THE SLIDE RULE SIMPLIFIED

BY
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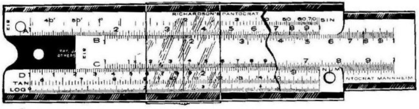

SIXTH EDITION
Revised and Enlarged

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PREFACE

Methods of solving mathematical problems by the use of logarithmic scales have been in use ever since shortly after the invention of logarithms, about 300 years ago. Originally, calculations were performed with the aid of a scale (or scales) and a pair of dividers, a notable example being the mathematical instrument known as the sector. Instruments manufactured at the present time, however, consist of a fixed part or body containing several scales, called the rule, and a sliding or rotating part, also containing several scales, called the slide, the entire instrument being called a slide rule.

Except when extreme accuracy is required, the slide rule can be used for all purposes of draftsmen, electricians, designers, and engineers, and it is also a useful instrument for merchants and business men. By its use, the most difficult and complicated formulas can be solved with ease and with a rapidity that cannot even be approached by any other method of calculation. It is invaluable for checking purposes; and when the very few principles governing its operation (and which are easily and quickly learned) have been mastered, it is practically impossible to make a mistake.

Many persons have been deterred from learning to use a slide rule owing to the erroneous idea that it is difficult to understand and that a knowledge of logarithms is a preliminary necessity. As a matter of fact, its operation is very quickly acquired and no knowledge whatever of logarithms is needed, except that a very slight understanding of the principles of logarithms may be necessary in order to get the full benefits of the special instrument known as the Logometric Slide Rule. In the case of all the other rules and interchangeable slides here described, no knowledge of logarithms is required.

The instructions in this book are clear and explicit, and they are so worded that they can be readily understood by anyone having a fair knowledge of arithmetic. The many illustrations show the beginner how to set the slide for practically every calculation he is likely to make, and the text matter explains in detail the reasons for every setting. The latter part of the book, which describes the use of the special rules and interchangeable slides, was written by Mr. J. J. Clark, M.E., who is the author of many books and pamphlets on engineering and mathematical subjects, and who has also written a book entitled "The Slide Rule," which has been very successful. Special attention is called to the diagrams illustrating the settings; these show at a glance the various settings and the order in which they are to be performed. Nothing similar to these has heretofore been printed.

GEO. W. RICHARDSON.

The Slide Rule Simplified

NOTE.—Pages 3 to 63, inclusive, relate to our engineer's Slide Rule No. 812 non add or subtracting.

Pages 64-65 Relate to Add and Subtracting No. 1812.
Pages 66-71 Relate to Polymetric No. 1776.
Pages 72-81 Relate to Logometric No. 1860-LL.
Pages 82-86 Relate to Binary Polymetric No. 1865-O.
Pages 3-35; 45-56; 58-63; 66-71 Relate to EDUCATOR No. 1917.
Pages 90-101 Relate to MILITARY No. 1918.

All slide rule divisions are to be read decimally, for all spaces are, or should be, divided and subdivided into tenths, the visible marks being fifths, or halves, or even multiples of tenths.

Where spaces do not admit of subdivision, the fractions must be estimated, and after a little practice the eye grows so accustomed to the scale that tenths of a division may be read with sufficient accuracy for all practical purposes.

The divisions on the rule are marked simply with the numbers 1, 2, 3, and etc., **but these numbers are arbitrary** and any value required by the problem in hand may be assigned to them thus, the 3 may be called 3, or 30, or even 300, provided it is borne in mind that the figures on the whole line are affected in the same ratio during the calculation.

The slide rule consists of four scales marked A, B, C, and D, the A and B scales being generally spoken of as the upper set, and the C and D as the lower set. The A and B scales are exactly alike, and the C and D scales are also alike, except that they are divided off twice the length of the former.

The slide has the B and C scales mounted thereon and is free to slide adjacent to the A and D scales.

The **frameless** runner consists of a piece of transparent material, formed to fit the stock of the rule, with a hair line engraved parallel with the lines of the scales, and used for making extensions, and transferring readings from one scale to another. Different users of slide rules require different degrees of tension on their runner. Our frameless runner can be adjusted for any desired tension by the application of a little heat, such as from an electric bulb.

Reference will frequently be made to the **left** or **right** index 1, and it must be borne in mind that the latter, whether on the A, B, C, or D scale, is an arbitrary figure; for example, if a value of 1 is assigned to the left index of the A or B scales, the middle 1 will denote 10, and the right-hand index will be 100, while, on the other hand, if a value of 10 is assigned to the **left**-hand figure 1 on the A and B scales, the center (or middle index 1) will be **100, and the extreme right** index will denote 1,000.

Likewise, we may call the left index 1 of the A scale 1, while we call the left index 1 of the B scale 10, or "vice versa," but it must be remembered that the figures on that whole line are affected in the same ratio during the calculation in hand.

GRADUATIONS

All numbers and divisions on the rule are to be read in decimals, for all the spaces are, or should be, divided and subdivided into tenths, the visible marks may describe fifths, or halves, which are even multiples of tenths, where the spaces do not admit of subdivision, the proportions ¼, ½ and ¾ must be estimated, and when the eye grows accustomed to the scale, tenths of a division may be judged with sufficient accuracy for all practical purposes.

The divisions on the rule are marked simply with the numbers 1, 2, 3, etc., but these numbers are **arbitrary,** and any required value by the problem in hand may be assigned.

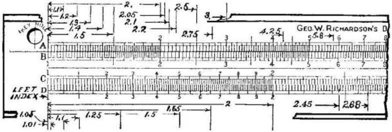


FIG. 1.

The illustration Fig. 1 has been drawn and dimensioned to aid the uninitiated in reading the divisions more readily. The **left** index is shown to be the extreme left of either the A, B, C, or D scales, and in this case it is set for value of 1. But as this 1 is an arbitrary figure, it may, where needed, be called .001, .01, or .1, or any tenth multiple of 1, such as 10, 100, 1,000, 10,000, etc., depending upon the requirements of the problem in hand.

With the value 1 given to the **left** index on the A and B scales the next longest division towards the right is marked 1.5, the longest next following 2, etc. As the distance between the figure 1 and 2 is divided in 10 parts, each one of these divisions are tenths, and are marked in the above illustration as such, thus: 1.1, 1.2, 1.3, 1.4, etc.

These tenths are further subdivided into five parts, which are therefore read as 1.02, 1.04, 1.06, 1.08, that is, of course, when a value of 1 is given the **left**-hand index. Had a value of 10 been given this index, the point 1.5 would be read 15; the 2, as 20, and so forth, while similarly all divisions should be read off at a value increased in the same ratio; the decimal point must in this case be shifted one place to the right, while, on the other hand, if the left index had been given a value of 100, the 1.5 would be read 150; the 2, 200, etc., or the decimal point moved two places to the right. **Always remember that the**

figures are arbitrary, and that when a value is once assigned, all the figures on that line are affected in the same ratio during the calculation.

When the left index value is 1, the right index is 100, and the middle index of the rule is 10. But if the left index is assigned a value of 10, then the right-hand one is 1,000, and the middle is 100.

This applies only to the A and B scales. As the C and D, or lower, scales, have no middle index, the right of the latter is read as 100, when the left has a value of 10, and correspondingly for other values adopted for the convenient solution of any particular problem.

MULTIPLICATION

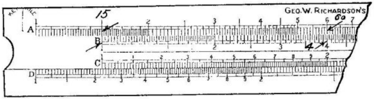


FIG. 2.

Multiply 15 by 4 (15 × 4 = 60). Set the **left** index of the B scale at 15 on the A scale, as illustrated in Fig. 2, and note over 4 on the B scale 60 on the A scale as the answer.

In this example the A scale has a value of 10, while B scale is only given a value of 1. The lower set of scales C and D could also have been employed in the example, and the answer read on same by simply placing the **left** index of C at 15 on D, then opposite 4 on C read the answer 60 on D. As the lower set of scales, C and D, have been laid out to a larger scale, they are to be recommended, as more accurate results can be obtained than on the upper set, A and B.

It may be well to mention here that in the example if the value of 1 instead of 10 was assigned to the A scale, the answer would be 1.5 × 4 = 6. On the other hand, if the value of 10 was assigned to both the A and B scales, or their left indexes, the problem would be 15 × 40 = 600. Hence the importance of assigning a value to the indexes of sufficient magnitude to cover the problem under consideration.

To multiply 18 by 2—set the left index B or C to 18 on A or D and at 2 on B or C read the answer 36 on A or D. The illustration Fig. 3 shows the solution of this problem on the A and B scales only. Thus, left index B set at 18 A, over 2 on B read 36 on A. Also with the slide in this position 18 may be multiplied by any other number if so desired, such as $18 \times 3 = 54$, $18 \times 4 = 72$, and $5 \times 18 = 90$, etc.

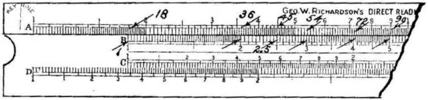


FIG. 3.

CONTINUOUS MULTIPLICATION

Suppose the problem is $18 \times 2 \times 2.5 = 90$. Multiply $18 \times 2 = 36$ as explained in the last example, but leave the hair line of the runner at 36 on A, then pull out the slide to the **right** until the **left** index of B comes under the hair line. This makes the setting to multiply 36 by any given number, and as 2.5 is the one required in this case, simply move the hair line of the runner to 2.5 B and read the answer 90 on A, as shown in Fig. 4.

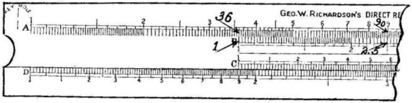


FIG. 4.

If it is desired to multiply $36 \times 63 = 2{,}268$, note that the slide rule will give the answer to only the third figure, and as the fourth is required it may be ascertained mentally thus: as in the above example the rule gives the answer only to the third figure, viz: 226, in order to ascertain the fourth figure simply multiply mentally the units of the two figures, that is $3 \times 6 = 18$, and the required fourth figure in the answer is found to be 8, so that the correct answer is 2,268.

It is very important to know how to proceed when, by reason of the continuous multiplications, it would be necessary to pull the slide from the

stock of the rule, or move the runner off the end of the stock to reach the figures required. This difficulty may be overcome as shown in Fig. 5.

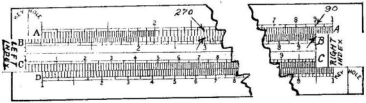


FIG. 5.

In Fig. 4 it was shown how to multiply 2.5×36. Now it is evident that if another extension had to be made, the above obstacles would present themselves. Therefore suppose another extension was to be made, or, in other words, that the last product 90 should be multiplied by 3, set the **right** index of B against 90 on A as shown in Fig. 5, then starting over again (apparently) on the **left** index B, setting the hair line of the runner to 3 on B, and read the answer 270 on A scale.

In this kind of a problem it was shown that the **right** index A (before the slide was shifted to left) equals 100. But as it becomes necessary to frequently change from **right** to **left** index, it should not be forgotten that the **left** index also becomes 100 instead of 1, hence the answer is read as 270 on A.

COMBINED MULTIPLICATION AND DIVISION

Multiply 36×2.5 and divide by 4.5. Proceed to multiply the same as described, setting the left index of B to 36 on A, and place the hair line of the runner to 2.5 on B (without looking at the 90 on A) as shown in Fig. 6. Next as per Fig. 7.

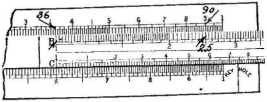


FIG. 6.

Move the slide towards the **left** until 4.5 on B shows under the runner as illustrated in Fig. 7, and read the answer 20 on A over the **left** index of B.

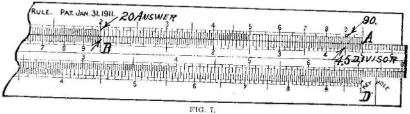


FIG. 7.

It makes no difference if the division is made first or last, and quite frequently it is more convenient to make the division first.

Take the last example given to divide first, place 4.5 on B adjacent to 36 on A and over 2.5 on B read the answer 20 on A.

As stated before, either the upper set of scales A and B or the lower set C and D may be used, and as we have just explained the setting of the former, it may not be out of place to explain the solution on the lower set. Therefore 36 on C against 4.5 on D and read 20 on C against 2.5 on D.

DIVISION

As division is the inverse operation of multiplication, Figs. 6 and 7 will suffice for an explanation.

To divide 90 by 2.5, set 2.5 on B adjacent to 90 on A, as per Fig. 6, and over the left index of B scale read on A scale 36 as the answer.

Likewise in Fig. 7 it is shown that with 90 on A against 4.5 on B, directly over the middle index B, the answer 20 is found on A.

In solving problems where both multiplication and division are required, it is best to multiply a few and then divide them, as by following this routine the slide or runner will not be moved in one direction constantly, and changing ends of the slide is avoided, which in itself is very confusing until one becomes more accustomed to operating the rule.

TO SQUARE A NUMBER

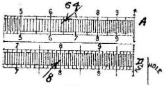


FIG. 8.

Use the A and D scales, and the square of any number on D will be found directly opposite on the scale A. The hair line of the runner is used in reading

from one scale to the other. Thus, set the runner to 8 on D and read on A the answer 64, as shown in Fig. 8.

TO FIND THE SQUARE ROOT OF A NUMBER

The finding of the square root of any number, being the inverse operation of squaring a number, the square root of 64 on A is, therefore, with aid of the runner, found to be 8 on D, as shown in Fig. 8.

TO CUBE A NUMBER

The four scales A, B, C, and D must be used. For example, find the cube of 3.

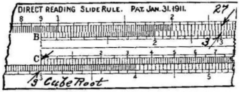


FIG. 9.

With the left index set at 3 on D, read the answer 27 on A adjacent to 3 on B, Fig. 9.

TO FIND THE CUBE ROOT OF A NUMBER

The finding of the cube root of a number is the inverse operation of cubing a number. Example: Find the cube root of 64. This also requires the use of the four scales.

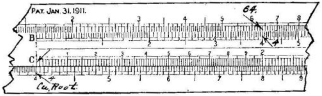


FIG. 10.

Set the hair line of the runner to 64 on A, as in Fig. 10, move the slide back and forth until a setting is found where the unit or figure is the same on the B scale under the hair line as it is on D at the left index C. In Fig. 10, 4 on B is opposite to 64 on A, and 4 on D is also opposite the left index of C therefore 4 is the cube root of 64.

The cube root in the previous problems was found under the **left index.**

This does not hold good in all cases. Therefore, if it cannot be found on the left, try the **right** index. How to find the cube root of 729 is shown in Fig. 11.

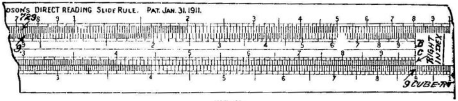


FIG. 11.

When the number opposite to 729 on A is the same as the number opposite to the **right** index C the cube root is found. In this case 9 on D.

TO REDUCE VULGAR FRACTIONS TO A DECIMAL OR "VICE VERSA"

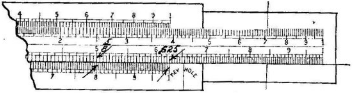


FIG. 12.

Fig. 12 illustrates the simplicity of changing the vulgar fraction ⅝ to a decimal: Set 5 on C to 8 on D and over the right index of D read the decimal equivalent .625. Likewise the fraction may be found when the decimal is known.

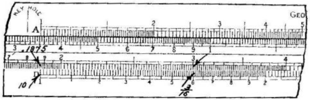


FIG. 13.

Fig. 13 illustrates the manner of ascertaining the decimal equivalent of $\frac{3}{16}$. Set 3 on C opposite to 16 on D and opposite the **left** index of D read the answer decimally, .1875, on C.

TO EXTRACT CUBE ROOT ON RICHARDSON'S 10″ SLIDE RULE No. 812

Notation—Call left end-mark on all scales 1; denote any number on left half of scales A and B by giving its figures and the letter of the scale (thus, 27B means 27 on left half of scale B); Similarly, 27D means 27 on scale D; denote the numbers 2.1544 and 4.6416 on scale C by c^1 and c^2, respectively.

Cube Root.—To cube a number, as 2, bring 1C to 2D, and opposite 2B read 8 on A. To find the cube root of a number, as 8, move the slide until the number opposite 8 on B is the same as the number on D opposite 1C; this is evidently 2. Now retaining this setting, bring the runner to c^1, and the number on D under the hair line will be the cube root of 80, since $\sqrt[3]{80}=\sqrt[3]{8\times10}=\sqrt[3]{8}\times\sqrt[3]{10}=\sqrt[3]{8}\times2.1544$. Bringing the runner to c^1, the number on D under the hair line will be the cube root of 800, since $\sqrt[3]{800}=\sqrt[3]{8}\times\sqrt[3]{100}=\sqrt[3]{8}\times4.6416$.

Example.—Find the cube root of 78.5.

Solution.—78.5=7.85×10. Bring runner to 785A; move slide until number on B under hair line equals number on D opposite 1C; this is apparently 199, but the reading on B is 198; hence, the true value is more nearly $\frac{2\times199+198}{3}=$ 1987, and $\sqrt[3]{7.85}=1.987$. Now bring 1C to 1987 D and move runner to c^1; under the hair line on D read 428; hence, $\sqrt[3]{78.5}=4.28$. Bringing the runner to c^2, read on D 923; whence, $\sqrt[3]{785}=9.23$.

Procedure.—Point off the number into periods of three figures each, in the usual way, in order to determine the position of the decimal point in the root. Shift the decimal point until it occupies the same relative position in the left-hand period that contains one or more digits; there will then be 1, 2, or 3 digits to the left of the decimal point; if 1, proceed as for $\sqrt[3]{8}$; if 2, proceed as for $\sqrt[3]{80}$; if 3, proceed as for $\sqrt[3]{800}$. Thus, 0.7854 will be regarded as 785.4; 0.007854, as 7.854; and 78540, as 78.54.

The reason for proceeding as above in obtaining the number 1987 is that two of the factors are 199, the third is 198, and the arithmetical mean is (199+199+198)÷3=(2×199+198)÷3=1987, neglecting decimal points.

QUESTION.

Multiply 27 by 20?

Set runner to 27 on the "D" scale, bring left hand 1 of "C" scale to the hair line and under 2 of "C" scale, read on the "D" scale, 54. As in this case you have used the left hand 1 of the "C" scale, the slide projects to the right, and the number of figures in the product will be ONE LESS than the sum of the figures in the multiplier and multiplicand. The answer is therefor, 540.

QUESTION.

Multiply 3000 by 1800?

You read 54 on the "D" scale; as slide projects to the right, you mark off 8 less 1, or 7 figures, adding cyphers if necessary, and the answer becomes 5,400,000.

QUESTION.

Multiply 4150 by 825?

On the "D" scale read 342, and as the slide projects to the left you place 7 figures in the answer adding 4 cyphers, and the answer becomes 3,420,000 correct to three significant figures.

RULE.

If there is a decimal point in either multiplier or multiplicand, or in both, first treat them as if they were whole numbers, i. e., had no decimal points, and then move the decimal point of the product to the left a number of places equal to the sum of the decimal places in both the multiplier and the multiplicand.

EXAMPLE.

Multiply 41.5 by 8.25?

Slide projects to the left, therefore their product as whole numbers would have 6 figures, and this less the number of decimal places in both numbers, or 3, gives three figures in the product. The answer is therefore, 342.

QUESTION.

Multiply 1.85 by 3.45.

On the "D" scale read 6375, and as the slide projects to the right, there would be five places in the product, if the numbers had no decimal points. As there are 4 decimal places in both of the numbers, the correct number of places in the product will be 5 less 4, or 1 place, and the answer becomes 6.37.

QUESTION.

Multiply 2850 by 925? Answer 2,635,000.
Multiply 28.50 by 925? Answer 26,350.
Multiply 265 by 270? Answer 71,500.

MULTIPLICATION OF A CHAIN OF FIGURES.

RULE.

NOTE HOW MANY TIMES THE SLIDE PROJECTS TO THE LEFT IN GETTING THE PRODUCT, TO THIS NUMBER ADD THE SUM OF ALL THE FIGURES IN ALL THE FACTORS AND THEN SUBTRACT THE NUMBER OF FACTORS **LESS ONE.**

EXAMPLE.

Multiply 275, 85, 125 and 7 together?

Read 223 on the "D" scale, and as the slide projected to the left 2 times in getting the product, the number of figures in the answer will equal the number of figures in all the factors or 9, plus the number of slides to the left or 2, minus the number of factors less one or 3. The answer will therefore contain 9 plus 2 minus 3 places or 8 figures. Your answer is therefore, 22,300,000.

EXAMPLE.

Multiply 450 by 28 by 42?

You will get 528 on the "D" scale, and as the slide projected once to the left, you will have 1 plus 7 minus 2, or 6 figures in the answer. Your answer will therefore be 528,000.

EXAMPLE.

Multiply 69 by 50 by 91?

Read 313 on the "D" scale, and as the slide projected twice to the left, the number of figures will equal 2 plus 6 minus 2, or 6, and the answer becomes 313,000.

Where any or all of the factors contain decimals, first treat them as if they were integers and then after getting the number of places or figures in the answer, as integers, subtract the sum of the decimal places in all the factors, or after getting the number of places as integers, move the decimal point of the product to the left a number of places equal to sum of decimal places in all the factors.

EXAMPLE.

Multiply 48 by .165 by 7.8?

Read on the "D" scale 617, as slide has projected to the left once, the number of figures in the answer if all the factors had been whole numbers, would have been 1 plus 7 minus 2, or 6, and the answer would have been 617,-000, you then move the decimal point to the left, a number of places equal to the number of decimal places in all the factors, or 4, and you get the answer, 61.7.

EXAMPLE.

Multiply .67 by 42 by .0029?

On the "D" scale read 817, and as the slide projected once to the left, regarding the factors as integers, the answer would have 5 figures in it, or 81,700. The sum of the number of decimal places in the factors is 6, you therefore move the decimal point 6 places to the left, adding one cypher to do this, and your answer becomes .0817.

EXAMPLE.

Multiply .047 by .0068 by .0088 by 2.7?

Read on the "D" scale 76, and regarding the factors as integers, the answer would have 8 plus 2 minus 3, or 7 figures, it would be therefore, 7,600,-000. Making off to the left the sum of the number of decimal places in all the factors, adding the necessary cyphers, you get as the answer, .0000076.

DIVISION.

In division the important thing to remember and note is the number of times the slide projects **to the right.**

RULE FOR DIVIDING ONE NUMBER BY ANOTHER.

WHEN SLIDE PROJECTS TO THE RIGHT NUMBER OF FIGURES IN ANSWER TO LEFT OF DECIMAL POINT WILL EQUAL NUMBER OF FIGURES IN DIVIDEND **PLUS ONE** MINUS THE NUMBER OF FIGURES IN THE DIVIDEND. IF SLIDE PROJECTS TO THE LEFT THE NUMBER OF FIGURES WILL BE EQUAL TO NUMBER OF FIGURES IN THE DIVIDEND LESS NUMBER OF FIGURES IN THE DIVISOR.

QUESTION.

Divide 480 by 24?

Read 2 on the "D" scale; slide projects to the right, so the number of figures will be 3 plus 1 minus 2, or 2, and answer is 20.

QUESTION.

Divide 5680 by 85?

Read 668 on the "D" scale; slide projects to the left and there will be 4 less 2, or 2 figures in the answer which is accordingly 66.8.

WHERE EITHER DIVISOR OR DIVIDEND CONTAIN A DECIMAL POINT.

A decimal point in the divisor makes the answer larger as you are dividing by a smaller number than if the figures of it were an integer. Likewise a decimal point in the dividend makes the answer smaller because you are dividing into a smaller number than if the figures represented an integer.

From this you derive the following rule:

IN THE DIVISION OF ONE NUMBER BY ANOTHER, IF THERE IS A DECIMAL PLACE IN EITHER OR BOTH NUMBERS, OBTAIN AN ANSWER AS THOUGH THEY WERE INTEGERS AND THEN MOVE THE DECIMAL POINT TO THE LEFT, THE NUMBER OF DECIMAL PLACES IN THE DIVIDEND, AND THEN MOVE IT BACK TO THE RIGHT, THE NUMBER OF DECIMAL PLACES IN THE DIVISOR.

EXAMPLE.

Divide 6850 by 26.5?

Read 259 on the "D" scale; slide projects to the right so that regarding both numbers as integers, the answer would contain 4 plus 1 minus 3, or 2 places, and would therefore be 25.9. Pointing off to the right the number of decimal places in the divisor, that is 1 place, your answer becomes 259.

EXAMPLE.

Divide 2.87 by 276?

Read 104 on the "D" scale; slide projects to the right so that regarding both numbers as integers, the answer would contain 3 plus 1 minus 3 or 1 figure to left of decimal point, and would therefore be 1.04. Marking off to the left the two decimal places in the dividend, you get as the final answer, .0104.

EXAMPLE.

Divide 28.75 by 30850?

Read 933 on "D" scale; slide projects to left, therefore number of figures will equal 4 minus 5 minus 2, or — 3; you will therefore add 3 cyphers to the left of first significant figure and answer becomes .000932.

COMBINED MULTIPLICATION AND DIVISION.

Examples of this sort involve a combination of the previous rules for division and multiplication, and can be solved by the following rule:

RULE.

NOTE EVERY TIME IN AN OPERATION OF MULTIPLYING THAT THE SLIDE PROJECTS TO THE LEFT AND ALSO EVERY TIME IN AN OPERATION OF DIVISION THAT THE SLIDE PROJECTS TO THE RIGHT AND KEEP ACCOUNT OF THE NUMBER OF THESE OPERATIONS; THEN TO THIS SUM ADD THE SUM OF ALL THE FIGURES IN ALL FACTORS ABOVE THE LINE AND SUBTRACT THE SUM OF ALL THE FIGURES IN ALL THE FACTORS BELOW THE LINE, AND THEN SUBTRACT THE NUMBER OF FACTORS ABOVE THE LINE **LESS ONE.** IN GETTING THE SUM OF THE FIGURES IN ALL FACTORS ABOVE OR BELOW THE LINE DON'T COUNT CYPHERS BETWEEN FIRST SIGNIFICANT FIGURE AND DECIMAL POINT.

EXAMPLE.

$$\text{Solve } \frac{27 \times 75 \times 150 \times 42}{16 \times 9 \times 115} = 770$$

Read 77 on the "D" scale; in the multiplication the slide has projected to the left twice, and in the division the slide has projected once to the right, you therefore have for the number of slides, 3, and this plus 9, the number of figures in the factors above the line, minus 6, the number of figures in all the factors below the line, minus 3, one less than the number of factors above the line, gives 3. And your answer will therefore be 770.

EXAMPLE.

$$\text{Solve } \frac{22 \times 22 \times 650 \times 75}{33000} = 716$$

Read 716 on the "D" scale, slide has projected twice to the left in the multiplication and no times to the right in division, you will therefore have in the answer 2 plus 9 minus 5 minus 3, or 3 figures, and the answer will be 716.

RULE.

Where any or all the factors, either above or below the line, contain decimal points, proceed as though all the factors were integers, and then point off to the right the sum of the decimal places below the line, and then point back to the left the sum of all the decimal places above the line, adding cyphers if necessary.

EXAMPLE.

$$\text{Solve } \frac{245 \times .68 \times .0075 \times 14}{1.15 \times .09 \times .55} = 309$$

Read 309 on the "D" scale; slide projects twice to the left in multiplication and once to the right in division; you therefore add 3 to the number of figures above the line, or 9, minus 6 (the number of figures below the line) plus 6 (the number of decimal places below the line) minus 6 (the number of decimal places above the line), all minus one less than the number of factors above the line, or 3, which will all give you 3 figures in the answer which will therefore be 309.

This could be expressed in this way:

Add 3, Number of projections of slide.
Add 9, Number of figures in all factors above line.
Add 6, Number of decimal places below the line.
Subtract 6, Number of figures below line.
Subtract 6, Number of decimal places above the line.
Subtract 3, ONE less than number of factors above line.

The sum of the adds is 18 and the sum of the subtracts is 15; the difference between these two sums is 3, and this result is plus as the adds are larger. You will therefore have 3 figures to the left of the decimal point.

If the sum of the subtracts had been larger than the sum of the adds, then you would add cyphers to the left of the first significant figure, in number equal to this difference. If the sums of the adds and subtracts are equal, then the decimal point adjoins the first significant figure.

DECIMAL POINT IN USE OF A. & B. SCALE.

The operation of placing the decimal point in using the "A" and "B" scale is essentially the same as in using "C' and "D" scales but it is rendered somewhat more difficult by the fact that the scales are of half length and it is consequently more difficulty to lay down any rule for counting the slide projections. The following rule covers this point as well as any.

RULE FOR MULTIPLICATION.

IF THE NEAREST LEFT HAND INDEX TO THE MULTIPLIER ON THE "B" SCALE, EXTENDS TO THE LEFT OF THE NEAREST LEFT HAND INDEX TO THE PRODUCT ON THE "A" SCALE COUNT IT, OTHERWISE DO NOT.

RULE FOR DIVISION.

IF THE NEAREST RIGHT INDEX TO THE DIVISOR ON THE "B" SCALE, EXTENDS TO THE RIGHT OF THE NEAREST RIGHT HAND INDEX TO THE QUOTIENT ON THE "A" SCALE, COUNT IT, OTHERWISE DO NOT.

PROPORTION

Either the upper or lower set of scales may be used for proportion, but the lower being drawn to a larger scale they are to be preferred. It will be noted that by moving the **slide** to the **right** until the **left** index of C is opposite to any number, say 2, on D, that any number on D coinciding with a number on C gives a proportion equal to 2 to 1.

To find the unknown quantity, or, in other words, the fourth term, of the proportion 180 : 25 :: 28.8 : X, set the slide as follows:

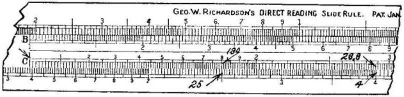


FIG. 14.

The illustration Fig 14 shows 180 on C opposite to 25 on D, and at 28.8 on C is the unknown quantity, or fourth term, 4 on D. This applies to all similar calculations. It is well to remember, however, that the first and third terms are always found on one scale, while the second and fourth terms are found on the other scale adjacent thereto.

MENSURATION OF SUPERFICIES

Tiling—A problem: Find the cubic contents in feet of a piece of tiling 2¼ feet (2.25 ft.) long, 17 inches wide, and 1¾ (1.75) feet thick.

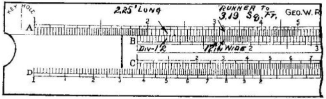


FIG. 15.

First calculate the sq. ft. as shown above by setting 12 on B at 2.25 (feet) on A and by placing the runner to 17 (inches in width) on B, read the sq. ft., 3.19, on A.

Second, the cubic contents may be ascertained by bringing the left index of the slide under the runner to the 3.19 on A and at 1.75 on B read the cubic contents 5.6 on A. This is illustrated in Fig. 16.

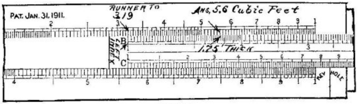


FIG. 16.

Find the area in square feet of a strip of tiling 65 ft. long and 14 in. wide

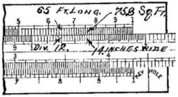


FIG. 17.

Set 12 on B to 65 on A and over 14 (inches) on B read the answer 75.8 sq. ft. on A, as shown in Fig. 17.

Tiling—How many squares contained in a piece of tiling 57 ft. long by 50 ft. wide?

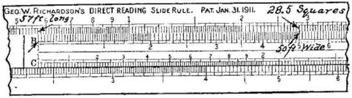


FIG. 18.

Set the left index B at 57 ft. on A and opposite to 50 ft. wide on B read 28.5 squares as the answer on A scale, as shown in Fig. 18.

Brick Work—Find the number of square rods in a brick wall 765 ft. long and 8 ft. high.

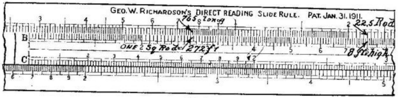


FIG. 19.

Set the number 272 on B adjacent to 765 on A and opposite 8 on B read the answer, 22.5 sq. rods, on A. See Fig. 19.

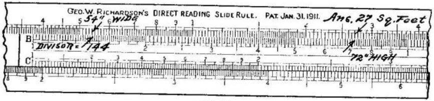


FIG. 20.

Glass, Sheet Material, Etc.—How many sq. ft. of glass in a door 72 in. high and 54 in. wide? In this example the length and width are both in inches, therefore the divisor is 144.

Set 54 on A opposite to 144 on B, and against 72 on B read the answer, 27 sq. ft., on A. (Fig. 20.)

Paviors, Plasterers, Painters, Walls—Measurements by the square yard or 9 sq. ft.

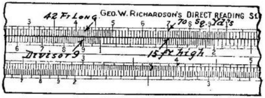


FIG. 21.

How many square yards in a wall 15 ft. high and 42 ft. long? Set 9 on **B** opposite to 42 on A, and over 15 on B read the answer, 70 sq. yd., on A.

Paviors' Sq. Yds.—A piece of paving 18¾ (18.75) ft. long and 15½ (**15.5**) ft. wide contains how many square yards?

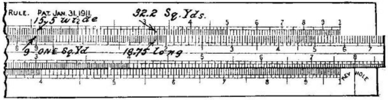


FIG. 22.

Set 15.5 on A adjacent to 9 (1 sq. yd.) on B. Then against 18.75 on B read 32.2 sq. yd. on A.

Lumber, Board Measure, Sq. Ft.—If the width is given in **inches** and the length in **feet**, set 12 on B to the length (in feet) on A and at the width in inches on B read the answer in square feet on A, as shown in Fig. 23.

How many sq. ft. in a board 14 in. wide and 18 ft. long? Place 12 on B against 18 on A, and over 14 on B read 21 sq. ft. on A.

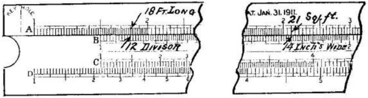


FIG. 23.

To find the square feet of, say, 39 boards of the size in the last example, simply set 1 on B against 21 on A, and opposite to 39 on B read the answer, 820 sq. ft., nearly, on A. This is shown in Fig. 24.

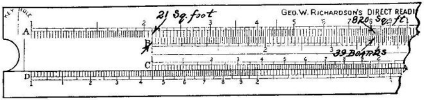


FIG. 24.

Note—This applies only to boards 1 in. To find the board measure for any other thickness, multiply according to the rules given for multiplication.

Board Measure—Wheel Strips, and Small Pieces—Find the number of sq. ft. board measure of 311 pieces of wheel strips 1½ in. × 1⅝ in. × 6½ ft. long. As explained before, the slide rule is scaled decimally, therefore it becomes necessary to change the above vulgar fraction into decimals, which may be done either mentally or upon the slide rule. Instructions for this operation are explained elsewhere under its heading.

Briefly, we may say when the above numbers are thus changed the problem would read 1.5 × 1.625 × 6.5 × 311.

First—Find the sq. ft. in one piece by placing 12 on the C scale opposite to 6.5 on the D scale, as shown in Fig. 25, and at 1.5 on C read .81 sq. ft. on the D scale.

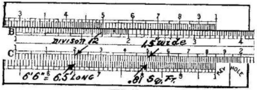


FIG. 25.

Second—To multiply .81 × 311, proceed as in Fig. 26, by setting the **right** index of C to .81 on D, then under 311 on C read the answer, 252 sq. ft., on D. (Fig. 26.)

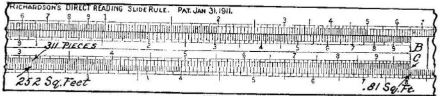


FIG. 26.

Third—The examples are for boards 1 in. thick. For material 1⅝ in. thick, which equals 1.625, leave the runner at 252 on D and bring the **left**

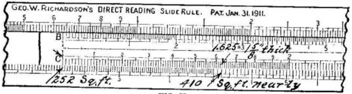


FIG. 27.

index C opposite to it. Then at 1.625 on C read the answer on D as almost 410 sq. ft., the correct answer being 409½ sq. ft. (Fig. 27.)

Paper Mills, Printers, Box Manufacturers—Find the number of sheets of cardboard to cut a given number of cards. Suppose the sheet measures 20 in. × 30 in. and the cards are to be cut 3 in. × 5 in.

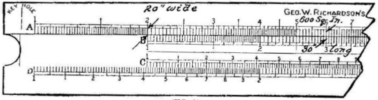


FIG. 28.

Set the slide as shown in Fig. 28, with left index B at 20 on A, then at 30 on B read 600 sq. in. on A.

A card 3 in. × 5 in. contains 3 × 5 = 15 in. Therefore, if the rule is set as shown in Fig. 29 with 15 on B opposite to 600 on A, the answer can be read off at random for the required sheets to cut any given amount of cards. Fifty sheets make 2,000 cards, 90 sheets make 3,600, or for any number of cards wanted, on the A scale, the required number of sheets is opposite thereto on the B scale.

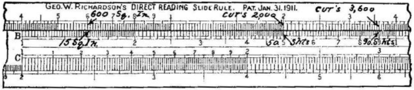


FIG. 29.

Land Measuring—Acreage—In measuring land the divisors are generally 10 sq. chains, 160 sq. perches, or 4,840 sq. yds. per acre.

How many acres in a piece of land 26 chains and 20 links in length, and 3 chains and 50 links in width?

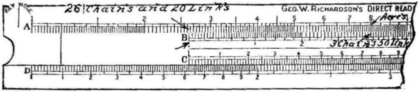


FIG. 30.

Set the rule as shown in Fig. 30 with left index of B at 26.2 and on A read 8 acres opposite to 3 chains and 50 links on B.

How many acres in a strip of land 28 perches wide and 40 perches long? Set 160 on B opposite to 28 on A and at 40 on B read the answer, 7 acres, on A, as shown in Fig. 31.

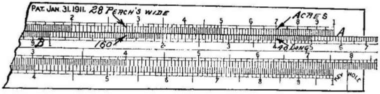


FIG. 31.

How many acres in a piece of land 420 yards long and 75 yards wide?

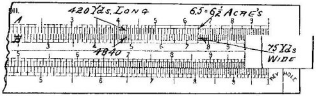


FIG. 32.

Set the rule as shown in Fig. 32 with 420 on A opposite to 4,840 on B, and read the answer, 6.5 = 6½ acres, on A opposite to 75 on B.

MISCELLANEOUS PROBLEMS

Timekeepers and Pay-Roll Accountants—A workman receives $22.50 per week of 50 hours, but he only works 7 hours; how much is due him; also what is the rate per hour?

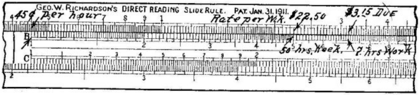


FIG. 33

The illustration here shown, Fig. 33, exemplifies the value of the slide rule, because, when the rate per week is set on the A scale opposite to the number of

hours which constitute a week's work on the B scale, the amount due for any other number of hours worked set on the B scale will be found opposite thereto on the A scale, and in addition to this, the rate per hour is shown at either the left or right index on the A scale.

In the above example set $22.50 on the A scale oposite 50 (hours) on the B scale, and over 7 (hours worked) on the B scale read the amount due opposite thereto on the A scale, or $3.15.

If the rate per hour is required, it can be found on the A scale opposite to the **left** index B. Yet under certain conditions the rate per hour will be shown opposite to the **right** index B, depending upon the magnitude of the problem, Fig. 33 shows it at the **left.**

Bankers, Brokers—A customer receives an annual interest of $770 on a loan for $22,000; what is the rate of interest?

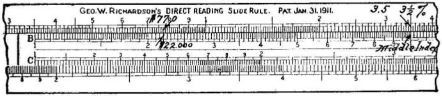


FIG. 34.

Place 22 on the B scale (which is the same as 22,000) at 77 on the A scale (which is the same thing as 770), and opposite the middle index 1 on B is shown 3.5 = 3½% on the A scale. Other problems are worked in a similar manner.

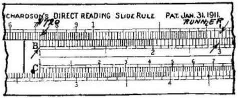


FIG. 35.

Example—Find the interest due on $600 at 6% for 2 years and 5 months.

Solution—2 years and 5 months equals 29 months. First, set left index of B scale to 6 on the A scale, and move the hair line of runner to 6 (6%) on the B scale. Second, move slide until 12 comes under the runer, moving the hair line again to 29 on the B scale, and under the hair line on the A scale read 87 or $87.

Example—A merchant purchases a bill of goods for $18.00 and sells same for $27.00. Find what percentage of profit he makes on the sale.

Solution—The difference equals 27 less 18 is 9, therefore set 9 of the C scale adjacent to 18 on the D scale, and against the (left) index D read 5 (which represents 50%) net profit.

Example—The list price of an article is $27.00. It is sold to dealers at $18.00. What discount in percentage does he receive?

This is liking to the last problem, except you set 27 (instead of 9) on C adjacent to 18 on D scale, and opposite the **right** index C read 66 2-3%, and this deducted from 100% leaves 100 — 66 2-3 = 33 1-3%.

You could however get the same answer without subtracting by counting 33 1-3 **backwards** from the **right D** scale index to the **right C** scale index, or in other words the spaces or remainder left between these two points.

How much interest will be due on $720 at 3% for 5 months?

Set the left index of B to 72 on A, also on the left half of A scale as shown in Fig. 35. Place runner 3 on B and leave it at this position. Move the slide B to the **left** until 12 comes under the hair line of the runner as shown in Fig. 36, and opposite to 5 on B read the answer, $9.00 interest, on the A scale.

To find the rate per day, use 365 days for divisor, instead of 12 months.

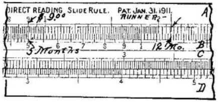


FIG. 36.

Merchants—If 850 articles cost $6.30, what will 270 articles cost? Or, if 850 pounds cost $6.30, what will 270 pounds cost?

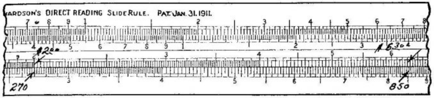


FIG. 37.

Set 850 on D opposite to 6.30 on C, as shown in Fig. 37, and at 270 on D read the answer, $2.00, on A.

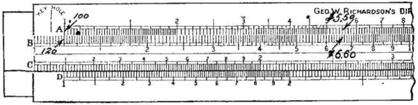


FIG. 38.

A bill of goods was bought for $5.50; what price must they be sold at to make a profit of 20%? The cost price is in proportion to the sale price, as 100 : 120. Therefore, place 120 on B under left index of A scale and at 5.5 ($5.50), also on the A scale read the answer $6.60 on the B scale. (See Fig. 38.) This applies to all similar problems.

A bill of goods was sold amounting to $745. It cost $26.60 to sell them; what is the cost of selling in per cent?

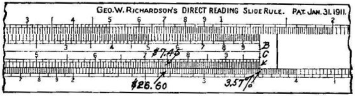


FIG. 39.

Set 745 on C opposite to 26.60 on D, and read 3.57% on D scale opposite to the **right** hand end, or **right** index, of the C scale. (Fig. 39.)

Trade Discounts—A bill of goods cost $6.35 with 37½% discount; what is the net cost? (A slight mental calculation is required in this problem, thus: 100 — 37.5 = 62.5.)

Setting rule as shown in Fig. 40 with **right** index of the C scale against 62.5 on D, the net amount, $3.96, on D can be read at $6.35 on the C scale.

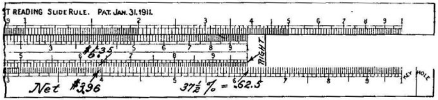


FIG. 40.

Note—If more than one discount is required, say, for instance, 5% off from $3.96, simply repeat the operation by setting the hair line of the runner to 396 on D, bringing the **right** index C to the hair line (and recall that 100 — 5 = 95), and move runner again to 95 on C, and read the net, $3.76, on D as illustrated in Fig. 41. Additional discounts may be calculated in a similar manner.

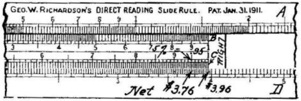


FIG. 41.

ELECTRICAL

THE SLIDE RULE A COMPLETE WIRE TABLE.

Knowing any **one** of the following values, five others may be read off, viz: diameter in mils, area in circular mils, square mil area, pounds per 1,000 ft.; resistance in ohms per 1,000 ft., size wire B. and S. gauge.

An annealed copper wire, at 20° C. or 68° F., and 80.8 mils in diameter required the other equivalents as above.

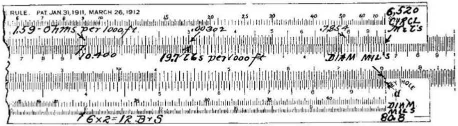


FIG. 42

Proceed to set 80.8 on the C scale directly over the right index of the D scale as shown in Fig. 42.

Note—Then directly under and in line with the **right** index of the A scale read on the B scale 6,520 circular mils area.

To find the square inch or square mil area, leave the slide as per first setting (Fig. 42) and place the hair line of the runner to the special graduation mark .7854 (if such a graduation mark is on the rule) on the A scale, and read opposite same 51, which means .0051 sq. in. area, on the B scale.

To find the weight per 1,000 ft., set the runner on the A scale to 302 (.00302) and adjacent to same read on the B scale 19.7 pounds per 1,000 ft., as shown in Fig. 42.

To find the resistance in ohms per 1,000 ft., place the hair line of the runner to 104 (10,400) on the B scale, as shown in Fig. 42, and read the answer, 1.59 ohms res., under the runner of the A scale.

To find size wire B. and S. gauge set runner as per resistance in ohms 1.59 per (1000 ft.) on A scale and under hair line on lower log scale read 6 (double this) equals No. 12 B. and S. gauge.

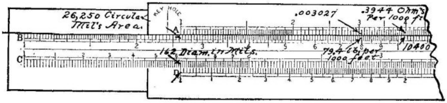


FIG. 43.

Using the middle index of B scale (instead of 10.4) corresponds to a temperature 10° C. or 50° F.

The above holds good only for wires under 100 mils in diameter. For larger wires it is necessary to use the left index, as illustrated in Fig. 43. The diameter in mils, 162, is set on C at the left index of D and its circular-mils area is read off similarly as before on the B scale as 26,250 at the left index of A. The other values are read of as per Fig. 43. Thus, 79.4 pounds is found opposite to 302 on A, also opposite 10,400 on B read on A .3944 ohms per 1,000 ft. With hair line of runner set as above, i. e. at .3944 on (A) scale read under the same hair line on the LOG scale three (3) double this gives you (6) or number 6 B. and S. gauge.

TESTING A RECORDING WATT-METER

One of two methods are generally employed. The first consists of comparing under load a recording watt-meter known to be correct with the one to be tested. The percentage of slow or fast may be ascertained by the use of the slide rule by referring to examples under the head of proportion explained elsewhere in the text.

The second method, which is the one to be explained and illustrated in Fig. 44, involves the use of a standard portable, direct-reading watt-meter, and a stop watch and some suitable resistance to give a load, such as lamp bank or a water rheostat.

It will be necessary to explain briefly what is meant by a meter constant. The constant is generally stamped on the dial or the disc of the meter and the results of the reading of the meter must be multiplied by it to get the true watts, and it is based on the number of seconds in one hour, which is 60 seconds times 60 minutes = 3,600 seconds in one hour. Therefore, if a meter constant is stamped 1, the setting on the slide rule will be 3,600 on the D scale, while if it is stamped ½, use 1,800; likewise, if it is stamped 2, use 7,200, etc.

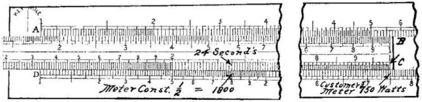


Fig. 44.

The illustration Fig. 44 shows the setting of the slide rule from data obtained from a test, in which the customer's meter was under test 24 seconds, and the meter constant = ½ (1,800), set 24 on C against 1,800 on D and directly under the right index C note 750 watts on D.

By comparison of the company's meter it is found to be 770 watts and to ascertain the percentage of slow or fast, set 770 on C at 750 on D, as shown in Fig. 45, and read 2.5% or 2½% slow. To distinguish whether the meter is slow or fast, note, if the slide projects to the left of index D, as in Fig. 45, which indicates slow, while if the slide projected to the right of index D, the meter would be fast. The percentage is read by the number of divisions between the end of the right index C and D, each main division being 1% and the smaller ones .5 or ½%.

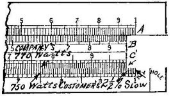


FIG. 45.

Find the electrical energy expended in delivering 18 amperes through a circuit, the resistance of which is 210 ohms.

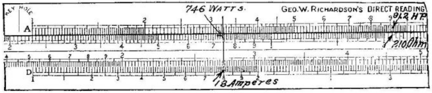


FIG. 46.

Use the four scales A, B, C, and D, set the hair line of the runner to 18 on D and move the slide to the left until 746 on B is under the hair line of the runner; then opposite to 210 on B read on the A scale 91.2 H. P.

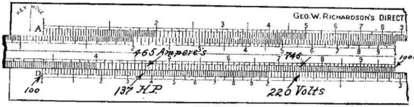


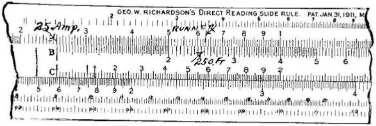
FIG. 47.

A motor absorbs, on the average load, 465 amperes at 220 volts; find the horsepower. Set the rule as shown in Fig. 47: 220 (volts) on D opposite to 746 (watts) on C and at 465 (amperes) on C read the answer, 137 H. P., on D.

CALCULATING SIZE OF WIRE CIRCULAR MIL AREA

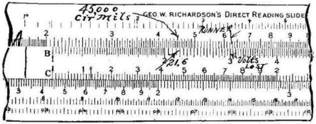
Example—Find the size of wire **circular mil area** to transmit 25 amperes 250 feet, with a **drop** not to exceed (3) three volts.

Solution—First—Set the **left** index of the B scale to 25 on the A scale, placing the hair line of the runner at 250 B, as per cut.



Second—Move 3 on the B scale under the hair line, and over 21.6 on the B scale read the answer on the A scale as 45,000 circular mil area, see cut.

Note—The reason for using the constant 21.6 is because it represents the resistance of 2 mil feet of copper wire at normal temperature and thus saves the labor of multiplying the distance by 2, as in a two-wire circuit. To find size B. and S., refer to Figs. 42 and 43.



HEAT LOSSES OR SQ. OF THE CURRENT TIMES RESISTANCE.

Find the **heat loss**, or what is known as the current squared multiplied by the resistance, in a circuit of 5.6 ohms' resistance, carrying 25 amperes. Set the **left** index of C at 25 on D and at 5.6 on B read the answer, 3,500 watts, on A.

SIZE OF GEAR AND PULLEYS FOR VARIOUS SPEEDS

A pair of gear wheels, one having 58 teeth and making 23½ revolutions per minute, meshes with another that has 34 teeth; how many revolutions per minute does the latter make?

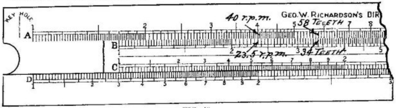


FIG. 48.

Set 58 on A at 34 on B, and opposite to 23.5 on B read the answer on A as 40 r.p.m., as per Fig. 48.

Also note that, knowing the number of revolutions required of each gear, setting them opposite to one another on the A and B scales and observing the latter scales, the required number of teeth of any other set of gears that would perform the same operation are shown. **Furthermore, that diameters of pulleys may be substituted** for number of gear teeth and same results be obtained.

CHANGE GEARS FOR SCREW CUTTING LATHES.

Find the right size gears to cut a thread of $\frac{5}{16}$-in. pitch (as the rule is read decimally, the decimal equivalent should be found as explained under the proper heading, being in this case .3125) when the guide screw of the lathe is ½-in. or 5-in. pitch.

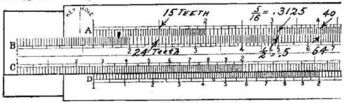


FIG. 49.

Set the rule as shown in Fig. 49 with .3125 on A at .5 on B. Then any number of teeth required in each gear will be found as illustrated. Thus, 15 at A at 24 on B will do the work. Also 40 on A against 64 on B will suffice, or any other pair of gears found opposite one another on A and B or C and D scales may be selected.

THE RICHARDSON DIRECT READING SLIDE RULE

On the Richardson **Direct**-Reading Slide Rule there is a sight opening, or what is termed a key hole, in the upper left and lower right-hand corners of the A and D scales. In these key holes red letters appear as the slide is moved back and forth. These red letters are for the solution of problems, reference to which is on the back of the rule and also in the text of this book. The following examples will explain this feature of the rule. The direct reading rule does not prevent the solution of any problems by its adoption. It simply gives you this feature in addition to what all other rules have not.

Gallons and Cubic Inches—The illustrations under this head (Figs. 50-71) will show the solution of the different problems, and therefore they will be only briefly explained, as the operation is very simple indeed.

The red letters, as explained before, are those printed on the slide and show through the key holes, but are printed in black in the cuts. They are designated as **keys,** and the sight openings in the A and D scales as **key holes.** They will be referred to by these names throughout the text.

Take, for an example, the problem of converting cubic inches into gallons. Find the proper heading on the back of the rule and note that the red key in this instance is S, and that as indicated in the column of scales the number of gallons are found on the D scale, while the number of cubic inches should be on the C scale.

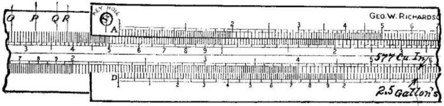


FIG. 50.

Now turn to the face of the rule and move the slide to the right or left until the key S appears in the upper key hole of the A scale. Suppose it is required to know how many gallons are contained in 577 cubic inches. (As explained before, remember that the figures on the scales are arbitrary.) Place runner over 577 on C and read 2.5, or which is the same thing, 2½ gallons, on the D scale. This is shown in Fig. 50.

Cubic Feet and Cubic Inches—Proceed as before. The key being O, place it in the key hole and note that 6,050 cubic inches on the C scale equals 3.5 cu. ft. on the D scale. Similar problems are calculated likewise. (See Fig. 51.)

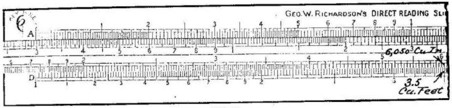


FIG. 51.

Area and Diameter—What is the diameter of a circle the area of which is 755 sq. in.?

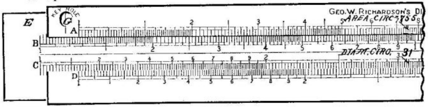


FIG. 52.

Refer to the back of the rule and note the key is G. Place it in the key hole and use the A and C scales, as shown in Fig. 52. Opposite to 755 on the A scale will be found 31 (in. in diameter) on the C scale.

Circumference and Diameter—The small key, j, on the lower right end of

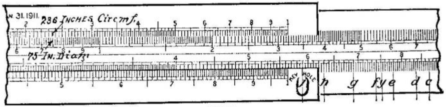


FIG. 53.

slide, is in this case the proper key. Also in the scale column on the back of rule we note that the A and B scales are to be use. Therefore, as shown in Fig. 53, at 236 (in. in circumference) on A we find 75 (in. in diameter) on the B scale.

Gallons and Cubic Feet—How many U. S. gallons in 3 cu. ft.?

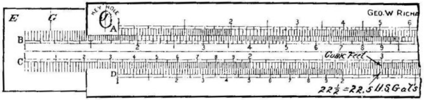


FIG. 54.

Place the capital J in the upper left-hand key hole and opposite to 3 on C read the answer, 22.5 or 22½ gallons, on the D scale, as in Fig. 54.

Evaporation from and at 212° F. and Std. Boiler H. P.—In an evaporation test on a steam boiler it was shown that the boiler evaporated 2,070 lb. water, from and at 212° F., per hour; what was the boiler horsepower?

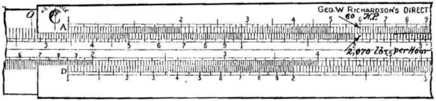


FIG. 55.

Set rule as shown in Fig. 55 with key P in the right-hand key hole and read, opposite to 2,070 on the B scale, 60 horsepower on the A scale.

Meters and Inches—How many inches in 4½ (4.5) meters?

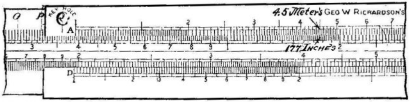


FIG. 56.

Place the key Q in the key hole as shown in Fig. 56 and opposite to 4.5 on A, read the answer, 177 in., on B.

Piston Speed in Feet Per Minute—What will be the piston speed in ft. per minute of an engine making 200 r.p.m. with a stroke of 15 in? First multiply 200 × 15 (= 3,000) mentally or according to the rules of multiplication previously explained under that heading.

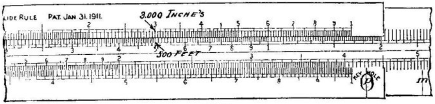


FIG. 57.

Set rule as in Fig. 57, with small letter p in the right-hand key hole, and opposite to 3,000 on A scale read 500 (feet of piston speed) on the B scale.

Horsepower of an Engine—The horsepower of an engine may be ascertained by first calculating the piston speed in feet per minute as explained in the preceding problem.

Let us assume the m.e.p. (mean effective pressure) to be 40 lb. per sq. in., and the diameter of piston 20 in., the square of which will be 20 × 20 = 400 sq. in., and the piston speed in ft. per min. 420.

As explained under the head of continuous multiplication, these values would equal 40 × 400 × 420 = 6,720,000, but for our convenience we need for the present consider it as only 672.

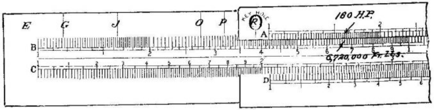


FIG. 58.

Place R in the-left-hand key hole, as shown in Fig. 58, and over 672 on B read the answer, 160 h.p., on the A scale. To decide what value to give the answer it is only necessary to consider that an engine of these dimensions could neither develop as high as 1,600 h.p., nor as little as 16 h.p., consequently the only other possible value of the answer is 160 h.p.

Ventilating Ducts—The velocity of air in a duct was found to be 540 ft. per min. And it is desired to find the area in square feet of a duct that will supply 1,200 cu. ft. of air per minute.

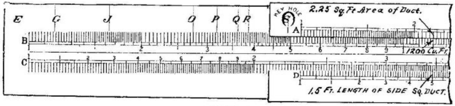


FIG. 59.

Place the key S in the key hole, as in Fig. 59, and opposite to 1,200 on the B scale read the answer, 2.25 (sq. ft. area), on the A scale. Also, if the duct is to be **square,** the length of one side will be given by placing the hair line of the runner to 2.25 on A, as above, and under the same line read 1.5 linear feet per side on D.

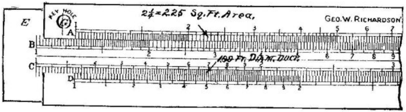


FIG. 60.

If the duct is to be **round,** leave the hair line of the runner as before at 2.25 on A, and move the **slide** until the key G comes into the key hole, as shown in Fig. 60, and read the diameter of the duct, 1.69 ft., on C under the runner line.

GALLONS AND POUNDS OF WATER.

The fresh water in a tank weighs 541 lb.; how many gallons does it contain? Set rule as in Fig. 61 with the key c in the right-hand key hole and against

541 on the scale C, read 65 (gallons), on D. Similar problems may be read off along these two scales in the same manner.

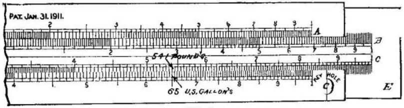


FIG. 61.

Pressure and Height—In a standpipe, the level of the water above a gauge tapped into its side is 90 ft.; how much pressure will the gauge show?

Set the rule as shown with the key, g, in the right-hand key hole and opposite to 90 (feet) on A read 39 (pounds) on B.

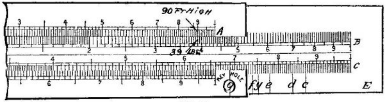


FIG. 62.

Diagonal and Side of a Square—The diagonal of a square is 7.5 ft.; what is the side of the square? Set rule as shown in Fig. 63, with key, f, in the key hole, and against 7.5 on D read the answer, 5.3 on C.

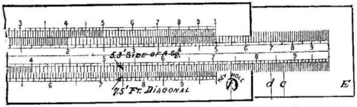


FIG. 63.

Area and Side of an Equal Square—Place the hair line of the runner over the number indicating the area on the A scale, and the side of the square will be found on D under the hair line, or vice versa, the runner is placed over the number indicating the side of a square on D, the area will be given opposite on A.

Kilowatts and Horsepower—Place the small letter e in the key hole, as

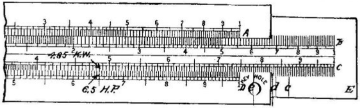


FIG. 64.

shown in Fig. 64, and against any kilowatts on C read the horsepower on D, as, for instance, placing the runner at 4.85 (k.w.) on C, the corresponding horsepower will be found to be 6.5, on D.

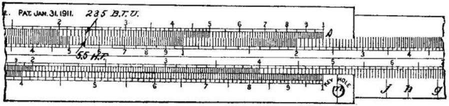


FIG. 65.

B. T. U. and Horsepower—If the rule is set as in Fig. 65, with m in the key hole and the runner placed on 235 (heat units [B.T.U.] expended per minute) on the A scale, 5.5 (H.P.) may be read opposite thereto on the B scale. Similar problems may be solved in the same manner.

Direct Radiation and Size of Grate—What should be the grate surface in sq. ft. of a boiler that would supply 1,800 sq. ft. of direct radiation, including the mains?

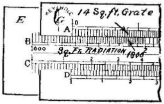


FIG. 66.

Set the key G in the left-hand key hole, as in Fig. 66, and opposite to 1,800 on B read 14 (sq. ft. of grate surface) on A.

Diameter Safety Valve and Square Feet of Grate—Wanted to find the diameter of a safety valve for a grate area of 36 sq. ft. As in Fig. 67, set d in the key hole, and against 36 on A read 4.8 (in. in diam.) on C.

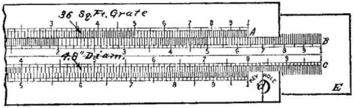


FIG. 67.

Diameter Steam Mains and Square Feet Radiation—To find, for instance, the diameter of a steam main to supply 1,600 sq. ft. of direct radiation, set key, G, in the key hole, using right half of B and C scales, and place the hair line of the runner over 1,600 on B. The diameter 4 (inches) may be read opposite on C, and the area, if desired, on A. (Fig. 68.)

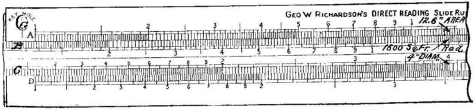


FIG. 68.

For Hot Water—With the same key, G, in the key hole, set 1,600 on A instead of B, and read the diameter on C as in the last example.

For Indirect Radiation—Add 75% to the area, or the radiation, before calculating the diameter.

Horsepower of a Waterfall—Suppose the area in sq. ft. of the water flowing over a dam is 4 ft.; the velocity of the water 150 ft. per min., and the height of the fall 12 ft.; what is the horsepower of the fall? Proceed as with continuous multiplication, explained under that head, and find the product of

150 × 4 × 12 = 7,200. Next place the key, S, in the key hole and over 72 on the B scale (which is the same as 7,200), read the answer on the A scale, 13.5 (horsepower). (Fig. 69.)

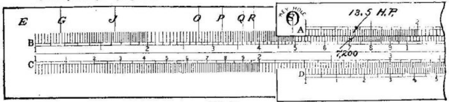


FIG. 69.

Horsepower and Belting—To find the horsepower a **single**-ply belt will transmit when the diameter of pulley is 23 in., the r.p.m. 100, and the width of the belt is 3.5 inches. As explained under the head Continuous Multiplication, find the product of 23 × 100 × 3.5 = 8,050. With h in the key hole and the hair line of the runner set at 8,050 on the A scale, as shown in Fig. 70, the answer, 2.93 h.p., may be read under the hair line on the B scale.

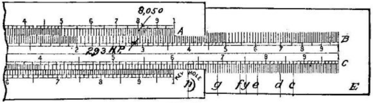


FIG. 70.

FRACTIONS—MULTIPLICATION AND DIVISION OF

To **multiply** a given number, say 275 by $\frac{3}{16}$, set 16 on B adjacent to 275 on A (see Fig. 71), and over 3 on B read 51.5 as the answer on A.

To **divide**, reverse the operation, as 51.5 divided by $\frac{3}{16}$. Set 3 on B (Fig. 71) adjacent to 51.5 on A scale, and read the answer 275 on A over 16 on B.

This applies to all similar problems.

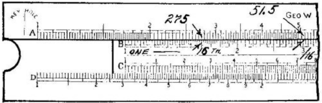


Fig. 71

STEAM BOILER SAFE WORKING PRESSURE

Given—A boiler of the following dimensions: Thickness of plate, $\frac{7}{16}''$; tensile strength, 60,000 lbs. per sq. in.; efficiency of joint, 70%; factor of safety, 5; radius, 36 in. Required the safe working of the boiler?

The above put in the form of an equation would be $P = \frac{\frac{7}{16} \times 60{,}000 \times .70}{5 \times 36}$

This requires three settings of the slide rule.

First—Set 16 on **left** half of B scale adjacent to 6 (60,000 lbs.) on **left** half of A scale, as per Fig. 72, and place hair line of runner to 7 ($\frac{7}{16}$) on B.

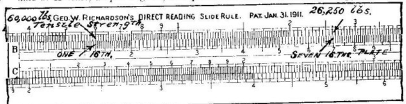


Fig. 72

Second—Set 5B (factor of safety) under the hair line (Fig. 73).

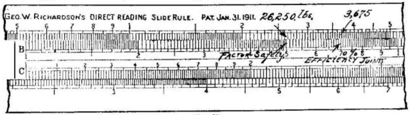


Fig. 73

Third—Move hair line of runner to 7 (efficiency of joint) on B (Fig. 73).

Fourth—Move slide B until 36 (radius) shows under the runner (Fig. 74). Then over the **left** index B1 read 102 lbs. adjacent on A scale, as the safe working pressure.

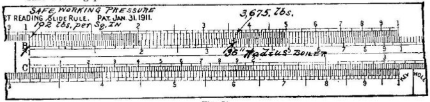


Fig. 74

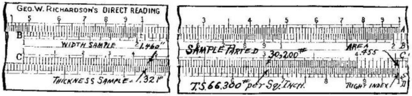
SAMPLE TESTING

A piece of sample steel was to be tested, it measured .321" thick, by 1.460" width at its least cross section.

When put in the testing machine and a stress of 30,200 lbs. applied **the sample parted,** find the ultimate tensile strength of sample.

Set 146 on **C** to 321 on **D,** and opposite (30,200) that is, 302 **on C** read 66,300 lbs. T. S. per square inch on **D.**

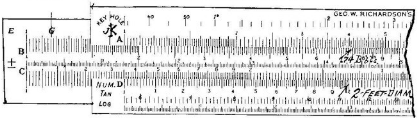
The area of sample is shown on **C adjacent to right** index **D.** (See fig.)



TANK BARRELS PER FOOT OF DEPTH

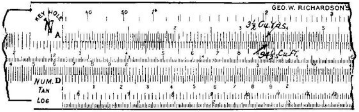
Example—How many U. S. Barrels per each foot of depth in a tank 2 feet in diameter?

Place the red key (K) in the keyhole and set hair line of the runner to 2 on **D** and read .74 of a barrel on the B scale. (See cut.)



CUBIC YARDS AND CUBIC FEET

How many cubic yards in 94½ cubic feet? Place the key **N** in the key hole. and adjacent to 94½ on the **B** scale read 3.5 = 3½ cubic yards. (See cut.)



DIAMETER CIRCLES INCHES VS. AREA SQUARE FEET

Example—How many square feet in a circle 24 inches in diameter? Use the key **K** in the key hole and by aid of the hair line set at 24 **C** read under the hair line on **A** 3.15 square feet.

CYLINDER DIAMETER, MEAN EFFECTIVE PRESSURE, FEET PER MINUTE, AND HORSEPOWER

Example—Find the cylinder diameter of an engine to develop 154 horsepower with a piston speed of 600 feet per minute, and a mean effective pressure of 42 pounds per square inch.

First—Set 154 H. P. on the **right** half of the A scale adjacent to 6 (600 ft. per minute) on the **left** half of the B scale (Fig. 76), with the hair line of the runner set to 33 (33,000 ft. pounds) on the left half of the B scale as shown also in Fig. 76.

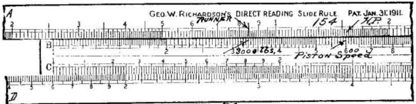


Fig. 76

Second—Move the slide B to the left until 42 comes under the runner (Fig. 77) and adjacent to the special graduation mark (x) on left end of C scale, read 16 inches on the D scale as the required cylinder diameter for such an engine.

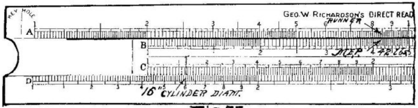


Fig. 77

Rule 1—To find the horsepower set the special graduation mark (x) on the C scale to the cylinder diameter (16) on D scale (Fig. 77) and place the runner to 42 (M. E. P.) on the B scale. Moving the slide to the right (Fig. 76) until 33 on the B scale comes under the runner. Then read 154 H. P. on the A scale over 6 (600 ft. piston speed) on the B scale (Fig. 76).

Rule 2—To find the piston speed in feet per minute of the above engine proceed the same as you do in Rule 1, except you read the answer under 154 H. P. on the A scale, read 600 ft. per minute on the B scale (Fig. 76).

Rule 3—To find the M. E. P. of the above engine, set 154 H. P. on the A scale (Fig. 76) against 6 (600 ft. per minute) on the B scale, and place the hair line of the runner to 33 (33,000 ft. pounds) on the B scale, and leaving the runner at this point move the slide until the special graduation mark (x) on the C scale is adjacent to 16 inches on the D scale (Fig. 77), and under the hair line of the runner on the B scale read the answer, 42 pounds mean effective pressure.

It will be noted with the setting as last explained that if you move the slide B back and forth the mean effective pressure is increased or decreased; likewise you will observe that the cylinder diameters are increased or decreased in the same proportion to the mean effective pressure. The same ratio will be noted with a variation in piston speed.

RATIO OF COMPOUND CYLINDERS

Example—Required to find the ratio of two cylinders 15 in. × 25 in.

First—Set the left index on the C scale to 15 in. on the D scale. Next place the hair line of the runner to 25 on the D scale and under the same line read 2.78 on the B scale (Fig. 78), which is the ratio of the two cylinders.

Suppose we desire to have a triple expansion with a high and intermediate cylinder of the above dimensions and a ratio of 2.78.

First—Perform the above operation. Then move the left index of the B scale to the hair line of the runner and move the latter to 2.78, the given ratio, on the B scale. The diameter of the low pressure cylinder will be found under the hair line of the runner on the D scale, which is 41.7 diameter.

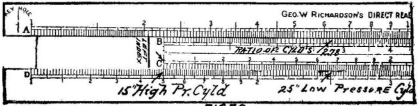


FIG 78

CYLINDRICAL CONTENTS

Given—A cylindrical tank 14 ft. mean diameter and 23 feet high. Required its capacity in cubic feet? Set the special graduation mark × on the left half of the C scale adjacent to 14 on the D scale, as per Fig. 79. Next set the hair line of the runner to 23 on B scale and read adjacent thereto on the A scale 3,540 cubic feet capacity.

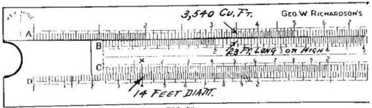


FIG. 79

SLIDE-RULE ACCURACY

In regard to slide-rule accuracy, it is to be remembered that a chain is no stronger than its weakest link, and a scientific result no more accurate than the most uncertain factor used in its calculation. No matter with how many decimals the result may be given, any figures beyond those which are justified by the accuracy of the most uncertain step of the process are meaningless and should be disregarded.

Take, for example, an indicator card. According to the best information, indicator springs are seldom to be relied upon closer than 1 per cent, with errors varying at different pressures, and sometimes as high as 5 per cent.

To read a slide-rule to 1/10 of 1 per cent, and not to have tested the spring or made spring corrections, would certainly be most unnecessary, and, in general, it might be said that for indicator work, where no spring tests or corrections are used, which is the case ninety-nine times out of a hundred, a slide-rule reading to one-half of 1 per cent would easily cover all requirements.

The 10-inch slide rule will give but three or, at the most, four significant figures of the product. At first thought it might seem that the results obtained were not accurate enough for practical purposes; but, as a matter of fact, few empirical constants employed in engineering practice are correct to more than three significant figures, and if they are, the conditions under which they are used are such that they are in most cases valueless beyond the third significant figure.

Consequently, the result will be just as accurate if calculated by means of the slide rule; and the slide rule may be used for all calculations of any nature whatsoever where mathematical exactness is not required.

It is invaluable in checking calculations performed in some other way, either by logarithms, by arithmetical processes, or by calculating machines. If the user of a slide rule exercises care and discrimination in applying it and becomes proficient in its use, he will find it the greatest labor- and brain-saving instrument he could possibly procure.

LOGARITHMS.

The lowest scale on the rule, marked "Log," is the scale of **logarithms**, applicable to line values on the "D" scale and to be used therewith directly.

Without going into extended definitions, logarithms are a series or system of numbers by means of which the long and tedious operations of multiplication and division are transformed into simple addition and subtraction.

To work a table of logarithms, one must keep in mind three rules:

1. To multiply numbers, you add the logs of the numbers.
2. To divide numbers. you substract the logs of the numbers.
3. Look out you get your index correctly.

RULE OF INDEX OR CHARACTERISTICS.

The index of the log of a number is equal to the number of places in the number, to the left of the decimal point, **minus one.**

If the number is entirely decimal the index is equal to the number of cyphers following the decimal point, **plus one,** but in this latter case, the index is negative and is designated by placing the minus sign over it, or by subtracting the index from 10, thus making it apparently positive, and placing after the log the suffix minus 10.

In practice the minus 10 is never written after the log, as it will be readily apparent whether the resulting log is negative or positive.

EXAMPLES.

What is the log of—

4212	answer, 3.62449
421.2	answer, 2.62449
42.12	answer, 1.62449
4.212	answer, 0.62449
.4212	answer, $\bar{1}.62449$ / 9.62449
.04212	$\bar{2}.62449$ / 8.62449
.004212	$\bar{3}.62449$ / 7.62449
.0004212	$\bar{4}.62449$ / 6.62449

Multiply 4212 by 2760 and divide the result by 426, using logs. Log of 4212 is 3.62449 and the log of 2760 is 3.44091 and the log of 426 is 2.62941: the sum of the first two logs, less the log of the last equals 4.43599. The number corresponding to 43599 is 27290 and as the index of the log is 4, you mark off one more place, that is 5 places, and you get the answer 27290.

Rule for getting the index corresponding to a certain log. Neglect the index for the moment and find the number in the log table that corresponds to the mantissa, that is, the portion of the log that is to the right of the decimal point; after you get this number, you then use the index for pointing off, pointing off one more place than the figure in the index.

In the question just given, the log tables will give 27290 opposite mantissa 43599 and you will mark off 5 places as there is a 4 in the index.

To find a log by the use of slide rule.

Set the hair line of the runner to the number desired on the "D" scale, directly below on the log scale read off the log of the number. You will note that the log scale only gives the mantissa and the proper index figure must be applied according to the index rule above.

Find log of 4212.

Set runner to 4212 on the "D" scale below on the log scale to read 624 which will be the value of the mantissa; as there are four places in the number, the index will be 3 and the entire log of 4212 becomes 3.624.

Find log .0564.

Set runner to 564 on the "D" scale, under it on the log scale read 751 for the mantissa; by index rule as the number is entirely decimal, index is negative and one more than the number of zeros between the decimal point and the first figure of the number, that is minus 2. The log of decimal .0564, therefore, is 2.751, or 8.751, minus 10 being remembered.

Find logs of 2780	3.444
Find logs of 45.65	1.660
Find logs of .785	9.894
Find logs of .00045	6.653

TO RAISE A NUMBER TO ANY POWER.

Place the runner to the number on the "D" scale, read its log on the log scale, being careful to give it the proper index. Transfer the entire log index and all to the "D" scale, multiply this log by the power wanted and the result is the log of the number required. To find the number place the runner to the mantissa on the log scale and above it on the "D" scale read the number wanted. Then point off one more place than the number of the index of the log and you have the answer.

Example: Find the 6th power of 18.

Place runner on the "D" scale to 18. under it on the log scale read log mantissa of 18, which is 255, giving it its proper index, one less than the number of places in the number wanted, you get, 1.255. Set 1 of "C" scale to 1255 on the "D" scale and under 6 on the "C" scale, read 7.53 the log of 18 to the 6th power. Then place the runner to 53 on the log scale and above it read 339. As your index is 7, you point off 8 places and get 33,900,000, which is the 6th power of 18, correct to three places.

To get any root of a number is of course just the reverse of raising to any power.

RULE.

Place the runner to the number on the "D" scale, below on the log scale read the log mantissa of the number. Give this the proper index according to the index rule above. Then place the runner to the log, index and all, on the "D" scale, and set the root required on the "C" scale to the hair line of the runner; then under 1 on the "C" scale read on the "D" scale the log of the number required. Then set the runner to the mantissa of this log on the log scale and above it on the "D" scale read the number. But be careful to watch your index.

EXAMPLE.

Find the 5th root of 68000.

Place the runner to 68 on the "D" scale and under it read the mantissa of its log on the log scale, 833. Giving it its proper index, 1 less than the number places in the number to the left of the decimal point, the entire log becomes 4.833. Place 5 on the "C" scale, over 483 on the "D" scale and under 1 on the "C" scale read .967 on the "D" scale, which is the log of the number required. Be careful when making the division to get the correct position of the decimal point. Then place the runner to 967 on the log scale and over it on the "D" scale read the number 93. Then marking off one more place than the index of the log, in this case 1, as there are no places in the index, you get 9.3 as the number wanted. These answers are as close as you can get on a small slide rule.

EXAMPLE.

Find the 8th root of 32500? Answer, 3.65.
Find the 6th power of 7.8? Answer, 225000.
Find the 24th root of 11500? Answer, 1.476.
Find the 15th power of .55? Answer, .000126.
Find the 6 root of .0098? Answer, .464.

In the extraction of roots of numbers that are entirely decimal, express the negative decimal as though subtracted from 10, in other words express —1 as 9, or —2 as 8. Then when dividing by the required root increase the index of the log by adding to it a number of tens, which shall be one less than the number of the root. For instance, in the last question above, the log of .0098 is 7.99 —10; before you divide by 6, you add 5 tens or 50 to the index and make it 57.99 and this divided by 6, gives the log of the number required, i. e., 9.666, the log of .464. If it had been the 10th power, you would have expressed 7.99 as 97.99 before dividing.

SINES, TANGENTS, ETC.

Above the "A" scale and marked SIN is the scale of natural sines from which can also be easily derived the natural cosine. Both sines and cosines are to be used in connection with scale "A." The log sine or log cosine can be found by transferring the value of the natural function as found on the "A" scale to the "D" scale, and then reading below this on the log scale, the log corresponding to this natural function value or in other words the log sine or log cosine as the case may be.

In reading the value of the natural sine or cosine on the "A" scale, care must be taken to get the correct position of the decimal point, by the following rule.

RULE.

All values of natural sines or cosines found on left scale "A," that is, on the left half between 1 and 10, must have a cypher between the decimal point and the first figure. When the functions are found on the right hand scale the decimal point adjoins the first figure.

In other words, when the angle is less than 5° 45′, the natural sine or cosine is less than .10, but on the right hand half, it is expressed in tenths.

EXAMPLES.

What is the natural sine of 4° 30′?

Set the runner to 4° 30′ on the sine scale, directly under it on the "A" scale read 784; as this is found on the left hand half of the "A" scale, we prefix zero between 7 and the decimal point and the function becomes .0784.

What is the cosine of 87° 30′?

Set runner to 48° on the sine scale and under it on the "A" scale read 743; as the number is found on the right hand "A" scale, the value is in tenths and you write it .743.

To find the cosine use the following rule:

RULE.

COSINE X equals SINE (90°-X).

EXAMPLE.

What is the natural sine of 48°?

By substituting in the above rule, you get—
Cos 87° 30′ equals sine (90°—87° 30′).
Cos 87° 30′ equals sine 2° 30′.

Then set the runner to the sine of 2° 30′ and below it on the "A" scale read 436; as it is on the left hand "A" scale, you prefix zero and it becomes .0436.

EXAMPLES.

Find Nat sine of 47°? Answer .732.
Find Nat cosine of 28°? Answer .883.
Find Nat cosine of 15°? Answer .97.
Find Nat sine of 66°? Answer .913.
Find Nat cosine of 85° 30′? Answer .0785.

TO FIND THE LOG SINE OR COSINE.

RULE.

Place the runner to the required angle on the sine scale and read the value of the natural function; transfer this last to the "D" scale, and below it on the log scale read the mantissa of the log required, placing the index by the following:

RULE FOR INDEX OF LOG SINE OR COSINE.

All natural sines or cosines are less than 1, they will therefore have a negative index. If the natural function of the angle is found on the left hand "A" scale, the index will be —2 and will be written 8. If it is found on the right hand "A" scale, then index will be —1 and will be written 9. The suffix —10 is in all cases presumed.

QUESTION.

What is the log sine of 27° 30′?

Place the runner to 27° 30′ on the sine scale and then read the natural sine on the "A" scale, .462. Set the hair line to 462 on the "D" scale and under it on the log scale read the mantissa of the log or .664; as the natural function was found on the right hand "A" scale, the index will be —1, and the entire log will read as 9.664.

What is the log cosine of 38°?

Subtract 38° from 90° and you get 52°, therefore the cosine of 38° equals the sine of 52°; place the runner to the sine of 52°, and under it on the "A" scale read the value of the natural function or .788; transfer this 788 to the "D" scale, and under it on the log scale read the mantissa of the log, 896; as the natural function was found on the right hand "A" scale, the whole log becomes 9.896.

What is the log cosine of 84° 30′?

Subtract 84° 30′ from 90° and you get 5° 30′, therefore log cosine of 84° 30′ equals log sine of 5° 30′.

Place the runner to 5½° on the sine scale and under it on the "A" scale read 96 as the value of the natural function; transfer this 96 to the "D" scale, and under it on the log scale read the log mantissa 982; as the natural function was found on the left hand "A" scale, the entire log becomes 8.982 as the index is —2.

EXAMPLES.

Find log sine of 27° 30′? Answer 9.664.
Find log cosine of 87° 15′? Answer 8.681.
Find log cosine of 68°? Answer 9.573.
Find log sine of 1° 15′? Answer 8.339.

TANGENTS.

Just below the "D" scale is the tangent scale, to be used in connection with the "D" scale. The natural tangents are only given on this scale from 6° to 45°, corresponding to values on the "D" scale for the natural functions of from .1 to 1.

If the tangent of an angle less than 6° is wanted, use the sine scale as directed above, as up to 6° there is no such difference between the sine and tangent, that it may not be considered negligible so far as slide rule work is concerned.

EXAMPLE.

What is the natural tangent of 27° 30′?

Place the runner on 27° 30′ on the tangent scale and above it on the "D" scale read .522.

What is the natural tangent of 4° 30′?

Place the runner on 4° 30′ on the sine scale and below it on the "A" scale 785 and as it is found on the left hand "A" scale we prefix a zero and the tangent becomes .0785.

To get tangent of an angle greater then 45°, you can use these two formulae:

$$\text{tangent X equals } \frac{1}{\tan(90°-X)}$$

$$\text{or tangent X equals } \frac{\text{sine X}}{\text{cosine X}}$$

EXAMPLE.

What is the tangent of 50°, using both methods?

$$\text{tangent } 50° \text{ equals } \frac{\text{sine } 50°}{\cos 50°} \text{ equals } \frac{\text{sine } 50°}{\text{sine } 40°}$$

Place the runner over sine 40° on the sine scale and note its value on the "B" scale. Move the "B" scale till this value of sine 40° is in line with sine 50° on the sine scale, then over 1 on the "B" scale read the value of the tangent on the "A" scale or 119. To place the decimal point remember that the tangents of angles over 45° are greater than 1, you therefore point off 1 place and the natural tangent of 50° becomes 1.19.

Using the other method.

$$\text{Tangent } 50° \text{ equals } \frac{1}{\tan(90°-50°)} \text{ or } \frac{1}{\text{tangent } 40°}$$

After expressing the tangent according to the formula, which will make it 1 over the tangent of 40°, place the runner on tangent 40° and note its value on the "C" scale, then move the "C" scale to the right till this value of tangent 40° already noted coincides with the right hand 1 on the "D" scale, then read the value of the tangent required on the "D" scale, below the 1 on the "C" scale. In the case in question you will set 84 on the "C" scale over right hand 1 on the "D" Scale and under left hand 1 on "C" scale read 119 on the "D" scale. As the tangents over 45 are greater than 1, you point off 1 place and you have the tangent of 50° again, 1.19.

EXAMPLES.

What is the natural tangent of 62° 30'? Answer 1.92.
What is the natural tangent of 71°? Answer 2.90.
What is the natural tangent of 55° 30'? Answer 1.46.

TO FIND LOG TANGENTS.

RULE.

As the scale of logarithms corresponds to line values on the "D" scale and as this same "D" scale is to be used in connection with the tangent scale, it is apparent that for angular values of tangents within the scope of the "D" scale, i. e., from 6° to 45°, the log tangents can be read directly under the value of the natural tangent.

Therefore to find the log tangent of an angle between 6° to 45°, set the runner to the angle on the tangent scale and under it on the log scale, read the mantissa of the log tangent.

If the angle is not included between 6° and 45°, find the value of the natural tangent as directed above, i. e., if less than 6° use the sine scale, and if greater than 45° use one of the expression on the preceeding page for an equivalent value.

After finding the mantissa of the log, give it its proper index by the following rules:

Tangents from 1° to 5° 42', index is 8.
Tangents from 5° to 43' to 45°, index is 9.
Tangents from 45° to 84° 17', index is 10.
Tangents from 84° 17' to 89° 25', index is 11.

The two divisions that are in the brackets are the important ones to keep in mind.

EXAMPLE.

What is the log tangent of 27°?

Set the runner to 27° on the tangent scale, and under the hair line on the log scale read the mantissa of the log or .707; the index by the general index rule, and also by the short rules above will be 9, so that the entire log becomes 9.707.

EXAMPLE.

What is the log tangent of 75°?

Express the tangent as being equal to sine 75° over the cosine of 75°, and this is equal to the sine of 75° over the sine of 15°, as follows:

$$\text{tangent } 75° = \frac{\text{sine } 75°}{\text{cosine } 75°} = \frac{\text{sine } 75°}{\text{sine } 15°}$$

The reasons for this have been explained above.

Place the runner to the sine of 15° and note its value on the "A" scale. Move the slide to the right until this value of sine of 15° on the "B" scale is in coincidence with sine 75° on the "A" scale, then above the left hand 1 of the "B" scale, read on the "A" scale the natural tangent of the angle wanted, or 3.73. Set the runner to this value, 3.73, on the "D" scale, and under it on the log scale read the log mantissa, or .572. As the angle is between 45° and 84°, the proper index is 10 and the entire log becomes 10.572.

IMPERIAL GALLONS vs CUBIC INCHES.

Set the slide with (h) in the key-hole similar to Fig. 70 and read Imperial Gals., on the B scale adjacent to Cubic Inches on the A scale.

IMPERIAL GALLONS vs U. S. GALLONS.

Set slide with (c) in key-hole, similar to Fig. 61 and read Imperial Gals., on (D) adjacent to U. S. Gals. on (C).

EXAMPLES.

Find the log tangent of 37°? Answer 9.88.

Find the log tangent of 59°. Answer 10.22.

Find the log tangent of 35°. Answer 9.845.

MULTIPLICATION AND DIVISION OF FUNCTIONS.

These are performed precisely like any other such calculation and the methods will readily suggest themselves. Moreover in the preceding pages, many of our questions have taken in multiplication and division.

SOLUTION OF RIGHT ANGLED TRIANGLES.

Let us first express the relation of the sides of a right angled triangle to the angles.

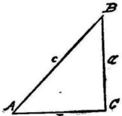


FIG. 80.

Let figures A B and C represent any right angled triangle and let the sides opposite the angles be represented by the same small letters.

Then $\frac{a}{c}$ equals sine angle A.

Then $\frac{b}{c}$ equals cosine angle A.

Then $\frac{a}{b}$ equals tangent angle A.

Then $\frac{b}{a}$ equals cotangent angle A.

These equations together with use of the fact that in any right angled triangle, the sum of the two acute angles is equal to 90°, is sufficient to solve any right angled triangle.

RULE.

Choose from the above equations that define the functions of an angle, one in which the desired part will be the only unknown part and solve it. Also remember that any acute angle on a right angled triangle is equal to 90° less the other acute angle.

In the solution of the right triangles 5 cases can arise.

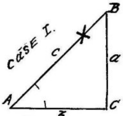


Fig 81

CASE 1.

Given the hypothenuse and one acute angle.

Formulae { side a = sine A times side c.
side b = sine (90-A) times side c.
angle B = 90° less angle A.

EXAMPLE.

Given an angle of 55° and the hypothenuse of 28, solve triangle.

Other angle equals 90° minus 55° = 35°.

Side a = Sine A times side c = sine 55 times 28 = 22.9.

Place runner to sine 55° on sine scale, bring right hand 1 into line with the sine of 55 and over 28 on the "B" scale, read 22.9 on the "A" scale, which is the value of side wanted.

Side b = sine (90°-A) times side c = sine 35 times 28 = 16.1.

Place right hand 1 of "B" scale in line with the sine of 35° then read over 28 on the "B" scale, 16.1 on the "A" scale.

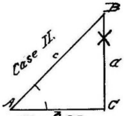


Fig 82

CASE 2.

Given one of the sides about the right angle and an acute angle opposite it.

Formulae { angle B = 90° less angle A
side $c = \frac{\text{side a}}{\text{Sine A}}$
side $b = \frac{\text{side a}}{\text{Tangent A}}$

EXAMPLE.

Given an angle of 28° and a side opposite it of 21, solve triangle.

90° minus 28° = 62° or the other angle.

21 divided by sine 28° = 44.8.

Place runner to sine 28° and note its value on the "B" scale, place this value of sine 28° on "B" scale under 21 on the "A" scale, over left hand 1 on the "B" scale read 44.8 on the "A" scale.

21 divided by tangent 28° = 39.5.

Transfer tangent of 28° from the tangent scale to the "B" scale, placing it under 21 of the "A" scale, then read over left hand 1 of the "B" scale on the "A" scale the value wanted, 39.5, or this division can be made directly with the "C" and "D" scales.

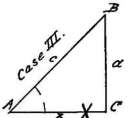


Fig 83

CASE III.

Given an acute angle and the adjacent side.

Formulae
$$\begin{cases} c = \dfrac{b}{\text{sine } (90\text{-A})} \\ a = b \text{ tangent A} \\ B = 90\text{-A} \end{cases}$$

EXAMPLE.

Given an angle of 11° and an adjacent side of 35. Solve the triangle.
90° — 11° = 79° or other angle.
35 divided by the sine of 79° = 35.7 = C.
35 times tangent 11° = 6.80 = a.

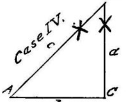


Fig 84

CASE IV.

Given the Hypothenuse and side.

Formulae
$$\begin{cases} \text{Sine A} = \dfrac{a}{c} \\ B = 90\text{-A} \\ b = c \text{ sine } (90\text{-A}) \end{cases}$$

EXAMPLE.

Given hypothenuse of 92 and side of 56, solve the triangle.
56 divided by 92 = .609 = sine of 37½° = A.
90°—37½ = 52½ = B.
92 times sine of 52½° = 73 = b.

CASE V.

Given the two sides about the right angle.

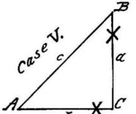


Fig 85

Formulae
$$\begin{cases} \text{Tan A} = \dfrac{a}{b} \\ B = 90\text{—A} \\ c = \sqrt{a^2+b^2} \end{cases}$$

Question: 2 sides of a right angled triangle are 58 and 43, solve the triangle.

Tan A = $\frac{58}{43}$ = 1.35 = 53½°
B = 90 — A = 36½°
$C = \sqrt{58^2 + 43^2} = \sqrt{3364 + 1849} = 72.2$

OBLIQUE ANGLED TRIANGLES.

To solve oblique triangles, three rules are necessary.

1. Law of sines; the sides of a triangle are proportional to the sines of the opposite angles.

2. Law of cosines; cosine of an angle equals the sum of the squares of the including sides, diminished by the square of the opposite side, all divided by 2 times the product of the including sides.

3. Law of tangents; the difference of the two sides of a triangle is to the sum of the two sides as the tangent of one-half of the difference of the opposite angles is to the tangent of the half sum of the opposite angles.

CASE I.

Given one side a and two angles A and B.

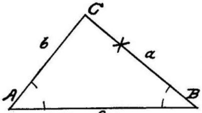


Fig. 86

Formulae
$$\begin{cases} \text{Angle C} = 180\text{—(A plus B)} \\ \text{Side b} = \dfrac{\text{side a}}{\text{sine A}} \text{ X sine B.} \\ \text{side c} = \dfrac{\text{side a}}{\text{sine A}} \text{ X sine C.} \end{cases}$$

EXAMPLES.

Angle A = 38°; angle B = 58°; side a = 400. Solve triangle.
180° — (38° plus 58°) = angle C = 84°.

$$\frac{\text{400 times sine 58°}}{\text{sine 38°}} = \text{side b} = 550.$$

$$\frac{\text{400 times sine 84°}}{\text{sine 38}} = \text{side c} = 646.$$

CASE II.

Given 2 sides, a and b, and angle A opposite side a.

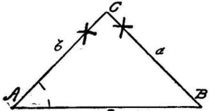


Fig 87

Formulae
$$\begin{cases} \sin B = \dfrac{\text{side b sine A}}{\text{side a}} \\ \text{angle } C = 180 - (A + B) \\ \text{side } c = \dfrac{\text{side a sine C}}{\text{sine A}} \end{cases}$$

When an angle is determined by its sine, it admits of 2 solutions in some cases.

Rules for number of solutions:

Two solutions when A is acute and value of side a lies between side b and b sine A.

No solution: When A is acute and side a is less than b sine A.

No solution: When A is obtuse and side a is less side b.

All other cases have one solution.

Example: a = 55, b = 66, B = 77°.

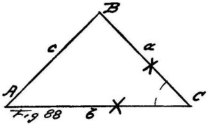
$$\text{Sin A} = \frac{a \text{ Sin B}}{b} = \frac{55}{66} \times \sin 77° = .82 = 55°.$$

$$\text{Angle C} = 180 - (A + B) = 180 - 132 = 48°$$

$$\text{Side c} = \frac{a \times \text{sine C}}{\text{sin A}} = \frac{55 \times \sin 48°}{\sin 55°} = 50$$

CASE III.

Given 2 sides a and b and the included angle C.



Formale:

$$\text{Side c} = \sqrt{a^2 + b^2 - 2ab \cos C}$$

$$\text{Sin A} = \frac{\sin C}{c} \times a$$

$$\text{Sin B} = \frac{\sin C}{c} \times b$$

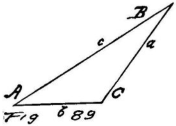
Or angles A and B can be found by tangent formula $\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \times \tan \frac{1}{2}(A + B)$ in which everything on right of equation is known.

Question: a = 78; b = 82; angle C = 72°

$$\text{Side C} = \sqrt{12800 - 3965} = 94$$

$$\text{Sin A} = \frac{\sin 72° \times 78}{94} = .81 = 54°$$

$$\text{Sin B} = \frac{\sin 72° \times 82}{94} = .829 = 56°$$



CASE IV.

Give the 3 sides a, b, and c.

Formulae:

$$\text{Cos C} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\text{Sin B} = \frac{\text{side b sin C}}{\text{side C}}$$

$$\text{Sin A} = \frac{\text{Side a sin C}}{\text{side C}}$$

Example: a = 43; b = 50; c = 57

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{1075}{4300} = .248 = 75°$$

As such gives only sine values, look for .248 on sine scale and then subtract the angle found from 90 and that will give you the angle whose cosine is .248. In this case we find .248 is sine of 15° and .'. the cosine of 75°.

$$\text{Sin B} = \frac{50 \times \sin 75°}{57} = 58° \qquad \text{Sin A} = \frac{43 \sin 75°}{57} = 47°$$

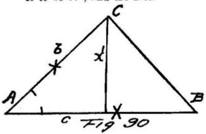
TO FIND THE AREAS OF TRIANGLES.

There are various methods of doing this, but only the general idea will be here given.

AREA always equals ½ product of the base times the altitude.

Taking any side as the base, it is readily seen that by dropping a perpendicular to that base, a right triangle is formed in which one of the angles of the original triangle is the angle opposite the altitude and one of the sides of the original triangle is the hypothenuse. You can thus express the value of the altitude in terms of a side and angle of the original triangle. By then taking ½ half the product of this altitude and the base the area of the triangle is found.

Example: In a triangle given side b to be 22, side c to be 37 and angle A to be 66°, find the area.



Let X equal the perpendicular. Then in terms of the triangle's parts X over 22 or over side b equals the sine of 66°.

X^1 therefore equals 22 times sine 66, and area equals ½ base times the altitude.

$$\text{Area} = \frac{22 \times \text{sine } 66° \times 37}{2} = 370$$

The same method applies to all other triangles. Take one of the side as a base and get the value of the perpendicular representing the altitude in terms of a side and angle of the original triangle.

DIRECTION FOR USING RICHARDSON'S ADDING AND SUBTRACTING SLIDE RULE No. 1812

So far as the writer knows, The first attempt to make possible addition and subtraction, as well as multiplication and division and **combining** them on the face of **one** instrument, developed itself in the Richardson Direct Reading Slide Rule No. 1812.

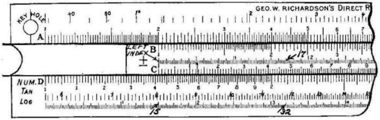


FIG. 91

This is accomplished by using **two** LOG scales, (see cut above) one below the tangent scale on the **D** scale, and the other on the **slide** between the **B** and **C** scales. The latter having the prefix + to distinguish it from the other scales. The numerals of these log scales run as follows beginning at the left index 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 they can also be read thus 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, or 0, 100, 200, 300, 400, 500, etc., depending upon the value give the **left** index.

The space between each of the numerals are again divided into **ten** main divisions thus permitting ten units between each numeral for example let it be required to add 15 and 17. Calling the **left index one,** the first red numeral 1, would signify 10, therefore placing the hair line of the runner to 10 on the lower log scale, plus five main divisions further towards the right, making 15 (see cut), **next** bring the **left** index of the + scale under the hair line. Now move the runner to 17 on the + scale and read on the lower log scale 32 as the sum of 15+17=32. (Please note that the number 3 on the lower log scale denotes 30, and the two main divisions denote 2, hence the answer, 32.)

Example.—Find the sum of 15, 17, and 41, this being in part a repetition of the former example except adding 41 to the sum of 32 leaving the hair line of runner to 32 on the lower log scale, bring the **left** index of the slide to it, moving the hair line of the runner to 41 on the + scale, and under this same hair line on the lower log scale read 73 as the sum of 15+17+41=73. Now let it be required to add 78 more to this sum of 73, proceeding same as before we bring the left index of the + scale under the hair line of the runner

as at 73 on lower log scale. Then if we proceed to move the runner to the **right** on the + scale **we find that is is necessary to take the runner off the rule to place it at 78.** It is evident that this would not do. Then what will do? **Just leave the hair line of the runner at 73** on the lower log scale, and bring the **right** hand index (instead of the **left**) under the hair line, next move this hair line to 78 on the + scale and read under the hair line on the lower log scale 51, and as it required the changing from right index to left index **once** we simply prefix 1 to the 51 making a total of 151 as the sum of 15+17+41+78=151.

We only need to consider the last **two** figures, as every time we change indexes we add 100, therefore in this case we changed indexes but once. We conclude that the last two figures 51+100=151, or the sum of 15+17+41+78.

IMPORTANT.—In adding numbers the sum of which is over 100 you must **remember** how many times you changed this 100 from **right** to **left** index, adding 100 for each such change. It is not necessary to read any but the **last** result.

As **Subtraction** is the reverse of addition it is not necessary to make much of an explanation, more than to say if you desire to subtract 51 from 73 simply place hair line of runner to 73 on lower log scale bringing 51 on + scale to it, then place hair line of runner to **left** index of + scale and read 22 on lower log scale.

If you set **left** index of + scale for example at 19 on lower log scale, any other number can be added to this 19 without changing the positions of the scales, such as 19 on lower log, runner to 21 on + scale, read 40 adjacent on the lower log scale, or 19 and 32=51; 19 and 73=92. Likewise in subtracting 73 on + scale to 92 on lower log scale, read 19 adjacent to left index of + on the lower log scale. The addition of **three** numbers may be performed by using the smaller **sub-divisions** (each one of these indicate 2. or 20. or .2), depending upon the problem at hand.

Example.—Find the sum of 274 and 618. Set hair line of runner to 274 on lower log scale, bringing the left + index to it. **(in this case we assume the left index to have a value of 0, same as before, except the first numeral to the right of this 0 must be assigned a value of 100).** The numeral 2 represents 200 while the **main** divisions represent tens, and the **sub-divisions** units of 2, 4, 6, 8, etc.

Having set the hair line to 274 on lower log scale, and left index + to it, move the hair line to 618 on the + scale and under this same hair line read 892 as the sum of 274 and 618.

As this scale is divided in tenths and multiples thereof 2.74+6.18=8.92 or 27.4+61.8=89.2 permits of the addition or subtraction of decimal parts, equally as well as whole numbers.

DIRECTIONS

for using the

Richardson Polymetric Slide Rule

No. 1776

(By J. J. Clark, M. E. Lehigh)

The CI Scale—The polymetric slide rule differs from the regular engineer's slide rule in that it has an inverted C scale extending along the middle of the slide. By **inverted** is meant turned upside down, thus causing the right index on this scale to indicate the same number as the left index of the C scale, and vice versa. This scale is marked CI (CI means C inverted), and the figures on it are printed in red, to call attention to the fact that numbers on the CI scale increase from **right** to **left** instead of from left to right, as on the other scales. Thus, to locate a number as 463, find the red figure 4, glance to the left and locate the long division mark that indicates 6, then just a little to the left of this, locate the position of 3, which will be three-fifths of the distance between 460 and 465. This setting is shown in Fig. 1. Note that it is always necessary to employ the runner when using the CI scale in making settings.

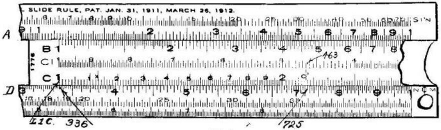


Fig. 1

Reciprocals—In order to understand properly the relations between the CI scale and scale C (or D), it is necessary to consider a few of the properties of reciprocals. The **reciprocal** of a number is 1 divided by the number; thus, the reciprocal of a is $\frac{1}{a}$, a being any number whatever. Denoting the **value** of the reciprocal by b, $b = \frac{1}{a}$, or $a \times b = 1$. From the last equation, $a = \frac{1}{b}$;

in other words, a and b are said to be **mutually reciprocal,** or **reciprocals of each other.** From the second of the above equations, it is evident that the product of any two numbers that are reciprocals of each other is always 1; hence, one number must be greater than 1 and the other less than 1. For example, the reciprocal of 4 is $\frac{1}{4} = 0.25$, and $4 \times 0.25 = 1$; the reciprocal of 6 is $\frac{1}{6} = 0.16\frac{2}{3}$, and $6 \times 0.16\frac{2}{3} = 1$; the reciprocal of 0.4 is $\frac{1}{0.4} = 2.5$, and $0.4 \times 2.5 = 1$. The reciprocal of a common fraction is the fraction inverted; thus, the reciprocal of $\frac{4}{7}$ is $\frac{1}{4/7} = \frac{7}{4}$, and $\frac{4}{7} \times \frac{7}{4} = 1$.

If the reciprocal of a number is known and the number is used as a divisor, division may be changed into multiplication; or, if the number is used as a factor, multiplication may be changed into division. Thus, $\frac{a}{b} = a \times \frac{1}{b}$. For example, $\frac{216}{12} = 216 \times \frac{1}{12} = 216 \times 0.08\frac{1}{3} = 18$; $27 \times 16 = 27 \div \frac{1}{16} = 27 \div 0.0625 = 432$; also, $27 \times 16 = 16 \div \frac{1}{27} = 16 \div 0.037\ 1/27 = 432$.

Set the slide so that the indexes will be in line with those on the rule; bring the runner to some number on the CI scale; under the hair line, it will be noted that the number on the C (or D) scale is the reciprocal of that on the CI scale. Or, if the runner be brought to some number on C (or D), the number under the hair line on CI will be the reciprocal of that under the hair line on C (or D). The reason for this is plain when it is noted that the distance from the left index to the number on C added to the distance from the right index to the number under the hair line on CI is always exactly equal to the length of the slide, or 10, which may be regarded as 1. Since these spaces are logarithmic, this sum is really the product of the two numbers; and it has just been shown that if the product of two numbers is 1, the numbers are reciprocals of each other. For example, neglecting the decimal point, 2 on C is opposite 5 on CI, and $2 \times 5 = 10$; 8 on CI is opposite 125 on C, and $8 \times 125 = 1000$; 45 on CI is opposite 222 on C, and $222 \times 45 = 99990$, which is equivalent to 10000 in slide rule calculations.

Multiplication with CI and D Scales—To find the product of two numbers, bring runner to one of the numbers (factors) on D, bring the other number (factor) on CI to hair line, and opposite index on CI, read product on D.

Example—Multiply 7.25 by 463.

Analysis—Referring to Fig. 1, set runner to 725 D (this means 725 on scale D); move slide until 463 CI comes under the hair line; opposite L1C (this means left index on scale C), read 336 on D. This operation is evidently equivalent to dividing 725 by the reciprocal of 463; and, since there are obviously 4 integral places in the product, $7.25 \times 463 = 3360$. Observe also that 336 CI is opposite R1D.

Note that when the CI and D scales are used, the setting (insofar as the numbers are concerned) is exactly the same as for division, when using the C and D scales. The position of the decimal point is determined by the regular rule for multiplication, except that we note whether the slide projects to the **right.**

One great advantage of using the CI and D scales for multiplication is that we are never in doubt as to which index to use; one or the other (and only one) comes opposite the product on the D scale.

Division with CI and D Scales—As may be supposed, the setting for division when using the CI and D scales is the same as for multiplication when using the C and D scales. Bring one of the indexes to the dividend on D; set runner to divisor on CI; under the hair line, read quotient on D. To locate the decimal point, apply the regular rule for division, except that we note whether the slide projects to the **left.**

Example—Divide 21.75 by 3.14.

Analysis—Set L1C to 2175D; bring runner to 314 CI; under the hair line, read 693 on D. There is evidently 1 integral place in the quotient; hence, $\frac{21.75}{3.14} = 6.93$. This operation is evidently equivalent to multiplying 21.75 by the reciprocal of 3.14.

SPECIAL OPERATIONS WITH CI SCALE

Remark—The operations now to be described all involve the use of the CI scale. While it may be that in some cases it would be more convenient to use the C scale, the reader is advised to work on his rule all the problems according to the settings here given; he will, in this way, get practice in using the CI scale, and can readily determine when it should be given preference over the C scale; he is also advised to study out the reasons for the settings.

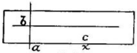
Diagrams of Settings—The setting for the various operations are most conveniently presented by means of diagrams, since these show at a glance just what procedure is to be followed, and are much better for this purpose than a detailed explanation in words.

A necessary adjunct to a diagram is a formula to accompany it and indicate what operations are to be performed; the diagram then shows how the result is obtained. In the formula (which is printed at the left of the diagram), the letters a, b, c, d, etc., are used to represent the numbers, which may have

any value, and the letter x is used to represent the answer sought, and the position of x on the diagram shows the scale on which the answer will be found.

The diagram represents the slide only, see Fig. 2, and the space above and below it represents the rule. The upper line represents the B scale, and immediately above this is the A scale; the middle line is the Cl scale; the lower line is the C scale, and immediately below this is the D scale. When the figure 1 occurs on the diagram, it indicates one of the indexes on the slide, either right or left, according to which has to be used. A straight line across the diagram indicates the hair line of the runner. The settings are supposed to occur in regular order, from left to right, in the diagrams. When the runner is used and it is brought to a number (letter), the straight line across the diagram comes **before** the number (letter); but when the runner is stationary and the number on the slide is brought to the hair line, the letter that represents the number is placed before the hair line. Bearing this in mind, the diagram in Fig. 2 is interpreted as follows: Bring runner to a on D; bring b

$x = a \times b \times c$



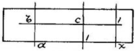
$3.14 \times 62.75 \times 144 = 28370$

Fig. 2

on CI to runner; opposite c on C, read x on D. The formula shows the object of the setting, and the example affords practice in making the setting. Note that three numbers are here multiplied together with one setting of the slide.

If the third number c lies beyond the end of the rule, we can proceed in one of two ways: (1) Shift the slide until the other index comes under the hair line; or (2) proceed as shown in Fig. 3. Here runner is brought to a on D, b on CI is brought to hair line, runner is shifted to the index on slide, c on CI is brought to hair line, and opposite index on CI read x on D. The first method involves but one setting of the slide (changing the index is not a setting), while the second method requires two settings of the slide. The writer prefers the second method, principally because the slide always moves through a shorter distance.

$x = a \times b \times c$



$3.14 \times 62.75 \times 864 = 170{,}200$

Fig. 3

The following diagrams illustrate the solution of various formulas with one setting of the slide.

$x = \frac{a}{b \times c}$

Fig. 4

$\frac{628000}{432 \times 25.7} = 56.6$

$x = \frac{a}{b}, \frac{a}{c}, \frac{a}{d}$, etc.

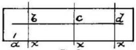


Fig. 5

$\frac{1728}{24} = 72, \quad \frac{1728}{36} = 48,$ etc.

The setting in Fig. 5 may be used when it is desired to find a series of quotients, the dividend remaining constant.

$m = x_1y_1 = x_2y_2 = x_3y_3$, etc.

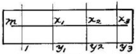


Fig. 6

$1728 = 24 \times 72 = 64 \times 27$, etc.

The setting in Fig. 6 may be used when it is desired to find a series of sets of two factors of a number.

$x = a^3$

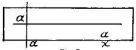


Fig. 7

$3.14^3 = 31.0$

$x = a^4$

Fig. 8

$3.14^4 = 97.2$

$x = ab^4$

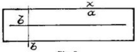


Fig. 9

$25.6 \times 3.14^4 = 2490$

Note that in the last two diagrams x is found on the A scale.

$x = a^5$

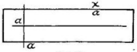


Fig. 10

$3.14^5 = 305$

$x = \frac{1}{a^4}$

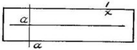


Fig. 11.

$\frac{1}{3.14^4} = 0.01029$

$x = \frac{1}{ab^2} = \frac{a}{a^2b^2}$

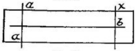


Fig. 12

$\frac{1}{2.56 \times 3.14^2} = 0.397$

$x = \frac{1}{a^3} = \frac{a}{a^4}$

Fig. 13

$\frac{1}{3.14^3} = 0.0323$

$x = \frac{1}{a^2b^2}$

Fig. 14

$\frac{1}{2.56^2 \times 3.14^2} = 0.1548$

$x = \frac{1}{a\sqrt{b}} = \frac{\sqrt{b}}{a \times b}$

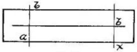


Fig. 15

$\frac{1}{2.56\sqrt{3.14}} = 0.220$

$x = a^2\sqrt{b}$

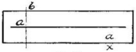


Fig. 16

$2.56^2\sqrt{3.14} = 11.61$

$x = \frac{\sqrt{a}}{b^2}$

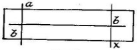


Fig. 17

$\frac{\sqrt{2.56}}{3.14^2} = 0.1623$

$x = \frac{b^2}{a^3}$

Fig. 18

$\frac{2.56^3}{3.14^2} = 1.70$

The second forms of the formulas in Figs. 12, 13, and 15 indicate the reasons for the settings shown by the diagrams.

DIRECTIONS

for using the

Richardson Logometric Slide Rule

No. 1860-LL

By J. J. Clark, M. E. (Lehigh)

The Log Log Scale.—Many of the formulas used in engineering calculations contain quantities affected with fractional (either common fractions or decimals) exponents, or the exponents may be integral but greater than 3; hyperbolic logarithms also frequently occur in certain classes of formulas. In all such cases, it is very difficult, if not impossible, to solve such formulas with the aid of the ordinary engineer's slide rule. For instance, the formula for the resistance to sliding of a belt on a pulley is $F = S_1 - S_2 = S_2\,(e^{f\phi} - 1)$, in which S_1 = the tension (pull) on the tight side of the belt, S_2 = the tension on the slack side, e = the base of the hyperbolic system of logarithms = 2.71828+, f = the coefficient of friction, and ϕ = the angle of contact of the belt on the smaller pulley, measured in radians. The ordinary slide rule is not adapted to calculating formulas of this kind; but, with the aid of the log log scale, the calculation is very simple.

Referring to your log log scale, bring the right and left indexes on the slide into coincidence with those on the rule. The bottom scale on the rule (marked Log) gives the mantissas of the common logarithms of the numbers directly above it on scale D^3, and scale D^3 gives the hyperbolic logarithms of the numbers directly above it on the scales marked LL1, LL2, and LL3, situated on the slide. Thus, with the slide in the position mentioned above, bring the runner to 20 on LL3; under the hair line, read 3 on D^3, and read 477 on the Log scale; in other words, the common logarithm of the hyperbolic logarithm of 20 is 0.477. In mathematical notation, this would be written $\log_{10} \log_e 20 = 0.477$. The hyperbolic logarithm of 20 is 2.99573, or 3.00, and the common logarithm of 2.99573 is 0.476503, or 0.477, both being expressed to three significant figures.

Reason for Using Scale of Hyperbolic Logarithms.—When common logarithms are used, the characteristics are not given, neither in tables nor on slide rules, the mantissas only being recorded. But, when the logarithm of a logarithm is taken, the characteristic of the logarithm must be included as a part of the logarithm; and this makes calculations very awkward when the base is 10 and the characteristic is determined by inspection. Of course, some base other than e might have been selected; but, since logarithms to this base are frequently used, it is the most convenient base to employ. The ordinary rules governing operations with logarithms apply in all cases, whatever the base may be (provided it is greater than 1); hence, to find the value of an expression like $44^{\frac{1}{41}}$, we may use either of the two following methods:

(1) $\text{Log } 44^{\frac{1}{41}} = 1.41 \times \log 44 = 1.41 \times 1.64345 = 2.31726 = \log 207.61.$

(2) $\text{Log}_e \log 44^{\frac{1}{41}} = \log_e (1.41 \times \log 44) = \log_e 1.41 + \log_e \log 44 = \log_e 1.41 + \log_e 1.64345 = 0.34359 + 0.49680 = 0.84039 = \log_e 2.31726 = \log 207.61.$

When using the LL (log log) scale, no attention is paid to the kind of logarithms employed; in the case just cited, we simply add the space on the rule (which represents the logarithm of the exponent) to the space on the slide (which represents the log log of the number) in exactly the same manner that we would obtain the product of two numbers on the slide when using scales C and D. In the present case, bring 44 on LL3 to L1D³ (this means left index on D^3), and opposite 141 on D^3, read 208 on LL3.

Construction of Log Log Scale.—The three LL scales (see Fig. 1) really constitute a single scale divided into three parts; they might have been placed in line, end to end, and the D^3 and other scales repeated three times, but the rule would then be three times as long and would be difficult to handle. The first part of the scale, marked LL1, extends from $\sqrt[100]{e} = e^{\frac{1}{100}} = 1.01005$ to $\sqrt[10]{e} = e^{\frac{1}{10}} = 1.10517$; the second part, marked LL2, extends from $\sqrt[10]{e} = 1.10517$ to $e = 2.71828$; the third part, marked LL3, extends from $e = 2.71828$ to $e^{10} = 22026.5$. Hence, speaking roughly, the numbers that can be handled on this scale are included between the limits 1.01 and 22,000. If a number is less than 1.01 (this includes all proper fractions and decimals that are not mixed numbers) or is greater than 22,000, special methods must be employed, which will be described later.

Relations of Numbers on the Three LL Scales.—If the runner be brought to any number on LL1, the numbers on the LL scales under the hair line bear the following relations to one another: the number on LL2 is the 10th power of that on LL1 and the 10th root of that on LL3; the number on LL3 is the 100th power of that on LL1 and the 10th power of that on LL2; the number on LL1 is the 10th root of that on LL2 and the 100th root of that on LL3. Thus, bringing the runner to 1.04 on LL1, read under the hair line 1.48 ($= 1.04^{10}$) on LL2, and 50.5 ($= 1.04^{100}$) on LL3; also read 392 on scale C, the first three significant figures of the hyperbolic logarithms of three numbers under the hair line on the LL scales. The decimal points in these logarithms are located as follows: for any number on the LL1 scale, the hyperbolic logarithm has one cipher between the decimal point and the first digit; for any number on the LL2 scale, the decimal point immediately precedes the first digit of the logarithm; and for any number on the LL3 scale, the hyperbolic logarithm has one integral place. Hence, in the above case, $\log_e 1.04 = 0.0392$; $\log_e 1.48 = 0.392$; $\log_e 50.5 = 3.92$.

Note that when using the LL scale, the position of the decimal point in the number must always be considered; it cannot be disregarded, as is done when using the ordinary slide rule. To find the hyperbolic logarithm of a number, note the scale on which the number is located, bring runner to number, read the significant figures of the logarithm on scale C, and place the decimal point according to the foregoing rule. Thus, to find $\log_e 2.14$, bring the runner to 2.14 on LL2, and read 761 on C; since the number is on LL2 $\log_e 2.14 = 0.761$. To find the number corresponding to a hyperbolic logarithm (i. e., the antilogarithm), determine from the position of the decimal point which of the LL scales contains it; then bring runner to logarithm on scale C, and under the hair line on the proper LL scale, read the antilogarithm. Thus, $\text{antilog}_e 2.36 = 10.59$; $\text{antilog}_e 0.236 = 1.266$; and $\text{antilog}_e 0.0236 = 1.0239$.

Reading and Setting Numbers on LL Scale.—The distance between any two consecutive short marks is called the **interval.** When setting or reading a number, the first step is to ascertain the relation between the interval and the space included by the two long marks to the right and left of the interval. This relation, or ratio, varies very much for different parts of the LL scale. From 1.01 to 1.02, the interval represents 0.1; from 1.02 to 1.05, it represents 0.2; and from 1.05 to the right end of LL1, it represents 0.5. The first three figures of the number are printed at convenient distances apart, and the intermediate long marks give the fourth figures, the fifth figure being obtained by eye. Thus, to set runner to 1.0347, bring it to 1.03, move it to the fourth long mark to the right, which is 1.034, and then move it still farther to the right, so that the hair line falls between the third and fourth short marks, that is, midway between 0.6 and 0.8. No difficulty should be experienced in setting and reading numbers on LL1 and LL2.

It is necessary, however, to exercise particular care with large numbers on LL3, especially for numbers greater than 30. From 30 to 50, each short mark represents 1, the first following 30 being 31, the next 32, the next 33, etc.; this is indicated by the fact that between the printed numbers 30 and 40 (and between 40 and 50) there are 10 divisions. Between 50 and 100, each interval represents 2, since it is one-fifth of the space between two long marks, which represents 10. From 200 to 500, each interval represents 20; from 500 to 1000, it is 50; from 1000 to 2000, it is 100; from 2000 to 5000, it is 200; from 5000 to 10,000, it is 500; and from 10,000 to 22,000, it is 1000. To set runner to 745, for example, count two long marks beyond 500 (this is the mark for 700);

note that the interval here is 50; divide (mentally) 745 — 700 = 45 by 50, obtaining 0.9; hence, bring the hair line to 0.9 of the interval beyond the 700 mark, or just the merest trifle short of the half way mark between 700 and 800. To read a number, we reverse the above proceeding: estimate the fraction of the interval, as 1/2, 2/3, 3/5, etc.; multiply it by the value of the interval; and add to the number indicated by the long mark. These operations should all be performed mentally.

To Find any Power of a Number.—Let a be the number and b the exponent, then we are to find x in $a^b = x$. Find number on LL scale, bring it to one of the indexes on D^3 (using runner, if necessary), bring runner to exponent on D^3; under hair line, read the power (x) on LL.

Example.—What is the fifth power of 2.14? that is, $2.14^5 = x$.

Analysis.—Set runner to R1D^3; bring 2.14 LL2 to hair line; move runner to 5 D^3, and read 44.9 on LL3. We know that the result must come on LL3, since even the square of 2.14 is greater than 4, and 4 is greater than 2.72, the smallest number on LL3.

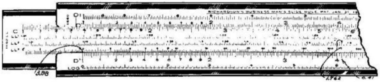


Fig. 1

Example.—What is the value of $3.98^{0.41}$?

Analysis.—Bring 3.98 LL3 to L1D^3; bring runner to 41 D^3; under hair line, read 1.762 on LL2. The result must come on LL2, since it is smaller than $4^{0.5} = 4^{\frac{1}{2}} = 2$. This setting is shown in Fig. 1.

No general rule can be given for determining on which part of the LL scale the result will come; but, by exercising common sense and remembering that a number on any scale is the 10th power of the number on the scale above it and the 10th root of the number on the scale below it, no difficulty should be experienced in selecting the proper scale.

When the Number is Not on the LL Scale.—Let a be the number and n the exponent; then. $a^n = x$, and $\frac{1}{x} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$. Hence, if the number is less than 1, find its reciprocal by using the DI and D^1 scales and the runner; raise the reciprocal to the nth power; and, again using the DI and D^1 scales, find the reciprocal of the power, which will be the value sought.

Example.—$0.342^{2.19} = x$.

Analysis.—Set runner to 342 on DI, and read 292 on D^1; hence, $\frac{1}{0.342} = 2.92$. Bring 2.92 on LL3 to $L1D^3$, and opposite 219 D^3, read 10.45 on LL3. Then $x = \frac{1}{10.45} = 0.0957$.

If the number is greater than 22,000, $a^n = (b \times c)^n = b^n \times c^n$. Hence. resolve a into two factors, the nth power of either being less than 22,000; raise both factors to the given power; and then multiply the results.

Example.—$34520^{1.41} = x$.

Analysis.—$34,520 = 345.2 \times 100$; $345.2^{1.41} = 3800$; $100^{1.41} = 660$; $3800 \times 660 = 2,508,000$, using scales C and D^1.

If the number is less than 1.01 and greater than $\frac{1}{1.01} = 0.99$, neither of the two preceding methods can be applied. For such cases, we write $a^n = \frac{a^n \times b^n}{b^n} = \frac{(a \times b)^n}{b^n}$. Hence, multiply a by some convenient number b; raise the product to the nth power; and divide the result by the nth power of b.

Example.—$0.995^5 = x$.

Analysis.—$0.995^5 = \frac{(0.995 \times 1.2)^5}{1.2^5} = \frac{1.194^5}{1.2^5} = \frac{2.43}{2.49} = 0.976$.

To Find any Root of a Number.—We reason out the steps of this operation as follows: $\sqrt[n]{a} = a^{\frac{1}{n}} = x$; $\frac{1}{n} \times \log a = \log x = \frac{\log a}{n}$; and $\log_e \log x = \log_e \log a - \log_e n$. Hence, bring a on LL to n on D^3, and over 1 on D^3 read x on LL. The setting is in all respects similar to division with the C and D scales when the quotient comes on the slide.

Example.—$\sqrt[5]{79.25} = x$.

Analysis.—Bring 79.25 LL3 to 5 D^3; bring runner to $R1D^3$, and under hair line read 2.398 on LL2. It is obvious that the root could not come on LL3, since the number on this scale under the hair line is over 6000; hence, it must come on the preceding scale, or LL2.

If the number whose root is to be found is not on the LL scale, we employ methods similar to those used in finding the powers of such numbers. For example, $\sqrt[3.46]{68000} = 68^{\frac{1}{3.46}} \times 1000^{\frac{1}{3.46}} = 3.39 \times 7.36 = 24.9$.

Example.—$\sqrt[5]{0.679} = x$.

Analysis.—$\sqrt[5]{0.679} = \frac{1}{\sqrt[5]{\frac{1}{0.679}}} = \frac{1}{\sqrt[5]{1.473}} = \frac{1}{1.0805} = 0.925$.

Using the DI and D^1 scales, $\frac{1}{0.679} = 1.473$. Bring runner to 5 D^3; bring 1.473 LL2 to hair line; bring runner to $R1D^3$, and under hair line, read 1.0805 on LL1. The root cannot come on LL2, since the number under the hair line is greater than 2, which is greater than 1.473, the number whose fifth root is sought. The reciprocal of 1.0805 is $0.925 = \sqrt[5]{0.679}$.

Important.—It is well to keep in mind that whenever the LL scale is used, the numbers are always taken on this scale and the exponents are always taken on the D^3 scale. Note this fact in connection with the preceding examples and in those that follow.

Exponent a Common Fraction.—When the exponent is a common fraction, as expressed by the equation $a^{\frac{b}{c}} = x$, $\log x = \frac{b}{c} \times \log a = \frac{b \times \log a}{c}$; and the setting is therefore similar to that for finding the product of two numbers and dividing by a third number, when using scales C and D. Bring the number on LL to the denominator on D^3, and over the numerator b on D^3, read x on LL.

Example.—$24.6^{\frac{3}{2}} = x$.

Analysis.—Bring 24.6 LL3 to 2 D^3; over 3 D^3, read $122 = x$ on LL3. This setting is evidently equivalent to finding the square root of 24.6 and then cubing the result.

When the Exponent Is Unknown.—There are two cases: (1) $a^x = b$; (2) $x^x = b$. For the first case, the setting is exactly the same as for finding the power of a number, except that the result comes on D^3 (since it is an exponent) instead of on LL.

Example.—What is the value of x in $3.88^x = 19.6$?

Analysis.—Bring 3.88 LL3 to $L1D^3$; under 19.6 LL3, read 2195 on D^3. Evidently, the decimal point must be placed between the first and second figures, since 19.6 falls between 3.88^2 and 3.88^3. Therefore, $x = 2.195$, and $3.88^{2.195} = 19.6$.

The second case, $x^x = b$, cannot be solved directly; we shift the slide until the number on LL over 1 D^3 is the same as the number on D^3 under b on LL; this number will be the value of x.

Example.—What is the value of x in $x^x = 100$?

Analysis.—Since $3^3 = 27$ and $4^4 = 256$, x lies between 3 and 4. Now trying 3.3, 3.4, etc., it will be found that when 3.6 on LL3 is over $L1D^3$, the number on D^3 under 100 on LL3 is also 3.6 (i. e., 36). Hence, $3.6^{3.6} = 100$. The exact value to 5 significant figures is 3.5973.

Logarithms to Any Base.—Consider the equation $a^x = b$, in which a is any number greater than 1. Taking the logarithms of both sides, x log a = log b. If a be made the base of the system, then (since $\log_a a = 1$) $x = \log_a b$. If, therefore, we wish to find the logarithm of a number b to any base a, we may always obtain it by finding the value of x in $a^x = b$.

Suppose, for example, we wanted the common logarithm of 3.14; we simply find the value of x in $10^x = 3.14$. Bring 10 on LL3 to $R1D^3$; under 3.14 on LL3, read 497 on D^3; hence, $\log_{10}3.14 = 0.497$. Had the number been 31.4, bring 10 LL3 to $L1D^3$, and under 31.4 LL3, read 1.497 on D^3. Note that when the number is greater than 10, but not greater than 22,000, the characteristic is 1, 2, 3, or 4, and is the first figure of the logarithm as read on scale D^3. If the number lies between $10^{\frac{1}{10}}$ and 10, the characteristic is 0, and the decimal point precedes the first (left-hand) digit of the logarithm. If the number lies between $10^{\frac{1}{100}}$ and $10^{\frac{1}{10}}$, a cipher must be placed between the decimal point and the first digit of the logarithm. Finally, if the number lies between the left index on $LL1 = e^{\frac{1}{100}}$ and $10^{\frac{1}{100}}$, two ciphers must be placed between the decimal point and the first figure of the logarithm. These limits are easily found, since, as before explained, if the runner be brought to 10 on LL3, the number under the hair line on LL2 will be $10^{\frac{1}{10}}$, and on LL1, it will be $10^{\frac{1}{100}}$. Bearing this in mind, $\log_{10}30 = 1.477$; $\log_{10}1.52 = 0.182$; $\log_{10}1.067 = 0.0282$; and $\log_{10}1.0156 = 0.00672$.

Diagrams of Settings.—In the following diagrams, the rectangular outlines represent the slide; the upper line represents scale C, and the space above it scale D^1; the lower line represents the LL scale, and the space below it scale D^3. The formula at the side shows the setting to which the diagram applies. For example, Fig. 5 is interpreted as follows: bring a on LL to 1 (either right or left index) on D^3; over b on D^3, read x on LL. Fig. 2 is read: bring runner to a on LL, and under hair line, read x on scale C. The straight line here represents the hair line of the runner.

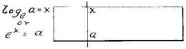


Fig. 2

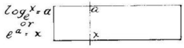


Fig. 3

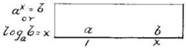


Fig. 4

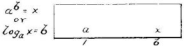


Fig. 5

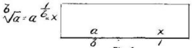


Fig. 6

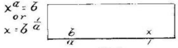


Fig. 7

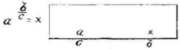


Fig. 8

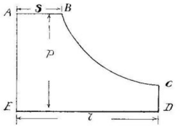
Fig. 9

Solution of Formulas.—A few formulas will now be given, together with their solutions, that will illustrate the use of the LL scale.

(1) In the formula $F = S_1 - S_2 = S_2\,(e^{f\phi} - 1)$, what are the values of S_2 and S_1 when $F = 86.4$, $f = 0.42$, and $\phi = 180° = \pi$ radians? Here $S_2 = \frac{F}{e^{f\phi}-1} = \frac{86.4}{e^{0.42\times\pi}-1} = \frac{86.4}{e^{1.319}-1} = \frac{86.4}{3.74-1} = \frac{86.4}{2.74} = 31.5$. From the formula, $S_1 = S_2 e^{f\phi} = 31.5 \times 3.74 = 117.8$. From the formula, S_1 is also equal to $F + S_2 = 86.4 \times 31.5 = 117.9$, the two results agreeing almost exactly. Having found the value of the exponent $(0.42 \times 3.14 = 1.319)$, we use the setting of Fig. 3 to find the value of $e^{1.319}$; the remainder of the work is evident.

(2) According to Weisbach, the diameter in inches of a wrought-iron shaft that will transmit H horsepower at N r. p. m. is determined by the formula $d = 3.65\sqrt[3]{\frac{H}{N}}$, when stiffness is considered, and by the formula $d = 4.8\sqrt[4]{\frac{H}{N}}$, when strength is considered; for what diameter will both formulas give

the same result? Let d_0 be the required diameter, then $d_0=3.65\sqrt[3]{\frac{H}{N}}=4.8\sqrt[4]{\frac{H}{N}}$, or $\frac{H}{N}=\left(\frac{4.8}{3.65}\right)^{12}=1.315^{12}=26.8$. Therefore, $d_0=3.65\sqrt[3]{26.8}=3.65\times 2.99=4.8\sqrt[4]{26.8}=4.8\times 2.275=10.92$ inches.



(3) In the figure, BC is a part of an equilateral hyperbola, and here corresponds to the expansion line of an indicator diagram, when the height $EA=p$ is the steam pressure and $ED=l$ is the length of the diagram. The distance AB represents the volume of steam in the cylinder at cut-off, including clearance. Let $r=$ number of expansions $=\frac{l}{s}$; then the mean ordinate (or m. e. p.) $p_m=p\left(\frac{1+\log_e r}{r}\right)$. Find the value of p_m when $p=124.7$, and $r=4.75$. Substituting the given values in the formula, $p_m=124.7\left(\frac{1+\log_e 4.75}{4.75}\right)=124.7\left(\frac{1+1.56}{4.75}\right)=67.2$. We use the setting of Fig. 2 to find $\log_e 4.75=1.56$; the remainder of the work is obvious.

(4) Using the United States rule for partial payments, how much must be paid each month to clear off a debt of \$2000 in 7 years at 6% per annum? Let $x=$ one of the equal payments, $r=$ rate for 1 month $=\frac{.06}{12}=0.005$, $p=$ principal $=\$2000$, and $n=$ number of equal payments $=12\times 7=84$; then

$$x=\frac{pr}{1-\left(\frac{1}{1+r}\right)^n}=\frac{2000\times 0.005}{1-\left(\frac{1}{1.005}\right)^{84}}=\frac{10}{1-\frac{1.1^{84}}{(1.1\times 1.005)^{84}}}=\frac{10}{1-\frac{3000}{4560}}=\frac{10}{1-0.658}$$

$=\frac{10}{0.342}=\$29.20$. Since the number 1.005 does not occur on the LL scale, we multiply both terms of the fraction by 1.1, and then proceed as outlined above. The actual value of x to even cents is \$29.22.

(5) According to the laws of mine ventilation, if the cross-sections of two airways are similar, and if they have the same length and the ventilating pressure is the same in both, the quantity of air passing through the airways in equal times is directly proportional to the square root of the fifth power of any two homologous lines of the cross-sections. In the case of two circular airways, $q_1 : q_2 = d_1^{\frac{5}{2}} : d_2^{\frac{5}{2}}$, where d_1 and d_2 are the diameters and q_1 and q_2 are the corresponding quantities of air. If 10,000 cubic feet of air per minute pass through an airway 12 feet in diameter, how many cubic feet per minute will pass through an airway of equal length and 8 feet in diameter, the pressure being the same in both airways? Substituting the values given in the proportion given above, $10000 : q_2 = 12^{\frac{5}{2}} : 8^{\frac{5}{2}}$, or $q_2=10000\times\frac{8^{\frac{5}{2}}}{12^{\frac{5}{2}}}=10000\times\frac{181}{499}=3630$ cubic feet per minute. In finding the values of the terms of the fraction, we use the setting of Fig. 8.

DIRECTIONS

for using the

Richardson Binary Polymetric Slide

No. 1865-0

By J. J. Clark, M. E. (Lehigh)

DESCRIPTION OF SLIDE

The Richardson Binary Polymetric slide is intended for use with any of the following Richardson slide rules: Nos. 812, 1812 and 1776. Simply take out the regular slide and replace it with the binary polymetric slide.

This slide contains three scales, the upper being a regular C scale, the middle a CI (inverted C) scale, and the lower an inverted scale on which the divisions indicate binary fractions ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.) combined with integers running from 1 to 10.

By means of the runner, the C scale can be used with the D scale in the regular way for multiplying and dividing; and by using the CI scale also, the product of three factors can be obtained with one operation. Thus, to find the product of $3.14 \times 62.75 \times 144$, set runner to 314 D, bring 6275 CI to hairline, move runner to 144 C, and under hairline read 284 on D; as there are 5 integral places, the product is 28,400. For further information regarding use of the CI scale, see directions for the use of the Richardson Polymetric Slide Rule No. 1776.

By having the C scale adjacent to the A scale, squares, cubes, and square roots can be found without the aid of the runner. Thus, to find the square or square root, bring the end indexes of the rule and slide into line; then any number on scale A will be the square of the number opposite to it on scale C, and any number on scale C will be the square root of the number opposite to it on scale A. To cube a number, bring one of the indexes on the slide to the number on scale A, and opposite the number on scale C read the cube on scale A. Thus, to cube 256, bring L1C (this means left index on scale C) to 256 A, and opposite 256 C read 168 on scale A. As there are 8 integral places, $256^3 = 16{,}800{,}000$.

THE LOWER SCALE ON THE SLIDE.

The lower scale, marked R, is an inverted scale, i. e., the numbers on it increase from right to left; it differs from the CI scale in two respects:

(1) the division marks indicate mixed numbers, the denominators of the fractions being 2, 4, 8, etc.; (2) there is only one index mark, and that is situated near the middle of the scale. This index is, in fact, opposite 360 on the C scale, being placed there to facilitate the computation of interest, as will be explained later. The operations of multiplying and dividing are performed with the R and D scales in exactly the same way as with the CI and D scales. For example, $15 \times 3\frac{1}{4} = 48.75 = 48\frac{3}{4}$. Bring $3\frac{1}{4}$ R to 15 D, and opposite 1 R, read 4875 on D; as there are 2 integral places, the product is $48.75 = 48\frac{3}{4}$.

To multiply 15 by $3\frac{3}{4}$, set $3\frac{3}{4}$ R to 15 D; move runner to R1C; bring L1C to hair line, and opposite 1 R read $56\frac{1}{4}$ on D.

DIAGRAMS OF SETTINGS.

The settings for the various operations are most conveniently presented by means of diagrams. For explanation of diagrams, see Directions for Using the Richardson Polymetric Slide Rule. In the diagrams now to be given, the upper line represents the C scale, and the space above it the A scale; the middle line represents the CI scale; the lower line represents the R scale, and the space below it the D scale. In the diagrams that follow, the letters m, n, p, etc., indicate numbers on scale R, while the letters a, b, c, etc., indicate numbers on the other scales of the rule and slide. Thus, the product of two numbers, one being on scale R, will be indicated by the formula $a \times m$, as in Fig. 1, the diagram being interpreted as follows: bring m on scale R to a on scale D; opposite 1 R, read x on D.

Fig. 1

TO FIND THE PRODUCT OF TWO NUMBERS ON SCALE R

The setting is shown in Fig. 2, which is read as follows: bring m R to 1 D; bring runner to 1 R; bring n R to hair line; and opposite 1 R, read x on D. For example, $4\frac{3}{16} \times 6\frac{3}{8} = 30.7$. Referring to the diagram, set $4\frac{13}{16}$ R to R1D; bring runner to 1 R; bring $6\frac{3}{8}$ R to hair line; opposite 1 R read 307 on D. As there are two integral places, the product is 30.7.

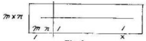


Fig. 2

DIVISION

All numbers on scale R will be called **mixed numbers** to distinguish them from ordinary numbers taken on the other scales. Fig. 3 gives the setting for dividing a number by a mixed number. For example, $30.7 \div 6\frac{3}{8} = 4.81 = 4\frac{13}{16}$. Set 1 R to 307 D, and opposite $6\frac{3}{8}$ R read 481 on D. The quotient is evidently 4.81. To express the decimal as a fraction having any denominator,

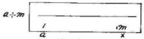


Fig. 3

multiply it by the denominator, and the product will be the numerator. Thus, to express 0.81 as a fraction having 16 for the denominator, bring runner to 16 D; bring 81 CI to hair line; opposite L1C1, read 13 on D. Of course, the numerator will not always be an integer; in such case, take the nearest number on D, expressed as an integer.

TO DIVIDE A MIXED NUMBER BY A NUMBER

Fig. 4 gives the setting for m÷a. As an example, what per cent of $6\frac{3}{8}$ is 51.34? Here $6\frac{3}{8}$ is to be divided by 51.34. Set $6\frac{3}{8}$ R to R1D; bring runner to 1 R (the number under the hair line on D is 6.375=$6\frac{3}{8}$); bring 5134 C to hair line, and opposite L1C read 1242 on D. The quotient is evidently 0.1242, and the rate per cent is 0.1242×100=12.42.

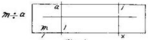


Fig. 4

TO DIVIDE A MIXED NUMBER BY A MIXED NUMBER

The setting is shown in Fig. 5. Set dividend on R to 1D, and opposite divisor on R read quotient on D. As an example, $7\frac{3}{4} \div 3\frac{5}{16} = 2.34 = 2\frac{11}{32}$. Here set $7\frac{3}{4}$ to R1D, and opposite $3\frac{5}{16}$ R read 2.34 on D. Had it been desired to obtain the quotient as a mixed number, use the setting shown in Fig. 6, which is interpreted as follows: set m R to 1 D; bring runner to n R; bring 1 R to hair line; opposite 1 D read x on R. In the last example, set $7\frac{3}{4}$ to R1D; bring runner to $3\frac{5}{16}$ R; bring 1 R to hair line, and opposite L1D read $2\frac{11}{32}$ on R. Note that each short division mark on scale R represents $\frac{1}{16}$th, 32ds and 64ths being obtained by eye.

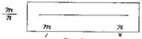


Fig. 5

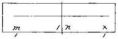


Fig. 6

BOARD MEASURE

If the length of a piece of timber is given in feet and its width and thickness in inches, the number of board feet is found by multiplying the length, breadth and thickness and dividing the product by 12. As an example, how many board feet in 311 strips $1\frac{1}{2}'' \times 1\frac{5}{8}'' \times 6\frac{1}{2}$ ft. long? Let m, n, and o represent the mixed numbers, then the formula is $\frac{m \times n \times o \times a}{12} = x$. The setting is shown in Fig. 7. Set $1\frac{1}{2}$ R to L1D; bring runner to 1 R; bring $1\frac{5}{8}$ R to hair line; bring runner to 1 R; bring $6\frac{1}{2}$ R to hair line; bring runner to 1 R; bring 12 C to hair line; bring runner to 311 C, and under hair line read 410 on D. There are 3 integral places; hence, in the 311 strips, there are 410 board feet.

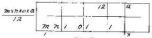


Fig. 7

INTEREST

Let p = the principal, r = the rate per cent per annum, t = the time in days, and I = the interest. Business men usually allow 30 days for each month and 360 days for a year, in which case $I = \frac{p \times r \times t}{360}$. Since 1 R is placed at a point equivalent to 360 on scale C, and since scale R is inverted, numbers on scale C are automatically divided by numbers opposite them on scale R. Hence, to find the interest for any number of days, simply find the product of p, r, and t. For example, what is the interest of $188.20 for 63 days at $5\frac{1}{2}$ per cent? The setting is shown in Fig. 8. Set $5\frac{1}{2}$ R to 1882 D; bring runner to 63 C, and under hair line read 181 on D. The interest is evidently $1.81.

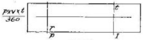


Fig. 8

Bankers and brokers use the actual number of days that the loan is drawing interest and the actual number of days in the year; therefore, the divisor, in such cases, must be either 365 or 366, according to whether the year is a common year or a leap year. This interest is called **exact interest**, and is readily found, as indicated by the diagram, Fig. 9. Set r on R to p on D; bring runner to 1 R; bring number of days in year (365 or 366) on C to hair line; move runner to t on C, and under the hair line, read I on D. Thus, in the last example, had the exact interest for a common year been required, set $5\frac{1}{2}$ R to 1882 D; bring runner to 1 R; bring 365 C to hair line; bring runner to 63 C, and under the hair line read 179 on D. The exact interest is $1.79.

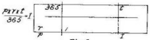
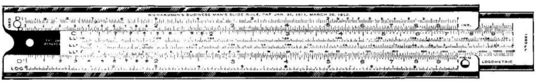


Fig. 9

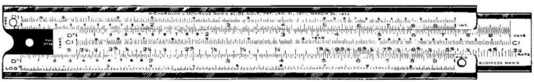
CUBE ROOT

Note that on the C scale, there are two gauge points indicated by c^1 and c^2, c^1 being situated at $\sqrt[3]{10}=2.154$ C and c^2 at $\sqrt[3]{100}=4.642$ C. If the cube root of any number between 1 and 10 is known, call it a, the cube root of 10 times that number will be 2.154×a, and the cube root of 100 times the number will be 4.642×a. To find the cube root of a number between 1 and 10, set runner to number on scale C; move slide until the number on C under the

No. 1865-O—Richardson's Binary Polymetric with CI scale for handling three factors with one setting, also an inverted scale divided **not** decimally, but in halves, quarters, eighths, sixteenths and but one index mark located near the middle of the scale, this scale permits one to multiply or divide mixed numbers not greater than 10, such as for instance: $4\frac{13}{16} \times 6\frac{3}{8}$ or $7\frac{3}{4} \div 3\frac{5}{16}$ or $3.14 \times 7\frac{5}{8}$, with book, each **$4.00**



No. 1860-LL—Richardson's Logometric (log log) for handling fractional powers and roots, also natural or hyperbolic logarithms so frequently met with in electrical, mechanical and hydraulic formulas, such as $\sqrt[3.46]{68000} = 68^{\frac{1}{3.46}} \times 1000^{\frac{1}{3.46}} = 3.39 \times 7.36 = 24.9$; likewise $\text{Log } 44^{1.41} = \log 207.61$; with book, each **$4.00**



No. 1860—Richardson's Business Man's Slide Rule. Will give the amount of interest due on any principal, at any rate, for any number of days or year, especially designed for computing commercial problems, cost estimation, division of freight rates, wages by the month, day or week for any number hours at any rate per hour, unit-cost and selling price, string discounts off the list price, percentages, proportion, the rule of three, etc., with 20-page book especially written and illustrated with 24 cuts showing **just how** to set the rule for solving **commercial** problems, each **$5.00**

Richardson's PANTOCRAT Slide Rules

Made in Chicago

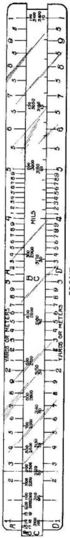
These extra slides are interchangeable with rule. Thus changing a Mannheim into a Polymetric, Logometric, Binary Polymetric or adding and subtracting slide rule, etc. Complete (with leather case and 100-page book, 135 illustrations) price $10.00

Military Slide Rule.

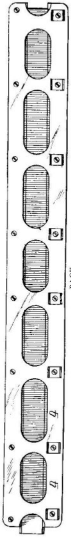
Used for the Solution of Triangle in Computing Range Target

Size 2 in. x 18 in. x ¼ in.

Scales engraved on white celluloid and mounted on Mexican mahogany. The scale members are secured to a $\frac{1}{16}$ in. brass metal back, with seven elliptic holes to reduce weight. The A scale however is provided with eight adjustable clamps (see lower cut of back) to compensate for changes in atmospheric condition and proper alignment of the scales.



FRONT



BACK

The slide is graduated in mils on front and back. All Scales have subdivisions not shown in cuts on this page (see figure 2, page 96). This rule adopted by U. S. Field Artillery. Model 1917.

This rule is constructed very rigid to meet the hard service of which it will be called upon to perform. Price with complete directions $20.00

DIRECTIONS

for using the

Military Slide Rule

By J. J. Clark, M. E. (Lehigh)

Purpose of the Military Slide Rule.—The purpose of the military slide rule is to solve quickly, almost instantly, the problem illustrated in Fig. 1. Here G represents the position of a gun, T is the target, and O is the position of the observer. The distance between G and O is known, and the angles C and A (or C′ and A′) are also known, being readily measured; it is required to find the distance between the gun and target. In other words, given the triangle GTO, in which the side GO and the angles C and A are known, it is required to find the side GT. This problem would be solved by trigonometry in the following manner: represents the side GT opposite the angle A by *a*, the side TO opposite the angle C by *c*, and the side GO opposite the angle B by *b*. Angle $B = 180° - (C + A)$; or, if the angle C′ is known, $B = C' - A$, since, as shown in geometry, the exterior angle C′ of a plane triangle is equal to the sum of the two opposite interior angles B and A; whence, $C' = B + A$, and $B = C' - A$. It is shown in trigonometry that the sines of the angles are proportional to the sides opposite them; in other words, $\frac{a}{b} = \frac{\sin A}{\sin B}$, from which

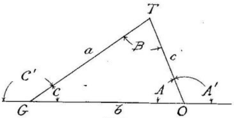


Fig. 1

$$a = \frac{b \sin A}{\sin B}. \quad (1)$$

By means of equation (1), *a* can be readily calculated, but it requires time and a table of trigonometric functions; by the use of the military slide rule, the time is reduced to almost nothing, and the table of trigonometric functions is eliminated. The angles used in computation with the military slide rule are not expressed in degrees, minutes, and seconds, as is customary; instead, they are expressed in mils—a single unit—which greatly facilitates the calculation. Before proceeding further, itis necessary to explain this method of measuring angles.

Measuring Angles in Mils.—The arc of a quadrant is supposed to be divided into 1600 equal parts, each of which is called a mil. Since a quadrant contains 90°, a semicircle, or 180°, contains $2 + 1600 = 3200$ mils, and a circle, or 360°, contains $4 \times 1600 = 6400$ mils. The relations between degrees, minutes, and seconds and mils are easily established. Thus, $1° = \frac{1600}{90} = 17\frac{7}{9} = 17.7778$ mils. Also, 1 mil $= \frac{90 \times 60 \times 60}{1600}$ 202.5 seconds $= 3'\ 22.5''$. Further, 1 minute $= 0.2963$ mil, and 1 second $= 0.00494$ mil.

To convert mils into degrees, minutes, and seconds, the easiest way is to proceed as follows: Multiply the number of mils by 200, and add to the product 1/80th of the product; the result will be in seconds, which can then be reduced to degrees, minutes and seconds. Thus, to convert 1444 mils into degrees, minutes and seconds, $1444 \times 200 = 288800$; $288800 \div 80 = 3610$; $288800 + 3610 = 292410$ seconds $= 81°\ 13'\ 30''$. The work would be performed as shown in the margin. This conversion is useful in those cases where it is desired to check the results obtained with the military slide rule by making a direct trigonometric calculation. If it is desired to express in mils an angle given in degrees, minutes, and seconds, multiply the degrees by 17-7/9, the minutes by 0.2963, the seconds by 0.00494 and add the several products. Thus, $81°\ 13'\ 30'' = 1444$ mils, since $81 \times 17\text{-}7/9 = 1440$ mils; $13 \times 0.2963 = 3.8519$ mils; $30 \times 0.0494 = 0.1482$ mil; and $1440 + 3.8519 + 0.1482 = 1440.0001 = 1440$ mils.

	1444	
	200	
80	288800	
	3610	
60	292410"	
60	4873'	30"
	81°	13'

Description of Scales on Military Slide Rule.—The military slide consists of two parts—a fixed part, called the **rule**, and a movable (or sliding) part, called the **slide**. The rule (see Fig. 2) contains two scales, the upper one being called scale A and the lower one scale D. These two scales are exactly alike, and the corresponding divisions on each are exactly opposite each other. These scales are divided in practically the same manner as the C and D scales on the ordinary slide rule,

and no special description of them is necessary. To locate a number, such as 387, find the long division mark numbered with the large figure 3; this is the first figure of the number; glancing to the right, note the division mark numbered with a small figure 8; this is the second figure of the number; since there are five spaces between 8 and 9, each interval represents two-tenths of the space between 8 and 9, and each unnumbered division mark within this space represents 2; hence the third mark represents 6 and the fourth represents 8, while half way between represents 7, the third figure of the number. With this explanation no difficulty should be experienced in locating the position of any number on scales A and D.

Scales on the Slide.—The upper scale on the slide is called scale B, and the lower scale is called scale C. These two scales are straight sine scales, but graduated from *right* to *left* instead of from left to right, as in the case of scales A and D. As a consequence of graduating scales B and C in the reverse direction, it follows that when the left-hand division mark on scale B (marked 16, but representing 1600) is placed in line with the left index mark on scale A, the numbers on scales A and D opposite the adjacent divisions on Scales C and B are the reciprocals of the sines of the angles taken on scales C and B, when the angles are expressed in mils; in other words, they are cosecants of those angles. (See paragraph, "To Find the Sine of an Angle".) Thus, the number on D opposite 54 on C is 1885, and the cosecant (abbreviated to csc) of 54 mils is 18.85; the number on A opposite 400 on B is 2613, and csc 400 mils is 2.613. The last reading is obtained as follows: the 400-mark comes about ⅔ ds of the way between 260 and 262 on A; $2 \times ⅔ = 4/3 = 1.3$; and $260 + 1.3 = 261.3$, or 2613.

As before stated, the numbers on scales B and C increase from *right* to *left*, instead of from left to right, as on scales A and D. Therefore, in taking numbers on these scales, after the first figure has been found, the second and subsequent figures for all numbers less than 1600 must be found by going to the *left*. No difficulty should be experienced in locating numbers on these scales. For example, to locate 725 mils, find the 700 mark on B; note that between 700 and 800 there are 10 long marks, and one short mark between two consecutive long ones; hence, the second long mark to the left of 700 represents 2, the second figure of the number, and the next short mark represents 5, the third figure of the number, 725.

It will be noted that there are two sets of numbers printed over the main division numbers; the sum of any two numbers of a set is always 3200. The reason for this is that the cosecant of an angle is equal to the cosecant of its supplement. For instance, csc 27° = csc 180° — 27° = csc 153°; and 153° + 27° = 180°. Since 180° = 3200 mils, csc 700 mils = csc 3200 — 700 = csc 2500 mils, and $700 + 2500 = 3200$. Hence, if an angle is greater than 1600 mils, use the outside row of numbers to determine its position on B or C.

With the slide in the position previously mentioned, that is with 1600 on B opposite the left index on A, note that the right index on D is opposite about 10.2 on C; this is because csc 10.18 (= 10.2 nearly) mils is 100. Note further that the right index on A is opposite 102 on B; this is because csc 102.03 mils = 10. Therefore the cosecant of any number on scale C will have 2 integral places; and the cosecant of any number on scale B will have 1 integral place. Hence, as previously determined, csc 54 mils is 18.85, and csc 400 mils is 2.613. It is well to note that the cosecant of 1600 mils ($= 90°$) is 1; that it can never be less than 1, and that its value varies between 1 and

For angles between 1600 and 3200 mils, the cosecant increases as the angle increases; consequently, for angles greater than 1600 mils and less than 3200 mils, the readings must be taken from *left* to *right*, as on scales A and D. Thus, csc 2816 mils = 2.715, the mark representing 2816 on scale B coming three-quarters of the way between 270 and 272 on scale A; the space between two short marks here represents 2; hence, $\frac{3}{4} \times 2 = 3/2 = 1.5$; and $270 + 1.5 = 271.5$. Consequently, csc 2816 mils = 2.715.

Scales B and C Form One Scale.—Scales B and C are in reality two halves of a single scale beginning at 10.18 mils on C and extending to 1600 mils on B. In order to keep the length of the rule within reasonable limits, the scale has been divided in the middle, the first half forming scale C and the second half forming scale B. The range of cosecants is, as shown above, from 100 to 1, the lower scale being from 100 to 10, and the upper from 10 to 1.

Hereafter, unless otherwise especially stated, the mil will not be used in connection with angles and cosecants; thus, an angle of 1250 will mean 1250 mils, and csc 358 will mean csc 358 mils.

Index Marks.—The owner of a military slide rule will find it greatly to his advantage to set the slide so that the 1600 mark on B is directly in line with the left index (1) on scale A; then, with a sharp instrument (a scriber or the blade of a penknife) and straightedge, cut a line across the slide from 1 to 1; do the same thing at the other end of the slide. Now, with a sharp lead pencil, go over these lines, so as to blacken them. The left line should pass through and form a continuation of line indicating 1600 mils, and the right line should pass through and be a continuation, the line indicating 102 mils. In the directions that follow, it will be assumed that these two lines have been drawn; the left line will be called the **left index** on the slide or L1S, and the right line will be called the **right index** on the slide or R1S. The lines at the left end of the rule marked 1 will be called the left index on rule or L1R, and those at

the right end of the rule will be called the right index on rule or R1R. If it is desired to indicate any particular scale in connection with the indexes, the letter of the scale will be used in place of S and R; thus, L1A means left index on A, R1C means right index on C, etc.

Length of Arc of One Mil.—It is useful to know the relation between the length of an arc of one mil and the radius of the arc; this relation is easily determined. For instance, the length of an arc of a semicircle is $\pi r = 3.1416 \times r$; since a semicircle contains 3200 mils, $1 \text{ mil} = \frac{3.1416 \times r}{3200} = 0.000982\, r = 0.001\, r$, very nearly. In other words, an arc of 1 mil is very nearly equal to $\frac{1}{1000}$th of the radius.

To Find the Sine of an Angle.—The **reciprocal** of a number is 1 divided by the number; thus, the reciprocal of 3.1416 is $\frac{1}{3.1416} = 0.3183$. It is shown in trigonometry that the cosecant is the reciprocal of the sine; consequently, the reciprocal of the sine is the cosecant. It is also shown in works treating on the slide rule that if the slide be inverted, that is, turned upside down, and the index marks on the rule and slide are placed in line, the numbers on the rule will be the reciprocals of those on the slide, and vice versa. Hence, if it is desired to use the military slide rule to find the sine of an angle, remove the slide, turn it upside down, and replace it in the rule. Bring the index marks into line with those on the rule; the index mark passing through 1600 will then be in line with R1D, and the numbers on the slide will be upside down. The number opposite 50 on C is 491 on A, and sin 50 = 0.0491; the number opposite 400 on B is 383, and sin 400 = 0.383; etc. The decimal point is easily determined, since as was previously shown, numbers on scale C have two integral places and those on scale B have one integral place; therefore, the reciprocal of a number on scale C will have one cipher between the decimal point and the first digit (a cipher is not a digit), while the decimal point comes immediately before the first digit in the case of reciprocals of numbers on scale B.

The sine may also be found by setting the angle on scale B or C, as the case may be, to one of the index marks on scale A or D; then, opposite the index mark on scale B or C, according to which scale is used, read the sine on A r D. Thus, to find the sine of 50 mils, set 50-C to L1D (or R1D), and opposite R1C (or L1C) read 491 on A or D; hence, sin 50 mils = 0.0491. Similarly, to find sin 400 mils, set 400-B to L1A (or R1A), and opposite R1B (or L1B), read 383 on A or D; hence, sin 400 mils = 0.383. This method is to be preferred when the sine of only one angle is desired.

Restatement of Formula (1).—It was stated above that the reciprocal of the sine was the cosecant; bearing this in mind, Formula (1) may be written as follows:

$$a = \frac{b \sin A}{\sin B} = \frac{b \times \frac{1}{\sin B}}{\frac{1}{\sin A}} = \frac{b \csc B}{\csc A}.$$

For purposes of calculation on the military slide rule, this last expression may be written

$$a = \frac{b}{\csc A} \times \csc B \qquad (2)$$

The sequence of operations is: divide the distance b by csc A and multiply the quotient by csc B; the result is the value of *a*, which will be obtained with a single setting on the military slide rule. The following examples will illustrate the process. The reader is advised to read very carefully the explanations and to work out each example on his rule, in accordance with the directions.

Example 1.—Referring to Fig. 1, b = 200 yards, A = 1440, and B = 1260; what is the range *a*?

Solution.—Bring 1440-B (this means 1440 on scale B) to line with 2-A (= 200-A); opposite 1260-B, read 209 on A. Hence, *a* = 209 yards.

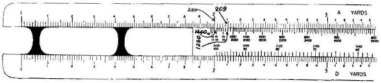


Fig. 2

This setting is shown in Fig. 2. Note that between 1200 and 1300, and between 1300 and 1400, the spaces between the long division lines is divided into 4 parts; whence, each small space represents 25. Therefore, 1260 will be 10 ÷ 25 = 0.4 of the space to the *left* of the second short mark that lies to the *left* of the 1200 mark on B. These two spaces are the only ones divided into 4 parts. The position of 1440 on B lies 40 ÷ 50 = $\frac{4}{5}$ths of the space between the 1400 mark and the next short mark to the left.

Example 2.—When b = 300, A = 1060, and B = 260, what is the range *a*?

Solution.—Set 1060-B to 3-A (= 300-A); note that 260-B is beyond R1A, and does not come opposite any number on A. To provide for this case, take the reading on A (or D) opposite L1S, in this case 159, and shift the slide to the left until R1S comes opposite 159-A (or 159-D); then opposite 260-B read 1025 on A. The range *a* is then 1025 yards, if b is in yards.

Locating the Decimal Point.—In practice, the position of the decimal point will usually be evident, that is, it can be determined by observation whether the value obtained for the range is reasonable or whether it is 10 times too long or too short. It is, however, an easy matter to determine definitely the position of the decimal point without regard to practical considerations, as will now be shown.

The product of any two numbers will contain as many integral places as there are integral places in the two numbers (factors) or that sum less 1. For example, 34.2 × 67.45 = 2306.79, and 2 + 2 = 4, there being 2 integral places in each factor and 4 in the product. Again, 142 × 67.45 = 9577.9, and 3 + 2 − 1 = 4, there being 3 integral places in the first factor and 2 in the second; the sum is 5, and 5 − 1 = 4, the number of integral places in the product. In order to determine whether to subtract 1 or not, note whether the slide projects to the right or left; that is, whether R1S is to the right or left of R1A (or R1D); if it projects to the **right**, subtract 1; otherwise, simply add, as in the first case.

The number of integral places in the quotient is equal to the number of integral places in the dividend minus the number of integral places in the divisor or that difference plus 1. For example, 2306.79 ÷ 34.2 = 67.45, and 4 − 2 = 2, there being 4 integral places in the dividend, 2 in the divisor, and 4 − 2 = 2 in the quotient. Again, 9577.9 ÷ 142 = 67.45, and 4 − 3 + 1 = 2 there being 4 integral places in the dividend, 3 in the divisor, and 4 − 3 + 1 = 2 in the quotient. In order to determine whether or not to add 1, note whether the slide projects to the right or left; if it projects to the **right**, add 1; otherwise, simply subtract, as in the first case.

Referring to example 1, the first operation is to divide 200 by the cosecant of 1440; the slide projects to the right, and as previously pointed out, any number on scale B contains 1 integral place; there will be, therefore, 3 − 1 + 1 = 3 integral places in the quotient. The second operation is to multiply this quotient by csc 1260, which contains 1 integral place, as it comes on scale B. The slide projects to the right and the product contains 3 + 1 − 1 = 3 integral places.

Referring to example 2, the first operation is to divide 300 by csc 1060; the slide projects to the right, and the quotient contains 3 − 1 + 1 = 3 integral places. Multiplying this quotient by csc 260, the slide projects to the left, and the product contains 3 + 1 = 4 integral places.

In practice, write down the number of integral places in b, subtract the number in csc A, add 1 if the slide projects to the right in making the setting, add the number of integral places in csc B, subtract 1 if the slide projects to the left when taking the reading on A or D. Thus, for example 1, 3 — 1 + 1 + 1 — 1 = 3 integral places in *a*; and for example 2, 3 — 1 + 1 + 1 = 4 integral places in *a*.

Example 3.—When b = 300, A = 2456, and B = 544, what is the range *a*?

Solution.—Here 2456 comes on B; setting it opposite 3 (= 300) on A, find 393 on A opposite 544 on B. The slide projects to the right for both operations; hence, there are 3 — 1 + 1 + 1 — 1 = 3 integral places in the result, and the range is 393.

Example 4.—When b = 200, A = 82, B = 724, what is the range?

Solution.—Here A comes on scale C and B on scale B. Set 82-C opposite 2 (= 200) on D; opposite 724-B read 2466 on A. The slide projects to the right for both operations, and since csc 82 contains 2 integral places, the number of integral places in the result is 3 — 2 + 1 + 1 — 1 = 2; therefore the range is 24.66.

It may here be remarked that the range will be in the same units as the distance b, which may be measured in feet, yards, meters, or any other convenient unit.

Example 5.—When b = 400, A = 77, B = 235, what is the range?

Solution.—Set 77-C opposite 4 (= 400) on D; since 235-B lies beyond R1A, it is necessary to shift the slide. The number on A opposite L1S is 321; hence, move slide to left until R1S comes opposite 321 on A or D; the number on A opposite 235-B is 1321. The slide projects to the right in division and to the left in multiplication; therefore, the number of integral places in the product is 3 — 2 + 1 + 1 = 3, and the range is 132.1.

Example 6.—When b = 200, A = 800, B = 12, what is the range?

Solution.—Set 800-B opposite 2 (= 200) on A; since 12-C lies beyond R1D, shift the slide and read 12 on D opposite 12-C. Here the slide projects to the right in division and to the left in multiplication; hence, the number of integral places in the result is 3 — 1 + 1 + 2 = 5, and the range is 12000.

Example 7.—When b = 200, A = 60, B = 20, what is the range?

Solution.—Set 60-C opposite 2 (= 200) on D; opposite 20-C, read 6 on D. The number of integral places in the result is 3 — 2 + 1 + 2 — 1 = 3, and the range is 600. Here the slide projects to the right for both operations, and the cosecants of both angles contain 2 integral places.

Example 8.—When b = 1500, A = 63, and B = 30, what is the range?

Solution.—Set 63-C opposite 15 (= 1500) on D; opposite 30-C read 315 on D. Here the slide projects to the left for both operations; hence, the number of integral places in the range is 4 — 2 + 2 = 4, and the range is 3150.

Example 9.—What is the range when b = 200, A = 2840, and B = 3125?

Solution.—Setting 2840-B opposite 2 (= 200) on D, it is found that 3125-C fails to the left of L1D; hence, it is necessary to shift the slide to the right. The number on D (or A) opposite R1S is 6925; bringing L1S to 6925-D, the number on D opposite 3125-C is 94. Since the slide projects to the left in division and to the right in multiplication, the number of integral places in the range is 3 — 1 + 2 — 1 = 3, and the range is therefore 940.

Shifting Slide without Using Index Marks.—If the lines have not been drawn across the slide as previously described, the slide may be shifted by making use of the 100 marks on B and C. Placing the slide so that 100-C is in line with L1D, 100-B will be in line with R1A; that is, the distance between 100-C and 100-B is equal to the distance between L1R and R1R, as it should be. Therefore, when it is necessary to shift the slide, note the reading on the rule opposite one of the 100 marks, and then shift the slide until the other 100 mark comes opposite the same reading on the rule. Thus, in example 9, when 2840-B is set to 200-D, 100-B comes opposite 706-A; hence, shift the slide until 100-C comes opposite 706-D, and the number on D opposite 3125-C is 94, as before.

Scales on Back of Slide.—If the slide be turned over, it will be seen that there are two scales on the under side, or back. These scales are exactly the same as the B and C scales on the face, with the exception that they are numbered differently, and are used when the angle is greater than 3210 mils. These scales are not really necessary, since when the angle is greater than 3200, all that is required is to subtract 3200 from the angle. However, by using the scales on the back of the slide, this operation of subtraction is avoided.

To understand how an angle can be greater than 3200 (= 180°), refer to Fig. 3. An angle in surveying and in field operations is always measured in one direction—*counter-clockwise.* Thus, if the instrument is pointing from O towards A and it is desired to find the angle which OB makes with OA, the instrument will be turned counter-clockwise to the position OB, and the angle measured; suppose it equals 850. If, however, the instrument were pointing in the direction OB and it was desired to find the angle BOA, which AO makes with BO, the instrument would still be turned counter-clockwise until it pointed from O towards A, and the angle through it turned would be 6400 — 850 = 5550. In other words, the angle AOB is 850, but the angle BOA is 5550.

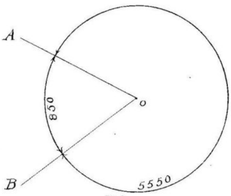


Fig. 3

Index Marks on Back of Slide.—If desired, the index marks may be scribed on the back of the slide in the same manner as on the front. Place 4800-B (note that 4800 = 1600 + 3200) in line with L1A; then with a straight-edge and sharp instrument, draw lines across the slide that will be continuations of the right and left index marks on the rule, and blacken with a sharp lead pencil, as before. If these lines are not drawn, use two 3300 marks when shifting the slide. Note that 3300 = 100 + 3200. As these two scales are used in exactly the same manner as those on the face of the slide, no special instructions regarding their use are necessary.

Multiplication and Division.—Multiplication and division of ordinary numbers may be performed on the military slide rule, though not readily. For this purpose, use scales A and B, the face of the slide being up.

Example 10.—Find the product of 24.3 and 368.

Solution.—Place the index marks on the rule and slide in line. Opposite 243-A, read 2767 on B. (Use the higher numbers on B, as both scales then read the same way—from left to right.) Now move the slide until L1B comes opposite 368 (the other factor) on A, and opposite 2767, the number previously noted, read 894 on A. To locate the decimal point, apply the rule previously given. Here the slide projects to the right; hence, there are $2 + 3 - 1 = 4$ integral places, and $24.3 \times 368 = 8940$. The exact product is 8942.4.

Example 11.—Find the product of 57.3 and 3.14.

Solution.—With the index marks in line, the number on B opposite 314-A is 2870. Bring R1B to 573 A, and the number on A opposite 2870-B is 18. Since the slide projects to the left, the number of integral places in the product is $2 + 1 = 3$, and $57.3 \times 3.14 = 180$. The exact product is 179.922.

Example 12.—Find the value of $180 \div 57.3$.

Solution.—With the index marks in line, the number on B opposite 573-A is 30214. Bring 30214-B to line with 18-A, and the number on A opposite R1B is 314. Since the slide projects to the left, the number of integral places in the quotient is $3 - 2 = 1$, and the quotient is 3.14.

Example 13.—Find the value of $8940 \div 368$.

Solution.—With the index marks in line, the number on B opposite 368-A is 2920. Bring 2920-B to line with 894-A, and the number on A opposite L1B is 243. Since the slide projects to the right, the number of integral places in the quotient is $4 - 3 + 1 = 2$, and the quotient is 24.3.

It will be noted that in example 1, setting 1440-B against 200-A means dividing 200 by csc 1440, and the quotient, 1975, is on A opposite L1B. To multiply this quotient by csc 1260, the slide is already in correct position, and all that is necessary is to read the number on A opposite 1260-B.

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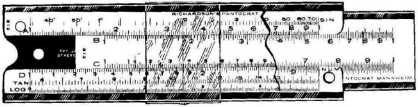

SIXTH EDITION
Revised and Enlarged

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