HOW TO USE THE

DECI·LOG LOG SLIDE RULE

DECI• LOG LOG

Trade Mark

by

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SPECIAL FEATURES
of the
DECI • LOG LOG RULE

Among the special features of the DECI • LOG LOG Slide Rule are the following:

1—The Log Log scales *all* refer to a single D scale, and are so arranged that logarithms or cologarithms of numbers to base 10 may be read directly from this scale.

2—Powers (including those of base *e*) with both positive and negative exponents can be read with one setting of the hairline. Hyperbolic functions are easily computed.

3—Reciprocals can be read with the decimal point placed.

4—Exponential equations in which the unknown exponent is a negative number are easily solved without transferring readings from one scale to another.

5—The scales have a greater range of direct reading (10^{-10} to 10^{+10}) than those of conventional rules.

6—A table to aid in determining the scale on which the power is found, or the decimal point of the exponent, is printed on the slide.

7—Square roots may be found on a scale which is *double* the length of the C scale itself, with resulting increased accuracy and convenience.

8—Cube roots may be found on a scale which is *three* times the length of the C scale, instead of on the C scale itself, with greater accuracy and convenience.

9—A tangent scale for angles from 45° to 84.3° is provided.

10—The scale divisions are unusually sharp and readable, and the rule is made of metal, which gives dimensional stability and makes it relatively indestructible.

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PART I—SLIDE RULE OPERATION

INTRODUCTION

The table below shows some of the mathematical operations which can be done easily and quickly with an ordinary slide rule.

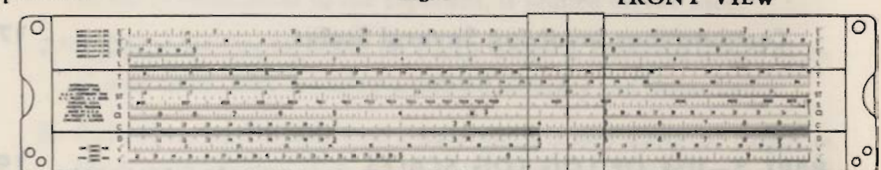
OPERATIONS	INVERSE OPERATIONS
Multiplying two or more numbers	Dividing one number by another
Squaring a number	Finding the square root of a number
Cubing a number	Finding the cube root of a number
Finding the logarithm of a number	Finding a number whose logarithm is known
Finding the sine, cosine, or tangent of an angle	Finding an angle whose sine, cosine, or tangent is known

Various *combinations* of these operations (such as multiplying two numbers and then finding the square root of the result) are also easily done. Numbers can be added or subtracted with an ordinary slide rule, but it is usually easier to do these operations by arithmetic.*

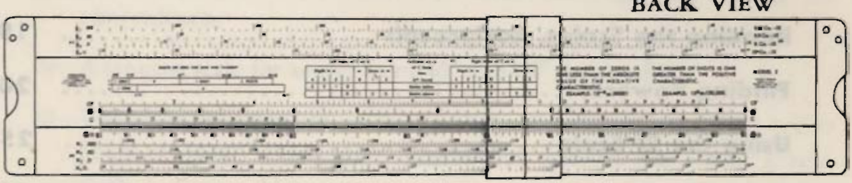
The slide rule consists of three parts: (1) the rule; (2) the slide; (3) the "runner" or indicator. On the rule and the slide several number scales are printed.

Fig. 1

FRONT VIEW



BACK VIEW



Each scale is named by a letter (C, D, L, S, T) or other symbol ($\sqrt{\ }$, $\sqrt[3]{\ }$) at both ends.

In order to use a slide rule, a computer must know: (1) how to read the scales; (2) how to "set" the slide and runner for each operation to be done; and (3) how to determine the decimal point in the result.

It is best to learn how to multiply first.

*By putting special scales on a slide rule, these and certain other operations much more difficult than those shown in the table above can be done easily.

MULTIPLICATION

The scale labeled C (on the slide) and the scale D (on the rule itself) are used for multiplication. These two scales are exactly alike. The total length of these scales has been separated into many smaller parts by fine lines called "graduations."

If these scales were long enough the total length of each would be separated into 1000 parts. First they would be separated into 10 parts. Then each of these parts would be again separated into 10 parts. Finally each of these smaller parts would be separated into 10 parts, making 1000 parts in all. On the C and D scales the parts are not all equal. They are longer at the left-hand end than at the right-hand end. At the left end there is enough space to *print* all of the fine graduations. Near the right end of a short rule there is not enough room to print all the graduations. In using the rule, however, you soon learn to *imagine* that the lines are all there, and to use the *hairline* on the indicator to help locate where they would be.

Reading the Scales

The marks which first separate the entire D scale into ten parts are called the *primary* graduations. The points of separation are labeled **2**, **3**, **4**, etc., and the end points are both labeled **1**. These are the largest numerals printed on the rule. Do not confuse these with the smaller numerals 1, 2, 3, etc., to 9 which are found at the left end between the large **1** and **2**. The line above the **1** on the left end is called the *left index;* the line above **1** on the right is the *right index*.

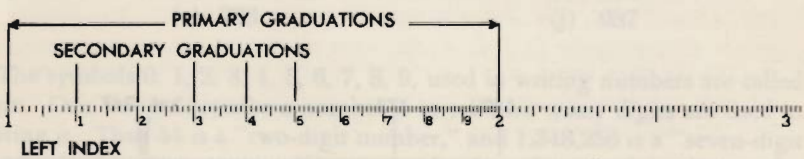


Fig. 2

Simple examples of multiplication can now be done. Numbers that are to be multiplied are called *factors*. The result is called the *product*. Thus in the statement $6 \times 7 = 42$, the numbers 6 and 7 are factors, and 42 is the product.

EXAMPLE: Multiply 2×3.

Setting the Scales: Set the left index of the C scale on **2** of the D scale. Find **3** on the C scale, and below it read the product, **6**, on the D scale.

Think: The length for 2 plus the length for 3 will be the length for the product. This length, measured by the D scale, is 6.

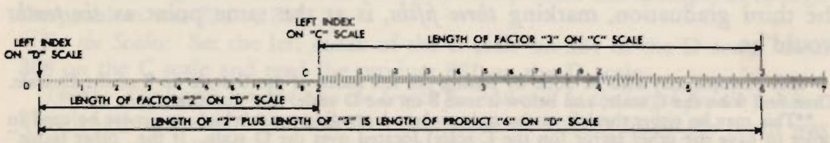


Fig. 3

EXAMPLE: Multiply 4 × 2.

Setting the Scales: Set the left index of the C scale on **4** of the D scale. Find **2** on the C scale, and below it read the product, **8**, on the D scale.

Think: The length for 4 plus the length for 2 will be the length for the product. This length, measured by the D scale, is 8.*

***Rule for Multiplication:* Over one of the factors on the D scale, set the index of the C scale.** Locate the other factor on the C scale, and directly below it read the product on the D scale.**

Next notice again that the distance between **1** and **2** on the D scale has been separated into ten parts, marked with smaller numerals 1, 2, 3, etc. These are *secondary* graduations. Each of the spaces between the large numerals **2** and **3**, between **3** and **4**, and between the other primary graduations is also divided into ten parts. Numerals are not printed beside these smaller secondary graduations because it would crowd the numerals too much.

The space between each secondary graduation at the left end of the rule (over to primary graduation **2**) is separated into ten parts, but these shortest graduation marks are not numbered. In the middle part of the rule, between the primary graduations **2** and **4**, the smaller spaces between the *secondary* graduations are separated into five parts. Finally, the still smaller spaces between the secondary graduations at the right of **4** are only separated into two parts.

To find 173 on the D scale, look for primary division **1** (the left index), then for secondary division 7 (numbered) then for smaller subdivision 3 (not numbered, but found as the 3rd very short graduation to the right of the longer graduation for 7).

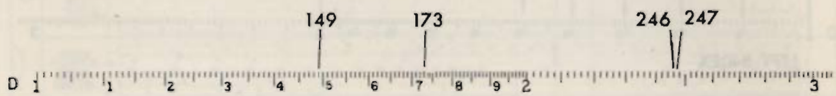


Fig. 4

Similarly, 149 is found as the 9th small graduation mark to the right of the 4th secondary graduation mark to the right of primary graduation **1**.

To find 246, look for primary graduation **2**, then for the 4th secondary graduation after it (the 4th long line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are fifths. Since $\frac{3}{5} = \frac{6}{10}$, then the third graduation, marking *three fifths*, is at the same point as *six tenths* would be.

*This example may also be done by setting the left index of the C scale on **2** of the D scale. Then find **4** on the C scale, and below it read **8** on the D scale. See drawing above.

**This may be either the left or the right index, depending upon which one must be used in order to have the other factor (on the C scale) located over the D scale. If the "other factor" falls outside the D scale, the "other index" is used.

The number 247 would be half of a small space beyond 246. With the aid of the *hairline* on the runner the position of this number can be located approximately by the eye. The small space is mentally "split" in half.

The number 685 is found by locating primary graduation 6 and then secondary graduation 8 (the 8th long graduation after 6). Between secondary graduations 8 and 9 there is one short mark. Think of this as the "5 tenths" mark. The location of 683 can be found approximately by mentally "splitting" the space between 680 and 685 into fifths, and estimating where the 3rd "fifths" mark would be placed. It would be just a little to the right of halfway between 680 and 685.

On the scale below are some sample readings.

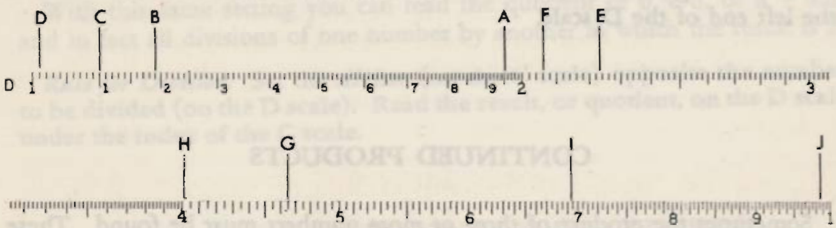


Fig. 5

(a) 195
(b) 119
(c) 110
(d) 101
(e) 223
(f) 206
(g) 465
(h) 402
(i) 694
(j) 987

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used in writing numbers are called *digits*. One way to describe a number is to tell how many digits are used in writing it. Thus 54 is a "two-digit number," and 1,348,256 is a "seven-digit number." In many computations only the first three or four digits of a number need to be used to get an approximate result which is accurate enough for practical purposes. Usually only the first three digits of a number can be "set" on the slide rule scales. If the first digit of a number is 1, however, the number is located near the left end of the rule and the first four digits can be "set." In the majority of practical problems this degree of accuracy is sufficient.

Multiplication of numbers having three digits can now be done.

EXAMPLE: Multiply 2.34 × 36.8.

Estimate the result: First note that the result will be roughly the same as 2 × 40, or 80; that is, there will be two digits to the left of the decimal point. Hence we can ignore the decimal points for the present and multiply as though the problem was 234 × 368.

Set the Scales: Set the left index of the C scale on 234 of the D scale. Find 368 on the C scale and read the product 861 on the D scale.

Think: The length for 234 plus the length for 368 will be the length for the product. This length is measured on the D scale. Since we already knew the result was somewhere near 80, the product must be 86.1, approximately.

EXAMPLE: Multiply 28.3 × 5.46.

Note first that the result will be about the same as 30 × 5, or 150. Note also that if the left index of the C scale is set over 283 on the D scale, and 546 is then found on the C scale, the slide projects so far to the right of the rule that the D scale is no longer below the 546. When this happens, the *other* index of the C scale must be used. That is, set the *right* index of the C scale over 283 on the D scale. Find 546 on the C scale and below it read the product on the D scale. The product is approximately 154.5.

This illustrates how in simple examples the decimal point can be placed by use of the estimate (the result was *estimated* to be near 150), and also shows how "four-digit accuracy" can often be obtained when the result falls at the left end of the D scale.

CONTINUED PRODUCTS

Sometimes the product of three or more numbers must be found. These "continued" products are easy to get on the slide rule.

EXAMPLE: Multiply 38.2 × 1.65 × 8.9.

Estimate the result as follows: 40 × 1 × 10 = 400. The result should be, very roughly, 400.

Setting the Scales: Set left index of the C scale over 382 on the D scale. Find 165 on the C scale, and *set the hairline* on the indicator on it.* Move the index on the slide under the hairline. In this example if the *left* index is placed under the hairline, then 89 on the C scale falls outside the D scale. Therefore move the *right* index under the hairline. Move the hairline to 89 on the C scale and read the result under it on the D scale (561).

Below is a *general rule* for continued products: $a \times b \times c \times d \times e \cdots$

Set hairline of indicator at a on D scale.

Move index of C scale under hairline.

Move hairline over b on the C scale.

Move index of C scale under hairline.

Move hairline over c on the C scale.

Move index of C scale under hairline.

Continue moving hairline and index alternately until all numbers have been set.

Read result under the hairline on the D scale.

*The product of 382 × 165 could now be read under the hairline on the D scale, but this is not necessary.

DIVISION

In mathematics, division is the opposite or *inverse* operation of multiplication. In using a slide rule this means that the process for multiplication is reversed. To help in understanding this statement, set the rule to multiply 2×4 (see page 6). Notice the result 8 is found on the D scale under 4 of the C scale. Now to divide 8 by 4 these steps are reversed. First find 8 on the D scale, set 4 on the C scale over it, and read the result 2 on the D scale under the index of the C scale.

Think: From the length for 8 (on the D scale) *subtract* the length for 4 (on the C scale). The length for the difference, read on the D scale, is the result, or quotient.

With this same setting you can read the quotient of $6 \div 3$, or $9 \div 4.5$, and in fact all divisions of one number by another in which the result is 2.

Rule for Division: **Set the *divisor* (on the C scale) opposite the number to be divided (on the D scale). Read the result, or quotient, on the D scale under the index of the C scale.**

COMBINED MULTIPLICATION AND DIVISION

Many problems call for both multiplication and division.

EXAMPLE: $\dfrac{42 \times 37}{65}$.

First, set the division of 42 by 65; that is, set 65 on the C scale opposite 42 on the D scale.* Move the hairline on indicator to 37 on the C scale. Read the result 239 on the D scale under the hairline. Since the fraction $\frac{42}{65}$ is about equal to $\frac{2}{3}$, the result is about two-thirds of 37, or 23.9.

EXAMPLE: $\dfrac{273 \times 548}{692 \times 344}$

Set 692 on the C scale opposite 273 on the D scale. Move the hairline to 548 on the C scale. Move the slide to set 344 on the C scale under the hairline. Read the result on the D scale under the C index.

In general, to do computations of the type $\dfrac{a \times c \times e \times g \cdots}{b \times d \times f \times h \cdots}$, set the rule to divide the first factor in the numerator a by the first factor in the denominator b, move the hairline to the next factor in the numerator c; move the slide to set next factor in denominator, d, under the hairline. Continue moving hairline and slide alternately for other factors (e, f, g, h, etc.). Read the result on the D scale. If there is one more factor in the numerator than in the denominator, the result is under the hairline. If the number of factors in numerator and denominator is the same, the result is under the C index.

*The quotient, .646, need not be read.

Sometimes the slide must be moved so that one index replaces the other.

EXAMPLE: $\dfrac{2.2 \times 2.4}{8.4}$

If the rule is set to divide 2.2 by 8.4, the hairline cannot be set over 2.4 of the C scale and at the same time remain on the rule. Therefore the hairline is moved to the C index (opposite 262 on the D scale) and the slide is moved end for end to the right (so that the *left* index falls under the hairline and over 262 on the D scale). Then the hairline is moved to 2.4 on the C scale and the result .63 is read on the D scale.

If the number of factors in the numerator exceeds the number in the denominator by more than one, the numbers may be grouped, as shown below. After the value of the *group* is worked out, it may be multiplied by the other factors in the usual manner.

$$\left(\frac{a \times b \times c}{m \times n}\right) \times d \times e$$

PROPORTION

Problems in proportion are very easy to solve. First notice that when the index of the C scale is opposite 2 on the D scale, the ratio 1 : 2 or $\frac{1}{2}$ is at the *same time* set for all other opposite graduations; that is, 2 : 4, or 3 : 6, or 2.5 : 5, or 3.2 : 6.4, etc. It is true in general that for any setting the numbers for *all pairs of opposite graduations have the same ratio.* Suppose one of the terms of a proportion is unknown. The proportion can be written as $\frac{a}{b} = \frac{c}{x}$, where a, b, and c, are known numbers and x is to be found.

***Rule:* Set a on the C scale opposite b on the D scale. Under c on the C scale read x on the D scale.**

EXAMPLE: Find x if $\frac{3}{4} = \frac{5}{x}$.

Set 3 on C opposite 4 on D. Under 5 on C read 6.67 on D.

The proportion above could also be written $\frac{b}{a} = \frac{x}{c}$, or "inverted," and exactly the same rule could be used. Moreover, if C and D are interchanged in the above rule, it will still hold if "under" is replaced by "over." It then reads as follows:

Set a on the D scale opposite b on the C scale. Over c on the D scale read x on the C scale. In solving proportions, keep in mind that the position of the numbers as set on the scales is the same as it is in the proportion written in the form $\frac{a}{b} = \frac{c}{d}$.

Proportions can also be solved *algebraically*. Then $\frac{a}{b} = \frac{c}{x}$ becomes $x = \frac{bc}{a}$, and this may be computed as combined multiplication and division.

PART 2. USE OF CERTAIN SPECIAL SCALES

In the discussion which follows, it will occasionally be necessary to refer to the number of "digits" and number of "zeros" in some given numbers.

When numbers are greater than 1 the number of *digits* to the left of the decimal point will be counted. Thus 734.05 will be said to have 3 digits. Although as written the number indicates accuracy to *five* digits, only three of these are at the left of the decimal point.

Numbers that are less than 1 may be written as *decimal fractions.** Thus .673, or six-hundred-seventy-three thousandths, is a decimal fraction. Another example is .000465. In this number three zeros are written to show where the decimal point is located. One way to describe such a number is to tell how many zeros are written to the right of the decimal point before the first non-zero digit occurs.

In scientific work a zero is often written to the left of the decimal point, as in 0.00541. This shows that the number in the units' place is definitely 0, and that no digits have been carelessly omitted in writing or printing. The zeros will *not* be counted unless they are (a) at the *right* of the decimal point, (b) before or at the *left* of the first non-zero digit, and (c) are not between other digits. The number 0.000408 will be said to have 3 zeros (that is, the number of zeros between the decimal point and the 4).

EXAMPLES:

Number	Number of digits to be counted	Number of zeros to be counted
64523.	5	0
802.7	3	0
0.00457	0	2
3.05	1	0
.00206	0	2
65.023	2	0
.00006	0	4

Fig. 6

In working with logarithms, if the number is greater than 1 the *characteristic* of the logarithm is one less than the number of digits in the number. If the number is less than 1, the characteristic is a negative number, and numerically is one greater than the number of zeros.

*Only positive real numbers are being considered in this discussion.

RECIPROCALS

The CI scale on the slide is used for finding reciprocals. Notice that it *increases from right to left.* When any number is set under the hairline on the C scale, its reciprocal is found under the hairline on the CI scale, and conversely.

EXAMPLES:

(a) Find 1/2.4. Set 2.4 on C. Read .417 directly above on CI.

(b) Find 1/60.5. Set 60.5 on C. Read .1652 directly above on CI. Or, set 60.5 on CI, read .1652 directly below on C.

The CI scale is useful in replacing a division by a multiplication. Since $\frac{a}{b} = a \times 1/b$, any division may be done by multiplying the numerator (or dividend) by the reciprocal of the denominator (or divisor). This process may often be used to avoid settings in which the slide projects far outside the rule.

EXAMPLES:

(a) Find 13.6 ÷ 87.5 Consider this as 13.6 × 1/87.5. Set left index of the C scale on 13.6 of the D scale. Move hairline to 87.5 on the CI scale. Read the result, .155, on the D scale.

(b) Find 72.4 ÷ 1.15. Consider this as 72.4 × 1/1.15. Set right index of the C scale on 72.4 of the D scale. Move hairline to 1.15 on the CI scale. Read 62.9 under the hairline on the D scale.

An important use of the CI scale occurs in problems of the following type:

EXAMPLE: Find $\dfrac{13.6}{4.13 \times 2.79}$

This is the same as $\dfrac{13.6 \times (1/2.79)}{4.13}$.

Set 4.13 on the C scale opposite 13.6 on the D scale. Move hairline to 2.79 on the CI scale, and read the result, 1.180, on the D scale.

By use of the CI scale, factors may be transferred from the numerator to the denominator of a fraction (or vice-versa) in order to make the settings more readily. Also, it is sometimes easier to get $a \times b$ by setting the hairline on a, pulling b on the CI scale under the hairline, and reading the result on the D scale under the index.

SQUARE ROOTS AND SQUARES

When a number is multiplied by itself the result is called the *square* of the number. Thus 25 or 5 × 5 is the square of 5. The factor 5 is called the *square root* of 25. Similarly, since 12.25 = 3.5 × 3.5, the number 12.25 is called the square of 3.5; also 3.5 is called the square root of 12.25. Squares and square roots are easily found on a slide rule.

Square Root. Just below the D scale is another scale marked with the square root symbol, √.

Rule. **The square root of any number located on the D scale is found directly below it on the √ scale.**

EXAMPLES: Find $\sqrt{4}$. Place the hairline of the indicator over 4 on the D scale. The square root, 2, is read directly below. Similarly, the square root of 9 (or $\sqrt{9}$) is 3, found on the √ scale directly below the 9 on the D scale.

Reading the Scales. The square root scale directly below the D scale is an enlargement of the D scale itself. The D scale has been "stretched" to double its former length. Because of this the square root scale seems to be cut off or to end with the square root of 10, which is about 3.16. To find the square root of numbers greater than 10 the bottom √ scale is used. This is really the rest of the stretched D scale. The small figure 2 near the left end is placed beside the mark for 3.2, and the number 4 is found nearly two inches farther to the right. In fact, if 16 is located on the D scale, the square root of 16, or 4, is directly below it on the *bottom scale* of the rule.

In general, the square root of a number between 1 and 10 is found on the upper square root scale. The square root of a number between 10 and 100 is found on the lower square root scale. If the number has an odd number of digits or zeros (1, 3, 5, 7, . . .), the upper √ scale is used. If the number has an even number of digits or zeros (2, 4, 6, 8, . . .), the lower √ scale is used.

On the Deci-Point Slide Rule, the first three (or in some cases even four) figures of a number may be set on the D scale, and the first three (or four) figures of the square root are read directly from the proper square root scale.

The table below shows the number of digits or zeros in the number *N* and its square root.

ZEROS					or	DIGITS				
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.

Fig. 7

EXAMPLES:

(a) Find $\sqrt{248}$. Set the hairline on 248 of the D scale. This number has 3 (an *odd* number) digits. Therefore the figures in the square root are read from the upper √ scale as 1575. The result has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000}$. Set the hairline on 563 of the D scale. The number has 6 (an *even* number) digits. Read the figures of the square root on the bottom scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. Set the hairline on 1362 of the D scale. The number of *zeros* is 4 (an *even* number). Read the figures 369 on the bottom scale. The result has 2 zeros, and is .00369.

***Squaring* is the opposite of finding the square root. Locate the number on the proper bottom scale (marked $\sqrt{}$) and with the aid of the hairline read the square on the D scale.**

EXAMPLES:

(a) Find $(1.73)^2$ or 1.73×1.73. Locate 1.73 on the $\sqrt{}$ scale. On the D scale find the approximate square 3.

(b) Find $(62800)^2$. Locate 628 on the bottom scale. Find 394 above it on the D scale. The number has 5 digits. Hence the square has either 9 or 10 digits. Since, however, 628 was located on the bottom $\sqrt{}$ scale, the square has the *even* number of digits, or 10. The result is 3,940,000,000.

(c) Find $(.000254)^2$. On the D scale read 645 above the 254 of the $\sqrt{}$ scale. The number has 3 zeros. Since 254 was located on the scale for "odd zero" numbers, the result has 7 zeros, and is .0000000645.

CUBE ROOTS AND CUBES

At the top of the rule there is a cube root scale marked $\sqrt[3]{}$. It is a D scale which has been stretched to three times its former length, and then cut into three parts which are printed one below the other.

***Rule.* The cube root of any number on the D scale is found directly above it on the $\sqrt[3]{}$ scales.**

At the left end of the cube root scales a small table serves as a guide as to which scale to use.

	ZEROS				or	DIGITS			
N	11, 10, 9	8, 7, 6	5, 4, 3	2, 1	0	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12
$\sqrt[3]{N}$	3	2	1	0	0	1	2	3	4

EXAMPLES:

(a) Find $\sqrt[3]{8}$. Set the hairline over the 8 of the D scale. On the topmost scale of the rule read 2 under the hairline.

(b) Find $\sqrt[3]{27}$. Set the hairline over 27 of the D scale. On the middle $\sqrt[3]{}$ scale, find 3 under the hairline.

(c) Find $\sqrt[3]{372}$. Set the hairline over 372 of the D scale. On the bottom $\sqrt[3]{}$ scale find 719, or 7.19.

Cubing is the opposite of finding the cube root.

***Rule.* The cube of any number located on the $\sqrt[3]{\ }$ scale is found directly below it on the D scale.**

EXAMPLE: (a) Find $(32.8)^3$. Locate 32.8 on the middle $\sqrt[3]{\ }$ scale. On the D scale read directly below it the figures of the cube 353. Since 32.8 is a two digit number, found on the *middle* $\sqrt[3]{\ }$ scale, the number of digits is 5. The result is 35300 approximately.

LOGARITHMS

The L scale just above the slide is used for finding the logarithm (to the base 10) of any number.

***Rule.* Locate the number on the D scale, and read the mantissa of its logarithm (to base 10) directly above it on the L scale.**

EXAMPLE: Find log 425. Set the hairline over 425 on the D scale. Read the mantissa of the logarithm (.628) on the L scale. Since the number 425 has three digits, the characteristic is 2 and the logarithm is 2.628. As an aid to memory, the rule for finding the characteristic is printed on the back of the Deci-Point slide.

If the logarithm of a number is known, the number itself may be found by reversing the above process.

EXAMPLE: If $\log x = 3.248$, find x. Set the hairline over 248 of the L scale. Below it read the number 177 on the D scale. Then $x = 1770$ approximately.

EXAMPLE: Find log .000627. Opposite 627 on the C scale find .797 on the L scale. Since the number has 3 zeros, the characteristic is -4, and the logarithm is usually written 6.797–10, or 0.797–4.

TRIGONOMETRY

Sines and Cosines

The scale marked S is used in finding the approximate sine or cosine of any angle between 5.73 degrees and 90 degrees. Since $\sin x = \cos(90 - x)$, the same graduations serve for both sines and cosines. Thus $\sin 6° = \cos (90 - 6)° = \cos 84°$. The numbers printed at the right of the longer graduations are read when sines are to be found. Those printed at the left are used when cosines are to be found. Angles are divided decimally instead of into minutes and seconds. Thus sin 12.7° is represented by the 7th small graduation to the right of the graduation marked 78|12.

***Rule:* To find the sine or cosine of an angle on the S scale, set the hairline of the indicator on the graduation which represents the angle. Read the sine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the mantissa of the logarithm of the sine (log sin) may also be read on the L scale.**

EXAMPLES:

(a) Find $\sin x$ and also $\log \sin x$ when $x = 15° 30'$. Set left index of C scale over left index of D scale. Set hairline on 15.5° (i.e., 15° 30′). Read $\sin x = .267$ on the C scale. Read .427 on the L scale. Then the $\log \sin x = 9.427–10$.

(b) Find cos x and log cos x when $x = 42° 15'$ (or $x = 42.25°$). Observe that the cosine scale decreases from left to right, or *increases from right to left*. Set the hairline over 42.25 on the S scale (reading from the right). Find cos 42.25 = .740 on C scale. Find .869 on L scale. Hence log cos 42° 15' = 9.869–10.

Tangents

The upper T scale is used to find tangents of angles between 5.70° and 45°. These tangent ratios are all between 0.1 and 1; that is, the decimal point is at the left of the number as read from the scale. The lower T scale is used in finding tangents of angles between 45° and 84.3°. These tangents are between 1 and 10; that is, they all have one digit to the left of the decimal point.

***Rule.* Set the angle value on the graduation which represents the angle and read the tangent on the C scale. If the C and D scale have their indices exactly together, the mantissa of the logarithm of the tangent may also be read on the L scale.**

EXAMPLES:

(a) Find tan x and log tan x when $x = 9° 50'$. First note that $50' = \frac{50}{60}$ of 1 degree = .83°, approximately. Hence 9° 50' = 9.83°. Set the left index of the C scale and of the D scale opposite each other. Locate $x = 9.83°$ on the upper T scale. Read tan x = .173 on the C scale, and read .239 on the L scale. Then log tan x = 9.239–10.

(b) Find tan x when $x = 68.6°$. Use the lower T scale. Read 255 on the C scale. Since all angles on the lower T scale have tangents greater than 1 (that is, have one digit as defined above), tan x = 2.55.

Sines and Tangents of Small Angles

The sine and the tangent of angles of less than about 5.7° are so nearly equal that a single scale, marked ST, may be used for both. The graduation for 1° is marked with the degree symbol (°). To the left of it the primary graduations represent tenths of a degree. The graduation for 2° is just above the graduation for 35 on the C scale. The graduations for 1.5° and 2.5° are also numbered.

A small scale on the back of the rule shows the number of zeros in the sine of angles between 0 and 90°, and the number of zeros or digits in the tangents of most of these angles. Sines or tangents of angles on the ST scale have one zero. Sines (or cosines) of all angles on the S scale have no digits or zeros—the decimal point is at the left of figures read from the C (or D) scale. All angles located on the upper T scale also have the decimal point of the tangents at the left of the numbers. Angles located on the lower T scale have one digit in their tangents. Tangents of angles larger than 84.3° are not read from the rule; they increase rapidly and have at least two digits.

Two seldom used special graduations are also placed on the ST scale. One is marked with the symbol for minutes (') of angle, and is found just to the left of the graduation for 2°. When this graduation is set opposite any number of minutes on the D scale, the sine (or the tangent) of an angle of that many minutes may be read on the D scale under the C index.

Sin 0° = 0, and sin 1′ = .00029, and for small angles the sine increases by .00029 for each increase of 1′ in the angle. Thus sin 2′ = .00058; sin 3.44′ = .00100, and the sines of all angles between 3.44′ and 34.4′ have two zeros. Sines of angles between 34.4′ and 344′ (or 5.73°) have one zero. The tangents of these small angles are very nearly equal to the sines.

EXAMPLE: Find sin 6′. With the hairline set the "minute graduation" (′) opposite 6 located on the D scale. Read 175 on the D scale under the C index. Then sin 6′ = .00175.

The second special graduation is marked with the symbol for seconds of angle (″) and is located near the graduation for 1.2°. It is used in exactly the same way as the graduation for minutes. Sin 1″ = .0000048, approximately, and the sine increases by this amount for each increase of 1″ in the angle, reaching .00029 for sin 60″ or sin 1′.

COMPUTATIONS INVOLVING SEVERAL SCALES

Many calculations are simplified by using several different scales. In a brief manual it is impossible to describe more than a few examples.

Suppose it is necessary to compute the areas of many circles. Since the formula $A = \pi r^2$ can be written as a proportion (that is, $\frac{1}{\pi} = \frac{r^2}{A}$, the following rule will hold. Set π of the C scale opposite the index of the D scale. Locate the hairline over the value of the radius r on the $\sqrt{\ }$ scale. Read the area under the hairline on the C scale. If the diameters are known, instead of the radii, then $A = \frac{\pi}{4}d^2$ or $A = .7854d^2$. Hence set .785 on the C scale opposite the index (usually right-hand) of the D scale. Locate the hairline over the value of the diameter on the $\sqrt{\ }$ scale, and read the area under the hairline on the C scale. In similar fashion, but using the cube root scales, the volume of spheres may readily be found.

Many formulas involve both trigonometric ratios and other factors. By using several different scales such computations are easily done.

EXAMPLE: Find the length of the legs of a right triangle in which the hypotenuse is 48.3 ft. and one acute angle is 25° 20′.

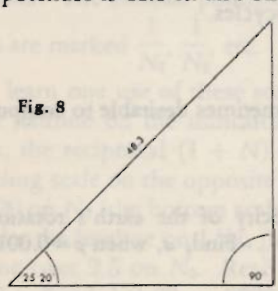


Fig. 8

The side opposite the given acute angle is equal to 48.3 sin 25° 20′. Hence we compute 48.3 × sin 25.3°. Set the index (right-hand index in this example) of the C scale on 48.3 of the D scale. Move the hairline over 25.3° on the S scale. Read 20.7 under the hairline on the D scale. Another method is to set the left index of the C scale and D scale opposite each other. Set the hairline over 25.3° on the S scale. Move the slide so that (right) index of the C scale is under the hairline. Read 20.7 on the D scale under 48.3 of the C scale. The length of the other leg is equal to 48.3 cos 25.3° or 48.3 sin 64.7° = 43.7.

Powers involving the fractional exponents 2/3 and 3/2, or in other words, combinations of squares of cube roots, and of cubes of square roots, may be done with one setting of the hairline.

Rule to compute $a^{2/3}$: Set a on the $\sqrt{\ }$ scale. Read $a^{2/3}$ on the $\sqrt[3]{\ }$ scale.

Rule to compute $a^{3/2}$: Set a on the $\sqrt[3]{\ }$ scale. Read $a^{3/2}$ on the $\sqrt{\ }$ scale.

EXAMPLE: Find the surface area of a cube which has a volume of 64 cu. in. Since $V = e^3$, then $e = \sqrt[3]{V} = V^{1/3}$. Also $S = 6e^2$ or $S = 6V^{2/3}$. If $V = 64$, then $S = 6 \times 64^{2/3}$. To find $64^{2/3}$, set the hairline over 64 on the $\sqrt{\ }$ scale. Read on the $\sqrt[3]{\ }$ scale, $64^{2/3} = 16$.

EXAMPLE: A formula sometimes used in aeronautical computations is $P_n = \left(\frac{W_n}{W_o}\right)^{\frac{3}{2}} P_o$. It is used to help answer questions like the following: If the weight of a plane is increased 15%, what effect has this on the required horsepower, P_n?

Solution: In this case $\frac{W_n}{W_o} = 1.15$. Then $(1.15)^{3/2}$ must be computed. Set 1.15 on the $\sqrt[3]{\ }$ scale. Read 1.23 on the $\sqrt{\ }$ scale. Hence the horsepower must be increased by 23%.

EXAMPLE: In radio theory, the *resonance frequency* is given by the formula $f = \frac{1}{2\pi\sqrt{LC}}$. Find f when $L = 253$ microhenries and $C = 90$ micro-micro farads. Then $f = \frac{1}{2\pi\sqrt{.000,253 \times .000,000,000,090}}$. For slide rule computation it is more convenient to write this in the equivalent form:

$$f = \sqrt{\frac{1}{4\pi^2 \times 0.000,253 \times .000,000,000,090}}$$

Using C and D scales, divide 1 by 4, the result by π, this by π again, this result by 253, and finally this by 90. This may also be done by using the CI scale. After the last division the square root of the result is found on the upper $\sqrt{\ }$ scale, since the number has 13 (an odd number) digits. The result is 1,060,000 and the frequency is about 1060 kilocycles.

EXAMPLE: In the study of meteorology it is sometimes desirable to compute

$$a = \sqrt{\frac{\rho\omega \sin\phi}{\mu}}$$

where ρ = density of the air, ω = angular velocity of the earth's rotation, ϕ=latitude, and μ=coefficient of eddy viscosity . Find, a, when ρ=0.0011, ω = 0.0000729, ϕ=40°, and μ = 116.

The ease with which such a calculation can be done on the Deci. Log Log rule is shown below. We have to compute:

$$\sqrt{\frac{0.0011 \times 0.0000729 \times \sin 40^\circ}{116}}$$

Next, set 116 on C scale over 11 on D scale, move runner to 729 on C scale. Move right index under runner; then move hairline of runner over sin 40 on S scale. The number under the square root symbol has 9 (an odd number) zeros. Hence read the result on the upper square root scale, as 211. Point off the decimal place in the result 0.0000211.

Other values, such as log *a* could be read from the same setting but would not usually be found in this example; log *a* = 5.648 − 10.

PART 3. USE OF LOG LOG SCALES

To find the value of expressions like 1.3^7, $5.6^{3.21}$, $\sqrt[5]{38}$, $\sqrt[3.5]{84}$, and of many other types of expressions, Log Log scales are used. The method of computing such expressions will be explained in a later section. First, the Log Log scales will be described.

READING THE SCALES. RECIPROCALS.

Below the slide is an ordinary D scale. Just below it is one continuous scale about 40 inches long cut in 4 parts which are placed one under the other. The top scale, marked N_1, begins at about 1.00230. Set the hairline of the indicator on this at the left end, then move it slowly to the right, reading 1.0025, 1.003, etc., ending at 1.0232. When the end of the scale is reached, move the indicator to the left and continue on the N_2 scale below, reading 1.03, 1.04, etc. to about 1.259. The scale marked N_3 begins at 1.259 and ends at 10. Finally, the scale N_4 begins at 10 and ends at 10^{10} or 10,000,000,000 (ten billion). Note that the decimal points of the numbers on these scales are shown.

Above the slide is a scale reading from about .9977 to 10^{-10} or .000,000,000,1 (one ten-billionth). This scale *decreases* from left to right. It is about 40 inches long and is cut into 4 pieces which are placed one under the other. The decimal points of the numbers on these scales are shown. The scales are marked $\frac{1}{N_1}$, $\frac{1}{N_2}$, etc.

To learn one use of these scales, and for practice in using them, note that if the hairline on the indicator is set over any number N on one of these scales, the reciprocal $(1 \div N)$ may be read under the hairline on the corresponding scale on the opposite side of the slide. For example, set the hairline over 20 on N_4 (the bottom scale *below* the slide). The reciprocal $(1/20) = .05$ is under the hairline on $1/N_4$ (the bottom scale *above* the slide). Similarly, set hairline over 2.5 on N_3. Read $1/2.5 = .40$ on $1/N_3$. Set hairline over 1.03 on N_2. Read $1/1.03 = .9708$ on $1/N_2$. Set hairline over 1.0124 on N_1. Read $1/1.0124 = .9878$ on $1/N_1$. Conversely, for any number set on a scale *above* the slide, the reciprocal is located on the corresponding scale *below* the slide. Although reciprocals can also be read using only the CI and C scales, this method is less accurate for many numbers, and moreover, the location of the decimal point must be found by other methods.

Numbers on the N_4 scale from 10^{+3} up to 10^{+4} (that is, 1000 to 10,000) have 4 digits to the left of the decimal point. From 10^{+4} up to 10^{+5}, they have five digits. In general, for $n > o$ and $10^{+n} \leq N < 10^{n+1}$, the numbers have $n + 1$ digits. Numbers on the $1/N_4$ scale from 10^{-3} down to 10^{-4} have three *zeros* to the right of the decimal point. From 10^{-4} down to 10^{-5}, they have four *zeros*. In general, for $10^{-n} > N \geq 10^{-(n+1)}$, they have n zeros to the right of the decimal point followed by the scale reading, which decreases toward the right. Thus the reciprocal of 13,500 (or 1.35×10^4) is .0000740.

FINDING POWERS

To find, for example, the value of 2^5, set the right index of the C scale on the slide over 2 on scale N_3. Place the hairline of the indicator over 5 on the C scale; read 32 under the hairline of scale N_4.

Rule: **To find b^m, set an index of the C scale, over *b* on the Log Log scale. Move hairline to *m* on the C scale. Read result under hairline on Log Log scale.**

Sometimes it is difficult to decide on which scale the value of the power, or b^m, is to be read. To help make this decision, a small table nas been printed on the slide. The left-hand part is read as follows. When the *left* index of the C scale is set over *b* of a Log Log scale, and there are 4 digits in *m*, then b^m is found 3 scales below (if there are that many on the rule). If there are 3 digits in *m*, then b^m is found 2 scales below. If there is 1 digit in *m*, then b^m is found 0 scales below (i.e., on the same scale as *b*). If there are 2 zeros in *m*, then b^m is found 3 scales *above* the one on which *b* is located (if there are that many on the rule), etc.

Left index of C on b →							Indicator on m of C Scale then	← Right index of C on b						
Digits in m				or	Zeros in m			Digits in m			or	Zeros in m		
4	3	2	1	0	1	2	b^m found	3	2	1	0	1	2	3
3	2	1	0				Scales below	3	2	1	0			
			0	1	2	3	Scales above				0	1	2	3

Fig. 9

EXAMPLES:

(a) Find $(1.00342)^{1780}$. Set left index of the C scale on 1.00342 (on N_1). Move hairline to 178 on C. Since 1780 is a four digit number, the result is found under the hairline and 3 scales below, namely 435, approximately, on N_4.

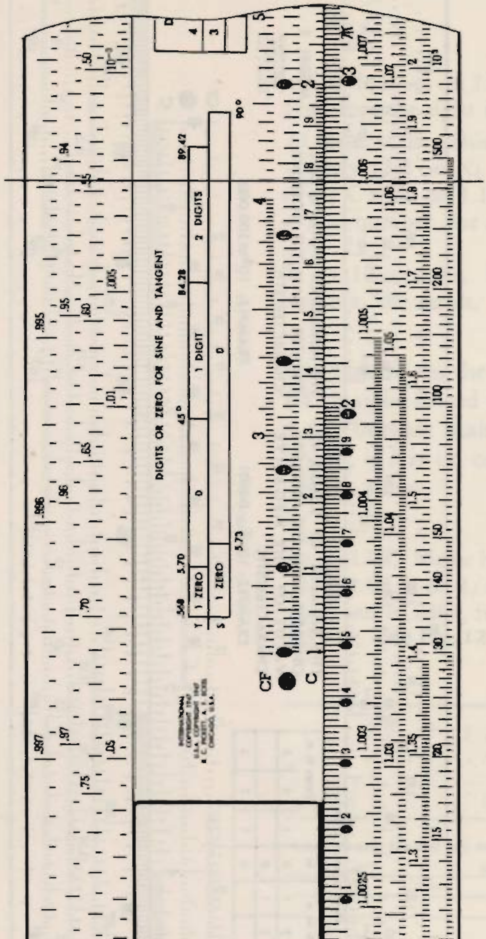


Fig. 10

(b) Find $(1.046)^{23.5}$. Set left index of C scale on 1.046 on N_2. Move hairline to 23.5 of C scale. Since 23.5 is a two-digit number, the result is found 1 scale below N_2; that is, on N_3. It is 2.88, approximately.

(c) Find $(1.352)^{.0378}$. Set left index of C on 1.352 on N_3. Move hairline to 378 on C. Since .0378 is a one-zero number, the result is found 2 scales *above* N_3, or on N_1. It is 1.0114, approximately.

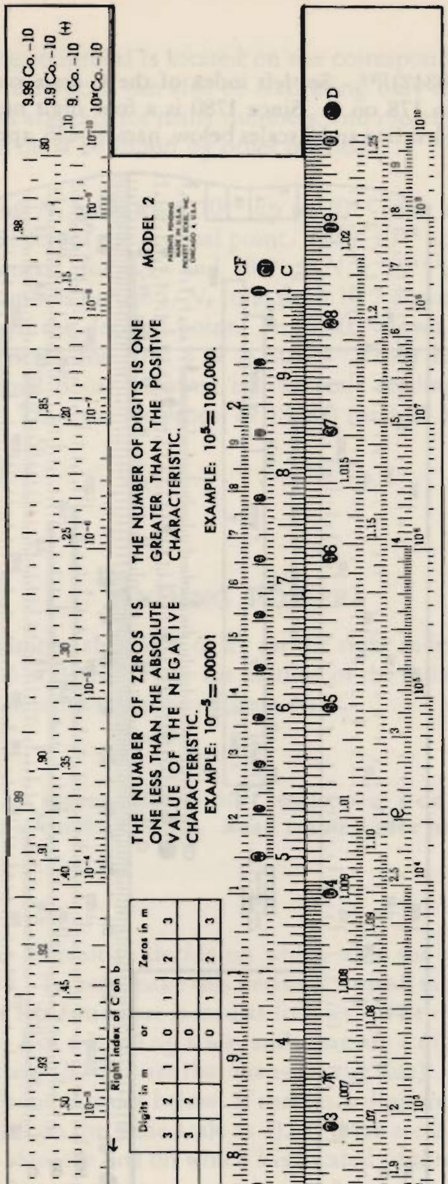

THE NUMBER OF ZEROS IS ONE LESS THAN THE ABSOLUTE VALUE OF THE NEGATIVE CHARACTERISTIC.
EXAMPLE: 10−5 = .00001
THE NUMBER OF DIGITS IS ONE GREATER THAN THE POSITIVE CHARACTERISTIC.
EXAMPLE: 105 = 100,000.
MODEL 2
9.99 Co.-10
9.9 Co.-10
9. Co.-10
10×Co.-10
CF
C
D

Fig. 11

(d) Find $(.97)^{14}$. Set left index of C on .97 on $1/N_2$. Move hairline to 14 on C. Since 14 is a two digit number, the result is found one scale below on $1/N_3$. It is .653, approximately.

When the *right* index of C must be used to set b, the *right* side of the table is read in the same manner as above.

EXAMPLES:

(a) Find $(1.163)^{16.8}$. If the left index of the C scale is set on 1.163, the number 16.8 is too far to the right to be used. Therefore the right index is set on 1.163 on N_2, and the hairline moved to 16.8 of the C scale. This is a two digit number, and the result is two scales below, namely 12.6 on N_4.

(b) Find $(1.163)^{.0168}$. Set the right index of the C scale on 1.163 on N_2, and the hairline on 16.8 of C. Since .0168 is a one-zero number, the result is found 1 scale above N_2, and is 1.00254, read on N_1.

(c) Find $(.15)^{.027}$. Set right index of C scale on .15 on $1/N_3$. Move hairline to 27 on C scale. Since .027 is a one-zero number, the result, .95, is 1 scale above on $1/N_2$. See Fig. 11

One of the great advantages of the scale arrangement on the *Deci-Log* rule is that, if m is a *negative* exponent, the value of b^m may be found without intermediate settings. If b is set on any one of the bottom scales, b^m (for m negative) is on one of the top scales. Conversely, if b is on one of the top scales, b^m (for m negative) is on one of the bottom scales.

EXAMPLES:

(a) Find $(1.03)^{-1.75}$. Set left index of C scale on 1.03. Move hairline to 1.75 of C scale. Since 1.03 is on N_2 the reciprocal, or $1/1.03$, is on $1/N_2$. Since 1.75 is a one digit number, the result is 0 scales below, and is .9496, found on $1/N_2$. Recall that $(1.03)^{-1.75} = 1/(1.03)^{1.75} = (1/1.03)^{1.75}$. See Fig. 12

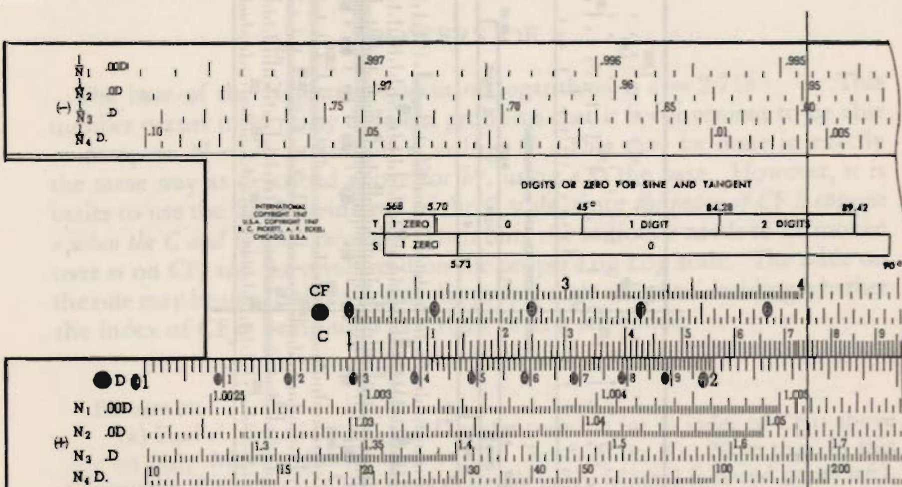


Fig. 12

(b) Find $(.05)^{-.0028}$. Set left index of C on .05 on $1/N_4$. Move hairline to 28 on C. The reciprocal of .05 is on N_4. Since .0028 is a negative two zero number, the result is read 3 scales above N_4, and is 1.00842 on N_1.

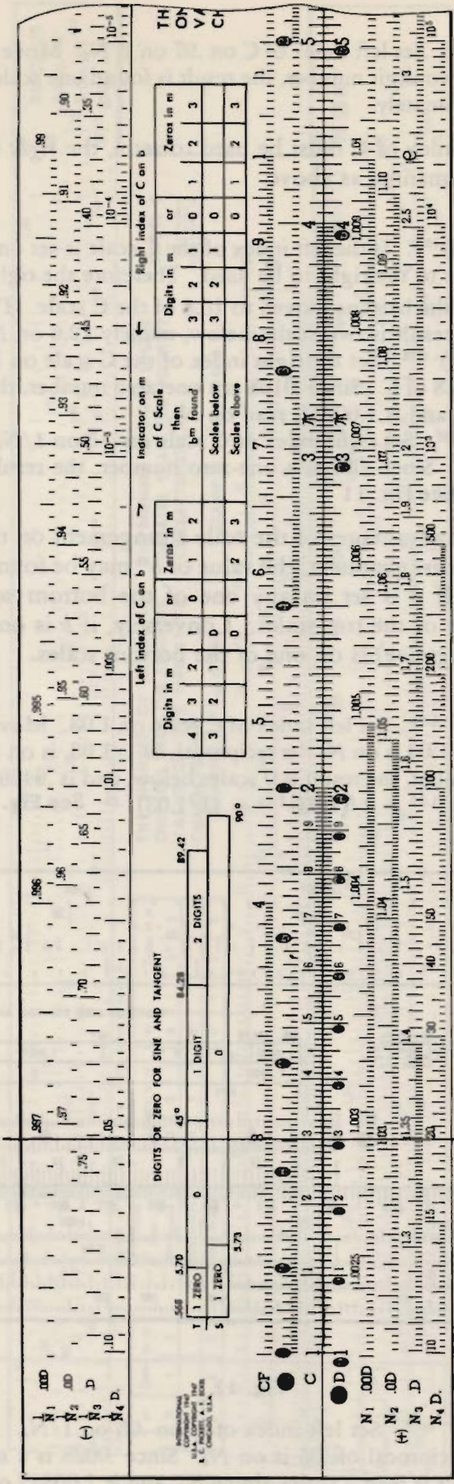

DIGITS OR ZERO FOR SINE AND TANGENT
Left Index of C on b
Indicator on m of C Scale then
b^m found
Scales below
Scales above
Right Index of C on b
Digits in m
Zeros in m
CF
C
D

Fig. 13

USING THE CF SCALE

When the C scale is used for some settings, the slide extends far outside the stock or fixed part of the rule. For example, when finding $7^{1.12}$, it extends far to the right. This may be avoided by using the CF scale. Set the index of CF (i.e. the scale mark under the large 1 on CF) on 7 (on N_3), move the hairline to 1.12 on CF, and read the result, 8.84, on N_3. Similarly, to find $(1.03)^{.091}$, if the C scale is used, the slide projects far to the left. It need not be moved so far if the index of CF is used instead of the right index of C.

To determine on which scale the result must be read, decide whether the index of the CF scale is being used as a left index or as a right index, and use the table printed on the rule. In the first example above, the index is used as a left index; in the second, it is being used as a right index. Since .091 is a one-zero number, the result for $(1.03)^{.091}$ is read one scale *above* N_2, and is 1.0027, approximately.

The CI scale may also be used in this way. In this case, the hairline is set over b on a Log Log scale, the slide is moved until m on CI is under the hairline, and the result is read opposite the CI index on a Log Log scale. Since the CI scale is a C scale reading in the opposite direction, to use the table on the slide, the right index of CI must be regarded as a left index, and the left index of CI regarded as a right index.

POWERS OF e

The base of the Napierian system of logarithms is $e = 2.718 \cdots$ This number occurs in so many different problems that it is convenient to be able to compute $N = e^m$, or to find $m = \log_e N$. This may be done in exactly the same way as described above for b^m, using e as the base. However, it is easier to use the CF scale instead of the C scale, since *the index of CF is opposite e when the C and D scales coincide*. Then only the indicator needs to be moved over m on CF, and the result read on the proper Log Log scale. The table on the rule may be used to help decide the proper scale after first deciding whether the index of CF is being used as a right or as a left index.

EXAMPLES:

(a) Find e^3. Set slide so C and D scales coincide (or so index of CF is above e on N_3). Move indicator to 3 on CF. Read 20 on N_4. Note that, in this example, the index of CF is used as a *right* index (since 3 is found on its left), and 3 is a one-digit number. The result is 1 scale below N_3. See Fig. 13

(b) Find $e^{.0135}$. Move indicator to 135 of CF. Since .0135 is a one-zero number, and the index is being used as a left index (135 is on its right), the result is 2 scales above e on N_1. It is 1.0136, approximately.

The scale arrangement of the Deci • Log Log rule makes the evaluation of hyperbolic functions easy. Note that the e^x and e^{-x} are found with one setting of the hairline. Thus to compute cosh x for $x = 1.5$, set the slide so the C and D scales coincide, move hairline to 1.5 on CF, read $e^{1.5} = 4.48$ on N_3, read $e^{-1.5} = .223$ on $1/N_3$. Since $\cosh x = (e^x + e^{-x})/2$, $\cosh 1.5 = (4.48 + .223)/2 = 2.352$. Also, note $\sinh x = (e^x - e^{-x})/2 = (4.48 - .22)/2 = 2.13$. Since $\tanh x = \sinh x/\cosh x$, $\tanh 1.5 = .905$.

ROOTS

The finding of roots is the *inverse* of finding powers.

Rule: **To find $\sqrt[m]{b}$, set hairline over b on a Log Log scale, pull m on C scale under hairline, read result on Log Log scale at the index.**

EXAMPLES:

(a) Find $\sqrt[5]{6.3}$. Set hairline over 6.3 on N_3, move slide so 5 of C scale is under hairline, read 1.445 under left index on N_3.

(b) Find $\sqrt[4]{.56}$. Set hairline over .56 on $1/N_3$, move slide so 4 of C scale is under hairline, read .865 at right index of C on $1/N_2$.

The proper scale on which to read the root may be determined by the table on the slide if the top legend is read "Left index of C on *root*" (or Right index of C on *root*) and the center is read with the words *below* and *above interchanged.* Thus in example (b) above, the right index is used and since 4 is a one-digit number, the root is found one scale *above* $1/N_2$. The explanation for this is simple. By definition, $\sqrt[m]{b}$ is a number which raised to the mth power produces b. That is $(\sqrt[m]{b})^m = b$. If, for example, the *right* index is on $\sqrt[m]{b}$, and m has one digit, the table shows that b is one scale *below* $\sqrt[m]{b}$, or reversing the language, $\sqrt[m]{b}$ is one scale *above* b.

The CF scale may be used instead of the C scale to avoid having the slide project far beyond the rule in some problems.

A second method of finding roots makes use of the CI scale.

Rule: **To find $\sqrt[m]{b}$ or $b^{\frac{1}{m}}$, set index of C scale on b on Log Log scale, move indicator to m on CI, read result on Log Log scale under hairline.**

This method uses the theory of exponents to express a root by using a fractional exponent (e.g.= $1/m$). This fraction can be divided out and the result used as an exponent as described under finding powers. Thus $\sqrt[4]{3} = (3)^{1/4} = 3^{.25}$. However, the CI scale does the division automatically, since it gives reciprocals of numbers on the C scale.

EXAMPLES:

(a) Find $\sqrt[4.2]{8.5}$ or $(8.5)^{1/4.2}$. Set right index of C scale over 8.5 on N_2. Move hairline to 4.2 on CI scale, read 1.664 on N_2 under hairline.

(b) Find $\sqrt[.03]{.964}$ or $(.964)^{1/.03}$. Set left index of C scale on .964 on $1/N_2$. Move hairline to 3 on CI. Note that when .03 is on $1/N_4$, the reciprocal $1/.03$ on N_4 is a two-digit number. The table on the slide shows that the result for a two-digit exponent is 1 scale below. Hence the result in this problem is under the hairline on $1/N_3$, and is .295, approximately.

To compute $b^{\frac{m}{n}}$, set index of C scale on b, move hairline over m of C scale, pull n of C scale under hairline, read result under index of C scale.

Examples:

(a) Find $16^{3/4}$. Set left index of C scale on 16 of N_4 scale. Move hairline to 3 of C scale, pull slide so 4 of C scale is under hairline. Read 8 under right index of C scale on N_3. This is the same as finding $16^{.75}$, and the table on the slide shows the result is 1 scale above N_4.

(b) Find $(.615)^{2.3/4.7}$. Set left index of C scale on .615 of $1/N_3$. Move hairline to 2.3 on C scale, pull 4.7 of C scale under hairline. Note that the exponent is about 1/2 or .5, and to set $(.615)^{.5}$ properly, the right index of C would be on .615, and .5 of C would be near the left end of the rule. The table on the slide shows the result is on the same scale as .615. Read the result .788 on $1/N_3$.

This example can be solved more readily by use of the CF scale as follows: Set index of CF on .615, move hairline to 2.3 on C, pull slide until 4.7 of C is under hairline, read result above index of CF.

COMMON LOGARITHMS, BASE 10

When the hairline of the indicator is set on any number on one of the four bottom scales (N_1, N_2, N_3, or N_4), the logarithm of the number (to the base 10) can be read on the D scale under the hairline. Thus (a) $\log_{10} 100 = 2$, (b) $\log_{10} 20 = 1.301$, (c) $\log_{10} 200 = 2.301$. For numbers set on the N_4 scale, the primary graduations on D give the characteristics, and the secondary and other graduations give the mantissas. The decimal point of the logarithm is at the right of the first digit read. Note that this is indicated by the notation D. at the left end of the N_4 scale. For numbers set on scale N_1, (or .00D scale), place two zeros at the left of the reading on the D scale, then the decimal point. Thus log 1.004 = .001733. On scale N_2 (.0D), place one zero at the left of the reading under the hairline of the D scale, then the decimal point. Thus log 1.04 = .01703. In other words, for each lower scale the decimal point is moved one place to the right. Thus log 1.5 = .1761, and log 15 = 1.176. The table below summarizes this discussion.

If number is on scale	Read figures of logarithm on D and	Example	Logarithm
N_1	put two zeros on the left, then point	$N_1 = 1.00644$	.00279
N_2	put one zero on the left, then point	$N_2 = 1.0662$	0.0279
N_3	put point on the left	$N_3 = 1.9$	0.279
N_4	put point after first digit	$N_4 = 601$	2.790

The cologarithm of a number set on one of the bottom scales may also be read on the D scale. The cologarithm may be read with a *positive* mantissa on the D scale by *reading from right to left* using the *white on black* numerals. The symbol "Co." is printed at each end of the D scale as a reminder of this fact. For example, colog 20 = −log 20 = −1.301. When the Co. scale is used, the reading is 8.699, and the complete cologarithm is 8.699 − 10.

If numbers are set on the N_4 or on the $1/N_4$ scale, the decimal point is at the right of the first digit read on the Co scale. However, it is useful to read as though it was on the left, then multiply by 10 and subtract 10. Thus, read .8699 and rewrite 8.699-10.

EXAMPLES:

(a) Find colog 1.71. Set hairline on 1.71 on scale N_3, read .767 on Co. scale. Refer to the upper right-hand corner of the rule, at the end of the $1/N_3$ scale, where the symbol 9.Co−10 is printed. This helps to show that the complete cologarithm of 1.71 is 9.767−10.

(b) Find colog 1.128. Set hairline on 1.128 on N_2 scale, glance at right end of $1/N_2$ scale, and write 9.9___−10. Read 477 on Co. scale under the hairline, and enter these figures on the blanks. Thus, colog 1.128 = 9.9477−10.

The logarithms for all numbers on the scales above the slide ($1/N_1$, $1/N_2$, etc.) are negative. The small minus sign (−) at the left end of the upper scales is a reminder of this fact. When the hairline of the indicator is set on one of the numbers of these scales, the figures of its logarithm can be read under the hairline on the D scale. Thus $\log_{10} .50 = -.301$.

If the logarithm is to be expressed with a *positive* mantissa, the Co. scale is used. Glance at the right end of the upper scales, and write down the symbol at the end, replacing the "Co." by the reading of the Co. scale.

EXAMPLES:

(a) Find log .50. Set hairline on .50 of $1/N_3$ scale. Write 9.___−10, and read 699 on Co. scale. Then log .50 = 9.699−10.

(b) Find log .006. Set hairline on .006 on $1/N_4$. Read .778 on Co. scale. Multiply by 10 and subtract 10. log .006 = 7.78−10, approximately.

(c) Find log .9952. Set hairline on .9952 on $1/N_1$ scale. Write 9.99___−10. Read 792 on Co. scale, and enter these figures after the 9, obtaining 9.99792−10.

(d) Similarly, log .005 = −2.301 = 7.699−10, and log .945 = −0.0246 = 9.9754−10.

From a table of logarithms, log .997 = 9.99870−10 and log .9997 = 9.999870−10; that is, the insertion of an extra 9 between the decimal point and the original number yields a logarithm which also has an extra 9 in the same place. Consequently, the left part of the uppermost scale can also be used for numbers which have three 9's to the right of the decimal point followed by other digits. The logarithms of these numbers, written in the form 9.___−10, have three 9's following the decimal point, followed by the reading of the Co. scale. Thus, log .9991 = 9.99961−10, approximately. Their cologarithms have an extra zero to the right of the decimal point. Their reciprocals, read on scale N_1, also have an extra zero. Thus, if $N_1 = .9994$, log N_1 = 9.99974−10 = −.00026, and $1/N_1 = 1.000603$.

LOGARITHMS TO BASE e

The process of finding logarithms to the base *e*, or natural logarithms, is the opposite of the steps explained above for finding powers of *e*.

Rule: **To find $\log_e N$, set the hairline over N on the Log Log scale. Read $\log_e N$ on the CF scale.**

EXAMPLES:

(a) Find $\log_e 20$. Set hairline over 20 on N_4 scale. Read 3, approximately, on CF scale. To verify that the result is 3, and not .3 or 30, etc., the table on the slide may be used. In this example, the index of CF is being used as a *right* index. Since 20 is one scale *below e* (i.e., *b*), the top line of numbers in the right part of the table shows there is 1 digit in the exponent, or logarithm.

(b) Find $\log_e 1.08$. Set hairline on 1.08 on N_2. The index of CF is used as a *right* index, and 1.08 is one scale above *e* on N_2. Hence, from the table, there is 1 zero in *m*, the logarithm, which is read as .077 on CF.

(c) Find $\log_e .25$. Set hairline on .25 of $1/N_3$. The index of CF is used as a *left* index. The logarithm is *negative*, since .25 is on an upper scale. Note .25 is on $1/N_3$, which corresponds to N_3, or the scale on which *e* is found. That is, it is "0" scales above or below. Enter the second line of the left side of the table on the slide at 0, and read from the line above that there is 1 digit in *m*. The logarithm is -1.386, read on the CF scale. To express this with a positive mantissa, set 1.386 on the D scale, and read from the Co. scale $8.614-10$. Thus, $\log_e .25 = -1.386 = 8.614-10$.

(d) Find $\log_e .035$. Move hairline to .035 on $1/N_4$. Read 335 on CF. Since .035 is on $1/N_4$, the reciprocal is on N_4 (at about 29), and since the logarithm of the reciprocal is 3.35, $\log_e .035 = -3.35$. Also, set 3.35 on D, and read $6.65-10$ on the Co. scale. To use the table on the slide to determine the decimal point in 335, note that the index of CF is being used as a *right* index in this example. Since $1/.035 = 29$ is one scale below *e*, enter the second line of the right side of the table at 1, read 1 in the line above as the number of digits in *m*, the logarithm or exponent.

SOLVING EXPONENTIAL EQUATIONS

The method of solving equations of the type $b^m = N$, where *b* and *N* are known and *m* is unknown, is very similar to the process of finding $b^m = N$ when *m* is known and *N* unknown. (See Finding Powers, above.)

***Rule:* Set the index of the C scale (or CF scale) on *b*. Move the hairline to *N* on a Log Log scale. Read *m* under the hairline on the C scale (or CF scale, if it was used).**

EXAMPLES:

(a) Solve $1.37^m = 8.43$. Set left index of C on 1.37 on N_3. Move hairline to 8.43 on N_3. Note that both 1.37 and 8.43 are on the same scale. The table on the slide shows that the exponent *m* has 1 digit. Read 6.77 under the hairline on the C scale.

(b) Solve $(.75)^x = .872$. Set left index of C scale on .75 on $1/N_3$. Move hairline to .872 on $1/N_2$, which is 1 scale above. Hence, there are 0 digits in *x*, which is read as .476 on the C scale.

(c) Solve for *y* if $(.94)^y = 2.37$. Set left index of C scale on .94 on $1/N_2$. Move hairline to 2.37 on N_3. Since these numbers are on opposite sides of the slide, the exponent is negative. The reciprocal of .94 is on the N_2 scale, and 2.37 on N_3, is one scale below. Hence there are two digits in *y*, which is read as 13.9 on the C scale.

(d) Solve for *p* if $(5.27)^p = .818$. Since the slide would project far to the right if the left index is set on 5.27, it is better to use the CF scale. Set index of CF on 5.27 on N_3. Move hairline to .818 on $1/N_2$. Read 1209 on the CF scale under the hairline. The two known numbers are on opposite sides of the slide, so the exponent is negative. In this example, the index of CF is being used as a *left* index. The reciprocal of .818 on N_2 is one scale above 5.27 on N_3, and hence there are 0 digits in the exponent. The result is, therefore, $-.1209$.

It is useful to notice that when powers are being found, the logarithmic solution may often be directly observed on the C and D scales. As a simple example, consider finding $x = 2^3$. Then $\log x = 3 \log 2$. When the left index of the C scale is set on 2 of the N_3 scale, the logarithm of 2, or .301, is visible on the D scale under the index of the C scale. When the hairline is moved to 3 on the C scale, one may think of this operation as multiplying .301 by 3 by use of the C and D scales. The result is .903, read on the D scale, and this in turn is the logarithm of 8, read below it on the N_3 scale.

READINGS BEYOND THE SCALES

Occasionally there is need to compute an expression which involves values not on the scales. To compute b^m for b less than 1.0023, note that by the binomial expansion $(1 + xy)^{\frac{m}{y}} = 1 + mx + \ldots$, and if xy is sufficiently small, these first two terms will give a good approximation.

EXAMPLES:

(a) Find $(1.0004)^{2.7}$. Since 1.0004 cannot be set on the scales, compute $1 + (2.7)(.0004) = 1.00108$, approximately.

(b) Find $53^{0.00008}$. Although 53 can be set, the result cannot be read on the scales. Write the expression in the equivalent form $\left[53^{\frac{0.00008}{0.02}}\right]^{0.02} = [53^{0.004}]^{0.02}$. The expression in brackets is found in the usual manner to be 1.016. Then $(1.016)^{0.02} = 1 + 0.02 \times 0.016 = 1.0003$, approximately.

(c) Find 30^8. The usual setting leads to a result beyond the N_4 scale. Write the expression as $5^8 \times 6^8$. Now $5^8 = 3.9 \times 10^5$, approximately, and $6^8 = 1.7 \times 10^6$ approximately. Hence $30^8 = 3.9 \times 10^5 \times 1.7 \times 10^6 = 3.9 \times 1.7 \times 10^{11} = 6.6 \times 10^{11}$. Moreover, note $30^8 = 30^4 \times 30^4 = 8.1 \times 10^5 \times 8.1 \times 10^5 = 66 \times 10^{10} = 6.6 \times 10^{11}$, approximately. Thus, by breaking up the expression into factors, and computing each separately, the approximate results are obtainable. These results are also readily obtained by logarithms.

Also, it may be noted that if greater accuracy is desired in the logarithms of any numbers set on the N_4 scale to the right of 10^3, these numbers may be set on the scale above (the N_3 scale), and the sequence of digits in the mantissa read from the D scale. The characteristic is given by the primary scale division at the left of the setting on the N_4 scale. Thus, to find the logarithm of 2,430,000, or 2.43×10^6, note that this number could be set on N_4 between 10^6 and 10^7. Set the hairline over 2.43 on N_3, and read 385 on the D scale. Then, $\log 2{,}430{,}000 = 6.385$, approximately. A slightly more accurate value may be found by setting 243 on the D scale on the other side of the rule, and then reading the mantissa as 386 on the L scale.

The logarithms of numbers on the $1/N_4$ scale between 10^{-3} and 10^{-10} may also be obtained in this way. Thus, to find log .000000437, or $\log 4.37 \times 10^{-7}$, set 4.37 on the N_3 scale and read 640 on the D scale, or set 437 on the D scale on the other side of the rule, and read 640 + on the L scale. The complete logarithm is 3.640—10, approximately.

1. A volume of 1.2 cu. ft. of air at 60° F (or 520° absolute) and atmospheric pressure (14.7 lbs./sq. in.), is compressed adiabatically to a pressure of 70 lbs./sq. in. What is the final volume and final temperature?

(a) Compute: $V = 1.2\left(\frac{14.7}{70}\right)^{\frac{1}{1.4}}$ Ans. 0.394 cu. ft.

Set 70 on C opposite 14.7 on D, read .21 on D under the C— index. By means of the hairline, transfer .21 to the $1/N_3$ scale, and pull the right index under the hairline. Move hairline to 1.4 on CI, read .328 on $1/N_3$ under the hairline. Multiply 1.2 × .328 by the C and D scales, reading .394 on the D scale.

(b) Compute: $T = 520\left(\frac{70}{14.7}\right)^{\frac{0.4}{1.4}}$ Ans. 812° Absolute or 352°F.

Divide 70 by 14.7, and set the result, 4.76, under the hairline on N_3. Move right index of the C scale under the hairline, then move hairline over 0.4 on the C scale, then pull the slide so 1.4 of the C scale is under the hairline. Read 1.564 on N_3. Multiply this by 520, obtaining 812, the final temperature in degrees absolute. Subtract 460° to obtain 352°F.

2. (a) Find the compound amount on an investment of $1200 at 3½% compounded annually for 20 years. The formula is $A = P(1+i)^n$, or, in this example, $A = 1200(1.035)^{20}$. Set left index of the C scale on 1.035 on N_2. Move hairline over 20 on the C scale, read 1.99 on N_3. Multiply this by 1200, obtaining $2390, approximately.

(b) In how many years does money double itself at 4.2% compounded annually? This problem requires finding n in the expression $(1.042)^n = 2$. Set the left index of the C-scale over 1.042 on N_2, move the hairline over 2 on N_3, read 17 years, approximately, on the C scale under the hairline.

3. The formula $y = \frac{k}{1 + be^{-at}}$ is the so-called "logistic of population."

For the United States, the time t is measured in years from 1780. From studies by the statistician Hotelling, $a = 0.0315$, $b = 64.5$, $k = 195.9$ (millions). Estimate the population for the year 1960 when the value of t will be 180.

Here $y = \frac{195.9}{1 + 64.5 \times e^{-0.0315 \times 180}}$

First compute $-.0315 \times 180 = -5.65$. Set C and D scales in coincidence, move the hairline over 5.65 on CF, read .00345 on $1/N_4$. Multiply this by 64.5, obtaining .223, approximately. Add 1, and then divide 195.9 by 1.223, obtaining 160 (million) approximately, as the estimated population for 1960.