SUN

INSTRUCTIONS

FOR THE USE OF

THE "HEMMI " BAMBOO SLIDE RULES

(No, 153,)

"UNIVERSAL"

DUPLEX

PUBLISHED BY

HEMMI BAMBOO SLIDE RULE MFG. CO., LTD.

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CONTENTS

Hemmi Bamboo "Universal" Duplex Slide Rule

Fig. 1 Electrical Engineer's 10″ (Front face).

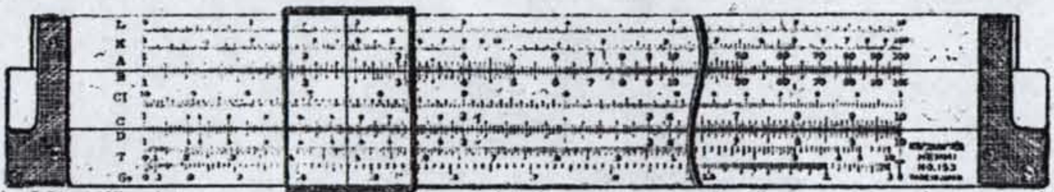


Fig. 2 Electrical Engineer's 10″ (Back face).

PREFACE

It is a great pleasure to see that the invention of the slide rule has made it possible to solve a wide variety of complicated arithmetical and other calculations with ease and rapidity. There have been invented also special slide rules for engineers, designers, contractors, statisticians and commercial people. The number of inventions in slide rules may be taken as the measure of the civilization of a country.

The wide use of slide rules has brought forth inventions of different scales, and the variety of scales induced improvements in the construction of the instrument. The "Rietz" and the "Stadia" slide rules are really prominent inventions; and there has finally come the "Duplex" slide rule. This book is to explain the duplex type: Electrical Engineers' "Universal" slide rule, Hemmi No. 153, the very best of the type.

Electrical engineers, especially those who are engaged in research, experiment, or design, find their own work mostly consist of calculations. And these calculations are not limited to multiplication and division, but extend to trigonometrical functions, logarithms, complex numbers, &c. &c. That is why electrical engineer's slide rules, classified as special slide rules, are the most popular among slide rule users. But the old electrical engineer's slide rule does not show much improvement or addition to the ordinary Mechanical Engineer's slide rule. It had the log-log scales which we admit, were of some use; it had the efficiency scale for generators and motors, and the voltage drop scale; and these were but quite dispensable. The questions of vectors and trigonometrical, functions were still to be solved.

Our Electrical Engineer's "Universal" slide rule has been invented for these aims. It calculates the complicated complex numbers, vector functions, circuit calculations, &c. &c. with ease and rapidity. It is of the duplex type. It has our patent non-logarithmic scales of (P),, (Q) for complex numbers an angle scale (θ), a radian scale $(R\theta)$, tangent scale (T), a Gudermanniaan Scale, $(G\theta)$ and a set of log-log scales in three parts. It is of course for electrical engineers, but at the same time, it is good for vector calculations as in Applied Dynamics, and angle calculations.

CHAPTER I.

SLIDE RULE LACONISM AND INTERNATIONAL SLIDE RULE DIAGRAM

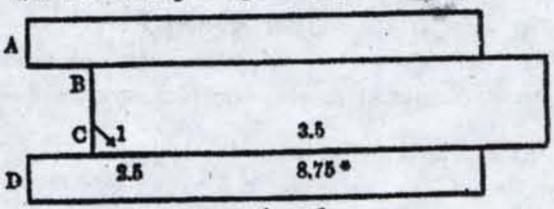
A Chemical Equation, *e.g.* $2H_2+O_2=2H_2O$, or an Arithmetical Equation, *e.g.* $2.5\times3.5=8.75$ is international and laconic; and boys of any nationality understand their meanings quite concretely: even better than the prosaic expression in his own mother tongue. If the readers had a convention, similarly international and laconic as well, given to them to guide them in their study of slide rules, it would no doubt assist them very much. Mr. E. Hirano, of the Third Commercial School of Tokyo Prefecture, has made two such proposals which the author of this book has accepted.

One of the proposals is a Laconism and the other an International Diagram. Either of the two is quite sufficient, and independent of the other. Here we shall give examples of both:—

Example 1. $2.5\times3.5=8.75$

Laconism:—Set 1 *C* to 2.5 *D* against 3.5 *C* read 8.75 *D*

Diagram

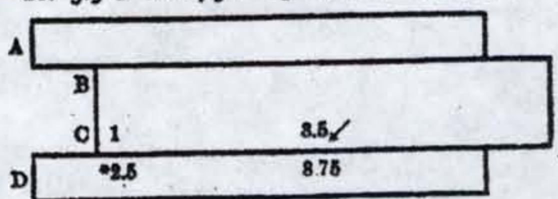


∴ *Ans.* 8.75

Example 2. $8.75\div3.5=2.5$

Laconism:—Set 3.5 *C* to 8.75 *D* against 1 *C* read 2.5 *D*

Diagram

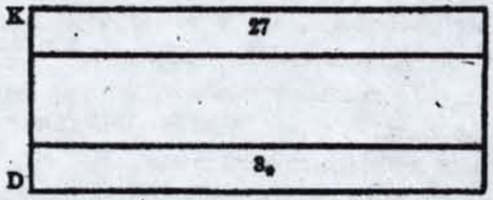


∴ *Ans.* 2.5

Regarding the Laconism, in cases where either "no setting is necessary" or "setting the slide end on end" or "the hairline only is to be employed" the left hand half of the laconism may be entirely omitted.

Example 3. $\sqrt[3]{27} = 3$
Laconism: against 27 *K* read 3 *D* *

Diagram



∴ *Ans.* 3.

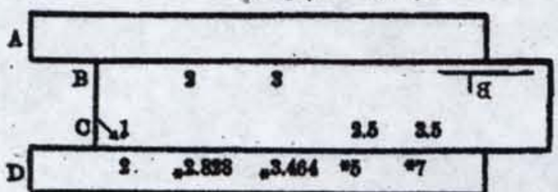
Regarding the Diagram, the lettering on the left side of the Diagram is to denote the names of the scales. But (*A*), (*B*), (*C*), (*D*) are so familar on the ordinary slide rule, that they may be omitted and understood for the sake of simplicity.

The arrow in the diagram points out the point of setting the slide; and the asterisk the answer required.

When one setting is used for more than one reading, all the Laconisms may be combined, also the diagrams as well.

Example 4. $2\times\sqrt{2}$, $2\times\sqrt{3}$, 2×2.5, 2×3.5
Laconism:—Set 1 *C* to 2 *D* against 2 *B* read 2.828 *D*
" 3 *B* " 3.464 *D*
" 2.5 *C* " 5 *D*
" 3.5 *C* " 7 *D*

Diagram



∴ *Ans.* 2.828, 3.464, 5, 7 respectively.

When the meaning will be clear and obvious to the reader, part of the diagram might be omitted or cut off.

Example 5. $2 \times 37 = 74$

∴ *Ans.* 74

For teaching or writing about a slide rule, the Diagram is better as it gives a fuller idea than the Laconism; but for the student to write down on his examination papers, the Laconism would be better as it is simpler. Simple as it is, the Laconism makes it clear, whether or not the student has made a correct calculation, whether or not he has correctly used the slide rule, whether or not he has used it in the wisest way. The teacher can instruct his students to solve their problems specifying the use of such and such scales, rather than others.

If you like to internationalize the Laconism, you might as well write down as:—

1 *C*→2.5 *D*=3.5 *C*→8.75 *D* (Example 1)
3.5 *C*→8.75 *D*= 1 *C*→2.5 *D* (" 2)
27 *K*→ 3 *D* (" 3)
1 *C*→ 2 *D*= 2 *B*→2.828 *D*
= 3 *B*→3.464 *D*
=2.5 *C*→5 *D*
=3.5 *C*→7 *D* (" 4)

CHAPTER II.

ELECTRICAL ENGINEER'S "UNIVERSAL" DUPLEX SLIDE RULE WITH PATENT VECTOR AND LOG-LOG SCALES

(NO. 153)

Section A. SCALES DESCRIBED

The type of this slide rule is Duplex, just like the Mechanical Engineer's Slide Rule; see Figs. 1 and 2 Fig. 1 shows the front face of this slide rule, and Fig. 2 the back face. The

As you see in the figures the slide rule has on its front face, (*L*), (*K*), (*A*), (*D*), (*T*) and (*Gθ*) on the rule, and (*B*), (*CI*) and (*C*) on the slide.

Among the nine scales, (*T*) and (*Gθ*) one entirely a new scales, but other seven scales are familiar with old slide rules. (*A*), (*B*) and (*C*), (*D*) are all logarithmic scales, the former two of two-sections, the latter two of one-section each. They do all serve for Multiplication, Division, Proportion, Squares and Square Roots, Circles, etc. (*K*) is a logarithmic scale of three-sections; and works with (*C*) or (*D*) to give x^3, $\sqrt[3]{x}$ or ax^3, $\sqrt[3]{x}$.

(*L*) is the equi-division scale and good for obtaining the logarism of a given number.

(*CI*) is the inverted scale of (*C*); it works with (*D*) to give the reciprocal of a number It also gives *abc* at one setting when it works with (*C*) and (*D*) in cooporation.

(*T*) is to give *tanθ* with respect to θ° or *Rθ* on (*θ*) or on (*Rθ*) respectively. It is entirely different from the (*T*) scale that was familiar with the old slide rules. For further particulars, see Section B, [3] "Trigonometrical Functions," p. 14.

(*Gθ*) is called the Gudermanian Scale," and will give you at once *sinhθ*, *tanhθ* and *sechθ* in a simple and rapid way.

On the back face, this slide rule has (θ), ($R\theta$) (P), LL_3), (LL_2) and (LL_1) on the rule, and (Q) (Q') and (C) on the slide.

Among the nine scales, (C) is entirely the same as (C) on the front face, a one-section longarithmic scale.

(LL_1), (LL_2), (LL_3) are dived as per *log-log* x, x ranging from 1.01 to 22000; which is cut off into three pieces at $\varepsilon^{0.1}$ and ε. Like the *log-log* scales in the old slide rules they are to serve with (C) to do involution and evolution.

But as they are cut off at $\varepsilon^{0.1}$ and ε, they can give natural logarithms of a given number without having the slide set in any way.

By setting the index of (C) to 10 on (LL_3) you can get the common logarithm of a given number, or $log_{10}x$. Thus you can get the logarithm of any number on any base.

Another advantage of this slide rule is this. The old slide rule with *log-log* scales had its scales divided only from 1.1 and upward, and naturally was of little use for the calculation of compound interest while this slide rule has its *log-log* scales divided for the range of x 1.01–22,000.

(θ), ($R\theta$), (P), (Q) and (Q') are entirely new scales, and in them the uniqueness of this slide rule does lie.

Here we shall give the outline of the scales, but for particulars we shall dwell upon later.

(P) and (Q) are the most important and prominent scales of all; they are both divided as per x^2, x ranging from 0 to 10. They are non-logarithmic. They are called either "vector scales" or "square scales." (Q') is nothing but the extention of (Q), x ranging from 10. to 14.14.

When you know two sides of a right triangle, you are to get the value of the third side by the formula $\sqrt{a^2 \pm b^2}$, which could be calculated by means of (*P*) and (*Q*) at one setting with simplicity and rapidity. Also this calculation is very valuable for the absolute value of a vector.

(*P*) gives the sine of any angle on (θ) or ($R\theta$); here θ is the angle in degrees and decimals, while $R\theta$ the same in circular measure. Please note that θ is in degrees and *decimals*, and not in degrees, *minutes* and *seconds*.

If $\sqrt{a^2 \pm b^2}$ can be calculated very easily, $\cos\theta = \sqrt{1 - \sin^2\theta}$ must also be calculated very easily: and even so, it can be had on (*Q*). We shall discuss later how to do this.

(*T*) is divided as per $\tan(\sin^{-1}\sqrt{x})$, where x the distance of a point on the scale from left end, and ranges from 0 to 1.0.

(*T*) is so divided as to give $\tan\theta$ with respect to an angle, θ on (θ) or the angle in circular measure on ($R\theta$). Thus if you ever have but one of the five functions, $\sin\theta$, $\cos\theta$, $\tan\theta$, $\theta°$, $R\theta$, you can have all the other four functions at once by a single setting either of the hairline alone or of the slide and hairline. Besides $\cot\theta$, $\sec\theta$, $\operatorname{cosec}\theta$ are not very hard to be had as they are the reciprocals of $\tan\theta$, $\cos\theta$, $\sin\theta$ respectively.

($R\theta$) is divided as per $\sin^{-1}\sqrt{x}$ where x is the same as in (*T*).

(θ) is divided as per $\frac{180°}{\pi}\sin^{-1}\sqrt{x}$ on the same basis as above.

The Gudermanian Scale ($G\theta$) in divided with reference to the Radian Scale ($R\theta$); a reading x on ($G\theta$) is equivalent to gdx on ($R\theta$). So by the Gudermanian theorem:—

$$\tanh x = \sin gdx, \quad \sinh x = \tan gdx, \quad \operatorname{sech} x = \cos gd\theta$$

and each of them is very easy to get.

As you have just had the idea, the Electrical Engineer's "Universal" slide rule has no less than seven scales entirely free from logarithmic nature—they are non-logarithmic—so that they enable us to calculate a vector at one setting, and also to read out all trigonometrical functions or all hyperbolic functions at a

Section B. THE USE OF SCALES

[1] Multiplication, Division, Proportion, Squares and Square Roots, Cubes and Cube Roots.

All these could be disposed of among (*A*), (*B*), (*C*), (*D*), (*K*) and (*CI*) scales just as in the ordinary slide rules. And we shall not dwell upon these subjects here.

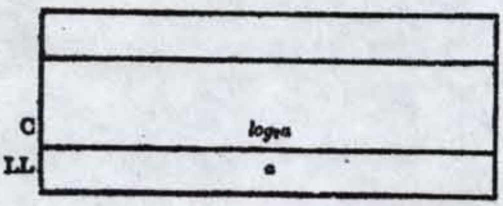
[2] Logarithms.

The common logarithm could be of course obtained by means of the (*L*) scale just as in the ordinary Slide Rules. But with the "Universal" slide rule we could compute out generally the logarithm on any base among (*C*) and (LL_3), (LL_2), (LL_1). Here we shall explain the Natural and the Common Logarithmus only.

(a) **Natural Logarithms** (on base, $e=2.71828$).

Set the slide end by end with the rule, put the hairline at a number on any of (LL_3), (LL_2) or (LL_1) according to the magnitude of the number, and read the natural logarithm of the number on (*C*).

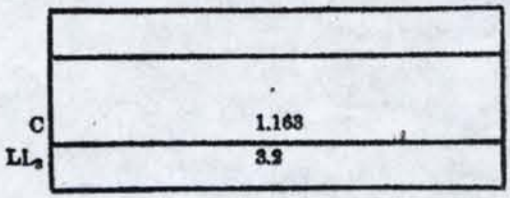
Against *a-LL* read $log_e a$ *C*



Here keep in mind that the readings on (LL) must be taken just as they are described, and not universal to the congruent numbers of similar digit value, as it is the case with the ordinary logarithmic scales. Another point: "LIC" or the left index of (C) corresponds with 1.0, 0.1, 0.01 on LL_3, LL_2, LL_1 respectively.

Example 1. $log_e 3.2$=[1.163 as below]

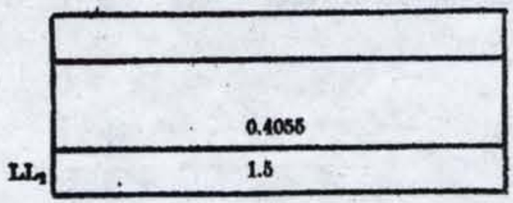
Against 3.2 LL_3 read 1.163 C



∴ *Ans.* 1.163

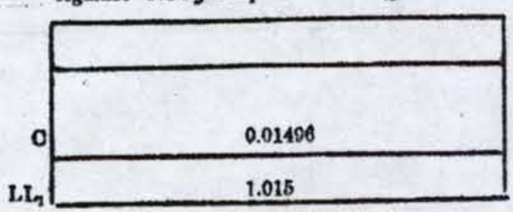
Example 2. $log_e 1.5$=[0.4055 as below]

Against 1.5 LL_2 read 0.4055 C



Example 3. $log_e 1.015$=[0.01496 as below]

Against 1.015 LL_1 read 0.01496 C



(b) **Common Logarithms** (with 10 for the base).

Set "*LIC*" or "*RIC*" to 10 (LL_3); against the given number on one of the *log-log* scales you can read on (C) the common *log.* required.

Here keep in mind that the "*LIC*" corresponds with 1.0, 0.1, 0.01 on (LL_3), (LL_2), (LL_1) respectively when the slide is drawn out to your right, and "*RIC*" does so or stands in its place when the slide is drawn to your left.

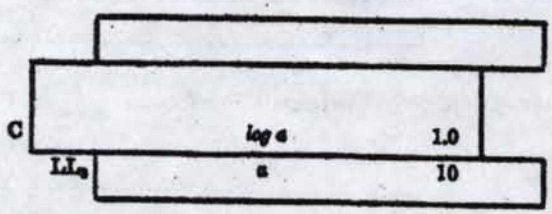
Another important point is that when $a > 10$ or $a < 1$, make previous transformation

$$a = \frac{a}{10^n} \times 10^n \qquad \text{so as } 10 > \frac{a}{10^n} > 1$$

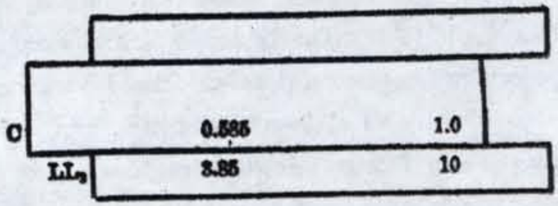
Or $$\log a = n + \log \frac{a}{10^n}$$

where a is any number, and n a suitable positive or negative integer. This is also good for the sake of accuracy. And it means that we should employ LL_3 scale only when a does not exceed 10.

Set 1.0 C to 10 LL_3 against a-LL_3 read $\log_{10} a$-C



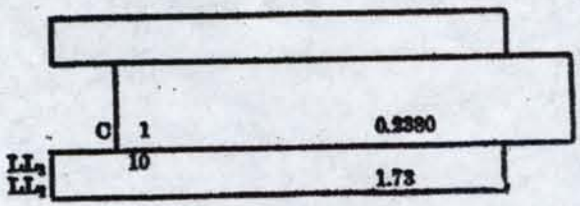
Set 1.0 C to 10 LL_3 against 3.85 LL_3 read 0.585 C



$$\therefore \ \log 385 = 2 + 0.585 = 2.585$$

Example 5. $\log_{10} 1.73 = 0.2380$

Set 1 C to 10 LL_3 against 1.73 LL_2 read 0.2380 C

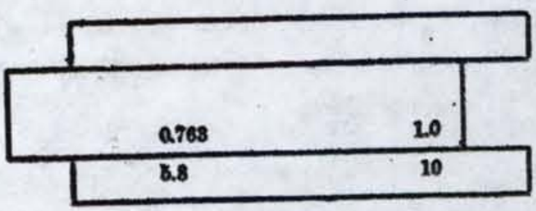


$$\therefore \ \log 1.73 = 0.2380$$

Example 6. $\log_{10} 0.58 = \bar{1}.763 \qquad (n = -1)$

$$\log_{10} 0.58 = -1 + \log_{10} 5.8$$

Set 1.0 C to 10 LL_3 against 5.8 LL_3 read 0.763 C



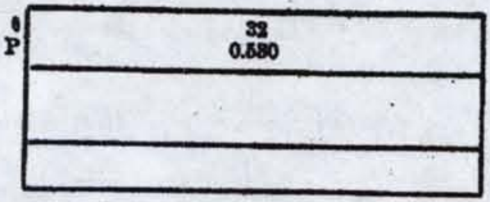
$$\therefore \ \log 0.58 = -1 + 0.763 = \bar{1}.763$$

[3] Trigonometrical Functions.

In the old slide rule, you found only the sine scale, (S), and the tangent scale, (T). The cosine, which is exceedingly important for electrical engineers, was calculated indirectly by $\cos\theta = \sin(90° - \theta)$. With this slide rule you can get it all at once with $\sin\theta$, $\tan\theta$. You know electrical engineers are compelled to compute out $\cos\theta$, when you have $\tan\theta$ only. It is really a hard and complicated job with the ordinary slide rule, and naturally inaccuracy is the result. With this slide rule, as we have already stated you can get $\sin\theta$, $\cos\theta$, $\tan\theta$, θ, $R\theta$ all at once or simultaneously if you ever have but one of the five values given. So naturally you can get them with rapidity and accuracy.

Example 7. $\sin 32° = 0.530$

Against 32 θ read 0.530 P

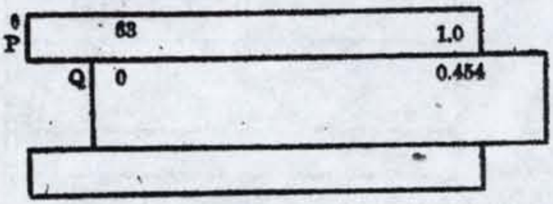


∴ *Ans.* 0.530

Note, for Trigonometrical functions, take the whole length of (P), (Q), (Q') as unit so that the highest figure read on the scale is on the first decimal place.

Example 8. $\cos 63° = 0.454$

Set 0 Q to 63 θ against 1.0 P read 0.454 Q



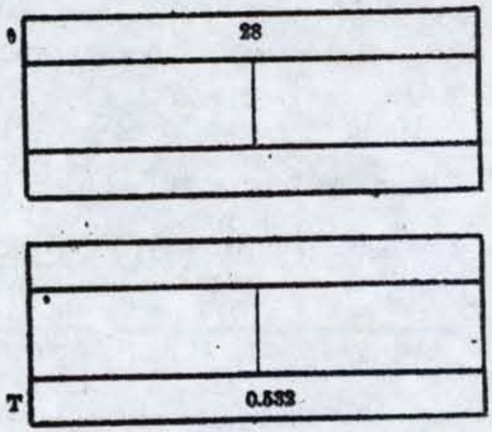
∴ *Ans* 0.454

Example 9. *tan* 28° = 0.532

Against 28 θ read 0.532 T on the other side of the slide rule

Back Face

Front Face



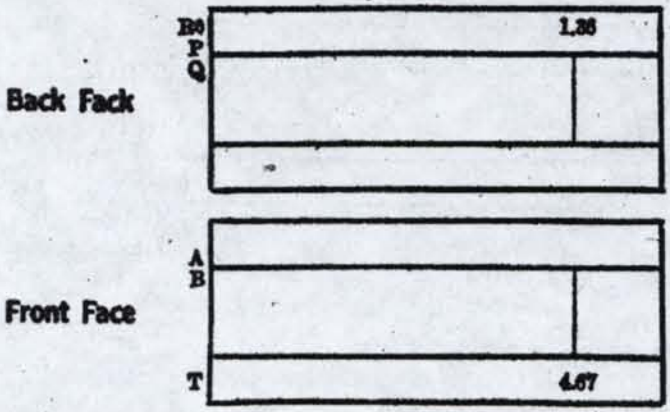
Ans. 0.532

The above cases are for the degrees and decimals of angle, but for angles in radians, only you have got to take $(R\theta)$ in place of (θ).

Example 10. *tan* (1.36) = 4.67

Where ( ) means that the angle is in radians.

Against 1.36 $R\theta$ read 4.67 T on the other side of the slide rule

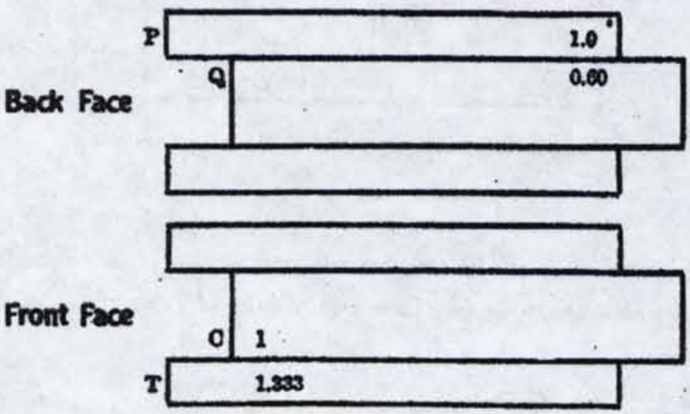


$\therefore$ *tan* (1.36)=4.67

When the angle is obtuse, transform first either $\sin\theta = \cos(\theta - 1.57)$ or $\cos\theta = \sin(\theta - 1.57)$, as (1.57)=90°.

Example 11. To get $\tan\theta$, when $\cos\theta = 0.6$

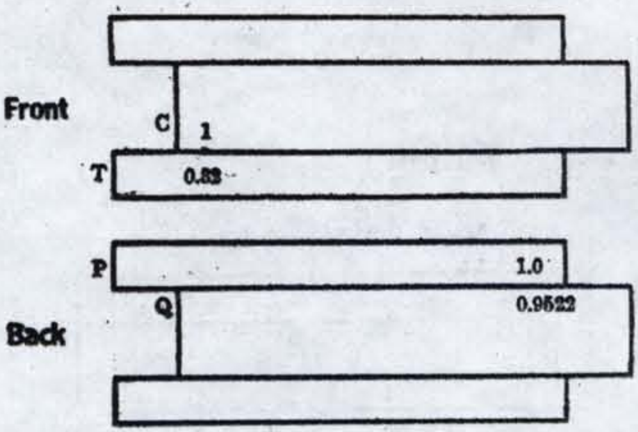
Set 0.6 Q to 1.0 P against 1 C read 1.333 T



$\therefore$ *Ans.* $\tan\theta = 1.333$

Example 12. To calculate $\cos\theta$, when $\tan\theta = 0.32$

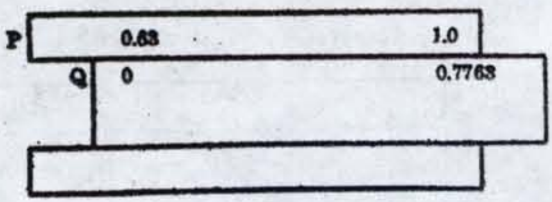
Set 1.0 *C* to 0.32 *T* against 1.0 *P* read 0.9522 *Q*



$\therefore\ \cos\theta = 0.9522$

Example 13. To get $\cos\theta$ when $\sin\theta = 0.63$

Set 0 *Q* to 0.63 *P* against 1.0 *P* read 0.7763 *Q*



$\therefore\ \cos\theta = 0.7763$

This slide rule is also good for hyperbolic functions, as what (Q') is to $\cosh\theta$ is (Q) to $\sinh\theta$.

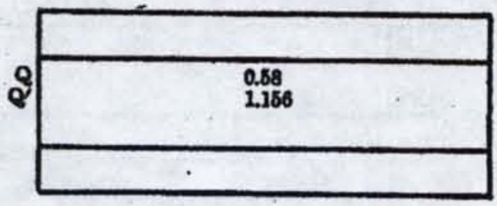
Here you cannot calculate out the angle, but you can get the relation between *sinhθ* and *coshθ* within the limit of some 0°–80°. In practice, keep in mind the formula:—

$$\cosh^2\theta = 1 + \sinh^2\theta$$

or $$\cosh\theta = \sqrt{1 + \sinh^2\theta}$$

Example 14. To get *coshθ*, when *sinhθ*=0.58

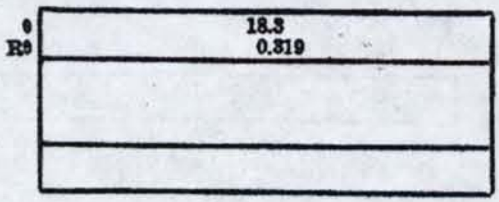
Against 0.58 *Q* read 1.156 *Q'*



∴ $\cosh\theta = 1.156$

Example 15. Convert 18.3° into radians.

Against 18.3 *θ* read 0.319 *Rθ*



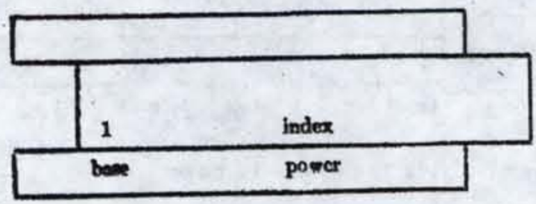
∴ *Ans.* 0.319 radian.

You could do this between (*C*) and (*D*) quite as well.

[4] Involutions and Evolutions.

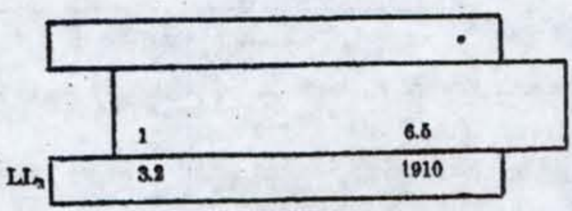
In the old slide rule, we had the *log-log* scales only for a limit 1.1—20,000. This new slide rule has a wide limit of 1.01—22,000. So with the old slide rules you could not calculate such functions as the compound interests when the rate is lower than 10%. Here you can do it with this new slide rule for the rate as low as 1%.

In this slide rule, the *log-log* scale is in three parts; and graduated as per $log_e(log_{10}x)$ or $log_e(x\text{-}C)$. Here $(x\text{-}C)$ means a reading, x, on the scale (C). Hence, e^n, which so often occurs in electricians' computation, could be had without setting the slide. Generally :—



Example 16. $3.2^{6.5} = 1910$

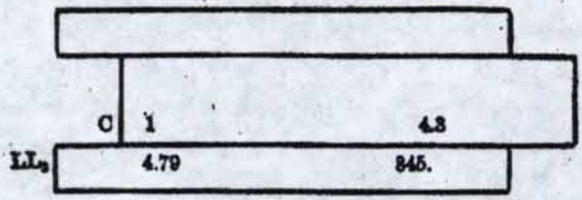
Set 1 C to 3.2 LL_3 against 6.5 C read 1910 LL_3



∴ Ans. 1910

Example 17. $\sqrt[4.3]{845} = 4.79$

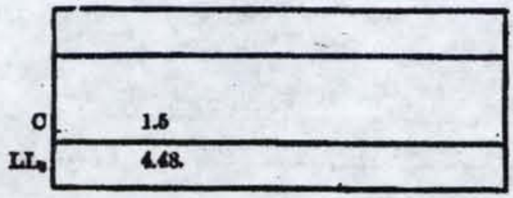
Set 4.3 C to 845 LL_3 against 1 C read 4.79 LL_3



∴ *Ans.* 4.79

Example 18. $e^{1.5} = 4.48$

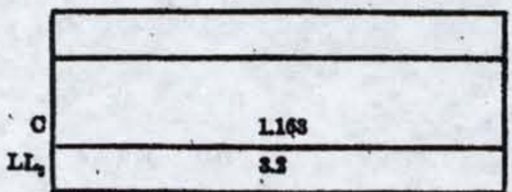
Against 1.5 C read 4.48 LL_3



∴ *Ans.* 4.48

Example 19. $\log_e 1.163 = [3.2$ as helow]

Against 1.163 C read 3.2 LL_3



∴ *Ans.* 3.2

For decimalization of e^n, keep in mind that

Left end of $LL_3 = e^{1.0} = 2.717$

" " " $LL_2 = e^{0.1} = 1.105$

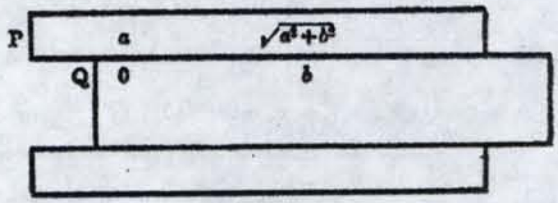
" " " $LL_1 = e^{0.01} = 1.010$

[5] Calculations on a Right Triangle.

The sides of a right triangle, is calculated by $\sqrt{a^2 \pm b^2}$ according to the Pythagorian theorem; and for the calculation of such functions there is nothing like this patent slide rule. The calculation is done between (P) and (Q), (Q').

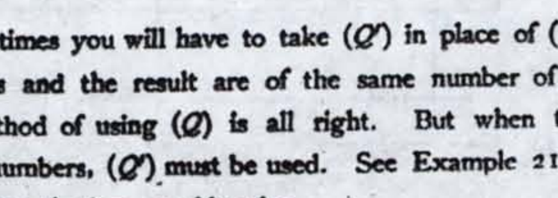
For $\sqrt{a^2+b^2}$.

Set 0 Q to a P against b Q read $\sqrt{a^2+b^2}$ P



For $\sqrt{a^2-b^2}$.

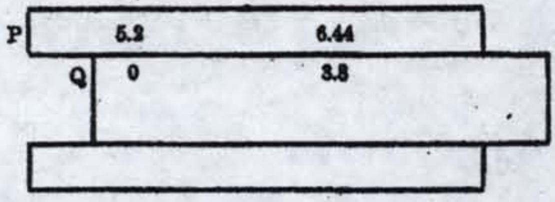
Set b Q to a P against 0 Q read $\sqrt{a^2-b^2}$ P



Sometimes you will have to take (Q') in place of (Q). When the factors and the result are of the same number of places, the above method of using (Q) is all right. But when they are of different numbers, (Q') must be used. See Example 21. Extreme cases require further concideration.

Example 20. $\sqrt{5.2^2+3.8^2}=6.44$

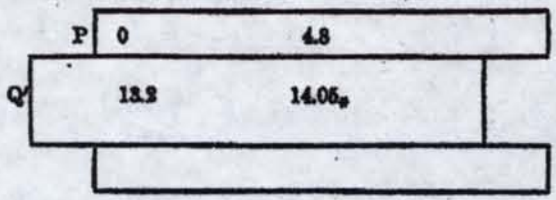
Set 0 *Q* to 5.2 *P* against 3.8 *Q* read 6.44 *P*



Ans. 6.44

Example 21. $\sqrt{13.2^2+4.8^2}=14.05$

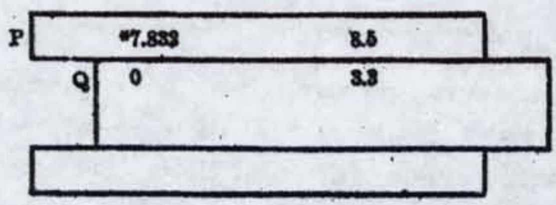
Set 13.2 *Q'* to 0 *P* against 4.8 *P* read 14.05 *Q'*



Ans 14.05

Example 22. $\sqrt{8.5^2-3.3^2}=7.832$

Set 3.3 *Q* to 8.5 *P* against 0 *Q* read 7.832 *P*

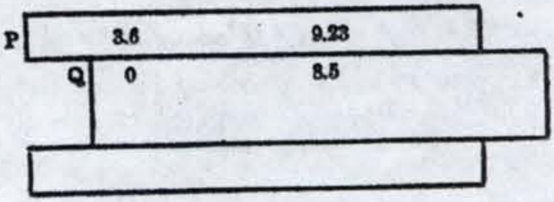


Ans. 7.832

Example 23. In a right triangle, $a=8.5$ ft.; $b=3.6$ ft.; what is the length of the hypotenuse, c in feet?

As $c=\sqrt{a^2+b^2}$.

Set 0 Q to 3.6 P against 8.5 Q read 9.23 P



Ans. 9.23 ft.

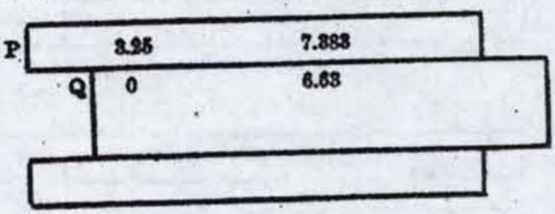
Example 24. What is the absolute value of the compound number,

$3.25+j6.63$ (where $j=\sqrt{-1}$)

The absolute value of $(a+jb)$ is $\sqrt{a^2+b^2}$.

Hence

Set 0 Q to 3.25 P against 6.63 Q read 7.383 P



Ans. 7.383

Section C. APPLICATIONS TO HIGHER MATHEMATICS

[I] On Compound Numbers.

(a) The absolute value, $\sqrt{a^2+b^2}$, of a compound number $(a+jb)$.

$$\circ\ Q \text{ to } a\ P \text{ against } b\ Q \text{ read } \sqrt{a^2+b^2}\ P$$

just as per Example 23 in the preceding Section.

(b) To convert a compound number $(a \pm jb)$ into the form of $A(\cos\theta \pm j\sin\theta)$.

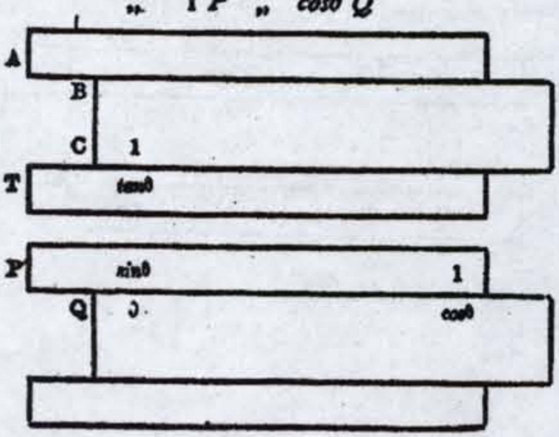
By the formulae, $A=\sqrt{a^2+b^2}$, $\tan\theta=\dfrac{b}{a}$

A is obtained by the method already stated in (a).

For $\sin\theta$ and $\cos\theta$, first get $\dfrac{b}{a}$ or $\tan\theta$ between (C) and (D).

Set 1 C to $\tan\theta$ T against $\circ$ Q read $\sin\theta$ P

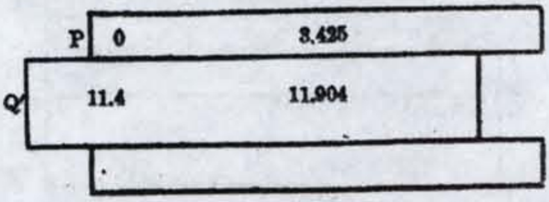
" 1 P " $\cos\theta$ Q



xample 1. To convert $(6.85+22.8j)$ into the form of $A(\cos\theta+j\sin\theta)$.

(i) As $\sqrt{6.85^2+22.8^2}$ is rather awkward to attack it should be onverted first into $\sqrt{4}\,\sqrt{3.425^2+11.4^2}$.

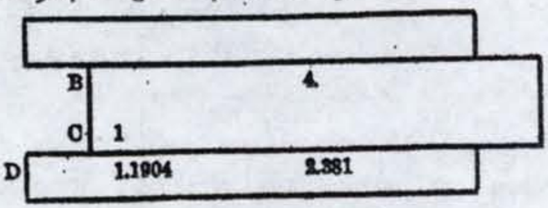
Set 11.4 Q' to 0 P against 3.425 P read 11.904 Q'



$\therefore\ \sqrt{3.425^2+11.4^2}=11.904$ or by inspection $A=23.808$

For $11.904\times\sqrt{4}=A$ on the slide rule :—

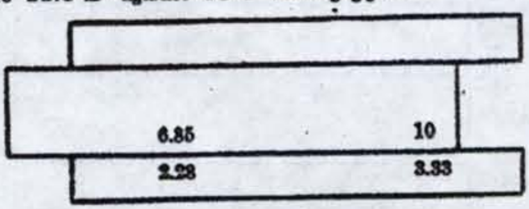
Set 1 C to 11.904 D against 4 B read 23.81 D



$\therefore$ *Ans.* $A=23.81$

(ii) For $\frac{22.8}{6.85}$ [or $=\tan\theta$]

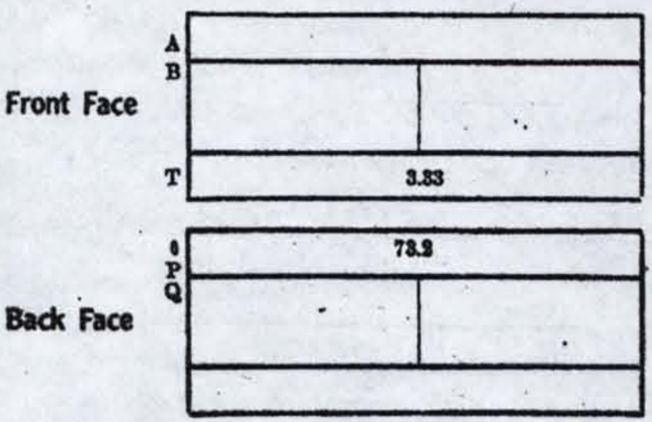
Set 6.85 C to 22.8 D against 10 C read 3.33 D



$\therefore\ \tan\theta=\frac{22.8}{6.85}=3.33$

(iii) For $sin\theta$ and $cos\theta$, we are to calcurate $tan^{-1}3.33$:

Against 3.33 T read 73.2 θ on the other side of the slide rule.



$\therefore$ *Ans.* 23.81 ($cos\ 73.2° + jsin\ 73.2°$)

(c) $A(cos\theta \pm jsin\theta)$ to be converted into the form $(a+jb$

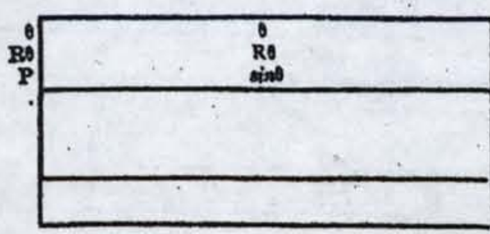
This is very simple for $a = A \times cos\theta$, $b = A \times sin\theta$

(d) $A(cos\theta \pm jsin\theta)$ to be converted into the form $A_{\underline{/\theta°}}$.

(d′) When $cos\theta$ and $sin\theta$ are expressed in terms of θ: Yo can very simply write down $A_{\underline{/\theta°}}$.

(d″) When $cos\theta$ or $sin\theta$ are given instead of θ:

Against $sin\theta$ P read θ-θ; and also read Rθ-$R\theta$



Hence you have either θ or Rθ. Then treat exactly as (d′).

(e) To get $(a_1+jb_1)(a_2+jb_2)$

Say $(a_1+jb_1)=A_1\varepsilon^{j\theta_1}$

$(a_2+jb_2)=A_2\varepsilon^{j\theta_2}$

Then $(a_1+jb_1)(a_2+jb_2)=A_1A_2\times\varepsilon^{j(\theta_1+\theta_2)}$

If you should have the result in the form of a compound number, then you can have

$$(a_1+jb_1)(a_2+jb_2)=A_1A_2[cos(\theta_1+\theta_2)+jsin(\theta_1+\theta_2)]$$

Example 2. $(5.8+j3.2)\times(6.8+j2.6)$

Similarly as Example 1 :—

$5.8+j3.2=6.622(cos\ 28.85^\circ+jsin\ 28.85^\circ)$

$6.8+j2.6=7.28(cos\ 20.9^\circ+jsin\ 20.9^\circ)$

$\therefore$ $A=6.622\times7.28=48.2$

$\theta=28.85^\circ+20.9^\circ=49.75^\circ$

$\therefore$ *Ans.* $48.2\varepsilon^{j49.75^\circ}$ Q.E.I.

$48.2\ cos\ 49.75^\circ=31.2$

$48.2\ sin\ 49.75^\circ=36.8$

$\therefore$ *Another Ans.* $(31.2+j36\ 8)$ Q.E.I.

(f) The quotient of complex numbers can easily be calculated by the reverse method.

To get $\dfrac{a_1+jb_1}{a_2+jb_2}$

$$\frac{a_1+jb_1}{a_2+jb_2}=\frac{A_1}{A_2}\times\varepsilon^{j(\theta_1-\theta_2)}$$

Example 3. $\dfrac{(6.336+j5.22)}{(5.8\ \ +j0.56)}$

Just as in the previous example :—

$$6.336+j5.22=8.207\varepsilon^{j(0.69)}$$

(The angle here has been computed out in radians)

$$5.8+j0.56=5.83\varepsilon^{j(0.096)}$$

$$A=\frac{8.207}{5.83}=1.407$$

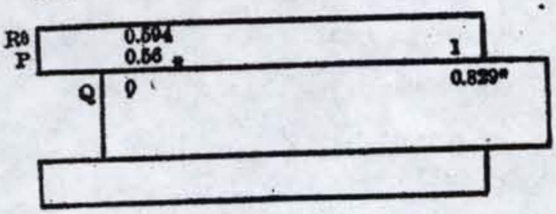
$$\theta=0.69-0.096=(0.594)$$

$$\therefore \quad \textit{Ans.} \quad 1.407\varepsilon^{j(0.594)}$$

To convert this form of a vector into the form of $(a+jb)$:—
This is nothing but (c).

Against 0.594 $R\theta$ read 0.56 P

Set 0 Q to 0.594 $R\theta$ against 1.0 P read 0.829 Q



$$\therefore \quad sin\theta=0.56$$

$$cos\theta=0.829$$

$$\therefore \quad a \ =1.407\times0.829=1.166$$

$$b \ =1.407\times0.56\ =0.788$$

$$\therefore \quad \textit{Ans.} \quad 1.166+j0.788$$

Thus you can get generally :—

$$(a_1+jb_1)\times(a_2+jb_2)^{\pm1}=A_1A_2^{\pm1}\times\varepsilon^{j(\theta_1\pm\theta_2)}=(a+jb)$$

[2] Examples of Application to Electrical Calculations.

Example 4. There is a voltage wave with higher frequencies.

The effective value of the fundamental wave, $Ee_1=82$,

Ditto of the third hermonic wave, $Ee_3=25$,

Ditto of the fifth hermonic wave, $Ee_5=12$.

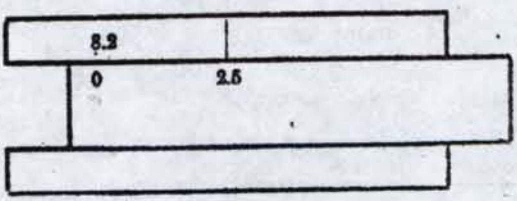
What is the effective value of the distorted wave?

$$Ee_0=\sqrt{Ee_1^2+Ee_3^2+Ee_5^2}$$

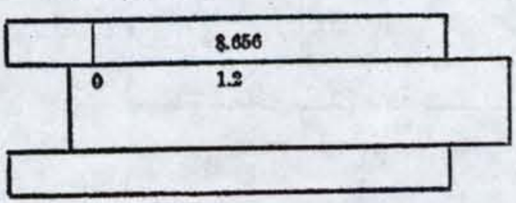
$$=\sqrt{82^2+25^2+12^2}$$

Set 0 Q to 82 P against 25 Q put Hairline
Set 0 Q to Hairline against 12 Q read 86.56 P

Stage I



Stage II



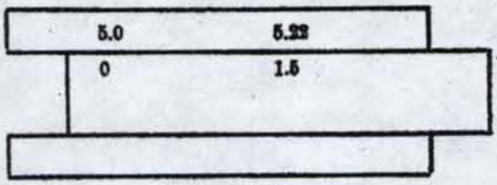
$\therefore\ Ee_0=86.56$

Example 5. A circuit whose Impedance is $(2+j3.2)$, is subjected to an $\dot{E}.M.F$, $(50+j15)$ between both ends. What is the current ?

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{50+j15}{2+j3.2}$$

(i) $(50+j15)$ to be converted into the form of $A_1 \underline{|\theta_1}$.

Set 0 Q to 5 P against 1.5 Q read 5.22 P

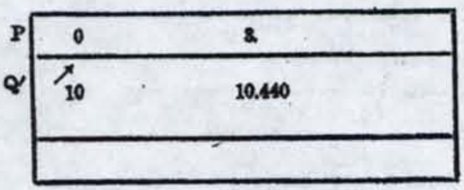


$\therefore \; A_1 = 5.22$

[Note : for accuracy, the following method is preferred :

$$\tfrac{1}{2}\sqrt{100^2+30^2}$$

Against 3 P read 10.440 Q'

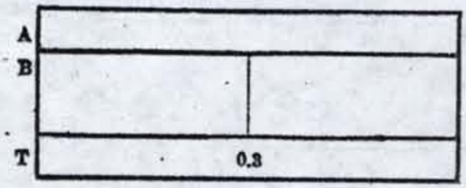


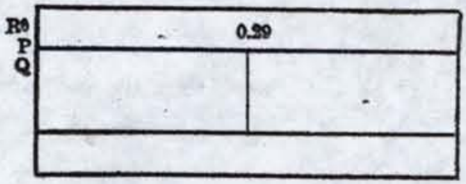
$A_1 = 104.40 \div 2 = 52.20$

Four places have been made clear.]

As $\tan\theta_1 = \frac{15}{50} = 0.3$

Against 0.3 T read 0.29 $R\theta$



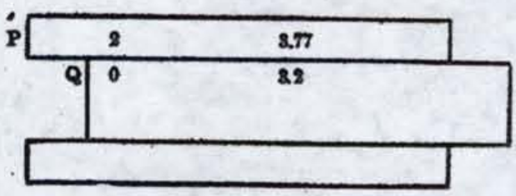


$$\therefore \quad R\theta = (0.29)$$

$$\therefore \quad 50 + j15 = 52.2 \underline{\lfloor 0.29} \qquad (\dot{E})$$

(ii) **Similarly we can get** $(2 + j3.2) = A_2 \underline{\lfloor \theta_2}$

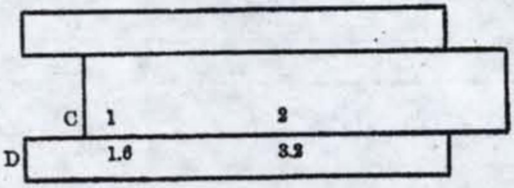
Set 0 Q to 2 P against 3.2 Q read 3.77 P



$$\therefore \quad A_2 = 3.77$$

To get $\tan\theta_1 = \frac{3.2}{2} = 1.6$

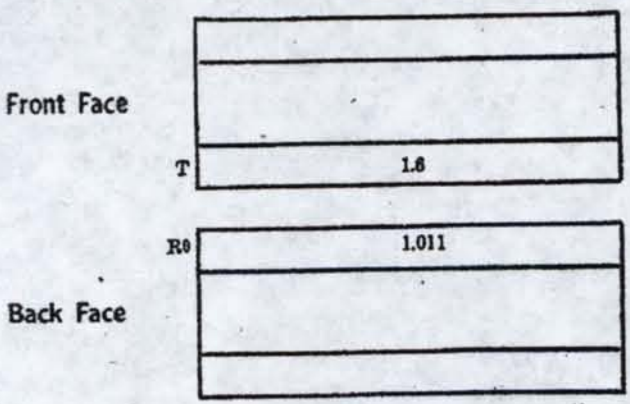
Set 2 *C* to 3.2 *D* against 1 *C* read 1.6 *D*



$$\therefore \quad \tan\theta_1 = 1.6$$

To get Rθ :

Against 1.6 *T* read 1.011 *Rθ* on the other side of the slide rue



$$\therefore 2 + j3.2 = 3.77 \underline{\lfloor 1.011} \qquad (\dot{Z})$$

Thus we have the values of $\dot{E}$ and $\dot{Z}$ separately; so now we shall proceed for $\dot{I}$ or $\frac{\dot{E}}{\dot{Z}}$:—

$$\therefore \quad \dot{I} = \frac{2.2 \;\underline{|0.29}}{3.77 \;\underline{|1.011}} = A_0 \underline{|\theta_0} \qquad \frac{\dot{E}}{\dot{Z}},$$

$$A_0 = \frac{52.2}{3.77} = 13.85$$

$$\theta_0 = 0.29 - 1.011 = -0.721$$

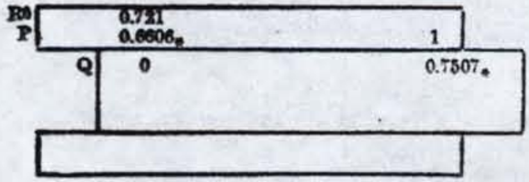
$\therefore$ The vector required is $13.85\varepsilon^{-j0.721}$.

This can be transformed into the form $(a+jb)$.

$$13.85\varepsilon^{-j0.721} = 13.85[\cos(0.721) - j\sin(0.721)]$$

Set 0 Q to 0.721 $R\theta$ against 0 Q read 0.6606 P and also

" 1 P " 0.7507 Q



$$\therefore \quad \sin(0.721) = 0.6606$$

$$\cos(0.721) = 0.7507$$

$$\therefore \quad 13.85 \times 0.7507 = 10.4$$

$$13.85 \times 0.6606 = 9.15$$

$$\therefore \quad \dot{I} = 10.4 - j9.15 \qquad \text{Q.E.I.}$$

The calculations look complicated, but in practice they can be done with simplicity, easiness and rapidity.

[3] Supplementary.

(I) $\sqrt{a^2 \pm b^2}$

The vector scales (P) and (Q) are quite different from logarithmic scales, and a and b must not differ too much from each other; or they must be of alike places in digits. When $a=38$, $b=6$, you could not do

Set 0 Q to 38 P against 6 Q read $\sqrt{38^2+6^2}$ P

as you will find $\sqrt{38^2+6^2}$ off (P). If you take 0.6 in place of 6, you will get the result only inaccurately.

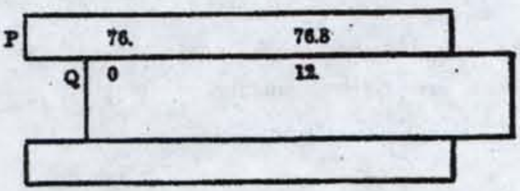
In such a case, you can do in the follywing way:—

$$\sqrt{a^2+b^2}=\frac{1}{n}\sqrt{(na)^2+(nb)^2}$$

where n is to be chosen a very simple integer or a fraction like 2 or $\frac{1}{2}$, &c.

Example 6. $\sqrt{38^2+6^2}=\frac{1}{2}\sqrt{76^2+12^2}$

Set 0 Q to 76 P against 12 Q read 76.8 P



$$\therefore \sqrt{38^2+6^2}=76.8 \div 2=38.4$$

[4] Hyperbolic Functions.

This Slide rule has a patent "Gudermanian Scales" ($G\theta$) for hyperbolic functions, and you can get all hyperbolic functions at once with rapidity and accuracy.

Example 7. *sinh* $0.32 = 0.325$

Against $0.32G\theta$, read $0.325T$

Example 8. *tanh* $0.83 = 0.68$

Against $0.83G\theta$, read $0.68P$

Example 9. *cosh* $0.55 = 1.155$

Method 1.

Against $0.55G\theta$, read *sinh* $0.55 = 0.578T$

Against $0.578Q$, read $1.155Q'$

Method 2.

Set $0Q$ to $0.55G\theta$, against right end or $1P$ read *sech* $0.55 = 0.866Q$

Set $0C$ to $0.866D$, against right end D read $\frac{1}{0.866} = 1.155C$

Thus the addition of ($G\theta$) to the patent vector scales (P) and (Q), and the tangent scale (T), has simplified the computations of hyperbolic functions. Also the hyperbolic functions of complex number can be obtained easily by the following formurae :—

$$\sinh(a+jb) = \sqrt{\sinh^2 a + \sin^2 b} \ \Big|\, \tan^{-1}\left(\frac{\tan b}{\tanh a}\right)$$

$$\cosh(a+jb) = \sqrt{\sinh^2 a + \cos^2 b} \ \Big|\, \tan^{-1}(\tan b.\ \tanh a)$$

$$\tanh(a+jb) = \sqrt{\frac{\sinh^2 a + \sin^2 b}{\sinh^2 a + \cos^2 b}} \ \Big|\, \tan^{-1}\left(\frac{\sin 2b}{\sinh 2a}\right)$$

Example 10.

$$\sinh(0.32 + j\ 1.22) = 0.994 \ \underline{|\ 1.460}$$

$\sinh 0.32=0.326 \qquad \sin 1.22=0.939$

$\tanh 0.32=0.310 \qquad \tan 1.22=2.73$

$$\sinh(0.32+j\,1.22)=\sqrt{0.326^2+0.939^2}\;\underline{\Big|\tan^{-1}\frac{2.730}{0.310}}$$

$$=0.994\;\underline{|\,1.460}$$

Example 11.

$$\cosh(0.257+j\,0.652)=0.836\;\underline{|\,0.190}$$

$\sinh 0.257=0.260 \qquad \tanh 0.257=0.250$

$\cos 0.652=0.795 \qquad \tan 0.652=0.763$

$$\cosh(0.257+j\,0.652)=\sqrt{0.260^2+0.795^2}\;\underline{|\,\tan^{-1}(0.250\times0.763)}$$

$$=0.836\;\underline{|\,0.190}$$

Example 12.

$$\tanh(1.25+j\,0.28)=0.87\;\underline{|\,0.0885}$$

$\sinh 1.52=1.60$

$\sin 0.28=0.276$

$\cos 0.28=0.961$

$\sinh(2\times1.25)=6.0$

$\sin(2\times0.28)=0.531$

$$\tanh(1.25+j\,0.28)=\sqrt{\frac{1.60^2+0.276^2}{1.60^2+0.961^2}}\;\underline{\Big|\tan^{-1}\left(\frac{0.531}{6.0}\right)}$$

$$=\sqrt{\frac{0.80^2+0.138^2}{0.80^2+0.4805^2}}\;\underline{|\,\tan^{-1}0.0885}$$

$$=0.87\;\underline{|\,0.0885}$$

Example 13.

Calculate $Y_\infty=\dfrac{\tanh(0.511+j\,0.04)}{0.88\times10^3\;\underline{|\,0.033}}$

First obtain the hyperbolic tangent by the following formura :—

$$\tanh(a+jb)=\sqrt{\frac{\sinh^2a+\sin^2b}{\sinh^2a+\cos^2b}}\;\underline{\Big|\tan^{-1}\left(\frac{\sin 2b}{\sinh 2a}\right)}$$

$sinh\ a = sinh\ 0.511 = 0.534$

$sinh\ 2a = sinh\ 1.022 = 1.210$

$sin\ b = sin\ 0.04 \doteq 0.04$

$sin\ 2b = sin\ 0.08 \doteq 0.08$

$cos\ b = cos\ 0.04 \doteq 0.9995$

$$tanh\ (0.511 + j\,0.04) = \frac{0.536}{1.133} \left| tan^{-1} \frac{0.08}{1.210} \right.$$

$$= 0.473 \left| 0.066 \right.$$

$$Y_\infty = \frac{0.473}{0.88 \times 10^3} \left| 0.066 + 0.033 \right. = 0.538 \left| 0.099 \right. \times 10^{-3}$$

Example 14.

Calculate $V_\infty = \dfrac{cosh\,(1.191 + j\ 2.485)}{cosh\,(2.230 + j\ 2.551)} \times (-56.9 \times 10^3 \overline{|0.180})$

$sinh\ 1.191 = 1.492$

$sinh\ 2.230 = 4.60$

$cos\ 2.485 = -0.611$

$cos\ 2.551 = -0.557$

$tanh\ 1.191 \doteq 0.831$

$tanh\ 2.230 = 0.977$

$tan\ 2.485 = -1.293$

$tan\ 2.551 = -1.490$

$$cosh\ (1.191 + j\ 2.485) = \sqrt{1.492^2 + 0.611^2} \left| tan^{-1}(1.293 \times 0.831) \right.$$

$$= 1.72 \left| 0.83 + \pi \right.$$

$$cosh\ (2.230 + j2.551) = \sqrt{4.60^2 + 0.557^2} \left| tan^{-1}(0.977 \times 1.49) \right.$$

$$= 4.63 \left| 0.969 + \pi \right.$$

$$V_\infty = \frac{1.72}{4.63} \times (-56.9 \times 10^3) \left| 0.82 + \pi - 0.969 - \pi - 0.18 \right.$$

$$= -22.13 \times 10^3 \overline{|0.32}$$