MATHEMATICS MADE EASY

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MATHEMATICS MADE EASY

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CONTENTS

ARITHMETIC

By far the most used form of Mathematics is Arithmetic, the science of numbers and computations, which you have been using daily in its simpler forms even if only to count your change after buying a package of cigarettes.

Fundamental Operations

The fundamental operations of arithmetic are addition, subtraction, multiplication, division, and the finding of squares, square roots, cubes, and cube roots.

We assume that you know how to add, subtract, multiply, and divide whole numbers. However, it is quite possible that you have forgotten how to perform these operations with fractions, mixed numbers, and decimals, and so, you are asked to give special attention to these items in the following paragraphs, and practice using fractions and decimals until you can handle them with facility.

Symbols

In discussing the arithmetical operations, it will be convenient to use certain symbols and terms. These are therefore given here:

$=$ equals	$\sqrt{\ }$ square root of
$+$ plus	$\sqrt[3]{\ }$ cube root of
$-$ minus	$(\ )^2$ squared
$\times$ times	$(\ )^3$ cubed
$\div$ divided by	

The Sum = all the parts added.

The Difference = the Minuend — the Subtrahend.

The Product = the Multiplicand × the Multiplier.

The Quotient = the Dividend ÷ the Divisor.

FRACTIONS

The use of fractions implies that a certain unit has been divided into a number of parts and that one or more of these parts are being considered. Suppose that a board five feet long is sawed into five equal lengths of one foot each. Then, the amount of wood in two such pieces would be two-fifths of the original amount of wood. The two-fifths is a fraction and is usually expressed as 2/5 or $\frac{2}{5}$ where the number below the line (5), called the **Denominator,** expresses the number of parts into which the whole is divided and the number above the line (2), called the **Numerator,** expresses the quantity of divisions being considered. The diagonal or horizontal line is called the **Fraction Line.**

The value of a fraction does not change if both its numerator and its denominator are multiplied or divided by the same amount. Thus

$$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$$

To reduce a fraction to its lowest terms, divide both numerator and denominator by the greatest number both can be divided by without a remainder, i. e., by the greatest common divisor. Thus to reduce $\frac{8}{20}$ to its lowest terms, divide both numerator and denominator by 4 (the greatest common divisor) obtaining $\frac{2}{5}$.

Mixed Numbers

Frequently, fractions are combined with whole numbers. For example, suppose you had two boards, each five feet long and the two one foot lengths that you considered in the preceding paragraphs. Expressing the amount of lumber, then, in terms of the five foot length, you would have two and two-fifths lengths, expressed as 2 2/5 lengths. This is known as a **Mixed Number** because it involves a whole number called the **Integer** or **Integral Part** and a fraction. In many of your calculations you will find it desirable to change Mixed Numbers into Pure Fractions. This can be done if we consider that a unit could be expressed as five-fifths, in which case then the above mixed number contains two units of five-fifths each, or ten-fifths, plus two-fifths, making a total of twelve-fifths. You say, then, that $2\frac{2}{5}$ is the same as $\frac{12}{5}$. For measuring purposes, the mixed number is more desirable but for calculations the pure fraction is desirable.

A fraction whose numerator is greater than its denominator is called an **Improper Fraction.**

To change an improper fraction to a mixed number, simply divide the numerator by the denominator. Thus

$$\frac{25}{12} = 25 \div 12 = 2\frac{1}{12}$$

Adding and Subtracting Fractions

In adding or subtracting pure fractions whose denominators are the same, we add or subtract the numerator of the fraction in the usual manner. Thus $\frac{3}{8}$ and $\frac{2}{8}$ are expressed in the same fractional unit (eighths). To add these two fractions, simply add their numerators and you obtain $\frac{5}{8}$. In subtracting, subtract the one numerator from the other and you obtain a difference of $\frac{1}{8}$.

If the fractions to be added or subtracted do not have the same denominator, the denominator must be changed so that they are the same. This may be done by increasing the denominator of one (or both) of the fractions, but it is important for you to remember that whenever you increase the denominator of a fraction, you must also increase the numerator if you do not want to change the value of the fraction. If you had a fraction of $\frac{3}{4}$ and you wanted to change that fraction so that it would have a denominator of 8, you would have to multiply the numerator by the same amount as you would multiply the denominator so that $\frac{3}{4}$ becomes $\frac{6}{8}$.

Now, suppose that you wanted to add the two fractions $\frac{3}{4}$ and $\frac{7}{8}$ which, as you see, do not have a common denominator. By multiplying both numerator and denominator of $\frac{3}{4}$ by 2 it becomes $\frac{6}{8}$ which then has the same denominator as $\frac{7}{8}$ and can be easily added or subtracted. The sum of $\frac{6}{8}$ and $\frac{7}{8}$ is $\frac{13}{8}$ or $1\frac{5}{8}$ while the difference between $\frac{7}{8}$ and $\frac{3}{4}$ is $\frac{1}{8}$.

Now, suppose you wish to add $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$ In this case all the denominators (and numerators) will have to be multiplied by a number such as to make all three changed denominators alike. You should select the smallest number which will be a multiple of 3, 4, and 5. In this case 60 is such a number and the fractions will be changed to

$$\frac{40}{60}+\frac{45}{60}+\frac{48}{60}=\frac{133}{60}=2\frac{13}{60}$$

In the above addition the 60 is known as the **Least Common Denominator.**

In adding mixed numbers, simply add the fractions and the integers separately. If the sum of the fractions results in an improper fraction, this may be changed to a mixed number whose integer will be added to the sum of the given integers. Thus

$$3\frac{5}{8}+4\frac{1}{8}+3\frac{3}{8}=10\frac{9}{8}=10+1\frac{1}{8}=11\frac{1}{8}$$

Multiplying and Dividing Fractions

To multiply a fraction by a whole number, the numerator **only** is multiplied by that number and the product is then written as the fraction whose numerator is the product of the original numerator and the whole number, and whose denominator remains the same. For example, suppose you multiply $\frac{5}{8}$ by 3. The product has for its numerator 5×3 or 15, while the denominator of the product remains as 8; thus, the answer is $\frac{15}{8}$ or $1\frac{7}{8}$.

To multiply one fraction by another fraction, multiply the numerators together and multiply the denominators together; thus,

$$\frac{3}{5}\times\frac{4}{7}=\frac{12}{35}$$

If you were to multiply $\frac{3}{5}\times\frac{5}{3}$ you would get $\frac{15}{15}$ or 1. When the product of two fractions is 1, one fraction is said to be the **reciprocal of the other.**

To divide by a fraction, you need merely multiply by its reciprocal; thus,

$$\frac{5}{8}\div\frac{3}{4}=\frac{5}{8}\times\frac{4}{3}=\frac{20}{24}\text{ or }\frac{5}{6}$$

Since the denominator is actually the number of parts that something has been divided into, the fraction line represents the same thing as a division sign. The numerator is the same as the dividend while the denominator is the divisor. For example, $5 \div 2 = \frac{5}{2}$.

Below you see various ways in which an identical calculation is indicated:

$$7\div 3\times 4\div 9=\frac{28}{27}=1\frac{1}{27} \qquad \frac{7}{3}\times\frac{4}{9}=\frac{28}{27}$$

$$(7\times 4)\div(3\times 9)=\frac{28}{27} \qquad \left(\frac{7}{3}\right)\left(\frac{4}{9}\right)=\frac{28}{27}$$

$$7\times 4\div 3\div 9=\frac{28}{27} \qquad \frac{7}{3}\cdot\frac{4}{9}=\frac{28}{27}$$

$$\frac{7\times 4}{3\times 9}=\frac{28}{27} \qquad (7\div 3)(4\div 9)=\frac{28}{27}$$

Exercises:

1. Change the following to pure improper fractions.

   (a) $3\frac{7}{8}$ (b) $4\frac{3}{4}$ (c) $6\frac{3}{5}$

2. Change the following to mixed numbers.

   (a) $\frac{7}{2}$ (b) $\frac{19}{15}$ (c) $\frac{16}{9}$

3. $\frac{3}{4}+\frac{7}{8}+\frac{4}{5}=?$

4. $1\frac{3}{4}+1\frac{5}{8}+4\frac{15}{16}=?$

5. (a) $\frac{3}{7}\times\frac{8}{9}=?$ (b) $\frac{4}{7}\div\frac{8}{9}=?$ (c) $\frac{7\times 12\times 56}{9\times 42\times 8}=?$

The answers to these exercises will be found on the last page of this book.

DECIMALS

Expressing Fractions with Decimals

Fractions may also be written in decimal form. In our monetary system, we use a decimal system with the dollar as the unit. We signity the dollar by the number 1. If we wish to designate a PART of this dollar, say 5 cents, it is not as convenient to write it as 5/100 of a dollar as it would be to write .05.

Fractions written in this manner are called **decimal fractions,** or simply **Decimals,** and the period placed before the figure is called the decimal point. In this system, all numbers placed to the **Left** of the decimal point are whole numbers, or **Integers,** and the numbers placed to the **Right** of the decimal point are the **Decimals.** The decimals form the numerator of the fraction while the denominator is equal to 10 times the number of places behind the decimal point. Thus, $.7 = \frac{7}{10}$, $.07 = \frac{7}{100}$, $.235 = \frac{235}{1000}$, and $.0003 = \frac{3}{10,000}$.

To Convert a Common Fraction into a Decimal

Suppose that you wanted to change the fraction $\frac{7}{13}$ into a decimal correct to three places behind the decimal point. This is the same as converting it to a fraction whose denominator is 1000. Now remember that **if you multiply both the numerator and denominator by the same amount, the value of the fraction doesn't change.** Therefore $\frac{7}{13} = \frac{7 \times 1000}{13 \times 1000} = \frac{7000}{13 \times 1000}$. Now, divide 7000 by 13 and you have the number of thousandths: $\frac{7000}{13 \times 1000} = \frac{538}{1000} = .538$.

In connection with measurements on blueprints, you will frequently have occasion to convert fractional parts of an inch into decimals, and vice versa. This can always be done by the method just described. However, it is very handy to have a table of decimal equivalents especially if you are in one of those industries where blueprints are so extensively used. This you will find in Table I of this book. Tables such as these are invaluable for making mathematics easy.

To Add or Subtract Decimal Numbers

In adding and subtracting numbers with decimals, they should be written one below the other **with their decimal points in line** and the addition or subtraction can then be carried out in the usual way. In the sum or remainder, the decimal point is then placed in line with the decimal points above.

To illustrate this, suppose we wish to add 13.46, 2.625, and 230.8 together. This is written down and carried out as shown at the right:

Notice that in this operation the decimal point is retained in its position in line with the rest of the decimal points.

```
 13.46
  2.625
230.8
-------
246.885
```

Subtraction is carried out in the same manner and, from this, it should be realized that addition and subtraction of decimals will give no trouble when the numbers are written **with the decimal points in line, one below the other.**

To Multiply or Divide Decimal Numbers

To multiply two decimal numbers, first carry out the multiplication without regard to the position of either of their decimal points. You must then, however, very carefully determine the location of the decimal point by strict adhesion to the following:

RULE: The number of places to the right of the decimal point in the product of two decimal numbers equals the sum of the numbers of places to the right of the decimal points in the multiplicand and the multiplier.

As an example, suppose you want to multiply 1.025 by 3.04. Dropping the decimal points, the product is 311600. To determine the location of the decimal point, you observe that the multiplicand has 3 figures to the right of the decimal point and that the multiplier has 2 figures. Applying the above rule, you place the decimal point to the **LEFT** of the last five figures in the product, and you find the product to be 3.11600, or, what is the same thing, 3.116.

The rule for determining the decimal point position in the quotient of the division is as follows:

RULE: Subtract the number of places to the right of the decimal point in the divisor from the number in the dividend to get the number in the quotient.

For example, divide 3.804 by 1.2. Dividing without regard to decimal points you get 317. Now, applying the above rule, you subtract the 1 place in the divisor from the 3 places in the dividend and you get 2 places for the quotient. Therefore, the correct answer is 3.17.

Percentage

A method of indicating fractions whose denominators are 100 is by means of percentage. Thus, twenty-seven percent (27%) means $\frac{27}{100}$ or .27. In all calculations, the percentage is reduced to a decimal after which you may proceed exactly as in the preceding paragraph.

To Multiply Mixed Numbers

Suppose you had to multiply two figures together, such as $1\frac{7}{8}$ by $1\frac{3}{4}$. To perform this multiplication you have two methods available; one is to reduce each of the factors to a common fraction and the other is to change each to decimals.

Using the first method, $1\frac{7}{8}$ becomes $\frac{15}{8}$ and $1\frac{3}{4}$ becomes $\frac{7}{4}$.

Then, $\frac{15}{8} \times \frac{7}{4} = \frac{105}{32}$ or $3\frac{9}{32}$.

Using the second method, $1\frac{7}{8}$ becomes 1.875 and $1\frac{3}{4}$ becomes 1.75.

Then, $1.875 \times 1.75 = 3.28125$.

Exercises:

6. Change the following to three place decimals.

(a) $\frac{17}{63}$ (b) $\frac{8}{9}$ (c) $\frac{7}{8}$ (d) $\frac{9}{13}$

7. $10.31 + 6.98 + .432 + 18.9 = ?$

8. (a) $6.98 \times 4.32 = ?$
   (b) $8.92 \div 2.13 = ?$

9. 68% of 439 = ?

10. Multiply $6\frac{7}{8}$ by $4\frac{10}{13}$.

The answers to these exercises will be found on the last page of this book.

POWERS AND ROOTS

If a quantity is multiplied by itself one or more times, the product is known as a **Power** of the quantity, which is specified by the number of times the quantity is taken as a factor. The power to which a number is raised is indicated by writing a small figure, called the **Exponent,** to the right of and a little above the number. Thus:

$3^2 = 3 \times 3 =$ the second power of 3 or the square of 3.

$3^3 = 3 \times 3 \times 3 =$ the third power of 3 or the cube of 3.

$3^4 = 3 \times 3 \times 3 \times 3 =$ the fourth power of 3, etc.

The finding of a quantity which when multiplied by itself a given number of times equals the given quantity is known as **Extracting the Root,** and the process is indicated by a radical sign $\sqrt{\ }$ in the following way:

$\sqrt{\ }$ indicates square root

$\sqrt[3]{\ }$ indicates cube root

$\sqrt[4]{\ }$ indicates fourth root, etc.

You will readily understand the meanings of these from the following examples:

$\sqrt{9} = 3$ since $3 \times 3 = 9$

$\sqrt[3]{27} = 3$ since $3 \times 3 \times 3 = 27$

$\sqrt[4]{81} = 3$ since $3 \times 3 \times 3 \times 3 = 81$

Often the process of extracting a root is indicated by a fractional exponent; thus:

$\sqrt{9} = 9^{1/2}$ $\sqrt[4]{81} = 9^{1/4}$

Finding the Square Root of a Number

Problem: Find the square root of 1892.25.

```
          43 . 5
        | 1892. 25
          16
    83  | 292
        | 249
    865 | 43 25
        | 43 25
```

Procedure: 1. Beginning from the decimal point, point off as many groups of 2 digits* as possible both to the right and to the left of the decimal point.

2. Find the greatest number whose square is equal to or less than the first left-hand group and place it as the first figure in the root (in this case 4).

3. Subtract its square (16) from the first group (18) and annex the remainder (2) to the second group (92).

4. Double the first figure of the root for a partial trial divisor (8), divide it into the remainder (29 omitting the right hand digit); and place the integer (3) of the quotient as the next digit of the root.

5. Annex the root digit just found (3) to the trial divisor (8), multiply the complete divisor (83) by the root digit (3), and subtract the product (249) from the dividend; then proceed as before.

NOTE: If, in steps 4 and 5, the product of the second figure in the root by the completed divisor is greater than the dividend, erase the second figure both from the root and from the divisor, and substitute the next lower digit until the product can be subtracted from the dividend.

Exercises:

11. Find the square root of

(a) 776161 (b) 85.1929 (c) .555025

The answers to these exercises will be found on the last page of this book.

RATIO AND PROPORTION

Ratio is the relation between two magnitudes of the same kind. Thus, if a cubic foot of cast iron weighs 450 pounds and a cubic foot of steel weighs 490 pounds, the ratio of the weight of cast iron to the weight of steel is 450 to 490, or 45 to 49. It is frequently written 45:49 but it may also be written $\frac{45}{49}$ since a ratio is, after all, a fraction.

* A digit is any one of the numerals from 0 to 9.

When two different ratios are expressed as being equal, a **Proportion** is formed. This equality is expressed either by the ordinary equality sign (=) or by a special proportional sign (::).

Thus a proportion may be expressed as

$$x : 14 :: 45 : 49$$

$$\text{or } x : 14 = 45 : 49$$

$$\text{or } \frac{x}{14} = \frac{45}{49}$$

In these expressions the x and 49 are called the **Extremes** and the 14 and 45 are the **Means.** The x is the symbol for the **Unknown Quantity.** It appears here as an extreme but it may also be a mean.

A proportion is easily solved for the unknown x by applying the following rules:

RULE 1. The product of the extremes is equal to the product of the means.

RULE 2. The product of the extremes divided by either mean gives the other mean.

RULE 3. The product of the means divided by either extreme gives the other extreme.

Example 1: If a welder can make a five foot weld in 13 minutes, how long will it take him to make 28 feet of weld at the same rate of speed?

Note: This is an example of a **Direct Proportion** since the longer the time taken the greater will be the amount of weld.

Solution: $5 : 28 :: 13 : x$

From Rule 3, $x = \frac{28 \times 13}{5} = 72.8$ minutes.

Example 2: If 3 men can lay a roof in 7 days, how long will it take 5 men?

Note: This is an example of an **Inverse Proportion,** since the greater the number of men employed, the less will be the amount of time consumed.

Solution: $\frac{3}{5} = \frac{x}{7}$

From Rule 2, $x = \frac{7 \times 3}{5} = 4\frac{1}{5}$ days.

Exercises:

12. An iron casting weighs 236 lbs. How much would the same casting weigh if made of aluminum, assuming that cast iron weighs 450 lbs. per cu. ft. and aluminum weighs 168 lbs. per cu. ft.?

13. If a machinist must run a drill at 284 R.P.M. for drilling steel at a cutting speed of 65 feet per minute, at how many R.P.M. must the drill run for the same size hole in aluminum at a cutting speed of 195 feet per minute.

The answers to these exercises will be found on the last page of this book.

ARITHMETICAL SHORT-CUTS

Cancellation

A very valuable device for making mathematics easy is the process of **Cancellation.** This is the "rejecting" or "cancelling out" of factors which are common to both the numerator and denominator, i. e., both the dividend and divisor.

Example 1. Divide 8 × 50 by 4 × 25.

Solution: $\frac{\overset{2}{\cancel{8}} \times \overset{2}{\cancel{50}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{25}}} = \frac{4}{1} = 4$

Here 4 is a common factor of 8 and 4. Therefore, when both are divided by 4, 8 becomes 2 and 4 becomes 1. Similarly, $50 \div 25 = 2$ and $25 \div 25 = 1$.

Example 2. Divide 96 × 42 × 40 by 84 × 60 × 21.

Solution: $\frac{\overset{8}{\cancel{96}} \times \overset{2}{\cancel{42}} \times \overset{2}{\cancel{40}}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{60}} \times \underset{1}{\cancel{21}}} = \frac{32}{21} = 1\frac{11}{21}$

Here 12 is rejected by 96 and 84; 20 from 40 and 60; and 21 from 42 and 21.

To Multiply Numbers between 11 and 99 by 11.

(a) When the sum of the digits of the multiplicand is 9 or less, the product is written directly by simply placing in between the two digits of the multiplicand their sum. Notice that

FIRST DIGIT / SECOND DIGIT OF MULTIPLICAND — $45 \times 11 = 495$ — SUM OF 2 DIGITS

(b) When the sum of the digits of the multiplicand is greater than 9, the second digit of the sum is placed before the second digit of the multiplicand and is preceded by the first digit increased by one. Thus:

$67 \times 11 = 737$ — 1st DIGIT PLUS 1; 2nd DIGIT OF MULTIPLICAND; 2nd DIGIT OF SUM OF GIVEN DIGITS

To Multiply by ¾

The ordinary way of multiplying a number by ¾ is to multiply by 3 and divide by 4. You will find it much easier however to write ½ the number and then ½ of this half and add the two. Thus to multiply 92 by ¾,

write 46
and 23
and add, obtaining 69

To Multiply by 25

The quick way is to multiply by 100 and divide by 4, all of which can be done mentally. Thus,

$$68 \times 25 = \frac{6800}{4} = 1700.$$

Of course, this method is also applicable to multiplication by 250, 2½, .25, etc., providing you lay off the correct number of places.

To Multiply by 75

Here the best way is to multiply by $100 \times \frac{3}{4}$. Thus, to multiply 86 by 75,

write	4300
and	2150
and add, obtaining	6450

To Add Two Numbers when One is just under 100

In adding a number such as 97 it requires less mental strain to consider it as adding 100 and subtracting 3. Thus you can add 58 to 97 mentally by thinking of it as $158 - 3 = 155$.

To Subtract a Number just Less than 100

To subtract, say, 97, add 3 and subtract 100. Thus

$$185 - 97 = 85 + 3 = 88.$$

You should observe that this method is also applicable to adding or subtracting numbers like 996, 9997, 99.98, etc.

To Multiply by a Number just under 100

Multiplying by, say, 98, is the same as multiplying by $100 - 2$. Therefore, multiply the multiplicand by 100 and then by 2 (which can be done mentally) and subtract. Thus to multiply 87 by 98

$$87 \times 100 = 8700$$
$$87 \times 2 = 174$$
$$\text{Subtracting we get} = 8526 = \text{Product}$$

Now see if you can apply this trick to multiply (a) $88 \times .998$ (b) 47×9.97 (c) 79×999. Check your answers by the long method.

To Square a Number of Two Digits

A rule that is very useful for shortening the labor of finding squares is the following:

(1) Square the first digit and place in front of the Square the second digit
(2) Multiply the product of the digits by 20
(3) Add the results of 1 and 2

Example: Find the square of 87.

Solution:

USUAL LONG METHOD	SHORT CUT METHOD	
87	(8^2 alongside 7^2)	6449
87	($20 \times 8 \times 7$ mentally) =	1120
609		7569
696		
7569		

To Square a Number just under 100

$(98)^2$ is the same as $(100 - 2)^2$ which in turn is equal to $(100)^2 + (2)^2 - (2 \times 100 \times 2)$. Thus $(98)^2$ can be done very rapidly by

writing	10004	(the sum of 100^2 and 2^2)
and	$-$ 400	(which is $2 \times 100 \times 2$)
and subtracting to get	9604	

$$\text{Similarly } 998^2 = 1000^2 + 2^2 - 4000$$
$$9.97^2 = 10^2 + .03^2 - .60$$

There are other numbers where this principle can be usefully applied. Examples are:

$$(148)^2 = 150^2 + 2^2 - 2 \times 2 \times 150$$
$$= 22504 - 600 = 21904$$

$$(198)^2 = 200^2 + 2^2 - 2 \times 2 \times 200$$
$$= 40004 - 800 = 39204$$

UNITS OF MEASUREMENT AND WEIGHT

If you have ever been employed in construction or mechanical work where blueprint reading and estimating were involved, you will realize how important it is to be thoroughly familiar with geometric shapes and expressions and formulae for their mensuration. It is often necessary to convert lengths given in certain units to lengths in other units. Or, you may be called upon to calculate areas, volumes, and weights from the measurements given on blueprints. Before you can begin to do this you must know the relationship that exists between the various units used for measurement. This important information will now be given to you in a series of tables.

Long Measure:

The following Long Measure Table expresses the relationship between the usual units of linear measurement:

12 inches (″) = 1 foot (′)
3′ = 1 yard (yd.)
5½ yds. or 16½′ = 1 rod
40 rods = 1 furlong
8 furlongs or 320 rods = 1 statute mile

Surveyor's or Old Land Measure

In reading blueprints of old Land Maps, you are likely to find lengths expressed in the units of the Surveyor's or Old Land Measure, a table of which is given here:

7.92 inches = 1 link
25 links = 1 rod
4 rods or 66′ = 1 chain
80 chains = 1 mile

Note: Rods are seldom used in this measure, distances being usually in chains and links.

Square Measure:

When the area of a surface is to be indicated, the units of a Square Measure are used. Area represents a product of 2 linear dimensions both of which are in the same unit. The product of the 2 lengths is expressed by the word "square". Thus:

inches × inches = square inches (sq. ins.)

The Table of Square Measure follows:

144 sq. ins. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
30¼ sq. yds. = 1 sq. rod
43,560 sq. ft. or 160 sq. rods = 1 acre
640 acres = 1 sq. mile

Surveyor's Square Measure:

Here is the table for Square Measure corresponding to the Old Land Long Measure:

625 sq. links = 1 pole
16 poles = 1 sq. chain
10 sq. chains = 1 acre
640 acres = 1 sq. mile
36 sq. miles = 1 township

Cubic Measure:

The units of this measure are used to express the volume of material in a solid or the space within a container. Volume represents a product of three dimensions, such as length, width, and thickness, either of which may be expressed in any of the linear units, such as inches, provided the units are the same for all three. The word "cubic" in conjunction with the word expressing the linear unit denotes the product of three dimensions using this unit. For example:

feet × feet × feet = cubic feet

The Cubic Measure Table follows:

1,728 cubic ins. = 1 cubic ft.
27 cubic ft. = 1 cubic yd.
24¾ cubic ft. = 1 perch (of stone masonry)

Liquid Measure:

If, in your vocation, you handle containers for liquids, you should know the Liquid Measure Table:

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
31½ gallons = 1 barrel
2 barrels = 1 hogshead

Note: U. S. gallon = 231 cubic ins.; approximately 7½ such gallons = 1 cubic ft.

Board Measure:

This is the system in general use for measuring lumber. A Board Foot or Foot Board Measure (F.B.M.) equals 1 ft. × 1 ft. × 1 in. Thus, a board 5 ft. long × 18 ins. wide × 3 ins. thick would contain 5 × 1.5 × 3 = 22.5 F.B.M.

Avoirdupois Weight:

Three measures of weight are used in this country, namely Troy, Apothecaries, and Avoirdupois. Only the last mentioned, however, is of sufficient industrial importance to be included here:

437.5 grains = 1 ounce
16 ounces = 1 pound
100 pounds = 1 hundred weight (cwt.)
2000 pounds = 1 short ton
2240 pounds = 1 long ton

Angular Measure:

An Angle is the space between two intersecting lines and is measured in degrees, minutes, and seconds. If the two lines coincide the angle is zero, while if one of these lines is made to revolve about the point of intersection until it again coincides, the angle generated is 360 degrees. Thus, a degree is $\frac{1}{360}$th of this revolution and this, in turn, is further divided into minutes and seconds in accordance with the following table:

60 seconds (60″) = 1 minute
60 minutes (60′) = 1 degree
90 degrees (90°) = 1 right angle

THE METRIC SYSTEM

In the Metric System, a length of a number of specified units can be expressed in terms of any other unit by multiplying or dividing by the appropriate power of ten (i. e., 10, 100, 1000, etc.). The same is also true of areas, volumes, and weights. For that reason, calculations are greatly simplified when this system is used. In continental Europe, the Metric System is used universally, whereas in England and the United States it is confined more or less to scientific weights and measurements.

The base of the Metric System is the **Meter,** which is fixed as one ten-millionth of the distance on the earth's surface from the Equator to either the North or the South Pole.

Tables of Long, Square, and Cubic Measure in the Metric System with the English (or U. S.) equivalents, follow:

Metric Long Measure Table:

1 Millimeter (mm.)	=	.001	Meter	=	.03937	in.
1 Centimeter (cm.)	=	.01	Meter	=	.3937	in.
1 Decimeter (dm.)	=	.1	Meter	=	3.937	ins.
		1.	Meter	=	39.3707	ins.
1 Dekameter	=	10.	Meters	=	32.809	ft.
1 Hectometer	=	100.	Meters	=	328.09	ft.
1 Kilometer	=	1000.	Meters	=	.62137	mi.
1 Myriameter	=	10000.	Meters	=	6.2137	mi.

Metric Square Measure Table:

100 sq. mms.	= 1 sq. Centimeter	=	.155	sq. in.
100 sq. cms.	= 1 sq. Decimeter	=	15.5	sq. ins.
100 sq. dms.	= 1 sq. Meter (sq. m.)	=	10.7369	sq. ft.
1 Centiare	= 1 sq. Meter	=	1.196034	sq. yds.
100 Centiares	= 1 Are	=	119.6034	sq. yds.
100 Ares	= 1 Hectare	=	2.47114	acres
100 Hectares	= 1 sq. Kilometer	=	.3861	sq. mi.

Metric Cubic Measure Table:

1000 cu. mms.	= 1 cu. cm.	=	.061	cu. in.
1000 cu. cms.	= 1 cu. dm.	=	61.023	cu. ins.
1000 cu. dms.	= 1 cu. meter	=	35.314	cu. ft.

Metric Capacity Table:

In the Metric System, the measurement of capacity is the same whether for liquids or solids. The basic unit of this measure is the **Liter,** which is equal to one cubic decimeter or 1.0567 U. S. liquid quarts. The table follows:

1 Milliliter	=	1 cu. cm.	=	.00845	gill
1 Centiliter	=	10 cu. cms.	=	.0845	gill
1 Deciliter	=	100 cu. cms.	=	.845	gills
1 Liter	=	1 cu. dm.	=	1.0567	qts.
1 Dekaliter	=	10 cu. dms.	=	2.6417	gals.
1 Hectoliter	=	.1 cu. meter	=	26.417	gals.
1 Kiloliter	=	1 cu. meter	=	264.17	gals.
1 Myrialiter	=	10 cu. meters	=	2641.7	gals.

Metric Weight Measure Table:

The Metric basic unit of weight is the Gram, which is equal to the weight of a cubic centimeter of distilled water. A Gram equals .35273 ounces Avoirdupois. A Kilo, or Kilogram equals 2.20462 pounds Avoirdupois. A Tonneau, or Ton equals 2204.62125 pounds. The complete table follows:

10 Milligrams	(mg.)	= 1 Centigram
10 Centigrams	(cg.)	= 1 Decigram
10 Decigrams	(dg.)	= 1 Gram
10 Grams	(g.)	= 1 Dekagram
10 Dekagrams	(Dg.)	= 1 Hectogram
10 Hectograms	(hg.)	= 1 Kilogram or Kilo
10 Kilograms	(kg.)	= 1 Myriagram
10 Myriagrams (Mg.) or 100 Kilograms		= 1 Quintal
10 Quintals or 1000 Kilos		= 1 Tonneau, or 1 Ton

DECIMALS OF AN INCH FOR EACH 64TH

WITH MILLIMETER EQUIVALENTS

Fraction	1/64ths	Decimal	Millimeters	Fraction	1/64ths	Decimal	Millimeters
....	1	.015625	0.39688	....	33	.515625	13.09690
1/32	2	.03125	0.79375	17/32	34	.53125	13.49378
....	3	.046875	1.19063	....	35	.546875	13.89065
1/16	4	.0625	1.58750	9/16	36	.5625	14.28753
....	5	.078125	1.98438	....	37	.578125	14.68440
3/32	6	.09375	2.38125	19/32	38	.59375	15.08128
....	7	.109375	2.77813	....	39	.609375	15.47816
1/8	8	.125	3.17501	5/8	40	.625	15.87503
....	9	.140625	3.57188	....	41	.640625	16.27191
5/32	10	.15625	3.96876	21/32	42	.65625	16.66878
....	11	.171875	4.36563	....	43	.671875	17.06566
3/16	12	.1875	4.76251	11/16	44	.6875	17.46253
....	13	.203125	5.15939	....	45	.703125	17.85941
7/32	14	.21875	5.55626	23/32	46	.71875	18.25629
....	15	.234375	5.95314	....	47	.734375	18.65316
1/4	16	.25	6.35001	3/4	48	.75	19.05004
....	17	.265625	6.74689	....	49	.765625	19.44691
9/32	18	.28125	7.14376	25/32	50	.78125	19.84379
....	19	.296875	7.54064	....	51	.796875	20.24067
5/16	20	.3125	7.93752	13/16	52	.8125	20.63754
....	21	.328125	8.33439	....	53	.828125	21.03442
11/32	22	.34375	8.73127	27/32	54	.84375	21.43129
....	23	.359375	9.12814	....	55	.859375	21.82817
3/8	24	.375	9.52502	7/8	56	.875	22.22504
....	25	.390625	9.92189	....	57	.890625	22.62192
13/32	26	.40625	10.31877	29/32	58	.90625	23.01880
....	27	.421875	10.71565	....	59	.921875	23.41567
7/16	28	.4375	11.11252	15/16	60	.9375	23.81255
....	29	.453125	11.50940	....	61	.953125	24.20942
15/32	30	.46875	11.90627	31/32	62	.96875	24.60630
....	31	.484375	12.30315	....	63	.984375	25.00318
1/2	32	.5	12.70003	1	64	1.	25.40005

Courtesy of the
Bethlehem Steel Company, Inc.

DECIMALS OF A FOOT FOR EACH 32ND OF AN INCH

Inch	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″
0	0	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167
1/32	.0026	.0859	.1693	.2526	.3359	.4193	.5026	.5859	.6693	.7526	.8359	.9193
1/16	.0052	.0885	.1719	.2552	.3385	.4219	.5052	.5885	.6719	.7552	.8385	.9219
3/32	.0078	.0911	.1745	.2578	.3411	.4245	.5078	.5911	.6745	.7578	.8411	.9245
1/8	.0104	.0938	.1771	.2604	.3438	.4271	.5104	.5938	.6771	.7604	.8438	.9271
5/32	.0130	.0964	.1797	.2630	.3464	.4297	.5130	.5964	.6797	.7630	.8464	.9297
3/16	.0156	.0990	.1823	.2656	.3490	.4323	.5156	.5990	.6823	.7656	.8490	.9323
7/32	.0182	.1016	.1849	.2682	.3516	.4349	.5182	.6016	.6849	.7682	.8516	.9349
1/4	.0208	.1042	.1875	.2708	.3542	.4375	.5203	.6042	.6875	.7708	.8542	.9375
9/32	.0234	.1068	.1901	.2734	.3568	.4401	.5234	.6068	.6901	.7734	.8568	.9401
5/16	.0260	.1094	.1927	.2760	.3594	.4427	.5260	.6094	.6927	.7760	.8594	.9427
11/32	.0286	.1120	.1953	.2786	.3620	.4453	.5286	.6120	.6953	.7786	.8620	.9453
3/8	.0313	.1146	.1979	.2812	.3646	.4479	.5313	.6146	.6979	.7813	.8646	.9479
13/32	.0339	.1172	.2005	.2839	.3672	.4505	.5339	.6172	.7005	.7839	.8672	.9505
7/16	.0365	.1198	.2031	.2865	.3698	.4531	.5365	.6198	.7031	.7865	.8698	.9531
15/32	.0391	.1224	.2057	.2891	.3724	.4557	.5391	.6224	.7057	.7891	.8724	.9557
1/2	.0417	.1250	.2083	.2917	.3750	.4583	.5417	.6250	.7083	.7917	.8750	.9583
17/32	.0443	.1276	.2109	.2943	.3776	.4609	.5443	.6276	.7109	.7943	.8776	.9609
9/16	.0469	.1302	.2135	.2969	.3802	.4635	.5469	.6302	.7135	.7969	.8802	.9635
19/32	.0495	.1328	.2161	.2995	.3828	.4661	.5495	.6328	.7161	.7995	.8828	.9661
5/8	.0521	.1354	.2188	.3021	.3854	.4688	.5521	.6354	.7188	.8021	.8854	.9688
21/32	.0547	.1380	.2214	.3047	.3880	.4714	.5547	.6380	.7214	.8047	.8880	.9714
11/16	.0573	.1406	.2240	.3073	.3906	.4740	.5573	.6406	.7240	.8073	.8906	.9740
23/32	.0599	.1432	.2266	.3099	.3932	.4766	.5599	.6432	.7266	.8099	.8932	.9766
3/4	.0625	.1458	.2292	.3125	.3958	.4792	.5625	.6458	.7292	.8125	.8958	.9792
25/32	.0651	.1484	.2318	.3151	.3984	.4818	.5651	.6484	.7318	.8151	.8984	.9818
13/16	.0677	.1510	.2344	.3177	.4010	.4844	.5677	.6510	.7344	.8177	.9010	.9844
27/32	.0703	.1536	.2370	.3203	.4036	.4870	.5703	.6536	.7370	.8203	.9036	.9870
7/8	.0729	.1563	.2396	.3229	.4063	.4896	.5729	.6563	.7396	.8229	.9063	.9896
29/32	.0755	.1589	.2422	.3255	.4089	.4922	.5755	.6589	.7422	.8255	.9089	.9922
15/16	.0781	.1615	.2448	.3281	.4115	.4948	.5781	.6615	.7448	.8281	.9115	.9948
31/32	.0807	.1641	.2474	.3307	.4141	.4974	.5807	.6641	.7474	.8307	.9141	.9974

Courtesy of the Bethlehem Steel Company, Inc.

THE SLIDE RULE

By means of a Slide Rule, such as the one we have given you, all sorts of problems involving multiplication, division, squares, cubes, and square and cube roots can be correctly solved with much less effort and in much faster time than by the usual methods.

After you read the following paragraphs and learn how to use a Slide Rule through careful study and practice, you will be convinced that it is definitely a most satisfactory device for making mathematical calculations easy.

You can use a Slide Rule to advantage even if you are not familiar with the higher forms of mathematics which follow. All you need is a thorough knowledge of fractions and decimals, which subjects you are strongly urged to review before attempting the study of the Slide Rule.

Significant Figures

An important consideration when using a Slide Rule is its degree of accuracy. To understand any discussion of this, you must understand what is meant by Significant Figures. Probably you will best understand this from a discussion of a series of examples:

Suppose you multiply 284 by 346. You will find that the product is 98,264. There are 5 so-called Digits in this product, each of which is considered a Significant Figure. In many multiplication operations it is not necessary to get the product as accurately as this. For instance, the product could be expressed as 98,260 which would be correct to 4 Significant Figures, or as 98,300 which would be correct to 3 Significant Figures, or simply as 98,000 which would be correct to 2 Significant Figures.

In performing the operation just described by the Slide Rule, the answer 98,300 would have been obtained since the Slide Rule is accurate for the most part to only 3 Significant Figures, although in one portion of the Slide Rule accuracy to 4 Significant Figures can be obtained. For most engineering calculations, it is not necessary to work any closer than this.

Assuming, now, that you understand the application of Significant Figures to whole numbers, let us take an example involving decimals. Suppose that we are to multiply 1.36 by 1.26. The correct answer is 1.7136, which is correct to 5 Significant Figures. This would, ordinarily, be read on the Slide Rule, however, as 1.714 (correct to 4 Significant Figures) or 1.71 (correct to 3 Significant Figures).

As a further example, suppose we multiply .000123 by .0032. The product is .0000003936. Although there are 10 Digits here behind the decimal point, the figure is correct to only 4 Significant Figures. In other words, a zero is only a Significant Figure of a decimal when it appears between two other digits. On a Slide Rule, this product would, ordinarily, be read as .000000394 (correct to 3 Significant Figures).

READING THE C AND D SCALES

Probably the most difficult part of using a Slide Rule is becoming familiar with the Scales, of which there are six on the Slide Rule you possess; namely, the A, B, CI, C, D, and K. The C and D scales are alike, while the CI scale is inverted, or opposite hand to the C and D scales. Since the C and D scales are used more than the others, let us begin by studying one of these.

You will observe that at each end there is a line, or graduation, numbered 1. Since the Slide Rule gives no indication of the number of digits, either before or after the decimal point, the 1 on the left, called the **Left Index,** may be read as .001, 0.1, 1, 10, 100, 1000, etc. However, when the left hand 1 is read as 1, then the right hand 1, called the **Right Index,** must be read as 10; in other words, the right hand 1 is always 10 times as great as the 1 on the left end.

Now, beginning at the left end index, skip over the small, **secondary** graduations 1, 2, 3 . . . 8, 9, until you come to the next main or **prime** graduation marked 2 (about 3 inches to the right of 1). Following still further you find additional prime graduations labeled 3, 4 . . . 8, 9, the space between getting smaller and smaller. Taking the graduation 4 as an example, this should be read as 0.4, 4, 40, or 400, depending on whether the left hand 1 is read as 0.1, 1, 10, or 100.

Consider next the **secondary** divisions labeled 1, 2 . . . 9, between the left index and the prime 2. If the left index is read as 1, these will be read as 1.1, 1.2, 1.3, 1.4 . . . . 1.9; while if the left index is called 100, the secondary graduations will be read 110, 120, 130 . . . 190. Note that the secondary graduations to the right of the prime 2 are not numbered. Therefore, to determine their values, you will have to count the spaces from the nearest prime graduation, until, after sufficient practice, you will recognize them at sight.

Now notice that the spaces between the secondary lines or graduations between prime 1 and prime 2 are subdivided into 10 parts each; the secondary spaces between prime 2 and prime 4 are subdivided into five spaces each; and those from prime 4 to the right index into two spaces each.

To read a number in the range between prime 1 and 2 (see Fig. 1), the first digit is taken as 1, the second is the secondary figure to the left of the point being read, the third digit is the number of spaces between the secondary line and the subdivision line nearest to and to the left of the point read, while a fourth digit can be approximately determined by estimating the proportional distance between the subdivision to the

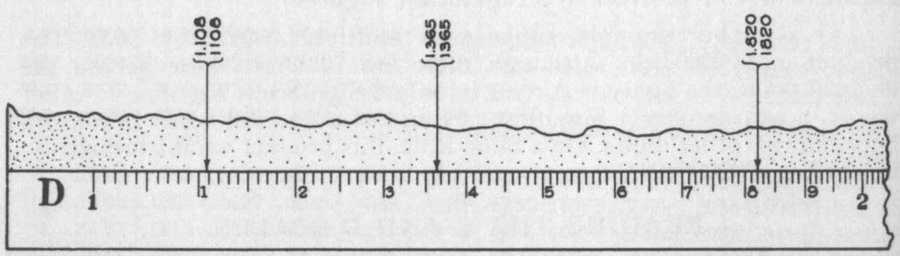


FIG. 1

left and that to the right. Suppose these digits are 1, 3, 6, and 5. The number could be read as 1.365 or 136.5 or 1,365. However, since the decimal point is usually determined as a later step, the number is best read as merely one, three, six, five and written as either 1365 or 1.365. Note that in this range, numbers can be accurately read to three Significant Figures, and very closely approximated to the fourth Significant Figure. For examples of reading numbers in other ranges, see Figs. 2 and 3.

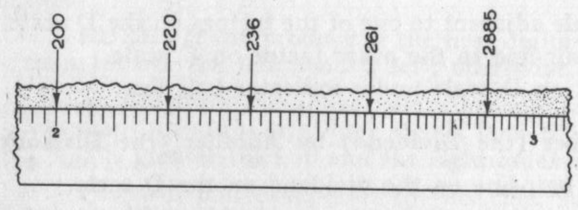


FIG. 2

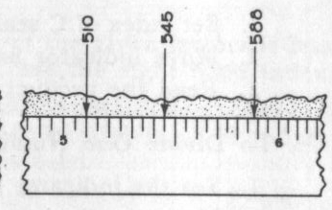


FIG. 3

You should practice reading and setting until you can do so accurately and without hesitation. Then you are ready to give attention to the solution of simple multiplication problems.

EXAMPLES OF SIMPLE MULTIPLICATION AND DIVISION

In order that you may become familiar with the Slide Rule and have confidence in your ability to use it, let us begin with the multiplication of two simple numbers of which you know the product, such as $2 \times 4 = 8$. (The 2 and 4 in this operation are known as **Factors**).

Set the left index of the C scale at the prime 2 line of the D scale. Now, slide the indicator until its hair-line coincides with the prime 4 of the C scale. The product 8 is then read along the indicator hair-line on the D scale.

Division is the reverse of this process. To divide 8 by 4, set the indicator hair-line at 8 on the D scale. Now bring the prime 4 of the C scale to coincide with this, and the quotient, 2, can be read on the D scale opposite the left C index.

Now, for practice, try a few more simple examples of this sort, such as $2 \times 3 = 6$, $3 \times 3 = 9$, $9 \div 3 = 3$, etc.

If you try to find 3 times 4 by setting the left C index at 3, you will find that 4 on the C scale comes off the right end of the rule because the product is greater than ten. In all such cases, you should set the right index of the C scale at 3 of the D scale and move the indicator until the hair-line coincides with C4, whereupon you will read the product, 12, on the D scale.

For further practice in multiplication, find, in the manner just described, the products of 4×8, 9×5. You will then find that multiplication on the Slide Rule is really a very simple procedure. Now, you will probably want to know when you should set the left index and when the right. This cannot always be known offhand, but by following a rule for approximation, which will be given later, and by practicing with the Slide Rule as often as possible, you will soon be able to guess right most of the time.

What has been said in regard to Multiplication and Division on the Slide Rule may be summarized in the two following rules:

A. To Multiply Two Factors Together

1. **Set index of C scale adjacent to one of the factors on the D scale.**
2. **Move indicator hair-line to the other factor on C scale.**
3. **Read the product on D scale under indicator hair-line.**

B. To Divide One Number (the Dividend) by Another (the Divisor)

1. **Set the indicator hair-line on the dividend on the D scale.**
2. **Move the C scale until the divisor on it coincides with the hair-line.**
3. **Read the quotient on the D scale on line with the index of the C scale.**

PLACING THE DECIMAL POINT WHEN USING THE SLIDE RULE

Let us consider the following problems:

(a) $261 \times 3.43 = ?$ (d) $261 \times .343 = ?$

(b) $.261 \times 3.43 = ?$ (e) $2.61 \times 3.43 = ?$

(c) $261 \times 343 = ?$ (f) $.0261 \times .0343 = ?$

As far as the Slide Rule work is concerned, all of these problems are solved identically. The left index of scale C is set at 261 of the D scale, (i. e. prime 2, secondary 6, subd. 1), and the product is read on the D scale opposite 343 (i. e. prime 3, secondary 4, and subd. 3) on the C scale. The result is read as prime 8, secondary 9, subd. 5 (or merely eight, nine, five). But does this mean 8.95, 895000, 895, or what? Evidently it is different for each of the six problems.

The method of arriving at the correct location of the decimal point is to make a rough calculation, after the Slide Rule work, using round numbers. For instance, in example (a), approximate 3.43 as 3 and 261 as 300. Then, 3×300 is mentally determined as 900, and the correct answer to problem (a) is 895 because that is nearer 900 than either 89.5 or 8950.

Similarly, the answers to the other problems of the group are:

(b) .895 (d) 89.5

(c) 89,500. (e) 8.95

(f) .000895

When you have operated the Slide Rule for some time, you will learn to make these approximations mentally and almost instantaneously.

WHICH INDEX SHOULD YOU USE?

In the six preceding examples the left index was used. However, if you were to multiply 362 by 458, you would find that by setting the left index at D362, the C458 would come off the right end of the D scale. Therefore, the right index would have to be set at D362 to get the desired result. It isn't always possible to tell beforehand which is the proper index to use, but the following rule will be found very helpful in most cases:

RULE: If the product of the first digits of the given factors is less than 10, use the left hand index; otherwise, use the right hand index.

To illustrate the above rule, consider 2.36×1.45. Then, 2×1 is **less** than 10 and the **left** index should be used. In multiplying 3.34×5.14, 3×5 is **greater** than 10 and the **right** index is used.

You will find exceptions to this rule, such as 3.43×3.12, where 3×3 is less than 10 but where the actual product (10.70) is greater than 10. However, you will find the rule a great time-saver in the majority of cases.

Exercises in Multiplication and Division:

14. $1.416 \times .0625 = ?$
15. $891 \times 45 = ?$
16. $*14154 \times 31.2 = ?$
17. $3.14 \times 14 = ?$
18. $.205 \times .317 = ?$
19. $.0023 \times .069 = ?$
20. $81 \times 64 = ?$
21. $649 \div 18 = ?$
22. $.742 \div .152 = ?$
23. $1055 \div .276 = ?$
24. $\frac{1}{36} \times 452 = ?$
25. $1.655 \div 455 = ?$

All of these problems should be worked out on the Slide Rule and at least some of them by ordinary arithmetical multiplication. By doing some of them two ways you will be able to observe:

1. The saving in time by using the Slide Rule.
2. The relative degree of accuracy of the two methods.

You will find the answers to the above problems (to the number of significant figures as determined by the slide rule) on the last page of this book.

MULTIPLICATION OF THREE OR MORE FACTORS

Suppose you had a problem, such as multiplying 3.5 by 642 by .0164. To perform this multiplication, you proceed with the first two factors as you would in your other problems, that is, you will set the C index at the 3.5 of the D scale and then move the indicator until the hair-line coincides with the 642 of the C scale. At this point, however, it is not necessary to read the product of these two numbers since we are only interested in the final result. Then, keeping the indicator as just set, move the C index until it coincides with the hair-line of the indicator. Now, move the indicator to .0164 on the C scale and the digits of the product will be found on the D scale under the hair-line as 369.

* This should be set as 1415 since the 5th Significant Figure is lost on the Slide Rule.

You may determine the position of the decimal point in the usual manner of substituting approximate round numbers. Thus, $600 \times 4 \times .01 = 24$ from which you know that the final answer is 36.9 since that is closer to 24 than either 369 or 3.69.

Any number of factors can be multiplied together in a similar manner. Later, when you learn to use the CI scale you will learn of a still quicker method of multiplying three or more factors.

SOLUTION OF PROBLEMS INVOLVING BOTH MULTIPLICATION AND DIVISION

Problems involving both Multiplication and Division can be worked out on the Slide Rule with a tremendous saving in time over the ordinary method. In solving a problem of this type, it is not necessary to read the answer for each step when we are interested only in the final result. Take as an example:

$$\frac{840 \times 648 \times 426}{790 \times 611} = ?$$

In the above example, the long fraction line, as you already know, stands for division. This problem could be read as the product of 840 by 648 by 426 divided by the product of 790 by 611 or it could be read as 840 divided by 790 multiplied by 648 divided by 611 multiplied by 426. For Slide Rule calculations it is better to consider the problem as stated in the latter manner, since alternating the processes of division and multiplication on the Slide Rule saves time by requiring fewer settings of the index and indicator.

The problem can best be worked out in the following steps:

1. Set the indicator at 840 on D.
2. Move the Slide until 790 on C coincides with indicator hair-line. (Division).
3. Move indicator to 648 on C. (Multiplication).
4. Move 611 on C to indicator line. (Division).
5. Move indicator to 426 on C. (Multiplication).
6. Read 480 on D, which is the answer, not considering the proper number of digits.

The correct number of digits must be determined by the usual method of approximation; thus,

$$\frac{800 \times 700 \times 400}{800 \times 600} = \text{about } 500.$$

Therefore, the correct answer is 480 and not 48.0 or 4800.

Exercises:

26. $\frac{248 \times 1.141}{38.3} = ?$

27. $\frac{3.14 \times 19.11 \times 16.42}{9.87 \times 13.14} = ?$

28. $\frac{1}{25} \times \frac{3}{83} \times \frac{41}{7} = ?$

29. $.0034 \times \frac{7}{97} = ?$

30. $\frac{421}{18.4} \times \frac{639}{1412} = ?$

31. $\frac{181 \times 324}{16.4} = ?$

The answers to these exercises will be found on the last page of this book.

THE A AND B SCALES

If you will now look at your Slide Rule, you will see on the top two scales labeled A and B. Careful observation will point out to you that these scales are the same as the C and D scales except that they are only half-size and that there are two of them in the same space that is occupied by one of the C or D scales. The A and B scales can be used in the same manner as the C and D scales to perform multiplication or division. However, they will not be as accurate for the simple reason that the lengths of them are so much shorter.

There is one advantage in using the A and B scales for multiplication and division and that is that the left index can always be the one set in performing multiplication. For example, if you were to multiply 3×4, you could set the left index at the 3 of the left half of the A scale and when you go to set the indicator at B4 you will find that, instead of coming off at the end of the Slide Rule, as happened when you used the C and D scale, the answer 12 can be read on the right half of the A scale.

The A and B scales are used most advantageously, however, in determining the squares and square roots of numbers and in determining areas of circles.

TO FIND THE SQUARE OF A NUMBER

If a number is multiplied by itself, the product is said to be the **Square** of the number. The operation of squaring a number is indicated by a small number 2 to the upper right of the number to be squared, known as the **exponent** of the number. Thus, we can write:

$$2^2 = 4 \qquad 3^2 = 9 \qquad 4^2 = 16$$

To find the square of 2 on the Slide Rule, set the indieator at 2 on the D scale and read the answer 4 on the A scale, under the indicator hairline.

Now, take another example: Find the square of 4.36 by setting the indicator at 436 on the D scale and reading the answer 190 on the A scale. To determine the correct decimal point in the answer, the following rule will guide you:

RULE: The number of places to the left of the decimal point in the answer is equal to TWICE the number of places before the decimal in the original number if the answer is found on the RIGHT HALF of the A scale.

If the answer is found in the LEFT HALF of the A scale, then the number of places to the left of the decimal point is equal to TWICE THE NUMBER OF PLACES IN THE ORIGINAL NUMBER, MINUS 1.

Thus, the correct answer to the problem that we just worked out would be 19.0 since the answer was found on the **Right Half** of the A scale.

If there are no places to the left of the decimal point, the following rule can be used to determine the position of the decimal point with respect to the 1st significant figure:

RULE: If the square is found on the RIGHT HALF of the A scale, then the number of zeros between the decimal point and the 1st significant figure equals TWICE the number of zeros between the decimal point and the 1st Significant Figure of the original number.

If the answer appears on the LEFT HALF of the A scale, then the number of zeros is GREATER BY 1 than that determined by the preceding rule.

For example,

$$(.0451)^2 = .00204$$
$$(.31)^2 = .096$$

TO FIND THE SQUARE ROOT OF A NUMBER

The Square root of a number is that factor which, when multiplied by itself, will give you the number. The process of finding the square root of a number is indicated by the radical sign $\sqrt{\ }$ over the number. For example, the $\sqrt{9} = 3$. Another, but less common, way of indicating this operation is by showing the exponent as a fraction; thus $(9)^{1/2} = 3$. The procedure of carrying out the operation on the Slide Rule is just **the reverse of finding the square.**

Thus, to find the square root of 7.5, you set the indicator at 7.5 on the A scale and read the root on the D scale as 2.74. In doing this, however, you may be at a loss to know whether you should set the indicator at the left half of the A scale or on the right half of the A scale and, therefore, it would be well for you to remember the following rule:

RULE: If the number of places to the left of the decimal point is EVEN, then the RIGHT HALF of the A scale should be used.

If the number of places to the left of the decimal point is ODD, then the LEFT HALF of the A scale should be used.

If there are NO PLACES to the left of the decimal point, then count the number of zeros between the decimal point and the 1st Significant Figure. If this number of zeros is ODD, then use the LEFT HALF of the A scale. If the number of zeros is EVEN, then use the right half of the A scale.

TO FIND THE AREA OF A CIRCLE

A very important use for the A and B scales is in determining the area of a circle. The formula by which such an area is found can be expressed as:

Area $= 3.1416 \times (R)^2$ where R is the radius of the circle.
Or, **Area** $= .7854 \times (D)^2$ where D is the diameter of the circle.

Suppose you wanted to find the area of a circle whose radius is 2.58 feet. Set the left index of C at 2.58 on D. Then, the left index of B would be opposite the square of 2.58 (on A). It is not necessary, however, to read this square but, instead, we continue with the multiplication by setting the indicator at 3.14 on the B scale and the required area will then be read on the A scale as 20.8 square feet.

TO FIND THE CUBE OF A NUMBER

If you will now look at the bottom the your Slide Rule you will find a scale marked K which, again, is similar to the other scales we discussed except that there are three such scales within the space occupied by one on the D scale. This K scale indicates the **Cube** of the numbers on the D scale.

The Cube of a number is the product of the number multiplied by itself twice and is represented by the exponent 3. Thus, $3^3 = 3 \times 3 \times 3 = 27$.

Now, suppose you want to find the cube of 3.42. You set the indicator at 3.42 on the D scale and read the answer as 40.8 on the K scale. How do you know where to place the decimal point? The rules are as follows:

If the cube falls in the RIGHT HAND THIRD of the K scale, then the number of places to the left of the decimal point will be THREE TIMES that of a given number.

If the cube falls in the MIDDLE THIRD of the K scale, then the number of places to the left of the decimal point is equal to THREE TIMES that of the given figure, MINUS 1.

If the cube falls in the LEFT HAND THIRD of the K scale, then the number of places to the left of the decimal point, is equal to THREE TIMES that of the given figure, MINUS 2.

Where there are no digits to the left of the decimal point, the rules are as follows:

When the cube falls in the RIGHT HAND THIRD of the K scale, the number of zeros between the decimal point and the 1st significant figure is equal to THREE TIMES that of the given figure.

When the cube falls within the MIDDLE THIRD of the K scale, the number of zeros between the decimal point and the 1st significant figure is equal to THREE TIMES that of the given figure, PLUS 1.

When the cube falls on the LEFT HAND THIRD of the K scale, the number of zeros between the decimal point and the 1st significant figure is equal to THREE TIMES that of the given figure, PLUS 2.

TO FIND THE CUBE ROOT OF A NUMBER

The Cube Root of a number is that factor which, when multiplied by itself twice, would produce the given number. The Cube Root is indicated by a very small 3 in the upper portion of the groove of the radical sign. For example, $\sqrt[3]{27} = 3$ or, sometimes, the cube is written with a fractional exponent as $(27)^{1/3} = 3$. The process of finding the Cube Root is the reverse of finding the Cube.

To find the Cube Root of 8.3, you set the indicator at 8.3 on the left hand third of the K scale and read the root as 2.3 on the D scale. Again, the most difficult part of this operation is in determining which third of the K Scale you are to use and the rules for this follow along the same lines as those given for determining the number of places in a Cube. The rules are as follows:

When the number of digits to the left of the decimal point is a MULTIPLE OF 3 (i. e., 3, 6, 9, 12, etc.) use the RIGHT HAND THIRD of the K scale.

When the number of digits to the left of the decimal point is 1 LESS THAN A MULTIPLE OF 3 (i. e., 2, 5, 8, 11, etc.) use the MIDDLE THIRD of the K scale.

When the number of digits to the left of the decimal point is 2 LESS than a multiple of 3 (i. e., 1, 4, 7, 10, etc.) use the LEFT HAND THIRD of the K scale.

When there are no digits to the left of the decimal point, the following rules apply:

When the number of zeros between the decimal point and the 1st significant figure is a MULTIPLE OF 3, use the RIGHT HAND THIRD of the K scale.

When the number of zeros between the decimal point and the 1st significant figure is 1 MORE THAN A MULTIPLE OF 3, use the MIDDLE THIRD of the K scale.

When the number of zeros between the decimal point and the 1st significant figure is 2 MORE THAN A MULTIPLE OF 3, use the LEFT HAND THIRD of the K scale.

Exercises:

32. $(6.24)^2 = ?$
33. $(14.18)^2 = ?$
34. $\sqrt{81.6} = ?$
35. $\sqrt{.0000931} = ?$
36. $(16.38)^3 = ?$
37. $(.0029)^3 = ?$
38. $\sqrt[3]{187.4} = ?$
39. What is the area of a circle whose diameter is 28 feet?

The answers to these exercises will be found on the last page of this book.

THE CI SCALE

If you will look at your Slide Rule you will see between the B and C scales another scale on the sliding portion which is labeled CI and, after close inspection, you will realize that the scale is exactly opposite hand to the C scale. In other words, it is an inverted C scale, for which reason it is often called a **Reciprocal Scale.**

Before you can appreciate the use of such an inverted scale it is necessary for you to know what is meant by the reciprocal of a number. This may be defined as a number which, when multiplied by the given number, produces 1. For example, 1/5 is the reciprocal of 5, or .4 is the reciprocal of 2.5. Notice that every reading on the CI scale is opposite its reciprocal on the C scale.

Now, of what use could this reciprocal be? It so happens that multiplying by any number is the same as dividing by its reciprocal and we can make use of this relationship to find a **shortcut method** of multiplying several factors together. As an example, suppose we want to multiply $2 \times 3 \times 7 \times 16$. This would be the same as $2 \times 3 \div \frac{1}{7} \times 16$. A quick method of performing this operation is to set the C index at 2 on the D scale, move the indicator to 3 on the C scale, move the slide until the 7 on the CI scale coincides with the hair-line and then move the hair-line to 16 on C. The product is then read on the D scale, the significant figures being 672 and the decimal point being determined by approximation in the usual way, fixing the product at 672. You will notice that in determining this product only 4 settings were required, whereas in the orthodox method that you previously learned, 6 settings would be required.

For practice in using this CI scale, see if you can find the products of the following, using the method just described:

40. $2.8 \times 32.8 \times 1.615 = ?$
41. $.062 \times .274 \times .0667 = ?$
42. $10350 \times 645 \times .310 = ?$
43. $0.113 \times 42.6 \times .0069 = ?$
44. $9.8 \times 23.4 \times .643 = ?$

The answers to these exercises will be found on the last page of this book.

SOLVING PROBLEMS WITH THE SLIDE RULE

Illustrative Problems:

1. (For the Draftsman). A gear has 42 teeth and a diametral pitch of 1¼. What is its pitch diameter?

Solution: According to Page 18, Lesson 8 of "A Home Study Course in Blueprint Reading":

$$\text{Diametral Pitch (D.P.)} = \frac{\text{Number of Teeth } (n)}{\text{Pitch Diameter (P. D.)}}$$

Therefore

$$\text{Pitch Diameter (P. D.)} = \frac{n}{\text{D.P.}} = \frac{42}{1.25} = 33.6 \text{ inches}$$

2. (For the Machinist). At what speed (R. P. M.) should a ⅞″ drill be run for a cutting speed of 65 feet per minute?

Solution:

$$\text{R.P.M.} = \frac{\text{cutting speed}}{\text{perimeter of drill}}$$

$$= \frac{65 \times 12}{.875 \times 3.1416} = 284 \text{ R.P.M.}$$

3. (For the Contractor). What will be the cost of materials on a job requiring 148 cubic yards of concrete at $9.55 per C.Y. and 51,800 Ft. B.M. of lumber at $38.00 per thousand FBM?

Solution:

$$148 \times \$9.55 + 518 \times \$38.0 = \$1413 + \$1968 = \$3381$$

Exercises:

Now see if you can solve the following practical problems with the Slide Rule. Check your answers with those given on the last page of this book.

45. (For the Plumber). A plumber needs 64 feet of ½″ pipe at .850 pounds per foot, 39 feet of ¾″ pipe at 1.130 lbs./ft. and 43 feet of 1″ pipe at 1.678 lbs./ft. If the pipe costs 9.4c per pound, what will be the total cost of the pipe?

46. (For the Sheet Metal Worker) What will be the total weight of 32 sq. ft. of #12 ga. sheet metal at 4.462 pounds per sq. ft. and 48 sq. ft. of #16 ga. at 2.55 pounds per sq. ft.?

47. (For the Electrician). How many kilowatt hours are required to run a D. C. motor developing 5 Horse Power for 19 hours if the efficiency of the motor is 85%? (Note that 1 H. P. = .746 KW.)

Hint: $K.W.H. = \frac{100}{85}(5 \times 19 \times .746)$

48. (For the Carpenter). How many board feet (F. B. M.) in 69—3″ × 8″ boards 22′ 0″ long?

49. (For the Ordnance Man). What is the diameter in inches of the bore of a 75 mm. gun?

50. (For the Welder). What will be the cost of electrodes for 438 feet of a weld between sheets of 10 gauge metal, if .051 pounds of electrode are required per foot of weld, and electrodes cost $.095 per pound?

51. (For the Machine Designer). According to the American Standards Association, a Medium Fit (Class 3) should have an allowance of $0.0009 \sqrt[3]{d^2}$ between hole and shaft, where d is the diameter of the shaft. For a $4\frac{15}{16}$ inch shaft, what should be the allowance?

The answers to these exercises will be found on the last page of this book.

GEOMETRY

Geometrical Magnitudes and Shapes

If you have ever read a blueprint, whether of a machine part, tool, or house, you know how important it is to recognize the Geometrical Magnitudes or Shapes shown on these prints. For the purpose of identifying these Shapes and applying formulas for calculations of their lengths, areas, volumes and weights, it is highly desirable that you become familiar with the names and properties of the common geometrical figures.

There are four kinds of Geometrical Magnitudes. First, there is the Point which has neither length, breadth, nor thickness. The Line has length but no breadth or thickness. Areas have length and breadth but no thickness. Solids have length, breadth and thickness.

Lines:

There are two classes of Lines, namely Straight and Curved. A Straight Line is one which never changes its direction, whereas a Curved Line continually changes its direction.

Two Lines which have exactly the same direction are said to be **Parallel.**

When one line intersects another, four angles are formed. If these angles are all equal they are called **Right Angles** and the lines are said to be **Perpendicular** to each other. A Right Angle is equal to 90°.

Plane Figures:

A surface in which a straight line can be placed in any position and lie completely within the surface is known as a **Plane.** A **Plane Figure** is a portion of a Plane bounded by either straight or curved lines.

When the bounding lines of a Plane Figure are **straight,** the figure is known as a **Polygon.** Polygons are classified according to the number of sides that bound them.

Triangles (Fig. 4):

Polygons which have three sides are known as **Triangles.**

An **Equilateral Triangle** is one with three equal sides.

An **Isosceles Triangle** is one with only two of its sides equal.

A **Scalene Triangle** has each of its three sides of different length.

A **Right Triangle** is one that has a right angle between two of its sides.

Quadrilaterals (Fig. 5):

A polygon with four sides is known as a **Quadrilateral.**

A **Parallelogram** is a quadrilateral having its opposite sides parallel.

A **Rectangle** is a parallelogram having four right angles.

A **Square** is a rectangle having four equal sides.

An **Oblong** is a rectangle with adjacent sides not equal.

A **Rhombus** is a parellelogram not having right angles but with all four sides equal.

A **Rhomboid** is a parallelogram without right angles and with adjacent edges not equal.

A **Trapezoid** is a quadrilateral with two sides parallel but not equal.

A **Right Trapezoid** is a trapezoid having two right angles.

A **Trapezium** is a quadrilateral with no two opposite sides parallel.

A **Right Trapezium** is a trapezium having at least one right angle.

Additional Polygons:

A **Pentagon** is a polygon having five sides.

A **Hexagon** is a polygon having six sides.

A **Heptagon** is a polygon having seven sides.

An **Octagon** is a polygon having eight sides.

A **Dekagon** is a polygon having ten sides.

Plane Figures with Curved Boundaries:

The **Circle** (Fig. 6) is a plane figure bounded by a curved line, every point of which is equidistant from the center.

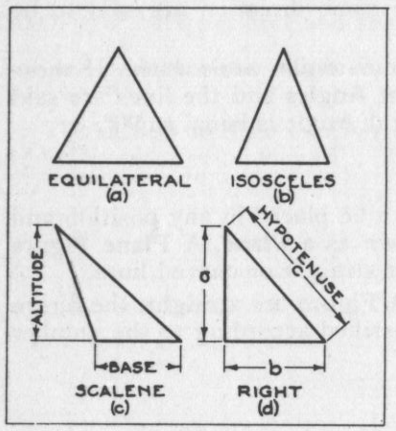


Fig. 4
Triangles

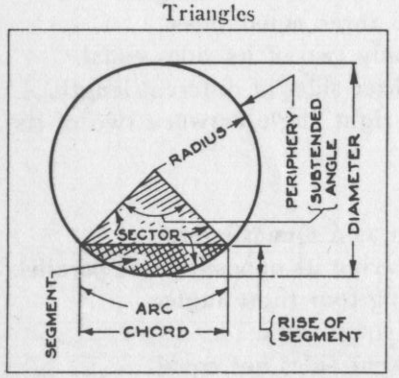


Fig. 6
Parts of a Circle

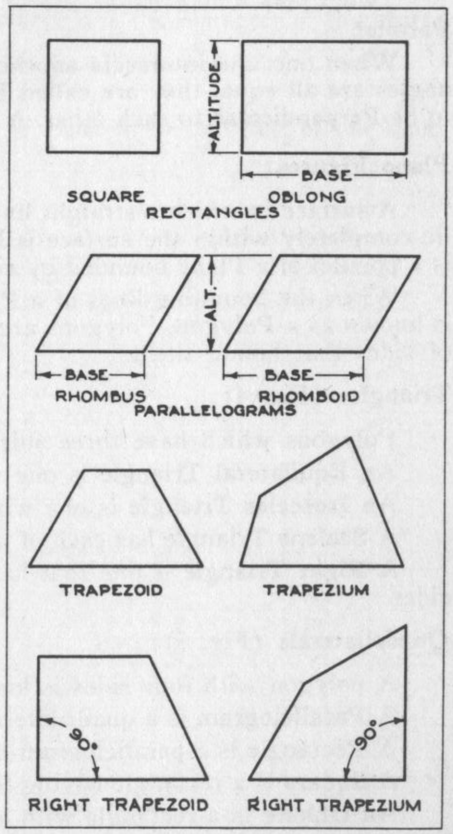


Fig. 5
Quadrilaterals

The **Radius** of the circle is the distance from the center to the curved boundary.

The **Diameter** of the circle is the widest dimension of the circle and is equal to twice the radius.

The **Circumference** or **Periphery** of a circle is the length of the boundary.

A **Sector** of a circle is that part of the circle bounded by two radii and a portion of the periphery, such as the hatched area shown in Fig. 6.

The **Segment** of a circle is that portion of a sector which remains after the Isosceles Triangle between the two bounding radii has been deducted. The area with double line cross-hatching in Fig. 6 is a Segment.

The **Arc** of a circle is a portion of the circumference.

The **Ellipse** is a plane figure bounded by a curve drawn so that the sum of the distances from any point on it to two fixed points, known as the Foci, is constant and equal to the Major Axis of the Ellipse. The Ellipse looks like a flattened circle, the largest diameter of which is called the Major Axis, and the smallest the Minor Axis.

Solids:

Solids are three dimensional magnitudes bounded by Planes, Single-curved Surfaces, Double-curved Surfaces, or combinations of these. As an example of one bound by a double-curved surface, you have the Sphere, which is like a round ball. Cylinders and Cones have single-curved surfaces but plane bases. Fig. 7 illustrates a number of geometric solids.

A **Polyhedron** is a solid bound by plane surfaces.

A **Prism** is a polyhedron that has two parallel and equal polygons as bases and whose other sides (lateral faces) are parallelograms. A

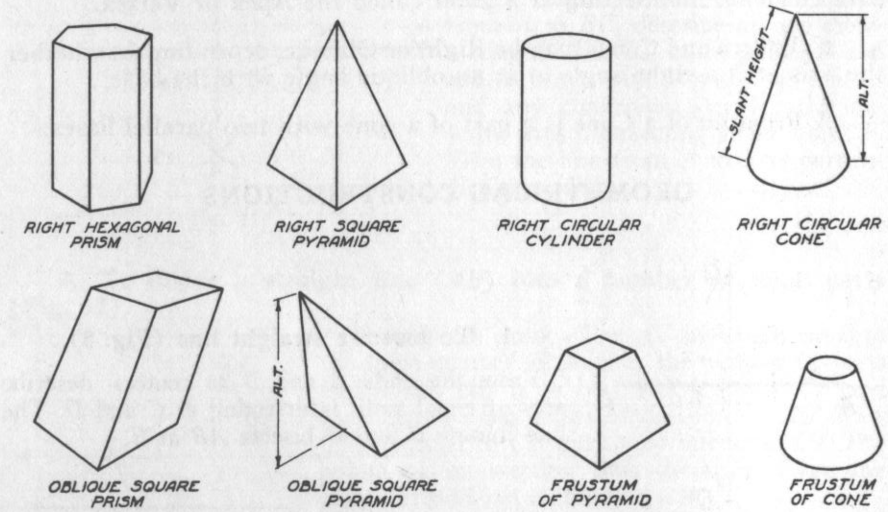


Fig. 7
Geometric Solids

Prism is known according to the polygon which forms the base. Thus, a Prism with a triangle as a base is known as a **Triangular Prism** and a Prism with a hexagon as a base is known as a **Hexagonal Prism**, etc.

A **Lateral Edge** of a Prism is the line of intersection between two of the lateral faces.

The **Axis** of a Prism is a line joining the centers of the bases.

A **Right Prism** is one whose lateral edges, and therefore axis, are perpendicular to the bases.

An **Oblique Prism** is one in which the axis is not perpendicular to the bases.

A **Pyramid** is a polyhedron bounded by a base which is a polygon and by triangular lateral faces which meet at a common point known as the **Vertex** or **Apex.**

The **Axis** of a Pyramid is the line joining the apex with the center of the base.

A **Right Pyramid** is one whose axis is perpendicular to the base.

An **Oblique Pyramid** is one whose axis is oblique to the base.

A **Frustum** of a Pyramid is a portion of a pyramid having two parallel bases.

A **Cylinder** is a solid with a singly curved surface having parallel elements, and two parallel plane bases. The most common form of cylinder is that having a circle as a base.

A **Cone** is a solid having a plane base and a singly curved surface with elements intersecting at a point called the **Apex** or **Vertex.**

Cylinders and Cones may be Right or Oblique, depending on whether the axis is at a right angle or at an oblique angle with the base.

A **Frustum** of a Cone is a part of a cone with two parallel bases.

GEOMETRICAL CONSTRUCTIONS

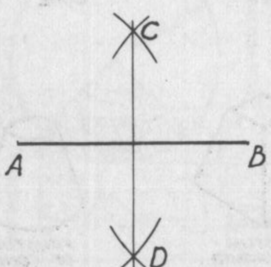


Fig. 8

1. To bisect a straight line (Fig. 8).

From the ends A and B as centers, describe arcs of equal radii intersecting at C and D. The line joining C and D bisects AB at E.

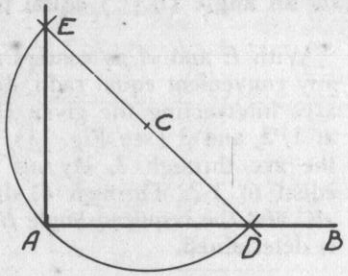


Fig. 9

2. To erect a perpendicular to a straight line (AB) from a given point (A) in that line.

First Method (Fig. 9)

From a convenient center, C, draw an arc through A intersecting AB at D. Through C and D draw a line intersecting the arc at E. AE is then the required perpendicular.

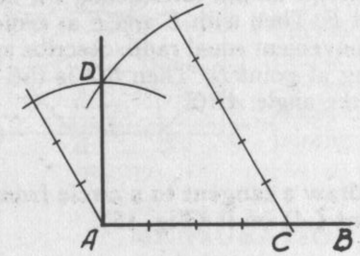


Fig. 10

Second Method (Fig. 10)

With A as a center and a radius of 3 units, describe an arc. On the line AB, lay off AC equal to 4 units. With C as a center and 5 units as a radius, describe an arc intersecting the arc previously drawn at point D. Then the line AD is the required perpendicular.

3. To construct a perpendicular to a straight line (BC) from a point (A) without it (Fig. 11).

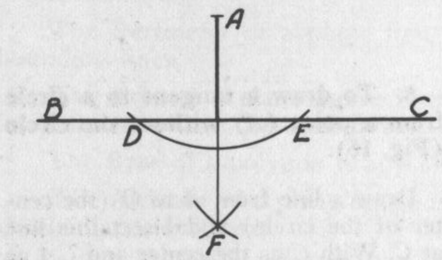


Fig. 11

With A as a center and any convenient radius greater than the distance from A to BC, describe an arc intersecting the line BC at the points D and and E. Now, with D and E as centers and any convenient equal radii, describe arcs intersecting at the point F. Then the line from A to F is perpendicular to BC.

4. To divide a straight line (AB) into a number of equal parts (Fig. 12).

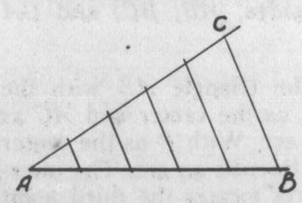


Fig. 12

Through A, draw a line AC of length equal to the same number of units as the number of parts into which AB is to be divided. Draw CB and then through the points of division on AC draw lines parallel to CB until they intersect AB. The points of intersection thus determined are the points of division of the line AB.

By a similar process, a line may also be divided unequally, as required.

5. Upon a straight line (AB), to draw an angle (BAC) equal to a given angle (DEF) (Fig. 13).

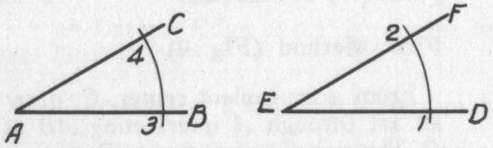


Fig. 13

With E and A as centers and any convenient equal radii, draw arcs intersecting the given sides at 1, 2, and 3 (see Fig. 13). On the arc through 3, lay off 3-4 equal to 1-2. Through 4, draw AC and the required angle BAC is determined.

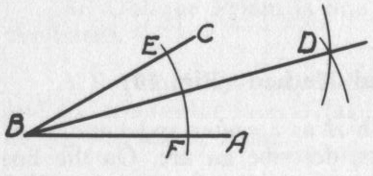


Fig. 14

6. To bisect an angle (ABC) (Fig. 14).

With B as a center and any convenient radius, describe an arc intersecting BC at E and BA at F. Then with E and F as centers and any convenient equal radii, describe arcs intersecting at point D. Then BD is the bisector of the angle ABC.

7. To draw a tangent to a circle from a given point (A) on it (Fig. 15).

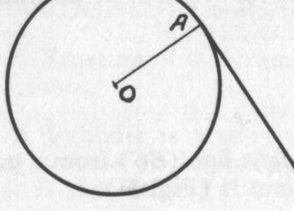


Fig. 15

Through A, draw a line through the center of the circle O. Then at A, erect a perpendicular to AO which is then the required tangent.

8. To draw a tangent to a circle from a point (A) without the circle (Fig. 16).

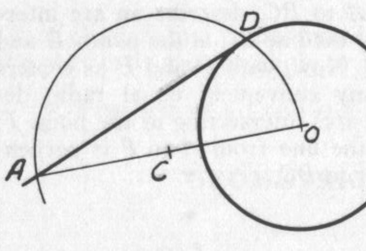


Fig. 16

Draw a line from A to O (the center of the circle) and bisect this line at C. With C as the center and CA as the radius, describe an arc intersecting the circle at D. A line joining A with D is the required tangent.

9. To construct a triangle, knowing the length of the three sides, AB, BC, and CA (Fig. 17).

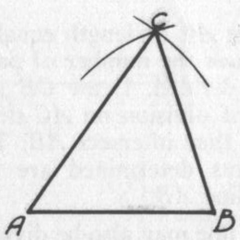


Fig. 17

Draw the side of the triangle AB with the known length. With A as the center and AC as the radius, describe an arc. With B as the center and BC as the radius, describe an arc. The intersection of these two arcs locates the third point of the triangle.

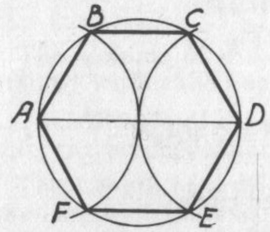


Fig. 18

10. To construct a regular hexagon knowing its diagonal AD (Fig. 18).

With the diagonal AD as a diameter, construct a circle. With A and D as centers and the radius of the circle AO as radii, draw arcs intersecting the circle at B, C, E, and F. Lines joining A to B, B to C, etc., form the hexagon.

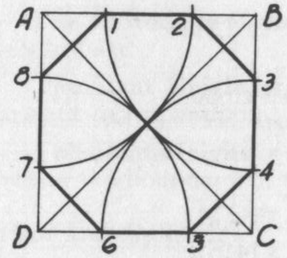


Fig. 19

11. To change a square to an octagon (Fig. 19).

Draw the diagonals of the square BD and AC, intersecting at E. With A as center and AE as radius, describe an arc intersecting AB at 2 and AD at 7. Determine points 1, 4 similarly from B, 3 and 6 from C, and 5 and 8 from D. The lines joining 1, 2, 3, 4, 5, 6, 7 and 8 form the octagon.

MENSURATION OF PLANE FIGURES

The relationship that exists between lengths, areas, and volumes of geometrical magnitudes is known as **Mensuration.** Mensuration is continually being used for estimating the quantities of materials and costs of construction and mechanical work on buildings, bridges, airplanes, ships, tanks, etc.

The **Perimeter** of a plane figure is the sum of the lengths of the bounding lines.

The **Area** of a plane figure is the number of unit squares that can be included within the boundaries.

The **Base** of a polygon is any one of the edges.

The **Altitude** of a plane figure is the overall distance measured perpendicular to the base.

Triangle:

$$\text{Area} = \text{Base} \times \tfrac{1}{2}\ \text{Altitude} \quad \text{(see Fig. 4)}$$

Right Triangle:

Referring to Fig 4(d)

$$c^2 = a^2 + b^2$$

where c is the hypotenuse and a and b are the other two legs. From this, it follows that

$$c = \sqrt{a^2 + b^2}$$
$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$

Parallelogram:

$$\text{Area} = \text{Base} \times \text{Altitude (See Fig. 5)}$$

Regular Hexagon:

$$\text{Area} = 2.598 \times S^2 = 2.598\,R^2 = 3.464\,r^2$$

where S = length of a side
R = radius of circumscribed circle
r = radius of inscribed circle

Octagon:

$$\text{Area} = 4.828 \times S^2 = 2.828\,R^2 = 3.314\,r^2$$

where S = length of a side
R = radius of circumscribed circle
r = radius of inscribed circle

Circle:

$$\text{Area} = \pi r^2 = 3.1416 \times r^2 = .7854\,d^2$$

$$\text{Circumference} = 2\pi r = 6.2832 \times r = 3.1416\,d$$

where r = radius
d = diameter

Circular Sector:

$$\text{Length of Arc} = \frac{3.1416\,ra}{180} = .01745 \times r \times a$$

$$\text{Area} = \frac{3.1416\,r^2 a}{360} = .008727 \times a \times r^2$$

where r = radius
a = angle in degrees

Circular Segment:

$$c = 2\sqrt{2br - b^2}$$

$$b = r - \tfrac{1}{2}\sqrt{4r^2 - c^2}$$

where c = length of chord
b = rise of segment
r = radius

Circular Ring:

$$\text{Area} = 3.1416\,(R^2 - r^2) = 3.1416\,(R + r)\,(R - r)$$

where R = outside radius
r = inside radius

Ellipse:

$$\text{Area} = 3.1416 \times a \times b$$

where a = ½ major axis
b = ½ minor axis

MENSURATION OF SOLIDS

The **Volume** of a solid is the number of unit cubes that could be contained within the boundaries.

The **Altitude** of a prism, or cylinder, or frustum of a cone or pyramid, is the perpendicular distance between the bases.

The **Length** of a prism, cylinder, or frustum of a cone or pyramid, is the length of the axis.

In the case of a Right Prism or Cylinder, the Altitude and Length are the same.

The **Altitude** of a cone or pyramid is the perpendicular distance from apex to base.

The **Slant Height** of a Right Regular Pyramid is the distance from the apex to the midpoint of one of the edges of the base.

The **Slant Height** of a Right Circular Cone is the distance from the apex to the boundary of the base.

Prism and Cylinder:

Lateral Surface Area = Length × Base Perimeter
Total Surface Area = Lateral Surface + (2 × Base Area)
Volume = Altitude × Base Area

Pyramid or Cone:

Volume = ⅓ × Altitude × Base Area

Right Pyramid or Cone:

Lateral Surface Area = ½ × Slant Height × Base Perimeter
Total Surface Area = Lateral Surface + Base Area

Frustum of Pyramid or Cone:

$$\text{Volume} = \tfrac{1}{3} \times h \times (a + A + \sqrt{aA})$$

where h = altitude
a = area of small base
A = area of large base

Frustum of Right Circular Cone:

$$\text{Volume} = .2618\,h\,(d^2 + dD + D^2)$$

$$\text{Lateral Surface Area} = 1.5708\,(D + d)\,\sqrt{\tfrac{1}{4}\,(D - d)^2 + h^2}$$

where h = altitude
d = diameter of small base
D = diameter of large base

Hollow Right Cylinder:

$$\text{Volume} = .7854 \times h \times (D^2 - d^2)$$

$$= .7854 \times h \times (D - d)\,(D + d)$$

where h = altitude
D = outside diameter
d = inside diameter

Sphere:

Surface Area $= 3.1416 \times d^2$
Volume $= .5236 \times d^3$
where $d =$ diameter

Hollow Sphere:

Volume $= .5236\ (D^3 - d^3)$
where $D =$ outside diameter
$d =$ inside diameter

Circular Ring (Torus):

Surface Area $= 9.8696 \times D \times d$
Volume $= 2.4674 \times D \times d^2$
where $D =$ mean diameter of ring
$d =$ diameter of section

Exercises:

Here are some practical problems involving the formulae just given. See if you can solve them.

52. Find the number of cubic yards of concrete in a footing, if you read on a blueprint that it is 6′ 10″ long, 6′ 10″ wide, and 2′ 6″ deep.

Suggestion: Reduce inches to decimals of a foot by means of Table II.

53. Suppose that sand has been dumped in a pile shaped like a ight circular cone. The height is 6 feet and the base diameter is 12 feet. Find the number of cubic yards in the pile.

54. A piece of round steel stock 1½″ diameter is 3′ 0″ long. Find its weight if the steel weighs 490 lbs. per cu. ft.

Note: A piece of this steel 1 in. square × 1′ 0″ would weigh 490 ÷ 144 = 3.4 pounds.

55. A conical piece of a duct of #20-gauge metal (U.S.S.) is 2′ 0″ long, has a diameter of 1′ 0″ at one end, and a diameter of 6″ at the other. Find its weight.

Note: #20 U.S.S. gauge is equivalent to a weight of 1.5 pounds per square foot.

56. What is the length of diagonal pipe necessary to join two points when their elevations are + 28′ 0″ and 16′ 5¼″, and the distance between them is scaled on a plan (horizontal projection) as 13′ 8″?

57. Find the weight of a hollow cylindrical iron casting (right circular) when the outside diameter is 6″, the inside 4″, and the length 8″. Cast iron weighs 450 lbs. per cu. ft.

58. How many gallons will a cylindrical tank hold whose diameter is 10 feet and whose altitude is 10 feet?

59. Change 13.245 meters to feet and inches.

60. What is the weight of a steel ball 2¼ inches in diameter if steel weighs 490 lbs. per cu. ft.

The answers to these exercises will be found on the last page of this book.

ALGEBRA

Literal Numbers

In Algebra, letters as well as Arabic numerals are used to represent numbers. Thus, in the statement that the Area of a rectangle equals the Base times the Altitude, the Area may be expressed by the letter A, the Base by b and the Altitude by h, and the statement may be expressed as

$$A = b \times h$$

Symbols

The expression $A = b \times h$ can also be expressed as

$$A = b \cdot h$$
$$\text{or } A = bh$$

Division is usually expressed as a fraction in Algebra. Thus,

$$h = A \div b \quad \text{is usually written} \quad h = \frac{A}{b}$$

Addition, subtraction, and equality symbols are the same as those used in arithmetic, as are also the symbols for powers and roots.

∴ means "therefore".
± means "plus or minus".

Coefficients

An Arabic number prefixed to a literal quantity to indicate how many times the quantity is to be taken is called a **Coefficient.** Thus, $3ax$ means $ax + ax + ax$, the coefficient being 3.

ALGEBRAIC EXPRESSIONS, FORMULAE AND EQUATIONS

A quantity expressed by one or more literal numbers with symbols to indicate how they are combined is called an **Algebraic Expression.**

Thus if m and n stand for certain numbers, $7\,m$, $n + 2$, $m + n$, $2\,mn$, etc., are Algebraic Expressions.

These Algebraic Expressions can be readily evaluated if the value of the literal numbers are known.

Thus, if $m = 2$ and $n = 4$,

$$7\,m = 14 \qquad n + 2 = 6$$
$$m + n = 6 \qquad 2\,mn = 16$$

A **Term** of an algebraic expression is any combination of symbols and coefficients not separated by a plus or minus sign. Thus, in the expression $3\,bx + 2\,ay$, the Terms are $3\,bx$ and $2\,ay$.

Similar or **Like Terms** are those which differ only in their coefficients. Thus, $4\,ax$ and $3\,ax$ are Like Terms.

The number of **literal factors** contained in a term determines the **Degree** of the term. Thus,

$2a$ is of the first degree
$3ab$ is of the second degree
$6a^3b^2$ is of the fifth degree

When one algebraic expression is indicated as being equal to a number or another algebraic expression, there results an **Equation** or **Formula.**

Examples of equations are:

$$m + n = 6$$

$$2\,mn = a + 2b$$

The formula for the area of a rectangle is:

$$A = b \cdot h$$

The numbers on the right side of the equality sign comprise the **Right Member** or **Right Side** of the equation, while those on the left side make up the **Left Member** or **Left Side** of the equation.

Exercises:

Given $a = 3$ and $b = 4$, find the values of:

61. $a \cdot b$
62. $\frac{a}{b}$
63. $6a + 4b$
64. $\sqrt{b}$
65. $3a - b$
66. $(a + b)a$
67. $a + ba$
68. $(b - a)^2$
69. b^2a
70. a^3

The answers to these exercises will be found on the last page of this book.

Note: In exercise 66 above, the symbol ( ) is called **Parentheses.** The number inside must be combined before using them in any other addition, subtraction, multiplication, or division. When no parentheses are used, multiplications and divisions should be performed before additions or subtractions. Thus,

in exercise 66, $(a + b)a = (3 + 4)3 = 7 \quad 3 = 21$
while in exercise 67, $a + ba = 3 + 4 \cdot 3 = 3 + 12 = 15$

Many problems which cannot easily be solved by the ordinary methods of arithmetic can be readily solved by translating the statement into an algebraic equation and then solving the equation for the unknown quantities. That is one reason why Algebra is a valuable tool for making calculations easy.

Before you can solve equations, however, you must learn the significance of the various parts of an equation and how to manipulate them. You must learn how to add, subtract, multiply, divide, factor, and find the powers and roots of algebraic expressions. When you accomplish this you will be surprised how easy the solution of algebraic equations becomes.

ALGEBRAIC MANIPULATION

Positive and Negative Quantities

Positive and Negative numbers are used in Algebra to distinguish between opposite quantities. The + or — signs are used respectively, to denote whether terms are positive or negative.

Examples of the Use of Negative Numbers

1. The number of degrees below zero as recorded on a thermometer may be described as —8°.

2. A \$100 loss may be written —\$100 as contrasted with a \$100 gain.

3. The elevation of a certain point on a structure may be indicated as +80′ 4″ while that of the cellar floor may be indicated as —10′ 2″, the ground level being taken as elevation 0.

The minus sign must *always* be written before a **Negative Number,** for a number with *no sign* before it is assumed to be **Positive.**

The number itself, without regard to the Sign, is known as the **Absolute Value** of the number. Thus the Absolute Value of either +8 or —8 is 8.

Rules for Adding Positive and Negative Numbers:

1. To add two numbers of LIKE SIGNS, add the Absolute Values and prefix the common sign to the result.

2. To add two numbers of UNLIKE SIGNS, subtract the smaller Absolute Value from the larger and prefix the sign of the LARGER to the result.

Examples:

$$(+5)+(+4) = +9$$
$$(-5)+(-4) = -9$$
$$(+5)+(-4) = +1$$
$$(-5)+(+4) = -1$$

Rules for Subtracting Positive and Negative Numbers.

In Subtraction, the quantity to be subtracted is the **Subtrahend;** the quantity from which it is to be subtracted is the **Minuend;** the result is the **Remainder.**

RULE: Change the sign of the SUBTRAHEND and add the CHANGED SUBTRAHEND to the Minuend.

Examples:

$$(+6)-(+4) = (+6)+(-4) = +2$$
$$(+5)-(-3) = (+5)+(+3) = +8$$
$$(-8)-(+4) = (-8)+(-4) = -12$$

ADDITION AND SUBTRACTION OF ALGEBRAIC TERMS

If we add three dimes to five dimes we get eight dimes. Expressing this numerically, we have

$$3 \times 10 + 5 \times 10 = 8 \times 10 \text{ or } 80$$

To express this algebraically, let d equal dime or the number ten.

Then $3d + 5d = 8d$

In that way you can see how easy it is to add algebraic expressions involving **Like Terms.** In the above expressions, the *d*'s, or the 10's are called **Common Factors.**

You can make the subtraction of Like Terms just as easy by just memorizing the rule, "Change the sign of the Subtrahend and add the Changed Subtrahend to the Minuend".

Example:

Subtract $-8ab$ from $+10\,ab$

Solution: $10\,ab-(-8ab)=10\,ab+8\,ab=18\,ab$

Rules for Removing Parentheses:

1. If the parentheses are removed from an expression preceded by a plus sign and the enclosed terms are left with their original signs, the value of the expression is unchanged.

Thus:

$$(a+b)+(-a-c)+(a-d)$$
$$=a+b-a-c+a-d$$
$$=a+b-c-d$$

2. To remove the parentheses from an expression preceded by a minus sign, without changing its value, the signs of the enclosed terms must be changed.

Thus:

$$a+b-(-a-c)-(a-d)$$
$$=a+b+a+c-a+d$$
$$=a+b+c+d$$

In the above examples note how the Like Terms (*a*'s) were combined in the last step.

When parentheses are included between parentheses, you should proceed step by step in removing them. *Brackets* [ ] or *Braces* { } are used instead of parentheses in such cases.

Thus:

$$32-\{3+7-5[3-(5+2)]-8\}$$
$$=32-\{3+7-5[3-7]-8\}$$
$$=32-\{10+20-8\}$$
$$=32-22$$
$$=10$$

Addition and Subtraction of Polynomials:

A **Monomial** is an algebraic expression of one term.

A **Polynomial** is an expression of two or more terms, such as

$$a^2+a+ab+3$$

In adding or subtracting Polynomials it is necessary simply to add all like terms, changing the signs of the terms of those Polynomials which are to be subtracted.

Example: To the expression $a^2+a+ab+3$ add $3a^2+4a+15$ and subtract $2a^2+3a-5ab+4$.

This may be written:

$$(a^2+a+ab+3)+(3a^2+4a+15)-(2a^2+3a-5ab+4)$$
$$=a^2+a+ab+3+3a^2+4a+15-2a^2-3a+5ab-4$$
$$=a^2+3a^2-2a^2+a+4a-3a+ab+5ab+3+15-4$$
$$=2a^2+2a+6ab+14$$

Multiplication and Division of Positive and Negative Numbers

RULE: The PRODUCT of two factors is POSITIVE if the signs of the factors are alike. Otherwise the PRODUCT is NEGATIVE.

The above rule also applies to division which is, as you have already observed, the same as multiplying by the reciprocal.

Examples:

$$(+4)\times(+3)=+12$$
$$(-4)\times(-3)=+12$$
$$(-4)\times(+3)=-12$$
$$(+12)\div(-3)=(+12)\times(-\tfrac{1}{3})=-4$$

Exponent, Base, and Power

When a quantity is multiplied by itself one or more times, the number of times it is taken as a factor is called the **Exponent,** the quantity itself is called the **Base,** and the result the **Power** of the number.

The Exponent is written as a small integer to the right of and higher than the base.

Thus x^4 is read as "*x* fourth" or "*x* fourth power" where *x* is the base and 4 the exponent.

Every power of a **positive** number is **positive.**

Every **even** power of a **negative** number is **positive,** while every odd power is **negative.**

For example:

$$(+3)^2=(+3)\cdot(+3)=+9$$
$$(+3)^3=(+3)\cdot(+3)\cdot(+3)=+27$$
$$(-3)^2=(-3)\cdot(-3)=+9$$
$$(-3)^3=(-3)\cdot(-3)\cdot(-3)=-27$$

Rules of Algebraic Multiplication

1. The product of two terms containing both numbers and letters is written as the product of the numerals followed by the product of the letters; for example,

$$3a\cdot 4b=12ab$$

2. The exponent of the product of two factors having the same base, is equal to the sum of the exponents of the factors.

You can readily understand this rule by considering the multiplication of a^4 by a^3. For then,

$$\begin{aligned} a^4 \cdot a^3 &= (a \cdot a \cdot a \cdot a) \times (a \cdot a \cdot a) \\ &= a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \\ &= a^7 \end{aligned}$$

The rule for multiplying Positive and Negative Numbers applies equally well to Literal Numbers.

For example,

$$(-a) \cdot (+3b) = -3ab$$

$$(-a^3) \cdot (-a^4) = +a^7$$

3. To multiply a POLYNOMIAL by a MONOMIAL, multiply the Monomial by EACH of the terms of the Polynomial and write the resulting terms in order with their proper signs.

$$\begin{aligned} \text{Thus:} \quad & [a^2b^2 + 2ab + b^2 - 3] \cdot [3a] \\ &= 3a \cdot a^2b^2 + 3a \cdot 2ab + 3a \cdot b^2 - 3a \cdot 3 \\ &= 3a^3b^2 + 6a^2b + 3ab^2 - 9a \end{aligned}$$

4. To multiply a POLYNOMIAL by another POLYNOMIAL, multiply each term of the one by each term of the other in turn, and add the partial products.

To understand this, first consider the multiplication of $(2 + 5)$ by $(3 + 6)$. This, of course is $7 \times 9 = 63$. The multiplication could also be carried out in this way:

$$\begin{aligned} (2+5)(3+6) &= 2(3+6) + 5(3+6) \\ &= 2 \cdot 9 + 5 \cdot 9 \\ &= 18 + 45 \\ &= 63 \end{aligned}$$

Example: Multiply $3y^2 - 8y + 6$ by $y - 4$

$$\begin{aligned} (3y^2 - 8y + 6) \cdot (y - 4) &= y(3y^2 - 8y + 6) - 4(3y^2 - 8y + 6) \\ &= 3y^3 - 8y^2 + 6 - 12y^2 + 32y - 24 \\ &= 3y^3 - 20y^2 + 38y - 24 \end{aligned}$$

Algebraic Division:

Division is indicated by the sign $\div$ or as a fraction.

$$\text{Thus:} \quad 8 \div 2 = \frac{8}{2} = 4$$

$$\text{or} \quad a \div b = \frac{a}{b}$$

RULE: The Exponent of any letter in a Quotient is Equal to its Exponent in the Dividend MINUS its Exponent in the Divisor.

This is easily seen by considering the division of a^6 by a^4.

$$\begin{aligned} \frac{a^6}{a^4} &= \frac{a \cdot a \cdot a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a} \\ &= \frac{a}{a} \cdot \frac{a}{a} \cdot \frac{a}{a} \cdot \frac{a}{a} \cdot a \cdot a \\ &= 1 \cdot 1 \cdot 1 \cdot 1 \cdot a^2 \\ &= a^2 \end{aligned}$$

When the Dividend and the Divisor both have the Same Exponent, the Quotient is 1; e. g.,

$$\frac{a^2}{a^2} = \frac{a \cdot a}{a \cdot a} = \frac{a}{a} \cdot \frac{a}{a} = 1 \cdot 1 = 1$$

You will probably best understand the method of dividing Monomials by the following examples:

1. Divide $5a^3$ by a^2.

$$\frac{5a^3}{a^2} = 5 \cdot \frac{a^3}{a^2} = 5 \cdot a = 5a$$

2. $\dfrac{a^2b^3c}{ab^3} = \dfrac{a^2}{a} \cdot \dfrac{b^3}{b^3} \cdot c = a \cdot 1 \cdot c = ac$

3. $\dfrac{48ax^6}{-8bx^4} = \dfrac{48}{-8} \cdot \dfrac{a}{b} \cdot \dfrac{x^6}{x^4} = -6 \cdot \dfrac{a}{b} \cdot x^2 = -\dfrac{6ax^2}{b}$

4. $\dfrac{-27a^3b^2}{-9a^2b^2} = \dfrac{-27}{-9} \cdot \dfrac{a^3}{a^2} \cdot \dfrac{b^2}{b^2} = +3 \cdot a \cdot 1 = 3a$

The following rule should be used for dividing a Polynomial by a Monomial:

RULE: Divide each term of the Dividend by the Divisor and write the results in succession.

Example: Divide $9x^3 - 12x^2 + 3x$ by $-3x$

$$\frac{9x^3 - 12x^2 + 3x}{-3x} = \frac{9x^3}{-3x} - \frac{12x^2}{-3x} + \frac{3x}{-3x} = -3x^2 + 4x - 1$$

The following example will illustrate to you how to divide a Polynomial by another Polynomial. You will see from this that the method is the same as that used in Arithmetic.

Example: Divide $6x^2 - 22x + 12$ by $x-3$

Solution:

1. $6x^2 \div x = 6x$
2. $(x-3)6x = 6x^2 - 18x$; subtract
3. $-4x \div x = -4$
4. $(x-3) - 4 = -4x + 12$; subtract

$$\begin{array}{rl} & \quad 6x - 4 \\ \text{Divisor } x-3 & \overline{)\, 6x^2 - 22x + 12} \text{ Dividend} \\ & \quad \underline{6x^2 - 18x} \\ & \qquad -4x + 12 \\ & \qquad \underline{-4x + 12} \\ & \qquad\qquad 0 \end{array}$$

Check:

$$\begin{aligned} (x-3)(6x-4) &= 6x^2 - 18x - 4x + 12 \\ &= 6x^2 - 22x + 12 \end{aligned}$$

Factoring

To Factor an expression is to find other expressions which, when multiplied by each other, result in the given expression.

Example: The factors of $3x^2 + 6x - ax$ are x and $3x + 6 - a$ since $x(3x+6-a) = 3x^2 + 6x - ax$.

In general, Factors cannot always be readily found, but you should know of a few types of expressions that are easily factorable:

1. Polynomials with Monomial Factors such as the example just given.
2. The difference of two squares, e. g.,
$$a^2 - b^2 = (a+b)(a-b)$$
3. The sum of two cubes, e. g.,
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
4. The difference of two cubes, e. g.,
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
5. Trinomial squares, e. g.
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

A noteworthy example of the use of Factoring in making mathematics easy is in the finding of a leg of a triangle, given the hypotenuse and the other leg. You will recall that $b = \sqrt{c^2 - a^2}$ where a and b are the legs and c the hypotenuse.

Suppose $c = 3.25$ and $a = 1.25$ and you wish to find b.

Then $b = \sqrt{(3.25^2 - (1.25)^2}$

Factoring $b = \sqrt{(3.25 - 1.25)(3.25 + 1.25)}$

$= \sqrt{2 \times 4.50} = \sqrt{9} = 3$

Determine for yourself if this isn't easier than finding first $(3.25)^2$, then $(1.25)^2$ and then subtracting.

SOLVING AN EQUATION

When $x = 4$,
$$3x + 12 = 24$$
since $3 \cdot 4 + 12 = 12 + 12 = 24$.

The number 4 is then said to satisfy the equation $3x + 12 = 24$ or to be the Root of that equation.

Note that the equation is not satisfied when x has any other value.

To solve an equation such as $3x + 10 = 22$ is to find the value of the unknown quantity, (in this case x) which will satisfy the equation. The solution of equations is made very easy if you learn the following facts:

1. Both sides of the equation may be multiplied or divided by the same number without changing the equality.

2. If a number is added to one side of the equation, the same number must be added to the other side to preserve the equality.

3. If a number is subtracted from one side of the equation, the same number must be subtracted from the other side to preserve the equality.

4. Both sides of the equation may be raised to the same power or have the same root extracted without destroying the equality.

5. Quantities which are equal to the same quantity are equal to each other.

Examples:

1. Solve the equation $4y = 48$

Solution: Dividing both sides by 4 we get $y = 12$.

2. Solve the equation $\frac{x}{3} = 11$

Solution: Multiplying by 3 we get $x = 33$.

3. Solve the equation $x + 16 = 28$

Solution: Subtracting 16 from each side $x = 28 - 16 = 12$.

The process of transferring the 16 from the left side to the right side is known as **Transposition** and is done in accordance with the following

RULE: Any term may be transposed from one side of an equation to the other by changing its sign.

4. From the formula $V = \frac{1}{3}hb$, find b when $V = 20$ and $h = 10$.

Solution:

Substituting $20 = \frac{1}{3} \cdot 10 \cdot b$

or $\frac{10}{3}b = 20$

Dividing by 10, $\frac{b}{3} = 2$

Multiplying by 3, $b = 6$

5. Solve the equation $\frac{x}{4} - \frac{3}{5} = \frac{x}{3} + \frac{5}{8}$

Solution: Multiplying by 120, the least common denominator,

$$\frac{120x}{4} - \frac{360}{5} = -\frac{120x}{3} + \frac{600}{8}$$

Clearing of fractions $30x - 72 = -40x + 75$

Transposing $70x = 147$

$$x = \frac{147}{70} = 2\frac{7}{70} = 2\frac{1}{10}$$

6. Solve the equation $ax + 7a = a^2 + 4x + 12$

Solution:

Transposing	$ax - 4x = a^2 - 7a + 12$
Collecting coefficients of x	$x(a - 4) = a^2 - 7a + 12$
Dividing by $a - 4$	$x = \dfrac{a^2 - 7a + 12}{a - 4}$
Performing the division	$x = a - 3$

Exercises:

Solve the following equations for x or y

71. $15x = 225$

72. $\frac{1}{3}y = 4$

73. $2x + 14 = 28$

74. $3y - 4 = 5$

75. $\frac{1}{2}x - 9 = 5$

76. $3ax - 2a = 7a$

77. $5x - 2a = 10a + 3x$

78. $xa + 3a = a^2 - 2x - 10$

79. $ax - bc = bx - ac$

80. From the Formula $S = \pi 2rh$, find r when $\pi = \frac{22}{7}$, $h = 3$, and $S = 66$

The answers to these exercises will be found on the last page of this book.

PROBLEMS SOLVED BY EQUATIONS

Algebra becomes extremely useful in the solution of many types of problems. You can make the solution of these problems very easy if you first translate the wording of the problem into an equation, letting the unknown quantity be represented by some letter, such as x or y.

Examples:

1. The sum of two numbers is 60. The larger is 4 times the smaller. What are the numbers?

Solution: Let	$x =$ the smaller number
Then	$4x =$ the larger number
Then	$x + 4x = 60$
Or	$5x = 60$
Dividing by 5,	$x = 12$
	$4x = 48$

Check: The sum of 48 and 12 is 60 and 48 is 4 times 12.

2. An airplane covered 1000 miles at the rate of 200 miles per hour. How long did it travel?

Solution: In problems of this type $d = r \times t$, where

$d =$ distance

$r =$ rate

$t =$ time

$$\therefore t = \frac{d}{r} = \frac{1000}{200} = 5 \text{ hours}$$

3. Two cars started out at the same time and place, for the same destination. Car A traveled at 50 miles an hour and reached its destination 1 hour before car B which averaged 35 miles an hour. How far did they travel?

Solution: Let	$t =$ time taken by Car A
Then	$t + 1 =$ time taken by Car B
Then the distance	$50t = 35 \quad (t + 1)$
	$= 35t + 35$
Transposing	$15t = 35$
	$t = \dfrac{35}{15} = \dfrac{7}{3}$
Then	$d = 50t = \dfrac{350}{3} = 116\dfrac{2}{3}$ miles

4. A square 25 ft. $\times$ 25 ft. is to be divided into two rectangles so that the area of one is 50 sq. ft. more than the other. What will be the widths of the rectangles?

Solution: Let	$x =$ the width of the larger rectangle
And	$25 - x =$ the width of the smaller rectangle
Then	$25 \cdot x - 25\,(25 - x) = 50$
	$25x - 625 + 25x = 50$
	$50x = 675$
	$x = 13.5$

Now try to solve the following problems, remembering to first translate the wording into an equation:

81. The perimeter of a rectangle is 150 feet. The length is four times the width. Find the dimensions.

82. What is the area of a triangle whose base is one-half as long as the altitude, and the sum of whose base and altitude is twelve inches.

83. The perimeter of a quadrilateral *ABCD* is 168 feet. The side *CD* is twice as long as *AB*; the side *AD* is three times as long as *AB*; the side *BC* is one and one-half times as long as *AD*. How long is each?

84. A, traveling 40 miles per hour, starts at 7 A. M. towards B, who is 570 miles away. At 10 A. M., B sets out to meet A at 50 miles per hour. At what time will they meet?

85. A steel rod 135 inches long is to be cut into three pieces so that the second is 7″ longer than the first and the third 13″ longer than the second. What will be the lengths?

86. Find a number whose difference from 96 is three times as great as the number is larger than 48.

87. In a triangle whose sides are *AB, BC,* and *CA,* and whose perimeter is 24 inches, *BC* is 3 times as large as *AB*, and *CA* is $\frac{4}{3}$ as large as *BC*. Find the lengths of the three sides.

The answers to these exercises will be found on the last page of this book.

SIMULTANEOUS EQUATIONS

When the conditions of a problem are stated by two or more separate equations, the equations are said to be simultaneous. Simultaneous equations can be solved only if the number of unknowns does not exceed the number of equations. The solution of simultaneous equations of two unknowns is very important, and you should practice with them until you find that you can solve them readily.

In the solution of 2 simultaneous equations, one of the unknowns is eliminated either by altering one or both of the equations until the numerical coefficient of one of the unknowns is the same in each equation. Then, this unknown is eliminated by either addition or subtraction, depending on the signs.

Examples:

1. Solve the equations:

$$4x + 3y = 18 \qquad (1)$$
$$2x - y = 4 \qquad (2)$$

Solution:

Multiplying equation (2) by 2 and subtracting we get:

$$\begin{array}{r} 4x + 3y = 18 \\ 4x - 2y = 8 \\ \hline 5y = 10 \\ y = 2 \end{array}$$

Multiplying equation (2) by 3 and adding we get:

$$\begin{array}{r} 4x + 3y = 18 \\ 6x - 3y = 12 \\ \hline 10x = 30 \\ x = 3 \end{array}$$

2. Solve the equations:

$$4x + 3y = 25 \qquad (1)$$
$$3x - 2y = 6 \qquad (2)$$

Solution: Multiplying equation (1) by 2 and equation (2) by 3 and adding we get:

$$\begin{array}{r} 8x + 6y = 50 \\ 9x - 6y = 18 \\ \hline 17x \quad = 68 \end{array}$$

Substituting in equation (1) we get:

$$4(4) + 3y = 25$$
$$3y = 9$$
$$y = 3$$

Now see if you can solve the following pairs of simultaneous equations:

88. $4x + 5y = 84$
    $3x + 4y = 64$

89. $11x - 2y = 69$
    $2x - 3y = 2$

90. $2x + \frac{y}{3} = -7$
    $-3x - 2y = 6$

91. $\frac{1}{4}x + \frac{2}{3}y = 4\frac{2}{3}$
    $2x - 3y = 4$

The answers to these exercises will be found on the last page of this book.

Simultaneous Equation Problems:

Problem: Two weights A and B of 15 and 25 pounds respectively balance each other on a lever at unknown distances from the fulcrum. If 5 pounds are added to A, B must be moved one foot further from the fulcrum to maintain the balance. What was the original distance from the fulcrum to each of the weights?

Solution: Let x be the distance from A to the fulcrum and y the distance from B to the fulcrum.

Then, according to the Laws of Equilibrium, from the first condition:

$$Ax = By$$

Or

$$15x = 25y \qquad (1)$$

From the second condition:

$$(15 + 5)x = 25(y + 1)$$

Or

$$20x = 25y + 25 \qquad (2)$$

Subtracting (1) from (2):

$$5x = 25$$
$$\therefore x = 5 \text{ feet}$$

Substituting in (1):

$$15(5) = 25y$$

Or

$$y = \frac{75}{25} = 3 \text{ feet}$$

QUADRATIC EQUATIONS

A Quadratic Equation is one that contains the square of the unknown quantity but no higher power.

All Quadratic Equations have two possible values of the unknown. Thus, in the equation $x^2 = 4$, the roots are $+2$ and -2 since either of these satisfy the equation.

In the equation $x^2 + x - 2 = 0$, the roots are easily obtained because the left hand term can be factored into $(x - 1) \cdot (x + 2)$, therefore $x - 1 = 0$ and $x + 2 = 0$ and $x = 1$ or -2.

Not all Quadratics are easily factorable, however. A method employing the fact that $\pm(x + a)^2 = x^2 + 2ax + a^2$ and $\pm(x - a)^2 = x^2 - 2ax + a^2$ is available however and is illustrated below:

1. Solve the equation $3x^2 + 2x - 6 = 0$

Solution:

Dividing each term by 3 and transposing:

$$x^2 + \frac{2}{3}x = +2$$

Adding $(\frac{1}{3})^2$ to each side to get it into the form $x^2 + 2ax + a^2$ (a perfect square):

$$x^2 + \frac{2}{3}x + \frac{1}{9} = +2 + \frac{1}{9} = \frac{19}{9}$$

Taking the square root of each side:

$$(x + \frac{1}{9}) = \pm\sqrt{\frac{19}{9}}$$

from which

$$x = -\frac{1}{9} \pm \sqrt{\frac{19}{9}}$$

This method is known as "completing the square", the rule for which follows:

RULE: 1. Group all terms in x^2 and x on the left side of the equation, and the known quantities on the right.

2. Divide all terms by the coefficient of x^2.

3. To both sides of the equation, add the square of ½ the coefficient of the x term.

4. Find the square root of both sides of the equation and then solve as in a simple equation.

Problems Involving Quadratic Equations:

1. Find the dimensions of a rectangle if its perimeter is to be 28 feet and its area 45 square feet.

Solution: Let $x = \text{width}$

Then $\frac{28-2x}{2} = \text{length} = 14 - x$

Then

$$x(14-x) = 45$$
$$-x^2 + 14x = 45$$
$$x^2 - 14x = -45$$
$$x^2 - 14x + (7)^2 = -45 + (7)^2$$
$$(x-7)^2 = +4$$
$$x - 7 = \pm\sqrt{4}$$
$$x = 7 \pm 2 = 9 \text{ or } 5$$

Therefore Length $= 14 - 9 = 5$
or $14 - 5 = 9$
Thus the Length should be 9 or 5
and the Width 5 or 9

Now see if you can solve the following problems, the answers of which will be found on the last page of this book.

92. What must be the lengths of the sides a and b of a right triangle if a plus b are to be 10 feet and the hypotenuse c is to be 7.33′?

93. The difference in volume between two cylindrical tanks four feet high is 10.21 cu. ft. The difference between their diameters is 6 inches. Find their diameters.

FRACTIONAL AND NEGATIVE EXPONENTS

You have seen that:

1. $a^2 \cdot a^3 = a \cdot a \cdot a \cdot a \cdot a = a^5$
2. $a^3 \div a^2 = \frac{a \cdot a \cdot a}{a \cdot a} = a$
3. $(a^3)^2 = (a \cdot a \cdot a)(a \cdot a \cdot a) = a^6$
4. $(ab)^3 = ab \cdot ab \cdot ab = a^3b^3$
5. $\left(\frac{a}{b}\right)^3 = \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} = \frac{a^3}{b^3}$

Or, in general terms:

1. $a^m \cdot a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^m = a^mb^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Heretofore, you have considered only Positive Whole Number Exponents. It is possible, however, to have **Fractional, Zero,** and **Negative Exponents** which satisfy the above conditions.

1. In a **Fractional Exponent,** the **Numerator** indicates the **power** to which the number is raised while the **Denominator** indicates the **root** to be extracted. Thus:

$$a^{1/2} \cdot a^{1/2} = \sqrt{a} \cdot \sqrt{a} = a^{1/2 + 1/2} = a$$
$$(a^{2/3})^3 = (\sqrt[3]{a^2})^3 = a^2$$

2. Any quantity, regardless of its value, with an **exponent of zero,** attains a value of 1.

Since $a^o \cdot a^m = a^{o+m} = a^m$,
a^o must equal 1

3. Any quantity with a **Negative Exponent** is equal to the **Reciprocal** of the quantity raised to the corresponding positive power.

Since $a^{-m} \cdot a^m = a^o = 1$
a^{-m} must equal $\frac{1}{a^m}$

LOGARITHMS

If $a^x = b$, then x is known as the logarithm of b to the base a and is expressed as

$\log_a b$.

The anti-logarithm of x, to the base a, is b.

As an example, consider the equation of $2^4 = 16$. Here 4 is the logarithm of 16, and 16 the anti-logarithm of 4 to the base 2, or $4 = \log_2 16$.

Exercises:

94. $\log_2 32 = ?$
95. $\log_{10} 1000 = ?$
96. $\log_a a^4 = ?$
97. $\log_a 1 = ?$

The answers to these exercises will be found on the last page of this book.

Important Relationships Involving Logarithms

1. The logarithm of the product of two or more numbers equals the sum of the logarithms of these numbers.

For example,

$$\log_{10} 1000 = \log_{10} 100 + \log_{10} 10$$
or $\log_a bc = \log_a b + \log_a c$

2. **The logarithm of the quotient of two numbers equals the difference between the logarithms of the numbers.**

For example,

$$\log_{10} 10 = \log_{10} 1000 - \log_{10} 100$$

or $$\log_a \frac{b}{c} = \log_a b - \log_a c$$

3. **The logarithm of a number to the n^{th} power, equals n times the logarithm of the number.**

For example,

$$\log_{10} 100^2 = 2 \log_{10} 100$$

or $$\log_a b^n = n \log_a b$$

COMMON LOGARITHMS

When logarithms are used for numerical calculations, the base 10 is generally used, in which case they are known as **Common Logarithms.** In writing common logarithms the base is frequently omitted. Thus

$\log_{10} 139$ is written log 139.

The common logarithm of 100 is evidently 2, and 1000 is 3. But what about the logarithms of the intermediate values such as the log139? It is extremely difficult to calculate such logarithms, but you needn't worry about that because mathematics has been made easy for you through the medium of logarithm tables. In Table III you will find an abbreviated form of logarithm table, which gives logarithms to four decimal places and can be used for numbers of four significant figures. Volumes giving tables to many more places have been published and should be used where greater accuracy is required.

USE OF LOG TABLES

The Logarithms are usually expressed as decimal numbers of which the integer is called the **Characteristic** and the decimal portion the **Mantissa.** Thus, in considering 1.4771 which is the approximate logarithm of 30, the characteristic is 1 and the mantissa .4771.

The use of Table III is best illustrated by some examples:

1. Find the logarithm of 683.

Here the mantissa is obtained by looking in the first column for 68 and then over on this row to the column headed by 3. The mantissa is then read as .8344 and represents the log of 6.83.

The characteristic is obtained without the use of the table, but by an inspection of the number itself.

Note that

$$10^4 = 10{,}000, \text{ or } \log 10{,}000 = 4$$
$$10^3 = 1{,}000, \text{ or } \log 1{,}000 = 3$$
$$10^2 = 100, \text{ or } \log 100 = 2$$
$$10^1 = 10, \text{ or } \log 10 = 1$$
$$10^0 = 1, \text{ or } \log 1 = 0$$

From the tables of pages 60 and 61, it is obvious that the log of 683 must lie between 2 and 3 and hence the part of the logarithm in front of the decimal, i. e., its characteristic, must be 2. Another way to think of this is that $\log 683 = \log (100 \times 6.83)$

$$= \log 100 + \log 6.83 = 2 + .8344 = 2.8344$$

You will find that memorizing the rules of characteristics will greatly aid in making the use of logarithms easy to learn.

RULE: The characteristic of the logarithm of a number greater than one is one less than the number of digits to the left of the decimal point.

2. Find the logarithm of .00683.

Now note that $10^{-1} = .1$, or $\log .1 = -1$

$$10^{-2} = .01, \text{ or } \log .01 = -2$$
$$10^{-3} = .001 \text{ or } \log .001 = -3, \text{ etc.}$$

Then note that $\log .00683 = \log (6.83 \times .001)$

$$= \log 6.83 + \log .001$$

But the log of 6.83 is 0.8344, and hence $\log .00683 = .8344 - 3$

Thus the characteristic is -3.

In calculations using logarithms it is convenient to express this as $7 - 10$. Thus:

$$\log 6.83 = 7.8344 - 10$$

The following rule will guide you in determining the logarithms of numbers less than 1:

RULE: The characteristic of the logarithm of a number less than 1 is equal to 9—n—10 where n is the number of zeros between the decimal point and the first significant figure. This characteristic is written in two parts. The first part, 9—n, is written at the left of the mantissa, and the —10 at the right.

3. Find the logarithm of 683.4.

Here the characteristic is 2. The mantissa cannot be obtained directly from the table but must be obtained by **interpolation.**

From the table, $\log 683.0 = 2.8344$

$$684.0 = 2.8351$$

The difference between these logarithms is .0007, from which we can write

$$\log 683.2 = 2.8344 + (0.4 \times .0007)$$
$$= 2.8344 + 0.00028$$
$$= 2.8344 + 0.0003$$
$$= 2.8347$$

Don't write logarithms to five decimal figures when using four-place tables. If the fifth figure is 5, 6, 7, 8 or 9, omit it and increase the fourth figure by 1.

TABLE III

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

TABLE III

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Exercises:

Find the logarithms of the following numbers:

98.	1429	100.	.00745	102.	62.47
99.	.6928	101.	428.9	103.	300.8

The answers to these exercises will be found on the last page of this book.

Antilogarithms

1. Find the antilog 1.3032.

Here the mantissa is found directly in the table corresponding to the number 2.01. But since the characteristic is 1 the number of places in front of the decimal point must be 2.

Hence antilog $1.3032 = 20.1$.

2. Find antilog 1.3042.

The mantissa of this logarithm lies between .3032, the log of 2.01 and .3054, the log of 2.02.

Therefore, the antilog 1.3042 must lie between 20.1 and 20.2. The difference between .3032 and .3054 is .0022, and the difference between .3032 and $.3042 = .0010$. Hence the required antilogarithm is $\frac{10}{22}$ of the way from 20.1 and 20.2.

$$\text{Antilog } 1.3042 = 20.1 + \frac{10}{22}(.1)$$
$$= 20.1 + .045$$
$$= 20.15$$

RULE: (a) Find two consecutive mantissae in the table between which the given one lies.

(b) Find their difference.

(c) Find the difference between the lower of the two tabular mantissae and the given mantissae and divide this by the difference (b), expressing the quotient to the nearest digit.

(d) Annex this digit as a fourth place to the three digits corresponding to the smaller tabular mantissa.

(e) Place the decimal point as indicated by the characteristic.

Exercises: Find the antilogarithm of the following:

104.	.6561	106.	9.7841—10
105.	3.7531	107.	7.1345—10

The answers to these exercises will be found on the last page of this book.

Multiplication and Division by Logarithms

Logarithms are extensively used for performing multiplication, division raising to a given power, or for extracting roots. The work is less laborious than that involved in the ordinary arithmetical methods but at the same time more accuracy can be obtained than with the slide rule.

Naturally, the more places the logarithm tables are carried to, the greater will be the accuracy of the work.

In all work involving logarithms, you are advised to carefully arrange and lay out the work before looking up any logarithms.

Examples:

1. If $x = \frac{6.28 \times 39.42}{18.31 \times 42.19}$, find x.

Solution: Here $x = \log 6.28 + \log 39.42 - \log 18.31 - \log 42.19$

$\log 6.28 = 0.7980$
$\log 39.42 = 1.5957$
$\log 6.28 \times 39.42 = 2.3937$
$\log 18.31 \times 42.19 = 2.8879$
$\log x = 9.5058—10$
$x = 0.3205$

$\log 18.31 = 1.2627$
$\log 42.19 = 1.6252$
$\log 18.31 \times 42.19 = 2.8879$

2. If $x = \frac{(3.12)^4}{\sqrt[5]{.681}}$, find x.

Solution: Here $x = 4 \log 3.12 - \frac{1}{5} \log .681$

$\log 3.12 = .4942$
$\times 4$
$\log (3.12)^4 = 1.9768$
$\log (.681)^{1/5} = 9.9666—10$
$\log x = 2.0102$
$x = 102.4$

$\log .681 = 49.8331—50$
$\frac{1}{5} \log .681 = 9.9666—10$

In order that you may appreciate the use of logarithms, ask your mathematical friends to solve problem 2 without them.

Exercises:

Find numerical values of x by logarithms when x equals:

108. $\frac{322.6 \times 14.18}{162 \times 13.6}$ 110. $\frac{(14.28)^4}{(6.29)^3}$

109. $\frac{481.0 \times 35.14 \times 16.7}{182.9}$ 111. $\frac{(\sqrt[3]{4228})(9.68)}{16.18}$

The answers to these exercises will be found on the last page of this book.

TRIGONOMETRY

Trigonometry is that branch of mathematics which deals with the functions of angles called **Trigonometric Functions.** These functions are called the Sine, Cosine, Tangent, Cotangent, Secant, and Cosecant and for any angle A are abbreviated *sin A, cos A, tan A, cot A, sec A,* and *csc A,* respectively.

If ABC in Fig. 20 be any right angle triangle with the sides a and b, hypotenuse c, acute angles A and B, and right angle C, the Trigonometric Functions are defined as follows:

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$$

$$\cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{1}{\tan A} = \frac{b}{a}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{1}{\cos A} = \frac{c}{b}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{1}{\sin A} = \frac{c}{a}$$

To these six Functions are sometimes added:

versed sine $A = 1 - \cos A$, written vers A
coversed sine $A = 1 - \sin A$, written covers A

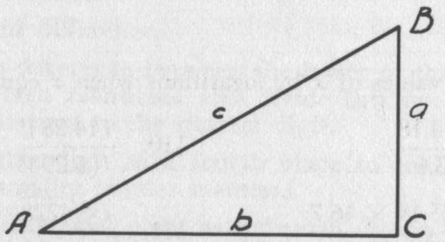


Fig. 20

In Fig. 20, angle B is the **Complement** of angle A, i. e., it has the value of $90° - A$ (since C is 90° and the three sides of any triangle add up to 180°) and hence:

$$\sin A = \cos (90° - A)$$
$$\tan A = \cot (90° - A)$$
$$\sec A = \csc (90° - A)$$

Trigonometric Tables:

The values of the Trigonometric Functions for all angles have been computed to many decimal places and are available in many forms. In

Table IV you will find the values of the Sine, Cosine, Tangent, and Cotangent, of every degree from zero to ninety, correct to four decimal places.

Problem: In the right triangle ABC (Fig. 20), a is 35′ and angle A is 35°. Find the hypotenuse c.

Solution:

$$\frac{c}{a} = \csc A = \frac{1}{\sin A}$$

Therefore $$c = \frac{a}{\sin A} = \frac{35}{.5736} = 61.01'$$

For functions of angles involving fractions of degrees (i. e., minutes) you will have to interpolate, as you did for Logarithms, unless you have a more complete table available.

Examples:

1. Find $\sin 24° 32'$.

Solution: From the Table, *sin* 24° is .4067 and *sin* 25° is .4226 the difference being .0159. *Sin* 24° 32′ lies $\frac{32}{60}$ of the way between the tabular values. Therefore $\frac{32}{60} \times .0159$, or .0085 must be added to .4067, obtaining .4152 as the required sine.

2. Find A if $\sin A = .6098$.

Solution: *Sin A* lies between .6018 and .6157, the sines of 37° and 38°, respectively. The difference between .6157 and .6018 is .0139. Since .6098 is greater than .6018 by .0080, it follows that the required angle is $\frac{.0080}{.0139}$ of a degree or $\frac{.0080}{.0139} \times 60 = 35$ minutes greater than 37°.

Thus the required angle A is 37° 35′.

You will find the Slide Rule very helpful in the process of interpolation.

LOGARITHMIC TABLES OF TRIGONOMETRIC FUNCTIONS

The solution of trigonometrical problems requires a considerable amount of multiplication and division, for which reason it is usually more convenient to use Logarithms of Functions rather than the functions themselves. In Table V you will find this information in brief form. In using this Table note that the sines and cosines of all angles less than 90° and greater than 0° are less than one, and that the tangents of all angles less than 45° are less than one. The logarithms of these functions are therefore negative and minus 10 is understood after each in the table.

Problems Involving Right Triangles:

1. A Parallelogram, as shown in full lines in Fig. 21, is to be cut from sheet metal. What must be the size of the rectangular sheet from which it is cut.

Solution: The dotted lines indicate the rectangular plate from which it will be cut. It is seen that $a = 16\ sin\ (90° - 64°) = 16\ sin\ 26°$ and that $W = 16\ cos\ 26°$.

$$\begin{aligned} \text{Log } 16 &= 1.2041 \\ \text{Log } sin\ 26° &= 9.6418\text{—}10 \\ \text{Log } a &= 10.8459\text{—}10 \\ a &= 7.01 = 7'' \end{aligned} \qquad \begin{aligned} \text{Log } 16 &= 1.2041 \\ \text{Log } cos\ 26° &= 9.9537\text{—}10 \\ \text{Log } W &= 11.1578\text{—}10 = 1.1578 \\ W &= 14.38 = 14\tfrac{3}{8}'' \end{aligned}$$

Thus the width W must be $14\frac{3}{8}''$ and the length L must be $16 + a = 16'' + 7'' = 23''$.

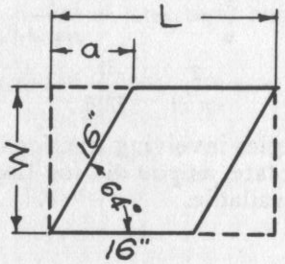


Fig. 21

2. At two shore observation posts 200 feet apart, the angles A and B (Fig. 22) to a ship are observed. How far is the ship from shore? $A = 59°$, $B = 51°$.

Solution:

Let x be the distance from the shore

Then $x\ cot\ A + x\ cot\ B = 200'$

Or $x\ (cot\ A + cot\ B) = 200'$

Using natural functions:

$$\begin{aligned} cot\ 59° &= 0.6009 \\ \text{And} \quad cot\ 51° &= .8098 \\ \text{And} \quad cot\ 59° + cot\ 51° &= 1.4107 \end{aligned}$$

$$\text{Therefore} \quad x = \frac{200}{1.4107} = 141.8 \text{ ft.}$$

Note that this is really the solution of an oblique triangle. Almost any oblique triang'e can be solved by breaking it into appropriate right triangles. This usually requires, however the addition and subtraction of functions which requires the use of natural functions. Later, you will be given formulae for directly solving oblique triangles.

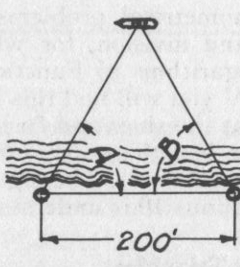


Fig. 22

Now see if you can solve the following problems by yourself, the correct answers to which will be found on the last page of this book.

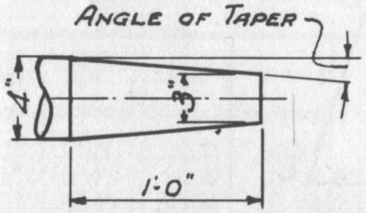


Fig. 23

112. A machinist has to make a shaft having a taper 1 foot long. The large end of the taper is 4 inches in diameter and the small end 3 inches. What is the angle of taper? (Fig. 23).

113. How long must a guy wire be to help brace a mast 60 feet high if the wire is to make an angle of 50° with the mast?

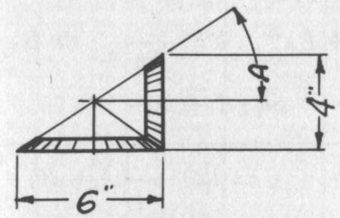


Fig. 24

114. Two bevel gears have dimensions as shown in Fig. 24. What is the cone angle A?

115. At a point on the ground 260 feet from a building, the line of sight to the top of the building from the observer makes an angle of 36° with the horizontal. How high is the building above the elevation of the observer's eye?

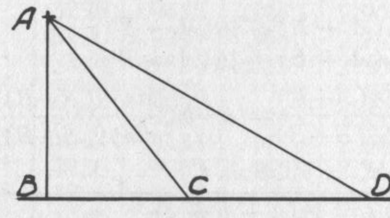


Fig. 25

116. Referring to Fig. 25, an aviator (A) observes that his altitude (distance to B) is 1500 feet, that the angle BAC is 40° and that angle BAD is 65°. What is the distance from C to D?

117. The radius of a circle is 5 inches and the length of a chord is 4 inches. Find the angle subtended by the chord.

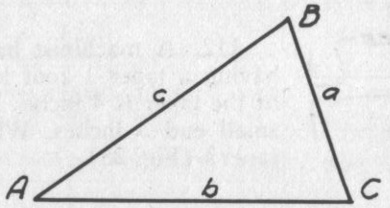
118. A rectangle is 48×22. Find the angle made by a diagonal with the longer side.

Functions of Obtuse Angles:

In the solution of oblique triangles you will occasionally have to find the Functions of Obtuse Angles. This should give you no trouble, however, if you remember these simple relationships:

$$\begin{aligned} sin\ A &= +sin\ (180° - A) \\ cos\ A &= -cos\ (180° - A) \\ tan\ A &= -tan\ (180° - A) \end{aligned} \qquad \begin{aligned} cot\ A &= -cot\ (180° - A) \\ sec\ A &= -sec\ (180° - A) \\ csc\ A &= +csc\ (180° - A) \end{aligned}$$

FORMULAE FOR THE SOLUTION OF OBLIQUE TRIANGLES



GIVEN	TO FIND	FORMULAE
A, B, a	C, b, c	$C = 180° - (A + B)$ $\quad b = \frac{a}{\sin A} \cdot \sin B$ $c = \frac{a}{\sin A} \cdot \sin (A + B)$
A, a, b	B, C, c	$\sin B = \frac{\sin A}{a} \cdot b$ $\quad C = 180° - (A + B)$ $c = \frac{a}{\sin A} \cdot \sin C$
C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$
	$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B)$
	A, B	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$ $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$
	c	$c = (a+b) \frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)} = (a-b) \frac{\sin\frac{1}{2}(A+B)}{\sin\frac{1}{2}(A-B)}$
	Area	$K = \frac{1}{2} ab \sin C$
a, b, c	A	Let $s = \frac{1}{2}(a + b + c)$ $\sin \frac{1}{2} A = \frac{\sqrt{(s - b)(s - c)}}{bc}$ $\cos \frac{1}{2} A \frac{\sqrt{s(s - a)}}{bc}$ $\tan \frac{1}{2} A = \frac{\sqrt{(s - b)(s - c)}}{s(s - a)}$ $\sin A = \frac{2\sqrt{s(s - a)(s - b)(s - c)}}{bc}$ $\text{vers } A = \frac{2(s - b)(s - c)}{bc}$
	Area	$K = \sqrt{s(s - a)(s - b)(s - c)}$
$A, B, C\ a$	Area	$K = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$

TABLE IV

NATURAL TRIGONOMETRIC FUNCTIONS

Angle	sin	cos	tan	cot	Angle	sin	cos	tan	cot
0°	.0000	1.0000	.0000	∞	45°	.7071	.7071	1.0000	1.0000
1°	.0175	.9998	.0175	57.290	46°	.7193	.6947	1.0355	.9657
2°	.0349	.9994	.0349	28.636	47°	.7314	.6820	1.0724	.9325
3°	.0523	.9986	.0524	19.081	48°	.7431	.6691	1.1106	.9004
4°	.0698	.9976	.0699	14.300	49°	.7547	.6561	1.1504	.8693
5°	.0872	.9962	.0875	11.430	50°	.7660	.6428	1.1918	.8391
6°	.1045	.9945	.1051	9.5144	51°	.7771	.6293	1.2349	.8098
7°	.1219	.9925	.1228	8.1443	52°	.7880	.6157	1.2799	.7813
8°	.1392	.9903	.1405	7.1154	53°	.7986	.6018	1.3270	.7536
9°	.1564	.9877	.1584	6.3138	54°	.8090	.5878	1.3764	.7265
10°	.1736	.9848	.1763	5.6713	55°	.8192	.5736	1.4281	.7002
11°	.1908	.9816	.1944	5.1446	56°	.8290	.5592	1.4826	.6745
12°	.2079	.9781	.2126	4.7046	57°	.8387	.5446	1.5399	.6494
13°	.2250	.9744	.2309	4.3315	58°	.8480	.5299	1.6003	.6249
14°	.2419	.9703	.2493	4.0108	59°	.8572	.5150	1.6643	.6009
15°	.2588	.9659	.2679	3.7321	60°	.8660	.5000	1.7321	.5774
16°	.2756	.9613	.2867	3.4874	61°	.8746	.4848	1.8040	.5543
17°	.2924	.9563	.3057	3.2709	62°	.8829	.4695	1.8807	.5317
18°	.3090	.9511	.3249	3.0777	63°	.8910	.4540	1.9626	.5095
19°	.3256	.9455	.3443	2.9042	64°	.8988	.4384	2.0503	.4877
20°	.3420	.9397	.3640	2.7475	65°	.9063	.4226	2.1445	.4663
21°	.3584	.9336	.3839	2.6051	66°	.9135	.4067	2.2460	.4452
22°	.3746	.9272	.4040	2.4751	67°	.9205	.3907	2.3559	.4245
23°	.3907	.9205	.4245	2.3559	68°	.9272	.3746	2.4751	.4040
24°	.4067	.9135	.4452	2.2460	69°	.9336	.3584	2.6051	.3839
25°	.4226	.9063	.4663	2.1445	70°	.9397	.3420	2.7475	.3640
26°	.4384	.8988	.4877	2.0503	71°	.9455	.3256	2.9042	.3443
27°	.4540	.8910	.5095	1.9626	72°	.9511	.3090	3.0777	.3249
28°	.4695	.8829	.5317	1.8807	73°	.9563	.2924	3.2709	.3057
29°	.4848	.8746	.5543	1.8040	74°	.9613	.2756	3.4874	.2867
30°	.5000	.8660	.5774	1.7321	75°	.9659	.2588	3.7321	.2679
31°	.5150	.8572	.6009	1.6643	76°	.9703	.2419	4.0108	.2493
32°	.5299	.8480	.6249	1.6003	77°	.9744	.2250	4.3315	.2309
33°	.5446	.8387	.6494	1.5399	78°	.9781	.2079	4.7046	.2126
34°	.5592	.8290	.6745	1.4826	79°	.9816	.1908	5.1446	.1944
35°	.5736	.8192	.7002	1.4281	80°	.9848	.1736	5.6713	.1763
36°	.5878	.8090	.7265	1.3764	81°	.9877	.1564	6.3138	.1584
37°	.6018	.7986	.7536	1.3270	82°	.9903	.1392	7.1154	.1405
38°	.6157	.7880	.7813	1.2799	83°	.9925	.1219	8.1443	.1228
39°	.6293	.7771	.8098	1.2349	84°	.9945	.1045	9.5144	.1051
40°	.6428	.7660	.8391	1.1918	85°	.9962	.0872	11.430	.0875
41°	.6561	.7547	.8693	1.1504	86°	.9976	.0698	14.300	.0699
42°	.6691	.7431	.9004	1.1106	87°	.9986	.0523	19.081	.0524
43°	.6820	.7314	.9325	1.0724	88°	.9994	.0349	28.636	.0349
44°	.6947	.7193	.9657	1.0355	89°	.9998	.0175	57.290	.0175
45°	.7071	.7071	1.0000	1.0000	90°	1.0000	.0000	∞	.0000

TABLE V

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

	SIN	COS	TAN	COT			SIN	COS	TAN	COT	
0°	— ∞	10.0000	— ∞	∞	**90°**	**11°**	9.2806	9.9919	9.2887	0.7113	**79°**
15′	7.6398	.0000	7.6398	2.3602	45′	15′	.2902	.9916	.2987	.7013	45′
30′	.9408	.0000	.9409	.0591	30′	30′	.2997	.9912	.3085	.6915	30′
45′	8.1169	.0000	8.1170	1.8830	15′	45′	.3089	.9908	.3181	.6819	15′
1°	8.2419	9.9999	8.2419	1.7581	**89°**	**12°**	9.3179	9.9904	9.3275	0.6725	**78°**
15′	.3388	.9999	.3389	.6611	45′	15′	.3267	.9900	.3367	.6633	45′
30′	.4179	.9999	.4181	.5819	30′	30′	.3353	.9896	.3458	.6542	30′
45′	.4848	.9998	.4851	.5149	15′	45′	.3438	.9892	.3546	.6454	15′
2°	8.5428	9.9997	8.5431	1.4569	**88°**	**13°**	9.3521	9.9887	9.3634	0.6366	**77°**
15′	.5939	.9997	.5943	.4057	45′	15′	.3602	.9883	.3719	.6281	45′
30′	.6397	.9996	.6401	.3599	30′	30′	.3682	.9878	.3804	.6196	30′
45′	.6810	.9995	.6815	.3185	15′	45′	.3760	.9874	.3886	.6114	15′
3°	8.7188	9.9994	8.7194	1.2806	**87°**	**14°**	9.3837	9.9869	9.3968	0.6032	**76°**
15′	.7535	.9993	.7542	.2458	45′	15′	.3912	.9864	.4048	.5952	45′
30′	.7857	.9992	.7865	.2135	30′	30′	.3986	.9859	.4127	.5873	30′
45′	.8156	.9991	.8165	.1835	15′	45′	.4059	.9855	.4204	.5796	15′
4°	8.8436	9.9989	8.8446	1.1554	**86°**	**15°**	9.4130	9.9849	9.4281	0.5719	**75°**
15′	.8699	.9988	.8711	.1289	45′	15′	.4200	.9844	.4356	.5644	45′
30′	.8946	.9987	.8960	.1040	30′	30′	.4269	.9839	.4430	.5570	30′
45′	.9181	.9985	.9196	.0804	15′	45′	.4337	.9834	.4503	.5497	15′
5°	8.9403	9.9983	8.9420	1.0580	**85°**	**16°**	9.4403	9.9828	9.4575	0.5425	**74°**
15′	.9614	.9982	.9633	.0367	45′	15′	.4469	.9823	.4646	.5354	45′
30′	.9816	.9980	.9836	.0164	30′	30′	.4533	.9817	.4716	.5284	30′
45′	9.0008	.9978	9.0030	0.9970	15′	45′	.4597	.9812	.4785	.5215	15′
6°	9.0192	9.9976	9.0216	0.9784	**84°**	**17°**	9.4659	9.9806	9.4853	0.5147	**73°**
15′	.0369	.9974	.0395	.9605	45′	15′	.4721	.9800	.4921	.5079	45′
30′	.0539	.9972	.0567	.9433	30′	30′	.4781	.9794	.4987	.5013	30′
45′	.0702	.9970	.0732	.9268	15′	45′	.4841	.9788	.5053	.4947	15′
7°	9.0859	9.9968	9.0891	0.9109	**83°**	**18°**	9.4900	9.9782	9.5118	0.4882	**72°**
15′	.1011	.9961	.1045	.8955	45′	15′	.4958	.9776	.5182	.4818	45′
30′	.1157	.9963	.1194	.8806	30′	30′	.5015	.9770	.5245	.4755	30′
45′	.1299	.9960	.1338	.8662	15′	45′	.5071	.9763	.5308	.4692	15′
8°	9.1436	9.9958	9.1478	0.8522	**82°**	**19°**	9.5126	9.9757	9.5370	0.4630	**71°**
15′	.1568	.9955	.1614	.8387	45′	15′	.5181	.9750	.5431	.4569	45′
30′	.1697	.9952	.1745	.8255	30′	30′	.5235	.9743	.5491	.4509	30′
45′	.1822	.9949	.1873	.8127	15′	45′	.5288	.9737	.5551	.4449	15′
9°	9.1943	9.9946	9.1997	0.8003	**81°**	**20°**	9.5341	9.9730	9.5611	0.4389	**70°**
15′	.2061	.9943	.2118	.7882	45′	15′	.5392	.9723	.5669	.4331	45′
30′	.2176	.9940	.2236	.7764	30′	30′	.5443	.9716	.5727	.4273	30′
45′	.2288	.9937	.2351	.7649	15′	45′	.5494	.9709	.5785	.4215	15′
10°	9.2397	9.9934	9.2463	0.7537	**80°**	**21°**	9.5543	9.9702	9.5842	0.4158	**69°**
15′	.2503	.9930	.2573	.7427	45′	15′	.5592	.9694	.5898	.4102	45′
30′	.2606	.9927	.2680	.7320	30′	30′	.5641	.9687	.5954	.4046	30′
45′	.2707	.9923	.2784	.7216	15′	45′	.5689	.9679	.6009	.3991	15′
	COS	SIN	COT	TAN	°		COS	SIN	COT	TAN	°

TABLE V

LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

	SIN	COS	TAN	COT			SIN	COS	TAN	COT	
22°	9.5736	9.9672	9.6064	0.3936	**68°**	**34°**	9.7476	9.9186	9.8290	0.1710	**56°**
15′	.5782	.9664	.6118	.3882	45′	15′	.7504	.9173	.8331	.1669	45′
30′	.5828	.9656	.6172	.3828	30′	30′	.7531	.9160	.8371	.1629	30′
45′	.5874	.9648	.6226	.3774	15′	45′	.7559	.9147	.8412	.1588	15′
23°	9.5919	9.9640	9.6279	0.3721	**67°**	**35°**	9.7586	9.9134	9.8452	0.1548	**55°**
15′	.5963	.9632	.6331	.3669	45′	15′	.7613	.9120	.8493	.1507	45′
30′	.6007	.9624	.6383	.3617	30′	30′	.7640	.9107	.8533	.1467	30′
45′	.6050	.9616	.6435	.3565	15′	45′	.7666	.9093	.8573	.1427	15′
24°	9.6093	9.9607	9.6486	0.3514	**66°**	**36°**	9.7692	9.9080	9.8613	0.1387	**54°**
15′	.6135	.9599	.6537	.3463	45′	15′	.7718	.9066	.8652	.1348	45′
30′	.6177	.9590	.6587	.3413	30′	30′	.7744	.9052	.8692	.1308	30′
45′	.6219	.9582	.6637	.3363	15′	45′	.7769	.9038	.8732	.1268	15′
25°	9.6259	9.9573	9.6687	0.3313	**65°**	**37°**	9.7795	9.9023	9.8771	0.1229	**53°**
15′	.6300	.9564	.6736	.3264	45′	15′	.7820	.9009	.8811	.1189	45′
30′	.6340	.9555	.6785	.3215	30′	30′	.7844	.8995	.8850	.1150	30′
45′	.6379	.9546	.6834	.3166	15′	45′	.7869	.8980	.8889	.1111	15′
26°	9.6418	9.9537	9.6882	0.3118	**64°**	**38°**	9.7893	9.8965	9.8928	0.1072	**52°**
15′	.6457	.9527	.6930	.3070	45′	15′	.7918	.8950	.8967	.1033	45′
30′	.6495	.9518	.6977	.3023	30′	30′	.7941	.8935	.9006	.0994	30′
45′	.6533	.9508	.7025	.2975	15′	45′	.7965	.8920	.9045	.0955	15′
27°	9.6570	9.9499	9.7072	0.2928	**63°**	**39°**	9.7989	9.8905	9.9084	0.0916	**51°**
15′	.6607	.9489	.7118	.2882	45′	15′	.8012	.8890	.9122	.0878	45′
30′	.6644	.9479	.7165	.2835	30′	30′	.8035	.8874	.9161	.0839	30′
45′	.6680	.9469	.7211	.2789	15′	45′	.8058	.8858	.9200	.0800	15′
28°	9.6716	9.9459	9.7257	0.2743	**62°**	**40°**	9.8081	9.8843	9.9238	0.0762	**50°**
15′	.6752	.9449	.7302	.2698	45′	15′	.8103	.8827	.9277	.0723	45′
30′	.6787	.9439	.7348	.2652	30′	30′	.8125	.8810	.9315	.0685	30′
45′	.6821	.9429	.7393	.2607	15′	45′	.8148	.8794	.9353	.0647	15′
29°	9.6856	9.9418	9.7438	0.2562	**61°**	**41°**	9.8169	9.8778	9.9392	0.0608	**49°**
15′	.6890	.9408	.7482	.2518	45′	15′	.8191	.8761	.9430	.0570	45′
30′	.6923	.9397	.7526	.2474	30′	30′	.8213	.8745	.9468	.0532	30′
45′	.6957	.9386	.7571	.2429	15′	45′	.8234	.8728	.9506	.0494	15′
30°	9.6990	9.9375	9.7614	0.2386	**60°**	**42°**	9.8255	9.8711	9.9544	0.0456	**48°**
15′	.7022	.9364	.7658	.2342	45′	15′	.8276	.8694	.9582	.0418	45′
30′	.7055	.9353	.7701	.2299	30′	30′	.8297	.8676	.9621	.0379	30′
45′	.7087	.9342	.7745	.2255	15′	45′	.8317	.8659	.9659	.0341	15′
31°	9.7118	9.9331	9.7788	0.2212	**59°**	**43°**	9.8338	9.8641	9.9697	0.0303	**47°**
15′	.7150	.9319	.7831	.2169	45′	15′	.8358	.8624	.9735	.0265	45′
30′	.7181	.9308	.7873	.2127	30′	30′	.8378	.8606	.9772	.0228	30′
45′	.7212	.9296	.7916	.2084	15′	45′	.8398	.8588	.9810	.0190	15′
32°	9.7242	9.9284	9.7958	0.2042	**58°**	**44°**	9.8418	9.8569	9.9848	0.0152	**46°**
15′	.7272	.9272	.8000	.2000	45′	15′	.8437	.8551	.9886	.0114	45′
30′	.7302	.9260	.8042	.1958	30′	30′	.8457	.8532	.9924	.0076	30′
45′	.7332	.9248	.8084	.1916	15′	45′	.8476	.8514	.9962	.0038	15′
33°	9.7361	9.9236	9.8125	0.1875	**57°**	**45°**	9.8495	9.8495	0.0000	0.0000	**45°**
15′	.7390	.9224	.8167	.1833	45′						
30′	.7419	.9211	.8208	.1792	30′						
45′	.7447	.9198	.8249	.1751	15′						
	COS	SIN	COT	TAN	°		COS	SIN	COT	TAN	°

ANSWERS

1. (a) $\frac{31}{8}$ (b) $\frac{19}{4}$ (c) $\frac{33}{5}$
2. (a) $3\frac{1}{2}$ (b) $1\frac{4}{15}$ (c) $1\frac{7}{9}$
3. $2\frac{17}{40}$
4. $8\frac{5}{16}$
5. (a) $\frac{8}{21}$ (b) $\frac{32}{63}$ (c) $1\frac{5}{9}$
6. (a) .270 (b) .839 (c) .875 (d) .692
7. 36.622
8. (a) 30.1536 (b) 4.188
9. 298.52
10. $32\frac{41}{52}$ or 32.789
11. (a) 881 (b) 9.23 (c) .745
12. 88.11 lbs.
13. 852 R.P.M.
14. .0885
15. 40,100
16. 442,000
17. 44.0
18. .0650
19. .0001587
20. 5,180
21. 36.1
22. 4.88
23. 3.820
24. 12.56
25. .00364
26. 7.39
27. 7.60
28. .00847
29. .000245
30. 10.36
31. 3,580
32. 38.9
33. 201
34. 9.03
35. 00965
36. 4,390
37. .0000000244
38. 5.72
39. 616 sq. ft.
40. 148.3
41. .001133
42. 2,070,000
43. .0332
44. 147.5
45. $16.04
46. 265.2 lbs.
47. 83.4 KWH.
48. 3,040 FBM.
49. 2.96 inches
50. $2.12
51. .0026
52. 4.32 C.Y.
53. 8.38 C.Y.
54. 18.0 lbs.
55. 7.13 lbs.
56. 17 ft. $10\frac{13}{16}$ in.
57. 32.7 lbs.
58. 5,880 gals.
59. 43′—$5\frac{15}{32}$″
60. 1.69 lbs.
61. 12
62. ¾
63. 34
64. 2
65. 5
66. 21
67. 15
68. 1
69. 48
70. 27
71. 15
72. 12
73. 7
74. 3
75. 28
76. 3
77. $6a$
78. $a - 5$
79. $-c$
80. 3½
81. 15′ by 60′
82. 16 sq. in.
83. 16, 32, 48 and 72
84. 3 P. M.
85. 36, 43, 56
86. 60
87. 3, 9, 12
88. $x = 16$, $y = 4$
89. $x = 7$, $y = 4$
90. $x = -4$, $y = 3$
91. $x = 8$, $y = 4$
92. 6.37 and 3.63
93. 3 ft. and 3 ft. 6 in.
94. 5
95. 3
96. 4
97. 0
98. 3.1550
99. 9.8406 — 10
100. 7.8722 — 10
101. 2.6324
102. 1.7957
103. 2.4783
104. 4.530
105. 5,664
106. .6083
107. .001363
108. 2.076
109. 1543
110. 167.1
111. 9.674
112. 1° 12′
113. 93.34 ft.
114. 33° 42′
115. 118.9 ft.
116. 395 ft.
117. 47° 9′
118. 24° 37′