FOWLER'S "UNIVERSAL" CALCULATOR

INSTRUCTIONS

with Supplementary Chapters on Logarithmic Calculation, and Circular Calculators, in addition to numerous worked-out examples

By

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FOWLER & Co. ESTABLISHED 1898

HAMPSON STREET, SALE, MANCHESTER

'Phone: SALe. 1433 'Grams: Scientific, Manchester

SEE REVISED PRICE

Price

33/-

Postage:

Gt. Brit. 6d,
Abroad 2/-

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To my Son

Harold Fowler.

Wh. Ex., A.M.I.Mech.E.,

Whose co-operation through 25 years has contributed greatly to the successful Evolution of

Accurate Circular Calculators.

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JOHN NAPIER.

JOHN NAPIER.

The Discoverer of Logarithms.

Born at Merchiston Castle near Edinburgh, 1550.
Discovery of Logarithms Announced, 1614.
Tercentenary Celebrated at Edinburgh, 1914.

John Napier, "the author and inventor of logarithms," to quote the appellation given on the title page of his "Admirable Canon of Logarithms," published in 1614, was born at Merchiston Castle, near Edinburgh, in 1550, and was the eighth member of the Napier family to succeed to the Merchiston estates.

Of his early life not much is known except that he was educated at St. Salvators College, St. Andrews, and was boarded within the college under the special charge of the principal, but it would appear that his stay was not of long duration, though his mind during that period evidently received a strong impetus towards theological studies, for theology throughout his life was as great an attraction as the subject of mathematics that eventually made him world-famous.

His subsequent studies at the University appear to have been followed by several years of travel and study on the Continent, though little is known of his actual experiences at the places he visited. There is evidence, however, that he immersed himself in the troublous ecclesiastical affairs of the time and acquired some reputation, both at home and abroad, as a scholar and a theologian.

It was not until 1614, in his sixty-fourth year, that he announced to the world his invention of the numbers which he called "logarithms," a discovery that not only constituted one of the greatest forward steps in the practical application of pure mathematics to arithmetical computation, but stamped its author as one of the foremost men of his time.

In 1914 posterity paid tribute to his greatness by celebrating his tercentenary, under the auspices of the Royal

Society of Edinburgh, in which city a most interesting collection of personal relics, books, instruments and devices for facilitating calculation were gathered together, and a record of permanent value established by the publication of an exhaustive illustrated handbook, edited by E. M. Horsburgh, M.A., Assoc.M.Inst.C.E., giving an instructive illustrated account of the exhibits, accompanied with a brief biography of Napier's life and work by Professor George A. Gibson, M.A., LL.D.

To this book we beg to express our indebtedness, and at the same time our obligation to the Royal Society of Edinburgh for their kind permission to reproduce the accompanying portrait of John Napier.

Although Napier's discovery was not published until 1614, three years before his death, there is evidence that he had been engaged for many previous years on the studies relating to it, and that when made known it excited profound interest and respect, both here and on the continent, amongst those best capable of appreciating its magnitude.

Amongst the earliest to recognise it in this country was Henry Briggs, Professor of Mathematics at Gresham College, London. So deep was his admiration excited by its importance, that he made a special journey from London to Edinburgh—no light task in those days—to make Napier's personal acquaintance, and the account of the interview is one of the pleasing reminiscences of science. When first brought together the two men gazed into each other's face for a time without speaking. At length Briggs began: "My Lord, I have undertaken this long journey purposely to see your person and to know by what engine of wit or ingenuity you came first to think of this most excellent help unto astronomy, viz., the logarithms; but, my Lord being found, I wonder nobody else found it out before, since now it appears so easy."

Napier and Briggs were obviously congenial spirits, for the intimacy thus begun ripened into a permanent and warm affection. Briggs spent a month at Merchiston, returned for a second visit the following summer, with the further intention

of making a third visit the next year, but was prevented by the untimely death of Napier.

Briggs' admiration for his friend's "invention" was so great that he began the calculation of a set of tables, which is essentially the system now in use, and to this labour devoted some years of his life. A tribute that deserves to be borne in mind when recognising the genius of his friend.

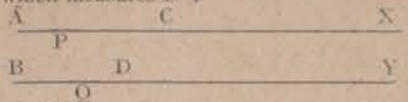
On the Continent recognition of the importance of Napier's work was as great as at home. The earliest tables were printed in 1617, and roused Kepler, the astronomer, to an admiration second only to that of Briggs.

Napier's conception of the logarithm, as his biographer remarks, "cannot fail to suggest to the student of mathematics Newton's treatment of the fluxional calculus; not that Newton borrowed from Napier, but that the fundamental ideas of both were so much alike." The great generality of Napier's conception has been more clearly understood in recent years, and there is a strong tendency in the advanced stages of mathematical study to return to a definition of the logarithm very much akin to that of Napier.

Present admiration of Napier's genius is enhanced when it is considered how little the field of algebra in his day was explored. It must be remembered that the modern conception of a logarithm as the index of a base, did not exist until long afterwards. Napier makes no use of a base and the whole conception of indices in the modern sense of fractional and negative values was then unknown. Napier's conception of a logarithm is based on the following comparison of the velocities of two points moving along two straight lines.

Suppose, for example, one point P to move from A along the line A X with the uniform velocity V; and suppose a second point Q to move from B along the line B Y (of given length *r*) at the same time as P, and with the same velocity, but to move *not uniformly* but with a velocity, at any point D, proportional to the distance D Y.

If now P reaches C when Q reaches D, then Napier defines the number which measures A C to be the "logarithm" of the number which measures D Y



Napier had the needs of trigonometry primarily in mind when discussing the matter and speaks of B Y (or *r*) as "the whole sine" and D Y as "a sine," and it is to be further remembered that in Napier's day the sine was regarded as *a line* and not *a ratio* as it is now.

Referring to Napier's diagram: When Q is at B, P is at A, and the logarithm of the "whole sine" B Y is zero.

The logarithms of numbers less than B Y are positive.

If Q were to the left of B then P would be to the left of A, and A P would be negative so that in Napier's system the logarithms of numbers greater than the whole sine are negative.

The circumstance that log of *r* is not zero in Napier's system, as in the modern convention, is awkward, but as he had the needs of trigonometry mainly in view, it is less so than appears and as the matter is only of historical interest need not here be dwelt upon.

Suffice it that having established his conception of a logarithm Napier next proceeded to show that if

$$\frac{a}{b} = \frac{c}{d}$$

then log a—log b=log c—log d, and from this rule established all the rules of logarithms required for ordinary calculations.

How greatly these have lightened and simplified the mental work of computation for succeeding generations of men, only those who have had great quantities of calculating work to perform can adequately appreciate, and it is in this spirit of thankfulness this slight tribute is here offered to the memory of one of the world's great men.

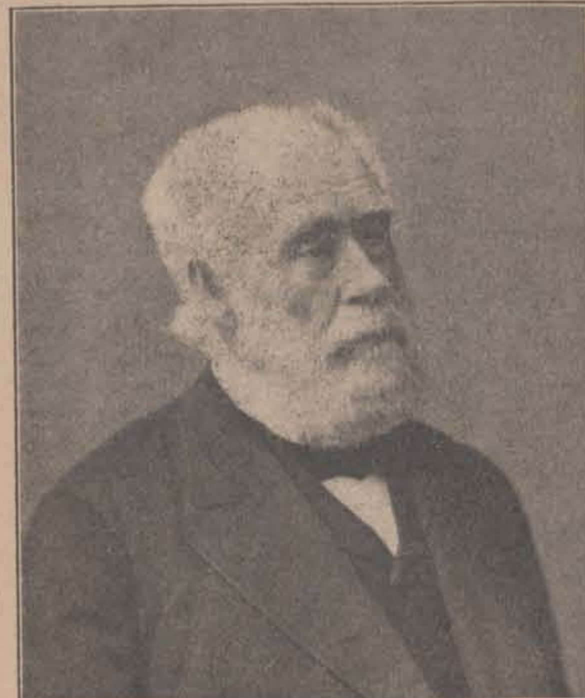


Photo by Elliott & Fry Ltd.

SIR JOSEPH WHITWORTH, BART.

A Pioneer of Mechanical Accuracy and Efficiency.

SIR JOSEPH WHITWORTH, Bart.

A Pioneer of Mechanical Accuracy and Efficiency.

The increasing adoption of Napier's logarithmic system of computation has been due in large measure to the application to it of graphic principles; the substitution of scales for figures, and the use of mechanical methods of manipulation or of making comparisons between one scale and another, coupled with the fact that since man first counted with his fingers, the decimal notation has become almost universal for purposes of reckoning, though its obvious lack of convenience for fractional division has led to a duo-decimal base being adopted in special cases for commercial reasons.

Engineering has, not inaptly, been described as the art of measurement, either of matter, space, force or time, etc., and when extremely accurate measurement is required the art is capable of taxing man's ingenuity and constructive skill to the utmost.

Rough approximations often do not present much difficulty but when we get from the domain of rough estimation to measured accuracy, difficulties increase with amazing rapidity.

The truth of this is impressed on every practical engineer, who in the pursuit of some ideal of perfection is brought into close contact with the imperfections of his materials, and the difficulties of fashioning them to his ends. Up to a point they yield to his skill and experience, but as higher degrees of perfection are sought, and stores of existing knowledge are exhausted, he finds that to make progress he must contribute to them by personal research, and so learn how much each generation is indebted to the past, and how few are the really great pioneers in any field of human knowledge.

It is this consciousness which impels the writer to pay a tribute of respect and admiration to Sir Joseph Whitworth, whose high standards of mechanical perfection and workmanship did so much to raise the reputation of British engineering during the last century.

A perfect physical straight line, a physical flat surface, an accurate screw, seem trivial details to the unthinking mind, but they are the foundations of mechanical perfection and standards of human measurement of all other things.

It is easy to express accuracy to any degree in figures as an abstract idea; it is impossible to do so in terms of matter in the way all measurements in the practical world have to be finally translated. One is a symbol, the other is an actual construction; and when Sir Joseph Whitworth, the great Manchester Engineer, showed that dimensions of the order of one-hundred-thousandth of an inch could be physically measured and applied in a workshop, he did a great deal more than talk about it.

Apart from his pre-eminence as an engineer, Sir Joseph Whitworth was a great public benefactor, and in this respect his memory specially appeals to engineers for his munificent founding and endowment of the scheme of scholarships bearing his name, and by the aid of which a number of young engineers in workshops are annually enabled to enter upon a university course of scientific and technical education, and to qualify for the highest posts in the engineering profession.

It may be asked: What has the construction of an accurate physical straight line, or standard screw, or flat surface to do with the construction of a logarithmic scale? This! That while the value of a logarithm may be expressed in figures with any degree of refinement, and used as such without difficulty in an arithmetical operation, it is otherwise when used as an actual physical dimension on a logarithmic scale for purpose of mechanical measurement and calculation under varying conditions of temperature, moisture, etc.

A logarithmic scale it must be remembered is not a uniformly divided one. No two adjoining divisions are alike, and when several such scales are required in circular form with the added feature that they must be capable of strict concentric rotation in conjunction with three radial lines (a zero, a datum, and a cursor), each having independent "dead-beat" movement and capable of being read with little, or no, parallax, it will be manifest that the problem of mecanical construction is not an easy one.

It is, of course, impossible to bring the refinement of accuracy of a logarithmic scale up to the standard of a seven figure table of logarithms, and where such accuracy is required, as it is in certain astronomical and physical calculations, the printed table of definite values with its longer arithmetical processes will always be preferred.

The greater convenience and rapidity of manipulation of a mechanically divided scale, however, atones largely for its lack of accuracy, and accounts for its universal adoption for the calculations of daily life. At the same time the potential effect of errors of construction emphasises the necessity of the highest standards of workmanship, to insure corresponding efficiency of operation.

This has throughout been the aim of Messrs. Fowler in the design and construction of their Circular Calculators, and they trust their efforts have not proved unworthy of their aims.

FOWLER'S

"UNIVERSAL" CALCULATOR.

Fowler's "Universal" Calculator, like the well-known waistcoat pocket instrument consists of a series of concentric circular scales, logarithmically divided and mounted on a dial capable of rotation by a thumb nut outside the containing case. The scales are equipped with a fixed radial datum line, and a radial cursor line rotated by a second thumb nut. The rotating scales, cursor line, and operating mechanism are enclosed in an airtight electro- plated metal containing case, fitted with a glass face so that the scales are always kept clean, and the instrument is preserved from external injury in a handsome leather wallet which fits easily into a side pocket. The large size of the "Universal" enables all the scales to be mounted on one dial, and so of being synchronised and read concurrently. It also permits of the use of larger figures and easier reading an advantage to persons of weak eyesight. Another important feature is that the scales are longer and admit of finer graduation and more accurate reading. This is specially noticeable in the "Long-Scale" which gives a lenght of 19½ ins. as compared with 10 ins. in the ordinary slide-rule and permits of calculations being made to three, and sometimes four, significant figures. The motions of the scales and cursor being effected by gearing adjustments can be made with great ease and nicety. There is none of the objectionable sticking or slackness often so troublesome with the straight slide rule owing to changes of climate or temperature, nor is there any of the "end switching" often necessary with that instrument when working with the full length scale and which induces many users to work habitually with the half length. In circular calculators the scales are continuous and there is no half length.

LOGARITHMS.

What they are. What they will do.

Logarithms are certain numbers, first discovered by the great mathematician, John Napier, which allow the ordinary arithmetical operations of multiplication and division to be replaced by the simpler operations of addition and subtraction. They also allow difficult calculations of powers and roots and the making of trigonometrical calculations to be simply effected with great saving of time and mental labour.

Definition of a Logarithm.—A logarithm (usually written Log) expresses a certain relationship between any given number and another number, called the base, and may be defined as the power to which the base must be raised to be equal to the given number.

For example, in the expression $N = A^x$

x is the log of N to the base A.

Any number may be used as a base, but 10 is usually employed, and is the most conveient, because it is the base or standard of measurement used in ordinary arithmetical notation.

In the expression $100 = 10^2$ we say 2 is the log of 100, because 10 would have to be raised to the 2nd power to equal 100, the base 10 being always understood unless the contrary is stated.

Considering successive powers of 10, the relation between numbers and their logs is as follows :—

Number :	1,	10,	100,	1,000,	10,000,	etc.
Log :	0,	1,	2,	3,	4,	etc.

Obviously, for any number

over 1, and	under	10, the log=	a decimal quality.	
„ 10	„	100 „ „	=1+ „	„
„ 100	„	„ 1,000 „ „	=2+ „	„
„ 1,000	„	„ 10,000 „ „	=3+ „	„
		etc.	=x etc.	

Characteristic and Mantissa of a Logarithm.—The integral part of a log is called the characteristic. The decimal part is called the mantissa. Obviously, numbers that are exact multiples of 10 have no mantissa.

A little consideration of the preceding shows that for any number the characteristic can be read off at sight. To distinguish numbers above 0 from numbers below 0, the characteristic is marked positive or negative, the latter quality being indicated by a minus sign (—) placed over the characteristic.

Learners are often puzzled by the fact that the mantissa of a logarithm is always positive, while the characteristic changes sign when the number gets less than 1, although the number itself may be more than 0 (zero), and therefore positive. The following explanation may help to clear the matter :—

Consider the number 0·75 the decimal equivalent of $\frac{3}{4}$, a positive quantity, greater than 0 but less than 1.

Without going into an algebraic demonstration, it may be stated that

$$\log \tfrac{3}{4} = \log 3 - \log 4$$
$$= (\cdot 4771213) - (\cdot 6020600)$$
$$= - \cdot 1249387$$

There are two ways of expressing this minus quantity—

(1) As a minus quantity, below 0, as shown above.

or (2) As a positive quantity above — 1.

Expressing it in the second manner, *i.e.*, by subtracting ·1249387 from 1 and adding — 1,

we get $\bar{1} + \cdot 8750613$ written $\bar{1}\cdot 8750613$ as the log of $\frac{3}{4}$, instead of $- \cdot 1249387$.

Written in this way, the Characteristic or integral part is minus and the Mantissa or decimal part is positive, the value being the same, of course, in both cases.

Further, the Mantissa expressed thus is the same for ·075, ·75, 7·5, 75, 750, or any multiple of 10 thereof, i.e., whether the number be more or less than O.

A table of logs so constructed is greatly simplified.

The only part of the logarithm that changes sign is the Characteristic, and its magnitude as well as its sign can be determined at a glance.

The Mantissa of the logarithm is always positive, and always the same so long as the digits composing the number are the same, and in the same order.

This feature of logarithms is shown in the following table where, it will be seen, the only changes that occur are the position of the decimal point in the number and the sign of of the Characteristic :—

Number.		Logarithm
750	......	2·8750613
75	......	1·8750613
7·5	......	0·8750613
·75	......	$\bar{1}$·8750613
·075	......	$\bar{2}$·8750613
·0075	......	$\bar{3}$·8750613

The preceding will enable the learner to understand the following rules regarding the characteristic :—

For numbers greater than 1 the characteristic is one less than the digits in the numbers, and is positive.

For numbers less than 1 the characteristic is one more than the number of cyphers following the decimal point, and is negative.

The mantissa of a logarithm is always positive, and is the only part of a logarithm given in tables of logarithms.

Confusion in the mind of the learner arises sometimes from a failure to recognise that the zero position on the logarithmic scale does not coincide with the zero position on the arithmetical scale, of notation, and that when a number falls below 1 the characteristic takes a minus sign,

so that the mantissa may be always positive and be of the same value for the same digits in the same order, i.e., independent of the position of the decimal point.

Calculation of Mantissa.—While the characteristic, or integral part of a logarithm can be determined by a mere inspection of the number, the determination of the mantissa or decimal part involves a great deal of aithmetical labour, and the compilation of a complete table is a big task.

Space does not here permit of a description of the methods employed. Those interested are referred to the article in the *Encyclopoedia Britannica* or to special works dealing with the subject.

How Logarithms Save Labour.—The labour-saving properties of logarithms may be best illustrated by a few practical examples showing how multiplication can be replaced by additions and division by subtraction. Afterwards it will be shown how, by the use of Logarithmic Scales, further saving may be effected by performing addition and subtraction mechanically, and finally how, by combinations of circular scales, as in FOWLER'S CALCULATORS, complex calculations can be further simplified with fewer mechanical movements and less mental effort.

It can be shown algebraically that if A and B are two factors requiring multiplication, this operation can be reduced to addition by the aid of a Table of Logarithms.

Since the log of the product equals the sum of the logs of the factors, that is to say,

$$\log (A \times B) = \log A + \log B.$$

If it be desired to divide A by B, this operation can be reduced to subtraction, that is to say,

$$\log \frac{A}{B} = \log A - \log B.$$

If it be desired to find the value of a number A raised to a given power say n (roots are only powers in decimal or fractional form) the operation, by the aid of logarithms, is reduced to simple multiplication, since

$$\log A^n = n \log A.$$

For the learner not familiar with the algebra of indices, it may be explained that

A^2 means A squared $A^{\frac{1}{2}}$ means square root of A,
A^3 means A cubed, $A^{\frac{1}{3}}$ means cube root of A,
etc. etc.

A^{-2} means $\frac{1}{A}$ $A^{-\frac{1}{2}}$ means $\frac{1}{\sqrt{A}}$

$A^{\frac{3}{4}}$ (*i.e.* $A^{\cdot 75}$) means the Fourth root of A cubed, or A raised to the ·75th power.

Generally, $A^{\frac{m}{n}}$ means the *n*th root of the *m*th power of A

In the evaluation of powers of numbers the Reciprocal Scale in Fowler's Calculators often proves extremely useful in reducing movements and saving time and labour.

Short Table of Logs for numbers 1 to 30 :—

No.	Log.	No.	Log.	No.	Log.
1......	·0000	11......	1 ·0413	21......	1 ·3222
2......	·3010	12......	1 ·0791	22......	1 ·3424
3......	·4771	13......	1 ·1139	23......	1 ·3617
4......	·6020	14......	1 ·1461	24......	1 ·3802
5......	·6990	15......	1 ·1761	25......	1 ·3979
6......	·7782	16......	1 ·2041	26......	1 ·4149
7......	·8451	17......	1 ·2304	27......	1 ·4313
8......	·9031	18......	1 ·2553	28......	1 ·4471
9......	·9542	19......	1 ·2787	29......	1 ·4624
10......	1 ·000	20......	1 ·3010	30......	1 ·4771

Examples of the Use of Logarithms.—The relationship between numbers and their logs and its bearing on the arithmetical operations of multiplication, division, and evaluation of powers and roots, may be demonstrated with the appended short table of logs from 1 to 30.

The brevity of the table limits the demonstrations to very simple examples, but they suffice for illustration.

Multiplication.—Taking first the application of logarithms to the operation of multiplication, we have to choose examples to come within the range of the table, and the results in each case are obvious at sight, viz. :—

6 × 3 = 18 5 × 4 = 20 7 × 4 = 28

Taking now the logs of each of these pairs of factors and adding them together, we get the following results :—

Log 6 = ·7782	Log 5 = ·6990	Log 7 = ·8451
Log 3 = ·4771	Log 4 = ·6020	Log 4 = ·6020
= 1 ·2553	= 1 ·3010	= 1 ·4471

Comparing these logs in the table we find that

1 ·2553 = log 18 1 ·3010 = log20 1 ·4471 = log 28

In other words, Multiplication has been reduced to Addition and it should be added, results accurate to seven significant figures could be obtained with just as little mental labour with a more complete table.

Division.—Let us now take two examples of division. Again very simple, owing to the small limits of our log table, and the results of which are obvious at sight, viz. :—

28 ÷ 7 = 4 26 ÷ 2 = 13

Subtracting in each case the log of the divisor from the log of the dividend we get

Log 28 = 1 ·4471	Log 26 = 1 ·4149
Log 7 = ·8450	Log 2 = ·3010
= ·6020	= 1 ·1139

Comparing these results with our table we find

·6020 = log 4 1 ·1139 = log 13.

Thus the operation of Division is reduced to Subtraction.

Powers and Roots of Numbers.—We will now give one or two illustrations showing the application of logs to the evaluation of powers, or roots of numbers.

Ex. 1.—

If A = 3^3, find value of A

We know log A = 3 × log 3

and from table, log3 = ·4771

Therefore log A = ·4771 × 3

= 1 ·4313

Referring to table we find 1·4313 is log 27.

Therefore we say 3^3 = 27

Ex. 2.—

If $A = \sqrt{25}$; *i.e.* $= 25^{\frac{1}{2}}$, find value of A

We know $\log A = \frac{1}{2} \log 25$

and from table $\log 25 = 1.3979$

Therefore $\frac{1}{2} \log 25 = .6989$

Referring to table we find $.6989 = \log 5$.

Therefore we say $\sqrt{25}$; *i.e.* $25^{\frac{1}{2}} = 5$.

The examples given are simple, but with a fuller table, and compound fractional indices, or powers, answers could be arrived at just as easily.

If, for instance, we had the expression

$$x = 93.7^{\frac{7}{9}}$$

x would be the 9th root of the 7th power of 93.7

As an ordinary arithmetical problem this would be practically impossible, but with a table of logarithms it would present no difficulty.

We should say $\log x = \frac{7}{9} \log 93.7$.

It would only be necessary to find value of log 93.7 from a table and to tak e ths of that value.

This would be the log of *x*, and the number corresponding would be the answer required.

The above examples suffice to show how greatly arithmetical labour can be reduced by the aid of logarithms. In succeeding chapters it will be shown how by the aid of Logarithmic Scales time and labour can be still further saved.

The Mechanical Operation of Logarithms.

Straight Logarithmic Scales.—The earliest form of logarithmic scale for calculating purposes was the straight slide-rule. Essentially it consists of two straight scales, similarly graduated to represent the logarithms of the prime number, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and with the intervals between these numbers, subdivided in a logarithmic manner as far as space will permit. The two scales are arranged so that they can slide past and overlap each other, and are fitted with an independent sliding Cursor-line at right angles to the scales.

By means of this contrivance additions or subtractions of the logarithmic values marked on the scales can be performed mechanically. The scales are in fact miniture representations of a table of logarithms, but as such printed tables are available giving values of logarithms to seven places of decimals for numbers from 1 to 100,000, it is manifest that no contrivance of this kind, unless of inordinate length, can give calculated results with the same degree of accuracy as those obtainable by ordinary arithmetical methods with the aid of a printed table.

Nevertheless, mechanically operated scales give results accurate enough for all ordinary calculations, and the enormous advantage of being able to do this rapidly by simple manipulation of a scale, with little or no mental labour, has made the logarithmic scale in some form a time-saving and indispensable instrument for all who have calculating work to perform.

Fortunately the mathematical structure of logarithms enables a comparatively short scale to give fairly accurate readings, though the length of the scale is an important factor when accuracy is essential.

It has been shown that the characteristic or integral part of the logarithm of any number can be read at sight, and the prime numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 may represent simple intergers or any multiple of 10 thereof. It is only in respect to the mantissa or decimal part of a logarithm that the value on a scale is differentiated from that of a printed table, and by suitable construction an adequate length of scale can be provided to allow this to be largely overcome.

Defects of the Straight Slide-Rule.—The straight slide-rule has many inherent defects.

It is not capable of affording readings of much practical value unless it is about 10 in. (254mm.) in length, and this is inconvenient for the pocket.

The slide is liable to get dirty and stick, or get too slack, and make setting and reading troublesome.

Changes of climate or temperature produce errors.

The result of an operation often cannot be read unless the slide is drawn out completely and reversed or "end switched."

This causes confusion and waste of time, and leads many users to work habitually with the "half-length" scale, which reduces the accuracy of the instrument, because logarithmic graduation is not uniform, and the intervals between the prime numbers of the basic scale, (No. 1) diminish rapidly as one progresses from 1 to 10. This is shown by the following figures which represent the percentage of the total length of the scale occupied by the respective intervals :—

Intervals...	1-2,	2-3,	3-4,	4-5,	5-6,	6-7,	7-8,	8-9,	9-10,
Per cent of total length.	30,	17·6,	12·6,	9·7,	7·9,	6·9,	5·8,	5·1,	4·7.

The distance between 9 and 10, it will be seen, is only one-seventh of that between 1 and 2, and cannot be subdivided to the same extent. The inconvenience of this is intensified by the fact that subdivision of the intervals must be of a decimal character and therefore by 2, 5 or 10.

It is easy to see from these facts why logarithmic scales for multiplication and division require length to allow of reasonable graduation and of setting and reading. Sacrifice of length means sacrifice of accuracy and clarity.

These defects of the straight logarithmic slide-rule are recognised, and many efforts have been made to produce

circular ones on account of their greater convenience, but the difficulties of constructing an accurate and reliable pocket instrument of this kind at a reasonable price are great, and until Messrs. Fowler & Co. turned their attention to the subject they had never been satisfactorily overcome.

Circular Logarithmic Scales can be constructed and operated exactly in the same way as straight logarithmic scales, that is to say with one scale rotating inside another concentric one, and the two being equipped with an independent radial, rotating Cursor-line.

A second method may also be adopted with circular scales. In this there is only a single rotating dial on which one or more concentric circular scales are marked; the operation of addition and subtraction being performed by the aid of a fixed radial Datum-line in conjunction with a rotating radial Cursor-line.

Each method has advantages, but on the whole the second is the best, and is adopted in the construction of Fowler's Calculators, as it permits of the adoption of a "Long-scale" extending round several circles, and there is less liability of confusion in setting factors and reading results. A calculation begins at the Datum-line and ends there.

One special advantage of a circular scale over a straight scale, apart from its greater compactness, and therefore convenience, is that the ends of the scale (the 1 and the 10) coincide, *i.e.*, the scale is continuous.

No "end switching" is necessary to obtain a reading, and there is no "half-length" scale with its resultant limitations of subdivision and accurate reading.

On the contrary, the length of the scale can be increased by extending it round the circumferences of several circles in spiral fashion, a feature that has been taken advantage of in Fowler's "Long-scale" Calculators.

How greatly this contributes to accurate calculation, as compared with the ordinary 10 in. straight slide-rule, may be judged from the following particulars :—

Fowler's Circular Long-Scales.

In Fowler's vest pocket "Long-scale" instrument with two dials, and only 2¾ in. in diameter, the Long-scale (6 circles) has a length of 30 ins.

In Fowler's "Magnum," a single dial instrument, 4½ in. in diameter, the Long-scale (6 circles) has a length of 50 in.

In Fowler's "Universal" a single dial instrument 3¼ in. in diameter, the Long-scale (3 circles) has a length of nearly 20 in.

The "Long-scale," it should be noted, is supplementary to the single circle or "Short-scale" in each of the three types mentioned, and is intended for use when results of greater accuracy are desired.

A further advantage of circular scales is that scales with different functions can be more conveniently worked in conjunction if desired.

In all Fowler's Calculators the scales are graduated with extreme accuracy, protected with glass faces, and enclosed in protecting cases to seal them as far as possible from dirt and moisture. The dial and cursor are operated with gearing, and can be set with ease and rapidity, even in a moving train. The instruments work equally well in any climate, while their workmanship and construction secures a degree of accuracy that has never been previously reached in instruments of this kind.

"UNIVERSAL" CALCULATOR.

Description of Scales.

The instrument contains six separate circular scales arranged concentrically on a single dial, so that they all rotate together.

The scales are rotated by the knurled nut at the top.

The scales have a common Zero line, which rotates with the scales, and also a common "cursor" line which is rotated by the nut at the side. See illustrations at front of book.

Commencing with the largest circle, and proceeding inwards to the smallest, these scales are as follows:—

Scales No. 1 (The "Short-Scale" for Multiplication or Division).—This (the Primary-Scale of the instrument) consists of a single circle, graduated clockwise, and logarithmically to represent the nine primary spaces between the numbers 1 to 10, or any multiple or division of 10 thereof.

The scale, being a circle, the beginning and the end of it coincide. If this point be designated "Zero" the scale may be regarded

as beginning at 0 and ending at 1
or, as beginning at 1 and ending at 10
or, as beginning at 10 and ending at 100, etc.

Learners are often a little puzzled by this.

The point to grasp is that the scale s logarithmic, both as regards the principal divisions and the subdivisions, and is subdivided decimally for reckoning by 10, 5 or 2, as space permits.

The user may assign to the prime numbers any value he pleases, provided it be a multiple of 10, and that the subdivisions are given proportionate values.

The space between the first pair of principal numbers here marked 10 and 20 and which occupies 30 per cent of the whole circumference is subdivided into 100, each equivalent to ·01 of the value attaching to the primary number, i.e., the first subdivision may represent 1·01, 10·1, 101 or 1010, etc., according as unity is taken to represent 1, 10, 100, 1000, etc. Every tenth division is figured for convenience of reference, 11, 12, 13, etc.

In the second primary space there are 50 subdivisions, each equivalent to ·02 of the value attaching to the primary number, so the first subdivision may represent 2·02, 20·2, 202, etc.

The even subdivisions in this space are figured 22, 24, etc., for convenience of reference, and the odd ones (23, 25, etc.) distinguished with a longer line.

In each of the four spaces 3–4, 4–5, 5—6, 6–7, there are 20 subdivisions, each equivalent to ·05 of the value assigned to the prime numbers, so that the graduations following 3 may read ·305, 3·05, 30·5, 305, etc. The half of these four spaces are marked 35, 45, 55, 65.

In each of the prime spaces 7–8, 8–9, 9–10, there are 10 subdivisions, each equivalent to ·1 of the prime number, so that graduations following 7 might read ·71, 7·1, 71, or 710, etc.

Owing to the diminishing distances between the prime numbers it is impossible to subdivide the scale uniformly, but every effort has been made to produce a clear well-illuminated scale by avoiding excessively close division lines, or by overcrowding the reference figures.

Changes in magnitude of subdivision have been kept as few as possible, consistent with their decimal character. The values of the subdivisions in the primary spaces are as follows :—

Primary Spaces ...	1 to 2.	2 to 3.	3 to 7.	7 to 10
Subdivision equals	0·01 ($\frac{1}{100}$)	0·02 ($\frac{1}{50}$)	0·05 ($\frac{1}{20}$)	0·1 ($\frac{1}{10}$)

Experience shows that spaces which can be split with the Cursor-line or the Datum-line are preferable for accurate setting and reading to finer subdivisions that produce a pallisading effect difficult to read.

It has been pointed out that the prime divisions and numbers indicating them along the scale may represent any multiple of 10 thereof—that is to say, 3 may represent ·03, ·3, 3, 30, 300, etc. but the subdivisions must have proportionate values.

If, for example, the number 30 on the scale represented 300, the readings following would be 305, 310, 315, etc. but if the same prime number were taken to represent 3, the readings immediately following would be 3·05, 3·1, 3·15, 3·2, etc.

Learners should further bear in mind, when setting or reading with the Cursor-line or Datum-line that the scale is logarithmic throughout, and that 30·5 on the scale is rather more than half-way between 30 and 31. Attention to this point makes for accuracy when setting, or reading, positions on the scales.

Scale No. 2 (The Reciprocal Scale).—This is exactly like Scale No. 1 except that it is graduated *contra-clockwise* (a point to be remembered when setting or reading values). The scale gives by aid of Cursor-line or Datum-line the reciprocal value of any reading on Scale No. 1. and can be used with any other scale provided the value, whose reciprocal is desired, is read from Scale No. 1. This Reciprocal Scale often permits the work of calculation to be shortened. (See pages 22, 23 and 24.)

Scale No. 3 (The Logarithm Scale).—This scale consists of a single circle graduated clockwise. Further, the graduations are uniform, not logarithmic, and numbered from 0 to 1·00 at intervals of 10 divisions, thus :—

·05, ·10, ·15, ·25.............. ·90, ·95, 1·00.

This gives 200 graduations in the circumference, and as each of these graduations can be split with the Cursor-line values can be read accurately to four figures at least.

Scale No. 3 is used with Scale No. 1 as follows :—

If the logarithm of a number is required, the Cursor-line is set over the number on Scale No. 1, and the mantissa of the log of the number is read on Scale No. 3 under the Cursor. The characteristic of the logarithm is determined as explained in the chapter on logarithms.

Scale No. 4 for Cubes and Cube Roots, also "Long-Scale Multiplication and Division."

This extends round three circles and is graduated clockwise.

Scale No. 1 gives cubes of readings on Scale No. 4.

Scale No. 4 gives cube-roots of readings on Scale No. 1.

For locating positions of cubes and cube-roots the following tables, constructed mentally, are convenient :—

For Numbers between	The Cube is between
1 and 10	1 and 1,000
10 and 20	1,000 and 8,000
20 and 30	8,000 and 27,000
30 and 40	27,000 and 64,000
etc.	etc.

The converse is of course true, viz :—

For Numbers between	The Cube-root is between
1 and 1,000	1 and 10
1,000 and 8,000	10 and 20
8,000 and 27,000	20 and 30
27,000 and 64,000	30 and 40
etc.	etc.

To construct above tables it is only necessary to remember that the cubes of 2, 3 and 4, are 8, 27 and 64, then to add three cyphers.

When multiplying and dividing with Scale No. 4, don't forget that Factors must be set on that Scale, and answers sought for on that scale. Don't mix Scale No. 1 and Scale No. 4.

Squares and Square-Roots.—Squares and square-roots can be obtained by means of a two-circle scale and in Fowler's "Vest Pocket Long-Scale" and "Magnum Long-Scale" instruments a special two-circle scale is provided, which allows these values to be read directly with a single movement and greater accuracy.

In the "Universal" Calculator space considerations have prevented the inclusion of a two-circle scale, and the Square of a number can be found by multiplying the number by itself on Scale No. 1.

The Square Root of a number can be found by using Sca'e No. 1 (a single circle) in conjunction with Scale No. 2, as follows :—

(1) Set the number (whose square root is required) on

Scale No. 1 under the Datum, and set the Cursor over the Unity line of the dial.

(2) Rotate the dial until the reading of the Datum on Scale No. 1 is the same as the reading of the Cursor on Scale No. 2.

(3) The reading so found is the Square Root required.

The above operations amount to finding the half-logarithm of the number, *i.e.*, the logarithm of the square root, since Scales No. 1 and No. 2 are graduated in opposite directions and rotation of the dial divides thé allotted value between Datum and Cursor (i.e., the whole logarithm) into two equal parts when the readings are equal.

This method may be employed with all types of Fowlers' Calculators fitted with a Reciprocal Scale.

The magnitude of the squares and square roots can be approximately located mentally by the following tables :—

For Numbers between	The Square is between
1 and 10	1 and 100
10 and 20	100 and 400
20 and 30	400 and 900
30 and 40	900 and 1,600
40 and 50	1,600 and 2,500
50 and 60	2,500 and 3,600
etc.	etc.

The converse of the above is equally true.

For Numbers between	The Square root id between
1 and 100	1 and 10
100 and 400	10 and 20
400 anc 900	20 and 30
900 and 1,600	30 and 40
1,600 and 2,500	40 and 50
2,500 and 3,600	50 and 60
etc.	etc.

Note.—For numbers less than unity—

The Square becomes Less than the Number.

The Square-root becomes Greater than the Number.

All powers and roots can of course be got by aid of logs as explained in chapter on logs.

NOTE.—It will occasionally be found that the square root given by above method is an imaginary root, but there is no difficulty in detecting this, and the second correct value should then be sought.

Scale No. 5 (Sines of Angles).—This is a scale of angles graduated clockwise round the inner, and then continued round the outer, circumference of a common circle. The scale ranges from 35 minutes to 90 deg.

Angles are read on Scale No. 5.
Natural Sines are read on Scale No. 1.
Logarithmic Sines are read on Scale No. 3.

The range of angles on the circles are as follows:—
Inner Circle from 35 mins. to 5 deg. 45 mins.
Outer Circle from 5 deg. 45 mins. to 90 deg.

NOTE.—The scale of angles is not divided "decimally" like all the other scales, but into degrees and minutes, as angles are always expressed in terms of degrees, minutes and seconds, though seconds are seldom used.

360 deg. = a circle, 60 mins. = 1 deg., 60 secs. = 1 min.

The graduations of the scale are as follows:—

From	35 mins.	to	1 deg.	the intervals	=	1 min.
,,	1 deg.	to	3 ,,	,,	=	2 mins.
,,	3 ,,	to	8 ,,	,,	=	5 mins.
,,	8 ,,	to	12 ,,	,,	=	10 mins.
,,	12 ,,	to	30 ,,	,,	=	20 mins.
,,	30 ,,	to	60 ,,	,,	=	1 deg.
,,	60 ,,	to	80 ,,	,,	=	2 deg.

Scale No. 6 (Tangents of Angles).—This is a scale of angles graduated clockwise round the inner circumference of a single circle.

The scale ranges from 5 deg. 45 mins. to 45 degs.
Angles are read on Scale No. 6.
Natural Tangents are read on Scale No. 1.

Logarithmic Tangents are read on Scale No. 3.

The graduations are as follows:—

From	5 degs. 45 mins.		8 degs.	Intervals	=	5 mins.
,,	8 ,,	to	12 ,,	,,	=	10 ,,
,,	12 ,,	to	30 ,,	,,	=	20 ,,
,,	30 ,,	to	45 ,,	,,	=	30 ,,

Gauge Points.—In making practical calculations there are certain multiplying factors which often occur, and to save time and trouble in setting them, it is convenient to have them marked on Scale No. 1 and Scale No. 4.

Only a limited number of factors or "gauge points" can be given in this way. A great many must inevitably be omitted and sought for in the Book of Instructions relating to the "Universal" Calculator. It is hoped however that the selection shown below will prove generally convenient.

Gauge Point.		Multiplier.
$\sqrt{2}$	=	1·4142
$\sqrt{3}$	=	1·7321
Dia. of Circle of Area 1 = $\sqrt{4 \div \pi}$	=	1·1284
π	=	3·14159
$\pi \div 4$	=	0·7854
Degrees in Arc equal to radius	=	57·298
To convert com. logs to hyperbolic logs	=	2·30258
Metres to yards...............	=	1·0936
Yards to metres...............	=	0·9144
Inches to centimetres	=	2·54
Centimetres to inches	=	0·3937
Kilogrammes to lbs.	=	2·2046
Lbs. to kilogrammes...............	=	0·4536
E.H.P.	=	746

FOWLER'S "UNIVERSAL" CALCULATOR

Operation of Scales.

Multiplication Scale No. 1 alone.—

To find product of a series of factors—A, B, C, D . . etc.—
Set dial till A comes under datum.
Set cursor to unity.
Set dial till B comes under cursor.
(Read product A × B under datum).

Continuing the operations for further factors—
Set cursor to unity.
Set dial till C comes under cursor.
(Read product A × B × C under datum).

Continuing—
Set cursor to unity.
Set dial till D comes under cursor.
(Read product A × B × C × D under datum.)

Repeat as many times as there are factors.

The movements of the scales and cursor described above have been equivalent practically to the adding together of the logs of A, B, C, and D: seven movements altogether: three for the first pair of factors and two for each factor afterwards.

The number of movements can be reduced by using Scales No. 1 and No. 2 in conjunction, as will be shortly shown but whatever the method adopted, the final effect amounts to adding together the logs, of the multipliers, and subtracting the logs of the divisors.

Division Scale No. 1 alone.—Consider the fraction $\frac{A}{B}$ with one numerator and one denominator—

Set dial till A comes under datum.
Set cursor to B.
Set unity to cursor.
Read answer under datum.

Next consider the fraction $\frac{A \times B}{C}$ with two factors in the numerator and one in the denominator—
Set dial till A comes under datum.
Set cursor to C.
Set dial till B is under cursor.
Read answer under datum.

Now consider a f action with several factors in the Numerator as well as in the denominator:—

It makes no difference to the answer whether all the top factors are multiplied together, and then divided by all the bottom factors multiplied together; or, whether the top and bottom factors are worked in pairs as single f actions, and finally united in a group.

Obviously, also the insertion of the factor 1, in the numerator, or the denominator, can make no difference. Learners, however, will find it easier, at first, to work such a compound fraction by taking the factors alternately from numerator and denominator, and to enable them to do this in a routine manner the factor 1 should be inserted in the fraction as often as may be necessary to make the numerator contain one more factor than the denominator

Any compound fraction can then be worked as follows:—

Fraction $\frac{A \times B \times C}{M}$ work as $\frac{A \times B \times C}{M \times 1}$

Set factor A under datum.
Set cursor to M.
Set factor B to cursor.
Set cursor to 1.
Set factor C to cursor.
Read answer under datum.

Fracrion $\frac{A \times B}{M \times N}$ work as $\frac{A \times B \times 1}{M \times N}$

Set factor A under datum.
Set cursor to M.
Set factor B to cursor.
Set cursor to N.
Set factor 1 to cursor.
Read answer under datum.

Fraction $\frac{A \times B}{M \times N \times P}$ work as $\frac{A \times B \times 1 \times 1}{M \times N \times P}$

Set factor A under datum.
Set cursor to M.
Set factor B to cursor.
Set cursor to N.
Set factor 1 to cursor.
Set cursor to P.
Set factor 1 to cursor.
Read answer under datum.

In all these examples the routine is alike.

The factors are taken alternately from the numerator and the denominator beginning with the numerator.

The Dial is always turned for Multipliers.

The Cursor is always turned for Divisors.

The Datum used only for First Factor and Answer.

Rapid Action with Fowler's Calculator.—If the reader has followed the previous routine methods of manipulation, he will be able to understand some shorter ones, and devise further ones for himself, according to circumstances; recognising that they all depend on the same principles, viz., the the addition of logarithms of factors for purpose of multiplication, and their substraction for purpose of division. A few examples are here given for illustration.

As there are several ways of working a problem with arithmetic, so there are several ways of working with a Fowler's Calculator, and when the user becomes familiar with its manipulation he will discover how movements can be curtailed and time saved by using the Reciprocal Scale (No. 2) in conjunction with the Primary Scale (No. 1).

Ex. 1.—Multiplying three factors.
Find value of A × B × C.
Set A on Scale 1 under datum.
Set cursor to B on Scale 2.
Set C on scale 1 under cursor.
Read answer on Scale 1 under datum. (3 movements.)

Ex 2.—Multiplying five factors.
Find value of A × B × C × D × E.
Set A on Scale 1 under datum.
Set cursor to B on Scale 2.
Set C on Scale 1 under cursor.
Set cursor to D on Scale 2.
Set E on Scale 1 under cursor.
Read answer on Scale 1 under datum. (5 movements.)

Ex. 3.—Operate similarly for any odd number of factors

It is interesting to compare the above example with the 9 movements necessary when using Scale No. 1 alone (see page 20), or by comparing it with the movements of an ordinary Straight Slide-rule with its intermittent "end-switching." This is only one of many illustrations that could be given.

Ex. 4.—**To Multiply any Even Number of Factors.**—Suppose product of four factors A, B, C, D, or any other even number, is required. It can be obtained quickly with Scales No. 1 and No. 2 by adding a factor 1 to make the even number of factors into an odd number, thus :—
A × B × C × D × 1. Then proceed exactly as in Example 2 above :—

Set A on Scale 1 under datum.
Set cursor to B on Scale 2.
Set C on Scale 1 under cursor.
Set cursor to D on Scale 2.
Set 1 under cursor.
Read answer on Scale 1 under datum. (5 movements.)

Rapid Division with Scales No. 1 and No. 2 with Even Number of Factors in the denominator.—

Example 1.—Find value of $\frac{A}{B \times C}$

Set A on Scale 1 under datum.
Set Cursor to B on Scale 1.
Set C on Scale 2 under cursor.
Read answer on Scale 1 under datum. (3 movements.)

EXAMPLE 2.—Find value of $\frac{A}{B \times C \times D}$

Here the artifice may be adopted of inserting an extra, factor 1, into the denominator, to make it contain an even number of factors, thus :—

$$\frac{A}{B \times C \times D \times 1}$$

Then proceed as follows :—

Set A on Scale 1 under datum.

Set cursor to B on Scale 1.

Set C on Scale 2 under cursor.

Set cursor to D on Scale 1.

Set 1 under cursor.

Read answer on Scale 1 under datum. (5 movements).

FOWLER'S "UNIVERSAL" CALCULATOR.

Practical Worked-out Examples.

Multiplication of Two Factors on Short Scale No. 1.

Ex. 1.—Multiply 12 ·8 by 5 ·62.
Set 12 ·8 on Scale 1 under datum.
This is the 8th graduation past the 12.
Set Cursor to 1.
Set dial till 5 ·62 on Scale 1 comes under Cursor.
This is between the 55 and the 60 ; the exact point being 2 divisions past the 55 to make the 56, and two-fifths of the next division to make the 2 of 562.
Read answer just under 72 on Scale 1 under Datum.
We should estimate this as 71 ·9.

Ex. 2.—Multiply ·0347 by 2 ·8 (on the Short Scale No. 1).
Set 347 on Scale 1 under Datum.
This lies between the 30 and 35 ; the exact point being the 9th division past the 30 to make the 345 and two-fifths of the next division to make the 347.
Set the Cursor to 1.
Set dial till 28 comes under Cursor.
Read answer (just over 97) on Scale 1 under Datum.
By visual inspection it will be seen that the answer must be in the neighbourhood of ·09.
Therefore we write our answer as given by the Calculator as ·097.
By actual multiplication, the correct answer is ·09716, showing how close is the approximation by the instrument.

Multiplication of Two Factors on the Long Scale (No. 4).

Ex. 3.—Multiply 12 ·8 by 5 ·62.
Set 128 under Datum.
Set Cursor to 1.
Set dial till 562 comes under the Cursor (this is the first small division after the 56 on the Long Scale).
Read answer just over 71 ·9 under Datum.

Ex. 4.—Multiply ·0347 by 2 ·8.
Set 347 on the Long Scale under Datum.
This is the 7th graduation past the 34.

Set Cursor to 1.
Set dial till 28 on the Long Scale comes under Datum.
Read answer 972 on the outer of the three circles forming the Long Scale, under the Datum.

This is really ·0972, as shown in Ex. 2, and is a very close approximation to the true value ·09716.

From the above it will be seen that, where greater accuracy is desired, it is often advantageous to use the Long Scale.

Multiplication of 3 Factors on the Short or Long Scale.

The method is precisely the same whichever Scale is used, so it will be described only for the Short Scale.

Ex. 5.—Find product of ·0347 × 2 ·8 × 63 ·5.

Set 347 on Scale 1 under Datum.
Set Cursor to 1.
Set dial till 2 ·8 on Scale 1 comes under Cursor.
All the above settings as shown in Ex. 2.
Set Cursor to 1.
Set dial till 635 on Scale 1 comes under Cursor.
This is the 7th division past the 60.
Read answer, 6 ·17, on Scale 1 under Datum.
The position of the decimal point in the answer is judged by inspection.

By actual multiplication the correct answer is 6·16966, showing a close approximation by the use of the instrument.

Multiplication of 4 or more factors on the Short or Long Scales.

Ex. 6.—Find product of ·0347 × 2 ·8 × 63 ·5 × 4 ·9.

Set 347 under Datum.
Set Cursor to 1.
Set dial till 28 comes under Cursor.
Read product ·347 × 2 ·8 under Datum.
Next set Cursor again to 1.
Set dial till 635 comes under Cursor.
Read product ·0347 × 2 ·8 × 63 ·5 under Datum.
Again set Cursor to 1.
Set dial till 49 comes under Cursor.
Read product ·0347 × 2 ·8 × 63 ·5 × 4 ·9 under Datum.

This, if using the Short Scale, comes just short of midway between the 30 and the first division past the 30; and we should estimate the answer as 30 ·23 (midway being 30 ·25).

The process after setting the first factor under the Datum is a succession of settings of Cursor and of factors on the scale, and of finally reading the product under the Datum.

The whole operation begins at the Datum and ends there. It consists in sum of turning the several factors past a fixed point, and reading the total of angular movements at the end. It matters not whether the angular movements of the dial and Cursor are made clockwise or contra-clockwise for the individual settings. So long as the sequence is in the order stated, the reading of the final result is the same.

If there are decimal points in the factors, the position of the point in the final product is to be decided by inspection and mental consideration, as with all logarithmic work. Actual examples of this will be given in the course of the exercises.

If Scale No. 4 (Long Scale) is used instead of Scale No. 1 (Short Scale), the succession of operations is precisely the same, but the setting calls for a little more care, as the factors are spread over a scale extending round three circles, and the answer may also be on any one. The particu'ar circle must be determined by a mental consideration of the factors.

Multiplication of an odd number of factors, using Scales Nos. 1 and 2 in conjunction. (See pages 22 and 23 for explanation of principle).

Ex. 7.—Find product of 8 ·42 × 16 ·16 × ·422 (3 factors).

Set 842 on Scale 1 under Datum.
Set Cursor to 1616 on Scale 2.
Set 422 on Scale 1 under Cursor.
Read answer, 57 ·4, on Scale 1 under Datum. By actual multiplication the correct answer is 57 ·42036. The decimal point is fixed mentally in this way: ·422 is roughly ·5; ·5 × 8 ·42 is roughly 4, and 4 × 16 ·16 is roughly 56. Therefore there are two whole numbers in the answer.

Ex. 8.—Find product of $\cdot354 \times 29\cdot4 \times 63\cdot6 \times \cdot862$ (4 factors)
This will be worked as $\cdot354 \times 29\cdot4 \times 63\cdot6 \times \cdot862 \times 1$ to make it into an odd number of factors.

Set 354 on Scale 1 under Datum.
Set Cursor to 294 on Scale 2.
Set 636 on Scale 1 under Cursor.
Set Cursor to 862 on Scale 2.
Set 1 on Scale 1 under Cursor.
Read answer, 571 on Scale 1 under Datum.

The correct answer by actual multiplication is 570·578.

The decimal point is fixed mentally in this way—·354 is roughly one-third; one-third of 29·4 is roughly 9; nine times 63·6 is roughly 560; 560 multiplied by ·8 is roughly 500. Therefore the answer must contain 3 whole numbers, and is 571.

Division on the Short Scale No. 1.

Ex. 9.—Divide 7,256 by 13·85.

Set 7256 on Scale 1 under Datum.
Set Cursor to 13·85.
Set 1 to Cursor.
Read answer, 524, under Datum.

It is obvious by inspection that the answer will have three whole numbers, and so we fix the decimal point after the 4.

The correct answer by actual multiplication is 523·9 and when the example was worked out on the Long Scale this correct answer was obtained.

Fractions.—Consider first a faction with two factors in the numerator and one in the denominator and worked out on the Short Scale.

Ex. 10.—Solve $\dfrac{676\cdot9 \times 364}{114\cdot2}$

Set dial till 6769 comes under Datum.
Set cursor to 1142.
Set dial till 364 comes under Cursor.
Read answer, 2158, under Datum.

The correct answer by actual multiplication and division is 2157·5. When worked out on the Long Scale the answer came barely 2158.

Consider now fractions with several factors in the numerator and denominator. (See notes on pages 21 and 22.)

Ex. 11.—**Solve** $\dfrac{19\cdot5 \times 66\cdot6 \times \cdot0042}{8\cdot9}$

Work this as $\dfrac{19\cdot5 \times 66\cdot6 \times \cdot0042}{8\cdot9 \times 1}$ taking the factors alternately from the numerator and the denominator.

Set 19·5 under Datum.
Set Cursor to 8·9.
Set 66·6 to Cursor.
Set Cursor to 1.
Set 42 to Cursor.

Read answer, ·613, under Datum, the decimal point being fixed by a rough mental calculation as previously described.

The correct answer worked out by actual multiplication and division is ·61287.

Ex. 12.—Solve $\dfrac{13\cdot8 \times 723\cdot6}{15\cdot8 \times 176 \times 2\cdot42}$

Work this as $\dfrac{13\cdot8 \times 723\cdot6 \times 1 \times 1}{15\cdot8 \times 176 \times 2\cdot42}$

taking the factors alternately as in the previous example.

Set 13·8 under Datum.
Set Cursor to 15·8.
Set 7236 to Cursor.
Set Cursor to 176.
Set 1 to Cursor.
Set Cursor to 142.
Set 1 to Cursor.
Read answer, 1·487, under Datum.

The correct answer is 1·484 (a close approximation).

If the reader has followed the previous worked-out examples carefully he will be in a position to solve in a routine way any compound fraction presented to him, and also to apply the more rapid method of division permitted by means of Scales 1 and 2 used in conjunction, which will now be shown, and which was explained in principle on pages 23 and 24.

Ex. 13.—Solve $\frac{6734}{9 \cdot 6 \times 142 \cdot 5}$ where there is an *even* number of factors in the denominator.

Set 6734 on Scale 1 under Datum.
Set Cursor to 96 on Scale 2.
Set 1425 on Scale 2 under Cursor.
Read answer, 4·92, on Scale 1 under Datum (3 movements), the position of the decimal point being fixed mentally.

The correct answer worked out by multiplication and division is 4·923.

Ex. 14.— Solve $\frac{4276}{3 \cdot 42 \times 18 \cdot 7 \times 32 \cdot 62}$

Here the artifice may be adopted of inserting an extra factor, 1, into the denominator to make it contain an even number of factors, thus :—

$$\frac{4276}{3 \cdot 42 \times 18 \cdot 7 \times 32 \cdot 62 \times 1}$$

Set 4276 on Scale 1 under Datum.
Set Cursor to 342 on Scale 1.
Set 187 on Scale 2 under Cursor.
Set cursor to 3262 on Scale 1.
Set 1 under Cursor.
Read answer, 2·050, on Scale 1 under Datum (5 movements).

Exercises with the Reciprocal Scale No. 2.

Ex. 15.—Find the decimal equivalent of $\frac{1}{6 \cdot 456}$

Set Cursor over 6456 on Scale No. 1. Read under Cursor on Scale No. 2, 0·1548.

In setting the Cursor to 6456 on Scale 1, we note that between 60 and 70 there are 20 graduations, the reading advancing clockwise—605, 610, 615, 620, etc. and 6456 is between 64 and 65, its exact position being estimated. Conceive this space to be divided into 100 parts, and advance 56 of these parts past 64, i.e., just a little more than half way.

Reading Scale 2 anti-clockwise, we make the value under the Cursor as near as may be 1548.

From inspection of the fraction its value is obviously between one-sixth and one-seventh, and without hesitation write down the decimal valne as 0·1548.

Ex. 16.—Find decimal equivalent of $\frac{1}{3475}$

Set Cursor over 3475 on Scale 1.
Read 2878 on Scale 2.

The fraction is manifestly less than $\frac{1}{3000}$ and expressed decimally will require 3 cyphers after the decimal point, so we write the answer 0·0002878.

In setting 3475 under the Cursor we note it falls between the graduations 34 and 35, and that between 34 and 35 there are 2 graduations, each advancing 5, thus 340, 345, 350. About half-way between 345 and 350 is 347, and a shade past this is 3475.

Reading Scale No. 2 the Cursor is just short of the value 288. We estimate it as 2878, and the answer, therefore, as 0·0002878.

Ex. 17.—Find the decimal equivalent of $\frac{1}{0 \cdot 0284}$

Set Cursor over 284 on Scale 1. (It is the second graduation line past 28, and the spaces count 2 each.)

The reading on Scale 2 is just past the graduation following the 35 mark, reading anti-clockwise, and where each space counts 5. We estimate the reading as 3520.

By inspection, the value of the fraction is seen to be more than $\frac{1}{36}$ and we write down the answer as 35·21.

The three preceding examples are good illustrations of the care required in scale reading in noting the value of the graduations, and whether the advance of the Scale is clockwise or anti-clockwise. Scale No. 2, the Reciprocal Scale it may be noted, is the only one graduated anticlockwise.

Ex. 18.—Find the decimal value of $\frac{1}{37}$

Set 37 on Scale 1 under Datum.
Read ·027 on Scale 2 under Datum.

Note that in reading decimal values of fractions less than $\frac{1}{10}$ there will be one cypher placed after the decimal point, and preceding the number as read from the Reciprocal Scale.

With values less than $\frac{1}{100}$ and greater than $\frac{1}{1000}$ two cyphers will precede the number and so on.

Ex. 19.—Find the decimal value of $\frac{5}{7}$

Set 5 on Scale 1 under Datum.
Set Cursor to 7 on Scale 1.
Set dial till 1 comes under Cursor.
Read ·714 under Datum.

Ex. 20.—Find the fractional value of ·1428.

Set (anti-clockwise) 1428 on Scale 2 under Datum.
Read 7 on Scale 1 under Datum.

Fractional value is therefore $\frac{1}{7}$

Ex. 21.—Find the fractional value of ·00653.

Set 653 on Scale 2 under Datum.
Read 153 on Scale 1 under Datum.

Fractional value is therefore $\frac{1}{15300}$

Note that as many cyphers must follow the 153 as there are cyphers following the decimal point in the given number.

Examples of the Use of the Scale of Logarithms (No. 3).

See also pages 2 to 7.

Ex. 22.—Find logarithm of 2675.

Set Cursor over 2675 on Scale No. 1. Read Mantissa of log., viz., 427 on Scale 3 under Cursor. As there are four figures in the number all to the left of the decimal point, the characteristic of the log. is positive, and its value is 3.

The complete log. is 3·427.

Ex. 23.—Find logarithm of 50·75.

Set Cursor over 5075 on Scale No. 1 (it lies between the first and second graduation line after 50).

Read Mantissa of log. on Scale 3 under Cursor, viz., 7055.

The characteristic of the log. (as there are two figures to left of decimal point) is 1.

The complete log. is 1·7055.

Ex. 24.—Find logarithm of 0·024076.

Set Cursor over 24076 on Scale No. 1. This is about one-third of the way between 24 (which represents 240) and the first graduation after it, which represents 242.

Read Mantissa of log. on Scale No. 3.

We make the reading 3815.

As the number is less than unity, the characteristic is negative and as there is a cypher to the right of the decimal point, its value is 2.

Therefore the logarithm of 0·024076 = $\bar{2}$·3815.

Hyperbolic Logarithms.—These which are to the base e = 2·71828 are much used in calculations relating to the expansion of gases. They can be easily derived by multiplying the common logarithm (i.e., the log. to the base 10) by 2·30258.

The exact position of this multiplier denoted by $\log_e 10$

is indicated both outside Scale No. 1 and on Scale No. 4, but for purpose of finding common logarithms Scale No. 1 must be used.

Ex. 25.—Find hyperbolic log. of 14·35.

First find common log. of 14·35.

Set cursor over 1435 on Scale No. 1.

Read Mantissa of common log., viz., 1575 on Scale 3.

As there are two figures to the left of the decimal point in the number and the number is greater than unity, the characteristic is 1, and positi e.

Therefore the log. of 14·35 is 1·1575.

Now multiply 1·575 by $\log_e 10$.

Set $\log_e 10$ on Scale No. 1 under Datum.

Turn Cursor to 1 and turn dial till 11575 comes under Cursor.

Read answer 266=hyperbolic log. under Datum.

Examples of Powers and Roots.

Ex. 26.—Find the value of $(36·7)^2$

This can be done with the Calculator in two ways, either by multiplying 36·7 by itself as an ordinary multiplication sum, as previously described, or by the method shown below.

Set 36·7 on Scale 1 under Datum.

Set Cursor to 36·7 on Scale 2.

Turn dial till 1 comes under Cursor.

Read 1347 under Datum on Scale 1.

Ex. 27.—Find the value of $(16·4)^3$.

This can be done by extended multiplication, 16·4 × 16·4 × 16·4 on Scale 1, or by the method shown below, in which we first find the square of 16·4, as in Example 26, and then multiply this result on Scale 1 by 16·4.

Thus set 16·4 on Scale 1 under Datum.

Set Cursor to 16·4 on Scale 2.

Turn dial till 16·4 on Scale 1 comes under Cursor.

Read 441 under Datum on Scale 1.

NOTE.—The result is obtained in 3 movements.

Finding Nth Powers and Nth Roots of Numbers—

with logarithms (whether N be a whole number or a fraction)

Let A be a number and suppose $x = A^n$.

Where n may be a whole number or a fraction.

Then log. x = n log. A.

Ex. 28.—Find 5th root of 51·53 (i.e., find $51·53^{\frac{1}{5}}$).

Here n=1-5th and A=51·53.

Set Cursor over 51·53 on Scale No. 1.

This is between the 3rd and 4th graduations after 50.

Read Mantissa of log. on Scale No. 3, viz., 713.

The number is more than unity, therefore the log. is positive. There are two figures to left of decimal point therefore the value of the characteristic is 1.

Therefore the log. of 51·53=1·713.

One-fifth of log. of 51·53=0·3426.

Set Cursor over 3426 on Scale No. 3.

Read 5th root of 51·53 on Scale No. 1, viz., 2·2.

Ex. 29.—Find the value of $(2·8)^4$

i.e., 2·8 × 2·8 × 2·8 × 2·8.

This may be worked out in several ways.

1st Method—By taking logs.

Set 28 on Scale 1 under Cursor.

Read Mantissa of log. on Scale No. 3, viz., ·447.

As there is only one figure to the left of the decimal point in 2·8, there will be no characteristic.

Now multiply ·447 by 4 mentally to get 4 (log 2·8).

This equals 1·788.

·788 is therefore the Mantissa of the log of $(2·8)^4$.

Set 788 on Log Scale under Cursor.

Read 614 on Scale 1 under Cursor. The answer will have two whole numbers in front of the decimal point.

Therefore $(2·8)^4$=61·4.

2nd Method.—Multiply 2·8 by itself four times on Scale 1 by the method shown in Example 6.

3rd Method.—Multiply 2·8 by itself four times and then by 1, using Scales No. 1 and 2 in conjunction, as shown in Example 8.

"**Square Roots**"—

Ex. 30.—Find the square root of 1849.

Set 1849 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial until the same number comes simultaneously under the Datum on Scale 1, and the Cursor on Scale 2.

This number, 43, is the square root of 1849.

Opposite 43 on either Scale 1 or Scale 2 will be found the value of $\frac{1}{\sqrt{1849}}$ viz.: ·02326.

It will be observed that two values may be obtained when setting in this manner to find the value of the square root of a number. For instance in Ex. 30 above we could either get 43 coming on Scales 1 and 2, when the zero line on the dial comes opposite the mid point between the datum and cursor, or we could get 13·6 when the zero line falls midway between the Datum and the Cursor.

The point to note is, that it is the first number which appears simultaneously under the Datum and Cursor on Scales 1 and 2 respectively, *when the Dial is revolved in a clockwise direction* after the initial settings that is the correct square root.

The other value, for example the 13·6 given above, is the square root of the original number (1849) multiplied by the square root of 10.

Thus $13{\cdot}6 = \sqrt{1849} \times \sqrt{10}$.

Ex. 31.—Find the 4th root of 1849.

Proceed as in Example 29 above to find the square root (43), and then obtain the square root of this number.

Set 43 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial until the same number comes simultaneously under the Datum on Scale 1 and the Cursor on Scale 2.

This (6·56) is the 4th root of 1849.

"**Cube Roots**" of numbers on Scale 1 can be read directly on one of the 3 circles which comprise the "Long-scale" No. 4.

This is more particularly described on page 16.

Ex. 32.—Find the cube root of 964.

Set 964 on Scale 1 under Datum.

Read 9876 on the outer of the 3 circles comprising the "Long-scale."

The cube root of 964 is therefore 9·876.

In fixing the magnitude of the cube root and on which circle to read the answer, the reader is referred to the notes on page 16.

Ex. 33.—Find the cube root of 1430.

Set 143 on Scale 1 under Datum.

Read 11·275 on the inner of the 3 "Long-scale" circles. It is obvious that the cube root lies between 10 and 20 and therefore must be read on the inner circle.

Other roots can be obtained by taking logarithms, as in Example 28, or if a sixth root was required it could be obtained by taking the square root of the cube root of the number.

Trigonometrical Scales.

Sines, Tangents, etc.—The values of sines, tangents, etc., are read from the Scale of angles No. 5 and No. 6 by means of the Cursor.

Read Natural Sin. or Natural Tan. on Scale No. 1.

Read Log. Sin. or Log. Tan. on Scale No. 3.

Cosine, Cotangent, Secant and Cosecant are deduced from Sine and Tangent through the following relationships.

For any given angle A:—

$$\text{Cos. } A = \text{Sin.}(90 - A); \qquad \text{Cot. } A = \frac{1}{\text{Tan } A}$$

$$\text{Sec. } A = \frac{1}{\text{Cos. } A} \qquad \text{Cosec. } A = \frac{\text{Sine } A}{1}$$

The Scale of Sines, No. 5, extends twice round the circumference of a circle. The inner circle gives angles between 35 mins. and 5 degs. 45 mins. and the value of the sine increases from 0·01 to 0·10. The outer circle gives angles between 5 degs. 45 mins. and 90 degs., and the value of the Sine increases from 0·10 to 1·0.

Ex. 34.—Find value of Natural Sine of 4° 40′.
Set cursor over over 4° 40′ on Scale No. 5.
Read Natural Sine 0·0813 on Scale No. 1.

N.B.—The number on the Scale is 813, but as the sines of all angles on the inner circle of Scale No. 5 are between 0 and 0·01, we write down the value 0·0813.

Ex. 35.—Find value of Natural Sine of 20° 30′.
Set Cursor over 20° 30′ on Scale No. 5.
Read value of Natural Sine 0·3500 on Scale No. 1.

N.B.—The angle being on the outer circle of Scale No. 5, and the angle exceeding 5° 45′, the value of the sine is between 0·1 and 1·0.

Between 20° and 25° the scale is graduated at intervals of 20′ so that 20° 30′ falls midway in the second interval following 20°.

Ex. 36.—Find the value of cosecant 20° 30′.
Set Cursor as in Ex. 35 above.

Read value $\frac{1}{\sin\ 20^\circ\ 30'}$, which is cosecant 20° 30′, on Scale 2.

This value is (reading anti-clockwise) 2·855.

Ex. 37.—Find the value of Cosine 48°.
This equals sin. (90°—48°)=sin. 42°.
Set Cursor over 42° on Scale 5.
Read cosine 48° on Scale 1 under Cursor= ·669.
On the Reciprocal Scale under Cursor is shown the value of the secant of 48°, which equals 1·494. (Read contra-clockwise).

Ex. 38.—Find the value of natural tangent 25°.
Set 25° on Scale 6 under Cursor.
Read tan 25°= ·466 on Scale 1 under Cursor.
Cotangent 25° will be read under Cursor on the Reciprocal Scale, and equals 2·145.

NOTE.—In the above examples the natural sines, cosines, tangents, etc., are given. If the log. values of these functions are required they must be read on the Log. Scale (No. 3).

In reading the values of log. sines of angles, the characteristic of the logs. for all angles between **35** mins. and **5** degs. **45** mins. is **8**, and for all angles between **5** deg. **45** mins. and **90** degs. is **9**.

The Mantissa only of the log. is read on Scale No. 3.

Ex. 39.—Find value of Log. Sine of 27° 20′.
Set Cursor over 27° 20′ on Scale No. 5.
Read Mantissa of log. sine on Scale No. 3=662.
As the angle is on the outer circle of the Sine Scale, the log sine is 9·662.

Ex. 40.—Find value of Log. Sine of 4° 25′.
Set Cursor over 4° 25′ on Scale No. 5.
Read Mantissa of log. sine on Scale No. 3=8865.
As the angle is on the inner circle of the Sine Scale, the complete log. is 8·8865.

Mensuration of Circles.

Ex. 41.—Find the area of a circle 3½ inches diameter.

$$\text{Area} = d^2 \times \frac{\pi}{4}$$
$$= 3{\cdot}5 \times 3{\cdot}5 \times {\cdot}7854.$$

Set 3·5 on Scale 1 under Datum.
Set Cursor to 3·5 on Scale 2.

Turn dial till $\frac{\pi}{4}$ (gauge point on outer circle) comes under Cursor.

Read area, 9·62 square inches on Scale 1 under Datum.

Ex. 42.—Find circumference of a circle 9·3 inches diameter.
Set 93 on Scale 1 under Datum.
Set Cursor to 1.
Turn dial till π (gauge point on outer circle) comes under Cursor.

Read circumference, 29·2, under Datum on Scale 1.

This example may, of course, be worked out on the Long-scale (No. 4) in a similar manner; the value of π as well as the diameter being taken on the Long-scale. This will give a closer approximation, viz., ~~29·33~~. 27·92

Ex. 43.—Find the diameter of a circle whose area is 227 square inches.

Dia. $=\sqrt{\text{Area}}\times$ C.

C = 1·12838, and is marked as a gauge point on outer circle.

Set 227 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial till same number comes under Datum on Scale 1 as comes under Cursor on Scale 2. This is the square root of 227.

Turn Cursor to 1.

Turn dial till C comes under Cursor.

Read answer, 17, under Datum on Scale 1.

Examples using Conversion Gauge Points on Scale No. 1.

Ex. 44.—How many yards are there in 396 metres?

Set 396 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial till gauge point "Metres to Yards" comes under Cursor.

Read answer, 433, on Scale 1 under Datum.

Ex. 45.—How many metres are there in 660 yards?

Set 660 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial till gauge point "Yards to Metres" comes under Cursor.

Read answer, 603, on Scale 1 under Datum.

Ex. 46.—How many lbs. are there in 86 Kilogrammes?

Set 86 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial till gauge point "Kg. to Lbs." comes under Cursor.

Read answer, 189·3, on Scale 1 under Datum.

Ex. 47.—How many kilogrammes are there in 56 lbs.?

Set 56 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial till gauge point "Lbs. to Kg." comes under Cursor.

Read answer 25·4 on Scale 1 under Datum.

Ex. 48.—How many centimetres are there in 24 inches?

Set 24 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial till gauge point "Ins. to Cm." comes under Cursor.

Read answer, 60·9, on Scale 1 under Datum.

Ex. 49.—How many inches are there in 80·5 centimetres?

Set 80·5 on Scale 1 under Datum.

Set Cursor to 1.

Turn dial till gauge point "Cm. to Ins." comes under Cursor.

Read answer, 31·7, on Scale 1 under Datum.

Examples in Percentages and Proportion are given on pages 46 and 47.

DISCOUNT.

Ex. 50.—What is the wholesale price of an article subject to a discount of 20 per cent., the retail price of which is 15/-

Set 1. to Datum.

Set Cursor to 15.

Turn dial to 80 (20 backwards) representing 20 per cent.

Read 12/- under Cursor.

Ex. 51.—What is the wholesale price of an article subject to 12½ per cent, the retail price of which is 52/6?

Set 1. to Datum.

Set Cursor to 52·5 (52/6).

Turn dial to 87·5 (12·5 divisions backwards, representing 12½ per cent).

Read nearly 46 under Cursor, which we should estimate as 45/11.

TRIGONOMETRY.

Mathematical Principles.

If in a triangle the angles are denoted by A, B, C, of which C is a right angle (90°) ; and the sides opposite the angles are denoted by *a*, *b*, *c*, the letters being arranged clockwise.

Then the following relations exist between the several angles and sides.

$\text{Sin } A = \frac{a}{c}$ $\quad \sin B = \frac{b}{c}$ $\quad \cos A = \frac{b}{c}$ $\quad \cos B = \frac{a}{c}$

$\tan A = \frac{a}{b}$ $\quad \text{cotan } A = \frac{b}{a}$ $\quad \text{secant } A = \frac{c}{b}$

$$\text{cosecant } A = \frac{c}{a}$$

If C is not a right angle, the sine, cosine, etc., still have same values, but *a* and *b* are not now the sides of the actual triangle, but of an imaginary triangle with B C perpendicular to C A, and the following relationships hold for triangle A, B, C.

$$A + B + C = 180 \text{ degrees.}$$

$$\frac{a}{b} = \frac{\sin A}{\sin B} \quad \frac{b}{c} = \frac{\sin B}{\sin C} \quad \frac{c}{a} = \frac{\sin C}{\sin A} \quad \ldots \; (1)$$

$$a^2 = b^2 + c^2 - 2bc \text{ cosine } A \ldots\ldots\ldots (2)$$

Cosine A is itself minus, and the whole of the last factor of equation (3) becomes plus, if A is greater than 90°.

If A is 90° this last factor disappears.

Sine (180—A) = Sine A
Cosine (180—A) = — Cosine A
Cotangent A = Tangent (90—A)

The area of a triangle is .... $\frac{ab \sin C}{2}$

$$\text{Tangent } \frac{A-B}{2} = \frac{a-b}{a+b} \text{ Cotan } \frac{C}{2} \ldots\ldots\ldots (3)$$

$$\text{Cotangent } \frac{C}{2} = \text{Tan. } \frac{A+B}{2} \ldots\ldots\ldots (4)$$

Formulae (3) and (4) are much used for solving triangles when two sides and the included angle are known.

The following method is also used when two sides say *b* and *t* of a triangle, B T R, and the included angle R are known or can be calculated from other known data.

The third side *r* (usually the range in an artillery problem) can then be found as follows :—

$$\text{First calculate the ratio } \quad m = \frac{b}{t} \ldots\ldots\ldots (5)$$

$$\text{Now } \frac{b}{t} = \frac{\sin B}{\sin T} \quad \text{Also } B = 180 - T - R \ldots\ldots (6)$$

$$\text{And } \quad \sin B = \sin (T+R) \ldots\ldots\ldots (7)$$

Set the Ratio *m* under Datum, and set Cursor over 1 both on Scale 1.

This is automatically the result of the process of dividing *b* by *t*

Turn Dial until, on the Sine Scale (No. 5), the angle under Datum, is the sum of the angle under the Cursor and the angle R.

Then the angle under the cursor is T.

The angles T and R and the side *t* being now known, the side r is calculated by the usual formula. See Ex. 9.

Thus, if *b* and *t* are 5,000 yds. and 900 yds. respectively, and R is 40°.

The ratio $m = 5 \cdot 555$, and T = 7° 38′.

The range *r* is therefore 4,350 yds.

HINTS ON CALCULATIONS.

Simplifying a Decimal Quantity.—Regard should be paid to the value of the terminal figures which are struck off. If it be desired to contract 15·647 to four significant figures, then 15·65 is nearer than 15·64, because 7 is nearer 10 than 1; but if the original number had been 15·642, then 15·64 would have been the closer approximation.

Fractional Value of Decimals.—A misconception of the fractional value of decimals sometimes causes mistakes, especially if there are cyphers between the decimal point and the first digit. To avoid this, remember that when expressed as a fraction the number of cyphers in the denominator is the same as the number of figures after the decimal point in the number.

For example :—

$$3{\cdot}04 = 3\frac{4}{100} \text{ or } \frac{304}{100} \qquad {\cdot}96 = \frac{96}{100} \qquad {\cdot}002 = \frac{2}{1000}$$

Locating Position of Decimal Point.—When factors containing decimals are multiplied by ordinary arithmetic there is no difficulty in locating the decimal point; one simply ticks off the same number of decimal figures in the answer as there are in the factors; but this cannot always be done when the operation is performed with a scale or scales.

The number of significant figures in the answer (i.e., the number which can be written down) cannot be stated beforehand. It will depend on several things, viz., the size of the factors; the degree of subdivision of the scale; the accuracy of the scale or scales, and the accuracy of the operator in setting and reading them.

The position of the decimal however determines the accuracy of the answer, and its location is important.

Usually the answer is known with some degree of approximation, or can be determined from inspection of the factors, and when the operator is familiar with a logarithmic scale (straight or circular), he generally decides in this way, as being the safest and quickest.

The writer is not enamoured of Rules however and prefers, as nearly every user of a logarithmic rule does, to work out the position of the decimal point from first principles by mental arithmetic and rough cancelling. In his opinion arbitrary rules are a tax on memory, and liable to mislead.

In such a case as the following, which may be taken as an ordinary calculation, rules would be of little help in deciding the position of the decimal when writing down the answer.

$$\frac{6{\cdot}92 \times 746 \times 19{\cdot}2 \times 9}{2876 \times 92{\cdot}5}$$

Whereas, we could reason mentally about it thus :—

6·92 is practically 7.
7 into 2876 is roughly 400.
400 into 746 is roughly 2.
2 into 92·5 is roughly 45.

This would be in the denominator, and in the numerator there would still be left 19·2 × 9, which is roughly 170; and 170 divided by 45 would obviously have a value less than 10 though more than unity.

Therefore, in writing down the three or four significant figures given by the Calculator as the answer, the operator would only write one figure in front of the decimal point.

A rough estimate like the above takes less time to make than to describe, and, in the writer's opinion, is safer than a hard rule, which may be imperfectly remembered and liable to mislead.

Use of Proper Units.—When making calculations, the quantities should be expressed in proper units.

Different things often require to be multiplied together but the answer has only *one quality*. It may be money, weight, force, area, etc.

If an area is desired in square feet as a product of linear dimensions, these dimensions must be expressed in feet. If this is not done, account must be taken of the fact in the answer.

It does occasionally happen that some quantities are in one unit, and others in another, e.g., in certain formulae depth in inches. Weights of metal bars are generally given in lbs. per foot run, although sectional dimensions are in inches. These points should be borne in mind.

Problems in Percentages.—In speaking of percentages, confusion often arises through inattention to the basis on which it is measured. If A's salary is £75 and B's £50, it would be true to say A's salary was 50 per cent greater than B's, and equally true to say that B's salary was 33 per cent less than A's. The fact is only expressed in two different ways.

There can be no misapprehension in any case if the quantity representing the 100 is made clear. Set the question as a problem in fractions, thus :—

Example.—In an examination 27 scholars pass 1st class ; 35, 2nd class ; and 63, 3rd class. Express the various numbers as percentages of the whole.

Here 27 + 35 + 63 = 125

and this total must be regarded as a 100 base which has to be divided into three similar proportions.

Therefore if x, y, z are the three percentages, we have the following relationship :—

$$\frac{27}{125} = \frac{x}{100} \text{ and } x = \frac{100 \times 27}{125} = 21 \cdot 6 \text{ per cent.}$$

$$\frac{35}{125} = \frac{y}{100} \text{ and } y = \frac{100 \times 35}{125} = 28 \text{ per cent.}$$

$$\frac{63}{125} = \frac{z}{100} \text{ and } z = \frac{100 \times 63}{125} = 50 \cdot 4 \text{ per cent.}$$

For this class of question the instrument is very convenient. Set 1·0 on Scale No. 1 under datum line and set cursor to 125 (i.e., 12·5). Rotate dial until the several figures 27, 35, 63, come under the cursor and read the several percentages under the datum.

Problems in Proportion.—Set the question in simple fractional form as follows : Where A, B, C are certain known quantities and x is the unknown quantity.

$$\frac{A}{B} = \frac{C}{x}$$

Each of these quantities may be in the numerator or the denominator, as the operator finds it convenient in setting down their relationship, but must, of course, be done correctly. Then by cross-multiplication we have :—

$$A \times x = B \times C \text{ and } x = \frac{B \times C}{A}$$

Example 1.—If 15 men do a task in 28 days, in how many days will 21 men do it, assuming they do it at the same rate ?

Obviously more men will take less time in the ratio of 15 to 21, and if x is the number of days, then

$$\frac{x}{28} = \frac{15}{21} \text{ and } x = \frac{28 \times 15}{21} = 20 \text{ days.}$$

EXAMPLE 2.—If a task takes 18 men 36 days, how many men will be required to do it in 27 days.

Obviously more men will be required in proportion to the increased speed at which the task must be done, and therefore—

$$\frac{36}{27} = \frac{x}{18} \text{ and } x = \frac{36 \times 18}{27} = 24 \text{ men.}$$

Practice in Reading and Setting Scales.—To get accustomed to reading the Primary Scale No. 1 and the Long-Scale No. 4, and their graduations, the learner will find it good practice to work through a multiplication table, thus :—

Set 2 on Scale 1 under Datum.

Set Cursor to 1.

Turn, in succession, all figured graduations past Cursor

Note the values which pass the Datum are twice those which pass the Cursor.

Thus 2 × 11 = 22 ; 2 × 12 = 24 ; 2 × 13 = 26 ; etc.

Do the same for 3 or other simple number, and proceed to such multipliers as 3·1, etc. This teaches the learner to read parts of the scale not figured, or where the graduations are counted as 1, 2, or 5, according to their nature.

Metrical Equivalents of British Imperial Weights and Measures.

MEASURES OF LENGTH.

BRITISH.		FRENCH.	
Inch	= 2·5399 centimetres	Millimetre	= 0·03937in
Foot	= 3·0479 decimetres.	Centimetre	= 0·3937in.
Yard	= 0·9143 metre.	Decimetre	= 3·937in.
Fathom	= 1·8287 metre	Metre	= 39·37079in.; 3·2808ft.; 1·0936yd.
Pole	= 5·0291 metres.	Kilometre	= 1093·633yds.
Furlong	= 201·1643 metres.	Myriametre	= 6·2138 miles.
Mile	= 1609·3146 metres.	Noeud	= Eng. nautical mile
Nautical Mile	= 1855·020 metres.		

SUPERFICIAL MEASURES.

Sq. inch	= 0·000645 sq. metre.	Acre	= 0·4047 hectare.
Sq. foot	= 0·0929 sq. metre.	Sq. metre	= 1·1960 sq. yd.
Sq. yard	= 0·836 sq. metre.	Are	= 0·0988 rood.
Rod	= 25·2915 sq. metre.	Hectare	= 2·4711 acres.
Rood	= 10·1167 ares		

VOLUME.

1 cu. inch	= 16·387 cu. centimetres.	1 cu. centimetre = ·061 cu. inch
1 cu. foot	= ·0283 cu. metres.	1 cu. decimetre = 61·024 cu. ins.
	= 28·317 litres.	1 litre = 1000 cu. centimetres.
1 cu. yard	= ·7645 cu. metres.	1 litre = 1·7598 pints.
	= 764·558 litres.	1 cu. metre = 35·3148 cu. feet.
1 gallon	= 4·5459 litres.	1 cu. metre = 1·3079 cu. yards
	= ·1605 cu. ft.	
	= 277·27 cu. inches.	
1 U.S.A. gallon	= 231 cu. inches	
	= ·8325 imperial gallon	

WEIGHTS.

(Troy) Grain	= 0·065 gramme.	Gramme	= 15·433 troy grains; 0·643dwt
Pennyweight	= 1·555 gramme.	Kilogrm.	= 15,433·0 troy grains; 2·679 troy lb.; 2·205 avoir. lb.
Ounce	= 31·103 grammes.	Myriagramme or 10 kilos.	= 22·0462 lb.
Pound (5760grs.)	= 373·220 grammes.; 0·3732 kilogr'me.	Quintal or 100 kilos.	= 220·4621 lb.
(Avrdps.) Dram.	= 1·77 gramme.	Tonneau or Millier 1000 kilos.	= 0·9842 of a ton
Ounce	= 28·35 grammes.		
Pound (7000 grs.)	= 453·57 grammes.; 0·4536 kilogr'me.		
Cwt.	= 50·8 kilogrammes.		
Ton	= 1016·0 kilogr'mes.		

Useful Electric Formulæ.

1 H.P. = 33,000 ft. lbs. per minute.

$$\text{Torque} = \frac{\text{H.P.} \times 33{,}000}{\text{Revs. p. min.} \times 2\pi}$$

1 H.P. = 746 Watts = ·746 K.W.

1 B.O.T. unit = 1,000 Watt hours or 1 K.W. hour.

$$\text{Input of Motor in K.W.} = \frac{\text{H.P.} \times \cdot 746}{\text{efficiency}}$$

$$\text{Current in Amps.} = \frac{\text{Watts}}{\text{Volts}}$$

$$\text{Output of Motor in H.P.} = \frac{\text{Input in K.W.} \times \text{Efficiency}}{\cdot 746}$$

Apparent Power of Single-phase Circuit in

$$\text{K.V.A.} = \frac{\text{Volts} \times \text{Amps.}}{1{,}000}$$

Real Power of Single-phase Circuit in

$$\text{K.W.} = \frac{\text{Volts} \times \text{Amps.}}{1{,}000} \times \text{Power factor}$$

Apparent Power of Two-phase Circuit in

$$\text{K.V.A.} = 2 \times \frac{\text{Volts} \times \text{Amps.}}{1{,}000}$$

Real Power of Two-phase Circuit in

$$\text{K.W.} = 2 \times \frac{\text{Volts} \times \text{Amps.}}{1{,}000} \times \text{Power factor}$$

Apparent Power of Three-phase Circuit in

$$\text{K.V.A.} = 1{\cdot}73 \times \frac{\text{Volts} \times \text{Amps.}}{1{,}000}$$

Real Power of Three-phase Circuit in

$$\text{K.W.} = 1{\cdot}73 \times \frac{\text{Volts} \times \text{Amps.}}{1{,}000} \times \text{Power factor}$$

Input of Single-phase, Two-phase, or Three-phase Motor in

$$\text{K.V.A.} = \frac{\text{H.P.} \times \cdot 746}{\text{Efficiency} \times \text{Power factor}}$$

Output of Single-phase, Two-phase, or Three-phase Motor in

$$\text{H.P.} = \frac{\text{Input in K.V.A.}}{\cdot 746} \times \text{Efficiency} \times \text{Power factor}$$

Current Input of Single-phase Motor

$$= \frac{\text{K.V.A.} \times 1{,}000}{} = \frac{\text{H.P.} \times 746}{\text{Efficiency} \times \text{Power factor} \times \text{Volts}}$$

Current Input of Two-phase Motor

$$= \frac{\text{K.V.A.} \times 1{,}000}{2 \times \text{Volts}} = \frac{\text{H.P.} \times 746}{\text{Efficiency} \times \text{Power factor} \times 2 \times \text{Volts}}$$

Current Input of Three-phase Motor

$$= \frac{\text{K.V.A.} \times 1{,}000}{1{\cdot}73 \times \text{Volts}} = \frac{\text{H.P.} \times 746}{\text{Efficiency} \times \text{Power factor} \times 1{\cdot}73 \times \text{Volts}}$$

Output of Alternators and Transformers in

$$\text{K.V.A.} = \frac{\text{Output in K.W.}}{\text{Power factor}}$$

Compound Conversion Factors.

ENGLISH TO METRICAL.

Pounds per lineal foot	×	1·488	= kilos. per lineal metre.
Pounds per lineal yard	×	0·496	= kilos. per lineal metre.
Tons per lineal foot	×	3333·33	= kilos. per lineal metre.
Tons per lineal yard	×	1111·11	= kilos. per lineal metre.
Pounds per square inch	×	0·0703	= kilos. per square centimetre
Tons per square inch	×	1·575	= kilos. per square millimetre
Pounds per square foot	×	4·883	= kilos. per square metre.
Tons per square foot	×	10·937	= tonnes per square metre.
Tons per square yard	×	1·215	= tonnes per square metre.
Pounds per cubic yard	×	0·5933	= kilos. per cubic metre.
Pounds per cubic foot	×	16·020	= kilos. per cubic metre.
Tons per cubic yard	×	1·329	= tonnes per cubic metre.
Grains per gallon	×	0·01426	= grammes per litre.
Pounds per gallon	×	0·09983	= kilos. per litre.
Gallons per square foot	×	48·905	= litres per square metre.
Foot-pounds	×	0·1382	= kilogrammetres.
Foot-tons	×	0·328	= tonne-metres.
Horse-power	×	1·0139	= force de cheval.
Pounds per H.P.	×	0·477	= kilos. per cheval.
Square feet per H.P.	×	0·0916	= square metre per cheval.
Cubic feet per H.P.	×	0·0279	= cubic metre per cheval.
Heat units	×	0·252	= calories.
Heat units per square foot	×	2·713	= calories per square metre.

METRICAL TO ENGLISH.

Kilos. per lineal metre	×	0·672	= pounds per lineal foot.
Kilos. per lineal metre	×	2·016	= pounds per lineal yard.
Kilos. per lineal metre	×	0·0003	= tons per lineal foot.
Kilos. per lineal metre	×	0·0009	= tons per lineal yard.
Kilos per square centimetre	×	14·223	= pounds per square inch
Kilos. per square millimetre	×	0·635	= tons per square inch.
Kilos. per square metre	×	0·2048	= pounds per square foot.
Tonnes per square metre	×	0·0914	= tons per square foot.
Tonnes per square metre	×	0·823	= tons per square yard.
Kilos. per cubic metre	×	1·685	= pounds per cubic yard.
Kilos. per cubic metre	×	0·0624	= pounds per cubic foot.
Tonnes per cubic metre	×	0·752	= tons per cubic yard.
Grammes per litre	×	70·1	= grains per gallon.
Kilos. per litre	×	10·02	= pounds per gallon.
Litres per square metre	×	0·0204	= gallons per square foot
Kilogrammetres	×	7·233	= foot-pounds.
Tonne-metres	×	3·09	= foot-tons.
Force de cheval	×	0·9863	= horse-power.
Kilos. per cheval	×	2·235	= pounds per H.P.
Square metre per cheval	×	10·913	= square foot per H.P.
Cubic metre per cheval	×	35·806	= cubic feet per H.P.
Calories	×	3·968	= heat units.
Calories per square metre	×	0·369	= heat units per square foot