NUCLEAR–WEAPONS–TRAINING–CENTER–ATLANTIC

REVIEW OF MATHEMATICS AND SLIDE RULE

NUCLEAR WARFARE SCHOOL

FOREWORD

The Weapons Employment Course that you have been nominated to attend includes a number of problems which require a certain minimum knowledge of basic algebra and a degree of familiarity with the slide rule. This booklet is designed to assist prospective weapons employment students in preparing themselves for the course in these areas.

A student thoroughly familiar with the principles involved in this booklet should have little difficulty with the mathematics and slide rule portions of the weapons employment course. The problems should be completed prior to commencing the course. Section 1.06 (Factoring) is optional and is included solely for information purposes.

PART I
MATHEMATICS

CHAPTER I
ALGEBRA

1.01. THE LAWS OF SIGNS.

a. General. Numbers are either *positive* or *negative* in character and are indicated by a plus (+) or minus (–) sign, respectively. When a number is written without a sign, it is understood to be positive. Positive and negative numbers are referred to as *signed numbers*.

b. Law of addition. To add numbers having like signs, find the sum of their absolute values and prefix to this sum their common sign. To add numbers having unlike signs, find the difference between their absolute values and prefix to it the sign of the number having the greater absolute value. The sum of signed numbers is called their *algebraic sum*.

c. Law of subtraction. To subtract one signed number from another, change the sign of the subtrahend and proceed as in addition. In algebra, the minus sign has three meanings: Subtraction, as in $8 - 2 = 6$; shortage, as in $5 - 9 = -4$; and direction, as in −5 −4 −3 −2 −1 0 1 2 3 4 5

d. Law of multiplication. To multiply positive numbers, find the product of their absolute values and make it positive. In multiplying negative, or negative and positive numbers, the sign of the product is positive when the number of negative quantities is an even integer and negative when the number of negative quantities is an odd integer.

e. Law of division. To divide two signed numbers, find their absolute quotient which is positive when their signs are alike, and negative when their signs are unlike.

1.02. COLLECTIVE SYMBOLS.

The collective symbols, parentheses ( ), brackets [ ], and braces { } are used to segregate groups of related quantities. Thus, a multiplication sign before a collective symbol indicates that the combined value of the quantities within the symbol is multiplied by the preceding quantity. When a collective symbol is preceded by a plus or minus sign, the collective symbol may be removed by changing all the signs within the symbol according to the laws for addition or subtraction. When a collective sign appears within a collective sign, the innermost signs are removed first. For example:

$$a - \{5 + b - [2c - d + (6 - a)]\} =$$
$$a - \{5 + b - [2c - d + 6 - a]\} =$$
$$a - \{5 + b - 2c + d - 6 + a\} =$$
$$a - 5 - b + 2c - d + 6 - a = 1 - b + 2c - d$$

Parentheses are also used to indicate multiplication: $a(a + b) = a \times (a + b)$.

1.03. RULE OF TRANSPOSITION.

a. General. The two members of an equation may be interchanged without changing the value of the unknown. Thus, $4c + 6 - b = 2a$ is identical to $2a = 4c + 6 - b$. The methods of addition and subtraction given in *b* and *c* below are the same as transposing a quantity from one side of the equation to the other by changing the sign of the quantity.

b. Addition. The equality of an equation is not changed by adding the same quantity to both members of the equation:

$$\begin{array}{l} 4c + 6 - b = 2a \\ \quad\quad\quad\;\; b = \;\; b \\ \hline [4c + 6 = 2a + b] \equiv [4c + 6 - b = 2a] \end{array}$$

c. Subtraction. The equality of an equation is not changed by subtracting the same quantity from both sides of the equation:

$$\begin{array}{l} 4c + 6 = 2a + b \\ \underline{-4c \quad\;\; = \quad -4c} \\ [6 = 2a + b - 4c] \equiv [4c + 6 = 2a + b] \end{array}$$

d. Multiplication. The equality of an equation is not changed by multiplying both sides of the equation by the same quantity:

$$\frac{a + b}{c} = x$$

$$\left[c\left(\frac{a + b}{c}\right) = cx\right] \equiv [a + b = cx] \equiv \left[\frac{a + b}{c} = x\right]$$

e. Division. The equality of an equation is not changed by dividing both sides of the equation by the same quantity:

$$ab + c = x$$
$$\left[\frac{ab + c}{a} = \frac{x}{a}\right] \equiv \left[b + \frac{c}{a} = \frac{x}{a}\right] \equiv [ab + c = x]$$

1.04. FUNDAMENTAL OPERATIONS.

a. Addition. To add, similar terms should be algebraically combined, retaining their common factor. For example, when $ax - c + 2ax + 5c - z + d$ is added to $ax + 2c - 2z - 4c - 4$, similiar terms are combined and the sum is:

$$(3ax + 4c - z + d) + (ax - 2c - 2z - 4) =$$
$$3ax + 4c - z + d \;+\; ax - 2c - 2z - 4 \;= 4ax + 2c - 3z + d - 4$$

b. Subtraction. To subtract, similar terms should be combined and subtraction performed according to the law of subtraction. For example, when $a + 5 + b - 4a - 2 + 2c$ is subtracted from $2a - 6 + 7b + 4 + 3c$, similar terms are combined and the difference is:

$$(2a - 2 + 7b + 3c) - (-3a + 3 + b + 2c) =$$
$$2a - 2 + 7b + 3c \;+\; 3a - 3 - b - 2c \;= 5a - 5 + 6b + c$$

c. Multiplication. Multiplication is indicated by a cross (×), a dot (·), parentheses, or the absence of a sign:

$$a \times b = a \cdot b = (a)(b) = ab$$

When two or more numbers, called *factors*, are multiplied, the result is called the *product*. Thus, a, b, and c are the factors of the product abc and the factors may be written in any order:

$$abc = bac = bca = cba = acb = cab$$

(1) To multiply fractions, find the product of the numerators and the product of the denominators separately:

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)\left(\frac{e}{f}\right) = \frac{ace}{bdf}$$

(2) To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial:

$$x(a + b + c) = ax + bx + cx$$

d. Division. Division is indicated by the division sign (÷), an oblique line (/), or by placing the dividend over the divisor separated by a line (–): $6 \div 3$, $6/3$, $\frac{6}{3}$. In division, the number to be divided is the *dividend*, the number by which the dividend is divided is the *divisor*, and the result is the *quotient*.

(1) The division of a number by 1 does not change its value and any number divided by itself is 1.

(2) Division by zero is not possible:

$$\frac{n}{0} = m \quad \text{then } n = m0 \quad \text{but } m \times 0 = 0 \quad \text{and not } n$$

(3) To divide monomials, consider the divisor as a fraction, invert the divisor, and multiply the inverted quantities:

$$6 \div 3 = 6 \div \frac{3}{1} = 6 \times \frac{1}{3} = \frac{6}{3} = 2$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

(4) To divide a polynomial by a monomial, divide each term of the polynomial by the monomial:

$$(3ax - 2ay) \div a = \frac{3ax}{a} - \frac{2ay}{a} = 3x - 2y$$

$$\left(\frac{xy}{4} + \frac{x-y}{8}\right) \div \frac{xy}{4} = \left(\frac{xy}{4} + \frac{x-y}{8}\right)\left(\frac{4}{xy}\right) = \frac{4xy}{4xy} + \frac{4(x-y)}{8xy} = 1 + \frac{x-y}{2xy}$$

(5) The sequence of operations is first to perform the multiplications and divisions in the order in which they occur, and then perform the additions and subtractions in any order. Thus, $4 + 5 \times 2$ is ambiguous in that by multiplying first, the quantity is $4 + 10$; but by adding first, the equation becomes $9 \times 2 = 18$. Note that by use of parentheses, $4 + (5)(2)$, 5 and 2 appear as a single quantity. The word *result* refers to the quantity derived from any mathematical operation or series of operations.

1.05. EXPONENTS.

a. General. An exponent is a term written above and to the right of another quantity, denoting how many times the latter is repeated as a factor: $2^3 = (2)(2)(2) = 8$. It is read as the *power* of the number; thus, 2^3 is read "two to the third power" or simply "the cube of two." A number raised to a power is referred to as an *exponential number*. The exponent 1 is not indicated in that it does not change the value of the quantity: $3^1 = 3$. The *exponent* is called the *power* and the number is called the *base*. Thus, for a^x the exponent is x and the base is a.

b. Addition and subtraction. When adding or subtracting quantities with exponents, the exponent does not change:

$$2a^2 + a^2 = 3a^2 \qquad\qquad 2a^2 - a^2 = a^2$$

The coefficient 1 is understood in the case of a^2, since $1 \times a^2 = a^2$.

c. Multiplication. The first law of exponents states that the exponent of any number in a product equals the sum of its exponents in the factors of the product:

$$x^7 = (x^3)(x^4) \qquad y^{x+2} = (y^x)(y^2)$$

(1) To multiply two like quantities, each raised to some power, the exponents are added and the common quantity is retained:

$$(x^3)(x^2) = (x)(x)(x)(x)(x) = x^5$$

(2) When unlike quantities, each raised to a power, are multiplied, the results of each quantity must first be determined or the quantities must be expressed by a common base:

$$(4^2)(3^3) = (16)(27) \qquad (4^2)(2^2) = (2^4)(2^2) = 2^6$$

(3) To multiply monomials, find the product of their numerical coefficients and multiply this product by the product of the literal figures according to the law of exponents for multiplication:

$$(3x^2y)(-4yz^3) = -12x^2y^2z^3$$

(4) To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial:

$$(x + y - a)(3a) = 3ax + 3ay - 3a^2$$

d. Division. To divide two like quantities, each raised to some power, the exponents are subtracted and the common quantity is retained:

$$\frac{x^5}{x^2} = \frac{(x)(x)(x)(x)(x)}{(x)(x)} = x^{(5-2)} = x^3$$

(1) To divide monomials, find the quotient of their numerical coefficients and multiply the result by the quotient of their literal factors:

$$\frac{-12x^2y^3}{2xy^3} = -6x$$

(2) To divide a polynomial by a monomial, divide each term of the polynomial by the monomial:

$$(6a^3 - 4a^2 - 2a) \div 2a = 3a^2 - 2a - 1$$

e. Negative exponents. A negative exponent denotes division: $3^{-2} = \frac{1}{3^2}$. Hence, a quantity raised to a negative power is the reciprocal of the quantity to a positive power. *The second law of exponents* states that the exponent of any number in a quotient equals its exponent in the dividend minus its exponent in the divisor:

$$\frac{3^2}{3^4} = \frac{1}{3^2} = 3^{-2} \qquad \frac{y^x}{y^z} = y^{x-z}$$

f. Powers. The third law of exponents states that any exponential quantity raised to a power is equal to the quantity raised to the product of the exponent and the power of the exponential quantity:

$$(3^2)^3 = 3^6 \qquad (y^x)^n = y^{(x)(n)}$$

(1) Any quantity, except zero, raised to the zero power is 1; $6^0 = 1$. This relation of the zero power to any quantity is clearly seen by taking the number to the power of 1 and −1:

$$6^1 = 6 \text{ and } 6^{-1} = \frac{1}{6}$$

$$(6^1)(6^{-1}) = 6^{(1-1)} = 6^0 = \frac{6^1}{6^1} = 1$$

$$10^1 = 10,\ 10^{-1} = \frac{1}{10},\ \text{and } 10^0 = 1$$

(2) When a product is raised to a power, each member is raised to that power:

$$(2xy)^4 = 16x^4y^4$$

(3) To find the result of an expression such as $2x^3$, first raise the term to the power indicated, then multiply the quantity by the coefficient:

If $x = 3$, $\quad 2 \times 3^3 = 2 \times 27 = 54$

g. Roots. A root indicates that a quantity is raised to a fractional power: $\sqrt[2]{16} = 16^{\frac{1}{2}}$. *The fourth law of exponents states* that the radicand is expressed as an exponential quantity to a power equal to the exponent divided by the index of the root:

$$\sqrt[2]{4^3} = 4^{\frac{3}{2}} \qquad \sqrt[n]{y^x} = y^{\frac{x}{n}}$$

The symbol indicating a root is a *radical sign* ($\sqrt{\ }$). When a radical sign appears in an expression, the expression is a *radical* ($2\sqrt{5}$), the number above the radical sign is the *index* ($\sqrt[3]{\ }$), and the number under the radical sign is the *radicand* ($\sqrt{7}$). The absence of an index above the radical sign indicates an index of 2, or simply the square root.

(1) To raise a radical to a power, find the root and raise it to the power indicated by applying the third law of exponents:

$$(\sqrt{4^3})^4 = (4^{\frac{3}{2}})^4 = 4^6 \qquad \left(\sqrt[n]{y^x}\right)^a = \left(y^{\frac{x}{n}}\right)^a = y^{\frac{ax}{n}}$$

(2) The root of a root is taken by reducing the dual radical to the power of the radicand:

$$\sqrt{\sqrt[3]{2}} = \left(\sqrt[3]{2}\right)^{\frac{1}{2}} = 2^{\frac{1}{6}}$$

(3) Addition or subtraction of quantities within radicals is possible only when the quantities and indices are similar. Thus, $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$ but $\sqrt[3]{2} + \sqrt{2}$ cannot be further combined without changing their forms. For example:

$$\sqrt{8} + \sqrt{2} = \sqrt{(4)(2)} + \sqrt{2} = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$$

(4) Multiplication can be performed directly when radicals have the same index:

$$(2\sqrt{x})(3\sqrt{y}) = 6\sqrt{xy}$$

When the indices are dissimilar, the radical is changed to exponential form and the product found is returned to radical form:

$$(\sqrt{3})(\sqrt[3]{3}) = (3^{\frac{1}{2}})(3^{\frac{1}{3}}) = 3^{\frac{1}{2}+\frac{1}{3}} = 3^{\frac{5}{6}} = \sqrt[6]{3^5}$$

(5) Division of quantities within a radical sign or of a radical by a radical, can be performed by changing to exponential form:

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{9^{\frac{1}{2}}}{16^{\frac{1}{2}}} = \frac{3}{4}$$

Note that the root of a fraction is equal to the quotient of the roots of the numerator and denominator. Polynomials are divided by rationalizing the denominator:

$$\frac{3}{\sqrt{4}} = \left(\frac{3}{\sqrt{4}}\right)\left(\frac{\sqrt{4}}{\sqrt{4}}\right) = \frac{3\sqrt{4}}{4} = \frac{3 \times 2}{4} = \frac{3}{2} \qquad \frac{a}{\sqrt{b}} = \left(\frac{a}{\sqrt{b}}\right)\left(\frac{\sqrt{b}}{\sqrt{b}}\right) = \frac{a\sqrt{b}}{b}$$

(Since $\frac{\sqrt{4}}{\sqrt{4}} = 1$, and $\frac{\sqrt{b}}{\sqrt{b}} = 1$, multiplication by such a factor does not alter the value of the original quantity.)

To *rationalize* means to reduce an expression to an integer, or quotient of two integers; that is, any whole number or quantity which may be expressed as a whole number is a rational number or quantity:

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right) = \frac{(\sqrt{3})(\sqrt{5}) + (\sqrt{2})(\sqrt{5}) + (\sqrt{3})^2 + (\sqrt{2})(\sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$\frac{\sqrt{15}+\sqrt{10}+3+\sqrt{6}}{5-3} = \frac{3+\sqrt{15}+\sqrt{10}+\sqrt{6}}{2}$$

(6) The *degree* of a radical is the same as its index.

During the radiological defense course, exponents will be used extensively. Such values as Avogadro's number, velocity of light, Planck's constant, and electronic charge are readily expressed as a small number times 10 raised to some power. For example, Planck's constant is

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 006\ 624 \pm \text{ or } 6.6 \times 10^{-27}$$

Multiplication or division in this form is simplified:

$$(6.03 \times 10^{14})(1.01 \times 10^{-16}) = 6.09 \times 10^{-2}$$

$$\frac{6.03 \times 10^{14}}{1.01 \times 10^{-16}} = 5.97 \times 10^{30}$$

When expressing quantities in this form, the decimal point is usually placed after the first significant figure:

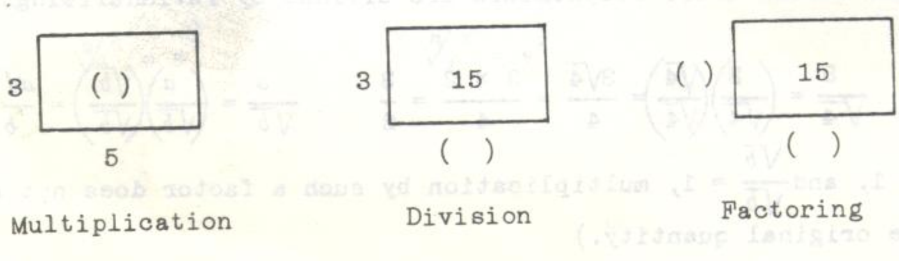
$$6.03 \times 10^{14} = 603{,}000{,}000{,}000{,}000$$

$$1.01 \times 10^{-16} = 0.000\ 000\ 000\ 000\ 000\ 101$$

Note that a positive power of 10 indicates the number is multiplied by 10 to the power indicated and that a negative power of 10 indicates that the number is divided by 10 to the power indicated.

1.06. FACTORING.

a. General. The relationships of multiplication, division, and factoring are shown by the following diagrams:



Multiplication   Division   Factoring

In multiplication, the factors are given and the product is to be found; in division, the product and one factor are given and the other factor is to be found; in factoring, the product is given and the factors are to be found. That is, factoring is the process of finding two or more quantities whose product is the given quantity and the factors may be positive or negative.

$$ab = a \times b$$

$$16 = 4^2 = 2^2 \times 4 = 2^4 = 8 \times 2 \quad (2,4,8,2^2,2^4, \text{ and } 4^2 \text{ are all factors of } 16)$$

A *prime factor* is a factor which is not further factorable into any other factors except itself and 1.

b. Type products. Factoring in elementary algebra depends mainly upon the recognition of certain type products.

(1) Common monomial factor:

$$ax + ay = a(x + y)$$

(2) Difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$

(3) Trinomial square:

$$a^2 \pm 2ab + b^2 = (a \pm b)^2$$

(4) Trinomial of the form:

$$x^2 + ax + bx + ab = x^2 + (a + b)x + ab = (x + a)(x + b)$$

(5) Trinomial of the form:

$$ax^2 + bx + c$$

Certain expressions of this form can be factored by inspection:

$$2x^2 + 5x - 3 = (2x - 1)(x + 3)$$

(6) Sum of two cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

(7) Difference of two cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

(8) Factors found by grouping:

$$ax + ay + bx + by = (a + b)(x + y)$$

This method of expressing a ratio in formulas is very convenient as,

$$A = \pi r^2 \text{ or } A = Kr^2$$

Another example of the use of a ratio is that of showing relationships such as distance, time, and rate:

$$\frac{\text{distance}}{\text{time}} = \text{rate}$$

(2) Ratios may be expressed in any one of four forms, as follows:

$$x = ry$$

$$\frac{x}{y} = r$$

$$\frac{x}{r} = y$$

$$\frac{1}{r} = \frac{y}{x}$$

A ratio may also be indicated by a colon as $x:y$.

Note that the law of division precludes the use of $\frac{0}{0}$ as a ratio.

b. Proportion. A proportion is a statement of the equality of two fractions or ratios:

$$\frac{x}{y} = \frac{a}{b}$$

Any two of a series of ratios, as, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{7}{14} = \frac{18}{36}$ constitute a proportion, as: $\frac{1}{2} = \frac{3}{6}$, $\frac{2}{4} = \frac{18}{36}$, and $\frac{18}{36} = \frac{3}{6}$.

(1) The rule of proportion states that the cross products are equal: $\frac{1}{2} = \frac{4}{8}$ or $1 \times 8 = 2 \times 4$. When using the form $x:y$, the rule states that the product of outer terms (extremes) equals the product of the inner terms (means): $1:2 :: 4:8$ or $1 \times 8 = 2 \times 4$.

(2) When applied to proportions, the K form is a valuable aid to simplification. For example, the pendulum formula, $l = Kt^2$, expressed without the K form, is

$$\frac{l}{l_1} = \frac{t^2}{t_1^2} \qquad \text{where } \frac{l_1}{t_1^2} \text{ is a constant}$$

Note: A proportion expresses the relation between two pairs of values, while the K form expresses the relation between the variables for any values.

(3) It may be observed that the product of corresponding terms of two or more proportions are themselves in proportion:

When $\frac{a}{b} = \frac{c}{d}$ and $\frac{e}{f} = \frac{g}{h}$, then $\frac{ae}{bf} = \frac{cg}{dh}$.

(4) The relationships of ratios and proportions are summarized below.

If $\frac{a}{b} = \frac{c}{d}$, then:

$ad = bc$	by multiplication
$\frac{a}{c} = \frac{b}{d}$	by alteration
$\frac{b}{a} = \frac{d}{c}$	by inversion
$\frac{a+b}{b} = \frac{c+d}{d}$	by composition
$\frac{a-b}{b} = \frac{c-d}{d}$	by division
$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	by composition and division

c. Variation. Variation means the amount or rate of change. The equation $x \propto y$ is read "x varies directly as y." The variation sign $\propto$ represents both the equality sign and the constant K (called the constant of variation). A *variable* is a literal number which has any one of several arithmetical values during a particular discussion. The variable (or variables) which changes first is the *independent variable* and the one whose change is caused by a change in the independent variable (or variables) is the *dependent variable*.

(1) Variation is expressed by the relationships below.

Constant variation	$x = Ky$	"x varies as y" and "x is proportional to y" or "x varies directly as y" and "x is directly proportional to y"
Inverse variation	$x = \frac{K}{y}$	"x varies inversely as y" and "x is inversely proportional to y"
Joint variation	$z = Kxy$	"z varies jointly as x and y"
Combined variation	$z = K\frac{x}{y}$	"z varies directly as x and inversely as y"

(2) Proportional variation may be expressed in six ways:

By the constant of variation

By proportion

By the word *proportional*

By the word *ratio*

By the *variation sign*

By the words *vary as*

d. Function. Function is discussed here, not as a part of higher mathematics, but as an aid in further clarification of variation. By definition, *if two variables are so related that when a value of one is given, a corresponding value of the other is determined, the second variable is called a function of the first.* In the equation $2x - 3y = 4$, y is a function of x; by this method, reference to x is made by "function of x" instead of "expression involving the variable x." "Function of x" is referred to as $f(x)$.

Note: To say that x is dependent on y is the same as saying x is a function of y:

$$x = f(y)$$

1.08. Equations.

a. General. An equation is an expression of equality between two quantities or operations: $2x + y = 0$ or $a + b = x - y$. The two parts, separated by the equality sign, are called the *sides* or *members* of the equation. A second definition is "An equation is an equality that asks a question." Algebraic equations contain unknown quantities represented by letter symbols and to solve an equation for these unknowns is to find its root or roots. The solving of the equation is the process of determining values for the unknown which will satisfy the equation. The degree of an equation corresponds to the value of the highest exponent appearing in the equation when all exponential quantities in the denominator or under a radical sign have been removed. As methods of solving simple equations have been discussed under fundamental operations, only additional methods of solution will be considered in this section.

b. Linear equation. A linear equation is one of the first degree, so-called because its graph is a straight line (see par. 3.02). When the unknown quantities are expressed as a relationship between two linear equations, the values of the unknowns may be determined by solving the combined equations. This is called solving a system of equations.

A system of two equations in two unknowns is solved by making the values of one unknown equivalent in both equations, and then by addition or subtraction of the system, eliminating one unknown. The value of the other unknown is determined by substituting the value found in either of the equations and solving for the other unknown. For example, solve the system $2x - y = 2$ and $x + 3y = 15$.

$$
\begin{aligned}
2x + 6y &= 30 \qquad \equiv (x + 3y = 15)(2) \\
2x - y &= 2 \\
\hline
7y &= 28 \\
y &= 4 \\
2x - 4 &= 2 \qquad \text{(substituting value of } y\text{)} \\
2x &= 6 \\
x &= 3
\end{aligned}
$$

c. *Quadratic equations.* A quadratic equation is one of the second degree. A quadratic equation in one unknown has two roots, both real or both complex. A quadratic equation may be solved by factoring (par. 1.06), by completing the square, or by formula.

(1) Solution by completing the square is carried out as indicated by the following example for $2x^2 + x - 3 = 0$. First transpose the terms not containing the unknown to the right side of the equation, $2x^2 + x = 3$, and divide the equation by the coefficient of the term containing the square of the unknown, $x^2 + \frac{1}{2}x = \frac{3}{2}$; then complete the square by adding the square of half the coefficient of the term containing the unknown to the first power:

$$x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 = \frac{3}{2} + \left(\frac{1}{4}\right)^2$$

$$\left(x + \frac{1}{4}\right)^2 = \left(\frac{5}{4}\right)^2$$

The left side has both positive and negative roots: $x + \frac{1}{4} = \pm\left(\frac{5}{4}\right)$ and $x = 1$ or $x = -\frac{3}{2}$.

(2) Solution by formula is accomplished by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

in which a is the coefficient of the term with the unknown to the second power, b is the coefficient of the term with the unknown to the first power, and c is the term in which the unknown does not appear. For example, in solving equation

$$2x^2 + x - 3 = 0,\ x = \frac{-1 \pm \sqrt{1^2 - (4 \times 2)(-3)}}{(2 \times 2)} = \frac{-1 \pm \sqrt{25}}{4} = \frac{-1 \pm 5}{4} = 1 \text{ or } -\frac{3}{2}.$$

(3) The nature of the roots can be determined by the radicand of the formula as follows:

When $b^2 - 4ac > 0$,	the roots are real and unequal
When $b^2 - 4ac = 0$,	the roots are real and equal
When $b^2 - 4ac < 0$,	the roots are complex and unequal
When $b^2 - 4ac$ is {a positive perfect square or zero}	the roots are rational
When $b = 0$,	{the roots are equal in absolute value, but opposite in sign
When $c = 0$,	one root is zero

1.09. PROBLEMS.

a. $E = mc^2$

$E = 1.5 \times 10^5$

$c = 3 \times 10^{10}$ Ans: $m = 1.67 \times 10^{-16}$

b. $E = \frac{1}{2}mv^2$

$m = 9 \times 10^{-28}$

$v = 3 \times 10^8$ Ans: $E = 4.05 \times 10^{-11}$

c. $E = \frac{hc}{\lambda}$

$E = 1.6 \times 10^{-9}$

$h = 6.6 \times 10^{-27}$

$c = 3 \times 10^{10}$ Ans: $\lambda = 1.24 \times 10^{-7}$

d. $m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$

$m_0 = 9 \times 10^{-28}$

$v = 2.9 \times 10^{10}$

$c = 3 \times 10^{10}$ Ans: $m = 3.5 \times 10^{-27}$

e. $I = \frac{0.001 \times 24 \times M}{s^2}$

$M = 246.66$

$I = 0.3$ Ans: $s = 4.44$

f. $R = \frac{E \times 0.526 - 0.094}{0.0013}$

$E = 1.8$ Ans: $R = 656$

g. $It^2 = K$

$I = 1.2$

$K = 120$ Ans: $t = 10$

h. $t = \frac{s}{60v}$

$s = 9.3 \times 10^7$

$v = 1.86 \times 10^5$ Ans: $t = 8.33$

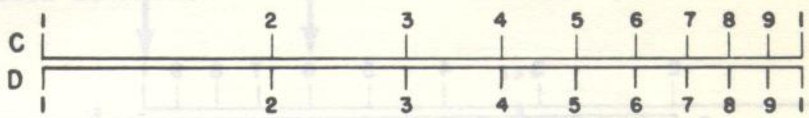
i. $2\pi \times 10^{-8} = \frac{1 \times 10^9}{R}$ Ans: $R = 1.59 \times 10^{16}$

2.10. REVIEW OF THE SLIDE RULE.

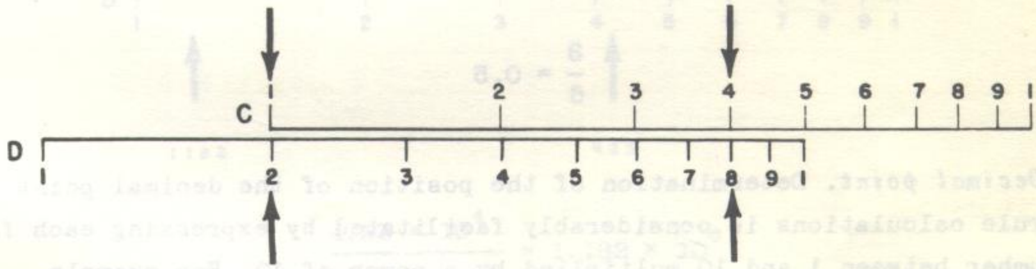
This paragraph on the use of the slide rule is merely a review of some of the more important rules applicable to the solution of problems in this course. It is not intended as a complete course of instruction.

Multiplication and division with the slide rule are accomplished by use of the *C* and *D* scales, each of which is identically scaled in logarithmic lengths. Thus, multiplication and division are actually the physical addition and subtraction of the logarithms of the numbers on the slide rule. Squares and square roots are found by use of the *A* and *D* scales; cubes and cube roots, by use of the *K* and *D* scales.

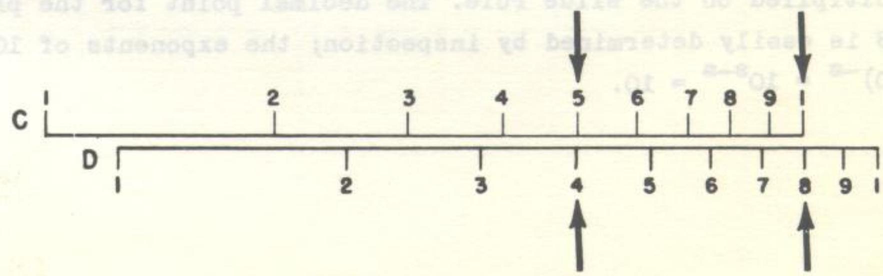
a. Multiplication. To multiply two numbers, place either the right or left index of the *C* scale opposite the first factor on the *D* scale; and read the result on *D* directly under the second factor on *C* using the hairline on the indicator. (Use the index which keeps the second factor on *C* within the range of *D*.)



SLIDE RULE

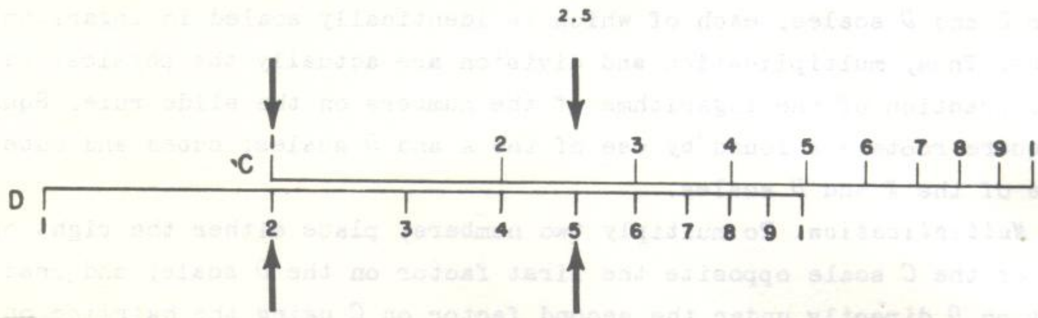


2 × 4 = 8

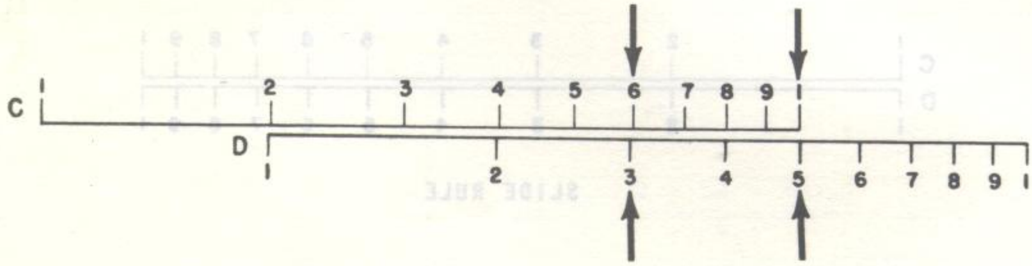


5 × 8 = 40

b. Division. Division with the slide rule is the reverse of multiplication. To divide one number by another, place the hairline of the indicator over the dividend on *D*, move the divisor on *C* to the hairline, and directly under the index of *C* read the quotient on *D*.

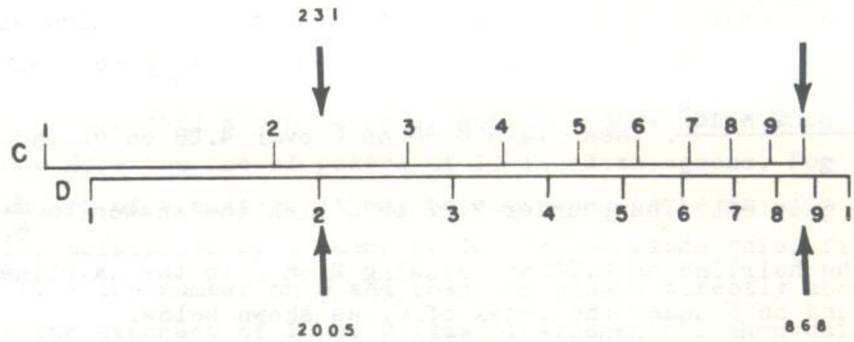


$$\frac{5}{2.5} = 2$$



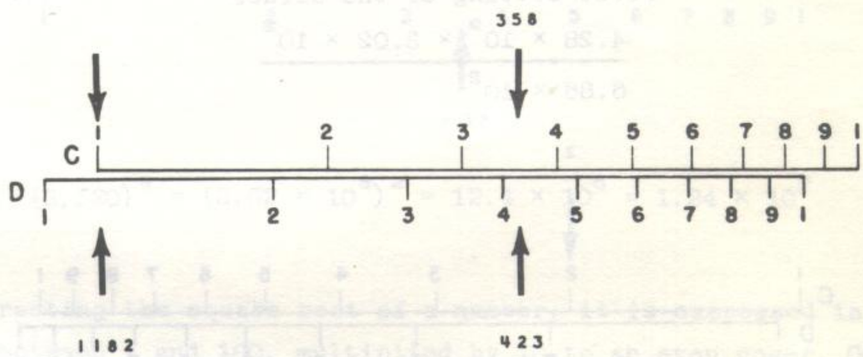
$$\frac{3}{6} = 0.5$$

c. Decimal point. Determination of the position of the decimal point in slide-rule calculations is considerably facilitated by expressing each factor as a number between 1 and 10 multiplied by a power of 10. For example, $420 = 4.2 \times 10^2$. Similarly, if 8,682.3 is to be multiplied by 0.0231, the two factors may be expressed as 8.68×10^3 and 2.31×10^{-2}. The significant figures are then multiplied on the slide rule. The decimal point for the product of 2.31×8.68 is easily determined by inspection; the exponents of 10 are added: $(10)^3 \times (10)^{-2} = 10^{3-2} = 10$.



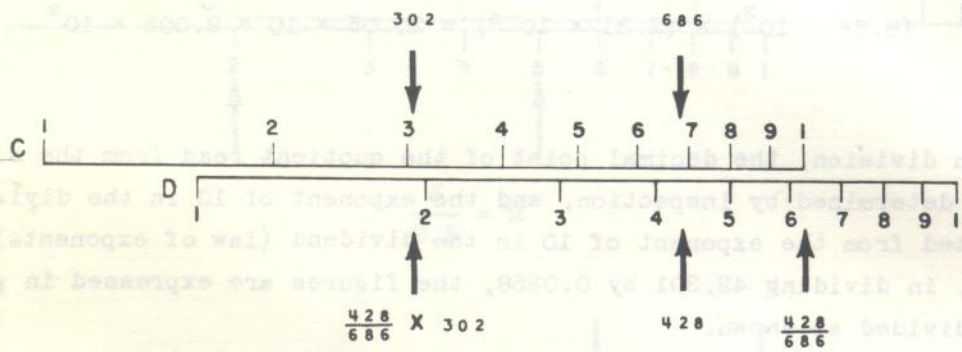
$$(8.68 \times 10^3) \times (2.31 \times 10^{-2}) = 20.05 \times 10 = 2.005 \times 10^2$$

In division, the decimal point of the quotient read from the slide rule is also determined by inspection, and the exponent of 10 in the divisor is subtracted from the exponent of 10 in the dividend (law of exponents). For example, in dividing 42,301 by 0.0358, the figures are expressed in powers of 10 and divided as shown.



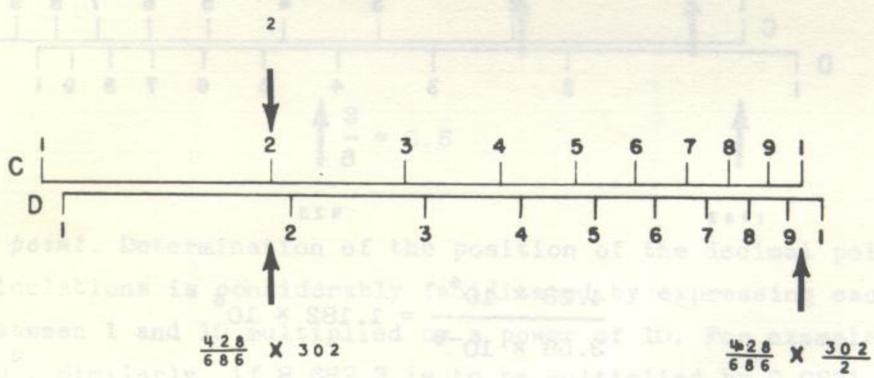
$$\frac{4.23 \times 10^4}{3.58 \times 10^{-2}} = 1.182 \times 10^6$$

d. Multiple operations. Any number of factors can be handled with the slide rule. For example, to calculate $\frac{428 \times 302}{686 \times 2}$, first express as $\frac{4.28 \times 10^2 \times 3.02 \times 10^2}{6.86 \times 10^2 \times 2}$, then place 6.86 on *C* over 4.28 on *D*. The index is now over the quotient. Thus, under 3.02 on *C* lies the answer to $\frac{4.28}{6.86} \times 3.02$ By placing the hairline on 3.02 and drawing 2 on *C* to the hairline, the final answer is found on *D* under the index of *C*, as shown below.



First setting of the slider

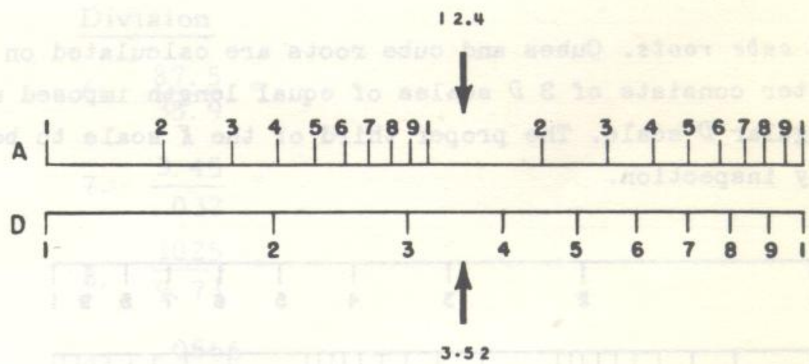
$$\frac{4.28 \times 10^2 \times 3.02 \times 10^2}{6.86 \times 10^2}$$



Second setting of the slider

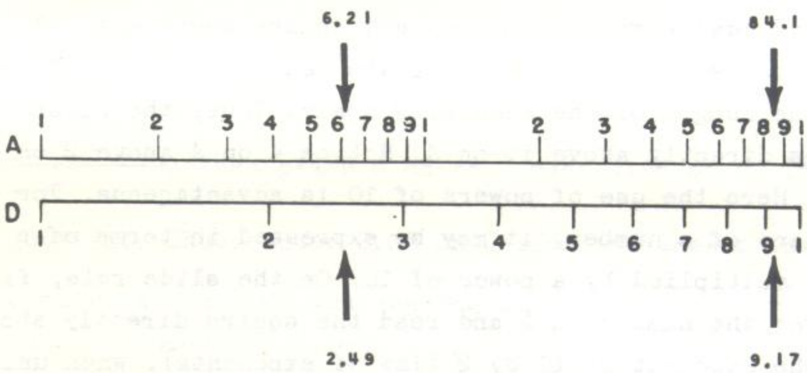
$$\frac{4.28 \times 10^2 \times 3.02 \times 10^2}{6.86 \times 10^2 \times 2} = 0.944 \times 10^2 = 9.44 \times 10^1 = 94.4$$

e. Squares and square roots. Squares and square roots are calculated by use of the *A* and *D* scales. Note that the *A* scale really consists of two *D* scales, each one-half the length of the regular *D* scale. Thus, the square of every number on *D* lies directly above it on *A*. Notice 4 on *A* above 2 on *D*; and 9 on *A* above 3 on *D*. Here the use of powers of 10 is advantageous. For example, in finding the square of a number, it may be expressed in terms of a number between 1 and 10, multiplied by a power of 10. On the slide rule, first place the hairline over the number on *D* and read the square directly above on *A*. Then multiply the exponent of 10 by 2 (law of exponents). When using this exponential method with squares and square roots, the *D* scale represents numbers between 1 and 10 and the *A* scale represents numbers between 1 and 100. For example, $(3{,}520)^2$ is calculated below.



$$(3{,}520)^2 = (3.52 \times 10^3)^2 = 12.4 \times 10^6 = 1.24 \times 10^7$$

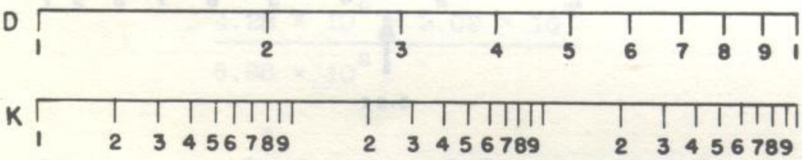
In extracting the square root of a number, it is expressed in the form of a number between 1 and 100, multiplied by 10 to an even power. On the slide rule, first place the hairline over the number on *A* and read its root on *D*. Then divide the exponent of 10 by 2 (law of exponents). For example, to extract the square root of 124,000, express it as 12.4×10^4. Under 12.4 on *A*, read 3.52 on *D*. Divide the exponent of 10 by 2. The answer is 3.52×10^2. To extract the square root of decimal fractions, the method is the same.



$$\sqrt{0.0000841} = \sqrt{84.1 \times 10^{-6}} = 9.17 \times 10^{-3}$$

$$\sqrt{0.000621} = \sqrt{6.21 \times 10^{-4}} = 2.49 \times 10^{-2}$$

f. Cubes and cube roots. Cubes and cube roots are calculated on the *D* and *K* scales. The latter consists of 3 *D* scales of equal length imposed upon the range of the regular *D* scale. The proper third of the *K* scale to be used can be determined by inspection.



In finding the cube of a number, it is expressed in the form of a number between 1 and 10, multiplied by a power of 10. For example, to find the cube of 86,810, express it as 8.68×10^4. Under 8.68 on *D*, read 654 on *K*. Multiply the exponent of 10 by 3 (law of exponents). The answer is 654×10^{12} or 6.54×10^{14}

In finding a cube root, the process is the reverse of that above. For example, in extracting the cube root of a number, it is expressed as a number between 1 and 1,000, multiplied by 10 to a power whose exponent is a multiple of 3 (the exponent must later be divided by 3). For example, to extract the cube root of 86,400,000, express it as 86.4×10^6, and below 86.4 on *K*, read 4.42. Then the exponent of 10 is divided by 3. The answer is 4.42×10^2.

2.11 SLIDE RULE EXERCISE

Multiplication

1. $2.45 \times 31 =$

2. $345 \times 3.46 =$

3. $972 \times 0.45 =$

4. $1.035 \times .081 =$

5. $23.1 \times 1.03 =$

Division

6. $\frac{87.5}{35.9} =$

7. $\frac{3.45}{.032} =$

8. $\frac{1025}{9.71} =$

9. $\frac{.0566}{5.47} =$

10. $\frac{3.42}{3.27} =$

Proportions

Determine the value of the unknown quantities in each of the following.

11. $\frac{Y}{6.73} = \frac{81}{109}$

12. $Y = \frac{(14)(0.787)}{3.45}$

13. $\frac{X}{2.81} = \frac{3.92}{5.41} = \frac{4.32}{Z} = \frac{Y}{8.92}$

14. $407 = \frac{71.2X}{48.3}$

Squares and Square Roots

Perform following operations and square answers:

15. $\frac{3.67 \times 7.34}{15.89} =$

16. $\frac{79.67 \times 3.45}{5.35} =$

17. $\frac{5.81 \times 9.89}{689.7} =$

Find square roots of each of following numbers:

18. 3, 30, 785, 78.5, 9.8, 98

Cubes and Cube Roots

19. $\pi (63.2)^3 =$

20. $\sqrt[3]{63.2}\ (\pi) =$

21. $7.81 (2.31)^3 =$

22. $\sqrt[3]{0.0785} =$

Answers to above exercises

1.	76.0	12.	Y = 3.19
2.	1194	13.	X = 2.04; Z = 5.96; Y = 6.46
3.	437	14.	X = 277
4.	0.0838	15.	2.87
5.	23.8	16.	2640
6.	2.44	17.	0.00694
7.	107.8	18.	1.732, 5.48, 28.0, 8.86, 3.13, 9.90
8.	105.6	19.	794,000
9.	.01035	20.	12.52
10.	1.046	21.	96.3
11.	Y = 5.01	22.	0.428