	Size of grains	File which leaves corresponding marks
8 to 10	Wood rasp	46 to 60	Second-cut
16 to 20	Coarse-rough	70 to 80	Smooth
24 to 30	Rough	90 to 100	Superfine
36 to 40	Bastard	120, F and FF	Dead smooth

A wheel of hard **bond** is retentive of the grains, while in the softer bonds the grains are more easily broken out. In a wheel of proper grade for the work being done, the grains are automatically replaced when dull. A **wheel glazes** when it is too hard. Dressing a wheel breaks off the surface and exposes fresh, sharp grains.

Table 42 gives a comparison of the selection of grades and number as used by both the Norton Co. and Carborundum Co. for the general run of work that is ground.

Table 42. Abrasive Wheels for Various Kinds of Work

Class of work	Norton Co.		Carborundum Co.	
	Number usually furnished	Grade usually furnished	Number usually furnished	Grade usually furnished
Large cast-iron and steel castings	16 to 20	Q to R	16 to 24	G to H
Small cast-iron and steel castings	20 to 30	P to Q	20 to 30	G to H
Large malleable-iron castings	16 to 20	Q to R	16 to 24	G to H
Small malleable-iron castings	20 to 30	P to Q	20 to 30	H to I
Chilled-iron castings	16 to 20	Q to R	16 to 24	H
Wrought iron	16 to 30	P to Q	16 to 24	F to H
Brass castings	16 to 30	O to P	20 to 36	H to I
Bronze castings	16 to 30	P to Q	20 to 30	I
Rough work in general	16 to 30	P to Q	20 to 30	H
General machine shop use	30 to 46	O to P	24 to 36	G to J
Lathe and planer tools	30 to 46	N to O	30 to 36	I to J
Small tools	36 to 100	N to P	50 to 80	I to J
Wood-working tools	36 to 60	M to N	40 to 60	L to M
Twist drills (hand grinding)	36 to 60	M to N	60	I to J
Twist drills (special machines)	46 to 60	K to M	50	L to O
Reamers, taps, milling cutters, etc. (hand grinding)	46 to 100	N to P	50 to 80	K to N
Reamers, taps, milling cutters, etc. (special machines)	46 to 60	H to K	50 to 60	L to M
Edging and jointing agricultural implements	16 to 30	Q to R	141 to 24	G to I
Grinding plow points	16 to 30	P to Q	20 to 24	H
Surfacing plow bodies	20 to 30	N to O	16 to 20	G
Stove mounting	20 to 36	P to Q	24 to 30	G
Finishing edges of stoves	30 to 46	O to P	24 to 30	G
Drop-forgings	20 to 30	P to Q	24 to 36	G to I
Gumming and sharpening saws	36 to 60	M to N	403—603	J to L
Planing-mill and paper-cutting knives	30 to 46	J to K	202—60 to 80	M to R
Car-wheel grinding	20 to 30	O to P	16 to 24	H

Combination Grit Wheels, as their name implies, are composed of grits of varying degrees of fineness. For finishing purposes these wheels have the compactness and smooth face of a wheel made solely from their finest grit, and for roughing the ability to take a depth of cut determined by the capacity of the largest grains. As a general standard, the harder the material to be ground the softer should be the bond of the wheel. This is modified by the fact that cast iron and hardened steel require the same grades.

Too large an assortment of wheels is liable to lead to confusion in their use. With a Norton plain cylindrical grinder, **four grades**, J, K, L and M, of a 24 combination grit are found **sufficient** for all the materials **usually** ground. These include chilled iron, cast iron, high-and low-carbon steels, bronzes and the common compositions.

In the grinding process cutting is done with the lightest tool pressure of all cutting methods. This advantage may be lost if too hard a wheel is used, that is, one in which the bond will not crumble under the pressure of the cut but must be forced with a pressure sufficient to displace the work. Destructive vibrations and inaccurate work are often chargeable to too hard wheels. Grinding is a **true cutting process.** Under the microscope, the particles of metal removed by an abrasive wheel are seen to have the shape and character of chips cut by a lathe tool from the same material.

Alden (*Trans. A. S. M. E.*, 1914) comes to the following conclusions: (*a*) Increase of work speed increases grain depth of cut, and makes a wheel appear softer; (*b*) decrease of wheel speed increases grain depth of cut; (*c*) diminishing the diameter of the grinding wheel increases grain depth of cut, and (*d*) diminishing the diameter of work increases grain depth of cut.

Wheel Speeds. Surface speeds of 5000 to 6000 ft. per min. are commonly employed, although lower speeds are sometimes used. F. B. Jacobs gives the following **data for wheel speeds,** work speeds, traverse feeds and depth of cut. The proper surface speed for a carborundum wheel is 5000 to 5500 ft. per min. Wheels made of alumina abrasives, corundum, aloxite, alundum, etc., give better results when run at a higher speed, say, 5500 to 6000 ft. per min. A general rule is to reduce the wheel speed if glazing occurs frequently, and to increase the speed if the wheel does not hold its shape well and wears away too readily.

Work Speeds. For roughing cast iron, 40, and for finishing, 50 ft. per min. surface speed; for roughing steel, 20 to 30, and for finishing, 30 to 40 ft. per min. surface speed are good practice. These are rules for grinders equipped with the coarse traverse feeds that are used with wide-faced grinding wheels. With the older types of machines having fine traverse feeds, it is necessary to use higher speeds. High work speeds cause chattering, glazing, and undue wearing of wheels.

Traverse Feeds depend entirely on the width of the wheel. In roughing, the work should travel past the wheel ¾ to ⅞ of the width of the wheel for each revolution of the work. As the work, or rather cut, travels past the wheel with a helical motion, the above rule allows a slight overlap. In finishing, a finer feed is used, generally ⅛ to ¼ of the width of the wheel for each revolution of the work.

Depth of Cut. In the roughing operation the depth of cut should be all the wheel will stand without crowding. This varies with the hardness of the material and the diameter of the work; the operator's experience is the only guide. In the finishing operation the depth of cut is always slight, from 0.001 in. to 0.002 in. Excellent results as regards finish are obtained by letting the wheel run over the work several times without cross-feeding. This practice of letting the wheel "grind out" has been found by the majority of expert operators to give satisfactory results even with a comparatively coarse wheel.

Grinding Allowances. On heavy work and heavy machines, grinding allowances are commonly from $\frac{1}{64}$ to $\frac{1}{32}$ in. On fine work, from 0.003 to 0.007 in. is sufficient.

Grindstones are still used for some kinds of work, particularly for the rough grinding of forgings, but the tendency is to replace them with steel wheels carrying segmental abrasive blocks. The Ohio sandstones can be safely run up to a peripheral **speed** of 2500 to 3000 ft. per min. and Huron stone up to 4000 ft. per min. These speeds presuppose careful setting up, large flanges and only moderate wedging. The older form of square shaft hole is dangerous, in that it furnishes points from which fractures may easily start. The strength of grindstones is reduced by wetting, some investigations indicating by as much as 40 to 50 per cent.

The Sand Blast consists of particles of sand, powdered quartz, chilled iron globules, emery or other hard granular material blown by a jet of compressed air or of steam against a hard surface which it is desired to abrade. It is commonly used for cleaning metal, castings, frosting smooth surfaces, etc. Portions of the surface which are not to be abraded can be protected by coating with a soft material such as wax, lead or rubber. For cleaning castings compressed air at about 10 lb. pressure is used; with a 2-h.p. compressor 2 sq. ft. of surface can be cleaned per minute, or 100 lb. of small castings in a slowly rotating barrel can be cleaned in 10 to 15 min. To completely remove the hard skin of the castings takes about twice as long.

Polishing Wheels are run at the following peripheral **speeds** (ft. per min.): Walrus hide. 8000; wood, leather-covered, 7000; rag wheels, 4 to 8 in. in diam., 7000.

Exhaust Fans for Grinding Wheels. The exhaust pressure commonly used for removing dust in polishing, buffing and light grinding is 3 oz. per sq. in.; in heavy grinding, 4 to 4½ oz.

Hood inlets for buffing, polishing and grinding wheels up to 26 in. in diam. should be made for the following sizes of pipe:

Wheel diam., in.	4 or less	4 to 6	8 to 10	12 to 16	18	20	22 to 26
Pipe diam., in.	2	3	3½	4	4½	5	6

The **hoods and piping** of dust-exhausting systems are usually made of galvanized iron. The **size of fan inlet** is usually made equal to the sum of the areas of the hood inlets or from 10 to 20 per cent. less.

Files are designated by their length, longitudinal and sectional contours, the kind of cut and the spacing of the teeth. Longitudinally, files are **blunt** (with parallel sides) or **taper**. The **length** is the distance (in in.) from the cutting end of the file to the **tang**, or pointed end for fitting into a handle. **Safe-edge** files have one of the narrow edges uncut, so that one surface near a corner may be filed without marring the other surface. Standard **shapes** of files in general use are as follows:

Metal saw—Blunt; triangular section (equal sides).
Three-square—Taper; triangular section (equal sides).
Barrette—Taper; thin trapezoidal section.
Slitting—Blunt; rhombic section, diagonals in ratio 1 : 3 or more.
Square—Blunt or taper.
Round—Blunt or taper (rat-tail).
Knife—Taper; width = thick edge × 3 or more.
Half-round—Blunt or taper; section, segment of circle.
Crossing—Taper; section, two half-rounds with flat faces back to back.
Crochet and Warding—Taper, thin rectangular sections.
Mill, Hand and Pillar—Blunt or taper; medium to thick rectangular section.

Files are either **single-cut** (with teeth at about 25 deg. to direction of width), **double-cut** (single-cut with a second cut crossing first at angle of 45 to 50 deg. to direction of width), or **rasp-cut** (individual teeth forced up from the surface by a blunt-ended punch). They are ordinarily **graded** as follows (the finer cuts for machinists' use being numbered 00 to 8):

Single-cut:	Rough;	middle;	halfway;	bastard;	second cut;	smooth.	
Double-cut:	Rough;	middle;	halfway;	bastard;	second cut;	smooth,	dead smooth.
By number:				00	1	2–3	6–8

The **tooth spacing** varies according to the length and kind of file. For 12-in. files the number of teeth per in. are approximately 17 (rough), 30 (bastard), 96 (smooth). Taper files run from 64 teeth per in. for 2½-in. length, to 34 teeth for 10-in. length; mill and flat bastard files, from 48 to 50 for 4-in. length, to 18 to 22 for 20-in. length.

Files cut better after a little use, as the burrs left on some of the tooth edges become worn away, thus permitting other teeth to come into action. They may be **resharpened** by directing a sand blast against the backs of the teeth; there is a wide difference of opinion as to the value of a resharpened file.

Broaching Machines

Broaching machines are used to produce irregular-shaped holes, keyways, the teeth of internal gears, square, hexagonal and other polygonal holes, and similar work. The cutting tool—the broach—is long and is provided with many teeth so graded in size that each takes a small chip when the tool is (usually) pulled or (sometimes) pushed through a drilled leader hole. The employment of sliding gears in automobiles has extended the use of broaching machines, for these are used to produce the square holes or multiple keyways in such gears. These machines are made in numbered sizes from 1 to

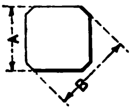
4, inclusive. The capacity of the Lapointe machines is as follows, the second dimension being the length of hole or keyway:

		1	2	3	4
Size Number		1	2	3	4
Capacity	Keyway, in.	$\frac{1}{4} \times 6$	$\frac{5}{8} \times 6$	$\frac{3}{4} \times 12$	$1\frac{1}{4} \times 14$
	Square hole, in.	$\frac{1}{2} \times 2$	$1\frac{1}{8} \times 5$	2×6	3×8

Broaching is also sometimes substituted for reaming in finishing round holes to size. Where a great deal of metal is to be removed, two or more broaches are employed, in series, the smallest tooth of the second being slightly larger than the largest tooth of the first, and so on.

The Lapointe Machine Tool Co., manufactures **standard broaches for round-cornered square holes**, such as are used in automobile gears. Table 43 gives the sizes of holes produced by these broaches.

Table 43. Dimensions of Lapointe Standard Round-cornered Square Holes



Size No.	*A* inches	*B* inches	Size No.	*A* inches	*B* inches	Size No.	*A* inches	*B* inches
1	1	$1\frac{1}{4}$	7	$1\frac{3}{8}$	$1\frac{3}{4}$	13	$1\frac{3}{4}$	$2\frac{1}{4}$
2	$1\frac{1}{16}$	$1\frac{5}{16}$	8	$1\frac{7}{16}$	$1\frac{13}{16}$	14	$1\frac{13}{16}$	$2\frac{5}{16}$
3	$1\frac{1}{8}$	$1\frac{3}{8}$	9	$1\frac{1}{2}$	$1\frac{7}{8}$	15	$1\frac{7}{8}$	$2\frac{3}{8}$
4	$1\frac{3}{16}$	$1\frac{15}{32}$	10	$1\frac{9}{16}$	$1\frac{15}{16}$	16	$1\frac{15}{16}$	$2\frac{7}{16}$
5	$1\frac{1}{4}$	$1\frac{9}{16}$	11	$1\frac{5}{8}$	2	17	2	$2\frac{1}{2}$
6	$1\frac{5}{16}$	$1\frac{5}{8}$	12	$1\frac{11}{16}$	$2\frac{1}{8}$	......	....	....

In *Am. Mach.*, vol. 33, part 2, p. 688, some **records are given for broaching** to finish size $2\frac{3}{32}$-in. holes in bronze bearings. The roughing broach roughs from 3000 holes up, and must be honed after each 1000. The corresponding finishing broach finishes from 1000 to 2000 pieces, and must be honed after each 400 to 500 holes. These broaches are made with six sets of teeth, of which the lower three increase successively 0.002 in. each. The last three teeth are all of the same size, 0.002 in. larger than the immediately preceding tooth, and of the finished diam. wanted. With broaches of this kind, it is customary to grind them 0.0004 in. large, leaving this amount to hone to finished size to produce a smooth, keen edge. Mr. Lapointe states that 3-in. internal gears of alloy steel have been broached at the rate of 25 to 30 per hour.

The making of broaches depends to a great extent upon experience. The **number and spacing of teeth** depends upon the length of the hole, the longer the hole the more chips each tooth will cut and therefore the more chip space necessary. The **steel for broaches** is usually high-grade tool steel which will not warp in hardening. There is more difficulty with hardening broaches with fine teeth than with the larger ones, as in the former case the teeth cool more quickly than the body, thus tending to the formation of water cracks.

Cutting-off Machines

These machines—used to cut up merchant bar and rolled shapes—are made in **three types**: the lathe type, using single-pointed cutting-off tools; friction sawing machines, using notched disks of soft steel running at a high speed and cutting by the heat generated by friction; and cold-sawing machines, using circular saws. As regards production, there is but little choice between the first and third types.

The **lathe-type cutting-off machines** are graded into inch sizes, according to the largest diameter of round stock that the machine will handle. These are:

Size of machine, in.	2	3	4	5	6
Capacity: size of bars, in.	$\frac{1}{4}$ to 2	$\frac{3}{8}$ to 3	$\frac{1}{2}$ to 4	$\frac{3}{4}$ to 5	1 to 6
Horse power of motor	1	2	3	3	3

The **friction sawing machine** (fusing disk machine) is largely used on structural shapes. A 15-in. beam can be cut through in 28 to 38 sec., and smaller sections in proportionate times. A machine carrying disks from 44 to 52 in. in diam. is provided with a 52-h.p. motor. Peripheral speeds of about 25,000 ft. per min. are used. **Cold saws** are made in a wide variety of styles and sizes. Some makers list them in numbered sizes; others, according to the diameter of saw blade which the machine will carry. Table 44 gives the sizes of standard inserted-tooth saws made by the Tindel Morris Co., and also sizes of saws to cut structural shapes. With all these machines it is necessary to have a special grinder to grind blades automatically.

Table 44. Dimensions of Inserted-tooth Cold-saw Blades
(Tindel Morris Co.)

Diam., in.	Thickness, in.	Kerf, in.	No. of cutters	Diam., in.	Thickness, in.	Kerf, in.	No. of cutters
Standard Sizes							
12	0.250	0.312	36	30	0.437	0.530	54
14	0.270	0.332	38	32	0.458	0.551	56
16	0.291	0.353	40	36	0.500	0.593	60
18	0.312	0.375	42	38	0.520	0.613	62
20	0.333	0.395	44	42	0.562	0.687	66
22	0.354	0.416	46	48	0.625	0.750	72
24	0.375	0.437	48	52	0.666	0.791	76
26	0.396	0.458	50	62	0.750	0.906	86
28	0.417	0.510	52	....	....	....	....
Sizes of Blades for Cutting Structural Shapes							
22	0.302	0.365	88	34	0.364	0.426	136
24	0.312	0.375	96	36	0.375	0.437	144
26	0.323	0.385	104	38	0.385	0.447	152
28	0.333	0.395	112	40	0.396	0.458	160
30	0.344	0.406	120	42	0.406	0.468	168
32	0.354	0.416	128	....	....	....	....

Table 45. Cutting Speeds and Feeds for Cold Saws

Diam. of stock, in.	Diam. of saw, in.	Machinery and Soft Steel (Cutting speed, 50 ft. per min.)				Tool Steel (Cutting speed, 70 ft. per min.)				High-speed Tool Steel (Cutting speed, 50 ft. per min.)			
		Number of teeth in saw	Feed of saw, in. per min.	Revolutions of saw per min.	Approx. min. to cut off one piece	Number of teeth in saw	Feed of saw, in. per min.	Revolutions of saw per min.	Approx. min. to cut off one piece	Number of teeth in saw	Feed of saw, in. per min.	Revolutions of saw per min.	Approx. min. to cut off one piece
2	16	76	0.88	11.93	2¼	108	0.666	16.71	3	108	0.250	11.93	8
2½	16	76	0.833	11.93	3	108	0.555	16.71	4½	108	0.250	11.93	10
3	16	76	0.750	11.93	4	108	0.500	16.71	6	108	0.250	11.93	12
3½	16	76	0.70	11.93	5	108	0.466	16.71	7½	108	0.225	11.93	15½
4	18	76	0.66	10.61	6	108	0.444	14.85	9	108	0.210	10.61	19
4½	18	76	0.642	10.61	7	108	0.409	14.85	11	108	0.195	10.61	23
5	18	76	0.588	10.61	8½	108	0.384	14.85	13	108	0.178	10.61	28
5½	18	76	0.55	10.61	10	108	0.366	14.85	15	108	0.166	10.61	33
6	22	86	0.500	8.68	12	120	0.333	12.15	18	120	0.153	8.68	39
6½	22	86	0.464	8.68	14	120	0.309	12.15	21	120	0.138	8.68	47
7	24	86	0.411	7.95	17	120	0.280	11.14	25	120	0.126	7.95	55
7½	24	86	0.357	7.95	21	120	0.250	11.14	30	120	0.116	7.95	64
8	24	86	0.307	7.95	26	120	0.228	11.14	35	120	0.108	7.95	74

The **sizes of stock** for which **blades of different diameters** are **suitable** are as follows:

Diam. of blade, in.		13½	15	18	20	24
Thickness of blade, in.		⅛	⅛	5/32	3/16	7/32
Capacity	Rounds (diam. in in.)	4	4½	6	6½	8½
	I-beams, size in in.	6	7	12	12	12

The practice of the Brown & Sharpe Mfg. Co. for **cutting speeds and feeds** of cold saws is given in Table 45. **Motor sizes,** according to Charles Robbins (*Trans. A. S. M. E.*, vol. 32, p. 215) are given in Table 46.

Table 46. Sizes and Speeds of Motors for Cold Saws

Diam. of blade, in.	12	15	18	20	24	32	36
Thickness of blade, in.	5/32	5/32	3/16	3/16	3/16	3/16	3/16
Horse power	2	2	3	3	5	7½	10

Hack Saws for use in **hand frames** are made in lengths ranging by inches from 6 to 14 in., ½ to ⅝ in. in width, and either 0.026 or 0.028 in. in thickness. Number of teeth per inch: For cutting soft steel or cast iron, 14; tool steel and angle iron; 22; brass, copper, heavy tubing, 28; sheet metals and thin tubing, 32.

Blades for **power hack saws** are made in 10, 12, 14, 16, 17, 18, 20, 22 and 24-in. lengths, 0.033 (light), 0.05 and 0.065 in. in thickness, and with 14 teeth per in. for regular work (10 for coarse sawing). In power hack saws, the frame in which the blade is strained is reciprocated above the work which is held at the desired angle in a vise. Feed downward is effected by weighting the frame. On the (quick) return stroke the blade is automatically lifted to clear the bottom of cut. The larger sizes will cut off bars or rolled sections up to 12 in. across. The power requirements are inconsiderable.

Turret Lathes

Flat Turret Lathes. The flat turret lathe has a low, rigid turret using ordinary turning tools. Special tools are unnecessary. Representative dimensions are 2¼ × 24 in. (2¼-in. hole through spindle and 24-in. diameter turret) and 3 × 36 in., the first requiring a 2½-h.p. and the second a 3-h.p. constant-speed motor. The head is made cross-sliding, thus adapting it to chuck work.

Semi-automatic Turret Lathes are used on both bar and chuck work. The piece of work must be chucked and unchucked by hand. The cutting operations are automatic.

Screw Machines

Screw machines are turret lathes usually equipped with automatic chucks, bar feeds, oil pans and oil pumps. They were originally designed to make screws and similar parts at one setting from bar stock, the principal operations being turning and threading under a flood of lubricant. At present they are not confined to bar stock, for forgings and parts made from the bar and requiring second operations are handled with a magazine feed. They are operated either by hand or automatically.

Sizes, Capacities and Motor Requirements of Hand Screw Machines

Size number		1	2	3	4	5
Capacity	Diam., in.	⅝	1	1½	2¼	2⅝
	Length, in.	7	13½	18	25	26
Horse power of motor		1	1¼	2	2¾	5

Screw-machine Tools. **Spring collets** and **feed chucks** grip the bar while feeding and machining. **Box tools and hollow mills** are used for external cutting. Box-tool practice favors carbon steel for radial-type cutters, and high-speed steel for tangent cutters for roughing. Recommended **sizes of the cutters** for box tools are: For stock up to 5/16 in. diam., 3/16 in. sq.; for 5/16 to 3/8 in. diam., 7/32 in. sq.; for 3/8 to 1/2 in. diam., 1/4 in. sq.; for 1/2 to 3/4 in. diam., 5/16 in. sq.; 3/4 to 1 in. diam., 3/8 in. sq.; 1 to 1 1/4 in. diam., 1/2 in. sq.

Table 47. Cutting Speeds and Feeds for Screw Stock

For brass the cutting speed should be three times as great and the feed per revolution from 25 to 50 per cent. greater.

For hollow mills on screw stock the cutting speed should be as in the table and the feed per revolution 30 per cent. greater than the tabulated values.

1/32-in. Chip				1/16-in. Chip				1/8-in. Chip			
Diam. of stock, in.	Surface speed, ft. per min.	Rev. per min.	Feed per rev., in.	Diam. of stock, in.	Surface speed, ft. per min.	Rev. per min.	Feed per rev., in.	Diam. of stock, in.	Surface speed, ft. per min.	Rev. per min.	Feed per rev., in.
1/8	80	2445	0.002	1/4	60	916	0.0035	3/8	55	500	0.004
3/16	70	1426	0.003	3/8	60	611	0.004	1/2	55	420	0.005
1/4	70	1069	0.004	1/2	60	458	0.005	3/4	55	280	0.006
3/8	70	713	0.005	3/4	55	280	0.006	1	50	191	0.007
1/2	60	458	0.006	1	55	210	0.007	1 1/4	50	152	0.007
3/4	60	305	0.007	1 1/4	55	168	0.007	1 1/2	45	114	0.007
1	60	229	0.008	1 1/2	50	127	0.008	1 3/4	45	98	0.007
1 1/4	60	183	0.008	1 3/4	50	109	0.008	2	40	76	0.008
1 1/2	50	127	0.009	2	45	86	0.009	2 1/4	40	68	0.008
1 3/4	50	109	0.010	2 1/4	45	76	0.009	2 1/2	40	61	0.008
2	50	95	0.010	2 1/2	45	68	0.009	3	40	51	0.008
2 1/4	50	85	0.010	3	45	57	0.009	3 1/2	40	44	0.008
3/16-in. Chip				1/4-in. Chip				3/8-in. Chip			
1/2	50	382	0.004	3/4	50	254	0.004	1 1/4	45	137	0.005
3/4	50	254	0.005	1	50	191	0.005	1 1/2	45	114	0.005
1	50	191	0.005	1 1/4	45	137	0.005	1 3/4	45	98	0.005
1 1/4	45	137	0.006	1 1/2	45	114	0.006	2	40	76	0.006
1 1/2	45	114	0.006	1 3/4	45	98	0.006	2 1/4	40	68	0.006
1 3/4	45	98	0.006	2	40	76	0.006	2 1/2	40	61	0.007
2	40	76	0.007	2 1/4	40	68	0.007	3	40	51	0.007
2 1/4	40	68	0.007	2 1/2	40	61	0.007	3 1/2	40	44	0.007
2 1/2	40	61	0.007	3	40	51	0.007	4	40	38	0.008
3	40	51	0.007	3 1/2	40	44	0.007	4 1/2	40	34	0.008

In **tapping and threading**, the hole should be a little larger and the part to be threaded a little smaller than the theoretical root and outside diameters, respectively, of the thread to be cut. Suitable allowances are given in Table 53.

Speeds and Feeds for Screw-machine Tools. Tables 47 to 53, inclusive, give speeds and feeds for automatic screw-machine stocks and tools, and represent good practice. They are taken from "Automatic Screw Machines and Their Tools," by Goodrich and Stanley, and used here by permission. With the exception of Table 52 for dies, the values are based on carbon-steel tools. The die table is based on the use of high-speed steel dies. For

Table 48. Speeds and Feeds for Forming Tools

Diam. of work, in.	Screw stock		Brass rod		Cast iron		Tool steel	
	Surface speed, ft. per min.	Rev. per min.	Surface speed, ft. per min.	Rev. per min.	Surface speed, ft. per min.	Rev. per min.	Surface speed, ft. per min.	Rev. per min.
1/8	75	2292	200	6112	....	....	....	....
3/16	75	1528	200	4074	....	....	....	....
1/4	70	1069	185	2827	75	1146	45	688
3/8	65	662	185	1885	70	713	40	407
1/2	65	497	185	1414	70	535	40	306
3/4	60	305	175	882	65	331	35	178
1	60	229	175	667	65	248	35	134
1 1/2	60	153	170	432	60	153	30	76
2	50	96	170	324	60	115	30	57

FEEDS FOR FORMING TOOLS

Width of form, in.	Smallest diameter of form in inches							
	1/16	1/8	3/16	1/4	3/8	1/2	3/4	1 1/2
1/16	0.0007	0.0008	0.001	0.0012	0.0012	0.0012	0.0012	0.0012
1/8	0.0005	0.0008	0.001	0.0012	0.0015	0.0020	0.0025	0.0025
1/4	....	0.0007	0.001	0.001	0.0015	0.0015	0.0018	0.0018
3/8	....	....	0.0009	0.001	0.001	0.0012	0.0015	0.0015
1/2	....	....	0.0008	0.0009	0.001	0.001	0.0015	0.0015
3/4	....	....	....	0.0008	0.0009	0.001	0.0011	0.0012
1	....	....	....	....	0.0008	0.0009	0.001	0.0012
1 1/2	....	....	....	....	0.007	0.0007	0.0009	0.0011
2	....	....	....	....	....	....	0.0007	0.001

Table 49. Drilling Speeds and Feeds in Screw Machines

Diam. of drill, in.	Screw stock and cast iron		Brass rod		Tool steel	
	Feed per rev., in.	R.p.m. at 60 ft. peripheral speed	Feed per rev., in.	R.p.m. at 175 ft. peripheral speed	Feed per rev., in.	R.p.m. at 35 ft. peripheral speed
1/32	0.0006	7,333	0.0008	21,390	0.0005	4,278
3/64	0.0008	4,889	0.0010	14,260	0.0006	2,852
0.059	0.0010	3,884	0.0013	11,329	0.0008	2,265
1/16	0.0013	3,667	0.0017	10,696	0.0010	2,139
5/64	0.0016	2,093	0.002	8,555	0.0012	1,658
3/32	0.0018	2,445	0.0023	7,130	0.0014	1,430
0.105	0.002	2,186	0.0026	6,366	0.0015	1,305
1/8	0.0025	1,833	0.0033	5,348	0.0018	1,070
0.150	0.003	1,528	0.0039	4,456	0.0022	891
5/32	0.003	1,421	0.0039	4,144	0.0022	828
3/16	0.004	1,222	0.0052	3,565	0.003	713
7/32	0.004	1,048	0.0052	3,050	0.003	611
1/4	0.0045	916	0.0058	2,674	0.0033	535
9/32	0.0045	815	0.0058	2,377	0.0033	475
5/16	0.0045	733	0.0058	2,139	0.0035	427
3/8	0.0045	611	0.0061	1,783	0.0035	356
7/16	0.005	524	0.0065	1,528	0.0037	305

Diam. of drill, in.	Screw stock and cast iron		Brass rod		Tool steel	
	Feed per rev., in.	R.p.m. at 55 ft. peripheral speed	Feed per rev., in.	R.p.m. at 165 ft. peripheral speed	Feed per rev., in.	R.p.m. at 33 ft. peripheral speed
1/2	0.005	420	0.0065	1,260	0.0037	252
9/16	0.0057	373	0.0074	1,120	0.0043	224
19/32	0.0057	353	0.0074	1,067	0.0043	212
5/8	0.0059	336	0.0077	1,008	0.0044	202
11/16	0.006	305	0.0078	917	0.0045	183
3/4	0.0065	280	0.0084	840	0.0049	169
13/16	0.007	258	0.0091	776	0.0052	155
7/8	0.0075	240	0.0097	702	0.0056	144
15/16	0.008	224	0.0104	672	0.006	134
1	0.0085	191	0.0110	573	0.0064	115
1 1/8	0.009	169	0.0117	509	0.0067	102
1 1/4	0.0095	152	0.0123	458	0.0071	92
1 3/8	0.010	* 139	0.0130	† 416	0.0075	‡ 83
1 1/2	0.011	127	0.0143	382	0.0082	76
1 5/8	0.012	118	0.0156	352	0.009	70
1 3/4	0.013	109	0.0169	327	0.0097	65
2	0.014	96	0.0182	294	0.0105	57

* At 50 ft. peripheral speed. † At 150 ft. ‡ At 30 ft.

carbon-steel dies the values should be reduced from 25 to 50 per cent. The **values for tapping** are the **same as for threading.**

Table 50. Cutting Speeds and Feeds for Finishing Box Tools

Finished diam. of work, in.	Screw stock			Brass rod			Cast iron			Tool steel			Amount advisable to remove on a side, in.
	Surface speed, ft. per min.	Rev. per min.	Feed per rev., in.	Surface speed, ft. per min.	Rev. per min.	Feed per rev., in.	Surface speed, ft. per min.	Rev. per min.	Feed per rev., in.	Surface speed, ft. per min.	Rev. per min.	Feed per rev., in.	
1/16	80	4889	0.003	180	11000	0.0030	......	......	......	40	2445	0.002	0.002
1/8	80	2445	0.0045	180	5500	0.0045	......	......	......	40	1222	0.003	0.0025
3/16	70	1426	0.0055	180	3668	0.0055	70	1426	0.0055	40	815	0.003	0.0025
1/4	65	993	0.0075	180	2750	0.0075	70	1069	0.0075	35	531	0.004	0.0045
1/2	60	458	0.011	180	1375	0.011	65	496	0.011	35	267	0.005	0.006
3/4	60	305	0.012	180	917	0.012	65	331	0.012	35	178	0.007	0.006
1	60	229	0.012	175	668	0.012	60	229	0.014	30	115	0.009	0.0065
1½	55	140	0.014	170	433	0.014	60	153	0.016	30	76	0.009	0.007
2	50	95	0.014	170	325	0.014	60	115	0.016	30	57	0.009	0.008

Table 51. Reaming Feeds and Speeds in Screw Machines

Diam. of reamer, in.	Feed per rev., in.	Amount to remove on diam. in.	R.p.m. at various peripheral speeds				Diam. of reamer, in.	Feed per rev., in.	Amount to be removed on diam. in.	R.p.m. at various peripheral speeds			
			Screw stock at 40 ft.	Brass rod at 130 ft.	Cast iron at 45 ft.	Tool steel at 25 ft.				Screw stock at 40 ft.	Brass rod at 130 ft.	Cast iron at 45 ft.	Tool steel at 25 ft.
1/8	0.005	0.0045	1222	3972	1375	764	1¼	0.018	0.010	122	397	138	76
3/16	0.006	0.0045	815	2648	917	509	1½	0.020	0.010	102	331	115	63
1/4	0.007	0.006	611	1986	688	382	1¾	0.022	0.010	87	284	98	54
3/8	0.0085	0.006	407	1324	458	254	2	0.024	0.013	76	248	86	48
1/2	0.0105	0.008	306	993	344	191	2¼	0.026	0.013	68	220	76	42
5/8	0.012	0.008	245	795	275	153	2½	0.028	0.013	61	199	69	38
3/4	0.014	0.008	204	662	229	127	2¾	0.030	0.013	56	181	63	35
1	0.016	0.010	153	497	172	95	3	0.032	0.013	51	165	57	32

Table 52. Speeds for High-speed-steel Dies in Screw Machines
(Standard Threads)

Diam. of thread, in.	Screw stock		Brass rod		Cast iron		Tool steel		Cast brass	
	Surface speed, ft. per min.	Rev. per min.	Surface speed, ft. per min.	Rev. per min.	Surface speed, ft. per min.	Rev. per min.	Surface speed, ft. per min.	Rev. per min.	Surface speed, ft. per min.	Rev. per min.
1/8	40	1222	135	4126	40	1222	25	764	120	3666
1/4	40	611	125	1909	40	611	25	382	110	1680
3/8	35	356	120	1222	35	356	20	204	100	1019
1/2	35	267	120	917	35	267	20	153	100	764
3/4	35	178	115	586	30	153	20	102	100	509
1	30	115	110	420	30	115	20	76	90	344
1¼	30	92	100	306	25	76	15	46	90	275
1½	30	76	90	229	25	64	15	38	90	229
2	25	48	85	162	20	38	15	29	90	172

Table 53. Allowances for Threading in Screw-machine Work

No. of threads per in.	Undersize allowance on external work, in.	Oversize allowance on holes, in.	No. of threads per in.	Undersize allowance on external work, in.	Oversize allowance on holes, in.
6	0.005	0.010	13	0.0035	0.007
7	0.0045	0.0095	14	0.003	0.0065
8	0.0045	0.009	16	0.003	0.006
9	0.004	0.0085	20	0.0025	0.0055
10	0.004	0.008	22	0.0025	0.005
11	0.0035	0.0075	24	0.002	0.0045
12	0.0035	0.007	28	0.002	0.004

Forging, Hardening and Tempering High-speed-steel Tools

A high-speed-steel bar ought always to be nicked hot on all four sides for cutting off. If this is not done there is danger of splitting at the ends. A temperature of 1400 to 1500 deg. fahr., or a dull cherry red, is best. The following instructions for forging round-nose roughing tools such as have been introduced into the United States Navy are given by W. H. Taylor (*Am. Mach.*, vol. 33, part 1, p. 432). Set stop to length of tool, *L*, (see Table 24). Heat slowly to forging heat (1800 deg. fahr.). Cut-off piece in gage (see *A*, Fig. 9). Heat slowly to forging heat *X* in. back from end (see Table 24). Put bending die on anvil. Put tool in die and drive down (*D* and *E*, Fig. 9). Flatten sides at bend (*F*). Draw down heel (*G* and *H*). Straighten bottom (*J*). Straighten front (*K*) and spread nose to width *B* as given in Table 24. Heat slowly to forging heat. Mark off height *H* (Table 24) with 20 deg. height gage (*L*). Cut back slope and side slope roughly (about ⅛ in. higher than finished height) as in *M*. Trim nose to shape. Offset to suit limit gage (*P*). Keep base of tool straight. Test clearance angle, back and side slope with cone and limit gages.

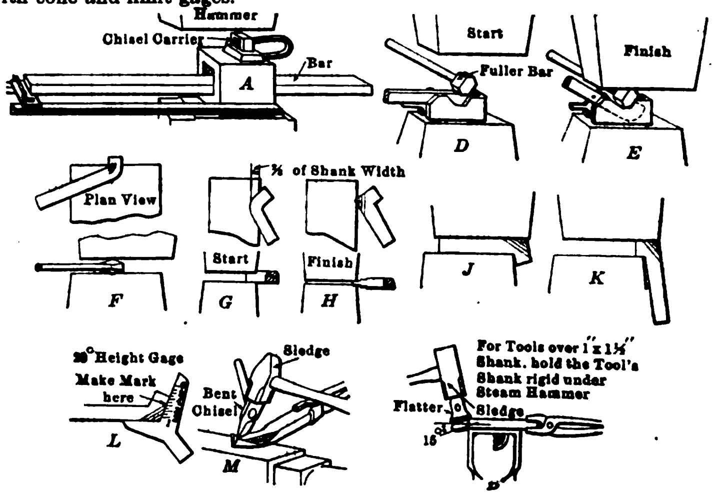


Fig. 9.—Forging Operations for Straight Round-nose Roughing Tool.

The rules governing the heating of high-speed-steel bars preparatory to cutting off and for forging are as follows:

In forging high-speed steel, care must be used not to heat the steel too rapidly nor to work it too cold. A yellow heat, or 1800 to 1900 deg. fahr., is the temperature at which best results will be obtained. Tools should not be left in the furnace more than a minute or two after reaching the forging heat, as the steel is injured if it is allowed to soak at a temperature above 1500 deg. fahr. Below this temperature high-speed steel does not work readily and there is danger of its breaking or developing cracks if heavy work is done on it.

In working, when a tool cools to a cherry red it should be put back in the furnace and another taken that is heated to the forging temperature. From 4 to 6 tools should be kept heating at a time. When one is finished, another should be placed in the fire and still another on the hearth to preheat.

Hardening High-speed-steel Tools. (See also p. 482.) To get the best and most uniform results (F. W. Taylor, *Trans. A. S. M. E.*, vol. 28, p. 227) the tools should first be heated to a temperature between 1500 and 1700 deg. fahr. and then cooled rapidly to a point *below* the breaking-down point (1550 deg.), preferably in a bath of molten lead kept below a temperature of 1550 deg. The quality of the tool is but little affected whether cooled rapidly or slowly from this point down to the temperature of the air. After all parts of the tool have reached a uniform temperature below the breaking-down point, it may be laid in some place free from moisture and allowed to cool in the air or in an air blast. The temperature of a tool should *never* be allowed to rise after the cooling-down process has started; failure to observe this caution will result in injury to the tool.

The second, or low-heat, treatment consists in reheating the tool which has had the high-heat treatment in a lead bath to a temperature between 700 and 1240 deg. fahr., preferably near the higher temperature. To avoid the danger of cracks if the tool is a complicated shape, it is well to heat it slowly before it is plunged into the lead.

The good effects of the second treatment can be obtained by running the tool under cut until its nose has been raised to between 700 and 1240 deg. fahr. and then allowing it to cool in the air; the next time it is used it will have a materially increased degree of red-hardness.

Higher temperatures are now used for heating than those originally recommended by Mr. Taylor. The newer high-speed steels are treated by heating them slowly to 1500 deg. fahr., and then rapidly to a temperature between 1850 and 2450 deg.; they are then cooled rapidly in a blast down to 1500 deg., and then cooled either slowly or rapidly to the temperature of the air. There must be no reversal of the cooling process, once it has commenced.

The hardening and tempering temperatures for ordinary forms of cutting tools in deg. fahr. are as follows:

Lathe, planer, shaper and slotter tools: hardening temperature, just below melting point (2450 deg. down) depending on the steel. Milling cutters and similar tools for heavy cuts: hardening temperature, 2375 to 2450; drawing temperature, 400. Milling cutters for light work, screw-machine tools, brass-working tools: hardening temperature, 2100 to 2300; drawing temperature, brass-working tools, 450; others, 400. Drills, reamers, dies and taps: hardening temperature, 2150 to 2200; drawing temperature, large drills and reamers, 440; small drills and reamers, 460; dies and taps, 490. Punches and dies, shear blades and tools subjected to blows: hardening temperature, 1750 to 1900; drawing temperature, tools subject to blows, as snaps and chisels, 570; others, 530. Woodworking tools: hardening temperature, 2100 to 2300; drawing temperature, 525 to 625.

Quenching. Air and oil are used for cooling mediums. Cold water should be avoided, but hot water has been successfully used in special cases to get extreme hardness. Oil is now preferred, as it is believed that its use results in more uniform hardening and greater toughness. Linseed, cottonseed, whale, rape, paraffin and kerosene oils are all used, the necessary properties being thinness and freedom from gumming. The disagreeable odor of whale oil can be minimized by covering the surface with a layer of tempering oil.

Barium Chloride Bath. Because of trouble from oxidization on thin edges of cutting tools, experiments have been carried out with baths covering the heated portion with a protecting coating. The most extensively used salt is barium chloride mixed with about 2 per cent. of soda ash. Its melting point is 1635 deg. fahr.—considerably higher than that of lead. A vertical gas-fired furnace is generally used, with the chloride in a graphite crucible. A temperature in the bath of 2200 deg. is easily obtained.

This bath is used particularly for milling cutters and other tools having keen, thin edges. It is not used for heavy roughing tools.

Occasionally there is trouble from pitting, which seems to be due to melting in spots. This can be largely avoided by not allowing the temperature to run too high.

Care must be exercised in grinding high-speed steel cutters to prevent checking. The best practice is to heat the tools before grinding to a temperature of 200 to 250 deg. fahr., and grind under a copious supply of water. Intense local heating must be avoided in grinding.

Heating Furnaces. Coke, anthracite, oil and gas-fired furnaces are used to heat high-speed steel tools for hardening. In general, oil fuel is not well suited to obtain and regulate the high temperatures required, and considerable oxidation results. Coke and anthracite fuels are satisfactory, but the furnace must have a floor or hearth to keep the tools away from the fire bed. Gas is the most commonly used fuel, and is unquestionably the most convenient.

The minimum equipment for hardening high-speed steel tools is a combined forging and heating furnace, an air blast and an oil quenching bath.

The flame should be directed around the interior of the furnace chamber rather than directly into it, so as to prevent the impinging of the flame upon the tool. The air supply should be at a pressure of 1½ to 2 lb. Both gas and air must be controll-

Table 54. Sizes and Gas Consumption of Furnaces for Tool Hardening

Kind of furnace	Dimensions of heating chamber, in.	Gas consumption,* cu. ft. per hour	Work for which furnace is intended
Bench forge	2½ × 2 × 6	50	Dressing small tools.
Tool room No. 1	3 × 2 × 4	60	Dressing tools.
Tool room No. 2	6 × 3 × 6	100	Dressing tools.
Tool room No. 3	8 × 10	100	Heating tools and blanks.
Tool room No. 4	3 × 15	90	Rapid forging and hardening of small tools.
Tool room No. 9	3½ × 3½ × 3½ preheating slot, 9½ × 1½	200	Hardening large lathe tools.
Oil tempering furnace No. 3	Pot 11½ diam. × 10¾ deep	30	General oil quenching and tempering.
Oil tempering furnace No. 4	Pot 10½ diam. × 10 deep	40	General oil quenching and tempering.
Oil tempering furnace No. 30	23 × 15 × 12	50	General oil quenching and tempering.
Lead hardening furnace No. 9	Black-lead crucible No. 9, 18 lb. lead.	75	General lead-bath hardening.

* Based on the heating value of city illuminating gas. The air pressure should be about 1 lb. per sq. in., and in the nature of a positive blast.

able. Table 54 gives data on a few sizes of hardening furnaces made by the American Gas Furnace Co.

The air nozzle should not be less than ¾ in. in diameter. **Milling cutters** are mounted on a revolving spindle to bring all parts under the action of the blast.

The **oil tempering bath** is held in a metal tank or pot in a simple furnace which is usually provided with a hood connected with an exhaust system to remove the fumes. Small parts are suspended in a wire-mesh basket or perforated pan; large tools are suspended by wires, or in any convenient manner. Whale and black cylinder oils and tallow are used.

WOOD-WORKING MACHINES

Wood-working machines are characterized by the high speed of the cutters, and this peculiarity requires careful fitting of journals, their bearings and cutting tools, with balancing of all rotating parts. The greater part of the motive power is needed to overcome frictional resistance. Ample lubrication is important and is most satisfactorily obtained by using dust-proof ring oilers; grease should be avoided. The frictional resistances are greater at starting than during normal running, and considerable power is required to accelerate the moving parts; consequently the starting power is much greater than that needed for ordinary operation.

Because of the rapidity with which operations are performed, the supply of lumber, removal of chips and finished material should be adequately provided for; chips are usually handled by a suction or exhaust system through galvanized iron pipes into a dust separator, then into a storage room. The uncut material and finished material are stacked on trucks or transported by some form of mechanical conveyor. Considerable floor space is needed around wood-working machines for stock.

Motor Drive. Both group and individual electric motor drives are used. The general comparison is the same as for metal-cutting machines (see p. 1467). Because of the high speeds used, direct connection can often be made from cutter shaft to armature shaft.

Saws

Circular Saw Blades are of two kinds, solid and with inserted teeth. Tables 55 and 56 give particulars regarding commercial sizes of **solid-tooth saws** and Table 57 of **saws with inserted teeth.**

Table 55. Dimensions of Solid-tooth Circular Saws

(Simonds Mfg. Co.)

Diam., in.	Thickness, B.W.G.	Diam. of hole, in.	Diam., in.	Thickness, B.W.G.	Diam. of hole, in.	Diam., in.	Thickness, B.W.G.	Diam. of hole, in.
1	24	⅜	9	17	⅞	28	10	1½
1½	24	⅜	10	16	1	30	10	1½
2	23	⅜	11	16	1	32	10	1⅝
2½	22	⅜	12	15	1	34	9	1⅝
3	21	½	14	14	1⅛	36	9	1⅝
3½	20	½	16	14	1⅛	38	9	1⅝
4	19	¾	18	13	1¼	40	9	2
5	19	¾	20	13	1 5/16	42–48	8	2
6	18	¾	22	12	1 5/16	*50–58	7	2
7	18	¾	24	11	1⅜	*60–66	6	2
8	18	⅞	26	11	1⅜	*68–84	5	2

* By 2-in. intervals.

Table 56. Standard Number of Teeth in Solid-tooth Circular Saws

Diam., in.	Splitting	Cut-off	Resaws	Edgers
4–8	38 to 40	100 to 120	....	....
9–10	36 to 38	90 to 110	....	....
12–14	36 to 38	90 to 100	....	....
16	36 to 38	80 to 90	....	18 to 30
18	34 to 36	80 to 90	36 to 48	18 to 30
20	34 to 36	80 to 90	36 to 48	20 to 32
22	34 to 36	72 to 80	36 to 50	22 to 34
24	34 to 36	72 to 80	36 to 52	24 to 36
26	32 to 34	72 to 80	40 to 60	26 to 36
28	32 to 34	72 to 80	40 to 60	
30	32 to 34	80 to 90	40 to 60	Shingle Reg.
32–34	32 to 34	80 to 90	44 to 66	60 to 80
36	34 to 38	80 to 90	48 to 72	80 to 100
38	34 to 38	80 to 100	48 to 72	80 to 120
40	36 to 40	80 to 100	....	80 to 120
42–46	36 to 40	80 to 100	....	....

Diam., in.	550 r.p.m. or less, 4-in. feed or less	600-700 r.p.m., 6-in. feed	700–800 r.p.m., over 6-in. feed	Cut-off
	Splitting			
48	24 to 36	34 to 40	48 to 60	80 to 100
50	26 to 38	34 to 42	50 to 70	80 to 100
52	28 to 38	36 to 44	52 to 80	80 to 100
54	30 to 40	36 to 48	54 to 80	90 to 120
56	34 to 44	36 to 50	56 to 90	90 to 120
58	36 to 46	40 to 52	58 to 90	90 to 120
60	36 to 48	42 to 60	60 to 100	90 to 120
62	....	44 to 60	60 to 100	100 to 140
64	....	44 to 60	60 to 100	100 to 140
66	....	48 to 66	72 to 100	100 to 140
68	....	48 to 68	80 to 100	100 to 160
70	....	48 to 68	90 to 100	100 to 160
72	....	48 to 72	90 to 100	100 to 160

Diameters increase by 1-in. intervals from 4 to 10 in.; by 2-in. from 10 to 72 in.

Table 57. Dimensions and Standard Number of Teeth of Inserted-tooth Saws

Diam., in.	Thickness, B.W.G.	No. of teeth	Diam., in.	Thickness, B.W.G.	No. of teeth	Diam., in.	Thickness, B.W.G.	No. of teeth	Diam., in.	Thickness, B.W.G.	No. of teeth
12	11	10	28	10	20	44	7	30	60	5	42
14	11	10	30	10	20	46	7	30	62	5	44
16	11	12	32	9	22	48	7	32	64	5	44
18	11	14	34	9	22	50	7	34	66	5	48
20	11	14	36	8	24	52	6	36	68	5	48
22	11	16	38	8	24	54	6	38	70	4	52
24	11	18	40	8	26	56	6	40	72	4	52
26	10	18	42	8	28	58	6	42	....	....	....

Band-saw Blades. Dimensions of narrow band-saw blades are given in Table 58. Band saws 2 to 18 in. wide increase in width by steps of ½ in. up to 6 in., and by 1-in. steps thereafter. The usual B.W.G. thicknesses are as follows: 2 to 3½ in. wide, 18 to 20; 4 to 6 in., 17 to 19; 7 in., 16 to 18; 8 to 11 in., 14 to 16; 12 to 14 in., 13 to 15; 15 to 18 in., 12 to 14.

Table 58. Dimensions of Narrow Band-saw Blades

Width, in.	Usual thickness, B.W.G.	Number of teeth per in.	Width, in.	Usual thickness, B.W.G.	Number of teeth per in.
⅛	22 or 23	6 or 7	⅞	20 or 21	2½ or 3
3/16	21 or 22	6	1	20 or 21	½" or ⅝" pitch
¼	21 or 22	5 or 6	1⅛	19 or 20	½" or ⅝" pitch
⅜	21 or 22	4 or 5	1¼	19 or 20	½" or ⅝" or ¾" pitch
½	21 or 22	3½ or 4	1⅜	19 or 20	½" or ⅝" or ¾" pitch
⅝	20 or 21	3–3½ or 4	1½	19 or 20	1" or 1¼" pitch
¾	20 or 21	2½ or 3	1¾	19 or 20	1" or 1¼" pitch

Notation.

h = thickness of cut, in.
H = stroke of gang saw, in., $\geq h$
t = pitch of saw teeth, in.
s = thickness of blade, in.
u = peripheral speed of blade, ft. per min.
n = r.p.m.
f = feed per min., in.
f_1 = feed per stroke, in.
D = diam. of circular saw blade or band-saw wheel, in.
L = free length of gang-saw blade, in.
P = cutting resistance, lb.

Pitch of Teeth in Saw Blades. For solid blades with set teeth, $t = 0.97\sqrt{kh}$, and for swaged or inserted teeth cutting their full width, $t = 1.38\sqrt{kh}$. For band saws, $k = 0.02$ to 0.1; for gang saws, $k = 0.08$ to 0.4; for circular saws, $k = 0.1$ to 0.6.

Speed-Feed Ratios for Saws. For the maximum permissible rate of feed the following relation holds: $f/u = f_1/H = 0.0072(t/h)$.

Practical values of these ratios are:

For heavy circular saws for logs	f/u =	0.00083	to 0.000083
For circular rip saws	f/u =	0.00023	to 0.000047
For fine-toothed circular saws	f/u =	0.000083	to 0.000012
For heavy band saws for logs	f/u =	0.00023	to 0.000023
For gang saws	f_1/H =	0.03	to 0.003
For veneer saws	f_1/H =	0.002	to 0.0005

Cutting Speeds. The cutting speeds commonly employed (ft. per min.) are as follows:

Circular rip saws for hard, knotty wood	3,000 to 6,000
Circular rip saws for oak	4,000 to 8,000
Circular rip saws for soft woods	6,000 to 12,800
Circular cross-cut saws	3,000 to 6,000
Band rip saws	4,000 to 8,300
Band cross-cut saws	2,000 to 5,000
Gang saws (when $u = 2Hn$):	
Heavy frames	500 to 700
Light vertical frames	700 to 800
Horizontal frames	800 to 1,400

According to *Wood Craft* (Oct., 1912), satisfactory speeds (r.p.m.) for other types of saws are: For resawing machines, 250 to 300; scroll saws, 200 to 400; jig saws with spring tension, 500 to 800; jig saws with unstrained blades, 800 to 1500.

Cutting Resistance. The cutting resistance of a saw in lb. $= P = Cshf/u = Csf_1$, where $C = 1{,}200{,}000$ to $2{,}900{,}000$.

Thickness of Saw Blades. For circular saws, $s = 0.02\sqrt{D}$; for gang and band saws, $s = L/800$. The **tension** in gang saws (lb.) should equal $1{,}138{,}000s^2$; for band saws, when $D \geq 1000s$, tension $= 569{,}000s^2$.

Resistance to Feeding. For circular saws, resistance $= 1.25P$; for gang or band saws, resistance $= P$, where P = cutting resistance, lb. The force (lb.) required to feed a log to a circular saw (including that required to move the timber carriage and the log) $= 0.006W + 1.25P$, where W = weight of log, lb.; for a band or gang saw, force $= 0.006\,W + P$.

Power Requirements of Saws. The horse power required by a saw $= N = N_0 + N_1$, where N_0 = h.p. required to overcome the friction of the machine and N_1 = h.p. expended in cutting. In the following formulæ b = width of saw cut, in., and F = area (length cut × thickness of lumber) cut per hour, sq. ft.

For circular saws,

$$N = \frac{nD}{32{,}000} + \frac{2.36bF}{(14 \text{ to } 28)}$$

For **band saws** N_0 = 0.3 to 3 h.p. (up to 9 h.p. for large log saws);

$$N_1 = \frac{37 + 0.01b(u/f)}{10{,}764}F \text{ for pine}$$

For oak, substitute 52 for 37 and 0.0124 for 0.01. For red beech, substitute 62 for 37 and 0.0148 for 0.01.

For **gang saws**, N_0 = 0.25 to 3 h.p. and

$$N_1 = \frac{46 + 5.7b(H/f_1)}{10{,}764}F \text{ for air-dry pine}$$

According to Hermann's investigations, for **gang saws with freshly sharpened teeth,**

$$N_1 = K\left[1 + \frac{b + 0.118}{t}\left(4 + \frac{H}{100f_1}\right)\right]F$$

where K = 1/315 for pine, 1/361 for fir and 1/347 for deciduous trees. After z hours of operation the value of N_1 should be multiplied by $(1 + 0.145z - 0.005z^2)$.

According to Dodge, for circular rip saws running at from 7000 to 9000 ft. per min., h.p. (approx.) = $D^2/40$, and for band saws (at 3500 ft. per min.), h.p. = $D^2/300$.

Drum or Cylinder Saws are used for sawing staves and work having a concave or convex surface. They are made up to about 3 ft. in length and are run at a maximum peripheral speed of 6000 ft. per min.

Planers

Jointers and Buzz Planers are common pattern-shop tools used to surface, to edge, and for general work. Some styles admit of the use of special cutters, so that surfacing and beading, surfacing and molding, and surfacing and grooving may be done at one time. The feed is by hand. Common **sizes** and horse power required are:

Size, table width in inches	8	12	16	20	24	30	36
Horse power required	1	1½	2	2	3	4	4

The table is in two sections adjustable for height with reference to the cutter to admit of different thicknesses of chip. Table lengths range from 6 to 7 ft. **Cutting speeds,** 3500 to 5000 ft. per min.

Cylinder Planers or **surfacers** are built as single and double machines. The first has one cutter arbor or cylinder and two feed rolls set close to it, the upper one being fluted. The second has two cylinders, an upper and lower, and four, six or eight feed rolls in different sizes and designs. The roll pressure should be from 27 to 55 lb. per inch of roll length. Cylinder planers are used for surfacing the heavier sizes in planing mills, box shops, sash, door and blind factories, furniture and cabinet factories and general jobbing. The smaller sizes are used on patterns, furniture, chairs, and fine door and panel work. Common **sizes** (maximum width and height planed) and horse power required for single machines:

Sizes, inches	16 × 16,	20 × 6,	24 × 6,	24 × 8,	26 × 8,	30 × 7,	36 × 7
Horse power	2 to 4,	2 to 4,	2 to 4,	5,	5,	5 to 7,	5 to 7

Common sizes (maximum width and height planed) and horse-power required for double machines:

Size, inches	24 × 8,	30 × 8,	30 × 10,	36 × 7
Horse power	7,	5 to 8,	6 to 8,	7 to 10

Heavy-type machines may take 50 per cent. more power than the above values. An approximate rule is: h.p. = 1/4 × width of cut in inches.

The rates of **feed** in the single machines range from 16 to 50 lin. ft. per min. usually in two, three or four rates. The double machines usually feed at a somewhat higher rate, from 20 to 80 lin. ft. per min. being a normal range, with four or six rates. Four-cylinder planers for squaring timbers have feeds up to 150 ft. per min. **Cutting speeds** range from 3500 to 10,000 ft. per min. The diameter of the path of planer knives ranges from 3.5 to 10 in. Planers to be used chiefly on walnut, ash, oak or other hardwoods will give better results if run at about 75 per cent. of the speeds used on softwoods.

Shapers

Shapers are made with one or two vertical spindles carrying knives ground to any desired contour. The work is slid over the table and drawn past the revolving cutter heads, the depth of cut being controlled by a collar under the knives. When two spindles are used they rotate in opposite directions, and at speeds of from 5000 to 7000 r.p.m. According to Dodge, h.p. (approx.) = width of cut (in.) + depth of cut (in.).

Lathes

Pattern-makers' Lathes differ from metal-turning lathes in being simpler in construction, lighter in design and in usually having provision to swing a large face plate from the outer end of the spindle across the end of the bed. The ways are frequently made flat to provide a convenient place to lay tools. These machines are used for all kinds of hand and power turning in wood where a single or only a few pieces of a kind are needed. Common **sizes** are 16, 20, and 24 in. swing, requiring respectively 1 1/2, 2 and 2 1/2 h.p. The standard bed length is 8 ft., but longer ones (10, 12, 14 and 16 ft.) can be furnished. Spindle **speeds**, up to 2000 r.p.m.

Automatic Back Knife Lathes are used in manufacturing cylindrical parts of wood of both simple and complicated shapes. A formed knife set at an angle with the axis of the work is fed downward to form it. Sizes and horse-power requirements are as follows:

Length turned, in.	Diameters turned, in.	Size of squares that can be left, in.	H.p.
28 and 36	3/16 to 2 3/4	2 × 2	3 to 5
40 and 50	1/4 to 3	2 1/4 × 2 1/4	4 to 6
40 and 50	3/4 to 5	3 1/2 × 3 1/2	5 to 8
45 and 52	3/16 to 2 3/4	2 × 2	3 to 5

Boring Machines

Boring machines for wood are lighter in construction than metal-drilling machines and are built for higher spindle speeds.

Bench Machines consist of a frame carrying a single pulley, and arranged to feed by sliding the spindle through its bearings. A common size will bore holes up to 1 1/2 in. in diam.

Vertical Machines are made in the form of simple machines to fasten to posts and taking from 1/2 to 1 h.p.; as machines resembling sensitive drilling machines for metal, taking from 1 to 1 1/2 h.p.; as heavy machines with universal tilting tables, requiring from 2 to 3 h.p.; and as multiple-spindle machines designed to bore heavy timbers in bridge and car work, and taking from 2 to 5 h.p. In most types the spindle is fed down by foot pressure, though some have a hand lever arranged for a direct pull.

Horizontal Boring Machines are used for small work where a number of holes are wanted closely spaced. In general, the table is mounted on a knee and arranged to tilt for angle boring, and has an adjustable fence. The feed is by a foot lever. Smaller sizes capable of driving a 2-in. bit require up to 1 h.p. to drive; larger sizes to drive up to a 3-in. bit require up to 2 h.p. Ordinary spindle speeds range from 2500 to 4000 r.p.m.

Bits and Drills. Both auger bits and twist drills are used. The bits are listed in sixteenth sizes running from 3/16 to 24/16 in. and in any lengths desired. Short dowel bits are made 3 in. long over all, and long dowel bits 4½ in. over all. They have a gimlet point to draw into the wood and aid in feeding.

Mortising Machines

Mortising machines have a vertical spindle which carries a short chisel at its lower end and is given a rapid reciprocating motion from an overhead crank disk. Provision is made for reversing the chisel face so that cutting can be done in either direction of the table motion. The work is clamped to the table, which is moved to the right or left by a hand wheel. A boring spindle is usually provided to make a hole in which the chisel may start cutting. In another type a square hollow chisel is used, in which an auger bit revolves at a high speed and removes the chips as the chisel is slowly forced into the wood. The following speeds for reciprocating mortisers are recommended in *Wood Craft* (Oct., 1912): Machines with movable tables, 300 to 450 r.p.m.; machines with chisel-feed tables, 250 to 350 r.p.m.; heavy machines for car work, 200 to 300 r.p.m.

Pattern-making Machine

There are several machines known as **mechanical or universal wood workers** that are extensively used in large pattern shops. Each style is universal in its adjustment and motions and is adapted to produce irregular and regular sections as gear patterns, worm patterns, core boxes, pulley patterns, and for mortising, tenoning, radius cutting, forming bends, etc. The spindle can be used in a vertical, horizontal or any intermediate inclined position. Speeds range from 1250 to 3600 r.p.m. The feed movements are ndependent of any movement of the head, and are obtained through either quick-acting lever or sensitive feed screw.

Pattern-makers' Disk Grinders

These are a type of abrasive disk grinders for smoothing, forming and sizing wooden patterns and their parts. They will work knots, cross grain, end grain, and even projecting nails and screws. They are also used on metal patterns. Both single- and two-disk machines are made, for motor drive and for overhead or through-floor drive. The grinding wheel is a metal disk to which an abrasive-coated cloth or paper is pasted.

Size (disk diam.), inches	24	30	40
Disk speed, r.p.m	950	750	525
Horse power	2	3	5

ELECTRIC DRIVES

By

C. DAY

Advantages of the Electric Drive. The general advantages possessed by the electric drive are as follows:

It imposes no restriction on the location of equipment, but permits the best location of machinery from the standpoints of sequence of operations, handling of materials, light and facility of operation.

It imposes no restrictions as to character of buildings and permits of the adoption of the least expensive type of building construction and the minimum headroom consistent with the efficient housing of the machinery, equipment and process work.

The use of mechanical transmission frequently involves additional cost of building construction, owing to the loads imposed on ceilings, roof trusses or columns, through hanging line shafting and countershafts from same. In addition, it is necessary in certain cases to provide greater headroom than would otherwise be required in order to obtain proper belt centers or for the purpose of securing greater window area in order to compensate to as great an extent as possible for the light obstructed by belts and countershafts.

It permits of the most efficient overhead handling facilities. Machines can be located under traveling cranes, jib cranes, etc.

It imposes no restrictions in connection with the construction of independent buildings.

It permits of the most efficient operation of machinery from the standpoints of power and speed.

By means of the direct-current, adjustable-speed motor it is possible to drive any type of machinery to its maximum power capacity and to obtain any gradations of speed desired. In the case of machine-tool equipment, however, the problem of adjustable-speed control has been accomplished with fair satisfaction by means of change gearing.

It permits of the greatest facility in starting, stopping and adjustment of the speed of machinery.

It permits of more continuous operation.

It permits of the operation of one or more machines while the rest of the plant is shut down.

For general data on the **application of the electric drive to machine tools,** see p. 1418. See also pp. 1617, 1629 and 1640 for **motor characteristics.**

Cost of Drive. When making a comparison of the cost of drives there should be included the cost not only of the entire equipment required in connection with the generation and distribution of power, but all collateral costs such as those incurred through special building construction; for example, belt or rope towers, necessary members to carry line shafting, additional headroom to secure adequate natural lighting, etc.

Cost of Maintenance. The cost of maintenance should include all expenditures for supplies such as oil, grease, waste, etc., and for labor in lubricating transmission devices, keeping shafting in alignment and in making repairs. Records covering 8 years show that the annual maintenance cost of mechanical installations varies from 2 per cent. to 15 per cent. of the first cost of equipment; for electrical installations the range is from ½ per cent. to 8¼ per cent.

Individual Drive vs. Group Drive. The use of the individual drive is imperative when variable or adjustable speed is required. It is also needed in connection with portable equipment and for the operation of machinery which is so located as to be inaccessible to the motor-driven lineshaft. When these conditions do not apply, the group drive is entirely satisfactory and ordinarily involves a smaller investment.

Selection of Motors for Electric Drive. The types of motors available as regards speed conditions are (*a*) constant speed, (*b*) multi-speed, (*c*) adjustable speed and (*d*) varying speed (see p. 1640). Torque conditions met with are either (*e*) constant or (*f*) varying; and load conditions are either those of (*g*) frequent overload or of (*h*) no overload. The following speed-torque-load combinations are frequently encountered: *aeg*, *aeh*, *beg*, *beh*, *bfg*, *bfh*, *cfg*, *cfh*, etc.

Thus, the machine to be driven may require a constant-speed, constant-torque motor capable of withstanding heavy overload (*aeg*); or it may require an adjustable-speed motor where the torque will vary in direct proportion to the speed and will never require any overload capacity (*cfh*); or it may require a motor that will exert a constant torque at only two definite speeds (*be*), etc.

Standard Practice. In general, the following statements may be considered as representative of standard practice:

1. Machines of present design for comparatively small work requiring constant-speed drive should be grouped in most instances and operated from motor-driven lineshafts.

2. For group driving both direct- and alternating-current motors give satisfactory results. In certain industries (textile mills, for example) the induction motor has decided advantages on account of close speed regulation with varying loads and lessened fire risk, but for machine shops these features are unimportant.

3. Mechanical means of speed control, by step cone pulleys, variable-speed countershafts or by gearing, is usually costly, if the machinery must be stopped to change from one speed to another and if it cannot be controlled from an independent point. It may also be inefficient.

4. For adjustable-speed work, direct-current motors of the field-weakening type are now used almost exclusively. It is hardly practicable to secure by this means a range greater than six to one, and in most cases the best economy is attained by limiting the range to three to one. In many instances it is necessary to resort to a combination of mechanical and electrical control, the disadvantages of each method being largely done away with by this means. Even where machines are handling a very general line of work, the greater part of it will be covered by a range of three to one, so that if this is obtained electrically, gear changes seldom will be necessary and at the same time comparatively inexpensive motors may be used.

5. The necessity of long-distance transmission lines may make alternating current desirable from the standpoint of line losses. When such conditions exist it is customary to use alternating-current motors for constant-speed requirements, and direct-current, adjustable-speed motors operated by current from a motor-generator or rotary converter for the adjustable-speed requirements. If but one kind of current is available, decision should be governed by the number of individual adjustable-speed drives required. In many instances, while group drives may be desirable at the start, new equipment should be purchased with individual motors for the sake of adjustable speed, ease of control, etc.

6. The most desirable voltage at which to transmit current through the various buildings and departments of an industrial plant is about 230.

7. If alternating current is to be used, 60-cycle, three-phase is recommended, as it meets satisfactorily both the light and power requirements and has been adopted as standard by a majority of the central station companies.

INDUSTRIAL MANAGEMENT

By

C. M. SAMES

REFERENCES: Diemer, "Factory Organization and Administration;" Emerson, "Efficiency as a Basis for Operation and Wages," and "Twelve Principles of Efficiency;" Ennis, "Works Management;" Evans, "Cost Keeping and Scientific Management;" Gantt, "Work, Wages and Profits;" Gilbreth, "Motion Study;" Going, "Principles of Industrial Engineering;" Hartness, "The Human Factor in Works Management;" Kimball, "Principles of Industrial Organization;" Lodge, "Rules of Management;" Parkhurst, "Applied Methods of Scientific Management;" Taylor, "Shop Management," and "Principles of Scientific Management;" files of *American Machinist*, *The Engineering Magazine* and *Industrial Engineering*.

Types of Organization. Under the **ordinary** or traditional **type** of organization groups of workers are in charge of foremen, who assign to them their tasks, supply whatever instructions are considered necessary, and exercise a general supervision of the output and of the methods employed in production. The foremen report to and receive orders from superintendents, who in turn are responsible to the general manager, the organization being **military in form** as regards the allocation of authority. Under this type of organization whatever economy in production has been effected has been largely that due to the introduction of labor-saving machinery and processes. Improvements are also possible in the methods employed in the management of industries, which should result in still further increase in economy of production. Dr. Frederick W. Taylor first formulated a system of management having this end in view, in a paper read before the American Society of Mechanical Engineers, in June, 1903. For the military type of organization he substituted a **functional type,** foremen and superintendents being replaced by eight functional officials, each of whom came into direct contact with and exercised authority over individual workmen within strictly defined limits. At a little later date Mr. Harrington Emerson put into practice a system in which the **military type** of organization was **supplemented by a staff** of counselors made up of **experts** in each department of the industry. Variants of these two systems have also been installed. In all of them each step in production is directed by an expert having special knowledge of his particular field, the activities of these specialists being co-ordinated by the system employed.

Planning Department. The important feature of the Taylor system of functional management (generally known as **"scientific management"**) is the **separation of the planning** of how production shall be accomplished **from the performance** of the work. The former is carried out by a planning department, the duties of which are to determine *how* and *when* work is to be done. Taking a machine shop as an example, this includes laying out the route each piece is to travel through the shop and specifying the machines upon which the work is to be done and the workmen who are to do it. With a knowledge of the course of work obtained by means of daily reports from the shop, the planning department, by proper changes in the **routing** of various jobs (standardization of equipment permitting this), is enabled to reduce the time that machines and men are idle to a minimum. All jobs are given out to workmen accompanied by **instruction cards** prepared by the planning department, which specify among other things the special tools or fixtures that must be used, the speeds, feeds, etc., to be employed, the time within which each operation should be completed, and the extra compensation to be received by the workmen if he performs his task within that time. The information embodied in these instruction cards is derived from data accumulated and classified by the department from the results of engineering investigations, experiments and time studies. For example, the determination of the speed, feed and depth of cut to be used in removing metal is not left to the judgment of the operator of the machine, but these are specified by the planning department at rates which their experiments or the records of experiments of others show will result in the metal being removed with maximum economy, wear and tear of machinery, cost of power, cost of regrinding tools, etc., all being taken into consideration. The various operations employed in production are subjected at one time or another to **time study,** by means of which each operation is resolved into a number of elementary operations, and a fair average time for performing each is determined. Such analyses make it nearly always possible to reduce the gross time required for an operation, by indicating where a given number of movements or elementary operations may be replaced by fewer, simpler, more quickly performed or less fatiguing movements, this often being accomplished by merely altering the relative positions of the man, the work and the machine (**motion study**). From the accumulated and classified records of elementary time requirements the department is generally able to predict the probable time required for the production of a given piece of new work, by synthesis of its data. From returns made by the workmen the department determines their compensation and also the cost of work. Records of stores are also kept by the planning department.

Following is an illustrative **time study** of a simple operation (drilling a hole 1 in. deep in an iron casting with a ⅜-in. high-speed drill). The drill press is assumed to be ready for use. The timing is done with a stop watch reading in hundredths of a minute.

	Time in minutes 1st trial	2d trial	3d trial	Average
*1. Place drill in spindle	0.40	....	....	....
*2. Place jig on table	0.40	....	....	....
*3. Center jig bushing with drill	0.45	....	....	....
*4. Bolt jig to table	1.20	....	....	....
*5. Adjust height of table	0.75	....	....	....
*6. Change speed and start machine	0.50	....	....	....
7. Pick up piece and place in jig	0.05	0.04	0.05	0.05
8. Clamp piece in jig	0.20	0.22	0.21	0.21
9. Drill hole at 1000 r.p.m.	0.20	0.21	0.22	0.21
10. Unclamp drilled piece and place in removal box	0.05	0.05	0.04	0.05
*11. Remove drill and jig from machine	1.50			0.52
Total machine time (items marked *) =	5.20 min.			
Average time for drilling each piece =	0.52 min.			

A study of the elementary machine times often indicates possible simplifications of procedure which will result in a decrease of the total machine time.

An instruction card for the preceding operation on a lot of 30 pieces would embody, in a printed heading, data as to the drawing number, the shop number of the machine, the order number, the material to be worked upon, the number of pieces in the lot, the time allowed for completing the work, the bonus to be paid for completing the lot in the given time, and the following detailed instructions:

	Time per piece, min.	Time for entire lot, min.
1. Obtain instruction card and learn what is to be done	....	4.00
2. Place drill in spindle	....	0.40
3. Place jig on table	....	0.40
4. Center jig bushing with drill	....	0.45
5. Bolt jig to table	....	1.20
6. Adjust height of table	....	0.75
7. Change speed of machine and start	....	0.50
8. Pick up piece and place in jig	0.05	....
9. Clamp piece in jig	0.21	....
10. Drill hole at 1000 r.p.m.	0.21	....
11. Unclamp drilled piece and place in removal box	0.05	....
12. Remove drill and jig from machine	....	1.50
Time allowances:		
Handling time, 50 per cent., of items 8, 9, and 11	0.16	....
Machine time, 10 per cent. of item 10	0.02	....
Totals	0.70	9.20

Time for lot = (30 pieces × 0.70) + 9.20 = 30.2 min.

The **tool list** furnished to the gang boss by the planning department would indicate the jig (by symbol-number) and drill to be delivered to the workman by the tool-room messenger, the former in good working order and provided with bolts having easy-running nuts and smooth threads, the latter properly sharpened.

Production Department. Under the Taylor system of management the orders issuing from the planning department are executed in the shops under the directions of four functional foremen (and their assistants), known respectively as the gang boss, the speed boss, the repair boss, and the inspector. The **gang boss** sees that all materials, tools, fixtures, special supplies, etc., are delivered to the workmen at exactly the time they are needed, and that finished pieces are promptly removed after they have been inspected. The information upon which he works is furnished to him by the planning department in the form of route sheets, tool lists, etc. The function of the **speed boss** is to see that work is performed exactly in accordance with the specifications of the instruction cards, and, in case of inability on the part of the workman to comprehend the instructions or to do the work in the allotted time, to teach him the proper procedure. The **repair boss** is required to keep all machinery and power-transmitting devices in first-class working order, seeing to it that they are well cleaned and properly lubricated, that belts run at proper tensions, etc. The **inspector** accepts or rejects finished work after examination and measurements made thereon. An additional member of the planning department staff is the **disciplinarian,** an official removed from all shop influences, to whom all disputes and differences between foremen and men are referred for adjustment. In Mr. **Emerson's system** the same general ends are sought as in the Taylor system, but through the activities of a line organisation supplied with information and counseled by a staff of experts guided by certain principles of efficiency which he has formulated.

Bonus Wage Systems. Where the co-operation of workmen can be secured, modern management methods make it possible greatly to increase the output of a given industry without increasing the number of its employees —in some cases to more than double it. Workmen, however, generally view the introduction of strange or novel methods with apprehension, and are reluctant to adopt them. To enlist their co-operation various methods of bonus wage payment have been devised, whereby a part of the money savings made possible by increased production is given to the workmen, who receive from 20 to 40 per cent. greater returns for their labor than they could obtain in competing establishments where efficient methods have not been installed. It is insisted upon by those advocating the new systems of management that the utmost good faith must always be maintained with the workmen, and that rates once set must not be lowered so long as the original method of working is followed, the management bearing all losses due to errors in calculations which may in isolated instances result in abnormally high wages.

In the **Taylor differential-piece-rate system** of compensation the workman receives a certain (low) rate per piece for production falling below the standard set, and a much higher rate provided his production attains or exceeds the standard. In the **Gantt task-and-bonus plan** a definite task for a given time is prescribed; if the workman accomplishes his task within the allotted time he is given a bonus in the form of an extra time allowance (25 to 50 per cent. usually); should he fail to accomplish his task as set, however, the management guarantees that he shall still receive his day rate. In the **Emerson plan** the workman is assured initially that he will receive his day wage, whatever the amount of his production. As soon, however, as he attains a production of 66⅔ per cent. of the standard set, he begins to receive a small bonus, which increases rapidly with an increase in the percentage of his efficiency, and substantially as follows:

Effy., per cent.	Bonus per \$1 of wages, cents	Effy., per cent.	Bonus per \$1 of wages, cents	Effy., per cent.	Bonus per \$1 of wages, cents	Effy., per cent.	Bonus per \$1 of wages, cents
67	0.01	75	1.31	87	7.56	100	20
68	0.04	77	1.99	90	9.91	105	25
69	0.11	80	3.27	92	11.62	110	30
70	0.22	82	4.33	95	14.53	120	40
72	0.55	85	6.17	97	16.62	140	60

The following table, arranged from three published in Kimball's "Principles of Industrial Organization," serves to illustrate the application of the bonus systems described; the rates and bonuses, however, are arbitrarily chosen, and no attempt is made to compare the systems. The task assumed is the machining of pieces of metal, bonus beginning at the standard output of 30 pieces a day. A machine rate of \$2.50 per day is also assumed. In the Taylor system the day wage is not assured to the workman; in the Gantt and Emerson systems he is assured (in the illustration) of a daily wage of \$3 regardless of his output. (It is to be understood of course that men would not be retained under any system on work where they proved to be hopelessly inefficient.)

		Taylor			Gantt			Emerson			
Number of pieces a day	Cost of material, \$	Piece rate, cents	Daily earnings, \$	Cost per piece, cents*	Bonus, \$	Daily earnings, \$	Cost per piece, cents*	Efficiency, per cent.	Bonus, \$	Daily earnings, \$	Cost per piece, cents*
10	1.00	10	1.00	45.0	0.00	3.00	65.0	33⅓	0.00	3.00	65.0
15	1.50	10	1.50	36.7	0.00	3.00	46.7	50	0.00	3.00	46.7
20	2.00	10	2.00	32.5	0.00	3.00	37.5	66⅔	0.03	3.03	37.5
30	3.00	15	4.50	33.3	1.00	4.00	31.7	100	0.60	3.60	30.3
40	4.00	15	6.00	31.3	1.33	5.33	29.5	133⅓	1.59	4.59	27.8
50	5.00	15	7.50	30.0	1.66	6.66	28.3	....	......	......	....

* Cost per piece = (daily earnings + machine rate + cost of material) ÷ number of pieces produced in a day.

Other Wage Systems. Where **day work** is employed, the time for performing a given task is not specified. It is the rule where the nature of the work varies from day to day to such an extent that it would be unprofitable to make time and motion studies of tasks that might never occur again. **Straight piece work** (wages = number of pieces × piece-work rate), disliked by workmen because of the tendency toward rate-cutting, is employed in repetition work where large quantities are dealt with. Where delays due to causes beyond the workman's control are liable to occur frequently, a **piece rate with guaranteed day rate** is fairer. In this case the hourly rate is fixed at a lower amount than the workman could ordinarily earn, to prevent recourse to the day rate except when absolutely necessary. A **differential piece rate** is advantageous where fixed charges are high. The workman is paid at a certain piece rate until his daily output reaches about 80 per cent. of the amount a good workman can regularly accomplish without injury to his health. Further output is paid at a higher piece rate.

In the **premium system** devised by F. A. Halsey the workman receives his hourly rate and in addition extra pay every time he performs the task in less than the standard time. The standard time is usually from 20 to 50 per cent. in excess of the time required by a skilled workman under correct conditions, and is established from the results of time and motion studies. Wages = (time taken × hourly rate) + (time saved × a fraction of the hourly rate), the fraction customarily taken being one-half. The field for a premium system would be in establishments where part of the work is repetitive, part interchangeable, and part that which is performed only once or at long intervals.

In the **bonus system on individual tasks** the workmen is paid an hourly rate and in addition a bonus on each task completed in the standard time. Its field is in industries in which the standards have been fairly well established under the original Gantt bonus system. A further development of this system is that of paying to the foreman a differential bonus varying with the number of workmen receiving bonus in his department. In a **combined bonus and premium system** devised by Hugo Diemer, the workman is paid a 10 per cent. increase in his hourly rate if he reduces the time for performing the task below past averages, and 20 per cent. increase if the task is performed in a specified standard time (based on time and motion studies). A further bonus on the Halsey premium system is allowed if the task is performed in less than standard time. In addition, a record is kept of the percentage of success of each man during each pay period, and this is made the basis of his promotion to a higher hourly rate.

It is impossible to realize to the fullest extent the economies attainable under modern management systems in industrial establishments working less than several hundred men. But a modification of the Taylor system, devised and described by F. A. Parkhurst in "Applied Methods of Scientific Management," shows excellent results in shops where as few as 100 men are employed.

It is stated by Dr. Taylor that an important fact to be noted in shops where an approved modern system of management has been installed, is the change in the mental attitude of both the men and the managerial staff, suspicion and open antagonism being replaced by cordial good-will and the earnest desire to co-operate; that the impartial justice of the code of laws upon which the system is based is recognized, and that its operation automatically does away with practically all of the subjects of dispute which under the old type of management were settled by argument or by collective bargaining.

COST AND OTHER FACTORY ACCOUNTS

By

HUGO DIEMER

The three fundamental cost accounts employed prior to the introduction of modern methods were designated as Material, Labor and Expense. The more modern accounts deal respectively with Raw Materials, Material Burden, Direct Labor, Expense Burden, Work in Process, and Finished Stock.

Raw Material Account. As material is received this account is **debited** with the amount of its cost as shown by the invoices, and is **credited** with the total charges to Work in Process for all material drawn for work in process through orders on the storeroom.

Material Burden is a general account which is **debited** with the monthly or periodic totals of all costs of carriage on materials received, cost of buying (including purchasing department salaries and expenses) and cost of storing and handling (including all storeroom salaries and expenses).

The Material Burden account is **credited** with the burden amount charged to the Work in Process account. These charges are usually on the basis of percentage-on-raw-material-cost, based on the ratio of the Material Burden account during the past period or several periods to the Raw Materials account during the same time.

Work in Process Account. This account is **debited** with all materials drawn for work in process per orders on the storeroom which have been credited to the Raw Materials account, as well as with all Material Burden items which have been credited to Material Burden. It is also debited with all Direct Labor charged to production orders during a given period, as well as with the Expense Burden. This Expense Burden may be determined by using one of the methods described below. If the Expense Burden includes in addition to Indirect Labor all other expense factors, there will be no further burden charges to Work in Process, but if the Expense Burden is divided into different kinds of expense factors, such as Indirect Labor Burden, Machine Burden and General Expense Burden, these will all have to be charged to Work in Process. Work in Process is **credited** with the total of all orders as soon as finished, or, in other words, just as soon as they become Finished Stock.

Finished Stock Account. This account is **debited** with the total of all work in process just as soon as an order is transformed from work in process into a finished order. The Work in Process account is credited with the same amount. Finished Stock is **credited** with the Manufacturer's Cost price as established for the various articles to be made, whenever any articles are shipped. This Manufacturer's Cost price is then made the basis of operations of the selling department. It may be higher than the sum of the Raw Material, Material Burden, Direct Labor, Indirect Labor, Machine Burden and General Expense Burden charges, in order to allow for fluctuations in cost. Any apparent profits accumulated in this way can be transferred to reserves for meeting depreciation of plant and equipment.

Methods of Allotting Expense Burden

Percentage on Wages. The oldest method and the one still in most general use is the percentage-on-wages method, established by finding the ratio in percentage which the sum total of Indirect Labor and all expense accounts in a given period bears to the sum total of Direct Labor in the same period. Its greatest weakness lies in the fact that it overcharges high-priced labor which may use shop and equipment and share running expenses only a short time, and undercharges low-priced labor which may use shop and equipment and share running expenses a long time, in creating a direct-labor cost of the same dollars-and-cents value.

Hourly Expense or Man-hour Rate. A more recent method, and the one which is in use by all machine-tool builders using the uniform cost-accounting methods recommended by the National Machine Tool Builders' Association, is the hourly expense rate obtained by dividing the sum total of all indirect labor and expenses during a given period by the sum total of direct-labor hours in the same period.

Separate Man-hour Departmental and General Rates and Hourly Machine Rates. A third method, probably in the least general use, is one which establishes (*a*) an hourly rate for each department, local to that department, (*b*) a general hourly shop overhead rate, and (*c*) an hourly rate for each machine or group of machines. Rates (*a*), (*b*) and (*c*) are mutually exclusive and the sum total of all expenses allotted from these three channels must approximate the total of all indirect labor and all other expenses in the same period. Inasmuch as the charges to Work in Process and corresponding credits to the various expense accounts during a given period are the results of calculations based on averages of preceding periods, the exact balancing in order to close the expense accounts must be left to an adjustment entry which may result in either a debit or credit to the expense account at the opening of the next period. If a considerable item, this adjustment entry may demand a modification of the rates for allotting expense burden during the next new period.

The establishment of local departmental burden rates affords the proper basis for comparing local expenses from month to month, as well as holding down to their true value costs on product which passes through departments having low local burdens.

Method of Establishing the Hourly Machine Rate. A number of different considerations enter into the establishment of the hourly machine rate. Among these are the following: (1) The interest and depreciation on the purchase price of the machine; (2) the annual cost of repairing the machine and keeping it supplied with lubricants, cutting tools, etc.; (3) the floor space occupied by the machine and the stock lying about it; (4) the power consumed by the machine.

With the previous year's expense accounts as a basis, an estimate is made of the total of the above-mentioned items. Out of this grand total, or budget, each machine tool is given a certain definite allowance or allotment for the coming year. A card is then written out for each separate machine tool, and after determining approximately the number of hours that the machine will run during the year, its hourly operating rate is established for the next year by dividing the total budget by the estimated hours that the machine is to be used. Every time that the machine is used it is credited with the amount it earns as its hourly rate, the amounts being posted on the card record for that machine tool from the cost department's records showing the hours which that machine was in operation. If at the end of an annual or semi-annual period it is found likely that the postings on any card will overrun or fall short of the amount allotted the given machine, changes must be made in the individual machine rate. The exact balancing at the close of a year will be left to adjustment entries, which may result in either a debit or credit to the machine expense account at the opening of the next period. With careful allotment of rates these adjustment entries will be relatively small and required only to make an exact balance at closing.

Segregation of Works Expenses from Selling Expenses. All distinctly selling expenses should be kept separate and the sales department made to show a profit on the basis of the difference between Manufacturer's Cost and Selling Price. Administrative expense which is neither a direct charge to Works Management nor Selling, should be equally apportioned against Manufacturer's Cost and Selling Cost.

Detailed Cost Statistics and Records. The nature of the business will determine whether these records are to be based on unit prices per quantity of output, per variety of output, per department, or per individual lot order or item, or whether each individual item is to have its cost figured regularly or at intervals.

In general, material charges on cost records are taken care of by posting on to the general cost records the various material withdrawals, from withdrawal slips which are exchanged for the goods designated at the time of withdrawal. In establishments having well-organized planning or production departments, these withdrawal slips are written out in advance so that no time is lost in writing them when the materials designated are needed.

Labor postings on cost records are transferred from time slips written out by time keepers or on job cards stamped with time stamps. With a well-organized planning department the job cards can be written out in advance.

In addition to the material and labor postings, there remain the postings covering expense burden. These postings are usually made in the cost department in accordance with whichever of the above-outlined systems of distributing expense burden is in use.

Cost Accounts a Part of Balance Sheets. If the fundamental principles above enunciated are observed, the totals of the money value of Material in Store Room, Work in Process, and Finished Stock can be obtained in a very short time, and the profit and loss account can be closed at as frequent financial intervals as may be desired.

SECTION 13

PUMPS AND COMPRESSORS

BY

L. C. LOEWENSTEIN, E. E., Ph. D., Engineer with General Electric Co., Mem. A. S. M. E., A. I. E. E., Etc.

F. F. NICKEL, Engineer with Henry R. Worthington.

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CONTENTS

PUMPS

BY

F. F. NICKEL

REFERENCES: Bach, "Pumpenventile," Springer, Berlin. Berg, "Die Pumpen," Springer. Dahme, "Die Kolbenpumpe," Oldenbourg, Berlin. Greene, "Pumping Machinery," Wiley. Haeder, "Pumpen," Otto Haeder, Wiesbaden. Hague, "Pumping Engines," McGraw-Hill. Mueller, "Das Pumpenventil," Felix, Leipzig. Nickel, "Direct-acting Steam Pumps," McGraw-Hill.

PISTONLESS PUMPS

Pulsometers

Pulsometers (Fig. 1) are pistonless steam-operated displacement pumps with two chambers, *AA*. The steam enters these chambers alternately and is controlled by a ball or flap valve *C*. It enters one chamber and forces the water contained therein out through a check valve *F* to the discharge pipe *H* until the steam enters the discharge pipe where it is condensed and produces a suction effect which throws the valve *C* and draws up the water through the suction valves. *J* is an air chamber. Notwithstanding their low efficiency they are used extensively in mines, at railroad watering stations, for draining building pits, etc. Suction lift, up to 26 ft., max., preferably 7 to 14 ft.; discharge head, up to 150 ft. The steam pressure must be about 50 per cent. higher than the total water pressure. Low heads give a lower efficiency than high heads. The discharge water is heated 3.5 deg. fahr. for a head of 30 ft. For every additional 30 ft., add 1.5 deg. For water-temperatures in excess of 120 deg. fahr., no suction lift is obtainable. The steam consumption by volume is from 2 to 3 times the water displacement. Wood finds **duties** varying from 10.5 to 13.4 million ft.-lb., the higher figure corresponding to the higher head. This corresponds to a steam consumption of 190 to 150 lb. per h.p. per hour. A duty of 21,345,000 ft.-lb. has been found for total lifts of 102.6 ft.

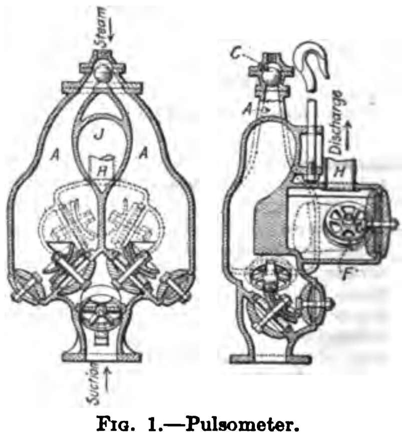


FIG. 1.—Pulsometer.

Jet Pumps

Water-jet Pumps are made in capacities up to 700 gal. per min., and may be driven by water entering under a head of 10 to 2500 ft. The efficiency is dependent upon the relation of supply head to pump head and ranges from 15 to 30 per cent. Suction lift up to 26 ft., preferably not more than 10 to 14 ft. They are used in mines for sinking purposes, in foundation work, for cellar drainage, etc. Because of the absence of moving parts, dirty and muddy water can be handled. Their special field is for high drive heads and low pump heads.

The Hydraulic Ram is used for the lifting of small quantities of water against high heads and where an abundance of drive water is available; capacities, 1 to 30 gal. per min. It works by a mild water-hammer action. (For theory of water hammer, see p. 277.) The clack valve closes at regular intervals, stopping the column of moving water in the drive pipe suddenly. This changes the kinetic energy of the water into pressure which is relieved by the forcing of part of the water through the check valve into the air chamber against the discharge head, and by the pressure wave traveling back up the drive pipe. The **drive pipe** should be unobstructed and nearly straight and have a length equal to $(h_d + 2)(h_d - h_s)/h_s$ and not less than $5h_s$, where h_d and h_s are respectively the delivery and supply heads, ft. The diam. of the drive pipe should be $d(\text{ft.}) = \sqrt{1.63 \times \text{cu. ft. of water supplied per sec.}}$ The volume of the **air chamber** should equal that of the delivery pipe. Hydraulic rams are generally used only under low supply heads, as the maximum pressure due to the water-hammer action increases directly with the velocity of the water in the supply pipe, or as the square root of the supply head. Their **efficiency** as pumps is dependent upon the relation of pump head to supply head or the ratio of lift to fall, as follows:

Ratio of lift to fall	4	6	8	12	16	20	24	26
Efficiency, per cent	72	61	52	37	25	14	4	0

A large ram designed by Prof. D. W. Mead for the village of West Dundee, Ill., delivers water into a stand pipe 115 ft. above the ram. The 10-in. drive pipe is 2200 ft. long and under a head of 55 ft. The duration of one cycle of this ram is 4⅓ sec. and the maximum resistance of the check valve is less than 2½ lb. According to indicator cards taken the maximum pressure in the drive pipe lasts only about 1 sec., which is equal to the time required for the wave of compression, or sound wave, to make a "round trip" from the valve to the reservoir and back.

According to Joukovsky's experiments on water hammer, the velocity, C, of the compression wave is about 4670 ft. per sec. and the time for the "round trip" would equal $t_r = 2l/C = 4400/4670 = 0.95$ sec., neglecting the distension of the pipe.

Tests of a Rumsey hydraulic ram (Prof. R. C. Carpenter, *Eng. Mechanics*, 1894) with a discharge head of 19.75 ft. of water through a ½-in. pipe, an average supply head of 5.67 ft. of water through a 1½-in. pipe having an equivalent length of 65 ft., and with full travel of the clack valve = $^{15}/_{16}$ in., gave the following results:

Length of stroke, per cent	100	80	50	46
Number of strokes per min	52	56	61	66
Efficiency, per cent	64.9	66	74.9	70

At maximum efficiency the travel of the clack valve was $^{9}/_{16}$ in. The weight of water pumped was practically constant throughout these tests.

Tests of a Rife hydraulic ram (*Stevens Indicator*, April, 1898) give the highest efficiency as 75.6 per cent.

Steam-jet Pumps

Injectors. An injector consists of a **steam nozzle** a (Fig. 2), in which the steam acquires kinetic energy; a **combining tube** b, at the entrance to which the steam impinges on the feed water entering at c; and a **delivery tube** d, in which the velocity head of the feed water is reduced and its static pressure increased.

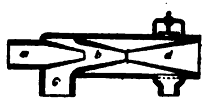


Fig. 2.—Injector.

The velocity acquired by the steam in the steam nozzle can be calculated as indicated on p. 355. The pressure at the entrance to the combining tube is found to be 3 to 4 lb. per sq. in. abs. for usual feed temperatures. The quantity of feed water that can be handled is obtainable from the equation

$$(W + 1)v_m = C(v_s + Wv_w)$$

where W = weight of water discharged per lb. of steam, lb.; v_s = velocity of steam before impact, ft. per sec.; v_w = velocity of feed water before impact, ft. per sec.; v_m = velocity of mixture after impact, ft. per sec.; and C = coefficient of impact. The value of v_w is calculable when the head and lift of the feed water, the vacuum at the entrance to the combining tube and the frictional resistances are known. Its amount is generally negligible, so that the equation becomes $(W + 1)v_m = Cv_s$. The value of C averages about 0.5.

The **actual weight of feed water handled per lb. of steam,** W, is easily determined in an injector from a heat balance. The heat given up by the steam is equal to the heat given to the feed water plus the external work done. The external work is usually about 2 per cent. of the heat given up by the steam. The heat-balance equation becomes

$$0.98(H - h_2) = W(h_2 - h_1)$$

where H, h_1 and h_2 are respectively the total heats (in B.t.u.) of 1 lb. of steam, 1 lb. of feed water, and 1 lb. of the mixture. Table 1 gives results of tests of an injector and shows its general characteristics. The weight of feed water handled per lb. of steam is generally found to diminish as the steam pressure increases, and usually varies from about 21 lb. at 20 lb. gage pressure to 10 lb. at 100 lb. pressure; the maximum temperature of feed water which can be handled does not vary much with the steam pressure, and averages from 120 to 140 deg. fahr. The pressure against which the injector will deliver water is usually from 50 to 80 lb. above the steam pressure.

Table 1. Test of Sellers Injector

(From "Theory and Practice of the Injector," by S. L. Kneass)

Mean steam pressure, lb. per sq. in.	30	60	121	150	200
Temperature of supply water, deg. fahr.	67	67	54	54	50
Maximum capacity:					
Gal. water handled per hour	1912	2535	3517	3765	4005
Temp. of delivered water, deg. fahr.	113	125	134	135	154
Weight of delivered water per lb. of steam used, lb.	25.9	19.1	13.6	12.6	10.3
Minimum capacity:					
Gal. water handled per hour	765	937	1290	1432	1732
Temp. of delivered water, deg. fahr.	171	212	238	250	263
Ratio of min. to max. capacity	0.4	0.37	0.37	0.38	0.43
Limiting restarting temp., deg. fahr	130	135	122	120	......
Limiting operating temp., deg. fahr	139	144	137	133	......

The **efficiency** of an injector considered merely as a pump is very low, about 1 to 2 per cent.; as a boiler feed pump, where the heat of the steam is utilized in the boiler, the efficiency is nearly 100 per cent. It is commonly not the most economical means for feeding a boiler, since it can handle only cold or moderately warm water, and the effect is equivalent to heating the feed water by live steam, while a pump can handle water which has been heated by exhaust steam from the main and auxiliary engines, which would otherwise be wasted. Steam consumption about 400 lb. per water h.p. per hour; a small direct-acting steam pump consumes from 100 to 200 lb.

The Ejector is a modified type of injector for handling large volumes. Discharge heads with usual steam pressures range up to about 120 ft. for suction lift of less than 5 ft., and up to 75 ft. for suction lift of 22 ft. With no suction lift the discharge pressure for cold water may rise to twice the steam pressure; water as hot as 175 deg. fahr. may be raised 6 to 10 ft. Ejectors are used as well pumps, bilge pumps, as fire pumps in factories, and for the lifting of muddy liquids, acids, alkaline solutions, and other liquids where admixture with the condensing steam is unobjectionable.

The Air Lift

The air lift consists of a drop pipe placed in a well with its lower end submerged. The depth of submergence measured from the level at which the water stands during operation to the air entrance is called the **submergence.** An air pipe delivers air at the bottom of the drop pipe and forms a mixture of air and water which is lighter than the column of solid water in the well; consequently the mixture rises above the surrounding water. The necessary **percentage of submergence** (the percentage of the total length of pipe which is submerged in "solid" water when pumping) decreases as the lift increases, ranging from 66 per cent. for a lift of 20 ft. to 41 per cent. for a lift of 500 ft. The average will usually lie between 50 and 60 per cent. A submergence of 60 per cent. would require a submergence of 60 ft. for a lift of 40 ft. and a total height of discharge pipe of 100 ft. from air inlet to water discharge. The **efficiency** (ratio of the water h.p. to the indicated air h.p. of the compressor) varies between 20 and 50 per cent. The low efficiency is offset by the advantage of having the whole operating mechanism above ground and by the capacity for handling dirty, gritty or acidulous mine water, slimes from reduction plants, sewage, etc. The theoretical consumption (v) of free air in cu. ft. per gal. of water pumped is given by Goodman's formula:

$v = l/585 \log [1 + (s/34)]$, where l = total lift, ft.; s = submergence, ft. The displacement of the air compressor = $[3 + (l/700)]v$.

The air pressure required for starting is higher than that for operating and is equivalent to the height of the water level at rest above the end of the air pipe. The actual pumping level and yield of a well are seldom known in advance. After the piping is installed the submergence is altered to suit by raising or lowering the pipe in the well. The quantity of water a well will yield depends upon its diameter. The water pipe should have a cross-sectional area (sq. in.) equal to discharge in gal. per min. ÷ 12 to 15. Too large a pipe lets the air slip by, and too small a pipe means excessive friction and inefficient expansion of the air bubbles.

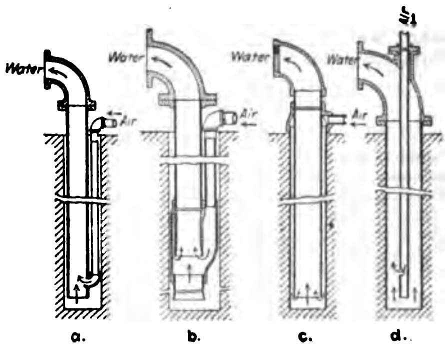


FIG. 3.—Arrangements of Air Lifts.

The arrangements most generally used, where conditions permit, are shown at *a* and *b*, Fig. 3. This is the **Pohlé** or "side-inlet" **method,** where the discharge and air pipes are placed side by side in the well and joined by a suitable foot-piece. In *b*, compressed air fills the annular space or ring surrounding the uptake pipe and is free to enter the rising column at all points of its periphery, at the same time acting without obstructing or contracting the discharge pipe anywhere. The **Saunders system** is shown at *c*. A central discharge pipe is suspended in the well, the air passing down between it and the well casing. If the well is not cased a second pipe must be used outside of the main discharge pipe, the air as before filling the annular space between the two pipes. The **central air pipe system** is shown at *d*. This method is generally used where the lift is low. The air pipe is suspended in the well without the usual discharge pipe, thereby making the well its own discharge pipe. Data on air-lift pumps are given in Table 2.

Table 2. Pipe Sizes and Capacities of Air Lifts

Side air inlet (Pohlé)				Concentric air pipe (Saunders)		Central air pipe		
Air pipe diam., in.	Water pipe diam., in.	Well diam., in.	Max. economical capacity (moderate lift), gal. per min.	Height of lift, ft.	Gal. per min. per sq. in. of water pipe cross-section	Diam. of casing, in.	Air pipe diam., in.	Capacity, gal. per min.
½	1	3	7	25	15–20	3½	1¼	80– 100
¾	1½	4	20	50–125	12–15	4	1½	100– 150
1	2	4½	35			5	2	150– 250
1	2½	5	60			6	2	275– 375
1¼	3	6	90			8	2½	500– 665
1½	3½	7	120			10	2½	775–1000
1½	4	8	160					
1½	5	9	250					
2	6	10	350					

Tests of Air Lifts operating in oil wells (Ivens, *Trans. A. S. M. E.*, 1909) give the following results (Weight of 1 gal. of fluid pumped, at 116.3 deg. fahr., 8.67 lb. Pipe diameters: Casing, 6 in.; discharge, 4 in.; air, 1¼ in. Total length of vertical discharge line, 1500 ft.; air line, 12 ft. shorter):

	Well No. 32 Old System	Well No. 32 New System	Well No. 2 Old System	Well No. 2 New System
Pumping head, ft.	1160	1080	866	987
Submergence, ft.	350	411	633	502
Submergence, per cent	23	27.6	42.3	33
Mean air h.p.	107.4	79.16	82.5	50.4
Mean water h.p.	9.97	10.36	9.85	13.36
Fluid pumped per hour, gal.	1953.6	2188.2	2499.6	3056.4
Efficiency on the air h.p., per cent	9.3	13.1	11.9	26

Tests of eleven wells at Atlantic City, N. J. (*Eng. News*, June 18, 1908) showed the following results: At capacity (3,544,900 gal. per 24 hr. through a mean lift of 26.88 ft. with air pressure of 31 lb.), duty of whole plant, 19,900,000 ft.-lb. per 1000 lb. of steam used by compressors; at ¾ capacity (2,642,900 gal. through mean lift of 25.43 ft. with 26 lb. air pressure), duty, 24,207,000 ft.-lb. The wells were 10 in. in diam., water pipes 4 to 5¼ in. and air pipes ¾ to 1¼ in. Maximum lift of the several wells ranged from 26 to 40 ft.; submergence 37 to 49 ft.; ratio of submergence to lift, 0.9 to 1.8; submergence in percentage of length of pipe, 53 to 64.

RECIPROCATING PUMPS

Efficiencies, Suction Lift, Dimensions, Speeds

Reciprocating vs. Centrifugal Pumps. The advantages of reciprocating pumps over centrifugal pumps are their flexibility in regard to capacity, head and speed, and their nearly uniform efficiency under a wide range of conditions. The advantages of centrifugal pumps over reciprocating pumps lie in their lower initial cost, smaller floor-space requirements, less need for attendance, quiet operation, absence of excessive stresses in the pipe line owing to the uniform discharge, and adaptability for being driven by motors of high rotative speed.

Pump Diagram. In Fig. 4, h = head pumped against; h_i = indicated head; h_s = static suction lift; h_d = static discharge head; s = losses through suction pipe and valves; d = losses through discharge valves and force chamber. $h_i = h + s + d$, and $h = h_s + h_d$.

Efficiencies

Volumetric Efficiency = $E_v = Q_e/Q$, where Q_e = actual volume discharged, and Q = plunger displacement. This efficiency gives information as to the loss of capacity due to defective piston packing, leaky stuffing boxes or valves and the delayed closing of the valves. It is generally stated as loss in percentage of the displacement and is then called **slip** ($= 1 - E_v$). The slip is very little in a new pump, actual tests having demonstrated that it rarely exceeds 2 per cent. in a plunger-and-ring pump. In packed pumps where there is no leakage past the piston or plunger, it is less. Pumps handling large amounts of water have shown slips of ½ per cent., no slip or even negative slip, which latter is a result of the tendency of the water to continue to flow after the plunger has stopped. The average slip for good pumps is probably from 3 to 5 per cent.; under unfavorable conditions it may amount to as much as 10 or 15 per cent. Small pumps are tested for tightness by closing the discharge valve and admitting sufficient steam to produce the required water pressure. The pump will then make a certain number of strokes per min., called **lost actions.** If the lost actions are about 1 per cent. of the normal speed, the pump is passed as satisfactory. At normal speed the leakage will be less than this amount.

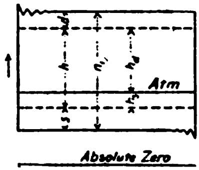


Fig. 4.—Pump Diagram.

Hydraulic Efficiency = $E_h = h/h_i$, where h = total head pumped against and $h_i = h$ + hydraulic losses. These losses consist of (1) the losses in passing from the well to the pump chamber, and (2) the losses in passing from the pump chamber to the point where the discharge gage is attached. The losses under (1) are usually small. They comprise: (*a*) The velocity head, (*b*) entrance head, (*c*) friction in the suction pipe, (*d*) losses in the bends, and (*e*) loss in passing through the suction valves. The losses under (2) comprise: (*a*) The loss in passing through the discharge valves, which consists of the head set up by the spring pressure and the friction through the valves. The former should be ½ per cent. of the discharge pressure and the whole loss may be assumed to be equal to 1 per cent. of this pressure. (*b*) The velocity head, the friction in the passages, etc. At the low velocity used these losses are insignificant. The friction in the main discharge pipe is part of the useful work of the pump and is not included in the hydraulic efficiency. The pressure indicated by a gage at the discharge nozzle plus the height of this gage above the level in the suction well constitutes the total head h. The hydraulic efficiency is greater in a high-service pump than in a low-service pump. It is very difficult to measure this efficiency owing to the impossibility of obtaining reliable indicator cards from the pump end (due to the disturbing influence of the inertia of the water), and it is therefore generally included in the expression for the mechanical efficiency.

Indicated Efficiency = $E_{pi} = N_w/N_i$, where N_w = the water horse power calculated from the amount of water actually pumped ($= Q_e$) and the total head h, and N_i = the h.p. calculated from the pump indicator diagram.

$$E_{pi} = Q_e h/Q h_i = E_v \times E_h.$$

This efficiency comprises all losses in the pump end.

Mechanical Efficiency $= E_m = N_i/N_b$, where N_b = the brake horse power or the indicated steam horse power. This efficiency gives information as to the mechanical friction in the mechanism transmitting the power to the pump. It can only be determined by actual experiments with the different types of pumps, and depends upon the size of the pump and the service. In Table 3 the pumps are therefore classified according to stroke and type. The figures are conservative.

Table 3. Mechanical Efficiencies of Direct-acting Pumps
(Efficiencies in per cent.)

Stroke, in.	Plunger pumps	Piston pumps	Outside-packed pumps	Pressure pumps	Stroke, in.	Plunger pumps	Piston pumps	Outside-packed pumps	Pressure pumps
3	50	50	47	45	12	77.5	77.5	74	70
4	55	55	52	50	15	80	80	76	72
5	60	60	57	54	18	82.5	82.5	78	74
6	65	65	61	58	24	85	85	81	77
8	70	70	66	63	36	87.5	87.5	83	79
10	75	75	71	67	48-60	90	90	85	81

Total Efficiency $= E = E_v \times E_h \times E_m$. This efficiency comprises all losses in the driving mechanism and the pump and indicates the economy of the whole installation.

Suction Lift. The velocity of the water in the suction pipe is usually about 3 ft. per sec., making the losses enumerated under "hydraulic efficiency" as follows:

(*a*) Velocity head $= h = v^2/2g = 0.14$ ft. water column.

(*b*) Entrance head $= h/2 = 0.07$ ft.

(*c*) and (*d*) Friction in suction pipe and loss in bends: small at the low velocity, usually not over 1 ft.

(*e*) Loss through suction valves: this may be assumed as equal to 1.5 ft., so that the total loss will rarely exceed 3 ft.

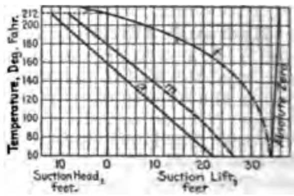


Fig. 5.—Suction Lifts for Various Water Temperatures.

The inertia of the water in the suction passages may assume serious proportions in a single single-acting pump. It can, however, be disregarded in a pump with vacuum chamber, and especially in a duplex pump.

The temperature of the water determines the theoretical suction lift from which all the suction losses should be deducted. The theoretical suction lift = atmospheric pressure minus the vapor pressure of the water. The air contained in the water is the main cause for the difference between the theoretical and the actually attainable suction lift. Fig. 5 gives the theoretical suction lift *t*, the attainable suction lift *a*, and the maximum *m* possible under favorable conditions. This diagram is based on a velocity of 3 ft. per sec. and a short suction pipe with one bend. The suction lift is the vertical distance from the water in the well to the top of the discharge valve deck. A head of 12 ft. is sufficient for any temperature, because water cannot exist at a temperature above 212 deg. fahr., except under pressure, the effect of which is added to the static head.

Air Chamber. To take up irregularities and induce a uniform flow in the discharge pipe, an air chamber is generally used. This is a necessity in single pumps and also in crank and flywheel pumps of any type, but may be dispensed with in duplex direct-acting low-service pumps (75 lb. pressure). In duplex pumps its volume should be 4 times the displacement of one plunger per stroke; in the other types, 8 times.

Plunger Diameters—Piston Speeds. The diameter d (in.) of one double-acting plunger for a given displacement G (U. S. gal. per min.) may be obtained from the formula $d^2P = 24.51G$, where P = piston speed, ft. per min. For the diameter d of each of two double-acting plungers for a given displacement, substitute $\frac{1}{2}G$ for G; for that of each of three single-acting plungers, substitute $\frac{2}{3}G$ for G, etc. The displacement of the piston rod must be deducted from G and the proper allowance made for slip. The proper **speeds** for different types of pumps are given in Table 4.

Table 4. Normal Speeds of Different Types of Pumps

Stroke, in.	Direct-acting pumps: Piston pumps: Plunger and ring pumps, and outside center-packed: Simple cylinder and compound		Triple-expansion		Pressure pumps, wet vacuum pumps, geared power pumps		Boiler feed pumps and pumps for thick liquids		Crank and flywheel pumps	
	r.p.m.	ft. per min.	r.p.m.	ft. per min.	r.p.m.	ft. per min.	r.p.m.	ft. per min.	r.p.m.	ft. per min.
3	80	40	....	....	64	32	40	20	....	....
4	75	50	....	....	60	40	38	25	....	....
5	72	60	....	....	58	48	36	30	....	....
6	65	65	....	....	52	52	33	33	....	....
8	56	75	....	....	45	60	29	38	....	....
10	48	80	54	90	38	64	24	40	....	....
12	45	90	50	100	36	72	23	45	90	180
15	40	100	44	110	32	80	20	50	80	200
18	37	110	40	120	29	88	18	55	73	220
24	30	120	33	130	24	96	15	60	60	240
30	25	125	28	140	20	100	13	63	50	250
36	23	135	25	150	18	108	11	68	45	270
48	18	145	20	160	15	116	9	73	36	290
60	15	150	17	170	12	120	7.5	75	30	300

Arrangements of Pumps

1. **Single Single-acting Pumps** are seldom used; they cannot run without large air chambers. Exception: Deep well pumps.

2. **Single Double-acting Pumps,** either of the direct-acting or crank-and-flywheel type, are used extensively for boiler feeding, elevator service and in waterworks. They must be equipped with large air chambers and especial attention must be paid to keeping the water free from air.

3. **Duplex Double-acting Pumps,** either direct-acting or with crank and flywheel, deliver a very steady flow of water, and are used for all services. Only small air chambers required.

4. **Triplex Single-acting Pumps** deliver an almost uniform flow of water. This arrangement is used in power pumps and triple-expansion crank-and-flywheel pumps.

5. **Triplex Double-acting Pumps.** The triplex double-acting arrangement is preferred for horizontal pumps because the load on the frame and shaft is only one-half of that of a single-acting pump.

Fig. 6 shows the irregularity in discharge of types (2), (3) and (4) when driven by a crank motion.

Methods of Driving

Belt-driven Pumps are built as horizontal or vertical, duplex, triplex or quintuplex pumps; very small house pumps as single single-acting. The shaft carries one or two gear wheels driven by a pinion on a countershaft carrying a pulley.

Steam-driven Pumps are either direct-acting or operated by crank and flywheel. **Direct-acting pumps** are used extensively for all purposes, especially as duplex pumps with simple cylinders (so-called high-pressure pumps), compound, or triple-expansion steam-ends. Advantages: Low initial cost, small attendance, few repairs, reliability, safety of operation, flexibility and inexpensive foundations. Disadvantages: High steam consumption, difficulty in regulating the length of stroke, slow speed. Regulation is effected either by hand, by a pressure regulator or by a Mason speed

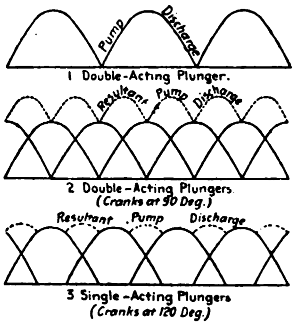


Fig. 6.—Discharge of Crank Pumps.

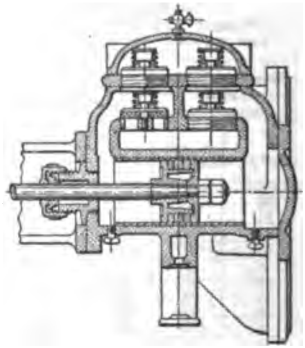


Fig. 7.—Submerged Piston Pump.

governor. An automatic stop valve is necessary to stop the pump if the discharge main breaks.

Crank-and-flywheel Pumps are power pumps driven from the cross head of a steam engine. The governor should not be very sensitive. Watt's slow-speed governor with heavy balls is considered the best, since extreme uniformity is not desired and the load is constant. For the same reason, engines with hand-operated cut-off and a throttling governor give excellent satisfaction. Advantages: High economy, high speed. Disadvantages: High initial cost, expensive foundations, high cost of attendance and repairs.

Pump Ends

Submerged Piston Pumps (Fig. 7) are pumps with both valve decks located above the piston. Small pumps, such as boiler feed and tank pumps, also wet vacuum pumps, small or large, are of this type.

General Service and Boiler Feed Pumps have a ratio of piston areas of about 2¼ to 1, and are either fitted with packed pistons and driven linings, packed pistons and removable linings, or plunger and ring. The plungers are usually cast iron and the rings bronze. Pumps are designed for 150 lb. water

pressure. Sizes and capacities of boiler feed pumps are given in Table 5. To find the size of pump to supply a given boiler, multiply the boiler horse power by 45, which will give the pounds of water required per hour.

Table 5. Boiler Feed Pumps

Size, in.	Rev. per min.	Gal. per min.	Lb. per hour	Size, in.	Rev. per min.	Gal. per min.	Lb. per hour
2 × 1¼ × 2¾	40.0	2.2	1,075	8 × 5 × 10	24.0	76	38,000
3 × 2 × 3	40.0	6.0	3,000	9 × 5¼ × 10	24.0	84	42,000
3½ × 2¼ × 4	37.5	9.6	4,800	10 × 6 × 10	24.0	110	55,000
4½ × 2¾ × 4	37.5	14.4	7,200	12 × 7 × 12	22.5	170	85,000
5¼ × 3½ × 5	36.0	28.0	14,000	14 × 8½ × 12	22.5	250	125,000
6 × 4 × 6	32.5	39.6	19,800	16 × 10¼ × 12	22.5	360	180,000
7½ × 4½ × 6	32.5	50.6	25,300	18 × 12 × 12	22.5	500	250,000
7½ × 4½ × 10	24.0	60.0	30,000	20 × 14 × 12	22.5	690	345,000

Tank Pumps are fitted with packed pistons, either with driven or removable linings. Ratio of piston areas, 1½ to 1. **Wet vacuum pumps** are used for removing air and water from jet or surface condensers, and operate at moderate speeds (see Table 4). They are of the submerged piston type and are very effective on account of the absence of clearance. Deaerated water fills the pump barrel and ports and acts as a water piston. They are suitable for 27 in. vacuum.

Straightway Pumps (Fig. 8) are pumps with the suction valve deck below and the discharge valve deck above the plunger or piston. They are fitted with piston and removable sleeve or with plunger and ring, the latter type being used only with clean water. For **general service** they are designed for 150 lb. pressure, and for **low service**, for 75 lb. On account of the large clearance spaces this type is not suitable for wet vacuum pumps.

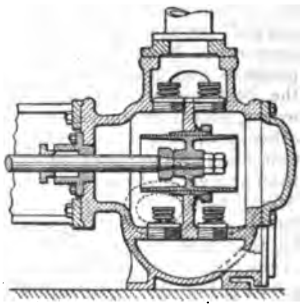


FIG. 8.—Straightway Pump.

Table 6. Sizes and Capacities of National Standard Fire Pumps

Pump sizes, in.			Approximate ratio of piston areas	Capacity at 100 lb. pressure at pump			Boiler power required		Full speed	
Steam cyl.	Water cyl.	Stroke		Number of 1⅛-in. streams	Nominal gal. per min.	Actual gal. per min.	Horse power	Steam pressure at pump, lb. per sq. in.	R.p.m.	Piston speed, ft. per min.
14 ×	7 ×	12	4:1	2	500	483	100	40	70	140
14 ×	7¼ ×	12				520				
16 ×	9 ×	12	3:1	3	750	806	115	45	70	140
18 ×	10 ×	12	3:1	4	1000	999	150	45	70	140
18½ ×	10¼ ×	12				1050				
20 ×	12 ×	16	2:1	6	1500	1655	200	50	60	160

Data on National Standard pumps for **fire service, as developed by John R. Freeman in 1890, are given in Table 6. These pumps are designed to be** rust-proof, reliable and durable; they have extra-large valve areas and are fitted with hose nozzles. (See Specifications published by National Board of Fire Underwriters.)

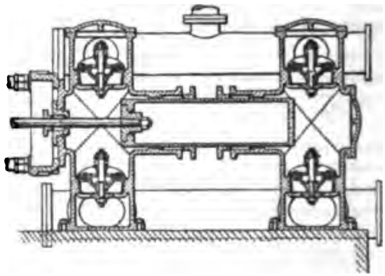


FIG. 9.—Outside Center-packed Pump.

The plunger-and-ring type is suitable for **elevator service** if water is circulating in the system. Elevator pumps operate non-condensing because no cooling water is available, and must be provided with a pressure regulator to adjust the speed to the varying demand for water. There are two methods of regulation: (1) Regulation between minimum and maximum speed, and (2) automatic stop and start.

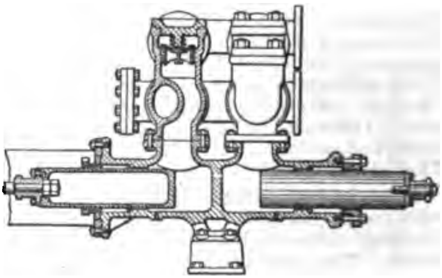


FIG. 10.—Outside End-packed Pump.

Outside Center-packed Pumps (Fig. 9) may be of the straightway or the submerged plunger type. The former are built for 300 lb. pressure and are considered the best pumps for waterworks, elevators, mines, and other general service. They cost about 50 per cent. more than plunger-and-ring pumps. Advantages: The plunger packing is accessible and can be tightened while pump is in operation; plungers can be watched and cutting can be detected; no leakage past the plunger is possible. Disadvantages: Greater friction in the large stuffing boxes. The mechanical efficiency is in consequence about 5 per cent. lower than in plunger-and-ring pumps.

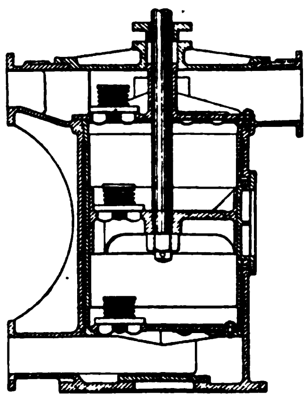


FIG. 11.—Bucket Pump.

Outside End-packed Pumps (Fig. 10) are used for pressures over 300 lb. per sq. in., and are subdivided into small parts. Cast iron is used for 1000 lb. pump pressure, cast steel for 3000 lb. and forged steel for pressures over 3000 lb. These pumps are used for mine service, accumulators, hydraulic presses, etc. The pump barrels are plain cylinders, and the valves (usually of the conical wing type) are located in separate pots containing either one valve, one pair of valves, or two pairs of valves. For moderate pressures large pots may be used containing a greater number of valves. The water should come to the pump under a head, as a slight amount of air

compressed to such high pressures would, in expanding, destroy the suction action.

Bucket Pumps are always vertical. A bucket is a piston containing valves. Bucket pumps are excellent **for condenser service** (Fig. 11), and for such work should be provided with three sets of valves, viz., foot valves, bucket valves and discharge valves. The bucket may be packed and is accessible through a hand hole in the cylinder wall. They are generally arranged duplex and the two sides are connected by a beam, giving the effect of a double-acting pump. They may be driven by a single double-acting steam cylinder, a duplex steam end or a compound steam end. The displacement of bucket should be from 0.5 to 0.7 cu. ft. per lb. of steam coming from the (surface) condenser.

In the **Edwards air pump** (Fig. 12) the piston has a conical undersurface that exactly fits the lower cylinder cover. There are no bucket or foot valves. On the up stroke the condensate flows into the space below the piston while the water of the previous stroke is lifted and delivered through head valves into an upper chamber. On the down stroke the piston almost reaches the bottom cover and forces the condensate through ports into the barrel and above the piston. These pumps may be run at higher speeds than older forms. An Edwards pump with three 14-in. barrels (stroke, 12 in.), single-acting, at 150 r.p.m. will handle 45,000 lb. of steam per hour from a surface condenser, which is equivalent to a cylinder volume of 0.64 cu. ft. per lb. of boiler feed water.

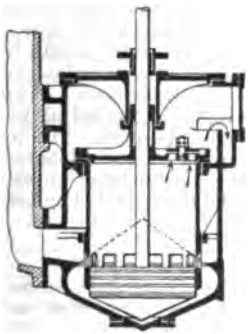


FIG. 12.—Edwards Air Pump.

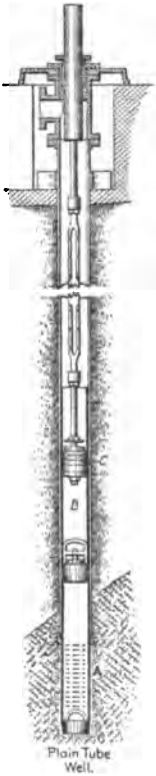


FIG. 13.

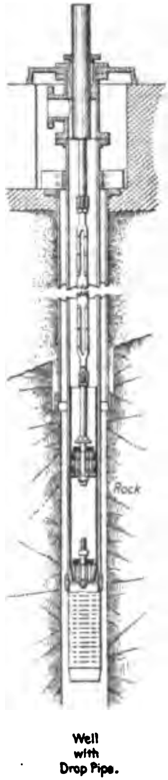


FIG. 14.

Deep-well Pumps.

Deep-well Pumps of the bucket type are designed for use on non-flowing wells where the water does not stand within suction distance. Two methods are employed: In the **plain tube well** (Fig. 13) the well pipe with open ends is sunk to the proper depth where a sufficient water-bearing stratum is penetrated. The strainer *A* is then lowered to bottom of the well and the well pipe drawn back far enough to expose the slotted portion. The working barrel *B* is then lowered into the gum packer on top of strainer and tapped firmly into place. The top of the well pipe is provided with a tee for discharge connection. The bucket *C* is driven by means of wooden rods of convenient length (18 ft.), coupled together. The top of the well pipe is provided with a stuffing box through which a brass plunger works. If the well discharges at the surface this plunger is made one-half the area of the bucket, resulting in a uniform discharge. If the pump discharges into an elevated tank the top plunger is made larger so as to throw the effect of the additional head upon the plunger and thereby assist in the up stroke. The **well with drop pipe** (Fig. 14) differs from the plain tube well in that the brass working barrel is attached to a pipe and is lowered into the well pipe. This is a better arrangement in case of a small supply, as it allows a head of water to collect above the bucket during the down stroke. It is used in wells that are either wholly or partly drilled into rock and in wells where the casing does not extend low enough to reach the water supply; also in old wells with defective or bent casing.

Vertical End-packed Plunger Pumps are only used as multiplex power pumps or with triple-expansion crank-and-flywheel engines. In the latter case the plunger is weighted to counterbalance one-half of the water pressure so as to equalize the loads on the up and down strokes.

Vertical Double-acting Pumps are used with duplex direct-acting steam ends and cross-compound crank-and-flywheel engines, or are power-driven. They may be fitted with either piston, plunger-and-ring or outside center-packed plungers. A design in which the **valves** are **arranged around** the **plunger** is excellent for moderate sizes. For large capacities and heavy pressures the castings become too large, and must be subdivided into several chambers which are connected together by pipes. When the **pump barrels and valve chambers** are **separate,** one valve chamber is placed in each corner for moderate pressures, and one or more for higher pressures. Either piston or outside center-packed plungers may be used. **Differential plunger pumps** have a large piston and a small single-acting plunger. The pump is single-acting on the suction side and double-acting on the discharge side. Plunger area = ½ piston area, so as to give uniform discharge. Irregularity in load resulting from single-acting suction can be counteracted by balancing weights.

Pump Valves

Disk Valves (Fig. 15). The valves generally used are plain rubber disks 1½ to 9 in. in diam. The 4½-in. size is considered the most suitable, considering quiet working and initial cost. The valve seat is screwed or driven into the valve deck (see Fig. 20). In the latter case the seat should be expanded below the deck to withstand the hammering of the valve against the guard. The stem is screwed into the hub of the seat on a fine taper. A number of ribs connect the hub and rim and form a support for the rubber disk. The valve disks are made of **rubber** of a **composition** suitable for the service. For wet vacuum pumps, soft rubber

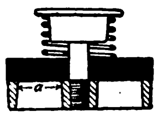


Fig. 15. Disk Valve.

Fig. 16.—Metal Valve Disk.

is used; for low service pumps (75 lb.), medium soft; for general service pumps (150 lb.), medium hard; and for pressure of 300 lb., hard rubber. Where hot water is handled a special composition is employed. For pressures over 125 lb., **metal valve disks** (Fig. 16) are preferable to those of rubber.

The **thickness of the valve disk** (in.) $= t = 0.05\,a\sqrt{p}$, where a = span, in. (see Fig. 15), and p = pressure, lb. per sq. in.

The **area of the valve seat** of a plain disk valve $= F = pA/b$, in which F = area of seat, sq. in.; p = pressure on the back of valve, lb. per sq. in.; A = area of valve on which p is acting, sq. in.; and b = bearing pressure of valve on seat, lb. per sq. in. = 10,000 for steel, 2000 for red brass, bronze and gun metal, 2500–3000 for phosphor bronze, 1000 for cast iron, 750 for leather and 375 for rubber. For high-speed pumps these values should be reduced to allow for shock. The **valve area** is the free area through the valve seat, and equals the area at the inner circle minus the areas of ribs and hub. The water velocity through this valve area should not be over 222 ft. per min., which requires a valve area of 45 per cent. of the plunger area at 100 ft. piston speed. Frequently areas of 50 per cent. and 60 per cent. are demanded by water-works' specifications, corresponding to velocities of 200 and 167 ft. per min., respectively.

The **number of valves** N of a given size required in a pumping engine, according to Nagle (*Trans. A. S. M. E.*, vol. 31, p. 974) and Bach, is $N = QV_m/CLv_mk$, where Q = area of pump plunger, sq. in.; V_m = maximum velocity of plunger, ft. per sec. (= 1.6 × mean plunger velocity for flywheel pumps); C = net circumference of valve seat (subtracting parts occupied by ribs), in.; L = lift of valve (= 0.1 to 0.2 × outside diam. D of valve seat), in.; v_m = maximum velocity of water at lift L, ft. per sec.; k = coefficient of contraction at the point of discharge with a given lift.

The **spring pressure** per sq. in. of inside seat area usually runs from 0.3 to 0.6 lb. at the beginning of the lift, and is about 40 per cent. of the pressure when the valve is at full lift. This final pressure, plus the weight of the valve in water, is designated by p in Table 7. This quantity is the determining factor for the velocity of the issuing stream. The relation of v_m to p and the ratio of lift to valve diam., according to Bach's experiments, are given in Table 7. The **discharge area** of the valve = CLk.

Table 7. Pump Valve Ratios

$\frac{L}{D}$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.50
k	0.65	0.60	0.56	0.53	0.50	0.47	0.44	0.41	0.37
$\frac{100p}{v_m^2}$	0.69	0.72	0.74	0.77	0.80	0.83	0.86	0.89	0.92

Approximate Rules for Spring Pressures on Disk Valves. For discharge valves make p = 0.005 to 0.01 × water pressure, with a maximum of 5 lb. per sq. in. For a suction valve under suction lift, p = 0.25 to 0.5 lb. per sq. in. For a suction valve under suction head, p = 0.5 to 1 lb. per sq. in.

Leather-faced Valves have the form shown in Fig. 17. **Conical wing valves** (Fig. 18) are used for high pressures because they can be ground in more tightly than flat valves. As regards operation they are inferior to flat disk valves because they must lift higher to give the same opening. Lift $h_1 = h/\cos a$ (see Fig. 18). For a = 45 deg., $h_1 = 1.41h$, or the conical valve must lift 41 per cent. higher than the flat disk and therefore close with a greater shock. **Leather-faced wing valves** are made as shown in Fig. 19.

FIG. 17. Leather-faced Valve.

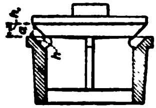


FIG. 18. Conical Wing Valve.

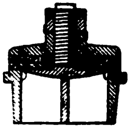
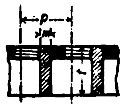


FIG. 19. Leather-faced Wing Valve.



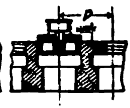


FIG. 20. FIG. 21 Valve Decks.

Valve Decks are perforated plates, and can be calculated by the formula, $t = cd\sqrt{Pp/m}$, where t = thickness, in.; d = diam. of round plate or smallest dimension of rectangular plate, in.; P = pressure, lb. per sq. in.; p = pitch = distance between centers of valves, in.; m = thickness of metal between the valves, in.

	Rectangular	Square	Round
For a plain valve deck (Fig. 20) c = ...	0.013	0.009	0.008
For reinforced valve deck (Fig. 21) c = ...	0.010	0.007	0.006

Mechanically Operated Valves. In a crank-and-flywheel pump the valve lags behind the plunger so that the valve is open a certain amount at the end of the stroke. Attempts to close the valve by mechanical means have not met with success, resulting in breakages. To overcome this difficulty springs are inserted which produce an extra-heavy spring pressure at the end of the stroke, thus reducing the shock greatly without throwing too heavy a load on the valve during the period of opening. Pumps with mechanically operated valves were developed by Prof. A Riedler and are successfully used abroad, especially for pumping water carrying large impurities, such as sewerage, where a high valve lift is desired.

Valves for High-speed or Express Pumps. The attempts at producing pumps with high piston speeds have not led to very satisfactory results. The valve area remains the same for a given capacity. The reduction in the size of the plunger reduces the plunger load without cheapening the frame, which, owing to the high speed, must be made extra strong and rigid. Fig. 22 shows a design of valve for a high-speed pump. Rubber rings are used in place of springs, giving a very strong spring pressure and a very low lift with high velocity through the discharge area. Table 8 (due to Hæder) gives the possible speeds in relation to the suction lift.

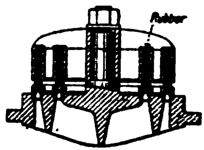


FIG. 22.—Valve for High-speed Pumps.

Table 8. Possible Rev. per Min. of High-speed Pumps
(Temperature, 70 deg. fahr.)

Suction lift, ft.	Stroke in inches							
	4	5	6	7	8	10	12	14
0	600	500	435	380	325	265	235	210
3	565	480	415	360	300	250	220	200
6	535	445	390	340	280	235	210	185
9	500	420	360	315	265	220	195	175
12	460	385	335	290	250	205	180	160
15	420	350	305	265	225	185	165	145
18	370	310	270	235	200	165	145	130
21	320	270	240	200	170	140	125	110
24	255	215	185	160	135	110	100	85

Ball Valves for pumps operating in impure liquids, paper pulp, etc., are made to the form shown in Fig. 23. The balls are made of bronze, and when large, of rubber with lead or iron cores. The diam. d_k of the ball should be from 1.2 to 1.6 times the diam. d of the outlet. The angle a should be equal to or less than 45 deg. The lift of the valve should be from 0.10d to 0.25d, while the mean velocity of the liquid through the valve should be from 3 ft. per sec. for suction valves to 8 ft. per sec. for discharge valves, determined by the equation $QV_m = Lv_mC$, in which the symbols have the meanings given on p. 1491. **Annular valves** are shown in Fig. 9.

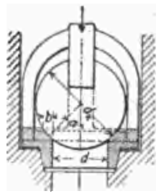


FIG. 23.—Ball Valve.

Clack or Flap Valves. Figs. 24 and 25 show leather-faced flap valves provided with strainers, for pump suction pipes. The hinge shown in Fig. 24 is of metal, while that in Fig. 25 is of leather. When the leather forms its own hinge, it is liable to rupture. The **size** of clack or flap valves depends on the shape of the seat opening, which is determined for a stated velocity of flow of the liquids by taking into consideration the narrowing of the opening by ribs, wings and screws, and by contraction. In the case of rubber clacks or flaps the grid bars will take up from 0.3 to 0.5 the total area of the opening. The coefficient of contraction is about 0.9 for circular, and 0.8 for rectangular, openings. The **lift** h is dependent on the method of hinging and on the shape of the opening. If the passage is **rectangular,** with a width a and a length c, and the hinge is parallel to c, then the **cross-section** through which the liquid can escape when the clack is open (at an angle b with the seat) is approximately equal to $h(c + a \cos b)$, where $h = a \sin b$.

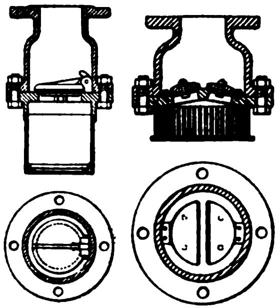


FIG. 24. FIG. 25.
Leather-faced Flap Valves.

The volume of liquid Q (cu. ft. per sec.) flowing through the valve with a velocity v (ft. per sec.) is expressed by the equation

$$Q = kvh(c + a \cos b)/144,$$

where h, c, and a are in in. and k = 0.85 to 0.90. For practical purposes, assume that v is equal to the velocity through the unobstructed cross-section ac.

For a passage of **circular cross-section** (diam. = d), with the valve hinged as in Fig. 24,

$$Q = \frac{\pi d}{4 \times 12}(1 + \cos b)\frac{hv}{12} = \frac{\pi d^2 v}{4 \times 144} \text{ approximately.}$$

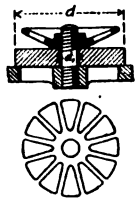


FIG. 26.—Rubber Flap Valve.

Fig. 26 shows a circular rubber flap valve, a type which can only be made with a thin and very soft rubber disk. Such valves will generally be found to bend on one diameter.

The **Gutermuth flap valve** for pumps consists of a thin sheet of tempered steel or bronze, one end of which is coiled to form a spiral spring hinge. Fig.

27 shows such valves mounted in a laterally removable cylindrical casing of and inserted between the suction and compression air chambers *C* and *D A* the pump.

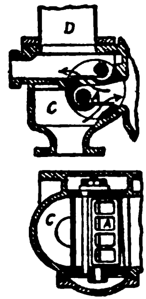


Fig. 27. Gutermuth Flap Valve.

Materials for Pumps. The **allowable stresses** (lb. per sq. in.) in pumps are lower than in most machinery owing to the shocks and water hammer, and the tensile stress may be taken as follows: Cast iron, 1500 to 1800; malleable iron, 3000; steel castings, plain, 8000; complicated, 5000; bronze (government metal), 3000; Tobin bronze, 5000; steel, 7000; forged steel, 10,000. The materials to be used for pump parts vary with the liquids handled, and are specified in Table 9.

Steam Ends

Direct-acting Pumps have the steam piston connected to the pump piston by means of a rod without the intervention of a crank motion. Here the steam end forms an important part of the machine and must be given especial care in the design. As there is no moving mass to store up energy, no early cut-off is possible.

Table 9. Materials for Pump Parts for Handling Various Liquids

Liquid pumped	Material for pump parts	Liquid pumped	Material for pump parts
Ammonia	All iron*	Magma	Brass fitted, large openings‡
Beer	Brass fitted†	Milk	All bronze
Bichloride of mercury	All iron	Milk of lime	All iron
Brine	Brass fitted	Molasses	Brass fitted, large openings
Calcium brine	Brass fitted	Naphtha	Regular fitted[1]
Calcium chloride	All iron	Oil	Brass fitted
Cane juice	Brass fitted	Salt water	Brass fitted
Caustic soda	All iron	Sewage	Brass fitted, large openings
Chlorate of lime	Regular fitted[1]	Soap	All iron
Citric acid	Regular fitted[1]	Sulphuric acid	All bronze or wood-lined cast iron, no zinc
Copper liquors	Brass fitted		
Glue (hot)	Brass fitted	Syrup	Brass fitted
Glycerine	Brass fitted	Tan liquor	All bronze
Hydrochloric acid	Lead lined, no zinc	Tar	All iron, large openings
Hyposulphite of soda	Lead lined, no zinc	Vinegar	All bronze
Lard (hot)	Brass fitted	Whiskey	All bronze

[1] Regular fitted: cast-iron plunger or piston in bronze sleeve, steel rods, rubber or bronze valves.
* Cast iron, steel or malleable cast iron only, bronze not to be used.
† Rods and pistons to be of bronze; stuffing-box throats and glands to be bronze-lined.
‡ Valve seats must not be obstructed by ribs; ball valves or clapper valves are suitable.

Single Steam Pumps. The steam end of a single steam pump consists of four elements: the main piston, the main valve, the auxiliary piston and the auxiliary valve. They may be combined in various ways or split up into several units. The pumps may be fitted with external or internal valve motions and the valves may be of the "D" or the "B" type. Fig. 28 shows a plain D-valve with neither inside nor outside lap, to avoid the danger of stoppage. The B-valve, Fig. 29, takes steam at the opposite end from that of the D-valve. With the D-valve the piston follows its valve; with the B-valve the piston travels in the opposite direction; in other words, the B-valve reverses the mo-

Fig. 28.—D-Valve.

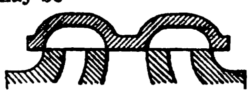


Fig. 29.—B-Valve.

tion, while with a D-valve this must be accomplished by a special reversing mechanism. Figs. 30–32 show the three combinations diagrammatically.

Duplex Steam Pumps. In a duplex pump the same four elements are found as in a single pump, but they appear as two main pistons and two main valves. The piston of one side is mechanically connected to the valve of the opposite side. As it is desirable to have both cylinders alike, D-valves are used, and the reversal of the motion is accomplished by a reversing mechanism. Fig. 33 shows a common design of the steam end of a duplex pump. The valve is not rigidly connected to the crank pin but a certain clearance or lost motion is allowed in order to retard the movement of the valve. This gives the desired pause at the end of the stroke. Figs. 33 and 34 show common types of lost-motion devices, inside for small and outside for large pumps. The following formulæ for valve motion (see Fig. 34) are based on the width of the port A. The port areas (= width × length) are calculated for a velocity of steam through the steam port of 6000 ft. per min., and 5000 ft. through the exhaust port; travel of crank pin $= L = 3.3A$; lost motion $= 2J = A = 0.3\,L$; clearance in chest to avoid danger by improper adjustment $= I = \frac{1}{2}L + J$; to avoid danger of reopening, make $\frac{1}{2}L + \frac{1}{8}$ in. $< (A + D)$; width of inner bridge $= B = (9A + 4)/16$; width of outer bridge $= D = B + \frac{1}{8}$ in.; width of main exhaust port $= C = 2A$. The sectional areas of the pipes are made equal to

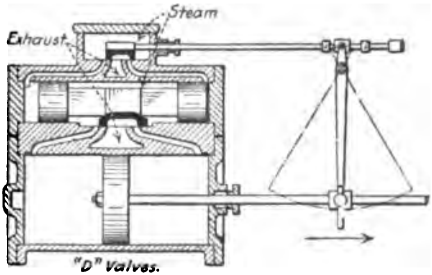


FIG. 30.

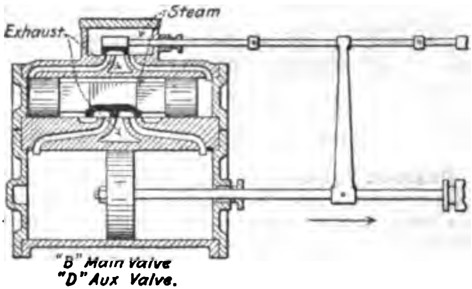


FIG. 31.

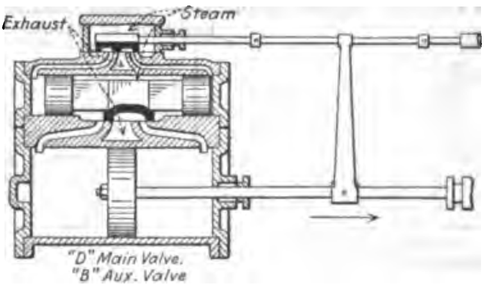


FIG. 32.

FIGS. 30–32.—Valve Combinations in Steam Ends of Single Steam Pumps.

the areas of the ports, using the next larger commercial size. The area of the main pipe is 1½ times that of the side pipe; the exhaust pipe is taken one size larger than the steam pipe.

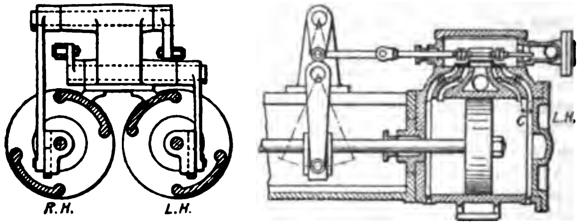


Fig. 33.—Steam End of a Duplex Pump.

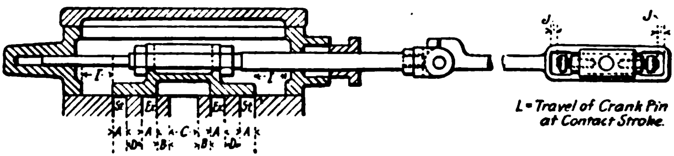


Fig. 34.—Dimensions for Duplex Valve Motions.

Cushion Valves. To stop the motion of the piston the cylinder is made with five ports (see Fig. 33), the outer ones being used for steam admission, the inner ones for exhaust. A communicating port between the two, provided with a regulating valve called the cushion valve or **dash relief valve,** offers a means for regulating the cushion. The cushion is the distance *c* (Fig. 33) from the cylinder head to the outer edge of the exhaust port. The cushion valve opening should be 0.5 per cent. of the area of the piston.

Table 10. Proper Amount of Steam Cushion for Pumps of Various Lengths of Stroke

Nominal stroke, in.	Contact stroke, in.*	Cushion, in.	Nominal stroke, in.	Contact stroke, in.*	Cushion, in.	Nominal stroke, in.	Contact stroke, in.*	Cushion, in.
3	3.5	0.625	8	8.75	1.25	18	19.5	2.25
4	4.5	0.750	10	11	1.50	24	26	2.75
5	5.75	0.875	12	13	1.75	30	32	3.125
6	6.75	1.125	15	16	2.00	36	38	3.5

* Extreme length of stroke possible = internal length of cylinder (from head to head) minus width of piston.

Compound Direct-acting Steam Ends. Single pumps and duplex pumps may be compounded by arranging a high- and a low-pressure cylinder tandem with each pump cylinder. No cross-compound arrangement is possible in a duplex pump, because there is no mechanical connection between the two sides, which must therefore be symmetrical and duplicates. No early cut-off is possible. On non-condensing engines the low-pressure cylinders are fitted with cushion valves; on condensing engines both cylinders should be so fitted on account of the poor cushioning effect of the low-pressure exhaust steam. The two high-pressure exhaust pipes are connected by a communicating port with valve for the purpose of straightening out the back-pressure line. This is called the **cross exhaust.**

Triple-expansion Direct-acting Steam Ends. In this arrangement the three steam cylinders are arranged tandem on each side. The high-pressure cylinders are equipped with cut-off valves for the purpose of regulating the length of stroke. Cushion valves are inserted in one of the larger cylinders and cross exhausts on the h.-p. and i.-p. exhaust pipes. A live-steam connection is made to the intermediate-pressure cylinder for starting.

The above-mentioned pumps all belong to the class of **low-duty** pumps, the term **high duty** being applied to pumps using an early cut-off in the steam cylinders.

Calculation of the Steam Ends of Direct-acting Pumps. Let a_s = area of steam piston, sq. in.; p = absolute initial pressure in cylinder, lb. per sq. in.; b = absolute back pressure in cylinder, lb. per sq. in.; L = plunger load, lb.; S = total pressure exerted by the steam, lb. $= a_s p_{si}$. Then $E_m S = L$, where E_m is the mechanical efficiency of the pump. Since no cut-off is possible in a direct-acting pump, the steam diagram is composed of plain rectangles. In **simple steam cylinder pumps** (so-called high-pressure pumps) $p_{si} = p - b$. In **compound pumps** $p_{si} = 2p - (p/R) - bR$, where R = ratio of the cylinder areas, the best value of this ratio being $R = \sqrt{p/b} \leq 4$. In **triple-expansion pumps** $p_{si} = 3p - (2p/\sqrt{R_1}) - bR_1$, where R_1 = area of low-pressure cylinder + area of high-pressure cylinder. The best values of these ratios are $R_1 = R^2 = \sqrt[3]{(p/b)^2} \leq 9$, where R = area of intermediate-pressure cylinder + area of high-pressure cylinder. In compound and triple-expansion pumps p is the absolute initial pressure in the high-pressure cylinder, and p_{si} is the mean effective pressure referred to the area of the high-pressure cylinder.

PERFORMANCE

Duty. Pumps are tested for economy by determining the duty in foot-pounds of work done by 1000 lb. of dry steam, or per 1,000,000 B.t.u. furnished by the boiler. Although the latter unit is more rational, it has not found favor among the manufacturers because it gives a slightly lower duty.

The work is found by measuring the amount of water pumped, and the height through which it is lifted. The former may be done either by direct measurement or, as recommended by the Committee, by calculating the plunger displacement, deducting a certain amount for slip.

The data and results should be reported in accordance with the form given herewith, adding lines for data not provided for, or omitting those not required, as may conform to the object in view. In the case of a pumping engine of the reciprocating class for which a record of the complete performance is desired, the additional engine data and results given in the Steam Engine Code may supplement those here given. In trials for maximum duty, air should be prevented from snifting into the pump cylinders. In all cases where air is thus admitted, the results should be corrected accordingly. For duration, starting and stopping, and records, see Steam Engine Code, p. 1756.

Data and Results of Duty Trial of Steam Pumping Machinery
(A. S. M. E. CODE OF 1915)

(For a short report the items designated by letters of the alphabet may be omitted.)

(1) Test of..........pump located at..........

To determine..........Test conducted by..........

Dimensions, etc.

(2) Type of machinery..
(3) Rated capacity in gallons per 24 hr..gal.
(4) Size of engine or turbine.......... (5) Size of pump..
(6) Auxiliaries (steam- or electric-driven)..
(a) Type and make of condenser equipment..
(b) Rated capacity of condenser equipment..
(c) Type of oil pump, jacket pump, and reheater pump, (direct or independently driven)..

Date and Duration

(7) Date........................ (8) Duration..hr.

Average Pressures and Temperatures

(9) Pressure in steam pipe near throttle by gage..lb.
(10) Barometric pressure..in.
(a) Steam chest pressure..lb.
(b) Pressure in receivers and reheaters by gage..lb.
(c) Pressure in turbine stages by gage..lb.
(11) Pressure in exhaust pipe near engine or turbine by gage..lb.
(12) Vacuum in condenser..in.
(a) Corresponding absolute pressure..lb.
(b) Absolute pressure in exhaust chamber..lb.
(13) Temperature of steam, if superheated, at throttle..deg.
(a) Normal temperature of saturated steam at throttle pressure..........deg.
(b) Temperature of steam leaving receivers, if superheated..............deg.
(14) Temperature of steam in exhaust pipe near engine or turbine..............deg.
(a) Temperature of circulating water entering condenser................deg.
(b) Temperature of circulating water leaving condenser................deg.
(15) Pressure in force main by gage..lb.
(16) Vacuum or pressure in suction main by gage........................in. or lb.
(a) Correction for difference in elevation of the two gages................lb.
(17) Total head expressed in lb. pressure per sq. in..lb.
(a) Total head expressed in ft..ft.

Quality of Steam

(18) Percentage of moisture in steam near throttle, or number of degrees of superheating..............................per cent. or deg.

Total Quantities

(19) Total water fed to boilers..lb.
(20) Total condensed steam from surface condenser (corrected for condenser leakage)..lb.
(21) Total dry steam consumed (Item 19 or 20 less moisture in steam)............lb.
(22) Total gal. of water discharged, by measurement..gal.
(a) Total gal. of water discharged, by plunger displacement, uncorrected....gal.
(b) Percentage of slip [100(Item 22a–Item 22)/Item 22a.]..........per cent.
(c) Leakage of pump*..gal.
(d) Total gal. of water discharged, by calculation from plunger displacement, corrected for leakage..gal.
(e) Total weight of water discharged, as measured..lb.
(f) Total weight of water discharged, by calculation from plunger displacement, corrected for leakage..lb.

*Leakage of an inside plunger may best be determined by removing the cylinder head and bolting a wide board over the lower part of cylinder end to form a dam, in which an overflow pipe is inserted. The plunger is then blocked (preferably at some intermediate point of the stroke), and water admitted behind it from the force main, at full pressure. The leakage is caught from the overflow pipe in barrels and measured. If possible, tests should be made with the plunger in various positions. If the cylinder head is difficult to remove, the leakage may be measured through one of the openings provided for inspecting the suction valves. Any leakage of valves should be remedied

HOURLY QUANTITIES

(23) Total water fed to boilers or drawn from surface condenser per hour............lb.
(24) Total dry steam consumed for all purposes per hour (Item 21 ÷ Item 8).......lb.
(25) Steam consumed per hour for all purposes foreign to main engine ....lb.
(26) Dry steam consumed by engine or turbine per hour (Item 24 − Item 25)......lb.
(*a*) Circulating water supplied to condenser per hour..................lb.
(27) Weight of water discharged per hour, by measurement.....................lb.
(*a*) Weight of water discharged per hour, calculated from plunger displacement, corrected..lb.

HOURLY HEAT DATA

(28) Heat units consumed by engine or turbine per hour [Item 26 × (total heat of 1 lb. of steam at pressure of Item 9, less heat in 1 lb. of water at temperature of Item 14)]..B.t.u.

INDICATOR DIAGRAMS

(29) Mean effective pressure, each steam cylinder..............................lb.
(*a*) Mean effective pressure, each water cylinder, if any.................lb.

SPEED AND STROKE

(30) Revolutions per minute..r.p.m.
(*a*) Number of single strokes per min......(*b*) Average length of stroke......ft.

POWER

(31) Indicated horse power developed......................................i.h.p.
(*a*) Brake horse power consumed by pump..................................h.p.
(32) Water horse power†..h.p.
(33) Friction horse power (Item 31 − Item 32)...............................h.p.
(34) Percentage of i.h.p. lost in friction...............................per cent.

CAPACITY‡

(35) Number of gal. of water discharged in 24 hr., as measured...................gal.
(*a*) Number of gal. of water discharged in 24 hr., calculated from plunger displacement, corrected...................................gal.
(*b*) Gal. of water discharged per min., as measured........................gal.
(*c*) Gal. of water discharged per min., calculated from plunger displacement, corrected..gal.

ECONOMY RESULTS

(36) Heat units consumed per i.h.p.-hr. ....B.t.u.
(37) Heat units consumed per water h.p.-hr..................................B.t.u.
(*a*) Dry steam consumed per i.h.p.-hr.......................................lb.
(*b*) Dry steam consumed per water h.p.-hr...................................lb.

EFFICIENCY RESULTS

(38) Thermal efficiency referred to i.h.p. [(2546.5 ÷ Item 36) × 100]........per cent.
(*a*) Thermal efficiency referred to water h.p. [(2546.5 ÷ Item 37) × 100].per cent.
(*b*) Mechanical efficiency [(100 × Item 32)/Item 31]................ per cent.
(*c*) Pump efficiency [(100 × Item 32)/Item 31*a*]..................per cent.

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before making the plunger test. Leakage of discharge valves will be shown by water passing down into the empty cylinder at either end when they are under pressure; and of suction valves by the disappearance of water which covers them. If valve leakage is found which cannot be remedied, the water thus lost should also be measured. One method consists in measuring the amount of water required to maintain a certain pressure in the pump cylinder when this is introduced through a pipe temporarily erected, no water being allowed to enter through the discharge valves of the pump.

† Water h.p. = ft.-lb. of work per min. ÷ 33,000. Work done per min., ft.-lb. = net area of plunger A (sq. in.) × total head H, lb. per sq. in. (= pressure on force main + pressure on suction main + pressure equivalent to the head or vertical distance between the centers of the two gages) × length of stroke S, ft. × number of single strokes per min., N; corrected for leakage.

‡ Capacity in gal. per 24 hr. = 74.8 ASN, corrected for leakage; in direct-connected engines, S = average length of stroke.

Duty

(39) Duty per 1,000,000 heat units..ft.-lb.

Work Done per Heat Unit

(40) Ft.-lb. of work per B.t.u. (1,980,000 ÷ Item 37).....................ft.-lb.

Sample Diagrams

(41) Sample indicator diagrams from each steam and pump cylinder................

Note. The items relating to indicator diagrams and indicated horse power are to be used only in the case of reciprocating machines. The i.h.p. on which the economy results are based is that of the main engine, given by Item 31.

Table 11. Probable Duty of High-pressure Direct-acting Steam Pumps

(Duty in millions of foot-pounds per 1000 lb. of steam. Steam cylinders lagged; 4.7 lb. wire-drawing)

Stroke, in.	Simple non-condensing, not jacketed, 16-lb. back press				Compound non-condensing, not jacketed, 16-lb. back press				Compound condensing, jacketed, 6-lb. back press				Triple-expansion condensing, jacketed, 4-lb. back press			
	Boiler pressure, lb. per sq. in.															
	50	80	120	150	50	80	120	150	50	100	150	180	70	120	160	200
10	15	18	20	20	21	25	29	30	35	40	43	44	64	76	80	82
15	18	21	23	24	24	30	34	36	41	47	50	51	73	87	92	95
18	20	23	25	25	26	32	36	38	44	51	54	56	78	93	98	101
24	21	25	27	28	28	35	40	42	46	54	58	59	83	99	104	108
36	23	26	29	29	30	37	42	45	50	58	62	63	88	105	111	114
48	25	28	31	32	33	40	46	48	53	62	66	68	93	111	117	121

Triple-expansion non-condensing pumps give approximately the same duty as compound condensing pumps.

Card Duty. The duty can be calculated approximately from the indicator diagram (either actual or theoretical):

$$\text{Card Duty} = 144{,}000pv, \text{ and Duty} = \text{Card Duty} \times E_sE_m$$

where p = mean effective pressure of whole steam end referred to the area of any one cylinder; v = specific volume of steam at the terminal pressure in this same cylinder, E_m = mechanical efficiency, and E_s = steam efficiency. Table 12 gives practical values for the steam efficiencies when the card duty is calculated from a theoretical diagram and is based on the high-pressure p and v. If the card duty is calculated from actual diagram, the steam efficiency must be divided by the diagram factor.

Cost of Pumping Engines, complete with foundations, piping and appurtenances per million gallons per 24 hr. capacity: Compound condensing, low duty, $2300; triple-expansion condensing, low duty, $2800; triple-expansion vertical, high duty, $4800.

Cost of Complete Pumping Stations (triple-expansion high-duty engines and high-pressure boilers) per million gallons per 24 hr., including a reserve:

Total water pressure, lb. per sq. in.	30	40	60	80	100	120	130
Cost	$6750	7000	7500	8000	8500	9000	10,000

These costs include building of a good quality of brick or stone, steel-trussed and slated roofs and adequate chimneys. Cost of land not included. With cheaper types of engines more boiler capacity is required, which should be taken into account.

Table 12. Steam Efficiencies of Direct-acting Pumps

Stroke, in.	Simple cylinders	Compound pumps			Triple-expansion pumps	
	Non-condensing	Non-cond.	Condensing		Non-cond.	Condensing
			Non-jacketed	Jacketed		
	Steam efficiency, per cent.					
4	37.5					
6	40.0	40.0	45.0			
8	42.5	42.5	47.5			
10	45.0	45.0	50.0	55.0	55.0	67.5
12	47.5	47.5	52.5	57.5	57.5	70.0
15	50.0	50.0	55.0	60.0	60.0	72.5
18	52.5	52.5	57.5	62.5	62.5	75.0
24	55.0	55.0	60.0	65.0	65.0	77.5
36	57.5	57.5	62.5	67.5	67.5	80.0
48	60.0	60.0	65.0	70.0	70.0	82.5

For mechanical efficiencies, see Table 3, p. 1484.

Rotary Pumps

Various types of rotary pumps may be distinguished:

1. Pumps with two rotating elements, called pistons, impellers or lobes. These impellers are designed with one, two or more teeth shaped for continuous contact at uniform velocity. In the Roots rotary pump (Fig. 35) two teeth are used. The revolution of the shafts and impellers traps the water between the lobes and the case and delivers it to the discharge side; the rolling together of the impellers prevents the return of the water. These pumps handle any liquid not containing grit under any head from 10 to 200 ft., with an efficiency which is claimed to range from 75 per cent. to 85 per cent. Pumps of this type give a continuous but not a uniform discharge and therefore require air chambers. (See *Trans. A. S. M. E.*, 1930, p. 387.)

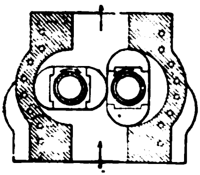


FIG. 35.—Roots Rotary Pump.

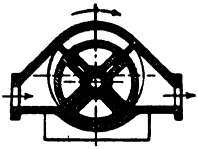


Fig. 36.

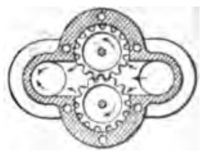


FIG. 37.—Gear-wheel Pump.

Rotary Pumps.

2. Pumps with one rotating element eccentrically located in the casing and provided with a number of packing blades which are free to slide radially in the rotor (Fig. 36). The blades can never wear down to a tight fit owing to the variation of the curvature. Either 3 or 4 blades may be used and they must be arranged so as to avoid short-circuit losses. Packing strips cannot be used at the ends of the rotor and blades, and, consequently, there is considerable slip. These pumps have a uniform discharge, but the speed is limited by the effect of the reciprocating blades.

3. Gear-wheel pumps for high rotative speeds (Fig. 37). Volumetric efficiencise range from 90 to 95 per cent. (less for thin liquids); overall efficiencies 50 to 60 per cent. for small and up to 70 per cent. for large pumps. Can be built for pressures up to 350 lb. per sq. in.; capacities from ⅛ to 60 gal. per min.

Table 13. Results of Duty Trials of Large Pumping Engines

Number of engine	1	2	3	4	5	6	7	8
Type	Triple-expansion, vertical, crank and flywheel					Hor. high-duty direct-acting		Quad. exp. vert. crank and flywheel
Extent of jacketing: Cylinders (C.), Receivers (R.), Heads (H.)	C. R.	C. R. H.	C. R.	C. R.	C. R. H.	C. R.	C. R.	C. R.
Date of test	1893	1895	1895	1896	1898	Nov., '08	Sept. '09	1900
Diameters of steam cylinders, in.	28, 48, 74	13.8, 24.4, 39	28, 48, 74	37, 63, 94	29, 52, 80	21, 33, 60	21, 33, 60	19.5, 29.5, 49.5, 57.5
Diameters of water plungers, in.	32	17.5	36	42	33	29.5	24.5	14.75
Number, single- or double-acting	3-SA	3-DA	3-SA	3-SA	3-SA	2-DA	2-DA	2-DA
Stroke, in. (S.E. = steam end; P.E. = pump end.)	60	S. E. 60 P. E. 48	60	60	60	36	36	42
Capacity, million gallons in 24 hr.	18	20	24	30	20	14.4	10	6.23
Total head, lb.	70.4	59.4	53.4	86.1	88.7	91	100	260
Piston speed, ft. per min.	203	607	210	208	215	138	149	256
Steam pressure at throttle, gage	122	176	125	167	156	150; 209°S.H.	125	200
Cut-off in h.-p. cylinder, decimal part of stroke	0.337	0.384	0.338	0.323	0.315	0.36	0.325	0.52
Total number of expansions by vol.	20.4	21	20.3	19.6	23.8	22.5	25	17.5
Low-pressure, terminal, abs., lb. per sq. in.	5.3	6.9	5.8	7.4	6.4	6.25	5.4	8
M.e.p. ref. to area of l.-p. cylinder	21.77	26.36	21.03	27.19	23.65	23.62	17	35.47
Back pressure, absolute, lb. per sq. in.	1.6	1.5	2.8	2.2	2.5	2	2	1.5
Indicated horse power	573.9	575.7	573.7	1185.5	775.5	560	435	712.2
Water horse power	515	480	518	1050	715	530	405	659
Mechanical efficiency (vol. eff. not included)	90.8	89.5	89.8	94.9	95.4	96	95	93.9
Steam condensed in jackets and receiver, per cent.	9.2	10.5	10.2	5.1	4.6	(Superheated steam)	....	....
Duty in ft.-lb. per 1,000,000 B.t.u.	137	141.9	129.7	135.4	150.1	150*	127.5*	163
Duty in ft.-lb. per 1,000 lb. steam	154	154.9	142.4	152	167.8	177.5	136.5	150.3
Card duty based on l.-p. terminal = 144,000 pv.	217	205	196	199	197	203	167	‡
Duty efficiency, † per cent.	71	75.5	72.5	77	85	87.5	81.5	....

Builders: Nos. 1 and 3, E. P. Allis Co.; No. 2, E. D. Leavitt, Jr.; No. 4, Lake Erie Engineering Works; No. 5, Snow Steam Pump Works; Nos. 6 and 7, Henry R. Worthington; No. 8, Nordberg Manufacturing Co. Locality: No. 1, Milwaukee; No. 2, Chestnut Hill, Mass.; No. 3, Detroit; No. 4, Buffalo; No. 5, Indianapolis; No. 6, Montreal; No. 7, Fall River, Mass.; No. 8, Pittsburgh. Tested by: Nos. 1 and 8, Prof. R. C. Carpenter; No. 2, Prof. E. F. Miller; Nos. 3 and 4, G. H. Barrus; No. 5, Prof. W. F. M. Goss; No. 6, W. H. Laurie; No. 7, A. T. Safford.

* Estimated and based on feed water at 150 deg. fahr.

† Includes mechanical efficiency, steam efficiency and allowance due to compression.

‡ Cannot be calculated, as the heaters are fed by steam taken from the cylinders.

CENTRIFUGAL PUMPS

BY

L. C. LOEWENSTEIN

REFERENCES: Loewenstein and Crissey, "Centrifugal Pumps," Van Nostrand. Greene, "Pumping Machinery," Wiley. Lorenz, "Neue Theorie und Berechnung der Kreiselräder," Oldenbourg, Berlin.

A centrifugal pump consists of one or more impellers (wheels carrying blades) mounted on a shaft. These impellers are surrounded by sets of discharge vanes (stationary reversed nozzles) or by a spiral casing. The rotating impellers generate centrifugal force, which is utilized for the compression of liquids and for transmitting them against resistance. The discharge vanes, or the spiral casing, serve to convert the high velocity of the liquids as they leave the impeller into additional pressure.

The centrifugal pump and the centrifugal compressor differ only in the kind of fluid that each handles. Consequently, much of the subject matter under centrifugal pumps applies to centrifugal compressors as well, and *vice versa*.

Classification. Centrifugal pumps are classified

1. According to the magnitude of pressure developed: (*a*) Low-pressure pumps for heads of 50 ft. or less. (*b*) High-pressure pumps for heads above 50 ft.

2. According to position of shaft: (*a*) Vertical pumps, having their shafts vertical. (*b*) Horizontal pumps, having their shafts horizontal.

3. According to the number of inlets per impeller: (*a*) Single-inlet, in which the liquid enters each impeller on one side or face only. (*b*) Double-inlet, in which the liquid enters each impeller on both sides in opposite directions.

4. According to the direction of the impeller tips at impeller exit: (*a*) Radial discharge, (*b*) backward discharge and (*c*) forward discharge, according as the impeller tips at exit are radial, bent backward or bent forward, respectively, with reference to the direction of rotation.

5. According to the number of stages in the pump: (*a*) Single-stage pumps, in which the required head or pressure rise is generated by a single impeller. (*b*) Multi-stage pumps in which the required head or pressure rise is generated by two or more impellers in series.

Low-pressure pumps are usually built without a suction pipe, being placed below the level of the water supply. They are also generally built without discharge vanes, and are designated as **volute pumps.** Vertical pumps are chiefly used in mine shafts, and for small heads are built without a casing. Single-inlet impellers are easier to construct, but they are subject to an axial thrust in the direction of the liquid flow. Double-inlet impellers are practically free from axial thrust, but the leading of the liquid from stage to stage in multi-stage pumps is somewhat complicated. The backward-discharge impeller is the one most commonly used, especially where the pump is liable to be suddenly overloaded. The radial-discharge impeller also finds wide application, particularly in high-pressure pumps.

THEORY

Fundamental Equation. Referring to Fig. 1, the water enters the impeller inlet with an absolute velocity (in space) w_e, and leaves the impeller with an absolute velocity w_a. The inner peripheral velocity of the impeller is u_e, and the outer peripheral velocity u_a. All velocities are in ft. per sec.

Let H_b be the theoretical head (in ft.) against which the pump would deliver if there were no losses. Then $H_b = (1/g)(u_a w_a \cos d_a - u_e w_e \cos d_e) = (1/2g) \times (u_a^2 - u_e^2 + w_a^2 - w_e^2 + v_e^2 - v_a^2)$, where v_e and v_a are the inlet and exit relative velocities, respectively.

Axial Impeller Inlet. Unless special inlet guide vanes are provided to introduce the water into the impeller at a certain angle (sometimes done in high-pressure pumps to reduce inlet friction losses), the water approaches the impeller in an axial direction. In that case $d_e = 90°$; $v_e^2 = w_e^2 + u_e^2$, and $H_b = (1/g)(u_a w_a \cos d_a) = (1/2g) \times (u_a^2 - v_a^2 + w_a^2)$. Of this, $w_a^2/2g$ is available for the discharge vanes or the volute casing.

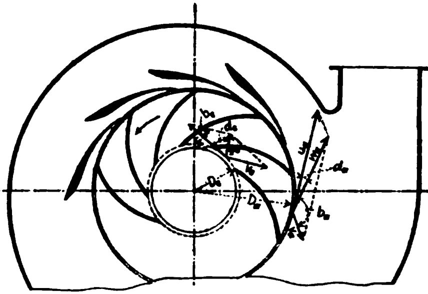


FIG. 1.

Radial Impeller Exit. For radial impeller exit, or $b_a = 90°$, $w_a^2 = u_a^2 + v_a^2$. If, in addition to this, $d_e = 90°$, as is usually the case, the theoretical head is $H_b = u_a^2/g$, while $(u_a^2 + v_a^2)/2g$ is available for the discharge vanes or the casing.

Pressure, Quantity, and Impeller Exit Angle. For radial impeller discharge ($b_a = 90°$) the pressure head H_b is independent of the quantity Q of liquid passing through the pump. For forward discharge, $b_a < 90°$, H_b increases with Q. For backward discharge, $b_a > 90°$, H_b decreases as Q increases. A backward-discharge pump is thus in a way self-regulating, being unlikely to overload its driver under circumstances where Q is liable to vary.

Hydraulic Losses. The theoretical head H_b is never realized on account of the hydraulic losses in the pump. A certain part is always wasted in friction losses, in losses due to sudden changes in velocity or direction of flow and, finally, in the discarded velocity head at the pump exit. These losses may be subdivided as follows: The **suction-pipe losses** due to friction may be computed from formula on p. 269. The liquid velocity in the suction pipe w_s (ft. per sec.), is so chosen that $w_s^2/2g = mH_b$, where m varies from 0.01 for high-pressure and small-quantity pumps to 0.08 (and sometimes greater) for low-pressure and large-quantity pumps. The clear area in the strainer at the suction-pipe inlet should be about two or three times the pipe area. The **impeller losses** can be subdivided into three parts: (1) Impeller inlet loss, caused by the sudden reduction of clear passage area by the thicknesses of the impeller blades. This can be reduced by sharpening the blades at inlet for a length equal to about twice their thickness, and by having the number of blades as small as possible, consistent with the proper guiding of the liquid. (2) Friction and shock losses along the blades. These can be reduced by having the impeller castings very smooth, the passages between blades of ample area, and the blades so shaped that the reduction of liquid velocity is gradual and constant. (3) Impeller exit loss, caused by the sudden emerging of the liquid from the impeller into a clear space. This can also be reduced by sharpening the blade exit tips. The **discharge-vane losses** (at the inlet, along the vanes, and at their exit) and their remedies are of the same nature as those in the impeller. With long, scroll-shaped vanes, the number of vanes can be made very small. The **casing loss**, due to the fact that the jet issuing from every discharge vane has to be carried around a part of a circumference till all the jets are brought to the pump outlet, and resulting in shocks and eddy currents, is best minimized by employing a spiral casing. To carry the liquid away from the pump, an unutilizable velocity head $w_d^2/2g$ is necessary. This **discarded velocity head** varies from 0.005 H_b for high heads, to 0.10 H_b for low heads and large quantities of liquid.

Hydraulic Efficiency. The hydraulic losses enumerated above reduce the theoretical head H_b to a smaller head H_n; the hydraulic efficiency $e_h = H_n/H_b$. Unless the number of impeller blades is very large, the angle d_a in Fig. 1 is frequently greater than would be computed from the *apparent* impeller exit angle b_a. Consequently, the hydraulic efficiency as computed from tests on actual pumps at about or above full load is slightly *below* its true value, since the theoretical head H_b is not really as high as it is supposed to be.

The **theoretical horse power** necessary to raise Q cu. ft. per sec. of a liquid of density d lb. per cu. ft. through a height of H_n ft. $= QdH_n/550 = Qde_hH_b/550$.

Power Losses. The power losses in a centrifugal pump consist mainly of bearing and stuffing-box losses, of rotation loss, and of short-circuit losses. The **bearing loss** for ring-oiled bearings with proper lubrication $= F = 0.002 \times dNW$, where F = work lost in bearing friction, ft.-lb. per min.; d = shaft diam., in.; N = r.p.m., and W = weight of shaft and impeller plus belt pull (if any), lb. For a properly designed and well-lubricated thrust bearing, $F = 0.005P_a(d + d_1)N$, where P_a = axial thrust, lb., and d_1 = outer diam. of collar, in. With proper design and arrangement for water-cooling, the **stuffing-box losses** can be made negligibly small. The friction loss caused by the rotation of the impeller in the liquid can be expressed by the formula: **Rotation loss** (in h p.) $= 1.25\ (u_a/100)^3D^2Ss$, where u_a = wheel speed (ft. per sec.), D = outer diam. of impeller (ft.), S = number of stages, and s = specific gravity of the liquid, referred to water as unity. This loss is seen to be proportional to the number of stages and to the 5th power of the impeller diameter. (In numerous tests worked up by the writer, nearly the entire power loss at full load and beyond has been accounted for by the rotation loss alone.) The problem, therefore, of building a small centrifugal pump for high pressure and of good efficiency at a low cost is frequently perplexing. **Short-circuit losses** result from the return of part of the liquid raised through the impeller to the impeller inlet instead of proceeding to the pump exit. Part of this short-circuiting takes place through the axial clearance between the impeller and the casing under the impeller pressure head $(u_a^2 - v_a^2)/2g$, and evidently decreases as v_a, which is a measure of the quantity of liquid passed by the pump, increases. Another part passes up the backs of the vanes, where the static pressure is lowest and the relative velocity is highest, and, at very light loads, passes down the front or driving faces of the vanes, where the static pressure is highest and the relative velocity is very low and may be even negative. At no load the short-circuit loss is usually about 25 or 30 per cent. of the rated power of the pump, while at full load it is fairly negligible.

Shaft Efficiency. The shaft efficiency of a pump is the ratio of the theoretical power to the power delivered to the shaft of the pump. In the best modern pumps it is between 70 and 80 per cent. In a few cases even higher efficiencies have been reported.

Efficiencies of Centrifugal Pumps. In general, the efficiency of a centrifugal pump will depend on its brake horse power, or on the product of its rated gallons per minute into the head in feet. The table below gives a fair idea of current practice, but it must be remembered that first cost and efficiency are very closely related, and that in smaller machines efficiency is very likely to be sacrificed to first cost, while larger machines have been designed with shaft efficiency as high as 88 per cent.

In the table the shaft efficiencies are for rated head, volume and r.p.m. Fig. 2, shows typical interrelations of head, volume, r.p.m. and efficiency for good commercial pumps. There is no sharp division line between high head and low head; low head is assumed to mean less than 50 ft.; high head above 50 ft. Low speed is up to 600 r.p.m.; moderate speeds from 600 to 1800 r.p.m.; high speeds above 1800 r.p.m.

The most general drive for low-speed centrifugal pumps is the reciprocating

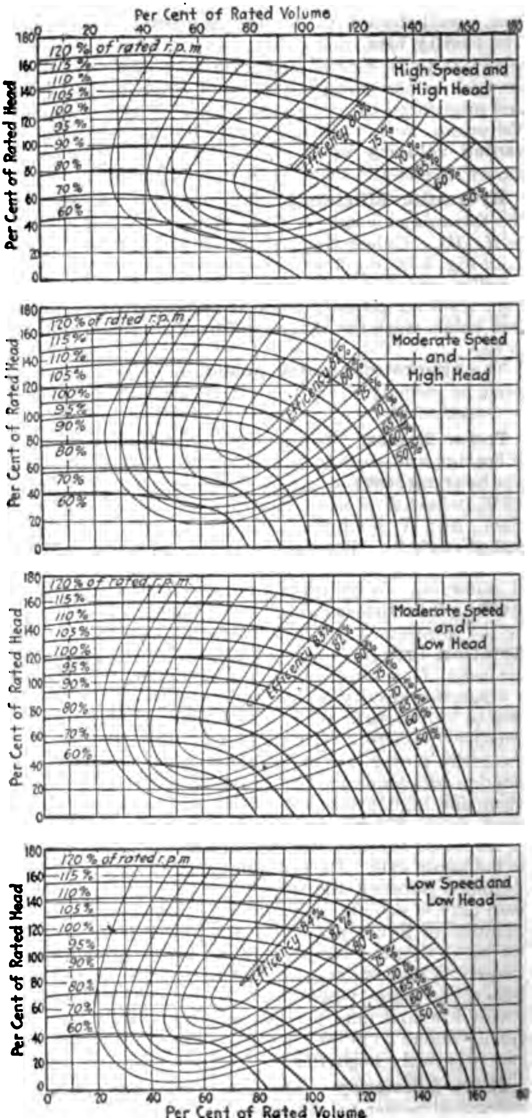


Fig. 2.—Typical Interrelations of Head, Volume, R.P.M., and Efficiency for Good Commercial Centrifugal Pumps.

steam engine, although low-speed motors are sometimes used. The most general drive for moderate-speed pumps is the electric motor, although high-speed engines and low-speed steam turbines are also occasionally used. For high-speed pumps, steam turbines are generally best, although high-speed motors are occasionally used. The curves represent general practice with commercial machines; that is, units that are built in quantity. In most cases special machines can be designed to give better economies than those quoted in the curves, but the cost of such units is proportionately higher.

Gal. per min. × Head (ft.)/1000	50	100	200	300	400	600	800	1000
Shaft efficiency, per cent	70	73	78	81	82	82	80	78

CENTRIFUGAL-PUMP CONSTANTS AND CHARACTERISTICS

Constant-head Characteristics (Fig. 1). Let D_e and D_a be respectively the inlet and exit diameters of the impeller, ft.; F_e and F_a the inlet and exit areas, sq. ft.; w_r the radial component of the impeller inlet velocity, ft. per sec., $= w_e \sin d_e = Q/F_e$ where Q is in cu. ft. per sec.; other quantities as previously defined. Then, if

$$C_1 = \left(\frac{F_e}{F_a \tan b_a} + \frac{D_e}{D_a \tan b_e}\right) \Big/ 2\left[1 - \left(\frac{D_e}{D_a}\right)^2\right], \text{and } C_2 = 1 \Big/ \left[1 - \left(\frac{D_e}{D_a}\right)^2\right],$$

the pump characteristic for constant head will be

$$u_a = -w_r C_1 - \sqrt{w_r^2 C_1^2 + gH_b C_2}.$$

This equation gives for a pump of given dimensions the variation of u_a (r.p.m.) against w_r (discharge quantity) for a fixed total head H_b if the inlets into the impeller and into the discharge vanes are to be shockless.

Constant-speed Characteristics. Letting $C_3 = (u_a^2 - u_e^2)/g$ and $C_4 = [(u_a/\tan b_a) + (u_e/\tan b_e)(F_a/F_e)]/g$, the pump characteristic for constant speed will be $H_b = C_3 + v_r C_4$, where $v_r = w_a \sin d_a = Q/F_a$. The total head H_b will therefore for a constant r.p.m. increase with increasing values of Q if C_4 is positive (forward discharge), and will decrease with increasing values of Q if C_4 is negative (backward discharge).

Constant-efficiency Characteristics. A pump has its best hydraulic efficiency when the sizes and shapes of its passages have been correctly designed for the particular head and quantity desired. If the characteristic of the satisfactory pump design be written as $C_5 = Q/\sqrt{H_b}$, then the same pump patterns can be used for various values of Q with the same hydraulic efficiency if H_b is so varied that C_5 remains constant. The relation between H_b and H_n, or the hydraulic efficiency for the particular model, is, of course, supposed to be known.

The expression $C_5 = Q/\sqrt{H_b}$ may also be written nearly enough as $C_5 = Q/N$, where N = r.p.m., so that if the r.p.m. are not prescribed, the same model can be used for various heads by changing the r.p.m. to correspond.

Speed and Shock vs. Efficiency. The various hydraulic losses are directly proportional to the total head, so that changing the speed and the head will have no effect on the hydraulic efficiency. Of the power losses, all, except the rotation loss, are fairly constant for various heads, and (especially at full load) their variations may be quite neglected. The rotation loss however, varying with the cube of the r.p.m. and with the fifth power of the outer impeller diameter, must always be very carefully considered. The foregoing discussion of characteristics and efficiencies assumes shockless entrance into the impeller and into the discharge vanes, in other words, that the angles b_e and d_e are adjusted to suit the varying quantity of liquid. Adjustable discharge vanes have actually been built for single-stage, large-quantity pumps; but for multi-stage or small-quantity pumps they are impracticable. Shock losses are therefore usually present, but, as they can not be evaluated mathematically, the exact characteristic curves for any particular pump can be determined only by experiment.

Behavior of Centrifugal Pumps with Changes in Quantity, Speed and Head Regulation. From the fundamental equation the theoretical pressure head for zero quantity ($Q = 0$) is u^2/g. With increase in quantity, the head remains constant with radial blades, increases with forward-discharge blades and decreases with backward-discharge blades (see Fig. 3). The actual pressure lines pursue a materially different course; because even with very reduced quantities, and especially for zero quantity, the fundamental equation is no longer applicable. For zero quantity, when practically the centrifugal force alone is effective, the generated head is $u^2/2g$; or, since even with the blast gate closed there are water currents within the pump, it is slightly larger, on account of the conversion of velocity into pressure. With increase in quantity, the pressure rises, with forward-discharge blades faster than with backward-discharge blades; and then drops again, as the pressure drop within the pump is increasing. (See lower lines in Fig. 3.)

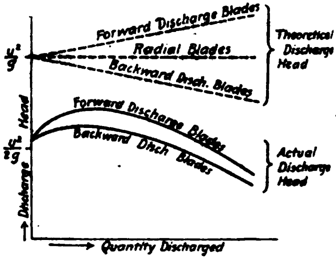


FIG. 3.

If the head to be overcome is plotted against the quantity Q, the intersection of the line of generated head with the line of head operated against

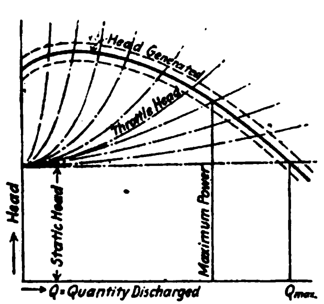


FIG. 4.

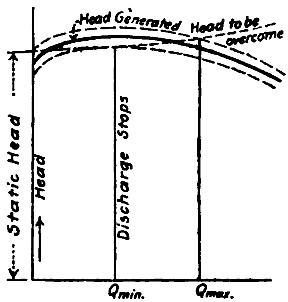


FIG. 5.

forms the point at which the pump is working at the time. Such a diagram also shows how the quantity varies with the r.p.m. In Figs. 4 and 5 the heavy line shows the head generated at normal speed with different rates of discharge. The two neighboring dotted lines give the heads for speeds 2 per cent. above and below, respectively. The curves marked "throttle head" in Fig. 4 show the variations in the total head to be overcome (static head + throttle head) with different amounts of throttling and with different rates of discharge. If the throttled head of the pump is large in comparison with the geometric or static head, as in the case of a mine-shaft pump when the head corresponding to the r.p.m. of the pump is greater than the depth of the shaft and the excess head has to be throttled down (see Fig. 4), or in the case of a condenser circulating pump with a small static head but with very long piping, the pump is not so sensitive to variations in the r.p.m. But in a pump where there is mainly a static head to be overcome, as in drainage work, the quantity of discharge varies considerably with variation in the r.p.m., and it decreases even with a moderate drop in the r.p.m.; that is, when the generated head becomes too small, the check valve of the pump closes and the discharge of water stops (see Fig. 5). If a drainage pump is to work with normal r.p.m. against the full discharge head and not have its discharge vary unduly with variations in r.p.m., it should work on the drooping part of the pressure curve, as indicated in Fig. 5. In actual drainage pumps, a change of 1 per cent. in speed involves a change of about 5 per cent. in discharge.

Regulation of Quantity or Output. In centrifugal pumps the quantity Q is proportioned to the r.p.m. only when flow resistances alone are to be overcome (zero static head). But if a static head is also to be overcome, the quantity changes faster than the r.p.m. If the head operated against is also variable, as in the case of water-works installations, the quantity of discharge no longer bears a definite relation to the r.p.m. To regulate for constant quantity while maintaining the r.p.m. constant, it is necessary to regulate for constant velocity of flow in the discharge pipe or in the suction pipe; for instance, by arranging that a change in velocity should cause a valve in the piping to open to a greater or lesser extent. In certain cases, as in water-works pumps, which operate against a gradually increasing head, so that the r.p.m. must be gradually increased, it is sufficient to admit a constant supply of steam; then the pump automatically maintains a speed and a quantity corresponding to the steam supply. Governors which hold the speed are a hindrance in this respect. In pumps where flow resistances alone are to be overcome, the quantity may be regulated down to zero. But where static pressure is mainly to be overcome, the quantity can be reduced only down to a certain limit, because if the pump works before the pressure-curve peak, the check valve closes.

DESIGN DATA

Suction Pipe. If A = atmospheric pressure, H_s = suction head, h_s = friction head, and $w_s^2/2g$ = velocity head in the suction pipe, in general $A - [H_s + h_s + (w_s^2/2g)] = 0$. In practice, it is advisable to take $H_s + h_s + (w_s^2/2g) = 23$ ft. In low-pressure pumps the loss of head in the suction pipe may affect materially the efficiency of the pump.

Impeller. After determining the shaft and the impeller hub diameters from mechanical considerations, the impeller inlet diameter is taken to allow an annulus inlet velocity of from 6 to 12 ft. per sec. The number of impeller blades is made from 6 to 12, with an equal number of half-blades extending inward from the outer periphery. Backward-discharge blades are more easily bent to the proper involute shapes, and render the pump proof against overloading. The tips, however, are subject to centrifugal forces normal to their axial surfaces, and they must be reinforced by shrouds. These shrouds, as well as the impeller web and blades, must be carefully computed for centrifugal stresses. The angle between blade and wheel tangent is usually about 30 deg. The impeller inlet is frequently provided with a labyrinth packing or some similar arrangement to reduce the short-circuit loss.

Discharge Vanes. The number of discharge vanes is somewhat smaller than that of the impeller blades. They are usually of the involute shape, with a gradual divergency of passage. Their inlet angle should be small so as to keep down the short-circuit losses and also the shock losses with varying quantities of liquid.

TYPES OF CENTRIFUGAL PUMPS

The methods usually employed for overcoming axial thrust with single-inlet impellers and other features of modern centrifugal pumps are shown below. Fig. 6 illustrates diagrammatically the **Sulzer type** (Sulzer Bros., Winterthur, Switzerland), in which the impellers are mounted alternately in a reverse position so that the directions of thrust of any two adjacent impellers are opposed. The projected areas at 1 and 4, taken in a plane perpendicular to the shaft axis, are equal; similarly, the projected areas at 2 and 3 are equal. By making the pressure heads of all impellers equal, substantially $p_2 - p_1 = p_4 - p_3$. Any small resulting axial thrust is taken care of by a rigid collar thrust bearing. R. D. Wood & Co. (Philadelphia) and the Buffalo Steam Pump Co. (Buffalo, N. Y.) make pumps of this type.

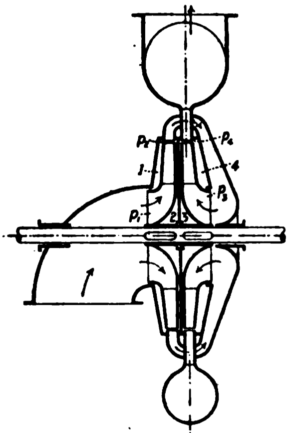


Fig. 6.—Sulzer Type of Pump.

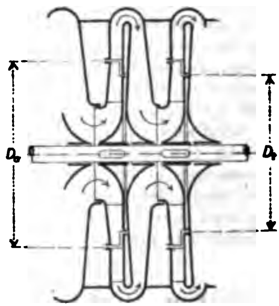


Fig. 7.—Rateau Type of Pump.

Fig. 7 illustrates diagrammatically the **Rateau type,** in which the end thrust is largely neutralised by shrouding the two sides of each impeller to different diameters D_a and D_t. The higher pressure corresponding to the larger diameter D_a acting on a smaller surface is fairly balanced by the lower pressure corresponding to the smaller diameter D_t acting on a larger surface. Complete balance is finally obtained by means of a balancing piston keyed to the shaft beyond the last impeller on the high-pressure side, and rotating in a cylindrical casing with very small clearances. On one side of the piston full water pressure exists, while the other side can be connected to any pressure stage or to the suction end of the pump.

Fig. 8 illustrates diagrammatically the **Jaeger type** (J. H. Jaeger & Co., Leipsig). In this design—the one most generally adopted—each impeller is balanced individually by means of holes in the impeller web which equalize the pressure in spaces 3 and 4. By making the clearance at the outer rim of the impeller sufficiently large, equal pressure will exist in spaces 1 and 2. By using an inner and an outer packing as indicated at *R*, the short-circuit

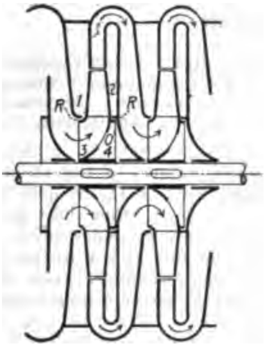


FIG. 8.—Jaeger Type of Pump.

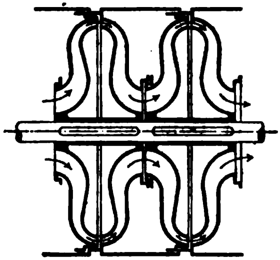


FIG. 9.—Kugel-Gelpke Type of Pump.

loss is greatly reduced and the maintenance of equal pressures in spaces 1 and 2 is made possible. Any remaining axial thrust is taken up by a marine-type thrust bearing on the suction side of the pump. Henry R. Worthington, of New York, the Byron Jackson Iron Works, of San Francisco, the Alberger Pump Co., of New York, and the De Laval Steam Turbine Co., of Trenton, N. J., build pumps embodying the basic Jaeger principle.

Fig. 9 illustrates diagrammatically the **Kugel-Gelpke type** (Escher, Wyss & Co., Zurich, Switzerland). As seen, both impeller and discharge vanes are S-shaped, the water entering and leaving both the impeller and the discharge vanes in an axial direction. The axial thrust is balanced by the admission of water under pressure in the spaces between the guide vanes and impellers, the water pressure being regulated by means of a valve. Balancing pistons and thrust bearings are also used to balance the axial thrust. The Allis-Chalmers Co., of Milwaukee, Wis., builds pumps of this type.

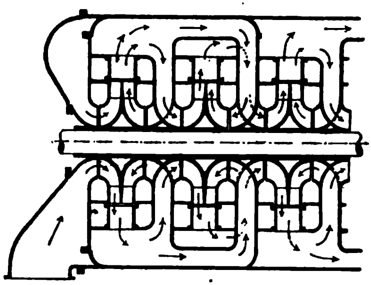


FIG. 10.—Double-inlet Pump without End Thrust.

Fig. 10 illustrates a double-inlet pump which has no end thrust and needs no special balancing arrangement excepting an ordinary thrust bearing for simply locating the rotating part.

AIR COMPRESSORS

BY

H. J. THORKELSON

REFERENCES: Simons, "Compressed Air," McGraw-Hill. Peele, "Compressed Air Plants," Wiley. Harris, "Compressed Air Theory and Computation," McGraw-Hill. Innes, "Air Compressors and Blowing Engines," Van Nostrand. Thorkelson, "Air Compression and Transmission," McGraw-Hill. Von Ihering, "Die Gebläse," Springer, Berlin. Hirsch, "Die Luftpumpen," Jänecke, Hanover.

Types of Compressors. For pressures below that of the atmosphere many types of compressors are available. **Fans** (see p. 1541) are usually used for exhaust systems for handling foul air, shavings or other light material. **Jet blowers** (p. 1516) are also used, and for the low pressures occurring in condensers, piston and special centrifugal or impeller types are employed.

For pressures varying from ⅛ oz. to 16 oz., various types of **fans** are used. For pressures from 1 to 5 or even 10 lb. above atmospheric pressure, **rotary blowers** (p. 1515) are available.

For compressing air to pressures of 5 or 10 lb., and in some cases pressures as high as 100 lb. and over, **centrifugal compressors** (turbo-blowers—p. 1530) have been recently introduced.

Piston Compressors are built for pressures as low as 1 lb. above atmospheric, and single cylinders are used with "single-stage" compression to pressures of 80 or 100 lb. per sq. in. gage. When **higher pressures** are desired, the compression is divided into stages, with intercoolers between the cylinders. Two-stage compressors are used for pressures from 80 or 100 to 500 lb., three-stage for pressures up to 1200 lb. (in small units up to 2000 or 2500 lb.), and four-stage for pressures of 2500 to 3000 lb. per sq. in. or even higher. All these types of piston compressors may be driven by steam or gas engines, electric motors or water wheels.

Table 1. Specific Weight of Air at Various Temperatures, Pressures and Degrees of Humidity

Temperature, deg. fahr.	Weight of 1 cu. ft. of dry air (at 14 lb. per sq. in. or 28.5 in. of Hg), lb.	Increase or decrease of weight for each 0.1 lb. change in pressure, lb.	Increase or decrease of weight for each 1 in. of Hg. change of pressure, lb.	Decrease of weight for each 10 per cent. increase in relative humidity, lb.
32	0.07688	0.000549	0.002698	0.000019
35	0.07642	0.000546	0.002681	0.000021
40	0.07565	0.000540	0.002654	0.000025
45	0.07490	0.000535	0.002628	0.000030
50	0.07417	0.000530	0.002602	0.000035
55	0.07340	0.000525	0.002580	0.000040
60	0.07272	0.000520	0.002554	0.000051
65	0.07203	0.000515	0.002530	0.000059
70	0.07134	0.000510	0.002506	0.000070
75	0.07068	0.000505	0.002482	0.000081
80	0.07003	0.000500	0.002457	0.000095
85	0.06938	0.000495	0.002432	0.000111
90	0.06875	0.000490	0.002408	0.000127
95	0.06811	0.000485	0.002384	0.000147
100	0.06752	0.000480	0.002359	0.000172
105	0.06694	0.000475	0.002334	0.000199

Several types of **compressors** have been designed to utilise water powers directly **without** the use of any mechanical **moving parts.** These are particularly adaptable when such natural resources are available near mines or other industries using large quantities of compressed air. An installation of this type is described on p. 1517.

DATA ON AIR

Specific Weight. Table 1 will be found useful in determining the specific weight of air at various temperatures, pressures and degrees of humidity.

Table 2 gives the specific weight of saturated air at various temperatures and pressures.

Table 2. Weight of Saturated Air in Lb. per Cu. Ft. at Different Barometric Pressures

Temp., deg. fahr.	Barometer readings, inches of mercury								
	28.5	29.0	29.5	29.7	29.9	30.1	30.3	30.5	31.0
30	0.07703	0.07839	0.07974	0.08028	0.08083	0.08137	0.08191	0.08245	0.08381
35	0.07621	0.07756	0.07890	0.07943	0.07997	0.08051	0.08104	0.08158	0.08292
40	0.07541	0.07674	0.07806	0.07859	0.07913	0.07966	0.08019	0.08072	0.08205
45	0.07461	0.07592	0.07724	0.07776	0.07829	0.07881	0.07934	0.07986	0.08118
50	0.07381	0.07512	0.07642	0.07694	0.07746	0.07798	0.07850	0.07902	0.08032
55	0.07302	0.07431	0.07560	0.07612	0.07663	0.07715	0.07766	0.07818	0.07947
60	0.07224	0.07352	0.07479	0.07530	0.07581	0.07632	0.07683	0.07734	0.07862
65	0.07145	0.07272	0.07398	0.07449	0.07499	0.07550	0.07600	0.07651	0.07777
70	0.07067	0.07192	0.07317	0.07367	0.07417	0.07467	0.07518	0.07568	0.07693
75	0.06988	0.07112	0.07236	0.07286	0.07335	0.07385	0.07434	0.07484	0.07608
80	0.06909	0.07032	0.07155	0.07204	0.07253	0.07302	0.07351	0.07400	0.07523
85	0.06829	0.06950	0.07072	0.07121	0.07170	0.07218	0.07267	0.07316	0.07437
90	0.06748	0.06868	0.06989	0.07037	0.07085	0.07133	0.07182	0.07230	0.07351
95	0.06665	0.06785	0.06904	0.06952	0.07000	0.07048	0.07095	0.07143	0.07263
100	0.06581	0.06700	0.06818	0.06866	0.06913	0.06960	0.07008	0.07055	0.07174

Specific Heat. The instantaneous specific heat of **dry air** is given by F. G. Swann as $0.24112 + 0.000009\,t$, and the specific heat of **water vapor** as $0.4423 + 0.00018\,t$, where t is the temperature in deg. fahr. The **mean specific heat** of air with any degree of saturation may then be found by multiplying the weight of air by its specific heat and adding to this the product of the weight of water vapor and its specific heat and dividing the sum by the weight of the mixture. Table 3 is calculated on that basis.

For dry air at high temperatures see p. 366.

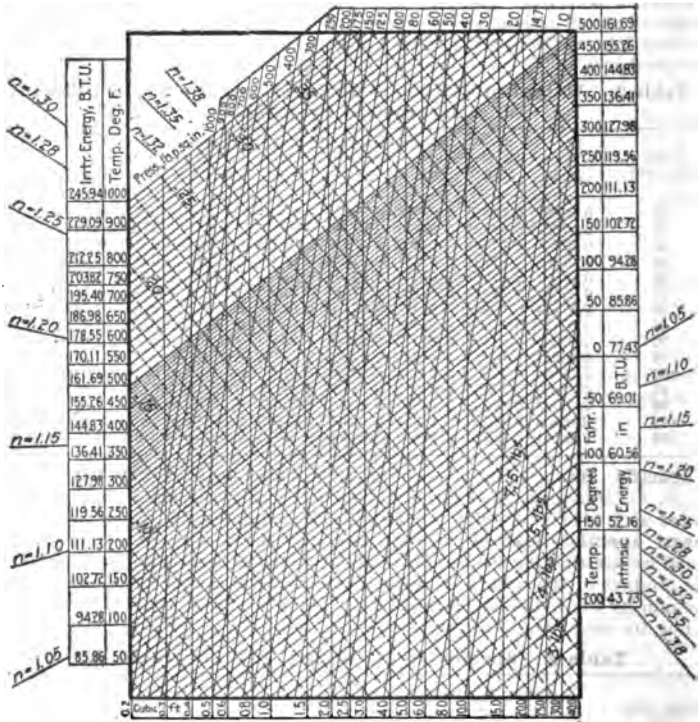
Table 3. Specific Heats of Dry and Saturated Air

Temp., deg. fahr.	Specific heat of		Temp., deg. fahr.	Specific heat of	
	Dry air	Saturated air		Dry air	Saturated air
60	0.2417	0.244	85	0.2419	0.2474
65	0.2417	0.2447	90	0.2419	0.2486
70	0.2417	0.2452	95	0.2420	0.2498
75	0.2418	0.2458	100	0.2420	0.2512
80	0.2418	0.2466			

The mean specific heat of air having any relative humidity may be found from Table 3 by interpolation. *E.g.*, for air of 40 per cent. relative humidity and at 80 deg. fahr. the mean specific heat will be $0.2418 + 0.40 \times (0.2466 - 0.2418)$, or 0.2437.

The variation of the specific heat of air with pressure has been investigated by Holborn and Jakob (*Z. V. D. I.*, vol. 58, p. 1436). The mean specific heat at constant pressure for the temperature range 20–100 deg. cent. (68–212 deg. fahr.) is given by the equation $10^4c_p = 2413 + 286p + 0.0005\,p^2 - 0.00001\,p^3$, where p is the pressure in kg. per sq. cm. The experimental values are given below:

Pressure, lb. per sq. in. abs.	14.2	356	711	1422	2133	2844
c_p	.2415	.2490	.2554	.2690	.2821	.2925



Adapted from Prof. C. R. Richards, Entropy-Log. Temp. Diagram, Univ. of Ill. Eng. Experiment Station Bulletin No. 63.

FIG. 1.—Diagram for Determining the Amount of Energy Stored in Compressed Air.

Calculations of the work done by the expansion of air are greatly facilitated by the diagram Fig. 1, adapted from the "Entropy-Log. Temperature Diagram for Air" by Prof. C. R. Richards (*Bulletin* No. 63, University of Illinois Experiment Station).

In Fig. 1 the vertical lines represent the volume in cu. ft. occupied by 1 lb. of air. The lines slightly inclined to the vertical represent the absolute pressure in lb. per sq. in. The lines at an angle of 45 deg. represent temperature in deg. fahr. and intrinsic (internal) energy in B.t.u., and the dotted inclined lines represent entropy. These last lines are paralleled in finding the results of an adiabatic change, while if the change follows a $p_1V_1^n = p_2V_2^n$ path, the effects are studied by paralleling lines indicated at the margin for various values of n. For example, an inspection of Fig. 1 shows that 1 lb. weight of air at a pressure of 125 lb. abs. per sq. in. and occupying a volume of 1.5 cu. ft. will have a temperature of 50 deg. fahr. and contain 85.86 B.t.u. or 66,800 ft.-lb. of internal energy.

Furthermore, the **adiabatic expansion** of 1 lb. of air from a pressure of 125 lb. abs. per sq. in. and a temperature of 70 deg. fahr to a pressure of 14.7 lb. abs. will result in a final temperature of -170 deg. fahr., and the work done during the expansion would be $(89.22 - 48.79 =)$ 40.43 B.t.u. or 31,454 ft.-lb. per lb. of air. This expansion follows the dotted line of the diagram, but if the expansion followed the equation $p_1V_1^{1.2} = p_2V_2^{1.2}$, the available intrinsic energy per lb. for any pressure range would be found by following a line parallel to the marginal line of the chart marked $n = 1.20$ from the upper to the lower pressure. *E.g.*, if such an expansion occurred between 125 lb. abs. per sq. in. and 70 deg. fahr. and 14.7 lb. abs., the resulting temperature would be -87 deg. fahr. and the work done would be $(89.22 - 62.76 =)$ 27.46 B.t.u. or 21,346 ft.-lb. per lb. of air.

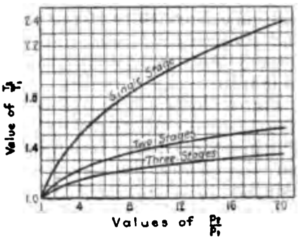


FIG. 2.—Compression Temperatures.

With **adiabatic compression**, the **final temperature** T_2 (deg. fahr. abs.) is given by the formula $T_2 = T_1\,(p_2/p_1)^{0.29}$ for single-stage compression; by $T_2 = T_1\,\sqrt{(p_2/p_1)^{0.29}}$ for two-stage compression, and by $T_2 = T_1\,\sqrt[3]{(p_2/p_1)^{0.29}}$ for three-stage. Values of T_2 may be obtained readily from the chart devised by F. W. O'Neill and shown in Fig. 2, in which the ordinates give the factors by which the initial absolute temperature T_1 (deg. fahr.) is to be multiplied to obtain the final absolute temperature T_2 for various ratios of p_2 to p_1.

BLOWERS AND COMPRESSORS

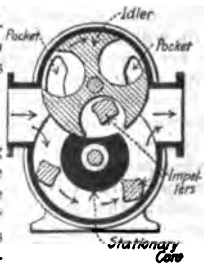


FIG. 3.—Sturtevant Rotary Blower.

Rotary Blowers are built for air pressures varying from 6 oz. to 10 lb. or even 12 lb. per sq. in. The best efficiencies of this type of blower, however, are usually secured below 5 lb. pressure, but the simplicity of the machine gives it an advantage over compressors of the piston type and frequently warrants its installation for the higher pressures indicated when designed for this purpose. As the machine operates by displacement, it is usually preferred for cupola practice because its positive action will not permit a reduction in air supply if the cupola tends to clog. For other uses of air at pressures below 8 oz. the fan is ordinarily more economical.

Blowers of this type may be arranged to give either **constant volume or constant pressure,** and to handle either liquids or gases. They consist of a casing containing one or more revolving impellers of various forms of design.

Fig. 3 represents a cross-section of the **Sturtevant high-pressure blower,** which is built in capacities ranging from 5 to 15,000 cu. ft. per min. at 8 oz. pressure (speeds, from 375 to 800 r.p.m. for the smallest size down to 160 to 220 r.p.m. for the largest; weights with sub-base range from 200 to 39,000 lb.). The smaller machines have a vertical arrangement of shafts, while the larger types usually have their shafts in the same horizontal plane. Two impeller blades are always in action, and leakage by one is caught by the other. The proper **size of blower for a cupola** may be calculated on the basis of 30,000 cu. ft. of air per ton of iron melted. Sturtevant machines are also used for handling gases. The capacities for the various sizes of **gas exhausters** range from 7500 to 900,000 cu. ft. per hour at 8 oz. pressure, making no allowance for shrinkage, which will vary from 10 to 20 per cent., depending on the gas and its pressure. The inlet and outlet diameters run from 3 in. in the smallest size to 30 in. in the largest; weights, from 400 to 33,000 lb.

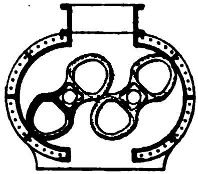


FIG. 4.—Roots Blower.

Fig. 4 illustrates a cross-section of a **Roots blower**. The two impellers are symmetrical and are driven in opposite directions by gears outside the casing. The impellers do not touch each other nor the casing, but the clearance is reduced to a minimum in order to reduce slip or leakage. The amount of this **slip or leakage** may be determined by operating the machine with a closed discharge, at a speed sufficient to maintain the required discharge pressure. The amount is usually largest in machines of smallest capacity, *i.e.*, a machine displacing 0.75 cu. ft. per rev. at a pressure of 1 lb. will have a slip of from 60 to 70 rev., while a machine having a capacity of 300 cu. ft. per rev. will have a slip of from 3 to 5 rev. For intermediate capacities the slip will vary proportionally and increase with higher pressures as the square root of the discharge pressure, *i.e.*, at 4 lb. pressure the slip will be approximately twice that at 1 lb.

In most blower work the so-called hydraulic formula for **horse power** will be found satisfactory: h.p. = $Q(p_2 - p_1)/33{,}000$, where Q is the cu. ft. of air compressed per min., p_1 the initial pressure and p_2 the final pressure, lb. per sq. ft. To get the actual horse power at the shaft, the horse power should be divided by the efficiency, which will vary from 0.80 to 0.90.

FIG. 5.

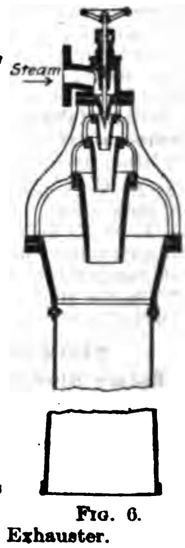


FIG. 6.

FIG. 5.—Körting Exhauster.

FIG. 6.—Körting Steam-jet Air Compressor.

Steam-jet Blowers. Steam jets have long been used for "blowing" or exhausting in order to maintain combustion in locomotive boilers, usually employing the exhaust from the engines through properly shaped "nozzles." This type of air compressor or exhauster also finds extended application for emergency use and at times in permanent installations for removing foul air from mines, factories, ship holds, and for gas exhausters, for securing forced draft, and for handling gases under low pressures in certain chemical industries. Among its **advantages** are simplicity, ease of operation, small space, minimum of repairs and ease of regulation.

Fig. 5 shows a **Körting exhauster** for removing air, and Fig. 6 a Körting compressor of the steam-jet type. These machines are built in **capacities** of from 10 to over 20,000 cu. ft. per min., and usually operate with steam at from 60 to 100 lb. gage pressure. In some cases they are operated by water under pressures from 1 to 45 lb. gage. A. von Ihering ("Die Gebläse") reports some tests as to **steam consumption** which indicate from 1.3 to 3.4 lb. of steam required per 1000 cu. ft. of air handled, the larger sizes being the more economical.

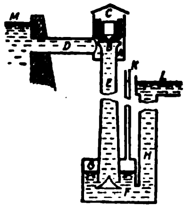


FIG. 7.—Taylor Hydraulic Air Compressor.

Hydraulic Compressors. Several devices have been made for utilizing falling water for the purpose of compressing air without the use of any mechanical moving parts. The most successful of these is the **Taylor compressor,** shown diagrammatically in Fig. 7. In the figure, air tubes are represented at *C*, all terminating at the conical entrance *B* to the down-flow pipe *E*. The water supply is furnished through the flume *D*. As the water falls it draws air through the small tubes, carrying it down to the separating tank *G*, where it is liberated at a pressure depending on the weight of water in the vertical pipe *H*. The compressed air is then conducted through the pipe *K* to the place where it is to be used. The distance from *M* to the tail race *L* represents the height or fall of water that is available.

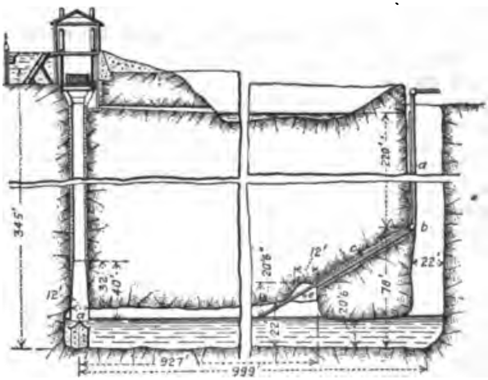


FIG. 8.—Taylor Hydraulic Compressor Installation near Cobalt, Ont.

In this system the compression is isothermal and the compressed air is saturated with moisture. The oxygen content of the air is reduced, which renders the air less beneficial for purposes of mine ventilation if the exhaust from the air tools is planned to assist ventilation. The system offers a very simple solution for

Air Measurements				Water Measurements			
	Absolute Pressures (lb. per sq. in.)						
Free Air, cu. ft. per min.	Free Air	Compressed Air	Horse power	Cu. ft. per min.	Head, ft.	Horse power	Efficiency, per cent.
10,580	14	128	1,430	13,057	70.5	1,741	82.17
11,930	14	128	1,623	14,820	70.0	1,961	82.27
9,238	14	128	1,248	12,710	70.6	1,700	73.50

utilising water powers when the market for compressed air justifies its installation. It has the advantage of simplicity with a minimum of operating expense, and very high efficiencies are secured. The first cost of the installation is likely to be high.

In 1906 a large plant of this type was installed at the Victoria Copper Mine, near Rockland, Mich., consisting of three units with a total capacity of from 34,000 to 36,000 cu. ft. of free air per min. A series of tests made on a single intake head by Prof. F. W. Sperr, gave the results shown on p. 1517.

Fig. 8 illustrates some of the dimensions of a Taylor hydraulic compressor installed near Cobalt, Ontario, Canada. This was designed for a capacity of 40,000 cu. ft. of free air per min. to be compressed to a gage pressure of 120 lb. The compressed air is conducted to mines through 9 miles of 20-in. pipe leading to two 12-in. lines with a total distributing line of 21 miles in length. The water is admitted through suitable gates to two "heads" each 16 ft. in diam. and containing 66 pipes 14 in. in diam. The size of the heads is reduced in diameter to about 8 ft. and the whole apparatus can be raised or lowered as required by operating conditions. A cone *a* assists in separating the air and water, and the long horizontal tunnel permits quite complete separation. The compressed air is removed through the pipe *c* and the water freed from the entrained air escapes through the vertical shaft *b*. Pipe *e* acts as a relief for a surplus of compressed air. Its end is normally below the surface of the water in the tunnel, but if the amount of air should accumulate it would be exposed and permit the escape of the surplus air without seriously affecting the normal air pressure of the distribution system.

Piston Compressors and Blowers

The large quantities of air required for blast-furnaces and Bessemer converters are usually supplied by piston compressors of large capacity, driven either by steam or gas engines. Turbo-blowers directly driven by steam turbines, however, have been recently developed for this work.

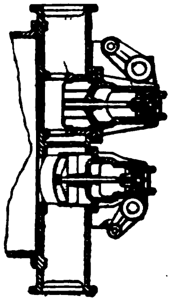


Fig. 9.—Valves of Allis-Chalmers Blower.

Piston compressors used as blowers for blast furnaces and Bessemer converters usually operate at **discharge pressures** of 15 or 20 lb. gage. The tendency in blower design is to secure increased capacity by higher speeds than formerly. The Allis-Chalmers Company now uses a maximum speed of 90 r.p.m. for 48-in. stroke and 85 r.p.m. for 60-in. stroke, giving **piston speeds** of from 720 to 850 ft. per min. With piston speeds approximating 750 ft. per min. the inlet area is approximately 13 per cent. and the outlet area 11 per cent. of the piston area. With increased speeds the tendency is to increase clearance, which will usually vary from 9½ to 11½ per cent.

Fig. 9 shows the **valves** of an Allis-Chalmers blower, in which the upper valve is for inlet and is mechanically operated. The lower discharge valve opens automatically but is closed mechanically. It is made of sheet steel so as to diminish inertia effect.

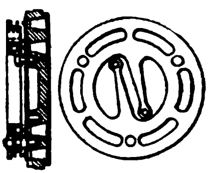


Fig. 10.—Borsig Valve.

The Slick blowing tub consists of a reciprocating cylinder on the outside of the compressing cylinder, arranged so as to open ports at the ends of the cylinders for inlet. This gives unobstructed inlet areas of from 18 to 20 per cent. with very small clearance.

Fig. 10 illustrates a European design of valve (the Borsig) which has been used quite successfully in this country. It is guided without friction by elastic deformation of part of the valve. To get enough valve area the cylinder heads must be extended to permit the insertion of a number of these valves in cages. The **Gutermuth flap valve** (Fig. 11) is also used in compressors. It consists of a thin sheet of tempered steel or bronze, formed at one end into a spring hinge.

FIG. 11.—Gutermuth Valve.

Figs. 12–14 show types of inlet and discharge valves made by the Ingersoll-Rand Co. Fig. 12 is a "Hurricane-Inlet" valve placed in the piston and communicating with the intake through a tube. These valves are made from a continuous ring of oil-treated high-carbon steel forged from the solid and turned to a T-section. No springs, pins or rods are used, and the valve is entirely automatic in its action. Fig. 13 shows a direct-lift inlet valve and Fig. 14 an automatic discharge valve.

The **ratio of inlet valve area to piston area** varies from 0.05 to 0.14. For ordinary types of valves, the inlet area should, as a rule, be not less than 8 or 10 per cent. of the piston area. Automatically operated inlet valves are apt to be irregular in their action and reduce the volumetric efficiency of the compressor. This disadvantage is overcome by using mechanically operated inlet valves, usually of a Corliss type, but on account of the variation of discharge pressure they are not used as often for discharge or for inlet on the high-pressure stages of a multi-stage compressor. The **area of the discharge valves** will usually vary from 10 to 15 per cent. of the piston area, the larger percentage being required for the higher piston speeds.

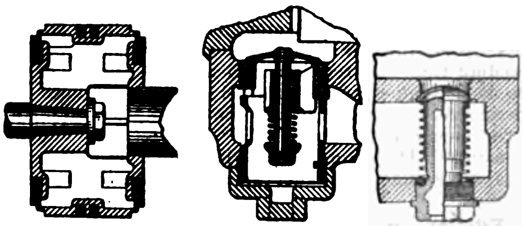


FIG. 12. FIG. 13. FIG. 14.
Ingersoll-Rand Inlet and Discharge Valves.

In the Laidlaw-Dunn-Gordon compressors a valve gear is used that mechanically controls the opening and closing of the suction and the closing of the discharge. The opening of the discharge is effected by means of vertically placed poppet valves. At the end of each stroke, however, the discharge is mechanically closed by means of a Corliss semi-rotary valve directly under the poppet valves, thus retaining under the latter a cushion of air under discharge pressure, enabling them to seat without violent and noisy impact. The construction is such that very small clearances are possible. In a 28 × 30-in. cylinder, for example, it is but 0.7 per cent. This type of gear has proved very successful on dry air pumps when very high vacuum is required.

AIR COMPRESSION

For the **theory of air compression**, see p. 321.

Mean Effective Pressure in Multi-stage Compression. The mean effective pressure, lb. per sq. in., with complete intercooling, referred to the l.-p. piston, is expressed by the formula

$$p_m = p_1\left\{\frac{Sn}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{Sn}} - 1\right]\right\}$$

where S represents the number of stages and n is the exponent of the compression curve. Fig. 15, from a chart plotted by F. W. O'Neill, shows the relation between the mean effective pressure and the initial pressure for various pressure ratios, p_2/p_1, with adiabatic compression and complete intercooling.

Wet vs. Dry Compression. The ideal method of compressing air when it is to be stored or allowed to cool before being used is the isothermal, and in the earlier types of compressors this was attempted by so-called wet compressors, by means of which it was possible to secure compressions approximating $p_1V_1^{1.2} = p_2V_2^{1.2}$. The mechanical difficulties involved and the necessary low speeds with consequent small capacity have led to the use of modern "dry compressors," which in small sizes have cylinders with cast-iron ribs for radiating heat, and in large sizes have water jackets surrounding the cylinder. The cooling thus secured is sufficient to keep temperatures from being excessive, but as a rule the compression curves are above $pV^{1.35}$ = constant. Dry compression has the advantage of higher speeds and larger capacities.

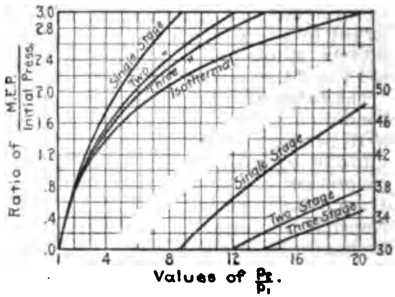


FIG. 15.—Mean Effective Pressures in Air Compression.

Efficiencies. The effect of clearance upon capacity is usually expressed in terms of **volumetric efficiency.**

The **apparent volumetric efficiency** is the apparent volume of free air drawn in (as shown by the indicator card) divided by the volume of the piston displacement. This is the term that is commonly used in speaking of volumetric efficiency, and in Fig. 16 it is GK/L. If the clearance expansion line follows the equation $p_JV_J^n = p_KV_K^n$, where the clearance $C = V_J/L$,

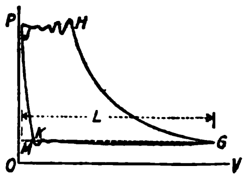


FIG. 16.—Air Compressor Card.

ıen $V_K/L = (p_J/p_K)^{\frac{1}{n}}C$. The volumetric efficiency may also be written:

$$1 - \frac{V_K - V_J}{L} = 1 - C\left[\left(\frac{p_J}{p_K}\right)^{\frac{1}{n}} - 1\right].$$

Example. If p_J is 94.7 lb. per sq. in. abs. and C is 2 per cent., the volumetric efficiency will be $1 - 0.02\left[\left(\frac{94.7}{14.7}\right)^{\frac{1}{1.4}} - 1\right] = 0.9444$.

The loss in capacity for stage compression will be represented by the following formulæ, in which p_1 is the initial and p_2 the final pressure:

Two-stage: $1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{2\times 1.4}} - 1\right]$; Three-stage: $1 - C\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{3\times 1.4}} - 1\right]$

Fig. 17 represents graphically the part within the brackets of the above equations. Knowing the actual clearance and the pressure range, the **effect of this clearance on capacity** can be found. For example, with a pressure ratio of $p_2/p_1 = 8$, the chart shows this value for single-stage compression to be 3.3. The capacity for these conditions and a 4-per cent. clearance will be $1 - (0.04 \times 3.3) = 1 - 0.132$, or 86.8 per cent. of the piston displacement. **Clearances** in the larger sizes of compressors approximate 1 per cent.; in smaller machines they are greater, being in some cases as high as 3 per cent.

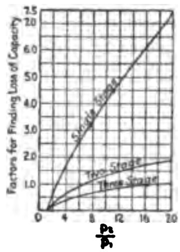


Fig. 17.

The indicator card would be a true method of measuring the volumetric efficiency if the temperature of the air after being drawn into the cylinder were the same as that of the atmosphere, and if the pressure at the end of the suction stroke were the same as that of the atmosphere. This is never the case.

The **true volumetric efficiency** is the ratio of the free air actually drawn in to the piston displacement.

The **cylinder efficiency** of an air compressor may be defined as the ratio of the work done in a complete cycle to compress isothermally a volume of of air at atmospheric pressure equal to the intake piston displacement, divided by the actual work done in the air cylinder. This would be the area *AKCG* (Fig. 18) divided by the shaded area or the actual work done in the air cylinder.

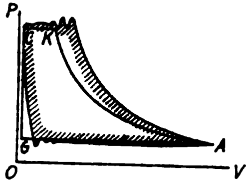


Fig. 18.

The **efficiency of compression** may be defined as the product of the cylinder efficiency and the true volumetric efficiency, or it is the work done in a complete cycle to compress isothermally (without clearance) a given volume of free air, divided by the work actually expended in compressing the same volume of free air.

The **mechanical efficiency** of an air compressor is the work done in the air cylinders divided by the work done in the engine cylinders if driven direct by steam or gas engine, or by the work delivered at the belt if the compressor is belt-driven.

Actual Values of Efficiencies. Tests of piston compressors show extreme variations of mechanical efficiency from 76 to 97 per cent., with approximate averages for the more common sizes of 85 per cent.

The true volumetric efficiency of piston air compressors will vary from 80 to 97 per cent., and the cylinder efficiency for water-jacketed compressors from 80 to 85 per cent. This will result in efficiencies of compression varying from 64 to 82 per cent.

Multiplying the mechanical efficiency by the efficiency of compression will show variations from 48 to 79 per cent. That is, the energy required to compress a certain amount of air isothermally is only from 48 to 79 per cent. of that actually expended in the steam cylinder. Tests show that with the great majority of compressors this product ranges from 50 to 60 per cent., while with some of the best compressors under test conditions it will reach as high as 78 per cent.

Horse Power Required to Compress Air. Disregarding clearance, the horse power required to compress air in a single-stage compressor is:

$$\text{h.p.} = \frac{144}{33{,}000}\,\frac{n}{n-1}\,p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

where V_1 represents the volume (cu. ft.) of free air compressed per min. and p_1 and p_2 the pressures in lb. per sq. in. abs.

Representing the intercooler pressure by p_i, the work done in both cylinders of a two-stage compressor will be

$$\text{h.p.} = \frac{144}{33{,}000}\,\frac{n}{n-1}\,p_1V_1\left[\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p_i}\right)^{\frac{n-1}{n}} - 2\right]$$

With perfect intercooling, $p_1V_1 = p_iV_i$. The above expression for the total work will be a minimum when $p_i = \sqrt{p_1p_2}$.

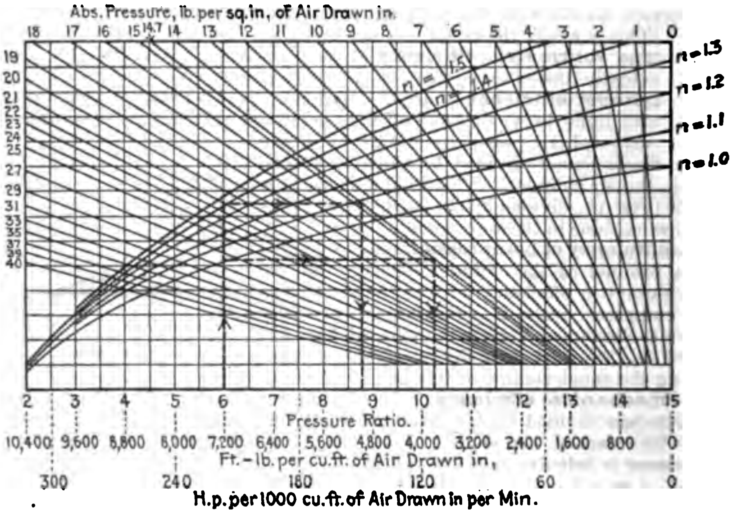


FIG. 19.—Chart for Determining the Work Done in Single-stage Air Compression (Lucke).

The proper intercooler pressures for three-stage compression are:

First intercooler, $p = \sqrt[3]{p_1^2p_2}$; second intercooler, $p = \sqrt[3]{p_1p_2^2}$

The minimum work done in compressing air is given by

$$\text{h.p.} = 2 \times \frac{144}{33{,}000}\,\frac{n}{n-1}\cdot p_1V_1\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{2n}} - 1\right] \text{ for two-stage compressors.}$$

$$\text{and h.p.} = 3 \times \frac{144}{33{,}000}\,\frac{n}{n-1}\cdot p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{3n}} - 1\right] \text{ for three-stage compressors.}$$

With perfect intercooling the volumes of the cylinders should be inversely as the pressures of the air admitted to them.

Professor Lucke ("Engineering Thermodynamics") gives a number of charts for solving graphically many of the problems of air compression, of which Fig. 19 is a very convenient one for determining the work per cu. ft. and the horse power per 1000 cu. ft. of free air compressed in a single stage to any pressure up to 15 times its initial pressure and for various kinds of compression curves from the isothermal to the adiabatic.

In Fig. 19 the diagonal lines represent various absolute pressures for the free air drawn in and the curved lines apply to various kinds of compression curves. The lower horizontal scale gives pressure ratios, work and horse power. In using the curves follow vertically from the pressure ratio to the n curve, horizontally to the inlet pressure line and vertically downward to the horizontal axis where the work and horse power may be read. For example, if the compression ratio is 6, the compression curve follows the equation $p_1V_1^{1.4} = p_2V_2^{1.4}$, and the free air is at 14.7 lb. per sq. in. abs., there will be required 4960 ft.-lb. of work per cu. ft. of free air compressed, or 152 h.p. per 1000 cu. ft. of free air per min.

If the compression for the same pressure range (or to 88.2 lb. per sq. in. abs.) had followed the isothermal compression curve it would have required 3880 ft.-lb. of work per cu. ft., or 120 h.p. per 1000 cu. ft. per min.

Fig. 20 enables calculations to be made for work and horse power for two- and three-stage compression, when used in connection with Fig. 19. The dotted lines represent two-stage and the full lines three-stage compression, and are marked according to the character of the compression curve. The horizontal scale shows pressure ratios and the vertical scale ratio of work or horse power for two- or three-stage compression to the work that would be required for single-stage compression as determined from Fig. 19. For example, for a pressure ratio of 8, or a discharge of 117.6 lb. per sq. in. abs. on a suction pressure of 14.7 lb., the work for a compression following the equation $p_1V_1^{1.4} = p_2V_2^{1.4}$ would be 85.2 per cent. of the work for the same conditions single-stage if two-stage were used, and 81 per cent. of it if three-stage compression were used.

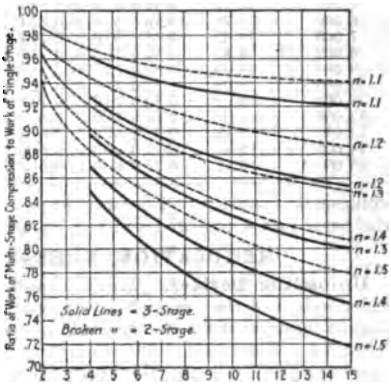


Fig. 20.—Chart for Determining the Work Done in Two- and Three-stage Air Compression (Lucke).

Effect of Altitude. As the density of the atmosphere decreases with the altitude, a compressor located at a high altitude will take in a smaller weight of air at each stroke. The reduction of pressure at the inlet affects the power expended in compressing the air, but the decrease in power required does not vary in the same ratio as the decrease in capacity. For this reason compressors to be used at high altitudes should have the steam and air cylinders properly proportioned to meet the varying conditions at different levels. Table 4, published by the Sullivan Machinery Co., of Chicago, Ill., shows the variation in capacity and horse power for various altitudes. The altitudes given are heights above mean sea level and are subject to correction for temperature and latitude. From the table it can be seen that for a two-stage compressor discharging at 100 lb. pressure when operating at an altitude of 8000 ft. the

Table 4. Volumetric and Horse-power Coefficients for Two-stage Air Compression

Altitude, ft.	Barom. press., lb. per sq. in.	Terminal gage pressure, pounds per sq. in.													
		70		80		90		100		120		140		150	
		H.p. coeff.	Volum. coeff.	H.p. coeff.	Volum. coeff.	H.p. coeff.	Volum. coeff.	H.p. coeff.	Volum. coeff.	H.p. coeff.	Volum. coeff.	H.p. coeff.	Volum. coeff.	H.p. coeff.	Volum. coeff.
Sea level	14.72	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1,000	14.17	0.98	0.97	0.98	0.97	0.98	0.97	0.98	0.97	0.98	0.97	0.98	0.97	0.98	0.97
2,000	13.64	0.97	0.94	0.96	0.94	0.96	0.94	0.96	0.94	0.96	0.93	0.96	0.93	0.96	0.93
3,000	13.13	0.95	0.91	0.95	0.91	0.94	0.91	0.94	0.91	0.94	0.90	0.94	0.90	0.94	0.90
4,000	12.64	0.93	0.88	0.93	0.88	0.93	0.88	0.92	0.88	0.92	0.87	0.92	0.87	0.92	0.87
5,000	12.17	0.91	0.85	0.91	0.85	0.91	0.85	0.91	0.85	0.90	0.84	0.90	0.84	0.90	0.84
6,000	11.71	0.90	0.82	0.89	0.82	0.89	0.82	0.89	0.82	0.88	0.82	0.88	0.81	0.88	0.81
7,000	11.27	0.88	0.80	0.88	0.79	0.87	0.79	0.87	0.79	0.86	0.79	0.86	0.78	0.86	0.78
8,000	10.85	0.86	0.77	0.86	0.77	0.85	0.77	0.85	0.76	0.85	0.76	0.84	0.76	0.84	0.76
9,000	10.45	0.85	0.75	0.84	0.74	0.84	0.74	0.83	0.74	0.83	0.73	0.82	0.73	0.82	0.73
10,000	10.06	0.83	0.72	0.83	0.72	0.82	0.72	0.82	0.71	0.81	0.71	0.81	0.71	0.80	0.70
11,000	9.69	0.82	0.70	0.81	0.70	0.80	0.69	0.80	0.69	0.79	0.68	0.79	0.68	0.79	0.68
12,000	9.33	0.80	0.68	0.79	0.67	0.79	0.67	0.78	0.67	0.78	0.66	0.77	0.66	0.77	0.66
13,000	8.98	0.78	0.65	0.78	0.65	0.77	0.65	0.77	0.64	0.76	0.64	0.75	0.63	0.75	0.63
14,000	8.64	0.77	0.63	0.76	0.63	0.76	0.62	0.75	0.62	0.74	0.62	0.74	0.61	0.74	0.61
15,000	8.32	0.75	0.61	0.74	0.61	0.74	0.60	0.74	0.60	0.73	0.59	0.72	0.59	0.72	0.59

volumetric capacity will be only 76 per cent. of that at mean sea level, while the horse power required will be 85 per cent. of that at mean sea level.

REGULATION, REHEATING, LUBRICATION

Unloading Devices. Many compressors operate at **constant speed,** independent of the demands for compressed air; and, in order to secure economy of operation for this condition, various types of "unloaders" have been designed. For small single-stage compressors the Sullivan Machinery Co. provides an unloading valve connected by piping to the inlet valves. When the predetermined pressure is exceeded, the unloader raises the inlet valves from their seats and prevents further compression of air until the pressure falls a few pounds, when the unloader allows the valves to resume their seats and the work of compression is again taken up. Other manufacturers use a similar device to keep the inlet closed when the predetermined pressure is reached. For larger compressors a double-beat valve is used, which is placed on the air inlet duct and controlled by air pressure from the air receiver. This valve is set to shut off all or part of the incoming air from the compressor when the receiver pressure rises above a predetermined point.

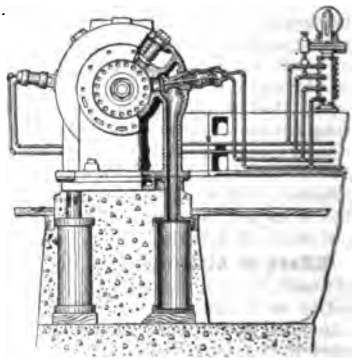


Fig. 21.—Ingersoll-Rand Automatic Clearance Unloader.

The Ingersoll-Rand automatic "clearance unloader" (Fig. 21) consists of a number of clearance pockets which are thrown automatically into communication with the ends of each air cylinder in proper succession, the process being controlled by predetermined variation in receiver pressure. Regulation is obtained in five stages, *viz.*, full load, three-quarter, half, quarter and no load. The effect of thus changing the clearance space is to change the volumetric efficiency without changing the suction or discharge pressures. Cards for the high-pressure cylinder of a two-stage compressor, showing the operation of this device, are reproduced in Fig. 22. The device is applied simultaneously

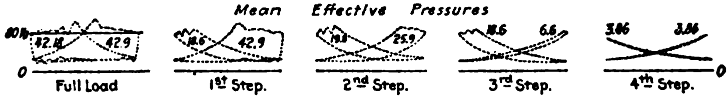


FIG. 22.—Cards from High-pressure Cylinder of a Two-stage Compressor with Clearance Unloader.

to all the cylinders of a multi-stage compressor. The performance of a 20¼-in. and 12¼-in. × 18-in. direct-connected motor-driven compressor (piston displacement 1087 cu. ft.) operating at sea level with this type of unloader is given by F. D. Longacre as follows (r.p.m., 175; discharge pressure, 100 lb.):

	Full load	¾ load	½ load
Actual capacity, cu. ft. free air per min.	940.0	698.0	458.0
Compression effy., isothermal base, per cent	80.0	76.5	73.0
Indicated horse power of air cylinder	164.5	129.0	91.0
Mechanical efficiency, per cent	92.0	89.0	86.0
Brake horse power of motor	179.0	145.0	105.8
Motor efficiency, per cent	92.5	91.5	90.0
Elec. horse-power input per 100 cu. ft. actual	20.53	22.6	25.78

He gives a comparison of the action of clearance and choking control between zero and full load for a 1000-cu. ft. compressor, as shown in Fig. 23.

Regulators. When the **speed is not constant**, other types of regulating devices are available. For simple belt-driven compressors a belt shifter can be designed to shift the belt from the tight to the loose pulley when the predetermined air pressure is reached, but for steam-driven compressors the device is usually one for governing the speed of the compressor to suit the demands for compressed air.

A combined air and speed governor manufactured by the Nordberg Mfg. Co. is shown in Fig. 24. In this type of governor the speed of the engine is controlled not only by the centrifugal action of the governor, but also by any variation of the air pressure. The arm *A* controls the point of cut-off for the steam cylinder and is operated by the movement of the bell-crank *C* about the fulcrum *D*. The rod *E* controls the bell-crank and is connected to a "floating lever" *B*. This is con-

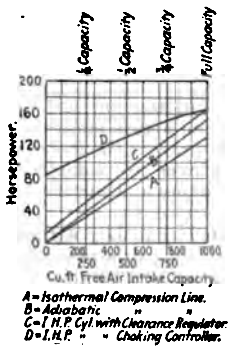


FIG. 23.

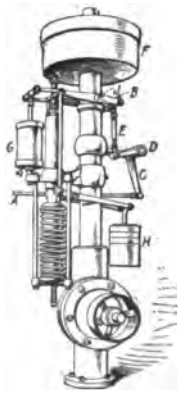


FIG. 24.

FIG. 23.—Comparison of Clearance and Choking Control.

FIG. 24.—Nordberg Combined Air and Speed Governor.

nected with the centrifugal governor F at J, and with a piston which is in communication with the air pressure at G and is held up by the weight H. With this arrangement, if the air pressure should rise above normal, the engine will have its supply of steam per stroke reduced, and if the air pressure should fall, the supply of steam will be increased; or, if the pressure of air remains constant, the governor will have the same control over the speed of the engine that the ordinary centrifugal governor has.

Water Jackets, Intercoolers, Receivers and Aftercoolers. The cooling surface in an intercooler is generally designed from the formula $S = Q/0.25(t_a - t_w)$, where S is the cooling surface in sq. ft., Q is the number of cu. ft. of free air per min., and t_a and t_w are the temperatures, in deg. fahr., of the air leaving and the water entering the intercooler, respectively.

The amount of cooling water required is given by Longacre as $G = 75 + 2.5t$ when the intercooler and jacket are in series; $G = 20 + 2t$ for a separate intercooler; and $G = 2t$ for l.-p. and h.-p. jackets only. In these formulæ G is in gallons of water per hour per 100 cu. ft. of free air per min. and t is the entering temperature, deg. fahr., of the cooling water.

Receivers are used to supply a reservoir of air; to equalize the pulsations in the air coming from the compressor; to collect the water and grease held in suspension by the compressed air as it leaves the compressor; to reduce the friction of air in the pipe system; and to cool the air as thoroughly as possible before entering the transmission system. To facilitate the removal of water from the compressed air, a receiver is frequently equipped with a coil of pipes through which cooling water flows; in this way it serves as an **aftercooler**, and by precipitating the water from the air reduces difficulties in transmission lines and tools. The Ingersoll-Rand Co. states the amount of cooling water required for aftercoolers for air at 80 to 100 lb. pressure to be as follows:

Temp. of cooling water, deg. fahr.	50	60	70	80	90
Gal. per min. per 1000 cu. ft. of free air per min.	120	140	160	180	200

When the transmission pipe line is long, receivers should be placed at both ends of the pipe.

Reheaters. Heating the air just before expansion may increase the efficiency of the system, and in addition will increase the temperature at the end of expansion and prevent the freezing up of the motor.

In quarry work **stoves** are sometimes used for preheating the air. In locomotive work for mines and surface use, hot water is frequently employed for this purpose.

Reheaters are usually capable of raising the **temperature** of the air to from 300 to 500 deg. fahr., although common practice shows temperatures of from 250 to 350. In figuring on reheaters it is usual to assume that 1 lb. of coal will give from 8000 to 10,000 B.t.u. to the air. As the specific heat of air is approximately 0.24, 1 lb. of coal will raise the temperature of approximately 100 lb. (or 1200 cu. ft.) of free air 300 deg.

The **increase in efficiency resulting from reheating** is greater with tools that use air expansively than with machines taking in air for full stroke. Sometimes it is not desirable to have the air entering a tool at a temperature above 300 deg., because of the effect of this high temperature on the lubrication. For these conditions small portable hot-water stove-type reheaters are available in capacities of from 62 to 800 cu. ft. of air per min.

Lubrication. If oil is fed too rapidly in the air cylinders there is a gradual accumulation of carbon, which interferes with the free movement of the valves and may actually choke the passages and produce high temperatures sufficient to produce ignition or **explosion.**

Explosions have taken place from the introduction of kerosene or naphtha into the air cylinder for the purpose of cleaning the valves and cutting away the carbon deposits. This is a very effective way of **cleaning valves and pipes,** but is a source of danger and should be absolutely prohibited. Soft soap and water is the best cleanser for the air cylinder and is recommended even when the best grades of cylinder oil are used, feeding once or twice a week in order to prevent any gumming of the valves.

In order to reduce the danger of excessive temperatures, fusible **safety alarm plugs** may be inserted in the discharge line. These are usually set for a temperature of 350 deg. for a single-stage compressor working at 40 lb. gage pressure, for a two-stage compressor at 100 lb. gage, and for a three- or four-stage compressor delivering at 1000 lb. gage. A 500-deg. plug is furnished for use with a single-stage compressor discharging at 100 lb. gage pressure.

AIR CONSUMPTION OF VARIOUS TOOLS

Air Consumption of Rock Drills. Table 5 gives the free-air consumption per min. of rock drills of various cylinder diameters for fair conditions in

Table 5. Cubic Feet of Free Air Required per Minute by One Rock Drill

(Ingersoll-Rand Co.)

Gage pressure, lb. per sq. in.	Cylinder diameter of drill, inches												
	2	2¼	2½	2¾	3	3⅛	3³⁄₁₆	3¼	3½	3⅝	4¼	5	5½
60	50	60	68	82	90	95	97	100	108	113	130	150	164
70	56	68	77	93	102	108	110	113	124	129	147	170	181
80	63	76	86	104	114	120	123	127	131	143	164	190	207
90	70	84	95	115	126	133	136	141	152	159	182	210	230
100	77	92	104	126	138	146	149	154	166	174	199	240	252

rock of ordinary hardness. To find the amount of free **air required by a number of drills** supplied from the same compressor, multiply the air required by one drill, as given in Table 5, by one of the following factors:

Number of drills..........	2	3	4	5	6	7	8	9	10
Factor (*a*)................	1.8	2.7	3.4	4.1	4.8	5.4	6	6.5	7.1
Number of drills..........	12	15	20	25	30	40	50	60	70
Factor (*a*)................	8.1	9.5	11.7	13.7	15.8	21.4	25.5	29.4	33.2

To find the **free air consumption at any altitude** above sea level, multiply the consumption at sea level as obtained above by the one of following factors:

Altitude, ft.......	1,000	2,000	3,000	4,000	5,000	6,000
Factor (*b*)........	1.03	1.07	1.10	1.14	1.17	1.20
Altitude, ft.......	7,000	8,000	9,000	10,000	12,000	15,000
Factor (*b*)........	1.23	1.26	1.29	1.32	1.37	1.43

Example. Required the amount of free air to operate thirty 5-in. drills at an altitude of 9000 ft., air being furnished at a gage pressure of 80 lb. per sq. in.

From Table 5, cu. ft. per min. for one 5-in. drill at 80 lb. gage pressure = 190. For 30 drills, factor (*a*) = 15.8, and for 9000 ft. altitude, factor (*b*) = 1.29. Whence, cu. ft. of free air per min. for thirty 5-in. drills at 9000 ft. altitude = $190 \times 15.8 \times 1.29 = 3872$. The capacity of the compressor should be larger than this, to provide for leakage and friction in the pipe lines.

Air Consumption of Pneumatic Tools. The Sullivan Machinery Co. gives the following data on the consumption of free air (cu. ft. per min.) of various industrial tools and machines with air pressures from 70 to 90 lb. gage. Small hand paint sprays, 2–3 cu. ft.; foundry jolting machines, platform type, 30–40 cu. ft. per ton lifting capacity; hand grinders, 20 lb. size, 20 cu. ft; molding machines and squeezers, ½ to 1½ cu. ft. per mold; riveting machines, reach 12 to 30 (36 to 45) in., 5 (6½) cu. ft. per rivet; surfacers, small size (35 lb. approx.), 30 to 40 cu. ft., large size (65 lb. approx.), 60–70 cu. ft.

Chipping hammers	Weight, lb	5	7	8	9	10	11	12	13	14	18	
	Cu. ft. per min	6	10	12	13	15	17	18	20	20	22	
Hoists, direct lift	Cylinder diam., in	6	8	10	12	14	17	19				
	Cu. ft. per ft. lift, 2-to-1 lift	0.8	1.5	2.2	3.3	4.7	6.6	8.1				
	Cu. ft. per ft. lift, 4-to-1 lift	0.4	0.7	1.5	1.7	2.4	3.0	4.0				
Geared hoists	Capacity, tons	1	1½	2	3	4	5	6	8	10	12½	
	Cu. ft. per ft. lift	3	5	6	8	10	15	20	25	30	40	
Jacks	Cylinder diam., in	8	10	12	14	16	20	24				
	Cu. ft. per ft. lift	1.8	2.8	4.0	5.4	6.9	11.1	15.8				
Motors	Horse power	2	4	5	8	15						
	Cu. ft. per min	40–50	60–75	90–100	125	240						
Pile hammers	Weight, lb	145	640	1500	5000	7500						
	Cu. ft. per min	100	150	200	350	600						
Hand riveters	Size, lb	13	14–15	16–17	19–20	21–22	23–25					
	Cu. ft. per min	16	18	20	22	24	25					
Rotary drills	Weight, lb	10	15	20	30	35	40	45	50	60	70	75
	Cu. ft. per min	15	18	20	25	27	30	35	40	43	45	48
Rotary flue rolling, reaming and tapping machines.	Weight, lb	30	40	50	55	65	130					
	Cu. ft. per min	20	25	30	35	35	45					
Sand blast (80 lb. pressure).	Size nozzle, in	3/16	¼	⅜	½							
	Sand per hour, lb	500	900	1700	3000							
	Cu. ft. air per min	45	85	190	340							
Hand sand rammers.	Weight of tool, lb	7	18	24	30							
	Cu. ft. per min	9	15	20	25							
Suspended sand rammers.	Weight, lb	280	325	370	925							
	Cu. ft. per min	50	90	100	130							
Stone-carving tools	Weight of tool, lb	1	2	3	4	5	9					
	Cu. ft. per min	3–6	4–7	5–8	6–9	7–10	12					
Wood-boring machines.	Weight, lb	10	15	30								
	Cu. ft. per min	15	20	25								

Compressor Dimensions. The Ingersoll-Rand Co. manufactures small single-acting compressors of the following sizes: 2½ × 3 in., running at from 450 to 700 r.p.m.; 3½ × 4 in., at from 350 to 550 r.p.m., and 4½ × 5 in., at from 325 to 500 r.p.m. Its double-acting compressors vary in speed from 100 to 225 r.p.m., direct-electric-driven compressors from 110 to 225 r.p.m., steam-driven compressors with simple or Meyer type of slide-valve gears from 150 to 225 r.p.m., and Corliss-valve-gear steam-driven compressors of from 1450 to 8000 cu. ft. per min. capacity at speeds of from 80 to 150 r.p.m.

The average **piston speeds** of commercial air compressors will vary approximately from 300 ft. per min. for the smaller sizes to 500 ft. per min. for the larger sizes.

Compressed-air Hoisting Engines. Compressed air can be used in any engine that will operate with steam and finds considerable application for hoisting engines for mining, or for work of a temporary or intermittent character. R. L. Streeter (*Eng. Mag.*) gives the following table of free-air consumptions for single-cylinder hoisting engines, the air being compressed to 60 lb. gage:

Cyl. diam., in.	Stroke, in.	R.p.m.	Nominal h.p.	Actual h.p.	Weight lifted by single rope	Free air per min., cu. ft.
5	6	200	3	5.9	600	75
5	8	160	4	6.3	1,000	80
6¼	8	160	6	9.9	1,500	125
8¼	10	125	15	12.1	2,000	151
7	10	125	10	16.8	3,000	170
8½	12	110	20	18.9	5,000	238
10	12	110	25	26.2	6,000	330

Compressed-air Locomotives are used in mines and quarries for handling excavated material, and are particularly adapted to the handling of explosives or other material where it is necessary to reduce the fire hazard to a minimum. Such locomotives are usually supplied with compressed air at a pressure of approximately 800 lb. per sq. in. The air is admitted to the cylinders at a reduced pressure varying from 150 to 250 lb. per sq. in., and in some cases is exhausted after a single expansion. In the more economical types of locomotives, the air after expansion in the first cylinder with consequent reduction of temperature passes to a second cylinder, and in doing so is reheated by the surrounding atmosphere before its final expansion and discharge. This reheating effects an increase of from 40 to 60 per cent. in the work done by the locomotive.

For the A. S. M. E. Code for testing steam-driven air-compressing machinery, see p. 1776.

CENTRIFUGAL COMPRESSORS

BY

L. C. LOEWENSTEIN

REFERENCES: Ostertag, "Kolben und Turbo-Compressoren," Springer, Berlin. Zerkowitz, "Thermodynamik der Turbomaschinen," Oldenbourg, Berlin.

Classification. Centrifugal compressors differ from centrifugal pumps (see p. 1503) only in handling gases instead of liquids, and are similarly classified as regards the number of inlets per impeller, the direction of the impeller tips at impeller exit, and the number of stages employed. They are further classified into low-pressure (1 to 5 lb. per sq. in.) and high-pressure (above 5 lb.) compressors, and also into the radial-inlet type and the axial-inlet type according as the gas enters at right angles to the shaft or in a direction parallel to the shaft.

Centrifugal compressors for pressures below 1 lb. per sq. in. are generally known as blowers or centrifugal fans (see p. 1541); in these the kinetic energy of the gas at the impeller exit is usually allowed to dissipate itself in eddies. For air pressures of 5 lb. per sq. in. and under, a single impeller is generally sufficient. For comparatively light gases, however, a pressure of 5 lb. may require two or more impellers in series, or a multi-stage compressor. Such a compressor is also frequently spoken of as a high-pressure gas compressor. For quantities of gas of 10,000 cu. ft. per min. and over, compressors of the radial-inlet type require shrouds or reinforcing rings at the inner ends of the impeller blades to prevent the wide blades from crumpling at the inlet under the action of centrifugal stresses. Impellers of the axial-inlet type are not subject to such crumpling, and are therefore generally used when large volumes are handled. As to single-inlet and double-inlet impellers, see remarks on p. 1503. The radial-discharge impeller is the one best adapted for high peripheral speeds, and is the type most commonly used. Both the backward-discharge and the forward-discharge impellers require shrouds at their outer peripheries. The former are frequently resorted to when large gas quantities (requiring a large impeller inlet) are to be raised to a comparatively low pressure with a direct-connected high-r.p.m. driver; the latter are but rarely used. Multi-stage compressors are usually provided with special means for cooling the gas during its passage through each impeller and from stage to stage, and also for preventing leakage from stage to stage and to the atmosphere.

Advantages and Applications. The centrifugal compressor occupies comparatively little room for its output; direct-connected to an electric motor or to a steam turbine, it forms a very compact unit. Besides its bearings it has no rubbing or wearing parts; it contains no moving valves or springs; and it requires a minimum of attendance and oiling. It is also fairly free from vibrations and requires comparatively light foundations. At constant r.p.m. it will maintain approximately constant pressure for widely varying quantities of gas, which makes it very desirable for general power transmission. If no gas is required temporarily the discharge pipe may be shut off without stopping the compressor or wasting the gas into the atmosphere. Its steadiness of blast makes it also very valuable for oil burning and for general forge work.

For blast furnaces, where the resistance to the flow of the air is likely to vary from time to time, while a uniform supply of oxygen or air regardless of such variation in resistance is important, the centrifugal compressor may be supplied with a constant-volume governor. This governor, actuated by the variation of air velocity in the compressor inlet, causes the speed of the driver to vary in accordance with the needs of the furnace, and thus maintains a constant volume (referred to atmospheric conditions) of air against widely varying pressures with fairly constant efficiency. The centrifugal compressor also finds wide application as an exhauster in ash conveying, sawdust conveying, and in the general pneumatic conveying of coal, cement, rice, starch, etc. For intermittent work, such as pneumatic cash and mail conveying, its prompt response to overloads allows the use of a comparatively small driver. In coke-oven-gas manufacture the centrifugal compressor maintains constant suction on the gas main, and then compresses the gas so that it will flow through the condensers, purifiers, and into the gas holders. It is frequently used as a booster to a high-pressure reciprocating compressor, the two compressors forming together a very compact and efficient set. By compressing the air, say, to 30 lb. per sq. in. gage in the centrifugal compressor, the volume of the air to be handled by the reciprocating compressor is only about one-third of what it would otherwise be, making it possible to employ a much smaller unit.

Limitations. While a centrifugal compressor will maintain a fairly constant pressure over a wide range of quantities, there is for every speed a certain range of quantity at which the discharge vanes cease to co-operate, causing a sudden drop in pressure. This "break-down" region can be pushed back toward lighter loads, and the drop in pressure made less abrupt by making the discharge vanes very few and their inlet angle small. Also, when working on that part of the pressure curve where the pressure increases with the quantity or remains constant, there are usually pressure surges or pulsations which, while slight in themselves, may be greatly intensified by a sort of resonance effect if the volume of the inlet and of the discharge piping happens to have a certain critical value. A slight throttling of the inlet will always stop these pulsations by making the pressure curve slightly drooping.

Theory

Notation. Referring to Fig. 1, let

$D_e(D_a)$ = impeller inlet (exit) diameter, ft.
$u_e(u_a)$ = impeller inlet (exit) peripheral velocity, ft. per sec.
$w_e(w_a)$ = absolute inlet (exit) velocity of gas, ft. per sec.
$v_e(v_a)$ = relative inlet (exit) velocity of gas, ft. per sec.
$b_e(b_a)$ = impeller inlet (exit) angle, deg.
d_e = angle between w_e and u_e, deg.
d_a = angle between w_a and u_a, deg., or inlet angle of discharge vanes, if any.

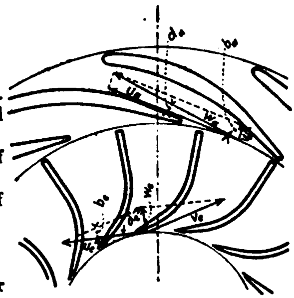


FIG. 1.

p_1 = initial pressure of the gas, including the velocity energy (if any), lb. per sq. in.
T_1 = temperature of gas corresponding to p_1, deg. fahr. abs.
d_1 = density of gas corresponding to p_1, lb. per cu. ft.
p_2, T_2, d_2 = corresponding values for the final conditions of the gas as it leaves the compressor.
K = ratio of specific heats of gas, taken as 1.41.

$A = (T_2/T_1) - 1 = (p_2/p_1)^{(K-1)/K} - 1 = (p_2/p_1)^{0.289} - 1.$

s = specific gravity of gas at inlet conditions, referred to that of "free air" as unity.

Q = quantity of inlet gas, cu. ft. per sec.

e_h = hydraulic efficiency, referred to adiabatic compression.

H = the theoretical (or total) head, or height against which the gas is raised, ft., including all hydraulic losses.

N = r.p.m.

Total Pressure Rise. The fundamental equation giving the value of H is the same as that for H_b (see Centrifugal Pumps, p. 1504). For single-stage compression,

$$A = \left(\frac{p_2}{p_1}\right)^{(K-1)/K} - 1 = \frac{e_h d_1 (K-1) H}{144 p_1 K} = \frac{e_h d_1 (K-1) u_a^2}{4631 p_1 K}$$

when $d_e = 90$ deg. (no inlet guide vanes) and $b_a = 90$ deg. (radial discharge). Assuming $p_1 = 14.7$, $T_1 = 520$, $K = 1.41$, $e_h = 0.72$, and substituting, $p_2 = 14.7[1 + (u_a^2 s/4{,}300{,}000)]^{3.44}$. For small pressure rise, with $d_e = 90$ deg., $p_2 - p_1 = 0.0000165 e_h u_a^2 s[1 + (v_a/u_a) \cos b_a]$.

Fluid Input Horse Power is the horse power applied to the gas and is independent of the actual pressure rise obtained. Fluid input h.p. = $Q d_1 H/550$. For $d_e = 90$ deg., this becomes $0.00000432\, Q s u_a^2 [1 + (v_a/u_a) \times \cos b_a]$.

Theoretical Horse Power. This is the horse power necessary to compress (and deliver) Q cu. ft. of gas per sec. from p_1 to p_2.

Theoretical horse power $= 0.901\, Q p_1 (p_2/p_1)^{0.289} - 1$; or, for small pressure rise $= 0.2618 Q p_e$, where $p_e = p_2 - p_1$. This formula may also be used for higher pressures if $p_1 = 14.7$ lb. per sq. in. and p_e ($= 50.6A$) is regarded as the mean effective pressure for adiabatic compression from p_1 to p_2. Table 1 gives values of the mean effective pressure p_e against values of $p_2 - p_1$, for the usual case of $p_1 = 14.7$.

Table 1

(p_2 − 14.7), lb. per sq. in.	5	10	15	20	25	30	35	40	45	50
p_e, lb. per sq. in.	4.5	8.2	11.4	14.3	16.9	19.3	21.5	23.5	25.4	27.2

Table 2 gives the theoretical horse power necessary to compress adiabatically (and deliver) 100 cu. ft. of air per min. from 14.7 lb. abs. to various gage pressures.

Table 2

Final gage pressure, lb. per sq. in.	5	10	15	20	25	30	35	40	45	50
Theoretical horse power per 100 cu. ft.	1.95	3.58	4.99	6.25	7.37	8.39	9.35	10.23	11.06	11.84

Hydraulic Losses. The hydraulic losses are the losses in pressure caused by the gas friction and by the sudden changes in the gas velocity or direction of flow. On the basis of Mr. D. W. Taylor's experiments on the flow of air in pipes, the pressure drop in the suction and in the discharge pipes (lb. per sq. in.) $= L = l v^2 s/400{,}000D$, where l is the length of pipe, ft., v the velocity of gas, ft. per sec., s the specific gravity of gas referred to free air (0.0764) as unity, and D the diameter of pipe, in. For pipes of first-class workmanship and in very best condition, this loss may be reduced by about 20 per cent. (See also p. 357 for further data on friction losses in pipes.) The other hydraulic losses are of the same nature and can be reduced in the same way as explained under Centrifugal Pumps, p. 1504.

Hydraulic Efficiency. The hydraulic efficiency is the ratio of the theoretical horse power to the fluid input horse power, or $e_h = 184.8AT_1/H$. For $d_e = 90$ deg.,

$$e_h = \frac{5955\ A\ T_1}{u_a^2[1 + (v_a^2/u_a^2)\cos b_a]}. \quad \text{Also, } e_h = \frac{60{,}600(p_2 - p_1)}{s\ u_a^2[1 + (v_a^2/u_a^2)\cos b_a]}$$

for small pressure rise, with $d_e = 90$ deg.

Other Losses. The **rotation loss,** or friction loss of the impeller considered as a flat disk of negligible axial width, as it rotates in the compressed gas, may be obtained from the formula: Rotation loss (h.p.) $= 0.0737 \times (u_a/1000)^3 D_a^2 d_m$, where d_m is the mean density of the gas (between p_1 and p_2), lb. per cu. ft. At full load this is practically the only loss to be considered, and it is this loss, more frequently than the centrifugal stresses, that determines the pressure rise that may be developed by a single impeller. The nature of the **short-circuit loss** has been explained under Centrifugal Pumps, p. 1505. It is highest when v_a, the relative velocity of the gas, is lowest, that is, at light loads. For the same reason, it is roughly proportional to the impeller outer diameter, the impeller exit width, and the passage height at the discharge-vane inlet. At no load the short-circuit loss is between two and four times the rotation loss, while at full load it is fairly negligible.

Centrifugal Compressor Constants and Characteristic Curves

Quantity Constant. The quantity of gas delivered by a centrifugal compressor is proportional to $u_a D_a b_a$, or, quantity constant $= u_a D_a b_a$.

Compressor Constant. A compressor model can be used with practically the same efficiency for various combinations of quantity and pressure such that $K = Q/\sqrt{p_e}$, where K is the compressor constant, Q and p_e are the desired quantity and mean effective pressure, respectively. The compressor constant can be more conveniently written as $K = QN^2/p_e^{3/2}$.

Similar Compressors. Two compressors are similar when they have the same compressor constant. In similar compressors all impeller and discharge-vane linear dimensions are in the same ratio as their impeller diameters, while their impeller and discharge vane angles are respectively equal. For the same r.p.m. the quantities delivered by similar compressors will vary as the cubes of their diameters; the pressures will vary as the squares of their diameters, and the shaft powers will vary as the fifth powers of their diameters. For the same wheel speed the quantity and the power will vary as the square of the diameter, while the pressure will remain constant.

Compressor Coefficients. In order to make the tests on different compressors, or on the same compressor under different circumstances, comparable on a common basis, various coefficients are computed corresponding to the given observations and these coefficients are plotted as **characteristic curves.** From these curves the pressure, power, and the hydraulic and shaft efficiencies can readily be computed for any quantity of gas and any r.p.m. The departure of the ratio v_a/u_a from that value for which the compressor was designed determines largely the efficiency of operation. Therefore, v_a/u_a, or its equivalent, Q/u_a, is generally used as the abscissa for the characteristic curves of a compressor, and it is designated as the **load coefficient,** C_q. It is also frequently represented by Q/N. The **fluid input coefficient** (C_i) represents the horse power corresponding to any given observation divided by the cube of the wheel speed. For the case of axial impeller inlet and radial impeller exit, $C_i = 0.00000432Q_s/u_a = 0.00000432C_s s$. Evidently the characteristic curve of C_i against C_s is a straight line making an angle with C_s whose tangent is $0.00000432s$; it may be drawn independently of the actual observations. In general, $C_i = 0.00000216s\,QV^2/u_a^3$, where $V^2 = 2gH = u_a^2 + w_a^2 - v_a^2 - u_e^2 - w_e^2 + v_e^2$. The **pressure coefficient** corresponding to the observed pressure rise $(p_2 - p_1)$, is $C_p = AT_1/u_a^2$, (for notation see p. 1531). The **theoretical power coefficient is** $C_t = 0.02571C_sC_p$. The characteristic curve for C_t should be computed and drawn from readings from the *smooth* curve of C_p against C_s. The **shaft power coefficient** is C_s = shaft h.p./u_a^3. The **rotation loss coefficient** is $C_r = 0.0737 \times 10^{-6} D_a d_m$, where d_m is the average density of the gas between p_1 and p_2, lb. per cu. ft. By adding the values of C_r to the values of C_i, a characteristic curve (practically a straight line) of fluid input plus rotation loss is obtained, and this curve will in a correctly designed compressor nearly touch the shaft power characteristic curve at the value of Q/u_a corresponding to the rated load of the compressor.

Hydraulic and Shaft Efficiencies. The ratio of C_t to C_i for any value of C_s gives the hydraulic efficiency e_h for that particular load. (For $C_s = 0$, the general formula $e_h = 5955\ C_p$ must be used). Similarly, ratios of C_t to C_s give values of e_s, the shaft efficiency. The efficiency curves thus obtained will of course be smoother and more reliable than if the efficiencies were computed directly from the individual observations of pressure and power.

Uses of Characteristic Curves. Besides affording smooth curves of hydraulic and shaft efficiencies, a set of characteristic curves as shown in Fig. 2 enables one readily to draw reliable pressure and power curves against quantity for any given wheel speed or r.p.m. In that case, the load-coefficient scale may be replaced by a quantity scale, while the readings of the C_p and the C_s curves will give the data for the corresponding pressures and powers. (In Fig. 2 the wheel speed has, in all the coefficients, been replaced by the r.p.m.) The system of characteristic curves is found to hold true for various sizes of centrifugal compressors, centrifugal blowers, and centrifugal pumps, giving in each case consistent curves regardless of the actual speeds, pressures and powers.

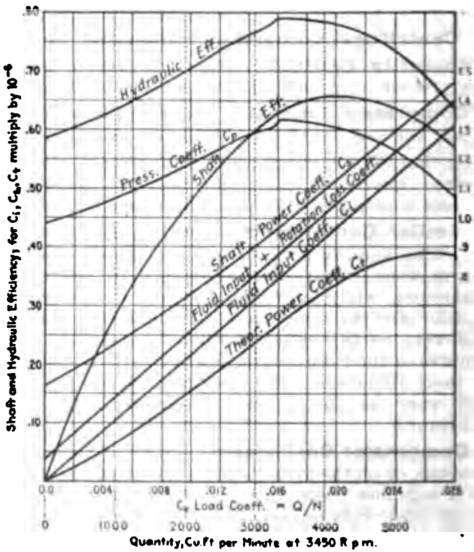


FIG. 2.—Characteristic Curves of Centrifugal Compressors.

Only in ventilating-fan blowers do there seem to be occasionally some serious discrepancies near the point of maximum efficiency.

Numerical Examples

Power Required. Find the power required to compress adiabatically 20,000 cu. ft of air per min. from atmosphere to 30 lb. per sq. in. gage, the shaft efficiency of the compressor being 75 per cent.

SOLUTION: From Table 2 the theoretical horse power is $8.4 \times 20{,}000/100 = 1680$, and the shaft h.p. is $1680/0.75 = 2240$.

Equivalent Suction Pressure. What suction can be obtained with a compressor rated to deliver 2500 cu. ft. of air per min. against 2 lb. per sq. in. gage?

SOLUTION: The compressor is rated for an *initial* pressure of 14.7 lb. per sq. in. Since the *pressure ratio* depends only on the wheel speed, the hydraulic efficiency and the initial temperature, all of which are supposed to remain the same, the initial suction pressure is $(14.7/16.7) \times 14.7 = 12.94$ lb. per sq. in. abs., and the suction obtained is $14.70 - 12.94 = 1.76$ lb. per sq. in.

Equivalent Pressure when Compressing Gas. What pressure can be obtained when compressing water gas with a standard unit rated 25,000 cu. ft. of air per min. and 15 lb. per sq. in. gage, and what power will be required if it requires 2000 h.p. to compress the air to 15 lb. pressure?

SOLUTION: The density of water gas is 0.05167, and its specific gravity (compared with air) is 0.677. The mean effective pressure (m.e.p.) corresponding to 15 lb. per sq. in. is 11.44 for air (see also Table 1). For water gas, everything remaining the same, m.e.p. $= 11.44 \times 0.677 = 7.74$ lb. per sq. in., and from Table 1 the corresponding final pressure is 9.3 lb. per sq. in. gage. The theoretical power for the air rating (from Table 2) is $5 \times 25{,}000/100 = 1250$ h.p. For water gas the theoretical horse power is $1250 \times 0.677 = 846$ h.p., and the actual power $= 2000 \times 0.677 = 1354$ h.p.

Equivalent Rating at Other Speeds. A standard centrifugal compressor rated 4500 cu. ft. of air and 15 lb. per sq. in. pressure is to be speeded up from 3450 r.p.m. to 4000 r.p.m. What increase of pressure and of quantity will result?

SOLUTION: The m.e.p. for 15 lb. per sq. in. is 11.44. At 4000 r.p.m. it will increase to $(4000/3450)^2 \times 11.44 = 15.38$, and the corresponding final pressure from Table 1 is 22.05 lb. per sq. in. abs. Also the new quantity will be $(4000/3450) \times 4500 = 5220$ cu. ft. per min. if the same ratio of v_a/u_a, and therefore the same hydraulic efficiency, is to be maintained. See "Quantity Constant" and "Load Coefficient," p. 1533.

General Problem. What standard compressor can be used to exhaust 18,500 cu. ft. of anthracite producer gas per min. against a suction of 7 lb. per sq. in.? The compressor is to be installed 2000 ft. above sea level. What horse power is required?

SOLUTION: The barometer at 2000 ft. altitude is 13.56. The compressor is therefore required to compress the gas from $13.56 - 7.00 = 6.56$ lb. per sq. in. abs. to 13.56 lb. This is equivalent to compressing the gas at sea level to a final pressure of $(13.56/6.56) \times 14.7 = 30.4$ lb. per sq. in. abs. or 15.7 lb. gage. The density of anthracite producer gas is 0.065 lb. per cu. ft. The m.e.p. corresponding to 15.7 final pressure is (Table 1) 11.9 lb. per sq. in. The corresponding m.e.p. for air is $(0.0764/0.065) \times 11.9 = 14.0$, and the final pressure from Table 1 is 19.4 lb. per sq. in. gage. Suppose the nearest standard compressor is rated 16,000 cu. ft. per min., and 15 lb. per sq. in. at 3200 r.p.m. The m.e.p. for 15 lb. is 11.44 and for 19.4 is 14.0. The standard compressor must therefore be speeded up to $3200 \times \sqrt{14/11.44}$, or 3540 r.p.m. For a constant "load coefficient" Q/N, the new quantity will be $16{,}000 \times 3540/3200 = 17{,}700$. So if it is desired to use the standard compressor and save the extra cost of a special size, then only 17,700 cu. ft. per min. of the gas can be exhausted with a suction of 7 lb. per sq. in. gage; or the compressor can be speeded up to handle 18,500 cu. ft. per min., but the suction will be somewhat above 7 lb. If the h.p. required by the standard compressor is 1300 (corresponding to a shaft efficiency of 0.615) the h.p. for the desired conditions will be $1300 \times (17{,}700/16{,}000) \times (11.9/11.44) = 1495$.

Multi-stage Centrifugal Compressors

General. A multi-stage compressor consists of a number of single-stage compressors connected in series. When the number of single-stage compressors is small they are usually enclosed in the same casing; but when the number exceeds eight or ten they are generally subdivided between two or more casings, separated by **intercoolers.** Multi-stage compressors are frequently built with single-inlet impellers, depending for the overcoming of the axial thrust on balancing pistons, on special grouping of the impellers, and on similar devices; also with backward-discharge impellers, when they are supplied with substantial shrouds to prevent the blades from bending or breaking under the action of centrifugal force. They are also built with all the impellers of the double-inlet type, thus obviating the need of all balancing means and requiring a much smaller number of impellers; the interstage passages, however, become rather complicated.

Similar Compressors. The successive impellers in a multi-stage compressor, handling smaller and smaller volumes of air, should be designed on the principle of similar compressors if all stages are to be equally efficient. Since the r.p.m. is the same for all impellers, the diameters and all the other dimensions should vary *inversely as the cube root of the density.* In practice, however, the impellers are divided into two or more groups, and each group designed for its own average conditions.

Cooling. The cooling of the gas during its passage through a multi-stage compressor is of paramount importance for high efficiency and low power consumption, and ample passages for cooling water must be provided in the diaphragms between the stages. For pressures below 50 lb. per sq. in., it is generally aimed to keep the temperature down to that corresponding to adiabatic compression; while for higher pressures, isothermal compression is more usually aimed at by the introduction of intercoolers between groups of stages. In the latter case, the hydraulic efficiency is given on the basis of isothermal compression.

Pressure Rise. For adiabatic compression, if the cooling is just enough to remove the heat above that corresponding to adiabatic compression:

$$p_2/p_1 = [1 + e_h d_1 (K - 1)\, u_a{}^2 S/144\, p_1 g K]^{K/(K-1)}$$

where S is the number of stages, while the other symbols have the same meaning as on p. 1531. If the cooling is such that each stage starts with the same temperature, the temperature corresponding to p_1, then

$$p_2/p_1 = [1 + e_h d_1 (K - 1) u_a{}^2/144\, p_1 g K]^{S(K-1)/K}$$

For strictly isothermal compression,

$$\log_{10}\frac{p_2}{p_1} = \frac{e_h d_1 u_a{}^2 S}{144\, p_1 g \times 2.302} = \frac{e_h d_1 u_a{}^2 S}{10{,}663\, p_1}.$$

In all these formulæ axial inlet flow and radial impeller exit are assumed; for all other cases $u_a{}^2/g$ should be replaced by the proper value for H.

Theoretical Power. The theoretical horse power required to compress adiabatically and deliver 100 cu. ft. of gas per min. (initial pressure, p_1 lb. per sq. in.), is $\text{h.p.}_a = 1.501 p_1[(p_2/p_1)^{0.29} - 1]$; and for isothermal compression it is: $\text{h.p.}_i = 1.004 p_1 \log_{10}(p_2/p_1)$. Table 3 gives the ratio of the theoretical work for isothermal compression to that for adiabatic compression for the largest range of pressures likely to be met with in practice.

Table 3

p_2/p_1	1.5	2.0	2.5	3	4	5	6	7	8	9	10
(Isoth. ÷ Adiab.)	.940	.904	.875	.85	.812	.784	.763	.744	.728	.715	.703

Leakage. To reduce the leakage between stages and from the inlet of the last stage to the atmosphere, labyrinth packings are usually provided. A formula for the amount of leakage is given on p. 989. In general, the leakage may be assumed as about 3 per cent. of the rated quantity, about two-thirds of this taking place between the inlet of the last stage and the atmosphere. This loss is fairly independent of the number of stages.

Details of Design

The **impeller hub diameter** is generally from 1 to 2 in. larger than the shaft diameter, and usually between $\frac{1}{4}D_a$ and $\frac{1}{3}D_a$. The **casing inlet diameter** (D_c) is determined by the velocity desirable to allow in the annulus between the casing inlet and the impeller hub and varies generally between $\frac{1}{3}D_a$ and $\frac{2}{3}D_a$. For an axial-inlet type D_c (in.) $= 12(Q/KN)^{\frac{1}{3}}$, where Q = quantity, cu. ft. per min., N = r.p.m. and $K = 1$ for single-inlet impellers, and 2 for double-inlet impellers. In radial-inlet-type impellers the **impeller inlet diameter** is from $\frac{3}{4}$ in. to $1\frac{1}{2}$ in. larger than D_c, to allow the air to turn into a radial direction before striking the impeller blades. In axial-inlet-type impellers the outer diameter of the impeller inlet blades is just sufficiently smaller than the casing inlet diameter to allow for mechanical clearance. The total **axial width of blade** at the inlet (on both sides of the impeller web in a double-inlet impeller) is from $\frac{1}{8}D_a$ to $\frac{1}{4}D_a$; at exit, from $\frac{1}{8}D_a$ to $\frac{3}{8}D_a$. Frequently the impeller exit width is designed to give a sufficiently small discharge-vane angle and then the blade is made of constant overall width (parallel edges). The **number of impeller blades** (z_i) should be as small as possible consistent with the proper guiding of the gas. The usual number is between 16 and 24, with an equal number of half-blades at the outer periphery, if necessary.

The absolute velocity of the gas, w_e, is computed from Q, D_e and the axial width of the blade (radial-inlet type). u_e is computed from D_e and the r.p.m. The ratio w_e/u_e gives the tangent of the impeller inlet angle b_e absence of impeller inlet guides, which is the usual case, is assumed). For the axial-inlet type, w_e is the *average* velocity in the annulus between the hub and the casing inlet, and u_e and b_e vary with the diameter, giving a helical edge. The **impeller inlet passage height** is the perpendicular from the tip of a blade to the surface of the next blade. Its theoretical value is $(\pi D_e \times \sin b_e)/z_i$, and this should be increased by from 10 to 30 per cent. The r.p.m. is generally determined by that of the driver or by other considerations. From the formulæ given before for pressure rise, the wheel speed u_a and the **outer impeller diameter** D_a can then be determined by assuming a reasonable value for the hydraulic efficiency e_h. The rotation loss should now be computed to make sure that it is not excessive in comparison with the theoretical horse power. The impeller stresses should also be computed to see that they are not excessive for the available material. Increasing the number or stages will reduce both the rotation loss and the stresses. The inner diameter of the **discharge vanes** should be from $1.1D_a$ to $1.2D_a$. Three to six vanes are ample for most purposes. The overlap of one vane on the other, or the definite passage between vanes, need not be very great. The **discharge vane angle** d_a is found in the same way as explained for b_e, the inlet gas quantity being multiplied by a compression factor according to the wheel speed u_a, as per Table 4. Generally, d_a is made between 3 deg. and 5 deg. The theoretical **passage height** of the discharge vanes (perpendicular to the axial width) is $(\pi D_a \sin d_a)/z_d$, where z_d is the number of discharge vanes (same axial width of impeller and discharge vanes is assumed). The actual passage height should be from 50 to 100 per cent. larger.

Table 4

Impeller wheel speed, ft. per sec.	200	250	300	350	400	450	500	550	600	650	700
Compression factor	.989	.982	.974	.965	.955	.944	.931	.917	.903	.888	.871

Centrifugal Compressor Tests

Acceptance Tests. In making acceptance tests, the compressor is tested with no piping at the inlet, the air being taken directly into the unrestricted compressor inlet, which is to be at a distance from the wall or from the floor equal to at least its own diameter. At the discharge end a length of pipe equal to about ten times the discharge diameter is attached. At about the middle of this pipe, a blast gate is inserted, while at the far end the measuring orifice, or a header containing a number of small orifices, is bolted on.

Size of Orifice. The orifice is usually selected to pass a quantity about 35 per cent greater than the rated quantity. The following rough formulæ are frequently used: $Q = 109d^2\sqrt{P} = 21d^2\sqrt{i}$, where Q = cu. ft. of air per min.; d = diam. of orifice, in.; P = pressure above atmosphere, lb. per sq. in., and i = inches of water. Readings are generally taken at about six or seven different loads, the readings taken being the inlet temperature, the discharge pressure, the orifice pressure and temperature and the r.p.m. The **discharge pressure** is taken with an impact tube inserted into the discharge pipe about midway between the compressor and the blast gate, pointing upstream and reaching a little more than half way into the pipe. A second impact tube nearer the blast gate is desirable. The **orifice pressure** is taken with an impact tube placed opposite the center of the orifice at a distance from the plane of the orifice equal to about one-fifth of the orifice diameter. The orifice temperature is taken in the straight part of the pipe before the orifice. The bare bulb of the thermometer should protrude for about 2 in. into the stream of air, the air velocity rarely exceeding 60 or 70 ft. per sec.

Efficiency. The efficiency of a centrifugal compressor is the ratio of the theoretical power, adiabatic or isothermal, corresponding to the rated quantity and pressure, divided by the power delivered to the compressor shaft. There is, however, considerable room for controversy and misunderstanding in this connection, owing to the difficulty of measuring accurately the output of the driver. The input of the driver, however, is generally easily determinable. It is therefore customary to specify that the input of the driver per 100 cu. ft. of air delivered by the compressor shall not exceed so many kilowatt-hours, or so many pounds of steam, the steam conditions being those of the customer's plant.

Centrifugal Compressor Types

General Electric Compressor. The principal characteristic of the compressor manufactured by the General Electric Co. is the double-inlet impeller, which admits air on both sides of its central web. This impeller, used both in the single-stage and in the multi-stage compressors, is free from axial thrust, and permits the number of impellers in a multi-stage compressor to be either even or odd. It also makes possible the handling of very large quantities of air without entailing the use of too wide blades. In the smaller sizes, the impellers are of the radial-inlet type, the blades being curved in a plane perpendicular to the shaft, and attached to the central web by a cast welding process. In the larger sizes, the impellers are of the axial-inlet type, and the blades are frequently milled out from the solid disk which forms the impeller, separate entrance buckets curved in a plane tangential to the shaft being supplied. With this latter construction, wheel speeds of 600 to 800 ft. per sec. may be safely used, so that compressors designed for pressures as high as 30 lb. per sq. in. have only three or four stages. Each impeller is surrounded by a set of stationary discharge vanes which connect either directly with the discharge pipe, or, in multi-stage compressors, with the opposite sides of the succeeding impeller by means of two sets of passages at right angles to each other in an intervening diaphragm. To reduce the temperature of the air and thereby also the work necessary for compression, large passages are cored out in the diaphragms to permit the efficient use of cooling water.

Commercial sizes of the single-stage compressors have pressure ratings from 0.75 to 4 lb. per sq. in. and capacities from a lower limit of 500 cu. ft. per min. to a higher limit which ranges from 12,000 cu. ft. at 0.75 lb. pressure down to 3000 cu. ft. at 4 lb. pressure. The multi-stage compressors are built of the following sizes:

Cu. ft. per min.	4,500	9,000	16,000	25,000	40,000	50,000
Pressure, lb. per sq. in.	6–35	6–25	6–25	12–30	12–30	12–30

Multi-stage compressors have also been built for pressures as high as 150 lb. per sq. in., the volumes being comparatively small.

With a driver allowing speed variation, like a steam turbine or direct-current motor, a centrifugal compressor of a given rating will operate with a number of different combinations of pressure and volume without excessive loss in efficiency. The different rated pressures of multi-stage compressors are usually obtained by the use of a different number of stages, all stages in a given size of machine being similar. A given compressor may, however, operate successfully between, say, 15 and 30 lb. per sq. in., if the driver will allow of the necessary speed variation.

Pressure and Power Variations. Without any special governing device a centrifugal compressor driven at constant speed will maintain a substantially constant pressure, regardless of the volume of air delivered. Should there be temporarily no demand for air while the compressor is being driven at full speed, the casing will become slightly hotter, but otherwise the machine will not be subjected to any particular strains. The power required by a centrifugal compressor between half-load and, say, 25 per cent. overload varies approximately as the load. The following table shows the usual variation of pressure and power with the volume handled by a single-stage compressor at constant speed.

Characteristics of Centrifugal Compressors at Constant Speed

Volume, per cent. of full load	0	20	40	50	60	70	80	100	120
Pressure, per cent. of full load	92	94	97.5	100	103	105	105	100	91
Power, per cent. of full load	50	53	58	62	66	73	81	100	120

For blast-furnace purposes, where a constant quantity of air is to be delivered against varying pressures, a constant-volume governor, Fig. 3, has been developed, which consists mainly of a disk suspended in a vertical, slightly conical pipe attached to the compressor inlet. This disk is connected to one end of a lever, the other end of which is connected to the throttle valve of the driving turbine or to the rheostat of the driving electric motor. The lever carries a sliding weight, which by its position on the lever determines the position of equilibrium of the disk in the conical pipe. If the pressure in the blast furnace drops, so that the compressor tends to pass more air than the quantity for which the sliding weight on the lever was set, the velocity of the air in the annular passage between the disk and the conical pipe increases, the disk rises, and the lever actuates the speed-varying mechanism of the driver so as to reduce the speed and the generated pressure. Should the pressure in the blast furnace rise, then the reduced velocity of the air in the annular passage will

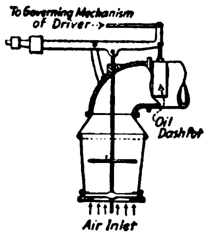


Fig. 3.—General Electric Co. Constant-volume Governor.

cause the disk to drop and the lever will speed up the driver till the compressor generates enough pressure to pass the desired quantity.

The Rateau Compressor, built by the Ingersoll-Rand Co., by the Southwark Foundry and Machine Co., and by others, has a single-inlet impeller, admitting the air on only one side of the web. In the single-stage compressor, the axial thrust is overcome by the use of a balancing piston or otherwise, while in multi-stage compressors the impellers, of which there is always an even number, are divided into two equal groups through which the air passes in opposite directions. The two groups of impellers are mounted in separate casings and on separate shafts, the two shafts being connected by a flange coupling. The air enters the impeller nearest the middle bearing, passes through the first group of impellers, into the receiver pipe which acts also as an intercooler, and then in a reversed direction through the second group of impellers which raise its pressure to that existing in the discharge pipe. The impeller is usually a steel plate, to which straight sheet-steel blades are riveted so as to give backward discharge; the main blades are interspersed at the outer periphery with short half-blades. On leaving the impeller, the air passes outward into a parallel-sided bladeless diffuser, and is then led inward between fixed blades to the inlet of the next impeller. The diaphragms carrying the fixed return blades are cored out to allow ample space for cooling water.

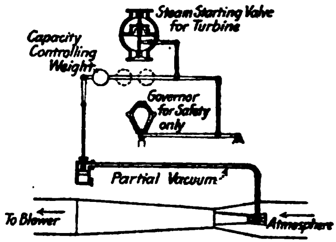


FIG. 4.—Rateau Constant-volume Governor.

The Rateau constant-volume governor (Fig. 4) for blast-furnace work is based on the Venturi meter principle. The increased velocity in the throat formed in the inlet pipe causes the pressure there to drop below atmospheric pressure, and this partial vacuum is conveyed to one side of a spring-controlled piston, the other side of the piston being open to the atmosphere. The piston rod is connected through a lever to the speed-varying device of the driver, and the position of a sliding weight on this lever determines the quantity of air which the compressor is to pass continuously. Should the resistance to air flow in the furnace rise or fall, tending to change the volume of the air from the normal, the vacuum in the throat of the governor will fall or rise correspondingly and the piston will actuate the control lever so as to speed up or to slow down the driver and the compressor.

CENTRIFUGAL FANS

BY

E. B. WILLIAMS

REFERENCES: Gustav Herrmann, "Die Graphische Theorie der Turbinen una Kreiselpumpen." Julius Ritter von Hauer, "Ventilationsmaschinen der Bergwerke" and "Die Wettermaschinen." Louis Ser, "Traité de Physique Industriale," pp. 668–723. J. Boulvin, "Cours de Mécanique Appliqué aux Machines." Bryan Donkin, "Experiments on Centrifugal Fans," *Proc. Inst. C. E.*, vol. cxxii, part 4, 1895. Charles H. Innes, "The Fan."

FUNDAMENTAL FORMULÆ

Notation

H	= head of air, ft.	B	= Barometer reading, inches of mercury
h	= velocity head, inches of water	a.h.p.	= air horse power
h_s	= static head, inches of water	b.h.p.	= brake horse power
h_i	= impact head, inches of water	E	= mechanical efficiency
v	= velocity, ft. per sec.	N	= revolutions per min.
V	= velocity, ft. per min.	D	= diameter of fan wheel, ft.
W	= weight of air, lb. per cu. ft.	r	= radius of fan wheel, ft.
P	= pressure, lb. per sq. ft.	b	= width of fan wheel, ft.
A	= area, sq. ft.	M	= manometric efficiency
Q	= volume, cu. ft. per min.	C_1	= tip speed, ft. per sec.
q	= volume, cu. ft. per sec.	T	= tip speed, ft. per min.
t	= temp., deg. fahr.		

Velocity. From the formula $v = \sqrt{2gH}$ the velocity of air at standard atmospheric conditions ($W = 0.075$) is given by $V = 4000\sqrt{h}$. For any given air temperature t and pressure B, neglecting the slight effect of a variable humidity, $V = 956\sqrt{(460 + t)h/B}$.

Pressure is generally expressed in inches of water or in ounces per sq. in. 1 oz. per sq. in. = 1.732 in. of water. $H = 5.2h/W$; $P = HW$, and $P = 5.2h$. For air at density $W = 0.075$, a head of 1 in. of water equals that of 69.3 ft. of air. Three pressures must be considered in a column of moving air, namely, static, velocity and impact. The first represents the compression, the second kinetic energy of the blast, and the third the total pressure or the sum of the static and velocity pressures. The static pressure in a system through which air passes is often referred to as **maintained resistance.**

Air Horse Power. The horse power in a column of moving air is a.h.p. = $VAP/33{,}000 = QP/33{,}000 = 5.2Qh_i/33{,}000 = 0.1728Ah_i\sqrt{h/W}$.

Mechanical Efficiency is the ratio of a.h.p. to b.h.p. Sometimes the a.h.p. is computed by use of only the static pressure—positive, negative or both—against which the fan operates; it being assumed that static pressure represents the useful pressure and that the kinetic energy or velocity pressure is thrown away on leaving the system and entering the atmosphere. Since in most fan systems the velocity at the point of delivery into the atmosphere is considerably lower than at the fan outlet, the kinetic energy or velocity pressure which can rightfully be deducted from the impact or total head produced by the fan is only that existing at the point of leaving the system, and not the velocity pressure at the fan outlet. The impact or total head produced by the fan is more generally used in computing the a.h.p.

Table 1. Conversion Tables for Air Pressures

PRESSURES IN INCHES OF WATER CORRESPONDING TO OUNCES PER SQ. IN.

Pressure oz. per sq. in.	Decimal parts of an ounce									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	......	0.17	0.35	0.52	0.69	0.87	1.04	1.21	1.38	1.56
1	1.73	1.90	2.08	2.25	2.42	2.60	2.77	2.94	3.11	3.29
2	3.46	3.63	3.81	3.98	4.15	4.33	4.50	4.67	4.84	5.01
3	5.19	5.36	5.54	5.71	5.88	6.06	6.23	6.40	6.57	6.75
4	6.92	7.09	7.27	7.44	7.61	7.79	7.96	8.13	8.30	8.48
5	8.65	8.82	9.00	9.17	9.34	9.52	9.69	9.86	10.03	10.21
6	10.38	10.55	10.73	10.90	11.07	11.26	11.43	11.60	11.77	11.95
7	12.11	12.28	12.46	12.63	12.80	12.97	13.15	13.32	13.49	13.67
8	13.84	14.01	14.19	14.36	14.53	14.71	14.88	15.05	15.22	15.40
9	15.57	15.74	15.92	16.09	16.26	16.45	16.62	16.79	16.96	17.14

PRESSURE IN OUNCES PER SQ. IN. CORRESPONDING TO INCHES OF WATER

Head, in.	Decimal parts of an inch									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	......	0.06	0.12	0.17	0.23	0.29	0.35	0.40	0.46	0.52
1	0.58	0.63	0.69	0.75	0.81	0.87	0.93	0.98	1.04	1.09
2	1.16	1.21	1.27	1.33	1.39	1.44	1.50	1.56	1.62	1.67
3	1.73	1.79	1.85	1.91	1.96	2.02	2.08	2.14	2.19	2.25
4	2.31	2.37	2.42	2.48	2.54	2.60	2.66	2.72	2.77	2.83
5	2.89	2.94	3.00	3.06	3.12	3.18	3.24	3.29	3.35	3.41
6	3.47	3.52	3.58	3.64	3.70	3.75	3.81	3.87	3.92	3.98
7	4.04	4.10	4.16	4.22	4.28	4.33	4.39	4.45	4.50	4.56
8	4.62	4.67	4.73	4.79	4.85	4.91	4.97	5.03	5.08	5.14
9	5.20	5.26	5.31	5.37	5.42	5.48	5.54	5.60	5.66	5.72

Table 2. Velocity of Air Due to Pressure

(Air at 65 deg. fahr.; barometer reading, 29.92 in.)

Pressure, in. of water	Velocity, ft. per min.	Pressure, in. of water	Velocity, ft. per min.	Pressure, in. of water	Velocity, ft. per min.	Pressure, in. of water	Velocity, ft. per min.
0.1	1,265	0.9	3,790	1.7	5,210	3.25	7,210
0.2	1,790	1.0	4,000	1.8	5,360	3.50	7,490
0.3	2,190	1.1	4,190	1.9	5,510	3.75	7,750
0.4	2,530	1.2	4,380	2.00	5,650	4.00	8,000
0.5	2,830	1.3	4,560	2.25	6,000	4.25	8,250
0.6	3,100	1.4	4,730	2.50	6,320	4.50	8,490
0.7	3,350	1.5	4,900	2.75	6,630	4.75	8,720
0.8	3,580	1.6	5,050	3.00	6,930	5.00	8,950

Manometric Efficiency or pressure efficiency is the pressure developed by the fan divided by the pressure against a plane surface due to a velocity equal to the peripheral speed of the fan wheel. $M = gH/C_1^2$. Manometric efficiency is defined by some engineers as $2gH/C_1^2$.

Volumetric Efficiency is not really an efficiency and might better be called **volumetric capacity**. It is defined as the quantity of air delivered per revolution divided by the overall cubical contents of the wheel, and is represented by the formula Vol. Eff. $= Q/\pi r^2 bN$.

Performance of Fans at Various Air Temperatures. In Table 3 are given factors for determining the influence of the air temperature on the volume of a given weight (col. 2) and on the weight of a given volume (col. 3) of air; on the pressures (cols. 2 and 3) corresponding to constant weight and constant volume, respectively, of air handled by a fan; on the speeds (cols. 2 and 4) corresponding to constant weight and constant pressure, respectively, of air handled by a fan; on the power (cols. 3 and 5) corresponding to constant volume and constant weight, respectively, of air handled by a fan; and the power (col. 2) necessary to handle a given weight at a given pressure with a fan proportioned to operate at a given efficiency. All these factors are to be applied to the quantities corresponding to the standard air temperature of 65 deg. fahr.

Table 3. Factors for Determining the Performance of Fans at Various Air Temperatures (see text)

Temp., deg. fahr.	2	3	4	5	Temp., deg. fahr.	2	3	4	5
30	0.94	1.07	0.97	0.87	325	1.50	0.67	1.22	2.24
40	0.96	1.05	0.98	0.91	350	1.55	0.65	1.24	2.38
50	0.97	1.03	0.99	0.95	375	1.59	0.63	1.26	2.54
60	0.99	1.01	0.99	0.98	400	1.63	0.61	1.28	2.69
65	1.00	1.00	1.00	1.00	425	1.68	0.60	1.30	2.85
70	1.01	0.99	1.01	1.02	450	1.73	0.58	1.32	3.02
80	1.03	0.97	1.02	1.06	475	1.78	0.57	1.33	3.18
90	1.05	0.95	1.03	1.11	500	1.83	0.54	1.35	3.38
100	1.07	0.93	1.04	1.15	525	1.88	0.53	1.37	3.52
125	1.12	0.89	1.06	1.25	550	1.93	0.52	1.39	3.72
150	1.17	0.86	1.08	1.36	575	1.98	0.50	1.41	3.90
175	1.21	0.83	1.10	1.47	600	2.02	0.49	1.42	4.10
200	1.26	0.80	1.12	1.58	650	2.12	0.47	1.46	4.50
225	1.30	0.77	1.14	1.71	700	2.21	0.45	1.49	4.90
250	1.35	0.74	1.16	1.83	750	2.31	0.43	1.52	5.32
275	1.40	0.71	1.18	1.96	800	2.41	0.41	1.55	5.78
300	1.45	0.69	1.20	2.10					

For the **weight of 1 cu. ft. of air** at any temperature, pressure and humidity within the usual range, see p. 1512.

FAN CHARACTERISTICS

The characteristics of any fan can best be shown graphically. The characteristic curves from any size of a series of similar fans suffice for the entire group. This follows from the fact that the area of the frictional surfaces varies directly with the area of the various passages through which air flows when passing through the fan. The different bases upon which these characteristic curves can be plotted are as stated in the following paragraphs.

Static No Delivery (S.N.D.). This gives a useful basis for calculating the performance of various fans. Fig. 1 is the chart of a fan known as the "steel-plate" or "paddle-wheel" type. Data for these curves are obtained by running the fan at some convenient constant speed and taking a set of readings at each of six to eight air deliveries produced by as many different restrictions placed on the discharge and varying from a condition of closed outlet to wide open or free discharge. All data are taken at the fan outlet. The abscissæ are the static pressures or maintained resistances in percentages of the pressure which the fan will exert with its outlet completely closed; this latter pressure with a few exceptions is the maximum pressure a fan will exert at any given speed, and is symbolized S.N.D. Four curves are plotted, using as ordinates (1) the ratio of the velocity through the fan outlet to the peripheral speed of the wheel and designated V/T; (2) the mechanical efficiency E; (3) the ratio of impact pressure to S.N.D., designated h_i/S.N.D.; and (4) the ratio of static pressure to impact pressure, designated h_s/h_i. From these curves can be computed the performance of any size of symmetrical fan under any conditions of pressure, volume or speed within its structural possibilities.

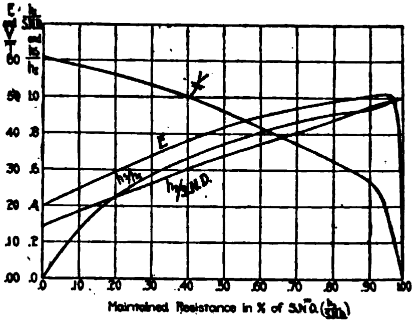


Fig. 1.—Characteristic Curves of a Steel-plate Paddle-wheel Fan.

As an example of how these curves are applied, let it be assumed that a size of fan is required which will operate most efficiently when delivering 10,000 cu. ft. per min. against 2 in. static pressure. Assume the fan to operate at a mechanical efficiency of 50 per cent. At this efficiency the abscissa h_s/S.N.D. is 0.857, from which S.N.D. = 2.0/0.857 = 2.33 in. For the same efficiency h_i/S.N.D. = 0.923, therefore h_i = 0.923 × 2.33 = 2.15 in.; $h_i = h_s + h$, hence h = 0.15 in.; and $V = 4000\sqrt{0.15}$ = 1548 ft. per min. For 10,000 cu. ft. per min. the area of the fan outlet must then be 6.47 sq. ft. Knowing that for this particular design of fan the outlet area is expressed by the equation $A = 0.306D^2$, the diam. D is found to be 4.59 ft. Corresponding to h_s/S.N.D. = 0.857 is found a value of V/T = 0.298, from which the peripheral speed = 1548/0.298 = 5200 ft. per min., and the speed of the fan is computed as 360 r.p.m. The other dimensions of the fan are found from the basis of design, in which these dimensions should be expressed as functions of the diam. of the wheel D. Air horse power = a.h.p. = 10,000 × 2.15 × 5.2/33,000 = 3.39; b.h.p. = 3.39/0.50 = 6.78.

As a second example suppose a fan with a wheel 3 ft. in diam. is to deliver 5000 cu. ft. per min. against 2 in. maintained resistance or static pressure, leaving r.p.m. and b.h.p. to be determined. From the equation $A = 0.306D^2$, A = 2.76 sq. ft., therefore V = 1810 ft. per min. From $V = 4000\sqrt{h}$, h = 0.205 in. making h_i = 2.205 in. Here h_s/h_i = 0.907, at which point on the curves of Fig. 1 V/T = 0.322; hence T = 5620 ft. per min. As D = 3 ft., r.p.m. = 597. a.h.p. = 5000 × 0.205 × 5.2/33,000 = 1.74. From the curves E = 49.2 per cent., hence b.h.p. = 3.54.

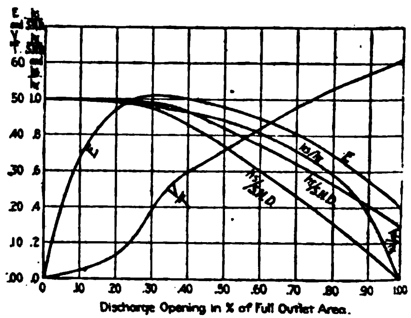


Fig. 2.

The curves of Fig. 1 may also be plotted using for abscissæ the static pressures or maintained resistances expressed in percentages of the pressure which would result from a velocity equal to the peripheral or tip speed of the blades and which is referred to as **tip-speed pressure.**

Discharge Opening in Percentage of Full Outlet. Fig. 2 is for the same fan as Fig. 1 and shows the same ordinates plotted against the discharge outlet area (during the test) expressed as a percentage of the full outlet. As the majority of fan problems require solution on a basis of static pressure or maintained resistance rather than on a percentage of full outlet (a laboratory condition), Fig. 1 proves of greater utility. A curve of "discharge opening in percentage of full outlet" could be plotted on Fig. 1 as another ordinate against maintained resistance if desired.

Reduced Orifice. This term is more common in European practice and is used as an abscissa against which are plotted volumetric, manometric and mechanical efficiencies. Reduced orifice is expressed by the equation $O_r = q/(r^2\sqrt{gH})$. Since this equation contains the factor r, it is evident that regardless of the size of fan of any given type the reduced orifice will be the same when the same relative restriction exists. This approaches the basis of Fig. 2, but instead of dealing with the percentage of opening of outlet, it deals with a value that is in reality some constant times the true percentage of full opening.

Equivalent Orifice. This term is also more common in European practice and differs from "reduced orifice" in that the equation contains no term bearing on the proportions or size of the fan; hence the equivalent orifice is an actual quantity in sq. ft. varying with the volume delivered. Equivalent orifice is expressed by the equation $O_e = q/(K\sqrt{2gH})$. It represents the area of a circular hole in a thin plate which will produce a restricting effect equal to that which exists in the system under consideration regardless of the nature of the actual restriction. The value of K generally used ranges from 0.60 to 0.65, making $O_e = q/0.65\sqrt{2gH}$. It is more of a laboratory characteristic than one adaptable to general use.

Volumetric and manometric efficiency curves could be plotted, but the usefulness of such curves is limited. Volumetric efficiency is a direct function of the V/T-curve and may be represented by the product of V/T and a constant, the latter depending on the design of the fan. As the volume delivered at a given r.p.m. varies as D^3, the volume per revolution $= K_1D^3V/T$, and since the overall cubical content of the wheel is K_2D^3, the volumetric efficiency $= KV/T$, where K equals K_1/K_2. K_1 and K_2 depend on the design and proportions of the fan. The volumetric efficiency is maximum with free intake and discharge and runs as high as 400 to 500 per cent. in fans of the multiblade type designed for high pressures at low peripheral speeds, while it runs as low as 100 per cent. in fans designed for low pressures at high peripheral speeds. The manometric efficiency is generally highest when the outlet is closed and lowest when maximum volume is delivered. There are, however, types where the maximum value occurs at light loads varying from ¼ to ½ maximum volumetric delivery. In cased fans of the multiblade type the maximum efficiency (gH/C_1^2) attained is about 150 per cent., while in open-running fans it is as low as 30 per cent. High mechanical efficiency (and generally high volumetric efficiency) is desirable, but with manometric efficiency it is usually the method of drive which determines the desirable value. The higher the speed of the driver the lower the manometric efficiency of the fan best adapted to the drive.

Effects of Variations in R.P.M., Volume, Pressure, Horse Power, Efficiency, Etc. Where a given fan is operating with a given restriction on its inlet or outlet, the volume delivered will vary directly with the r.p.m. (N), the various pressures as N^2, and the air horse power as N^3. The mechanical efficiency will remain the same, therefore the b.h.p. will also vary as N^3. The volume delivered by symmetrical fans will vary as D^2 for any given peripheral speed of the wheel and resistance or static pressure.

Under these conditions the mechanical efficiency remains constant, since the area of friction surfaces varies directly with the change in volume. The horse power will therefore vary as D^2. It is sometimes erroneously assumed that at very high speeds a given fan will show a marked decrease in efficiency due to slippage, or what with ship propellers is known as cavitation. This, however, is not the case.

METHODS OF TESTING FANS

Measuring the Flow of Air. For various methods of measuring the flow of air, see p. 1691.

Air-tight Room Test. In this test the fan discharges into a closed room, the outlet of which is a sliding door or a suitable discharge pipe. The static pressure in the room represents the resistance against which the fan is operating. The sliding door is used to vary the output of the fan, but as the coefficient of efflux varies with different openings as a result of their varying shape, it is better to measure the volume delivered through a discharge pipe of such length as will practically eliminate eddies. The room should be of such size that there are no heavy swirls, and even though extra caution be used in making it tight, a leakage test should be made. With this method of testing it is impossible to get data for a free and unrestricted discharge, yet if the room is relatively large this condition is approached with reasonable closeness. Prof. R. J. Durley (*Trans. A. S. M. E.*, vol. 27) lets the air discharge from a room or closed vessel through small circular orifices in thin plates of known coefficient of discharge. This, however, is essentially a laboratory method.

Discharge Pipe Test. The discharge pipe, which is preferably of the shape and size of the fan outlet, should have a length of 15 to 20 diameters or 15 to 20 times the mean of the rectilinear dimensions. The Pitot tube should be placed at about 10 diameters from the fan outlet. Traverse readings are taken at this point with a Pitot tube. To get a correct average of the velocity pressure, square-root-of-mean-square-values should be used, but, even with a wide variation in the readings, the inaccuracy of using a plain arithmetical mean is not over 0.5 per cent. Center readings only can be taken, corrected by a coefficient for the pipe if such procedure seems desirable. Credit should be given the fan for the friction loss between the fan outlet and point of measurement. Galvanized iron is the common material of the discharge duct, and the **friction loss** in inches of water can be computed from the formula $H = 4flv^2/d$, where H = head in feet of air, f = coefficient of friction = 0.0001 (about) for galvanized iron pipe, l = length (ft.), v = velocity (ft. per sec.), and d = diam. of discharge (ft.). This formula may be simplified and written as $h_f = lh/kd$, where k = a constant, h_f = head lost in friction, and h = velocity head in the pipe, both in inches of water. The value of k for smooth galvanized-iron piping ranges from 50 to 60, while for 12-in. swaged pipe it may be as low as 40 and in some cases is still lower. (See also pp. 357 and 1358.) Readings taken at conical mouthpieces at the end of this discharge pipe are not as accurate as those taken with a Pitot tube in the pipe.

Suction Pipe Test. A similar pipe can be placed on the inlet and Pitot tube readings taken therein. Here the true resistance against which the fan works will be the impact reading.

FAN DESIGN

Casings

Design is best based upon the characteristic curves of fans wherein the shape and proportions of the fan casings are varied. The principal dimensions of any design can be made functions of the diameter of the wheel, *i.e.*, some constant times the diameter to the first power for straight dimensions and to the second power for areas. This procedure should be adhered to in making calculations for fans of different sizes of a given type, and should always refer to the inside or effective dimensions.

The Spiral or Scroll. If the air is given off equally for each unit of length around the periphery of a centrifugal-fan wheel, the casing should have a uniform increase in area outside the wheel for the passage of air. The curve of the outer or scroll sheet of the casing is therefore that of the spiral of

Archimedes, and its equation is $R = r + Ka$, where R is the straight-line distance from the center of the wheel to any given point on the curve, r the radius of the wheel, a the angle of advance in radians, and K a constant. The air is not uniformly given off all around the circumference of the wheel except when the fan is properly loaded; when the load is too light the air will blow back between the blades at certain points while being delivered at other points. The curve of the spiral scroll is stopped some distance from the point of intersection with the wheel and formed into the cut-off piece. The value of the constant K in a casing for a given wheel is best determined by experiment. The point where the curve of the spiral would, if extended, intersect the circumference of the wheel, varies from about 30 deg. back of a point from which a tangent would be parallel to the axis of the discharge opening (and from which point the angle of advance is measured) to about 30 deg. ahead of this point. (See Figs. 3 and 4.) The former is found in some of the older designs of steel-plate or paddle-wheel fans, while the mean of the two extremes is more general in the multiblade types of recent development. Unless the curve of the spiral intersects the wheel at the point from which the angle of advance is measured, the equation becomes $R = r + Ka + K_1$; the value of K_1 depending on the point of intersection. In the spiral equation for a series of fans of a given design, the constant K varies with the diameter of the fan and can be written rK. A common commercial form of paddle-wheel fan, having eight flat radial blades of a depth about one-sixth the diameter of the wheel, has a spiral equation $R = r(1 + 0.118a + 0.074)$; a common fan of the multiblade type has the spiral $R = r(1 + 0.198a)$. The following table, from tests on a standard form of multiblade fan, shows the effect of varying the value of K.

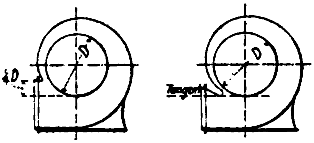


FIG. 3. FIG. 4.
Spirals of Fan Casings.

Scroll Equation	Maximum Efficiency, Per Cent. Mechanical	Volumetric	Manometric
$R = r(1 + 0.148\,a)$	48.5	367	130
$R = r(1 + 0.198\,a)$	52.0	452	128
$R = r(1 + 0.396\,a)$	36.0	493	117

Casings are often made with two discharges 180 deg. apart. In these the spiral is developed as for a single-discharge fan, stopping off at 180 deg. and starting at this point a second spiral. Although this gives the same total outlet area and the same velocity outside of the wheel with a given volume flowing as with a single-discharge fan having the same spiral equation, the mechanical and manometric efficiencies fall off to a marked degree and the volumetric efficiency more or less. The following table shows the effect on efficiencies of double discharge in a standard type of multiblade fan with a given spiral equation, and also the effect of changing the spiral equation in a double discharge casing.

Discharge	Spiral Equation	Maximum Efficiencies, Per Cent. Mechanical	Volumetric	Manometric
Single...........	$R = r(1 + 0.198\,a)$	52.0	452	128
Double..........	$R = r(1 + 0.198\,a)$	43.6	416	105
Double..........	$R = r(1 + 0.269\,a)$	45.5	490	98

With forward-curved-blade fans where the velocity of the air leaving the fan is high, a scroll whose radius increases in a geometrical ratio is sometimes used. By this means a more complete conversion of kinetic to static pressure is obtained within the fan casing, thereby eliminating the need for the use of an expansion piece at the outlet if high efficiency is desired.

Cut-off Point. The point where the scroll or spiral discontinues its approach to the circumference of the wheel is called the cut-off point. In some types of fan nearly half the diameter of the wheel is exposed when looking along the axis of discharge, while the other extreme is found where no part of the wheel is exposed, in which case a tangent to the wheel passing through the cut-off point is parallel to the axis of discharge. A fan with a large exposure of wheel will deliver, with very little or no restriction to the flow of air, a relatively greater volume at a given number of r.p.m. than the fan in which there is little or no exposure of the wheel, while the latter will deliver a relatively greater volume against average restrictions or resistances met with in practice. The following table, from tests on a multiblade fan, shows the difference in efficiencies at various pressures or resistances at a given speed in r.p.m. when 25 per cent. of the diameter of wheel is exposed [col. (*a*)] and when a tangent to the wheel passing through the cut-off point is also parallel to the axis of discharge [col. (*b*)]. The same wheel and casing, except for the cut-off piece, were used in each case. Figs. 3 and 4 show the feature being considered.

Static Pressure, in. of water	Efficiencies, Per Cent.					
	Mechanical		Volumetric		Manometric	
	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
0.6	40.0	40.0	396	383	79.4	76.8
1.0	47.2	49.5	335	340	94.6	94.6
1.1	48.2	51.0	305	322	96.2	98.5
1.2	48.5	51.5	262	301	96.0	100.7
1.3	47.0	51.5	189	271	94.7	102.4
1.4	41.0	50.0	116	215	96.0	103.7

When a fan operates against considerable restriction or resistance to the flow of air, there is a strong tendency in the air to leak back through the wheel at the point where it is about to leave the spiral to pass through the fan discharge. If the delivery is restricted, it leaks back through the wheel at the point of maximum pressure. The above table shows how the efficiencies drop off in the fan of Fig. 3 as compared with that of Fig. 4. The maximum pressure in the air is at the fan discharge, while from the bottom of the wheel to the cut-off point (that is, the exposed portion of the wheel) the air has only that pressure imparted to it by the blades in that section of the wheel. This produces leakage back into the wheel as soon as the pressure at the discharge builds up due to restriction. Experiment shows that cut-off points with very small clearances have a merit which exists only in fancy. The effect upon efficiency of a reasonably large clearance at this point is negligible; the amount of clearance may be 5 per cent. of the wheel diameter in medium-sized fans. Most of the noise in a fan originates at the cut-off, the function and form of which make it like a reed that will give a musical note unless this is guarded against. A blunt, rigid construction as in Fig. 4 will keep the noise down, whereas small clearance and a sharp, knife-like edge will give a loud, shrill noise. In very-high-speed fans the cut-off is sometimes made of a material less subject to vibrations than metal, such as soft wood.

Width of Spiral Casing. In general, low-pressure volume fans have a width of casing 20 to 40 per cent. greater than the width of the wheel at its periphery. As the relative width is increased beyond this the air must distribute itself laterally to such extent that serious eddy currents are produced in the casing, resulting in loss of efficiency. Fans with cast-iron casings for high pressures are often made with a casing of gradually increasing width; a section at any point along the spiral air passage being circular and of increasing diameter with increase in the angle of advance. The table following shows the effect of a change in the width of the casing. The tests were made on a multiblade wheel 17¼ in. in diam. and 9 in. wide. Casing A was 13½ in. wide and casing B 12 in. wide. The equation of the spiral was in each case such as to produce the same sectional area of air passage outside the wheel at any given angle of advance, and the inlet connections to the casings were the same.

		Casing A		Casing B	
Volume, cu. ft., per min.	Static pressure, in. of water	R.p.m.	Mech. effy., per cent.	R.p.m.	Mech. effy., per cent.
2800	1	796	47.5	728	53.5
3400	1	800	45.7	736	53.2

The position of the wheel relative to the width of the casing is important in single-inlet fans, owing to the tendency of the air to crowd to the back. The wheel should be placed centrally in the casing or nearer the inlet side so as to reduce this crowding effect. Casings are frequently made double-width, double-inlet, which, for a given volume, pressure and efficiency, gives a rotative speed 41 per cent. higher. This has decided advantages where direct-connected high-speed drivers such as electric motors and steam turbines are used.

Inlet of Casing. The following table (from "Fan Engineering," published by The Buffalo Forge Co.) shows how the dimensions, horse power and speed of a steel-plate paddle-wheel fan vary, for the same capacity and pressure, as D_1/D varies from the standard value, 0.625. D_1 is the diameter of the fan inlet.

D_1/D	Relative diameters		Relative widths, b	Relative horse power	Relative r.p.m., N
	Wheel, D	Inlet, D_1			
0.700	0.82	0.92	1.09	1.12	1.23
0.650	0.93	0.98	1.03	1.04	1.10
0.625	1.00	1.00	1.00	1.00	1.00
0.600	1.07	1.03	0.98	0.97	0.93
0.550	1.24	1.09	0.92	0.91	0.78
0.500	1.45	1.17	0.86	0.87	0.65
0.450	1.71	1.23	0.81	0.83	0.53
0.400	2.07	1.32	0.76	0.80	0.43
0.350	2.55	1.43	0.70	0.78	0.35

A common form of inlet still used on many casings has a diameter equal to (and in some cases less than) the inlet of the wheel, a casting or rolled angle being used to facilitate connection to a pipe, as in Figs. 5 and 6. With these forms the contraction of the entering stream of air constitutes a restricted area and is a source of loss, as it requires a higher velocity for a given volume of air. The shape of the inlet connection should be conformed to the natural path of the outer particles of air by means of a conical or bell-shaped inlet connection such as shown in Fig. 7. A connection in the form of a frustum of a cone is preferable from a manufacturing standpoint. Variation in the form of inlet connection produces change both in the mechanical efficiency and in the volumetric efficiency. The fol-

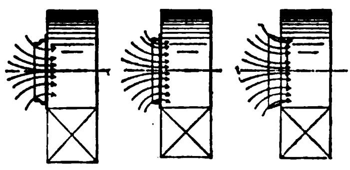


FIG. 5. FIG. 6. FIG. 7.
Inlets of Fan Casings.

lowing table gives results obtained with three inlet connections, *a*, *b*, and *c*, similar to Figs. 5, 6 and 7, respectively. The diameter of the wheel inlet was 14 in. Connection *a* was 12¼ in. in diam. at the entering end, *b* was 14 in. in diam., or the same as the wheel inlet, and *c* was 14 in. in diam. at the small end with a 16-deg. taper to its axis and an axial length of 4 in.

Inlet connection	Cu. ft. per min.	Static pressure, in. of water	R.p.m.	Mechanical efficiency, per cent.
a (Fig. 5)	1680	1	726	40.5
b (Fig. 6)	1680	1	698	45.5
c (Fig. 7)	1680	1	682	51.8

Another fan, of lower average efficiency, was tested with inlet cones identical in all respects except as to axial length. In the table below test *a* is with a plain inlet like that of Fig. 6, while tests *b*, *c* and *d* are with the cones just mentioned wherein the small diameter is 14 in. and the taper 16 deg. to the axis.

Test	Depth of cone, in.	Cu. ft. per min.	Static pressure, in. of water	R.p.m.	Mechanical efficiency, per cent.
a	....	2500	1	960	34.2
b	2	2500	1	950	36.3
c	4	2500	1	888	41.1
d	7	2500	1	884	42.8

The angle of convergence or taper of the inlet cone is best taken at about 15 deg. to the axis (under which condition the coefficient of influx is nearly unity), and should not exceed 30 deg. Its larger diameter should be at least 25 per cent. greater than the small diameter. (See also Fig. 18, full lines.) Where a fan draws its air direct from the atmosphere or from the room in which it is placed and does not have a pipe attached to its inlet, it is of material advantage to use a fan with two inlets. For a given volume delivered the inlet velocity where two inlets are used will be one-half (and therefore the inlet velocity pressure one-quarter) that with one inlet. The effect of this reduction in inlet losses is illustrated in the table below, where the two fans are identical except as to inlets.

Inlet	R.p.m.	Static pressure, in. of water	Cu. ft. per min.	Air h.p.	B.h.p.	Mechanical efficiency, per cent.
Single	265	1.5	39,200	12.50	25.0	50.0
Double	265	1.5	44,100	14.95	27.7	54.0

Another test, in which the volume and static pressure were held constant instead of the r.p.m. and static pressure, shows the decrease in r.p.m. and increase in mechanical efficiency of the double-inlet fan, as follows [see also Figs. 18 (full lines) and 19]:

Inlet	R.p.m.	Static pressure, inches of water	Cu. ft. per min.	Air h.p.	B.h.p.	Mechanical efficiency, per cent.
Single	265	1.5	39,200	12.50	25.0	50.0
Double	253	1.5	39,200	12.50	22.6	55.2

This gain in volumetric and mechanical efficiency holds only with a single-width fan. A double-width, double-inlet fan is in effect simply two single-width, single-inlet fans placed back to back. Obstructions to the flow of air into the inlets, such as bearings and bearing supports, are often important offenders. The inlet area is reduced and the inlet losses for a given volume increase as the square of the decrease in inlet area due to this cause, while the interference with stream lines adds further to these inlet losses. The table below gives the results of tests on a multiblade type of fan (*a*) with a free and unobstructed inlet, and (*b*) with a bearing and its support mounted directly in the inlet. The effect on the volumetric and mechanical efficiencies with a given number of r.p.m and static pressure is very noticeable.

Test	R.p.m.	Pressure, Inches of Water Static	Impact	Velocity	Cu. ft. per min.	Horse Power Air	Brake	Mechanical efficiency, per cent.
(*a*)	900	3.0	4.45	1.45	26,500	18.6	37.0	50.3
(*b*)	900	3.0	3.91	0.91	21,000	12.9	27.0	48.0

Outlet or Discharge of Casing. The area of the outlet of a spiral casing is seldom made as small as the area between the wheel and spiral at the point of maximum area or where the air passes the cut-off in leaving the casing. A relatively large velocity pressure exists at this point and it is general practice in fans of small and medium size to make no attempt whatever to conserve the kinetic energy in this blast. In most installations this velocity pressure will be from 25 to 50 per cent. of the total head developed. Fans in general have outlets varying from 25 per cent. to 75 per cent. larger than the area at the cut-off point. The expansion is abrupt, so that although some of the velocity pressure is transformed into static pressure, there is nevertheless considerable loss. If the fan discharges directly into the atmosphere the entire velocity pressure in the outlet is lost. Most of this can be saved, however, if the outlet is fitted with a proper evasé discharge piece. (See also Figs. 19 and 20.) In larger fans such as are used for mine ventilation the evasé chimney is started from the cut-off point and thereby conserves the larger portion of the velocity head at that point. If the fan is connected to a duct larger than the outlet it should have a long, tapered connection.

It is not practical to carry the expansion piece on a fan outlet to a point where the velocity is below 1000 to 500 ft. per min. The velocity head due to a velocity of 500 ft. per min. is but $\frac{1}{64}$ in., and is negligible. The angle between the sides and the axis of an efficient chimney or diffuser should be from 7 deg. to 10 deg. The use of a diffuser produces a lower static pressure at the cut-off point and consequently the fan may run at a lower speed and at higher volumetric efficiency. Actual tests on a multiblade fan for mine ventilation showed that, when fitted with a chimney whose larger end was twice the cut-off area and whose sides had a taper of 8 deg. from the axis, the volumetric capacity was increased 12 per cent. when operating against a given static pressure and at a given number of r.p.m. The horse power, however, increased only 5 per cent.

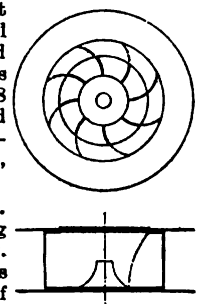


Fig. 8. Diffuser Casing.

Diffuser Casing Open Around Circumference. Similar results may be obtained with wheels discharging into circular diffusers open all around their periphery. Fig. 8 shows this form of diffuser, which incidentally has still another effect on the outflow of air. This type of fan must deliver its air against the pressure in the chamber into which it discharges, and this pressure acts in a radial direction at all points. It follows, therefore, that the resultant direction of flow should be as near radial as possible. This necessitates a blade curved backward relative to rotation, but it can be considerably helped by a radial diffuser. Fig. 9 shows that the velocity of the air decreases in this diffuser and at the same time the direction of the flow becomes more nearly radial; also, that the radial component of the air velocity becomes greater relative to the resultant as the distance from the center increases, thus utilizing a greater part of the resultant velocity. In Fig. 9, r_2 is the radius of the wheel or entrance to diffuser, r_3 the outer radius of diffuser, and A_2 is the circumferential area at entrance to the diffuser. The volume delivered is $u_2A_2 = v_2 \sin b_2A_2$. No force acts on the air as it passes through the diffuser and its direction is unchanged, but its radial component varies inversely as the radius, or $u_3/u_2 = r_2/r_3$. The direction of the air leaving the diffuser is known, and its velocity is determinable since u_3 is known. Both v_3 and w_3 decrease much more rapidly than inversely as the radius, and the ratio of u_3 to v_3 is greater than that of u_2 to v_2. This type of diffuser is generally made stationary and its sides are made radial, converging or diverging. With a slight convergence there can still be an increase in circumferential area with an increase in radius. When the sides diverge the angle of divergence should not exceed from 7 deg. to 10 deg., in order to conserve a full section of flow. (See Fig. 24.) A diffuser of this sort is also at times applied to a fan wheel running in a spiral casing, the air passing from the wheel to the diffuser and thence to the spiral, which must have small radial depth.

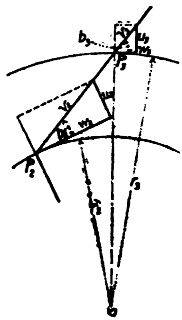


FIG. 9.

Fan Wheels

Blade Curvature, Diagrams, Losses. The losses in the wheel are caused by surface friction and by sudden changes in direction and velocity which produce wasteful eddies. The force necessary to overcome the surface friction is represented by the formula $P_fA = fWv^2S$, where P_f is the pressure loss, lb. per sq. ft.; A the sectional area, sq. ft.; f the coefficient of friction or loss in lb. per sq. ft. per ft. (velocity) per sec.; v the velocity, ft. per sec., and S the surface, sq. ft. For smooth iron f is approximately 0.00008. The formula may also be written as $h_f = fWv^2S/5.2A$, where h_f = pressure loss in inches of water. The loss due to a sudden change in velocity is computed from the formula $H_g = v_2(v_1 - v_2)/g$, where H_g = feet of air, or from $h_g = v_2 \times (v_1 - v_2)/69.3g$, where h_g = loss in inches of water. The losses due to change of direction are computed from the triangle of forces. In Fig. 10 let the velocity v represented by BA be changed to v_1, represented by BC, and the angle ABC be called a; then the loss of head $H_c = \overline{AC}^2/2g = (v^2 + v_1^2 - 2vv_1 \cos a)/2g$. This has its least value when AC coincides with BD the perpendicular on AC, that is, when $v_1 = v \cos a$. Let the static pressures before and after the change be represented by P and P_1, respectively; then $(P_1/w) + (v_1^2/2g)$ + loss of head $= (P/W) + (v^2/2g)$. The gain of static pressure $= (P_1 - P)/W = (v^2 - v_1^2)/2g - (v^2 + v_1^2 - 2vv_1 \cos a)/2g = (vv_1 \cos a - v_1^2)/g = BC \times BD/g$. This has a maximum value when $BC = CD$. Therefore, if it is afterward expected to convert a large part of the velocity head into static pressure by means of an expanding pipe, v_1 should equal $v \cos a$, but when no expanding pipe is to be used BC should equal ½ BD, or $v_1 = ½v \cos a$.

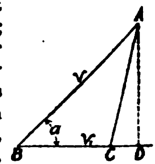


FIG. 10.

The performance of a wheel is dependent on the casing in which it is placed. The volume, pressure, and efficiency of a wheel are functions of the shape, relative size, length, depth, curvature, inclination and number of blades.

[See Figs. 18, 21, 22, (full lines) and 23 (broken lines).] Figs. 11, 12 and 13 give velocity diagrams for wheels with blades with their generatrices parallel to the axis, curved backward, radial and curved forward, respectively. Air enters the ports between the blades in a radial direction and follows a path similar to *ED*, which may differ slightly from the actual surfaces of the blades. The lighter the load the nearer will the path of air approach the curve of the blade. As the blade curvature changes from convex forward to concave forward the resultant velocity for a given tip speed increases in value and approaches more nearly a tangential direction. Where

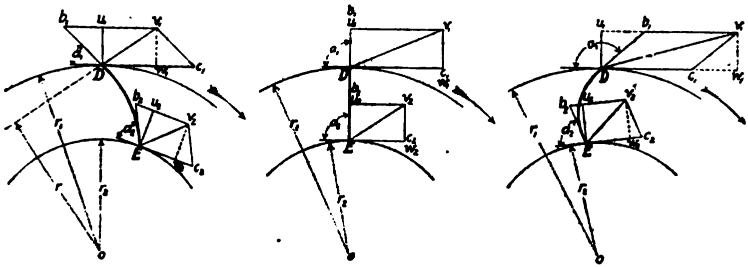


Fig. 11. Fig. 12. Fig. 13.

Velocity Diagrams for Fan Wheels.

a wheel is placed in a spiral casing, the blade curved forward is best; where the wheel is not fitted with a spiral casing but discharges directly into a chamber or the open atmosphere, whether through a radial diffuser or not, the convex forward blade is best, since the pressure against which it discharges acts in a radial direction.

The following symbols apply to Figs. 11–13, the subscript 1 being used for discharge from, and subscript 2 for entrance to, the blades. The angle between the tangent to an element of the blade and the tangent to a circumference through that element is denoted by a, b is the velocity in the direction of an element of the blade, c the tangential velocity of an element of the blade, u the radial component, v the resultant velocity relative to stationary objects, and r the radius. A particle of air weighing W and leaving the wheel at a velocity v_1, has a momentum of Wv_1/g, and if r is a perpendicular on the direction of v_1 drawn through the center of the wheel, the moment of momentum (or angular momentum) of the particle is $Wv_1\, r/g$. By a solution of triangles, $v_1\, r = w_1\, r_1$; therefore, the angular momentum equals $Ww_1\, r_1/g$. The particle had no angular momentum when entering the wheel, so $Ww_1\, r_1/g$ is the angular momentum of all the forces acting on it. Now if W_a is the total weight of air passing through the fan per second and o equals the angular velocity in radians, $W_a w_1\, r_1\, o/g$ equals the work done per second by the wheel on the air. Since $r_1\, o$ equals c_1, the peripheral speed per second, the work done per second is equal to $W_a w_1\, c_1/g$.

The loss of head at entrance to the blades, for instance in Fig. 11, is $L_1 = w_2{}^2/2g$, w_2 being the length from E to w_2. The length w_2c_2 equals $u_2 \cot a_2$, therefore $L_1 = (c_2 - u_2 \cot a_2)/2g$. To minimize this loss $\cot a_2$ should equal c_2/u_2. If the fan discharges directly into the atmosphere, all the kinetic head $v_1{}^2/2g$ is lost. If the fan discharges into a spiral casing, the loss of head here is $L_2 = (r_1\, v_3)^2/2g$, where $v_1\, r_3$ represents the length between those points in Fig. 14, and v_3 the velocity in the spiral, whose direction is nearly tangential to the wheel. $L_3 = [u_1^2 + (w_1 - v_3)^2]/2g$. To minimise this loss, v_3 should equal w_1 where the spiral is fitted with an expansion outlet, and equal ½ w_1 when an expanded outlet is not employed. The value of v_3 in terms of w_1 will vary with the restriction against which the fan operates. In Figs. 11–13 the same peripheral speed of wheel c_1 and radial component u_1 are used, so that the relative effect upon the resultant v_1 is shown graphically.

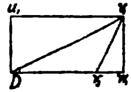


FIG. 14.

The blades in these figures have their generatrices parallel to the axis of the wheel. There are types, however, which include a compound curved blade taking the form of a screw or propeller at the intake and gradually changing into a true centrifugal blade at the discharge. Where the exact duty to be imposed upon this type of fan is known and the pitch of the screw portion of the blade is made accordingly, this form of blade has merit.

In order to equalize the flow of air from the ports of the blades or wheel periphery, different methods have been resorted to. One is to form the blade with spherical depressions along its axial length, in this way pocketing the air as it slides along the blade axially in its changing from axial flow at inlet to radial flow at discharge, and thereby reducing the slippage. Another is to allow the end of the blade nearest the intake to lag behind in rotation, that is, the outer peripheral edge of the blade is in a plane not parallel to the plane of the axis but advancing in the direction of rotation from the inlet end toward the back of the blade. Still another method is to make the diameter of the outer peripheral edge of the blade greatest at the end of the blade nearest the inlet and of gradually decreasing diameter as the rear or center of the wheel is approached, according as the fan may have one or two inlets. Whatever the form of curved blade—and especially with blades of relatively small radial depth, the inclination with respect to the axis is very important, it being found in a case where blades were curved forward that a variation of 1 deg. backward resulted in an increase of approximately 1 per cent. in speed to get the same volumetric delivery. This same rate held approximately up to a variation of 30 deg., but the mechanical efficiency began to suffer greatly after a few degrees departure from the inclination for which the blades were designed. Obviously, the inlet and outlet port areas between blades are seriously affected.

Axial Length of Blade. A common rule for maximum efficiency is to make the port area between blades less than or equal to the inlet area of the wheel, but practice has exceeded this limit in many instances. In a fan with radial blades, neglecting their thickness, this rule would limit the axial length of the tip of the blade to $d^2/4D$, where d is the diam. of the inlet and D

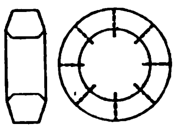


FIG. 15.

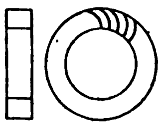


FIG. 16.

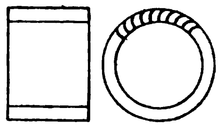


FIG. 17.

Fan-wheel Blade Constructions.

the diam. of the outer edge of the blades. Radial-blade wheels are almost invariably made with the axial length of the edge of the blade at intake materially greater than that of the outer or peripheral edge, as in Fig. 15. This is for the purpose of keeping up the area of the ports at the intake edge, yet in many fans the length of the inner edge is for reasons of construction not increased sufficiently to make the port area at intake to blades equal to that at discharge. In wheels with flat parallel sides in planes perpendicular to the axis, the port area can be kept constant from intake to discharge by the use of blades having the curve of an involute of a circle, as in Fig. 16, in which case the height of a normal from one blade to the next is constant for all points along the blade and the port area equals the inlet area when the axial length of blade is ¼ *d*. When the blade is curved to some other form, such as that of a circle, it is seen that the port area is reduced and depends on the length of a line drawn through the tip of one blade and normal to the next adjacent blade, this representing the minimum distance between blades. The axial length of the blade may, therefore, be considerably more than ¼*D*, depending on the curvature and radial depth of blade. Decreasing the radial depth of the blade augments the inlet area to the wheel, hence, in itself, allowing of greater axial length of blade. Following this method and by the use of shallow, curved blades, there is developed the multiblade construction of Fig. 17, in which the axial length of blade reaches values from 4 to 8 times the radial depth.

Radial Depth and Number of Blades. A blade of small radial depth permits a larger inlet area to the fan, thereby decreasing the inlet losses for a given volume and increasing the volumetric capacity of the wheel. [See Figs. 18 and 21 (full lines).] The inlet area will increase as d^2 and the loss of head at the inlet for a given volume will decrease as $1/d^4$. Again, the rotative speed is increased for a given duty, making the fan more applicable to the more modern direct-connected drives such as electric motors, turbines, etc. Decreasing the radial depth, however, reduces the blade area, which can only be obviated by increasing the number of the blades. When the number becomes too great, not only do constructional interferences arise but also the port area between the blades becomes reduced and surface friction increased to a point where both are detrimental. It therefore becomes a matter of experiment to find the most desirable number from the standpoint of both volumetric capacity and mechanical efficiency. As the number is increased the volumetric capacity will still be on the increase after the mechanical efficiency has passed its maximum. (See Fig. 19.) A fan with many shallow blades has considerably higher volumetric capacity and manometric efficiency than the older types with few deep blades. The extent to which the radial depth can be decreased is limited entirely by the strength of the blade to resist the stresses that come upon it in operation. Most of the stress arises from the centrifugal force due to the weight of the blade itself. The blade can be considered as a uniformly loaded beam, so that the radial blade is strongest, while the strength of the curved blade depends entirely on its curvature. In fans for very high pressures and speeds it is common practice to cut the length of the blade into parts and insert annular rings or sheets such as are used regularly for side plates on the wheel.

Support of Blades. Both deep-bladed and shallow-bladed wheels are generally built up in either of two forms. (1) All or some of the blades are attached to the outer ends of spider arms emanating from a central hub. In this case the driving effort is applied to the center of the wheel and the work is fairly well balanced on either side of the spider. The force F (lb.) exerted by each of the spider arms is small: $F = (\text{h.p.} \times 33{,}000)/(T \times \text{number of arms})$. Where there are more blades than spider arms, the intermediate blades are supported only from the end rings or side plates, to which are also fastened the ends of the driving blades. (2) The other common form of support is that in which all of the blades are fastened at one end to a driving disk connected to a hub, and at the other or inlet end to a sheet-metal annulus. All the driving effort being placed upon the rear end of the blades, it becomes necessary, except for low speeds, to run stay rods out from the central hub to the sheet-metal annulus or side plate nearest the inlet, thus counteracting the tendency of this portion of the wheel to lag behind the driven end.

The weight of the fan wheel is generally so great that the use of a flywheel for direct-connected engine drive is not necessary. The diameter of the fan wheel is always considerably larger than would be that of the flywheel for the engine, and in fans of the multiblade construction the radius of gyration is approximately 80 per cent. of the radius of the fan wheel.

Characteristics of Various Designs and Effect of Change in Elements of Design and Proportions. The effect of a change in a single element of design on the principal factors of fan performance is seen by comparison of the characteristic curves in Figs. 18 to 24. The curves are drawn for fans handling standard air at a density of 0.075 lb. per cu. ft., having wheels 1 ft. in diam. and operating at a peripheral speed of 4000 ft. per min. or at 1274 r.p.m. This facilitates comparison of one with another. The speed of 4000 ft. per min. is not only a good average speed, but for standard air represents the velocity producing a pressure of 1 in. of water gage. The diam. of 1 ft. was chosen in order to establish as a basis unit diameter and thereby simplify calculations. From these curves the performance at any other peripheral speed can be computed by considering the volume to vary as the 1st power, the pressures as the 2nd power and the h.p. as the 3rd power of the speed, while the efficiency remains constant for any given load point on the curves. The performance of a symmetrical fan of different size or the computation of size, h.p., r.p.m., etc. necessary to do a given work, is readily accomplished by use of the above rules and the further rule that the capacity of a given design of fan operating against a given static pressure at a given peripheral speed varies as the square of the diameter. All effective fan dimensions will vary as the 1st power and areas as the 2nd power of the diameter. Where the fan handles air or gas at a density different from standard, the pressure and horse power will vary directly with the density, and the mechanical efficiency will not be affected. For factors to apply with variations to the air (or gas) temperature, see Table 3.

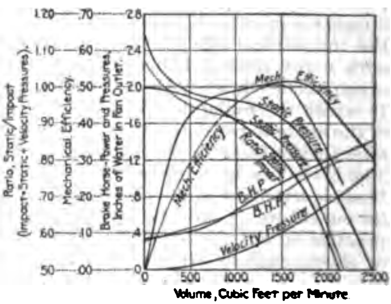


Fig. 18.—Characteristics of a Multiblade Fan with Single-inlet Spiral Casing.

Fig. 18 deals with a multiblade type of wheel fitted with a single-inlet spiral casing. The blades are of relatively small radial depth. The curves with solid lines are for blades having radially disposed corrugations equally spaced along the axial length of the blade; the broken-line curves are for similar blades without corrugations. They are curved so that the concave surface moves forward in rotation and are so inclined forward that a chord of the arc of the blade is about 18 deg. ahead of a radius through the tip of the blade, thereby making the outlet ports smaller than the inlet ports of the blades. The principal proportions, given as functions of the wheel diameter, are as follows: Number of blades, 60; radial depth of blades, $0.066D$; axial length of blades, $0.52D$; diam. at inlet of wheel, $0.868D$; equation of spiral, $R = r(1 + 0.198a)$; width of spiral, $0.69D$; number of inlets, 1; diam. of inlet cone, $1.05D$; area of cut-off point, $0.434D^2$; area of outlet, $0.59D^2$.

Method of Using Characteristic Curves.—Example. To illustrate the use of these characteristic curves a problem may be taken, as follows: What will be the size of a fan with corrugated blades of the type covered by Fig. 18 which will be required to deliver 50,000 cu. ft. per min. against 1.5 in. static pressure, the fan to operate at approximately maximum mechanical efficiency? The diam. of the fan found by calculation will vary according to the point on the curves of Fig. 18 taken as a basis, and will become less as the volume represented by that point increases. Assume as a basis the delivery of a fan 1 ft. in diam. at 51.5 per cent. mechanical efficiency where it delivers 1500 cu. ft. per min. against 1.67 in. static pressure at 1274 r.p.m., requiring 0.95 b.h.p. Operating at the same point on the mechanical efficiency curve but against 1.5 in. pressure would require the speed to be lowered in proportion to the square root of the decrease in pressure, or be multiplied by the factor 0.947. The volume would decrease in the same proportion and the b.h.p. as the 3rd power of 0.947, giving a delivery of 1412 cu. ft. per min. against 1.5 in. static pressure at 1200 r.p.m. and requiring 0.81 b.h.p. Since a delivery of 50,000 cu. ft. per min. is desired at this static pressure of 1.5 in., the peripheral speed will remain the same but the diam. will vary as the square root of the increase in volume. The speed in r.p.m. will vary inversely as the diam. and the b.h.p. directly as the volume, giving a fan 5.95 ft. in. diam. to deliver 50,000 cu. ft. per min. against 1.5 in. at 202 r.p.m. and requiring 28.7 b.h.p. Since the diam. obtained is an odd figure, it might be decided to use a fan of even diam., say 6 ft., in which case its performance would be calculated by considering that a similar fan 1 ft. in diam. at the same mechanical efficiency would deliver a volume in proportion to the square of the diameters, or 1/36 of 50,000, which is 1390 cu. ft. per min., against 1.5 in. pressure. Since the area of outlet is 0.59 sq. ft. the velocity in the outlet is 2350 ft. per min. and the velocity pressure 0.347 in., hence the ratio of static to impact pressure is then 0.813. Fig. 18 contains a curve of this ratio, which facilitates such computations and does away with cut-and-try methods. From Fig. 18 it is seen that when the ratio of static to impact pressure equals 0.813, the 1-ft.-diam. fan delivers 1465 cu. ft. per min. against 1.69 in. static pressure for 0.93 b.h.p. at 1274 r.p.m. Now to deliver 1390 cu. ft. per min. against 1.5 in., the fan speed would drop in direct proportion to the volume, or to 1209 r.p.m., and the b.h.p. as the cube, or to 0.79 b.h.p. From this the speed and horse power of the 6-ft. fan when delivering 50,000 cu. ft. per min. against 1.5 in. are found to be 201 r.p.m. and 28.5 b.h.p.

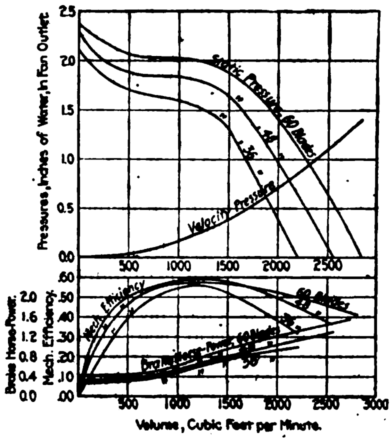


FIG. 19.—Characteristics of Multiblade Fans with Double-inlet Spiral Casings.

Fig. 19 covers a double-inlet fan with wheels and casing of the same proportions as the fan of Fig. 18; the three wheels tested in this casing had respectively 60, 48 and 36 blades. The one with 60 blades was the same wheel as used in Fig. 18. The effect of making the casing with a double inlet was to increase the volumetric capacity and mechanical efficiency by an appreciable amount. The effect of a reduction in the number of blades was a decrease in volumetric capacity and pressure developed, yet the mechanical efficiency increased somewhat with 48 blades. Fig. 20 is for the same fan as that having 48 blades in Fig. 19, and shows what an evasé (expanding) discharge on the fan will do in raising the resistance against which a given volume can be delivered or raising the volume delivered against a given resistance. The evasé discharge connection increased in area with a taper to its sides of about 7 deg. until it was twice that of the fan outlet. For any given volume the b.h.p. is the same as for the fan of Fig. 19, the gain being in static pressure produced.

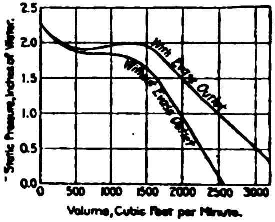


FIG. 20.—Effect of Using an Evasé Discharge on Fan.

Fig. 21 covers a steel-plate paddle-wheel fan with eight flat radial blades of relatively large radial depth and a single-inlet spiral casing. The side plates are inclined inwardly as in Fig. 26. The principal proportions are as follows: Number of blades, 8; radial depth of blades at intake, $0.145D$; maximum radial depth of blades, $0.29D$; axial length of blades (outer edges), $0.39D$; axial length of blades (at intake), $0.46D$; diam. at inlet of wheel, $0.71D$; equation of spiral, $R = r(1 + 0.106a + 0.125)$; width of spiral, $0.5D$; number of inlets, 1; diam. of inlet, $0.71D$; area at cut-off point, $0.306D^2$; area at outlet, $0.306D^2$. While there is not much difference in mechanical efficiency between this fan and the multiblade fan of Fig. 18, there is a great difference in the pressures and volumetric capacities; much less space is required for the multiblade type when performing a given duty.

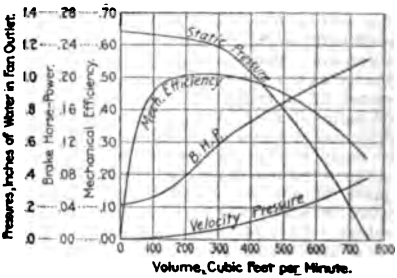


FIG. 21.—Characteristics of a Single-inlet Steel-plate Paddle-wheel Fan with Radial Blades.

Fig. 22 covers a fan designed for high rotative speeds at relatively low ressures. The wheel is fitted with single-inlet spiral casing and has 16 ep blades with radially disposed rrugations equally spaced along eir axial length. They are so urved that the concave surface moves forward in rotation and are so inclined rearwardly that a chord of the arc of the blade is about 35 deg. back of a radius through the tip of the blade, thereby making the outlet ports larger than the inlet ports of the blades. Reference to Fig. 18 will show the greater peripheral speed required of the fan of Fig. 22 for a given pressure.

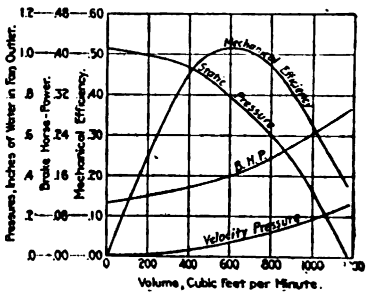


FIG. 22.—Characteristics of a Single-inlet Fan for High Speeds and Low Pressures.

The principal proportions are as follows: Number of blades, 16; radial depth of blades, 0.135D; axial length of blades, 0.52D; diam. inlet of wheel, 0.868D; equation of spiral, $R = r(1 + 0.198a)$; width of spiral, 0.69D; number of inlets, 1; diam. of inlet cone, 1.05D; area at cut-off point, 0.434D^2; area of outlet, 0.59D^2.

The full-line curves of Fig. 23 cover a fan designed for high rotative speeds at relatively high pressures such as are found in forced-draft applications. The wheel has no casing and discharges directly into the compartment in which the fan maintains a pressure. The wheel has 16 blades so curved that the convex surface moves forward in rotation. The blade is of variable depth, being most shallow at the intake side of the wheel and deepest at the part farthest from the intake side, forming a type known as a cone fan. The blades have a very decided back slope, which, as previously shown, is essential in an open-running wheel. The efficiency curves of Figs. 23 and 24 are those obtained by the use of static pressure in computing the air horse power, since obviously the kinetic energy at discharge from the fan is not useful as in a cased fan where the air generally travels along ducts. The principal proportions are as follows: Number of blades, 16; radial depth of blades, intake side, 0.052D; radial depth of blades, rear of wheel, 0.28D; axial length of blades, 0.38D; diam. of inlet of wheel, 0.896D; outlet port area of wheel, 0.56D^2. The broken curves of Fig. 23 cover the same fan, except that the wheel is fitted with a single-inlet spiral casing. The volumetric capacity and pressure are increased slightly and the mechanical efficiency considerably; the latter increase being due to the gain in pressure head incident with the expansion from the outlet ports of the wheel into the casing and from the cut-off of the casing to its outlet. This fan is particularly adapted to high-speed drives such as steam turbines. The principal proportions of the casing are as follows: Equation of spiral, $R = r(1 + 0.318a)$; width of spiral, 0.64D; number of inlets, 1; diam. of inlet cone, 1.2D; area at cut-off, 0.64D^2; area of outlet, 0.785D^2.

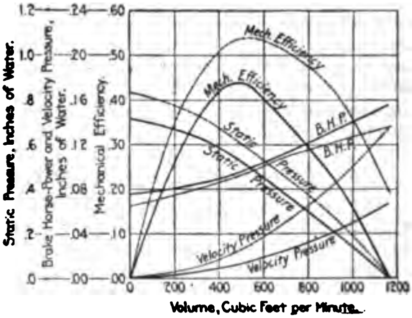


Fig. 23.—Characteristics of a Casingless Fan for Forced-draft Applications.

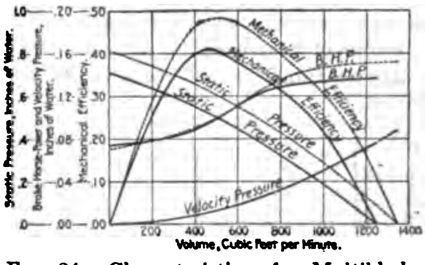


Fig. 24.—Characteristics of a Multiblade Casingless Fan for Forced-draft Work.

Fig. 24 covers a fan of practically the same design as Fig. 23, except that the wheel has 32 blades. The full-line curves show its performance when it is not fitted with a casing and discharges directly into a closed compartment. The greater number of blades gives it increased volumetric capacity and causes

the mechanical efficiency to hold up a little better on heavy load due to the increased blade area. The blade shape is the same as that in the fan of Fig. 23, as are also all the proportions. The broken-line curves show its performance when the wheel is fitted with a radial diffuser with its sides tapering toward each other, thereby reducing the circumferential area at the discharge of the diffuser below that it would have were its sides perpendicular to the axis of the fan. The volumetric capacity, pressure and mechanical efficiency are all materially increased due to the gain of head incident to the expansion of the air path from the blade ports to and through the diffuser. The principal proportions of the diffuser are as follows: Diam. of diffuser, $1.5D$; width at discharge, $0.24D$; area at discharge, $1.13D^2$.

CAPACITY TABLES

The accompanying tables give average performances of fans of the stated types, for a limited number of sizes and for given speeds. The performance at other pressures, speeds and diameters can be computed in the manner shown on p. 1545. The approximate height in col. 2 is the overall dimension for horizontal discharge. The abbreviation "c.f.m." is for cu. ft. per min.

Multiblade Fans

Makers and Trade Names. Am. Blower Co., Sirocco; Buffalo Forge Co., Conoidal; Keith & Blackman, Keith; N. Y. Blower Co., Seri-Vane; B. F. Sturtevant Co., Multivane.

Description. The wheels contain from 30 to 60 blades, generally of shallow radial depth and curved with concave surface forward in rotation. The inlet is exceptionally large compared to the wheel diam. A spiral casing encloses the wheel. (See Fig. 25.)

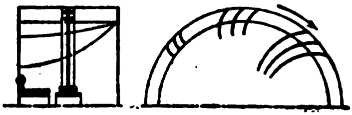


Fig. 25.—Multiblade Fan.

Range of Pressures. 0 to 5 in.

Applications. Forced draft, induced draft, heating, ventilating, cooling, drying, etc.

Wheel diam., ft.	Approx. height, ft.	Static pressure, in.											
		0.5			1			2			4		
		c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.
1	2	1,170	780	0.30	1,650	1100	0.85	2,340	1,560	2.4	3,300	2,200	6.8
2	4	4,650	390	1.2	6,600	550	3.4	9,300	780	9.6	13,200	1.100	27.0
4	8	18,700	195	4.8	26,400	275	13.6	37,400	390	38.5	52,800	550	109.0
6	12	42,000	130	10.8	59,400	184	30.5	84,000	260	86.0	118,800	368	244.0
8	16	75,000	98	19.1	105,600	138	54.0	150,000	196	153.0	211,200	276	432.0
10	20	117,000	78	30.0	165,000	110	85.0	234,000	156	240.0	330,000	220	680.0

Cast Casing Paddle-wheel Fans

Makers and Trade Names. American Blower Co., Type V; Buffalo Forge Co., Type B; Green Fuel Econ. Co., Cast-iron Volume Fan; N. Y. Blower Co., Peerless; B. F. Sturtevant Co., Monogram.

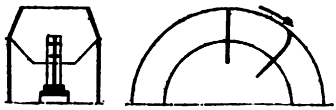


FIG. 26.—Paddle-wheel Fan with Cast Casing.

Description. Wheels are of paddle-wheel type—6 to 8 blades, and not quite so wide axially as the paddle wheels in "steel-plate" fans. Casings are spiral in form and of cast iron. (See Fig. 26.)

Range of Pressures. 0 to 6 oz.

Applications. Forges, oil furnaces, light dust collecting, etc.

Wheel diam., in.	Approx. height, in.	Static pressure, oz.											
		1			2			4			6		
		c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.
10	19	360	2,050	0.18	510	2,900	0.50	720	4,100	1.45	880	5,000	2.65
15	27	810	1,390	0.40	1,150	1,970	1.15	1,620	2,780	3.2	1,980	3,400	5.9
23	42	2,150	885	1.05	3,050	1,250	2.9	4,300	1,770	8.4	5,250	2,170	15.5
33	58	4,650	630	2.3	6,450	890	6.5	9,300	1,260	18.5	11,400	1,540	34.0

Cast-iron and Steel-plate Pressure Fans

Makers and Trade Names. American Blower Co., Type P; Buffalo Forge Co., Type P; B. F. Sturtevant Co., "Steel Pressure Blower."

Description. Wheels are of paddle type; number of blades from 6 to 24, width narrower than other types. Casing of spiral form. (See Fig. 27.)

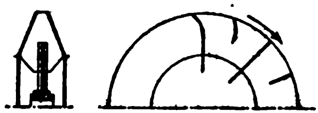


FIG. 27.—Cast-iron or Steel-plate Pressure Fan.

Range of Pressures. 4 oz. to 16 oz.

Applications. Forges, oil furnaces, cupolas, gas producers, boosters, etc.

Wheel diam., in.	Approx. height, in.	Static pressure, oz.											
		4			8			12			16		
		c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.
14	25	600	2,800	1.3	850	3,960	3.7	1,040	4,850	6.7	1,200	5,600	10.4
21	39	1,300	1,900	2.8	1,850	2,680	7.9	2,250	3,300	14.5	2,600	3,800	22.5
34	59	3,400	1,150	7.1	4,800	1,625	20.0	5,900	2,000	37.0	6,800	2,300	57.0
45	77	6,300	875	13.0	8,900	1,240	37.0	10,900	1,500	67.0	12,600	1,750	104.0

Steel-plate Planing-mill Fans (Slow-speed Type)

Makers. American Blower Co., Buffalo Forge Co., Green Fuel Econ. Co., Sterling Blower Co., B. F. Sturtevant Co., etc.

Description. Wheels have 12 to 18 paddle-type blades, curved forward. Casing is of spiral form, about the same as in common steel-plate fans. (See Fig. 28.)

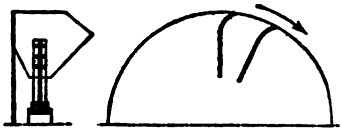


Fig. 28.—Steel-plate Planing-mill Fan.

Range of Pressures. 0 to 6 oz.

Applications. Conveying materials such as refuse from wood-working machinery, shoe machinery, buffing and emery wheels, etc.

Wheel diam., in.	Approx. height, in.	Static pressure, oz.											
		1			2			4			6		
		c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.
24	35	1,770	650	0.7	2,500	920	2.0	3,540	1,300	5.6	4,350	1,600	10.2
36	50	4,000	435	1.6	5,600	615	4.5	8,000	870	12.8	9,800	1,060	23.5
48	70	7,050	325	2.8	10,000	460	8.0	14,100	650	22.4	17,300	800	41.0
64	90	12,600	245	5 0	17,800	345	14.2	25,200	490	40.0	31,000	600	73.0

Steel-plate Paddle-wheel Fans

Makers. American Blower Co., Buffalo Forge Co., Garden City Fan Co., Green Fuel Econ. Co., N. Y. Blower Co., B. F. Sturtevant Co., etc.

Description. Wheels have 8 to 12 blades which are deep radially and connected to spider arms emanating from a central hub. Casings of spiral form. (See Fig. 26.)

Range of Pressures. 0 to 5 in.

Applications. Forced draft, induced draft, heating, ventilating, cooling, drying, etc.

Wheel diam., ft.	Approx. height, in.	Static pressure, in.											
		0.5			1			2			4		
		c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.
2	40	1,800	510	0.37	2,500	720	1.1	3,600	1,020	3.0	5,000	1,440	8.6
3	60	4,000	340	0.85	5,700	480	2.4	8,000	680	6.8	11,400	960	19.0
4	80	7,200	255	1.5	10,200	360	4.3	14,500	510	12.0	20,400	720	34.0
6	120	16,000	170	3.4	22,800	240	9.6	32,000	340	27.0	45,600	480	77.0
8	160	28.700	128	6.0	40,700	180	17.1	57,500	255	48.0	81,400	360	137.0
12	240	65,000	85	13.5	91,200	120	38.4	130,000	170	108.0	182,000	240	307.0

Steel-plate Cone Fans

Makers. American Blower Co., Buffalo Forge Co., Green Fuel Econ. Co., N. Y. Blower Co., B. F. Sturtevant Co.

Description. These fans are of the paddle-wheel type, but built on a back or supporting cone instead of a spider having arms emanating from a central hub; 8 to 12 blades are employed and no casing is used. The fan discharges directly into the atmosphere or a large room, as the case may be. (See Fig. 29.)

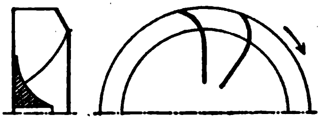


FIG. 29.—Steel-plate Cone Fan.

Range of Pressures. 0 to 1½ in.

Applications. Heating, ventilating, cooling, drying, etc.

Wheel diam., ft.	Static pressure, in.											
	0.25			0.5			1.0			1.5		
	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.
3	4,300	320	0.63	6,100	450	1.8	8,600	640	5.0	10,400	785	9.2
4	7,650	240	1.1	10,800	340	3.2	15,300	480	8.9	18,700	590	16.3
6	17,200	160	2.5	24,300	225	7.1	34,400	320	20.0	42,000	390	37.0
8	30,500	120	4.5	43,000	170	12.7	61,000	240	36.0	75,000	295	66.0
12	69,000	80	10.0	98,000	113	28.0	138,000	160	80.0	169,000	195	147.0

Propeller Fans

Makers and Trade Names. Buffalo Forge Co.; Green Fuel Econ. Co.; Keith & Blackman, Blackman; Davidson; N. Y. Blower Co.; B. F. Sturtevant Co.

Description. Wheels are of screw or propeller form, having 6 to 12 blades of curved formation.

Range of Pressures. 0 to 1 in.

Applications. Heating, ventilating, cooling, drying, etc.

Wheel diam., ft.	Free delivery			Static pressure, in.								
				0.25			0.5			1.0		
	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.	c.f.m.	r.p.m.	b.h.p.
2	4,000	700	0.24	2,100	710	0.53	3,000	1,000	1.5	4,250	1,420	4.2
3	9,000	465	0.54	4,800	470	1.2	6,750	670	3.4	9,500	940	9.6
4	16,000	350	0.95	8,500	355	2.1	12,000	500	6.0	17,000	710	17.0
6	36,000	233	2.15	19,000	235	5.1	27,000	335	14.5	38,000	470	41.0
8	64,000	175	3.8	34,000	175	8.5	48,000	250	24.0	68,000	355	68.0

SECTION 14

ELECTRICAL ENGINEERING

BY

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CONTENTS

ELECTRICAL ENGINEERING

BY

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AND

F. A. KARTAK

REFERENCES: Perrine, "Electrical Conductors," Van Nostrand. "National Electric Light Assn. Electrical Meterman's Handbook." Underhill, "Solenoids," Van Nostrand. Croft, "American Electrician's Handbook," McGraw-Hill. Knox, "Electric Light Wiring," McGraw-Hill. "Standardisation Rules," American Inst. Electrical Engineers. A. Russell, "Alternating Currents," Cambridge University. Christie, "Electrical Engineering," McGraw-Hill. A. Gray, "Electrical Machine Design," McGraw-Hill. Jones, "Electric Ignition," Wiley. "Wiring Diagrams of Electrical Apparatus and Installations," McGraw-Hill. Creedy, "Single-phase Commutator Motors," Van Nostrand. McAllister, "Alternating-current Motors," McGraw-Hill. D. C. & J. P. Jackson, "Alternating Currents and Alternating-current Machinery," Macmillan. Morse, "Storage Batteries," Macmillan.

MAGNETIC AND ELECTRICAL UNITS

Systems of Units. Two systems of electrical units exist, the electrostatic and the electromagnetic. The electrostatic system is based upon the force exerted between two unit charges of electricity at points 1 cm. apart in a medium of unit dielectric constant. The electromagnetic system is based upon the force exerted between two unit magnetic poles placed at points 1 cm. apart in a medium of unit magnetic permeability. The **practical units** are based upon the electromagnetic system.

Unit Magnetic Pole is one which will repel an equal and like pole with a force of one dyne at a distance of 1 cm. (A dyne is the force which, acting upon a mass of 1 gram during 1 sec., gives the mass a velocity of 1 cm. per sec.)

Magnetic Potential is measured by the work involved in moving a unit magnet pole from the boundary of the field to the point at which the potential is desired.

Unit Magnetic Field is that which acts upon a unit magnet pole with a force of one dyne. Unit field has one line of force per sq. cm., and the magnetic flux density in this case is one **gauss**. Magnetic flux density ($\mathcal{B}$) is expressed in lines of force Φ (**maxwells**) per sq. cm. Frequently, magnetic flux density is expressed in lines of force per sq. in.

Magnetizing Force ($\mathcal{H}$). Unit magnetizing force will establish in air one line of force per cm. cube; it is also the magnetic potential gradient or the change of magnetic potential per cm. length in air when the magnetic flux density is one maxwell per sq. cm.

Magnetomotive Force ($\mathcal{F}$). Magnetomotive force (m.m.f.) is the line integral of the magnetizing force, that is, the total magnetizing force integrated over the length of the complete magnetic circuit. The unit is the **gilbert**.

Magnetic Permeability (μ) expresses the ratio of the magnetic flux density to the magnetizing force. $\mathcal{B}/\mathcal{H} = \mu$. It is the specific magnetic conductance. The permeability of air is unity.

Reluctivity (ν) or specific magnetic reluctance expresses the magnetic reluctance of a cm. cube of the magnetic medium. Unit reluctivity is that through which unit magnetizing force will establish unit flux density. Reluctivity is the reciprocal of permeability.

Magnetic Reluctance ($\mathcal{R}$) expresses the total resistance to the passage of magnetic flux. In a homogeneous medium of constant cross-section, the reluctance is the reluctivity multiplied by the length and divided by the cross-section of the medium. The unit of reluctance is the **oersted.**

Ampere (I,i). The practical unit of current flow is the ampere, which is equal to one-tenth the absolute unit of current, and is the current that will flow through a conductor having a resistance of 1 ohm and a difference of potential of 1 volt between its ends. The absolute unit of current is that which flowing in a conductor perpendicular to the lines of force in unit field causes to be exerted upon each centimeter of the conductor a (unit) force of one dyne. One ampere (direct current) will deposit 0.001118 gram of silver per second from a silver solution.

Ampere-turn. Magnetic effects of current are proportional to the product of amperes flowing and the turns or convolutions of the electric circuit. The ampere-turn, coupled with the proper constant, is used as a unit of magnetomotive force.

Coulomb. The unit of quantity is the coulomb, and is that quantity which passes a cross-section of the conductor in 1 sec. when the rate of flow is 1 ampere.

Volt. The practical unit of electromotive force (e.m.f.) is the volt. An electrical conductor cutting lines of force at the rate of one per second has induced in it one absolute unit of e.m.f. The volt is 10^8 times the absolute unit.

Ohm. The practical unit of resistance is the ohm, and is that resistance through which the fall of potential is 1 volt when the current strength is 1 ampere. The ohm equals 10^9 absolute units of resistance.

Conductance. Conductance is the reciprocal of resistance and is expressed in reciprocal ohms or mhos ("mho" is ohm spelled backward).

Farad. The unit of electrical capacity is the farad, and is that capacity whose potential will be raised 1 volt by the addition of a charge of one coulomb. As the farad is too large a unit for practical purposes the microfarad, which is one-millionth of a farad, is generally used. The absolute unit of capacity is 10^9 farads.

Dielectric Constant. Air has a dielectric constant of unity. The dielectric constant is the ratio of the conductivity of a dielectric for electrostatic lines of force to that of air. Therefore, it is the ratio of the electrostatic capacity of a given condenser having a certain dielectric to the capacity of the same condenser with air as the dielectric.

Self-inductance. The practical unit of self-inductance is the **henry.** The henry is also called the **coefficient of self-induction.** An electric circuit possesses 1 henry when a rate of change of 1 amp. per sec. will induce a counter-pressure of 1 volt. It also follows that in such a circuit 1 ampere will set up 10^8 linkages of magnetic lines (product of turns and flux) in the circuit, since a change of 10^8 linkages per sec. is required to induce 1 volt. One henry is equal to 10^9 absolute units of self-inductance.

Mutual Inductance. Mutual inductance relates to the e.m.f. induced in an electric circuit by the mutual magnetic flux produced by current in a separate electrical circuit. The unit of mutual inductance is also the henry. When a change of current of 1 ampere per sec. in either of two separate circuits sets up an e.m.f. of 1 volt in the other circuit, their mutual inductance is 1 henry. Also, 1 ampere in one circuit induces 10^8 magnetic linkages in the other circuit when their mutual inductance is 1 henry. If M

denotes the mutual inductance of a pair of circuits and if there is no magnetic leakage, that is, when all the lines set up link both circuits, then $M^2 = L_1 L_2$, where L_1 and L_2 are the coefficients of self-induction of the two circuits, respectively.

Watt. The practical unit of power is the watt. One watt is produced when 1 ampere flows under an e.m.f. of 1 volt. One watt equals 10^7 ergs per sec. One kilowatt equals 1000 watts.

1 watt = 0.00134 h.p. = 44.25 ft-lb. per min. = 14.33 gram-calories per min. = 0.737 ft-lb. per sec., = 0.057 B.t.u. per min.

Watts = volts × amperes × cosine of angle of phase difference for an alternating current.

Table 1. Magnetic and Electrical Units

ELECTRICAL UNITS

Symbol	Quantity	Equation*	Practical unit	Value of practical units in electro-magnetic units
I, i	Current	$I = E/Z,\ I = Q/T$	Ampere	10^{-1}
	Current density	I/A	Amp. per sq. in.	1.55
			Amp. per sq. cm.	10^{-1}
Q, q	Quantity	$Q = IT$	Coulomb	10^{-1}
			Ampere-hour	360
R, r	Resistance	$R = P/I^2 = E/I$	Ohm	10^9
E, e	Electromotive force	$E = W/Q = d\Phi/dT$	Volt	10^8
ρ	Resistivity†	$\rho = RA/L$	Ohms per circular mil-foot	166.4
			Ohms per centimeter cube	10^9
			Ohms per sq. mm. per meter	10^5
C	Capacity	$C = Q/E$	Farad	10^{-9}
			Microfarad	10^{-15}
G, g	Conductance	$G = R/Z^2$	Mho	10^{-9}
B, b	Susceptance	$B = X/Z^2$	Mho	10^{-9}
Y, y	Admittance	$Y = \sqrt{G^2 + B^2} = 1/Z$	Mho	10^{-9}
λ	Conductivity	$\lambda = 1/\rho$	Mhos per unit volume.	....
	Time constant	L/R	Second	1
			Henry per ohm	1
T	Period or cycle	$T = 1/f$	Second	1
f	Frequency	$f = 1/T$	Cycles per second	1
ω	Angular velocity	$\omega = 2\pi f$	....	....
L	Coefficient of self-induction	$L = n\Phi/I$	Henry	10^9
X_L	Inductive reactance	$X_L = 2\pi fL$	Ohm	10^9
X_C	Condensive reactance	$X_C = 1/2\pi fC$	Ohm	10^9
X, x	Reactance	$X = X_L - X_C$	Ohm	10^9
Z, z	Impedance	$Z = \sqrt{R^2 + X^2}$	Ohm	10^9
P	Electric power	$P = EI = I^2R = EI \cos \phi$ (a.c.)	Watt	10^7
			Kilowatt	10^{10}
W	Electric energy	$W = PT$	Joule	10^7
			Watt-hour	36×10^9
			Kilowatt-hour	36×10^{12}
	Power factor	$\dfrac{EI \cos \phi}{EI} = \dfrac{\text{real } P}{\text{apparent } P}$		
	Reactive factor	$\dfrac{EI \sin \phi}{EI} = \dfrac{\text{reactive } P}{\text{apparent } P}$		

MAGNETIC UNITS

Symbol	Quantity	Equation*	Practical unit
m	Pole strength	$\mathfrak{F}/\mathfrak{R}$	
$\mathfrak{M}$	Magnetic moment	$\mathfrak{M} = mL$	
$\mathfrak{H}$	Field intensity	$\mathfrak{H} = \mathfrak{F}/m$	Gauss
$\mathfrak{I}$	Intensity of magnetization flux	$\mathfrak{I} = \mathfrak{M}/V$	
Φ	Flux	$\Phi = \mu\mathfrak{H}A$	Maxwell
$\mathfrak{B}$	Flux density	$\mathfrak{B} = \Phi/A$	Gauss
$\mathfrak{H}$	Magnetizing force	$\mathfrak{H} = 4\pi nI/10L$	
$\mathfrak{F}$	Magnetomotive force	$\mathfrak{F} = 4\pi nI/10$	Gilbert
ν	Reluctivity	$\nu = 1/\mu$	
$\mathfrak{R}$	Reluctance	$\mathfrak{R} = \nu L/A = \mathfrak{F}/\Phi$	Oersted
μ	Permeability	$\mu = \mathfrak{B}/\mathfrak{H}$	
$\mathfrak{P}$	Permeance	$\mathfrak{P} = 1/\mathfrak{R}$	
κ	Susceptibility	$\kappa = \mathfrak{I}/\mathfrak{H}$	
W	Magnetic energy	$W = \Phi\mathfrak{F}$	Erg
	Magnetic potential	$= W/m$	
P	Magnetic power	$P = \Phi\mathfrak{F}f$	

* L = length; V = volume; A = sectional area; n = number of turns; T = time; f = frequency.

† See also p. 1586 for the practical units.

Volt-amperes or apparent power = volts × amperes.

Watt-hour. Watt-hours and kilowatt-hours are the units of energy used in commercial electrical work.

1 watt-hour = 3600 joules = 2655.4 ft-lb. = 859.975 gram-calories = 3.412 B.t.u. = 0.001341 h.p-hr.

Joule. The practical unit of electrical energy is the joule. One watt-second is a joule. One joule is produced when a steady current of one coulomb per sec., *i.e.*, 1 ampere, passes through a resistance of 1 ohm for 1 sec. One joule is equivalent to 0.2388 gram-calorie, and also equals 10^7 ergs. (An **erg** is a dyne-centimeter.)

ELECTRIC AND MAGNETIC CIRCUITS

Electric Circuits

Ohm's Law states that the current in a circuit at any instant is $I = E/R$, where I is the current in amperes, E the e.m.f. impressed upon the circuit in volts, and R its resistance in ohms. The law as stated above is equally applicable to direct- and alternating-current circuits, but in the latter case effective values of e.m.f. and current are almost always used, and the above simple relation must in general be modified as explained later.

The following are the most important relations between the electrical units:

$$I = E/R; \quad Q = It; \quad C = Q/E; \quad W = \tfrac{1}{2}QE; \quad W = \tfrac{1}{2}LI^2; \quad P = IE$$

The following relations may be derived from the above by simple algebraic transformations:

$$Q = \frac{E}{R}t; \qquad C = \frac{I}{E}t; \qquad W = IEt = \frac{E^2}{R}t = I^2Rt = Pt$$

$$E = IR; \qquad R = \frac{E}{I}; \qquad P = \frac{E^2}{R} = I^2R = \frac{W}{t} = \frac{QE}{2t} = \frac{LI^2}{2t}$$

where Q is in coulombs (ampere-seconds), P in volt-amperes, W in joules (volt-coulombs), C in farads, and L in henrys.

Kirchhoff's Laws (derived from Ohm's Law) are of special value when dealing with networks of conductors. The first law states that at any point in a circuit the sum of the currents directed toward the point is equal to the sum of the currents directed away from the point. The second law states that in any closed circuit the algebraic sum of the e.m.f.'s and IR drops is equal to zero. By setting up suitable algebraic equations in accordance with these principles, otherwise complicated network problems can be solved without difficulty.

Series Circuits. The combined resistance of a number of series-connected resistances is the sum of the separate resistances. When batteries or other sources of direct e.m.f. are connected in series the total e.m.f. of the combination is the sum of the separate e.m.f.'s. The open-circuit e.m.f. of a battery is the total generated e.m.f., and may be measured at the battery terminals only when no current is being furnished by the battery. The internal resistance is the resistance possessed by the battery alone. The current which will flow in a circuit connected in series with a source of e.m.f. is $I = E/(R + r)$, where E is the open-circuit e.m.f., R the external resistance, and r the internal resistance of the source of e.m.f. Ohm's Law is similarly applicable to the magnetic circuit, as explained under Magnetic Circuit.

Parallel Circuits. The combined conductivity of a number of parallel-connected resistances is the sum of the separate conductivities.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ . \ . \ .$$

The equivalent resistance for three parallel resistances, R_1, R_2, R_3, is

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

and for four parallel resistances, R_1, R_2, R_3, R_4,

$$R = \frac{R_1R_2R_3R_4}{R_1R_2R_3 + R_2R_3R_4 + R_3R_4R_1 + R_4R_1R_2}$$

To obtain the resistance of combined series and parallel resistances, the equivalent resistance of each parallel portion is obtained separately and then these equivalent resistances are added to the series resistances according to the principles stated above.

Joule's Law. When an electric current passes through a resistance the number of heat units developed is proportional to the square of the current, directly proportional to the resistance, and directly proportional to the time that the current flows. $p = 0.2389i^2rt$, where p represents the number of gram-calories, i the current in amperes, r the resistance in ohms and t the time in seconds. p (in B.t.u. per sec.) $= 0.000948i^2rt$.

Magnetism

The Magnetic Circuit. In dealing with electromagnetism, a conception of the magnetic circuit quite analogous to that of an electric circuit is generally used. Ohm's Law of the magnetic circuit is stated thus:

$$\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}, \text{ or, in symbols, } \Phi \text{ (maxwells)} = \frac{\mathfrak{F} \text{ (gilberts)}}{\mathfrak{R} \text{ (oersteds)}}$$

The m.m.f. acting in any magnetic circuit is always $4\pi nI_a$, where n is the number of turns in the circuit and I_a the current in absolute units. Permeability expresses the ability of a medium to conduct magnetic flux. Unit magnetic permeability is the permeability of a cm. cube of air. Reluctivity or specific reluctance is the reciprocal of permeability and expresses the opposition to the passage of magnetic flux through a medium, and is the reluctance possessed by a cm. cube of air. As in electric circuits, the total reluctance is proportional to the length and inversely proportional to the cross-sectional area of the magnetic circuit. Reluctances in series are added to obtain their combined reluctance. Ohm's Law of the magnetic circuit becomes: flux $= \dfrac{0.4\pi nI}{(1/\mu)l/A + (1/\mu')l'/A' + (1/\mu'')l''/A'' + \ldots}$, where I is expressed in amperes, μ, μ', μ'' are the magnetic permeabilities and hence $1/\mu$, $1/\mu'$ and $1/\mu''$ are the reluctivities or specific magnetic reluctances, l, l', l'', etc., are the lengths and A, A', A'', etc., the cross-sectional areas of the various portions of the magnetic circuit, expressed in centimeter measure.

Magnetization and Permeability Curves. The magnetic permeability of air is a constant and is taken as unity. The permeability of iron and other magnetic substances is not constant but varies with the flux density. No satisfactory equation has been found to express the relation between magnetizing force and flux density, so it is usual to plot curves showing the variation of permeability with magnetic flux density (or permeability curves; see Fig. 1). It is more usual to express the magnetic properties by a magnetization curve, which shows the magnetic flux density and magnetizing force; see Fig. 2. The data must be experimentally determined for the iron it is proposed to use in constructing a magnetic circuit. The magnetization curve shows that in order to drive any value $\mathfrak{B}$ (flux density in lines of force per sq. cm.) through the iron whose magnetization curve is given, the corresponding value of $\mathfrak{H}$ must be applied for each cm. of length through which the flux is driven.

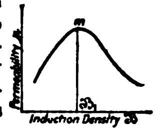


FIG. 1.—Permeability Curve.

The total magnetizing force, that is, the magnetomotive force, of a coil is $0.4\pi nI$, where I is given in amperes, so that $0.4\pi nI/l$ is the force per unit length of circuit to which the force $0.4\pi nI$ is applied. Or, one ampere-turn produces 1.257 gilberts of m.m.f. It is more convenient to express the magnetizing forces of the magnetization curves directly in ampere-turns per cm. of length, and it is usual in engineering work to plot the magnetization curves in terms of lines of force (lines of magnetic induction) per sq. in. and ampere-turns per inch of magnetic circuit length. To convert from one scale to another, use the relation $\mathfrak{H} = 0.4\pi nI/l$, where n is the number of turns, I the current in amperes, and l is in cm. One unit of magnetizing force or 0.796 ampere-turn is required to drive one line of force per sq. cm. through a length of 1 cm. of air, and 0.313 ampere-turn to drive one line per sq. in. through a length of 1 in. Since the permeability of air is constant, its magnetization curve is a straight line of constant slope. To determine the number of ampere-turns necessary to set up a given total flux in a magnetic circuit composed of various cross-sections and permeabilities, determine the flux density if the cross-section is fixed, or otherwise choose a cross-section to give a suitable flux density, and from the magnetization curve obtain the ampere-turns necessary to drive this flux density through unit length of the portion of the circuit considered and multiply by the length. Add together the ampere-turns required for each portion

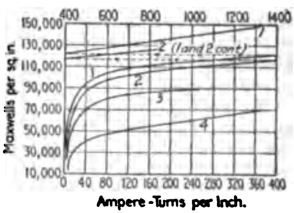


FIG. 2.—Magnetization Curves.

of the magnetic circuit to obtain the total ampere-turns necessary. Magnetisation curves of a number of commercial samples are given in Fig. 2.

Flux = $\Phi = \mathfrak{F}/\mathfrak{R}$; $\mathfrak{R}$ = 1/permeance; permeance = $\mu a/l$, in which μ = permeability, a = cross-section of path of lines of force, sq. cm., and l = length of path, cm. Hence $\Phi = \mathfrak{F}\mu a/l$. As one ampere-turn ($nI$) produces 1.257 gilberts of m.m.f., Φ in inch measure = $3.192\mu anI/l$; or, the ampere-turns necessary to produce a given flux Φ in a given circuit of dimensions a, l, will be $nI = \Phi l/3.192\mu a = 0.3133\Phi l/\mu a$. Also, since $\Phi/a = \mathfrak{B}$, the ampere-turns required to produce a density $\mathfrak{B}$ in lines of force per sq. in. of path section = $nI = 0.3133\mathfrak{B}l/\mu$.

Magnetic Leakage. It is impossible to completely confine magnetic lines of force to any desired path for the reason that there is no known non-conductor of lines of force. In the case of the magnetic circuit shown in Fig. 3, some lines of force will leak back across the air space separating the limbs of the magnet. The ratio of the maximum number of lines to the useful lines is called the **leakage coefficient.** This coefficient can never be as low as unity, and in modern dynamos ranges from 1.2 to 2.0. Accurate predetermination of magnetic leakage, except in the case of the simplest geometrical configurations, is a matter of great difficulty.

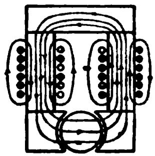


FIG. 3.

Magnetic Hysteresis. The magnetization curves shown in Fig. 2 are average curves. A different set of values of $\mathfrak{B}$ will result for each value of the magnetizing force when the magnetizing force is increasing from that when it is decreasing. A complete magnetization curve is shown in Fig. 4. It can be shown that the energy stored in the magnetic field is $\int \mathfrak{H}d\mathfrak{B}/4\pi$. In a magnetic field set up in air, all the energy stored in the form of magnetism is returned to the electric circuit when the magnetism disappears. When iron undergoes cyclic magnetization, as represented in Fig. 4, an amount of energy proportional to the area of the closed hysteresis loop, *GCDEFA*, appears as heat. The area of the hysteresis loop is dependent upon the final value of the flux density, and the hysteresis loss is represented by the Steinmetz Law: Loss in ergs per cu. cm. per cycle = $\eta\mathfrak{B}^{1.6}$, η being the hysteresis constant and $\mathfrak{B}$ the flux density. The loss in watts per cu. cm. is $W = \eta f\mathfrak{B}^{1.6} \times 10^{-7}$, f being the frequency, or number of complete cycles per sec. Steinmetz gives the following values for η:

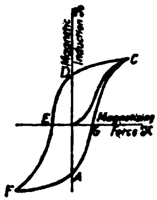


FIG. 4.—Hysteresis Loop.

Silicon steel	0.001	
Average sheet steel	0.002	
Wrought iron, sheet iron and sheet steel	0.0012–0.0055	(mean 0.003)
Cast iron	0.011 –0.016	(mean 0.013)
Soft cast steel	0.0032–0.012	(mean 0.060)
Hard cast steel . . . . up to	0.028	
Forged steel	0.015 –0.025	
Magnetic iron ore	0.020 –0.024	
Nickel	0.013 –0.039	
Cobalt	0.012	

A permanent increase in the hysteresis constant occurs if the temperature of operation remains for some time above 80 deg. cent. This phenomenon is known as **aging**, and may be much reduced by proper annealing of the iron. Silicon steels containing about 3 per cent. of silicon have a lower hysteresis loss, somewhat larger eddy-current loss, and are practically non-aging. **Eddy-**

current losses, also known as Foucault-current losses, occur in iron subjected to cyclic magnetisation. Eddy-current losses are minimised by laminating the iron, which subdivides the e.m.f. and increases greatly the length of path of the parasitic currents. Eddy currents have also a screening effect, that is, the full cross-section of a magnetic circuit cannot be utilised for an alternating flux unless it is laminated.

The watts loss in sheets per cu. cm. is: $P_e = (\pi t f \mathfrak{B}_{max})^2/6\rho 10^{16}$, where t = thickness in cm., f = frequency, cycles per sec., $\mathfrak{B}$ = flux density, lines per sq. cm., and ρ = specific resistance.

Directions of Lines of Force and of Induced E.M.F.'S. An electric current flowing in a conductor sets up lines of magnetic force in the directions indicated in Fig. 5*a*. The + sign indicates current flowing away from the observer and the negative sign current, toward the observer.

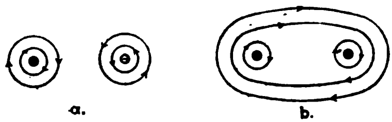


FIG. 5.

Lines of magnetic force are assumed to emanate from a north pole and to enter the magnet at a south pole. Current flowing in a conductor as shown in Fig. 6 will produce magnetic poles as there indicated.

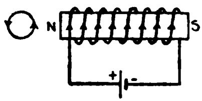


FIG. 6.

Currents flowing in the same direction in parallel conductors exert an attractive force between the conductors. The lines of magnetic force which combine and link both conductors appear to have the property of shortening themselves like rubber bands. See Fig. 5*b*. Currents flowing in opposite directions in parallel conductors exert a repulsive force between the conductors. The lines of force resist crowding, or an electric circuit always tends to change its geometrical shape so that its magnetic reluctance is a minimum.

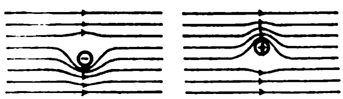


FIG. 7.

The following method may be used to determine the **direction of the e.m.f. induced when a conductor cuts a magnetic field.** If the property of elasticity be ascribed to lines of magnetic force, a conductor cutting them will distort them as shown in Fig. 7. From the direction of the lines of force surrounding the conductor the direction of the e.m.f. induced is determined at once from the relations stated above; that is, with the observer looking along the conductor, lines of force encircling the conductor in the direction of the hands of a clock mean that current is flowing away from the observer, and *vice versa*.

Alternating Currents

The fundamental equation of a circuit containing self-inductance L and resistance R upon which a direct e.m.f. E is impressed, is $E = Ri + Ldi/dt$, where i represents the current at any instant. By Kirchhoff's Law, the sum of all e.m.f.'s of a circuit must be zero, and the interpretation of the above equation is that part of the applied e.m.f. is consumed by resistance and the remainder by the counter e.m.f. of self-induction Ldi/dt. A solution of this differential equation is $i = (E/R)(1 - \epsilon^{\frac{-Rt}{L}})$, where ϵ is the base of the natural system of logarithms.

Time Constant. In the case of an inductive circuit, the time constant is the ratio L/R, where L is the coefficient of self-induction and R the resistance of the circuit. Theoretically, an infinite time is required for a current to reach its ultimate value, $I = E/R$, when a circuit containing self-inductance is connected to a source of direct e.m.f., but practically the ultimate value is reached after a very short time. The time constant expresses the time required in seconds for the current to attain 0.632 of its ultimate value, $I = E/R$. In a circuit containing self-inductance and resistance, the current rises according to the equation $i = (E/R)(1 - \epsilon^{\frac{-Rt}{L}})$. If a resistance r be introduced into the circuit at the time the generator is removed, the current will follow the equation $i = (E/R)\ \epsilon^{\frac{-(R+r)t}{L}}$ and the e.m.f. across the resistance r will be $e = ri = (Er/R)\ \epsilon^{\frac{-(R+r)t}{L}}$, and may be higher than E and will, of course, be very large when r is large as in the case of a spark coil and gap such as is used in gas-engine ignition.

If the equation $E = Ri + Ldi/dt$ is multiplied by idt and integrated between 0 and T, the energy dissipated $= \int_0^T Eidt = RI^2T + \frac{1}{2}LI^2$, where I is the value of i at time T. The term RI^2T represents the energy dissipated in the resistance and $\frac{1}{2}LI^2$ is the energy *stored* in the magnetic field. Likewise, when a voltage E is impressed upon a circuit containing resistance and capacity, at any instant,

$$E = iR + \frac{q}{C} = iR + \int \frac{idt}{C}$$

the solution of which is $i = \frac{E}{R}\ \epsilon^{-\frac{t}{CR}}$. $\frac{1}{CR}$ is the time constant of this circuit, and C must be expressed in farads. Also the energy $\int Eidt = I^2RT + \frac{1}{2}Q/E$. I^2RT is the energy dissipated in the resistance and $\frac{1}{2}Q/E$ is the energy stored in the electrostatic field.

Alternating Currents. In alternating-current theory, the current and e.m.f. are assumed to vary according to the sine law; thus, $i = I_m \sin x$, where i is the instantaneous value of current, I_m its maximum value, and x a variable angle. An **alternation** is a half cycle. A **cycle** or period is generated by the passage of an armature coil past two poles of the generator. Consequently, the **frequency** of a generator is expressed as the speed in revolutions per sec. multiplied by the number of *pairs* of poles. The standard frequencies now in use are 60 cycles for lighting and power and 25 cycles for power. Frequencies of 40 and 50 cycles are used in some localities and 12½ and 15 cycles are used abroad in railway work. One hundred and thirty-three cycles was extensively used some years ago, but is rarely used at the present time.

The Average Value of a Wave. The average value of a sine wave is $(2/\pi)I_m$, or 0.637 I_m, where I_m is the maximum value of the sine wave. The average value is of little practical importance. A direct-current measuring instrument gives the average values of a pulsating wave. The average value is of use only when the effects of the current are proportional to the number of coulombs passing, as in electrolytic work.

The Effective Value of a Wave (root mean square value, or virtual value) produces the same heating in a given resistance as a direct current of the same value. Since the heating effect of a current is proportional to ri^2, the effective value is obtained by squaring the ordinates, finding their average value, and extracting the square root, that is, the effective value

$$I = \sqrt{\frac{2}{t}\int_0^{t/2} i^2 dt}$$

where t is the time of a period. The effective value of a sine wave equals $(1/\sqrt{2})I_m = 0.707I_m$.

Form Factor. The form factor of a wave is the ratio $\frac{\text{effective value}}{\text{average value}}$. For a sine wave this is $\pi/(2\sqrt{2}) = 1.11$. A sine wave of e.m.f. applied to a circuit containing resistance and inductance only, is represented by $E_m \times \sin \omega t = Ri + \frac{Ldi}{dt}$. The solution of this equation is:

$$i = \frac{E_m \sin(\omega t - \phi)}{\sqrt{R^2 + \omega^2 L^2}} + \frac{E_m \sin(\phi \epsilon^{-\frac{Rt}{L}})}{\sqrt{R^2 + \omega^2 L^2}}, \text{ where } \phi = \tan^{-1}\frac{\omega L}{R}$$

The current wave is then made up of a pure sine wave and an exponential curve. This latter disappears a very short time after the closing of the switch.

Inductive Reactance, ωL or $2\pi fL$, opposes an alternating current in passing through an inductance L. It is expressed in ohms. Reactance is usually denoted by the symbol x.

A solution of the equation $E_m \sin \omega t = Ri + \int \frac{idt}{C}$ representing a sine wave of pressure applied to a circuit containing resistance and capacity only, is $i = \frac{E_m}{\sqrt{R^2 + 1/4\pi^2 f^2 C^2}} \sin(\omega t + \phi) + A\epsilon^{-\frac{t}{RC}}$, where $\phi = \tan^{-1}\frac{1}{C\omega R}$. The exponential portion disappears after a few cycles, leaving only a sine wave of current.

Capacity Reactance is $1/2\pi fC = 1/\omega C$. It is convenient to regard inductance as the analog of inertia in mechanics and capacity as the analog of elasticity.

Impedance is also expressed in ohms, and is the total opposition to the flow of an alternating current in a circuit possessing both resistance and reactance. Impedance $= Z = \sqrt{R^2 + \omega^2 L^2}$, for a circuit having resistance and inductance. Resistance and reactance may then be represented as the two legs of a right triangle, but cannot be combined algebraically. In a circuit containing resistance, inductance and capacity, the impedance

$$Z = \sqrt{R^2 + [2\pi fL - (1/2\pi fC)]^2}$$

Phase Difference. When an alternating e.m.f. is impressed upon resistance only, at each instant $i = e/R$. The current wave is therefore similar to the voltage wave and is in phase with it, for the two coincide in time position. When a sine e.m.f. wave is impressed upon a circuit containing inductance only, the current lags 90 deg. or a quarter period behind the pressure wave and they are said to be in quadrature. When a sine e.m.f. wave is impressed upon a condenser, and without resistance in the circuit, the current leads the impressed pressure by 90 deg. Thus inductance and capacity reactances have opposite effects and may be combined algebraically. When resistance is taken into account, as it generally must be, the current lags or leads by less than 90 deg. Symbolically, a lagging or leading current is expressed by $i = I_m \sin(\omega t \mp \phi)$, where ϕ is the angle of phase difference and is equal to $\tan^{-1}[(2\pi fL - 1/2\pi fC)/R]$. The negative sign represents a lag and the positive sign a lead, with reference to the impressed e.m.f.

Vector Representation. It is usually convenient to represent the magnitude and phase position of alternating e.m.f.'s and currents by vectors, which implies that the waves so represented follow the sine law. The vectors may be combined as forces are combined in mechanics. Both graphical methods and the methods of complex algebra are used. Impedances and also admittances may be similarly combined, either graphically or symbolically. In the latter case the usual method is to resolve the impedances, for example, into their component resistances and reactances, then combine all resistances and all reactances, from which the resultant impedance is obtained. Thus, $Z_1 + Z_2 = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}$, where r_1 and x_1 are the components of Z_1.

Resonance occurs when the inductive reactance equals the capacity reactance, *i.e.*, $2\pi fL = 1/2\pi fC$. In this case the circuit acts as though it contained nothing but resistance. The voltage across the inductance and the voltage across the capacity are opposite and equal, and may be many times greater than the circuit voltage.

The frequency, $f = 1/2\pi\sqrt{LC}$, is the "natural frequency" of the circuit and is the frequency at which it will oscillate if the circuit is not acted upon by some external frequency. This is the principle of the radio sending and receiving circuits. Resonant conditions of this type should be avoided in power circuits, as the "piling up" of voltage may endanger apparatus and insulation.

Power Factor. In alternating-current circuits, the power P is equal to $EI \cos\phi$, where ϕ is the angle of phase difference between E and I. $\cos\phi$ is called the power factor. EI is called the **apparent power.** The ratio $EI\cos\phi/EI$, which is the ratio of true power to apparent power, is the power factor of the circuit. When the pressure and current are sine waves, $\cos\phi$ may be given a trigonometrical interpretation, *i.e.*, it is the angle of phase difference. In general, the power factor is the ratio $\dfrac{\text{true power}}{\text{apparent power}}$.

Active Current is the projection of the total current on the voltage vector and is equal to $I_e = I\cos\phi$. Power $= EI_e$.

Reactive Current $I_w = I \sin\phi$ and represents the component of the current which contributes no power, but increases the I^2R losses of the system. In power systems it should be made as low as possible or eliminated entirely.

Effective Resistance. When power losses in an alternating-current circuit are greater than that due to resistance alone, this additional power must be taken into account in vector diagrams or computations involving impedance and resistance. Power expended because of iron losses as well as resistance brings the current and pressure more nearly in phase with each other. Since $P = I^2R$, $R = P/I^2$ is the effective resistance, or it is that resistance which would cause a power expenditure P (which may include iron losses) with a current I flowing in the circuit. These iron losses vary approximately as $I^{1.8}$ or nearly as I^2, so little error is introduced in assuming the effective resistance constant over considerable current range.

Parallel Circuits

Admittance is the reciprocal of impedance and is usually denoted by Y. $Y = \sqrt{G^2 + B^2}$; $I = EY$.

Conductance is the horizontal component of admittance and is generally denoted by the symbol G. $g_1 = \frac{r_1}{r_1^2 + x_1^2}$. $G = g_1 + g_2 + g_3 \ldots\ldots$ The active current, $I_a = EG$. Conductance in this case is not the reciprocal of resistance.

Susceptance is the vertical component of admittance and is generally denoted by B. $b_1 = \frac{x_1}{r_1^2 + x_1^2}$. $B = b_1 + b_2 + b_3 \ldots\ldots$ The reactive current $I_w = EB$. Susceptance is not the reciprocal of reactance.

The following additional relations hold:

$$\text{Power} = E^2G \qquad \text{Power factor} = G/Y$$

$$r = \frac{g}{g^2 + b^2} = \frac{g}{Y^2} \qquad x = \frac{b}{g^2 + b^2} = \frac{b}{Y^2}$$

$$g = \frac{r}{r^2 + x^2} = \frac{r}{Z^2} \qquad b = \frac{x}{r^2 + x^2} = \frac{x}{Z^2}$$

Ohm's Law for alternating-current circuits is $I = E/Z = EY$.

Solution of Circuit Problems. Alternating-current circuit problems may be solved graphically or analytically. For numerical computations the principles of complex algebra greatly facilitate the work. In **series circuits** containing resistances, inductances and capacities the joint impedance is the vector sum of the separate impedances. The vector diagrams are laid out with the current horizontally to the right to serve as the reference vector because the current has the same phase in every part of the circuit. On the other hand, the voltages across the various parts of the circuit have different phase positions. Resistances, or the voltages across them, are laid off horizontally to the right since the voltage across a resistance is in phase with the current. Reactances or the voltages consumed by them are laid off vertically, inductive reactances upward and capacity reactances downward.

In dealing with **parallel circuits** it is more convenient to take as the initial vector the voltage impressed across all the parallel branches, since this pressure is common to all. The other vectors then represent currents in the separate branches and may be combined vectorially to obtain the joint current, or the vectors may represent conductances, susceptances and admittances. The following examples illustrate the methods used in solving series and parallel circuit problems. The same methods are used for combined series and parallel circuits, the series and parallel portions of the circuit being solved separately and then combined.

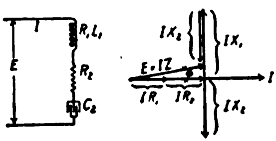


FIG. 8.

Examples. In the series circuit of Fig. 8, $E = 120$ volts, $R_1 = 3$ ohms, $R_2 = 4$ ohms, $L_1 = 0.02$ henry, and $C_2 = 400$ microfarads. Find the joint impedance, the current, power and power factor, if $f = 60$. Solution: $X_1 = 2\pi fL = 2\pi \times 60 \times 0.02 =$

7.54 ohms, $X_2 = 1/2\pi fC_2 = 1/(2\pi \times 60 \times 0.004) = 6.63$ ohms, and joint impedance $Z = \sqrt{(3+4)^2 + (7.54 - 6.63)^2} = 7.06$ ohms. The phase angle $\phi = \tan^{-1}\frac{7.54 - 6.63}{3+4} = 7° 24'$. The current $I = \frac{120}{7.06} = 17$ amperes. Power, $P = I^2R = EI \cos\phi = 2020$ watts. Power factor $= \cos\phi \frac{2020}{17 \times 120} = 99.1$ per cent.

In the parallel circuit of Fig. 9 it is desired to find the joint impedance, the total current, the power in each branch, the total power and the power factor, when $E = 100$,

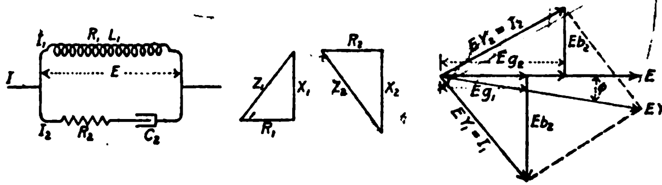


FIG. 9.

$f = 60$, $R_1 = 2$ ohms, $R_2 = 4$ ohms, $L_1 = 0.00795$ henry, $X_1 = 2\pi fL_1 = 3$ ohms, $C_2 = 1326$ microfarads, $X_c = 1/2\pi fC_2 = 2$ ohms, $Z_1 = \sqrt{2^2 + 3^2} = 3.6$ ohms, and $Y_1 = 1/3.6 = 0.278$ mhos. Solution: $g_1 = R_1/(R_1^2 + X_1^2) = 2/13 = 0.154$; $b_1 = 3/13 = 0.231$; $Z_2 = \sqrt{16+4} = 4.47$; $Y_2 = 1/4.47 = 0.224$; $g_2 = R_2/(R_2^2 + X_2^2) = 4/(16+4) = 0.2$; $b_2 = -2/20 = -0.1$; $G = g_1 + g_2 = 0.154 + 0.2 = 0.354$; $B = b_1 + b_2 = 0.231 - 0.1 = 0.131$; $Y = \sqrt{G^2 + B^2} = \sqrt{0.354^2 + 0.131^2} = 0.376$, and joint impedance $Z = 1/0.376 = 2.66$ ohms. Phase angle $\phi = \tan^{-1}\frac{0.131}{0.354} = 20° 18'$. $I = EY = 100 \times 0.376 = 37.6$ amp.; $P_1 = E^2g_1 = (100)^2 0.154 = 1540$ watts; $P_2 = E^2g_2 = (100)^2\, 0.2 = 2000$ watts; total power $= E^2G = (100)^2 0.354 = 3540$ watts. Power factor $= \cos\phi = \frac{3540}{100 \times 37.6} = 94$ per cent.

Circuits. Fig. 10 represents a two-phase generator and receiving apparatus. A two-phase generator has two independent windings *OA* and *OB* displaced 90 electrical degrees apart so that the e.m.f.'s generated are in quadrature. Two entirely independent single-phase circuits could be used to transmit the power to the receiving circuit, but it is simpler to use only three conductors, one conductor serving as a common return. If I_c be the coil current, $\sqrt{2}\, I_c$ will be the value of the current in the common conductor, *OO′*. If E_c be the e.m.f. across *OA* or *OB*, $\sqrt{2}\, E_c$ will be the voltage across *AB*. The power of a two-phase circuit is twice the power in either coil if the load is balanced. Normally, the voltages *OA* and *OB* are equal, and the current is the same in both coils. Due to non-symmetry and the high degree of unbalancing of this system even under balanced loads, it is not used at the present time for transmission and is little used for distribution.

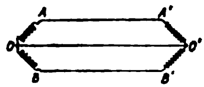


FIG. 10.—Two-phase Circuit.

Quarter-phase Circuit. A quarter-phase circuit is shown in Fig. 11. The windings *AC* and *BD* may be independent or connected at *O*. The e.m.f.'s *AC* and *BD* are 90 deg. apart as in two-phase circuits. If a neutral wire *OO′* be added, 3 different voltages may be obtained. Let

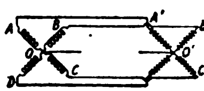


FIG. 11.—Quarter-phase Circuit.

E_1 = voltage $OA = OB = OC = OD$. Voltages $AB, BC, CD, DA = \sqrt{2}E_1$. Voltage $AC = BD = 2E_1$. Due to this multiplicity of voltages and the fact that polyphase power and lamps may be connected at the same time, this system is still used to some extent in distribution.

Three-phase Circuits. Alternating-current generators are usually wound with three armature circuits which generate e.m.f.'s 120 electrical degrees apart. The coils are joined either in Y (star) or in Δ (mesh) connection as shown in Fig. 12. Whether Y- or Δ-connected with a balanced load, the three coil e.m.f.'s and the three currents are equal. In the Y-connection the line and coil currents are equal, but the line e.m.f.'s AB, BC, CA equal $\sqrt{3}$ times the coil e.m.f.'s OA, OB, OC, for they are the vector sum of the coil e.m.f.'s. In the delta connection the line and coil e.m.f.'s are equal, but I, the line current, equals $\sqrt{3}\,I_c$, the coil current, that is, it is the vector sum of the currents in the two coils connected to the line. The power of a coil is $E_cI_c\cos\phi$, so that the total power is $3E_cI_c\cos\phi$. If ϕ is the angle between *coil* current and *coil* voltage, the angle between line current and line voltage will be $(30° \pm \phi)$. In terms of line current and e.m.f., the power is $\sqrt{3}\,EI\cos\phi$. A fourth or neutral conductor is sometimes used with the Y-connection. The neutral point O is frequently grounded in high-voltage transmission circuits. The coil e.m.f.'s are assumed to be sine waves, in which case they balance, so that in the delta connection, for example, the sum of the two coil e.m.f.'s at each instant is balanced by the third coil e.m.f. Even though the 3d, 9th, 15th. . . harmonics, $3(2n + 1)f$, where n = 0 or an integer, exist in the coil e.m.f.'s, they cannot appear on the external line of the three-phase circuit, except when the neutral conductor is used with the Y-connection. In the delta circuit, the same harmonics $3(2n + 1)f$ cause a local current to circulate around the mesh. This may cause a very appreciable heating in the apparatus.

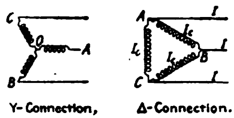


FIG. 12.—Three-phase Circuits.

Equivalent Single-phase Resistance. Three-phase power is expressed as $\sqrt{3}\,EI\cos\phi$, where E and I are the line pressure and current. $\sqrt{3}\,I$ may be called the total current. The equivalent single-phase resistance is that resistance which multiplied by the total current squared will equal the total copper loss of the circuit and is equal to one-half the measured resistance between any two lines of a delta- or Y-connected three-phase circuit. In a Y-connection the measured resistance between two lines is $2r$, where r is the resistance per coil or per phase winding. The equivalent single-phase resistance would be r. Expressed in terms of total current and equivalent resistance, the copper loss would be $(\sqrt{3}\,I)^2 r = 3I_c^2r$, where I_c is the coil current and in this case equal to I. The loss per coil is I_c^2r and for three coils $3I_c^2r$. The measured resistance between two lines of a delta-connected circuit is $1/[(1/r) + (1/2r)] = 2r/3$. The equivalent single-phase resistance is one-half this or $r/3$. The copper loss is then $(\sqrt{3}\,I)^2r/3 = I^2r$. But $I = \sqrt{3}\,I_c$, so the copper loss calculated by this method is again $3I_c^2r$. The practical utility of this method is that in many instances it permits the calculation of three-phase circuits by the methods already applicable to single-phase circuits and without the necessity of knowing whether the connections are delta or Y.

A simple method when calculations of balanced three-phase systems are made is to work with only one conductor to the neutral of the system, assuming that the neutral has zero resistance. In this case ⅓ the power is transmitted over one conductor, the actual line current is used, but the voltage to

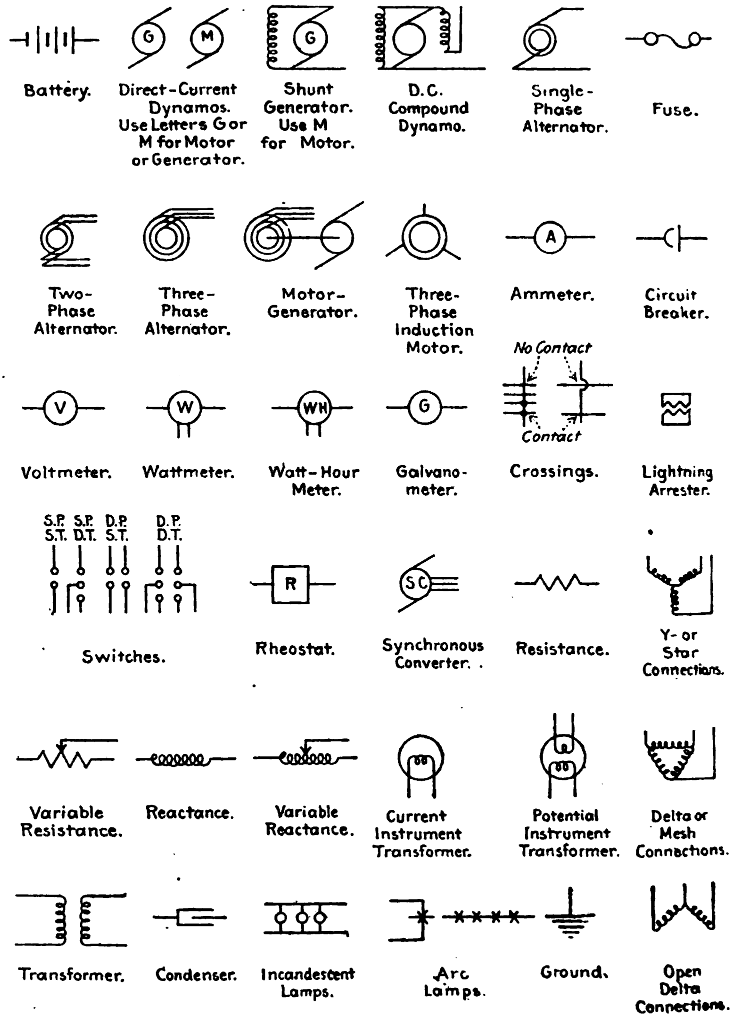


FIG. 13.—Diagrammatic Symbols for Electrical Machinery and Apparatus.

neutral is the voltage between lines divided by $\sqrt{3}$. Calculations of regulation and efficiency under these conditions are also true for the system.

Diagrammatic Symbols for Electrical Machinery and Apparatus have not been standardized to any great extent, but those given in Fig. 13 (taken from various sources) cover general practice. In the designation of switches, *S*, *D*, *P* and *T* are abbreviations of single, double, pole, and throw, respectively; thus, *S.P.D.T.* indicates that the apparatus is a single-pole, double-throw switch.

ELECTRICAL INSTRUMENTS AND MEASUREMENTS

See Circular 20 of the U. S. Bureau of Standards, for a thorough discussion of the various kinds of electrical instruments, their construction, uses and errors.

Direct-current Instruments. For the commercial measurement of direct current and voltage, the permanent-magnet, moving-coil instrument is the standard type. It consists of a light coil free to turn in the field of a fixed permanent magnet. The magnet is usually provided with soft iron pole pieces so shaped as to give a uniform radial field in which the coil moves. The motion of the coil is opposed by a counter force, usually that of two flat spiral springs which also serve to carry the current to and from the coil. The deflection given is proportional to the current flowing, and the instrument is always, in reality, an ammeter, whether used as such or as a voltmeter. When used as a voltmeter the coil is composed of many turns of fine wire, and a resistance coil of comparatively high resistance is connected in series with it. If small currents are to be measured, they may be passed directly through the moving coil; when the current to be measured exceeds that corresponding to full scale deflection, usually a few hundredths of an ampere, it becomes necessary to divert a definite portion of it through a circuit of low resistance connected in parallel with the coil. By this means it is possible to measure heavy currents of any desired magnitude, the heavy current being passed through a suitable low resistance, called a "shunt," from two points on which small leads run to the instrument.

Direct-current and Alternating-current Instruments. Instruments for use on both direct and alternating currents may be divided into four types, namely, electrodynamometer, electromagnetic, hot-wire, and electrostatic. Instruments of the **electrodynamometer** type (probably the most valuable) depend upon the force exerted by one circuit traversed by a current upon another, or by a portion of a given circuit upon another portion of the same circuit. They usually contain one or more fixed coils which set up a magnetic field directly proportional to the strength of the current flowing through them. Within this field is arranged a moving coil or system of coils through which current may be passed. If the two sets of coils are connected in series, the torque exerted upon the moving system, for a given relative position of the coil systems, is proportional to the square of the current strength, and is not dependent upon the direction of the current. Such an instrument constitutes an **ammeter** which is equally correct on direct current and on alternating or pulsating current of any frequency or wave form. Its value is very largely due to the fact that such an instrument calibrated with direct current is accurate with alternating, so may be used as a transfer standard. The electrodynamometer instrument, calibrated on direct current, may be used as a precision instrument for alternating current. The **electrodynamometer voltmeter** has fixed and moving coils of moderately fine wire, connected in series with each other and in series with a non-inductive resistance of low temperature coefficient. This latter resistance (for usual commercial voltages) brings the time-constant (ratio of inductance to resistance) down to a small value, and hence well-made voltmeters of this type, calibrated on direct current, show practically negligible errors on commercial alternating-current circuits. If the fixed coils are made of thick wire and are connected in series with a load, while the moving coils of fine wire are connected (with a series resistance as in a voltmeter) across the line supplying the load, the resulting instrument (a **wattmeter**) will measure the power expended upon the load, and affords a very convenient method of measuring the power taken by alternating-current lamps, motors, etc., and in testing alternating-current watthour meters. With these wattmeters, as usually constructed, it is necessary to take the mean of two readings, when using direct current, in order to eliminate the effect of the earth's field. By modifying the arrangement of the circuits, the wattmeter may be arranged to measure the power factor, or phase angle between current and voltage.

The difficulty of leading heavy currents to a moving coil has led to the development of the **electromagnetic instruments.** Instruments of this type depend upon the action of a coil traversed by a current upon one or more pieces of soft iron. These instruments are also designated as "moving iron," "soft-iron" and "iron vane" type.

The error in the voltmeter due to its inductance is not large over the commercial range of frequencies, and may be computed, for unusual frequencies, from the measured values of resistance and inductance. Ammeters and voltmeters of this type are well suited for commercial measurements on alternating-current circuits, and for direct current when only approximate results (within 2 or 3 per cent.) are required. When intended for use on alternating-current circuits they should be calibrated with alternating current, using suitable transfer instruments which may be checked with direct-current standards. They cannot be checked accurately on direct currents.

Instruments of the **hot-wire type** depend upon the expansion of a wire which is heated by the passage of a current. The more modern instruments have a working wire 6 or 8 in. in length. In the voltmeter this wire is quite fine and a series resistance is provided. In the hot-wire ammeter the working wire is larger, and is connected with several sections in parallel to reduce the required drop in the shunt. Hot-wire instruments except in wireless telegraphy are but little used in this country in practical work.

Instruments of the **electrostatic type** depend upon the attraction of oppositely charged bodies and the repulsion of similarly charged ones. As these forces are relatively small, such instruments cannot well be made as ammeters, and in fact it is difficult to construct satisfactory voltmeters on this principle for the ordinary 110-volt range. The great advantage of this type of voltmeter is that it takes no current when used on direct-current circuits, and an extremely small current when used on alternating-current circuits. Like the hot-wire instrument, it is not affected by frequency changes, wave form, or stray magnetic field. Frictional errors, however, are hard to avoid. For very high voltages the electrostatic voltmeter has some marked advantages over other types, if the use of potential transformers is not considered.

Alternating-current Instruments. Instruments operating only on alternating current depend upon the interaction of inducing and induced currents, and are usually described as induction instruments. The usual form contains a laminated iron core surrounded by one or more coils of wire; an alternating magnetic flux is set up in the air gap of this core when current flows through the coils. Provision is made for securing the effect of a rotating field, either by having more than one group of coils, the currents in them differing in phase, or one coil may be used, with fixed copper plates or bands, in which induced currents are set up; the resultant action of the flux due to the coil and that due to the induced currents in the fixed copper pieces gives the effect of a rotating field. In this field is pivoted a light disk or drum, usually of aluminum, which tends to rotate with the rotating field. If an indicating instrument is desired, the motion of the disk is opposed by a spring or other suitable counter force; an integrating meter is provided with drag magnets and a dial or register to read the total quantity that has passed through the meter. Such a meter tends to vary greatly in its readings with change of frequency. The induction watthour meter is a reliable commercial instrument, having the great advantages of absence of a commutator and brushes, and a very light moving element. Portable induction indicating instruments have a nearly closed magnetic circuit and fairly strong working fields; they are thus sensitive to external stray field. The scales are long, it being possible to make them cover 300 deg. or even more. Such instruments may be used for commercial testing on definite frequencies, after calibration under conditions as nearly as possible like those under which they are used; they should be checked by comparison with proper standards, for important work. They are not suitable for general service in the laboratory.

Watt-hour Meters record the energy of a circuit during some given time interval. The direct-current type consists of two fixed coils in series with one or both lines, and an armature connected across the line through a comparatively high resistance. In the armature circuit is a "starting coil" which compensates for the meter friction. An aluminum or copper disk on the shaft rotates between the poles of permanent magnets, so a retarding torque, proportional to the speed, is produced.

Alternating-current meters are usually of the induction type. The torque is produced by the inductive action of series and shunt coils acting on a disk or a cylinder. The friction is compensated by a small "shading coil" actuated by a lever. A low power-factor adjustment is made by changing the resistance of a small coil wound around the core of the potential coil. Retarding magnets act on the disk as in the direct-current meter. The general formula for the meters is,

$$W \times t = K \times N \times 3600$$

where W = watts; t = sec.; K = meter constant (usually marked on the disk); N = revolutions of the disk. The meters are usually calibrated by measuring the average watts supplied to a load during an interval, with indicating voltmeter and ammeter, or a wattmeter, and counting the disk revolutions with a stop watch. This value of power is then compared with that obtained from the formula. To speed the meter up at high loads, move the magnets nearer the shaft, and to slow down the meter move magnets nearer the disk periphery. Make the light-load adjustment by means of starting coil in direct current or by means of lever in alternating current.

Accessory Apparatus, Including Instrument Transformers. The series resistance of a voltmeter for one or more ranges is usually a part of the instrument. When higher voltages than the maximum thus provided for are to be measured, external series resistance boxes, called multipliers, are used to extend the range. These are used with alternating voltmeters for voltages below 600 volts. For voltages in excess of 600 a small transformer is ordinarily used. Its primary coil is wound for the line voltage to be measured, and the secondary voltage (usually 100 to 110 for normal working line voltage on the primary) is applied to the voltmeter. Such potential transformers are usually wound for some convenient integral value of the ratio of primary applied voltage to secondary terminal voltage. This arrangement has the great advantage of insulating the voltmeter from the high voltage of the line—a very important point in connection with safety to the operator, with voltages of 1000 or greater.

Alternating-current ammeters in excess of 100 amperes as a rule are not accurate, and further are not adapted to operate with shunts. Current transformers may therefore be used, which allow the use of a low-scale ammeter and insulate the instruments from the line voltage. The secondaries usually have a 5-ampere rating. Care should be taken not to connect more instruments than the transformer is designed to operate, especially when one of the instruments is a wattmeter or a watthour meter. For a given transformer the general performance will be better the smaller the load of instruments it is required to operate. As both ratio and phase-angle errors affect the reading of a wattmeter, care should be taken that the series transformer used with the wattmeter is of proper design and capacity and that it is not overloaded with instruments. The secondary circuit of a series transformer should never be opened while current is passing through the primary, not only on account of the rise of voltage at the secondary terminals, which may become dangerously high, but also because the flux in the iron rises to abnormal values. The resulting high flux density in the iron will cause excessive core loss and the temperature of the transformer may become so high as to cause deterioration of the insulation.

Power-factor Measurement. The usual method of determining power factor is by the use of voltmeter, ammeter, and wattmeter. The wattmeter gives the true watts of the circuit, and the product of the voltmeter reading and the ammeter reading gives the apparent power. From the ratio of the two, the power factor is directly determined.

Measurement of Resistance. A common method of measuring resistance, known as the fall of potential method, makes use of an ammeter and a voltmeter. In Fig. 14, the resistance to be measured is indicated by R. A current I is passed through the resistance and ammeter in series and the fall of potential across the resistance is measured by the voltmeter E. The amount of current shunted by the voltmeter is so small that it may generally be neglected. A correction may be applied if necessary, for the resistance of the voltmeter is generally given with the instrument. The potential difference divided by the current gives the resistance included between the voltmeter leads. As a check, determinations are generally made with several values of current, which may be varied by means of the controlling re-

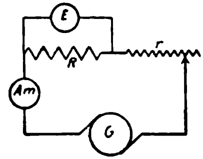


Fig. 14.

sistance r. If the resistance to be measured is the armature of a direct-current machine and the voltmeter leads are placed upon the brush holders, the resistance determined will include that of the brush contacts. To measure the resistance of the armature alone, the voltmeter leads should be placed directly upon the commutator segments under the brushes.

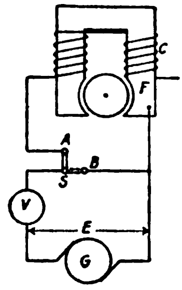


FIG. 15.

Insulation Resistance. A convenient method of measuring insulation resistance by means of a voltmeter is as follows: In Fig. 15 the insulation resistance to be measured is that from the conductor of the field winding C to the frame of the machine F. To measure the current flowing when a pressure E is impressed across the resistance R, a high-reading voltmeter V is connected in series with R. The current which flows under this condition with the switch connecting S and A, is $E/(R + r)$, where r is the resistance of the voltmeter. A high-reading voltmeter is necessary since its resistance is higher than that of a low-reading instrument, and as the method is in reality a comparison of the unknown insulation resistance with the known resistance of the voltmeter, the latter must be comparable to the former, else the deflection of the instrument will be so small that the results will be inaccurate. To determine the impressed pressure, the same voltmeter is used. The switch S connects S and B for this purpose. With these two readings, the unknown resistance is $R = r(E - e)/e$, where E is the impressed pressure and e is the deflection of the voltmeter when closed through the resistance to be measured at contact A. If a special voltmeter, having a resistance of 100,000 ohms per 150 volts, is available, a resistance of the order of 2 to 3 megohms may be measured very accurately.

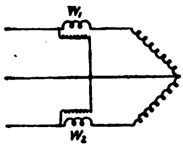


FIG. 16.

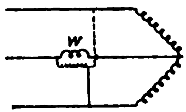
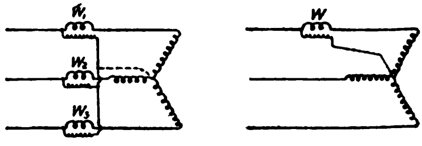


FIG. 17.

Measurement of Power of Polyphase Circuits. The power of polyphase alternating circuits may be measured by means of one meter, but these methods require special conditions. In a three-wire two-phase circuit, two wattmeters connected as shown in Fig. 16 will give the true power of the circuit whether it is balanced or not, since each of the wattmeters is in a practically independent circuit. If but one wattmeter is available, the power may be measured in one circuit and the result multiplied by 2 if the two circuits are balanced. If not balanced, the wattmeter may be successively connected to the positions of W_1 and W_2, and the sum of the readings taken. In two-phase circuits with a common return, as shown in Figs. 16 and 17, the current coil of a single wattmeter may be placed in the common return and if the pressure coil be connected successively from the common return to each of the outside conductors, the sum of the two readings will be the power of the circuit if it is in balance.

Three-phase power may be measured by means of three wattmeters, one in circuit with each coil so as to measure the power of that coil as in Fig. 18*a*. Unless the common connection of the three potential coils is connected to the neutral of the system O, the resistance of each of the three potential circuits should be equal. Under a balanced load the three wattmeters read the same regardless of power factor. This system is adapted to a three-phase system having a fourth or neutral wire. If the load is balanced, one watt-meter may be used as in Fig. 18*b* and its reading multiplied by 3. The common method for the measurement of three-phase power is the "two-wattmeter method." This method is convenient and is independent of the condition of balance of the load. The con-



(a.) (b.)

FIG. 18.

nections for this method are indicated in Fig. 19. The watt-meters may be placed in any two of the leads and the pressure coils simply connected from the leads in which the current coils are connected to the lead which contains no wattmeter current coil. The sum of the two wattmeter readings is the power of the circuit, regardless of the power factor or the condition of balance of the load. With the load balanced and the power factor 100 per cent., the two wattmeter readings will be equal. In general, however, the two readings will not be equal. With a power factor of less than 50 per cent., one of the wattmeters will be reversed, and in this case it is necessary to reverse the pressure or current coil of the instrument and obtain the true power by taking the difference of the readings. The power factor of a balanced system may be determined directly from the wattmeter readings. $\frac{W_1}{W_2} = \frac{\cos(30^\circ - \phi)}{\cos(30^\circ + \phi)}$ and also $\tan \phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}$, where ϕ is the coil phase angle. The two-wattmeter method cannot be used when current is flowing in a fourth or neutral wire. The total power of a balanced load may also be measured by one wattmeter. The current coil is connected in one line, Fig. 19, one end of the potential coil to the same line, and the other end of the potential coil touched to each of the other two wires. The sum of these readings gives the total power.

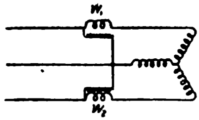


FIG. 19.

Tests for Dielectric Strength may be made either with a voltmeter or with a **spark gap** to measure the voltage. In the latter case, the spark gap is adjusted so that it will break down with a certain predetermined voltage and is connected in parallel with the insulation under test. For values of voltage between 10 kilovolts and 50 kilovolts a **needle spark gap** should be used, consisting of new sewing needles supported axially at the ends of linear conductors which are at least twice the length of the gap. There must be a clear space around the gap for a radius of at least twice the gap length. Needle gap root-mean-square (R. M. S.) sparking voltages at 25 deg. cent. and 760 mm. barometer with No. 00 sewing needles are given below.

R. M. S. kilovolts....................	10.0	15.0	20.0	25	30	35	40	45	50
Millimeters..........................	11.9	18.4	25.4	33	41	51	62	75	90

For the measurement of voltages in excess of 50 kilovolts, a gap between metal spheres should be used. **Sphere-gap** voltages are given below.

Sphere-gap Spark-over Voltages

(At 25 deg. cent. and 760 mm. barometric pressure)

Sparking distance in millimeters											
62.5-mm. spheres			125-mm. spheres			250-mm. spheres			500-mm. spheres		
Kilo-volts	One sphere grounded	Both spheres insulated	Kilo-volts	One sphere grounded	Both spheres insulated	Kilo-volts	One sphere grounded	Both spheres insulated	Kilo-volts	One sphere grounded	Both spheres insulated
10	4.2	4.2	30	14.1	14.1	60	29	29	100	52	51
20	8.6	8.6	40	19.1	19.1	70	35	35	120	63	62
30	13.5	13.5	50	24.4	24.4	80	41	41	140	74	73
40	19.2	19.2	60	30	30	90	46	45	160	85	83
50	25.5	25.0	70	36	36	100	52	51	180	97	95
60	34.5	32.0	80	42	42	120	64	63	200	108	106
70	46.0	39.5	90	49	49	140	78	77	220	120	117
80	62.0	49.0	100	56	55	160	92	90	240	133	130
90	....	60.5	120	79.7	71	180	109	106	260	148	144
			140	108	88	200	128	123	280	163	158
			160	150	110	220	150	141	300	177	171
			180	....	138	240	177	160	320	194	187
						260	210	180	340	214	204
						280	250	203	360	234	221
						300	....	231	380	255	239
						320	....	265	400	276	257.

The sphere gap is more sensitive than the needle gap to momentary rises of voltage, and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage or by slowly raising the voltage with a fixed setting of the gap. (For detailed specifications for the manipulation of spark gaps, see "Standardisation Rules of the American Institute of Electrical Engineers.")

CONDUCTORS, RESISTANCES, RHEOSTATS

Materials. The materials used for the transmission and distribution of energy are restricted to copper, aluminum, iron and steel. For resistors and heaters, carbon and commercial alloys are most used.

Resistance Units. The **resistivity** or the **specific resistance** of a conductor material is the resistance in ohms of a sample of the material of unit length and unit section. Thus, if l represents the length of the conductor, a its sectional area, r its resistance, and ρ its specific resistance, then $r = \rho l/a$ and $\rho = ra/l$. For purposes of comparison, l and a are usually taken in centimeters and square centimeters, respectively, and ρ thus expresses the resistance in ohms between opposite faces of a centimeter cube. Table 2 gives values of specific resistance for various materials. Since most conductors are drawn wires of circular section, it is customary in engineering work to express the resistivity of a material in **ohms per (circular) mil-foot,** where the mil-foot represents a cylinder of the material 1 mil or 0.001 in. in diam. and 1 ft. in length. A mil-foot of copper has a resistance of 10.4 ohms at 20 deg. cent. The circular mils of a circular conductor may be found by squaring its diameter in mils. Thus, a cylindrical conductor 1 in. in diam. has a section of 1,000,000 cir. mils, and each foot of length has a resistance of 1/1,000,000 of that of 1 mil-foot of the material. In telegraph and telephone work, a still different unit of resistivity is in use—the **weight per mile-ohm.** This is the weight of a conductor of the material 1 mile in length which has a resistance of 1 ohm. It is equal to the prod-

Table 2. Specific Resistances of Metallic Wires
(From Smithsonian Tables)

Substance	Microhms per cu. cm. at 0 deg. cent.	Ohms per mil-foot at 0 deg. cent.	Temperature coefficient at 20 deg. cent.
Silver (annealed)	1.460	8.781	0.00377
Silver (hard drawn)	1.585	9.538	....
Copper (annealed)	1.584	9.529	0.00388
Copper (hard drawn)	1.619	9.741	....
Gold (annealed)	2.088	12.56	0.00365
Gold (hard drawn)	2.125	12.78	....
Aluminum (annealed)	2.906	17.48	....
Zinc (pressed)	5.613	33.76	0.00365
Platinum (annealed)	9.035	54.35	....
Iron (annealed)	9.693	58.31	....
Nickel (annealed)	12.43	74.78	....
Tin (pressed)	13.18	79.29	0.00365
Lead (pressed)	19.14	115.1	0.00387
Antimony (pressed)	35.42	213.1	0.00389
Bismuth (pressed)	130.9	787.5	0.00354
Mercury (pressed)	94.07	565.9	0.00072
Platinum-silver, 2Ag + 1Pt by weight	24.33	146.4	0.00031
German silver	20.89	125.7	0.00044
Gold-silver, 2Au + 1Ag by weight	10.84	65.21	0.00065

uct of the weight per mile by the resistance per mile. If the weight per mile-ohm is 2000, the resistance of a wire weighing 20 lb. per mile is 100, or, for a wire weighing 200 lb. per mile, is 10 ohms per mile. The relations between the various constants are shown by the following equations:

Microhms per cm. cube	= 0.1662	× ohms per mil-foot.
Ohms per mil-foot	= 6.015	× microhms per cm. cube.
Pounds per mile-ohm	= 57.07	× microhms per cm. cube × sp. g.

Matthiessen's Standard of conductivity is the conductivity of a copper wire having the following characteristics at 0 deg. cent., and is taken as the standard at 100 per cent.:

Specific gravity....	8.89	Resistance...........	0.141729 ohm
Length...........	1 meter	Specific resistance.....	1.594 microhms/cm.^{3}
Weight...........	1 gram	Relative conductivity.	100 per cent.

Since the time of adoption of Matthiessen's Standard, advances in the refinement of copper have been made so that to-day it is possible to obtain copper of 102 per cent. conductivity. It is customary in specifications, in order to insure high conductivity, to specify a conductivity of at least 98 per cent. according to Matthiessen's Standard for annealed copper wire, and 97 per cent. for hardened copper.

Temperature Coefficient. For practically all conductors the resistivity is not constant but varies with the temperature. The general equation which shows this variation is $\rho_t = \rho_0(1 + \alpha t + \beta t^2)$, where ρ_t is the resistivity at t deg. cent., ρ_0 that at 0 deg. cent., t is the final temperature, deg. cent., and α and β are temperature coefficients. For practical work this equation may be taken as $\rho_t = \rho_0(1 + \alpha t)$. The average value of α for copper between 0 and 100 deg. may be taken as 0.0042. Table 2 gives α for various metals for temperature rises above 20 deg. cent. For all pure metals the temperature coefficient is very nearly the same and may be taken as 0.004 per deg. cent. or 0.0023 per deg. fahr. For alloys in general the temperature coefficient is less than the mean of the coefficients of the constituent metals. Electrolytes and carbon have negative coefficients.

Wire Gages. For a comparison of the various gages, see p. 498. The American or **Brown and Sharpe (B. & S.) Gage** is the standard for copper wire used for electrical purposes. The diameters in this system form a geometric series (in which No. 0000 has a diam. of 460 mils and No. 36 a diam. of 5 mils) and follow the law $d = 324.9/(1.123)^n$, where n is the gage number and d the diam. in mils. (It should be observed that in this equation No. 0000 = −3, No. 000 = −2, No. 00 = −1, and No. 0 = 0.) Since the sectional area varies as the square of the diameter, an increase of 3 in the gage number means a doubling of resistance with the sectional area and weight reduced by one-half. Also, an increase of 10 in gage number increases the resistance tenfold and reduces the section and weight to one-tenth. Also No. 10 has a diameter of 0.1 in. and a resistance of 1 ohm per 1000 ft. The **Roebling or Washburn and Moen Gage** is used almost entirely in this country for iron and steel wire. Except for iron wire, the **Birmingham or Stubs Gage** is used in America for wires made for other than electrical purposes.

Table 3. Copper Wire Table of the American Institute of Electrical Engineers

(Condensed. Calculated for a temperature of 20 deg. cent.)

Gage, B. & S.	Area, circular mils	Weight, lb. per 1000 ft.	Length, ft. per ohm	Resistance, ohms per 1000 ft.*	Gage, B. & S.	Area, circular mils	Weight, lb. per 1000 ft.	Length, ft. per ohm	Resistance, ohms per 1000 ft.*
0000	211,600.0	640.5	20,440.0	0.04893	19	1,288.0	3.899	124.4	8.038
000	167,800.0	508.0	16,210.0	0.06170	20	1,022.0	3.092	98.66	10.14
00	133,100.0	402.8	12,850.0	0.07780	21	810.1	2.452	78.24	12.78
0	105,500.0	319.5	10,190.0	0.09811	22	642.4	1.945	62.05	16.12
1	83,690.0	253.3	8,083.0	0.1237	23	509.5	1.542	49.21	20.32
2	66,370.0	200.9	6,410.0	0.1560	24	404.0	1.223	39.02	25.63
3	52,630.0	159.3	5,084.0	0.1967	25	320.4	0.9699	30.95	32.31
4	41,740.0	126.4	4,031.0	0.2480	26	254.1	0.7692	24.54	40.75
5	33,100.0	100.2	3,197.0	0.3128	27	201.5	0.6100	19.46	51.38
6	26,250.0	79.46	2,535.0	0.3944	28	159.8	0.4837	15.43	64.79
7	20,820.0	63.02	2,011.0	0.4973	29	126.7	0.3836	12.24	81.7
8	16,510.0	49.98	1,595.0	0.6271	30	100.5	0.3042	9.707	103.0
9	13,090.0	39.63	1,265.0	0.7908	31	79.7	0.2413	7.698	129.9
10	10,380.0	31.43	1,003.0	0.9972	32	63 21	0.1913	6.105	163.8
11	8,234.0	24.93	795.3	1.257	33	50.13	0.1517	4.841	206.6
12	6,530.0	19.77	630.7	1.586	34	39.75	0.1203	3.839	260.5
13	5,178.0	15.68	500.1	1.999	35	31.52	0.09543	3.045	328.4
14	4,107.0	12.43	396.6	2.521	36	25.0	0.07568	2.414	414.2
15	3,257.0	9.858	314.5	3.179	37	19.83	0.06001	1.915	522.2
16	2,583.0	7.818	249.4	4.009	38	15.72	0 04759	1.519	658.5
17	2,048.0	6.200	197.8	5.055	39	12.47	0 03774	1.204	830.4
18	1,624.9	4.917	156.9	6.374	40	9.888	0 02993	0.955	1,047.0

*For resistance at 0 deg. cent., multiply values by 0.9262; at 50 deg. cent., by 1.11723; and at 80 deg. cent., by 1.23815.

Table 4. Weights of Double- and Triple-covered Weatherproof Wire and Cable

Solid conductors			Stranded conductors					
Size, B. & S. gage	Pounds per 1000 ft.		Size, B. & S. gage	Pounds per 1000 ft.		Size, cir. mils	Pounds per 1000 ft.	
	Double-covered	Triple-covered		Double-covered	Triple-covered		Double-covered	Triple-covered
14	20	25	8	68	78	450,000	1601	1724
12	30	35	6	103	115	500,000	1765	1894
10	46	53	5	126	140	600,000	2093	2235
9	54	62	4	155	170	700,000	2471	2650
8	66	75	3	190	206	750,000	2635	2822
6	100	112	2	246	270	800,000	2799	2992
5	122	135	1	303	328	900,000	3127	3322
4	151	164	0	388	424	1,000,000	3456	3674
3	185	199	00	482	522	1,500,000	5098	5380
2	239	260	000	604	653	2,000,000	6690	7008
1	294	316	0000	745	800	....	....	....
0	377	407	cir. mils			....	....	....
			250,000	907	985	....	....	....
00	467	502	300,000	1083	1174	....	....	....
000	587	629	350,000	1248	1345	....	....	....
0000	723	767	400,000	1436	1553	....	....	....

Table 5. Tensile Strength of Copper Wire: Breaking Weight in Pounds

B. & S. gage No.	Hard-drawn	Annealed	B. & S. gage No.	Hard-drawn	Annealed	B. & S. gage No.	Hard-drawn	Annealed
0000	8310	5650	5	1559	883	13	244	138
000	6580	4480	6	1237	700	14	193	109
00	5226	3553	7	980	555	15	153	87
0	4558	2818	8	778	440	16	133	69
1	3746	2234	9	617	349	17	97	55
2	3127	1772	10	489	277	18	77	43
3	2480	1405	11	388	219	19	61	34
4	1967	1114	12	307	174	20	48	27

Stranding. In sizes larger than No. 0000 (B. & S.) and in many cases for sizes larger than No. 6, copper conductors are usually made stranded rather than solid, to facilitate handling and splicing. Wires may be laid up in a **rope** or in a **concentric strand.** A 7 × 7 rope is one made up of seven strands of seven wires each. To obtain a smooth and symmetrical cable, certain combinations of wires must be used which are laid over a center wire in layers wound spirally in opposite directions, giving what is known as a concentric strand. Starting with one conductor in the center, 6 can be laid around it, 12 around this layer, then 18, 24, etc., making total strands of 1, 7, 19, 37, 61, etc.

Table 6. Diameters and Weights of Small Sizes of Magnet Wire

(General Electric Co.: S.C.C. = single cotton covered; D.C.C. = double cotton covered; S.S.C. = single silk covered; D.S.C. = double silk covered)

Size, B. & S.	Diameters in mils						Weight in lb. per 1000 ft.				
	Bare	S.C.C.	D.C.C.	S.S.C.	D.S.C.	Enamel	S.C.C.	D.C.C.	S.S.C.	D.S.C.	Enamel
14	64.0	70.0	74.0	......	......	67.0	12.684	12.918	......	......	12.684
15	57.0	63.0	67.0	......	......	60.0	10.082	10.274	......	......	10.053
16	51.0	56.0	59.0	......	......	53.5	8.012	8.176	......	......	7.973
17	45.3	50.0	53.0	......	......	47.5	6.375	6.510	......	......	6.322
18	40.3	45.0	48.0	......	......	42.0	5.081	5.188	......	......	5.009
19	36.0	40.0	44.0	......	......	37.0	4.043	4.130	......	......	3.966
20	32.0	36.0	40.0	......	......	34.0	3.218	3.289	......	......	3.136
21	28.5	32.5	36.5	......	......	30.5	2.569	2.628	......	......	2.475
22	25.4	29.4	33.4	......	......	27.5	2.055	2.106	......	......	1.970
23	23.0	26.5	30.5	26.0	29.0	25.0	1.630	1.676	1.573	1.604	1.555
24	20.0	24.1	28.0	23.0	26.0	22.0	1.279	1.344	1.241	1.298	1.232
25	18.0	22.0	26.0	21.0	24.0	20.0	1.036	1.082	0.991	1.040	0.980
26	16.0	20.0	24.0	19.0	22.0	17.5	0.828	0.873	0.791	0.833	0.777
27	14.0	18.0	22.0	17.0	20.0	15.5	0.661	0.703	0.631	0.666	0.616
28	12.6	16.6	20.6	15.6	18.6	14.0	0.524	0.562	0.499	0.521	0.485
29	11.3	15.3	19.3	14.0	17.0	12.3	0.421	0.457	0.397	0.416	0.384
30	10.0	14.0	18.0	12.5	15.0	11.3	0.336	0.372	0.315	0.312	0.303
31	9.0	13.0	17.0	11.4	13.9	10.2	0.271	0.307	0.254	0.267	0.262
32	8.0	11.9	15.9	10.5	13.0	9.2	0.215	0.248	0.203	0.214	0.192
33	7.0	11.0	15.0	9.5	12.0	8.2	0.174	0.201	0.161	0.172	0.152
34	6.3	10.3	14.3	8.8	11.3	7.3	0.141	0.161	0.130	0.140	0.121
35	5.6	9.6	13.6	7.6	9.6	6.8	0.120	0.137	0.110	0.119	0.101
36	5.0	8.5	12.0	7.0	9.0	6.2	0.099	0.112	0.089	0.096	0.081
38	4.5	......	......	6.0	8.0	5.2	......	......	0.058	0.065	0.051
40	3.1	......	......	5.0	7.0	4.2	......	......	0.037	0.040	0.031

Magnet Wire is a soft copper wire of high conductivity insulated in a manner suitable for winding in as small a space as possible in coils and on spools and bobbins. Magnet wire may be obtained in square, rectangular and circular section, but the round or cylindrical wire is used most extensively. **Cotton covering** finds its greatest use on wires of large and medium size, and **silk covering** is used upon the smallest of wires because of the smaller space it requires. Both cotton and silk insulation char at about 260 deg. fahr. and so are unsuitable for high-temperature operation. **Enameled magnet wire** is used where it is necessary to have the thinnest insulation possible and still retain high insulation resistance. Because of the liability of abrasion, enamel insulation is used mostly on the smaller wires, and great care must be taken in winding. Enamel insulation will withstand a temperature of 400 deg. fahr. for long periods of time without deterioration. A simple test of the adhesion of enamel to wire may be made by drawing the wire between the nails of the thumb and forefinger—it should not be possible to remove the enamel in this manner. **Deltabeston wire** or magnet wire provided with an asbestos covering is used on magnets and spools operating at high temperature. Wire insulated in this manner may be operated at near a red heat without injuring the insulation. **Salamander wire** is very similar to Deltabeston in its properties and has approximately the same diameters as single-cotton-covered magnet wire.

Aluminum is next to copper in the order of importance as an electrical conductor. It has a conductivity of about 62 per cent. of that of hard-drawn copper, but weighs about 50 per cent. as much for the same conductivity. Its elastic limit in tension varies from 16,000 to 20,000 lb. per sq. in., and its average ultimate strength is about 25,000 lb. per sq. in. Thus for the same conductivity as copper, an aluminum conductor would have a section approximately 60 per cent. greater, and with this section the ultimate strength would be 64 per cent. of that of the copper conductor while the weight would be about 50 per cent. as great. For this reason aluminum is much used for conductors for high-tension transmission lines where moderate spans are necessary.

Telegraph and Telephone Wire. Aside from the hard-drawn copper wire which is now being used for telegraph and telephone lines (see p. 523 for specifications), there are three grades of galvanised steel wire in use. "Steel" has the highest tensile strength and the lowest conductivity and is used in telephone service for short lines where a light, strong wire is preferable to a heavier wire of greater conductivity; weight per mile-ohm, 6500 lb. "Best Best" (BB) has a somewhat lower tensile strength but higher conductivity and is used by telephone companies; weight per mile-ohm, 5700 lb. "Extra Best Best" (EBB) has the highest conductivity (see Table 7) and is used largely by telegraph and railway companies; weight per mile-ohm, 5000 lb.

Cables

Underground and Submarine Cables are often used for transmission and distribution purposes, where city ordinances or appearance require that the wires be installed underground, or where power must be transmitted across a bay or river and overhead construction is impossible. Power cables of pressures up to 60,000 volts are now in successful operation.

Three types of insulation are now in common use: rubber compounds, varnished cambric, and impregnated paper.

(a) Rubber compounds consist of pure Para rubber and such mineral ingredients as sulphur, whiting, talc and litharge, which are thoroughly mixed together; the compounds are forced upon the conductors and then vulcanised.

Table 7. Properties of Galvanized Telephone and Telegraph Wires
(American Steel & Wire Company)

Size B.W.G.	Diam. in mils	Area in circular mils	Approximate weight in pounds		Approximate breaking strain in pounds			Resistance per mile at 68 deg. fahr., international ohms		
			Per 1000 ft.	Per mile	E.B.B.	B.B.	Steel	E.B.B.	B.B.	Steel
0	340	115,600	313	1,655	4,138	4,634	4,965	2.84	3.38	3.93
1	300	90,000	244	1,289	3,233	3,609	3,867	3.65	4.34	5.04
2	284	80,656	218	1,155	2,888	3,234	3,465	4.07	4.85	5.63
3	259	67,081	182	960	2,400	2,688	2,880	4.90	5.83	6.77
4	238	56,644	153	811	2,028	2,271	2,433	5.80	6.91	8.01
5	220	48,400	131	693	1,732	1,940	2,079	6.78	8.08	9.38
6	203	41,209	112	590	1,475	1,652	1,770	7.97	9.49	11.02
7	180	32,400	87	463	1,158	1,296	1,389	10.15	12.10	14.04
8	165	27,225	74	390	975	1,092	1,170	12.05	14.36	16.71
9	148	21,904	60	314	785	879	942	14.97	17.84	20.70
10	134	17,956	49	258	645	722	774	18.22	21.71	25.29
11	120	14,400	39	206	515	577	618	22.82	27.19	31.55
12	109	11,881	32	170	425	476	510	27.65	32.94	38.23
13	95	9,025	25	129	310	347	372	37.90	45.16	52.41
14	83	6,889	19	99	247	277	297	47.48	56.56	65.66
15	72	5,184	14	74	185	207	222	63.52	75.68	87.84
16	65	4,225	11	61	152	171	183	77.05	91.80	106.55

(*b*) Cambric insulation consists of cotton cloth and a viscous filler. Each surface of the cloth is covered with multiple films of insulating varnish. The cloth is then applied to the conductor in the form of tape wound on spirally, the filler being applied between the successive layers.

(*c*) Impregnated paper insulation consists of Manila paper tapes applied spirally and evenly to the conductor, and then thoroughly impregnated with an insulating compound.

Cable Insulation. In underground practice it has been found necessary to protect the insulation by a lead sheath. For submarine installations, cambric and paper-insulated cables usually have lead sheaths armored with steel for protection, although, when rubber is used, the lead sheath is unnecessary. Varnished cambric will resist water to a certain extent, but paper is worthless as an insulator if moisture is allowed to enter the cable.

Sometimes two or more of the insulations are used in the same cable to good advantage. For alternating-current transmission it is advisable to have all the conductors of the circuit in one cable, as the currents neutralize the tendency to set up eddy currents in the sheath. In all high-tension cables, the sheaths should be well grounded to allow the high-voltage static charges to pass readily to earth. When high-voltage conductors pass from the air into the cable, or from the cable into air, a large pot-head should be used; this should have no sharp inside edges, and should be filled with some insulating substance. Use of the pot-head reduces to a minimum the high potential gradient that would otherwise occur at this point.

A cable system differs from an overhead system, in that the linear inductance is usually negligible and the electrostatic capacity much greater. Hence the charging current may be nearly equal to the load current itself. Surges and transient phenomena are much more common than on an overhead line of the same length.

Underground cables for high-tension work are ordinarily installed in standard ducts. Although all underground cables should be lead-covered, the rubber covering may last for years after the lead sheath has been eaten away by electrolysis or has been injured mechanically. The lead sheath may become heavily charged by electrostatic induction and consequently should be grounded at intervals.

Steel-taped cable is a lead-covered cable mechanically protected with steel tapes

and jute and may be buried directly in the ground. It is offered as a safe and reliable substitute for the more expensive conduit system.

Resistance Materials

For furnace resistors, a material having high resistivity and which does not disintegrate or corrode at high temperatures is best. For electrical instrument construction a material of medium-high resistivity together with low temperature coefficient and low thermoelectric power against copper is most valuable. For rheostats and other power-consuming devices, an inexpensive and non-corrosive material of high resistivity and which is suited to stand repeated heating and cooling serves the purpose best. Most of the materials listed in Table 8 may be obtained in ribbon form up to 1 in. in width.

Advance (a copper-nickel alloy) has a low temperature coefficient and is suited for electrical instrument work. **Therlo** (an alloy of copper, aluminum, and manganese) may be used for instrument shunts and coils, since aside from its low temperature coefficient, its thermoelectric power against copper is very low—0.3 microvolt per deg. cent. **Manganin** (copper, nickel, ferro-manganese alloy) is much used for standard resistance coils and units because of its low thermoelectric power with copper—1.5 microvolts per deg. cent.—and low temperature coefficient— − 0.00001 to + 0.000039 ohm per deg. cent. Manganin can be made to have permanent electrical properties by annealing for at least 5 hr. at a temperature of 140 deg. cent., but should not be subsequently heated without immersion in oil. **Nichrome** and Nichrome II are used for electric-furnace and high-temperature work, especially in ribbon form, having high operating temperatures. **Climax** (nickel-steel alloy) is often used for load rheostats and other similar resistors where first cost is the controlling factor in the choice of material. Like all steel alloys, it should be kept dry to prevent corrosion. **Ferronickel** (nickel-steel

Table 8. Properties of Resistance Materials

Name	Specific gravity	Resistivity		Temperature coefficient per deg. cent. at 0 deg. cent.	Melting point, deg. cent., approx.	Maximum safe operating temperature, deg. cent., approx.	Maker or agent*
		Microhms per cu. cm. at 20 deg. cent.	Ohms per mil-foot at 20 deg. cent.				
Advance	8.9	48.8	294	0.000018	1320	540	D-H
Therlo	8.15	46.7	280	−0.0000056	1200	....	D-H
Manganin	8.9	41.4–73.8	249–443	+0.000011 0.000039	....	....	....
Nichrome II	8.02	105.0	632	0.000162	1650	1090	D-H
Nichrome	8.15	95.5	575	0.000432	1480	870	D-H
Climax	8.13	87.2	525	0.00054	1430	650	D-H
Monel	8.9	*42.6	256	0.00198	1350	900	D-H
Superior	8.4	86.0	517	0.00072	1230	540	B
Ia Ia (hard)	8.4	50.2	304	0.000011	1175	575	B
Ia Ia (soft)	8.4	47.1	283	0.000005	1175	575	B
Constantan	9.73	50.0	300	0.000005	....	....	S
German silver, 18 per cent	8.5	33.3	200	0.00031	1095	500	....
German silver, 30 per cent	8.5	48.1	290	0.00025	1165	500	....
Krupp's	8.1	85.13	511	0.0007	....	600	P
Calido	....	94.2	567	0.000342	1540	760	E

* B, Hermann Boker & Co.; D-H, Driver-Harris Wire Co.; E, Electrical Alloy Co.; P, Thomas Prosser & Son; S, C. Schniewindt.

alloy) is inexpensive and is much used for rheostats and coils. It corrodes rather readily. **Monel metal** (copper-nickel alloy) is medium priced and varies somewhat in electrical properties. **Superior** (nickel-steel alloy) is much used for rheostat coils. **Ia Ia** (copper-nickel alloy) has low temperature coefficient and permanent electrical properties and is suited for use in instruments. **Constantan** (50 per cent. nickel, 50 per cent. copper) has a low temperature coefficient but its high thermoelectric effect against copper (40 microvolts per deg. cent.) prohibits its use in electrical instruments. **German silver** (alloy of copper, nickel, and zinc) is used extensively, but its properties are not permanent, as it undergoes a change in resistance due to the crystallization of the zinc. It has a high thermoelectric force against copper—20 to 30 microvolts per deg. cent. It is made of various compositions, but alloys of 18 per cent. and 30 per cent. nickel respectively are most common. **Krupp's resistance wire** (nickel-steel) is not suited for use in damp places and should preferably be operated in oil. It should not be insulated with asbestos.

Carbon withstands high temperatures and has a high resistance, while its temperature coefficient is negative; it will safely carry about 125 amperes per sq. in. Amorphous carbon has a resistivity of from 3800 to 4100 microhms per cu. cm., retort carbon about 720 microhms, and graphite about 812 microhms per cu. cm. The properties of any particular kind of carbon depend upon the temperature at which it was fired. Carbon for rheostats may best be used in the form of compression rheostats.

Rheostats

Carbon Compression Rheostats made up of columns of graphite disks are much used for heavy currents and for fine regulation. The resistance variation is obtained by varying the contact resistance between disks by adjusting the pressure on the column. Graphite carbon disks rather than "flint" or coke carbons should be used, as a wider range of resistance may be obtained. The heat-radiating surface may be increased by inserting plates of sheet iron between the carbon disks. Black sheet iron is better than brass or bright tin, since the contact between carbon and iron does not change due to the change of surface condition of the metal at high temperatures. Carbon plates are also used as the resistors of electric furnaces of the resistance type, and carbon electrodes are used for furnaces of the arc type. Table 9 gives data from tests by Prof. A. L. Goddard upon a rheostat composed of a column of 275 Columbia Graphite disks 1 in. square piled alternately with black sheet-iron disks of larger area.

Table 9. Resistance Variation with Temperature and Pressure in a Carbon Compression Rheostat (Goddard)

Temperature, deg. fahr.	Watts emissivity per pair of disks (carbon and iron)	Resistance in ohms per pair of disks at different pressures in pounds				
		0 lb.	5 lb.	10 lb.	15 lb.	20 lb.
100	....	0.65	0.22	0.16	0.12	0.10
200	0.66	0.56	0.18	0.12	0.10	0.08
300	1.33	0.46	0.15	0.11	0.09	0.06
400	2.33	0.37	0.12	0.09	0.07	0.05
500	3.00	0.27	0.10	0.06	0.05	0.04
600	4.00	0.18	0.06	0.05	0.04	0.03

Water Rheostats. In the testing of electrical machinery where large amounts of power are to be dissipated, some type of a water rheostat is often used as a load because of its comparatively low first cost. A rheostat of this kind consists essentially either of resistance material placed in water for cooling purposes or a set of electrodes immersed in water, the water acting as the conductor. For the dissipation of large amounts of energy at low voltage **coils of wire** or other metallic conductor may be submerged in barrels of water. Enough cold water should be allowed to circulate through the barrels to keep the temperature below 200 deg. fahr. In Table 10 are given the constants for iron wire for use in rheostats of this type. For continuous operation for long periods, especially on direct current, water-cooled coils of this kind are not suitable, since the coils become slimed over due to the heating of the water and must ordinarily be changed or cleaned at intervals of about 1 hr. In order to prevent excessive deposition of lime and magnesia upon the wire, the water should be well circulated. The arrangement shown in Fig. 20 may be used.

Table 10. Coil Proportions for Submerged Iron Wire Rheostats.*

Size, B.W.G.	Diam., mils	Ohms per ft., hot	Permissible current, amperes	Minimum length of wire to carry permissible current at 100 volts, ft.	Size, B.W.G.	Diam., mils	Ohms per ft., hot	Permissible current, amperes	Minimum length of wire to carry permissible current at 100 volts, ft.
20	35	0.08310	44	27.3	12	109	0.00886	240	47.1
19	42	0.06120	57	28.6	10	134	0.00587	327	52.1
18	49	0.04380	72	31.7	9	148	0.00481	381	54.6
17	58	0.03204	93	33.6	8	165	0.00388	447	57.7
16	65	0.02496	110	36.4	7	180	0.00325	512	60.1
15	72	0.02032	129	38.2	6	203	0.00256	610	64.3
14	83	0.01530	159	41.1	5	220	0.00218	691	66.4
13	95	0.01168	197	43.5	4	238	0.00186	778	69.1

* Based on the use of BB galvanized iron wire (or annealed iron in the smaller sizes) having a resistance of 75 ohms per mil-foot, at 68 deg. fahr., a temperature coefficient of 0.00289 per deg. fahr. from 68 deg., and an allowable expenditure of 125 watts per sq. in. of wire surface. The turns of wire should be spaced at least ½ in. c. to c. The minimum length of wire is proportional to the voltage; *e.g.*, for 500 volts the minimum length is 5 times that for 100 volts.

In the **concentric cylinder type** of rheostat, where water is employed as the conductor, use is made of concentric metal cylinders with water flowing between them. (See Fig. 21.) Adjustment is obtained by raising and lowering the inner electrode, the outer cylinder being fixed. Table 11 gives electrode dimensions for the values of specific resistance of water of 500, 1000, and 2000 ohms per in. cube. The cylinders considered are **standard wrought-iron pipes.** Water should be allowed to flow fast enough to prevent boiling, as boiling causes unsteadiness and fluctuations of the current. For operation at 110 volts, it is not economical to build a rheostat of this type of greater than 10 kw. capacity because of the relatively high cost of construction.

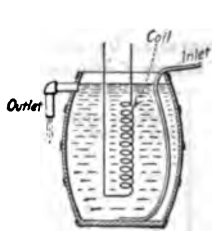


Fig. 20.

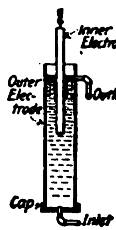


Fig. 21.

Water Rheostats.

Table 11. Dimensions for Concentric Cylinder Water Rheostats

Kw. capacity	Volts	Amperes	Resistance, ohms	Specific resistance, ohms/in.³	Nominal diameters of pipes: Inner, in.	Nominal diameters of pipes: Outer, in.	Depth of immersion, in.
5	110	45.5	2 42	500	¾	3	35
10	110	91.0	1.21	500	½	1¼	33
2	110	18.2	6.05	1000	¼	2	35
5	110	45.5	2.42	1000	½	1¼	33
10	110	91.0	1.21	1000	2	3	34
2	110	18.2	6.05	2000	¾	2	36
5	110	45.5	2.42	2000	2	3	34
10	110	90.0	1.21	2000	3	4	37
20	220	91.0	2.42	500	1	3	38
10	220	45.5	4.85	1000	¾	3	35
20	220	91.0	2.42	1000	1	2	30
5	220	22.7	9.70	2000	¾	3	35
10	220	45.5	4.83	2000	½	1¼	33
20	220	91.0	2.42	2000	2	3	34
50	500	100.0	5.00	500	⅛	4	37
50	500	100.0	5.00	1000	1	4	36
25	500	50.0	10.00	2000	1	4	36
50	500	100.0	5.00	2000	2	4	34

When large amounts of power are to be dissipated (up to 5000 kw. and over) plates placed in a tank or lowered into a stream or pond may be used. For voltages below 1000, the conductivity of the water is increased by the addition of a small amount of **common salt** or **sulphuric acid.** Table 12 gives the relations between solution density and resistance for salt and for sulphuric acid. When acid is used the plates should be made of lead, while iron plates are satisfactory for salt-water tanks. The capacity of a rheostat of this type for a given rise in temperature is determined by the volume of solution and not by the radiating area. Table 13 gives the permissible watts per cu. in. of solution for various temperature rises. In general, 400 to 800 cu. in. should be used per h.p. absorbed continuously. The **current density** should not exceed 1 ampere per sq. in. of electrode surface, as above this value there is wasting away of the electrode material and unsteadiness of the current due to the formation of steam. When water rheostats are used on high-voltage circuits and the tanks are not grounded, the

Table 12. Water Rheostats—Resistance in Ohms between Opposite Faces of a Cubic Foot of Electrolyte

(Merrill, *Am. Elecn.*, Dec., 1897, p. 468)

Per cent. of salt or acid by weight	0.20	0.40	0.60	0.80	1.00	1.20	1.40
Resistance in ohms:							
H_2SO_4 solution	3.78	1.98	1.30	1.00	0.87	0.79	0.75
NaCl solution	8.38	5.30	3.59	2.75	2.22	1.82	1.39

Table 13. Water Rheostats—Permissible Number of Watts per Cu. In. of Solution for Various Temperature Rises

(From chart in Dix-Whittaker's Handbook)

Temp. rise, deg. cent.	10	20	30	40	50	60	70
Watts per cu. in. of solution	0.26	0.60	0.90	1.40	2.10	3.00	4.75

circulating water must be conducted to and from the tanks by means of **rubber hose.** For voltages up to 2500, a hose 15 or 20 ft. in length is sufficient if the diam. is not greater than 1 in., while for higher voltages the hose must be correspondingly longer.

Wire Rheostats. For light and medium current values, resistances and rheostats made of metallic wire or ribbon are more satisfactory than water rheostats. An inexpensive load or starting rheostat for heavy currents may be made up of **poultry netting** (Dix-Whittaker's Handbook, p. 166) supported on iron rods mounted on insulators. Standard galvanised poultry netting of 1-in. mesh and 12 in. wide made up of No. 20 B.W.G. wires has a resistance of about 0.005 ohm per yd. and will carry 100 amperes when properly ventilated.

Enclosed Resistances must be more liberally designed than open-coil resistances because of the smaller radiating surface. In general, the size of the enclosing box should be so proportioned as to allow from 0.5 to 1.0 sq. in. of radiating surface per watt of energy to be dissipated. With good through draft of air, the air temperature should not rise to higher than 200 deg. fahr. under these conditions. For permanent service, a substantial form of rheostat is found in the **enameled type,** which is much used in the shunt-field circuits of generators. Here the wire or ribbon is embedded in enamel which is fused or baked on to one side of a ribbed cast-iron plate. Assuming one side of the plate only to be radiating energy, the allowable expenditure may rise to 5 watts per sq. in. of surface.

INSULATION

Insulation is applied to conductors for the purpose of preventing leakage of current therefrom. Various **insulating materials** are **described** on pp. 626–629 and their properties stated. The prime requisite of an insulating substance is that it shall be able to resist puncture when submitted to the maximum e.m.f. under working conditions. Breakdown potentials given in the tables following are effective values. The maximum instantaneous value of the voltage, however, is the measure of the dielectric strength, and, assuming a sine wave of voltage, the maximum instantaneous value is $\sqrt{2}$ times the values given.

Insulation Resistance is the resistance (measured in ohms or megohms) offered by an insulating coating, cover, material or support to an impressed voltage which tends to produce a leakage of current through it. High temperature and absorbed moisture (generally) lower the ohmic resistance; moisture deposited on the outside surface increases surface leakage.

Table 14. Insulating Materials Suitable for Various Service Conditions

(From the "Standard Handbook." T indicates suitability to resist high temperature; M, suitability for use in moist places; S, mechanically strong dielectrics)

Adit.....	T.M.S.	Berrite.....	T.	Horn fiber..	T.M.S.	Mineralite.....	T.S.
Aetna....	T.M.S.	Celluloid....	T.M.S.	Lava.......	T.S.	Presspahn (treated).....	M.S.
Ambroin.	T.M.S.	Ebonite.....	M.S.	Leatheroid...	T.M.S.	Psychiloid.....	S.
Armalac.	T.M.	Electro-enamel...	T.M.	Megomit....	M.S.	Rubber........	M.
Asbestos.	T.	Fullerboard.	S.	Mica.......	T.	Sterling varnish	T.
Asphalt..	M.	Gutta percha	M.	Micanite....	S.	Vulcanised fiber	S.

Dielectric Strength. The dielectric strength of an insulating wall, coating, cover or path is its ability to resist the passage of a disruptive discharge through it, and is measured by the voltage which must be applied in order to effect such disruptive discharge. High temperature lowers the dielectric strength, and many materials break down at comparatively low temperatures. Alternating electric stresses often cause dangerously high rises in temperature.

Table 15. Approximate Dielectric Strength of Various Substances, in Volts (Mean Effective) per Millimeter of Thickness
("Standard Handbook")

Substance	Volts per mm.	Substance	Volts per mm.	Substance	Volts per mm.
Aetna	700	Galith	6,000 8,500	Mica	28,000
Ambroin	7,000	Glass, ordinary	8,000	Oiled cloth	18,000 23,000
Asbestos paper	4,000	Glass, lead	5,500	Oiled paper	25,000 30,000
Berrite	5,000	Hard rubber	10,000 38,000	Para rubber	18,000
Bitumen	11,000	India rubber	18,000	Paraffin	11,500
Calico treated with rubber	1,500	Jute (impregnated)	800	Porcelain	9,000
Cambric oiled	17,000	Lava	3,000 10,000	Porcelain (Locke)	16,350
Canvas oiled	5,000	Leatheroid	5,000 12,000	Presspahn	4,000 10,000
Celluloid, clear	14,000	Linen cloth	5,000 8,000	Psychiloid	6,000
Celluloid, colored	19,000	Litholite	4,500	Rosin	11,000
Coal-tar pitch	2,000	Manila paper	5,000	Stabilit	9,500 17,500
Cotton cloth	3,500	Marble	6,500	Vulcabeston	2,300 4,000
Cotton drill	2,000	Mica	17,000	Wax	11,500
Cotton duck	2,300				
Ebonite	30,000				
Empire cloth (oiled)	10,000				
Fullerboard	16,000				

Table 16. Resistance and Dielectric Strength of Insulating Materials
(Fowler's "Electrical Engineer's Pocket Book," 1910)

Material	Thickness used in dynamo work, inches	Megohms per square inch-mil	Specific dielectric strength	
			Limits in volts per mil thickness	Practical average volts per mil
Asbestos paper	0.004–0.020	7	100–180	125
Asbestos and muslin, oiled	0.010–0.030	850	330–500	375
Cotton, single covering	0.005–0.012	10	260–340	275
Cotton, single covering soaked in paraffin.	0.006–0.015	11,800,000	380–480	400
Cotton, double covering	0.012–0.020	10	210–240	225
Cotton, double covering, shellacked	0.015–0.025	25	250–300	275
Fiber, red, vulcanised	0.030–0.075	470	150–325	200
Mica	0.001–0.125	33,000	2,000–8,000	3,000
Micanite cloth, flexible	0.008–0.020	440,000	175–310	200
Micanite paper, flexible	0.010–0.025	500,000	280–390	300
Micanite plate, flexible B	0.010–0.020	320,000	575–790	600
Oiled cloth	0.005–0.030	650	450–650	500
Oiled paper, double coat	0.006–0.010	1,600	600–950	700
Brown paper	0.005–0.010	2	160–200	175
Paraffined paper	0.002–0.008	11,800,000	800–1,000	900
Rubber sheet	0.015–0.060	3,000,000	350–600	400
Shellacked cloth	0.006–0.012	30	30–60	40
Silk, single covering	0.001–0.0025	50	350–565	475
Silk, single covering, shellacked	0.0015–0.004	75	500–570	525
Silk, double covering	0.0015–0.005	50	320–420	375
Silk, double covering, shellacked	0.002–0.007	75	420–510	450

MAGNETS

A Permanent Magnet is one which retains a constant amount of magnetism indefinitely. Magnets of this kind are used in electrical instruments, magnetos, etc., where a constant magnetic field is desired. They usually consist of hardened steel bars which, after having been placed in a strong magnetic field, retain a portion of the induced magnetism. The best permanent magnet is the one having the greatest hardness together with the greatest amount of free iron. The greater the hardness, the greater is the retentivity, which means permanency. For carbon steels, the greatest retentivity is obtained with 0.97 per cent. carbon. One of the best **permanent magnet steels** contains about 0.6 per cent. carbon and 5 per cent. tungsten and shows the greatest retentivity when quenched at a temperature of from 930 to 950 deg. cent. Another satisfactory steel contains 1 per cent. silicon and 5 per cent. chromium. (Mars, *Stahl u. Eisen*, Oct. 27, Nov. 10, 1909; Burgess and Aston, *El. Wld.*, Dec., 1910.) Unless permanent magnets are subjected to an **artificial aging** process, they gradually deteriorate or weaken. In the case of magnets for use in electrical instruments, where a constant field strength is imperative, artificial aging is accomplished by mechanical vibration or by immersion in boiling water for a period of a few hours.

An Electromagnet is one in which the magnetic field is produced by the flow of an electric current. The core of an electromagnet is usually made of soft iron or mild steel, because, the permeability being higher, a stronger pull may be obtained. When the circuit is opened there is little trouble due to the sticking of the parts of the magnetic circuit. Electromagnets may have the form of **simple solenoids, ironclad solenoids, plunger electromagnets,** and **electromagnets with external armatures.**

Magnetomotive Force and Flux in a Coil. The magnetic potential of a point is expressed numerically by the number of ergs required to bring a unit positive pole from the boundary of the field to the point in question. The magnetic potential of the plane of a loop of any shape carrying a current I_a may be deduced as follows: If a magnetic pole of strength m is moved from infinity to the plane of the loop, the number of lines of force which cut the loop during this process is $2\pi m$, since 4π lines emanate from unit pole. The e.m.f. induced in a circuit is $e = d\Phi/dt$, where Φ is the flux. The energy involved in moving it is $w = I_a\int edt$, but $\int edt = \Phi$, which in this case is $2\pi m$. Hence $w = 2\pi mI_a$, or when the current is expressed in amperes, $w = 0.2\pi mI$. The magnetic potential is numerically equal to w.

The m.m.f. linking an electrical circuit is $0.4\pi nI$. The **magnetizing force,** *i.e.*, potential gradient, is $0.4\pi nI/l$, where l is the length of path taken by the magnetic flux and n is the number of turns in the coil. In the case of a solenoid, the length l is taken to be the length of the solenoid, for it is assumed that all of the magnetomotive force is consumed within the length of the coil since the path outside has an infinite cross-section and hence no reluctance. The magnetizing force of a solenoid is thus $H = 0.4\pi nI$, and with an air core the flux density is numerically the same as H.

Direct-current Tractive Magnets. The **range** of an electromagnet is the distance through which the plunger will perform work when the winding is energized. The range for a solenoid is independent of the number of ampere-turns. After the core is saturated, the pull varies directly with the number of ampere-turns. For long range of operation, the **plunger type** of tractive magnet is best suited, since the length of core is governed practically by the range of action desired, while the **area of the core is determined**

by the amount of pull necessary. The curve in Fig. 22 shows that the maximum pull occurs when the end of the core is in the middle of the winding. The equation for **maximum uniform pull** is

$$P(\text{lb.}) = KAIN/l$$

where A is the area of the core, in sq. in., I the current in amperes, N the number of turns of wire, l the length of the winding in in., and K a constant which represents the pull per sq. in. of core per ampere-turn per in. and which depends upon the coil proportions, degree of saturation, chemical purity of iron, and length and physical properties of the core. Values of K are given in Table 17.

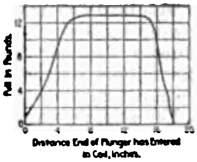


FIG. 22.—Pull of Plunger Magnet.

Table 17. Maximum Pull per Square Inch of Core for Solenoids with Open Magnetic Circuit

(From data by Underhill, *El. Wld.*, vol. 45, pp. 796, 881, 1906)

Length of coil, in., l	Length of plunger, in.	Core area, sq. in., A	Total ampere-turns, $I \times N$	Max. pull, lb. per sq. in., P	$1000 \times K$	Length of coil, in., l	Length of plunger, in.	Core area, sq. in., A	Total ampere-turns, $I \times N$	Max. pull, lb. per sq. in., P	$1000 \times K$
6	Long	1.0	15,000	22.4	9.0	12	Long	1.0	11,200	8.75	9.4
9	Long	1.0	11,330	11.5	9.1	12	Long	1.0	20,500	16.75	9.8
9	Long	1.0	14,200	14.6	9.2	18	36	1.0	18,200	9.8	9.7
10	10	2.76	40,000	40.2	10.0	18	36	1.0	41,000	22.5	9.8
10	10	2.76	60,000	61.6	10.3	18	18	1.0	18,200	9.8	9.7
10	10	2.76	80,000	80.8	10.1	18	18	1.0	41,000	22.5	9.8

Where a **strong pull** is desired at the end of the stroke, a stop may be used as shown in Fig. 23, which gives rise to the pull curve of Fig. 24. Here the pull is made up of two components—that due to the attraction between plunger and winding and that due to the attraction between plunger and stop. The equation for the pull is

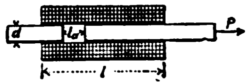


FIG. 23.

$$P = A\left[\frac{IN}{l_aK_1}\right]^2 + \frac{KAIN}{l} = AIN\left[\frac{IN}{l_a^2K_1^2} + \frac{K}{l}\right] \text{ in pounds,}$$

where A is the area of the core in sq. in., N the number of turns, l_a is the length of gap between core and stop, and K and K_1 are constants. At the beginning of the stroke the second member of the equation is predominant, while at the end of the stroke the first member represents practically the entire pull. Approximate values of K and K_1 are $K_1 = 2660$, and (for l greater than $10d$) $K = 0.0096$.

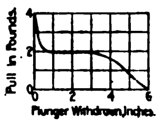


FIG. 24.

The **range of uniform pull** may be extended by the use of **conical ends** of stop and plunger, as shown in Fig. 25. The same effect may be obtained and a stronger magnet obtained mechanically by using an **ironclad solenoid** (Fig. 26), in which an iron return path is provided for the flux. Except for low flux densities and short air gaps the dimensions of the iron return path are of no practical importance, and the fact that an iron

FIG. 25.

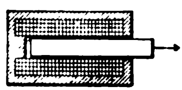


FIG. 26.—Ironclad Solenoid.

return path is used does not affect the pull curve except at short air gaps. A typical pull curve for an ironclad solenoid is shown in Fig. 27.

Mechanical jar at the end of the stroke may be prevented by boring through the end of the solenoid. The plunger then comes to equilibrium when its middle is at the middle of the winding, thus providing a magnetic cushion effect.

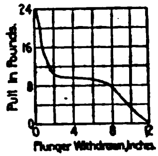


Fig. 27.—Pull of Ironclad Solenoid.

For **short-range work,** electromagnets with external armatures are best adapted, and the best type is the **horseshoe magnet.** The pull for short-range magnets is expressed by the equation $P(\text{lb.}) = B^2A/72{,}134{,}000$, where B is the flux density in lines per sq. in., and A the area of the core in sq. in. It should be noted that a stronger pull may be obtained if the surfaces of the armature and core are not machined and scraped to an absolutely smooth contact surface. Where the surface is slightly rough, the area of contact A is reduced but the flux density B is increased approximately in proportion (if the iron is being operated below saturation), and the pull is increased since it varies as the square of the density B. **Non-magnetic stops** should be used so that the core may release the armature readily when the current is cut off.

Lifting Magnets. It is difficult to state what the lifting capacity of a magnet is, for the reason that the weight that can be lifted depends upon the character and the length of the magnetic path. Table 18 gives the approximate **lifting capacities** of different sizes of commercial magnets with various kinds of load. For instance, the 62-in. Cutler-Hammer magnet will lift as much as 40,000 or 50,000 lb. where a large casting or skull-cracker ball affording a large magnetic contact surface is to be lifted. This same magnet might not lift over 1000 lb. of loose tin scrap. While commercial lifting magnets may be had in various forms for special purposes, those listed in Table 18 are known as **universal magnets,** as they are adapted to nearly all classes of work. See also p. 1110.

Rapid Action in a magnet may be obtained by reducing the time constant of the winding, and by subdividing the metal parts to reduce induced currents which have a demagnetizing effect. The movement of the plunger through the winding causes the winding and its bobbin to cut a field of force; if the bobbin is of metal and not slotted longitudinally, it acts as a short-circuited conductor cutting this field of force and hence consumes energy which would otherwise be available as mechanical pull. In cases where it is found impossible to reduce the time constant sufficiently, an electromagnet designed for a voltage much lower than normal is often used, which, at the completion of its stroke, mechanically connects in series with its winding a resistance of sufficient magnitude to limit the current flowing to a safe value. The solenoids on many automatic motor-starting panels are designed in this manner, as the extremely short time of overload does not injure the winding but causes a very rapid action. When **slow action** is desired, it can be obtained by using solid cores and yoke and by using a heavy metallic spool or bobbin for the winding. A separate winding short-circuited on itself is also used to some extent.

Sparking at switch terminals may be reduced or eliminated by neutralizing the inductance of the winding. This is accomplished by winding a separate short-circuited coil with its wires parallel to those of the active winding. (This method can be used with direct-current magnets only.) This is not economical, since one-half of the winding space is wasted. By connecting a condenser across the switch terminals, the energy of the inductive discharge upon opening the circuit may be absorbed. For the purpose of neutralizing the inductive discharge and causing a quick release, a small **reverse current** may be sent through the coil winding automatically upon opening the circuit. Tinfoil sleeves placed over the various layers of the winding absorb energy when the circuit is broken and reduce the energy dissipated at the switch terminals. This scheme can be used for direct-current magnets only. **Sticking** of the parts of the magnetic circuit due to residual magnetism may be prevented by the use of non-magnetic stops. In the case of lifting magnets subjected to rough usage and hard blows (as in a steel works), these stops usually consist of plates of manganese steel, which are extremely hard and non-magnetic.

Table 18. Dimensions, Weights and Lifting Capacities of Commercial Lifting Magnets

Diam., in.	Head* room, in.	Weight, lb.	Current required at 220 volts, d.c.	Lifting capacity, in pounds†				
				Skull-cracker balls or large castings	Ingots	Pig iron	Fine wire scrap	Maker
36	40	2,100	11.0	12,000	6,000	600	400	E-C-M‡
43	43	3,250	27.0	20,000	6,000	1,350	500	E-C-M
52	48	4,800	38.0	20,000	8,400	1,950	700	E-C-M
61	48	6,600	47.0	20,000	16,000	2,300	900	E-C-M
18	12	350	1.8					C-H‡
24	23	750	5.0					C-H
36	42	1,800	17.5	800–1,000				C-H
43	45	3,300	30.0*	1,300–1,500				C-H
52	50	5,200	40.0	1,800–2,000				C-H
62	56	7,500	55.0	2,400–2,800				C-H

* With three-chain suspension.

† The values given are conservative values. Those given for Cutler-Hammer magnets are for kinds of load varying from small scrap to pig iron; large castings of much greater weight can be handled.

‡ E-C-M: Electric Controller & Mfg. Co. C-H.: Cutler-Hammer Mfg. Co.

Alternating-current Tractive Magnets. Because of the iron losses due to eddy currents, the magnetic circuits of alternating-current electromagnets should be composed of laminated iron or steel. For large magnets the iron circuit is usually built up of thin sheets of varnished sheet metal held together by means of suitable clamps. For a small core of circular section a bundle of soft wires is often used. Since the iron losses increase with the flux density, it is not advisable to operate at as high a density as would be permissible with direct current. The current instead of being limited by the resistance of the winding is now governed practically by the inductive reactance, for, in all well-designed magnets, the resistance should be small. In a **single-phase magnet** the pull varies from zero to a maximum and back to zero twice every cycle, which may cause considerable **chattering** of the armature against the stop. This may be prevented by the use of a spring, or, in the case of a solenoid coil, by allowing the plunger to seek its position of equilibrium in the coil. In a **two-phase magnet** the pull is constant and equal to the maximum pull produced by one phase so long as the voltage is a sine function. Similarly, in a **three-phase magnet** under the same conditions the pull is constant and equal to 1.5 times the maximum instantaneous pull of one phase. Should the load become greater than the minimum instantaneous pull, there will be chattering as in a single-phase magnet.

Heating of Magnets. The lifting capacity of an electromagnet is limited by the permissible current-carrying capacity of the winding, which in turn is dependent upon the amount of heat energy which the winding can dissipate per unit time without exceeding a given temperature rise. Coils wound with wire having cotton insulation will in general be operating at a **safe temperature** if a watt expenditure in the coil of 0.5 watt per sq. in. of radiating surface be allowed.

Space Factor of Winding. The "space factor" is the ratio of the net volume of conductor in a given winding to the gross volume of the winding. Only in the theoretical case of uninsulated square or rectangular conductor may the space factor be 100 per cent. For wire of circular section with insulation of negligible thickness, wound as shown in Fig. 28, the space factor will be 78.5 per cent. When the turns of wire are "bedded," as shown in Fig. 29 (which is the case in most windings, particularly with smaller-sized wires), there is a theoretical gain of about 7 per cent. in space factor. (Underhill, *El. Wld.*, vol. 53, p. 155, 1909.) Experiments have shown that in most cases this gain is about neutralized in practice by the flattening out of the insulation of the wire due to the tension used in winding. When wound in a haphazard manner, the space factors of magnet wires vary according to size, substantially as follows:

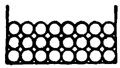


FIG. 28.

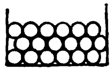


FIG. 29.

	—Double cotton covered—					Single cotton covered		
Size, B. & S. gage	0	5	10	15	20	25	30	35
Space factor, per cent	60	53.8	45.5	35.1	32.2	32	25.7	16

BATTERIES

Primary Cells

Each metal has what is termed its single potential, or the electromotive force of that metal when tested with a mercury or normal electrode. Elements having great differences of single potential, when used in batteries, will give a higher voltage than elements having single potentials which are nearly the same. If two plates, such as copper and zinc, are placed in an electrolyte of diluted sulphuric acid, the direction of the e.m.f. in the cell is from zinc to copper, and if these be electrically connected, hydrogen bubbles accumulate on the surface of the copper. This accumulation of hydrogen not only offers a resistance to the flow of current but also sets up a counter e.m.f. which may result in a lower terminal e.m.f. These two effects may eventually reduce the flow of current to practically nothing; the phenomenon is known as **polarization.**

Mechanical means of **preventing polarization,** such as agitation of the liquid, forcing air through it, and roughening of the plates, have all been tried, with varying degrees of success. Chemical means are used extensively at the present time, various chemicals which contain oxygen in loose combination being used to dispose of the hydrogen as it forms in the electrolyte. Soluble oxidizing agents extensively used are the bichromates of potassium and sodium. To confine the depolarizing agent to the vicinity of the electrode, a type of cell has been developed having a diaphragm (usually a porous cup of unglazed porcelain) which does not interfere with the passage of the electric current, but prevents the passage of electrolyte from one compartment to the other. Such cells give greater efficiency than those without the porcelain cup. The use of bichromates as depolarisers has given the name **bichromate cell** to a battery constructed of zinc and carbon with diluted H_2SO_4 as the electrolyte, which is extensively used for medical purposes. E.m.f., about 2.2 volts.

The **Leclanché cell** is a combination of zinc and carbon electrodes with sal ammoniac solution, and is used where only low values of current and intermittent service are desired. In the original type polarization was retarded by making the carbon electrode very large and porous. In the improved type a porous carbon cup electrode is used, in which are packed lumps of manganese dioxide to serve as an insoluble oxidizing agent. The rather high internal resistance has been reduced by casting or rolling a zinc cylinder which is placed as near the carbon as possible. The cell is set up by putting 3 or 4 oz. of sal ammoniac in the jar, filling it about one-third full with water, stirring until the salt is entirely dissolved, and then inserting the carbon cup and zinc. In the older type the carbon electrode is placed in the porous cup and packed around with manganese dioxide and crumbled carbon, the cup with its contents being then inserted in the jar with the zinc. The e.m.f. of this cell is about 1.4 volts.

Of electrochemical depolarizers, one common type, that of the ordinary **crowfoot cell** (gravity cell), is made up of electrodes of copper and zinc with zinc sulphate and copper sulphate solutions. In setting up this cell the copper electrode (−) is placed at the bottom of a jar, into which about 3 lb. of copper sulphate crystals are poured. The zinc electrode (+) is then suspended near the top of the jar and water poured in to cover it, a tablespoonful of sulphuric acid being added. To prevent "creeping," the electrolyte is then covered with a layer of pure mineral oil free from naphtha or acid and having a flash point above 400 deg. fahr., or the rim of the jar dipped in melted paraffin. When thus set up the cell should be short-circuited for a day or two to form zinc sulphate, which will protect the zinc electrode and also reduce the internal resistance. E.m.f., about 1 volt. Polarization is eliminated because copper is more easily deposited upon the copper electrode from the copper sulphate solution than is hydrogen. It is possible to draw current continuously from a cell of this type for a long period, or until the zinc sulphate solution reaches the copper electrode, and for this reason it is known as a **closed-circuit cell.** Closed-circuit batteries are best adapted to telegraph, railway signal and similar services, which demand a continuous draft of current. To recharge the cell, empty out one-half the solution, fill up with water, add more $CuSO_4$ (blue vitriol) crystals, and the cell is then ready for use. The zinc electrode gradually goes into solution and copper is deposited out on the copper electrode.

The **Edison primary cell** is suitable for either closed- or open-circuit service. The negative electrode is made of compressed oxide of copper, the surface of which is reduced to metallic copper to improve the conductivity, and also acts as a depolarizer. The positive electrode is a casting of pure zinc which is thoroughly amalgamated. The electrolyte is a solution of caustic soda. The surface of the solution is covered with a layer of heavy oil to keep it from evaporating. The cell has an initial e.m.f. of about 1 volt, which drops to about 0.7 volt when the circuit is closed. These cells are made in the following capacities:

Continuous capacity, amp.	1.5	2.5	4.0	6.0	7.0
Maximum capacity, amp.	7.49	9.53	15.51	26.68	33.35
Capacity, ampere-hours	100.00	150.00	300.00	500.00	600.00
Internal resistance, ohms	0.080	0.070	0.043	0.025	0.020

Batteries of the **open-circuit type** polarise quickly and are more suitable where only intermittent service is demanded, such as in gas-engine ignition, door-bell service, and a great variety of other uses.

Dry Batteries. The so-called dry battery is a development of the Leclanché battery and consists of a zinc container lined with paper to prevent the mixture of carbon and manganese, which surrounds the carbon electrode, from coming in contact with the zinc. The mixture is made active by impregnating with a solution of zinc chloride and sal ammoniac. Some types of cells are being made without a paper lining, but the core is held away from the zinc container. In the make-up of such a cell the space between the core and the container is filled with a heavy electrolyte. Commercial sizes range from those used in flash lights (1¼ in. or less in diam.) to the heavy-duty cells used for gas-engine ignition and small lighting sets (about 3 in. in diam.). Because of polarisation it is not possible to use a dry battery for service demanding a continuous draft of current.

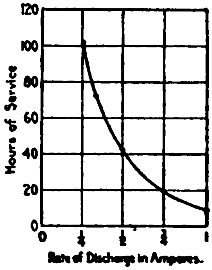


FIG. 30.—Discharge of Dry Cell.

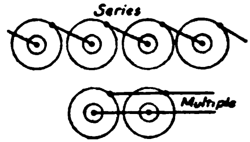


FIG. 31.—Cell Connections.

To get the **maximum efficiency** from a dry cell the discharge rate should be as low as possible. Fig. 30 shows the service and corresponding rates of discharge, and that the maximum output of energy is obtained when the rate of discharge is the lowest. Fig. 31 shows two **methods of connecting** cells. (For series-multiple connections, see p. 1610.) Dry cells can be **temporarily regenerated** by carefully puncturing the zinc and setting the cells in water, or, better, in a salt or sal ammoniac solution. If this water is heated the cell will revive more quickly.

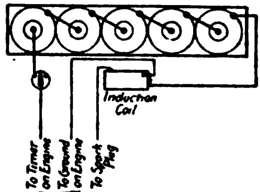


FIG. 32.—Connections of Dry Batteries for Gas-engine Ignition.

Fig. 32 is a diagram for connecting dry batteries in a **gas-engine ignition** system. As a rule, five cells are ample for such service, a greater number in series being likely to burn the platinum points on the coil. Almost all dry-battery ignition systems use 6 volts, which would theoretically require only four dry cells; in service, however, the voltage decreases to some extent, which makes it preferable to use five cells. Fig. 33 shows the dependence of **flash amperage** upon temperature, and indicates why difficulties are sometimes experienced in the use of dry cells in the winter.

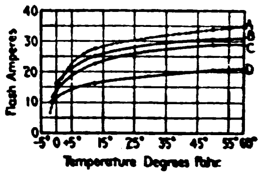


FIG. 33.—Flash-amperage Curves for Four Different Makes of Dry Cells.

Dry cells should be kept where it is cool but not damp. Heat dries out the cells. If space is available, use multiple connection so that the **life of the cells** will be increased and greater efficiency obtained.

Below are stated the elements, electrolytes and voltages of the more common primary cells.

Primary batteries			
Name	**Elements**	**Electrolyte**	**E.m.f., volts**
Daniell (or gravity)	Cu–Zn	$ZnSO_4 + CuSO_4$	1.07
Fuller	Cu–Zn	Dilute H_2SO_4 + $K_2Cr_2O_7$	2.1
Edison primary	Cu–Zn	Concentrated NaOH	0.7–0.95
Leclanché	C–Zn	NH_4Cl	1.4
Dry cell	C–Zn	NH_4Cl	1.0–1.8

All the preceding cells are termed **primary cells** because they are renewed by replacing the used-up elements of the cell.

Secondary Cells—Storage Batteries

In a **storage cell** electrical energy is commercially stored as chemical energy, but returns to electrical energy when the cell is connected to supply an external circuit. The desired voltage is obtained by connecting a sufficient number of cells in series, forming a **storage battery.** There are two general classes; the **lead-lead-acid cell,** and the **iron-nickel-alkaline cell.**

In the manufacture of the lead-lead-acid cells there are two general types of plates or electrodes. In the **Planté** type the active material is electrically formed of pure lead by repeated reversals of the charging current. In the **Faure** or pasted plate the active material is obtained by applying paste to supporting grids, lead peroxide to the positive and lead oxide to the negative plate.

The chemical reactions in a lead cell may be expressed by the following equation, based on the double sulphation theory:

$$\underset{\substack{\text{Positive}\\\text{plate}}}{PbO_2} + \underset{\substack{\text{Negative}\\\text{plate}}}{Pb} + \underset{\substack{\text{Electro-}\\\text{lyte}}}{2H_2SO_4} \underset{\text{Discharge}}{\overset{\text{Charge}}{\rightleftharpoons}} \underset{\substack{\text{Positive and}\\\text{negative plates}}}{2PbSO_4} + \underset{\substack{\text{Electro-}\\\text{lyte}}}{2H_2O}$$

Between the extremes of complete charge and discharge complex combinations of lead and sulphate are formed. After complete discharge a hard insoluble sulphate forms slowly on the plates and this is reducible only by slow charging. This sulphation is objectionable and should be avoided. The **weight of metallic lead actually required** to be reduced to sponge lead on the cathode plates to produce 1 amp-hr. of discharge at ordinary commercial rates is from 0.5 to 0.8 oz., and that converted into lead peroxide on the anode plates from 0.53 to 0.86 oz. These weights are from 4 to 6 times those theoretically required. The **quantity of electrolyte required** by a given cell may be computed from the formula $Q = (1290 - 10.53d)/(D - d)$, where Q = number of ounces (avoirdupois) of electrolyte per 100 amp-hr. of discharge, and $D(d)$ = percentage of sulphuric acid in the electrolyte at the beginning (end) of discharge. For other amounts, multiply Q by the actual ampere-hours and divide by 100.

Voltage. The voltage of a lead cell when fully charged and idle is 2.05 to 2.10 volts. Discharge lowers the voltage in proportion to the current flowing. Complete discharge is reached at 1.7 volts, at the normal discharge rate, fixed by the manufacturer. When charging at **constant current**

and normal rate, the voltage gradually increases from 2.2 to 2.35 volts then increases rapidly to between 2.5 and 2.7 volts. This latter interval is known as the gassing period and during this time the battery should be watched closely to avoid waste of power and unnecessary deterioration of the plates. The **constant potential** method of charging is to be preferred. A potential of 2.3 volts per cell is necessary. Under these conditions the above precautions are unnecessary, as the charging rate automatically drops as the cell voltage rises, and when the battery is entirely charged the current ceases to flow. Fig. 34 shows the change of voltage during charge and discharge. Fig. 35 shows the percentage of rated capacity available when lead batteries are charged under a constant potential of 2.3 volts per cell for various charging periods. Each curve relates to a different initial charge.

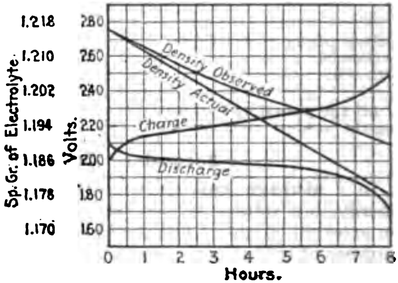


Fig. 34.—Change of Voltage of Lead Storage Cells During Charge and Discharge.

The **Edison storage cell** has a positive plate of iron and a negative plate of steel. The active material for the positive plate is nickel hydrate and for the negative plate iron oxide. The electrolyte is a 21 per cent. solution of potassium hydrate with a little lithium hydrate. Its initial e.m.f. is about 1.5 volts and its average e.m.f. about 1.1 volts throughout its discharge. On account of the high internal resistance of the cell the battery is not efficient from the energy standpoint, 60 per cent. being the efficiency usually attained in practice. It is compact and extremely light and strong, and for these reasons is particularly adapted for service in electric vehicles and for train-lighting systems. Its output is about 15 watthours per lb.; that of the lead storage cell ranges from 3 to 7 watthours for the Planté type to 8 to 14 watthours for the pasted or Faure type.

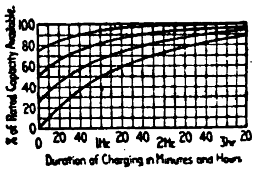


Fig. 35.—Available Capacity of Lead Storage Battery Charged at 2.3 Volts per Cell.

Hard rubber jars, glass-jars, and lead-lined tanks are generally used with lead cells. The first are generally used for portable and vehicle batteries, and the last two are used in power plants.

Installing a Lead Battery. Vehicle and portable batteries are generally shipped assembled (filled with the electrolyte) and ready for use. Plates for stationary batteries are usually received dry and the positive and negative plates separate. When installed they should be carefully handled and the separators placed carefully between the plates. Wood separators are shipped wet and should be kept wet until put in service. The jars should be set in trays filled with white bar sand, and these placed on glass or porcelain insulators. The battery room should be between 50 and 80 deg. fahr., fairly dry, and well ventilated to allow the explosive gases to escape.

When the plates are in place there should be enough space between the plates and the bottom of the jar to prevent the sediment from touching and short-circuiting the plates. The electrolyte is sulphuric acid (specific gravity, 1.21 at 70 deg. fahr.) and may be obtained by mixing one part of sulphuric acid of 1.84 specific gravity (oil of vitriol) to 4¼ parts of water by volume. The acid should be poured into the water, not *vice versa*. Distilled water is preferable, but ordinary tap water may often be used without harm being done. The **initial charge,** after installation, is from 40 to 60 hr. at the normal rate, and should continue 10 hr. after the specific gravity and voltage have ceased to rise. The cells should gas freely during this period. To take out of service for less than 9 months, charge once a month if possible, but if not, give a heavy overcharge before discontinuing service. To remove for a longer period, siphon off the acid (which may be used again) and fill with fresh water. Allow to stand 15 hr., and siphon off water. Remove and throw away the wood separators. The battery will now stand indefinitely. To put in commission again, install new separators, fill with acid (sp. gr. 1.210) and charge at normal rate 35 hr. or until gravity has ceased to rise over a period of 5 hr. Charge at a low rate a few hours longer.

Charging. Lead batteries should be charged at a normal 8-hr. rate until the voltage ceases to rise, and the charge continued from this point about half an hour longer. When fully charged the positive plate should have a chocolate-brown appearance and the negative a grayish color. If the positive plates have a whitish appearance, the sulphate has not been entirely reduced and it is advisable under such conditions to continue the charge at a low rate until the sulphate appearance has been entirely removed. To diminish **sulphation,** overcharging once a week for stationary cells and once every 2 months for portable or vehicle cells is recommended. Water, free from harmful substances, should be used to replace electrolyte lost through evaporation. Several **systems** are used **for charging:** the motor-generator, rheostat or lamp bank from a direct-current lighting system, electrolytic rectifier, mercury-arc rectifier and vibrator rectifier; in some cases for portable batteries a primary battery such as the crowfoot cell is used. With **portable batteries** the charging wires are likely to get confused, so that before charging it may be necessary to determine the polarity of the charging wires. If two wires from the generating system are placed in a container of salty water it will be noticed that one of the wires will generate gas more freely than the other. This is the negative wire and should be connected to the negative pole of the battery. By moistening a strip of litmus paper with salty water and holding the wires from the charging system close together on this strip of paper, an acid reaction develops a red spot under the positive terminal and an alkaline reaction under the other produces a blue spot. The **charging current** should be at the 8-hr. rate, which is from 5 to 10 amperes for the average portable cell. If, however, a more rapid charge is required, it is well to start off at twice the 8-hr. rate and charge until gassing develops freely. Then reduce to the 8-hr. rate and continue to charge until the cell has generated gas for half an hour. This high rate of charge is not as efficient as the lower rate.

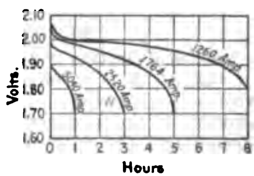


Fig. 36.—Output of Storage Battery at Various Rates of Discharge.

Rates of Charging and Discharging. Fig. 36 is a curve giving the **output at various rates of discharge,** which shows that as the rate of discharge is increased the output in ampere-hours or watthours is materially decreased.

It is desirable to use a battery of such size as to permit of the lower rates of discharge which result in higher efficiencies, but portable batteries have usually to sacrifice efficiency and durability in order to reduce weight and bulk. Portable batteries are made up of many thin plates whose thickness varies from ⅛ to ³⁄₁₆ in. The central-station type of battery (Planté) is developed primarily for long life. The plates are as a rule about ½ in. thick, and with proper care may give service for 10 years or even longer. To obtain maximum efficiency it is necessary to use low rates both of charging and of discharge. Cells fully charged up to 2.5 volts and discharged down to 1.75 volts have an **efficiency** varying from 75 to 80 per cent. according to the type of cell and the rates of charge and discharge. Under ordinary conditions the rates of charge and discharge should be from 6 to 10 amperes per sq. ft. of *positive* plate surface; the surface of one plate is measured as twice the product of height by breadth (omitting ribs and other irregularities of surface).

The **capacity** of a lead storage cell **varies with** the **rate of discharge**, substantially as follows:

Rate of discharge, hours......	8	5	3	1
Percentage of capacity at 8-hr. rate:				
Planté type...............	100	88	75	50
Pasted type...............	100	93	83	60

Care of Batteries. Some of the precautions to be observed in the care of storage batteries are as follows: An ammeter should not be used to test the condition of a cell; a battery should not be left to stand in a discharged condition; a naked flame should not be brought in the vicinity of a battery that is being charged; charging should not be done at a high rate, and the battery should not be allowed to get hot when charging; water should never be added to the concentrated acid—always acid to the water; acid should never be equalized except when the battery is in a charged condition; a battery should never be exposed to the influence of external heat; voltmeter tests should be made when the current is flowing; batteries should always be kept clean. To replace acid lost through slopping, it is well to use a solution of two parts of concentrated sulphuric acid in five parts of water by weight, unless a hydrometer is at hand to enable the solution to be made up according to the specifications of the makers of the cell.

Applications. In connection with central stations as a standby or to level the load peaks; floating on railway systems; electric vehicles, pleasure and commercial; storage battery cars (N. Y. City); automobiles, ignition, lighting and starting; submarines; gun-firing and sighting on battleships; wireless emergency; private lighting plants; telephone centrals; telegraph; fire-alarm; and in testing where a constant voltage is necessary.

ELECTRICAL IGNITION SYSTEMS

Electrical ignition systems utilize electromagnetic phenomena (see p. 1573). In Fig. 37, B is a source of current, N' a coil of wire and S a switch. The magnetic flux is represented by the arrowed lines. If switch S be opened and the current falls to zero, N' is surrounded by a magnetic field which is decreasing in value. Similarly, when S is closed, N' is surrounded by a rising magnetic field. In both cases a pressure of self-induction is generated. Different results occur in N' when S is closed and when it is opened. In making contact no current can flow until the switch is closed. The opposition to the rise of current, or the self-inductive pressure, and the resistance of the circuit are the only op-

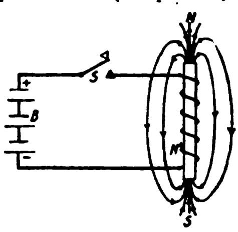


FIG. 37.—Inductance or Spark Coil.

posing forces, and the resistance is comparatively low. When S is opened, however, an air gap is introduced into the circuit, and, as air offers great resistance to the current flow, this resistance is many times greater than when switch S is closed. The current decreases very rapidly and with it the flux present in N'. This causes the generation of a high pressure sufficient to jump the small air gap formed as S opens and to establish an arc across the gap. The resistance of this arc is considerably less than that of air and the current flow will last for a short time after the switch gap is opened. Since the pressure generated in N' is proportional to the rate of change of the flux, and as the resistance of an air gap is practically proportional to the length, the arc established will be long if the gap be opened rapidly.

In closing, the source of power is B, which has a constant pressure. In opening, the energy for the arc at the gap must come almost entirely from the magnetic flux in the coil. A definite amount of energy is stored in the magnetic field, and is available at the gap when the switch is opened.

If an additional coil N'' be wound concentric with N', as in Fig. 38, any magnetic changes in one will occur also in the other. Hence, pressures may be developed in N'' independently from N' as the current is varied in N'. Since the pressures developed are dependent on the number of turns in the coil, any desired value may be generated by winding a sufficient number of turns on N''. If a sufficient number be wound, a pressure sufficient to jump an air gap such as a, b will be produced. Such a pressure is said to be of **high tension.** A coil wound as in Fig. 37 is called an **inductance or a spark coil.** A coil with a double winding as in Fig. 38 is called an **induction coil** or a **jump-spark coil.** The winding N' in an induction coil is called the **primary,** while the N'' winding is called the **secondary.** Whenever a spark jumps between **points,** the metal of which they are made is vaporized or burned. Unless the polarity of the battery or other source of power is alternated frequently, the wear is uneven, one point pitting and the other building up like a cone, resulting in variable action of the coils. Points should be kept flat with a very fine file or stone. The material for points that best resists wear and oxidation is an alloy of platinum and iridium, and this is used in high-grade apparatus. Nickel steel and other cheap alloys need renewal frequently and require considerable adjustment to keep them in good order. Tungsten contacts are now coming into use.

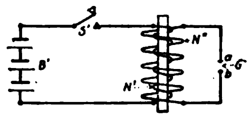


Fig. 38.—Induction or Jump-spark Coil.

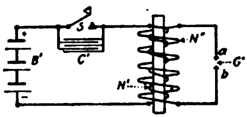


Fig. 39.—Induction Coil with Condenser.

To reduce the arcing at the contact points of the primary circuit, a condenser is shunted around the switch. A **condenser** consists of two conductors separated by some insulating material. It is usual to make it of a large number of sheets of a very thin metal like tinfoil with thin paraffined paper sheets separating them. Every other sheet of metal extends to one side and the balance to the other. By connecting all of one side to one terminal and the remainder to another, the condenser is complete. By connecting the condenser across switch S, Fig. 39, the energy of what would otherwise be an arc is absorbed. This prevents the contacts from being burned away rapidly and increases the induced voltage of the secondary by making a sharper break.

Electrical ignition systems are frequently operated upon low voltages, from primary batteries, storage batteries or small, low-pressure generators.

The power required is very small and on this account batteries are much used. The **primary voltage** is generally 6 volts. Series-multiple connections are much used, as shown in Fig. 40. Sometimes additional cross connections are made as shown in in Fig. 41, so that a loose connection in one series of batteries will not render the entire series useless.

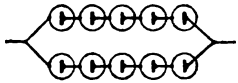


FIG. 40.—Series-multiple Connections.

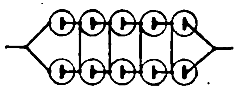


FIG. 41.—Series-multiple with Cross Connections.

The generators used are either **magnetos** or dynamos. A magneto is a generator having permanent magnets for the fields. Referring to Fig. 42, *N* and *S* are the permanent magnet poles. Lines of force pass from *N* to *S* and then through the iron back to *N*, as shown. A set of soft iron poles is attached to these two pole pieces. The fall of magnetic potential is the same by any path, so that with longer air gaps the flux density must be less.

Fig. 42 also shows a simple two-pole or shuttle armature made in the form of an *H* which is turned to fit the bore of the poles with a very small clearance. End plates are provided and a shaft extension added to enable the armature to rotate freely. The air reluctance is reduced to the short length of the air gap between the armature surface and the pole pieces. The total magnetic reluctance is therefore low. If a coil of wire now be wound on the core of the armature, as shown in Fig. 42, and one terminal be connected to the armature metal or **grounded** to the machine and the other be insulated and provided with a sliding connection to an insulated terminal, an elementary magneto results. In the position (*a*) of the armature the coil encloses all of the magnetism passing through the core of the armature. In position (*b*), the armature has rotated one-fourth of a revolution and the magnetic flux cannot now pass through the core and coil, but takes another path across the pole pieces of the armature, as indicated. The magnetic flux threading the armature coil is now zero. Rotating the armature another quarter turn brings it into the same position as in (*a*), with the exception that now the coil is reversed and the flux passes in full strength through the coil but in the opposite direction to that in (*a*). By rotating a second half revolution, the flux will fall to zero and then back to full value as before. In a complete revolution, the flux changes from full value to zero and back again twice. Since the pressure generated in a coil is proportional to the rate of change of flux, the pressure must also go through the same cycle. The cycle of pressure is alternating. A **dynamo** or generator differs from a magneto only in the use of a field excited by a direct current.

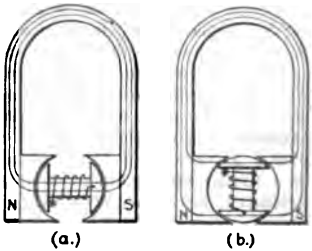


FIG. 42.—Simple Magneto.

The generator when driven by a variable-speed motor will naturally generate a pressure varying with the speed, since the pressure is dependent on the rate of change of flux through the armature. Some form of regulator is therefore required in this case to keep either the speed or the voltage constant. Some friction-driven generators make use of a centrifugal device in which the armature is disconnected the moment the speed, and therefore the voltage, rises above a certain fixed value. Such devices, unless very well made, are not very reliable. Electric clutches are used also and operated either by the load current or the pressure.

The most satisfactory systems are those in which the generator is constantly supplying the circuit with a current. This may charge a storage battery or furnish power for the electric-lighting circuit. A number of successful systems have been developed. The storage battery is kept constantly charged and this insures a steady pressure for operating the ignition system. A switch electrically or mechanically operated disconnects the generator when its voltage is below that of the battery or when the battery is receiving too much current. It is to be noted that any generator takes a certain amount of power to run it and this load is a constant one on the engine.

Electrical Ignition Systems may be classed under two general heads, **low-tension** and **high-tension**. In the former, there is only one circuit, and the device for making and breaking the circuit is applied to the cylinder so that the circuit is opened in the combustion chamber by mechanical or electromechanical means. The spark thus produced is the inductance spark. In the latter, a high-tension pressure is generated by apparatus outside of the engine and conducted to a fixed spark gap which is carefully insulated and screwed with its mounting into the combustion chamber.

Mechanically Operated Low-tension Systems. To produce a hot spark in breaking a circuit, it is necessary to make this break very rapidly. In low-tension systems this is accomplished almost entirely by a spring and cam escapement device. Fig. 43 illustrates the principle of all such apparatus. *K* is a cam rotating in synchronism with the engine shaft. As the cam revolves it moves a follower *F* which turns the shaft *R* in its bearing located in the cylinder wall. Attached to this shaft is the lever *L* which carries one of the sparking points *P*. As the cam advances further, *P* touches *P′* and closes the circuit. *F* is then suddenly pulled by the spring *M* as *F* runs over the cam lip, *P* is as suddenly drawn away from *P′*, and the spark produced. *P′* is located on an insulated stem which is rigidly fastened into the cylinder wall.

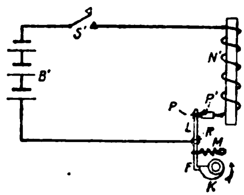


FIG. 43.—Mechanically Operated Low-tension System.

Electrically Operated Low-tension Systems. Fig. 44 shows a system in which the additional device *T* is installed for closing the circuit outside of the cylinder. The points *P* and *P′* are normally held in contact by the spring *M*, and when *T* completes the circuit *N′* attracts an armature *F* which causes the points to separate and draw out the arc. The timer *T* must break contact before *P* and *P′* come together or the action will be repeated. This system admits of ready adjustment of the timing of the spark, but the use of an electromagnet to operate the moving parts in the tignt-ntting bearing through the cylinder wall requires more power than can be furnished by primary batteries advantageously. In the magnetic spark-plug system the timer operates a small electromagnet in the plug, which in turn operates the contact points.

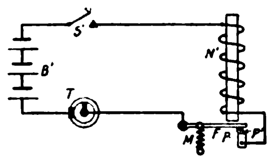


FIG. 44.—Electrically Operated Low-tension System.

Low-tension System Using Magneto. A magneto geared positively to the engine at the proper speed relation can be used as a source of power in a low-tension system, but for high-speed work its use is difficult. The magneto has an appreciable inductance, and if wound properly does not need a spark coil.

High-tension Systems are more numerous and more complex than the low-tension systems. There are, however, only a few basic types. The simplest system is that of the **single break** of the **DeDion** type. **Fig.** 45 shows this mechanism. A cam K, rotating in synchronism with the engine shaft, strikes the fixed spring strip V and forces the contacts P, P' together, after which P is allowed to snap away from P' and thus break the circuit. The high-tension spark produced at a, b is within the cylinder.

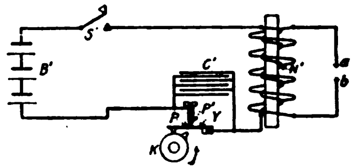


FIG. 45.—Single-break High-tension System.

Fig. 46 shows a **vibrator coil system.** At one end of the coil is placed a spring of steel having an iron head V. One end is firmly fixed while the free or head end may be attracted by the core of the coil. On the spring or vibrator is placed one of the contact points P, while opposite it the other point P' is held by an adjustable screw. When the timer T makes contact the core of the coil attracts the vibrator and pulls the contact points apart, thus breaking the circuit quickly. This induces in the secondary of the coil a high pressure, which can jump the spark gap in the plug. As soon as the circuit is interrupted the core loses its magnetism and the spring returns into contact with P. This re-establishes the current in the coil and the process is repeated. The speed of the vibrator depends on its stiffness; usually it is from 150 to 250 per sec., but in some coils as high a speed as 500 per sec. is attained. At low speed with a given vibrator it is evident that the timer makes a longer contact than at high speed and that the number of vibrations per timer contact decreases with increasing speed of engine. Hence, at low speed a large number of sparks are produced in the plug at each contact, while at very high speed it may reduce even to one. If there is not a sufficient rest between timer contacts for the vibrator to resume its normal state of rest, the timer may make contact before or after it has come into contact with the fixed point; this will disturb its natural period and will result in giving irregular sparks at the plug.

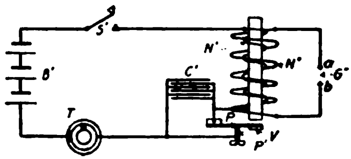


FIG. 46.—Vibrator-coil High-tension System.

If dry cells are used for vibrator coils, each cell should be first tested on short-circuit to see if all cells are uniform, and then all in series should be tested with the coil in circuit with its vibrator screwed down so that it will not vibrate. If the cells then do not test at least 4 to 5 amperes they should be discarded. A vibrator coil is more efficient with a high voltage supply than with low voltage, because the rapidity of the break in the former case is faster than in the latter. Hence, a coil testing ½ ampere on four new cells is delivering more spark energy than the same coil testing the same with four low cells. A slow-speed vibrator will deliver a greater power per spark than a high-speed vibrator when both are tested to the same ampere reading.

High-tension Magneto Systems. A magneto may be substituted for the battery in Fig. 45 if its motion is properly synchronised with that of the timer. A simpler and more effective arrangement of the high-tension magneto with separate coil is shown in Fig. 47. Here the timer remains closed or the circuit is short-circuited until a point is reached where the current flowing is a maximum. Then the contacts are suddenly knocked apart by the cam on the armature shaft. The armature coil will be linked with a strong magnetic flux at this moment. Now the timer, instead of being in the primary circuit of the coil, acts as a by-path or shunt to it, and, as its resistance is very low, nearly all of the current flows through it. When the cam opens the contact at *T*, the current is diverted suddenly through *N′* and a high pressure is generated in *N″*, producing the spark at the plug. If used on a constant-speed motor the operation is satisfactory, but when used with a variable-speed motor, such as an automobile engine, the change of speed makes the current vary. It is difficult to start an engine on a small magneto. The purely high-tension magneto is one which differs from that just described in that it has both the low-pressure coil, or primary, and the secondary on the same armature. Both coils link with the same magnetic circuit and therefore it becomes an induction coil. Fig. 48 shows a pure high-tension magneto. The two windings are shown with one terminal grounded to the machine. The smaller coil of few turns and heavy wire is short-circuited by *T* until the proper moment, when it is opened and *G″* gets the spark.

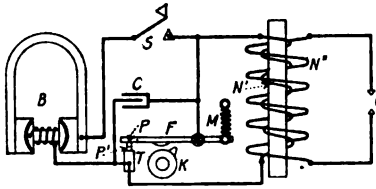


Fig. 47.

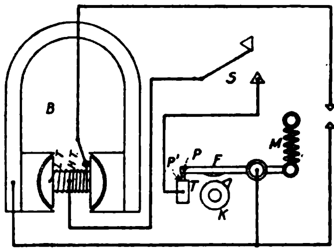


Fig. 48.
High-tension Magneto Systems.

Spark Plugs should be adjusted so that the gap is approximately 1/16 in. in length. A smaller gap will take less pressure, but this may allow carbon or dirt to bridge the points and thus short-circuit the plugs. A gap that is too large will not work at all when the spark is weak.

Voltage. The voltage required to jump a given gap in the cylinder varies with conditions of temperature and atmospheric pressure. In air it varies almost directly as the absolute atmospheric pressure. Since gas-air mixtures are mostly air, this holds practically true for the charge in an engine. Fig. 49 shows tests made for different pressures in compressed air. As the temperature rises the voltage drops.

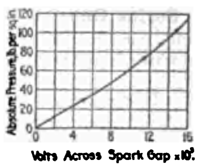


Fig. 49.—Spark Voltage Curve.

All ignition apparatus is designed to run on certain voltages. If a higher voltage is used better sparks may be produced at the expense of greater wear or damage to the apparatus. The condenser applied to the vibrator coils is adjusted for minimum sparking. If a high voltage is used more current flows, the sparking increases, or the condenser is constantly overloaded and finally breaks down.

GENERATORS MOTORS, TRANSFORMERS

General Considerations. The essential elements of a generator or motor are a magnetic field and conductors so arranged that they cut the magnetic lines of force. Either the field or the armature may rotate, or, as in some alternating-current generators, the equivalent of conductors cutting the magnetic field is accomplished by moving inductors which vary the amount of magnetic flux linking with the armature conductors. In alternating-current generators the armature conductors alternately cut lines of magnetic force emanating from a north and a south pole so that the induced e.m.f. reverses in direction as frequently as the armature coils pass by the field poles. In direct-current generators (except the unipolar type), an alternating e.m.f. is generated, but the commutator and brushes provided with this type of machine rectify the alternating e.m.f.'s to produce a unidirectional e.m.f. No fundamental differences exist between generators and motors. Direct-current dynamos may be separately excited, or the very small sizes may have permanent magnet fields, as in the case of magneto generators. The usual types are the series, shunt and the compound-wound machines, these terms indicating the mode of field excitation. The **fundamental equation** which expresses the relation between the generated e.m.f. in volts E_a, the total magnetic flux Φ cut by the armature conductors, the speed in r.p.m. and the number Z of armature conductors in series between brushes, is $E_a = \Phi NZ/(10^8 \times 60)$. The terminal volts $E = E_a - I_aR_a$, where I_a and R_a are the armature current and resistance, respectively. Under no load the generated e.m.f. appears at the terminals, but, under load, armature resistance and armature reactions generally make the terminal e.m.f. somewhat less than that indicated by the equation. A study of the fundamental equation enables one to predetermine much concerning the performance of a generator or motor under various conditions of service. Except in the case of arc-lighting circuits, electrical distribution systems are constant-voltage systems with the receiving units in parallel. The standard voltages commonly employed for direct-current distributions are 110, 220, 440, 550 and 1200 volts, the latter two chiefly for electric railway work. By **voltage regulation** is meant the per cent. variation in voltage from full load to no load at constant speed expressed in terms of the normal full-load voltage; speed regulation of motors is expressed in a similar manner.

Direct-current Generators

Series Generator. In the series generator (Fig. 50) the entire load current passes through the field winding, which consists of relatively few turns of wire of sufficient size to carry the entire load current without undue heating.

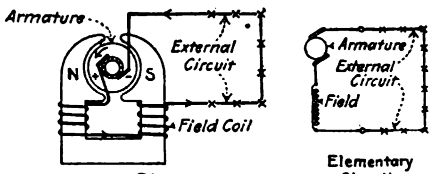
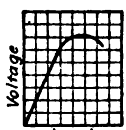


Diagram. Elementary Circuit.



Load Characteristic Curve.

FIG. 50. FIG. 51.

Series D.C. Generator and Its External Characteristic.

The field excitation, and hence the terminal e.m.f., depends upon the magnitude of the load current. An external characteristic showing the relation between load current and terminal e.m.f. is given in Fig. 51. Series generators now find no extended application, but were much used a few years ago to operate series arc-light systems.

Shunt Generator. The connections of a shunt generator are shown in Fig. 52. This machine maintains approximately constant terminal e.m.f.

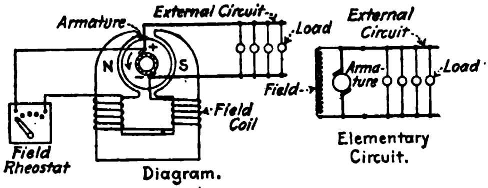


FIG. 52.

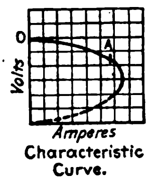


FIG. 53.

Shunt D.C. Generator and Its External Characteristic.

at all loads and is widely used. An external characteristic of a shunt generator is given in Fig. 53. Due to armature resistance and armature reactions the terminal e.m.f. decreases with increasing load. At point *A* the generator commences to "break down." Further attempts to apply load result in an abrupt decrease of voltage and a decrease of current even to short-circuit. *OA* is the operating portion of the characteristic. The breakdown point of large machines is many times their rated load. The field rheostat permits the field current to be adjusted so that constant terminal e.m.f. may be maintained.

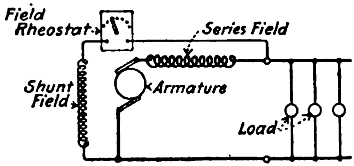


FIG. 54.—Compound-wound D.C. Generator.

Compound-wound Generators (Fig. 54). By the addition of a series winding to a shunt generator the terminal e.m.f. may be automatically maintained constant, or, by properly proportioning the series coil, the terminal e.m.f. may be made to increase with load to compensate also for loss of voltage in the line, so that approximately **constant voltage** is maintained at the load. Compound-wound generators are chiefly used for small isolated plants and for generators supplying a purely motor load subject to rapid fluctuations as in railway work. There is no advantage in using compound generators in central stations supplying lighting load, for the reason that either automatic regulators or manual control must be used to keep the voltage constant at the various centers of distribution, each supplied by separate feeders. Fig. 55 gives the characteristics of a 200-kw. compound-wound generator.

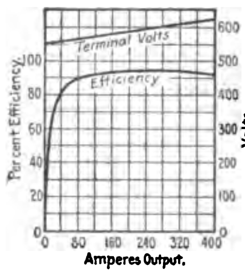


FIG. 55.—Characteristics of a 200-kw. Compound-wound D.C. Generator.

Parallel Operation of Direct-current Generators. To operate shunt generators in parallel it is necessary to see that the switches are so connected that like poles are connected to the same bus bars when the switches are closed. Suppose one generator to be operating; to connect another generator in parallel with it, the incoming machine is first brought up to speed and its terminal pressure adjusted to a value slightly higher than the bus-bar voltage, when they may be connected together without difficulty. The proper division of load between them is adjusted by the field rheostats and will be maintained automatically if the machines have similar voltage regulation **characteristics.** Shunt machines have drooping voltage characteristics and parallel operation is therefore stable. Compound-wound direct-current generators have flat characteristics or frequently rising voltage characteristics. To operate them successfully in parallel an equalizer connection must be provided, as shown in Fig. 56, connecting the generators at the junctions of the armature and series field leads. This connection must be of low resistance so that any increase of load current demanded of the machines will be proportionally divided between the series windings of the two machines. The equalizer switch should be closed first and opened last, if possible. In practice, the equalizer switch is often one blade of a 3-pole switch, the other two being the bus switch, as in Fig. 56. When compound generators are used on a 3-wire system, two series fields—one at each armature terminal and two equalizers, are necessary. It is equally feasible to operate three or more compound generators in parallel in the same way. The average values of **efficiencies** of compound-wound direct-current commutating-pole generators given in Table 19 are fairly representative of modern practice.

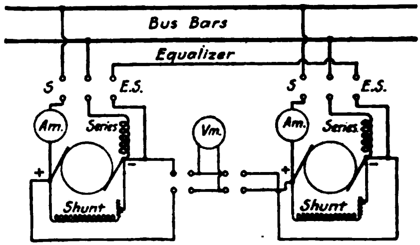


FIG. 56.—Connections for Compound-wound D.C. Generators Operating in Parallel.

Table 19. Approximate Data on Standard Compound-wound Direct-current Commutating-pole Generators

(Croft's "Am. Electrician's Handbook")

Kilowatts capacity	Efficiency			Kilowatts capacity	Efficiency		
	½ load	¾ load	Full-load		½ load	¾ load	Full-load
5	77.0	81.0	82.5	100	89.0	90.5	91.0
10	82.0	85.0	86.0	125	90.5	91.0	91.0
15	82.5	86.5	86.5	150	90.5	91.3	91.5
20	84.0	86.5	87.5	200	91.0	91.5	92.0
25	85.0	88.0	89.0	300	91.3	91.8	92.0
35	87.0	89.0	89.5	400	91.8	92.3	92.5
50	88.0	89.5	90.5	500	91.8	92.2	92.5
60	88.5	90.5	91.0	750	92.0	92.3	92.5
75	88.5	90.5	91.0	1,000	92.5	93.0	93.5
90	88.5	90.5	91.0				

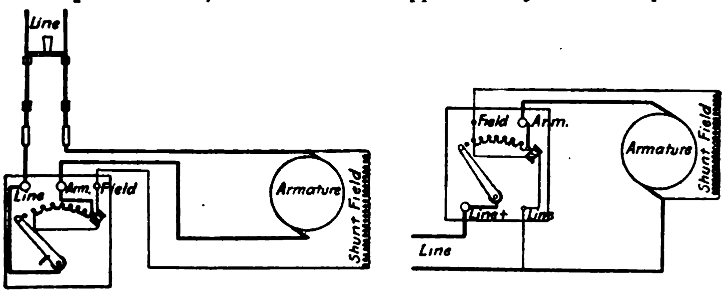
Direct-current Motors

A direct-current motor may be regarded as a **counter-voltage generator.** ts **fundamental** e.m.f. equation is $E = e + I_a R_a$, where E is the e.m.f.

impressed on the armature, e the counter e.m.f. generated by the armature, I_a the armature current and R_a its resistance. Let Z be the total number of conductors on the armature (of a 2-pole motor), Φ the total flux through the armature in c.g.s. lines, and N the speed of armature in rev. per. sec. Then $e = Z\Phi N/10^8$, or $N = e \times 10^8/Z\Phi$, which latter is the **fundamental speed equation** of the direct-current motor. The **torque** of a motor is its turning moment and is generally expressed in lb.-ft., or the number of pounds of effort exerted at a radius of 1 ft. It is proportional to the product of Φ and I_a and its value may be determined as follows: Let T = torque at the shaft in lb.-ft., and P the power in watts delivered to the shaft. Then $P = I_a e = 2\pi NT \times 1.356$, since 1.356 watts ($= 746/550$) are equivalent to 1 ft.-lb. per sec. In a machine with constant flux e is directly proportional to the speed and is equal to $\Phi ZN \times 10^{-8}$, whence $\Phi ZI_a = 2\pi T \times 1.356 \times 10^8$, or T (in lb.-ft.) $= \Phi ZI_a/(8.52 \times 10^8)$, which is the **fundamental torque equation** of the direct-current motor. Also, h.p. $= T \times$ r.p.m. $\times 2\pi/33{,}000 = I_a e/746$, whence $T = 7.04eI_a/$(r.p.m.).

Series Motor. The series motor is **adapted to variable-speed work** such as railway, crane and elevator service, on account of its speed-torque characteristic. Under heavy load the fields are strongly excited so that the **torque,** which is proportional to the product of the field flux and the armature current, is large. Assuming the fields are not saturated, the torque is proportional to the square of the motor current. Under heavy load (and hence strong field flux) the speed is low. The series motor develops almost any torque demanded of it, but with a reduction in speed, so that it is especially adapted to hard work where constant speed is not essential. On light loads it speeds up and may easily attain an unsafe speed if not controlled. For starting and speed control a series resistance is used, which results in loss of power as heat. In railway work, where two motors at least are used for each car, the series-parallel system of speed control is used. (See Speed Control of Series Motors, p. 1637.)

Shunt Motor. The shunt motor is almost universally **used for constant-speed service,** for it maintains approximately constant speed re-



(a) (b.)

FIG. 57.—Connections for Shunt D.C. Motors.

gardless of the load. The speed is such that the counter e.m.f. equals the impressed e.m.f. less the drop of voltage IR due to armature resistance. Methods of connecting a starting rheostat are shown in Fig. 57(a) and 57(b). The fields are first fully excited and then current, controlled by a resistance, is allowed to pass through the armature circuit. The armature resistance is permitted to remain in circuit only during the process of starting. The starting lever is held, against the force of a spring, in the running position by an electromagnet in series with the field circuit, so that if the field circuit is interrupted or the line voltage becomes too low, the lever is released and the armature circuit is opened automatically. The electromagnet is sometimes connected directly across the line, as shown in Fig. 57(*b*), in the four-point starting box. In this type the arm is released instantly upon failure of the line voltage, when in the other some time elapses before the field current drops enough to effect the release. Some **starting rheostats** are provided with an overload device so that the circuit is automatically interrupted if an unsafe current is demanded by the armature. The speed of a shunt motor decreases somewhat with load, because the larger load current causes a greater armature *IR* drop. This effect of armature resistance is somewhat diminished by the effect of armature reactions, as explained later. **Speed control** by means of armature resistance is not satisfactory because it involves a considerable loss of power, and the speed varies with the load because of the variation of the *IR* voltage drop across the resistance, resulting in a variable voltage applied to the armature. In the ordinary shunt motor the speed may be controlled to a limited extent by means of a shunt-field rheostat. Weakening the field increases the speed of the motor, for the decreased field flux means that the speed must increase to develop a counter e.m.f. equal to the applied armature e.m.f. This method of control is limited in range for ordinary shunt motors, as the decreased field flux increases commutation difficulties. Motors for adjustable-speed service are generally built with commutating poles. Ranges as high as 5 to 1 are obtained in many instances.

Compound Motors. A cumulative compound winding of a generator becomes a differential compound winding when the machine is used as a

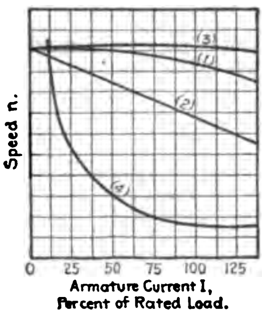


Fig. 58.

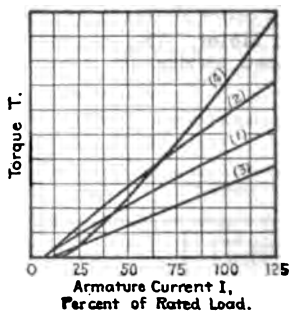


Fig. 59.

Speed and Torque Characteristics of Direct-current Motors.

(1) Shunt motor; (2) compound motor; (3) differential compound motor; (4) series motor.

motor. The differential motor, however, is not ordinarily used except to drive certain textile machines. However, its speed may be made more nearly constant than that of a shunt motor, or, if desired, it may be adjusted to increase in speed with increasing load. Cumulative compound motors have very poor speed regulation, since the field strength is increased with increasing load, and they are used where constant speed is not important and where larger starting torque is desired than would be developed by a shunt motor. Cumulative motors are used for elevators, to drive punches, etc., where a large instantaneous torque is necessary. Figs. 58 and 59 give **speed and torque characteristics** of the different types of direct-current motors.

Commutation. The proper position for the brushes upon the commutator is at the point where the current in the coil under the brushes is passing through zero. In a generator the brushes are given a slight lead in the direction of rotation to obtain sparkless or "black" commutation. In a motor the brushes are given a negative lead, that is, against the direction of rotation. Reversible motors must have the brushes placed in the theoretical neutral plane, so that commutation difficulties are especially difficult to overcome in such motors. The armature current sets up a magnetomotive force proportional to its strength. In shunt motors the armature current sets up a magnetomotive force which decreases the field flux and consequently reduces the decrease of speed with load. The proper position of the brushes then varies with the armature current or load. This undesirable result may be minimized in several days. Motors for adjustable-speed service may be improved by the addition of an **interpole** or commutating pole. In the ordinary motor the field flux performs two functions, that of reacting with the armature current to produce torque, and that of reversing the current in the short-circuited coil under the brush. In the **commutating-pole motor** this latter function is accomplished by auxiliary poles placed between the regular poles. These are excited by a few turns in series with the armature circuit so that their strength is proportional to the motor load. The main field flux strength may be altered without affecting the commutation, consequently such motors are better adapted for adjustable-speed service and counter e.m.f. control by varying the field strength. The development of the commutating pole has made it possible to design generators (see Table 19) and motors for 1200 volts and higher. The same device is also used to some extent in alternating-current commutating machines.

Table 20. Amperes per Motor at Different Voltages for Several Sizes of Motors at Efficiencies Obtained in Ordinary Practice

H.p.	Per cent. efficiency	Watts	Volts 110	Volts 220	Volts 550	H.p.	Per cent. efficiency	Watts	Volts 110	Volts 220	Volts 550
			Amperes						Amperes		
¾	65	860	7.82	3.91	1.56	25	90	20,722	188	94.2	37.7
1	65	1,148	10.4	5.22	2.09	30	90	24,867	226	113.0	45.2
2	65	2,295	20.8	10.4	4.17	40	90	33,155	301	151.0	60.3
2½	75	2,487	22.6	11.3	4.52	50	90	41,444	377	188.0	75.4
3½	75	3,480	31.6	15.8	6.33	70	90	58,022	528	264.0	106.0
5	80	4,662	42.4	21.2	8.48	90	90	74,600	678	339.0	136.0
7½	80	6,994	63.6	31.8	12.7	100	93	80,215	729	365.0	146.0
10	85	8,776	79.8	39.9	16.0	125	93	100,269	912	456.0	182.0
15	85	13,165	120.0	59.8	23.9	150	93	120,323	1,094	547.0	219.0
20	90	16,578	151.0	75.4	30.1						

In many instances commutation difficulties are encountered even in commutating-pole machines. The commutating poles may be connected incorrectly. In a motor, passing from a N main pole in the direction of rotation of the armature, a N interpole should be encountered, as shown in Fig. 60. In a generator under these conditions a S pole should be encountered. The test may be easily made with a compass. If poor commutation is caused by too strong interpoles, the winding may be shunted. If the poles are too weak they may be strengthened by inserting sheet-iron shims between the pole and the yoke.

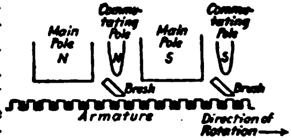


Fig. 60.

Alternating-current Generators

Alternating-current Generator. The usual type of alternating-current generator consists of a separately excited field winding mounted upon revolving field poles, and a stationary armature with coils embedded in slots. The direct current for excitation is carried to the revolving fields by means of slip rings and brushes. The earlier generators, were made with revolving armatures, following the design of direct-current generators, but alternators are generally built for higher voltages, so that the revolving-field type possesses the advantages that the armature insulation problem is more easily met and no slip rings are necessary to conduct the current from the armature. The armature and generally the fields are laminated to reduce eddy-current losses. The armature may have a wave winding or a coil winding. In **single-phase machines** one set of armature coils is provided, in **two-phase machines** two sets of armature coils are placed on the armature and spaced 90 electrical degrees apart, and in **three-phase machines** three sets are spaced 120 deg. apart and may be connected either in delta or in Y. The standard wave shape is the sine curve. With a single turn the wave shape would be that of the magnetic flux distribution under the poles, but where the conductors are in a belt, the voltage wave may differ materially from the flux wave. The designer may control the wave form by varying the flux distribution, by varying the distribution of the turns, by concentrating in two slots or distributing them among several slots. The standard voltages are 1100, 2200, and 6600, and machines can be built to deliver as high as 20,000 volts; however, the successful insulation of such machines imposes difficulties and added cost, which do not often justify their use. In connection with a transmission line of moderate length the elimination of step-up transformers may make the use of high-voltage generators desirable. The **frequency**, expressed in cycles per sec., is always the product of the number of pairs of poles and the number of revolutions per sec. The standard frequencies were formerly 125 and 133, and some of these machines are still in use, but the standards are now 60 for combined lighting and power service and 25 for power. The cost of apparatus is greater for the lower frequencies, but the operation of induction motors, synchronous apparatus and commutating motors is much more satisfactory at low frequencies. An additional reason for using low frequencies is that voltage regulation of machines and transmission lines is better. It is customary to use the **kilovolt-ampere** (kva.) as the unit in rating the capacity of generators rather than the kilowatt, for the reason that the heating of the machine is determined by the value of the current regardless of what the power factor of the load may be.

Fundamental Equation. If the useful flux per pole linking with the armature conductors is Φ and there are N conductors in series, the **average generated e.m.f.** in volts with a frequency of f will be $E = 2\Phi N f \times 10^{-8}$, since for each cycle 2Φ lines of force are cut by each armature conductor.

The effective value of the generated e.m.f. is $E = k \times 2\Phi Nf \times 10^{-8}$, where k is the form factor of the wave of e.m.f. and also of the flux distribution curve. It is assumed that there is no differential action, that is, that the armature conductors are concentrated so as to pass similar points of the flux curve simultaneously so that the e.m.f.'s generated in the armature conductors are all in phase. If this is not the case, a new constant k' must be introduced into the equation to express the magnitude of the differential action. The relation then is $E = k'k \times 2\Phi Nf \times 10^{-8}$.

Inductor Alternators. Especially before alternators were built with revolving fields and when higher frequencies were standard, certain advantages pertained to inductor alternators in which neither armature conductors nor field windings revolved. The flux linking with the armature conductors was varied by masses of iron passing the poles and armature so as to increase and decrease alternately the magnetic reluctance in the path of the flux. At the lower frequencies now used they do not seem to possess advantages over the revolving-field type and are not generally used.

Merits of Single-phase versus Polyphase Generators. In a single-phase machine not more than half of the armature can be utilized for armature conductors on account of differential action. In two- and three-phase machines the entire armature may be utilized, consequently a single-phase generator has not more than about 66 per cent. of the capacity of a polyphase machine using the same frame. It is possible to generate as polyphase and transform to single-phase power. Single-phase current may be taken from a three-phase generator either by simply using one phase, or the three-phase windings may be connected in series. In either case the phases will be equally unbalanced and no advantage is to be gained by the latter method. Three-phase generators are frequently used to supply three separate single-phase circuits, so that advantage may generally be taken of the greater weight-efficiency of polyphase generators even where single-phase power is desired.

Regulation. The factors entering into the voltage regulation of an alternating-current generator are the armature resistance, the armature leakage reactance, and the armature reactions. The leakage reactance is due to the lines of force generated by the armature current which link with the armature conductors only. The armature reactions are the effects produced by the magnetomotive force of the armature ampere-turns upon the field flux. The effect of the component of current ($I \cos \phi$) in phase with the generated e.m.f. is to strengthen the trailing pole tip and to weaken the leading pole tip of a generator. The effect of the quadrature component of current ($I \sin \phi$) is to oppose the field m.m.f. in the case of an inductive load and to aid it in the case of a leading current. These various reactions are represented in terms of e.m.f.'s in Fig. 61, where OA, AB, BC and CD are respectively the voltages consumed by load resistance, reactance, armature resistance and leakage reactance; EF, FD and ED respectively represent the effects on the voltage of the demagnetizing effect of armature, the cross-magnetization of armature ampere-turns and the combined effect of armature ampere-turns; and CE and BE are the voltages respectively consumed by synchronous reactance and by synchronous impedance. OB is the terminal voltage of the machine and OE is the voltage on open circuit with the same excitation. The open-circuit voltage OE may be determined trigonometrically as follows:

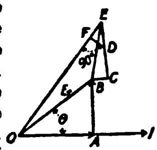


Fig. 61.

$$OE = \sqrt{(E_t \cos \theta + IR)^2 + (E_t \sin \theta + IX_s)^2}$$

where E_t = terminal volts = OB; θ = angle between load current and voltage; I = load current; R = effective armature resistance; X_s = synchronous reactance.

$$\text{Percentage regulation} = \frac{OE - E_t}{E_t} 100$$

The term **synchronous impedance** is a fictitious but useful one by which the combined effect of the armature resistance, reactance and armature reactions may be expressed. The synchronous impedance is not quite constant for all loads, but is frequently assumed to be so without serious error. The method of measuring it is to short-circuit the armature through an ammeter and then gradually increase the field excitation from zero until full-load current is indicated by the ammeter. A reading of the terminal voltage of the machine with the armature open, and with the same field excitation, is then taken. The assumption is made that the e.m.f. measured upon open circuit is that consumed by synchronous impedance when the current I flows, consequently impedance Z equals the ratio E/I. The synchronous reactance $X_s = \sqrt{Z^2 - R^2}$, but R is usually so small that $Z = X_s$ nearly. Due to slot iron losses the apparent alternating-current or effective armature resistance is from 1.2 to 1.6 times the resistance measured with direct current. The demagnetizing effect of the armature current may be somewhat different under the conditions described for determining synchronous impedance than it would be with the fields fully excited, especially if the magnetic density is above the knee of the magnetization curve under normal excitation. As a result, determinations of alternator regulation by this method may be unfavorable. An inspection of Fig. 61 shows that with a fixed armature current the regulation is much poorer when the current lags than when it is in phase with the terminal volts. A leading current may produce 0 regulation or even negative (no-load volts less than rated-load volts). The magnetomotive force method of determining regulation is as follows: Add the IR drop vectorially to the rated terminal voltage E_t. From the saturation curve look up field current I' (Fig. 62) corresponding to E'. I_2 is the field current necessary to send rated current through the armature under short-circuit conditions. Add I_2 and I' vectorially as shown, obtaining current I_0. The voltage corresponding to I_0 is the open-circuit voltage, E'.

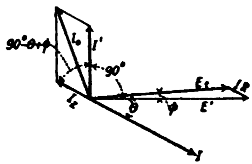


FIG. 62.

$$\text{Percentage regulation} = \frac{E' - E_t}{E_t} 100$$

The angle ϕ is usually negligible and can be assumed equal to 0. At unity power factor, $I_0 = \sqrt{I'^2 + I_2^2}$. This method yields results which are better than the actual regulation. The actual regulation lies somewhere between the values obtained with the synchronous impedance and the magnetomotive force methods. **Voltage regulators** have been perfected which maintain constant voltage satisfactorily under all conditions of load. In the **Tirrill regulator** a voltage relay operates to vary the resistance in the field of the exciter so that constant voltage is maintained at the switchboard. Similar regulators are also applied to direct-current generators.

Parallel Operation of Alternators. Successful parallel operation of alternating-current generators demands that the generators so coupled shall have similar characteristics; unless they have very nearly similar voltage wave forms, a prohibitive amount of current will circulate between the machines, causing needless heating and thereby reducing their useful output. For parallel operation, too close voltage regulation is not desirable, for, although close regulation may mean low armature impedance and therefore greater synchronizing force, there is greater danger of an excessive rush of current at the instant of synchronizing them and greater liability of excessive current if one machine is thrown out of step. The armature impedance should be large enough to prevent a dangerous current flowing when it is short-circuited across the bus bars. A machine having 8 per cent. regulation has sufficiently close voltage regulation and at the same time is rugged enough to insure immunity from burnout in case it falls out of step. Especially in the case of large generators there is an advantage in using machines of low armature impedance and inserting an air-core impedance in the external circuit to limit the short-circuit current, because the magnetic circuit is sluggish and armature reactions are slow to act to limit the short-circuit current. Successful parallel operation is as much a function of the prime-mover characteristics as of the generators. Since the generators, and consequently the prime movers, are in step, the division of load among them cannot be controlled by the field rheostats, as is the case with direct-current generators. The machine ammeters apparently indicate a shifting of the load when the field excitation is varied, because the wattless interchange current is changed, but the wattmeters indicate no appreciable change of load. The division of the load depends upon the adjustment of the engine governors. If the speed-load characteristics of the prime movers are identical, the machines will assume their proper proportion of the total load as it varies. If the speed-load characteristics differ as shown in Fig. 63, each will take the load indicated by the intersection of its characteristic with a horizontal line representing the speed. Very close speed regulation is not desirable on account of the greater difficulty of adjusting the governors to insure proper division of load, and 5 per cent. regulation is commonly used. The governors should be adjusted for proper division of load under full-load conditions, because unequal loading is not detrimental at partial loads. A more important requirement of the prime movers is that the angular velocity be uniform, otherwise the machines will alternately lead each other during a revolution, and if the natural period of mechanical oscillation of the revolving elements corresponds with the forced oscillations, troublesome hunting or mechanical resonance ensues. Water and steam turbines are inherently better adapted to parallel operation than reciprocating engines, on account of their uniform angular velocity. For successful parallel operation it is recommended that the rotating element shall not be allowed to deviate by more than one-sixtieth of a pole pitch or three electrical degrees from the mean position it would occupy if the angular velocity were exactly uniform. It is common practice to provide alternating-current machines with dampers or "amortisseur" windings to minimize the tendency to hunt. These consist of grids of conducting material surrounding or embedded in the pole pieces, so that any change of field flux, which must accompany hunting, will set up eddy currents in the dampers and oppose the action which induces them.

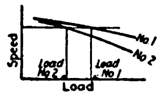


FIG. 63.

Stability in parallel operation results from having the leading machine always more heavily loaded than the lagging machine. Stable operation will result so long as the effect of the synchronizing current to load the leading machine is greater than that of the load current to load the lagging machine. On heavy inductive loads the machines may be thrown out of step. Capacity loads, on the other hand, tend to improve the stability.

Table 21. Approximate Performance Values of Alternating-current Generators

(From "American Electrician's Handbook," Terrell Croft)

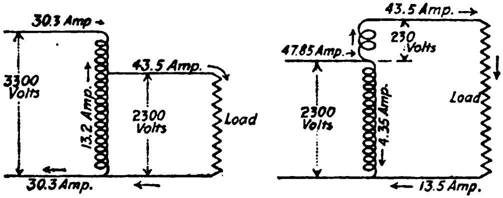
Values vary with speed and other conditions. Those given are approximate only. A slow-speed machine, *S*, makes 100 to 200 r.p.m.; a medium-speed machine, *M*, makes from 200 to 300 r.p.m., and a high-speed machine, *H*, makes 300 to 1200 r.p.m.

Kva. output		Current at 1200 volts		Efficiency			Exciter capacity required, kw.
		Three-phase	Two-phase	½ load	¾ load	Full-load	
50	S			....	....	....	....
	M	24.0	20.8	85.5[1]	88.0[1]	89.0[1]	7.0
	H			86.6	89.8	90.8	2.0
75	S			....	....	....	....
	M	36.1	31.3	88.0[1]	90.0[1]	91.3[1]	8.0
	H			87.1	89.7	90.8	3.0
100	S			....	....	....	....
	M	48.1	41.7	89.0[1]	91.0[1]	92.0[1]	9.0
	H			87.7	90.2	91.3	3.0
200	S			90.7[1]	92.3[1]	93.4[1]	12.0
	M	96.2	83.3	91.0[1]	93.0[1]	93.5[1]	11.0
	H			90.1	92.7	93.5	6.0
500	S			92.5[1]	94.0[1]	94.5[1]	23.0
	M	241.0	208.0	91.8	93.5	94.4	16.0
	H			90.8	93.5	94.5	13.0
1000	S			93.0[1]	94.0[1]	94.8[1]	35.0
	M	481.0	417.0	92.3	94.2	95.0	29.0
	H			92.5	94.0	94.6	25.0
2000	S			94.0[1]	95.0[1]	95.8[1]	50.0
	M	962.0	833.0	92.6	94.8	95.8	42.0
	H			92.3	94.7	95.7	38.0

[1] Engine-type machines—efficiencies do not include friction of bearings.

Transformers, Converters, Rectifiers

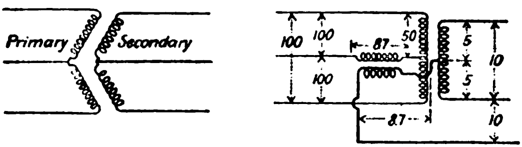
Transformers. Alternating currents may be transformed from one voltage to another by the use of a transformer consisting of a primary and a secondary electrical circuit, each linking with a common magnetic circuit. Since the same magnetic flux links both circuits the same voltage is induced per turn and therefore the voltages of the two coils are in proportion to the number of turns they contain. The **fundamental equation** of the transformer is $E = k \times 4\Phi_m f n 10^{-8}$, where E is the effective value of the impressed e.m.f. in volts, Φ_m the maximum value of the magnetic flux, f the frequency, n the number of primary turns, and k the form factor of the e.m.f. wave. When the e.m.f. is a sine curve, the equation reduces to $E = \sqrt{2}\pi f \Phi_m n 10^{-8}$. Transformers are built for a great variety of service and in capacities from a fraction of a kilovolt-ampere to 10,000 kva. and for voltages as high as 500,000. Ordinarily, transformers are used for the purpose of transforming constant voltage to another constant voltage. Assuming constant voltage applied to the primary, the secondary voltage will decrease slightly with increase of load due to the combined effect of resistance of the windings and the magnetic leakage, which, combined, make up the impedance of the transformer. The effect of the impedance is to cause a greater drop of voltage upon an inductive load than upon a non-inductive one, and a capacity load tends to counteract the effect of the impedance and may cause an actual rise of the secondary voltage with increased load. A load consisting of induction motors or other inductive apparatus having a low power factor, therefore, demands a lower transformer reactance than a non-inductive lighting load for the same percentage regulation. The energy **losses** involved in a transformer are the iron and the copper losses. The hysteresis losses constitute the major portion of the iron losses and are proportional to the 1.6 power of the maximum magnetic flux and directly proportional to the frequency. The eddy-current losses of the iron are proportional to the square of both the magnetic flux and the frequency. The maximum magnetic flux is directly proportional to the impressed voltage. The combined iron losses are approximately constant in **constant-potential transformers,** regardless of load. The copper losses are proportional to the square of the current. The efficiency of a transformer is best determined by measuring these losses and calculating the efficiency as the ratio of output/(output + losses). The iron losses are constant and may be measured by a wattmeter by applying normal voltage to the low-tension terminals and leaving the high-tension winding open. The copper losses can be calculated from the measured resistance of the windings, or they should be measured by applying to the high-tension terminals a low voltage sufficient to cause full-load current to flow through the short-circuited secondary. Low iron losses are especially important where, as in lighting loads, the transformer is only loaded for a few hours during the day, because the iron losses continue through the whole day. The **maximum efficiency** results when the iron losses and copper losses are equal. The all-day efficiency is the ratio of the total energy output for the day to the total energy input. It is customary to immerse the core and windings in oil, because of its greater dielectric strength and because the energy losses are more rapidly dissipated by oil, thus increasing the output without exceeding the safe limit of temperature. In the larger sizes the dissipation of heat is accelerated by a system of water circulation or by an air blast. Transformers for converting energy at constant potential to **constant current** are used for operating series arc-light circuits from constant potential circuits. This transformation is accomplished by having one of the coils movable with respect to the other and so balanced that the repulsion existing between the coils, due to their currents flowing in opposite directions, varies their distance apart and hence their mutual inductance. An **auto-transformer,** also called **compensator** or **balancing coil,** consists of a single coil linking with a magnetic circuit. Part of the energy is *transformed* and part flows through conductively. Suitable taps are provided so that if the primary voltage is applied to two of the taps, a voltage may be taken from any two taps which bears to the primary voltage the ratio of the turns included between the primary and secondary taps. It is desirable to install an auto-transformer when the ratio of transformation is not great. The ratio of energy transformed to total energy is $\frac{n-1}{n}$, where n is the ratio of transformation. This represents the saving over the ordinary transformer and is greatest when the ratio is small. Fig. 64(*a*) shows 100 kw. being transformed to from 3300 volts to 2300 volts; 30.3 kw. only are being actually transformed and the remainder of the power flows through conductively. Fig. 64(*b*) shows how an ordinary 10:1 10-kw. lighting transformer can be connected to boost 110 kw. 10 per cent. in voltage. The pressure may likewise be reduced by reversing the 230-volt coil. An auto-transformer should never be used when it is desired to keep dangerous primary potentials from the secondary. This type is used for starting induction motors and for a number of other similar uses.



(a.) (b.)

FIG. 64.

Polyphase Power Transformation. For the transformation of polyphase power single-phase transformers may be used. In the case of three-phase power it is frequently advisable to use three-phase transformers, for they cost less than an equal capacity in single-phase transformers. Trouble in one phase, however, means that all three phases must be shut down. The primaries and secondaries may be connected either in Δ or in Y; however, with a non-sine wave form it is important for satisfactory operation to use connections which will give similar secondary e.m.f.'s and current waves. For operation on high voltages the Y connection is frequently preferred, for the reason that the transformers need only be insulated to safely withstand $1/\sqrt{3} = 0.58$ times the line voltage when the neutral is grounded. In the



V-connection. T-connection.

FIG. 65.—Transformer Connections for Transforming Small Amounts of Three-phase Power.

case of high-tension transmission, the Δ connection possesses the advantage that if one of the transformers becomes disabled the other two will continue to carry the load until the third one can be replaced. For the transformation of small amounts of power from three-phase to three-phase, two transformers are frequently used as a permanent installation. Either the V- or the T- connection may be used. These are shown in Fig. 65. The capacities of these systems are only 58 per cent. of the capacity of the system using 3 similar transformers, one for each phase.

Table 22. Performance of Distributing Transformers

(From Croft's "Am. Electrician's Handbook")
Average values for 2200 to 220–110-volt, 60-cycle transformers

Kva.	Watts loss		Per cent. efficiency				Per cent. regulation				Per cent. exciting current
	Iron	Copper	Full-load	¾ load	½ load	¼ load	100 per cent. P.F.	90 per cent. P.F.	80 per cent. P.F.	70 per cent. P.F.	
½	15	13	94.7	94.4	93.2	88.8	2.6	2.73	2.62	2.5	8.0
1	20	24	95.8	95.7	95.0	92.0	2.4	2.51	2.41	2.25	5.5
1½	24	33	96.4	96.4	95.8	93.5	2.2	2.4	2.35	2.3	4.0
2	29	40	96.7	96.7	96.2	94.1	2.0	2.25	2.23	2.2	3.6
2½	32	51	96.8	96.9	96.5	94.7	2.05	2.42	2.45	2.4	3.3
3	33	57	97.1	97.2	96.9	95.4	1.92	2.31	2.38	2.35	3.0
4	37	70	97.4	97.5	97.4	96.0	1.81	2.55	2.75	2.85	1.9
5	43	82	97.5	97.6	97.5	96.3	1.7	2.35	2.51	2.6	1.8
7½	57	110	97.8	97.9	97.7	96.7	1.55	2.4	2.6	2.8	1.7
10	70	140	97.9	98.0	97.9	96.9	1.47	2.32	2.6	2.7	1.65
15	95	192	98.1	98.2	98.1	97.2	1.35	2.2	2.42	2.58	1.5
20	123	255	98.1	98.2	98.1	97.3	1.35	2.6	3.0	3.25	1.3
25	138	305	98.2	98.3	98.3	97.5	1.3	2.6	3.0	3.3	1.25
30	158	370	98.3	98.4	98.3	97.6	1.29	2.75	3.2	3.5	1.15
37½	175	415	98.4	98.5	98.5	97.9	1.18	2.9	3.43	3.8	1.05
50	239	520	98.5	98.6	98.5	97.8	1.14	2.7	3.22	3.57	1.0

To transform from two- to three-phase or *vice versa*, the T-connection shown in Fig. 66 is used. To make the secondary voltages symmetrical a tap (called a Scott tap) is brought out at 87 per cent. ($\sqrt{3}/2$) of the primary winding of the auxiliary transformer as shown in Fig. 66. With balanced no-load

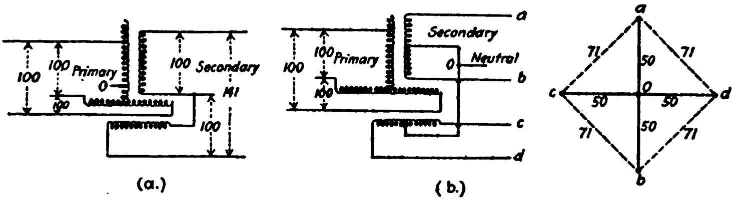


Fig. 66.—Connections for Transforming from Two-phase to Three-phase.

voltages the voltages become slightly unbalanced even under a symmetrical load, due to unequal phase differences in the individual coils. The three-phase neutral is ⅓ the distance along the auxiliary transformer from the junction.

Synchronous Converter. The synchronous converter is essentially a direct-current dynamo with the addition of collector rings connected to suitable points of the armature winding. This machine is generally used for converting polyphase alternating current to direct current. In large generating plants and where power is to be transmitted more than a few miles, alternating-current apparatus generally is employed and synchronous converters are used to transform the alternating current to direct current. The same machine may be used to convert direct current to alternating current or it may be used as a double-current generator to supply alternating and direct current simultaneously. It may be used as a three-wire direct-current generator by connecting the neutral wire to the center of an auto-transformer across the alternating-current end. The rotary converter is cheaper and much more efficient than a motor-generator set of the same capacity, as there is only one armature, one field, and the armature conductors need only carry a current equal to the *difference* of the motor and generator current. The capacity is increased materially by running six-phase, as shown in the following table:

Power factor	Continuous-current generator	Single-phase converter	Three-phase converter	Four-ring converter	Six-phase converter
100 per cent.	100	85	132	161	194
95.5 per cent.	100	78	120	145	170

Assuming a sine-voltage wave, the direct-current voltage is the peak of the diametrical alternating-current voltage waves. The voltage relations for sine waves are as follows: Direct-current volts, 141; single-phase, 100; two-phase, diametrical, 100; two-phase, adjacent taps, 71; three-phase, 87; six-phase, diametrical, 100; six-phase, adjacent taps, 50.

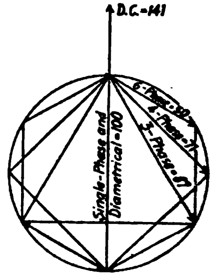


FIG. 67.

These relations are obtained by finding the sides of polygons inscribed in a circle having a diameter of 100 volts, as shown in Fig. 67. When used to convert alternating to direct current, the machine must be in synchronism with the alternating supply. Generally, the same methods of starting and synchronising are used as for synchronous motors, but occasionally the machine is brought to speed as a direct-current motor by supplying direct current to the commutator end. When operated inverted (direct current to alternating current) some centrifugal or electric device must be employed to prevent the rotary from running away, as it will if a highly inductive load is applied. To vary the voltage of the direct-current end, the applied alternating voltage must be varied or the wave shape must be altered. To vary the ratio of transformation by the latter method, special split-pole converters are built, in which the space distribution of the field magnetism is altered by subdividing the field poles into two or three sections. By varying the ampere-turns of the sections the wave form and ratio of transformation may be varied to a limited extent.

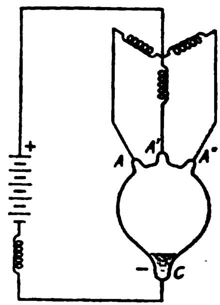


FIG. 68.—Mercury-arc Rectifier.

Rectifiers. In many cases, where only an alternating-current source of energy is available and it is desirable to use direct current of relatively small magnitude for special purposes, such as the charging of storage batteries, operation of arc lamps for projection lanterns, etc., it is found convenient to make use of some form of rectifying apparatus. The rectifiers available for this class of service which can be operated from an alternating source of supply and will deliver a rectified or direct current, are vibrating armature rectifiers, selective spark-gap rectifiers, and mercury-arc rectifiers. The **mercury-arc rectifier** makes use of the selective action of an arc in mercury vapor between mercury (—) and graphite or iron (+) electrodes, and consists (Fig. 68) of an evacuated glass bulb containing a pool of mercury, *C*, and as many graphite or iron electrodes, *A'*, *A″*, *A‴*, as there are leads in the supply circuit. The figure shows the circuits for a three-phase rectifier. These rectifiers may be had in capacities up to about 50 amperes direct current, with voltages of from about 5 to 110 volts direct current, and can be operated from 110- or 220-volt circuits. In Fig. 69 are given the performance characteristics of a General Electric 20-ampere rectifier set. The **Murphy rectifier** is a representative of a class of apparatus in which the rectifying action depends upon the selective characteristics of a particular kind of high-tension spark gap. The performance curves of a 10-ampere Murphy rectifier are given in Fig. 70.

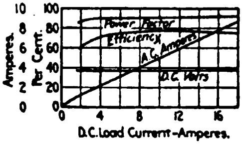


Fig. 69.—Performance of a 20-amp. G. E. Co. Rectifying Set.

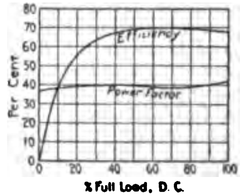


Fig. 70.—Performance of a 10-amp. Murphy Rectifier.

Alternating-current Motors

Induction Motor. In fundamental principle the induction motor is identical with the transformer. A two- or a three-phase e.m.f. is applied to phase windings displaced 90 or 120 electrical degrees from each other, depending upon whether the motor is two-phase or three-phase. A rotating magnetic field is set up which revolves at synchronous speed (r.p.m. = frequency × 60/number of pairs of poles) and cuts the conductors of the short-circuited secondary winding and produces a current therein. Since the secondary current finds itself in a magnetic field, torque is produced. The maximum e.m.f. is induced in the secondary at standstill or when the slip is 100 per cent. The motor can never attain full synchronous speed because at zero slip there is no induced secondary current. The torque is directly proportional to the magnitude of the rotating field and the component of the secondary current in phase with the field. The secondary has leakage reactance, and at the slower speeds and at starting the secondary frequency is high and the secondary current lags behind the secondary e.m.f., and the primary field, and consequently the torque, is ordinarily low under these conditions. The torque at starting can be improved by adding resistance to the secondary circuit so long as it increases the in-phase component of current. As the motor attains normal running speed the secondary resistance is reduced, for then the slip and secondary frequency are low. The induction motor is inherently a **constant-speed machine**, but where it is to be operated for variable-speed work it is customary to provide the secondary with slip rings so that the secondary resistance and power factor may be varied, and the maximum torque developed at all speeds. A great drawback to the induction motor is its poor power factor, necessitating added investment in machinery and distribution system to carry the reactive component of current. The effect of the poor power factor upon the voltage regulation of the system is an additional drawback. Its advantages are great simplicity and ruggedness, and it is finding extended application in a great variety of work in units of all sizes. The development of automatic controlling devices has considerably widened the field of application of induction motors.

Induction motors are made with two types of **rotors**, the **squirrel cage**, in which the inductors are copper bars embedded in longitudinal slots in the laminated steel rotor core and are connected in parallel to short-circuiting copper rings, one at each end of the rotor; and the **wound rotor,** in which the winding is polar, and the terminals of the windings are connected to a resistance carried on the rotor spider or through slip rings to an external controlling resistance, which resistance may be cut out when the proper speed is attained. Polyphase squirrel-cage induction motors are used for constant-speed service where starting and reversing are infrequent. Their starting torque is relatively small, and a large starting current (2 to 6 times the full-load current) is drawn from the lines if the motor must start full-load torque. Such motors have no moving electrical contacts (hence are sparkless) and are advantageous for use in places where exposed to inflammable gases or dust. By increasing the rotor resistance, small motors of this type may be built for high starting torque, rapid acceleration and frequent starting, for the operation of punches, presses and similar machinery, where simplicity is desirable. Motors with wound rotors and internal starting resistance give about 1½ times full-load torque with approximately 1½ times full-load current, making them suitable for use on lighting circuits and for other applications where a minimum starting current is desirable. They will develop a greater starting torque per ampere than those of the squirrel-cage type, but should not be applied to overcome great inertia or excessive static friction.

If, while an induction motor is connected to a source of power it is driven at a speed above synchronism, it becomes an **induction** or **asynchronous generator** and returns electrical energy to the line without any change of connections whatever, for, when driven above synchronous speed, the rotor conductors cut the lines of force of the rotating field in a direction opposite to that when the rotor speed is less than the speed of the field, and hence mechanical power applied to the shaft is converted into electrical power. The greater the negative slip, the greater is the load assumed by the induction generator, and for this reason practical application of the induction generator is sometimes made in connection with exhaust-steam turbines which are run without governors. In this way all the power available from the exhaust steam is converted to electrical energy by the induction generator, which is connected in parallel with the synchronous generators driven by the engines exhausting into the exhaust-steam turbine. Upon short-circuit, the induction generator has the desirable characteristic of not delivering any power. It must always be used in parallel with some synchronous apparatus. Induction motors are not infrequently **used in railway work,** especially in mountain electric railways, where it is advantageous to make use of the motors as induction generators on the down grades as electric brakes returning power to the circuit. When two motors are mechanically connected and the secondary of one motor is electrically connected through slip rings to the primary of the other motor, they are said to be **connected in concatenation or** in **cascade**; thus connected they operate efficiently at a little less than half speed, and if driven at more than half synchronous speed they return power to the circuit.

Single-phase Induction Motor. If from a polyphase motor which is running, one of the line wires is disconnected so that but one phase remains connected to the supply circuit, the motor will continue to run as a single-phase induction motor. If all the phases of another polyphase motor be connected to the terminals of the motor which is running as a single-phase motor, the second motor can be started as a polyphase motor. The first motor is in fact acting as a transformer, converting single-phase power into polyphase power, for the rotor currents of the single-phase motor combine with the supply current to produce a rotating magnetic field as in a true polyphase motor. The great drawback of the single-phase induction motor, aside from some disadvantage in the matter of weight, is the lack of any starting torque. This necessitates that the motor be started either as a polyphase motor by applying polyphase or split-phase power to it, or that it be provided with auxiliary windings to start it as a series or as a **repulsion motor.** All of these methods are used in practice, but the single-phase motor is only used in small sizes where polyphase power is not available.

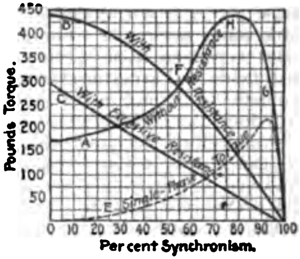


Fig. 71.—Torque Curves of an Induction Motor with Wound Rotor.

The torque curves of an induction motor with a wound rotor, from rest to synchronism, running both three-phase and single-phase with resistance and without resistance, are shown in Fig. 71. Curve *A* shows the torque from rest to synchronism without resistance in the rotor circuit. If resistance is inserted, curve *B* is obtained. Curve *C* indicates the torque where too much resistance is used in the rotor. Curve *E* illustrates the torque single-phase, which is zero at starting. An induction motor starts as shown on curve *B* until it reaches the point *F*, when the resistance is cut out and the motor adjusts itself to its operating position at *G*. Thus, if the torque required of the motor is greater than that shown at *H*, the motor will stall. With the resistance in the rotor, a starting torque *B* is available, but this load cannot be brought up to normal speed. The motor can only bring the torque represented by the point *F* up to normal speed. The torque is always zero at synchronism.

Table 23. Approximate Data on Standard Induction Motors
(220, 440 and 2200 volts,* two-phase and three-phase.—From "American Electricians' Handbook")

H.p.	No. of poles	Synchronous speed		Approx. full-load speed		†Efficiency, per cent.				Power factor, per cent.			
		25 cycles	60 cycles	25 cycles	60 cycles	½ load	¾ load	Full load	1¼ load	½ load	¾ load	Full load	1¼ load
1	4	750	1800	690	1700	74	77	77	76	60	72	80	83
2	6	500	1200	460	1120	82	84	84	83	60	72	78	80
5	6	500	1200	460	1120	84	86	86	85	78	84	86	87
10	6	500	1200	465	1135	85	86	85	84	80	86	89	90
20	6	500	1200	470	1135	87	88	87.5	87	82	89	91	90
50	8	375	900	360	850	87	88	88	88	78	86	89	90
100	10	300	720	288	690	89	90	90	90	83	89	91	91
200	12	....	600	......	575	91	92.5	92	91	85	91	92	91

* 2200-volt motors are seldom made in capacities under 20 to 30 h.p.

† Efficiencies of 25-cycle motors are slightly lower than those of 60-cycle motors, due to their lower speeds.

The **starting current** for full-load torque varies from 2.7 to 3 times the full-load current for sizes up to 50 h.p., and from 3 to 3.5 times for sizes from 50 h.p. to 200 h.p. The **starting torque** at rated voltage averages from 1.3 to 1.5 times the full-load torque. The **pull-out torque** varies from 2.3 to 3 times the torque at full-load current.

Alternating-current Commutator Motors. A number of different motors have been developed for operation upon single-phase circuits. The use of alternating-current motors in railway work makes it feasible to transmit the power at high voltage and to use it without intermediate substations and the accompanying synchronous converters. In the smaller sizes, single-phase commutator motors are chiefly used where polyphase power is not available. For railway work series motors or a modification of the series type are used. Such motors have both fields and armature laminated, and the problems to be met in the motor design are to reduce the self-inductance and sparking to a minimum. This has been accomplished by using low frequencies, compensating windings to neutralize armature reaction and preventive leads to still further reduce sparking. Alternating-current commutator motors with shunt characteristics are available in small units. These are generally single-phase induction motors or modifications which have starting torque and are compensated also to give a high power factor. Fig. 72 gives the operating characteristics of a 5-h.p. motor of this type.

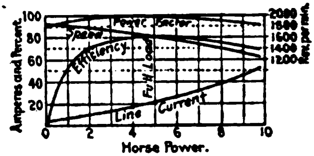


FIG. 72.—Characteristics of a 5-H.P. A.C. Commutator Motor.

Synchronous Motor. When alternating current is supplied to an alternator which is in synchronism with the supplied current, the alternator becomes a synchronous motor. Such motors find considerable application in power-transmission systems. They possess the disadvantage that they are not inherently self-starting, so that they have not been very generally adopted for ordinary power requirements. Their speed is constant regardless of load, which characteristic is often valuable. A characteristic which makes them particularly adapted to use in power-transmission systems is that their power factor may be controlled and varied through a wide range by varying the field excitation. Normal excitation which gives unity power factor produces a motor e.m.f. about equal to the applied e.m.f. Excitation below normal causes the machine to act like an inductive load and above normal causes it to act like a condenser in drawing a leading current. A synchronous motor may be started as an induction motor by opening the field circuit in several places and applying a reduced e.m.f to the armature until nearly synchronous speed is attained. The revolving field thus produced reacts upon the field poles to start the motor. The starting torque thus obtained is not high and a large inductive current is demanded. Another method is to use a small induction motor having one less pair of poles than the connected synchronous motor. If connected to a direct-current generator, this may be used as a shunt motor to start the synchronous motor.

Synchronous motors are often "floated" or run light, overexcited, at the end of transmission lines to balance the reactive current due to inductive loads. Under these conditions they are called **synchronous condensers.** The present long-distance lines require synchronous motors with field regulators at the load end to keep the line regulation within reasonable limits by supplying lagging or leading current.

Flywheel Equalizer. Conditions may be such that sudden overloads occur only intermittently some distance from the station, so that a large investment in copper is not justified as it would be if the maximum power were a steady load. If the load be intermittent and the copper loss of the transmission line large, the voltage regulation will be poor. If the load fluctuations are rapid, the regulation may be improved by the installation of a flywheel equalizer, which is nothing more than a shunt dynamo provided with a flywheel. At the periods of light load the voltage impressed upon the equalizer will be high and the speed will be accelerated. When the load is heavy the terminal voltage will fall and the equalizer machine will now return energy to the circuit at the expense of the momentum of the flywheel. Storage batteries floating upon the line would accomplish the same purpose, but, on account of their cost, are only superior to the flywheel method of storing energy when the fluctuations are so infrequent or of such long duration that the flywheel scheme becomes impracticable.

Rating of Electrical Apparatus—Efficiency

Rating. All electrical apparatus should be rated by output and not by input, and electrical power should generally be expressed in kilowatts. The rating of electrical apparatus is as a rule chiefly determined by the maximum temperature at which the materials in the machine, especially those employed for insulation, may be operated for long periods without deterioration. The effect of too great a temperature may also cause the hysteresis loss of the iron to increase permanently. The aging of iron and injury to the insulating materials usually employed in electromagnetic machinery begin at a temperature of about 85 deg. cent. Since the temperature does not rise immediately to its ultimate value, it is permissible, as far as temperature is concerned, to overload the apparatus so long as the safe temperature is not exceeded. Extracts from the Standardization Rules of the American Institute of Electrical Engineers, covering these matters, are as follows:

Temperature Limits. The highest temperatures attained in any machine corresponding to the output for which it is rated must not exceed the following values: For cotton, silk, paper and other fibrous materials, not so treated as to increase the thermal limit, 95 deg. cent.; for cotton, silk, paper and other fibrous material treated or impregnated and including enameled wire, 105 deg. cent.; for mica, asbestos or other material capable of resisting high temperatures, in which the previous materials, if used, are for structural purposes only, and may be destroyed without impairing the insulating or mechanical qualities, 125 deg. cent.; for fireproof and refractory materials, no limit specified. The limits of temperature rise permitted under rated-load conditions are found by subtracting 40 deg. cent. from the figures just given, whatever be the ambient temperature at the time of the test.

In commutator generators or motors the sparking limit may be below the heating limit under some conditions. Ordinarily, the sparking limit is higher than the heating limit, but in shunt motors, if the field strength is reduced to increase the motor speed, the sparking becomes excessive before the safe temperature limit has been exceeded. In distribution circuits the Underwriters' rules specify safe carrying capacities for the various sizes of conductors, see p. 1652. Except for very short circuits the voltage variation permissible on the receiving apparatus determines the size of conductors. Kelvin's law of economy (see p. 1651) is sometimes the determining factor in the size of conductor chosen.

Efficiency of Electrical Apparatus. The losses to be considered in electrical apparatus are iron losses, copper or I^2R losses, and, where there are revolving parts, friction losses. In constant-speed machinery or machinery operating under constant e.m.f., as is generally the case, the iron losses are constant regardless of load. The copper losses are proportional to the square of the load current. To determine the efficiency of electrical apparatus the method sometimes employed is to measure both output and input. This method is open to the objection that it is difficult to measure mechanical input or output with as great accuracy as electrical quantities may be measured. A more accurate and satisfactory method is to determine the efficiency by assuming different outputs and calculating the corresponding losses from simple measurements. In a dynamo the constant iron and friction losses may be measured by operating the machine as a motor upon the normal voltage and without load. The constant losses are bearing and brush friction, windage, field I^2R loss and armature hysteresis and eddy-current losses. From the measured resistance of the armature (including brushes and brush contact resistance), the armature I^2R losses may be calculated for any assumed output or input. The efficiency is then calculated from the ratio Output/(Output + Losses) if efficiency as a generator is being considered, and from the ratio Input/(Input − Losses) if motor efficiency is desired. The maximum efficiency occurs at the point at which the variable copper losses and the constant losses are equal.

Control of Direct-current Motors

Starting Direct-current Motors. The fundamental e.m.f. equation of the direct-current motor (see p. 1616) may also be written as

$$E = (\Phi ZN/10^8) + I_aR_a \qquad (1)$$

Since the armature resistance I_a is made as small as possible, in order to limit the heating effect in the motor and to maintain high efficiency, the quantity I_aR_a will be small and the counter pressure $\Phi ZN/10^8$ will be very nearly equal to the impressed pressure E. When the armature is not rotating, however, the counter pressure is zero, since the speed N is zero. If the motor is connected to the source of power under these conditions, the relation will be expressed by $E = 0 + I_aR_a$, and it is evident that the armature current I_a will necessarily be abnormally high until the armature has reached such a speed as to generate an appreciable counter pressure. It is, therefore, necessary to introduce additional resistance into the armature circuit of the motor while it is being started. This resistance is usually made up of several steps so that, as the speed of the motor increases, the total resistance can be decreased and gradual acceleration of the motor obtained.

In some motor applications it is desirable to have the starting operations automatically controlled or to have the motor started by pushing a button or closing a switch at some remote point. The two most important methods used in controlling the operations of **automatic motor starters** are (*a*) by a dash pot, and (*b*) by series or current relays. Automatic starters of the **dash-pot type** consist of a solenoid and dash-pot attachment applied to the starting arm of a common manually operated starting box. Closing the main switch energizes the solenoid, which is connected across the line in shunt with the motor, and causes its plunger to move the starting arm upward, cutting out the starting resistance. The rate at which the arm moves is controlled by the dash pot. When the main switch is opened, or the power supply is off, the starting arm falls to its zero position. If the motor is to be started from a distant point, it is not necessary to extend the main current carrying lines to this point. The solenoid circuit can be run to this point and an auxiliary switch installed to control it. This type of starter is used very frequently with motor-driven pumps controlled by float switches and motor-driven air compressors controlled by pressure gages.

The **control of a starter by series or current relays** is the method most generally used in the automatic starting of motors. In this scheme the operation of the switches is controlled by the current in the armature circuit of the motor. Fig 73 shows diagrammatically a controller of this type. The operation in starting is as follows: The power is supplied to the controller by the closing of the main switches S_2 and S_1. The closing of the master switch *MS* closes the circuit through the shunt field and through the electromagnet M_1. Electromagnet M_1 closes contactor 1, allowing current to flow through relay A_1, the total starting resistance, and the motor armature, and causing the motor to start. The plunger of relay A_1 is drawn up, due to the flow of armature current through its winding, and breaks contact at P_1. As the motor accelerates, the armature current decreases until it reaches a value that allows the plunger of A_1 to drop and close the contacts at P_1. This completes a circuit through contact P_4 and electromagnet M_2, and energizes this electromagnet so that contactor 2 is closed. The armature current now flows through relay A_2 and resistance R_2, the first step in the starting resistance being short-circuited. The plunger of relay A_2 is held up until the current again drops to the minimum value, which allows contact P_2 to be closed. The operation is repeated until the total starting resistance has been cut out of the circuit and the motor runs with its armature directly across the line. It will be noted that the armature current varies between maximum and minimum values, the maximum current being determined by the total resistance in the armature circuit at the instants that the contactors close and the minimum values by the adjustment of the series relays. To give the best starting conditions, the contactors should close for the same minimum value of current, and the different inrushes of current should reach the same maximum value. In practice, the relays are set to close at about full-load current, while the resistances are adjusted to give about 50 per cent. overload current. To stop the motor it is only necessary to open the master switch *MS*. It should also be noted that if the power mains become dead, the motor will stop and automatically start again, without any attention whatever from the operator, when the power returns. When a manually operated starter is used, the rapidity with which the starting resistance is cut out is left to the judgment of the operator. If the resistance is cut out too rapidly damage to motor or machine may result. On the other hand, if the resistance is cut out too slowly, there result waste of time and possibly burning out the rheostat.

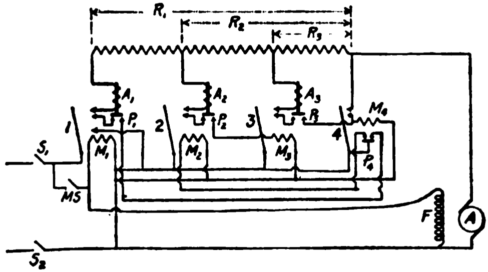


FIG. 73.—Automatic Starting Controller for D.C. Motor.

Speed Control of Shunt Motors. If Eq. (1) be solved for the speed N, the relation obtained is

$$N = 10^8(E - I_aR_a)/\Phi Z \tag{2}$$

From this equation it is evident that if either E, R_a, or Φ is changed the speed of the motor will be changed. There are three general methods of controlling the speed of a shunt (or compound) motor, depending upon which of these factors is changed.

Control by Armature Resistance. When the load I_a on a motor remains constant, the speed of the motor can be reduced by inserting resistance into the armature circuit. If the load on the motor varies, the speed of the motor changes, since the numerator of the above equation changes with each new value of I_a. If the speed of the motor is reduced to a low value while the motor is loaded, the speed will be very greatly increased when the load is thrown off. This makes this type of speed control unsuitable for motors which have fluctuating loads. Moreover, the method is very inefficient. The same amount of power is supplied at all speeds (assuming I_a constant), but at low speeds only a part of it is converted into useful work, the rest being wasted as heat. These objections are in part offset by the fact that speed reductions by armature resistance may be applied to any motor of standard design and require only the simplest speed-regulating rheostat. This method of control is often used to regulate the speed of motor-driven ventilating fans and in many cases where the power demand decreases rapidly with decrease in speed.

Control by Changing Field Magnetism. It is evident from Eq. (2) that the speed of a motor is inversely proportional to the magnetic flux Φ. The flux can be changed either by changing the shunt-field current or by varying the reluctance of the magnetic circuit. The insertion of resistance into the shunt-field circuit affords the simplest and most easily controlled of all methods of speed variation. The chief limitation to this method of control is the sparking at the commutator. More or less satisfactory operation of almost any standard shunt motor can be obtained by this method, providing that a speed ratio of not more than 1.5 to 1 is required. To attain greater ratios, it is necessary to have the motor equipped with commutating poles or interpoles (see p. 1619) which produce a magnetic field of the right polarity and intensity for commutation. Then each armature coil as its corresponding commutator bar passes under the brushes has induced in it an e.m.f. just sufficient to arrest the current in the coil and build up a current of the same strength in the opposite direction during the time the coil is short-circuited by the brushes. Commutating-pole motors are regularly manufactured with adjustable-speed ranges of 4 and 5 to 1. Another important characteristic of this type of motor is its excellent speed regulation with changes of load. The field rheostat can be manipulated to give the desired initial speed, and this speed will be maintained to within 5 per cent. when the load is varied from full load to zero. The only way of obtaining speed **control by varying the magnetic reluctance** of a motor is to mechanically move the iron parts so as to change the length of air gap or the effective area of the magnetic circuit. The Lincoln motor is designed so that both the length of the air gap and the effective area of the pole face are changed to give speed adjustments. The armature is movable in the direction of its axis, so that as it is withdrawn the area covered by the poles is decreased. Moreover, the armature is slightly tapered, so that the length of the air gap is increased simultaneously with the decrease of area. In both of these methods there are no distinct "steps" in the speed variations, all values of speed within the range of the motor being passed through in going from maximum speed to slow speed. Moreover, the only electrical controlling device necessary is the ordinary starting box. The regulation and efficiency of these motors is good at all speeds, but they are expensive.

Control by Changing Impressed Voltage. From Eq. (2) it is evident that the speed of a motor will be changed if its armature is connected to sources of power of different voltages. Speed control by this method is obtained by having current supplied over a number of mains (usually four) between which constant electrical pressures of different values are maintained by generators. The shunt field of the motor is generally permanently connected to one pair of mains and the armature circuit is provided with a controller which the operator can adjust at will to quickly connect the armature to any pair of mains. Such a system gives a series of distinct and widely separated speeds and generally necessitates the use of field-resistance control, in combination, to obtain intermediate speeds. This method, known as the "multi-voltage" method, has the disadvantage that the system is expensive, since it requires several generating machines, an elaborate switchboard, and a complicated system of wiring. The system is extensively used in machine shops.

Another application of variable armature voltage to motor speed control is found in the **Ward Leonard method.** This system makes use of an auxiliary motor-generator set which runs at constant speed from the supply mains. The field circuit of the generator is excited from the main power circuit and has a field rheostat which has sufficient resistance to vary the field current from its full value to nearly zero. The working motor which is to have its speed changed has its armature connected to the variable-voltage generator while its excitation is maintained constant from the supply mains. This arrangement gives any speed from zero up to full speed by the adjustment of the generator field current. On account of its cost and complications, the practical applications of this system are limited to very special conditions. It has been employed in the operation of large newspaper printing presses and the turning of gun turrets on battleships.

Speed Control of Series Motors. The series motor is fundamentally a variable-speed motor. Since the amount of current drawn from the lines by the motor depends upon the load which it is carrying, it follows that the strength of the fields will depend on the load, and, as the speed of the motor depends inversely upon the field strength, the speed of the series motor is inversely proportional to the load. Since the speed of the motor depends upon the voltage impressed upon its armature, the speed of a series motor may be controlled by introducing resistance in series with its armature. This method, which is practically the same as the armature resistance control method of shunt motors, has the same objections of low efficiency and poor regulation with fluctuating loads. It is extensively used, however, in controlling the speed of hoist and crane motors. At least two series motors, controlled by the **series-parallel system,** are usually used in traction work. The two motors are first placed in series with each other and with the starting resistance. The starting resistance is gradually cut out and the motors reach approximately half speed. On the next step the motors are placed in parallel with each other and again in series with the starting resistance. Full speed of the motors is then obtained by gradually cutting out the resistance. This scheme of putting the two motors in series at the start, allows the motors to be started on half the current it would take to start them if they were always connected in parallel to the line. The surges of current drawn from the trolley are thus less and the efficiency of starting higher.

Control of Alternating-current Motors

Starting Alternating-current Motors. Polyphase **induction motors of the squirrel-cage type** are generally started by means of an auto-transformer or compensator (see p. 1625). From the winding of each coil a number of taps are taken off to give a reduced potential at the motor terminals.

A double-throw switch is provided and so arranged that the motor, after it has been started, can be thrown onto the full line potential and the auto-transformer disconnected from the circuit. The double-throw switch, which is equipped with sliding, self-wiping contacts, is immersed in oil. Means are provided for compelling the operator to throw the switch first to the starting position and then to the running position. The taps are generally brought out from the windings so as to give potentials approximately equal to 40, 58, 70, and 80 per cent. of the line voltage. Approximately full-load starting torque will be developed in a motor at starting when 70 per cent. of the line voltage is used. In order to limit the current to as low a value as possible, the lowest taps that will give the motor sufficient voltage to supply the required starting torque should be used. As the torque of an induction motor varies as the square of the voltage, the compensator produces a very low starting torque.

A typical starting compensator is shown in Fig. 74. On starting, the switch cylinder contacts are connected to the corresponding ones on the front finger block. The three compensator coils are then connected in Y across the line and the motor voltage is tapped from some point on the coils. When the handle is thrown into the running position, the motor takes its power directly from the line through the fuses, and the compensator coils are entirely disconnected.

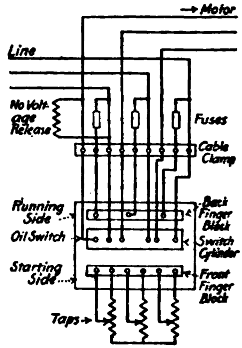


FIG. 74.—Typical Starting Compensator for A.C. Motor.

Resistances can also be used to start motors which have squirrel-cage rotors. These are inserted in each phase and are gradually cut out as the motor comes up to speed. These resistors are generally made of wire-type resistance units or of graphite disks inclosed within heat-resisting porcelain-lined iron tubes. The disadvantage of resistance starters is that the current is not limited. The handle operating the starter may be moved directly to the line contact before the motor reaches full speed, thus causing many times full-load current to flow, or the handle may be moved over so slowly that the resistor becomes very hot and possibly burns out. Resistor starters are less expensive than auto-transformers. Their application is to motors that start light loads at infrequent intervals.

The **wound-rotor type of induction motor** is always started by inserting resistance in the secondary circuit. In one type of this motor the resistances are mounted on the rotor in the space between the iron core and the shaft, and are controlled by operating a lever so placed as to engage a collar free to move longitudinally along the shaft. When the collar is moved in, the resistance is gradually cut out until the winding is short-circuited. Such a motor gives satisfactory operation where frequent starting is not necessary. Extreme heating occurs, however, if heavy loads are started at frequent intervals. For this class of service the windings of the rotor are connected to slip rings and the variable resistance is mounted externally to the machine. Putting resistance into the rotor of a polyphase induction motor at starting reduces the primary currents and increases the starting torque. Slip-ring motors are particularly adapted to starting heavy loads with a minimum amount of current.

Single-phase motors, as such, are not self-starting but require currents differing in time phase to produce a rotating field. This result is often accomplished by providing the stator with a two-phase winding and connecting these in parallel to the single-phase supply mains with a resistance or condenser in series with one of the windings. The currents in the two windings are sufficiently out of phase with each other to produce a revolving field which gives the required starting torque. After the motor reaches a moderate speed, one of the windings is disconnected and the machine operates as a single-phase induction motor. Such motors have very low starting torque, so are sometimes equipped with an automatic centrifugal clutch pulley which only allows the load to be applied after the motor has come up to a fairly high speed itself.

Another method of starting single-phase motors is to provide an armature of the ordinary direct-current drum type, so arranged that the commutator is short-circuited when nearly full speed is attained. During the starting period, carbon brushes which are electrically connected short-circuit the armature in one direction, producing a magnetic field in the rotor at an angle with the primary field in the stator. A high starting torque is thus developed due to the repulsion between the two fields. An automatic centrifugal device converts the machine into an induction motor just before full speed is reached. Such a machine can be adjusted to start with about twice normal full-load torque without requiring excessive current.

A **series alternating-current commutator motor** can be started by means of a rheostat placed in series with it, or by reducing the voltage impressed on its terminals by means of an auto-transformer. With the resistance method the efficiency of the system is low, but the cost of the controlling equipment is much less. Series motors operating cranes and hoists are generally controlled by rheostats, while such motors used in traction work are operated from the secondaries of auto-transformers.

Speed Control of Alternating-current Motors. The synchronous speed of an induction motor is given by the equation $S = 60f/P$, where f is the frequency of the supply and P the number of pairs of poles. Any change in the frequency of the applied voltage or in the number of poles on the motor, causes a change in the speed of the rotating field. No practical method has been developed whereby the frequency of the supply power can be varied. Induction motors are built, however, in which the stator windings are arranged so that they may be changed quickly from say a four-pole winding to an eight-pole winding by means of double-throw switches. In some motors, two independent windings, corresponding to different numbers of poles, each of which may be halved by a pole-changing switch, are provided so that four different speeds are available. When a speed ratio of only 2 to 1 is required, the connections of this type of motor are relatively simple, but if a greater number of speed ratios are necessary, and, especially if machines with wound rotors are used, the connections are very complicated. The power factor of this type of machine is not greatly affected by a change in the number of poles. This method of speed adjustment is very satisfactory as regards efficiency and constancy of speed. The disadvantages of the system are that the motor must be disconnected from the line to effect speed changes, and that the first cost is high.

The most common method employed in varying the speed of a polyphase induction motor is to introduce resistance into the rotor circuit. This method is essentially the same as that used in starting slip-ring motors. The introduction of non-inductive resistances in series with the rotor windings has the effect of reducing the speed while leaving the torque unaltered. At constant torque, the total power drawn from the line will be constant, and the loss of mechanical power due to reduced speed appears as heat dissipated in the regulating rheostat. This method, therefore, although giving any desired change of speed, is very wasteful, especially at the low speeds. Moreover, the speed changes over a wide range when the torque varies from full-load value to zero. This method is used where the periods requiring speed variation are comparatively short and when the motor runs at its normal speed most of the time.

Application of Motors

Alternating- or Direct-current Power. The question as to whether alternating- or direct-current power is more suitable to an industrial plant depends on the size of the plant and whether the kind of work the motors are to do requires adjustable speed. In large manufacturing plants covering a considerable area, alternating current is generally preferable because it permits the use of high voltage with the corresponding saving in copper in the distribution system. The chief disadvantage of the alternating-current system is that the motors do not have the adjustable-speed characteristics required in many machine-tool drives. In plants which consist of a few buildings grouped close together and where the saving in copper in the distributing system is not so important, the direct-current system, possessing the adjustable-speed possibilities, is more advantageously installed. Even where transmission from a distance makes the use of alternating current necessary, motor-generators are often installed to furnish direct current to the adjustable-speed motors. Where the installation uses direct-current power, the general practice is to distribute at a pressure of 230 volts, while with alternating current a three-phase transmission voltage of 2300 or 6600 volts is employed, and this is transformed down to 440 or 220 volts for distribution to the motors.

The application of motors to industrial plants depends upon their **operating characteristics,** and of these characteristics probably the most important is the speed-torque characteristic. Motors may be classified according to their speed-torque characteristics as follows:

(*a*) Motors with constant-speed characteristics.
(*b*) Motors with compound characteristics.
(*c*) Motors with variable-speed characteristics.
(*d*) Motors with adjustable-speed characteristics.

The **constant-speed** motor gives practically constant speed under all conditions of load. Shunt-wound direct-current and single-phase and polyphase induction motors are made which give a speed regulation of less than 5 per cent. when the load is varied from full load to no load. Constant-speed motors are used to drive line shafting to which a number of machines are belted, woodworking machines, lathes, boring mills, circular and band saws, milling machines, and in general in all cases where the work of cutting is continuous. Their characteristics also correspond to the requirements of fans, blowers, and air compressors.

The speed variation with change of load is greater in a **motor with compound characteristics** than that for a constant-speed motor and less than that for a variable-speed motor. Also the maximum starting torque is greater than that for the constant-speed and less than that for the variable-speed motor. The compound-wound direct-current motor is the only motor that has strictly compound characteristics, although polyphase induction motors can be made which have characteristics approaching those of the direct-current machine. The compound type of motor has good starting torque and at the same time gives average speed variations for different loads. It is, therefore, applicable for work requiring fairly heavy starting torque or frequent starting and reversing and also for operation at fairly constant speed for a certain length of time. Good examples of its application to this class of work are found in the electric-driven elevator and in rolling mills. The compound type of motor is also best suited to driving printing presses and reciprocating-motion machine tools, such as planers, shapers, slotters, etc. This type of motor is also used to operate shears and other tools having heavy flywheels. In such machines, when the motor slows down during the cutting stroke, the inertia of the flywheel aids in doing the work. Such a combination is especially useful in keeping down the rush of current from the line at the time of sudden heavy cutting.

Series-wound direct-current and single-phase series-wound alternating-current motors are **variable-speed motors.** At no load the field is very weak and the motor races and tends to run away, while under heavy load the field is strong and the speed is greatly reduced. This type of motor is never run without load. The characteristics of these motors are heavy starting torque and speed dependent upon the load. Polyphase induction motors can be made to have varying speed characteristics somewhat like those of the series-wound machine by the introduction of resistance into their rotor circuit, but the efficiency of the induction machine is considerably less. Variable-speed motors are used for work requiring frequent starts and large torques, such as hoisting, hauling, and traction work. In these fields they are superior to all other types of motors. They are also used for auxiliary purposes, such as raising the cross rails of planers and boring mills and traversing the carriages of large lathes.

An **adjustable-speed motor** is a machine the speed of which can be adjusted over a very wide range, and which, after being adjusted, remains practically constant for all changes of torque throughout its working range. Shunt-wound direct-current motors with commutating poles and motors in which the magnetic flux is varied by changing the reluctance of the magnetic circuit, fulfill these requirements. Such motors are **constant-power machines,** for, since the speed varies inversely and the torque directly with the field strength, the product of the two for the same armature current will be constant. The average class of machine tools, such as lathes, boring mills, etc., requires motors of such characteristics. When the tool is taking a light cut it is run at a high speed, and when taking a heavy cut the speed is low, thus requiring practically constant horse power throughout its working range.

The **choice of** the **correct size** of motor for a particular machine is of great importance. If the machine is already installed and is to be furnished with a new motor drive, it is customary to set up a test motor and make careful runs to determine the maximum and average loads. The motor is then generally selected on the basis of the average load, provided its overload capacity will take care of the maximum loads. All good motors have a 25 per cent. overload capacity for a period of at least 2 hr. When such a test is made, the efficiency of the test motor should be determined for the conditions under which it is working. It is not safe to make an estimate of this, for, as in the case of a large motor operating under light load, the efficiency may be rather low. In many instances the size of the motor required can be obtained from the builder of the machine, but it should be noted that many manufacturers recommend motors that are altogether too large. The expenditure of time and money in the accurate determination of the amount of power required for a machine installation is wisely made.

SWITCHBOARDS

Switchboards may in general be divided into two classes: direct-control and remote-control boards. On or near **direct-control boards** are mounted the switches, bus bars, meters, and other apparatus for handling directly the current supplied, while in the case of the **remote-control boards,** all bus bars, switches, etc., are removed to a distance and only the control-circuit apparatus finds place on the board. For low-tension circuits up to and including 750 volts direct-control panels are generally used. For circuits of 1100 volts and over, no "live" switch parts or apparatus should appear on the face of the board and all switches should be of the remote-control type. Most switchboard panels are now standardized as to **size** and thickness. Standard switchboards for central stations are usually 90 in. high and each panel consists of two or three slabs. The lengths of the General Electric Co. standard slabs for a two-piece panel are 62 in. and 28 in., respectively, for upper and lower slabs, while the corresponding dimensions for Westinghouse slabs are 65 in. and 25 in. **Marble** is extensively used for the slabs of switchboard panels, because of its appearance and high insulating qualities. For many installations slate is satisfactory and less expensive; it may be made moisture- and oil-proof by black enameling or by finishing in black marine.

Switchboards should be erected at least 3 or 4 ft. from the wall to permit of ready access to the connections at the rear. For panels supplying circuits of 750 volts or less the frames should be insulated from ground, while for higher voltages all frames should be grounded. For low-potential work, the **conductors** on the rear of the switchboard are usually made up of flat copper strip, known as **bus-bar copper.** The size required is based upon a current density of about 1000 amp. per sq. in. Where heavier currents are to be carried than can be accommodated by the sizes given in Table 24, a laminated bar is built up of thin strips insulated from each other to give a greater radiating surface.

Table 24. Copper Bar Data*

(The Cutter Company)

Size, in.	Amperes	Amp. per sq. in.	Ohms per 1000 ft.	Wt., lb. per ft.	Size, in.	Amperes	Amp. per sq. in.	Ohms per 1000 ft.	Wt., lb. per ft.
1 × ¼	433	1732	0.0336	0.97	2½ × ½	1500	1200	0.00672	4.86
1¼ × ¼	530	1696	0.0269	1.21	2½ × ⅝	1715	1097	0.00537	6.07
1½ × ¼	626	1669	0.0223	1.45	2 × ½	1222	1222	0.00840	3.89
1¾ × ¼	725	1657	0.0192	1.70	No. 0000 B.& S.	267	1606	0.0505	0.64
1¼ × ⅜	676	1442	0.0179	1.82	½ in. round	305	1552	0.0428	0.76
1½ × ⅜	798	1418	0.0149	2.18	⅝ in. round	426	1388	0.0273	1.18
1¾ × ⅜	916	1395	0.0128	2.54	¾ in. round	560	1267	0.0190	1.71
2 × ⅜	1035	1380	0.0112	2.91	1 in. round	861	1096	0.0107	3.05
2¼ × ⅜	1154	1367	0.00995	3.27	....	......	......	....	....

* The current-carrying capacity is calculated on the basis of 50 per cent. load factor for densities which under average conditions of radiation would give a temperature rise of about 10 deg. cent. With a load factor of 100 per cent. the current densities given should be halved.

Switches. The current-carrying parts of switches are usually designed for a current density of 1000 amperes per sq. in. At contact surfaces, the current density should be kept down to about 50 amperes per sq. in. of contact surface. **Knife switches** are used on low-tension circuits, and in most cases are mounted on the front of the board. They should be mounted to "throw" vertically, with the blade side of the switch "dead" or disconnected from the source of power when open, to lessen the danger of accidental contact. **Plug switches** are used on high-voltage circuits where the current to be carried is small, as in the case of arc-lighting circuits. **Copper-brush switches** substitute a leaved copper brush with a wiping contact for the knife-blade contact and make use of an auxiliary break between carbon blocks to prevent burning of the copper leaves due to arcing. This type of switch is much used as a circuit breaker, being rendered automatic in its action by the addition of a tripping coil. **Oil-break switches** are used in high-voltage work where the oil is necessary to extinguish the arc.

Circuit Breakers. Any of the above switches equipped with a tripping device constitutes a circuit breaker. The tripping device usually takes the form of a solenoid which may be energized in various ways. In **overload circuit breakers** a solenoid coil connected

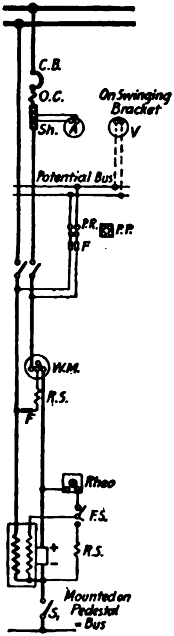


Fig. 75.

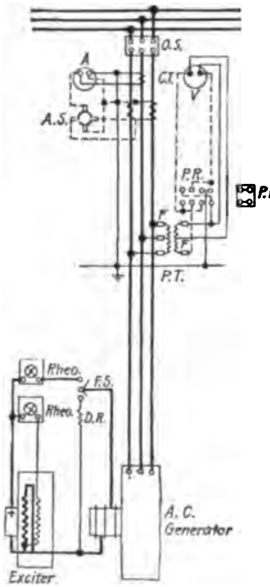


Fig. 76.

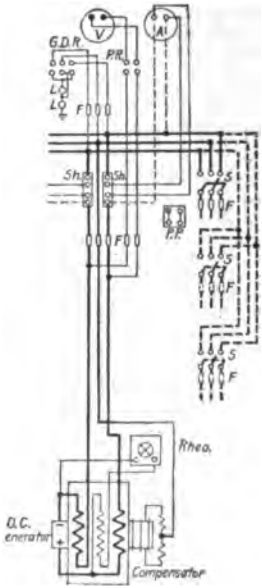


Fig. 77.

Switchboard Wiring Diagrams for Generators.

Fig. 75.—Diagram for 125-volt or 250-volt D.C. Generator. Fig. 76.—Diagram for Three-phase Alternator and Exciter for Small or Isolated Plant. Fig. 77.—Diagram for Three-wire D.C. Generator for Use in Small or Isolated Plant.

Symbols: *A*, ammeter; *A.S.*, 3-way ammeter switch; *C.B.*, circuit breaker; *C.T.*, current transformer; *D.R.*, discharge resistance; *F.*, fuse; *F.S.*, field switch; *G.D.R.*, ground detector receptacle; *L.*, ground detector lamp; *O.C.*, overload coil; *O.S.*, oil switch; *P.P.*, potential plug; *P.R.*, potential receptacle; *P.T.*, potential transformer; *Rheo.*, rheostat; *R.S.*, resistance; *S.*, switch; *Sh.*, shunt; *V.*, voltmeter; *W.M.*, watt-hour meter.

into the main circuit trips the breaker when the current exceeds a certain value. If the coil is either connected in series with or across the circuit and is designed to trip the breaker when the current decreases beyond a certain value, the arrangements are respectively known as underload and reverse-current circuit breakers. In the case of a switchboard panel for the control of a motor, where the load on the motor seldom or never exceeds its maximum rating, it is found satisfactory and less expensive to install **fuses** rather than a circuit breaker to protect the motor from overloads. **Wiring diagrams** are shown in Figs. 75–77.

Equipment of Standard Panels. Following are enumerated the various parts required in the equipment of standard panels for varying services:

Generator or Synchronous Converter Panel, Direct-current, 2-wire System: 1 circuit breaker; 1 ammeter; 1 handwheel for rheostat; 1 voltmeter; 1 main switch (3-pole s.t. or d.t.) or 2 s.p. switches.

Generator or Synchronous Converter Panel, Direct-current, 3-wire System: 2 circuit breakers; 2 ammeters; 2 handwheels for field rheostats; 2 field switches; 2 potential receptacles for use with voltmeter; 3 switches; 1 four-point starting switch.

Generator or Synchronous Motor Panel, Three-phase, 3-wire System: 3 ammeters; 1 three-phase wattmeter; 1 voltmeter; 1 field ammeter; 1 d.p. field switch; 1 handwheel for field rheostat; 1 synchronizing receptacle (4-pt.); 1 potential receptacle (8-pt.); 1 field rheostat; 1 triple-pole oil switch; 1 power-factor indicator; 1 synchroniser; 2 series transformers; 1 governor control switch.

Synchronous Converter Panel, Three-phase: 1 ammeter; 1 power-factor indicator; 1 synchronizing receptacle; 1 triple-pole automatic oil switch; 2 series transformers; 1 shunt transformer; 1 watthour meter (polyphase); 1 governor control switch.

Induction Motor Panel, Three-phase: 1 ammeter; series transformers; 1 oil switch.

Feeder Panel, Direct-current, 2-wire and 3-wire: 1 s.p. circuit breaker; 1 ammeter; 2 s.p. main switches; potential receptacles (1 four-point for 2-wire panel; 1 four-point and 1 eight-point for 3-wire panel).

Feeder Panel, 3-wire, Three-phase and Single-phase: 3 ammeters; 1 automatic oil switch (3-pole for three-phase, 2-pole for single-phase); 2 series transformers; 1 shunt transformer; 1 wattmeter; 1 voltmeter; 1 watthour meter; 1 handwheel for control of potential regulator.

Exciter Panel (for 1 or 2 exciters): 1 ammeter (2 for 2 exciters); 1 field rheostat (2 for 2 exciters); 1 four-point receptacle (2 for 2 exciters); 1 equalizing rheostat for regulator.

DISTRIBUTION AND WIRING

Distribution

Distribution Systems. The choice of a particular system of power distribution is determined by the nature of the load. It is more economical in copper to transmit electric energy at high voltage than at low voltage. To transmit a given power over a given distance with a given percentage power loss (I^2R), or a given percentage voltage drop (IR), the **amount of copper** required varies inversely as the square of the voltage between wires. Because of the danger to life and because of the requirements of lamps, motors, and other apparatus, this power may not in general be utilized at high voltage. Incandescent electric lamps cannot be built for economical operation at voltages higher than 110, while commercial power for direct-current motor service has found its most satisfactory operating pressure at about 500 volts. When power for lighting is to be distributed in a district where the consumers are relatively far apart, alternating current is ordinarily used at high voltage (1100, 2200 volts, or higher), and transformed to 110 volts at the consumer's premises. In congested city districts, where the draft of heavy currents necessitates the use of large cables, direct-current distribution from substations is advisable and necessary, as the use of heavy alternating currents is prohibited by the reactive voltage drop in the cables, which interferes with voltage regulation.

Series Circuits. Where the devices to be supplied with power are all of about the same capacity, are located relatively far apart, and are ordinarily used simultaneously, it is often more economical to supply power at **constant**

current than at constant potential. To operate at constant current, the power-consuming devices as well as the generator are all connected in series, as the same current passes through them all. In cutting any individual device out of service, an equivalent resistance must be inserted in its place to maintain the same current in the circuit, or else some means must be provided at the generator to adjust automatically the total voltage so as to maintain the constant current. If, in a series circuit of the kind under consideration, the resistance of each of the power-consuming devices is R_d, the resistance of the line is R, the current is I, the generator voltage when all of the devices are operating is E, and the number of the receiving devices is n, then $E = nIR_d + IR$, where $IR = e$ is the voltage consumed as resistance drop in the line. Now $e = \rho lI/A$, where ρ is the specific resistance of the material in ohms per mil-ft., l the length of the conductor in feet and A the sectional area of the conductor in circular mils. Having decided upon the permissible voltage drop e, the proper **sectional area of conductor to use** is

$$A = e/\rho lI$$

This method of distribution finds almost exclusive use in the case of arc-lighting systems for street illumination where the lamps are scattered over a large area. Here it is found more economical to connect the lamps in series and cause the constant-current transformer or other apparatus in the station to adjust the total voltage as the number of lamps in service changes. The high voltage used is not objectionable in this case, as circuits placed on poles or underground can be safely and satisfactorily operated at these voltages (1100 to 2300 volts per circuit).

Parallel Circuits. Power is usually delivered by the generator at constant potential, and all the devices or receivers in the circuit are connected in parallel, making a constant-potential system, as in Fig. 78. If conductors of constant cross-section are used, and all the lamps, L_1, L_2, etc., are operating, there will be a greater **voltage drop** or IR drop in the portion of the circuit AB and CD than in any other portion, and the voltages at the terminals of the different lamps will not all be the same.

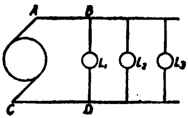


FIG. 78.
Parallel Circuit.

Loop Circuits. A more nearly constant voltage may be obtained by the use of the loop system of wiring shown in Fig. 79. The electrical distance from one generator terminal to the other through any receiver is the same as that through any other receiver, and the terminal voltage at the receivers may be maintained more nearly constant at the expense of additional conductor material.

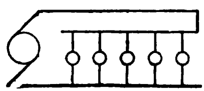


FIG. 79.—Loop Circuit.

Three-wire Circuits. In the operation of the incandescent electric lamp, power must be delivered at the lamp terminals at low voltage (110 volts) and with good voltage regulation, *i.e.*, small voltage fluctuation. For transmission to any considerable distance or for large loads per circuit, this necessitates a large investment in conductor material. In some special cases where it is permissible, lamps may be operated in groups of two in series as shown in Fig. 80. The voltage may thus be doubled, and, for a given number of lamps, the current is halved, the permissible voltage drop (IR) in conductors doubled, the conductor resistance quadrupled, and the amount of conductor material reduced by nearly 75 per cent.

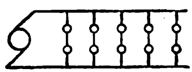


FIG. 80.

The lamps must be used in pairs. Independence in lamp operation, however, may be secured by the use of two generators and a third or **neutral wire** as shown in Fig. 81. When an equal number of lamps is burning on each half of the system, there will be no current flowing in the middle or neutral wire, and the condition is the same as that shown in Fig. 80. When there is a difference in the number of lamps in use on the two sides, the neutral wire will carry a current equal to the difference of the currents in the outside wires. Assuming each lamp to carry 1 amp., the currents in the various parts of the circuit would be as indicated by the ammeters in Fig. 81.

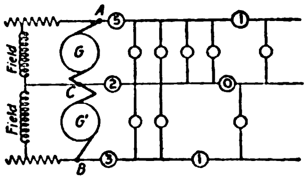


FIG. 81.—Three-wire Circuit.

Where there is a possibility of the load on one-half of the circuit being thrown entirely off, the neutral wire should be of the same size as the outside wires. In practice, it is usually possible so to lay out the circuits that the amount of unbalancing will never be very great, and the neutral wire is often made with one-half the sectional area of the outside wires. This is not always good practice, for if the fuse in one of the outside lines should open the circuit while the load were all on, the neutral wire would have to carry full-load current. Again, if the load were unbalanced, as for instance in Fig. 81, and the fuse or circuit breaker in the neutral wire should "blow," the voltage of the outside lines would be impressed across a combination of a group of 5 parallel lamps in series with a group of 3 lamps in parallel. This would give rise to an unbalanced voltage condition, whereby the lamps on the lightly loaded side of the system would be operated at an abnormally high voltage. For example, if the outside line voltage is 220, the resistance of each lamp is 150 ohms, and the fuse in the neutral wire has been accidentally "blown," the resistance of the lamp load between main A and the neutral wire will be 30 ohms and between main B and the neutral 50 ohms. The total voltage will be divided between the two halves of the load in proportion to the resistances, and hence the voltage across AC will be $(30/80) \times 220 = 82.5$ volts, while that across BC will be $(50/80) \times 220 = 137.5$ volts. Where the amount of unbalancing is slight, independence in the use of lamps may be obtained by the use of a third wire in the receiver part of the circuit only, as shown in Fig. 82. As the amount of unbalancing increases, this case becomes the one discussed above with the fuse blown in the neutral.

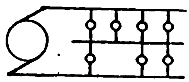


FIG. 82.

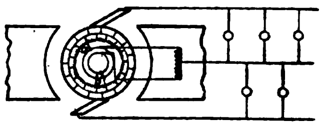


FIG. 83.—Three-wire Generator.

Three-wire Generator. Another method of obtaining a neutral point is by means of a three-wire generator. This machine consists of a direct-current generator wound for outside line voltage with points on the winding 180 electrical degrees apart connected through slip rings to the ends of a compensator or reactance coil, the connections being made as shown in Fig. 83. The **compensator** draws a small amount of reactive alternating current by way of the slip rings continuously. The mid-point of its winding is always at neutral potential with respect to the outside lines, and any unbalanced or neutral current will flow in the middle wire to the compensator and there divide in proportions depending upon the instantaneous position of the armature with respect to the field. Thus, when the slip-ring tap connections (a, b) are directly under the brushes, the pressure at the slip rings is a maximum and all of the unbalanced-load current flows through one-half of the compensator. With the same load conditions and the armature rotated through 180 deg., the neutral current flows through the other half of the compensator, while at intermediate positions in the revolution the current divides in the two halves in varying proportions. In some types of machines, to eliminate some of the slip rings, the reactance coil is mounted within the spider of the armature.

Motor-generator Balancer. Where close voltage regulation is desirable in a three-wire system, it can be obtained by providing a single generator supplying power at 220 volts and "floating" across the line a motor-generator set or balancer of sufficient current-carrying capacity to carry the maximum neutral or unbalanced current. (See Fig. 84.) The armatures of the two machines are connected in series across the line. When the system is balanced, the voltage across each machine of the balancer set is the same, and only enough energy is drawn from the line to supply machine losses and cause the armature to revolve. As the system becomes unbalanced the voltage on the side with the heavier load decreases, the voltage M becomes higher than that across G, M acts as a motor and drives G as a generator, which in turn supplies the additional current necessary on the heavily loaded side and thus maintains good voltage regulation. This method of balancing is most desirable where the amount of neutral current is not large and where it is not necessary to operate the generator continuously. On the other hand, there is a constant loss due to the energy required to drive the balancer set.

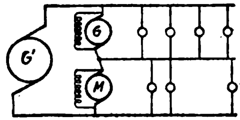


Fig. 84.—Motor-generator Balancer.

Feeders and Mains. Where power is to be supplied to a large district, improved voltage regulation is obtained by establishing **centers of distribution,** to which power is supplied by heavy circuits known as feeders and from which power is distributed to the consumers by circuits known as mains. As there are no load wires connected at any point on the feeders between the generating station and the centers of distribution, the voltage at the latter points may be kept very constant, and thus the consumer is supplied with power at better voltage regulation than would otherwise be possible. Pilot wires from the centers of distribution often run back to the station voltmeter so that the operator can keep the potential at the center constant.

Alternating-current Three-wire Distribution. With the exception of the congested districts of large cities, where power is more economically distributed as direct current, practically all energy for lighting and small motor work (such as dental motors, fan motors, etc.) is transmitted at 1100 or 2200 volts alternating current to the centers of distribution and from there to transformers which step the voltage down to 220 and 110 volts suitable for use in three-wire lighting systems. The transformers are so designed that the secondary or low-tension winding will deliver power at 220 volts and the middle or neutral wire is obtained by connecting to the center or mid-point of this winding.

Grounding. The neutral wire of the secondary circuit of the transformer should be grounded on the pole (or in the manhole) and at the service switch in the building supplied. If, as a result of a lightning stroke, the transformer primary circuit becomes grounded at *a* (Fig. 85) and the transformer insulation between primary and secondary windings is broken down at *b*, then, if there is no permanent ground connection provided in the secondary neutral wire as shown, the potential of wire No. 1 is raised 2200 volts above ground potential. The National Electrical Code requires the use of a ground wire not smaller than No. 6 B. & S. copper; on secondary circuits grounds should be provided at least every 500 ft.

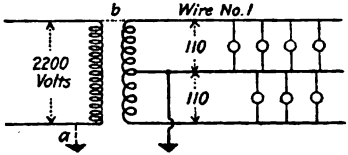


FIG. 85.

Wiring

Copper Requirements of Power Transmission Lines. The amount of copper required to transmit power by any given system, as a three-phase system, with a given percentage power loss, or a given percentage drop in voltage in the ohmic resistance, varies directly with the amount of power, directly as the square of the distance of transmission, and inversely as the square of the voltage used. The cross-sectional area of the conductors for transmitting power with a given percentage loss or voltage drop varies directly with the amount of power, directly with the distance of transmission, and inversely as the square of the voltage. If the percentage loss in transmission is not fixed but if the cross-sectional area of the copper is proportioned to make the annual expenditures for lost power plus interest, depreciation and taxes on the copper a minimum, the weight of the copper required varies directly with the amount of power, directly as the first power of the distance, and inversely as the first power of the voltage used. With the conductors proportioned for minimum annual expenditures, the cross-sectional area of the conductors varies directly with the amount of power and inversely as the voltage, and, for a given voltage, is independent of distance of transmission.

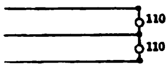
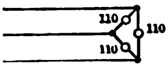
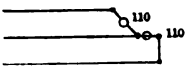
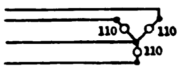
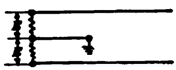
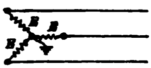
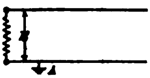
For two systems of the same length transmitting the same amounts of power at different voltages and with the same power loss for both systems, the cross-sectional area and weight of the conductors will vary inversely as the square of the voltage. The foregoing relations between the cross-section or weight of the copper and transmission distance and voltage hold for any system whatsoever, whether direct current, single-phase, three-phase, or quarter-phase.

The copper required at different voltages or by different systems is often determined by the use to which the circuit is put. In lighting mains, the drop in the conductors must not be allowed to exceed a small percentage (4 to 6 per cent.) of the total voltage, in order to maintain a fairly uniform voltage across the lamp throughout the length of the circuit. By reason of this fact, a conductor much larger than the most economical size must in many instances be used. When this is the case the amount of copper required varies inversely as the square of the voltage. In transmission lines and in feeders provided with means for regulating the voltage impressed across the feeders at the generator end, the drop is not limited to this low value and the conductors may be proportioned for economy.

In comparing the different systems for copper economy, the voltage across the load units should be assumed equal in each case. If the systems are high-oltage transmission systems, the line and station insulation may limit the oltage. The comparison of the amount of copper required by two systems hould be made in this case on the basis of equal voltage stress on the systems' isulation. The relative amounts of copper required in transmitting the same amount of power the same distance by different systems are shown in Table 25.

Table 25. Relative Amounts of Copper Required in Transmitting Power by Different Systems

(Power and distance the same in all systems. Table based on equal receiver voltage)

		Conductors proportioned	
		for same percentage of power loss in all systems	for economy or minimum annual expenditure
	Lighting Circuits	Per cent.	Per cent.
	2-wire (copper required taken as 100 per cent.)	100.0	100.0
	3-wire Edison:		
	Neutral same size as outer wire	37.5	61.3
	Neutral half size of outer wire	31.2	55.8
	No neutral	25.0	50.0
	Three-phase delta	75.0	86.6
	Inverted three-phase	56.2	75.0
	Three-phase 4-wire star, neutral full size	33.3	57.9
	Transmission Circuits (Based on same maximum stress on the insulation)		
	Single-phase, neutral grounded (Copper required taken as 100 per cent.)	100.0	100.0
	Three-phase, neutral grounded	100.0	100.0
	Single-phase, ungrounded, to operate with accidental grounds, as at *A*	400.0	200.0
	Three-phase, neutral ungrounded, to operate with accidental grounds	300.0	173.2

The **economical cross-section** may be determined from the curves of Figs. 86 and 87, relating to copper and aluminum, respectively. The ordinates of the curved lines are given in cents and give the value of power

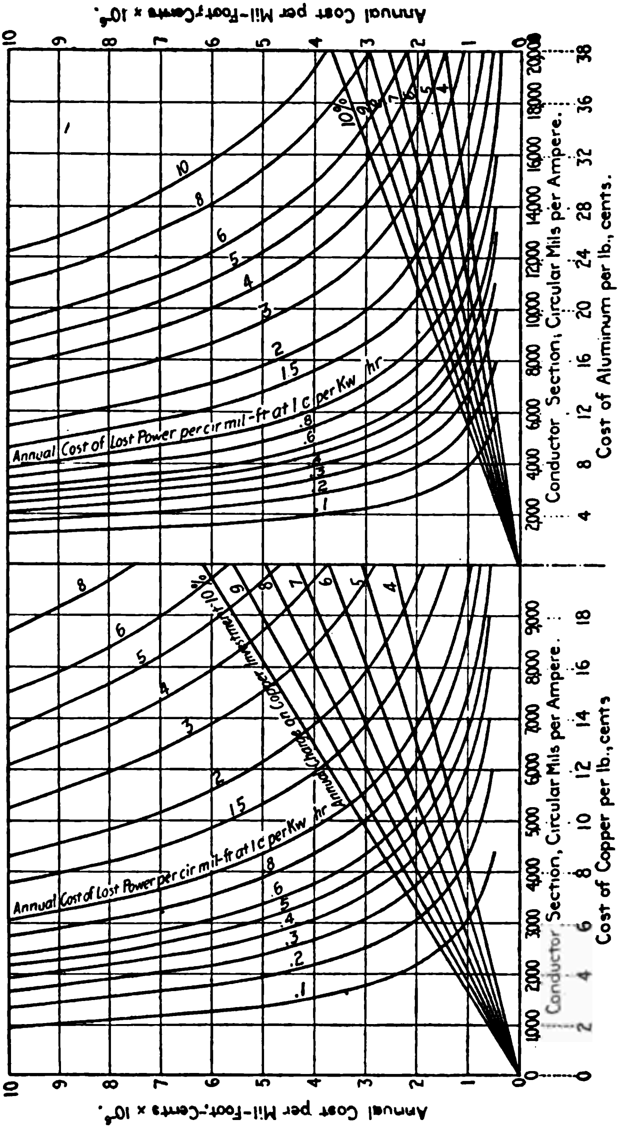


FIG. 86.

FIG. 87.

Charts for Determining the Economical Cross-sections of Copper and Aluminum Conductors.

lost annually in 1 circular mil-foot of conductor for each current density plotted as abscissæ (expressed in circular mils per ampere) when the value of power per kw.-hr. is that given on the curve. Each straight line gives for each of several interest rates the annual cost of interest, depreciation, taxes, etc., upon 1 cir. mil-foot of conductor for the various costs of conductor metal per lb. as abscissæ. The economical conductor is that at which the current density is such as to make the annual interest on the value of each circular mil-foot equal to the annual value of the power lost in each circular mil-foot (Kelvin's Law).

Example. To determine the economical size of copper conductor when power is worth 1 cent per kw.-hr., copper conductor is worth 16 cents per lb. and interest, etc., is equal to 7 per cent., refer to the 7-per cent. curve, where the annual interest is found to be 3.5×10^{-6} cents when copper is 16 cents. A horizontal line at this value intersects the curve giving the annual value of power loss in 1 cir. mil-foot at approximately 5250 cir. mils per ampere. This is the economical section to use. The value of current used in expressing current density in circular mils per ampere should be the square root of the mean square of the current ordinates taken over a year, that is, the **effective value.** With constant load this annual effective value would equal the average value of the current. With a varying load current, the annual effective value will be greater than the average value.

The maximum and minimum values of effective current for various load factors may be determined as follows: Assume a load current as shown by the area *OABC* in Fig. 88, which has a load factor of 50 per cent. A load having the same maximum value and the same average value and therefore the same load factor, is outlined in the figure as *ODEFG*. The rate of loss in the first case will be four times that of the second case, but for half the time; therefore, the loss for the total time period will be twice as great. The effective value of the current in the first case is then twice the average value. The **form factor,** which is the ratio of effective value to average value, is therefore two. In the following tables are set down the maximum possible form factors for various load factors, the minimum form factor being unity in each case:

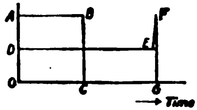


FIG. 88.

Load factor...........	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Max. form factor......	10.0	5.0	3.33	2.5	2.0	1.66	1.428	1.25	1.11	1.0

Wiring Calculations for Direct-current Circuits. The determination of the proper size of conductor to use in supplying a certain electrical load with power is influenced by a number of factors. In general, it is not sufficient to use the minimum size of wire recommended by the National Board of Fire Underwriters in Table 26 based upon the maximum permissible current for the kind of insulation used, but the size of conductor must be so chosen that the voltage drop (IR) along the conductor is small. In the case of **feeders and branch circuits** for an incandescent lamp load, this **drop** may not be more than a small percentage of the voltage between wires. Good practice in interior wiring allows a drop of not more than 3 volts in feeder circuits from the main switchboard to the farthest tablet board, and a drop of not more than 1 volt from any tablet board to the farthest lamp.

The resistance of a mil-foot of copper is 10.8 ohms, and the resistance of a copper conductor may be expressed as $R = 10.8l/A$, where l is the length in feet and A the area in circular mils. Expressing the length in terms of the transmission distance d (since the two wires are usually run parallel), the voltage drop IR to the end of the circuit is

$$e = 21.6Id/A \qquad (1)$$

and the size of conductor in circular mils necessary to give the permissible voltage drop e is

$$A = 21.6Id/e \tag{2}$$

If e is expressed as a percentage of the voltage between conductors and this percentage is represented by P, then

$$A = 2160Id/P \tag{3}$$

Example. Find the size of conductor to supply a 10-h.p. 220-volt direct-current motor with power with 5 volts drop at 500 ft. from the switchboard. Assume a motor efficiency of 86 per cent. The motor will then require a current of [(10 × 746)/(0.86 × 220) =] 39.4 amperes. From Equation (2), $A = 21.6 \times 39.4 \times 500/5 = 85{,}200$ c.m. The next largest wire is No. 0 (B. & S.). Should a voltage drop of slightly more than 5 volts be permissible, No. 1 wire might be used.

Table 26. Permissible Currents for Different Sizes of Copper Wire*

(National Board of Fire Underwriters)

B. & S. gage No.	Area, circular mils	Amperes: Rubber insulation	Amperes: Other insulations	Area, circular mils	Amperes: Rubber insulation	Amperes: Other insulations
18	1,624	3	5	200,000	200	300
16	2,583	6	10	300,000	275	400
14	4,107	15	20	400,000	325	500
12	6,530	20	25	500,000	400	600
10	10,380	25	30	600,000	450	680
8	16,510	35	50	700,000	500	760
6	26,250	50	70	800,000	550	840
5	33,100	55	80	900,000	600	920
4	41,740	70	90	1,000,000	650	1,000
3	52,630	80	100	1,100,000	690	1,080
2	66,370	90	125	1,200,000	730	1,150
1	83,690	100	150	1,300,000	770	1,220
0	105,500	125	200	1,400,000	810	1,290
00	133,100	150	225	1,600,000	890	1,430
000	167,800	175	275	1,800,000	970	1,550
0000	211,600	225	325	2,000,000	1,050	1,670

* The current is limited by the deterioration of the insulation due to temperature rise. For insulated **aluminum wire** the safe carrying capacity is 84 per cent. of that given above.

The calculation of wire size for direct-current **three-wire circuits** is made in practically the same manner. With a perfectly balanced circuit no current flows in the neutral wire and the current in each outside wire will be equal to one-half the sum of the currents taken by all of the receiving devices connected between neutral and outside wires plus the sum of the currents taken by the receivers connected between the outside wires. Using this total current and neglecting the neutral wire, make calculations for the size of the outside wires according to equation (2). The neutral wire should be given the same sectional area as the outside wires.

Example. What size wire should be used for the 3-wire main of Fig. 89? Allowable drop is 3 volts and the distance to the load center 40 ft.; circuit loaded with two groups of receivers each taking 60 amp. connected between the neutral and the outside wires, and one group receivers taking 20 amperes connected across e outside wires. Solution: Load = (60 + 60)/2 20 = 80 amperes. Substituting in Equation), cir. mils = 21.6 Id/e = 21.6 × 80 × 40/3 = ,030 cir. mils.

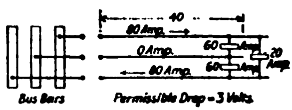


FIG. 89.

Referring to Table 26, 23,030 cir. mils correspond most nearly to No. 6 wire, which has an area of 26,250 cir. mils. This size wire would satisfy the voltage drop requirements, but rubber insulated No. 6 has a safe carrying capacity of but 50 amperes. The current in the circuit is 80 amperes. Therefore, with rubber-insulated wire No. 2 should be used, which will safely carry 90 amperes. The neutral wire is made the same size as the outside wires. For exposed wiring with slow-burning or weather-proof insulation, three No. 4 wires, each of which safely carries 90 amperes, could be used.

Wiring Calculations for Alternating-current Circuits. While these are essentially the same as for direct-current circuits, the problems are often complicated by other factors which have to be considered, such as power factor and line reactance. Skin effect becomes pronounced only when very large conductors are used for alternating current. For interior wiring, conductors larger than 700,000 c.m. should not be used, and many engineers will not use conductors larger than 300,000 c.m. For sectional areas larger than these conductor sizes, a number of smaller conductors in parallel should be used. At voltages under 2000 the effect of line capacity may be neglected.

In the case of ordinary **single-phase interior wiring,** where the effect of line reactance may be neglected and where the power factor of the load (incandescent lamps) is nearly 100 per cent., the calculations are made the same as for direct-current circuits. **Three-wire alternating-current circuits** of ordinary length with incandescent lamp loads are also determined as outlined for direct current.

When the electrical load is other than that of incandescent lamps, it is necessary to know the **power factor** of the load before commencing calculations. When the exact power factor cannot be determined, the following approximate values may be used: Incandescent lamps, 0 95 to 1.00; lamps and motors, 0.85; motors, 0.80. While not strictly accurate, the following equation may be used for the calculation of alternating-current circuits (single-phase) of moderate length where the effect of line reactance may be neglected:

$$I = \frac{\text{kw.} \times 1000}{E \times \text{p.f.}} \qquad (4)$$

where I is the current in amperes, kw. the kilowatts consumed by the load, E the load voltage, and p.f. the power factor of the load. The size of conductor to use is then determined by substituting this value of I in (2) or (3) above.

Example. In Fig. 90, load = 10 kw.; voltage of circuit = 220; power factor = 0.85; distance = 180 ft.; allowable drop = 4 volts. Substituting these values in (4), the current flowing = I = (10 × 1000)/(220 × 0.85) = 53.5 amperes. Substituting again in (2), cir. mils of conductor section = 21.6 × 53.5 × 180/4 = 52,000. The next larger standard size of wire is No. 3 (52,630 c.m.), will safely carry 80 amperes (see Table 26), and is therefore ample for 53.5 amperes.

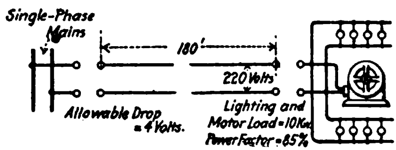


FIG. 90.

For **three-phase three-wire alternating-current circuits** where line reactance may be neglected, the following equations may be used:

$$I = \frac{\text{kw.} \times 1000}{E \times \text{p.f.} \times \sqrt{3}} = \frac{\text{kw.} \times 580}{E \times \text{p.f.}} \qquad (5)$$

and

$$A = \frac{10.8 \times I \times d \times \sqrt{3}}{e} = \frac{18.7 \times I \times d}{e} \qquad (6)$$

Example. In Fig. 91, load = 10 kw.; voltage of circuit = 220; power factor = 0.85; distance = 180 ft.; allowable drop = 4 volts. Substituting in (5), $I = (10 \times 580)/(220 \times 0.85) = 31$ amperes. Substituting in (6) to find the conductor size, $A = 18.7 \times 31 \times 180/4 = 26{,}100$ c.m. The next larger standard size wire is No. 6 (26,250 c.m.), will safely carry with rubber insulation 50 amperes and with other insulations 70 amperes (Table 26), and is therefore ample in section for 31 amperes. Three No. 6 wires would be used for this circuit.

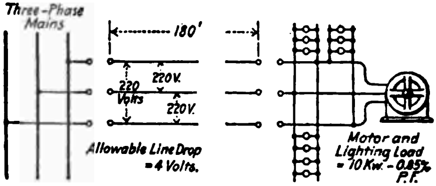


Fig. 91.

Practically all alternating-current circuits have some line reactance The **effect of line reactance** is to cause a drop in voltage somewhat similar to that caused by resistance. Where all of the wires of a circuit, two wires for a single-phase, four wires for a two-phase and three wires for a three-phase circuit, are carried in the same conduit or where the wires are separated less than an inch between centers, the effect of line (inductive) reactance may ordinarily be neglected. Where circuit conductors are large and widely separated from one another and the circuits are long, the effect of inductive reactance may increase the volts line loss considerably over that due to resistance alone. Every such case should be investigated with the Mershon diagram. With aerial circuits on pole lines, where the wires are widely separated, the effect of inductive reactance may be too large. Line reactance decreases somewhat as the size of wire decreases, and decreases as the distance between wires decreases.

The **inductance per mile of single conductor** in henrys is $L = \left(0.74 \times \log_{10} \dfrac{D - R}{R} + 0.0805\right) \times 10^{-3}$; R = radius of conductor, cm., D = distance between centers, cm. R is usually small compared with D and in the numerator may be neglected. The reactance $X = 2\pi fL$, where f is the frequency.

Line or circuit reactance with a conductor of given area can be reduced in two ways. One of these is to diminish the distance between wires. The extent to which this can be carried is limited, in the case of a pole line, to the least distance at which the wires are safe from swinging together in the middle of a span. In inside wiring, knob or cleat work, it is limited by the separation distances required by the underwriters. In conduit work the conductors lie so close together that there is very little effect from inductive reactance under ordinary conditions. The other way of reducing reactance is to divide the copper into a greater number of circuits. Voltage drop in lines due to inductive reactance is best diminished by subdividing the copper or by bringing the conductors closer together. It is little affected by changing the size of conductor.

The **Mershon diagram** (*Am. Elecn.*, June, 1897) offers a "cut and try" method of ascertaining voltage drop. The distance between the wires and the frequency being known, a conductor of the size that appears to be about right selected for trial. With the known current owing, the volts line loss in this conductor can be determined with the diagram. If the volts line loss with this conductor is found to be excessive, a conductor of different size is tried.

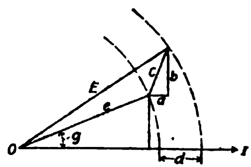


Fig. 92.

Fig. 93 is simply an extension of the vector diagram in Fig. 92, giving the relations of the e.m.f.'s of line, load, and generator. In Fig. 92, *E* is the

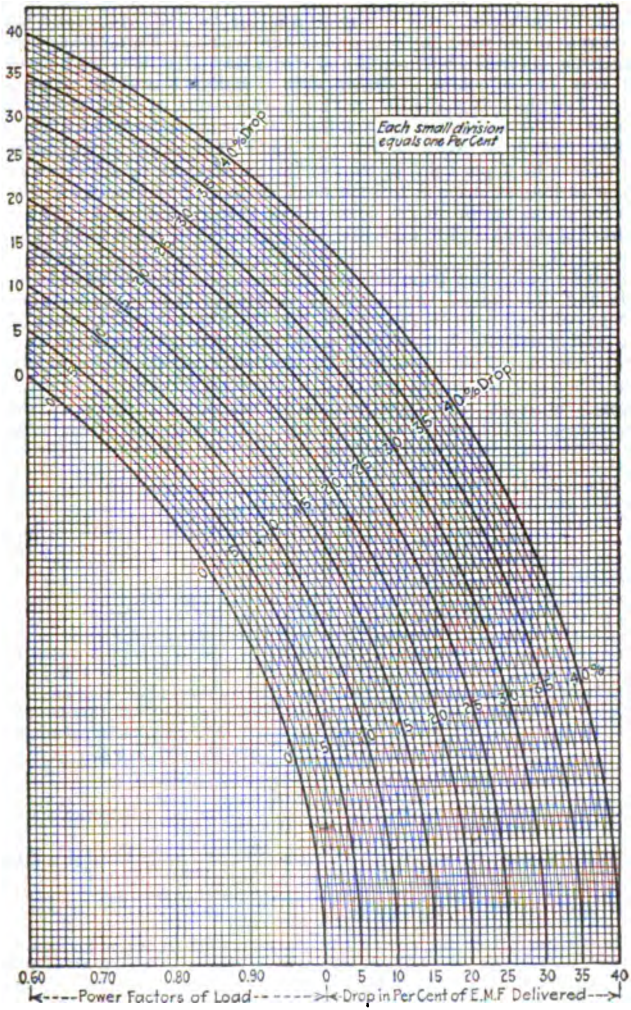


Fig. 93.—Mershon Diagram for Determining the Voltage Drop in A.C. Circuits.

generator e.m.f., *e* the e.m.f. impressed upon the load, and *c* that component of *E* which overcomes the back e.m.f. due to the impedance of the line. *c* is

made up of two components at right angles to each other. One is a, the component overcoming the IR or back e.m.f. due to the resistance of the line, and is in phase with I. The other is b, the component overcoming the reactance e.m.f. or back e.m.f. due to the alternating field set up around the wire by the current in the wire, and is in quadrature with I. Numerically, the drop is the algebraic difference between E and e, or d the radial distance between two circular arcs, one of which is drawn with a radius E and the other with a radius e. The chart is made by striking a succession of circular arcs with 0 as a center. The radius of the smallest circle corresponds to e, the e.m.f. of the load, which is taken as 100 per cent. The radii of the succeeding circles increase by 1 per cent. of that of the smallest circle, and as the radius of the last or largest circle is 140 per cent. of that of the smallest, the chart answers for drops up to 40 per cent. of the voltage delivered. Thus, since the vector e is chosen as 100 per cent., the cosine of the angle g represents the p.f. of the load, and knowing the value of this p.f., the vector position of e is determined by projecting from this p.f. value on the horizontal axis to the 0 arc. Having determined the values of resistance drop and reactance drop in per cent. of load voltage (a and b in Fig. 92), these values are laid off horizontally and vertically respectively as shown, which locates the vector of generator voltage. This value may be read off from the series of arcs and gives the generator voltage in per cent. of the load voltage.

Example. What size wire should be used for the branch to the 50-h.p., 60-cycle, 250-volt, single-phase induction motor of Fig. 94? The name-plate current rating of the motor is 195 amperes and its full-load power factor is 0.85. The wires are run open and separated 4 in.; length of circuit, 600 ft. The volts line loss must not exceed 7 per cent. or 0.07 × 250 = 17.5 volts.

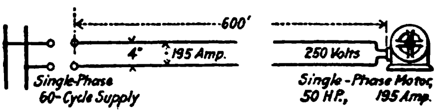


FIG. 94.

Solution. To ascertain approximately what size the conductor must be, substitute in (2), giving cir. mils = 21.6 × 195 × 600/17.5 = 144,500. Referring to Table 27, the next larger standard size wire is No. 000 or 167,800 cir. mils. This size would be ample if there were no line reactance, but as it is known that there is line reactance, select a larger conductor and find what the volts loss with it will be, using the Mershon diagram (Fig. 93). Find the resistance and reactance drops in the line, using the values from Table 27 for a 250,000-cir. mil conductor for 60 cycles and a 4-in. separation. From the table, *resistance volts* = 0.085 and *reactance volts* = 0.139.

$$\text{Resistance drop} = \frac{I \times \text{resistance volts} \times d}{1000} = \frac{195 \times 0.085 \times 600}{1000} = 9.9 \text{ volts}$$

$$\text{Per cent. of resistance drop} = \frac{\text{resistance drop}}{\text{receiver volts}} = \frac{9.9}{250} = 3.96 \text{ per cent.}$$

$$\text{Reactance drop} = \frac{I \times \text{reactance volts} \times d}{1000} = \frac{195 \times 0.139 \times 600}{1000} = 16.3 \text{ volts.}$$

$$\text{Per cent. of reactance drop} = \frac{\text{reactance drop}}{\text{receiver volts}} = \frac{16.3}{250} = 6.5 \text{ per cent.}$$

Now, in Fig. 93, follow the vertical line corresponding to the power factor, 0.85, upward until it intersects the smallest circle. From this point lay off horizontally the percentage resistance drop, 3.96. From this last point lay off vertically the percentage actance drop, 6.5. This last point lies about on the 7-per cent. circle, indicating that e volts line loss in this circuit with 195 amperes flowing will be 0.07 × 250 = 17.5 volts. e conditions of the example are satisfied by a 250,000-cir. mil. conductor. Actually, e line loss will be somewhat less than 7 per cent., as the last point does not quite touch e 7-per cent. circle. Inasmuch as this is a motor branch, the code rules require that

its safe carrying capacity be sufficient for a 25 per cent. overload. Therefore the conductor should be capable of safely carrying 195 × 1.25 = 244 amperes. Referring to Table 26, a 300,000-cir. mil. conductor, rubber-insulated cable would be required to safely carry this 244 amperes. In a problem in practice one would, therefore, immediately try a 300,000 conductor for volts line loss.

Table 27. For Calculating Drop in Alternating-current Lines with the Mershon Diagram—60 Cycles*

B. & S. gage and size of wire (cir. mils)	Resistance volts in 1000 ft. of line (2000 ft. of wire) for 1 amp.	Reactance volts in 1000 ft. of line (2000 ft. of wire) for 1 amp. at 60 cycles per sec. for the distance given in inches between centers of conductors.										
		½	1	2	3	4	5	6	9	12	18	24
14- 4,107	5.06	0.138	0.178	0.218	0.220	0.233	0.244	0.252	0.271	0.284	0.302	.....
12- 6,530	3.18	0.127	0.159	0.190	0.210	0.223	0.233	0.241	0.260	0.273	0.292	.....
10- 10,380	2.00	0.116	0.148	0.180	0.199	0.212	0.223	0.221	0.249	0.262	0.281	.....
8- 16,510	1.26	0.106	0.138	0.169	0.188	0.201	0.212	0.220	0.238	0.252	0.270	0.284
6- 26,250	0.790	0.095	0.127	0.158	0.178	0.190	0.210	0.209	0.228	0.241	0.260	0.272
4- 41,740	0.498	0.085	0.117	0.149	0.167	0.180	0.190	0.199	0.217	0.230	0.249	0.262
2- 66,370	0.312	0.074	0.106	0.138	0.156	0.169	0.180	0.188	0.206	0.220	0.238	0.252
1- 83,690	0.248	0.068	0.101	0.132	0.151	0.164	0.174	0.183	0.201	0.214	0.233	0.246
0-105,500	0.196	0.063	0.095	0.127	0.145	0.159	0.169	0.177	0.196	0.209	0.228	0.241
2/0-133,100	0.156	0.057	0.090	0.121	0.140	0.153	0.164	0.172	0.190	0.204	0.222	0.236
3/0-167,800	0.122	0.052	0.085	0.116	0.135	0.148	0.158	0.167	0.185	0.199	0.217	0.230
4/0-211,600	0.098	0.046	0.079	0.111	0.130	0.143	0.153	0.161	0.180	0.193	0.212	0.225
250,000	0.085	......	0.075	0.106	0.125	0.139	0.148	0.157	0.175	0.189	0.207	0.220
300,000	0.075	......	0.071	0.103	0.120	0.134	0.144	0.153	0.171	0.185	0.203	0.217
350,000	0.061	......	0.067	0.099	0.118	0.128	0.141	0.149	0.168	0.182	0.200	0.213
400,000	0.052	......	0.064	0.096	0.114	0.127	0.138	0.146	0.165	0.178	0.197	0.209
500,000	0.042	......	.....	0.090	0.109	0.122	0.133	0.141	0.160	0.172	0.192	0.202
600,000	0.035	......	.....	0.087	0.106	0.118	0.128	0.137	0.155	0.169	0.187	0.200
700,000	0.030	......	.....	0.083	0.102	0.114	0.125	0.133	0.152	0.165	0.184	0.197
800,000	0.026	......	.....	0.080	0.099	0.112	0.122	0.130	0.148	0.162	0.181	0.194
900,000	0.024	......	.....	0.077	0.096	0.109	0.119	0.127	0.146	0.159	0.178	0.191
1,000,000	0.022	......	.....	0.075	0.094	0.106	0.117	0.125	0.144	0.158	0.176	0.188

* For other frequencies the reactance will be in direct proportion to the frequency.

In the calculation of **three-phase three-wire circuits** where line reactance must be considered, the Mershon diagram may also be used. After a conductor of the most economical section has been determined by following the procedure indicated in Table 28, the drop along each of the three conductors may be considered separately, as follows: Assuming a line pressure *E*, the pressure between any line and the neutral or imaginary zero point will

Table 28. Illustrative Tabulation of Relative Economies of Conductors of Different Sizes

	No. 6 wire	No. 4 wire	250,000 c.m.	300,000 c.m.	400,000 c.m.	500,000 c.m.	600,000 c.m.
Cost of 400 ft. of conductor..	$16.80	$22.50	$102.40	$118.40	$150.40	$184.00	$218.20
Interest on above cost at 5 per cent..............	0.84	1.13	5.12	5.92	7.52	9.20	10.91
Cost of energy lost in conductor at 8 cents per kw.-hr...	101.64	66.00	10.56	8.84	6.60	5.28	4.49
Total annual cost of conductor......................	102.48	67.13	15.68	14.76	14.12	14.48	15.40

be $e = E/\sqrt{3}$. Knowing the line resistance (r) and reactance ($2\pi fL$) per conductor and the current per wire, the resistance and reactance drops may be computed and laid off on the Mershon diagram as for a single-phase circuit. Using the value e as a base pressure, the pressure value found from the Mershon diagram will be the phase or neutral pressure at the generator end of the line, whence the line pressure at the generator end will be $E' = e'\sqrt{3}$.

National Electrical Code. The rules governing the installation of electric wiring vary in different parts of the country, but the rules and requirements of the National Board of Fire Underwriters, as published in the National Electrical Code, should be followed in all cases where they are not modified by existing requirements. The rules are revised biennially and a copy of them may be obtained gratis from the Underwriters' Laboratories. A list of **approved fittings,** a supplement to the Code, is revised semi-annually, and only fittings therein recommended should be used. The approved methods of wiring are given below.

Rigid conduit consists of standard sizes of iron, gas or water pipes which have been galvanized or covered with an insulating enamel ("loricated"). It is recommended for all classes of work and fills the requirements of a mechanically strong, waterproof and fireproof duct for electrical conductors. It is installed as a complete system with iron outlet boxes for all switches and fixtures before the wire is drawn in. In the case of defective wiring, the conductors may be withdrawn at any time and new ones inserted. In cases where it is embedded in concrete in modern fireproof buildings, the fact that it may in time corrode and disintegrate is no objection, since the duct left in the concrete is still satisfactory for electrical conductors. Double-braid rubber-covered wire must be used in rigid conduit. Table 29 shows the proper sizes of conduit to use with the various sizes of wire. **Flexible conduit** consists of a continuous flexible steel tube composed of concave and convex metal strips wound spirally upon each other in such a manner that their concave surfaces interlock, thus leaving smooth interior and exterior surfaces. While it is not waterproof, it possesses great mechanical strength and is ventilated, and due to its flexibility may be installed easily and cheaply in places where it would be impossible to install rigid conduit. It is particularly well adapted to the wiring of old buildings. It can be obtained in coils of from 50 to 200 ft. in length, depending upon the diameter. **Armored cable** consists of either one or two insulated wires with the steel strips used in flexible conduit permanently wound upon them. For alternating current, a pair of wires or "twin conductor" must be used to avoid inductive effects. When installed in old or completed buildings, it is not necessary to draw in the conductor after the conduit is installed. **Flexible tubing** is being used less as a conduit every year, because, as this tubing (or flexible loom, as it is also known) is made up of cotton and fiber, it may not be used where there is a possibility of the presence of moisture. It is much cheaper than any of the other kinds of conduit. **Knob and tube work** is used mostly in houses of frame construction where first cost of the installation is the controlling factor in the choice of a system. The wires are run concealed under floors and in partitions, mounted on porcelain knobs, and where the conductors pass through beams they are insulated by means of porcelain tubes. This method is not to be recommended strongly, because, unless properly installed, there is much danger due to the possibility of mechanical injury, sagging of wires between supports, and gnawing of the insulation by rats and mice. **Cleat work** for open wiring is much used in factories and similar places, and when properly installed has much to recommend it. The wires are supported in porcelain cleats or holders, and where necessary these cleats may be mounted in tiers to economize space. When mounted in a dry place excellent insulation and ventilation are secured. **Wooden moldings** with grooves to receive the wires are often used for exposed and ceiling work, although the general tendency in practice is away from this method of wiring. Wooden moldings may not be installed where there is the possibility of the presence of moisture, but they have the advantage of the protection of wires from mechanical injury and present a neater appearance than open wiring.

Table 29. Sizes of Conduits to be Used with Different Sizes of Wires and Cables*

B. & S. gage No.	Area, circular mils	Amperes, rubber insulation	Sizes of pipe, in.			Area, circular mils	Amperes, rubber insulation	Sizes of pipe, in.		
			1 wire	2 wire	3 wire			1 wire	2 wire	3 wire
18	1,020	3	½	½	½	500,000	390	2	3	3½
16	2,583	6	½	½	½	550,000	420	2	3½	4
14	4,107	12	½	½	¾	600,000	450	2	3½	4
12	6,530	17	½	¾	¾	650,000	475	2	3½	4
10	10,380	24	½	¾	1	700,000	500	2	3½	4
8	16,510	33	½	1	1	750,000	525	2	3½	4
6	26,250	46	¾	1	1¼	800,000	550	2	3½	4
5	33,100	54	¾	1¼	1¼	850,000	575	2½	4	4
4	41,740	65	¾	1¼	1½	900,000	600	2½	4	4½
3	52,630	76	¾	1¼	1½	950,000	625	2½	4	4½
2	66,370	90	¾	1½	2	1,000,000	650	2½	4	4½
1	83,690	107	1	1½	2	1,100,000	690	2½	4	5
0	105,500	127	1	2	2	1,200,000	730	2½	4	5
00	133,100	150	1	2	2	1,300,000	770	2½	4½	5
000	167,800	177	1¼	2	2½	1,400,000	810	3	4½	6
0000	211,600	210	1¼	2	2½	1,500,000	850	3	5	6
	200,000	200	1¼	2	2½	1,600,000	890	3	5	6
	250,000	235	1½	2½	2½	1,700,000	930	3	5	6
	300,000	270	1½	2½	3	1,800,000	970	3	6	7
	350,000	300	1½	2½	3	1,900,000	1,010	3	6	7
	400,000	330	1½	3	3	2,000,000	1,050	3	6	7
	450,000	380	2	3	3½					

* Allowance has been made in the table for the easy pulling of wires around three elbows, so that in straight, short runs conduits a size smaller may be used, excepting that ½ in. is the smallest size permitted by the underwriters' rules.

Costs of Wiring. The **cost of installing a system** of wiring based upon the number of outlets for various systems of wiring is given below (Whitehead, "Lectures on Illuminating Engineering," vol. 1, p. 266, 1911.) The values given represent the costs for work done for all classes of buildings in the larger cities of the eastern part of the United States, and it is believed that the values stated will be subject to but little variation.

	Per Outlet
Exposed wiring	$1.50 to 1.60
Wooden molding	2.00 to 2.50
Concealed knob and tube work	2.50 to 3.00
(Add $1.00 for each switch outlet)	
Iron conduit in new building	$4.50 to 5.00
Iron conduit in concrete buildings	5.00 to 6.00

Switch and base board plugs are considered outlets when the iron box is included.

These values cover that portion of the work from the source of supply at the building line to all outlets, but include no switches, lamps, or fixtures.

According to Knox ("Electric Light Wiring," p. 40), the **relative costs** (approximate) of the various methods based upon the cost of installing a rigid conduit system as 100, are as follows:

Rigid conduit...........	100	Armored cable........	70	Flexible tubing.......	40
Greenfield flexible conduit	80	Molding (hardwood)..	65	Cleat work...........	40
Molding (fireproof wood)	80	Molding (soft wood)..	50	Knob and tube work..	35

Rubber-covered Wire consists of a copper conductor, either solid or stranded, covered with one or two layers of rubber and then protected from abrasion by one or two layers of woven cotton sleeving. Rubber-covered wire is required by the Code for all interior wiring, except in places subject to high temperature. For permissible current-carrying capacities, see Table 26. The values there given for rubber-covered wire are based upon a temperature rise of 15 deg. cent.

Weatherproof Wire. The insulation on this wire consists of two or more layers of cotton braid impregnated with asphaltum or some similar compound. Weatherproof wire is recommended for use out of doors only where it withstands weather conditions better than rubber-covered wire, even though it absorbs more moisture. Weights of weatherproof wire are given on p. 1588.

Slow-burning or Underwriters' Wire. The insulation on this wire is made up of cotton braid impregnated with a heat-resisting compound containing white lead or zinc oxide and chalk. It is used for interior wiring in places where high temperature prohibits the use of rubber-covered wire.

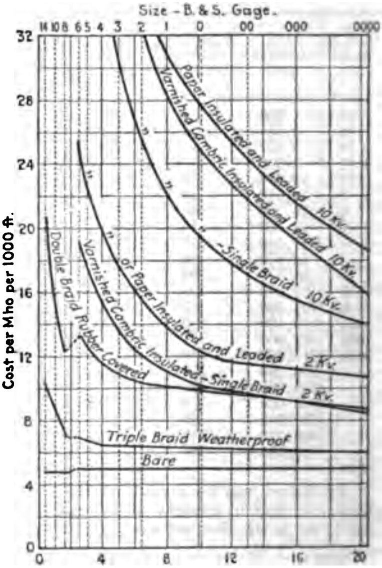


Fig. 95.—Costs of Wires and Cables. (Based on bare copper conductor at 15 cents per lb.)

Cost of Wire. Bare copper wire is sold by the pound, based on the market quotation on bar copper (usually from $0.13 to $0.21) with about 1 cent advance for drawing solid wires and ½ cent additional for stranding. Fig. 95 shows the costs of various kinds of wire and cable based upon a quotation of 15 cents per lb. for bare conductor. In sizes larger than No. 6 (B. & S.), conductors are stranded. To use Fig. 95, proceed as follows: Assume for instance that it is desired to determine the cost per 1000 ft. of a No. 0 triple-braid weather-proof wire. Referring from No. 0 on the horizontal axis to the T.B.W.P. curve, the cost per ohm per 1000 ft. is found to be $6.40. From the copper wire table on p. 1588 the resistance of 1000 ft. of a No. 0 wire is found to be 0.0981 ohm, which means a conductivity of 1/0.0981 or 10.19 mhos. The cost per 1000 ft. will thus be 10.19 × 6.40, or $65.22.

Systems of Wiring. A diagrammatic scheme for laying out the lighting circuits for a building for both a two-wire system and a three-wire system

is shown in Fig. 96. Here the power is brought through the main fuse

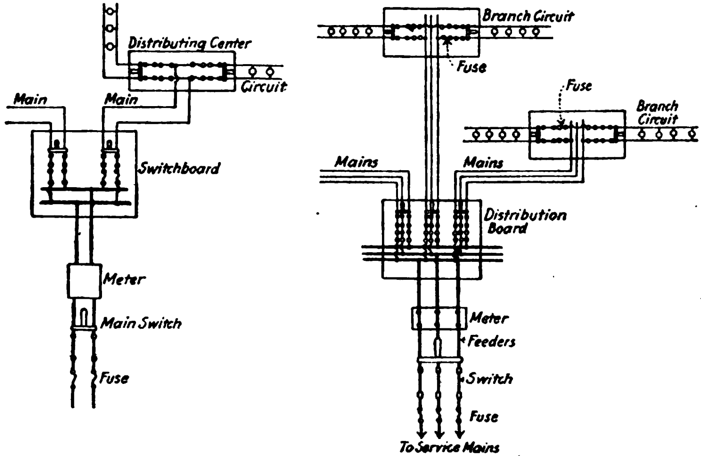


FIG. 96.—Layouts of Two-wire and Three-wire Lighting Circuits for Buildings.

block, switch, and meter to the main switchboard or distribution board as shown. The power is fed through feeder switches and fuses to the distribution panels or tablet boards on the various floors, from which it is supplied to the load on the branch circuits through branch-circuit switches and fuses. The only difference in the wiring for the two systems is that in the latter three wires are carried through to the tablet boards instead of two, and this for a given load permits of the use of a smaller wire for the feeders. Care must be taken, however, to connect the load so that it is divided as evenly as possible between the two halves of the system.

Location of Switches. Although single-pole wall switches are used in most installations for the control of lighting fixtures, double-pole switches (those which open both sides of a circuit) should be used, especially on grounded three-wire systems, unless special precautions are taken to place the single-pole switches in the outside wires.

Multi-switch Control. Where a group of lamps is to be controlled independently from two points, two three-way switches must be used as shown in Fig. 97, while if control is to be had from more than two points, the end switches of the series must be three-way

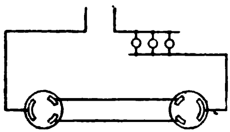


FIG. 97.

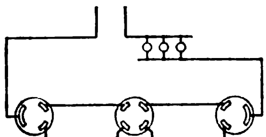


FIG. 98.

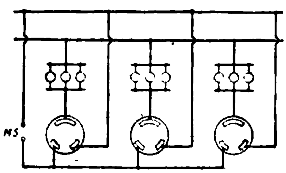


FIG. 99.

FIGS. 97–99.—Switch Control of Lamp Groups.

switches and all others four-way switches, as shown in Fig. 98. Fig. 99 shows a "burglar" circuit, by means of which, through a master switch located at some convenient point, all of the lamps in the building may be thrown on. The diagrams are self-explanatory.

Fuses. Fuses are ordinarily placed in the circuit where power is drawn from the feeders, and also at the main cut-out where the service wires enter the building. They consist of strips, rods, or tubes of some easily fusible metal, so proportioned that they will melt and thus open the circuit at some predetermined value of current. Fuses exposed to the air are known as **open fuses**; fuse links enclosed in insulating cartridges filled with porous or powdered insulating material are known as **enclosed fuses.** With open fuses and heavy currents, the explosive force at the instant of rupture is dangerous, and the location of the fuse block should be such that the possibility of fire is eliminated. For currents of 1000 amperes and over, **copper fuses** are often used. There is objection to using copper as a fuse due to its high melting point. At 75 per cent. of their rated capacities, copper fuses often become so hot as to greatly heat the connecting blocks to which they are attached. **Commercial fuses** are stamped or rated at 80 per cent. of the maximum currents which they will carry continuously, thus allowing for about a 25 per cent. overload. Table 30 gives the diameters of wires of various materials which will fuse with a current of given strength.

Table 30. Diameters of Wires of Various Materials Which Will Fuse with a Given Current (Preece)*

Amperes	Diam. in mils						Amperes	Diam. in mils			
	Copper	Aluminum	Platinum	German silver	Iron	Lead		Copper	Aluminum	Platinum	German silver
1	2.1	2.6	3.3	3.3	4.7	8.1	70	36.0	44.0	56.8	56.4
2	3.4	4.1	5.3	5.3	7.4	12.8	80	39.4	48.1	62.1	61.6
3	4.4	5.4	7.0	6.9	9.7	16.8	90	42.6	52.0	67.2	66.7
4	5.3	6.5	8.4	8.4	11.7	20.3	100	45.7	55.8	72.0	71.5
5	6.2	7.6	9.8	9.7	13.6	23.6	120	51.6	63.0	81.4	80.8
10	9.8	12.0	15.5	15.4	21.6	37.5	140	57.2	69.8	90.2	89.5
15	12.9	15.8	20.3	20.2	28.3	49.1	160	62.5	76.3	98.6	97.8
20	15.6	19.1	24.6	24.5	34.3	59.5	180	67.6	82.6	106.6	105.8
25	18.1	22.2	28.6	28.4	39.8	69.0	200	72.5	88.6	114.4	113.5
30	20.5	25.0	32.3	32.0	45.0	77.9	225	78.4	95.8	123.7	122.8
35	22.7	27.7	35.8	35.6	49.8	86.4	250	84.1	102.8	132.7	131.7
40	24.8	30.3	39.1	38.8	54.5	94.4	275	89.7	109.5	141.4	140.4
45	26.8	32.8	42.3	42.0	58.9	102.1	300	95.0	116.1	149.8	148.7
50	28.8	35.2	45.4	45.0	63.2	109.5	$k =$	10,244	7585	5172	5230
60	32.5	39.7	51.3	50.9	71.4	123.7					

* The values in the table have been calculated by means of the equation $d = (I/k)^{2/3}$. Values of k for different materials are given in last line of table for values of diameter d in inches. For iron, $k = 3148$; for lead, 1379.

COST OF ELECTRICAL APPARATUS

The data given in this article are based upon the average retail prices, f.o.b. factory, to small consumers or to small central stations. Jobbers, large central stations and other large consumers usually obtain contract prices averaging about 10 per cent. less. All the following cost values, however, are necessarily approximate, and variations of as much as 20 or 25 per cent. may be expected. The cost of rotating electrical machinery varies inversely as the speed, and hence the curves in Figs. 100, 101 and 102 are plotted with machine ratings as abscissæ against costs per unit of rating at a speed of 100 r.p.m., *i.e.* [cost in dollars per kilovolt-ampere (kva.), kilowatt, or horse power of rating × 100's of r.p.m.], as ordinates in order to obtain smooth curves. A **general rule**, therefore, is: Cost of generator or motor = rating of apparatus in kilovolt-amperes, kilowatts or horse power (K) × cost reading from curve (k) × 100/speed of machine in r.p.m. (N), or

$$\text{Cost of generator or motor} = 100Kk/N \tag{1}$$

For example, the cost of a 750-kva. low-speed waterwheel-type alternator will be 100 × 750 × \$14.90/150 = \$7450, where \$14.90 is the cost reading (curve XI, Fig. 100) for a rating of 750 kva., and 150 is the number of r.p.m. assumed as standard for an alternator of this size and type. Were a different speed demanded, say 200 r.p.m., the cost would be lower, namely, 100 × 750 × \$14.90/ 200 = \$5587. All costs for three-phase machinery apply also to two-phase machinery.

Costs of Polyphase Alternators, 250 to 2300 Volts

Turbo-alternators. For 60-cycle alternators (3600 r.p.m.) below 800 kva. rating, take value of k from curve I of Fig. 100 and **multiply it by 100** before insertion in formula (1). For larger sizes use the following **net** values of k:

Rating of alternator in kva.	750	1000	1500	2000	2500	5000	7500
	Values of k						
60-cycle { 3600 r.p.m. below 2500 kva. / 1800 r.p.m. above 2500 kva. }	\$660	590	510	450	220	200	190
60-cycle { 1800 r.p.m. below 1500 kva. / 900 r.p.m. above 1500 kva. }	\$560	540	270	230	210	170	...
25-cycle (1500 r.p.m.)	\$360	320	270	240	220	190	170

Up to 750-kva. rating the costs include a direct-connected exciter.

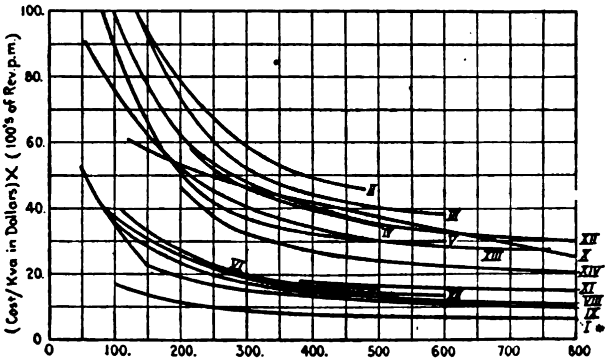


FIG. 100.—Costs of Polyphase Alternators.
(*Multiply cost by 100)

Belted-type Alternators. Costs include bed plate, rails, belt tightener and pulley for exciter, but not the exciter. Obtain value of k in formula (1) from the curves of Fig. 100, using

Curve II for 25-cycle, high-speed (750 r.p.m. below 300 kva. and 500 r.p.m. above).
Curve III for 60-cycle, high-speed (1200 r.p.m. below 300 kva. and 600 r.p.m. above).
Curve IV for 25-cycle, low-speed (500 r.p.m. below 300 kva. and 375 r.p.m. above).
Curve V for 60-cycle, low-speed (600 r.p.m. below 300 kva. and 300 r.p.m. above).
Curve V for 60-cycle, low-speed (900 r.p.m. below 150 kva.).

Thus, a 400-kva. 25-cycle high-speed alternator (500 r.p.m.) would cost (see Curve II) 100 × 400 × \$49/500 = \$3920.

Engine-type Alternators. Costs include brushes and holders and field rheostat, but do not include bed plate, shaft or exciter. The irregular point in Curve IX is due to a sudden drop in the rated speed. Obtain value of k in (1) from Fig. 100, using

Curve VI for 25-cycle, high-speed (150 r.p.m. above 300 kva.).
Curve VII for 60-cycle, high-speed (150 r.p.m. above 400 kva.).
Curve VIII for 25-cycle, low-speed (125 r.p.m. above 300 kva.).
Curve IX for 60-cycle, low-speed (100 r.p.m. above 150 kva.).

The costs for low-speed alternators (25 or 60 cycle) are practically the same at 1000-kva. ratings as at 800 kva.

Waterwheel-type Alternators (60-cycle). Costs do not include exciter. For ratings up to 800 kva., obtain value of k in formula (1) from Fig. 100, using Curve X for high-speed (600 r.p.m. up to 800 kva.; 510 r.p.m. to 1000 kva.); Curve XI for low-speed (150 r.p.m.).
For larger sizes use the following values of k:

Rating of alternator in kva.	1000	1200	1400	1500
High-speed (400 r.p.m. above 1000 kva.).	\$20	18	17	16
Low-speed (150 r.p.m.)	\$15	14	13	11

Direct-driven-type Alternators. Costs include bearings with supporting base and pulley for exciter, but do not include exciter. Obtain value of k in formula (1) from Fig. 100, using

Curve XII for 25-cycle (750 r.p.m. below 300 kva.; 500 r.p.m. above).
Curve XII for 60-cycle, high-speed (600 r.p.m.).
Curve XIII for 25-cycle (500 r.p.m. below 300 kva.; 375 r.p.m. above).
Curve XIV for 60-cycle (450 r.p.m. below 300 kva.; 300 r.p.m. above).

For higher ratings use the following values of k:

Rating of alternator in kva.	1200	1400	1600	1800	2000
25- and 60-cycle, high speed	\$29	27	24	21	18
60-cycle, low-speed	\$20	18	16	13	10

Costs of Direct-current Generators and Motor-generator Sets

Turbo-generators for Exciters, Etc. (125 to 250 volts). Obtain value of k from Curve VI of Fig. 101 and **multiply it by 100** before insertion in formula (1). k is constant from 180- to 250-kw. ratings. Speeds (r.p.m.): 4500 below 25 kw.; 3600 below 75 kw.; 2400 below 125 kw.; 2000 above 125 kw.

Belted-type Generators, Compound-wound (125 to 250 volts). Costs given are for 2-wire generators and include slide rails, pulley and field rheostat. 250-volt 3-wire generators cost from 12 to 15 per cent. more. For high-speed generators (1950 to 980 r.p.m., inversely as size), obtain k from Curve V of Fig. 101; for low-speed (1650 to 720 r.p.m., inversely as size) from Curve VII of Fig. 101.

Engine-type Interpole Generators. Costs include field rheostat, but do not include bed plate, sole plate, bearings, shaft or turning tool. Obtain value of k in formula (1) from Fig. 101, using

Curve I for high-speed (330 r.p.m. down to 200 r.p.m. above 300 kw.).
Curve II for low-speed (300 r.p.m. down to 100 r.p.m. above 200 kw.).

At higher ratings use the following values of k:

Rating of generator in kw.	200	300	400	500	700	1000
High-speed	\$23	19	17	15	...	....
Low-speed	\$19	14	12	11	8	6

Motor-generator Exciter Sets (125 to 250 volts). These consist of 60-cycle polyphase induction motors (220-2200 volt) direct-connected to 125- to 250-volt compound-wound direct-current generators. Costs are for complete sets. Speeds range from 1130 r.p.m. down to 840 r.p.m. Obtain value of k in (1) from Fig. 101, using

Curve III for 2200-volt alternating-current motors.
Curve IV for 220–440-volt alternating-current motors.

Generating Sets (Gasoline and steam-engine). Costs given are for combinations of engines and generators, but without switchboards. The gasoline-engine sets up to

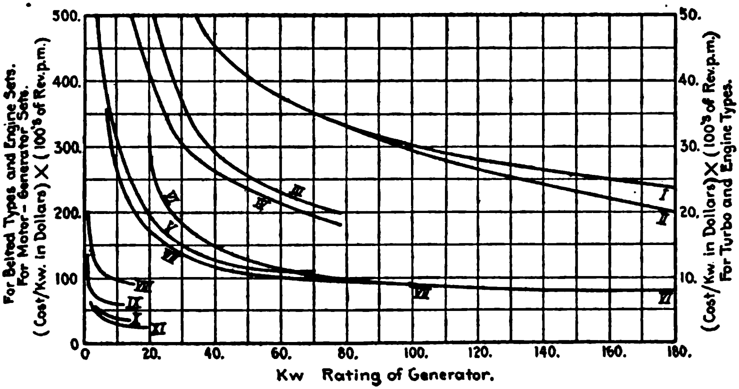


FIG. 101.—Costs of Direct-current Generators and Motor-generator Sets.

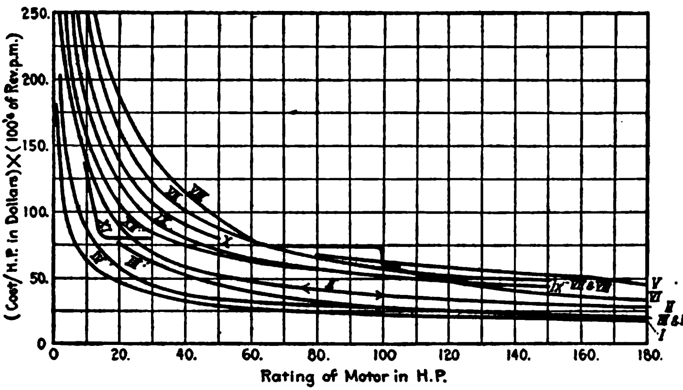


FIG. 102.—Costs of Electric Motors.

7.5 kw. are furnished with cooling tanks and 110-volt generators. The steam-engine sets comprise a standard-type automatic engine direct-connected to a 125- or 250-volt generator. Obtain value of k in (1) from Fig. 101, using

Curve VIII for direct-connected gasoline-engine sets.
Curve IX for belted gasoline-engine sets.
Curve X for low-speed direct-connected steam-engine sets.
Curve XI for high-speed direct-connected steam-engine sets.

Costs of Alternating-current Motors

Belted-type Polyphase Motors, 25-cycle. Costs are for open-type machines with squirrel-cage rotors, and include bed plate, pulley and auto-starter, except for sizes under 5 h.p., where no starter is required. Obtain value of k in formula (1) from Fig. 102, using

Curve I for 110–220-volt, low-speed (360 r.p.m.).
Curve I for 2200-volt, low-speed (360 r.p.m.) above 75 h.p.
Curve II for 110–220-volt, high-speed (720 r.p.m.) above 100 h.p.
Curve XII for 110–220-volt, high-speed (1440 r.p.m.) below 30 h.p.
(For 110–220-volt, high-speed (1440 r.p.m.) between 30 h.p. and 100 h.p., average value of k = $75).
Curve II for 2200-volt, high-speed (720 r.p.m.) above 75 h.p.

Belted-type Polyphase Induction Motors, 60-cycle. Costs include same equipment as for 25-cycle above. Obtain value of k from Fig. 102, using

Curve III for 2200-volt, low-speed (575 r.p.m. below 100 h.p.; 500 r.p.m. above).
Curve IV for 220–550-volt, low-speed (575 r.p.m. below 100 h.p.; 500 r.p.m. above).
Curve V for 2200-volt, high-speed (1730 r.p.m.) between 80 and 180 h.p.
Curve VII for 2200-volt, high-speed (1730 r.p.m.) between 30 and 80 h.p.
Curve XII for 220–550-volt, high-speed (1730 r.p.m.) up to 40 h.p.
Curve VI for 220–550-volt, high-speed (1730 r.p.m.) between 100 and 180 h.p.
(For 220–550-volt, high-speed (1730 r.p.m.) between 40 and 100 h.p., the average value of k is $70.

Costs of Direct-current Motors

Shunt-wound Motors. Costs are for open-type motors and include pulley, starting box and rails. Compound-wound motors cost from 1 to 3 per cent. more. Obtain value of k in formula (1) from Fig. 102, using

Curve VII for 110-volt, high-speed (1300 down to 450 r.p.m.).
Curve VIII for 550-volt, high-speed (1400 down to 430 r.p.m.).
Curve IX for 110-volt, low-speed (400 down to 300 r.p.m.).

Series-wound Motors (Open-type 550-volt motors for indoor service only). Costs of enclosed motors for crane service are not greatly different. Obtain value of k in formula (1) from Curve II of Fig. 102. Speeds decrease from 800 r.p.m. at 10 h.p. to 425 at 75 h.p.

Adjustable-speed Motors (Speed ratio, 1 : 2). Costs are computed from normal or minimum speeds and do not include starting or field rheostats. Obtain value of k from Fig. 102, using

Curve X for 110-volt, high-speed (average 1100 r.p.m. below 20 h.p.; 550 r.p.m. above).
Curve X for 550-volt, high-speed (530 r.p.m.).
Curve XI for 110-volt, low-speed (average 550 r.p.m.) above 10 h.p.
Curve XII for 110-volt, low-speed (average 550 r.p.m.) below 10 h.p.

Adjustable-speed Motors (Speed ratio, 1 : 4). Costs are for 220-volt motors only and do not include starting or field rheostats. Obtain value of k from Curve II, Fig. 102, up to 20 h.p. Minimum speeds average 500 r.p.m. below 5 h.p., 400 between 5 and 15 h.p., and 300 above 15 h.p.

Costs of Transformers

Pole-type Transformers (25- and 60-cycle). Pole-type lighting and power transformers up to 50-kva. ratings are available in two classes, a high-efficiency type and a standard type costing from 75 to 90 per cent. of the former, for use where high core loss is of no great moment. The costs below are for the high-efficiency type, 2300 volts primary, 110/220 secondary, and single-phase unless otherwise stated.

Rating of transformer in kva	2.5	5	7.5	10	15	30	50
Cost per kva. of rating:							
60-cycle, three-phase	$26	18	15.0	13.5	11.5	9.5	8.0
60-cycle, single-phase	15	12	10.5	9.5	9.0	7.5	6.5
25-cycle, 2300-volt	26	18	15.0	13.0	11.5	10.0	8.0
25-cycle, 6600-volt	40	26	20.0	17.5	14.5	11.5	10.0

Oil-insulated Self-cooled Transformers (Single-phase). High-duty power transformers are built as standard apparatus up to 66,000 volts. Cost data are given for 2200- and 44,000-volt transformers, and those for intermediate sizes may be obtained roughly by interpolation. For the smaller sizes the secondary winding is usually designed for 110, 220 or 440 volts; in the very large sizes for 220 or even 660 volts.

Rating of transformer in kva.	80	125	150	200	300	400	500	750
Cost per kva. of rating:								
44,000-volt, 25-cycle	$11.5	8.7	7.9	6.9	5.9	5.2	4.7	...
44,000-volt, 60-cycle	$ 9.6	7.2	6.6	5.8	4.8	4.1	3.6	...
2200-volt, 25-cycle	$ 6.3	5.0	4.7	4.2	3.8	3.5	3.3	3.1
2200-volt, 60-cycle	$ 4.8	3.7	3.5	3.2	2.8	2.6	2.5	2.3

Oil-insulated Water-cooled Transformers (single-phase) are less expensive than self-cooled transformers, as they do not have to be as liberally designed. About ¼ gal. of water per kw. of loss is made to pass through a cooling coil immersed in the oil, which results in a temperature rise of about 15 deg. cent. in the cooling water.

Rating of transformer in kva.	100	150	200	300	400	750	1000
Cost per kva. of rating:							
44,000-volt, 25-cycle	$8.7	6.7	5.8	4.8	4.0	3.2	2.7
44,000-volt, 60-cycle	7.0	5.3	4.6	3.8	3.2	2.4	2.2
2,200-volt, 25-cycle	4.5	3.8	3.4	3.0	2.6	2.2	2.0
2,200-volt, 60-cycle	3.4	3.0	2.8	2.5	2.3	2.0	1.8

Oil-insulated Water-cooled Transformers (Polyphase). Costs given below are for delta connection; for star (Y) connection they are 3 to 4 per cent. less.

Rating of transformer in kva.	125	175	250	300	400	700	1000
Cost per kva. of rating:							
44,000-volt, 25-cycle	$15.7	12.3	8.9	7.8	6.4	4.8	4.4
44,000-volt, 60-cycle	11.7	9.3	7.0	6.2	5.0	3.6	3.0
2,200-volt, 25-cycle	6.2	5.2	4.4	4.0	3.6	3.0	2.2
2,200-volt, 60-cycle	4.5	3.8	3.3	3.0	2.8	2.4	2.0

Air-blast Transformers (polyphase) are ordinarily used only in the larger sizes. No oil is provided either for cooling or insulation, a reasonable operating temperature being obtained by forcing a blast of air through the windings (about 150 cu. ft. per min. per kw. of loss). Costs below are for delta connection; those for Y connection are about 4 per cent. less.

Rating of transformer in kva.	220	400	600	800	1200	2200
Cost per kva. of rating:						
33,000-volt, 25-cycle	$10.4	6.5	4.9	4.1	3.6	3.1
33,000-volt, 60-cycle	8.7	5.6	4.3	3.5	3.0	2.4
2,200-volt, 25-cycle	5.5	4.2	3.5	3.2	2.7	2.3
2,200-volt, 60-cycle	5.1	3.6	2.9	2.6	2.2	2.0

Cost of Electrical Machinery in Cents per Pound

Name of machine	New or second-hand	Values of kw. ÷ r.p.m. 0.001	0.01	0.1	1.0	10.0
Direct-current generators and motors	New	65	35	27	25	......
Induction motors	New	125	51	30	23	......
Alternators	New	......	......	29	23	18
Turbo-alternators	New	......	......	......	22	21
Low-speed engines, compound	New	......	......	......	28	11
High-speed engines, compound	New	......	......	36	15	......
Low-speed engines, simple	New	......	......	......	17	9
High-speed engines, simple	New	......	......	15	8	......
Direct-current generators and motors	Second-hand	31	15	11	7	......
Induction motors	Second-hand	56	33	20	17	......
Alternators	Second-hand	......	17	11	11	9
Engine-driven d.c. and a.c. generators	Second-hand	......	14	9	6	5

Approximate values of *cost per pound* of electrical machinery are given in the table just preceding (see L. A. Doggett, *El. Wld.*, vol. 66, p. 746, Oct. 2, 1915). It will be noted that the average cost of all the new machinery tabulated is 32 cents per pound while that for second-hand machinery is 17 cents per pound, indicating a depreciation of about 50 per cent. for such machinery when in good condition and not obsolete.

Depreciation of Electrical Apparatus, Equipment and Distribution Systems. The values employed by the Railroad Commission of Wisconsin in making appraisals of estimated life in years and minimum service value in per cent. (the latter in parentheses), are as follows:

Generators, motors and rotaries, modern type, 20 (20), obsolete type, 15 (15); static transformers, including regulators and compensators, station type, 20 (40); steam turbo-generators, 20; switchboard and wiring complete, modern type, 20 (45), obsolete type, 15 (35); switchboard instruments alone, modern type, 20 (80), obsolete type, 15 (60); storage batteries, (35); lightning arresters, station type, 10 (50).

Electric Shock and Resuscitation

The effects of the passage of electrical currents through the human body vary with almost every individual case. Generally speaking, 1/10 ampere may be considered dangerous and 1/2 ampere of current prohibitive. With the same conditions, it is probable that alternating current is less dangerous than direct current. With high-frequency alternating currents, it is found that above 50,000 cycles per sec. no muscular contraction or pain is experienced. (Kennelly and Alexander, *El. Wld.*, July 21, 1910.)

Resuscitation from Electrical Shock. If the victim of electrical shock is still in contact with the line conductors, he should be freed at once by cutting off the power, if that is possible. Pulling him off by grasping the clothing is usually dangerous and should be attempted only as a last resort. If the patient is unconscious, the work of **artificial respiration** should be commenced at once, as a few seconds delay may be fatal. The prone pressure method of artificial respiration should be practised, as follows:

1. Place the patient upon his stomach with face turned to one side so that the mouth and nose do not touch the ground.

2. The operator should kneel, straddling the patient's hips, or kneel by either side of the hips, facing the patient's head.

3. The operator should place his spread hands upon the lower ribs of the patient and throw his body and shoulders forward so as to bring his weight heavily upon the lower ribs of the patient. The downward pressure of the hands should take about 3 sec., when the pressure should suddenly be released. This act should be repeated at the rate of 12 times a minute, and should be continued regularly. Rapid and irregular operations are of no avail. Continue for at least 1 hr. or preferably longer, even though the patient is apparently dead.

4. Send for a physician. Have patient's tight clothing loosened, keep him warm and provide plenty of fresh air. Do not give patient liquids by mouth until he is fully conscious.

Cases are on record where the victim has been revived after 2 hr. of artificial respiration treatment.

SECTION 15

ENGINEERING MEASUREMENTS, MECHANICAL REFRIGERATION, ETC.

BY

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* Staff Contribution.

ENGINEERING MEASUREMENTS

MEASURING INSTRUMENTS

BY

J. A. MOYER

REFERENCES: Moyer, "Power Plant Testing," McGraw-Hill. Carpenter and Diederichs, "Experimental Engineering," Wiley. Gramberg, "Technische Messungen," Springer.

TEMPERATURE MEASUREMENTS

Instruments for measuring temperature are classified in the following table, which also gives the temperature range and the degree of accuracy usually obtainable.

	Range, deg. fahr.	Probable accuracy, deg. fahr.
1. Mercury thermometers:		
(a) Ordinary type	− 38 to + 575	From 1 deg. in common instruments up to 0.01 deg.
(b) Jena glass, capillary tube filled with nitrogen	− 38 to +1000	Higher ranges accurate to 1 deg.
(c) Quartz glass, capillary tube filled with nitrogen	− 37 to +1500	Higher ranges accurate to 1 deg.
2. Alcohol or petrol ether	− 325 to + 100	Accurate to 1 deg.
3. Electrical resistance ("bridge" and galvanometer)	− 400 to +2200	Accurate to 0.01 deg., for range 0–500 deg.
4. Thermo-electric	− 400 to +3500	Reliable to nearest 5 deg.
5. Metallic (mechanical)	+ 300 to +1000	Uncertain.
6. Vapor (usually recording)	+ 95 to +1350	Reliable to nearest 2 to 10 deg.
7. Radiation:		
(a) Thermo-couple (in focus of mirror—Féry)	+ 300 to +4000	Reliable to about nearest 20 deg.
(b) Bolometer	− 400 to Sun	Reliable to about nearest 20 deg.
8. Optical	+1100 to Sun	Reliable to about nearest 20 deg.
9. Seger cones	+1100 to +3600	Reliable to about nearest 20 deg.

Mercury Thermometers. The ordinary type has a vacuum in the capillary tube above the mercury and is generally applicable to about 575 deg. fahr. (300 deg. cent.). For higher temperatures the capillary is filled with nitrogen or carbonic acid gas under high pressure, making the thermometer serviceable up to 1000 deg. fahr., and if quartz glass is used it is applicable up to about 1500 deg. fahr. **Quartz thermometers** are much stronger than those of glass but are too expensive for ordinary commercial uses. The lower limit for mercury thermometers is −38 deg. fahr. (−39 deg. cent.). For lower temperatures alcohol and other thermometers are used.

When the thread of mercury in the stem is not at the same temperature as the bulb, as in the case of partial immersion of the stem in a thermometer cup, a **correction for "stem exposure"** must be added. In practice, this correction K is generally calculated from the equation $K = 0.000088D \times (t - t')$, where D is the number of degrees on the scale exposed, t the observed reading, and t' the temperature observed by an auxiliary thermometer having its bulb about 4 in. away from and on a level with the mid-point of the exposed stem (all in fahr. deg.). The work of Reimbach (*Zeit. f. Inst.*, vol. 10, 1900) shows that this equation is not very accurate for short stem exposures. His determinations are given in deg. fahr. for Jena glass thermometers with degree intervals about 1/30 in. long in Figs. 1 and 2. Fig. 1 is for thermometers with the ordinary type of solid stem, that is, with the scale on the stem. Fig. 2 is for "sleeve" thermometers with a capillary tube enclosed in an outer glass tube, which covers also the scale. Beyond the limits of the curves the equation given above should be used.

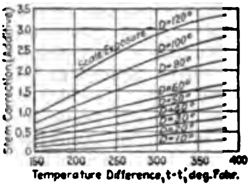

FIG. 1.

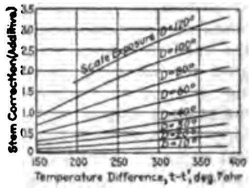

FIG. 2.

Stem Exposure Corrections.

When the **temperature of superheated steam** is to be determined, care must be taken that the thermometer cup is well protected with some non-conducting covering. It is desirable to insert the thermometer into a cork stopper fitting the thermometer cups. Calibrations of the scales of thermometers are usually made by comparing them with mercury thermometers which have been standardized by comparison with a primary standard. The freezing point is usually tested in melting ice, and calibrations above the boiling point of water can be readily made by comparing the readings of the thermometer with the temperatures of saturated steam as given in steam tables for observed pressures.

Electrical-resistance Thermometers are based on the increase of electrical resistance of certain metals with the temperature. Platinum is usually selected because for a given temperature it has a very constant resistance and does not deteriorate at high temperatures. A resistance thermometer is made of a coil of pure annealed platinum wire wound upon a mica framework. The variation of resistance is measured by a small Wheatstone bridge. Commercial instruments are usually arranged so that the galvanometer indicates the temperature in degrees. Up to 1000 deg. fahr. the error should be less than 1/50 deg. and at 2400 deg. fahr. not more than 1/2 deg. A delicate galvanometer is required so as to be sensitive for small currents. For many classes of work, particularly where there is liability to rough usage, the platinum coil must be protected by a porcelain or iron tube, which introduces a time lag. The junctions of the platinum wire of the thermometer with the wires going to the resistance-measuring device, must be placed in the cooler part of the circuit, where the temperature should be the same as when the instrument was calibrated; or compensators may be used. Electric-resistance thermometers are readily calibrated for temperatures of melting ice, steam at varying pressures (212 to 350 deg. fahr.), and boiling sulphur (832.5 deg. fahr.) Intermediate temperatures are computed (*Bulletin* No. 7, U. S. Bureau of Standards).

Thermo-electric Thermometers, or Pyrometers, measure temperatures by the electric current generated in a "couple" made by joining two different metals. The voltage of this current increases as the temperature to which the couple is subjected. A sensitive galvanometer or voltmeter, usually graduated in degrees of temperature, measures the voltage. For thermo-couples to be used not above 1500 deg. fahr., a wire of nickel is usually the positive element and an alloy of nickel and chromium the negative. A couple made of one wire of pure platinum and another of 90 per cent. platinum and 10 per cent. rhodium is not injured at 3500 deg. fahr. There are two general types of such instruments, (1) high-resistance and (2) low-resistance. The **high-resistance type** has a couple formed of wires of small diameter of platinum and an alloy of platinum and rhodium. To protect the wires against breakage and also because platinum deteriorates when acted on by various hot gases, the couples of the high-resistance type are always protected by porcelain or iron tubes. If the temperature does not exceed 1500 deg. fahr., iron tubes are satisfactory. For higher temperatures, porcelain tubes are used, but they must be handled more carefully. Formerly iridium was used for alloying, but it volatilizes rapidly at over 1500 deg. fahr., causing a gradual lowering of the voltage produced. **Base-metal or low-resistance types** of couples are usually made of alloys of nickel, iron and copper. A couple used very largely in America is made of nickel steel and copper. Such couples made of cheap metals can be made of larger rods and at less cost than those of the high-resistance type. They are not quite as accurate, but are generally preferred for industrial work. There is also the advantage that a broken couple can be readily replaced by two rods of nickel steel and copper fused together at one end, and a calibration curve for the couple can be quickly made to be accurate enough for most practical purposes. The principal consideration in selecting rods for such couples is to get them of uniform chemical composition. They should be annealed preferably in an electric furnace at a temperature higher than that to which they are to be exposed. If rods in a couple are not of uniform composition, parasitic currents are produced which oppose that produced by the couple at the junction. Since these wires can be made comparatively large, usually about 3/32 in. in diameter, the current generated will be large compared with that in the high-resistance types and its change in resistance with change in temperature will be small, so that a cheaper low-resistance galvanometer can be used.

Low-resistance couples are usually protected by an iron tube, mainly because steel deteriorates in the presence of sulphur gases, and the asbestos insulation needed for separating the rods along their full lengths is likely to last longer with this protection. Low-resistance pyrometers often have the leads of the same metals as the couples, so that the so-called **"cold junction"** is at the terminals of the galvanometer, and the leads are usually long enough to permit the instrument being placed where the temperature can be maintained at about normal "room" temperatures. Variation in the "cold-junction" temperature from the calibration temperature produces more error in low-resistance than in high-resistance types. Correction is best made with compensators placed in the circuit at the "cold junction." When iron is a constituent of a couple, it should not be used for temperatures above 1300 (to 1400) deg. fahr., as this temperature is a "transition point" for this metal, and its physical and also its thermo-electric properties are changed.

Thermo-couples of platinum and platinum-nickel alloy produce twice the voltage of a platinum-rhodium combination, but should not be subjected to temperatures above 2000 deg. fahr. Thermo-electric couples are particularly adaptable for measuring temperatures in restricted spaces. The couples can be made thin enough to be inserted into spaces less than 1/50 in. wide. They are adaptable also for indicating temperatures of objects at a distance from the observer.

Metallic or Mechanical Pyrometers consist of two rods made of substances having different rates of expansion connected by gears and levers to rotate a pointer on a graduated dial. Generally the rods are made of iron and brass, or of graphite and iron. Although the use of such instruments is quite common, they are generally very unreliable and should never be used for temperatures above 1000 deg. fahr. There is always a tendency for the zero of the instrument to get higher with use, even at moderate temperatures. Beckert and Weinhold found that in a number of cases the zero changed 200 to 400 deg. fahr. in two months. In order to obtain readings corresponding with the graduations, the entire length of the tube enclosing the rods should be placed in the chamber of which the temperature is being measured.

Recording Thermometers. The greater number of recording thermometers operate by the expansion of the vapor of ether, mercury or other liquids confined in a bulb and capillary tube connected to a pressure-measuring device. When the bulb is heated the vapor tension increases and actuates the pressure-recording device. The following **liquids are used,** depending on the range of temperature (deg. fahr.): Liquid sulphur dioxide (SO_2), 15 to 200; ether (free of water), 95 to 250; water, 212 to 450; heavy hydrocarbons, 410 to 700; mercury, 650 to 1350. The capillary tube may be made 100 ft. long, and such instruments are suitable for "**distant reading,**" but varying the temperature of the capillary tube by exposure will alter the observations. The whole length of the bulb must be exposed to the temperature to be measured.

Thermometer Cups should be filled with cylinder oil or mercury. If cylinder oil is used the cups must be kept free from water, as the steam that forms will produce explosions violent enough to throw out the thermometer. The thermometer should be kept from contact with the sides of the cup by wrapping it with a little waste or fitting a cork loosely over it.

The Féry Radiation Pyrometer and the **Wanner optical pyrometer** are instruments devised to measure temperature by means of the radiation from incandescent bodies. In the Féry instrument heat rays are focused upon a miniature thermo-couple and the temperature is indicated by the use of a sensitive galvanometer. These instruments are particularly serviceable for determining the temperature of a bed of fire and of the parts of the settings of boiler furnaces. Optical and radiation pyrometers are calibrated in terms of the radiation from a so-called "black body," which is approximately realized by a uniformly heated enclosure. It is only for "black bodies" that the temperature is exactly proportional to the radiation. Observations made with such pyrometers of incandescent bodies or gases do not give the true temperature. It is generally assumed, however, that they can be used to measure fairly accurately the temperature of heated chambers when focused upon the walls, because of the reflection going on in all directions. In most cases the flame temperature can be taken as the same as that of the surrounding walls. A relatively large area is usually required to sight Féry pyrometers. The distance from the telescope to the hot body can be as much as 30 times the diameter of the hot body, and the telescope can be taken as much nearer as desired without changing the reading of the instrument. Both the Féry and Wanner pyrometers have a satisfactory range from 800 to 4000 deg. fahr. At the lower temperature the average error of such instruments is about 3 deg. fahr., and the maximum error at temperatures above 3000 has been shown to be not more than 20 deg. fahr.

Furnace Temperatures (in deg. fahr.) can be determined approximately from the values corresponding to the color of the fire; see p. 290.

High-reading thermometers and pyrometers are best calibrated by comparison with the known melting points of tin, lead, zinc, antimony, silver, gold, nickel and platinum. Seger cones may also be used.

Seger Cones are pyramids about 2 in. high made of several different oxides mixed to give a definite melting point. Melting points range from 590 to 2000 deg. cent. by steps of from 20 to 30 deg., each having a definite number. The temperature corresponding to a number has been reached when the tip of the cone turns over and touches the surface on which its base is resting. Following are the Centigrade and approximate Fahrenheit temperatures corresponding to a few numbers (These cones indicate only the maximum temperature):

No.	Deg. cent.	Deg. fahr.	No.	Deg. cent.	Deg. fahr.	No.	Deg. cent.	Deg. fahr.
022	600	1110	7	1270	2320	30	1670	3040
016	750	1380	15	1435	2615	35	1770	3220
010	950	1740	20	1530	2790	39	1880	3420
02	1110	2030	26	1580	2880	42	2000 (about)	3600

Relative Accuracy of Thermometers. Mercury thermometers are usually preferred for low-temperature work. They can be made of any degree of accuracy and cost. Electric-resistance thermometers can be used for low and fairly high temperatures and can be made for observing at a distance. Accurate thermometers of this type are, however, expensive; but with them temperature *differences* can be determined much more accurately than with mercury thermometers. Thermo-electric pyrometers are generally more sensitive than those of the optical and radiation types.

Prices of Thermometers and Pyrometers. Mercury thermometers, \$0.50 to \$35, depending largely on accuracy and number of graduations. Ether-bulb recording thermometers, \$40 to \$60; electric-resistance thermometers, \$100 to \$300 (Leeds & Northrup Co., Philadelphia); thermo-electric pyrometers, \$50 to \$100 (Bristol Co., Waterbury, Conn. and Hoskins Instr. Co., Detroit, Mich.); Féry optical pyrometer, \$350 (Taylor Instrument Co., Rochester, N. Y.); Seger cones, 2 to 3 cents each (Eimer & Amend, New York).

The Bureau of Standards, at Washington, D. C., makes a charge of \$0.50 for two points and 10 cents for each additional point for calibrating mercury thermometers. A certificate of calibration is provided by the Bureau for each instrument. Charges for calibrating pyrometers are also nominal.

PRESSURE MEASUREMENTS

Mercury Barometers are ordinarily used for measuring atmospheric pressure. Standard barometric pressure for comparison is usually taken as 30 in. at 32 deg. fahr. Some observers have adopted 62 deg. fahr. instead of 32 as a standard for the brass scale.

Vacuum measured with a mercury manometer must be corrected to standard temperature conditions (see p. 1675, 1). To correct observations of vacuum to **equivalent vacuum** compared to (or "referred to") 30-in. barometer, the difference between the corrected barometer and 30 in. is *added* to the observed vacuum when the barometer is less than 30 in., and is *subtracted* when greater than 30 in. Temperature corrections are avoided when the barometer and the vacuum manometer are hung very near each other. Furthermore, if there is much difference in level, mercury columns should be corrected also for elevation. The correction (to be subtracted) is 1.15 in. per 1000 ft. (see p. 1748).

In using mercury barometers and vacuum gages it is necessary to correct for temperature, gravity (g) and capillarity if the height of the mercury column must be known closer than 2 or 3 mm. The method of correcting the mercury barometer is as follows:

Let h be the observed height of the mercury column at t deg. read on a brass scale which has an expansion coefficient of b, a be the latitude (in degrees) and H be the elevation of the barometer above sea level. Then,

1. If the scale be graduated into millimeters and temperatures are in deg. cent., subtract 0.000182 ht to correct for the density of the mercury, and add 0.000019 ht to correct for the scale.

If the scale is graduated into inches and fractions and temperatures are in deg. fahr., subtract 0.000101 $h(t - 32)$ to correct for the density of the mercury, and add 0.0000105 $h(t - 32)$ to correct for the scale.

2. To correct for capillary depression of the mercury column, add the amount in the accompanying table corresponding to the known internal diameter of the tube and the observed height of the meniscus of the barometer.

Capillary Depression of Mercury for Tubes of Different Diameters, and for Different Meniscus Heights
(Kohlrausch, 1910)

Diam. of tube, mm.	Height of meniscus in millimeters							
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
5	0.47	0.65	0.86	1.19	1.43	1.80	....	....
6	0.27	0.41	0.56	0.78	0.98	1.21	1.43	....
7	0.18	0.28	0.40	0.53	0.67	0.82	0.97	1.13
8	....	0.20	0.29	0.38	0.46	0.56	0.65	0.77
9	....	0.15	0.21	0.28	0.33	0.40	0.46	0.52
10	....	....	0.15	0.20	0.25	0.29	0.33	0.37
11	....	....	0.10	0.14	0.18	0.21	0.24	0.27
12	....	....	0.07	0.10	0.13	0.15	0.18	0.19
13	....	....	0.04	0.07	0.10	0.12	0.13	0.14

3. Pressure of mercury vapor: The correction is less than 0.01 mm. for temperatures under 40 deg. cent., and hence may be disregarded.

4. To correct for latitude and elevation: Multiply height of the column by $g_a/g_{45} = (1 - 0.0026 \cos 2a - 0.0000002H)$, H being in meters (see p. 84).

Aneroid Barometer. Atmospheric pressure is sometimes measured by an aneroid, which is a very delicate and portable pressure gage. Aneroids are frequently used in power-plant testing and for determining elevations. They must be frequently calibrated by comparison with a mercury barometer.

Prices: Mercury barometers, $15 to $35. Aneroids, $4 to $25, according to size, finish and accuracy.

Pressures are usually measured with **Bourdon gages**, consisting of an elastic brass tube of oval cross-section, bent into an arc. In another type of gage the pressure is applied to a diaphragm or plate in place of the Bourdon tube. The tube type is generally considered the more accurate. Gages of the tube type can be used for determining the pressure of steam, water, air, ammonia, etc. For ammonia, steel must be used instead of brass. For low pressures, **U-tubes** filled with water, mercury, gasoline, or other liquids are often used. A light liquid, such as gasoline or water, permits a greater degree of accuracy than mercury. For accurate work, vacuum is nearly always measured with a U-tube, although Bourdon and diaphragm gages are also adapted for measuring vacuum. Gages of either the Bourdon or diaphragm types when arranged for measuring on the same dial both pressure and vacuum are called **compound gages.**

Bourdon gages must never be exposed to temperatures over 150 deg. fahr. If the tube is heated above this limit it is likely to lose some of its temper. Above this temperature some form of siphon or water seal must always be used to prevent the heated fluid from entering the gage. Hydrostatic pressure must be taken into account. If the gage is located below the surface of the water in a boiler, it will read too high; the error being the pressure equivalent to the head of water between the surface in the boiler and the center of the gage. The correction in lb. per sq. in. is 0.433 times the head in feet.

When a **water column** accumulates on top of a mercury column, the length of the mercury column must be corrected. This **correction** is most conveniently made by dividing the length of the water column by the specific gravity of mercury (13.6) and adding this equivalent length to the mercury column on which the water rests. For capillarity corrections, see above. Mercury columns should be read at the top of the meniscus and water columns should be read at the bottom. Except in very small tubes, the errors due to capillarity may then be regarded as negligible.

Draft in flues and chimneys, as well as that produced by **fans** when the velocity of the air exceeds 2000 ft. per min., is conveniently measured with a U-tube manometer filled with water or some very light oil. The difference in level of the liquid in the two legs, expressed in inches of water, is the conventional measure of the force of the draft. Draft gages with inclined tubes can be read with greater accuracy than those with vertical tubes.

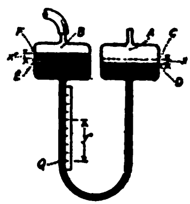

FIG. 3.—Draft Gage.

A very delicate **draft gage** can be made by connecting two glass vessels A and B by a glass U-tube of small diameter, as in Fig. 3. Water is put into A and oil into B, with the surface of separation at Q when A and B are both subjected to the same pressure. When B is connected to a vacuum chamber the levels are changed from E to F in B (x in.) and from C to D in A (also x in.), and the surface of separation moves y in. If s = specific gravity of the oil, then the change of pressure p in in. of water in A and B is $p = x + sx + y(1 - s)$. If the areas of A and B are equal and are n times the area of the connecting tube, then $y = nx$, and $p = \frac{y + sy}{n} + y(1 - s) = y\left[\frac{1 + s}{n} + 1 - s\right]$.
The smaller the factor in the brackets the greater will be the magnification or sensitiveness. The value of n should therefore be made as large as possible, say about 100, and the two liquids should be preferably nearly of the same specific gravity. Gasoline and brandy are frequently used for the two liquids. They are of nearly the same specific gravity and the line of separation is readily observable. If S_b is the specific gravity of the brandy and S_g that of the gasoline,*

$$p = y\left[\frac{S_b + S_g}{n} + S_b - S_g\right].$$

Another method of using the same apparatus is illustrated on p. 285.

Calibration of Gages. Gages are commonly calibrated either by comparison with a **test gage** or by use of the **gage tester.** Test gages must be calibrated from time to time with some standard apparatus to insure their accuracy. A vacuum gage is usually calibrated by comparison with a mercury column. The gage and column should both be connected to the suction of an air pump or to an aspirator.

Prices of Gages and Testers. Pressure and vacuum gages of good quality and of standard makes cost from $5 to $75, according to size and finish. Gages with a 6¾-in. dial—a suitable size for use on small engines—cost from $8 to $10; with 10-in. dial, from $15 to $25. Standard test and ammonia gages cost about twice as much. Recording gages cost from $40 to $75. (Bristol Co., Waterbury, Conn., Precision Instrument Co., Detroit, and Schaeffer & Budenburg, Brooklyn, N. Y.) Dead-weight gage testers, intended for the usual boiler pressures, cost about $50. (Crosby Steam Gage & Valve Co., Boston, Mass.) Ellison's differential pressure and draft gages (modified U-tubes) cost from $10 to $15. (American Steam Gauge and Valve Mfg. Co., Boston, Mass.)

DETERMINATION OF THE MOISTURE IN STEAM

The Throttling Calorimeter is most generally used for determining the moisture in steam. It consists of a vessel C (Fig. 4), into which steam enters through a small orifice O. If there is not too much moisture in the steam this throttling produces superheating. Temperature and pressure after expansion are indicated by a thermometer T and by a manometer attached to the cock V_2. Many throttling calorimeters are made with a large free opening on the exhaust side so that the pressure in the calorimeter is atmospheric. The valve V_2 may then be omitted. All the connecting piping and valves between the calorimeter and the steam main should be as short as possible and carefully covered with non-conducting material to reduce radiation to a minimum. For the usual range of boiler pressures throttling calorimeters cannot be used to determine the moisture in steam when it amounts to more than from 3 to 5 per cent. By connecting the vessel C to a condenser this range may be extended. The sampling pipe A should be made, as recommended by the American Society of Mechanical Engineers, of ½-in. pipe, extending to within ½ in. of the opposite side of the steam pipe, and should be closed at its end. At least twenty ⅛-in. holes (uniformly distributed and preferably drilled in spiral rows) are to be provided for the entrance of steam into the sampling tube. The first of these holes is to be not less than ½ in. from the wall of the pipe.

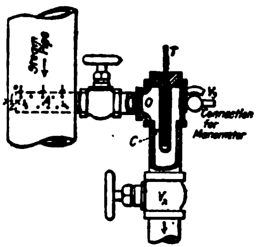

Fig. 4.—Throttling Calorimeter.

The **quality or relative dryness of wet steam** may be easily calculated from the following formula. Let p_1 = steam pressure in main, lb. per sq. in., abs.; p_2 = steam pressure in calorimeter, lb. per sq. in., abs.; t_c = temperature in calorimeter, deg. fahr.; l_1 and h_1 = heat of vaporization and heat of liquid corresponding to pressure p_1, B.t.u.; H_2 and t_2 = total heat (B.t.u.) and temperature (deg. fahr.) corresponding to pressure p_2; c_p = specific heat of superheated steam (assume 0.47 for approximately atmospheric pressures existing in calorimeters); x_1 = initial quality of steam; and $1 - x_1$ = initial moisture in steam; then $x_1 = [H_2 + 0.47(t_c - t_2) - h_1]/l_1$.

Chart for Moisture Determinations. The diagram in Fig. 5 provides a convenient and accurate means for determining the quality of steam without using the formula given above. Horizontal lines represent the process in a throttling calorimeter. As an example of its use, let initial pressure of the steam be 165 lb. per sq. in. abs., the temperature in the calorimeter 270 deg. fahr. and the pressure therein 15.2 lb. per sq. in. abs. Starting at the intersection of the temperature line for 270 deg. fahr. with the 15.2-lb. pressure line and going across the chart horizontally to the 165-lb. pressure line, the "lines of constant quality" indicate that the quality for this case is 0.979.

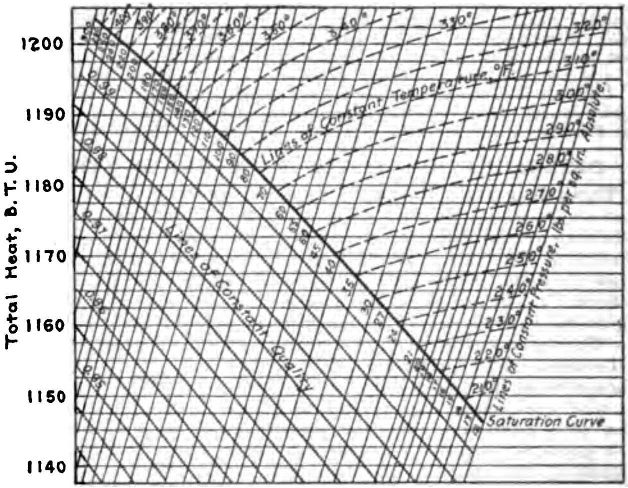

FIG. 5.—Chart for Determining the Quality of Steam.

A throttling calorimeter may be easily made up of pipe fittings, inserting a flat disk with a 3⁄32-in. hole for an orifice. The cost of an instrument made in this way should not be more than $5.

Separating Calorimeters may be used for determining the quality of steam which contains more moisture than can be determined by a throttling calorimeter. A simple form is shown in Fig. 6. The moisture is removed from the sample of steam by mechanical separation just as in the ordinary steam separator installed in the steam mains of a power plant. Steam enters at *A*, passes down through the vertical pipe into the perforated basin *B*, from which the dry steam escapes through a narrow slot near the top into the jacket *J*, while the moisture is deposited at the bottom of the vessel *V*. The volume or weight of the moisture can be determined from the height of the water in the gage glass *G*. Dry steam from the jacket *J* is discharged from the orifice *O* and must be condensed and weighed in a vessel containing cold water. The percentage of moisture is found by dividing the weight of water collected in the vessel *V* by the sum of the weight of steam condensed and the weight of water collected in *V*.

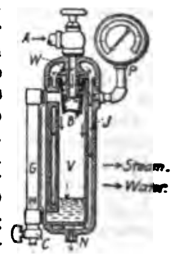

FIG. 6.—Separating Calorimeter.

Combined Separating and Throttling Calorimeters. The separating calorimeters already described are effective in removing practically all of the moisture in steam when the pressure is not lower than 25 lb. per sq. in.

gage pressure. For lower pressures, such calorimeters may not take out more than 80 per cent. of the moisture (*Trans. A. S. M. E.*, 1910, p. 76). Consequently, for determinations of moisture in low-pressure steam, a throttling calorimeter should be attached to the discharge of the separating calorimeter. A throttling calorimeter discharging into the atmosphere has very little capacity when used with low steam pressures. By making it discharge into a receiver in which a high vacuum is maintained, the throttling portion of the calorimeter will evaporate 2 to 3 per cent. of moisture. A combination separating and throttling calorimeter for low-pressure steam is shown in Fig. 7. The ½-in. brass nozzle of the sampling tube is directed against the flow of the steam. The lip of this nozzle is filed to a knife-edge to reduce the disturbance of the current to a minimum. This sampling orifice should be located at a distance of one-sixth of the diameter of the pipe from its side. The lever cock between the separating and the throttling calorimeters produces the necessary throttling action. In the throttling portion steam is condensed in a vacuum receiver, from which it flows to a volumetric measuring tank. A spyglass is used for observing whether moisture is passing through. As the spyglass is not large enough to carry all the steam, a by-pass connection is provided. All parts are covered with magnesia covering 2 in. thick. Radiation can be made less than 0.1 per cent.

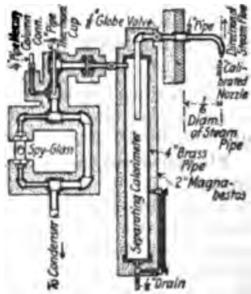

FIG. 7.—Combined Separating and Throttling Calorimeter.

Calculation of Percentage Moisture for Combination Separating and Throttling Calorimeter. The quality of steam x_1 is calculated for a combination calorimeter as follows: Let w_1 = weight of moisture collected in the separating calorimeter in a given time, lb.; w_2 = weight of dry steam condensed after passing through the throttling calorimeter, lb.; and x_2 = quality of steam discharged from separating portion as determined by the throttling calorimeter. Then, without sensible error, $x_1 = w_2x_2/(w_1 + w_2)$.

If radiation is appreciable and R is weight of condensation due to radiation in pounds in a given time corresponding to the other units, then $x_1 = (w_2 + R)x_2/(w_1 + w_2)$.

Superheating Calorimeters. A number of different types of calorimeters have been devised which depend for their action on superheating the steam by heat from an external source. The most successful of these uses electrical heating. Such an apparatus is serviceable for steam of high quality as well as low. Throttling calorimeters are usually preferred on account of greater simplicity, lower cost and because electric current is often not available where tests are to be made.

MEASUREMENT OF AREAS

The areas of irregular figures such as indicator diagrams are generally determined either by measuring the lengths of ordinates drawn on the figure and inserting their values in certain formulæ or "rules," or from readings of planimeters.

In the **ordinate method**, the figure (Fig. 8) is divided by parallel lines into an even number of strips of equal width w, and the ordinates $y_0, y_1, y_2 \ldots\ldots y_n$ measured.


FIG. 8.

Letting n = number of strips (the greater the value of n the greater the accuracy of the method), the area A may be approximately determined by using one of the following formulæ:

(1) **Trapezoid Rule:**

$$A = w(\tfrac{1}{2}y_0 + y_1 + y_2 + \ldots\ldots\ldots y_{n-1} + \tfrac{1}{2}y_n)$$

(2) **Durand's Rule:**

$$A = w(0.4y_0 + 1.1y_1 + y_2 + y_3 + \ldots\ldots\ldots y_{n-2} + 1.1y_{n-1} + 0.4y_n)$$

(3) **Simpson's Rule** (see also p. 106):

$$A = \tfrac{1}{3}w(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots\ldots + 2y_{n-2} + 4y_{n-1} + y_n)$$

The various lengths required by the foregoing methods can be conveniently added by laying them off with dividers one after the other along a straight line and finally measuring the total length of the line.

Planimeters are instruments for measuring areas. The **Amsler planimeter** (Fig. 9) consists of two arms pivoted at O. At the end of one arm is a tracing point T, and at the end of the other a "fixed point" P. Attached to the tracing arm is a small graduated wheel W. In using the instrument, mark a starting point for the tracing point T on the contour of the figure to be measured and observe the reading of the graduated wheel W. If the figure is traced in a **clockwise** direction, back to the starting point, the area measured is found by subtracting the first reading from the last; but if the tracing point is moved around in a **counter-clockwise** direction, the last reading must be subtracted from the first. The foregoing applies to small areas of only a few square inches, such as indicator diagrams, etc. For large figures the area of the zero circle of the instrument enters into the calculation of areas. This correction has to be added whenever the fixed point is inside the area which is being measured. Its value can be determined by measuring a circle or other figure of known area. The difference between the known area and that recorded by the instrument is the area of the zero circle. The area of the zero circle is stamped conspicuously on many planimeters. For measuring figures of indefinite length and limited breadth, **roller planimeters** must be used. They are expensive and are not much used in America.

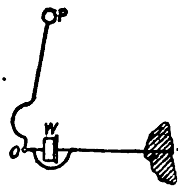

Fig. 9. Planimeter.

Integrators for Circular Charts. Instruments have been developed for measuring the mean ordinate of circular charts having a constant radial scale. Such instruments are made by the Bristol Co.

Prices of Planimeters. Amsler's "polar," \$7 to \$25 (Keuffel & Esser Co., and Eugene Dietzgen Co., New York); Coffin's "averager," \$20 to \$35 (Ashcroft Mfg. Co., New York); Coradi's roller, \$25 to \$50; Bristol-Durand integrator, \$15 to \$25 (Bristol Co., Waterbury, Conn.).

INDICATORS AND REDUCING MOTIONS

Indicators. Outside-spring indicators are coming into general use, and are particularly desirable for use with steam at high temperatures and for internal-combustion engines. In this type of indicator the elasticity of the spring cannot be affected by the steam temperature and the springs can be readily changed without removing parts of the indicator usually uncomfortably heated.

The **essential requirements for a good indicator** are (1) an accurately calibrated and elastic spring; (2) a light piston; (3) a small and light pencil mechanism and drum; (4) a "free" and practically frictionless movement of the pencil on the paper. The **scale or "number"** of a spring indicates the pressure in lb. per sq. in. producing a vertical movement of .1 in. at the pencil point. For high-speed engines the makers of the Thompson indicator recommend the following scales:

Maximum pressure, lb. per sq. in.	6	12	20	30	45	60	70	80	100	125	150	180	200	250
Scale or "number" of spring....	12	16	20	24	30	40	50	50	60	80	100	120	150	175

For slow-speed Corliss engines the following scales may be used:

Maximum pressure, lb. per sq. in..	8	12	18	30	40	50	70	90	125	150	200
Scale or "number" of spring.....	8	10	12	16	20	24	30	40	50	60	80

For engines operating at not over 200 r.p.m. the Crosby Co. recommends the selection of a spring making the height of the diagram for the maximum steam pressure at the engine about 1¾ in. For high-speed engines it is desirable to take smaller diagrams to avoid excessive inertia effects of the indicator piston and pencil mechanism. It is also recommended that the length of the diagram should not exceed 2½ to 3 in.

Indicator **springs** are never quite accurate for all heights of diagrams, even when new, so that for accurate work, as for example the making of guarantee tests of engines, the springs should be calibrated before and after a test. Calibrations of indicator springs that are good enough for rough checking in ordinary service can be made by comparing the vertical pencil movement with an accurate "test" steam gage at about 5 "steps" of pressure above and 3 below the atmospheric line. For accurate work comparison must be made with the indications of a mercury column or of some other special calibrating device.

There are other **errors** in indicator **diagrams. The drum** of the indicator is usually moved by means of a cord, the stretch of which varies throughout the reciprocation. When steel wires are used instead of a cord, errors due to stretching are rendered negligible; but the wire has the disadvantages of being stiff and easily tangled, while its weight introduces an objectionable element. The **indicator cord** used should be of the very best quality and its length should be made as short as possible. For lengths greater than 3 ft., wire should be used. The stretching of the cord is due to (1) inertia of the drum, causing the tension in the cord to be variable and the indicator diagram too long; (2) friction of the drum, causing stretching of the cord so as to make the position of points in the diagram lag behind the true position by an amount equal to the stretch.

Errors due to varying tension in the cord caused by inertia can be made very small by using a drum spring of a stiffness suitable for the speed of the engine. For indicating a high-speed engine a very stiff drum spring is required. The distortion and variation in length of the indicator diagram can be shown by turning the engine over by hand and taking the length of the stroke of the drum when inertia is negligible, and afterward running the engine at various speeds and taking diagrams. Such tests will show the suitableness of a drum spring for high speeds. If there is a large variation, a stiffer spring should be obtained. This error is also proportional to the length of the diagram, so that shorter diagrams are advisable at high speeds.

Drum friction and its effect in stretching the cord have been shown by Prof. O. Reynolds (*Proc. Inst. C. E.*, vol. 83) to have an appreciable effect on the calculated value of indicated horse power. He shows that the error in mean effective pressure is proportional to the stretching of the cord caused by drum friction, and that the least amount of stretch in a good indicator cord is 0.4 per cent. of its length. In the case of a cord 3 ft. long, the stretching under the best conditions would be about 0.14 in.

The errors due to inertia and friction have opposite effects, the one making the diagram longer and the other making it shorter. The indicated horse power at moderate speeds should be not more than 2 or 3 per cent. in error if the indicator drum spring and quality of cord are properly selected for the pressure and speed, assuming that the piston and drum move freely and are well lubricated (*Proc. Inst. M. E.*, 1905, pp. 785–788) and the piston spring has been carefully calibrated. At speeds exceeding 300 to 350 r.p.m. the probable error increases rapidly.

Automobile, aeroplane and other engines operating at speeds exceeding 500 to 600 r.p.m. require special indicators. The best known is the **manograph** or diaphragm type, in which the pressure in the engine cylinder deflects a diaphragm which in turn moves a small mirror about an axis in a vertical plane, while a somewhat complicated reducing motion deflects the mirror about an axis in a horizontal plane. A beam of light reflected from the mirror traces the diagram on a ground glass or a photographic plate. An instrument of this kind shows practically no inertia effects and there is no cord used to cause errors by its stretching. It is liable to very large errors. Another type of **optical indicator** devised by Prof. Hopkinson is also used in England. It differs essentially from the manograph in having a piston instead of a diaphragm. It can be made to give excellent results.

Precautions in the Use of Indicators. Unless an engine indicator is carefully handled the diagrams taken with it may be in error from 5 to 10 per cent. The following are the most important considerations:

1. Springs must be calibrated frequently.

2. The tension of the spring in the drum should be adjusted so that the drum revolves without slackness in the cord on the return stroke. If the cord sags on the return stroke the error may be considerable, but if the tension of the cord does not vary a great deal, errors due to stretching of cords of ordinary lengths are not serious.

3. Before an indicator is used all working parts should be cleaned and oiled. The piston and its rod should be examined before attaching the spring to determine by lifting the pencil lever and letting it fall whether these parts move easily. Attach the spring firmly and observe carefully that there is no lost motion. This precaution is most important in indicators having a bead on the spring for making a ball-and-socket joint. To put this kind of spring in place properly, the piston rod should be screwed tightly into the piston when the lower adjusting nut is loose. Then screw up this nut just tight enough to permit a slight movement. If the nut in the piston has been properly adjusted there should be no lost motion between the rod and the piston, and still there should be flexibility in this joint permitting the piston to adjust itself in the indicator cylinder.

4. Oil the piston with cylinder oil every time it is taken from the cylinder. Many careful engineers oil the piston regularly after taking about ten diagrams. A new piston usually requires more lubrication than one that is well worn.

5. Adjust the handle on the pencil motion so that when the pencil is sharp it will draw a very fine line. If the pencil presses heavily on the paper, friction and shifting of the paper may distort the diagrams. Prepared metallic paper if used with blunt brass points increases pencil friction considerably above what it is when sharp lead pencils or ordinary "household" pins are used.

6. Adjust the length of the indicator cord. If too short, it will be broken when the engine is started and may also injure the indicator; and if too long, the drum will strike against its stops and give a deformed diagram. After adjusting the length of the cord the engine should be turned over by hand to make sure the cord is of proper length.

7. Immediately after a diagram has been taken it should be examined and any irregularities or faults noted and corrected.

8. Mark diagrams plainly as regards head and crank ends of double-acting engines, the time, scale of spring, and name or initials of the person taking them.

9. Best results are always obtained with an indicator at each end of the cylinder of a double-acting engine. Tests show that sharp bends, long pipes and restrictions of bore may cause errors in indicator diagrams as great as 20 per cent. (W. F. M. Goss, *Trans. A. S. M. E.*, 1896.) This is a particularly important consideration in air and ammonia

compressors having very small clearance, and in long-stroke engines where the piping for a 3-way cock for a single indicator would add considerably to the clearance.

10. The indicator cock should be kept closed and the cord to the reducing motion should be unhooked except when a diagram is being taken.

Prices of Indicators. Crosby, $80 (Crosby Steam Gage & Valve Co., Boston); Thompson, $50 to $75 (American Steam Gauge and Valve Mfg. Co., Boston, and Trill Indicator Co., Corry, Pa.); Tabor, $65 (Ashcroft Mfg. Co., New York); Manograph (optical), $300 (Hugo C. Gibson, W. 135th St., New York); Hopkinson's (optical), $100 (Dobbie-McInnes Co., Ltd., Glasgow and London).

Reducing Motions

To obtain the conventional indicator diagram the drum of the indicator must be moved so as to give on a smaller scale an exact reproduction of the motion of the piston. A device accomplishing this is called a perfect reducing motion. Many devices used quite generally do not give a perfect motion.

The Pantagraph (Fig. 10) is a theoretically perfect device, and is actually accurate so long as there is no slackness at the numerous pinned joints. The point of attachment of the indicator cord *B* must be in the straight line joining the fixed point *C* with the connection on the cross head *A*. Fig. 11 is a simple reducing motion that is nearly perfect when properly laid out. The pin *D* is attached to the cross head *C* and the link *BD* connects the cross head to the oscillating arm *AB*. To convenient points, as 1, 2, or 3, the indicator cord is attached. If the arc *XY* in its extreme positions at the ends of the stroke reaches as much above the center line of motion of the pin *D* as it does below, the movement of the drum will be almost an exact duplicate of that of the piston. If the indicator motion is taken from the heavy dotted circular segment *G* instead of from a pin, the motion is more accurate. Fig. 12 is also a simple device and is fairly accurate if *AB* is made equal to *BC*. The cord is attached to *D*.

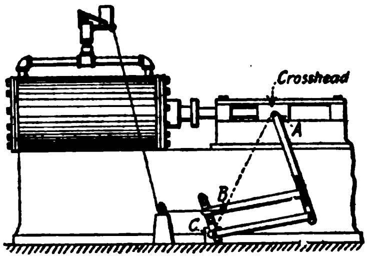

Fig. 10.—Pantagraph Reducing Motion.

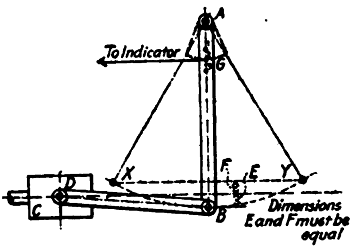

Fig. 11.

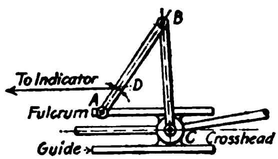

Fig. 12.

Simple Indicator Reducing Motions.

The method illustrated in Fig. 13 consists merely in attaching the indicator cord to a pin on the crank shaft and then changing its direction by passing it over a pulley not far from the shaft. In such an ar-

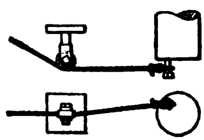

Fig. 13.

rangement the motion of the drum will not be even approximately proportional to that of the engine piston. Indicator diagrams taken with this device will be very much in error unless the pulley changing the direction of the cord is located at a considerable distance from the shaft. To reduce the error due to a long cord, wire should be used in such cases. Fig. 14 shows an accurate and simple reducing motion for attachment to the crank shaft. The ratio of the lengths of the connecting rod and stroke of this device must be the same as the corresponding ratio in the engine. A similar device is shown in Fig. 15.

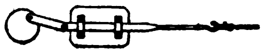

FIG. 14.

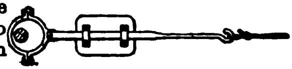

FIG. 15.

Reducing Wheels are coming into very general use. They may either be attached to the drum of the indicator or to some convenient part of the engine. Fig. 16 shows a typical device. The large wheel receives the cord direct from the cross head and the cord on the small wheel is connected direct to the drum of the indicator. Ratio of diameters gives the reduction. Each reducing wheel is provided with a nest of rings to be put on the small wheel to increase its diameter so that various reductions are possible to suit the stroke of the engine. The indicator drum is started and stopped by turning the knurled nut on top of the small wheel. The large wheel has a spring at its center to bring it back on the inward stroke. In engine testing for long periods it is desirable to disconnect the cord connecting the large pulley with the cross head during the intervals between taking diagrams. Hooks like those shown in Figs. 17 and 18 are very convenient for this purpose, particularly if attachment of the cords can be made to a pin on the cross head. The Trill hook (Fig. 17) is intended to be held between the thumb and finger, about an inch from either end of the stroke, so that the pin or standard on the cross head strikes the straight part of the hook, and immediately the pin or standard will be caught. The hook shown in Fig. 18 is used in the same way. The hook is of spring brass and is held so that the pin or standard *A* of the cross head strikes the crotch when about 1 in. from the end of its stroke and opens the hook and assumes the dotted position when in operation. In disengagement the hook is slipped off the end of the pin. Unless some special form of hook is used, as described, it is difficult to connect the cord when a diagram is to be taken, and the cord is likely to get tangled in moving parts of the engine or to be broken. Such difficulties can often be avoided, particularly in the case of such devices as Figs. 14 and 15, by continuing the cord from its point of attachment to the reducing motion closely past the indicator drum to a pipe or simple bracket that may serve as a stationary support. Between the indicator and this last support a spiral or helical spring of rather light wire or a heavy rubber band is attached to the cord, and will keep it taut and in motion. If now a ring is attached to the cord close to the indicator and between it and the reducing motion, this ring will be continually in motion but it will not be difficult to hitch into it the hook on the portion of cord

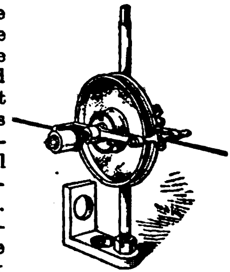

FIG. 16.—Reducing Wheel.

FIG. 17.

FIG. 18.

connected to the indicator drum. This method is particularly recommended in every case where wire is used instead of cord.

Some indicators are provided with a detent, or device for engaging a pawl in teeth on the circumference of the drum near the bottom and stopping the drum. Obviously, the pawl must engage when the cord is pulled out to the end of its stroke. On the return stroke the cord will flap about and possibly catch on something and be broken on the next outward stroke. The best way to prevent this is to keep the string always taut by using a helical spring or rubber band attached to some fixed point, as explained above. In all types of reducing motions except those in which the cord is taken off on a tangent to an arc of a circle, the direction of the cord as it moves back and forth at its point of attachment to the reducing motion must be parallel to the movement of the engine piston. Reducing motions are sometimes made inaccurate by not locating auxiliary pulleys properly to give the cord its proper direction.

WEIGHING DEVICES

The following devices are ordinarily used in engineering work to determine weight:

	Usual capacity, lb.	Probable sensitiveness, lb.
Platform scales	100 to 2,000	1/16 to 1/4
Spring balances	5 to 200	1/16 to 1
Torsion balances	10	1/32 to 1/4
Automatic scales:		
1. Pendulum type	50	1/32
2. Spring type	200	1/16
Chemical balances	50 (grams)	1 in 100,000

Platform Scales are best suited for most weighing operations. This type is quite sensitive when new, but its original sensitiveness is soon lost if the knife edges are allowed to become rusty from exposure to dampness, or become dulled from careless and excessive loading. All scales should be tested with standard weights and adjusted whenever used in important work. **Spring and torsion balances** are usually not very reliable and are used chiefly because they are easily portable and compact. Zero reading is not dependable. Careful calibrations are essential. **Automatic indicating scales,** particularly of the pendulum ("Toledo") type, are the best for weighing the fuel consumption of oil engines. A pointer on a dial indicates continuously the weight, so that observations can be made rapidly and without balancing. Devices of this type, operating with a spring mechanism, are subject to the same faults as the usual types of spring balances. **Automatic scales** for coal bunkers are filled under the chute from the bunker till a predetermined weight has accumulated. The supply is then shut off, and the scoop trips and discharges. Such devices are usually integrating, while automatic scales for weighing fuel being conveyed to power houses and furnaces are recording (see p. 1183).

Prices of Scales. Small platform (portable, for floor use), $15 to $50; (for counter use) $7 to $25; (for weighing wagon and car loads) $125 to $500; spring balances, $0.50 to $10; automatic pendulum scales, $50 to $100 (Toledo Scale Co., Toledo, O.); chemical balances, $25 to $100; integrating scales for coal bunkers, etc., $200 to $1000 (Fairbanks Co., New York and Chicago; or Streeter-Amet Co., Chicago).

MEASUREMENT OF POWER

Dynamometers, or instruments for measuring force or "power," are in general of two kinds: (1) Those **absorbing** the power by friction and dissipating it as heat; (2) those **transmitting** or passing on the power they measure, and wasting only a small part in friction. Devices for measuring power may be classified for convenience as follows:

Type	Approx. limit of speed, r.p.m.	Usual power limit, h.p.	Probable error, per cent.
Prony brakes:			
Block	1,000	10	1
Band	1,000	5	1
Wooden cleats on bands	1,000	100	1
Rope (¾-in.) with wooden cleats	1,000	50	1
Fluid friction dynamometers:			
Froude (ordinary "water" brake)	2,500	500	1
Westinghouse (turbine)	4,000	5,000	0.2
Alden	300	3,000	1
Fan brake	2,000	200	1 to 5
Electric eddy-current brake	2,000	100	1
Electric generator	750 to 4,000	30,000	0.2
Transmission dynamometers:			
Torsion	1,000 to 3,000	50,000	2 to 5
Emerson	300	50	2 to 5
Kenerson	1,500	100	2

The speed limits given above are approximately the highest allowable. Limits of horse power refer to the largest sizes made commercially. Any of these types (with the exception of the Westinghouse turbine) are made in sizes to absorb from 1 h.p. up. The probable error stated is very approximate and refers to the apparatus in fair adjustment.

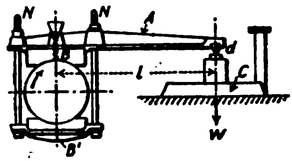

Fig. 19.

Absorption Dynamometers

A Prony Brake consists of a lever A (Fig. 19) and blocks, B, B' supported on a revolving drum or pulley. The blocks are held in place and tightened by the thumb nuts N, N. The tendency of the arm A to revolve is prevented by the resistance of a platform scales C as shown, or by weights attached. If the pressure on the pedestal at d due to its own weight and that of the lever arm A is W_0 lb. (determined by the weight on the scales when the brake block is supported at B on a three-cornered prism or on a small rod of circular section), the gross weight at the end of the brake arm with load is W lb., the length of the brake arm in ft. is l, and the number of revolutions per minute is n, then brake horse power (b.h.p.) $= 2\pi ln(W - W_0)/33{,}000$.

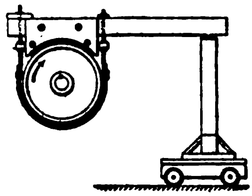

Fig. 20.

According to Bach, suitable **dimensions for a brake** of this type are given by the formula $bd =$ b.h.p. $\times 12/k$, where d is the diameter of the brake pulley in in., b the breadth of the brake blocks in in. (usually about 1.5 times the diam. of the shaft), and $k = 1/2$ for air cooling and varies from 2.5 to 5 for water-cooling as

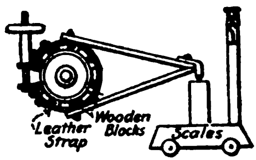

Fig. 21.

Fig. 19–21.—Types of Prony Brakes.

the speed increases. F. A. Halsey (*Am. Mach.*, April 4, 1912) states that the brake drum surface for a water-cooled Prony brake should not be less than 0.09 sq. ft. per b.h.p., to avoid the danger of the blocks taking fire.

In Fig. 20 the lower block in the preceding figure is replaced by two steel bands and narrow cleats of maple or oak attached by wood screws inserted from the outside through the bands; the upper block is lined with similar cleats, the screws being countersunk. At least ¼-in. spaces should be allowed between cleats for air circulation. In all such constructions for dynamometers, screws and nails should not touch the friction surface, as they are likely to cause the friction to be variable and the sound produced is objectionable. Grooves may be cut in the inside surface of a few of the cleats, and these grooves filled with grease to provide a little lubrication. A similar brake is shown in Fig. 21, where maple cleats are screwed to a leather belt. This type is more easily adjustable to different sizes of pulleys than the preceding designs, but it is not as durable. Washers should be placed on the heads of the screws fastening the cleats to the belt. If a brake wheel similar to Fig. 22 is used so that it can be satisfactorily cooled with water on its inside surface, the cleats should provide 5 to 10 sq. in. of friction surface per b.h.p. according as the speed ranges from low to high.

Fig. 22.

Band and Rope Brakes are also frequently used. The simplest form is shown in Fig. 23. If P is the reading of the spring balance in lb., r the wheel radius in ft., Q the applied weight in lb. (both P and Q must be net), and n the r.p.m., then b.h.p. = $2\pi rn(Q - P)/33{,}000$. This type of brake is very **accurate** and **sensitive**, but it is suitable only for low powers. About the same friction surface must be allowed as given for wooden blocks by Bach's formula (see Prony Brakes, *ante*). A convenient type of **rope brake** with "stay" cleats is shown in Fig. 24, the rope being passed around the circumference of the pulley. In this arrangement the cord or rope supporting the spring balance must have some point of attachment overhead. It is advisable to provide an anchoring rope or wire securely attached to the weights P, and its weight (or that part of it suspended) must then be added to P. Similarly, the weight of the rope between the spring balance and the point where it touches the pulley should be deducted from the readings of the spring balance. Usually 4 to 6 cleats are used on rope brakes. These cleats are often attached to the ropes by strong copper wire passed through a strand of the rope and over the backs of the cleats. The best arrangement is to place the rope on

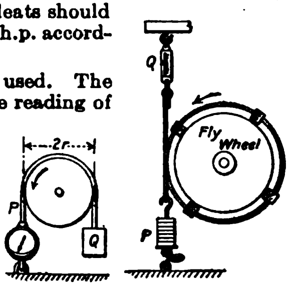

Fig. 23. Fig. 24.
Rope Brakes.

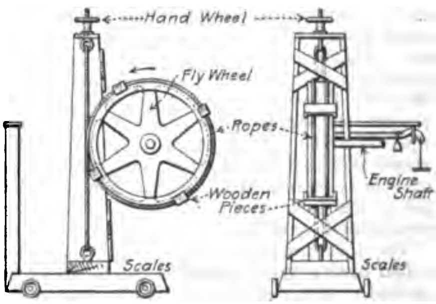

Fig. 25 Rope Brake.

the wheel double. In Fig. 25 an arrangement is shown in which both ends of the rope are attached to a rigid frame supported on weighing scales. The reading of the scales (corrected for the weight of the frame) gives $Q - P$ directly. A double rope ¾ in. in diameter with 6 cleats each of about 12 sq. in. on the "friction" side will absorb 50 b.h.p. For engines of smaller power ½-in. rope with 4 cleats is used. By steeping the rope in a mixture of melted tallow and graphite the frictional properties are improved. Manila or cotton ropes are used.

Fluid Friction Dynamometers are of the absorption type. There are three kinds. One arrangement is similar in principle to a centrifugal pump, the casing being balanced and supported on the pump shaft. An attached brake arm supported on scales measures the turning moment. This is the **Froude** or ordinary **"water" brake,** which differs from a centrifugal pump in that the runner is made with deep recesses to increase the resistance. The tendency is to carry the casing around in the direction of rotation. If $(W - W_0)$ is the net pressure at the end of the brake arm and l is the length of the arm, then the brake horse power is calculated in the same way as for a Prony brake (see *ante*). The **Westinghouse turbine hydraulic dynamometer** is a second type. It is essentially a double-flow steam turbine with blades shaped to produce great resistance to flow. The casing is balanced on the shaft and provided with a brake arm in the same way as the Froude dynamometer. Power is also calculated in the same way.

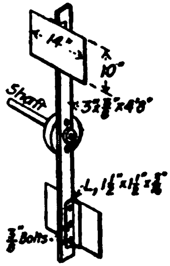

Fig. 26.—Fan Brake Dynamometer.

Alden's Dynamometer has for its rotor simply a flat cast-iron disk which revolves between two flexible copper plates attached to the inside of the casing. The iron rotor revolves in heavy cylinder oil. There is a small clearance on each side between the ends of the cylindrical casing and the copper plates, and water under pressure is forced into these spaces. By increasing the pressure the copper plates will be brought nearer the rotor and the friction will be increased. The turning moment is measured with a lever arm resting on a platform scales. (For a more detailed description of absorption dynamometers, see Royd's "Testing of Motive Power Engines," pp. 89–98; Moyer's "Power Plant Testing," pp. 127–131.) If the temperature of the water is raised 50 deg. fahr., dynamometers of this type require about 50 lb. water per b.h.p. per hour; but if the water is entirely evaporated, only about ⅒ as much.

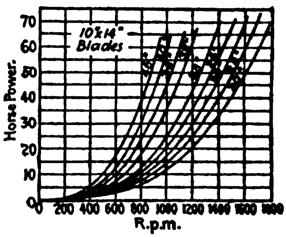

Fig. 27.—Power Absorbed by Fan Brake Dynamometer.

For the approximate determination of the horse power of high-speed engines a **fan brake dynamometer** consisting of two fan plates (Fig. 26) to be attached to the shaft is very convenient but is less reliable than any of the other methods described. The power is absorbed by the "fan" action of the plates on the surrounding air, which depends on the size of the plates, their distance from the center of rotation, and upon the cube of the r.p.m. Fig. 27 shows curves for determining the b.h.p. for varying speeds of a fan with rectangular blades 10 in. wide in a radial direction, 14 in. wide axially, and ¼ in. thick. These curves are for atmospheric pressure of 14.7 lb. per sq. in. and a temperature of 70 deg. fahr. Errors are introduced by other pressures and temperatures. The numbers on the curves indicate inches from the center of one plate to the center of the other. This distance is varied by shifting the bolts. This is probably the simplest as regards construction and most compact of all the types of dynamometers.

Eddy-current Brakes are made with a number of electromagnets and one or more copper disks. Either the electromagnets may be rotated with the shaft while the copper disks are held stationary, or *vice versa*. Eddy currents generated by rotation with the magnets excited produce a resistance, of which the moment is measured by a lever arm and scales as with similar forms of dynamometers. (For further information regarding the design of this type see "Eddy-current Brakes," *Jour. Inst. E. E.*, vol. 35.)

Dynamos as Power Dynamometers. One of the most convenient means for measuring the power of high-speed engines and turbines is to connect an electric generator to the main shaft. Then, if the efficiency of the generator is known at the particular speed and output at which it is operated, a very accurate method of measuring the power of the engine or of any other type of motor becomes readily available. For direct-current generators, b.h.p. = (volts × amperes)/(746 × efficiency of generator).

The load on the generator should be maintained uniform by absorbing electrical output in lamp or wire resistances, but for larger powers a water resistance (see p. 1593) is generally used. In some places the dynamo has been used as a brake, by supporting the dynamo frame on its shaft and measuring the resulting torque by a lever arm and scales.

Transmission Dynamometers

Torsion Dynamometers. When a shaft is subjected to a twisting moment an angular twist is produced which is proportional to that moment. Thus, if the angle of twist is produced by a twisting moment T in in.-lb. and n is the r.p.m., then b.h.p. $= 2\pi Tn/(12 \times 33{,}000)$.

Torsion meters, although applicable to large as well as small powers, have their most important applications for measuring shaft horse power of marine turbines and engines. (For descriptions of various types of transmission dynamometers, see Carpenter and Diederichs, "Experimental Engineering," pp. 293–318.)

Prices of Dynamometers. Prony brake, \$5 to \$25; rope or band brakes, \$2 to \$5; electric brakes (50 h.p.), \$1000 to \$2000 (Sprague Electric Co., New York; Diehl Mfg. Co., Elizabethport, N. J.); water brakes (25 h.p.), \$200 to \$500 (Standard Motor Co., Jersey City, N. J.; Michigan Motor Specialty Co., Detroit, Mich.); Kenerson torsion dynamometer, \$150 to \$400 (Builders' Iron Fdy. Co., Providence, R. I.); Emerson power scales (Florence Machine Co., Florence, Mass.).

MEASUREMENT OF FLUIDS

The following tabulation serves to indicate the methods used for the measurement of the volume, weight, or velocity of fluids:

	Measurement	Permissible velocity, ft. per sec.	Probable error, per cent.
Gas meters:			
"Wet" meter	Volume	(100 r.p.m.)	½
"Dry" meter	Volume		5 to 10
Pitot tube	Velocity	8 to 400	1
Anemometer	Velocity	1 to 40	1 to 5
Electric meter	Weight	3 to 500	½
Receiver	Volume	Unlimited	1
Orifice	Weight	Unlimited	2
Venturi tube	Velocity	Unlimited	1
Steam meters:			
Pitot tube	Velocity	10 to 400	1
Orifice	Weight	Unlimited	½
Float	Volume	50 to 200	2
Water meters:			
Impulse wheel	Velocity	5 to 100	1
Rotary disk	Volume	(100 r.p.m.)	1
Piston	Volume	(150 strokes per min.)	1 to 5
Rectangular weir	Volume	Unlimited	½ to 1
V-shaped weir	Volume	Unlimited	½ to 1
Flow-proportional-to-head weir	Volume	Unlimited	½ to 1
Automatic tank	Volume	(Up to 20 lb.)	1
Venturi tube	Velocity	Varying	½
Pitot tube	Velocity	10 to 400	1

Gas Meters

"Wet" Gas Meters are used when accurate determinations of volume of gas are required. A meter of this kind consists of a drum somewhat resembling a paddle wheel revolving on a horizontal axis in an enclosed cylindrical casing partly filled with water. The principle of operation is shown in Fig. 28. There are four compartments *A*, *B*, *C*, and *D*, and passages *a*, *b*, *c*, and *d* at the center are made to operate as valves by the water seal, admitting gas to the compartments out of the water. Gas is discharged through similar valves *a′*, *b′*, *c′* and *d′* at the periphery. Smallest rates of flow are accurately measured. It is essential that the water level be maintained at the standard level of its calibration and that the apparatus be properly adjusted by the spirit levels. Fluctuations of velocity can produce errors only when large enough to cause surging of water level so that the valves are not at all times sealed. If the flow is intermittent, as in the suction line of an engine or compressor taking gas under pressure, a gas bag is generally used as a regulator. When as in suction-producer plants there is a vacuum in the suction line, a regulator consisting of a diving bell hung on springs is used.

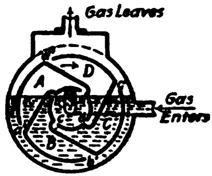

Fig. 28.—Wet Gas Meter.

"Dry" Gas Meters are used for the usual "house metering" of gas. No high degree of accuracy is claimed, but they are more adaptable for such service than the "wet" types because a water level need not be maintained. Such meters consist of two chambers separated by a vertical partition, each chamber containing an interior measuring receiver having a flexible shell. Gas is admitted to these measuring receivers alternately and automatically by means of slide valves. Reciprocating movements of filling and emptying these receivers actuate the counting mechanism.

Pitot Tubes (see pp. 255 and 285) are the most satisfactory devices for measurement of gas in large volumes. Let p = velocity pressure, in inches of water; v = velocity of flow, ft. per sec.; h = equivalent height (head) in ft. of a column of air to produce p; w_w = weight of 1 cu. ft. of water, lb. (62.4 nearly); w_a = weight of 1 cu. ft. of air at conditions of test; then $h = (p/12)(w_w/w_a) = 5.02p/w_a$, and $v = 18.3\sqrt{p/w_a}$. Tables of the weight of air for varying temperatures and humidities are given on p. 1512.

In **fan testing** the B. F. Sturtevant Co. employs a Pitot tube similar to that shown in Fig. 45, p. 285, in which the concentric brass tubes are ¼ in. and ⅜ in. in diameter and the overall dimension from nozzle tip to heel is 10 in. In place of the small holes in the outer tube of Fig. 45, however, there are two $1/16 \times 2\frac{1}{4}$-in. diametrically opposite slots beginning 3¾ in. back from the nozzle top and extending toward the heel. The form shown in Fig. 45, with small holes in the outer tube, indicates a greater volume and less static pressure than does the slit-tube type.

For finding average velocity in a circular pipe, see p. 286. For a pipe of radius R divided into 5 rings, the "stations" or points of observation would be at the following distances from the center: (1) $0.316R$, (2) $0.548R$, (3) $0.707R$, (4) $0.837R$, (5) $0.949R$. Ducts of square or rectangular section are usually divided up similarly into a series of elementary squares or rectangles.

A detailed discussion of the relative accuracy of different types of Pitot tubes is given in *Trans. A. S. M. E.*, vol. 33, p. 1137 (1911).

Anemometers are used for approximate work in measuring the velocity of air and gas. These instruments consist of a light vane wheel fixed on a shaft to which a counting mechanism is attached. Readings of the counter are taken at the beginning and end of a suitable time interval, usually ½ to 1 min. The axis of rotation must be in the direction of the flow of air. Upper and lower velocity limits beyond which they should not be used are generally stated on the cases of the better grade of these instruments. They are delicate and require frequent calibration and careful handling. Above a velocity of 2000 ft. per min. their use is not advocated. For the higher velocities a Pitot tube is the best means for calibration, and for low velocities a "wet" type of gas meter can be used. A common method of calibration, having many objections on account of the difficulty in preventing eddy currents, is to mount the anemometer at the end of a rotating arm. In such tests the anemometer records a higher value than when used for measuring a "straight-line" flow.

Prices of Meters, Pitot Tubes, and Anemometers. "Wet" gas meter (small laboratory sizes), $25 to $65 (American Meter Co., Philadelphia); "dry" meters (20 "lights"), $15 to $20 (Helme and McIlhenney, Philadelphia); Pitot tubes, $15 (B. F. Sturtevant Co., Hyde Park, Mass.; American Blower Co., Detroit), Burnham, $20 (C. F. Gebhardt, Chicago); anemometers, $18 to $35 (Keuffel & Esser Co., New York).

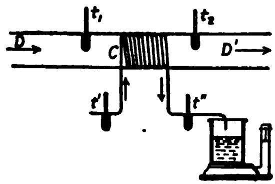

Fig. 29.

Measurements of Flow by Heating or Cooling. This method depends on taking from or adding to the gas to be measured a quantity of heat and observing its change in temperature. Fig. 29 shows a typical arrangement in which a coil C for either heating or cooling is well distributed over the section of the duct DD'. Thermometers are arranged as shown. Initial and final temperatures of the air are t_1 and t_2. Entering and leaving temperatures of the heating or cooling medium are t' and t''. If W_a = weight of air flowing per sec., lb.; W_e = weight of heating or cooling medium per sec., lb.; and s = specific heat of this medium, then $sW_e(t' - t'') = 0.2375\ W_a(t_2 - t_1)$, where 0.2375 is specific heat of air at constant pressure.

The **Thomas meter** employs this method of measuring gas. The gas flows past a series of electrical resistances enclosed in a pipe and the temperature of the gas is raised a few degrees. Temperature before and after heating is measured by resistance thermometers arranged to record temperature difference on a chart. If the electrical energy used for heating be kept constant and if the specific heat of the gas does not vary, the flow is inversely proportional to the rise in temperature.

Receiver Method of Measuring Air. None of the preceding methods are adaptable for measuring the volume of air at high pressures, as in the case of measuring the discharge in tests of air compressors. Pumping air into a suitably strong receiver is a method often used. The compressor is made to pump against any desired pressure, which is kept constant by a regulating valve on the discharge pipe.

Let P_1 and P_2 = absolute initial and final pressures for the test, lb. per sq. in.; T_1 and T_2 = mean absolute initial and final temperatures, deg. fahr.; W_1 and W_2 = initial and final weight of air in the receiver, lb.; and V = volume of receiver, cu. ft. Then, as $P_1V = W_1RT_1$, and $P_2V = W_2RT_2$, where R is the constant 53.3 for air, weight of air pumped $= W_2 - W_1 = (V/53.3)\ (P_2/T_2 - P_1/T_1)$.

In accurate laboratory tests the humidity of the air entering the compressor should be measured in order to reduce this weight of air to the corresponding equivalent volume at atmospheric pressure and temperature (see p. 339). The principal errors in this method are due to a difficulty in measuring the average temperature in the receiver. Whenever practicable, the final pressure should be maintained in the receiver at the end of the test until the final temperature is fairly constant.

Flow of Air Through an Orifice. Discharge from compressors at high pressure is also frequently measured by observing the pressures and temperatures on the high- and low-pressure sides of the orifice, see p. 353.

Durley (*Trans. A. S. M. E.*, vol. 27, 1905) gives the following coefficients for discharge into the atmosphere for circular orifices with sharp corners in a plate 0.057 in. thick where there is appreciable contraction:

Coefficients of Discharge for Sharp-edged Orifices

Diam. of orifice, in.	Pressure Difference on the Two Sides of Orifice (In inches of water column) 1	2	3	4	5
5/16	0.603	0.606	0.610	0.613	0.616
1/2	0.602	0.605	0.608	0.610	0.613
1	0.601	0.603	0.605	0.606	0.607
2	0.600	0.600	0.600	0.600	0.600
3	0.599	0.598	0.597	0.596	0.596
4	0.598	0.597	0.595	0.594	0.593
4 1/2	0.598	0.596	0.594	0.593	0.592

For accurate work the orifice should be carefully calibrated, measuring the discharge for the range of pressures at which it is to be used. For coefficients for orific meter disks in 8-in. and 10-in. pipe lines see Hickstein, *Jour. A. S. M. E.*, March, 1916. (For a discussion of nozzle and orifice coefficients for the flow of fluids, see pp. 353 to 357; also "Steam Turbines" (3d

Ed.), by J. A. Moyer, pp. 29–53, and Bendemann, in *Mitteilungen über Forschungsarbeiten*, No. 37.)

Steam Meters

Pitot-tube Steam Meters are made by the General Electric Co. The nozzle plug which is inserted into the steam pipe has two sets of holes, each set connecting into a separate pipe which is joined by unions to an indicator or recorder. The "leading" set of holes is subjected to velocity plus static pressure, and the "trailing" holes to velocity pressure less static. The principles of operation are essentially the same as for the measurement of air by Pitot tubes.

The Orifice Steam Meter (Fig. 30) requires the insertion into the steam pipe of a special flanged fitting F in which there is an orifice as shown. Pressure difference between the two tubes t_1 and t_2 located in this flange (produced by velocity) is measured by a differential mercury manometer. Without changing the orifice the apparatus is not adapted to a large range in the rate of flow. The spiral coils C_1 and C_2 are inserted for maintaining by condensation constant water levels in each of the legs of the manometer, irrespective of variations of pressure.

Float Steam Meters are designed so that a float (usually a disk or a cone) moves against a constant resistance in a passage in which the unrestricted area for the flow of steam varies with the height of the float. This principle is applied in the St. John and Sargent meters, in each of which there is a conical float connected to the registering device showing the rate of flow. In the St. John meter (Fig. 31) the float V rises with increased flow, carrying with it the arm N connected to the registering device. The accuracy of steam meters is usually not greater than ±.5 per cent.

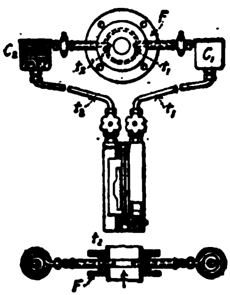

FIG. 30.—Orifice Steam Meter.

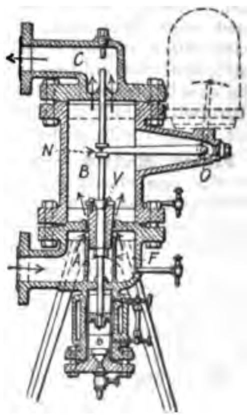

FIG. 31.—St. John Float Steam Meter.

Prices of Steam Meters: "G. E" Pitot tube, \$50 to \$165 for indicating types, \$260 to \$270 for recording type (General Electric Co., Schenectady, N. Y.); St. John recorder, \$250 for 2-in. pipe, \$550 for 6-in. pipe (G. C. St. John, New York); Sargent, \$200 for 2-in. pipe, \$450 for 6-in. pipe (Pittsburgh Supply Co., Pittsburgh, Pa.).

Water Meters

Impulse-wheel or "Velocity" Meters (Fig. 32) are arranged to receive the water at one end, where it passes through a screen, is deflected downward, and a central baffle plate distributes the water horizontally and radially into the row of guide vanes leading the water to the vanes on the rotating impulse wheel which is connected to the counting mechanism of the meter. After discharging from the vanes of the wheel the water flows out as indicated by the arrows. This type is particularly suitable for measuring large quantities at low pressure. A comparatively large flow is required to start such a meter.

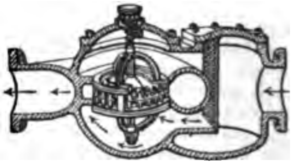

Fig. 32.—Impulse-wheel Water Meter.

Rotary-disk Meters (Fig. 33) have a measuring chamber which is divided into two equal compartments by a hard rubber disk, as shown. While one compartment is filling the other is emptying. There is thus an unbalanced water pressure always moving the disk and giving the pin *P* at the center a circular motion which is transferred to the counting mechanism. This type is not suitable for irregular flows because of excessive leakage, nor for use with hot water, as the rubber disk is likely to be cracked; but it is probably used more than any other in America. For a fairly constant flow it is generally quite accurate. Such meters should be calibrated frequently under the exact conditions of flow, pressure, and temperature with which they are operated. They should not be used with water above 105 deg. fahr.

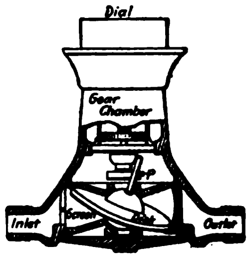

Fig. 33.—Rotary-disk Water Meter.

Piston Meters have two pistons side by side. In the one shown in Fig. 34, water is admitted alternately at the two ends by a slide valve moving on seats in the bottom plate *P*, containing the inlet and discharge ports. One of the pistons moves the counting mechanism by means of a lever. Piston meters are not as reliable as those of the disk type, but they can be used for measuring hot feed water, for which work the rubber-disk type is useless.

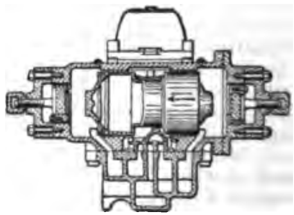

Fig. 34.—Piston Water Meter.

Weir Meters. For a general discussion of the flow of water over weirs, see pp. 263 to 268. Rectangular weirs are generally used for large quantities and "notch" weirs of various shapes for small quantities. The instrument used for measuring the head should be not less than 2 ft. from the crest. The distance from the crest to the bottom of the weir tank should be not less than three times the head. For a tank with two contractions the width of the tank should be not less than three times the width of the weir. Baffle plates, preferably of perforated sheet metal, should be judiciously located to eliminate ripples.

V-notches (see p. 265) are used in many services for measuring water. The head is usually measured by means of a float attached to a vertical shaft. In the **Lea recorder** the rise and fall of the water in a tank provided with a V-notch actuates a float which operates a drum in the recording mechanism through a rack and pinion. This drum has a helical groove whose constantly changing pitch varies with the amount of water flowing over the weir. An arm sliding on a rod parallel to the drum carries on one end a stylus engaging with the drum groove and on the other a pencil the point of which rests on a paper record wound around a clockwork-actuated drum. The groove is so cut that the ordinates of the record drawn by the pencil are proportional to the flow of water over the weir. In **Moyer's weir meter** one side is without contraction and the other is a logarithmic curve of a shape to give a flow proportional to the head. Charts for both the Lea and the Moyer recorders are graduated in lb. of water flowing per hour. This is made possible by an automatic temperature correction: for a given weight of flow, as the temperature varies the head varies correspondingly, but at the same time the displacement of the float changes so as to compensate the change in head. Error due to temperature variation is always less than ½ per cent. No weir device is very accurate for low rates of flow.

The Wilcox Tank Meter is a cylindrical vessel divided by a horizontal partition into two chambers. Water enters the top chamber, which, when filling, compresses air in a trip valve designed to open and discharge the water into the lower chamber after a given weight has accumulated. The discharge from the lower chamber is through a siphon-shaped pipe which is always water-sealed. A weigher of this kind is easily calibrated by weighing several fillings, and it can be used with as great a degree of accuracy as can be expected with rapid weighings in tanks on platform scales. An error of not more than 1 per cent. should be expected if the tank has been calibrated with water at the temperature prevailing in service.

For **Venturi Meters,** see p. 262.

Prices of Water Meters. Velocity, rotary or disk types (3 in.), $80 to $90; Worthington piston meter (3 in.), $125; Lea's V-notch recorder, $400 to $500 (Yarnall-Waring Co., Philadelphia); Moyer's weir recorder, $90 to $125 (J. C. Moyer & Co., Philadelphia); Wilcox tank weigher, $150 to $600 (Wilcox Co., Battle Creek, Mich.); Venturi meter, recording, $250 to $1000 (Builders' Iron Foundry, Providence, R. I.).

APPARATUS FOR FLUE-GAS ANALYSIS

For the analysis of flue gases in a power plant by chemical means an **Orsat apparatus** is generally used. It consists of a graduated tube or burette surrounded by a water jacket to receive and measure the volume of the gas. This burette is connected by a manifold of glass or hard rubber to "pipettes" containing the liquids for absorbing CO_2, O_2 and CO. The pipettes are each a little more than half filled with the absorbing liquid. Water well saturated with CO_2 is ordinarily used for displacement of gases. For accurate work brine or mercury should be used. To absorb CO_2 the gases are first passed into a pipette containing a solution of one part by weight of KOH (caustic potash) in two parts by weight of water. Free oxygen is similarly absorbed by a mixture of pyrogallic acid and KOH, prepared by pouring 5 grams of powdered pyrogallic acid into the pipette and then adding 100 c.c. of the KOH solution as prepared for the first pipette. Some engineers use solutions containing a much larger proportion of pyrogallic acid to make the absorption more rapid, but this is not recommended, as stronger solutions are likely to evolve CO in the presence of free oxygen. Phosphorus sticks may be used in the place of pyrogallic acid for absorbing oxygen. Phosphorus acts more rapidly than the pyrogallic solution; it must be always handled under water and should be at a temperature of about 100 deg. fahr.

The solution for absorbing CO is cuprous chloride (Cu_2Cl_2), which should be made in a glass bottle having a well-fitting glass stopper, greased to made it air-tight. It can be made by pouring 25 grams of copper oxide into the bottle and adding about 500 c.c. of commercial hydrochloric acid (sp.

gr. about 1.1, being obtained by adding *to water* an equal volume of chemically pure acid). To this solution is added from 150 to 200 grams of copper wire cut in lengths to extend from the bottom to the top of the bottle. The solution is then allowed to stand in the bottle till clear, when it is ready for use in gas analysis. By filling the bottle from time to time with acid and adding copper wire as needed to replace that dissolved, a supply of the solution will always be on hand. The KOH solution will absorb 40 times its volume of CO_2 before requiring renewal, while the pyrogallic solution will absorb efficiently only twice its volume of oxygen, and cuprous chloride only an equal volume of CO. The solution for absorbing oxygen, therefore, requires very frequent renewals.

The most common **errors** in the use of gas apparatus are **due to leakage.** The apparatus should be carefully tested before starting an analysis by filling each pipette with the solution up to a mark on its capillary tube, and then, after closing all stoppers and pinch cocks, carefully observing the volume of gas at atmospheric pressure on the scale of the burette. The water bottle is then placed on top of the case so that the gas in the burette and in the yoke will be under pressure for about 5 min., after which the volume of gas is again measured. If there is no leakage the volume will remain constant. If there is leakage a drop of water should be put at each place where there might be leakage. Bubbles of gas will indicate where the apparatus is not air-tight. Extreme care should be taken in arranging for and in collecting the samples of flue gas for analysis. A continuous sample should be collected at a uniform rate during the whole period of the test. Many engineers use for a collecting tube a ½-in. pipe open at the end, which is carefully located so that the open end will be in the unobstructed flow of the gases. Others prefer a perforated pipe closed at the end. The latter method will generally give a more nearly average sample. A small steam or water jet "ejector" or aspirator may be used for securing a continuous flow of gas into the collecting vessel. In most cases it is most convenient to collect the gas over water, which must be saturated with the gas in order to avoid differential absorption of the constituents of the gas.

If by accident a little of the KOH or pyrogallic solution has been drawn into the manifold and into the water in the burette, the analysis is not necessarily spoiled but may be continued by flushing out the tubes with water and adding a few drops of HCl to the water in the bottle. To make sure that all the gas has been absorbed by each reagent, the process should be repeated until readings agree within $^1/_{10}$ per cent. With fairly fresh solutions 2 to 3 min. are required to absorb all the CO_2 or CO, while from 5 to 7 min. may be required to determine the oxygen. It is desirable to replace the pyrogallic solution frequently. One type of automatic apparatus in common use absorbs CO_2 continuously and registers the percentage content. In practically all such apparatus KOH is the absorbent. Another type depends for its action on the difference between the specific weight of air and of flue gas, which is approximately proportional to the percentage by volume of CO_2 in the gas ("econometer" method). Of the two methods, that of absorption is the better.

Prices of Apparatus. Fisher's Orsat, $30 to $50 (Eimer & Amend, N. Y.); Allen & Moyer's, $25 (Bausch & Lomb Co., Rochester, N. Y.); "Sarco" CO_2 recorder, $300 (Sarco Co., 90 West St., N. Y.); "Precision" CO_2 recorder, $250 (Precision Instrument Co., Detroit); Uehling's "composimeter," $350 (Uehling Instrument Co., Passaic, N. J.) Arndt's "econometer," $100.

TACHOMETERS AND SPEED COUNTERS

Hand Speed Counters consist of a small steel shaft pointed at one end and geared to operate the counting device. If handled carefully and well made, these are the most reliable type of instrument, particularly for speeds of from 200 to 2000 r.p.m. To prevent slipping of the head at high speeds, a rubber tip is sometimes provided.

Continuous Speed Counters consist of a series of gears operating a set of dials indicating, by the successive digits, totals of revolutions. Such instruments, although operating by a reciprocating motion, can be arranged ɪo **rotary** motion by any simple "crank" or eccentric device. :s **are not suited** for speeds much over 250 r.p.m.

ers give the instantaneous speed of a machine. At any instant **shown by** an indicating dial in r.p.m. In the operation of **ents rotating** parts usually connected to the shaft by gearing **es which,** reacting against a spring, or in some cases against **.ace the indicating** device in proportion to the speed. Utiliza- **entrifugal force** of rotating masses, eddy currents in a magnetic **e pumping** of liquids so as to indicate the speed by the height **› discharge** rises in a glass tube, are the most common means **. instruments.**

Resonance Tachometer is designed on the principle that bodies **same** "period" of vibration will be in resonance; that is, they will **ether.** It consists of a series of steel reeds, L-shaped at their ends. **of each reed** is adjusted so that its vibration is of a definite period. **atus** is to be set upon the frame of the machine of which the speed

.'s magneto type of electric generator which indicates speeds by **' graduated** voltmeter, is probably the best apparatus available for **ɛ high** speeds. It should preferably be attached directly to the **which** the speed is to be measured. By any application of belting **.** be some error due to slipping.

of Tachometers. Hand counters: Starrett's, $2 to $3; Veeder's, "V-P," $2; ıs counters, $5 to $15. Tachometers: Centrifugal (according to size of dial), 0 (Schaeffer & Budenburg, Brooklyn, N. Y., and Schuchardt & Schütte, West dg., N. Y.); eddy current, $35 to $50 (Warner Instr. Co., Beloit, Wis.); liquid 5 (Veeder Co., Hartford, Conn.); resonance, $25 to $60 (James G. Biddle, phia).

SURVEYING

BY

WILLIAM G. RAYMOND

REFERENCES: Johnson-Smith, "Theory and Practice of Surveying," Wiley. Breed and Hosmer, "Principles and Practice of Surveying," Wiley. Tracy, "Plane Surveying," Wiley. Raymond, "Plane Surveying," American Book Company. Pence and Ketchum, "Surveying Manual," McGraw-Hill. Gillespie-Staley, "A Treatise on Surveying," Appleton.

LINEAR MEASUREMENTS

Tapes. The linear measuring instrument most used is the **steel ribbon tape.** For **surveying,** the **tape** is 100 or more ft. long, graduated to ft., with 1 ft. at the end graduated to tenths and sometimes hundredths. The **builder's tape** is graduated to ft., in., and 8ths. On the reverse side the engineer's tape is usually graduated to links of Gunter's chain, which is 66 ft. long, divided into 100 **links** of 7.92 in. This is used only in land surveying, and was devised so that 10 square chains are 1 acre. **Linen tapes,** even when wire strands are woven in them, are useless for work of precision, but answer for laying out road and railroad earthwork and the like. They are not sufficiently precise for laying out foundations or placing machinery.

Variations in Tape Measurements Due to Temperature, Tension and Sag. Steel tapes are usually of **standard length** at 62 deg. fahr. They change 0.0000065 of their length per fahr. deg. change in temperature. A 100-ft. tape standard at 62 deg. fahr. will be short 0.04 ft. at 0 deg. and long 0.025 ft. at 100 deg.

A steel tape is usually standard for the pull necessary to straighten it when supported throughout its length. Additional pull stretches it at about the rate $1/28{,}000{,}000S$ of its length per lb. of pull, S being its cross-sectional area in sq. in. For the common **sectional area** of 0.002 sq. in. a pull of 10 lb. will produce an elongation of 0.002 ft. in 100 ft. The sectional area of a tape may be determined by dividing the weight of the tape in lb. by its length in in. and the quotient by 0.284.

When a tape is unsupported throughout its length, the measured distance is less than that indicated by about $C_s = l\,(wl/P)^2/24$, in which l is the nominal length of unsupported tape, w the weight of the tape in lb. per unit of length, and P the pull in lb. A 100-ft. tape weighing 0.624 lb. pulled with 10 lb. and unsupported throughout its length would in effect be shortened by 0.016 ft. Calling the correction for temperature C_t, for pull C_p and for **sag** C_s, the distance between two points would be given by the measured length $\pm C_t + C_p - C_s$. The **temperature correction is positive or negative** according as the temperature is *higher* or *lower* than standard. If a line of given length is to be measured from a given point, all of the signs of the corrections are reversed. (See also theory and tables for catenary curves, pp. 147 to 151.)

To Measure a Horizontal Line on Sloping Ground. Most engineering linear measurements are horizontal or vertical. All land measurements mentioned in descriptions are horizontal. Vertical measurements may be made with tape or leveling instrument. When horizontal measurements are to be made on sloping ground, one of the two following methods is used: (1) The low end of the tape is raised to make the tape horizontal, the proper point on the tape or ground being transferred to the ground or tape by a plumb line held by the tapeman. When the slope is so steep that a full tape cannot be used, part of it is used—called "breaking the tape"—and it is better to pull

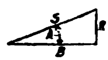

FIG. 1.

ape clear out, raising the several necessary sections consecutively, rather than to ne particular part of the tape several times. (2) The measurement is made along **slope,** the angle of which or its rise in a tape length is determined, and the slope urement reduced to the horizontal. In Fig. 1, the horizontal distance $B = S - S$, approximately, with an error of about 0.0013 of 1 per cent. for a 10 per cent. :, and less for flatter slopes.

Measure or Lay Out an Angle With a Tape. In Fig. 2, from the apex of the A measure equal distances d along its sides and the distance a be- n the ends of these measurements. Then $\sin \frac{1}{2}A = a/2d$. For ll angles A (in degrees) $= 57.3\ a/d$, approximately, or, if d is 100 s, A (in degrees) $= 0.573a$ (approx.) $= 4a/7$ (approx.). Two lines rating at the rate of n deg. separate in distance at the approximate of $7n/4$ units per hundred.

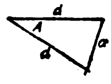

FIG. 2.

o lay out an angle, reverse the process. From the apex measure 100 units along one ; from the point obtained describe an arc with a radius of 200 $\sin \frac{1}{2}A$ or (approxi- ely) $7A/4$ units long; and from the apex with a radius of 100 units, ribe an arc intersecting the first.

'o Lay Out a Right Angle. In Fig. 3, from the point A where the le is to be, measure along one side a distance of $4n$ units; from the nt thus obtained swing an arc with radius of $5n$ units; and from A ng an arc with a radius of $3n$ units to intersect the first arc at B; line joining A and B is perpendicular to the $4n$ line.

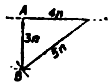

FIG. 3.

LEVELING

The Level, shown in Fig. 4, consists of a telescope EO resting in supports ' attached to a bar B and carrying a level bubble L. The bar B is attached a spindle D which rests in a socket carrying the ball, J, of a ball-and-socket int. The bearing of the spindle carries the upper leveling plate P and the cket of the ball joint is part of the lower leveling plate P' which screws on the tripod head T. By the veling screws S working through ate P and resting on plate P', e upper part of the instrument ay be tipped with respect to P'. the telescope, whose objective at O and eyepiece at E, there is ring R carrying two fine wires t right angles. This may be ad- usted so that the intersection of he wires is in the optical axis of he telescope, and one wire may e made vertical when the other vill be horizontal. Fig. 4 shows **Y-level,** the supports YY being n the form of Y's (wyes) in which the telescope rests and from which it may e easily removed by opening clips on the Y's. In the Y-level the tele- scope is usually an erecting one, *i.e.*, it shows objects as they are. In another form of level, the supports are part of the telescope tube or are fastened rigidly to it and are held to the bar by screws. This is called a **dumpy level** and its (short) telescope (of large aperture) is usually inverting, *i.e.*, it shows objects upside down and right side left. When the axis of the bubble tube and the line of sight of the telescope are parallel, the line of sight will be horizontal when the bubble is brought to the center of its tube. The level is **used with** some form of **graduated rod** read by a movable tar- get or slide and vernier.

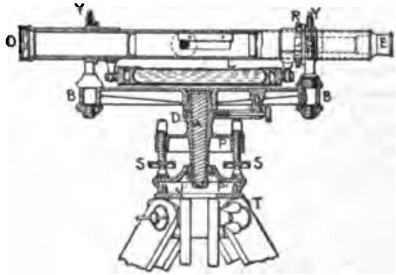

FIG. 4.—Y-level.

To Set Up the Level, the legs are planted firmly, with the lower leveling plate as nearly horizontal as practicable. The telescope and frame are swung over one diagonally opposite pair of leveling screws by which the bubble is brought to the middle of the tube. The frame is then brought over the other pair of screws and leveled; back to the first pair; to the second pair, etc., until the bubble remains in the middle of its tube for any position of the telescope. The screw which clamps the spindle motion should not be used in ordinary leveling operations. The eyepiece should be focused so that the wires are sharp against a blank ground as the sky or side of a light-colored building. It will not need changing so long as the same eye uses it. The objective is focused for each object looked at if the distance varies.

To Determine the Difference in Level Between Two Points, set the level nearly midway between the points, hold a rod on one, look through the level and see where the line of sight as defined by the eye and horizontal cross-wire cuts the rod, called **rod reading.** Move the rod to the second point and read. The difference of the readings is the difference in level of the two points. If it is impossible to see both points from a single setting of the level, one or more intermediate points, called **turning points,** are used. The readings taken on points of known or assumed elevation are called **plus sights,** those taken on points whose elevations are to be determined are called **minus sights.** The elevation of a point plus the rod reading on it gives the elevation of the line of sight; the elevation of the line of sight minus the rod reading on a point of unknown elevation, gives that elevation. In Fig. 5, I_1 and I_2 are intermediate points between A and B. The set-ups are numbered. Assuming A to be of known elevation, the reading on A is a + sight; the reading on I_1 from 1 is a − sight; the reading on I_1 from 2 is a + sight and on I_2 is a − sight. The algebraic sum of the plus and minus sights is the difference of level between A and B. Target rods may usually be read by vernier to thousandths of a foot. In grading work the nearest tenth of a foot is good; in lining shafting the finest possible reading is none too good. It is desirable that the sum of the distances to the plus sights shall approximately equal the sum of the distances to the minus sights to insure compensation of errors of adjustment. On a side hill this may be accomplished by "zigzagging." If, when the direction of pointing is changed, the bubble leaves the middle of its tube, the instrument should be releveled with the telescope in the direction of sight. If adjustments have been properly made little releveling will be necessary, but it must be remembered that **the bubble must be in the middle of its tube whenever a rod reading is taken.**

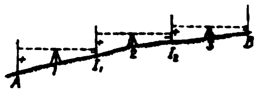

FIG. 5.

To Make a Profile of a Line. A **"bench mark"** is a point of reasonably permanent character whose elevation above some surface—as sea level—is known or assumed and used as a reference point for levels. The level is set up either on or a little off the line some distance—not more than about 300 ft.—from the beginning end or a convenient bench mark (B.M.), as at K in Fig. 6. A reading is taken on the B.M. and added to the known or assumed elevation to get the height of the instrument, called H.I. Readings are then taken at regular intervals (or **stations**) along the line and at such irregular points as may be necessary to show change of slope, as at B and

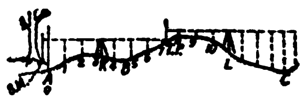

FIG. 6.

between the regular points. The regular points are marked by stakes previ-ısly set "on line" at distances of 100 ft., 50 ft. or other distance suitable to ıe character of the ground and purpose of the work. When the work has ·oceeded as far as possible—not more than about 300 ft. from the instru-ent for good work—a **turning point** (T.P.) is taken at a regular point · other convenient place, the instrument moved ahead and the operation ›ntinued. The first reading on the B.M. and the first reading on a T.P. ter a new set-up are plus sights (+S); readings to points along the line ıd the first reading on a T.P. to be established are minus sights (− S).
The **notes** are taken in the following form. The elevation of a given point, ›th sights taken on it and the H.I. determined from it all appear on a line ith its station (Sta.) designation. In plotting the **profile**, the vertical scale usually exaggerated from 10 to 20 times.

Left-hand page

This space for a heading, telling what the work is, who does it, and the date on which it is done.				
Sta.	+ S	H. I.	− S	Elev.
B.M.	6.42	506.42	....	500.0
A − 0	....	....	10.4	496.0
1	....	....	8.2	98.2
2	....	....	6.1	500.3
+30	....	....	5.5	0.9
3	....	....	6.1	0.3
4	....	....	7.9	498.5
B − + 40.	....	....	8.4	98.0
5	....	....	7.5	98.9
6	....	....	5.1	501.3
7	....	....	3.2	3.2
+ 10 T.P..	4.27	509.13	1.56	504.86
8	....	....	2.2	506.9

Right-hand page

This page is for remarks describing B.M.s and T.P.s or other important particulars.

Adjustment of the Y-level. (1) Plumbing the Wire. Set up the level and bring he vertical wire to cover a suspended plumb line or the vertical corner of a building by wisting the wire ring if necessary after loosening its screws.

(2) Line of Sight. Loosen the Y clips and by means of the leveling screws and the lamp and slow motion of the spindle bring the intersection of the wires to cover a ninute distant point; carefully turn the telescope upside down in the Y's, keeping the ye at the glass; if the intersection remains on the point, the line of sight is in adjustment; f not, bring the intersection half way back to the point by the screws carrying the wire ing, and repeat till the distant point is covered by the intersection of the wires in either ›osition of the telescope.

(3) The Bubble Tube. Level the bubble over two sets of screws and carefully ›ver one set; lift the telescope from the Y's, turn it end for end and replace in the Y's; f the bubble returns to the center of the tube, its axis is parallel to the lower side of the elescope barrel and to its axis if the bearing rings are of equal diameter. For practically ıll work they are nearly enough so. If the bubble moves from the center of its tube, ›ring it halfway back by the adjusting nuts at the ends of the bubble tube, relevel and repeat until the bubble remains in the center of its tube for both positions of the telescope.

(4) The Y's. With the instrument leveled and more carefully over one set of screws, turn end for end on the vertical spindle and note whether the bubble remains in the middle of its tube; if not, adjust by the capstan nuts on the Y's at one end of the bar, bringing the bubble halfway back to the middle; relevel and repeat till the bubble remains in the middle through a complete revolution on the spindle. This last adjustment of the Y's is not essential to correct leveling, but is convenient in that when once set up the level requires no releveling for a change in the direction of pointing.

Adjustment of the Dumpy Level. The dumpy level is adjusted by adjusting the bubble and afterward the line of sight to it.

(1) THE BUBBLE TUBE. Set up and, having the instrument level over one set of screws, swing through 180 degrees on the vertical axis; if the bubble moves from the center, bring it halfway back by the adjusting screws, relevel and repeat the test and adjustment until complete.

(2) LINE OF SIGHT. Set the instrument midway between two stakes from 200 to 400 ft. apart, as in Fig. 7. With the bubble in the center of the tube, read a rod on each stake. The difference in readings is the difference in level, d, of the two stakes. Remove the level and set it up near one of the stakes and in line with both. If set between the two stakes and close to the near stake so that the eye end will just clear a rod held on the stake, look through the object end at the rod and with a pencil point get the reading in the middle of the small spot of light that will be seen; remove the rod to the distant stake, set a target to read the reading on the near rod + allowance for earth's curvature $\pm d$ according as the far stake is the lower or higher; turn the telescope line of sight toward the distant rod and adjust the horizontal wire till it reads on the target when the bubble is centered. Earth's curvature is approximately 0.001 ft. for 200 ft. distance and varies with the square of the distance. If the set-up is a distance b outside the stakes which are a ft. apart, read on the near stake, add earth's curvature e and add or subtract d for the trial reading r on the distant rod; if the reading is not r but r', move the target from r' by a distance $(r - r')(a + b)/a$, up if the result is plus, down if minus; by the wire adjusting screws bring the wire to read on the target, the bubble being kept in the middle of its tube. It is convenient to make $b = a/10$, so that $(a + b)/a = 11/10$.

FIG. 7.

TRANSIT WORK

The Transit. The essential parts of a surveyor's transit are shown in Fig. 8. The telescope T swings on axis A in standards S resting on plate P carrying verniers seen through openings in its upper side which permit readings on a graduated circle on plate P' which may be turned on a spindle in a socket in the leveling head and clamped in position by the clamp H. The clamp D clamps the plates P and P' together, but it is still possible to move one on the other by the slow-motion or tangent screw F. A similar slow-motion screw attached to the clamp H serves to move the whole upper part of the instrument a little in a horizontal plane when the two plates are clamped together. The leveling head is like that of the level, but to the bottom of the spindle a hook is attached, from which a plumb line may be suspended for centering the transit over a point. For doing leveling and measuring vertical angles, there is a level under the telescope and a vertical circle C attached to the horizontal axis of the telescope and read by a vernier V. There is a clamp and slow-motion screw for the telescope axis of revolution. The telescope, though of shorter focal length, is like that of the level.

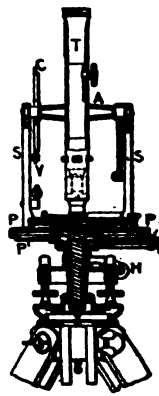

FIG. 8.—Surveyor's Transit.

To Set Up the Transit Over a Point, plant the legs firmly in the ground with the plumb swinging as nearly as possible over the point. At the same time, the lower plate on the leveling head should be nearly level, as judged by eye. Loosen the leveling screws of the leveling head and shift the upper part on the lower plate till the bob swings over the required point. Bring the screws again to bearing—never tight—swing the upper part of the instrument so that the plate bubbles are respectively parallel to the two diagonally opposite sets of leveling screws, and by the screws bring the bubbles to the center by first leveling one and then the other in turn until both are level. Focus the eyepiece of the telescope so that the cross wires are distinct against the sky or a light ground. Set the zeros of the verniers and graduated circle together by the clamp and slow-motion screws of the plates.

To Produce a Straight Line. Set up the transit over one end of the line; with the lower motion clamp and tangent screw bring the telescopic line of sight to the other end of the line marked by a flag, a pencil, a pin or other object; transit the telescope, *i.e.*, plunge it by revolving on its horizontal axis, and set a point (drive a stake and "center" it with a tack or otherwise) a desired distance ahead in line with the telescopic line of sight; loosen the lower motion clamp and turn the instrument in azimuth until the line of sight may be again pointed to the other end of the line, which do; again transit and set a point beside the first point set. If the instrument is in adjustment the two points will coincide; if not, the point marking the projection of the line lies midway between the two established points.

To Measure a Horizontal Angle. Set up the transit over the apex of the angle; with the lower motion bring the line of sight to a distant point in one side of the angle; unclamp the upper motion and bring the line of sight to a distant point in the second side of the angle, clamp and set exactly with the tangent screw; read the circle by the vernier for the angle turned.

To Measure a Vertical Angle. Set up the transit over a point marking the apex of the angle A (see Fig. 9); by the lower motion and the motion of the telescope on its horizontal axis bring the intersection of the vertical and horizontal wires of the telescope in line with a point as much above the point defining the lower side of the angle as the telescope is above the apex; read the vertical circle; turn the telescope to a point which is the height of the instrument above the point marking the upper side of the angle and read the vertical circle. How to combine the readings to find the angle will be obvious.

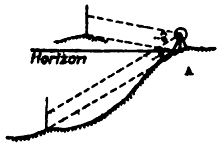

FIG. 9.

To Run a Traverse. A traverse is a broken line marking the line of a road, bank of a stream, fence, ridge or valley, or it may be the boundary of a piece of land. The bearing or azimuth and length of each portion of the line are determined, and this constitutes "running the traverse."

The **bearing** of a line is the angle it makes east or west of a north and south line either true, magnetic or assumed for the purpose of the survey. The bearing is read north or south so many degrees east or west, and never east or west so many degrees north or south. Thus, a line running only 1° north of east would have a bearing N 89° E, and one running 1° south of east would have a bearing S 89° E, etc.

To determine the bearing, set up over one end of the line, loosen the needle clamp, turn the telescope with its object end over the *fleur de lis* or north side of the compass box toward the farther end of the line and read the needle, using the two letters between which its north end lies. It should be noticed that the compass box letters E and W are reversed to make the reading agree with the telescope pointing.

The **azimuth** of a line is the angle the line makes with a north and south line, true, magnetic, or assumed, and differs from bearing in being measured always in one direction through 360°, while bearing is measured in each of four directions through 90°. Azimuth is measured to the right or clockwise. Astronomers use the south for zero azimuth. Surveyors, with some exceptions, use the north. Lines whose bearings are N 88° E, S 38° E, S 70° W, and N 60° W, have azimuths of 88°, 142°, 250°, and 300°, respectively.

To determine the azimuth of a line, set up over one end, set the horizontal circle to read the azimuth of a known line through the point of set-up (as the meridian or the preceding line of a traverse), and by the lower motion turn the line of sight in the direction of the known line; loosen the upper motion and set the line of sight in the direction of the required azimuth and read the circle. Always read the same vernier and the same row of figures—those inclined to the left—since the vernier reads with these when the telescope is turned clockwise. When the preceding line of the traverse is used for orienting the transit, the "back azimuth" should be set on the circle. The **back azimuth** is the azimuth read in a direction opposite to that in which the survey proceeds, and is the forward azimuth plus 180 deg. If this gives more than 360 deg., subtract 360 deg. The distance may be measured with the tape or with the stadia, as explained on p. 1705.

In work with the transit, bearings are not usually read by the needle except for checking. Instead, the deflection angles from one course produced to the next are measured; one course—as the initial course—is taken as a meridian, and the bearings of the other courses with respect to the assumed meridian are calculated. The true or magnetic bearing of the first course may be determined, from which the bearings of all courses will be calculated from the true or magnetic meridian. To determine the **magnetic bearing** of the first course and to establish a meridian of reference, set up the transit over the initial point, let the needle swing free; with the zeros together turn the instrument on its vertical axis by the lower motion till the needle reads north, and set a point some distance away in the line of sight. The line ranged will be the magnetic meridian. By the upper motion set the telescope in the line of the initial course; the vernier will read the angle with the meridian, from which the bearing is calculated.

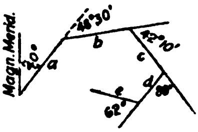

FIG. 10.

In Fig. 10, the bearing of a is N 40° E, of b is N 88° 30′ E, of c is S 49° 20′ E $= 180° - (40° + 48°\ 30' + 42°\ 10')$, of d is S 36° 40′ W $= 86° - 49°\ 20'$, or $40° + 48°\ 30' + 42°\ 10' + 86° - 180°$, of e is N 81° 20′ E $= 180° - (36°\ 40' + 62°)$. A meridian is established because the needle cannot be depended on to give exactly the same line twice. The needle pointing varies as much as 10 min. or more during the day.

To Adjust a Transit. The adjustment of the transit consists in: (1) Making the plate bubbles parallel to the plates, *i.e.*, perpendicular to the vertical axis; (2) making the line of sight perpendicular to the horizontal axis of revolution; (3) adjusting horizontal axis horizontal so that the line of sight may revolve in a vertical plane; (4) making the telescope bubble parallel to the line of sight; and (5) making the vernier of the vertical circle read zero when the line of sight is horizontal or determining the index error.

(1) THE PLATE BUBBLES. Set up the transit; when both plate bubbles are in the centers of their tubes turn the instrument on its vertical axis 180 deg., thus reversing the bubbles. If the bubbles remain in the centers of their tubes they are in adjustment; if not, raise or lower one end of one bubble tube with a small adjusting pin till the bubble seems to move halfway back to the center; do the same with the other tube; relevel with leveling screws, turn 180 deg. to test correctness of work and repeat till perfect.

(2) LINE OF SIGHT. Set up and fix the vertical wire on a suspended plumb line or corner of a vertical building. If the wire does not coincide with the vertical line, loosen all capstan screws carrying the wire ring, twist the ring in the barrel by the screws

until the wire is vertical, and then tighten the screws. By the lower motion and the vertical swing of the telescope fix the line of sight, as defined by the vertical wire, on a distant point about on a level with the instrument or the ground under it (do not clamp the horizontal axis); transit the telescope, *i.e.*, plunge it on its horizontal axis, and, finding a minute point in the line of sight, note it carefully; turn in azimuth, *i.e.*, on the vertical axis, until the line of sight covers the first point sighted; transit and note whether line of sight covers the second point; if not, adjust the wires, moving the ring right or left as the case may be by the capstan-headed screws that carry it till the vertical wire seems to pass over ¼ the distance between the two distant points; again set on the first point, transit and note a point in line—it will be neither of the points previously noted, but if the work has been completed at the first trial, the new point will lie midway between the two previously noted points—reverse in azimuth to the first point, transit, adjust if necessary, and repeat till the adjustment is complete. Stakes centered with pins or pencils 200 or more ft. either side of the transit may be used for points.

(3) The Horizontal Axis. With the transit set up near a tall building, turn the line of sight on a plumb corner near the top; plunge the telescope and note if the line of sight follows down the edge of the building; if not, raise or lower one end of the horizontal axis—one end is adjustable—until the line of sight will revolve in a vertical plane. If no vertical line is available, set the line of sight on some high point, plunge and set a point on the ground; reverse in azimuth, transit and set again on the high point and plunge; if the line of sight cuts the point set on the ground the horizontal axis is in adjustment; if not, adjust the axis until the line of sight cuts the same point below when plunged from the high point both direct and reversed.

(4) Level under Telescope. This is adjusted by the "peg" method described for the dumpy level (see p. 1701), except that—the plate bubbles being centered—the wires are brought to the correct target reading by tipping the telescope, and the bubble is adjusted to bring it to the center, the wires being undisturbed.

(5) Vertical Circle. If the bubble tube is parallel to the line of sight and the latter is horizontal, the bubble will be in the center of its tube and the vertical circle should read zero. If it does not, the vernier may be moved slightly after loosening the screws that hold it. If not convenient, the reading may be noted and used as an index error. If the reading indicates a small angle of elevation, all angles of elevation will be read too large, *i.e.*, the index error is to be subtracted; while depression angles will be read too small—the index error is to be added.

(6) Line of Sight for Leveling. For good leveling, the horizontal wire should be in the center of the telescope tube. If the eyepiece is non-adjustable, it will generally be sufficient to adjust the wire so that it appears to be in the field of view. This is not true of all instruments, and the only way to make the adjustment with certainty is to remove the telescope with its horizontal axis, place it in a pair of Y's made, for instance, by cutting notches in the ends of a starch box or other box of suitable size, and adjust the wires as in the Y-level. (See p. 1701.)

To Measure Distances with the Stadia. In the transit telescope are two extra horizontal wires so spaced (when fixed by the maker) that they are 1/100 of the focal length of the objective apart. When looking through the telescope at a rod held in a vertical position, 100 times the rod length S intercepted between the two extra horizontal wires plus an instrumental constant (C) is the distance (D) from the center of the instrument to the rod if the line of sight is horizontal, or $D = 100\,S + C$, see Fig. 11. If the line of sight is inclined by a vertical angle A, as in Fig. 12, then if S is the space intercepted on the rod and C is the instrumental constant, the distance is given by the formula $D = 100\,S\cos^2 A + C\cos A$. For angles between 5 deg. and 6 deg. the distance is given with sufficient exactness by $D = 100\,S$. Although theory would indicate that distances may be thus determined to within 0.2 ft., yet in practice it is not well to rely on a precision greater than the nearest foot for distances of 500 ft. or less.

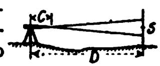

Fig. 11.

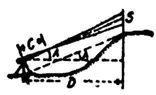

Fig. 12.

The instrumental constant is usually stated on a paster in the transit box. When not so given, it may be determined by measuring several distances on level ground, reading the distances with the stadia, as $D_1 = 100\ S_1$, $D_2 = 100\ S_2$, etc., subtracting the readings from the measurements and averaging the remainders. For most transits, the so-called instrumental constant is not constant, but has an extreme variation of perhaps 0.1 ft. With different instruments its average value varies from about 0.75 ft. to 1.25 ft. It is made up of the focal length of the objective and the distance from the objective to the horizontal axis.

For good work, the wire interval, *i.e.*, the coefficient of the rod space, should be determined daily, as it may change slightly with atmospheric changes and may not always be 100. This may be done, if C is known, by measuring $(100 + C)$ ft., $(200 + C)$ ft., etc., from the instrument, noting the rod intercepts. When C is not known, measure two distances on level ground and read the intercepts S and S_1; then, if K be the coefficient of S, C the instrumental constant, and D and D_1 the two distances, $K = (D - D_1)/(S - S_1)$, and $C = D - KS$ or $D_1 - KS_1$. Several sets of readings should be taken and average results used.

The stadia wires are sometimes adjustable as to the space between them. When so, they are not in the same plane with the line and level cross wires, and hence are not seen with the same focusing of the eyepiece. Adjustable stadia wires should be tested daily and so set that 100 shall be the coefficient of the rod intercept. This may be done by laying off $(100 + C)$ ft. from the center of the instrument and adjusting the wire to cover 1 ft. on a rod held at the further end of the line.

To Measure Differences of Level with Transit and Stadia. Measure the angle of elevation from the point of set-up to the distant point required, according to the method already described, and read the rod intercept on a rod held *vertical* at the distant point. The rod intercept being S, its coefficient K, the instrumental constant C, angle of elevation or depression A, and difference of level H,

$$H = KS \cos A \sin A + C \sin A.$$

If the rod be held normal to the line of sight,

$$H = (KS + C) \sin A.$$

Tables of horizontal distances and differences of level for various vertical angles and a 100-unit rod, are found in all surveying text-books and in most railway surveyors' field-books.

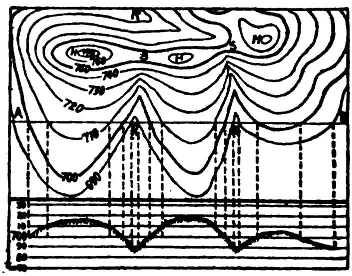

FIG. 13.—Contour Map.

Contour Maps. A contour map is one on which the configuration of the surface is shown by lines of equal elevation called **contour lines.** In Fig. 13, contour lines varying by 10 ft. in elevation are shown. *H*, *H* are hill peaks, *R*, *R* ravines, *S*, *S* saddles or low places in the ridge *HSHSH*. The horizontal distance between adjacent contours shows the distance for a fall or rise of the contour interval—10 ft. in the figure. A profile of any line as *AB* can be made from the contour map as shown in the lower part of the figure. Conversely, a contour map may be made from a series of profiles properly chosen. Thus, a profile line run along the ridge *HSSH* and radiating profile lines from the peaks down the hills and from the saddles down the ravines would give data for projecting points of equal elevation which could be connected for contour lines. This is the best method for making contour maps of very limited areas, such as city squares or very small parks. If the ground is not too much broken, the small tract is divided into squares and levels are taken at each square corner, and between two corners on some lines if necessary to get correct profiles.

Contours with the Transit and Stadia. When a large area of several hundred or more acres is to be contoured, or a long belt within which a railroad line is to lie, the best method is the transit and stadia method. Referring to Fig. 13, a traverse line would be run along the ridge by transit and stadia, establishing points in the saddles and on the peaks; from these, radial lines would be run, establishing points on the slopes of the hills; from each of these points a number of readings would be taken to slope-governing points, the azimuth, distance and vertical angle being read, from which each point could be located in place and elevation.

THE PLANE TABLE

Topography with the Plane Table. The plane table is the standard instrument of the United States Geological Survey and the United States Coast and Geodetic Survey for filling in topography. It consists essentially of a drawing board with a suitable leveling device mounted on a tripod (see Fig. 14). Accompanying the plane table is a telescope or open line of sight attached to a ruler which may be moved over the drawing board. The combined line of sight and ruler is called the **alidade.** The purpose of the plane table is to make a map directly from the measurements made in the field, without notes. The general method of use in locating points is as follows:

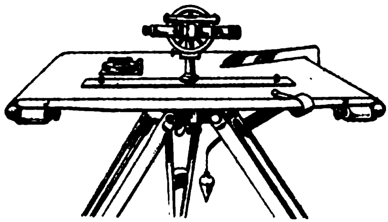

Fig. 14.—Plane Table.

A line is drawn on the paper to scale to represent a known line on the ground. The table is set over one end of the line on the ground, the ruler brought to the line on the paper representing the line on the ground, and the table is swung in azimuth until the line of sight is in the direction of the distant end of the line. The table is then clamped and is said to be **oriented.** The ruler is now moved about the plotted point on the paper representing the occupied point on the ground as a center, and various points in the surface to be mapped are located by directing the line of sight to them, measuring the distance and vertical angle as in the stadia method and plotting the points observed to scale along the edge of the ruler. When the various points that can be conveniently located from the first setting have been mapped, the table is moved either to the other end of the line with which the work began or to one of the plotted points which has been marked by a stake, and the table is set up and oriented on the point previously occupied in the same manner as described for the first set-up, and further points are located and mapped as before.

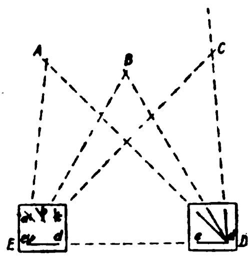

Fig. 15.

Location by Intersections. A method much used with the plane table is known as the method of intersections. In Fig. 15, let it be required to locate the points *A*, *B*, *C* from the points *D* and *E*. Measure *DE*, and lay off to scale on the paper a line equivalent to DE in proper position so that the points desired will fall on the paper. Set the table over D and orient on E. Swinging the alidade about d, draw lines toward C, B, and A. Set the table over E and orient on D. Swinging the alidade about e, draw lines toward C, B, and A, and note the intersections of these lines with those drawn from d to corresponding points. These intersections locate the points.

Adjustments. The bubbles for leveling the table are adjusted to be parallel to the base of the alidade. The line of collimation (*i.e.*, the axial line of the telescope) and the axis of the telescope bubble are made parallel by the "peg" adjustments as applied to the transit. The horizontal axis is adjusted like that of the transit.

SPECIAL PROBLEMS IN SURVEYING AND MENSURATION

Volume of Earth in Foundation and Area Grading. The volume of earth removed from a foundation pit or in grading an area may be computed in several ways, of which two follow.

(1) The area (Fig. 16) is divided into squares or rectangles, levels are taken at each corner before and after grading, and the volumes are computed as a series of prisms. If A be the area (in sq. ft.) of one of the squares or rectangles—all being equal—and h_1, h_2, h_3, h_4 be corner heights (in ft.) equal to the differences of level before and after grading, the subscripts referring to the number of prisms of which h is a corner, then the volume in cubic yards is

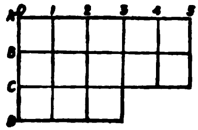

FIG. 16.

$$Q = A(\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4)/(4 \times 27)$$

In Fig. 16 the h's at A_0, D_0, D_3, C_5, and A_5 would be h_1's; those at B_0, C_0, D_1, D_2, C_4, B_5, A_4, A_3, A_2 and A_1 would be h_2's; that at C_3 an h_3; and the rest h_4's. The rectangles or squares should be of such size that their tops and bottoms are practically planes.

(2) A large-scale profile of each line one way across the area is carefully made, as the A, B, C, and D lines of Fig. 16, the final grade line is drawn on it and the areas in excavation and embankment are separately measured with a planimeter or by estimation from the drawing. The excavation area of profile A is averaged with that of profile B, and the result multiplied by the distance AB and divided by 27 to reduce to cu. yd. Similarly, the material between B and C is found.

To Pass an Obstacle. Four cases are shown in Fig. 17. If the obstacle be large, as a building, (1) turn right angles at B, C, D, and E, making $BC = DE$ when $CD = BE$. All distances should be long enough to insure sufficiently accurate sighting. (2) At B turn the angle K and measure BC to a convenient point. At C turn $\angle = 2K - 180$ deg; measure $CD = BC$. At D turn K for line DE. $BD = 2BC \cos(180 \text{ deg.} - K)$. (3) At B lay off a right angle and measure BC. At C measure any angle to clear object and measure $CD = BC/\cos C$. At D lay off $K = 90$ deg. $+ C$ for the line DE. $BD = BC \times \tan C$. If the obstacle is small, as a tree, (4) at A, some distance back, turn the small angle a necessary to pass the obstacle and measure AB. At B turn the angle $2a$ and measure $BC = AB$. At C turn the small angle a for the line AC, and transit, or turn the large angle $K = 180$ deg. $- a$. If a is but a few minutes,

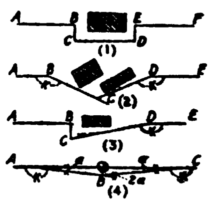

FIG. 17.

$AC = AB + BC$ with sufficient exactness. If only a tape is available, the right-angle method (1) above given may be used; or, an equilateral triangle ABC (Fig. 18) may be laid out, AC produced a convenient distance to F, the similar triangle DEF laid out, FE produced to H making $FH = AF$, and the similar triangle GHI then laid out for the line GH. $AH = AF$.

To Measure the Distance Across a Stream. To measure AB (Fig. 19), B being any established point, tree, stake, or building corner: 1. Set a transit over A; turn a right angle from AB and measure any distance AC; set over C and measure the angle ACB. $AB = AC \tan ACB$. 2. Set over A, turn any convenient angle BAC' and measure AC'; set over C' and measure $AC'B$. Angle $ABC' = 180$ deg. $- AC'B - BAC'$. $BA = AC' \times \sin AC'B/\sin ABC'$. 3. Set up on A and produce BA any measured distance to D; establish a convenient point C about opposite A and measure BAC and CAD; set over D and measure ADC; set over C, and measure DCA and ACB; solve ACD for AC, and ABC for AB. For best results the acute angles of either method should lie between 30 deg. and 60 deg.

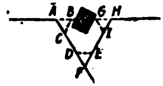

FIG. 18.

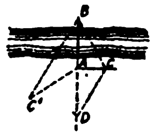

FIG. 19.

To Measure a Visible but Inaccessible Distance, as AB in Fig. 20. Measure CD. Set a transit at C and measure angles ACB and BCD; set at D and measure angles CDA and ADB. $CAD = 180$ deg. $- (ACB + BCD + CDA)$. $AD = CD \times \sin ACD/\sin ACD$. $CBD = 180$ deg. $- (BCD + CDA + ADB)$. $BD = CD \times \sin BCD/\sin CBD$. In the triangle ABD, $\frac{1}{2}(B + A) = 90° - \frac{1}{2}D$; where A, B and D are the angles of the triangle; $\tan \frac{1}{2}(B - A) = \cos \frac{1}{2}D(AD - BD)/(AD + BD)$; $AB = (AD + BD) \sin \frac{1}{2}D/\cos \frac{1}{2}(B - A)$, $= (AD - BD) \cos \frac{1}{2}D/\sin \frac{1}{2}(B - A)$.

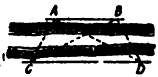

FIG. 20.

Setting Stakes for Trenching. A common way to give line and grade for trenching (see Fig. 21) is to set stakes K ft. from the center line, driving them so that the near face is the measuring point and the top is some whole inch or tenth of a foot above the bottom grade or grade of the center or top of the pipe to be laid. The top of the pipe barrel is perhaps the better line of reference. If preferred, two stakes may be driven on opposite sides and a board nailed across, on which the center line is marked and the depth to pipe line given. When only one stake is used, a graduated pole sliding on one end of a level board at right angles is convenient for workmen and inspectors. On long grades the grade stakes are set by "shooting in." Two grade stakes are set, one at each end of the grade, a transit is set over one, its height above grade determined, and a rod reading calculated for the distant stake such as to make the line of sight parallel to the grade line; the transit line of sight is then set at this rod reading; when the rod is taken to any intermediate stake, the height of instrument above grade less the rod reading will be the height of the top of the stake above grade. If the ground is uniform, the stakes may all be set at the same height above grade by driving them so as to give the same rod readings throughout.

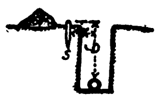

FIG. 21.

To Reference a Point. The point P (Fig. 22) which must be disturbed during construction operations and will be again required as a line point in a railway, pipe line, or other survey, is referenced as follows: 1. By setting the transit over it and setting four points, A, B, and C, D, on two intersecting lines. When P is again required, the transit is set over B and with foresight on A two temporary points close together near P but on opposite sides of the line DC are set; the transit is then set on D and, with foresight on C, a point is set in the lines DC and BA by setting it in DC under a string stretched between the two temporary points on BA. 2. Points A and E and C and F may be established instead of A, B, C, D. 3. If the ground is fairly level and is not to be much disturbed, only points A and C need be located, and these by simple tape measurement from P. They should be less than a tape length from P. When P is wanted, arcs struck from A and C with the measured distance for radii will give P at their intersection.

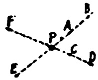

FIG. 22.

Foundations. The corners and lines of a foundation are preserved by setting stakes outside the area to be disturbed, as in Fig. 23. Cords stretched around nails in the stakes marking the reference points, will give the referenced corners at their intersections and the main lines of the building. These corners can be plumbed down to the level desired if the height of the stakes above grade is given. It is well to nail boards across the stakes at AB, putting nails in the top edge of the board to mark the points A and B, and if the ground permits, to put all the boards at the same level.

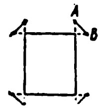

FIG. 23.

To Test the Alignment and Level of a Shaft. Having placed the shaft hangers as closely in line as possible by the use of a chalk line, the shaft is finally adjusted for line by hanging plumb lines over one side of the shaft at each hanger and bringing these lines into a line found by stretching a cord or wire or by setting a transit instrument at one end and adjusting at each hanger till its plumb line is in the line of sight. The position of the line will be known either on the floor or on the ceiling rafters or beams to which the hangers are attached. If the latter, the transit may be centered over a point found by plumbing down, and sighted to a plumb line at the farther end.

To level the shaft, an ordinary carpenter's level may be used near each hanger, or, better, a pole with an improvised sliding target may be hung over the shaft at each hanger by a hook in one end. The target is brought to the line of sight of a leveling instrument set preferably about under the middle of the shaft, by adjusting the hanger.

When the hangers are attached to inclined roof rafters, the two extreme hangers may be put in a line at right angles to the vertical planes of the rafters by the use of a square and cord. The other hangers will then be put as nearly as possible without instrumental test in the same line. The shaft being hung, the two extreme hangers, which have been attached to the rafters about midway between their limits of adjustment, are brought to line and level by trial, using a transit instrument with a well-adjusted telescope bubble, a plumb line and inverted level rod or target pole. Each intermediate hanger is then tested and may be adjusted by trial.

To Determine the Verticality of a Stack. If the stack is not in use and its top is accessible, a board may be fitted across the top, the center of the opening found and a plumb line suspended to the bottom, where its deviation from the center will show any leaning. If the stack is in use or its top not accessible and its sides are battered, the following procedure may be followed: Referring to Fig. 24, set up a transit at any point T and measure the horizontal angles between vertical planes tangent respectively to both sides of the top and the base and also the angle a to a second point T_1. On a line through T approximately at right angles to the chimney diam., set the ıd perform the same operations as at T, measuring also K and n the drawing board, lay off K to as large a scale as convenient, ›lotted T and T_1 lay off the shown in the figure. By cumferences tangent to the rals formed by the intersect- ›f the base and top, respec- ine joining the centers of rences will be the deviation ;al in direction and amount. square, T and T_1 should be

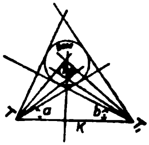

FIG. 24.

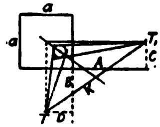

FIG. 25.

posite the middle points of two adjacent sides, as in Fig. 25.

MECHANICAL REFRIGERATION

BY

FRANK L. FAIRBANKS

REFERENCES: Siebel, "Compend of Mechanical Refrigeration," Nickerson and Collins. Lorenz-Heinel, "Neuere Kühlmaschinen," Oldenbourg. Macintire, "Mechanical Refrigeration," Wiley. *Proc. Am. Soc. Refrigerating Engineers. Ice and Refrigeration. Zeit. für die Gesamte Kälte-industrie.*

REFRIGERATING MACHINES AND PROCESSES

Means of Producing Refrigeration

Refrigeration may be produced (1) by transferring heat from a warmer body to a colder one (*e.g.*, refrigeration by cooled brine, etc.); (2) by the absorption of heat by a compressed gas when expanding, as in the air machine; (3) by melting or dissolving solid bodies, as in the melting of ice, the solution of salts in water, and in chemical machines; (4) by evaporating liquids which have a low boiling point, as liquid ammonia, water, carbonic acid, sulphurous acid, ether, etc.

Air Machines (For theory, see p. 346). Compressed-air machines are but little used except on shipboard, on account of their low efficiency. Table 1 gives results of tests of several of these machines. See also Table 4, p. 1718.

Table 1. Test Results on Cold-air Machines

(Linde, *Trans. A. S. M. E.*, vol. 14, p. 1416)

	System		
	Bell-Coleman	Lightfoot	Haslam
Air pressure in receiver, lb. per sq. in. abs	61.0	65.0	64.0
Temperature of air entering compression cylinders, deg. fahr	65.5	62.0	....
Temperature of air after expansion, deg. fahr	−52.6	−82.0	−85.0
I.h.p. in compression cylinder	124.5	43.1	346.4
I.h.p. in expansion cylinder	58.5	28.0	176.2
I.h.p. in steam cylinder	84.4	24.6	332.7
B.t.u. abstracted per hour per i.h.p. of steam cylinder, at 20 deg. fahr	668.0	1554.0	954.0

Chemical Machines. In most of these the temperatures of water and brine are successively lowered by dissolving a salt (usually nitrate of ammonia) in water, the salt being recovered later by evaporating the water. They have no commercial importance. For **freezing mixtures,** see p. 297.

Vaporization Machines. The machines now used for the production of refrigeration on a large scale, make use of the evaporation of liquids. Ammonia, sulphurous acid, carbonic acid, or a mixture of the latter two (Pictet's fluid) have been employed. These machines may be classified as vacuum machines, absorption machines, and compression machines.

Vacuum Machines. In these, water is generally employed as the refrigerating medium, its volatilization at a sufficiently low temperature being effected by means of vacuum pumps, with or without the assistance of strong sulphuric acid. The sulphuric acid may be reconcentrated and used over and over again.

35. The comparative efficiency of the ammonia compression machine increases with the suction pressure, and decreases slightly with increased condenser pressure. The efficiency of the carbon-dioxide machine decreases only slightly with the lowering of the suction pressure, within the ordinary temperature limits, but quite rapidly with the increase of the condenser temperature, especially after the temperature of the condensing water exceeds 80 deg. fahr. The compressed-air machine is affected adversely in its efficiency by the moisture content of the air, decreasing very rapidly with the increase of moisture, but it is affected only moderately by changes of temperature on either side of the machine.

Basis of Rating of Refrigerating Machines. The commercial **unit of capacity** of a refrigerating machine is taken as the abstraction of an amount of heat equal to the heat of fusion of 1 ton (2000 lb.) of ice per day (24 hr.). The determinations of the heat of fusion of ice by the Bureau of Standards show slight deviations between plate ice, can ice, and natural ice. The mean of twenty-one determinations is 79.63 cal. per gram, or 143.33 B.t.u. per lb. avoirdupois. This is equivalent to 286,600 B.t.u. per ton, or the taking up of heat in a machine of unit capacity at the rate of 199.028 B.t.u. per min. This is so close to the convenient round figures of 200 B.t.u. per min., 12,000 B.t.u. per hour, or 288,000 B.t.u. per ton, that these latter figures are always used except where extremely close calculations are necessary.

The rating of the tonnage of a refrigerating machine taken in connnection with the plane of temperatures at which heat is to be taken up and that at which it is to be discharged (*i.e.*, temperature of the refrigerant in the evaporating or refrigerator coils and the highest temperature in the condenser), has not been finally agreed upon internationally. European engineers suggest +14 deg. fahr. at the refrigerating coil and +77 deg. fahr. at the condenser, but in America, where the cooling water is as a rule much warmer than in Europe and the demand for lower temperatures in the refrigerator is greater, it is the custom to rate the tonnage of the machine at 0 deg. fahr. for the temperature of the refrigerant leaving the refrigerator coil, and 95.5 deg. fahr. in the condenser. This is equivalent in ammonia machines to practically 200 lb. abs. for condenser pressure, and 30 lb. abs. leaving the refrigerator piping. For the influence of change in the plane of temperatures on the capacity and horse power of ammonia compressors, see Table 8, p. 1720.

The refrigerating capacity of a machine is different from the actual **ice-making capacity** of a plant; the latter is considerably less, being 50 per cent. and upward of the refrigerating capacity, according to temperature of water, etc.

The Refrigeration Research Committee of the (British) Institution of Mechanical Engineers has adopted (1914) the **calory per sec.** (= 342,860 B.t.u. per 24 hr.) as the unit of refrigeration, and has adopted as "standard conditions" a temperature range of the cooling water from 15 deg. cent. (59 deg. fahr.) at inlet to 20 deg. cent. (68 deg. fahr.) at outlet, and a temperature range of the brine from 0 deg. cent. (32 deg. fahr.) to −5 deg. cent. (23 deg. fahr.). For direct-expansion systems the standard vapor temperature in the refrigerator is to be taken as −10 deg. cent. (14 deg. fahr.). The **rated capacity** of a machine is the number of units of refrigeration developed under the above standard conditions.

Compression Machines

The **compression system** should be selected for mild-temperature refrigerating systems. It is particularly economical where the exhaust steam from the compressor can be used for heating, dyeing and other industrial purposes. It is the only system which can be favorably considered where electric current, and especially direct current, is low in price. Direct-current

efficiency of 80 to 85 per cent. in machines of 50 tons and over; under good average working conditions, with a reasonable amount of care to maintain tight valves and tight piston rings, it should run from 75 to 80 per cent.

Table 3. Theoretical Horse Power to Produce 1 Ton of Refrigeration

(Add 50 per cent. for probable actual horse power; see also Table 41, p. 849)

Suction pressure (gage) and corresponding temperature		Condenser pressures (lb. per sq. in. gage), and corresponding temperatures (deg. fahr.)								
Lb. per sq. in, *P*	Deg. fahr., *T*	103 (65°)	115 (70°)	127 (75°)	139 (80°)	153 (85°)	168 (90°)	184 (95°)	200 (100°)	218 (105°)
4	− 20	1.058	1.130	1.205	1.283	1.361	1.443	1.525	1.609	1.691
6	− 15	0.997	1.069	1.145	1.222	1.300	1.410	1.461	1.546	1.630
9	− 10	0.903	0.978	1.045	1.118	1.193	1.260	1.347	1.435	1.509
13	− 5	0.818	0.883	0.954	1.023	1.094	1.168	1.244	1.321	1.396
16	0	0.735	0.801	0.865	0.933	1.002	1.072	1.147	1.219	1.255
20	5	0.666	0.731	0.795	0.859	0.928	0.998	1.066	1.138	1.212
24	10	0.592	0.663	0.726	0.789	0.854	0.921	0.991	1.060	1.129
28	15	0.541	0.600	0.664	0.728	0.792	0.855	0.922	0.994	1.060
33	20	0.474	0.534	0.592	0.672	0.715	0.780	0.842	0.903	0.974
39	25	0.410	0.466	0.523	0.580	0.599	0.702	0.767	0.829	0.892
45	30	0.351	0.406	0.461	0.518	0.576	0.635	0.694	0.759	0.817
51	35	0.300	0.355	0.410	0.467	0.521	0.580	0.640	0.701	0.763

Wet Compression vs. Dry Compression. The considerable superheat of the ammonia vapor at the end of compression may be reduced or prevented either by injecting refrigerated oil or by carrying liquid ammonia into the compressor. If the latter method is used and sufficient liquid is admitted to keep the vapor always in a saturated condition, the operation is known as **wet compression.** If the ammonia gas is dry or superheated when admitted to the compressor and there is no liquid injected during the compression stroke, the operation is known as **dry compression.** The form of a compressor should be especially adapted either to wet or dry compression. The **advantages of wet compression** over dry compression are as follows: (1) A higher back pressure can be maintained; (2) the surface of the refrigerator can be materially reduced; (3) the volumetric efficiency is higher; (4) the oil taken in through the stuffing box remains in the liquid state; (5) no water jacketing is necessary. The regulation for wet compression is simple, because the temperature of the delivering pipe, as indicated by touch, shows whether a sufficient amount of ammonia is passed through the system.

There are two means of producing wet compression. In the **wet-suction-gas method** liquid ammonia is fed to the cooler in such quantity that a part of it remains unvaporized and passes to the compressor in the form of finely divided liquid suspended in the gas. With this method of wet compression it is impossible in practice to maintain a definite condition of the gas at the compressor; at times there will be comparatively dry gas and at other times an excess of liquid, which is left in the clearance and produces a heavy re-expansion loss which more than nullifies the gain made on the compression stroke. With this wet-gas method the capacity of a machine has been found to be much less than with dry compression, and the horse power per ton of refrigeration correspondingly greater.

Tests made by the York Mfg. Co. on a horizontal double-acting compressor gave a tonnage with dry compression of nearly 3 times that made with wet compression when the suction pressure was 5 lb. gage, and of 1.5 times that made by wet compression when the suction pressure was 25 lb. The increased re-expansion at low suction pressure causes the considerable falling off in volumetric efficiency and capacity.

Table 4. Actual Performance of Refrigerating Machines
(Denton and Jacobus, *Trans. A. S. M. E.*, vol. 13)

Class of machine*	Absolute pressure, lb. per sq. in.		Temperature corresponding to pressure, deg. fahr.		Temperature of brine, deg. fahr.		Per cent. of indicated power of steam cylinder lost in friction	Ice-melting capacity, in tons per 24 hr.	Tons refrigeration per i.h.p.	Difference between theoretical and actual ice-melting capacity, per cent.†	Loss due to heating,‡ per cent. of theoretical amount with friction	Actual mean effective pressure, lb. per sq. in.
	Condenser	Suction	Condenser	Suction	Inlet	Outlet						
A	59	15	65‖	−53‖	....	....	21.9	10.3	0.123	71.7	56.9	26.6
B	175	54	81‖	−40‖	....	....	32.1	4.9	0.108	80.0	63.0	89.2
C	166	43	84	15	37	28	22.7	73.9	0.87	32.8	11.7	65.9
C	167	23	85	−11	6	2	18.6	37.9	0.52	37.4	22.7	57.6
C	162	28	83	−3	14	2	19.3	46.5	0.63	34.9	18.6	59.9
C	176	42	88	14	36	28	19.7	74.4	0.84	30.5	13.5	70.5
D	135	55	72	27	43	37	14.4	26.2	1.46	30.8	19.1	54.8
D	131	42	70	14	28	23	16.7	19.5	1.08	33.5	20.2	53.4
D	128	30	69	1	14	9	16.0	13.3	0.79	37.1	25.2	50.3
D	126	22	68	−12	30	−5	19.5	9.0	0.58	42.9	29.1	44.7
D	200	42	95	14	28	23	10.5	16.5	0.63	36.0	28.5	77.0
D	136	60	72	30	44	37	10.7	20.8	1.66	28.5	19.9	56.8
D	131	45	71	18	28	23	12.1	21.6	1.19	31.3	21.9	56.4
D	126	24	68	−9	0	−6	18.0	9.9	0.63	41.1	28.3	46.1
D	117	41	64	13	28	23	13.5	20.0	1.23	33.1	22.9	50.6
D	130	60	70	31	43	37	14.8	19.5	1.62	35.2	23.8	52.0
E§	152	40	79	13	21	16	....	42.2	0.72	47.8	....	....

* Class of machine and authority: A, Bell-Coleman air (Schröter); B, closed-cycle air (Renwick, Jacobus); C, ammonia, dry compression (Denton); D, ammonia, wet compression (Schröter); E, ammonia, absorption (Denton).

† Difference between theoretical ice-melting capacity (no cylinder heating or friction) and actual capacity, in per cent. of theoretical with no friction.

‡ Loss due to heating during aspiration of gas in the compression cylinder and to radiation and superheating at the brine tank.

§ Evaporation of 11.1 lb. of water per lb. of combustible from and at 212 deg. fahr. in absorption machine.

‖ Temperature of air at entrance and exit of expansion cylinder.

Table 7. Influence of Clearance on Ammonia Compressors

(From tests on a 12½ × 18-in. compressor at 70 r.p.m.; 185 lb. gage condensing pressure; 95.5 deg. fahr. liquid at expansion valve.—York Mfg. Co.)

Linear clearance, in.	Clearance volume in per cent. of displacement*	Discharge temperature, deg. fahr.			Refrigeration, tons per 24 hr.			Compressor i.h.p. per ton		
		Gage pressure in suction pipe, lb. per sq. in.								
		5	15.67	25	5	15.67	25	5	15.67	25
Single-acting Compressor										
1/32	0.24	251	230	213	22.7	38.0	50.4	1.75	1.30	1.09
1/8	0.76	251	233	212	22.6	37.2	50.1	1.77	1.32	1.10
1/4	1.46	242	232	214	21.0	35.6	49.1	1.81	1.34	1.11
1/2	2.85	245	230	212	19.7	34.4	47.0	1.82	1.36	1.12
1	5.63	230	223	209	15.5	29.7	42.6	1.83	1.39	1.13
Double-acting Compressor										
3/64	0.42	321	287	253	19.2	33.0	47.4	2.18	1.60	1.26
1/8	0.85	338	292	259	17.3	32.1	45.1	2 [illegible]	[illegible].52	1.28
1/4	1.55	335	285	255	16.0	30.0	44.8	2.45	1.64	1.30
1/2	2.93	341	293	261	14.3	28.9	42.3	2.56	1.72	1.35
1	5.71	329	300	265	10.6	22.9	36.5	2.89	2.01	1.44

* Clearance volume includes indicator connections, valve shut.

Table 8. Variation in Capacity and Horse Power Required in Ammonia Compression Machines with Variation in Operating Conditions

(The values in this table are the ratios of the capacity and horse power under the stated operating conditions as compared with capacity and horse power when operating between 0 deg. and 95.5 deg. fahr.—York Mfg. Co.)

Condenser pressure, lb. per sq. in., gage (and corresponding temperature, deg. fahr.)	Suction gage pressure, lb. per sq. in. (and corresponding temp., deg. fahr.)											
	5 (− 17.5°)		10 (− 8.5°)		15.67 (0°)		20 (5.7°)		25 (11.5°)		30 (16.8°)	
	Tons per 24 hr.	H.p.	Tons per 24 hr.	H.p.	Tons per 24 hr.	H.p.	Tons per 24 hr.	H.p.	Tons per 24 hr.	H.p.	Tons per 24 hr.	H.p.
145 (82°)...	0.665	0.738	0.855	0.803	1.070	0.857	1.240	0.885	1.437	0.908	1.632	0.924
165 (89°)...	0.642	0.790	0.828	0.866	1.035	0.930	1.200	0.966	1.392	1.000	1.580	1.021
185 (95.5°)..	0.620	0.836	0.800	0.922	1.000	1.000	1.163	1.041	1.347	1.082	1.532	1.115
205 (101.4°).	0.600	0.882	0.775	0.977	0.972	1.063	1.125	1.110	1.307	1.160	1.485	1.256

Efficiency of Compression and Absorption Machines at the Quincy Market Cold Storage and Warehouse Co. A number of tests, each of 24 hr. duration, with a 1000-ton compressor doing 750 tons of refrigeration showed an average result of a ton of refrigeration at 0.9 h.p., with a steam consumption of 228 lb. per ton in 24 hr. In these tests the gas came to the compressor at + 10 deg. fahr., and was discharged into shell condensers with 70 deg. fahr. condensing water, corresponding to a head pressure of approximately 115 lb. The engine driving this compressor was a cross-compound Corliss, using steam at 150 lb. pressure, superheated 100 deg., and exhausting into a vacuum of approximately 28 in.

Similar tests of a 400-ton machine under the same conditions gave the following results at rated load: With the gas coming to the compressor at −10 deg. fahr., and a head pressure of 115 lb., a ton of refrigeration was produced for 1.25 h.p. and a con-pipe line to the condenser may be based on a velocity of 8000 to 10,000 ft. per min., depending upon the length of line, fittings, etc.

Water Jackets. The water jackets on the compressor of a dry-compression machine change the compression curves only slightly. In single-acting machines the compression line will usually fall slightly under the adiabatic, making a card about 4 per cent. smaller than the adiabatic. The control of the **superheat** is not attempted, the jacket water used being merely enough to keep down the temperature of the cylinder, valves and gaskets.

Liquid Receiver. If the shell type of condenser is not used, a liquid receiver should be provided. This receiver acts as an oil separator. It should be vented to the condenser at the top, the liquid entering at the bottom. Where the piping to the condenser is long, the flow of liquid from the condenser to the receiver may be interrupted by gas formed in this liquid line; in such cases a cooling coil with water circulation may be inserted. The receiver is sometimes made large enough to store the entire ammonia charge of the plant. For average conditions a reasonable **receiver capacity** will be obtained by allowing ½ cu. ft. per ton up to 20 tons, ⅓ cu. ft. for 25 to 50 tons, ¼ cu. ft. for 60 to 100 tons, ⅙ cu. ft. for 150 to 200 tons, and 1/10 cu. ft. above 200 tons.

Weight of Ammonia in System. The amount of anhydrous ammonia needed in a compression plant may be estimated by assuming the whole space of the condenser to be filled with ammonia vapor at 165 lb. pressure, the ammonia expansion pipes to be filled with ammonia vapor at 20 lb. pressure, and allowing besides 25 lb. of liquid ammonia for every cubic foot of liquid receiver capacity. The following table, derived from the averages of a number of commercial installations, gives the weight of anhydrous ammonia required for the **compression side** of refrigerating plants:

Tons of refrigeration	5	10	20	40	100	200	300	500
Ammonia, lb.	110	150	230	300	440	620	840	1215

The anhydrous ammonia required per 100 running feet of pipe on the **expansion side** is approximately as follows:

Size of pipe, in.	1	1¼	1½	2
For refrigerating plants, direct expansion and brine cooling coils, lb.	14	18	20	25
For ice plants, expansion coils for can and plate use, lb.	8	11	12	15

The total ammonia required in the system in **ice-making plants** is as follows:

Tons ice per 24 hr.	5	10	15	25	50	100
Ammonia, lb.	100	250	500	1000	2000	4000

Operation of Ammonia Compression Plant

Preliminary Leakage Tests. To prove a new plant before it is charged with ammonia, it should be filled with compressed air (at about 100 lb. pressure) by working the compressor while the suction valves provided for this purpose are opened. Thick soap lather spread over the pipes, etc., indicates leaks by the formation of bubbles. The coils in brine tanks, filled with water, show leaks by the bubbles of air escaping through the water. After the pressure is equalized throughout the whole system, the pressure shown by the gage will remain stationary if the plant is absolutely air-tight. A similar test should be made under vacuum.

Charging the Plant. After obtaining the best possible vacuum, a drum of ammonia is connected to the charging valve and the expansion valve is closed. The liquid ammonia is then admitted into the system, while the compressor is kept running at a very slow speed with suction and discharge valves opened and water running in the condenser. If the air is not completely exhausted from the plant, **charging by degrees** is advisable. First, about one-half of the total amount of ammonia is charged, and after this has thoroughly circulated in the system, most of the remaining air will have collected in the top of the condenser, whence it can be blown off. The balance of the ammonia is then charged in a similar way, in one or two operations.

Operation of Plant. The working of a compression machine is regulated chiefly by the expansion valve. The pipe conveying the compressed ammonia to the condenser should not get hotter than 300 deg. fahr., and the temperature of the brine should be also recommended for closing leaks. When so used the cement must be fortified by the application of sheet rubber and sheet-iron sleeves, kept in position by iron clasps.

Absorption Machines

For the theory of the absorption system of refrigeration, see p. 348. In this system there is no positive action of the machine, and the capacity and efficiency depend not only upon details of design, but also to a much larger extent upon the skill and watchfulness of the operator. An absorption machine, under best conditions of design and operation, using low gas pressure, with a temperature in the refrigerator of not more than 0 deg. fahr., and of a capacity of 50 to 100 tons, gives refrigerating results practically equal to those of a compression machine also under favorable conditions. For capacities exceeding 100 tons the compression machine is not only more positive but becomes more efficient because of the greater efficiency of the larger steam engine, which is commonly compound-condensing and may use superheated steam. The absorption machine gives best comparative results as the suction pressures are reduced; the compression machine as they are raised. Absorption machines have been confined to comparatively small units (up to about 225 tons), while compression machines are now in successful operation with capacities as high as 1000 tons.

Where local conditions are favorable, a **combination of the two machines** will result in an increased efficiency, the absorption machine being run with the exhaust steam from the compression machine—which in this case may be simple, non-condensing, and which would handle the refrigerating load of a high-temperature system—while the absorption machine handles the load of the low-temperature system. This is seldom done in practice, for the reason that the two systems are never constant with relation to each other, making necessary some manipulation of the steam supply to either machine, which, together with the lack of interchangeability of coolers and condensers, tends to make this method of combining the two systems undesirable. The coolers and condensers after having been used on a compression system cannot be used on an absorption system because the lubricating oil of the compressor which deposits in these devices upsets the operation of an absorption system. Cleaning is possible but not practicable.

Advantages and Disadvantages of Absorption System. The absorption system would be favorably indicated where refrigeration is to be accomplished in a plant in which there are steam engines operated for power purposes, as in small electric-light plants and in some manufacturing plants, the machine being run with the exhaust steam from these other units.

The **repair costs** of the absorption machine may be heavy when overdriven and when using a corrosive water in the absorber and rectifier. Slight quantities of oil have a very bad effect upon the efficiency of the absorption machine. Faulty **rectification,** due either to faulty design or operation, is one of the greatest sources of loss in the use of the absorption system. This defect has been known to be so serious as to nullify 85 to 90 per cent. of the total capacity of the machine. The best systems of rectification should consist of an efficient analyzer, a mechanical separator, a cooling coil and preferably a second separator with liquid drip after the cooling coil for the purpose of testing the results of rectification. At the full capacity of the machine from 1 to 5 per cent. of the water vapor is usually carried through the rectifier to the condenser and cooler. The analyzer, first separator and cooling coils should discharge their drips as returns to the generator; the drip from the final separator gives an indication of the condition of the rectified gas, which should be used as a basis for the regulation of the machine.

the same conditions when driven by a simple non-condensing engine (S.C.) using 28 lb. of steam per engine i.h.p. per hour, and when driven by a compound condensing engine (C.C.) using 15 lb. per i.h.p-hour. The variation in the calculated capacity of an absorption plant with change of condenser and suction pressures is much less than with a compression machine, as is shown by the last two lines of Table 9.

Comparison of Compression and Absorption Plants. Denton and Jacobus (*Trans. A. S. M. E.*, vol. 13) have made a theoretical comparison of compression and absorption plants under various assumed conditions of operation, showing that the refrigeration per lb. of coal does not vary greatly with variation in the temperature range of the ammonia in an absorption system, whereas it decreases rapidly in a compression machine as the temperature range widens. The range of temperatures at which the two systems are theoretically of equal efficiency depends upon the steam consumption of the engine driving the compressor. In Table 10, 15 per cent. is added to the theoretical work of compression for compressor and engine friction. Each pound of coal is assumed to give 10,000 B.t.u. to the boiler. It is assumed that no water goes over to the condenser of the absorption machine; if 5 per cent. is entrained it will lower the efficiency of the machine 15 to 20 per cent.

Table 10. Comparison of Absorption and Compression Plants Under Various Conditions of Operation

Temperatures, deg. fahr.			Pounds of ice-melting effect per lb. of coal				B.t.u. per lb. of ammonia circulated through the system				
			Compression machine		Absorption machine						
Condenser	Refrigerating coils	Absorber	Using 3 lb. of coal per steam i.h.p.-hour.	Using 1.6 lb. of coal per steam i.h.p.-hour.	Strong liquor pump exhausts to the generator	Strong liquor pump exhausts to feed heater; feed water at 212 deg. fahr.	Heat furnished to generator of absorption machine.	Refrigerating effect (same for both systems)	Heat abstracted from condenser (same for both systems)	Equivalent of work of compression (same for both systems)	Heat abstracted from absorber of absorption machine
61.2	5	61.2	38.1	71.4	38.1	33.5	969	524	596	72	897
59.0	5	59.0	39.8	74.6	38.3	33.9	967	526	595	69	898
59.0	5	130.0	39.8	74.6	39.8	35.1	931	526	595	69	862
59.0	−22	59.0	23.4	43.9	36.3	31.5	1000	516	631	115	885
86.0	5	86.0	25.0	46.9	35.4	28.6	988	497	601	104	884
86.0	5	130.0	25.0	46.9	36.2	29.2	966	497	601	104	862
86.0	−22	86.0	16.5	30.8	33.3	26.5	1025	486	640	154	871
86.0	−22	130.0	16.5	30.8	34.1	27.0	1002	486	640	154	848
104.0	5	104.0	19.6	36.8	33.4	25.1	1002	476	603	127	875
104.0	−22	104.0	13.5	25.3	31.4	23.4	1041	466	646	180	861

Physical Data on Aqua Ammonia. For the solubility of ammonia in water, see Table 17, p. 300. For the heat of absorption and weight of strong liquor circulated in an absorption machine, see Table 43, p. 351. For the properties of saturated and superheated ammonia, see Tables 34 and 35, pp. 333–5. Table 11 given the heat of solution of liquid ammonia. Fig. 1 shows the temperature, specific gravity and concentration relations of aqua ammonia, and Fig. 2 the pressure, temperature and concentration relations.

Table 11. Heat of Solution of Liquid Ammonia
(B.t.u. given up per lb. of ammonia dissolved).

Concentration*	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution	Concentration	Heat of solution
0	347.4	11	302.8	21	253.8	31	197.6	41	135.0	51	63.0
1	343.8	12	298.2	22	248.4	32	191.9	42	127.8	52	55.8
2	340.2	13	293.6	23	243.0	33	186.1	43	120.6	53	48.6
3	336.6	14	289.0	24	237.6	34	180.4	44	113.4	54	41.4
4	333.0	15	284.4	25	232.2	35	174.6	45	106.2	55	34.2
5	329.4	16	279.4	26	226.4	36	168.1	46	99.0	56	27.4
6	325.0	17	274.3	27	220.7	37	161.6	47	91.8	57	20.5
7	320.6	18	269.2	28	214.9	38	155.2	48	84.6	58	13.7
8	316.2	19	264.2	29	209.2	39	148.7	49	77.4	59	6.8
9	311.8	20	259.2	30	203.4	40	142.2	50	70.2	60	0.0
10	307.4										

* Average concentration; per cent. of ammonia by weight.

Charging an Absorption Plant. Pump a vacuum in the system and close all the valves when a vacuum of 25 in. is obtained. Then admit aqua ammonia through the charge pipe in the generator until the vacuum in the machine is gone, and pump in the balance with the ammonia pump until nearly the requisite charge is in; then heat the ammonia slowly by turning steam through the heater coils. When the pressure gage indicates about 100 lb., purge the condenser, turn on the water into the condenser and absorber, and apply the steam until liquefied gas shows in the glass gage of the condenser. Open the expansion valve and turn the poor liquor into the absorber; the ammonia pump may be started as soon as liquor has filled the glass gage. An absorption machine, if charged when cold to its working level, will be **overcharged** under working conditions, so that the liquor may rise out of sight in the gage glass of the generator.

To recharge an absorption plant when the gas has leaked out or the liquor has become impoverished, add anhydrous ammonia to the required amount. When the liquor is lacking it is best to recharge the machine with strong aqua ammonia at 26 to 28 deg. Baumé until the original level is reached. The weight of anhydrous ammonia, x (lb.), which must be added to m lb. of ammonia liquor of the percentage strength a to raise it to the percentage strength b, is given by the equation: $x = m\ (b - a)/(100 - b)$. The drum of anhydrous ammonia may be mounted on scales and connected by a long ½-in. pipe to the low-pressure side of the expansion valve until the desired weight has entered the system.

Ammonia Piping and Fittings

Three general types of fittings are now furnished by a number of manufacturers (see also p. 832): (1) The **gland end,** in which the fitting is tapped with the regular pipe thread, outside of which is a recess for a rubber washer, which is pressed into the recess and against the pipe at the joint by a gland which slips over the pipe and bolts to the end of the fitting; (2) the **flanged end,** which usually makes a tongue-and-groove joint with its companion flange; (3) the **threaded end** fitting, which may or may not have a recess at the outer end for soldering. The defects of the soldering method are: (*a*) When used on lines larger than 3 in., the vibration, expansion and contraction ultimately break the soldered joint, allowing the fitting to leak; (*b*) when used on the discharge gas line of a compressor operating at a pressure of from 175 to 200 lb., the temperature of the gas, which often reaches 300 deg. fahr. or more, brings the solder to a plastic state where it is of little or no value for maintaining a tight joint.

It is customary on high-grade work to use extra-heavy pipe. Vibration, expansion and bending stresses may produce excessive stresses at the root of the thread, just outside of the fitting. The heavier the piping the more trouble from this cause. In the practice of the Quincy Market Cold Storage and Warehouse Co. no threaded pipe over 3 in. in diam. is used. Up to 3 in. a plain screwed fitting is used and the joint is coated with a cement made of litharge and glycerine before screwing together. For **piping over 3 in.** the Fairbanks method is employed, and, as no threads are used, only standard-speed of the water should be from 250 to 300 ft. per min. for the best results and the heat exchange about 170 B.t.u. per hr. per deg. fahr. mean difference between the ammonia and the water. This type is badly affected by corrosive and muddy water, making it necessary to clean often (either by scraper or compressed air) through the end of each section. Leakages can be detected only by regular tests of the outlet water. Table 12 shows the rate of heat exchange with varying velocities of fluid for double-pipe (2-in. and 3-in.) condensers and brine coolers. Table 13 gives results of tests by the York Mfg. Co. on a 1¼-in. × 2-in. double-pipe condenser, showing the effects of varying the amount (and velocity) of the condensing water (at 70 deg. fahr.); gage suction pressure, 15.66 lb. per sq. in.

Table 12. Heat Transmission through 2-in. × 3-in. Double-pipe Condensers and Brine Coolers

Velocity of fluid in pipe, ft. per min.		100	200	300	400	500	600	700
B.t.u. per sq. ft. per deg. fahr. per hr.	Condenser	150	235	290	340	...	...	...
	Brine cooler	95	130	160	180	190	205	215

Table 13. Effect of Varying the Amount and Velocity of Condensing Water in a 1¼-in. × 2-in. Double-pipe Condenser

Velocity of condensing water, ft. per min.	Constant head pressure (185 lb. per sq. in., gage)		Constant capacity		
	Relative capacity, tons per 24 hr.	Condensing water, gal. per min. per ton of refrigeration	Head pressure, lb. per sq. in., gage	Horse power per ton of refrigeration	Condensing water, gal. per min. per ton refrigeration
100	0.67	1.160	225	2.04	0.78
150	1.00	1.165	185	1.71	1.17
200	1.34	1.165	165	1.54	1.55
250	1.64	1.180	155	1.46	1.94
300	1.88	1.240	148	1.40	2.33
400	2.40	1.300	140	1.33	3.11

Flooded or Circulating Condensers are made both atmospheric and double-pipe. In these condensers advantage is taken of the fact that the rate of heat transfer between a liquid and a liquid is much greater than between a liquid and a gas. As usually constructed, the gas is first passed through sufficient pipe to remove the superheat. It is then fed by an injector action into the lower part of the condenser proper, and the piping is so arranged that the condenser is kept full of liquid ammonia moving rapidly and subjected to the cooling effect of the condensing water so that the liquid ammonia when arriving at the injector is cold enough to condense the incoming gas. The liquid ammonia is drawn off from the rapidly circulating liquid at a point just before the injector and conveyed to the receiver. For 70 deg. fahr. condensing water and a head pressure of 200 lb. absolute, a coil of 12 pipes high, atmospheric type, 2-in. pipe 20 ft. long, *i.e.*, with 150 sq. ft. of pipe surface, will have a capacity of 39.3 tons, using 60 gal. of cooling water per min. or 1.53 gal. per min. per ton per day. The surface is 150/39.3 = 3.81 sq. ft. per ton, or only about 60 per cent. of that required for the regular double-pipe type or about 15 per cent. of the pipe required in a regular atmospheric condenser. The flooded condenser is adversely affected by oil, air and foul gases, and for this reason 25 to 30 per cent. additional surface should be added.

Shell-and-tube Condensers are practically the same as steam surface condensers. The water passes back and forth through tubes and the am-the tubes. The surface required is from 12 to 20 sq. ft. per on. These condensers must be shut down for cleaning. With igh vertical shells and plenty of water, this type is very e pass, the water flowing upward through the tubes. The t open so that tubes can be cleaned by blowing with com- scraping and brushing, without interfering with the opera- e makes a good oil, air and foul-gas separator and acts as a eceiver; leaks are easily detected and repaired and tubes can aced.

oil Condensers usually have a circular shell into which the e cooling coils are helical concentric coils through which the ows at a speed usually about 100 to 150 ft. per min. The coils o a header at each end. About 16 sq. ft. of surface per ton is y have the advantage of being compact and clearing the sur- ammonia as fast as formed. With clean water and where is obtainable for blowing out the coils, this type is satisfactory. **d condensers** the ammonia coils are submerged in a tank the cooling water flows. Except in very small machines they on account of the large water area and consequent slow speed ver the coils calling for large coil surface. When used, from 30 surface should be provided for each ton of refrigeration.

ndenser is similar to the steam jet condenser and consists of a el into which the ammonia gas is discharged from the compres- of cool liquid anhydrous ammonia is sprayed upon the gas as it th pass through a combining cone and the resulting liquid flows rough a liquid cooler. This type gives a minimum condenser is suitable only for large plants.

Surface and Cooling-water Requirements of Different Types of Refrigerating Condensers*

	Surface, sq. ft. per ton of refrigeration	Transmission, B.t.u. per sq. ft. per min. per ton	Pipe per ton of refrigeration		Water, gal. per min. per ton
			Size most used, in.	Lineal ft.	
le-pipe...	5	40.0	1¼	14	1½–2
spheric...	6	33.3	2	11	1½–2
....	8	25.0	1¼	22	1½–2
....	16	12.5	2 1½ 1¼	30 38 44	2–2½
e........	18	11.0	1½	55	2–3
....	24	8.3	2	45	1½–3
....	35	5.7	2	65	3–7

water at 70 deg. fahr., condenser pressure 200 lb. abs. Where the condens- oul and local conditions do not admit of frequent cleaning, both surface and should be increased from 10 to 25 per cent.

Methods of Applying Refrigerants

ation is carried out either by direct expansion or by the use of th **direct expansion** the liquid at approximately the condenser fed through an expansion valve directly into the piping which is for cooling. The liquid spray from the expansion valve passes e coils at high velocity, maintaining a wetted inner surface, and is vaporized by the absorption of heat through the pipe walls; the vapor is removed and liquefied by the refrigerating machine and delivered to the expansion valve again.

In the **brine system** the direct-expansion system is used for cooling brine and the cooled brine is pumped through pipe lines to the point where the cooling is to be done, the heat being absorbed by the brine and the brine returned to be again cooled. The brine-cooling vessel is usually either a double or triple pipe coil or of the shell type.

In refrigeration by **compressed air** the exhaust from the motor cylinder is passed through the room to be cooled and allowed to escape (open system) or is passed through piping and returned to the compressor (closed system); the latter system is the more efficient and used for lower temperatures, the open system being used principally where mild temperatures are required and where ventilation is a factor.

Piping of Rooms. The **size of pipe** usually employed for piping rooms varies from 1 to 2 in. If a room is to be held at a temperature of 34 deg. fahr. and the temperature of the expanding ammonia is 10 deg., it will take only half as much pipe to convey a certain amount of refrigeration as with an ammonia temperature of 22 deg. It is generally advisable to use the larger amount of pipe in order to enable the compressor to work with a higher back pressure.

Allowing a difference of 8 to 15 deg. fahr. between the temperatures inside and outside of the pipes, it is usually assumed that 1 sq. ft. of pipe surface will absorb 2500 to 4000 B.t.u. in 24 hr. in direct expansion. Assuming a **rate of heat transmission** of 10 B.t.u. per hour per sq. ft. of surface per deg. fahr. difference between the temperatures inside and outside of pipe, in case of **direct expansion,** if R is the amount of refrigeration (tons) required for a room in 24 hr., and t and t_1 the temperatures of the room and within the ammonia coils (deg. fahr.), respectively, then the **pipe surface required** (sq. ft.) $= S = 288{,}000R/240(t - t_1)$. In **brine circulation** the brine, with the same back pressure, has a higher temperature than the ammonia, and consequently from one to one and one-half times as much pipe is used in brine circulation as in direct expansion for a given back pressure.

Practical Rules for Piping. The following rules, which are commonly used, give surfaces in excess of those determined by the foregoing equation. They are likely to yield satisfactory results under very adverse conditions.

One running foot of 2-in. pipe (direct-expansion) will take care of 10 cu. ft. of space in rooms which are to be kept between 32 deg. and 10 deg. fahr.; of 40 cu. ft. of space in rooms to be kept at or above the 32 deg. fahr., and of 60 cu. ft. of space in rooms to be kept at 50 deg. fahr. and above. These rules are intended to cover rooms of 50,000 cu. ft. capacity and less, poorly insulated, and operated with small differences in temperature. Still more liberal is the common assumption that 1 ton of refrigerating capacity will take care of 4500 cu. ft. of cold-storage capacity to be held at 32 to 35 deg. fahr., and that from 260 to 300 ft. of 1¼-in. pipe should be used to distribute 1 ton of refrigeration. The extra cost of liberal piping allowance will often be offset by the consequent improvement in the efficiency of operation of the compressor. An expansion valve should be provided for every 400-ft. length of 1-in. pipe, every 500 ft. of 1¼-in. pipe, and every 1000 ft. of 2-in. pipe. The **pipes in storage rooms** should be placed where they are least in the way, and should be arranged in independent sections connected by manifolds in such a way that each section can be shut out to throw off the frost which gathers on the surface.

Brine Circulation is preferred by many to direct expansion, in consequence of the fear of danger from escaping ammonia in case the pipes should leak. This danger has probably been exaggerated, as but few accidents of this kind have been known, the pressure in the ammonia pipe being generally ing point than common salt. It is, however, higher in price. It is supplied in 600-lb. drums, either solid or granulated, and in tank cars containing 6000, 8000 or 10,000 gal. of liquid at 1.350 sp. gr. The properties of sodium chloride solutions are given in Tables 6 and 11, pp. 296–9, and of calcium chloride in Table 11, p. 299, and Fig. 4. The specific gravities in Fig. 4 are measured at 60 deg. fahr. The Solvay Co. calcium chloride crystals are of approximately the following percentage composition: Calcium chloride, 73.6; sodium chloride, 1.45; water, 24.9; constituents insoluble in water, 1.07. Table 15 gives the densities of Solvay calcium chloride solutions.

The density of brine is sometimes measured by a **salinometer**. This is a simple hydrometer the indications on which are 4 times greater than on the corresponding Baumé scale (see p. 85).

It is undesirable to use a **strength of solution** of salt greater than is necessitated by its freezing temperature, as the specific heat (Table 6, p. 296 and Table 11, p. 299) decreases as the concentration of the brine increases and consequently the stronger the brine the less heat a given amount of it will be able to convey between certain definite temperatures. Moreover, brine which is too strong may cause clogging of pipes, etc., by depositing salt. On the other hand, if the solution is too weak it may not be able to withstand the temperature existing in the expansion coil, so that a layer of thin ice will form around the latter and interfere with the absorption of heat from the brine. The surface of the expansion coils in the brine tank should be inspected from time to time to see if any ice has formed on them. In larger plants it is customary to use a solution with a freezing point not less than 10 deg. below the lowest temperature which will be obtained in the operation of the plant. In smaller isolated plants and where careful supervision is not insured, it is customary to make the solution as strong as possible without being unstable, usually 1.240 to 1.250 sp. gr. A solution of 1.250 sp. gr. at 60 deg. fahr. becomes unstable at approximately −37 deg. fahr.

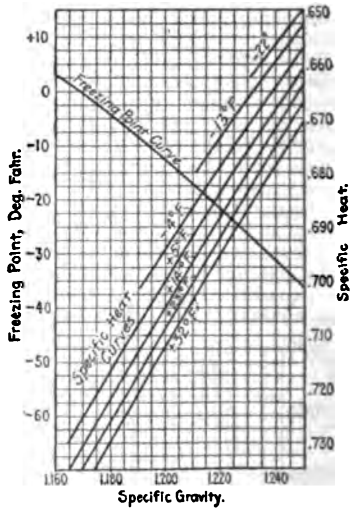

FIG. 4.—Properties of Calcium Chloride Solutions.

Table 15. Density of Calcium Chloride Solutions (at 64 deg. fahr.)

Specific gravity	$CaCl_2$, per cent.	Specific gravity	$CaCl_2$, per cent.	Specific gravity	$CaCl_2$, per cent.	Specific gravity	$CaCl_2$, per cent.
1.007	0.943	1.065	8.487	1.131	16.031	1.205	23.575
1.014	1.886	1.073	9.430	1.140	16.974	1.215	24.518
1.021	2.829	1.081	10.373	1.149	17.917	1.225	25.461
1.028	3.772	1.089	11.316	1.158	18.860	1.236	26.404
1.035	4.715	1.097	12.259	1.167	19.803	1.246	27.347
1.043	5.658	1.105	13.202	1.176	20.746	1.257	28.290
1.050	6.601	1.114	14.145	1.186	21.689	1.268	29.233
1.058	7.544	1.112	15.088	1.196	22.632	1.279	30.176

intervals, with expansion joints at definite distances from each other, confining the expansion and contraction to those distances and to calculable limits; and (2) welding the pipes in a continuous length from manhole to manhole and putting expansion joints at the manhole or "U" bends on the run.

Brine Circulation System. In large installations the ammonia should be confined to the power house, where it can be controlled readily, and the transmission of refrigeration accomplished by means of brine circulation. The **brine coolers** used are either of the tank-and-submerged-coil type or shell coolers of either the coil or straight-tube type. For other than small capacities coolers of the shell type with straight tubes are more efficient, offer less frictional resistance to flow, and are cleaned and repaired more readily. The **brine pumps** are preferably of the reciprocating type. The centrifugal pump is not well adapted for the larger work, not only on account of its low mechanical efficiency, but also because the work lost in fluid friction in the pump heats up the brine, sometimes as much as 20 per cent. of the total cooling work being done on the brine. This is especially true when the flow from the centrifugal pump is restricted by line friction or by the valves used to regulate the brine supply.

In general practice, the **amount of brine required** per min. per ton of refrigeration is from 5 to 6 gal. Assuming 5 gal. per ton at 1.2 sp. gr., or 50 lb. per min. per ton, and a specific heat of 0.700, the difference in temperature of the incoming and outgoing brine will be 200 B.t.u./(50 $\times$ 0.700) = 5.7 deg. fahr. With 6 gal. per min. per ton the temperature difference would be 4.8 deg.; an average of 5 deg. is good practice. A fair-sized street system supplying brine to boxes in the third and fourth stories of a building will require (including line friction) a pressure of about 75 lb. (= 172.8 ft. head) on the discharge side of the pump. The useful work done on the brine will be: Head (ft.) $\times$ lb. brine per min./33,000 = 172.8 $\times$ 50/33,000 = 0.26 h.p. per ton of refrigeration for 24 hr. With a pump of 50 per cent. mechanical efficiency, the **horse power required** would therefore be 0.26/0.50 = 0.52; with 85 per cent. efficiency, which is obtainable with pumping engines of the type used in the largest installations, the pump will use 0.3 h.p. per ton. The **discharge pressure** of the Quincy Market Cold Storage & Warehouse Co.'s street system averages 100 lb., with a suction pressure varying from 15 to 35 lb., pumping 6,000,000 gal. of brine in 24 hr. This will supply brine to the tenth story of a building with ample reserve pressure.

The **street mains** for brine circulation may be either of cast iron of the bell-and-spigot type, the ordinary flanged and screwed type, or of the gland-end type. The Fairbanks joint (p. 1728) gives the necessary tightness and flexibility, but costs about 10 per cent. more than the other types; it lacks, however, the quality which the bell-and-spigot type has, namely, that of varying the alignment at each joint to get around obstructions.

Mains, as far as possible, should be **laid below the frost line,** and expansion joints, either of the packed-sleeve type or the accordion type, should be provided at least every 500 ft.; where large branch mains are taken off expansion joints should be not more than 250 ft. apart. All **expansion joints** should be placed in manholes for easy inspection and repair. As a further protection against expansion a 1-in. by-pass should be connected between the supply and return main at the end of each section of not more than 500 ft. when laid, and brine circulation set up so that the lines may be brought down to the average service temperature before being anchored, insulated and covered in. The destructive effects of expansion and contraction are best safeguarded against by the provision of power-plant equipment pipe in the middle of each expansion section so that the cork and pitch will weld to the pipe as well as to the box itself.

Cold Storage

Insulation. For general theory and tables of conductivities, see p. 304. Insulation used in refrigeration is usually compound. The insulating value of filling material is nearly inversely as its specific gravity. This goes down to a point where the material is so loose as to permit air circulation. With most material a specific gravity of 0.160 seem to be the limit. When any fibrous material so far in use is arranged so that it weighs much under this, the effect of an open air space is obtained. This does not apply to **material arranged in layers** of different densities transversely to the direction of the heat. **Dryness of insulation** is of great importance; moist shavings pass 70 per cent. more heat than dry shavings. Paper insulation is often valuable in keeping out moisture.

Refrigerating-coil Pressures, according to Matthews, should be as follows for the room temperatures given:

Temperature of room, deg. fahr.	10	15	20	28	32	36	40	50	60
Coil pressure, lb. per sq. in., gage	10	12	15	22	25	27	30	35	40

Cold-storage Temperatures. Modern cold-storage warehouses of the larger concerns are cooled by brine which is furnished at two different temperatures only. The higher temperature for the mild-temperature warehouses is from 10 to 12 deg. above zero (fahr.), and the low-temperature brine for the freezers is from 10 to 12 deg. below zero. All temperatures above that of the brine are obtained by regulating the amount of brine circulated in any particular set of coils. In the low-temperature warehouses the piping is arranged for two classes of service: (1) **Sharp freezers,** where the goods which are to be frozen are kept while their temperature is brought down quickly to the holding temperature (say in from 24 to 48 hr.), after which they are stored in (2) **holding rooms** where the desired temperature is maintained.

Sharp freezers are now piped with 2 sq. ft. of pipe surface per cu. ft. of room space for fish, meats, game, etc., where the material is open to a more ready absorption of the heat, and with 1 sq. ft. per cu. ft. for packaged goods such as poultry, pork, butter or other goods in boxes or barrels. Holding rooms are usually piped with 1 sq. ft. of pipe surface per 7 cu. ft. of room space; this will allow for an ordinary amount of removal and replacement of goods without materially raising the temperature of the room. Where the warehouse has a high grade of insulation and the goods are not disturbed for a considerable period of time, a piping ratio of 1 sq. ft. of pipe surface to 10 cu. ft. of room space may be used.

The system of cooling which is now being installed in the highest type of warehouses consists of a coil room containing the necessary brine coils, through which the air from the different rooms is circulated by a pressure blower. The inlet and outlet of each room are so arranged that the cooled circulating air will cover the entire room in its transit; this is usually accomplished by having the cold-air inlet in the center of the room and two return outlets—one at each end of the room. The piping ratio for the coil room in this system, assuming a high-grade insulation with 2 to 4 B.t.u. transmission per sq. ft. per 24 hr. per degree difference in temperature, should be sq. ft. of external pipe surface to 15 cu. ft. of space to be cooled (with brine at 10 to 12 deg. below zero) for warehouses carrying temperatures of zero and below, and 1 sq. ft. of pipe surface to 24 cu. ft. of space to be cooled for freezing different thicknesses of ice is proportional to the square of the thickness.

Size of Cans. Commercial sizes of galvanized-iron cans as made by the Triumph Ice Machine Co. are as follows:

Size of can, in..	6×12×24	8×18×32	8×16×40	11×22×32	11×22×44	11×22×57
Weight of cake, lb.	50	100	150	200	300	400
Time to freeze, hr.	20	36	36	55	60	60

Cans are made to hold about 5 per cent. more than their rated capacity, to allow for thawing. The freezing times given are based on a bath temperature of 14–18 deg. fahr.

Brine Tanks are made of sheet iron or steel, of wood, or of cement. Sheet steel is commonly considered the best material and will last from 10 to 12 years. Wooden tanks are built of 2× 4 or 2 × 6-in. planks, according to size of tank, lined with ¼-in. matched flooring. All the 2 × 4-in. plank or the matched flooring is laid and bedded in pure hot asphaltum before being nailed together. Cedar or cypress and hard yellow pine are the woods recommended. Cement tanks must be made of the best cement, thoroughly hardened and dried, and coated with hot asphaltum before being used. A 2-in. space is left between molds and a 3-in. space where the pipes pass between them. Three feet additional length of tank should be allowed for the agitator.

Insulation of Brine Tank. The insulation of the brine tank should be 12 to 18 in. thick at the sides and at least 12 in. under the bottom if granulated cork, sawdust or shavings are used. An excellent insulation consists of not less than 6 in. of waterproof-lith or cork board or a combination of the two in two layers with broken joints thoroughly sealed, and a water- and air-proof material between the two. The intermediate material may be any high grade of waterproof felt paper, but is preferably a heavy coating of cement composed principally of asphalt which is laid with a brush as heavily and smoothly as possible on the outer surface of the first layer of insulation; the second layer is applied to the first as soon as possible so that the asphalt cement will unite the layers of insulation. The cork board is preferably used for the outer layer and should be plastered with a coat of waterproof Portland cement plaster thick enough to stand ordinary wear and tear and to prevent moisture reaching the insulation from the outside.

Top-and-bottom-feed Brine Coil. The expansion coils in brine tanks are fed from the bottom in dry compression systems, and from the top in wet compression systems. The disadvantages of both methods of feeding can be avoided by using "top feed and bottom expansion." In this system alternate coils in the tank are connected to a liquid manifold at the top of the tank, and the ammonia is evaporated downward through one-half of the coils in the tank. All of the coils in the tank are connected to a large bottom manifold from which the gas is returned up through the remaining half of the coils to a gas suction manifold at the top of the tank, located behind and a little above the liquid manifold.

Brine Circulation is effected by the use of propeller wheels, paddles, or pumps (preferably centrifugal pumps). For a 10-ton plant and smaller, a 12-in. propeller wheel run at 200 r.p.m. is recommended, while for a larger plant an 18-in. wheel should be employed.

Economical Method of Operation. According to Voorhees, considerable saving is possible in can-system plants by operating at two different back pressures; the low pressure for freezing the ice and a higher pressure for cooling the water before it goes to the cans and for cooling the liquid ammonia on its way to the expansion valve. By using a multiple-effect system (see p. 1717) and receiver the following (unverified) results are claimed:

Temperature of condensing water, deg fahr...	100	90	80.0	70	60.0	50
Increase in ice-making capacity, per cent......	80	70	58.0	48	38.0	29
Decrease in horse power per ton, per cent....	35	25	22.5	20	17.5	15

Water and Coal Required. For every ton of ice about 2400 lb. of water are required, allowing for loss and leakage, and with good coal (evaporating about 8½ lb. of water per lb. of coal) it takes an average of about 1 lb. of coal to produce 7 lb. of distilled-water ice by the can system (9 lb. with the

PATENTS FOR INVENTIONS

BY

ODIN ROBERTS

REFERENCES: Albert H. Walker, "Patents," Baker Voorhis & Co. William C. Robinson, "Patents," Little, Brown & Co., Boston.

UNITED STATES OF AMERICA

What Subject-matter is Patentable. Any original and useful art (*i.e.*, process or method), machine, article of manufacture, or composition of matter, or any improvement on either, which has not been

(1) Known to or used by others in the United States prior to the invention or origination;

(2) Described anywhere in any patent or printed publication prior to the invention or origination, or more than 2 years before application is made for patent in the United States;

(3) In public use or on sale in the United States more than 2 years before application is made for patent in the United States;

(4) Patented to the inventor in some other country, which by treaty or convention has established reciprocal relations with the United States in patent matters, upon an application filed more than 1 year prior to the patent application in the United States. (This applies to practically all civilized countries.)

Who May Apply for Patent. The original inventor, or inventors jointly, if more than one. The existence of joint invention can be determined only by the facts of each case. The only general rule is that if two or more have worked and consulted together in the development of an invention they are properly joined as applicants for patent. The *owner* of an invention, by assignment and sale from the inventor, may not *apply* for patent, but may receive the patent as assignee, provided the deed of assignment has been recorded in the Patent Office with request by the inventor-applicant that the patent be granted to the assignee.

Term of Patent. Seventeen years from the date of issue. No extensions are granted under the general law.

The Patent Grant gives to the patentee for the term of the patent, the sole and exclusive right to manufacture, sell and use the alleged invention patented.

The Patent Application should be prepared by a competent solicitor. For information concerning the forms and rules, obtain from the Commissioner of Patents, Washington, D. C., a copy of the Rules of Practice of the United States Patent Office.

Cost of Obtaining a Patent. The Patent Office fees are: Filing fee, $15; final fee to obtain patent after allowance, $20. The cost of preparing drawings and specifications will, of course, vary with the subject-matter.

›r **Assignment of Patent,** to be binding, must be by an instrument g. Acceptable forms are found in the Rules of Practice of the ffice. To provide for constructive notice to all, an assignment must led in the Patent Office within 3 months of its date.

of Fractional Interests in Patents. Undivided fractions or l divisions of patent rights are advisable only in special cases, with the best intentions the interests of joint or territorial owners are interfere.

ses are permissions by the patentee to make, use or sell the thing , and, broadly speaking, a license may carry any imaginable provi- parties agree on. The "Act to Supplement Existing Laws Against l Restraints and Monopolies, etc.," approved October 15, 1914, ; unlawful for any person engaged in interstate or interterritorial :e, to lease or sell, or make contract for sale of, *patented* or unpatented or fix prices, discounts, or rebates, on the condition that the lessee ıaser thereof shall not use or deal in commodities of a competitor ›titors of the lessor or seller, where the effect may be to "substantially ›mpetition or tend to create a monopoly in any line of commerce." :ontracts should be reduced to writing. License by oral arrangement ıplication from circumstances, is as binding on the parties as if re- › writing, the difference lying in difficulty of proof.

ions of Employer and Employee, when the latter makes an n. Unless by express contract, or by implication from circumstances, loyee agrees that his patentable inventions shall be the property of loyer, the employer has no title or claim to the invention or to the or it. Even if the employee has developed his invention in the time h materials belonging to the employer, the claim of the employer is the value of time and materials and does not extend to the invention. the employer, with the employee's consent, express or implied, put ›ntion into use, license under the patent to continue use to the extent l is implied, but no such implication extends to any enlargement se.

ngements of Patent. As the grant purports to give the exclusive › make, sell or use, so an unlicensed manufacture, sale or use of the atented is an infringement, and the maker, seller, or user may be a District Court of the United States. **Suit** must be **brought** in rict wherein the defendant resides, or where the defendant, though not ;, has a regularly established place of business, and has committed the nfringement. In all but very exceptional cases, the courts will not ›**reliminary injunctions** in a suit on a patent which has not previ- een held valid in another contested litigation. In the first suit, the r of the patent is only *prima facie*, and is open to contest.

sues of Patent. If, through inadvertence, accident or mistake the l patent was defective, or claimed too much or too little, the error may aired by surrender of the patent and reissue, provided the reissue tion be filed without unreasonable delay and will not, if granted, be cial to rights of others which have been established in the interval n issue and application for reissue. The term of a reissued patent on the day when the original patent would have expired.

king Patented Articles. Unless the patentee has given particular of his patent, or general notice by marking the patented articles ıted," or "Pat.," with the date of his patent, he may not recover damages for infringement. If the patented articles themselves cannot be marked, the mark may be affixed to the packages in which they are contained.

False Marking. Any person who, with intent to deceive the public, falsely marks an article or parcel with a patentee's mark, without leave or authority, or so marks an unpatented article or parcel with patent marks, is liable to a fine of $100 for each offense, one-half to the use of the United States, and one-half to the person bringing the information to a District Court by proper action.

Proceedings in the U. S. Patent Office. Each application for patent found to be correct in form, is examined in its turn. Rejections of claim or requirements of amendment must be answered by the applicant (or his attorney) within 1 year after the date of the official communication, or the application will be held to be abandoned. It can be renewed only by filing a fresh application.

After allowance of an application the applicant has 6 months in which to pay the final fee. Lapse of 6 months without payment of the fee makes forfeiture of the application, which may, however, be renewed within 2 years from the date of allowance in the original case.

Interferences. When an application for patent is found to interfere with, *i.e.*, to present or claim substantially the same inventions as (1) another pending application, (2) a patent issued on a date less than 2 years prior to the date of the application in question, (3) a reissued patent, of which the original was granted on a date less than 2 years prior to the date of the application in question, the two interfering cases are impleaded in an action called an interference, the purpose of which is to determine, in the Patent Office, which of two or more rival claimants is the first inventor. The practice before the Patent Office in Interference is of such a character that an interference should be conducted only by legal counsel equipped with special experience in that practice.

FOREIGN COUNTRIES

International Convention for the Protection of Industrial Property. The important provision of the Convention is that any person who has duly applied for a patent in one of the contracting states shall have a right of priority for 12 months in making application in the other states. Such subsequent application is unaffected by any acts accomplished in the interval, as for example, by publication of the invention, or by the working of it.

The Convention (subject to war conditions) is in force in Australia, Austria, Belgium, Brazil, Ceylon, Cuba, Denmark, with the Faröe Islands, Dominican Republic, France, with Algeria and other colonies, Hungary, Germany, Great Britain, Italy, Japan, Mexico, Netherlands, with their East and West India Colonies, Dutch Guiana (Surinam), New Zealand, Norway, Portugal, with the Azores and Madeira, Servia, Spain, Sweden, Switzerland, Tunis, Tobago, Trinidad and the United States of America.

PRINCIPAL PROVISIONS OF THE PATENT LAWS OF FOREIGN COUNTRIES

Who May Obtain Protection. GREAT BRITAIN. (1) The inventor. (2) The inventor jointly with one or more persons, firms or corporations. (3) A person, firm or corporation residing in Great Britain, on a communication from a person, firm or corporation residing abroad, in which case the communicator need not be the inventor. GERMANY, FRANCE, BELGIUM, AUSTRIA, ITALY, SWITZERLAND, HUNGARY. Any person, firm or corporation, whether inventor or not. The patent office does not require the applicant to establish his right to apply. DENMARK, SWEDEN, JAPAN. The inventor duly authorised assignee. CANADA. The inventor. MEXICO. The inventor or orney.

n of Patent. GREAT BRITAIN. 14 years. GERMANY, FRANCE, SWEDEN, SWITD, HUNGARY, JAPAN, DENMARK, ITALY, AUSTRIA, 15 years. BELGIUM. 20 years. i. 18 years. MEXICO. One year or 20 years. A patent for 1 year is extensible ears during the first year.

uirements as to Novelty. Barring the Provisions of the International Conven- › invention is held to be new: GREAT BRITAIN, SWITZERLAND;—if prior to the date ication, it has become known within the country by printed publication or other- CANADA;—if it was known or used by others before the applicant's invention , or has been in public use with his consent for more than 1 year previous to his tion for patent thereof in Canada. But an application may be filed in Canada 1 year from the date of the first foreign patent therefor. FRANCE, SWEDEN, ITALY, ›, JAPAN;—if prior to the date of application it has been described in print or made in any other way in any country. This rule is subject to the following excep- In Italy, officially printed copies of foreign patents are not a bar; in Sweden re in an International Exposition is not a bar if the application is filed in Sweden 6 months after the opening of the Exposition; in Mexico, exposure in an Inter- l Exposition is not a bar if a description is first filed in Mexico and an application ed within 3 months after the close of the Exposition; or if a foreign patentee in Mexico within 3 months after the first publication of the invention by a foreign office. GERMANY;—if the invention has been described in print in any country the last 100 years prior to the application, or if it has been publicly used in ny before the German application. BELGIUM;—if the invention has been used rcially in Belgium prior to the application, or has become known by a printed tion in any country, unless such prior publication was officially prescribed. A, DENMARK;—if the invention has been described in a printed publication in untry prior to the application, or has been in public use within the country prior application. HUNGARY;—if the invention has been described in print in any y within the last 100 years prior to the application, or if it has been publicly l or exhibited in any country prior to the application in such a manner as to en- ose skilled in the art to use it.

king. (In Great Britain and Canada working must be on an industrial scale. other countries named the requirement as to working is usually met by adver- nts with a view of finding a purchaser or a licensee.) GREAT BRITAIN;—within s from date of grant. CANADA, FRANCE;—within 2 years from date of grant. M;—within 1 year of working elsewhere. AUSTRIA, DENMARK, SWEDEN, HUNGARY, —within 3 years from date of grant. GERMANY;—within 3 years from publica- the grant in the Official journal. Patentees who are citizens of the United States t required to work their patents. SWITZERLAND;—within 3 years from date of tion. This provision has no application to citizens of the United States, pro- the invention covered by the Swiss patent is being worked in the United States. O;—no requirement as to working.

es. GREAT BRITAIN. Progressive annual taxes, beginning before the end of urth year. GERMANY, FRANCE, AUSTRIA, SWEDEN, DENMARK, ITALY, HUNGARY, RLAND, BELGIUM, JAPAN. Annual taxes. CANADA, MEXICO. No taxes.

ortation. Importation of the patented invention is permitted in all the coun- amed in this summary; but importation into Canada after the expiration of 1 om the date of the Canadian patent, invalidates the importer's interest in the .

npulsory Licenses. A patentee may be required to grant licenses in GREAT N, if the reasonable requirements of the public with respect to the invention have en satisfied. In Sweden, if the patentee fails substantially to work the invention. NGARY, if the patentee fails to work the invention to an extent sufficient to supply me demand. In AUSTRIA, (1) when public interest demands; (2) when the holder ter patent cannot employ his invention without using that of an earlier patent, h cases each patentee may be ordered to license the other,

INSTRUCTIONS TO LAYMEN FOR FIRST-AID TREATMENT OF COMMON INJURIES AND DISORDERS

The following instructions have been prepared by a board of 20 physicians in industrial practice organized for co-operative effort in introducing into industrial establishments the most effective measures for the treatment of injuries or ailments of employees. They have been approved by the Conference Board on Safety and Sanitation of the National Founders' Association, the National Association of Manufacturers, the National Metal Trades Association and the National Electric Light Association, and are reproduced here by permission.

Wounds That Bleed. (Abrasions, Cuts, Punctures.) Drop 3 per cent. alcoholic iodine into wound freely, then apply dry sterile gauze to wound and bandage it. Do not otherwise cleanse wound.

Severe Bleeding. Place patient at rest and elevate injured part. Apply sterile gauze pad large enough to allow pressure *upon*, *above* and *below* wound. Bandage *tightly*. If severe bleeding continues, apply tourniquet *between* wound and heart and secure doctor's services at once. Use tourniquet with caution and only after other means have failed to stop bleeding.

Nose Bleeding. Maintain patient in upright position with arms elevated. Have him breathe gently through mouth and not blow nose. If bleeding continues freely, press finger firmly on patient's upper lip close to nose or have him snuff diluted white-wine vinegar into nose.

Injuries Which Do Not Bleed. (Bruises and Sprains.) Cover injury with several layers of sterile gauze or cotton, then bandage tightly. Application of heat or cold may help; other means are unnecessary. If injury is severe, place patient at rest and elevate injured part until doctor's services are secured.

Eye Injuries. (Except Eye Burns.) For ordinary eye irritations flood eye with 4 per cent. boric acid solution. Remove only loose particles which can be brushed off gently with absorbent cotton wrapped around end of toothpick or match. Do not remove foreign bodies stuck in the eye. In that case and for other eye injuries drop castor oil freely into eye, apply sterile gauze, bandage loosely and send patient to doctor.

Splinters or Slivers Embedded in Skin. (Except in Eyes.) If easily reached, withdraw with tweezers, then treat same as "Wounds that Bleed;" otherwise let doctor attend to it.

Fire Burns, Electrical Burns and Sunburn (also **Scalds.**—Ed.). Do not open blisters. Use burn ointment (3 per cent. bicarbonate of soda in petrolatum) freely on sterile gauze applied directly to burn. Cover with several thicknesses of flannel or other soft material, then bandage, *but not tightly*.

Acid Burns. Thoroughly flush wound with water, then dry wound, apply burn ointment and bandage as above.

Alkaline Burns. Thoroughly flush wound with water, then flood with white-wine vinegar to neutralize (dilute vinegar for alkaline *eye* burns), dry wound, apply burn ointment and bandage as above.

Eye Burns. Treat in the same manner as other burns.

Dislocations. In case of dislocation of finger (except second joint of thumb), grasp finger firmly and pull it gently to replace joint; then place finger in splint and bandage it. In all other cases place dislocated part at rest and promptly secure doctor's services.

The jars may be obtained from the Executive Secretary of the Conference Board on Safety and Sanitation, 928 Western Avenue, West Lynn, Mass., or from the Secretary of any of the four Associations named at the head of this article. They cost $6.50 each. In Massachusetts, the Board of Labor and Industries requires three extra articles which cost 40 cents.

MISCELLANEOUS

Lenses

Lenses are transparent bodies which, from the curvature of their surfaces, cause light waves traversing them to converge or diverge. Optical lenses have one or both surfaces of spherical curvature. Biconvex (), plano-convex (| and concavo-convex ((lenses are thicker at the center than at the edges and have a convergent effect. Biconcave)(, plano-concave)| and convexo-concave ((lenses are thinner at the center than at the edges and have a divergent effect. The distance from the center of a lens to the point at which incident plane light waves are brought to a focus, is called the **focal length** (f). If u is the distance from the source of light to the lens, and v the distance from the lens at which the image is formed, then, for a biconvex lens, $1/f = 1/u + 1/v = (n - 1)[(1/r_1) + (1/r_2)]$, where $r_1(r_2)$ is the radius of curvature of the lens surface nearer to (farther from) the source of light, and n is the index of refraction of the material of the lens (= 1.5 to 2.0 for glass). For a biconcave lens, minus signs should precede the reciprocals of f, v, r_1 and r_2 in the formula. For a plano-convex or plano-concave lens, $r_2 = \infty$ and $1/r_2 = 0$. When $u = \infty$ (*i.e.*, the incident waves are plane), $f = v$; when $u = f$, $v = \infty$ (*i.e.*, the transmitted rays are parallel). The **magnifying power** of a lens is measured by $1/f$, the practical unit being that of a lens for which $f = 1$ meter. The magnifying power of a number of lenses in contact is the algebraic sum of their individual powers, the powers of diverging lenses being considered negative. Thus, for two *convex* lenses of focal lengths f_1 and f_2, $1/f = 1/f_1 + 1/f_2$; if the second lens be concave (*i.e.*, divergent), $1/f = 1/f_1 - 1/f_2$.

Approximate Altitudes Corresponding to Different Heights of the Barometer

Reading of barometer		Altitude		Reading of barometer		Altitude	
Millimeters	Inches	Meters	Feet	Millimeters	Inches	Meters	Feet
762	30.0	0	0	660	25.98	1147	3763
760	29.92	21	69	650	25.59	1269	4163
750	29.53	127	417	640	25.20	1393	4568
740	29.13	234	768	630	24.80	1519	4983
730	28.74	342	1122	620	24.41	1647	5403
720	28.35	453	1486	610	24.02	1777	5830
710	27.95	564	1850	600	23.62	1909	6243
700	27.56	678	2224	590	23.23	2043	6703
690	27.16	793	2600	580	22.83	2180	7152
680	26.77	909	2962	570	22.44	2318	7605
670	26.38	1027	3369	560	22.05	2460	8071

Velocity of Sound and Light

The **velocity v of sound in air** is independent of the pressure and, in consequence, of the altitude above the surface of the earth. It is, however, affected by the direction of the wind and by the moisture in the air.

A. S. M. E. TESTING CODES

RULES FOR CONDUCTING EVAPORATIVE TESTS OF BOILERS

Object of Test. Ascertain specific object of test, as determination of capacity and efficiency, comparison of different conditions or methods of operating and of different kinds of fuel, effect of changes in design or capacity or efficiency, etc., and keep same in view in the work of preparation as well as throughout test.

Preparations. Obtain dimensions of principal parts of apparatus to be tested and note general features. Heating surfaces are those in contact with the fire or hot gases. The submerged surfaces in boilers at mean water level are to be considered as water-heating surfaces, and other surfaces which are exposed to the gases as superheating surfaces.

Examine boilers and settings thoroughly for steam and air leakages, and heating surfaces for deposits of soot, mud or scale, remedying such defects as are found in case the test is to determine the highest efficiency or capacity obtainable. The steam main should be so arranged that condensed and entrained water cannot flow back into the boiler.

In steam tests make sure that there is no leakage through blow-offs, drips, etc., or through any steam or water connections of plant or apparatus under test, which would in any way affect the results. Such connections should preferably be blanked off.

Fuel. For tests of maximum efficiency or capacity the coal selected should be the best of its class, especially free from slagging and unusual clinker-forming impurities, and of some kind locally regarded as a commercial standard. In the Eastern States such standards are: Pocahontas (Va. and W. Va.) and New River (W. Va.) semi-bituminous coals; fresh-mined No. 1 buckwheat size anthracite coals containing not over 13 per cent. of ash by analysis; Youghiogheny and Pittsburg bituminous coals. Also, in some sections east of the Allegheny Mountains, Clearfield (Pa.) and Cumberland (Md.) semi-bituminous coals. In the Western States the best coal obtainable in any particular locality is used as a standard.

For guarantee and other tests with a specified coal containing not more than a certain amount of ash and moisture, the coal selected should not be higher in this amount, because any increase is liable to reduce the efficiency and capacity more than the equivalent proportion of such increase.

Apparatus and Instruments Required: (*a*) Platform scales for weighing coal and ashes; (*b*) graduated scales attached to the water glasses; (*c*) tanks and platform scales for weighing water (or water meters calibrated in place), single tanks having a capacity such as to require emptying not oftener than every 5 min. (If two or more are used, interval should be not less than 3 min.); (*d*) pressure gages, thermometers and draft gages; (*e*) calorimeters for determining the calorific value of fuel and the quality of steam; (*f*) furnace pyrometers; (*g*) gas-analysing apparatus. All properly calibrated. (For description of apparatus see "Engineering Measurements," pp. 1670–1697.)

Operating Conditions. Determine proper operating conditions and method of firing, and maintain them throughout test as nearly as possible. Where uniformity in the rate of evaporation from a single boiler is required, this can be accomplished by discharging steam through a waste pipe and regulating the amount by means of a valve. In a battery of boilers, in which only one is tested, the draft may be regulated on the remaining boilers to meet the varying demand for steam, leaving test boiler to work under a steady rate of evaporation.

Duration of Tests. Efficiency tests: Hand-fired boilers and stoker-fired boilers which permit quality and condition of fuel bed to be accurately estimated at beginning and end of test, 10 hr. of continuous running, or sufficient time to burn 250 lb. of coal per sq. ft. of grate. Stoker-fired boilers, 24 hr. Commercial tests, where the service

ous operation night and day, should continue at least 24 hr., for both **toker-fired** boilers. Tests to determine maximum evaporative capacity **be less than** 3 hr. in duration.

l Stopping. The temperature of furnace and boiler, the quantity and **.ve coal and** ash on the grates, the water level, and the steam pressure, **›arly as possible** the same at the end as at the beginning of test. To **ility of conditions** with hand-fired boilers, heat furnace well by a prelimi- **fire low, and** thoroughly clean it, leaving a 2- to 4-in. layer of evenly spread **.te (1 to 2 in.** for small anthracite coals) as a foundation for the new fire. **;hickness** of fuel bed, water level, steam pressure, and time, recording **arting time.** Fire fresh coal from that weighed for test, thoroughly clean **:oceed with test.**

›f test, burn fire low and clean so as to leave same amount of live coal on **rt.** Observe quickly the water level, steam pressure and (stopping) time. **is not the same** as at beginning, make correction therefor by computa- **·t blow down** water-glass column for at least 1 hr. before initial and final **·aken.)** Remove ashes and refuse from ash pit.

containing several boilers where it is not practicable to clean them simul- **›** fires should be cleaned one after the other as rapidly as may be, and each **ıning charged** with enough coal to maintain a thin fire in good working **\fter the last** fire is cleaned and in working condition, burn all the fires **› 6 in.),** note quickly the thickness of each, also the water levels, steam pres- **ıe.** At the time for closing the test the fires should be quickly cleaned one then burned low the same as at the start, and the various observations made **'hese** directions also apply in the case of a large boiler having several **rs** requiring the fire to be cleaned in sections one after the other.

ıechanical stoker other than a chain grate is used, regulate coal feed so as to the low condition required for cleaning. Shut off coal-feeding mechanism pers level full. Clean ash or dump plate, note quickly the depth and condi- on grate, water level, steam pressure, and (starting) time. Then start coal- :hanism, clean ash pit, and proceed with test.

e time arrives for closing test, shut off coal-feeding mechanism, fill hoppers re to same low point as at beginning. Then note water level, steam pressure ing) time. Finally clean ash plate and haul ashes.

ase of chain-grate stokers, the desired operating conditions should be main- 30 min. before starting a test and for a like period before its close, height of te and speed of grate being the same during both of these periods.

s of data should so be made that the test may be divided into periods of 1 hr. ı order that the degree of uniformity obtained may be seen. Half-hourly f instruments are usually sufficient. If there are sudden and wide fluctuations, ings every 15 min. or even oftener. The coal should be weighed and delivered n in amounts sufficient for 1 hr. run, thereby ascertaining the degree of y of firing. The records should be such as to show the consumption of feed :h hour, thereby determining the degree of uniformity of evaporation.

:y of Steam. If the boiler does not produce superheated steam, the percentage ıre should be determined with a throttling or separating calorimeter. If the s superheating surface, the steam temperature should be determined by use of ometer inserted in a thermometer well.

ıturated steam insert a sampling pipe (see p. 1677) in the steam main at a point he entrained moisture is probably most thoroughly mixed. This pipe should ›laced near a point where water may pocket, or where such water may affect the of moisture contained in the sample. If it is necessary to attach sampling ıear the end of a long horizontal run, provide a drip pipe near the front of nozzle ıbly at a pocket) to draw off the water running along bottom of main; add the of this water to that determined by the calorimeter, or, better, install a steam ›or at the point noted.

›sting a stationary boiler, locate sampling nozzle as near boiler as possible, ermometer well in same vicinity when the steam is superheated. For complete ation, provide also sampling nozzle and thermometer well close to throttle of engine or turbine.

Sampling and Drying Coal. Select a representative shovelful from each barrow load as fired, and store samples in a covered metal receptacle in a cool place. When all sampled, break up the lumps, mix intimately and reduce to a sample (of pea size) weighing about 5 lb. by repeated quartering and crushing. Then fill two 1-qt. air-tight fruit jars or other vessels for subsequent use in determinations of moisture, calorific value and chemical composition.

When the sampled coal has been reduced by quartering to, say, 100 lb., withdraw, say, 15 lb., place it in a shallow iron pan and dry on the hot iron boiler flue for 12 hr. or more, weighing before and after drying on scales reading to ¼ oz. The moisture thus determined is approximately reliable for anthracite and semi-bituminous coals, but not for coals containing much inherent moisture. For these, one of the jar samples should be thoroughly air-dried in a thin layer in a warm room, being weighed before and after to determine the surface moisture. Then crush into somewhat coarse grains (less than 1/16 in.), thoroughly mix, and withdraw, say, ½ to 2 oz., weigh in a delicate balance and dry for 1 hr. in an air or sand bath at 240–280 deg. fahr. Then weigh sample and record the loss, and heat and weigh again until the minimum weight is reached. The difference between original and minimum weight is the moisture in the air-dried coal. This added to the surface moisture gives the total moisture.

Ashes and Refuse withdrawn from furnace and ash pit during test and at its close should be weighed as far as possible in a dry state. If wet, the amount of moisture should be ascertained and allowed for, a sample being taken for this purpose, as well as for use in analysis and in determining the unburned carbon and for fusing tests.

Calorific Tests and Analyses of Coal. The quality of the fuel should be determined by calorific tests and an analysis of the sample above referred to. (See p. 603 for methods employed.)

Analyses of Flue Gases. For approximate determinations, use the Orsat or similar apparatus. If momentary samples are obtained, the analyses should be made very frequently, say every 15 to 30 min. If the sample drawn is a continuous one, the intervals may be longer. (For methods of analysis, see p. 1695.)

Smoke Observations. In tests of bituminous coals requiring a determination of the amount of smoke produced, observations should be made regularly throughout the trial at intervals of 5 min. (or if necessary every minute), noting at the same time the furnace and firing conditions. (See p. 885.)

Calculation of Results

Corrections for Moisture in Steam. When the percentage is less than 2 per cent. deduct same from the weight of water fed. If greater than 2 per cent., or if extreme accuracy is required, the factor of correction equals $1-[P(H-h_1)/(H-h)]$, in which P is the proportion of moisture, h_1 the total heat of water at the temperature of the steam, h the total heat of the feed water, and H the total heat of saturated steam of the given temperature. For superheated steam, correction factor $= (H_s-h)/(H-h)$, where $H_s =$ total heat of the steam at the observed temperature and pressure.

Correction for Live Steam Used for Aiding Combustion. If live steam is admitted into the furnace or ash pit for producing blast, injecting fuel, or aiding combustion, it is to be deducted from the total evaporation, and the net evaporation used in the various calculations.

Equivalent Evaporation. (See p. 893.)

Efficiency. For "efficiency of boiler, furnace and grate," and "efficiency of boiler and furnace," see p. 896.

The "combustible burned" is determined by subtracting from the weight of coal supplied to the boiler the moisture in the coal, the weight of ash and unburned coal withdrawn from the furnace and ash pit, and the weight of dust, soot, and refuse, if any, withdrawn from the tubes, flues, and combustion chambers, including ash carried away in the gases, if any, determined from the analyses of coal and ash. The "combustible" used for determining the calorific value is the weight of coal less the moisture and ash found by analysis.

The "heat absorbed" per lb. of coal, or combustible, is calculated by multiplying the equivalent evaporation from and at 212 deg. per lb. of coal or combustible by 970.4.

ıt **Balance.** (See p. 898.)

al **Heat of Combustion of Coal, by Analysis.** (See p. 596.)

for **Combustion.** (See p. 369.)

ı **and Results** should be reported in accordance with the Form given below, *lines* for data not provided for, or omitting those not required, as may conform to ect in view. For a short report, the items designated by letters of the alphabet omitted.

t. For the determination and exposition of the complete boiler performance, the ıg of readings and data should be plotted on a chart and represented graphically.

with Oil and Gas Fuels. Tests of boilers using oil or gas fuel should accord ı rules here given, excepting as they are varied to conform to the particular ristics of the fuel. The duration in such cases may be reduced, and the "flying" of starting and stopping employed. The table of data and results should conıs stating character of furnace and burner, quality and composition of oil or ›erature of oil, and data regarding performance of apparatus supplying the fuel.

Data and Results of Evaporative Test

CODE OF 1915

of............boiler located at...

› determine.............Test conducted by............................

DIMENSIONS

ber and kind of boilers....... (3) Kind of furnace.....................

› surface (width ——— length ———)*..............................sq. ft.

Approx. width of air openings in grate................................in.

Percentage of area of air openings to grate surface...............per cent.

· heating surface,...........sq. ft. (6) Superheating surface.........sq. ft.

heating surface...sq. ft.

Ratio of water heating surface to grate surface...................(–) to 1

Ratio of total heating surface to grate surface...................(–) to 1

Ratio of minimum draft area to grate surface.....................1 to (–)

Volume of combustion space between grate and heating surface......cu. ft

Distance from center of grate to nearest heating surface................ft

DATE, DURATION, ETC.

.... (9) Duration..hr.

ıd size of coal...

AVERAGE PRESSURES, TEMPERATURE, ETC.

ıressure by gage..lb.

arometric pressure...in.

ıture of steam, if superheated.....................................deg.

ormal temperature of saturated steam...............................deg.

ture of feed water entering boiler.................................deg.

ımperature of feed water entering economizer.......................deg.

crease of temperature of water due to economizer...................deg.

ture of escaping gases leaving boiler..............................deg.

mperature of gases leaving economizer..............................deg.

crease of temperature of gases due to economizer...................deg.

nperature of furnace...deg.

lraft between damper and boiler.....................................in.

ft in main flue near boiler...in.

ft in main flue between economizer and chimney......................in.

ft in furnace.......in. (*d*) Draft or blast in ash pit..............in.

eather...

perature of external air...deg.

perature of air entering ash pit†..................................deg.

tive humidity of air entering ash pit..........................per cent.

wise designated this is the total area enclosed within the furnace walls ıtally.

r should be protected from direct radiation of boiler and furnace.

Quality of Steam

(17) Percentage of moisture in steam or number of degrees of superheating .... per cent. or deg.
(18) Factor of correction for quality of steam ....

Total Quantities

(19) Total weight of coal as fired* .... lb.
(20) Percentage of moisture in coal as fired .... per cent.
(21) Total weight of dry coal (Item 19 × [1 − Item 20] ÷ 100) .... lb.
(22) Ash, clinkers and refuse (dry) ....
  (*A*) Withdrawn from furnace and ash pit .... lb.
  (*B*) Withdrawn from tubes, flues and combustion chamber .... lb.
  (*C*) Blown away with gases .... lb.
  (*D*) Total .... lb.
  (*a*) Weight of clinkers contained in total ash .... lb.
(23) Total combustible burned (Item 21 − Item 22 (*D*)†) .... lb.
(24) Percentage of ash and refuse based on dry coal .... per cent.
(25) Total weight of water fed to boilers‡ .... lb.
(26) Total water evaporated, corrected for quality of steam (Item 25 × Item 18) .... lb.
(27) Factor of evaporation based on temperature of water entering boiler ....
(28) Total equivalent evaporation from and at 212 deg. (Item 26 × Item 27) .... lb.

Hourly Quantities and Rates

(29) Dry coal per hour .... lb.
(30) Dry coal per sq. ft. of grate surface per hour .... lb.
(31) Water evaporated per hour, corrected for quality of steam .... lb.
(32) Equivalent evaporation per hour from and at 212 deg.§ .... lb.
(33) Equivalent evaporation per hour from and at 212 deg. per sq. ft. of water heating surface§ .... lb.

Capacity

(34) Evaporation per hour from and at 212 deg. (same as Item 32) .... lb.
  (*a*) Boiler horse power developed (Item 34 ÷ 34½) .... bl.h.p.
(35) Rated capacity per hour, from and at 212 deg .... lb.
  (*a*) Rated boiler horse power .... bl.h.p.
(36) Percentage of rated capacity developed .... per cent.

Economy

(37) Water fed per lb. of coal as fired (Item 25 ÷ Item 19) .... lb.
(38) Water evaporated per lb. of dry coal (Item 26 ÷ Item 21) .... lb.
(39) Equivalent evaporation per lb. of coal as fired (Item 28 ÷ Item 19) .... lb.
(40) Equivalent evaporation per lb. of dry coal (Item 28 ÷ Item 21) .... lb.
(41) Equivalent evaporation per lb. of combustible (Item 28 ÷ Item 23) .... lb.

Efficiency

(42) Calorific value of 1 lb. of dry coal by calorimeter‖ .... B.t.u.
  (*a*) Calorific value of 1 lb. dry coal by analysis .... B.t.u.
(43) Calorific value of 1 lb. of combustible by calorimeter .... B.t.u.
  (*a*) Calorific value of 1 lb. combustible by analysis .... B.t.u.
(44) Efficiency of boiler, furnace, and grate (100 × Item 40 × 970.4) /Item 42. See p. 896 .... per cent.
(45) Efficiency based on combustible (100 × Item 41 × 970.4)/Item 43 .... per cent.

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* The term "as fired" means actual condition including moisture, corrected for estimated difference in weight of coal on the grate at beginning and end.
† If either of the two Items 22 (*B*) and 22 (*C*) is omitted, the fact should be so stated.
‡ Corrected for inequality of water level and of steam pressure at beginning and end.
§ The symbol "U. E.," meaning Units of Evaporation, may be substituted for the expression, "Equivalent evaporation from and at 212 deg."
‖ If the calorific value is desired per lb. of coal "as fired," multiply Item 42 by (100 − Item 20) ÷ 100.

If it is desired that the heat balance be based on coal "as fired" or on "combustible burned," the items in first column are multiplied by the proportion (100 − Item 20)/100 for coal "as fired," or by the proportion (100 − Item 20)/[100 − (Item 20 + Item 24)] for "combustible burned."

The Principal Data and Results of a Boiler Test are given by Items (4), (7), (8), (9), (10), (11), (13), (17), (20), (29), (30), (32), (33), (35), (36), (40), (41), (42), (43), (44) and (45).

RULES FOR CONDUCTING TESTS OF RECIPROCATING STEAM ENGINES

The code for steam engine tests applies to tests for determining the performance of the engine alone (including reheaters and jackets, if any) apart from that of steam-driven auxiliaries which are necessary to its operation. For tests of engine and auxiliaries combined, and tests of multiple-expansion engines from which steam is withdrawn for heating feed water or otherwise, refer to the Code for Complete Steam-power Plants, p. 1763.

Object and Preparations. Determine the object of the test, take the dimensions, and note the physical condition not only of the engine but of all parts of the plant that are concerned in the determinations, and examine for leakages.

Apparatus and Instruments. The determination of the heat and steam consumption of an engine by feed-water test requires the measurement of the various supplies of water fed to the boiler; of the water wasted by separators and drips on the main steam line; of steam used for other purposes than the main-engine cylinders; and of water and steam which escape by leakage of the boiler and piping; all of these last being deducted from the total feed water measured.

Where a surface condenser is provided and the steam consumption is determined from the water discharged by the air pump, no such measurement of drips and leakage is required, but assurance must be had that all the steam passing into the cylinders finds its way into the condenser. If the condenser leaks, the defects causing such leakage should be remedied, or suitable correction should be made. The water of condensation from jackets and reheaters, if not included in the air-pump discharge, should be added thereto.

When no other method is available the steam consumption may be determined by the use of a steam meter, which should be calibrated under the exact conditions of use.

The steam consumed by steam-driven auxiliaries which are required for the operation of the engine should not be included in the total steam from which the heat consumption is calculated, but the quantity of steam thus used should be determined and reported.

Duration. A test for steam or heat consumption, with substantially constant load, should be continued for such time as may be necessary to obtain a number of successive hourly records, during which the results are reasonably uniform. For a test involving the measurement of feed water for this purpose, 5 hr. is sufficient. Where a surface condenser is used, and the measurement is that of the water discharged by the air pump, the duration may be somewhat shorter. In this case, successive half-hourly records may be compared and the time correspondingly reduced. When the load varies widely at different times of the day, the duration should be such as to cover the entire period of variation.

Starting and Stopping. The engine and appurtenances having been set to work and thoroughly heated under the prescribed conditions of test (except in cases where the object is to obtain the performance under working conditions), note water levels in boilers and feed reservoir, take the time, and consider this the starting time. Then begin the measurements and observations and carry them forward until end of period determined on, when the water levels and steam pressure should be brought as near as practicable to the same points as at the start. If there are differences in the water levels, proper corrections are to be applied.

Where a surface condenser is used, the collection of water discharged by the air pump begins at the starting time, and the water is thereafter measured or weighed until the end of the test.

Care should be taken in cases where the activity of combustion in the boiler furnaces affects the height of water in the gage glasses that the same conditions of fire and drafts n at the beginning and end of the test. For this reason it is best to start and stop ; without interfering with the regularity of the operation of the feed pump, pro- the latter can be regulated to run so as to supply the feed water at a uniform rate. ne cases where the supply of feed water is irregular, as, for example, where an in- of excessive capacity is used, the supply of feed water should be temporarily ff. Suitable care should be observed in noting the average height of the water glasses, taking sufficient time to satisfactorily judge of the full extent of the ation of the water line, and thereby its mean position.

ords. Half-hourly readings of the instruments are sufficient, excepting where re wide fluctuations. A set of indicator diagrams should be obtained at intervals : 20 min., and oftener if the nature of the test makes it necessary. Mark on each e cylinder and the end on which it was taken, also the time of day. Record on d of each set the readings of the steam and vacuum gages. These records should iently be entered on the general log, together with the areas, pressures, lengths, asured from the diagrams, when these are worked up.

Calculation of Results

Steam. The quantity of dry steam consumed is determined by deducting the , if any, found by calorimeter test from the total amount of feed water (the latter rected for leakages and other losses) or from the amount of air-pump discharge, ise may be. If the steam is superheated, no correction is to be made for the t.

Consumption. The number of heat units consumed by the engine is found by ng the weight of feed water consumed, corrected for moisture in the steam, if for plant leakages and other exterior losses, by the total heat of 1 lb. of steam l or superheated) at the pressure in the steam pipe near the throttle, less the b. of water at the temperature corresponding to the pressure in the exhaust pipe engine.

ıcy. The thermal efficiency, that is, the proportion of the total heat consump- is converted into work, is found by dividing 2546.5 (B.t.u. equivalent of one y the number of heat units actually consumed per h.p.-hr.

ciency of the corresponding Rankine cycle is to be calculated between the ıd temperature in the steam pipe near the throttle and the pressure and tem- the exhaust pipe near the engine. See p. 344.

kine cycle ratio (or the efficiency ratio referred to the Rankine cycle) is found ; the efficiency of the actual engine (referred to the i.h.p. or b.h.p., as the e) by the efficiency of the Rankine cycle.

Accounted for by Indicator Diagrams at Points Near Cut-off and The steam accounted for, expressed in lb. per i.h.p. per hour, may readily be ing the formula

$$(13{,}750/\text{m.e.p.})[(C+E)Wc-(H+E)Wh]$$

e.p. = mean effective pressure; C = proportion of direct stroke completed expansion line near cut-off or release; E = proportion of clearance; H = f return stroke uncompleted at point on compression line just after exhaust = weight of 1 cu. ft. steam at pressure shown at cut-off or release point; eight of 1 cu. ft. steam at pressure shown at compression point.

-expansion engines the mean effective pressure to be used in the above form- gregate m.e.p. referred to the cylinder under consideration. In a com- the aggregate m.e.p. for the h.-p. cylinder is the sum of the actual m.e.p. of inder and that of the l.-p. cylinder multiplied by the cylinder ratio. aggregate m.e.p. for the l.-p. cylinder is the sum of the actual m.e.p. of the and the m.e.p. of the h.-p. cylinder divided by the cylinder ratio.

d Ratio of Expansion. To find the percentage of cut-off, or what may ! the "commercial cut-off," the following rule should be observed: point of maximum pressure during admission draw a line parallel to the ıe. Through a point on the expansion line where the cut-off is complete, olic curve. The intersection of these two lines is the point of commercial proportion of cut-off is found by dividing the length measured on the this point by the total length.

To find the ratio of expansion divide the volume corresponding to the piston displacement, including clearance, by the volume of the steam at the commercial cut-off, including clearance.

In a multiple-expansion engine the ratio of expansion is found by dividing the volume of the l.-p. cylinder, including clearance, by the volume of the h.-p. cylinder at the commercial cut-off, including clearance.

Data and Results should be reported in accordance with the Form given herewith, adding lines for data not provided for, or omitting those not required, as may conform to the object in view. If a shorter form is desired the items designated by letters of the alphabet may be omitted; or if the principal data and results pertaining to steam consumption only are desired, those indicated in paragraph following the Form may be used. Unless otherwise indicated, the items should be the averages of the data.

(See note following Item 55 for reference to engines driving electric generators and other machinery.)

Data and Results of Steam-Engine Test

Code of 1915

(1) Test of........................engine located at..............................
To determine......................Test conducted by..................

Dimensions, etc.

(2) Type of engine (simple or multiple-expansion)..................................
(3) Class of service (mill, marine, electric, etc.).................................
(4) Auxiliaries (steam- or electric-driven)..
(a) Type and make of condenser equipment..................................
(b) Rated capacity of condenser equipment............................ h.p.
(c) Type of oil pump, jacket pump, and reheater pump (direct or independently driven)...
(5) Rated power of engine...
(a) Name of builders...
(b) Kind of valves................(c) Type of governor..................

	1st	2d	3d
(6) Diameter of cylinders......................in.	......	......	......
(7) Stroke of pistons..........................ft.	......	......	......
(a) Diameter of piston rod, each end........in.	......	......	......
(8) Clearance (average) in per cent. of piston displacement...............................	......	......	......
(9) H.p. constant 1 lb. 1 rev....................h.p.	......	......	......
(a) Cylinder ratio (based on net piston displacement)..........................1 to—	......	......	......
(b) Area of interior steam surface........sq. ft.	......	......	......
(c) Area of jacketed surfaces.............sq. ft.	......	......	......
(10) Capacity of generator or other apparatus consuming power of engine.........................h.p.	......	......	......

Date and Duration

(11) Date.............................. (12) Duration.... ....hr.

Average Pressures and Temperatures

(13) Pressure in steam pipe near throttle, by gage............................. lb.
(14) Barometric pressure...in.
(a) Pressure at boiler, by gage...lb.
(15) Pressure in 1st receiver, by gage...................................lb.
(16) Pressure in 2d receiver, by gage ....lb.
(17) Pressure in exhaust pipe near engine by gage..........................lb.
(18) Vacuum in condenser...in.
(a) Corresponding absolute pressure......................................lb.
(19) Pressure in jackets and reheaters.....................................lb.
(20) Temperature of steam near throttle...................................deg.
(a) Temperature of saturated steam at throttle pressure................deg.
(b) Temperature of steam leaving 1st receiver, if superheated............deg.
(c) Temperature of steam leaving 2d receiver, if superheated.............deg.

ımperature of steam in exhaust pipe near engine..........................deg.
(*a*) Temperature of injection or circulating water entering condenser......deg.
(*b*) Temperature of injection water leaving condenser...................deg.
(*c*) Temperature of air in engine room...................................deg.

Quality of Steam

centage of moisture in steam near throttle or number of degrees of
ıperheating..per cent. or deg.

Total Quantities

al water fed to boilers..lb.
al condensed steam from surface condenser (corrected for condenser leakage) lb.
al dry steam consumed (Item 23 or 24 less moisture in steam)*...........lb.

Hourly Quantities

l water fed to boilers or drawn from surface condenser per hour..........lb.
l dry steam consumed for all purposes per hour (Item 25 + Item 12).....lb.
n consumed per hour for all purposes foreign to the main engine.........lb.
steam consumed by engine per hour (Item 27 − Item 28)................lb.
Circulating water supplied to condenser per hour.........................lb.

Hourly Heat Data

units consumed by engine per hour [Item 29 × (total heat of steam per
ınd at pressure of Item 13 minus heat in 1 lb. of water at temperature
tem 21)]..B.t.u.
Heat converted into work per hour...........................B.t.u.
Heat rejected to condenser per hour (Item 29*a* × [Item 21*b* − 21*a*])
(approximate)...B.t.u.
Heat rejected in form of uncondensed steam withdrawn from cylinders† B.t.u.
Heat lost by radiation......................................B.t.u.

Indicator Diagrams

	1st cyl.	2d cyl.	3d cyl.
ırcial cut-off in per cent. of stroke...per cent.	....	....	....
pressure above atmosphere...............lb.	....	....	....
ressure at lowest point above or below atmos-..................................lb.	....	....	....
Iean back pressure above atmosphere or zero ....lb.	....	....	....
ffective pressure......................lb.	....	....	....
quivalent m.e.p. referred to 1st cylinder..lb.	....	....	....
juivalent m.e.p. referred to 2d cylinder..lb.	....	....	....
juivalent m.e.p. referred to 3d cylinder..lb.	....	....	....
e m.e.p. referred to each cylinder......lb.	......	......	....
counted for per i.h.p.-hr. at point on expan- e shortly after cut-off.................lb.	....	....	....
counted for per i.h.p.-hr. at point on expan- e just before release.................lb.	....	....	....
ssure at selected point near cut-off‡....lb.	....	....	....
ssure at selected point near release....lb.	....	....	....
ssure at point on compression curve shortly fter exhaust closure................. lb.	....	....	....
portion of direct stroke completed at se- cted point near cut-off...............lb.	....	....	....
portion of direct stroke completed at se- cted point near release...............lb.	....	....	....
portion of return stroke uncompleted at lected point on compression line......lb.	....	....	....

ated or superheated. If steam contains moisture, the moisture is to be
erheated, no correction is tc be made. See also paragraph on Dry

xpansion engines.
referred to zero.

(*g*) Ratio of expansion....................... ....
(*h*) M.e.p. of hypothetical diagram (p. 938)...lb. ....
(*i*) Diagram factor (p. 939).................. ....

Speed

(38) Revolutions per minute.. r.p.m.
(39) Piston speed per minute...ft.
(*a*) Variation of speed between no load and full load..................per cent.
(*b*) Momentary fluctuation of speed on suddenly changing from full load to half-load.. per cent.

Power

(40) Indicated horse power developed, whole engine.......................i.h.p.
(*a*) I.h.p. developed by 1st cylinder....; (*b*) by 2d cyl.....; (*c*) by 3d cyl......
(41) Brake horse power...b.h.p.
(42) Friction of engine (Item 40 − Item 41)..............................h.p.
(*a*) Friction expressed in percentage of i.h.p. (Item 42 ÷ Item 40 × 100).. per cent.
(*b*) Indicated horse power with no load, at normal speed................i.h.p.

Economy Results

(43) Dry steam consumed by engine per i.h.p. per hr.........................lb.
(44) Dry steam consumed by engine per brake h.p.-hr.........................lb.
(45) Percentage of steam consumed by engine accounted for by indicator at point near cut-off.. per cent.
(46) Percentage of steam consumed near release...........................per cent.
(47) Heat units consumed by engine per i.h.p.-hr. (Item 30 ÷ Item 40)........B.t.u.
(48) Heat units consumed by engine per b.h.p.-hr. (Item 30 ÷ Item 41)........B.t.u.

Efficiency Results

(49) Thermal efficiency of engine referred to i.h.p. [(2546.5 ÷ Item 47) × 100]..per cent.
(50) Thermal efficiency of engine referred to b.h.p. [(2546.5 ÷ Item 48) × 100]... per cent.
(51) Efficiency of Rankine cycle between temperatures of Items 20 and 21..........
(52) Rankine cycle ratio referred to i.h.p. (Item 49 ÷ Item 51)..................
(53) Rankine cycle ratio referred to b.h.p. (Item 50 ÷ Item 51)..................

Work Done per Heat Unit

(54) Ft.-lb. of net work per B.t.u. consumed by engine (1,980,000 ÷ Item 48)...ft.-lb.

Sample Diagrams

(55) Sample diagrams from each cylinder.......................................
(*a*) Steam-pipe diagrams.

Note. For an engine driving an electric generator the Form should be enlarged to include the electrical data, embracing the average voltage, number of amperes each phase, number of watts, number of watt-hours, average power factor, etc.; and the economy results based on the electric output embracing the heat units and steam consumed per electric h.p.-hr. and per kw.-hr., together with the efficiency of the generator. (See Steam Turbine Code, p. 1761).

Likewise, in a marine engine having a shaft dynamometer, the Form should include the data obtained from this instrument, in which case the b.h.p. becomes the shaft h.p.

The Principal Data and Results of a Reciprocating Engine Test are given by Items (6), (7), (11), (12), (13), (15), (16), (18), (22), (29), (34), (38), (40), (43), (45) and (47).

RULES FOR CONDUCTING TESTS OF STEAM TURBINES AND TURBO-GENERATORS

The code for steam-turbine tests applies to tests for determining the performance of the turbine alone, apart from that of steam-driven auxiliaries which are necessary to its operation. For tests of turbine and auxiliaries combined, and tests of turbines from which steam is withdrawn for heating feed water or other purposes, refer to the Code for Complete Steam Power Plants, p. 1763. For methods of conducting tests of generators, etc., and for general information bearing on the subject, reference may be made tandardisation Rules of the A. I. E. E.

t and Preparations. See Steam Engine Code, p. 1756.

'atus and Instruments. For the determination of the heat and steam con- of a turbine or turbo-generator, employ the methods described in the Steam Code, p. 1756. If the steam consumption is determined from the water dis- by the wet vacuum or hot-well pump, correction should be made for water through the packing glands of the turbine shaft, for condenser leakage, and for foreign supply of water.

on, Starting and Stopping, Records, Calculation of Results. See re- headings in the Steam Engine Code, pp. 1756, 1757 (with the exception of lating to indicator diagrams and results computed therefrom).

nd Results should be reported in accordance with the form given herewith, es for data not provided for, or omitting those not required, as may conform ct in view. For a shorter form of report, omit the items designated by letters abet; or if only the principal data and results are desired, those indicated in following the Form may be used. Unless otherwise indicated, the items the averages of the data.

Data and Results of Steam Turbine or Turbo-generator Test

Code of 1915

f......................turbine located at..............................
letermine..................Test conducted by.........................

Dimensions, etc.

of turbine (impulse, reaction, or combination)...........................
Number of stages..............(b) Condensing or non-condensing.......
Diameter of rotors.............(d) Number and type of nozzles.........
Area of nozzles................(f) Type of governor...................
f service (electric, pumping, compressor, etc.)...........................
ries (steam- or electric-driven)..
Type and make of condensing equipment...................................
Rated capacity of condensing equipment..................................
Type of oil pumps (direct or independently driven)......................
Type of exciter (direct or independently driven).........................
ype of ventilating fan, if separately driven...............................
apacity of turbine..
ame of builders...
r of generator or other apparatus consuming power of turbine..........

Date and Duration

.... (8) Duration..............................hr.

Average Pressures and Temperatures

in steam pipe near throttle by gage..................................lb.
ric pressure...in.
essure at boiler by gage....lb. (b) Pressure in steam chest by gage....lb.
ssure in various stages...lb.
n exhaust pipe near turbine, by gage......................................lb.
n condenser...in.
responding absolute pressure...lb.
olute pressure in exhaust chamber of turbine...........................lb.
ure of steam near throttle...deg.
nperature of saturated steam at throttle pressure......................deg.
nperature of steam in various stages, if superheated..................deg.
ure of steam in exhaust pipe near turbine.................................deg.
perature of circulating water entering condenser.......................deg.
perature of circulating water leaving condenser........................deg.
perature of air in turbine room...deg.

Quality of Steam

(15) Percentage of moisture in steam near throttle, or number of degrees of superheating..per cent. or deg.

Total Quantities

(16) Total water fed to boilers..lb.
(17) Total condensate from surface condenser (corrected for condenser leakage and leakage of shaft and pump glands)..lb.
(18) Total dry steam consumed (Item 16 or 17 less moisture in steam)............lb.

Hourly Quantities

(19) Total water fed to boilers or drawn from surface condenser per hour...........lb.
(20) Total dry steam consumed for all purposes per hour (Item 18 + Item 8)........lb.
(21) Steam consumed per hour for all purposes foreign to the turbine (including drip and leakage of plant)..lb.
(22) Dry steam consumed by turbine per hour (Item 20 − Item 21)..............lb.
(a) Circulating water supplied to condenser per hour......................lb.

Hourly Heat Data

(23) Heat units consumed by turbine per hour [Item 22 × (total heat of steam per pound at pressure of Item 9 less heat in 1 lb. of water at temperature of Item 14)]..B.t.u.
(a) Heat converted into work per hour..............................B.t.u.
(b) Heat rejected to condenser per hour (Item 22a × [Item 14b − Item 14a]) (approximate)..B.t.u.
(c) Heat rejected in the form of steam withdrawn from the turbine......B.t.u.
(d) Heat lost by radiation from turbine, and unaccounted for...........B.t.u.

Electrical Data

(24) Average volts, each phase....volts. (25) Average amperes, each phase.....amp.
(26) Average kilowatts, first meter..kw. (27) Average kilowatts, second meter...kw.
(28) Total kilowatts output........kw. (29) Power factor.......................
(30) Kilowatts used for excitation, and for separately driven ventilating fan........kw.
(31) Net kilowatt output..kw.

Speed

(32) Revolutions per minute..rev.
(33) Variation of speed between no load and full load..........................rev.
(34) Momentary fluctuation of speed on suddenly changing from full load to half load..rev.

Power

(35) Brake horse power, if determined..b.h.p.
(36) Electrical horse power..el.h.p.

Economy Results

(37) Dry steam consumed by turbine per b.h.p.-hr..............................lb.
(38) Dry steam consumed per net kw.-hr..lb.
(39) Heat units consumed by turbine per b.h.p.-hr. (Item 23 ÷ Item 35).........B.t.u.
(40) Heat units consumed per net kw.-hr..B.t.u.

Efficiency Results

(41) Thermal efficiency of turbine (2546.5 ÷ Item 39) × 100..............per cent.
(42) Efficiency of Rankine cycle between temperatures of Items 13 and 14.....per cent.
(43) Rankine cycle ratio (Item 41 ÷ Item 42)..

Work Done per Heat Unit

(44) Ft.-lb. of net work per B.t.u. consumed by turbine (1,980,000 ÷ Item 39)...ft.-lb.

The Principal Data and Results of Turbine Test are given by Items (2a) to (2f), (7), (8), (9), 12), (15), (22), (31), (32), (35), (37), (38), (39) and (40).

FOR CONDUCTING TESTS OF COMPLETE STEAM-POWER PLANTS

team-power plants to which this code applies are assumed to be plants embrac- or more boilers using coal for fuel, one or more engines or turbines, and the es concerned, the power being utilized for any industrial purpose, such as mill generation of electricity, pumping water or compressing air. The object of such a plant is the determination of the performance of the plant as a whole of its component parts, and the efficiency of the plant based on coal and steam tion.

plant contains a number of power-generating units (especially if of different classes), the test when practicable should determine the performance of each ddition to that of the plant as a whole. If the boilers also supply steam for r other industrial purposes, the various quantities of steam thus used should red and the complete distribution of the steam output ascertained.

thods to be followed in testing the component parts of a complete plant, such engines, turbines, pumping machinery, and air machinery, reference may be he respective codes which apply thereto.

See Steam Boiler Code, p. 1750.

n. The duration of a plant test should be not less than 1 day of 24 hours, ably in some cases, such as ice-making plants, a full week of 7 days, including

determine the steam consumption of the individual parts of a plant should be gth as to secure a number of consecutive hourly or half-hourly records show- nity within the desired limits of accuracy. In cases where the engine or r-developing machine is in operation only a part of the day, the duration on ourly results are computed should be considered the length of time that the ther power-developing machine is in operation at its working speed.

and Stopping. In a plant operating continuously day and night, the times rting and stopping should follow the regular periods of cleaning the fires. ure, **see 3rd and 2nd** paragraphs under "Starting and Stopping," p. 1751. unning only a part of the day, and with fires banked during the balance of time selected for the beginning and end of the test should be that following the day's run, when the fires have been burned low preparatory to cleaning . The amount of live coal left on the grates under these circumstances is the beginning of the test, and the fires brought to the same condition, as be, at the close of the test the next day. If the two quantities differ, a ection is made in the weight of coal fired, as found by calculation.

See Steam Engine Code, p. 1757.

and Drying Coal, Ashes and Refuse, Calorific Tests and Analyses e Steam Boiler Code, p. 1752.

Calculation of Results

umption. The number of heat units consumed by the engine and its hour is found by multiplying the total weight of the feed water consumed e total heat of 1 lb. of steam supplied to the engine (corrected for moisture less the heat in 1 lb. of water at the temperature of the water entering conomizer, if any). If the water is supplied from a number of sources and peratures, the weight of each supply is to be taken and the heat in the temperature of supply, the various individual quantities thus obtained gether to obtain the total heat consumption.

esults should be reported in accordance with the following Form; the ne topics on p. 1761 also apply. The **report** should clearly set forth the t findings and conclusions bearing on the work, taking care that these d by minor considerations and details. It is desirable to plot the princi- **hart,** especially the indicated horse power, kilowatt or other load, and consumption.

Data and Results of Steam Power Plant Test

CODE OF 1915

....................plant located at.............................

mine..................Test conducted by.............................

Date, Duration, etc.

(2) Number and kind of boilers (superheaters, if any), engines, turbines, etc.........
(3) Rated capacity of boilers in lb. of steam per hour from and at 212 deg...........lb.
(a) Kind of furnace................(b) Grate surface................sq. ft.
(c) Percentage of area of openings to area of grate..................per cent.
(d) Water heating surface........sq. ft. (e) Superheating surface.....sq. ft.
(4) Rated power of engines or turbines..
(a) Dimensions of cylinders of engine.....(b) Dimensions of turbine..........
(c) Type of engines or turbines and class of service..........................
(d) Name of builders..
(5) Type of auxiliaries*..
(a) Dimensions of auxiliaries*..
(6) Type and capacity of condenser..
(7) Capacity of generators, pumps, or other apparatus consuming power of engine or turbine..

Date, Duration, etc.

(8) Date..
(9) Duration. Length of time engine or turbine was in motion with throttle open...hr.
(a) Length of time engine or turbine was running at normal speed...........hr.
(b) Elapsed time from start to finish.......................................hr.
(10) Kind and size of coal..

Average Pressures, Temperatures, etc.

(11) Boiler pressure by gage...lb.
(a) Steam pipe pressure near throttle, by gage..............................lb.
(b) Barometric pressure...in.
(c) Steam chest pressure by gage..lb.
(d) Pressure in receivers and reheaters by gage.............................lb.
(e) Pressure in turbine stages by gage......................................lb.
(f) Pressure in exhaust pipe near engine or turbine.........................lb.
(12) Vacuum in condenser...in.
(a) Corresponding absolute pressure...lb.
(b) Absolute pressure in exhaust chamber....................................lb.
(13) Temperature of steam, if superheated (taken at boiler or superheater).......deg.
(a) Temperature of steam, if superheated (taken at throttle).............deg.
(b) Normal temperature of saturated steam at boiler pressure............deg.
(c) Normal temperature of saturated steam at throttle pressure...........deg.
(d) Temperature of steam leaving receivers, if superheated...............deg.
(e) Temperature of steam in exhaust pipe near engine or turbine..........deg.
(f) Temperature of condensed water in hot-well or feed tank.............deg.
(g) Temperature of circulating water entering condenser.................deg.
(h) Temperature of circulating water leaving condenser..................deg.
(i) Temperature of air in boiler room ....deg.
(j) Temperature of air in engine or turbine room.........................deg.
(14) Temperature of feed water entering boilers (average)....................deg.
(a) Temperature of each feed supply (if more than one)..................deg.
(b) Temperature of feed water entering economizer, if any...............deg.
(c) Increase in temperature of water due to economizer..................deg.
(15) Temperature of escaping gases leaving boiler............................deg.
(a) Temperature of escaping gases leaving economizer....................deg.
(b) Decrease in temperature of gases due to economizer..................deg.
(c) Temperature of furnace...deg.
(16) Force of draft in main boiler flue.......................................in.
(a) Force of draft at base of chimney.......................................in.
(b) Force of draft at each end of economizer................................in.
(c) Force of draft at individual boiler dampers ....in.
(d) Force of draft in individual furnaces...................................in.
(e) Force of draft or blast in individual ash pits†.........................in.

* For full particulars see text of Report.
† If artificial draft or blast is employed, the force of draft or blast at the fan should also be given.

State of weather..
(a) Temperature of external air....................................deg.

QUALITY OF STEAM

Percentage of moisture in steam, or number of degrees of superheating
....per cent. or deg.
(a) Factor of correction for quality of steam.............................

TOTAL QUANTITIES OF COAL AND WATER

Total weight of coal as fired..lb.
(a) Percentage of moisture in coal...............................per cent.
(b) Total weight of dry coal...lb.
(c) Total ash, clinkers, and refuse (dry)...............................lb.
(d) Weight of clinkers contained in total ash..........................lb.
(e) Percentage of ash and refuse in dry coal......................per cent.
(f) Total combustible burned (Item 19b − 19c).........................lb.
Total weight of water fed to boiler from all sources*........................lb.
(a) Total water evaporated corrected for quality of steam (Item 20 × Item 18a).lb.
(b) Factor of evaporation based on average temperature of water entering boiler.
(c) Total equivalent evaporation from and at 212 deg. (Item 20a × Item 20b).lb.

HOURLY QUANTITIES OF COAL, WATER, AND STEAM, AND RATES

Coal, as fired, per hour (Item 19 + Item 9)...............................lb.
(a) Dry coal per hour (Item 19b + Item 9)..............................lb.
(b) Dry coal per sq. ft. of grate surface...............................lb.
Water evaporated per hour (Item 20 + Item 9)...............................lb.
(a) Equivalent evaporation per hour from and at 212 deg..................lb.
(b) Equivalent evaporation per sq. ft. of water heating surface.............lb.
Dry steam generated per hour (sum of sub-items a to g) (Item 20 less moisture in steam + Item 9)...lb.
(a) Moisture formed per hour between boiler and engine..................lb.
(b) Dry steam consumed per hour by engine cylinders or turbine...........lb.
(c) Dry steam consumed per hour by reheaters and jackets, if any.........lb.
(d) Dry steam consumed per hour by air and circulating pump of condenser..lb.
(e) Dry steam consumed per hour by boiler feed pump.....................lb.
(f) Dry steam consumed per hour by other steam driven auxiliaries........lb.
(g) Dry steam consumed per hour to supply leakage of boilers and piping between boilers and engine (including steam supplied for foreign purposes, if any)..lb.
(h) Live steam supplied for heating, or miscellaneous purposes.............lb.
(i) Injection or circulating water supplied condenser per hour.............lb.

CALORIFIC VALUE OF COAL

Calorific value of 1 lb. of coal as fired, by calorimeter test..................B.t.u.
(a) Calorific value of 1 lb. of dry coal.................................B.t.u.
(b) Calorific value of 1 lb. of combustible..............................B.t.u.

HOURLY HEAT DATA

Heat units in coal as fired generated per hour (Item 21 × Item 24)........B.t.u.
Heat units consumed by engine and auxiliaries per hour (Item 22 × total heat of 1 lb. of steam at pressure of Item 11 less heat in 1 lb. of water at temperature of feed water supplied to boiler, or economiser, if any).......B.t.u.
(a) Heat converted into work per hour..................................B.t.u.
(b) Heat rejected to condenser per hour.................................B.t.u.
(c) Heat rejected in steam withdrawn from receivers or turbine-stages not used by feed water...B.t.u.
(d) Heat lost by radiation from engine and auxiliaries, including piping between boilers and condensers................................B.t.u.
(e) Heat units lost in operation of boiler, including economiser (if any) (Item 25 − Item 26)..B.t.u.

* If there are a number of supplies of feed water, the weight and temperature of each is to be given, and total weight and average temperature ascertained.

Indicator Diagrams

(27) Mean effective pressure, each cylinder.......................................lb.
 (a) Commercial cut-off (in per cent. of stroke) each cylinder..........per cent.
 (b) Initial pressure, above atmosphere, each cylinder......................lb.
 (c) Back pressure at lowest point above or below atmosphere, each cylinder...lb.
 (d) Steam accounted for per i.h.p. per hour at point near cut-off, each cylinder.lb.
 (e) Steam accounted for per i.h.p. per hour at point near release...........lb.

Electrical Data

(28) Average kilowatt output, gross...kw.
 (a) Volts each phase.............volts. (b) Amperes each phase.........amp.
 (c) Kilovolt-amperes.................kva. (d) Power factor..............
(29) Current used by exciter...kw.
(30) Net kilowatt output (Item 28 − Item 29)....................................kw.

Speed

(31) Revolutions per minute..rev.
 (a) Variation of speed between no load and full load.......................rev.

Power

(32) Indicated horse power........i.h.p. (33) Brake horse power.............b.h.p.

Capacity

(34) Water evaporated per hour from and at 212 deg. (same as Item 22a)...........lb.
 (a) Percentage of rated boiler capacity developed (Item 34 ÷ Item 3 × 100). per cent.
(35) Percentage of rated engine or turbine capacity developed (Item 32 ÷ Item 4 × 100) ....per cent.

Economy Results

(36) Coal as fired per i.h.p. of engine per hour....................................lb.
(37) Coal as fired per b.h.p. of engine or turbine per hour........................lb.
 (a) Dry coal per i.h.p. per hr...lb.
 (b) Dry coal per b.h.p.-hr...lb.
 (c) Dry coal per kw.-hr..lb.
(38) Heat units in coal consumed per i.h.p. of engine per hour..................B.t.u.
(39) Heat units in coal consumed per b.h.p. of engine or turbine per hour (Item 37 × Item 24)...B.t.u.
 (a) Heat units consumed by engine (including auxiliaries) per i.h.p.-hr....B.t.u.
 (b) Heat units consumed by engine or turbine (including auxiliaries) per h.p.-hr. (Item 26 ÷ Item 33).....................................B.t.u.
 (c) Heat units consumed by engine per kw.-hr..........................B.t.u.
(40) Heat units in coal consumed per kw.-hr..
(41) Water evaporated per lb. of coal as fired.......................................lb.
 (a) Water evaporated per lb. of dry coal...lb.
 (b) Equivalent evaporation from and at 212 deg. per lb. of dry coal.........lb.
 (c) Equivalent evaporation from and at 212 deg. per lb. of combustible.... ..lb.
(42) Dry steam consumed by engine along per i.h.p.-hr...........................lb.
 (a) Dry steam consumed by auxiliaries per i.h.p.-hr............................lb.
 (b) Dry steam consumed by combined engine and auxiliaries per i.h.p.-hr...lb.
(43) Dry steam consumed by engine or turbine alone per b.h.p.-hr..................lb.
 (a) Dry steam consumed by auxiliaries per b.h.p.-hr............................lb.
 (b) Dry steam consumed by combined engine or turbine and auxiliaries per b.h.p.-hr...lb.

Efficiency Results

(44) Thermal efficiency of plant referred to i.h.p. [(2546.5 ÷ Item 38) × 100]..........
(45) Thermal efficiency of plant referred to b.h.p. [(2546.5 ÷ Item 39) × 100].........
 (a) Efficiency of boilers (Item 41b × 970.4 × 100 ÷ Item 24a)..............
 (b) Efficiency of engine referred to i.h.p. [(2546.5 ÷ Item 39a) × 100].........
 (c) Efficiency of engine or turbine referred to b.h.p. [(2546.5 ÷ 39b) × 100].....

duration should be that of the regular commercial operating cycle, or the time elapsing between two successive renewals of the fuel bed.

Starting and Stopping. The conditions regarding the temperature of the producer and its contents, and the quantity and quality of the latter, should be as nearly as possible the same at the end as at the beginning of the trial. To secure this equality, the starting and stopping should occur at times of regular cleanings, and they should be preceded for a period of not less than 10 hours by the same regular working conditions as are intended to characterize the test as a whole. The operations of starting and stopping should then be carried on as follows:

Continuous Producers with Grate and No Ash Bed. Remove ash and clinkers from grate and lower part of furnace space, taking care that the crust or closely united layer which supports the coal above is not unduly disturbed. Then break open crust and allow mass to drop into space left vacant. Introduce a poker rod through poke holes in upper head and stir up coal within, thereby causing it to settle and fill remaining spaces. As a final step, quickly replenish producer with coal to the working depth, fill hopper level-full, take time, and consider this the starting time. Then clean ash pit, and thereafter proceed with the regular work of test, using weighed coal. When the time arrives for bringing trial to a close, the cleaning operations described above are repeated, ending with filling hopper, taking time, and considering this the stopping time; finally hauling the ashes and refuse from ash pit.

Continuous Producers with Supporting Ash Beds. Remove ashes until top of ash-bed is lowered to normal working point. Introduce poker-rod and break down any bridge or crust that may have formed, at the same time closing up channels that run through fuel bed, thereby making bed homogeneous. Then replenish producer with coal to working depth, fill hopper level-full, take time, and consider this the starting time. Thereafter proceed with regular work of test, using weighed coal. When the time approaches for closing test, the operations above described are repeated, ending with replenishing producer and filling hopper with weighed coal, taking time, and considering this the stopping time. The ashes and refuse finally removed are to be dried before weighing, unless already dried, or a sample should be taken and the moisture, as determined therefrom, allowed for.

Intermittent Producers. Thoroughly clean producer of its entire contents. Introduce a weighed supply of coke or coal, start fire, and build up fuel bed to its working condition, using weighed coal. When this point is reached, take time, and consider this the starting time. Thereafter proceed with regular work of test. When the time approaches for closing test, burn fuel bed as low as practicable to prepare for cleaning, note time, and consider this the stopping time. Then completely empty producer, quench fire remaining in the live coals, separate and weigh coke and ash, and deduct weight of former from that of coke as charged. Finally dry the ash and refuse, or take a sample and allow for the moisture determined therefrom.

Note. To gain some idea of the depth of the ash bed, insert a long poker-rod through fuel bed, and determine how many minutes it takes to become red-hot. Cool it, and insert it again through two or more poke holes successively for the determined time, cooling it after each trial. Measure the distance from top of the producer to the lower end of red portion in each case, and subtract average of distances thus found from total depth of producer. The result gives the approximate depth of ash bed. The length of red portion furnishes also some idea of the depth of zone of burning fuel. The distance from top of producer to surface of fuel bed may be found by direct measurement with the poker-rod, noting by sense of touch when end of rod reaches fuel.

Records, Sampling and Drying Coal, Ashes and Refuse, Calorific Tests and Analyses of Coal. See same topics in Steam Boiler Code, pp. 1751, 1752.

Calorific Tests and Analyses of Gas Output. The quality of the gas should be determined by calorific tests and analyses, continuous samples for this purpose being taken from the delivery pipe at a point near the producer and at other points as may be needed. A satisfactory sample can be obtained by tapping into the main or delivery pipe a ¼-in. pipe, extending it to the center, and leaving its end open. The thermometer for obtaining temperature of gas should be located near the sampling pipe.

The calorific test should be made with a Junker calorimeter or its equivalent. Unless otherwise required the "higher value" should be employed in calculating the results of the test. If the lower value is used, the fact should be so stated. For an approximate determination of the composition of the gas, the Orsat apparatus (p. 1695) may be used, The "combustible" is determined by subtracting from the weight of coal charged the moisture in the coal and the weight of ash refuse and unburned coal withdrawn from the producer or ash pit during the progress of the trial. The "combustible" used for determining the calorific value is the weight of the coal less the moisture and ash found by analysis.

The efficiency of "conversion and cleaning" or "gross efficiency" in the above calculation is found by using the total volume of gas delivered. The "efficiency of the plant" or "net efficiency" is found by using the net volume of gas delivered.

Heat Balance. The various quantities showing the distribution of heat in the heat balance, given in the Form below, are computed in the following manner:

Calorific value of dry gas = cu. ft. of gas at 60 deg. and 30 in. per lb. of dry coal × calorific value of 1 cu. ft. of gas at 60 deg. and 30 in. (higher value).

Sensible heat in the dry gas = weight of gas per lb. of coal × mean specific heat of gas × gas temperature measured above 60 deg.

Heat carried away by scrubber = weight of water fed to scrubber × number of degrees rise of temperature + total weight of dry coal consumed.

Heat contained in moisture leaving producer = total weight of dry gas per lb. of dry coal × proportion of moisture in the gas × total heat of 1 lb. of superheated steam at temperature of gas leaving producer, reckoned from 60 deg.

Loss due to combustible matter in ash = proportion that this combustible bears to the whole amount of dry coal × 14,600 B.t.u.

Data and Results should be reported in accordance with the following Form, to which the remarks on same topics on p. 1761 also apply. If a preliminary trial of the fuel is made in a test-producer, add to the table the general results obtained. In trials having for an object the determination and exposition of the complete performance from beginning to end, the entire log of readings and data should be plotted on a **chart** and represented graphically.

Data and Results of Gas Producer Test
CODE OF 1915

(For a short report the items designated by letters of the alphabet may be omitted)

(1) Test of.................producer located at.....................................
To determine...............Test conducted by.............................

DIMENSIONS, ETC.

(2) Outside diameter and height of producer.....................................ft.
(3) Inside diameter of producer.....................................ft.
(4) Area of grate. (diameter = ...) ....sq. ft.
(*a*) Percentage of air space in grate.............................per cent.
(*b*) Area of blast inlet..........sq. ft. (*c*) Area of exit flue.............sq. ft.
(5) Area of fuel bed (at maximum diameter)...............................sq. ft.
(*a*) Area of water heating surface in vaporizer..........................sq. ft.
(6) Rated capacity of producer in lb. of coal* per hour.........................lb.

DATE, DURATION, ETC.

(7) Date........................... (8) Duration..........................
(9) Kind and size of coal*...

AVERAGE PRESSURES, TEMPERATURES, ETC.

(10) Steam pressure in vaporizer by gage.....................................lb.
(11) Gas pressure in main at point where gas is measured..................in. water
(*a*) Pressure at top of producer..................................in. water
(*b*) Pressure beyond scrubber...
(*c*) Pressure beyond purifier..
(12) Force of blast or draft in ash pit, or bottom of producer ....in. water
(*a*) Barometric pressure....in merc. (*b*) Relative humidity of air...per cent.
(*c*) Depth of fuel bed.........(*d*) Intervals between cleaning..............
(*e*) Intervals between poking...
(13) Temperature of feed water entering vaporizer..............................deg.
(*a*) Temperature of gas in exit flue at producer..........................deg.

* If other fuel than coal is used, change line to read accordingly.

Economy Results

(57) Equivalent cu. ft. of dry gas at 60 deg. and 30 in. per lb. of dry coal.......cu. ft.
(a) Equivalent cu. ft. of dry gas at 60 deg. and 30 in. per lb. of combustible...cu. ft.
(58) Net cu. ft. of dry gas at 60 deg. and 30 in. per lb. of dry coal.............cu. ft.
(a) Net cu. ft. of dry gas at 60 deg. and 30 in. per lb. of combustible....cu. ft.

Efficiency

(59) Gross efficiency of producer, based on dry coal.........................per cent.
(a) Net efficiency of producer, based on dry coal.....................per cent.
(60) Gross efficiency of producer, based on combustible.....................per cent.
(a) Net efficiency of producer, based on combustible.................per cent.

Cost of Coal

(61) Cost of coal per ton of ——— lb. delivered...........................dollars
(62) Cost of coal required for producing 1000 net cu. ft. of gas at 60 deg. and 30 in. ....dollars
(a) Cost of coal for producing 1,000,000 B.t.u...........................dollars

Heat Balance Based on 1 Lb. of Dry Coal

	B.t.u.	Per cent.
(63) Total calorific value of 1 lb. of dry coal, same as Item 54	....	....
(a) Calorific value of dry gas....................	....	....
(b) Sensible heat in hot dry gas above 60 deg. fahr.	....	....
(c) Total heat of moisture in gas above 60 deg......	....	....
(d) Heat lost in scrubber..........................	....	....
(e) Heat lost by combustible in ash................	....	....
(f) Heat lost by radiation, and unaccounted for (difference between the sum of items *a*, *b*, *c*, *d*, *e* and item 63)...........................	....	....

Note. If steam is supplied to the producer from an outside source the data and results should be modified accordingly.

RULES FOR CONDUCTING TESTS OF GAS AND OIL ENGINES

Duration. The test of a gas or oil engine with substantially constant load should be continued for such time as may be necessary to obtain a number of successive records covering periods of half an hour or less during which the results are found to be uniform. In such cases a duration of 3 to 5 hours is sufficient for all practical purposes.

Starting and Stopping. The engine having been set to work under the prescribed conditions, the test is begun by commencing to weigh the oil, or measure the gas, as the case may be, and take other data concerned; after which the regular measurements and observations are carried forward until the end.

Records. See same topic under Steam Engine Code, p. 1757.

Calorific Tests and Analyses. The quality of the oil or gas should be determined by calorific tests and analyses made on representative samples.

Calculation of Results

Volume of Gas at 60 deg. and 30 in. See method for obtaining equivalent volume, p. 1769.

Heat Consumption. The number of heat units consumed by the engine = heat units per lb. of oil or per cu. ft. of gas (higher value) as determined by calorimeter test × total weight of oil in lb. or volume of dry gas in cu. ft. consumed.

Efficiency. See methods used in Steam Engine Code, p. 1757.

Heat Balance. The various quantities showing the distribution of heat in the heat balance given in the following Form are computed as below:

Heat rejected in the cooling water = weight of water supplied × number of degrees rise of temperature ÷ indicated horse power.

Total Quantities

(12) Gas (or oil) consumed..cu. ft.(lb.)
(13) Moisture in gas, in per cent. by weight, referred to dry gas..............per cent.
(14) Equivalent dry gas at 60 deg. and 30 in......................................cu. ft.
(a) Air supplied in cu. ft...cu. ft.
(15) Cooling water supplied to jackets..lb.
(a) Water or steam fed to cylinder...lb.
(16) Calorific value of oil per lb., or of dry gas per cu. ft. at 60 deg. and 30 in. by calorimeter test (higher value)..B.t.u.

Hourly Quantities

(17) Gas (or oil) consumed per hour.......................................cu. ft.(lb.)
(18) Equivalent dry gas per hour at 60 deg. and 30 in..............................cu. ft.
(19) Cooling water supplied per hour...lb.
(20) Heat units consumed per hour (Item 16 × Item 18)..........................B.t.u.

Analysis of Oil

(21) Carbon (C)...........per cent. (22) Hydrogen (H)..................per cent.
(23) Oxygen (O)...........per cent. (24) Sulphur (S)...................per cent.
(a) Moisture.........per cent. (b) Result of fractional distillations..per cent.

Analysis of Fuel Gas by Volume

(25) Carbon dioxide (CO_2).....per cent. (26) Carbon monoxide (CO).....per cent.
(27) Oxygen (O)................per cent. (28) Hydrogen (H).............per cent.
(29) Marsh gas (CH_4)..per cent.
(30) Heavy hydrocarbon C_nH_m..per cent.
(a) Sulphur dioxide (SO_2)...per cent.
(b) Hydrogen sulphide (H_2S)...per cent.
(c) Nitrogen (N) by difference..per cent.

Analysis of Exhaust Gases by Volume

(31) Carbon dioxide (CO_2)...per cent. (32) Carbon monoxide (CO)........per cent.
(33) Oxygen (O)...........per cent. (34) Nitrogen (N).................per cent.

Indicator Diagrams

(35) Pressure in lb. per sq. in. above atmosphere.......................................lb.
(a) Maximum pressure........... (b) Pressure at beginning of stroke......
(c) Pressure at end of expansion... (d) Exhaust pressure at lowest point.....
(36) Mean effective pressure in lb. per sq. in...lb.

Speed

(37) Revolutions per minute..rev.
(38) Average number of explosions or firing strokes per min..........................
(a) Variation of speed between no load and full load..............................rev.
(b) Momentary fluctuation of speed on suddenly changing from full load to half load...

Power

(39) Indicated horse power...i.h.p.
(40) Brake horse power..b.h.p.
(41) Friction horse power by difference (Item 39 − Item 40)*.................fr.-h.p.
(a) Friction horse power by friction diagrams......................................fr.-h.p.
(42) Percentage of indicated horse power lost in friction Item 41 ....per cent.

Economy Results

(43) Heat units consumed by engine per i.h.p. hour†..................................B.t.u.
(44) Heats units consumed by engine per b.h.p..B.t.u.
(45) Pounds of oil or cubic feet of dry gas at 60 deg. and 30 in. consumed per i.h.p.-hr...lb., cu. ft.
(46) Pounds of oil or cubic feet of dry gas per b.h.p.-hr.........................lb., cu. ft.

* In two-cycle engines this includes the power required for compression.
† If these results, in the case of a gas engine, are based on the low value of the heat of combustion, that fact should be so stated.

case of float measurement, repeated observations should be made one after the other throughout the whole period of the trial.

Data and Results should be reported in accordance with the following Form, adding lines for data not provided for or omitting those not required, as may conform with the object in view.

Data and Results of Water-wheel Test with Brake Measurement of Power

CODE OF 1915

(1) Test of........................water wheel located at........................
To determine..
Test conducted by...
(2) Type of wheel and class of service...
(3) Type of generator, if any, kind of current, etc...........................
(4) Rated power of wheel...h.p.
(5) Cross-section of stream where velocity of water is measured..................sq. ft.

GENERAL DATA

(6) Date...
(7) Duration of period covered by test.......................................hr.
(8) Average net weight on brake arm..lb.
(9) Average revolutions per minute...r.p.m.
(10) Total average head of water on wheel....................................ft.
(11) Average velocity of water per second in measuring canal..................ft.
(12) Cu. ft. of water flowing per second.....................................cu. ft.
(13) Weight of water flowing per second (Item 12 × 62.35)......................lb.
(14) Leakage per second..lb.
(15) Net water discharged by wheel per second (Item 13 − Item 14)..............lb.

POWER

(16) Total power of water available..h.p.
(17) Brake horse power developed by wheel....................................b.h.p.

EFFICIENCY

(18) Efficiency of wheel (Item 17 ÷ Item 16) × 100...........................per cent.

RULES FOR CONDUCTING TESTS OF STEAM-DRIVEN COMPRESSORS, BLOWERS AND FANS*

Duration, Starting and Stopping, Records. See same items under Steam Engine Code, p. 1756.

The quantity of air discharged may be measured by a gasometer, or by delivery into tanks of known capacity. Where these means are not available the Pitot tube may be used, or some other means which is subject to calibration. If the air end is of the reciprocating type, indicator diagrams should be regularly taken from this end as well as from the steam end.

Calculation of Results

For rules pertaining to dry steam, heat consumption, and indicated horse power of the steam end, see Steam Engine Code, p. 1757.

Air Horse Power. The gross work done at the air end of a reciprocating machine is obtained from the indicator card.

The net work at the air end of either reciprocating or rotary machines (ft.-lb. per min.) = corrected volume of compressed air (cu. ft.) delivered into main delivery pipe per min. × the impact or total pressure (lb. per sq. ft.) × hyp. log. of ratio of total pressure to the atmospheric pressure (all pressures absolute). Net air horse power (a.h.p.) = above product ÷ 33,000. Corrected volume of compressed air = sectional area of delivery main (sq. ft.) × mean velocity (ft. per min., determined by Pitot tube or other measurement) × $(460 + T_1) / (460 + T_2)$, in which T_1 = temperature of air supplied to machine and T_2 = temperature of air in delivery main.

* When some other prime mover than a steam engine or turbine is employed, the code may be modified to meet the particular requirements.

(16) Pressure in delivery main by gage (impact pressure)*........................lb.
(a) Pressure in each stage, if more than one................................lb.

QUALITY OF STEAM

(17) Percentage of moisture in steam near throttle, or number of degrees of superheating.. per cent. or deg.

TOTAL QUANTITIES

(18) Total water fed to boilers..lb.
(19) Total condensed steam from surface condenser (corrected for condenser leakage)...lb.
(20) Total dry steam consumed (Item 18 or 19 less moisture in steam)............lb.
(21) Total volume of compressed air delivered, as measured..................cu. ft.
(a) Total volume of compressed air delivered, reduced to atmospheric temperature and pressure ....cu. ft.
(b) Total weight of air delivered ....lb.

HOURLY QUANTITIES

(22) Total water fed to boilers, or drawn from surface condenser, per hour..........lb.
(23) Total dry steam consumed for all purposes (Item 20 + Item 8)..............lb.
(24) Steam consumed per hour for all purposes foreign to main engine.............lb.
(25) Dry steam consumed by engine or turbine per hour (Item 23 − Item 24)......lb.
(a) Circulating water supplied to condenser per hour....................lb.
(26) Volume of compressed air delivered per hour, as measured.................cu. ft.
(a) Volume of compressed air delivered per hour, reduced to atmospheric temperature..cu. ft.
(b) Volume of compressed air delivered per hour, reduced to atmospheric temperature and pressure.............................cu. ft.
(c) Weight of air delivered per hour......................................lb.

HOURLY HEAT DATA

(27) Heat units consumed by engine or turbine per hour (Item 25 multiplied by total heat of 1 lb. of steam at pressure of Item 9, less heat in 1 lb. of water at temperature of Item 14..B.t.u.

INDICATOR DIAGRAMS

		1st cyl.	2d cyl.
(28) Mean effective pressure, each steam cylinder............	lb.	....	....
(a) Mean effective pressure, each air cylinder..........	lb.	....	....

SPEED

(29) Revolutions per minute ....r.p.m.
(a) Number of single strokes per minute...........................strokes

POWER

(30) Indicated horse power of steam end.....................................i.h.p.
(31) Gross air horse power as indicated in air cylinders..........................h.p.
(a) Brake horse power consumed by blower or fan ....b.h.p.
(32) Net air horse power (see p. 1776)...h.p.
(33) Friction horse power (Item 30 − Item 31)....................................h.p.
(34) Percentage of i.h.p. lost in friction of machine...........................per cent.

CAPACITY

(35) Compressed air delivered per min. as measured..........................cu. ft.
(a) Compressed air delivered per min., reduced to atmospheric temperature ....cu. ft.
(b) Compressed air delivered per min. at 100 lb. pressure, reduced to 62 deg... cu. ft.
(36) Compressed air delivered per min., reduced to atmospheric temperature and pressure (free air)..................................... cu. ft.

* In the case of compressors or blowers having more than one stage, additional data should be given covering pressures and temperatures in the different stages, the quantity of water used for cooling, and temperatures of the air and water entering and leaving the intercooler.

ECONOMY RESULTS

ɛat units consumed per i.h.p.-hr.................................B.t.u.
ɛat units consumed per net h.p.-hr. of Item 32........................B.t.u.
·y steam consumed per i.h.p.-hr.......................................lb.
:y steam consumed per net air h.p.-hr. of Item 32.........................lb.

EFFICIENCY RESULTS

ıermal efficiency referred to i.h.p. [(2546.5 + Item 37) × 100]........per cent.
ıermal efficiency referred to net air h.p. [(2546.5 + Item 38) × 100]...per cent.
ficiency of compression [(Item 32 + Item 31) × 100]................per cent.
(*a*) Mechanical efficiency of machine [(Item 31 + Item 30) × 100]....per cent.
(*b*) Volumetric efficiency [(Item 36 + displacement in cu. ft. per min. of first compressor) × 100]..................................per cent.

WORK DONE PER HEAT UNIT

:.-lb. of net work per B.t.u. (1,980,000 + Item 38).....................ft.-lb.

SAMPLE DIAGRAMS

ımple indicator diagrams from each cylinder...............................

ᴛᴇ. The items relating to indicator diagrams and indicated horse power are to be ıly in cases where the machine is of the reciprocating type.

Rules for Conducting Tests of Steam Pumping Machinery, . 1497.

INDEX

(For abbreviations used, see p. xix)

IMPORTANT REFERENCE TABLES