A lucid explanation of the use of the Slide Rule for beginners in Science and Engineering which caters adequately for the trigonometrical needs of the sixth form.

It has been tested at SANDHURST and at WELBECK COLLEGE and is particularly suited to the needs of the Technical branches of the Army.

PRICE

3/6

Published by
THE ENGLISH UNIVERSITIES PRESS LTD.
LONDON

THE SLIDE RULE — T. G. C. WARD & G. W. BLAKEY

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for Students of Science and Engineering

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ENGLISH UNIVERSITIES PRESS LTD.
IN THE CITY OF
LONDON

First printed 1955

PRINTED IN ENGLAND
FOR THE ENGLISH UNIVERSITIES PRESS LTD.
BY ELLIOTT BROS. AND YEOMAN, LTD., LIVERPOOL

FOREWORD

ALTHOUGH the routine drudgery of calculations in elementary Arithmetic and Trigonometry by means of four figure tables has a disciplinary value for the fifth former which is certainly not underestimated by the authors, there yet comes a time when a sixth form beginner in Science and Engineering needs to be freed from laborious calculation, as far as possible, in order to concentrate on Principles and Methods.

A reasonable ability to use a Slide Rule is then of great value.

Of the many excellent books of instruction on the Slide Rule the authors have noted, in general, that either they are written primarily for the engineer, and hence over-detailed for the schoolboy, or else they are of the "popular" type and do not cater adequately for the trigonometrical needs of a sixth form.

This book, though small in content, will, the authors hope, fill the school need.

It has already been tested at SANDHURST and at WELBECK COLLEGE, and although it is particularly suited to the needs of the Technical branches of the Army, the authors believe it may be of equal value to science and mathematical VIth forms of all schools.

Although much care has been taken to check the answers, the authors will be grateful for correction of any errors which may have been made.

T.G.C.W.
G.W.B.

ACKNOWLEDGMENT

We gratefully acknowledge the courtesy of the Commandant, The Royal Military Academy, Sandhurst, Major-General R. G. S. Hobbs, D.S.O., O.B.E., and his Director of Studies, T. S. J. Anderson, Esq., C.B.E., T.D., M.A., who kindly allowed us to use examples from former question papers at the Academy.

We are also deeply indebted to G. R. Sisson, Esq., O.B.E., M.A., Head of the Mathematical Department at Sandhurst, first for his encouragement, without which the book would not have been written, and secondly for his valuable suggestions and criticisms during the course of the work.

We take the opportunity of thanking those of our colleagues, both at Sandhurst and Welbeck College, who have offered criticism and help.

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PART I

THE SLIDE RULE

CHAPTER I

Mechanical Aids

The slow steady growth of mathematical lore has often been accompanied by mechanical aids, to reduce the labour of calculations, to assist visually (as the square and compasses in Geometry) and further to illustrate new ideas and principles.

As a simple example of mechanical calculation consider a pair of ordinary straight edge rulers placed side by side (as in Fig. 1), with their figure marks exactly opposite.

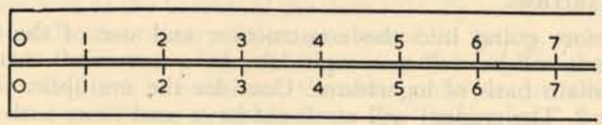


Fig. 1

If the lower be now moved along so that its "beginning" (i.e. the cypher "0") be placed opposite the cypher "2" of the other, then the lower may be used to add any number to "2." For instance, if now the finger be run along the lower to the mark "3" then opposite thereto on the upper will be read the figure "5." This is a mechanical method of performing the simple addition "2+3=5."

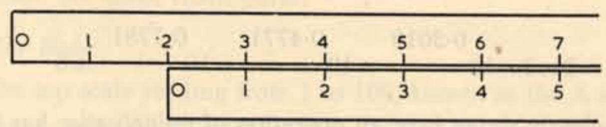


Fig. 2

Further if the rulers be made long enough, the process may be used to add any two numbers together and find the answer as the reader will readily see. If the two numbers are not integral then the subdivisions of the two straight edges will have to be sufficiently small to enable the numbers to be read accurately, but this is a matter of detail which does not interfere with the simple principle.

In the example the mental work involved is sufficiently simple to make such an "adding machine" quite unnecessary.

However, when the problem of multiplication is considered, it will be realised that the mental work involved in multiplying is of a much harder order, and it is at this stage that a mechanical aid is of value.

This mechanical aid is called a SLIDE RULE.

Logarithms

Before going into the construction and use of the Slide Rule it will be well to recapitulate, by a numerical example, the main basis of logarithms. Consider the multiplication of 2 by 3. The student will no doubt have used some such form as:—

No.		*Log.*
2	→	0·3010
3	→	0·4771
6	←	0·7781

This should be thought of in the more fundamental form

$$2\times3=10^{0\cdot3010}\times10^{0\cdot4771}=10^{0\cdot7781}=6$$

and clearly shows how an operation of *multiplication* has been transformed with aid of tables into one of *addition.*

Returning now to our simple adding machine: if instead of 2 (inches) on the ruler we had a distance of 0·3010″ (i.e. if the number were a measure of *log 2* instead of 2 etc. then the same operation that we performed before would now yield the answer 6.

Equally well the "2" could be replaced by any length provided the "3" were of such a length that the ratio

$$\frac{\text{length for "2"}}{\text{length for "3"}}=\frac{\log 2}{\log 3}=\frac{0\cdot3010}{0\cdot4771}$$

i.e. the length for 2 might well be 3·010″
and then the length for 3 would have to be 4·771″
and the multiplication, mechanically, would give
2×3 = that number whose log is represented by (3·010″ + 4·771″) = 7·781″, i.e. the number 6.

This, basically, is the principle of the slide rule.

Now look at the bottom of the slide rule. By actual measurement with an ordinary ruler, graduated in inches, you will find that the distance from the beginning of the figures marked on the scale are in each case the same multiple (usually 10) of their logarithms, in inches from the "beginning," usually called the INDEX of the scale.

(Note.—Some rulers may be "25 cm" rather than "10 inch" rulers; in this case the MEASURE will be in different units, the PRINCIPLE will be the same.)

Description of the Slide Rule

There are three main parts:

I. The Body

The top scale reading from 1 to 100, known as the A scale.

The bottom scale reading from 1 to 10, known as the D scale.

II. The Slide

Having along its top edge the B scale, a facsimile of the A scale, and along its bottom edge the C scale, a facsimile of the D scale. On its reverse side are various trigonometrical scales to be described later.

III. The Cursor

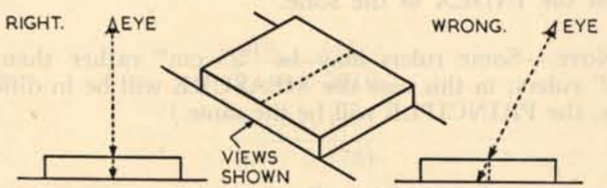
A movable "window" which can slide along the whole length of the body, having a hair line inscribed upon it perpendicular to the scales. The cursor enables figures to be read and to be placed opposite to one another with accuracy. This last operation is known as "setting" the slide rule.

The whole instrument, though stoutly constructed, is liable to be thrown out of true by careless or clumsy handling. It should be kept in its case when not actually in use.

Two important points in using the instrument are:—

(*a*) To seek a comfortable position in a good light.

(*b*) To maintain the gaze perpendicular to the cursor and the faces. The diagrams show the result of looking askew.



The spring inside the frame of the cursor should be placed so that it is at the top of the body of the rule.

Drill

The secret of success in using the slide rule, exactly as

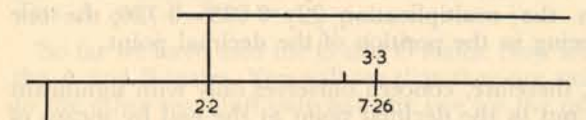
for other instruments, lies in continuous drill. Ten minutes a day for a week is worth far more than two hours on only one day of that week. So important is this aspect that you will find at the end of every set of exercises, a reminder of the fact.

Use

Let us now examine the C and D scales in more detail. For our previous example ($2 \times 3 = 6$) we may try out the appropriate settings once more.

Shown diagrammatically:—

Now suppose a little variation be attempted, e.g. $2{\cdot}2 \times 3{\cdot}3$. We observe that the space between 2 and 3 is divided into ten main divisions, each with five sub-divisions; similarly for the space between 3 and 4; and hence the setting becomes



the "·2" being 2 of the main divisions,

where the answer is easily seen to be between 7·2 and 7·3; but it is a matter of approximation by eye to see that the cursor is 6/10 of the distance between 7·2 and 7·3 (only 5/10 of the way being marked).

Initially the student should aim at 1% accuracy; but a little practice should enable him to reach the full "slide rule accuracy" of $\frac{1}{3}$%.

EXERCISE I

Simple Multiplication

1. 1·3×7·2
2. 3·9×2·3
3. 4·25×2·15
4. 8·43×1·02
5. 1·12×1·53
6. 5·43×1·49
7. 6·34×1·08
8. 1·135×3·04
9. 2·76×3·29
10. 1·023×8·77

Remember: "10 minutes a day."

Position of the Decimal Point

You will now notice that the multiplication cited above, i.e., 2·2×3·3=7·26, is in all respects save one the same problem as the multiplication 22×0·033=0·726, the one difference being in the position of the decimal point.

We shall, therefore, concern ourselves only with significant figures and put in the decimal point at the end by means of a rough check; e.g., in the example 22×0·033, we set as if the figures were simply 22 (or 2·2) and 33 (or 3·3) and read off the answer as the significant figures "726."

Now a rough approximation gives 20×0·03=0·6. Hence the answer (which, apart from the decimal point might have been 7260000,726,7·26,0·726,0·00000726, etc.) is now seen to be 0·726. The approximation can be very rough indeed. Suppose, for example, we took the "22" as 30 and the "0·033" as 3/100, then the approximation gives 90/100, or 0·9, and

of the various possibilities it is obvious that 0·726 is much nearer to 0·9 than 7·26, say, or 0·0726.

EXERCISE II

Simple multiplication

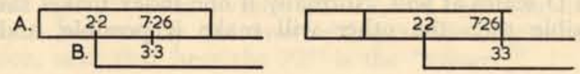
1. 12·7×3·71
2. 1·64×12·6
3. 0·0243×16·2
4. 148×112
5. 46·7×0·0212
6. 64·5×0·152
7. 0·0109×706
8. 0·0361×0·0216
9. 169×137
10. 0·00536×0·0142

Remember: "10 minutes a day."

The A and B Scales

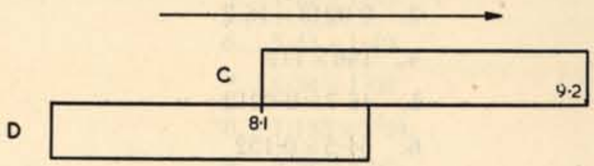
So far we have used the C and D scales. Now let us examine the A and B scales. You will see that they are exactly similar to the other two; but only half the size for the range 1 to 10, having the identical continuation for 10 to 100. This fact emphasises how immaterial to the *mechanical* handling is the position of the decimal point.

For, trying out the example of the last paragraph, we see that the alternative settings shown give the *same significant figures*.



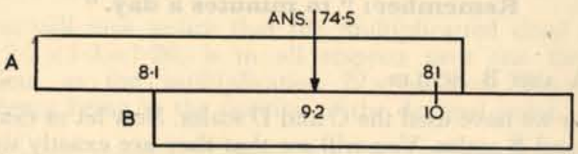
The work with the A and B scales will not normally be so accurate as that with the C and D scales. The A and B scales, of course, have an important use as we shall see later, but for simple multiplication (and division) we normally use the C and D scales. However, many will prefer to use the A and B scales for continued multiplication and division.

The point of examining the scales will now be appreciated if we try another example on the C and D scales, viz., $8{\cdot}1\times9{\cdot}2$.



If we try tô apply the method so far learnt we are unable to slide the cursor over 9·2 as it will then be "off the scale."

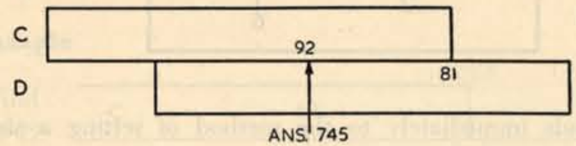
However, trying it on the A and B scales we can deal with it comfortably enough as in the diagram.



We can, however, see that by setting the 10 mark (i.e., the equivalent of a *right hand index* for a 1—10 scale) opposite to 81 (on right half A scale) and moving the cursor LEFTWARDS over 9·2 we get the identical setting as before.

Hence it becomes clear that we can use *either* index of the C and D scales at will. Normally if one index makes the task impossible then the other will make it possible, and vice versa.

Returning, therefore, to the C and D scales, we once more make the setting for $8{\cdot}1\times9{\cdot}2$ as in the diagram



the requisite approximation (10×8) giving 80, and showing the answer as 74·5 (in fact, the answer is 74·52, so our answer is well within $\frac{1}{3}\%$, i.e. 2 in approximately 7000).

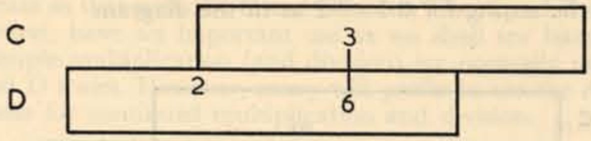
EXERCISE III

1. $3{\cdot}71\times4{\cdot}62$
2. $0{\cdot}741\times0{\cdot}0432$
3. $39{\cdot}4\times0{\cdot}628$
4. $705\times0{\cdot}0915$
5. $642\times0{\cdot}0735$
6. $0{\cdot}195\times0{\cdot}0822$
7. $121{\cdot}9\times0{\cdot}037$
8. $5{\cdot}01\times0{\cdot}0701$
9. $0{\cdot}00378\times79{\cdot}5$
10. $35{\cdot}7\times29{\cdot}3$

Remember: "10 minutes a day."

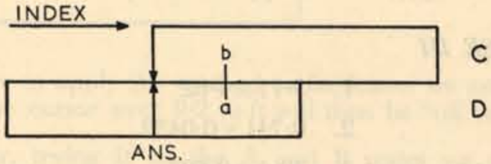
Division

The example $2\times3=6$ can, of course, be looked at in a different light; by cross division $6/3=2$, and hence the division of 6 by 3 has exactly the same setting as the original multiplication, save that here the "2" is the "answer,"



and leads immediately to the method of setting a simple division.

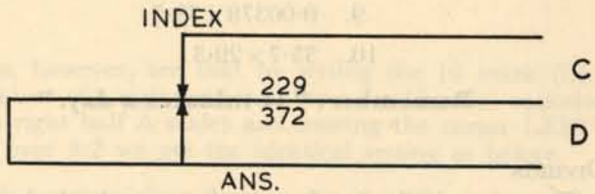
Generally a/b is "set" as in the sketch:—



the answer being read on the D scale opposite the index of the C scale (either index; if one works the other doesn't, of course): once again the position of the decimal point is a matter of rough approximation.

Example

Find $\frac{3{\cdot}72}{22{\cdot}9}$



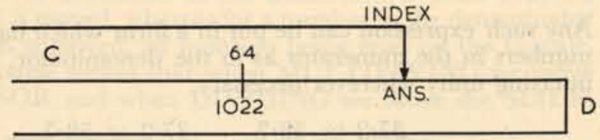
The setting shows the significant figures 1625 against the index (in this case the left hand index).

Approximation gives 4/20, or approximately 0·2.

Hence $\frac{3{\cdot}72}{22{\cdot}9} = 0{\cdot}1625$

Example

Find $\frac{10{\cdot}22}{640}$



The setting shows the significant figures 16 against the right hand index.

Approximation gives 10/500 = 0·02
Therefore 10·22/640 = 0.016

EXERCISE IV

Simple division

1. 17·21÷5·23
2. 72·7÷0·00169
3. 726÷3·58
4. 2·468÷0·0536
5. 3·05÷695
6. 158·5÷0·124
7. 374÷12·7
8. 4·55÷785
9. 196·5÷0·108
10. 31·9÷0·736
11. π ÷16·43
12. 23÷955
13. 0·0773÷0·00927

Remember: "10 minutes a day."

Continued Multiplication and Division

We are now in a position to deal with the calculation of expressions such as:—

$$\frac{37{\cdot}2 \times 58{\cdot}7 \times 293 \times \text{etc.}}{15{\cdot}5 \times 172 \times 1991 \times \text{etc.}}$$

Any such expression can be put in a form which has as many numbers in the numerator as in the denominator, simply by inserting unity wherever necessary.

e.g., $$\frac{37{\cdot}2 \times 58{\cdot}7}{15{\cdot}5} = \frac{37{\cdot}2 \times 58{\cdot}7}{15{\cdot}5 \times 1}$$

and e.g., $$\frac{1}{15{\cdot}5 \times 172 \times 1991} = \frac{1 \times 1 \times 1}{15{\cdot}5 \times 172 \times 1991}$$

Hence the rule can be given and applied, quite generally, for expressions of the type

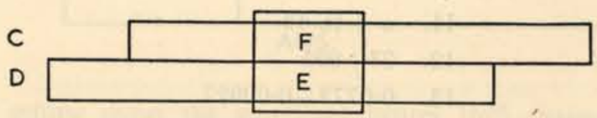
$$\frac{E \times G \times \ldots . \times M}{F \times H \times \ldots . \times N}$$

where there are the same number of factors in the numerator and denominator.

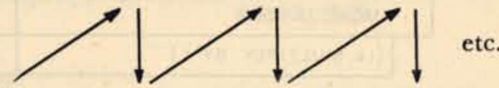
Setting

E is set on the *D* scale.
This is the last time the D scale is used until the answer is read thereon.

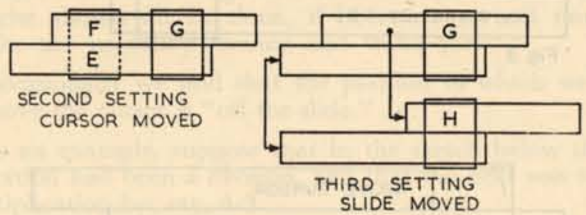
F, on the C scale, is set opposite to E.



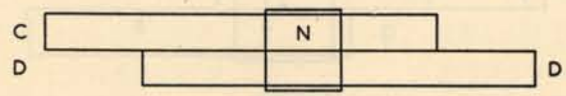
Thereafter the numbers are dealt with alternatively (numerator — denominator) according to diagram



and whenever a number in the numerator is dealt with the CURSOR is moved, whereas for a number in the denominator the SLIDE is moved, *all* numbers being read on the C scale. This, of course means that when MULTIPLYING we move the CURSOR and when DIVIDING we move the SLIDE.



The final operation will, of course, be a move of the *slide*.



The answer is then read on the D scale opposite the index of the C scale.

An example is now shown in full, stage by stage:—

$$\frac{235}{24 \times 7{\cdot}8 \times 90} \text{ (Rewrite) } = \frac{235 \times 1 \times 1}{24 \times 7{\cdot}8 \times 90}$$

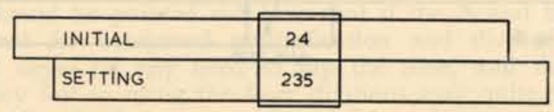


Fig. I

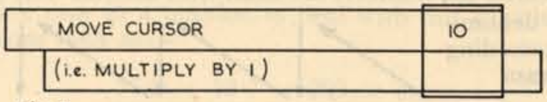


Fig. 2

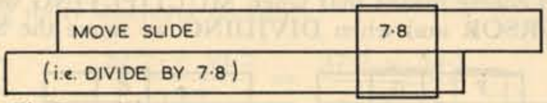


Fig. 3

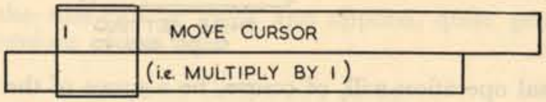


Fig. 4

Fig. 5

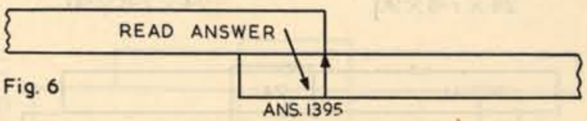


Fig. 6

Hence the significant figures are read as 1395.

Once more the rough approximation $\dfrac{250}{25\times5\times100}$ gives $1/50=0{\cdot}02$.

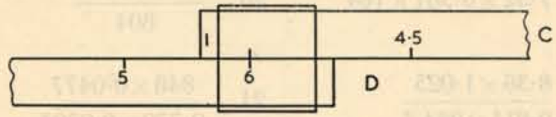
Therefore $\dfrac{235}{24\times7{\cdot}8\times90}=0{\cdot}01395$

This rule must be learnt *by heart* so that calculations may be quite automatic.

For this reason the large number of examples in the following exercise should all be done, if necessary several times, to obtain the necessary fluency and technique.

Occasionally we find that the position to which we want to move the cursor is "off the slide."

As an example, suppose that in the sketch below the last operation had been a division, and that the next was to be a multiplication by, say, 4·5.



Operations are "suspended" while the slide (C) is moved leftward "through itself." This is done by putting the Cursor over the left index of the SLIDE (C) and moving the slide leftwards until its right index is under the CURSOR.

In fact all that has been done is to perform an interim division by 10, and since the decimal point is not in question until the end, this will in no way affect the significant figures nor indeed the answer. We call this process "SLIPPING THE SLIDE."

It should be pointed out here that if the A and B scales are used for continued multiplication and division there should never be any need to slip the slide, and that any accuracy lost in using the finer divisions may quite well be made up by the fewer movements of slide and cursor necessary.

EXERCISE V.

Continued Multiplication and Division

1. $\dfrac{15{\cdot}62\times 0{\cdot}0826}{43{\cdot}7}$

2. $\dfrac{13{\cdot}3\times 495}{67{\cdot}5\times 0{\cdot}722}$

3. $\dfrac{63{\cdot}8\times 1230}{15{\cdot}3}$

4. $\dfrac{71{\cdot}2}{4{\cdot}62\times 0{\cdot}0137}$

5. $7{\cdot}32\times 0{\cdot}361\times 164$

6. $\dfrac{8{\cdot}36\times 1{\cdot}025}{0{\cdot}815\times 234{\cdot}5}$

7. $\dfrac{13{\cdot}2\times 243\times 35{\cdot}4}{4{\cdot}7\times 17{\cdot}4\times 82{\cdot}3}$

8. $5{\cdot}62\times 0{\cdot}231\times 981$

9. $\dfrac{21500\times 72{\cdot}6}{163}$

10. $\dfrac{0{\cdot}425}{9{\cdot}36\times 0{\cdot}0217}$

16. $\dfrac{10{\cdot}08\times 9{\cdot}45}{0{\cdot}055\times 448}$

17. $\dfrac{69{\cdot}7\times 0{\cdot}425}{1{\cdot}83\times 139}$

18. $\dfrac{43{\cdot}1\times 48{\cdot}3}{0{\cdot}477\times 175}$

19. $\dfrac{933}{45{\cdot}6\times 8{\cdot}95}$

20. $\dfrac{75{\cdot}8\times 27{\cdot}9}{804}$

21. $\dfrac{848\times 0{\cdot}0477}{0{\cdot}770\times 0{\cdot}0707}$

22. $\dfrac{0{\cdot}497\times 892}{0{\cdot}359}$

23. $\dfrac{1}{0{\cdot}945\times 0{\cdot}00588\times 922}$

24. $\dfrac{45{\cdot}8\times 7{\cdot}06}{753\times 0{\cdot}931}$

25. $\dfrac{0{\cdot}165\times 0{\cdot}05309}{4{\cdot}60}$

11. $\dfrac{7{\cdot}92\times 1{\cdot}525}{0{\cdot}93\times 212{\cdot}5}$

12. $\dfrac{1{\cdot}82\times 0{\cdot}64}{52{\cdot}7}$

13. $\dfrac{2{\cdot}47}{21{\cdot}4\times 0{\cdot}98}$

14. $\dfrac{0{\cdot}153\times 7{\cdot}62}{42{\cdot}1\times 89{\cdot}1}$

15. $\dfrac{12{\cdot}6\times 545}{46{\cdot}3\times 0{\cdot}926}$

26. $\dfrac{0{\cdot}734\times 50{\cdot}8}{22{\cdot}5\times 316}$

27. $\dfrac{0{\cdot}0354\times 86{\cdot}2\times 4{\cdot}76}{109{\cdot}2}$

28. $\dfrac{781}{15{\cdot}6\times 432\times 0{\cdot}026}$

29. $\dfrac{0{\cdot}00824\times 0{\cdot}097}{15{\cdot}2\times 0{\cdot}0634\times 0{\cdot}0743}$

30. $\dfrac{21{\cdot}4\times 0{\cdot}386\times 17{\cdot}2}{98{\cdot}1\times 520\times 0{\cdot}00764}$

Remember: "10 minutes a day."

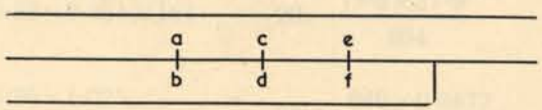
CHAPTER II

Proportion

The reader is now in a position to appreciate the following most useful and important property of the slide rule:—

$$\text{If } a/b = c/d = e/f = \ldots \text{ etc.,}$$

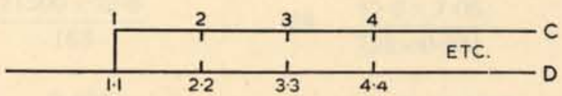
then, on either (A & B) or (C & D) the setting of one ratio (e.g., the setting of c opposite to d, say) will automatically "set" the rest.



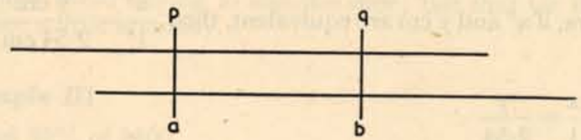
To see this, set 1 on the C scale opposite 1·1 on the D scale:

then the set of equal ratios $\frac{1}{1\cdot 1} = \frac{2}{2\cdot 2} = \frac{3}{3\cdot 3} = \frac{4}{4\cdot 4} =$ etc.

is completely exhibited on the C & D scales.



In general we see that a setting



means

"Dist. for log q—Dist. for log p" = "Dist. for log b—Dist. for log a."

That is q/p=b/a, or p/a=q/b.

Thus, if any simple problem, involving the finding of an unknown quantity, can be put into a RATIO form, then the solution becomes only a matter of reading.

Example I

Solve $x = \frac{1\cdot 95 \times 2\cdot 515}{\pi}$

This may be put in ratio form as $\frac{x}{1\cdot 95} = \frac{2\cdot 515}{\pi}$

and the setting $\frac{2\cdot 515}{\pi}$ automatically sets $\frac{x}{1\cdot 95}$

thus:— $\frac{x}{1\cdot 95} \quad \frac{2\cdot 115}{\pi} \quad \frac{C}{D}$

enabling x to be read off as 1·56.

Example II

Given 1″ = 2·54 cm, make a setting on the (C & D) scales to enable inches and centimetre equivalents to be read off.

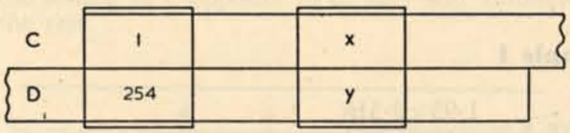
Here, if x″ and y cm are equivalent, then $\frac{x''}{1''} = \frac{y\text{ cm}}{2{\cdot}54\text{ cm}}$

or $\frac{x}{1} = \frac{y}{2{\cdot}54}$

$\therefore \frac{x}{y} = \frac{1}{2{\cdot}54}$

and the right hand ratio "sets" any pair of x and y

C being the "inch" scale
D the "centimetre" scale

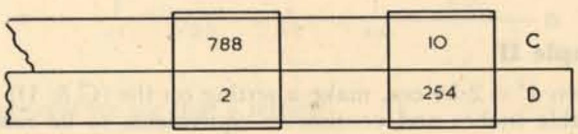


For instance, on the above setting when x = 32·5
y = 82·5

∴ 32·5″ converts to 82·5 cm.

Similarly, 98·4 cm is equivalent to 38·75 inches.
For significant figures "off the scale" we have to "slip" the slide, i.e., move the right hand index into the place set for the left hand index.

In this example, to find the number of inches in 2·0 cm we must "slip" the slide and read, opposite 2 on the D scale, 0·788.



It should again be noted that by using the A and B scales we could avoid having to slip the slide; but that we should sacrifice a little accuracy.

Example III

Find 35% of 1400.

Here, if the answer is x, then $\frac{x}{1400} = \frac{35}{100}$ or $\frac{x}{35} = \frac{1400}{100}$

and in any of the possible settings x is 490.

e.g. $\frac{C\ 14}{D\ 1}\ \frac{49}{35}$ or $\frac{C\ 35}{D\ 1}\ \frac{49}{14}$ or $\frac{1\ \ 14\ C}{35\ \ 49\ D}$ etc.

Example IV

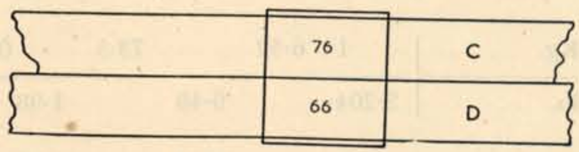
Complete the table of equivalence:—

Statute miles	76	17·5	36·1	56·4		32·1
Nautical miles	66				102	

Here s statute miles is equivalent to n nautical miles.

Where $\frac{76}{66} = \frac{s}{n}$ (all pairs of s and n)

the setting gives



C being the "stat. mile" scale
D being the "naut. mile" scale

and the gaps are filled in by reading direct,

e.g.,	17·5	on	C	gives	15·2	on	D
	36·1	,,	C	,,	31·5	,,	D
	56·4	,,	C	,,	48·9	,,	D
	102	,,	D	,,	117·5	,,	C
	32·1	,,	C	,,	27·85	,,	D

We shall come to further illustrations when dealing with the trigonometrical scales.

EXERCISE VI

Complete the following tables:—

1.	m.p.h.	60	22·5	47·8	92·1	107	362
	ft. per sec.	88					

2.	Feet	3·281	625	83·6	421	2560	1348
	Metres	1					

3.	m.p.h.	60	92·5	3·62	450		
	ft. per sec.	88				625	29·4

4.	Kg.	1	6·52		73·5		0·3650
	lb.	2·204		9·48		1·98	

5.	cu. ft.	0·16		0·164		84·5	
	Litres	4·54	88		216		0·9

6.	sq. in.	1		0·0137		63·8	
	sq. cm.	6·45	11·2		0·156		258

7.	Kilowatts	1	0·973		39·4		629
	Horse power	1·341		18·6		204	

8.	Mtrs. per sec.	1	8·35		19·7		450
	m.p.h.	2·24		12·4		65·2	

9.	Knots	66		6·71		29·7	
	m.p.h.	76	3·24		15·4		49·3

10.	Miles	1		6·32		49·3	
	Km.	1·609	0·157		15·6		7380

11.	Ounces	1		91·3		84·3	
	Grammes	28·35	12·6		420		1000

12.	Watt-hours	1	0·073		59·5		840
	B.Th.U.	3·41		16·2		29·6	

Remember: "10 minutes a day."

CHAPTER III

Squares and Square Roots

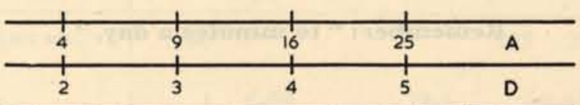
Let us now remove the slide altogether and fix our attention on the relationship between the A and D scales.

Observe that the physical distance of a number, say 9, on the A scale, being proportional to its logarithm, will be double the physical distance of 3 (its square root) on the same scale. But since the scale of the D scale is twice as big as that of the A scale, then the physical distance of 3 *on the D scale* will also be double the physical distance of 3 on the A scale.

Hence directly opposite to 9 on the A scale will lie 3 (its square root) on the D scale.

Squares

Hence, to find the square of any number all we need do is to set that number on the D scale and read the answer opposite on the A scale. Trial with simple numbers will convince the reader.



The decimal point is, of course, fixed as before by approximation, e.g. $(2{\cdot}15)^2 = 4{\cdot}62$

But $21{\cdot}5^2 = 462$

and $(0{\cdot}123)^2 = 0{\cdot}151$

EXERCISE VII

1. $23{\cdot}7^2$
2. $0{\cdot}312^2$
3. $31{\cdot}5^2$
4. $0{\cdot}0192^2$
5. $43{\cdot}5^2$
6. $0{\cdot}0234^2$
7. $3{\cdot}95^2$
8. $0{\cdot}0155^2$
9. $52{\cdot}5^2$
10. $0{\cdot}0324^2$

Remember: "10 minutes a day."

Square Roots

Since

$$\text{if } y = x^2 \text{ then } x = \sqrt{y}$$

the problem of finding square roots is plainly the reverse of finding squares. It would appear simple to put the cursor over the required number on the A scale and to read the answer on the D scale. There is, however, a difficulty.

Suppose we wish to find the square root of 9·25. There are two places on the A scale whereon the cursor may be placed for this series of significant figures: which shall we choose?

In this example the left hand position gives a reading of 304, and the right hand position a reading of 962, so that by approximation we can see which to choose, i.e., the left side, to get an answer of 3·04.

On the other hand, had the number been 92·5, then the approximation would have clearly indicated the right hand reading as 9·62.

This difficulty arises from the fact that numbers with the same significant figures do not necessarily have the same significant figures in their square roots. But their square roots do at any rate fall into one of two classes, which we will call the left class and the right class.

B

To see this, observe that $(3{\cdot}04)^2 = 9{\cdot}25$ and $(9{\cdot}62)^2 = 92{\cdot}5$

$(30{\cdot}4)^2 = 925$ and $(96{\cdot}2)^2 = 9250$

$(0{\cdot}304)^2 = 0{\cdot}0925$ and $(0{\cdot}962)^2 = 0{\cdot}925$

etc. etc.

the left class being numbers which are an *even* number of decimal places away from 9·25 and the right class being numbers which are an *odd* number of decimal places away from 9·25. It is convenient to think of the left class as being the class of 9·25 and the right class as being the class of 92·5.

To recall a process probably familiar to the reader in his original and elementary solving of square roots, the class will be immediately recognisable if the number is "marked off in pairs" from the decimal point,

e.g., if the number were $\sqrt{925000000}$; this, marked off, gives 9 25 00 00 00, showing that the square root belongs to the "9" or left class, whereas $\sqrt{0.00\ 00\ 00\ 92\ 5}$ manifestly belongs to the "92," or right, class.

Hence, $\sqrt{925{,}000{,}000} = 30{,}400$

and $\sqrt{0{\cdot}0000000925} = 0{\cdot}0000962$.

Hence, before finding a square root we first determine to which class it belongs and then take the appropriate half of the A scale to make the setting.

EXERCISE VIII

1. $\sqrt{5720}$
2. $\sqrt{0{\cdot}0425}$
3. $\sqrt{3950}$
4. $\sqrt{0{\cdot}0214}$
5. $\sqrt{23{\cdot}50}$
6. $\sqrt{3{\cdot}260}$
7. $\sqrt{0{\cdot}1952}$
8. $\sqrt{0{\cdot}000697}$
9. $\sqrt{2060}$
10. $\sqrt{0{\cdot}000758}$
11. $\sqrt{0{\cdot}89 \times 3{\cdot}43}$
12. $22{\cdot}5\sqrt{19{\cdot}6}$

13. $\sqrt{\dfrac{0{\cdot}159}{0{\cdot}0725}}$
14. $\dfrac{463}{\sqrt{117{\cdot}5}}$
15. Calculate t from the formula

$$t = 2\pi\sqrt{\frac{l}{g}}, \text{ where}$$

(a) $l = 207{\cdot}6$ $g = 981$

(b) $l = 136{\cdot}8$ $g = 32$

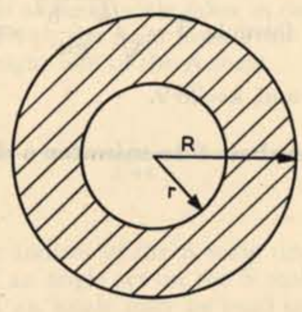
Remember: "10 minutes a day."

The Difference of Two Squares

It is often convenient to use a formula of algebra in computations involving numbers of the form $(a^2 - b^2)$.

The formula is $a^2 - b^2 = (a-b)(a+b)$ or, in words, the difference of the squares of two numbers is equal to the product of the sum and difference of the numbers.

For instance, when finding the annular area (the area shaded in the figure) between two concentric circles of radii R and r units



the area $= \pi\ (R^2 - r^2)$ sq. units of area and (by the formula above) $= \pi\ (R-r)\ (R+r)$ sq. units of area, a form much more suitable to slide rule computation.

e.g. If $\left.\begin{array}{l} R = 7{\cdot}324' \\ r = 5{\cdot}112' \end{array}\right\}$ then $\begin{array}{l} R+r = 12{\cdot}436' \\ R-r = 2{\cdot}212' \end{array}$

and Area $= \pi\,(12{\cdot}436')\,(2{\cdot}212') = 86{\cdot}5$ sq.′

The reader should verify that this is an easier method than squaring 7·324 and 5·112 separately, performing the subtraction and finally multiplying by π.

Unhappily no such formula exists for the SUM of two squares, and calculations such as $(3{\cdot}52)^2+(7{\cdot}33)^2$ have to be done the "hard way," by calculating each square separately and adding the results.

EXERCISE IX

1. $9{\cdot}78^2-2{\cdot}32^2$
2. $18{\cdot}42^2+9{\cdot}47^2$
3. $\pi\,(46{\cdot}2^2-29{\cdot}3^2)$
4. $\pi\,(7{\cdot}08^2-6{\cdot}32^2)$
5. $\pi\,(0{\cdot}592^2-0{\cdot}274^2)$
6. $\sqrt{47{\cdot}5^2-36{\cdot}5^2}$
7. From the formula $T=\sqrt{\dfrac{k^2+h^2}{gh}}$ calculate T when

$g=980$, $h=45{\cdot}7$ and $k=28{\cdot}9$.

Remember: "10 minutes a day."

PART II

THE SLIDE RULE APPLIED TO TRIGONOMETRY

CHAPTER IV

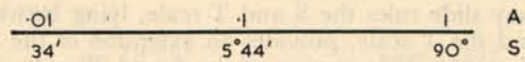
The Scales

On the reverse side of the slide will be found the S, or sine, Scale and the T, or tangent, Scale, the former being used in conjunction with the A Scale and the latter with the D scale.

The Sine Scale

You will notice that this runs from about 34′ (the sine of which is 0·01) to 90° (the sine of which is, of course, 1).

At the half-way mark is 5°44′ (the sine of which is 0·1), hence the left half of the A scale takes in only the values from sine 34′ to sine 5°44′, the sines from 5°45′ to 90° being compressed into the right half of the A scale.



By placing the indices of the S scale under those of the A scale, the sine of an angle set on the S scale may be read on the A scale; and an angle may be read on the S scale if its sine is set on the A scale. It will be clear that the requisite significant figures for the sines of angles on the left half must be preceded by 0·0——— and for those on the right half by 0·———, e.g., sin 3° = 0·0523 and sin 13° = 0·2250.

It will be further noticed that for angles greater than 60° it is difficult to obtain much accuracy in reading the sine. For angles smaller than 34′ we use the APPROXIMATE rule $\sin x = \frac{1}{10} \sin 10x$.

$$\text{e.g., } \sin 22' \simeq \frac{1}{10} \sin 220' \simeq \frac{1}{10} \sin 3°40' \simeq \text{etc.}$$

$$\text{or } \sin 2' \simeq \frac{1}{10} \sin 20' \simeq \frac{1}{100} \sin 200' \simeq \frac{1}{100} \sin 3°20' \simeq$$

etc.

This approximation is true for other values than multiples of 10, but the latter are obviously the most useful in practice.

We stress that this APPROXIMATE rule can only be used for SMALL angles. It would be patently *untrue*, for instance,

to say that $\sin 30° = \frac{1}{10} \sin 300°$, or that $\sin 30° = \frac{1}{3} \sin 90°$,

even approximately.

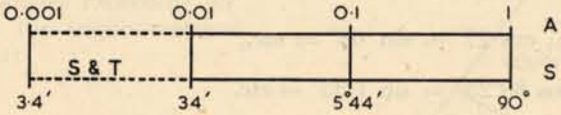
The S and T Scale

On many slide rules the S and T scale, lying between the S scale and the T scale, provides an extension of the S scale from 3·4′ to 34′. This portion of the S and T scale, indeed, may easily be constructed by dividing the readings of the S scale by ten from 34′ to 6° and inscribing the results underneath.

Many S and T scales run from 3·4′ to approximately 6°; but it is usually only necessary to use the portion between 3·4′ and 34′.

This portion of the S and T scale may be looked upon as

an extension to the left of the S scale for another power of ten, as illustrated in the diagram below.



For compactness the S and T scale is placed directly below the S scale. It is read, of course, in conjunction with the A scale.

Thus to find the sine of an angle between 3·4′ and 34′ we set the angle on the S and T scale and read the answer on the A scale, remembering that the answer will lie between 0·001 and 0·01.

Conversely, to find an angle whose sine lies between 0·001 and 0·01 we set the figures of the sine on the left hand half of the A scale and read the angle on the S and T scale.

It is useful to remember the following results:—

$\sin 3{\cdot}4' = 0{\cdot}001$
$\sin 34' = 0{\cdot}01$
$\sin 5°44' = 0{\cdot}1$
$\sin 90° = 1$

EXERCISE X(a)

1. sin 18°25′	7. $\sin^{-1} 0{\cdot}327$
2. sin 2°16′	8. $\sin^{-1} 0{\cdot}518$
3. sin 62°34′	9. $\sin^{-1} 0{\cdot}0724$
4. sin 21°06′	10. $\sin^{-1} 0{\cdot}00395$
5. sin 9′30″	11. $\sin^{-1} 0{\cdot}112$
6. sin 6′30″	12. $\sin^{-1} 0{\cdot}023$

Remember: "10 minutes a day."

The Cosine

The relation cosine θ = sine $(90-\theta)$ enables any cosine value to be read off,

e.g., $\cos 27° = \sin 63° =$ etc.,

and $\cos 88°25' = \sin 1°35' =$ etc.

EXERCISE X (b)

1. $\cos 61°30'$	5. $\cos 89°54'$
2. $\cos 32°15'$	6. $\cos^{-1} 0{\cdot}793$
3. $\cos 2°07'$	7. $\cos^{-1} 0{\cdot}826$
4. $\cos 89°10'$	8. $\cos^{-1} 0{\cdot}0243$

The Tangent Scale

Here you will notice that the scale runs from 5°44′ (whose tangent is approx. 0·1) to 45° (whose tangent is 1).

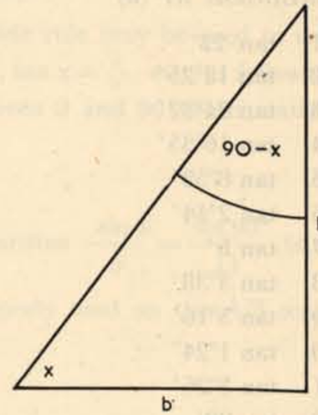
Thus if the indices of the T scale are set over those of the D scale we may read the tangent of an angle on the D scale by setting the angle on the T scale; conversely we may find an angle by setting its tangent on the D scale and reading the angle on the T scale. In both cases it is assumed that the angle lies between 5°44′ and 45°. The decimal point is fixed by remembering that for this range of angle the tangent lies between 0·1 and 1.

Thus, for example, $\tan 25° = 0{\cdot}466$.

For angles less than 5°44′ we use the fact that in such cases the tangent and the sine are very nearly the same; hence we switch to the A and S scales, e.g., $\tan 4° \simeq \sin 4° = 0{\cdot}699$.

or $\tan 2' \simeq \sin 2' \simeq \frac{1}{100} \sin 200' = \frac{1}{100} \sin 3°20' =$ etc.

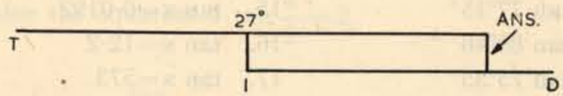
For angles exceeding 45° we use the trigonometrical fact that



$$\tan x = \frac{1}{\tan (90-x)}$$

since each = p/b in the diagram

Hence, we write $\tan 63° = \frac{1}{\tan 27°} = 1{\cdot}963$



In this example we need to realise that since tangents of angles between 5°44′ and 45° lie between 0·1 and 1, then tangents of angles greater than 45° must exceed 1.

Hence, $\tan 63° = \frac{1}{\tan 27°}$ (giving a reading of 1963 on the D scale) is, in fact, 1·963; and $\tan 83° = \frac{1}{\tan 7°} = 8{\cdot}14$.

EXERCISE XI (a)

1. tan 23°
2. tan 12°25′
3. tan 24°32′
4. tan 16°35′
5. tan 8°52′
6. tan 2°14′
7. tan 5′
8. tan 3°38′
9. tan 5°16′
10. tan 1°24′
11. tan 3°26′
12. tan 20′
13. tan 4°30′
14. tan 61°
15. tan 72°35′
16. tan 89°42′
17. tan 77°15′
18. tan 89°48′
19. tan 75°35′
20. tan 69°05′
21. tan 49°32′
22. tan 56°43′

EXERCISE XI (b)

In each case find the angle x between 0° and 90°.

1. tan x=0·46
2. tan x=3·20
3. cot x=1·75
4. tan x=2·18
5. tan x=2·67
6. tan x=0·395
7. cot x=0·345
8. tan x=3·41
9. tan x=2·45
10. cot x=0·294
11. tan x=0·34
12. tan x=0·445
13. cot x=3·333
14. tan x=0·0053
15. tan x=0·0192
16. tan x=12·2
17. tan x=573
18. cot x=17·3
19. cot x=143·2
20. cot x=0·0029

Remember: "10 minutes a day."

Examples

1. Use the formula $\sin\theta = 1-2\sin^2\frac{90-\theta}{2}$ to evaluate sin 80°.

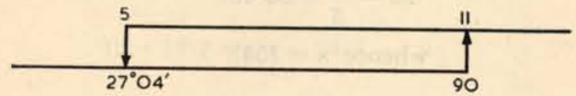
2. Use the formula $\sin\theta = 2\sin\frac{\theta}{2}\sin\left(90-\frac{\theta}{2}\right)$ to evaluate sin 80°.

THE USE OF THE RATIO PROPERTY

The ratio property of the slide rule may be used to solve examples of the type $\sin x = \frac{p}{q}$, $\tan x = \frac{r}{s}$, etc., as follows:—

1. "Find the value of x between 0 and 90° which satisfies the equation $\sin x = \frac{5}{11}$."

This equation may be re-written $\frac{\sin x}{5} = \frac{\sin 90°}{11}$ (since sin 90°=1) and the ratio property used on the A/S scales, the angle being read as 27°04′



2. Solve the equation $3\tan\frac{x}{2}=2$,

$$\text{re-writing as } \frac{\tan\frac{x}{2}}{2} = \frac{\tan 45°}{3} \quad (\text{since } \tan 45° = 1)$$

$$\text{gives } \frac{x}{2} = 33°41'$$

$$\text{or } x = 67°22'$$

N.B.—We cannot do any doubling until the LAST stage, for $\tan\frac{x}{2}$ is NOT $\frac{1}{2}$ tan x.

If, in the equation to be solved, the unknown tangent is greater than tan 45°, we proceed as follows:—

3. Solve the equation $4 \tan \frac{x}{3} = 5$ for values of x between 0° and 270° (i.e. values of $\frac{x}{3}$ between 0° and 90°).

Re-write the equation in the form

$$\frac{\tan\left(90° - \frac{x}{3}\right)}{4} = \frac{\tan 45°}{5}$$

From the slide rule we find that

$$90° - \frac{x}{3} = 38°40'$$

whence $x = 154°$.

EXERCISE XI (c)

1. $\sin x = \frac{4{\cdot}13}{12{\cdot}35}$

2. $\cos x = \frac{36{\cdot}8}{73{\cdot}6}$

3. $\sin x = \frac{3{\cdot}28}{5{\cdot}95}$

4. $19{\cdot}7 \tan x = 32{\cdot}2$

5. $\operatorname{cosec} x = \frac{68{\cdot}4}{47{\cdot}6}$

6. $72{\cdot}9 \cos x = 35{\cdot}4 \sin 31°30'$

7. $\sin x = \frac{7{\cdot}94}{11{\cdot}66}$

8. $\cot x = \frac{65}{28}$

9. $\sin x = \frac{43{\cdot}2 \sin 64}{93{\cdot}5}$

10. $\sin x = \frac{19{\cdot}7}{29{\cdot}7}$

11. $\tan x = \frac{29}{18{\cdot}5}$

12. $17{\cdot}2 \sec x = 24{\cdot}3$

13. $\tan \frac{A}{2} = \frac{5{\cdot}6}{4{\cdot}7} \tan 24°30'$

14. $\sin \frac{A}{2} = \frac{5{\cdot}12}{13{\cdot}75}$

15. $\tan \frac{x}{2} = \frac{2{\cdot}4}{26{\cdot}5}$

16. $4 \tan (3x) = 5$

17. $19 \tan \left(\frac{x}{2}\right) = 208$

18. $0{\cdot}631 \cot \left(\frac{x}{2}\right) = 0{\cdot}296$

19. $0{\cdot}734 \tan (2x) = 1{\cdot}93$

20. $3{\cdot}78 \cot\left(\frac{x}{3}\right) = 0{\cdot}026$

21. $\cos^{-1} \frac{31}{47}$

22. $\cos^{-1} \frac{36{\cdot}7}{83{\cdot}4}$

23. $\cos^{-1} \frac{43{\cdot}6}{104{\cdot}3}$

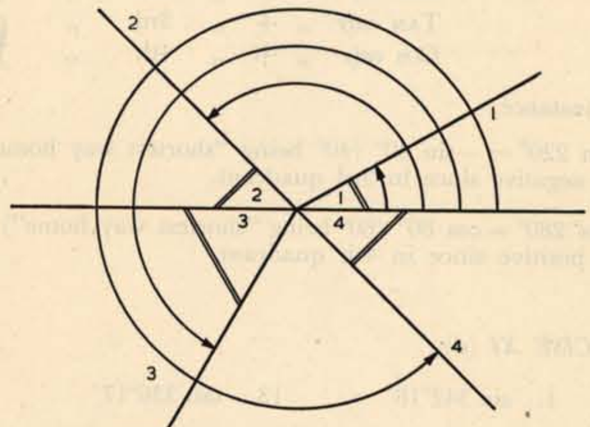
24. $\cos^{-1} 0{\cdot}00236$

Remember: "10 minutes a day."

CHAPTER V

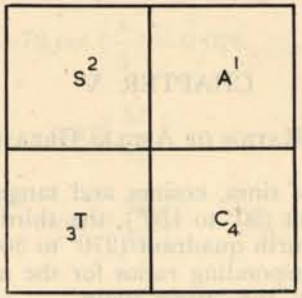
Trigonometrical Ratios of Angles Greater than 90°

The *magnitudes* of sines, cosines and tangents of angles in the second quadrant (90° to 180°), the third quadrant (180° to 270°) and the fourth quadrant (270° to 360°) are obtained by taking the corresponding ratios for the angle made with the "0°—180°" line, the "basic angle."



The basic angles for an angle of each type (1st, 2nd, 3rd and 4th quadrant type) are shown by a double line in the diagram and may be thought of as the "shortest way home," "home" being the 0°—180° line, horizontal in the diagram.

The signs to be given to the ratios are best remembered by the MNEMONIC in the diagram, the word CAST.



This shows that ALL ratios are + in the 1st quadrant
SIN *only* is + ,, 2nd ,,
TAN *only* ,, + ,, 3rd ,,
COS *only* ,, + ,, 4th ,,

For instance,

sin 220° = −sin 40° (40° being "shortest way home") negative since in 3rd quadrant.

cos 280° = cos 80° (80° being "shortest way home") positive since in 4th quadrant.

EXERCISE XI (d)

1. sin 342°18′	13. tan 336°17′
2. cos 137°20′	14. sin 97°08′
3. tan 208°26′	15. tan 120°14′
4. sin 136°42′	16. cos 192°53′
5. sin 195°18′	17. cos 321°42′
6. tan 248°30′	18. tan 259°16′
7. sin 210°36′	19. sin 164°32′

8. cos 94°42′	20. cos 108°17′
9. cos 108°16′	21. sin 314°22′
10. sin 315°21′	22. sin 359°42′
11. sin 137°14′	23. tan 270°24′
12. tan 177°41′	24. cos 90°05′

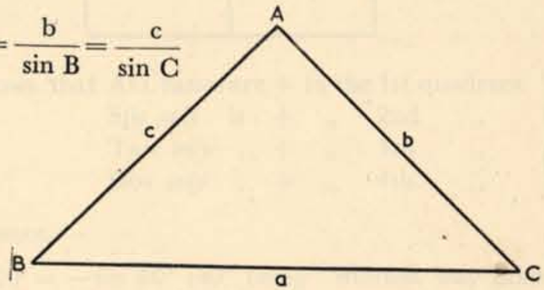
Remember: "10 minutes a day."

CHAPTER VI

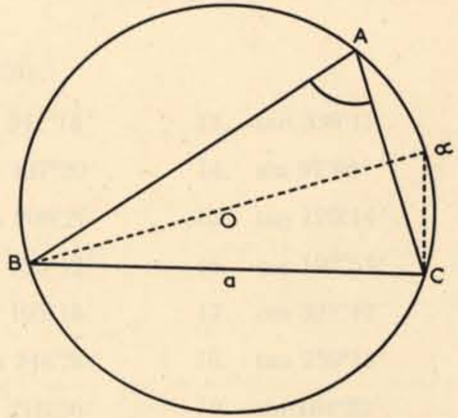
The Sine Rule

In any triangle the three sides are proportional to the sines of the opposite angles,

i.e., $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



We shall prove this.



Let O be the centre of the circum circle of the triangle ABC.

Join BO.

Produce BO to cut the circle at α.

Then $\angle B\alpha C = \angle A$ (angles subtended by same segment) and $\angle BC\alpha = 90°$ (angle in a semi-circle), also $B\alpha = 2R$ (R being the circum-radius).

Hence, $\frac{a}{2R} = \sin A \therefore \frac{a}{\sin A} = 2R.$

By similar argument $\frac{b}{\sin B} = 2R$

and $\frac{c}{\sin C} = 2R.$

Hence, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$

This formula, being of a ratio type, is particularly suited to the slide rule.

For instance, if A is known to be 53°
,, B ,, 61°
,, a ,, 7″

then the triangle may be "solved" in one setting. For since the three angles add to 180° then C must be 66° and the single setting

A Scale	7	768	800	878
S Scale	53°	61°	66°	90°

gives b = 7·68″
c = 8·00″

Note also that the same setting gives 2R = 8·78 or the circum-radius = 4·39″.

Solution of Triangles

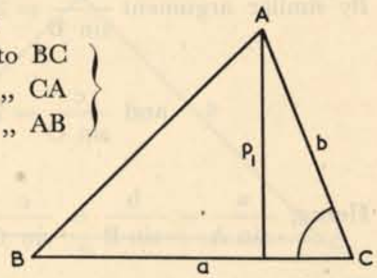
The foregoing is a simple application of the slide rule to the "solution of a triangle." Before going into the solution of triangles in more detail, we must recapitulate certain trigonometrical formulae, necessary in any type of solution.

I The Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} (=2R)$

II The Area formulae

(a) $\triangle = \frac{1}{2}ap_1 = \frac{1}{2}bp_2 = \frac{1}{2}cp_3$

where p_1 is $\perp^r$ from A to BC
p_2 ,, $\perp^r$,, B ,, CA
p_3 ,, $\perp^r$,, C ,, AB



This is the basic formula "$\frac{1}{2}$ (base $\times$ height)".

(b) $\triangle = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$.

This is obtainable from (a) by putting $p_1 = b\sin C$, etc.

(c) $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$: "s" in this formula is the SEMIPERIMETER of the triangle; the formula being due to HERO of Alexandria (c. 120 B.C.); this is sometimes known as the "s" form of the area.

III The Cosine formulae

This is the trigonometrical form of the extensions of Pythagoras' theorem.

$$\left.\begin{aligned} a^2 &= b^2+c^2-2bc\cos A \\ b^2 &= c^2+a^2-2ca\cos B \\ c^2 &= a^2+b^2-2ab\cos C \end{aligned}\right\}$$

IV The "half angle" formulae

The only type of these we shall need are

$$\left\{\begin{aligned} \tan\frac{B-C}{2} &= \frac{b-c}{b+c}\cot\frac{A}{2} \\ \tan\frac{C-A}{2} &= \frac{c-a}{c+a}\cot\frac{B}{2} \\ \tan\frac{A-B}{2} &= \frac{a-b}{a+b}\cot\frac{C}{2} \end{aligned}\right.$$

and are proved in books of trigonometry.

The "elements" of a triangle, ABC, namely, the three angles A, B, and C and the three sides a, b, and c, are not entirely free to take any values we wish.

The nature of a triangle imposes two restrictions:—

(i) $A+B+C$ must be 180°.

(ii) The sum of any pair of sides must exceed the third side.

In general, given three independent elements, the determination of the other three is known as "solving the triangle." It is obvious that three angles can only fix the shape of a triangle and not its size: and of course three angles are not independent elements.

In some cases there will be one unique solution. In others there may be two possible solutions, and in others there may be no possible solution at all.

We shall consider, systematically, the various combinations. Firstly, two angles and one side, known briefly as (SAA).

[*SAA*]

Here, since $A+B+C = 180°$, the third angle can at once be found, and the Sine Rule gives a unique solution. The uniqueness of the result can also be seen by drawing the triangle from the data.

Example

Given a = 131 cm.

B = 73°

C = 57°

By calculation A = 50° and the slide rule setting is

$$\frac{131 \quad 1435 \quad 1635}{50° \quad 57° \quad 73°} \quad \frac{A}{S}$$

Giving A = 50°

b = 163·5 cm.

c = 143·5 cm.

EXERCISE XII

1. A = 31°17′, B = 35°21′, c = 3·72
2. B = 32°39′, C = 121°14′, a = 5·43
3. A = 75°18′, C = 40°40′, b = 7·24

Remember: "10 minutes a day."

Next we deal with two sides and one angle, subdividing into the case

(i) when the angle is not included [SSA],

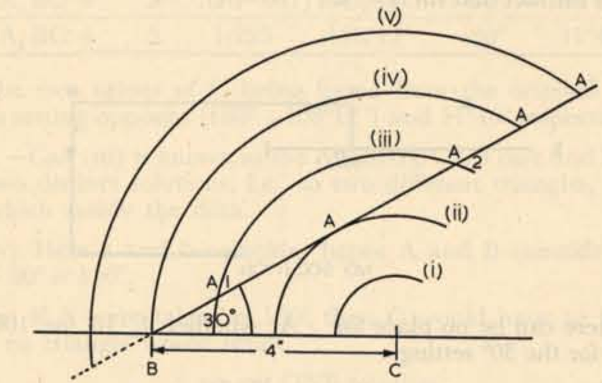
(ii) when the angle is included [SAS].

[*SSA*]

As a preliminary the student should draw the triangles whose data are

a = 4″
b = 1″, 2″, 3″, 4″, 5″
∠B = 30°

} a and ∠B being the same throughout each of the five constructions.



The constructions are numbered from (i) to (v) for the different values of b.

Draw BC = 4″ and direction BA at 30° to BC.

The construction is completed by taking centre C and radius "b" to find cutting point A.

In case (i) we see there is NO solution, arc not cutting.

" (ii) " " ONE " arc *touching* and giving a Right-angled Triangle.

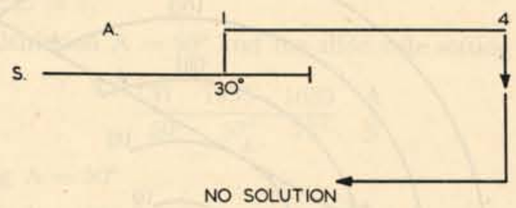
" (iii) " " are TWO solutions. (In this case A_1BC and A_2BC both accord with data).

In case (iv) we see there is ONE solution (an isosceles triangle).

" (v) " " ONE " (the other cutting point giving B as 150°, NOT 30°).

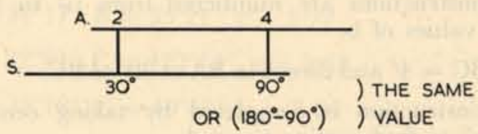
Now we can apply the Sine Rule to each of these cases in turn (since we have 'b' to set opposite to ∠B, there is no difficulty in the SETTING), such ambiguity as there may be lies in the fact that sin A = sin (180 − A).

(i)

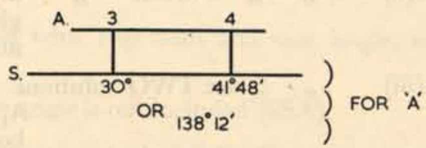


There can be no place for ∠A, whether 1, 10, or 100 be used for the 30° setting.

(ii)



(iii)



Considering each possibility in turn C is either:—

$$180° - (30° + 41°48') = 108°12'$$

or $180° - (30° + 138°12') = 11°48'$

Both these values of C are perfectly possible and there are thus TWO SOLUTIONS:—

	a	b	c	A	B	C
△ A_1BC	4	3	5·7	41°48′	30°	103°12′
△ A_2BC	4	3	1·225	138°12′	30°	11°48′

The two values of C being found from the original Sine Rule setting opposite (180° − 108°12′) and 11°48′ respectively.

N.B.—*Case (iii)* is known as the AMBIGUOUS case and leads to two distinct solutions, i.e., to two different triangles, both of which satisfy the data.

(iv) Here a and b coincide; hence A and B coincide and A = 30° *or* 150°.

But if A were taken as 150° then C would have to be 0° and no triangle would result.

∴ we get ONE solution.

(v) Here the setting gives

$$\frac{4}{23°35'} \quad \frac{5}{30°} \text{ exactly as in case (iii)}$$

or

156°25′

but although A = 23°35′ gives C = 126°25′

the value A = 156°25′ gives C = 180° − 186°25′,

which is NOT a possible angle of a triangle.

Hence, ONE solution,

so that the slide rule always reflects, easily and faithfully, the non-existence, uniqueness, or ambiguity of the data (SSA).

In practice we expect at least one solution. If the given angle is opposite the smaller of the two given sides we should look for two solutions; but if the given angle is opposite the larger of the two given sides there can be only one solution. Thus if the given angle is obtuse (or right-angled) one solution only can be found.

EXERCISE XIII

1. A = 72, b = 81, B = 50°00′
2. a = 4·70, c = 3·72, A = 44°04′
3. b = 3·92, c = 5·89, C= 125°14′
4. a = 61·2, b = 129·4, A = 24°13′
5. a = 316, c = 432, C = 129°40′
6. b = 831, c = 992, B = 41°16′
7. a = 9·81, b = 6·31, A = 103°00′
8. a = 76, b = 101, B = 46°21′
9. a = 37·2, b = 47·4, A = 56°21′
10. b = 18·4, c = 13·2, C = 7°14′

Remember: "10 minutes a day."

[*SAS*]

Here there are four methods open to us:—

(i) First principles.

(ii) The formula IV for tan $\frac{B-C}{2}$

(iii) The "Guesswork" method.

(iv) The use of the Cosine Rule.

The Cosine Rule is not easily adaptable to use with a slide rule. The "Guesswork" method will be explained by an

example, though as its name implies it is a method of scientific guessing. The formula IV gives a good degree of accuracy and is well adapted to slide rule work.

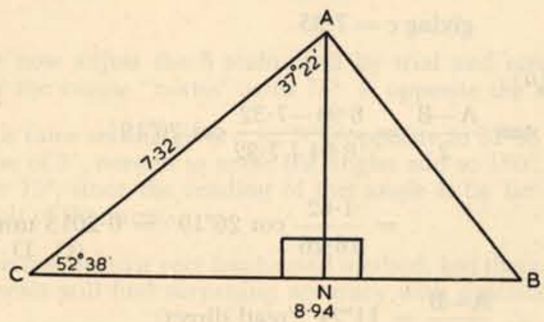
We will consider each of the methods applied to the example.

"Solve the triangle in which a = 8·94

b = 7·32

C = 52°38′."

Method (*i*)



Drop the perp$^{r\cdot}$ AN from A to BC.

Solving the right-angled triangle ACN by the sine rule we find:—

AN = 5·82

CN = 4·45

Therefore BN = 8·94 − 4·45

= 4·49

We now find the smaller of the two remaining angles of the right-angled triangle ANB using the T and D scales.

In the example under consideration

$$\tan \angle NAB = 4{\cdot}49/5{\cdot}82$$

$$\text{giving } \angle NAB = 37°40'$$

$$\text{whence } \angle B = 52°20'$$

$$\text{and } \angle A = 37°22' + 37°40'$$

$$= 75°02'$$

Solving triangle ANB by the sine rule we have

$$\frac{c}{\sin 90°} = \frac{4{\cdot}49}{\sin 37°40'}$$

$$\text{giving } c = 7{\cdot}35$$

Method (ii)

$$\tan\frac{A-B}{2} = \frac{8{\cdot}94-7{\cdot}32}{8{\cdot}94+7{\cdot}32}\cot 26°19'$$

$$= \frac{1{\cdot}62}{16{\cdot}26}\cot 26°19' = 0{\cdot}2015 \text{ using (C, D \& T)}$$

$$\therefore \frac{A-B}{2} = 11°24' \text{ (read direct)}$$

(NOTE.—We CANNOT say tan (A−B) = 2 × 0·2015 as this is NOT true)

$$\therefore A-B = 22°48'$$

But A+B = 127°22′ since C = 52°38′

$\therefore$ A = 75°05′, B = 52°17′ } and the sine rule setting gives c = 7·35

Method (iii)

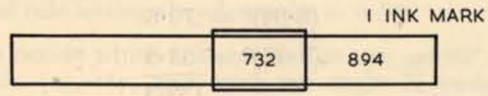
We know that (A+B) has to be 127°22′, and since a and b are not very unequal, then A and B are likely to be acute angles.

If we could set cursors over 8·94 and 7·32 on the A scale, it is then a matter of trial to find an 'A' and a 'B' which will

(i) be simultaneously under each cursor,

(ii) add to 127°22′

In practice, for this particular example, the cursors are too wide to do this, so we set a cursor over 7·32 (say) and make a small ink mark opposite 8·94.



We now adjust the S scale until by trial and error 52°17′ under the cursor "mates" with 75° + opposite the ink mark.

This same setting gives c = 7·35 opposite to 52°38′ and the residue of 5′, needed to make the angles add to 180°, is added to the 75°, since the reading of this angle is by far the most difficult of the three.

This may seem a very haphazard method, but those to whom it appeals will find surprising accuracy with practice.

To make the method more systematic the trials should be tabulated as below, making as much use as possible of "bracketing."

Trial	A	B	127°22′
1	70°00′	50°10′	120°10′
2	80°00′	54°00′	134°00′
3	74°00′	52°00′	126°00′
4	75°00′	52°15′	127°15′

As before, we take $B = 52°15'$, $A = 75°07'$, and find that $c = 7{\cdot}35$.

If either A or B were *obtuse*, we should have to proceed as shown later, though on the same principle.

Method (iv)

We use $c^2 = (8{\cdot}94)^2 + (7{\cdot}32)^2 - 2 \times 8{\cdot}94 \times 7{\cdot}32 \cos 52°38'$, each of the three separate parts needing separate slide rule treatment.

$$(8{\cdot}94)^2 = 79{\cdot}8$$
$$(7{\cdot}32)^2 = \underline{53{\cdot}5}$$
$$\underline{133{\cdot}3}$$

$$2 \times 8{\cdot}94 \times 7{\cdot}32 \times \cos 52°38' = \underline{79{\cdot}3}$$
$$c^2 = 54{\cdot}0$$
$$c = 7{\cdot}35$$

the solution being completed by applying the Sine Rule on (A, S) now that c is known.

It is clear that the last method is cumbersome, but some may prefer it.

Method (iii) Another example.

Now, as a final example of the guesswork type, consider

$$\text{Solve } \left.\begin{aligned} b &= 9{\cdot}88 \\ c &= 4{\cdot}78 \\ A &= 32°03' \end{aligned}\right\}$$

Here b and c differ considerably and it is likely that B will be obtuse.

In this case write $B_1 = 180° - B$, so that B_1 is acute; then, since $B + C = 147°57'$, we have

$$180 - B_1 + C = 147°57'$$
$$\text{or} \quad B_1 - C = 32°03'$$

and of course, B_1 and B have the same sine, so the problem becomes "find B_1 and C so that they will lie opposite 9·88 and 4·78 respectively and at the same time DIFFER by 32°03′."

By trial, tabulating as before,

$$\begin{aligned} B_1 &\text{ is } 55°34' \\ C &\text{ is } 23°31' \end{aligned} \quad \therefore \left\{\begin{aligned} B &= 124°26' \\ C &= 23°31' \end{aligned}\right.$$

and by the sine rule setting already used $a = 6{\cdot}36$.

In order to decide which trial method to use, set 90° under the larger side (a), say, and read the angle X under the smaller side (b).

If $90° + X$ is greater than $180° - C$, then the first method must be used.

If $90° + X$ is less than $180° - C$, then the second method must be used.

EXERCISE XIV

1. $b = 16{\cdot}8$, $c = 24{\cdot}1$, $A = 51°13'$
2. $a = 8{\cdot}91$, $b = 7{\cdot}54$, $C = 36°42'$
3. $a = 36{\cdot}7$, $b = 20{\cdot}7$, $C = 99°24'$
4. $a = 4{\cdot}08$, $c = 7{\cdot}24$, $B = 117°36'$
5. $b = 168$, $c = 5396$, $A = 112°00'$
6. $b = 4{\cdot}5$, $c = 1{\cdot}5$, $A = 30°00'$
7. $b = 15{\cdot}2$, $c = 7{\cdot}10$, $A = 29°24'$
8. $a = 98{\cdot}4$, $c = 31{\cdot}2$, $B = 37°16'$
9. $b = 7{\cdot}95$, $c = 4{\cdot}31$, $A = 18°24'$
10. $a = 16{\cdot}3$, $b = 2{\cdot}10$, $C = 6°14'$

Remember: "10 minutes a day."

[*SSS*]

Here, provided the sum of any two sides exceeds the third, we shall find a unique solution by drawing. We shall thus expect a unique solution by calculation.

Method (*i*)

We first find s, thence $\triangle$ by Hero's formula, and equate this to $\frac{1}{2}ab\sin C$, etc., to get C, etc. We choose the smallest angle of the three (opposite the smallest side), this being necessarily acute.

As an example:—

	s = 28·13″
a = 24·76″	s−a = 3·37″
b = 16·38″	s−b = 11·75″
c = 15·12″	s−c = 13·01″
2s = 56·26″	2s = 56·26″*

whence $\triangle = \sqrt{28{\cdot}13 \times 3{\cdot}37 \times 11{\cdot}75 \times 13{\cdot}01}$ sq.″

$= 120{\cdot}4$ sq.″

* This addition being made as a *check* on the work.

Now C is the smallest angle,

$\therefore \frac{1}{2}(24{\cdot}76)(16{\cdot}38)\sin C = 120{\cdot}4$

$$\therefore \sin C = \frac{2 \times 120{\cdot}4}{24{\cdot}76 \times 16{\cdot}38}$$

$\therefore C = 36°26'$

The setting now is:—

1512	1638	2476	A
36°26′	40°	76°26′	S

so that B = 40°; and the reading opposite 2476 is 76°26′. But A *may* be obtuse and hence we assume the POSSIBILITY

$A = 76°26'$

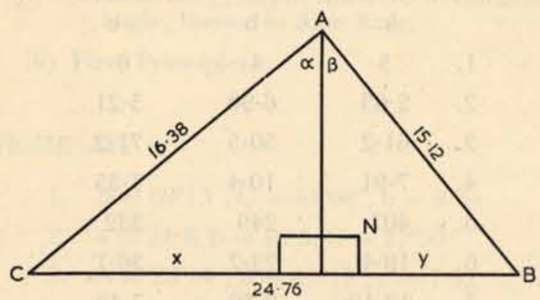
OR $A = 103°34'$

and the sum of the angles (180°) shows that it is the obtuse angle value of A which we must take.

Hence, A = 103°34′, B = 40°00′, C = 36°26′

Method (*ii*) *First Principles*

Drop the perp$^{r\cdot}$ to the longest side from its opposite vertex as in the diagram.



We now have $x^2 - y^2 = 16{\cdot}38^2 - 15{\cdot}12^2$

Therefore $(x+y)(x-y) = 31{\cdot}50 \times 1{\cdot}26$

and $$x - y = \frac{31{\cdot}50 \times 1{\cdot}26}{24{\cdot}76}$$

C

since	$x+y = 24{\cdot}76$
Thus	$x+y = 24{\cdot}76$
and	$x-y = 1{\cdot}60$
Adding	$2x = 26{\cdot}36$, giving $x = 13{\cdot}18$
Subtracting	$2y = 23{\cdot}16$, giving $y = 11{\cdot}58$

From the right-angled triangle CAN $\left(\text{since } \sin\alpha = \frac{13{\cdot}18}{16{\cdot}38}\right)$ $\alpha = 53°34'$.

From the right-angled triangle BAN $\left(\text{since } \sin\beta = \frac{11{\cdot}58}{15{\cdot}12}\right)$

$$\text{Whence} \begin{cases} \beta = 50°00' \\ A = 103°34' \\ B = 40°00' \\ C = 36°26'. \end{cases}$$

EXERCISE XV

	a	b	c
1.	5	4	6
2.	2·65	6·90	5·21
3.	61·2	50·5	71·2
4.	7·91	10·4	7·35
5.	407	249	332
6.	18·4	23·7	36·7
7.	12·10	8·30	7·40
8.	140	178	139
9.	17·4	21·2	9·52
10.	2·54	7·62	8·23

Remember: "10 minutes a day."

Summary
of the Methods to be Applied on the Slide Rule

[SAA] Sine Rule—NO AMBIGUITIES POSSIBLE.

[SSA] Sine Rule—watching always the "OTHER" (supplementary) angle.

[SAS] (i) First Principles.

(ii) $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$ etc.

(If $C>B$ then $\tan\frac{C-B}{2} = \frac{c-b}{c+b}\cot\frac{A}{2}$ etc.)

(iii) Guesswork.

(iv) Cosine Formula.

[SSS] (i) Use Hero to calculate $\triangle$.

Thence use $\triangle = \frac{1}{2}bc\sin A$, etc. to calculate smallest angle, thence to Sine Rule.

(ii) First Principles.

EXERCISE XVI

1. $B = 69°13'$, $C = 43°08'$, $b = 8{\cdot}54$
2. $a = 24{\cdot}8$, $b = 16{\cdot}6$, $C = 37°50'$
3. $A = 28°14'$, $B = 49°17'$, $c = 7{\cdot}52$
4. $a = 8{\cdot}72$, $b = 7{\cdot}24$, $B = 49°37'$
5. $a = 27{\cdot}6$, $b = 18{\cdot}3$, $c = 16{\cdot}1$
6. $a = 2{\cdot}49$, $c = 3{\cdot}72$, $B = 42°40'$
7. $b = 8{\cdot}51$, $c = 11{\cdot}41$, $A = 57°02'$
8. $b = 6{\cdot}23$, $c = 4{\cdot}17$, $A = 141°16'$

9. $a = 9{\cdot}36, b = 8{\cdot}24, C = 54°12'$

10. $A = 34°15', C = 108°14', b = 9{\cdot}83$

11. $a = 72{\cdot}4, c = 51{\cdot}2, B = 129°13'$

12. $a = 6{\cdot}41, b = 5{\cdot}65, c = 7{\cdot}72$

Remember: "10 minutes a day."

An Illustration from Mechanics

There is a type of problem, met with in the treatment of relative velocity in kinematics, known as the "Interception Problem," which furnishes an excellent illustration of the value of the slide rule in the solution of triangles. An example will illustrate the point:—

Example

A destroyer, capable of 25 knots, is located 10 sea miles South East of an enemy vessel which is moving steadily at 12½ knots on a bearing of 100° (East of North).

Find how quickly the destroyer can intercept the enemy and on what bearing she should move, assuming her path to be a straight line.

Solution

Let the required course make θ° with the line joining the original positions.

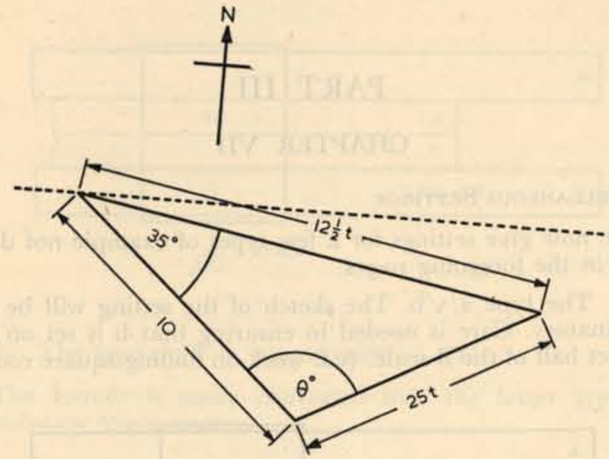
Let the time required for the interception be t hours.

Then the problem becomes, in effect, "solve the triangle shown in the diagram."

By the Sine Rule $\dfrac{25t}{\sin 35°} = \dfrac{12\frac{1}{2}t}{\sin \theta°} = \dfrac{10}{\sin (35+\theta)°}$

which, on dividing throughout by 12½t, gives

$$\frac{2}{\sin 35°} = \frac{1}{\sin \theta°} = \frac{0{\cdot}8/t}{\sin (\theta+35)°}$$



This can now be set directly on the A and S scales and gives a SINGLE solution for θ of 16°40′, which in turn gives

$$0{\cdot}8/t = 2{\cdot}73$$
$$\text{or } t = 0{\cdot}293 \text{ hours.}$$

Hence the destroyer must set a course of 331°40′ and will intercept in about 17 minutes 35 seconds.

It is to be noted that there might have been NO solution for θ (c.f. the case SSA) in which case the destroyer would be unable to intercept. Or there might have been TWO solutions for θ, the basic angle θ_1, say, and the supplementary angle $\theta_2 = 180° - \theta_1$. In this case there would have been two possible courses and two corresponding times of interception.

Even when such problems are solved by drawing, the above method gives a useful and rapid check on the work.

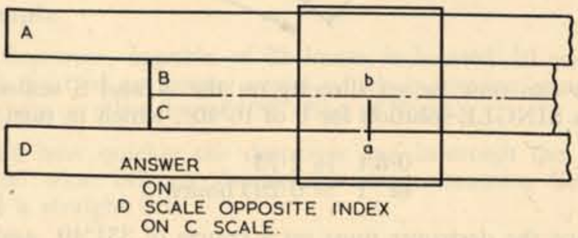
PART III

CHAPTER VII

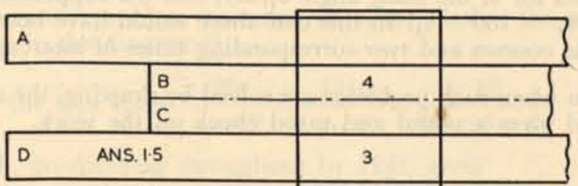
MISCELLANEOUS SETTINGS

We now give settings for a few types of example not dealt with in the foregoing pages.

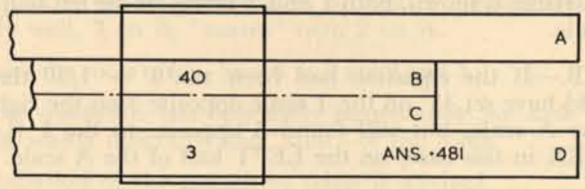
1. The type $a/\sqrt{b}$. The sketch of the setting will be self explanatory. Care is needed in ensuring that b is set on the correct half of the B scale. (c.f. work on finding square roots).



The sketch settings for the two calculations $\frac{3}{\sqrt{4}}$ and $\frac{3}{\sqrt{40}}$ are shown for purposes of illustration.



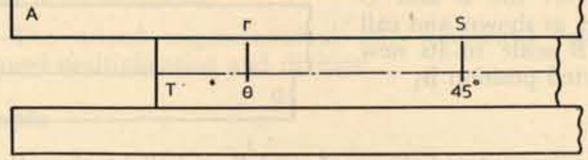
$\frac{3}{\sqrt{4}} = 1{\cdot}5$



$\frac{3}{\sqrt{40}} = 0{\cdot}481$

2. The type $\sin^2\theta = p/q$ or $\tan^2\theta = r/s$.

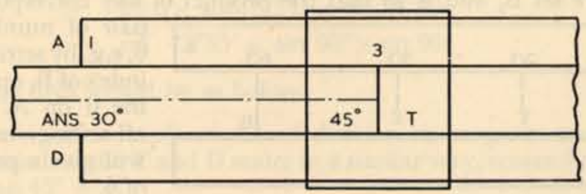
The former is easily converted into the latter type by elementary trigonometry.



We use the T scale in conjunction with the A scale. The setting is shown for $\tan^2\theta = r/s$. Care must again be exercised in selecting the positions of r and s.

Example

Solve the equation $\sin^2\theta = 1/4$.



Elementary trigonometry gives the equivalent form $\tan^2\theta = 1/3$. The setting is shown, both 1 and 3 being on the left half of the A scale.

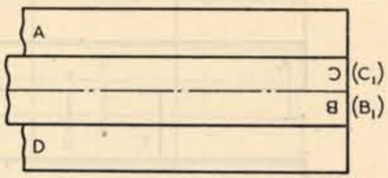
N.B.—If the equation had been $\tan^2\theta = 1/30$ then we should have set 45° on the T scale opposite 3 on the right half of the A scale, but still found θ opposite to the 1 (i.e. the INDEX in this case) on the LEFT half of the A scale.

3. *The Quadratic Equation Setting*

Here the problem is "Find two numbers whose product is known and whose (algebraic) sum is also known."

We therefore make use of the following device:—

Invert the B and C scales as shown and call the B scale in its new inverted position B_1

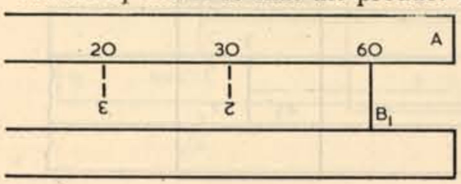


In any setting between A and B_1 it will be clear that the product of any corresponding pair of numbers will be constant for that setting. That is, for instance, if the index marks are set opposite, then any reading on A will give its reciprocal on B_1

Let us apply this property to the example:—

Solve $x^2 - 5x + 6 = 0$

We set B_1 and A so that the product of any corresponding pair of numbers is 6, e.g. by setting the index of B_1 opposite the 6 on A. Then all settings on A, B_1 will give a product of 6.



We now find, by trial and error, a cross pairing which will add up to 5; we find either 2 on B_1 "mates" with 3 on A or, equally well, 3 on B_1 "mates" with 2 on A.

Hence the roots of $x^2 - 5x + 6 = 0$ are 2 and 3.

If the quadratic has oppositely signed roots we look for a pairing which differs by the "sum" of the roots.

Tabulation of the successive trials is advised.

Example

$$x^2 - 1{\cdot}23x - 0{\cdot}496 = 0$$

Set A and B_1 to the product 0·496, and find 1·55 and 0·32 to DIFFER by 1·23. Giving the roots 1·55 and − 0·32.

Note.—On some rules there is engraved a C_1 scale; this of course would be used with the D scale in exactly similar manner to the foregoing.

4. The mixed trigonometrical and numerical type of continued multiplication and division.

Example

$$\frac{23{\cdot}6 \times \sin 15°21' \times \sin 26°30'}{\sin 42°35'}$$

Here the procedure is exactly as that given on page 20 except that the S Scale replaces the B Scale and for extra factors of unity we automatically take sin 90°.

The example would be re-written (mentally)

$$\frac{23{\cdot}6 \times \sin 15°21' \times \sin 26°30'}{\sin 42°35' \times \sin 90° \times \sin 90°}$$

and the drill would be as before.

Continued multiplication and division by tangents may be performed on the T and D scales in a similar way, remembering that $\tan 45° = 1$.

EXERCISE XVII

Solve the equations :—

1. $x^2-3{\cdot}09x+2{\cdot}327=0$
2. $x^2-x-0{\cdot}24=0$
3. $x^2-25{\cdot}5x+162{\cdot}5=0$
4. $x^2+0{\cdot}52x-0{\cdot}1533=0$
5. $x^2-28x+195=0$
6. $x^2-0{\cdot}25x-0{\cdot}75=0$
7. $2x^2-5{\cdot}5x+3{\cdot}63=0$
8. $11x^2-12{\cdot}87x-24{\cdot}926=0$
9. $x^2-6{\cdot}673x+1=0$
10. $x-1/x=8{\cdot}899$
11. $x^2-170x+7225=0$
12. Solve the equation

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

if $a=2b=4c=1{\cdot}234$

Remember: "10 minutes a day."

CHAPTER VIII

GAUGE MARKS

On many slide rules, particularly on C/D scales, the value corresponding to π is engraved.

For any number which is of frequent occurrence in the calculations of a particular user it may be of great convenience to have a mark (usually lettered with a distinctive or apt letter) engraved on the scale for that number.

As an instance; suppose an engineer had much recourse to the formula $d=\dfrac{C}{\pi}$ for the diameter-circumference relationship of varying-sized cog wheels. Then a mark for $\dfrac{1}{\pi}$ (usually marked M) might be of great assistance.

His slide rule, once set with the index of the B scale opposite to M on the A scale, would give reading after reading for the diameter needed to produce a given circumference, needing only a cursor move each time.

A surveyor might well "mark" a setting to transform yards to metres in a similar manner and further instances will no doubt occur to the reader.

We are little concerned here with technical gauge marks except insofar as they illustrate the methods available to any user.

Those commonly marked are

(i) π (found often on both A/B and C/D)

(ii) $M=1/\pi$ (found on A/B at about 319)

(iii) $C_1 = \sqrt{\dfrac{4}{\pi}}$ (found on C/D)

(iv) $C = \sqrt{\dfrac{40}{\pi}}$ (found on C/D)

EXERCISE XVIII (Miscellaneous)

1. (a) $\dfrac{513}{\sqrt{71 \cdot 3}}$ (b) $(4 \cdot 3 \times 18 \cdot 6 \times 0 \cdot 0672)^2$

(c) $13 \cdot 7 \times \sqrt{82 \cdot 7}$ (d) $\sqrt{47 \cdot 5^2 - 36 \cdot 5^2}$

(e) $\sqrt{\dfrac{0 \cdot 159}{0 \cdot 0725}}$ (f) $\dfrac{463}{\sqrt{117 \cdot 5}}$

(g) $\sqrt{0 \cdot 89 + 3 \cdot 43}$ (h) $9 \cdot 78^2 - 2 \cdot 32^2$

(i) $1 + \dfrac{2}{\pi} - \dfrac{128}{9\pi^2}$ (j) $\dfrac{1}{7 \cdot 2} + \dfrac{1}{25 \cdot 8}$

2. (a) Calculate r from the formula $r = 2\pi\sqrt{\dfrac{l}{g}}$ when $l = 153 \cdot 4$; and $g = 32 \cdot 2$.

(b) Calculate r from the formula $r = \sqrt{\dfrac{V}{\pi h}}$ when $V = 123 \cdot 4$; $h = 15 \cdot 7$

(c) Given $F = \dfrac{EI\ \pi^2}{4L^2}$, find F when $E = 3 \cdot 68 \times 10^7$; $I = 0 \cdot 02$; $L = 62$.

(d) Given that $R = \dfrac{p\,l^2}{3\pi g}$, find R when $g = 32 \cdot 2$, $p = 1 \cdot 41 \times 10^5$; $l = 0 \cdot 178$

(e) Calculate f from the formula $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ when $u = 34 \cdot 5$ and $v = 83 \cdot 3$

(f) From the formula $T = \sqrt{\dfrac{h^2 + k^2}{gh}}$, calculate the value of T when $g = 980$; $h = 54 \cdot 7$; $k = 31 \cdot 2$.

3. (a) $\sqrt{\sin 37°15'}$ (b) $[\tan 55°]^2$

(c) $46 \cdot 9 \sin x = 23 \cdot 3 \sin 25°30'$

(d) $\tan^2 x = 0 \cdot 69$ (e) $72 \cdot 9 \cos x = 35 \cdot 4 \sin 31°30'$

(f) $\tan x = \sqrt{2 \cdot 15}\ \tan 29°50'$

(g) $\sin x = \dfrac{43 \cdot 2 \sin 64°}{93 \cdot 5}$

(h) $\dfrac{37 \cdot 3 \sin 29°30'}{\sin 3°15'}$

(i) $42 \cdot 1 \tan 57°20' \tan 10°15'$

(j) $2 \cdot 58 \sin 11°20' \operatorname{cosec} 160°10'$.

4. (a) Given that $\dfrac{a}{\sin 22°30'} = \dfrac{32 \cdot 3}{\sin B} = \dfrac{19 \cdot 5}{\sin 28°46'}$ find a and B.

(b) Given that

$$\frac{x}{\sin 24°15'} = \frac{14 \cdot 6}{\sin 36°14'} = \frac{19 \cdot 2}{\sin A} = \frac{y}{\sin 116°} = \frac{1 \cdot 6}{\sin B}$$

find x, y, A and B when A and B lie between 0° and 90°.

(c) Given that

$$\frac{15\cdot 2}{\sin 37°10'} = \frac{x}{\sin 20°30'} = \frac{11\cdot 1}{\sin A} = \frac{1\cdot 9}{\sin B} = \frac{y}{\sin 115°}$$

find x, y, A & B when A & B lie between 0° and 90°.

5. (a) $\sqrt{\cos 48°30' \times \text{cosec } 159°15'}$

(b) $4\cdot 7 \sin 146° - 2\cdot 5 \cos 108°$

(c) $360 \cot 12°30' \sec 67°30'$

(d) If $\frac{L}{\tan 20°10'} = 5 \tan 73°15' = \frac{0\cdot 138}{\tan x} = 28\cdot 7 \cot y$

find L, x and y

(e) $\sqrt{54\cdot 2 \sec 42°45' \cos 84°10'}$

(f) $\frac{2\cdot 73 \times 4\cdot 61}{2} \sin 59°42'$

(g) Find A from the formula $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

where $2s = a+b+c$ and $a = 6$, $b = 2$, $c = 5$

(h) By using the formula $\sec x = \sqrt{1+\tan^2 x}$ find the value of sec 8° as accurately as possible.

(i) $\frac{16\cdot 4 \sin 148°}{\cos 105°}$

(j) $\sqrt{61\cdot 3 \sin 15° \div \sin 42°30'}$

(k) Find the least positive value of A which satisfies the equation $\tan \frac{A}{2} = \frac{5\cdot 6}{4\cdot 7} \tan 24°30'$.

(l) Use the formula $\cos A = 1 - 2\sin^2 \frac{A}{2}$ to calculate cos 2° as accurately as possible.

(m) Use the formula $\sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}$

to calculate sin 89° − sin 88° as accurately as possible.

(n) Use the formula $\tan (45+A) = \frac{1+\tan A}{1-\tan A}$

to calculate tan 88°50′ as accurately as possible.

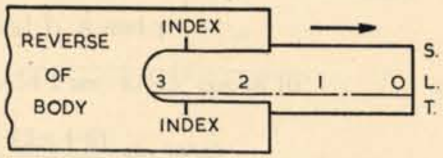
Remember: "10 minutes a day."

APPENDIX

I The Log (or L) Scale

On the "trigonometrical scale" face of the slide there is sometimes found, between the S and the T scales, a scale marked equidistantly from 10 to 0. This is the logarithmic or L Scale.

This scale is used and read in the small inset at the right hand end of the back of the whole rule, as indicated in the diagram:—

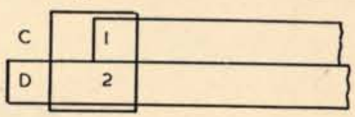


The inset (cut out portion of the body) is sometimes called the WINDOW.

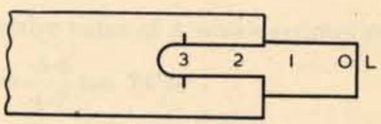
In this position, as the L scale is drawn out to the right, the readings pass successively across the INDEX (notched on the window) in the "forward order," 0, 1, 2, etc.

If, now, the INDEX of the C scale be set opposite any value, say x, on the D scale, then the reading on the L scale will be the MANTISSA part of the logarithm of the number x.

For instance, the setting



gives, on reversing the rule, the reading "301" on L.



This setting gives us the fact that

$$\log_{10} 2 = 0{\cdot}301$$

Let it be stressed that the reading on L gives *only* the MANTISSA of the logarithm.

For instance, had we set C/D for 200 then the reading "301" would have had to be written as 2·301, since $\log_{10} 200 = 2{\cdot}301$.

Bearing this in mind we are now in a position to use the scale for the calculation of such expressions as, e.g. $\left(\dfrac{7{\cdot}32}{2{\cdot}09}\right)^{0{\cdot}75}$

In such a case, ordinary division on C/D gives the value 3·502 for $\dfrac{7{\cdot}32}{2{\cdot}09}$ and the problem is immediately reduced to "Find $(3{\cdot}502)^{0{\cdot}75}$."

Keep now a mental picture of the solution when logs are used, i.e. $\log (3{\cdot}502)^{0{\cdot}75} = 0{\cdot}75 \times \log 3{\cdot}502$.

We can seek log 3·502 direct from the setting which gave us 3·502, the INDEX of C being already opposite to 3·502 on D. Reverse the rule and read 0·5444 as the MANTISSA of log 3·502. The characteristic is in this case zero. Hence the logarithm of the answer we require will be $0{\cdot}75 \times 0{\cdot}5444$. Performing this on C/D gives 0·408 on D.

We now set 0·408 on the L scale and read our final answer on the D scale (opposite to the index of the C scale for the setting) as 2·56.

Whence $\left(\dfrac{7{\cdot}32}{2{\cdot}09}\right)^{0{\cdot}75} \simeq 2{\cdot}56$

Had the calculation required been $\left(\dfrac{73{\cdot}2}{2{\cdot}09}\right)^{0{\cdot}75}$ then we should have used 1·5444 instead of 0·5444 at the appropriate stages.

The method is rather cumbersome and inferior to the use of the Log Log Scales, a brief treatment of which is added for the sake of completeness, although, generally speaking, only the more expensive Slide Rules are engraved with these types of scale.

II. The Log Log Scales

$$\text{If } a^x = y, \qquad \text{(i)}$$

Then, taking logs to the base p,

$$x \log_p a = \log_p y. \qquad \text{(ii)}$$

Taking logs again to the base 10,

$$\log_{10} x + \log_{10} \log_p a = \log_{10} \log_p y \qquad \text{(iii)}$$

If we construct a scale such that a point marked *a* on it is at a distance proportional to $\log_{10} \log_p a$ from its left hand index; then by placing the left hand index of the C scale over *a* and moving the cursor over x on the C scale we shall reach a point on the new scale at a distance proportional to $\log_{10}\log_p a + \log_{10} x$ from its left hand index. This reduces to

$$\log_{10} \log_p a^x.$$

The reading on the new scale will thus be a^x i.e. y in the relation (i) above.

Since $\log_{10} \log_p p = \log_{10} 1 = 0$ we see that the left hand index of the new scale must be marked p.

This new scale is known as the log log or LL scale.

The base p is usually taken as e (2·7183). Equation (iii) then becomes $\log_{10} x + \log_{10} \log_e a = \log_{10} \log_e y$.

Unlike the A, B, C, and D scales the LL scale does not repeat itself and a given number can only be set in one way. This limits the range of the scale. To overcome this difficulty various devices are used, and the modern slide rule may have as many as six LL scales, three for numbers less than 1 and three for numbers greater than 1.

Here we describe only one simple form of the LL scale; but it is hoped that this will be sufficient to enable the student to make use of more complicated types should he come across them.

Description of a simple combination of LL Scales.

At the bottom of the body of the rule we have the LL2 scale running from e^1 to e^{10} i.e. 2·718 to 22,000.

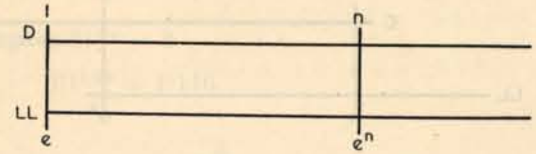
At the top edge we have the LL1 scale running from $e^{1/10}$ to e^1 i.e. from 1·105 to 2·718.

Both are used in conjunction with the C and D scales.

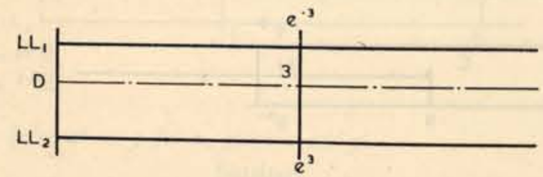
Uses of the LL Scales

(a) *To raise* e *to any power*

Setting



Examples e^3 and $e^{\cdot 3}$



We read $e^3 = 20{\cdot}09$ on LL2 and $e^{\cdot 3} = 1{\cdot}35$ on LL1.

The range of the scales is too small to evaluate e^{30} or $e^{\cdot 03}$.

(b) *Evaluation of the Hyperbolic Functions* $\sinh x = \dfrac{e^x - e^{-x}}{2}$

$$\cosh x = \frac{e^x + e^{-x}}{2} \text{ and } \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

e^x is found as in (a) and its reciprocal e^{-x} determined on the C and D scales.

(c) *To find the natural logarithm of any number n, say.*

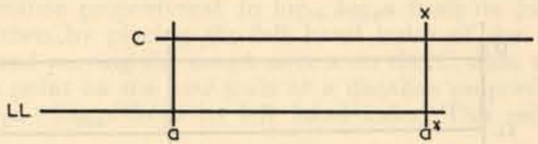
The process is the reverse of that in (a). That is, *n* is set on the LL scale and $\log_e n$ is read on the D scale.

Examples $\log_e 4560 = 8{\cdot}425$ (LL2)

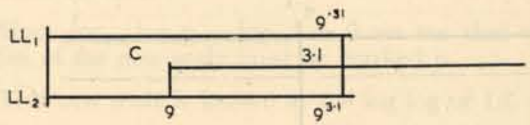
$\log_e 1{\cdot}2 = 0{\cdot}182$ (LL1).

(d) *To raise a number to a power*

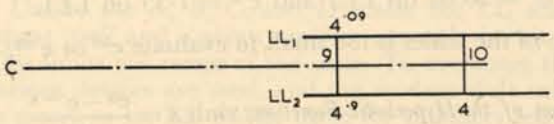
Setting



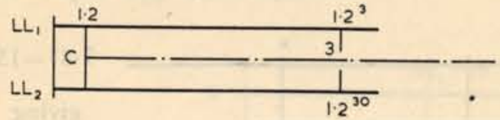
Examples



$9^{\cdot 31} = 1{\cdot}976$
$9^{3\cdot 1} = 908$



$4^{\cdot 9} = 3{\cdot}48$
$4^{\cdot 09} = 1{\cdot}132$

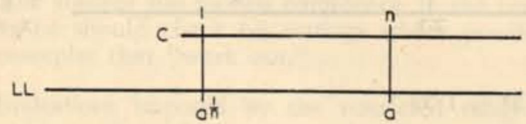


$1{\cdot}2^3 = 1{\cdot}728$
$1{\cdot}2^{30} = 238$

(e) *To Extract a Root*

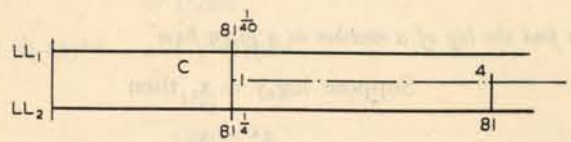
The process is the reverse of (d).

Setting



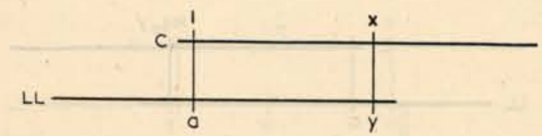
Examples $81^{1/4} = 3$

$81^{1/40} = 1{\cdot}116$

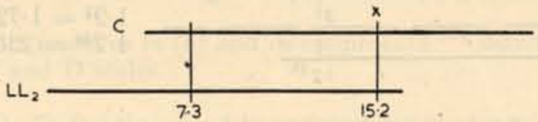


(f) *To Solve $a^x = y$ for x, given a and y*

Setting



Examples



$7 \cdot 3^x = 15 \cdot 2$

giving $x = 1 \cdot 37$

$7 \cdot 3^x = 1 \cdot 52$

giving $x = 0 \cdot 211$

$0 \cdot 73^x = 1 \cdot 52$

Here x is clearly negative $= -y$, say.

The reciprocal of 0·73 is 1·37, thus $1 \cdot 37^y = 1 \cdot 52$

giving $y = 1 \cdot 33$, and thus $x = -1 \cdot 33$

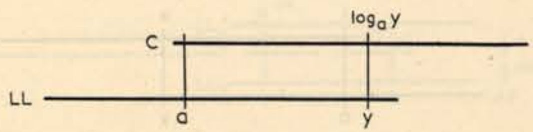
(g) *To find the log of a number to a given base*

Suppose $\log_a y = x$, then

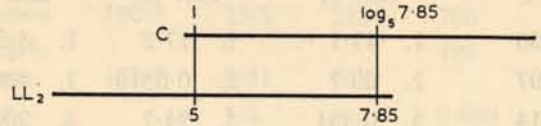
$a^x = y$.

Using the method in (f) we can thus find the log of a number to a given base.

Setting



Example $\log_5 7 \cdot 85 = 1 \cdot 28$



General

Until the student has gained confidence in the use of the LL scales he should check his settings where possible with simple examples that "work out."

The limitations imposed by the restricted range of the scales may sometimes be overcome by the use of factors.

Thus $25000^{1/40} = 250^{1/40} \times 100^{1/40}$

$= 1 \cdot 1475 \times 1 \cdot 122$

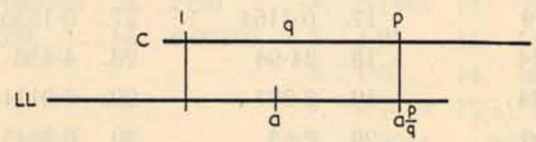
$= 1 \cdot 288$

and $(0 \cdot 43)^{3 \cdot 2} = 4 \cdot 3^{3 \cdot 2} / 10^{3 \cdot 2}$

$= \dfrac{106}{1580}$

$= 0 \cdot 067$

$a^{p/q}$ may be solved as in the following diagram



ANSWERS

EX. I		*EX. II*		*EX. III*		*EX. IV*	
1.	9·36	1.	47·1	1.	17·2	1.	3·295
2.	8·97	2.	20·7	2.	0·0319	2.	43000
3.	9·14	3.	0·394	3.	24·7	3.	203
4.	8·60	4.	16600	4.	64·5	4.	46·0
5.	1·714	5.	0·99	5.	47·2	5.	0·00439
6.	8·09	6.	9·8	6.	0·01602	6.	1278
7.	6·85	7.	7·695	7.	4·51	7.	29·5
8.	3·45	8.	0·00078	8.	0·351	8.	0·00580
9.	9·08	9.	23150	9.	0·3005	9.	1820
10.	8·97	10.	0·0000761	10.	1046	10.	43·35
						11.	0·191
						12.	0·0241
						13.	8·34

EX. V

1.	0·0295	11.	0·0611	21.	743·0
2.	135·1	12.	0·02211	22.	1235
3.	5130	13.	0·1178	23.	0·1952
4.	1126	14.	0·0003109	24.	0·4613
5.	433·3	15.	160·2	25.	0·001904
6.	0·04483	16.	3·864	26.	0·005244
7.	16·9	17.	0·1164	27.	0·1330
8.	1274	18.	24·94	28.	4·458
9.	9574	19.	2·287	29.	0·01116
10.	2·09	20.	2·63	30.	0·3645

EX. VI

1.	ft./sec.	33	70·1	135	156·6	531
2.	metres	190·5	25·5	128	780	411
3.	m.p.h.				426	20·05
	f.p.s.	135·7	5·31	660		
4.	Kg.		4·3		0·899	
	lb.	14·37		162		0·804
5.	cu. ft.	3·101		7·612		0·0317
	litres		4·654		2389	
6.	sq. inches	1·736		0·0242		40
	sq. cm.		0·0884		411·5	
7.	Kw.		13·87		152·2	
	H.p.	1·305		52·82		844
8.	metres/sec.		5·537		29·11	
	m.p.h.	18·7		44·13		1008
9.	Knots	2·813		13·38		42·81
	m.p.h.		7·727		34·21	
10.	Miles	0·09759		9·696		4587
	Km.		10·16		79·3	
11.	oz.	0·4444		14·81		35·27
	Gm.		2589		2390	
12.	Watt-hours		4·75		8·68	
	B.Th.U.	0·249		202·9		2865

EX. VII				*EX. VIII*			
1.	561	6.	0·000547	1.	75·63	9.	45·39
2.	0·0973	7.	15·6	2.	0·206	10.	0·02753
3.	992	8.	0·00024	3.	62·85	11.	1·747
4.	0·000369	9.	2760	4.	0·1463	12.	99·6
5.	1893	10.	0·00105	5.	4·85	13.	1·481
				6.	1·81	14.	42·7
				7.	0·4418	15(a).	2·89
				8.	0·0264	(b)	12·97

EX. IX

1.	90·26
2.	428·98
3.	4007
4.	32·0
5.	0·8650
6.	30·4
7.	0·2556

EX. X(a)

1.	0·3159
2.	0·0396
3.	0·8875
4.	0·3600
5.	0·00276
6.	0·0019
7.	19°05′
8.	31°12′
9.	4°09′
10.	13·5′
11.	6°26′
12.	1°19′

EX. X(b)

1.	0·4772
2.	0·847
3.	0·9993
4.	0·0145
5.	0·00174
6.	37°32′
7.	34°18′
8.	88°36′

EX. XI(a)

1.	0·4245	8.	0·0635	15.	3·19
2.	0·2202	9.	0·0922	16.	191
3.	0·4564	10.	0·0244	17.	4·42
4.	0·2978	11.	0·0600	18.	286·5
5.	0·1560	12.	0·0058	19.	3·89
6.	0·0390	13.	0·0787	20.	2·617
7.	0·00146	14.	1·804	21.	1·172
				22.	1·523

EX. XI(b)

1.	24°42′	8.	73°40′	15.	01°06′
2.	72°39′	9.	67°48′	16.	85°18′
3.	29°45′	10.	73°36′	17.	89°54′
4.	65°21′	11.	18°47′	18.	03°18′
5.	69°28′	12.	24°00′	19.	00°24′
6	21°33′	13.	16°42′	20.	89°50′
7.	70°58′	14.	00°18′		

EX. XI(c) (*Only Basic solutions given*).

1.	19°32′	7.	42°54′	13.	57°	19.	34°35′
2.	60°	8.	23°19′	14.	43°40′	20.	268°49′
3.	33°27′	9.	24°32′	15.	10°22′	21.	48°40′
4.	58°32′	10.	41°33′	16.	17°07′	22.	63°53′
5.	44°05′	11.	57°28′	17.	169°34′	23.	65°17′
6.	75°18′	12.	44°57′	18.	129°44′	24.	89°52′

EX. XI(d)

1.	−0·3040	7.	−0·509	13.	−0·4393	19.	+0·2667
2.	−0·7353	8.	−0·0819	14.	+0·9923	20.	−0·3137
3.	+0·5415	9.	−0·3134	15.	−1·7158	21.	−0·7149
4.	+0·6858	10.	−0·7028	16.	−0·9748	22.	−0·00523
5.	−0·2639	11.	+0·6791	17.	+0·7848	23.	−143·2
6.	+2·5386	12.	−0·0405	18.	+5·2772	24.	−0·001455

EX. XII

1.	a = 2·104	2.	A = 26°07′	3.	a = 7·787
	b = 2·344		b = 6·655		B = 64°02′
	C = 113°22′		c = 10·54		c = 5·247

EX. XIII

1.	A = 42°55′	2.	C = 33°24′	3.	B = 32°58′
	C = 87°05′		B = 102°32′		A = 21°48′
	c = 105·5		b = 6·597		a = 2·677

4. B = 60°09′ or 119°51′ / C = 95°38′ or 35°56′ / c = 148·5 or 87·54

5. A = 34°17′ / B = 16°03′ / b = 155·2

6. C = 51°57′ or 128°03′ / A = 86°47′ or 10°41′ / a = 1257 or 233·5

7. B = 38°48′ / C = 38°12′ / c = 6·227

8. A = 32°33′
C = 100°39′
c = 137·2

9. No Solution.

10. B = 10°06′ or 169°54′
A = 162°40′ or 2°52′
a = 31·24 or 5·243

EX. XIV

1. B = 43°57′
C = 84°50′
a = 18·87

2. A = 85°44′
B = 57°34′
c = 5·34

3. A = 53°36′
B = 27°00′
c = 45·00

4. C = 40°48′
A = 21°36′
b = 9·817

5. C = 66°22′
B = 1°38′
a = 4853

6. B = 136°49′
C = 13°11′
a = 3·29

7. B = 129°28′
C = 21°08′
a = 9·665

8. A = 128°26′
C = 14°18′
b = 76·08

9. B = 142°11′
C = 19°25′
a = 4·093

10. A = 172°52′
B = 00°54′
c = 14·29

EX. XV

	A	B	C	△
1.	55°45′	41°24′	82°51′	9·92
2.	19°36′	119°08′	41°16′	6·03
3.	57°25′	44°04′	78°31′	1515
4.	49°18′	85°54′	44°48′	29·0
5.	87°43′	37°41′	54°36′	41300
6.	25°30′	33°41′	120°49′	187·3
7.	100°41′	42°23′	36°56′	30·18
8.	50°36′	79°18′	50°06′	9560
9.	54°00′	99°42′	26°18′	81·64
10.	17°54′	67°19′	94°47′	9·642

EX. XVI

1. A = 67°39′
a = 8·45
c = 6·25

2. A = 101°07′
B = 41°03′
c = 15·51

3. C = 102°29′
a = 3·64
b = 5·84

4. A = 66°33′ or 113°27′
C = 63°50′ or 16°56′
c = 8·53 or 2·77

5. A = 106°31′
B = 39°29′
C = 34°00′
△ = 141·2

6. A = 41°46′
C = 95°34′
b = 2·53

7. a = 9·845
B = 46°29′
C = 76°29′

8. a = 9·83
B = 23°21′
C = 15°23′

9. c = 8·08
A = 70°00′
B = 55°48′

10. B = 37°31′
a = 9·09
c = 15·32

11. b = 112·0
A = 30°02′
C = 20°45′

12. A = 54°42′
B = 46°00′
C = 79°18′
△ = 17·78

EX. XVII

1.	1·3	and 1·79	7.	1·65	and 1·1
2.	1·2	and −0·2	8.	2·2	and −1·03
3.	13	and 12·5	9.	6·52	and 0·153
4.	−0·73	and 0·21	10.	9·01	and −0·111
5.	13	and 15	11.	85	and 85
6.	1	and −0·75	12.	0·9917	and 0·4476

EX. XVIII

1. (a) 60·74 (b) 28·9 (c) 124·5 (d) 30·4 (e) 1·481 (f) 42·7 (g) 2·08 (h) 90·26 (i) 0·195 (j) 0·1777

2. (a) 13·7 (b) 1·581 (c) 472 (d) 14·72 (e) 24·4 (f) 0·272

3. (a) 0·778 (b) 2·04 (c) 12°21′ (d) 39°43′ (e) 75°18′ (f) 40°03′ (g) 24°32′ (h) 324 (i) 11·88 (j) 1·494

4. (a) a = 15·51, B = 52°51′ or 127°09′ (b) 10·1; 22·2; 51°; 3°43′ (c) 8·8; 22·8; 26°11′; 4°20′

5. (a) 1·368 (b) 3·4 (c) 4,243 (d) 6·101, 28·5′, 59°56′ (e) 2·74 (f) 5·433 (g) 110°30′ (h) 1·01 (i) −33·57 (j) 4·85 (k) 57° (l) 0·99939 (m) 0·000456 (n) 49·12