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MANNHEIM AND MULTIPLEX SLIDE RULES

BY
L. W. ROSENTHAL

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MANNHEIM

AND

MULTIPLEX SLIDE RULES

THEORY AND PRACTICAL APPLICATION

THIRD EDITION
REVISED

BY

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PART I.

MANNHEIM SLIDE RULE.

Part I.—Mannheim Slide Rule

I. INTRODUCTION.

1. **APPLICATION.**—A slide rule has fixed and movable logarithmic scales, by means of which arithmetical, algebraic and trigonometrical calculations may be performed mechanically. The instrument is applicable to nearly all forms of calculation, and is becoming recognized with increased rapidity in almost all branches of commerce and engineering. Although slide rules have been employed by professional men but for a comparatively short time, yet their utility is so clearly marked that their use is now demanded in many places. To the engineer and student the slide rule is invaluable, while the merchant, manufacturer, accountant, statistician and almost everyone connected in any way in a business undertaking will find it an instrument of material service. In the following text an attempt has been made to include all that is useful in a slide rule to the engineer and student. Other readers will find it necessary to understand only those parts of the book dealing with the ordinary examples of multiplication, division and proportion.

2. **QUALIFICATIONS.**—Let the reader clearly understand at the outset that the principles which underlie the theory and practical application of the slide rule are so few and so simple that its proficient use may be easily understood and mastered by almost everyone. The theory of the slide rule lies in the elementary principles of logarithms, and its practical application involves the ability to read graduated scales.

3. **ACCURACY.**—The degree of exactness to which results may be found depends upon the skill of the operator, the length of the scales and the accuracy of their division. Proficiency in setting and reading comes naturally to the operator together with confidence in the results. The principles upon which the slide rule is based are infallible, but a slight error enters into computations involving numbers of many figures. due to the fact that interpolation is then necessary in setting and reading. Roughly speaking, the accuracy obtainable with the common ten-inch slide rule is equivalent to that of a three-

place logarithm table, while a rule twenty inches long will generally add another figure to the results. This degree of precision is sufficient for almost all engineering calculations and is of material value, at least as a check, for the most part of all other computations of an ordinary nature. With the full-length scales of the ten-inch rule, results should always be accurate within three-tenths of one per cent., while a little experience and care will better this to two-tenths and often less even in rapid working. For the upper scales of the ten-inch rule the error may amount to one-third per cent., while with a twenty-inch rule it is proportionately decreased.

4. **SAVING IN TIME AND LABOR.**—The fact that the slide rule will just as readily solve problems involving any number of factors with any combination of figures in each, results in a time and labor saving device of much importance. Furthermore, it is almost as easy to operate upon numbers of many figures as it is to perform the same operations upon those of the simplest kind. Let the reader but understand the following text and then with a little practice, a considerable amount of time, labor and mental strain will be eliminated from his daily calculations.

II. THEORY OF LOGARITHMS.

5. **DEFINITION.**—To understand the theory and action of a slide rule it is necessary to be familiar with the elementary principles of logarithms. These principles are primarily based upon the fact that every number is equal in value to some power of every other number, the exponent or index of the power being greater or less than one. For example, any number, as 49, is equal to any number, as 10, raised to a certain power, the exponent of the power in this case being approximately 1.69. If 10 be chosen as the fixed number which is to be raised to a power to produce any other number, then 10 becomes the base of this system, the exponent in any case being the logarithm. In general, the logarithm of any number is the exponent of the power to which the base of the system must be raised to produce that number. Thus 2 is the logarithm of 100 to the base 10, since $10^2=100$. The whole part of the logarithm which precedes the decimal point is called the characteristic, while the decimal part following it is the mantissa. In the logarithm 1.69, 1 is the characteristic and .69 is the mantissa.

6. **COMMON LOGARITHMS.**—The system of logarithms having 10 for its base is called the common system. In this system the characteristic simply determines the position of the decimal point in the number corresponding to the logarithm, while the mantissa of the logarithm is identical for the same series of figures no matter where the decimal point in the number is placed. Therefore if the position of the decimal point be neglected, only the mantissa of the common logarithm need be considered. Herein lies the peculiar advantage of the common system of logarithms, and for this reason it is always applied to the slide rule. In the following text wherever logarithms are mentioned the common system will be understood.

7. **RELATION BETWEEN NUMBERS AND LOGARITHMS.**—As stated, the mantissa or decimal part of the logarithm in any case depends only upon the figures comprising the number. The mantissæ of the numbers 1 to 10 are given in Table I.

TABLE I.

Number........	1	2	3	4	5	6	7	8	9	10
Logarithm......	0	.301	.477	.602	.699	.778	.845	.903	.954	1.000

The logarithm of any number composed of 2 and as many zeros as you please will always have .301 for its mantissa, but its characteristic will depend upon the number of figures before the decimal point; and similarly for any other of the above numbers.

8. **MULTIPLICATION.**—It will be observed from Table I that the sum of any two logarithms is the logarithm of the product of the two corresponding numbers. For example, the sum of the logarithms of the numbers 2 and 3 is $.301+.477$ or .778, which is the logarithm of 6 or 2×3. Similarly the sum of the logarithm of 3, 5 and 6 is 1.954, of which the mantissa .954 is the logarithm of 9, the characteristic 1 indicating that there are two figures in the result before the decimal point. Hence their product is 90 or $3\times5\times6$. This same relation will be observed between any two or more numbers and their logarithms. Therefore, by adding the logarithms of any two or more numbers the logarithm of their product is obtained, from which the product itself may be easily found.

9. **DIVISION.**—From Table I it will also be apparent that the difference between any two logarithms is the logarithm of the quotient of the corresponding numbers. The difference between the logarithms of 8 and 4 is $.903-.602$ or .301, which is seen to be the logarithm of 2 or $8\div4$; and similarly for any two or more numbers and their logarithms. The general rule then follows that by subtracting the logarithm of one number from the logarithm of another number, there results the logarithm of the quotient of the two numbers, from which the quotient itself may be readily obtained.

10. **POWERS.**—If the logarithm of any number in Table I be multiplied by 2, the resulting logarithm corresponds to the second power or square of the number. Thus the logarithm of 3 multiplied by 2 is $.477\times2$ or .954, which is the logarithm of 9 or 3^2. Also if the logarithm of 2 be multiplied by 3 there

results .903, the logarithm of 8 or 2^3. Since this relation is true for any number to any power, the rule follows that the logarithm of a number multiplied by any factor gives as a product the logarithm of that power of the number of which the factor is the index. The power itself may then be observed.

11. **ROOTS.**—Again referring to Table I, it will be seen that by dividing the logarithm of any number by 2, the logarithm corresponding to the square root of the number is obtained. Thus the logarithm of 9 divided by 2 is .954 ÷ 2 or .477, which is the logarithm of 3 or $\sqrt{9}$. Also the logarithm of 8 divided by 3 is .903 ÷ 3 or .301, which is the logarithm of 2 or $\sqrt[3]{8}$. Since the principle is similar for the fourth, fifth or any other whole or decimal root of any number, the rule may be stated that by dividing the logarithm of any number there is obtained the logarithm of that root of the number of which the divisor is the index. The root itself may then be found.

12. **APPLICATION TO SLIDE RULES.**—It has been seen that multiplication and division reduce to addition and subtraction of logarithms. Hence if an instrument is capable of mechanically adding and subtracting these logarithms, it may perform all computations of multiplication and division. Likewise the second or third powers and roots may be mechanically determined if the instrument is capable of multiplying and dividing the logarithms by 2 or 3.

The slide rule accomplishes these results by applying the principles of logarithms. Instead of tabulating the logarithms the slide rule carries them in the form of scales or graduated lengths, each unit length representing equal parts of the logarithm table. Thus if the logarithm of 10 be chosen as the unit, then the logarithm of 2, or .301, will be represented by .301 of that unit; 3 by .477 of the unit; 4 by .602; and so on, as shown in Table I. The numbers between 1 and 2, 2 and 3, 3 and 4, etc., are represented on this logarithmic scale by intermediate divisions, the entire scale being graduated as closely as is convenient for reading. It will be observed, however, that the values of the logarithms themselves are not shown on the scales, but instead there will be found the numbers corresponding to those logarithms. At $\frac{301}{1000}$th part along the scale on the slide rule will be found 2, not .301; similarly at the $\frac{845}{1000}$th part is found 7. The graduations are not equally spaced since the numbers and not the logarithms are noted on the scales. In this way the process of finding the logarithm corresponding to the numbers and the numbers corresponding to the logarithms is entirely eliminated from all calculations.

III. MECHANICAL CONSTRUCTION.

13. **MECHANICAL PRINCIPLES.**—If two scales be drawn so that their divisions are equal throughout, the sum of any number of units of one scale and any number of units of the other may be determined mechanically. In Fig. 1, any number on scale C is opposite the sum of itself and 3 on scale D.

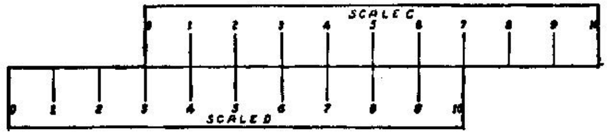


FIG. 1. ADDITION AND SUBTRACTION WITH LINEAR SCALES.

Thus 3 of C and 6 of D are opposite one another; and similarly for 4 and 7, 5 and 8, and so on for the full range of the scales. Conversely for subtraction, any two numbers whose difference is 3 will be found opposite one another on C and D. Furthermore, if scale C be turned upside down with respect to D, the above conditions for addition and subtraction are reversed.

These principles will be obvious to the reader, and they embody everything that is necessary for completely understanding the mechanical principles of the slide rule. It will be remembered, however, that in adding logarithms of numbers, the logarithm of their product results; and in subtracting them the difference is the logarithm of the quotient.

14. **BODY.**—The slide rule is composed of the body, slide and runner, the first two being made of well-seasoned selected wood. The body is the fixed part, consisting of a base upon which are rigidly mounted two graduated rules exactly parallel to each other and separated by an opening for the reception of the slide. The rules are faced with celluloid and have logarithmic scales engraved upon them. The under side of the base carries reference tables, while along one side is a scale of inches for linear measurements and along the other a scale of centimeters or a cube scale.

15. **SLIDE.**—This is a comparatively thin slip of wood faced on the top and bottom with celluloid upon which logarithmic and other scales are engraved. Along each edge of the slide is an extended tongue which accurately fits corresponding grooves in the body of the rule. This construction allows the slide to move lengthwise within the base in either normal or inverted position and with either face uppermost. The scales of the fixed rules and slide should lie absolutely in one plane and no appreciable opening should appear at any point between the edges of the scales for any position of the slide. The slide should move just freely enough so as to neither bind nor stick and still be secure in every position.

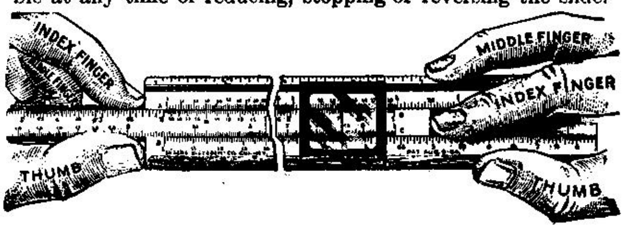
16. **RUNNER.**—The runner comprises a small aluminum frame enclosing a piece of glass. On the underside, almost in contact with the scales, is a fine transverse hair-line. The runner slides in grooves on the sides of the base, while a spring on one side permits free movement along the length of the rule, but always holds the hair-line truly at right angles to all scales. The hair-line should be about as fine as the graduations, and uniform throughout. The runner is used for settings and readings and for referring from one scale to the other; it also eliminates the necessity of reading intermediate results.

17. **LENGTH OF RULES.**—Slide rules are made in five standard sizes, the full length of the logarithmic scales being 12.5, 20, 25, 40 and 50 centimeters respectively. The rule generally used has the 25-centimeter scale and is known as the ten-inch slide rule. This length combines accuracy and convenience, although the larger sizes are more accurate while the smaller ones are more convenient to carry around. The body and slide project beyond the scales to secure a firm setting of the slide and runner at the ends, and in some forms the slide is longer than the body for convenience in operation.

18. **GRADUATION OF SCALES.**—The scales are engine divided and engraved, the divisions being automatically spaced by means of a logarithmic screw. This process results in scales of highest accuracy and greatest durability. The lines are black on a clear white celluloid background.

19. **CARE OF RULES.**—It is important that slide rules be kept in places where excessive changes of temperature or humidity are not likely as all varieties of wood and celluloid are liable to shrink and warp under unfavorable conditions.

20. **METHOD OF OPERATION.**—It is most convenient and accurate to operate the slide rule flat down on a table. The projecting end of the slide is held along both tongues at the end of the base, between the thumb and index finger. The index finger of the other hand is kept between the fixed rules and against the end of the slide, with the thumb and middle finger of that hand along the sides of the base, as in the cut below. The slide may then be easily and gradually moved between the fingers of the first hand, which are capable at any time of reducing, stopping or reversing the slide.



IV. NOTATION OF SCALES.

21. **DESIGNATION OF SCALES.**—There are six complete logarithmic scales on both the fixed rules and the upper face of the slide. The upper fixed rule carries two scales which are identical with the two scales along the upper edge of the slide. Along the lower edge of the slide and the lower fixed rule are two complete scales. The upper scales are usually designated A and B, while the lower ones are marked C and D, as shown in Fig. 2. Scale A includes the two complete logarithmic scales A′ and A″, while B includes B′ and B″. The initial graduations at the left-hand end of the scales A, B, C and D are each termed the left index of the respective scale, while the corresponding lines at the right-hand end are called the right indices. The middle graduations of scales A and B are called their center indices.

22. **RELATION OF DIVISIONS.**—Each logarithmic scale is laid off from its left index, which, being the starting-point, is marked 1 since the logarithm of 1 is 0. (See Table I.) Each other number is located at a point corresponding to that part of the chosen unit length that the logarithm of the number bears to the

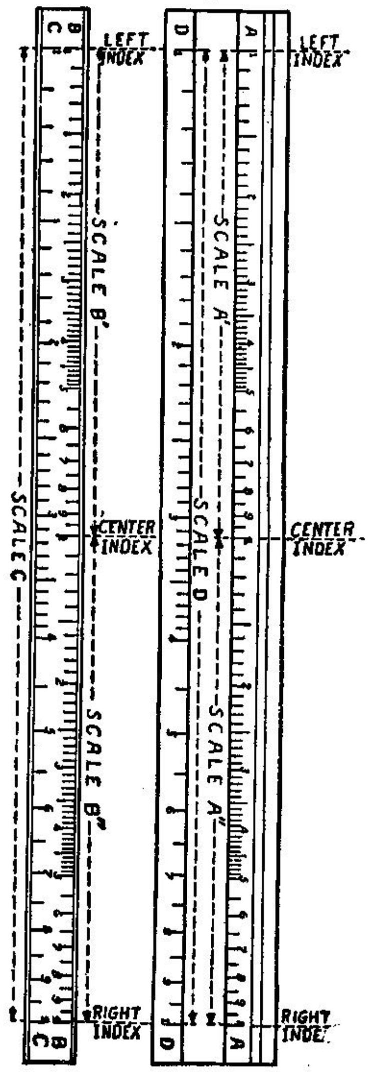


FIG. 2. DESIGNATION OF SCALES AND INDICES.

logarithm of the chosen unit. Hence, since 10 is taken as the chosen unit or the base of the system of logarithms used on the slide rule, the number 2 will be found at the distance .301 from the left index of the scale; 3 is located at .477; 4 at .602; etc. In this way the complete logarithmic scales are laid out, it being observed that each number is represented by a certain length in terms of the length of the scale.

From what has been said in reference to the common system of logarithms it will be apparent that the left index of any scale may represent 1, 10, 100, .1, .01, .001, etc., in which cases the division marked 2 will represent 2, 20, 200, .2, .02, .002, etc., respectively; and similarly for all other numbers.

23. SCALES C AND D.—These scales are similar to each other in every respect and progress from left to right. In order that they may be graduated as minutely at all parts as is convenient for reading, it is necessary to have three different values for the smallest division. With the left index equal to 1, the value of all divisions on scales C and D is shown in Table II for the five sizes of rules.

TABLE II. VALUE OF DIVISIONS ON SCALES C AND D.

SIZE OF RULE	PART OF SCALE	EACH MAIN DIVISION	EACH MINOR DIVISION	EACH SMALLEST DIVISION
5 Inch	1 to 2	1.00	.10	.02
	2 to 6	1.00	.10	.05
	6 to 10	1.00	..	.10
8 or 10 Inch	1 to 2	1.00	.10	.01
	2 to 4	1.00	.10	.02
	4 to 10	1.00	.10	.05
16 or 20 Inch	1 to 2	1.00	.10	.005
	2 to 5	1.00	.10	.01
	5 to 10	1.00	.10	.02

24. SCALES A AND B.—These comprise the four complete logarithmic scales A′, A″, B′ and B″ on the upper fixed rule and along the upper edge of the slide. Each is exactly one-half the length of the C or D scale, and the left indices of A′ and B′ and the right indices of A″ and B″ are engraved accurately in line with the corresponding indices of the lower scales. In the Mannheim slide rules all these scales progress from left to right. The value of the divisions of the upper scales with their left index equal to 1 is shown in Table III.

TABLE III. VALUE OF DIVISIONS ON SCALES A AND B.

SIZE OF RULE	PART OF SCALE	EACH MAIN DIVISION	EACH MINOR DIVISION	EACH SMALLEST DIVISION
5 Inch	1 to 3	1.00	.10	.05
	3 to 6	1.00	..	.10
	6 to 10	1.00	..	.20
8 or 10 Inch	1 to 2	1.00	.10	.02
	2 to 5	1.00	.10	.05
	5 to 10	1.00	..	.10
16 or 20 Inch	1 to 2	1.00	.10	.01
	2 to 4	1.00	.10	.02
	4 to 10	1.00	.10	.05

On scales A′ and B′ will be found a graduation at 3.1416 which is marked π in each case, and on scales A″ and B″ is located .7854 or $\pi \div 4$. These graduations are for convenience in the frequently recurring calculations of areas and diameters of circles. Scales A and B are usually employed for computations where rapidity is the main consideration, and for finding powers and roots.

25. READING SCALES C AND D.—For proper working with the slide rule it is necessary that the operator be accurate and rapid in setting and reading on the various scales. The C and D scales are divided and subdivided as shown in Table II, and for further figures it is necessary to estimate the decimal part of the smallest divisions by eye. Various examples of settings and interpolations appear in Fig. 3, along different parts of the scale of a ten-inch rule with the left index equal to 1,000. Besides these, the reader should make the following and many other settings and readings, using both the upper and lower scales: Set 8 on C to 17 on D, and read 2.125 on D at the right index of C; set 4 on C to 53 on D, and at the left index of C read 13.25 on D; set 68 on C to the right index of D, and by means of the runner read 8.25 on D at 561 on C.

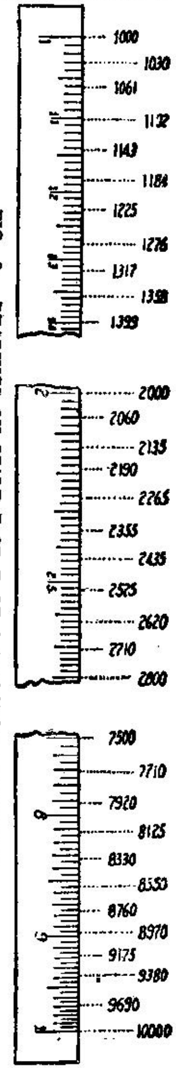


FIG. 3. READINGS ON SCALE C OR D OF A TEN-INCH SLIDE RULE.

26. READING SCALES A AND B.—The value of the various divisions on these scales is different from that of scales C and D, as noted in Table III. Various readings along the upper scales are noted in Fig. 4 (see page 16) for a ten-inch rule, and after these have been carefully compared the reader should make many other settings and readings both with and without the runner, until he is thoroughly familiar with all parts of the upper and lower scales.

27. TEST OF ACCURACY OF GRADUATION.—Although the scales are engine divided, the reader should test their accuracy. First set the left indices of scales C and D together and note that all other corresponding indices and graduations are in exact contact. Then by means of the runner see that the four left indices are exactly in line, and similarly for the four right indices, which test will simultaneously check the alignment of the hair-line of the runner. Now set 2 on scale C to the left index of D and note that all graduations between 2 and 4 on scale C exactly coincide with the graduations on D between 1 and 2. Then set 4 on C to 8 on D and observe exact coincidence of divisions from 4 to 5 on C with 8 to 10 on D. Finally set 5 on C to the left index of D and see that all divisions between 5 and the right index of C are in absolute contact with the graduations of scale D.

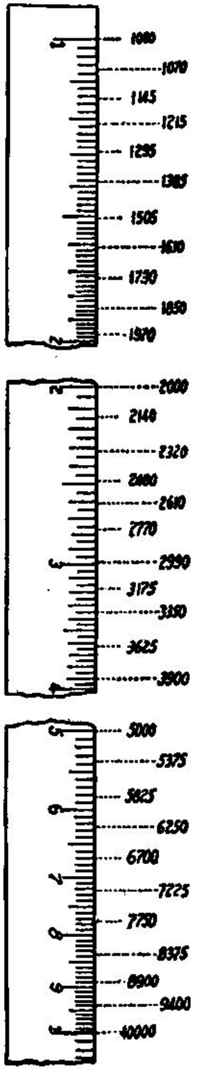


FIG. 4. READINGS ON SCALE A OR B OF A TEN-INCH SLIDE RULE.

To completely check scales A and B, set the left index of B′ to the left index of A″ and observe alignment of all corresponding graduations throughout these scales. Then set 2 on B″ to 4 on A′ and note contact for all divisions between 2 and the right index of B″.

V. MULTIPLICATION.

28. TWO FACTORS.—It has been explained that the sum of the logarithms of two or more numbers is the logarithm of the product of those numbers; also that the logarithms of numbers are mechanically added by means of the slide and fixed rules. Hence to multiply any two numbers, using scales C and D, one index of C is set directly above either factor on D and under the other factor on C is read their product on D. This statement may be expressed in tabular form as follows for both the C and D scales and A and B, the latter including A′ or A″ with either B′ or B″.

C	Set 1	At other number	B
D	to one number	read product	A

The tabular statement reads thus: Set 1 on C to one number on D, and at the other number on C read the product on D. Or, using scales A and B, set 1 on B to one number on A, and at the other number on B read the product on A.

It will be observed that but one of the two indices of C can be used with D for any given problem. If 25 is to be multiplied by 31, the left index of C must be set to either 25 or 31, as desired, in order that the other factor on C will be above scale D; and if 635 is to be multiplied by 2, the right index of C must be used in order to read the product 1270 under scale C. The following examples should be solved, using both the upper and lower sets of scales, and then checked by multiplying the factors out by hand:

C	Set 1	At 25	B	C	Set 1	At 3	B
D	to 636	read product 15900	A	D	to 259	read product 777	A

With the slide projecting on the right the logarithm of one factor on one scale is added to the logarithm of the other factor on the other scale, but if the slide projects on the left of the rule, 10 minus the logarithm of the factor on the slide is subtracted from the logarithm of the factor on the fixed rule. However, the results of these two operations are exactly equivalent, since for the latter case,

$$\log. a - (10 - \log. b) = \log. a + \log. b - 10,$$

wherein a and b represent any two numbers while 10 represents the full length of the scale.

29. ALTERNATIVE METHOD.—The following method of multiplication is sometimes convenient, although the preceding form is usually adopted:

C	Set one number	read product	B
D	to 1	At other number	A

In the above method the two logarithms of the numbers are added together, but in a different way from the method of paragraph 28.

30. CONTINUED MULTIPLICATION.—The method of multiplying more than two numbers together is exactly similar to the method for two factors, the product of the first two being in turn multiplied by the third number, and that product by the fourth, and so on. In such cases it will be found necessary to use the runner in order to avoid reading off the intermediate products. The following tabular statement for the multiplication of three factors will be apparent:

C	Set 1	Runner to second number	1 to runner	At third number	B
D	to first number			read final product	A

31. CONSTANT MULTIPLIER.—By setting the index of C to any number on D, all products of the factor on D with all numbers on C may be read off directly on D without resetting the slide. This property of the slide rule is convenient in a series of multiplications involving a constant factor. For such problems it is generally preferable to use A and B since complete scales are always in contact, thus eliminating the necessity of shifting the slide, as may be required for the lower scales.

32. **DECIMAL POINT.**—The string of figures in the product having been determined, it then becomes necessary to locate the decimal point. It is always advisable for the operator to mentally check the problem, thus locating the decimal point at the same time. In many cases this may be done by inspection. However, rules will be given which are simple and cover all cases of multiplication. First it must be understood that the number of digits in any number greater than 1 is the same as the number of figures preceding the decimal point, while for numbers less than 1, the number of digits is minus and equal to the number of zeros which directly follow the decimal point. Thus, 2.693 has one digit; 149.06 three digits; 14836, five digits. For decimal numbers .103 has zero digits; .09, minus one digit; .0000238, minus four digits. Where more than two factors are to be multiplied together, each setting is to be considered separately. The following rules apply only to the method of multiplication outlined in paragraph 28:

FOR SCALES C AND D.—*The number of digits in the product is equal to the sum of the number of digits in the two factors, except where the slide projects on the right, in which case one must be subtracted from the sum of the number of digits.*

FOR SCALES A AND B.—*The number of digits in the product is equal to the sum of the number of digits in the two factors, except where that scale of B on which one factor is taken extends to the right of that scale of A on which the product is read, in which case one must be subtracted from the sum of the number of digits.*

In applying the above rules to a decimal factor, remember that subtracting from a minus number of digits increases the number of minus digits, while adding to a number of minus digits has the opposite effect. For example, 2 subtracted from — 3 digits gives — 5 digits, and 2 added to — 3 digits gives — 1 digit.

The position of the decimal point may be determined for all scales and methods by remembering that where the first significant figure of the product is less than the first significant figure of either factor, or equal to it, in which case the succeeding figures are to be likewise compared, the number of digits in the product is equal to the sum of the number of digits of the two factors; and where it is greater, the number of digits in the product is one less than the sum of the number of digits in the two factors. By the first significant figure is meant the first figure from the left other than zero. The first significant figure of 309.6 is 3, and of .00298 is 2.

33. **EXAMPLES.**—The following examples are appended so that the reader may become familiar with the method of performing multiplication and of locating the decimal point. The solutions are shown for scales C and D, but scales A and B should also be used by the reader in the solution of each problem.

EXAMPLES	SUM OF DIGITS	NO. OF TIMES SLIDE PROJECTS ON RIGHT	NO. OF DIGITS IN PRODUCT	ANSWER
55 × 4.6	3	0	3	253
.00039 × 1.41 × 41.5	0	1	—1	.0228
4.75 × 1.28 × 83.3 × .0351	3	1	2	17.78
.0017 × .029 × .111 × 68	—1	2	—3	.000372

VI. DIVISION.

34. **TWO NUMBERS.**—The slide rule mechanically performs division by subtracting the logarithm of the divisor from the logarithm of the dividend. Using the C and D scales, the process consists in setting the divisor on C over the dividend on D, and under one index of C finding the quotient on D. To use scales A and B, set the divisor on B under the dividend on A, and over 1 on B read the quotient on A.

C	Set divisor	At 1	B
D	to dividend	read quotient	A

Where the quotient is found at the left index of either C or B the logarithms are subtracted directly. In using the right index, 10 minus the logarithm of the divisor is added to the logarithm of the dividend, which, however, is exactly equivalent to the first case, since

$$\text{log. a} + (10 - \text{log. b}) = \text{log. a} - \text{log. b} + 10,$$

wherein a is the dividend, b the divisor and 10 represents the full length of the scale.

The reader should perform the following divisions, using in turn all sets of scales, and then check the answer by hand:

C	Set 81	At 1	B	C	Set 26	At 1	B
D	to 28500	read quotient 352	A	D	to 4550	read quotient 175	A

35. **ALTERNATIVE METHOD.**—It is sometimes convenient to divide in the following manner:

C	Set divisor	At dividend	B
D	to 1	read quotient	A

36. **CONTINUED DIVISION.**—This operation consists of a series of divisions, the runner being used to avoid reading off the intermediate quotients:

C	Set first divisor	Runner to 1	Second divisor to runner	At 1	B
D	to dividend			read final quotient	A

37. **RECIPROCALS.**—The reciprocal of a number is equal to 1 divided by the number. Thus the reciprocal of 4 is .25; of 500 is .002; and of .0625 is 16. The process is simply one of division, and may be performed by either of the following methods:

C	Set number	At 1	B
D	to 1	read reciprocal	A

C	Set 1	read reciprocal	B
D	to number	At 1	A

Although the method is seldom used, the above principles may be applied to division as follows:

C	Set dividend	read quotient	B
D	to divisor	At 1	A

By this process the divisor is divided by the dividend and the reciprocal of that quotient found, all at one setting.

38. **CONSTANT DIVIDEND.**—With the ordinary position of the slide, such problems are solved by bringing the runner to the constant dividend on D, and successively setting the divisors on C to the hair-line. The quotients are read in turn on D under the index of C.

39. **CONSTANT DIVISOR.**—Much time is saved in problems of this kind by multiplying the reciprocal of the constant divisor by the series of dividends. The entire set of quotients may then be read off without shifting the slide. Where the accuracy of the upper scale suffices, they should generally be used so that all numbers will be in contact for any divisor.

C	Set constant divisor	At dividends	B
D	to 1	read quotients	A

40. **DECIMAL AND COMMON FRACTIONS.**—Common fractions may be converted into decimals by dividing the numerator by the denominator according to the ordinary method of division. Decimals may be changed to common fractions by setting 1 on the slide to the decimal and finding two numbers in contact. If the common fraction is to have a certain denominator or numerator, the corresponding numerator or denominator, respectively, is then found in contact with the given term.

41. **DECIMAL POINT.**—After the string of figures in the quotient has been found, the decimal point may be located for the method of paragraph 34, as follows:

FOR SCALES C AND D.—*The number of digits in the quotient is equal to the number of digits in the dividend less the number of digits in the divisor, except where the slide projects on the right, in which case one must be added to the difference of the number of digits.*

FOR SCALES A AND B.—*The number of digits in the quotient is equal to the number of digits in the dividend less the number of digits in the divisor, except where that scale of B on which the divisor is taken extends to the right of that scale of A on which the dividend is taken, in which case one must be added to the difference of the number of digits.*

The decimal point may also be located in a quotient for any method and either set of scales from the rule that where the first significant figure of the divisor is greater than that of the dividend, the number of digits in the quotient is equal to the number of digits of the dividend less those of the divisor. If the first significant figure of the divisor is less than that of the dividend, then one must be added to the difference. Where the first significant figures are the same, the succeeding figures must be likewise compared.

42. **EXAMPLES.**—The reader should solve the following problems, using both sets of scales in turn:

DIVIDEND	FIRST DIVISOR	SECOND DIVISOR	THIRD DIVISOR	DIFFERENCE OF DIGITS	NO. OF TIMES SLIDE PROJECTS ON RIGHT	NO. OF DIGITS IN QUOTIENT	ANSWER
546	32.9	.....	.....	1	1	2	16.6
182.9	11.11	.0042	.....	3	1	4	3920
.0458	.1746	7.21	1.246	—3	2	—1	.0292
.387	.938	4.94	.0333	0	1	1	2.51

VII. PROPORTION.

43. **DEFINITION.**—Proportion is an equality of ratios. The statement that 3 is to 6 as 4 is to 8 is a proportion in which 3 bears the same relation to 6 as 4 does to 8. The solution of problems in proportion for any of the unknown quantities is an example in combined multiplication and division, and is conveniently and readily solved with the slide rule.

44. **DIRECT PROPORTION.**—The general form is,

1st term: 2d term:: 3d term: 4th term.

From well-known principles the product of the two outer terms, 1st and 4th, equals the product of the two inner terms, 2d and 3d. In examples of this kind three of the quantitites are given and the remaining term is determined from the following:

C	1st term	3d term	B
D	2d term	4th term	A

If one of the terms equals 1, then the process reduces to simple multiplication or division of the other two terms.

By setting two terms together as indicated above, it will be noted that all other numbers which have the same relation are in contact with each other. As an example, consider the following: At the rate of 60 miles in 2 hours, how far will a train travel in 15 hours; 25 hours; 30 hours?

C	Set 2 hours	At 15 hours	At 25 hours	At 30 hours	B
D	to 60 miles	read 450 miles	read 750 miles	read 900 miles	A

It will be observed that 60 is first divided by 2 and the quotient found at one index of the slide, which is the required setting for the multiplication of this quotient by any number within the range of contact of the scales.

45. INVERSE PROPORTION.—Where more requires less or less requires more, there exists an inverse form of proportion. With the ordinary position of the slide, such problems are solved in a manner similar to examples in direct proportion, provided the problem is stated inversely, so that the product of the outer terms is equal to the product of the inner terms. For instance, assume that 8 men perform a piece of work in 3 days, how long will it take 6 men working at the same rate to do the work? This is a case of inverse proportion in which more men require less time, and vice versa.

C	Set 8 men	find 4 days	B
D	to 6 men	At 3 days	A

46. SOLUTION OF $\frac{a}{b} \times x$.—Problems of this kind wherein a, b and x represent any numbers whatsoever, are problems in proportion, and may be solved in a single setting by the following method:

C	Set b	At x	B
D	to a	read answer	A

If in a set of calculations a and b are each constant numbers and x has a series of values, this setting will be found convenient, especially if scales A and B are used.

In using a constant multiplier, such as given in the Conversion Ratios, Part III, and in the reference table on the back of the rule, an equivalent ratio is noted, rather than the multiplier itself which is usually a long decimal. For example, instead of stating that the diameter of a circle multiplied by 3.1416 equals its circumference, the relation between diameter and circumference is given as 226 : 710, since the quotient of these numbers very closely equals 3.1416, and since they are points of graduation on scales C and D. The determination of any circumference from its diameter, or vice versa, is then as follows:

C	Set 226	At diameter	B
D	to 710	read circumference	A

47. SOLUTION OF $\frac{a\times b\times c}{d\times e\times f}$.—Problems in this form are solved by a series of multiplications and divisions. Instead of multiplying the factors of the numerator together and then dividing in turn by the quantities in the denominator, greater rapidity is usually obtained by the following method:

C	Set d	Runner to b	e to runner	Runner to c	f to runner	At 1	B
D	to a					read answer	A

48. DECIMAL POINT.—The decimal points in the preceding problems are located by the rules for multiplication and division, each operation being considered separately. The following method of keeping record in multiplication and division is recommended for long problems: For each time that an extra digit is to be added the sign | is noted, and for each time an extra digit is to be subtracted the sign — is set down, the two opposite signs being allowed to cancel each other as far as they will. In the problem $\frac{.042\times 36.9\times 147}{32.6\times .00186\times 232}$, performed on the C and D scales, the slide projects on the right twice in multiplying and three times in dividing, giving the record + + |, which indicates that the answer contains one more than the number of digits in the numerator less the number of digits in the denominator. Hence the final answer has (−1+2+3)−(2−2+3) + 1 or 2 digits, and equals 16.2.

49. MULTIPLICATION, DIVISION AND PROPORTION WITH A AND B.—The upper scales may be used for all examples of this kind, according to the methods outlined under the corresponding operations for the lower scales. In fact, the preceding text has been made general so as to apply to either set of scales. Due to the fact that all numbers are always in contact, the upper scales should be used for such examples where rapid working is desired and where the greater error due to the decreased length and subdivision of the scales is permissible.

VIII. POWERS AND ROOTS.

50. RELATION OF UPPER AND LOWER SCALES.—Each of the upper scales is exactly one-half the length of a lower scale, and the corresponding indices are accurately in line. Hence any logarithm on the D scale multiplied by 2 equals the logarithm directly above it on A; and similarly for scales C and B. Therefore, since multiplying or dividing the logarithm of a number by 2 gives the logarithm of the second power or root, respectively, of the number, the slide rule gives a direct means of determining squares and square roots of all numbers.

51. SQUARES.—The squares of all numbers on D will be found directly above on A, and similarly for scales on C and B.

A	read square	B
D	Over number	C

Either the runner or an index line of the slide may be used in referring from D to A. The square of any number may also be found by multiplying the number by itself, preferably using the lower scales for the purpose. By any of these methods the square of 2 is seen to be 4; 5 squared is 25; $\overline{17.2^2}=296$; and $\overline{.0831^2}=.0069$.

If a square is read on A′ or B′, the number of its digits is one less than twice the number of digits in the given number; and if read on A″ or B″, it is twice those in the number.

52. SQUARE ROOTS.—The method of finding square roots is exactly opposite to that for finding squares. However, it must be observed that any string of figures has two roots, the proper one for any number depending on its digits. The square root of a number having an odd number of digits, as 144, 16,000 or .000,25, is found on the lower scales under A′ or B′, while for an even number of digits, as 14.4, 1600 or .0025, the root is under A″ or B″.

FOR AN ODD NUMBER OF DIGITS

A′	Under number	B′
D	find square root	C

FOR AN EVEN NUMBER OF DIGITS

A″	Under number	B″
D	find square root	C

If the square root is found under A′ or B′, the number of digits equals (number of digits in given number + 1) $\times \frac{1}{2}$; and for a square root under A″ or B″, the number of digits is one-half that of the given number.

The square root of numbers may also be determined by setting the runner to the number on the fixed rule and moving the slide until the number at the index of the slide equals the number on the slide under the runner.

53. CUBES.—The third power or cube of a number may be found by either of the following methods:

A		find cube
B		At number
C	Set 1	
D	to number	

A	to number	find cube
B	Set 1	
C		Over number
D		

Rules may be given for locating the decimal point in a cube, but it is better to determine its position by inspection of the given number.

54. CUBE ROOTS.—Cube roots are best determined with the slide inverted, but may be determined by setting the runner to the given number on A and noting the number on D under the index of C equal to the number on B under the runner. In this way three cube roots of any string of figures may be found, the correct one in any case depending on the number of digits in the given number. For numbers containing — 8, — 5, — 2, 1, 4, 7, etc., digits, scales A″ and B″ and the left index of C are used; for numbers of — 7, — 4, — 1, 2, 5, 8, etc., digits, scales A′ and B″ and the right index of C are used; and for numbers having — 6, — 3, 0, 3, 6, 9, etc., digits, the A″ and B″ scales are used with the right index of C. There is one digit in the cube root for each period of three figures, or less in the extreme period, contained in the given number, counting from the decimal point toward the left for numbers greater than 1, and toward the right for numbers wholly decimal. For example, the cube root of 2,700 is 13.92; $\sqrt[3]{27{,}000}=30$; and $\sqrt[3]{.000{,}27}=.0647$. It will be observed from the last case that the periods of digits in decimal numbers indicate minus digits.

55. HIGHER POWERS.—The fourth power of a number is equal to the square of its square, and the sixth power is the square of its cube, or the cube of its square. Other powers may be found in this way, but for those greater than the fourth it is better to use the scale of logarithms, as will be explained later. The setting for the fourth power is as follows, the decimal point being located by remembering the rules for squares:

A		find fourth power
C	Set 1	Over number
D	to number	

56. FRACTIONAL POWERS.—In almost all cases fractional powers should be solved by means of the scale of logarithms, although there are a few exceptions. The one-fourth power of a number, which is the same as its fourth root, may be determined by extracting the square root of its square root, due attention being paid to the number of digits in each setting.

The two-thirds power is determined by setting the runner to the number on D, and then finding the cube root of the square on A by the method of paragraph 54, or with the slide inverted, as will be shown.

The three-halves power is obtained as follows, care being taken that the answer is found under the proper scale of B:

B		Under number
C	Set 1	
D	to number	read three-halves power

57. POWERS WITH PROPORTIONAL DIVIDERS.—Where a series of numbers is to be raised to the same power, whether whole or decimal, this method may be found useful.

Set the pair of proportional dividers so that the ratio of its openings in linear measure is equal to the exponent of the required power. Then by opening one side of the dividers from the left index to any number on the logarithmic scale, the opening of the other side measured along the same scale will give the required power.

IX. INVERTED SLIDE.

58. **RECIPROCALS.**—The slide may be turned around end for end so that scale C is in contact with A, and B with D. It will be observed that all the scales on the slide now progress from right to left. With this arrangement and with the indices in line, all numbers on C inverted (CI) are reciprocals of those directly below on D, and similarly for B inverted (BI) and A.

59. **MULTIPLICATION AND DIVISION.**—By setting any logarithm on CI (scale C inverted) over any logarithm on D, their sum is found on D under the index of CI, and similarly for A and BI (B inverted). Also the difference of logarithms is obtained by placing the index of CI over one logarithm on D and reading on D under the other logarithm on CI. The following will now be obvious:

MULTIPLICATION.

CI	Set one number	Opposite 1	BI
D	opposite other number	read product	A

DIVISION.

CI	Set 1	Opposite divisor	BI
D	opposite dividend	read quotient	A

Where a constant quantity is to be successively divided by a series of numbers, this method of division is of value, since all quotients may then be read off without resetting the slide.

60. **INVERSE PROPORTION.**—This operation is best accomplished with the slide inverted. For example, how many teeth must a gear wheel have if it is to turn 300 revolutions per minute when engaging with a gear of 48 teeth making 50 revolutions per minute?

CI	Set 50 R. P. M.	Opposite 300 R. P. M.	BI
D	opposite 48 teeth	read 8 teeth	A

61. **CUBE ROOTS.**—The method with the slide inverted differs from that of paragraph 54 in that the index of the slide is set to the given number on A, and the two equal numbers are found in contact on BI and D. The scales and indices to be used are the same for each of the three cases, it being kept in mind, however, that B″ now occupies the left-hand side of (BI), and that the right and left indices are to be considered as interchanged. The method of locating the decimal point is the same as given in paragraph 54.

X. COMBINED SETTINGS.

62. **LIST OF SETTINGS.**—The adaptability of the slide rule to the ordinary calculations is exemplified by the following list of settings. Only those operations requiring a single setting are given, although the list might be increased indefinitely by using the runner for intermediate results. The reader will observe that alternative methods of procedure are possible for almost every setting, and he is urged to become familiar with them.

In the following settings a, b and c stand for any numbers whatsoever, while x represents the required number. Careful attention must be paid to the number of digits in the numbers of which roots are found. The decimal point may be located in each problem by properly combining the preceding rules, or by inspection in most cases.

SETTINGS FOR ONE NUMBER.

1. $x = a^2$—Over a on D and read x on A.
2. $x = a^3$—Set 1 on C to a on D; at a on B read x on A.
3. $x = a^4$—Set 1 on C to a on D; over a on C read x on A.
4. $x = 1 \div a$—Set a on C to 1 on D; at 1 on C read x on D.
5. $x = 1 \div a^2$—Set on C to 1 on D; over 1 on C read x on A.
6. $x = \sqrt{a}$—Under a on A read x on D.
7. $x = \sqrt{a^3}$—Set 1 on C to a on D; under a on B read x on D.
8. $x = \sqrt[3]{a}$—See paragraphs 54 and 61.
9. $x = 1 \div \sqrt{a}$—Set 1 on B to a on A; at 1 on D read x on C.

SETTINGS FOR TWO NUMBERS.

10. $x = a \times b$—Set 1 on C to a on D; at b on C read x on D.
11. $x = a \div b$—Set b on C to a on D; at 1 on C read x on D.
12. $x = a \times b^2$—Set 1 on B to a on A; over B on C read x on A.
13. $x = a \div b^2$—Set b on C under a on A; at 1 on B read x on A.
14. $x = a^2 \div b$—Set b on B over a on D; at 1 on B read x on A.
15. $x = a^2 \times b^2$—Set 1 on C to a on D; over b on C read x on A.
16. $x = a^2 \div b^2$—Set b on C to a on D; over 1 on C read x on A.
17. $x = a^3 \div b$—Set b on B to a on A; over a on C read x on A.
18. $x = a^3 \div b^2$—Set b on C to a on D; at a on B read x on A.
19. $x = a^4 \div b^3$—Set b on C to a on D; over a on C read x on A.
20. $x = \sqrt{a} \times b$—Set 1 on B to a on A; under b on B read x on D.
21. $x = \sqrt{a} \div b$—Set b on B to a on A; under 1 on B read x on D.
22. $x = a \times \sqrt{b}$—Set 1 on C to a on D; under b on B read x on D.
23. $x = a \div \sqrt{b}$—Set b on B over a on D; at 1 on C read x on D.
24. $x = \sqrt{a} \div b$—Set b on C under a on A; at 1 on C read x on D.
25. $x = \sqrt{a^3} \div b$—Set b on B to a on A; at a on C read x on D.
26. $x = \sqrt{a^3} \div b$—Set b on C to a on D; under a on B read x on D.

SETTINGS FOR THREE NUMBERS.

27. $x = a \times b \div c$—Set c on C to a on D; at b on C read x on D.
28. $x = a^2 \times b \div c$—Set c on B to b on A; over a on C read x on A.
29. $x = a \times b \div c^2$—Set c on C under a on A; at b on B read x on A.
30. $x = a^2 \times b^2 \div c$—Set c on B over a on D; over b on C read x on A.
31. $x = a^2 \times b \div c^2$—Set c on C to a on D; at b on B read x on A.
32. $x = a^2 \times b^2 \div c^2$—Set c on C to a on D; over b on C read x on A.
33. $x = \sqrt{a \times b \div c}$—Set c on B to a on A; under b on B read x on D.
34. $x = \sqrt{a \times b} \div c$—Set c on C under a on A; under b on B read x on D.
35. $x = a \times \sqrt{b \div c}$—Set c on B to b on A; at a on C read x on D.
36. $x = \sqrt{a} \times b \div c$—Set c on C to b on D; under a on B read x on D.
37. $x = a \times b \div \sqrt{c}$—Set c on B over a on D; at b on C read x on D.

XI. SCALE OF SINES.

63. **NOTATION OF SCALE.**—On the back of the slide are three scales, each extending the full graduated length. Two of these are trigonometrical scales and the other is the scale of logarithms. The scale of sines is arranged above the other two and is marked for identification by the letter S. This scale is graduated from approximately 0° 35′ to 90°, as shown in Table IV.

TABLE IV.—SCALE OF SINES.

10-INCH RULE	35′ TO 10°	10° TO 20°	20° TO 40°	40° TO 70°	70° TO 80°	80° TO 90°
Number of divisions......	113	60	40	30	5	2
Value of each division....	5′	10′	30′	1°	2°	5°

64. **NATURAL SINES.**—If the slide be turned over so that the sine scale is adjacent to scale A of the rule, then with corresponding indices in line any angle on S is in contact with its natural sine on A. The maximum value of the sine is 1, the sines read on A′ having — 1 digit, while those on A″ have zero digits. Thus sine 3° 35′ is .0625, and sine 20° 30′ is .350.

The sines may also be found with the slide in its normal position by setting the angle to the upper index mark in the right-hand recess in the base of the rule. The sine is then read under either index of A″, and the decimal point is located by remembering that angles between 35′ and 5° 44′ have — 1 digit, and those between 5° 45′ and 90° have zero digits. Sines

of angles greater than 90° may be determined from the following:

From 90° to 180°, sine a° = sine (180°—a°).
From 180° to 270°, sine a° = —sine (a°—180°).
From 270° to 360°, sine a° = —sine (360°—a°).

65. **SINES OF ANGLES 40° TO 90°.**—It will be seen that the divisions rapidly diminish in length, making it impossible to obtain accurate readings toward the end of the scale. Hence in most cases it is desirable to calculate the sines of angles between 40° and 90° using the relation,

$$\text{sine } a^\circ = 1 - 2 \times \text{sine}^2 \frac{90^\circ - a^\circ}{2}.$$

The angle $\frac{90^\circ - a^\circ}{2}$ is set to the index of the recess and on B under the index of A″ is read the corresponding sine, or x, which is squared without shifting the slide by reading on B under x on A. This final value on B is then doubled and subtracted from 1.

As an example, find the sine of 78° 14.′ Set $\frac{90^\circ - 78^\circ\ 14'}{2}$ or 5° 53′ to the upper index of the right-hand recess and under either index of A″ read .1025 on B, which squared gives .0105 on B′ under .1025 on A″. Hence sine 78° 14′ = 1—2×.0105 or .979.

66. **SINES OF SMALL ANGLES.**—The sines of angles less than 35′ are almost exactly proportional to the corresponding angles, so that they may be read off directly. For this purpose there is a graduation on the sine scale designated (″) and another one marked (′). (See Plate I, Setting 4.) For the sine of a small angle expressed in seconds, the graduation (″) is placed in contact with the number on A which is numerically equal to the given angle, and then over the index of S is read the sine. For small angles expressed in minutes the graduation (′) is used in the same way. In locating the decimal points of these small angles, it must be noted that the sine of angles less than 2″ has —5 digits; sine 3″ to sine 20″ has —4 digits; sine 21″ to sine 3′26″ has —3 digits; and sine 3′27″ to sine 34′23″ has — 2 digits.

67. **MULTIPLICATION AND DIVISION OF SINES.**—The sines of angles may be multiplied or divided by the ordinary method for numbers, having scale S in contact with A.

TO MULTIPLY BY SINE.

A	to number	read product
S	Set index	At angle

TO DIVIDE BY SINE.

A	to number	read quotient
S	Set angle	At index

To divide the sine of an angle by any number, proceed as if dividing the number by the sine, but then read the quotient on B at the index of the recess. The decimal point should be located by inspection.

68. **NATURAL COSINES.**—The cosine of any angle a° is equal to sine (90°—a°). Hence by setting (90°—a°) on the sine scale cosine a° is determined directly; and it may be multiplied or divided by the ordinary methods for sines. For accurately finding the cosines of angles less than 50°, it is usually desirable to employ the following relation:

$$\text{cosine } a^\circ = 1 - 2 \times \text{sine}^2 \frac{a^\circ}{2}.$$

The cosines of angles between 89° 25′ and 90° may be found by means of the special graduations (″) and (′) on scale S, as noted in paragraph 66 for sines of small angles.

Cosines of angles less than 84° 15′ have zero digits, while for angles between 84° 16′ and 89° 25′ there is one zero directly following the decimal point. The cosines of angles greater than 90° may be determined by means of the following:

From 90° to 180°, cosine a° = —sine (a°—90°).
From 180° to 270°, cosine a° = —sine (270°—a°).
From 270° to 360°, cosine a° = sine (a°—270°).

69. **NATURAL SECANTS.**—The secant of an angle is equal to the reciprocal of the corresponding cosine. Hence for any angle a°, (90°—a°) on S is set to the index of the recess, and over 1 on B is read secant a° on A; or with (90°—a°) on S set to 1 on A, secant a° is read on A over the index of S. It is advisable to fix the decimal point by inspection.

70. **NATURAL COSECANTS.**—The cosecant of an angle is equal to the reciprocal of the sine of that angle, which may be found directly over the index of B with the angle set to the index of the recess. If scale S is placed in contact with A, then the angle on S is set to the index of A, and at the index of S is read the cosecant on A. The decimal point should be located by inspection, remembering the rules for decimals in sines.

71. **NATURAL VERSED SINES AND COVERSED SINES.**—The versed sine of a° equals (1—cosine a°), while the coversed sine of a° equals (1—sine a°). These functions are found by obtaining the cosine and sine respectively, and subtracting the results from 1.

XII. SCALE OF TANGENTS.

72. **NOTATION OF SCALE.**—Along the lower edge of the under side of the slide is a scale of tangents marked T. It is used in conjunction with the lower scales, and with them gives directly the natural tangents of all angles from approximately 5° 43′ to 45°. From 5° 45′ to 20° there is a total of 171 divisions on the ten-inch slide rule, each having a value of 5′, and from 20° to 45° there are 150 divisions, each being equivalent to 10′.

73. **NATURAL TANGENTS.**—By placing scale T adjacent to D of the rule and setting corresponding indices in line, the natural tangent of any angle on T is found in contact on D. Tangents may also be read with the slide in its normal position by setting the angle on T to the lower index of the left-hand recess of the rule and reading on C at the index of D. Thus the tangent of 13° 30′ is .24, the result always having zero digits for angles between 5° 43′ and 45°. Tangents of angles greater than 90° may be determined as follows:

From 90° to 180°, tangent a° = —tangent (180°—a°).
From 180° to 270°, tangent a° = tangent (a°—180°).
From 270° to 360°, tangent a° = —tangent (360°—a°).

The tangents of angles less than 5° 43′ are very closely equal to the corresponding sines, and hence may be determined directly from scale S. For very small angles expressed in seconds or minutes, the special graduations (″) or (′) respectively, on scale S give accurate results for tangents also.

74. **TANGENTS OF ANGLES 45° TO 90°.**—For these angles the natural tangents may be obtained by means of the relation,

$$\text{tangent } a^\circ = \frac{1}{\text{tangent } (90^\circ - a^\circ)}.$$

This operation is performed in one setting by placing (90°—a°) on T to 1 on D, and at the index of T reading tangent a° on D. Tangents of angles between 45° and 84° 17′ have 1 digit; between 84° 18′ and 89° 25′, 2 digits; and at 90° the tangent is infinite.

75. **MULTIPLICATION AND DIVISION OF TANGENTS.**—Problems of this kind are solved by the ordinary methods of multiplication and division as explained for natural sines in paragraph 67.

76. **NATURAL COTANGENTS.**—The cotangent of an angle equals the reciprocal of the corresponding tangent. With the slide in normal position the angle on T is set to the index of the left-hand recess, and at 1 on C is read the cotangent on D; or with the slide turned over, the angle on T is brought in contact with one on D, and at the index of T is read the cotangent on D. The decimal point may be located by inspection, remembering the rules for tangents.

77. **SOLUTION OF TRIANGLES.**—The angles, sides and areas of right and obtuse angle triangles are readily determined by the slide rule. The relation of the parts are given in books

on trigonometry, while the methods of solution for the unknown quantities will be apparent from the preceding text.

XIII. SCALE OF LOGARITHMS.

78. **GENERAL.**—On the back of the slide between S and T is a scale of logarithms progressing from right to left the full graduated length of the slide, and having equally spaced graduations throughout. On the ten-inch slide there is a total of 500 divisions, each representing .002. This scale used in conjunction with the lower scale gives directly the mantissa or decimal part of the logarithm of any number. The index of C is set to the number on D and at the proper index of either recess is read the mantissa on the scale of logarithms. In this way the logarithm of 2 is seen to be .301; logarithm of 3 = .477; and similarly for all numbers. Table 1 may be derived in this way.

In finding a series of logarithms it is often advisable to set scale D in contact with the scale of sines with the indices in line. The scale of logarithms then progresses from left to right. By means of the runner the mantissæ are read on the scale of logarithms directly above the corresponding numbers on D.

79. **CHARACTERISTIC.**—In determining powers and roots attention must be paid to the characteristic or whole part of the logarithm. According to the common system the characteristic of the logarithm of any number is one less than the number of its digits. Thus the characteristic of 11.83 is 1; of 487 is 2; of .00691 is — 3. The complete logarithm of a number is the sum of its characteristic and mantissa, the latter always being positive, while the characteristic may be either positive or negative. For example, the logarithm of 80,000 is 4.903; of 90 is 1.954; of .004 is $\bar{3}.602$, the — sign above the 3 in the last case indicating the negative characteristic.

80. **POWERS AND ROOTS.**—From the principles of logarithms explained in Chapter II, a number may be raised to any power by multiplying the complete logarithm of the number by the index of the required power, and then finding the number corresponding to the resulting logarithm. For roots the complete logarithm of the number is divided by the index, and for negative powers or roots the logarithm of the reciprocal of the number is multiplied or divided respectively, by the index. Having found the resulting logarithm in any case, the mantissa on the scale of logarithms is set to the index of either recess, and at 1 on C is read the corresponding number on D. The decimal point in the power or root is fixed by the resulting characteristic. As an example solve $\sqrt[3]{24.6^5}$. Set 1 on C to 24.6 on D, and read .391 on the scale of logarithms at the index of the recess. The complete logarithm is 1.391, which

multiplied by 5, the index of the power, and divided by 3, the index of the root, gives 2.318 as the resulting logarithm. The mantissa .318 on the scale of logarithms is then set to the index of the recess, and at 1 on C is found 2080. . . .on D. Since the characteristic of the resulting logarithm is 2, the answer to the example is 208.

In raising a number less than 1 to a power of which the exponent involves a decimal, care must be taken to keep the mantissa of the logarithm of the power, which is positive, separate from its characteristic, which is negative.

XIV. EXAMPLES.

The following examples are intended to illustrate the application of the Mannheim slide rule to common problems, and should be carefully noted by the beginner. The settings on pages 27 and 28 should then be studied, numbers at random being substituted for the letters.

MULTIPLICATION (See Par. 28 to 33).

1. Find the circumference of a wheel 31 inches in diameter. Since circumference = 3.1416 × diameter, set the left index of scale C to 3.1416 on D, and at 31 inches on C read 97.4 inches on D. (See Par. 28.) This example may also be solved on the upper scales, where a special graduation is marked at $\pi = 3.1416$. The solution may be further simplified by using the geometrical ratio between diameter and circumference given on page 64. Thus by setting 226 on C to 710 on D, the required circumference 97.4 inches is read on D under 31 inches on C. Any other circumference within the range of contact of the scales may be read without further shifting the slide. The decimal point may be located by inspection of the problem or by the rules stated in Par. 32.

2. What is the cost of 9¼ yards of cloth at $.45? Set the right index of C to $.45 on D, and by means of the runner read $4.16 on D at 9.25 on C. (See Plate I, Setting 1, Reading a, and Par. 28 and 32.)

3. Determine the capacity of a tank 7.3 feet long by 3.9 wide by 4.6 high. Set the right index of C to 7.3 on D; bring the hair-line of the runner to 3.9 on C; shift the right index of C to the hair-line; then, by means of the runner, read 131 cubic feet on D at 4.6 on C. (See Par. 30.)

4. Find the interest for 1 year at 4.5% on $2,470; $33,100; $463; $5,580; $6,560; $72,200; $895; $995,000. Set the right index of C to .045 on D and by means of the runner read on D at the given amounts on C, $111.20; $1,489; $20.83; $251.10; $295.20; $3,249; $40.30; $44,800; respectively. (See Plate I, Setting 1, Readings b, and Par. 31 and 32.)

DIVISION. (See Par. 34 to 42.)

5. At a schedule speed of 34.2 miles per hour, in what time does a train travel 648 miles? Set the hair-line to 648 on D;

bring 34.2 on C to the hair-line and at the left index of C read 18.95 hours on D. (See Plate I, Setting 2, Readings a, and Par. 34 and 41.)

6. Calculate the brake horse-power of an engine required to drive a dynamo delivering 96.5 kilowatts at an efficiency of 89%. The problem may be solved as follows:

Brake H. P. $= \frac{96.5}{.746 \times .89}$. Set the hair-line to 96.5 on D; bring .746 on C to the hair-line; shift the runner to the left index of C; set .89 on C to the hair-line and at the right index of C read 145.2 brake horse-power. (See Par. 36 and 41.)

7. Convert to miles: 1,230 feet; 16,900 feet; 214 feet; 3,220 feet; 49,800 feet; 5,230 feet. Since 5,280 feet = 1 mile, set 5,280 on C to the right index of D and under the given number of feet on C read on D, .233; 3.2; .0405; .61; 9.44; .99; respectively. (See Plate I, Setting 2, Readings b, and Par. 39).

8. Find the reciprocals of 129; 25.6; 3.17; 4,320; 563; 68.2; .0776; 84.9; 9,620. Since the reciprocal of a number equals 1 divided by the number, the reciprocal of 129 may be found by bringing 129 on C to the left index of D, and then at the right index of C is read .00775 on D. Similarly the other reciprocals are found to be .0391; .316; .0002315; .001778; .01468; 12.89; .0118; .000104; respectively. (See Par. 37.)

PROPORTION. (See Par. 43 to 49.)

9. If a pump delivers 5,300 gallons of water in 28 minutes, how many gallons will it deliver at the same rate in 1.5 minutes; 20; 382; 40; .45? Set 28 minutes on C to 5,300 gallons on D, and under the given times on C read: 284; 3,790; 72,300; 7,580; 85.2; respectively. (See Plate I, Setting 2, Readings c, and Par. 44.)

10. A pinion turns a 40-tooth gear wheel 36 revolutions per minute. At what rate would the gear revolve if it had 18 teeth? This is a problem in inverse proportion in which a less number of teeth produces a higher rate of rotation. Set 40 teeth on C to 18 teeth on D; over 36 R. P. M. on D read 80 R. P. M. on C. (See Plate I, Setting 1, Readings c, and Par. 45.)

11. Convert to meters: 982 feet; 87.3 feet; 7.64 feet; 6,560 feet; 546 feet; 43.7 feet; 328 feet. From the Ratios of Length, page 64, set 82 on C to 25 on D and under the given feet on C read the following number of meters: 299.5; 26.6; 2.33; 2,000; 166.5; 13.33; 100; respectively. (See Par. 46.)

POWERS AND ROOTS. (See Par. 49 to 57.)

12. Find the area of circles having diameters of 9.5; .842; 73.4; 68.7; .0581; .426; 3.79; 24.3; 1.336. Since the area of a circle $= \frac{\pi}{4}$ (diameter)^2 = .7854 (diameter)^2, set the right index of B to the special graduation .7854 on A, and by means of the runner read in line with the diameter on C the areas on A, respectively as follows: 71; .558; 4,230; 3,700; .00265; .1425; 11.25; 463; 1.40. (See Plate I, Setting 3, Readings a, and Par. 51.)

13. Find the square root of 13.33; 24.5; 3.17; 497; 5.09; 63.1; .764; .0832; .00989. Set the hair-line over each number on A′ or A″, and under the hair-line on D read the square roots, respectively, as follows: 3.655; 4.95; 1.78; 22.3; 2.255; 7.95; .875; .2885; .0995. (See Par. 52.)

14. Calculate the cube root of 46,700. Invert the slide so that scales A and C are in contact; bring the right index of C (now at the left end) to 46,700 on A′; read the cube root 36 on D in contact with the same number on B″I. (See Par. 61.)

ANGLES AND LOGARITHMS. (See Par. 63 to 80.)

15. Find the natural sines of the following angles: 38° 30′; 17° 50′; 6° 15′. Set the given angles on scale S to the index line in the right recess of the base, and under the right index of A″ read on B: .623; .306; .109. (See Par. 64.)

16. Find the natural tangents of 6° 53′; 21° 40′; 42° 10′. Set the given angles on scale T to the index line in the left recess of the base, and at the left index of D read in turn on C the required tangents: .121; .397; .906; respectively. (See Par. 73.)

17. Raise 13.6 to the 1.83 power. Set the left index of C to 13.6 on D, and at the index line in the right recess of the base read the mantissa .134. The characteristic is 1 and the complete logarithm is 1.134, which, multiplied by 1.83, gives 2.075. Set the mantissa .075 to the index line of the right recess, and at the left index of C read 1189 on D. Since the characteristic of the product is 2, the answer is 118.9. (See Par. 80.)

18. A right-angle triangle has a hypothenuse of 6.4 inches with an angle of 25° between it and the base. Find the base and altitude. Reverse the slide so that scales A and S are in contact. Since base = 6.4 × cosine 25° = 6.4 × sine (90°—25°), set the right index of S to 6.4 inches, and at 65° on S read 5.8 inches on A. The altitude = 6.4 inches × sine 25° = 2.7 inches. (See Plate I, Setting 4, Readings a, and Par. 67.)

19. Calculate 8 × tangent 26° 15′. Set the right index of Scale T to 8 on D and at 26° 15′ on T read 3.94 on D. (See Plate I, Setting 4, Reading b, and Par. 75.)

20. Find the logarithm of the following numbers: 1253; 238.6; 3.97; .628; .00913. Reverse the slide so that scales S and D are in contact with the corresponding indices of fixed rule and slide in line. By means of the runner read the mantissæ on the scale of logarithms opposite the numbers on D. The complete logarithms are, respectively, as follows: 3.098; 2.378; 0.599; $\bar{1}.798$; $\bar{3}.96$. (See Par. 78.)

PART II.

THE MULTIPLEX SLIDE RULE

Part II—Multiplex Slide Rule

PATENTED AUG. 9, 1904; NO. 767,170.

I. INTRODUCTION.

1. APPLICATION.—As its name implies, the Multiplex slide rule is a calculating instrument of many uses. Not only does it broaden the field of application, but it offers a convenient means of more rapid working and greater accuracy. The theoretical and mechanical principles upon which the Multiplex slide rule is based are identical in all respects to those underlying the action of the ordinary Mannheim rules. The operator has but little more to learn to gain a great deal.

Not only does the Multiplex solve all the arithmetical trigonometrical and logarithmic examples which are possible with the Mannheim, and in the same convenient and rapid manner, but it further possesses the following characteristic advantages:

1. It multiplies three numbers in one setting.
2. It divides one number by two numbers in one setting.
3. It solves inverse proportion more conveniently.
4. It solves in a single setting a series of divisions having a constant dividend.
5. It gives cubes and cube roots directly.
6. It gives three-halves and two-thirds powers directly.
7. It solves in a single setting many combined operations which require the shifting of the slide of a Mannheim rule.

2. ACCURACY.—The division of all scales is practically perfect, so that the only errors which enter into calculations are those due to setting and reading. Hence by reducing the number of times that the slide must be set and the scales read, the result of any calculation becomes more accurate, even where shorter scales are employed.

3. SAVING IN TIME.—Next to accuracy of results the calculator is most concerned with rapidity in working. Here again the Multiplex rule is of advantage, since it eliminates some of the settings and readings required in many of the common problems.

4. **MECHANICAL ADVANTAGES.**—Due to improved design the Multiplex is a perfect acting and durable slide rule. A study of the following text will reveal to the reader a simple instrument, of wide application, capable of rapid and accurate results, and one of superior and lasting mechanical properties.

5. **NOTE.**—All processes of multiplication, division, proportion, powers, roots and logarithms, together with the determination of trigonometrical functions and combined operations, may be solved with the Multiplex just as with the Mannheim. Hence in the following text only those settings which are applicable to the Multiplex alone will be considered, the other features and principles common to both types of rules being understood from Part I.

II. CONSTRUCTION.

6. **GENERAL.**—The mechanical construction of the Multiplex is in general similar to that of the Mannheim, there being the same arrangement of body, slide and runner. Five sizes of each of two forms of the Multiplex are now manufactured, one form having a reciprocal scale but no cube scale, while the other has both of these features. The cube scale is located on the side of one fixed rule, where it cannot confuse the operator in examples not requiring its use, but still is conveniently accessible when needed. The addition of the cube scale and the substitution of the reciprocal scale for B′ are the only differences between the scales of the Multiplex and the Mannheim slide rules. The scales of sines, tangents and logarithms are identical in arrangement and use for both types.

7. **RECIPROCAL SCALE.**—Along the left-hand side of the upper edge of the slide there is a complete logarithmic scale progressing in the reverse direction, from the center index of B

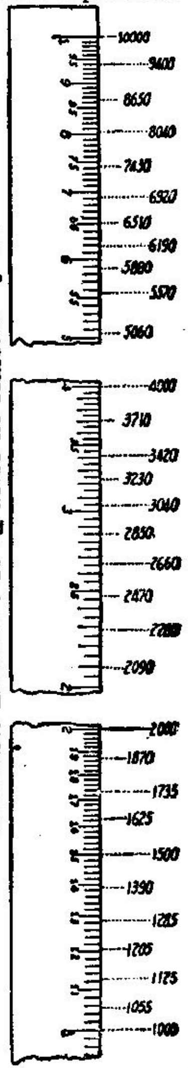


FIG. 1. READINGS ON SCALE Br OF A TEN-INCH SLIDE RULE.

toward the left. This scale will be called the reciprocal scale of B, or Br. It is exactly one-half the length of C or D, and hence equal to that of A′, A″ or B″. Scales Br and B″ are also identical in every other respect except that they progress in opposite directions from the center index of B. In order to avoid any possible confusion, the numbers on scale Br are colored red. In this way the attention of the operator is immediately drawn to the fact that the scale in question progresses in the reverse direction, from right to left.

8. **CUBE SCALE.**—This scale is located on the side of that fixed rule which carries scale D. It is composed of three identical and complete logarithmic scales, E′, E″ and E‴, progressing from left to right. Each is exactly one-third the length of C or D and two-thirds that of any one of the upper scales. The outer indices of E are accurately in line with those of the fixed rules, while the indicator line (I. L.) of the cube scale is carried on a depending lip of the runner and is accurately in line with the hair-line.

9. **READING SCALE Br.**—The division of the reciprocal scale is identical with that of the other upper scales, as shown in Table I for the right-hand index equal to 1.

TABLE I.—VALUE OF DIVISIONS ON SCALE Br.

SIZE OF RULE	PART OF SCALE	EACH MAIN DIVISION	EACH MINOR DIVISION	EACH SMALLEST DIVISION
5 Inch	10 to 6 6 to 3 3 to 1	1.00 1.00 1.00	.. .. .10	.20 .10 .05
8 or 10 Inch	10 to 5 5 to 2 2 to 1	1.00 1.00 1.00	.. .10 .10	.10 .05 .02
16 or 20 Inch	10 to 4 4 to 2 2 to 1	1.00 1.00 1.00	.10 .10 .10	.05 .02 .01

A little practice will accustom the reader to accurately and quickly read the reversed graduations and to readily pass from this scale to others progressing in the opposite direction. Fig. 1 (see page 40) should be carefully examined and compared with the ten-inch rule, and then the following settings and readings should be performed with the reciprocal scale. Set 3 on Br to 4 on A′ and at 1 on Br read 12 on A; note that 3 on A′ and 4 on Br are likewise in contact, and that 12 on Br is at the center index of A. Set 125 on Br to 36 on A′ and at 1 on Br read 4500 on A′. Set 1 on Br to 884 on A″ and at 425 on Br read 2.08 on A″.

10. **READING SCALE E.**—Scale E consists of the three parts E′, E″ and E‴, the first being taken on the extreme left and E‴ on the extreme right. Each of these scales progresses from left to right and is divided in exactly the same way as the upper scales of the slide and rule.

III. MULTIPLICATION.

11. **MECHANICAL PRINCIPLES.**—By drawing two linear scales with equal divisions throughout, but progressing in opposite directions, the addition of any number of units on one with those on the other may be obtained mechanically by setting the two quantities together, as shown in Fig. 2. With

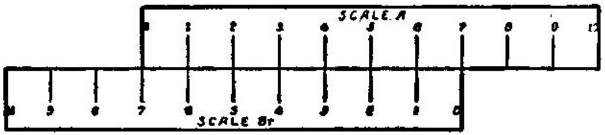


FIG. 2. ADDITION AND SUBTRACTION WITH REVERSED SCALE.

0 on Br set to 7 on A, it will be observed that those numbers whose sum is 7 are in contact.

12. **TWO FACTORS.**—From the simple principle of the preceding paragraph it will be evident that by setting the logarithm of any number on Br to the logarithm of any number on A, their sum, which is the logarithm of the corresponding product, is found on A at the index of Br. In the manner shown in the tabular statement below, the logarithms are added exactly as with scales progressing in the same direction.

A	to second number	read product	Br
Br	Set one number	At 1	A

13. **THREE FACTORS.**—One of the many useful advantages of the Multiplex rule is multiplication of three numbers with but one setting of the slide. It will be observed that the product of two factors, using scale Br, is found on A at the index of Br, which is then the required setting for the multiplication of that product by any third factor on B″, within the range of contact.

A′	to second number	A	read final product
Br	Set one number	B″	At third number

The first and second factors are always taken on Br and A′, from which it will be observed that the logarithms of the three numbers are directly added.

14. **CONSTANT PRODUCT.**—If the index of Br or B″ be set to a given number on A, all combinations of two factors whose products equal the given number will be found in contact between Br and A. Similarly if the product of three factors is to be a constant quantity, then all sets of the factors may be determined by setting the runner to the given quantity on A″ and bringing any number on B″ to the hair line, the other two corresponding factors then being in contact on Br and A. These principles are of much importance in engineering design work and are characteristic features of the Multiplex slide rule.

15. **PROPORTION.**—Direct proportion is best solved with the scales C and D or A and B″. Inverse proportion is most conveniently performed by means of the reciprocal scale of the Multiplex rule, the method being similar to that for the Mannheim rule with its slide inverted. As an example, assume that the electrical resistance of 1,000 feet of copper wire having a cross-section of 350,000 circular mils in .030 ohm. What is the resistance of 1,000 feet of copper wire of 500,000 c. m.; 800,000 c. m.?

A	to .030 ohm	read .021 ohm	read .0131 ohm
Br	Set 350,000 c. m.	At 500,000 c. m.	At 800,000 c. m.

By this method the problem is solved as if dealing with direct proportion on the other scales.

16. **DECIMAL POINT.**—The rules for locating the decimal point in a product obtained by using the reciprocal scale are simple and may be remembered as follows:

If the first significant figure of the product of two numbers is greater than the first significant figures of either factor, the number of digits in the product is one less than the sum of the number of digits in the two factors; if less, the number of digits in the product is equal to the sum. Where the first significant figures are the same, the following figures must likewise be compared.

The number of digits in the product of three factors obtained in one setting is two less than the sum of those in the three factors if the final product is read on A′, *and is one less than their sum if read on* A″.

In multiplying more than three factors together the above rules are combined, or the decimal point is located by inspection.

17. **EXAMPLES.**—The reader should work the following examples, using scale Br, and then compare the operation in each case with the process required for the lower scales:

EXAMPLES	SUM OF DIGITS	FINAL PRODUCT READ ON SCALE	NO. OF DIGITS IN PRODUCT	ANSWER
24×1.42×18.2......	5	A′	3	620
182×2.95×.087......	3	A″	2	46.7
.0024×.56×7.1......	—1	A″	—2	.00954

IV. DIVISION.

18. **MECHANICAL PRINCIPLES.**—By using linear scales progressing in opposite directions, any number may be subtracted from another by setting one index to the latter number on the other scale, and reading at the number to be subtracted. In Fig. 2 (see page 42) the index of Br is set to 7 on A, and at 2 on Br is 5 on A; and similarly for 1 and 6, 3 and 4, etc. In the same way any two logarithms may be subtracted, giving the logarithm of the quotient of the corresponding numbers. Hence for division,

A	to dividend	read quotient	Br
Br	Set 1	At divisor	A

19. **CONSTANT DIVIDEND.**—If a constant quantity is to be divided by a series of numbers, the entire set of quotients may be read off directly without shifting the slide. The index of B is set to the constant dividend on A, and at the divisors on Br are read in turn the corresponding quotients on A.

20. **RECIPROCALS.**—The reciprocals of all numbers may be read directly on the Multiplex rule without shifting the slide. For this operation the index of Br is set to the index of A and at any number on Br is read its reciprocal on A, or vice versa.

21. **CONTINUED DIVISION.**—Any number may be divided by two numbers in one setting of the Multiplex rule. One divisor on B″ is set to the dividend on A″ and at the other divisor on Br is read the final quotient on A.

A″	to dividend	A	read final quotient
B″	Set one divisor	Br	At other divisor

By this method the sum of the logarithms of the two divisors is directly subtracted from the logarithm of the dividend.

22. **DECIMAL POINT.**—The number of digits in a quotient may be determined as follows:

If the first significant figure of the divisor is greater than that of the dividend, the number of digits in the quotient of the two numbers is equal to the number of digits in the dividend less the number in the divisor; if less, then one digit must be added to this difference. The following significant figures are to be compared where the first ones are alike.

When one number is divided by two divisors, the number of digits in the final quotient is two more than the difference between the number of digits in the dividend and divisors if the final quotient is read on A″, and one more than the difference if read on A′.

23. **EXAMPLES.**—The reader should solve the following problems using scale Br, and then compare this method with the operations required for the lower scales:

DIVIDEND	FIRST DIVISOR	SECOND DIVISOR	DIFFERENCE OF DIGITS	FINAL QUOTIENT READ ON SCALE	NO. OF DIGITS IN FINAL QUOTIENT	ANSWER
985	.023	168	1	A″	3	255
.0046	.325	.199	—2	A′	—1	.0712
6200	8.4	3.95	2	A′	3	187

V. POWERS AND ROOTS.

24. **GENERAL.**—Since the scales A′, A″, B″, C and D of the two types of rules are identical, all problems and settings involving them may be performed in exactly the same way. Hence almost all the cases given in Chapter VIII of Part I covering powers and roots apply equally well to the Multiplex rule. The following solutions are best determined with the Multiplex slide rule:

25. **SOLUTION OF** $\frac{1}{a^2}$.—Problems in this form may be solved directly by setting the reciprocal scale over a on D, having the proper indices of Br and A in line and then reading on Br above a on D. For numbers less than 3162 ..... $\frac{1}{a^2}$ may be read on Br directly above a on C. The decimal point in such problems should be located by inspection.

26. **SOLUTION OF** $\frac{1}{\sqrt{a}}$.—For numbers having an odd number of digits, the left index of Br is set to the center index of A, and under a on Br is read $\frac{1}{\sqrt{a}}$ on D. For an even number of digits, $\frac{1}{\sqrt{a}}$ may be found on C directly under a on Br.

27. **CUBES.**—Since scale D is three times the length of each of the scales comprising E, the logarithm of any number on D is directly in line with three times that logarithm on E. Hence all numbers on D are in line with their cube or third power on E.

D	Set hair line to number
E	at I. L. read cube

If read on E′, the cube has two less than three times the number of digits in the given number; if read on E″, it has one digit less; and where read on E‴, the number of digits in the cube equals three times the number of digits in the given number. Hence the cube of .16 is .0041; $\overline{42}^3 = 74{,}100$; and $\overline{76}^3 = 439{,}000$.

28. **CUBE ROOTS.**—The cube root of a number may be read directly by referring from E to D as follows:

D	at hair line read cube root
E	Set I. L. to number

There are three cube roots of any string of figures, the proper one for any given number depending on the number of its digits. For numbers containing — 8, — 5, — 2, 1, 4, 7, etc., digits, scale E′ is used; for numbers of —7, —4, —1, 2, 5, 8, etc., digits, scale E″ is used; and for numbers having — 6, — 3, 0, 3, 6, 9, etc., digits, the given number is taken on E‴. There is one digit in the cube root for each period of three figures, or less in the extreme period, contained in the given number, counting from the decimal point toward the left for numbers greater than 1, and toward the right for decimals. The periods in decimals indicate minus digits. With these principles in mind the cube root of 9 is seen to be 2.08; $\sqrt[3]{43{,}000} = 35$; $\sqrt[3]{.000{,}125} = .05$.

29. **THREE-HALVES POWERS.**—Since the length of scale A′ or A″ is exactly three-halves times that of either E′, E″ or E‴, the three-halves power of any number may be determined directly with the Multiplex rule by passing from A to E.

A	Set hair line to number
E	at I. L. read three-halves power

In this process numbers having an odd number of digits are taken on A′, while those with an even number of digits are taken on A″.

The decimal point in the three-halves power may be located as follows: Representing the number of digits in the given number by Nn and those in the power by Np,

For power on E′, $Np = \left(Nn \times \frac{3}{2}\right) - \frac{1}{2}$.

For power on E″ and number on A′, $Np = \left(Nn \times \frac{3}{2}\right) + \frac{1}{2}$.

For power on E″ and number on A″, $Np = \left(Nn \times \frac{3}{2}\right) - 1$.

For power on E‴, $Np = Nn \times \frac{3}{2}$.

As examples, the three-halves power of 4 is 8; $(933)^{3/2} = 28.500$; $\sqrt{.0015^3} = .000{,}058$; and $\sqrt{66^3} = 536$.

30. **TWO-THIRDS POWERS.**—This operation is the reverse of the preceding, as shown below:

A	at hair line read two-thirds power
E	Set I. L. to number

Numbers having — 8, — 5, —2, 1, 4, 7, etc., digits, are taken on E′; for — 7, — 4, — 1, 2, 5, 8, etc., digits, use scale E″; and for — 6, — 3, 0, 3, 6, 9, etc., digits, the number is taken on E‴.

By referring to paragraph 28 it will be seen that the proper scale of E to use in finding the cube root of a given number is the same as for its two-thirds power. Hence both the one-third and two-thirds powers of any number may be determined from the same setting; and similarly for the second and third powers, and for the one-half and three-halves powers.

The number of digits in the two-thirds power may be found as follows, wherein the same notation is used as in the preceding paragraph:

For number on E′, $Np = \left(Nn \times \frac{2}{3}\right) + \frac{1}{3}$.

For number on E″, and power on A″, $Np = \left(Nn \times \frac{2}{3}\right) - \frac{1}{3}$.

For number on E″, and power on A″, $Np = \left(Nn \times \frac{2}{3}\right) + \frac{2}{3}$.

For number on E‴, $Np = Nn \times \frac{2}{3}$.

31. **OTHER POWERS AND ROOTS.**—By properly combining the scales of the Multiplex rule and using the runner for intermediate results, a great variety of powers and roots may be readily determined. For a series of examples involving the same process such methods are recommended, but for a single problem it is sometimes better to resort to the scale of logarithms on the back of the slide, using the method outlined in Chapter XIII of Part I.

VI. SETTINGS FOR THE MULTIPLEX SLIDE RULE.

32. **LIST OF SETTINGS.**—The following settings are intended to supplement those of Chapter X Part I, all of the latter being applicable to the Multiplex rule also. The additional value of the Multiplex slide rule may be judged from the great variety of important operations which may be solved in a single setting. By using the runner for intermediate results and settings, the list may be enlarged indefinitely.

The operator must pay attention to the number of digits (see Page 18, Par. 32) in the given number and the intermediate results, in order to determine the part of the upper and cube scales to be used. The decimal point in the final result should usually be fixed by inspection, although the preceding rules for digits may be combined for the purpose.

The letters a, b and c represent any numbers whatsoever, while x is used for the required result. It will be remembered that I.L. stands for the indicator line of the cube scale, while CI, B″I and BrI designate scales C, B″ and Br, respectively, with the slide inverted.

SETTINGS FOR ONE NUMBER.

1. $x = a^2$—Over a on D read x on A.
2. $x = a^3$—Under a on D read x on E at I. L.
3. $x = a^4$—Set 1 on C to a on D; over a on C read x on A.
4. $x = a^5$—Set a on CI over a on D; over a on BrI read x on A.
5. $x = a^6$—Set 1 on C to a on D; under a on C read x on E.
6. $x = 1 \div a$—Set 1 on Br to 1 on A; at a on Br read x on A.
7. $x = 1 \div a^2$—Set 1 on B to 1 on A; over a on D read x on Br.
8. $x = 1 \div a^3$—Set a on Br over a on D; at 1 on A read x on B″.
9. $x = 1 \div a^4$—Set a on CI over a on D; under 1 on A read x on BrI.
10. $x = \sqrt{a}$—Under a on A read x on D.
11. $x = \sqrt{a^3}$—Under a on A read x on E at I. L.
12. $x = \sqrt{a^5}$—Set a on CI over a on D; at a on BrI read x on D.
13. $x = \sqrt{a^7}$—Set 1 on B over a on D; under a on B″ read x on E.
14. $x = \sqrt{a^{15}}$—Set a on CI over a on D: under a on BrI read x on E.
15. $x = \sqrt[3]{a}$—Over a on E read x on D with runner.
16. $x = \sqrt[3]{a^2}$—Over a on E read x on A with runner.
17. $x = \sqrt[3]{a^4}$—Set 1 on C over a on E; at a on C read x on D.
18 $x = \sqrt[3]{a^5}$—Set 1 on B over a on E; at a on B″ read x on A.
19. $x = \sqrt[3]{a^8}$—Set 1 on B over a on E; over a on C read x on A.
20. $x = \sqrt[3]{a^{11}}$—Set a on Br over a on E; over a on C read x on A.
21. $x = \sqrt[6]{a^5}$—Set 1 on C over a on E; under a on B″ read x on D.
22. $x = \sqrt[6]{a^{11}}$—Set a on CI over a on E; at a on BrI read x on D.
23. $x = 1 \div \sqrt{a}$—Set 1 on Br to 1 on A; under a on Br read x on D.
24. $x = 1 \div \sqrt{a^3}$—Set a on Br over a on D; under 1 on A read x on C.
25. $x = 1 \div \sqrt[3]{a}$—Set 1 on C over a on E; at 1 on D read x on C.
26. $x = 1 \div \sqrt[3]{a^2}$—Set 1 on B over a on E; at 1 on A read x on B″.
27. $x = 1 \div \sqrt[3]{a^4}$—Set 1 on CI over a on E; at a on CI read x on A.
28. $x = 1 \div \sqrt[3]{a^5}$—Set a on Br over a on E; at 1 on A read x on B″.
29. $x = 1 \div \sqrt[6]{a}$—Set 1 on C over a on E; under a on Br read x on D.
30. $x = 1 \div \sqrt[6]{a^5}$—Set a on Br over a on E; at 1 on D read x on C.

SETTINGS FOR TWO NUMBERS.

31. $x = a \times b$—Set 1 on C to a on D; at b on C read x on D.
32. $x = a \div b$—Set b on C to a on D; at 1 on C read x on D.
33. $x = a \times b^2$—Set 1 on B to a on A; over b on C read x on A.
34. $x = a \div b^2$—Set b on C under a on A; at 1 on B read x on A.
35. $x = a^2 \div b$—Set 1 on C to a on D; at b on Br read x on A.
36. $x = a^2 \times b^2$—Set 1 on C to a on D; over b on C read x on A.
37. $x = a^2 \div b^2$—Set b on C to a on D; at 1 on B read x on A.
38. $x = a^3 \times b$—Set a on Br over a on D; at b on B″ read x on A.
39. $x = a^3 \div b$—Set b on B to a on A; over a on C read x on A.
40. $x = a \div b^3$—Set b on C under a on A″; at b on Br read x on A.
41. $x = a^3 \times b^2$—Set a on Br over a on D; over b on C read x on A.
42. $x = a^3 \div b^2$—Set b on C to a on D; at a on B″ read x on A.
43. $x = a^2 \div b^3$—Set b on C to a on D; at b on Br read x on A.
44. $x = a^3 \times b^3$—Set 1 on C to a on D; under b on C read x on E.
45. $x = a^3 \div b^3$—Set b on C to a on D; under 1 on C read x on E.
46. $x = a^4 \times b$—Set a on CI over a on D; over b on BrI read x on A.
47. $x = a^4 \div b$—Set a on CI over a on D; over b on B″I read x on A.
48. $x = a \div b^4$—Set b on CI over b on D; under a on A read x on BrI.
49. $x = a^4 \div b^2$—Set b on C to a on D; over a on C read x on A.
50. $x = 1 \div (a \times b)$—Set a on Br to b on A; at 1 on A read x on B″.
51. $x = 1 \div (a^2 \times b)$—Set b on Br over a on D; at 1 on A read x on B″.
52. $x = 1 \div (a^2 \times b^2)$—Set a on CI over b on D; under 1 on A read x on BrI.
53. $x = \sqrt{a \times b}$—Set 1 on B to a on A; under b on B″ read x on D.
54. $x = \sqrt{a \div b}$—Set b on B″ to a on A; under 1 on B read x on D.
55. $x = a \times \sqrt{b}$—Set 1 on C to a on D; under b on B″ read x on D.
56. $x = a \div \sqrt{b}$—Set b on B″ over a on D; under 1 on B read x on D.
57. $x = \sqrt{a \div b}$—Set b on C under a on A; under 1 on B read x on D.
58. $x = a \times \sqrt{b^3}$—Set 1 on B over a on E; under b on B″ read x on E.
59. $x = a \div \sqrt{b^3}$—Set b on B″ over a on E; under 1 on B read x on E.
60. $x = a^3 \times \sqrt{b^3}$—Set b on Br over a on D; under 1 on B read x on E.
61. $x = a^3 \div \sqrt{b^3}$—Set b on B″ over a on D; under 1 on B read x on E.
62. $x = \sqrt{a^3} \div b^3$—Set b on C under a on A; under 1 on B read x on E.
63. $x = \sqrt{a^2 \times b}$—Set a on Br to b on A; at a on C read x on D.
64. $x = \sqrt{a^3 \div b}$—Set b on B″ to a on A; at a on C read x on D.
65. $x = \sqrt{a^3 \times b^3}$—Set 1 on B to a on A; under b on B″ read x on E.
66. $x = \sqrt{a^3 \div b^3}$—Set b on B″ to a on A; under 1 on B read x on E.
67. $x = a \times \sqrt[3]{b}$—Set 1 on C over b on E; at a on C read x on D.
68. $x = a \div \sqrt[3]{b}$—Set a on C over b on E; at 1 on D read x on C.
69. $x = \sqrt[3]{a \div b}$—Set b on C over a on E; at 1 on C read x on D.
70. $x = a \times \sqrt[3]{b^2}$—Set 1 on B over b on E; at a on B″ read x on A.
71. $x = \sqrt[3]{a^2 \div b}$—Set b on B″ over a on E; at 1 on B read x on A.
72. $x = a \div \sqrt[3]{b^2}$—Set a on B″ over b on E; at 1 on A read x on B″.
73. $x = a^2 \times \sqrt[3]{b^2}$—Set 1 on B over b on E; over a on C read x on A.
74. $x = \sqrt[3]{a^2 \div b^2}$—Set b on C over a on E; at 1 on B read x on A
75. $x = a^2 \div \sqrt[3]{b^2}$—Set a on C over b on E; at 1 on A read x on B″.
76. $x = 1 \div \sqrt{a \times b}$—Set a on Br to b on A; at 1 on D read x on C.
77. $x = 1 \div (a \times \sqrt{b})$—Set a on C to 1 on D; under b on Br read x on D.

SETTINGS FOR THREE NUMBERS.

78. $x = a \times b \times c$—Set a on Br to b on A′; at c on B″ read x on A.
79. $x = a \times b \div c$—Set c on C to a on D; at b on C read x on D.
80. $x = a \div (b \times c)$—Set b on B″ to a on A″; at c on Br read x on A.
81. $x = a \times b \times c^2$—Set a on Br to b on A; over c on C read x on A.
82. $x = a^2 \times b \div c$—Set b on Br over a on D; at c on Br read x on A.
83. $x = a \times b \div c^2$—Set c on C under a on A; at b on B″ read x on A.
84. $x = a^3 \div (b \times c)$—Set b on B″ over a on D; at c on Br read x on A.
85. $x = a^2 \times b^2 \div c$—Set c on B″ over a on D; over b on C read x on A.
86. $x = a^2 \times b \div c^2$—Set c on C to a on D; at b on B″ read x on A.
87. $x = a^2 \div (b^2 \times c)$—Set b on C to a on D; at c on Br read x on A.
88. $x = a^2 \times b^2 \div c^2$—Set c on C to a on D; over b on C read x on A.
89. $x = a^3 \times b^3 \div c^3$—Set c on C to a on D; under b on C read x on E.
90. $x = \sqrt{a \times b \times c}$—Set a on Br to b on A′; under c on B″ read x on D.
91. $x = \sqrt{a \times b \div c}$—Set a on Br to b on A; under c on Br read x on D.
92. $x = \sqrt{a \div (b \times c)}$—Set b on B″ to a on A″; under c on Br read x on D.
93. $x = a \times \sqrt{b} \times c$—Set b on Br to c on A; at a on C read x on D.
94. $x = \sqrt{a \times b} \div c$—Set c on C under a on A; under b on B″ read x on D.
95. $x = a \times \sqrt{b} \div c$—Set c on B″ to b on A; at a on C read x on D.
96. $x = a \times b \times \sqrt{c}$—Set c on Br over a on D; at b on C read x on D.
97. $x = a \times b + \sqrt{c}$—Set c on B″ over a on D; at b on C read x on D.
98. $x = \sqrt{a} \times b \div c$—Set c on C to b on D; under a on B″ read x on D.
99. $x = \sqrt{a} \div (\sqrt{b} \times c)$—Set b on BrI under a on A; under c on CI read x on D.
100. $x = \sqrt{a^3 \times b^3 \times c^3}$—Set a on Br to b on A′; under c on B″ read x on E.
101. $x = \sqrt{a^3 \times b^3 \div c^3}$—Set c on B″ to a on A; under b on B″ read x on E.
102. $x = \sqrt{a^3 \div (b^3 \times c^3)}$—Set b on B″ to a on A″; under c on Br read x on E.
103. $x = \sqrt{a^3 \times b^3} \div c^3$—Set c on C under a on A; under b on B″ read x on E.
104. $x = a^3 \times \sqrt{b^3 \div c^3}$—Set c on B″ to b on A; under a on C read x on E.
105. $x = a^3 \times b^3 \times \sqrt{c^3}$—Set a on CI over b on D; under c on BrI read x on E.
106. $x = a^3 \times \sqrt{b^3} + c^3$—Set c on C to a on D; under b on B″ read x on E.
107. $x = a^3 \times b^3 \div \sqrt{c^3}$—Set c on B″ ove a on D under b on C read x on E.

VII. EXAMPLES.

The following examples are appended in order to familiarize the beginner with the operation of the Multiplex slide rule and to demonstrate its useful and characteristic properties. The problems illustrating the application of the Mannheim rule (page 33) may be solved in exactly the same way with the Multiplex and should be studied in conjunction with these examples.

MULTIPLICATION AND PROPORTION.

(See Par. 11 to 17.)

1. Calculate the cost of 526 tons of steel rails at $29.50. Set the hair line of the runner to $29.50 on A′ and bring 526 on the reciprocal scale (Br) to the hair line; at the left index of Br read $15,500 on A′. (See Plate II, Setting 1, Readings a, and Par. 12 and 16.)

2. Determine the capacity of a tank 29.5 feet long, 3.25 wide and 5.75 high. Set the runner to 29.5 on A′ and bring 3.25 on Br to the hair line; at 5.75 on B″ read 551 cubic feet on A″. (See Plate II, Setting 2, Readings a, and Par. 13 and 16.)

3. A steel plate is to have a cross section of 1.55 square inches; what combinations of width and thickness can be used? By setting the left index of Br to 1.55 on A′, some of the required combinations of the two dimensions are found in contact between Br and A as follows: 2 and .775; 3 and .517; 4 and .388; 6 and .258; 7 and .222; 8 and .194; 9 and .1722 (See Plate II, Setting 1, Readings b, and Par. 14.)

4. A certain piece of work requires the services of 12 men for 8 days. At this rate how many days would be required for 4 men, 5, 6, 7, 9, 10, or 15? This is a case of inverse proportion in which having more men requires less time. Set the runner to 12 on A′ and bring 8 on Br to the hair line. At the different number of men read the following number of days: 24; 19.2; 16; 13.7; 10.65; 9.6 and 6.4; respectively. (See Plate II, Setting 2, Readings b and Par. 15.)

DIVISION. (See Par. 18 to 23.)

5. Calculate the electric current which produces a drop of 7.3 volts over a resistance of .22 ohm. Since current in amperes equals drop in volts divided by resistance in ohms, set the left index of Br to 7.3 volts and by means of the runner read 33.2 amperes on A″ at .22 ohm on Br. (See Plate II, Setting 2, Readings a and Par. 18 and 22.)

6. Find the capacity of a pump required to fill a water tank holding 1,250 gallons in 4; 5; 6; 7; 8; and 9 minutes. Set the left index of Br to 1,250 on A′, and at the given times on Br read on A′; 312.5; 250; 208.5; 178.6; 156.3; and 138.9 gallons per minute, respectively. (See Par. 19 and 22.)

7. Find the reciprocals of 15.8, 2.37, 38.5, .495, 547, 6.79, .0763, 8.84 and 94.9. Set the left index of Br to the left index of A′, and at the given number on Br read the corresponding reciprocals on A′ as follows: .0633; .422; .026; 2.02; .00183; .1475; 13.1; .1132; .01053; respectively. (See Par. 20.)

8. Solve $\frac{950}{13 \times 84}$. Set the runner to 950 on A″; bring 13 on B″ to the hair line; at 84 on Br read .87 on A′. (See Plate II, Setting 3, Readings b, and Par. 21 and 22.)

9. Divide 4,320 by several combinations of two factors so as to obtain 25 as the quotient in each case. Set the runner to 4,320 on A″; move the slide along, reading one factor on B″ under the hair line and at the same time the corresponding factor on Br at 25 on A′, as 9 and 19.2; 8 and 21.6; .7 and 247; 600 and .288; .5 and 345; 40 and 4.32; etc. (See Par. 21.)

POWERS AND ROOTS. (See Par. 24 to 31.)

10. Find the cubes of 19.3, 3.54 and .671. Set the hair line to the numbers on D and at the indicator line (I. L.) on the runner read on scale E, 7190, 44.4 and .302, respectively. (See Par. 27.)

11. Find the three-halves power of 169, 8.68 and 43.7. Set the hair line over 169 on A′ and at I. L. read 2,190 on E′; similarly set to 8.68 on A′ and read 25.5 on E″; set to 43.7 on A″ and read 288 on E‴. (See Par. 29.)

12. Determine the one-third (cube root) and two-thirds powers of 3,860, 53.7 and .749. Set I. L. to 3,860 on E′ and under the hair line read the one-third power on D, 15.7, and the two-thirds powers on A′, 246; similarly set I. L. to 53.7 on E″ and read 3.775 and 14.25 under the hair line; then bring I. L. to .749 on E‴ and under the hair line read .909 and .827. (See Plate II, Setting 4, Readings a and Par. 28 and 30.)

In the following examples, numbers are substituted for the letters in some of the settings shown on pages 48, 49 and 50. The student should become familiar with the entire list of settings as most of them are applicable to ordinary problems.

13. Solve the following: $\frac{1}{(1.063)^2}$; $\frac{1}{(13.83)^2}$; $\frac{1}{(.1879)^2}$; $\frac{1}{(22.8)^2}$ $\frac{1}{(.03073)^2}$. Over the given denominators on C read the answers on Br as follows: .884; .00522; 28.4; .00192; 1060; respectively. (See Par. 25 and Setting 7, Page 48.)

14. Solve $\frac{1}{\sqrt{2.15}}$; $\frac{1}{\sqrt{36.9}}$; $\frac{1}{\sqrt{.437}}$; $\frac{1}{\sqrt{.684}}$; $\frac{1}{\sqrt{.0876}}$

For denominators having an odd number of digits (see Page 18, Par. 32), the left index of Br is set to the center index of A, and under the given denominator on Br is read the answer on D. For an even number of digits the answer may be read on C under the denominator on Br. Hence the above are equal to .683; .1647; .0478; 1.209; 3.38; respectively. (See Par. 26 and Setting 23, Page 48.)

15. Divide 7,490 by the cube of 7.67. Set the runner to 7,490 on A″ and bring 7.67 on C to the hair line; at 7.67 on Br read 16.6 on A′. (See Setting 40, Page 49.)

16. Multiply the cube of 2.78 by the cube of 18.3. Set the left index of C to 2.78 on D, and bring the runner to 18.3 on C; at I. L. of runner read 132,000 on E‴. (See Setting 44, Page 49.)

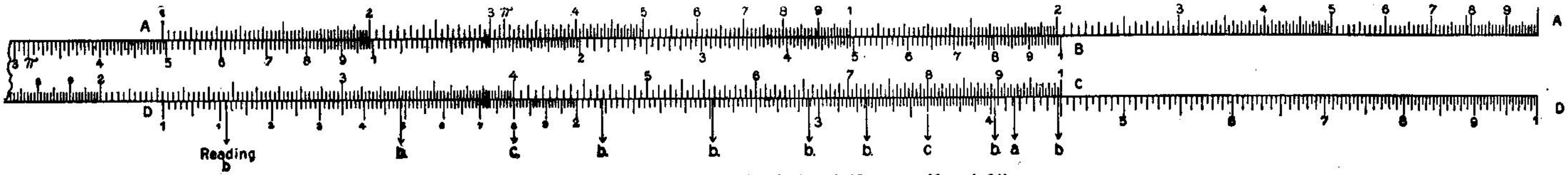
17. Solve $\frac{1}{(3.415 \times 5.07)^2}$. Invert the slide so as to have scales CI (C inverted) and A in contact. Now set the runner to 3.415 on D and bring 5.07 on CI under the hair line; at the right index of D read .00334 on BrI. (See Setting 52, Page 49.)

18. Calculate $\frac{1}{\sqrt{6.03 \times .298}}$ and $\frac{1}{\sqrt{7.81 \times 189.5}}$. For the former set the runner to 6.03 on A′ and bring .298 on Br to the hair line; at the right index of D read .745 on C. In the second case 189.5 on Br is set to 7.81 on A″, giving .026 for the answer. A′ is used in the first case since the product in the denominator has an odd number of digits, and hence its root must be found under A′. In the second case the product has an even number of digits and its root is under A″. (See Setting 76, Page 49.)

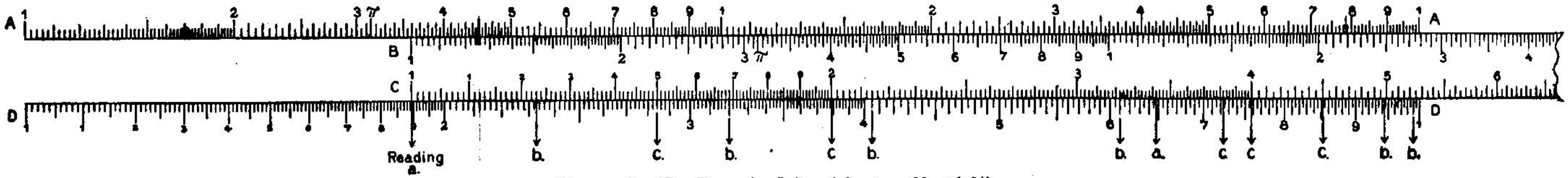
19. Solve $\sqrt{3.73 \times 2.91 \times 405}$. Set the runner to 3.73 on A′ and bring 2.91 on Br to the hair line; under .405 on B″ read 66.4. If the last factor were 40.5 the final product under the root sign would contain an odd number of digits instead of an even number, and then it would be further necessary to bring the right index of B″ to the last position of the runner and read the answer, .21, under the center index of B. (See Setting 90, Page 50.)

20. Solve $\frac{(\sqrt{83.1})^3}{\sqrt{(3.19 \times 2.87)^3}}$. Set the runner to 83.1 on A″ and bring 3.19 on B″ to the hair line; shift the runner to 2.87 on Br and at I. L. of runner read 27.4 on E″. If the number of digits in the final quotient on A is even, the reading to E is made under A″. (See Setting 101, Page 50.)

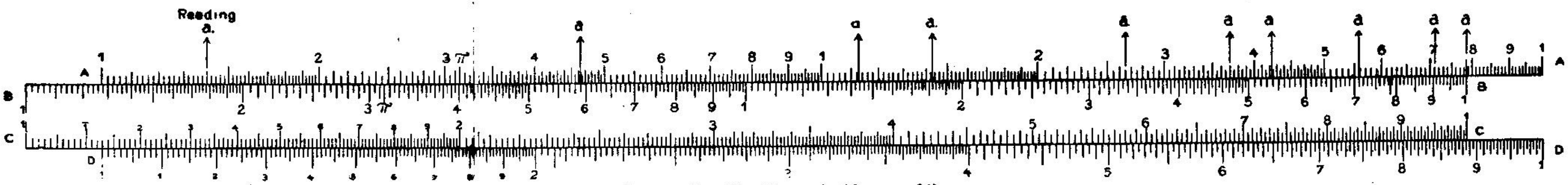
PLATE I.



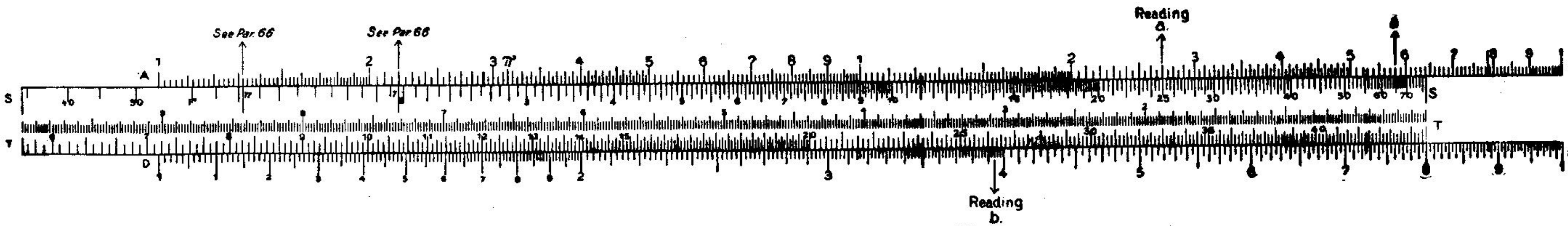
SETTING. 1. (See Examples 2, 4 and 10, pages 33 and 34).



SETTING 2. (See Examples 5, 7 and 9, pages 33 and 34).

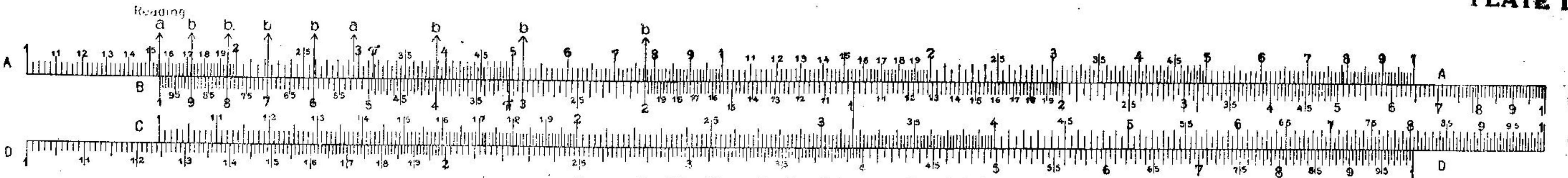


SETTING 3. (See Example 12, page 34).

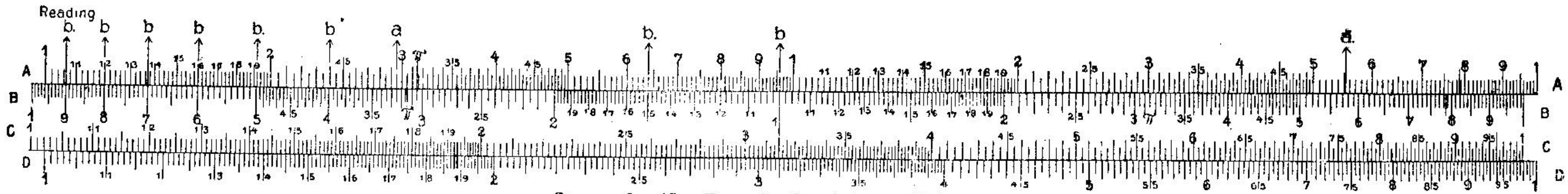


SETTING 4. (See Examples 18 and 19, page 35).

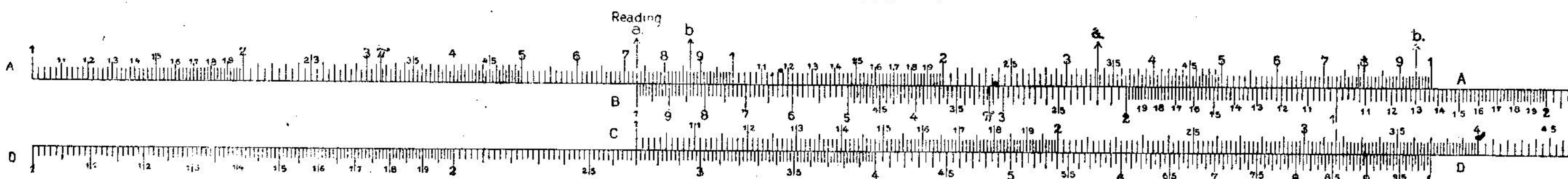
PLATE II.



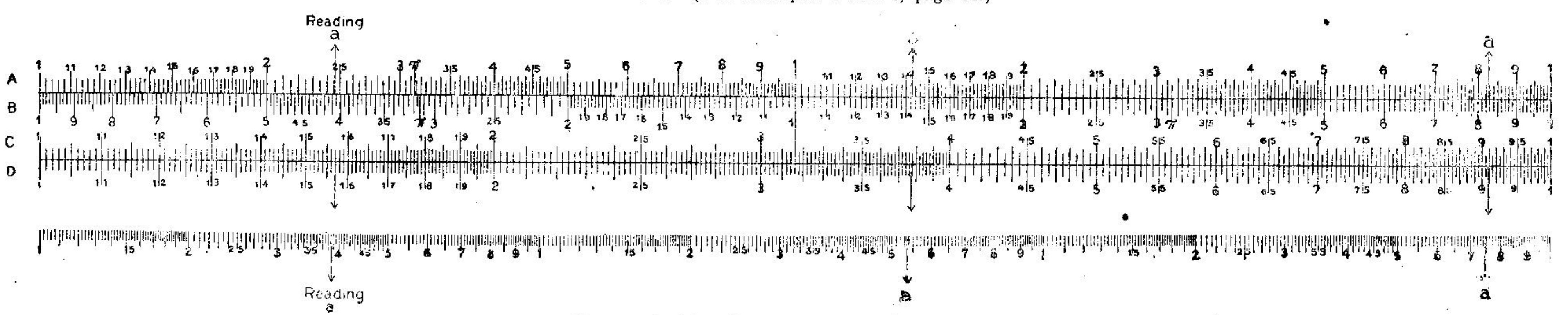
SETTING 1. (See Examples 1 and 3, pages 50 and 51.)



SETTING 2. (See Examples 2 and 4, page 51.)



SETTING 3. (See Examples 5 and 8, page 51.)



SETTING 4. (See Example 12, page 52.)