
Lafayette

BAMBOO SLIDE RULES

INSTRUCTIONS for the Use of

MODEL 99-7055
99-7099

LAFAYETTE RADIO ELECTRONICS CORPORATION

Instrument Division

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INSTRUCTIONS

FOR THE USE OF YOUR SLIDE RULE

1. GENERAL DESCRIPTION OF SCALES

The following is a brief description of the scales.

(a) C and D

These are exactly alike and are the fundamental scales of the slide rule. They are used for general fundamental calculations.

(b) DF and CF

These are the same as the D and C scales, except that they are "folded" at π . Therefore, any number on the D scale is automatically multiplied by π on the DF scale. In order to avoid resetting when the answer runs off the scale, they are used with the C and D scales.

(c) CI

This is an inverted C scale it is used with the C scale in reading directly the reciprocal of a number. It permits multiplication of three factors with just one setting of the slide.

(d) DI

This is an inverted D scale and is used just the same as the CI scale.

(e) CIF

This is an inverted CF scale and is used with the DF scale in the same relation as the CI scale to the D scale.

(f) **A and B**

These are exactly alike and are used with the C and D scales to find the square and square root.

(g) **K**

This is used in finding the cube and cube root.

(h) **S**

This scale gives the sine and cosine of an angle.

(i) **T or T₁**

This scale gives the tangent of an angle.

(j) **T₂**

This scale gives the tangent of an angle greater than 45°.

(k) **ST**

This scale gives the sine and tangent of an angle smaller than 6°.

(l) **L**

This scale is used with the D scale in finding directly the mantissa of the common logarithms of a number.

(m) **LL₀, LL₁, LL₂ and LL₃**

These are used to find the values of the type form of a^n , e^x , and give the natural logarithms of a number.

(n) **LL₀, LL₁, LL₂ and LL₃**

These are used to find the values of the type form of a^{-n} , e^{-x} and give the natural logarithms of a number.

(o) **P**

This is a special scale. When used with the S scale, it gives the value of the cosine. When used with the D, DI and A scales, it gives $\sqrt{1-x^2}$, $\sqrt{1-(1/x)^2}$ and $\sqrt{1-x}$ only by the hairline.

2. MULTIPLICATION AND DIVISION**(1) How to Use C and D Scales**

In the following Explanation the left hand 1 of the C or D scale is called its Left Index, the right hand 1 is called its Right Index.

(Multiplication)

The process of multiplication may be performed by using the C and D scales.

To multiply a by b ,

opposite a on D, set left or right index of C,
move hairline to b on C,
under hairline read the product on D.

Example 1. $2.4 \times 3 = 7.2$

Opposite 2.4 on D, set left index of C,
move hairline to 3 on C,
under hairline read answer as 7.2 on D.

Example 2. $262 \times 33.2 = 8700$

Opposite 2.62 on D, set left index of C,
move hairline to 3.32 on C,
under hairline find 8.70 on D,
read answer as 8700.

The position of the decimal point is made by rough mental calculation as follows:

$$262 \times 33.2 \doteq 300 \times 30 = 9000$$

Comparing 9000 with 8.7 which obtained by the slide rule, it is apparent the answer is 8700.

Example 3. $4.5 \times 3.2 = 14.4$

Opposite 4.5 on D, set right index of C,
move hairline to 3.2 on C,
under hairline find 1.44 on D,

read answer as 14.4.

Note in above case that the reading would have been off scale, if the left index had been used.

(Division)

The process of division is performed by using the C and D scales

To divide a by b ,

move hairline to a on D,

set b on C under hairline,

opposite left or right index of C read the quotient on D.

Example 4. $32 \div 8 = 4$

Move hairline to 3.2 on D,

set 8 on C under hairline,

opposite right index of C read answer as 4 on D.

(Combined Multiplication and Division)

To calculate the type form of $\frac{a \times c}{b}$, first divide a by b , and then multiply the quotient by c .

Example 5. $\frac{1.57 \times 32}{44} = 1.142$

Move hairline to 1.57 on D,

set 4.4 on C under hairline,

move hairline to 3.2 on C,

under hairline read answer as 1.142 on D.

(2) How to Use CI Scale

(Reciprocal)

The reciprocal of a number is 1 divided by the number. Thus, the reciprocal of 2.5 is $\frac{1}{2.5}$ or 0.4. Opposite any number on the C scale, read its reciprocal on the CI scale. The

number on the CI scale is given in red figures.

Example 6. Opposite 2.5 on C, read $\frac{1}{2.5} = 0.4$ on CI.

Opposite 125 on C, read $\frac{1}{125} = 0.008$ on CI.

(Multiplication)

The process of multiplication also may be performed by using the CI and D scales. By using of the CI scale, computations can be made quickly.

To multiply a by b ,

move hairline to a on D,

set b on CI under hairline,

opposite left or right index of C read the product on D.

Example 7. $2.3 \times 3.4 = 7.82$

Move hairline to 2.3 on D,

set 3.4 on CI under hairline,

opposite right index of C read answer as 7.82 on D.

Example 8. $4.77 \times 0.613 = 2.92$

Move hairline to 4.77 on D,

set 6.13 on CI under hairline,

opposite left index of C read answer as 2.92 on D.

To multiply three factors, first multiply two of them, and then multiply the result by the third.

Example 9. $1.5 \times 3.2 \times 8 = 38.4$

Move hairline to 1.5 on D,

set 3.2 on CI under hairline,

move hairline to 8 on C,

under hairline find 38.4 on D,

read answer as 38.4.

(Division)

The process of division also may be performed by us-

ing the CI and D scales.

To divide a by b ,

opposite a on D, set left or right index of C,
move hairline to b on CI,
under hairline read the quotient on D.

Example 10. $1.2 \div 4 = 0.3$

Opposite 1.2 on D, set left index of C,
move hairline to 4 on CI,
under hairline find 3 on D,
read answer as 0.3.

Example 11. $2.42 \div 2.03 = 1.192$

Opposite 2.42 on D, set right index of C,
move hairline to 2.03 on CI,
under hairline read answer as 1.192 on D.

Note in above example that the reading would have been off scale, if the left index had been used.

To divide three factors, first divide two of them, and then divide the answer by third.

Example 12. $15.7 \div 3.2 \div 4.4 = 1.115$

Move hairline to 1.57 on D.
set 3.2 on C under hairline,
move hairline 4.4 on CI,
under hairline read answer as 1.115 on D.

(Combined Multiplication and Division)

The calculation of the type form of $\frac{a \times b}{c}$ follows out the procedure of multiplication and division.

Example 13. $\frac{8 \times 2}{5} = 3.2$

Move hairline to 8 on D,
set 2 on CI under hairline,

move hairline 5 on CI,

under hairline read answer as 3.2 on D.

(3) How to Use DF and CF Scales

The DF and CF scales are similar to the D and C scales folded at π . As π is very near $\sqrt{10}$, so 1 of the DF and CF scales lies about in the middle and π on both ends of a scale. These scales often can be used in calculations in order to avoid resetting when the answer runs off the scale. When the slide is in any position with the number x on the D scale appearing opposite the number y on the C scale, then this same number x appears also on the DF scale opposite y on the CF scale. If the reading is off scale on the C-D scale it may be found on the CF-DF scale. Thus, the DF and CF are extensions of scales D and C.

Moreover scales DF and CF can be used in problems requiring multiplication by π . Opposite any number on the D scale, read π times of this number on the DF scale. Thus, by taking any number on the D scales as the diameter of circle, its circumference can be found on the DF scale.

These folded scales may be used to perform multiplication and division just as the C and D scales are used.

(Multiplication)

Example 14. $2.5 \times 5 = 12.5$

Opposite 2.5 on D, set left index of C,
(or opposite 2.5 on DF, set middle index of CF),
move hairline to 5 on CF,
under hairline find 1.25 on DF,
read answer as 12.5.

As in above example, when the reading is off scale on the D scale, it may be found on the DF scale.

Example 15. $2.5 \times 2.4 \times 1.2 = 7.20$

Move hairline to 2.5 on D,
set 2.4 on CI under hairline,
move hairline to 1.2 on CF,
under hairline read answer as 7.20 on DF.

Example 16. $2 \times 7 \times 8 = 112$

Move hairline to 2 on D,
set 7 on CI under hairline,
move hairline to 8 on CF,
under hairline find 1.12 on DF,
read answer as 112.

(Division)

Example 17. $9 \div 6 = 1.5$

Move hairline to 9 on DF,
set 6 on CF under hairline,
opposite left index of C read answer as 1.5 on D.
(or opposite middle index of CF read answer as 1.5
on DF.)

Example 18. $9 \div 1.5 \div 4 = 1.5$

Move hairline to 9 on DF,
set 1.5 on CF under hairline,
move hairline to 4 on CI,
under hairline read answer as 1.5 on D.

(Combined Multiplication and Division)

Example 19. $11.2 \div 8.4 \times 2.7 = 3.6$

Move hairline to 1.12 on DF,
set 8.4 on CF under hairline,
move hairline to 2.7 on C,
under hairline read answer as 3.6.

Example 20. $\frac{3.2 \times 1.2 \times 5.8}{8.2 \times 9.5} = 0.286$

Move hairline to 3.2 on D,
set 8.2 on C under hairline,
move hairline to 1.2 on CF,
set 9.5 on CF under hairline,
move hairline to 5.8 on C,
under hairline find 2.86 on D,
read answer as 0.286.

(Computation involving π)

By using the DF scale which is folded to begin and end with π , if the diameter of a circle d is set on the D scale, the circumference πd is given on the DF scale, opposite d on the D scale.

Example 21. (A) Find the circumference of circle with its diameter of 5 cm. (B) Find the diameter of a circle with its circumference of 46.8 cm. Answer (A) 15.7 cm, (B) 14.9 cm.

(A) Move hairline to 5 on D,
under hairline find 1.571 on DF,
read answer as 15.71 cm.

(B) Move hairline to 4.68 on DF,
under hairline find 1.49 on D,
read answer as 14.9 cm.

Example 22. Find the value of 3π . Answer 9.42
Move hairline to 3 on D,
under hairline read answer as $9.42 = 3\pi$ on DF.

Example 23. Find the value of $\frac{7}{\pi}$. Answer 2.23
Move hairline to 7 on DF,
under hairline read answer as $2.23 = \frac{7}{\pi}$ on D.

(4) How to Use CIF Scale

The CIF scale is an inverted CF scale. It may be used to perform multiplication and division just as the CI scale is used. Thus, by the use of six scales of the C, D, CI, CF, DF and this CIF, the fundamental calculations are performed very quickly and efficiently, without resetting the scale or running off scale.

(Multiplication)

Example 24. $3.1 \times 1.6 \times 1.5 \times 8.8 = 65.5$

Move hairline to 3.1 on D,
set 1.6 on CI under hairline,
move hairline to 1.5 on CF,
set 8.8 on CIF under hairline,
opposite right index of C find 6.55 on D,
read answer as 65.5.

Example 25. $9 \times 8 \times 3 \times 7 = 1512$

Move hairline to 9 on DF,
set 8 on CIF under hairline,
move hairline to 3 on C,
set 7 on CI under hairline,
opposite left index of C find 1.512 on D,
read answer as 1512.

(Division)

Example 26. $27 \div 5 \div 7.5 = 0.72$

Move hairline to 2.7 on D,
set 5 on C under hairline,
move hairline to 7.5 on CIF,
under hairline find 7.2 on DF,
read answer as 0.72.

(Combined Multiplication and Division)

Example 27. $3.8 \times 6.5 \div 2 \times 1.96 \div 1.45 = 167$

Move hairline to 3.8 on D,
set 6.5 on CI under hairline,
move hairline to 2 on CIF,
set 1.96 on CIF under hairline,
move hairline to 1.45 on CIF,
under hairline find 1.67 on DF,
read answer as 1.67.

Example 28. $\frac{236000 \times 0.0065 \times 188 \times 715}{0.00032 \times 7800000} = 82600$

Move hairline to 2.36 on D,
set 3.2 on C under hairline,
move hairline to 6.5 on C,
set 7.8 on C under hairline,
move hairline to 1.88 on CF,
set 7.15 on CIF under hairline,
opposite right index of C find 8.26 on D,
read answer as 82600.

The Position of the decimal point is made by rough mental calculation as follows:-

$$\begin{aligned} & \frac{236000 \times 0.0065 \times 188 \times 715}{0.00032 \times 7800000} \\ &= \frac{2.36 \times 6.5 \times 1.88 \times 7.15}{3.2 \times 7.8} \times \frac{10^5 \cdot 10^{-3} \cdot 10^2 \cdot 10^2}{10^{-4} \cdot 10^6} \\ &\doteq \frac{2 \times 7 \times 2 \times 7}{3 \times 8} \times \frac{10^6}{10^2} = \frac{2 \times 7 \times 2 \times 7}{3 \times 8} \times 10^4 \\ & \qquad \qquad \qquad \frac{4}{2} \\ &\doteq \frac{7 \times 7}{6} \times 10^4 \doteq \frac{50}{6} \times 10^4 \doteq 8 \times 10^4 = 80000 \end{aligned}$$

Comparing 80000 with 8.26 which is obtained by the slide rule, the answer clearly is 82600.

3. PROBLEMS OF PROPORTION

Proportion is a special case of multiplication and division. Generally, when the slide scale is set at one point on the stock scale, there exists certain ratio relation between the scales on the slide and stock. So, the answer is given by removing of the hairline only.

There are two kinds of proportional problems, one is proportion and another inverse proportion.

(1) Proportion

The problems of proportion are widely applied to conversion, indexes, proportional division, percentage, etc.

Proportional calculation is performed by referring to each other scales C and D or CF and DF.

Example 29 Fill the following blanks, given $1\text{kt}=1.852\text{ km/h}$.

kt	27	3.24	44
km/h	50*	60*	81.5*

*.....answer shown

Opposite 1.852 on D, set left index of C, move hairline to 2.7 on C, under hairline find 5 on D, read answer as 50.

Move hairline to 3.24 on C, under hairline find 6 on D, read answer as 60

Move hairline to 4.4 on C, under hairline find 8.15 on C, read answer as 81.5.

Example 30. Find % in the following table.

	Amount	%
A	\$2,410	14.6
B	\$2,640	16.0
C	\$3,400	20.6
D	\$8,050	48.8
Total	\$16,500	100.0

*.....shown answer

Opposite left index of D, set 1.65 on C, move hairline to 2.41 on C, under hairline find 1.46 on D, read answer as 14.6.

Move hairline to 2.64 on C, under hairline find 1.60 on D, read answer as 16.0.

Move hairline to 3.4 on C, under hairline find 2.06 on D, read answer as 20.6.

Move hairline to 8.05 on C, under hairline find 4.88 on D, read answer as 48.8.

Example 31. $1.43 : 3.39 = 8.65 : x$ $x=20.5$

Move hairline to 3.39 on D, set 1.43 on C under hairline, move hairline to 8.65 on CF, under hairline find 2.05 on DF, read answer as $x=20.5$.

(2) Inverse Proportion

Inverse proportion is calculated by referring to each other scales D and CI, or scales DF and CIF.

Example 32. A job requires 12 days for completion 45 with workmen. How many days will it take to complete it with 30 workmen?

Answer 18 days

Move hairline to 1.2 on D,
set 4.5 on CI under hairline,
move hairline to 3 on CI,
under hairline find 1.8 on D,
read answer as 18.

Example 33. A pulley with 7" diameter revolves 360 times per minute. How many revolutions per minute are there when diameter of pulley is 9" Answer: 280 rpm

Move hairline to 3.6 on D,
set 7 on CI under hairline,
move hairline to 9 on CI,
under hairline find 2.8 on D,
read answer as 280.

4. SQUARE AND SQUARE ROOT

(1) Square Root

Opposite any number on the D scale, read its square on the A scale. Similarly, opposite any number on the C scale, read its square on the B scale.

Example 34. $2^2=4$, $5.63^2=31.7$

Move hairline to 2 on D,
under hairline read answer as 4 on A.
Move hairline to 5.63 on D,
under hairline read answer as 31.7 on A.

(2) Square Root

To find the square root of a number between 1 and 10, use scale A left. To find the square root of a number between 10 and 100, use scale A right. Or a given number is divided into several groups of two digits per group, counting from decimal point to the right or left. If the first group consists of only one figure, a given number is set on the scale A left. If it consists of two figures, a given number is set on the scale A right. Place the decimal point of the answer, taking one digit per group.

Example 35. $\sqrt{6.56}=2.56$, $\sqrt{6'56'00}=256$,
 $\sqrt{0.00'06'56}=0.0256$

Move hairline to 6.56 on A left,
under hairline find 2.56 on D,

Example 36. $\sqrt{65.6}=8.1$, $\sqrt{65'60}=81$,
 $\sqrt{0.00'00'65'6}=0.0081$

Move hairline to 65.6 on A right,
under hairline find 8.1 on D.

(3) Multiplication and Division including Square

Example 37. $2 \times 4^2=32$

Move hairline to 2 on A left,
set 4 on CI (back face) under hairline,
opposite right index of C read answer as 32 on A right.

Example 38. $1.3^2 \times 6.5^2=71.4$

Move hairline to 1.3 on D,
set 6.5 on CI (back face) under hairline,
opposite right index of C read answer as 71.4 on A right.

Example 39. $30 \div 5^2=1.2$

Move hairline to 30 on A right

set 5 on C under hairline,

opposite left index of C read answer as 1.2 on A left.

Example 40. $25^2 \div 50 = 12.5$

Move hairline to 2.5 on D,

set 50 on B right under hairline,

opposite right index of C read answer as 12.5 on A right.

Example 41. $2.4^2 \div 3.82^2 = 0.395$

Move hairline to 2.4 on D,

set 3.82 on C under hairline,

opposite right index of C find 39.5 on A right,
read answer as 0.395.

(4) Multiplication and Division including Square Root

Example 42. $\sqrt{20} \times 2 = 8.95$

Move hairline to 20 on A right,

set 2 on CI (back face) under hairline,

opposite right index of C read answer as 8.95 on D.

Example 43. $\sqrt{120 \times 36} = 65.7$

Opposite 1.2 on A left, set left index of C,

move hairline to 36 on B right,

under hairline find 6.57 on D,

read answer as 65.7.

Example 44. $\sqrt{36} \div 0.5 = 12$

Move hairline to 36 on A right,

set 5 on C under hairline,

opposite left index of C find 1.2 on D,

read answer as 12.

Example 45. $6 \div \sqrt{16} = 1.5$

Move hairline to 6 on D,

set 16 on B right under hairline,

opposite left index of C read answer as 1.5 on D.

Example 46. $\sqrt{2880 \div 8.35} = 18.57$

Move hairline to 28.8 on A right,

set 8.35 on B left under hairline,

opposite left index of C find 1.857 on D,
read answer as 18.57.

(5) How to Use Gauge Mark "c"

area of circle $A = \frac{\pi}{4} d^2$ $d = \text{diameter of circle}$

$$= \left(\sqrt{\frac{\pi}{4}} \cdot d \right)^2 = \left(d / \sqrt{\frac{4}{\pi}} \right)^2$$

$\sqrt{\frac{4}{\pi}}$ corresponds to the value of "c". These gauge

marks are graduated on the C and D scales, which are used for calculation of the area of a circle.

Example 47. Find the area of a circle with its diameter of 1.5 cm. Answer 1.77 cm²

Move hairline to 1.5 on D,

set c on C under hairline,

opposite left index of B read answer as 1.77 on A left.

Example 48. Find the volume of a cylinder with 1.2 m in diameter and 5 m in length.

Answer 5.66 m³

Move hairline to 1.2 on D,

set c on C under hairline,

move hairline to 5 on B left,

under hairline read answer as 5.66 on A left.

5. CUBE, CUBE ROOT, $\frac{3}{2}$ POWER AND $\frac{2}{3}$ POWER

(1) Cube

Opposite any number on the D scale, read its cube on

the K scale

Example 49. $2^3=8$, $3^3=27$, $7^3=343$

Move hairline to 2 on D,
under hairline read answer as 8 on K.
Move hairline to 3 on D,
under hairline read answer as 27 on K.
Move hairline to 7 on D,
under hairline read answer as 343 on K.

(2) Cube Root

To find the cube root of a number between 1 and 10, use scale K left. To find the cube root of a number between 10 and 100, use scale K middle. To find the cube root of a number between 100 and 1000, use scale K right.

Or a given number is divided into several groups with three digits per group, counting from the decimal point to right or left direction. If the first group consists of only one figure, a given number is set on the scale K left. If it consists of two figures, a given number is set on the scale K middle. If three figures, on the scale K right.

Place the decimal point of the answer, taking one digit per group.

Example 50. $\sqrt[3]{2.62}=1.379$, $\sqrt[3]{2.620'000}=137.9$

$$\sqrt[3]{0.002'62}=0.1379$$

Move hairline to 2.62 on K left,

under hairline find 1.379 on D.

Example 51. $\sqrt[3]{26.2}=2.97$, $\sqrt[3]{26'200}=29.7$

$$\sqrt[3]{0.000'026'2}=0.0297$$

Move hairline to 26.2 on K middle,

under hairline find 2.97 on D.

Example 52. $\sqrt[3]{262}=6.4$, $\sqrt[3]{262'000}=64$,
 $\sqrt[3]{0.000'000'262}=0.0064$

Move hairline to 262 on K right,
under hairline find 6.4 on D

(3) $\frac{3}{2}$ Power and $\frac{2}{3}$ Power

$\frac{3}{2}$ power and $\frac{2}{3}$ power are calculated by the use of the reference scales A and K.

Example 53. $4^{\frac{3}{2}}=8$

Move hairline to 4 on A left,

under hairline read answer as 8 on K left,

Example 54. $29.5^{\frac{2}{3}}=9.55$

Move hairline to 29.5 on K middle,

under hairline read answer as 9.55 on A left.

6. LOGARITHMS

(1) Common Logarithms

Common Logarithms are calculated to the base 10 including the integer (characteristic) and the decimal part (mantissa).

A slide rule gives only the mantissa of common logarithms; the characteristic can be calculated by the following formula:

(the number of places exceeding decimal point of a given number—1)

viz. the characteristic of any number greater than unity is always 1 less than the number of figures at the left of decimal point.

If given number is of n places under the decimal point, the characteristic is also negative; this is indicated by placing the negative sign or bar over the figure as $-n$ or \bar{n} .

The mantissa of a given number may be found by L and D scales in reference relation.

Example 55. $\log_{10} 3.14 = 0.497$

Move hairline to 3.14 on D,
under hairline read answer as 0.497 on L.

Example 56. $\log_{10} 0.000342 = 4.534$

Move hairline to 3.42 on D,
under hairline find 0.534 on L,
add characteristic $\bar{4}$, read answer as $\bar{4}.534$.

Example 57. $\log_{10} 887 = 2.948$

Move hairline to 8.87 on D,
under hairline find 0.948 on L,
add characteristic 2, read answer as 2.948.

(2) Natural Logarithms

Logarithms to the base of e (2.718) are called natural logarithms; they be determined by multiplying by 2.3026 common logarithms according to the following formula:

$$\log_e N = 2.3026 \times \log_{10} N$$

(3) Anti-Logarithms

The type form of $\log_{10}^{-1} x = a$ is calculated by the exact reverse procedure of the above examples. Take the mantissa only of a given number x and set it on the L scale. The significant figure of number of logarithm a is given on the D scale, opposite the mantissa on the L scale. When the characteristic is n , there will be $(n+1)$ figures before the decimal point in the answer. When the characteristic is \bar{n} , the answer is of n places under the de-

cimal point.

Example 58. $\log_{10}^{-1} 1.398 = 25$

Move hairline to 0.398 on L,
under hairline find 2.5 on D,
characteristic is 1,
there will be two figures before the decimal point,
thus read answer as 25.

(4) Exponent

Calculation of the type form of a^n is made by the following procedures:

$$\begin{aligned} \text{Now, } a^n = x & \quad \therefore \log_{10} x = n \times \log_{10} a \\ & \quad \therefore x = \log_{10}^{-1}(n \times \log_{10} a) \end{aligned}$$

Find $\log_{10} a$,
calculate $n \times \log_{10} a$,
answer is given by $\log_{10}^{-1}(n \times \log_{10} a)$.

Example 59. $3.14^{2.5} = 17.46$

Move hairline to 3.14 on D,
under hairline read 0.497 ($= \log_{10} 3.14$) on L,
move hairline to 4.97 on D,
set 2.5 on CI under hairline,
opposite left index of C read 1.242 ($= 2.5 \times \log_{10} 3.14$) on D,
move hairline to 0.242 on L,
under hairline read 1.746 ($= \log_{10}^{-1}(2.5 \times \log_{10} 3.14)$) on D,
characteristic is 1, therefore there will be two figures before the decimal point,
then read answer as 17.46.

7. TRIGONOMETRIC FUNCTIONS

These computations usually can be obtained by the co-

operation of S, T, ST and C scales.

S scale gives the sines of angles ranging from 5.5° or 5.8° to 90° .

T scale gives the tangents of angles ranging from 5.5° or 5.75° to 45°

ST scale gives the sines or tangents of small angles ranging from 0.54° or 0.58° to 6° .

T_2 scale gives the tangents of angles ranging from 45° to 84.2° .

(1) Sin θ

Example 60. $\sin 1.5^\circ = 0.259$

Move hairline to 15 on S,
under hairline find 2.59 on C,
read answer as 0.259.

When using the S scale to read the value of $\sin \theta$, read the left index of C as 0.1 and the right index as 1.

Example 61. $\sin 1.5^\circ = 0.0262$

Move hairline to 1.5 on ST,
under hairline find 2.62 on C,
read answer as 0.0262.

When using the ST scale to read the value of $\sin \theta$, read the left index of C as 0.01 and the right index as 0.1.

Example 62. $\sin 30^\circ \times 5 = 2.5$

Opposite 5 on D, set right index of C,
move hairline to 30 on S,
under hairline read answer as 2.5 on D.

Example 63. $\sin 24.4^\circ \div 1.67 = 0.247$

Opposite left index of D, set left index of C,
move hairline to 24.4 on S,
set 1.67 on C under hairline,

opposite left index of C find 2.47 on D,
read answer as 0.247.

Example 64. $3.06 \div \sin 43^\circ = 4.49$

Move hairline to 3.06 on D,
set 43 on S under hairline,
opposite right index of C, read answer as 4.49 on D.

(2) How to Use Gauge Mark ρ°

The sines and tangents of angles smaller than 6° can be found by the following approximation.

$$\sin \theta^\circ \approx \tan \theta^\circ \approx \theta \text{ (in radians).}$$

Therefore, to find the sine and tangent of an angle smaller than 6° , find the value of the angle in radians, and obtain by use of ρ° on the C scale.

$$\rho^\circ \dots\dots\dots \frac{180^\circ}{\pi} = 57.2958^\circ = 1 \text{ radian}$$

Example 65. $\sin 1.5^\circ = 0.0262$

Move hairline to 1.5 on D,
set ρ° on C under hairline,
opposite right index of C, find 2.62 on D,
read answer as 0.0262.

The position of the decimal point is estimated as follows:

$$1^\circ \approx 0.02$$

(3) Cos θ

The value of $\cos \theta$ is found by reading the sines of the complementary angles as $\cos \theta = \sin(90^\circ - \theta)$.

The red numbers on the S and ST scales represent the complements of the angles as shown by the corresponding black numbers on these scales. The red numbers on the S and ST scales are used with C scale to find the values of

the cosines of angles. When using the red S scale to read the value of $\cos \theta$, read the left index of C as 0.1 and the right index as 1, and using the red ST scale, read as 0.01 and 0.1.

Example 66. $\cos 30^\circ = 0.866$

Move hairline to 30 on S (red),
under hairline find 8.66 on C,
read answer as 0.866.

Example 67. $\cos x^\circ = 0.413$ $x = 65.6^\circ$

Move hairline to 4.13 on C,
read answer as 65.6 on S (red) under hairline.

(4) Sine Proportions

The calculation of sine proportion is worked out by referring to S and D scales. In Fig. 1, there is the following formula.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Therefore, opposite a , b and c on the D scale, set A , B and C on the S scale respectively.

Example 68. In Fig. 1, find a and c , given $A=80^\circ$, $B=35^\circ$, $C=65^\circ$ and $b=25$.

Answer $a=42.9$, $c=39.5$

Move hairline to 2.5 on D,
set 35 on S under hairline,
move hairline to 80 on S,
under hairline find 4.29 on D,
read answer as 42.9.

Move hairline to 65 on S,
under hairline find 3.95 on D,
read answer as 39.5.

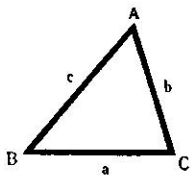


Fig. 1

(5) Tan θ

Example 69. $\tan 32.5^\circ = 0.637$

Move hairline to 32.5 on T,
under hairline find 6.37 on C,
read answer as 0.637.

Example 70. $\tan 1.5^\circ = 0.0262$

Move hairline to 1.5 on ST,
under hairline find 2.62 on C,
read answer as 0.0262.

When using the T scale to read the value of $\tan \theta$, read the left index of C as 0.1 and the right index as 1, and using the ST scale to read the value of $\tan \theta$, read the left index of C as 0.01 and the right index as 0.1.

Tangents of angles greater than 45° are calculated by the use of the T_2 scale.

Example 71. $\tan 62^\circ = 1.88$

Move hairline to 62 on T_2 ,
under hairline read answer as 1.88 on C.

When using the T_2 scale to read the value of $\tan \theta$, read the left index of C as 1 and the right index as 10.

Or to find the value of the tangents of angles greater than 45° read the reciprocals of the tangents of the complementary angles as $\tan \theta = \frac{1}{\tan(90^\circ - \theta)}$.

The red numbers on the T scale represent the complements of the angles as shown by the corresponding black numbers on this scale. Therefore, the tangents of angles greater than 45 degrees can be read directly between the red numbers on the T scale and DI or CI scale. When using the red T scale to read the value of $\tan \theta$, read the right index of DI as 1 and the left index as 10, and using red ST

scale, read as 10 and 100.

Example 72 $\tan 62^\circ = 1.88$

Set C index even with D index,

move hairline to 62 on T (red),

under hairline read answer as 1.88 on DI.

(6) Other Trigonometric Functions

To get $\cot \theta$, $\sec \theta$ and $\operatorname{cosec} \theta$, use the following formulae:

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

(7) Solutions of Right Triangle and Vector Problems

By the co-operation of S, T and DI scales, solutions of the right triangle and vector problems can be obtained.

In the right triangle $\triangle ABC$ (in Fig. 2), there is the following formula:

$$c \cdot \sin \theta = a \cdot \tan \theta = b$$

$$\therefore \frac{\sin \theta}{\frac{1}{c}} = \frac{\tan \theta}{\frac{1}{a}} = \frac{1}{\frac{1}{c}}$$

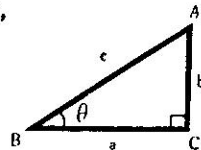


Fig. 2

Therefore, move the hairline to b on the DI scale, set the right or left index of the C scale under the hairline, move the hairline to a on the DI scale, under the hairline read θ on the T scale, move the hairline to θ on the S scale, under hairline read c on the DI scale.

Example 73. In Fig. 2, find c and $\angle B$, given $a=2.3$ and $b=1.8$.

Answer $c=2.92$, $\angle B=38.08^\circ$

Move hairline to 1.8 on DI,

set right index of C under hairline,

move hairline to 2.3 on DI,

under hairline read answer as $38.08 = \angle B$ on T,

move hairline to 38.08 on S,

under hairline read answer as $2.92 = c$ on DI.

Example 74. $\sqrt{2.54^2 + 1.65^2} = 3.03$

$$2.54 + j 1.65 = 3.03 \angle 33^\circ$$

Move hairline to 1.65 on DI,

set right index of C under hairline,

move hairline to 2.54 on DI,

under hairline read answer as 33 on T,

move hairline to 33 on S,

under hairline read answer as 3.03 on DI.

Example 75. Analyze the following vector.

Answer $x=5.2$, $y=3.38$ $6.20 \angle 33^\circ = 5.20 + j 3.38$

Move hairline to 6.20 on DI,

set 33 on S under hairline,

move hairline to 33 on T,

under hairline read answer as

$5.20 = x$ on DI,

move hairline to right index of C,

under hairline read answer as $3.38 = y$ on DI.

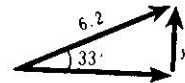


Fig. 3

8. HOW TO USE P SCALE

The P scale is called "Pythagorean scale". When used with the S scale, it gives the value of the cosine directly by the hairline.

(1) $\cos \theta$

Against the value of $\sin \theta$ on the D scale, the value of $\cos \theta$ is shown on the P scale.

Example 76. Find $\cos \theta$, given $\sin \theta = 0.5$.

Answer 0.866

Move hairline to 5 on D,
under hairline read answer as 0.866 on P.

Example 77. $\cos 15^\circ = 0.966$

Set C index even with D index,
move hairline to 15 on S,
under hairline read answer as 0.966 on P.

(2) Type Form of $\sqrt{1-x^2}$

By referring to the D and P scales, compute the type form of $\sqrt{1-x^2}$.

Example 78. $\sqrt{1-0.4^2} = 0.9165$

Move hairline to 4 on D,
under hairline read answer as 0.9165 on P.

9. HOW TO USE LL SCALES

(1) Explanation of LL Scales

LL represent that the scale is a logarithm of a logarithm. There are two groups of LL scales. One is LL_n scales (LL_0 , LL_1 , LL_2 and LL_3) ranging from 1.001 to 10^5 or 2×10^4 for the computation of the type form of a^n . The other is $LL_{\bar{n}}$ scales ($LL_{\bar{0}}$, $LL_{\bar{1}}$, $LL_{\bar{2}}$ and $LL_{\bar{3}}$) called Reciprocal LL_n scales ranging from 10^{-5} or 5×10^{-5} to 0.999 for the computation of the type form of a^{-n} .

Construction of LL scales $\left\{ \begin{array}{l} \uparrow LL_n \text{ group} \dots LL_0, LL_1, LL_2, LL_3 \\ LL_{\bar{n}} \text{ group} \dots LL_{\bar{0}}, LL_{\bar{1}}, LL_{\bar{2}}, LL_{\bar{3}} \end{array} \right.$

These LL scales are really divisions of one long Log Log scale. And the next scale to LL_0 or $LL_{\bar{0}}$ scale might be

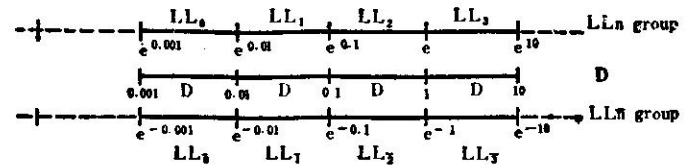


Fig. 4

considered common logarithmic scale D. Therefore, eight LL scales as shown in Fig. 4 are considered to cover the complete range of the LL scales.

As shown in Fig. 4, the LL_n and $LL_{\bar{n}}$ scales are in a reciprocal relation respectively. This arrangement can be used very conveniently for calculations of powers, roots of numbers, and hyperbolic functions. Usually, general slide rules have three LL scales (LL_1 , LL_2 and LL_3).

(2) Natural Logarithms

Set the hairline to the given number N on the LL scale, $\log_e N$ will be found out under the hairline on the D scale.

The position of the decimal point is determined as follows:

When N is set on $\left\{ \begin{array}{l} \text{the } LL_3 \text{ scale} \\ \text{the } LL_2 \text{ scale} \\ \text{the } LL_1 \text{ scale} \\ \text{the } LL_0 \text{ scale} \end{array} \right. \dots \left\{ \begin{array}{l} \text{one digit at the left} \\ \text{one digit at the right} \\ \text{two digits at the right} \\ \text{three digits at the right} \end{array} \right\}$ of the decimal point,

when N is set on $\left\{ \begin{array}{l} \text{the } LL_{\bar{3}} \text{ scale} \\ \text{the } LL_{\bar{2}} \text{ scale} \\ \text{the } LL_{\bar{1}} \text{ scale} \\ \text{the } LL_{\bar{0}} \text{ scale} \end{array} \right. \dots \left\{ \begin{array}{l} \text{one digit at the left} \\ \text{one digit at the right} \\ \text{two digits at the right} \\ \text{three digits at the right} \end{array} \right\}$ of the decimal point and place the negative sign before the figures.

Example 79. $\log_e 5 = 1.609$

Move hairline to 5 on LL_3 ,
under hairline read answer as 1.609 on D.

Example 80. $\log 2 = 0.693$

Move hairline to 2 on LL_2 ,
under hairline find 6.93 on D,
read answer as 0.693.

Example 81. $\log 1.03 = 0.0296$

Move hairline to 1.03 on LL_1 ,
under hairline find 2.96 on D,
read answer as 0.0296.

Example 82. $\log 1.006 = 0.00598$

Move hairline to 1.006 on LL_0 ,
under hairline find 5.98 on D,
read answer as 0.00598.

Example 83. $\log .023 = -1.47$

Move hairline to 0.23 on LL_3 ,
under hairline find 1.47 on D,
read answer as -1.47.

Example 84. $\log 0.625 = -0.47$

Move hairline to 0.625 on LL_2 ,
under hairline find 4.7 on D,
read answer as -0.47.

Example 85. $\log 0.955 = -0.0461$

Move hairline to 0.955 on LL_T ,
under hairline find 4.61 on D,
read answer as -0.0461.

Example 86. $\log 0.995 = -0.00501$

Move hairline to 0.995 on LL_0 ,
under hairline find 5.01 on D,
read answer as -0.00501.

(3) Powers and Roots

The type form of $a^{\pm n}$ or $a^{\pm \frac{1}{n}}$ is calculated by the use of the LL scales in an operation similar to multiplication and division.

Example 87. $4.25^{2.12} = 21.5$, $4.25^{-2.12} = 0.0466$

Move hairline to 4.25 on LL_3 ,
set left index of C under hairline,
move hairline to 2.12 on C,
under hairline read answer as 21.5 on LL_3 ,
under hairline read answer as 0.0466 on LL_3 .

Example 88. $1.96^{2.3} = 4.70$, $1.96^{-2.3} = 0.213$

Move hairline to 1.96 on LL_2 ,
set right index of C under hairline,
move hairline to 2.3 on C,
under hairline read answer as 4.70 on LL_3 ,
under hairline read answer as 0.213 on LL_3 .

Example 89. $1.02^{24.5} = 1.624$, $1.02^{-24.5} = 0.616$

Move hairline to 1.02 on LL_1 ,
set left index of C under hairline,
move hairline to 2.45 on C,
under hairline read answer as 1.624 on LL_2 ,
under hairline read answer as 0.616 on LL_3 .

Example 90. $11.4^{0.7} = 5.50$, $11.4^{-0.7} = 0.182$

Move hairline to 11.4 on LL_3 ,
set 7 on CI under hairline,
move hairline to left index of C,
under hairline read answer as 5.50 on LL_3 ,
under hairline read answer as 0.182 on LL_3 .

Example 91. $330^{\frac{1}{6.2}} = 2.55$, $330^{-\frac{1}{6.2}} = 0.392$

Move hairline to 330 on LL_3 ,
 set 6.2 on C under hairline,
 move hairline to right index of C,
 under hairline read answer as 2.55 on LL_2 ,
 under hairline read answer as 0.392 on LL_2 .

Example 92. $28.5^{\frac{2.31}{3.41}}=17.4$, $28.5^{-\frac{2.31}{3.41}}=0.0574$

Move hairline to 28.5 on LL_3 ,
 set 3.41 on C under hairline,
 move hairline to 2.91 on C,
 under hairline read answer as 17.4 on LL_3 ,
 under hairline read answer as 0.0574 on LL_3 .

Example 93. $0.795^{1.4}=0.725$, $0.795^{-1.4}=1.379$

Move hairline to 0.795 on LL_2 ,
 set left index of C under hairline,
 move hairline to 1.4 on C,
 under hairline read answer as 0.725 on LL_2 ,
 under hairline read answer as 1.379 on LL_2 .

Example 94. $0.795^{14}=0.0402$, $0.795^{-14}=24.9$

Move hairline to 0.795 on LL_2 ,
 set left index of C under hairline,
 move hairline to 1.4 on C,
 under hairline read answer as 0.0402 on LL_2 ,
 under hairline read answer as 24.9 on LL_2 .

Example 95. $0.99676^{\frac{1}{2.2}}=0.99854$, $0.99676^{-\frac{1}{2.2}}=1.001462$

Move hairline to 0.99676 on LL_0 ,
 set 2.2 on C under hairline,
 move hairline to left index of C,
 under hairline read answer as 0.99854 on LL_0 ,
 under hairline read answer as 1.001462 on LL_0 .

Example 96. $e^{1.96}=7.10$, $e^{-1.96}=0.1408$

Move hairline to 1.96 on D,
 under hairline read answer as 7.10 on LL_3 ,
 under hairline read answer as 0.1408 on LL_3 .

Example 97. $e^{0.94}=2.56$, $e^{-0.94}=0.39$

Move hairline to 9.4 on D,
 under hairline read answer as 2.56 on LL_2 ,
 under hairline read answer as 0.39 on LL_2 .

Example 98. $e^{0.056}=1.0576$, $e^{-0.056}=0.9455$

Move hairline to 5.6 on D,
 under hairline read answer as 1.0576 on LL_1 ,
 under hairline read answer as 0.9455 on LL_1 .

Example 99. $e^{0.004}=1.00401$, $e^{-0.004}=0.996$

Move hairline to 4 on D,
 under hairline read answer as 1.00401 on LL_0 ,
 under hairline read answer as 0.996 on LL_0 .

(4) Hyperbolic Functions

Hyperbolic functions are computed from the following formulas; in these cases, we use the LL_n and $LL_{\bar{n}}$ scales.

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$$

Example 100. $\sinh 0.802=0.891$

$\cosh 0.802=1.339$

$\tanh 0.802=0.665$

Move hairline to 8.02 on D,
 under hairline find $2.23=e^{0.802}$ on LL_2 ,
 under hairline find $4.48=e^{-0.802}$ on LL_2 .
 Move hairline to $1.604=2 \times 0.802$ on D,

under hairline find $4.975 = e^{2 \times 0.802}$ on LL_3 .

$$\therefore \sinh 0.802 = \frac{2.23 - 0.448}{2} = 0.891$$

$$\cosh 0.802 = \frac{2.23 + 0.448}{2} = 1.339$$

$$\tanh 0.802 = \frac{4.975 - 1}{4.975 + 1} = 0.665.$$

10. HOW TO USE GAUGE MARKS

There are the following gauge marks on our slide rules

(1) $c \doteq 1.128$

The c mark is placed on the C and D scales at 1.128, and is used for the calculation of a circle area.

Example 101. Find the area of a circle with its diameter of 1.5 cm. Answer 1.77 cm²

Move hairline to 1.5 on D,
set c on C under hairline,
opposite left index of C read answer as 1.77 on A left.

(2) $\rho^\circ \doteq 57.2958$

The ρ° mark is placed on the C and D scales at 57.2958, and is used for converting an angle from degrees to radians.

Example 102. $2.4^\circ = 0.0419$ radians

Move hairline to right index of D,
set ρ° on C under hairline,
move hairline to 2.4 on C,
under hairline find 4.19 on D,
read answer as 0.0419 radians.

$$\left[\begin{array}{l} 1^\circ \doteq 0.02 \text{ radians (approximately)} \\ \therefore 2.4^\circ \doteq 0.02 \times 2.4 = 0.048 \text{ radians.} \end{array} \right]$$

(3) $\pi \doteq 3.141$

The π mark is placed on the C, D, DF and CF scales at 3.1416, and shows the ratio of the circumference of a circle to its diameter.

(4) $\epsilon \doteq 2.718$

The ϵ mark is placed on the LL_2 and LL_3 scales at 2.718, and shows the value of the base of natural logarithms.

(5) $1/\epsilon \doteq 0.3679$

The $1/\epsilon$ mark is placed on the LL_2 and LL_3 scales at 0.3679, and shows the value of the reciprocal of ϵ .

— THE END —