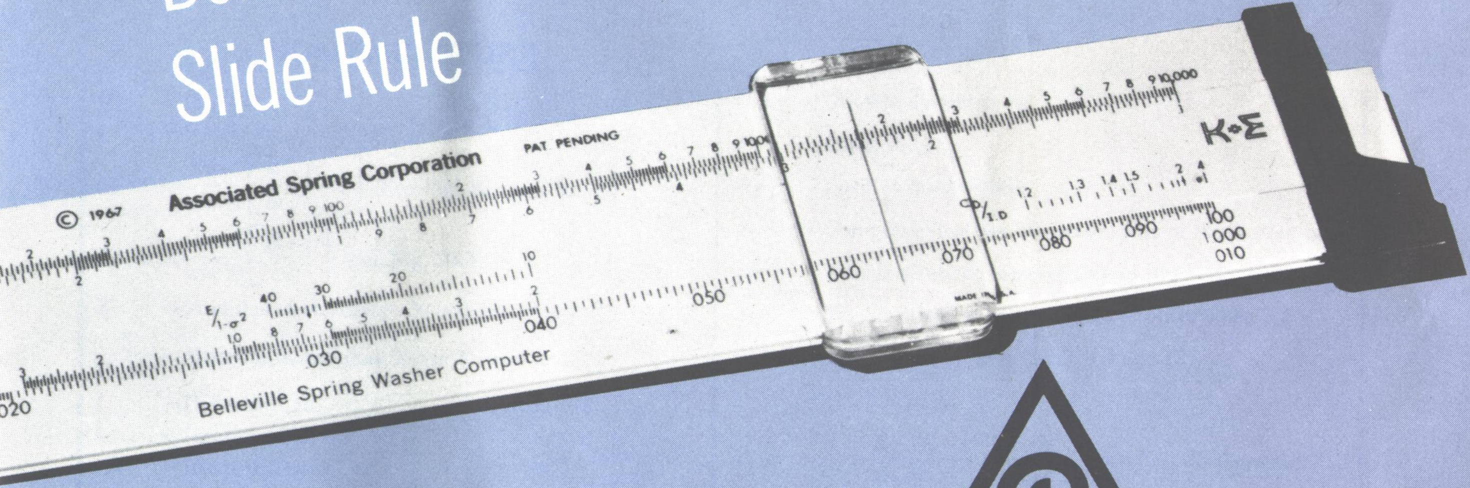
How to use the A.S.C. Belleville Spring Washer Slide Rule



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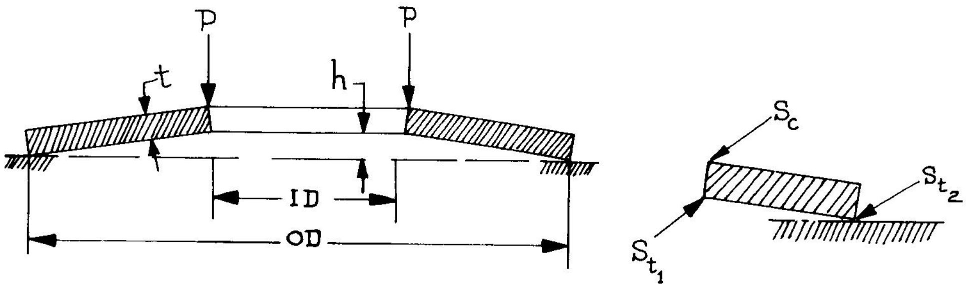


Figure 1

NOMENCLATURE

R $= OD/ID$

a $= OD/2$, in.

C_1 $= \dfrac{6}{\pi \ln R}\left[\dfrac{R-1}{\ln R}-1\right]$ (See Fig. 22 Page 9)

C_2 $= \dfrac{6}{\pi \ln R}\left[\dfrac{R-1}{2}\right]$ (See Fig. 22 Page 9)

E = Modulus of elasticity (30,000,000 for steel)

$\dfrac{E}{1-\sigma^2}$ = Material factor (See Table 1 Page 12)

f = Deflection, in.

h = Inside height, in.

ID = Inside diameter, in.

M $= \dfrac{6}{\pi \ln R}\ \dfrac{(R-1)^2}{R^2}$ (See Fig. 22 Page 9)

OD = Outside diameter, in.

P = Load, lb.

P_F = Load at flat position

S_c = Compressive stress on convex side of inner edge, psi

S_{t_1} = Tensile stress at concave side of inside diameter, psi

S_{t_2} = Tensile stress at concave side of outer edge, psi

σ = Poisson's ratio (See Table 1 Page 12)

t = Thickness, in.

T_1 $= \dfrac{R \ln R-(R-1)}{\ln R}\times\dfrac{R}{(R-1)^2}$ (See Fig. 23 Page 10)

T_2 $= \dfrac{0.5\,R}{R-1}$ (See Fig. 23 Page 10)

How to Use the A.S.C. Belleville Spring Washer Slide Rule

The A.S.C. Belleville Spring Washer Slide Rule is designed to simplify the design of this type of part. It deals with load at the flat position as computed by the following formula:

$$P = \frac{Ef}{(1-\sigma^2)\ Ma^2}\ [(h-f)\ (h-f/2)\ t+t^3]$$

when $h=f$

$$P_F = \frac{Ef\ t^3}{(1-\sigma^2)\ Ma^2}$$

Loads at intermediate deflections should be determined from the typical load/deflection curves shown in Fig. 2.

The principal scales are based on steel and a ratio of OD/ID of 2. Auxiliary scales are shown for converting to other materials and other ratios of OD/ID. Thickness is labeled from 0.010 in. to 0.100 in., but the scales may be read from 0.100 to 1.0 or 0.001 to 0.010 by

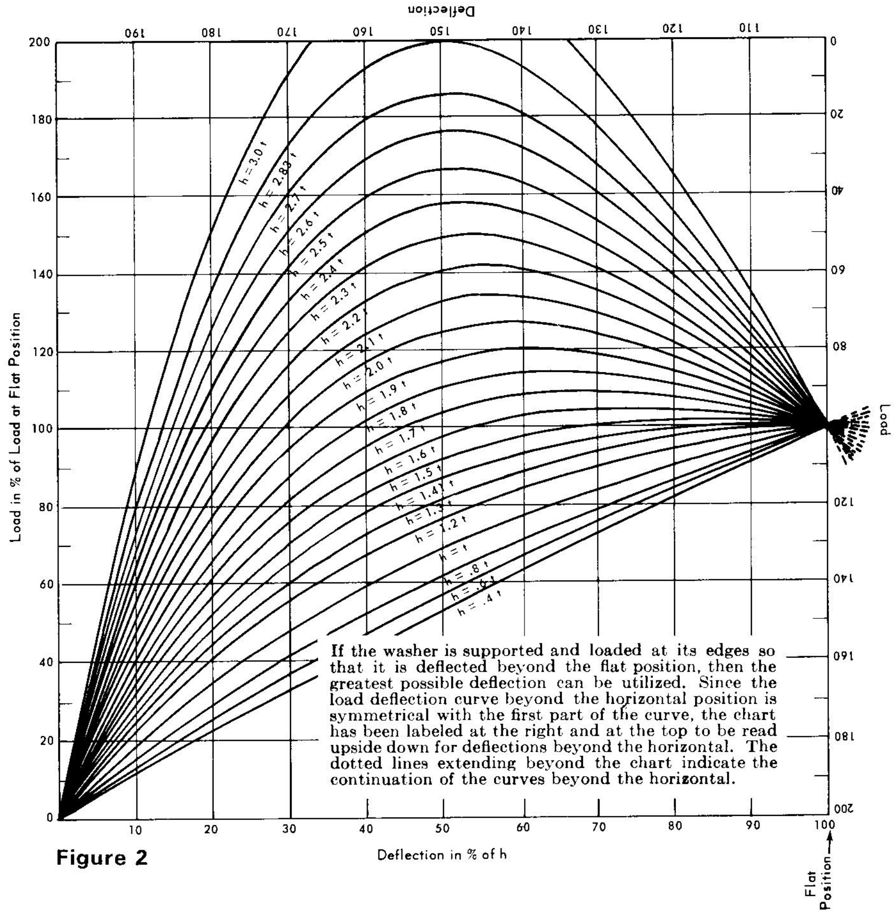


Figure 2

moving the slide its full length in either direction.

The stress side of the slide rule also deals with stress at the flat position. The scale printed in green solves the formula for compressive stress at the convex side of the inside diameter.

$$S_c = \frac{Ef}{(1-\sigma^2)\ Ma^2}\ [C_1\ (h-f/2)+C_2t]$$

The formula for tensile stress at the concave side of the inside diameter is:

$$S_{t_1} = \frac{Ef}{(1-\sigma^2)\ Ma^2}\ [C_1\ (h-f/2)-C_2t]$$

The value for this stress at flat may be taken from Fig. 3 after finding the value of S_c on the slide rule.

The scales in red solve the formula for the tensile stress at the concave side of the outside diameter.

$$S_{t_2} = \frac{Ef}{(1-\sigma^2)\ a^2}\ T_1\ (h-f/2)+T_2\, t$$

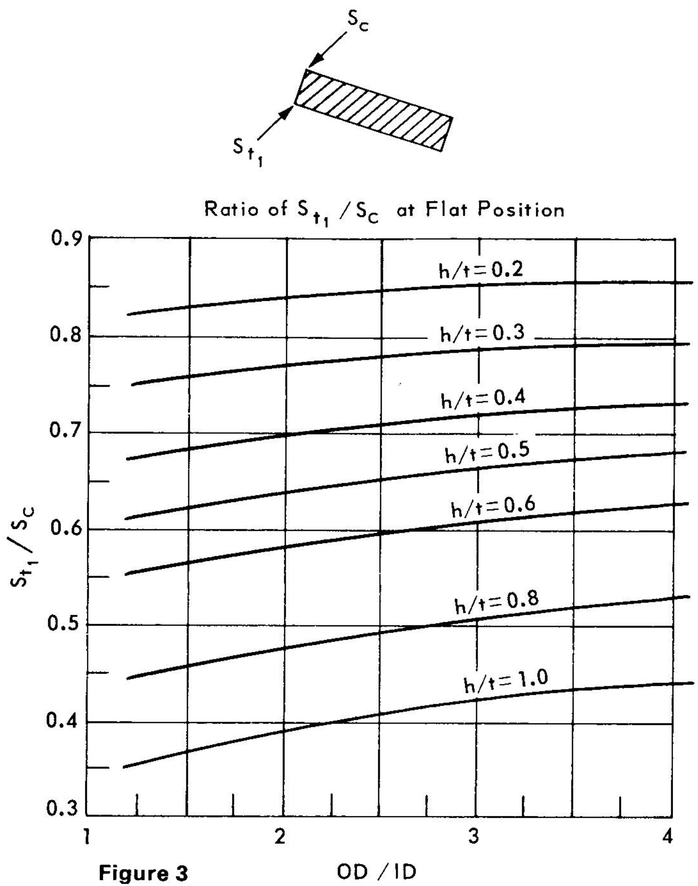


Figure 3

Special Notations

On the OD/ID scales on the thickness side of the slide rule, the dot between 2 and 4 represents 2.5. This notation is used because the line for 3.0 would be practically superimposed on 4.

On the green OD/ID scale on the stress side there are three lines between 1.3 and 1.7, representing 1.4, 1.5, and 1.6. Between 1.7 and 2 there are two lines representing 1.8 and 1.9. A magnified illustration (Fig. 4) shows the numbering of this scale.

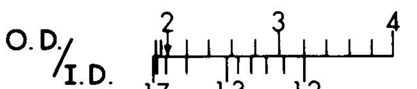


Figure 4

If the slide is removed, it should be returned to position so that the two red half circles match to make a complete circle.

Each of the auxiliary scales has an arrow to show the value of the factor on which the principal scales are designed. For example, the arrow at 2 on the OD/ID scales shows that the formulas will be solved for this ratio if no adjustment is made on this scale.

The following examples illustrate the use of the slide rule.

Example 1. $OD/ID = 2$; material—steel

Given: $OD = 1.0$ in. $ID = 0.5$ in.
$t = 0.050$ in. $h = 0.025$ in.

Find: load at flat position
stress at flat position

Solution: $h/t = \dfrac{0.025}{0.050} = 0.5$

On the thickness scale, set 0.5 on h/t scale over 0.050 (Fig. 5). Opposite 1.0 on the OD scale, read 600 on the load flat scale.

On the stress side, set 1.0 on the OD scale opposite 600 on the load flat scale. Opposite 0.5 on the green h/t scale, read 405,000 on the stress scale (Fig. 6). This is the compressive stress.

Determine S_{t_1} from Fig. 3. From the intersection of the line for $OD/ID = 2$ and the curve for $h/t = 0.5$, read at the left $S_{t_1}/S_c = 0.637$. Then $S_{t_1} = \dfrac{S_{t_1}}{S_c} \times S_c = 0.637 \times 405{,}000 = 268{,}000$ psi.

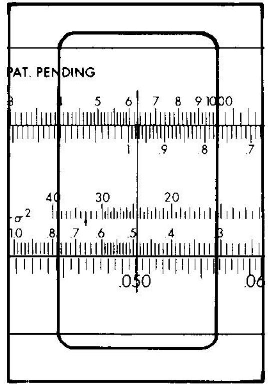


Figure 5

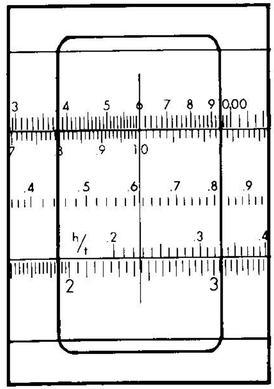


Figure 6

The tensile stress as shown by S_{t_2} in Fig. 1 is computed by using the scales with the red figures. Set 1.0 on the OD scale opposite 600 on the load scale (Fig. 7). Opposite 0.5 on the red h/t scale read 210,000 psi on the stress scale.

Example 2.

Given: $OD = 1.50$ in. $ID = 1.25$ in.
$t = 0.050$ in. $h = 0.025$ in.

Find: load at flat position
stress at flat position

Solution: $OD/ID = 1.5/1.25 = 1.2$
$h/t = 0.025/0.050 = 0.5$

On the thickness scale, set 0.5 on h/t scale opposite 0.050 (Fig. 8). Set the hair line on 2 on OD/ID scale (Fig. 9). Move the slide to 1.2 under the hair line (Fig. 10). Opposite 1.5 on the OD scale read 630 on the load flat scale.

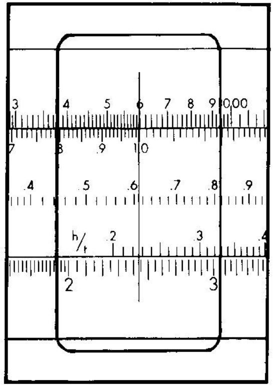


Figure 7

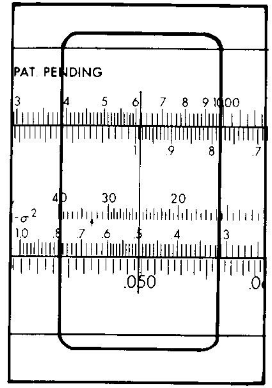


Figure 8

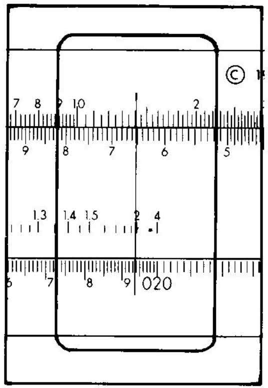


Figure 9

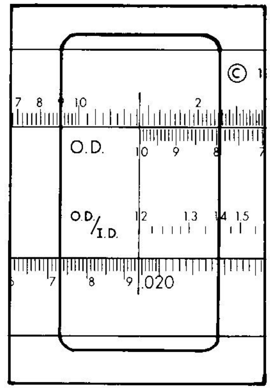


Figure 10

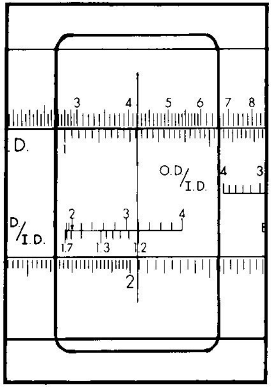


Figure 11

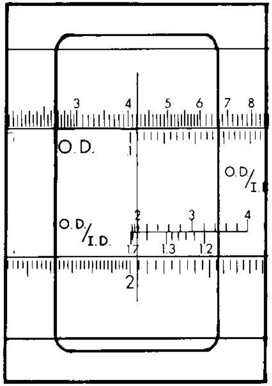


Figure 12

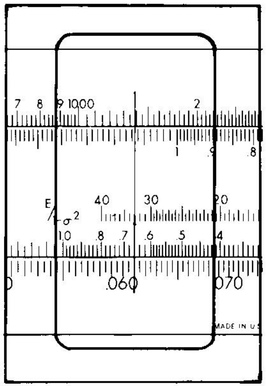


Figure 13

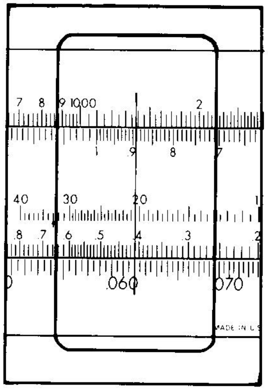


Figure 14

On the stress side, set 1.5 on the *OD* scale opposite 630 on the load flat scale. Set hair line at 1.2 on the green *OD*/*ID* scale (Fig. 11). Move the slide so that 2 is under the hair line (Fig. 12). Opposite 0.5 on the green *h*/*t* scale read 333,000 on the stress scale.

Note that the slide is moved from $OD/ID =$ 2 to the new value when progressing from lower to upper scales, and it is moved from the new value back to 2 when progressing from upper to lower scales. *OD*/*ID* scales are shown at each end of the slide so that one scale will always be in position to see under the hair line.

Example 3. $OD/ID = 2$; material other than steel

Given: $OD = 2$ in. $ID = 1$ in.
$t = 0.052$ in. $h = 0.065$ in.
Material—beryllium copper

Find: load at flat position
stress at flat position

Solution: $h/t = 0.065/0.052 = 1.25$

$$\frac{E}{1-\sigma^2} = 20.8$$ (See Table I Page 12)

On the thickness scale set 1.25 on the *h*/*t* scale opposite 0.052. Set the hair line over 33 on the $\frac{E}{1-\sigma^2}$ scale. (Fig. 13). Move the slide so that 20.8 is under the hair line (Fig. 14). Opposite 2 on the *OD* scale read 277 on the load flat scale.

On the stress side, set 2 on the *OD* scale opposite 277 on the load scale. Set the hair line over 20.8 on the $\frac{E}{1-\sigma^2}$ scale (Fig. 15). Move the slide so that 33 is under the hair line (Fig. 16). Opposite 1.25 on the green *h/t* scale read 219,000 on the stress scale.

If the $\frac{E}{1-\sigma^2}$ value is off the slide rule, read the value of stress for steel, then move the slide and make the correction. In this example, set the hair line over 1.25 on the *h/t* scale opposite a stress of 275,000 for steel (Fig. 17). Move the slide so that 33 on the $\frac{E}{1-\sigma^2}$ scale is under the hair line (Fig. 18). Then read the stress opposite 20.8.

The same scale for $\frac{E}{1-\sigma^2}$ is used in conjunction with the red scales for computing the tensile stress.

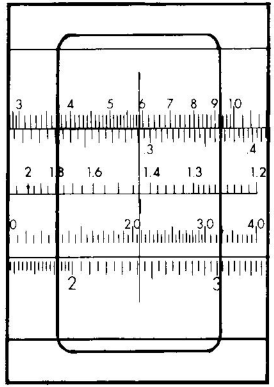


Figure 15

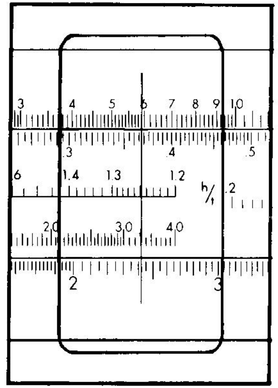


Figure 16

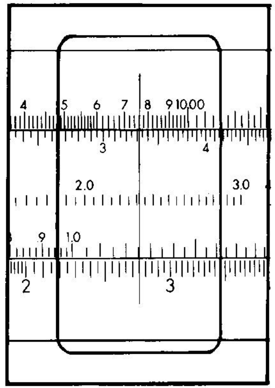


Figure 17

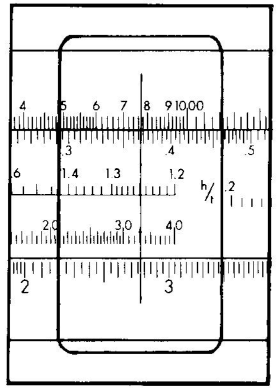


Figure 18

Example 4. Thickness outside of the range 0.010 in. to 0.100 in.

Given: $OD = 6.0$ in. $ID = 3.0$ in.
$h/t =$ approximately 0.4 in.
Load flat = 8,000 lb.
Material—steel

Find: Thickness

Solution:

On the thickness side, set 6 on the OD scale opposite 8,000 on the load flat scale. Set the hair line over 10 on the OD scale (Fig. 19). Move the slide so that 0.1 is under the hair line (Fig. 20). Opposite 0.4 on the h/t scale read 0.0247 but change this to 0.247 because moving the slide to the left was equivalent to extending the thickness scale one cycle to the right (Fig. 21). With a thickness of 0.250 the ratio of h/t would be 3.8.

Example 5. Load at intermediate position

Given: Load at 50% of deflection is 150 lb.
$h/t = 1.3$

Find: Load at flat position

Solution: From Fig. 2 load at 50% of deflection is 82% of load at flat

$$\text{Load at flat} = \frac{150}{0.82} = 183 \text{ lb.}$$

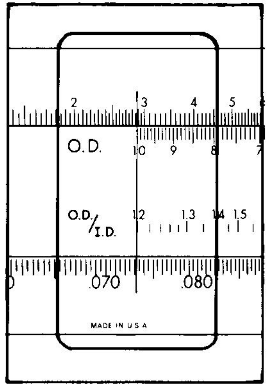


Figure 19

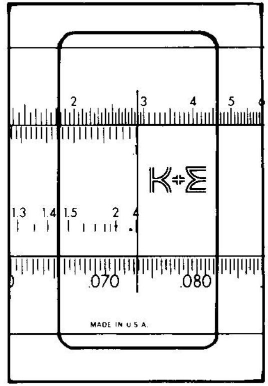


Figure 20

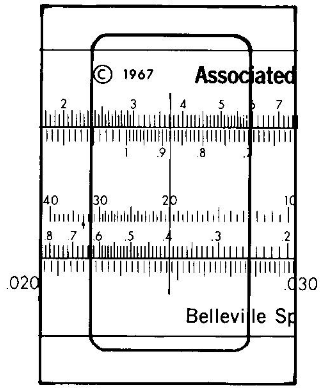


Figure 21

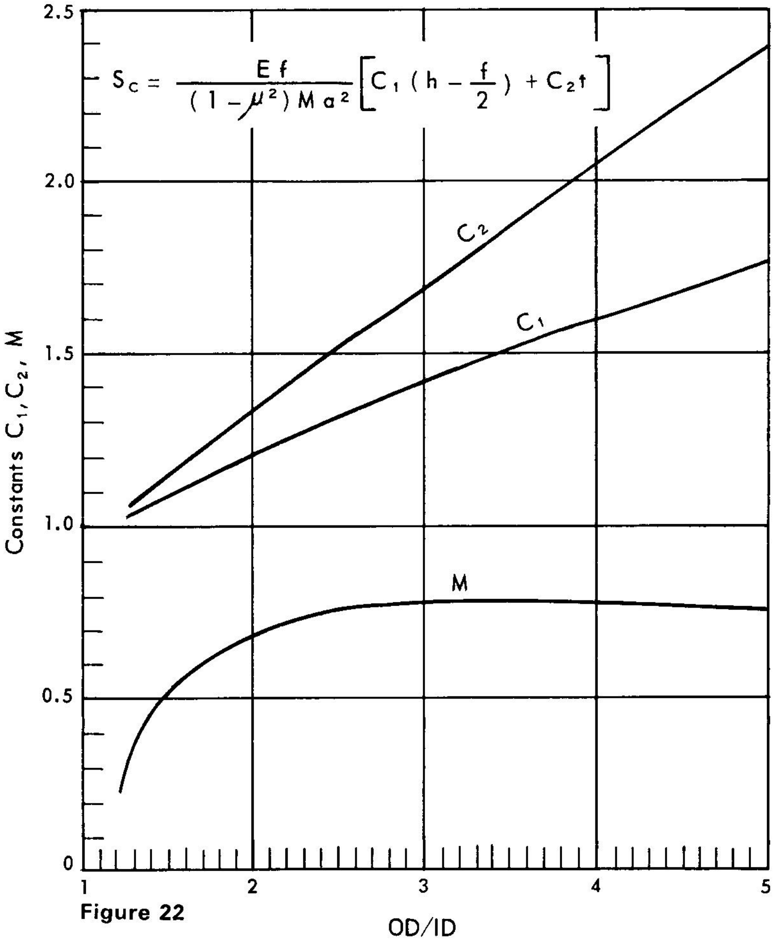


Figure 22

Stress Levels

Static Applications

Stress level for static application is judged according to the formula for S_c which is determined by using the green figures on the slide rule or the formula for S_c. A belleville spring washer should be designed so it can be compressed to the flat position without setting. This can be accomplished by either using such a low stress that the spring will not set, or making the spring higher than the design height and taking out set by compressing flat. The following stress levels are suitable for belleville spring washers made of carbon steel or alloy steel.

	S_c
Will not set more than 2%	250,000
Maximum apparent stress with all set out	700,000

These values should be reduced for plated parts, elevated temperatures, or for other materials.

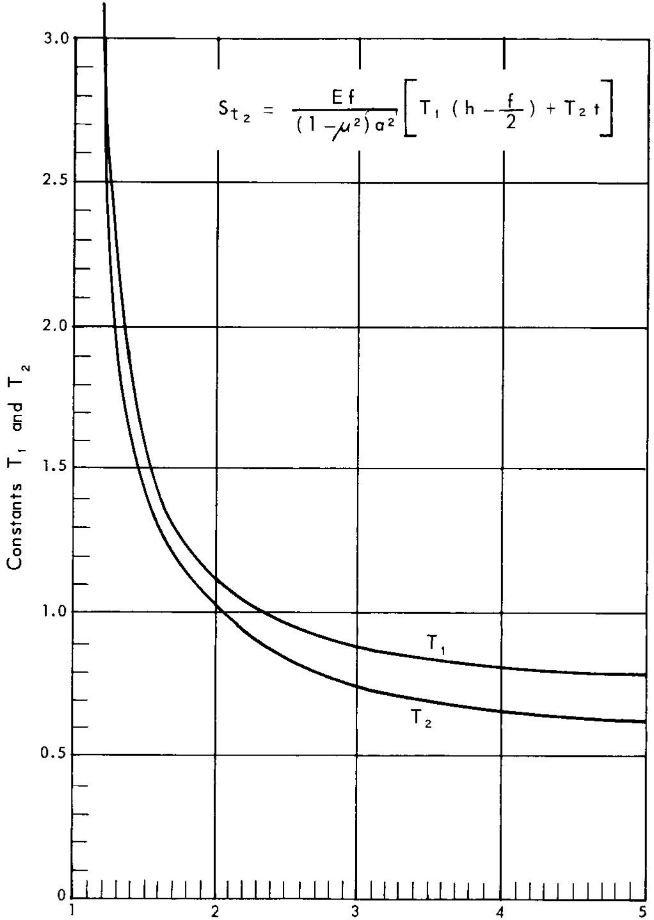


Figure 23

Fatigue Applications

Stress level for fatigue application is judged according to the tensile stresses S_{t_1} and S_{t_2} as indicated in Fig. 1. The value for S_{t_1} at the flat position may be taken from Fig. 3 after finding S_c on the slide rule. The value for S_{t_2} is read directly from the slide

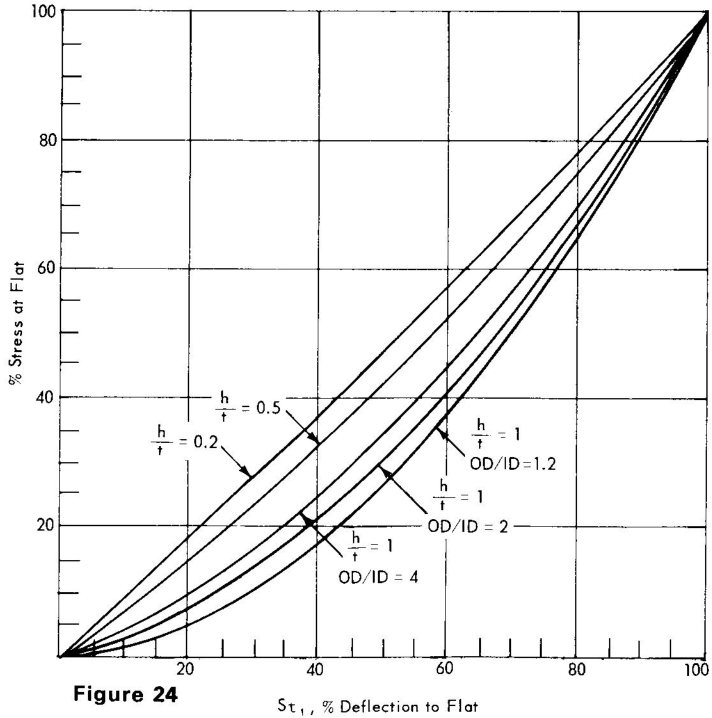


Figure 24

rule by using the red scales. Stresses at intermediate deflections may be taken from Fig. 24 for S_{t_1} or Fig. 25 for S_{t_2}. The effect of OD/ID in these charts is insignificant except in Fig. 24, for $h/t = 1$, where three curves are shown for three different ratios of OD/ID.

Fatigue life depends on either S_{t_1} or S_{t_2} according to which is numerically higher. The relation of these depends on deflection as well as h/t and OD/ID. Fig. 26 enables the designer to determine which will be greater in a given design. In general, S_{t_1} is higher for

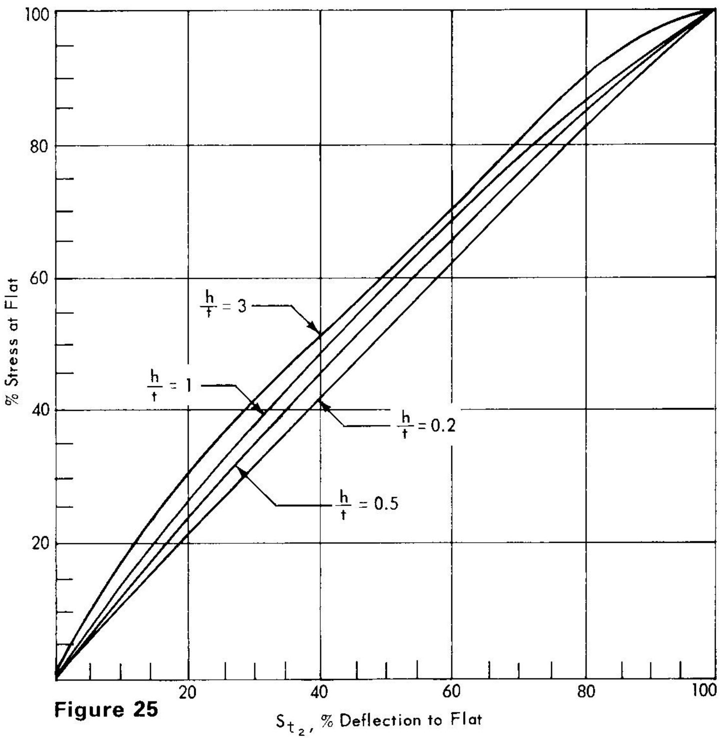


Figure 25

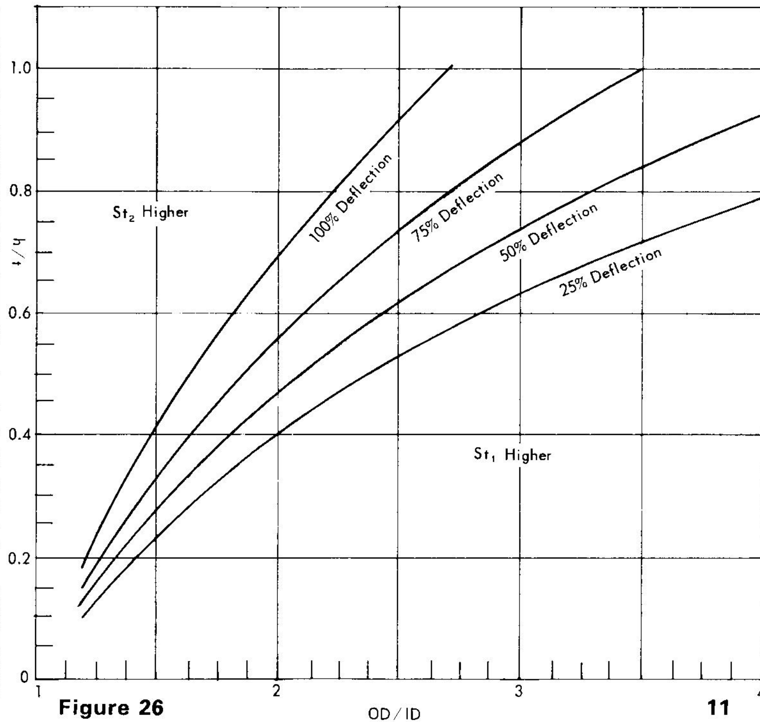


Figure 26

small ratios of h/t and large ratios of OD/ID. Note that it is possible for the location of highest stress to change from one position to the other at various stages of deflection.

Fig. 27 shows a chart for judging fatigue life vs. tensile stress. The following example will illustrate the use of these charts and stress data:

Given:

$OD = 0.750$ in., $ID = 0.375$ in.,
$t = 0.028$ in., $h = 0.023$ in.

Is it necessary and possible to take out set? Will this spring stand up for a million cycles if deflected a minor deflection of 0.005 in. and a major deflection of 0.012 in?

Solution:

$OD/ID = 2$ $\qquad h/t = 0.82$

Using the green scales, the compressive stress S_c is 410,000 psi at flat. Therefore, the spring would set at flat if not set out. It is possible to take out set as indicated on page 9.

Examine Fig. 26 to see which is the higher—S_{t_1} or S_{t_2}. With $OD/ID = 2$ and $h/t = 0.82$, S_{t_2} will be higher because the intersection of these values lies to the left of all stages of deflection. Compute S_{t_2} at flat using the red scales. S_{t_2} at flat $= 220{,}000$ psi. Now use Fig. 25 to find the intermediate stresses—0.005 in. and 0.012 in. are 22% and 52%, respectively, of the deflection to flat. These correspond to 27% and 60%, respectively, of the stress at flat.

Minor stress $S_{t_2} = 0.27 \times 220{,}000 =$ 60,000 psi

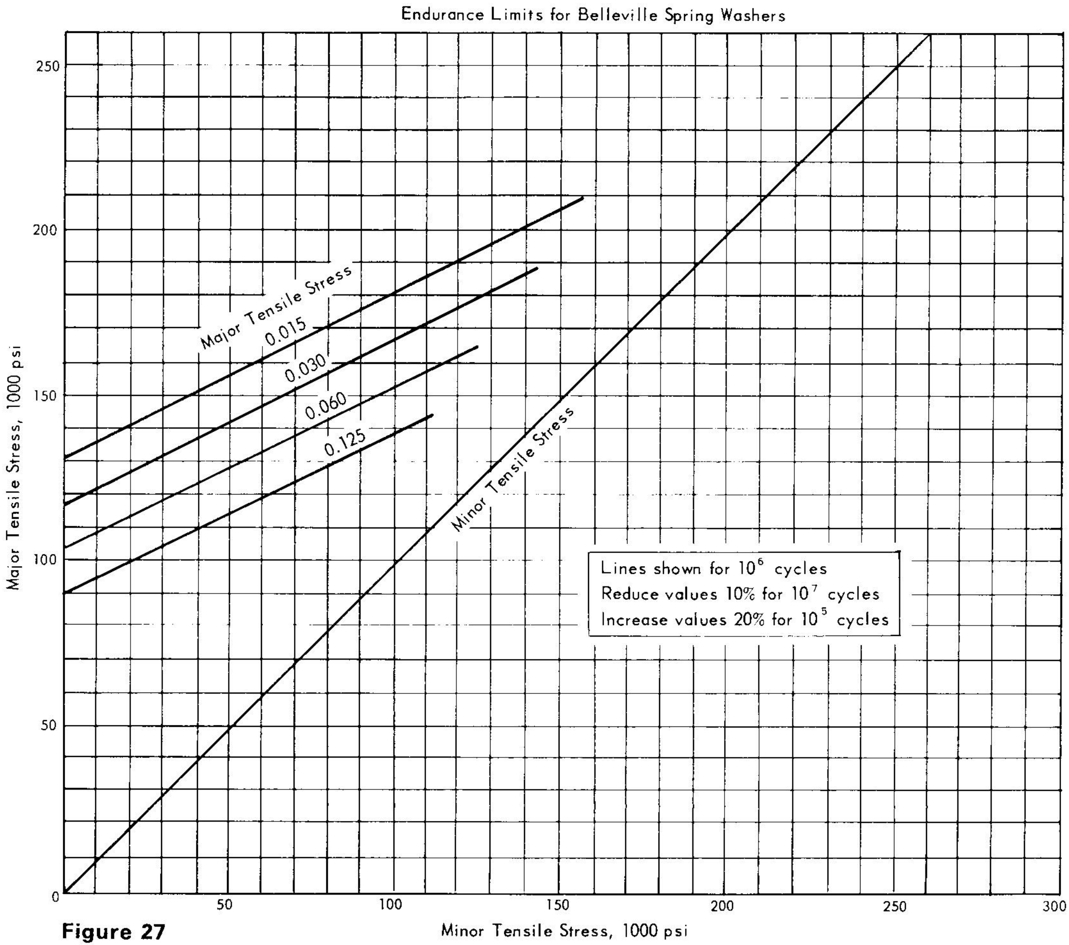
Major stress $S_{t_2} = 0.60 \times 220{,}000 =$ 132,000 psi

Now refer to Fig. 27. Plot the 60,000 point on the diagonal line labeled minor tensile stress. Draw a vertical line to the intersection of 130,000 on the major stress axis. This represents the stress range and is below the limiting line for a thickness of 0.028 in. It is, therefore, okay for a life of 1,000,000 cycles.

Table I—Values for $\dfrac{E}{1-\sigma^2}$

Metal	E	σ	$\dfrac{E}{1-\sigma^2}$
Steel	30×10^6*	0.30	33×10^6*
Phosphor bronze	15×10^6	0.20	15.6×10^6
17-7 PH stainless	29×10^6	0.34	33.0×10^6
302 stainless	28×10^6	0.30	30.8×10^6
Beryllium copper	18.5×10^6	0.33	20.8×10^6
Inconel	31×10^6	0.29	33.8×10^6
Inconel X	31×10^6	0.29	33.8×10^6

*The factor 10^6 is omitted on the slide rule.



Endurance Limits for Belleville Spring Washers
Major Tensile Stress, 1000 psi
250
200
150
100
50
0
50
100
150
200
250
300
Minor Tensile Stress, 1000 psi
Major Tensile Stress
0.015
0.030
0.060
0.125
Minor Tensile Stress
Lines shown for 10^6 cycles
Reduce values 10% for 10^7 cycles
Increase values 20% for 10^5 cycles

Figure 27