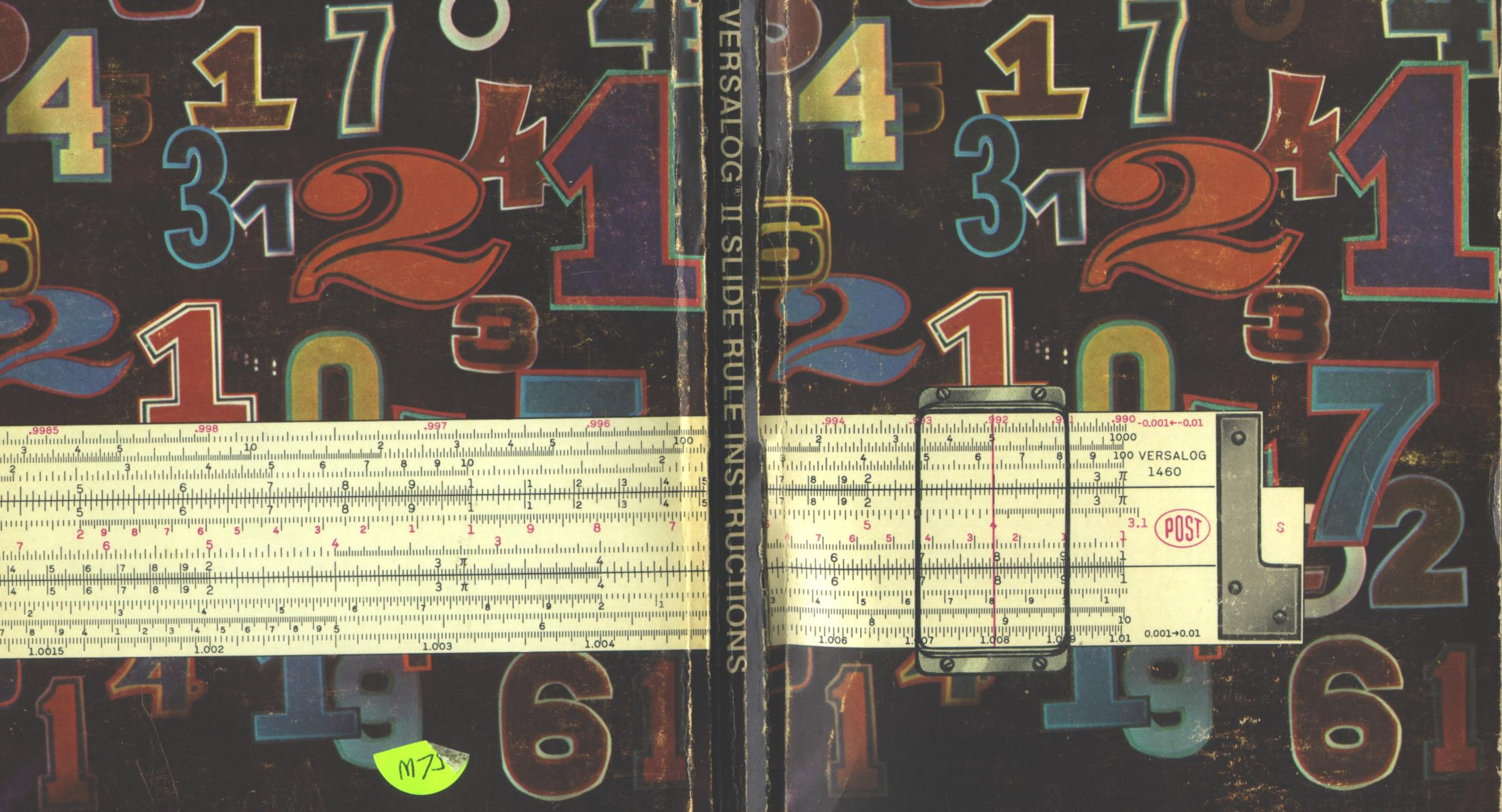
POST

# VERSALOG® II

VERSALOG® II SLIDE RULE INSTRUCTIONS




SLIDE RULE INSTRUCTIONS

TELEDYNE POST

# VERSALOG II
## SLIDE RULE INSTRUCTION MANUAL AND APPLIED TEXT
### by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**WITH CHAPTERS ON**

**CIVIL ENGINEERING APPLICATIONS**


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*Professor of Civil Engineering*
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**MECHANICAL ENGINEERING APPLICATIONS**


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## YOUR NEW VERSALOG II SLIDE RULE

The original VERSALOG slide rule was designed by engineers, for engineers. Not merely an instrument for general computation, the VERSALOG was designed around the *problems of engineering*. It has successfully provided both the practicing engineer and the student with a highly efficient and practical slide rule to match the high tempo of modern engineering development. In keeping pace with the rapid advancement of engineering sophistication, this fine rule has been further improved to provide even greater efficiency and convenience. The VERSALOG II improvements include the following:

- The addition of an A scale which, combined with the $R_1$ and $R_2$ scales, facilitates the computation of many combined operations involving squares and square roots.
- The extension of the $R_2$ scale to reduce the re-setting of indices.
- The addition of symbols at the left of each scale to provide immediate, simple identification of the use of the scale.
- The designation of the related range of the trigonometric functions at the right of each trigonometric scale.
- The addition of radian identification on a trigonometric scale allowing direct conversion of radians to degrees and degrees to radians.
- The rearrangement of scales to provide greater efficiency in overall slide rule operations.
- Improved functional positioning of all numerals on the three trigonometric scales for better legibility, thereby reducing reading errors.

## INSTRUCTION TEXT

The original VERSALOG instruction manual has been carefully edited and rewritten to parallel the improvements in the VERSALOG II slide rule. Continuing in the spirit of the original manual of breaking with conventional "instruction pamphlets," the author has placed a renewed emphasis on the efficient use of the slide rule scales. The value of this approach will be appreciated by students, teachers, and practicing engineers who have experienced difficulty in the transition between the abstract mathematical approach of contemporary slide rules and "instructions"... and the practical application of those mathematical principles to every day engineering problems.

The author retained from the original text the now highly proven successful practical application sections on the three basic fields of engineering. Revisions were made only wherever necessary to up-date the subject matter to embrace present day engineering developments.

These sections (Chapters 8, 9, and 10) make a long step forward toward the complete use of the slide rule and all its scales by graphically illustrating the application of your VERSALOG II to three separate and distinct engineering fields. Each section presents a practical and easy-to-comprehend guide to the use of the VERSALOG II in these specialized activities. By so doing, eminently qualified authorities in the fields of Civil, Electrical, and Mechanical Engineering have solved one of the great problems of slide rule technique and use.

## CONSTRUCTION

While many features are sought for in a slide rule, one is foremost above all others – unquestioned accuracy at all times, no matter what the conditions. The owner and user of a new VERSALOG II slide rule will have extra confidence in knowing that the ultimate in craftsmanship, care and exactness in manufacture has been followed to produce the very finest, most accurate slide rule available today.

To insure accuracy, your VERSALOG II slide rule is constructed from carefully selected and laminated bamboo. Bamboo is tough, and was chosen because of its ability to resist contraction and expansion under varying climatic conditions. Bamboo has natural oils, imperceptible to the touch, constantly lubricating the bearing surfaces and allowing a smoothness of action not found in any other wood or metal. It operates more easily over the years, due to this self-lubricating characteristic. White plastic faces are used for



easy reading, and all scale graduations and figures are deeply cut into the face to insure a lifetime of accurate calculations.

In your POST VERSALOG II slide rule you have truly one of the finest and most exact instruments this century's ingenuity is able to produce.

Our deepest gratitude is extended to George John Zanotti for his untiring efforts throughout the writing and editing of this text book of instructions. A special thanks is due again to the original designers of the Versalog and many of its unique features to Professor E. I. Fiesenheiser, Professor R. A. Budenholzer, and Associate Professor B. A. Fisher for their specialized chapters on applied engineering.

It is a tribute to the engineering profession, and to the never ending efforts of such men who are devoting their lives educating and training the engineer of the future.

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# PREFACE

This instructional manual is intended to be used as an accompanying text or reference book for the Post VERSALOG II slide rule. Its main purpose is to provide essential instructions in the use of the slide rule as clearly and completely as possible in a straightforward language and with ample examples and illustrations.

It has been written for study without the aid of a teacher. However, a knowledge of basic elementary mathematics is assumed. The student engineer will probably have this knowledge when he acquires the slide rule. Although the manual contains many examples of mathematical problems as well as engineering problems, no effort is made to teach either mathematics or engineering in this book.

The material divides rather naturally into three parts.

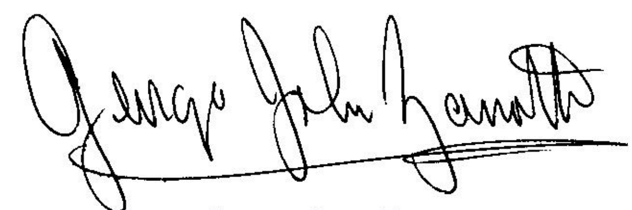
1. Chapter 1 provides the necessary description and scale familiarization. In addition, three decimal locating methods are introduced early in the text providing the background for subsequent subject matter.
2. Chapters 2-5 deal with direct slide rule instruction and manipulation. Every one of the twenty-four scales is amply described, and detailed instruction in their efficient use is fully covered with over one hundred examples. Sections on the extended use of each of these scales has been included in,each chapter. The user will find these techniques extremely useful in future slide rule applications.
3. Chapters 6-10 present the methods of directly applying the slide rule for the solution of practical problems. Specifically, these chapters deal with slide rule methods for the solution of mathematical, business, civil engineering, mechanical engineering, and electrical engineering applications.

Although the book can be utilized in a variety of situations, it will serve best as a continuing reference for the most efficient use of the slide rule. The user should practice to develop mastery of *all* the scales and their most *efficient* uses. Persistent usage and continued investigation of the versatile scale arrangements will yield many dividends over the years.



It is almost impossible to give proper credit to everyone who helped in one way or another to make this book possible. For the technical review of the manuscript and their many helpful suggestions, my appreciation is particularly due to Frank Heurich, Post Slide Rule Consultant and members of the Post marketing staff. Assistance from Prof. Milton D. Eulenberg and Prof. Theodore S. Sunko of Chicago City College, especially in the Applications to Mathematics chapter, is also acknowledged. I am very grateful to Judy Vee, my secretary, for her untiring and patient efforts to transcribe my innumerable difficult notes.

To Terry, my loving and ever-devoted wife, I owe the most, for her continued loving faith and confidence of my efforts to complete this work. Without her none of this would have been possible.



GEORGE JOHN ZANOTTI

CHICAGO, ILLINOIS
*July 1970*

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## ADJUSTMENT AND CARE OF THE VERSALOG II SLIDE RULE

### ADJUSTMENT

Your VERSALOG II slide rule should come to you in perfect adjustment. However, in case it is dropped or severly jarred, the precise adjustment may be lost. In any case, it is advisable to check the adjustments occasionally to make sure that the scale readings are as accurate as the instrument will allow.

In order to check and adjust the slide rule, the following procedure may be followed. With the rule held so that the shorter body member is uppermost, move the slide until the C and D scales coincide perfectly. The DF scale on the upper body should now be in alignment with the identical CF scale of the slide. If it is not, the upper body member must be moved to the right or left. In order to adjust this member, loosen the two screws in the metal end bars about one-half turn and move the upper body member as required, then tighten the screws.

The hairline should now be moved to coincide with the left index (the 1 mark) of the D scale. In this position, the hairline should also coincide with the symbol $\pi$ of the DF scale. If it does not, the hairline is not perpendicular to the slide rule scales. It may be adjusted by loosening the four screws of the metal frame which surrounds the glass. The glass may then be moved until perfect alignment is obtained, after which the screws should be carefully retightened.

The hairline on the reverse side should be in perfect alignment with the left index of the D scale, as well as with the 1/e mark of the LL/3 upper scale. If it is not, this hairline must be moved. This is again done by adjusting the glass and retightening the screws. When properly set, both hairlines should align simultaneously. In making this adjustment, care must be exercised not to disturb the position of the hairline previously adjusted.

If it is difficult to push the slide, the body parts may be gripping it too tightly. To adjust for easy operation, loosen a screw at one end *only* of the adjustable part of the body. This end may then be pulled away from the slide. The screw may then be retightened, and the operation repeated at the other end. By adjusting one end at a time, the alignment of the scales is not affected.

## CARE

It is important to keep the slide rule as clean as possible. Keeping the hands clean and keeping the rule in its case when not in use will help. To clean the scales, a slightly moist cloth may be used. To remove particles from under the glass, a narrow strip of paper may be placed over the scales. The glass may then be run over the paper to pick up the dirt particles.

With proper adjustment and care, your VERSALOG II slide rule will provide a lifetime of accurate service. The property of the bamboo construction is that the operation of the slide becomes easier and smoother with age and usage.

## MANIPULATION

In setting the hairline, the cursor is generally pushed with one hand to the area of the desired setting. It may then be set accurately by placing the thumbs of both hands against either side of the cursor frame, pushing a little more with one thumb than the other to set the hairline.

In setting the slide, it may be moved to the area of the desired setting with one hand. Usually one end of the slide projects beyond the body of the rule. Should the right end project, the right hand is then used to make the exact setting. The thumb and forefinger of the hand grasp the slide and at the same time press against the end of the body of the rule. By this control an exact setting of the slide may be made very quickly, the forefinger and thumb doing the precise work. In case the left end of the slide projects, the left hand is used in the same manner to make the setting.

## POCKET VERSALOG II

The VERSALOG II is available in two sizes: the standard, full size VERSALOG II, and the handy pocket version, a precise miniature of the full size VERSALOG II. Since the scales on the POCKET VERSALOG II are half as long as the scales on the full size VERSALOG II (4.92" rather than 9.84"), space limitations permit only half the number of graduations. The illustrations and references to the scale length throughout the manual are based on the full size VERSALOG II, which should be kept in mind when using the manual. When using the slide rule, however, every operation, simple or complex, is identical whether you are using the full size VERSALOG II or the POCKET VERSALOG II.

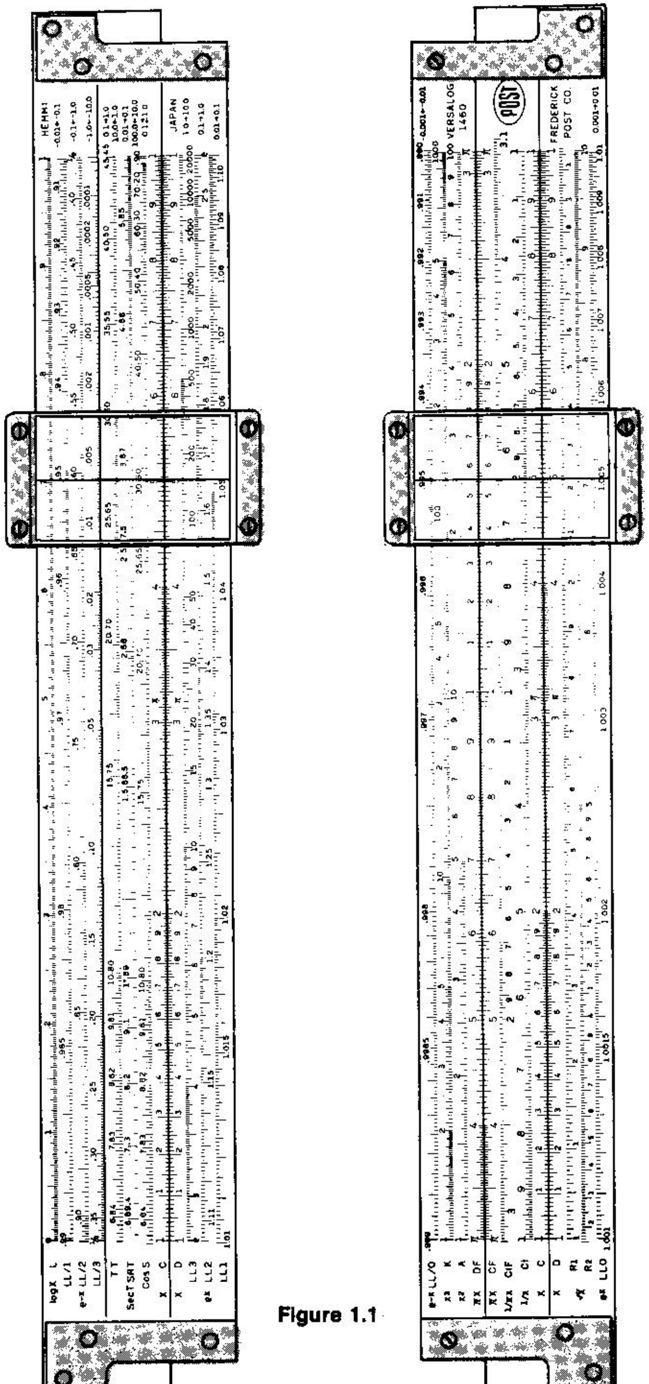


Figure 1.1

# THE VERSALOG II SLIDE RULE



## THE SCALES OF THE SLIDE RULE

### 1.1 GENERAL DESCRIPTION OF THE SLIDE RULE

The slide rule consists essentially of three parts as illustrated in Figure 1.2. The part fixed between the end plates is called the *body*, the long movable portion is called the *slide*, and the glass runner the *indicator*. The fine vertical line on the indicator is called the *hairline*.

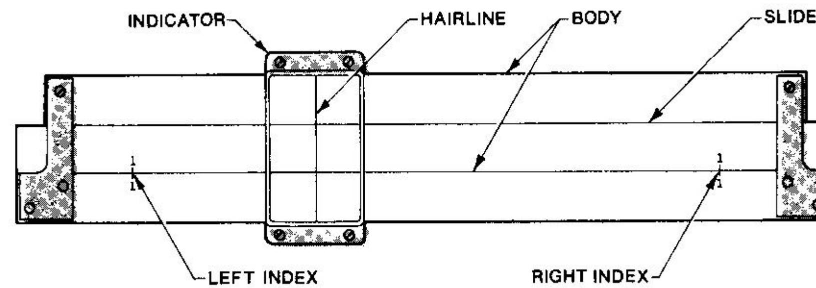



**Figure 1.2—Parts of Slide Rule.**

The Post Versalog II slide rule has 24 scales, located and arranged in a convenient and logical manner; see Figure 1.1. These scales will permit the solution of *any kind* of arithmetical problem except adding and subtracting. Each scale is designated on the rule by a letter or a combination of letters and symbols which appear at the left end of the scale. Only one of these, the L scale, is a *uniformly divided* scale, that is, the spaces between graduations are the same throughout the scale. All the other scales are non-uniform, that is, the distance between consecutive markings does not remain the same on all parts of the scale. The reason for this will become apparent as the construction and use of the various scales is developed in subsequent chapters.

In order to appreciate the scope of computations which are possible with the Versalog II slide rule, it is useful, at the onset, to consider a brief overview of the various scales. Each of the listed scales will, of course, be discussed in appropriate detail in the balance of this book.

## 1.2 DESCRIPTION OF THE SCALES

### C and D Scales

Probably the scales most often used are those marked C and D. For convenience, they appear on both sides of the rule. They are identical in markings and length, the D scale appearing on the body of the rule, and the C scale on the movable slide. The scale length is 25 cm. or 9.84 in. although the instrument is commonly called a 10 inch slide rule. The C and D scales are used for multiplication and division and in conjunction with all of the other scales on the rule. These scales are discussed in Chapter 2 and in subsequent chapters.

### CI Scale

The CI, or C inverted, scale is exactly the same as the C or D scales, except that it is graduated from right to left. Numbers appearing on the CI scale are reciprocals of numbers appearing directly opposite on the C scale. Chapter 2 includes a discussion of the use of the CI scale for rapid and efficient multiplication and division, and for the solution of certain problems in repeated division, continued multiplication, and combined operations.

### CF, DF, and CIF Scales

These are the so-called folded scales. The DF scale located on the body of the rule is of the same construction and length as the D scale, but begins and ends at $\pi$. This places the 1 mark very near the midpoint of the scale. The CF scale is identical to the DF scale, but is located on the slide. The CIF scale is merely an inverted CF scale, that is, the numbers on the CIF scale are reciprocals of those directly opposite on CF. The folded scales, discussed in Chapter 2, are designed to eliminate certain unnecessary moves which are often required when using the C, D, and CI scales alone.

### A, $R_1$, and $R_2$ Scales

These scales are used for finding squares and square roots and for combined operations which include squares and square roots. When the hairline is set on any number on the D scale, its square appears under the hairline on the A scale, and its square root appears under the hairline on either the $R_1$ and $R_2$ scales. The A and R scales are discussed in Chapter 3.

### K Scale

The K scale is used for obtaining cubes and cube roots. The readings



on the K scale represent the cubes of the corresponding readings on the D scale, and, conversely, readings on the D scale represent cube roots of the corresponding readings on the K scale. The K scale is discussed in Chapter 3.

### L Scale

The L scale is used to obtain common logarithms, that is, logarithms to the base 10. When the hairline is set to any number on the D scale, the mantissa of the common logarithm is read under the hairline on the L scale. Applications of the L scale are discussed in Chapter 4.

### LL Scales

The LL0, LL1, LL2, and LL3 scales, called the log log scales, are used to find powers and roots of numbers from 1.001 to 22,000. Fractional and decimal powers are easily handled with these scales. Powers of e (the base of natural logarithms) are also obtained directly on the LL scales by setting the hairline to the power desired on the D scale.

The scales LL/0, LL/1, LL/2, and LL/3 are reciprocal log log scales and are used in the same manner as are the LL scales, but for numbers less than 1. Their range extends from 0.00005 to 0.999 and the numbers and graduations extend from right to left. When the hairline is set to a number on the D scale, the reciprocal of e raised to the power of this number is read directly on a reciprocal log log scale.

The log log scales are one-quarter lengths of a single long scale. Thus the LL1 scale begins where the LL0 scale ends, the LL2 scale begins where the LL1 scale ends, and so on. If these four scales, or their reciprocal scales, could be placed end to end, a single continuous scale one meter in length would result.

An important property of the log log scales is that they represent powers designated as $e^x$ whereas the reciprocal log log scales represent the reciprocals $1/e^x$, which are the same as $e^{-x}$. Hence any number on an LL scale has its reciprocal directly opposite on the corresponding reciprocal log log scale. Many of the other advantages and uses of the log log scales will be explained in Chapter 4.

### Cos S

The Cos S scale is used to obtain sine and cosine functions of angles and is graduated in degrees and decimals of degrees. With the hairline set at the angle on the S scale, the sine of the angle is read on the C scale. For sines, the scale is graduated from left to right from 5.74 degrees to 90 degrees and the numerals are black. To obtain the cosine of an angle, the hairline is set at the angle on the Cos scale and its cosine function is read on the C scale. For cosines, the scale is graduated from right to left from zero to 84.3 degrees, and the numerals are green.

### TT Scale

The T scale is used to find the tangent of angles from 5.71 degrees to 84.3 degrees. For angles in the range of 5.71 degrees to 45 degrees, the scale is graduated from left to right, and the numerals are black. When the hairline is set to an angle in this range, its tangent function is read on the C scale. For angles from 45 degrees to 84.3 degrees, the scale is graduated from right to left, and the numerals are red. When the hairline is set to an angle in this range, its tangent function is read on the CI scale.

### Sec T SRT

An additional scale marked Sec T SRT is provided for determining the tangent function of small angles varying from 0.57 to 5.74 degrees. This scale is graduated from left to right in this range, numbered in black, and is used with the C scale. It may also be used for determining the sine function of small angles since the sine and tangent functions are nearly equal for small angles. For large angles, the scale is graduated from right to left and numbered in red for use with the CI scale. In the range of 84.26 to 89.43 degrees, with the hairline set to the angle on this scale, either tangents or secants are read at the hairline on the CI scale. In this range the tangent and the secant are nearly equal. The R (radian) scale is read from left to right numbered in black and is used with the C scale. Respective equivalent radian values of angles set on the R scale can be obtained directly on the C scale. These scales are discussed in Chapter 5.

## 1.3 READING THE SCALES

The construction and reading of the D scale only will be explained here, since with this information the student will be able to read any of the other scales. It is readily noted that the D scale has 10 primary marks which are numbered with the large numerals



1, 2, 3, 4, 5, 6, 7, 8, 9, 1. The mark corresponding to each large numeral 1 at either end of the scale is called an index of the scale, hence the D scale has a left index and a right index. Figure 1.3 shows the 10 primary marks and the indices.

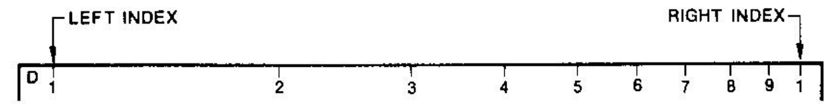



**Figure 1.3—Primary Marks and Indices.**

It is immediately apparent that the distances between successive primary markings are non-uniform, that is, the distance between the primary marks 1 and 2 is greater than that between 2 and 3; the distance between the primary marks 2 and 3 is greater than that between 3 and 4; and so on. This is because distances are proportional to the logarithms of the corresponding numbers.

The scale length, from left to right index, is 25 cm or 9.84 in. The scale equation then becomes $x = 9.84 \log_{10} N$, where x is the distance in inches from the left index to any number N appearing on the scale. For example, the distance from the left index to 2 on the rule is $9.84 \log_{10} 2 = 2.96$ in. It should be noted that the scale begins with the number 1, because in the scale equation $x = 9.84 \log_{10} N$, it is clear that $x = 0$ when $N = 1$ ($\log_{10} 1 = 0$). Figure 1.4 illustrates the relation between the primary markings and the logarithmic related distances from the left index.

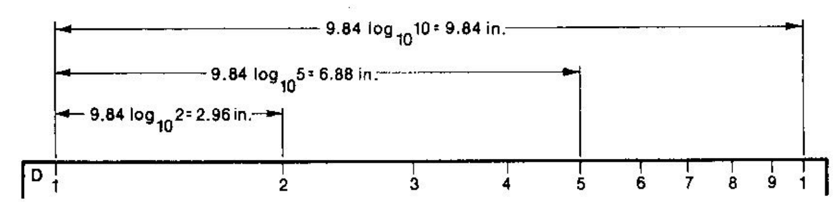



**Figure 1.4—Logarithmic Related Distances.**

Figure 1.5 shows several examples of numbers on the D scale. In reviewing these, it should be noted that the secondary divisions become more closely spaced when moving from left to right along the scale. Proportionally, the accuracy of the reading diminishes.

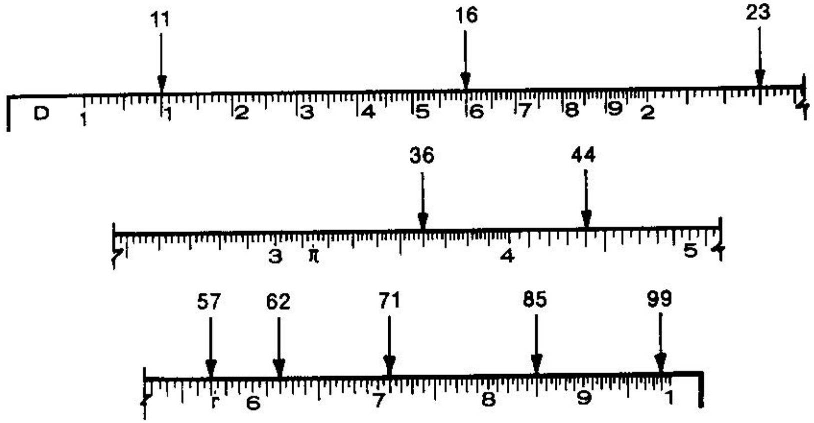



Figure 1.5 – Numbers on the D Scale.

Between the left index and the large numeral 2, there are ten secondary divisions which are numbered with the small numerals 1, 2, 3, 4, 5, 6, 7, 8, 9. Their positions are determined by distances proportional to the logarithms of the respective numbers 1.1, 1.2, 1.3, . . . 1.8, 1.9, 2.0. These secondary divisions are further subdivided into ten parts. Each of these smallest intervals may be taken to represent one unit. These divisions, as well as several representative readings, are illustrated in Figure 1.6.

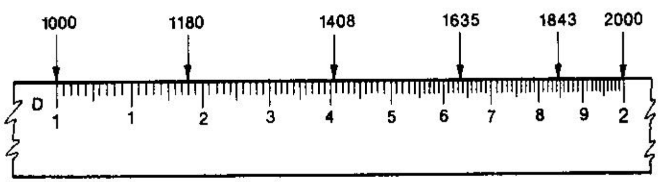



Figure 1.6 – Divisions Between 1 and 2.

It is important to recognize, at this point, that the setting corresponding to a particular number on the D scale is not affected by the position of the decimal point which is another advantage of the logarithmic basis of the construction of the scale. Therefore the readings shown in Figure 1.6 are merely illustrative; the setting which corresponds to the number 1635 could also represent the number 0.01635, 0.1635, 1.635, 16.35, and so on.



In the range between the primary marks 2 and 4, there are again ten unnumbered secondary divisions between the successive primary marks of 2 and 3 and also 3 and 4. Each of these in turn is subdivided into five parts. These final intervals, therefore, may be taken to represent *two* units, as illustrated in Figure 1.7.


Figure 1.7 – Divisions Between 2 and 4.

In the final section of the scale, between the primary mark 4 and the right index, there are also ten unnumbered secondary divisions between the successive marks of 4 and 5, 5 and 6, etc. Each of these is further subdivided into only two parts. Therefore each of the smallest intervals in this range represents *five* units. Examples of readings within this portion of the rule are illustrated in Figure 1.8.

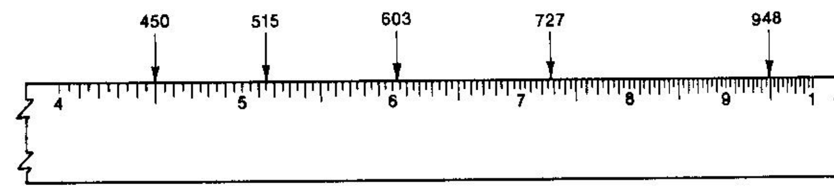



Figure 1.8 – Divisions Between 4 and 10.

In using a 10 inch slide rule, the fourth digit is ordinarily estimated for readings between the left index and the primary mark 2, that is, those numbers whose first significant digit is 1. All other settings are normally limited to three digits.

An excellent procedure for checking ability to read the D scale is to practice the pairing of readings on the D and L scales. Since the L scale is a uniform scale, it can be read directly in much the same way one reads a ruler or a thermometer. The following examples illustrate the procedure.

Example 1.1 Find the D scale reading which corresponds to a setting of 0.366 on the L scale.

Operation The L scale is divided into ten equal primary divisions marked 0, .1, .2, . . . .9, 1. It is easily verified, therefore, that each secondary division represents 0.01, and each final or tertiary division represents 0.002.

A reading of 0.366 on the L scale is found by first moving the hairline to the primary mark .3, continuing beyond it to the sixth secondary division (which represents .36) and continuing finally to the third tertiary division. The corresponding reading on the D scale, as illustrated at the hairline setting in Figure 1.9 is 232.

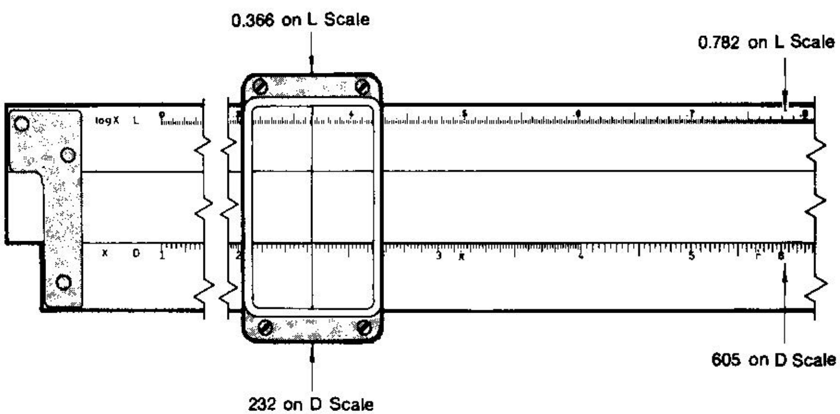



**Figure 1.9—Corresponding D and L Scale Readings.**

Example 1.2 What reading on the L scale corresponds to a setting of 605 on the D scale?

Operation The D scale setting of 605 is found by moving the hairline to mark immediately following the primary division 6, since each of the smallest intervals in this range represents five units. The corresponding reading on the L scale is 0.782. See Figure 1.9.



**Exercise 1.1**
**Reading the D Scale**

Complete the following tables by listing the readings which correspond to the given settings.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | L | 0.233 | 0.480 | 0.549 | 0.850 | 0.658 | 0.911 | 0.300 | 0.088 |
| | D | | | | | | | | |
| 2. | L | | | | | | | | |
| | D | 542 | 984 | 873 | 955 | 673 | 798 | 404 | 1865 |
| 3. | L | 0.617 | | 0.935 | | 0.943 | | 0.219 | |
| | D | | 520 | | 618 | | 984 | | 333 |
| 4. | L | | 0.936 | | 0.090 | | 0.686 | | 0.668 |
| | D | 798 | | 975 | | 274 | | 1905 | |
| 5. | L | 0.305 | 0.008 | 0.083 | 0.100 | | | | |
| | D | | | | | 947 | 506 | 292 | 1175 |

## 1.4 ACCURACY

The graduations of the VERSALOG II are highly accurate, but the accuracy is limited to the ability of the user to see, set, and read the desired numbers. Settings can be made as accurate as 4 significant digits for numbers having one as the first digit on the D scale. For other numbers, the scales can only be read to an accuracy of three significant digits. Since the entire scale must be used, the accuracy as a whole is limited to three digits, or 99.9%. Such accuracy is all that is required for ordinary design calculations since seldom are all the elements of a computation accurate to the degree that no one contains an error of at least one part in one thousand parts.

## 1.5 EFFECTS OF ERRORS IN READING THE SCALE

In case the hairline is set incorrectly or the reading is made incorrectly, the effect may be evaluated by use of the scale equation previously stated as $x = 9.84 \log_{10} N$, in which x is the distance in inches from the left end of the scale to any number N appearing on the scale. Taking the derivative of both sides with respect to N, the following equation results: $\frac{dN}{N} = 2.3026\left(\frac{dx}{9.84}\right)$. The term $\frac{dN}{N}$ is the relative error in the number N, while $\frac{dx}{9.84}$ is the relative error in reading or setting the hairline. Therefore the relative error in the number is independent of the size of the number or its location on the scale and is 2.3026 times the relative error in reading the scale.

### 1.6 DECIMAL POINT LOCATION

Thus far, the topic of the location of the decimal point has been just briefly mentioned. As has been noted before, since the slide rule has no provision for "carrying along" the decimal point in a given problem, some method must be adopted. Three methods are suggested. In all probability, all will be used at one time or another, depending on the complexity of the particular problem. Therefore, all three methods should be thoroughly understood.

#### THE INSPECTION AND COMMON SENSE METHOD

For many problems, the combination of factors is simple enough that by inspection the location of the decimal point is obvious.

Example 1.3 $\frac{80.5}{35.0} = 230$ (slide rule reading)

Operation By merely inspecting the numbers involved shows the answer to slightly above "2," so the answer is 2.30.

Even for other problems, which may be comprised of many factors and be more complicated mathematically, the result may also be reasonably interpreted in this way.

Example 1.4 Find the average speed of an automobile in miles per hour that has traveled 2050 miles in 39 hours.

Operation $\frac{2050}{39} = 525$ (slide rule reading)

Common sense tells us that the answer is 52.5 MPH and not 5.25 MPH or 525 MPH.



#### THE APPROXIMATION METHOD

This method covers a greater range of problems and in all probability is the most common method in use. Essentially it requires an estimate using rounded numbers.

Example 1.5 $137.2 \times 41.2 = 565$ (slide rule reading)

Operation We would round the numbers to read $100 \times 40$ resulting in an answer equal to 4000 approximating our original problem's answer. Our answer can be correctly interpreted as 5,650.

For more involved calculations, it may be more convenient to jot down the rounded numbers and cancel.

Example 1.6 $\frac{96 \times 55 \times 63 \times 57}{46 \times 2.7 \times 10{,}320 \times 688} = 215$ (slide rule reading)

Operation Rewrite the original problems using rounded numbers.

$$\frac{1\cancel{0}\cancel{0} \times \overset{2}{\cancel{6}}\cancel{0} \times 6\cancel{0} \times 6\cancel{0}}{50 \times \cancel{3} \times 10{,}\cancel{0}\cancel{0}\cancel{0} \times 7\cancel{0}\cancel{0}} = \frac{72}{3500} = \frac{7.2}{350}$$

Since 7.2 is divided by a number greater than 100 but less than 1,000, the result is a number less than 0.072 but greater than 0.0072. Using our slide rule reading of 215, the resulting answer must be 0.0215.

#### THE SCIENTIFIC NOTATION METHOD

This is the most exact method and is recommended for problems too complicated to be easily handled through either of the preceding methods. It is particularly helpful when dealing with numbers of very large or very small magnitude.

Placing a number in its scientific notation form entails placing the decimal point after the first non-zero digit of the number and indicating the true location of the decimal point by multiplying it by the appropriate power-of-ten. The magnitude of the appropriate power-of-ten is determined by the number of digits that the decimal point was moved to place it after the first non-zero digit of the number. If the number is larger than 10, the decimal point is moved to the *left*, and the power-of-ten is positive. If the number is larger than 1, but less than 10, the decimal point is not relocated, and the power-of-ten is therefore zero, ($10^0 = 1$). If the number is smaller than 1, the decimal point is moved to the *right*, and the power-of-ten is *negative*. The following examples are presented for further clarification.

Example 1.7

| Number | | Scientific Notation |
|---|---|---|
| 735 | = | $7.35 \times 10^2$ |
| 4,360,000 | = | $4.36 \times 10^6$ |
| 0.0001354 | = | $1.354 \times 10^{-4}$ |
| 0.0862 | = | $8.62 \times 10^{-2}$ |
| 7.1 | = | $7.1 \times 10^0$ |

When multiplying these powers of ten, the exponents are added algebraically; when dividing, they are subtracted algebraically. Combining the scientific notation and approximation methods, the location of the decimal point in more complicated problems should present no difficulty.

To locate the decimal point when the decimal location is not obvious, use scientific notation. When the computation is reduced to the scientific form, the decimal point placement becomes obvious. The slide rule reading will always yield a number larger than one (*ie.* 0 to 1) or smaller than 1 (*ie.* 0 to .1) which is multiplied by the appropriate power of ten.

Example 1.8 $\dfrac{495000}{384} = 1289$ (slide rule reading)

Operation Rewrite the original problem using the scientific notation method.

$$\frac{4.95 \times 10^5}{3.84 \times 10^2} \approx \frac{(5) \times 10^5}{(4) \times 10^2}$$

$$\approx 1.25 \times (10^5 \times 10^{-2})$$

$$\approx 1.25 \times 10^3$$

$$\approx 1250$$

Our answer must then be 1289.

When applying the scientific notation method to a division problem of two numbers as in example 1.8, there are just two possibilities for your estimate.

If numerator > denominator, your quotient > 1; in fact, $1.0 < Q < 10$.

If numerator < denominator, your quotient < 1; in fact, $0.1 < Q < 1.0$.



This is so, because you have converted your original problem numbers to $1.0 < N < 10$.

Example 1.9 $\dfrac{26 \times 79{,}800 \times 0.00633}{0.0081 \times 7{,}800{,}000} = 208$ (slide rule reading).

Operation Rewrite the original problem using the scientific notation method.

$$\frac{(2.6 \times 10^1)(7.98 \times 10^4)(6.33 \times 10^{-3})}{(8.1 \times 10^{-3})(7.8 \times 10^6)} \approx \frac{(3)(\cancel{8})(6) \times 10^2}{(8)(\cancel{8}) \times 10^3}$$

$$\approx \frac{18}{8} \times (10^2 \times 10^{-3})$$

$$\approx 2.25 \times 10^{-1}$$

$$\approx 0.225$$

Our answer must then be 0.208.

## MULTIPLICATION AND DIVISION

Versalog II provides a selection of scales for optimum speed in any multiplication or division operation or series of computations. A familiarity of the alternates available can save time and steps in simple everyday computations, and a thorough understanding of the proper use of the scales is essential for efficient handling of sequences or series of computations. The most efficient technique of slide rule operation is emphasized.

Multiplication and division are performed on the slide rule by the simple process of adding or subtracting logarithms. The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers;

$$\log a\,b = \log a + \log b.$$

The logarithms of the quotient of two numbers is equal to the difference of their logarithms;

$$\log \frac{a}{b} = \log a - \log b.$$

Since the scales used are logarithmic scales, with markings corresponding to their antilogarithms, products and quotients are obtained by merely mechanically adding or subtracting logarithmic lengths.

### 2.1 MULTIPLICATION USING THE LOWER SCALE COMBINATION

The D and CI Scale combination is, for the majority of cases, the most efficient. The greater efficiency of the D and CI scales for multiplication is mainly due to the fact that the product always lies within the body of the rule, making it unnecessary to determine in advance the proper index to use. This is because the numbers on the CI scale, being the reciprocals of the corresponding numbers on the C scale, permit the conversion of the product $a \times b = c$ to the quotient $a \div \frac{1}{b} = c$. When using the CI scale, remember that it is an inverted scale and the values increase toward the left.

This multiplication should start by setting the hairline to the number (a) on the D scale (either D or the DF scale). The slide should then be moved until the other number (b) on the corresponding CI scale (the CI scale if the D scale is used, or the CIF scale if the DF scale is used) coincides with the hairline. The product (c) is then read on the D scale directly adjacent to the index of the C scale.

The D scale is on the lower part of the body and the CI scale is on the lower part of the slide. Hence the D and CI scale combination is called a *lower scale combination.* (To avoid confusion only the scales being used are shown in Figure 2.1 and in the figures which follow.)

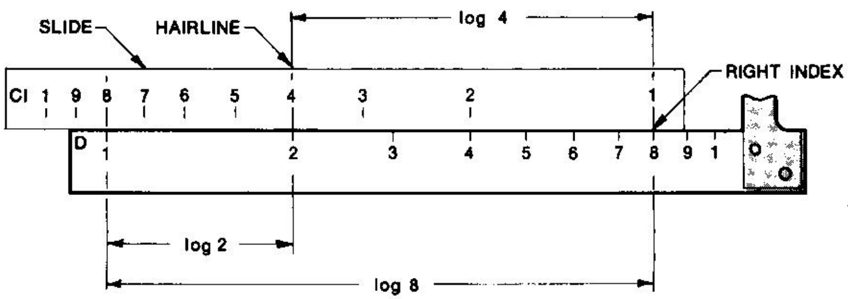



**Figure 2.1 — Use of D and CI Scale Combination for Efficient Multiplication of 2 by 4.**

In figure 2.1, the D and the CI scales are used to multiply 2 by 4. The hairline is set to the number 2 on the D scale. The slide is then moved until the number 4 on the CI scale coincides with the hairline. The answer, 8, is read on the D scale opposite the 1 mark of the CI scale, which is the right index of the scale.

The addition of log 2 and log 4 to obtain log 8 is shown in Figure 2.1. The distances along the scales are proportional to the logarithms of the number. This is the reason why adding these distances automatically adds the logarithms.

The multiplication of 2 by 4 in Figure 2.1 required a slide movement to the *left* so that the answer was read at the *right* index of the slide. The multiplication of two numbers such as 2 and 8 requires a



movement of the slide to the *right* so that the answer is read at the *left* index of the slide, as in Figure 2.2. The hairline is set on 2 of the D scale and the number 8 on the CI scale is moved to the hairline. Here it is noted that the scale length log 2 − (log 10 − log 8) = $\log \frac{2(8)}{10}$ = log 1.6, whereas we know that the product of 2 and 8 is 16. Hence the result 1.6 is correct except for the decimal point, which is easily found by methods already described in Chapter 1. Thus the product 16 is read on the D scale at the *left* index of the slide.

It is emphasized that either the left or the right index, whichever is in contact with the D scale, is used to read the result on the D scale.

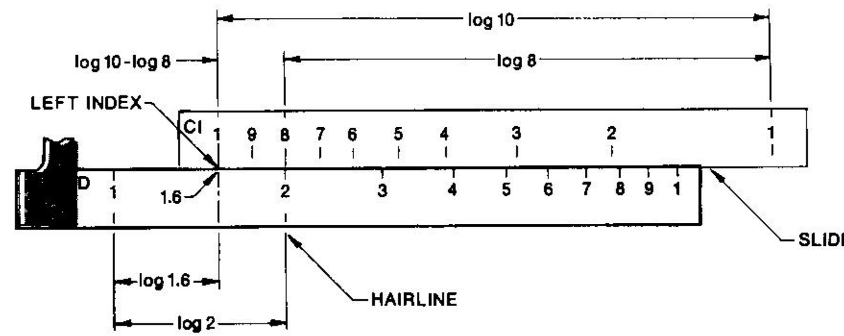



**Figure 2.2 — D and CI Scale Combination for Multiplying 2 by 8.**

**Exercise 2.1**
**Multiplication Using the D and CI Scales**

| | | |
|---|---|---|
| 1. 24.2 × 46.5 | 6. 8.62 × 5.88 | 11. 4.65 × 0.355 |
| 2. 0.765 × 0.136 | 7. 3.06 × 6.92 | 12. 0.00000484 × 57 |
| 3. 1.08 × 13.6 | 8. 0.00291 × 53 | 13. 62.4 × 13.5 |
| 4. 3.14 × 24.6 | 9. 68.3 × 0.047 | 14. 3.14 × 34.4 |
| 5. 5510 × 0.065 | 10. 2.22 × 360 | 15. 875 × 15.45 |

## 2.2 MULTIPLICATION USING THE D AND C SCALES

Although the D and C scales are the most fundamental scales of the slide rule and are used extensively for multiplication and division, they are less efficient than the D and CI scales when applied to multiplication.

The multiplication of 2 by 4 might have been performed *less efficiently* by using the D and C scales as shown in Figure 2.3.

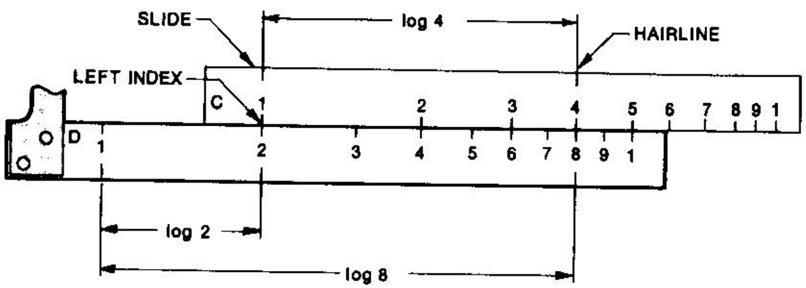



**Figure 2.3 – D and C Scale Combination Used Inefficiently to Multiply 2 by 4.**

By this method, the hairline is set on 2 of the D scale, the slide is moved until the 1 mark or left index of the C scale coincides with the hairline; then the hairline is moved to 4 on the C scale, and the result, 8, is read at the hairline on the D scale. This method of multiplication requires two movements of the hairline instead of one and generally requires greater movement of the slide. It is not recommended for multiplication of two numbers. However, this method should be kept in mind for it greatly simplifies operations such as multiplying or dividing a series of numbers, multiplying a series of numbers by a single factor, solving proportions, etc., as explained later in this chapter.

**Exercise 2.2**
**Multiplication on the D and C Scales**

| | | |
|---|---|---|
| 1. $2.46 \times 3.52$ | 6. $0.957 \times 208$ | 11. $0.01757 \times 254$ |
| 2. $1.27 \times 0.224$ | 7. $0.724 \times 195$ | 12. $777.5 \times 46.2$ |
| 3. $1.49 \times 1.32$ | 8. $0.0086 \times 22.4$ | 13. $1728 \times 24$ |
| 4. $2.25 \times 2720$ | 9. $10.5 \times 0.542$ | 14. $33{,}000 \times 24.6$ |
| 5. $7.12 \times 9.60$ | 10. $0.162 \times 0.075$ | 15. $45 \times 1.467$ |

## 2.3 DIVISION USING LOWER SCALE COMBINATIONS

In dividing two numbers, a D and C scale combination should be used for the most efficient operation. Dividing involves subtracting



logarithms. The example of Figure 2.4 indicates the division of 9 by 6. In this case log 9 − log 6 = log 1.5. The slide has been moved to the *right*. Therefore the quotient 1.5 is read on the D scale at the *left* index of the C scale.

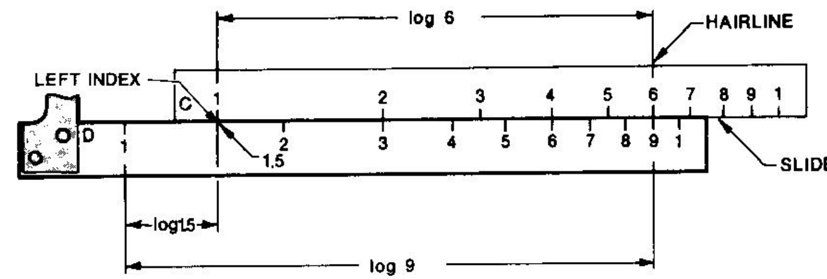



**Figure 2.4 – D and C Scale Combination for Dividing 9 by 6.**

Another example is shown in Figure 2.5, in which 1.8 is divided by 2.5. The hairline is set to 1.8 on the D scale. Then 2.5 on the C scale is moved to the hairline. The result, 0.72, is read on the D scale at the *right* index of C since the slide was moved to the *left*.

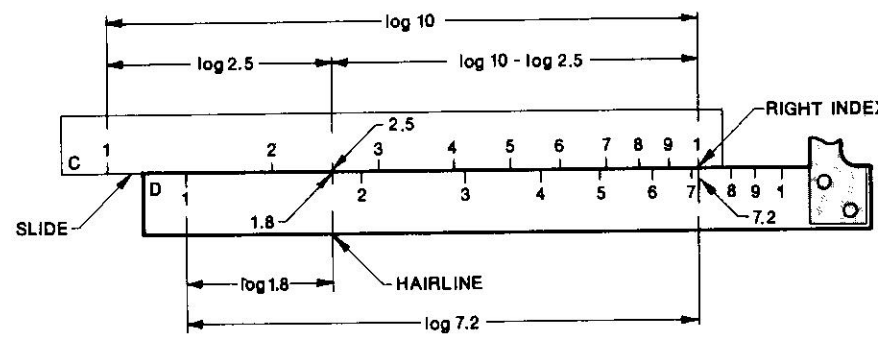



**Figure 2.5 – D and C Scale Combination for Dividing 1.8 by 2.5.**

From the figure the scale distances are as follows: $\log 1.8 + \log 10 - \log 2.5 = \log \dfrac{1.8(10)}{2.5} = \log 7.2$. The number 7.2 is correct, except for the location of the decimal point. This is located by mental calculation, since dividing 1.8 by 2.5 obviously results in a number less than 1.

**Exercise 2.3**

**Division Using the D and C Scales**

1. 9.30 ÷ 3.08
2. 7.48 ÷ 2.63
3. 6.30 ÷ 0.27
4. 1950 ÷ 435
5. 8.55 ÷ 10.5
6. 6.30 ÷ 14.2
7. 1950 ÷ 94.5
8. 9.30 ÷ 6.50
9. 7.48 ÷ 3.54
10. 450 ÷ 57.2
11. 54.2 ÷ 2.24
12. 26.8 ÷ 12.3
13. 0.832 ÷ 1.45
14. 16.65 ÷ 0.0363
15. 0.0346 ÷ 0.000291

## 2.4 DIVISION OF THE SPECIAL FORM 1/N

In problems where the dividend is 1, that is, problems such as 1/3, 1/4.5, and so forth, where we are finding reciprocals, the division is most efficiently done using the CI and C scales. As has been stated in Chapter 1, numbers on the CI scale are the reciprocals of numbers directly opposite them on the C scale, and vice versa. We may then state that for any quotient 1/N, opposite N on the C scale, read 1/N on the CI scale. We can also, opposite N on the CI scale, read 1/N on the C scale.

For example, the reciprocal of 0.72 can be found to be 1.39. Set the hairline to 72 on the C scale and read 139 on the CI scale.

**Exercise 2.4**

**Division Using CI and C Scales**

1. 1 ÷ 31
2. 1 ÷ 3.01
3. 1 ÷ 310
4. 1 ÷ 27
5. 1 ÷ 2.7
6. 1 ÷ 0.27
7. 1 ÷ 355
8. 1 ÷ 0.005
9. 1 ÷ 0.326
10. 1 ÷ 0.85
11. 1 ÷ 24.6
12. 1 ÷ 1.21

## 2.5 MULTIPLICATION OR DIVISION USING THE UPPER SCALE COMBINATIONS

The upper group of scales are designated DF, CF and CIF. These are also referred to as the folded scales, and are so indicated by the letter F because they begin and end at values other than 1. They may also be used for multiplying and dividing, since they are identical to the D, C and CI scales, except that the index of each is very near the middle of each scale.



Figure 2.6 shows the use of the upper scales in multiplying. Here 1.1 is multiplied by 1.2 by setting the hairline to 1.1 on DF and moving 1.2 on CIF to the hairline. The product 1.32 is read on DF at the 1 mark or index of the CIF scale.

Let it be assumed that the slide was centered, with all indices in line, before beginning the calculation. From Figure 2.6 the total movement of the slide from its centered position was proportional to log 1.1 + log 1.2 = log 1.32. The left index of the C scale has moved exactly the same distance, so that the answer might also be read on the D scale, at the left index of C.

Either the DF or D scale may be used, but it is often faster and more convenient to use the D scale. There are graduations on only one size of the indices of the C and CI scales and the use of the hairline is unnecessary. The lower C and CI scales have two indices, and one will always be adjacent to the D scale; while the upper CF and CIF scales each have only one index. Notice that whenever the slide is moved the lower index reading on D is the same as the upper index reading on DF.

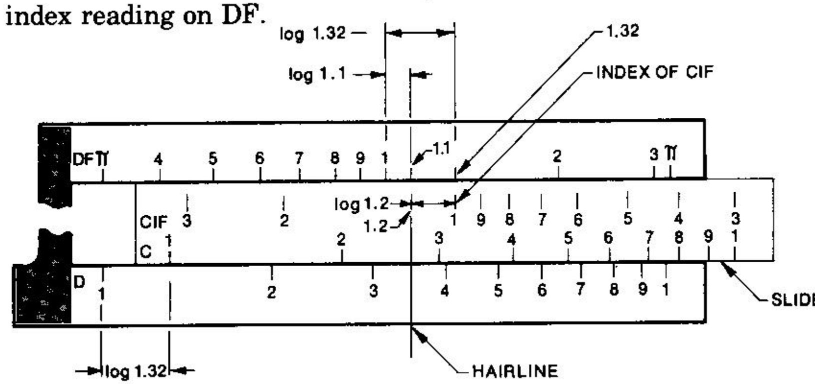



**Figure 2.6—DF and CIF Scale Combination for Multiplying 1.1 by 1.2.**

For multiplying 1.1 by 1.2 there is a definite advantage in using the upper scales. The total movement of the slide from its centered position in Figure 2.6 was 1.18 in. Had the D and CI scale combination been used, a slide movement of 8.66 in. would have been required. The advantage of the upper group of scales for certain operations is therefore obvious.

Division may be performed on the upper group of scales as follows: to divide 8 by 2 efficiently, set hairline to 8 on DF and move 2 on

CF to hairline, read 4 on D at right index of C. This requires a slide movement of about 3.9 in. If the same operation were performed by using the C and D scale combination, a slide movement of about 5.9 in. would be required.

### 2.6 CHOICE OF UPPER OR LOWER SCALE COMBINATIONS

For some operations the choice of scale combinations makes no difference in efficiency, and so either the upper or lower group may be chosen. For other operations a lower scale combination is more advantageous, while for still other operations an upper scale combination is best. To determine which combination is best to use, a general rule is desirable. Such a rule may be stated definitely as follows: *Either lower or upper scale combinations may be used for multiplication and division, but whenever one scale combination requires moving the slide more than one-half its length, use the other.* By following this rule the maximum slide movement required is one-half of 9.84 in. or about 5 in.

The student should perform the following exercises in multiplication and division in order to familiarize himself with the six scales and their most efficient uses. No attempt should be made to read results more accurately than the instrument allows. In this connection it will be remembered that accuracy is limited to four significant figures for numbers whose first digit is 1 but to only three significant figures for numbers beginning with the digits 2 to 9. The D and CI or the DF and CIF scale combinations should always be used when multiplying, whereas the D and C or the DF and CF scale combinations should always be used when dividing.

#### Exercises 2.5, 2.6
#### Multiplication or Division Using Upper or Lower Scale Combinations

Multiplication:
Perform the operation and indicate the most advantageous scale combination.

| | | |
|---|---|---|
| 1. 2.4 × 3.02 | 7. 2.13 × 12.11 | 13. 4.15 × 26.2 |
| 2. 1.52 × 2.95 | 8. 1.49 × 1.32 | 14. 29.2 × 7.68 |
| 3. 6.12 × 3.4 | 9. 9.12 × 8.25 | 15. 20.8 × 95.7 |
| 4. 1.57 × 2.2 | 10. 7.1 × 9.6 | 16. 42.5 × 14.24 |
| 5. 3.24 × 7.22 | 11. 5.13 × 9.08 | 17. 2.25 × 3720 |
| 6. 9.18 × 3.32 | 12. 3.3 × 9.8 | 18. 392 × 10.33 |



Division:
Perform the operation and indicate the most advantageous scale combination.

| | | |
|---|---|---|
| 19. 9.3 ÷ 3.08 | 25. 9.3 ÷ 2.18 | 31. 9.3 ÷ 6.5 |
| 20. 8.55 ÷ 2.96 | 26. 8.55 ÷ 10.5 | 32. 8.55 ÷ 5.12 |
| 21. 7.48 ÷ 2.63 | 27. 7.48 ÷ 115 | 33. 7.48 ÷ 3.54 |
| 22. 6.3 ÷ 0.27 | 28. 6.3 ÷ 14.2 | 34. 6.3 ÷ 7.5 |
| 23. 450 ÷ 19.2 | 29. 450 ÷ 10.4 | 35. 450 ÷ 57.2 |
| 24. 1950 ÷ 435 | 30. 1950 ÷ 94.5 | 36. 1950 ÷ 10.6 |

### 2.7 MULTIPLYING A SERIES OF NUMBERS

A great advantage in slide rule calculation is that any number of factors may be multiplied together in one continuous operation to obtain their product. In terms of logarithms, the addition of the logarithms of a series of numbers is equal to the logarithm of the product of the numbers. If more than two factors are to be multiplied together, the logarithms of the first two are added automatically on the slide rule scales and to this sum the logarithm of the next factor is added by the next setting, to this sum is added the logarithm of the next factor, and so on, for any number of factors. Therefore it is *not* necessary, as in long hand multiplication, to multiply the factors two by two and then to multiply these separate products. A simple example will serve to illustrate the procedure.

Example 2.1 $1.41 \times 7.25 \times 2.02 \times 8.1 = 167.3$

Operation
Set the hairline to 1.41 on D
Move 7.25 on CI to the hairline
Move hairline to 2.02 on C
Move 8.1 on CI to the hairline
Read 167.3 on the D scale at left index of C.

Only the final result needs to be set down on paper. About thirty seconds are required to do the entire operation. By either long hand or electric calculator multiplication, we would first multiply 1.41 by 7.25 to obtain 10.2225; then we would multiply 2.02 by 8.1 to obtain 16.362; then 10.2225 would be multiplied by 16.362 to obtain 167.2605450. The numbers 10.2225 and 16.362 would need to be set down on paper or transferred to another dial, even if a calculator were used. Of course the final result 167.2605450 is accurate to ten significant figures. However, in ordinary design calculations such accuracy is unnecessary and time is therefore wasted in doing unnecessary work. Our slide rule result 167.3 is accurate to four significant figures, a degree of accuracy usually sufficient.

**Exercise 2.7**
**Multiplying a Series of Numbers**

1. $3 \times 2 \times 1.5$
2. $2.1 \times 3.2 \times 1.3$
3. $7.6 \times 53 \times 0.08$
4. $0.096 \times 8.15 \times 18.4$
5. $6.48 \times 0.0266 \times 0.039$
6. $4.3 \times 16.4 \times 19.7 \times 1.35$
7. $3.4 \times 1.6 \times 0.062 \times 74$
8. $72.2 \times 0.108 \times 1.46$
9. $10.9 \times 0.103 \times 6.15 \times 92$
10. $3.45 \times 54.7 \times 106.8$

### 2.8 DIVIDING BY A SERIES OF NUMBERS

Division by a series is just as easy as multiplication by a series. One computation follows another as in multiplication, but instead of multiplying, the scale combinations resulting in division are used. When performing a series of division computations, it is helpful to use the D and CI scale combination for division; that is, for subtracting logarithms. Here is an example.

Example 2.2 $\dfrac{1}{4.7 \times 5.24 \times 10.12} = 0.00401$

Operation
Set 4.7 on C to right index of D
Move hairline to 5.24 on CI
Move 10.12 on C to hairline
Read 0.00401 on D at index of C

Once again, only the final result will be recorded. However, the intermediate steps of the computation are available if they are required.

Due to the presence of two methods of division (D and C or D and CI scales) and two sets of scales (lower combination or upper combination), we may choose the most convenient method and scales to perform either multiplication or division when a series of factors is involved. It will be well to keep in mind that in multiplying, one must add logarithms and in dividing, one must subtract logarithms. By noting, always, the direction of the numbering and graduations of the scales one plans to use, errors will be avoided.



**Exercise 2.8**
**Dividing by a Series of Numbers**

1. $\dfrac{1}{3.25 \times 4.28 \times 6.13}$
2. $\dfrac{1}{1.04 \times 1.71 \times 9.25}$
3. $\dfrac{1}{0.02 \times 0.164 \times 3.6}$
4. $\dfrac{1}{0.098 \times 6.82 \times 1.103}$
5. $\dfrac{1}{0.0021 \times 7.53 \times 0.923}$
6. $\dfrac{1}{36.5 \times 0.0095 \times 1.3 \times 12.43}$

### 2.9 COMBINED MULTIPLICATION AND DIVISION SERIES

A great advantage of the slide rule is that a number of calculations can be performed in one continuous operation. It is not necessary to record the answer to intermediate steps of a compound problem.

The process of combining multiplication and division in a series of computations is as simple as combining the operations just illustrated. In the example that follows, two sequences of operations are described.

Example 2.3 $\dfrac{2.5 \times 5.85 \times 16.4}{4.35 \times 13.9 \times 3.36} = 1.18$

Operation (A)
Set hairline to 2.5 on D
Move 5.85 on CI to hairline
Move hairline to 16.4 on C
Move 4.35 on C to hairline
Move hairline to 13.9 on CI
Move 3.36 on C to hairline
Read 1.18 on D at left index of C.

Operation (B)
Set hairline to 2.5 on D
Move 4.35 on C to hairline
Move hairline to 5.85 on C
Move 13.9 on C to hairline
Move hairline to 16.4 on C
Move 3.36 on C to hairline
Read 1.18 on D at left index of C.

While there is no difference in the efficiency of the two sequences of operations, the first sequence, Operation A, is recommended. It is believed that fewer errors result by first using all factors in the numerator, then next using all factors in the denominator. In this way, one first concentrates on continuous multiplication, then on continuous division without alternating from one process to the other. Therefore, the first method shown for solving the last example is followed in the next examples.

Example 2.4 $\dfrac{120 \times 8.25 \times 19.1 \times 9.6}{40.5 \times 3.24 \times 50.4 \times 25} = 1.098$

Operation
Set hairline to 120 on D
Move 8.25 on CI to hairline
Move hairline to 19.1 on C
Move 9.6 on CI to hairline
Move hairline to 40.5 on CI
Move 3.24 on C to hairline
Move hairline to 50.4 on CI
Move 25 on C to hairline
Read 1.098 on D at left index of C.

Example 2.5 $\dfrac{30.6 \times 41.2 \times 5.41}{(40.8)^2 \times 7.3} = 0.561$

Operation
Set hairline to 30.6 on D
Move 41.2 on CI to hairline
Move hairline to 5.41 on C
Move 40.8 on C to hairline
Move hairline to 40.8 on CI
Move 7.3 on C to hairline
Read 0.561 on D at right index of C.

Example 2.6 $\dfrac{100 \times (60.5)^3}{48 \times 3(10)^4 \times 655} = 0.0235$

Operation
Set hairline to 60.5 on DF
Move 60.5 on CIF to hairline
Move hairline to 60.5 on C
Move 48 on C to hairline
Move hairline to 3 on CI
Move 655 on C to hairline
Read 0.0235 on D at right of index C.



**Exercise 2.9**

**Combined Multiplication and Division Series**

1. $\dfrac{916 \times 0.752}{5.6}$
2. $\dfrac{42.6 \times 1.935}{750.3}$
3. $\dfrac{56.7 \times 0.00336}{15.06 \times 8.23}$
4. $\dfrac{0.916}{90.5 \times 13.06}$
5. $\dfrac{755 \times 1.15}{51.4 \times 0.093}$
6. $\dfrac{1573 \times 4618}{3935 \times 97}$
7. $\dfrac{258 \times 4.8}{64.2}$
8. $\dfrac{58.7 \times 0.0125}{9.65}$
9. $\dfrac{685 \times 2.36}{195 \times 0.0625}$
10. $\dfrac{8.15 \times 94.5}{24.8 \times 7.25}$
11. $\dfrac{0.324 \times 48{,}500 \times 5.95}{88.5 \times 1920}$
12. $\dfrac{12{,}800 \times 0.0425 \times 785}{36.8 \times 495}$
13. $\dfrac{75.4 \times 6.35 \times 3.78}{5.85 \times 8.45 \times 268}$
14. $\dfrac{368 \times 0.0525 \times 975}{23.5 \times 3.65 \times 8.36}$
15. $\dfrac{372 \times 0.000234 \times 8.72}{145.3 \times 96.6 \times 0.00247}$
16. $\dfrac{37.2 \times 0.08 \times 192.3}{85 \times 63 \times 8.63}$

### 2.10 MULTIPLICATION OF A SINGLE FACTOR BY A SERIES OF NUMBERS

In engineering calculations it is frequently necessary to obtain the products of several different numbers each multiplied by the same single factor. In this type of problem the best procedure is to set the index of the C scale to the single factor on the D scale and to use the D and C or the DF and CF scale combination for multiplying. By this method only the hairline needs to be moved to perform the successive multiplications.

Example 2.7 Multiply 1.27 by each of the following numbers: 3.16, 4.28, 6.55, 8.4 and 9.85

Operation
Set the left index of C to 1.27 on D
Move hairline to 3.16 on C, reading 4.01 on D
Move hairline to 4.28 on C, reading 5.44 on D
Move hairline to 6.55 on C, reading 8.32 on D
Move hairline to 8.4 on CF, reading 10.67 on DF
Move hairline to 9.85 on CF, reading 12.51 on DF.

### 2.11 DIVISION OF A SINGLE FACTOR BY A SERIES OF NUMBERS

In this type of problem it is best to use the reciprocal scales CI and CIF. Division of a single factor by a series of numbers is illustrated by the following example.

Example 2.8 Divide 41.5 by each of the following numbers: 12.4, 20.8, 44.5 and 92.

Operation
Set right index of CI to 41.5 on D
Move hairline to 12.4 on CI, reading 3.35 on D
Move hairline to 20.8 on CI, reading 1.995 on D
Move hairline to 44.5 on CIF, reading 0.933 on DF
Move hairline to 92 on CIF, reading 0.451 on DF.

### 2.12 DIVISION OF A SERIES OF NUMBERS BY A SINGLE FACTOR

Likewise, a series of numbers can be divided by a single factor, simply by multiplying each number in the series by the reciprocal of the single factor.

Example 2.9 Divide each of the following numbers by 0.561: 3.65, 30.5, 95.2 and 6.95.

Operation
Set 0.561 on C to the right of index D
Move hairline to 3.65 on C, reading 6.51 on D
Move hairline to 30.5 on C, reading 54.4 on D
Move hairline to 95.2 on CF, reading 169.7 on DF
Move hairline to 6.95 on CF, reading 12.39 on DF.

In effect, we have multiplied the series of numbers by $\frac{1}{0.561} = 1.784$, which is read on D at the left index of C.

In setting an index of the slide in the above operations, either the left or the right index of the slide might have been used. It should be remembered, however, that the slide need not be moved more than one-half of the scale length. The number 3.16 of the D scale is located approximately at its mid point. Therefore, *for a single factor less than 316, set the left index; for one greater than 316, set the right index of the* slide. If this rule is followed, the single factor may be either *multiplied* or *divided* by *any* number without again moving the slide. It is only necessary to move the hairline to perform the successive operations.



**Exercises 2.10, 2.11 and 2.12**

Multiplication or Division of a Single Factor by a Series of *Numbers; Division of a Series of Numbers by a Single Factor.*

1. Multiply 320 successively by 1.15, 2.42, 3.18, 4.5, 5.42, 6.88, 7.96, 8.05, and 9.6.
2. Divide 7.18 successively by 1.02, 2.15, 3.29, 4.18, 5.67, 6.41, 7.85, 8.76, and 9.34.
3. Divide 107, 181, 257, 294, 352, 671, 707, 775, 988 each by 358.

### 2.13 MULTIPLICATION AND DIVISION USING $\pi$

Since the folded scales (DF, CF, and CIF) are folded at $\pi$ (CIF at $1/\pi$), the value of $\pi$ and $1/\pi$ on the three scales is opposite the indexes of the corresponding D, C and CI scales. Since the D scale is in a fixed position relative to the DF scale with its indexes lined up with $\pi$ on the DF scale, it is easy to multiply by $\pi$. Simply set the hairline on a number on the D scale and read $\pi$ times the number on the DF scale.

Example 2.10 $\pi \times 2.4 = 7.54$

Operation
Set hairline on 2.4 on the D scale
Read 7.54 on the DF scale.

Note the scale instruction symbols at the extreme left end of the D and DF scales. Approximately, the D scale is symbolized by an X while the DF scale has an $\pi$ X symbol. The C and CF have similar symbols indicating the multiplication by $\pi$ can also be carried out on these scales in the same manner as the D and DF scales. Since CIF is the reciprocal of CI, it carries the symbol $1/\pi$ X and CI has 1/X because it is the reciprocal of the C scale.

To divide by $\pi$ simply set the hairline on the number to be divided by $\pi$ on the DF scale and read the answer on the D scale.

Example 2.11 $\frac{13}{\pi} = 4.14$

Operation
Set hairline on 13 on DF scale
Read 4.14 on the D scale.

Complete use of these scales using $\pi$ should be understood. The following examples will serve to introduce several popular problems using $\pi$.

Example 2.12 $\dfrac{5.9 \times 2.2\pi}{25} = 1.63$

Operation
Set hairline to 5.9 on D
Move 2.2 on CI to hairline
Move hairline to 25 on CI
Read 1.63 on DF at hairline.

Example 2.13 $\dfrac{21.2 \times 7.7}{8\pi} = 6.5$

Operation
Set hairline 21.2 on D
Move 7.7 on CI to hairline
Move hairline to 8 on CIF
Read 6.5 on D at hairline.

**Exercise 2.13**
**Multiplication and Division Using $\pi$**

1. $89.2\pi$
2. $\pi \times \pi$
3. $6 \div \pi$
4. $0.0246 \div \pi$
5. $\dfrac{37\pi}{93}$
6. $\dfrac{37}{9.3\pi}$
7. $\dfrac{2.6 \times 39.8}{116 \times \pi}$
8. $\dfrac{16.9 \times 1.14 \times 7.05\pi}{50.2 \times 2.6 \times 2.17}$

## 2.14 RATIO AND PROPORTION

The principle of proportion is convenient in solving simple equations without having to solve the equations explicitly for the unknown. The use of proportion in this manner is perhaps best illustrated by the use of simple algebraic expressions. Let x be the unknown quantity which is to be solved for when the known quantities are C′, D′, C, or D. In a proportion such as $\frac{x}{C'} = \frac{D}{C}$; C′, D and C are known and x is to be determined. If we set the number D on a D scale (D or DF) opposite C on a C scale (C or CF), x may be read directly on the D scale opposite C′ on the C scale.



Example 2.14 Solve for x in $\frac{x}{5.1} = \frac{3}{2}$; x = 7.65

Operation
Set hairline to 3 on DF
Move 2 on CF to hairline
Move hairline to 5.1 on CF
Read 7.65 on DF at hairline

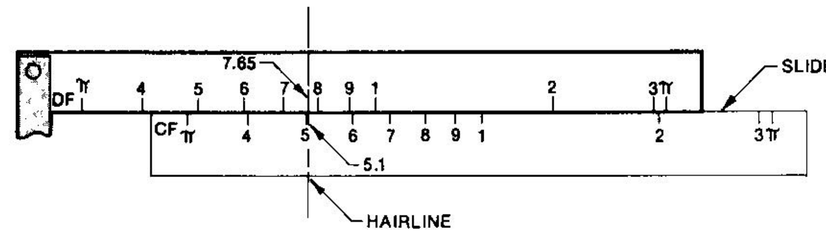



**Figure 2.7—Ratios using the DF and CF scales**

In example 2.14, notice that the physical form of the proportion is exactly duplicated on the slide rule when DF and CF scales are used. The D and C scales could also be used to solve this ratio problem. Some problems lend themselves to only these scales for solution. It should be noted in figure 2.7 that at any position of the slide, the ratio of any number on the C or CF scale to its opposite on the D or DF scale is the same ratio of any other pairs of numbers on these scales. The ratio of 3 to 2 is the same as 6 to 4, 9 to 6, and so forth.

Example 2.15 $\dfrac{8.7}{15.2} = \dfrac{x}{27.6} = \dfrac{44.4}{y} = \dfrac{z}{39.3}$

Operation
Set hairline at 8.7 on DF.
Move 15.2 on CF to hairline.
Move hairline to 27.6 on CF.
Read x = 15.8 on DF.
Move hairline to 44.4 on DF.
Read y = 77.5 on CF.
Move hairline to 39.3 on C.
Read z = 22.5 on D.

**Exercise 2.14**
**Ratio and Proportions**

1. $\frac{21.4}{195} = \frac{x}{12.1}$
2. $\frac{71}{705} = \frac{18.25}{x}$
3. $\frac{7.18}{x} = \frac{32.4}{17.9}$
4. $\frac{356}{51} = \frac{42.5}{x} = \frac{x}{y} = \frac{y}{z}$

### 2.15 QUADRATIC EQUATION SOLUTION BY FACTORING

The slide rule may be used for the rapid factoring of a quadratic equation. All that is required is a single setting of the slide and mental summation of factors. Any quadratic equation may be reduced to the form: $x^2 + Ax + B = 0$

The factors, or roots, of the equation are designated as $-r_1$ and $-r_2$, and must satisfy the following conditions.

(1) $r_1 + r_2 = A$
(2) $r_1r_2 = B$

The general procedure of finding two factors, whose sum is A and whose product is B, on the slide rule follows:

(1) Set the appropriate index of C opposite the location of B on the D scale. Now the slide is in such a position that for any setting of the indicator, the *product* of the reading at the hairline on the D and CI scales, or on the DF and CIF scales, is equal to the number B.

(2) Move the hairline so that the *sum* of the readings on the D and CI, or the DF and CIF scales is equal to the number A.

The following examples will clarify this procedure.

Example 2.16 $x^2 + 10x + 15 = 0$

Operation: Set left index of C at 15 on D.
Move hairline until $r_1 + r_2 = 10$.

(They will be positive as both 10 and 15 are positive. This occurs with hairline at 8.15 on CI and 1.84 on D. Thus the roots $-r_1$ and $-r_2$ are $-8.15$ and $-1.84$.)

Sum = A = 10 = 8.15 + 1.84
Product = B = 15 = (8.15)(1.84)



Example 2.17 $x^2 - 12.2x - 17.2 = 0$

Operation: Set left index of C at 17.2 on D.
Move hairline until $r_1 + r_2 = -12.2$ which is when the hairline is at $-13.5$ on DF and 1.275 on CIF. (Since the product is negative, one of the factors must be negative. Also, since the sum is negative the larger factor must be negative.) The values of $r_1$ and $r_2$ are $-13.5$ and 1.275, therefore the roots $-r_1$ and $-r_2$ are 13.5 and $-1.275$.

Sum = A = −13.5 + 1.275 = −12.225
Product = B = (−13.5)(1.275) = −17.2

Hence, it is obvious that this method involves trial-and-error in setting the hairline, and a little practice is necessary to master the technique.

**Exercise 2.15**
**Solving Quadratic Equations by Factoring**

1. $x^2 - 34.5x + 18 = 0$
2. $x^2 - 21.1x + 32 = 0$
3. $x^2 - 20.2x - 120 = 0$
4. $2x^2 + 82.8x + 840 = 0$
5. $1.2x^2 - 13.38x + 36 = 0$

## SQUARE ROOTS AND SQUARES, CUBE ROOTS AND CUBES

The Versalog II permits solving any power or root of a number. The powers and roots most commonly encountered can be rapidly calculated with just one setting of the hairline. For squares and square roots, a choice of scales is available to permit greater speed and accuracy in these calculations. For operations involving exponents other than 2, 3, 4, 6 and $\frac{3}{2}$ (and $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{2}{3}$), the LL (log log) scales are used. Uses of the $R_1$, $R_2$, A and K scales are discussed in this chapter.

### 3.1 SQUARES AND SQUARE ROOTS USING $R_1$ AND $R_2$ SCALES

The $R_1$ and $R_2$ scales are called the "root" scales. Actually, they are the two halves of one 20 inch scale (50 cm.) similar to a D scale, but twice as long. The $R_1$ scale is 25 cm. long, graduated and numbered from left to right, and ranges from 1 to $\sqrt{10}$, which is 3.16; while the $R_2$ scale, also 25 cm. long, and graduated and numbered from left to right, ranges from $\sqrt{10}$, (or 3.16) to 10. These scales yield unusual accuracy in solving for squares and square roots.

The simple mathematical relationship of the R and D scales may be expressed as follows: $R^2 = D$. Taking logarithms of both sides of the equation, $2 \log R = \log D$. Therefore the scale distance, from the index to any number on the R scale, is twice the scale distance to the same number on the D scale. This means that readings of the R scales are *twice* as accurate as readings of the D scale.

#### SQUARES

In squaring a number, we set the hairline on the number on the $R_1$ or $R_2$ scale and read its square directly on the D scale.

To square a number whose digits fall between 1 and 3.16:

1. Set the hairline to the number on $R_1$.
2. Read the square of the number on D at the hairline.

To square a number whose digits fall between 3.16 and 10:

1. Set the hairline to the number on $R_2$.

2. Read the square of the number on D at the hairline.

Example 3.1 $3^2 = 9$.

Operation Set hairline on 3 on the $R_1$ scale.
Read square, 9, on the D scale at hairline.

Example 3.2 $(7.1)^2 = 50.4$

Operation Set hairline on 7.1 on the $R_2$ scale.
Read square, 50.4, on the D scale at hairline.

When squaring larger or smaller numbers than used in the examples above, the location of the decimal point requires more consideration. Use of the scientific notation method (as outlined in Section 1.6) is recommended. In squaring numbers by use of the scientific notation method,

$$N^2 = (N \times 10^n)^2 = N^2 \times 10^{2n}.$$

The scientific notation method is precise. However, a short cut is available for quick placement of the decimal point, and it may be well to remember the following simple rules.

Squares of numbers greater than 1 (> 1):

1. The square of a number greater than 1 on the $R_1$ scale will have an *odd* number of digits to the left of the decimal point, one less than twice the number of digits to the left of the decimal point in the number being squared.
2. The square of a number greater than 1 on the $R_2$ scale will have an *even* number of digits to the left of the decimal point, exactly twice the number of digits to the left of the decimal point in the number being squared.

Squares of numbers less than 1 (< 1):

Zeros appearing to the right of the decimal point and before the first non-zero digit in a number less than 1 may be defined as significant zeros.

1. The square of a number less than 1 on the $R_1$ scale will have an *odd* number of significant zeros, one more than twice the number of significant zeros in the number being squared.
2. The square of a number less than 1 on the $R_2$ scale will have an *even* number of significant zeros, exactly twice the number of significant zeros in the number being squared.



The following examples show both methods of decimal point location.

Example 3.3 $(26.53)^2 = 704$

Operation Set hairline to 26.53 on $R_1$.
Read 704 on D at hairline.
Locate decimal point by either,
a) $(2.653 \times 10^1)^2 = 7.04 \times 10^2 = 704$.
b) $R_1$, $(2 \times 2) - 1 = 3$ digits.

Example 3.4 $(6{,}110)^2 = 37{,}300{,}000$

Operation Set hairline to 6,110 on $R_2$.
Read 37,300,000 on D at hairline.
Locate decimal point by either,
a) $(6.11 \times 10^3)^2 = 37.3 \times 10^6 = 37{,}300{,}000$.
b) $R_2$, $2 \times 4 = 8$ digits.

Example 3.5 $(0.1575)^2 = 0.0248$

Operation Set hairline to .1575 on $R_1$.
Read .0248 on D at hairline.
Locate decimal point by either,
a) $(1.575 \times 10^{-1})^2 = 2.48 \times 10^{-2} = 0.0248$.
b) $R_1$, $(2 \times 0) + 1 = 1$ significant zero.

Example 3.6 $(0.00917)^2 = 0.0000841$

Operation Set hairline to .00917 on $R_2$.
Read .0000841 on D at hairline.
Locate decimal point by either,
a) $(9.17 \times 10^{-3})^2 = 84.1 \times 10^{-6} = 0.0000841$.
b) $R_2$, $2 \times 2 = 4$ significant zeros.

### SQUARE ROOTS

The square root of a number N is that number whose square is N. For example,

$$\text{if } N = 9$$
$$\text{then } \sqrt{9} = 9^{\frac{1}{2}} = 3$$
$$\text{and } 3^2 = 9.$$

The superscript after 9 is $\frac{1}{2}$, the reciprocal of 2. This inverse relationship indicates a reversal of the slide rule procedure. Thus, the square root of numbers on the D scale are read directly opposite on the $R_1$ or $R_2$ scale. Since the relationship between the R and D scales is $R^2 = \sqrt{D}$, conversely $R = D$. It should be noted that the R scales have a scale instruction symbol $\sqrt{x}$ indicating that the R scales represent the square roots of numbers on the D scale.

A scientific notation method is recommended to correctly perform the operation. The square root required is expressed as $(N \times 10^n)^{\frac{1}{2}}$, when the exponent n *must* be evenly divisible by 2, while the number N can be between 1 and 100. Using the short cut rules for positioning the decimal point, odd numbers of digits are again associated with the $R_1$ scale, while even numbers of digits are associated with the $R_2$ scale. The rules are as follows:

Square roots of numbers greater than 1 (> 1):

1. The square root of a number greater than 1 with an *odd* number of digits to the left of the decimal point is found on the $R_1$ scale. To obtain the number of digits to the left of the decimal point in the square root, add one to the number of digits to the left of the decimal point in the number and divide by two.
2. The square root of a number greater than 1 with an *even* number of digits to the left of the decimal point is found on the $R_2$ scale. The number of digits to the left of the decimal point in the square root is exactly half the number of digits to the left of the decimal point in the number.

Square roots of numbers less than 1 (< 1):

1. The square root of a number less than 1 with an *odd* number of significant zeros to the right of the decimal point is found on the $R_1$ scale. To obtain the number of significant zeros in the square root, subtract one from the number of significant zeros in the number and divide by two.
2. The square root of a number less than 1 with an *even* number of significant zeros or no significant zeros to the right of the decimal point is found on the $R_2$ scale. The number of significant zeros in the square root is exactly half the number of significant zeros in the number.



Example 3.7 $\sqrt{196} = 14$

Operation
Set hairline to 196 on D.
Read square root, 14, on $R_1$ at hairline.
Locate decimal point by either,

a) $(196 \times 10^2)^{\frac{1}{2}} = 1.4 \times 10^1 = 14.$

b) $R_1, \frac{3+1}{2} = 2$ digits.

Example 3.8 $\sqrt{124{,}600} = 353$

Operation
Set hairline to 124,600 on D.
Read 353 on $R_2$ at hairline.
Locate decimal point by either,

a) $(12.46 \times 10^4)^{\frac{1}{2}} = 3.53 \times 10^2 = 353.$

b) $R_2, \frac{6}{2} = 3$ digits.

Example 3.9 $\sqrt{0.43} = 0.656$

Operation
Set hairline to .43 on D.
Read .656 on $R_2$ at hairline.
Locate decimal point by either,

a) $(43 \times 10^{-2})^{\frac{1}{2}} = 6.56 \times 10^{-1} = .656.$

b) $R_2, \frac{0}{2} = 0$ significant zeros.

Example 3.10 $\sqrt{0.00097} = 0.03115$

Operation
Set hairline to .00097 on D.
Read .03115 on $R_1$ at hairline.
Locate decimal point by either,

a) $(9.7 \times 10^{-4})^{\frac{1}{2}} = 3.115 \times 10^{-2} = .03115.$

b) $R_1, \frac{3-1}{2} = 1$ significant zero.

Selection of the $R_1$ or $R_2$ scale on which to read the square root of a number is determined by the number of digits to the left of the decimal point for numbers greater than 1, and the number of significant zeros in numbers less than 1. This is further illustrated by the following examples.

Example 3.11 $\sqrt{0.5} = 0.707$

Operation Set hairline to .5 on D.
Read .707 on $R_2$ at hairline.

Example 3.12 $\sqrt{0.05} = 0.2236$

Operation Set hairline to .05 on D.
Read .2236 on $R_1$ at hairline.

Example 3.13 $\sqrt{0.005} = 0.0707$

Operation Set hairline to .005 on D.
Read .0707 on $R_2$ at hairline.

Example 3.14 $\sqrt{0.0005} = 0.02236$

Operation Set hairline to .0005 on D.
Read .02236 on $R_1$ at hairline.

Example 3.15 $\sqrt{0.00005} = 0.00707$

Operation Set hairline to .00005 on D.
Read .00707 on $R_2$ at hairline.

Example 3.16 $\sqrt{0.000005} = 0.002236$

Operation Set hairline to .000005 on D.
Read .002236 on $R_1$ at hairline.

**Exercise 3.1**

**Squares and Square Roots Using the $R_1$ and $R_2$ Scales**

1. $(20.4)^2$
2. $(715)^2$
3. $(1{,}070)^2$
4. $(125.4)^2$
5. $(0.85)^2$
6. $(0.000157)^2$
7. $(0.094)^2$
8. $(0.0076)^2$
9. $\sqrt{27}$
10. $\sqrt{925}$
11. $\sqrt{820{,}000}$
12. $\sqrt{1{,}265}$
13. $\sqrt{71{,}500}$
14. $\sqrt{51{,}000{,}000}$
15. $\sqrt{1{,}970{,}000}$
16. $\sqrt{660}$
17. $\sqrt{0.424}$
18. $\sqrt{0.0875}$
19. $\sqrt{0.00097}$
20. $\sqrt{0.00725}$



## 3.2 SQUARES AND SQUARE ROOTS USING THE A SCALE

The A scale can also be used for square and square root calculations. It ranges from 1 to 100. When the greater accuracy provided by the $R_1$ and $R_2$ scales is not required, some series of computations are more rapidly solved using the A scale. The A scale is, in effect, two short (12.5 cm.) D scales placed end to end, and that is approximately how it is used. In finding squares and square roots, the left half of the A scale is used as a D scale, and the left half of the D scale is used as an $R_1$ scale. Likewise, the right half of the A scale is used again as a D scale, and the right half of the D scale is used as an $R_2$ scale. The simple mathematical relationship of the A and D scales may be expressed as, $D^2 = A$. Scale instruction symbol on the A scale is $X^2$.

### SQUARES

The square of numbers on the D scale are read directly opposite on the A scale. The procedure only requires the use of the hairline. The short cut rules for the placement of the decimal point are the same as when using the $R_1$ and $R_2$ scales, except that the left half of the A scale is associated with odd numbers of digits (as the $R_1$ scale) and the right half associated with even numbers of digits (as the $R_2$ scale).

Example 3.17 $(24.8)^2 = 615$

Operation Set hairline to 24.8 on D.
Read 615 on A at hairline.
Locate decimal point by either,
a) $(2.48 \times 10^1)^2 = 6.15 \times 10^2 = 615.$
b) A left, $(2 \times 2) - 1 = 3$ digits.

Example 3.18 $(417)^2 = 174{,}000$

Operation Set hairline to 417 on D.
Read 174,000 on A at hairline.
Locate decimal point by either,
a) $(4.17 \times 10^2)^2 = 17.4 \times 10^4 = 174{,}000.$
b) A right, $2 \times 3 = 6$ digits.

Example 3.19 $(0.0196)^2 = 0.000384$

Operation Set hairline to .0196 on D.
Read .000384 on A at hairline.

Locate decimal point by either,
a) $(1.96 \times 10^{-2})^2 = 3.84 \times 10^{-4} = 0.000384$.
b) A left, $(2 \times 1) + 1 = 3$ significant zeros.

Example 3.20 $(0.822)^2 = 0.676$

Operation Set hairline to .822 on D.
Read .676 on A at hairline.
Locate decimal point by either,
a) $(8.22 \times 10^{-1})^2 = 67.6 \times 10^{-2} = 0.676$.
b) A right, $2 \times 0 = 0$ significant zeros.

### SQUARE ROOTS

The square roots of numbers on the A scale are read directly opposite on the D scale. The procedure is essentially the reverse of finding squares using the A scale, but in locating numbers on the A scale, the position of the decimal point is important since the A scale ranges from 1 to 100. For numbers outside of this range, the use of the scientific notation method is recommended. The number is rewritten in the form of $N \times 10^n$, where n *must* be evenly divisible by 2, and N is between 1 and 100. The number N is then located on the A scale, $\sqrt{N}$ read on the D scale, and the number rewritten in its ordinary form.

A short cut rule can also be used. The rule for locating the decimal point is the same as when using the $R_1$ and $R_2$ scales, but in locating numbers on the A scale, the following can be used.

Square roots of numbers greater than 1 (> 1):

1. For the square root of a number greater than 1 with an *odd* number of digits to the left of the decimal point, use the *left* half of the A scale.
2. For the square root of a number greater than 1 with an *even* number of digits to the left of the decimal point, use the *right* half of the A scale.

Square roots of numbers less than 1 (< 1):

1. For the square root of a number less than 1 with an *odd* number of significant zeros to the right of the decimal point, use the *left* half of the A scale.



2. For the square root of a number less than 1 with *no* significant zeros or an *even* number of significant zeros to the right of the decimal point, use the *right* half of the A scale.

Example 3.21 $\sqrt{5{,}480} = 74.0$

Operation Set hairline to 5,480 on right half of A.
Read 74.0 on D at hairline.
Locate decimal point by either,

a) $(54.80 \times 10^2)^{\frac{1}{2}} = 7.40 \times 10^1 = 74.0$.

b) A right, $\frac{4}{2} = 2$ digits.

Example 3.22 $\sqrt{54{,}800} = 234$

Operation Set hairline to 54,800 on left half of A.
Read 234 on D at hairline.
Locate decimal point by either,

a) $(5.48 \times 10^4)^{\frac{1}{2}} = 2.34 \times 10^2 = 234$.

b) A left, $\frac{5+1}{2} = 3$ digits.

Example 3.23 $\sqrt{0.0000176} = 0.0042$

Operation Set hairline to .0000176 on right half of A.
Read .0042 on D at hairline.
Locate decimal point by either,

a) $(17.6 \times 10^{-6})^{\frac{1}{2}} = 4.2 \times 10^{-3} = .0042$

b) A right, $\frac{4}{2} = 2$ significant zeros.

Example 3.24 $\sqrt{0.000176} = 0.01327$

Operation Set hairline to .000176 on left half of A.
Read .01327 on D at hairline.
Locate decimal point by either,

a) $(1.76 \times 10^{-4})^{\frac{1}{2}} = 1.327 \times 10^{-2} = 0.01327$.

b) A left, $\frac{3-1}{2} = 1$ significant zero.

**Exercise 3.2**

**Squares and Square Roots Using the A Scale**

| | |
|---|---|
| 1. $4.76^2$ | 7. $\sqrt{4.83}$ |
| 2. $0.149^2$ | 8. $\sqrt{0.775}$ |
| 3. $13.7^2$ | 9. $\sqrt{48.3}$ |
| 4. $0.037^2$ | 10. $\sqrt{0.028}$ |
| 5. $238^2$ | 11. $\sqrt{483}$ |
| 6. $0.00785^2$ | 12. $\sqrt{0.00186}$ |

It is suggested Exercise 3.2 be re-worked using the $R_1$ and $R_2$ scales. Notice the greater degree of accuracy of the R scales.

## 3.3 COMBINED OPERATIONS WITH SQUARES AND SQUARE ROOTS

Combined operations are a series of operations which include the square or square root of a number. The inclusion of the A scale together with the $R_1$ and $R_2$ scales on the VERSALOG II provides a unique advantage of mathematical and slide rule technique in the solution of this type of computations.

### CHOICE of $R_1$ and $R_2$ or A SCALES

It is evident from above that the R scales provide greater accuracy in determining squares and square roots than does the A scale. When the increased accuracy of the R scales is not required, however, a choice of using either the R or A scales is possible. For combined operations, this choice should be exercised in such a way as to minimize the number of settings necessary to solve the given problem. For example, an operation of the form

$$k\sqrt{x}$$

lends itself to the use of the A scale since the setting of the number x on A projects the reading $\sqrt{x}$ directly onto the D scale, where it can readily be multiplied by k. On the other hand, an operation of the form

$$kx^2$$

is best performed by using the R scales since setting the number x on R projects the reading $x^2$ directly onto the D scale, where it can be readily multiplied by k. The following examples illustrate some common variations of these general principles.



Example 3.25 $\dfrac{\sqrt{47.2} \times 7.85}{13.51 \times 6.11} = 0.653$

Operation
Set hairline to 47.2 on A.
Move right index of C to hairline.
Move hairline to 7.85 on CF.
Move 13.51 on CF to hairline.
Move hairline to 6.11 on CIF.
Read .653 on DF at hairline.

Example 3.26 $\dfrac{(3.485)^2 \times 9.44}{0.777 \times 3.9} = 37.8$

Operation
Set hairline to 3.485 on $R_2$.
Move 9.44 on CI to hairline.
Move hairline to .777 on CI.
Move 3.9 on C to hairline.
Read 37.8 on D at right index of C.

Example 3.27 $\sqrt{\dfrac{9.7 \times 14}{2.35}} = 7.6$

Operation
Set hairline to 9.7 on D.
Move 14 on CI to hairline.
Move hairline to 2.35 on CI.
Read 7.6 on $R_2$ at hairline.

Example 3.28 $\left(\dfrac{4.1}{6.95 \times 0.233}\right)^2 = 6.4$

Operation
Set hairline to 4.1 on D.
Move 6.95 on C to hairline.
Move hairline to .233 on CI.
Read 6.4 on A at hairline.

### AREA OF CIRCLES

Finding the area of a circle when the radius is given is a commonly encountered problem. Solution requires only a single setting of the hairline, which is another decided advantage of the $R_1$ and $R_2$ scales. Set the radius, r, on the $R_1$ or $R_2$ scale, and read the area, $\pi r^2$, on the DF scale. (The value $r^2$ is available at the hairline on the D scale.)

Example 3.29 Find the area of a circle whose radius is 4.82 feet.
Area = 73 square feet: $\pi(4.82)^2 = 73$

Operation — Set hairline to 4.82 on $R_2$.
Read area, 73, on DF at hairline.

When the area of a circle is known and the radius required, the inverse of the above procedure can be used. Set the area on the DF scale and read the radius on either the $R_1$ or $R_2$ scale, whichever is appropriate. The same rules as stated in Section 3.1 apply to the selection of the proper root scale.

Example 3.30 Find the radius of a circle whose area is 2,670 sq. in. Radius = 29.15 in.:

$$\sqrt{\frac{2{,}670}{\pi}} = 29.15$$

Operation — Set hairline to 2,670 on DF.
Read 29.15 on $R_1$ at hairline.

When the diameter of a circle is given, rather than radius, the area can be found by solving for $\frac{\pi d^2}{4}$, where d is the diameter. The operation is similar to Example 3.29, but a setting of the slide is necessary for the division.

Example 3.31 Find the area of a circle whose diameter is 2.437 in. Area = 4.66 sq. in.:

$$\frac{\pi(2.437)^2}{4} = 4.66$$

Operation — Set hairline to 2.437 on $R_1$.
Move 4 on C to hairline.
Move hairline to left index of C.
Read area, 4.66, on DF at hairline.

**Exercise 3.3**

**COMBINED OPERATIONS WITH SQUARES AND SQUARE ROOTS; AREAS OF CIRCLES**

1. $\frac{17\sqrt{676}}{3.19 \times 12}$
2. $\frac{(14)^2 \times 3.2}{225 \times 6.4}$
3. $\sqrt{27^2 \times 41.3}$
4. $\frac{\pi(3.955)^2}{4.1 \times 2.49}$
5. $\left(\frac{11.45 \times 6.8\pi}{1.605 \times 5.35}\right)^2$
6. $\left(\frac{\sqrt{811} \times 4}{2.36}\right)^2$
7. $(\pi(6.69)^2 \times 1.19)^2$



8. Find the area of circles whose radii are known:
a) 6; b) 4.2; c) .0581; d) 31; e) 1.314.
9. Find the area of circles whose diameters are known:
a) 7.1; b) .42; c) 1.09; d) .0495; e) 1,700.
10. Find the radius of circles whose areas are known:
a) 116.5; b) .0491; c) .601; d) 760; e) 80.4.

## 3.4 CUBES AND CUBE ROOTS USING THE K SCALE

The K scale is used with the D scale for finding cubes and cube roots. Since these two scales are of equal length and their mathematical relationship is $D^3 = K$, it follows that $\log D = \frac{\log K}{3}$. Therefore, the K scale is divided into three equal segments, each segment graduated and numbered from left to right. The first segment extends from 1 to 10, the second from 10 to 100, and the third from 100 to 1,000. Since the scale distance from the index to a number on K is only one-third the scale distance to the same number on D, the accuracy of K scale readings is only one-third that of the D scale readings.

### CUBES

Cubes of numbers on the D scale are read directly opposite on the K scale. The K scale has a self instruction symbol $x^3$ relating that $D^3 = K$. For numbers on the D scale, between 1 and 10, the location of the decimal point is indicated by the K scale, since it ranges from 1 to 1,000.

Example 3.32 $(6.1)^3 = 227$

Operation — Set hairline to 6.1 on D.
Read .227 on K at hairline.

For numbers larger than 10 and smaller than 1, the location of the decimal point is not as obvious and the use of the scientific notation method again is recommended. Briefly, since $(N \times 10^n)^3 = N^3 = 10^{3n}$, the K scale is used for finding the cube of N and the power of ten for relocating the decimal point. If preferred in such cases, a definite rule may be followed:

If the decimal point is moved n number of places in a number set on D, it must be moved back 3n places in the cube, which is read on K.

Example 3.33 $(1{,}214)^3 = 1{,}790{,}000{,}000$

Operation
Express problem as $(1.214 \times 10^3)^3$.
Set hairline on 1.214 on D.
Read 1.79 on K at hairline.
Answer, $1.79 \times 10^9 = 1{,}790{,}000{,}000$.

Example 3.34 $(0.0721)^3 = 0.000375$

Operation
Express problem as $(7.21 \times 10^{-2})^3$.
Set hairline to 7.21 on D.
Read 375. on K at hairline.
Answer, $375 \times 10^{-6} = .000375$.

### CUBE ROOTS

Cube roots of numbers on the K scale are read directly opposite on the D scale. Therefore, the cube root of numbers, on the K scale, between 1 and 1,000 range between 1 and 10 as read on the D scale.

Example 3.35 $\sqrt[3]{5.2} = 1.733$

Operation
Set hairline to 5.2 on K.
Read 1.733 on D at hairline.

For numbers beyond the range of 1 to 1,000, use of a modified scientific notation form is recommended to assure correct placement of the decimal point. In the modified scientific notation method, the number is expressed as $N \times 10^n$, where n *must* be evenly divisible by 3 while N can range from 1 to 1,000 (since the K scale ranges from 1 to 1,000). The power of 10(n) must be evenly divisible by 3 to enable removal from the inside of the cube root radical. Again, if preferred, a definite rule may be followed:

If the decimal point is moved n number of places in a number set on K, it must be moved back $\frac{n}{3}$ places in the cube root, which is read on D.



Example 3.36 $\sqrt[3]{26{,}400} = 29.8$

Operation
Express problem as $(26.4 \times 10^3)^{\frac{1}{3}}$.
Set hairline to 26.4 on K.
Read 2.98 on D at hairline.
Answer, $2.98 \times 10^1 = 29.8$.

Example 3.37 $\sqrt[3]{0.0052} = 0.1732$

Operation
Express problem as $(5.2 \times 10^{-3})^{\frac{1}{3}}$.
Set hairline to 5.2 on K.
Read 1.732 on D at hairline.
Answer, $1.732 \times 10^{-1} = .1732$.

Example 3.38 $\dfrac{\sqrt[3]{0.000475}}{4.6} = 0.01696$

Operation
Express problem as $\dfrac{\sqrt[3]{475}}{4.6} \times (10^{-6})^{\frac{1}{3}}$.

Set hairline to 475 on K.
Move 4.6 to C to hairline.
Read 1.696 on D at index of C.
Answer, $1.696 \times 10^{-2} = 0.01696$.

**Exercise 3.4**
**Cubes and Cube Roots Using the K Scale**

1. $(1.26)^3$
2. $(2.715)^3$
3. $(5.85)^3$
4. $(41)^3$
5. $(750)^3$
6. $(3.2)^3$
7. $\sqrt[3]{6}$
8. $\sqrt[3]{24}$
9. $\sqrt[3]{270}$
10. $\sqrt[3]{1{,}720}$
11. $\sqrt[3]{29{,}000}$
12. $\sqrt[3]{560{,}000}$
13. $(0.245)^3$
14. $(0.036)^3$
15. $(0.0048)^3$
16. $\sqrt[3]{0.32}$
17. $\sqrt[3]{0.041}$
18. $\sqrt[3]{0.0075}$

### 3.5 SPECIAL POWERS AND ROOTS USING A, K AND $R_1$ AND $R_2$ SCALES

Since the A, K and R scales are all related to the D scale, an interrelation exists between each of these scales. Using the $R_1$, $R_2$, A and K scales, powers of 4, 6 and $\frac{3}{2}$ (and roots of $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{2}{3}$) can be found with a single setting of the hairline. As was previously mentioned, the mathematical relationship between these scales may be expressed as: $R^2 = D$, $D^2 = A$, and $D^3 = K$, and therefore, $R^4 = A$, $R^6 = K$ and $A^3 = K^2$.

#### POWERS OF 4 AND ROOTS OF $\frac{1}{4}$

The $R_1$ and $R_2$ scales are used with the A scale for powers of 4 and $\frac{1}{4}$. The square of numbers on an R scale are directly opposite on the D scale, and the square of numbers on the D scale are directly opposite on the A scale. Therefore, the 4th power of numbers on an R scale are directly opposite on the A scale.

Symbolically, $N^4 = (N^2)^2$; $N^{\frac{1}{4}} = (N^{\frac{1}{2}})^{\frac{1}{2}}$.

Example 3.39 $(22)^4 = 234{,}000$

Operation Express numbers as $(2.2 \times 10^1)^4$.
Set hairline to 2.2 on $R_1$.
Read 23.4 on A at hairline.
Answer, $23.4 \times 10^4 = 234{,}000$.

Example 3.40 $(0.06)^{\frac{1}{4}} = 0.495$.

Operation Express number as $(600 \times 10^{-4})^{\frac{1}{4}}$.
Set hairline to 600 on A (left half).
Read 4.95 on $R_2$ at hairline.
Answer, $4.95 \times 10^{-1} = 0.495$

#### POWERS OF 6 AND ROOTS OF $\frac{1}{6}$

The $R_1$ and $R_2$ scales are used with the K scale for finding powers of 6 and $\frac{1}{6}$. The $\frac{1}{3}$rd power of numbers on the K scale are opposite the D scale, and the square root ($\frac{1}{2}$ power) of numbers on the D scale are opposite on an R scale. Symbolically, $N^6 = (N^3)^2$; $N^{\frac{1}{6}} = (N^{\frac{1}{3}})^{\frac{1}{2}}$.

Example 3.41 $(2)^6 = 64$

Operation Set hairline to 2 on $R_1$.
Read 64 on K at hairline.



Example 3.42 $(2{,}000)^{\frac{1}{6}} = 3.55$

Operation Set hairline to 2,000 on K.
Read 3.55 on $R_2$ at hairline.

#### POWERS OF $\frac{3}{2}$ AND ROOTS OF $\frac{2}{3}$

The A and K scales are used since the $\frac{1}{3}$rd power of numbers on the K scale are opposite on the D scale and squares of numbers on the D scale are opposite on the A scale. Thus, the $\frac{2}{3}$ power of numbers on the K scale are opposite on the A scale. Symbolically, $N^{\frac{2}{3}} = (N^{\frac{1}{3}})^2$; $N^{\frac{3}{2}} = (N^3)^{\frac{1}{2}}$.

Example 3.43 $(0.875)^{\frac{2}{3}} = 0.915$

Operation Set hairline to .875 on K.
Read .915 on A at hairline.

Example 3.44 $(18.2)^{\frac{3}{2}} = 78.$

Operation Set hairline to 18.2 on A.
Read 78 on K at hairline.

# OPERATIONS INVOLVING POWERS, ROOTS, EXPONENTIAL EQUATIONS AND RECIPROCALS USING LOG LOG SCALES. LOGARITHMS USING LOG SCALE.

The log log scales are exceptionally useful in engineering calculations which involve powers and roots. As previously explained, square roots and squares, cube roots and cubes may be found by using the special scales A, $R_1$, $R_2$, and K. However, *any* power or root of a number may be found by using the log log scales. For numbers close to one, powers and roots are determined in this way with considerable accuracy.

One important feature of the log log scales is that the decimal point is always given by the scale reading, so that it is unnecessary to determine its location by additional calculation. This feature reduces the chance of error. However, because of frequent changes in sub-dividing along the scales and because of the extremely wide range of numbers (from 0.00005 to about 22,000), care must be used in reading the scales. The sub-dividing should be carefully checked by eye for that portion of any log log scale being used. Distances along the log log scales are proportional to $\log_{10}\log_e N$ where N is any number appearing on a log log scale.

## 4.1 POWERS AND ROOTS OF NUMBERS

It can be shown mathematically that the scale equation $\log_{10}\log_e N$ lends itself to a convenient relationship to other logarithmically graduated scales on the slide rule for raising numbers to given powers. Let's consider the following example to outline scale construction and relationships between them.

It is desired to raise the number N to the P power and obtain the answer A.

$$N^P = A$$

Take the natural log of both sides of the equation.

$$\text{Log}_e N^P = \text{Log}_e A$$

since:

$$\text{Log}_e N^P = P\ \text{Log}_e N$$

we have

$$P\ \log_e N = \text{Log}_e A$$

Take the common log of both sides of the expression.

$$\text{Log}_{10}(P\ \text{Log}_e N) = \text{Log}_{10}\text{Log}_e A$$

and since

$$\text{Log}_{10}(P\ \text{Log}_e N) = \text{Log}_{10} P + \text{Log}_{10}\text{Log}_e N$$

we have

$$\text{Log}_{10} P + \text{Log}_{10}\text{Log}_e N = \text{Log}_{10}\text{Log}_e A$$

We should recognize the $\text{Log}_{10}P$ is the scale length expression for any number P on such scales as D, C or CI. Also, we have noted that $\text{Log}_{10}\text{Log}_e N$ is the scale length expression for any number N on the LL (Log Log) scales.

Therefore, from the above expression, we see that raising numbers to powers by using the log log scales is as simple as multiplication. Now if we wish to raise a number N to the power P to obtain $N^P$, we must add the $\log_{10}P$ by use of one of the scales on the slide, either C or CI, to the number N on a log log scale. The answer will be read on the log log scale.

Example 4.1 $(3)^4 = 81$

Operation Set hairline to 3 on LL3.
Move left index of C to hairline.
Move hairline to 4 on C.
Read 81 at the hairline on LL3.

In this example (see Figure 4.1), N corresponds to the number 3 and p to the exponent 4. We have added $\log_{10}4$ to $\log_{10}\log_e{}^3$, because the distance moved by the hairline from the left index to 4 of C was proportional to $\log_{10}4$. The result is $\log_{10}\log_e(3)^4 = \log_{10}\log_e 81$. Hence, $(3)^4 = 81$. Other examples follow in this chapter.



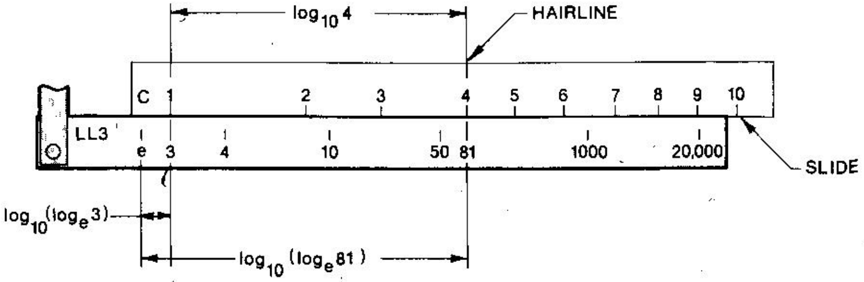



**Figure 4.1 – LL3 and C Scales for Raising 3 to the 4th Power.**

Scale C was used in example 4.1 and shown in Figure 4.1 for its convenience and clarity only. For greater slide rule efficiency, as in multiplication, the CI scale is recommended.

## POWERS AND ROOTS OF NUMBERS GREATER THAN 1.001

The four log log scales LL0, LL1, LL2, and LL3 are used with CI or C scales in raising numbers *greater* than 1.001 to a power. If the log log scales were placed end to end they would form one continuous scale from 1.001 to 22,026. See Figure 4.2. The log log scales are in black and read from left to right.

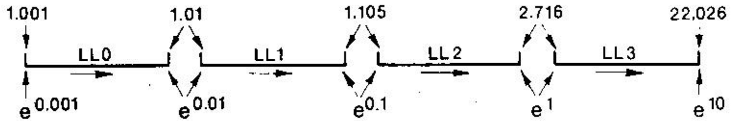



**Figure 4.2 – Log Log Scales LL0, LL1, LL2, and LL3 Placed End to End.**

Reconsidering our problem in Example 4.1, $(3)^4 = 81$ after having moved the left index of C to the hairline, observe the following readings on the LL3 scale as we move the hairline to various values on the C scale:

$$(3)^2 = 9$$
$$(3)^{2.5} = 15.6$$
$$(3)^{2.78} = 21.2$$
$$(3)^4 = 81$$

It is evident that as the powers of 3 increase, the hairline is moved to the right "up the scale," and readings are made on the LL3 scale.

Conversely, as powers of 3 decrease, the hairline moves to the left, "down the scale," and since the LL scales are continuous, the readings are continued on the LL2 scale after the lower limit of LL3 is reached. This leads to another important principle. The relationship of successive scales is that of *one-tenth* powers of any number on successive scales. For example, if we set the hairline to 4 on C in Example 4.1, we observe the following readings on the various LL scales.

$3^{4} = 81$ (LL3 scale)
$3^{0.4} = 1.552$ (LL2 scale)
$3^{0.04} = 1.0449$ (LL1 scale)
$3^{0.004} = 1.004405$ (LL0 scale)

Summarizing, we can establish two working rules.

1. Powers *increase* when moving "up" the log log scales.
2. Powers *increase* by tenths when moving "up" successive log log scales.

"Up" refers to a higher mathematical value and not necessarily to scale numbers.

Example 4.2 Verify
$(1.15)^{0.015} = 1.0021$ (LL0 scale)
$(1.15)^{0.058} = 1.00815$ (LL0 scale)
$(1.15)^{0.13} = 1.01835$ (LL1 scale)
$(1.15)^{0.6} = 1.0875$ (LL1 scale)
$(1.15)^{1.0} = 1.15$ (LL2 scale)
$(1.15)^{4.0} = 1.75$ (LL2 scale)
$(1.15)^{11.5} = 5.0$ (LL3 scale)
$(1.15)^{51.5} = 1340.0$ (LL3 scale)

Operations
Set hairline on 1.15 on LL2.
Move left index of C to hairline.
Move hairline to 0.015 on C.
Read 1.0021 at hairline on LL0.
Move hairline to 0.058 on C.
Read 1.00814 at hairline on LL0, etc.

Continue verification by moving hairline to power reading on the C scale and reading the answer on the LL scales. This example serves to exhibit rule one, that the powers *increase* as we move up the LL scales, or as we increase powers, we are moving up the LL scales.



Example 4.3 Verify
$(1.11)^{0.041} = 1.00429$ (LL0 scale)
$(1.11)^{0.41} = 1.0437$ (LL1 scale)
$(1.11)^{4.1} = 1.534$ (LL2 scale)
$(1.11)^{41.0} = 72.0$ (LL3 scale)

Operations
Set hairline on 1.11 on the LL2.
Move left index of C to hairline.
Move hairline to 0.041 on C.
Read 1.00429 at hairline on LL0.
Observe hairline set at 0.41 on C.
Read 1.0437 at hairline on LL1.
Observe hairline set at 4.1 on C.
Read 1.534 at hairline on LL2.
Observe hairline set at 41.0 on C.
Read 72.0 at hairline on LL3.

This example serves to exhibit rule two, that powers *increase* by tenths as we move up successive LL scales, or as we move up successive LL scales, the powers increase by tenths.

We note further that 1) by continuing "up the scale" and 2) by continuing to increase the magnitude of the power, we extend beyond the range of the LL scales. Consideration of the solution to this condition is discussed in this chapter in the section on logarithms using the L scale.

Example 4.4 $(1.00555)^{1.72} = 1.00957$

Operations
Set hairline at 1.00555 on LL0.
Move 1.72 on CI to hairline.
Move hairline to right index of CI.
At hairline read 1.00957 on LL0.

Example 4.5 $(650)^{0.5} = 25.5$

Operations
Set hairline at 650 on LL3.
Move right index of C to hairline.
Move hairline to .5 on C.
Read 25.5 at hairline on LL3.

Example 4.6 $(220)^{0.000352} = 1.0019$

Operations
Set hairline on 220 on LL3.
Move right index of C to hairline.
Move hairline to 0.000352 on C.

Turn over slide rule to reverse side.
Read 1.0019 at hairline on LL0.

Example 4.7 $(1.0125)^{492} = 450$

Operations Set hairline on 1.0125 on LL1.
Move left index of C to hairline.
Move hairline to 492 on C.
Read 450 at hairline on LL3.

### POWERS AND ROOTS OF NUMBERS LESS THAN 0.999

The four reciprocal log log scales LL/0, LL/1, LL/2, and LL/3 are used with CI or C scales in raising numbers *less* than 0.999 to a power. If the reciprocal log log scales were placed end to end, they would form one continuous scale from approximately 0.000045 to 0.999. See Figure 4.3. The reciprocal log log scales are in red and read from right to left.

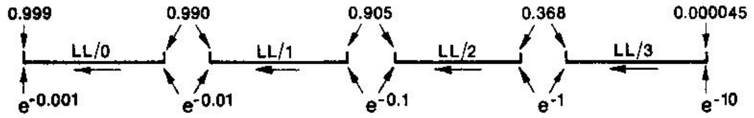



**Figure 4.3 – Reciprocal Log Log Scales LL/0, LL/1, LL/2, and LL/3 Placed End to End.**

As with the log log scales, the reciprocal log log scales have similar working rules.

1. Powers *decrease* when moving "up" the reciprocal log log scales.
2. Powers *decrease* by tenths when moving "up" successive reciprocal log log scales.

"Up" refers to a higher mathematical value and not necessarily to scale numbers.

Example 4.8 Verify
$(0.876)^{51.5} = 0.0011$ (LL/3 scale)
$(0.876)^{11.5} = 0.218$ (LL/3 scale)
$(0.876)^{4.0} = 0.589$ (LL/2 scale)
$(0.876)^{1.0} = 0.876$ (LL/2 scale)
$(0.876)^{0.6} = 0.9236$ (LL/1 scale)
$(0.876)^{0.13} = 0.98294$ (LL/1 scale)
$(0.876)^{0.058} = 0.99235$ (LL/0 scale)
$(0.876)^{0.015} = 0.99802$ (LL/0 scale)



Operations Set hairline on 0.876 on LL/2.
Move left index of C to hairline.
Move hairline to 51.5 on C.
Read 0.0011 at hairline on LL/3.
Move hairline to 11.5 on C.
Read 0.218 at hairline on LL/3, etc.

Continue verification by moving hairline to power reading on the C scale and reading the answer on the reciprocal LL scales. This example serves to exhibit rule one, that the powers *decrease* as we move up the reciprocal LL scales, or as we decrease powers, we are moving up the reciprocal LL scales.

Example 4.9 Verify
$(0.272)^{4.1} = 0.0048$ (LL/3 scale)
$(0.272)^{0.41} = 0.5862$ (LL/2 scale)
$(0.272)^{0.041} = 0.948$ (LL/1 scale)
$(0.272)^{0.0041} = 0.99468$ (LL/0 scale)

Operations Set hairline on 0.272 on LL/3.
Move left index of C to hairline.
Move hairline to 4.1 on C.
Read 0.0048 at hairline on LL/3.
Observe hairline set at 0.41 on C.
Read 0.5862 at hairline on LL/2.
Observe hairline set at 0.041 on C.
Read 0.948 at hairline on LL/1.
Observe hairline set at 0.0041 on C.
Read 0.99468 at hairline on LL/0.

This example serves to exhibit rule two, that powers *decrease* by tenths as we move up the reciprocal LL scales or as we move up successive reciprocal LL scales, the powers decrease by tenths.

Example 4.10 $(0.99646)^{1.54} = 0.99456$

Operations Set hairline at 0.99646 on LL/0.
Move 1.54 on CI to hairline.
Move hairline to right index of CI.
At hairline read 0.99456 on LL/0.

Example 4.11 $(0.554)^{8.65} = 0.0060$

Operations Set hairline at 0.554 on LL/2.
Move right index of C to hairline.

Move hairline to 8.65 on C.
At hairline read 0.0060 on LL/3.

Example 4.12 $(0.00016)^{0.54} = 0.0089$

Operations
Set hairline at 0.00016 on LL/3.
Move right index of C to hairline.
Move hairline to 0.54 on C.
At hairline read 0.0089 on LL/3.

Example 4.13 $(0.9435)^{22.2} = 0.275$

Operations
Set hairline at 0.9435 on LL/1.
Move right index of C to hairline.
Move hairline to 22.2 on C.
At hairline read 0.275 on LL/3.

### RECIPROCALS OF NUMBERS

The log log and reciprocal log log scales are additionally related. Any number on a log log scale has its reciprocal directly opposite on the corresponding reciprocal log log scale. Conversely any number on a reciprocal log log scale has its reciprocal on the corresponding log log scale.

Example 4.14 Reciprocal of 2 is $\frac{1}{2}$ or 0.5

Operations
Set hairline on 2 on LL2.
At hairline read 0.5 on LL/2.

Example 4.15 Reciprocal of 0.99738 is 1.00263

Operations
Set hairline on 0.99738 on LL/0.
At hairline read 1.00263 on LL0.

### NEGATIVE POWERS AND ROOTS OF NUMBERS

Negative powers of numbers may be obtained by use of reciprocals. Remembering that $N^{-P} = \frac{1}{N^P}$, we may use the operations necessary to determine $N^P$, then by reading the corresponding reciprocal scale, $N^{-P}$ is obtained.

Example 4.16 $(25)^{-0.67} = 0.116$

Operations
Set hairline at 25 on LL3.
Move right index of C to hairline.



Move hairline to 0.67 on C.
Read 0.116 at hairline on LL/3.

Observe that the hairline is set to also read $(25)^{0.67}$ on the LL3 scale.

Example 4.17 $(0.985)^{-37.2} = 1.755$

Operations
Set hairline on 0.985 on LL/3.
Move right index of C to hairline.
Move hairline to 37.2 on C.
Read 1.755 at hairline on LL2.

Observe that the hairline is set to also read $(0.985)^{37.2}$ on the LL/2 scale.

#### Exercise 4.1

**Powers and Roots of Numbers Greater Than 1.001 and Less Than 0.999, Reciprocals and Negative Powers and Roots of Numbers.**

Determine the following powers or roots of numbers.

| | | |
|---|---|---|
| 1. $1.015^{4.38}$ | 8. $1.0247^{1.32}$ | 15. $1.164^{2.08}$ |
| 2. $1.00261^{3.92}$ | 9. $2.96^{0.737}$ | 16. $1.1235^{0.139}$ |
| 3. $2.453^{0.75}$ | 10. $1.0633^{0.83}$ | 17. $1.0071^{0.5}$ |
| 4. $0.9985^{3.74}$ | 11. $0.9816^{1.19}$ | 18. $0.875^{3.94}$ |
| 5. $0.868^{0.117}$ | 12. $0.114^{0.303}$ | 19. $0.9812^{11.7}$ |
| 6. $0.301^{0.0166}$ | 13. $1.00164^{3.2}$ | 20. $1.0446^{2.54}$ |
| 7. $0.922^{4.1}$ | 14. $0.568^{9.1}$ | 21. $0.114^{0.252}$ |

Find the reciprocals of the following numbers.

| | | |
|---|---|---|
| 22. 1.0248 | 25. 1.1503 | 28. 24.5 |
| 23. 1.00347 | 26. 0.99438 | 29. 0.998515 |
| 24. 0.9529 | 27. 0.8035 | 30. 0.00305 |

Determine the following negative powers or roots of numbers.

| | | |
|---|---|---|
| 31. $415^{-0.75}$ | 34. $1.31^{-3.2}$ | 37. $0.877^{-2.5}$ |
| 32. $0.99245^{-1.2}$ | 35. $1.22^{-0.33}$ | 38. $1.0745^{-0.95}$ |
| 33. $1.0063^{-0.25}$ | 36. $0.841^{-0.29}$ | 39. $10^{-0.063}$ |

### 4.2 POWERS AND ROOTS OF e

Since e appears as the left index of the LL3 scale, also the right index of the LL2 scale, all numbers on the log log and reciprocal log log represent powers of e. Also, since all of the log log scales are located on the body of the rule and are used with the D scale, the powers are read by simply setting the hairline. If x represents a number to which the hairline is set on the D scale, values of $e^x$ appear at the hairline on the log log scales. LL0, LL1, LL2, and LL3 are used for positive powers of e; whereas LL/0, LL/1, LL/2, and LL/3 are used for negative powers of e. To aid the operator in remembering this relationship, the symbol x appears at the left end of the D scale, the symbol $e^x$ at the left end of the LL scales, and $e^{-x}$ at the left end of the reciprocal log log scales.

Additional aid in reading powers of e on the log log and reciprocal log log scales is provided in the scale instruction symbols opposite the right ends of the scales. The scale instruction symbols indicates the range of x in $e^x$ or $e^{-x}$ covered by each scale. The arrows indicate the direction of scale numberings.

#### POSITIVE POWERS AND ROOTS OF e

The four log log scales LL0, LL1, LL2, and LL3 are used with the D scale to raise e to a positive power or root. A direct one step procedure is all that is needed. Since e appears as an index of the LL3 scale, we merely set the hairline on the value of x on the D scale and read the answer on the appropriate log log scale. It should be noted that since e is the left index of LL3, it follows that the left index of LL2 is $e^{0.1}$; LL1 is $e^{0.01}$ and LL0 is $e^{0.001}$. Refer to Figure 4.2.

Example 4.18 $e^{1.44} = 4.22$

Operations
Move hairline to 1.44 on D.
Select appropriate scale as indicated by scale instruction symbols at right.
Read 4.22 at hairline on LL3.

Example 4.19 $e^{0.00623} = 1.00625$

Operations
Move hairline to 0.00623 on D.
Select appropriate scale as indicated by scale instruction symbols at right.
Read 1.00625 at hairline on LL0.



#### NEGATIVE POWERS AND ROOTS OF e

The four reciprocal log log scales LL/0, LL/1, LL/2, and LL/3 are used with the D scale to raise e to a negative power or root. Again, only a direct one step procedure is needed. The right index of LL/0 is $e^{-0.01}$; LL/1 is $e^{-0.1}$; LL/2 is $e^{-1}$ and LL/3 is $e^{-10}$. Refer to Figure 4.3.

Example 4.20 $e^{-1.44} = 0.237$

Operations
Move hairline to 1.44 on D.
Select appropriate scale as indicated by scale instruction symbol at right.
Read 0.237 at hairline on LL/3.

Example 4.21 $e^{-0.00623} = 0.99378$

Operations
Move hairline to 0.00623 on D.
Select appropriate scale as indicated by scale instruction symbol at right.
Read 0.99378 at hairline on LL/0.

**Exercise 4.2**

POWERS AND ROOTS OF e

| | | |
|---|---|---|
| 1. $e^{2.04}$ | 5. $e^{0.204}$ | 9. $e^{0.0204}$ |
| 2. $e^{0.00204}$ | 6. $e^{-2.04}$ | 10. $e^{-0.204}$ |
| 3. $e^{-0.0204}$ | 7. $e^{-0.00204}$ | 11. $e^{-3}$ |
| 4. $e^{-0.665}$ | 8. $e^{-0.0352}$ | 12. $3^{-0.0332}$ |

### 4.3 SPECIALIZED OPERATIONS USING LOG LOG SCALES

The Log Log scales are so constructed that they conveniently lend themselves to other specialized mathematical operations. These computations are somewhat typical of problems found in engineering and mathematics. They will serve as a useful reference and operational tool to both engineer and mathematicians.

#### EXPONENTIAL EQUATIONS

Equations of the form $N^p = A$, in which N and A are known quantities, may be solved for the unknown exponent p. The problem may be stated thus; to what exponent p must N be raised so that the result is A? Steps in the process may be described as follows:

1. Set the hairline to the number N on a log log scale,
2. Set an index of C to the hairline,

3. Move the hairline to the number A on a log log scale,
4. Read the exponent p on C.

This process is the reverse of that used for determining powers of numbers.

Example 4.22 $(25.5)^p = 17.5$

Operations Set hairline to 25.5 on LL3.
Move right index of C to hairline.
Move hairline to 17.5 on LL3.
At hairline read p = 0.884 on C.

Example 4.23 $(2.4)^p = 185$

Operations Set hairline to 2.4 on LL2.
Move right index of C to hairline.
Move hairline to 185 on LL3.
At hairline read p = 5.96 on C.

Example 4.24 $e^p = 7.7$

Operations Set hairline to 7.7 on LL3.
Observe range of scale instruction symbol.
At hairline read p = 2.04 on D.

### HYPERBOLIC FUNCTIONS

Certain combinations of powers of e, which occur frequently in engineering and applied mathematics, are known as hyperbolic functions. The three hyperbolic functions most commonly used are defined as follows:

hyperbolic sine of x: $\sinh x = \frac{1}{2}(e^x - e^{-x})$

hyperbolic cosine of x: $\cosh x = \frac{1}{2}(e^x + e^{-x})$

hyperbolic tangent of x: $\tanh x = \frac{\sinh x}{\cosh x} \quad \frac{e^{2x}-1}{e^{2x}+1}$

The hyperbolic functions may be determined by substituting the powers of e read from the log log and reciprocal log log scales. For example:



$$\sinh 0.434 = \frac{1}{2}(e^{0.434} - e^{-0.434}) = \frac{1}{2}(1.544 - 0.648) = 0.448$$

$$\cosh 0.434 = \frac{1}{2}(e^{0.434} + e^{-0.434}) = \frac{1}{2}(1.544 + 0.648) = 1.096$$

$$\tanh 0.434 = \frac{e^{0.868}-1}{e^{0.868}+1} = \frac{2.382-1}{2.382+1} = \frac{1.382}{3.382} = 0.408$$

It should be noted that, once the sinh and cosh have been found, the tanh may be found by the expression, $\tanh x = \frac{\sinh x}{\cosh x}$. Values $e^x = e^{0.434} = 1.544$ and $e^{-x} = e^{-0.434} = 0.648$ were taken from the LL2 and LL/2 scales by only a single setting of the hairline to 0.434 on the D scale; $e^{2x} = e^{0.868} = 2.382$ was read on LL2 with the hairline set at 0.868 on the D scale.

The inverse of the hyperbolic functions may also be evaluated by use of the log log scales. If the value of a hyperbolic function such as sinh x, cosh x, or tanh x is given or known, the value of x may then be found by substituting the known value into the formulas given below, in which A, B or C are known:

If sinh x = A, then $e^x = A + \sqrt{A^2 + 1}$.

If cosh x = B, then $e^x = B + \sqrt{B^2 - 1}$.

If tanh x = C, then $e^x = \sqrt{\frac{1+C}{1-C}}$.

The recommended procedure is to first substitute the known values into the formulas, thus solving for $e^x$. (The $R_1$ and $R_2$ scales are extremely convenient for this work.) Then set the hairline to $e^x$ on the appropriate log log scale and read x at the hairline on the D scale. For example, if sinh x is given as 2.12, x may be evaluated as follows: since A = 2.12, $A + \sqrt{A^2 + 1} = 2.12 + \sqrt{4.50 + 1} = 4.46$. Now set the hairline to 4.46 on LL3, and read x = 1.496 at the hairline on D.

### LOGARITHMS TO ANY BASE

The log log scales are so constructed that logarithms to *any* base may easily be determined. By this method complete logarithms including both characteristic and mantissa are obtained directly. Let a number "a" represent the base of logarithms which is to be used. Mathematically then, $a^p = N$ or $p = \log_a N$, where the exponent or logarithm p is to be determined for number N to the base "a". Taking logarithms of both sides to the base e, we obtain $p \log_e a$

$= \log_e N$ or $p = \dfrac{\log_e N}{\log_e a}$. The numerator $\log_e N$ is determined by setting

the hairline to the number N on a log log scale, the numerator then appearing directly opposite on the D scale. The denominator appears directly opposite the base "a" set on a log log scale. To obtain p the $\log_e N$ is simply divided by $\log_e a$.

As an example, in the expression $\log_{10} 9.1$, the base $a = 10$. Setting the hairline to 10 on LL3, the right index of C is set to the hairline. The hairline is then moved to 9.1 on LL3 and $\log_{10} 9.1 = 0.959$ is read at the hairline on C.

$\text{Log}_{10}$ 800 may be determined as follows: with left index of C aligned with 10 on LL3, move the hairline to 800 on the LL3 and read 2.903 at the hairline on C. Since the characteristic must be 2, the complete logarithm to the base 10 is 2.903.

If many computations of the above type are to be made, it may be advantageous to remove the slide and to reinsert it reversed. Then use the CF scale instead of the C scale. This will eliminate the possibility of being "off scale", without having to turn the rule over during the computations.

To obtain logarithms, for example, to the base 8, we may set the index of CF opposite 8 on LL3. Moving the hairline to a number on an LL scale, its logarithm to the base 8 is read at the hairline on CF. For example $\log_8 200 = 2.55$.

The log log scales are especially useful for determining the values of logarithms to the base e (natural logarithms). The natural logarithm of a number is readily found by merely setting the hairline to the number on the log log scale and reading the logarithm to the base e on the D scale. For example, the log 7.7 is found by simply moving the hairline to 7.7 on the LL3 scale and read the natural log of 2.04 on the D scale.



**Exercise 4.3**

**Exponential Equations, Hyperbolic Functions and Logarithms to Any Base**

Solve for the exponent p in the following equations:

1. $(9.1)^p = 16.4$
2. $(3.25)^p = 71.5$
3. $(0.915)^p = 0.614$
4. $(0.425)^p = 0.0174$

Determine the values of the following hyperbolic functions:

5. sinh 0.2
6. sinh 3.0
7. cosh 0.45
8. tanh 0.35
9. tanh 2.1

Evaluate x, given the following values of the hyperbolic functions:

10. sinh x = 9.82
11. sinh x = 0.625
12. cosh x = 3.73
13. cosh x = 1.32
14. tanh x = 0.917
15. tanh x = 0.300

Determine the logarithm of the following:

16. $\log_e 1.0242$
17. $\log_{10} 61.8$
18. $\log_4 160$
19. $\log_e 270$

## 4.4 LOGARITHMS, ANTILOGARITHMS, POWERS AND ROOTS USING THE L SCALE

The uniformly divided L scale serves as a simple table of common logarithms. The antilogarithms may be found by reversing the procedure. The L scale also provides a means of determining the powers of numbers which fall outside the range of the LL scales.

### COMMON LOGARITHMS

Common logarithms (logarithms to the base 10) may be found directly by use of the L scale. If the hairline is set to a number on the D scale, the mantissa of the common logarithm of the number may be read on the L scale. Both D and L scales are located on the body of the rule, hence no slide movement is required. The characteristic of the logarithm must be determined mentally, keeping in mind that $\log_{10}1$ is zero, $\log_{10}10$ is 1, $\log_{10}100$ is 2, etc. Any number between 1 and 10 will therefore have a characteristic of 0, and any number between 10 and 100 will have a characteristic of 1, etc.

Example 4.25 $\log_{10}79.8 = 1.902$

Operations
Set hairline at 79.8 on D.
At hairline read mantissa .902 on L.
Introduce characteristic 1.902

Example 4.26 $\log_{10}0.00207 = \bar{3}.316$

Operations
Set hairline at 0.00207 on D.
At hairline read mantissa .316 on L.
Introduce characteristic $\bar{3}.316$

### ANTILOGARITHMS

Finding the antilogarithms is just the reverse of finding a logarithm, that is, given $\log_{10}N$, we must find N. Thus "antilogarithm N" means the number whose logarithm is N.

Example 4.27 Antilogarithm 2.086 = 121.9

Operations
Set hairline at mantissa 0.086 on L.
At hairline read 1219 on D.
Consider characteristic 121.9

Example 4.28 Antilogarithm $\bar{1}.835 = 0.684$

Operations
Set hairline at mantissa 0.835 on L.
At hairline read 684 on D.
Consider characteristic 0.684

### POWERS AND ROOTS

The L scale serves as a simple table of common logarithms to determine powers and roots of numbers which fall outside the range of the LL scales. In the expression, $A = N^P$, we can operate as follows:

$$A = N^P$$

$$\text{then } \log_{10}A = P \log_{10}N$$

$$A = \text{antilogarithm } (P \log_{10}N)$$



Example 4.29 $1.074^{200} = 1{,}585{,}000$

Operations
Set hairline at 1.074 on D.
At hairline read mantissa .031 on L.
Introduce characteristic 0.031.
Multiply $200 \times 0.031 = 6.2$.
Set hairline at mantissa 0.2 on L.
At hairline read 1585 on D.
Consider characteristic 1,585,000.

**Exercise 4.4**

**Logarithms, Antilogarithms, Powers and Roots Using the L Scale**

Determine the common logarithms of the following:

| | | |
|---|---|---|
| 1. $\log_{10}12.6$ | 3. $\log_{10}0.0312$ | 5. $\log_{10}1.675$ |
| 2. $\log_{10}61.8$ | 4. $\log_{10}1894$ | 6. $\log_{10}0.815$ |

Determine the common antilogarithm of the following:

| | | |
|---|---|---|
| 7. antilog 2.047 | 9. antilog 0.865 | 11. antilog $\bar{3}.214$ |
| 8. antilog $\bar{1}.835$ | 10. antilog 1.391 | 12. antilog $\bar{1}.281$ |

Determine the values of the following:

| | | |
|---|---|---|
| 13. $(1.055)^{300}$ | 14. $(35.4)^4$ | 15. $(0.000025)^{1.3}$ |

# TRIGONOMETRIC OPERATIONS

Finding trigonometric functions on the VERSALOG II is a direct and rather simple process. The angles are read on the TT, Sec T SRT or Cos S scales. Their respective functions may be read at the hairline on C or CI scales. The angles on the trigonometric scales of the VERSALOG II slide rule are expressed in degrees and decimal parts of a degree, conforming to modern practice.

## 5.1 TRIGONOMETRIC FUNCTIONS

There are six basic trigonometric functions, or relations between the sides of a right triangle. Each angular function is expressed as the ratio of a particular pair of sides of the triangle. These six ratios are the sine, cosine, tangent, cosecant, secant, and cotangent of an angle, and are stated for convenient reference.

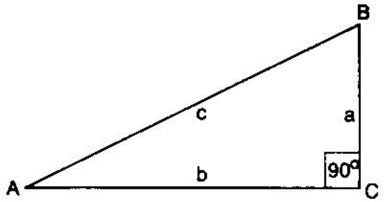



**Figure 5.1 – The Right Triangle.**

Referring to Figure 5.1, the six basic trigonometric functions may be written as:

$$\text{sine A} = \sin A = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{1}{\text{cosec A}}$$

$$\text{cosine A} = \cos A = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{1}{\sec A}$$

$$\text{tangent A} = \tan A = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{1}{\cot A}$$

$$\text{cosecant A} = \csc A = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{1}{\sin A}$$

$$\text{secant } A = \sec A = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{1}{\cos A}$$

$$\text{cotangent } A = \cot A = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{1}{\tan A}$$

Note the reciprocal nature between the first and last three functions. The first set of three (sine, cosine, and tangent) find the most frequent application and are most commonly used by engineers. Consequently scales for finding values of these functions for any given angle are found on the slide rule. Should the other ratios (cotangent, secant, cosecant) be required, they may be obtained through the use of a convenient reciprocal scale.

### 5.2 THE TRIGONOMETRIC SCALES

The three trigonometric scales are located on the slide and are designated Cos S, Sec T SRT, and TT. Each set of graduations is actually two scales in one; for each is numbered in two directions (left to right, and right to left). The angular values at the *left* of the graduations increase toward the right (left to right). Likewise, the angular values at the *right* of the graduations increase toward the left (right to left).

#### COLOR CODING

The unique color coding system used on the trigonometric scales is consistent and compatible with all of the other scales on the VERSALOG II. It is easiest to learn for beginners, and helps experienced users to avoid errors, and speeds calculations.

Black Scales: The black T, SRT and S trigonometric scales have angles numbered in black and read from left to right. These scales can be thought of and used basically like a C scale.

Red Scales: The red T and Sec T trigonometric scales have angles numbered in red and read from right to left. These scales can be thought of and used basically like a CI scale.

Green Scale: The Cos trigonometric scale has angles numbered in green and read from right to left and should be thought of and used basically like a C scale. The Cos scale is unique in its color coding in that it is the only green scale on the VERSALOG II. This coding has been adopted for values of the cosine angles so that they can be more readily distinguished from the sine angles on the same scale. Also, although the angles are read from right to left, the corresponding cosine functions are similar to the C scale and read from left to right.



Summarizing the above observations, it can be stated:

Angles are numbered in *black* or *green* when the scale is to be used like a C scale.

Angles are numbered in *red* when the scale is to be used like a CI scale.

The fundamental nature of these scales makes their use quite as simple as the C and CI scale.

#### SCALE INSTRUCTIONAL SYMBOLS

Located at the right end of each trigonometric scale are a pair of scale instructional symbols. These symbols are very helpful in placing the decimal point of the trigonometric functions. The related angular values of the trigonometric function and the scale symbols are summarized in the following table.

| Scale | Symbol: Decimal Point Range | Trigonometric Function | Angular Range |
|---|---|---|---|
| TT | 0.1 → 1.0 (Black) | tangent | 5.7° to 45° |
| | 10.0 ← 1.0 (Red) | tangent | 45° to 84.3° |
| SecT SRT | 0.01→0.1 (Black) | sine or tangent | 0.57° to 5.74° |
| | 100.0 ← 10.0 (Red) | sec or tangent | 84.26° to 89.43° |
| Cos S | 0.1 → 1.0 (Black) | sine | 5.7° to 90° |
| | 0.1 ← 1.0 (Green) | cosine | 0° to 84.3° |

The ranges of values in the above table should be memorized. A review of this section may be desirable before continuing to the next. With the use of the VERSALOG II, note particularly the color coding, numerical placement, direction of scale numbering, and the ranges of the scales as summarized above.

### 5.3 THE SINE FUNCTION OF ANGLES FROM 0° TO 90°

The sine functions of angles from 0° to 90° are found by the use of three different scales. The R values of the Sec T SRT scale are used for extremely small angles. The S values of the Sec T SRT scale are used for angles between 0.57° and 5.74°. For angles between 5.7° and 90°, the S values of the Cos S scale are used.

### SINE 0° TO 0.574° (Using R of Sec T SRT Scale)

The sine of extremely small angles is approximately equal to the size of the angle expressed in radians. Radian values of any angle may be read directly on the C scale by setting the hairline to magnitude of the angle on the Sec T SRT scale (hereafter referred to the radian scale). The decimal point can be placed mentally.

57.3° = 1 radian (approximately)
5.73° = 0.1 radian (approximately)
0.57° = 0.01 radian (approximately)
0.057° = 0.001 radians (approximately) etc.

Therefore, we can allow the radian scale to be read in any decimal multiple of the angle shown to obtain the equivalent radian value. For angles from 0° to 0.57° the radian value will be approximately equal to the value of the sine function.

Example 5.1 Find the sine of the angles 0.41°, .041°, and .0041°.

Operations Set hairline on 4.1° on the radian scale.
For 0.41° read 0.00715 radian on C,
sin 0.41° = 0.00715.
For 0.041° read 0.000715 radian on C,
sin 0.041° = 0.000715.
For 0.0041° read 0.0000715 radian on C,
sin 0.0041° = 0.0000715.

### SINE 0.574° TO 5.74° (Using S of Sec T SRT Scale)

To find the sine of an angle from 0.574° to 5.74°, the Sec T SRT scale is used with the C scale. The desired angle is set on the S scale (black) and the sine function will be found on the C scale. The decimal range can be found at the right hand scale instruction symbol, (0.01 → 0.1).

Example 5.2 Find sin 2.14° = 0.0374

Operations Opposite 2.14° on S (black) of Sec T SRT read 0.0374 on C.



### SINE 5.74° TO 90° (Using S of Cos S Scale)

To find the sine of an angle from 5.74° to 90°, the Cos S scale is used with the C scale. The desired angle is set on the S scale (black) and the sine function is read on the C scale. The decimal range can be found at the right hand scale instruction symbol, (0.1 → 1.0).

Example 5.3 Find sin 32° = 0.530

Operations Opposite 32° on S (black) of Cos S read 0.530 on C.

It follows that the angle value can be found if the sine function value is known. Merely set the sine value on the C scale and read the angular value on the radian, on Sec T SRT or on Cos S scale, the choice being made depending upon the decimal point location.

**Exercise 5.3**

**The Sine Functions of Angles from 0° to 90°**

Find the sine values of the following angles:

| | | |
|---|---|---|
| 1. Sin 34.5° | 6. Sin 5.3° | 11. Sin 33° 15′ |
| 2. Sin 0.3° | 7. Sin 8.5° | 12. Sin 60° |
| 3. Sin 1.2° | 8. Sin 13° 40′ | 13. Sin 0.25° |
| 4. Sin 12.4° | 9. Sin 20° 30′ | 14. Sin 80° |
| 5. Sin 2° 50′ | 10. Sin 25° 50′ | 15. Sin 3.5° |

Find the angle values of the following sine functions:

| | | |
|---|---|---|
| 16. Sin ___ = 0.0160 | 20. Sin ___ = 0.104 | 24. Sin ___ = 0.852 |
| 17. Sin ___ = 0.0320 | 21. Sin ___ = 0.231 | 25. Sin ___ = 0.902 |
| 18. Sin ___ = 0.000291 | 22. Sin ___ = 0.358 | 26. Sin ___ = 0.020 |
| 19. Sin ___ = 0.00450 | 23. Sin ___ = 0.759 | 27. Sin ___ = 0.000582 |

## 5.4 THE COSINE FUNCTION OF ANGLES FROM 0° TO 90°

The cosine function of angles from 0° to 90° are found by the use of three different scales. For angles between 0° and 84.26° the Cos values of the Cos S scale are used. The complement of the angle is read as S values of the Sec T SRT scale is used for angles between 84.26° and 89.4°. The complement of the angle is read as the R values of the Sec T SRT scale is used for angles between 89.4° and 90°.

### COSINE 0° TO 84.26° (Using Cos of Cos S Scale)

The cosine of angles from 0° to 84.26° are found on the C scale opposite the angle setting on the Cos scale (green). The decimal range is found at the right hand scale instruction symbol, (0.1 ← 1.0)

Example 5.4 Find Cos 23.5° = 0.917

Operation Opposite 23.5 on Cos (green) of Cos S read 0.917 on C.

### COSINE 84.26° TO 89.43° (Using S of Sec T SRT Scale)

The cosine of angles from 84.26° to 89.43° are found on the C scale opposite the complement of the angle setting on the S values of the Sec T SRT scale. The cosine of an angle is equal to the sine of its complement. For this range of angles, determine the complement of the angle and find the value of the sine of this angle as described in Section 5.3.

Example 5.5 Find Cos 86.5° = 0.061

Operation Opposite 3.5° (the complement of 86.5°) on S (black) of Sec T SRT read 0.061 on C.

### COSINE 89.43° TO 90° (Using R of Sec T SRT Scale)

The cosine of angles from 89.43° to 90° are found on the C scale opposite the complement of the angle setting on R (radian) values of the Sec T SRT scale. As stated before, the cosine of the angle is equal to the sine of its complement. The complement of this range of angles is the sine of extremely small angles. Since the sine of extremely small angles is approximately equal to the size of the angle expressed in radians, the cosines are found as described in Section 5.3.

Example 5.6 Find Cos 89.6° = 0.00698

Operation Opposite 0.4° (the complement of 89.6°) on R of Sec T SRT scale read 0.00698 on C.

The angle value can be found if the cosine function value is known. Merely set the cosine value on the C scale and read the angular value on the radian, on Sec T SRT or on Cos S scale, the choice being made depending upon the location of the decimal point.



**Exercise 5.4**

**The Cosine Function of Angles from 0° to 90°**

Find the cosine values of the following angles:

| | | |
|---|---|---|
| 1. Cos 34.5° | 6. Cos 6.4° | 11. Cos 46.5° |
| 2. Cos 0.4° | 7. Cos 8.4° | 12. Cos 60° |
| 3. Cos 1.4° | 8. Cos 12° 42′ | 13. Cos 80° |
| 4. Cos 13.5° | 9. Cos 20° 30′ | 14. Cos 87.5° |
| 5. Cos 2° 48′ | 10. Cos 24° 36′ | 15. Cos 80° 18′ |

Find the angle values of the following cosine functions:

| | | |
|---|---|---|
| 16. Cos ___ = 0.052 | 20. Cos ___ = 0.809 | 24. Cos ___ = 0.00200 |
| 17. Cos ___ = 0.201 | 21. Cos ___ = 0.913 | 25. Cos ___ = 0.00350 |
| 18. Cos ___ = 0.400 | 22. Cos ___ = 0.200 | 26. Cos ___ = 0.800 |
| 19. Cos ___ = 0.588 | 23. Cos ___ = 0.020 | 27. Cos ___ = 0.900 |

## 5.5 THE TANGENT FUNCTION OF ANGLES FROM 0° TO 90°

The tangent function of angles from 0° to 90° are found by the use of five different scales. The radian scale is used to find tangent functions of extremely small angles and angles extremely close to 90°. The red and black T values of the Sec T SRT scale are used to find tangent functions of angles between 0.574° and 5.74° as well as those between 84.27° and 89.43°. All remaining tangent functions of angles between 5.74° and 84.27° are found by using the TT scale. Each scale and technique is described in detail below.

### TANGENT 0° TO 0.574° (Using R of Sec T SRT Scale)

The technique for finding tangent of angles between 0° and 0.574° is the same as described in Section 5.3, since the tangent and sine of extremely small angles are considered equal. The tangent is approximately equal to the size of the angle expressed in radians.

Example 5.7 Find tan 0.5° = 0.00872

Operation Opposite 0.5° on R of Sec T SRT scale read 0.00872 on C.

### TANGENT 0.574° TO 5.74°. (Using T (black) of Sec T SRT)

To find the tangent of an angle from 0.574° to 5.74°, the Sec T SRT scale is used with the C scale. The desired angle is set on the T (black) scale and the tangent is read directly opposite on the C scale. The decimal range can be found at the right hand scale instruction symbol, (0.01 → 0.1).

Example 5.8 Find $\tan 3° = .0524$

Operation Opposite 3° on T (black) of Sec T SRT scale read 0.0524 on C.

### TANGENT 5.74° TO 45° (Using T (black) of TT Scale)

To find the tangent of an angle from 5.74° to 45°, the TT scale is used with the C scale. The desired angle is set on the T (black) scale and the tangent is read directly opposite on the C scale. The decimal range can be found at the right hand scale instruction symbol, $(0.1 \rightarrow 1.0)$.

Example 5.9 Find $\tan 16.9° = 0.304$

Operation Opposite 16.9° on T (black) of TT scale read 0.304 on C.

### TANGENT 45° TO 84.27° (Using T (red) of TT Scale)

To find the tangent of an angle from 45° to 84.27°, the TT scale is used with the CI scale. The desired angle is set on the T (red) scale and the tangent is read directly opposite on the CI scale. The decimal range can be found at the right hand scale instruction symbol, $(10.0 \leftarrow 1.0)$.

Example 5.10 Find $\tan 75° = 3.73$

Operation Opposite 75° on T (red) of TT scale read 3.73 on CI.

### TANGENT 84.27° TO 89.43° (Using T (red) of Sec T SRT)

To find the tangent of an angle from 84.27° to 89.43°, the Sec T SRT scale is used with the CI scale. The desired angle is set on T (red) scale and the tangent is read directly opposite on the CI scale. The decimal range can be found at the right hand scale instruction symbol, $(100.0 \leftarrow 10.0)$.

Example 5.11 Find $\tan 89° = 57.3$

Operation Opposite 89° on T (red) of Sec T SRT scale read 57.3 on CI.

### TANGENT 89.43° TO 90° (Using R of Sec T SRT)

To find the tangent of an angle from 89.43° to 90°, the Sec T SRT scale is used with the CI scale. For this range of angles use the formula $\tan A = \frac{1}{\sin (90° - A)}$. The expression $\sin (90° - A)$ is



merely the sine of the complement of our desired angle. Since the complement angle is extremely small, we can use the angle expressed in radians. The CI scale is the reciprocal of the C scale and will give values of the tangent of angles between 89.43° to 90°.

Example 5.12 Find $\tan 89.5° = 114.8$

Operation Opposite 0.5° (the complement of 89.5°) on the R of Sec T SRT scale read 114.8 on CI.

The angle values can be found if the tangent function is known. Set the tangent function on the C or CI scale, whichever applies, and read the angular value on the radian, Sec T SRT or TT scale, the choice being made depending upon the decimal point location.

**Exercise 5.5**

**The Tangent Function of Angles from 0° to 90°**

Find the tangent values of the following angles:

| | | |
|---|---|---|
| 1. tan 43.5° | 6. tan 17° 54′ | 11. tan 84.5° |
| 2. tan 0.2° | 7. tan 23.2° | 12. tan 89° |
| 3. tan 1.6° | 8. tan 42° 30′ | 13. tan 30′ |
| 4. tan 4.3° | 9. tan 48° | 14. tan 1° 24′ |
| 5. tan 6° 24′ | 10. tan 64° | 15. tan 44.7° |

Find the angle values of the following tangent functions:

| | | |
|---|---|---|
| 16. tan ___ = 0.00873 | 20. tan ___ = 0.121 | 24. tan ___ = 57.3 |
| 17. tan ___ = 0.0349 | 21. tan ___ = 0.784 | 25. tan ___ = 143 |
| 18. tan ___ = 0.000291 | 22. tan ___ = 1.170 | 26. tan ___ = 0.00349 |
| 19. tan ___ = 0.0750 | 23. tan ___ = 3.73 | 27. tan ___ = 0.102 |

## 5.6 THE SECANT, COSECANT, AND COTANGENT FUNCTIONS OF ANGLES

Although secants of angles from 84.27° to 89.43° may be read from the Sec (red) of the Sec T SRT scale. The following relationships lead to a more general method for finding secant, cosecant, and cotangent of an angle.

since: $\sec A = \frac{1}{\cos A}$, solve for cos A on C, read sec A on CI.

since: $\csc A = \frac{1}{\sin A}$, solve for sin A on C, read csc A on CI.

since: $\cot A = \frac{1}{\tan A}$, solve for tan A on C or CI, read tan A on CI or C respectively.

Care must be exercised in locating the decimal point for the reciprocal functions.

### 5.7 SMALL ANGLES EXPRESSED IN MINUTES OR SECONDS

Special marks on the Sec T SRT scale simplify the process of finding sines and tangents of small angles when they are expressed in minutes or seconds, or of converting these angles to radian measure. The operations are based on the fact, previously discussed, that the sine, tangent, and radian measure of small angles are approximately equal. Since one minute equals approximately 1/3436 radians, we merely divide the number of minutes by 3436 to obtain the sine, tangent, and radian measure of the angle. To facilitate this division, a small dot is placed on the Sec T SRT scale opposite 3436 on the C scale.

Example 5.13 Convert 49.2′ into radians = 0.0143

Operation
Move the hairline to 49.2′ on D.
Set simple dot (on Sec T SRT) at hairline.
Read 0.0143 radians at left index of C.

To convert an angle expressed in seconds to radian measure, a similar mark, but with two dots, is placed on the Sec T SRT scale opposite 206 on the C scale. This is because one second is approximately equal to 1/206,240 radians.

Example 5.14 Conver 29″ to radians = 0.0001406

Operation
Move hairline to 29″ on D.
Set double dot (on Sec T SRT) at hairline.
Read .0001406 radians at left index of C.



In placing the decimal points in examples 5.13 and 5.14, it is helpful to recall that one minute equals 0.000291 radians, and one second equals 0.00000485 radians.

### 5.8 ANGLES IN RADIANS

In addition to the direct reading radian scale of Sec T SRT, angles in radians may be converted to degrees and vice versa by use of a multiplication factor. Since one radian is equal to $\frac{180}{\pi} = 57.3°$, the angle in radians multiplied by 57.3 equals the angle in degrees. For convenience, a graduation designated r has been placed at this point on the C and D scales on one face of the rule.

Example 5.15 Convert to degrees; 0.53, 2.19 and 1.43 radians.

Operation
Set right index of C to r on D.
Move hairline to 0.53 on C.
Read 30.4° on D at hairline.
Move hairline to 2.19 on C.
Read 125.5° on D at hairline.
Move hairline to 1.43 on CF.
Read 82° on DF at hairline.

Example 5.16 Convert to radians; 30.4°, 125.5°, an 82°.

Operation
Set r on C to right index of D.
Move hairline to 30.4° on C.
Read 0.53 on D at hairline.
Move hairline to 125.5° on C.
Read 2.19 on D at hairline.
Move hairline to 82° on CF.
Read 1.43 on DF at hairline.

The following table is a summary of the above sections on the trigonometric functions and should serve as a useful reference.

| Angle Range | Trigonometric Function | Angle Read On | Function Read On | Comments |
|---|---|---|---|---|
| 0° to 0.574° | Sine | Sec T SR T | C | Radians |
| | Cosine | Cos S | C | Direct |
| | Tangent | Sec T SR T | C | Radians |
| 0.574° to 5.74° | Sine | Sec T SR T | C | Direct |
| | Cosine | Cos S | C | Direct |
| | Tangent | Sec T SR T | C | Direct |
| 5.74° to 45° | Sine | Cos S | C | Direct |
| | Cosine | Cos S | C | Direct |
| | Tangent | T T | C | Direct |
| 45° to 84.27° | Sine | Cos S | C | Direct |
| | Cosine | Cos S | C | Direct |
| | Tangent | T T | CI | Direct |
| 84.27° to 89.43° | Sine | Cos S | C | Direct |
| | Cosine | Sec T SR T | C | Use Compliment |
| | Tangent | Sec T SR T | CI | Direct |
| 89.43° to 90° | Sine | Cos S | C | Direct |
| | Cosine | Sec T SR T | C | Use Compliment |
| | Tangent | Sec T SR T | CI | Use $\frac{1}{\sin(90° - P)}$ |



### 5.9 COMBINED OPERATIONS WITH THE TRIGONOMETRIC SCALES

Calculations involving products and quotients of trigonometric functions may be performed by using the trigonometric scales without actually reading the functions from the C or CI scales. It is only necessary to remember to use any scale as a C scale when the angles are numbered black or green and as a CI scale when the angles are numbered red. Examples of this type of computation follow:

Example 5.17 $9.2 \sin 43° \cos 70.46° = 2.10$

Operations
Set right index of C at 9.2 on D.
Move hairline to 43° on S.
Set right index of C to hairline.
Move hairline to 70.46° on Cos.
Read 2.10 on D at hairline.

Example 5.18 $10.1 \tan 18.5° \tan 48° = 3.75$

Operations
Set left index of C at 10.1 on D.
Move hairline to 18.5° on T.
Move 48° on T to hairline.
Read 3.75 on D at right index of C.

Example 5.19 $\frac{12.8 \tan 19° \sin 47°}{\cos 25° \tan 32°} = 5.69$

Operations
Set left index of C at 12.8 on D.
Move hairline to 19° on T.
Set right index of C to hairline.
Move hairline to 47° on S.
Move 25° on Cos to hairline.
Move hairline to right index of C.
Move 32° on T to hairline.
Read 5.69 on D at right index of C.

**Exercise 5.9**

**Combined Operations with the Trigonometric Scales**

Solve the following expressions:

1. $14 \sin 28°$
2. $2.5 \tan 43°$
3. $\dfrac{18}{\cos 65°}$
4. $2.7 \cos 20°$
5. $4.2 \cot 27°$
6. $\dfrac{27 \sin 30° \sin 45°}{2}$
7. $\dfrac{28 \sin 40°}{\sin 60°}$
8. $\dfrac{64.5}{\sin 32.5°}$
9. $18 \sin 28° \cos 42.5°$
10. $\dfrac{16 \cos 50°}{\sin 24°}$
11. $\dfrac{34 \sin 30° \sin 65°}{\sin 85°}$
12. $\dfrac{14 \tan 34° \sin 60°}{\sin 30°}$

## 5.10 SOLUTION OF TRIANGLES

Many practical problems involving the trigonometric functions are efficiently and rapidly calculated with a slide rule. In this section, a number of typical trigonometric applications involving right triangles and oblique triangles are illustrated. A familiarity of the trigonometric functions defined in Section 5.1 and a thorough understanding of the numerical ranges of the trigonometric scales as shown in the table in Section 5.2 is essential.

### RIGHT TRIANGLES

Example 5.20 Find the length of the hypotenuse (side c) of the triangle in Figure 5.2.

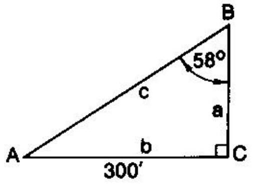



Figure 5.2.

Operations $\sin 58° = \dfrac{300'}{c}$

Set hairline to 300′ on D.
Move 58° on S to hairline.
Read c, 354′ on D at right index of C.

Example 5.21 Solve the triangle in Figure 5.3 for A, B, and a.



Operations $\sin B = \dfrac{2.33''}{9.6''} = \cos A$

$c \sin A = a$
Set right index of C to 9.6″ on D.
Move hairline to 2.33″ on D.
Read B = 14.05° on S at hairline.
Read A = 75.95° on Cos at hairline.
Move hairline to 75.95° on S.
Read 9.3″ on D at hairline.

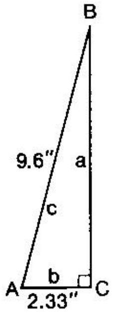



Figure 5.3.

Example 5.22 Solve the triangle in Figure 5.4 for a, b, and B

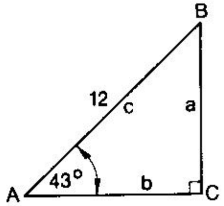



Figure 5.4.

Operations $a = c \sin A$
$b = c \cos A$
$B = 90° - A$
Set left index of C to 12 on D.
Move hairline to 43° on S.
Read a = 8.18 on D at hairline.
Move hairline to 43° on Cos.
Read b = 8.76 on D at hairline.
$B = 90° - A = 90° - 43° = 47°$.

Example 5.23 Solve the triangle in Figure 5.5 for A, B, and c

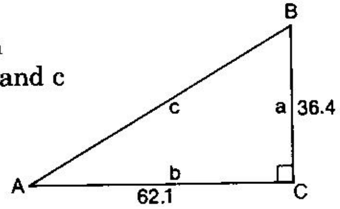



Figure 5.5.

Operations $\text{Tan } A = \dfrac{36.4}{62.1}$
$B = 90° - A$
$c = \dfrac{36.4}{\sin A}$
Set right index of C to 62.1 on D.
Move hairline to 36.4 on D.
Read A = 30.4° on T (black) at hairline.
Read B = 59.6° on T (red) at hairline.
Move 30.4° on S to hairline.
Read c = 72 on D at right index of C.

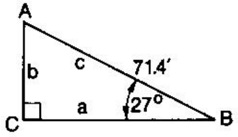



Figure 5.6.

Example 5.24 Solve the triangle in Figure 5.6 for A, a, and b

Operations Applying the law of sines, which may be written $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C'}$ solve as a proportion.

$$\frac{a}{\sin (90° - 27°)} = \frac{b}{\sin 27°} = \frac{71.4'}{\sin 90°}$$

Set hairline to 71.4′ on D.
Move 90° on S to hairline.
Move hairline to A, 63° (90° − 27°) on S.
Read a = 63.6′ on D at hairline.
Move hairline to 27° on S.
Read b = 32.4′ on D at hairline.

## OBLIQUE TRIANGLES

The two basic formulas for the solution of oblique triangles are the law of sines, as previously stated in example 5.24

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The other is the law of cosines. When the three sides are given, the angles can be determined using the law of cosines.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

While the law of cosines can be written in three forms (one for each angle), it is preferable to use it to find just one angle and use the law of sines, which is easier to calculate, for finding the other angles.

Figure 5.7

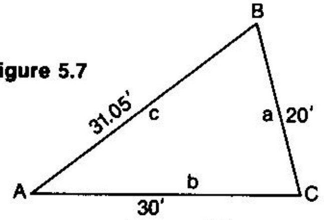



Figure 5.7

Example 5.25 Solve the triangle in Figure 5.7 for A, B, and C



Operations $\text{Cos } A = \frac{900 + 964 - 400}{2(30)31.05} = \frac{1464}{1863}$

Set right index of C to 1863 on D.
Move hairline to 1464 on D.
Read A = 38.2° on Cos at hairline.

$$\frac{20}{\sin 38.2°} = \frac{30}{\sin B} = \frac{31.05}{\sin C}$$

Set hairline to 20 on D.
Move 38.2° on S to hairline.
Move hairline to 30 on D.
Read B = 68.1° on S at hairline.
Move hairline to 31.05 on D.
Read C = 73.7° on S at hairline.
C can be accurately determined also by 180° − A − B = 73.7°.

Example 5.26 Solve the triangle in Figure 5.8 for B, C, and a

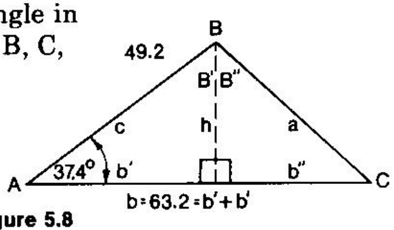



Figure 5.8

Operations A perpendicular, h, is drawn from B to the base to form two right triangles.

$$\frac{h}{\sin 37.4°} = \frac{49.2}{\sin 90°} = \frac{b'}{\sin B'}.$$

Set right index of C to 49.2 on D.
Move hairline to 37.4° on S.
Read h = 29.8 on C at hairline.
Move hairline to 52.6° on S.
Read b′ = 39.1.
Then, b″ = 63.2 − 39.1 = 24.1.

$$\tan C = \frac{29.8}{24.1}$$

Set right index of C to 29.8 on D.
Move hairline to 24.1 on D.
Read C = 51° on T.

$$\text{Then, } \frac{29.8}{\sin 51^\circ} = \frac{24.1}{\sin B''} = \frac{a}{\sin 90^\circ}$$

Set hairline to 29.8 on D.
Move 51° on S to hairline.
Move hairline to 24.1 on D.
Read B″ = 39° on S at hairline.
Read a = 38.4 on D at right index of C (90° on S).
B = 52.6° + 39° = 91.6°.

**Exercise 5.10**
**Solution of Triangles**

Solve the following triangles:

1. c = 60, a = 40, C = 90°
2. a = 80, A = 75°, C = 90°
3. A = 50°, B = 55°, c = 25
4. a = 20, b = 25, C = 60°
5. a = 12.3, b = 20.2, C = 90°
6. a = 20, b = 25, c = 30
7. a = 24, A = 45°, C = 90°
8. b = 0.46, c = 0.58, C = 90°
9. a = 13.2, B = 22°40′, C = 90°
10. a = 14, A = 9°, C = 90°
11. a = 101, b = 116, C = 90°
12. a = 50, b = 23.3, C = 90°
13. a = 621, b = 227, C = 90°
14. a = 15, b = 18, C = 36°
15. A = 68°, B = 59°, a = 25
16. A = 28.6°, a = 52.8, c = 12.4
17. a = 75, c = 100, B = 42°
18. b = 28.3, c = 36.7, A = 37.3°
19. a = 30, b = 50, c = 60
20. a = 35.8, b = 47.2, c = 54.3

## 5.11 VECTOR ANALYSIS AND COMPLEX NUMBERS

From the standpoint of the mechanics of the computation, vector analysis is essentially an application of right triangle analysis and solution. The theory of complex numbers deals with subject matter only slightly more complicated, but conveniently solved with the VERSALOG II slide rule.

### VECTOR ANALYSIS

A vector is a quantity having both magnitude and direction. Using rectilinear coordinates, x and y, a vector quantity is completely described by an x component and a y component which are combined (i.e., the vector sum is taken) to form resultant R, oriented at some angle Θ with the horizontal.



$$x = R \cos \Theta$$
$$y = R \sin \Theta$$
$$R = \sqrt{x^2 + y^2}$$
$$\tan \Theta = \frac{y}{x}$$

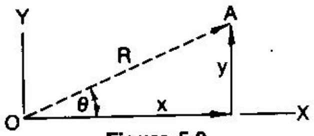



Figure 5.9

Example 5.27 If R = 8 and Θ = 19°, solve for the x and y components.

Operations
Set the right index of C to 8 on D.
Move hairline to 27° on Cos.
Read x = 7.13 on D at hairline.
Move hairline to 27° on S.
Read y = 3.63 on D at hairline.

### COMPLEX NUMBERS

A complex number of the form x + jy where $j = \sqrt{-1}$ may be represented in a complex plane, using a coordinate system, with x being the real axis and jy the imaginary axis.

The number may be presented graphically in Figure 5.10 by the point R, whose coordinates are x and y.

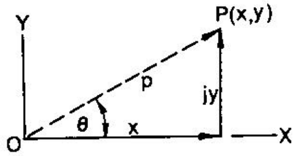



Figure 5.10

It may be shown that $^{j\Theta} = \cos \Theta + j \sin \Theta$ and therefore $\rho e^{j\Theta} = \rho \cos \Theta + j\rho \sin \Theta$. Here the complex number $\rho e^{j\Theta}$ (which is commonly simplified, $\rho e^{j\Theta} = \rho/\underline{\Theta}$) consists of two parts, $\rho \cos \Theta$ being the real part, and $j\rho \sin \Theta$, the imaginary part.

The above expression may be simplified by noting that $\rho \cos \Theta = x$ and $\rho \sin \Theta = y$. Then $\rho/\underline{\Theta} = x + jy$. The x and y values are determined as vector components of ρ as previously explained. This operation is called changing from exponential form to component form.

Example 5.28 Change to exponential form: $7.2 + j4.5 = 8.49\underline{/32^\circ}$

Operations $\tan \Theta = \frac{4.5}{7.2}$

Set right index of C to 7.2 on D.
Move hairline to 4.5 on D.
Read $\Theta = 32^\circ$ on T at hairline.

$\rho = \frac{y}{\sin 32^\circ}$

Move 32° on S to hairline.
Read $\rho = 8.49$ on D at right index of C.

Example 5.29 Change to coordinate form: $5.83\ \underline{/39.4^\circ} = 4.5 + j3.7$

Operations Set right index of C to 58.3 on D.
Move hairline to 39.4° on S.
Read $y = 3.7$ on D at hairline.
Move hairline to 39.4° on Cos.
Read $x = 4.5$ on D at hairline.

**Exercise 5.11**
**Vector Analysis and Complex Numbers**

Determine the x and y components of the following vectors:

| | | |
|---|---|---|
| 1. $R = 5, \Theta = 60^\circ$ | 3. $R = 2, \Theta = 30^\circ$ | 5. $R = 4, \Theta = 24^\circ$ |
| 2. $R = 3, \Theta = 90^\circ$ | 4. $R = 5, \Theta = 30^\circ$ | 6. $R = 7, \Theta = 35^\circ$ |

Change to exponential form:

| | | |
|---|---|---|
| 7. $3 + j4$ | 8. $4 + j3$ | 9. $14 + j8.9$ |

Solve for x and y:

10. $21\underline{/27^\circ} = x + jy$ 11. $8.49\underline{/32^\circ} = x + jy$



# APPLICATIONS TO TYPICAL PROBLEMS IN MATHEMATICS

In the chapters that follow, applications to specific areas of applied mathematics will be discussed. In this chapter we shall give examples of problems found in mathematics courses from arithmetic to calculus and differential equations. The number and types of problems are, of course, almost endless; the examples selected here are merely typical problems in each area. Although we have indicated the formulas to be used, no attempt is made to explain the derivation of these formulas, since our prime interest is the application of the slide rule to evaluate the results. The interested student may consult any good textbook on the subject, not only for explanations of method, but for obtaining a larger selection of problems for practice. It is suggested that an estimate of each answer be made, where possible, not only for decimal point location, but as a check on the method of solution. Thus every solution of a mathematical problem should end with the question, "Is this a reasonable answer?" For some problems an accurate sketch will provide a very reasonable estimate of the answer, and thus errors in calculations or in methods of solution will be quickly noted.

## 6.1 ARITHMETIC

Example 6.1 An order calls for making 5,000 pieces, each 1.56″ long on a screw machine. Assuming 8% waste, how many 12′ bars of material would be required for this job?

$$\frac{5{,}000 \times 1.56'' \times .92}{144''} = 49.8, \text{ or } 50 \text{ bars}$$

References: Section 2.9, Combined Multiplication and Division Series.

Example 6.2 When wheat is ground into flour, the flour weighs 17/19 as much as the wheat. How much wheat must be used to produce 340 pounds of flour? Since

$$\frac{\text{lbs of flour}}{\text{lbs of wheat}} = \frac{17}{19}$$

$$\frac{340}{\text{lbs of wheat}} = \frac{17}{19}$$

$$\text{lbs of wheat} = 380$$

It takes 380 lbs of wheat to produce 340 lbs of flour.

Reference: Section 2.14 Ratio and Proportion

Example 6.3 A manufacturer of belting uses the following formula for estimating the number of feet in a roll of belting.

$$f = n \times \pi\left(\frac{D + d}{2}\right)$$

where $f$ = number of feet in the roll
$D$ = outside diameter of the roll in feet
$d$ = inside diameter of the roll in feet
$n$ = number of coils in the roll

Use this formula to estimate the number of feet in a roll with $D = 4.5'$, $d = \frac{1}{2}'$, and $n = 50$.

$$f = \frac{50\pi(4.5 + 0.5)}{2}$$

$$f = 393'$$

There are approximately 393 feet of belting in the roll.

Reference: Section 2.13 Multiplication and Division using $\pi$

Example 6.4 A bag of fertilizer will treat 2500 square feet of grass. How many bags are needed for a park with 4 acres of grass? One acre = 43,560 square feet.

$$\text{Number of bags} = \frac{4 \times 43{,}500}{2500}$$

$$= 69.6 \text{ or } 70 \text{ bags}$$

It will require 70 bags to treat the 4 acres of grass.



Reference: Section 2.9 Combined Multiplication and Division Series

Example 6.5 If Noah was directed to make the ark 300 cubits long, 50 cubits wide, and 30 cubits high, find the dimensions in feet. Take 1 cubit = 18.24 inches.

$$\text{length} = \frac{18.24}{12} \times 300 = 456$$

$$\text{width} = \frac{18.24}{12} \times 50 = 76$$

$$\text{height} = \frac{18.24}{12} \times 30 = 45.6$$

The dimensions of the ark are 456 feet long, 76 feet wide, and 45.6 feet high.

Reference: Section 2.10 Multiplication of a Single Factor by a Series of Numbers

## 6.2 ALGEBRA

Example 6.6 Solve the exponential equation

$$75(1.03)^{4x} = 225$$
$$(1.03)^{4x} = 3$$
$$4x = 37.16$$
$$x = 9.29$$

Therefore $x = 9.29$

Reference: Section 4.3 Specialized Operations Using Log Log Scales, Exponential Equations

Example 6.7 Solve the logarithmic equation

$$\log_2 4x = 2.4$$
$$4x = 5.28$$
$$x = 1.32$$

Therefore $x = 1.32$

Reference: Section 4.3 Specialized Operations Using Log Log Scales, Logarithms to Any Base

Example 6.8 The first term of a geometric progression is 3, the common ratio is $\frac{2}{3}$, and the nth term is $\frac{512}{6561}$. Find n, the number of terms.

$n = ur^{(n-1)}$ where $n$ = the number of terms
$u$ = the first term
$r$ = the common ratio

$$3\left(\frac{2}{3}\right)^{n-1} = \frac{512}{6561}$$

$$(.667)^{n-1} = .026$$

$$n - 1 = 9.0$$

Since $n - 1 = 9$, $n = 10$, the number of terms is ten.

Reference: Section 4.3 Specialized Operations Using Log Log Scales, Exponential Equations

Example 6.9 A man deposits $250 in a savings bank at the end of each six months for 20 years. If the interest rate is 5% per year compounded semi-annually, what will be the amount of his account at the end of 20 years?

The formula for the amount of an annuity F, is:

$$F = A\left[\frac{(1+r)^n - 1}{r}\right]$$

where $A$ = amount of each deposit
$r$ = the rate of interest per period
$n$ = number of interest periods

$$F = 250\left(\frac{(1.025)^{40} - 1}{0.025}\right)$$

$$= \frac{250 \times 1.685}{0.025}$$

$$= 16{,}850$$

Therefore the amount after 20 years equals $16,850

Reference: Section 4.1 Powers and Roots of Numbers, Powers and Roots of Numbers Greater than 1.001



Example 6.10 A family buys a home for $40,000, with a down payment of $10,000 and a 15 year mortgage at 6%. What are the monthly payments?

The formula for the present value of an annuity P, is:

$$P = A\left[\frac{1 - (1+r)^{-n}}{r}\right]$$

$$\$30{,}000 = A\left[\frac{1 - (1.005)^{-180}}{.005}\right]$$

$$\$30{,}000 = A\ (118.5)$$

$$A = \$253$$

The payments are $253 per month.

Reference: Section 4.1 Powers and Root Numbers, Negative Powers and Roots of Numbers

#### 6.3 GEOMETRY

Example 6.11 Find the area of a 60° sector of a circle of radius 5″.

$$A = \frac{A^\circ}{360}\pi r^2$$

$$= \frac{60 \times \pi \times 5^2}{360} = 13.09$$

The area is 13.09 square inches.

Reference: Section 3.3 Combined Operations with Squares and Square Roots, also Section 2.13, Multiplication and Division using $\pi$

Example 6.12 Find the volume of a pyramid of height 10″ and with the base an equilateral triangle of side 6″.

$$V = \frac{s^2\sqrt{3}\,h}{12}$$

$$= \frac{6^2\sqrt{3} \times 10}{12} = 52.0$$

The volume is 52.0 cubic inches.

Reference: Section 3.3 Combined Operations with Square and Square Roots

Example 6.13 How many gallons of water are contained in a cylindrical tank 36″ high and with radius 18.5″? There are 231 cubic inches in 1 gallon.

$$G = \frac{\pi r^2 h}{231}$$

$$= \frac{\pi \times (18.5)^2 \times 36}{231} = 168$$

There are 168 gallons in the tank.

Reference: Section 3.3 Combined Operations with Square and Square Roots, also Section 2.13, Multiplication and Division using $\pi$

Example 6.14 Find the length of BE in Figure 6.1 if AE = 12.5, CE = 14.3, DE = 2.6.

By geometry

$$\frac{BE}{DE} = \frac{CE}{AE}$$

$$\frac{BE}{2.6} = \frac{14.3}{12.5}$$

$$BE = 2.97$$

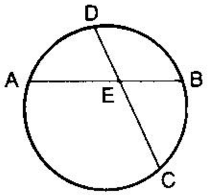


Figure 6.1

R ference: Section 2.14 Ratio and Proportion

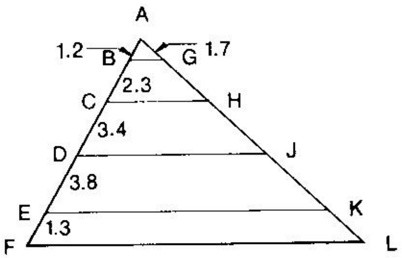


Figure 6.2



Example 6.15 BG, CH, DJ, EK, and FL are parallel lines in Figure 6.2. Find GH, HJ, JK, and KL if AG = 1.7, and the line segments on the left are as shown in Figure 6.2
Each pair of segments has the same ratio as $\frac{AG}{AB}$.

$$\frac{1.7}{1.2} = \frac{GH}{2.3} = \frac{HJ}{3.4} = \frac{JK}{3.8} = \frac{KL}{1.3}$$

GH = 3.26 HJ = 4.82 JK = 5.38 KL = 1.84

Reference: Section 2.14 Ratio and Proportion

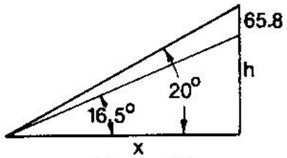


Figure 6.3

### 6.4 TRIGONOMETRY

Example 6.16 The flag pole in Figure 6.3 is 65.8 feet high, and stands on the top of a hill. From a point at the foot of the hill the angle of elevation of the top of the pole is 20° and that of the foot of the pole is 16.5°. Find the height of the hill.

From the figure, $x = h \cot 16.5°$
$x = (h + 65.8) \cot 20°$

Solving the pair of equations,

$$h = \frac{65.8 \cot 20°}{\cot 16.5° - \cot 20°} = 288.$$

The hill is 288 feet high.

Reference: Section 5.6 The Secant, Cosecant and Cotangent Functions of Angles, Also Section 5.9 Combined Operations with the Trigonometric Functions

Example 6.17 The angles of depression of two boats in line with a cliff are 10.5° and 12.6°. If the boats are 980 feet apart, find the height of the cliff. See Figure 6.4.

By the law of sines

$$a = \frac{980 \sin 10.5°}{\sin 2.1°}$$

From right triangle ABC

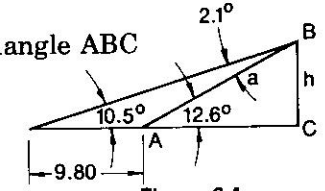



Figure 6.4

$$h = a \sin 12.6°$$

Hence

$$h = \frac{980 \sin 10.5° \sin 12.6°}{\sin 2.1°}$$

$$= 1063.$$

The height of the cliff is 1063 feet.

Reference: Section 5.3 The Sine Function of Angles from 0° to 90°, also Section 5.9 Combined Operations with the Trigonometric Scales

### 6.5 CALCULUS

Example 6.18 Evaluate the definite integral by the Trapezoidal Rule. Use $n = 10$.

$$\int_0^5 x\sqrt{25 - x^2}\,dx$$

$$\text{Area} = h\left(\frac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}y_n\right)\Delta x$$

$$= 0.5(0 + 0.5\sqrt{24.75} + \sqrt{24} + 1.5\sqrt{22.75} + 2\sqrt{21}$$
$$+ 2.5\sqrt{18.75} + 3\sqrt{16} + 3.5\sqrt{12.75} + 4\sqrt{9}$$
$$+ 4.5\sqrt{4.75} + 2.5\sqrt{0})$$

$$= 40.42$$



The value of the definite integral is 40.42, which may be checked by ordinary integration.

Reference: Section 3.1 Squares and Square Roots using $R_1$ and $R_2$ Scales

### 6.6 DIFFERENTIAL EQUATIONS

Example 6.19 If the rate of increase in population of a city is proportional to the population, and if the population doubles in 50 years, in how many years will it be 4 times as great?

The differential equation is $\frac{dy}{dt} = ky$

where y is the population at any given time t.

The solution is

$$y = C\,e^{kt}$$

where C is the initial population, since $y = C$ when $t = 0$.

When the population doubles

$$2C = C\,e^{50k}$$
$$e^{50k} = 2$$
$$k = 0.014$$

Therefore $y = Ce^{0.014t}$

When the population is 4 times as great

$$4C = C\,e^{0.014t}$$
$$e^{0.014t} = 4$$
$$t = 99.$$

The population will be 4 times as great in 99 years.

Reference: Section 4.2 Powers and Roots of e

Example 6.20 A heavy cylindrical buoy 1.5 feet in diameter floats in fresh water (density 62.4 pounds per cubic foot). When it is slightly depressed it vibrates with a period of 2.8 seconds. Find the weight of the buoy.

When the buoy is depressed a distance x, the upward buoyant force F equals the weight of the displaced water. Hence

$$F = \frac{\pi(1.5)^2\ 62.4}{4}\ x$$

Since $F = ma = \frac{W}{g}\frac{d^2x}{dt^2}$, where W is the weight of the buoy, and since the restoring force is opposite in direction to the displacement,

$$\frac{W}{g}\frac{d^2x}{dt^2} = -\frac{\pi(1.5)^2 62.4}{4}\ x$$

Taking g = 32.2,

$$\frac{d^2x}{dt^2} + \frac{3550}{W}\ x = 0 \qquad (1)$$

It is shown in any textbook of mechanics or differential equations that equation (1) represents simple harmonic motion, with period equal to $\frac{2\pi\sqrt{W}}{\sqrt{3550}}$. Hence

$$\frac{2\pi\sqrt{W}}{\sqrt{3550}} = 2.8$$

$$W = \frac{(2.8)^2 \times 3550}{4\pi^2}$$

$$= 705 \text{ pounds}$$

Reference: Section 3.3 Combined Operations with Squares and Square Roots, also Section 2.13 Multiplication and Division using $\pi$

**Exercises 6.1 thru 6.6**

**Applications to Typical Problems in Mathematics**

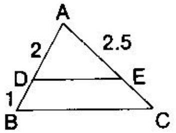
1. A piece of tin with an area of 46 square inches is cut from a larger rectangular piece 8 inches by 7 inches. What per cent of the larger sheet is wasted?
2. 4000 pounds of milk are delivered to a dairy. If the milk is 4.7% butterfat and the farmer is paid 75 cents per pound of butterfat, what amount is he paid?

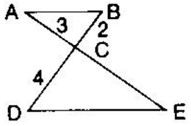


3. A map of the United States is drawn to a scale of 1 inch to 240 miles. If the airline distance from New York to Chicago is 713 miles, how far apart are these cities on the map?
4. A box of bolts weighs 37.4 pounds. The empty box weighs 3.3 pounds. If 100 bolts weigh 1.6 pounds, what is the approximate number of bolts in the box?
5. The shadow of a church with a tall steeple is 160 feet long at the same time of the day that the shadow of a 60-foot flag pole is 76 feet long. Find the height of the top of the steeple.
6. The specific gravity of a substance is the ratio of the weight of the substance to the weight of an equal volume of water. A cubic foot of water weighs 62.4 pounds. A cubic foot of aluminum weighs 168.5 pounds. Find the specific gravity of aluminum.
7. Cottonseed oil has a specific gravity of 0.926. What is the weight of one gallon of cottonseed oil if one gallon of water weighs 8.34 pounds?
8. A recording tape plays at a speed of $1\frac{7}{8}$ inches per second. What is the playing time, in hours, of a 1200-foot tape?
9. If a baseball is dropped from a height of 1000 feet, how long will it take to reach the ground, and with what velocity? (If t is the time, v the velocity, and s the distance, then $t = \sqrt{\frac{s}{16}}$ and $v = 5.45\sqrt{s}$.
10. In the formula $k = \frac{Mv^2}{2g}$, find k if M = 240, v = 64, and g = 32.2.
11. Find the future value of $1500 at 4.5% compounded quarterly for 8 years. The formula is $A = P(1 + r)^n$, where F is the future value, P is the principle, r is the interest rate per period, and n is the number of periods.
12. Find the discounted value of $5000 due in 15 years at interest 4% compounded quarterly. The formula is $P = F(1 + r)^{-n}$. See exercise 11.
13. The distance that a person can see depends upon his height above the surface of the earth. To find the distance a person

can see from a height of 1000 feet, use the formula $r = 1.065\sqrt{\frac{3h}{2}}$, where r is the approximate distance in miles and h is the height in feet.

14. The elongation of a spring varies directly as the weight applied. If a weight of 50 pounds causes an elongation of 4 inches, find the elongation caused by a weight of 35 pounds.
15. What is the length of the side of a square whose area is 2.66 square centimeters?
16. Find the length of AC in the figure. See Example 6.15, Section 6.3




17. Find the length of CE in the figure.
18. Find the area of a circular sector of 135° with radius 16 inches. See Example 6.11 Section 6.3.
19. How many gallons will a cylindrical can hold if its diameter is 14 inches and its height is 10 inches. One cubic foot equals 7.5 gallons.
20. Find the diagonal of a cube with edge equal to 4.43 inches. Use the formula $D = e\sqrt{3}$, where e is edge length.
21. Find the diagonal of one face of the cube of problem 20. Use the formula $d = e\sqrt{2}$, where e is edge length.
22. A wheel is driven by a belt. The angular velocity of the wheel is $\frac{5\pi}{3}$ radians per second and the radius of the wheel is 1.25 feet. Find the linear velocity of the belt in feet per minute. $v = r\Theta$.
23. The tires of an automobile have a diameter of 30 inches. Find the angular velocity of the wheels when the car is driven at 50 miles per hour. How many revolutions per minute do the wheels make? $\Theta = \frac{v}{r}$
24. Find the highest point reached by a projectile with initial velocity of 1400 feet per second and angle of projection $(\Theta) = 30°$. Use the formula

$$h_{max.} = \frac{v_o^2 \sin^2 \Theta}{2g}$$

and take $g = 32$.



25. Find the range of the projectile of problem 24, using the formula

$$R = \frac{v_o^2 \sin^2 2\Theta}{g}$$

26. The centrifugal force acting on a car going around a circular curve is given by the formula

$$F = \frac{w\,v^2}{g\,r},$$

where w = weight of car in pounds, v = velocity in feet per second, r = radius in feet, and g = 32.
Find the force on a 3200 pound car going around a curve of radius 100 feet at 60 miles per hour (88 feet per second).

27. The angle of elevation of the top of an inaccessible cliff is 48° and 150 feet farther away it is 27°. Find the height of the cliff.
28. From a building 284 feet high, on the edge of a lake, the angle of depression of a boat is 16.2°. Beyond this boat and in line with it is another boat with angle of depression 12.5°. Find the distance between the boats.
29. Solve the equation $2^{5x} = 77.8$.
30. Solve the equation $\log_3 5x = 3$.
31. Evaluate the integral by means of the trapezoidal rule, using $n = 4$

$$\int_0^4 \frac{dx}{\sqrt{4 + x^3}}.$$ See Example 6.18, Section 6.5.

## APPLICATIONS TO BUSINESS

The Versalog II slide rule is ideal for performing computations typically encountered in business. The purpose of this chapter is to illustrate some of these applications. Most business calculations simply involve multiplication and division, and the C, CI, and D scales and their folded counterparts can save considerable time when they are used in the most efficient manner. Since in business the cost of using money is basic, compound interest and present value computations are important. Here, the log log and reciprocal log log scales are particularly helpful. Many problems of statistical analysis and statistical inference can also be easily solved with the Versalog. Several examples of representative problems and their solutions follow.

### 7.1 PERCENTAGE

The use of percentages is very common in business. The use of the slide rule is a natural choice, since typically, percentages are not carried beyond slide rule accuracy. In pricing decisions, for example, these formulas may be used.

$$\frac{c}{(1-r)} = s$$

when: c = cost
s = selling price
r = gross profit rate

and $l(1-d) = s$

when: s = net selling price
l = list price
d = discount rate

#### MARK UP

Example 7.1 What should be the unit selling price of an item to yield a 35% gross profit, if it costs \$6.95 per dozen?

$$\frac{\$6.95}{12 \times (1-.35)} = \$0.89$$

Operation Set the hairline at \$6.95 on DF (the upper scale combination is chosen because less slide movement is required); move 12 on CF to the hairline, and move the hairline to .65 on CIF. Read \$0.89 on

DF at the hairline. The unit cost of \$0.58 can be read on D at the right index of C.

#### DISCOUNT

Example 7.2 For a particular quantity of goods, a chain discount of 25% + 10% is offered off of the list price of \$289.00. What is the net selling price?

$$\$289\ (.75)\ (.9) = \$195$$

Operation The formula is extended to accommodate the chain discount. Set the hairline to \$289 on D, move .75 on CI to the hairline. Move the hairline to .9 on CF and read \$195 on DF at the hairline.

#### PER CENT OF TOTAL

The percentage breakdown of a total is frequently used for comparison, analysis, allocation of costs, etc.

Example 7.3 Determine the percentage that each revenue source represented of the total revenue for March.

| Source | March | % of total |
|---|---|---|
| SALES | \$ 84,600 | 52.0% |
| RENTALS | 39,900 | 24.5 |
| PARTS | 21,300 | 13.1 |
| SERVICE | 16,900 | 10.4 |
| TOTAL | \$162,700 | 100.0% |

Operation Opposite the left index of D, set 162,700 on C. Move the hairline to 84,600 on C and read 52.0% on D at the hairline. Move hairline to 39,900 on C; read 24.5% on D. Move hairline to 21,300 on CF, read 13.1% on DF. Move hairline to 16,900 on CF, read 10.4 on DF.

#### Exercise 7.1
#### Percentages

Find the selling price of an item costing \$14.76 if a;

1. 26% gross profit is required.
2. 29% gross profit is required.

What is the net price of an article listing for \$70.00 if a;



3. 15% discount is offered?
4. 25% discount is offered?
5. What percentage of the total sales is represented by each product?

| | |
|---|---|
| Product a | 18.4 thousand units |
| Product b | 23.9 |
| Product c | 7.6 |
| Product d | 5.0 |
| Total | 54.9 thousand units |

### 7.2 COMPOUND INTEREST—FUTURE AND PRESENT VALUE

The following equations apply to interval compound interest and present value respectively.

$$F = P\ (1 + r)^n$$

$$P = F\ (1 + r)^{-n}$$

when: F = the future value in period n
P = the present value
n = the number of periods
r = the interest rate per period

#### FUTURE VALUE OF AN INVESTMENT

Example 7.3 How much will \$50 invested at $4\frac{1}{2}\%$ interest compounded semi-annually amount to in 5 years?

$$r = \frac{0.045}{2} = 0.0225; \qquad n = 2(5) = 10$$

$$F = \$50\ (1.0225)^{10} = \$50\ (1.249) = \$62.50$$

Operations Set the hairline to 1.0225 on LL1, and read approximately 1.249 at the hairline on LL2. Set the hairline to 1.249 on DF, align 50 on CIF with the hairline and read \$62.50 on D at the index of C.

#### PRESENT VALUE OF AN INVESTMENT

Example 7.4 What is the present value of an investment that, in three years, will pay \$75,000 if the cost of capital is 6%?

$$P = \$75{,}000\ (1.06)^{-3} = \$75{,}000\ (0.84) = \$63{,}000$$

Operations Set the hairline to 1.06 on LL1, slide the index of C to the hairline, slide the hairline to 3 on C, and read approximately 0.8395 on LL/2. Set 0.84 on DF, slide \$75,000 on CIF to the hairline, and read \$63,000 on D.

### APPRECIATION—RATE OF RETURN OF AN INVESTMENT

The rate of appreciation can be quickly and accurately determined by rewriting the above relationship:

$$r = \left(\frac{F}{P}\right)^{\frac{1}{n}} - 1$$

Example 7.5 If land is purchased for speculation for \$4,850 and sold for \$9,800 5½ years later, what was the average annual rate of return on the investment?

$$r = \left(\frac{\$9,800}{\$4,850}\right)^{\frac{1}{5.5}} - 1 = (2.02)^{\frac{1}{5.5}} - 1 = .1363 = 13.63\%$$

Operations Divide \$9,800 by \$4,850 to find 2.02. Set the hairline to 2.02 on LL2, slide 5.5 on C to the hairline, and read .1363 on LL2 at the index of C, or a 13.63% average annual rate of return on the investment.

**Exercise 7.2**
**Compound Interest—Future and Present Value**

What rate of compound interest is necessary to double the value of an investment in the following period of time?

1. 5 years
2. 15 years
3. 25 years
4. How much must a person invest today so that it will amount to \$1,000 in 25 years at 6% compounded annually?

## 7.3 ANNUITIES—FUTURE AND PRESENT VALUE

An annuity is a series of equal payments (or receipts) to be paid (or received) at the beginning (or end) of successive periods of equal length. The appropriate equations follow, when A is the amount of the annuity payment (or receipt), and when F, P, n, and r remain as previously defined.

Future value of an annuity (in arrears): $F = A\left(\frac{(1+r)^n - 1}{r}\right)$

Present value of an annuity (in arrears): $P = A\left(\frac{1-(1+r)^{-n}}{r}\right)$



### RETIREMENT FUND

Example 7.6 If a company sets aside \$100 a year in a retirement trust fund for an employee 30 years old, how much will it amount to when the employee retires at age 65 if the retirement trust fund earns 4% per annum?

$$F = \$100\left(\frac{(1.04)^{35} - 1}{.04}\right) = \$100\left(\frac{3.95 - 1}{.04}\right) = \$7,370$$

Operations The value $(1.04)^{35}$ was found by setting the hairline at 1.04 on LL1, sliding 35 on CI to the hairline and reading 3.95 on LL3 at the index of CI.

### INSTALLMENT BUYING

Example 7.7 Suppose the buyer of an automobile wishes to pay \$2,000 of its cost in monthly installments and has obtained a loan with an annual interest rate of 6 per cent. What monthly payment must be made over a period of two years in order to obtain title to the automobile?

$$r = \frac{0.06}{12} = 0.005; \qquad n = 2(12) = 24$$

$$\$2,000 = A\left[\frac{1-(1.005)^{-24}}{.005}\right] =$$

$$A\left[\frac{1-.8872}{.005}\right] = A\ (22.6)$$

$$A = \frac{\$2,000}{22.6} = \$88.50$$

Operations The value $(1.005)^{-24}$ was obtained by setting the hairline to 1.005 on LL0, moving 24 on CI to the hairline, moving the hairline to the left index of CI, and reading .8872 at the hairline on LL/2. The monthly payment will be approximately \$88.50 for two years. The total interest paid will be 24(\$88.50) − \$2,000 = \$124.

### LEASING DECISION

Example 7.8 How much would a firm be willing to accept in one payment in advance as settlement of a 3 year lease, instead of the usual monthly rent of \$500 if the firm's cost of capital was 9% per year? The present value of subsequent rent is:

$$P = \$500\left(\frac{1-(1.0075)^{-35}}{.0075}\right) =$$

$$\$500\left(\frac{1-.77}{.0075}\right) = \$15{,}330$$

Operations To find the value $(1.0075)^{-35}$, set the hairline at 1.0075 on the LL0 scale, slide 35 on CI to the hairline and read .77 on LL/2 at the index of CI. $15,830 ($15,330 + $500) is the present value of the entire three year lease. The rent for the first month would be paid at the same time under either alternative and the final rent is 35 months away. Thus, n = 35 and the first month's rent is added to the present value of the subsequent rent to determine the present value of the entire three year lease. In effect, this is the present value of an annuity in advance and the equation could also be rewritten as

$$P = A\left[\frac{1-(1+r)^{-(n-1)}}{r} + 1\right]$$

### CAPITAL EQUIPMENT DECISION

Example 7.9 What price could profitably be paid for a machine that would last 5 years (and become worthless thereafter) and would save $42,500 a year over present production methods, if the minimum acceptable return on capital of the firm was 12% per year?

Here $e^{-rn}$ replaces $(1 + r)^{-n}$ in the equation, since the income stream is assumed to be continuous rather than at regular intervals, and the solution is:

$$P = \$42{,}500\left(\frac{1-e^{-(0.12)(5)}}{0.12}\right) =$$

$$\$42{,}500\left(\frac{1-.549}{.12}\right) = \$159{,}700$$

Operations To find the value $e^{-(0.12)(5)}$, simply set the hairline to 0.60 on the D scale and read 0.549 on LL/2.



**Exercises 7.3**

**Annuities—Future and Present Values**

If interest is compounded annually, find the final amount of each of the following ordinary annuities.

| | Annuity payment | No. of years | Annual interest rate |
|---|---|---|---|
| 1. | 480 | 15 | 4% |
| 2. | 360 | 20 | 5½% |
| 3. | 2,400 | 10 | 6¼% |
| 4. | 1,450 | 38 | 4½% |

5. If a company sets aside $250 per year in a retirement trust fund for a 45 year old employee, how much will it amount to at age 65 if the fund earns 6% per annum?

## 7.4 STATISTICAL ANALYSIS

Statistical analysis is a body of methods enabling more informal decisions in the face of uncertainty. The slide rule can be used to readily solve many problems in this area and Versalog II is particularly well suited for these applications, because of logical scale arrangements, the incorporation of the $R_1$ and $R_2$ scales, and the wide range of the log log scale. For example, probabilities as small as 0.00005, which may be encountered during an analysis, can be easily determined on the LL/3 scale.

### BINOMIAL DISTRIBUTION

Example 7.10 The probability of having r successes in a random sample of N from a population with the parameter P is expressed in the Binomial Distribution as:

$$Pr = C\binom{N}{r}P^r(1-P)^{N-r}$$

What is the probability of finding exactly 2 defects in a random sample of 10 parts from a lot that contains 5% defective parts? In this case:

N = the sample size
r = the number of defects in the sample
P = the proportion of defects in the population

By substitution, the probability is expressed as:

$$Pr = \frac{10!}{8!\ \ 2!} (0.05)^2(0.95)^8 =$$

$$45\ (0.0025)\ (0.6635) = 0.0746$$

Operations — Set the hairline at 0.95 on LL/1, slide right index of C to the hairline, move the hairline to 8 on C, and read 0.6635 on LL/2. Set hairline at 0.05 on LL/3, slide 2 on CI to the hairline, move the hairline to the right index of C and read 0.0025 on LL/3. Cancelling 8! in both the numerator and denominator, 9 on D multiplied mentally by 10 and divided by 2 on CI equals 45 on D, slide 0.0025 on CI to hairline and read 0.0746 on D opposite approximately 0.664 on C. Thus, the probability of finding exactly 2 defects in this lot is 0.0746, or about a 7½% chance.

## NORMAL DISTRIBUTION

Example 7.11 In testing the hypothesis that the sample mean $\overline{X}$ is not significantly greater than the population mean $\mu$ when the population variance $\sigma^2$ is known, it is necessary to solve the following equation for z.

$$z = \frac{\overline{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}}$$

If $z > z_{1-\alpha}$, the hypothesis is rejected at the $1 - \alpha$ level of significance. A particular type of machine produces an average of 18.5 units per hour. The variance of this output is 2.0 units. A modification becomes available that will improve production, but it will be uneconomical unless production is increased to at least 20 units per hour. Management is willing to take a 20% risk of accepting the modification when it is not economical ($\alpha = .20$). From a table of the Normal Distribution, $z_{.80} = 0.842$. A sample of twelve modified machines is tested and produce a mean of 20.3 units per hour. The hypothesis being tested is that the average output of *all* modified machines is not greater than 20 units per hour.



$$z = \frac{20.3 - 20}{\sqrt{\frac{2.0}{12}}} = \sqrt{\frac{(0.3)^2(12)}{2.0}} = 0.734$$

Operations — Set the hairline on 0.3 on $R_1$, move 12 on CI to the hairline, move the hairline to 2.0 on CI, and read 0.734 on $R_2$. Since z, 0.734, is not greater than $z_{.80}$, 0.842, the hypothesis is accepted and the modification is not purchased.

### Exercise 7.4
### Statistical Analysis

1. A quality control engineer examines 10 parts selected at random from the assembly line in a manufacturing plant. By experience, it has been found that 5% of the items turned out are defective. What is the probability that exactly 3 of the selected items are defective?

2. If a manufacturing process results in a "defect rate" of 10%, what is the probability that 2 of 5 parts selected at random from a lot of 50 parts will be defective?

3. A mutual fund has an extensive holding of common stocks, 80% of which pay some dividends. What is the probability that all of a random sample of 10 stocks pay dividends?

4. In a certain city, one of every ten auto licenses issued are for a foreign made car. In a random sample of eight cars, what is the probability that there will be at least six domestic cars? (Consider the separate probabilities of six, seven, or eight domestic cars in the sample).

5. In testing its job applicants over a long period of time, a sales firm has established a normal distribution of test scores with a mean of 10 and a variance of 4. If a random sample of 25 scores is taken, what is the probability that the sample mean exceeds 10.6?

# APPLICATIONS TO CIVIL ENGINEERING

The purpose of this chapter is to illustrate some of the many applications of the slide rule to civil engineering problems. (Other chapters illustrating other fields of engineering follow.) No attempt is made to cover the entire field of civil engineering since to do so would require many volumes. Only a few typical problems from various branches of this field are discussed. Equations, where used, are given without derivation.

## 8.1 SURVEYING PROBLEMS

The slide rule is always useful for checking even though its accuracy is not always sufficient for a particular problem solution. In calculating dimensions and long lengths with precision, it is often necessary to resort to the use of five or seven place logarithms, or to refer to extensive tables of natural functions for use with a mechanical calculator. This is particularly true in surveying problems. In such cases, errors may sometimes be discovered by approximate slide rule checking of a precisely calculated result.

### EARTHWORK QUANTITIES

The slide rule is useful in calculating the amount of earthwork to be moved for the construction of a highway. The contour of the ground is determined by leveling along the proposed line of the road and the area of the cross sections perpendicular to this line are calculated at various stations along the road. V, the volume of earth to be moved, may then be determined by either of two methods. The first method is called the "average end area" method in which $V = \frac{1}{2}(A_1 + A_2)L$, where $A_1$ and $A_2$ are cross sectional areas and L is the distance between them.

Example 8.1 Determine the volume of earthwork to be moved if $A_1$ is calculated as $162(\text{ft.})^2$ and $A_2$ as $184(\text{ft.})^2$, with the distance L between the sections 54 ft.

Solution $V = \frac{1}{2}(162 + 184)54 = 173(54) = 9{,}340(\text{ft.})^3$ by slide rule.

Solving for the sum of 162 + 184 and setting 346 on the D scale, we can divide by 2 with the C scale, and moving hairline to 54 on C scale, our answer 9,340 is found on the D scale. One setting of the slide is all that is necessary.

While the above formula is simple, it is not quite exact enough. When greater precision is desired the "prismoidal" formula is used. This method involves an additional area $A_m$, the area of a cross section half-way between $A_1$ and $A_2$. By the prismoidal formula $V = \frac{1}{6}(A_1 + 4A_m + A_2)L$.

Example 8.2 What volume of earthwork is to be moved if $A_1 = 500(\text{ft.})^2$, $A_m = 684(\text{ft.})^2$, $A_2 = 896(\text{ft.})^2$, and $L = 92$ ft.

Solution $V = \frac{1}{6}(500 + 2736 + 896)92 = \frac{1}{6}(4{,}132)92 = 63{,}400(\text{ft.})^3$ by slide rule.

## TAPING

When measuring distances by tape in the field, many times it is necessary to measure along a slope in hilly country, although the horizontal distance is dêsired. It is then necessary to correct the slope measurement. This may be done if the angle of slope is determined by use of a transit. If S is the slope measurement or taped distance, h the horizontal length and A the angle of slope, then $h = S \cos A$.

Example 8.3 What is the horizontal distance between two points 131 ft. apart when measured along a 15° slope?

Solution $h = 131 \cos 15° = 126.5$ ft. by use of the cos, D, and D scales.

A common source of error is the use of a tape too long or too short. However, if the tape being used is compared with a standard and the error in its length determined, a correction may be made.

Example 8.4 In measuring a line by use of a 100 ft. tape the measured distance was 864.91 ft. The tape was found to be 0.14 ft. too short. What is the true length of the line?

Solution The correction is $\frac{864.91}{100}(0.14) = 1.21$ by slide rule.

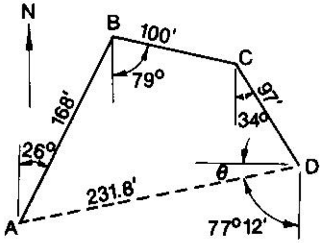


This error of 1.21 ft. must be added to 864.91 ft. to give the correct length and 866.12 ft. (Had the tape been 0.14 ft. too long, the correction would have been subtracted.)

## LATITUDES AND DEPARTURES

In locating a point with reference to a previously located point from field survey data, the method of latitudes and departures is often used.

The latitude is defined as the component of a given distance in the north-south direction whereas the departure is the component in the east-west direction. The bearing of a line is the angle between the line and the true north.




| Line | Dist. | Bearing | LAT. | | DEP. | |
|---|---|---|---|---|---|---|
| | | | N. | S. | E. | W. |
| AB | 168 | N. 26° E. | 151.0 | | 73.6′ | |
| BC | 100 | S. 79° E. | | 19.1′ | 98.1′ | |
| CD | 97 | S. 34° E. | | 80.5′ | 54.3′ | |
| DA | | | 51.4′ N. | | 226.0′ E. | |

**Figure 8.1 — Tape Distances and Bearings.**

Figure 8.1 represents a plot of a field traverse made by taping distances, AB, BC, and CD. At each point a bearing was taken. These and the taped distances are recorded in the table. The line AD was not measured in the field. Nevertheless its length and bearing are desired.

It may be seen from the figure that the latitude of each distance is the length multiplied by the cosine of the bearing angle and the departure is the length multiplied by the sine of the bearing. The necessary multiplications, performed by slide rule, are set down in the appropriate columns of the table. After summing the distances we observe that point D is 51.4′ north and 226.0′ east of A. Length DA is therefore $\sqrt{(51.4)^2 + (226.0)^2} = 231.8'$. Angle $\Theta$ in the figure is then arc sin $\frac{51.4}{231.8} = 12.8°$ or $12° - 48'$. The same angle may be determined from the relationship $\Theta = \text{arc tan} \frac{51.4}{226}$ which also yields

12.8°. The bearing of point A from point D is then 90° − Θ and since A is south and west of D, the bearing is designated S.77° − 12° W. The same calculations, performed with 5-place logarithmic tables, result in a bearing of S.77° − 09′ − 54″ W and a length DA = 231.84′. Line DA represents the closing line of the traverse. It is obvious that the slide rule calculations provide an accurate check.

### INACCESSIBLE DISTANCES

In running a survey line, obstacles may occur on the line of sight or it may be impossible to measure certain lengths such as the distance across a river. In such cases it is necessary to extend the line by indirect methods. Figure 8.2 illustrates a method for passing an obstacle by use of angular deflections. Point B is a point visible

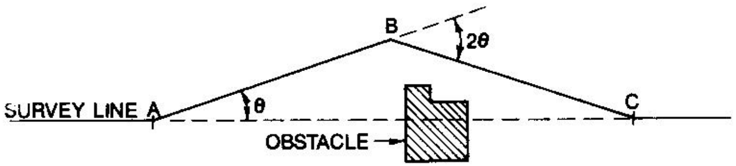



**Figure 8.2 – Inaccessible Distances Through Obstacle.**

from A. The procedure then is to measure distance AB and the angle Θ. The angle at B is then taken as 2 Θ and the length BC as equal to AB. By sighting along BC point C may be located. Distance AC is then 2(AB) cos Θ. For example, if AB measures 94′ and Θ = 21.8°, then AC = 2(94′) cos 21.8° = 174.6′.

Figure 8.3 illustrates a method for extending a survey line across a river when it is not practical to measure directly across. Point C is visible from either A or B.

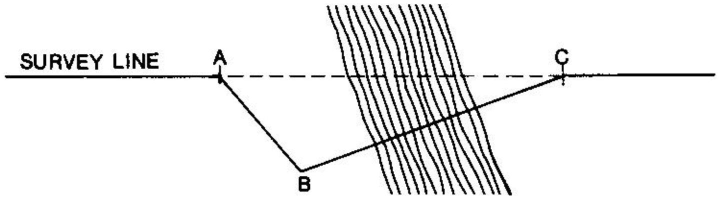



**Figure 8.3 – Inaccessible Distances Over River.**



Angles at A and B are measured by the use of a transit and distance AB is accurately measured by tape. The angle at C will then be 180° − A − B. Then by the law of sines $\frac{AC}{AB} = \frac{\sin B}{\sin C}$.

Example 8.5 In Figure 8.3, find the length of the inaccessible distance AC if A = 73°18′, B = 101°, and AB = 54 ft.

Solution $C = 180° - (101° + 73.3°) = 5.7°$

$$\frac{AC}{54'} = \frac{\sin 101°}{\sin 5.7°} = \frac{\cos 11°}{\sin 5.7°}$$

AC = 534 ft., by slide rule using the proportion principle previously explained.

### STADIA CALCULATIONS

A stadia transit is an instrument for determining horizontal and vertical distances from the observer to a point by taking readings on a rod held vertically at the point. By this method it is unnecessary to tape the distance from the observer to the point. The transit telescope is provided with an upper, lower, and a middle horizontal cross hair. The upper and lower cross hairs are equidistant from the middle cross hair which represents the line of sight of the telescope. A special stadia rod is held at the point to be located, and the transit is focused on this rod. Rod readings are taken at the upper and lower cross hairs, and the rod length between these points, called the rod intercept r, is determined. Also the vertical angle Θ, between the line of sight and the horizontal, is read at the transit.

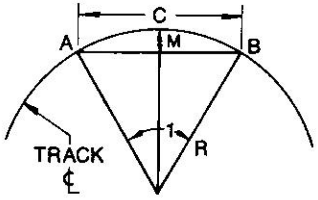
The horizontal distance H from the observer to the point is then calculated by the equation $H = a \cos \Theta + kr \cos^2\Theta$, in which a and k are instrumental constants, known for any particular transit. The vertical distance from the telescope to the center of the rod intercept is $V = a \sin\Theta + \frac{1}{2}kr \sin 2\Theta$.

Example 8.6 The upper and lower cross hair readings of a stadia transit are 4.32′ and 1.14′ respectively, with a vertical angle Θ = 26′. The rod intercept r = 4.32′ − 1.14 = 3.18′. Assume instrumental constants a and k to be 1′ and 100 respectively. Determine (a) the horizontal distance between the transit and the stadia rod, and (b) the vertical distance between the transit and the stadia rod.

Solution (a) $H = 1.\cos 26° + 100(3.18)\cos^2 26° = 0.9' + 257' = 258'$. (b) $V = 1.\sin 26° + \frac{1}{2}(100)3.18 \sin 52° = 0.4' + 125.1' = 125.5'$. (Ordinarily stadia distance calculations are made only to the nearest foot, for which the slide rule provides ample accuracy.)

### RADIUS AND DEGREE OF CURVE OF A CURVED TRACK

In railroad track surveying it is possible to calculate the radius and the degree of curve from measured lengths of a straight chord C and the mid-ordinate M.




$$R = \frac{C^2 + 4M^2}{8M}.$$

$$D = \frac{5730'}{R} \text{ (approx.) and}$$

$$D = 2 \text{ arc sin } \frac{50}{R} \text{ (exact).}$$

$$I = 2 \text{ arc sin } \frac{C}{2R}.$$

**Figure 8.4—Curved Track.**

In Figure 8.4 formulas for calculating the radius R and degree of curve D are given. The degree of curve is the central angle subtended by a 100′ chord. As an example, suppose the distance C to be 300′ and at 150′ from A the ordinate M is found to be 2.15′. The radius is then $\frac{90{,}000 + 18.5}{17.2} = 5230'$ approx. and the degree of curve is $D = \frac{5730'}{5230'} = 1.095° = 1° - 5.7'$. The central angle $I = 2$ arc sin $\frac{300}{10{,}460} = 2(1.644°) = 3.288°$.

**Exercise 8.1**

**Surveying Problems**

1. Calculate the volume of earthwork to be moved between two stations 73.4 ft. apart if $A_1 = 124(\text{ft.})^2$, $A_m = 136(\text{ft.})^2$, and $A^2 = 154(\text{ft.})^2$. (a) By the average end area method; (b) by the "prismoidal" formula.



2. A distance measured by a 50 ft. steel tape was found to be 484.15 ft. If the tape used was actually 0.028 ft. too short what is the true length of the line?

3. Solve for the length of the closing line DA in the traverse shown in Figure 8.5. Calculate its bearing.

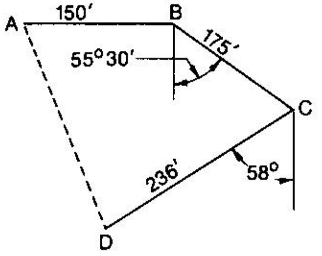



**Figure 8.5—Tape Distances and Bearings.**

4. If, in passing an obstacle (see Figure 8.2) the deflection angle measured by transit at A was 37° and the taped distance AB was 86 ft., determine the length AC.

5. Referring to Figure 8.3, determine the distance AC across a river if AB = 75 ft. and angles at A and B are 47° and 115° respectively.

6. If the instrumental constants a and k for a stadia transit are 1′ and 100 respectively, determine the vertical and horizontal distances V and H from the following stadia readings:

| | Vertical Angle Θ | Rod Intercept |
|---|---|---|
| (a) | 10°–15′ | 5.42 ft. |
| (b) | 7°–30′ | 2.14 ft. |
| (c) | 19°–45′ | 4.25 ft. |

7. Referring to Figure 8.4, determine the radius R, degree of curve D, and central angle I for the following values of the chord C and the mid-ordinate M:

| | C | M |
|---|---|---|
| (a) | 208 ft. | 6.4 ft. |
| (b) | 147 ft. | 5.65 ft. |
| (c) | 61.5 ft. | 1.27 ft. |

### 8.2. STRUCTURAL DRAFTING

The structural steel draftsman is concerned mainly with the calculation of the lengths of members and the details of their connections to other members in the structure. These lengths and details are shown on drawings which are used in the shop for fabrication of the various members.

#### LENGTHS AND BEVELS

Many times it is necessary for members to be skewed and to connect to other members at an angle. The skew of a connection is ordinarily indicated on the drawing by a bevel which is a figure calculated to the nearest 1/16 of an inch for the distance perpendicular to a base line 12" long. Thus in Figure 8.6 the distance R is known as the bevel.

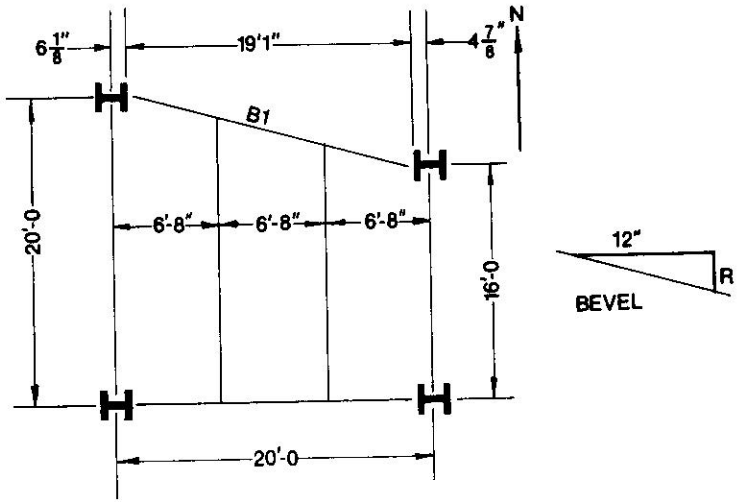



**Figure 8.6 – Beam and Column Floor Plan.**

Figure 8.6 represents a floor plan in which columns and beams are shown. The beams are indicated by heavy lines and are marked B1 to B6 inclusive. Due to the skew of beam B1 it will be necessary to calculate the lengths of B1, B3 and B4 and the bevel due to the skew. Beam B1 will be connected at the mid point of the flanges of column C and D. All distances must be figured to the nearest 1/16.

The bevel can be calculated or checked by slide rule, using the principle of proportion. Column D is 4'0" south of column C and its flange



face is 19'1" or 229" east of that of column C. The bevel R is found as follows:

$$\frac{R}{12''} = \frac{4'0''}{19'1''} = \frac{48''}{229''}$$

$$R = 2\tfrac{1}{2}''$$

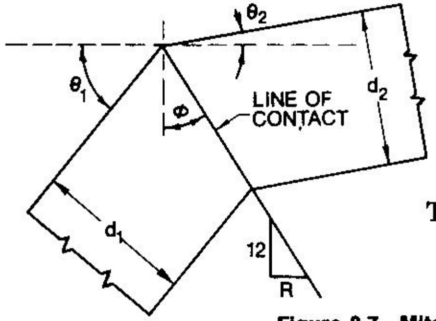
The length of B1 will be $\sqrt{(229)^2 + (48)^2} = \sqrt{52{,}500 + 2{,}300} = \sqrt{54{,}800} = 234'' = 19'6''$. The $R_1$ and $R_2$ and A scales were used for determining the squares and the square root of their sum.

The length of B4 will be greater than 16'0". The increase in length will be termed D' and will be determined by the proportion $\frac{D'}{6'3\frac{1}{8}''} = \frac{48''}{229''}$ or $\frac{D'}{75.1''} = \frac{48''}{229''}$ from which D' = 15.73" or $1'3\frac{3}{4}''$ by proportion. Then the length of B4 will be $16'0'' + 1'3\frac{3}{4}'' = 17'3\frac{3}{4}''$.

The proportion for determining the increased length of B3 will be $\frac{D'}{12'11\frac{1}{8}''} = \frac{48''}{229''}$ or $\frac{D'}{155.1} = \frac{48''}{229''}$ from which D' = 32.5" or $2'8\frac{1}{2}''$. Hence the length of B3 is $16'0'' + 2'8\frac{1}{2}'' = 18'8\frac{1}{2}''$.

#### MITER JOINTS

A miter joint is one in which intersecting members meet on a common line of contact. In detailing the top chord members of a bridge truss, and in other cases, it is important to know the angle or bevel of the line of intersection. Figure 8.7 indicates the manner in which this angle, designated $\phi$, may be determined. The depths $d_1$ and $d_2$ and angles of slope $\Theta_1$ and $\Theta_2$ are known for the intersecting members. The tangent of angle $\phi$ may be determined from the formula.




$$\tan\phi = \frac{d_1\cos\Theta_2 - d_2\cos\Theta_1}{d_2\sin\Theta_1 - d_1\sin\Theta_2}$$

**Figure 8.7 – Miter Joint.**

**Example 8.7** Referring to Figure 8.7, determine the angle and the bevel R if $d_1 = 12\frac{3}{4}''$, $d_2 = 12\frac{1}{2}''$, $\Theta_1 = 48°$, and $\Theta_2 =$ 9° 30′45″ or 9.51°.

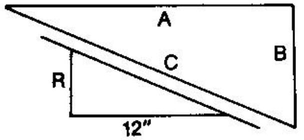
**Solution** $\tan\phi = \dfrac{12.75\cos 9.51° - 12.5\cos 48°}{12.5\sin 48° - 12.75\sin 9.51°} = \dfrac{12.58 - 8.36}{9.28 - 2.10}$

$= \dfrac{4.22}{7.18} = 0.588.\ \phi = \text{arc tan } 0.588 = 30.4°.$

The bevel $R = 12\tan\phi = 7.06''$ or $7\frac{1}{16}''$. This bevel, calculated by slide rule, was also calculated by 5-place logarithmic tables. Both methods give the same result to the nearest 1/16″.

**Exercise 8.2**
**Structural Drafting**

1. Determine the lengths C and bevels R for values A and B given below:




| | A | B |
|---|---|---|
| (a) | 4′5¼″ | 2′8″ |
| (b) | 8′7¾″ | 7′7″ |
| (c) | 14′9″ | 4′9⅝″ |

Figure 8.8—Lengths and Bevels.

2. Determine the bevel R for miter joints having the following properties: (See Figure 8.7)

| | $\Theta_1$ | $\Theta_2$ | $d_1$ | $d_2$ |
|---|---|---|---|---|
| (a) | 40° | 11° | 14″ | 12″ |
| (b) | 49° | 8° | 11½″ | 9½″ |
| (c) | 52° | 0° | 7½″ | 7½″ |

### 8.3 STRUCTURAL ANALYSIS

In the stress analysis of structural members, it is necessary to work constantly with the applied forces. A force, being a vector quantity having both magnitude and direction, may be resolved into components (as in Chapter 5). If convenient, the components may be used separately. As an example, Figure 8.9 (a) shows a force of 250# acting on a two member truss. The forces in members A and B are to be determined. For convenience the force A and the applied



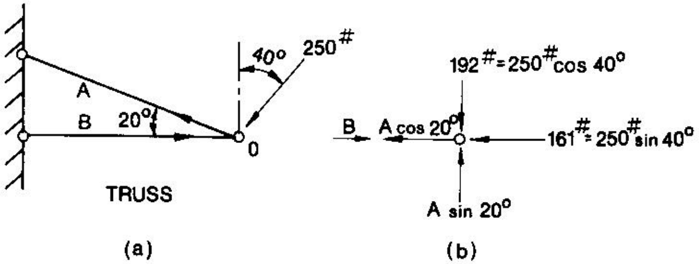


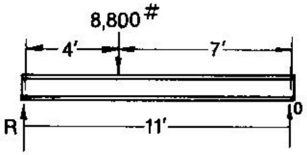

Figure 8.9—Two Member Truss.

250# force are resolved into their horizontal and vertical components. Force B is horizontal and has no vertical component. All forces and components acting at point 0 are shown in Figure 8.9 (b).

To maintain equilibrium, the sum of vertical forces must be zero and the sum of horizontal forces must also be zero. Hence from Figure 8.9 (b), $192 - A\sin 20° = 0$, from which $A = \dfrac{192}{\sin 20°} = 562\#$. Also $B - A\cos 20° - 161 = 0$, from which $B = 161 + 562\cos 20° = 161 + 527 = 688\#$. All of the operations are performed using the Cos S scales.

#### A STEEL BEAM

In determining the stresses in a beam, it is necessary to determine the forces acting, the shear, and the bending moment. The internal stresses in the material are then determined from the theory of the strength of materials. As an example, Figure 8.10 illustrates a steel beam designed to carry a concentrated load of 8,800# on a span of 11′.




8WF17
$S = 14.1(\text{in.})^3$
$Aw = 1.84(\text{in.})^2$

Figure 8.10—Steel Beam.

Its section modulus S and web area Aw are given for use in determining the moment and shear stresses in the material.

The left reaction force R may be determined by proportion, utilizing the distances from point 0 at the right end of the span $\frac{R}{8,800} = \frac{7'}{11'}$, from which R = 5,600#. This force is the shear acting at the left of the load. It will be resisted primarily by the web area. Hence the shearing unit stress $\frac{5,600}{1.84} = 3,040\#$ per sq. in.

The moment under the 8,800# load will be the product R × 4' = 5,600(4) = 22,400'#. The unit stress due to this bending moment will be $\frac{22,400(12)}{14.1} = 19,060\#$ per sq. in.

Stress analysis problems having to do with steel beams are many and varied. The above example is intended to serve merely as an illustration of this type of calculation to which the slide rule is well adapted.

### A REINFORCED CONCRETE BEAM

For a simple concrete beam, reinforced by steel to resist tension, it is necessary to determine the location of the neutral axis of the cross section before the bending moment stresses can be determined. The strength of the concrete in tension is ignored. In Figure 8.11 the cross section of a beam is shown, for which the concrete compressive unit stress $f_c$ and unit stress in the steel $f_s$ are to be determined when the moment is 440,000#.

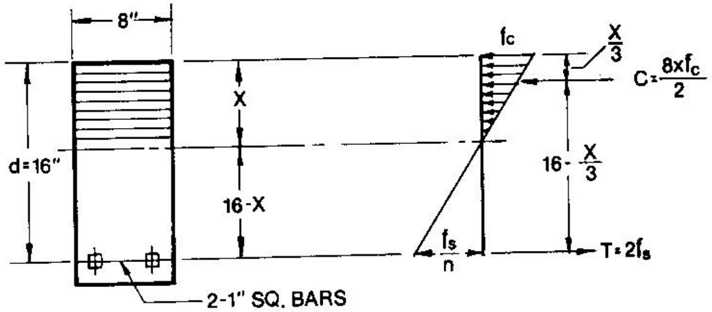



Figure 8.11 – Concrete Beam.

The effective area of steel is equivalent to n times its actual area, where n is the modular ratio, assumed equal to 10 in this case. The effective steel area is therefore 2 × 10 = 20 sq. in. The neutral axis



may be located by equating the moments of effective tensile and compressive areas about this line. Thus $8X\left(\frac{X}{2}\right) = 20(16 - X)$ or $X^2 + 5X - 80 = 0$. This is a quadratic equation which may be solved for X by the factoring method. Setting the right index of the C scale at 80 on D, the hairline is moved to 6.78 on CI where the simultaneous hairline reading on D is 11.78, the difference of these two readings being 5. The neutral axis is therefore located at X = 6.78". The lever arm between the forces C and T is $16 - \frac{X}{3} = 13.74''$, and the moment is either C or T times this lever arm. Hence $\frac{8(6.78)}{2} \cdot f_c \cdot (13.74) = 440,000$ and $f_c = 1,180\#$ per sq. in. From the equation $2 f_s(13.74) = 440,000$, $f_s = 16,000\#$ per sq. in.

### A FILLED ARCH

Arches are frequently used to carry loads over long spans. They are economical structures provided the end supports are capable of withstanding the thrusts transmitted by the arch rib and provided the curve of the arch axis is properly designed. A well designed arch curve will be such that the applied loads produce primarily forces or thrusts, with little or no bending moment. For structures of this type, the log log scales are very useful in evaluating logarithms and hyperbolic functions.

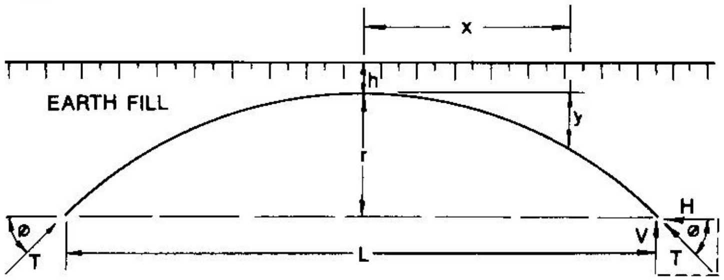



Figure 8.12 – A Filled Arch.

In Figure 8.12 the primary load to be carried consists of an earth fill. The weight of this fill is termed w lb. per cubic foot. In order to carry this load without introducing bending moment, the arch axis curve must be determined by the equation $y = h\left(\cosh\sqrt{\frac{w}{H}}(x) - 1\right)$, where H is the horizontal component of the thrust to be resisted by a section of arch rib one foot in width. If the ratio of depths of fill is

$$g = \frac{h+r}{h}, \text{ then } H = \frac{wL^2}{4[\text{Log}_e(g + \sqrt{g^2 - 1})]^2}.$$

The angle of slope $\phi$, at the end of the arch, may be determined from the expression $\tan \phi = h\sqrt{\frac{w}{H}} \cdot \sinh\left(\frac{L}{2}\sqrt{\frac{w}{H}}\right)$. Then the vertical component of thrust $V = H \tan \phi$ and the maximum thrust $T = \frac{H}{\cos \phi}$.

Example 8.8 Given a filled arch such that L = 200′, w = 120 lbs./cu. ft., r = 40′ and h = 10′ determine:

(a) The horizontal component of thrust H.
(b) The vertical component of thrust V.
(c) The maximum thrust T.
(d) The equation of the arch curve.
(e) The vertical distance y at a quarter point of the span.

Solution (a) $g = \frac{10 + 40}{10} = 5$

$$H = \frac{120(200)^2}{4[\text{Log}_e\,(5 + \sqrt{24})]^2} = \frac{1.2(4)(10^6)}{4(\log_e 9.9)^2} = \frac{1.2(10^6)}{(2.294)^2}$$

$= 228{,}000$ lbs.

(b) $\tan \phi = 10\sqrt{\frac{120}{228{,}000}} \cdot \sinh\left(\frac{200}{2}\sqrt{\frac{120}{228{,}000}}\right)$

$= 0.2294 \sinh 2.294)$

$\sinh(2.294) = \frac{1}{2}(e^{2.294} - e^{-2.294}) = 4.899$

$\tan \phi = 1.124$

$V = H \tan \phi = 228{,}000\ (1.124) = 256{,}500$ lbs.

(c) $T = \frac{H}{\cos \phi} = \frac{228{,}000}{\cos 48.4^\circ} = 343{,}000$ lbs.

(d) $y = 10\left(\cosh \frac{120}{228{,}000}(x) - 1\right)$



$$y = 10\left(\cosh \frac{x}{43.6} - 1\right)$$

(e) At the quarter point of the span, x = 50 ft. At this point $y = 10\left(\cosh \frac{50}{43.6} - 1\right) = 10$

$\left(\cosh 1.147 - 1\right) = 7.31$ ft.

### A GRAVITY DAM

Gravity dams are structures in which the weight of the dam itself is utilized to balance the pressure of water to prevent overturning. In calculating the pressures on the base and in investigating the stability of such structures, the weight of each part of the dam is calculated separately. Then the moment of all forces about a common point is determined and the resultant force acting on the base of the dam is located.

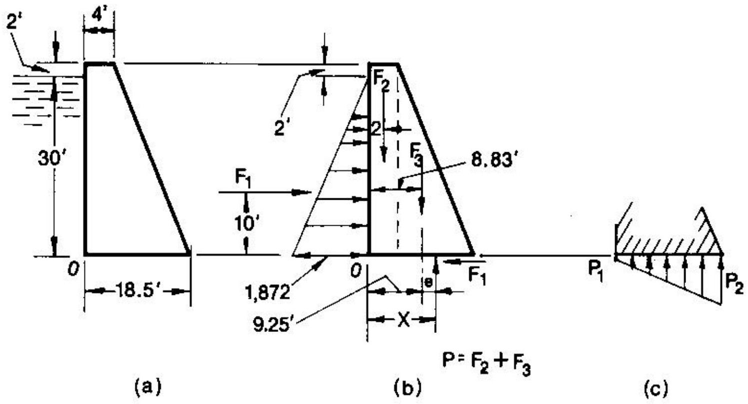



**Figure 8.13—A Gravity Dam.**

In Figure 8.13 (a) a simple dam is shown which is to retain a 30′ head of water weighing 62.4 lbs. per cu. ft. For analysis a width of dam of one foot perpendicular to the figure, is used. The dam is to be constructed of concrete assumed to weigh 145 lbs. per cu. ft. Figure 8.13 (b) indicates the separate forces and their locations. Distance x, locating the resultant vertical component P, is to be determined.

The water pressure acts horizontally and at the bottom of the dam has an intensity of 62.4(30′) = 1,872 lb. per foot. The total force due to water pressure is the area of the force triangle and is termed $F_1$. $F_1 = \frac{30}{2}(1{,}872) = 28{,}100$ lb. The forces $F_2$ and $F_3$ are due to the weight of concrete, $F_2$ being equal to 145(4)32 = 18,600 lb. and $F_3$ being $\frac{145(14.5)32}{2} = 33{,}600$ lb. The moment of forces $F_1$, $F_2$, and $F_3$ about point 0 is balanced by the moment $P \cdot x$, shown in Figure 8.13 (b). Therefore (18,600 + 33,600)x = 28,100(10) + 18,600(2) + 33,600(8.83).

$$x = \frac{615{,}000}{52{,}200} = 11.78'.$$

The eccentricity e, measured from the center line of the base, is then 11.78′ − 9.25′ = 2.53′. Pressures $p_1$ and $p_2$ indicated in Figure 8.13 (c) may now be determined from the equations $p_1 = \frac{P}{L}\left(1 - \frac{6e}{L}\right) = \frac{52{,}200}{18.5}\left(1 - \frac{6(2.53)}{18.5}\right) = 2{,}820(1 - 0.821) = 506$ lb. per sq. ft.; and $p_2 = \frac{P}{L}\left(1 + \frac{6e}{L}\right) = 2{,}820(1 + 0.821) = 5{,}140$ lb. per sq. ft. The resultant is located within the base, since $x < 18.5'$, which indicates that the dam is stable and will not overturn.

**Exercise 8.3**

**Structural Analysis**

1. Calculate the stresses in the members of trusses shown below in Figures 8.14 (a) and (b). Indicate whether the stresses calculated represent tension or compression.

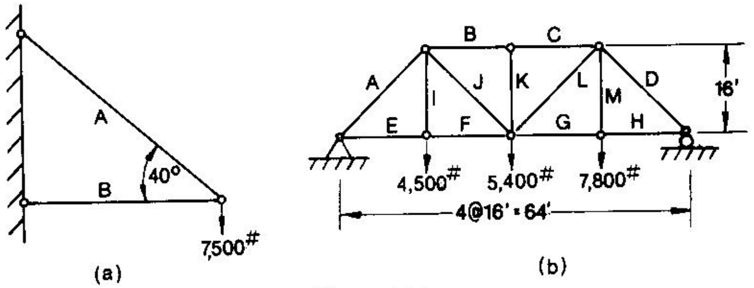



**Figure 8.14**



2. Solve for the end reactions $R_L$ and $R_R$ and for the moment in the steel beam at each load point. Calculate the maximum bending stress for the loads shown if the section modulus of the beam is 107.8(in.)$^3$.

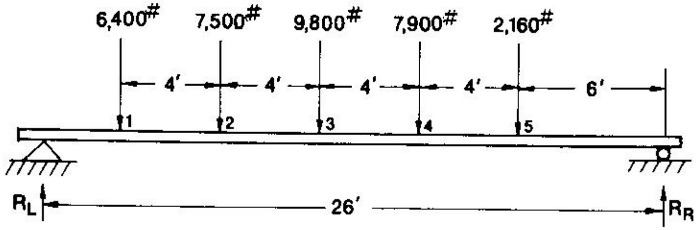



**Figure 8.15**

3. A concrete beam 12½ in. wide, with an effective depth d of 27 in., is reinforced by four 1-inch dia. rods. If the value of n is 10, locate the neutral axis and determine $f_c$ and $f_s$ for a bending moment of 1,500,000 in. lb.

4. Determine the economical curve equation for a 300 ft.-span filled arch if the depth of fill h is 6 ft. at the center and the rise r is 50 ft. If the fill weighs 120 lbs./cu. ft., determine the force H, V, and T for a one-foot width of arch rib.

5. Determine the base pressures $p_1$ and $p_2$ for a dam 30 ft. high if the width is 3 ft. at the top and 14 ft. at the bottom. The depth of water retained is to be 24 ft. and the weight of masonry is 145 lbs./cu. ft.

## APPLICATIONS TO MECHANICAL ENGINEERING

The subject of mechanical engineering is so diversified that it would be quite impossible to cover even a small portion of the many types of problems which may readily be solved with the aid of the slide rule. Although many of these problems are simple and require only the use of the basic scales designed for multiplication and division, there are others which require considerable skill in the use of the more complicated scales. It is the purpose of this chapter to acquaint the student with some of the latter and to illustrate their solution with the Versalog II Slide Rule.

There are several fields of mechanical engineering in which an abundance of problems exist whose solutions are particularly suited to the slide rule. Of these, perhaps the most important are thermodynamics, heat transfer, and machine design. In this chapter, each of these branches will be treated separately, and a few representative examples, together with their solutions, will be included. The treatment assumes that the student already possesses a basic knowledge of the use of all scales, and that he is familiar with the technique of setting decimal points and of performing other commonly employed operations. In studying the illustrative examples, the student is urged to follow the operations listed, and at each step to call to mind the reason why the particular operation was employed, and why it accomplishes its objective.

### 9.1 THERMODYNAMICS

The science of thermodynamics is related to the behavior of gases, liquids, and solids when under the influence of the interchange of heat and mechanical energy. A large number of problems in which the use of the log-log scales are particularly valuable are those involving the behavior of perfect gases undergoing changes in state. The derivation of the equations expressing the various relationships existing between such properties as pressure, absolute temperature, volume, internal energy, enthalpy, and entropy can be found in any standard textbook on thermodynamics. These equations are usually derived for certain commonly employed processes such as constant volume, constant pressure, isothermal, isentropic,

**TABLE I**

Thermodynamic Equations For Perfect Gases (Non-Flow Processes)

| Name of Process | Value of n | P–V–T Relationships | | | Heat Added ${}_1Q_2$ Btu | Work Done By Gas ${}_1W_2$ Btu | Change In Internal Energy $U_2-U_1$ Btu | Change In Enthalpy $H_2-H_1$ Btu | Change In Entropy $S_2-S_1$ Btu/°R |
|---|---|---|---|---|---|---|---|---|---|
| | | P–V | T–P | T–V | | | | | |
| Constant Volume V = Const. | ∞ | — | $\frac{T_1}{T_2}=\frac{P_1}{P_2}$ | — | $wc_v(T_2-T_1)$ | 0 | $wc_v(T_2-T_1)$ | $wc_p(T_2-T_1)$ | $wc_v \log_e \frac{T_2}{T_1}$ |
| Constant Pressure P = Const. | 0 | — | — | $\frac{T_1}{T_2}=\frac{V_1}{V_2}$ | $wc_p(T_2-T_1)$ | $\frac{P(V_2-V_1)}{J}$ | $wc_v(T_2-T_1)$ | $wc_p(T_2-T_1)$ | $wc_p \log_e \frac{T_2}{T_1}$ |
| Isothermal T = Const. | 1 | $\frac{P_1}{P_2}=\frac{V_2}{V_1}$ | — | — | $\frac{wRT}{J}\log_e\frac{P_1}{P_2}$ | $\frac{wRT}{J}\log_e\frac{P_1}{P_2}$ | 0 | 0 | $\frac{wR}{J}\log_e\frac{P_1}{P_2}$ |
| Isentropic S = Const. | k | $\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^k$ | $\frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}}$ | $\frac{T_1}{T_2}=\left(\frac{V_2}{V_1}\right)^{k-1}$ | 0 | $wc_v(T_1-T_2)$ | $wc_v(T_2-T_1)$ | $wc_p(T_2-T_1)$ | 0 |
| Polytropic $PV^n$ = Const. | n | $\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^n$ | $\frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}$ | $\frac{T_1}{T_2}=\left(\frac{V_2}{V_1}\right)^{n-1}$ | $wc_n(T_2-T_1)$ | $\frac{wR}{J(n-1)}(T_1-T_2)$ | $wc_v(T_2-T_1)$ | $wc_p(T_2-T_1)$ | $wc_n \log_e \frac{T_2}{T_1}$ |

$c_v$ = Specific heat at constant volume, Btu/lb.
$c_p$ = Specific heat at constant pressure, Btu/lb.
$c_n$ = Specific heat for a polytropic process $= c_v\left(\frac{k-n}{1-n}\right)$, Btu/lb.
H = Enthalpy, Btu.
J = Mechanical Equivalent of heat, 778 Ft. lbs./Btu.
k = Isentropic exponent, $c_p/c_v$.
n = Polytropic exponent.

P = Pressure, lbs./ft.$^2$
R = Gas constant, ft./°R.
S = Entropy, Btu/°R.
T = Absolute temperature, °R = (460 + F)
U = Internal Energy, Btu.
V = Volume, ft.$^3$
w = Weight of gas, lbs.

**Figure 9.1**



and polytropic. In Table I, Figure 9.1 are presented those equations which have been found to be useful to the mechanical engineer. The nomenclature employed is expressed in engineering units.

A study of Table I shows that all equations listed fall into one or more of the following three categories: equations involving powers of numbers; equations involving the natural logarithms of numbers; and equations involving simple multiplication or division.

Of these, only the first two are sufficiently difficult to require special treatment in this chapter.

### EQUATIONS INVOLVING POWERS OF NUMBERS

Solution of these equations is accomplished by the use of the log-log scales, LL3 to LL0 and LL/3 to LL/0. In many cases, several alternative solutions of equal accuracy and speed will suggest themselves. At first the student should employ more than one method using the other as a check. After proficiency in all methods is achieved, the student should be able to select for himself the one best suited to the particular circumstances.

Example 9.1 For a polytropic process, solve for $P_2$ if $n = 1.21$, $P_1 = 120$ psia, and $V_2 = 3.17V_1$. Answer, 29.7 psia.

Solution From Table I write $P_2 = \frac{P_1}{\left(\frac{V_2}{V_1}\right)^n} = \frac{120}{3.17^{1.21}}$

Set left index of C opposite 3.17 on LL3. Move hairline to 1.21 on C and read 4.04 on LL3. Set 120 on D and slide 4.04 on C. Read 29.7 at the right index of C.

Alternate Solution Write, $P_2 = P_1\left(\frac{V_1}{V_2}\right)^n = 120\left(\frac{1}{3.17}\right)^{1.21} =$

$120 \times 0.315^{1.21}$

Set hairline over 3.17 on LL3 and read its reciprocal 0.315 on LL/3. Slide the left index of C to the hairline and move hairline to 1.21 on C. Read 0.247 on LL/3. This is $0.315^{1.21}$. Set left index of C to 120 on D and move hairline to 0.247 on C. Read 29.7 on D.

Example 9.2 For a polytropic process, find n if $\frac{P_1}{P_2} = 7.23$ and $\frac{V_2}{V_1} = 5.63$. Answer 1.145

Solution From Table I write, $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$ or $7.23 = 5.63^n$

Set left index of C opposite 5.63 on LL3. Move hairline to 7.23 on LL3 and read n = 1.145 on C.

Alternate Solution An alternate, but much less rapid solution, may be obtained by using natural logarithms. Write

$$\log_e 7.23 = n \log_e 5.63 \text{ or } n = \frac{\log_e 7.23}{\log_e 5.63}$$

Find $\log_e 5.63$ by setting hairline to 5.63 on LL3 and reading 1.73 on D. Find $\log_e 7.23$ by setting hairline to 7.23 on LL3 and reading 1.98 on D. Slide 1.73 on C to hairline and read 1.145 on D opposite left index of C.

The same result could have been obtained using common logarithms and reading their values on the L scale. Thus

$$n = \frac{\log_{10} 7.23}{\log_{10} 5.63} = \frac{0.859}{0.75} = 1.145$$

Example 9.3 For a polytropic process find $\frac{V_2}{V_1}$ if $\frac{T_2}{T_1} = 0.94$ and $n = 1.037$. Answer 5.30

Solution $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1}$. Therefore, $\frac{1}{0.94} = \left(\frac{V_2}{V_1}\right)^{0.037}$

The simplest solution is to find the number which, when raised to the 0.037 power, will give $\frac{1}{0.94}$. Set hairline to 0.94 on LL/1 and read its reciprocal 1.0637 on LL1. Slide 0.037 on C to the hairline and read 5.30 at the left index of C on LL3. The choice of the correct LL scale on which to read the answer is governed by the position of the decimal point. In this case, since the decimal point is in the second place to the left of 3.7, it is necessary to move upward two scales to the LL3, in order to obtain the correct result.



(Preferred) Alternate Solution

$$\frac{V_2}{V_1} = \left(\frac{1}{0.94}\right)^{\frac{1}{0.037}} = \left(\frac{1}{0.94}\right)^{27}$$

Opposite 0.94 on LL/1 set right index of C. Move hairline to 27 on C and read 5.30 on LL3. The first operation above was equivalent to setting the right index of C opposite 1.0637 (the reciprocal of 0.94) on D. The second operation raised 1.0637 to the 27 power. Again, the choice of LL scale on which the answer is read is determined by the position of the decimal point. In this case, it is clear that the answer would not be on the LL2 scale because this would give 1.1815, which would be the answer had the exponent been 2.7 instead 27.

Example 9.4 Solve example 9.3 if n = 1.37. Answer 1.1815

Solution The solution in this case is identical with that of example 9.3 except that the exponent by the first method becomes 0.37 instead of 0.037. By the second method it becomes 2.7 instead of 27. The answer is 1.1815 instead of 5.30 and is read on LL2 instead of LL3.

Example 9.5 Find the change in internal energy for air undergoing the following isentropic compression. $P_1 = 15$ psia, $P_2 = 60$ psia, $T_1 = 520$ deg. R, w = 13 lbs, $c_v = 0.1715$ btu/lb, F, k = 1.40. Answer 564 btu.

Solution From Table I write $U_2 - U_1 = wc_v(T_2 - T_1)$ and $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$$\text{Then } U_2 - U_1 = wc_vT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right]$$

$$= 13 \times 0.1715 \times 520\left[\left(\frac{60}{15}\right)^{\frac{1.4-1}{1.4}} - 1\right]$$

$$= 1160\left[4^{0.286} - 1\right]$$

### EQUATIONS INVOLVING NATURAL LOGARITHMS

Most thermodynamic equations involving logarithms can be reduced to one number multiplied by the natural logarithm of another. The solution of this type of problem is quite simple, since the natural log of a number can be read directly on the D scale opposite the number on one of the LL scales. If the number is greater than one, the logarithm will be positive and will have the decimal point indicated by the symbol at the right of the scale. If the number is less than one, the logarithm will be negative with the decimal point also indicated by the symbol. The multiplication process which follows is one of simply setting the left or right index of the C scale (whichever is appropriate) opposite the value of the logarithm on the D scale and moving the hairline to the number by which the logarithm is to be multiplied. The final result is read on the D scale and given the appropriate sign and decimal point.

In the interest of accuracy and ease of computation, it is often an advantage to reduce the problem to its simplest form before performing the final operations. This will result in a minimum of effort in obtaining the solution. An example illustrates this point.

Example 9.6 Find the change in entropy per lb. of gas resulting from a polytropic expansion for which $n = 1.32$ if $V_2 = 6V_1$. Assume $c_v = 0.18$ btu/lb F, $w = 1$ and $k = 1.39$. Answer 0.0226 btu/deg. R.

Solution Without reducing to its simplest form, the solution could be found as follows:

$$S_2 - S_1 = wc_n \log_e \frac{T_2}{T_1} = wc_v\left(\frac{k-n}{1-n}\right)\log_e\left(\frac{V_1}{V_2}\right)^{n-1}$$

$$= 0.18\left(\frac{1.39 - 1.32}{1 - 1.32}\right) \log_e(\tfrac{1}{6})^{0.32} = -0.0394 \log_e(\tfrac{1}{6})^{0.32}$$

Set left index of C opposite 6 on LL3 and move hairline to 0.32 on C. Read 0.5635 on LL/2. This is equal to $\frac{1}{6}$ raised to the 0.32 power. The logarithm of this is read at the hairline on the D scale, but with a negative sign, since it is for a number less than one.

From the symbol at the right of LL/2, it is clear that the logarithm read on D is −0.574. Set the right index of C to the hairline and move the hairline to −0.0394 on C. Read 0.0226 on D.



Alternate Solution. By further mathematical manipulation, the solution can be reduced to the following, which is the preferred method.

$$S_2 - S_1 = -0.0394 \log_e (\tfrac{1}{6})^{0.32} = 0.32 \times 0.0394 \log_e$$

$$\tfrac{1}{6} = 0.0126 \log_e 6$$

Set left index of C opposite 6 on LL3. Move hairline to 0.0126 on C and read 0.0226 on D.

Example 9.7 Find the work of an isothermal expansion of 7 lbs. of hydrogen gas from a volume of 500 ft³ to 10,000 ft³. The temperature is 80°F (540 deg. R.). The gas constant for hydrogen is 772. Answer 11,250 btu.

Solution Referring to Table I and noting that $\frac{P_1}{P_2} = \frac{V_2}{V_1}$ one may write

$${}_1W_2 = \frac{wRT}{J} \log_e \frac{P_2}{P_1} = \frac{7 \times 772 \times 540}{778} \log_e \frac{V_2}{V_1} = \frac{7 \times 772 \times 540}{778} \log_e 20.$$

Set 778 on C opposite 20 on LL3. Move hairline to 772 on C. Turn rule over and move 540 on CI under hairline. Move hairline to 7 on CF and read 11,250 on DF.

Example 9.8 Find the change in entropy for a constant pressure process in which 4 lbs. of air are compressed at constant pressure from a volume of 50ft³ to 10 ft³. $c_p$ for air = 0.24 btu/lb F. Answer −1.545 btu/deg. R.

Solution Noting from Table I that $\frac{V_2}{V_1} = \frac{T_2}{T_1}$, the following can be written

$$S_2 - S_1 = wc_p \log_e \frac{T_2}{T_1} = 4 \times 0.24 \log_e \frac{10}{50} = -4 \times 0.24 \log_e 5.$$

Set 4 on CI opposite 5 on LL3. Move hairline to 0.24 on C and read −1.545 on D.

**Exercise 9.1**

**Thermodynamics**

1. Solve the following exercises, using alternate methods when feasible. Find $T_2$ for an isentropic process for which $P_1 = 14.7$, $P_2 = 49.25$, $k = 1.40$.
2. Find $P_2$ for an isentropic process for which $P_1 = 15$, $T_1 = 520$, $T_2 = 360$, $k = 1.30$.
3. Find $P_2$ for a polytropic process for which $P_1 = 400$, $T_1 = 625$, $T_2 = 500$, $n = 1.05$.
4. Find n if $\frac{P_2}{P_1} = 7.5$ and $\frac{V_1}{V_2} = 4.4$.
5. Compute the heat added to 1 lb. of air which undergoes a polytropic expansion with $n = 1.16$ from a pressure of 200 psia to 42 psia. The initial temperature is 900 deg. R. For air $c_v = 0.1715$ btu/lb F and $k = 1.40$.
6. Find $V_2$ for a polytropic compression of a gas if $n = 124$, $T_1 = 600$, $T_2 = 800$ and $V_1 = 16$.
7. Find the change in entropy per lb. of air resulting from a polytropic expansion for which $n = 1.12$ if $V_2 = 18V_1$. Assume $c_v = 0.1715$ and $k = 1.4$.
8. Find the work of isothermal compression of 10 lbs. of nitrogen from a volume of 36 $ft^3$ to a volume of 4 $ft^3$. The temperature is 60° F. Gas constant for nitrogen = 55.2.
9. Find the heat added per lb. of air undergoing an isothermal expansion from a pressure of 140 psia to 40 psia. The temperature is 600 deg. R. $R = 53.3$.
10. Find the change in entropy for a constant volume extraction of 1000 btu of heat from 15 lbs. of oxygen originally at 760 deg. R. $c_v$ for Oxygen = 0.155 btu/lb F.
11. For an isothermal compression the change in entropy of 3 lbs. of carbon dioxide is −0.37 btu/deg. R. If the initial pressure is 15 psia, what is the final pressure? R for carbon dioxide is 35.1 ft/deg. R.

## 9.2 HEAT TRANSFER

The mechanisms by which heat may be transferred are three, conduction, convection, and radiation. In this section each of these will be treated separately for the case of steady flow. The case of transient flow requires a high degree of mathematical training and is beyond the scope of this chapter.



### CONDUCTION

Conduction may be defined as the flow of heat through a substance, the particles of which remain in a fixed position relative to each other. It is usually associated with the flow through solids, although, in the absence of convection currents, heat can also be said to flow by conduction through liquids and gases. The flow of heat by conduction is directly proportional to a constant called the thermal conductivity multiplied by the temperature gradient and the cross sectional area perpendicular to flow, and inversely proportional to the distance through which it flows.

For a slab, the flow may be expressed by the simple equation:

$$Q = \frac{kA\Delta t}{\tau} \qquad (1)$$

where

$Q$ = rate of flow of heat through the slab, btu/hr
$k$ = thermal conductivity of the slab material, btu/hr F ft
$A$ = cross sectional area of slab perpendicular to the flow of heat, $ft^2$
$\Delta t$ = temperature difference across the slab, F
$\tau$ = thickness of slab, ft

A problem of frequent occurrence in mechanical engineering is the the determination of the flow of heat from an insulated pipe. For this case equation (1) must be modified to conform to the fact that the insulation is curved and that the area perpendicular to flow is greater at the outer surface.

The equation for this case is:

$$Q = \frac{2\pi kL\Delta t}{\log_e \frac{D_2}{D_1}} \qquad (2)$$

where

$Q$ = heat loss, btu/hr
$L$ = length of pipe, ft
$\Delta t$ = temperature difference between inner and outer surface of the insulation, F
$D_2$ = outer diameter of insulation, ft
$D_1$ = inner diameter of insulation, ft

Example 9.9 Find the heat loss in btu per hr from a pipe of 8 inches outside diameter if it is 50 ft. long and covered with 2 inches of insulation having a thermal conductivity of 0.035 btu/hr F ft. The inner temperature is 850°F and the outer temperature is 150°F. Answer 18,950 btu/hr.

Solution From equation 2, write

$$Q = \frac{2\pi \times 0.035 \times 50(850 - 150)}{\log_e \frac{12}{8}}$$

Divide 12 by 8 mentally to obtain 1.5. Set hairline at 1.5 on LL2 and read $\log_e 1.5 = 0.406$ on D. Subtract 150 from 850 mentally to obtain 700. Noting that $2 \times 50 = 100$, the problem reduces to $\pi\left(\frac{0.035 \times 70{,}000}{0.406}\right)$. Set hairline to 70,000 on D and move 0.406 on C to the hairline. Move hairline to 0.035 on C and read 6030 on D. This may be multiplied by $\pi$ by simply reading 18,950 on DF at the hairline.

## CONVECTION

Heat flow by convection is an extremely complex subject since the mechanism of transfer is largely one of heat being conveyed from one portion of a fluid to another by physical mixing. The interaction of forces creating mixing and the consequent transfer of heat depends on many factors such as the density, specific heat, viscosity, thermal conductivity, temperature, and velocity of the fluid, as well as upon the geometry of the apparatus in which the fluid is contained. Many cases of practical importance have been studied, but perhaps the most useful to the mechanical engineer is the rate of flow of heat to or from a fluid flowing inside a pipe or circular conduit. This problem has been studied by Dittus and Boelter of the University of California. Their work indicates that the rate of heat interchange between the inner surface of pipe and a fluid flowing inside the pipe is proportional to a coefficient of conductance h. The rate of heat interchange in btu per hr may be computed by multiplying h by the inner surface area of the pipe and by the temperature difference between the inner surface and the fluid.



The value of h is given by the equation

$$h = 0.023\frac{k}{D}\left(\frac{DV\rho}{\mu}\right)^{0.8}\left(\frac{\mu c_p}{k}\right)^{n} \qquad (3)$$

where

- $h$ = coefficient of conductance, btu/hr F ft$^2$
- $k$ = thermal conductivity of the fluid, btu/hr F ft$^2$
- $D$ = inside diameter of pipe, ft
- $V$ = mean velocity of fluid inside pipe, ft/hr
- $\rho$ = density of fluid, lbs/ft$^3$
- $\mu$ = viscosity of fluid, lbs/hr ft
- $c_p$ = specific heat of the fluid at constant pressure, btu/lb F
- $n$ = an exponent equal to 0.4 if the fluid is being heated and 0.3 if the fluid is being cooled.

The term $\frac{DV\rho}{\mu}$ is a dimensionless group called Reynold's number and occurs frequently in heat transfer and fluid flow calculations. It is sometimes very large and for this reason falls beyond the range of the LL3 scale on the slide rule making it necessary to apply special methods when raising it to a power. The term $\frac{\mu c_p}{k}$ is called Prandtl's number. It is usually quite small; often less than unity. Another group called Nusselt's number can be formed as $\frac{hD}{k}$. The use of such dimensionless groups is widely employed in the theory of heat transfer and fluid flow. These groups usually occur raised to some power, thus making the slide rule particularly applicable to their solution.

Example 9.10 Find the coefficient of conductance of superheated steam flowing to a turbine with a velocity of 150 ft/sec. The inside diameter of the pipe is 6 inches. The steam is under a pressure of 1,000 psia and temperature of 800°F. The constants needed for the problem are $k = 0.065$ btu/hr F ft, $\rho = 1.451$ lbs/ft$^3$, $\mu = 0.104$ lbs/ft hr, $c_p = 0.61$ btu/lb F. Answer 548 btu/hr F ft$^2$.

Solution Since the steam is losing heat and therefore being cooled, the value of n will be 0.3. Hence, equation (3) becomes

$$h = 0.023\frac{0.065}{\left(\frac{6}{12}\right)}\left(\frac{6}{12} \times \frac{150 \times 3600 \times 1.451}{0.104}\right)^{0.8}$$

① ②

$$\left(\frac{0.104 \times 0.61}{0.065}\right)^{0.3}$$

③

The solution can be obtained by treating the individual terms of the equation indicated by the encircled numbers separately. Then solve by the usual methods.

$$h = 0.00299 \times 3{,}770{,}000^{0.8} \times 0.976^{0.3}$$

① ② ③

Since 3,770,000 is beyond the range of the LL3 scale, it must be divided into parts of the operation of raising to the 0.8 power. The choice of division is not important so long as it is convenient and will accomplish the desired result. The method which involves the least work is to find the square root of the number, raise this to the 0.8 power, and square. Thus, $3{,}770{,}000^{0.8} = 1940^{0.8} \times 1940^{0.8} = 430^2 = 184{,}900$. To perform this operation, move the hairline to 3,770,000 on D and read its square root 1940 on $R_1$. Set right index of C to 1940 on LL3 and move hairline to 0.8 on C. Read 430 on LL3. Set 430 on $R_2$ and read 184,900 on D. To find $0.976^{0.3}$ set left index of C opposite 0.976 on LL/1 and move hairline to 0.3 on C. Turn rule over and read 0.99275 on LL/0. Substituting these values into the equation for h gives
$h = 0.00299 \times 184{,}900 \times 0.99275 = 548$ btu/hr F ft$^2$

**RADIATION**

Heat may also be transferred from one surface to another by radiation. The mechanism of radiation differs from that of convection and conduction inasmuch as the heat is transferred without benefit of any intervening substance. It is radiated just as light except that the wave lengths are usually much greater. The general relation expressing the interchange of heat between two surfaces may be expressed by the equation.



$$Q = 0.173\ F_A\ F_E\ A\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \quad (4)$$

where

- $Q$ = heat transferred by radiation, btu/hr
- $F_A$ = an angle factor which depends upon the geometry of the surfaces and their relative positions, dimensionless
- $F_E$ = an emissivity factor which depends upon the ability of the surfaces to absorb and emit energy, dimensionless
- $A$ = area of one of the surfaces, the choice of which depends upon the method of evaluating $F_A$, ft$^2$.
- $T_1$ = absolute temperature of the warmer surface, deg. R. = 460 + F
- $T_2$ = absolute temperature of the cooler surface, deg. R. = 460 + F

An illustration will show the method of solution for problems of this type.

Example 9.11 Find the heat transferred per square ft of surface of one of two parallel plates if the angle factor is unity and if the emissivity factor is 0.154. The temperature of the two surfaces are 400°F and 60°F respectively. Answer 126.5 btu/hr ft$^2$

Solution

$$Q = 0.173 \times 1 \times 0.154\left[\left(\frac{460 + 400}{100}\right)^4 - \left(\frac{460 + 60}{100}\right)^4\right] = 0.02665\left[8.60^4 - 5.20^4\right]$$

The values of $8.60^4$ and $5.20^4$ can be obtained by the use of the log log scales, by direct multiplication, or by the use of the R scales in conjunction with the C and D scales. The use of the log log scales is fast, but not always as accurate.

Solution using the LL scales:

Set left index of C opposite 8.6 on LL3 and move hairline to 4 on C. Read 5500 on LL3. This is the value of $8.60^4$. By similar methods, find $5.20^4 = 730$. Then $Q = 0.02665(5500 - 730) = 0.02665 \times 4{,}770 = 127.2$ btu/hr ft$^2$.

Solution using the R and A scales:

Set hairline on 8.6 on $R_2$. Read 5480 on A. This is $8.60^4$. Notice that 74 on D is $8.6^2$. Similarly, set hairline on 5.2 on $R_2$ and read 731 on A. Then $Q = 0.02665(5480 - 731) = 0.02665 \times 4749 = 126.5$ btu/hr ft$^2$.

The latter solution is the more accurate of the two and to be preferred.

### HEAT EXCHANGES

Various types of heat exchanger equipment are frequently employed in mechanical engineering applications. The most important of these are surface condensers, feedwater heaters, refrigeration condensers and evaporators, and counter and parallel flow heat exchangers. Their primary purpose is to transfer heat from one fluid to another across a barrier such as a pipe wall or some other separating surface. If the over-all coefficient of heat transfer is known, it is possible to compute a logarithmic mean temperature difference between the two fluids that can be multiplied by the surface area separating the fluids, and by the over-all coefficient, to obtain the rate of heat transfer.

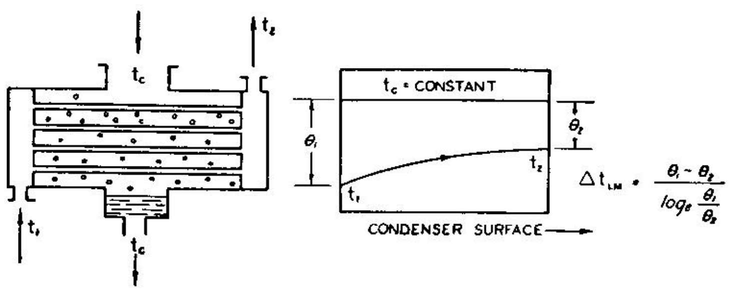
Thus

$$Q = UA\Delta t_{LM} \tag{5}$$

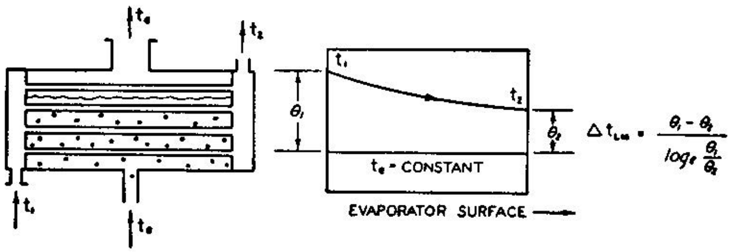
where

$Q$ = rate of flow of heat from one fluid to the other, btu/hr
$U$ = over-all coefficient of heat transfer between the two fluids, btu/hr F ft$^2$.
$\Delta t_{LM}$ = logarithmic mean temperature difference between the two fluids, F.

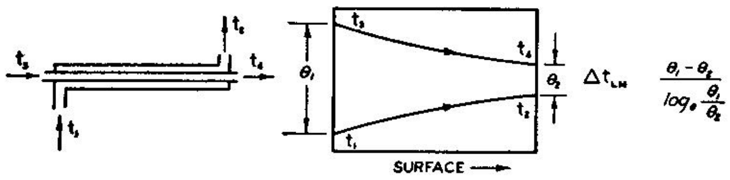




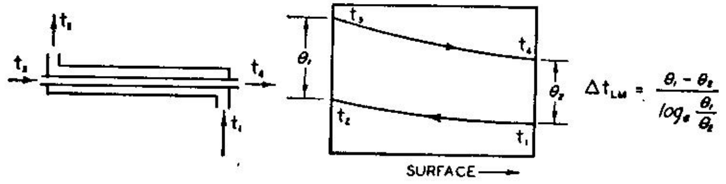

(a) *Steam condensers, feedwater heaters, refrigeration condensers.*




(b) *Refrigeration evaporators.*




(c) *Parallel flow heat exchangers.*




(d) *Counter flow heat exchangers.*

**Figure 9.2—Temperature-surface curves for fluids passing through various types of heat exchanger equipment. $\Delta t_{LM}$ = logarithmic mean temperature difference.**

The relation between the two fluid temperatures as a function of the surface area for the various types of heat exchanger equipment is illustrated in Figure 9.2. The equation expressing the logarithmic mean temperature difference is the same in all cases and is given by the relation $\Delta t_{LM} = \dfrac{\Theta_1 - \Theta_2}{\log_e\left(\dfrac{\Theta_1}{\Theta_2}\right)}$ (6)

where

$\Theta_1$ = temperature difference between the two fluids at inlet as indicated in Figure 9.2, F.

$\Theta_2$ = temperature difference between the two fluids at outlet as indicated in Figure 9.2, F.

For surface condensers, feedwater heaters, and refrigeration condensers and evaporators, the fluid which is condensing or evaporating remains at a constant temperature. Hence, only one of the fluids changes temperature as is clearly indicated in Figure 9.2, (a) and (b). Since equation (6) holds for all cases, it is important in engineering work. Its use in conjunction with equation (5) is illustrated by the three following examples.

Example 9.12 In a large steam surface condenser, 5,000,000 lbs/hr of circulating water is raised in temperature from 60°F to 70°F. If the over-all coefficient of heat transfer is 720 btu/hr F ft² and if the condensing steam temperature is 79°F, what will be the required surface area? The specific heat of the water may be taken as 1 btu/lb F. Answer 5190 ft².

Solution From equations (5) and (6)

$$A = \frac{Q}{U\Delta t_{LM}} = \frac{Q \log_e \dfrac{\Theta_1}{\Theta_2}}{U(\Theta_1 - \Theta_2)}$$

Since the condensing steam temperature is constant, the value of $\Theta_1 - \Theta_2$ will be equal to the rise in temperature of the water. Hence, $Q = 5{,}000{,}000\,(\Theta_1 - \Theta_2)$. Substitution of this value into the above equation gives:

$$A = \frac{5{,}000{,}000}{720} \log_e \frac{79-60}{79-70} = \frac{6{,}000{,}000}{720} \log_e \frac{19}{9}$$



Opposite 19 on DF, set 9 on CF and read 2.11 on D at left index of C. Move hairline over 2.11 on LL2 and slide 720 on C to hairline. Move hairline to 5,000,000 on C and read 5190 on D.

Example 9.13 A feedwater heater raises the temperature of 216,000 lbs of water per hr from 92° to 175°F. If the over-all coefficient of heat transfer is 528 btu/hr F ft² and if the surface area is 769 ft², what will be the temperature of the condensing steam $t_c$?

Solution From equations (5) and (6)

$$\Delta t_{LM} = \frac{\Theta_1 - \Theta_2}{\log_e\left(\dfrac{\Theta_1}{\Theta_2}\right)} = \frac{Q}{UA} \text{ or } \log_e \frac{\Theta_1}{\Theta_2} = \frac{UA(\Theta_1 - \Theta_2)}{Q}$$

Since the specific heat of water is unity, and since the condensing steam temperature is constant, $Q = 216{,}000(\Theta_1 - \Theta_2)$. Hence

$$\log_e \frac{\Theta_1}{\Theta_2} = \log_e \frac{t_c - 92}{t_c - 175} = \frac{528 \times 769}{216{,}000} = 1.88$$

or

$$\frac{t_c - 92}{t_c - 175} = e^{1.88}$$

The value of $e^{1.88}$ may be read directly on LL3 opposite 1.88 on D to obtain 6.55. Then

$$\frac{t_c - 92}{t_c - 175} = 6.55 \text{ or } t_c = \frac{(6.55 \times 175) - 92}{5.55} = 190°\text{F}.$$

Example 9.14 A fluid having a specific heat of 0.65 btu/lb F flows through a counter flow heat exchanger at a rate of 520 lbs/hr. A second fluid having a specific heat of 0.72 btu/lb F flows through the exchanger at a rate of 714 lbs/hr. (a) If the first fluid enters at 560°F and leaves at 318°F, what will be the temperature of the leaving fluid if it enters at 194°F? (b) What will be the logarithmic mean temperature difference? Answer 353°F and 162°F.

Solution (a) The heat absorbed by the cooler fluid must equal that surrendered by the warmer fluid. Hence, the following heat balance can be written

$$520 \times 0.65(560 - 318) = 714 \times 0.72(t_2 - 194)$$

or

$$t_2 = \frac{520 \times 0.65 \times 242}{714 \times 0.72} + 194 = 159 + 194 = 353°F.$$

(b) Referring to Figure 9.2, it is clear that $\Theta_1 = 560 - 353 = 207°F$, and $\Theta_2 = 318 - 194 = 124°F$. Then

$$\Delta t_{LM} = \frac{\Theta_1 - \Theta_2}{\log_e\left(\frac{\Theta_1}{\Theta_2}\right)} = \frac{207 - 124}{\log_e\left(\frac{207}{124}\right)} = \frac{83}{\log_e\left(\frac{207}{124}\right)}$$

Opposite 207 on D, set 124 on C, and read 1.669 opposite left index of C. Set hairline to 1.669 on LL2 and read 0.512 on D. This is $\log_e\frac{207}{124}$. Set hairline to 83 on D and move 0.512 on C to the hairline. Read 1.62 on D opposite the left index of C.

**Exercise 9.2**
**Heat Transfer**

1. Find the heat loss in btu/hr from a pipe 42.8 ft long covered with insulation 1.5 inches thick having a thermal conductivity of 0.032 btu/hr F ft. The outside diameter of the insulation is 6 inches and the temperature drop across the insulation is 227°F.
2. Compute the coefficient of conductance of water flowing through a condenser if the tubes are $\frac{3}{4}$ inch inside diameter. The velocity of flow is 8 ft/sec. The physical constants are $k = 0.35$ btu/lb F ft, $\rho = 62.3$ lbs/ft$^3$, $\mu = 2.37$ lbs/ft hr, and $c_p = 1.00$ btu/lb F.
3. Seven hundred lbs/hr of a fluid having a specific heat of 0.85 btu/lb F are passed in a heat exchanger counter flow to 600 lbs/hr of a fluid having a specific heat of 0.94 btu/lb F. If the first fluid enters at 500°F, and leaves at 200°F, what will be the leaving temperature of second fluid if it enters at 100°F? Compute the logarithmic mean temperature difference and the required area if the over-all coefficient of heat transfer $U = 473$ btu/hr F ft$^2$.



4. A surface condenser having a surface area of 40,000 ft$^2$ circulates 43,000,000 lbs of water per hr. The water increases in temperature from 70°F to 82°F. If the over-all coefficient of heat transfer $U = 638$ btu/hr F ft$^2$, what will be the temperature of the condensing steam?
5. A bare steam pipe passes through a room whose walls are at a temperature of 70°F. If the surface temperature of the pipe is 325°F, find the rate at which heat is lost to the walls per square ft of pipe surface as a result of radiation. For this case assume $F_A = 1.00$ and $F_E = 0.90$.

## 9.3 MACHINE DESIGN

In this section a few selected examples will be used to illustrate typical problems encountered in machine design practice. The problems are selected on the basis of their illustration of certain points regarding the operation of the slide rule rather than on frequency of occurrence.

### RECTANGULAR AND POLAR MOMENTS OF INERTIA, RADII OF GYRATION

An important problem in machine design is the calculation of the stress induced in beams and machine members by the application of bending moments and torsional forces. The methods required for the complete solution of these problems are beyond the scope of this chapter. However, an important item that often enters into the solution, and which must be computed, is the moment of inertia of the cross section of the beam or machine member. When taken about a horizontal axis lying in plane of the cross sectional area, and passing through its center, one obtains the rectangular moment of inertia I. When taken about an axis passing through the center of the cross sectional area, but perpendicular to the plane of the area, one obtains the polar moment of inertia Ip.

Also of importance is the radius of gyration. It is that radius which, when squared and multiplied by the cross sectional area, gives the moment of inertia. In Figure 9.3, formulas for computing the two moments of inertia and their corresponding radii gyration for several widely employed cross sections are presented. A few examples of their solution, illustrating principally the use of the K and R scales follow.

**TABLE II**

**Rectangular and polar moment of inertia of plane cross sections. Radii of gyration.**

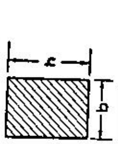
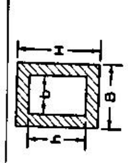
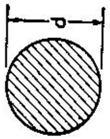
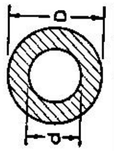
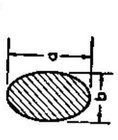
| Section | Rectangular Moment of Inertia I | Rectangular Radius of Gyration R | Polar Moment of Inertia $I_P$ | Polar Radius of Gyration $R_P$ |
|---|---|---|---|---|
|  | $\frac{bh^3}{12}$ | $\frac{h}{\sqrt{12}}$ | $\frac{bh(b^2+h^2)}{12}$ | $\sqrt{\frac{b^2+h^2}{12}}$ |
|  | $\frac{BH^3-bh^3}{12}$ | $\sqrt{\frac{BH^3-bh^3}{12(BH-bh)}}$ | $\frac{B^3H+BH^3-b^3h-bh^3}{12}$ | $\sqrt{\frac{B^3H+BH^3-b^3h-bh^3}{12(BH-bh)}}$ |
|  | $\frac{\pi d^4}{64}$ | $\frac{d}{4}$ | $\frac{\pi d^4}{32}$ | $\frac{d}{\sqrt{8}}$ |
|  | $\frac{\pi(D^4-d^4)}{64}$ | $\frac{\sqrt{D^2+d^2}}{4}$ | $\frac{\pi(D^4-d^4)}{32}$ | $\sqrt{\frac{D^2+d^2}{8}}$ |
|  | $\frac{\pi a^3 b}{64}$ | $\frac{a}{4}$ | $\frac{\pi a^3 b}{32}$ | $\frac{a}{\sqrt{8}}$ |

**Figure 9.3**



Example 9.15 Find the polar moment of inertia and polar radius of gyration of the rectangular cross section in Figure 9.3 if h and b are 2.22 and 1.50 inches respectively. Answer 1.99 $in^4$, 0.774 in

Solution Write the polar moment of inertia as

$$I_p = \frac{bh(b^2+h^2)}{12} = \frac{b^3h}{12}\left[1+\left(\frac{h}{b}\right)^2\right] =$$

$$\frac{1.50^3 \times 2.22}{12}\left[1+\left(\frac{2.22}{1.50}\right)^2\right]$$

Opposite 2.22 on D set 1.50 on C. Opposite left index of C read 1.48 on D. Move hairline to left index of C and read 2.19 on A.This is $\left(\frac{2.22}{1.50}\right)^2$. Add one to 2.19 mentally obtaining 3.19 and set hairline to 3.19 on D. Move slide so that 12 on C is at the hairline and then move hairline to 2.22 on C and read 0.59 on D. Find $1.50^3$ by setting hairline to 1.5 on D and reading 3.375 on K. Opposite 0.59 on D, set 3.375 on CI. At left index of C read the answer 1.99 on D.

Write the polar radius of gyration as

$$R = \sqrt{\frac{b^2+h^2}{12}} = b\sqrt{\frac{1+\left(\frac{h}{b}\right)^2}{12}} = 1.50\sqrt{\frac{1+\left(\frac{2.22}{1.50}\right)^2}{12}}$$

By the same methods as above find $1+\left(\frac{1.22}{1.50}\right)^2 =$ 3.19. Set hairline to 3.19 on D. Move 12 on C to the hairline and move hairline to the left index of C. Read 0.5155 on $R_2$. Set the hairline of C to 0.5155 on D, slide 1.5 on C to the hairline. Read the answer 0.774 on D at the right index of C. As a check $R_p^2 \times bh = I_p$. Hence, $0.774^2 \times 2.22 \times 1.50 =$ 1.99.

Example 9.16 Find the rectangular radius of gyration of the hollow rectangular section if B, H, b and h are 3.2, 4.6, 1.8 and 2.6 respectively. Answer 1.524 $in^2$

Solution For this problem it is easiest to solve for each member under the radical separately using the R scales in conjunction with CI and D scales. Thus $BH^3 = 3.2 \times 4.6^3 = 3.2 \times 4.6 \times 4.6^2 = 312$

To perform the above operations, set hairline to 4.6 on $R_2$ and read 21.16 on D. This is $4.6^2$. Slide 4.6 on CI to the hairline and then move hairline to 3.2 on C. Read 312 on D.
$bh^3 = 1.8 \times 2.6 \times 2.6^2 = 31.65$

Set hairline to 2.6 on $R_1$ and read 6.76 on D. This is $2.6^2$. Slide 2.6 on CI under hairline and move hairline to 1.8 on C. Read 31.65 on D. The denominator under the radical is found in the usual manner. $12(BH - bh) = 12(3.2 \times 4.6 - 1.8 \times 2.6) = 12(14.72 - 4.68) = 120.5$ then

$$R = \sqrt{\frac{312 - 31.65}{120.5}} = \sqrt{\frac{280.35}{124.8}} = 1.50$$

Opposite 280.4 on D, set 120.5 on C. Move the hairline to left index of C and read 1.524 on $R_1$.

Example 9.17 Find the width of an elliptical section of height 2.9 inches which will give a rectangular moment of inertia equal to 0.584 in$^4$. Answer 1.60 in

Solution

$$I = \frac{\pi a^3 b}{64} \text{ and } a = \sqrt[3]{\frac{64\, I}{\pi b}} = \sqrt[3]{\frac{64 \times 0.584}{\pi \times 2.9}} = 1.60$$

Set hairline to 64 on DF and shift to lower group of scales to divide by $\pi$. Slide 0.584 on CI to hairline, move hairline to 2.9 on C. Read 4.1 on D at hairline. Set hairline to 4.1 on K and read $\sqrt[3]{4.1} = 1.60$ on D.

### BELT LENGTH AND TENSION

The use of the S scale together with certain other manipulations may be illustrated by the equations for belt length and belt tension. Figure 9.4 is an illustration of two pulleys over which an open belt is stretched.

Figure 9.4 is an illustration of two pulleys over which an open belt is stretched.



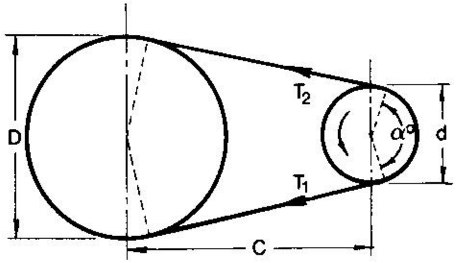



**Figure 9.4 – Open Belt Over Two Pulleys.**

The equation for length of belt required is

$$L = \sqrt{4C^2 - (D - d)^2} + \pi\left(\frac{D + d}{2}\right) + (D - d)\sin^{-1}\frac{D - d}{2C} \qquad (7)$$

where $\sin^{-1}\frac{D - d}{2C}$ must be in radians and

$L$ = belt length, in
$D$ = diameter of larger pulley, in
$d$ = distance of smaller pulley, in
$C$ = distance between pulley centers, in

The angle of contact between belt and smaller pulley is given by the expression

$$\alpha^\circ = 180 - 2\sin^{-1}\frac{D - d}{2C} \qquad (8)$$

Expressed in radians $\alpha = \frac{\alpha^\circ}{57.3}$ radians.

The tension developed by the tight side of the belt in terms of that on the loose side is

$$T_1 = T_2 e^{\mu\alpha} \qquad (9)$$

In equation (9)

$T_1$ = tension on tight side, lbs
$T_2$ = tension on loose side, lbs
$\mu$ = coefficient of friction between belt and pulley
$\alpha$ = angle of contact between belt and smaller pulley, radians

The effective torque for producing power will be $(T_1 - T_2)\frac{d}{2}$ inch lbs so the horsepower developed will be

$$hp = \frac{2\pi N(T_1 - T_2)d}{12 \times 33{,}000 \times 2} = \frac{Nd(T_1 - T_2)}{126{,}000} \qquad (10)$$

where

N = revolutions per minute of the smaller pulley
d = diameter of the smaller pulley, in

Example 9.18 (a) Compute the required length of an open belt to stretch between two pulleys 60 inches apart if their diameters are 22 and 8 inches. (b) Compute the angle of contact of the belt on the smaller pulley in degrees and in radians. (c) If the belt is to transmit 20 hp and if the smaller pulley is to operate at 800 rpm, what will be the tension on the tight and loose sides of the belt. Assume $\mu = 0.30$.

Answer 168 in, 166.6 deg., 2.91 radians, 676 lbs, 282 lbs

Solution

(a) $$L = \sqrt{4 \times 60^2 - (22-8)^2} + \pi\left(\frac{22+8}{2}\right) + (22-8)\sin^{-1}\left(\frac{22-8}{2 \times 60}\right)$$

this may easily be reduced to the following

$$L = \underset{①}{14\sqrt{\left(\frac{120}{14}\right)^2 - 1}} + \underset{②}{\pi \times 15} + \underset{③}{14 \sin^{-1}\frac{14}{120}}$$

① Opposite 120 on D, set 14 on C and read 73.5 on A opposite right index of C. This is $\left(\frac{120}{14}\right)^2$. Subtract one from this mentally to obtain 72.5. Set hairline to 72.5 on D and read 8.51 on $R_2$. Set right index of C opposite 8.52 on D and move hairline to 14 on C. Read 119.2 on D. This is the value of the first term.

② Set hairline to 15 on D and read $\pi \times 15 = 47.1$ on DF.



③ Divide 14 by 120 and obtain 0.1167. Set hairline to 0.1167 on C and read 6.70 degrees on S. This is $\sin^{-1}\frac{14}{120}$ in degrees. Now multiply 14 by the angle 6.70° expressed radians. Set left index of C at 14 on D. Move hairline to 6.7 (.67 on rule) on Sec T SRT scale and read 1.64 on D. Note, reading on C scale is .117 radians. This is the third term. Adding the three terms gives

$$L = 119.2 + 47.1 + 1.64 = 167.94 \cong 168 \text{ in.}$$

(b) The angle of contact will be

$$\alpha^\circ = 180 - 2\sin^{-1}\frac{D-d}{2C} = 180 - 2\sin^{-1}\frac{14}{120} = 180 - 2 \times 6.70 = 166.6 \text{ deg.}$$

Set hairline to 166.6 on D and move 57.3 (r on C scale) to hairline. Read $\alpha = 2.91$ radians on D opposite right index of C.

(c) From equation (10) the difference in belt tensions can be computed

$$T_1 - T_2 = \frac{126{,}000 \text{ hp}}{Nd} = \frac{126{,}000 \times 20}{800 \times 8} = 394 \text{ lbs}$$

also

$$\frac{T_1}{T_2} = e^{\mu\alpha} = e^{0.30 \times 2.91} = e^{0.873}$$

Set hairline to 0.873 on D and read $e^{0.873} = 2.393$ on LL2. Then

$$T_1 = 2.393T_2 = 2.393(T_1 - 394)$$

or

$$T_1 = \frac{2.393 \times 394}{1.393} = 676 \text{ lbs}$$

and

$$T_2 = 676 - 394 = 282 \text{ lbs}$$

### DISPLACEMENT AND VELOCITY OF THE PISTON OF A RECIPROCATING ENGINE

In Figure 9.5 is represented a crank and connecting rod similar to that employed on reciprocating engines for the conversion of rectilinear motion to rotary motion. With this mechanism two important problems arise. These are the determination of piston displacement and piston velocity as a function of crank angle Θ. The two quantities may be expressed by the equations:

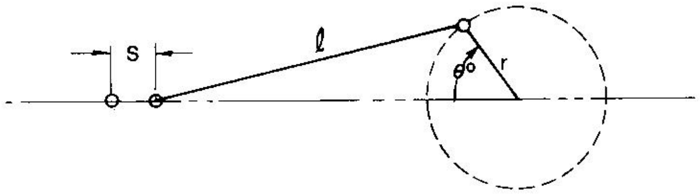



**Figure 9.5 – Reciprocating Engine Connecting Rod.**

$$S = r\left(1 - \cos\Theta + \tfrac{1}{2}\frac{r}{l}\sin^2\Theta + \tfrac{1}{8}\left(\frac{r}{l}\right)^3 \sin^4\Theta + \cdots\right)$$

$$V = 2\pi N r\left(\sin\Theta + \frac{r}{l}\sin\Theta\cos\Theta + \tfrac{1}{2}\left(\frac{r}{l}\right)^3 \sin^3\Theta\cos\Theta + \right)$$

where

- S = piston displacement, ft or inches
- l = length of piston rod, same units as S
- r = radius of crank, same units as S
- Θ = crank angle, degrees
- V = piston velocity, ft/min or in/min depending on units chosen for l and r
- N = revolutions per minute

The above equations may be solved with a high degree of accuracy by including the last term, but in general this may be neglected.

Example 9.19 (a) Find the piston displacement in inches and piston velocity in ft/min for internal combustion engine operating at 3000 rpm if Θ = 68 degrees, l = 8 in, and r = 3 in. (b) Solve the same problem, neglecting the last term, if Θ = 185 degrees. Answer; 2.37 in, 5020 ft/min, 5.99 in, –257 ft/min.



Solution (a) $S = 3\,[1 - \cos 68° + \tfrac{1}{2} \times \tfrac{3}{8} \sin^2 68° + \tfrac{1}{8}(\tfrac{3}{8})^3 \times \sin^4 68° + \cdots]$ and $V = \frac{2\pi \times 3000 \times 3}{12}[\sin 68° + \tfrac{3}{8} \sin 68° \cos 68° + \tfrac{1}{2}(\tfrac{3}{8})^3 \sin^3 68° \cos 68° + \cdots]$

Using the S scale in conjunction with C scale both the sin and cos of 68° are found to be 0.926 and 0.3745 respectively. The remaining steps are simple and need not be explained in detail.

$$S = 3\,[1 - 0.3745 + \tfrac{3}{16} \times (0.926)^2 + \tfrac{1}{8}(\tfrac{3}{8})^3 \times (0.926)^4]$$

$$= 3\,[1 - 0.3745 + 0.161 + 0.00484] = 2.37 \text{ in.}$$

$$V = 4715\,[0.926 + \tfrac{3}{8} \times 0.926 \times 0.3745 + \tfrac{1}{2} \times (\tfrac{3}{8})^3 \times (0.926)^3 \times 0.3745]$$

$$= 4715\,[0.926 + 0.13 + 0.00784] = 5{,}020 \text{ ft/min.}$$

(b) Since sin 185° = –sin 5° and since cos 185° = –cos 5° one may find the following from the SRT and Cos S scales in conjunction with the C scale: Sin 185° = –sin 5° = –0.0871 (from SRT scale)

Cos 185° = –cos 5° = –0.996 (from Cos scale)

then,

$$S = 3\,[1 + 0.996 + \tfrac{1}{2} \times \tfrac{3}{8} \times (0.0871)^2] = 5.99 \text{ in.}$$

$$V = 4715\,[-0.0871 + \tfrac{3}{8} \times 0.0871 \times 0.996] = -257 \text{ ft/min.}$$

The negative sign for velocity in this case simply means that the piston in Figure 9.5 is traveling from right to left.

### SCREW MECHANISMS

Screw mechanisms are used to provide large mechanical advantages. The following equation gives the relation between the torque applied, load raised and the characteristics of the mechanism.

$$T = Lr_m\left[\frac{\tan\alpha + \left(\dfrac{f}{\cos\beta}\right)}{1 - \left(\dfrac{f\tan\alpha}{\cos\beta}\right)}\right] \qquad (13)$$

where

- T = torque applied to the mechanism, in. lbs.
- L = load parallel the mechanism's axis, lbs.
- $r_m$ = mean radius of the screw thread, inches
- $f$ = coefficient of friction between screw and nut threads
- $\alpha$ = helix angle of the screw thread at the mean radius

$$\tan \alpha = \frac{\text{screw lead}}{2\pi r_m} \qquad (14)$$

$\beta$ = an angle measured from a plane tangent to a cylinder formed by revolving the mean diameter and a plane perpendicular to the teeth at the pitch diameter

Answer 106.2 in. lbs.

Example 9.20 Find the torque required to raise a 2,500 lbs load if the lead is 1/18, the mean radius is 0.250 in., the coefficient of friction is 0.125, and $\beta = 21°$.

Solution

$$T = 2{,}500 \text{ lbs} \times .250 \text{ in} \left[ \frac{\dfrac{\frac{1}{18}}{2\pi \times .250 \text{ in}} + \dfrac{.125}{.934}}{1 - \dfrac{\left(.125 \times \dfrac{\frac{1}{18}}{2\pi \times .250 \text{ in}}\right)}{.934}} \right]$$

Using the Cos scale in conjunction with the C scale, the cosine of 21° is found to be .934. The remaining steps are simple and need not be explained in detail.

$$T = 625 \left[ \frac{\dfrac{1}{9\pi} + \dfrac{.125}{.934}}{1 - \dfrac{\left(\dfrac{.125 \times 1}{9\pi}\right)}{.934}} \right] \qquad T = 106.2 \text{ in lbs.}$$

## CLUTCHES

Clutches are used to transmit power. Below is the formula used in obtaining the capacity of a cone clutch.

$$T = F\left[\frac{R_1^3 - R_2^3}{3R_3 H \sin^2\alpha}\right] \qquad (15)$$

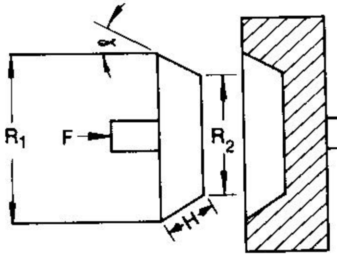



Figure 9.6.



where

$T$ = torque, in. lbs.
$F$ = axial load, lbs.
$f$ = coefficient of friction
$R_1$ = outside contact radius, in.
$R_2$ = inside contact radius
$R_3 = \dfrac{R_1 + R_2}{2}$
$H$ = face width, in.
$\alpha$ = cone angle

Example 9.21 How large a torque would a cone clutch with $R_1$ = 13.75 in. $R_2$ = 13.2 in. $f$ = .120 in. H = 5.00 in. $\alpha$ = 18° and an axial load of 150 lbs. be able to withstand. Answer; 279.5 in. lbs.

Solution

$$T = 150 \text{ lbs.} \times 0.120\left[\frac{(13.75)^3 - (13.2)^3}{3(13.48)\,5.00\,(0.309)^2}\right]$$

$$T = 18\left(\frac{300}{19.32}\right) = 279.5 \text{ in. lbs.}$$

**Exercise 9.3**
**Machine Design**

1. Compute the rectangular moment of inertia and rectangular radius of gyration of a circular annulus if D = 4.5 in and d = 3.44 in. Check moment of inertia by using radius of gyration and area.

2. Compute the polar moment of inertia and polar radius of gyration of the hollow rectangular cross section if B = 4.3, H = 6.4, b = 2.7 and h = 4.8. Check moment of inertia using radius of gyration and area.

3. Find the length of belt required for two pulleys 72 inches apart if one pulley is 48 inches in diameter and the other is 8 inches in diameter.

4. Find the angle of contact for the smaller pulley in problem 3 both in degrees and in radians. If the pulley is to transmit 15 hp at 800 rpm, what will be the tension on the two sides of the belt assuming $\mu$ = .020?

5. Find the piston displacement and velocity for a steam engine operating at 150 rpm if $\Theta = 80$ degrees, $l = 3$ ft and $r = 0.75$ ft. Solve the same problem for $\Theta = 12$ degrees.
6. In a screw mechanism, find the load that a torque of 350 ft lbs will raise if $f = .150$, $\beta = 33°$, lead $= 1/12$, $r_m = .66$ in.
7. If the torque to be transmitted by a cone clutch is to be 2,000 in lbs, and $R_1 = 6.88$ in, $R_2 = 6.6$ in, $f = 01.80$, $H = 2.50$ in, $\alpha = 34.1°$, what is the minimum axial load that must be applied?



# APPLICATIONS TO ELECTRICAL ENGINEERING

The purpose of this chapter is to present a few of the situations in which the Post Versalog II Slide Rule offers unusual advantages to the electrical engineer, and in certain cases to describe the methods to be used in order that the maximum advantage may be realized. Detailed attention will be given to the uses of the trigonometric scales. It is in this area that the electrical engineer will find his greatest satisfaction with this slide rule, but the benefits can be realized only if the proper operational procedures are mastered. A small investment in time spent at the outset in learning such procedures will pay off handsomely in the long run.

## 10.1 THE C AND D SCALES

In electrical engineering, as in other fields, the bulk of the every day routine work is done with the C and D scales. It is worthwhile to devote considerable attention to the procedures outlined in the earlier chapters of this manual for their most economical use, including combined operations with the CF and DF scales. Facility in handling proportions is also of great value in electrical engineering. Illustrations follow.

### USE OF PROPORTION METHODS IN PROBLEMS OF RESISTANCE CHANGES RESULTING FROM TEMPERATURE CHANGES

Resistances of metallic conductors increase with increasing temperature. The formula representing this change is most conveniently expressed as a proportion, as follows:

$$\frac{R_2}{R_1} = \frac{234.5 + t_2}{234.5 + t_1}$$

where $R_2$ is resistance at centigrade temperature $t_2$ and $R_1$ is resistance at $t_1$. The constant 234.5 is suitable for "standard annealed copper." Other constants are required for other materials. The slide rule C and D scales are very convenient for the solution of any proportion.

Example 10.1 The field winding of a motor has 56 ohms resistance at an ambient temperature of 25° C. After full load operation for two hours the resistance is found to be 74.3 ohms. What average temperature was reached by the winding?

Solution

$$\frac{74.3}{56} = \frac{234.5 + t_2}{259.5}$$

The slide rule is used to find $234.5 + t_2$. The procedure using proportion is to bring 74.3 on the C scale opposite 56 on the D scale. Then set the hairline to 259.5 on D and read $234.5 + t_2 = 344.5$ on C. $t_2 = 344.5 - 234.5 = 110°$ C. Any of the four quantities $R_2$, $R_1$, $t_2$, or $t_1$, may be the unknown.

For slide rule users who have to make this type of calculation frequently, it is recommended that auxiliary scales be etched on the slide rule adjacent to the C and D scales as follows:

At 214.5 on C and D, put a mark and label it −20°; at 234.5, another mark labeled 0°; at 254.5, a mark labeled 20° C; and on up every 20° to temperatures as high as required. With such auxiliary scales it becomes possible to make the calculations direct in centigrade degrees without adding and subtracting 234.5.

For materials other than standard annealed copper, the constant 234.5 must be replaced as follows:

| | |
|---|---|
| Hard drawn copper | 242 |
| Commercial aluminum | 236.5 |
| Silver | 243 |
| Platinum | 313 |
| Nickel | 230 |
| Mercury | 236.5 |
| Tungsten | 202 |

### USE OF THE FOLDED SCALES FOR CIRCULAR-MIL AREAS OF RECTANGULAR CONDUCTORS

The cross-sectional area expressed in circular-mils of a rectangular conductor is found as follows:



Area $= \frac{4\,a\,b}{\pi}$ circular mils, where a and b are the cross-section dimensions in mils.

Example 10.2 A rectangular rotorconductor in an induction motor has a cross-section $\frac{1}{4}$ inch by $\frac{1}{2}$ inch. Find the cross-section in circular mils.

Solution

$$\text{Area} = \frac{4 \times 250 \times 500}{\pi} = 159{,}100 \text{ circular mils.}$$

Here the important thing is to make economical use of the folded scales. In this case it is only necessary, after noting that $4 \times 250 = 1000$, to set the hairline to 5 on the DF scale and read 1591 on D under the hairline.

**Exercise 10.1**
**The C and D scales**

1. A 100 watt tungsten filament lamp operating at 2,200° C has a resistance of 132 ohms. What is its resistance just after switching on, before the temperature has had a chance to rise above room temperature of 20° C?
2. The "cold" (30° C) resistance of an armature winding of copper is 0.0345 ohms. If, under full load operation, the temperature is expected to rise 50°, what is the expected operating resistance?
3. What is the cross-section in circular mils of a bus bar 0.25 inches thick and 3.5 inches wide?

## 10.2 THE R AND A SCALES

The square root scales are of particular value to the electrical engineer. Examples of their uses follow:

### COPPER LOSS IN WIRES AND MACHINES WHEN THE CURRENT AND THE RESISTANCE ARE KNOWN

Example 10.3 For a current of 120 amperes in a resistance of 0.076 ohms, find the power dissipated, using $P = I^2R$.

Solution

Set hairline to 120 on $R_1$; move 0.076 on CI to hairline; read result on D at the left index of C. $P = 1094$ watts.

### COPPER LOSS IN WIRES AND MACHINES WHEN THE POTENTIAL DROP AND THE RESISTANCE ARE KNOWN

Example 10.4 For a voltage drop of 9.11 volts in a resistance of 0.076 ohms, to find the power dissipated using $P = \frac{E^2}{R}$.

Solution Set hairline to 9.11 on $R_2$; set 0.076 on C at the hairline; read result on D at the left index of C. P = 1092 watts.

As in Example 10.3, the R scales provide greater efficiency than the A scale.

### CALCULATIONS RELATING TO CIRCUITS POSSESSING RESONANT QUALITIES

It is frequently necessary to evaluate $\sqrt{LC}$, $\sqrt{L/C}$, and $\sqrt{C/L}$, where L and C are inductance and capacitance (sometimes per unit length of circuit). Here the quantity under the radical is evaluated by the usual methods using the C and D scales. A final setting of the hairline transfers this quantity to the R scale where the square root is read. The slide rule settings are simple, but the magnitudes of the quantities involved require care in locating the decimal point.

Example 10.5 Find $\sqrt{LC}$, given L = 150 microhenries and C = 80 micro-micro-farads.

Solution $\sqrt{LC} = \sqrt{1.5 \times 10^{-4} \times 0.8 \times 10^{-10}} =$

$\sqrt{1.2 \times 10^{-14}} = 1.095 \times 10^{-7}$.

It should be noted that even powers of ten were factored from the numbers to facilitate location of the decimal point.

### ROOT-MEAN-SQUARE VALUE OF NON-SINUSOIDAL CURRENT OR VOLTAGE

When the r.m.s. values of the harmonic components are known, the r.m.s. value of the non-sinusoidal function may be found from the equation

$$E = \sqrt{E^2{}_1 + E^2{}_2 + E^2{}_3 + \text{etc.}}$$



Here the R scales may be used with the D scale. Full advantage is gained from the superior accuracy of this slide rule over those having only A and B scales.

### POWER FACTORS FOR PHASE ANGLES LESS THAN 10 DEGREES

For small angles we may use the approximation

$$\cos x = 1 - \frac{x^2}{2}$$

where x is the angle in radians.

The cosine scale on the slide rule is so condensed below 10 degrees that accurate interpolation is difficult. When a cosine of an angle in this range must be known accurately, as is often the case in power factor problems, this approximation may be used to advantage. The upper limit at which this approximation should be applied is 10° = 0.1745 radians. Let us calculate cos 0.1745 according to the approximation and compare the results with a five-place table. The error made will be the maximum, since for smaller values of x the approximation is more accurate.

$x = 0.1745$ radians.
$x^2 = 0.03045$, using the R and D scales.

$$\frac{x^2}{2} = 0.01523$$

$$1 - \frac{x^2}{2} = 0.98477 = \cos 10°, \text{ approximately.}$$

From a five-place table, cos 10° = 0.98481. The difference is 0.00004.

### CIRCULAR-MIL AREAS OF ROUND CONDUCTORS

Area = $D^2$ circular mils where D is the diameter of the wire in mils.

Example 10.6 A micrometer caliper shows the diameter of a round wire to be 0.1019 inches. Find the area in circular mils.

Solution Area = $101.9^2$ = 10,380 circular mils. The R scale is used in the usual way.

**Exercise 10.2**
**The R and A Scales**

1. What capacitance C in micro-micro-farads is required to tune a

200 micro-henry coil to a frequency of one million cycles per second?

$$\left(C = \frac{1}{(2\pi f)^2 L}\right)$$

2. Measurements with a "wave analyzer" on a nonsinusoidal voltage wave indicate the following components to be present: $E_1 = 287$, $E_2 = 57$, $E_3 = 22$, $E_4 = 9$, $E_5 = 0$, $E_6 = 0$, $E_7 = 2$, all being root-mean-square voltages. Find the root-mean-square value of the wave.
3. Find cos 1.62 degrees.
4. Find the circular-mil area of a stranded wire made of 7 strands of circular conductor, each strand having a diameter of 0.0808 inches.
5. The potential drop across a load is indicated by a voltmeter reading to be 232 volts. The voltmeter resistance is 30,000 ohms, as is the resistance of the potential coil of the wattmeter. What "potential coil loss" error must be subtracted from the wattmeter reading?
6. Calculate the copper loss in a field winding of 57 ohms resistance if the current is 0.89 amperes.
7. Determine the surge impedance of a radio-frequency transmission line whose inductance per foot of line is L = 304,500 micro-micro-henries and whose capacitance per foot is C = 3.385 micro-micro-farads. ($Z_0 - \sqrt{L/C}$).

### 10.3 THE L SCALE

The L scale is useful for calculation of logarithmic power ratios in terms of decibels by either of the formulas:

$$\text{d.b.} = 10 \log_{10}\frac{P_2}{P_1}$$

$$\text{or d.b.} = 20 \log_{10}\frac{V_2}{V_1}$$

Example 10.7 Let $\frac{P_2}{P_1} = 460$. Calculate the decibels.



Solution: Opposite 460 on D read the mantissa of $\log_{10}460$ on L, 0.663. The characteristic is 2, so that $\log_{10}460$ is 2.663. Then d.b. = 26.63.

If the data from the same physical situation has been in terms of voltage ratio, this would have been $\frac{V_2}{V_1} = 21.45$. Proceeding as before, $\log_{10}21.45 = 1.3315$ or d.b. = 26.63.

Sometimes it is necessary to calculate the power ratio corresponding to a known number of decibels change in power level. This relationship is expressed by the equation

$$\text{Log}_{10}\left(\frac{P_2}{P_1}\right) = \frac{\text{d.b.}}{10}$$

Example 10.8 Given d.b. = 26.63. Find the power ratio $\frac{P_2}{P_1}$.

Solution: Using the first formula, $\frac{\text{d.b.}}{10} = 2.663 = \log_{10}\frac{P_2}{P_1}$.

Opposite .663 on L read 460 on D, the value of $\frac{P_2}{P_1}$.

The decimal point is placed after the third digit because the characteristic of the logarithm 2.663 is 2.

If the voltage ratio is desired from the given data, then $\frac{V_2}{V_1} = \sqrt{\frac{P_2}{P_1}} = \sqrt{460} = 21.45$. The R and D scales are used as usual.

**Exercise 10.3**
**The L Scale**

1. In carrier-frequency telephone repeater input circuits, one-half of the received power is lost in a line-matching resistor. What is the d.b. power loss in this case?
2. In a radio frequency amplifier the input voltage is 0.2 volts. The output voltage is 45 volts. Find the d.b. voltage gain.
3. A 600 ohm low pass filter designed to "cut off" at 2,000 cycles per second accepts 6 microwatts power at this frequency, whereas

a termination of 600 ohms would accept 1 milliwatt. What loss in d.b. is introduced by the filter, at this frequency?

### 10.4 THE LL SCALES

The unique log-log scales of the Post Versalog slide rule are of great value in a variety of electrical problems. These scales have an arrangement and coverage that makes them unsurpassed for the following calculations.

#### EXPONENTIAL DECAY TERMS IN THE SOLUTION OF TRANSIENT PROBLEMS

These terms take the form $e^{-kt}$ where the function must be evaluated for a series of values of the time t. The exponent kt is first determined for different values of the time t. The hairline is then successively set to the values of kt on the D scale and the corresponding results for $e^{-kt}$ are read from the appropriate level of the reciprocal log log scales as determined from the right end zone symbols. The following examples show the calculations of $e^{-kt}$ for several values of kt.

| | kt | | $e^{-kt}$ | |
|---|---|---|---|---|
| Example 10.9 | 0.008 | on D gives | 0.99204 | on LL/0 |
| Example 10.10 | 0.08 | on D gives | 0.9231 | on LL/1 |
| Example 10.11 | 0.8 | on D gives | 0.4495 | on LL/2 |
| Example 10.12 | 8.0 | on D gives | 0.00034 | on LL/3 |

Note that with this slide rule the exponential term may be found with good accuracy from 0.999 down to 0.00005, for values of the exponent from 0.001 to 10.0. For times on the transient earlier than kt = 0.001 it is possible with a maximum error of about 5 parts in one million to use the LL/0 scale for the range kt = 0.001 to 0.0001. This is done by assuming another 9 to be inserted between the decimal point and the numerals in the numbering of the LL/0 scale. Thus

$$e^{-0.0008} = 0.999204$$
$$e^{-0.00008} = 0.9999204, \text{ etc.}$$

Thus there is no limit to the re-cycling on the LL/0 scale toward unity. Two digits beyond the 9's will be accurate.



#### HYSTERESIS LOSS IN IRON

This loss is expressed as

$$P_h = K_h f B_m^x$$

where: $P_h$ = hysteresis loss in watts per pound of iron;
$K_h$ = a coefficient;
f = frequency in cycles per second;
$B_m$ = maximum flux density in kilo-lines per square inch;
x = the "Steinmetz exponent."

Example 10.13 Given $P_h = 0.6$, $K_h = 1.2 \times 10^{-5}$, $B_m = 65$, f = 60. Find the Steinmetz exponent, x

Solution $0.6 = 1.2 \times 10^{-5} \times 60 \times 65^x$.
$65^x = 833$.

Set the left index of C to 65 on LL3. Opposite 833 on LL3 read x = 1.61 on C.

Example 10.14 Given x = 1.61, $B_m = 70$, $K_h = 1.2 = 10^{-5}$, f = 60. Find $P_h$

Solution $P_h = 1.2 \times 10^{-5} \times 60 \times 70^{1.61}$
$= 1.2 \times 10^{-5} \times 60 \times 937 = 0.675$ watts.

Note that it was first necessary to evaluate $70^{1.61}$ using the LL3 and C scales.

#### EMISSION OF ELECTRONS FROM CATHODES

Calculations in this field frequently require raising a number to a power. The exponent is very often 1.5 or 4.0. Since the method of solution is the same as that given in the discussion of hysteresis loss in iron, details will not be repeated.

**Exercise 10.4**
**The LL Scales**

1. A 3 micro-farad capacitor charges through an 800,000 ohm resistor from a 400 volt source. Find the current at t = 2.4 seconds.

$$\text{Formula: } i = \left(\frac{E}{R}\right) e^{-\frac{t}{RC}}$$

2. Repeat the previous exercise when t = 4.8 seconds.
3. Repeat for t = 0.048 seconds.
4. An iron core has a hysteresis loss of 0.5 watts per pound at 60 cycles and $B_m = 65$. x is known to be 1.6. Find $K_h$.
5. The plate current in a certain vacuum tube follows the law: $I = 1.2 \times 10^{-5} E^{1.5}$. If the voltage E is 200, find I.

### 10.5 THE TRIGONOMETRIC SCALES

The Post Versalog II slide rule includes trigonometric scales which have been designed with special attention to the needs of the electrical engineer. In the past considerable resistance to the use of so-called "vector scales" has existed on the part of students of electrical engineering, and even among instructors in this field. With slide rules existing prior to the Post Versalog II rule, this resistance was well founded because there was no simple way to keep track of basic operations of multiplication and division by sin Θ, cos Θ, and tan Θ. So much care was required to avoid operational errors due to misuse of the scales that the many advantages possible with properly designed trigonometric scales were greatly reduced.

Any user of this slide rule who has mastered the use of the C and CI scales for multiplication and division can multiply and divide by sin Θ, cos Θ, or tan Θ with the same assurance he feels in using the C and CI scales. Only one simple rule has to be observed: *If a trigonometric scale is black or green, use it as you would a C scale; if red, use it as you would a CI scale.* Electrical engineers will find that their Post Versalog II slide rules permit solution of alternating current problems with a freedom from operational errors not possible with other slide rules.

The non-specialized uses of the trigonometric scales have been treated in Chapter Five. The reader should review this chapter before proceeding with the applications of the trigonometric scales to electrical engineering problems.

The reader should cultivate the habit of thinking of the black or green trigonometric scales as C scales, and of the red scales as CI



scales. This is the fundamental nature of these scales, a simple fact which makes their uses quite as simple as those of the C and CI scales. An example will illustrate the point.

Example 10.15 A load of 4,000 kilowatts draws current at a lag angle of 25 degrees. Find (a) the number of kilovolt-amperes, and (b) the reactive power drawn from the line.

Solution (a) $\text{kva} = \dfrac{\text{kw}}{\cos \Theta} = \dfrac{4{,}000}{\cos 25^\circ} = 4{,}415.$

Here cos Θ is 0.906 as may be verified by setting the hairline to 25° on the Cos scale and reading 0.906 on C. It is unnecessary to take the additional step of evaluating cos Θ, and then dividing 4,000 by 0.906. Instead, the hairline is set to 4,000 on D, 25° on the Cos scale is set under the hairline, and the result, 4415, is read on D at the right index of C. Note that the setting used is exactly the same as that used in evaluating $\dfrac{4{,}000}{0.906}$ with the C scale. Thus the division was performed by using the Cos scale as though it were a C scale.

(b) $\text{kvar} = (\text{kw}) \tan \Theta = 4{,}000 \tan 25^\circ = 1{,}865.$

Without evaluating tan 25°, we may set right index of T (black) to 4,000 on D; opposite 25° on T (black) read 1,865 on D. Here the (black) tangent scale has been used as if it were a C scale, to perform a multiplication.

Part (b) could have been solved in another way:

$$\text{kvar} = (\text{kva}) \sin \Theta = 4{,}415 \sin 25^\circ = 1{,}865.$$

Here the S scale has been used like a C scale in multiplication.

Another example will further illustrate the complete consistency possible in viewing the trigonometric scales as equivalent to C or CI scales.

Example 10.16 During a zero power factor test on an alternator, a phase angle of 89° was actually attained. The power delivered by the machine was 520 kw when operated at rated current and voltage. Find (a) the rated kva and (b) the reactive kva during the test.

Solution (a) $\text{kva} = \frac{\text{kw}}{\cos \Theta} = \frac{520}{\cos 89^\circ} = 520 \sec 89^\circ = 29{,}800.$

In this solution, the slide rule operator observes that his cosine scale ends at 84.27°. He finds instead a scale Sec T (red) covering this region. In place of dividing by cos 89°, he multiplies by sec 89°, since $1/\cos \Theta = \sec \Theta$. It is interesting to note that the settings employed are identical with the settings which would be required to divide by cos 89° had the secant scale been made green and called "cosine." This scale is made red and called "secant" in the design of the slide rule because it is desirable to utilize the same scale for tangents. The tangent and the secant are nearly equal in this range, and the tangent scale requires the red color.

(b) $\text{kvars} = (\text{kw}) \tan \Theta = 520 \tan 89^\circ = 29{,}800$

Here it is observed that the same setting is used as in (a), since for angles near 90° tan Θ is approximately equal to sec Θ.

**Exercise 10.5**

**The Trigonometric Scales**

1. A load of 5,000 kilowatts draws current at a lag angle of 32°. Find (a) the number of kilovolt amperes and (b) the reactive power drawn from the line.
2. Solve Example 10.16 with a phase angle of 87.5° if the power delivered is 600 kw.



### 10.6 THEORY AND PROCEDURES

In electrical engineering the principle applications will be in the solution of alternating current problems where it is necessary to make frequent conversions between the polar and the rectangular forms of the phasor (often but improperly called vector) quantities. In electrical engineering, these quantities are symbolized in the following two forms:

| Polar Form | | Rectangular Form | |
|---|---|---|---|
| $A\underline{\vert\Theta}$ | = | $a + jb$ | (1) |

The angle Θ may have any value from zero to 360 degrees and quite frequently is close to zero degrees or to 90 degrees. The process of conversion from polar to rectangular form will be discussed first.

#### POLAR PHASOR TO RECTANGULAR PHASOR

For purposes of illustration, let the phasor be an impedance

$$Z\underline{\vert\Theta} = R + jX \qquad (2)$$

Where: Z is magnitude in ohms;
Θ is phase angle in degrees;
R is resistance in ohms;
j is $\sqrt{-1}$, called in mathematics i;
X is reactance in ohms.

The problem is: given Z and Θ, to find R and X. The relations are analytically:

$$Z\underline{\vert\Theta} = Z \cos \Theta + jZ \sin \Theta \qquad (3)$$

The solution takes the form:

$$R = Z \cos \Theta \qquad X = Z \sin \Theta \qquad (4)$$

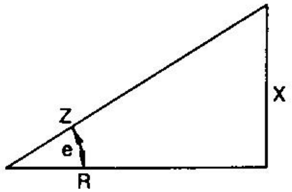



**Figure 10.1 – Graphically.**

which for convenience is usually applied in one of the following equivalent forms:

$$R = X/\tan \Theta \qquad X = Z \sin \Theta \qquad (5)$$
$$R = Z \cos \Theta \qquad X = R \tan \Theta \qquad (6)$$

Equation (4), while simple to visualize, requires additional labor under certain circumstances. Equations (5) and (6) cover all situations with equal economy of effort.

Equations (5) and (6) suggest the following rules:

**RULE A:**

When $\Theta < 45°$, use (5). First find $X = Z \sin \Theta$, then divide this result by $\tan \Theta$ to get R. For example:

$$1.2\underline{|7°} = 1.2 \sin 7°/\tan 7° + j\ 1.2 \sin 7° = 1.19 + j\ 0.1462$$

**RULE B:**

When $\Theta > 45°$, use (6). First find $R = Z \cos \Theta$, them multiply this result by $\tan \Theta$ to get X. For example:

$$1.2\underline{|70°} = 1.2 \cos 70° + j\,(1.2 \cos 70°) \tan 70° = 0.410 + j\ 1.128$$

The reader, having recognized that the trigonometric scales are used exactly as C or CI scales for multiplication and division, will check the above examples without difficulty. He will observe that such problems are solved with three motions: set slide, set hairline, set slide. As a check on proper procedure, he should have worked as follows:

**PROCEDURE A:**

Note that the angle is less than 45°. Therefore, find the imaginary component first. Set the index of C to 1.2 on D; set the hairline to 7° on S; read $X = 0.1462$ under hairline on D; move slide to bring 7° on T (black) under hairline; read $R = 1.19$ on D under index of C.

**PRODECURE B:**

Note angle is greater than 45°. Therefore, find real component first. Set the index of C to 1.2 on D; set the hairline to 70° on Cos; read $R = 0.410$ under hairline on D; move slide to bring 70° on T (red) under hairline; read $X = 1.128$ on D under index of C.

In practice, procedures A and B are almost identical. It is only necessary to watch the first multiplication, using the S scale in the one case and the Cos scale in the other.

Rules A and B may now be summarized in a single inclusive rule:

**RULE C:**

To convert a polar phasor to complex form, find first the smaller component by multiplying Z by $\sin \Theta$ or $\cos \Theta$ as the case may require; then divide or multiply by $\tan \Theta$ as the case may require.



The application of Rule C is extremely easy to master since the slide rule settings take the same form whether $\Theta$ is less than or greater than 45°. The slide rule settings will be: Set an index of C to Z on D. Set hairline to $\sin \Theta$ ($\Theta < 45°$) or to $\cos \Theta$ ($\Theta > 45°$). Read X ($\Theta < 45°$) or R ($\Theta > 45°$) on D under hairline. Move slide until $\Theta$ on T (black or red), is under hairline. Read R ($\Theta < 45°$) or X ($\Theta > 45°$) on D at an index of C.

**POLAR PHASOR TO RECTANGULAR PHASOR FOR ANGLES LESS THAN 5.73°**

The concepts expressed in rule C may be applied unchanged. Hence, find the short side by multiplying Z by $\sin \Theta$, using angles on scale ST (black). However, when the second step is taken, i.e., the division of X by $\tan \Theta$, it will be apparent that the result will be $R = Z$. That is to say, for angles less than 5.73°, the real or resistive component of Z is equal to Z. It is only necessary, then, to calculate X, the short side of the triangle.

Example 10.17 $1.2\underline{|5°} = 1.2 + j\ 1.2 \sin 5° = 1.2 + j\ 0.1045$

It should be remembered that the range of the ST scale is from 0.01 on the left to 0.1 on the right. Hence, the X component lies between 0.01 Z and 0.1 Z.

The lower limit of ST in terms of angle is 0.573°, found near the left end. The nature of this scale is such that we can begin again at the right end with 0.573° and range on down to 0.0573° at the left end, merely by moving the decimal point one place to the left, in both $\Theta$ and $\sin \Theta$. In this way, the conversion from polar form to rectangular form may be made for angles as near zero as we please. This cyclic feature of the ST scale results from the fact that it is based on the approximation (valid to slide rule accuracy for angles less than 5.73°) that

$$\Theta \text{ (in radians)} = \sin \Theta = \tan \Theta$$

The scale gives correct values of $\Theta$ in radians when used with the C scale. Consequently, there is a small but innocuous error in the values of $\sin \Theta$ and $\tan \Theta$ as read from the C scale for angles near the 5.73° limit of ST. The reader should insure his own confidence in the ST scale by comparing values of $\sin \Theta$ and $\tan \Theta$ taken from it with corresponding values found in trigonometric tables.

Example 10.18 $1.2\underline{|5^\circ} = 1.2 + j\,1.2 \sin 5^\circ = 1.2 + j\,0.1045$
(The first is repeated for comparison)

Example 10.19 $1.2\underline{|0.7^\circ} = 1.2 + j\,1.2 \sin 0.7^\circ = 1.2 + j\,0.01465$

Example 10.20 $1.2\underline{|0.5^\circ} = 1.2 + j\,1.2 \sin 0.5^\circ = 1.2 + j\,0.01045$

Example 10.21 $1.2\underline{|0.07^\circ} = 1.2 + j\,1.2 \sin 0.07^\circ =$

$$1.2 + j\,0.001465, \text{ etc.}$$

The decimal point in X is moved to the left as many places as the decimal point in Θ is moved. Another way of expressing this relationship is —the range of the ST scale is multiplied by $10^{-1}$ every time the decimal point is moved one place to the left in the angular markings of this scale.

### POLAR PHASOR TO RECTANGULAR PHASOR FOR ANGLES GREATER THAN 84.27°

Here again, the long side of the triangle, in this case X, is to be taken equal to Z. The short side is calculated according to Rule C from:

$$R = Z \cos \Theta = Z/\sec \Theta$$

Example 10.22 $9\underline{|88^\circ} = 9/\sec 88^\circ + j\,9 = 0.314 + j\,9$

Here the setting employed is: Right index of slide to 9; hairline to 88° on Sec T (red); read R = 0.314 under hairline on C. The secant scale is essentially a CI scale, hence it is employed for division like a CI scale. In other words, the proper view point to hold for angles greater than 84.27° is still to find the real component by multiplying Z by cos Θ. When the attempt is made on the slide rule, a secant scale is found in place of a cosine scale in this range of angles. So we divide by sec Θ as the equivalent of multiplying by cosine Θ.

The cosine scale covers the range of angles from 0° to 84.26° and of cosines from 1.0 to 0.1. The left end values of 84.26° and cosine = 0.1, are equivalent to 84.26° and secant = 10.0. The Sec T (red) scale begins with 84.27° at its right end and extends to 89.427° and secant = 100.0 (cosine = 0.01) at the left end. Like the ST scale, this scale can be used repeatedly for angles nearer and nearer to 90°. For each recycling, the fractional part 427 is to be moved one decimal place to the right and the vacated place replaced by a nine (9) as summarized in the following table:



Sec T (red) Scale:

| | Left end | Right end |
|---|---|---|
| Given range: | sec 89.427° = 100 | sec 84.27° = 10.0 |
| Second range: | sec 89.9427° = 1000 | sec 89.427° = 100 |
| Third range: | sec 89.99427° = 10000 | sec 89.9427° = 1000 |

Example 10.23 $1.2\underline{|85^\circ} = 1.2/\sec 85^\circ + j\,1.2 = 0.1045 + j\,1.2$

Example 10.24 $1.2\underline{|89.3^\circ} = 1.2/\sec 89.3^\circ + j\,1.2 = 0.01465 + j\,1.2$

Example 10.25 $1.2\underline{|89.5^\circ} = 1.2/\sec 89.5^\circ + j\,1.2 = 0.01045 + j\,1.2$

Example 10.26 $1.2\underline{|89.93^\circ} = 1.2/\sec 89.93^\circ + j\,1.2 =$

$$0.001465 + j\,1.2$$

### PHASORS NOT IN FIRST QUADRANT

(Conversion from polar to rectangular form.) In electrical problems phasors frequently appear at angles greater than 90°, i.e., in the second, third, and fourth quadrants. Line potential differences and currents at various points along a transmission line may lag several quadrants behind the input voltage. Transfer impedances may have any angle whatever. (A transfer impedance is defined as the ratio of a source potential difference applied in one branch of a network to the current in some other branch.) Such problems are brought within the scope of the preceding discussion of the first quadrant by the method illustrated in the following example.

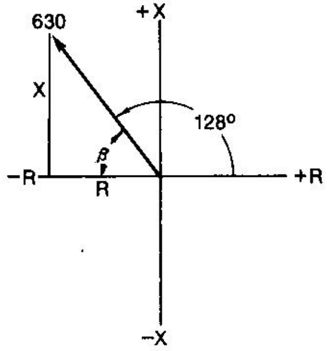



**Figure 10.2—Transfer Impedance—128°.**

Example 10.27 A transfer impedance is known in polar form as 630 ohms at angle 128°. Find its real and reactive components.

The recommended procedure is to draw a sketch in polar form as shown in the diagram. Calculate the angle $\beta$, the smaller angle made by the phasor with the horizontal axis.

Determine R and X by the methods explained for the first quadrant and give these components the proper sign as indicated in the sketch.

Solution $\beta = 180^\circ - 128^\circ = 52^\circ$

$630\,\underline{|52^\circ} = 630 \cos 52^\circ + j(630 \cos 52^\circ)(\tan 52^\circ) =$

$388 + j\,497$

$630\,\underline{|128^\circ} = -388 + j\,497$

Example 10.28 $630\,\underline{|218^\circ} = R + jX$. Find R and X.

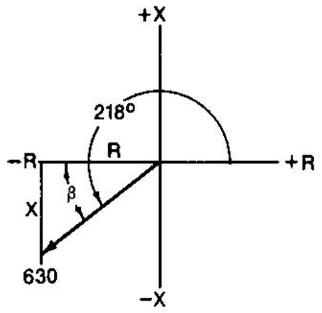



Figure 10.3—Transfer Impedance—218°.

Solution $\beta = 218^\circ - 180^\circ = 38^\circ$

$630\,\underline{|38^\circ} = (630 \sin 38^\circ)/\tan 38^\circ + j\,630 \sin 38^\circ =$

$497 + j\,388$

$630\,\underline{|218^\circ} = -497 - j\,388$

Example 10.29 $630\,\underline{|308^\circ} = R + jX$. Find R and X.



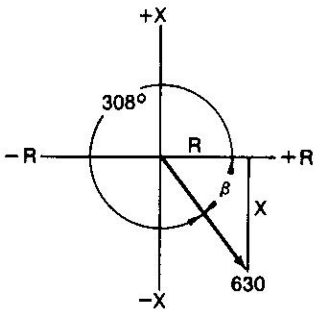



Figure 10.4—Transfer Impedance—308°.

Solution $\beta = 360^\circ - 308^\circ = 52^\circ$

$630\,\underline{|52^\circ} = 630 \cos 52^\circ + j(630 \cos 52^\circ) \tan 52^\circ =$

$388 + j\,497$

$630\,\underline{|308^\circ} = 388 - j\,497$

### RECTANGULAR PHASOR TO POLAR PHASOR

The problem is the inverse of that stated at the beginning of this section, and will be handled by the same relations, i.e., equations (5) and (6), rearranged as follows:

$$X/R = \tan \Theta/1 = \tan \Theta/\tan 45^\circ;\ Z = X/\sin \Theta \qquad (7)$$
$$R/X = 1/\tan \Theta = \tan 45^\circ/\tan \Theta;\ Z = R/\cos \Theta \qquad (8)$$

Equation (7) is to be used when $\Theta < 45^\circ$.
Equation (8) is to be used when $\Theta > 45^\circ$.

Two examples carried through in parallel form will illustrate the two cases:

To find $Z\,\underline{|\Theta}$ when

$R + jX = 4.33 + j\,2.5$ $\qquad$ $R + jX = 2.5 + j\,4.33$

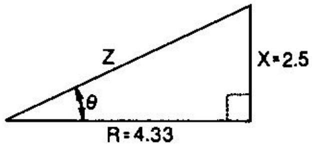



Figure 10.5—Rectangular < 45°.

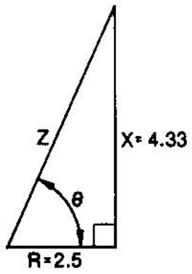



Figure 10.6—Rectangular > 45°.

Find Θ using the proportion

$$X/R = \tan \Theta / \tan 45^\circ$$

Settings: Hairline to 2.5 on D. Index of slide to 4.33.

The hairline is now located at 60°/30° on T; choose $\Theta = 30^\circ$ because $X < R$.

Find Z using $Z = X/\sin \Theta$.

Settings: The hairline is already on X on the D scale. Move slide to bring sin 30° under hairline. Read Z = 5 on D under index of slide.

Find Θ using the proportion

$$R/X = \tan 45^\circ / \tan \Theta$$

Settings: Hairline to 2.5. Index of slide to 4.33.

The hairline is now located at 60°/30° on T; choose $\Theta = 60^\circ$ because $X > R$.

Find Z using $Z = R/\cos \Theta$.

Settings: The hairline is already on R on the D scale. Move slide to bring cos 60° under hairline. Read Z = 5 on D under index of slide.

If the reader will carry out the operations just described, he will see that *the two cases resulted in identical slide rule settings* throughout. Differences appear only in the interpretation of the settings. Thus, to determine the angle, a choice had to be made between 30° and its complement, 60°. This choice should be made solely as a result of visualization of the triangle: If $X < R$, choose the smaller angle. If $X > R$, choose the larger angle.

Again, in finding Z after Θ was known, a choice had to be made between dividing the hairline setting by sin Θ or by cos Θ. Again, the choice is made by visualizing the trigonometry involved. If hairline is on X, divide by sin Θ, (sin 30°). If hairline is on R, divide



by cos Θ, (cos 60°). Since $\sin 30^\circ = \cos 60^\circ$, these two settings were identical. The reader is now in a position to appreciate the following:

#### RULE D:

To find $Z\angle\Theta$ when the two components R and X are given:

1. Set the hairline to the *smaller* component on D.
2. Set an index of the slide to the *larger* component on D. Under the hairline, read the angle on T (black or red) selecting the proper angle by visualization of the triangle.
3. Leaving the hairline on X or on R as the case may be, divide X by sin Θ or R by cos Θ to get Z. (Bearing in mind that the S and Cos scales are fundamentally C scales the user can readily determine how to perform the division.)

Example 10.30 $0.863 + j\,0.834 = 1.20\angle 44^\circ$

Example 10.31 $1.19 + j\,0.1462 = 1.20\angle 7^\circ$

Example 10.32 $0.411 + j\,1.128 = 1.20\angle 70^\circ$

Example 10.33 $0.1253 + j\,1.193 = 1.20\angle 84^\circ$

#### RECTANGULAR PHASOR TO POLAR PHASOR FOR SMALL AND FOR LARGE ANGLES

In the foregoing problems the ratio $X/R = \tan \Theta$ or $R/X = 1/\tan \Theta$ is limited to the range 1.0 to 10.0, corresponding to angles between 5.71° and 84.3°. It is important to extend the range toward 0° and 90°. The slide rule user must be constantly alert for the following cases:

When $X/R < 0.1$, the angle must be read on ST instead of on T.

When $R/X < 0.1$, the angle must be read on Sec T (red) instead of on T (red).

In either case the magnitude of Z is taken equal to the larger of the two components.

Example 10.34 $1.2 + j\,0.1045 = 1.2\angle 5^\circ$

Solution Here $\Theta = 5^\circ$ is read from the ST scale since $0.01R < X < 0.1\,R$, indicating that $0.01 < \tan \Theta < 0.1$. The highest range of ST is from 0.01 to 0.1. For all values of Θ on the ST scale, the approximation is made that sin Θ = tan Θ. Therefore, Z equals larger component, 1.2.

**Example 10.35** $1.2 + j\,0.01465 = 1.2\,\underline{|0.7^\circ}$

Solution: Here again 0.01 R < X < 0.1 R, hence the ST scale is read without change of decimal point. This angle is near the end of what might be called a first cycle over ST.

**Example 10.36** $1.2 + j\,0.01045 = 1.2\,\underline{|0.5^\circ}$

Solution: Here 0.001 R < X < 0.01 R. Hence Θ is read from the ST scale but with the decimal point moved one place to left. This angle is in and near the beginning of a second cycle over ST.

**Example 10.37** $1.2 + j\,0.001465 = 1.2\,\underline{|0.07^\circ}$

Solution: This is similar to example 10.36. The angle is in and near the end of the second cycle over ST.

**Example 10.38** $1.2 + j\,0.001045 = 1.2\,\underline{|0.05^\circ}$

Solution: Here the ST scale decimal points will be moved two places to the left. This angle is in and near the beginning of a third cycle over ST.

**Example 10.39** $1.2 + j\,0.0001465 = 1.2\,\underline{|0.007^\circ}$

Solution: In this example the ST scale decimal points will again be moved two places to the left. This angle is in and near the end of a third cycle over ST.

**Example 10.40** $0.1045 + j\,1.2 = 1.2\,\underline{|85^\circ}$

Solution: In examples 10.34 thru 10.39, X < R, requiring the use of ST. Here X > R which requires the use of Sec T (red). Angles are read without change of decimal point when 0.01 X < R < 0.1 X. In this example the angle is in and near the beginning of a first cycle over Sec T (red) approaching 90°.



**Example 10.41** $0.01465 + j\,1.2 = 1.2\,\underline{|89.3^\circ}$

Solution: This is similar to example 10.40. The angle is in and near the end of a first cycle over Sec T (red), approaching 90°.

**Example 10.42** $0.01045 + j\,1.2 = 1.2\,\underline{|89.5^\circ}$

Solution: Here the hairline will be near the beginning of a second cycle over Sec T (red). In example 10.40 we were 5° short of 90°. Here we are 0.5° short of 90°.

**Example 10.43** $0.001465 + j\,1.2 = 1.2\,\underline{|89.93^\circ}$

Solution: In this example the hairline will be near the end of a second cycle over Sec T (red). In example 10.41 we were 0.7° short of 90°. Here we are 0.07° short of 90°.

For rectangular form to polar form for angles not in the first quadrant, the reader should refer back to the corresponding problem in conversion from polar form to rectangular form. The angle β is to be found by the method just developed. Inspection of the diagram will then reveal how to find Θ.

**Exercise 10.6**

**Theory and Procedures**

Convert the following polar form phasors to rectangular form:

1. $1.2\,\underline{|44^\circ}$
2. $9\,\underline{|30^\circ}$
3. $9\,\underline{|-30^\circ}$
4. $0.02\,\underline{|29.2^\circ}$
5. $0.02\,\underline{|-29.2^\circ}$
6. $36.2\,\underline{|10^\circ}$
7. $1.2\,\underline{|46^\circ}$
8. $9\,\underline{|60^\circ}$
9. $9\,\underline{|-60^\circ}$
10. $0.02\,\underline{|60.8^\circ}$
11. $0.02\,\underline{|-60.8^\circ}$
12. $36.2\,\underline{|80^\circ}$

Convert to complex form:

13. $4{,}200\angle 2.5^\circ$
14. $4{,}200\angle 0.25^\circ$
15. $4{,}200\angle 0.025^\circ$
16. $4{,}200\angle 87.5^\circ$
17. $4{,}200\angle 89.75^\circ$
18. $4{,}200\angle 89.975^\circ$
19. $220\angle 62^\circ$
20. $220\angle 152^\circ$
21. $220\angle 242^\circ$
22. $220\angle 332^\circ$
23. $220\angle -28^\circ$

Convert the following phasors to polar form:

24. $0.863 + j\,0.834$
25. $7.8 + j\,4.5$
26. $7.8 - j\,4.5$
27. $0.01745 + j\,0.00976$
28. $0.01745 - j\,0.00976$
29. $35.6 + j\,6.29$
30. $0.834 + j\,0.863$
31. $4.5 + j\,7.8$
32. $4.5 - j\,7.8$
33. $0.00976 + j\,0.01745$
34. $6.29 + j\,35.6$
35. $4{,}200 + j\,183.2$
36. $4{,}200 + j\,18.32$
37. $4{,}200 + j\,1.832$
38. $0.314 + j\,9$
39. $183.2 + j\,4{,}200$
40. $18.32 + j\,4{,}200$
41. $1.832 + j\,4{,}200$
42. $103.3 + j\,194.2$
43. $-194.2 + j\,103.3$
44. $-103.3 - j\,194.2$
45. $194.2 - j\,103.3$



## ANSWERS TO EXERCISES

**EXERCISE 1.1**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. 171 | 302 | 354 | 708 | 455 | 815 | 1995 | 1225 | |
| 2. 0.734 | 0.993 | 0.941 | 0.980 | 0.828 | 0.902 | 0.606 | 0.271 | |
| 3. 414 | 0.716 | 861 | 0.791 | 877 | 0.993 | 166 | 0.525 | |
| 4. 0.902 | 863 | 0.989 | 123 | 0.438 | 485 | 0.280 | 466 | |
| 5. 202 | 102 | 121 | 126 | 0.976 | 0.704 | 0.465 | 0.070 | |

**EXERCISE 2.1**

| | | |
|---|---|---|
| 1. 1125 | 6. 50.7 | 11. 1.65 |
| 2. 0.104 | 7. 21.2 | 12. 0.000276 |
| 3. 14.69 | 8. 0.1542 | 13. 842 |
| 4. 77.2 | 9. 3.21 | 14. 108 |
| 5. 358 | 10. 799 | 15. 13,520 |

**EXERCISE 2.2**

| | | |
|---|---|---|
| 1. 8.66 | 6. 199 | 11. 4.46 |
| 2. 0.284 | 7. 141.2 | 12. 35,900 |
| 3. 1.967 | 8. 0.1926 | 13. 41,500 |
| 4. 6120 | 9. 5.69 | 14. 812,000 |
| 5. 68.4 | 10. 0.01215 | 15. 66 |

**EXERCISE 2.3**

| | | |
|---|---|---|
| 1. 3.02 | 6. 0.444 | 11. 24.2 |
| 2. 2.84 | 7. 20.6 | 12. 2.18 |
| 3. 23.3 | 8. 1.431 | 13. 0.574 |
| 4. 4.48 | 9. 2.11 | 14. 459 |
| 5. 0.814 | 10. 7.87 | 15. 118.9 |

**EXERCISE 2.4**

| | | |
|---|---|---|
| 1. 0.0322 | 5. 0.370 | 9. 3.07 |
| 2. 0.332 | 6. 3.70 | 10. 1.176 |
| 3. 0.00322 | 7. 0.00282 | 11. 0.0405 |
| 4. 0.0370 | 8. 200 | 12. 0.826 |

**EXERCISE 2.5, 2.6**
**Multiplication**

| | | |
|---|---|---|
| 1. 7.25 | 7. 25.8 | 13. 108.7 |
| 2. 4.48 | 8. 1.967 | 14. 224 |
| 3. 20.8 | 9. 75.2 | 15. 1,990 |
| 4. 3.45 | 10. 68.2 | 16. 605 |
| 5. 23.4 | 11. 46.6 | 17. 8,370 |
| 6. 30.5 | 12. 32.3 | 18. 4,050 |

Use D and CI scales, exercises 1 to 6.

Use DF and CIF scales, exercises 7 to 12.

Use D and CI scales or DF and CIF, exercises 13 to 18.

**Division**

| | | |
|---|---|---|
| 19. 3.02 | 25. 4.27 | 31. 1.431 |
| 20. 2.89 | 26. 0.814 | 32. 1.670 |
| 21. 2.84 | 27. 0.0650 | 33. 2.11 |
| 22. 23.3 | 28. 0.444 | 34. 0.840 |
| 23. 23.4 | 29. 43.3 | 35. 7.87 |
| 24. 4.48 | 30. 20.6 | 36. 184.0 |

Use D and C scales, exercises 19 to 24.

Use DF and CF scales, exercises 25 to 30.

Use D and C scales or DF and CF, exercises 31 to 36.

**EXERCISE 2.7**

| | |
|---|---|
| 1. 9.00 | 6. 1876 |
| 2. 8.74 | 7. 25.0 |
| 3. 32.2 | 8. 11.39 |
| 4. 14.40 | 9. 636 |
| 5. 0.00672 | 10. 20,150 |

**EXERCISE 2.8**

| | |
|---|---|
| 1. .01174 | 4. 1.355 |
| 2. .0608 | 5. 68.5 |
| 3. 84.8 | 6. .1785 |

**EXERCISE 2.9**

| | |
|---|---|
| 1. 123. | 9. 132.6 |
| 2. 0.1098 | 10. 4.28 |
| 3. 0.001537 | 11. 0.550 |
| 4. 0.000775 | 12. 23.4 |
| 5. 181.6 | 13. 0.1366 |
| 6. 19.03 | 14. 26.3 |
| 7. 19.29 | 15. 0.0219 |
| 8. 0.076 | 16. 0.01238 |

**EXERCISE 2.10, 2.11, 2.12**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 368 | 774 | 1,018 | 1,440 | 1,734 | 2,200 | 2,550 | 2,580 | 3,070 |
| 2. | 7.04 | 3.34 | 2.18 | 1.718 | 1.266 | 1.120 | 0.915 | 0.820 | 0.769 |
| 3. | 0.299 | 0.506 | 0.718 | 0.821 | 0.983 | 1.874 | 1.975 | 2.16 | 2.76 |

**EXERCISE 2.13**

| | |
|---|---|
| 1. 280 | 5. 1.25 |
| 2. 9.86 | 6. 1.267 |
| 3. 1.91 | 7. 0.284 |
| 4. 0.00783 | 8. 1.507 |

**EXERCISE 2.14**

| | |
|---|---|
| 1. 1.328 | 3. 3.97 |
| 2. 181.2 | 4. X = 6.09; Y = .872; Z = .125 |



**EXERCISE 2.15**

| | |
|---|---|
| 1. 34.0 and 0.53 | 4. −23.6 and −17.8 |
| 2. 19.5 and 1.64 | 5. 6.6 and 4.55 |
| 3. 25.0 and −4.8 | |

**EXERCISE 3.1**

| | |
|---|---|
| 1. 416 | 11. 906 |
| 2. 511,000 | 12. 35.57 |
| 3. 1,145,000 | 13. 267.4 |
| 4. 15,730 | 14. 7,140 |
| 5. .722 | 15. 1,404 |
| 6. .0000000246 | 16. 25.7 |
| 7. .00884 | 17. 0.651 |
| 8. .0000578 | 18. 0.2958 |
| 9. 5.20 | 19. 0.03115 |
| 10. 30.41 | 20. 0.0851 |

**EXERCISE 3.2**

| | |
|---|---|
| 1. 22.7 | 7. 2.20 |
| 2. 0.0222 | 8. 0.88 |
| 3. 188 | 9. 6.95 |
| 4. 0.00137 | 10. 0.167 |
| 5. 56,600 | 11. 22 |
| 6. 0.0000616 | 12. 0.0431 |

**EXERCISE 3.3**

| | |
|---|---|
| 1. 11.53 | 5. 811 |
| 2. .436 | 6. 2,330 |
| 3. 173.5 | 7. 28,000 |
| 4. 4.81 | |

| | | | | | |
|---|---|---|---|---|---|
| 8. | a) 113.1 | b) 55.4 | c) .0106 | d) 3,020 | e) 5.42 |
| 9. | a) 39.6 | b) .1385 | c) .8 | d) .01924 | e) 2,270,000 |
| 10. | a) 6.09 | b) .125 | c) .4375 | d) 15.55 | e) 5.06 |

**EXERCISE 3.4**

| | |
|---|---|
| 1. 2 | 10. 11.98 |
| 2. 20 | 11. 30.7 |
| 3. 200 | 12. 82.4 |
| 4. 69,000 | 13. .0147 |
| 5. 422,000,000 | 14. .0000467 |
| 6. 32.8 | 15. .000000111 |
| 7. 1.817 | 16. .684 |
| 8. 2.88 | 17. .345 |
| 9. 6.46 | 18. .1957 |

**EXERCISE 4.1**

| | | |
|---|---|---|
| 1. 1.0675 | 8. 1.0329 | 15. 1.372 |
| 2. 1.01027 | 9. 2.23 | 16. 1.0163 |
| 3. 1.96 | 10. 1.0523 | 17. 1.00354 |
| 4. 0.9454 | 11. 0.9781 | 18. 0.591 |
| 5. 0.9836 | 12. 0.518 | 19. 0.8005 |
| 6. 0.9803 | 13. 1.00526 | 20. 1.1172 |
| 7. 0.717 | 14. 0.0058 | 21. 0.578 |

22. 0.9758
23. 0.99654
24. 1.0494
25. 0.869
26. 1.00565
27. 1.245
28. 0.0408
29. 1.001485
30. 328

31. 0.0109
32. 1.00915
33. 0.99843
34. 0.421
35. 0.9365
36. 1.0515
37. 1.387
38. 0.934
39. 0.865

**EXERCISE 4.2**

1. 7.69
2. 1.00204
3. 0.9798
4. 0.514
5. 1.226
6. 0.130
7. 0.99796
8. 0.9654
9. 1.0206
10. 0.8155
11. 0.0498
12. 1.0338

**EXERCISE 4.3**

1. 1.267
2. 3.62
3. 5.50
4. 4.74

5. 0.201
6. 10.02
7. 1.103
8. 0.336
9. 0.971

10. 2.98
11. 0.59
12. 1.99
13. 0.78
14. 1.57
15. 0.31

16. 0.0239
17. 1.791
18. 3.66
19. 5.60

**EXERCISE 4.4**

1. 1.100
2. 1.791
3. $\bar{2}.494$
4. 3.277
5. 0.224
6. $\bar{1}.911$

7. 111
8. 0.684
9. 7.33
10. 24.6
11. 0.001637
12. 0.191

13. 11,220,000
14. 1,550,000
15. 0.000,001,041

**EXERCISE 5.3**

1. 0.5664
2. 0.00524
3. 0.0209
4. 0.215
5. 0.0494
6. 0.0924
7. 0.148
8. 0.236
9. 0.350
10. 0.436
11. 0.548
12. 0.866
13. 0.00436
14. 0.985
15. 0.0610

16. 55′
17. 1°50′
18. 1′
19. 0.258°
20. 6°
21. 13°20′
22. 21°
23. 49.3°
24. 58.4°
25. 64.4°
26. 1.15°
27. 2′



**EXERCISE 5.4**

1. 0.824
2. 1.0
3. 1.0
4. 0.972
5. 0.999
6. 0.994
7. 0.989
8. 0.976
9. 0.937
10. 0.909
11. 0.688
12. 0.500
13. 0.1736
14. 0.0466
15. 0.1685

16. 87.02°
17. 78.4°
18. 66.4°
19. 54°
20. 36°
21. 24.1°
22. 78.48°
23. 88.855°
24. 89.99°
25. 89.8°
26. 36.9°
27. 25.8°

**EXERCISE 5.5**

1. 0.949
2. 0.00349
3. 0.0279
4. 0.0752
5. 0.112
6. 0.323
7. 0.429
8. 0.916
9. 1.11
10. 2.050
11. 10.42
12. 57.3
13. 0.00873
14. 0.0244
15. 0.99

16. 0.5°
17. 2°
18. 1′
19. 4.3°
20. 6.9°
21. 38.1°
22. 49.5°
23. 75°
24. 89°
25. 89.6°
26. 0.2°
27. 5.8°

**EXERCISE 5.9**

1. 6.57
2. 2.33
3. 42.6
4. 2.54
5. 8.23
6. 4.77
7. 20.8
8. 120.1
9. 6.22
10. 25.3
11. 15.5
12. 16.34

**EXERCISE 5.10**

1. A = 41.8°, B = 48.2°, b = 44.7
2. B = 15°, b = 21.4, c = 82.8
3. a = 19.8, b = 21.2, C = 75°
4. A = 49°, B = 71°, c = 22.9
5. A = 31.3°, B = 58.7°, c = 23.7
6. A = 41.5°, B = 56°, C = 82.5°
7. B = 45°, b = 24, c = 33.9
8. A = 37.5°, B = 52.5°, a = 0.353
9. A = 67°20′, b = 5.51, c = 14.3
10. B = 81°, b = 88.4, c = 89.5
11. A = 41°, B = 49°, c = 153.8
12. A = 65°, B = 25°, c = 55.2
13. A = 69.9°, B = 20.1°, c = 661
14. A = 56.4°, B = 87.6°, c = 10.6
15. C = 53°, b = 23.1, c = 21.5
16. C = 6.5°, B = 144.9°, b = 63.3
17. A = 48.5°, C = 89.5°, b = 67
18. B = 50.5°, C = 92.2°, a = 22.3
19. A = 30°, B = 56.5°, C = 93.5°
20. A = 40.7°, B = 59°, C = 80.3°

**EXERCISE 5.11**

1. (2.5, 5.71)
2. (0, 3)
3. (1.73, 1)
4. (4.33, 2.5)
5. (3.65, 1.63)
6. (5.73, 4.02)

7. $5e^{0.925j}$
8. $5e^{0.645j}$
9. $16.6\underline{/32.4°}$

10. 18.71 + j9.54
11. 7.2 + j4.5

**EXERCISE 6.1 THRU 6.6**

1. 17.86%
2. $141
3. 2.97 in
4. 2,130 bolts
5. 126 ft
6. 2.7
7. 7.72 lbs
8. 2.13 hrs
9. 7.9 sec
10. 15,280
11. $2,150
12. $2,752
13. 41.2 miles
14. 2.8 in
15. 1.63
16. 3.75
17. 6
18. 301 $in^2$
19. 6.68 gal
20. 7.67 in.
21. 6.26 in
22. 392 ft/min
23. 560 RPM
24. 7,610 ft
25. 45,700 ft
26. 7,740 lbs
27. 141 ft
28. 302 ft
29. 1.25 ft
20. 5.40
31. 1.23

**EXERCISE 7.1**

1. $19.95
2. $20.80
3. $59.50
4. $52.50
5. a. 33.5%
   b. 43.6%
   c. 13.8%
   d. 9.1%
   TOTAL 100 %

**EXERCISE 7.2**

1. 14.9%
2. 4.73%
3. 2.81%
4. $233

**EXERCISE 7.3**

1. $9,760
2. $12,550
3. $32,000
4. $139,500
5. $9,200

**EXERCISE 7.4**

1. 0.0105
2. 0.0729
3. 0.1074
4. 0.961
5. 0.0668

**EXERCISE 8.1**

1. (a) 10,200 cubic feet
   (b) 10,060 cubic feet
2. 483.88 feet
3. 243 feet; N.22.77°W
4. 137.4 feet
5. 220 feet
6. (a) V = 95.2 feet; H = 526 feet
   (b) V = 27.8 feet; H = 211 feet
   (c) V = 135.5 feet; H = 378 feet
7. (a) R = 848 feet; D = 6.76°; I = 14.1°
   (b) R = 481 feet; D = 11.94°; I = 17.58°
   (c) R = 373 feet; D = 15.40°; I = 9.47°

**EXERCISE 8.2**

1. (a) C = 5′2$\frac{1}{8}$″; R = 7$\frac{3}{16}$″
   (b) C = 11′6″; R = 10$\frac{1}{2}$″
   (c) C = 15′6$\frac{1}{8}$″; R = 3$\frac{29}{32}$″
2. (a) R = 10$\frac{13}{16}$″ (b) R = 11$\frac{1}{8}$″ (c) R = 5$\frac{7}{8}$″



**EXERCISE 8.3**

1. (a) A = 11,660 lbs T
   (b) A = 11,350 lbs C
   B = 11,550 lbs C
   C = 11,550 lbs C
   D = 13,680 lbs C
   E = 8,030 lbs T
   F = 8,030 lbs T
   G = 9,670 lbs T
   B = 8,940 lbs C
   H = 9,670 lbs T
   I = 4,500 lbs T
   J = 5,000 lbs T
   K = 0
   L = 2,640 lbs T
   M = 7,800 lbs T
2. $R_L$ = 19,430 lbs
   $R_R$ = 14,330 lbs
   $M_1$ = 77,700 ft lbs
   $M_2$ = 129,800 ft lbs
   $M_3$ = 152,000 ft lbs
   $M_4$ = 134,900 ft lbs
   $M_5$ = 86,000 ft lbs
   $S_{b_{MAX}}$ = 16,900 lbs/$in_2$
3. $X = 9.40$ in; $f_c = 1{,}070 \frac{lbs}{in^2}$; $f_s = 20{,}000 \frac{lbs}{in^2}$
4. $y = 6 (\cosh 0.0195x - 1)$
   H = 316,000 lbs; V= 343,000 lbs; T = 466,000 lbs
5. $p_1 = 680 \frac{lbs}{ft^2}$; $p_2 = 4{,}600 \frac{lbs}{ft^2}$

**EXERCISE 9.1**

1. 792° R
2. 4.15 psia
3. 3.68 psia
4. 1.36
5. 44.8 btu
6. 4.81 $ft^3$
7. 0.139 btu/deg R
8. −811 btu
9. 51.6 btu/lb
10. −1.935 btu/deg R
11. 230 psia

**EXERCISE 9.2**

1. 2820 btu/hr
2. 1420 btu/hr F $ft^2$
3. 416° F, 92° F, 4.1 $ft^2$
4. 96.8° F
5. 467 btu/hr

**EXERCISE 9.3**

1. 13.25 $in^4$; 1.415 in
2. 103.6 $in^4$; 2.67 in
3. 237.7 in
4. 147.7°; 2.58 Rad
   734 lbs; 438 lbs
5. 0.711 ft.; 727 ft/min
   0.0214 ft.; 183 ft/min
6. 2660 lbs
7. 4330 lbs

**EXERCISE 10.1**

1. 12.2 ohms
2. 0.0410 ohms
3. 1,114,000 CM

**EXERCISE 10.2**

1. 126.7 $\mu\mu f$
2. 293.7 volts
3. 0.9996
4. 45,700 CM
5. 3.59 watts
6. 45.1 watts
7. 300 ohms

**EXERCISE 10.3**

1. 3.01 db loss
2. 47 db
3. 22.2 db loss

**EXERCISE 10.4**

1. $1.84\ (10)^{-4}$ amps
2. $6.75\ (10)^{-5}$ amps
3. $4.90\ (10)^{-4}$ amps
4. $1.048\ (10)^{-5}$
5. 0.0342 amps

**EXERCISE 10.5**

1. (a) 5,900 kva
   (b) 3,120 kvars
2. (a) 13,760 kva
   (b) 13,740 kvars

**EXERCISE 10.6**

1. 0.863 + j 0.834
2. 7.80 + j 4.50
3. 7.80 − j 4.50
4. 0.01745 + j 0.00976
5. 0.01745 − j 0.00976
6. 35.6 + j 6.29
7. 0.834 + j 0.863
8. 4.50 + j 7.80
9. 4.50 − j 7.80
10. 0.00976 + j 0.01745
11. 0.00976 − j 0.01745
12. 6.29 + j 35.6

13. 4,200 + j 183.2
14. 4,200 + j 18.32
15. 4,200 + j 1.832
16. 183.2 + j 4,200
17. 18.32 + j 4,200
18. 1.832 + j 4,200
19. 103.3 + 194.2
20. −194.2 + 103.3
21. −103.3 − j 194.2
22. 194.2 − j 103.3
23. 194.2 − j 103.3

24. $1.2\angle 44^\circ$
25. $9.0\angle 30^\circ$
26. $9.0\angle -30^\circ$
27. $0.02\angle 29.2^\circ$
28. $0.02\angle -29.2^\circ$
29. $36.2\angle 10^\circ$
30. $1.2\angle 46^\circ$
31. $9\angle 60^\circ$
32. $9\angle -60^\circ$
33. $0.02\angle 60.8^\circ$
34. $36.2\angle 80^\circ$
35. $4{,}200\angle 2.5^\circ$
36. $4{,}200\angle 0.25^\circ$
37. $4{,}200\angle 0.025^\circ$
38. $9\angle 88^\circ$
39. $4{,}200\angle 87.5^\circ$
40. $4{,}200\angle 89.75^\circ$
41. $4{,}200\angle 89.975^\circ$
42. $220\angle 62^\circ$
43. $220\angle 152^\circ$
44. $220\angle 242^\circ$
45. $220\angle 332^\circ$