The Common Logarithms

With the aid of the cursor-line C 1 or C 10 is placed above LL_3-10, and this provides a table of common logarithms.

The cursor-line can also be used for the setting operation and the readings.

log 10 = 1; log 100 = 2; log 1000 = 3; log 200 = 2·301
log 20 = 1·301; log 2 = 0·301; log 1·1 = 0·0414.

The Care of the Slide Rule:

CASTELL slide rules are valuable precision implements and require careful handling. They are made of an ideal material known as special plastic material. This is highly elastic and thus unbreakable provided it is competently handled. It will stand up to climate conditions; it is moisture-proof and non-inflammable and will resist the majority of chemicals.

Special plastic material slide rules should nevertheless not be allowed to come in contact with corrosive liquids or powerful solvents, which are at all events liable to attack the colouring-agents applied to the graduations-marks even if they do not actually harm the material itself. If necessary, the smooth movement of the slide can be improved by the use of vaseline or silicon oil. In order not to detract from the accuracy of the readings, the scales and the cursor should be protected from dirt and scratches and should be cleaned with the special cleaning agents CASTELL No. 211 (liquid), or No. 212 (cleaning paste).

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BOOTS RING PLAN

Instructions

for the use of Ringplan 10" Log-Log Slide Rule

Brief Explanation of Slide Rule:

The slide rule consists of **three** parts:

(1) The fixed main part or actual **body** of the rule consisting of the two faces, held together by bonded stays;
(2) The movable **slide** which fits within the grooves of the body;
(3) The **cursor** which extends across both faces of the entire rule.

Preliminary Remark

The most important main scales, A, B, C and D on the front and DF, CF C and D on the back, are identified by green panelling. This green enables them to be used continuously without eye-strain and also serves for quick location when the use of the slide rule is being studied and practised during the initial stage.

Scales on Front:

Mantissa scale	L	. . log x . . 0·0 to 1·0	upper section slide rule,
Cube scale	K	. . x^3 . . . 1 to 1000	Body of
Fixed square scale	A	. . x^2 . . . 1 to 100	
Movable square scale	B	. . x^2 . . . 1 to 100	Top of slide
Reciprocal scale for B	BI	. . 100÷x^2	Centre of slide
Reciprocal scale for C	CI	. . 10÷ x 10 to 1	
Movable basic scale	C	. . x . . . 1 to 10	Bottom of slide
Fixed basic scale	D	. . x . . . 1 to 10	Body of
Exponential scales	LL_1	. $e^{0\cdot01x}$ 1·0095 to 1·115	slide rule,
for positive	LL_2	. $e^{0\cdot1x}$ . . 1·095 to 3	lower section
exponents	LL_3	. e^x . . 2·5 to 60,000	

Scales on Back:

1 st tangent scale	T_1	. . tan 0·1 x 5·5° to 45°	Body of
2nd tangent scale	T_2	. . tan x 45° to 84·5°	slide rule,
Fixed basic scale displaced by π	DF	. . π x . . 3·14 to 3·14	upper section
Movable basic scale displaced by π	CF	. . π x . . 3·14 to 3·14	Top of slide
Reciprocal scale for CF	CIF	. . 1÷π x 3·2 to 3·2	Centre of slide
2nd sine scale	S'	. . sin, cos 5,5° to 90°	
Movable basic scale	C	. . x . . . 1 to 10	Bottom of slide
Fixed basic scale	D	. . x . . . 1 to 10	Body of
Sine scale	S	sine 0·1 x 5·5° to 90°	slide rule,
Arc scale	ST	arc 0·01 x 0·55° to 6°	lower section
Pythagorean scale	P	$\sqrt{1-(0{\cdot}1x)^2}$	

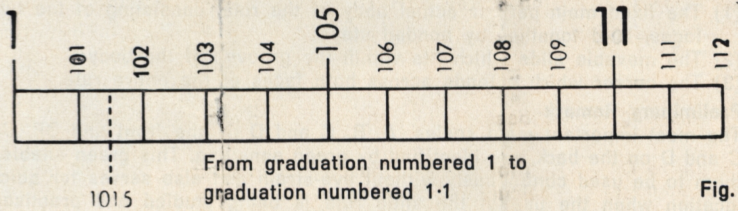
The Decimal Point

Since the upper scales extend from 1-100 and the lower scales from 1-10, it might appear to the beginner that calculations on the slide rule are confined to figures within this range. This is not so inasmuch as number values are disregarded initially. The decimal point is inserted in the appropriate position when the final result is determined, e.g. if a scale reading is 3 this can also indicate 0·03, 0·3, 3, 30, 300, 3,000. The placing of the decimal point is the important factor although this should present no difficulty as the approximate answer can usually be determined at the outset.

The slide rule can thus be used for any desired number.

Reading the Scales

There is insufficient space for every graduation mark to be given a number. Consequently only the principal numbers are given for purposes of guidance. The values represented by the other lines can be ascertained by relation to the figured graduation marks.
It should be noted, however, that the subdivision is not uniform over the entire length of the graduation, since the marks get closer together as one proceeds towards the right.

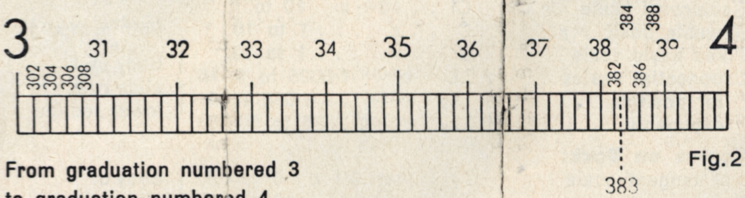


From graduation numbered 1 to graduation numbered 1·1

Fig. 1

Section of the Graduated Zone from 1 to 2 (Fig. 1)

Having 10 sub-sections, each with 10 intervals (= 1/100 or 0·01 per mark). This enables immediate readings to be taken of 3 different places (e.g. 1-0-1). By **halving** the distance between 2 graduation marks, the rule can be accurately set to 4 figures (e.g. 1-0-1-5). The last figure in such cases is always a 5.

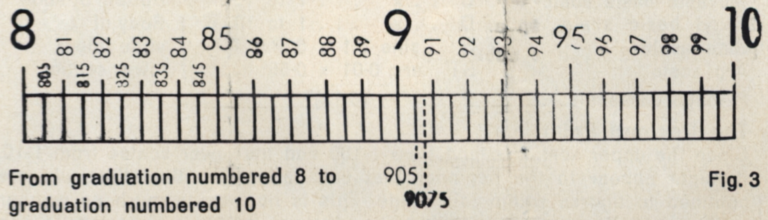


From graduation numbered 3 to graduation numbered 4

Fig. 2

Section of the Graduated Zone from 2 to 4 (Fig. 2)

Having 10 sub-sections, each with 5 intervals (= 1/50 or 0·02 per mark). This enables accurate readings of 3 figures to be taken (e.g. 3-8-2). The final figure is always an even number (2, 4, 6, 8). If the intervals are halved we also obtain the odd numbers 1, 3, 5, 7, 9 (e.g. 3-8-3).



From graduation numbered 8 to graduation numbered 10

Fig. 3

Section of the Graduated Zone from 4 to 10 (Fig. 3)

Having 10 sub-sections, each with 2 intervals (= 1/20 or 0·5 per mark). This enables accurate readings to be taken to 3 places, when the last figure is a 5 (e.g. 9-0-5). By halving the intervals one can even obtain 4 exact places. Here again the final figure is always a 5 (e.g. 9-0-7-5). Other intermediate values have to be estimated.

Once we have mastered the system of subdivision adopted for the basic scales C and D, it will be immediately evident to us that scales CI, CF, DF and CIF are subdivided in exactly the same manner, and the system on which the remaining scales are based will likewise be clear.

The Marks π, M, $\frac{\pi}{4}$, ϱ, e, C and C_1

A number of constants continually required are specially marked:

π = approx. 3·1416 on scales A, B, CI, C, D, CF, DF, CIF

$M = \frac{1}{\pi} = 0{\cdot}318$ on scales A and B

$\varrho = \frac{\pi}{180} = 0{\cdot}01745$; $\frac{\pi}{4} = 0{\cdot}785$

e = marking for the base of the natural logarithm e = 2·71828 on LL_2 and LL_3

The marks C and C_1 (not to be confused with the index figure 1 on the slide) facilitate the calculation of cross sections from given diameters.
Example: If the cursor-line is used to set C above 2·82 cm on scale D (by first bringing the cursor-line into position above the 2·82 on D, then moving the mark C into position underneath it) the cross section (6·24 cm²) can be found on scale A above the initial 1 of the upper slide scale B (henceforward invariably termed B 1).

Instead of Mark C, use could also have been made of Mark C_1 (not to be confused with the index 1 on the lower slide scale C, henceforward always termed C 1). The result is then to be found above B 100 (100 on scale B) on scale A. The setting is always carried out by means of the mark C or C_1 whichever involves lesser extension of the slide.

Preliminary Remark: For setting and reading numerical values, use is always made of the continuous main mark (hereinafter called the "cursor-line" for short) and the initial 1 or the final 10 or final 100 of the scales A, B, CI, C, D in the case of CF, DF and CIF the ←1→

On what System are Calculations with the Slide Rule based?

If two ordinary rules with centimetre graduations are placed one above the other, as in the accompanying diagram, then proceeding from left to right we obtain the result

3·5 + 4·5 = 8 (i.e. an **addition**) or

8 — 4·5 = 3·5 (i.e. a **subtraction**) (Fig. 4a)

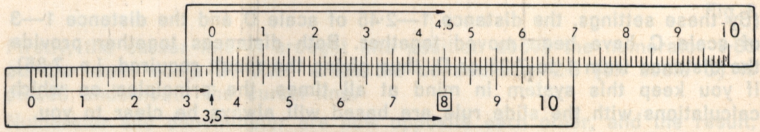


Fig. 4a

The "calculation" has thus been made by regarding the figures 3·5 and 4·5 as "distances" and adding them — or, in the second case, by taking the distance 4·5 away from the distance 8.

The slide rule operates in exactly the same way, with the sole difference that the graduations are built up in such a manner as to enable the **product** instead of the **sum** to be obtained by moving them apart — or in the second case, the **quotient** instead of the **difference.**

If, therefore, 2 scales of a slide rule are placed against each other, in the same manner as the aforementioned ordinary rulers, then the result reads

3·5 × 4·5 = 15·75 (i.e. a **multiplication**), or

15·75 ÷ 4·5 = 3·5 (i.e. a **division**) (Fig. 4b)

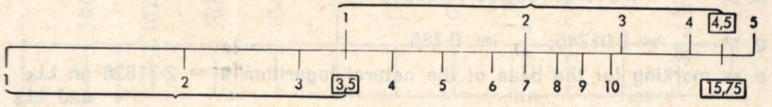


Fig. 4b

Conclusion:

If we add two "distances" on a slide rule, this results in a multiplication, whilst if we take one away from the other this results in a division.

Multiplication

The scales mainly used are the basic scales C and D. (For scales CF and DF, see page 8).

Example: 2·45 × 3 = 7·35 (Fig. 5)

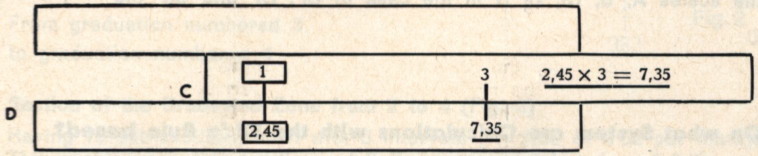


Fig. 5

The index figure 1 at the end of the slide (C 1) is placed above the 2·45 of the lower graduation of the slide rule (D 245) and the cursor-line is moved into position above the 3 on the lower slide graduation (C 3); the product, 7·35 can then be found underneath the cursor-line on the lower slide rule graduation (D 735).

(By these settings, the distance 1—2·45 of scale D and the distance 1—3 of scale C have been moved together. Both distances together provide the distance from 1 to 7·35 on D, thus giving the result required, i.e. 7·35). If you keep this system in mind at all times, the principles on which calculations with the slide rule are based will always be clear to you.

In calculations on scales C and D it may happen that the slide, when C 1 is placed above the first factor on Scale D, projects too far to the right, so that it is no longer possible to set C to the second factor. In this case the slide is pushed sufficiently far to the left to bring the right end of the slide (C 10) instead of the first index figure of the slide (C 1) underneath the cursor-line. This is referred to as transposing the slide.

This can be avoided if, when necessary, C 10 is placed above the first factor right away. Experienced users know immediately which setting is preferable.

Example: 7·5 × 4·8 = 36 (Fig. 6)

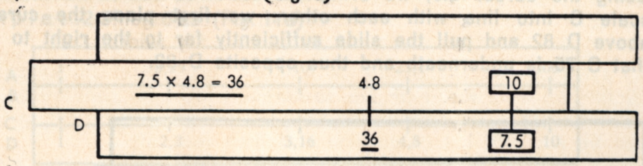


Fig. 6

We place C 10 above D 7·5, bring the cursor-line into position above the second factor, 4·8 on C, and find the result, 36 underneath it, on scale D. The C 10 setting is generally adopted in cases where the product of the first two figures is greater than 10.

The transposing of the slide (C 1 to C 10 or vice versa) is not necessary when using the π-displaced scales CF and DF.

In continuous calculations, e.g. when the number has first of all been squared, the further multiplication can be carried out on A and B.

Example: 2·5 × 3 = 7·5 (Fig. 7)

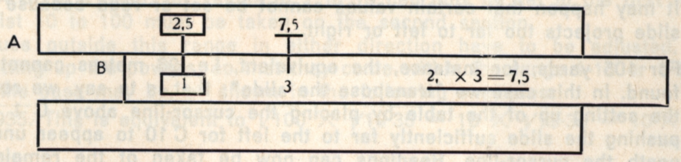


Fig. 7

Exercises: C 1 setting: 1·82 × 3·9 = 7·1; 0·246 × 0·37 = 0·091
C 10 setting: 4·63 × 3·17 = 14·7; 0·694 × 0·484 = 0·336.

Division

Using the cursor-line, the numerator and denominator on C and D are moved together, and the result can be found underneath the index figure C 1 of the slide or, at the other end, beneath C 10.

Example: 9·85 ÷ 2·5 = 3·94 (Fig. 8)

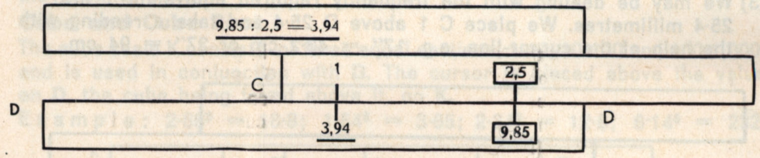


Fig. 8

Firstly, the cursor line is brought into position above the numerator 9·85 on the lower scale D, after which the denominator 2·5 (on scale C) is placed underneath the cursor-line.

Numerator and denominator are now opposite each other, and the result, 3·94 can be found on scale D underneath the index figure C 1 of the slide. Division can likewise be carried out on A and B. Here again, the cursor-line is used to place the numerator (on A) and the denominator (on B) opposite each other, and the result can then be found on scale A, above B 1 or B 100.

Exercises: 970 ÷ 26·8 = 36·2; 285 ÷ 3·14 = 90·7; 0·685 ÷ 0·454 = 1·51

Formation of Tables

(1) To convert yards into metres. Formula: 82 yards = 75 metres. Using the cursor-line, we bring the 82 on scale D and the 75 on scale C into line with each other: we first place the cursor-line above D 82 and pull the slide sufficiently far to the right to ensure that C 75 is underneath and thus opposite D 82.

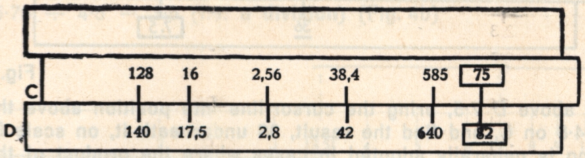


Fig. 9

The cursor-line is now placed above the known number of yards, on D, and the number of metres can then be found above it, on C, and vice versa.

For example, 17·5 yards are 16 metres and 140 yds are 128 m; conversely, 38·4 m are 42 yds, 2·56 m are 2·8 yds and 585 m are 640 yds. It may happen that certain values cannot be set or read because the slide projects too far to left or right.

For 105 yards, for instance, the equivalent, i.e. 96 metres cannot be found. In this case we "transpose the slide"; that is to say, we retain the setting up of the table by placing the cursor-line above C 1 and pushing the slide sufficiently far to the left for C 10 to appear underneath the cursor-line. Readings can now be taken of the remaining values.

The method of "transposing the slide indices" is not necessary if use is being made of the π-displaced scales CF and DF (see page 8).

(2) If, instead of the general formula, the unit-value is known, e.g. 1 yard = 0·914 metres, we place C 1 or C 10 (for 1 yd) above 0·914 on scale D. By the aid of the cursor-line, yards and metres can again be found on C and D.

(3) We may be dealing with the frequently required equivalent 1 inch = 25·4 millimetres. We place C 1 above D 25·4 and take a reading with the help of the cursor-line, e.g. 17″ = 43·2 cm or 37″ = 94 cm.

Fig. 10

At 42″ for example, we again find that the setting operation and the reading are not possible, so we bring C 10 into position in place of C 1.

(4) See that in all settings the unit-value or the equivalent can be read at the ends of the scales, under C 1 and above D 10, and vice versa. Thus, if C 1 is above D 25·4 (for 1 inch = 25·4 mm), then we find above D 10 the value 0·3937 on scale C (for 1 cm = 0·3937″).

Square and Square Root

Since the upper scales **A** and **B** are subdivided from 1 to 100, with the lower scales being subdivided from 1 to 10, it means that the square of any number on **D**, can be found on **A**.

Example: $2{\cdot}3^2 = 5{\cdot}29$ (Fig. 11a)

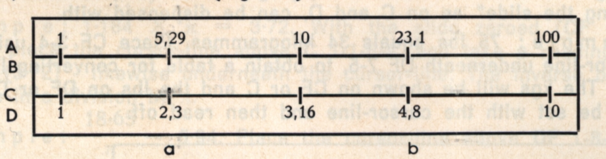


Fig. 11

The cursor-line is placed above the 2·3 on D, and the result, 5·29, is found under the cursor-line on A.

Examples for practice: $1{\cdot}345^2 = 1{\cdot}81$; $4{\cdot}57^2 = 20{\cdot}9$; $0{\cdot}765^2 = 0{\cdot}585$

The square root is obtained by setting the basic number on **A**, the result then being the number shown underneath it, on **D**.

Example: $\sqrt{23{\cdot}1} = 4{\cdot}8$ (Fig. 11b)

The cursor-line is placed above 23·1 on **A**, the result, 4·8, being found underneath the cursor-line, on **D**.

In the extraction of square roots, it is **not** immaterial which scale section of A or B is used, for values of 1 to 10 can only be set in the first half, whilst 10 to 100 must be taken on the second section.

Values outside this range in either direction have to be adjusted, by splitting up the powers, so that they come within the range 1-10 or 1-100, as the case may be, as shown in the following examples:

$\sqrt{1935}$. This is equivalent to $\sqrt{100} \times \sqrt{19{\cdot}35} = 10 \times \sqrt{19{\cdot}35} = 10 \times 4{\cdot}4 = 44$

$\sqrt{145{\cdot}8}$. This is equivalent to $\sqrt{100} \times \sqrt{1{\cdot}458} = 10 \times \sqrt{1{\cdot}458} = 10 \times 1{\cdot}207 = 12{\cdot}07$

If we wish to avoid "splitting off" the powers of 10, the following purely "mechanical" method of setting may be noted:

On the left-hand half, the figures must be set which have one, three, five, etc. digits in front of the decimal point, or one, three, five etc. noughts after the decimal point; on the right-hand half, those figures must be set which have two, four, etc. digits in front of the decimal point, or two, four etc. noughts (or no noughts at all) after the decimal point.

Cube and Cube Root

The cube scale K consists of three equal sections, 1-10, 10-100 and 100-1,000, and is used in conjunction with **D**. The cursor is placed above the value on **D**, the cube being found above it, on **K**.

Example: $2{\cdot}66^3 = 18{\cdot}8$; $1{\cdot}54^3 = 3{\cdot}65$; $2{\cdot}34^3 = 12{\cdot}8$; $6{\cdot}14^3 = 232$.

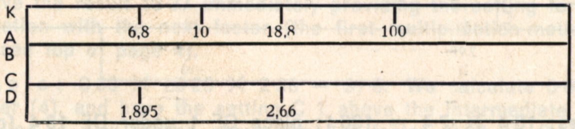


Fig. 12

If the cube root is to be extracted, the converse process is adopted. The setting is made on **K** and the reading taken from **D**.

Example: $\sqrt[3]{6{\cdot}8} = 1{\cdot}895$; $\sqrt[3]{4{\cdot}66} = 1{\cdot}67$; $\sqrt[3]{29{\cdot}5} = 3{\cdot}09$; $\sqrt[3]{192} = 5{\cdot}77$.

If the basic number is below 1 or above 1000, it must be adjusted (by "splitting off" appropriate powers) so that it falls within the 1-1000 range, as when extracting square roots.

Calculations with the π-displaced Scales CF and DF

(1) Formation of Tables

Since, in the case of the π-displaced scales CF and DF, the value 1 is roughly in the middle, it can be used with advantage when calculating with tables and when multiplying so that the operation of "transposing the slide" as on C and D, can be dispensed with.

Example: 75 lbs equals 34 kilogrammes. Place CF 3-4 using the cursor-line underneath DF 7-5, to obtain a table for converting lbs into kgs. The kgs will be shown on CF or C and the lbs on DF or D. They can be set with the cursor-line and then read off.

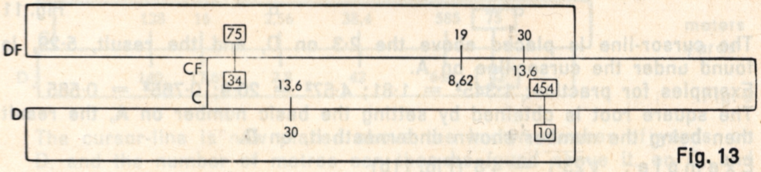


Fig. 13

See that with the above setting the unit value 4-5-4 at the same time appears on C, above D 10 (1 lb = 0·454 kgs).

Exercises: Using the set formula "CF 34 over DF 75" (kgs to lbs): 30 lbs. = 13·6 kg; place the cursor-line over DF 3 (or D 3) and find the value 13·6 underneath it on CF (or above it on C).

19 lbs = 8·62 kgs; place the cursor over DF 19 (in this case D cannot be used for the setting) and find the value 8·62 underneath it on CF. Thus, when passing from the lower to the upper scales, and vice versa, we always have the full graduation range at our disposal.

With the setting "CF 3-4 under DF 7-5" it runs from C 1 to C 4-5-4 (1 lb = 0·454 kgs) and continues at the top from CF 3-1-4 via CF-1 to CF-1-4-2-5. For purpose of practice you should also examine the range of DF.

(2) Multiplication

If the second factor cannot be set with the cursor-line when multiplying on C and D, or if we have to "transpose the slide", this can be avoided by performing the rest of the operation on CF and DF.

Example: 2·91 × 4 = 11·64. Place C 1 above D 2·91 (or CF 1 underneath DF 2·91), move the cursor into position above CF 4, and find the result, i.e. 11·64 above it, on DF.

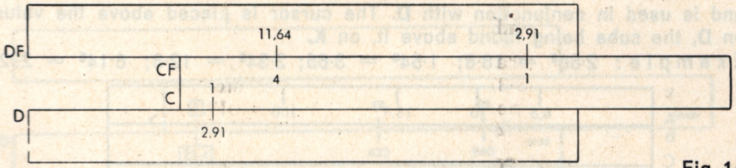


Fig. 14

Exercises: 18·4 × 7·4 = 136·1; place CF 1 under DF 18·4 (or C 1 above D 18·4), move the cursor into position above CF 7·4 — C cannot be used for the setting in this case — and find the value, i.e. 136·1, above it, on DF.

42·25 × 3·7 = 156·3; CF 1 underneath DF 42·25 (C 10 will also be found above D 42·25). Then place the cursor over C 3·7 and find the value, 156·3 below it, on D. (The setting 3·7 cannot be carried out on CF in this case).

Multiplication and Division by π

The transition from the **C** scale to the **CF** or **DF** scales can be carried out direct with the cursor and results in a multiplication by π; conversely, when we change over from the **CF** and **DF** scales to the **C** scale, we perform a division by π.

Example: $1·184 \times \pi = 3·72$. With the slide zeroed (**CF 1** above **DF 1**) place the cursor-line above **C** 1-1-8-4, and take a reading of the result (3·72) - likewise underneath the cursor-line. The inverse operation results in a division by π.

Example: $\frac{18·65}{\pi} = 5·94$. Place the cursor-line above **DF** 1-8-6-5 and read off the result (5·94) on **C.**

Calculation with the Reciprocal Scale CI

This scale is subdivided from 1 to 10, and its system of graduations corresponds to that on scales C and D, but takes the opposite direction.

(1) If, for a given value a, the reciprocal 1 ÷ a is required, the former is set on C or CI, and the reciprocal can then be found above it on CI or underneath it on C. The reading can be taken merely by setting the cursor, without adjusting the slide.

Examples: 1 ÷ 8 = 0·125; 1 ÷ 2 = 0·5; 1 ÷ 4 = 0·25; 1 ÷ 3 = 0·333

(2) If $1 \div a^2$ is required, the cursor-line is moved to the value a on scale CI, and the result may be found above it, on B, likewise underneath the cursor-line.

Example: $1 \div 2·44^2 = 0·168$ Quick guide to position of decimal point: Less than 1/5th = 0·2

(3) If $1 \div \sqrt{a}$ is required, the cursor-line is placed at a on scale B, and the result is found on CI, likewise beneath the cursor-line.

Example: $1 \div \sqrt{27·4} = 0·191$ Quick guide to position of decimal point: Less than 1/5th = 0·2

(4) **Scales D and CI also enable multiplications to be carried out.** (Division by the reciprocal = multiplication). This method is popular with many users.

Example: 0·66 × 20·25. Proceed as in division, i.e. first place the cursor-line above 0·66 on D, the 20·25 on CI then being placed under the cursor-line; the product, 13·37 can then be found on D under C 1.

(5) **Products with a number of factors** can thus be found very simply.

The first two factors are multiplied, as in (4) in the foregoing, C 1 above the result 13·37 immediately providing the setting for the multiplication with the next factor (the first multiplication-method studied — see top of page 4).

Example: 0·66 × 20·25 × 2·38 = 31·8. We calculate 0·66 × 20·25 as under (4), and have the setting C 1 above the intermediate result; the cursor-line is now placed above the 3rd factor, 2·38 on C. The result, 31·8 may be found underneath it, on D.

This could now be immediately followed by a further multiplication, by placing the next factor on CI underneath the cursor-line, the result being found on D, beneath C 1, (or C 10 as the case may be).

Multiplications can thus be carried out alternately by the aid of D and CI followed by the use of C and D, in accordance with the first method.

Calculations with the Reciprocal Scale CIF

The CIF graduation operates in conjunction with CF and DF just as CI does with C and D scales.

Examples for multiplication by a number of factors:

2·23 × 16·7 × 1·175 × 24·2 = 1059. Solution: CI 2·23 placed above D 16·7 by the aid of the cursor-line; the latter is placed above CF 1·175. CIF 24·2 under the cursor line. Read result, 1059, on DF, above CF 1.

0·53 × 0·73 × 39·1 × 0·732 = 11·07. Solution: CI 0·53 placed above D 0·73 with the aid of the cursor-line; the latter is placed above CF 39·1. CIF 0·732 underneath cursor-line. Read the result 11·07, on DF, above CF 1.

Pythagorean Scale P

This scale represents the function $y = \sqrt{1-(0{\cdot}1x)^2}$; it operates together with D (= x). The graduations run in the opposite direction, i.e. right to left, this being the reason why it is coloured red.

Trigonometrical calculations

The P Scale offers the advantage of enabling more accurate readings to be obtained for **high** sine and **small** cosine angles. While on Scale D, for example, the reading obtained for sin 67° is only 0·92, the P Scale provides under cos 67° = sin (90—67) the more accurate result 0·9204.

Examples: sin 72·3° = 0·9526   cos 12·3° = 0·9771
sin 81·2° = 0·98821   cos 6·3° = 0·99397

For every **sine** value on Scale D, above the angle on S, we find the **cosine value on the P Scale,** and vice versa. The two values, sine and cosine, are always provided, so that we can proceed straight from the former to the latter without taking a reading of the angle.

Example: sin = 0·134   cos = 0·991

Evolution

For numbers which are nearly 1, nearly 100 etc., the use of the P Scale results in a high degree of accuracy.

Example: $\sqrt{0{\cdot}925} = \sqrt{1-0{\cdot}075} = \sqrt{1-(0{\cdot}274)^2} = 0{\cdot}9618$

The radicand is subtracted from the next power of ten above. The difference is set with the cursor line on the A Scale and a reading of the required set taken in P; this root is then multiplied by that of the power of ten from which the radicand had been subtracted.

Analysis of right-angled triangle

The P Scale enables problems of this kind to be solved even when there is a considerable difference between the length of the hypotenuse and that of the other sides, and practically always without reversing the position of the slide. If, for example, the hypotenuse is 13 cm and the side in question 5 cm, we set the slide rule to "C 13" under D 1 and obtain the reading 0·923 on P, under C 5. This reading is transferred to DF with the cursor. The length of the other side, i.e. 12 cm. is then found on Scale CF.

Calculations with the reciprocal scale BI

The reciprocal square scale, **BI**, represents the reverse of scale B, operating as a square scale with **CI** and as a reciprocal scale with **A** and **B.** This is of advantage in compound calculations.

Here the same computations can be carried out as under 1 to 6 on pages 12 to 13, but scales **A, B, CI, C** and **D** are replaced by scales **D, C, BI** and **A.**

Example No. 1 from p. 12: 1 ÷ 8 = 0·125. Place the cursor line above 8 on **BI** or **A** and read off the reciprocal (0·125) — above it, on **A**, or below it, on **BI.**

Here is an example of a compound calculation in which one has the special advantage of starting from **A** and **B** and continuing on **BI.**

Example: $(2{\cdot}45 \times 3)^2 \times 2{\cdot}27 = 122{\cdot}6$.

Place C 1 above D 2-4-5, move the cursor until its line is above C 3: no reading need be taken, on D, of the intermediate result 7·35, and the square (54·02) can be found on A, again underneath the cursor line. (For squaring, see page 14). This is multiplied by 2·27 by moving BI 2-2-7 into position under the cursor line, above B 1, the result (122·) is found on A.

Example: To find the area of a sphere in which r = 7·2 cm.

$A = 4\pi r^2 = 652$ cm². Place BI 4 under A π and take the reading of the area (652 cm²) above C 7·2 on A.

The Trigonometrical Scales S, ST, T_1 and T_2

The trigonometrical scales S, ST, T_1 and T_2 are subdivided decimally, and show, in conjunction with the basic scale D, the angular functions; when the converse process is adopted, they indicate the angles.

Use as Tables

When using the scales S, ST, T_1 and T_2 in conjunction with scale D, as a **trigonometrical table,** the following should be noted:

The S scale, in conjunction with the **D scale,** provides a **sine table.**

The S scale with the values of the complementary angles (increasing from right to left) provides — in conjunction with the **D scale** — a **cosine table.**

The two **T scales,** in conjunction with the **D scale,** provide a **tangent table,** up to 84·28°.

The two **T scales,** with the values of the complementary angles (increasing from right to left) provide — in conjunction with the **D scale** — a **cotangent table.**

To find:	Setting:	
sin 13° = cos 77° = 0·225	S 13° — D 0·225	Only the long cursor-line is required for these settings.
sin 76° = cos 14° = 0·97	S 76° — D 0·97	
cos 28° = sin 62° = 0·883	S 62° — D 0·883	
cos 78° = sin 12° = 0·208	S 12° — D 0·208	
tan 32° = cot 58° = 0·625	T_1 32° — D 0·625	
tan 57° = cot 33° = 1·54	T_2 57° — D 1·54	
cot 18° = tan 72° = 3·08	T_2 72° — D 3·08*	
cot 75° = tan 15° = 0·268	T_1 15° — D 0·268*	
	* or:	
cot 18° = tan 72° = 3·08	T_1 18° — CI 3·08	Set with the long cursor-line with the slide rule set to **zero.**
cot 75° = tan 15° = 0·268	T_2 75° — CI 0·268	

The **ST scale** provides, with **D scale,** a table of the **arc function (circular measure of an angle)** and — when the correction-marks are used — a sine to tangent scale for the angles 0·55°—6°.

As an Arc scale (for circular measurement of angles):

Set the angle value on ST and find the functional values on D (by the aid of the cursor-line).

Examples: arc 2·5° = 0·0436; arc 4·02° = 0·07; and conversely:

$\widehat{0{\cdot}04} = 2{\cdot}29°$; $\widehat{0{\cdot}021} = 1{\cdot}205°$.

The arc scale also applies to the ten-fold angle values, but the function must then be multiplied by 10.

Examples: arc 31° = 0·541; $\widehat{0{\cdot}64} = 36{\cdot}7°$.

As a tangent scale or sine scale for small angles, up to 3° in the case of the tangent or up to 5° in the case of the sine, in accordance with the equation tan α ≈ sine α ≈ arc α.

Examples: tan 2·5° ≈ sine 2·5° = 0·0436
tan 4° ≈ sine 4° = 0·0697

For an **exact** reading of tangent 4°, the correction-mark to the **right** of the graduation-mark for 4° is used. The reading taken is 0·0697.

The following should be noted as regards the correction-marks for the tangent:
Tangent **greater** than arc, therefore use correction-mark to the **right** of the graduation-mark.
Example: tan $5^0 = 0{\cdot}0875$.
If the angle is between the full graduations provided with correction-marks, the correction-interval must be transferred accordingly.
Example: tan $3{\cdot}5^0 = 0{\cdot}0612$; tan $4{\cdot}2^0 = 0{\cdot}0734$; tan $5{\cdot}33^0 = 0{\cdot}0934$.
If the functional value is known, and the angle is required, the correction-mark to the **left** is used.
For the sine, the correction-mark is provided to the **left** of graduation-mark 6^0. It applies to the range 5^0—6^0.
Here the operation is carried out as above, but in the opposite direction.

Calculations with Trigonometrical Scales S, ST, T_1 and T_2

As every function is a ratio between one side and the other, all that is necessary in each case is to place the **relevant graduated part of the D scale** alongside that of the **CI scale.** By dropping a perpendicular from the final point of this combination of scales to the relevant angular-function scale (ST for 0·01 x; S and T_1 for 0·1 x and T_2 for x), we can immediately take a reading of the angular value.

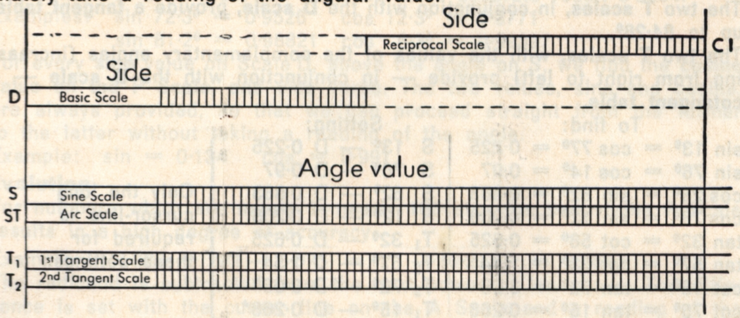


Fig. 15

But even in cases where the angle and one side are known, the same system of calculation can be adopted, but here we first have to trace the angular value with the aid of the cursor-line, and the relevant side of the triangle has to be taken into consideration on the scales D or CI.

Examples for the right-angled Triangle:

(1) Given: a = 3, b = 4. Find α and c.
Set C 1 above D 3, cursor-line on CI 4, and find the angle for α, i.e. $36{\cdot}9^0$ on the T_1 scale. With the cursor at S $36{\cdot}9^0$, we now find the hypotenuse 5 on CI.
(2) a = 30, b = 4. Find α and c.
The setting is carried out as in (1), i.e. C 1 above D 3, cursor-line on CI 4, but find the angle, i.e. $82{\cdot}4^0$ for α, on the T_2 scale (since $30 \div 4$ is more than 1). To find c, move cursor to S $82{\cdot}4^0$; the value for c, i.e. 30·3 is now found on CI.
(3) a = 3, b = 40. Find α and c.
The setting is carried out as above, but the angle i.e. $4{\cdot}28^0$ is found on ST (the first reading being $4{\cdot}3^0$ and a "correction towards the right" giving $4{\cdot}28^0$). Using this "corrected setting" $4{\cdot}28^0$, we find the value for c, i.e. 40·2 on CI.
(4) a = 8·2, b = 21·6. To find c and α.
C 10 above D 8·2, cursor at CI 21·6, value of α ($20{\cdot}78^0$) found on T_1 scale. Place cursor at $20{\cdot}78^0$ of S scale, and find value of c (23·1) on CI.
(5) a = 21·6, b = 8·2. To find c and α.
C 1 above D 21·6, cursor at CI 8·2, value of α ($69{\cdot}22^0$) shown on T_2 scale.

Place cursor at 69,22 of S scale, and find value of c (23·1) on CI.
One further example with the use of the correction-mark:
(6) a = 51·2, c = 612. To find α and b.
C 1 above D 51·2, cursor at CI 612. Reading $4{\cdot}8^0$ taken from ST scale. Now move to the right by the distance of the tangent correction-interval, and take the reading b = 610 on CI.

Examples for Scalene Triangle:

This is governed by the equation $a \div b \div c = \dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

(1) a = 38·3; $\alpha = 52^0$; $\beta = 59^0$; $\gamma = 69^0$. To find b and c.
Place C 383 above S 52^0. With the aid of the cursor-line, the results (b = 41·7 and c = 45·4) can be found on C, above S 59^0 and 69^0.
(2) $\alpha = 6^0$; $\beta = 5^0$; c = 165. To find a and b.
It is known that $\gamma = 180^0 - (\alpha + \beta) = 169^0$, and
$\sin\gamma = \sin(180^0 - \gamma) = \sin 11^0$.
We thus place C 165 above S 11^0 and can then find the angles, with the aid of the cursor-line, on the arc scale and with the use of the correction-mark, the values for a and b (90·4 and 75·4 respectively) then being shown on the C scale.
Cosine and Cotangent are obtained by the aid of the complementary angles $\cos\alpha = \sin(90^0 - \alpha)$; $\cot\alpha = \tan(90^0 - \alpha)$.
Examples:
(1) b = 1·17; a = 2·23. To find α and c.
Place C 1 above D 1·17 and place cursor at CI 2·23. Above, on the T_1 scale, the reading $62{\cdot}3^0$ is given for α (inverse, red figures). Now place cursor at (inverse, red figures) $62{\cdot}3^0$ on the S scale. The reading for c (2·52) is given above, on CI.
(2) b = 4·42; c = 46·2. To find α and a.
Place C 1 at D 4·42. Place cursor at CI 46·2. On ST (inverse) the reading $84{\cdot}52^0$ is given for α. (If account is taken of the "correction-value", i.e. one "graduation-mark width" to the **right,** we obtain the exact reading 84·5). Now place cursor to (inverse) $84{\cdot}5^0$ of ST scale (take tangent correction into account) and obtain reading 46 for a on CI, above.

Use of ϱ-Mark

The ϱ-mark can also be used to determine the circular measurement or arc function, in accordance with the equation

$$\varrho \times \alpha = 0{\cdot}01745 \times \alpha = \text{arc}\,\alpha.$$

If C 1 is placed above ϱ on D, or CF 1 under ϱ on DF, this provides an arc table on D (angle-value on C) or on DF (angle-value on CF).
Examples: arc $2{\cdot}5^0 = 0{\cdot}0436$; arc $0{\cdot}4^0 = 0{\cdot}00698$; arc $0{\cdot}0052^0 = 0{\cdot}0000907$.
Use cursor for setting and reading.

The Mantissa Scale L for Common Logarithms

This operates in conjunction with scale D and enables readings to be taken of common logarithms.
E x a m p l e : log 1·35 = 0·1303; log 13·5 = 1·1303; place the cursor-line above 1·35 on scale D and find the result, i.e. 1303, above it on L.
The user determines the characteristic himself, in the normal manner.
Conversely, when the logarithm is known, we can find the number itself, by setting the cursor on L and taking the reading underneath it, on D.
E x e r c i s e s : log 3 = 0·4772; log 36·2 = 1·5587; log 1·479 = 0·170 alternatively log sine 25^0 = log 0·4225 (on D) = 0·626 — 1 (on L) = 9·626 — 10; the cursor thus enables us to proceed direct from S 25^0 to L-626 to take the reading.

Calculations with the Exponential Scales LL_1, LL_2 and LL_3

The exponential scales are to be found on the lower edge of the front of the slide rule and run from 1·0095 to 60,000, operating in conjunction with C and D.

On the LL-scales the position of the decimal point must be taken into account.

(1) When changing over from LL_1 to LL_2 and from LL_2 to LL_3 (with the cursor-line) we obtain powers of 10.

Examples: $1{\cdot}02^{10} = 1{\cdot}219$; $1{\cdot}035^{10} = 1{\cdot}4105$;
$1{\cdot}204^{10} = 6{\cdot}4$; $1{\cdot}443^{10} = 39{\cdot}15$.

or vice versa from LL_3 to LL_2 and from LL_2 to LL_1 we obtain the 10th roots.

Examples: $\sqrt[10]{75} = 1{\cdot}54$; $\sqrt[10]{6{\cdot}4} = 1{\cdot}204$;
$\sqrt[10]{52} = 1{\cdot}458$; $\sqrt[10]{3{\cdot}4} = 1{\cdot}1302$.

(2) When changing over from LL_1 to LL_3 we obtain powers of 100.

Examples: $1{\cdot}025^{100} = 11{\cdot}8$; $1{\cdot}05^{100} = 131$.

or vice versa from LL_3 to LL_1 we obtain the 100th roots.

Examples: $\sqrt[100]{6} = 1{\cdot}0182$; $\sqrt[100]{300} = 1{\cdot}0587$.

The Power of e (= approx. 2·71828)

Place the cursor above the exponents on D.

The power of e can be found on the LL-scales; the graduation range for the scale D is 1 to 10 in the case of LL_3; 0·1 to 1 in the case of LL_2 and 0·01 to 0·1 in the case of LL_1.

Examples: $e^{1{\cdot}61} = 5$. We move the cursor-line into position above D 1-6-1 and find the result, i.e. 5, on LL_3.
$e^{0{\cdot}161} = 1{\cdot}175$. Cursor-line on D 1-6-1, which, however must now be treated as 0·161, result (on LL_2): 1·175.
$e^{0{\cdot}0161} = 1{\cdot}01625$. Cursor-line on D 1-6-1, result (on LL_1): 1·01625.

Exercises: $e^{6{\cdot}22} = 5 \times 10^2 = 500$; $e^{0{\cdot}622} = 1{\cdot}862$; $e^{2{\cdot}64} = 14$.

If the exponent of the power is negative, we use e^{-n}, thus first calculating with a positive n and then tracing the reciprocal.

Roots of e

We write down the root in the form of a power with a reciprocal exponent and then proceed as above.

Examples: $\sqrt[4]{e} = e^{0{\cdot}25} = 1{\cdot}284$; $\sqrt[0{,}25]{e} = e^4 = 54{\cdot}6$;
$\sqrt[12{,}5]{e} = e^{0{\cdot}08} = 1{\cdot}0834$; $\sqrt[140]{e} = e^{0{\cdot}00715} = 1{\cdot}0141$.

The Natural Logarithms

are found when changing over from the LL-scales to the basic scales. Here again, the graduated range for the D scale is 1 to 10 when LL_3 is used, 0·1 to 1 when LL_2 is used and 0·01 to 0·1 when LL_1 is used.

Examples: ln 25 = 3·22; we place the cursor above LL_3-25 and find the result, i.e. 3·22 above it, on D.
ln 1·3 = 0·262; we place the cursor above LL_2-1·3 and find the result, i.e. 0·262, above it, on D.
ln 1·05 = 0·04875; the cursor above LL_1-1·05; the result on D: 0·04875.

Exercises: ln 145 = 4·97; ln 36 = 3·58; ln 1·84 = 0·61; ln 2·36 = 0·86.

The natural logarithms of the numbers below 1 can be found in accordance with the equation $\ln a = -\ln \frac{1}{a}$

The 2nd Sine Scale S'

is on the back of the slide.

As the scale is movable, multiplications and divisions of angular functions can be carried out simply without having to take readings of the functional values.

Example: sn 41° × sin 23° = 0·2562

Place the final mark of the scale over S 41 (lower body of rule) with the aid of the cursor line. Below S' 23 (centre of slide) we find the result 0·2562 on scale D.

For multiplications $a \times \sin\alpha \times \sin\beta$ we always start with a on scale D.

Exercises: $\tan b = \tan 40^0 \times \cos 12^0 = 0{\cdot}82$; $b = 39{\cdot}35^0$.

$$\tan\beta = \frac{\tan 37^0}{\sin 14^0} = 3{\cdot}117;\ \beta = 72{\cdot}2^0$$

$$\sin\beta = \frac{\cos 33^0}{\cos 48^0} = 0{\cdot}1254;\ \beta = 7{\cdot}2^0$$

Powers of any Numbers required

Powers of the form a^n are obtained by using the cursor-line to bring C 1 into position above the basic value a of the relevant LL-scale, after which the cursor line is set to C-n. Here again the decimal-place rule applies, i.e. the range of the D scale is 1 to 10 in the case of LL_3, 0·1 to 1 in the case of LL_2 and 0·01 to 0·1 in the case of LL_1.

Example: $3{\cdot}75^{2{\cdot}96} = 50$; place the cursor above LL_3-3·75; bring C 1 under the cursor-line, then place the latter above C 2·96, and find the result, i.e. 50 underneath it on LL_3.

Exercises: $4{\cdot}2^{2{\cdot}16} = 22{\cdot}2$; $4{\cdot}2^{0{\cdot}216} = 1{\cdot}364$; both examples show that the decimal-place rule has to be observed.

Further exercises: $1{\cdot}03^{2{\cdot}22} = 1{\cdot}0678$; $1{\cdot}282^{1{\cdot}66} = 1{\cdot}51$; $11{\cdot}5^{2{\cdot}53} = 483$.

If C for n can no longer be set to the right, then C 10 instead of C 1 is placed above the basic value. The result is to be read on the next LL-scale.

Exercises: $1{\cdot}665^{5{\cdot}13} = 4{\cdot}98$; $1{\cdot}966^{6{\cdot}65} = 87{\cdot}8$; $2{\cdot}46^{2{\cdot}32} = 8{\cdot}07$.

If the exponent is over 10, the power can frequently be calculated by changing over from LL_2 to LL_3.

Example: $1{\cdot}196^{25{\cdot}3} = (1{\cdot}196^{10})^{2{\cdot}53} = 92{\cdot}6$; $1{\cdot}254^{13} = (1{\cdot}254^{10})^{1{\cdot}3} = 18{\cdot}96$.

No reading need be taken of the intermediate result.

Roots of any Numbers required

The root exponent is converted into a power exponent, in accordance with the formula $\sqrt[n]{a} = a^{\frac{1}{n}}$ or else the scale CI is used immediately for the setting operation.

Example: $\sqrt{23} = 2{\cdot}04$; place CI 10, with the help of the cursor-line, above LL_3-23, and read the result, i.e. 2·04, on LL_2, likewise with the cursor-line at CI 4·4.

Exercises: $\sqrt{15{\cdot}2} = 93{\cdot}5$ (CI-10 to be placed above LL_3-15·2; result to be read on LL_3).

$\sqrt[1{,}95]{23{\cdot}5} = 5{\cdot}05$ (CI 1 above LL_3-23·5; read the result, i.e. 5·05 underneath CI 1·95 on LL_3)

$\sqrt[2{,}36]{15} = 3{\cdot}15$ (CI 1 above LL_3-15; read the result, i.e. 3·15, underneath CI 2·36 on LL_3)

$\sqrt[7{,}15]{8{\cdot}75} = 1{\cdot}354$ (CI 10 above LL_3-8·75; read the result, i.e. 1·354, underneath CI 7·15 on LL_2)

Logarithms to any Base required

Either the left index figure on the slide, C 1, or the right, C 10, is placed above the base, on the LL-scale, thus providing a table of the relevant logarithms.

Example: ${}^2\log 200 = 7{\cdot}65$; ${}^2\log 22 = 4{\cdot}46$; ${}^2\log 1{\cdot}89 = 0{\cdot}918$.

We place C 1 or C 10, with the aid of the cursor-line above LL_2-2, obtain a table, and can now take the readings with the help of the cursor; at LL_3-200 we obtain the value 7·65 on C; at LL_2-1·89 the value 0·918 on C.

Exercises: ${}^5\log 25 = 2$; ${}^5\log 60 = 2{\cdot}54$; ${}^5\log 800 = 4{\cdot}15$
${}^{10}\log 20 = 1{\cdot}301$; ${}^{10}\log 2 = 0{\cdot}301$; ${}^{10}\log 800 = 2{\cdot}9$