Things of science

COMPUTATION UNIT No. 273

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This unit of THINGS of science consists of four specimens illustrating gadgets for adding and subtracting, multiplying and dividing, and this explanatory leaflet.

Ever since man first started to use numbers, he has sought ways to make computation with them easier. Before the adoption of our modern decimal system around 1200 A.D., computation of any kind was extremely difficult. More complicated problems could be done only by experts.

Today, when simple addition and subtraction, multiplication and division can be learned by a child, computation is still burdensome and errors easily made. So man has invented various computation devices such as the slide rule, adding machine and, more recently, electronic computers.

This unit of THINGS of science was designed to illustrate several simple computing devices. Scales printed on paper each illustrate a different device.

Identify the specimens included in this unit:

ADDITION-SUBTRACTION SCALES -- Add and subtract numbers.

SLIDE RULE SCALES -- Multiply and divide numbers.

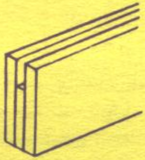
NAPIER'S RODS -- Multiply numbers.

SLIDE RULE -- Wood rule with cursor.

ADDITION-SUBTRACTION RULE

Experiment 1 -- To construct your addition-subtraction rule you will need a piece of sturdy cardboard 11 inches long and 3 5/8 inches or more wide. From the cardboard cut with scissors, razor or knife three strips each an inch wide, and a fourth strip only 5/8 inch wide. Measure these widths exactly and be careful not to bend the cardboard as you work with it.

Place two wide and one narrow strip on top of each other, with the narrow strip sandwiched between two of the one-inch ones so that one edge of all three is lined up. Glue or staple the three cardboard strips securely in place. You have now made the base of your rule. Notice you have a slot on the other edge through which the remaining strip can slide, as shown in the diagram.



Experiment 2 -- Cut the addition-subtraction scales from the paper, and cut the scales apart along the line running lengthwise of the scales. This will give you two parts with identical markings facing each other. Use household cement to stick the narrow scale on the upper part of the sliding cardboard strip and the wide scale to the base, aligning them so the numbers match exactly when the sliding strip is put into the slot. Be sure to glue them down smoothly so your answers will be correct -- stretched places or overlapped parts will give you the wrong answer. Slide the top scale along the lower scale to be sure it moves freely.

Experiment 3 -- Test your rule to see if it can add and subtract as accurately as you can in your head. First notice that the numbers of each scale start at the center of the scale or zero point, and run both ways. Call the zero point the "index" of each scale.

To add +3 and +4, place the index of the top scale over the +3, find +4 on the top scale and your answer will then be directly beneath it. Do you find the +7 as you expected?

It will help in using this device and in understanding the slide rule later if you think of the numbers on your scale as representing distances. Plus three, for example, represents the distance from the index to the point marked +3, since this distance is 3 units long. Thus, in adding +3 to +4, you add the distance for +3 to the distance for +4; the sum of the two is the distance +7.

Experiment 4 -- Try adding a negative number to a positive number, -8 to +5, for example. When you place the top index over +5 on the base, -8 on the sliding scale is just over -3 on the lower scale. The sum is then -3. Again you have added distances. To the distance +5 you have added the distance -8. This latter distance is, of course, 8 units long, but since the distance is negative, it extends to the left instead of to the right.

Experiment 5 -- You can subtract numbers with this rule as well as add them. To subtract +4 from +9, take the distance +4 from the distance +9. To do this, place +4 on the upper scale over the +9 on the base scale, and the upper index will indicate your remainder.

Experiment 6 -- Subtract -4 from -8: this involves two negative distances. Place -4 of the upper scale above -8 on the lower scale, and again the remainder is under the upper index. To do this problem, you went from the lower index to the left to -8 to get this negative distance, then on the upper scale you went to the right to reach the upper index which indicated your answer. In reality to subtract -4 you actually added +4. Thus we usually say that the rule for subtraction is to change the sign of the number being subtracted and add.

SLIDE RULE

Nearly everyone has heard of the slide rule. We know of the speed with which calculations can be done with it. But to most people it is a complicated instrument suitable only for experts. Many try to use the slide rule by memorizing rules. But rules are easily forgotten and confused so that a mathematical idea based upon rules alone seldom remains clear in our minds. The principles behind the construction of the slide rule are not difficult to grasp, and if you understand them, the way to use the rule will always be clear and reasonable.

The addition-subtraction scale is a foundation upon which the slide rule can be built. In one you add numbers, in the other you add logarithms or exponents of numbers. But first assemble the slide rule and do a few simple problems with it, then try to understand just why it works so effectively.

Experiment 7 -- Look carefully at your two scales. In the addition- subtraction scale you have just been using, notice that the numbers are evenly spaced whereas in the slide rule, scale 9 and 10 are much closer together than 1 and 2, for instance. Yet here again the spacing of the numbers follows a definite rule.

Actually the numbers are spaced according to their logarithms. This is why we can use a slide rule for multiplication, division and other operations. We multiply and divide simply by adding and subtracting distances, as with your addition-subtraction scale. But since the distances represent logarithms, adding two distances gives a product;

subtracting distances gives a quotient.

Experiment 8 -- Assemble the slide rule in the same way you did the addition-subtraction rule, again fastening the narrow cardboard strip between two of the inch-wide ones so that a slot is left in which the remaining piece will slide. Glue the narrow scale to the upper part of the sliding strip and the wider one to the base so they face each other and the numbers match exactly when the sliding strip is put into the slot. In this scale 1 is called your left index and 10 your right index.

Experiment 9 -- First try a simple problem such as multiplying 2 by 4. To do this, add the logarithmic distance for 2 to the logarithmic distance for 4, which you can do by placing the left index or 1 on the upper scale above 2 on the lower scale. What number do you find on the base scale beneath 4 of the upper scale? Is this the correct answer? The idea of thinking in terms of distances makes it easy to "set up" the multiplication on the slide rule and to locate the answer.

Experiment 10 -- To divide with the slide rule, you simply subtract. Try dividing 8 by 5. This means that from the distance 8 on the upper scale you are to take away 5 on the lower scale, thus you set 8 over 5. The distance remaining, as indicated by the number on the upper scale directly above the lower index (1 on your scale), is somewhere between 1 and 2. By using the finer divisions on the scale you can read it as 16, or more correctly as 1. 6.

The question thus arises as to how you obtain the decimal point. The simplest way is to get it mentally. It is always well to check the results you obtain with a slide rule by working out an approximate answer in your head.

Experiment 11 -- Next divide 8. 7 by 2. 3. You know at once to expect a quotient between 3 and 4, thus there will be one digit before the decimal point. Since the numbers on your slide rule do not go over 10, select 87 as a point somewhere between 8. 5 and 9 on the upper scale; 23 can be located exactly between the outstanding numbers 2 and 3 on the lower scale. When you thus subtract the lower distance from the upper, your base index indicates approximately 38, and putting in the decimal point gives the correct quotient, 3. 8.

To find the decimal point, you should do a quick mental computation, rounding off the numbers if necessary to make this easier. By thus divorcing the decimal point from the slide rule, you can let 6 on the scale, for example, represent 6, .6, .06, 6000, etc., thus extending indefinitely the range of numbers that you can handle.

Experiment 12 -- Sometimes computations involving several steps can be done with only one setting of the slide rule. Given $\frac{23 \times .39}{1.4}$. Think of this as $\frac{23}{1.4} \times .39$. Set 23 over 14 and the quotient is seen to be a little more than 16. Don't change the setting; you want to multiply this quotient by .39. To the distance for 16 (which you now have set up) on the upper scale add the distance for 39 on the lower scale. This gives the answer 64, but you know mentally that the quotient is between 1 and 10, thus the correct answer is 6.4.

Experiment 13 -- Try to multiply 3 x 7 and notice that 7 on the upper scale extends far beyond the lower right index. It would appear that the lower scale should be extended farther for problems of this type.

If the lower scale were extended, however, the extension would look like the scale we have with numbers multiplied by 10. Since the slide rule does not give the decimal point anyway, we would probably omit the zeros, and the extension would then be exactly like the original scale. Therefore why not pretend that the lower scale is the extended scale. Then the right index of the upper scale (10 at the right) would be set over 3 on the lower scale, and 7 is above the product, 21. Thus you see we can set the slide rule using either 1 or 10 as an index, whichever is the more convenient.

By this time you may be aware of a major disadvantage of your slide rule. You cannot locate accurately on a slide rule numbers which involve many digits. When you wanted to locate 87 in Experiment 11, for instance, you had to estimate the position of the digit 7.

WHAT ARE LOGARITHMS?

After using so successfully your slide rule based on logarithms, it is well to consider what a logarithm is and how it behaves. Since logarithms are closely related to exponents, first let us discuss them a bit.

Often we wish to multiply a number by itself, as $\underline{a} \times \underline{a}$. Or we may wish to use $\underline{a}$ in multiplication (as a "factor") three or four times.

To keep from writing such a long expression as $\underline{a} \times \underline{a} \times \underline{a} \times \underline{a}$, mathematicians adopt the abbrevaited form $\underline{a}^4$. The number 4, written above the line, is called an "exponent," and tells how many times a (the "base") is used in the multiplication process. The whole expression is called the "fourth power of a."

Suppose now we wish to multiply $\underline{a}^2 \times \underline{a}^3$. Obviously this means $(\underline{a} \times \underline{a})(\underline{a} \times \underline{a} \times \underline{a})$, so that a is now used as a factor 2 + 3, or 5 times. Thus when we multiply powers having like bases, we actually get the product by adding the exponents. This is the secret of the slide rule.

Let us divide $\underline{a}^6$ by $\underline{a}^4$. In fraction form this can be written $\frac{\underline{a} \times \underline{a} \times \underline{a} \times \underline{a} \times \underline{a} \times \underline{a}}{\underline{a} \times \underline{a} \times \underline{a} \times \underline{a}}$. Now we can divide the numerator and denominator by a four times, reducing the fraction to $\underline{a} \times \underline{a}$, or $\underline{a}^2$. The exponent is the same as if we had subtracted 4 from 6, or $\underline{a}^6 \div \underline{a}^4 = \underline{a}^2$. Thus we now have done a division by means of a subtraction.

To illustrate how computations can be done by means of exponents, let us make a short table of powers of 2.

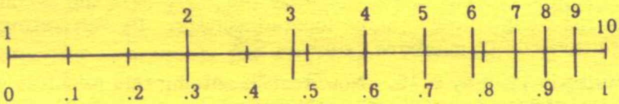
$2^0 = 1 \quad 2^1 = 2 \quad 2^2 = 4 \quad 2^3 = 8 \quad 2^4 = 16 \quad 2^5 = 32 \quad 2^6 = 64 \quad 2^7 = 128$

To multiply 4 x 8, use the table which shows: $4 \times 8 = 2^2 \times 2^3 = 2^{2+3} = 2^5$. Referring back to the table, we see that 2^5 is 32, the number we get by multiplying 4 x 8 in our heads. In like manner, $2 \times 32 = 2^1 \times 2^5 = 2^6 = 64$.

When exponents are used in this way to simplify multiplication and division, they are called logarithms. Since 2^3 equals 8, we say that the logarithm of 8 to the base 2 is 3, or in mathematical symbols, $\log_2 8 = 3$. We have then been doing multiplication and division by adding and subtracting logarithms. In our table above, the base of our logarithms was 2. For most computation, the base 10 is used. Any positive number greater than 1 can be used as the base for a system of logarithms.

The above description of logarithms is of course brief and incomplete, but it may help give some meaning to the numbers on the slide rule.

Experiment 14 -- To figure how the scale on the slide rule is divided off, first divide a line as shown on the scale on p. 7 into ten equal parts. Number these divisions 0, .1, .2, .3, etc., up to 1.



Now locate on this scale the logarithms of numbers from one to ten, using 10 as the base. Since $10^0 = 1$, the logarithm of one to the base 10 is 0. Therefore 1 is placed over 0 of the uniform scale. $10^{(.3)} = 2$ approximately. Thus the logarithm of 2 is approximately .3, and 2 is placed above .3 on the uniform scale. In like manner the logarithm of 3 is .48, etc., and the logarithm of 10 is 1. The result is a scale of numbers running from 1 to 10. The distances from the left index are now the actual logarithms of the numbers 1 through 10. Notice that the numbers get closer together as we read from left to right.

Experiment 15 -- Use your ruler to check that the distance from 1 to 2 is exactly the same as the distance from 2 to 4, 3 to 6, 4 to 8, and 5 to 10. This is as you would expect because each of these distances may be thought of as representing the same quotient, 2.

Experiment 16 -- Use your slide rule to multiply and divide again, this time thinking through the mathematical reasoning involved in the process. Redoing Experiment 9 will help.

Logarithms were invented by John Napier in 1614. He did not, however, relate his logarithms to exponents as we have done. That came later. It was only six years after Napier's invention that Edmund Gunter made the first slide rule. He plotted the logarithms of numbers on a two-foot straight line, and added and subtracted his distances by using dividers. In 1621 William Oughtred used two scales arranged so that one could slide along the other. Thus Oughtred's slide rule was like ours. For quick computations, the slide rule is perhaps one of the handiest tools man has invented.

Experiment 17 -- Examine your wood slide rule. Notice the runner attached to your slide rule which is called a cursor or indicator. The fine black line on the cursor is called the hair line. You will see that your slide rule contains several scales. The C and D scales are like the scales on your paper slide rule. Notice that the scales on the wood slide rule are more finely divided. In Experiment 11, in order to locate the number 87 you had to estimate the position of the second digit. On the wood rule 87 can be located exactly. By estimating, you can locate three-digit numbers, such as 873.

Multiply 12. 6 by 9. 15. Note that in solving this problem you must set the right index of the C scale over 9. 15. Review Experiment 13 if necessary. The approximate answer, 115. 3, can be read easily from your slide rule. On the paper slide you would have had to round these numbers off to two digits, with a resulting loss of accuracy. The answer by longhand multiplication is, however, 115. 290, which shows the limited accuracy of the slide rule.

Various refinements can be added to the basic slide rule. Notice that your wood slide rule contains A, B, CI and K scales in addition to the C and D scales. Compare the A and D scales; you will find that the A scale is made up of two identical scales half the length of the D scale. Compare the D and K scales. The K scale is composed of three scales, each 1/3 the length of the D scale.

The A and B scales are identical and, therefore, can also be used for multiplication and division. You will find that there are three indexes on each of these scales. Values can be assigned to the indexes. For example, if the index at the extreme left of the A or B scales is assigned a value of 1, the index in the middle of the scale could be 10 and the 1 at the extreme right could be 100.

It might seem inconvenient to use these A and B scales for larger numbers. If you are skillful in mentally locating the decimal point, however, you can use numbers of any size on the C and D scales. Furthermore, since the numbers on the A and B scales are very close together, it is impossible to read them as accurately as you can read the C and D scales. Therefore experienced computers usually do all ordinary multiplications and divisions with the C and D scales.

SQUARES AND SQUARE ROOTS

By arranging suitable scales it is possible to simplify many types of computations, to find squares, square roots, cubes, and cube roots.

<u>Experiment 18</u> -- The A and B scales are useful in finding squares and square roots of numbers. You may pair together for this purpose either the A and D scales, or the B and C scales. The hairline on the

indicator helps you jump from one scale to the other. If you will set the indicator over 9 at the right of scale D, you will find the corresponding number on scale A to be 81, the square of 9. Remembering that the numbers on scale A run from 1 to 100, you will see that the numbers on scale A are the squares of the numbers below them on scale D.

Experiment 19 -- In order to determine which half of the A scale to use to find the square root of a number, mark off in groups of twos the numbers to the left of the decimal, for example 1 32.5. If the first group contains only one number, use the left half of the A scale. Use the right half of the scale if the first group has more than one number.

Find the square root of 36. Since the decimal is understood to follow the 6, the right half of the A scale would be used. Set the hair line on 36 and read your answer below on the D scale. To find the square root of 144, the left half of the A scale would be used. Locate 144 with the hair line and read the answer, 12, on the D scale.

CUBES AND CUBE ROOTS

The K scale is used with the D scale to determine cubes and cube roots.

Experiment 20 -- Set the hair line over 3 on the D scale and read its cube on the K scale. Find the cube of 9. The placing of the decimal should be determined by mental calculations. $10^3 = 1,000$, therefore, 9^3 must be less than 1,000 so the answer is 729.

Experiment 21 -- In order to obtain the cube root of a number, first you must determine which of the K scales to use. Mark off the numbers in groups of threes, beginning at the decimal. If there is one number in the first group, the left scale is used; with two numbers in the first group, use the middle K scale; and for three numbers use the right scale. Find the cube root of 125 by using the right K scale. Place the hair line on 125, and read the answer above on the D scale. To get the cube root of 27, use the middle K scale.

Experiment 22 -- Notice that the numbers of your CI scale decrease instead of increase. They are the C scale numbers reversed, and this scale is read from right to left. The CI scale is called an inverted C scale or a reciprocal scale. By comparing the CI and C scales, you can read the reciprocals of numbers. Place the indicator so that the fine line is over 5 on the CI scale. Then on the C scale you find 2. Supply the decimal point and read .2; that is, the reciprocal of 5 is 1/5 or .2. Now see whether you can read the reciprocals of 25 and 6. (Remember that the reciprocal of a number is the result of dividing the number into one. Thus the reciprocals of 25 and 6 are respectively .04 and approximately .17.)

Experiment 23 -- To multiply 2 x 3 x 4.5 involves two steps, but you can do it in one step by using the CI scale. Think of this as $(2 \div 1/3)$ x 4.5. By this procedure, you actually do a division for the first step, which means to subtract distances. Remember that 3 on the CI scale corresponds to 1/3 on the C or D scale. Therefore, use your indicator to set 3 on the CI scale over 2 on the D scale. Then the first product, 6, is on the D scale under the right index of the CI scale. To multiply this result by 4.5, you must place an index of the C scale over 6, but the above operation has already done that. So without changing the setting you simply read on the D scale the final product, 27, under 4.5 on the C scale.

Use this method to multiply with just one setting 4. 3 x .32 x 18. Check you answer by using paper and pencil.

The wood slide rule in this unit was sent to you with the cooperation of Engineering Instruments, Inc., Peru, Ind., manufacturers of many types of slide rules and other calculators.

Experiment 24 -- The next time you are in an office supply store, examine the various slide rules for sale. Notice how carefully they are marked off. Of what material are most of them made? Multiply a few numbers and see how easy the answer is to read. The degree of accuracy a person needs determines how elaborate and expensive a slide rule he would buy.

NAPIER'S RODS

John Napier invented another service for computation. A method of multiplication that preceded the one illustrated here was the "lattice" method. Napier suggested that the lattice work be put on rods made of wood or bone. Napier made such a set of "rods" or "bones," and in 1617 published an article describing them.

Experiment 25 -- Glue your third set of scales, labeled "Napier's

Rods," to a large piece of cardboard. Cut around the outside lines and along the vertical lines, so that you have ten strips. Nine of these strips contain a lattice effect, the remaining one is the index strip. These strips are a simplified form of Napier's rods. The actual rods were made from solid pieces, each with four faces, each face containing a lattice.

Experiment 26 -- Use these strips to multiply 854 by 7. To do this, assemble the strips headed 8, 5 and 4 in this order next to the index strip, the 8 being nearest to the index. Run your finger down the index strip until you come to space 7. Your answer is given by the squares across from space 7, but you must read from right to left. Jot them down as you figure them out. The first number in your answer is 8. The second number is contained in the diagonal space to the left of 8, part being in the triangle above 8 and part in the lower triangle adjacent to it. The numbers 2 and 5 when added together give 7. In the next diagonal space you find 3 and 6, which when added together give 9. The remaining number is 5. Reading these numbers in reverse order, from left to right, you get the product 5978. Multiply 7 by 854 to see if the answer you obtained is correct.

Experiment 27 -- Try multiplying 854 by 6. The first number on the right is 4, the sum in the next diagonal space is 2, and that in the following diagonal is 11. Now, as in ordinary multiplication, put down 1 and carry the remaining 1 into the next space, getting 5 as your answer. Thus the product is 5124.

Experiment 28 -- To multiply 854 by 763, just get three partial products. First multiply 854 by 3, then by 6 and finally by 7. Write these partial products on paper as in ordinary multiplication by indenting one space more to the left each time, and add the columns. Check by multiplying it out on paper.

Experiment 29 -- Try a problem with the multiplicand containing zeros, as in 8054 times 6. To solve this, you will merely leave a space between the 8 and 5 strips, this strip representing the zero strip which would contain all zeros. Proceed as before, adding in two zeros in the zero strip into the diagonal spaces in which they fall.

Experiment 30 -- Try multiplying 7 by a number in which one of the digits is duplicated, as in 8545. Can you figure out how to do this?

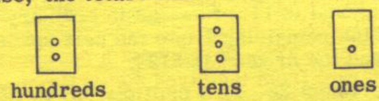
Napier's rods are not in common use today. For most of us it is more convenient to have our multiplication tables in our heads rather than on rods. But after the rods have been assembled, products can be read quickly. Many persons interested in mathematical curiosities have enjoyed "playing" with Napier's bones.

BINARY SYSTEM

Another system of computation used in electronic computers is based on the binary system, a method of counting by grouping.

The decimal system as you know is based on ten, or groups of ten. The binary system uses two as the base.

When man first began to understand the significance of numbers, his fingers were a convenient means of counting. However, when numbers became larger and transactions more complex, fingers were not enough to do calculations and pebbles were substituted. Pebbles were arranged in piles in grooves in the sand. In the right hand groove pebbles were placed to indicate a single unit. When ten pebbles were counted into the groove, a single pebble was placed in a groove to the left to indicate the ten pebbles in the first groove. When ten pebbles were counted in the second groove, a single pebble was placed into a third groove to the left to represent the ten pebbles in the middle. Thus, if a tradesman had 231 pieces of merchandise, the total number was shown in pebbles as follows:



As the pebble is moved to the left it is in actuality multiplied by 10.

How would the number 578 be represented by the above system?

The first calculating machine was based on ten or the decimal system but when electricity was used for computing machines, the binary system was more convenient because electrical systems have just two states, on and off, and a much faster, more efficient system was developed using base two.

Scientists devised the system of using only two numerals, 1 to represent on and 0 to represent off, or no current. Practically all electronic computers use the binary system.

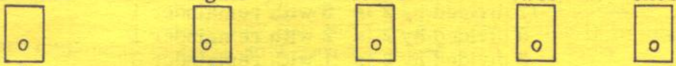
The binary system is not new and has been used since about

4,000 years ago and is still in use in some primitive tribes.

It is easy to understand the binary system if you will think in terms of two instead of ten. The numeral 1, and the symbol for zero are the tools with which you work.

In the binary system, each of the pebbles as it is shifted to the left is multiplied by two instead of ten as in the decimal system. Thus,

two x two x two x two	two x two x two	two x two	one x two	one
sixteens	eights	fours	twos	ones



1	1	1	1	1

Since each position to the left is represented by the symbol 1, the above figure is written 11111_{two} in the binary system, or 31 in the decimal system.

$$16 + 8 + 4 + 2 + 1 = 31$$

Thus, $100111_{two} = 1(\text{thirty-two}) + 0(\text{sixteen}) + 0(\text{eight}) + 1(\text{four}) + 1(\text{two}) + 1 = 32 + 0 + 0 + 4 + 2 + 1 = 39$

Experiment 31 -- Translate the binary numerals 1001110, 10101, 111001 into decimal numerals.

Experiment 32 -- To convert a decimal numeral into a binary numeral, subtract from the given decimal numeral the highest possible power of two. For example, decimal numeral 89 becomes 1011001_{two} by the following procedure:

89		
-64	(two x two x two x two x two x two, or two^6)	1
25		
	(two^5)	0
-16	(two^4)	1
9		
- 8	(two^3)	1
	(two^2)	0
	(two^1)	0
1		1

$$1(two^6) + 0(two^5) + 1(two^4) + 1(two^3) + 0(two^2) + 0(two) + 1 = 1011001_{two}$$

Binary numerals are longer than decimal numerals, as you can see. For instance, 2 in the decimal system is 10_{two} in the binary system. The binary numeral for the decimal 9 is 1001_{two}.

Another method of converting decimal numerals into binary numerals is to take the decimal numeral 1 and divide by 2 and note down the remainder as follows:

89 divided by 2 is 44 with remainder 1
44 divided by 2 is 22 with remainder 0
22 divided by 2 is 11 with remainder 0
11 divided by 2 is 5 with remainder 1
5 divided by 2 is 2 with remainder 1
2 divided by 2 is 1 with remainder 0
1 is remainder 1

Take the bottom numeral and read up to form the horizontal numeral 1011001_{two}. The answer is the same as that at which you arrived by the previous method: 1011001_{two}, or in the decimal system 64 + 0 + 16 + 8 + 0 + 0 + 1 = 89.

Convert decimal numerals 72, 95, and 47 into binary numerals.

Computers must add, subtract, multiply and divide to obtain answers to questions fed into them. By arranging switches and wiring them according to their needs, computers can be made to solve intricate mathematical problems or to answer questions to select persons for a particular assignment. A great variety of networks are possible with the computer answering only two questions at each step of the circuit, on or off, yes or no, 1 or 0.

<u>Experiment 33</u> -- Addition in the binary system is easier than adding in the decimal system. Thus:

$$\begin{array}{rr} 1001_{two} & (9) \\ +\ 101_{two} & (5) \\ \hline 1110_{two} & (14) \end{array}$$

1 + 1 in the far right column is equal to 10. Write down the 0 and carry the 1 to the next column. In the second column 0 + 0 is 0. Add the 1 from the previous column and write down 1. Add the other two columns and write down 1 in each. The answer 1110_{two} is equivalent to 8 + 4 + 2 + 0 or 14 in the decimal system.

Convert to the binary system and add 16 + 8; 25 + 13; 63 + 42.

Experiment 34 -- To subtract using the same numerals as in the above example, we get

$$\begin{array}{r} 1001_{two} \\ -101_{two} \\ \hline 100_{two} \end{array} \quad (4)$$

In subtracting, 1 - 1 = 0 in the first column at the right. 0 - 0 = 0 in the second column. In the next column you must regroup the eight into two groups of fours since you have no fours and place them in the fours column. Subtract one group of four and you still have one left so write down 1. Since you have regrouped the eight from the last column you now have a 0 there instead of the 1, and the answer is 100_{two}, or decimal 4. First translating the figures into binary numerals, subtract 36 - 24; 72 - 12; 18 - 5. Check your results.

Experiment 35 -- Division of binary numerals also is not difficult. In fact, it is simpler than with the decimal system since it is never necessary to try multiples of the divisor to find the largest number that will give a positive answer when subtracted. Divide 1010100_{two} by 100_{two}.

$$\begin{array}{rrl} & 10101_{two} & (21) \text{ Quotient} \\ \text{Divisor } (4) \ 100_{two} & \overline{)1010100_{two}} & (84) \text{ Dividend} \\ & \underline{100} & \\ & 101 & \\ & \underline{100} & \\ & 100 & \\ & \underline{100} & \end{array}$$

Another example demonstrates the result when a whole number is not obtained in the quotient.

$$\begin{array}{rl} & 1_{two} \\ 11_{two} & \overline{)100_{two}} \\ & \underline{11} \\ & 1 \text{ (remainder)} \end{array}$$

The answer to this problem is 1 1/3 in the decimal system.

When dividing in the decimal system, digits to the right of the decimal point are figured to the powers of one-tenth. In the binary system, the places to the right of the binary point represent the powers of one-half; i.e., one-half, one-half^2, one-half^3, one-half^4......, or one-half, one-fourth, one-eighth, one-sixteenth ......

Thus when dividing 8 into 5 by the binary method, the following result is obtained:

$$\begin{array}{r} 0.101_{two} \\ 1000_{two} \overline{)\,101.000_{two}} \\ \underline{100\ 0} \\ 1\ 000 \\ \underline{1\ 000} \end{array}$$

$$0.101_{two} = 1/2 + 0 + 1/8 = 5/8$$

Divide 11100_{two} by 101_{two}; 10110_{two} by 1000_{two}; 1110_{two} by 10000_{two}.

Experiment 36 -- To multiply, proceed as you would in the decimal system.

$$\begin{array}{rl} 1010_{two} & (10) \\ \times\ 101_{two} & (\ 5) \\ \hline 1010 & \\ 0000\ & \\ 1010\ \ & \\ \hline 110010_{two} & (50) \end{array}$$

In the fourth column from the right, the sum of 1 + 0 + 1 = 0 because there you have two groups of eight which are equal to one group of sixteen. This group of sixteen is placed in the sixteen column (fifth from the right) making the sum there 1.

Multiply in the binary system: 36 x 24; 72 x 12; 18 x 5. Check answers.

There are various ways in which the arithmetic processes can be done by computers according to their design, but here we have shown the methods most closely related to the decimal system.

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