THE SLIDE RULE

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PREFACE

The value of the slide rule as a time saver in the solution of problems cannot be overestimated. A single setting of the slide rule will give a result that would require a tedious multiplication, division, or other arithmetical operation; and two or three successive settings may give the answer that could be obtained only by a page of long-hand calculations. Moreover, the slide rule, when properly manipulated, has a degree of accuracy that makes it applicable to all ordinary calculations.

Unfortunately, the slide rule seems to be a thing of mystery to many who would be benefited by a knowledge of how to use it; yet there is no real mystery about it, for the methods by which it is used are simple, direct, and easy to comprehend.

The purpose of this book is to present, in a clear, concise manner the information required to enable the average person to learn how to use the slide rule for making calculations involving multiplication, division, proportion, finding powers and roots, and so on. In one respect this book differs from all others on this subject. It not only tells how the settings should be made, but it shows the actual settings by a profusion of illustrations made from photographs of the slide rule as arranged for the solution of the various problems stated. In this way the learner is left in no doubt as to the meaning of the instructions. The illustrations show clearly the relative positions of the slide, the runner, and the rule, after the given instructions have been correctly followed.

No attempt has been made to show all the various forms of slide rules or to explain all the ways in which they may be applied. Instead, the aim throughout has been to present a brief, yet thorough, treatise on the use of the ordinary rule for those kinds of calculations that are most frequently encountered. After a person has learned how to handle the ordinary slide rule, he will have no difficulty in learning how to manipulate one or more of the special rules manufactured for special purposes.

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CONTENTS

NOTE.—This book is made up of separate parts, or sections, as indicated by their titles, and the page numbers of each usually begin with 1. In this list of contents, the titles of the parts are given in the order in which they appear in the book, and under each title is a full synopsis of the subjects treated.

THE SLIDE RULE

CONTENTS

THE SLIDE RULE
(Continued)

THE SLIDE RULE

Serial 5364 Edition 1

CONSTRUCTION OF SLIDE RULES

GENERAL FEATURES OF SLIDE RULES

1. **Use of Slide Rule.**—The slide rule is an instrument by means of which various mathematical calculations may be performed mechanically. The number of significant figures that can be set or read on a slide rule, or the accuracy of the computations that can be made with it, is limited by the features of construction of the instrument. A slide rule can, therefore, be used only for calculations in which a reasonable degree of exactness is sufficient; but, in such calculations, it saves much time and labor.

The purpose of this text is to explain how to use a slide rule in performing the usual calculations made with it. In order to understand the various explanations fully, the student should provide himself with a slide rule.

2. **Types of Slide Rules.**—There are various types of slide rules. Some types are intended for general use in performing the computations in all lines of work, whereas others apply specially to the computations in a particular line of work, such as surveying, electricity, or chemistry. The explanations in this text will be limited to the ordinary slide rules for general use. These rules are used most widely. Also, one who learns how to operate them finds little difficulty in learning how to operate a special type for a particular line of work.

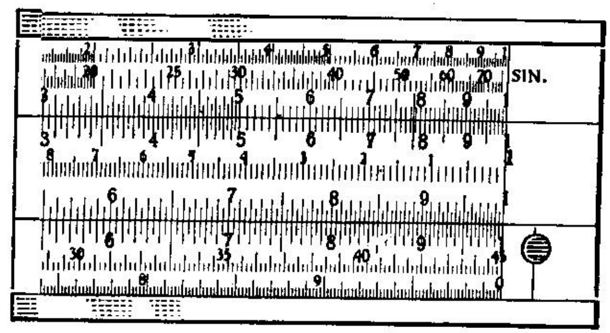
An ordinary slide rule is used most often for multiplication and division, for finding squares and cubes of numbers, and for extracting square roots and cube roots of numbers. It is also used for performing certain other operations, but not for the operations of addition and subtraction.

Slide rules vary, according to the manner in which they are constructed and the materials of which they are made, from very cheap instruments that can be procured for only a few cents to expensive ones that cost as much as $35.00.

The explanations given in this text apply in a general way to all ordinary slide rules, and some of the more common types of rules are described and illustrated. The primary concern of the student is to learn how to use the particular instrument that he has; therefore, in studying this text, he should endeavor to see how the various explanations apply to his rule and, whenever possible, he should work the examples with it. However, in order to derive the most benefit from the text, he should aim to obtain an intelligent understanding of the general features of slide-rule work.

3. Graduations on Slide Rules.—Front views of three typical slide rules are shown in Figs. 1, 2, and 3. At first glance, these rules seem to resemble ordinary rulers. However, an ordinary ruler consists of a single piece, and the graduated scale or scales on it are generally placed along beveled edges. In the case of a slide rule, one part slides within the other part, and the graduated scales used for performing calculations are placed on flat faces of the rule. The slide rules illustrated in Figs. 1 and 2 and most of the other types are made of wood. The particular rule shown in Fig. 2 is designated by the trade-mark name *polyphase slide rule*. The rule shown in Fig. 3, which is known as the *Richardson slide rule*, is made of metal.

The lines of the scales on a slide rule are called *graduations,* and the spaces between graduations are called *divisions* or *subdivisions.* On some inexpensive boxwood slide rules, the graduations are stamped directly in the wood. Usually, however, they are marked on strips of celluloid, which are fastened to the wood or metal parts of the rule. Every slide rule is equipped with a transparent runner, as *a* in Fig. 1, which can be moved along the rule without concealing the graduations underneath it.



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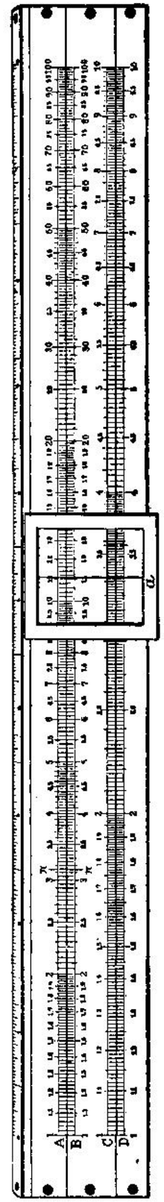


Fig. 1

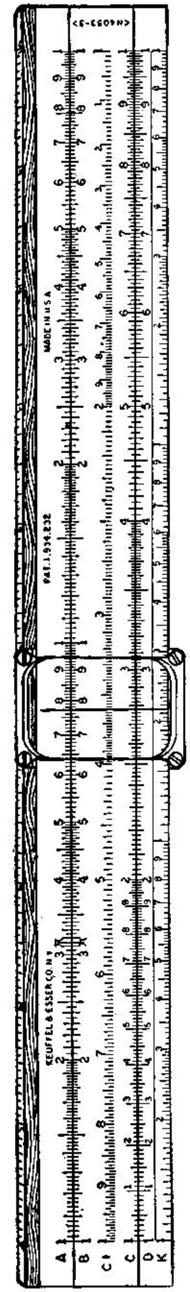


Fig. 2

(Courtesy Keuffel & Esser Co., New York)

All but a few types of slide rules have straight edges, as do the rules here illustrated. The graduated scales on a straight rule are usually about 10 inches long, but rules with scales having lengths of about 20, 16, 8, or 5 inches are also manufactured. A few types of slide rules are either circular or cylindrical in shape; these rules are not used extensively and will not be considered in this text.

4. Character and Arrangement of Scales.—The number of scales and the manner in which each is graduated are not the same on all types of slide rules, but similar scales on different rules are used in the same way. Practically all rules for general purposes contain scales like those marked *A*, *B*, *C*, and *D* on the types illustrated in Figs. 1, 2, and 3. The simplest types of slide rules contain only those four scales, whereas the more elaborate types contain numerous additional scales.

The manner in which the scales are arranged also varies for different types of slide rules, as the best arrangement of the scales on any rule depends to some extent on the number and characteristics of the scales used on that rule. On some rules, all the scales are on the front face. On other rules, there are also scales either on the back face or on the under side of the sliding part. The graduation marks on all scales of a rule are parallel to each other and at right angles to the long edges of the rule.

PARTS OF SLIDE RULE

5. Body and Slide.—An essential feature of a slide rule is that some of the scales may be moved with respect to the other scales. The part of the rule that is considered stationary is called the body of the rule, and the part that is considered movable is called the slide. The scales marked *A* and *D* in Fig. 1, 2, or 3 are on the body of the rule, and the scales marked *B* and *C* are on the slide.

In slide rules like those shown in Figs. 1, 2, and 3, each long edge of the slide is set in a groove in the body; only the front face of the body is graduated; and either one or both faces of the slide are graduated. Such slide rules are known as *Mannheim rules*.

In some slide rules, the two faces of the slide are flush with the two faces of the body, and the two parts of the body that are on opposite sides of the slide are held together by metal pieces in such a manner that the slide can move between those parts. Also, both faces of the body and both faces of the slide are graduated, and the runner is provided with two transparent pieces. Such rules are known as *duplex slide rules*; they are described and are illustrated later in this text.

6. Runner and Hair-Line.—On any slide rule, the runner should be attached to the body in such a way that it will not fall off, will remain in any desired position, and yet can be readily moved along the body. The runner is sometimes called a *cursor* or an *indicator*. Since the runner must be suitable for the particular slide rule on which it is used, the details of construction of runners differ. However, every runner consists essentially of a small plate that is made of glass or other transparent material and has on its under side a fine black line called the hair-line.

The runner shown in Fig. 1 has a metal frame around the glass, and the top and bottom edges of this frame are provided with tongues which fit into grooves in the body of the rule. In the runner on the slide rule shown in Fig. 2, the glass is surrounded by a metal frame that does not overlap the face of the glass and does not, therefore, hide any number on the rule. This frame is screwed to celluloid guides, which are provided with tongues that fit into longitudinal grooves in the body of the rule. The runner for the Richardson rule, shown in Fig. 3, consists of a single piece of transparent material that is hooked around the top and bottom edges of the rule. The tongue-and-groove or hook arrangement keeps the runner from falling off the rule. Also, the runner is usually equipped with a steel spring that bears against the upper edge of the body of the rule. The runner, therefore, remains in any desired position because of the friction between the spring and the body of the rule, and yet very little force is needed to move the runner.

7. The hair-line is so placed on the runner and the runner is so held in position on the rule that the hair-line is parallel to the graduations on all scales for any position of the runner. Thus, for any given settings of the slide and the runner, all values that are indicated by the hair-line on the various scales are directly opposite each other. A required setting or reading may sometimes be made readily without the aid of the hair-line, but it is usually advisable to use the hair-line for marking the position of a desired value on any scale.

8. Magnifying Devices.—The runners shown in Figs. 1, 2, and 3 do not magnify the sizes of the divisions between the graduation marks. Such runners are usually found satisfactory for slide rules because a slight inaccuracy in a setting or reading is of no practical importance in computations for which a slide rule is suitable. However, some types of runners are designed to magnify the portion of the rule in the vicinity of the hair-line. One form of magnifying runner is shown in Fig. 4. The metal frame *a* carries a glass strip *b*, which is flat on the side next to the face of the rule and is convex on the other side. There are also several types of magnifying devices that can be attached to the runner shown in Fig. 2.

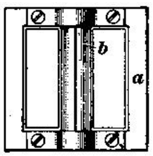


FIG. 4

9. Indexes of Scales.—On the slide rules shown in Figs. 2 and 3, and on most other rules, the graduations at both ends of the *A*, *B*, *C*, and *D* scales are numbered 1. Also, the graduations at the ends of the *CI* scale, at the centers of the *A* and *B* scales, and at the third-points of the *K* scale are numbered 1. On some slide rules, as on the rule shown in Fig. 1, only the graduations at the left-hand ends of the various scales are numbered 1. The graduations at the right-hand ends of the *C* and *D* scales of that rule and also those at the centers of the *A* and *B* scales are numbered 10, and the graduations at the right-hand ends of the *A* and *B* scales are numbered 100. Some rules have scales, called *folded scales*, which do not begin or end with a graduation numbered 1; but each folded scale has a graduation numbered 1 near its center. The graduations that are numbered 1, 10, or 100 on any scale are generally

known as indexes. In the case of a scale that has an index at each end, the index at the left end is the *left-hand index* and the index at the right end is the *right-hand index*.

GRADUATIONS OF SCALES

BASIS OF GRADUATION

10. Logarithmic Basis for Graduating Scales.—In order to understand the theory underlying the operation of a slide rule, it is necessary to have a knowledge of logarithms. However, one can learn how to use a slide rule for the various arithmetical calculations that are performed by means of it without understanding why a slide rule operates as it does. Therefore, all references to logarithms in Arts. 10 to 16, inclusive, may be disregarded by persons who are not familiar with the use of logarithms.

A glance at any of the scales on a slide rule will show that no two adjacent divisions on the scale have the same length. The divisions are not uniform because the scales are graduated on the basis of the mantissas of the logarithms of the numbers represented by the graduations. Except in the case of the numbers 2 and 4, the mantissas for any two numbers do not bear the same ratio to each other as do the numbers themselves. This is evident from the following tabulation, which gives the five-place logarithms of the whole numbers from 1 to 10, inclusive:

$\log 1 = 0.00000$	$\log 4 = 0.60206$	$\log 7 = 0.84510$
$\log 2 = 0.30103$	$\log 5 = 0.69897$	$\log 8 = 0.90309$
$\log 3 = 0.47712$	$\log 6 = 0.77815$	$\log 9 = 0.95424$
	$\log 10 = 1.00000$	

Thus, the ratio of 8 to 4 is 2, but the ratio of log 8 to log 4 is only $0.90309 \div 0.60206$ or 1.5.

11. Graduation of C and D Scales.—The *C* and *D* scales of any slide rule are exactly alike. Either of these scales may be assumed to represent the entire range of values between 1 and 10. Thus, on the type of slide rule shown in Fig. 1, the graduation at the left-hand end of the *C* or *D* scale is marked 1 and the graduation at the right-hand end is marked 10. Since the logarithm of 1 is zero and the logarithm of 10 is 1, the distance between the end graduations on the *C* or *D* scale may be divided logarithmically in the following manner:

The left-hand index is marked 1 and is considered to be the zero mark for locating the other graduations. Also, the right-hand index, which is marked 10 in Fig. 1, is placed at a convenient distance from this zero mark, and this distance is assumed to represent a unit distance or a difference of unity in the characteristics of logarithms of numbers. Then, the graduation mark representing any number between 1 and 10 is located by measuring from the zero mark the distance that represents the mantissa of the logarithm of the number.

In accordance with the foregoing explanations, the distance from the left-hand index of the *C* or *D* scale in Fig. 1 to the graduation numbered 2 would be made equal to 0.30103 times the unit distance between the left-hand and right-hand indexes; the distance from the left-hand index to the graduation marked 3 would be 0.47712 times that unit distance; and so on. The graduations between those numbered 1, 2, 3, 4, etc. would be located so that the distance from the left-hand index to each such graduation would correspond to the mantissa of the logarithm of the value represented by the respective graduation. For example, the distance from the left-hand index to the graduation marked 1.5 would correspond to the value of log 1.5, which is 0.17609. The *C* and *D* scales on the rules shown in Figs. 2 and 3 are graduated in the same way as are the corresponding scales on the rule in Fig. 1. The difference in the numbering of some of the graduations is unimportant, as the corresponding graduations have the same values on all types of rules.

12. Graduation of A and B Scales.—The *A* and *B* scales on any slide rule are also duplicates, but they differ from the *C* and *D* scales. The *A* or *B* scale is divided into two parts that are exactly alike as far as the arrangement of the graduations is concerned. The left-hand half is called the *left-hand A scale* or the *left-hand B scale*, as the case may be, and the right-hand half is called the *right-hand A scale* or the *right-hand B scale*. On most rules, as on those shown in Figs. 2 and 3, the numbering is also the same on both halves of the *A* or *B* scale. In Fig. 1, however, the graduations of the left half are numbered from 1 to 10 and the graduations of the right half are numbered from 10 to 100.

The graduations on the left-hand *A* or *B* scale are located by taking the zero point at the graduation at the left-hand end or at the left-hand index; by assuming the length of the half to represent a unit distance; and by making the distance from the zero point to any intermediate graduation correspond to the mantissa of the logarithm of the value represented by that intermediate graduation. The right-hand *A* or *B* scale is graduated in a similar manner, the zero point for that half being taken at the middle of the complete *A* or *B* scale.

13. Graduation of Other Scales.—The scale marked *K* on the rule shown in Fig. 2 or 3 consists of three similar parts. The length of each of these parts, or the unit distance for the scale, is one-third of the length or unit distance for the *C* or *D* scale. However, with this shorter length as a unit, the distance from the graduation marking the left-hand end of each part of the *K* scale to any graduation in that part corresponds to the mantissa of the logarithm of the number represented by the graduation.

If the slide of the rule shown in Fig. 2 or 3 is taken out of the body, is turned end for end so that the numbers appear upside down, and is replaced in the body in the reversed position, it will be seen that the scale marked *CI* is graduated in exactly the same way as the *C* and *D* scales. In its normal position, which is shown in Fig. 2 or 3, the *CI* scale is an inverted *C* scale.

The other scales on the rule shown in Fig. 3 and also additional scales on various types of rules are described later on in this text.

PRINCIPLES OF OPERATION OF SLIDE RULE

14. Principle of Multiplication With Slide Rule.—The primary purpose of this text is to teach the use of the slide rule rather than the theory underlying its operation. Little attention is therefore given to the reasons for performing the various operations in the manner indicated. However, in order to show that the operation of the slide rule is based on very simple principles, a brief explanation is here given of the application of the properties of logarithms in performing multiplication and division with the slide rule.

If the logarithms of two numbers are added, the sum represents the logarithm of the product of those numbers. It has been shown that the *C* and *D* scales are so divided that the distances to the various graduations from the left-hand index represent the mantissas of the logarithms of the numbers indicated by those graduations. Therefore, the slide-rule work involved in multiplication of two numbers consists in adding mechanically the distances representing the mantissas of their logarithms. When multiplication or division is performed by means of the slide rule, the position of the decimal point in the result is established by a separate process, and the characteristics of the logarithms are not considered in the slide-rule work. The method of finding the product of two numbers is illustrated in Fig. 5, in which are shown the settings that are made for finding the product of 1.5 and 2. In this and most of the following illustrations, the runner is represented by two rectangles, one of which encloses the other. The hair-line is represented by a fine line at the center of the width of the inner rectangle.

The graduation representing the value 1.5—which is marked 1.5 in Fig. 1, or 5 in Fig. 2, 3, or 5—is located on the *D* scale, and the *left-hand* index of the *C* scale is set opposite it by moving the slide in the body of the rule. Then the graduation representing 2 on the *C* scale is located, and the hair-line of the runner is set on it. The runner is left in this position while the product, which is 3, is read from the *D* scale under the hair-line.

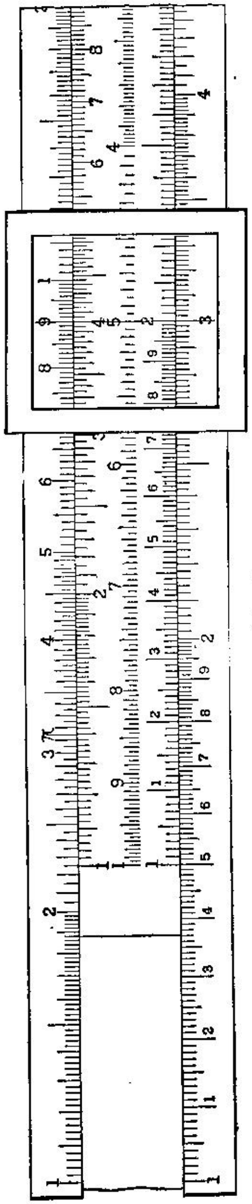


FIG. 5

The explanation is simple. The distance from 1 to 1.5 on the *D* scale is the mantissa of the logarithm of 1.5 and the distance from 1 to 2 on the *C* scale is the mantissa of the logarithm of 2. By setting the slide as shown, these two distances are placed end to end, and their sum is then the distance from 1 to 3 on the *D* scale. Since this distance is the mantissa of the logarithm of 3, the product of 1.5 and 2 is 3. In this case, it is obvious that the product is 3.

15. As another example, let it be required to determine the product of 6.5 and 8. If the *left-hand* index of the *C* scale were set opposite 6.5 on the *D* scale, the graduation representing 8 on the *C* scale would be outside the body of the rule and the product could not be read on the *D* scale. However, the problem can be readily solved if the *right-hand* index of the *C* scale is set to 6.5 on the *D* scale. When this is done and the hair-line is set to 8 on the *C* scale, the product will be found on the *D* scale under the hair-line.

When the right-hand index of the *C* scale is used, the following four distances are involved:

(1) The distance from the left-hand index of the *D* scale to the right-hand index of the *C* scale, or the mantissa of the logarithm of 6.5.

(2) The distance from the right-hand index of the *C* scale to the left-

hand index of that same scale, which distance is the entire length of the scale, or unity.

(3) The distance from the left-hand index of the *C* scale to the hair-line, or the mantissa of the logarithm of 8.

(4) The distance from the left-hand index of the *D* scale to the hair-line, or the mantissa of the logarithm of the product.

The fourth distance is obtained by starting from the left-hand index of the *D* scale, laying off the first distance to the right, then laying off the second distance back to the left, and finally laying off the third distance to the right. In other words, the fourth distance is obtained by adding the first and third distances and subtracting the second distance from the sum. Since the third distance is 1, the fourth distance is the decimal part of the sum of the first and second distances. But, such decimal part is the mantissa of the logarithm of the product, and it therefore follows that, in this case also, the slide-rule work is equivalent to adding logarithms.

16. Principle of Division With Slide Rule.—When logarithms are used, the division of one number by another is performed by first subtracting the logarithm of the divisor from the logarithm of the dividend, and then finding the number corresponding to the difference. With the slide rule, the subtraction is performed mechanically. For example, if it is desired to divide 3 by 2, the slide-rule settings may be made as shown in Fig. 5. The hair-line is set to 3 on the *D* scale and the slide is moved so that 2 on the *C* scale is under the hair-line. Then 1.5, which is the value on the *D* scale opposite the index of the *C* scale, is the quotient.

The explanation is as follows: On the *D* scale, the distance from 1 to 3 is the mantissa of the logarithm of the dividend 3. On the *C* scale, the distance from 2 to 1 (which is the same as that from 1 to 2) is the mantissa of the logarithm of the divisor 2. By placing the slide so that the graduations representing 2 on the *C* scale and 3 on the *D* scale coincide, the mantissa of the logarithm of 2 is subtracted from the mantissa of the logarithm of 3 and their difference is the distance from 1 on the *C* scale to 1 on the *D* scale. This distance is equal to that from 1 to 1.5 on the *D* scale, and is therefore the mantissa of the logarithm of 1.5.

VALUES OF GRADUATIONS

17. Importance of Reading the Scales.—It is very important that the user of the slide rule be able to read values rapidly and correctly from the various scales, and to locate any desired value on a scale. The beginner will probably have more difficulty with these matters than with anything else connected with the use of the rule, but accuracy and speed can be attained by constant practice. Before the methods of performing calculations with the slide rule are taken up, the reading of the scales will be explained fully.

18. Same Setting for Same Significant Figures.—While the slide-rule operations involved in multiplication or division are being performed, the position of the decimal point may be disregarded. In other words, in the operations in which the slide rule is used, the numbers are treated as whole numbers. After the product or the quotient of the whole numbers is obtained, the position of the decimal point in the result is established by a separate process.

In performing the slide-rule work in multiplication or division, the ciphers at the end of any number or at the beginning of a number that is wholly decimal may be disregarded. The figures of a number that remain after such ciphers and the decimal point have been dropped are the significant figures. Thus, each of the numbers 3, 30, 300, 0.3, 0.03, and 0.003 has only one significant figure, namely, 3. Similarly, each of the numbers 256, 25.6, 25,600, and 0.0256 has the same three significant figures, namely, 256. In the case of the number 1,072, all four given figures are significant figures, because a cipher must be considered as a significant figure when it occurs between two other significant figures. The number 107,200 likewise has the four significant figures 1072, as neither of the ciphers following the 2 is a significant figure.

It is important to remember that, in multiplication or division, all numbers with the same significant figures have the same position on a given scale of the slide rule; and the location of the decimal point in the given number is of no consequence in making a setting or a reading on the scale. In some operations, it is necessary to consider the number of figures preceding the decimal point in a given value that is greater than 1, or to consider the number of ciphers between the decimal point and the first significant figure in a given value that is less than 1.

19. Accuracy of Scale Readings.—The part of the *C* or *D* scale between 1 and 2 contains a greater number of divisions than any other part of any scale between two consecutive main graduations, and consequently readings or settings can be made with greater accuracy on this part than on any other part of any scale. No reading can be made accurately beyond four significant figures; and, in most cases, it is not possible to obtain an accurate reading to more than three significant figures. Whenever the fourth significant figure is used, it must be obtained by estimating by eye a fraction of a subdivision. It is, therefore, only an approximation whose closeness to the true value depends on the skill and carefulness of the person reading the rule. In general, then, it may be said that the results obtained by the slide rule are not correct to more than three significant figures; but, if a fourth significant figure can be approximated, that figure should be used.

20. Values of Graduations on C and D Scales.—Since settings on a slide rule are usually made to three significant figures, it is convenient to consider that the graduations on the various scales represent whole numbers of three figures. To aid the beginner in determining the value of a particular graduation or the position of the particular graduation that designates a given value, such values for all the graduations on the *C* and *D* scales are shown in Fig. 6. In order that the entire length of the rule may be represented to a large size, the rule is shown broken into three sections.

21. It should be noted that the distance between the left-hand and the right-hand indexes of the *C* or *D* scale in Fig.

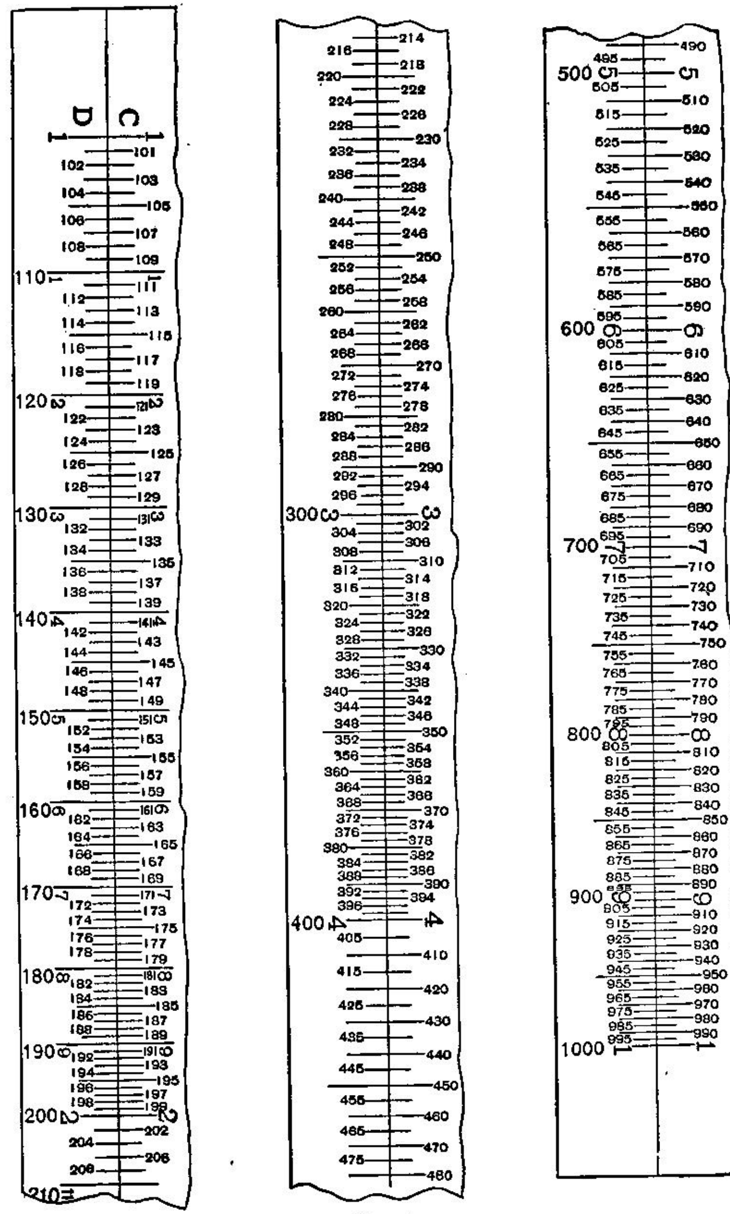


Fig. 6

1, 2, or 3 is divided into ten unequal spaces by main graduations marked with large figures. The left-hand index, which is marked 1 in all three illustrations, may be considered as 100, as indicated in Fig. 6. The next main graduation, which is marked with a large figure 2 in Fig. 1, 2, or 3, may be considered as 200, as indicated in Fig. 6. Likewise, the other main graduations, which are marked with the large figures 3, 4, 5, 6, 7, 8, and 9, may be considered as 300, 400, 500, 600, 700, 800, and 900. The right-hand index, which is marked 10 in Fig. 1 and 1 in Fig. 2 or 3, may be taken as 1,000.

22. The space between the left-hand index and the main graduation marked 2 in Fig. 1, 2, or 3 is divided by the intermediate numbered graduations into ten divisions. In Fig. 1, these intermediate graduations are marked 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, and 1.9. In Figs. 2 and 3, they are marked 1, 2, 3, 4, 5, 6, 7, 8, and 9, as are the main graduations, but the figures for the intermediate graduations are somewhat smaller than are the figures for the main graduations. Since the total distance between the two main graduations marked 1 and 2 represents $200-100$, or 100, each of the ten divisions between two consecutive intermediate numbered graduations represents $\frac{1}{10}\times 100$, or 10. Thus, the intermediate graduations are numbered 110, 120, 130, 140, 150, 160, 170, 180, and 190 in Fig. 6.

In Fig. 1, 2, or 3, each of the ten divisions between the main graduations marked 1 and 2 is subdivided into ten subdivisions by short graduations that are not numbered; the fifth of these graduations is somewhat longer than the others. The space between two consecutive short graduations represents $\frac{1}{10}\times 10$, or 1. Hence, in Fig. 6, the short graduations near the left-hand index are marked 101, 102, 103, 104, 105, etc., and the other short graduations representing values between 100 and 200 are marked with consecutive whole numbers.

23. The space between the main graduations marked with the large figures 2 and 3 in Fig. 1, 2, or 3 is divided by long intermediate graduations into ten divisions. The graduation at the end of the fifth division is somewhat longer than the other intermediate graduations; in Fig. 1 this graduation is marked 2.5, but in Fig. 2 or 3 none of the intermediate graduations are marked. Since each of the ten divisions represents 10, the long intermediate graduations are marked 210, 220, 230, 240, 250, etc. in Fig. 6.

Each division between the main graduations marked 2 and 3 in Fig. 1, 2, or 3 is further subdivided into five subdivisions by four short graduations of equal length. Hence, the space between two consecutive short graduations represents $\frac{1}{5}\times 10$, or 2. In Fig. 6, the short graduations in the first division beyond the main graduation numbered 2 are marked 202, 204, 206, and 208; and the other short graduations representing values between 200 and 300 are also marked with numbers that are multiples of 2.

On the rules shown in Figs. 1, 2, and 3, the intermediate and short graduations in the space between the main graduations marked with the large figures 3 and 4 are arranged in the same manner as are those in the space between the main graduations numbered 2 and 3, but the various graduations are closer together. Accordingly, in Fig. 6, the long graduations dividing the space into ten divisions are marked 310, 320, 330, 340, 350, etc., and the short graduations between the subdivisions are marked with values that are multiples of 2, as 302, 304, 306, etc.

24. Each of the spaces between the large figures 4 and 5, 5 and 6, 6 and 7, 7 and 8, 8 and 9, and 9 and the right-hand index is divided by the long intermediate graduations into ten divisions, each of which represents 10. Also, each division is subdivided by the short graduations into only two subdivisions, and each such subdivision represents $\frac{1}{2}\times 10$, or 5. Therefore, in Fig. 6, the various intermediate graduations beyond the main graduation numbered 4 are marked 405, 410, 415, 420, 425, etc.; the intermediate graduations beyond the main graduation numbered 5 are marked 505, 510, 515, 520, 525, etc.; and so on.

25. Reading C and D Scales.—A study of Fig. 1, 2, or 3 and the preceding four articles will show that the first significant figure in any reading on the *C* or *D* scale of any slide rule is the number that designates the main graduation immediately preceding, or to the left of, the reading. Also, the second significant figure in the required reading is equal to the number of long intermediate graduations between that numbered main graduation and the reading. On the part of the scale between the main graduations numbered 1 and 2, these intermediate graduations are numbered; but, on other parts of the rule, it is necessary to establish the number corresponding to the intermediate graduation by counting. For convenience in counting, the fifth intermediate graduation is somewhat longer than the others and on some rules, as on the rule in Fig. 1, is numbered. The third significant figure in the required reading is established according to the position of the reading on the scale. There are three general conditions: (1) When the reading is on the part of the scale between the main graduations marked 1 and 2, that is, when the first significant figure is 1; (2) when the reading is on the part of the scale between the main graduations 2 and 4, or when the first significant figure is 2 or 3; and (3) when the reading is on the part of the scale between the main graduation marked 4 and the right-hand index, or when the first significant figure is 4 or more.

In order to illustrate the procedure in reading a *C* or *D* scale under these three conditions, various readings on the *C* and *D* scales in Fig. 7 will be considered. For convenience, the scales are shown in three sections, as in Fig. 6.

26. When the reading is between the main graduations marked 1 and 2, the third significant figure of the number is equal to the number of short graduations or small subdivisions between the preceding main or long intermediate graduation and the reading. For example, if the position of the hair-line is as indicated by the line *A-A* in Fig. 7, the first significant figure in the reading is 1 because the main graduation preceding the hair-line is the left-hand index, which is marked 1. The second significant figure is 3, because the long intermediate graduation that precedes the reading is marked with the small figure 3. The third significant figure is 4, because the reading is at the fourth short graduation beyond the intermediate graduation marked 3. Hence, the reading at *A-A* is 134.

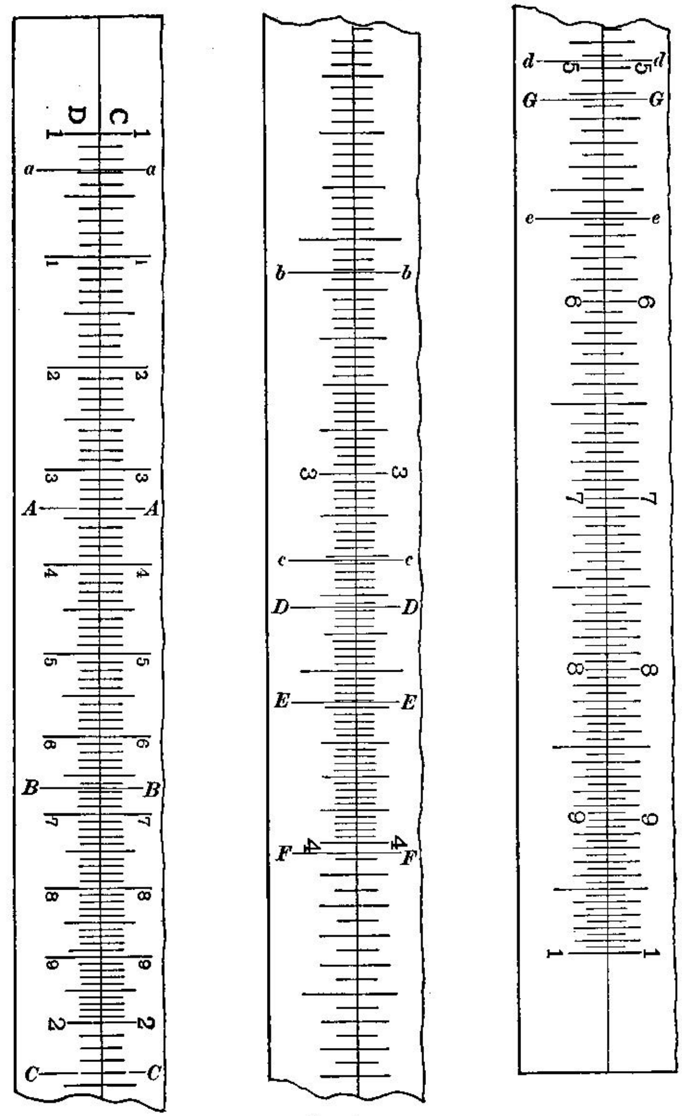


FIG. 7

In case the reading does not coincide with a short graduation, the fourth significant figure can be determined approximately by estimating the fraction of the subdivision between the preceding short graduation and the reading. Thus, at line *B-B*, the first significant figure is 1 and the second significant figure is 6. The third significant figure is 6, because there are six small subdivisions between the reading and the long intermediate graduation marked 6. However, the line *B-B* does not coincide with the sixth short graduation, but is located at a distance from that graduation which may be estimated as about 0.6 of the subdivision. Hence, the fourth figure is about 6, and the entire reading is 1666 if the decimal point is disregarded as it should be.

27. When the reading is between the main graduations numbered 2 and 4, the third significant figure in the reading is determined as follows: It is first necessary to decide whether the reading is nearer a graduation mark or nearer the middle of a space between graduations. If the reading is nearer a graduation, the third significant figure is equal to twice the number of subdivisions between the preceding long graduation and the short graduation nearest to the reading. If the reading is nearer the middle of a space, the third significant figure is found by doubling the number of short graduations or subdivisions between the preceding long graduation and the reading and adding 1.

If the position of the hair-line is as represented by line *C-C* in Fig. 7, the hair-line lies to the right of the main graduation numbered 2, and the first significant figure is 2. Also, the hair-line lies to the left of the first long intermediate graduation beyond that main graduation, and the second significant figure is therefore 0. In this case, the hair-line coincides with a graduation mark. Since there are four subdivisions between the main graduation numbered 2 and the hair-line, the third significant figure in the reading is twice 4 or 8. Thus, the complete reading is 208.

The first and second significant figures in the reading at the line *D-D* are readily seen to be 3 and 3. This reading is at the middle of a space, and there is one short graduation or subdivision between the preceding long graduation and the reading. Hence, the third significant figure is twice 1 plus 1, or $2\times1+1=3$, and the complete reading is 333.

It is often found that a reading on the part of the scale between the main graduations numbered 2 and 4 is located about a quarter of a subdivision from a graduation mark. It is then customary to read the value to four significant figures and to call the fourth figure 5. For example, the reading at the line *E-E* would be called 3585.

28. When the reading lies to the right of the main graduation numbered 4, the procedure for determining the third significant figure is as follows: If the reading coincides with one of the long graduations, the third significant figure is 0; and, if the reading coincides with one of the short graduations, the third significant figure is 5. For a reading between graduations, it is necessary to estimate by eye the number of fifths of a subdivision in the distance from the preceding graduation to the reading. When the preceding graduation is a long one, this estimated number of fifths is the required third significant figure. When the preceding graduation is a short one, the estimated number of fifths is added to 5 in order to obtain the third significant figure.

The first significant figure in the reading at the line *F-F* in Fig. 7 is 4 because the main graduation immediately preceding the reading is numbered 4. Also, the second significant figure in the reading is 0 because there are no long intermediate graduations between that main graduation and the reading. The third significant figure is less than 5 because the graduation preceding the reading is a long one. Since the distance from the preceding graduation to the reading is slightly more than half of a subdivision, the distance is taken as three-fifths of a subdivision, and the third significant figure in the reading is called 3. The complete reading is, therefore, 403.

If a reading beyond the main graduation numbered 4 happens to be exactly midway between two graduation marks, the reading may be expressed to four significant figures. The third and fourth figures are then taken as 2 and 5 when the graduation preceding the reading is a long one; and as 7 and 5 when such graduation is a short one. For example, the reading at the line *G-G* might be called 5125.

29. Reading of CI Scale.—The method of procedure in reading the *CI* scale is exactly the same as that described for the *C* or *D* scale. However, the user of the slide rule must bear in mind that the numbers on the *CI* scale increase from right to left. Therefore, the graduations that precede any reading on the *CI* scale lie to the right of that reading.

30. Reading of A and B Scales.—The procedure to be followed in reading the *A* and *B* scales of a slide rule is, in a general way, similar to that described for the *C* and *D* scales. The *A* and *B* scales on most rules are numbered like those shown in Fig. 2 or 3. For a reading on the part of either half of the *A* or *B* scale between the graduations numbered 1 and 2, the procedure is like that described for the part of the *C* or *D* scale between the main graduations numbered 2 and 4. Also, for a reading between the graduations numbered 2 and 5 on either half of the *A* or *B* scale, the procedure is like that described for the part of the *C* or *D* scale to the right of the main graduation numbered 4.

For a reading on the *A* or *B* scale between the main graduation numbered 5 and the following graduation numbered 1, the procedure is as follows: The first significant figure is the number that designates the main graduation immediately preceding the reading. The second significant figure is equal to the number of short graduations or small subdivisions between that main graduation and the reading. The third significant figure is determined approximately by estimating the fraction of the subdivision between the preceding short graduation and the reading.

When the *A* and *B* scales are numbered like those shown in Fig. 1, it is usually best for a beginner to disregard the ciphers and to imagine that both halves of these scales are numbered like the left-hand half.

31. Reading of K Scale.—In the case of the rule shown in Fig. 3, each third of the *K* scale is read in the same manner as either half of the *A* or *B* scale.

On the rule shown in Fig. 2, the part of each third of the *K* scale between an index and the following main graduation numbered 6 is read as is the part of the *A* or *B* scale between the main graduation numbered 2 and the following index. For a reading between the main graduation numbered 6 and the following index of any third of the *K* scale, the number that designates the main graduation preceding the reading is the first significant figure, and each subdivision represents a difference of 2 in the second significant figure.

32. Settings on Slide Rule.—The principles explained for reading the slide rule are applied also in making settings to indicate given values. In order to illustrate the procedure, it is assumed that each of the following settings is to be made on the *C* or *D* scale in Fig. 7: (*a*) 1,028, (*b*) 2,565, (*c*) 321, (*d*) 0.0497, and (*e*) 56.25. For the purposes of this explanation, the decimal point and any ciphers at the beginning of a number that is wholly decimal are neglected, and only the significant figures are considered. In other words, the given values are treated as the following whole numbers: (*a*) 1028, (*b*) 2565, (*c*) 321, (*d*) 497, and (*e*) 5625. In order to derive the most benefit from the following explanations, the student should first endeavor to make each setting on his slide rule by proceeding in the manner here described without referring to Fig. 7; he should then check his setting by comparing it with the corresponding position shown in that illustration.

Since the first significant figure in 1028 is 1 and the second figure is 0, this setting is between the left-hand index, which is numbered 1, and the first long intermediate graduation following that index. The number of short graduations between the index and the setting is the same as the third significant figure 2 in 1028. The presence of a fourth significant figure indicates that the required setting lies between two short graduations. Since in this case the fourth significant figure is 8, the required setting is eight-tenths of a subdivision beyond the second short graduation, as shown in Fig. 7 by the line *a-a*.

The setting for 2565 is made in the following manner: First, the main graduation numbered 2 and the fifth long intermediate graduation beyond it are located. Then, since the third significant figure is 6 and each small subdivision represents a difference of 2 in the third figure, it is necessary to count off three such subdivisions. Finally, the fourth significant figure 5 in the given number indicates that the required setting is a quarter of a subdivision from a graduation mark, and so the required setting is as shown by the line *b-b*.

To make the setting for 321, the first step is to locate the main graduation numbered 3. The next step is to count two of the long intermediate graduations beyond that main graduation. In this case, the third significant figure, which is 1, is an odd number and the required setting will be midway between two short graduation marks. Since each small subdivision represents a difference of 2 in the third significant figure, the setting is in the first small subdivision beyond the long intermediate graduation that was previously located. Thus, the required setting is indicated by the line *c-c*.

The setting for 497 obviously must lie very close to the main graduation numbered 5, but must precede that graduation because the first significant figure 4 is the number of the main graduation preceding the setting. Also, since the second significant figure is 9, there must be nine long intermediate graduations between the main graduation numbered 4 and the required setting. Finally, since the third significant figure is 7, which is greater than 5, the setting must be beyond a short graduation and the distance from that graduation to the setting must represent 7 − 5, or 2, fifths of a subdivision. Hence, the required setting is located by the line *d-d*.

The main graduation preceding 5625 is the one that is numbered 5, and there must be six long intermediate graduations between that main graduation and the setting. Also, the required setting is midway between the sixth long intermediate graduation and the following short graduation, because the third and fourth significant figures in 5625 are 2 and 5, respectively. The required setting is therefore located by the line *e-e*.

33. Approximation in Making Settings.—It will often be necessary for the user of a slide rule to disregard some of the figures in a given value because the value contains more significant figures than can be used in making the setting on the slide rule. For example, if the given value is 798,125, it is necessary to neglect the last three figures and to make the setting by considering only the significant figures 798. Again, if the given number is 102,483, the setting is made for the value 1025. Similarly, 45.083 is taken as 451; 214.48 is taken as 2145; 27,658, as 2765; and 0.0563175, as 563.

Every setting should be made as accurately as possible. After the user of a slide rule has had a little experience, he will be able to estimate fractions of subdivisions with a high degree of precision. For example, if the given value is 27,668, an experienced slide-rule user would realize that the setting should be practically one-third of the way between the short graduation representing 276 and the adjacent graduation representing 278. Likewise, the setting for the value 5633 should be two-thirds of the way between the graduations representing 560 and 565, respectively. However, this refinement is not usually expected in slide-rule work and, for ordinary calculations, the accuracy indicated by the explanations in Arts. 23 to 31, inclusive, is satisfactory.

USUAL OPERATIONS WITH MANNHEIM SLIDE RULES

LOCATION OF DECIMAL POINT

34. In performing a calculation with the slide rule, there are usually two distinct parts to the work. In one part, the significant figures in the required result are determined. In the other part, the decimal point is located in that result. The order in which the steps should be performed depends on the conditions in the particular case.

It is possible to locate the decimal point by keeping track of the operations that are performed with the slide rule and applying certain rules or principles, which depend on the nature of the calculations. However, if time is to be saved by the use of the slide rule, it is not usually convenient to attempt to apply a rule for locating the decimal point. For this reason, such rules are not given in this text. The simplest, and in most cases also the quickest, method of locating the decimal point is by inspection and approximation. This is the method employed by engineers who use the slide rule regularly. The details of the procedure by inspection or approximation are given in the examples that follow.

Proficiency in the use of a slide rule can be attained only by practice. Therefore, the beginner should solve numerous problems of various kinds by means of the slide rule. If he is not sure of his answer to a given problem, he should check his work by solving the problem by the usual arithmetical method. By such continual practice and checking, he will become so familiar with the methods to be used that he will be able to combine accuracy with speed.

MULTIPLICATION WITH SLIDE RULE

PRODUCT OF TWO NUMBERS

35. Product of Two Numbers by Means of C and D Scales. The product of two numbers may be found with the same degree of accuracy by using either the *C* and *D* scales or the *CI* and *D* scales. If the slide rule does not have a *CI* scale, it is necessary to use the *C* and *D* scales; but, if the rule has a *CI* scale, it is usually preferable to use the *CI* and *D* scales. When the *C* and *D* scales are used, the significant figures in the product of any two numbers may be found by applying the following rule:

Rule.—*To determine the significant figures in the product of any two numbers, set either the left-hand index or the right-hand index of the C scale opposite one of the given factors on the D scale; move the hair-line to the other given factor on the C scale; and read the result under the hair-line on the D scale.*

The *A* and *B* scales may also be used for multiplication by substituting *A* for *D* and *B* for *C* in the preceding rule. However, since the *C* and *D* scales can be read with greater precision than the *A* and *B* scales, it is usually preferable to use the *C* and *D* scales in order to obtain greater accuracy.

Whether the left-hand index or the right-hand index of the *C* scale should be used depends on the given numbers. If the product of the first significant figure in one number and the first significant figure in the other number is 10 or more, the right-hand index must be used. This index is also used in a great many cases in which the product of the first figures is less than 10. After the user of the slide rule has had some experience, he will be able to recognize the relatively few cases in which the left-hand index is to be used. If the wrong index is tried, the hair-line will be off the *D* scale when it is set to the second factor on the *C* scale. It is then necessary to use the other index.

The given numbers may be taken in either order. Thus, since the product of 1.5 and 2 is the same as the product of 2 and 1.5, the product of these numbers can be found either by setting the left-hand index of the *C* scale to 2 on the *D* scale and setting the hair-line to 1.5 on the *C* scale, or by setting the left-hand index of the *C* scale to 1.5 on the *D* scale and setting the hair-line to 2 on the *C* scale. When the right-hand index is used, or in most cases, it is preferable to set the index of the *C* scale to the number having the larger first significant figure. As much as possible of the slide will then be within the body of the rule and the position of the slide will not be disturbed so readily by accident. In the comparatively few cases when the left-hand index is used, it is more advantageous to set the index to the number having the smaller first significant figure.

36. It is usually most convenient to locate the decimal point in the product of two numbers after the significant figures in the product have been determined by means of the slide rule. A rough calculation is generally best for this purpose. In this computation, each of the given numbers is replaced by a number that is composed of only one significant figure and the proper number of ciphers to make the new value approximately equal to the given number. Usually, the numbers may be rounded off by taking the nearest approximate values. For instance, the number 463 would be called 500; the number 0.382 would be called 0.4; and the number 37.65 would be called 40. However, considerable leeway in choosing the approximate numbers is permissible, because the purpose of the rough computation is to establish the position of the decimal point in the product and not to determine the approximate value of the product. Thus, although 0.182 is much nearer to 0.2 than to 0.1, it may be more convenient in some cases to take the approximate value as 0.1. Also, in order to simplify the rough calculation, a value like 418 might be called 500 rather than 400, and a value like 79 might be called 100 rather than 80. If one factor is increased considerably, it might be well to decrease the other factor. The slide-rule user soon acquires good judgment in making these approximations. The product of the approximate numbers can be found by a simple calculation, which can be performed either mentally or on scrap paper. The decimal point is then located in the required product so as to make that product nearly equal to the product of the approximate numbers.

37. Illustrative Examples.—The procedure in multiplying two numbers by means of the slide rule is illustrated by the following examples.

Example 1.—Find the product of 25.6 and 33.

Solution.—In this case, it is necessary to use the left-hand index of the *C* scale, and the slide-rule settings are made as shown in Fig. 8. The left-hand index of the *C* scale is set to 25.6 or 256 on the *D* scale, and the hair-line is set to 33 or 330 on the *C* scale. The significant figures in the product are read from the *D* scale under the hair-line and are 845.

To locate the decimal point in the product, 25.6 is called 30 and 33 is called 30. Since $30 \times 30 = 900$, the required product must be 845. Ans.

It would obviously be impossible for the product to be 84.5 or 8,450. If the actual multiplication is performed by the usual arithmetical method, it is found that the product of 25.6 and 33 is 844.8.

Example 2.—Find the product of 3.75 and 0.088.

Solution.—For these factors, the right-hand index of the *C* scale must be used, and the slide-rule settings are made as shown in Fig. 9. Here, the first step is to set the right-hand index of the *C* scale opposite 0.088 or 880 on the *D* scale. The next step is to set the hair-line

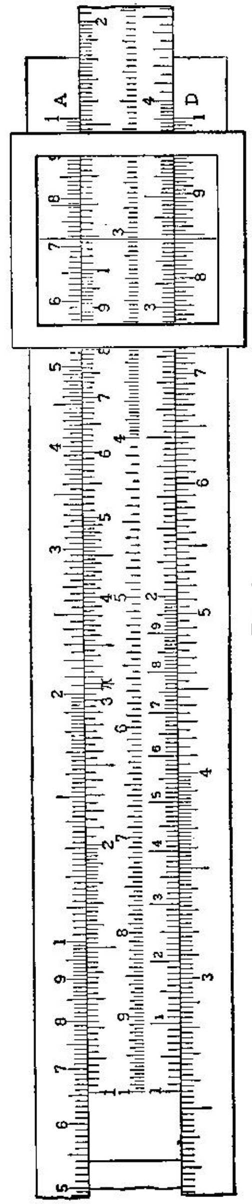


Fig. 8

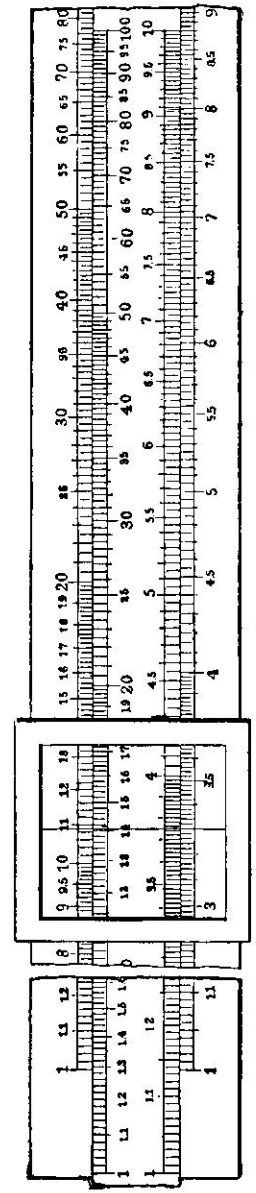


Fig. 9

to 3.75 or 375 on the *C* scale. Then, the significant figures in the reading on the *D* scale under the hair-line are 330 or 33.

The decimal point may be located in the product in the following manner: The given number 0.088 is taken as 0.1, and 3.75 is taken as 3. Since $0.1 \times 3 = 0.3$, the required product is obviously 0.33. Ans.

The value 0.09 is closer to 0.088 than is 0.1, but it is somewhat simpler to find the product of 0.1 and 3 than to find the product of 0.09 and 4. In general, the most convenient values should be used in the rough calculation.

38. Product of Two Numbers by Use of CI Scale.—If the slide rule has a *CI* scale, the product of two numbers can be conveniently found by use of the *D* and *CI* scales. The rule to be applied in this case is as follows:

Rule.—*To determine the significant figures in the product of any two numbers, set the hair-line to either of the given factors on the D scale; move the slide so that the other given factor on the CI scale is under the hair-line; and read the result on the D scale opposite whichever index of the C scale happens to be over the D scale.*

For example, the significant figures in the product of 25.6 and 33 can be found by setting the hair-line to 25.6 or 256 on the *D* scale, bringing 33 or 330 on the *CI* scale under the hair-line, and reading 845 on the *D* scale opposite the right-hand index of the *C* scale. Also, to find the product of 3.75 and 0.088, the hair-line is set to 3.75 or 375 on the *D* scale, the value 0.088 or 880 on the *CI* scale is brought under the hair-line, and the result 330 or 33 is read on the *D* scale opposite the left-hand index of the *C* scale. In any case, the decimal point is located in the product by performing an approximate calculation, as explained for the use of the *C* and *D* scales. When the *D* and *CI* scales are used, the slide will be in the same position no matter which of the two given numbers is located on the *D* scale. Hence, either factor may be so located.

39. Comparison of Methods.—The use of the *CI* scale instead of the *C* scale for multiplication has the advantage that it is not necessary to consider whether the left-hand index or the right-hand index of the *C* scale is to be used. Also, the

position of the slide will be the same no matter which factor is taken first. The *CI* scale is used to even greater advantage in problems involving the multiplication of more than two factors and in various other types of problems that are given in this text.

Even if the slide rule has a *CI* scale, the *C* and *D* scales are used advantageously for the multiplication of two factors when it is necessary to determine a series of products and the same number is used as a factor in each case; that is, when a constant factor is to be multiplied successively by several other factors. The proper index can then be set to the constant factor on the *D* scale and the hair-line can be moved successively to each of the other factors on the *C* scale. For each position of the runner, the required product is read from the *D* scale under the hair-line. If all the products that require the use of the same index of the *C* scale are read first and then the products that require the use of the other index are determined, only two settings of the index will be necessary. In some cases, a single setting of the slide will suffice for all the products.

If it were required to multiply 25.6 successively by 12.7, 21.4, 33, 42, 56, and 79, the slide-rule work might be performed in the following manner: The left-hand index of the *C* scale is set to 25.6 on the *D* scale. Then, the slide is left in this position while the hair-line is moved to 12.7 on the *C* scale and 325 is read on the *D* scale; the hair-line is moved to 21.4 on the *C* scale and 5475 is read on the *D* scale; and the hair-line is moved to 33 on the *C* scale and 845 is read on the *D* scale. The hair-line cannot be set to any of the other given factors without going outside the *D* scale. Therefore, the right-hand index of the *C* scale must be set to 25.6 on the *D* scale in order to find the remaining products. With the slide in this second position, the hair-line is set to 42 on the *C* scale and 1075 is read on the *D* scale; the hair-line is set to 56 on the *C* scale and 1434 is read on the *D* scale; and the hair-line is set to 79 on the *C* scale and 202 is read on the *D* scale.

It is preferable to use both the *C* and *CI* scales in conjunction with the *D* scale for multiplication in problems in which it is required to find the product of three or more factors.

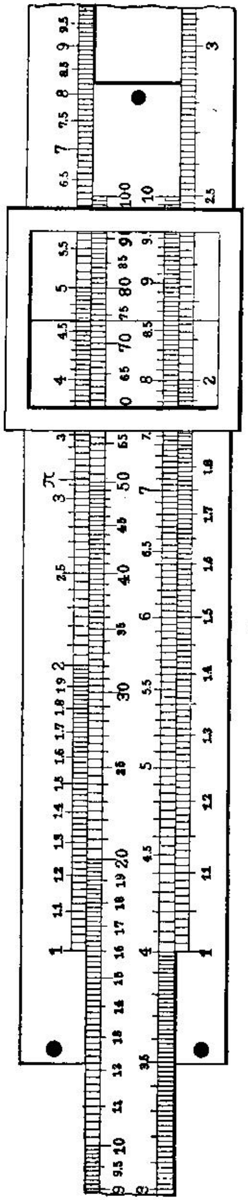


Fig. 10

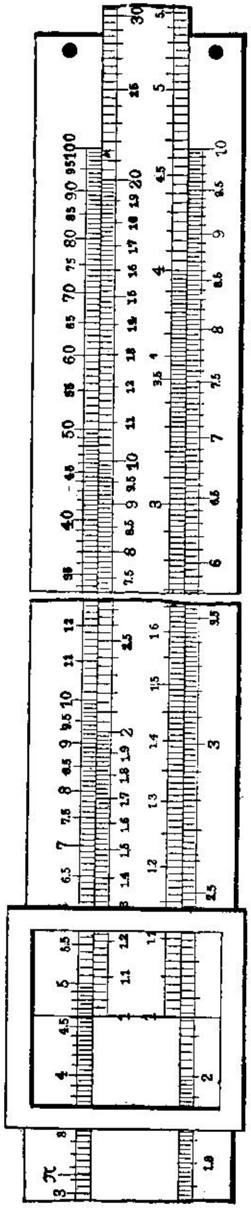


Fig. 11

EXAMPLES FOR PRACTICE

Use the slide rule for performing each of the following multiplications:

		Ans.
(a)	3.7×462.	(a) 1,709
(b)	$4{,}211 \times 7{,}838$.	(b) 33,000,000
(c)	$1{,}728 \times 0.065$.	(c) 112.3
(d)	18.17×124.	(d) 2,250
(e)	287.5×0.302.	(e) 86.8
(f)	$5{,}682 \times 543$.	(f) 3,085,000
(g)	$178.19 \times 1{,}004$.	(g) 179,000
(h)	0.000492×4.1418.	(h) 0.00204
(i)	0.3685×0.042.	(i) 0.0155

CONTINUOUS MULTIPLICATION

40. Product of Several Factors by Means of C and D Scales.—A common problem is to find the product of three or more factors. The slide-rule work in such a problem consists in performing a series of multiplications in each of which two numbers are involved. To illustrate the procedure, it will be assumed that the slide rule does not have a *CI* scale and that the following multiplication is to be performed: $0.25 \times 8.6 \times 0.107 \times 75{,}186 \times 39.6$. If only the *C* and *D* scales are used and the factors are taken in the order here given, the operations are as follows:

The multiplication of 0.25 and 8.6 is performed by setting the index of the *C* scale and the hair-line as shown in Fig. 10. After the right-hand index of the *C* scale is properly set to 0.25 or 250 on the *D* scale, the hair-line is moved so that it is over 8.6 or 860 on the *C* scale. The reading on the *D* scale under the hair-line then indicates the product of 0.25 and 8.6. However, the reading of this product is not needed, and it is only the position of the hair-line that is important.

The next step is to multiply the indicated product of 0.25 and 8.6 by 0.107. Since the product of the first two numbers is marked by the hair-line in the position shown in Fig. 10, the runner should be left undisturbed in that position while the left-hand index of the *C* scale is brought to the hair-line, as shown in Fig. 11. Then, the slide is left stationary and the hair-line is set over 0.107 or 107 on the *C* scale, as shown in Fig. 12.

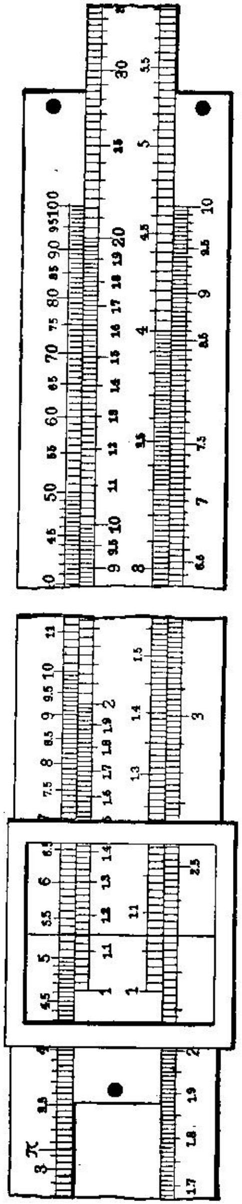


Fig. 12

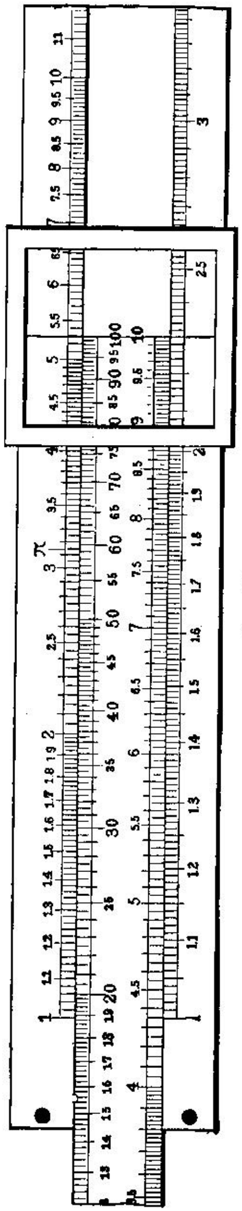


Fig. 13

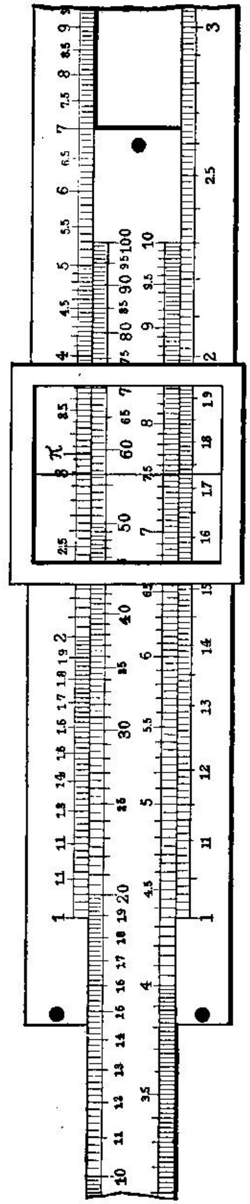


Fig. 14

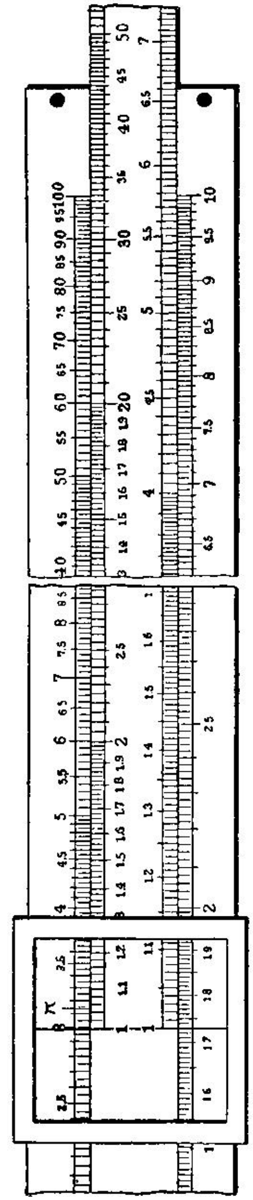


Fig. 15

The reading on the D scale under the hair-line now represents the product of 0.25, 8.6, and 0.107.

With the hair-line still marking the product of 0.25, 8.6, and 0.107, that product is now multiplied by the fourth factor 75,186 by making the settings shown in Figs. 13 and 14. First, the right-hand index of the C scale is brought under the hair-line, as indicated in Fig. 13. Then, the hair-line is set to 75,186 or 752 on the C scale, as shown in Fig. 14, so that it marks on the D scale the product of the first four factors.

The final settings are shown in Figs. 15 and 16. With the hair-line left in the position shown in Fig. 14, the left-hand index of the C scale is brought under the hair-line, as shown in Fig. 15. The hair-line is then set over 39.6 or 396 on the C scale, as indicated in Fig. 16. The significant figures in the required product are read from the D scale under the hair-line, and are found to be 685.

The decimal point is located in the product by taking 0.25 as 0.2; 8.6 as 10; 0.107 as 0.1; 75,186 as 70,000; and 39.6 as 50. Then, since $0.2 \times 10 \times 0.1 \times 70{,}000 \times 50 = 700{,}000$, the required product is 685,000. Although 8.6 is nearer 9 than 10 and 39.6 is nearer 40 than 50, the numbers 10 and 50 are used to simplify the approximate calculation, which may be performed best as follows: $0.2 \times 50 = 10$; $10 \times 0.1 = 1$; $1 \times 10 = 10$; and $10 \times 70{,}000 = 700{,}000$. The choice of the most convenient values often saves time, and the user of the slide rule can simplify his work considerably by exercising a little thought in this operation.

41. In the solution in Art. 40, the given factors were used in the order in which they were listed in the statement of the example. However, as it is easier to move the runner than the slide, the total movement of the slide could be reduced by using the factors in a different order. The best order in any case depends on the sizes of the first significant figures in the various factors. To reduce the movement of the slide to a minimum, the factors in the example in Art. 40 should be used as if the indicated calculation were $8.6 \times 75{,}186 \times 39.6 \times 0.107 \times 0.25$. The slide-rule operations would then be performed in the following manner:

The right-hand index of the C scale is set to 8.6 or 860 on the D scale.

The hair-line is set to 75,186 or 752 on the C scale.

The right-hand index of the C scale is brought under the hair-line.

The hair-line is set to 39.6 or 396 on the C scale.

The left-hand index of the C scale is brought under the hair-line.

The hair-line is set to 0.107 or 107 on the C scale.

The left-hand index of the C scale is brought under the hair-line.

The hair-line is set to 0.25 or 250 on the C scale.

The significant figures in the required product are read from the D scale under the hair-line and are found to be 685.

The amount of movement of the slide that may be saved by rearranging factors will be apparent if the total movement of the slide in this second method is compared with the total movement required in the first method, which is described in Art. 40. However, such rearrangement of factors constitutes one of the fine points in slide-rule operation, and skill in using the factors in the best order is acquired with practice. The beginner should concentrate his efforts mainly on learning how to perform the operations correctly.

42. Use of CI Scale for Finding Product of Three Factors. The CI scale is used to greatest advantage in finding the product of more than two factors, because fewer settings are needed and a considerable amount of movement of the slide can be saved by the use of that scale. If a slide rule with a CI scale is used, the significant figures in the product of three factors may often be found by applying the following method: The hair-line is set to one factor on the D scale, the slide is moved so that another factor on the CI scale is under the hair-line, and then the hair-line is set to the third factor on the C scale. The significant figures in the required result are read on the D scale under the hair-line.

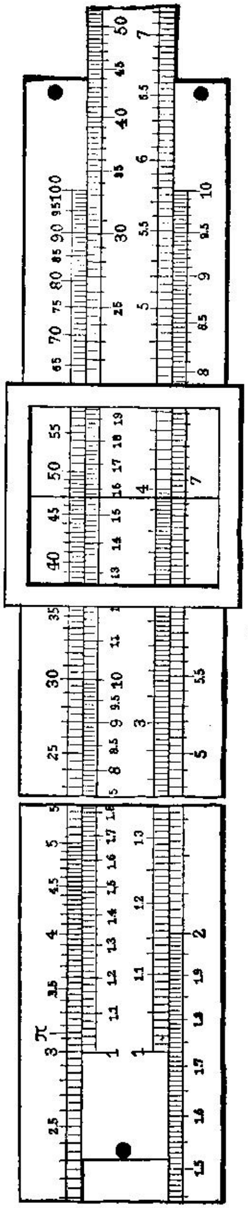


Fig. 16

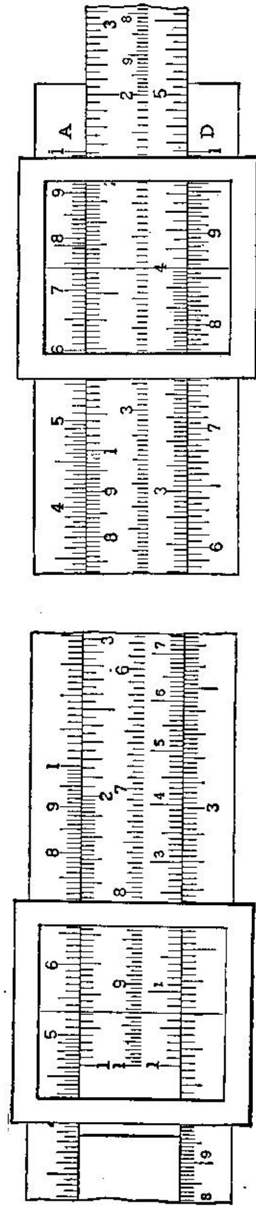


Fig. 17

For example, if it is required to find the product of 0.25, 8.6, and 0.107, the settings could be made as shown in Fig. 17. Thus, as shown in the right-hand half of the illustration, the hair-line is first set to 86 on the *D* scale, and 25 on the *CI* scale is brought under the hair-line. Then, as shown in the left-hand half of the illustration, the slide is kept stationary while the hair-line is set to 107 on the *C* scale. The reading on the *D* scale under the hair-line is found to be 230. Since $0.2\times 10 \times 0.1 = 0.2$, the required product is 0.23.

43. In the example in the preceding article, the last factor on the *C* scale is over the *D* scale and the required reading can be made as described. However, if the given problem is $0.25\times 8.6\times 5.4$ and the settings for 0.25 and 8.6 are made as shown in the right-hand half of Fig. 17, it will be found that 54 on the *C* scale is outside the body of the rule. Therefore, the required result cannot be read from the *D* scale opposite 54 on the *C* scale. In order to complete the solution in this case, the left-hand index of the *C* scale is left in the position shown in Fig. 17 and the hair-line is set at that index; the slide is then moved so that 54 on the *CI* scale comes under the hair-line; and the required product is read on the *D* scale at the left-hand index of the *C* scale. In this case, the significant figures in the product are 1161 and, since $0.2\times 10\times 5 = 10$, the required product is 11.61.

44. Product of More Than Three Factors by Use of CI Scale.—If a slide rule with a *CI* scale is used for solving the problem given in Art. 40, or for finding the product of 0.25, 8.6, 0.107, 75,186, and 39.6, the slide-rule work is best performed as follows:

The hair-line is set to 25 on the *D* scale.

The slide is moved so that 86 on the *CI* scale comes under the hair-line.

The hair-line is set to 107 on the *C* scale.

The slide is moved so that 752 on the *CI* scale comes under the hair-line.

The hair-line is set to 396 on the *C* scale.

The significant figures 685 in the required product are read from the D scale under the hair-line.

In this method, the C and CI scales are used alternately. Thus, one factor is used by setting the runner to that value on the C scale, and the next factor is used by setting that value on the CI scale under the hair-line. Whether the CI scale is used or not, the decimal point is located by approximation in the manner explained in Art. 40.

45. The factors in the example in Art. 40 are such numbers that the CI and C scales can be used alternately in the manner described in Art. 44. However, this is not always the case. In some problems the values of the factors are such that the hair-line cannot be set to the proper factor on the C scale without moving the runner off the body of the rule. It is then necessary to apply the following method in order to proceed with the slide-rule work. The hair-line is set to the index of the C scale that is within the body of the rule, and the slide is moved so that the factor on the CI scale comes under the hair-line.

For example, if it is required to determine the value of the expression $0.25 \times 8.6 \times 752 \times 0.54$, the first steps might be to set the hair-line to 0.25 on the D scale and to move the slide so that 8.6 on the CI scale comes under the hair-line. When the slide is in this position, 752 on the C scale lies outside the body of the rule. Hence, the slide-rule work is continued by setting the hair-line over the left-hand index of the C scale and moving the slide so that 752 on the CI scale comes under the hair-line. Finally, the hair-line is set to 0.54 on the C scale and the significant figures in the required product are read from the D scale under the hair-line. These figures are 873 and, since $0.2 \times 10 \times 800 \times 0.5 = 800$, the required product is 873.

EXAMPLES FOR PRACTICE

Use the slide rule for performing each of the following multiplications:

(a) $48 \times 375 \times 0.0056$.	Ans.	(a) 100.8
(b) $0.076 \times 25{,}486 \times 81.1 \times 360$.		(b) 56,600,000
(c) $1{,}728 \times 8.5 \times 24 \times 36 \times 0.0008375$.		(c) 10,630
(d) $150 \times 3.5 \times 452.4 \times 118 \times 0.0000303$.		(d) 849

DIVISION WITH SLIDE RULE

46. Division With C and D Scales.—Even when the slide rule is provided with a CI scale, the division of one number by another is performed most readily by using the C and D scales. The significant figures in the quotient may then be determined by applying the following rule:

Rule.—*To determine the significant figures in the quotient in division, set the hair-line to the dividend on the D scale; move the slide so that the divisor on the C scale comes under the hair-line; and read the value on the D scale opposite whichever index of the C scale happens to be over the D scale.*

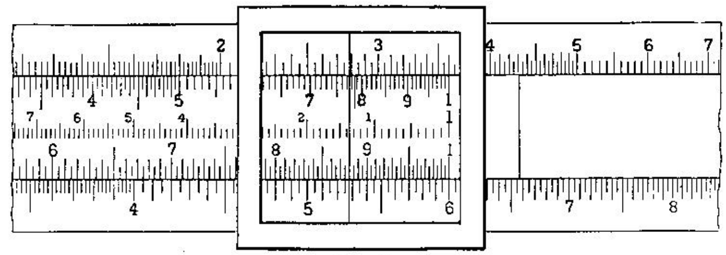


FIG. 18

These operations are similar to those performed in finding the product of two numbers by using the D and CI scales, but here the C scale is used instead of the CI scale. The decimal point is located in the quotient by making an approximate calculation.

EXAMPLE 1.—Divide 5,280 by 88.

SOLUTION.—The slide-rule setting is made as shown in Fig. 18. First, the hair-line is set to 5,280 or 528 on the D scale. Then, the slide is moved so that 88 or 880 on the C scale comes under the hair-line. The significant figures in the quotient are read from the D scale opposite the right-hand index of the C scale. In this case, the index coincides with the main graduation numbered 6.

To locate the decimal point in the quotient, 5,280 is called 5,000 and 88 is called 100. Then, since $5{,}000 \div 100 = 50$, the required result must be 60. Ans.

EXAMPLE 2.—Divide 26 by 0.152.

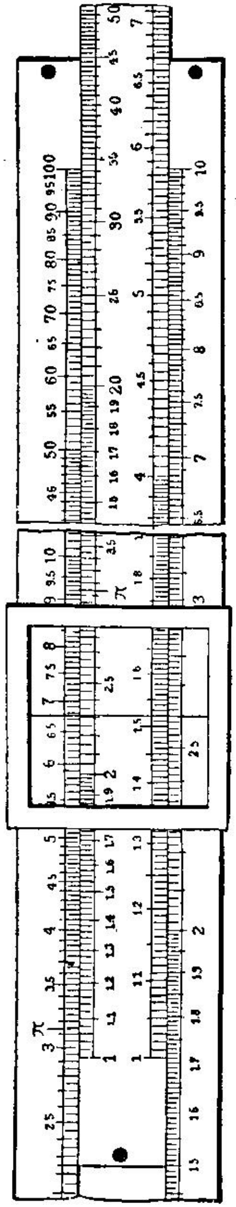


Fig. 19

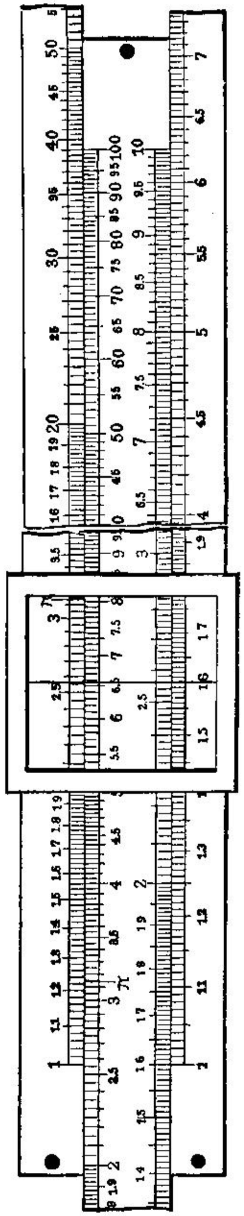


Fig. 20

Solution.—As shown in Fig. 19, the hair-line is set to 26 or 260 on the *D* scale and the value 0.152 or 152 on the *C* scale is brought under the hair-line. For this setting, the reading of the *D* scale at the left-hand index of the *C* scale is 171.

In the approximate calculation, 20 is used instead of 26 and 0.1 is used instead of 0.152. Since $20 \div 0.1 = 200$, the required quotient is 171. Ans.

Example 3.—Divide 16 by 256.

Solution.—The setting shown in Fig. 20 applies to this case. The hair-line is set to 16 or 160 on the *D* scale, the value 256 on the *C* scale is brought under the hair-line, and the figures 625 are read on the *D* scale at the right-hand index of the *C* scale.

The approximate calculation may be conveniently made by taking 16 as 20 and 256 as 400 (although it is much nearer 300). Thus, $20 \div 400 = 0.05$ and $16 \div 256 = 0.0625$. Ans.

Example 4.—Divide 0.1564 by 728.

Solution.—When the slide-rule setting is made as in Fig. 21, page 46, the significant figures in the quotient are found to be 215. The approximate result may be determined by taking 0.1564 as 0.2 and 728 as 1,000. Since $0.2 \div 1{,}000 = 0.0002$, the required quotient must be 0.000215. Ans.

47. It is often necessary to use the same number as a divisor in a series of calculations, the dividend being different in each calculation. Movement of the slide can then be avoided by setting the proper index of the *C* scale to the constant divisor on the *D* scale and moving the hair-line successively to each of the dividends on the *D* scale. For each position of the runner, the significant figures in the quotient are read from the *C* scale under the hair-line.

If the whole number consisting of the first three significant figures in the divisor is in every case greater, or in every case smaller, than the whole number composed of the first three significant figures of the dividend, one setting of the slide will suffice for all the calculations. If the divisor is greater than some of the dividends and smaller than the other dividends, two settings of the slide will be necessary. The right-hand index of the *C* scale must be used for the dividends in the first group, and the left-hand index must be used for the dividends in the second group.

48. Division With CI Scale.—It is also possible to divide one number by another by setting an index of the *C* scale to the dividend on the D scale, moving the hair-line to the divisor on the CI scale, and reading the product under the hair-line on the D scale. However, this method is not convenient for the usual case of division, because it is then necessary to decide which index should be used. The method given in Art. 46 is therefore preferable.

The use of the CI scale for performing division may be advantageous when it is necessary to divide the same number by a series of other numbers; that is, when a constant dividend is to be divided successively by several different divisors. In this case, the proper index of the C scale should be set to the dividend on the D scale, and the hair-line should be set in succession to the various divisors on the CI scale. For each position of the runner, the significant figures in the required quotient are read from the D scale under the hair-line. Either one or two settings of the slide will be sufficient for determining all the quotients. The required number of settings of the slide will depend on whether or not the same index of the C scale can be used for all the divisors.

49. Reciprocals.—The reciprocal of any given number is the quotient obtained by dividing 1 by the given number. On a slide rule with a CI scale, opposite values on the C and CI scales are reciprocals of each other as far as significant figures alone are concerned. Therefore, to find the significant figures in the reciprocal of any number by means of such a slide rule, it is merely necessary to set the hair-line to the given number on the C or CI scale and to read the result under the hair-line on the other of these scales. For example, if it is required to find the reciprocal of 126, the hair-line is set to 126 on the C scale and the significant figures in the required reciprocal are read from the CI scale under the hair-line. These figures are found to be 794. Since $1 \div 100 = 0.01$ and the required result is a little smaller than 0.01, the reciprocal of 126 is 0.00794.

From the relation between the values on the C and CI scales, it follows that multiplying by a given value on the C scale is equivalent to dividing by the corresponding reciprocal on the CI scale, or vice versa. It will now be clear why the same result is obtained whether multiplication is performed by applying the rule given in Art. 35 or by applying the rule given in Art. 38; and also why the same result is obtained whether division is performed according to the rule given in Art. 46 or according to the method given in Art. 48.

If the reciprocal of a number is to be found by means of a slide rule that does not have a CI scale, the usual procedure is as follows: The given number on the C scale is set over either index of the D scale, and the significant figures in the required reciprocal are read from the D scale under whichever index of the C scale happens to be inside the body of the rule.

EXAMPLES FOR PRACTICE

Find the quotient in each of the following examples:

Example		Ans.
(*a*)	$166 \div 13$.	(*a*) 12.77
(*b*)	$287 \div 45$.	(*b*) 6.38
(*c*)	$3{,}125 \div 125$.	(*c*) 25
(*d*)	$78 \div 16.1$.	(*d*) 4.84
(*e*)	$13.9 \div 6.72$.	(*e*) 2.07
(*f*)	$160 \div 936$.	(*f*) 0.171
(*g*)	$54.27 \div 25$.	(*g*) 2.17
(*h*)	$1.05 \div 0.18$.	(*h*) 5.83
(*i*)	$2{,}075 \div 8.38$.	(*i*) 247.5
(*j*)	$0.854 \div 0.1065$.	(*j*) 8.02
(*k*)	$3.635 \div 272.4$.	(*k*) 0.0133
(*l*)	$0.12 \div 0.192$.	(*l*) 0.625
(*m*)	$0.0212 \div 0.706$.	(*m*) 0.03
(*n*)	$542.7 \div 7{,}236$.	(*n*) 0.075
(*o*)	$63.3 \div 128.75$.	(*o*) 0.492
(*p*)	$1 \div 0.515$.	(*p*) 1.942

MULTIPLICATION AND DIVISION COMBINED

50. Problems Solved by Slide Rule.—A slide rule may be used to solve numerous types of practical problems. However, the purpose of this text is to explain the general use of the slide rule and not to present the various practical applications to which it can be put. Hence, no attempt will be made to show how to work out each of the various types of practical problems that can be solved by means of a slide rule; nor will general methods be given for all possible combinations of multi-

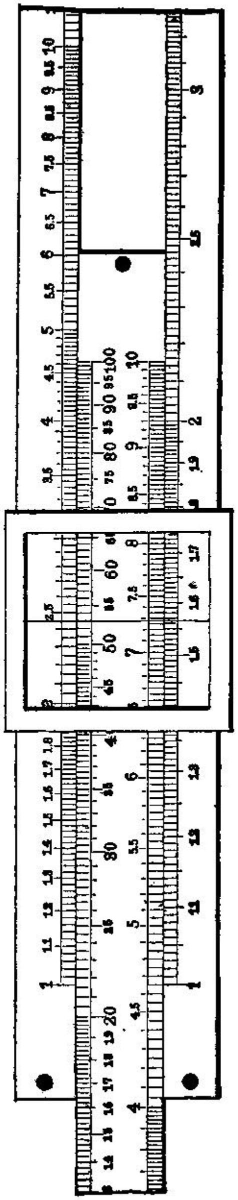


FIG. 21

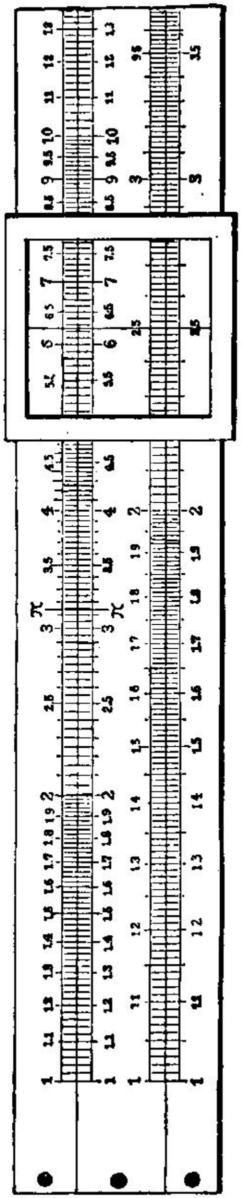


FIG. 22

plication and division involving simple numbers, indicated powers of numbers, and indicated roots of numbers. Only a few simple types of problems that occur frequently and illustrate the advantageous features of the slide rule will be considered.

51. Fractions Involving Multiplication and Division.—In practical work, an indicated calculation frequently consists of a fraction in which the numerator or the denominator or each of these terms is the product of two or more factors. For example, it might be required to determine the value of a fraction like $\frac{128\times 0.375\times 11\times 9.27}{360\times 16}$. When a slide rule is used for solving such a problem, the movement of the slide is reduced to a minimum by alternating multiplication and division. Thus, the steps in this example would be as follows:

The product of 128 and 0.375 is found by setting the left-hand index of the *C* scale to 128 on the *D* scale and setting the hair-line to 0.375 on the *C* scale.

The indicated product is divided by 360 by leaving the hair-line undisturbed and moving the slide so that 360 on the *C* scale comes under the hair-line.

This quotient, which is indicated on the *D* scale at the left-hand index of the *C* scale, is multiplied by 11 by moving the hair-line to 11 on the *C* scale.

This second product, which is indicated on the *D* scale under the hair-line, is divided by 16 by moving the slide so that 16 on the *C* scale comes under the hair-line.

This second quotient is multiplied by 9.27 by moving the hair-line to 927 on the *C* scale.

The significant figures in the required result are read from the *D* scale under the hair-line and are found to be 850.

The decimal point is located in the result by performing the following approximate calculation:

$$\frac{100\times 0.4\times 10\times 10}{400\times 10}=1$$

Hence, the value of the given fraction is 0.85.

There would be no advantage in using the *CI* scale in solving the preceding example. However, this scale is often useful in saving one or more settings in evaluating a fraction containing several factors when it is not possible merely to alternate multiplication and division.

52. Proportion.—In one type of problem that occurs frequently, three terms of a proportion are given and it is required to determine the fourth term. For example, it may be required to find the value of the fourth term x in the proportion $26:73 = 5.6:x$. From this relation, $x = \frac{73 \times 5.6}{26}$, and the calculation may be considered to be a combination of multiplication and division.

A method that can be applied for determining the fourth term of a proportion is as follows: The proper index of the *C* scale is set to either the second or the third term of the proportion on the *D* scale; the hair-line is set to the other of these two terms on the *C* scale; the slide is moved so that the first term on the *C* scale comes under the hair-line; and the required result is read from the *D* scale under whichever index of the *C* scale lies within the body of the rule.

EXAMPLE.—What is the value of the fourth term x in the proportion $26:73 = 5.6:x$?

SOLUTION.—The right-hand index of the *C* scale is set to 73 on the *D* scale, the hair-line is set to 5.6 on the *C* scale, and the slide is moved so that 26 on the *C* scale comes under the hair-line. The reading on the *D* scale under the left-hand index of the *C* scale is then found to be 1572. Since 73 is about 3 times 26, x must be about 3 times 5 or 15. Hence, the required value is 15.72. Ans.

53. In some cases there are several proportions and the first two terms are the same in each proportion, as in the series $26:73 = 5.6:a$; $26:73 = 8.1:b$; $26:73 = 15:c$; etc. When it is required to find the fourth term in each proportion of such a series, it is preferable to consider that each calculation is a combination of division and multiplication, as in the relations

$a = \frac{73}{26} \times 5.6$, $b = \frac{73}{26} \times 8.1$, $c = \frac{73}{26} \times 15$, etc. The slide-rule work may be performed advantageously in the following manner: The hair-line is set to the common second term in the proportions on either half of the *A* scale, and the slide is moved so that the common first term in the proportions on the same half of the *B* scale is under the hair-line. Then the hair-line is moved successively to the third term of each proportion on the *B* scale, and the respective fourth term is read from the *A* scale under the hair-line. Either half of the *B* scale can be used in the last part of the method. The *A* and *B* scales are usually employed in this type of problem in preference to the *C* and *D* scales, in order to avoid the possible necessity of resetting the slide.

If it is desired to use the *C* and *D* scales instead of the *A* and *B* scales in order to obtain a little greater accuracy, the procedure is similar to that described for the *A* and *B* scales. It is merely necessary to substitute the *D* scale for the *A* scale and the *C* scale for the *B* scale in applying the method outlined in the preceding paragraph. In case the third term of a given proportion on the *C* scale lies outside the body of the rule, it is necessary to set the hair-line to the index of the *C* scale that is inside the body and to set the other index of the *C* scale under the hair-line. The hair-line can then be set to the third term of the proportion on the *C* scale.

EXAMPLES FOR PRACTICE

Find the values of the following expressions:

(a) $\frac{0.012 \times 375 \times 86.4 \times 80.9}{72 \times 1{,}525}$.

(b) $\frac{0.0083 \times 1{,}785 \times 824}{103} \div \frac{257}{26}$.

(c) $\frac{36.72}{13.9} \times \frac{4{,}556}{2{,}780} \times \frac{36.3}{127.85} \times \frac{27}{28}$.

(d) $1{,}675 \times \frac{87.3}{12} \times 126 \times \frac{0.566}{42.83}$.

Ans. (a) 0.286; (b) 11.99; (c) 1.185; (d) 20,300

POWERS AND ROOTS

SQUARES AND SQUARE ROOTS

54. Square of Any Number.—The significant figures in the square of any number can be readily found by setting the hair-line to the given number on the *D* scale and reading the result on the *A* scale under the hair-line. The decimal point may be located by approximation. For example, if it is required to find the square of 2.5, the setting is made as shown in Fig. 22, page 46. When the hair-line is over 2.5 or 250 on the *D* scale, the reading on the *A* scale is 625. To locate the decimal point in the square, 2.5×2.5 is taken 2×3, which is 6. Hence, the required result is 6.25.

55. Square Root of Any Number.—Since finding the square root of a number is the reverse of squaring the required root, the slide-rule operations for getting the significant figures in the square root of any number are as follows: The hair-line is set to the given number on the proper half of the *A* scale, and the result is read on the *D* scale under the hair-line. However, it is important to know which half of the *A* scale must be used. A single procedure may be applied for determining whether the left-hand or right-hand half of the *A* scale should be used and also for locating the decimal point in the square root. It is convenient to consider two cases: (1) When the given number is greater than 1; and (2) when the given number is wholly decimal.

56. When the given number is greater than 1, the first step is to point off the figures to the left of the decimal point into periods of two figures each, the count being started from the decimal point; the figures to the right of the decimal point are disregarded in this part of the solution. Then, if the left-hand period contains only one figure, the left-hand half of the *A* scale is used. If the left-hand period contains two figures, the right-hand half of the *A* scale is used. In either case, there is one figure preceding the decimal point in the root for each period to the left of the decimal point in the given number. The

left-hand period is counted as a full period whether it contains one or two figures.

EXAMPLE 1.—Find the square root of 30,625.

SOLUTION.—When this number is pointed off by starting at the decimal point, which follows the last given figure, the result is 3′06′25. Since the first period contains only one figure, the left-hand half of the *A* scale is used. The slide-rule setting is shown in Fig. 23. The hair-line is set to 306 on the left-hand half of the *A* scale, and the significant figures in the required square root, or 175, are found on the *D* scale under the hair-line.

The decimal point is located in the root in the following manner: Since there are three periods to the left of the decimal point in the given number, namely, 3, 06, and 25, there are three figures to the left of the decimal point in the root. Hence, the square root of 30,625 is 175. Ans.

EXAMPLE 2.—What is the square root of 3,062.5?

SOLUTION.—In pointing the number off into periods of two figures, the figure to the right of the decimal point is neglected. The part of the number preceding the decimal point is divided into periods as follows: 30′62. Since the first period contains two figures, the hair-line is set to 306 on the right-hand half of the *A* scale. The reading on the *D* scale under the hair-line is then 553.

In the given number, there are two periods to the left of the decimal point. Hence, the required square root will contain two figures before the decimal point. Thus, $\sqrt{3{,}062.5}=55.3$. Ans.

It should be noted that the given numbers in the two preceding examples have the same significant figures. However, the finding of square root is one type of problem in which the position of the decimal point in the given number must be considered when making the setting on the slide rule.

57. In case it is required to find the square root of a number that is wholly decimal, the procedure for determining which half of the *A* scale should be used and for locating the decimal point in the root is as follows: The given number is pointed off, to the right from the decimal point, into periods of two figures each only until the period containing the first significant figure is passed. If this period contains one significant figure and a cipher preceding that figure, the left-hand half of the *A* scale is used. If this period contains two significant figures, the right-hand half of the *A* scale is used. In

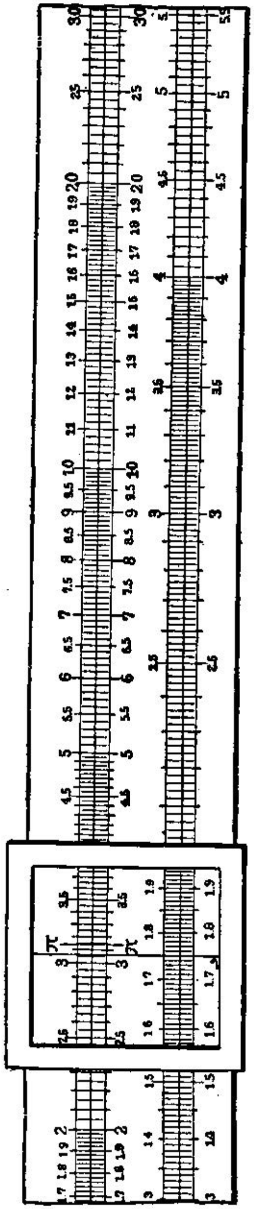


Fig. 23

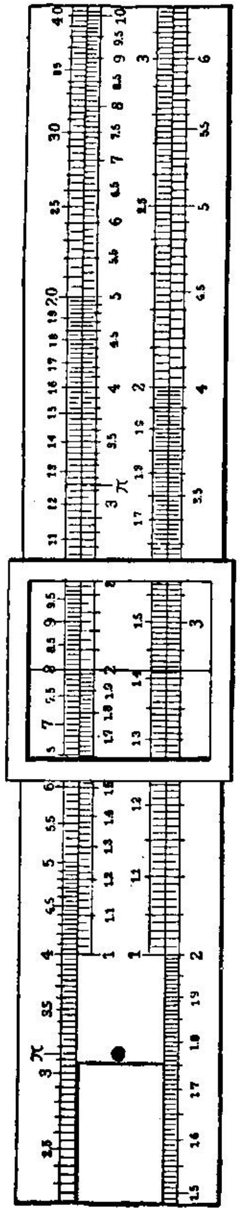


Fig. 24

either case, the square root is wholly decimal and there should be one cipher between the decimal point and the first significant figure in the root for each period of two ciphers that is pointed off in the given number.

Example.—Find the square root of 0.000306.

Solution.—The first part of the given number is pointed off as follows: .00′03. Since the period that includes the first significant figure contains a cipher in front of that figure, the left-hand *A* scale is used and the slide-rule setting is the same as that for example 1, Art. 56. As shown in Fig. 23, the significant figures in the required square root are 175.

In the given number there is one period of two ciphers between the decimal point and the period containing the first significant figure. Therefore, there is one cipher in the root between the decimal point and the first significant figure, and the required square root is 0.0175. Ans.

58. When the first significant figure in the given number is in the first period following the decimal point, the first significant figure in the square root comes immediately after the decimal point. It is sometimes required to find the square root of a decimal quantity which contains a single significant figure and in which either no ciphers or an even number of ciphers intervene between the decimal point and the significant figure. It is then necessary to add a cipher after the significant figure in order to complete the period containing the significant figure. For example, if it is required to find the square root of 0.003, the number should be pointed off as .00′30 and the right-hand half of the *A* scale should be used.

EXAMPLES FOR PRACTICE

Find the value of each of the following:

(*a*) 26^2.	Ans.	(*a*) 676
(*b*) 19.65^2.		(*b*) 386
(*c*) $\sqrt{498.}$		(*c*) 22.3
(*d*) $\sqrt{0.04365.}$		(*d*) 0.209

CUBES AND CUBE ROOTS

59. Finding Cube of Any Number by Use of K Scale. When a slide rule that is provided with a *K* scale is used, the significant figures in the cube of any number can be found merely by setting the hair-line to the given number on the *D* scale and noting the reading on the *K* scale under the hair-line. The decimal point may be located by approximation.

For example, the cube of 76.25 is found by setting the hair-line to 7625 on the *D* scale, reading 444 on the *K* scale under the hair-line, and locating the decimal point in the result by making the following calculation: $70 \times 70 \times 100 = 490{,}000$. Thus, the cube of 76.25 is 444,000.

60. Finding Cube Without Using K Scale.—The significant figures in the cube of any number can be found by using the *A*, *B*, *C*, and *D* scales. The general procedure consists in setting the proper index of the *C* scale to the given number on the *D* scale; moving the hair-line to the given number on the *B* scale; and reading the result on the *A* scale under the hair-line. When the first three significant figures in the given number are less than 216, the left-hand index of the *C* scale must be used; when the first three figures are greater than 464, the right-hand index must be used; for intermediate numbers, either index may be used.

Example 1.—Find the cube of 20.

Solution.—Since the first three significant figures of the given number, or 200, are less than 216, the slide-rule setting is made as shown in Fig. 24. First, the left-hand index of the *C* scale is set to 200 on the *D* scale. Then, the hair-line is set to 200 on the left-hand half of the *B* scale (the right-hand half could have been used instead), and the significant figures in the cube are read on the *A* scale under the hair-line. These figures are 800.

The approximate calculation that is made for locating the decimal point in the cube is $20 \times 20 \times 20 = 8{,}000$; and the required value is, therefore, 8,000. Ans.

Example 2.—Find the cube of 76.25.

Solution.—In this case, the first three significant figures are greater than 464 and the right-hand index of the *C* scale is used. As shown in Fig. 25, the right-hand index of the *C* scale is set to 7625 on the *D* scale, and the hair-line is set to 7625 on the left-hand half of the *B* scale (again the right-hand half could have been used instead). The reading on the *A* scale under the hair-line is 444 and, as found in Art. 59, the required cube is 444,000. Ans.

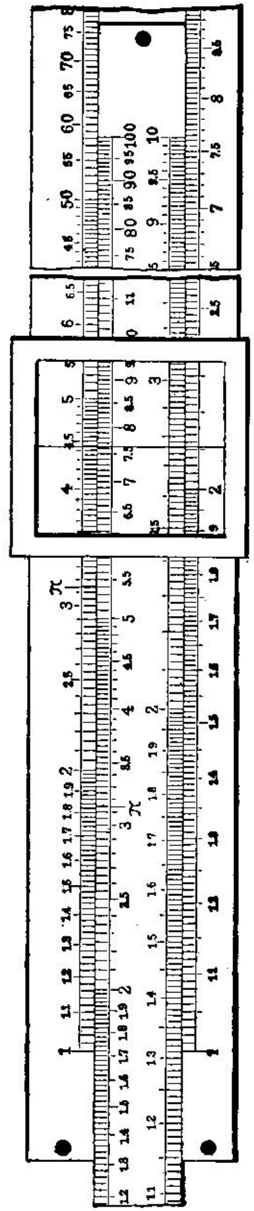


Fig. 25

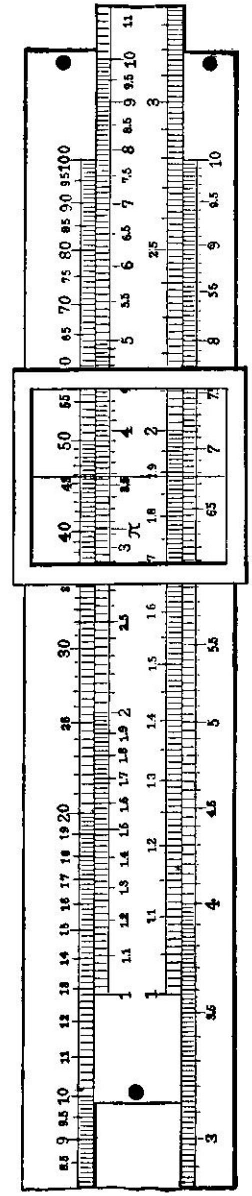


Fig. 26

EXAMPLE 3.—What is the cube of 3.58?

SOLUTION.—The significant figures in the given number lie between 216 and 464, and either index of the *C* scale may be used. In Fig. 26 is shown the slide-rule setting when the left-hand index is used. In this case, the hair-line must be set to 358 on the left-hand half of the *B* scale, because 358 on the right-hand half is beyond the end of the *A* scale. The reading on the *A* scale under the hair-line is 458 and, since $3 \times 3 \times 4 = 36$, the required cube is 45.8. Ans.

If the right-hand index of the *C* scale is used, the hair-line must be set over 358 on the right-hand half of the *B* scale, because 358 on the left-hand half will then be outside the *A* scale.

61. Cube Root by Use of K Scale.—When a slide rule with a *K* scale is used, the procedure in finding the cube root of a number is similar to that described for square root. The general method is to set the hair-line to the given number on the proper part of the *K* scale, to read the significant figures in the required root from the *D* scale under the hair-line, and to locate the decimal point in the root by inspection. The part of the *K* scale to be used and the position of the decimal point are determined by the same investigation. In the case of cube root also, it is convenient to consider separately numbers that are greater than 1 and numbers that are wholly decimal.

62. To find the cube root of a number greater than 1, the first step is to point off the figures to the left of the decimal point into periods of three figures each. If the first, or left-hand, period contains one figure, the left-hand third of the *K* scale is used when setting the hair-line to the given number; if the first period contains two figures, the middle third of the *K* scale is used; if the first period contains three figures, the right-hand third is used. In any case, there is one figure preceding the decimal point in the root for each period or part of a period to the left of the decimal point in the given number.

EXAMPLE.—Find the cube root of 45.5.

SOLUTION.—The part of the given number to the left of the decimal point consists of a single incomplete period containing two figures. Therefore, to determine the significant figures in the cube root, the hair-line is set to 455 on the middle third of the *K* scale. The reading on the *D* scale under the hair-line is then 357.

Since there is only a part of one period to the left of the decimal point in the given number, one figure precedes the decimal point in the required root and $\sqrt[3]{45.5}$ is 3.57. Ans.

63. When the number whose cube root is required is wholly decimal, the number is pointed off, from the decimal point, into periods of three figures each until the period containing the first significant figure is passed. If this period contains only one significant figure, the left-hand part of the *K* scale is used; if it contains two significant figures, the middle part of the scale is used; if it contains three significant figures, the right-hand part of the scale is used. The cube root is wholly decimal in any case and contains one cipher between the decimal point and the first significant figure for each period of three ciphers that is pointed off in the given number.

EXAMPLE.—What is the cube root of 0.00455?

SOLUTION.—When the given number is pointed off, it is found that the first period is .004, which contains one significant figure. Hence, the hair-line is set to 455 on the left-hand part of the *K* scale, and the significant figures in the cube root, which are found on the *D* scale under the hair-line, are 1657. Also, the first significant figure in the cube root immediately follows the decimal point, because there are no complete periods of ciphers in the given number. The required root is therefore 0.1657. Ans.

64. If a given number that is wholly decimal contains only one or two significant figures, it may sometimes be found that the period containing the first significant figure does not contain three figures. It is then necessary to add one or two ciphers to the right of the given number in order to complete that period. For example, when the number 0.0002 is pointed off, the result is .000′2. In such a case, it is necessary to complete the second period by adding two ciphers so that the result will be .000′200. Ciphers that are added in this manner are counted as significant figures in determining which part of the *K* scale is to be used. Thus, the right-hand part of that scale would be used for finding the cube root of 0.0002, because the second period would be taken as 200, which contains three significant figures.

65. Cube Root Without Using K Scale.—When the cube root of a number must be found by means of a slide rule without a *K* scale, the significant figures in the root may be determined in the following manner: The hair-line is set to the given number on the proper half of the *A* scale, and then the slide is moved until the reading on the left-hand half of the *B* scale under the hair-line is exactly the same as the reading on the *D* scale under the proper index of the *C* scale. The half of the *A* scale and the index of the *C* scale that are to be used, and also the position of the decimal point in the root, are determined by pointing off the given number into periods in the manner explained in Art. 62 for numbers greater than 1 and in Art. 63 for decimals. The left-hand half of the *B* scale should be used in all cases.

There are three types of settings. If the first period of a number greater than 1 or the period containing the first significant figure of a decimal quantity contains only one significant figure, the left-hand half of the *A* scale and the left-hand index of the *C* scale are used. If such period contains two significant figures, the right-hand half of the *A* scale and the left-hand index of the *C* scale are used. If the period contains three significant figures, the left-hand half of the *A* scale and the right-hand index of the *C* scale are used.

EXAMPLE 1.—Find the cube root of 729.

SOLUTION.—The given number consists of one period containing three significant figures. Therefore, the hair-line is set to 729 on the left-hand half of the *A* scale, and the slide is moved until the reading on the left-hand half of the *B* scale under the hair-line is the same as the reading on the *D* scale under the right-hand index of the *C* scale. As shown in Fig. 27, which illustrates the required setting, the reading is found to be 900 or 9. Since the root must contain one figure to the left of the decimal point, $\sqrt[3]{729}=9$. Ans.

EXAMPLE 2.—Find the cube root of 18,225.

SOLUTION.—When the number is pointed off, it is found that the first period is 18, which contains two significant figures. Therefore, the slide-rule setting is made as shown in Fig. 28. The hair-line is first set to 182 on the right-hand half of the *A* scale, and the slide is moved until the same value is read both on the left-hand half of the *B* scale under the hair-line and on the *D* scale under the left-hand index of the *C* scale.

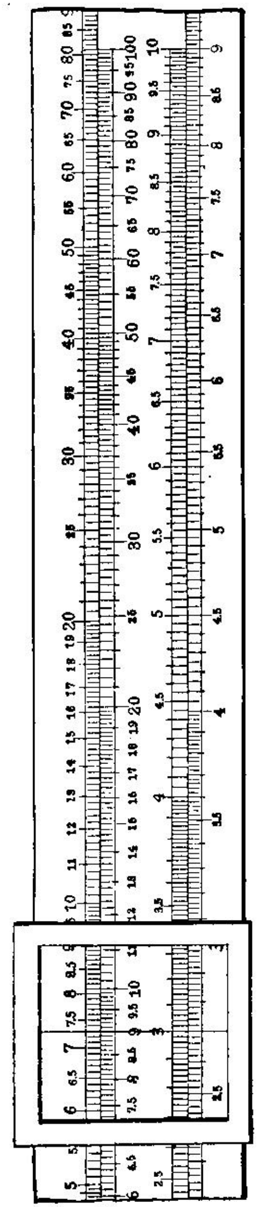


FIG. 27

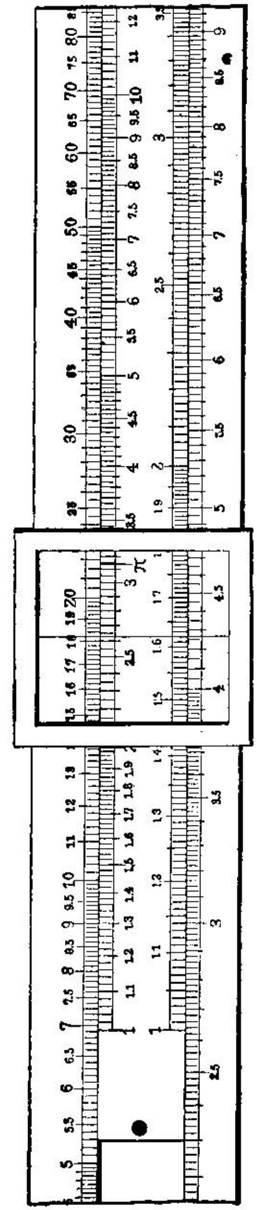


FIG. 28

This value is found to be 263. Since the given number contains one complete period and a part of a second period to the left of the decimal point, two figures precede the decimal point in the required cube root and $\sqrt[3]{18,225}=26.3$. Ans.

EXAMPLE 3.—What is the cube root of 0.00256?

SOLUTION.—The first period in the given number contains one significant figure. Therefore, the significant figures in the required cube root are determined by setting the hair-line to 256 on the left-hand half of the *A* scale and moving the slide so that the reading on the left-hand *B* scale under the hair-line is the same as the reading on the *D* scale at the left-hand index of the *C* scale. This reading is found to be very nearly 137. Since the first period in the given number contains a significant figure, the first significant figure in the root follows the decimal point directly, and $\sqrt[3]{0.00256}=0.137$. Ans.

EXAMPLES FOR PRACTICE

Find the value of each of the following:

(*a*) 129^3.	Ans.	(*a*) 2,150,000
(*b*) 4.72^3.		(*b*) 105
(*c*) 0.0894^3.		(*c*) 0.000715
(*d*) $\sqrt[3]{6,125}$.		(*d*) 18.3

COMBINATION OF OPERATIONS

66. Area of Circle of Given Diameter.—In the solutions of some problems that are of common occurrence, it is necessary to find a power or a root of a number and also to perform multiplication or division either before or after the other operation. For example, the area of a circle is calculated from its diameter by finding the square of the diameter and multiplying that square by 0.7854. The entire calculation can be performed very easily on a slide rule in the following manner: The index of the *C* scale is set to the given diameter on the *D* scale, the hair-line is set to 7854 or 785 on the *B* scale, and the significant figures in the required area are read from the *A* scale under the hair-line. On most slide rules, there is a special mark on the right-hand half of the *B* scale, and also on the right-hand half of the *A* scale, to indicate the value 7854. The types of rules shown in Figs. 2 and 3 have this mark. When the slide rule has the special mark, the right-hand index of the *C* scale should be used for all cases except when the significant figures in the diameter are less than 1128.

67. Diameter of Circle of Given Area.—To compute the diameter of a circle when the area is known, it is necessary to divide the area by 0.7854 and then to take the square root of the quotient. These operations can be best performed with a slide rule in the following manner: The hair-line is set to the area on the *A* scale, as if it were merely required to find the square root of the area. Then, the slide is moved so that 7854 or 785 on the right-hand half of the *B* scale is under the hair-line, and the significant figures in the required diameter are read from the *D* scale under the index of the *C* scale.

EXAMPLE.—Find the diameter of a circle, the area of which is 90 square inches.

SOLUTION.—If it were required to find the square root of 90, the right-hand half of the *A* scale would be used. Hence, in the given problem, the hair-line is set to 9 on the right-hand half of the *A* scale, and 7854 on the *B* scale is brought under the hair-line. The significant figures in the required diameter are found on the *D* scale under the left-hand index of the *C* scale, and are 107. Since $\sqrt{90}$ is approximately 10 and $\frac{10}{0.8}$ is a little greater than 10, the required diameter is 10.7 inches. Ans.

SPECIAL USES OF MANNHEIM SLIDE RULES

MANTISSAS OF LOGARITHMS

68. Features of Scale for Finding Logarithms.—Most slide rules have a scale for finding mantissas of logarithms of numbers and also antilogarithms, which are numbers corresponding to given logarithms. Therefore, Arts. 68 to 71, inclusive, deal with the use of the slide rule for finding logarithms and antilogarithms. These articles may be disregarded by anyone who is not familiar with logarithms.

On some types of slide rules, the scale used for finding logarithms and antilogarithms is on the body; and on other types of rules this scale is on the slide. On the rule shown in Fig. 3, the scale is placed along the lower edge of the body and is marked *LOG*. However, on the rule shown in Fig. 2, the scale for logarithms is on the under face of the slide. As shown in Fig. 29, there is a small cut-out in the end of that rule and in this cut-out is a piece of transparent celluloid in which is marked a hair-line *a*. This hair-line is exactly under the right-hand index of the *D* scale and is used for reading the scale of logarithms, which is marked *L* in Fig. 29. It will be noticed that the subdivisions on the scale for logarithms are uniform in size, and that each subdivision represents a difference of 2 in the third decimal place.

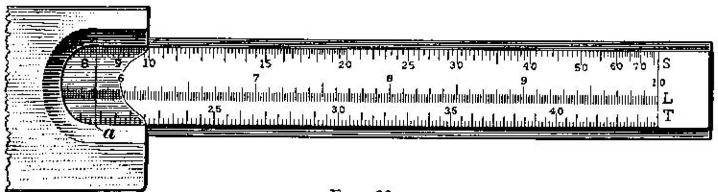


FIG. 29

69. Method of Finding Mantissa.—When the scale for logarithms is on the body of the rule, as in Fig. 3, the mantissa of the logarithm of any number is found by setting the hair-line to the given number on the *D* scale and reading the required value on the logarithm scale under the hair-line. For example, if it is required to find the logarithm of 381, the hair-line is set to 381 on the *D* scale and the mantissa of the logarithm is found on the logarithm scale under the hair-line to be .581. The characteristic of the logarithm is determined by applying the usual principles of logarithms. In this case, the characteristic is 2, and the complete logarithm is 2.581.

On the logarithm scale, the graduation at the left-hand end is marked 0. Therefore, if the hair-line lies to the left of the main graduation numbered 1, the first figure in the mantissa is 0. Thus, the extra-long intermediate graduation midway between the main graduations numbered 0 and 1 represents .05.

70. If a slide rule of the type shown in Fig. 2 is used, the procedure in determining the mantissa of the logarithm of any number is as follows: The slide is moved so that the given number on the *C* scale is over the right-hand index of the *D* scale. The required mantissa is then read from the *L* scale under the hair-line in the cut-out at the end of the rule. Thus, if the number whose logarithm is required is 381, the slide is set so that 381 on the *C* scale is over the right-hand index of the *D* scale. The reading of the *L* scale is then as shown in Fig. 29, and the required mantissa is .581.

71. Finding Antilogarithm.—A slide rule with a scale for logarithms may also be used to find an antilogarithm. If the rule is of the type shown in Fig. 3, the significant figures in the required antilogarithm are determined by setting the hair-line to the mantissa of the given logarithm on the scale for logarithms and reading the value on the *D* scale under the hair-line. In the case of a rule of the type shown in Fig. 2, it is necessary to set the slide so that the hair-line in the cut-out is at the mantissa of the given logarithm on the *L* scale, and to read the *C* scale at the right-hand index of the *D* scale. In any case, the decimal point is located in the required antilogarithm by considering the characteristic of the given logarithm and applying the usual principles of logarithms.

TRIGONOMETRIC FUNCTIONS

METHODS OF FINDING FUNCTIONS OF ANGLES

72. Graduations on Scales for Trigonometric Functions. Most slide rules have scales for finding the trigonometric functions of given angles or the angles corresponding to given functions. The use of such scales is explained in Arts. 72 to 78, inclusive, which may be omitted by those who are not familiar with the principles of trigonometry. The positions of the scales that are used for trigonometric work are different on different types of slide rules. On the rule shown in Fig. 3, the second scale from the top, or the scale marked *SIN.* (for sine) at the right-hand end, and the second scale from the bottom, or the scale marked *TAN.* (for tangent) at the left-hand end, are provided for the solution of problems involving trigonometry. Similar scales, which are marked *S* and *T* in Fig. 29, are placed on the under face of the slide of the rule shown in Fig. 2. Although the trigonometric scales are designated on the slide rule as the sine and tangent scales, they can also be used for finding cosines, cotangents, secants, and cosecants.

With the exception of the notations 40′ and 50′ near the left-hand end of the sine scale, the numbers on the sine and tangent scales represent degrees. The values of the smallest subdivisions are different on various parts of the scales. Up to 3 degrees on the sine scale of the rule shown in Fig. 2 or 29, each subdivision represents $\frac{1}{30}$ degree, or 2 minutes; and, between 3 and 10 degrees, each subdivision represents $\frac{1}{12}$ degree, or 5 minutes. On the sine scale of the rule shown in Fig. 3, each subdivision to the left of the graduation for 10 degrees represents 5 minutes. Between 10 and 20 degrees on the sine scale of either rule, each subdivision represents $\frac{1}{6}$ degree or 10 minutes; between 20 and 40 degrees, each subdivision represents $\frac{1}{2}$ degree or 30 minutes; between 40 and 70 degrees, each subdivision represents 1 degree; and the six graduations beyond 70 degrees have the values 72, 74, 76, 78, 80, and 90 degrees, respectively.

On the tangent scale, each subdivision up to 20 degrees represents 5 minutes, and each subdivision between 20 and 45 degrees represents 10 minutes.

73. Finding Sine of Angle.—When the sine scale is on the body of the rule, as in Fig. 3, the significant figures in the sine of any angle between 0°35′ and 90° can be found by setting the hair-line to the given angle on the sine scale and reading the *A* scale under the hair-line.

In using a slide rule of the type shown in Fig. 2, the slide is moved so that the given angle on the sine scale is under the hair-line in the cut-out in the end of the rule; and the required value is found on the *B* scale under the right-hand index of the *A* scale. If the reading is on the right-hand half of the *A* or *B* scale, the first significant figure in the sine follows the decimal point directly. In case the reading is on the left-hand half of the scale, there is one cipher between the decimal point and the first significant figure in the sine.

Example.—Find the sine of 8° 20′.

Solution.—If the type of rule shown in Fig. 2 is used, the setting at the hair-line in the cut-out is as shown in Fig. 29. For this setting, the reading on the *B* scale at the right-hand index of the *A* scale is 145. Since this reading is on the right-hand half of the *B* scale, sin 8° 20′ =0.145. Ans.

If the rule shown in Fig. 3 is used, the hair-line is set to 8° 20′ on the sine scale and 145 is read from the right-hand half of the *A* scale under the hair-line.

74. Finding Tangent of Angle.—To find the significant figures in the tangent of an angle between 5° 43′ and 45° by means of the rule shown in Fig. 3, it is merely necessary to set the hair-line to the given angle on the tangent scale and to read the result on the *D* scale under the hair-line. If the rule is of the type shown in Fig. 2, the slide is set so that the given angle on the tangent scale is under the hair-line in the cut-out in the end of the rule and the result is read from the *C* scale over the right-hand index of the *D* scale. For any angle between 5° 43′ and 45°, the first significant figure in the tangent comes immediately after the decimal point.

To find the significant figures in the tangent of an angle between 45° and 84° 17′ by means of the rule shown in Fig. 3, the indexes of the *C* and *D* scales are lined up, the complement of the given angle is found by subtracting the given angle from 90°, the hair-line is set to this complementary angle on the tangent scale, and the result is read from the *CI* scale under the hair-line. If the type of rule shown in Fig. 2 is used, the slide is set so that the complementary angle on the tangent scale is under the hair-line in the cut-out and the result is read from the *D* scale at the left-hand index of the *C* scale. In the tangent of any angle between 45° and 84° 17′, the decimal point follows the first significant figure.

If the given angle is between 0° 35′ and 5° 43′, the tangent may be assumed to be equal to the sine, which is found as described in Art. 73. In case the given angle is between 84° 17′ and 89° 25′, the procedure consists in subtracting the given angle from 90°, finding the sine of this complementary angle, and then getting the reciprocal of that sine.

Example 1.—Find the tangent of 20° 50′.

SOLUTION.—In this case, the given angle is between 5° 43′ and 45°. If the type of rule shown in Fig. 2 is used, the value representing 20° 50′ on the tangent scale is brought under the hair-line in the cut-out, as shown in Fig. 29. The required reading on the C scale at the right-hand index of the D scale is then found to be 3805 and tan 20° 50′=0.3805. Ans.

If the type of rule shown in Fig. 3 is used, the hair-line is set to 20° 50′ on the tangent scale and 3805 is read on the D scale under the hair-line.

EXAMPLE 2.—What is the tangent of 60°?

SOLUTION.—Since the given angle is between 45° and 84° 17′, the first step is to find its complement, which is 90°−60° or 30°. With the type of rule shown in Fig. 2, the slide is set so that 30° on the tangent scale is under the hair-line in the cut-out. The reading of the D scale under the left-hand index of the C scale is then found to be 1732. The decimal point in the required tangent comes after the first significant figure, and tan 60°=1.732. Ans.

With the type of rule shown in Fig. 3, the indexes of the C and D scales are lined up, the hair-line is set to 30° on the tangent scale, and the required significant figures 1732 are read on the CI scale under the hair-line.

75. Finding Other Functions.—The cosine of an angle may be found by subtracting the given angle from 90° and finding the sine of the complementary angle thus determined.

The cotangent of an angle may be found by subtracting the given angle from 90° and determining the tangent of this complementary angle.

The secant of an angle is the reciprocal of the cosine of the angle, and the cosecant is the reciprocal of the sine of the angle.

EXAMPLES FOR PRACTICE

Find the value of:

(a)	sin 16° 30′.	(a)	0.284
(b)	cos 38° 15′.	(b)	0.785
(c)	tan 23° 50′.	(c)	0.442
(d)	cot 7° 35′.	(d)	7.51
(e)	sec 32° 45′.	(e)	1.19
(f)	cosec 10° 30′.	(f)	5.49
(g)	tan 78° 15′.	Ans. (g)	4.81
(h)	cos 66° 30′.	(h)	0.399
(i)	cot 80° 45′.	(i)	0.163
(j)	sin 57° 30′.	(j)	0.843
(k)	sin 49° 15′.	(k)	0.758
(l)	cos 52° 45′.	(l)	0.605

METHODS OF FINDING ANGLE CORRESPONDING TO FUNCTION

76. With the type of slide rule shown in Fig. 3, an angle can be found from its sine by setting the hair-line to the given value on the A scale and reading the required angle from the sine scale under the hair-line.

When the given function is a tangent whose value is between 0.01 and 0.1, the angle is found as if the function were the sine. For a tangent that is between 0.1 and 1, the hair-line is set to the given value on the D scale and the required angle is read on the tangent scale under the hair-line. For a tangent that is between 1 and 10, the indexes of the C and D scales are lined up; the hair-line is set to the given function on the CI scale; the complement of the required angle is read from the tangent scale under the hair-line; and the desired angle is found by subtracting from 90° the angle read from the tangent scale.

77. In the case of a slide rule of the type shown in Fig. 2, an angle can be found from its sine by setting the slide so that the given value on the B scale is at the right-hand index of the A scale, and reading the angle on the sine scale under the hair-line in the cut-out.

If the given function is a tangent and its value is between 0.01 and 0.1, the angle is found as if the given function were the sine. For a tangent between 0.1 and 1, the given value on the C scale is set to the right-hand index of the D scale, and the required angle is read on the tangent scale under the hair-line in the cut-out. For a tangent between 1 and 10, the left-hand index of the C scale is set to the given value on the D scale, the complement of the required angle is read on the tangent scale under the hair-line in the cut-out, and the desired angle is found by subtracting from 90° the angle read from the tangent scale.

78. When the given function is the cosine of the angle, the complement of the required angle is found first by proceeding as if the given function were the sine. Then, the angle that is read from the sine scale is subtracted from 90° to give the proper angle.

If the cotangent of the angle is given, the complement of the required angle may be found by assuming that the given function is a tangent. This complementary angle is then subtracted from 90° to give the desired angle.

When the given function is a secant or a cosecant, the first step is to find the reciprocal of the given function. This reciprocal is either the cosine or the sine of the required angle, as the case may be, and the angle can be found by the method previously described for that function.

EXAMPLES FOR PRACTICE

Find the angle a corresponding to each of the following functions:

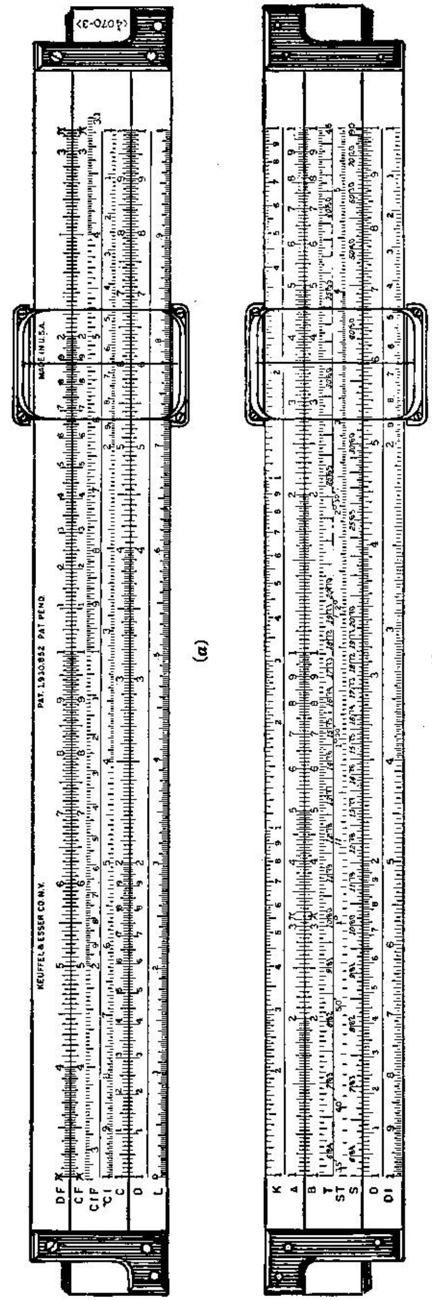
(*a*)	$\sin a = 0.458$.	Ans. (*a*)	27° 15′
(*b*)	$\tan a = 0.613$.	(*b*)	31° 30′
(*c*)	$\tan a = 1.228$.	(*c*)	50° 50′
(*d*)	$\cos a = 0.0944$.	(*d*)	84° 35′
(*e*)	$\cot a = 3.375$.	(*e*)	16° 30′

SLIDE RULES WITH FOLDED SCALES AND LOG LOG SCALES

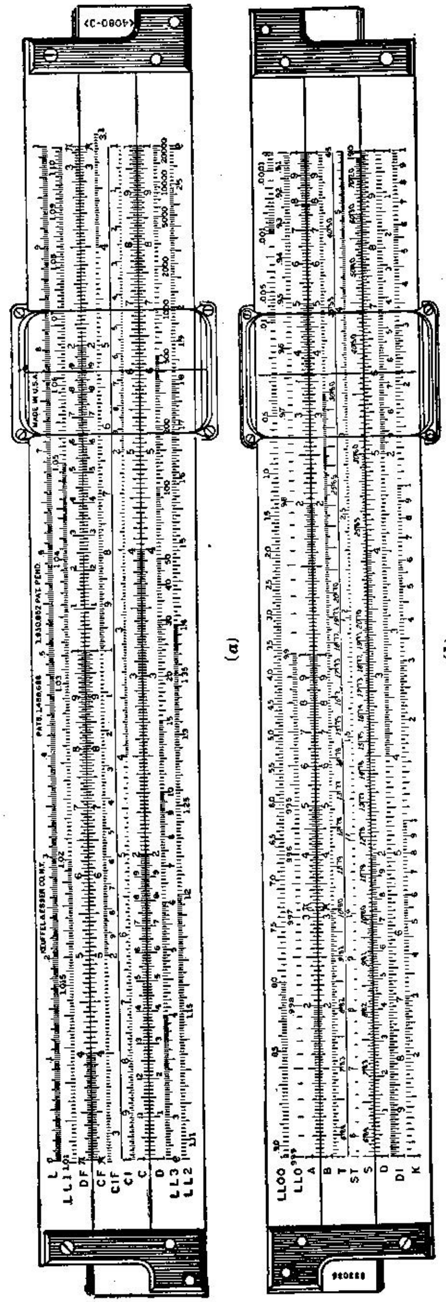
FOLDED SCALES

79. Types of Rules With Folded Scales.—There are several types of slide rules with folded scales, or scales having the index near the center. Two such rules are illustrated in this text. In Fig. 30 (*a*) and (*b*) are shown, respectively, the front and back of the type of rule that has the trade-mark name *polyphase duplex trig slide rule*. In Fig. 31 (*a*) and (*b*) are shown similar views of the type of rule that has the trade-mark name *LL decitrig duplex slide rule;* the notation *LL* is the abbreviation for log log. There are also slide rules that are known, respectively, by the trade-mark names *polyphase duplex decitrig slide rule* and *LL trig duplex slide rule*. The explanations in Arts. 79 to 84, inclusive, need not be studied by one who does not have a rule with folded scales.

A trig rule differs from a decitrig rule of the same type merely in the graduation of the scales for trigonometric functions. On a trig rule, these scales are graduated in degrees and minutes; whereas, on a decitrig rule, they are graduated in



(*a*) (*b*)
FIG. 30
(Courtesy Keuffel & Esser Co., New York)



(a) (b)

Fig. 31

(Courtesy Keuffel & Esser Co., New York)

degrees and decimals of a degree. The difference can be readily seen by comparing the scales marked *T*, *ST*, and *S* on the rule shown in Fig. 30 with the corresponding scales in Fig. 31.

80. Scales on Polyphase Duplex and LL Duplex Rules. As shown in Fig. 30 (*a*), the front of the polyphase duplex trig or polyphase duplex decitrig rule has the usual *D*, *C*, *CI*, and *L* scales and also three other scales, which are marked *DF*, *CF*, and *CIF*. On the back of the rule, as shown in view (*b*), are the usual *D*, *A*, *B*, and *K* scales and also four other scales marked *DI*, *T*, *ST*, and *S*.

On the front of the LL duplex rule shown in Fig. 31 are the usual *D*, *C*, and *CI* scales; the scales marked *DF*, *CF*, and *CIF*, which are like those on the polyphase duplex rule; the ordinary scale for logarithms, which is marked *L*; and three other scales marked *LL1*, *LL2*, and *LL3*. On the back of the LL duplex rule are the usual *D*, *A*, *B*, and *K* scales; the scales marked *DI*, *T*, *ST*, and *S*, which are like those on the polyphase duplex rule; and the two scales marked *LL0* and *LL00*.

The methods described in the previous articles for performing multiplication and division and for finding squares, cubes, square roots, and cube roots of numbers can also be used with the polyphase duplex rule or the LL duplex rule. However, many problems involving multiplication, division, or a combination of multiplication and division are solved much more easily by using the *DF*, *CF*, and *CIF* scales instead of, or in addition to, the *D*, *C*, and *CI* scales. The methods of finding the logarithm of a number and of finding an antilogarithm are the same as those explained for the type of rule shown in Fig. 3.

Readings and settings on the *DF* and *CF* scales of the polyphase duplex or LL duplex rule are made in the same way as are those on the *D* and *C* scales; and values on the *DI* and *CIF* scales are found in the same way as are those on the *CI* scale. However, on each folded scale, which contains the letter *F* in its designation, the index, or the graduation representing the number 1, is near the center of the scale. Only the use of the folded scales and the LL scales and the special features of the scales for trigonometric functions need be explained here.

81. Use of Folded Scales in Multiplication or Division of Two Numbers.—The *DF*, *CF*, and *CIF* scales bear the same relation to one another as do the *D*, *C*, and *CI* scales. When a polyphase duplex or an LL duplex slide rule is used and it is required to multiply a certain number by a series of other numbers, one setting of the slide will suffice for finding all the products. The index of the *C* scale that will leave the larger part of the slide in the body of the rule is set to the constant factor on the *D* scale. Some of the products are read from the *D* scale under the other given factors on the *C* scale, and the remaining products are read from the *DF* scale opposite the given factors on the *CF* scale.

One setting of the slide is also sufficient when a constant divisor is to be used with a series of dividends. In this case, the index of the *C* scale that will leave the larger part of the slide in the body of the rule is set to the given divisor on the *D* scale. Then some of the quotients are read from the *C* scale opposite the dividends on the *D* scale, and the other quotients are read from the *CF* scale opposite the dividends on the *DF* scale.

All the quotients that are required when a constant dividend is to be used with a series of divisors can also be found with one setting of the slide. First, the index of the *C* scale that will leave the larger part of the slide in the body of the rule is set to the constant dividend on the *D* scale. Then some of the required quotients are read on the *D* scale opposite the given divisors on the *CI* scale and the other quotients are read on the *DF* scale opposite the divisors on the *CIF* scale.

82. Use of Folded Scales for Finding Product of Three Factors.—If it is required to find the product of any three factors with a polyphase duplex or an LL duplex rule, the first step is to set the hair-line to one of the given factors on either the *D* scale or the *DF* scale, and to bring a second factor on the *CI* or *CIF* scale under the hair-line. The *CI* scale must be used with the *D* scale, and the *CIF* scale must be used with the *DF* scale. The better setting in any particular case is the one in which the larger part of the slide will be within the body of the rule. To complete the slide-rule work, the hair-line is set to the third factor on either the *C* scale or the *CF* scale, and the significant figures in the required product are read under the hair-line on the *D* scale if the *C* scale is used or on the *DF* scale if the *CF* scale is used. The *C* scale may be used whenever the third factor on that scale lies within the limits of the *D* scale, and the *CF* scale may be used whenever the third factor on that scale lies within the limits of the *DF* scale. It is not necessary to use the *C* scale because the *D* and *CI* scales were used in the first step, or to use the *CF* scale because the *DF* and *CIF* scales were used previously.

EXAMPLE 1.—Find the product of 0.25, 8.6, and 5.4.

SOLUTION.—In this case, it is convenient to set the hair-line to 8.6 on the *D* scale, and to bring 25 on the *CI* scale under the hair-line, as shown in the right-hand half of Fig. 17 for a Mannheim slide rule. Then, since 54 on the *C* scale is outside the body of the rule, the hair-line is set to 54 on the *CF* scale and 1161 is read under the hair-line on the *DF* scale. The product of 0.2, 10, and 5 is 10, and the required result is 11.61. Ans.

EXAMPLE 2.—Find the product of 0.107, 115, and 2.04.

SOLUTION.—For the given factors, the best method is to set the hair-line to 107 on the *DF* scale, to bring 115 on the *CIF* scale under the hair-line, and to set the hair-line to 204 on either the *C* scale or the *CF* scale. The reading under the hair-line on the *D* scale or on the *DF* scale, as the case may be, is then 251. Since $0.1 \times 100 \times 2 = 20$, the required product is 25.1. Ans.

83. Use of Folded Scales for Finding Product of More Than Three Factors.—When a polyphase duplex or an LL duplex rule is used for finding the product of more than three factors, the first operations with the slide rule are similar to those for finding the product of any three of the factors. Then, the fourth factor is used in the following manner: If the hair-line is set to the third factor on the *C* scale, the slide is moved so that the fourth factor on the *CI* scale comes under the hair-line. On the other hand, if the third factor is set on the *CF* scale, the fourth factor on the *CIF* scale is brought under the hair-line. The product of the four factors is then indicated either on the *D* scale at an index of the *C* scale or on the *DF* scale at the index of the *CF* scale.

Multiplication by a fifth factor can be performed, if necessary, by setting the hair-line to that fifth factor on either the *C* scale or the *CF* scale, as described in Art. 82 for the third factor.

EXAMPLE.—Find the product of 8.6, 752, 39.6, and 0.54.

SOLUTION.—First, the hair-line is set to 86 on the *DF* scale and 752 on the *CIF* scale is brought under the hair-line. Then, the hair-line is set to 396 on the *C* scale and 54 on the *CI* scale is brought under the hair-line. The reading on the *D* scale under the left-hand index of the *C* scale is 1383 and, since $10\times700\times40\times0.5=140{,}000$, the required product is 138,300. Ans.

84. Use of Folded Scales in Multiplication and Division Combined.—With a polyphase duplex or an LL duplex slide rule, the first step in determining the fourth term of any proportion or the fourth terms of a series of similar proportions is as follows: The hair-line is set to the second term on either the *D* scale or the *DF* scale, and the first term on the *C* or *CF* scale is brought under the hair-line. The *C* and *D* scales must be used together, and the *CF* and *DF* scales must be used together. Use is made of that pair of scales which will keep the larger part of the slide within the body of the rule. Then, the fourth term of any proportion can be found by setting the hair-line to the third term on the *C* or *CF* scale, and reading the result under the hair-line on the *D* or *DF* scale, as the case may be. Either the *C* or the *CF* scale may be used for the third term, regardless of which scales were used for the second and first terms.

No general method need be outlined for finding, by means of a polyphase duplex or an LL duplex rule, the value of a fraction in which the numerator or the denominator or each of these terms is the product of two or more factors. By using the *D*, *C*, *CI*, *DF*, *CF*, and *CIF* scales properly, the operations may be performed in almost any order without making unnecessary movements of the slide. With practice, the user of a polyphase duplex or an LL duplex rule will naturally acquire proficiency in switching from the *D*, *C*, and *CI* scales to the folded scales or vice versa.

LOG LOG SCALES

85. Features of LL Scales.—The explanations in Arts. 85 to 93, inclusive, need not be studied by one who does not have a slide rule with log log scales. When the log log scales are used, it is not permissible to disregard the decimal point in a given number. On the contrary, the position of the decimal point is very important in carrying out the slide-rule work in which these scales are used. The relations between the values on the *D* scale and the values on the scales marked *LL1*, *LL2*, and *LL3* are as follows:

The values on the *LL3* scale range from the special number 2.7182818, which is commonly designated by the letter *e*, to 22,000. These are the numbers for which the *natural logarithms*, or the logarithms with *e* as a base, are between 1 and 10. If the values on the *D* scale are assumed to range from 1 to 10, any value on the *D* scale is the natural logarithm of the opposite number on the *LL3* scale. The characteristic of the logarithm, as well as its mantissa, is included in the reading on the *D* scale. Thus, the natural logarithm of 10 is about 2.3, and the natural logarithm of 100 is about 4.6.

The values on the *LL2* scale range from 1.105 to *e*, and the natural logarithms of these numbers are between 0.1 and 1. Hence, if the values on the *D* scale are assumed to range from 0.1 to 1, any value on the *D* scale is the natural logarithm of the opposite number on the *LL2* scale. Thus, the natural logarithm of 2 is 0.693.

Similarly, if the values on the *D* scale are assumed to range from 0.01 to 0.1, any value on the *D* scale is the natural logarithm of the opposite number on the *LL1* scale. For instance, the natural logarithm of 1.05 is 0.0488.

The scales marked *LL0* and *LL00* are for decimal quantities. These scales are related to the *A* scale in such a way that any value on the *A* scale, with the decimal point properly located, is the natural logarithm of the reciprocal of the opposite number on the *LL0* or *LL00* scale. For example, the natural logarithm of the reciprocal of 0.998 is 0.002, and the natural logarithm of the reciprocal of 0.75 is 0.288.

TABLE I

VALUES OF SUBDIVISIONS ON LL SCALES

Scale	Range on Scale	Value of Subdivision
LL00	0.000045 to 0.0001	0.00001
	0.0001 to 0.001	0.00005
	0.001 to 0.01	0.0005
	0.01 to 0.05	0.002
	0.05 to 0.80	0.005
	0.80 to 0.905	0.002
LL0	0.905 to 0.99	0.001
	0.99 to 0.999	0.0001
LL1	1.01 to 1.02	0.0001
	1.02 to 1.05	0.0002
	1.05 to 1.105	0.0005
LL2	1.105 to 1.2	0.001
	1.2 to 1.4	0.002
	1.4 to 1.8	0.005
	1.8 to 2.5	0.01
	2.5 to 2.718	0.02
LL3	2.718 to 4	0.02
	4 to 6	0.05
	6 to 10	0.1
	10 to 15	0.2
	15 to 30	0.5
	30 to 50	1
	50 to 100	2
	100 to 200	5
	200 to 500	10
	500 to 1,000	50
	1,000 to 2,000	100
	2,000 to 5,000	200
	5,000 to 10,000	500
	10,000 to 22,000	1,000

86. Values on LL Scales.—Great care must be exercised in reading an *LL* scale or in making a setting on such a scale. Enough of the graduations are numbered to serve as a guide. But, in order to determine the value represented by a particular intermediate graduation or the difference in value represented

by a particular subdivision, it is necessary to study carefully the part of the scale in the vicinity of the graduation or subdivision. The values of the subdivisions on the various parts of the five *LL* scales are shown in Table I.

87. Finding Powers and Roots by Use of LL Scales.—By means of the *LL* scales, any power or any root of any number can be determined just as easily as if it were required to find the product or quotient of two numbers. In any case, the required quantity should be expressed as a number with an exponent that either is an integer or is in decimal form. Thus, if it is required to find the fourth root of 16, the operation should be indicated as $16^{0.25}$ rather than as $\sqrt[4]{16}$. When this system is adopted, finding a root of a number is equivalent to finding a power of the number, the exponent for the power being the reciprocal of the index of the root.

To raise any number between 1.01 and 22,000 to any power for which the exponent is between 1 and 10, the hair-line is set to the given number on the *LL1*, *LL2*, or *LL3* scale; an index of the *C* scale is brought under the hair-line; the hair-line is set to the given exponent on the *C* scale; and the required power is read under the hair-line on one of the *LL* scales. If the left-hand index of the *C* scale is used, the power is read from the *LL* scale on which the given number is found. If the right-hand index of the *C* scale is used, the power is read from the *LL* scale having the next higher number.

EXAMPLE 1.—Find the value of $7.81^{3.2}$.

SOLUTION.—The hair-line is set to 7.81 on the *LL3* scale; the left-hand index of the *C* scale is brought under the hair-line; the hair-line is set to 3.2 on the *C* scale; and the required power is found under the hair-line on the *LL3* scale to be 720. Ans.

EXAMPLE 2.—Find the value of 1.08^4.

SOLUTION.—First, the hair-line is set to 1.08 on the *LL1* scale, and the right-hand index of the *C* scale is brought under the hair-line; this index must be used in order that 4 on the *C* scale will be inside the body of the rule. Then the hair-line is set to 4 on the *C* scale. Since 1.08 is on the *LL1* scale and the right-hand index of the *C* scale is used, the required power is read from the *LL2* scale. This number is 1.36. Ans.

88. In case the given number and its exponent are so large that the exponent on the *C* scale is outside the body of the rule when the left-hand index of the *C* scale is set to the given number on the *LL3* scale, it is necessary to proceed as follows: The given number is divided into two or more factors; the required power of each of these factors is found; and these results are multiplied together. Thus, if it is required to find the value of $600^{2.5}$, the problem may be solved by first finding $30^{2.5}$ and $20^{2.5}$, and then taking the product of these two results.

89. If it is required to raise a number between 1.01 and 22,000 to a power for which the exponent is between 0.1 and 1, the hair-line is set to the given number on the *LL1*, *LL2* or *LL3* scale; the given exponent on the *CI* scale is brought under the hair-line; the hair-line is set to the index of the *CI* scale that lies within the body of the rule; and the required result is read under the hair-line on one of the *LL* scales. If the left-hand index of the *CI* scale lies inside the body of the rule, the result is taken from the *LL* scale on which the given number is found. If the right-hand index of the *CI* scale lies inside the body of the rule, the result is read from the *LL* scale having the next lower number.

EXAMPLE.—What is the value of $15^{0.3}$?

SOLUTION.—The hair-line is set to 15 on the *LL3* scale; 0.3 on the *CI* scale is brought under the hair-line; and the hair-line is set to the right-hand index of the *CI* scale, which index lies within the body of the rule in this case. The required value is read from the *LL2* scale and is 2.253. Ans.

90. The *LL0* and *LL00* scales on a log log duplex rule are used for finding a power of a number that is wholly decimal. If the exponent is greater than 1, the hair-line is first set to the given number on the *LL0* or *LL00* scale, and an index of the *B* scale is brought under the hair-line. Then, the hair-line is set to the given exponent on the left-hand half of the *B* scale for an exponent between 1 and 10 or on the right-hand half for an exponent between 10 and 100. The required result is read under the hair-line on an *LL* scale. If the left-hand index of the *B* scale is used, the result is taken from the *LL* scale on which the given number is found. If the right-hand index of the *B* scale is used with a given number on the *LL0* scale, the result is read from the *LL00* scale.

EXAMPLE.—Find the value of $0.95^{3.5}$.

SOLUTION.—The hair-line is set to 0.95 on the *LL0* scale; the right-hand index of the *B* scale is brought under the hair-line; the hair-line is set to 3.5 on the left-hand half of the *B* scale; and the required result is read under the hair-line on the *LL00* scale. The reading is 0.8355. Ans.

91. When a number that is wholly decimal has an exponent that is between 0.1 and 1, the procedure for finding the required result by use of the *LL* scales is as follows: In any case, the first step is to set the hair-line to the given number on the *LL0* or *LL00* scale. If the number is on the *LL0* scale, the right-hand index of the *B* scale is brought under the hair-line; the hair-line is set to the given exponent on the right-hand half of the *B* scale; and the required result is read under the hair-line on the *LL0* scale.

If the number is on the *LL00* scale, it is necessary to bring either the left-hand index or the right-hand index of the *B* scale under the hair-line and to set the hair-line to the given exponent on the right-hand half of the *B* scale. That index is used which will bring the exponent within the body of the rule. When the right-hand index is used, the required result is read under the hair-line on the *LL00* scale. When the left-hand index is used, the reading is made on the *LL0* scale.

EXAMPLE.—Find the value of $0.8^{0.3}$.

SOLUTION.—The hair-line is set to 0.8 on the *LL00* scale; and, in order that the value 0.3 on the right-hand half of the *B* scale will lie within the body of the rule, the left-hand index of that scale is brought under the hair-line. When the hair-line is set to the exponent, the reading on the *LL0* scale is 0.9352. Ans.

92. Richardson Log Log Rule.—The slide of the Richardson rule shown in Fig. 3 may be replaced by a different slide containing a *C* scale and *LL1*, *LL2*, and *LL3* scales. To raise a number between 1 and 22,000 to a power indicated by an exponent between 0.1 and 10, with a slide of this type and the body of the rule shown in Fig. 3, the general procedure is as follows: The hair-line is set to an index of the D scale, and the given number on one of the LL scales is brought under the hair-line. Then the hair-line is set to the given exponent on the D scale, and the required result is read from the proper LL scale.

In any case, that index of the D scale is used which will bring the given exponent on the D scale under the part of the slide that remains within the body of the rule. There are four cases to be considered in determining the position of the reading. (1) If the given exponent is between 1 and 10 and the left-hand index of the D scale is used, the required power is read from the same LL scale on which the given number is found. (2) If the given exponent is between 1 and 10 and the right-hand index of the D scale is used, the required power is read from the LL scale having the number just greater than the LL scale on which the given number is found. (3) If the given exponent is between 0.1 and 1 and the right-hand index of the D scale is used, the required power is read from the same LL scale on which the given number is found. (4) If the given exponent is between 0.1 and 1 and the left-hand index of the D scale is used, the required power is read from the LL scale having the number just smaller than the LL scale on which the given number is found.

EXAMPLE 1.—Find the value of $2.05^{3.1}$.

SOLUTION.—In this case, the hair-line must be set to the right-hand index of the D scale, and 2.05 on the $LL2$ scale is brought under the hair-line. When the hair-line is then set to 3.1 on the D scale, the reading on the $LL3$ scale is 9.26. Ans.

EXAMPLE 2.—Find the value of $2.05^{0.31}$.

SOLUTION.—The hair-line is set to the right-hand index of the D scale, and 2.05 on the $LL2$ scale is brought under the hair-line. Then the hair-line is set to 0.31 on the D scale. Since, in this case, the exponent is between 0.1 and 1, the required result is read from the $LL2$ scale and is 1.249. Ans.

93. If the given number and the given exponent are so large that the required result exceeds 22,000, the given number is first divided into two or more factors. The required power of each factor is found, and these powers are multiplied.

In case a number that is wholly decimal is to be raised to a power, the first step is to find the reciprocal of the given number. Then the required power of this reciprocal is determined by applying the method of Art. 92, and the reciprocal of that result is found.

TRIGONOMETRIC FUNCTIONS WITH DUPLEX RULE

94. Graduations on T, ST, and S Scales.—The explanations in Arts. 94 to 98, inclusive, need not be studied by one who either does not have a duplex rule or has no knowledge of trigonometry. The T scale of a polyphase duplex trig rule or an LL trig duplex rule is graduated like the scale marked TAN. on the rule shown in Fig. 3 and like the T scale on the rule shown in Figs. 2 and 29. On the LL decitrig duplex rule shown in Fig. 31, or on the polyphase duplex decitrig rule, the graduations representing degrees on the T scale are located in the same positions as are the graduations representing degrees on the other similar scales. The intermediate graduations on the T scale of a decitrig rule are established as follows: For angles less than 10° or greater than 80°, the space between two consecutive degree-graduations is divided into tenths of a degree by the longer intermediate graduations, and each of the smallest subdivisions represents 0.05 degree. For angles between 10° and 30° or between 60° and 80°, each subdivision represents 0.1 degree. For angles between 30° and 60°, each subdivision represents 0.2 degree.

The scale marked ST on a polyphase duplex or an LL duplex rule is equivalent to the left-hand half of the SIN. scale on the rule shown in Fig. 3 or to the left-hand half of the S scale on the rule shown in Figs. 2 and 29. On the decitrig rule shown in Fig. 31, the numbered graduations represent, respectively, 0.6, 0.7, 0.8, 0.9, 1, 1.5, 2, 2.5, 3, 4, and 5 degrees. The space between two consecutive degree-graduations is divided into tenths of a degree by the longer intermediate graduations, and each of the smallest subdivisions represents 0.02 degree. The graduations on the ST scale of a polyphase duplex trig rule or an LL trig duplex rule are like those on the ST scale shown in Fig. 30. Thus, for angles less than 4°, each subdivision represents 1 minute; and, for angles greater than 4°, each subdivision represents 2 minutes.

95. The scale marked *S* on a polyphase duplex or an LL duplex rule is equivalent to the right-hand half of the *SIN*. scale shown in Fig. 3 or to the right-hand half of the *S* scale shown in part in Fig. 29. In the case of the decitrig rule shown in Fig. 31, the degree-spaces on the *S* scale are subdivided as follows: For angles less than 10°, as determined by the numbers to the left of the degree-graduations, the space between degree-graduations is divided into tenths of a degree by the longer intermediate graduations, and each of the small subdivisions represents 0.05 degree. For angles between 10° and 20°, each small subdivision represents 0.1 degree. For angles between 20° and 30°, a small subdivision represents 0.2 degree. For angles between 30° and 60°, a small subdivision represents 0.5 degree. For angles between 60° and 80° (the 80° graduation is not numbered on the rule), each division represents 1 degree. The graduation between the 80° mark and the 90° mark represents 85°.

The *S* scale on a duplex trig rule is graduated as shown for the polyphase duplex rule in Fig. 30. For angles less than 16°, each small subdivision represents 5 minutes. For angles between 16° and 30°, each small subdivision represents 10 minutes. For angles between 30° and 60°, each small division represents 30 minutes. The graduations to the right of the 60° mark are like those on the decitrig rule.

96. Finding Functions of Given Angles.—To find the sine of an angle with a polyphase duplex or an LL duplex rule, the hair-line is set to the given angle on the scale marked *ST* or on the scale marked *S*, and the required value is read from the *C* scale under the hair-line. When the *S* scale is used for finding a sine, it is necessary to use the system of numbering in which the values increase from left to right; the number of degrees then lies to the left of its corresponding graduation mark. If the angle whose sine is required is on the *S* scale, the first significant figure in the sine comes immediately after the decimal point. If the angle is on the *ST* scale, a cipher intervenes between the decimal point and the first significant figure in the sine.

To find the cosine of an angle less than 84°17′, the hair-line is set to the given angle on the *S* scale and the required value is read from the *C* scale under the hair-line. For a cosine, it is necessary to use the system of numbering on the *S* scale in which the values increase from right to left and for which the numbers lie to the right of the corresponding degree-graduations. The first significant figure in the cosine comes immediately after the decimal point. The cosine of an angle between 84°17′ and 89°25′ may be found by determining the sine of the complementary angle.

The secant of an angle may be found by setting the hair-line to the angle on the *S* or *ST* scale, as if the cosine were to be found, and reading the required value under the hair-line on the *CI* scale. The cosecant of an angle is found by setting the hair-line to the angle on the *S* or *ST* scale, as for the sine, and reading the required value under the hair-line on the *CI* scale. If the angle is on the *S* scale, the decimal point follows the first figure in the function. In case the angle is on the *ST* scale, the decimal point follows the second figure.

97. To find the tangent or cotangent of an angle with a polyphase duplex or an LL duplex rule, the first step is to set the hair-line to the given angle on the *T* scale or the *ST* scale. For an angle between 84°17′ and 89°25′, the complementary angle on the *ST* scale is used.

If it is required to find the tangent of an angle less than 45° or the cotangent of an angle greater than 45°, the required value is read from the *C* scale under the hair-line. In case the angle is on the *T* scale, the first significant figure in the required function comes immediately after the decimal point. When the angle or its complement is on the *ST* scale, a cipher intervenes between the decimal point and the first significant figure in the required function.

If it is required to find the tangent of an angle greater than 45° or the cotangent of an angle less than 45°, the reading is made on the *CI* scale. When the angle is on the *T* scale, the decimal point follows the first significant figure in the required function. In case the angle is on the *ST* scale, the decimal point follows the second significant figure in the required function.

98. Finding Angle Corresponding to Given Function.—The angle corresponding to a given sine or tangent whose value is less than 0.1 is found by setting the hair-line to the given value on the *C* scale and reading the angle on the *ST* scale. If the given function is a cosine or cotangent less than 0.1, the procedure consists in finding the angle on the *ST* scale, as just described for a sine or tangent, and taking the complement of that angle.

When the given function is a sine, cosine, tangent, or cotangent between 0.1 and 1, the hair-line is set to the given function on the *C* scale and the angle is read from the *S* or *T* scale, as the case may be. For a sine or tangent, the number of degrees lies to the left of the degree-graduation and the angle increases from left to right. For a cosine or cotangent, the number of degrees lies to the right of the degree-graduation and the angle increases from right to left.

When the given function is a tangent, cotangent, secant, or cosecant between 1 and 10, the hair-line is set to the given function on the *CI* scale and the angle is read from the *T* or *S* scale. For a tangent, the angle is greater than 45°; for a cotangent, the angle is less than 45°; for a secant or cosecant, the angle is read as described in the preceding paragraph for a cosine or sine.

In case the given function is a cotangent or a cosecant greater than 10, the hair-line is set to the given function on the *CI* scale and the angle is read from the *ST* scale. For a tangent or secant greater than 10, the procedure consists in finding the angle on the *ST* scale as for a cotangent or cosecant, and taking the complement of that angle.

The *DI* scale on a polyphase duplex or an LL duplex rule can be used to advantage for finding an angle directly when two sides of a right triangle are given.

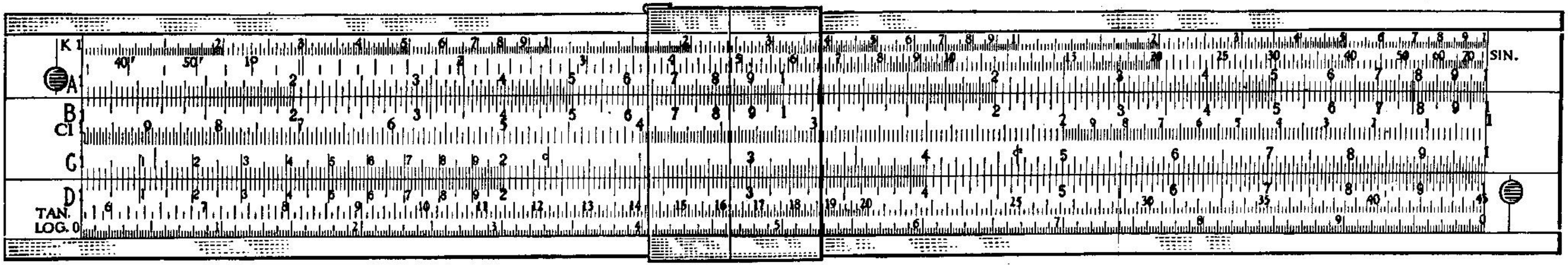


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Fig. 3