HOW TO ADJUST YOUR SLIDE RULE

A perfect slide rule, when out of adjustment, often appears defective. Each rule is accurately adjusted before it leaves the factory. However, handling during shipment, dropping the rule, or even a series of slight jars while laying the rule down during use, may loosen the adjusting screws and throw the rule out of alignment. Follow these simple directions for slide rule adjustment.

CURSOR-HAIRLINE ADJUSTMENT • Loosen the bottom two screws on both Cursor windows on spacer opposite tension spring. Press with left thumb to maintain constant contact with edge of rule; align hairline with left hand indices and tighten screws on that side. Turn rule over and check alignment of hairline on other Cursor window. If necessary, loosen all screws on this side and align with left hand indices as needed, and tighten screws carefully.

SLIDER TENSION ADJUSTMENT • Loosen adjustment screws on end brackets; regulate tension of slider, tighten the screws using care not to misalign the scales. The adjustment needed may be a fraction of a thousandth of an inch, and several tries may be necessary to get perfect slider action.

SCALE LINE-UP ADJUSTMENTS • (1) Move slider until indices of C and D scales coincide. (2) Move cursor to one end. (3) Place rule on flat surface with face uppermost. (4) Loosen end plate adjusting screws slightly. (5) Adjust upper portion of rule until graduations on DF scale coincide with corresponding graduations on CF scale. (6) Tighten screws in end plates.

REPLACEABLE ADJUSTING SCREWS • All Pickett All-Metal rules are equipped with Telescopic Adjusting Screws. In adjusting your rule, if you should strip the threads on one of the Adjusting Screws, simply "push out" the female portion of the screw and replace with a new screw obtainable from your dealer, or from the factory at a cost of $.06 each in stamps.

HOW TO KEEP YOUR SLIDE RULE IN CONDITION

Always hold your rule between thumb and forefinger at the ENDS of the rule. This will insure free, smooth movement of the slider. Holding rule at center tends to bind the slider.

LUBRICATION • *Do not use ordinary lubricating oil on your slide rule. It turns black and dirties your hands and work.* Your slide rule is treated with a light "silicone" lubricant at the factory. This oil, which works into the surface of the metal, is designed to lubricate your rule indefinitely.

If your rule should run dry, or if the slider begins to move hard or with a dry, rasping sound:

1. Lubricate with a light "Silicone" lubricant. Work in well by moving slider back and forth, then wipe off. OR –
2. If a light silicone is unobtainable, simply rub tongues and grooves with a very soft lead pencil. Move slider back and forth to work the graphite well into the metal, then wipe excess graphite off.

MAINTENANCE • The body of your rule is made of magnesium. The edges, not covered with plastic, may gradually darken (or oxidize) with age. This ageing, or darkening, is a common characteristic of metals like magnesium, German silver, silver, brass, copper, pewter, etc.

This normal ageing or darkening of the rule affects neither the accuracy of the scales nor ease of operation.

Extreme atmospheric exposure tends to warp and distort wood, and to rust steel, which is common knowledge. This is not true of magnesium. Such exposure may tend to deposit an oxidation film on the surface, causing the slider to stick or move hard.

If this happens to your rule, take out the Telescopic Adjusting Screws and remove both Top Rule Member and Slider without disassembling the Cursor. Clean the oxidized edges of the rule with a silver polish, Bon Ami, rubber ink eraser or other cleaning agent. Slide Top Rule Member and Slider back into position. Relubricate. Then make Scale Line-Up and Slider-Tension Adjustments.

WHY YOUR RULE OPERATES BETTER WITH CONSTANT USE • Being made of metal, the moving parts of your slide rule "lap in" with use. This process of wearing smooth means your slide rule will operate with increasing smoothness year after year.

CLEANING • Wash surface of the rule with non-abrasive soap and water when cleaning the scales. If Cursor window becomes dulled from long use, simply polish and brighten the window surfaces with a small rag and tooth powder.

FORM M 3
Printed in U.S.A.

HOW TO USE the Pickett Model 1000 SLIDE RULE

by

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PUBLISHED BY

Pickett & Eckel, Inc.

PRICE 50 CENTS

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PART I—SLIDE RULE OPERATION

INTRODUCTION

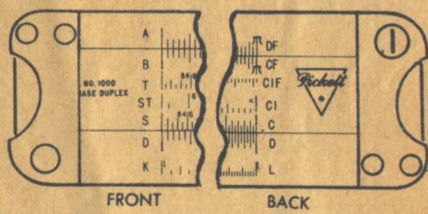
The table below shows some of the mathematical operations which can be done easily and quickly with an ordinary slide rule.

OPERATIONS	INVERSE OPERATIONS
Multiplying two or more numbers	Dividing one number by another
Squaring a number	Finding the square root of a number
Cubing a number	Finding the cube root of a number
Finding the logarithm of a number	Finding a number whose logarithm is known
Finding the sine, cosine, or tangent of an angle	Finding an angle whose sine, cosine, or tangent is known

Various *combinations* of these operations (such as multiplying two numbers and then finding the square root of the result) are also easily done. Numbers can be added or subtracted with an ordinary slide rule, but it is usually easier to do these operations by arithmetic.*

The slide rule consists of three parts: (1) the rule; (2) the slide; (3) the "runner" or indicator. On the rule and the slide several number scales are printed.

Fig. 1



Each scale is named by a letter (A, B, C, D, L, S, T) or other symbol at both ends.

In order to use a slide rule, a computer must know: (1) how to read the scales; (2) how to "set" the slide and runner for each operation to be done; and (3) how to determine the decimal point in the result.

It is best to learn how to multiply first.

*By putting special scales on a slide rule, these and certain other operations much more difficult than those shown in the table above can be done easily.

MULTIPLICATION

The scale labeled C (on the slide) and the scale D (on the rule itself) are used for multiplication. These two scales are exactly alike. The total length of these scales has been separated into many smaller parts by fine lines called "graduations."

If these scales were long enough the total length of each would be separated into 1000 parts. First they would be separated into 10 parts. Then each of these parts would be again separated into 10 parts. Finally each of these smaller parts would be separated into 10 parts, making 1000 parts in all. On the C and D scales the parts are not all equal. They are longer at the left-hand end than at the right-hand end. At the left end there is enough space to *print* all of the fine graduations. Near the right end of a short rule there is not enough room to print all the graduations. In using the rule, however, you soon learn to *imagine* that the lines are all there, and to use the *hairline* on the indicator to help locate where they would be.

Reading the Scales

The marks which first separate the entire D scale into ten parts are called the *primary* graduations. The points of separation are labeled **2**, **3**, **4**, etc., and the end points are both labeled **1**. These are the largest numerals printed on the rule. Do not confuse these with the smaller numerals 1, 2, 3, etc., to 9 which are found at the left end between the large **1** and **2**. The line above the **1** on the left end is called the *left index;* the line above **1** on the right is the *right index.*

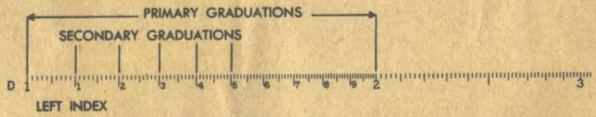


Fig. 2

Simple examples of multiplication can now be done. Numbers that are to be multiplied are called *factors*. The result is called the *product*. Thus in the statement $6 \times 7 = 42$, the numbers 6 and 7 are factors, and 42 is the product.

EXAMPLE: Multiply 2×3.

Setting the Scales: Set the left index of the C scale on **2** of the D scale. Find **3** on the C scale, and below it read the product, **6**, on the D scale.

Think: The length for 2 plus the length for 3 will be the length for the product. This length, measured by the D scale, is 6.

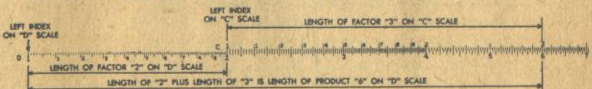


Fig. 3

EXAMPLE: Multiply 4×2.

Setting the Scales: Set the left index of the C scale on **4** of the D scale. Find **2** on the C scale, and below it read the product, **8**, on the D scale.

Think: The length for 4 plus the length for 2 will be the length for the product. This length, measured by the D scale, is 8.*

***Rule for Multiplication:* Over one of the factors on the D scale, set the index of the C scale.** Locate the other factor on the C scale, and directly below it read the product on the D scale.**

Next notice again that the distance between **1** and **2** on the D scale has been separated into ten parts, marked with smaller numerals 1, 2, 3, etc. These are *secondary* graduations. Each of the spaces between the large numerals **2** and **3**, between **3** and **4**, and between the other primary graduations is also divided into ten parts. Numerals are not printed beside these smaller secondary graduations because it would crowd the numerals too much.

The space between each secondary graduation at the left end of the rule (over to primary graduation **2**) is separated into ten parts, but these shortest graduation marks are not numbered. In the middle part of the rule, between the primary graduations **2** and **4**, the smaller spaces between the *secondary* graduations are separated into five parts. Finally, the still smaller spaces between the secondary graduations at the right of **4** are separated into only two parts.

To find 173 on the D scale, look for primary division **1** (the left index), then for secondary division 7 (numbered) then for smaller subdivision 3 (not numbered, but found as the 3rd very short graduation to the right of the longer graduation for 7).

Fig. 4

Similarly, 149 is found as the 9th small graduation mark to the right of the 4th secondary graduation mark to the right of primary graduation **1**.

To find 246, look for primary graduation **2**, then for the 4th secondary graduation after it (the 4th long line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are fifths. Since $\frac{3}{5} = \frac{6}{10}$, then the third graduation, marking *three fifths*, is at the same point as *six tenths* would be.

*This example may also be done by setting the left index of the C scale on **2** of the D scale. Then find **4** on the C scale, and below it read **8** on the D scale. See drawing above.

**This may be either the left or the right index, depending upon which one must be used in order to have the other factor (on the C scale) located over the D scale. If the "other factor" falls outside the D scale, the "other index" is used.

The number 247 would be half of a small space beyond 246. With the aid of the *hairline* on the runner the position of this number can be located approximately by the eye. The small space is mentally "split" in half.

The number 685 is found by locating primary graduation 6 and then secondary graduation 8 (the 8th long graduation after 6). Between secondary graduations 8 and 9 there is one short mark. Think of this as the "5 tenths" mark. The location of 683 can be found approximately by mentally "splitting" the space between 680 and 685 into fifths, and estimating where the 3rd "fifths" mark would be placed. It would be just a little to the right of halfway between 680 and 685.

On the scale below are some sample readings.

Fig. 5

A: 195	F: 206
B: 119	G: 465
C: 110	H: 402
D: 101	I: 694
E: 223	J: 987

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used in writing numbers are called *digits*. One way to describe a number is to tell how many digits are used in writing it. Thus 54 is a "two digit number," and 1,348,256 is a "seven-digit number." In many computations only the first three or four digits of a number need to be used to get an approximate result which is accurate enough for practical purposes. Usually only the first three digits of a number can be "set" on the slide rule scales. If the first digit of a number is 1, however, the number is located near the left end of the rule and the first four digits can be "set." In the majority of practical problems this degree of accuracy is sufficient.

Multiplication of numbers having three digits can now be done.

EXAMPLE: Multiply 2.34 × 36.8.

Estimate the result: First note that the result will be roughly the same as 2 × 40, or 80: that is, there will be two digits to the left of the decimal point. Hence we can ignore the decimal points for the present and multiply as though the problem was 234 × 368.

Set the Scales: Set the left index of the C scale on 234 of the D scale. Find 368 on the C scale and read the product 861 on the D scale.

Think: The length for 234 plus the length for 368 will be the length for the product. This length is measured on the D scale. Since we already knew the result was somewhere near 80, the product must be 86.1, approximately.

EXAMPLE: Multiply 28.3 × 5.46.

Note first that the result will be about the same as 30 × 5, or 150. Note also that if the left index of the C scale is set over 283 on the D scale, and 546 is then found on the C scale, the slide projects so far to the right of the rule that the D scale is no longer below the 546. When this happens, the *other* index of the C scale must be used. That is, set the *right* index of the C scale over 283 on the D scale. Find 546 on the C scale and below it read the product on the D scale. The product is approximately 154.5.

This illustrates how in simple examples the decimal point can be placed by use of the estimate (the result was *estimated* to be near 150), and also shows how "four-digit accuracy" can often be obtained when the result falls at the left end of the D scale.

PROBLEMS	ANSWERS
1. 15 × 3.7	55.5
2. 280 × 0.34	95.2
3. 753 × 89.1	67,100
4. 9.54 × 16.7	159.3
5. 0.0215 × 3.79	0.0815

DIVISION

In mathematics, division is the opposite or *inverse* operation of multiplication. In using a slide rule this means that the process for multiplication is reversed. To help in understanding this statement, set the rule to multiply 2 × 4 (see page 5). Notice the result 8 is found on the D scale under 4 of the C scale. Now to divide 8 by 4 these steps are reversed. First find 8 on the D scale, set 4 on the C scale over it, and read the result 2 on the D scale under the index of the C scale.

Think: From the length for 8 (on the D scale) *subtract* the length for 4 (on the C scale). The length for the difference, read on the D scale, is the result, or quotient.

With this same setting you can read the quotient of 6 ÷ 3, or 9 ÷ 4.5, and in fact all divisions of one number by another in which the result is 2.

Rule for Division: **Set the *divisor* (on the C scale) opposite the number to be divided (on the D scale). Read the result, or quotient, on the D scale under the index of the C scale.**

EXAMPLES:

(a) Find 63.4 ÷ 3.29. The quotient must be near 20, since 60 ÷ 3 = 20. Set indicator on 63.4 of the D scale. Move the slide until 3.29 of the C scale is under the hairline. Read the result 19.27 on the D scale at the C index.

(b) Find 26.4 ÷ 47.7. Since 26.4 is near 25, and 47.7 is near 50, the quotient must be roughly 25/50 = ½ = 0.5. Set 47.7 of C opposite 26.4 of D, using the indicator to aid the eyes. Read 0.553 on the D scale at the C index.

PROBLEMS	ANSWERS
1. 83 ÷ 7	11.86
2. 75 ÷ 92	0.815
3. 137 ÷ 513	0.267
4. 17.3 ÷ 231	0.0749
5. 8570 ÷ .0219	391,000

DECIMAL POINT LOCATION

In many, perhaps a majority, of the problems met in genuine applications of mathematics to practical affairs, the position of the decimal point in the result can be determined by what is sometimes called "common sense." There is usually only one place for the decimal point in which the answer is "reasonable" for the problem. Thus, if the calculated speed in miles per hour of a powerful new airplane comes out to be 4833, the decimal point clearly belongs between the 3's, since 48 m.p.h. is too small, and 4833 m.p.h. is too large for such a plane. In some cases, however, the data are such that the position of the point in the final result is not easy to get by inspection. The method described below may then be used to place the decimal point.

In the discussion which follows, it will occasionally be necessary to refer to the number of "digits" and number of "zeros" in some given numbers.

When numbers are greater than 1 the number of *digits* to the left of the decimal point will be counted. Thus 734.05 will be said to have 3 digits. Although as written the number indicates accuracy to *five* digits, only three of these are at the left of the decimal point.

Numbers that are less than 1 may be written as *decimal fractions.** Thus .673, or six-hundred-seventy-three thousandths, is a decimal fraction. Another example is .000465. In this number three zeros are written to show where the decimal point is located. One way to describe such a number is to tell how many zeros are written to the right of the decimal point before the first non-zero digit occurs.

In scientific work a zero is often written to the left of the decimal point, as in 0.00541. This shows that the number in the units' place is definitely 0, and that no digits have been carelessly omitted in writing or printing. The zeros will *not* be counted unless they are (a) at the *right* of the decimal point, (b) before or at the *left* of the first non-zero digit, and (c) are not between other digits. The number 0.000408 will be said to have 3 zeros (that is, the number of zeros between the decimal point and the 4).

In scientific work numbers are often expressed in *standard form*. For example, 428 can be written 4.28×10^2, and 0.00395 can be written as 3.95×10^{-3}. When a number is written in standard form it always has two factors. The first factor has one digit (not a zero) on the left of the decimal point, and usually other digits on the right of the decimal point. The other factor is a power of 10 which places the decimal point in its true position if the indicated multiplication is carried out. In many types of problems this method of writing numbers simplifies the calculation and the location of the decimal point.

When a number is written in standard form, the exponent of 10 may be called "the characteristic." It is the characteristic of the logarithm of the number to base 10. The characteristic may be either a positive or a negative number. Although the rule below appears long, in actual practice it may be used with great ease.

*Only positive real numbers are being considered in this discussion.

***Rule.* To express a number in standard form:**

(a) place a decimal point at the right of the first non-zero digit.*

(b) start at the right of the first non-zero digit in the original number and count the digits and zeros passed over in reaching the decimal point. The result of the count is the numerical value of the characteristic, or exponent of 10. If the original decimal point is toward the right, the characteristic is *positive* (+). If the original decimal point is toward the left, the characteristic is *negative* (−). Indicate that the result of (a) is to be multiplied by 10 with the exponent thus determined in (b).

EXAMPLES:

	Number	Number in standard form.
(a)	5,790,000	5.79×10^6
(b)	0.000283	2.83×10^{-4}
(c)	44	4.4×10^1
(d)	0.623	6.23×10^{-1}
(e)	8.15	8.15×10^0
(f)	461,328	4.61328×10^5
(g)	0.0000005371	5.371×10^{-7}
(h)	0.0306	3.06×10^{-2}
(i)	80.07	8.007×10^1

If a number given in standard form is to be written in "ordinary" form, the digits should be copied, and then starting at the right of the first digit the number of places indicated by the exponent should be counted, supplying zeros as necessary, and the point put down. If the exponent is positive, the count is toward the right; if negative, the count is toward the left. This converse application of the rule may be verified by studying the examples given above.

Consider now the calculation of $5{,}790{,}000 \times 0.000283$. From examples (a) and (b) above, this can be written $5.79 \times 10^6 \times 2.83 \times 10^{-4}$, or by changing order and combining the exponents of 10, as $5.79 \times 2.83 \times 10^2$. Then since 5.79 is near 6, and 2.83 is near 3, the product of these two factors is known to be near 18. The multiplication by use of the C and D scales shows it to be about 16.39, or 1.639×10^1. Hence, $5.79 \times 2.83 \times 10^2 = 1.639 \times 10^1 \times 10^2 = 1.639 \times 10^3 = 1639$. If, however, one has

$5{,}790{,}000 \div 0.000283$, the use of standard form yields

$$\frac{5.79 \times 10^6}{2.83 \times 10^{-4}} = 2.04 \times 10^{6-(-4)} = 2.04 \times 10.^{10}$$

In scientific work the result would be left in this form, but for popular consumption it would be written as 20,400,000,000. The general rule is as follows.

***Rule.* To determine the decimal point, first express the numbers in standard form. Carry out the indicated operations of multiplication or division, using the laws of exponents** to combine the exponents until a single power of 10 is indicated. If desired, write out the resulting number, using the final exponent of 10 to determine how far, and in what direction, the decimal point in the coefficient should be moved.**

*In using this rule, "first" is to be counted from the left; thus, in 3246, the digit 3 is "first."

**See any textbook on elementary algebra. The theory of exponents and the rules of operation with signed numbers are both involved in a complete treatment of this topic. In this manual it is assumed that the reader is familiar with this theory.

CONTINUED PRODUCTS

Sometimes the product of three or more numbers must be found. These "continued" products are easy to get on the slide rule.

EXAMPLE: Multiply $38.2 \times 1.65 \times 8.9$.

Estimate the result as follows: $40 \times 1 \times 10 = 400$. The result should be, very roughly, 400.

Setting the Scales: Set left index of the C scale over 382 on the D scale. Find 165 on the C scale, and *set the hairline* on the indicator on it.* Move the index on the slide under the hairline. In this example if the *left* index is placed under the hairline, then 89 on the C scale falls outside the D scale. Therefore move the *right* index under the hairline. Move the hairline to 89 on the C scale and read the result (561) under it on the D scale.

Below is a *general rule* for continued products: $a \times b \times c \times d \times e \cdots$

Set hairline of indicator at a on D scale.
Move index of C scale under hairline.
Move hairline over b on the C scale.
Move index of C scale under hairline.
Move hairline over c on the C scale.
Move index of C scale under hairline.

Continue moving hairline and index alternately until all numbers have been set.

Read result under the hairline on the D scale.*

PROBLEMS	ANSWERS
1. $2.9 \times 3.4 \times 7.5$	73.9
2. $17.3 \times 43 \times 9.2$	6,840
3. $343 \times 91.5 \times 0.00532$	167
4. $19 \times 407 \times 0.0021$	16.24
5. $13.5 \times 709 \times 0.567 \times 0.97$	5260

COMBINED MULTIPLICATION AND DIVISION

Many problems call for both multiplication and division.

EXAMPLE: $\frac{42 \times 37}{65}$

First, set the division of 42 by 65; that is, set 65 on the C scale opposite 42 on the D scale.** Move the hairline on indicator to 37 on the C scale. Read the result 239 on the D scale under the hairline. Since the fraction $\frac{42}{65}$ is about equal to $\frac{2}{3}$, the result is about two-thirds of 37, or 23.9.

EXAMPLE: $\frac{273 \times 548}{692 \times 344}$

Set 692 on the C scale opposite 273 on the D scale. Move the hairline to 548 on the C scale. Move the slide to set 344 on the C scale under the hairline. Read the result on the D scale under the C index.

*The product of 382×165 could now be read under the hairline on the D scale, but this is not necessary.
**The quotient, .646, need not be read.

In general, to do computations of the type $\frac{a \times c \times e \times g \cdots}{b \times d \times f \times h \cdots}$, set the rule to divide the first factor in the numerator a by the first factor in the denominator b, move the hairline to the next factor in the numerator c; move the slide to set next factor in denominator, d, under the hairline. Continue moving hairline and slide alternately for other factors (e, f, g, h, *etc.*). Read the result on the D scale. If there is one more factor in the numerator than in the denominator, the result is under the hairline. If the number of factors in numerator and denominator is the same, the result is under the C index. Sometimes the slide must be moved so that one index replaces the other.*

EXAMPLE: $\frac{2.2 \times 2.4}{8.4}$

If the rule is set to divide 2.2 by 8.4, the hairline cannot be set over 2.4 of the C scale and at the same time remain on the rule. Therefore the hairline is moved to the C index (opposite 262 on the D scale) and the slide is moved end for end to the right (so that the *left* index falls under the hairline and over 262 on the D scale). Then the hairline is moved to 2.4 on the C scale and the result .63 is read on the D scale.

If the number of factors in the numerator exceeds the number in the denominator by more than one, the numbers may be grouped, as shown below. After the value of the *group* is worked out, it may be multiplied by the other factors in the usual manner.

$$\left(\frac{a \times b \times c}{m \times n}\right) \times d \times$$

PROBLEMS	ANSWERS
1. $\frac{27 \times 43}{19}$	61.1
2. $\frac{5.17 \times 1.25 \times 9.33}{4.3 \times 6.77}$	2.07
3. $\frac{842 \times 2.41 \times 173}{567 \times 11.52}$	53.7
4. $\frac{1590 \times 3.64 \times 0.763}{4.39 \times 930}$	1.081
5. $\frac{0.0237 \times 3970 \times 32 \times 6.28}{0.00029 \times 186000}$	351
6. $\frac{231 \times 58.6 \times 4930}{182.5 \times 3770}$	97.0
7. $\frac{875 \times 1414 \times 2.01}{661 \times 35.9}$	104.8
8. $\frac{558 \times 1145 \times 633 \times 809}{417 \times 757 \times 354}$	2930
9. $\frac{0.691 \times 34.7 \times 0.0561}{91,500}$	0.0000147 or 1.47×10^{-5}
10. $\frac{19.45 \times 7.86 \times 361 \times 64.4}{32.6 \times 9.74}$	11,190

*This statement assumes that up to this point only the C and D scales are being used. Later sections will describe how this operation may be avoided by the use of other scales.

PROPORTION

Problems in proportion are very easy to solve. First notice that when the index of the C scale is opposite 2 on the D scale, the ratio 1 : 2 or $\frac{1}{2}$ is at the *same time* set for all other opposite graduations; that is, 2 : 4, or 3 : 6, or 2.5 : 5, or 3.2 : 6.4, etc. It is true in general that for any setting the numbers for *all pairs of opposite graduations have the same ratio.* Suppose one of the terms of a proportion is unknown. The proportion can be written as $\frac{a}{b} = \frac{c}{x}$, where a, b, and c, are known numbers and x is to be found.

***Rule:* Set a on the C scale opposite b on the D scale. Under c on the C scale read x on the D scale.**

EXAMPLE: Find x if $\frac{3}{4} = \frac{5}{x}$.

Set 3 on C opposite 4 on D. Under 5 on C read 6.67 on D.

The proportion above could also be written $\frac{b}{a} = \frac{x}{c}$, or "inverted," and exactly the same rule could be used. Moreover, if C and D are interchanged in the above rule, it will still hold if "under" is replaced by "over." It then reads as follows:

Set a on the D scale opposite b on the C scale. Over c on the D scale read x on the C scale. In solving proportions, keep in mind that the position of the numbers as set on the scales is the same as it is in the proportion written in the form $\frac{a}{b} = \frac{c}{d}$.

Proportions can also be solved *algebraically*. Then $\frac{a}{b} = \frac{c}{x}$ becomes $x = \frac{bc}{a}$, and this may be computed as combined multiplication and division.

PROBLEMS	ANSWERS
1. $\frac{x}{42.5} = \frac{13.2}{1.87}$	300.
2. $\frac{90.5}{x} = \frac{3.42}{1.54}$	40.7
3. $\frac{43.6}{89.2} = \frac{x}{2550}$	1247.
4. $\frac{0.063}{0.51} = \frac{34.1}{x}$	276.
5. $\frac{18}{91} = \frac{13}{x}$	65.7

PART 2. USE OF CERTAIN SPECIAL SCALES

USING THE CF AND DF SCALES

When π on the C scale is opposite the right index of the D scale, about half the slide projects beyond the rule. If this part were cut off and used to fill in the opening at the left end, the result would be a "folded" C scale, or CF scale. Such a scale is printed at the top of the slide. It begins at π and the index is near the middle of the rule. The DF scale is similarly placed. Any setting of C on D is automatically set on CF and DF. Thus if 3 on C is opposite 2 on D, then 3 on CF is also opposite 2 on DF. The CF and DF scales can be used for multiplication and division in exactly the same way as the C and D scales.

The most important use of the CF and DF scales is to avoid resetting the slide. If a setting of the indicator cannot be made on the C or D scale, it can be made on the CF or DF scale.

EXAMPLES:

(a) Find 19.2 × 6.38. Set left index of C on 19.2 of D. Note that 6.38 on C falls outside the D scale. Hence, move the indicator to 6.38 on the CF scale, and read the result 122.5 on the DF scale. Or set the index of CF on 19.2 of DF. Move indicator to 6.38 on CF and read 122.5 on DF.

(b) Find $\frac{8.39 \times 9.65}{5.72}$. Set 5.72 on C opposite 8.39 on D. The indicator cannot be moved to 9.65 of C, but it can be moved to this setting on CF and the result, 14.15, read on DF. Or the entire calculation may be done on the CF and DF scales.

These scales are also helpful in calculations involving π and $1/\pi$. When the indicator is set on any number N on D, the reading on DF is Nπ. This can be symbolized as (DF) = π(D). Then (D) = $\frac{(DF)}{\pi}$. This leads to the following simple rule.

***Rule:* If the diameter of a circle is set on D, the circumference may be read immediately on DF, and conversely.**

EXAMPLES:

(a) Find 5.6π. Set indicator over 5.6 on D. Read 17.6 under hairline on DF.

(b) Find 8/π. Set indicator over 8 on DF. Read 2.55 under hairline on D.

(c) Find the circumference of a circle whose diameter is 7.2. Set indicator on 7.2 of D. Read 22.6 on DF.

(d) Find the diameter of a circle whose circumference is 121. Set indicator on 121 of DF. Read 38.5 on D.

Finally, these scales are useful in changing radians to degrees and conversely. Since π radians = 180 degrees, the relationship may be written as a proportion $\frac{r}{d} = \frac{\pi}{180}$, or $\frac{r}{\pi} = \frac{d}{180}$.

***Rule:* Set 180 of C opposite π on D. To convert radians to degrees, move indicator to r (the number of radians) on DF, read d (the number of degrees) on CF; to convert degrees to radians, move indicator to d on CF, read r on DF.**

There are also other convenient settings as suggested by the proportion. Thus one can set the ratio $\pi/180$ on the CF and DF scales and find the result from the C and D scales.

EXAMPLES:

(a) The numbers 1, 2, and 7.64 are the measures of three angles in radians. Convert to degrees. Set 180 of C on π of D. Move indicator to 1 on DF, read 57.3° on CF. Move indicator over 2 of DF, read 114.6°. Move indicator to 7.64 of DF. Read 437° on CF.

(b) Convert 36° and 83.2° to radians. Use the same setting as in (a) above. Locate 36 on CF. Read 0.628 radians on DF. Locate 83.2 on CF. Read 1.45 radians on DF.

PROBLEMS	ANSWERS
1. 1.414×7.79	11.02
2. 2.14×57.6	123.3
3. $\frac{84.5 \times 7.59}{36.8}$	17.43
4. $2.65 \times \pi$	8.33
5. $\frac{.1955 \times 23.7}{50.7 \times \pi}$	.0291
6. $\frac{2.15 \times 16.35 \times 516 \times \pi}{.655 \times 9620}$	9.04

THE CI AND CIF SCALES

The CI scale on the slide is a C scale which *increases from right to left.* It may be used for finding reciprocals. When any number is set under the hairline on the C scale its reciprocal is found under the hairline on the CI scale, and conversely.

EXAMPLES:

(a) Find 1/2.4. Set 2.4 on C. Read .417 directly above on CI.

(b) Find 1/60.5. Set 60.5 on C. Read .1652 directly above on CI. Or, set 60.5 on CI, read .1652 directly below on C.

The CI scale is useful in replacing a division by a multiplication. Since $\frac{a}{b} = a \times 1/b$, any division may be done by multiplying the numerator (or dividend) by the reciprocal of the denominator (or divisor). This process may often be used to avoid settings in which the slide projects far outside the rule.

EXAMPLES:

(a) Find $13.6 \div 87.5$. Consider this as $13.6 \times 1/87.5$. Set left index of the C scale on 13.6 of the D scale. Move hairline to 87.5 on the CI scale. Read the result, .155, on the D scale.

(b) Find $72.4 \div 1.15$. Consider this as $72.4 \times 1/1.15$. Set right index of the C scale on 72.4 of the D scale. Move hairline to 1.15 on the CI scale. Read 62.9 under the hairline on the D scale.

An important use of the CI scale occurs in problems of the following type.

EXAMPLE: Find $\frac{13.6}{4.13 \times 2.79}$.

This is the same as $\frac{13.6 \times (1/2.79)}{4.13}$.

Set 4.13 on the C scale opposite 13.6 on the D scale. Move hairline to 2.79 on the CI scale, and read the result, 1.180, on the D scale.

By use of the CI scale, factors may be transferred from the numerator to the denominator of a fraction (or vice-versa) in order to make the settings more readily. Also, it is sometimes easier to get $a \times b$ by setting the hairline on a, pulling b on the CI scale under the hairline, and reading the result on the D scale under the index.

The CIF scale is a folded CI scale. Its relationship to the CF and DF scales is the same as the relation of the CI scale to the C and D scales.

EXAMPLES:

(a) Find 68.2×1.43. Set the indicator on 68.2 of the D scale. Observe that if the left index is moved to the hairline the slide will project far to the right. Hence merely move 14.3 on CI under the hairline and read the result 97.5 on D at the C index.

(b) Find $2.07 \times 8.4 \times 16.1$. Set indicator on 2.07 on C. Move slide until 8.4 on CI is under hairline. Move hairline to 16.1 on C. Read 280 on D under hairline. Or, set the index of CF on 8.4 of DF. Move indicator to 16.1 on CF, then move slide until 2.07 on CIF is under hairline. Read 280 on DF above the index of CF. Or set 16.1 on CI opposite 8.4 on D. Move indicator to 2.07 on C, and read 280 on D. Although several other methods are possible, the first method given is preferable.

It should be understood that the use of the CI, CF, DF, and CIF scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

PROBLEMS	ANSWERS
1. $\frac{1}{7}$	.143
2. $\frac{1}{35.2}$	.0284
3. $\frac{1}{.1795}$	5.57
4. $\frac{1}{6430}$	.0001556
5. $\frac{1}{\pi}$	.318
6. $\frac{1}{.00417}$	240

SQUARE ROOTS AND SQUARES

When a number is multiplied by itself the result is called the *square* of the number. Thus 25 or 5×5 is the square of 5. The factor 5 is called the *square root* of 25. Similarly, since $12.25 = 3.5 \times 3.5$, the number 12.25 is called the square of 3.5; also 3.5 is called the square root of 12.25. Squares and square roots are easily found on a slide rule.

Square Roots: To find square roots the A and D scales or the B and C scales are used.

***Rule:* The square root of any number located on the A scale is found below it on the D scale.**

Also, the square root of any number located on the B scale (on the slide) is found on the C scale (on the reverse side of the slide).

EXAMPLES: Find the $\sqrt{4}$. Place the hairline of the indicator over 4 on the left end of the A scale. The square root, 2, is read below on the D scale. Similarly the square root of 9 (or $\sqrt{9}$) is 3, found on the D scale below the 9 on the left end of the A scale.

Reading the Scales: The A scale is a contraction of the D scale itself. The D scale has been shrunk to half its former length and printed twice on the same line. To find the square root of a number between 0 and 10 the left half of the A scale is used (as in the examples above). To find the square root of a number between 10 and 100 the right half of the A scale is used. For example, if the hairline is set over 16 on the right half of the A scale (near the middle of the rule), the square root of 16, or 4, is found below it on the D scale.

In general, to find the square root of any number with an odd number of digits or zeros (1, 3, 5, 7, . . .), the left half of the A scale is used. If the number has an even number of digits or zeros (2, 4, 6, 8, . . .), the right half of the A scale is used. In these statements it is assumed that the number is not written in standard form.

The table below shows the number of digits or zeros in the number N and its square root.

	ZEROS					or	DIGITS				
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.	
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.	

This shows that numbers from 1 up to 100 have one digit in the square root; numbers from 100 up to 10,000 have two digits in the square root, etc. Numbers which are less than 1 and have, for example, either two or three zeros, have only one zero in the square root. Thus $\sqrt{0.004} = 0.0632$, and $\sqrt{0.0004} = 0.02$.

EXAMPLES:

(a) Find $\sqrt{248}$. This number has 3 (an *odd* number) digits. Set the hairline on 248 of the left A scale. Therefore the result on D has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000}$. The number has 6 (an *even* number) digits. Set the hairline on 563 of the right A scale. Read the figures of the square root on the D scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. The number of *zeros* is 4 (an *even* number.) Set the hairline on 1362 of the right half of the A scale. Read the figures 369 on the D Scale. The result has 2 zeros, and is .00369.

If the number is written in standard form, the following rule may be used. If the exponent of 10 is an even number, use the left half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the original. If the exponent of 10 is an odd number, move the decimal point one place to the right and decrease the exponent of 10 by one, then use the right half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the reduced exponent. This rule applies to either positive or negative exponents of 10.

EXAMPLES:

(1) Find the square root of 3.56×10^4. Place hairline of indicator on 3.56 on the left half of the A scale and read 1.887 on the D scale. Then the square root is 1.887×10^2.

(2) Find the square root of 7.43×10^{-5}. Express the number as 74.3×10^{-6}. Now place the hairline of the indicator over 74.3 on the right half of the A scale and read 8.62 on the D scale. Then the desired square root is 8.62×10^{-3}.

All the above rules and discussion can be applied to the B and C scales if it is more convenient to have the square root on the slide rather than on the body of the rule.

***Squares:* *To find the square of a number, reverse the process* for finding the square root. Set the indicator over the number on the D scale and read the square of that number on the A scale; or set the indicator over the number on the C scale and read the square on the B scale.**

EXAMPLES:

(a) Find $(1.73)^2$ or 1.73×1.73. Locate 1.73 on the D scale. On the A scale find the approximate square 3.

(b) Find $(62800)^2$. Locate 628 on the D scale. Find 394 above it on the A scale. The number has 5 digits. Hence the square has either 9 or 10 digits. Since, however, 394 was located on the right half of the A scale, the square has the *even* number of digits, or 10. The result is 3,940,000,000.

(c) Find $(.000254)^2$. On the A scale read 645 above the 254 of the D scale. The number has 3 zeros. Since 645 was located on the side of the A scale for "odd zero" numbers, the result has 7 zeros, and is 0.0000000645.

PROBLEMS	ANSWERS
1. $\sqrt{7.3}$	2.7
2. $\sqrt{73}$	8.54
3. $\sqrt{841}$	29
4. $\sqrt{0.062}$	0.249
5. $\sqrt{0.00000094}$	0.00097
6. $(3.95)^2$	15.6
7. $(48.2)^2$	2320
8. $(0.087)^2$	0.00757
9. $(0.00284)^2$	0.00000807
10. $(635000)^2$	4.03×10^{11}

CUBE ROOTS AND CUBES

Just below the D scale on the back of the rule is a scale marked with the letter K; this scale may be used in finding the cube or cube root of any number.

***Rule:* The cube root of any number located on the K scale is found directly above on the D scale.**

EXAMPLE: Find the $\sqrt[3]{8}$. Place the hairline of the indicator over the 8 at the left end of the K scale. The cube root, 2, is read directly above on the D scale.

Reading the scales: The cube root scale is directly below the D scale and is a contraction of the D scale itself. The D scale has been shrunk to one third its former length and printed three times on the same line. To find the cube root of any number between 0 and 10 the left third of the K scale is used. To find the cube root of a number between 10 and 100 the middle third is used. To find the cube root of a number between 100 and 1000 the right third of the K scale is used to locate the number.

In general to decide which part of the K scale to use in locating a number, mark off the digits in groups of three starting from the decimal point. If the left group contains one digit, the left third of the K scale is used; if there are two digits in the left group, the middle third of the K scale is used; if there are three digits, the right third of the K scale is used. In other words, numbers

containing 1, 4, 7, ··· digits are located on the left third; numbers containing 2, 5, 8, ··· digits are located on the middle third; and numbers containing 3, 6, 9, ··· digits are located on the right third of the K scale. The corresponding number of digits or zeros in the cube roots are shown in the table below.

	ZEROS				or	DIGITS			
N	11, 10, 9	8, 7, 6	5, 4, 3	2, 1	0	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12
$\sqrt[3]{N}$	3	2	1	0	0	1	2	3	4

EXAMPLES:

(a) Find $\sqrt[3]{6.4}$. Set hairline over 6.4 on the left most third of the K scale. Read 1.857 on the D scale.

(b) Find $\sqrt[3]{64}$. Set hairline over 6.4 on the middle third of the K scale. Read 4 on the D scale.

(c) Find $\sqrt[3]{640}$. Use the right most third of the K scale, and read 8.62 on the D scale.

(d) Find $\sqrt[3]{6{,}400}$. Use the left third of the K scale, but read 18.57 on the D scale.

(e) Find $\sqrt[3]{64{,}000}$. Use the middle third of the K scale, but read 40 on the D scale.

(f) Find $\sqrt[3]{0.0064}$. Use the left third of the K scale, since the first group of three, or 0.006, has only one non-zero digit. The D scale reading is then 0.1857.

(g) Find $\sqrt[3]{0.064}$. Use the middle third of the K scale, reading 0.4 on D.

If the number is expressed in standard form it can either be written in ordinary form or the cube root can be found by the following rule.

Rule: Make the exponent of 10 a multiple of three, and locate the number on the proper third of the K scale. Read the result on the D scale and multiply this result by 10 to an exponent which is ⅓ the former exponent of 10.

Examples: Find the cube root of 6.9×10^3. Place the hairline over 6.9 on the left third of the K scale and read 1.904 on the D scale. Thus the desired cube root is 1.904×10^1. 2) Find the cube root of 4.85×10^7. Express the number as 48.5×10^6 and place the hairline of the indicator over 48.5 on the middle third of the K scale. Read 3.65 on the D scale. Thus the desired cube root is 3.65×10^2. 3) Find the cube root of 1.33×10^{-4}. Express the number as 133×10^{-6} and place the hairline over 133 on the right third of the K scale. Read 5.10 on the D scale. The required cube root is 5.10×10^{-2}.

***Cubes:* To find the cube of a number, reverse the process for finding cube root. Locate the number on the D scale and read the cube of that number on the K scale.**

EXAMPLES:

(a) Find $(1.37)^3$. Set the indicator on 1.37 of the D scale. Read 2.57 on the K scale.

(b) Find $(13.7)^3$. The setting is the same as in example (a), but the K scale reading is 2570, or 1000 times the former reading.

(c) Find $(2.9)^3$ and $(29)^3$. When the indicator is on 2.9 of D, the K scale reading is 24.4. The result for 29^3 is therefore 24,400.

(d) Find $(6.3)^3$. When the indicator is on 6.3 of D, the K scale reading is 250.

PROBLEMS	ANSWERS
1. 2.45^3	14.7
2. 56.1^3	177,000
3. $.738^3$	.402
4. 164.5^3	4,450,000
5. $.0933^3$	.000812
6. $\sqrt[3]{5.3}$	1.744
7. $\sqrt[3]{71}$	4.14
8. $\sqrt[3]{815}$	9.34
9. $\sqrt[3]{.0315}$	.316
10. $\sqrt[3]{525{,}000}$	80.7
11. $\sqrt[3]{.156}$	.538

LOGARITHMS

The L scale is used for finding the logarithm (to the base 10) of any number.

Rule. **Locate the number on the D scale, and read the mantissa of its logarithm (to base 10) on the L scale.**

EXAMPLE: Find log 425. Set the hairline over 425 on the D scale. Read the mantissa of the logarithm (.628) on the L scale. Since the number 425 has three digits, the characteristic is 2 and the logarithm is 2.628.

If the logarithm of a number is known, the number itself may be found by reversing the above process.

EXAMPLE: If log X = 3.248, find X. Set the hairline over 248 of the L scale. Above it read the number 177 on the D scale. Then X = 1770 approximately.

EXAMPLE: Find log .000627. Opposite 627 on the D scale find .797 on the L scale. Since the number has 3 zeros, the characteristic is −4, and the logarithm is usually written 6.797−10, or 0.797−4.

PROBLEMS	ANSWERS
1. log 3.26	.513
2. log 735	2.866
3. log .0194	8.288–10
4. log 54800	4.739
5. log .931	9.969–10
6. log x = .357	x = 2.28
7. log x = 2.052	x = 112.7
8. log x = 1.598	x = 39.6
9. log x = 9.831–10	x = .678
10. log x = 7.154–10	x = .001426

TRIGONOMETRY

Sines and Cosines

The scale marked S is used in finding the approximate sine or cosine of any angle between 5.7 degrees and 90 degrees. Since sin x = cos (90 − x), the same graduations serve for both sines and cosines. Thus sin 6° = cos (90 − 6)° = cos 84°. The numbers printed at the right of the longer graduations are read when sines are to be found. Those printed at the left are used when cosines are to be found. Angles are divided decimally instead of into minutes and seconds. Thus sin 12.7° is represented by the 7th small graduation to the right of the graduation marked 78|12.

Sines (or cosines) of all angles on the S scale have no digits or zeros—the decimal point is at the left of figures read from the C (or D) scale.

Rule: **To find the sine or cosine of an angle on the S scale, set the hairline of the indicator on the graduation which represents the angle. Read the sine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the sine can also be read on a D scale, and the mantissa of the logarithm of the sine (log sin) may also be read on the L scale.**

EXAMPLES:

(a) Find sin x and also log sin x when x = 15°30′. Set left index of C scale over left index of D scale. Set hairline on 15.5° (i.e., 15° 30′). Read sin x = .267 on the C scale. Read .427 on the L scale. Then the log sin x = 9.427–10.

(b) Find cos x and log cos x when x = 42° 15′ (or x = 42.25°). Observe that the cosine scale decreases from left to right, or *increases from right to left.* Set the hairline over 42.25 on the S scale (reading from the right). Find cos 42.25 = .740 on C scale. Find .869 on L scale. Hence log cos 42° 15′ = 9.869–10.

Tangents and Cotangents

The T scale, together with the C or CI scales, is used to find the value of the tangent or cotangent of angles between 5.7° and 84.3°. Since tan x = cot (90 − x), the same graduations serve for both tangents and cotangents. For example, if the indicator is set on the graduation marked 60|30, the corresponding reading on the C scale is .577, the value of tan 30°. This is also the value of cot 60°, since tan 30° = cot (90° − 30°) = cot 60°. Moreover, tan x = 1/cot x; in other words, the tangent and cotangent of the same angle are reciprocals. Thus for the same setting, the reciprocal of cot 60°, or 1/.577, may be read on the CI scale as 1.732. This is the value of tan 60°. These relations lead to the following rule.

Rule: **Set the angle value on the T scale and read**
(i) tangents of angles from 5.7° to 45° on C,
(ii) tangents of angles from 45° to 84.3° on CI,
(iii) cotangents of angles from 45° to 84.3° on C
(iv) cotangents of angles from 5.7° to 45° on CI.

If the slide is set so that the C and D scales coincide, these values may also be read on the D scale. Care must be taken to note that the T scale readings for angles between 45° and 84.3° *increase from right to left.*

In case (i) above, the tangent ratios are all between 0.1 and 1.0; that is, the decimal point is at the left of the number as read from the C scale.

In case (ii), the tangents are greater than 1.0, and the decimal point is placed to the right of the first digit as read from the CI scale. For the cotangent ratios in cases (iii) and (iv) the situation is *reversed.* Cotangents for angles between 45° and 84.3° have the decimal point at the left of the number read from the C scale. For angles between 5.7° and 45° the cotangent is greater than 1 and the decimal point is to the right of the first digit read on the CI scale. These facts may be summarized as follows.

Rule: **If the tangent or cotangent ratio is read from the C scale, the decimal point is at the *left* of the first digit read. If the value is read from the CI scale, it is at the *right* of the first digit read.**

EXAMPLES:

(a) Find tan x and cot x when $x = 9°50'$. First note that $50' = \frac{50}{60}$ of 1 degree = .83°, approximately. Hence 9° 50′ = 9.83°. Locate $x = 9.83°$ on the T scale. Read tan $x = .173$ on the C scale, and read cot $x = 5.77$ on the CI scale.

(b) Find tan x and cot x when $x = 68.6°$. Locate $x = 68.6°$ on the T scale reading from right to left. Read 255 on the CI scale. Since all angles greater than 45° have tangents greater than 1 (that is, have one digit as defined above), tan $x = 2.55$. Read cot 68.6° = .392 on the C scale.

Finding the Angle

If the value of the trigonometric ratio is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value of the ratio is set on the C or CI scale, and the angle itself read on the S scale or T scale, depending upon which function is given.

EXAMPLES:

(a) Given sin $x = .465$, find x. Set indicator on 465 of C scale, read $x = 27.7°$ on the S scale.

(b) Given cos $x = .289$, find x. Set indicator on 289 on C scale. Read $x = 73.2°$ on the S scale.

(c) Given tan $x = .324$, find x. Set 324 on the C scale, read 17.95° on the T scale.

(d) Given tan $x = 2.66$, find x. Set 266 on the CI scale, read $x = 69.4°$ on the T scale.

(e) Given cot $x = .630$, find x. Set .630 on the C scale, read $x = 57.8°$ on the T scale.

(f) Given cot $x = 1.865$, find x. Set 1865 on the CI scale, read 28.2° on the T scale.

PROBLEMS	ANSWERS
1. sin 37°	.602
2. cos 79.3°	.1857
3. tan 18.6°	.3365
4. tan 66.4°	2.29
5. cot 31.7°	1.619
6. cot 83.85°	.1077
7. cos 11°	.982
8. sin 55.5°	.824
9. arc sin .438	26°
10. arc cos .1935	78.84°
11. arc tan 1.173	49.6°
12. arc cot .387	68.85°

Small Angles and Other Functions

The sine and the tangent of angles of less than about 5.7° are so nearly equal that a single scale, marked ST, may be used for both. The graduation for 1° is marked with the degree symbol (°). To the left of it the primary graduations represent tenths of a degree. The graduation for 2° is just about in the center of the slide. The graduations for 1.5° and 2.5° are also numbered.

Rule: **For small angles, set the indicator over the graduation for the angle on the ST scale, then read the value of the sine or tangent on the C scale. Sines or tangents of angles on the ST scale have one zero.**

EXAMPLES:

(a) Find sin 2° and tan 2°. Set the indicator on the graduation for 2° on the ST scale. Read sin 2° = .0349 on the C scale. This is also the value of tan 2° correct to three digits.

(b) Find sin 0.94° and tan 0.94°. Set the indicator on 0.94 of ST. Read sin 0.94° = tan 0.94° = .0164 on the C scale.

Since cot $x = 1/\tan x$, the cotangents of small angles may be read on the CI scale. Moreover, tangents of angles between 84.3° and 89.42° can be found by use of the relation tan x = cot $(90 - x)$. Thus cot 2° = 1/tan 2° = 28.6, and tan 88° = cot 2° = 28.6. Finally, it may be noted that csc $x = 1/\sin x$, and sec $x = 1/\cos x$. Hence the value of these ratios may be readily found if they are needed. Functions of angles greater than 90° may be converted to equivalent (except for sign) functions in the first quadrant.

EXAMPLES:

(a) Find cot 1.41° and tan 88.59°. Set indicator at 1.41° on ST. Read cot 1.41° = tan 88.59° = 40.7 on CI.

(b) Find csc 21.8° and sec 21.8°. Set indicator on 21.8° of the S scale. Read csc 21.8° = 1/sin 21.8° = 2.69 on CI. Set indicator on 68.2° of the S scale (or 21.8 reading from right to left), and read sec 21.8° = 1.077 on the CI scale.

When the angle is less than 0.57° the approximate value of the sine or tangent can be obtained directly from the C scale by the following procedure.

Read the ST scale as though the decimal point were at the left of the numbers printed, and read the C scale (or D, CI, etc.) with the decimal point one place to the left of where it would normally be. Thus sin 0.2° = 0.00349; tan 0.16° = 0.00279, read on the C scale.

Two seldom used special graduations are also placed on the ST scale. One is indicated by a longer graduation found just to the left of the graduation for 2° at about 1.97°. When this graduation is set opposite any number of minutes on the D scale, the sine (or the tangent) of an angle of that many minutes may be read on the D scale under the C index.

Sin 0° = 0, and sin 1′ = .00029, and for small angles the sine increases by .00029 for each increase of 1′ in the angle. Thus sin 2′ = .00058; sin 3.44′ = .00100, and the sines of all angles between 3.44′ and 34.4′ have two zeros. Sines of angles between 34.4′ and 344′ (or 5.73°) have one zero. The tangents of these small angles are very nearly equal to the sines.

Example: Find sin 6′. With the hairline set the "minute graduation" opposite 6 located on the D scale. Read 175 on the D scale under the C index. Then sin 6′ = .00175.

The second special graduation is also indicated by a longer graduation located at about 1.18°. It is used in exactly the same way as the graduation for minutes. Sin 1″ = .0000048, approximately, and the sine increases by this amount for each increase of 1″ in the angle, reaching .00029 for sin 60″ or sin 1′ = .00029.

COMBINED OPERATIONS

Many problems involve expressions like $\sqrt{ab}$, or $(a \sin \theta)^2$, etc. With a little care, many such problems involving combined operations may be easily computed. The list of possibilities is extensive, and it is no real substitute for the thinking which should suggest them. Consequently, only a few examples will be given.

The A and B scales may be used for multiplication or division in exactly the same way as the C and D scales. Since the scales are shorter, there is some loss in accuracy. Nevertheless, most computers employ the A and B scales (in conjunction with the C and D scales) to avoid extra steps which would also lead to loss of accuracy.

Example:

(a) Find $\sqrt{3.25 \times 4.18}$. First find the product 3.25 × 4.18 using the left A and B scales. Set left index of B on 3.25 of A. Move indicator to 4.18 of B. Read the square root of this product under the hairline on D. The result is 3.69 approximately.

(b) Find $(13.2 \sin 28°)^2$. Set S index on 13.2 of D. Move indicator to 28° on S. Read the result (after squaring) on A as 0.383.

(c) Find $5^{\frac{3}{2}}$ or $5^{1.5}$. This is the same as $(\sqrt{5})^3$. Hence set the indicator on 5 of the left A scale. Read 11.2 on the middle K scale under the hairline.

(d) Find $24^{\frac{2}{3}}$ or $(\sqrt[3]{24})^2$. Set the indicator on 24 of the middle K scale. Read the result after squaring as 8.3 on the A scale under the hairline.

Many formulas involve both trigonometric ratios and other factors. By using several different scales such computations are easily done.

Examples:

(a) Find the length of the legs of a right triangle in which the hypotenuse is 48.3 ft. and one acute angle is 25° 20′.

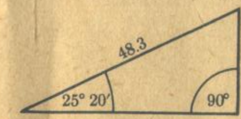
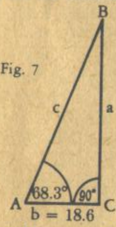


Fig. 6

The side opposite the given acute angle is equal to 48.3 sin 25° 20′. Hence we compute 48.3 × sin 25.3°. Set the index (right-hand index in this example) of the C scale on 48.3 of the D scale. Move the hairline over 25.3° on the S scale. Read 20.7 under the hairline on the D scale. Another method is to set the left index of the C scale and D scale opposite each other. Set the hairline over 25.3° on the S scale. Move the slide so that (right) index of the C scale is under the hairline. Read 20.7 on the D scale under 48.3 of the C scale. The length of the other leg is equal to 48.3 cos 25.3° or 48.3 sin 64.7° = 43.7.

(b) One angle of a right triangle is 68.3°, and the adjacent side is 18.6 ft. long. Find the other side and the hypotenuse.

Fig. 7



a = 18.6 tan 68.3° or 18.6/cot 68.3

c = 18.6/cos 68.3°.

To find *a*, set the indicator on 18.6 of the D scale, pull the slide until 68.3 of the T scale (read from right to left) is under the hairline, and read *a* = 46.7′ on the D scale under the right index of the C scale. To find c, pull the slide until 68.3° of the S scale (read from right to left) is under the hairline (which remains over 18.6), and read the result 50.3′ on the D scale at the right index.

This problem may also be solved by the law of sines, namely,

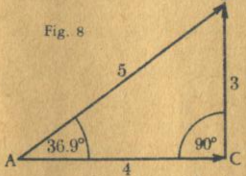
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}, \quad \text{or} \quad \frac{\sin 68.3}{a} = \frac{\sin 21.7}{18.6} = \frac{1}{c}$$

Set 21.7° on S opposite 18.6 on D. Read c = 50.3 on D under 1 of C. Move indicator to 68.3° on S, read 46.7 under the hairline on D.

The various computations used in solving triangles may be greatly simplified by use of the slide rule. Knowledge of various trigonometric relations is essential for trustworthy results, and these relations cannot be developed in this brief manual. It must suffice to say that once the proper relations are written down (for example, the law of sines) with the given data substituted, the general principles outlined above will suggest the proper settings to obtain the unknown parts. However, additional instruction with respect to *vector* calculations by use of the S and T scales is given below.

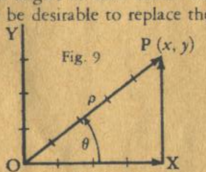
A *vector* quantity is one which has both *magnitude* and *direction*. For example, force and velocity are vector quantities. A quantity which has magnitude only is called a *scalar*. For example, mass is a scalar. Vector quantities are often represented by directed straight line segments. The length of the segment represents the magnitude in terms of a selected scale unit. The segment has an initial point A and a terminal point B, and direction is usually indicated by an arrowhead at B pointing in the same direction as the motion of a point which travels from A to B. In Fig. 10, three vectors are represented; namely AB of magnitude 5, AC of magnitude 4, and CB of magnitude 3. Vectors AB and AC have the same initial point, A, and form an angle, CAB, of 36.9°. The initial point of vector CB is at the terminal point of AC. Vectors CB and AB have the same terminal point.

Fig. 8



Operations with vectors (for example, addition and multiplication) are performed according to special rules. Thus in Fig. 10, AB may be regarded as the *vector sum* of AC and CB. AB is called the *resultant* of AC and CB; the latter are *components* of AB, and in this case are at right angles to each other. It is frequently desirable to express a given vector in terms of two such components at right angles to each other. Conversely, when the components are given, it may be desirable to replace them with the single resultant vector.

Fig. 9



In algebra, the complex number $x + iy$, where $i = \sqrt{-1}$, is represented by a point P (x, y) in the complex plane, using a coordinate system in which an axis of "pure imaginary" numbers, OY, is at right angles to an axis of "real" numbers, OX.

The same point can be expressed in terms of polar coordinates (ρ, θ) in which the radius vector OP from the origin of coordinates has length ρ and makes an angle θ with the X-axis. The two systems of representation are related to each other by the following formulas:

(1) $x = \rho \cos \theta$, (3) $\tan \theta = \frac{y}{x}$ or $\theta = \arctan \frac{y}{x}$

(2) $y = \rho \sin \theta$, (4) $\rho = \sqrt{x^2 + y^2}$

Finally, the complex number $x + iy$ may be regarded as a vector given in terms of its components x and y and the complex operator $i = \sqrt{-1}$. In practical work the symbol j is preferred to i, to avoid confusion with the symbol often used for the *current* in electricity.

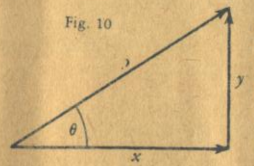
The "Euler identity" $e^{j\theta} = \cos \theta + j \sin \theta$ can be proved by use of the series expansions of the functions involved. Then $\rho e^{j\theta}$ is an *exponential* representation of the complex number $x + jy$, since $\rho e^{j\theta} = \rho \cos \theta + j\rho \sin \theta = x + jy$. The notation is often simplified by writing $\rho \,\underline{/\theta}$ in place of $\rho e^{j\theta}$.

If two or more complex numbers are to be added or subtracted, it is convenient to have them expressed in the form $x + jy$, since if $N_1 = x_1 + jy_1$, and $N_2 = x_2 + jy_2$, then $N_1 + N_2 = (x_1 + x_2) + j(y_1 + y_2)$. If, however, two or more complex numbers are to be multiplied, it is convenient to have them expressed in the exponential form. Then if $N_1 = \rho_1 e^{j\theta_1}$ and $N_2 = \rho_2 e^{j\theta_2}$, then $N_1 N_2 = \rho_1 \rho_2 e^{j(\theta_1 + \theta_2)}$, or $(\rho_1 \,\underline{/\theta_1})\,(\rho_2 \,\underline{/\theta_2}) = \rho_1 \rho_2 \,\underline{/\theta_1 + \theta_2}$.

It is therefore necessary to be able to change readily from either of these representations of a complex number to the other.

Changing from Components to Exponential Form

Fig. 10



If a complex number $x + jy$ (or vector in terms of perpendicular components) is given, the problem of changing to the form $\rho \,\underline{/\theta}$ is equivalent to finding the hypotenuse and one acute angle of a right triangle. The formulas $\tan \theta = \frac{y}{x}$ and $\rho = y/\sin \theta$, or $\rho = x/\cos \theta$, are the basis of the solution. Thus if $N = 4 + j3$, when 4 of C is set opposite 3 of D, the value of the ratio $\frac{y}{x}$, or $\frac{3}{4} = .75$ is read on D under the C index.

If the indicator is set at the index, and the slide moved so that .75 is under the hairline, the value of $\theta = 36.9°$ may be read on the T scale. Then $\rho = 3/\sin 36.9$ may be computed by moving the indicator to 3 on the D scale, pulling 36.9 on the S scale under the hairline, and reading $\rho = 5$ on the D scale opposite the left index of C. However, this method involves several unnecessary settings and is thus more subject to error than the method given in the general rule below.

Observe that if x and y are both positive and $x = y$, then $\tan \theta = 1$ and $\theta = 45°$. If $y < x$, then $\theta < 45°$; if $y > x$ then $\theta > 45°$. Thus if $y < x$, the T scale is read from *left* to *right*. If $y > x$, the T scale is read from *right* to *left*.

Rule: **(i) To the *larger* of the two numbers (x, y) on D set an index of the slide. Set the indicator over the smaller value on D and read θ on the T scale. If $y < x$, then $\theta < 45$. If $y > x$, then $\theta > 45$, and is read from right to left (or on the left of the graduation mark).**

(ii) Move the slide until θ on scale S is under the indicator, reading S on the same side of the graduation as in (i). Read ρ on D at the index of the C-scale.

Observe that the reading both begins and ends at an index of the slide. By this method the value of the ratio y/x occurs on the C (or CI) scale of the *slide* over the smaller of the two numbers, and the angle may be read immediately on the T scale without moving the slide. In using any method or rule, it is wise to keep a mental picture of the right triangle in mind in order to know whether to read θ on the T or on the ST scale. Thus if the ratio y/x is a small number, the angle θ is a small angle, and must be read on the ST scale. To be precise, if $y/x < 0.1$, the ST scale must be used. Similarly, if the ratio

$y/x > 10$, the angle θ will be larger than 84.3° and cannot be read on the T scale. The complementary angle $\varphi = (90 - \theta)$ will, however, then be on the ST scale, and then θ may be found by subtracting the reading on the ST scale from 90°, since $\theta = 90 - \varphi$.

EXAMPLES:

(a) Change $2 + j3.46$ to exponential or "vector" form. Note $\theta > 45$, since $y > x$ (or $3.46 > 2$). Set right index of S opposite 3.46 on D. Move indicator to 2 on D. Read $\theta = 60°$ on T at the *left* of the hairline. Move slide until 60° on scale S is under the hairline (numerals on the *left*), and read $\rho = 4$ on the D scale at the C-index. Then $2 + j3.46 = 4\rho e^{j60} = 4\ \underline{/60°}$.

(b) Change $3 + j2$ to exponential or vector form. Note that $\theta < 45°$ since $y < x$ (second component less than first). Set right index of S over 3 on D. Move indicator to 2 on D, read $\theta = 33.7°$ on T (use numerals on the right-hand side of graduations). Move hairline to 33.7° on S. Read $\rho = 3.60$ on D under index. Hence $3 + j2 = 3.60\ \underline{/33.7}$.

(c) Change $2.34 + j.14$ to exponential form. Since $y < x$, then $\theta < 45°$. Moreover, the ratio y/x is a small number (actually about .06). Since the tangent has one zero, the angle may be read on the ST scale. Set right index of S opposite 2.34 of D. Move indicator to .14 on D. Read $\theta = 3.43°$ on ST. The slide need not be moved. The value of ρ is approximately 2.34. In other words, the angle is so small that the hypotenuse is approximately equal to the longer side. Then $2.34 + j.14 = 2.34\ \underline{/3.43}$.

(d) Change $1.08 + j26.5$ to exponential form. Here $y > x$, so that $\theta > 45°$. But $\frac{y}{x} = \frac{26.5}{1.08} > 10$. Set right index of S on 26.5 of D. Move indicator to 1.08 of D. Read $\varphi = 2.34°$ on ST. The slide need not be moved. The value of ρ is approximately 26.5; $\theta = 90 - 2.34° = 87.66°$. Hence $26.5\ \underline{/87.66°}$ is the required form.

If x and y are both positive, $\theta < 90°$. If x and y are *not* both positive, the resultant vector does not lie in the first quadrant, and θ is not an acute angle. In using the slide rule, however, x and y must be treated as both positive. It is therefore necessary to correct θ as is done in trigonometry when an angle is not in the first quadrant.

EXAMPLES:

(a) Find the angle between the X-axis and the radius vector for the complex number $-4 + j3$. First solve the problem as though both components were positive. The angle θ obtained is 36.9°. In this case the required angle is $180° - \theta = 180° - 36.9° = 143.1°$. Hence $-4 + j3 = 5\ \underline{/143.1°}$. Similarly for $-4 - j3$, the required angle is $180 + \theta = 180 + 36.9° = 216.9°$, so $-4 - j3 = 5\ \underline{/216.9°}$. For $4 - j3$ the required angle is $360° - \theta = 323.1°$, so $4 - j3 = 5\ \underline{/323.1°}$, which may also be expressed in terms of a negative angle as $5\ \underline{/-36.9°}$.

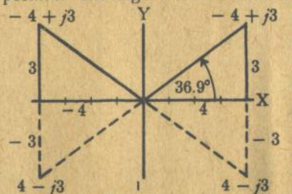


Fig. 11

(b) Change $17.2 - j6.54$ to exponential form. Here the ratio y/x is negative so θ can be expressed as a negative angle. In numerical value $y < x$, so the numerical or absolute value of $\theta < 45°$. Set left index of S opposite 17.2 on D. Move indicator over 6.54 of D, read $\theta = 20.8°$ on T. Pull 20.8 of S under hairline, read 18.4 on D at left index. Hence $17.2 - j6.54 = 18.4\ \underline{/-20.8°}$, or $18.4\ \underline{/339.2°}$.

Changing from Exponential Form to Components

The process of changing a complex number or vector from the form $\rho e^{j\theta} = \rho\ \underline{/\theta}$ to the form $x + jy$ depends upon the formulas $x = \rho \cos \theta$, $y = \rho \sin \theta$. These are simple multiplications using the C, D, and S (or ST) scales.

Rule: **Set an index of the S scale opposite ρ on the D scale. Move indicator to θ on the S (or ST) scale, reading from left to right (sines). Read y on the D scale. Moving indicator to θ on the S (or ST) scale, reading from right to left (cosines), read x on the D scale.**

If $\theta > 90°$ or $\theta < 0$, it should first be converted to the first quadrant, and the proper negative signs must later be associated with x or y.

EXAMPLES:

(a) Change $4\ \underline{/60°}$ to component form. Set right index of S on 4 of D. Move indicator to 60° on S (reading scale from left to right). Read 3.46 on D under hairline. Move indicator to 60° on S, reading scale from right to left (cosines). Read 2 on D under hairline. Hence $4\ \underline{/60°} = 2 + j3.46$.

(b) Change $16.3\ \underline{/15.4°}$ to the $x + jy$ form. Set left index of S on 16.3 of D. Move indicator to 15.4 of S, read 4.33 on D. Since 15.4° reading from right to left is off the D scale, exchange indices so the right index of C is opposite 16.3 of D. Move indicator to 15.4 of S, and read 15.7 on D. Hence $16.3\ \underline{/15.4°} = 15.7 + j4.33$.

(c) Change $7.91\ \underline{/3.25°}$ to component form. Set right index of S on 7.91 of D. Move indicator to 3.25 on ST. Read 0.448 on D. To determine the decimal point, observe that the angle is small, and hence the y component will also be small. Obviously, when the hypotenuse is near 8, 4.48 would be too large, and 0.0448 too small, to produce an angle of 3.25°. The cosine cannot be set on ST, but the angle is so small that the x-component is practically equal to the radius vector or hypotenuse. Hence 7.90 is a close approximation, and $7.91\ \underline{/3.25°} = 7.90 + j0.448$.

(d) Convert $263\ \underline{/160°}$ to the $x + jy$ form. Since $160° > 90°$, compute $180° - 160° = 20°$. Set left index of the S scale on 263 of D. Move indicator to 20° on S. Read 90.0 on D. Move the slide so that the right index of S is on 263 of D. Move indicator to 20 (reading from right to left) on S. Read 247 on D. Since the angle is in the second quadrant, $263\ \underline{/160°} = -247 + j90$.

ILLUSTRATIVE APPLIED PROBLEMS

1. Two forces of magnitude 28 units and 39 units act on the same body but at right angles to each other. Find the magnitude and angle of the resultant force.

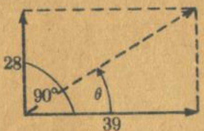


Fig. 12

In complex number notation, the resultant is $39 + j28$. Change this to exponential form. Since $28 < 39$, then $\theta < 45°$. Set the right index of S on 39 of D. Move indicator to 28 of D. Read $\theta = 35.6°$ on T. Move slide so 35.6° on S is under the hairline. Read $\rho = 48.0$ on D under the S-index. Hence the resultant has magnitude 48 units, and acts in a direction 35.6° from the larger force and $90 - 35.6°$ or 54.4° from the smaller force. This angle can be read on the T scale at the same time that θ is read.

2. A certain alternating generator has three windings on its armature. In each winding the induced voltage is 266.4 volts effective. The windings are connected in such a way that the voltages in each are given by the following vector expressions.

$$E_1 = 266.4\,(\cos 0° - j \sin 0°)$$

$$E_2 = 266.4\,(\cos 120° - j \sin 120°)$$
$$= 266.4 \cos 120° - j\,266.4 \sin 120°$$

$$E_3 = 266.4\,(\cos 240° - j \sin 240°)$$
$$= 266.4 \cos 240° - j\,266.4 \sin 240°$$

Express these numerically.

$$E_1 = 266.4\,(1 - j0) = 266.4 - j0$$

To find E_2, reduce the angles to first quadrant by taking $180° - 120° = 60°$. Set the right index of S on 266.4 of D. Move the indicator to 60° of S (reading right to left). Read 133.2 on D. Move indicator to 60° on S, read 230.7 on D. Then

$$E_2 = -\,133.2 - j230.7$$

To find E_3, reduce 240° to the first quadrant by noting $240° = 180° + 60°$. Hence, except for a negative sign, E_3 is the same as E_2, and

$$E_3 = -\,133.2 + j230.7$$

Suppose the first and second windings are so connected that their voltages subtract; that is,

$$E_0 = E_1 - E_2 = (266.4 - j0) - (-\,133.2 - j230.7) = 399.6 + j230.7$$

This may be changed to the $\rho \angle \theta$ form. Set the right index of S on 399.6 of D. Move the indicator to 230.7 of D. Read $\theta = 30°$ on T. Move slide so that 30° on S is under indicator, and read 461 on D at the S-index. Then $E_0 = 461 \angle 30°$, and hence the voltage is 461 volts and leads the voltage E_1 by 30°.

3. An alternating voltage of $104 + j60$ is impressed on a circuit such that the resulting current is $24 - j32$. Find the power and power factor. First convert each vector to exponential form.

$$E = 104 + j60 = 120 \angle 30° \text{ volts, approximately}$$
$$I = 24 - j32 = 40 \angle -53.1 \text{ amperes, approximately.}$$

Hence the voltage leads the current by $30° - (-53.1) = 83.1°$.
The power factor $\cos 83.1° = 0.120$.
The power $P = EI \cos \theta = (120)(40)(0.120) = 576$ watts, approximately.

4. The "characteristic impedance" of a section of a certain type of line is given by the formula $Z_0 = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$, where in each case, the symbol Z represents a vector quantity. Compute Z_0 when

$$Z_1 = 40 + j120,\ Z_2 = 220 - j110.$$

First convert to exponential form.

$$Z_1 = 40 + j120 = 126 \angle 71.6°$$
$$Z_2 = 220 - j110 = 246 \angle -26.6°$$

Hence

$$Z_1 Z_2 = (126)(246) \angle 71.6 - 26.6$$
$$= 31{,}000 \angle 45.0°$$
$$\frac{Z_1^2}{4} = \frac{126^2}{4} \angle 2(71.6)$$
$$= \frac{15{,}900}{4} \angle 143.2$$
$$= 3{,}975 \angle 143.2$$
$$Z = \sqrt{31{,}000 \angle 45.0° + 3{,}975 \angle 143.2}$$

Since vectors are to be added before the square root is found, it is now convenient to convert them to component form.

$$31{,}000 \angle 45.0° = 21{,}900 + j21{,}900$$
$$3{,}975 \angle 143° = -3{,}180 + j2{,}390$$

To compute the latter, take $180° - 143° = 37°$, compute the components using 37°, and observe that the x or real component must be negative since 143° is an angle in the second quadrant. Then

$$Z = \sqrt{(21{,}900 - 3180) + j(21{,}900 + 2390)}$$
$$= \sqrt{18{,}720 + j24{,}290}$$

In order to find the square root, it is convenient to change back to exponential form.

$$Z = \sqrt{18{,}720 + j24{,}290} = \sqrt{30{,}600 \angle 52.4°}$$
$$= 175 \angle 26.2° \text{ ohms.}$$

The final result is obtained by setting 30,600 on A and reading 175 on D; the angle 52.4° is merely divided by 2. This problem shows the value of being able to change readily from one form of vector representation to the other.

THE CI AND DI SCALES

It should be understood that the use of the CI, CF, DF, and DI scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

The CI scale on the slide is a C scale which *increases from right to left*. It may be used for finding reciprocals. When any number is set under the hairline on the C scale its reciprocal is found under the hairline on the CI scale, and conversely.

EXAMPLES:

(a) Find 1/2.4. Set 2.4 on C. Read .417 directly above on CI.

(b) Find 1/60.5. Set 60.5 on C. Read .0165 directly above on CI. Or, set 60.5 on CI, read .0165 directly below on C.

The CI scale is useful in replacing a division by a multiplication. Since $\frac{a}{b} = a \times 1/b$, any division may be done by multiplying the numerator (or dividend) by the reciprocal of the denominator (or divisor). This process may often be used to avoid settings in which the slide projects far outside the rule.

EXAMPLES:

(a) Find 13.6 ÷ 87.5. Consider this as 13.6 × 1/87.5. Set left index of the C scale on 13.6 of the D scale. Move hairline to 87.5 on the CI scale. Read the result, .155, on the D scale.

(b) Find 72.4 ÷ 1.15. Consider this as 72.4 × 1/1.15. Set right index of the C scale on 72.4 of the D scale. Move hairline to 1.15 on the CI scale. Read 63.0 under the hairline on the D scale.

An important use of the CI scale occurs in problems of the following type.

EXAMPLE: Find $\frac{13.6}{4.13 \times 2.79}$ This is the same as $\frac{13.6 \times (1/2.79).}{4.13}$

Set 4.13 on the C scale opposite 13.6 on the D scale. Move hairline to 2.79 on the CI scale, and read the result, 1.180, on the D scale.

By use of the CI scale, factors may be transferred from the numerator to the denominator of a fraction (or vice-versa) in order to make the settings more readily. Also, it is sometimes easier to get $a \times b$ by setting the hairline on a, pulling b on the CI scale under the hairline, and reading the result on the D scale under the index.

The DI scale (inverted D scale) below the D scale corresponds to the CI scale on the slide. Thus the D and DI scales together represent reciprocals. The DI scale has several important uses, of which the following is representative.

Expressions of the type $1/X$, where X is some complicated expression or formula, may be computed by first finding the value of X. If the result for X falls on D, then $1/X$ may be read under the hairline on DI.

Example:

(a) Find $\frac{1}{0.265 \times 138}$. Multiply 0.265×138 using the C and D scales. Read the reciprocal .0273 under the hairline on the DI scale. Or set the hairline on 265 of the DI scale, pull 138 of the C scale under the hairline, and read the result on the D scale under the left index of the C scale. This is equivalent to writing the expression as $\frac{(1/.265)}{138}$.

PROBLEMS	ANSWERS
1. $\frac{1}{7}$	.143
2. $\frac{1}{35.2}$	.0284
3. $\frac{1}{.1795}$	5.57
4. $\frac{1}{6430}$	.0001555
5. $\frac{1}{\pi}$.	.318
6. $\frac{1}{.00417}$	240