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M79

INSTRUCTIONS FOR USING

# SLIDE RULE

## LOG LOG, DUPLEX

## CONTENTS



# 1. INTRODUCTION.

The slide rule is an accurate mechanical device for rapidly and accurately making calculation. Problems involving multiplication, division, proportion, percentage, squares and square roots, cubes and cube roots, diameter, reciprocals, areas, logarithms and exponents, trigonometric formula and combination can be solved by means of a slide rule, regardless of one's mathematical knowledge. Accuracy can be obtained if the body, slide and indicator are carefully set, say, to three figures adequate for most practical application.

Speed in using this slide rule will follow as the result of practice. Read the instructions with the most care and attention to details over several times. The first important thing is to learn to locate numbers accurately upon the several scales. The numbers often fall between graduation lines on the rule when you read answer. In this case, we must judge the distance to obtain the last decimal place. Try problems which can be checked with simple arithmetic for the beginning. The calculations on the rule will check the numeral results within three places. You can master each operation quickly if you work out such "Check Problem" according to each manipulation.

# 2. GENERAL DESCRIPTION OF SCALES

The following is a brief description of the various scales of the log log rule;

## 1) C AND D SCALES

These scales are exactly alike and are the fundamental scales of the slide rule. They are used for multiplication and division, and are also used with the other scales in various operation.

## 2) DF AND CF SCALES

These are the same as the D and C scales "folded" at $\pi$ (=3.1416) and interchanging the two parts. This puts $\pi$ at the ends and 1 about in the middle. In order to avoid resetting when the answer runs off the scale, these scales are used with the C and D scales in multiplication and division. They are also useful in problems requiring the multiplication by $\pi$.

## 3) CI SCALE

This is an inverted C scale. The graduations run from right to left instead of from usual left to right. It is used for reading directly the reciprocal of a number and also gives us the answer of multiplication of three factors with just one setting of the slide.

## 4) CIF SCALE

This is an inverted CF scale. It bears the same relation to CF that CI bears to C and D.

## 5) A AND B SCALES

These scales are exactly alike and consist of two half size C and D scales. They are used with the C and D scales in finding the squares and square roots.

## 6) K SCALE

This is used in finding cubes and cube roots and consists of three one-third size C and D scales.

## 7) S AND ST SCALES

These scales give both the sines and cosines of angles.

## 8) T SCALE

This gives the tangents and cotangents of angles.

## 9) L SCALE

This scale is used with the D scale in finding directly the mantissa of the common logarithms of a number.

## 10) LL 1–2–3 SCALES

These scales constitute three sections of one long scale. This is used with C and D scales in evaluating expressions such as $(1.45)^{2.38}$ and $(7.32)^{2.4}$.

It also gives directly the values of the function $e^x$ ($e$=2.718) for values of X from 0.01 to 10, and is used in reading the natural logarithms of numbers.

## 11) LL 01–02–03 SCALES

These are three sections of one long scale. This is used with C and D in finding powers of decimal fractions - such as $(0.46)^{2.5}$ or $(0.39)^{0.34}$. It also gives directly the value of the function $e^x$ for negative powers from 0.01 to 10.0.

# 3. READING THE SCALES

## 12) HOW TO READ THE SCALES

Before attempting to operate the slide rule, the beginner must first learn how to read the scales. The various scales are not uniformly spaced (except L scale) and the marks on the scales do not measure lengths - the spacing is called "logarithmic" and it is based on the theory of logarithms. As the reading of all scales is done in much the same manner, it will be sufficient to illustrate the procedure with one scale. A slide rule only enables one to work with significant figures of a number. The significant figures are the ones that remain after the zeros to the right or left of a given number have been removed. For example: - The significant figures of the following numbers 0.0359, 3.59, 35.9, 35900 - are all the same, namely three-five-nine; making a total of three significant figures, due to the manner in which the slide rule is divided. To illustrate this, we will indicate the location of the three figure number 254 on the C and D scales in our explanation of the reading of the scales, as follows;



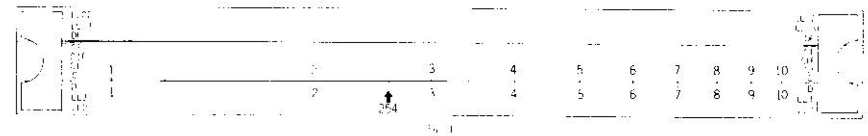



Fig. 1

**FIRST STEP:** The C and D scales are divided into ten major divisions, numbered from 1 to 10, giving us first significant figure. Figure 1 illustrates the major divisions of the C and D scales, however, the same explanations applies to the A and B scales. If the first significant figure of number is 1, the number will lie between the major division 1 and 2. If it is 2, the number will lie between 2 and 3, etc. The number 254 lies between the major division 2 and 3 as indicated by the arrow mark (Fig. 1) since the first significant figure of the number is 2.

**SECOND STEP:** Each of these major divisions are subdivided into ten parts or secondary divisions, giving our second significant figure (See Fig. 2)

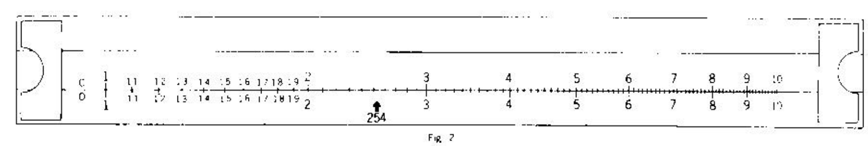



Fig. 2

In the number 254, the second significant figure -5- indicates that the location is between the 5th and 6th secondary division, as indicated by the arrow mark in Fig. 2.

**THIRD STEP:** Each of these secondary divisions are again subdivided into a third set of divisions, giving us our third significant figure (See Fig. 3)

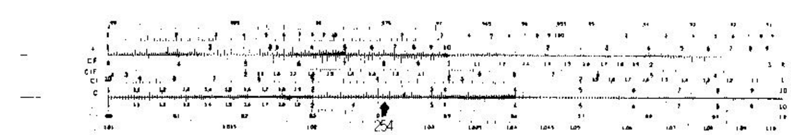



Fig. 3

In the number 254, the third significant -4- indicates that the location is the second tertiary division of 5th secondary division of the third major division as indicated by the arrow mark in Fig. 3.

The secondary subdivisions between 1 and 2 are each divided into 10 tertiary divisions. The shorter secondary divisions between 2 and 3 and between main 3 and 4 are divided only into 5 tertiary divisions. The still shorter secondary divisions between main divisions beyond 4 are divided into two tertiary division, because of short of space. Had the number been 2543, we should then have located it 3/10 of the tertiary division between 2540 and 2550. This last shift is made by estimating the 3/10 by eye as there are no fourth order divisions. The captioned procedure is the same in the any other number on the scale, excepted that it should be noted that the tertiary divisions toward the right end of scale represent fifths (between main 2 and 4) and halves (between main 4 and 10). For example, the number 853 lies, first on the main divisions between 8 and 9, second on the secondary division 850 and 860, and third 3/5 of the first tertiary division between 850 and 855, this 3/5 being estimated by eye.
The decimal point is ignored in operating the slide rule, because of being graduated with the theory of logarithms, but the location can be estimated without difficulty by estimating the answer from rounding off the factors and divisions.

# 4. SLIDE RULE OPERATION

## 13) MULTIPLICATION

Multiplication is performed on the C and D scales. The number 1 on the left end is called the "Left Index" and the number 10 in the right end is called the "Right Index". To multiply one number by another, set the Index of the C scale over either of the factors on the D scale. Move the hairline on the indicator to the second factor on the C scale. Read the answer on the D scale under the hairline. Determine the location of the decimal point-by rough mental calculation.

### EXAMPLE: MULTIPLY 2 X 4

Set the Left Index on C to 2 on D. Move the hairline to 3 on C. Under the hairline read the answer 8 on D.

### EXAMPLE: MULTIPLY 2.5 X 31

Set the Left Index on C to 2.5 on D. Move the hairline to 31 on C. Under the hairline read the answer 7.75 on D.

### EXAMPLE: MULTIPLY 3 X 8

If the Left Index of C is set to 3 on D, the number 8 on C will be off the rule. Therefore, to multiply 3 x 8, set the Right Index of C to 8 on D. Move the hairline to 3 on C and read the answer 24 under the hairline on D.



## 14) MULTIPLICATION OF THREE OR MORE FACTORS.

One of the biggest advantages of a slide rule is the product of any number of factors can be obtained by one continuous multiplication operation.

### EXAMPLE: MULTIPLY 12.5 x 34.4 x 0.32

The first two factors are multiplied together as the above explanation. Therefore, leave the indication where it is and bring Right Index of C under the hairline. Move the hairline to 0.32 on C and read the answer 137.6 under the hairline on D.

## 15) DIVISION

Division is generally performed on the C and D scales. To divide one number by another, set the numerator on the D scale and bring the denominator on the C scale opposite to it. Under the index on the C scale read the answer on the D scale. Determine of the decimal point by rough mental calculation. Again as in multiplication, either index of the C scale may be used and whichever index is within the body of the rule, is the one which is used.

### EXAMPLE: 9 ÷ 6

Set the hairline over 9 on D, bring 6 on C under the hairline.
Read the answer 1.5 under the Left Index of C.

### EXAMPLE: 2.7 ÷ 6.5

Set the hairline over 2.7 on D, bring 6.5 on C under the hairline and read the answer 0.416 under the Right Index of C.

## 16) PROBLEMS INVOLVING BOTH MULTIPLICATION AND DIVISION

Another distinct advantage of a slide rule is the speed with which problems involving both multiplication and division can be solved with the easy operation. The best way to solve the problem is to perform alternately, first a division, then a multiplication, then a division.....

### EXAMPLE: $\frac{16 \times 82}{75}$

The reasoning on a problem is to first divide 16 by 75 and then multiply the result by 82. That is, move the hairline to 1.6 on D. Bring 75 on C under the hairline, and move the hairline to 82 on C. Under the hairline, read the answer 17.5 on D.

**EXAMPLE:** $\frac{182}{14.5 \times 7}$

Set 14.5 on C opposite 182 on D. Move the hairline to 7 on CI and read the answer 1.795 on D under the hairline.

## 20) SQUARES

To find the square of a number, set the hairline over the number to be squared on the C or D scale and read the square of the number under the hairline on the B or A scale. It should be noted that on rules with graduations on both sides, and with an encircling indicator, the scales on one side can be read directly in connection with the scales on the opposite side.

**EXAMPLE:** $3^2$

Set the hairline over 3 on D and read the answer 9 under the hairline on A.

## 21) SQUARE ROOTS

Use the A–D combination of scales. "A" scale is divided into two parts called "A-Left" and "A-Right". To find the square root of a number greater than unity–if there are an odd number of figures before the decimal point, set the hairline over the number on "A-Left" and read the square root under the hairline on D scale. If the number has an ever number of figures before the decimal point, set the hairline over the number on "A-Right" and read the answer under the hairline on the D scale. Determine the location of decimal point by mental approximation.

**EXAMPLE:** $\sqrt{7.3}$

Set the hairline over 7.3 on "A-Left" and read the answer 2.7 under the hairline on D.

**EXAMPLE:** $\sqrt{0.0028}$

Move the decimal point 4 placees to the right to become $\sqrt{28}$. Set the hairline over 28 on "A-Right" and read the number 5.29 under the hairline on D. Move the decimal point 2 places to the left to make the correct answer 0.0529.

## 22) CUBES

Use the D–K combination of scales. "K" scale is so constructed that when the indicator hairline is set over a number on the D scale, the cube of the number is under the hairline on the K scale.

**EXAMPLE:** $7.4^3$

Set the hairline over 7.4 on D and read the answer 405 under the hairline on K.



## 23) CUBE ROOTS

To find the cube roots of a number, use the D–K combination in the reverse order. The "K" scale consists of three sections placed end to end, the combined length of which is equal to the length of the D scale. Those three sections will be referred to as "K-Left", "K-Middle" and "K-Right"..

In general to decide which part of the K scale to use in locating a number, mark off the digits in groups of three starting from the decimal point. If the left group contains one digit, "K-Left" is used, if there are two digits in the left group, the "K-Middle" is used, if there are three digits, "K-Right" is used. In other words, numbers containing 1, 4, 7, ..... digits are located on the "K-Left", numbers containg 2, 5, 8..... digits are located on the "K-Middle" and numbers containg 3, 6, 9, ..... digits are located on "K-Right". To find the cube root of a number, set the hairline over the number on "K-Left", "K-Middle" or "K-Right" according to the above rule, and read the answer under the hairline on D scale. For numbers less than 1 or greater than 1000, move the decimal point right or left three places at a time until a number between 1 and 1000 is obtained, find the cube root as explained above, then move the decimal point in the opposite direction one third as many places as it was originally moved to obtain the answer.

**EXAMPLE:** $\sqrt[3]{26}$

Set the hairline over 26 on "K-Middle" and read the answer 2.96 under the hairline on D scale.

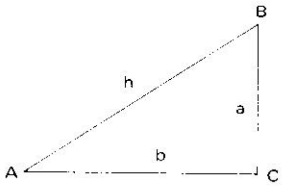
**EXAMPLE:** $\sqrt[3]{0.0315}$

Set the hairline over 31.5 on "K-Middle" and read the 3.16 under the hairline on D. then move the decimal point 1 place to the left to obtain the correct answer 0.316.

## 24) TRIGONOMETRY

Trigonometric functions are based on the ratios,sine, cosine, tangent, cotangent, secant and cosecant. These angular functions are based on the ratios of the lengths of the sides of a right triangle.

Some important formulas from trigonometry are listed here for ready reference.




Sine of angle A $= \dfrac{\text{side opposite}}{\text{hypotenuse}}$ (written Sin A $= \dfrac{a}{h}$)

Cosine of angle A $= \dfrac{\text{side adjacent}}{\text{hypotenuse}}$ (written Cos A $= \dfrac{b}{h}$)

Tangent of angles A $= \dfrac{\text{side opposite}}{\text{side adjacent}}$ (written Tan A $= \dfrac{a}{b}$)

Cotangent of angle A $= \dfrac{\text{side adjacent}}{\text{side opposite}}$ (written Cot A $= \dfrac{b}{a}$)

Secant of angle A $= \dfrac{\text{hypotenuse}}{\text{side adjacent}}$ (written Sec A $= \dfrac{h}{b}$)

Cosecant of angle A $= \dfrac{\text{hypotenuse}}{\text{side opposite}}$ (written Cosec A $= \dfrac{h}{a}$)

These ratios are functions of the angle. The definitions may be extended to cover cases in which the angle A is not an interior angle of a right triangle, and hence may be greater than 90 degrees. It should be remembered that the sine and cosecant are reciprocals, as are the cosine and secant, and the tangent and cotangent.

That is,

$$\text{Sine A} = \frac{1}{\text{cosec A}} \quad \text{and Cosec A} = \frac{1}{\text{sin A}}$$

$$\text{Tan A} = \frac{1}{\text{cot A}} \quad \text{and Cot A} = \frac{1}{\text{tan A}}$$

$$\text{Cos A} = \frac{1}{\text{sec A}} \quad \text{and Sec A} = \frac{1}{\text{cos A}}$$

Relations between the complementary angles;

Sin A = cos (90° − A)
Cos A = sin (90° − A)
Tan A = cot (90° − A)
Cot A = tan (90° − A)

Relations between supplementary angles;

sin (180° − A) = Sin A
cos (180° − A) = −Cos A
tan (180° − A) = −Tan A



## 25) THE S SCALE — SINES AND COSINES

This scale shows that it is divided left to right from 5.74° to 90°, and from right to left from 0° to 84.26°. The former graduated angle represents the sine and later represents the cosine. When the hairline is set to an angle on the S scale, the value of the sine is read under the hairline on the C and D scales. When the hairline is set to an angle on the cosine scale, the value of the cosine is read under the hairline on the C and D scales. Within the angular range of the S scale (5.74° to 90°), the numeral sine value varies from 0.1 to 1.0. Therefore, the legend 0.1 X is placed on the left and of the scale and 1.0 X is placed on the right end. The cosecant is the reciprocal of the sine, and the secant is the reciprocal of the cosine. With this relationship, it is possible to solve directly for either the secant or the cosecant by using S scale in conjunction with the reciprocal CI scale.

### EXAMPLE: SIN 22.4°

Set the hairline over 22.4° on S and read the answer 0.381 under the hairline on D. The decimal point position is determined by noting the range of the legend, from 0.1 to 1.0. Place it so the answer is within this range.

### EXAMPLE: COSEC 12.5°

Set the hairline over 12.5° on S and read the answer 4.63 under the CI scale in the reverse side.

## 26) THE ST SCALE—SINES AND TANGENTS (SMALL ANGLES)

This is a scale of sines and tangents from 0.57° to 5.74° and can be also used for cosines from 84.26° to 89.43°. When the hairline is set over an angle on ST scale, sine or tangent of angle is under the hairline on the C and D scale. The legends at the right and left end of the ST scale give the numerical range of the functions on the scale as in the case of S scale. The largest angles whose cosine can be obtained on S scale is 84.26°. To find the cosine of angles, larger than 84.26°, the relationship cos A = sin (90° − A) is used, giving the result which can be solved on ST scale.

### EXAMPLE: SINE 2.04°

Set the hairline over 2.04° on ST and read the answer 0.0356 under the hairline on D. The decimal point position is determined by noting the range of the legend from 0.01 to 0.1. Place it so the answer is within this range.

### EXAMPLE: COS 88.4°

Change the form to cos 88.4° = sin (90° − 88.4°) = sin 1.6°. Set the hairline over 1.6° on ST and read the answer 0.0279 under the hairline on D scale.

## 27) THE T SCALE–TANGENTS AND COTANGENTS

This scale is used with the combination of C and D or CI scale and it gives the tangents directly for the angles between 5.72 and 84.28 . The relationship explaind in the paragraph 23) should be reverted when using the T scale. Because tan x = cot (90—x), the same graduations serve for both tangents and cotangents. For instance, if the hairline is set on the graduation marked 70/20, the corresponding reading on the D scale is 0.364, this is the value of tan 20 . This is also the value of cot 70 , since tan 20 = cot (90 — 20 ) = cot 70 . Moreover, tan x = 1/cot x, in other words, the tangent and cotangent of the same angle are reciprocal. Therefore, for the same setting, the reciprocal of cot 70 or 1/0.364 is read on the CI scale in the reverse side as 2.745. This is the value of tan 70 . The followings are the list showing those relations. Set the hairline over an angle value on the T scale and read

(1) tangents from 5.7 to 45 on C and D scale
(2) tangents from 45 to 84.28 on CI scale
(3) cotangents from 5.7 to 45 on CI scale
(4) cotangents from 45 to 84.28 on C and D scale.

### EXAMPLE: TAN 42.4

Set the hairline over 42.4 on T scale and read the answer 0.913 under the hairline on D scale. The decimal point position is determined by noting the range of the legend, from 0.1 to 1.0 and place it so the answer should be within this range.

### EXAMPLE: COT 36.2

There are two ways of solutions.
(1) Set the hairline to 36.2 on T and read the answer 1.368 under the hairline on CI in the reverse side.
(2) Think of the problem as cot 36.2 = tan (90 − 36.2 ) = tan 53.8 . Set the hairline to 53.8 on T and read the answer 1.368 under the hairline on CI in the reverse side.

## 28) ILLUSTRATIVE PROBLEMS USING THE TRIGONOMETRIC SCALES

When the user has a sufficient knowledge of the slide rule and of the trigonometric scales on the slide rule, most types of angular problems can be solved. The following examples illustrate some of angular problems. The next right angle shows the case in which two sides are given.

### FIND ANGLES $\alpha$ AND $\gamma$ AND SIDE b.



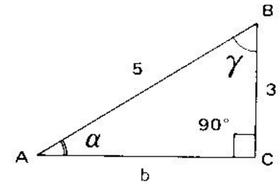



Fig. 4

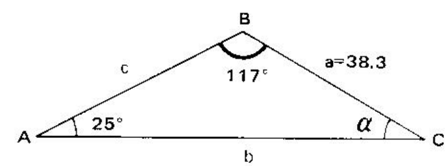



Fig. 5

### TO FIND ANGEL $\alpha$;

In Fig. 4, it is written sin $\alpha$ = 3/5, therefore, sin $\alpha$ = 0.60. Set the hairline on 60 on D and read the angle $\alpha$ as 36.9° under tha hairline on S.

### TO FIND $\gamma$;

As $\gamma$ is the complement of angle $\alpha$, $\gamma$ = 90 — 36.9° = 53.1°. Or, angle $\gamma$ can be obtained by nothing that the cos $\gamma$ = 3/5 = 0.60. Namely, set the hairline to 0.60 on D, and read the angle $\gamma$ as 53.1° on cosine scale of S.
(graduation from right to left)

### TO FIND SIDE b ;

Use the pythagorean Theory; $a^2 + b^2 = c^2$. Or, use cos $\alpha$ = b/c, which is rewritten as 5 cos $\alpha$ = c. In order to prove this, set the Right Index of C opposite 5 on D, then reverse the rule and move the hairline to 36.9° on the cosine scale of S. The answer of b, 4, will be read under the hairline on D scale.

### Oblique Triangle;

### EXAMPLE;

The left oblique triangle (Fig. 5) shows the case in which two angles and a side are given.

### FIND ANGLES $\alpha$ AND SIDES b AND c.

### TO FIND ANGLE $\alpha$ ;

Angle $\alpha$ = 180° − 117° − 25 = 38

### TO FIND SIDES b AND c;

By using the Law of Sines;

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

these sides can be solved by means of proportions. In this case, the proportions are

$$\frac{38.3}{\sin 25^\circ} = \frac{b}{\sin 117} = \frac{c}{\sin 38}$$

However, sin 117° = sin (180° −117°) = sin 63 . Therefore, the proportions become 38.3/sin 25 = b/sin 63° = c/sin 38°

To solve for b, move the hairline to 38.3 on D and bring sin 25° on the sine scale of S. Then, move the hairline to sin 63° on the sine scale of S and read the value of b as 80.7 under the hairline on D scale.

To solve for c, move the hairline to 38° on the sine scale of S and read the value of C as 55.8 under the hairline on D scale.

## EXAMPLE;

The left oblique triangle (Fig. 6) shows the case in which all three sides are given.

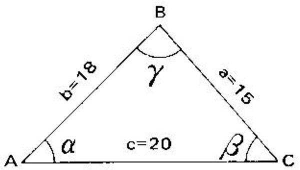



Fig. 6

### FIND ANGLES $\alpha$ AND $\gamma$

### TO FIND ANGLE $\alpha$ ;

By using the Law of cosines; $\cos A_{(\alpha)} = \frac{b^2+c^2-a^2}{2bc}$

In this example, that is,

$$\cos A = \frac{18^2+20^2-15^2}{2\times18\times20} = \frac{499}{720} = 0.693$$

To solve, set the hairline to 0.693 on D and read the angle $\alpha$ as 46.1° on the cosine scale of S

### TO FIND ANGLE $\gamma$ ;

By using the Law of cosines; $\cos B_{(\gamma)} = \frac{a^2+c^2-b^2}{2ac}$

In this example, that is,

$$\cos B_{(\gamma)} = \frac{15^2+20^2-18^2}{2\times15\times20} = \frac{301}{600} = 0.502$$

To solve, set the hairline to 0.502 on D and read the angle $\gamma$ as 59.9° on the cosine scale of S.

### TO FIND ANGLE $\beta$ ;

Angle $\beta$ = 180° − 46.1° − 59.9° =74°

## 29) TRIGONOMETRIC FUNCTION AND ITS MULTIPLICATION & DIVISION.

In many problems, products or quotients involving trigononetric functions are required. These answers can be obtained directly without actually reading the value of the trigonometric function.

### EXAMPLE; 2.55 ÷ sin 50°

Set the hairline over 2.55 on D and bring 50° on the sine scale of S under the hairline. The answer 3.33 is obtained under the Right Index on D scale.

### EXAMPLE; 10.8 x sin 22° x cos 55°

Set the Left Index over 10.8 on D and move the hairline to 22° on the sine scale of S. Bring the Right Index under the hairline and move the hairline to 55° on the cosine scale of S. The answer 23.2 is obtained under the hairline on D scale.

## 30) LOGARITHMS—L SCALE.

The L scale represents logarithms. The most common base for logarithms is 10. The logarithm of a number is the exponent to which a given base must be raised to produce the number. For example, Log $10^3$= 3.00, Log $10^4$= 4.00 etc.,

Logarithms, to the base 10, can be found on the slide rule. However, there are two parts to a logarithm, namely,



(1) The mantissa (the decimal fraction part on the right of the decimal point)
(2) The characteristic (the part on the left of the decimal point)
To find logarithms, to the base 10, and the following two rules should be noted.

(1) When the hairline is set over a number on the D scale, the mantissa of its logarithm is under the hairline on the L. Conversely, when the hairline is set over a mantissa on the L, the antilogarithm is under the hairline on the D scale.
(2) As the slide rule does not give the characteristic of a logarithm, the following rules should be applied in determing it.

(a) Of a number greater than 1, is positive, and is one less than the number of digits to the left of the decimal point

(b) Of a number less than 1, is negative, and is one greater than the number of zero immediately following the decimal point

### EXAMPLE; FIND LOG 15

Set the hairline over 15 on C and read 0.176 on L scale under the hairline. As 15 has two digits to the left of the decimal point, the characteristic is 1. Therefore, the Log 15 is 1.176

### EXAMPLE; FIND LOG 0.0187

Set the hairline over 187 on D and read 0.272 on L under the hairline. As 0.0187 has one zero immediately following the decimal point, the characteristic is−2. Therefore, the Log 0.0187 is−2.272.

### EXAMPLE; FIND THE NUMBER WHOSE LOGARITHM IS 2.675

Set the hairline over 0.675 on L and read 473 on D under the hairline. As the characteristic is 2, the antilogarithm should have 3 digits to the left of the decimal point. Therefore, the answer is 473.

## 31) LOG LOG SCALES

To find the value of powers, roots or logarithm of numbers, and many other types of expression, such as $1.8^7$, $6.3^{1.5}$, $\sqrt[5]{35}$, Log Log scales are used. There are six Log Log scales in the slide rule, namely, "$LL_1$", "$LL_2$", "$LL_3$","$LL_{01}$", "$LL_{02}$" and "$LL_{03}$" . The "$LL_1$" "$LL_2$" and "$LL_3$" scales (hereafter called as "LL" scales) are used in solving problems for number greater than one and the range of numbers is from 1.010 to 22.026. "$LL_{01}$", "$LL_{02}$" and "$LL_{03}$" scales (hereafter called as "$LL_0$" scales) are used in solving problem for number smaller than 1 and the range of numbers is from 0.00005 to 0.9905.

## 32) RELATIONSHIP OF "LL" AND "$LL_0$" SCALES TO C AND D SCALES.

"LL" and "$LL_0$" scales are one continuous scale. Thinking of "LL" and "$LL_0$" scales in this manner will be helpful in determing on which scale the answer is to be read.

## 33) RECIPROCAL RELATIONSHIP BETWEEN LOG LOG SCALES.

"$LL_0$" scales are the reciprocal of "LL" scales with the corresponding number designation. Namely, "$LL_{01}$" scale is the reciprocal of "$LL_1$" scale; "$LL_{02}$" is the reciprocal of "$LL_2$" scale and the same in between "$LL_{03}$" and "$LL_3$".

## 34) "$LL_1$", "$LL_2$" AND "$LL_3$" SCALES – FOR NUMBER GREATER THAN UNITY.

"LL" scales are used to solve the problems of the following types where "m" is greater than 1 in the following form.

(1) $E^y = m$, ........ either "m" or "y" is known
(2) $X = m^y$ ........ any two of the three unknowns are known
(3) $X = \sqrt[z]{m^y}$ ........ any three of the four unknowns are known
(4) $Log_e m = y$ ........ either "m" or "y" is known
(5) Log X = y ........ any two of the three unknowns are known.

## A. PROBLEMS INVOLVING POWER "E"

(Approximately 2.178)

occur so often in engineering and scientific calculation. Power of "e" from 0.01 to 10 can be solved directly by "LL" scale. In case where the power "e", is negative, "$LL_0$" scales are used.
For raising "e" to positive power from 0.01 to 10.0 set the hairline over the value of the exponent on D scale and read the answer on "LL" scale which has the value of the exponent within the range of its legend.

### EXAMPLE; $e^{0.07}$

Set the hairline over 0.07 on D and read the answer 1.0725 on "$LL_1$" under the hairline. At the right end of Log Log scales, there are legends similar to those at the end of Trigonometric Scales. The legends provide an instant reference as to which "LL" the answer is to be read on. In the above example, as the exponent of "e" is 0.07, the answer should be read on "$LL_1$" whose range we know to be, from the legend, $e^{0.01}$ to $e^{0.105}$

### EXAMPLE; $e^{0.65}$

Set the hairline over 65 on D and read the answer 1.916 on $LL_2$ under the hairline.
Because of the legends, we know the answer should be read on $LL_2$ .

### EXAMPLE; $e^x = 14.6$

Set the hairline over 14.6 on $LL_3$ and read the answer, 2.68 on D under the hairline. When it is required to raise "e" to a power larger than 10, for instance $e^{16}$ is required, apply the law of exponents and rewrite it as $e^{10} \times e^6$. Now the exponents of "e" are within the range of "LL" scales. The answer is obtained by multiplying the two answers, $e^{10}$ and $e^6$ together.



## B. POWERS OF NUMBERS GREATER THAN UNITY.

When solving exponential problems the continuous relationship of "LL" scales should be remembered.
If a given number greater than unity is raised to a power greater than unity, the answer is usually greater than the original number given. Considering "LL" scales as one continuous scale, the answer will be to the right of the original setting of the indicator. If a given number greater than unity to be raised to a power less than unity, the answer is usually less than the original number given. In this case, the answer will be to the left of the original setting of the indicator. To raise a number greater than unity to any power, set one index of C scale over the number of the power "LL" scale and then move the hairline to the value of the exponent on C scale. Read the answer on the proper "LL" scale under the hairline.

### EXAMPLE; $3.9^{3.4}$

Set the hairline over 3.9 on "$LL_3$" and bring the Left Index of C under hairline. Then, set the hairline over 3.4 on C and read the answer 102 on "$LL_3$" under the hairline.

### EXAMPLE; $4.6^{0.52}$

Set the hairline over 4.6 on "$LL_3$" and bring the left index of C under hairline. Then, set the hairline over 52 on C and read the answer 2.21 on "$LL_2$" under the hairline.
Because the exponent in this example is less than one, the answer should be less than the original number given. In determine which of "LL" scales the answer should be read on, it is necessary to estimate the answer mentally by noting that $4.6^{0.52}$ is roughly equal to $5^{0.5}$ which is written as $\sqrt{5}$. Therefore, you will note that the answer should be read on the "$LL_2$".

## NEGATIVE EXPONENTS;

When solving problems involving negative exponents, the "$LL_0$" scales are extremely useful as it is the reciprocal of "LL" scale. The numbers greater than one raised to a negative power can be read directly on "$LL_0$" scales. The "$LL_0$" scale on which the answer should be read is easily found by determing which scale the answer would be read on if the exponent were positive.

### EXAMPLE; $2.15^{-4.6}$

Set the hairline over 2.15 on "LL" and bring the Right Index of C under the hairline. Set the hairline over 4.6 on C. Had the exponent been positive, the answer would have been read on "$LL_3$". However, the exponent is negative in this example, the answer should be read on the "$LL_0$" scale with the corresponding number designation, namely "$LL_{03}$". Therefore, read the answer 0.0298 on "$LL_{03}$" under the hairline.

## C. POWERS OF NUMBERS GREATER THAN UNITY WITH FRACTIONAL EXPONENTS.

By the law of exponents in elementary Algebra, $\sqrt{x}$ is rewritten as $X^{\frac{1}{2}}$, $\sqrt[3]{X}$ is rewritten as $X^{\frac{1}{3}}$ and so forth.

### EXAMPLE; $X = \sqrt[8.9]{1.16^{4.2}}$

The above can be rewritten as $1.16^{\frac{4.2}{8.9}}$. Therefore, set the hairline over 1.16 on "$LL_2$" and bring the left index of C under the hairline. Move the hairline to 4.2 on C and bring 8.9 on C under the hairline. Then, move the hairline over the Right Index of C and read the answer 1.0726 on "$LL_1$" of the reverse side under the hairline.

## D. LOGAITHMS TO ANY BASE GREATER THAN UNITY.

In common logarithms, the base 10 is most used. A thorough understanding of the nature of logarithms will be found very helpful in determining the decimal point location when finding logarithms to any base on a slide.

## E. LOGARITMS TO THE BASE "E"

In engineering and scientific calculations, logarithms to the base "e", called natural logarithms, are frequently required. The natural logarithms of a number are actually the power to which "e" (approximately 2.718) should be raised to yield the original number. To find logarithms to the base "e", set the hairline over the number on the proper "LL" scale, and read the answer under the hairline on D scale. The decimal point is determined by placing it so the answer is within the range of the legend on the "LL" used.

### EXAMPLE; $Log_e$ 1.07

Set the hairline over 1.07 on "$LL_1$" and read 676 under the hairline on D. Place the decimal point so the answer is within the range of the legend on "$LL_1$". The answer is 0.0676.

To find logarithms to any base greater than unity. i, e, $log_n X$, set the hairline over the value of "n" on the proper "LL" scale, bring either Right or Left Index of C under the hairline, then, move the hairline to "X" on the proper "LL" scale. Read the answer under the hairline on "C" scale.

### EXAMPLE; $LOG_8$ 49

Set the Left Index opposite 8 on "$LL_3$" and set the hairline over 49 on "$LL_2$". Read 1871 under the hairline on C. Determine the decimal point location by noting mentally that $8^2 = 64$. Therefore, the answer is 1.871



### EXAMPLE; $LOG_{1.37}$ X = 5.2

Set the Right Index over 1.37 on "$LL_2$" and move the hairline to 5.2 on C. Read 5.14 on "$LL_3$" under the hairline.

## 35) "LL01" "LL02" AND "LL03" SCALES

Those "$LL_0$" scales are used primarily for solving problems of the following cases where "n" is less than one.

(1) $e^{y} = n$, ……. either "n" or "y" are known
(2) $X = n^{y}$ ……. any two of the three unknowns are known
(3) $X = \sqrt[z]{ny}$ ……. any three of the four unknowns are known
(4) $Log_n X = y$ ……. any two of the three unknowns are known

## A. POWER OF "E"

To raise "e" to negative powers from 0.01 to 10.0, set the hairline over the value of the exponent on D scale, and read the answer on "$LL_0$" scales which has the value of the exponent within the range of its legend.

### EXAMPLE; $e^{-0.65}$

Set the hairline over 0.65 on D and read 0.522 on "$LL_{02}$" under the hairline.

### EXAMPLE; $e^x = 0.705$

Set the hairline over 0.705 on "$LL_{02}$" and read the answer 349 on D. As the number 0.705 is on "$LL_{02}$" scale, the value of the exponent should be between 0.1 and 1.0. Therefore, the answer is 0.349

## B. POWERS OF NUMBERS LESS THAN UNITY.

The numbers less than one can be raised to a power in the same manner as the numbers greater than one, except that when raising numbers of less than one to a power "$LL_0$" scales are used.

### EXAMPLE; $0.46^{0.25}$

Set the hairline over 0.46 on "$LL_{02}$" and bring the Right Index of C under the hairline. Then, set the hairline over 0.25 on C and read the answer 0.8235 on "$LL_{02}$" under the hairline.

## NEGATIVE EXPONENTS.

As "$LL_0$" scales are the reciprocal of the corresponding "LL" scales. "LL" scales must also be the reciprocal of the corresponding "$LL_0$" scales. Therefore, in raising a number less than one to a negative power, the answer can be read directly on "LL" scales.

"LL" scales on whcih the answer should be read is found by determining which scale the answer would be read on if the exponent were positive.

## EXAMPLE; $0.215^{-0.23}$

Bring the Left Index of C under 0.215 on "$LL_{03}$" and move the hairline to 2.3 on C. In this example, as the exponent is negative, the answer must be read on "LL" scales with the corresponding number designation, "$LL_3$". Therefore, read the answer 1.424 on "$LL_2$" under the hairline.

## C. POWERS OF NUMBERS LESS THAN UNITY WITH FRACTIONAL EXPONENTS.

The procedure in this case is the same as explained for numbers greater than unity with fractional exponents.

## EXAMPLE; $0.108^{\frac{6.8}{8.6}}$

Set the hairline over 0.108 on "$LL_{03}$" and bring the Right Index of C under the hairline. Then, move the hairline to 6.8 on C and bring 8.6 on C under the hairline.
Again move the hairline over the Right Index of C and read the answer 0.172 on "$LL_{03}$" under the hairline.

## D. LOGARITHMS TO ANY BASE LESS THAN UNITY.

This procedure is the same as used for finding logs to a base greater than unity, except that the "$LL_0$" scales are used for the logs to a base less than unity.
To find the logarithm to any base less than unity, i, e, : $\log_n x$, set the hairline over the value of "n" on the proper "$LL_0$" scale, bring either Right or Left Index of "C" under the hairline, then, move the hairline to "X" on the proper "$LL_0$" scale. Read the answer on "C" under the hairline.

## EXAMPLE; $LOG_{0.57}$ 0.58

Set the Right Index of "C" opposite 0.57 on "$LL_{02}$" and move the hairline to 0.58 on "$LL_{02}$". Read the answer 0.968 on C under the hairline. In same problems, it is required to use both "$LL_0$" and "LL" scales by the combined operation.

## EXAMPLE; $LOG_{0.96}$ 3.48

Set the hairline over 0.96 on "$LL_{01}$" scale, bring Right Index of C (the reverse side) under the hairline. Move the hairline to 3.48 on "$LL_3$" and read the answer −3.059 under the hairline on C scale.