STERLING

DECIMAL TRIG LOG-LOG SLIDE RULE

INSTRUCTION MANUAL

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Sterling Decimal Trig Log-Log is an accurate and convenient instrument for use in solving multiplication, division, proportion, square and cube root problems, as well as computing logarithms, log-log functions, and trigonometric functions such as sine, cosine, and tangent.

The reading of any slide rule is accurate to the second place in decimal work, therefore, approximation of the third place number can be done by estimating the position of the hairline with respect to the neighboring markings. Accurate figures beyond this can be obtained by manual calculations. Three place accuracy, however is more than adequate for most problems.

The Sterling Decimal Trig Log-Log Slide Rule has the following standard scales: A, B, C, D, CI, DI, CF, DF, CIF, S, T, ST, K, L, LL1, LL2, LL3, LLO1, LLO2 and LLO3. The C, CI, L, CIF, CF, B, S, T, and ST, scales are on the **slide,** while the remaining scales are on the **body. The hairline crosses** these scales for direct comparison. The hairlines on both sides are aligned allowing for a reading from one scale to any other, even if they are on the opposite sides.

C & D Scales. Basic Logarithmic scales used for multiplication and division, used also in conjunction with other scales for other operations.

CF & DF Scales. Scales wherein the unit 1 is at the center, but otherwise identical to the C & D scales. F stands for "folded" at π. Therefore CF is C scale folded at π, and DF is D scale folded at π. They are used in conjunction with C & D in order to minimize the operations when multiplying and dividing.

CI Scale. Reversed or Inverted C scale. Scale reads from right to left, and is in italics to increase legibility. It is used also for directly reading the reciprocal of numbers.

CIF Scale. "Folded" CI scale. Used in the same manner with CF & DF as CI is used with C & D.

T Scales. Scales used for finding tangents and cotangents of angles. Cotangents are shown in italics, and the top scale is for angles of less than 45° while the lower scale is for angles of more than 45°.

S Scale. The S scale is used for finding the sines of angles and has an italicized scale for direct reading of cosines of angles.

ST Scale. This scale is used for finding sines or tangents of angles less than 5.74°.

K Scale. This consists of 3 individual scales of lesser dimensions, but similar to the D scale, of a length when placed end to end is equal to the length of the single D scale. The K scale is designed to give you cube readings of all numbers directly, or to find cube roots.

A & B Scales. Identical scales, each consisting of 2 individual scales half as long as the D scale, and used for squaring or finding square root of other scale readings.

DI Scale. A scale showing the D scale inverted or reversed to reduce operations. "It is printed in italics."

L Scale. A scale equally divided and the same length as the other scales. The L scale is so designed that when the hairline is placed to any number on the C scale the mantissa of the logarithm of that number is read on the L scale.

LL1-2-3. Each in their order, are parts of one scale. It is used in evaluating and reading the natural logarithms of numbers (base e). It gives directly the values of the function e^x ($e=2.718$) for values of x from 0.01 to 10.

LLO1-2-3. *LLO1* appears on the back at the top, while *LLO2* and *LLO3* appear on the face of the rule. All are italicized since they represent the negative values of x from -0.01 to $-.10$. It reads directly the function e^x for negative values of x.

General: It is assumed that the user is familiar with the reading of logarithmic scales and with the various values of the divisions for the purposes intended. Briefly, it will be observed that from 1 to 2 on D divisions read as single units while from 2 to 4 each division represents 2 units, and from 4 to 10, each division represents 5 units or decimal equivalents of same. This is typical of all scales, so care must be taken in reading and translating the divisions into the proper unit readings. L is an exception to the rule, here each division represents 2 units.

The Cursor: The Cursor of the Sterling Slide Rule is adjustable. Alignment of scales on the body and slide has been held in manufacture to closest tolerances, but should variations occur between back and front side with respect to the line on the cursor, simply loosen the screws and set so that readings taken when using both sides of the rule will be accurate. Adjustment can be made to a total of .020″ for your convenience.

Care of the Rule: Clean the rule with plain soap and water. Do not use abrasive or volatile cleaning fluids. Remove the slide and clean it separately when cleaning, being sure the grooves do not retain water or foreign particles. Should you wish to lubricate the slide, use a minimum amount of silicone spray for the purpose. Do not use oil or graphite. The material used is a special plastic copolymer which retains its dimension and provides a bright finish for maximum legibility and appearance. We suggest that when not in use the rule be kept in its convenient scabbard.

BASIC NUMERICAL FUNCTIONS OF THE RULE:

Multiplication: Use C & D scales. Either end 1 is used as index on the slide. Place left 1 over 2 on D, and read 4 below 2 on C, also 6 below 3 on C, etc. All numbers along the scale D are 2 $\times$ the number on C above them, with this setting.

Division: Use C & D scales. Place 2 on the slide (C) over 4 on the D scale, read 2 on D scale, at the index of C. If you so desire, multiples of a proper fraction can be read directly.

Thus:				
(C)—	1	2	3	5
(D)—	2	4	6	10

Square or Square root: Use A & D scales on opposite side of rule. Disregard the slide, and using cursor only, bring line to 3 on D—against line read 9 on A; thus 9 is the square of 3. The square root of 9 is 3, etc. If number of digits in the squared number is **ODD,** use left half of A scale. If number of digits is **EVEN,** use right half. Square root of 25 (even) is 5 on D. Square root of 144 (odd) reads 12 on D. Square root of 9 (odd) reads 3 on D. To find the square root of a number less than 1.

Example: $\sqrt{.000144}$

Move the point four places to the right obtaining $\sqrt{1.44}$. Find $\sqrt{1.44}=1.2$. Move the point half the number of places to the left obtaining .012.

Cube or Cube root: Use K & D scales. Use the K & D scales in much the same way as the A & D scales are used for square root. For numbers from 1 to 10 set cursor on left K scale; if between 10 and 100 use center K scale; if between 100 and 1000 use right K scale; then read the values on the D scale. Against 27 on K read 3 on the D scale, similarly the cube of 3 is 27. To find the cube root of 0.0729, move the decimal point to the right three places; the resulting value, 72.9, is between 10 and 100, therefor the cursor is set to 72.9 on K center scale, and reading on the D scale is found 4.18. Since in the original number the decimal point was moved 3 places to the right, in the number from the D scale the decimal must be moved one place to the left giving 0.418.

Reciprocals: Against any scale, where shown as an I scale and italicized, the reciprocal of that scale can be read directly. Therefore, DI is the reciprocal of D scale, and CI is the reciprocal of C. D and DI are printed on the body for non-adjustable readings. C and CI appear on the slide so that if desired either one may be used in conjunction with D for multiplications by reciprocals or fractions. If the basic scale represents x, then the

reciprocal or I scale represents $\frac{1}{x}$ (see small letters at right of scale being used).

The decimal point is fixed by the rule that if a number which is not a power of 10 has x digits, then its reciprocal has 1-x digits. Therefor 37.2 has 2 digits, and its reciprocal has 1−2 digits or −1 digit, and is read .0269 (approx.).

The sine of an angle. If an angle is between 5.74° and 90°, its sine is between 0.1 and 1. The S scale is used for the sine of angles in this range.

Opposite the angle on S (regular numbers) read its sine on D. Put the decimal point before the first figure. Thus:

Opposite 14.5° on S, read 0.2504 on D.

Opposite 31.4° on S, read 0.5209 on D.

Opposite 58.5° on S, read 0.8526 on D.

If an angle is between 0.57° and 5.74°, its sine is between 0.01 and 0.1.

The ST scale is used for the sines of angles in this range.

Opposite the angle on ST, read its sine on D. Put one zero between the decimal point and the first significant figure. Thus,

Opposite 1.75° on ST, read 0.0305 on D.

Opposite 4.25° on ST, read 0.0741 on D.

The cosine of an angle. We find the cosine of an angle by reading the sine of its complement. Thus cos 40° = sin 50°, etc. In order to eliminate the necessity for subtracting the given angle from 90°, the complement of each angle on S is given by the italicized number. Thus the mark that is numbered 40° is also numbered 50° in italics. Hence: Opposite an angle on S (the italic scale) read its cosine on D. Put a decimal point at the left of the first significant figure.

Opposite 58° on S (italics), read cos 58° = 0.530 on D.

Opposite 53° on S (italics), read cos 53° = 0.602 on D.

The tangent of an angle. The T scales give the tangents and cotangents of angles between 5.71° and 84.29°. Cotangents are italicized.

For an angle between 5.71° and 45°, the tangent is between 0.1 and 1. Opposite the angle on T, read its tangent on D. Put the decimal point before the first figure. Thus,

Opposite 14° on T, read 0.249 on D.

Opposite 25.25° on T, read 0.472 on D.

If an angle is between 45° and 84.29° its tangent is between 1 and 10. In this range use the plain numbers on T. Put the decimal point after the first figure. Thus,

Opposite 58° on T, read 1.60 on D.

The cotangent of an angle is the reciprocal of its tangent. Cotangents can be read against D by using the reciprocal or reversed scale T.

Opposite 13° on T, read cot 13° = 4.33 on D.

Logarithms. Only the mantissa or decimal part of the common logarithm of a number is read from the slide rule. The characteristic is supplied by rules with which the reader is assumed to be familiar. The scales used are C and L.

Opposite any number on C read the mantissa of its common logarithm on L. Thus,

Opposite 17 on C, read .230 on L.
Opposite 326 on C, read .513 on L.
Opposite 728 on C, read .862 on L.

log 17 = 1.230; log 256 = 2.408; log 570 = 2.756.

Raising a number to a power without using LL scales. To find the value of N^x we must take log N, multiply by x, and then find the number having this last result for its logarithm.

Example 1. Evaluate $(23.5)^{1.37}$

Step 1. Opposite 235 on C read .371 on L. Then log 23.5 = 1.371.

Step 2. Multiply 1.37 × 1.371 = 1.878.

Step 3. Find the number whose logarithm is 1.878. Opposite .878 on L; read 755 on C. Since the characteristic is 1, the required number is 75.5.

Natural logarithms without using LL scales. Logarithms to the base e, where e = 2.71828, are called natural logarithms. We may denote the natural logarithm of a number N by symbol **ln N** in order to distinguish it from the common logarithm or logarithm to the base 10, which we denote by log N. The relation between these logarithms is

$$\ln N = 2.303 \log N.$$

Thus we find the natural logarithm of N by multiplying its common logarithm by 2.303.

Example. Find ln 48.5.

Step 1. Opposite 485 on D, read .686 on L. Then log 48.5 = 1.686.

Step 2. Multiply 2.303 × 1.686 = 3.88. Then ln 48.5 = 3.88.

The LL1-2-3 scales. As mentioned previously these are three sections of one long scale running from $e^{0.01}$ up to e^{10} as follows:

LL1 runs from $e^{0.01}$ = 1.011 to $e^{0.1}$ = 1.105
LL2 runs from $e^{0.1}$ = 1.105 to e = 2.718
LL3 runs from e = 2.718 to e^{10} = 22.026

The major difficulty in reading the LL scales results from the fact that the amount represented by a smallest division changes quickly. The whole LL1 scale covers only the range from 1.011 to 1.105. At its left end the smallest division represents 0.0001. The LL3 scale covers the range from 2.718 to 22.026; between 10,000 and 20,000 each smallest division represents 1000.

A direct use of these scales is to give values of e^x for values of x from 0.01 to 10:

Opposite x on D, read e^x on,

LL1 if x is between 0.01 and 0.1;
LL2 if x is between 0.1 and 1;
LL3 if x is between 1 and 10.

Example: Opposite 27 on D read $e^{2.7}$ = 14.9 on LL3
Opposite 58 on D read $e^{0.58}$ = 1.786 on LL2
Opposite 8 on D read $e^{0.08}$ = 1.0833 on LL1

Conversely, if we read from LL1-2-3 to D, we get the natural logarithms of numbers between 1.011 and 22.026:

Opposite N on LL1, 2 or 3, read log N or ln N on D. If N is on LL1, ln N is between 0.01 and 0.1; if N is on LL2, ln N is between 0.1 and 1; if N is on LL3, ln N is between 1, and 10. This fixes the decimal point.

Examples:

Opposite 9 on LL3, read ln 9 = 2.20 on D;
Opposite 75 on LL3, read ln 75 = 4.32 on D;
Opposite 1.7 on LL2, read ln 1.7 = 0.531 on D;
Opposite 1.06 on LL1, read ln 1.06 = 0.0583 on D.

The *LLO1-O2-O3* scales are italicized and show negative exponents. These are three sections of one long scale running from $e^{-0.01}$ to e^{-10} as follows:

LLO1 runs from $e^{-0.01}$ = 0.990 to $e^{-0.1}$ = 0.905;
LLO2 runs from $e^{-0.1}$ = 0.905 to e^{-1} = 0.368;
LLO3 runs from e^{-1} = 0.368 to e^{-10} = 0.00005.

It gives directly the values of e^x for x from 0.01 to 10 as follows:

If x is between 0.01 and 0.1: opposite x on D read e^{-x} on *LLO1*.
If x is between 0.1 and 1: opposite x on D read $^{-x}$ on *LLO2*.
If x is between 1 and 10: opposite x on D read e^{-x} on *LLO3*.

Examples:

Opposite 6 on D, read $e^{-0.06} = 0.9418$ on *LLO1*.

Opposite 6 on D, read $e^{-0.6} = 0.549$ on *LLO2*.

Opposite 6 on D, read $e^{-6} = 0.0025$ on *LLO3*.

Opposite 193 on D, read $e^{-1.93} = 0.145$ on *LLO3*.

By reading from *LLO1-O2-O3* to D one finds, conversely, the natural logarithms of numbers. Thus we found above that $e^{-1.93} = 0.145$. It follows that $\ln 0.145 = -1.93$. Of course we actually read only the numbers 193 on the slide rule and supply the negative sign and decimal point. The decimal point is fixed as follows:

If N is on LLO1 then ln N is between —.01 and —.1 so there is one zero between the decimal point and the first significant digit.

Example: (Opposite .9512 on *LLO1* read 5 on D. Then ln .9512 = —.05.)

If N is on *LLO2* then ln N is between —.1 and —1 so the decimal point immediately precedes the first significant digit.

Example: Opposite .81 is on *LLO2* read 211 on D. Then ln .81 = —.211.

If N is on *LLO3* then ln N is between —1 and —10 so the decimal point follows the first significant digit.

Example: Opposite .175 on *LLO3* read 174 on D. Then ln .175 = —1.74.

The Hyperbolic Functions. The hyperbolic functions sinh x and cosh x are defined as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2} ; \cosh x = \frac{e^x + e^{-x}}{2} .$$

They are thus just new names given to half the sum and half the difference of e^x and e^{-x}. In order to find the value of sinh x or cosh x for a given value of x one simply reads off the values of e^x and e^{-x} and then takes half the sum or half the difference of these numbers.

Example: To evaluate sinh x and cosh x for x = 1.6 we proceed as follows:

Opposite 1.6 on D, read $e^{1.6} = 4.95$ on LL3 and $e^{-1.6} = .202$ on **LLO3.** Then,

$\sinh 1.6 = ½ (4.95 - 0.20) = 2.38$

$\cosh 1.6 = ½ (495 + 0.20) = 2.58$

Note that in the final calculation we "rounded off" the reading because one of the numbers involved 4.95 could be read only to two decimal places.

Raising a number to a power using the LL scales. We have already seen that an expression of the form b^x can be evaluated by multiplying the common logarithm of b by x and then reading the antilogarithm. A more convenient method employs the LL scales as follows: If b is between 1.01 and 22.026, and if b^x lies also in this range, then,

Step 1. Opposite b on LL1, 2 or 3 set an index of C.

Step 2. Opposite x on C, read b^x on LL1, 2 or 3.

The LL1-2-3 group is here regarded as one long scale, LL1 running from 1.011 to 1.105, LL2 from 1.105 to 2.718, and LL3 from 2.718 to 22.026.

Example 1. Evaluate $(4.95)^{2.25}$.

Step 1. Opposite 4.95 on LL3, set left index of C.

Step 2. Opposite 2.25 on C read 36.55 on LL3.

In this case both the number 4.95 and the answer 36.55 are in the range covered by LL3.

Example 2. Evaluate $(1.9)^{3.16}$.

Step 1. Opposite 1.9 on LL2, set right index of C.

Step 2. Opposite 3.16 on C, read 7.59 on LL3.

In this case the number 1.9 is in the range covered by LL2 but the answer 7.59 is in that covered by LL3.

Example 3. Evaluate $(54)^{0.0355}$.

Step 1. Opposite 54 on LL3, set right index of C.

Step 2. Opposite 355 on C, read 1.155 on LL2.

Observe that the value of $(54)^{0.355} = 4.12$ would be read on LL3 with this same setting.

The *LLO1-O2-O3* combination is used with C in the same manner to find powers of decimal fractions between 0.00005 and 0.9905:

Example Evaluate $(0.84)^{1.95}$.

Step 1. Opposite 0.84 on *LLO2,* set the left index of C.

Step 2. Opposite 1.95 on C, read 0.712 on LLO2. Thus $(0.84)^{1.95} = 0.712$.

Observe that the value of $(0.84)^{19.5} = 0.33$ can be read on the *LLO3* opposite 19.5 on C with the same setting — and that $(0.84)^{1.95} = 0.9666$ can be read on *LLO1*.

SPECIAL GRADUATIONS

The marks $\pi/4$ appear at 0.7854 in the A and B scales. They are used in determining the areas of circles, the area being the diameter squared times $\pi/4$.

Example. If the diameter of a circle is 3 feet, set the right index opposite 3 of the D scale, slide cursor to $\pi/4$ (0.7854) mark of the B scale, read 7.07 on A scale.

The "minute" mark on the ST scale, is at 1.667°, and the "second" mark at 2.778°. The value of 1′ is read on D = 0.000291. The value of 1″ read on D = 0.00000485. To obtain the radian value for any given angle expressed in minutes or seconds, multiply the number of minutes or seconds times the number of radians in one minute, use gauge mark.

Example: 18′ in radians.

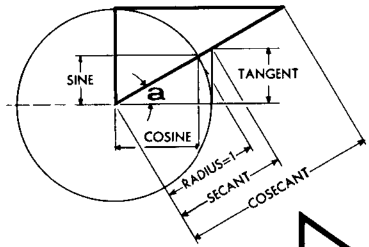
Set left index of slide at 18 on D.

Opposite "minute" mark on ST read 0.00523 radians on D.

Example: Find tan 23″.

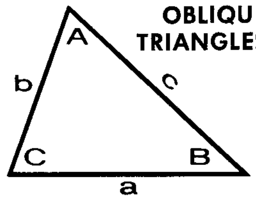
Set right index of slide at 23 on D.

Opposite "second" mark on ST read tan 23″ = 0.0001112 on D.



FORMULAS FOR FINDING FUNCTIONS OF ANGLES
$\frac{\text{Side opposite}}{\text{Hypotenuse}}$ = SINE
$\frac{\text{Side adjacent}}{\text{Hypotenuse}}$ = COSINE
$\frac{\text{Side opposite}}{\text{Side adjacent}}$ = TANGENT
$\frac{\text{Side adjacent}}{\text{Side opposite}}$ = COTANGENT
$\frac{\text{Hypotenuse}}{\text{Side adjacent}}$ = SECANT
$\frac{\text{Hypotenuse}}{\text{Side opposite}}$ = COSECANT

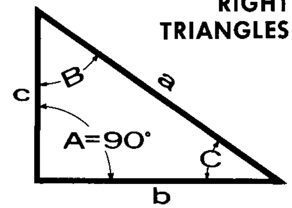
OBLIQUE TRIANGLES



TO FIND	KNOWN	SOLUTION
C	A-B	180° − (A + B)
b	a-B-A	$\frac{a \times \text{Sin. } B}{\text{Sin. } A}$
c	a-A-C	$\frac{a \times \text{Sin. } C}{\text{Sin. } A}$
Tan. A	a-C-b	$\frac{a \times \text{Sin. } C}{b - (a \times \text{Cos. } C)}$
B	A-C	180° − (A + C)
Sin. B	b-A-a	$\frac{b \times \text{Sin. } A}{a}$
A	B-C	180° − (B + C)
Cos. A	a-b-c	$\frac{b^2 + c^2 - a^2}{2bc}$
Sin. C	c-A-a	$\frac{c \times \text{Sin. } A}{a}$
Cot. B	a-C-b	$\frac{a \times \text{csc}C}{b} - \text{Cot. } C$
c	b-C-B	b × Sin. C × cscB

FORMULAS FOR FINDING THE LENGTH OF SIDES FOR RIGHT-ANGLE TRIANGLES WHEN AN ANGLE AND SIDE ARE KNOWN	
Length of side opposite	Hypotenuse × Sine Hypotenuse ÷ Cosecant Side adjacent × Tangent Side adjacent ÷ Cotangent
Length of side adjacent	Hypotenuse × Cosine Hypotenuse ÷ Secant Side opposite × Cotangent Side opposite ÷ Tangent
Length of Hypotenuse	Side opposite × Cosecant Side opposite ÷ Sine Side adjacent × Secant Side adjacent ÷ Cosine

RIGHT TRIANGLES



TO FIND ANGLES	FORMULAS	
C	$\frac{c}{a}$ = Sine C	90° − B
C	$\frac{b}{a}$ = Cosine C	90° − B
C	$\frac{c}{b}$ = Tan. C	90° − B
C	$\frac{b}{c}$ = Cotan. C	90° − B
C	$\frac{a}{b}$ = Secant C	90° − B
C	$\frac{a}{c}$ = Cosec. C	90° − B
B	$\frac{b}{a}$ = Sine B	90° − C
B	$\frac{c}{a}$ = Cosine B	90° − C
B	$\frac{b}{c}$ = Tan. B	90° − C
B	$\frac{c}{b}$ = Cotan. B	90° − C
B	$\frac{a}{c}$ = Secant B	90° − C
B	$\frac{a}{b}$ = Cosec. B	90° − C

TO FIND SIDES	FORMULAS	
a	$\sqrt{b^2 + c^2}$	
a	c × Cosec. C	$\frac{c}{\text{sine } C}$
a	c × Secant B	$\frac{c}{\text{Cosine } B}$
a	b × Cosec. B	$\frac{b}{\text{Sine } B}$
a	b × Secant C	$\frac{b}{\text{Cosine } C}$
b	$\sqrt{a^2 - c^2}$	
b	a × Sine B	$\frac{a}{\text{Cosecant } B}$
b	a × Cos. C	$\frac{a}{\text{Secant } C}$
b	c × Tan. B	$\frac{c}{\text{Cotangent } B}$
b	c × Cot. C	$\frac{c}{\text{Tangent } C}$
c	$\sqrt{a^2 - b^2}$	
c	a × Cos. B	$\frac{a}{\text{Secant } B}$
c	a × Sine C	$\frac{a}{\text{Cosecant } C}$
c	b × Cot. B	$\frac{b}{\text{Tangent } B}$
c	b × Tan. C	$\frac{b}{\text{Cotangent } C}$

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