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EVERYTHING FOR DRAFTING
SURVEYING & PRINTMAKING

Self-teaching

INSTRUCTION

MANUAL

NO. 1749

MANIPHASE MULTIPLEX

DUPLEX TYPE

METAL Style M

Slide Rule

IMPROVED
DECIMAL TRIG TYPE
MANIPHASE SLIDE RULE

PUBLISHED BY

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METAL STYLE "M"

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CHAPTER I
THEORY AND CONSTRUCTION

1. Introduction. A slide rule is a mechanical device for rapidly and accurately making calculations. Although to a beginner, even the simplest scale arrangement may appear very complicated, a slide rule is actually a relatively easy instrument to use.

Problems involving multiplication, division, proportions, square roots and cube roots can be solved by means of a slide rule, regardless of one's mathematical background.

Although the principle of a slide rule is based on logarithms, and a knowledge of mathematics is necessary to know **WHY** a slide rule works, it is not necessary to have any special knowledge of mathematics to be able to **use** a slide rule for many of its simpler applications.

This manual has been written to serve as a self-teaching manual on the operation of a slide rule. By studying the text and examples, the user can become proficient in the operation of a slide rule without the aid of a teacher or other assistance.

The user should have his slide rule before him while reading the text and should work the Examples given, as well as the practice Exercises. There is no substitute for practice in learning to operate a slide rule accurately, rapidly and efficiently.

2. General Description and Theory. There are three main parts to a slide rule, **Fig. 1:**

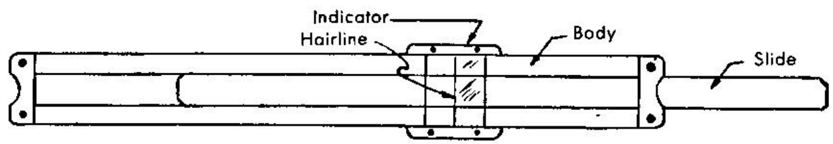

Fig. 1

(1) The Body.
(2) The Slide.
(3) The Indicator, which has a fine hairline etched on the glass.

On both sides of the Body and Slide there are many scales—each with a specific purpose. Each of these scales will be explained in subsequent chapters, and the user should concern himself only with the scale, or scales, being explained for the particular application he is studying.

The theory of the slide rule is based on logarithms. Scales are divided logarithmically and the distance to any number on the scale is in proportion to logarithm of that number. In **Multiplication** by logarithms, the logarithms of the numbers are added. In **Division** by logarithms, the logarithms of the numbers are subtracted. The slide rule performs the required addition or subtraction of logarithms **mechanically,** thereby saving time and work. It should be understood, however, that ordinary addition or subtraction of numbers cannot be performed on a slide rule.

3. How to Read the Scale. Reading a Slide Rule is no more difficult than reading a ruler. An explanation of how to read the "D" scale will be made, since, with this knowledge the user will be able to read any of the other scales.

In the operation of a slide rule, the user is concerned entirely with significant figures. To illustrate, the significant figures of the numbers 3.86, .000386, 38,600 and 0.00386 are all the same, namely 3—8—6; **and** the setting on the slide rule would be the same for all of these numbers.

Slide rules can be read accurately to three significant figures, and the fourth significant figure can be estimated with reasonable accuracy.

To illustrate the reading of the "D" scale to three significant figures, the number 3.86 will be considered.

FIRST STEP: The "D" scale on a slide rule is divided into ten major divisions, numbered from 1 to 10. **Fig. 2.** If, in the example, the first significant figure were 1, the number would lay between 1 and 2; if the first significant figure were 2, the number would lay between 2 and 3, etc. In our example, the first significant figure is 3, therefore, the number lays between 3 and 4. Fig. 2.

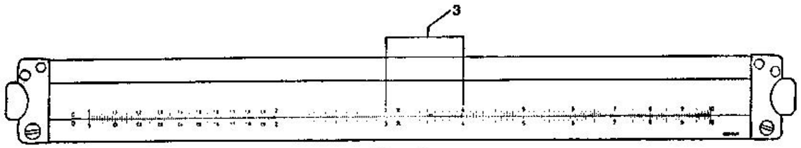

Fig. 2

SECOND STEP: Each of the ten major divisions are sub-divided into ten secondary divisions, thereby giving the second significant figure. Fig. 3. In the number 386, the second significant figure is 8. Therefore, the number in the example will lay between the eighth and ninth secondary divisions. Fig. 3.

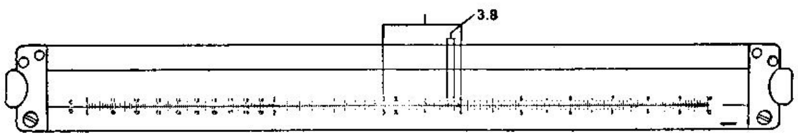

Fig. 3

THIRD STEP: Each of the secondary divisions are again divided into a third set of divisions (tertiary divisions), thereby giving the third significant figure. Fig. 4. In this example it will be noted there are five tertiary divisions between the secondary divisions representing 8 and 9. This means the value of each tertiary division is 2. Therefore, the third significant figure- -6—is represented by the third tertiary division marking. Fig. 4.

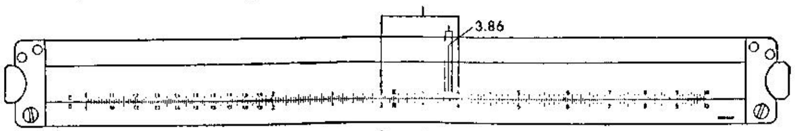

Fig. 4

It should be noted that there are three sections of the scale where the manner of sub-dividing between the prime numbers (the first significant figures) is different, i.e., between 1 and 2; 2 and 4; and 4 and 10.

Between the prime numbers 1 and 2, the secondary divisions are sub-divided into ten tertiary divisions. This means each tertiary division has a value of one. **Fig.** 5.

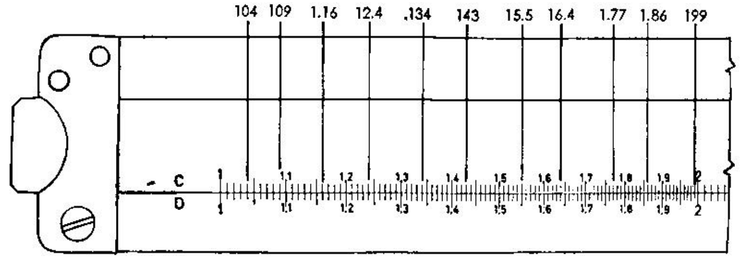

Fig. 5

Between prime numbers 2 and 4, the secondary divisions are sub-divided into five tertiary divisions. This means each tertiary division has a value of two. Fig. 6.

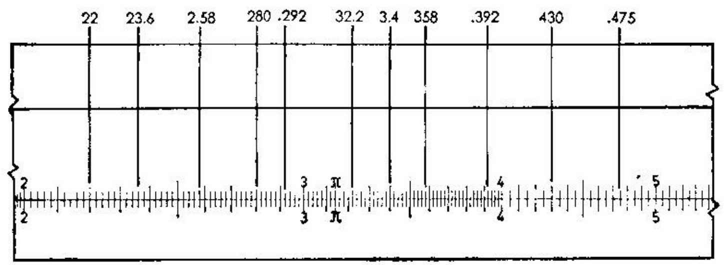

Fig. 6

Between the prime numbers 4 to 10, the secondary divisions are sub-divided into two tertiary divisions. This means each tertiary division has a value of five. Fig. 7.

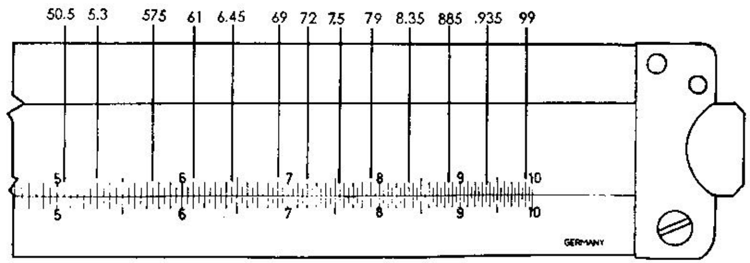

Fig. 7

Regardless of the value of the tertiary divisions, the **method** of reading the scale is the same as explained above.

CHAPTER II
MULTIPLICATION AND DIVISION

4. Multiplication. As previously stated, multiplication is performed on a slide rule by **mechanically** adding the logarithms of the numbers to be multiplied together.

Generally, multiplication is performed on the "C" and "D" scales. The number "1" on the left end is called the "Left Index" and the number "10" on the right end is called the "Right Index."

EXAMPLE: Multiply 2 × 3.

Set the Left Index on "C" to 2 on **"D."**
Move the hairline to 3 on "C."
Under the hairline read the answer **6** on "D."

EXAMPLE: Multiply 15 × 5.

Set the Left Index of "C" to 15 on "D."
Move the hairline to 5 on "C."
Under the hairline read the answer **75** on "D."

EXAMPLE: Multiply 15.5 × 5.4.

Disregard the decimal point and set the Left Index of "C" to 155 on "D."
Move the hairline to 54 on "C."
Under the hairline read **837** on "D."
To determine the location of the decimal point, approximate the answer by mentally noting 16 × 5 = 80. Hence, the answer in the example is **83.7.**

Rule for Multiplication. Set the Index of the "C" scale over either of the factors on the "D" scale. Move the hairline on the indicator to the second factor on the "C" scale. Read the answer on the "D" scale under the hairline. Determine the location of the decimal point by rough mental calculation.

To place the decimal point, "round off" the numbers to be multiplied, to the nearest, easily multiplicable number and mentally multiply the two "rounded off" numbers.

In multiplication, either Index may be used, and the determination of which Index to use is made by simply using whichever Index will place the answer on the rule.

EXAMPLE: Multiply 2 × 8.

If the Left Index of "C" is set to 2 on "D," the number 8 on "C" will be off the Rule.
Therefore, to multiply 2 × 8, set the Right Index of "C" to 8 on "D."
Move the hairline to 2 on "C."
Under the hairline on "D" read the answer **16.**

5. Multiplication of Three or More Factors. One important advantage of a slide rule is that the product of any number of factors can be obtained by one continuous multiplication operation.

EXAMPLE: Multiply 13.5 × 54.6 × 0.024.

The first two factors are multiplied together as previously explained, that is:
Set the Left Index of "C" to **13.5** on "D."
Move the hairline to 54.6 on **"C."**
Leave the Indicator where it is and bring the Right Index of "C" under the hairline.
Move the hairline to 0.024, on "C."
Under the hairline on "D" read the answer **17.69.**

The procedure is the same regardless of the number of factors involved, and since only the final result is required, it is not necessary to record any intermediate products.

EXERCISES

1.	2 × 4	5.	1.76 × 4.3	9.	7.15 × 1.58
2.	1.5 × 3	6.	2.12 × 4.15	10.	12 × 4 × 3
3.	5 × 2	7.	22.6 × 3.48	11.	10.5 × 3.6 × 7.05
4.	2 × 4.15	8.	8 × 3	12.	7.85 × 2.41 × 2.06

ANSWERS TO ABOVE EXERCISES

1.	8	5.	7.56	9.	11.3
2.	4.5	6.	8.8	10.	144
3.	10	7.	78.6	11.	266.5
4.	8.30	8.	24	12.	39.00

6. Division. As previously stated, division is performed on a slide rule by **mechanically** subtracting the logarithms of respective numbers involved.

As in multiplication, division is generally performed on the "C" and "D" scales.

EXAMPLE: Divide 6 × 3.

Set the hairline over 6 on "D."
Bring 3, on "C," under the hairline.
Read the answer **2** under the Left Index of "C."

Again as in multiplication, either Index of the "C" scale may be used, and whichever Index is within the body of the rule, is the one which is used.

EXAMPLE: Divide 600 × 9.4.

Set the hairline over 600 on "D."
Bring 9.4, on "C," under the hairline.
Move the hairline to the Right Index on "C."
Under the hairline read 638 on "D."

To locate the decimal point approximate the answer by mentally noting $\frac{600}{10} = 60.$

Therefore, the answer is **63.8.**

Rule for Division. Set the numerator (the dividend) on the "D" scale and bring the denominator (divisor) on the "C" scale opposite to it. Under the Index on the "C" scale read the answer on the "D" scale. Determine the position of the decimal point by rough mental calculation.

EXERCISES

1.	16 ÷ 4	5.	0.64 ÷ 0.15	9.	16.4 ÷ 0.45
2.	8.65 ÷ 3.7	6.	847 ÷ 13	10.	955 ÷ 54
3.	.47 ÷ 2.2	7.	10,500 ÷ 456	11.	2.7 ÷ 6.5
4.	3.75 ÷ .3	8.	1820 ÷ 4630	12.	18.3 ÷ 16.4

ANSWERS TO ABOVE EXERCISES

1.	4	5.	4.27	9.	36.45
2.	2.34	6.	65.2	10.	17.69
3.	.2137	7.	23.03	11.	.416
4.	12.5	8.	.3934	12.	1.115

7. Problems Involving Both Multiplication and Division. Another distinct advantage of a slide rule is the speed with which problems involving both multiplication and division can be solved. The best way to approach a problem of this type is to perform alternately, first a division, then a multiplication, then a division, then a multiplication until the problem is solved.

EXAMPLE: Find the value of $\dfrac{3.2 \times 64.}{2.46}$

As pointed out above, the reasoning on a problem of this type is to first divide 3.2 by 2.46 and then multiply the result by 64.

To accomplish this:
Move the hairline to 3.2 on "D."
Bring 2.46 on "C" under the hairline.
Move the hairline to 64 on "C."
Under the hairline read **8325** on "D."
To locate the decimal point approximate the answer by mentally noting $\dfrac{3 \times 60}{2} = .90.$
Therefore, the answer is **83.25.**

The fundamental procedure for combined operations is the same regardless of the number of factors involved.

EXAMPLE: Evaluate $\dfrac{3.46 \times 6.8 \times 4.}{4.4 \times 9}$

Move the hairline to 3.46 on "D."
Bring 4.4 on "C" under the hairline.
Move the hairline to 6.8 on "C."
Bring 9 on "C" under the hairline.
Move the hairline to 4 on "C."
Under the hairline read 2379 on "D."
Locate the decimal point by noting $\dfrac{3 \times 7 \times 4}{4 \times 9} = 2$ approx.
Therefore, the answer is **2.379.**

If there are more factors in the denominator than in the numerator, the problem is still performed by alternately dividing and multiplying.

EXAMPLE: Evaluate $\dfrac{2.4 \times 7}{3 \times 6.8 \times 9.5}$

Move the hairline to 2.4 on "D."
Bring 3 on "C" under the hairline.
Move the hairline to 7 on "C."
Bring 6.8 on "C" under the hairline.
Move the hairline to the Right Index of "C."
Bring 9.5 of "C" under the hairline.
Read the answer **.0868** under the Right Index of "C."

EXERCISES

1. $\dfrac{3.6 \times 18}{4.7}$	5. $\dfrac{2.5 \times 7.1 \times 8}{3.2 \times 6}$	9. $\dfrac{6.4 \times 2.6}{8. \times 4.7 \times 3.6}$
2. $\dfrac{51 \times .42}{60}$	6. $\dfrac{16 \times .6 \times 4}{12.5 \times 5}$	10. $\dfrac{7.3 \times 3.6}{5.4 \times 9 \times 2.6}$
3. $\dfrac{16 \times 82}{75}$	7. $\dfrac{.15 \times 25 \times .3}{42 \times .15}$	11. $\dfrac{6.9 \times 14}{18 \times 6 \times .7}$
4. $\dfrac{2.4 \times .35}{.4}$	8. $\dfrac{8.4 \times 6.2 \times 1.7}{6.7 \times .4}$	12. $\dfrac{92.7 \times 108}{89 \times 114}$

ANSWERS TO ABOVE EXERCISES

1. 13.79	5. 7.4	9. .123
2. .357	6. .615	10. .208
3. 17.5	7. .1785	11. 1.278
4. 2.1	8. 33.0	12. .988

8. Proportions. A proportion is an equation stating that two ratios are equal. They are written, for example, $\dfrac{A}{B} = \dfrac{C,}{D}$ and are usually read as "A is to B as C is to D." They are very convenient in solving simple equations without solving for the unknown explicitly. Problems involving proportions are encountered almost daily. They can be rapidly and accurately solved with a slide rule—generally using the "C" and "D" scales.

The use of proportions can be best explained by an example.

EXAMPLE: Assume it is known 47 lbs. is equal to 21.37 kilos and it is desired to know how many pounds there are in 15 kilos. The problem may be written mathematically as $\dfrac{47}{21.37} = \dfrac{X.}{15}$ To solve this proportion:

Move the hairline to 47 on "D."
Bring 21.37 on "C" under the hairline.
Move the hairline to 15 on "C."
Under the hairline read the answer 33.04 on "D."

The fundamental procedure is the same regardless of which side of the equality sign the unknown is on—or whether the unknown is in the numerator or denominator.

EXAMPLE: Suppose it is desired to know how many feet there are in 1.22 meters—and is known there are 8.2 feet in 2.5 meters. This may be solved by the following proportion: $\dfrac{1.22}{X} = \dfrac{2.5.}{8.2}$ To solve this proportion:

Move the hairline to 2.5 on "D."
Bring 8.2 on "C" under the hairline.
Move the hairline to 1.22 on "D."
On "C" read the answer 4 under the hairline.

Rule for Proportions. Opposite the numerator of the known relationship on the "D" scale, set the denominator of the known relationship on the "C" scale. If the unknown is in the numerator, opposite the remaining "C" scale, read the value of the unknown on the known quantity on the "C" scale. If the unknown is in the denominator, opposite the remaining "D" scale, read the value of the unknown on the known quantity on the "D" scale, read the value of the unknown on the "C" scale.

It should be remembered, proportion problems can be solved by straight multiplication and division if it wished to do so. The advantage of proportions is the equation need not be solved explicitly, and proportions are generally much faster and more convenient.

EXERCISES

Solve for "X" in the following examples, using only one setting of the slide.

1. $\frac{15}{12} = \frac{X}{35}$	4. $\frac{21}{56} = \frac{X}{47}$	7. $\frac{3.4}{6.7} = \frac{4.8}{X}$	10. $\frac{11.45}{X} = \frac{8.47}{9.6}$
2. $\frac{18.6}{26.4} = \frac{X}{64}$	5. $\frac{17}{35} = \frac{28}{X}$	8. $\frac{2.1}{19} = \frac{1.4}{X}$	11. $\frac{X}{17.4} = \frac{3.6}{4.7}$
3. $\frac{42}{18} = \frac{X}{24}$	6. $\frac{9}{7.4} = \frac{17}{X}$	9. $\frac{1.75}{X} = \frac{1.4}{18}$	12. $\frac{X}{19.7} = \frac{8.4}{6.2}$

ANSWERS TO ABOVE EXERCISES

1. 43.7	3. 56	5. 57.6	7. 9.46	9. 22.5	11. 13.32
2. 45.1	4. 17.63	6. 13.97	8. 12.67	10. 12.99	12. 26.7

9. The Folding Scales—"CF" and "DF."

Actually, the "CF" and "DF" scales are "C" and "D" scales cut in half (at $\pi = 3.1416$), and the two halves switched.

This arrangement has two important advantages:

1. Any number on the "C" and "D" scales can be easily multiplied or divided by π. To multiply any number by π, bring the hairline to the number on "D" or "C," and read the answer under the hairline on the "DF" scale (or the "CF" scale if the setting were made on the "C" scale).

   EXAMPLE: Multiply 4 by π.

   Bring the hairline to 4 on "D."
   Under the hairline read the answer 12.58 on "DF."

   EXAMPLE: Divide 16.7 by π.

   Bring the hairline to 16.7 on "DF."
   Under the hairline read the answer **5.31** on "D."

2. The other important advantage of the folded scales is that, without resetting the slide, numbers can be read which are "off the rule" on the "C" or "D" scale. The effect is the same as if the "C" and "D" scales were extended half a length on each end.

   EXAMPLE: Multiply 13.8 × 7.4.

   Set the Left Index of "C" over 13.8 on "D."
   It will be noted 7.4 on the "C" is beyond the graduations on the "D" scale.
   Therefore, move the hairline to 7.4 on "CF."
   Read the answer **102.1** under the hairline on "DF."

The "DF" and "CF" "folded" scales can also be used in proportions.

EXAMPLE: Suppose it is known a speed of 60 miles per hour is equivalent to 88 feet per second, and it is desired to know how many feet per second a speed of 86 miles per hour is equivalent to. The proportion may be written $\frac{88}{60} = \frac{X}{86}$ and to solve:

Move the hairline to 88 on "D."
Bring 60 on "C" under the hairline.
It will be noted 86 on "C" is "off" the rule.
Therefore, move the hairline to 86 on "CF."
Under the hairline read the answer 126.1 on "DF."

It should be remembered that the "CF" and "DF" scales can be used interchangeably with the "C" and "D" scales. If the value on the "C" scale is "off" the rule, make the setting on the "CF" scale and read the answer on the "DF" scale.

EXERCISES

Solve the following problems using the "CF" and "DF" scales.

1. $3 \times \pi$	4. $\pi \times 4.6$	7. 14.2 × .8 × 3	10. $\frac{95}{80} = \frac{X}{105}$
2. $16.2 \times \pi$	5. 14 × 7.6	8. 22 × 7 × .5	11. $\frac{16}{11} = \frac{X}{7.6}$
3. $9.7 \times \pi$	6. 4.2 × 4.3	9. 14.6 × 3.3 × .4	12. $\frac{.22}{.2} = \frac{X}{.945}$

ANSWERS TO ABOVE EXERCISES

1. 9.44	3. 30.5	5. 106.5	7. 34.05	9. 19.29	11. 11.05
2. 50.99	4. 14.48	6. 18.07	8. 77	10. 124.8	12. 1.04

10. The Reciprocal Scales—"CI," "CIF" and "DI."

The reciprocal of a number is 1 divided by the number. For example, the reciprocal of 3 is 1/3 or 0.333. The "CI," "CIF" and "DI" scales are reciprocal scales. They are graduated the same as the "C," "CF" and "D" scales respectively, except they are divided and numbered from right to left instead of left to right.

Because of this arrangement, the hairline can be brought to any number on the "C" scale and the reciprocal of that number can be read under the hairline on the "CI" scale. The same relationship exists between the "CF" scale and the "CIF" scale; and the "D" and "DI" scales. To illustrate, bring the hairline to 2 on "C," under the hairline read the reciprocal of 2, namely, 0.5 on "CI."

In addition to finding the reciprocal of numbers, the "CI," "CIF" and "DI" scales are extremely useful in problems involving multiplication and division.

For example, $\frac{25}{5}$ can be considered as 25 × 1/5 and to solve, by means of the Reciprocal Scales, set the Left Index of "C" opposite 25 on "D," move the hairline to 5 on "CI," under the hairline read the answer 5 on "D."

Also, for example, 15 × 4 can be considered as 15 ÷ 1/4 and to solve by means of the Reciprocal Scales, move the hairline to 15 on "D," bring 4 on "CI" under the hairline, opposite the Right Index of "C" read the answer 6.0 on "D."

The value of the Reciprocal Scales can best be shown by the following examples:

EXAMPLE: Multiply, $14 \times 6 \times 3$.

Move the hairline to 14 on "D."

Bring 6 on "CI" under the hairline.

Opposite 3 on "C" read the answer 252 on "D."

Note that only one setting of the slide was required and that two settings would have been required if only the "C" and "D" scales had been used.

EXAMPLE: Solve $\frac{14.5}{3.98 \times 1.86}$, this is the same as $\frac{14.5 \times (1/1.86)}{3.98}$

Opposite 14.5 on "D," set 3.98 on "C."

Move the hairline to 1.86 on "CI."

Under the hairline read the answer **1.96** on "D."

Here again it will be noted that only one setting of the slide was required.

EXAMPLE: Evaluate the following fractions: $\frac{874}{21}, \frac{76}{21}, \frac{135}{21}$

Set the Right Index on "C" over 21 on "DI."

Set the Indicator Hairline over 874 on "C."

Under the Hairline read the quotient of $\frac{874}{21}$, namely **41.6** on "D."

Next move the Hairline to 76 on "C."

Under the Hairline read the quotient of $\frac{76}{21}$, namely **3.62** on "D."

Next, move the Hairline to 135 on "CF."

Under the Hairline read the quotient of 135/21, namely **6.43** on "DF."

Any series of fractions having the same denominator can be solved by proceeding as explained above. Note that only one setting of the slide was required.

EXAMPLE: Evaluate $\frac{1}{14.7 \times 6}$

Set Left Index of "C" over 14.7 on "D."

Set Indicator Hairline over 6 on "C."

Under the Hairline read the answer **.01132** on "DI."

Note that by using the "DI" Scale only one setting of the slide was necessary.

EXERCISES

Perform the following exercise using only one setting of the slide.

1. $17 \times 2 \times 4$	5. $9.7 \times 6 \times 4.5$	9. $\frac{8.6}{12.2 \times .3}$ (Use "CIF" Scale)
2. $22 \times 6 \times 7$	6. $8.4 \times 4.3 \times 2.1$	10. $\frac{95.2}{18.4}, \frac{846}{18.4}, \frac{115}{18.4}$
3. $14 \times 6 \times 3.6$	7. $\frac{16.4}{4.2 \times 1.96}$	11. $\frac{6.42}{.2 \times 4}$
4. $19.5 \times 2.6 \times 1.3$ (Use "CF" Scale)	8. $\frac{182}{14.5 \times 7}$	12. $\frac{1}{8.6 \times 3.7}$

ANSWERS TO ABOVE EXERCISES

1. 136	3. 302.1	5. 261.8	7. 1.992	9. 2.35	11. 8.02
2. 924	4. 65.8	6. 75.9	8. 1.795	10. 5.17, 46, 6.25	12. .0314

CHAPTER III
SQUARES & SQUARE ROOTS— CUBES & CUBE ROOTS

11. Squares. To square a number, the number is multiplied by itself. For example: 6 squared $= 6 \times 6 = 6^2 = 36$. The factor 6 is called the square root of 36, and is written mathematically as $6 = \sqrt{36}$.

Problems involving squares and square roots are solved on the "A" and "B" scales, in conjunction with the "C" and "D" scales. An examination of the "A" scale will reveal it is actually two half-sized "D" scales placed end to end. The two parts of the scale are identical, and will be referred to as "**A left**" and "**A right.**"

The "A" and "B" scales are designed so that when the hairline is placed over a number on the "D" scale, the square of the number is under the hairline on the "A" scale.

Also, when the hairline is placed over a number on the "C" scale, the square of the number is under the hairline on the "B" scale. It should be noted that on rules with graduations on both sides, and with an encircling indicator, the scales on one side can be read directly in connection with the scales on the opposite side.

A few examples will familiarize the user with these scales:

EXAMPLE: Evaluate 2^2

Set the hairline over **2** on "D" and

Read the answer **4** under the hairline on "A."

EXAMPLE: Evaluate 6^2

Set the hairline over **6** on "D" and

Read the answer **36** under the hairline on "A."

EXAMPLE: Evaluate 396^2

Set the hairline over **396** on "D" and

Read the number **157** under the hairline on "A." Determine the location of the decimal point by mentally noting $400 \times 400 = 160{,}000$. Therefore, the answer is **157,000.**

Rule for Squaring Numbers. Set the hairline over the number to be squared on the "D" scale and read the square of the number under the hairline on the "A" scale. OR set the hairline over the number to be squared on the "C" scale and read the square of the number under the hairline on the "B" scale.

12. Square Roots. To find the square root of a number, the proceedure is essentially the reverse of squaring a number.

As explained above, the "A" scale is divided into two parts: "A left" and "A right." When finding the square root of a number with an **odd** number of figures **before the decimal point,** "**A left**" is used; with an **even** number of figures **before the decimal point,** "**A right**" is used.

EXAMPLE: Find the $\sqrt{9}$

Set the hairline over **9** on "**A left.**"

Read the answer **3** under the hairline on "D."

EXAMPLE: Find the $\sqrt{25}$

Set the hairline over **25** on "**A right.**"

Read the answer **5** under the hairline on "D."

To simplify the mental calculation in determining the location of a decimal point when finding the square root of numbers less than unity, or of numbers having three or more figures before the decimal point, the following procedure is recommended:

Numbers less than unity:

Move the decimal point an **even** number of places to the **right** to obtain a number between 1 and 100. Take the square root, as explained above, of the number thus obtained. Then, to secure the answer, move the decimal point to the **left** one-half as many places as it was originally moved to the right.

EXAMPLE: Find the $\sqrt{.00685}$

Move the decimal point **4** places to the **right,** thereby changing the number to $\sqrt{68.5}$.

Set the hairline over **68.5** on "**A right.**"

Read the number 8.27 under the hairline on "D."

Move the decimal point **2** places to the **left,** making the correct answer **.0827.**

Numbers with three or more figures before the decimal point:

Move the decimal point an **even** number of places to the **left** to obtain a number between 1 and 100. Take the square root, as explained above, of the number thus obtained. To secure the answer, move the decimal point to the **right** one-half as many places as it was originally to the left.

EXAMPLE: Find the $\sqrt{7{,}800{,}000}$

Move the decimal point **6** places to the **left,** thereby changing the number to $\sqrt{7.8}$.

Set the hairline over **7.8** on "**A left,**" and

Read the number 2.79 under the hairline on "D."

Move the decimal point **3** places to the **right,** making the correct answer **2790.**

Rule for Finding Square Roots. To find the square root of a number with an ODD number of figures before the decimal point, set the hairline over the number on "A left" and read the answer under the hairline on "D." To find the square root of a number with an EVEN number of figures before the decimal point, set the hairline over the number on "A right" and read the answer under the hairline on "D."

To simplify locating the position of the decimal point, move it an even number of places right or left so as to obtain a number between 1 and 100; take the square root of the resulting number; then, to obtain the answer, move the decimal point in the OPPOSITE direction half as many places as it was originally moved.

EXERCISES

1. $(3.1)^2$	5. $\sqrt{8.4}$	9. $\sqrt{175}$
2. $(7.5)^2$	6. $\sqrt{22}$	10. $\sqrt{.0028}$
3. $(14.2)^2$	7. $\sqrt{7.3}$	11. $\sqrt{217{,}000}$
4. $(37)^2$	8. $\sqrt{86}$	12. $\sqrt{.0000073}$

ANSWERS FOR ABOVE EXERCISES

1. 9.61	5. 2.9	9. 13.23
2. 56.1	6. 4.69	10. .0529
3. 202.	7. 2.7	11. 466.
4. 1370.	8. 9.28	12. .0027

13. Cubes.

To cube a number, the number is multiplied by itself three times. For example, 6 cubed $= 6 \times 6 \times 6 = 6^3 = 216$. The factor 6 is called the cube root of 216 and is written mathematically as $6 = \sqrt[3]{216}$.

Problems involving cubes and cube roots afe solved on the "K" scale. This scale is so constructed that when the hairline is set over a number on the "D" scale, the cube of the number is under the hairline on the "K" scale.

A few examples will familiarize the user with this scale.

EXAMPLE: Evaluate 2^3

Set the hairline over **2** on "D."

Read the answer **8** under the hairline on "K."

EXAMPLE: Evaluate 4^3

Set the hairline over **4** on "D."

Read the answer **64** under the hairline on "K."

EXAMPLE: Evaluate 7^3

Set the hairline over **7** on "D."

Read the answer **343** under the hairline on "K."

Rule for Cubing Numbers. Set the hairline over the number to be cubed on the "D" scale and read the cube of the number under the hairline on the "K" scale. Determine the location of the decimal point by means of mental approximation.

14. Cube Roots.

To find the cube root of a number, the procedure is essentially the reverse of cubing a number.

The "K" scale consists of three identical parts placed end to end, the combined length of which is equal to the length of the "D" scale. The three parts of this scale will be referred to as "**K left,**" "**K middle,**" and "**K right.**"

To find the cube root of a number between 1 and 10, "K **left**" is used; between 10 and 100, "K **middle**" is used; and between 100 and 1000, "K **right**" is used.

EXAMPLE: Find $\sqrt[3]{8}$

Set the hairline over **8** on "K left."

Read the answer **2** under the hairline on "D."

EXAMPLE: Find $\sqrt[3]{64}$

Set the hairline over **64** on "K **middle.**"

Read the answer **4** under the hairline on "D."

EXAMPLE: Find $\sqrt[3]{512}$

Set the hairline over **512** on "K **right.**"

Read the answer **8** under the hairline on "D."

In problems involving the cube root of numbers less than unity (one) or greater than 1000, the simplest solution is to move the decimal point in the direction required, three places at a time, until a number between 1 and 1000 is obtained. Find the cube root of the number thus obtained, then move the decimal point in the **opposite** direction one-third as many places as it was originally moved.

EXAMPLE: Find $\sqrt[3]{.0015}$

Move the decimal point **3** places to the **right**, thereby changing the number to $\sqrt[3]{1.5}$.

Set the hairline over **1.5** on "K **left.**"

Read the number 1.143 under the hairline on "D."

Move the decimal point **1** place to the **left**, making the correct answer **0.1143.**

EXAMPLE: Find $\sqrt[3]{28{,}000{,}000}$

Move the decimal point **6** places to the **left**, thereby changing the number to $\sqrt[3]{28}$.

Set the hairline over **28** on "K **middle.**"

Read the number 3.035 under the hairline on "D."

Move the decimal point **2** places to the **right**, making the correct answer **303.5.**

Rule for Finding Cube Roots. To find the cube root of a number between 1 and 10, set the hairline over the number on "K left" and read the answer under the hairline on "D"; of a number between 10 and 100, set the hairline over the number on "K middle" and read the answer under the hairline on "D"; of a number between 100 and 1000, set the hairline over the number "K right" and read the answer under the hairline on "D."

For numbers less than 1 or greater than 1000, move the decimal point right or left three places at a time until a number between 1 and 1000 is obtained; find the cube root of the resulting number as explained above; then move the decimal point in the OPPOSITE direction one-third as many places as it was originally moved to obtain the correct answer.

EXERCISES

1. $(2.7)^3$	5. $\sqrt[3]{9.3}$	9. $\sqrt[3]{474}$
2. $(7.4)^3$	6. $\sqrt[3]{26}$	10. $\sqrt[3]{10029}$
3. $(14.6)^3$	7. $\sqrt[3]{740}$	11. $\sqrt[3]{218{,}000}$
4. $(5.2)^3$	8. $\sqrt[3]{86}$	12. $\sqrt[3]{.000076}$

ANSWERS TO ABOVE EXERCISES

1. 19.7	5. 2.1	9. 7.8
2. 405	6. 2.96	10. 21.75
3. 3110	7. 9.05	11. 60.1
4. 141	8. 4.41	12. .0423

CHAPTER IV
PLANE TRIGONOMETRY

15. Trigonometric Functions. A brief review of some of the fundamental trigonometric functions is given below to help the user in understanding the operation of the "S," "T," and "ST" scales on the slide rule.

Trigonometric functions are based on the ratios sine, cosine, tangent, cotangent, secant, and cosecant. These angular functions are based on the ratios of the lengths of the sides of a right triangle.

The relationships of the right triangle may be written as follows:

Definition of Sine, Cosine, and Tangent:

$$\text{Sin A} = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{Cos A} = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{Tan A} = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}$$

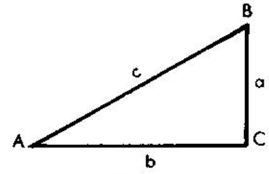

Fig. 8

Reciprocal relations:

$$\text{Cosec A} = \frac{c}{a} = \frac{1}{\text{Sin A}}$$

$$\text{Sec A} = \frac{c}{b} = \frac{1}{\text{Cos A}}$$

$$\text{Cot A} = \frac{b}{a} = \frac{1}{\text{Tan A}}$$

Relations between the complementary angles:

Sin A = cos (90° − A)

Cos A = sin (90° − A)

Tan A = cot (90° − A)

Cot A = tan (90° − A)

16. Trigonometric Scales—S (Sine), ST (Sine-Tangent), and T (Tangent). All of these scales are located on the reverse side of the slide.

Because of the manner in which the scales are divided, and because of the reciprocal relationship between the trigonometric functions, it is possible to obtain the value of all trigonometric functions by using the "S," "ST," and "T" scales in conjunction with the "C," "CI" and "DI" scales. This will be explained more fully as each scale is discussed in detail.

As with all other scales on the slide rule, the user should carefully examine the trigonometric scales to determine the value of the graduation for each part of each scale.

It will be noted that all trigonometric scales are divided into degrees and decimals of a degree—**not** degrees and minutes. For example, on the "S" scale there are ten major divisions between 7° and 8°, each of which therefore represents 0.1°. Each of these ten major intervals is divided into two secondary intervals, which gives the secondary intervals a value of 0.05°. This manner of reasoning will enable the user to determine the value of any interval on the trigonometric scales.

17. "S" (Sine) Scale. An examination of this scale will show that it is divided left to right from 5.74° to 90° (indicated by black numbers), and from right to left from 0° to 84.26° (indicated by the red numbers). It is actually, therefore, two scales in one, with the left to right graduations (black numbers) representing the sine, and the right to left graduations (red numbers) representing the cosine.

When the hairline is set to an angle on the "S" scale (black numbers), the value of the sine of the angle is read under the hairline on the "C" scale. Also, when the hairline is set to an angle on the cosine scale (red numbers on the "S" scale), the value of the cosine of the angle is read under the hairline on the "C" scale.

A few examples will familiarize the user with the use of the "S" scale.

EXAMPLE: Find the value of sine 17.6°.
Set the hairline over **17.6°** (black) on "S."
Read the answer **0.302** under the hairline on "C."

EXAMPLE: Find the value of cosine 52.50°.
Set the hairline over **52.50°** (red) on "S."
Read the answer **0.609** under the hairline on "C."

It was previously noted that the cosecant of an angle is the reciprocal of the sine, and the secant of an angle is the reciprocal of the cosine.

With this relationship in mind, it is possible to solve directly for either the secant or the cosecant by using the "S" scale in conjunction with the reciprocal "CI" scale.

EXAMPLE: Solve for cosec 34.0°.
Set the hairline over **34.0°** on "S,"
Read the answer **1.79** under the hairline on "CI."
The position of the decimal point can be determined by noting the value of the sine, 34.0° and calculating the reciprocal of this number.

EXERCISES

Evaluate the following trigonometric expressions:

1. sin 14.3°	5. cos 15°	9. cosec 12.5°
2. sin 47°	6. cos 36.5°	10. cosec 34.5°
3. sin 22.4°	7. cos 80.45°	11. sec 53.5°
4. sin 8.15°	8. cos 66.4°	12. sec 74.4°

ANSWERS TO ABOVE EXERCISES

1. .2466	5. .966	9. 4.63
2. .731	6. .804	10. 1.768
3. .381	7. .1659	11. 1.681
4. .1419	8. .4	12. 3.72

18 "ST" (Sine-Tangent) Scale. This is a scale of sines and tangents from 0.57° to 5.74°. It can also be used for cosines of angles from 84.26° to 89.43°.

The reason for a combined sine and tangent scale is that, for all practical purposes, the value of the sine and tangent are the same for angles of 5.74° or less.

When the hairline is set over an angle on the "ST" scale, the value of the sine or tangent of the angle is under the hairline on the "C" scale.

EXAMPLE: Find the value of sine 2.34°.
Set the hairline over **2.34°** on "ST."
Read the answer **0.0408** under the hairline on "C."

EXAMPLE: Find the value of tan 3.16°.
Set the hairline over **3.16°** on "ST."
Read the answer **.0551** under the hairline on "C."

The largest angle whose cosine can be found on the "S" scale is 84.26°. To find the cosine of angles larger than 84.26°, the relationship cos A = sin (90° − A) is used, giving a result which can be solved on the "ST" scale.

EXAMPLE: Find cos 87.6°.
Change the form to cos 87.6° = sin (90° − 87.6°) = sin 2.4°.
Set the hairline over **2.4°** on "ST."
Read the answer **0.0419** under the hairline on "C."

EXERCISES

Evaluate the following trigonometric expressions:

1. sin 4.62°	4. sin 2.04°	7. tan 3.52°	10. cos 84.96°
2. sin .66°	5. tan 2.64°	8. tan .82°	11. cos 88.4°
3. sin 1.58°	6. tan 1.04°	9. cos 86.5°	12. cos 87.9°

ANSWERS TO ABOVE EXERCISES

1. .0806	3. .0314	5. .0461	7. .0614	9. .061	11. .0279
2. .01151	4. .0356	6. .01813	8. .0143	10. .08	12. .0366

19. "T" (Tangent) Scale. This scale gives the values of tangents directly for angles between 5.72° and 84.28°.

When using the "T" scale, the following relationships should be remembered:

$$\text{tangent A} = \frac{1}{\text{Cot A}} \; ; \quad \tan A = \cot (90° - A)$$

$$\text{cotangent A} = \frac{1}{\text{Tan A}} \; ; \quad \cot A = \tan (90° - A)$$

On the "T" scale, the black numbers represent angles from 5.72° to 45°, and the red numbers represent angles from 45° to 84.28°.

Therefore, when the hairline is set over an angle on "T" black, the numerical value of the tangent is under the hairline on "C." When the hairline is set over a number on "T" red, the numerical value of the tangent is under the hairline on "CI."

EXAMPLE: Find the value of tan 27.2°.

Set the hairline over **27.2**° on "T" black.

Read the answer **0.514** under the hairline on "C."

EXAMPLE: Find the value of tan 79.7°.

Set the hairline over **79.7**° on "T" red.

Read the answer **5.51** under the hairline on "CI."

To find the cotangent of an angle, one of the following two indentities must be used:

$$\cot A = \frac{1}{\tan A} \quad ; \quad \text{or } \cot A = \tan\,(90^\circ - A)$$

EXAMPLE: Find the value of cot 36°.

Think of the problem as $\frac{1}{\tan 36^\circ}$

Set the hairline to **36**° on "T" black.

Read the answer **1.379** under the hairline on "CI."

OR

Think of the problem as cot 36° = tan (90° − 36°) = tan 54°.

Set the hairline to **54**° on "T" red.

Read the answer **1.379** under the hairline on "CI."

EXERCISES

Evaluate the following trigonometric expressions:

1. tan 14°	4. tan 8.45°	7. tan 71.2°	10. cot 36.2°
2. tan 42.4°	5. tan 68.1°	8. tan 56.8°	11. cot 52.8°
3. tan 25.6°	6. tan 83.45°	9. cot 17.6°	12. cot 76.4°

ANSWERS TO ABOVE EXERCISES

1. .249	3. .479	5. 2.49	7. 2.94	9. 3.152	11. .759
2. .913	4. .1485	6. 8.71	8. 1.53	10. 1.368	12. .2419

20. Illustrative Problems Using the Trigonometric Scales. The following problems are provided to acquaint the user with some of the practical applications of the slide rule as related to trigonometric problems. If the user has a thorough understanding of trigonometry and of the trigonometric scales on the slide rule, most types of angular problems can be solved rapidly and accurately with the slide rule.

Right Triangle. In Fig. 9, the hypotenuse and opposite side are given. Find angles A and B, and the length of the adjacent side.

To find angle A—

from Fig. 9 we can write sin A = 3/5, therefore, sin A = 0.60. Set the hairline on 0.60 on "C" and read the value of angle A as **36.9**° under the hairline on "S."

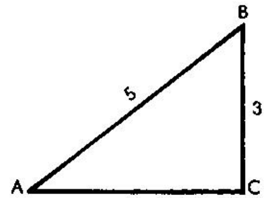

Fig. 9

To find angle B—

note that B is the complement of angle A, therefore B = 90° − 36.9° = 53.1°. Or, angle B can be determined by noting that the cos B = 3/5 = 0.60. Set the hairline to 0.60 on "C," and read the value of angle **B** as **53.1**° on "S" **red.**

To find side "b"—

use the Pythagorean Theorem: $a^2 + b^2 = c^2$. Or, more simply, note that cos A = b/c, which may be rewritten as 5cosA = b.
To evaluate this expression, set the right index of "C" opposite 5 on "D," move the hairline to 36.9° on "S" red, and read the answer 4 under the hairline on "D."

Oblique Triangle. Fig. 10 shows an oblique triangle in which two angles and a side are given. Find angle C and sides b and c.

To find angle C—

Angle C = 180° − 102° − 36° = **42**°.

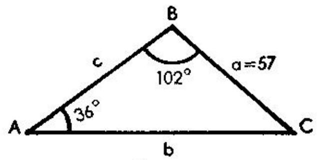

Fig. 10

To find sides b and c—

using the Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, these sides can be solved by means of proportions, as previously explained. In this example, the proportions are $\frac{57}{\sin 36^\circ} = \frac{b}{\sin 102^\circ} = \frac{c}{\sin 42^\circ}$. But it should be remembered that the sin 102° = sin (180° − 102°), and therefore sin 102° = sin 78°. Consequently, the proportions become 57/sin 36° = b/sin 78° = c/sin 42°

To solve for **b,** move the hairline to 57 on "D."
Bring sin 36° on "S" black under the hairline.
Move the hairline to sin 78° on "S" black.
Read the value of **b** as **94.9 feet** under the hairline on "D."

To solve for **c,** move the hairline to 42° on "S" black.

Read the value of **c** as **64.9 feet** under the hairline on "D."

If, instead of one side and two included angles, all three sides are known, an oblique triangle can be solved by making use of the Law of Cosines. Fig. 11 shows an oblique triangle, all three sides of which are known.

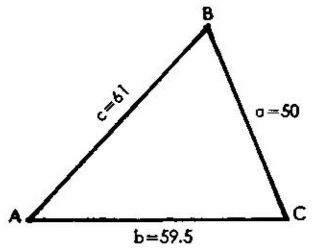

Fig. 11

To find angle A—

using the Law of Cosines: $\cos A = \dfrac{-a^2 + b^2 + c^2}{2bc}$. For this example, we would write—

$$\cos A = \frac{-2500 + 3550 + 3720}{2 \times 59.5 \times 61} = \frac{4770}{7260} = 0.656$$

To solve, set the hairline to 0.656 on "C".
Read the value of angle A as **49°** on "S" red.
Therefore, angle A = **49°**.

To find angle B—

also using the Law of Cosines: $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$. For this example, we would write—

$$\cos B = \frac{2500 + 3720 - 3550}{2 \times 50 \times 61} = \frac{2670}{6100} = .438$$

To solve, set the hairline at 0.438 on "C".
Read the value of angle **B** as **64°** on "S" red.
Therefore, angle **B** = **64°**.

To find angle C—

simply note that angle C = 180° − 49° − 64° = **67°**.

21. Combined Operations. In many types of problems, products or quotients involving trigonometric functions are required. These values can be obtained directly without actually reading the value of the trigonometric function involved.

EXAMPLE: Evaluate 8.32 sin 33°.
Set the Right Index over 8.32 on "D."
Move the hairline to 33° on "S" black.
Read the answer **4.53** under the hairline on "D."

EXAMPLE: Evaluate 11.3 × sin 24° cos 72.1.
Set the Left Index over 11.3 on "D."
Move the hairline to 24° on "S" black.
Bring the Right Index under the hairline.
Move the hairline to 72.1 on "S" red.
Read the answer **1.41** under the hairline on "D."

EXAMPLE: Evaluate 13.6 tan 28.2 tan 57°.
Set the Left Index over 13.6 on "D."
Move the hairline to 28.2 on "T" black.
Bring 57° on "T" red under the hairline (thereby dividing by the reciprocal which is the same as multiplying by the number), and
Read the answer **11.22** under the Left Index on "D."
Note: It should be observed that this illustration could have been solved by using another method similar to the previous example, but this would have involved an additional setting of the slide.

EXAMPLE: Evaluate $\dfrac{\tan 30°}{6 \sin 10°}$
Set 10° on "S" over 6 on "DI."
Move the Hairline to 30° on "T."
Under the Hairline read **.554** on "D."
Note that by using the "DI" Scale only one setting of the slide was required.

EXERCISES

Evaluate the following trigonometric expressions:

1. 4.6 × sin 22.6°
2. 12.5 × tan 18.4°
3. 25.2 cos 47°
4. 18.5 × cot 15°
5. 10.8 × sin 22° × cos 55°
6. 55 × tan 35° × tan 59°
7. 19.5 tan 14° × cot 18.5°
8. 47 cot 14° × sin 22°
9. 15 × sec 73° × tan 28°
10. 48.6 cosec 37° × cot 25°
11. 195 tan 17.5° × sin 24°
12. 42 cos 35.6° × tan 14.7°

ANSWERS TO ABOVE EXERCISES

1.	1.768	5.	2.32	9.	2.725
2.	4.16	6.	64.1	10.	173.2
3.	17.19	7.	14.52	11.	25.0
4.	69.1	8.	70.6	12.	8.95

CHAPTER V
LOGARITHMS—THE "L" SCALE

22. Logarithms. Logarithms are merely exponents. Once a base is selected, the logarithm of any number to this base is merely the exponent to which the base must be raised to give the original number.

The most common base for logarithms is 10. Therefore, from the above definition, we know that if the Log of 30, to the base 10, equals 1.477, then $10^{1.477} = 30$. To repeat, the logarithm of a number to a given base is merely the exponent to which the base must be raised to yield the original number.

Logarithms, to the base 10, can be found on the slide rule. However, there are two parts to a logarithm, the mantissa and the characteristic. When finding logarithms, to the base 10, the following two statements should be remembered.

I. When the hairline is set over a number on the "C" scale, the mantissa (fractional part) of the logarithm is under the hairline on the "L" scale. Conversely, when the hairline is set over a mantissa on the "L," the antilogarithm is under the hairline on the "C" scale.

II. The slide rule does not give the characteristic (integral part) of a logarithm. Hence, to determine the characteristic of a logarithm, the following must be remembered.

(a) The characteristic of a logarithm, of a number greater than 1, is positive, and is one less than the number of digits to the left of the decimal point.

(b) The characteristic of a logarithm, of a number less than 1, is negative, and is one greater than the number of zeros immediately following the decimal point.

EXAMPLE: Find the logarithm of 45.

Set the hairline over 45 on "C."

Under the hairline, on "L," read .653.

Since 45 has two digits to the left of the decimal point, the characteristic is 1. Hence, the log of 45 is **1.653.**

EXAMPLE: Find the logarithm of .083.

Set the hairline over 83 on "C."

Under the hairline, on "L," read .919.

Since .083 has one zero immediately following the decimal point, the characteristic is −2. Therefore, the log of .083 is **−2.919.**

EXAMPLE: Find the number whose logarithm is 3.658.

Set the hairline over .658 on "L."

Under the hairline, on "C," read 455.

Since the characteristic is 3, the antilog must have 4 digits to the left of the decimal point. Therefore, the answer is **4550.**

EXERCISES

Evaluate the following:

1. Log 15	4. Log .0015	7. Log .0187	10. Antilog 1.468
2. Log 1.5	5. Log 180	8. Log 7	11. Antilog −2.254
3. Log 150	6. Log 4672	9. Log 27.8	12. Antilog 2.675

ANSWERS TO ABOVE EXERCISES

1. 1.176	4. −3.176	7. −2.272	10. 29.4
2. .176	5. 2.255	8. .845	11. .01795
3. 2.176	6. 3.67	9. 1.444	12. 473.0