XVI. Inverse Trigonometric Function of Complex Number

For finding Inverse Trigonometric Function of complex number, The following three expressions can be used. There, the a and b are known in the expressions.

(10) $\sin^{-1}(a+jb)=x+j\theta$

$$x=\sin^{-1}\left[\frac{\sqrt{b^2+(1+a)^2}-\sqrt{b^2+(1-a)^2}}{2}\right]$$

$$\theta=sh^{-1}\left(\frac{b}{\cos x}\right)$$

(11) $\cos^{-1}(a+jb)=x+j\theta$

$$x=\cos^{-1}\left[\frac{\sqrt{b^2+(1+a)^2}-\sqrt{b^2+(1-a)^2}}{2}\right]$$

$$\theta=sh^{-1}\left(\frac{b}{\sin x}\right)$$

(12) $tg^{-1}(a+jb)=x+j\theta$

$$x=\frac{1}{2}tg^{-1}\left[\frac{2a}{1-(a^2+b^2)}\right]$$

$$\theta=\frac{1}{2}th^{-1}\left[\frac{2b}{1+(a^2+b^2)}\right]$$

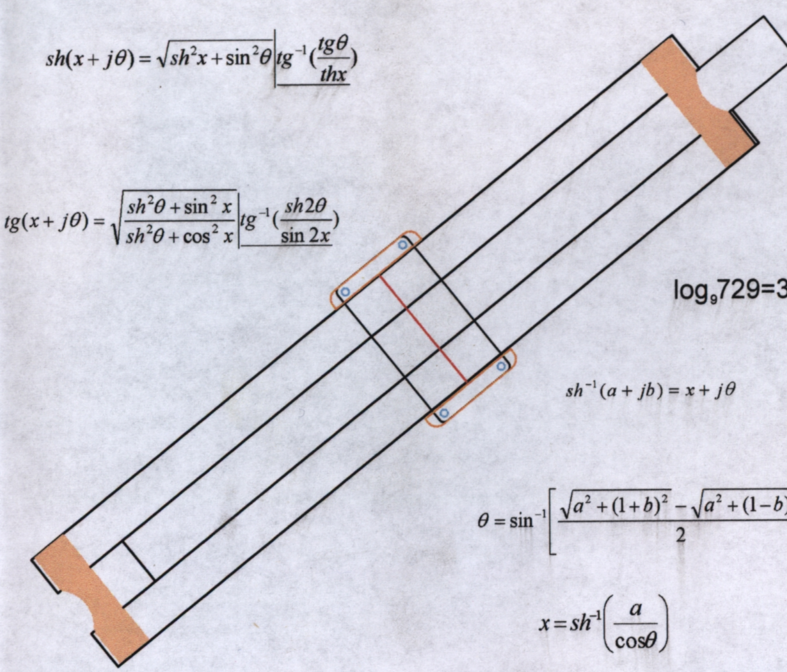
Flying Fish

TYPE 1003 Slide Rule

INSTRUCTION

$$sh(x+j\theta)=\sqrt{sh^2x+\sin^2\theta}\,\angle\, tg^{-1}\left(\frac{tg\theta}{thx}\right)$$

$$tg(x+j\theta)=\sqrt{\frac{sh^2\theta+\sin^2 x}{sh^2\theta+\cos^2 x}}\,\angle\, tg^{-1}\left(\frac{sh2\theta}{\sin 2x}\right)$$



$\log_9 729=3$

$sh^{-1}(a+jb)=x+j\theta$

$$\theta=\sin^{-1}\left[\frac{\sqrt{a^2+(1+b)^2}-\sqrt{a^2+(1-b)^2}}{2}\right]$$

$$x=sh^{-1}\left(\frac{a}{\cos\theta}\right)$$

SHANGHAI SLIDE RULE FACTORY

No.528 Nanchang Road
Shanghai, China

Instruction for The Use of

***Flying Fish* 1003 Slide Rule**

Instruction for The Use of
Flying Fish 1003 Slide Rule

Introduction

The type 1003 slide rule is a kind of powerful tool which can be used to find numerical answers to involved mathematical problems.

There are 4 scales for finding the natural logarithm of decimal fraction those less than 1, and 4 scales for finding the natural logarithm of number those more than 1. Combine using this 8 scales and C, D scales , you can directly find the reciprocal, root, power, logarithm of any base number etc. of from red 0.00002 to red 0.99905 and from 1.00095 to 50000. Add the scales Ln_{-2} and Ln_{2}, so, the limit point of natural logarithm is more close to 1.

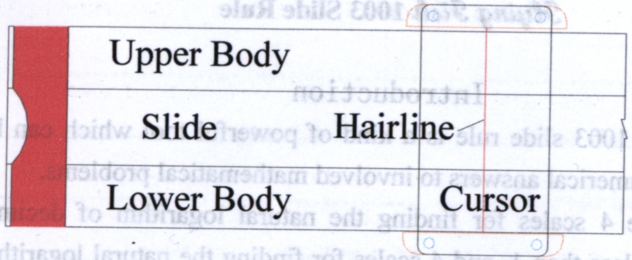
Add the scales srt_{-1} and cos ctg_{-1} , you can find the sinθ, tgθ, convert angle into radian that the angle is small, and, find the cosθ , ctgθ that the angle is large. Add the scales tg_2 and ctg_2 , you can find the tgθ that the angle is small, and, find the ctgθ that the angle is large.

Add the scales ch_1, you can find the chx directly but need not convert it by using shx/thx. It is more convenient for finding the formula that with chx.

Setting these scales sq_2, sq_1 and cu_1, cu_2, cu_3 instead of A and K scales, it will made the answer more precision for finding the square, square root, cubic, cubic root.

I. Structure and Scales

The slide rule consists of three parts: (1) The body (upper and lower fixed bars); (2) the slide; (3) the cursor or indicator. The scales on the body and slide are arranged to work together in solving problems. The hairline on the cursor is used to help the eyes in reading the scales and in adjusting the slide.



Each scale is named by letters or other symbol at the end. The 1003 slide rule 34 scales, there are 17 scales on the side 1: Ln_{-2}, Ln_{-1}, Ln_0, Ln_1, DF, CF, CIF, H_0, H_1, H_0', CI, C, D, Ln_1, Ln_0, Ln_{-1}, Ln_{-2}. There are 17 scales on side2 too: Sh_0, Sh_1, cu_1, cu_2, cu_3, lg^{-1}_1, tg_0ctg_0, tg_1ctg_1, tg_2ctg_2, sin_0cos_0, $s_{-1}r_{-1}t_{-1}cos_{-1}ctg_{-1}$, C, D, sq_2, sq_1, th_0, ch_1.

In order to use a slide rule, you should know: (1) how to read the scales; (2) how to "set the slide and cursor; (3) how to determine the decimal point in the result.

II. How to read the scales

The scales C and D are used most frequently. These two scales are exactly alike. The total length of the scales has been separated into many smaller parts by fine lines.

A line labeled 1 at the left end is called the **left index**. A line labeled 10 at the right end is called the **right index**. Between 1 and 2 has been separated into 10 parts by shorter lines. That is 1.1, 1.2, etc. These are secondary graduations. And the same, there are some tertiary graduations between 1.1 and 1.2. etc.. The graduations interval is at 0.01 between 1 and 2, The graduations interval is at 0.02 between 2 and 4, The graduations interval is at 0.05 between 4 and 10. On D or C scale, you can read 1 to 10, and, you can read 10 to 100, 100 to 1000, or, 0.1 to 1, and so on. By this way, you can judge other scales. Please pay attention to the red scales, some red scale is increasing from right to left, say, CI etc.

The black number is increase from left to right, the red number is increase from right to left.

In the instruction, C:3 means the figure 3 on the C scale, CI:4.2 Means the figure 4.2 on the CI scale, and so the rest.

III. Use C, D and CI Calculate Multiplication and Division

1. Use C, D scales calculate Multiplication

Example: 2×4=?

Set left index C:1 opposite D:2 (first factor), move hairline on C:4(Other factor), under hairline read D:8 is the Product.

i.e. 2×4=8

Example: 3×8=?

Set right index C:10 opposite D:3 (first factor) [If set left index C:1 opposite D:3 (first factor), then, C:8(Other factor) out off body.] Move hairline on C:8(Other factor), under hairline read D:24 is the Product.

i.e. 3×8=24

2. Use CI, D scales calculate Multiplication

Example: 2.5×7.2=?

Move hairline on D:2.5, move slide set CI:7.2 under hairline. Opposite C:1 read D:18 is the Product.

i.e. 2.5×7.2=18

3. Use CI, C, D scales calculate Continued Multiplication

Example: 1.85×6.2×4.75=?

Move hairline on D:1.85, move slide set **CI:6.2** under hairline. Temporality, C:1 opposite D:11.47 is the Product of 1.85×6.2. Remove hairline on C:4.75, under hairline read D:54.5 is the Product.

i.e. 1.85×6.2×4.75=54.5

4. Use C, D scales calculate Division

Example: 18÷3=?.

Move hairline on D:18, move slide set C:3 under hairline. Opposite C:10 read D:6 is the Result.

i.e. 18÷3=6.

Example: 7.68÷4.8=?.

Move hairline on D:7.68, move slide set C:4.8 under hairline. Opposite C:1 read D:1.6 is the Result.

i.e. 7.68÷4.8=1.6

5. Use CI, D scales calculate Division

Example: 7.68÷4.8=?.

Move slide set C:10 Opposite D:7.68, Move hairline on CI:4.8, under hairline read D:1.6 is the Result.

i.e. 7.68÷4.8=1.6

When dividend is fixed, divisor is change, use CI, D scales calculate Division, that is very convenient.

Example: 96÷12=?. 96÷19=?. 96÷25.6=?

Move slide set C:10 Opposite D:96, ordinal move hairline on CI:12, CI:19, CI:25.6, under hairline read D:8, D:5.05, D:3.75 are the Results.

i.e. 96÷12=8. 96÷19=5.05. 96÷25.6=3.75

6. Use CI, C and D scales calculate continued Division

Example 9 235÷3.8÷42=?

move hairline on D:2.35, Move slide set C:3.8 under hairline, remove hairline on CI:42, under hairline read D:1.472 is the Results.

i.e. 235÷3.8÷42=1.472

7. Use CI, C and D scales calculate combined continued multiplication and continued Division.

Example: (3.8×6.9×7.5)÷(8.4×3.2)=?

Move hairline on D:3.8, Move slide set C:8.4 under hairline (÷8.4).

Remove hairline on C:6.9 (×6.9), Move slide set C:3.2 under hairline (÷3.2). Remove hairline on C:7.5 (×7.5), under hairline read D:7.32 is the Results.

i.e. (3.8×6.9×7.5)÷(8.4×3.2)=7.32

Example: (28.7×5.35)÷(4.3×2.9×8.05)=?

Move hairline on D:28.7, Move slide set C:4.3 under hairline (÷4.3). Remove hairline on C:5.35 (×5.35), Move slide set C:2.9 under hairline (÷2.9). Remove hairline on CI:8.05 (÷8.05), under hairline read D:1.53 is the Results.

i.e. (28.7×5.35)÷(4.3×2.9×8.05)=1.53

IV. Folding Scales CF, DF and Red CIF

CF and DF scales are Folding Scales of C and D. Suppose both the length from C:1 to C:10 and from D:1 to D:10 are L, then, the length from C:1 to C: $\sqrt{10}$ and from D:1 to D: $\sqrt{10}$ will be L/2, scilicet, the point $\sqrt{10}$ (about 3.1623) will be the middle length point of the C and D scales. CF is Folding Scales of C, it with graduation started from $\sqrt{10}$ dead against C:1 and terminated at 10, and again in turn started from 1 and terminated at $\sqrt{10}$. And the same that DF is Folding Scales of D. For read easily the DF with graduation started from 3 and terminated at 3. 3. At both two end of DF scale marked π, it is for calculating some formula with π.

CIF is the inverted scale of CF. It can be used in combination with CF scale and DF scale. So as to prevent from the extreme shifting of the scale and diminish errors. Evidently, this methord is much convenient as compared with that by only making use of C and D scales. For multiplication and division include π, we need not shift the scale. Move the cursor and set the hairline directly over a given number on D scale, and correspondingly the product of that given number and π will be obtained on DF scale.

Example 7 22.7×6.45=(146.4)

Move the slide until C:1 exactly against the line D:22.7, then set the hairline directly over CF:6.45, read DF:146.4 under the hairline is the Result.

Example 8 2.5× π =(7.85)

Move slide let CF: $\sqrt{10}$ against DF: π, Set the hairline over C:2.5, read DF:7.85 under the hairline is the Result.

V. The sq_2, sq_1, scales: Square roots and squares

The 1003 slide rule have a special design for the square roots and squares. Differ from the K and C,D scale on other slide rule, the sq_2, sq_1 scales are the square roots of C and D scales. As the contrary, C and D scales is the squares of sq_2, sq_1 scales. which is to say that the scale is equivalent to prolong another rule length, so, the number on scale can be read more precision.

1. Find Squares

Example: 278^2=?

Move hairline on sq_1:2.78, under hairline read D:7.73. Before you give the answer, you should determine which scale should be used for the number.

When use the sq_1 scale	When use the sq_2 scale
Digits of power=digits of factor × 2 -1	digits of power=digits of factor × 2

278 is 3 places, hairline on the sq_1 scale, 3×2-1= 5,

i.e. 278^2=77300

Example: 0.0565^2=?

Move hairline on sq_2:5.65, under hairline read D:31.9. 0.0565 is –1 place, The hairline on the sq_2 scale, (-1) ×2=-2,

i.e. 0.0565^2=0.00319

2. Square Roots

In general, to find the square root of any number with an odd number of digits or zero (1,3,5,7,…), the sq_1 scale is used. And the digits of root=(digits of number +1) ÷ 2.

If the number with an even number of digits or zero (2,4,6,8,…), the sq_2 scale is used. And the digits of root=digits of number ÷ 2.

Example: $\sqrt{30,000} = ?$

30,000 is a 5 digits odd number, so, move the hairline on D:3, read 1.732 on sq_1 scale under hairline. (5+1) ÷ 2.=3, the digits of root is 3.

i.e. $\sqrt{30,000} = 173.2$

Example: $\sqrt{0.000585} = ?$

0.000585 is a -3 digits odd number, so, move the hairline on D:5.85, read 2.42 on sq_1 scale under hairline. (-3+1) ÷ 2.=-1, the digits of root is -1.

i.e. $\sqrt{0.000585} = 0.0242$

Example: $\sqrt{5300} = ?$

5300 is a 4 digits even number, so, move the hairline on D:5.3, read 7.28 on sq_2 scale under hairline. 4 ÷ 2.=2, the digits of root is 2.

i.e. $\sqrt{5300} = 72.8$

VI. The cu_1, cu_2, cu_3 scale: Cube roots and Cubes

The cu_1, cu_2, cu_3 scale is the cube roots of C and D scales. As the contrary, C and D scales are the cube of cu_1, cu_2, cu_3 scale. Different from the K scale on the other slide rules, The 1003 has 3 scales of cu_1, cu_2, cu_3 those can combine use with D, C scales for find the Cube roots and Cubes. which is to say that the scale is equivalent to prolong two other rule length, so, the number on scale can be read more precision.

1. Find Cubic

When find the power of cube, the way that determine the digits of the number is as follow,

(1) When use the cu_1 scale, the digits of power=digits of factor ×3-2.

(2) When use the cu_2 scale, the digits of power=digits of factor ×3-1.

(3) When use the cu_3 scale, the digits of power=digits of factor ×3

Example: 252^3=?

Move the hairline on D:2.52, read K:16 under hairline. Hairline on the 2nd sect of K scale, so, the digits of power is 3×3-1=8

i.e. 252^3=16000000

Example: 0.0575^3 =?

Move the hairline on D:5.75, read K:190 under hairline. Hairline on the 3rd sect of K scale, so, the digits of power is (-1)×3=-3

i.e. 0.0575^3=0.00019

2. Find Cube Roots

When find the **cube roots,** to decide which scale of the cu_1, cu_2, cu_3 scales to use in locating a number, mark off the digits in groups of three starting from the decimal point. If the left group contains one digit, the cu_1 scale is used; If the left group contains two digits, the cu_2 scale is used; If there are three digits in the left group, the cu_3 scale is used. In other words, numbers containing 1,4,7,… digits are located on the cu_1; numbers containing 2,5,8,… digits are located on the cu_2; and numbers containing 3,6,9,… digits are located on the cu scale. The root digits of integral is equal the number of groups of integral. The pure decimal fraction how many number of groups all in 0, the root has the number "0" after the decimal point.

Example: $\sqrt[3]{89600} = ?$

89600 can mark off 89'600, the left group contains two digits, so, use the cu_2 find the answer. Move the hairline over 89.6 on D scale, read 4.48 under the hairline on cu_2 scale. The integral is in 2 groups, so, the integral digits of the root is 2.

i.e. $\sqrt[3]{89600} = 44.8$

Example: $\sqrt[3]{0.00763} = ?$

0.00763 can mark off 0.007'630, the left group contains 1 digits, so, use the cu_1 find the answer. Move the hairline on D:7.63 , read cu_1:1.969 under the hairline . The all in 0 groups is 0 group, so, the root is in 0 digits.

i.e. $\sqrt[3]{0.00763} = 0.196$

VII. Use CI& C, CF&CIF scales Find reciprocal

CI scale is reciprocals of C scale; CIF scale is reciprocals of CF scale. CI and CIF are increasing from right to left. Put the hairline on C:n, read the CI:1/n under the hairline directly. Same way use CF and CIF scales.

Example: 1/25=?

Move hairline on C:2.5, Read CI:4 under hairline is the answer.

i.e. 1/25=0.04

Example: 1/0.0356=?

Move hairline on C:3.56, Read CI:2.81 under hairline is the answer.

i.e. 1/0.0356=28.1

VIII. Solve the Right Triangle

When known the size of an acute angle and the length of the hypotenuse of the right triangle. Find the length of other two sides. This sect is very useful, say, using this method can decompose a Vector into level and vertical two.

Example: Known the size of base acute angle is 36.9° and the length of the hypotenuse of the right triangle is 5. Find the length of other two sides.

Put the hairline over the D:5, move the slide let $\sin_0$:90° (or C:10) under the hairline too, remove the hairline over $\sin_0$:36.9°, read 3 on D scale is the length of opposite side. Remove the hairline over $\cos_0$:36.9°, read 4 on D scale is the length of base side.

This method show the way how to convert polar coordinates into rectangular coordinates. For this example, on alternating current circuit, we can show it as $5\underline{|36.9°} = 4 + j3$.

IX. Usage of H_0' Scale

This scale is for finding the base angle and opposite line when known the hypotenuse and base line of a right triangle.

The scales on **H'2** are engraved according to $\sqrt{1-(0.1C)^2}$, the 0 means it will combine use with the number on C scale divide by 10. When hairline on the **H_0':0.8,** meanwhile on C:6, $0.8^2+0.6^2=1$. Other numbers have the same relation. i.e. the number on C scales divide by 10 are the value of sin or cos, then the number on **H_0'** scale are the value of cos or sin of same angle.

Example A right triangle, known the hypotenuse is 5, base line is 4,

find base angle and opposite line.

Move the slide left until C:10 exactly against D:5, then set the hairline over C:4, read the D:8 under hairline, under the same hairline read **36.9°** on the **$\cos_0$** scales is the value of base angle, and the **H_0':.6**, move the hairline over C:6, read 3 on the D scale under hairline is the opposite line. Please pay attention, there, the **H_0'** scale just for read number.

X. Usage of H_0, H_1 scales

These two scales are for finding the base angle and hypotenuse when known the opposite line and base line of a right triangle. The 0 means it will combine use with the number on C scale divide by 10.

The scales on H_0 are engraved according to $\sqrt{1+(0.1C)^2}$, The scales on H_1 are engraved according to $\sqrt{1+C^2}$.

On the slide, When hairline on the H_0:1.044, meanwhile on C:.3, $1+0.3^2=1.044^2$. When hairline on the H_0:1.25, meanwhile on C:.75, $1+0.75^2=1.25^2$ When hairline on the H_1:3.16, meanwhile on C:3, $1+3^2=3.16^2$. Other numbers have the same relation. i.e. the number on C scales are the value of tgθ (or ctgθ) on tg_0 or tg_1 scale, then the number on H_0 or H_1 scale are the value of secθ (cscθ) of same angle. Because $1+tg^2\theta=sec^2\theta$, $1+ctg^2\theta=csc^2\theta$. Indeed, the H_0 and H_1 scale can be linked as a continuous scale.

Example A right triangle, known the base line is 4, opposite line is 3, find base angle and hypotenuse. (usage of H_0)

Use the side2 of the slide rule, move the slide left until C:10 exactly against D:4, then set the hairline over D:3, read the C:0.75 under hairline, under the same hairline read the **tg2:36.9°** is the value of base angle, turn off the rule to side1, under the same place hairline can read the H2:1.25, move the hairline over CF:1.25, read the DF:5 under hairline is the hypotenuse. For this example, on alternating current circuit, we can show it as $4 + j3 = 5\underline{|36.9°}$.

Example A right triangle, known the base line is 3, opposite line is 4, find base angle and hypotenuse. (usage of H_1)

Use the side2 of the slide rule, move the slide right until C:1 exactly against D:3, then set the hairline over D:4, read the C:1.33 under hairline, under the same hairline read the **tg_1:53.1°** is the value of base angle, turn off the rule to side1, under the same place hairline can read the H_1:1.667, sec53.1° =1. 667. move the hairline over C:1.667, read the D:5 under hairline is the hypotenuse. For this example, on alternating current circuit, we can show it as $3 + j4 = 5\underline{|53.1°}$.

From the two example, we can sum up a rule, when use the left index C:1 of C scale, use the tg_1 and H_1 scales. when use the right index C:10 of C scale, use the tg_0 and H_0 scales.

If above **Example** the opposite line is 40, still move the slide right until C:1 exactly against D:3, then set the hairline over D:40, read the C:13.3 under hairline, 13.3 is the value of tgθ for base angle. under the same hairline read **85.7° on** the **tg_2** scale is the value of base angle.The length of the hypotenuse is about 40 , about the same length with the opposite side. So, it is the reason why this slide rule has no need to set H_2 scale.

For this example, on alternating current circuit, we can show it as $3 + j40 = 40\underline{|85.7°}$.

If above **Example** the opposite line is 0.04, still move the slide right until C:1 exactly against D:3, then set the hairline over D:0.04, read the C:0.0133 under hairline, 0.0133 is the value of tgθ for base angle. under

the same hairline read **0.765° on** the t_{-1} scale is the value of base angle.The length of the hypotenuse is about 3 , about the same length with the base side. So, it is the reason why this slide rule has no need to set H_{-1} scale.

For this example, on alternating current circuit, we can show it as $3 + j0.04 = 3\underline{|0.765°}$.

XI. Usage of Log Log Scales Ln_1, Ln_0, Ln_{-1}, Ln_{-2} and Ln_{-2}, Ln_{-1}, Ln_0, Ln_1

In fact, These scales Ln_{-2}, Ln_{-1}, Ln_0, Ln_1 can be linked into one continuous scale. Folding it into 4 sect and put on the upper body. The D scale just has one sect put on the lower body. The number on D scale have the relationship with the Ln_1 scales directly; the number on D scale divide by 10 have the relationship with the Ln_0 scales; the number on D scale divide by 100 have the relationship with the Ln_{-1} scales; the number on D scale divide by 1000 have the relationship with the Ln_{-1} scales.

These scales Ln_1, Ln_0, Ln_{-1}, Ln_{-2} can be linked into one continuous scale. Folding it into 4 sect and put on the lower body. Like as above ,the number on D scale have the relationship with the Ln_1 scales directly; the number on D scale divide by 10 have the relationship with the Ln_0 scales; the number on D scale divide by 100 have the relationship with the Ln_{-1} scales; the number on D scale divide by 1000 have the relationship with the Ln_{-1} scales.

1. Find Reciprocals

Because the e^x and e^{-x} are reciprocals to each other, so, the scales Ln_1 and Ln_1 are reciprocals scales to each other, and so are Ln_0 and Ln_0, Ln_{-1} and Ln_{-1}, Ln_{-2} and Ln_{-2}.

Example: Find the Reciprocal of 0.75, Set hairline directly over **Ln_0:0.75**, read off the **Ln_0:1.333** is the answer.

2. Find the natural logarithm having positive characteristics or for a real number which is larger than 1.

Set hairline directly over any real number **X** on scales **Ln_1, Ln_0, Ln_{-1}, Ln_{-2}**, then read the value **LnX** on **D** scale under hairline.

Example: Ln20.1=?

Set fairline over Ln_1:20.1, Read D:3 under hairline.

i.e. Ln20.1=3

Example: Ln1.6=?

Set fairline over Ln_0:1.6, Read D:4.7 under hairline. When use Ln_0 and D, The number on D should divide by 10.

i.e.Ln1.6=0.47

Example: Ln1.032=?

Set fairline over Ln_{-1}:1.032, Read D:3.15 under hairline. When use Ln_{-1} and D, The number on D should divide by 100.

i.e. Ln1.032=0.0315

Same as above, When use Ln_{-2} and D, The number on D should divide by 1000.

3. Find the natural logarithm having negative characteristics or for a real number which is smaller than 1.

Set hairline directly over any real number **Y** on scales **Ln_{-2}, Ln_{-1}, Ln_0, Ln_1**, then read the value **LnY** on **D** scale under hairline.

For example:

Ln0.0497=(-3),

Ln0.67=(-0.4),

Ln0.9608=(-0.04)

4. Find a^x

Set hairline directly over **a** on **Ln_1** scale (or a on **Ln_1** scale), move the slide until C:1 is also set under the hairline, then set the hairline over C:x, read the value a^x on Ln_1 scale (or a^x on Ln_1 scale) under hairline. Meanwhile, the value a^{-x} can be read off on Ln_1 scale (or a^{-x} on Ln_1 scale) under hairline. The same way for Ln_0 and Ln_0, Ln_{-1} and Ln_{-1}, Ln_{-2} and Ln_{-2}.

For example:

3^4=(81), 3^{-4}=(0.0124);

0.25^2=(0.625), 0.25^{-2}=(16).

5. Find $a^{\frac{1}{x}}$

Set hairline directly over **a on Ln_1 scale** (or a on **Ln_1** scale), move the slide until C:x is also set under the hairline, then set the hairline over C:1, read the value $a^{\frac{1}{x}}$ on Ln_1 scale (or $a^{\frac{1}{x}}$ on Ln_1 scale) under hairline. Meanwhile, the value $a^{\frac{1}{x}}$ can be read off on Ln_1 scale (or a^{-x} on Ln_1 scale) under hairline. The same way for Ln_0 and Ln_0, Ln_{-1} and Ln_{-1}, Ln_{-2} and Ln_{-2}.

For example: $144^{1/2}$=(12), $144^{-1/2}$=(0.0833).

6. Find logarithm for any base

For example: find $lg_9 729$=(3)

Set hairline directly over **9 on Ln_1 scale,** move the slide until C:1 is also set under the hairline, then set the hairline over Ln_1:729, the value 3 on C scale under hairline is the answer.

7. Find e^X

Set the hairline directly over any know X on D, then, the value under hairline on **Ln_1, Ln_0, Ln_{-1}, Ln_{-2} and Ln_{-2}, Ln_{-1}, Ln_0, Ln_1** is the e^X.

8. Find $e^{1/X}$

Set the hairline directly over any know X on **CI,** then, the value under hairline on **Ln_1, Ln_0, Ln_{-1}, Ln_{-2} and Ln_{-2}, Ln_{-1}, Ln_0, Ln_1** is the $e^{1/X}$.

XII. Usage of Scales of Hyperbolic Function Sh_0, Sh_1, Ch_1 and Th_0

1. Find Shθ

Example: Sh0.39=(0.4), Set hairline directly over 0.39 on Sh_0 scale, read D:0.4 under the hairline is the answer.

Example: Sh 2.095=(4), Set hairline directly over 2.095 on Sh_1 scale, read D:4 under the hairline is the answer.

2. Find Thθ

Example: Th 0.424=(0.4), Set hairline directly over 0.424 on Th_0 scale, read D:0.4 under the hairline is the answer.

3. Find Chθ

Example: Ch 2.293=(5), Set hairline directly over 2.294 on Ch_1 scale, read D:5 under the hairline is the answer.

4. Find Cthθ

Cthθ=1/Thθ, Read the radian θ on Th, read the Cthθ on DI under the same hairline.

5.Find sechθ

sechθ=1/chθ, find the chθ first, then calculate sechθ.

6. Find cschθ

Cschθ=1/shθ, Read the radian θ on Sh1 or Sh2, read the Cschθ on DI under the same hairline.

XIII. Hyperbolic Function of complex number

For finding Hyperbolic Function of complex number, The following three expressions can be used. There, the x and θ are known in the expressions.

(1) $sh(x+j\theta)=\sqrt{sh^2x+\sin^2\theta}\,\Big|\,tg^{-1}(\frac{tg\theta}{thx})$

(2) $ch(x+j\theta)=\sqrt{sh^2x+\cos^2\theta}\,\Big|\,tg^{-1}(thx\,tg\theta)$

(3) $th(x+j\theta)=\sqrt{\frac{sh^2x+\sin^2\theta}{sh^2x+\cos^2\theta}}\,\Big|\,tg^{-1}(\frac{\sin 2\theta}{sh2x})$

Example: find th(0.256+j10.5)=?

Use above formula **(3).**

$$\sqrt{\frac{sh^2 0.256+\sin^2 10.5°}{sh^2 0.256+\cos^2 10.5°}}=\sqrt{\frac{0.2588^2+0.1822^2}{0.2588^2+0.9833^2}}=0.311$$

$$tg^{-1}\left[\frac{\sin 2(10.5°)}{sh2(0.256)}\right]=tg^{-1}\left(\frac{\sin 21°}{sh0.512}\right)=33.84$$

i.e. th(0.256+j10.5)=0.311 |33.84°

XIV. Inverse Hyperbolic Function of Complex Number

For finding Hyperbolic Function of complex number, The following three expressions can be used. There, the a and b are known in the expressions. First, find the imaginary number θ, then find the real number x.

(4) $sh^{-1}(a+jb)=x+j\theta$

$$\theta=\sin^{-1}\left[\frac{\sqrt{a^2+(1+b)^2}-\sqrt{a^2+(1-b)^2}}{2}\right]$$

$$x=sh^{-1}\left(\frac{a}{\cos\theta}\right)$$

(5) $ch^{-1}(a+jb)=x+j\theta$

$$\theta = \cos^{-1}\left[\frac{\sqrt{b^2+(1+a)^2}-\sqrt{b^2+(1-a)^2}}{2}\right]$$

$$x = sh^{-1}\left(\frac{b}{\sin\theta}\right)$$

(5) $th^{-1}(a+jb) = x + j\theta$

$$\theta = \frac{1}{2}tg^{-1}\left[\frac{2b}{1-(a^2+b^2)}\right]$$

$$x = \frac{1}{2}tg^{-1}\left[\frac{2b}{1+(a^2+b^2)}\right]$$

Example find $th^{-1}(0.2584 + j0.1732) = ?$

$$\theta = \frac{1}{2}tg^{-1}\left[\frac{2(.1732)}{1-(.2584^2+.1732^2)}\right] = \frac{1}{2}tg^{-1}\left(\frac{.3464}{1-.0965}\right)$$

$$= \frac{1}{2}tg^{-1}.384 = \frac{1}{2}\times 21° = 10.5°$$

$$x = \frac{1}{2}tg^{-1}\left[\frac{2(.2584)}{1+(.2584^2+.1732^2)}\right] = \frac{1}{2}tg^{-1}\left(\frac{.5168}{1+.0965}\right)$$

$$= \frac{1}{2}tg^{-1}.472 = \frac{1}{2}\times .512 = .256$$

So: $th^{-1}(0.2584 + j0.1732) = .256 + j10.5°$

XV. Trigonometric Function of Complex Number

For finding Trigonometric Function of complex number, The following three expressions can be used. There, the x and θ are known in the expressions.

(7) $\sin(x+j\theta) = \sqrt{sh^2\theta+\sin^2 x}\,\angle\, tg^{-1}\left(\frac{th\theta}{tgx}\right)$

(8) $\cos(x+j\theta) = \sqrt{sh^2\theta+\cos^2 x}\,\angle -tg^{-1}(tgx\,th\theta)$

(9) $tg(x+j\theta) = \sqrt{\frac{sh^2\theta+\sin^2 x}{sh^2\theta+\cos^2 x}}\,\angle\, tg^{-1}\left(\frac{sh2\theta}{\sin 2x}\right)$

Example find $tg(-.1835 + j14.64°) = ?$

$$x = [(-0.1835)\times 57.29°] = -10.5°$$

$$\theta = \frac{14.64°}{57.29°} = 0.256 radian$$

$$\sqrt{\frac{sh^2\theta+\sin^2 x}{sh^2\theta+\cos^2 x}} = \sqrt{\frac{sh^2 0.256+\sin^2(-10.5°)}{sh^2 0.256+\cos^2(-10.5°)}}$$

$$= \sqrt{\frac{0.2588^2+(-0.1822)^2}{0.2588^2+0.9833^2}} = \frac{0.317}{1.017} = 0.311$$

$$tg^{-1}\left(\frac{sh2\theta}{\sin 2x}\right) = tg^{-1}\left[\frac{sh(2\times 0.256)}{\sin[2\times(-10.5°)]}\right]$$

$$= tg^{-1}\frac{0.535}{-0.358} = 180° - 56.2° = 123.8°$$

So: $tg(-.1835 + j14.64°) = 0.311\angle 123.8°$

XVI. Inverse Trigonometric Function of Complex Number

For finding Inverse Trigonometric Function of complex number, The following three expressions can be used. There, the a and b are known in the expressions.

（10） $\sin^{-1}(a+jb)=x+j\theta$

$$x=\sin^{-1}\left[\frac{\sqrt{b^2+(1+a)^2}-\sqrt{b^2+(1-a)^2}}{2}\right]$$

$$\theta=sh^{-1}\left(\frac{b}{\cos x}\right)$$

（11） $\cos^{-1}(a+jb)=x+j\theta$

$$x=\cos^{-1}\left[\frac{\sqrt{b^2+(1+a)^2}-\sqrt{b^2+(1-a)^2}}{2}\right]$$

$$\theta=sh^{-1}\left(\frac{b}{\sin x}\right)$$

（12） $tg^{-1}(a+jb)=x+j\theta$

$$x=\frac{1}{2}tg^{-1}\left[\frac{2a}{1-(a^2+b^2)}\right]$$

$$\theta=\frac{1}{2}th^{-1}\left[\frac{2b}{1+(a^2+b^2)}\right]$$