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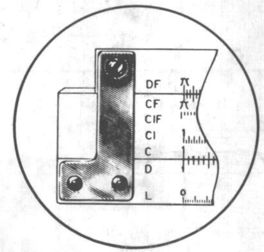
1776/ /ISB6

DIETZGEN REDIRULE® instruction manual

5 INCH 1776 POCKET SLIDE RULE

15 SCALES

FRONT/DF CF CIF CI C D L



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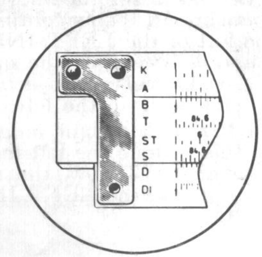

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Multiplication.

In using the "C" and "D" scales to multiply numbers, such as 8 × 5—where one or both of the numbers are on the right end of the scales, the right index can be used.

In Figure 1 is indicated the multiplication of 8 × 5. Set the right index of the "C" over 8 on the "D" scale. Move the indicator to 5 on the "C" scale and under the hairline read the answer—40—on the "D" scale.

NOTE: If you had used the left index of "C" over 8 on the "D" scale, the answer which is read under the 5 on the "C" scale would have been off the rule.

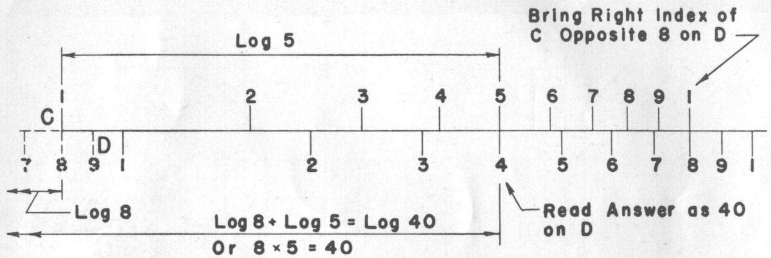

Fig. 1

Therefore, the right index and the left index of any of the scales can be used interchangeably, whichever will place the answer on the rule.

The reason for the above statement is that the "C" and "D" scales can be thought of as being continuous—or that they repeat themselves. In Fig. 1 to the left of the LEFT INDEX of "D" is shown in "dotted" the numbers 7, 8, and 9. These are the same numbers and are placed identically as those on the right end of the actual "D" scale. Therefore, you can think of an imaginary scale to the left of the LEFT INDEX of the "D" scale.

In Fig. 1, the right index of "C" is brought to 8 on "D". Notice that when this is done the left index of "C" is at 8 on the imaginary or "dotted" portion of "D". Now, the multiplication can be made as with any other numbers using the LEFT INDEX of "C". The answer is on "D" opposite 5 on "C".

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Division.

In dividing by logarithms one subtracts the logarithm of the divisor from the logarithm of the dividend in order to obtain the logarithm of the quotient or answer. This can be done by simple mechanical manipulation on the slide rule.

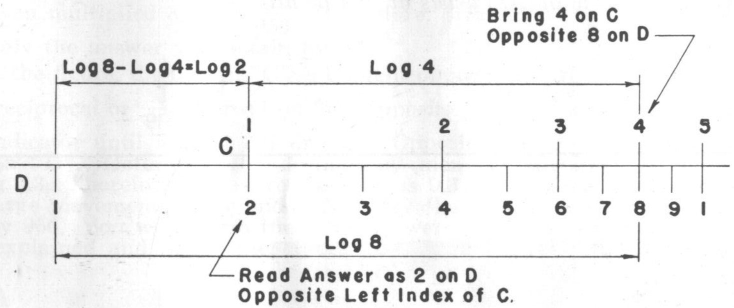


Fig. 2

In Figure 2 is indicated the division of 8 by 4. This is performed mechanically on the slide rule by the subtraction of the logarithm of 4 from the logarithm of 8.

Set the indicator at 8 on the "D" scale. Bring 4 on the "C" scale over 8 on the "D" scale and read the answer opposite the left index *of the "C" scale as 2 on the "D" scale.*

Note what you have actually done. In Figure 2 a distance equal to the logarithm of 8 (dividend) is located on the "D" scale, from which is subtracted a distance equal to the logarithm of 4 (divisor) on the "C" scale, leaving a distance equal to the logarithm of 2 (the quotient, or answer) on the "D" scale.

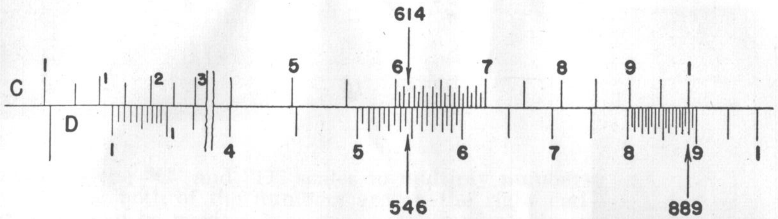


Fig. 3

The division of 546 by 614 is indicated in Figure 3. First, bring the indicator to the dividend, 546, on "D" and, second, bring to the hairline of the indicator 614 on the "C". You can then read your answer as 889 on "D" opposite the right index on "C". The left index would also be opposite the answer but no scale of "D" exists at this point.

Use of Reciprocals in Division.

The method of dividing 9 by 2 as explained above would be to bring the 2 on "C" opposite the 9 on "D". This could be done, but it requires that you bring the slide over to the right until it is almost out of the body of the rule. This division can be done in an easier manner by using the reciprocal of one of the numbers.

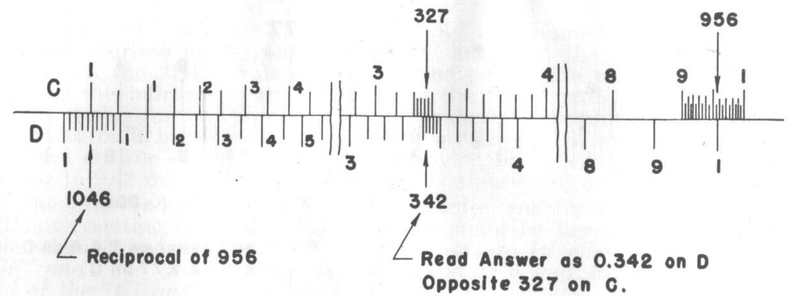


Fig. 4

In Figure 4, the number 327 is divided by 956. This division is the same as if you multiplied 327 by $\frac{1}{956}$. Therefore, divide 1 by 956 first and then multiply the answer you obtain by 327.

In the figure, the 956 on "C" is brought opposite the right index on "D". The reciprocal or $\frac{1}{956}$ is read on "D" opposite the left index on "C". Move the indicator until it is at 327 on "C". Opposite this, read the answer as 342 on "D". Determine the decimal by mentally dividing 300 by 1000 giving 0.3. Therefore, the correct answer is 0.342. This manipulation saves the large movement of the slide. Now try the regular method of dividing 327 by 956. You will obtain the same answer. Next, try again the method just explained and notice the saving in time and labor.

The Slide Rule is a tool for rapidly making calculations. It is an indispensable aid to the engineer and the scientist as well as to the accountant, statistician, manufacturer, teacher, and student or to anyone who has calculations to solve. Only when you are fully familiar with what it can do for you, can you reduce the amount of effort required in your calculations. The theory of the slide rule is quite simple and with a little practice proficiency in its operation may rapidly be developed.

When dividing 277 by 11.24 as in Figure 5, you can use the same scheme as above. First divide 1 by 11.24. This is done by bringing the 1124 opposite the left index on "D". The reciprocal could be read at the right index of "C" on "D" but instead move the indicator to 277 on "C". Opposite this, read 246 on the "D" scale, which is 277 ÷ 11.24.

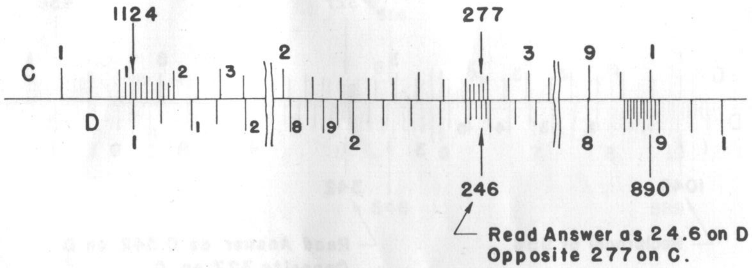


Fig. 5

The Folded Scales—"CF" and "DF".

The folded scales "CF" and "DF", in reality, are "C" and "D" scales cut in half at π(=3.1416) and the two halves switched, bringing the left and right indices to the middle as one index and π to each end in alignment with the left and right indices of the "C" and "D" scales. The "CF" and "DF" scales and their location with respect to the "C" and "D" scales are shown in Figure 6.

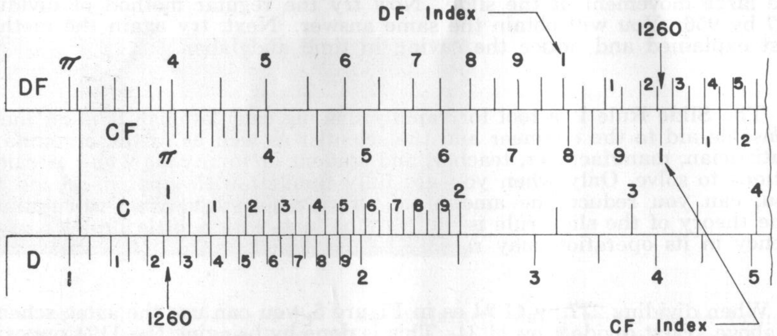


Fig. 6

This arrangement of scales has two distinct advantages:—Any number on the "C" and "D" scales is easily multiplied by π directly above on the "CF" and "DF" scales. Thus, to multiply any number by π, bring the indicator hairline to the number on "D" and read the answer under the hairline on the "DF" scale. Likewise, one can divide a number by π by bringing the hairline to the number on the "DF" scale and reading the answer under the hairline on the "D" scale. For instance, to multiply π (=3.1416) by 2, bring the indicator hairline to 2 on "D", and above on "DF" read the answer —6.28. To divide 9.42 by π, bring the hairline of the indicator to 9.42 on "DF" and below read the answer 3 on "D".

The other advantage of the folded scales enables factors to be read without resetting the slide, which factors might be beyond the end of the rule when using only the "C" and "D" scales. *In effect, the use of the "CF" and "DF" folded scales is like extending a half scale length to each end of the "C" and "D" scales.*

Looking again to the "DF" and "CF" scales in Figure 6, you will notice that no matter what number the left index of "C" is placed opposite on "D", the middle index (the only index) of "CF" is opposite the same number on "DF". Likewise, wherever the "D" indices are with respect to the "C" scale, the "DF" index is opposite the same number on the "CF" scale. In Figure 6, the left index of "C" is opposite 1260 on "D". Also, the index of "CF" is opposite 1260 on "DF". Set your slide rule as in Figure 6. Notice the numbers opposite the right index of "D" and the index of "DF". These should be the same.

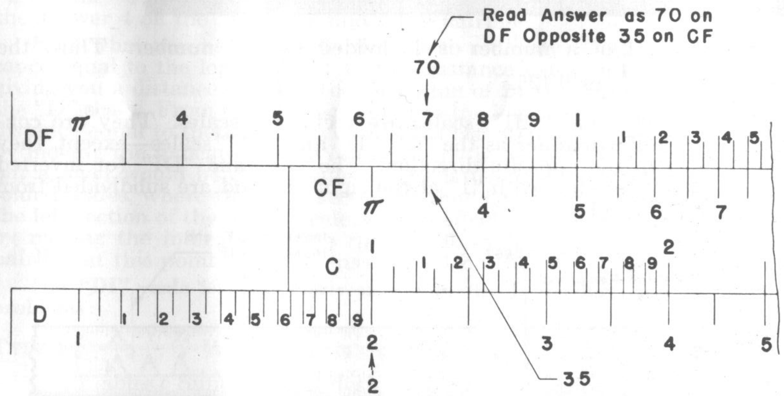


Fig. 7

In order to multiply by using the "CF" and "DF" scales, see the illustrated problem in Figure 7. Here 2 is multiplied by 35. *First set the left index of "C" opposite 2 on "D". The answer can be read on "D" opposite 35 on "C" as in the regular manner. Also, you can read the answer on "DF" opposite the 35 on "CF". Again, the answer is* **70.**

Using this same figure multiply 2 × 9. *The left index of "C" is placed opposite 2 on "D". Since 9 on "C" is off the scale on "D" you must read the answer as* **18** *on "DF" opposite 9 on "CF". This permits you to obtain the answer when it would otherwise be off the regular "D" scale.*

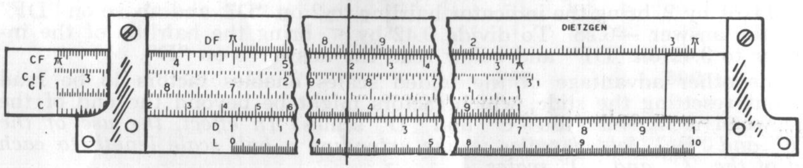


Fig. 8

To multiply 7 × 12 place the right index of "C" opposite 7 on "D", see Figure 8. Opposite the 12 on "CF" read **84** *on "DF", which is the answer.*

The Reciprocal Scales—"CI", "DI" and "CIF".

The reciprocal of a number is 1 divided by the number. Thus, the reciprocal of 8 is $\frac{1}{8}$ or 0.125.

The "CI", "DI" and "CIF" scales are reciprocal scales. They are constructed in a similar manner as the "C", "D" and "CF" scales—except, they are subdivided in the opposite direction. The "CI" and "DI" (or inverted "C" and "D" scales) start with "1" at the right end and are subdivided from 1 to 10 from right to left.

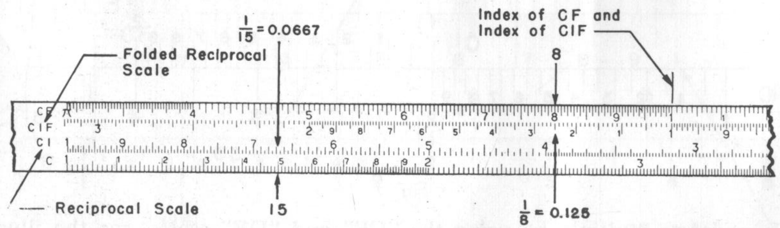


Fig. 9

Since the scales are inverted the indicator hairline can be brought to any number on the "C" scale and the reciprocal of that number can be read on the "CI" scale. Likewise, the reciprocal of any number shown on the "D" and "CF" scales can be found directly opposite the number on the "DI" or "CIF" scale.

For instance, determine the reciprocal of 15. *Bring the indicator hairline to 15 on the "C" scale and read directly above on the "CI" scale* **0.0667**, *see Figure 9. Also, for the reciprocal of 8, bring the hairline to 8 on the "CF" scale and directly below this read* **0.125** *on the "CIF" scale.*

These scales are used in connection with the "C", "D", "CF", and "DF" scales in multiplication and division. To multiply 12 × 2 × 15, one must add the logarithms of the three numbers together. This sum will be the logarithm of the answer. To do this on your slide rule, the indicator is first brought to 12 on the "D" scale. Second, the slide is moved until the "2" on the "CIF" is at the indicator. Third, the indicator is moved to the 15 on the "CF" scale. Fourth, read the answer as **360** on "D" directly under the hairline.

What has actually been done? A distance equal to the logarithm of 12 has been added to a distance equal to the logarithm of 2, which would bring you to the index on "CIF". Since the index on the "CF" scale is at the same point, you can then add a distance equal to the logarithm of 15 by sliding the indicator to the right until it is at 15 on the "CF" scale. The answer is then read on the "D" scale under the hairline of the indicator.

Using the same setting of your rule multiply 12 × 2 and divide the result by 6. Set the indicator at 12 on the "D" scale as before. To this, bring the "2" on the "CIF" scale. Slide the indicator to 6 on the "CIF" scale and read the answer 4 on the "D" scale under the hairline.

Note what you have actually done is to multiply 12 ×2 by adding a distance equal to the logarithm of 12 to a distance equal to the logarithm of 2 giving you a distance equal to the logarithm of 24 (the product of 12 × 2) on the "D" scale. From the distance equal to the logarithm of 24 is subtracted a distance equal to the logarithm of 6. This last step ordinarily would be done by moving the indicator to the *left* from the index of "CIF". However, since our answer would be off the rule on the left and since we are dealing with folded scales, where the right section of the "CIF" scale is a continuation of the left section of the "CIF" scale, we can effect this subtraction mechanically by moving the indicator to the *right* to 6 on the "CIF" scale. Under the hairline at this point read the answer as 4 on the "D" scale.

The "DI" scale is particularly useful in the following types of numerical problems:

TYPE I. $\frac{X}{A}, \frac{Y}{A}, \frac{Z}{A}$ Where "A" is constant, and "X", "Y", and "Z" are variables. Suppose it is desired to find the individual quotients of a series of fractions; all having the *same denominator* but *different numerators.* Without the "DI" scale, it would be necessary to move the slide and indicator to solve for each fraction. However, with the "DI" scale, the slide need only be set one time and successive quotients can be obtained by merely moving the indicator.

EXAMPLE: Evaluate the following series of fractions: $\frac{14}{5}, \frac{15}{5}, \frac{16}{5}.$

Set the Right Index of "C" over the common denominator, *5*, on "DI".
Set the Indicator Hairline over *14* on "C".
Under the Hairline read the quotient of $\frac{14}{5}$, namely *2.8* on "D".
Next, move the Hairline to *15* on "C".
Under the Hairline read the quotient of $\frac{15}{5}$, namely *3* on "D".
Next move the Hairline to *16* on "C".
Under the Hairline read the quotient of $\frac{16}{5}$, namely *3.2* on "D".

TYPE II. $\frac{1}{X \times Y}$
Without the "DI" scale, the solution to a problem of this type would require 2 settings of the slide, and 2 settings of the Indicator. If the problem were one of conventional multiplication, that is, simply $X \times Y$, the answer would be read on the "D" scale. However, since the reciprocal of the product is required, the answer may be read directly on the "DI" scale—which is the reciprocal of the "D" scale. By means of the "DI" scale, problems of this type can be solved directly with only one setting of the slide and one setting of the Indicator.

EXAMPLE: Evaluate $\frac{1}{2 \times 3}$
Set Left Index of "C" over 2 on "D";
Move Indicator Hairline to 3 on "C";
Under the Hairline, read .1666 on "DI".

Squares.

In solving problems, there are many occasions when a number must be multiplied by itself. Thus, the area of a square 4 ft. on each side is 4×4 (or 4^2) which equals 16 sq. ft. This operation is called squaring.

Instead of writing 4×4, or 35×35, or any other number multiplied by itself, the operation is indicated by writing 4^2, or $(35)^2$. This is read 4 squared, or 35 squared—sometimes read as 4 or 35 to the second power.

You will find that it is always possible to square a number by using the "C" and "D" scales. A shorter method is to use the "A" and "D" scales, or the "B" and "C" scales.

THE "A" AND "B" SCALES, which are exactly alike, are what are called two-unit logarithmic scales; that is, two complete logarithmic scales which, when placed end to end, equal the length of the single logarithmic scale "D" or "C", in connection with which they are usually used. You will note by the fact that these two-unit logarithmic sales "A" and "B" are directly above the single-unit logarithmic scale "D" that when the hairline of the indicator is set to a number on the "D" scale, the square of the number is found directly above under the hairline on the "A" scale. Likewise, if the hairline is set to a number on the "C" scale, the square of that number is found under the hairline on the "B" scale.

Note that dual faced rules, having graduations on both sides, have an encirling indicator permitting any one of the scales on one side to be read in connection with any of the scales on the opposite side. Thus, if the hairline of the indicator is set to 2 on "C", the square of 2, namely, 4, will be found under the hairline on the opposite side of the rule on scale "B".

Note also that since the "A" and "B" scales are each two complete logarithmic scales, they can be used for multiplication and division the same as the "C" and "D" scales; as, for example, to multiply 2×3, set either the left or the middle index of "B" under either the 2 on the first unit of "A" or under 2 on the second unit of "A" and above 3 on "B", read the answer 6 on "A" in either the first or the second unit.

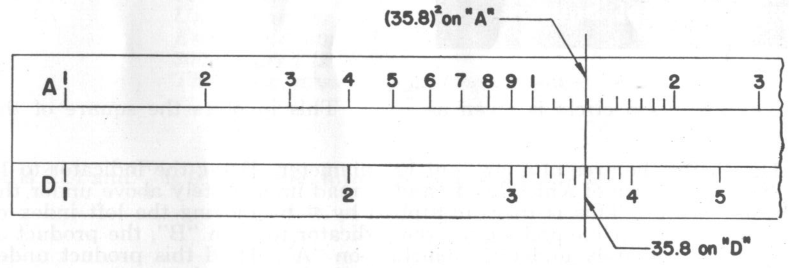


Fig. 10

ILLUSTRATION: What is the square of 35.8 or what is $(35.8)^2$?
See Figure 10.
Set indicator at 35.8 on the "D" scale
Read answer on the "A" scale under the hairline as **1282.**
(The fourth digit being estimated)
Obtain the decimal by estimation as 40 × 40 is 1600
Therefore read the answer as **1282.**

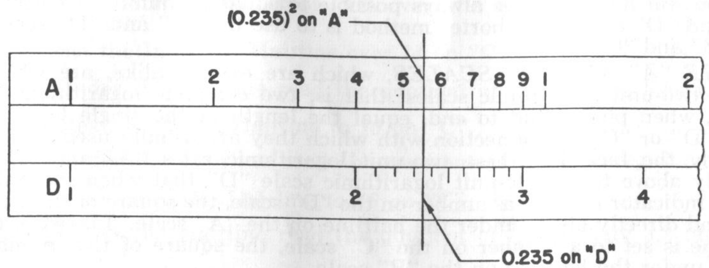


Fig. 11

ILLUSTRATION: What is 0.235 squared? See Figure 11.
Bring indicator to 235 on "D"
Read answer as **552** *on "A" under hairline*
Estimate decimal point by 0.2 × 0.2 which equals 0.04
Read answer as **0.0552.**

Applications of Squares.

The area of a circle is given as $\frac{\pi D^2}{4}$. This involves the square of the diameter.

Determine the area of a circle of 12″ diameter. Bring the indicator to 12 on "D", the square of which is 144 and is read immediately above under the hairline on "A". This is then multiplied by π by moving the left index of "B" under the hairline and sliding the indicator to π on "B", the product of which is immediately under the hairline on "A". Hold this product under the hairline on "A" and divide same by 4 which is done by moving 4 on the "B" scale under the hairline and reading the answer **113.0** on "A" immediately above the left index "B".

This operation could have been performed using the "A", "C", "D", and "DF" scales as follows: Use the "D" and "A" scales as above to obtain the square of 12. Since our calculation involves dividing by 4, we can effect this division by multiplying by the reciprocal of 4. Therefore, using the "C" and "D" scales, set 4 on "C" over the right index of "D" and read the reciprocal of 4 under the left index of "C". This reciprocal is then multiplied by $(12)^2$ or 144, by moving the indicator hairline to 144 on "C". This product can then be immediately multiplied by π by reading the answer **113.1** directly above under the hairline on "DF".

In this second method, you will be able to read to four significant figures (since you are between the prime numbers 1 and 2 of the rule), while in the first method, only three significant figures can be read on the "A" scale. It might be well that you do both of these operations again to familiarize yourself with the advantage of one method over the other.

In solving problems involving both multiplication and division, it is not necessary to read intermediate answers of each step in the calculation for all we are interested in is the final result. The best way to approach problems of this kind is to perform alternately—first, division; then multiplication; then division; then multiplication, and continue in this manner until the problem is solved. This minimizes the number of settings of the slide and the movement of the indicator.

ILLUSTRATION: Do the following indicated operation:

$$\left[\frac{45.8 \times 31.9}{5.6}\right]^2$$

Set the indicator at 45.8 on "D"
Bring 5.6 on "C" to hairline
Move indicator to 31.9 on "C"
Under hairline on "A" read **681**
Estimate decimal by 50 × 30 ÷ 5 equals 300
300 squared is 90,000
Therefore, answer should be **68,100**

π is 3.14 and $\frac{\pi}{4}$ is 0.785. Therefore, the area of a circle is the constant (0.785)

The area of a circle was given above as $\frac{\pi}{4}$ times the diameter squared. Toward the right end of both the "A" and "B" scales is a long mark at 0.785 or $\pi/4$.

To obtain the area of a 12″ circle, bring the 0.785 mark on "B" to the right index of "A". Move the indicator to 12 on "D" and read the answer on "B" under the hairline as 113.0. In this operation you are multiplying 0.785 by the square of 12. *Thus, to obtain the area of any circle, bring 0.785 mark on "B" to right index of "A". Bring indicator to the diameter on "D" and read answer on "B" under the hairline.*

Square roots.

The square root of any number is another number whose square is the first number. Five squared is 25 and the square root of 25 is 5. The symbol for the square root is $\sqrt{}$. Thus to indicate the square root of 25 the symbol is used as $\sqrt{25}$.

ILLUSTRATION:

$\sqrt{9} = 3$ $\sqrt{100} = 10$
$\sqrt{16} = 4$ $\sqrt{121} = 11$
$\sqrt{49} = 7$ $\sqrt{169} = 13$

The square root of a number is found on the slide rule by reversing the process used in finding the square of a number; namely, locating the number whose square root is desired on the "A" scale and reading the square root of the same under the indicator on the "D" scale.

The "A" scale has two parts that are identical. This scale is divided into divisions from 1 to 10 in one-half the length of the rule and again into divisions from 1 to 10 in the second half of the rule. The "B" scale is identical with the "A" scale. The first half of the "A" and "B" scales will be referred to as A-LEFT or B-LEFT and the other half as either A-RIGHT or B-RIGHT.

In order to find the square root of numbers with an *odd number of digits* to the left of the decimal point, use the A-LEFT scale in conjunction with the "D" scale.

Square Roots of Numbers Less Than Unity.

If the square root of a number less than unity is desired, move the decimal point to the RIGHT an even number of places until you have a number greater than 1. Thus to obtain $\sqrt{0.000347}$, change the number to read the $\sqrt{3.47}$. Obtain the $\sqrt{3.47}$ as before which is 1.864. Since the decimal point was moved 4 places to the right in the first operation, move it back to the left *half* of this amount. You would then read the answer as **0.01864.**

ILLUSTRATION: What is the square root of 0.0956?
Move the decimal 2 places to the right to obtain 9.56
Set the indicator at 9.56 on A-LEFT
Under the hairline read **3.09** *on "D".*
Finally, move the decimal $\frac{1}{2}$ *of 2 places to the left*
Read the answer as **0.309**

ILLUSTRATION: What is the square root of 0.0000158?
Move the decimal 6 places to the right to obtain 15.8
Set indicator at 15.8 on A-RIGHT
Under the hairline read **3.97**
Move the decimal $\frac{1}{2}$ *of 6 places to the left*
Finally the answer should be read as **0.00397**

Combined Operations Involving Squares and Square Roots.

The "B" and "C" scales can be used in the same manner as the "A" and "D" scales to obtain the square roots of numbers. This makes various combined operations easy with the slide rule.

For example, to obtain the result of $4 \times \sqrt{354}$, set the left index of "C" at 4 on "D" and move the indicator to 354 on "B-LEFT". Read the answer as **75.2** on "D" under the hairline. Likewise, to obtain the result of $8.6 \times \sqrt{34.8}$, set the right index of "C" at 8.6 on "D" and move indicator to 34.8 on "B-RIGHT". Read the answer as **50.7** on "D" under the hairline.

For simplicity the following general form will be used for all slide rule settings:

What is the value of $23.4\sqrt{7.86}$?
To 23.4 on "D", set 1 on "C"
Opposite 7.86 on B-LEFT read **65.6** *on "D"*

Cubes.

Just as 4^2 means 4×4, so 4^3 (read four-cubed) means $4 \times 4 \times 4$. The small number, 3, to the upper right indicates how many 4's (or whatever the number is) must be multiplied together. This small number is called the exponent or power of the number. To illustrate:

$$10^3 = 10 \times 10 \times 10$$
$$(4.7)^3 = 4.7 \times 4.7 \times 4.7$$

It is always possible to multiply these numbers out on the "C" and "D" scales—and in combined operations for complicated calculations, it is sometimes more convenient. However, the "K" scale on the slide rule is designed to give you the cubes of all numbers directly.

The "K" scale is what is called a three-unit logarithmic scale; that is, three complete logarithmic scales of a length which, when placed end to end, equal the length of the single logarithmic scale "D" with which it is usually used. You will note that this "K" scale is so arranged that when the indicator is set to a number on the "D" scale, the cube of that number is given under the hairline on the "K" scale.

ILLUSTRATION: What is the cube of 34.5?
Set indicator to 34.5 on "D"
Under hairline on "K" read **41,100**

To carry out this calculation on the full length scales, do the following:
To 34.5 on "D" set 34.5 on "CI"
Move indicator to 34.5 on "C"
Read **41,100** *under the hairline on "D"*

The reciprocal and folded scales are invaluable in shortening various calculations and one who expects to become proficient in the operation of the slide rule should use these scales as often as possible; as, for instance, dividing a product by the reciprocal of a number as illustrated in the above example gives the same result as multiplying by the number. A tool is of value only when it is used.

Cube Roots.

The cube root of a number is a number which when multiplied by itself three times gives the original number. Thus, the cube root of 27 is 3, because $3 \times 3 \times 3$ is 27. The symbol of cube root is $\sqrt[3]{}$ and the cube root of

8000 is written as $\sqrt[3]{8000}$.

The "K" scale is a triple scale, consisting of three identical sections, one following the other. In finding the cube roots of numbers, the "K" scale is considered as a single scale.

The first division of the "K" scale will be referred to as K-LEFT; the second division as K-MIDDLE; and the third division as K-RIGHT. To obtain cube roots of numbers, set the hairline on the number on the "K" scale (see unit below) and read the cube root at the hairline on "D" scale, using:

K-LEFT	for numbers between	1 and 10
K-MIDDLE	" " "	10 and 100
K-RIGHT	" " "	100 and 1000

For numbers greater than 1,000 or less than 1 (unity), proceed as follows:

FIRST: Move the decimal point to the left or right three places at a time until a number between 1 and 1000 is obtained.

SECOND: Take the cube root of this number using K-LEFT, K-MIDDLE, or K-RIGHT as explained above. Place the decimal point after the first figure of this reading.

THIRD: Now move the decimal point in the opposite direction one-third as many places as it was moved in (First) above.

ILLUSTRATION: What is the cube root of 34560?
Move the decimal point to the left three places (one group of three), thus obtaining 34.560.
Since the part to the left of the decimal point is between numbers 10 and 100, use the K-MIDDLE scale.
Set indicator to 346 on K-MIDDLE and
Read 3.26 under hairline on "D" scale.
Set decimal point one place $\left[\frac{1}{3}(3) = 1\right]$ *to the right to obtain the answer,* **32.6.**

Fundamental Ideas and Formulas of Plane Trigonometry.

A review of a few of the fundamental ideas and formulas of plane trigonometry is given here to help in understanding the explanation of the use of the "S", "T", and "ST" scales on your slide rule.

In the right triangle, Figure 12, the corners or angles are labeled A, B, and C. The triangle is referred to as triangle ABC. The sides are labeled *a*, *b*, and *c*, with *a* opposite angle A, *b* opposite angle B, and *c* opposite angle C. For right triangles the 90° angle is labeled C.

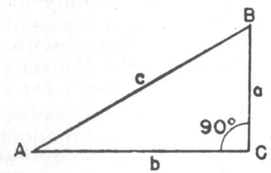


Fig. 12

Referring to this figure, the following definitions and relationships can be written.*

Definitions of the sine, cosine, and tangent:

$$\text{Sine A (written sin A)} = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{Cosine A (written cos A)} = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{Tangent A (written tan A)} = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}$$

Reciprocal relations:

$$\text{Cosecant A (written csc A)} = \frac{c}{a} = \frac{1}{\sin A}$$

$$\text{Secant A (written sec A)} = \frac{c}{b} = \frac{1}{\cos A}$$

$$\text{Cotangent A (written cot A)} = \frac{b}{a} = \frac{1}{\tan A}$$

*See any standard text on Plane Trigonometry.

RELATION BETWEEN FUNCTIONS OF ANGLES LESS THAN 90°:

$$\cos A = \sin (90° - A)$$
$$\cot A = \tan (90° - A)$$

Likewise,

$$\sin A = \cos (90° - A)$$
$$\tan A = \cot (90° - A)$$

From a table of functions of angles, the cosine of 35° is given as 0.819152. Looking up the sine of (90° — 35°) or the sine of 55°, we find that it is again 0.819152. You can check these relationships given above in a similar manner.

Complementary angles have their sum equal to 90°. Thus, in the above example, 35° and 55° are complementary angles since their sum is 90°.

RELATION BETWEEN FUNCTIONS OF ANGLES BETWEEN 90° AND 180°:

The definitions of the trigonometric functions given at the beginning of this article apply only to angles between 0° and 90°. More general definitions applying to angles of any size are given in texts on trigonometry. Since we will have to deal with functions of angles between 90° and 180°, a summary of these relationships only will be given here, and one is referred to any text on trigonometry for a complete statement of these definitions.

If A is an angle between 90°, and 180° then the following relationships hold between the functions of these angles:

$$\sin A = \sin (180° - A)$$
$$\cos A = -\cos (180° - A)$$
$$\tan A = -\tan (180° - A)$$

Thus, if the angle A is 123°, we may write:

$$\sin 123° = \sin (180° - 123°) = \sin 57°$$
$$\cos 123° = -\cos (180° - 123°) = -\cos 57°$$
$$\tan 123° = -\tan (180° - 123°) = -\tan 57°$$

From these relationships, the value of the functions of any angle between 90° and 180° can be obtained. These will be used later for the solution of oblique triangles.

RELATION BETWEEN ANGLES OF TRIANGLES

In a right triangle, the sum of the other two angles is 90°. Referring to Figure 12, the sum of A and B equals 90° and the sum of A, B, and C equals 180°.

In equation form:

For a right triangle:

$$A + B = 90° \text{ (where angle C is 90°)}$$

For any triangles:

$$A + B + C = 180°$$

In any triangle as Figure 13, the relation between the sides and the angles can be expressed as shown below:

Law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of cosines:

$$a^2 = b^2 + c^2 - 2bc (\cos A)$$
$$\text{Or } b^2 = a^2 + c^2 - 2ac (\cos B)$$
$$\text{Or } c^2 = a^2 + b^2 - 2ab (\cos C)$$

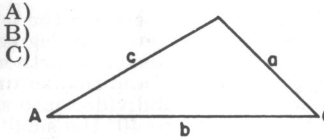


Fig. 13

The "S" (Sine) and "ST" (Sine-Tangent) Scales.

The "S" and "ST" scales are two sections of one long scale which, operating with "D", gives the sines of the angles between 34′ and 90°. The "S" scale represents the scale of sines of angles from 5°44′ to 90°. The "ST" scale represents the scale of sines and tangents of angles from 34′ to 5°44′. Since the value of the sine and tangent of angles below 5°44′ is for all practical purposes identical, we can use the same scale for finding either the sine or the tangent for angles below 5°44′ and above 34′. Thus, the reason for the "ST" scale.

The "S" and "ST" scales are so designed and arranged that when the indicator is set to a number (angle) on the "S" or "ST" scales, the sine of the angle is given under the hairline on the "D" scale. The cosine of any angle can be determined by subtracting the angle from 90° and following this same procedure. The cosine will be found under the hairline on the "D" scale.

When using the "S" scale to read the value of sines of any angle, read the left index of "D" as 0.1 and the right index as 1.0. When using the "ST" scale to read the value of the sine of any angle, read the left index of "D" as 0.01 and the right index as 0.1. This is illustrated in Figure 14.

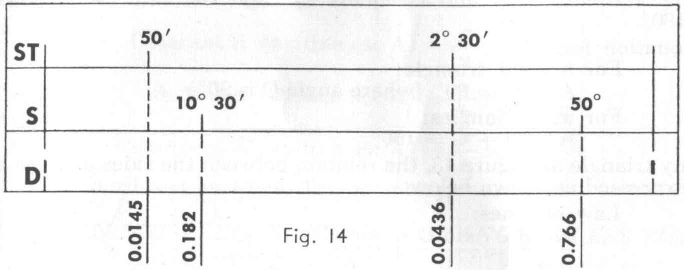


Fig. 14

The "S" scale between the left index (5°44′) and 10° is first divided into degrees and between each degree there are six subdivisions representing 10′ each. Each subdivision is further divided into two parts. Thus, the smallest direct reading is 5′ and smaller units must be estimated. Between 10° and 20°, the degrees are subdivided into six parts only and the smallest direct reading is 10′. From 20° to 40° the smallest direct reading is 30′ and from 40° to 70° it is only 1°; while between 70° and 80° the smallest subdivision is 2°. When obtaining the functions of angles requiring a reading smaller than the subdivision given, estimate the location as accurately as possible. Estimations can be made to 1′ between left index (5°44′) and 10°; to 5′ between 10° and 20°; to 10′ between 20° and 40°; to 30′ accurately and to 10′ fairly accurately between 40° and 70°; and to 1° between 70° and 80°.

Example 1. What is the sine 6°45′?
Bring indicator to 6°45′ on "S"
Under the hairline read **0.1175** *on "D"*

Example 2. What is the sine 27°35′ 06″?
Round this off to 27°35′
Set indicator to 27°35′ on "S"
Under hairline read **0.463** *on "D"*

Example 3. What is the sine 75°?
Set indicator to 75° on "S"
Under the hairline read **0.965** *on "D"*

The "T" (Tangent) Scale.

The "T" scale is designed to give directly the tangents and cotangents of angles between 5°43′ and 45°. When the indicator is set to any number (angle) on the "T" scale, the tangent of that angle is given on the "D" scale.

When the indicator is set to any number (angle) on the "T" scale, the cotangent of that angle can be read under the hairline on the "DI" scale.

For angles between 34′ and 5°43′, the tangent and the sine are for all practical purposes almost the same. We can therefore, use the "ST" (sine-tangent) scale in conjunction with the "D" scale to obtain the tangents and cotangents of angles between 34′ and 5°43′.

Thus, to determine the tangent of the angle 35°12′, set the indicator to 35°12′ on the "T" scale and under the hairline on "D", read *0.705* for the tangent. Under the hairline on "DI", read *1.418* as the cotangent.

You will note that the numbers (angles) on the "T" scale go from 5°43′ to 45° and their value reads directly on the "D" scale. For angles greater than 45°, use the relation of cotangent A = tan (90° − A). Therefore, in order to obtain the tangent of 62°30′, determine the cotangent of (90° − 62°30′) or the cotangent of 27°30′. Set the indicator to 27°30′ on "T" and under the hairline read *1.921* on the "DI". This is the cotangent of 27°30′ as well as the tangent of 62°30′.

Combined Operations.

Since the "S", "T", and "ST" scales are placed on the "slide" part of the rule, these scales can be used quite conveniently with the other scales of the rule to solve combined multiplication and division, etc., involving trigonometric functions.

The examples given below illustrate the various types of problems that can be solved using the "S", "T", and "ST" scales.

Example 1. Evaluate 4.53 sin 12°30′.
This indicates the multiplication of 4.53 times the sine of 12°30′.
Set left end of "S" to 4.53 on "D"
Bring indicator to 12°30′ on "S"
Under hairline read **0.982** *on "D"*

Example 2. Evaluate $\frac{23.5 \sin 34°40'}{\tan 15°18'}$

To 23.5 on "D" set 15°18' on "T"
Bring indicator to 34°40' on "S"
Under hairline read **48.8** *on "D"*

Example 3. Evaluate $\frac{8.34 \sqrt{34} \sin 63°}{4.23 \tan 42°24'}$

To 34 on "A-RIGHT" set 4.23 on "C"
Bring indicator to 8.34 on "CF"
Move slide so 42°24' on "T" is at hairline
Bring indicator to 63 on "S"
Under hairline read **11.20** *on "DF"*

What you have done in the above operations for the solution of Example 3 is this: First, you have divided $\sqrt{34}$ by 4.23 and multiplied this by 8.34 (this result would be at the index on "DF"); second, you have divided by tan *42°24'*; and third, you have multiplied by sin *63°*. The answer must, of course, be read on "DF" since the last two operations are done with respect to this scale.

Example 4. Evaluate $\frac{67.3 \csc 25° \cos 56°}{\sqrt{5.78} \tan 34°36'}$

Bring 5.78 on "B-LEFT" to left index of "A"
Move indicator to 67.3 on "C"
Bring sin **25°** *(this equals 1/csc 25°) on "S" to hairline*
Move indicator to 34 (90° − 56°) on "S" (this is to cos 56° on "S")
Bring 34°36' on "T" to hairline
Read **53.7** *on "D" opposite right index*

To Change Radians to Degrees or Degrees to Radians.

To find the sine or tangent of very small angles, it is convenient to write the angle in terms of radians. In a complete circle of 360°, there are 2 π radians. Therefore, the following relation can be set up:

$$\frac{2\pi}{360} \text{ or } \frac{\pi}{180} = \frac{\text{"r" (number of radians)}}{\text{"d" (number of degrees)}}$$

Example 1. How many radians is equivalent to 125°30'?
To π right on "DF" set 180 on "CF"
Opposite 125.5 on "C" (or "CF") read **2.19** *radians on "D" or "DF".*

The convenient setting to obtain the number of radians equivalent to the given number of degrees, or the number of degrees from the given number of radians, is as follows:

To π right on "DF" set 180 on "CF"
Opposite given degrees on "C" (or "CF") read radians on "D" (or "DF")
Opposite given radians on "D" (or "DF") read degrees on "C" (or "CF")

Example 2. How many degrees is equivalent to 5.46 radians?
To π right on "DF" set 180 on "CF"
Opposite 5.46 on "D" read **313°** *on "C"*

Example 3. How many radians is equivalent to 265°?
To π right on "DF" set 180 on "CF"
Opposite 265 on "C" read **4.63** *radians on "D"*

Exercises

1. Express the following angles in radians: 3°27', 76°30', 45°36', 0°48', 48°12', 346°, 320°, 201°, 308°, and 57°18'.

2. Express the following angles in degrees: 0.089, 2.345, 6.28, 6.34, 5.24, 0.896, 1.0894, 2.34, and 4.72. All given values are in radians.

Answers to the above exercises.

1. 0.0603, 1.336, 0.797, 0.01396, 0.842, 6.04, 5.58, 3.51, 5.38, and 1.00.
2. 5°6', 134°18', 360°, 363°, 300°, 51°18', 62°18', 134°, and 270°.

Sines and Tangents of Small Angles.

For smaller angles than are given on the "ST" (SINE AND TANGENT) scale, the following approximation from trigonometry can be used:

sin A = tan A = A (in radians) (approximately true)

The error in this assumption is quite small and is well within the accuracy of the slide rule.

Since 1° is equivalent to $\frac{\pi}{180}$ radians, set to π right on "DF", 180 on "CF"; opposite the angles expressed in degrees on either "C" or "CF", read the same angles expressed in radians on either "D" or "DF" as explained in the previous paragraph. Thus sin 0.2° equals $0.2 \times \frac{\pi}{180}$ which equals 0.00349 radians. Since 1' equals $\frac{\pi}{180 \times 60}$ radians, and since 1" equals $\frac{\pi}{180 \times 60 \times 60}$ radians, we can find the sine or tangent of any small angle expressed in minutes (or seconds) by multiplying it by the value of 1' (or 1") in radians.

Thus $\sin 16' = \frac{16 \times \pi}{180 \times 60} = 0.00466$

Or $\sin 23'' = \frac{23 \times \pi}{180 \times 60 \times 60} = 0.0001114$

For your convenience, the value of $\frac{180 \times 60}{\pi}$ has been marked by a *"minute"* sign point on the "ST" scale near the 2° division, and the value of $\frac{180 \times 60 \times 60}{\pi}$ has been marked by a *"second"* sign point near the 1.167° division. These marks will help you in the above multiplication.

To roughly obtain the decimal point, it is well to remember that the *sin 0.1°* is approximately *0.002;* that *sin 1'* is approximately *0.0003,* and that *sin 1"* is approximately *0.000005.*

Negative Exponents.

If 10^3 is divided by 10^5, the result would be 10^{3-5} or 10^{-2}. This indicates that the result is $\frac{1}{100}$. Thus, using 10 as a base and for any negative exponent, the result can be indicated by $1 \div 10^{+a}$, where "a" was the negative exponent.

ILLUSTRATION: What is $10^2 \div 10^7$?
$10^{2-7} = 10^{-5}$ or this may be written as
$10^{2-7} = 10^{-5} = \frac{1}{10^5}$

Notation Using the Base "10".

It is often convenient to change a number by either multiplying or dividing it by 10 to some exponent.

ILLUSTRATION: Change the number 30,000,000 to a more convenient form.
Divide this by 10^6 and write the number as 30×10^6
Or divide by 10^7 and write the number as 3×10^7
Change the number 0.000065 to a more convenient form.
Multiply this number by 10^5 and write the number as
6.5×10^{-5}

In the first illustration, the exponent of 10 is positive; and this indicates that the actual number of digits to the right of the number is the same as the exponent of 10. In the second illustration, the actual number of places to the left of the decimal as indicated by 10^{-5} is 5.

In each case, the number of places through which the decimal point moves is equal to the exponent of ten.

ILLUSTRATION: Evaluate $3450 \times 732 \times 0.032$
First, this can be changed to
$3.45 \times 10^3 \times 7.32 \times 10^2 \times 3.2 \times 10^{-2}$
Again, write it as
$3.45 \times 7.32 \times 3.2 \times 10^{3+2-2}$
To 7.32 on "D" set 3.2 on "CI"
Bring the indicator to 3.45 on "C" and
Read the answer as **80.8** *on "D" under hairline.*
Correct answer is then 80.8 × 10³ or **80,800**

Logarithms.

Logarithms are nothing more than exponents. A base is selected and the logarithm of any number to this base is just the *exponent* of the base that will give the original number. Usually the base 10 is selected and most all tables of logarithms are made to this base.

By definition the
Log 25 to the base 10 is 1.398
This being
$10^{1.398} = 25$

To either multiply or divide by logarithms, one adds or subtracts the logarithms of the numbers. From the above, one can see that this is the same as *adding* or *subtracting* the exponents of 10. This makes a convenient method of multiplying or dividing in complicated calculations.

The "L" (Logarithmic) Scale.

As stated in the first chapter, the logarithm of a number has two parts—the part to the *right* of the decimal called the *"mantissa"* and the part to the *left* of the decimal called the *"characteristic"*.

The characteristic of the logarithm of a number greater than one is obtained by inspection since it is defined as a number equal to one less than the number of digits in the original number. The characteristic for the logarithm of 456.0 is 3–1 or 2. The mantissa can be found on the "L" scale.

The "L" scale is so designed that when the hairline is placed to any number on the "D" scale, the mantissa of the logarithm of that number is shown on the "L" scale.

ILLUSTRATION: What is the logarithm of 456.0?
Set the indicator to 456 on "D"

Read 0.659 on the "L" scale—this is the mantissa.
By inspection, the characteristic is 2.
Therefore, the logarithm of 456 is **2.659.**

The characteristic of a number less than 1 is negative and is numerically one greater than the number of zeros immediately following the decimal point.

ILLUSTRATION: What is the logarithm of 0.0752?
The symbol for this is $\log_{10} 0.0752$ and is read as the logarithm of 0.0752 to the base 10.
The characteristic is -2, but is written as shown below.
Set indicator to 752 on "D"
Read 0.8761 on the "L" scale.
Logarithm is written **8.8761–10**

The characteristic is -2, but is written as 8–10 and the mantissa is written following the 8.

Calculations by Logarithms.

The logarithm of a number to the base 10 is defined as the exponent of 10 that will give the number. Thus,

$$10^2 = 100$$

Therefore, the logarithm of 100 is 2 because 10 raised to the second power gives 100.

Likewise, Log 34.5 = 1.5378. This means

$$10^{1.5378} = 34.5$$

Since the logarithms as given on the "L" scale are all to the base ten, one can multiply and divide by obtaining the logarithms of the numbers—then either add or subtract the logarithms depending upon whether or not you want to multiply or divide. The addition and subtraction of the logarithms is the same as the addition or subtraction of exponents as explained in article 39—the base being 10 in this case.

ILLUSTRATION: Evaluate $\dfrac{34.5 \times 9716}{3.24}$

Obtain the log 34.5 = 1.5378
Obtain the log 9716 = 3.9875
Their sum is 5.5253
Obtain the log 3.24 = 0.5106
Their difference is 5.0147
Set the indicator to 0.0147 on "L" scale
Under hairline read **1034**
Characteristic is 5; therefore, there should be 5 + 1 places to the left of the decimal.
Read answer as **103,400.**

This indicates a method of calculating problems as above, but as this can be done easier with the "C", "D", etc., scales, the "L" scale is used primarily when numbers with exponents are to be either multiplied or divided.

MATHEMATICAL FORMULAE

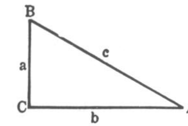
Plane Trigonometry.

Right Triangle

$\sin A = \dfrac{a}{c}$ $\qquad \cos A = \dfrac{b}{c}$

$\tan A = \dfrac{a}{b}$ $\qquad \cot A = \dfrac{b}{a}$

$\sec A = \dfrac{c}{b}$ $\qquad \operatorname{cosec} A = \dfrac{c}{a}$



$$\sin A = \cos\left(\frac{\pi}{2} - A\right) = -\cos\left(\frac{\pi}{2} + A\right)$$

$$\cos A = \sin\left(\frac{\pi}{2} - A\right) = \sin\left(\frac{\pi}{2} + A\right)$$

$$\tan A = \cot\left(\frac{\pi}{2} - A\right) = -\cot\left(\frac{\pi}{2} + A\right)$$

$$\cot A = \tan\left(\frac{\pi}{2} - A\right) = -\tan\left(\frac{\pi}{2} + A\right)$$

$$\sec A = \operatorname{cosec}\left(\frac{\pi}{2} - A\right) = \operatorname{cosec}\left(\frac{\pi}{2} + A\right)$$

$$\operatorname{cosec} A = \sec\left(\frac{\pi}{2} - A\right) = -\sec\left(\frac{\pi}{2} + A\right)$$

$\sin(-A) = -\sin A$ $\qquad \cos(-A) = \cos A$

$\tan(-A) = -\tan A$ $\qquad \cot(-A) = -\cot A$

$\sec(-A) = \sec A$ $\qquad \operatorname{cosec}(-A) = -\operatorname{cosec} A$

NUMERICAL VALUES

Angle	0°	30°	45°	60°	90°
sin	0	½	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	½	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

MATHEMATICAL FORMULAE

Plane Geometrical Figures

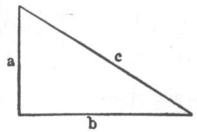
Right Triangle

$c = \sqrt{a^2 + b^2}$

$a = \sqrt{c^2 - b^2}$

$b = \sqrt{c^2 - a^2}$

area $= \frac{1}{2}\, ab$

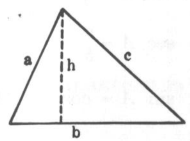


Any Triangle

area $= \frac{1}{2}\, bh$

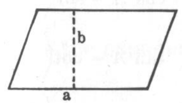
area $= \sqrt{s(s-a)\,(s-b)\,(s-c)}$

$s = \frac{1}{2}\,(a+b+c)$



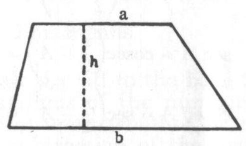
Parallelogram

area $= ab$



Trapezoid

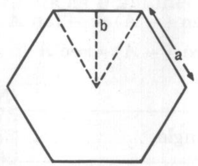
area $= \frac{1}{2}h\,(a+b)$



Regular Polygon

area $= \frac{1}{2}\, abn$

n = number of sides



Parabola

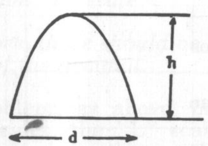
length of arc $= \dfrac{d^2}{8\,h}\Big[\sqrt{c\,(1+c)} +$

$2.3026 \log_{10}(\sqrt{c} + \sqrt{1+c})\Big]$

in which

$c = \left(\dfrac{4\,h}{d}\right)^2$

area $= \frac{2}{3}\, dh$



MATHEMATICAL FORMULAE

Plane Geometrical Figures.

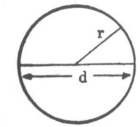
Circle

circumference $= 2\,\pi r$

$= \pi d$

area $= \pi r^2$

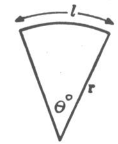
$= \pi \dfrac{d^2}{4}$



Sector of Circle

arc $= l = \pi r \dfrac{\Theta^\circ}{180^\circ}$

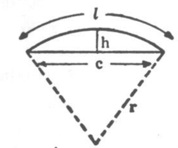
area $= \frac{1}{2}\, rl = \pi r^2 \dfrac{\Theta^\circ}{360^\circ}$



Segment of Circle

chord $= c = 2\sqrt{2\,hr - h^2}$

area $= \frac{1}{2}\, rl - \frac{1}{2}\, c\,(r-h)$



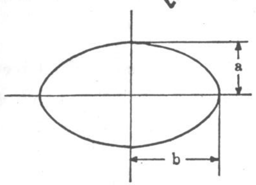
Ellipse

circumference =

$$\pi\,(a+b)\frac{64 - 3\left(\dfrac{b-a}{b+a}\right)^4}{64 - 16\left(\dfrac{b-a}{b+a}\right)^2}$$

(close approximation)

area $= \pi\, ab$

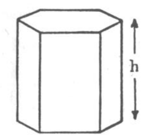


Solid Geometrical Figures.

Right Prism

lateral surface = perimeter of base $\times$ h

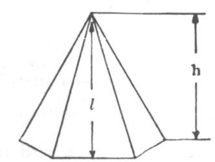
volume = area of base $\times\, h$



Pyramid

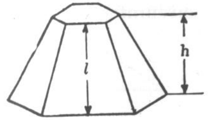
lateral area $= \frac{1}{2}$ perimeter of base $\times\, l$

volume = area of base $\times\, \dfrac{h}{3}$



Solid Geometrical Figures.

Frustum of Pyramid



lateral surface = $\frac{1}{2} l\,(P + p)$

P = perimeter of lower base

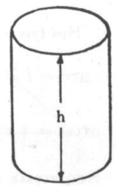
p = perimeter of upper base

volume = $\frac{1}{3}h\,[A + a + \sqrt{Aa}]$

A = area of lower base

a = area of upper base

Right Circular Cylinder

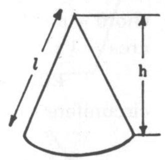


lateral surface = $2\,\pi r h$

r = radius of base

volume = $\pi r^2 h$

Right Circular Cone

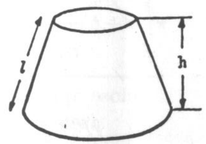


lateral surface = $\pi r l$

r = radius of base

volume = $\frac{1}{3}\,\pi r^2 h$

Frustum of Right Circular Cone



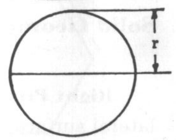
lateral surface = $\pi l\,(R + r)$

R = radius of lower base

r = radius of upper base

volume = $\frac{1}{3}\,\pi h\,[R^2 + Rr + r^2]$

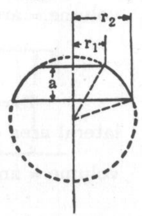
Sphere



surface = $4\,\pi r^2$

volume = $\dfrac{4}{3}\,\pi r^3$

Segment of Sphere

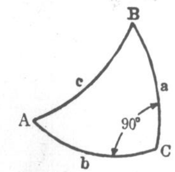


volume of segment

$$= \frac{1}{6}\,a\,\pi\,[3\,(r_1^2 + r_2^2) + a^2]$$

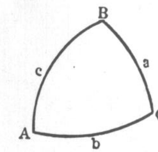
Spherical Trigonometry.

Right Spherical Triangles



$\cos c = \cos a \cos b$	$\cos A = \tan b \cot c$
$\sin a = \sin c \sin A$	$\cos B = \tan a \cot c$
$\sin b = \sin c \sin B$	$\sin b = \tan a \cot A$
$\cos A = \cos a \sin B$	$\sin a = \tan b \cot B$
$\cos B = \cos b \sin A$	$\cos c = \cot A \cot B$

Oblique Spherical Triangles



$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos A = \sin B \sin C \cos a - \cos B \cos C$$

$$\cot a \sin b = \cot A \sin C + \cos C \cos b$$

$$s = \tfrac{1}{2}\,(a + b + c)$$

$$S = \tfrac{1}{2}\,(A + B + C)$$

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{\sin\,(s-b)\,\sin\,(s-c)}{\sin b \sin c}}$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{\sin s \sin\,(s-a)}{\sin b \sin c}}$$

$$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{\sin\,(s-b)\,\sin\,(s-c)}{\sin s \sin\,(s-a)}}$$

$$\sin\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S \cos\,(S-A)}{\sin B \sin C}}$$

$$\cos\left(\frac{a}{2}\right) = \sqrt{\frac{\cos\,(S-B)\,\cos\,(S-C)}{\sin B \sin C}}$$

$$\tan\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S \cos\,(S-A)}{\cos\,(S-B)\,\cos\,(S-C)}}$$

$$\tan \tfrac{1}{2}\,(a-b) = \frac{\sin \frac{1}{2}\,(A-B)}{\sin \frac{1}{2}\,(A+B)} \tan \tfrac{1}{2}\,c$$

$$\tan \tfrac{1}{2}\,(a+b) = \frac{\cos \frac{1}{2}\,(A-B)}{\cos \frac{1}{2}\,(A+B)} \tan \tfrac{1}{2}\,c$$

$$\tan \tfrac{1}{2}\,(A-B) = \frac{\sin \frac{1}{2}\,(a-b)}{\sin \frac{1}{2}\,(a+b)} \cot \tfrac{1}{2}\,C$$

$$\tan \tfrac{1}{2}\,(A+B) = \frac{\cos \frac{1}{2}\,(a-b)}{\cos \frac{1}{2}\,(a+b)} \cot \tfrac{1}{2}\,C$$

$$\tan \tfrac{1}{2}\,c = \frac{\sin \frac{1}{2}\,(A+B)\,\tan \frac{1}{2}\,(a-b)}{\sin \frac{1}{2}\,(A-B)}$$