INSTRUCTIONS

FOR THE USE OF YOUR

LUTZ SLIDE RULE

MODELS
NO. 151
NO. 153
NO. 252
NO. 551


INTERNATIONAL
SLIDE RULE
MUSEUM

CONTENTS

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1. GENERAL DESCRIPTION OF SCALES

The following is the brief description of the scales.

(a) C and D

These are exactly alike and the fundamental scales of the slide rule. And they are used for general fundamental calculations.

(b) DF and CF

These are the same as the D and C scales, except that they are "folded" at π. Therefore, any number on the D scale is automatically multiplied by π on the DF scale. And in order to avoid resetting when the answer runs off scale, they are used with the C and D scales.

(c) CI

This is an inverted C scale and is used with the C scale in reading directly the reciprocal of a number. And it lets us do multiplication of three factors with just one setting of the slide.

(d) DI

This is an inverted D scale and just the same as the CI scale.

(e) CIF

This is an inverted CF scale and is used with the DF scale in the same relation as the CI scale with the D scale.

(f) A and B

These are exactly alike and are used with the C and D scales to find the square and square root.

(g) K

This is used in finding the cube and cube root.

(h) S

This scale gives the sine and cosine of an angle.

(i) T or T_1

This scale gives the tangent of an angle.

(j) T_2

This scale gives the tangent of an angle greater than 45°.

(k) ST

This scale gives the sine and tangent of an angle less than 6°.

(l) L

This scale is used with the D scale in finding directly the mantissa of the common logarithms of a number.

(m) LL_0, LL_1, LL_2 and LL_3

These are used to find the values of the type form of a^n, e^x, and give the natural logatithms of a number.

(n) $LL_{\overline{0}}$, $LL_{\overline{1}}$, $LL_{\overline{2}}$ and $LL_{\overline{3}}$

These are used to find the values of the type form of a^{-n}, e^{-x} and give the natural logarithms of a number.

(o) P

This is a special scale. When used with the S scale, it gives the value of the cosine and using with the D, DI and A scales, it gives $\sqrt{1-x^2}$, $\sqrt{1-(1/x)^2}$ and $\sqrt{1-x}$ only by the hairline.

2. MULTIPLICATION AND DIVISION

(1) How to Use C and D Scales

In the following, the left hand 1 of the C or D scale is called its Left Index, the right hand 1 is called its Right Index.

(Multiplication)

The process of multiplication may be performed by using the C and D scales.

To multiply a by b,

opposite a on D, set left or right index of C,
move hairline to b on C,
under hairline read the product on D.

Example 1. $2.4\times3=7.2$

Opposite 2.4 on D, set left index of C,
move hairline to 3 on C,
under hairline read answer as 7.2 on D.

Example 2. $262\times33.2=8700$

Opposite 2.62 on D, set left index of C,
move hairline to 3.32 on C,
under hairline find 8.70 on D,
read answer as 8700.

The position of the decimal point is made by rough mental calculation as follows:

$$262\times33.2\fallingdotseq300\times30=9000$$

Comparing with 9000 to 8.7 which obtained by the slide rule, the answer is decided as 8700.

Example 3. $4.5\times3.2=14.4$

Opposite 4.5 on D, set right index of C,
move hairline to 3.2 on C,
under hairline find 1.44 on D,

read answer as 14.4.

Note in above case that the reading would have been off scale, if the left index had been used.

(Division)

The process of division is performed by using the C and D scales.

To divide a by b,

move hairline to a on D,

set b on C under hairline,

opposite left or right index of C read the quotient on D.

Example 4. $32 \div 8 = 4$

Move hairline to 3.2 on D,

set 8 on C under hairline,

opposite right index of C read answer as 4 on D.

(Combined Multiplication and Division)

To calculate the type form of $\frac{a \times c}{b}$, first divide a by b, and then multiply the result by c. It is greatly that the increase of computation efficiency.

Example 5. $\frac{1.57 \times 32}{44} = 1.142$

Move hairline to 1.57 on D,

set 4.4 on C under hairline,

move hairline to 3.2 on C,

under hairline read answer as 1.142 on D.

(2) How to Use CI Scale

(Reciprocal)

The reciprocal of a number is 1 divided by the number. Thus, the reciprocal of 2.5 is $\frac{1}{2.5}$ or 0.4. Opposite any number on the C scale, read its reciprocal on the CI scale. The

number on the CI scale is given by the red figures.

Example 6. Opposite 2.5 on C, read $\frac{1}{2.5}=0.4$ on CI.

Opposite 125 on C, read $\frac{1}{125}=0.008$ on CI.

(Multiplication)

The process of multiplication may be performed by using the CI and D scales too. By using of the CI scale, computation can be made quickly.

To multiply a by b,

move hairline to a on D,

set b on CI under hairline,

opposite left or right index of C read the product on D.

Example 7. $2.3\times3.4=7.82$

Move hairline to 2.3 on D,

set 3.4 on CI under hairline,

opposite right index of C read answer as 7.82 on D.

Example 8. $4.77\times0.613=2.92$

Move hairline to 4.77 on D,

set 6.13 on CI under hairline,

opposite left index of C read answer as 2.92 on D.

To multiply three factors, first multiply two of them, and then multiply the result by third.

Example 9. $1.5\times3.2\times8=38.4$

Move hairline to 1.5 on D,

set 3.2 on CI under hairline,

move hairline to 8 on C,

under hairline find 3.84 on D,

read answer as 38.4.

(Division)

The process of division may be performed by using the

CI and D scales too.

To divide a by b,

opposite a on D, set left or right index of C,
move hairline to b on CI,
under hairline read the quotient on D.

Example 10. $1.2 \div 4 = 0.3$

Opposite 1.2 on D, set left index of C,
move hairline to 4 on CI,
under hairline find 3 on D,
read answer as 0.3.

Example 11. $2.42 \div 2.03 = 1.192$

Opposite 2.42 on D, set right index of C,
move hairline to 2.03 on CI,
under hairline read answer as 1.192 on D.

Note in above example that the reading would have been off scale, if the left index had been used.

To divide three factors, first divide two of them, and then divide the result by third.

Example 12. $15.7 \div 3.2 \div 4.4 = 1.115$

Move hairline to 1.57 on D,
set 3.2 on C under hairline,
move hairline 4.4 on CI,
under hairline read answer as 1.115 on D.

(Combined Multipilcation and Division)

The calculation of the type form of $\frac{a \times b}{c}$ is worked out the procedure of multiplication→division, too.

Example 13. $\frac{8 \times 2}{5} = 3.2$

Move hairline to 8 on D,
set 2 on CI under hairline,

move hairline 5 on CI,
under hairline read answer as 3.2 on D.

(3) How to Use DF and CF Scales

The DF and CF scales are similar to the D and C scales folded at π. As π is very near $\sqrt{10}$, so 1 of the DF and CF scales lies about in the middle and π on both ends of a scale. These scales can often be used in calculation in order to avoid resetting when the answer runs off scale. When the slide is in any position with a number x on the D scale appearing opposite a number y on the C scale, then this same number x appears also on the DF scale opposite y on the CF scale. If the reading is off scale on the C-D scale it may be found on the CF-DF scale. Thus, the DF and CF are the scales D and C extended.

Moreover we can use the DF and CF scales in problems requiring multiplication by π. Opposite any number on the D scale, read π times of this number on the DF scale. Thus if we take any number on the D scale as diameter of a circle, its circumference can be found on the DF scale.

These folded scales may be used to perform multiplication and division just as the C and D scales are used.

(Multiplication)

Example 14. $2.5\times5=12.5$

Opposite 2.5 on D, set left index of C,
(or opposite 2.5 on DF, set middle index of CF,)
move hairline to 5 on CF,
under hairline find 1.25 on DF,
read answer as 12.5.

As in above example, when the reading is off scale on the D scale, it may be found on the DF scale.

Example 15. $2.5\times2.4\times1.2=7.20$

Move hairline to 2.5 on D,
set 2.4 on CI under hairline,
move hairline to 1.2 on CF,
under hairline read answer as 7.20 on DF.

Example 16. $2\times7\times8=112$

Move hairline to 2 on D,
set 7 on CI under hairline,
move hairline to 8 on CF,
under hairline find 1.12 on DF,
read answer as 112.

(Division)

Example 17. $9\div6=1.5$

Move hairline to 9 on DF,
set 6 on CF under hairline,
opposite left index of C read answer as 1.5 on D.
(or opposite middle index of CF read answer as 1.5 on DF.)

Example 18. $9\div1.5\div4=1.5$

Move hairline to 9 on DF,
set 1.5 on CF under hairline,
move hairline to 4 on CI,
under hairline read answer as 1.5 on D.

(Combined Multiplication and Division)

Example 19. $11.2\div8.4\times2.7=3.6$

Move hairline to 1.12 on DF,
set 8.4 on CF under hairline,
move hairline to 2.7 on C,
under hairline read answer as 3.6.

Example 20. $\dfrac{3.2\times1.2\times5.8}{8.2\times9.5}=0.286$

Move hairline to 3.2 on D,
set 8.2 on C under hairline,
move hairline to 1.2 on CF,
set 9.5 on CF under hairline,
move hairline to 5.8 on C,
under hairline find 2.86 on D,
read answer as 0.286.

(Computation involving π)

By using the DF scale which is folded to begin and end with π, if the diameter of a circle d is set on the D scale, the circumference πd is given on the DF scale, opposite d on the D scale.

Example 21. Find the circumference of a circle with its diameter of 5cm. Find the diameter of a circle with its circumference of 46.8cm. Answer 15.71cm, 14.9cm

Move hairline to 5 on D,
under hairline find 1.571 on DF,
read answer as 15.71cm.
Move hairline to 4.68 on DF,
under hairline find 1.49 on D,
read answer as 14.9cm.

Example 22. Find the value of 3π. Answer 9.42

Move hairline to 3 on D,
under hairline read answer as $9.42=3\pi$ on DF.

Example 23. Find the value of $\frac{7}{\pi}$. Answer 2.23

Move hairline to 7 on DF,
under hairline read answer as $2.23=\frac{7}{\pi}$ on D.

(4) How to Use CIF Scale

The CIF scale is an inverted CF scale. It may be to perform multiplication and division just as the CI scale is used. Thus, by the use of six scales of the C, D, CI, CF, DF and this CIF, the fundamental calculations are performed very quickly and efficiently, without resetting the scale and running off scale.

(Multiplication)

Example 24. $3.1 \times 1.6 \times 1.5 \times 8.8 = 65.5$

Move hairline to 3.1 on D,
set 1.6 on CI under hairline,
move hairline to 1.5 on CF,
set 8.8 on CIF under hairline,
opposite right index of C find 6.55 on D,
read answer as 65.5.

Example 25. $9 \times 8 \times 3 \times 7 = 1512$

Move hairline to 9 on DF,
set 8 on CIF under hairline,
move hairline to 3 on C,
set 7 on CI under hairline,
opposite left index of C find 1.512 on D
read answer as 1512.

(Division)

Example 26. $27 \div 5 \div 7.5 = 0.72$

Move hairline to 2.7 on D,
set 5 on C under hairline,
move hairline to 7.5 on CIF,
under hairline find 7.2 on DF,
read answer as 0.72.

(Combined Multiplication and Division)

Example 27. $3.8 \times 6.5 \div 2 \times 1.96 \div 1.45 = 167$

Move hairline to 3.8 on D,
set 6.5 on CI under hairline,
move hairline to 2 on CIF,
set 1.96 on CIF under hairline,
move hairline to 1.45 on CIF,
under hairline find 1.67 on DF,
read answer as 1.67.

Example 28. $\dfrac{236000\times0.0065\times188\times715}{0.00032\times7800000}=82600$

Move hairline to 2.36 on D,
set 3.2 on C under hairline,
move hairline to 6.5 on C,
set 7.8 on C under hairline,
move hairline to 1.88 on CF,
set 7.15 on CIF under hairline,
opposite right index of C find 8.26 on D,
read answer as 82600.

The Position of the decimal point is made by rough mental calculation as follows:

$$\frac{236000\times0.0065\times188\times715}{0.00032\times7800000}$$

$$=\frac{2.36\times6.5\times1.88\times7.15}{3.2\times7.8}\times\frac{10^{5-3+2+2}}{10^{-4+6}}$$

$$\fallingdotseq\frac{2\times7\times2\times7}{3\times8}\times\frac{10^{6}}{10^{2}}=\frac{\cancel{2}\times7\times\cancel{2}\times7}{3\times\cancel{8}_{\cancel{4}_{2}}}\times10^{6-2}$$

$$\fallingdotseq\frac{7\times7}{6}\times10^{4}\fallingdotseq\frac{50}{6}\times10^{4}\fallingdotseq8\times10^{4}=80000$$

Comparing with 80000 to 8.26 which obtained by the slide rule, the answer is decided as 82600.

3. PROBLEMS OF PROPORTION

Proportion is a special case of multiplication and division. Generally, when the slide scale is set at one point on the stock scale, there exists certain ratio relation between the scales on the slide and stock. So, the answer is given by removing of the hairline only.

There are two kinds of proportional problems, one is proportion and another inverse proportion.

(1) Proportion

The problems of proportion are widely applied to conversion, indexes, proportional division and percentage etc.

Proportional calculation is performed by refering to each other C and D or CF and DF scales.

Example 29. Fill the following blanks, given 1kt=1.852 km/h.

kt	27	33.4	44
km/h	50*	60*	81.5*

*.....shown answer

Opposite 1.852 on D, set left index of C,
move hairline to 2.7 on C,
under hairline find 5 on D,
read answer as 50.
Move hairline to 3.34 on C,
under hairline find 6 on D,
read answer as 60.
Move hairline to 4.4 on C,
under hairline find 8.15 on C,
read answer as 81.5.

Example 30. Find % in the following table.

	Amount	%	
A	$2,410	14.6	*
B	$2,640	16.0	*
C	$3,400	20.6	*
D	$8,050	48.8	*
Total	$16,500	100.0	*

*.....shown answer

Opposite left index of D, set 1.65 on C,
move hairline to 2.41 on C,
under hairline find 1.46 on D,
read answer as 14.6.
Move hairline to 2.64 on C,
under hairline find 1.60 on D,
read answer as 16.0.
Move hairline to 3.4 on C,
under hairline find 2.06 on D,
read answer as 20.6.
Move hairline to 8.05 on C,
under hairline find 4.88 on D,
read answer as 48.8.

Example 31. $1.43 : 3.39 = 8.65 : x$ $x = 20.5$

Move hairline to 3.39 on D,
set 1.43 on C under hairline,
move hairline to 8.65 on CF,
under hairline find 2.05 on DF,
read answer as $x = 20.5$.

(2) Inverse Proportion

Inverse proportion is calculated by refering to each other D and CI or DF and CIF scales.

Example 32. There is a job which take 12 days by 45 men. How many days will it take by 30 men? Answer 18 days

Move hairline to 1.2 on D,
set 4.5 on CI under hairline,
move hairline to 3 on CI,
under hairline find 1.8 on D,
read answer as 18.

Example 33. A pulley with 7″ diameter revolves 360 per minute. Find number of revolution per minute, when diameter of pulley is 9″. Answer 280rpm

Move hairline to 3.6 on D,
set 7 on CI under hairline,
move hairline to 9 on CI,
under hairline find 2.8 on D,
read answer as 280.

4. SQUARE AND SQUARE ROOT

(1) Square

Opposite any number on the D scale, read its square on the A scale. Similarly, opposite any number on the C scale, read its square on the B scale.

Example 34. $2^2=4$, $5.63^2=31.7$

Move hairline to 2 on D,
under hairline read answer as 4 on A.
Move hairline to 5.63 on D,
under hairline read answer as 31.7 on A.

(2) Square Root

To find the square root of a number between 1 and 10,

the number would have to be set on the scale A left. To find the square root of a number between 10 and 100, the number would have to be set on the scale A right. Or a given number is divided into several groups with two digits per group, counting from decimal point to the right or left direction. If the first group consists of one figure, a given number is set on the scale A left. If it consists of two figures, a given number is set on the scale A right. Place the decimal point of the answer, taking one digit per group.

Example 35. $\sqrt{6.56}=2.56$, $\sqrt{6'56'00}=256$,
$\sqrt{0.00'06'56}=0.0256$

Move hairline to 6.56 on A left,
under hairline find 2.56 on D.

Example 36. $\sqrt{65.6}=8.1$, $\sqrt{65'60}=81$,
$\sqrt{0.00'00'65'6}=0.0081$

Move hairline to 65.6 on A right,
under hairline find 8.1 on D.

(3) Multiplication and Division including Square

Example 37. $2\times4^2=32$

Move hairline to 2 on A left,
set 4 on CI (back face) under hairline,
opposite right index of C read answer as 32 on A right

Example 38. $1.3^2\times6.5^2=71.4$

Move hairline to 1.3 on D,
set 6.5 on CI (back face) under hairline,
opposite right index of C read answer as 71.4 on A right.

Example 39. $30\div5^2=1.2$

Move hairline to 30 on A right.

set 5 on C under hairline,
opposite left index of C read answer as 1.2 on A left.

Example 40. $25^2 \div 50 = 12.5$

Move hairline to 2.5 on D,
set 50 on B right under hairline,
opposite right index of C read answer as 12.5 on A right.

Example 41. $2.4^2 \div 3.82^2 = 0.395$

Move hairline to 2.4 on D,
set 3.82 on C under hairline,
opposite right index of C find 39.5 on A right,
read answer as 0.395.

(4) **Multiplication and Division including Square Root**

Example 42. $\sqrt{20} \times 2 = 8.95$

Move hairline to 20 on A right,
set 2 on CI (back face) under hairline,
opposite right index of C read answer as 8.95 on D

Example 43. $\sqrt{120 \times 36} = 65.7$

Opposite 1.2 on A left, set left index of C,
move hairline to 36 on B right,
under hairline find 6.57 on D,
read answer as 65.7.

Example 44. $\sqrt{36} \div 0.5 = 12$

Move hairline to 36 on A right,
set 5 on C under hairline,
opposite left index of C find 1.2 on D,
read answer as 12.

Example 45. $6 \div \sqrt{16} = 1.5$

Move hairline to 6 on D,
set 16 on B right under hairline,

opposite left index of C read answer as 1.5 on D.

Example 46. $\sqrt{2880 \div 8.35} = 18.57$

Move hairline to 28.8 on A right,
set 8.35 on B left under hairline,
opposite left index of C find 1.857 on D,
read answer as 18.57.

(5) How to Use Gauge Mark "c"

area of circle $A = \frac{\pi}{4} d^2$ d = diameter of circle

$$= \left(\sqrt{\frac{\pi}{4}} \cdot d\right)^2 = \left(d \Big/ \sqrt{\frac{4}{\pi}}\right)^2$$

$\sqrt{\frac{4}{\pi}}$ corresponds to the value of "c". These gauge marks are graduated on the C and D scales, which are used for calculation of the area of a circle.

Example 47. Find the area of a circle with its diameter of 1.5cm. Answer 1.77 cm^2

Move hairline to 1.5 on D,
set c on C under hairline,
opposite left index of B read answer as 1.77 on A left.

Example 48. Find the volume of a cylinder with 1.2m in diameter and 5m in length. Answer 5.66 m^3

Move hairline to 1.2 on D,
set c on C under hairline,
move hairline to 5 on B left,
under hairline read answer as 5.66 on A left.

5. CUBE, CUBE ROOT, $\frac{3}{2}$ POWER AND $\frac{2}{3}$ POWER

(1) Cube

Opposite any number on the D scale, read its cube on

the K scale

Example 49. $2^3=8$, $3^3=27$, $7^3=343$

Move hairline to 2 on D,
under hairline read answer as 8 on K.
Move hairline to 3 on D,
under hairline read answer as 27 on K.
Move hairline to 7 on D,
under hairline read answer as 343 on K.

(2) Cube Root

To find the cube root of a number between 1 and 10, the number would have to be set on the scale K left. To find the cube root of a number between 10 and 100, the number would have to be set on the scale K middle. To find the cube root of a number between 100 and 1000, the number would have to be set on the scale K right.

Or a given number is divided into several groups with three digits per group, counting from the decimal point to right or left direction. If the first group consists of one figure, a given number is set on the scale K left. If it consists of two figures, a given number is set on the scale K middle. If three figures, on the scale K right.

Place the decimal point of the answer, taking one digit per group.

Example 50. $\sqrt[3]{2.62}=1.379$, $\sqrt[3]{2'620'000}=137.9$

$\sqrt[3]{0.002'62}=0.1379$

Move hairline to 2.62 on K left,
under hairline find 1.379 on D.

Example 51. $\sqrt[3]{26.2}=2.97$, $\sqrt[3]{26'200}=29.7$

$\sqrt[3]{0.000'026'2}=0.0297$

Move hairline to 26.2 on K middle,

under hairline find 2.97 on D.

Example 52. $\sqrt[3]{262}=6.4$, $\sqrt[3]{262'000}=64$,

$\sqrt[3]{0.000'000'262}=0.0064$

Move hairline to 262 on K right,
under hairline find 6.4 on D.

(3) $\frac{3}{2}$ Power and $\frac{2}{3}$ Power

$\frac{3}{2}$ power and $\frac{2}{3}$ power are calculated by the use of the reference scales A and K.

Example 53. $4^{\frac{3}{2}}=8$

Move hairline to 4 on A left,
under hairline read answer as 8 on K left.

Example 54. $29.5^{\frac{2}{3}}=9.55$

Move hairline to 29.5 on K middle,
under hairline read answer as 9.55 on A left.

6. LOGARITHMS

(1) Common Logarithms

Common Logarithms are calculated to the base 10 include integer (characteristic) and decimal part (mantissa).

A slide rule gives only the mantissa of common logarithms, and the characteristic can be calculated from the following formula:

(the number of places exceeding decimal point of a given number—1)

viz. the characteristic of any number greater than unity is always 1 less than the number of figures at the left of decimal point.

If given number is of n places under the decimal point, the characteristic is also negative; this is indicated by placing the negative sign or bar over the figure as $-n$ or $\bar{n}$.

The mantissa of a given number may be found out with L and D scales in the reference relation.

Example 55. $\log_{10} 3.14 = 0.497$

Move hairline to 3.14 on D,
under hairline read answer as 0.497 on L.

Example 56. $\log_{10} 0.000342 = \bar{4}.534$

Move hairline to 3.42 on D,
under hairline find 0.534 on L,
add characteristic $\bar{4}$, read answer as $\bar{4}.534$.

Example 57. $\log_{10} 887 = 2.948$

Move hairline to 8.87 on D,
under hairline find 0.948 on L,
add characteristic 2, read answer as 2.948.

(2) Natural Logarithms

Logarithms to the base of $e(\fallingdotseq 2.718)$ are called natural logarithms, they may be found out by multiplying 2.3026 to common logarithms from the following formula:

$$\log_e N = 2.3026 \times \log_{10} N$$

(3) Anti-Logarithms

The type form of $\log_{10}^{-1} x = a$ is performed by the perfect reverse procedure of the above examples. Take mantissa only of a given number x and set it on the L scale. Now, we find that the significant figure of number of logarithm a is given on the D scale, opposite the mantissa on the L scale. When the characteristic is n, there will be $(n+1)$ figures before the decimal point in the answer. And when the characteristic is $\bar{n}$, the answer is of n places under the de-

cimal point.

Example 58. $\log_{10}^{-1} 1.398 = 25$

Move hairline to 0.398 on L,
under hairline find 2.5 on D,
characteristic is 1,
there will be two figures before the decimal point,
thus read answer as 25.

(4) Exponent

Calculation of the type form of a^n is figured out by the following procedures.

Now, $a^n = x$ $\quad\therefore \log_{10} x = n \times \log_{10} a$

$\therefore x = \log_{10}^{-1}(n \times \log_{10} a)$

Find $\log_{10} a$,
calculate $n \times \log_{10} a$,
answer is given by $\log_{10}^{-1}(n \times \log_{10} a)$.

Example 59. $3.14^{2.5} = 17.46$

Move hairline to 3.14 on D,
under hairline read 0.497 ($=\log_{10} 3.14$) on L,
move hairline to 4.97 on D,
set 2.5 on CI under hairline,
opposite left index of C read 1.242($=2.5 \times \log_{10} 3.14$) on D,
move hairline to 0.242 on L,
under hairline read 1.746($=\log_{10}^{-1}(2.5 \times \log_{10} 3.14)$) on D,
characteristic is 1, therefore there will be two figures before the decimal point,
then read answer as 17.46.

7. TRIGONOMETRIC FUNCTIONS

These computations can be usually obtained by the co-

operation of S, T, ST and C scales.

S scale gives the sines of angles ranging from 5.5° or 5.8° to 90°.

T scale gives the tangents of angles ranging from 5.5° or 5.8° to 45°.

ST scale gives the sines or tangents of small angles ranging from 0.54° or 0.58° to 6°.

T_2 scale gives the tangents of angles ranging from 45° to 84.2°.

(1) Sin θ

Example 60. $\sin 15° = 0.259$

Move hairline to 15 on S,
under hairline find 2.59 on C,
read answer as 0.259.

When using the S scale to read the value of $\sin\theta$, read the left index of C as 0.1 and the right index as 1.

Example 61. $\sin 1.5° = 0.0262$

Move hairline to 1.5 on ST,
under hairline find 2.62 on C,
read answer as 0.0262.

When using the ST scale to read the value of $\sin\theta$, read the left index of C as 0.01 and the right index as 0.1.

Example 62. $\sin 30° \times 5 = 2.5$

Opposite 5 on D, set right index of C,
move hairline to 30 on S,
under hairline read answer as 2.5 on D.

Example 63. $\sin 24.4° \div 1.67 = 0.247$

Opposite left index of D, set left index of C,
move hairline to 24.4 on S,
set 1.67 on C under hairline,

opposite left index of C, find 2.47 on D,
read answer as 0.247.

Example 64. $3.06 \div \sin 43° = 4.49$

Move hairline to 3.06 on D,
set 43 on S under hairline,
opposite right index of C, read answer as 4.49 on D.

(2) How to Use Gauge Marks ρ'', ρ' and $\rho°$

The sines and tangents of angles less than 6° can be found by the following approximation.

$$\sin\theta° \fallingdotseq \tan\theta° \fallingdotseq \theta \text{ (in radians).}$$

Therefore, to find the sine and tangent of an angle less than 6°, find the value of the angle in radians, and get them by the use of $\rho°$, ρ' and ρ'' on the C scale.

$$\rho° \text{ .....} \frac{180°}{\pi} = 57.2958° = 1 \text{ radian}$$

$$\rho' \text{ .....} \frac{180 \times 60'}{\pi} = 3437.75' = 1 \text{ radian}$$

$$\rho'' \text{ .....} \frac{180 \times 60 \times 60''}{\pi} = 206265'' = 1 \text{ radian}$$

Example 65. $\sin 1.5° = 0.0262$

Move hairline to 1.5 on D,
set $\rho°$ on C under hairline,
opposite right index of C, find 2.62 on D,
read answer as 0.0262.

To determine the position of the decimal point is estimated as follows:

$1'' \fallingdotseq 0.000005$
$1' \fallingdotseq 0.0003$
$1° \fallingdotseq 0.02$

(3) Cos θ

We find the value of $\cos\theta$ by reading the sines of the

complementary angles as $\cos\theta = \sin(90°-\theta)$.

Example 66. $\cos 30° = 0.866$

Move hairline to $60(=90°-30°)$ on S,
under hairline find 8.66 on C,
read answer as 0.866.

Example 67. $\cos x° = 0.413$ $x = 65.6°$

Move hairline to 4.13 on C,
under hairline find 24.4 on S,
read answer as $90-24.4=65.6$.

(4) Sine Proportions

The calculation of sine proportion is worked out by refering to S and D scales. In Fig. 1, there is the following formula.

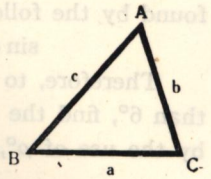

Fig. 1

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Therefore, opposite a, b and c on the D scale, set A, B and C on the S scale respectively.

Example 68. In Fig. 1, find a and c, given $A=80°$, $B=35°$, $C=65°$ and $b=25$.

Answer $a=42.9$, $c=39.5$

Move hairline to 2.5 on D,
set 35 on S under hairline,
move hairline to 80 on S,
under hairline find 4.29 on D,
read answer as 42.9.
Move hairline to 65 on S,
under hairline find 3.95 on D,
read answer as 39.5.

(5) Tan θ

Example 69. $\tan 32.5° = 0.637$

Move hairline to 32.5 on T,
under hairline find 6.37 on C,
read answer as 0.637.

Example 70. $\tan 1.5° = 0.0262$

Move hairline to 1.5 on ST,
under hairline find 2.62 on C,
read answer as 0.0262.

When using the T scale to read the value of $\tan\theta$, read the left index of C as 0.1 and the right index as 1, and using the ST scale to read the value of $\tan\theta$, read the left index of C as 0.01 and the right index as 0.1.

Tangents of angles greater than 45° are calculated by the use of the T_2 scale.

Example 71. $\tan 62° = 1.88$

Move hairline to 62 on T_2,
under hairline read answer as 1.88 on C.

When using the T_2 scale to read the value of $\tan\theta$, read the left index of C as 1 and the right index as 10.

Or we find the value of the tangents of angles greater than 45° by reading the reciprocals of the tangents of the complementary angles as $\tan\theta = \frac{1}{\tan(90°-\theta)}$.

Example 72. $\tan 62° = 1.88$

Set C index even with D index,
move hairline to 28(=90−62) on T,
under hairline read answer as 1.88 on DL

(6) Other Trigonometric Functions

To get $\cot\theta$, $\sec\theta$ and $\operatorname{cosec}\theta$, we use the following formulas.

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

(7) Solutions of Right Triangle and Vector Problems

By the co-operation of S, T and DI scales, solutions of the right triangle and vector problems can be obtained.

In the right triangle $\triangle ABC$ (in Fig. 2), there is the following formula:

$$c\cdot\sin\theta = a\cdot\tan\theta = b$$

$$\therefore \quad \frac{\sin\theta}{\frac{1}{c}} = \frac{\tan\theta}{\frac{1}{a}} = \frac{1}{\frac{1}{c}}$$

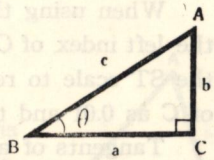

Fig. 2

Therefore, move the hairline to b on the DI scale, set the right or left index of the C scale under the hairline, move the hairline to a on the DI scale, under the hairline read θ on the T scale, move the hairline to θ on the S scale, under hairline read c on the DI scale.

Example 73. In Fig. 2. find c and $\angle B$, given $a=2.3$ and $b=1.8$.

Answer $c=2.92$, $\angle B=38.08°$

Move hairline to 1.8 on DI,
set right index of C under hairline,
move hairline to 2.3 on DI,
under hairline read answer as $38.08=\angle B$ on T,
move hairline to 38.08 on S,
under hairline read answer as $2.92=c$ on DI.

Example 74. $\sqrt{2.54^2+1.65^2}=3.03$

$2.54+j\,1.65=3.03\ \underline{/33°}$

Move hairline to 1.65 on DI,

set right index of C under hairline,
move hairline to 2.54 on DI,
under hairline read answer as 33 on T,
move hairline to 33 on S,
under hairline read answer as 3.03 on DI.

Example 75. Analyze the following vector. Answer $x=5.2,\ y=3.38$ $6.20\ \underline{/33°}=5.20+j3.38$

Move hairline to 6.20 on DI,
set 33 on S under hairline,
move hairline to 33 on T,
under hairline read answer as
$5.20=x$ on DI,
move hairline to right index of C,
under hairline read answer as $3.38=y$ on DI.

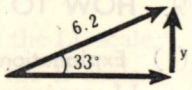

Fig. 3

8. HOW TO USE P SCALE

The P scale is called "Pythagorean scale". When used with the S scale, it gives the value of the cosine by the hairline directly.

(1) Cos θ

Against the value of $\sin\theta$ on the D scale, the value of $\cos\theta$ is shown on the P scale.

Example 76. Find $\cos\theta$, given $\sin\theta=0.5$. Answer 0.866

Move hairline to 5 on D,
under hairline read answer as 0.866 on P.

Example 77. $\cos 15°=0.966$

Set C index even with D index,
move hairline to 15 on S,
under hairline read answer as 0.966 on P.

(2) Type Form of $\sqrt{1-x^2}$

By refering to the D and P scales, we can compute the type form of $\sqrt{1-x^2}$.

Example 78. $\sqrt{1-0.4^2}=0.9165$

Move hairline to 4 on D,

under hairline read answer as 0.9165 on P.

9. HOW TO USE LL SCALES

(1) Explanation of LL Scales

LL represent that the scale is a logarithm of a logarithm. There are two groups of LL scales. One is LL_n scales (LL_0, LL_1, LL_2 and LL_3) ranging from 1.001 to 10^5 or 2×10^4 for the computation of the type form of a^{+n} and the other is $LL_{\bar{n}}$ scales ($LL_{\bar{0}}$, $LL_{\bar{1}}$, $LL_{\bar{2}}$ and $LL_{\bar{3}}$) called Reciprocal LL_n scales ranging from 10^{-5} or 5×10^{-5} to 0.999 for the computation of the type form of a^{-n}.

Construction of LL scales $\begin{cases} LL_n \text{ group...} LL_0, LL_1, LL_2, LL_3 \\ LL_{\bar{n}} \text{ group...} LL_{\bar{0}}, LL_{\bar{1}}, LL_{\bar{2}}, LL_{\bar{3}} \end{cases}$

These LL scales are really divisions of one long Log Log scale. And the next scale to LL_0 or $LL_{\bar{0}}$ scale might be

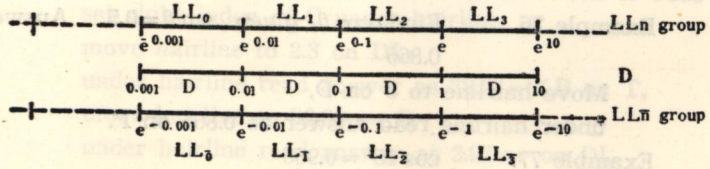

Fig. 4

considered common logarithmic scale D. Therefore, eight

LL scales as shown in Fig. 4 is considered to cover the complete range of the LL scales.

As shown in Fig. 4, the LL_n and $LL_{\bar{n}}$ scales are in a reciprocal relation respectively. This arrangement can be used very conveniently for calculations of powers, roots of numbers, and hyperbolic functions. Usually, general slide rule has three LL scales (LL_1, LL_2 and LL_3).

(2) Natural Logarithms

Set the hairline to the given number N on the LL scale, $\log_e N$ will be found out under the hairline on the D scale.

Determining the position of the decimal point is as follows:

When N is set on { the LL_3 scale / the LL_2 scale / the LL_1 scale / the LL_0 scale } ... { one digit at the left / one digit at the right / two digits at the right / three digits at the right } of the decimal point,

when N is set on { the $LL_{\bar{3}}$ scale / the $LL_{\bar{2}}$ scale / the $LL_{\bar{1}}$ scale / the $LL_{\bar{0}}$ scale } ... { one digit at the left / one digit at the right / two digits at the right / three digits at the right } of the decimal point and place the negative sign before the figures.

Example 79. $\log_e 5 = 1.609$

Move hairline to 5 on LL_3,
under hairline read answer as 1.609 on D.

Example 80. $\log_e 2 = 0.693$

Move hairline to 2 on LL_2,
under hairline find 6.93 on D,
read answer as 0.693.

Example 81. $\log_e 1.03 = 0.0296$

Move hairline to 1.03 on LL_1,
under hairline find 2.96 on D,

read answer as 0.0296.

Example 82. $\log_e 1.006 = 0.00598$

Move hairline to 1.006 on LL_0,
under hairline find 5.98 on D,
read answer as 0.00598.

Example 83. $\log_e 0.23 = -1.47$

Move hairline to 0.23 on $LL_{\overline{3}}$,
under hairline find 1.47 on D,
read answer as −1.47.

Example 84. $\log_e 0.625 = -0.47$

Move hairline to 0.625 on $LL_{\overline{2}}$,
under hairline find 4.7 on D,
read answer as −0.47.

Example 85. $\log_e 0.955 = -0.0461$

Move hairline to 0.955 on $LL_{\overline{1}}$,
under hairline find 4.61 on D,
read answer as −0.0461.

Example 86. $\log_e 0.995 = -0.00501$

Move hairline to 0.995 on $LL_{\overline{0}}$,
under hairline find 5.01 on D,
read answer as −0.00501.

(3) Powers and Roots

The type form of $a^{\pm n}$ or $a^{\pm \frac{1}{n}}$ is simply calculated by the use of the LL scales in an operation similar to multiplication and division.

Example 87. $4.25^{2 \cdot 12} = 21.5$, $4.25^{-2 \cdot 12} = 0.0466$

Move hairline to 4.25 on LL_3,
set left index of C under hairline,
move hairline to 2.12 on C,
under hairline read answer as 21.5 on LL_3,

under hairline read answer as 0.0466 on $LL_{\overline{3}}$.

Example 88. $1.96^{2.3} = 4.70$, $1.96^{-2.3} = 0.213$

Move hairline to 1.96 on LL_2,
set right index of C under hairline,
move hairline to 2.3 on C,
under hairline read answer as 4.70 on LL_3,
under hairline read answer as 0.213 on $LL_{\overline{3}}$.

Example 89. $1.02^{24.5} = 1.624$, $1.02^{-24.5} = 0.616$

Move hairline to 1.02 on LL_1,
set left index of C under hairline,
move hairline to 2.45 on C,
under hairline read answer as 1.624 on LL_2,
under hairline read answer as 0.616 on $LL_{\overline{2}}$.

Example 90. $11.4^{0.7} = 5.50$, $11.4^{-0.7} = 0.182$

Move hairline to 11.4 on LL_3,
set 7 on CI under hairline,
move hairline to left index of C,
under hairline read answer as 5.50 on LL_3,
under hairline read answer as 0.182 on $LL_{\overline{3}}$.

Example 91. $330^{\frac{1}{6.2}} = 2.55$, $330^{-\frac{1}{6.2}} = 0.392$

Move hairline to 330 on LL_3,
set 6.2 on C under hairline,
move hairline to right index of C,
under hairline read answer as 2.55 on LL_2,
under hairline read answer as 0.392 on $LL_{\overline{2}}$.

Example 92. $28.5^{\frac{2.91}{3.41}} = 17.4$, $28.5^{-\frac{2.91}{3.41}} = 0.0574$

Move hairline to 28.5 on LL_3,
set 3.41 on C under hairline,
move hairline to 2.91 on C,
under hairline read answer as 17.4 on LL_3,

under hairline read answer as 0.0574 on $LL_{\bar{3}}$.

Example 93. $0.795^{1.4}=0.725$, $0.795^{-1.4}=1.379$

Move hairline to 0.795 on $LL_{\bar{3}}$,
set left index of C under hairline,
move hairline to 1.4 on C,
under hairline read answer as 0.725 on $LL_{\bar{3}}$,
under hairline read answer as 1.379 on LL_3.

Example 94. $0.795^{14}=0.0402$, $0.795^{-14}=24.9$

Move hairline to 0.795 on $LL_{\bar{3}}$,
set left index of C under hairline,
move hairline to 1.4 on C,
under hairline read answer as 0.0402 on $LL_{\bar{3}}$,
under hairline read answer as 24.9 on LL_3.

Example 95. $0.99676^{\frac{1}{2.22}}=0.99854$, $0.99676^{-\frac{1}{2.22}}=1.001462$

Move hairline to 0.99676 on $LL_{\bar{0}}$,
set 2.22 on C under hairline,
move hairline to left index of C,
under hairline read answer as 0.99854 on $LL_{\bar{0}}$,
under hairline read answer as 1.001462 on LL_0.

Example 96. $e^{1.96}=7.10$, $e^{-1.96}=0.1408$

Move hairline to 1.96 on D,
under hairline read answer as 7.10 on LL_1,
under hairline read answer as 0.1408 on $LL_{\bar{3}}$.

Example 97. $e^{0.94}=2.56$, $e^{-0.94}=0.39$

Move hairline to 9.4 on D,
under hairline read answer as 2.56 on LL_2,
under hairline read answer as 0.39 on $LL_{\bar{2}}$.

Example 98. $e^{0.056}=1.0576$, $e^{-0.056}=0.9455$

Move hairline to 5.6 on D,
under hairline read answer as 1.0576 on LL_1,

under hairline read answer as 0.9455 on LL_T.

Example 99. $e^{0.004}=1.00401$, $e^{-0.004}=0.996$

Move hairline to 4 on D,
under hairline read answer as 1.00401 on LL_0,
under hairline read answer as 0.996 on $LL_{\bar{0}}$.

(4) Hyperbolic Functions

Hyperbolic functions are computed from the following formulas; in these cases, we use the LL_n and $LL_{\bar{n}}$ scales.

$$\sinh x=\frac{e^x-e^{-x}}{2}$$

$$\cosh x=\frac{e^x+e^{-x}}{2}$$

$$\tanh x=\frac{e^{2x}-1}{e^{2x}+1}$$

Example 100. $\sinh 0.802=0.891$
$\cosh 0.802=1.339$
$\tanh 0.802=0.665$

Move hairline to 8.02 on D,
under hairline find $2.23=e^{0.802}$ on LL_2,
under hairline find $0.448=e^{-0.802}$ on $LL_{\bar{2}}$.
Move hairline to $1.604=2\times0.802$ on D,
under hairline find $4.975=e^{2\times0.802}$ on LL_3.

$$\therefore \quad \sinh 0.802=\frac{2.23-0.448}{2}=0.891$$

$$\cosh 0.802=\frac{2.23+0.448}{2}=1.339$$

$$\tanh 0.802=\frac{4.975-1}{4.975+1}=0.665\ .$$

10. HOW TO USE GAUGE MARKS

There are the following gauge marks on our slide rules.

(1) $c \fallingdotseq 1.128$

The c mark is placed on the C and D scales at 1.128, and is used for the calculation of a circle area.

Example 101. Find the area of a circle with its diameter of 1.5 cm. Answer 1.77 cm^2

Move hairline to 1.5 on D,
set c on C under hairline,
opposite left index of C read answer as 1.77 on A left.

(2) $\rho'' \fallingdotseq 206265$

The ρ'' mark is placed on the C and D scales at 2.06265, and is used for converting an angle from seconds to radians.

Example 102. $46.2'' = 0.000224$ radians

Move hairline to left index of D,
set ρ'' on C under hairline,
move hairline to 4.62 on C,
under hairline find 2.24 on D,
read answer as 0.000224 radians.

$1'' = 0.000005$ radians (approximately)
$\therefore \quad 46.2'' \fallingdotseq 50'' = 0.000005 \times 50 = 0.00025$ radians.

(3) $\rho' \fallingdotseq 3437.75$

The ρ' mark is placed on the C and D scales at 3.43775, and is used for converting an angle from minutes to radians.

Example 103. $23.2' = 0.00675$ radians

Move hairline to right index of D,
set ρ' on C under hairline,
move hairline to 2.32 on C,
under hairline find 6.75 on D,
read answer as 0.00675 radians.

[$1' \fallingdotseq 0.0003$ radians (approximately)
$\therefore\ 23.2' \fallingdotseq 20' = 0.0003 \times 20 = 0.006$ radians.]

(4) $\rho^\circ \fallingdotseq 57.2958$

The ρ° mark is placed on the C and D scales at 5.72958, and is used for converting an angle from degrees to radians.

Example 104. $2.4^\circ = 0.0419$ radians

Move hairline to right index of D,
set ρ° on C under hairline,
move hairline to 2.4 on C,
under hairline find 4.19 on D,
read answer as 0.0419 radians.

[$1^\circ \fallingdotseq 0.02$ radians (approximately)
$\therefore\ 2.4^\circ \fallingdotseq 0.02 \times 2.4 = 0.048$ radians.]

(5) $\rho_{''} \fallingdotseq 63.662$

The $\rho_{''}$ mark is placed on the C and D scales at 6.3662, and is used for converting an angle from grades to radians.

Example 105. 2 grades = 0.0314 radians

Move hairline to right index of D,
set $\rho_{''}$ on C under hairline,
move hairline to 2 on C,
under hairline find 3.14 on D,
read answer as 0.0314 radians.

[1 grade $\fallingdotseq$ 0.015 radians (approximately)
$\therefore$ 2 grades $\fallingdotseq 0.015 \times 2 = 0.03$ radians.]

(6) $\pi \fallingdotseq 3.1416$

The π mark is plased on the C, D, DF, CF, A and B scales at 3.1416, and shows a ratio of the circumference of a circle to its diameter.

(7) $\varepsilon \fallingdotseq 2.718$

The ε mark is placed on the LL_2 and LL_3 scales at 2.718,

and shows the value of the base of natural logarithms.

(8) $1/\varepsilon \fallingdotseq 0.3679$

The $1/\varepsilon$ mark is placed on the LL_3 and LL_3 scales at 0.3679, and shows the value of the reciprocal of ε.

– THE END –


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