The Log Log Duplex

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Slide Rule

A Self Teaching Manual
with
tables of settings, equivalents and gauge points

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Published by

KEUFFEL & ESSER CO.

NEW YORK, 127 Fulton Street, General Office and Factories, HOBOKEN, N. J.

CHICAGO 516-20 S. Dearborn St. ST. LOUIS 817 Locust St. SAN FRANCISCO 30-34 Second St. MONTREAL 5 Notre Dame St. W.

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SLIDE RULE.

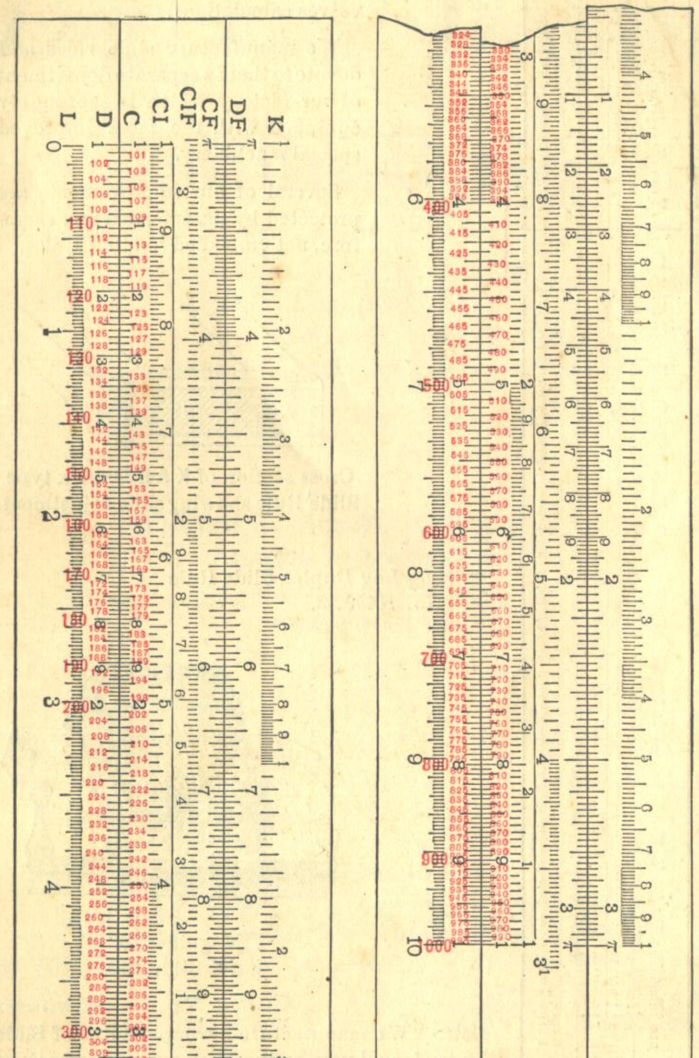
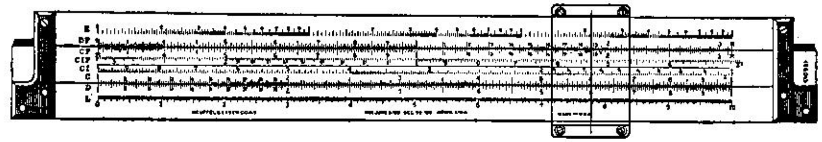


Diagram illustrating the reading of the graduations of the rule.



THE LOG LOG DUPLEX SLIDE RULE.

PREFACE.

This manual is designed to meet the needs of all who desire to learn the use of this slide rule.

Chapter I, through the use of numerous cuts and examples simply explained, is self-teaching. Some persons will learn all that they require from a few lessons in this chapter.

It is suggested that everyone learning to use the slide rule begin by working the problems in Chapter I.

In Chapters II, III, IV, and V, a simple explanation of the theory of the slide rule is followed by the advanced subjects of Cubes, Cube Root, Sines, Cosines, Tangents, Logarithms, and the Solution of Triangles.

Special work for technical men and typical problems from various occupations are presented in Chapters VI, VII, and VIII.

WHO SHOULD USE THE SLIDE RULE?

I. Teachers in the following types of schools:

1. Elementary Schools in the higher grades.
2. Junior High Schools for part of their practical mathematics.
3. High Schools in connection with logarithms, practical mathematics, or trigonometry.
4. Colleges in their courses in algebra or trigonometry. Most colleges have already made the slide rule a part of the trigonometry course.
5. Evening schools; since no subject holds the students so well as the teaching of the use of the slide rule.
6. Engineering and Trade Schools find the rule indispensable.

II. Engineers, Mechanics, Chemists, and Architects who have long understood its value.

III. Private Secretaries to check reports by the slide rule in a small fraction of the time required by ordinary calculation.

IV. Estimators, Accountants and Surveyors to make approximate calculations rapidly and with sufficient accuracy to check gross errors.

By means of the slide rule, all manner of problems involving multiplication, division and proportion can be correctly solved without mental strain and in a small fraction of the time required to work them out by the usual "figuring."

For instance, rapid calculation is made possible in the following everyday problems of office and shop: estimating; discounts; simple and compound interest; the conversion of feet into meters, pounds into kilograms and foreign money into U. S. money; the taking of a series of discounts from list prices; and adding profits to costs. Dozens of equivalents are instantly found, such as cubic inches or feet in gallons, and vice versa; centimeters in inches; inches in yards or feet; kilometers in miles; square centimeters in square inches; liters in cubic feet; kilograms in pounds; pounds in gallons; feet per second in miles per hour; circumferences and diameters of circles.

How much education is necessary?

Anyone who has a knowledge of decimal fractions can learn to use the slide rule.

How much time will it take?

The simplest operations may be learned in a few minutes, but it is recommended that at least the problems in Chapter I be worked thoroughly and checked by the answers, in order to gain accuracy and speed. This will take from one to ten hours, according to the previous training of the student.

How accurate is the Slide Rule?

The accuracy of the slide rule is about proportional to the unit length of the scales used.

The 10″ scale gives results correct to within about 1 part in 1000, or one tenth of one per cent.

The 20″ scale gives results correct to within one part in about 2000.

The Thacher Cylindrical slide rule gives an accuracy of about 1 part in 10000.

How to use this manual

For the man who desires to perform the simplest operations of multiplication and division, the first few lessons in Chapter I will be sufficient. Work the illustrative examples and as many problems for practice as seem necessary to obtain accuracy and speed.

For educational use, Chapter II furnishes the necessary theory and history of the rule, while Chapter I provides additional examples for practice. Chapters III, IV, and V may be used for advanced work.

CHAPTER I

ESSENTIALS OF THE SLIDE RULE SIMPLY EXPLAINED

The slide rule is an instrument that may be used for saving time and labor in most of the calculations that occur in the practical problems of the business man, mechanic, draftsman, engineer, or estimator.

On scales *C* and *D* (front face), if 1 at the extreme left is taken as unity, then 1 at the extreme right of these scales is 10.

On scales *A* and *B* (rear face), if 1 at the extreme left is taken as unity then 1 in the middle of the scale is 10 and 1 at the extreme right is 100.

In order that you may see how the rule is used on simple problems where you know the answers, let us take the following:

Example: 2 × 3. (See Fig. 1)

Opposite 2 on scale *D* set 1 on scale *C*. Then move the indicator or glass runner so that the hair line is over 3 on scale *C*. Directly below this 3 you will find 6, the answer.

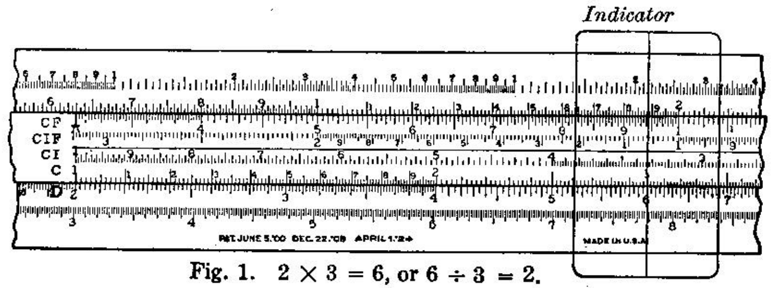


Fig. 1. 2 × 3 = 6, or 6 ÷ 3 = 2.

Example: 2×4. (See Fig. 2)

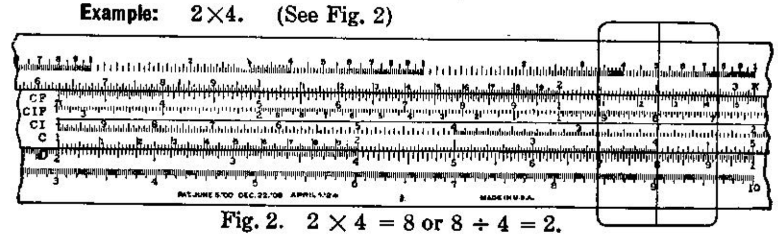


Fig. 2. 2 × 4 = 8 or 8 ÷ 4 = 2.

Example: 3×3. (See Fig. 3)

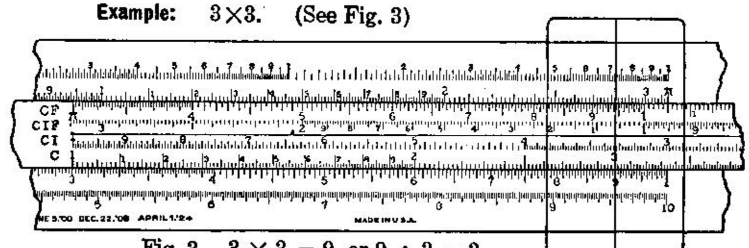


Fig. 3. 3 × 3 = 9, or 9 ÷ 3 = 3.

Example: 6 ÷ 3. (See Fig. 1)

Opposite 6 on scale *D*, set 3 on scale *C*. Look along *C* to the left, till you come to 1 at the end of the slide. Under this 1 you will find 2, the answer, on scale *D*.

Example: In the same way find 8 ÷ 4. (See Fig. 2)

Example: " " " 9 ÷ 3. (See Fig. 3)

It will be noted that the cuts shown are not in the same scale. This arrangement is for the purpose of illustrating various lengths of the rule.

SQUARES AND SQUARE ROOTS

Example: You will remember that to square a number means to multiply that number by itself; *e. g.*, 3^2 means $3 \times 3 = 9$. On the slide rule this is done as follows:

Method I. Using the face of the rule on which scale *B* appears, set the hairline of the indicator to 3 on scale *C*. Above, on scale *B*, under the hair line, you will find 9, the answer.

Method II. Bring the hairline of the indicator to 3 on scale *D*. Turn the rule over and on scale *A* on the opposite face, read 9 under the hairline of the indicator.

Method III. Set the left hand 1 on scale *C* to 3 on scale LL 3. Opposite 2 on scale *C* read 9 on scale LL 3.

Any of the above methods may be used to advantage in problems, depending upon the calculation that is made.

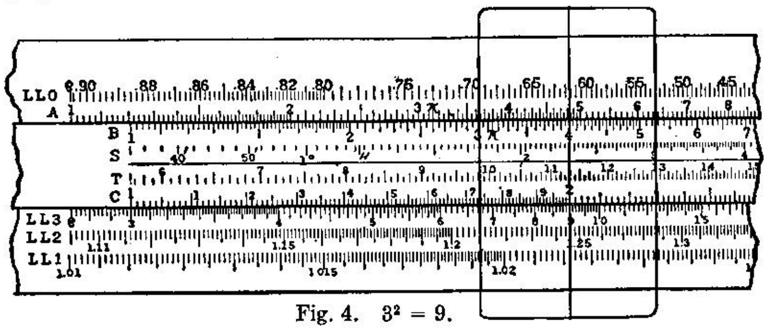


Fig. 4. $3^2 = 9$.

Example: In the same way find 2^2.

To find square roots simply do the work in the reverse order.

To find the square root of 9, find the number which multiplied by itself will give 9. The square root of 9 is indicated thus: $\sqrt{9}$.

Method I. Set the indicator to 9 on scale *B*, being careful to use the 9 on the left hand half of the rule, because the other 9 is really 90. Below, on scale *C*, opposite the indicator, find 3, the answer.

Method II. Bring the hairline of the indicator to the 9 on the left hand part of scale *A*. Turn the rule over and under the hairline of the indicator on scale *D* read 3.

Method III. Set 2 on scale *C* to 9 on scale LL 3. Opposite 1 on *C* read 3 on LL3. (Fig. 4).

Example: Find $\sqrt{4}$.

Methods I and II are precisely the same as in the preceding example. In Method III, 2 on scale *C* is set opposite 4 on scale LL 3, but the answer (2) is read opposite the right hand 1 of scale *C* on scale LL 2.

We shall now proceed to apply the same methods to numbers of two or more figures.

MULTIPLICATION OF TWO OR MORE FIGURES

Example: Find the value of 2×1.5.

Opposite 2 on *D* (front face) set 1 on *C*. Move the indicator to 1.5 on *C*. This will be between 1 and 2 at the division numbered 5; since the numbered divisions between 1 and 2 on *C* and *D* are the tenths. Under the indicator, find 3 on *D*. (Fig. 5)

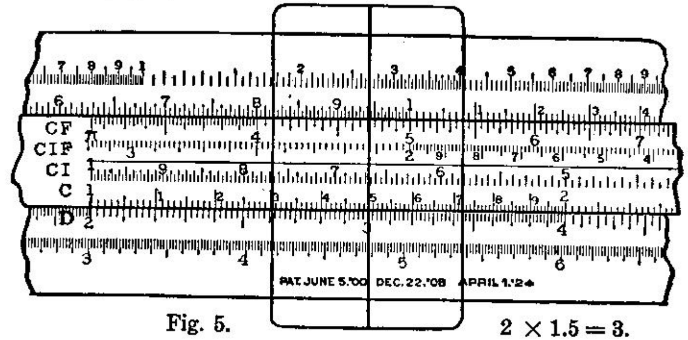


Fig. 5. $2 \times 1.5 = 3$.

Example: 2×1.8. Using Fig. 5, see if you can make it 3.6.

Example: 1.5×2.5. Opposite 1.5 on *D* set 1 on *C*. Move the indicator to 2.5 on *C*. Below 2.5, find 3.75, the answer, on *D*. Note that this answer is halfway between 3.7 and 3.8, which makes it 3.75. (Fig. 6).

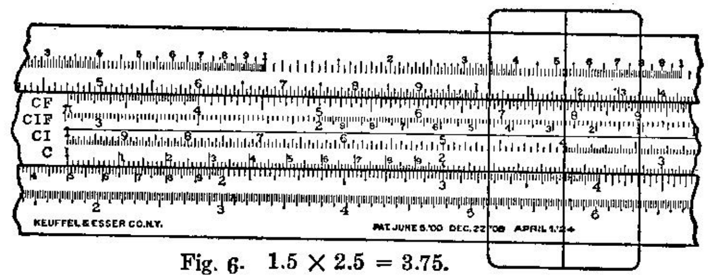


Fig. 6. $1.5 \times 2.5 = 3.75$.

HOW TO READ THE SCALES

Graduations on the slide rule are not measures of length, but represent figures.

On the 10″ slide rules, scales *C* and *D* consist of nine prime spaces of unequal length; the first line of each space is numbered, respectively, 1 (called left index), 2, 3, 4, 5, 6, 7, 8, 9, ; the last line is numbered 1, and is called the right index. The spaces 1-2, 2-3, 3-4, etc., decrease in length, the space from 1 to 2 being the longest; and every succeeding space being shorter than the one preceding it.

Each of these prime spaces is divided into ten (secondary) spaces, also decreasing in length, the nine lines between prime 1 and prime 2 being numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, in smaller figures than those of the prime graduations. Space does not permit the numbering of the other secondary lines.

Each of the spaces between these secondary lines is again subdivided. Thus, each secondary space between prime 1 and prime 2 is divided into ten (unequal) parts. The secondary spaces between prime 2 and prime 4 are subdivided into five (unequal) spaces.

The secondary spaces from 4 to the end are subdivided into two (unequal) parts by one line between the two secondary lines.

To find a number, always read the first figure to the left on the prime line, the second figure of the number on the secondary line to the right thereof, and the third figure on the subdivision; thus, to read 435 (say four, three, five, not four hundred and thirty-five) find prime 4, secondary 3 and sub. 5.

PLACING THE DECIMAL POINT

Example: 2×15.

This is worked on the rule exactly like the above examples, but you can see by looking at the problem that the answer is 30 and not 3.

Problems

1. 20×15.
2. 200×15.
3. 20×150.
4. $2 \times .15$.
5. $2 \times .015$.
6. $.2 \times 15$.
7. $.02 \times .015$.

All of these problems are worked like the above. As far as the slide rule is concerned, we multiply 2 by 1.5 and get 3. Then we place the decimal point by inspection. From arithmetic we remember that in multiplying decimals we first multiply as though there were no decimal points, then point off as many decimal places in the answer as there are total decimal places in the two numbers which were multiplied together. Thus, in Problem 7, there are two decimal places in .02 and three in .015. So in the answer, 30, we must have 2 + 3, or 5 places, making the result .00030. Of course the 0 at the right does not count and the final result is .0003.

From the above explanation it is evident that the decimal point is not considered in operating the slide rule. After the work of the rule has been done, the decimal point can usually be placed by inspection; i. e. through a mental survey of the influence of the involved factors upon the result. Where this is not feasible, a rough arithmetical calculation will serve to properly locate the decimal point.

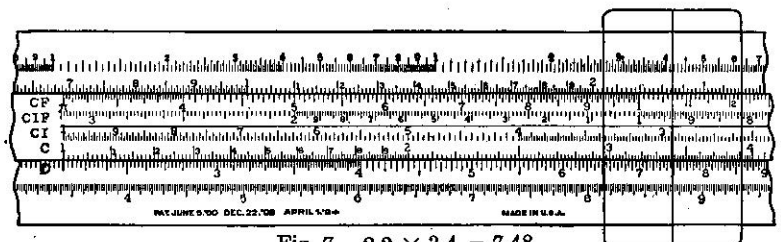


Fig. 7. $2.2 \times 3.4 = 7.48$.

Example: 2.2×3.4.

Opposite 2.2 on *D* set 1 on *C*. Move the indicator to 3.4 on *C*. Under the hair line on *D* find 748.

That the unit figure is 8 is further confirmed by observing that the product of the unit figures 2 and 4 in the example is 8.

Since 2.2×3.4 is roughly 2×3, or 6, place the decimal point in 748 so that the result will be as near 6 as possible. Evidently the answer is 7.48. (Fig. 7).

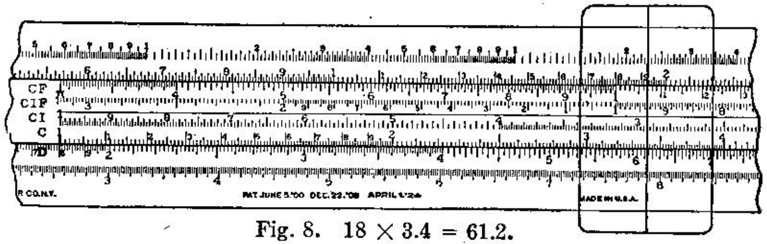


Fig. 8. $18 \times 3.4 = 61.2$.

Example: 18×3.4.

Using the same method as in the previous example, the slide rule gives 612.

By a rough calculation the problem is about equal to $20 \times 3 = 60$. Hence we make 612 look like 60 by placing the decimal point after the 1. The answer is 61.2.

Example: 16×2.4. Answer 38.4.
Example: 1.4×2.6. Answer 3.64.

Problem 8. Fill in the blanks in the following multiplication table, using the slide rule:

	21	22	23	24	25	26	27	28	29
31									
32									
33									
34									

Set left index of *C* to 31 on *D*. Note that the factors 21 to 29 can be taken without resetting the slide.

WHICH INDEX TO USE

If we attempt to multiply 30 by 45, using the preceding methods of setting the 1 on the left hand end of *C* to 30 on *D*, we shall find it impossible to move the indicator to 45, since 45 on scale *C* lies beyond the right hand end of scale *D*. In such a case, begin the work on the rule by setting the 1 on the right hand end of *C* to 30 on scale *D*. It is then possible to set the indicator to 45 on *C*. Opposite the 45 on *C* find 135 on *D*. Placing the decimal point by inspection, the result is 1350. Or, we may read the answer 1350 on *DF* opposite 45 on *CF*, the folded scales doing away with the necessity of shifting the slide.

We will now define the left hand 1 on scale *C* as the left index and the right hand 1 on scale *C* as the right index. In most examples, the following rule will be found useful in determining which index to use:

If the product of the first figures of the given numbers is less than 10, use the left index; if this product is greater than 10, use the right index.

Example 1. 2.13×3.33, $3 \times 2 = 6$. Use the left index.
Example 2. 7.23×4.71, $7 \times 4 = 28$. Use the right index.
Example 3. $.131 \times 4.6$, $1 \times 4 = 4$. Use the left index.

An exception to this rule will be found in such a case as 3.12×3.31. According to the rule the left index should be used. It will be found, however, that it is necessary to use the right index. This is due to the fact that while the product of the first figures of the two numbers is less than 10, the product of the complete numbers is greater than 10.

In most cases, the use of the above rule will save time.

PER CENT

Example: Suppose you are earning 56 cents per hour and you are given an increase of 8 cents. What per cent increase do you receive ?

Of course you will divide 8 by 56.

To divide one number by another on the slide rule we simply reverse the order of the work we have been doing in multiplication.

Set the indicator to 8 on scale *D*.

Move the slide so as to set 56 on *C* to the hair line of the indicator.

Fig. 9. $8 \div 56 = .14$.

Under 1 on *C* we find 14 and a little over. But the result is nearer 14 than 15. Hence the correct result to two figures is 14. By inspection the decimal point must be placed before the number, making the answer .14 or 14 per cent.

Example: A man earned 35 cents per hour. He learned a new trade which increased his earning power to 67 cents per hour. What per cent increase did he receive?

His increase is 32 cents per hour. The per cent of increase is found by dividing 32 by 35.

Set the indicator to 32 on *D*.

Set 35 on *C* to the indicator. The result cannot be found under the left index *i.e.* the 1 at the extreme left of scale *C*, since this projects beyond scale *D*. So we use the right index of *C*. Under this index, find 91 on scale *D*. (Fig. 10).

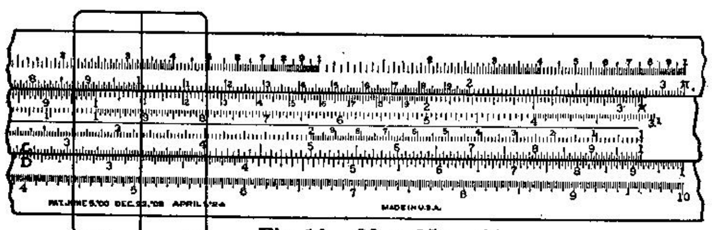


Fig. 10. $32 \div 35 = .91$.

In the same way, for practice, try the following, obtaining the result correct to two figures:

Problem 9. What per cent of 91 is 45?
(Divide 45 by 91)
10. What per cent of 73 is 24?
11. What per cent of 67 is 61?
12. What per cent of 53 is 31?
13. What per cent of 82 is 13?
14. What per cent of 42 is 9?

If you have a long report to make out in which a large number of per cents are to be calculated, why not use the slide rule?

A secretary to the president of a big corporation recently said: "The slide rule does my work in one-third of the time that would be required otherwise."

READING TO THREE FIGURES

Suppose you had to get per cents in a problem like the following:

Example: A baseball player made 57 hits out of 286 times at bat. What is his percentage?

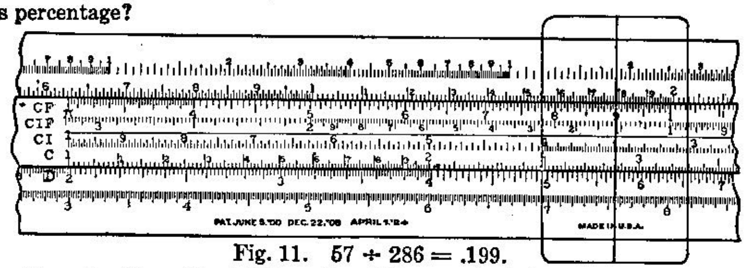


Fig. 11. $57 \div 286 = .199$.

Opposite 57 on *D* set 286 on *C*. When we look for 286 we observe that between 2.8 and 2.9 there are five spaces on the rule. Hence every space counts one-fifth of .1, which is .02. Since we want six points for the third figure, we have to use three spaces, every one worth .02. $3 \times .02 = .06$.

Under the left index of *C* look for the result on *D*. When we read this result, we see that it comes on the rule between 1.9 and 2.0. There are ten small spaces between 1.9 and 2.0. Hence every space counts one point. The index is close to the ninth of these divisions. Hence the reading is 199. Now we must place the decimal point. A rough calculation shows that $\frac{57}{286}$ is nearly $\frac{60}{300}$, or $\frac{1}{5}$. Hence the decimal point must be placed so as to make the result somewhere near one-fifth or .2. Evidently the result is .199. This may be read $19^{9}/_{10}$ per cent or $19^{9}/_{10}$ hundredths, or 199 thousandths.

Example: If your income is \$2,500 per year and you save \$451, what per cent do you save?

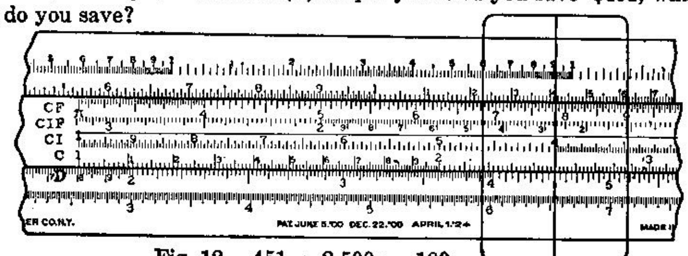


Fig. 12. $451 \div 2{,}500 = .180$.

Opposite 451 on *D* set 25 on *C*. Under the index find 180 on *D*. Hence the answer is .180, or 18 per cent. We note that when we look for the 1 in 451 on the rule, we find only two spaces between 45 and 46. Hence each space counts one-half of a hundredth or one-half of .01, which is .005 or five points for the third figure. We estimate one-fifth of the small space to obtain .001. (Fig. 12)

Example: If your salary is \$57.50 per week, and you are given an increase of \$12.40, what per cent increase do you receive?

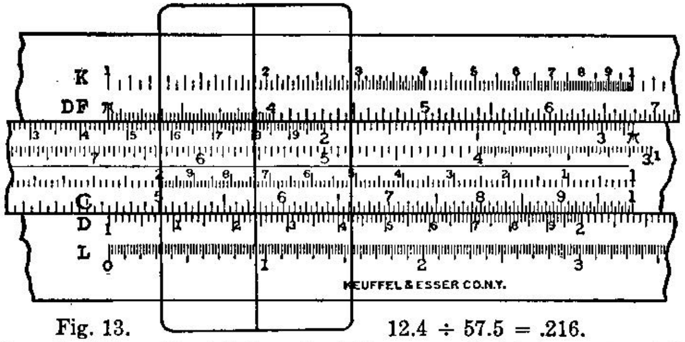


Fig. 13. $12.4 \div 57.5 = .216.$

Opposite 124 on *D* set 575 on *C*. This means that between 5 and 6 on *C* we must take 7 of the large divisions and one of the small divisions. Under the right-hand index read 216 on *D*. Hence the answer is $21\frac{6}{10}$ per cent.

Problem 15. $5.42 \div 2.42.$
16. $7.35 \div 3.14.$
17. $6.13 \div 4.61.$
18. $9.56 \div 7.26.$
19. $10 \div 3.14.$ For 10, use either the right or left index.

In the following problems the location of the decimal point is determined by working the problems in round numbers.

Problem 20.	$16.5 \div .245$	is approximately	$16 \div .2 = 80.$
21.	$.00655 \div .00034$	" "	$.0060 \div .0003 = 20.$
22.	$.00156 \div 32.8$	" "	$.0015 \div 30 = .00005.$
23.	$.375 \div .065$	" "	$.36 \div .06 = 6.$
24.	$.0385 \div .0014$	" "	$.038 \div .001 = 38.$

There is another method of placing the decimal point in division. Work the problem as though both dividend and divisor were integers (*i. e.*, not decimals), pointing off as usual. Move the decimal point to the left as many places as there are decimal places in the dividend. Then move it to the right as many places as there are decimal places in the divisor. For example in problem 20, $165 \div 245$ gives .673. Move the point one place to the left because there is one decimal place in the dividend, giving .0673. Then move it three places to the right because there are three places in the divisor, giving as a result 67.3. Try both methods and see which one you like the better. Let one check the other.

MORE THAN THREE FIGURES IN A FACTOR

Suppose we have more than three figures, as in the following example:

Problem 25. Multiply 28 by 3.1416.

The 10″ slide rule only reads to three figures. So cut off the fourth and fifth figures in 3.1416 and call it 3.14, since the number is nearer 3.14 than 3.15. It is, however, somewhat more convenient to work this problem on the *CF* and *DF* scales, where 3.1416 (π) is accurately marked. Use *DF* in place of *D*, and *CF* in place of *C*.

Problem 26. Multiply 26 by 8.149.
Call 8.149 equal to 8.15.

COMBINED MULTIPLICATION AND DIVISION

Example: If bell metal is made 25 parts of copper to 11 parts of tin, find the weight of tin in a bell weighing 402 pounds.

The tin is evidently eleven thirty-sixths of 402, or $\frac{11 \times 402}{36}$.

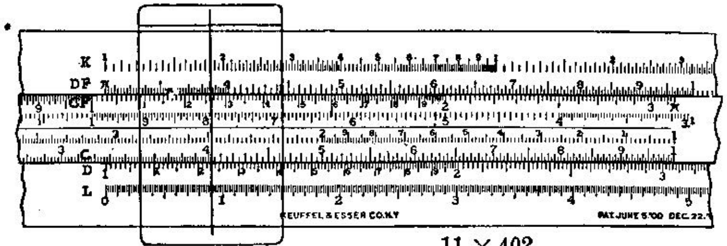


Fig. 14. $\frac{11 \times 402}{36} = 123.$

Opposite 11 on *D* set 36 on *C*. (Fig. 14)

Move the indicator to 402 on *C*.

Opposite 402 on *C* read 123 on *D*.

To place the decimal point, make a rough calculation as follows: The example is roughly equal to $\frac{10 \times 400}{40} = 100.$ So make 123 look as nearly like 100 as possible by placing the point after 3. The answer is 123 pounds of tin.

Problem 27. $\frac{14 \times 525}{47}$

Problem 28. $\frac{24.5 \times 43.4}{3620}$

Example: $\frac{1.35 \times 3.16}{6.2}$ (See **Fig. 15**)

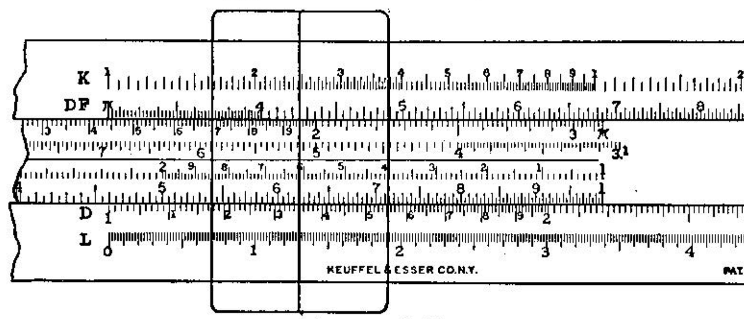


Fig. 15. $\frac{1.35}{6.2}$

Opposite 1.35 on *D*, set 6.2 on *C*. If we try to move the indicator to 316 on *C*, it is impossible because 316 lies beyond the extremity of *D*. In such a case proceed as follows: Move the indicator to the right-hand index of *C*. (See Fig. 15)

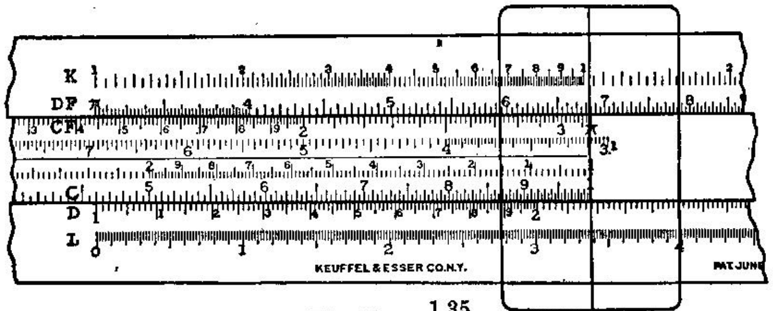


Fig. 15*a*. $\dfrac{1.35}{6.2}$

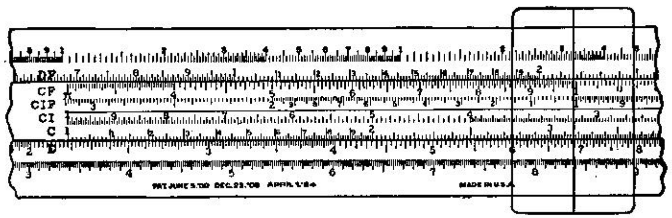


Fig. 16. $\dfrac{1.35 \times 3.16}{6.2}$

Then move the slide, setting the left-hand index of *C* to the indicator.

Now we can move the indicator to 316 on *C* and under 316 on *C* read the answer 688 on *D*. (Fig. 16.)

A rough calculation for the decimal point gives us $\dfrac{1 \times 3}{6} = \dfrac{3}{6}$, or .5. Making 688 look as much as possible like .5, we have .688.

Another Method:

Opposite 135 on *DF*, set 62 on *CF*.

Move the indicator to 316 on *C*.

Opposite 316 on *C* read 688 on *D*.

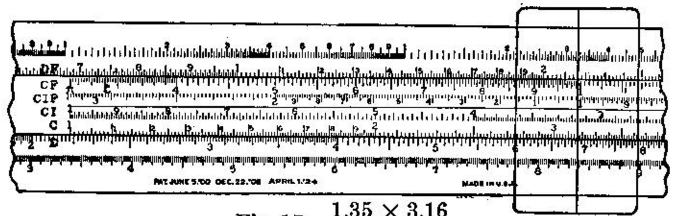


Fig. 17. $\dfrac{1.35 \times 3.16}{6.2}$

Example: $\dfrac{2.28 \times .0125}{4.36}$

To 2.28 on *D*, set 4.36 on *C*.

Move the indicator to 1.25 on *CF*.

On *DF*, opposite the indicator, read 654.

NOTE.—In problems like the two preceding examples the method given with the first may always be avoided provided the setting on the folded scales is so chosen as to keep at least half of the slide within the groove. For this purpose there is always a choice between starting on *DF*, as in the first example or or on *D*, as in the second example.

The rough calculation for the decimal point might be $\dfrac{2 \times .012}{4} = .006$. The answer is .00654.

Problem 29. $\dfrac{7.63 \times 2.34}{24.3}$

Work problems 30, 31, 32, using scales *CF* and *DF*.

Problem 30. $\dfrac{2.56 \times 1.78}{7.4}$

Problem 31. $\dfrac{82.5 \times 9.3}{56.5}$

Problem 32. $\dfrac{32.6 \times 22.1}{9.25}$

PROPORTION

Example: If an aeroplane flying 100 miles an hour travels 86 miles in a given time, how far will an automobile traveling 22 miles an hour go in the same time?

$$100 : 22 = 86 : x$$

which means that 100 is to 22 as 86 is to the answer.

The work on the rule is as follows:

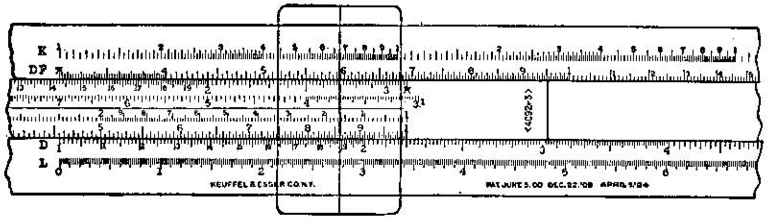


Fig. 18. 100 : 22 = 86 : 18.9.

Opposite 22 on *D*, set 100 on *C*. (Use right index for 100). Opposite 86 on *C* read the answer, 18.9 on *D*. An easy method of remembering this is:

C		*D*		*C*		*D*
100	:	22	=	86	:	18.9

In placing the decimal point, note that 100 has the same relation to 22 that 86 has to the answer. Since 22 is about one-fifth of 100, we must place the decimal point in 189 so that the answer shall be about one-fifth of 86. Hence, the answer is 18.9.

This problem may also be solved through the use of the *CF* and *DF* scales.

In the same way solve the following proportions.

Problem 33. 24 : 31 = 15.2 : x.
34. 1.4 : 2.5 = 12 : x.
35. 3.71 : 2.4 = 51.2 : x.

Problems 33 and 34 may be solved through the use of the *C* and *D* scales alone, or through the use of the *CF* and *DF* scales alone. Problem 35 may be solved through the use of the *C* and *D* scales alone; whereas, if the *CF* and *DF* scales are used, 51.2 must be divided by 3.71, and the answer read on *D* opposite 2.4 on *C*.

Example: 2.54 : 4.72 = 7.48 : x

To 472 on *D*, set 254 on *C*.

Above 748 on *CF*, find 139 on *DF*.

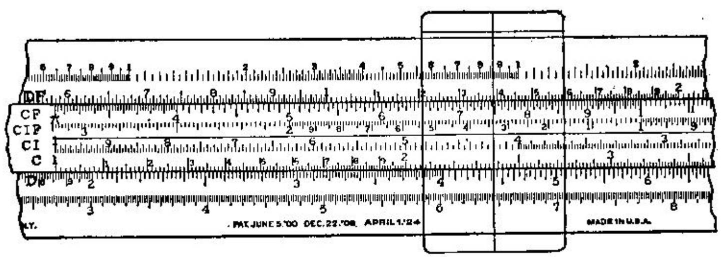


Fig. 19. 2.54 : 472 = 7.48 : 13.9.

The result is 13.9

Problem 36. If a post 13.2 feet high casts a shadow 27.2 feet long, how high is a tower which casts a shadow 116.8 feet long?

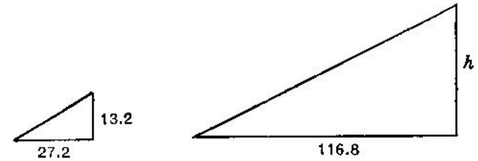


Fig. 20. 27.2 : 13.2 = 116.8 : *h.*

Problem 37. At 2,400 yards an increase of 1 mil in the elevation of a gun increases the range 25.0 yards. What change in elevation will increase the range 40 yards?

The mil is the unit of angle in the artillery. It is equal to $\frac{1}{6400}$ of 360°.

Example: The effects of wind on a shell are approximately proportional to the velocity of the wind. At 3,000 yards for a 3-inch gun, a rear wind of 10 miles per hour increases the range 30.1 yards. (a) What wind will increase the range 42.8 yards? (b) What wind will decrease the range 68.5 yards?

Answer (a) Rear wind of 14.2 miles per hour. (b) Head wind of 22.8 miles per hour.

SQUARES

Example: Find the area of a square plot of ground measuring 128 yards on a side.

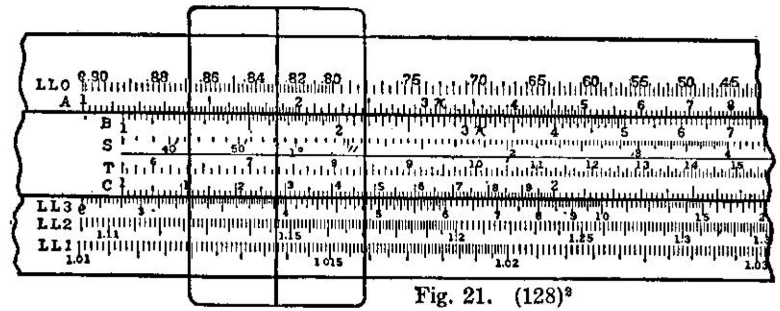


Fig. 21. $(128)^2$

Using the face of the rule with scale *B*, set the indicator to 128 on *C*, Directly above on *B* find the square required, 164. To place the decimal point make a rough calculation.

$(128)^2$ is roughly $(130)^2$ or 16900. Then make 164 look like 16900 by placing the point as follows: 16400. The result is only correct to three figures. The complete result is 16384.

The answer may be also be obtained by setting the indicator to 128 on *D*, turning the rule over, and reading the answer on *A*.

128 may also be squared by setting the left index of *C* to 128 on LL 3. Opposite 2 on *C* read the answer on LL 3. This is not as accurate as the other methods, since LL 3 cannot be read as closely as *A*.

It is well to know all three methods, since different problems may make any particular method of the three the most convenient to use.

If greater accuracy is desired, a number may be squared by the use of the longer scales *C* and *D*.

Example: Find the square of 128.

Regard this as an example in multiplication equivalent to:

Find 128 × 128.

To 128 on D set left index.

Opposite 128 on C read 1638 on D.

Placing the decimal point by a rough calculation, the result is 16380.

Example: Square 652.

Set the indicator to 652 on C reading the square 425 on B. Notice that here the arithmetic square would be 425104, but on the slide rule we can get only the first three figures, 425. This, however, is close enough for most practical purposes, such as estimating on contract work.

To place the decimal point,

$$652^2 > 600^2 = 360000.$$
$$< 700^2 = 490000.$$

since the value is between these limits the result is 425000.

Find the squares of the following numbers:

Problem 38.	3.2	**Problem 42.**	276.	**Problem 46.**	.0057
39.	4.65	**43.**	34.2	**47.**	.0244
40.	1.12	**44.**	.66	**48.**	2240.
41.	8.65	**45.**	.0625		

Example: Find the area of a circular plot of ground measuring 14.5 feet in diameter.

Use the formula $A = .7854\ d^2$, which means that the area of the circle is equal to .7854 multiplied by the square of the diameter. Set the indicator to 145 on D. The square is found directly above on A, but need not be read. Set the right-hand index of the slide to the indicator. Move the indicator to the constant, .7854 on B, and opposite find the result, 165 sq. ft. on A.

This constant, .7854, is so frequently used that it has been marked by a special line on the right-hand half of the A and B scales.

SQUARE ROOTS

Example: How long must one side of a square garden bed be made in order that it shall contain 8 square yards?

Here we have to find the square root of 8.

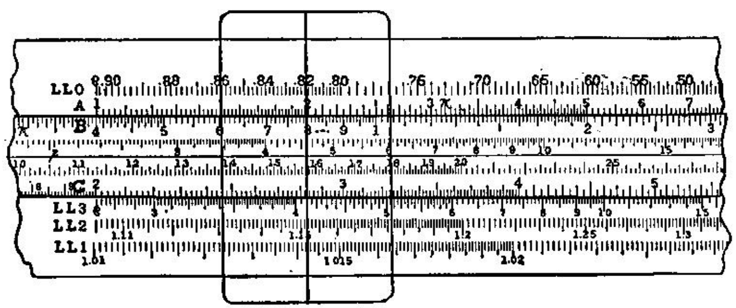


Fig. 22. $\sqrt{8} = 2.83$.

Method I. **Set** the indicator to 8 on scale B. Assume that scale B runs **from 1 to 100, so that 8 is found on the left-hand half of the rule.**

Now under the hair line on scale C, find 2.83, the square root.

Then the result is 2.83 yards.

Method II. Set the indicator to 8 scale A

Under hair line on scale D (other side of rule) find 2.83.

Method III. Set 2 on scale C to 8 on scale LL 3.

Opposite left index of C find 2.83 on scale LL 3.

Example: Find $\sqrt{3.}$

Set the indicator to 3 on B.

Under the hair line find 1.73 on C.

Example: Find $\sqrt{30.}$

Set the indicator to 30 on B, being careful to notice that 30 is indicated by 3 on the right-hand half of the rule. Opposite the indicator on C, find 5.48.

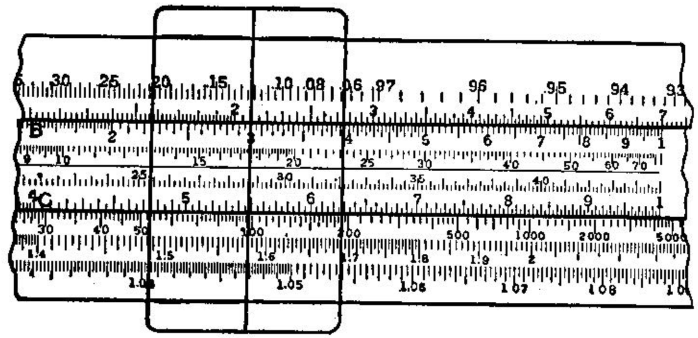


Fig. 23. $\sqrt{30} = 5.48$.

Example: Find $\sqrt{300.}$

Move the decimal point an even number of places in order to obtain a number that is between 1 and 100. This can be done by moving the point two places to the left, giving $\sqrt{3.00.}$

Find the $\sqrt{3,}$ which is 1.73. Then move the decimal point half as many places as it was moved in the first place, but in the opposite direction. In this case, move the point in 1.73 one place to the right, giving 17.3.

If 300 on scale LL 3 is divided by 2 on scale C, the answer is directly determined opposite the left index of C. This eliminates the necessity of determining which half of the A scale should be used.

Example: Find $\sqrt{.30.}$

Move the point two places to the right, obtaining 30.

Find $\sqrt{30} = 5.48$.

Move the point one place to the left, obtaining .548 for the result.

Or set 2 on scale B directly to 0.30 on scale LLO. Opposite left index of B find .548 on LLO.

Example: Find $\sqrt{.03.}$

Move the decimal point two places to the right, obtaining $\sqrt{3.}$

Find $\sqrt{3} = 1.73$.

Move the point one place to the left, obtaining .173.

None of the log log scales can be used directly where values less than 0.05 are involved (see page 49).

Example: Find $\sqrt{.003}$.

Move the point four places to the right, obtaining $\sqrt{30}$.

Find $\sqrt{30} = 5.48$.

Move the point two places to the left, obtaining .0548.

Find the square roots of the following numbers:

Problem 49. 1.42	**Problem 52.** .142	**Problem 55.** .365
50. 14.2	**53.** 2.43	**56.** .31416
51. 142	**54.** 85.4	**57.** 1450

Problem 58. Make a list of square roots of whole numbers between 110 and 130.

Problem 59. On a baseball field, find the distance from home plate to second base, measured in a straight line. (The distance between the bases is 90 feet).

Problem 60. Water is conducted into a tank through two lead pipes having diameters of ⅝ and 1¾ inches, respectively. Find the size of the lead waste pipe that will allow the water to run out as fast as it runs in.

Use ⅝ and 1¾ in the decimal form.

$$\text{Find} \quad \sqrt{(.625)^2 + (1.75)^2}.$$

NOTE:—Perform the addition by arithmetic. The slide rule cannot be used to advantage in addition.

The easiest way to solve this problem is to reduce to this form:

$$.625\sqrt{1+\left(\frac{1.75}{.625}\right)^2}$$

See Rectangular co-ordinates. (page 102).

This reduces the number of operations from 4 to 2, since 1 can be added mentally.

Problem 61. Two branch iron sewer pipes, each 6 inches in diameter, empty into a third pipe. What should be the diameter of the third pipe in order to carry off the sewage?

TEST PROBLEMS

Read carefully the following instructions:

a. Copy the test on your paper in the form given below.

b. Work the problems straight through, setting down the answers in the column at the extreme right.

c. Fold these answers underneath the paper.

d. Work the problems through again, setting down the answers in the other column.

e. Compare the two sets of answers.

f. If the answers to any problem do not agree (within one point in the third place), work the problem again.

g. The correct results are given on page 133.

TEST

		Answers Second Time	Answers First Time	Credits
Problem 62.	1.28×2.46			20
" **63.**	$84 \div 59.5$			20
" **64.**	$\frac{58.5 \times 15.2}{78}$			20
" **65.**	$6.25 : 24.2 = 9.5 : x.$			20
" **66.**	$\sqrt{182}$			20

CHAPTER II

THEORY OF THE SLIDE RULE

HISTORICAL NOTE

In 1614 John Napier, of Merchiston, Scotland, first published his "Canon of Logarithms."

Napier concisely sets forth his purpose in presenting to the world his system of Logarithms as follows:

"Seeing there is nothing (right well beloved Students of Mathematics) that is so troublesome to mathematical practice, nor doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances."

Napier builded better than he knew. His invention of logarithms made possible the modern slide rule, the fruition of his early conception of the importance of abbreviating mathematical calculations.

In 1620 Gunter invented the straight logarithmic scale, and effected calculation with it by the aid of compasses.

In 1630 Wm. Oughtred arranged two Gunter logarithmic scales adapted to slide along each other and kept together by hand. He thus invented the first instrument that could be called a slide rule.

In 1675 Newton solved the cubic equation by means of three parallel logarithmic scales, and made the first suggestion toward the use of an indicator.

In 1722 Warner used square and cube scales.

In 1755 Everard inverted the logarithmic scale and adapted the slide rule to gauging.

In 1815 Roget invented the log-log scale.

In 1859 Lieutenant Amèdèe Mannheim, of the French Artillery, invented the present form of the rule that bears his name.

In 1881 Edwin Thacher invented the cylindrical form which bears his name.

In 1891 Wm. Cox patented the Duplex Slide Rule. The sole rights to this type of rule were then acquired by Keuffel & Esser Co.

For a complete history of the Logarithmic Slide Rule, the student is referred to "A History of the Logarithmic Slide Rule," by Florian Cajori, published by the Engineering News Publishing Company, New York City. This book traces the growth of the various forms of the rule from the time of its invention to 1909.

ACCURACY

The accuracy of a result depends upon (*a*), accuracy of the observed data; (*b*), accuracy of mathematical constants; (*c*), accuracy of physical constants; (*d*), precision of the computation.

ACCURACY OF THE OBSERVED DATA

The precision of a measurement is evidently limited by the nature of the instrument, and the care taken by the observer.

Example 1. If a distance is measured by a scale whose smallest subdivision is a millimeter, and the result recorded 134.8 mm., evidently the result is correct to 134, but the .8 is estimated. Hence it is known that the actual measurement lies between 134 and 135 and is estimated to be 134.8.

The result 134.8 is said to be "correct to four significant figures."

If the result were desired correct to only three figures, it would be recorded 135, since 134.8 is nearer 135.0 than 134.0. This result is said to be "correct to three significant figures."

Example 2. If the distance is measured by a rule whose smallest subdivision is .1 inch, and found to be exactly 8. inches, the result would be recorded 8.00 inches. The zeros record the fact that there are no tenths and no hundredths, but the distance is exactly 8 inches. The result, 8.00 inches, is said to be "correct to three significant figures."

Example 3. If an object is weighed on a balance capable of weighing to .01 gram, then .001 gram can be estimated. Suppose several objects are weighed, with the following results:

1.	Seven grams	recorded	7.000 grams.
2.	Seven and a half grams	"	7.500 "
3.	Seven and 9/100 grams	"	7.090 "
4.	Seven and 6/1000 grams	"	7.006 "
5.	4/100 and 2/1000 grams	"	.042 "

Note that readings with the same instrument should show the same number of places filled in to the right of the decimal point, even if zero occurs in one or all of these places.

In number 5, the result, .042 grams is said to be "correct to two significant figures." The first significant figure is 4 and the second is 2.

Example 4. When we say that light travels 186,000 miles per second, we mean that the velocity of light is nearer 186,000 miles than 185,000 miles, or 187,000 miles. The result is said to be "correct to three significant figures."

Summarizing the preceding examples:

Example 1. 134.8 is correct to four significant figures.
Example 2. 8.00 is correct to three significant figures.
Example 3. .042 is correct to two significant figures.
Example 4. 186,000. is correct to three significant figures.

Counting from the left, the first significant figure is the first figure that is not zero.

After the first significant figure, zero may count as a significant figure, as in Example 2, where it represents an observed value; or it may not so count, as in Example 4, where the zeros merely serve to place the decimal point correctly, the number 186,000. being correct only to the nearest thousand miles.

Similarly in results derived from calculation, zero counts as a significant figure if it represents a definite value, *e. g.* $25 \times 36 = 900$.

Both zeros in 900 are significant figures. On the other hand, zero is not a significant figure if it does not represent a definite value, but merely serves to place the decimal point.

Find the cube of 234.

The complete result is 12,812,904.

On the slide rule only the first three significant figures can be found, and the result is 12,800,000. Here 128 are significant figures and the five zeros following are not significant, since they do not represent definite values, but merely serve to place the decimal point.

As far as calculation on the slide rule can determine, each of these five zeros might be any one of the numbers from 0 to 9. Arithmetical calculation shows that they are really, 12,904.

ACCURACY OF MATHEMATICAL CONSTANTS

A mathematical constant may be carried to any desired degree of accuracy, *e. g.*, the value of π usually given as 3.14159 has been calculated to 707 decimal places. For ordinary calculations 3.14 or $3\frac{1}{7}$ is sufficiently accurate.

ACCURACY OF PHYSICAL CONSTANTS

Most physical constants are only correct to three significant figures and some only to two figures.

e. g., The weight of a cu. ft. of water is 62.5 lb.
The weight of a cu. in. of cast iron is .26 lb.

LIMITS OF ACCURACY

Holman's rule states that if numbers are to be multiplied or divided, a given percentage error in one of them will produce the same percentage error in the result.

In other words, a chain is no stronger than its weakest link.

Since physical constants are not usually correct beyond three significant figures, and the observed data in an experiment are rarely reliable beyond this point, the slide rule reading to three figures gives results sufficiently accurate for most kinds of practical work.

PERCENTAGE OF ERROR

If a result is correct to three significant figures, the ratio of the error to the result is less than 1:100.

Suppose, for example, the result is 3527.6, which is known to be correct to three significant figures. Then the figures 352 are known to be correct and the figures 7.6 are doubtful.

Since 7.6 is less than 10 and 3527.6 is greater than 1000, the error must be less than 10:1000 or 1:100.

$$\frac{7.6}{3527.6} < \frac{10}{3527.6} < \frac{10}{1000}, \text{or } \frac{1}{100}.$$

A result read on the 10-inch slide rule to four significant figures is 1324, which is correct to three figures, 132, while the fourth figure, 4, is a close estimate not more than one point away from the correct reading.

The error here is less than $\frac{1}{1324}$, which is less than $\frac{1}{1000}$. Hence the error in this reading is less than one-tenth of one per cent.

It is evident that the per cent of error holds throughout the length of the slide rule, since the first significant figure increases from 1 to 10 as spaces decrease.

e. g., On the right end of the rule, a result read 998 might be really 999 making an error of 1 in 999 or approximately $\frac{1}{1000}$ or $\frac{1}{10}$ of 1%.

If greater accuracy is desired, a twenty-inch rule will give results correct to within one part in two thousand; while the Thacher Cylindrical Rule will give results correct to within one part in ten thousand.

LOGARITHMS

$$10^2 = 100.$$

Another form of making this statement is:
The logarithm of 100 is 2.
In the same way, $10^3 = 1{,}000$
or the logarithm of 1,000 is 3.

From these examples it is evident that the logarithm is the exponent which is given to 10.

Fill out the blanks in the following table:

$10^4 = 10{,}000$	Log 10,000 =
$10^5 = 100{,}000$	Log 100,000 =
$10^1 = 10$	Log 10 =
$10^0 = 1$	Log 1 =

LAW OF MULTIPLICATION

$$10^2 = 100.$$
$$10^3 = 1{,}000.$$
$$10^2 \times 10^3 = 100 \times 1{,}000.$$
$$10^5 = 100{,}000.$$

Log 100,000 is 5.

Since 5 is the sum of 2 and 3, log 100,000 = 2 + 3 = log 100 + log 1,000, or

The logarithm of a product is the sum of the logarithms of the multiplicand and the multiplier.

Hence to multiply one number by another, add their logarithms.

The construction of the rule allows this addition to be done easily.

The scales are divided proportionally to the logarithms of the numbers.

If the scale is considered as divided into 1,000 units, then any number—1, 2, 3, etc.,—is placed on the rule so that its distance from the left index is proportional to its logarithm.

Since log 1 = 0,	1 is found at the extreme left.	
" log 2 = .301,	2 is found	301 units from the left.
" log 3 = .477,	3 " "	477 " " "
" log 9 = .954,	9 " "	954 " " "
" log 10 = 1.000,	10 " "	1000 " " "

On the scale the number 8 is placed three times as far from the left index as 2, because the logarithm of 8 is three times the logarithm of 2.

MULTIPLICATION

When we multiply 2 by 4, we set the left index of the slide to 2 on scale *D* and under 4 on scale *C* find the product, 8 on scale *D*.

This is equivalent to adding log 2 to log 4 and finding log 8 (Fig. 24).

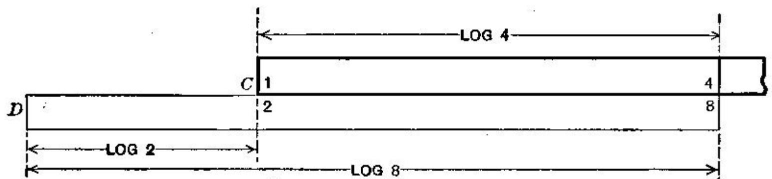


Fig. 24

Example: Multiply 2.45 by 3.52.

Opposite 2.45 on *D*, set 1 on *C* and under 3.52 on *C* find 862 on *D*.

Roughly calculating, 2.45 × 3.52 = 2 × 4 = 8.

Hence, we place the decimal point to make the result as near 8 as possible; and the result is 8.62.

Example: Multiply 24.5 by 35.2.

Working this like the preceding example, without regard to the decimal point, we obtain 862.

Roughly calculating, 24.5 × 35.2 = 25 × 36 = 900.

Placing the decimal point to make 862 as near 900 as possible, we obtain 862.

Example: Multiply 6.234 by 143.

Taking 6.234 correct to three significant figures we multiply 6.23 by 143.

Opposite 623 on *D* set 1 on *C*.

Under 143 on *C* find 891 on *D*.

Roughly calculating, 6 × 140 = 840.

Therefore the result is 891.

Example: Multiply 2.46 by 7.82.

Method I.

When the product of the given numbers is greater than 10, the sum of their logarithms will exceed the length of the rule. Hence if we set the left index of the slide to 246 on *D*, the other number 782 on *C* projects beyond the rule. In this case, think of the projection as wrapped around and inserted in the groove at the left, which would be the case in a circular slide rule. Now the right and left-hand indexes coincide.

Hence set the *right* index of the slide to 246 on *D*.

Under 782 on *C* find 192 on *D*.

Roughly calculating, 2 × 8 = 16.

Hence the result is 19.2.

Method II. By the use of the folded scales, *CF* and *DF*, the product of any two numbers may be found without any uncertainty regarding the index, provided that at least half of the slide remains in the groove.

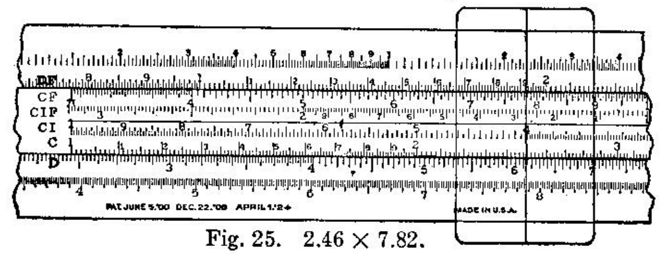


Fig. 25. 2.46 × 7.82.

Set the left index to 246 on *D*.

Since 782 on *C* is beyond the right-hand end of the rule, set the indicator to 782 on *CF* and read the product opposite the indicator on *DF*.

The result is 19.2.

Example: Multiply .146 by .0465.

Opposite 146 on *D* set 1 on *C*.

Under 465 on *C*, find 679 on *D*.

Roughly calculating, .1 × .05 = .005.

Hence the result is .00679.

Find the value of

Problem 67.	2.34 × 3.16.	**70.**	8.54 × 6.85.	**73.**	.023 × 2.35.
68.	3.76 × 5.14.	**71.**	34.2 × 7.55.	**74.**	.00515 × .324.
69.	1.82 × 4.15.	**72.**	4.371 × 62.47.	**75.**	.00523 × .0174.

Problem 76. Find the circumferences of circles having diameters of 4 ft., 6.5 ft., 14 ft.

Opposite π on *A*, set 1 on *B*.

Above 4, 6.5, and 14 read the circumferences on *A*.

Or opposite 4, 6.5, and 14 on *C*, read the circumferences on *CF*; in which case the results can be read to an additional significant figure, as 12.56.

DIVISION

In division, reversing the operation of multiplication,

$$8 \div 4 = 2.$$ (See Fig. 24)

We subtract log 4 from log 8 and obtain log 2.

The Use of Scales CF, DF and CIF

The folded scales, *CF*, *DF and CIF* were devised to eliminate the number of re-settings of the slide which result where only one straight scale with consecutive graduations is employed.

These folding scales have been split at π, since this is a useful constant; and by splitting the scale at this point the 1 or index is thrown near the middle of the rule.

The examples which follow bring out clearly the advantages of the folded scales.

Example: Find the value of 3×5.

To 3 on *D* set the left index of *C*.

Over 5 on *CF* find 15 on *DF*.

Explanation: Since we are measuring the result on *DF*, we have at the start, log π at the left end of *DF*.

When we set the left index of the slide to 3, we increase log π by the addition of log 3.

When we move the indicator to 5 on *CF*, we increased (log π + log 3) by the addition of (log 5 — log π).

The sum of these logs gives

$$\log \pi + \log 3 + \log 5 - \log \pi = \log 3 + \log 5 = \log 15.$$

This example shows how multiplication may be performed by the use of *D*, *CF* and *DF* when it would have required a re-setting of the slide if we had first tried the left index, then the right index and used only scales *C* and *D*.

This is especially valuable when one factor is a constant, as in the following example.

Example: Convert into centimeters:

1. 2.07 inches.
2. .858 inches.
3. 3.14 inches.
4. 6.83 inches.

Since 1 inch = 2.54 centimeters,
To 2.54 on *D* set the left index of *C*.

1. Under 2.07 on *C* find 5.25 on *D*.
2. Over 858 on *CF* find 218 on *DF*.

Placing the decimal point by inspection, we have 2.18.

3. Under 3.14 on *C*, find 7.98 on *D*.
4. Over 6.83 on *CF*, find 173 on *DF*.

Placing the decimal point by inspection, we have 17.3. After working 1 without the use of *CF* and *DF*, it would have been necessary to re-set the slide using the right index. Scales *CF* and *DF* save time in such operations as this.

Care should be taken to keep at least half of the slide in the groove, using the right or left index, as the case requires.

EXAMPLE: Find the circumference of a circle having a diameter of 2″.

Set the indicator to 2 on *C*.

On *CF*, opposite the indicator, find 6.28.

Since π on *CF* is opposite 1 on *C*, scales *C* and *CF* furnish a table of circumferences and corresponding diameters, a diameter on *C* having its circumference directly above it on *CF*.

Examples: With only one setting of the slide, find the following products:

1. $.26 \times 3.5$. Answer .91.
2. $.26 \times 5.6$. (Without re-setting the slide, opposite 5.6 on *CF* read 1.456 on *DF*).
3. $.26 \times 1.12$. Answer .291.
4. $.26 \times 8.4$. Answer 2.18.

Find the circumferences of circles having the following diameters:

5. 2.14″. Answer 6.72″.
6. 7.8″. Answer 24.5″.
7. 4.35″. Answer 13.7″.
8. 6.2″. Answer 19.47″.

PROPORTION

Problems in proportion are special cases of multiplication and division.

Example: Solve $16 : 27 = 17.5 : x$.

$$x = \frac{27 \times 17.5}{16}$$

Following the method on page 13, Fig. 14, we first divide 27 by 16 by setting 16 on *C* to 27 on *D*. We have subtracted the logarithm of 16 from the logarithm of 27. The result of this division, which is 169, is found on *D* under the left index. Now multiply by 17.5 by moving the indicator to 175 on *C*. On *D*, opposite the indicator, read 295.

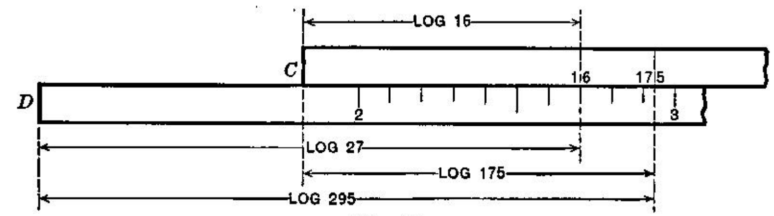


Fig. 26.

To place the decimal point, note that 16 has the same relation to 27 that 17.5 has to x. Since 27 is not quite twice 16, x will be not quite twice 17.5. Hence the decimal point must be placed so that the answer is 29.5.

The method of working a proportion is easily remembered as follows:

$$\begin{matrix} C & D & & C & D \\ 16 : 27 & = & 17.5 : x. \end{matrix}$$

Example: Solve $x : 24 = 11 : 18$.

$$\begin{matrix} C & D & & C & D \\ x : 24 & = & 11 : 18. \end{matrix}$$

To 18 on *D*, set 11 on *C*. Opposite 24 on *D* find x on *C*.

The significant figures of x are 147.

To place the decimal point, note that since 11 is a little more than half of 18, x will be a little more than half of 24, or 14.7.

SQUARES AND SQUARE ROOTS

$$(10^3)^2 = 10^3 \times 10^3.$$
$$= 10^6.$$
$$\text{Since } 6 = 2 \times 3,$$
$$\text{Log } (10^3)^2 = 2 \times \log 10^3.$$

Hence, to square a number, multiply its logarithm by 2.

The space given to each number on scale *C* is twice that given to the same number on scale *B*.

As an example, suppose we wish to square 3.

This can be done by doubling the space given to 3 on scale *B* and finding 9, or looking for 3 on scale *C* and finding its square above it on scale *B*.

Reversing the operation gives the square root.

As an example, find the square root of 9.

Look for 9 on scale *B*, and directly below it on *C* find 3, its square root.

THE LOG LOG (LL) SCALES.

Or, using the LL scales, if we wish to find the square of 5.

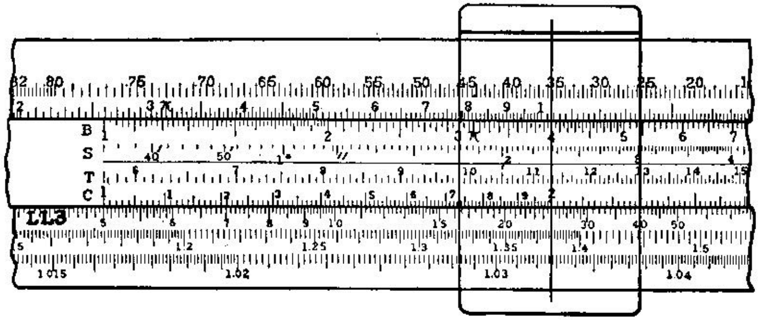


Fig. 27. $(5)^2$

Opposite 5 on LL 3 set the left index of *C*.

Opposite 2 on *C* read 25 on LL 3.

Explanation.

$$x = 5^2$$
$$\log x = 2 \log 5.$$
$$\log (\log x) = \log 2 + \log (\log 5)$$

Hence, we add log 2 on scale *C* to log (log 5) on scale LL 3, and find log (log *x*) on scale LL 3.

The scales LL 0, LL 1, LL 2, and LL 3 are graduated proportionately to the logarithms of the logarithms of the numbers shown on these scales.

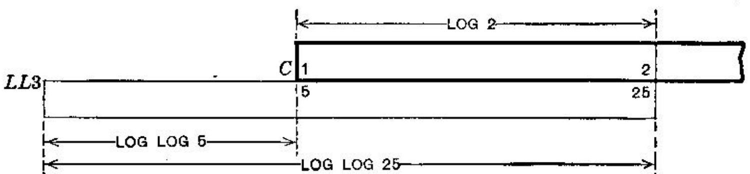


Fig. 28.

For square root: If we desire to find the square root of 25.

Opposite 25 on LL 3, set 2 on *C*.

Opposite the left index of *C* read 5 on LL 3.

Explanation.

$$x = \sqrt{25}$$
$$\log x = \tfrac{1}{2} \log 25$$
$$\log (\log x) = \log (\log 25) - \log 2.$$

Hence, we subtract log 2 on scale *C* from log (log 25) on scale LL 3 and find log (log *x*) on scale LL 3.

CUBES AND CUBE ROOTS.

$$4^3 = 4 \times 4 \times 4.$$

The most accurate way of doing this on the slide rule is to use the long units, or *C*, *D* and *CI* scales.

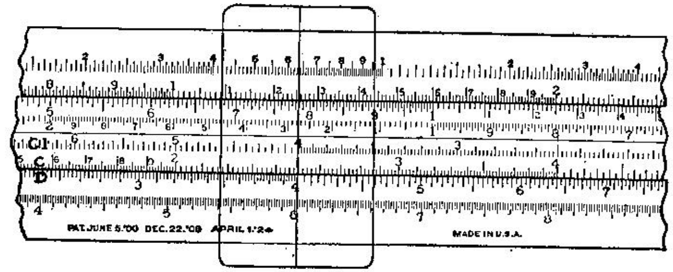


Fig. 29. $(4)^3$

To 4 on *D*, set 4 on *CI*.

Opposite 4 on *C*, find 64 on *D*.

Example: $x = 26^3$.

To 26 on *D*, set 26 on *CI*.

Opposite 26 on *C*, find 1,758 on *D*.

Placing the decimal point by a rough calculation, the result is 17,580.

The complete result by arithmetic is 17,576.

THE K SCALE.

$$(10^2)^3 = 10^2 \times 10^2 \times 10^2.$$
$$= 10^6.$$

Since $6 = 3 \times 2$,

$\text{Log } (10^2)^3 = 3 \times \log 10^2$.

Hence, to cube a number, multiply its logarithm by 3.

Scale *K* is graduated from 1 to 1,000, while scale *D* runs from 1 to 10. The space given to each number on scale *D* is three times that given to the same number on scale *K*.

Example: Find the cube of 2.

This could be done as follows:

Find 2 on scale *K* with the indicator. We have now measured log 2 from the left end of the rule.

Measure off 3 times this space on scale *K* and we have the indicator set at log 2^3 or 8.

But this measuring can be done by setting the indicator to 2 on scale *D*, since the space given to 2 on *D* is 3 times that given to 2 on *K*.

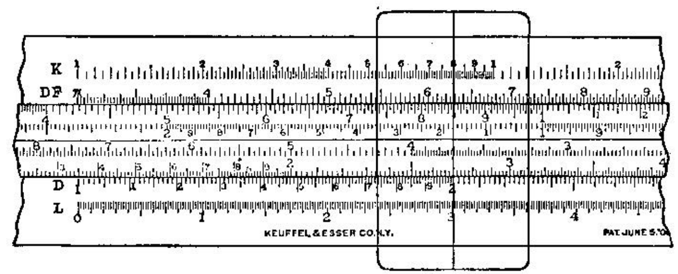


Fig. 30. $(2)^3$

Hence, to cube a number, using the face of the rule on which scale *K* appears, set the indicator to the given number on scale *D*.

On scale *K* opposite the indicator, find the result; which is 2^3, or 8.

THE LOG LOG (LL) SCALES.

Another method of finding cubes involves the use of the Log Log (LL) scales.

Example: $x = 4^3$.

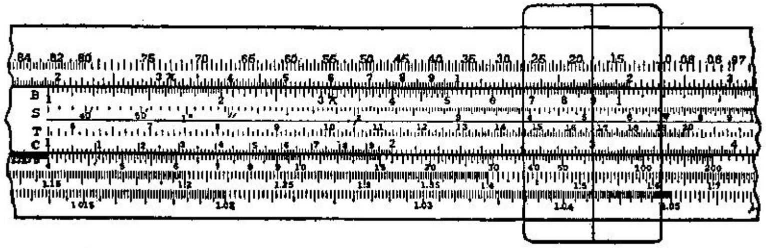


Fig. 31. $(4)^3$

Opposite 4 on LL 3 set left index of *C*.
Opposite 3 on *C* read 64 on LL 3.

Explanation

$$x = 4^3$$
$$\log x = 3 \log 4$$
$$\log (\log x) = \log 3 + \log (\log 4).$$

Hence, we add log 3 on scale *C* to log (log 4) on scale LL 3, and find log (log *x*) on scale LL 3. Since LL 3 is graduated proportionally to the logarithms of the logarithms of the numbers, opposite 3 on *C* we read $x = 64$ on LL 3.

Example: $x = 26^3$.
Set left index of *C* to 26 on LL 3.
Below 3 on *C* find 17,600 on LL 3.
The correct answer is 17576.
Example: $x = 1.25^3$.

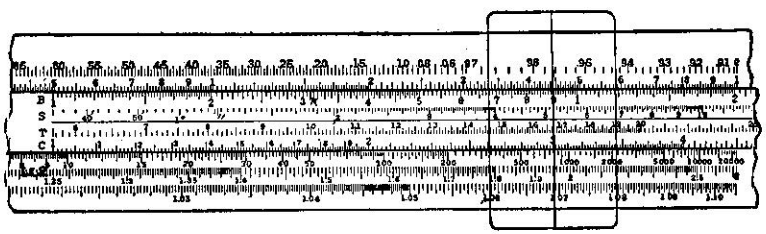


Fig. 32. $(1.25)^3$

Set left index of *C* to 1.25 on LL 2.
Below 3 on *C* read 1.953 on LL 2.
Example: $x = 1.101^3$.

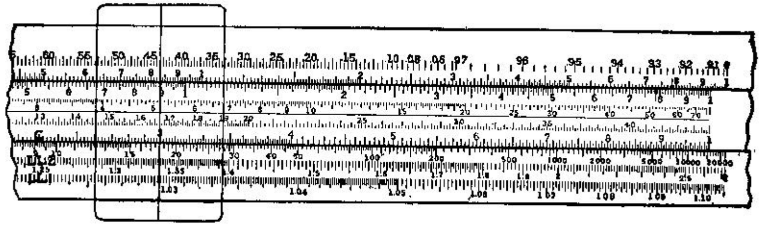


Fig. 33. $(1.101)^3$

Set right index of *C* to 1.101 on LL 1.
Below 3 on *C* read 1.335 on LL 2.

The correct scale on which to read the result is determined by inspection. It is readily seen that the number on scale LL1 is too small, and that on LL3 too large. Hence, we use LL 2.

Example: $x = .73^3$.

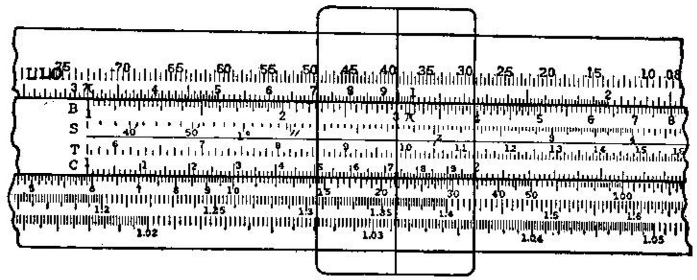


Fig. 34. $(.73)^3$

Set left index of *B* to .73 on LL 0.
Above 3 (left half of slide) on *B* read .389 on LL 0.

Note that the LL 0 scale reads from right to left; and that it is referred to the *B* scale.

Example: $x = .911^3$.
Set right index of *B* to .911 on LL 0.
Above 3 on *B* read .756 on LL 0.

Note that by the use of the Log Log Scales the cubes of quantities less than unity are correctly pointed off by the scale.

It is also well to remember that the cube of any quantity greater than unity is greater than the quantity; and that the cube of any quantity less than unity is less than the quantity. With this in mind there can be no doubt as to which of the LL scales contains the answer to any problem.

The range of the Log Log Scales is from .05 to 22,000. Hence, the finding of cubes upon them is limited to quantities between about 0.37 and 28. To handle the cube of any quantity not within the range of the Log Log Scales, multiply or divide the quantity by a factor that brings it within the range of the scale. Cube the multiplied or divided number, and cube the factor. Then divide or multiply the cube of the multiplied or divided number by the cube of the factor.

Example: x = quantity.
y = factor.

$$\frac{(xy)^3}{y^3} = x^3.$$

$$\left(\frac{x}{y}\right)^3 y^3 = x^3.$$

Example: $x = 60^3$.
The cubes of numbers above 28 are off the scale.
Divide 60 by 3 = 20.
Set left index of C to 20 on LL 3.
Below 3 on C, find 8,000 on LL 3.
$3^3 = 27$. Do this mentally.
Set left index of C to 27 on D.
Opposite 8 on CF, find 216 on DF.
Placing the decimal point by inspection, the answer is 216,000.

Example: $x = .023^3$.
The cube of any decimal below about 0.37 cannot be obtained directly.
Multiply .023 by 20 = .460.
Set left index of B to .46 on LL 0.
Above 3 on B read .097.
$20^3 = 8{,}000$.
To 97 on D, set 8 on C.
Below 1 on C read 121 on D.
Pointing off, the answer becomes .0000121.

Example: $x = 1.002^3$.
This quantity is not upon the scale.

$$1.002^3 = \frac{(1.002 \times 1.1)^3}{1.1^3}$$

To 1.002 on D, set left index of C.
Opposite 1.1 on C read 1.102 on D.
To 1.102 on LL 1, set right index of C.
Opposite 3 on C, read 1.338 on LL 2.
To 1.1 on LL 1 set right index of C.
Opposite 3 on C, read 1.331 on LL 2.
To 1.338 on D, set 1.331 on C.
Opposite left index of C read 1.006 on D.

This, like all quantities near 1, between .99 and 1.01, may be handled more simply by the application of the Binomial Theorem.

$$(a \pm b)^n = a^n \pm na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 \pm, \text{etc.}$$

When $a = 1$ and b is small, only two terms of this equation are required, which reduces to

$$(a \pm b)n = 1 \pm nb$$

The above example is then solved as follows:
$1.002^3 = 1 + .002 \times 3 = 1.006$.
The correct answer to four significant figures is 1.006.

Example: Find $x = .994^3$.

$$\left(\frac{.994}{1.5}\right)^3 \times 1.5^3$$

To .994 on D, set 1.5 on C.
Opposite left index of C read .663 on D.
To .663 on LL 0 set left index of B.
Opposite 3 on B read .291 on LL 0.
To 1.5 on LL 2 set right index of C.
Opposite 3 on C read 3.38 on LL 3.
To .291 on D set left index of C.
Opposite 3.38 on C read .982 on D.
Using the Binomial Theorem:
$(a - b)^n = 1 - nb$.
$a = 1$, $b = .006$
$(.994)^3 = (1 - .006)^3 = 1 - .006 \times 3$.
$= 1 - .018 = 0.982$.

Show that:

Example: $3^3 = 27$. [Try these by all methods explained previously so as to become familiar with the degree of accuracy obtained by the use of each].
" $4^3 = 64$.
" $5^3 = 125$.
" $6^3 = 216$.
" $7^3 = 343$.
" $8^3 = 512$.
" $9^3 = 729$.
" $11^3 = 1331$.

In the same way work the following:

Example: $12^3 = 1728$.

Find the cubes of the following numbers, correct to 4 significant figures:

Problem 77. Find the cube of 13.
78. " " " " 14.
79. " " " " 15.

Find the cubes of the following numbers, correct to 3 significant figures:

For a discussion of significant figures and per cent of error, read pp. 21-23.

Compare the speed and accuracy of the slide rule with that of three and four place log tables.

80. " " " " 16.
81. " " " " 17.
82. " " " " 18.
83. " " " " 19.
84. " " " " 20.
85. " " " " 21.
86. " " " " 31.
87. " " " " 46.

Problem 88. Find the cube of 47.
89. " " " " 53.
90. " " " " 64.
91. " " " " 758.
92. " " " " 232.
93. " " " " 425.6*.
94. " " " " 87.9.
95. " " " " 139.
96. " " " " .342.
97. " " " " .047.
98. " " " " .0068.
99. " " " " 1.03.
100. " " " " 2.12.

*In Problem 93, 425.6 is approximately 426. Roughly approximating the result $400^3 = 64{,}000{,}000$. The rule gives us 771. Hence the result is 77,100,000, correct to three significant figures. The complete result is 77,091,-209.

Problem 101. How many gallons will a cubical tank hold that measures 26 inches in depth? (1 gal. = 231 cu. in.).

CUBE ROOTS.

While any number or factor is readily cubed on the *CD* scales, or on the *CD* and *AB* scales in conjunction, the inverse operation of extracting the cube root, while it can be done, is neither an easy nor a practical one. Hence, it is best to confine the operation of extracting the cube root to the *K* and Log Log scales.

THE K SCALE.

Example: Find the cube root of 8.

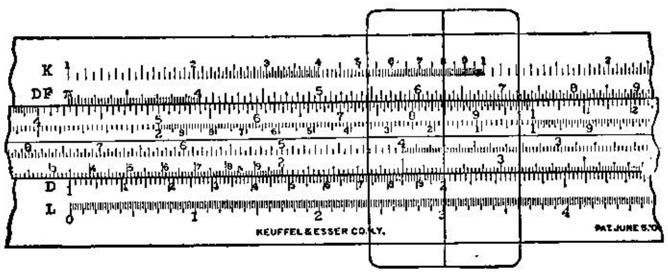


Fig. 35. $\sqrt[3]{8}$.

Set the indicator to 8 on the left-hand *K* scale, called K_1. On scale *D*, opposite the indicator, find 2.

Explanation. $\log \sqrt[3]{8} = \frac{1}{3} \log 8$.
Hence, to find the cube root of a number, divide the logarithm of the number by 3.
Using log 8 on scale *D* as a unit, set the indicator to 8 on *D*.
Measuring from the left index one-third of this space, we find 2 on scale *D*.

But one-third of log 8 on *D* may be found by setting the indicator to 8 on the left-hand *K* scale, since each *K* scale is one-third as long as the *D* scale.

Example: Find $\sqrt[3]{27}$.
Set the indicator to 27 on the middle *K* scale, called K_2.
On scale *D*, opposite the indicator, find 3.

Example: Find $\sqrt[3]{125}$.
Set the indicator to 125 on the right-hand *K* scale, called K_3.
On scale *D*, opposite the indicator, find 5.

Example: Find $\sqrt[3]{9}$.
Set the indicator to 9 on K_1.
On *D*, opposite the indicator, find 208, the significant figures of the result.
Placing the decimal point by inspection, we have $\sqrt[3]{9} = 2.08$.

Example: Find $\sqrt[3]{90}$.
Set the indicator to 90 on K_2.
On *D*, opposite the indicator, find 4.48, the cube root.

Example: Find $\sqrt[3]{900}$.
Set the indicator to 900 on K_3.
On *D*, opposite the indicator, find 9.66, the cube root.

Example: Find $\sqrt[3]{.9}$.
Point off the number into periods of three figures each, counting from the decimal point, adding zeros to fill out the three figures. This gives .900. Now we have the problem of finding the cube root of 900 as in the previous example.

The significant figures are 966, the setting of the indicator being the same as in the previous example.

In placing the decimal point, there is a decimal place in the cube root for every decimal period of three figures in the given problem.

Given number .900,000,000.

Cube root 9 6 6

The result is .966.

Example: Find $\sqrt[3]{.09}$.
Following the plan of the previous example, the first decimal period is .090.
Finding $\sqrt[3]{90}$ as before we have 448 for the significant figures.
Hence, the result is .448.

Rule for Placing the Decimal Point in Cube Root.

From consideration of the preceding eight examples, we derive a rule for placing the decimal point in finding the cube root of numbers that do not lie between 1 and 1,000.

a. Move the decimal point 3, 6, or 9 places, as may be necessary, in either direction to obtain a number between 1 and 1,000.

b. Find the cube root of this new number, using K_1 for a number of one integer.
K_2 for a number of two integers.
K_3 " " " " three "

c. In the result, move the decimal point one-third as many places as it was moved in *a*, and in the opposite direction.

Example: Find $\sqrt[3]{56{,}342}$.

a. Move the decimal point three places to the left, obtaining 56.342.

b. Find the cube root of 56.3, which is 3.83.

c. Move the decimal point one place to the right, obtaining 38.3.

Example: Find $\sqrt[3]{.00382}$.

a. Move the decimal point three places to the right, obtaining 3.82.

b. Find the cube root of 3.82, which is 1.563.

c. Move the decimal point one place to the left, obtaining .1563.

THE LOG LOG (LL) SCALES.

Example: Find the cube root of 8.

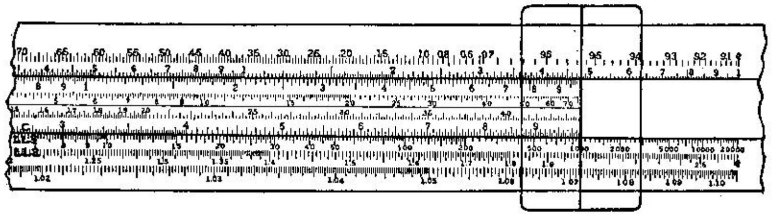


Fig. 36. $\sqrt[3]{8}$.

Set 3 on *C* to 8 on LL3.
Opposite right index of *C*, read 2 on LL2.

Explanation.

$$x = \sqrt[3]{8}.$$
$$\log x = \tfrac{1}{3} \log 8.$$
$$\log (\log x) = \log (\log 8) - \log 3.$$

Hence, we subtract log 3 on scale *C* from log (log 8) on scale LL3, and find log (log *x*) on scale LL2.

The scales LL1, LL2 and LL3 are graduated proportionally to the logarithms of the logarithms of the numbers shown on these scales.

Example: Find $\sqrt[3]{27}$.
Set 3 on *C* to 27 on LL3.
Opposite left index of *C*, read 3 on LL3.

Explanation.

$$x = \sqrt[3]{27}.$$
$$\log x = \tfrac{1}{3} \log 27.$$
$$\log (\log x) = \log (\log 27) - \log 3.$$

Example: Find $\sqrt[3]{125}$.
Set 3 on *C* to 125 on LL3.
Opposite left index of *C*, read 5 on LL3.

Example: Find $\sqrt[3]{9}$.
Set 3 on *C* to 9 on LL3.
Opposite right index of *C*, read 2.08 on LL2.

The proper scale on which to read the result is determined by a rough calculation. The first significant figure is evidently 2. Hence, we look for the scale on which 2 appears; *i. e.*, scale LL2. Also, since the cube root of any number greater than unity is less than the number, we must find the answer on that portion of the scale which is less than the number whose cube we are seeking.

Note that the correct pointing off is done by the scale itself.

Example: Find $\sqrt[3]{90}$.
Set 3 on *C* to 90 on LL3.
Opposite left index of *C*, find 4.48 (already pointed off) on LL3.

Example: Find $\sqrt[3]{900}$.
Set 3 on *C* to 900 on LL3.
Opposite left index of *C* read 9.65 on LL3.

Example: Find $\sqrt[3]{.9}$.

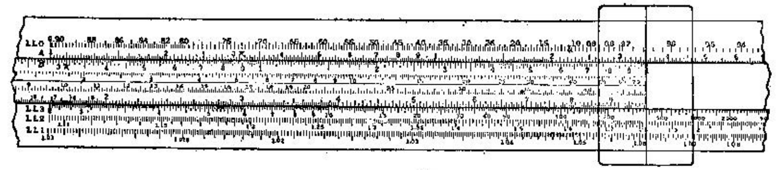


Fig. 37. $\sqrt[3]{.9}$

Set 3 on *B* (left half) to .9 on LL0
Opposite the right index of *B*, find .9655 (pointed off) on LL 0.

Example: Find $\sqrt[3]{.09}$.
Set 3 on *B* (left half) to .09 on LL 0.
Opposite left index of *B*, read .448 (already pointed off) on LL0.

Rule for Finding the Cube Roots of Numbers Off the Scale.

The cube roots of numbers between .05 and 22,000 may be found directly. For numbers above and below these limits the following rule must be applied:

When the number whose cube root is desired is above 22,000, divide it by a number whose cube root is easily determined, and one which will throw the quotient upon the scale. Thus:

y = number whose cube root is desired.
z = number whose cube root is easily determined, as 8, 27, or 1000.

$$\frac{y}{z} = x < 22{,}000.$$

Find the cube root of the quotient, and the cube root of the divisor. Multiply the cube root of the quotient by the cube root of the divisor.

$$\sqrt[3]{y} = \sqrt[3]{x} \times \sqrt[3]{z}.$$

Example: Find $\sqrt[3]{56{,}342}$.

(*a*) Move decimal point 3 places to left giving 56,342. This is the same as dividing by 1000.

(*b*) To 56.342 on LL3 set 3 on *C*.

(*c*) Opposite left index read 3.83 on LL3.

(*d*) Cube root of 1000 = 10.

(*e*) 10 × 3.83 = 38.3. Ans.

Where the number is less than .05, multiply it by a number whose cube root is easily determined, and which will throw the product upon the scale. Thus:

$$yz = x > .05.$$

Find the cube root of the multiplicand and the cube root of the multiplier. Divide the cube root of the multiplicand by the cube root of the multiplier.

$$y = \frac{\sqrt[3]{x}}{\sqrt[3]{z}}$$

Example: Find $\sqrt[3]{.00382}$.
Multiply .00382 × 27.
To 382 on *D*, set right index of *C*.
Opposite 27 on *C*, read 1031.
By inspection this becomes .1031
Set 3 on *B* (left half) to .103 on LL 0.
Opposite left index of *C*, read .469 on LL 0.
The cube root of 27 = 3 (done mentally).

$$\frac{.469}{3} = .1563$$

The correct answer to four significant figures is the same—.1563.

Problem

102. Find the cube root of 3.
103. " " " " " 30.
104. " " " " " 300.
105. " " " " " .3.
106. " " " " " .03.
107. " " " " " .003.
108. " " " " " 2613.
109. " " " " " 47.8.
110. " " " " " .784.
111. " " " " " 45083.

Problem

112. Find the cube root of 50.
113. " " " " " 7.35.
114. " " " " " .575.
115. " " " " " 241.
116. " " " " " 3840.
117. " " " " " 52076.
118. " " " " " .0163.
119. " " " " " .0094.
120. " " " " " 1.036.
121. " " " " " 108,723.

Problem 122. How deep should a cubical box be made to contain 8,500 cubic inches?

Find the answers to the above problems by both methods; *i. e.*, by use of the *K* scale and by means of the Log Log scales.

OTHER POWERS

The powers of numbers, other than the square and cube, cannot be found directly upon the slide rule, except by means of the Log Log scales.

Example: Find the value of $x = 2^4$.

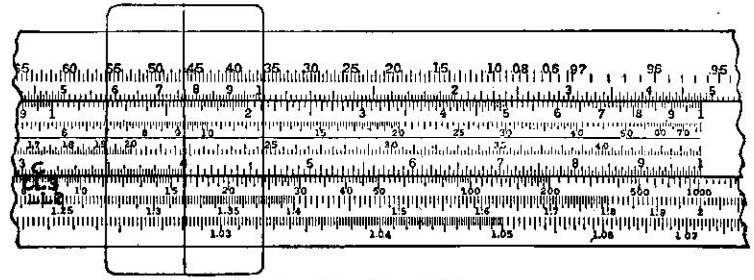


Fig. 38. $(2)^4$

To 2 on LL2 set the right index of *C*.
Opposite 4 on *C*, find 16 on LL 3.

Example: Find the value of $(4.54)^5$.
To 4.54 on LL 3, set the left index of *C*.
Opposite 5 on *C*, read 1930 on LL 3.

Example: Find the value of $(1.01666)^{13}$.
To 1.01666 on LL 1, set left index of *C*.
Opposite 13 on *C*, read 1.2395 on LL 2.

Example: Find the value of $(1.812)^8$.
To 1.812 on LL2, set right index of *C*.
Opposite 8 on *C*, read 116 on LL 3.

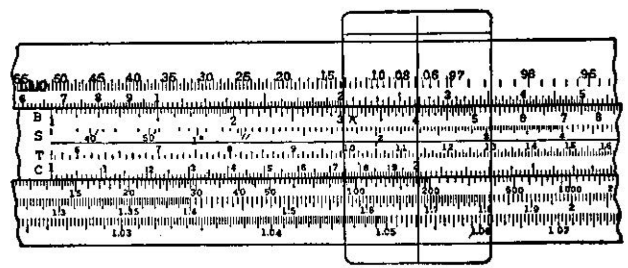


Fig. 39. $(.512)^4$

Example: Find the value of $(.512)^4$.
To .512 on LL0, set left index of *B*.
Opposite 4 on *B*, read .069 on LL 0.

Example: Find the value of $(.883)^9$.
To .883 on LL 0 set left index of *B*.
Opposite 9 on B read .326 on LL 0.

Example: Find the value of $(.768)^{10}$.
To .768 on LL 0 set left index of *B*.
Opposite 10 (middle 1) on *B*, find .071 on LL 0.

Where a number greater than unity is to be raised to any power, and the answer falls off the scale, divide the number by another sufficiently large to bring the quotient upon the scale. Raise the divisor and quotient to the required power; and multiply the power of the quotient by the power of the divisor, as follows:

x = quantity
y = factor

$$\left(\frac{x}{y}\right)^n y^n = x^n$$

Choose y so that $\left(\frac{x}{y}\right)^n$ and y^n (by trial with the rule) are on the scale.

For numbers greater than unity, the limits of the Log Log scales are as follows:

To find	x^4,	x must	be	less	than		12.
"	"	x^5 "	"	"	"	"	7.4.
"	"	x^6 "	"	"	"	"	5.3
"	"	x^7 "	"	"	"	"	4.15
"	"	x^8 "	"	"	"	"	3.48
"	"	x^9 "	"	"	"	"	3.03
"	"	x^{10} "	"	"	"	"	2.72
"	"	x^{11} "	"	"	"	"	2.48
"	"	x^{12} "	"	"	"	"	2.30

and etc.

Note that the highest number on the LL 3 scale—22,000—is approximately the 1,000th power of the lowest number on the LL 1 scale—1.01.

Example: Find the value of $(8.36)^6$.

The 6th power of any number greater than 5.3 does not lie upon the scale. Hence, divide 8.36 by a number which will produce a quotient less than 5.3, and which itself will be less than 5.3.

$$\frac{8.36}{3} = 2.787$$

To 2.79 on LL 3, set left index of C.
Opposite 6 on C, read 468 on LL 3.
To 3 on LL 3, set left index of C.
Opposite 6 on C, read 730.
To 468 on D, set right index of C.
Opposite 730 on C, read 341 on D.
To point off, $500 \times 700 = 350{,}000$.
Hence, $468 \times 730 = 341{,}000$, as close as the slide rule will read.
The correct answer is 341,380.
A shorter way to solve some problems which run off the scale, is as follows:

$$(8.36)^6 = (8.36)^3 \times (8.36)^3$$

Set indicator to 8.36 on D.
Under indicator on K, read 584.
To 584 on D, set right index of C.
Opposite 584 on C, read 341,000 on D.

Example: Find the value of $(3.91)^9$.

The 9th power of any number greater than 3.03 does not appear upon the scale.

$$\frac{3.91}{2} = 1.955$$

To 1.955 on LL2, set right index of C.
Opposite 9 on C, find 417 on LL3.
To 2 on LL2, set right index of C.
Opposite 9 on C, find 510 on LL3.
To 417 on D, set right index of C.
Opposite 510 on C, read 213 on D.

$$400 \times 500 = 200{,}000.$$

Hence, the answer is 213,000.
The correct answer is 213,595.
This is more readily performed as follows:

$$(3.91)^9 = (3.91)^3 \times (3.91)^3 \times (3.91)^3$$

Set indicator to 3.91 on D.
Under indicator on K, read 59.8.
Set right index of C to 59.8 on D.
Indicator to 59.8 on C.
Right index of C to indicator.
Opposite 59.8 on C, read 214,000 on D.

Example: Find the value of $(1.008)^4$.

Using the Binomial Theorem referred to on page 32:

$$(a + b)^n = 1 + nb$$
$$(1 + .008)^4 = 1 + .008 \times 4$$
$$= 1.032$$

Where a number less than unity is to be raised to any power, and the answer falls off the scale, multiply the number by another sufficiently large to bring the product upon the scale. Raise the product and multiplier to the required power and divide the power of the product by the power of the multiplier.

For numbers less than unity, the limits of the LL 0 scale are as follows:

Numbers below 0.05
Numbers above 0.97

For x^4,	x	must	be	over		.475
"	x^5, "	"	"	"		.550
"	x^6, "	"	"	"		.610
"	x^7, "	"	"	"		.65
"	x^8, "	"	"	"		.69
"	x^9, "	"	"	"		.72
"	x^{10}, "	"	"	"		.74
"	x^{11}, "	"	"	"		.76
"	x^{12}, "	"	"	"		.78

and etc.

The lowest number on the LL 0 scale (.05) is about the 100th power of the highest number on this scale(0.97).

Example: Find $x = (0.545)^9$

The 9th power of any number less than 0.72 cannot be found directly upon the LL 0 scale.

$$x = \frac{(0.545 \times 2)^9}{2^9}$$

$2 \times 0.545 = 1.090 > 0.72$
To 1.09 on LL 1, set right index of *C*.
Opposite 9 on *C*, read 2.172 on LL 2.
To 2.00 on LL2, set right index of *C*.
Opposite 9 on *C*, read 510 on LL 3.
To 2.172 on *D*, set 510 on *C*.
Opposite right index of *C*, read 426.

To point off: $\frac{2}{400} = \frac{1}{200} = .005$

Hence, the answer is .00426.
The correct answer is .00424.
Using the *K* scale solve as follows:

$$(.545)^9 = (.545)^3 \times (.545)^3 \times (.545)^3$$

Set the indicator to .545 on *D*.
Under the indicator on *K*, read 162.

$$.5 \times .5 \times .5 = .125$$

Hence, the result is .162.
To 162 on *D*, set left index of *C*.
Indicator to 162 on *C*.
Left index of *C* to indicator.
Indicator to 162 on *C*.
Under indicator on *D*, read 424.

$$.2 \times .2 \times .2 = .008$$

Making 424 look as nearly like .008 as possible, the answer becomes .00424

Example: Find $x = (0.982)^6$.

$$(0.982)^6 = \left(\frac{0.982}{1.5}\right)^6 \times (1.5)^6 = (0.655)^6 \times (1.5)^6.$$

To .655 on LL 0, set left index of *B*.
Opposite 6 on *B*, read .079 on LL 0.
To 1.5 on LL 2, set right index of *C*.
Opposite 6 on *C*, read 11.4 on LL 3.
Set index of *CF* to 79 on *DF*.
Opposite 11.4 on *CF* read .90 on *DF*.
The correct answer is 0.89675

Example: Find $x = (0.991)^5$.

This is most correctly determined by means of the Binomial Theorem:

$$(a-b)^n = 1 - bn$$
$$0.991 = 1 - .009 \times 5$$
$$= 0.955$$

Solving by means of the Binomial Theorem will give closer results for values between 0.97 and 1.01, which are missing from the rule, than any solution possible by the rule.

Problem.

123. Find the value of $(16)^4$.
124. " " " " $(8.4)^5$.
125. " " " " $(5.84)^6$.
126. " " " " $(4.62)^7$.
127. " " " " $(3.77)^8$.
128. " " " " $(5.57)^9$.
129. " " " " $(3.1)^{10}$.
130. " " " " $(.361)^4$.
131. " " " " $(.253)^5$.
132. " " " " $(.543)^9$.
133. " " " " $(.702)^{12}$.
134. " " " " $(1.0095)^6$.
135. " " " " $(1.0003)^5$.
136. " " " " $(0.998)^7$.
137. " " " " $(0.976)^8$.

FRACTIONAL POWERS

The fractional or decimal powers of numbers cannot be readily found by means of the *C*, *D*, *CF*, *DF*, *A*, *B* and *K* scales. The Log Log scales only are suitable for this purpose.

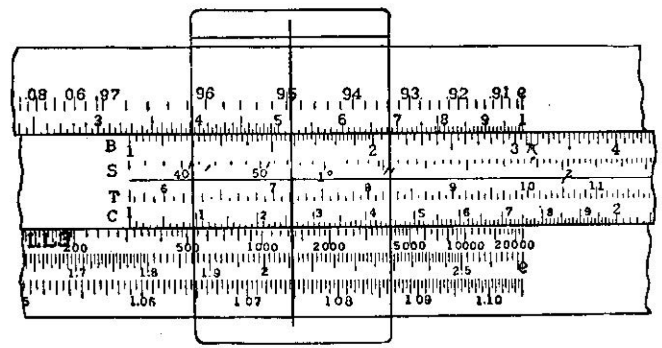


Fig. 40. $(310)^{1.26}$

Example: Find the value of $(310)^{1.26}$.
To 310 on LL 3, set left index of *C*.
Opposite 126 on *C* read 1380 on LL 3.
The true answer lies between 1377.5 and 1377.6.
Example: Find the value of $x = (16.6)^{3.31}$.

To 16.6 on LL 3, set left index of *C*.
Opposite 331 on *C*, read 10,900 on LL 3.
The correct answer is 10,928.

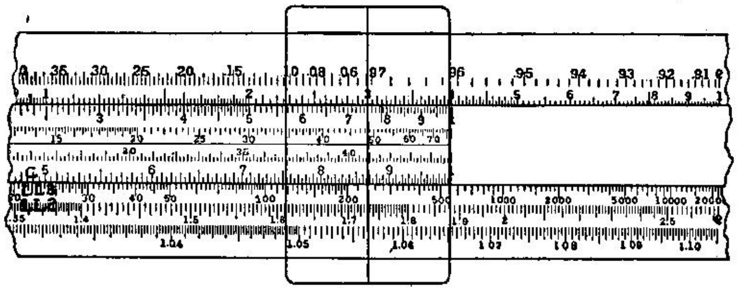


Fig. 41. $(1.885)^{8.66}$

Example: Find $x = (1.885)^{8.66}$
To 1.885 on LL 2, set right index of *C*.
Opposite 8.66 on *C*, read 242 on LL 3.
The correct answer is 242.2 +.

Example: Find $x = (1.885)^{13.2}$
This is the same number as in the preceding example.
To 1.885 on LL 2, set left index of *C*.
Opposite 132 on *C*, read 4300. on LL 3.
The correct answer is 4306.4

Where a number is greater than unity and the power is less than unity, the method is the same, except that the answer will be **less** than the number whose power is sought.

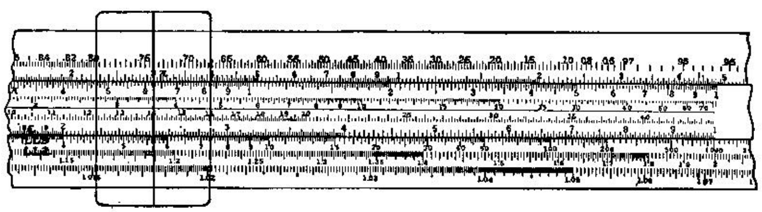


Fig. 42. $(2)^{0.25}$

Example: Find $x = (2)^{0.25}$.
To 2 on LL 2, set right index of *C*.
Opposite 25 on *C*, read 1.1892 on LL 2.
This example could be solved as follows:

$$x = (2)^{0.25} = 2^{\frac{1}{4}} = \sqrt[4]{2} \text{ or } \log x = \frac{\log 2}{4}$$

Set indicator to 2 on LL 2.
Set 25 on *CI* to indicator.

This automatically divides the log of 2 on LL2 by the reciprocal of .25 or 4 on *C*.

Example: Find the value of $(7.8)^{.36}$.
Set indicator to 7.8 on LL3.
Set 36 on *CI* to indicator.

Opposite right index of *C*, read 2.095 on LL2, since the answer must be less than 7.8, and the value on LL 1 is evidently too small.

Where a number and its power are both less than unity, the answer will be greater than the number. The method to be used is as follows:

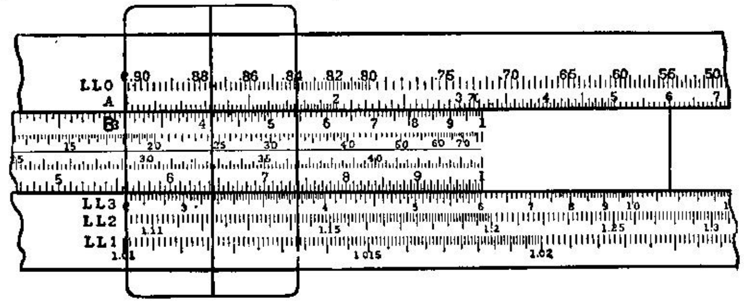


Fig. 43. $(.723)^{.41}$

Example: Find the value of $x = (.723)^{.41}$
Set right index of *B* to .723 on LL 0.
Opposite 41 (right half of scale) on *B*, read .8758 on LL 0.

Note that on the *B* scale powers less than unity are handled in exactly the same way as numbers less than unity. Thus .41 is found on the right hand half of the *B* scale, whereas .041 would be found on the left half.

When the answer runs off the scale, use the rule already stated on page 40.

Example: Find the value of $x = (50)^{2.82}$

$$\frac{50}{2} = 25 \qquad (25)^{2.82} = 8750 \qquad (2)^{2.82} = 7.06.$$

$$8750 \times 7.06 = 61800.$$

Correct answer is 61815.

Example: Find the value of $x = (.034)^{1.56}$

$$.034 \times 5 = .170 \qquad (.17)^{1.56} = .063 \qquad (5)^{1.56} = 12.32$$

$$\frac{.063}{12.32} = .00512$$

Example: Find the value of $x = (32000)^{.46}$

$$\frac{32000}{2} = 16,000 \qquad (16000)^{.46} = 86 \qquad (2)^{.46} = 1.375$$

$$86 \times 1.375 = 118.2$$

Correct answer = 118.13.

Example: Find the value of $x = (.026)^{.29}$.
Use right-hand half of B scale.

$.026 \times 2 = .052 \qquad (.052)^{.29} = .425$

$$(2)^{.29} = 1.2222 \qquad \frac{.425}{1.2222} = .3473.$$

Example: Find the value of $x = (1.0072)^{.71}$.
Use the Binomial Theorem, as explained on page 32.

$$(1.0072)^{.71} = 1 + .0072 \times .71 = 1.0051$$

Example: Find the value of $x = (.993)^{.53}$.

$$(.993)^{.53} = 1 - .007 \times .53 = 1.00371.$$

Problem.

138. Find the value of $(443)^{1.1}$.
139. " " " " $(38.4)^{2.72}$.
140. " " " " $(5.12)^{3.23}$.
141. " " " " $(2.263)^{4.18}$.
142. " " " " $(1.0341)^{5.71}$.
143. " " " " $(19100)^{.91}$.
144. " " " " $(123)^{.76}$.
145. " " " " $(16.4)^{.59}$.
146. " " " " $(4.57)^{.43}$.
147. " " " " $(1.1435)^{.38}$.
148. " " " " $(1.0221)^{.26}$.
149 " " " " $(0.47)^{2.3}$.
150. " " " " $(0.097)^{1.23}$.
151. " " " " $(0.998)^{.15}$.
152. " " " " $(1.0012)^{1.84}$.

It should be noted that where the 10th power of any number is desired, the following procedure may be adopted:

If the number is on the LL1 scale the 10th power will be found directly above it on the LL2 scale. The 10th power of any number on the LL2 scale will be found directly above it on the LL 3 scale.

The 100th power of any number on the LL1 scale is found directly above it on the LL 3 scale.

As stated before, the highest number on the LL3 scale is approximately the 1000th power of the lowest number on the LL1 scale.

Problem.

153. Find the value of $(306)^{1.92}$.
154. " " " " $(13.7)^{5.1}$.
155. " " " " $(41320)^{.19}$.
156. " " " " $(.315)^{3.3}$.
157. " " " " $(.026)^{.54}$.

OTHER ROOTS

In extracting roots, other than the square and cube roots, it is necessary to use the Log Log scales. The operation is exactly the reverse of finding powers.

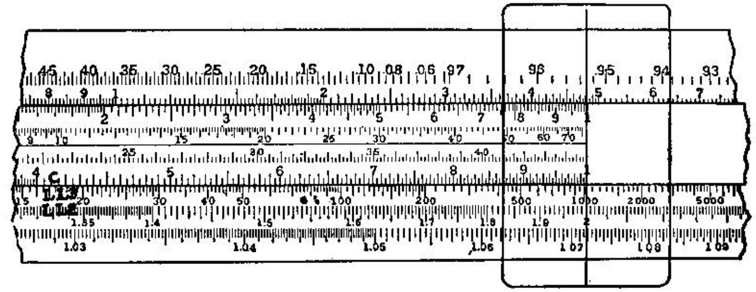


Fig. 44. $\sqrt[4]{16}$

Example: Find $x = \sqrt[4]{16}$.

$$\log x = \frac{\log 16}{4}$$

Indicator to 16 on LL 3.
4 on C to indicator.
Opposite right index of C, find 2 on LL 2.

Fig. 45. $\sqrt[5]{243}$

Example: Find $x = \sqrt[5]{243}$.
Indicator to 243 on LL 3.
5 on C to indicator.
Opposite left index of C find 3 on LL 3.

The root of any number greater than 22,000 cannot be found directly. If the number lies above unity, divide it by a number that will throw the quotient upon the scale. Extract the root of the quotient and the divisor and multiply the root of the quotient by the root of the divisor.

Example: Find the value of $x = \sqrt[7]{81{,}590}$.

$$\frac{81590}{5} = 16{,}318.$$

To 16,300 on LL 3, set 7 on C.
Opposite left index of C, read 4 on LL 3.
To 5 on LL 3, set 7 on C.
Opposite right index of C, read 1.2585 on LL 2.
To 1.259 on D, set left index of C.
Opposite 4 on C, read 5.03 on D.
The correct answer is 5.031 +.

Note that the lowest number on the LL1 scale is the 1,000th root of the highest number on the LL3 scale.

The root of any number less than unity may be found by means of the LL0 scale. It may be found directly if the number is not less than 0.05 and if—

$x^4 < .886$
$x^5 < .860$
$x^6 < .834$
$x^7 < .810$
$x^8 < .785$
$x^9 < .765$
$x^{10} < .740$
$x^{11} < .717$
$x^{12} < .695$
and etc.

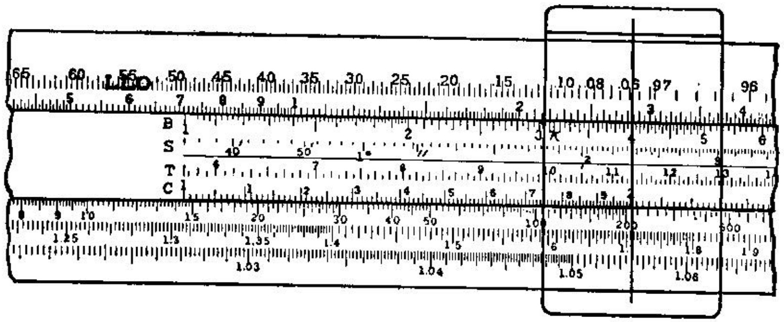


Fig. 46. $\sqrt[4]{.062}$

Example: Find the value of $x = \sqrt[4]{.062}$
To .062 on LL 0 set 4 on *B*.
Opposite left index of *B*, read 0.49 on LL 0.

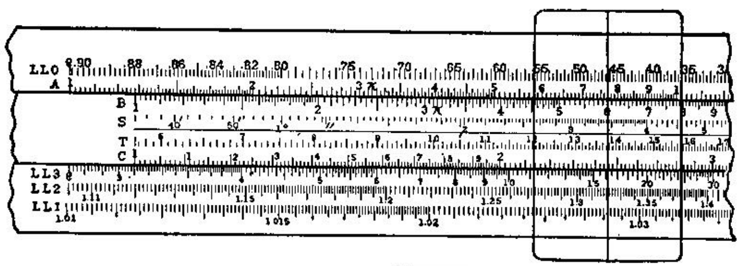


Fig. 47. $\sqrt[6]{0.463}$

Example: Find the value of $x = \sqrt[6]{0.463}$
To .463 on LL 0, set 6 on *B*.
Opposite left index of *B*, read .8795 on LL 0.

Where the number is less than 0.05, multiply it by a number which will bring the product upon the scale. Take the root of the product and the root of the multiplier; and divide the root of the product by the root of the multiplier.

Example: Find the value of $x = \sqrt[5]{.018}$

$$3 \times .018 = .054$$

To .054 on LL 0, set 5 on *B*.
Opposite left index of *B*, read .5580 on LL 0.
To 3 on LL 3, set 5 on *C*.
Opposite right index of *C*, read 1.246.
To .5580 on *D*, set 1.246 on *C*.
Opposite left index of *C*, read .448 on *D*.

When the answer falls between 0.97 and unity, divide the number whose root is desired by another which will bring the answer on the scale; take the root of the quotient and the root of the divisor and multiply the root of the quotient by the root of the divisor.

Example: Find the value of $x = \sqrt[6]{.911}$

$$\frac{.911}{2} = .4555$$

To .456 on LL 0, set 6 on *B*.
Opposite left index of *B*, find .877.
To 2 on LL 2, set 6 on *C*.
Opposite left index of *C*, find 1.1222.
To 1.122 on *D*, set left index of *C*.
Below .877 on *C*, find .985 on *D*.

When the number falls between 0.97 and unity, the Binomial Theorem can be conveniently used. The formula (see page 32) re-arranged for this case is

$$\sqrt[n]{b} = 1 - \frac{1-b}{n} = \frac{n-1+b}{n}$$

Example: Find the value of $x = \sqrt[9]{.984}$

$$\frac{n-1+b}{n} = \frac{9-1+.984}{9} = \frac{8.984}{9} = .9982$$

Problem.

158. Find the value of $\sqrt[4]{17,100}$.
159. " " " " $\sqrt[6]{1410}$.
160. " " " " $\sqrt[8]{138}$.
161. " " " " $\sqrt[10]{12.2}$.
162. " " " " $\sqrt[7]{4.76}$.
163. " " " " $\sqrt[5]{1.863}$.
164. " " " " $\sqrt[9]{1.0517}$.
165. " " " " $\sqrt[9]{.067}$.

166. “ “ “ “ $\sqrt[5]{.346.}$
167. “ “ “ “ $\sqrt[4]{.586.}$
168. “ “ “ “ $\sqrt[7]{.463.}$
169. “ “ “ “ $\sqrt[5]{94,800.}$
170. “ “ “ “ $\sqrt[7]{.019.}$
171. “ “ “ “ $\sqrt[6]{.9244.}$
172. “ “ “ “ $\sqrt[4]{.987.}$

FRACTIONAL ROOTS

For numbers greater than unity, fractional roots are extracted in the same manner as integral roots.

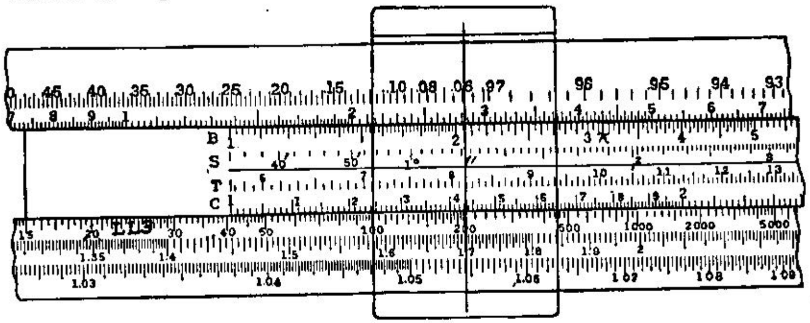


Fig. 48. $\sqrt[1.43]{201.}$

Example: Find the value of $\sqrt[1.43]{201.}$
To 201 on LL 3 set 1.43 on *C*.
Opposite left index of *C*, read 40.8 on LL 3.

Example: Find the value of $\sqrt[2.67]{1.976.}$
To 1.976 on LL 2, set 2.67 on *C*.
Opposite left index of *C*, find 1.291 on LL 2.

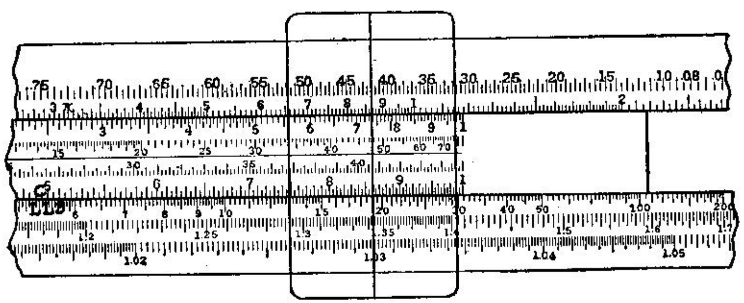


Fig. 49. $\sqrt[.86]{19.1.}$

Example: Find the value of $\sqrt[.86]{19.1.}$
To 19.1 on LL 3, set 86 on *C*.

Opposite right index of *C*, read 30.9 on LL 3.

This is the same as finding the value of $x = (19.1)^{1.163}$ where 1.163 is the reciprocal of .86.

Set indicator to .86 on *C*. Under indicator find 1.163 on *CI*.
To 19.1 on LL 3, set left index of *C*.
Opposite 1.163 on *C*, find 30.9.

If $x = \sqrt[a]{b}$
where a is less than 1
and b is greater than 1
then x is greater than 1.

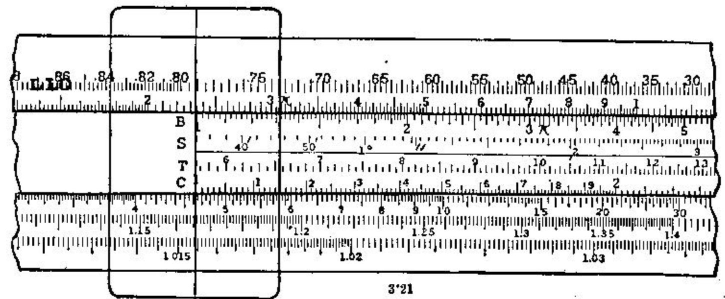


Fig. 50. $\sqrt[3.21]{0.47}$

Example: Find the value of $x = \sqrt[3.21]{0.47}$
To 0.47 on LL 0, set 3.21 on *B*.
Opposite left index of *B*, read .791 on LL 0.

Example: Find the value of $x = \sqrt[.83]{0.614}$
To 0.614 on LL 0, set .83 (right half of scale) on *B*.
Opposite right index of *B*, read .556 on LL 0.

For finding the roots of numbers which are not on the scale, proceed as in the extraction of integral roots of the same type.

Problem.

173. Find the value of $x = \sqrt[3.67]{18300.}$
174. “ “ “ “ $x = \sqrt[4.21]{1,140.}$
175. “ “ “ “ $x = \sqrt[2.76]{224.}$
176. “ “ “ “ $x = \sqrt[5.82]{21.3}$
177. “ “ “ “ $x = \sqrt[1.98]{2.576}$
178. “ “ “ “ $x = \sqrt[7.32]{1.3685}$
179. “ “ “ “ $x = \sqrt[4.19]{1.01463}$
180. “ “ “ “ $x = \sqrt[1.76]{.513}$
181. “ “ “ “ $x = \sqrt[3.42]{.073}$

182. Find the value of $x = \sqrt[.13]{5.06}$ *

183. " " " " $x = \sqrt[.29]{4.72}$ *

*Answer must be read opposite right index, which is off the scale.

184. Find the value of $x = \sqrt[95]{1.923}$

185. " " " " $x = \sqrt[36]{1.0778}$

186. " " " " $x = \sqrt[.87]{.081}$

187. " " " " $x = \sqrt[.51]{.422}$

188. " " " " $x = \sqrt[1,41]{28,962}$

189. " " " " $x = \sqrt[.92]{41,341}$

190. " " " " $x = \sqrt[2,21]{.962}$

191. " " " " $x = \sqrt[1.83]{.983}$

192. " " " " $x = \sqrt[3,16]{.014}$

193. " " " " $x = \sqrt[.64]{.992}$

MULTIPLICATION OF MORE THAN TWO NUMBERS

Using only scales C and D.

Example: Find the value of $4.1 \times 56 \times .26 \times .49$.

Using scales C and D, set the right index on C to 41 on D and move the indicator to 56 on C. We have now multiplied 41 by 56. The result thus far found on D, opposite the indicator is 2296, without regard to the decimal point.

Now set the left index of C to the indicator and move the indicator to 26 on C, thus adding the log of 26 to the former result. On D under the indicator is 597.

Set the right index to the indicator and move the indicator to 49 on C.

On D opposite the indicator, find 2925.

The position of the decimal point is determined by a rough calculation.

$4.1 \times 56 \times .26 \times .49$ is, roughly, $4 \times 60 \times \frac{1}{4} \times .5 = 30$.

Placing the decimal point so as to make 2925 read as near 30 as possible, it is evident that the result is 29.25.

Find the value of

Problem 194. $7.1 \times 31 \times .42$

195. $.64 \times 32 \times 5.6$

196. $16.3 \times 1210 \times 3.65 \times 243$

Problem 197. $8.25 \times .036 \times 1.07 \times 4.12$

198. $6.2 \times 37.8 \times .0052 \times 46$

On the Log Log Duplex Rule the operations in these problems may be considerably abridged by the method of page 25.

THE USE OF SCALES CI AND CIF

Important improvements found in the Log Log Duplex Slide Rule are the inverted scales CI and CIF.

These scales enable reciprocals of all numbers to be read at once without setting the slide. They also allow three factors to be taken at a single setting, thus saving one or more settings in many formulas, and increasing both speed and accuracy.

Scale CI (or C inverted) is like scale C, except that the numbers are placed on the rule in inverted order. Reading from left to right, the numbers on C run from 1 to 10, while those on CI run from 10 to 1.

Scale CIF is like scale CI, except that on CIF the number 1 is near the middle of the rule.

In the following problems the combination of scales is selected that will keep at least half of the slide within the groove.

RECIPROCALS

Two numbers are reciprocals if their product is equal to 1, or we may say that the reciprocal of a number is 1 divided by that number.

e. g., 5 and 1/5 are reciprocals since $5 \times 1/5 = 1$.

To find the reciprocal of a number:

Set the indicator to the given number on scale C.

Opposite the indicator on scale CI will be found the significant figures of the reciprocal.

The decimal point is placed by inspection.

Or:—

Set the indicator to the given number on scale CF.

Opposite the indicator on scale CIF will be found the significant figures of the reciprocal.

Since both CF and CIF are on the slide, no setting of the indexes is necessary.

Example: Find the reciprocal of 2.

Set the indicator to 2 on C.

Opposite the indicator on CI, find 5.

Placing the decimal point by inspection, the result is .5.

The same result can be found by moving the indicator to 5 on CF and reading the result directly on CIF.

Example: Find the reciprocal of .236.

Set the indicator to 236 on C.

On CI, opposite the indicator, find 424.

Roughly calculating $1/.236 = 1/.2 = 5$.

Hence, the result is approximately 5, or 4.24.

The same result can be found by moving the indicator to 236 on CF and reading the result directly on CIF.

Find the reciprocals of the following numbers:

Problem 199. 7.2

200. .41

201. 37.8

202. 68.2

203. .073

Problem 204. .182

205. 56.5

206. .85

207. 7.35

208. .0063

MULTIPLICATION

Example: Multiply 3 by 2, using the inverted scale.
Set the indicator to 3 on *D*.
To the indicator set 2 on *CI*.
Opposite the right index find 6 on *D*; or opposite 1 on *CIF*, find 6 on *DF*.

Note that 3 on scale *CI* is also in alignment with 2 on scale *D*. Hence, we may set the indicator to 2 on *D*. To the indicator set 3 on *CI*. Opposite the right index find 6 on *D*.

Explanation

log 3	+ log 2	= log 6
measured on *D* or *DF*	measured on *CI* or *CIF*	measured on *D* or *DF*

Example: Multiply 3 by 5, using the inverted scale.
Set the indicator to 3 on *D* or *DF*.
To the indicator set 5 on *CI* or *CIF*.
Opposite the left index find 15 on *D*, or opposite 1 on *CIF* find 15 on *DF*.

Exercise

Work problems on page 25, using scales *CI* and *CIF*, and compare with the previous method of using the *C* and *D* scales alone.

DIVISION

Example: Divide 28 by 7, using the inverted scale.
To 28 on scale *D*, set left index.
Set indicator to 7 on *CI*.
Opposite indicator on scale *D* read 4.
Or:—
To 28 on scale *D* set right index.
Set indicator to 7 on *CIF*.
Opposite indicator on scale *DF*, read 4.

SUCCESSIVE DIVISION.

The inverted scale is useful in problems of the type $x = \frac{a}{y}$, where a is a constant and y assumes successive values.

Example: A field rheostat on an electric generator is used to vary the resistance so as to give it the following values in ohms: 250, 298, 347, 401, 453, 496.

If the voltage is 125, what are the values of the field current?

$$I = \frac{E \text{ (Constant)}}{R \text{ (Varying)}}$$

Where I is the current in amperes; E the electromotive force in volts; and R the resistance in ohms.

$$I = \frac{125}{250}, \quad \frac{125}{298}, \quad \frac{125}{347}, \quad \frac{125}{401}, \quad \frac{125}{453}, \quad \frac{125}{496}.$$

To 125 on *D*, set 10 on *CI* (left index), or to 125 on *DF*, set 1 on *CIF*.

Opposite	250	on *CI*	or *CIF*,	read	.500	on *D*	or *DF*.
"	298	"	"	"	.419	"	"
"	347	"	"	"	.360	"	"
"	401	"	"	"	.312	"	"
	453	"	"	"	.276	"	"
"	496	"	"	"	.252	"	"

From one setting of the slide, all six values are read. By the use of scales *C* and *D* alone, six settings of the slide would have been required.

THREE FACTORS

$$a \times b \times c = x$$

Method I. Using *CI*.
To a on *D*, set b on *CI*.
a. At c on *CF* read x on *DF*.
or *b.* At c on *C* read x on *D*.

Example: $2 \times 3 \times 7 = x$.
To 2 on *D* set 3 on *CI*.
At 7 on *CF*, read 42 on *DF*.

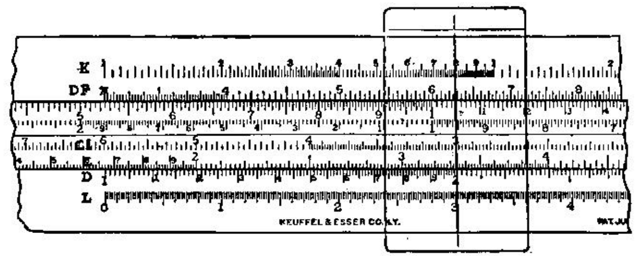


Fig. 51. $2 \times 3 \times 7$

Explanation

log 2	+ log 3	+ log 7	= log 42
measured on *D*	measured on *CI*	measured on *CF*	measured on *DF*

Note that the result is obtained with only one setting of the slide; and with no uncertainty as to whether the right or left index should be read. When the *C* and *D* scales alone are used, two settings of the slide are required, involving some uncertainty as to which index to use on steps 1 and 3.

Example: $2 \times 3 \times 4 = x$.
To 2 on *D*, set 3 on *CI*
At 4 on *C*, read 24 on *D*.

Try this example by Method I*a*, following the directions given in the example and note that the third factor, 4, on *CF* is beyond the end of the rule. Hence, instead of using *CF* and *DF*, we use *C* and *D*.

Explanation

log 2	+ log 3	+ log 4	= log 24
measured on *D*	measured on *CI*	measured on *C*	measured on *D*

Method II. Using *CIF*.
To *a* on *DF*, set *b* on *CIF*.
a. At *c* on *CF*, read *x* on *DF*.
or *b*. At *c* on *C*, read *x* on *D*.

Example: $1.2 \times 1.2 \times 2 = x$.

If we follow Method I, the third factor, 2, is beyond the end of the rule both on scales *CF* and *C*.

By Method II.
To 1.2 on *DF*, set 1.2 on *CIF*.
To 2 on *CF*, read 2.88 on *DF*.
Or at 2 on *C*, read 2.88 on *D*.

Explanation

log 1.2	+ log 1.2	+ log 2	= log 2.88
measured on *DF*	measured on *CIF*	measured on *CF* or *C*	measured on *DF* or *D*

Example: $\dfrac{72}{.75 \times a(=6.4)}$.

To 72 on *D*, set 75 on *C*.
Opposite any value of *a*, say 64, on *CI* or *CIF*, read 15 on *D* or *DF*.
The decimal point of the result can be placed by inspection.

Note that when the *C* and *D* scales alone are used in solving problems of this type, two settings will be required, including a separate setting for each value of the variable (*a*) of the denominator; whereas by employing the *CI* and *CIF* scales also, a single setting of the slide permits a solution for all values of the variable (*a*).

Using scale *CI*, find the value of:

Problem 209. $6.1 \times 24 \times 8.2$
210. $6.1 \times 24 \times .32$
211. $.53 \times 42 \times 6.5$
Problem 212. $53 \times 42 \times 1.6$
213. $54.3 \times 1.26 \times 2.3$

Using scale *CIF*, find the value of:

Problem 214. $1.3 \times 1.5 \times 2.1$
215. $6.1 \times 8.5 \times 2.5$
Problem 216. $6.25 \times 9.5 \times 1.5$
217. $6.25 \times 9.5 \times 1.8$
218. Find the area of a circle whose radius is 4.5 ft.

(Use the formula $A = \pi r^2$, and arrange the slide rule work in the form:

$$r \times r \times \pi$$

Set *r* on *CIF* to *r* on *DF*, read area at right index of *C* on *DF*.

Another method is to set the left index of the slide to 4.5 on *D* of the "*K*" face of the rule. Opposite π on *B* read 63.6 on *A*.

For multiplication and division of four numbers, one of which is π, see page 131.

FOUR FACTORS

Example: $1.43 \times 5.12 \times 1.76 \times 0.725 = x$.

Method I.
1. Set indicator to 143 on *D*.
2. To indicator set 512 on *CI*. Answer 7.32 on *D*, opposite right index.
3. Indicator to 176 on *C*. Answer 12.89 on *D* opposite indicator.
4. Set 725 on *CI* to indicator.
5. Opposite right index, read 9.34 on *D*.

This method requires only two settings of the slide.

Method II. Using only scales *C* and *D*.
1. Set left index to 143 on *D*.
2. Move indicator to 512 on *C*. Answer 7.32 on *D*, opposite indicator.
3. Set right index to indicator.
4. Indicator to 176 on *C*. Answer 12.89 on *D*, opposite indicator.
5. Left index to indicator.
6. Opposite 725 on *C*, read 9.34 on *D*.

This method involves three settings of the slide, and some uncertainty as to which index to use in steps 1 and 3. Hence, Method I will save considerable time in the solution of problems of this type.

FIVE FACTORS

Example: $2 \times 3 \times 4 \times 5 \times 6 = x$.

To 2 on *D*, set 3 on *CI*.
Indicator to 4 on *C*.
Set 5 on *CI* to indicator.
Opposite 6 on *C*, read 72 on *D*.
Placing the decimal point, the result is 720.

Note that the five factors are handled with only two settings of the slide. Without the *CI* scale, four settings of the slide would be required.

Occasionally, in finding the product of three or more numbers, using scale *CI*, it would be necessary to reset the index if the folded scales *DF* and *CF* were not present.

Example: $2 \times 3 \times 1.3 = x$.
To 2 on *D*, set 3 on *CI*.
Since 1.3 on *C* is beyond the left end of the rule:
Move indicator to 10 on *C*.
Set 1 on *C* to the indicator.
Opposite 1.3 on *C*, read 7.8 on *D*.

One setting of the slide can be saved, thus:
To 2 on *D*, set 3 on *CI*.
Opposite 1.3 on *CF*, read 7.8 on *DF*.

Using scales *CI* and *CIF*, find the value of:

Problem 219. $16.3 \times 3.65 \times 243 \times 1210$.
220. $8.25 \times .036 \times 1.07 \times 4.12$.
221. $37.8 \times .0052 \times 46 \times 6.2$.
222. $6.3 \times 2.5 \times .17 \times 5.4 \times 3.4$.

Example: Solve $x = \frac{\sqrt{78}}{y}$, where y has the series of values—1.2, 2.4, 3.6, 4.8 and 9.0.

Set right index to 78 on scale A; opposite any value of y on scale CI, read x on D; or opposite any value of y on scale CIF, read x on DF.

When y =	x =	
1.2	7.36.	Answer.
2.4	3.68	
3.6	2.45	
4.8	1.84	
9.0	9.81	

The last answer is read on DF opposite 9 on CIF, since 9 on CI is off the scale. The presence of the folded scale saves one resetting of the slide.

Example: Solve $x = \frac{y}{\sqrt{2.7}}$.

Set 2.7 on scale B to index.
Opposite any value of y on scale C, read x on D.
Example: Find the value of $4.3\ \sqrt[3]{25}$.
Using indicator, to 25 on scale K_2, set 4.3 on scale CI.
At index, read 12.57 on D.

Example: Find the value of $\frac{\sqrt[3]{760}}{94}$.

To 760 on K_3, set 94 on C.
At index of CF (at middle of scale), find .0973 on DF.
The decimal point may be placed by inspection.
The cube root of 760 has one integer, roughly 9; which, divided by 90, shows the magnitude of the answer.

Example: Solve $x = \frac{.27}{\sqrt[3]{.069}}$.

Set indicator to 69 on K_2.
Set 27 on scale C to indicator.
At index on D, read 658 on scale C.

Roughly calculating: $\frac{.27}{\sqrt[3]{.069}} = \frac{.28}{\sqrt[3]{.064}}$.

$$= \frac{.28}{.4} = .7$$

Hence, the result is .658
Example: Solve $x = 3\sqrt{23} \times \sqrt[3]{127}$.
To 127 on scale K_3, set 3 on scale CI.

Indicator to 23 on scale B_2.
Opposite indicator on scale D, read 723.
Roughly calculating: $3\sqrt{23} \times \sqrt[3]{127} = 3 \times \sqrt{25} \times \sqrt[3]{125}$.
$= 3 \times 5 \times 5$.
$= 75$.
Hence, the result is 72.3.

CUBES

Using the inverted scales CI and CIF, any number may be cubed at a single setting of the slide.

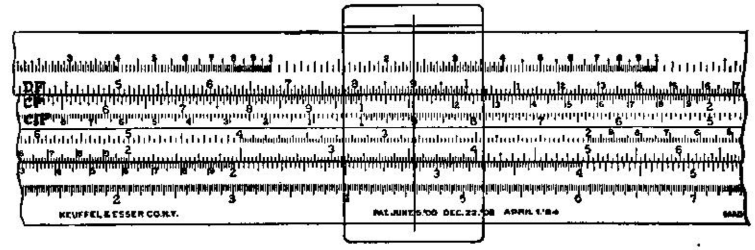


Fig. 52. $(9)^3$

Example: $x = 9^3$.
To 9 on DF, set 9 on CIF.
Opposite 9 on CF or C, find 729 on DF or D.

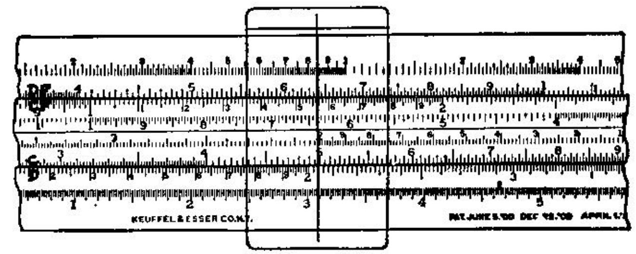


Fig. 53. $(6.4)^3$

Example: $x = 6.4^3$.
To 6.4 on DF, set 6.4 on CIF.
Opposite 6.4 on C, find 262 on D.

When using the C-D or CF-DF scales alone the slide must be set twice and the indicator once.

Through the use of the CI-CIF scales only one setting of the slide and one setting of the indicator is necessary.

Solve the examples and problems on pages 33 and 34, using the CI and CIF scales as described above.

The inverse operation of extracting the cube root, while it can be done, is neither an easy nor a practical one. Hence, it is best to confine the operation of extracting the cube root to the K and Log Log scales.

CHAPTER III

ADVANCED PROBLEMS

COMBINED MULTIPLICATION AND DIVISION

Example: Find the value of

$$\frac{23.5 \times 45.3}{2670}.$$

To 235 on *D*, set 267 on *C*. Opposite 453 on *C* find 399 on *D*. To obtain the decimal point make a rough calculation as follows:

$$\frac{23.5 \times 45.3}{2670} \text{ is roughly equal to } \frac{20 \times 50}{3000} = \frac{1}{3}.$$

Hence, we must place the decimal point so as to make 399 approximately equal to ⅓. The result is evidently .399.

Another method of placing the decimal point:

$$\frac{23.5 \times 45.3}{2670} = \frac{(2.35 \times 10) \quad (4.53 \times 10)}{2.67 \times 1000}$$

$$= \frac{2.35 \times 4.53}{2.67} \times \frac{1}{10}$$

$$= 3.99 \times \frac{1}{10}$$

$$= .399$$

The first method will be found preferable, but may be checked by the second.

Example. Find the value of $\frac{1.34 \times 2.15}{4.2}$.

Method I. To 1.34 on *D*, set 4.2 on *C*.
Above 2.15 on *CF*, find 686 on *DF*.
Method I is preferable.

Method II. To 1.34 on *D*, set 4.2 on *C*. When we attempt to move the indicator to 2.15 on *C*, it is impossible, because 2.15 projects beyond the left end of the rule. Bring the indicator to 10 on *C* and move the slide so as to set the left index to the indicator. This divides by 10, but is permissible, since dividing by 10 does not change the order of significant figures. Now move the indicator to 2.15 on *C*; and on *D*, opposite the indicator, read 686. A rough calculation shows that:

$$\frac{1.34 \times 2.15}{4.2} \text{ is approximately equal to } \frac{1 \times 2}{4} = \frac{1}{2}, \text{ or } .5.$$

Hence, the result is .686.

Example. Find the value of

$$\frac{30.5 \times 50.6 \times 835}{3.64 \times 380 \times 42.5} = x.$$

$$\frac{\overset{D}{30.5} \times \overset{C}{50.6} \times \overset{C}{835}}{\underset{C}{3.64} \times \underset{C}{380} \times \underset{C}{42.5}} = \overset{D}{x}.$$

The five operations are as follows: — Intermediate Results on *D*.

1. At 305 on *D* set 364 on *C*. — 838, opposite right index.
2. Move indicator to 506 on *C*. — 424, opposite indicator.
3. Set 380 on *C* to the indicator. — 111.6, opposite left index.
4. Indicator to 835 on *C*. — 931, opposite indicator.
5. Set 425 on *C* to the indicator, and opposite the index on *C*, find 219 on *D*.

Calculating roughly,

$$\frac{30 \times 50 \times 800}{3 \times 400 \times 40} = 25.$$

Hence 219 must be made to look as near as possible like 25, giving the result 21.9. It is not necessary to obtain the intermediate results, but with beginners it is an advantage to check the work at every step.

Example: Find the value of

$$\frac{25.4 \times 570 \times 26.8 \times 8.63 \times 1.3}{1.55 \times 8350 \times 4.15 \times 2.24}.$$

$$\frac{\overset{D}{25.4} \times \overset{C}{570} \times \overset{C}{26.8} \times \overset{C}{8.63} \times \overset{C}{1.3}}{\underset{C}{1.55} \times \underset{C}{8350} \times \underset{C}{4.15} \times \underset{C}{2.24}} = \overset{D}{x}.$$

	Intermediates on *D*
1. At 254 on *D*, set 155 on *C*	164
2. Move indicator to 570 on *C*.	934
3. Move the slide, setting 835 to indicator.	112
4. Indicator to 268 on *C*.	300
5. Move slide, setting 415 on *C* to indicator.	722
6. Indicator to 863 on *C*.	623
7. Move slide, setting 224 to indicator.	278
8. Indicator to 13 on *C*.	362

Find the answer 363 on *D* opposite the indicator.

Calculating roughly:

$$\frac{25 \times 600 \times 30 \times 8 \times 1}{1 \times 8000 \times 4 \times 2} = 60.$$

Making 363 look as much as possible like 60, we have 36.3.

Example: Find the value of

$$\frac{7.45}{3.65 \times .0267}$$

The preceding examples have had as many factors in the numerator as in the denominator or one more. This example can be changed to conform to these types by introducing unity as a factor in the numerator.

Method I. Using scales *CI*, *C*, and *D*.

1. To 745 on *D* set 365 on *C*.
2. Opposite 267 on *CI* read 764 on *D*.

Roughly calculating:

$$\frac{8 \times 1}{4 \times .02} = \frac{2.00}{.02} = 100.$$

Making 764 look as much as possible like 100, the result is 76.4.

The method is preferable, since it requires only one setting of the slide.

Method II. Using only scales C and D.

$$\frac{7.45}{3.65 \times .0267} = \frac{\overset{D}{7.45} \times \overset{C}{1}}{\underset{C}{3.65} \times \underset{C}{.0267}} = x.$$

	Check by Intermediates on D.
1. Divide 7.45 by 3.65.	204, opposite left index.
2. Move indicator to 1 on C.	204, opposite indicator.
3. Move slide, setting 267 to indicator.	764, opposite right index.

Example: Find the value of $\dfrac{1}{2.34 \times .33 \times 5.25}$.

	Check by Intermediates on D
1. To 1 on D, set 234 on C.	427, opposite right index.
2. Indicator to 33 on CI.	1295, opposite indicator.
3. 525 on C to indicator.	2467, opposite right index.

Rough calculation:

$$\frac{1}{2 \times \frac{1}{3} \times 6} = \frac{1}{4} = .25.$$

Making 2467 look as much as possible like .25 the result is .2467.

Example: Find the value of: $\sqrt{\dfrac{21.4 \times 3.45 \times 640}{4.15 \times .75 \times .08}}$.

Method I.—Work the example without regard to the square root, then find the square root of the result.

Method II.—Using scales A and B:

$$\frac{\overset{A}{21.4} \times \overset{B}{3.45} \times \overset{B}{640}}{\underset{B}{4.15} \times \underset{B}{.75} \times \underset{B}{.08}} = \overset{D}{x}.$$

	Intermediate on A.
1. To 21.4 on A set 4.15 on B.	516, opposite index.

Be careful to use 21.4 on the right half of A and not 2.14 on the left half, since the square root of 21.4 has different significant figures from the square root of 2.14. For the same reason use 4.15 on the left half of B.

2. Indicator to 3.45 on B (left half of rule)	178, opposite indicator.
3. Move slide setting .75 (right half) to indicator.	237, opposite index.
4. Indicator to 6.4 (left half) on B.	152, opposite indicator.

Change 640 to 6.4, by moving the decimal point an even number of places, in order not to change the square root.

5. Move slide, setting 8 (left half) on B to indicator. 190, opposite index.
6. Opposite right index of B find 436 on D.

Rough calculation

$$\sqrt{\frac{20 \times 3 \times 600}{4 \times 1 \times .1}} = \sqrt{90000} = 300.$$

Placing the decimal point so as to make 436 as near as possible to 300, the result is 436.

Example: Find the value of $\dfrac{254 \times 65 \times 24}{155 \times 45}$.

	Intermediate Results
1. Opposite 254 on D, set 155 on C.	164, Opposite left index on D.
2. Move indicator to 65 on CF.	1065, Opposite indicator on DF.
3. Set 45 on CF to indicator.	237 On DF, opposite middle index (10) of CF.
4. Opposite 24 on C, find 568 on D.	

Placing the decimal point by a rough calculation

$$\frac{254 \times 65 \times 24}{155 \times 45} \text{ is roughly } \frac{250 \times 60 \times 25}{150 \times 50} = 50.$$

Hence, the result is 56.8.

Find the value of

Problem 223. $\dfrac{3.26 \times .0235}{4.22}$.

224. $\dfrac{6.75 \times 1.35}{14.4}$.

225. $26.4 \times 4.8 \times 7.12$.

226. $6.2 \times 28 \times .35 \times 5.4$.

Problem 227. $.65 \times 24 \times 7.5 \times 9.5$

228. $\dfrac{6.45}{4.55 \times .0276}$

229. $\dfrac{1}{2.66 \times .75 \times 1.42}$

230. $\dfrac{3}{2.54 \times 7.45}$

Problem 231. $\dfrac{2.14 \times 4.6 \times .39}{24.3 \times .06 \times .575}$

232. $\dfrac{5.8 \times 4.5 \times 8.7 \times 132}{7.3 \times 6.2 \times 28 \times 14}$

233. $\sqrt{\dfrac{2.63 \times 82.5}{2450}}$

234. $\sqrt{\dfrac{48.6 \times 22.4}{56.5 \times 245}}$

235. $\sqrt{\dfrac{22.5 \times 12.2 \times 126 \times 405}{2760 \times 715 \times 6.16}}$

MISCELLANEOUS CALCULATIONS

Example: Find the value of $\dfrac{2.45 \times (76.5)^2 \times 625}{.55 \times .087}$.

Method I. Use scales A and B, but use C for 76.5.
At 245 on A set 55 on B.
Indicator to 765 on C.
Set 87 on B to the indicator.
Indicator to 625 on B.
Opposite the indicator on A, find 187.
A rough calculation shows:

$$\frac{2 \times 70 \times 80 \times 600}{.5 \times .1} = \frac{200 \times 70 \times 80 \times 600}{5 \times 1} = 134000000.$$

The result is 187,000,000.

Method II. Write the example:

$$\frac{2.45 \times 76.5 \times 76.5 \times 625}{.55 \times .087 \times 1}$$

Method III. Find $(76.5)^2$ as a separate problem, then work the example on C and D.

Example: Find the value of $\dfrac{135 \times \sqrt{475} \times 430}{26 \times 250 \times 638}$.

Use C and D, but use B for 475.
At 135 on D, set 26 on C.
Indicator to 4.75 on B (left half of slide, because the decimal point must be moved an even number of places).

Set 250 to the indicator.
Indicator to 430 on C.
Set 638 to the indicator.
On D, opposite the right-hand index, find 305.

Roughly calculating:

$$\frac{100 \times 20 \times 400}{25 \times 250 \times 600} = \frac{16}{75} = \text{about } \frac{1}{5}, \text{ or } .2$$

The result, then, is .305.

Example: Find the value of

$$\frac{\sqrt{260} \times \sqrt{3.80}}{\sqrt{1310}}.$$

Use A and B, but read the result on D.
At 2.6 on A set 13.1 on B (moving the decimal point an even number of places).

If we try to move the indicator to 3.8 on B, 3.8 projects beyond the end of the rule. Hence, move the indicator to the right index of the slide, then set the left index to the indicator. This operation divides by 100, but does not change the significant figures of the result.

Now move the indicator to 3.8 on B.
On D, opposite the indicator, read 869.
Roughly calculating:

$$\sqrt{\frac{300 \times 3}{1600}} = \frac{3}{4} = .75$$

Hence the result is .869.

Settings:
The result is denoted by x

Problem 236. $\dfrac{13.5 \times (14)^2}{82}$

A :	To 135 :	Find x
B :	Set 82 :	
C :	:	Over 14
D :		

237. $1.35 \times \sqrt{2}$

A :	:	
B :	:	Under 2
C :	Set 1 :	
D :	To 135 :	Find x

238. $\dfrac{42.3}{\sqrt{6720}}$

A :	:	:
B :	Set 67.2 :	
C :		Under 10
D :	To 42.3 :	Find x

239. $\dfrac{5.2}{(3.4)^2}$

A :	To 5.2 :	Find x
B :	:	Over 1
C :	Set 3.4 :	
D :	:	

240. $\dfrac{(16.2)^2 \times 45.2}{(2.7)^2}$

A :	:	Find x
B :	:	Over 452
C :	Set 27 :	
D :	To 162 :	

241. $\left\{\dfrac{.0347}{.0058}\right\}^2$

A :	:	Find x
B :	:	Over 100
C :	Set 58 :	
D :	At 347 :	

242. $\dfrac{2.31 \times (48.5)^2 \times 413}{.45 \times .087}$

243. $\dfrac{175 \times \sqrt{285} \times \sqrt{17} \times 410}{28 \times 228 \times 634}$

244. $\dfrac{\sqrt{8.32} \times \sqrt{56.5}}{\sqrt{2830}}$

245. $\dfrac{2.6}{(7.4)^3}$

246. $\left\{\dfrac{.0325}{.0075}\right\}^3$

247. $\sqrt[3]{\dfrac{420 \times 1.65}{2.64}}$

CHAPTER IV

PLANE TRIGONOMETRY

SINES

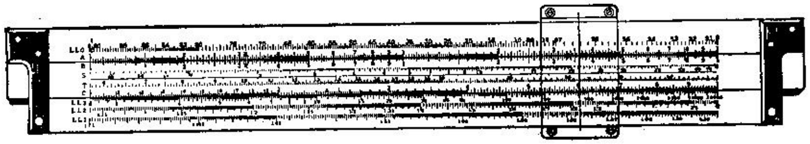


Fig. 54.

The scale marked S is a scale of sines. Angles are given on scale S, opposite their sines on scale B, and hence on scale A when the indexes of A and B coincide.

Example: Find sine 20°.

Opposite 20 on scale S is found its sine on scale B, and hence on scale A when the indexes of A and B coincide. This reads 342. To place the decimal point, a number read on the right half of scales A and B has the first significant figure in the first decimal place, except sine 90, which is 1; a number read on the left half of scales A and B has the first significant figure in the second decimal place.

Hence sine 20° = .3420.

Example: Find sine 2°.

The significant figures are 349.

The reading is on the left of scales A and B, hence the result is .0349.

COSINES

Since the cosine of an angle is equal to the sine of the complement of the angle, the cosine may be found on the slide rule.

Example: Find cos 30°.

Cos 30° = sin (90°—30°).
= sin 60°.
= .866.

Example: Find sin 5° 40′ × 35.

A : To 35	: Find 3.46	
S : Set Right Index	: Over 5° 40′	

Explanation

Log 35 is added to log sin 5° 40′, the sum being counted on scale A.

EXAMPLE: Find $\frac{35}{\sin 5° 40'}$.

A : To 35	: Find 354.
S : Set 5° 40′	: Over left index.

To place the decimal point, note that sin 5° 40′ is a trifle less than .1. This can be done just before setting down the answer by glancing at the value of the sine given under the indicator on the B scale. Hence, dividing 35 by .1, we have 350 for the rough calculation.

From log 35 we have subtracted log sin 5° 40′, the difference being counted on scale A.

EXERCISE

Problem 248. Find the sine of 90°.
249. " " " 45°.
250. " " " 30°.
251. " " " 3°.
252. " " " 40′.

Problem 253. Find the sine of 15° 20′.
254. " " " 1° 30′.
255. " " " 8° 30′.
256. " " " 2° 15′.
257. " " " 21° 30′.

Problem 258. Find the cosine of 80°.
259. " " " 65°.
260. " " " 42°.
261. " " " 14°.
262. " " " 12°.

Problem 263. Find the cosine of 75° 30′.
264. " " " 54° 10′.
265. " " " 20° 30′.
266. " " " 81° 45′.
267. " " " 88° 25′.

268. Sin 25° × 45.
269. Cos 56° × 27.

270. $\frac{18}{\sin 12° 30'}$.

271. $\frac{21.5}{\sin 42° 10'}$

Problem 272. A = 32°. (Fig. 55).
c = 65
Find a.

Problem 273. A = 70° 30′.
a = 15.4.
Find c.

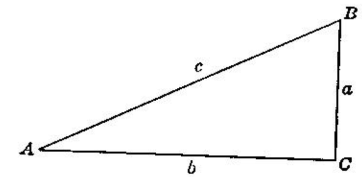


Fig. 55.

Problem 274. A disk is 21 inches in diameter. Find the distance necessary to set a pair of dividers in order to space off a,) 7 sides; b,) 8 sides; c,) 10 sides; d,) 13 sides.

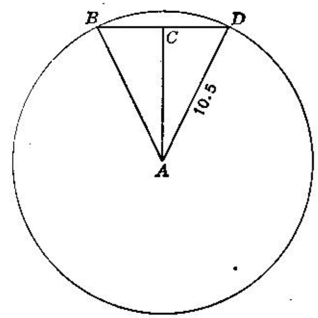


Fig. 56.

The angle $DAB = \frac{1}{7}$ of 360° = 51° 26′ (to the nearest minute).

The angle $DAC = \frac{1}{2}$ of 51° 26′ = 25° 43′.

$\frac{CD}{AD}$ = sine angle DAC.

$CD = AD \times$ sine angle DAC.

$BD = 2 \times CD = 2 \times AD \times$ sine angle DAC = d sine angle DAC where d = diameter of circle.

275. Holes A and C are to be drilled on the milling machine. After drilling C, in order to drill A, how much movement of the table will there be in each direction?

The table moves from C to B, then from B to A.

BC = 5 × cos 20°.
BA = 5 × sin 20°.

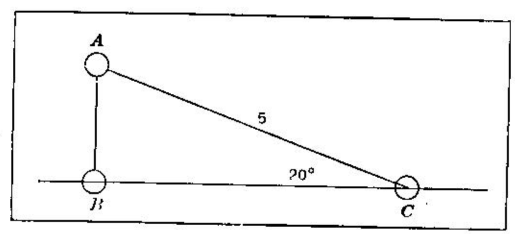


Fig. 57.

TANGENTS

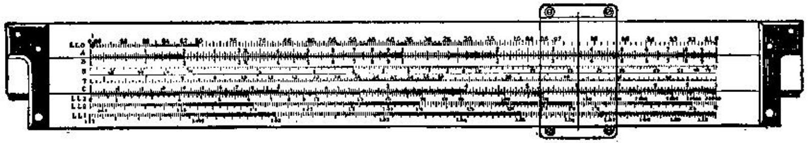


Fig. 58.

Scale T gives readings for angles whose tangents are found opposite on scale C.

The first significant figure comes in the first decimal place for all values found on the rule.

Example: Find tan 30°.

Opposite 30 on scale T, find 577 on C.

Pointing off, we have tan 30° = .5770, which is correct to three significant figures, the result correct to four figures being .5774.

Example: Find the value of tan 18° 30′ × 175.

Set left index of slide to 175 on D.

Opposite 18° 30′ on T, read 586 on D.

Since tan 18° 30′ is .334, the product must be roughly 1/3 of 175, making the result 58.6.

The scale gives tangents only as far as 45°.

For larger angles, use the formula:

$$\tan A = \frac{1}{\tan (90° - A)}.$$

Example: Find the tan of 75°.

$$\tan 75° = \frac{1}{\tan (90° - 75°)}.$$

$$= \frac{1}{\tan 15°}.$$

Opposite the right index of scale D, set 15° on scale T.

Opposite the left index of scale T, read 373 on scale D.

To place the decimal point, make a rough calculation, remembering that tan 45° = 1.

$$\frac{1}{\tan 15°} = \frac{1}{\frac{1}{3}} = 3.$$

Hence, the result is 3.73.

Owing to the presence of the CI scale, we may also obtain the answer directly by setting the indicator at 90° — A, and reading the tangent directly on the CI scale. This simplifies the operation, but is applicable only to angles less than 84°. For angles from 84° to 90° the formula

$$\tan A = \frac{1}{\tan (90° - A)} \text{ must be used.}$$

Tan (90°—A) can be obtained by finding sine (90°—A), as explained on page 66.

Example: Find the value of 565 ÷ tan 65°.

$$565 \div \tan 65° = 565 \div \frac{1}{\tan 25°}.$$

$$= 565 \times \tan 25°.$$

$$= 263.$$

Set right index of slide to 565 on D.

Opposite 25 on T find 263 on D.

Example: Find the value of 512 ÷ tan 22° 30′.

Method I. To 512 on D, set 22° 30′ on T.

Turn the rule over and opposite the index of C, find 1236 on D.

By rough calculation, remembering that tan 45° = 1, we place the decimal point, making the result 1236.

Method II. Set 22° 30′ on T to 1 on D.

Under 512 on C, find 1236 on D.

Method III. Set 22° 30′ on T to 10 on D.

Above 512 on CF, find 1236 on DF.

The tangent of an angle less than 5° 43′ cannot be obtained directly from the ordinary 10 in. rule, but the sine may be used in place of the tangent, since the sine and the tangent of any of these angles are identical to three significant figures.

$$\tan 1° 30' = \sin 1° 30' = .0262.$$

COTANGENTS

The cotangents of angles from 6° to 45° may be read directly upon the CI scale. In every case the first significant figure is a whole number.

Cotangents for angles greater than 45° may be found as follows:

$$\text{Cot } A = \tan (90° - A).$$

Example: Find cot 65°.

$$\text{Cot } 65° = \tan (90° - 65°).$$

$$= \tan 25°.$$

$$= .466.$$

SECANT AND COSECANT.

The secant and cosecant may be found by the formulas:

$$\sec A = \frac{1}{\cos A}.$$

$$\csc A = \frac{1}{\sin A}.$$

Problem 276. Find tangent of 25°.
277. " " " 14° 30′.
278. " " " 35° 30′.
279. " " " 26° 20′.
280. " " " 18° 30′.
281. " " " 55° 20′.

Problem 282. Find tangent of 75° 10′.
283. " " " 20° 10′.
284. " " " 15° 5′.
285. " " " 6° 25′.
286. " " " 1° 45′.
287. " " " 42° 20′.

Problem 288. Tan 15° x 18.

Problem 289. Tan 65° 30′ × 13.2 = $\frac{13.2}{\tan 24° 30'}$.

Problem 290. $\frac{5.62}{\tan 10°}$.

Problem 291. $\frac{8.5}{\tan 70° 20'} = 8.5 \times \tan 19° 40'$.

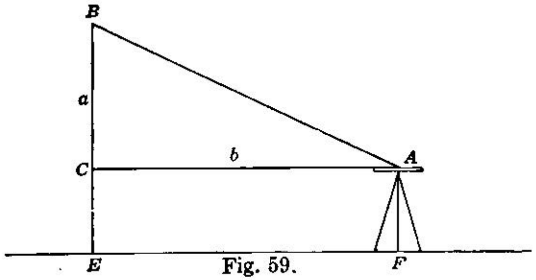


Fig. 59.

Example: To find BE, the height of a building, a transit is set up at A; a level line AC is sighted on a rod held at E.

CE is found to be 5.2 ft.

EF, which is equal to CA, is measured and found to be 138 ft.

The angle CAB is taken by the transit and found to be 28° 30′.

Find BE, the height of the building.

$$BE = BC + CE.$$
$$BC = CA \times \tan A.$$
$$BE = CA \times \tan A + CE.$$
$$= 138 \times \tan 28° 30' + 5.2.$$
$$= 74.9 + 5.2.$$
$$= 80.1 \text{ ft.}$$

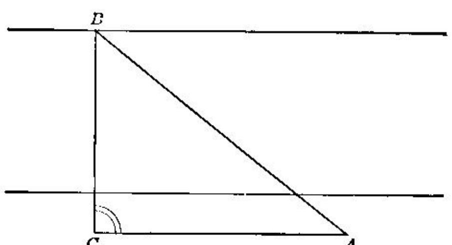


Fig. 60.

Example: To find CB, the width of a river.

A transit is set up at C and a right angle, BCA is laid off.

CA is measured and found to be 235 ft.

Then the transit is set up at A and the angle *A* found to be 75° 30′.

Find CB, the width of the river.

$$CB = CA \times \tan A.$$
$$= 235 \times \tan 75° 30'.$$
$$= \frac{235}{\tan 14° 30'}.$$
$$= 909 \text{ ft.}$$

SINES AND TANGENTS OF SMALL ANGLES

Gauge points are placed on the sine scale for reading sines of angles smaller than those given on the regular scale. Near the 1° 10′ division is the "second" gauge point and near the 2° division is the "minute" gauge point. By placing one of these gauge points opposite any number on the *A* scale, the corresponding sine of that number of minutes or seconds is read over the index of the sine scale on *A*. Or place the gauge point opposite the left index. Then for any value on scale *B* the corresponding sine may be read on scale *A* for angles from 4′ to 100′ or from 3″ to 100″, depending upon which gauge point is used. By placing the gauge point opposite the right index, sines for angles as small as 1″ may be read. In order to point off, it should be remembered that sine 1″ is about .000005 (5 zeros, 5), and sine 1′ is about .0003 (3 zeros, 3).

The sines and tangents of small angles being practically identical, these gauge points, as well as the portion of the sine scale below 5° 43′, may also be used for the tangents.

The tangents of angles greater than 89° 26′ are found as follows:

Determine 90°—*A*.

Set gauge point to 90°—*A* on *A*.

Opposite right index of *A* read the tangent of 90°—*A* on *B*.

This is true because the tangent (90°—*A*) is the reciprocal of tan *A*.

Example: Find sine 10″.

Opposite 10 on scale *A*, set the gauge point for seconds.

Opposite the left index find 485 on *A*.

Since sine 1″ = .000005,

sine 10″ is roughly 10 × .000005, or .00005.

Hence sine 10″ = .0000485.

Example: Find sine 12′.

Opposite 12 on scale *A*, set the gauge point for minutes.

Opposite the left index find 349 on *A*.

Since sine 1′ = .0003,

sine 12′ is roughly 12 × .0003 = .0036.

Making 349 look as near as possible like .0036,

sine 12′ = .00349.

Example: Find tan 89° 45′.

$$90° - 89° 45' = 15'.$$

Set minute gauge point to 15 on *A*.

Opposite right index of *A*, read 229 on *B*.

The tangent of 89° 45′ is actually 229.18 +.

Example: Find tan 89° 45′ 45″.

$$90° - 89° 45' 45'' = 14' 15'' = 855.''$$

Set "second" gauge point to 855 on *A*.

Opposite right index of A, read 241 on B.

The tangent of 89° 45′ 45″ is actually 241 .46 +.

Another method of finding sines and tangents of very small angles depends upon the fact that, for small angles, the sine or the tangent varies directly as the angle.

Example: Find tan 15′.

$$\begin{aligned} \text{Tan } 15' &= \sin 15'. \\ &= \frac{1}{10} \sin 150'. \\ &= \frac{1}{10} \sin 2^\circ\, 30'. \\ &= \frac{1}{10}\ .0436 \text{ by the slide rule.} \\ &= .00436. \end{aligned}$$

To Change Radians to Degrees or Degrees to Radians

$$\frac{\pi}{180} = \frac{\text{Radians}}{\text{Degrees}}.$$

A	Opposite π	Opposite Radians	or	Read Radians
B	Set 180	Read Degrees		Opposite Degrees

LOGARITHMS

The L scale is a scale of equal parts by which the common logarithm of any number may be found.

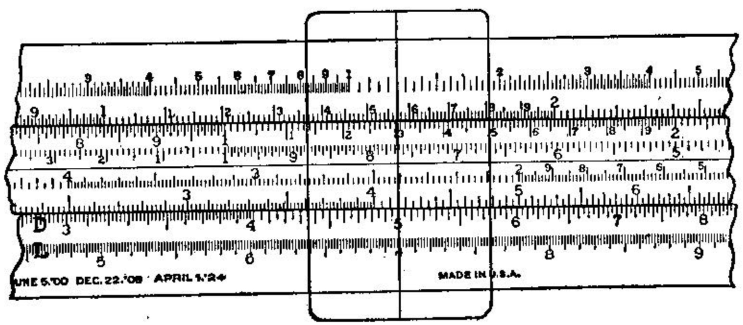


Fig. 61. Log 50.

Example: Find log 50.

Under 5 on scale D, find 699 on scale L.

This 699 is the mantissa. The characteristic is found by the usual rule, taking one less than the number of figures at the left of the decimal point. Since 50 has two such figures, its characteristic is 1.

Hence, log 50 is 1.699.

THE LOG LOG (LL) SCALES.

The logarithms of numbers to any base may also be found by means of the Log Log Scales.

The Log Log scale of quantities greater than unity is, for convenience, graduated in three sections—LL1, LL2 and LL3—which, if placed end to end, would form a continuous scale from the lower to the upper limit; *i. e.*, from 1.01005 to 22026.3. These sections are so arranged that "e"—2.71828—the base of Hyperbolic or Natural Logarithms, and its decimal powers and roots coincide with the left and right indexes of the other scales. Consequently scale LL1 is graduated from $e^{\frac{1}{100}}$ to $e^{\frac{1}{10}}$; LL2 from $e^{\frac{1}{10}}$ to e; and LL3 from e to e^{10}

It is not possible to have a continuous Log Log Scale through unity to fractional decimal quantities, since the curve passes through minus infinity at unity. Hence, since Log Log 1.01 is —2.364, and Log Log 1.00 is minus infinity, the decrease in the values of the Log Log scale is so very rapid as compared to the corresponding decrease in the series of numbers from which it is derived, that it would require an enormously long scale to make its employment practicable. Even if practicable, the employment of this section of the Log Log scale would not result in any distinct gain, since the solution of problems involving quantities between 1.01 and 0.97 is in general more simply performed by other means. These means are described elsewhere in the text.

For the reasons just given the LLO scale covers the range from 0.97 to .05 and is so placed that $e^{-.1}$ is in alignment with the right and left indexes of the A—B scales, and e^{-1} in alignment with the central index. It is so positioned that the hyperbolic co-logarithms of the numbers on it are read directly on scale A.

The location of "e" and its decimal powers and roots permits the hyperbolic logarithms of quantities greater than unity to be read on D without setting the slide; and the hyperbolic co-logarithms of quantities less than unity to be read on A without setting the slide. If the indexes of the slide and rule are aligned in the usual manner, the hyperbolic logarithms of quantities greater than unity may be read on C, and the hyperbolic co-logarithms of quantities less than unity on B.

Since scale LL3 runs from e to e^{10}, the characteristic of the hyperbolic logarithm of any quantity on this scale lies between 1 and 10. Hence, it is the first figure of the reading on D.

Example: Find Log_e 50.

Indicator to 50 on LL3.

At Indicator on *D*, read 391.

The first digit is the characteristic.

Log_e 50, therefore, is equal to 3.91.

Example: Find log_e 200.

Indicator to 200 on LL3.

At Indicator on *D*, read 5.30.

The quantities on scales LL1 and LL2 are greater than unity, but less than base "*e*". They have the characteristic zero.

Since the logarithm of a quantity on LL2 is equal to $^1/_{10}$ the logarithm of the quantity directly above it on LL3, any hyperbolic logarithm of a quantity on LL2 is preceded by a decimal point.

Example: Find log_e 1.48.

Indicator to 1.48 on LL2.

At Indicator on *D*, read 392.

The characteristic is zero and the decimal point immediately precedes the first digit.

Log_e 1.48 = 0.392.

Example: Find log_e 1.7.

Indicator to 1.7 on LL2.

At Indicator on *D*, read 0.531.

Since the logarithm of a quantity on LL1 is equal to $^1/_{100}$ of the logarithm of the quantity directly above it on LL3 any hyperbolic logarithm of a quantity on LL1 is preceded by a decimal point and one zero.

Example: Find log_e 1.04.

Indicator to 1.04 on LL1.

At Indicator on *D*, read 392.

The characteristic is zero and the decimal point and one zero precede the first digit.

Log_e 1.04 = 0.0392.

Example: Find log_e 1.0545.

Indicator to 1.0545 on LL1.

At Indicator on *D*, read 0.0531.

Logarithms to any other base than "*e*" may be found by bringing the index of *C* to the number on the LL scale representing the base.

Logarithms to the base 10 (common or Briggs logarithms) may be found directly by means of the *L* scale, or by setting the index of *C* to 10 on LL3.

Example: Find log 50.

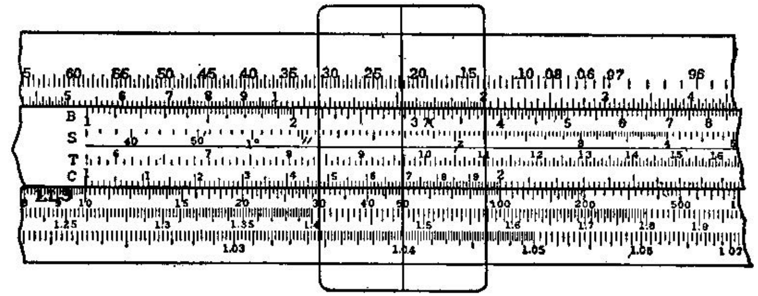


Fig. 62. Log 50.

Set left index of *C* to 10 (the base of common logarithms) on LL3. Then the log of any number on LL1, LL2 and LL3 may be read on scale *C*. Thus log 50 is read as 1.699; *the first figure on scale C being the characteristic for all numbers greater than 10.*

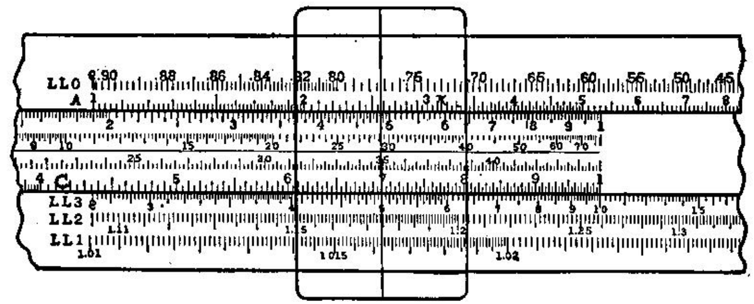


Fig. 63. Log 50.

Or log 50 may be found by setting the right index of *C* to 10 on LL3. Then opposite 5 on LL3 read .699 on *C*. In this case, the characteristic 1 must be supplied. Hence, the logs of any number from 1.01 to 10, and their multiples, may be obtained without their characteristics; or the logs of any numbers between 1.01 and 20,000 may be obtained with their characteristics. It is to be remembered that all numbers between 1 and 10 have zero for a characteristic.

When the left index of *C* is set at 10, the logarithm of any number on the LL1 scale, as read on *C*, must be divided by 100; and the log of any number on the LL2 scale, as read on *C*, must be divided by 10.

Example: Find common log of 1.04.

Characteristic, zero.

Set left index of *C* to 10 on LL3.

Opposite 1.04 on LL 1 read 1.7 on *C*.

$\frac{1.7}{100} = .017 = \log 1.04.$

Example: Find common log of 1.6.

Characteristic, zero.

Set left index of *C* to 10 on LL 3.
Opposite 1.6 on LL 2, read 2.04.

$\frac{2.04}{10} = .204 = \log 1.6.$

When the right index of *C* is set at 10 on LL 3, the logarithms of all numbers on the LL 3 scale, when read on *C*, have 1/10 of the values indicated; the logarithms of all numbers on the LL 2 scale, when read on *C*, have 1/100 of the values indicated; and the logarithms of all numbers on the LL 1 scale, as read on *C*, have 1/1000 of the values indicated. Thus:

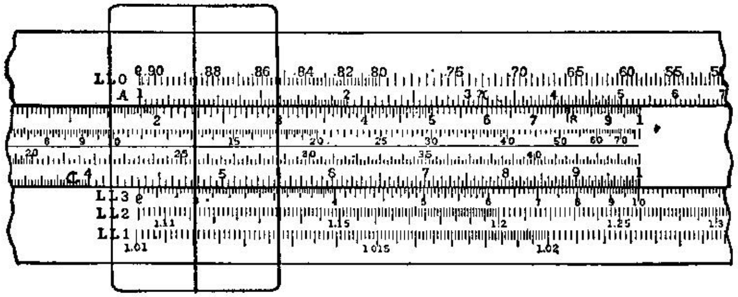


Fig. 64. Log 3.

Example: Find common log of 3.
Characteristic, zero.
Set right index of *C* to 10 on LL 3.
Opposite 3 on LL 3, read 4.77.

$\frac{4.77}{10} = 0.477 = \log 3.$

Example: Find common log of 1.15.
Characteristic, zero.
Right index of *C* to 10 on LL 3.
Opposite 1.15 on LL 2, read 6.07 on *C*.

$\frac{6.07}{100} = 0.0607 = \log 1.15.$

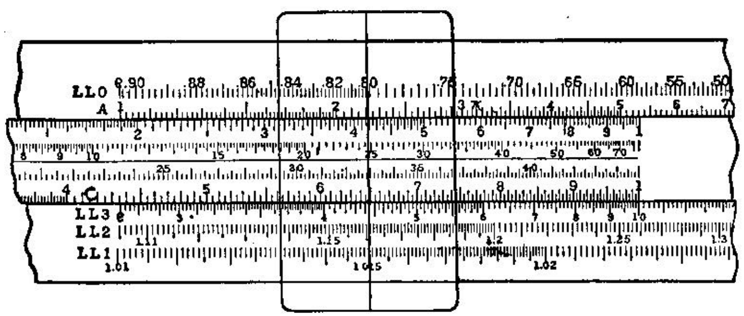


Fig. 65. Log 1.015.

Example: Find common log of 1.015.
Characteristic, zéro.
Right index of *C* to 10 on LL 3.
Opposite 1.015 on LL 1 find 6.47 on *C*.

$\frac{6.47}{1000} = .00647 = \log 1.015.$

The logs of numbers below unity must be obtained by using the portion of the Log Log scale between 1.01 and 10, and pointing off properly.

Example: Find common log of 0.35.
Characteristic is —1
Set right index of *C* to 10 on LL 3.
Opposite 3.5 on LL 3, read 544 on *C*.
Hence, log of .35 is 9.544—10.

For many purposes of computation it is better to use the co-log of a quantity less than unity. This is equal to 0 minus the log. Hence, the common co-logarithm of 0.35 is 0 — 9.544 = 0.456.

Hyperbolic co-logs, or co-logs to the base *e*, for numbers less than unity, are found on the *A* scale. Thus, to find the hyperbolic co-log of a number less than unity, set the indicator to that number on LL 0. Under the indicator on *A*, find the hyperbolic co-log.

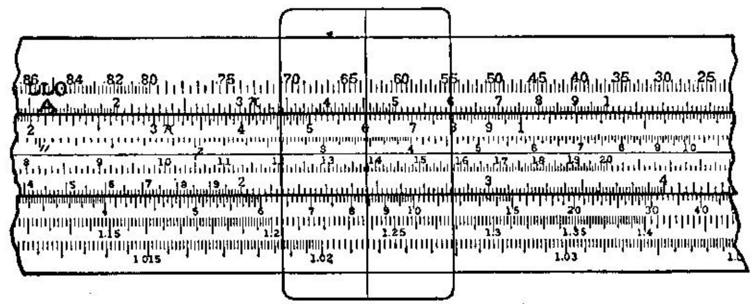


Fig. 66. Co-log .635.

Example: Find the hyperbolic co-logarithm of .635.
To .635 on LL 0, set indicator.
Under indicator on *A*, read .454.

Example: Find Log_e .635.
Proceed as in the preceding example.
$\text{Co-log}_e\ .635 = .454.$
$\text{Log}_e\ .635 = 0 - .454 = 9.546-10$

Where the decimal is off the scale, proceed as follows:

(*a*) If the decimal is less than .05, multiply it by 10, and find the co-log of the product. To the co-log of the product, add $\log_e 10$. This gives the co-log of the decimal which is off the scale.

(*b*) If the decimal is greater than 0.97, divide it by 10 and find the co-log of the quotient. From the quotient subtract $\log_e 10$. This gives the co-log of the decimal which is off the scale.

Try to remember that $\log_e 10$ is 2.3026.

The common logarithm of any number less than unity is found as follows:
Set index of B to 0.10 on LL 0.
Under any number on LL 0, find its common co-logarithm on B.

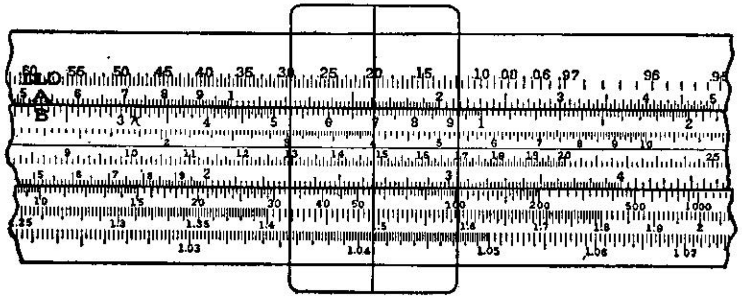


Fig. 67. Co-log .20.

Example: Find the common co-logarithm (*i.e.*, to base 10) of .20.
Set middle index of B to .10 on LL 0.
Under .20 on LL 0, find .699 on B.
The correct co-log is .69897.

When the middle index of B is set at .10 on LL 0, point off as follows:

The co-logarithms of numbers to the left of the index are the values given on the B scale, divided by 10.

The co-logarithms of numbers to the right of the index from the index to .05 are given on the B scale.

The co-logarithms of numbers to the right of the index from .97 downward are the values given on the B scale, divided by 100.

When the right index of B is set at .10 on the LL0 scale, point off as follows:

The co-logarithms of all numbers to the right of the middle index are the values given on the B scale, divided by 10.

The co-logarithms of all numbers to the left of the middle index are the values given on the B scale divided by 100.

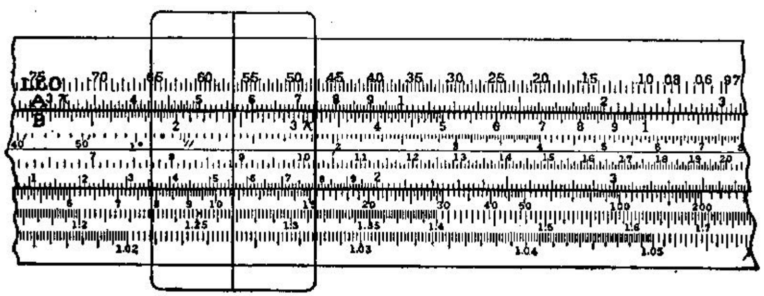


Fig. 68. Co-log .57.

Example: Find the common co-logarithm of 0.57.
Set middle index of B to .10 on LL 0.
Under .57 on LL 0, read 2.44 on B.

$\frac{2.44}{10} = .244.$ Ans.

The correct answer to five places is .24413.

Example: Find the common co-logarithm of .073.
Set middle index of B to .10 on LL 0.
Under .073 on LL 0, find 1.137.
The correct answer to five places is 1.1367.

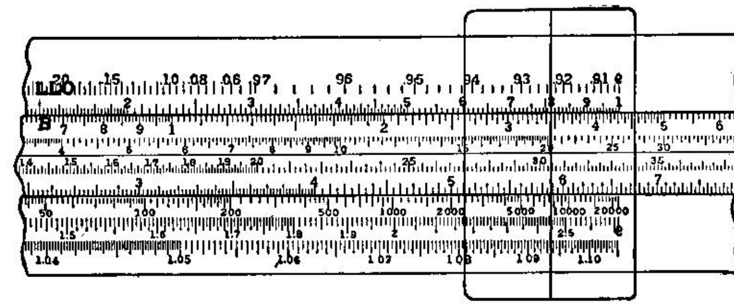


Fig. 69. Co-log .9235.

Example: Find the common co-logarithm of .9235.
Set middle index of B to .10 on LL 0.
Under .9235 on LL 0, read 3.46 on B.

$\frac{3.46}{100} = .0346.$ Ans.

The correct answer to five places is .03456.

Example: Find the common co-logarithm of .865.
Set right index of B to .10 on LL 0.
Under .865 on LL 0, read 6.30 on B.

$\frac{6.30}{100} = .0630.$ Ans.

The correct answer to five places is .06298.

The common co-logs of numbers from .05 to 0, and from .97 to 1.00 may be found as follows:

For a number less than .05 multiply the number by 10 and find the co-log of the product. Place the characteristic 1 before the result.

Example: Find the common co-logarithm of .045.

$$10 \times .045 = 0.45$$

Set middle 1 of B to .10 on LL 0.
Opposite 0.45 on LL 0, read 3.47 on B.

$\frac{3.47}{10} = .347 = $ co-log 0.45.

Hence, 1.347 = co-log .045.

For a number greater than 0.97, divide the number by 10, and find the co-log of the quotient. The initial number of the co-log thus found is discarded, and the remainder is the co-log of the original number.

Example: Find the common co-logarithm of .98.

$\frac{.98}{10} = .098.$

Set middle index of B to 10 on LL 0.
Opposite .098 on LL 0, read 1.009 on B.
Discarding the characteristic, we have .009, the common co-logarithm of 0.98.

Problem 292. Find the hyperbolic logarithms of the following numbers:

1.0212
1.672
5.36
30.5

Problem 293. Find the common logarithms of the following numbers by three methods:

2300
520
31.6

Problem 294. Find the common logarithms of the following numbers by two methods:

1.01785
1.1263
4.76

Problem 295. Find the common logarithms of the following numbers by two methods:

1.0734
1.413
111.

Problem 296. Find the common logarithms of the following numbers by two methods:

.142
.756
.491
.889

Problem 297. Find the logarithms of the following numbers to (*a*) base *e*; (*b*) base 4; and (*c*) base 15.

1.0462
2.63
9.72
55.0
5050.0

Problem 298. Find the hyperbolic co-logarithms of the following numbers:

.084
.823
.955
.012
.976

Problem 299. Find the common co-logarithms of the following numbers:

.054
.557
.912
.026
.981

ANTI-LOGARITHMS

An anti-logarithm is the number corresponding to a logarithm.

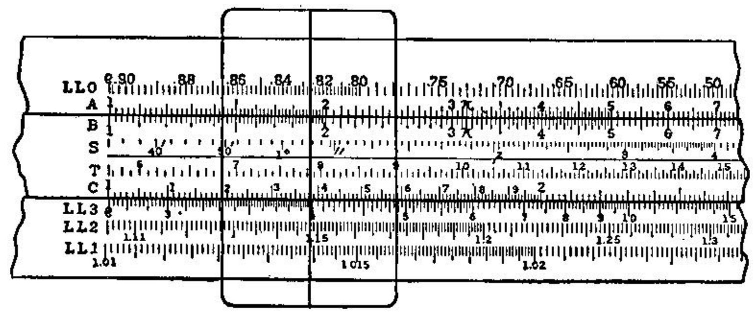


Fig. 70. Anti-log$_e$ 1.38.

Example: Find the anti-log$_e$ of 1.38.

Set left index of C to e on LL 3.
Below 1.38 on C, find 3.98 on LL 3.
Since the base of the hyperbolic logarithms is 2.71828 + which has the logarithm 1, the anti-log corresponding to 1.38 must be greater than 2.71828 +. Hence, it is found on scale LL 3.

Example: Find the anti-log of .212.

The decimal point indicates that we must divide 2.12 on C by 10. Hence, the anti-log will be found on scale LL 3.
Set left index of C to 10 on LL 3.
Opposite 2.12 on C, read 1.629 on LL 2.
The number corresponding to log .212 is 1.629.

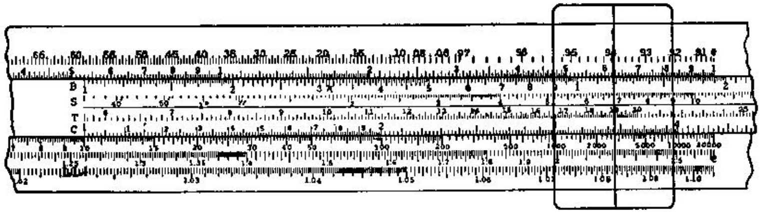


Fig. 71. Anti-log .0345.

Example: Find anti-log .0345.

The decimal point and zero before the first significant figure indicate that 3.45 on *C* must be divided by 100. The anti-log will then be found on scale LL 1.

Set left index of *C* to 10 on LL 3.
Opposite 345 on *C*, read 1.0827 on LL 1.

Example: Find anti-log of .0543.
5.43 on *C* must be divided by 100.
Hence, the anti-log will be found on LL 2.
Set right index of *C* to 10 on LL 3.
Opposite 5.43 on *C*, read 1.1332 on LL 2.

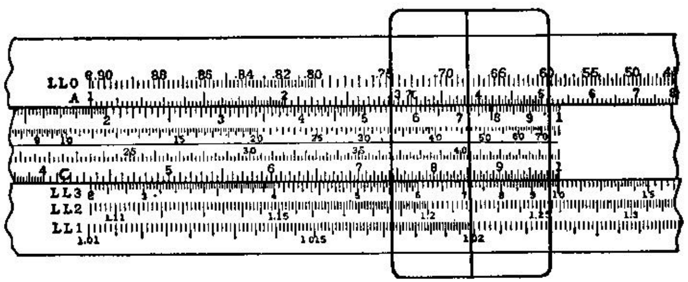


Fig. 72. Anti-log .00854.

Example: Find the anti-log of .00854.
Set right index of *C* to 10 on LL 3.

Inspection indicates that 8.54 on *C* must be divided by 1,000. Hence, the anti-log will be found on LL 1.
Opposite 854 on *C*, read 1.0199 on LL 1.

Example: Find anti-log of $\overline{8}.447$.
Set right index of *C* to 10 on LL 3.
Opposite 447 on *C*, read 280 on LL 0.
Hence, anti-log of $\overline{8}.447$ = .028.

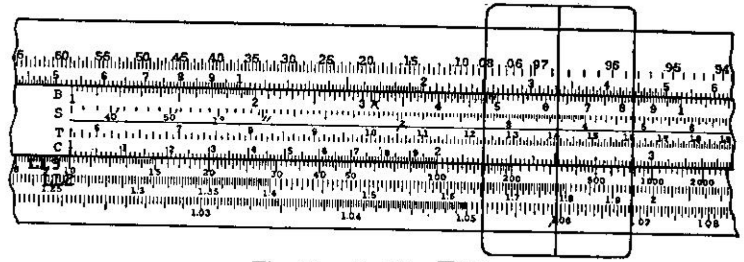


Fig. 73. Anti-log $\overline{9}.251$.

Example: Find anti-log of $\overline{9}.251$.
Set left index of *C* to 10 on LL 3.
Opposite 251 on *C*, read 1.783 on LL 2.
Hence, the anti-log of $\overline{9}.251$ is 0.1783.

Problem 300. Find the common anti-log of 1.34.
301. " " " " " 5.45
302. " " " " " .312
303. " " " " " .067
304. " " " " " 7.35
305. " " anti-log$_e$ of $\overline{8}.726$
306. " " " " " $\overline{9}.241$
307. " " " " " 0.943
308. " " " " " 0.0462
309. " " " " " 0.00697

Example: Find $(2.36)^5 = x$.
Log $x = 5 \times \log 2.36$.
Method I. Set indicator to 2.36 on *D*.
Below indicator on *L*, find .373.
Set right index of *C* to .373 on *D*.
Opposite 5 on *C*, read 1.865 on *D*.
Hence, log x = 1.865 and 1 is the characteristic.
Set indicator to 8.65 on *L*.
Under indicator on *D*, find 732.
The characteristic 1 indicates a whole number of two digits in the result.
Hence, x = 73.2.

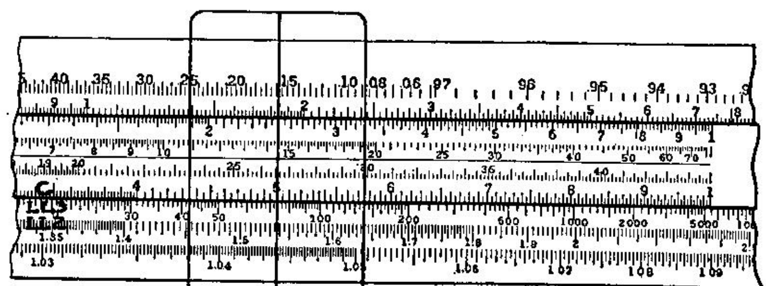


Fig. 74. $(2.36)^5$.

Method II. $\log x = 5 \times \log 2.36$.
To 2.36 on LL 2, set right index of C.
Below 5 on C, read 73.2 on LL3.

Example: Find $\sqrt[5]{187} = x$.

Method I. $\log x = \frac{\log 187}{5}$.

Below 187 on D, find 272 on L.
187 takes the characteristic 2.
Hence, log 187 = 2.272.
To 2.272 on D, set 5 on C.
Opposite right index of C, read 454.
Hence, log x = .454.
Opposite 454 on L, read 284 on D.
Hence, x = 2.84. Ans.

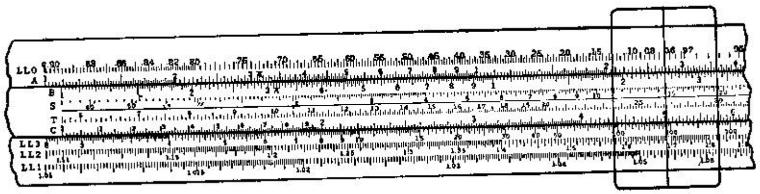


Fig. 75. $\sqrt[5]{187}$.

Method II. $\log x = \frac{\log 187}{5}$.

To 187 on LL 3, set 5 on C.
Opposite left index of C, read 2.84 on LL 3.

Example: $x = (2.7)^{1.41}$. Use both methods.

Method I.

$$\log x = 1.41 \log 2.7$$
$$= 1.41 \times 0.431$$
$$= 0.608$$
$$x = 4.05$$

Log 2.7 is found on slide rule as in first example. Multiply, using scales C and D.

Method II.

$$\log x = 1.41 \log 2.7$$
$$x = 4.05$$

Find 2.7 on LL 2.
Multiply by 1.41 on C. Ans. on LL3.

Example: $x = (41.5)^{0.23}$.

$$\log x = 0.23 \times \log 41.5$$
$$= 0.23 \times 1.618$$
$$= 0.372$$
$$x = 2.36$$

Find mantissa of log 41.5 = 0.618
Then prefix characteristic of 1, making 1.618.
Multiply, using scales C and D.

$$\log x = 0.23 \log 41.5$$
$$x = 2.36$$

Find 41.5 on LL 3.
Multiply by 0.23 on C. Ans. on LL2.

Example: $x = \sqrt[4.2]{51.3}$

Method I.

$$\log x = \frac{\log 51.3}{4.2}$$
$$= \frac{1.710}{4.2}$$
$$= 0.407$$
$$x = 2.56$$

Find log 51.3 = 1.710
Divide, using scales C and D.

Method II.

$$\log x = \frac{\log 51.3}{4.2}$$
$$x = 2.56$$

Find 51.3 on LL 3.
Divide by 4.2 on C.
Answer on LL 2.

CHAPTER V

SOLUTION OF TRIANGLES

By the Slide Rule a right triangle or an oblique triangle may be solved in a few seconds. On the 10″ Slide Rule a side of a triangle may be read to three significant figures, and the angles to within a few minutes. For many kinds of applied work this degree of accuracy is sufficient.

Where greater accuracy is required, as in surveying calculations, the work should be done by logarithms, and then checked by the slide rule. This check will show any gross error and will locate the error. For classes in Trigonometry it is recommended that the student proceed as follows:

a. Solve the triangle by logarithms.

b. Check by solving on the Slide Rule.

c. If the Slide Rule shows that there is an error, find the error and correct it.

d. If no error appears and it is desired to check to a greater degree of accuracy, apply the usual trigonometric check.

The use of the Slide Rule saves time and locates the error in a particular part of the work.

NOTE: In the following pages on right and oblique triangles the author has drawn freely upon the admirable treatment of this subject in a chapter of the former Mannheim Manual by Professor J. M. Willard, of the State College of Pennsylvania.

RIGHT TRIANGLES

Example: Given an Acute Angle and the Hypotenuse.
Let $A = 32° 30'$ and $c = 14.7$.
Find B, a, and b.
Solution: $B = 90° - A = 57° 30'$.

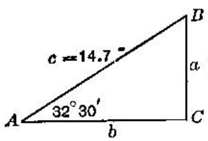


Fig. 76.

$$\frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}.$$

Substituting the given values,

$$\frac{14.7}{\sin 90°} = \frac{a}{\sin 32° 30'} = \frac{b}{\sin 57° 30'}.$$

Setting the rule as in proportion, using right half of scale A,

A	Opposite 14.7	Read $a = 790$	Read $b = 124$
S	Set 1 (sin 90°)	Opposite 32° 30′	Opposite 57° 30′

To place the decimal point, note that the sides will be in the same order of magnitude as their opposite angles.

$C = 90°$ $\quad c = 14.7.$
$B = 57° 30'$ $\quad b = 12.4.$
$A = 32° 30'$ $\quad a = 7.88.$

Where the S scale is involved, care should be taken to set the number on the proper half of scale A. The following diagram will make this clear. The numbers on the scale are continuous.

	Left End	Middle	Right End
	.01	.1	1.
Scale A	1.	10.	100.
	100.	1000.	10000.

Example: Given an Acute Angle and the Opposite Side.
Let $A = 55° 15'$ and $a = 465$.
Find B, b, and C.
Solution: $B = 90° - A = 90° - 55° 15' = 34° 45'$.

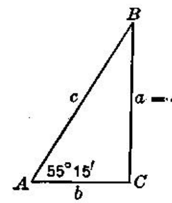


Fig. 77.

$$\frac{a}{\sin A} = \frac{c}{\sin C} = \frac{b}{\sin B}.$$

Using left half of scale A,

A	Opposite 465	Find $c = 566$	Find $b = 323$
S	Set 55° 15′	Opposite 1 (sin 90°)	Opposite 34° 45′

Placing the decimal point,

$C = 90°$ $\quad c = 566.$
$A = 55° 15'$ $\quad a = 465.$
$B = 34° 45'$ $\quad b = 323.$

Example: Given an Acute Angle and the Adjacent Side.
Let $A = 15°$, $b = 12.5$.
Find B, a, and c.
Solution: $B = 90° - A = 75°$.

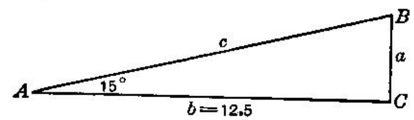


Fig. 78.

$$\frac{b}{\sin B} = \frac{a}{\sin A} = \frac{c}{\sin C}.$$

$$\frac{12.5}{\sin 75°} = \frac{a}{\sin 15°} = \frac{c}{1 \ (\sin 90°)}.$$

Using the right half of scale A,

A	Opposite 12.5	Find $a = 3.35$	Find $c = 12.9$
S	Set 75°	Opposite 15°	Opposite 90°

Placing the decimal point by arranging the angles and sides in order,

$C = 90°$ $\quad c = 12.9.$
$B = 75°$ $\quad b = 12.5.$
$A = 15°$ $\quad a = 3.35.$

Example: Given the Hypotenuse and a Side.
Let $a = 1.64$, $c = 2.55$
Find A, B and b.

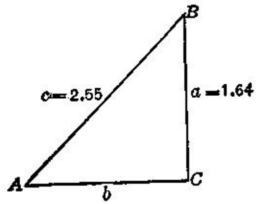


Fig. 79.

Solution: $\frac{c}{\sin C} = \frac{a}{\sin A} = \frac{b}{\sin B}$.

$$\frac{2.55}{1(\sin 90°)} = \frac{1.64}{\sin A} = \frac{b}{\sin B}.$$

A	Opposite 2.55	Opposite 1.64	Find $b = 1.95$
B	Set 1 (sin 90°)	Find $A = 40°$	Opposite B (90°-A)

B may be found after A is known.
$B = 90° — 40° = 50°$.
To place the decimal point in b:
Since B is a little larger than A, b will be a little larger than a.
Hence $b = 1.95$.

Example: Given the Two Sides.

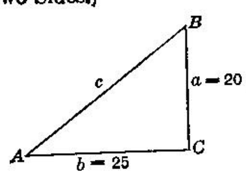


Fig. 80.

Case I. Where $\tan A$ or $\frac{a}{b}$ is less than 1.
Let $a = 20$ and $b = 25$.
Find A, B and c.

Solution: $\frac{a}{b} = \frac{\tan A}{1\ (\tan 45°)}$ or $\frac{b}{1} = \frac{a}{\tan A}$.

T	Set 1 (tan 45°)	Find $A = 38° 40'$
D	Opposite 25	Opposite 20

Or $\tan A = \frac{20}{25}$

C	Set 20	
D	Opposite 25	Opposite right index
T		Read 38° 40′

To find c, use the formula, $\frac{a}{\sin A} = \frac{c}{\sin C}$.

Case II. When $\tan A$ or $\frac{a}{b}$ is greater than 1.
Let $a = 30$, $b = 25$.
Find A, B and c.

Solution: Find B first in order to avoid finding the tangent of an angle greater than 45° since the T scale reads only to 45°.

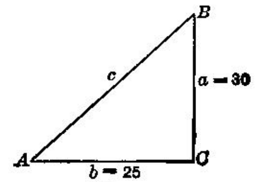


Fig. 81.

$$\frac{a}{1} = \frac{b}{\tan B} \qquad \frac{1}{a} = \frac{\tan B}{b}.$$

T	Set 1 (tan 45°)	Find $B = 39° 50'$
D	Opposite 30	Opposite 25

Or $\tan B = \frac{25}{30}$

C	Set 25	
D	Opposite 30	Opposite right index
T		Read 39° 50′

$A = 90° — 39° 50' = 50° 10'$.

Find c by the formula $\frac{b}{\sin B} = \frac{c}{\sin C}$.

OBLIQUE TRIANGLES

Example: Given Two Angles and a Side.
Let $a = 22.5$ $A = 44° 30'$ $B = 24° 15'$
Find C, b and c.

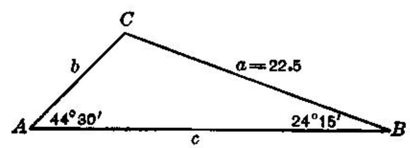


Fig. 82.

Solution: $C = 180° — (44° 30' + 24° 15')$
$= 180° — 68° 45'$
$= 111° 15'$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C \text{ or } \sin (A+B)}$$

$$\frac{22.5}{\sin 44° 30'} = \frac{b}{\sin 24° 15'} = \frac{c}{\sin 111° 15'\ (\sin 68° 45')}.$$

Using the right half of rule:

A	Opposite 22.5	Find $b = 132$	Find $c = 299$
S	Set 44° 30′	Opposite 24° 15′	Opposite 68° 45′

To place the decimal point, the sides will follow the same order of magnitude as their opposite angles.

$C = 111° 15'$	$c = 29.9$.
$A = 44° 30'$	$a = 22.5$.
$B = 24° 15'$	$b = 13.2$.

Example: Given Two Sides and the Angle Opposite One of these Sides. This example has two possible solutions, both of which are given below.

Let $a = 175.$, $b = 215.$, $A = 35° 30'$
Find B, C, and c, B' and c'

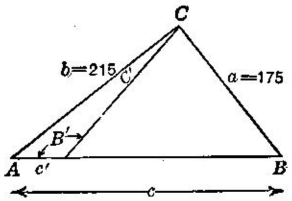


Fig. 83.

Solution: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$. Note: Sine C = Sine $(A + B)$.

Using left half of rule.

A	Opposite 175	Opposite 215	Find $c = 298$
S	Set 35° 30′	Find $B = 45° 30'$	Opposite $(A+B)$ or 81°

$$B' = 180° — 45° 30' (B).$$
$$= 134° 30'.$$
$$A+B' = 35° 30' + 134° 30'.$$
$$= 170°.$$
$$C' = 180° — 170°.$$
$$= 10°.$$

Indicator to right index	Find $c' = 52.2$
Left index to indicator	Opposite $C' = 10°$

To place the decimal point, arrange angles and sides in order of magnitude.

In triangle ABC,

$C = 99°$ $\quad c = 298.$
$B = 45° 30'$ $\quad b = 215.$
$A = 35° 30'$ $\quad a = 175.$

In triangle AB′C,

$B' = 134° 30'$ $\quad b = 215.$
$A = 35° 30'$ $\quad a = 175.$
$C' = 10° 0'$ $\quad c' = 52.2.$

Example: Given Two Sides and the Included Angle. The fact that the tangent scale runs only to 45° makes two cases.

Case I. When $\dfrac{C}{2}$ is greater than 45° whence ½ $(A + B)$ is less than 45°.

Example: $a = 5.14$, $b = 2.12$, $C = 112° 30'$.
Find A, B and c.

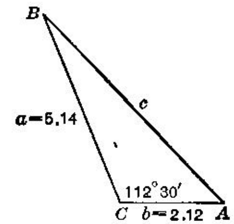


Fig. 84.

Solution:

$a = 5.14.$
$b = 2.12.$
$a+b = 7.26.$
$a—b = 3.02.$

$A+B = 67° 30'.$

Use the formula,

$$\frac{\tan \frac{1}{2}(A+B)}{a+b} = \frac{\tan \frac{1}{2}(A—B)}{a—b}.$$

T	Set ½ $(A+B)$	Find ½ $(A—B)$
D	Opposite $(a+b)$	Opposite $(a—b)$

$\frac{1}{2}(A+B) = 33° 45'.$
$\frac{1}{2}(A—B) = 15° 32'.$
$A = 49° 17'.$
$B = 18° 13'.$

T	Set 33° 45′	Find ½ $(A—B) = 15° 32'$.
D	Opposite 7.26	Opposite 3.02

c is found by the usual sine formula:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{5.14}{\sin 49° 17'} = \frac{c}{\sin 112° 30' \text{ or } \sin 67° 30'}.$$

A	Opposite 5.14 (Left half of scale A)	Find $c = 6.27$
S	Set 49° 17′	Opposite 67° 30′

Check: $\dfrac{c}{\sin C} = \dfrac{b}{\sin B}$.

A	Opposite 6.27 (Left half of scale A)	Find 2.12
S	Set 67° 30′	Opposite 18° 13′

NOTE:— In the mathematics classroom this check formula may be used after the student has solved the triangle by logarithms.

Example: $a = 154$, $b = 73.5$, $C = 120° 30'$.

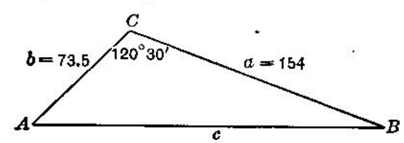


Fig. 85.

Solution:

$$\frac{\tan \frac{1}{2}(A+B)}{a+b} = \frac{\tan \frac{1}{2}(A—B)}{a—b}.$$

$$\frac{\tan 29° 45'}{227.5} = \frac{\tan \frac{1}{2}(A—B)}{a—b}.$$

T	Set 29° 45′	Indicator to right index	Find ½ $(A—B) = 11° 26'$
D	Opposite 227.5	Left index to indicator	Opposite 80.5

$\frac{1}{2}(A+B) = 29° 45'.$
$\frac{1}{2}(A—B) = 11° 26'.$
$A = 41° 11'.$
$B = 18° 19'.$

By the method of the preceding example, c is found to be 202.

Case II. When $\frac{C}{2}$ is less than 45°, whence $\frac{1}{2}(A+B)$ is greater than 45°.

Example: $a=75.5$, $b=42.5$, $C=65° 30'$.

$a+b=118$. $a-b=33$.

$\frac{1}{2}(A+B)=57° 15'$.

$$\frac{\tan \frac{1}{2}(A+B)}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{a-b}$$

$$\frac{\tan 57° 15'}{118} = \frac{\tan \frac{1}{2}(A-B)}{33}$$

Since tan 57° 15′ is not on the rule, we substitute for it

$$\frac{1}{\tan(90°-57° 15')} = \frac{1}{\tan 32° 45'}$$

The formula now reads:

$$\frac{1}{118 \times \tan 32° 45'} = \frac{\tan \frac{1}{2}(A-B)}{33}.$$

T	Set 1 (Left index)	Indicator to 32° 45′	Find ½ (A—B) = 23° 30′
D	Opposite 118	Right index to indicator	Opposite 33

$\frac{1}{2}(A+B)=57° 15'$.
$\frac{1}{2}(A-B)=23° 30'$.
$A=80° 45'$.
$B=33° 45'$.

Find c by the usual method.

Check by the sine formula.

Example: $b=83.4$, $a=78$, $C=72° 15'$.

$b+a=161.4$ $b-a=5.4$

$\frac{1}{2}(B+A)=53° 53'$.

T	Set 1 (Right index)	Indicator to 36° 7′	Find 24° 38′
D	Opposite 161	Left index to indicator	Opposite 5.4

$\frac{1}{2}(B-A)=24° 38'$.
$B=78° 31'$.
$A=29° 15'$.

Testing these results by the formula: $\frac{a}{\sin A} = \frac{b}{\sin B}$,

it will be found that the angles are incorrect. This results from the fact that the slide rule gives the significant figures of the tangent, but does not fix the decimal point. In this example, there are three values for $\frac{1}{2}(B-A)$ between 2° and 88°, corresponding to the natural tangent whose significant figures are 459.

1. $\tan^{-1} .0459 = 2° 38'$.
2. $\tan^{-1} .459 = 24° 38'$.
3. $\tan^{-1} 4.59 = 77° 43'$.

Other values may be found less than 2° or between 88° and 90°, but these will seldom be required.

Hence, in the solution of any problem in this case, it is necessary to test the results by the check formula.

An inspection of the example shows that a is slightly larger than b. Hence A will be only slightly larger than B. This would be possible if $\frac{1}{2}(A-B)$ were smaller than 24° 38′, which we obtained on the rule.

Find tan 24° 38′, which is .459.

Find $\tan^{-1} .0459$.

In order to secure this small angle, we use the sine scale, since the sine of an angle less than 5° 43′ is practically equal to the tangent.

Opposite .0459 on the left half of scale A, find 2° 37′ on S.

$\frac{1}{2}(B+A)=53° 53'$.
$\frac{1}{2}(B-A)=2° 37'$
$B=56° 30'$.
$A=51° 15'$.

Using the check formula, $\frac{a}{\sin A} = \frac{b}{\sin B}$ these results will be found to be correct.

Suppose it is desired to obtain the next larger angle than 24° 38′.

$\tan 24° 38' = .459$.

The next larger angle with the same significant figures for the tangent would be: $\tan x = 4.59$.

Since this angle is evidently greater than 45°, we may write:

$$\tan(90°-x) = \frac{1}{\tan x} = \frac{1}{4.59}.$$

Solving by the slide rule

T	Set 1 (tan 45°)	Find 12° 17′
D	Opposite 4.59	Opposite 1

$90°-x=12° 17'$.
$x=77° 43'$.

Or, opposite 4.59 on the CI scale read 12° 17′, which is equal to $90°-x$.

Example: $a=10$, $b=90$, $C=65°$.

$b+a=100$. $b-a=80$.

$\frac{1}{2}(B+A)=57° 30'$.
$\frac{1}{2}(B-A)=7° 10'$. by the first trial on the rule.
$B=64° 40'$.
$A=50° 20'$.

These results do not check.

Since b is nine times a, B must be considerably larger than A.

Using the method above,

$\tan 7° 10' = .126$.
$\tan x = 1.26$.
$\tan(90°-x) = \frac{1}{1.26}$.

T	Set 1 (Tan 45°)	Find 38° 32′.
D	Opposite 1.25	Opposite 1

Or read 38° 32′ opposite 1.26 on the CI scale.

$90°-x = 38° 32'$.
$x = 51° 28'$.
$\frac{1}{2}(B+A) = 57° 30'$.
$\frac{1}{2}(B-A) = 51° 28'$.
$B = 108° 58'$.
$A = 6° 2'$.

These results check by the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

ANOTHER METHOD:—

Example: $b = 83.4$, $a = 78$. $C = 72° 15'$.

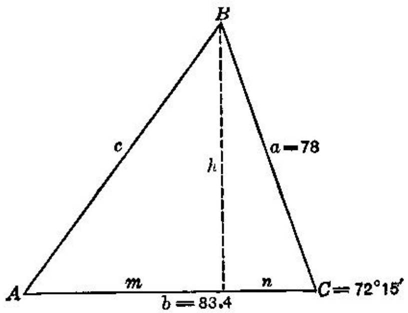


Fig. 86.

$h = a \sin C = 74.3$
$n = a \cos C = a \sin (90°—C) = 23.8$
$m = b—n = 59.6$

$90°—A = \tan^{-1} \frac{m}{h} = 90°—38° 45'$

$A = 51° 15'$
$B = 180°—(A+C) = 56° 30'$
$C = \frac{h}{\text{Sin } A} = 95.3$

Check, $\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$

Example: Given three sides.
Method I. Let $a = 32.0$. $b = 26.5$, $c = 14.7$.

Find A, B, and C.

$a = 32.0$.
$b = 26.5$.
$c = 14.7$.
$2s = 73.2$.
$s = 36.6$.
$s—a = 4.6$.
$s—b = 10.1$.
$s—c = 21.9$.

$\sin \frac{1}{2} A = \sqrt{\frac{(s—b)\ (s—c)}{bc}}$.

$= \sqrt{\frac{10.1 \times 21.9}{26.5 \times 14.7}}$.

$= 0.754$. By the slide rule.

Hence $\frac{1}{2} A = 49°$ (Using scales A and S).
$A = 98°$

Find B and C by the formula:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

$\frac{32}{\sin 98°(=\sin 82°)} = \frac{26.5}{\sin B} = \frac{14.7}{\sin C}$

A	Opposite 32	Opposite 26.5	Opposite 14.7
S	Set 82°	Find $B = 55°$	Find $C = 27°$

Method II. $\text{Cos } C = \frac{a^2 + b^2 — c^2}{2\,ab}$.

or $\text{Sin } (90° — C) = \frac{1024. + 702. — 216.}{1696.} = \frac{1510}{1696}$.

$\text{Sin } (90° — C) = .890$
$90° — C = 63°$ (to the nearest degree)
$C = 27°$

Find B from the formula $\frac{c}{\sin C} = \frac{b}{\sin B}$.

and A from the formula $\frac{c}{\sin C} = \frac{a}{\sin A}$.
Check: $A + B + C = 180°$.

Example: Given the three sides:—
$a = 20$, $b = 18$, $c = 15$.
Find the angles A, B and C.
An easy indirect solution suited to the slide rule is as follows:

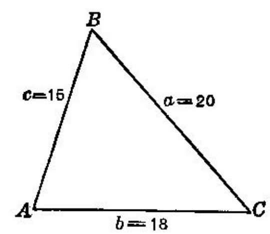


Fig. 87.

$\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$.
$A + B + C = 180°$

By inspection a is the longest side, hence angle A is the greatest angle and is greater than 60°.

Try $A = 65°$	Try $A = 75°$	Try $A = 74°$
$B = 55°$ Roughly	$B = 61°$	$B = 60°$
$C = 43°$	$C = 46°$	$C = 46°$
163°	182°	180°
Too small	Slightly too large	

To 20 on scale A set trial value of A on scale S; opposite sides b and c on A read corresponding angles on S. Only a few trials are necessary.

CHAPTER VI
TYPICAL EXAMPLES RELATING TO VARIOUS OCCUPATIONS
SECRETARIAL WORK

A secretary in checking a traveling man's expense account for one week found the following items:

Railroad fares	$27.50
Hotel bills	56.00
Total	83.50

Find what per cent of the total expense was used in hotel bills.

Solution: $56 \div 83.50 = 67$ per cent.

Opposite 56 on *D* set 835 on *C*.

Opposite the right index of *C*, find 67 on *D*.

EXCAVATING

What will be the cost of excavating rock for a cellar measuring 43 ft. × 28 ft. to an average depth of 6.5 ft. at $2.50 per cubic yard?

$$x = \frac{43 \times 28 \times 6.5 \times 2.5}{27}.$$

1. To 43 on *D* set 27 on *C*.
2. Indicator to 28 on *C*.
3. 65 on *CI* to indicator.
4. Opposite 25 on *C* read 725 on *D*.

Roughly calculating for the decimal point:—

$x = 800$.

Hence, the result is $725; which is correct to the nearest dollar.

PER CENT OF PROFIT

A merchant purchased a bill of goods for $318 and sold the same for $360. Find the per cent of profit reckoned,

a. On the cost.
b. On the selling price.

Solution: Profit = $360 — $318 = $42.

Per cent of profit reckoned on the cost $= \frac{42}{318} = 13.2$ per cent.

Per cent of profit reckoned on the selling price $= \frac{42}{360} = 11.7$ per cent.

DISCOUNT

Simple discount is set by reading the scales backwards, deducting from 100, thus, for a discount of 18%, set right hand (or middle) index at 82 (100 — 18 = 82) and the rule is set, so that opposite any number on *C*, the answer will be found on *D*, or opposite any number on *CF*, the answer will be found on *DF*. This is equivalent to multiplying by 82%.

For a combination of discounts, set by the use of the indicator, thus for 27½ — 15 — 5%, proceed as follows:

C	Right Ind.	Ind. to 85 (100-15)	R to Ind.	Ind. to 95 (100-5)	R to Ind.	Opp. any amount
D	To 72.5 (100-27½)					Find answer

For frequently occurring discounts, a gauge mark should be made.

COMPOUND INTEREST

How many years will it take a sum of money to double itself if deposited in a savings bank paying 4 per cent interest, compounded semi-annually.

Using the formula $A = P(1+r)^n$, where *A* is the amount, *P* the principal, *r* the interest on $1. for 6 months, and *n* the number of half years, if we take $1. as *P*, we have:

$$2 = (1 + .02)^n$$
$$\text{and } n = \frac{\log 2}{\log 1.02}.$$
$$= \frac{.301}{.0086}. \qquad \text{See page 73}$$
$$= 35 \text{ half years.} \qquad \text{See page 11}$$
$$\text{or } 17\tfrac{1}{2} \text{ years.}$$

NOTE:— It is advisable to use a 20-inch rule when solving the problem in this manner. On the 10-inch rule the result can be found only very roughly, unless the Log Log scales are employed. The example is then simply performed as follows:

$$2 = (1 + .02)^n$$
$$\sqrt[n]{2} = 1 + .02$$

Set 1 on *C* to 1.02 on LL 1.

Opposite 2 on LL 2, find 35 on *C*.

Example: Required the amount of $1500. at 6 per cent. compound interest for 5 years, compounded quarterly.

$$6 \div 4 = 1.5 \text{ per cent quarterly.}$$
$$A = P(1 + r)^n$$
$$A = 1500 (1 + .015)^{20}$$
$$\log A = \log 1500 + 20 \log (1 + .015).$$

C	Set Index	At 20	Set Index	At 1.3471
LL 1	To 1.015			
LL 2		Read 1.3471 $= (1.015)^{20}$		
D			To 1500	Read $2020. Ans.

PHYSICS

In a photometer a 16 c. p. lamp is used as a standard. The following distance readings are obtained in testing a nitrogen filled lamp.

D_s	D_x	
317 mm.	683 mm.	By experiment 1
304 mm.	696 mm.	" " 2
322 mm.	678 mm.	" " 3
248 mm.	570 mm.	" " 4

Using the following equation calculate the observed candle power of the unknown lamp.

$$\frac{D_s^2}{D_x^2} = \frac{\text{c. p. of standard}}{\text{c. p. of unknown}}.$$

$$\frac{(317)^2}{(683)^2} = \frac{16}{x}.$$

To 683 on scale D, set 317 on C.
Above 16 on B, find x on A.

$$x = 74.4$$

The operation of transferring from scales C and D to A and B squares the fraction $\frac{317}{683}$.

The first experiment gives	$x = 74.4$
The second " "	$x = 83.9$
" third " "	$x = 70.9$
" fourth " "	$x = 84.5$
	4)313.7
The result.	= 78.4

PHOTOMETRY AND LIGHT TRANSMISSION.

The general formula is as follows:

$$T' = T^{\frac{l'}{l}}$$

in which
T = transmission of the reference medium
l = length of path or thickness
T' = transmission sought for length l'.

Example: A particular glass 2.4 cm. thick transmits, exclusive of the reflection loss, 88 per cent of a certain wave length. What will be the transmission of 6.4 cm. of the same glass.

$$T' = .88^{\frac{6.4}{2.4}}$$

LL 0	To .88	Find .711. Answer
B	Set 2.4	Over 6.4

Example: A glass ray filter 3.2 mm. thick transmits, exclusive of the reflection loss, 52 per cent. of violet light. What must be the thickness to transmit 75 per cent. of the same light?

$$.75 = .52^{\frac{l'}{3.2}}$$

LL 0	To .52	Opposite .75
B	Set 3.2	Find 1.4 mm. Answer

The light transmission measured by the photometer is the total, which includes the reflection loss at the two surfaces.

From photometric readings of total the relative transmission only can be computed by the formula.

$$T' = \left(\frac{T_1\ (n+1)^4}{16n^2}\right)^{\frac{l'}{l}}$$

In which
n = Refractive index of medium.
T' = Transmission sought for length l'.
T_1 = Total transmission of length l.

Example: A glass plate 1.5 cm. thick, refractive index 1.52, has a total transmission of 72 per cent of visible light. What will be the transmission only of the same glass, but 2.5 cm. thick?

$$T' = \left(\frac{.72 \times 2.52^4}{16 \times 1.52^2}\right)^{\frac{2.5}{1.5}}$$

A	To 7.2			Read .785		
B	Set 16				Set 1.5	At 2.5
C		Ind. to 2.52	1.52 to Ind.	At 2.52		
LL 0					To .785	Find .668 Answer

CHEMISTRY.

By weight 80 parts of sodium hydroxide combine with 98 parts of sulphuric acid. How many grams of sodium hydroxide will neutralize 50 grams of sulphuric acid?

Solution: $98 : 50 = 80 : x$.
To 50 on D set 98 on C.
Under 80 on C find 40.8 on D.

SPEEDS OF PULLEYS

The diameter of the driving pulley is 9 inches and its speed is 1,300 R. P.M. If the diameter of the driven pulley is 7 inches, what is its speed?

Solution: The diameter of the driving pulley, multiplied by its speed, is equal to the diameter of the driven pulley, multiplied by its speed.

$$7 \times S = 9 \times 1300.$$

$$S = \frac{9 \times 1300}{7}.$$

See page 13.

S = 1670 correct to three significant figures

CUTTING SPEED

A certain grindstone will stand a surface or rim speed of 800 ft. per min. At how many R. P. M. can it run if its diameter is 57 in.?

Solution: The cutting speed is equal to the circumference of the stone in feet multiplied by the number of revolutions per minute.

$$\text{or } C = \frac{\pi d \times R.P.M.}{12} \text{ where d is expressed in inches.}$$

$$\text{Hence R. P. M.} = \frac{12\,C}{\pi d}.$$

$$= \frac{12 \times 800}{3.1416 \times 57}.$$

$$= 53.$$

See page 60.

GEARING

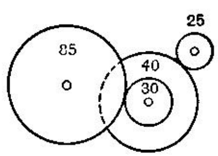


Fig. 88.

The gear with 85 teeth (Fig. 88) revolves 50 times per minute. Find the speed of the gear with 25 teeth.

Solution: The continued product of the R. P. M. of the first driver and the number of teeth in every driving gear is equal to the continued product of the R. P. M. of last driven gear and the number of teeth in every driven gear.

$$\text{Hence, } 50 \times 85 \times 40 = 30 \times 25 \times S.$$

$$S = \frac{50 \times 85 \times 40}{30 \times 25}.$$

See page 60.

$$S = 227.$$

LENGTH OF PATTERN

If window weights are 1½ inches in diameter, how long must we make the pattern for 8 lb. weights (1 cu. in. of cast iron weighs .26 lb.)?

Solution: The number of pounds in the window weight is equal to the volume of the cylindrical weight × .26 lb.

$$8 = \frac{\pi \times (1.5)^2 \times L \times .26}{4}.$$

$$\text{and } L = \frac{4 \times 8}{\pi \times (1.5)^2 \times .26}.$$

See page 61.

= 17.4 inches, or 17 and 7/16 inches to the nearest 16th.

COMPOSITION METAL MIXING

If bell metal is made of 25 parts of copper to 11 parts of tin by weight, find the weight of each metal in a bell weighing 1054 lbs.

Solution: The copper weighs $\frac{25}{36}$ of 1054 = 732 lbs. See page 13.

The tin weighs $\frac{11}{36}$ of 1054 = 322 lbs.

SURVEYING

The slide rule is used in surveying to check gross errors in computation, to reduce stadia readings, and to solve triangles.

See page 86 for the solution of triangles by the slide rule.

Example: Find the latitude and departure of a course whose length is 525 ft. and bearing N 65° 30′ E.

Latitude = length of course × cosine of bearing.
= 525 × cos 65° 30′.
= 525 × sin 24° 30′.
= 218.

Opposite the right index of *A*, set 24° 30′ on scale *S*.

Opposite 525 on *A*, find 218 on *B*.

The decimal point may be placed by inspection, since the sine and cosine are always less than one.

Departure = length of course × sine of bearing.
= 525 × sin 65° 30′.
= 478.

NOTE:– Keuffel and Esser Co. make a special rule for surveyors, known as the Surveyor's Duplex Slide Rule, which, has not only the *A*, *B*, *CI*, *C* and *D* scales on one face, but two full length stadia scales for computing horizontal distances and vertical heights. The other face is arranged for the determination of the meridian by direct solar observations, and carries the sine and cosine scales used in calculating latitudes and departures of the course. Hence, this rule reduces many complicated surveying calculations to mere mechanical operations.

For those who desire to calculate stadia reductions, and latitudes and departures, with a considerable degree of accuracy, the above mentioned company makes a special Stadia slide rule.

Rectangular Co—Ordinates

$$c=\sqrt{a^2+b^2}=a\sqrt{1+\frac{b^2}{a^2}}$$

C	Set a				At a	Read $\frac{a}{c}$
D	At b				Read $\sqrt{a^2+b^2}$	At 1
B		At 1	Mentally add 1 to $\left(\frac{b}{a}\right)^2$	Set 1		
A		Read $\left(\frac{b}{a}\right)^2$		at $1+\left(\frac{b}{a}\right)^2$		

or

C	Set b				At a	Read $\frac{a}{c}$
D	At a				Read $\sqrt{a^2+b^2}$	At 1
B		Read $\left(\frac{b}{a}\right)^2$	add 1 to $\left(\frac{b}{a}\right)^2$	Set 1		
A		At 1		at $1+\left(\frac{b}{a}\right)^2$		

or

C	Set b				Read $\sqrt{a^2+b^2}$	At 10
D	At a	Set Ind. at $\frac{a}{b}$			At a	Read $\frac{a}{c}$
B			Read $\left(\frac{b}{a}\right)^2$	Move $1+\left(\frac{b}{a}\right)^2$		
A			At 1	To 1		

Example:

Find the diagonal of a rectangle with sides $6\frac{1}{2}$ and $11\frac{1}{2}$ feet in length.

Diagonal $=\sqrt{(6\frac{1}{2})^2+(11\frac{1}{2})^2}=6\frac{1}{2}\sqrt{1+\left(\frac{11\frac{1}{2}}{6\frac{1}{2}}\right)^2}$

To $11\frac{1}{2}$ on *D* set $6\frac{1}{2}$ on *C*.
Opposite right index read 3.13.
Adding 1 = 4.13.
Set right index of slide to 4.13 on *A*.
Opposite 6.5 on *D* read 13.21. Answer.

This solution required only two settings of the rule. Compare this with the solution required if the equation had remained in its original form. This would have required 3 settings and an addition on paper.

CHAPTER VII

METHODS OF WORKING OUT MECHANICAL AND OTHER FORMULAS

Diameters and Areas of Circles

$A = .7854\ D^2$.

The *B* scale has .7854 $\left(\frac{\pi}{4}\right)$ marked by a long line on the left half.

A	R. Index	
B	Set .7854	Find Areas.
C		
D		Above Diameters

or

A	To 11	
B	Set 6	Find Areas in square feet
C		
D		Above Diameter in inches

To Calculate Selling Prices of Goods, with percentage of profit on Cost Price

C	Set 100	Below cost price
D	To 100 plus percentage of profit	Find selling price

To Calculate Selling Prices, of Goods, with percentage of profit on Selling Price

C	Set 100 less percentage of profit	Below cost price
D	To 100	Find selling price

Example: If goods cost 45 cents a yard, at what price must they be sold to realize 15 per cent profit on the selling price?

C	Set 85 (=100—15)	Below 45
D	To 100	Find 53. Ans.

To find the Area of a Ring.

$$A=\frac{(D+d)\times(D-d)}{1.2732}$$

D	To sum of the two diameters	Find area
C	Set 1.273	Under difference of the two diameters

Compound Interest [Log A = Log $P + n$ Log $(1+r)$]

Opposite 1 plus the rate of interest on *D*, find the corresponding number on the *L* scale and multiply it by the number of years.

Set the indicator to this product on the *L* scale.

Set index of *C* to the indicator.

Opposite the principal on *C*, read the amount for the given number of years at the given rate on *D*.

D	Set Ind. to 105			Read \$244.35 Ans.
L	Read .0212	Ind. to .2 (.0212×10)		
C			Left index to ind.	Opposite 150

This problem is solved simply on the Log Log scales as follows:

$$A = P\,(1 + r)^n$$
$$1 + r = 1.05$$
$$n = 10$$

To 1.05 on LL 1, set indicator.

Since the values on the LL2 scale are the 10th powers of the quantities on the LL 1 scale, read 1.63 under the indicator on the LL 2 scale.

$1.63 = (1 + r)^n$

To 1.63 on *D*, set left index of *C*.

Below 150 (*P*) on *C*, read 244.35 on *D*.

We thus obtain on *D*, below 1 on *C*, a gauge-point for 10 years at 5 per cent. and can obtain in like manner similar ones for any other number of years and rate of interest.

Levers

C	Set distance from fulcrum to power or weight transmitted	Below power or weight applied.
D	To distance from fulcrum to power or weight applied	Find power or weight transmitted

Diameter of Pulleys or Teeth of Wheels

C	Set diameter or teeth of **driving**	Revolutions of **driven**
D	To diameter or teeth of **driven**	Revolutions of **driving**

Diameter of two Wheels to work at given Velocities

C	Set distance between their centers	Find diameter
D	To half sum of their revolutions	Above revolutions of each

Strength of Teeth of Wheels $P = \dfrac{\sqrt{H}}{0.6V}$

A	To H. P. to be transmitted			
B		Indicator		
C	Set gauge point 0.6	to 1	Velocity in ft. per second to indicator	Under 1
D				Pitch in inches

Diameter and Pitch of Wheels $N = \dfrac{D \times \pi}{P}$

DF	To D	Find number of teeth
CF	Set P	Opposite π

Strength of Wrought Iron Shafting

$$D = \sqrt[3]{\frac{83\,H}{N}} \text{ for crank shafts and prime movers}$$

$$D = \sqrt[3]{\frac{65\,H}{N}} \text{ for ordinary shafting}$$

C	Set R. P. M.	Indicator to I. H. P.	
D	To 83 or 65	Read diameter³	Read dia.
K			Opposite dia.³

or

C	Set 3	Opposite index
D		
LL	To diameter³	Read diameter

NOTE.—In this, as in other cases, the coefficients (83 and 65) may be altered to suit individual opinions, without in any way altering the methods of solution.

To find the Change Wheel in a Screw-Cutting Lathe

$$N = T\frac{S \times W}{M \times P} \qquad W = N\frac{M \times P}{T \times S}$$

where
- N = Number of threads per inch to be cut.
- T = " " on traverse screw.
- M = " teeth in wheel on mandril.
- W = " " stud wheel (gearing in M).
- P = " " stud pinion (gearing in S).
- S = " " wheel on traverse screw.

C	Set T	Ind. to P	S to Ind.	Under M
D	To N			Find No. of teeth in W or stud wheel

Rules for Good Leather Belting $W = \dfrac{600 \text{ or } 375 \text{ H. P.}}{V \text{ ft. per min.}}$

D	To 600	Find width in inches	for **Single Belts**
C	Set velocity in feet per min.	Opposite actual H. P.	
D	To 375	Find width in inches	for **Double Belts**
C	Set velocity in feet per min.	Opposite actual H. P.	

Best Manila Rope Driving

A	To velocity in feet per min.	Find **Actual Horse Power**
B	Set 307	
C		Above diameter in inches
D		

Best Manila Rope Driving.

A	To 4	Find **Strength in Tons**
B		
C	Set 1	Above diameter in inches
D		

A	To 107	Find Working **Tension in Pounds**
B		
C	Set 1	Above diameter in inches
D		

A	To 0.28	Find **Weight per Foot in Pounds**
B		
C	Set 1	Above diameter in inches
D		

Weight of Iron Bars in Pounds per Foot Length

A	To 3	Weight of **Square Bars**
B	Set 1	
C		Above width of side in inches
D		

A	To 55	Weight of **Round Bars**
B	Set 21	
C		Above diameter in inches
D		

C	Set 0.3	Below thickness in inches
D	Breadth in inches	Weight of **Flat Bars**

Weight of Iron Plates in Pounds per Square Foot

C	Set 32	Below thickness in thirty-seconds of an inch
D	To 40	Find weight in pounds per square foot

Weights of other Metals

C	Set 1	Below G. P. for other metals
D	To weight in iron	Find weight in other metals

Gauge-points of other metals, and weight per cubic foot.

	W. I.	C. I.	Cast Steel.	Steel Plates.	Copper.	Brass.	Lead.	Cast Zinc.
G. P.......	1	.93	1.02	1.04	1.15	1.09	1.47	.92
Weight....	480	450	490	500	550	525	710	440 pounds

Example: What is the weight of a bar of copper, 1 foot long, 3 inches broad and 2 inches thick?

C	Set 0.3	Indicator to 2 inches thick	1 to Indicator	Below G. P. 1.15
D	To 3 inches broad			Find 23 pounds—Ans.

Weight of Cast Iron Pipes

C	Set .4075	Below **Difference** of inside and outside diameters in inches
D	To **Sum** of inside and outside diameters in inches	Find weight in pounds per lineal foot

G. P. for other metals........	Brass.	Copper.	Lead.	W. Iron
	.355	.333	.259	.38

Safe Load on Chains

A		Safe load in tons
B	Set 36 for open link or 28 for stud-link	Above 1
C		
D	To diameter in sixteenths of an inch	

Gravity

C	Set 1	Below 32.2
D	To seconds	Velocity in feet per second

A	Space fallen through in feet	
B		
C	Set 1	Under 8
D		Velocity in feet per second

A		Space fallen through in ft.
B		Above 16.1
C	Set 1	
D	To seconds	

Oscillations of Pendulums

A		
B	Set length pendulum in in.	
C		Below 1
D	To 375	Number oscillations per minute

Comparison of Thermometers

C	Set 5	Degrees **Centigrade**
D	To 9	Degrees **Fahrenheit** — 32

C	Set 4	Degrees **Reaumur**
D	To 9	Degrees **Fahrenheit** — 32

C	Set 4	Degrees **Reaumur**
D	To 5	Degrees **Centigrade**

Force of Wind

A	To 66	Find pressure in pounds per square foot
B	R. Index	
C		Velocity in **Feet** *per second*
D		

A	To 45	Find pressure in pounds per square foot
B	R. Index	
C		Velocity in **Miles** *per hour*
D		

Discharge from Pumps

A		Gallons delivered per stroke
B	Set 294	Stroke in inches
C		
D	To diameter in inches	

Diameter of Single-acting Pumps

A	To 294			
B	Set length of stroke in inches	R to gallons to be delivered per min.	No. strokes per min. to R	
C				Below 1
D				Diam. pump in inches

Horse Power required for Pumps

C	Set G. P.	Height in feet to which the water is to be raised
D	To cubic feet or gallons to be raised per minute	Horse power required

Gauge Points with different percentages of allowance.

Per Cent.	None	10	20	30	40	50	60	70	80
For Gallons Imp.	3300	3000	2750	2540	2360	2200	2060	1940	1835
" C. Feet	528	480	440	406	377	352	330	311	294
" U. S. Gallons	3960	3600	3300	3050	2830	2640	2470	2330	2200

Theoretical Velocity of Water for any Head

A	Head in feet	
B		
C	Set 1	Under 8
D		Velocity in feet per second

Theoretical Discharge from an Orifice 1 inch Square

A			If the hole is round and one inch dia. the G. P. is 2.62
B	Set 1	Under head in feet	
C			
D	To G. P. 3.34	Discharge in cubic feet per minute	

Real Discharge from Orifice in a Tank 1 inch Square

A			If the hole is round and 1 inch diam., the G. P. is 1.65
B	Set 1	Under head in feet	
C			
D	To 2.1 G. P.	Discharge in cubic feet per minute with coefficient .63	

Gauge Points for other coefficients.

Coefficient	.60	.66	.69	.72	.75	.78	.81	.84	.87	.90	.93	.96
G. P. Square	2.	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.	3.1	3.2
" Round	1.57	1.73	1.80	1.88	1.96	2.04	2.12	2.20	2.28	2.36	2.44	2.52

Discharge from Pipes when real velocity is known

Inverted	*A*		Discharge in cubic feet per min.
	B		Above 1.75
	C	Velocity in feet per second	
	D	Diameter in inches	

Delivery of Water from Pipes

$$W = 4.71\sqrt{\frac{D^5\ H}{L}}$$

Eytelwein's Rule

L	Read log of dia.	Log dia × 5 = *x*		
D	Opp. dia. in inch.			Read cu. ft. per min.
A		Opposite *x*		
B		Set length in feet	Opp. head in feet	
C			Set index	Opposite 4 71

When setting *x* on *A* do not include characteristic

Or:—

LL	To dia. in in.	Read dia.^{5}			
C	Set index	Below 5		Index to indicator	Opposite 4.71
A			Opposite dia.^{5}		
B			Set length in feet	Indicator to head in feet	
D					Read cubic feet per minute

Gauging Water with a Weir

Inverted	*A*		
Inverted	*C*		
Inverted	*B*	Depth in inches	Under 4.3
	D	Depth in inches	Discharge in cubic feet per minute from each foot width of sill

Discharge of a Turbine

$$\frac{\sqrt{H} \times V}{0.3} = D$$

A	Head in feet	
B		
C		Under square inches of water vented
D	0.3	Cubic feet discharged per minute

Revolutions of a Turbine

A	To head in feet			
B				
C	Set diameter in inches	Indicator to 1840	1 to Indicator	Under rate of peripheral velocity
D				Find revolutions per min.

Horse Power of a Turbine

C	Set 530	Indicator to discharge per c. ft. per min.	1 to Indicator	Percentage useful effect
D	Head in ft.			Horse power

or,

A	Under head in ft.					
B / *C*	Set 1	Indicator to head in ft.	158 to Indicator	Indicator to vent in sq. in.	1 to Indicator	Under useful effect
D						Horse power

Horse Power of a Steam Engine

C	Set 21,000	Indicator to diam.	1 to Indicator	Indicator to stroke in ft.	1 to Indicator	Indicator to rev. per min.	1 to Indicator	Mean pressure per sq. inch
D	To diam. in inches							Horse power

or,

A						H. P.
B	Set 21,000	Indicator	1 to	Indicator to	to	Mean pressure
C		to stroke in ft.	Indicator	revolutions	Indicator	
D	To diam. in inches					

CYLINDRICAL COLUMNS.

For solid cylindrical columns of cast iron, both ends rounded, the length of the column exceeding 15 times the diameter

$$P = 33{,}380\,\frac{d^{3.76}}{L^{1.7}}$$

Where P = crushing weight in pounds; d = exterior diameter in inches; L = length in feet.

LL	To *d*	Read $d^{3.76}$	To L	Read $L^{1.7}$		
C	Set 1	Below 3.76	Set 1	Below 1.7	Set $L^{1.7}$	Below 33,380
D					To $d^{3.76}$	Read P

RESISTANCE OF HOLLOW CYLINDERS TO COLLAPSE

Fairbairn's Empirical Formula

$$p = 9{,}675{,}000\,\frac{t^{2.19}}{ld}$$

Where p = pressure in lbs. per square inch; t = thickness of cylinder in inches; d = diameter in inches; l = length in inches.

LL	To *t*	Read $t^{2.19}$		
B or *C*	Set 1	Opposite 2.19		
D			To $t^{2.19}$	At Indicator read *p*
C			Set *d*	
CI			Indicator to *l*	Set 9,675,000 to Ind.

TORSIONAL STRENGTH

For hollow shaft:

$$Pa = .1963 \frac{d^4 - d_1^4}{d} S$$

Pa = moment of the applied force.
d = external diameter of shaft.
d_1 = internal diameter of shaft.
S = unit shearing resistance.

LL	To d_1	Read d^4	To d_1	Read d_1^4			
C	Set 1	Opp. 4	Set 1	Opp. 4	Set d	Ind. to .1963	Opp. Ind.
D					To d^4-d_1^4		Read Pa
CI						S to Ind.	

RADIUS OF GYRATION.

Spherical shell, radii R, r; revolving on its dia.

$$G = .6325 \sqrt{\frac{R^5 - r^5}{R^3 - r^3}}$$

LL	To R	Read R^5	Read R^3	To r	Read r^5	Read r^3		
C or *B*	Set 1	Opp. 5	Opp. 3	Set 1	Opp. 5	Opp. 3		Index to .6325
A							To R^5–r^5	
B							Set R^3–r^3	
D								Read G

FLOW OF AIR IN PIPES.

$$Q = 3.287 \sqrt{\frac{pd^5}{L}}$$

Q = Quantity in cubic feet per second.
p = Head or pressure in lbs. per square inch.
d = Diameter in inches.
L = Length in feet.

LL	To d	Read d^5					
C or *B*	Set 1	Opposite 5	*C*			1 to d	Below 3.287
A				To d^5			
			B	Set L	Ind. to p		
			D			Read Q	Read Q

WORK OF ADIABATIC COMPRESSION OF AIR

$$\text{Mean effective pressure during the stroke} = 3.463\, p_1 \left\{ \left(\frac{p_2}{p_1}\right)^{0.29} - 1 \right\}$$

Where p_1 and p_2 are absolute pressures above a vacuum in atmospheres or in pounds per square inch or per square foot.

Example: Required the work done in compressing one cubic foot of air per second from 1 to 6 atmospheres, including the work of expulsion from the cylinder.

$$\frac{P_2}{P_1} = 6$$

LL 3	To 6					
C	Set 1	Opp. 29	Set 10	Below 3.463		At 10
LL 2		Read 1.681				
D			To .681	Read 2.358 = atmos.	To 2.358	Read 34.66 lb. per sq. in. = mean effective pressure.
CI					Set 14.7	

(Continued)

CF	Above 144	Set 550	Above 1
DF	Read 4990 lbs. per square foot × 1 foot stroke = 4990 ft. lbs.	To 4990	Read 9.08 HP

AIRWAYS

To find the diameter of a round airway to pass the same amount of air as a square airway, the length and power remaining the same.

$$D^3 = \sqrt[5]{\frac{A^3 \times 3.1416}{.7854^3 \times O}}.$$

Where D = the diameter of the round airway.
A = the area of the square airway.
O = the perimeter of square airway.

For slide rule purposes this may be written:

$$D = \sqrt[15]{\frac{A^3 \times 3.1416}{(.7854)^3 \times O}}.$$

D	Under A	Under .7854		
K	Read A^3	Read $(.7854)^3$		Read $\frac{A^3 \times 3.1416}{(.7854)^3 \times O} = x$
DF			To A^3	
CF			Set $(.7854)^3$	
CI				Above O

(Continued)

LL	To x	Find D
C	Set 15	Below 1

FLOW OF STEAM IN PIPES.

$$d = 0.5374 \sqrt[5]{\frac{Q^2 l}{h}}$$

Where d = internal diameter of pipe in inches.
Q = quantity of steam in cubic feet per minute.
l = length of pipe in feet.
h = height of a column of steam, of the pressure of steam at entrance, which would produce a pressure equal to the difference of pressures at the two ends of the pipe.

D	Ind. to Q				To $\sqrt[5]{\frac{Q^2 l}{h}}$	Read Ans.
B	h to Ind.	At l				
A		Read $\frac{Q^2 l}{h}$				
C			Set 5	At index	Set index	At .5374
LL			To $\frac{Q^2 l}{h}$	Read $\sqrt[5]{\frac{Q^2 l}{h}}$		

EFFECTS OF BENDS AND CURVES IN PIPES.

$$\text{Loss of head in feet} = \left\{.131 + 1.847 \left(\frac{r}{R}\right)^{\frac{7}{2}}\right\} \times \frac{v^2}{64.4} \times \frac{a}{180}$$

r = internal radius of pipe in feet.
R = radius of curvature of axis of pipe in feet.
v = velocity in feet per second.
a = central angle, or angle subtended by bend.

C	Set R	Below index				
D	To r	Read $\frac{r}{R}$				
LL 0			To $\frac{r}{R}$	Read $\left(\frac{r}{R}\right)^7$	To $\left(\frac{r}{R}\right)^7$	Read $\left(\frac{r}{R}\right)^{\frac{7}{2}}$
B			Set index	Above 7	Set 2	Above index

(Continued)			
	C	Set index	Below 1.847
	D	To $\left(\frac{r}{R}\right)^{\frac{7}{2}}$	Read $1.847 \left(\frac{r}{R}\right)^{\frac{7}{2}} = x.$

(Continued)

D	Indicator to v			
B	Set 64.4 to Ind.	Ind. to .131 + x	180 to Ind.	Ind. to a
A				At Ind. read Ans.

LOSS OF PRESSURE AND HEAD IN RUBBER-LINED SMOOTH 2½ INCH HOSE.

$$P = \frac{lg^2}{4150\, d^5} \qquad h = \frac{lg^2}{1801\, d^5}$$

P = pressure lost by friction in lbs. per sq. in.
l = length of hose in feet.
g = gallons of water discharged per minute.
d = diameter of hose in inches.
h = friction-head in feet.

C	Set index	Below 5		Indicator to g		
LL	To d	Read d^5				
A			To l			Read Ans.
B			Set 4150 or 1801		d^5 to Indicator	At Index

RELATION OF DIAMETER OF PIPE TO QUANTITY OF WATER DISCHARGED.

$$d = .239 \left\{\frac{Q}{\left(\frac{h}{L}\right)^{\frac{1}{2}}}\right\}^{.387}$$

d = diameter of pipe in feet.
Q = quantity of water discharged in cubic feet per second.
h = head in feet.
L = length of pipe in feet.

A	To L					
B	Set h					
C		Below Q	Set Index	Under .387	Set index	Under .239
D		Find $\frac{Q}{\left(\frac{h}{L}\right)^{\frac{1}{2}}} = x$			To $x^{.387}$	Read Ans.
LL			To x	Find $x^{.387}$		

DENSITY AND VOLUME OF SATURATED STEAM.

$$V = \frac{330.36}{p^{.941}} \qquad D = \frac{p^{.941}}{330.36}$$

V = volume.
P = pressure in lbs. per square inch.
D = density.

C	Set Index	Below 941	Set $p^{.941}$	Below index	Read D
LL	To p	Read $p^{.941}$			
D			To 330.36	Read V	Opposite Index

TOTAL HEAT OF SUPERHEATED STEAM.

$$H = 0.4805\left(T - 10.38\,p^{\frac{1}{4}}\right) + 857.2.$$

H = total heat of superheated steam.
T = temperature in degrees F. + 460.7.
p = pressure in lbs. per square foot.

C	Set 4	Below 1	Set 1	Below 10.38	Set 1
LL	To p	Read $p^{\frac{1}{4}}$			
D			To $p^{\frac{1}{4}}$	Read $10.38\,p^{\frac{1}{4}}$	To $T - 10.38\,p^{\frac{1}{4}}$

(Continued)

C	Under 0.4805	To answer add 857.2
LL		
D	Read $.4805\,(T - 10.38\,p^{\frac{1}{4}})$	

LOSS OF PRESSURE OF STEAM DUE TO FRICTION.

Loss of power, expressed in heat units, due to friction:

$$H_f = \frac{W^3 f l}{10\,p^2 d^5}$$

W = weight in lbs. of steam delivered per hour.
f = the coefficient of friction of the pipe.
l = length of pipe in feet.
p = absolute terminal pressure.
d = diameter of pipe in inches.
f is taken as from .0165 to .0175.

D	Under W				
K	Read W^3				
C		Set index	At 5	Set p	
LL		To d	Read d^5		
A				To W^3	
B					Indicator to f

(Continued)

A		At Indicator read Answer.
B	Set d^5 to Indicator	Indicator to l

MEAN PRESSURE OF EXPANDED STEAM.

$$P_m = p_1\,\frac{1 + \log_e R.}{R}$$

P_m = absolute mean pressure.
p_1 = the absolute initial pressure taken. as uniform up to the point of cut-off.
R = absolute initial pressure.

C	Set index	Read $\log_e R$	Set R	Indicator to p_1
LL	To e	Above R		
D			To $1 + \log_e R$	At Indicator read Answer

Relative efficiency of one lb. of steam with and without clearance.

Back pressure and compression not considered.

$$\text{Mean total pressure} = p = \frac{P\,(l + c) + P\,(l + c)\log_e R - Pc}{L}$$

P = initial absolute pressure in lbs. per sq. in.

$$R = \text{actual ratio of expansion} = \frac{L + c}{l + c}$$

l = period of admission measured from beginning of stroke.
c = clearance in inches.
L = length of stroke in inches.

C	Set (l + c)	At index	Set index	Indicator to l + c	Index to Ind.
D	To (L + c)	Read R			
LL			To R		

(Continued)

C	Indicator to P	Index to (l + c)	At P
D	Read P (l + c) $\log_e$ R		Read P (l + c)

(Continued)

C	Set L	At Index
D	To P (l + c) + P (l + c) $\log_e$ R — Pc	Read p

DIAMETER OF PISTON RODS.

$$d = \sqrt[4]{\frac{D^2\,p\,L^2}{a}} + \frac{D}{80}$$

D = diameter of cylinder in inches.
p = maximum unbalanced pressure in lbs. per sq. in.
L = length in feet.
a = 10,000 and upward, increasing with decrease in speed of engine.

C				At L	Set 4	At Index
A				Read $\frac{D^2 p L^2}{a}$		
B	Set a	Ind. to p	Index to Indicat.			
D	To D					
LL					To $\frac{D^2 p L^2}{a}$	Read $\sqrt[4]{\frac{D^2 p L^2}{a}}$

(Continued)

C	Set 80	At index
A		
B		
D	To D	Read $\frac{D}{80}$
LL		

JOURNAL FRICTION.

Coefficient when shaft is revolving $= (0.2 \text{ to } 0.3) \frac{\sqrt[5]{\text{vel. in ft. per min.}}}{\sqrt[5]{\text{press. in lbs. per sq. in}}}$

NOTE.—This coefficient is for ordinary temperatures, pressures and speeds, with journals and bearing in good condition and well lubricated.

C	Set 5	Opposite index
LL	To vel. in ft. per min.	Find $\sqrt[5]{\text{vel. in ft. per min.}}$

(Continued)

C		At 0.2 or 0.3
D	To $\sqrt[5]{\text{vel. in ft. per min.}}$	Read Answer
A	Set press. in lbs. per sq. in.	

STORM FLOW-OFF.

$$\text{Cubic feet per second} = \underset{E}{\text{A coefficient according to judgment}} \times \underset{N}{\text{Average cu. ft. of rainfall per sec. per acre during heaviest fall}} \times \sqrt[4]{\frac{\text{Average slope of ground in feet per 1,000 feet} = S}{\text{No. of acres drained} = T}}$$

C	Set T	Opposite 1	Set 4	Opposite 1		At N
D	To S	Read $\frac{S}{T}$			To E	
LL			To $\frac{S}{T}$	Read $\sqrt[4]{\frac{S}{T}}$		
CI					Set $\sqrt[4]{\frac{S}{T}}$	Read Answer

TURBINE DISCS.

$$Y = Y_a e^{\frac{-uw^2x^2}{2t}} \quad \text{or } Y = \frac{Y_a}{e^{\frac{uw^2x^2}{2t}}} \text{ (according to Stodola).}$$

$Y =$ thickness at radial distance x
$Y_a =$ thickness of disc carried to shaft center.
$x =$ radial distance of a point from the axis.
$u =$ specific mass.
$w =$ angular velocity.
$t =$ radial and tangential stress per unit of area.

C	Set index	Ind. to w	1 to Ind.	Ind. to x
A	To u			
B				

(Continued)

C				Read Ans.
A				
B	2 to Index	Ind. to Index	t to Ind.	At Index

Any number of values are thus found for $\frac{uw^2x^2}{2t}$, by giving x a number of different values. With the indexes of *C* and *LL* coinciding, read under each value of $\frac{uw^2x^2}{2t}$ on *C* the corresponding value of $e^{\frac{uw^2x^2}{2t}}$ on LL.

From that point onward the solution is one of simple division on the *C* and *D* scales.

CATENARY CURVE

$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$$

C	Set a	At Index	Set 1	At $\frac{x}{a}$		
D	To x	Read $\frac{x}{a}$			To e	
LL 3			To e	Read $e^{\frac{x}{a}}$	Set 1	Read $e^{-\frac{x}{a}}$
LL 0						At $\frac{x}{a}$

Since $e^{-\frac{x}{a}}$ is the reciprocal of $e^{\frac{x}{a}}$, the corresponding value of $e^{-\frac{x}{a}}$ may be found on *CI* directly over the value of $e^{\frac{x}{a}}$ on *C*.

The remainder of the computation is a simple addition; coupled with a simple division, using the *CD* scales.

Example: Let $a = 8$, and $x = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

x	$\frac{x}{a}$	$e^{\frac{x}{a}}$	$e^{-\frac{x}{a}}$	$e^{\frac{x}{a}} + e^{-\frac{x}{a}}$	$\frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) = y$
0	0.	1.	1.	2.	8.
1	.125	1.133	.882	2.015	8.06
2	.25	1.284	.779	2.063	8.252
3	.375	1.455	.687	2.142	8.568
4	.5	1.649	.606	2.255	9.02
5	.625	1.869	.535	2.404	9.616
6	.75	2.12	.472	2.592	10.368
7	.875	2.4	.416	2.816	11.264
8	1.	2.718	.368	3.086	12.344

TRANSMISSION LINES

The capacity in Electro-static units of long transmission lines is given by:

$$C = \frac{l}{2 \operatorname{Log}_e \frac{l_1}{r}}$$

C	Set r	At index	Set 1			Read answer
D	To l_1	Read $\frac{l_1}{r}$				At l
LL			To e	Ind. to $\frac{l_1}{r}$		
CI					Set 2 to Ind.	

If length l is required the formula becomes

$$l = 2\,C \operatorname{Log}_e \frac{l_1}{r}.$$

C	Set r	At index		At 2
D	To l_1	Read $\frac{l_1}{r}$		Read Answer.
LL			To $\frac{l_1}{r}$	
CI			Set C	

Hysteresis loss: $W_h = nB^{1.6}$

C	Set 1	At 1.6	Set 1	At n
LL	To B	Read $B^{1.6}$		
D			To $B^{1.6}$	Read W_h = Answer.

The resistance of dielectrics is given by:

$$R = \frac{It}{C\,(\operatorname{Log}_e E_1 - \operatorname{Log}_e E_2)}$$

LL	At E_1	At E_2		
D	Read $\operatorname{Log}_e E_1$	Read $\operatorname{Log}_e E_2$	*To It*	Read R
C			Set C	
CI				At $\operatorname{Log}_e E_1 - \operatorname{Log}_e E_2$

Dynamometer; to Estimate the indicated H. P.

C	Set 5252	R to length of lever in feet from center of shaft	1 to R	Under rev. of shaft per min.
D	Weight applied at end of lever in pounds, including weight of scale			Actual horse power

To find the Geometric Mean, or Mean Proportional between two numbers, or $a : x :: x : b$

A		
B	Set less No. a	Below b
C		
D	To less No. a	Find X = G. M.

NOTE.—In operations involving square root care should be taken to move the decimal point an even number of places and to use the proper right or left half of A or B.

To reduce fractions to decimals:

C	Set numerator	Find equivalent decimal
D	To denominator	Above 1

To reduce decimals to fractions:

C	Set decimal	Find equivalent numerators
D	To 1	Find equivalent denominators

CHAPTER VIII

TABLE OF EQUIVALENTS OR GAUGE POINTS FOR SCALES C AND D

The following equivalents are in the form of proportions, which should be solved as such, thus

$$\text{Diameters of circles} = \frac{113}{355}\ \text{Circumferences of circles}$$

$$\text{Circumferences of circles} = \frac{355}{113}\ \text{Diameters of circles.}$$

This equation is for slide-rule purposes only and signifies that if 113 on *C* is set to 355 on *D*, then opposite any diameter on *C* the corresponding circumference will be found on *D*.

GEOMETRICAL

113 = Diameters of circles
335 = Circumferences of circles

79 = Diameter of circle
70 = Side of equal square

99 = Diameter of circle
70 = Side of inscribed square

39 = Circumference of circle
11 = Side of equal square

40 = Circumference of circle
9 = Side of inscribed square

70 = Side of square
99 = Diagonal of square

205 = Area of square whose side = 1
161 = Area of circle whose diameter = 1

322 = Area of circle
205 = Area of inscribed square

ARITHMETICAL

100 = Links
66 = Feet

12 = Links
95 = Inches

101 = Square links
44 = Square feet

6 = U. S. Gallons
5 = Imperial gallons

1 = U. S. gallons
231 = Cubic inches

800 = U. S. gallons
107 = Cubic feet

22 = Imperial gallons
6100 = Cubic inches

430 = Imperial gallons
69 = Cubic feet

METRIC SYSTEM

26 = Inches
66 = Centimeters

82 = Yards
75 = Meters

4300 = Links
865 = Meters

82 = Feet
25 = Meters

87 = Miles
140 = Kilometers

43 = Chains
865 = Meters

31 = Square inches
200 = Square Centimeters

140 = Square feet
13 = Square meters

61 = Square yards
51 = Square meters

42 = Acres
17 = Hectares

22 = Square miles
57 = Square kilometers

5 = Cubic inches
82 = Cubic centimeters

600 = Cubic feet
17 = Cubic meters

85 = Cubic yards
65 = Cubic meters

6 = Cubic feet
170 = Liters

14 = U. S. gallons
53 = Liters

46 = Imperial gallons
209 = Liters

108 = Grains
7 = Grams

75 = Pounds
34 = Kilograms

6 = Ounces
170 = Grams

63 = Hundredweights
3200 = Kilograms

63 = English tons
64 = Metric tons

PRESSURES

640 = Pounds per square inch
45 = Kilogs per square centimeter

51 = Pounds per square foot
249 = Kilogs per square meter

59 = Pounds per square yard
32 = Kilogs per square meter

57 = Inches of mercury
28 = Pounds per square inch

82 = Inches of mercury
5800 = Pounds per square foot

720 = Inches of water
26 = Pounds per square inch

74 = Inches of water
385 = Pounds per square foot

60 = Feet of water
26 = Pounds per square inch

5 = Feet of water
312 = Pounds per square foot

15 = Inches of mercury
17 = Feet of water

99 = Atmospheres
2960 = Inches of mercury

34 = Atmospheres
500 = Pounds per square inch

34 = Atmospheres
7200 = Pounds per square foot

30 = Atmospheres
31 = Kilogs per square centimeter

23 = Atmospheres
780 = Feet of water

3 = Atmospheres
31 = Meters of water

29 = Pounds per square inch
67 = Feet of water

1 = Kilogs per square centimeter
10 = Meters of water

COMBINATIONS

43 = Pounds per foot
64 = Kilogs per meter

127 = Pounds per yard
63 = Kilogs per meter

46 = Pounds per square yard
25 = Kilogs per square meter

49 = Pounds per cubic foot
785 = Kilogs per cubic meter

27 = Pounds per cubic yard
16 = Kilogs per cubic meter

89 = Cubic feet per minute
42 = Liters per second

700 = Imperial gallons per minute
53 = Liters per second

840 = U. S. gallons per minute
53 = Liters per second

38 = Weight of fresh water
39 = Weight of sea water

5 = Cubic feet of water
312 = Weight in pounds

1 = Imperial gallons of water
10 = Weight in pounds

3 = U. S. gallons of water
25 = Weight in pounds

50 = Pounds per U. S. gallon
6 = Kilogs per liter

10 = Pounds per Imperial gallon
1 = Kilogs per liter

30 = Pounds per U. S. gallon
25 = Pounds per Imperial gallon

3 = Cubic feet of water
85 = Weight in kilogs

46 = Imperial gallons of water
209 = Weight in kilogs

14 = U. S. gallons of water
53 = Weight in kilogs

44 = Feet per second
30 = Miles per hour

88 = Yards per minute
3 = Miles per hour

41 = Feet per second
750 = Meters per minute

82 = Feet per minute
25 = Meters per minute

340 = Footpounds
47 = Kilogrammeters

72 = British horse power
73 = French horse power

3700 = One cubic foot of water per minute under one foot of head
7 = British horse power

75 = One liter of water per second under one meter of head
1 = French horse power

In no case does the departure, in these equivalents, from the exact ratio attain one per thousand.

EXAMPLES

What is the pressure in pounds per square inch equivalent to a head of 34 feet of water.

C	Set 60	Under 34
D	To 26	Find 14.75 pounds—Answer

What head of water, in feet, is equivalent to a pressure of 18 pounds per square inch.

C	Set 26	Under 18
D	To 60	Find 41.5 feet—Answer

How many horse power will 50 cubic feet of water per minute give under a head of 400 feet.

C	Set 3700	Indicator to 400	1 to Indicator	Under 50
D	To 7			Find 37.8 H. P.—Answer

SUMMARY OF CONVENIENT SETTINGS

In the following problems and formulas a, b, and c represent any numbers and x the unknown quantity or answer sought. For the purpose of illustration only simple numbers are used in the examples. It will be noted that most of the problems may be solved in two ways, *i. e.*, different sets of scales may be used. However, the scales must be selected that will keep at least half of the slide within the scales on the body of the rule.

Multiplication and Division of Three Quantities, One of which is Variable

Example: $x=\frac{a\ c}{b}=\frac{4\ c}{16}$

Set 16 on *C* to 4 on *D*, at any value of *c* on $\begin{cases} C\text{, find } x \text{ on } D \\ CF\text{, find } x \text{ on } DF \end{cases}$

Example: $x=\frac{a\ b}{c}=\frac{75\times 8}{c}$

Set 8 on *CIF* to 75 on *DF*, at any value of *c* on $\begin{cases} CIF\text{, find } x \text{ on } DF \\ CI\text{, find } x \text{ on } D \end{cases}$

Example: $x=\frac{a}{b\ c}=\frac{84}{12\times c}$

Set 12 on *CF* to 84 on *DF*, at any value of *c* on $\begin{cases} CIF\text{, find } x \text{ on } DF \\ CI\text{, find } x \text{ on } D \end{cases}$

Multiplication and Division of Four Quantities, One of which is π with one setting of slide

Example: $x=\pi abc=\pi\times 5\times 8\times 3$

Set 8 on *CIF* to 5 on *D*, at 3 on *C* find 377 on *D*, or at 3 on *CF* find 377 on DF.

Any product or quotient which could be read on *CF* or *DF*, may be divided by π merely by reading on *D* or *C*.

Example: $x=\frac{abc}{\pi}=\frac{35\times 4\times c}{\pi}$

Set 4 on *CI* to 35 on *D*, at values of *c* on *CF* read *x* on *D*.

Example: $x=\frac{a\ b}{c\ \pi}=\frac{8\times 9}{c\times\pi}$

Set 9 on *CIF* to 8 on *DF*, at values of *c* on *CIF* read *x* on *D*.

EXPRESSIONS WHICH MAY BE READ DIRECTLY

By Means of the Indicator, without Setting the Slide

1. $x=a^2$, set Indicator to *a* on *D*, read *x* on *A*.
2. $x=a^3$, set Indicator to *a* on *D*, read *x* on *K*.
3. $x=a^{10}$, set Indicator to *a* on LL1, read *x* on LL2; or set Indicator to *a* on LL2, read *x* on LL3.
4. $x=a^{100}$, set Indicator to *a* on LL1, read *x* on LL3.
5. $x=\sqrt{a}$, set Indicator to *a* on *A*, read *x* on *D*.
6. $x=\sqrt[10]{a}$, set Indicator to *a* on LL3, read *x* on LL2; or set Indicator to *a* on LL2, read *x* on LL1.
7. $x=\sqrt[100]{a}$, set Indicator to *a* on LL3, read *x* on LL1.
8. $x=\sqrt[3]{a}$, set Indicator to *a* on *K*, read *x* on *D*.
9. $x=\sqrt{a^3}$, set Indicator to *a* on *A*, read *x* on *K*.
10. $x=\sqrt[3]{a^2}$, set Indicator to *a* on *K*, read *x* on *A*.
11. $x=\frac{1}{a}$, set Indicator to *a* on *CI*, read *x* on *C*.
12. $x=a\times\pi$, set Indicator to *a* on *D*, read *x* on *DF*.
13. $x=\frac{a}{\pi}$, set Indicator to *a* on *DF*, read *x* on *D*.
14. $x=\frac{\pi}{a}$, set Indicator to *a* on *CF*, read *x* on *CI*.
15. $x=\frac{1}{a\times\pi}$, set Indicator to *a* on *C*, read *x* on *CIF*.
16. $x=\pi\sqrt{a}$, set Indicator to *a* on *A*, read *x* on *DF*.
17. $x=\pi\sqrt[3]{a}$, set Indicator to *a* on *K*, read *x* on *DF*.
18. $x=\frac{a^2}{\pi^2}$, set Indicator to *a* on *DF*, read *x* on *A*.
19. $x=\frac{a^3}{\pi^3}$, set Indicator to *a* on *DF*, read *x* on *K*.
20. $x=\pi\sqrt{\sin a}$, set Indicator to *a* on *S*, read *x* on *CF*.
21. $x=\frac{1}{\pi\sqrt{\sin a}}$, setIndicator to *a* on *S*, read *x* on *CIF*.
22. $x=\pi\tan a$, set Indicator to *a* on *T*, read *x* on *CF*.
23. $x=\frac{1}{\pi\tan a}, =\frac{\cot a}{\pi}$, set Indicator to *a* on *T*, read *x* on *CIF*.
24. $x=\tan a$, set Indicator to *a* on *T*, read *x* on *C*.
25. $x=\frac{1}{\tan a}=\cot a$, set Indicator to *a* on *T*, read *x* on *CI*.
26. $x=\log a$, set Indicator to *a* on *D*, read *x* on *L*.
27. $x=\log_e a$, set Indicator to *a* on LL1, LL2 or LL3, read *x* on *D*.
28. $x=\text{colog}_e\ a$, set Indicator to a on LL0, read *x* on *A*.
29. $x=\log\sqrt{a}$, set Indicator to *a* on *A*, read *x* on *L*.
30. $x=\log\sqrt[3]{a}$, set Indicator to *a* on *K*, read *x* on *L*.
31. $x=\log\frac{a}{\pi}$, set Indicator to *a* on *DF*, read *x* on *L*.

WITH INDICES IN ALIGNMENT.

32. $x = \frac{1}{a^2}$, set Indicator to a on CI, read x on A.

33. $x = \frac{1}{a^3}$, set Indicator to a on CI, read x on K.

34. $x = \frac{1}{\pi^2 a^2}$, set Indicator to a on CIF, read x on A.

35. $x = \frac{1}{\pi^3 a^3}$, set Indicator to a on CIF, read x on K.

36. $x = \frac{1}{\sqrt{a}}$, set Indicator to a on A, read x on CI.

37. $x = \frac{1}{\sqrt[3]{a}}$, set Indicator to a on K, read x on CI.

38. $x = \frac{1}{\pi \sqrt{a}}$, set Indicator to a on A, read x on CIF.

39. $x = \frac{1}{\pi \sqrt[3]{a}}$, set Indicator to a on K, read x on CIF.

EXPRESSIONS SOLVED AT ONE SETTING OF SLIDE

40. $x = a^4$, set 1 to a on D, at a on C, read x on A; or set 1 to a on LL, at 4 on B or C, read x on LL.

41. $x = \frac{1}{a^5}$, set a on C to a on K, at a on CI, read x on K.

42. $x = a^6$, set 1 on C to a on D, at a on C read x on K. or set 1 to a on LL, at 6 on B or C read x on LL.

43. $x = a^7$, set a on CI to a on K, at a on C read x on K; or set 1 to a on LL, at 7 on B or C, read x on LL.

44. $x = a^9$, set a on CI to a on D, at a on C read x on K; or set 1 to a on LL, at 9 on B or C, read x on LL.

45. $x = a^n$, on set 1 to a on LL, at n on B or C read x on LL.

46. $x = \sqrt[n]{a}$, set n to a on LL, at 1 on B or C read x on LL.

47. $x = \sqrt{a^9}$, set 1 to a on A, at a on C read x on K.

48. $x = \sqrt[3]{a^4}$, set 1 to a on K, at a on C read x on D

49. $x = \frac{1}{\sqrt[3]{a^4}}$, set a on CI, to a on K. at 1 on D read x on C.

50. $x = \frac{1}{\sqrt[3]{a^5}}$, set a on C to a on K, at a on CI read x on D.

51. $x = \sqrt[3]{a^7}$, set a on CI to a on K, at a on C read x on D.

52. $x = \sqrt[3]{a^8}$, set 1 to a on K, at a on C read x on A.

53. $x = \frac{1}{\sqrt[3]{a^{10}}}$, set a on C to a on K, at a on CI read x on A.

54. $x = \sqrt[3]{a^{14}}$, set a on CI to a on K, over a on C read x on A.

55. $x = \frac{1}{ab}$, set a on CI to b on D, at 1 on D read x on C.

56. $x = a^5$, set a on CI to a on A, over a on C read x on A.

57. $x = \sqrt{a^5}$, set a on CI to a on A, at a on C read x on D.

58. $x = \frac{1}{\sqrt[3]{a^2}}$, set a on C to a on K, at 1 on C read x on D.

59. $x = \frac{1}{\sqrt[3]{a^5}}$, set a on C to a on K, at a on CI, read x on D.

60. $x = \log a$, set 1 on C to 10 on LL3, at a on LL1, LL2, or LL3 read x on C.

61. $x = \text{colog } a$, set 1 to 10 on LL0, at a on LL0 read x on B.

SETTING FOR TWO FACTORS

62. $x = \frac{1}{a^2 b^2}$, set a on C to 1 on D, over b on CI, read x on A.

63. $x = \frac{1}{a^3 b^3}$, set a on C to 1 on D, over b on CI, read x on K.

64. $x = \frac{a^2}{b^2}$, set b on C to a on D, over 1 on C, read x on A.

65. $x = b\sqrt{a}$, set 1 on C to a on A, under b on C, read x on D.

66. $x = b^3\sqrt{a^3}$, set 1 on C to a on A, over b on C, read x on K.

67. $x = \frac{a}{\sqrt{b}}$, set a on C to b on A, over 1 on D, read x on C.

68. $x = a^2 \sqrt{b}$, set a on CI to b on A, under a on C, read x on D.

69. $x = a^4 b$, set a on CI to b on A, over a on C, read x on A.

70. $x = a^6\sqrt{b^3}$, set a on CI to b on A, over a on C, read x on K.

71. $x = \frac{a^2}{\sqrt{b}}$, set a on C to b on A, over a on D, read x on C.

72. $x = a^2 \sqrt[3]{b^2}$, set 1 on C to b on K, over a on C, read x on A.

73. $x = \frac{\sqrt[3]{a^4}}{b}$, set b on C to a on K, under a on C, read x on D.

74. $x = \frac{\sqrt[3]{a^8}}{b^2}$, set b on C to a on K, over a on C, read x on A.

75. $x = \frac{a^4}{b^3}$, set b on C to a on K, over a on C, read x on K.

SETTINGS FOR TWO FACTORS AND π

76. $x = a\,b\,\pi$, set 1 on *C* to *a* on *D*, over *b* on *C*, read *x* on *DF*.

77. $x = \frac{\pi}{a\,b}$, set *a* on *CI* to *b* on *D*, over 1 on *D*, read *x* on *CF*.

78. $x = \frac{a\,b}{\pi}$, set *a* on *CI* to *b* on *D*, over 1 on *D*, read *x* on *CIF*.

79. $x = \frac{\pi^2 a}{b^2}$, set *b* on *CF* to *a* on *A*, over 1 on *C*, read *x* on *A*.

80. $x = \frac{\pi\sqrt{a}}{b}$, set *b* on *CF* to *a* on *A*, under 1 on *C*, read *x* on *D*.

81. $x = \frac{\pi^2}{a^2\,b^2}$, set *a* on *CF* to 1 on *D*, over *b* on *CI*, read *x* on *A*.

82. $x = \frac{\pi^3}{a^3\,b^3}$, set *a* on *CF* to 1 on *D*, over *b* on *CI*, read *x* on *K*.

83. $x = \frac{\pi^2\,a^2}{b^2}$, set *b* on to *CF* to 1 on *D*, over *a* on *C*, read *x* on *A*.

84. $x = \frac{\pi^3\,a^3}{b^3}$, set *b* on *CF* to 1 on *D*, over *a* on *C*, read *x* on *K*.

85. $x = \frac{a}{\pi\,b}$, set 1 on *C* to *a* on *D*, under *b* on *CIF*, read *x* on *D*.

86. $x = \frac{a^2}{\pi^2\,b^2}$, set 1 on *C* to *a* on *D*, over *b* on *CIF*, read *x* on *A*.

87. $x = \frac{a^3}{\pi^3\,b^3}$, set 1 on *C* to *a* on *D*, over *b* on *CIF*, read *x* on *K*.

88. $x = \pi\,a^2\sqrt{b}$, set *a* on *CI* to *b* on *A*, over *a* on *C*, read *x* on *DF*.

89. $x = \frac{\pi\,a}{\sqrt{b}}$, set *a* on *C* to *b* on *A*, over 1 on *D*, read *x* on *CF*.

90. $x = \frac{\sqrt{b}}{\pi\,a}$, set *a* on *C* to *b* on *A*, over 1 on *D*, read *x* on *CIF*.

91. $x = \frac{\pi\,a^2}{\sqrt{b}}$, set *a* on *C* to *b* on *A*, over *a* on *D*, read *x* on *CF*.

92. $x = \frac{\sqrt{b}}{\pi\,a^2}$, set *a* on *C* to *b* on *A*, over *a* on *D*, read *x* on *CIF*.

93. $x = \frac{\pi\sqrt[3]{a}}{b}$, set *b* on *C* to *a* on *K*, over 1 on *C*, read *x* on *DF*.

94. $x = \pi\sqrt{a^5}$, set *a* on *CI* to *a* on *A*, over *a* on *C*, read *x* on *DF*.

95. $x = \pi\sqrt[3]{a^4}$, set 1 to *a* on *K*, over *a* on *C*, read *x* on *DF*.

96. $x = \frac{\pi}{\sqrt[3]{a^4}}$, set *a* on *CI* to *a* on *K*, over 1 on *D*, read *x* on *CF*.

97. $x = \frac{\pi}{\sqrt[3]{a^5}}$, set *a* on *C* to *a* on *K*, over *a* on *CI*, read *x* on *DF*.

98. $x = \pi\sqrt[3]{a^7}$, set *a* on *CI* to *a* on *K*, over *a* on *C*, read *x* on *DF*.

99. $x = \frac{\pi}{\sqrt[3]{a^2}}$, set *a* on *C* to *a* on *K*, at 1 on *C*, read *x* on *DF*.

100. $x = \frac{\pi}{\sqrt[3]{a^5}}$, set *a* on *C* to *a* on *K*, at *a* on *CI*, read *x* on *DF*.

101. $x = \frac{\pi\sqrt[3]{a^4}}{b}$, set *b* on *C* to *a* on *K*, over *a* on *C*, read *x* on *DF*.

102. $x = \frac{\pi}{a\sqrt[3]{b}}$, set *a* on *CI* to *b* on *K*, over 1 on *D*, read *x* on *CF*.

103. $x = \frac{a\sqrt[3]{b}}{\pi}$, set *a* on *CI* to *b* on *K*, over 1 on *D*, read *x* on *CIF*.

SETTINGS FOR THREE FACTORS

104. $x = a\,b\,c$, set *a* on *CI* to *b* on *D*, under *c* on *C*, read *x* on *D*.

105. $x = a^2 \times b^2 \times c^2$, set *a* on *CI* to *b* on *D*, over *c* on *C*, read *x* on *A*.

106. $x = a^3 \times b^3 \times c^3$, set *a* on *CI* to *b* on *D*, over *c* on *C*, read *x* on *K*.

107. $x = \frac{a \times b}{c}$, set *c* on *C* to *a* on *D*, under *b* on *C*, read *x* on *D*.

108. $x = \frac{a^2\,b^2}{c^2}$, set *c* on *C* to *a* on *D*, over *b* on *C*, read *x* on *A*.

109. $x = \frac{a^3\,b^3}{c^3}$, set *c* on *C* to *a* on *D*, over *b* on *C*, read *x* on *K*.

110. $x = \frac{a}{b\,c}$, set *b* on *C* to *a* on *D*, under *c* on *CI*, read *x* on *D*.

111. $x = \frac{a^2}{b^2 \times c^2}$, set *b* on *C* to *a* on *D*, over *c* on *CI*, read *x* on *A*.

112. $x = \frac{a^3}{b^3 \times c^3}$, set *b* on *C* to *a* on *D*, over *c* on *CI*, read *x* on *K*.

113. $x = a\,b\sqrt{c}$, set *a* on *CI* to *c* on *A*, under *b* on *C*, read *x* on D.

114. $x = a^2\,b^2\,c$, set *a* on *CI* to *c* on *A*, over *b* on *C*, read *x* on *A*.

115. $x = a^3 b^3\sqrt{c^3}$, set *a* on *CI* to *c* on *A*, over *b* on *C*, read *x* on *K*.

116. $x = a\,b\,\sqrt[3]{c}$, set *a* on *CI* to *c* on *K*, under *b* on *C*, read *x* on *D*.

117. $x = a^2\,b^2\sqrt[3]{c^2}$, set *a* on *CI* to *c* on *K*, over *b* on *C*, read *x* on *A*.

118. $x = a^3\,b^3\,c$, set *a* on *CI* to *c* on K, over *b* on *C*, read *x* on *K*.

119. $x = \frac{\sqrt{a}}{b\sqrt[3]{c}}$, set *b* on *CI* to c on *K*, under *a* on *A*, read *x* on *C*.

120. $x = \frac{a\sqrt{b}}{\sqrt[3]{c}}$, set *a* on *C*, to *c* on *K*, under *b* on *A*, read *x* on *C*.

And scores of other combinations.

SETTINGS FOR THREE FACTORS AND π

121. $x = a\,b\,c\,\pi$, set *a* on *CI* to *b* on *D*, over *c* on *C*, read *x* on *DF*.

122. $x = \frac{a\,b\,\pi}{c}$, set *c* on *C* to *a* on *D*, over *b* on *C*, read *x* on *DF*.

123. $x = \frac{a\,\pi}{b\,c}$, set *b* on *C* to *a* on *D*, over *c* on *CI*, read *x* on *DF*.

124 $x = a\,b\,\pi\sqrt{c}$, set *a* on *CI* to *c* on *A*, over *b* on *C*, read *x* on *DF*.

125. $x = a\,b\,\pi\,\sqrt[3]{c}$, set *a* on *CI* to *c* on *K*, over *b* on *C*, read *x* on *DF*.

126. $x = \frac{\pi\sqrt{a}}{b\sqrt[3]{c}}$, set *b* on *CI* to *c* on *K*, under *a* on *A*, read *x* on *CF*.

127. $x = \frac{b\sqrt[3]{c}}{\pi\sqrt{a}}$, set *b* on *CI* to *c* on *K*, under *a* on *A*, read *x* on *CIF*.

128. $x = \frac{\pi\,a\,\sqrt{b}}{\sqrt[3]{c}}$, set *a* on *C* to *c* on *K*, under *b* on *A*, read *x* on *CF*.

129. $x = \frac{\sqrt[3]{c}}{\pi a\sqrt{b}}$, set *a* on *C* to *c* on *K*, under *b* on *A*, read *x* on *CIF*.

ANSWERS.

(Answers given with the problems are not given below.)

1. 300.
2. 3000.
3. 3000.
4. .3
5. .03
6. 3.
7. .0003

8.

	21	22	23	24	25	26	27	28	29
31	651	682	713	744	775	806	837	868	899
32	672	704	736	768	800	832	864	896	928
33	693	726	759	792	825	858	891	924	957
34	714	748	782	816	850	884	918	952	986

9. 49.4%
10. 32.9%
11. 91%
12. 58.5%
13. 15.9%
14. 21.4%
15. 2.24
16. 2.34
17. 1.33
18. 1.32
19. 3.18
20. 67.3
21. 19.3
22. .0000476
23. 5.77
24. 27.5
25. 87.9 In
26. 212
27. 156.
28. .294
29. .735
30. .615
31. 13.6
32. 77.9
33. 19.6
34. 21.4
35. 33.1
36. 56.7
37. 1.6 mils.
38. 10.2
39. 21.6
40. 1.254
41. 74.8
42. 76200.
43. 1170.
44. .436
45. .0039
46. .0000325
47. .000595
48. 5020000.
49. 1.1915
50. 3.76
51. 11.9
52. .376
53. 1.56
54. 9.24
55. .604
56. .560
57. 38.2
58. Square roots of numbers from 110 to 130.

Number	Square Roots
110.	10.5
111.	10.5
112.	10.6
113.	10.6
114.	10.7
115.	10.7
116.	10.8
117.	10.8
118.	10.9
119.	10.9
120.	11.0
121.	11.0
122.	11.0
123.	11.1
124.	11.1
125.	11.2
126.	11.2
127.	11.3
128.	11.3
129.	11.4
130.	11.4

59. 127 feet, 3 inches.
60. 1.9 inches. Use a 2-in. pipe, the nearest standard size.
61. 8.5 inches (Use a 9-in. pipe, the nearest standard size).

Answers to test problems on Page 20.

62. 3.15
63. 1.41
64. 11.4
65. 36.8
66. 13.5

ANSWERS.

Multiplication.

67. 7.39
68. 19.3
69. 7.55
70. 58.5
71. 258.
72. 273.
73. .0541
74. .00167
75. .0000910
76. 12.6, 20.4 **44.0.**

Cubes.

77. 2197.
78. 2744.
79. 3375.
80. 4096.
81. 4913.
82. 5832.
83. 6859.
84. 8000.
85. 9261.

(Three significant figures).

86. 29800.
87. 97300.
88. 104000.
89. 149000.
90. 262000.
91. 436,000,000.
92. 12,500,000.
93. 77,300,000.
94. 679,000.
95. 2,690,000.
96. .04
97. .000104
98. .000,000,314
99. 1.09
100. 9.53
101. 76.1 gal.

Cube Roots.

102. 1.44
103. 3.107
104. 6.69
105. .669
106. .3107
107. .144
108. 13.77
109. 3.628
110. .922
111. 35.59
112. 3.68
113. 1.94
114. .832
115. 6.22
116. 15.66
117. 37.34
118. .2535
119. .211
120. 1.012
121. 47.7
122. 20.4

Other Powers

123. 66,500
124. 41,800
125. 39,700
126. 44,900
127. 40,800
128. 516,000
129. 81,900
130. 0.01698
131. 0.001037
132. 0.004104
133. 0.01432
134. 1.058
135. 1.0015
136. 0.986
137. 0.823

Fractional Powers

138. 815
139. 20,400
140. 195
141. 30.4
142. 1.211
143. 7900
144. 38.8
145. 5.21
146. 1.922
147. 1.0523
148. 1.0057
149. 0.176
150. 0.057
151. 0.9997
152. 1.0022
153. 8.56
154. 627,000
155. 7.53
156. 0.0221
157. 0.1393

Other Roots

158. 11.43
159. 3.35
160. 1.851
161. 1.284
162. 1.2497
163. 1.1325
164. 1.006
165. 0.741
166. 0.809
167. 0.875
168. 0.896
169. 0.989
170. 0.568
171. 0.987
172. 0.997

Fractional Roots

173. 14.5
174. 5.32
175. 7.10
176. 1.691
177. 1.613
178. 1.0438
179. 1.003
180. 0.685
181. 0.465
182. 261,000
183. 211
184. 1.990
185. 1.231
186. 0.056
187. 0.184
188. 1460
189. 104,200
190. 0.983
191. 0.991
192. 0.259
193. 0.9875

Multiplication of More Than Two Numbers.

194. 92.4
195. 114.7
196, 17,490,000
197. 1.309
198. 56.1

Reciprocals

199. .139
200. 2.44
201. .0147
203. 13.7
204. 5.49
205. .0177
206. .136
208. 159.

Three Factors

209. 1200.
210. 46.8
211. 145.
212. 35.6
213. 157.
214. 4.1
215. 130.
216. 89.1
217. 107.
218. 63.6

Four and Five Factors

219. 17,490,000
220. 1.309
221. 56.1
222. 49.2

Combined Multiplication and Division

223. .01815
224. .633
225. 902.
226. 328.
227. 1111.
228. 51.4
229. .353
230. .1585
231. 4.58
232. 1.69
233. .298
234. .280
235. 1.073

Miscellaneous Calculations

236. 32.3
237. 1.91
238. .516
239. .45
240. 1627.
241. 35.8
242. 57,300,000.
243. 1.234
244. .408
245. .00642
246. 81.4
247. 6.4

Sines and Cosines

248. 1.
249. .707
250. .5
251. .0523
252. .0116
253. .264
254. .0262
255. .1478
256. .0393
257. .3665
258. .1736
259. .423
260. .743
261. .970
262. .978
263. .250
264. .585
265. .937
266. .1435
267. .0276
268. 19.
269. 15.1
270. 83.2
271. 32.0
272. 34.5
273. 16.3
274. $a = 9.11$, $b = 8.04$, $c = 6.49$, $d = 5.03$
275. $BC = 4.70$, $BA = 1.71$

Tangents

276. .466
277. .259
278. .713
279. .495
280. .335
281. 1.446
282. 3.78
283. .367
284. .270
285. .1125
286. .0306
287. .911
288. 4.82
289. 29.0
290. 31.9
291. 3.04

Logarithms

292. 0.02098
293. 3.362
2.716
1.500
294. 0.00768
0.0517
0.678
295. 0.0308
0.1501
2.045
296. 9.152
9.8785
9.691
9.9490

297.

	a	b	c
	0.0452	0.0326	0.0167
	0.967	0.698	0.357
	2.275	1.640	0.840
	4.01	2.89	1.479
	8.53	6.15	3.15

298. 2.48
0.195
0.046
4.44
0.02
299. 1.27
0.254
0.040
1.585
0.008

Anti-Logarithms

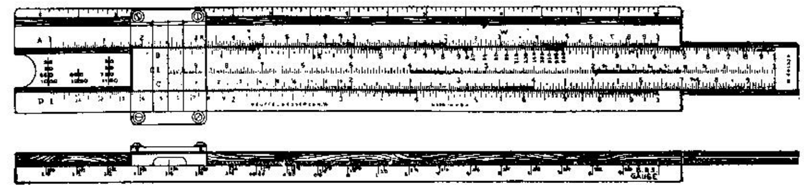
300. 21.9
301. 282,000
302. 2.051
303. 1.1668
304. 22,380,000
305. 0.280
306. 0.468
307. 2.57
308. 1.0473
309. 1.007

INDEX.

SLIDE RULES FOR MANY PURPOSES.

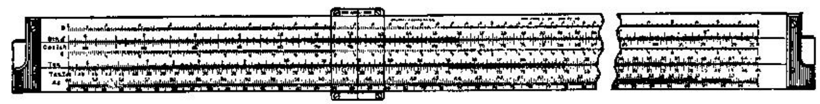
In addition to our extended line of regular Mannheim and Duplex Slide Rules, we manufacture a variety of rules for special purposes, of which we name a few below:

THE ROYLANCE ELECTRICAL SLIDE RULE



This rule is a modification of our regular Slide Rule No. 4035 and can be used for all the calculations made with the ordinary Slide Rule. In addition to the usual scales, it carries a series of scales or gauge marks by means of which the different properties of copper wire, such as size, conductivity, weight, etc., may be determined without the use of tables. It also carries the *CI*, or inverted *C* scale.

SURVEYOR'S DUPLEX SLIDE RULE



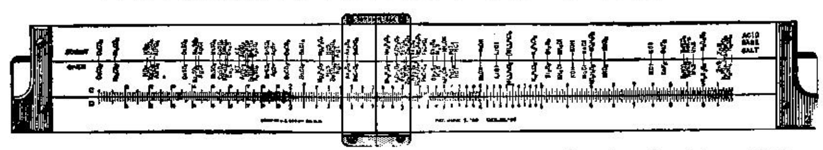
The fact that all astronomical data essential to surveying, such as azimuth, time, latitude, etc., can be ascertained by means of the usual type of Transit with vertical circle but without solar attachment, while generally known, is rather seldom utilized in this country. The main reason for this surprising condition is the difficulty of computing, in the field, by spherical trigonometry, the results of observations.

The new K & E Surveyors' Slide Rule entirely eliminates this difficulty by reducing the hitherto complicated calculations to mere mechanical operations, thereby rendering the method of field astronomy with the regular Engineer's Transit extremely simple and practical.

MERCHANT'S (CALCULATING) SLIDE RULE

Especially designed for the merchant, importer, exporter, accountant, manager, mechanic, foreman, etc. By means of it, all manner of problems involving multiplication, division and proportion can be correctly solved without mental strain and in a small fraction of the time required to work them out by the usual "figuring".

THE CHEMIST'S DUPLEX SLIDE RULE



For the rapid solution of problems in Stoichiometry, such as Gravimetric and Volumetric Analysis, Equivalents, Percentage Composition, Conversion Factors and many others.