

to the left, or to the right, must be carefully observed, and the divisions of the scale followed until the exact position is reached. For example, in the illustration of the rule, the cursor line is standing at 1.748 in LU, at 31.2 in A, at 37.7 in B, at 6.15 in C, at 5.6 in D, and at 270 in LL.

**MULTIPLICATION** is effected by using scale C in conjunction with scale D. Supposing multiplication of 15 and 45 is desired, the procedure is: move the slide so that the 1 on C is brought opposite 15 on D, and read the answer 675 in scale D, opposite 45 in scale C. In some cases when the 1 of scale C is used, the answer is off the scale, and the 10 of scale C must be used instead of the 1. For example, if  $25 \times 45$  is to be computed, the procedure is: set the 10 of C opposite 25 in D and, coincident with 45 in C, the result, 1125, will be found in D. Scales A and B may be used for multiplication if desired. The result will always be on the scale, and the slight delay occasioned by the double setting avoided. It is for this reason that the upper pair of scales is sometimes employed in multiplication, but greater accuracy will always be obtained when scales C and D are used, and their use in multiplication and division generally is strongly recommended.

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that repeated multiplication of the four numbers of the numerator, followed by division separately by the three numbers of the denominator, will give the result, but time is saved by dividing and multiplying alternately. Using scales C and D, find 862 on D, and bring 225 on C into coincidence; adjust X to 18 in C, then move slide to bring 8 on C under X; move X into position above 49 in C and adjust slide so that 1145 in C lies under X; read the answer, 627, in D opposite 17 in C. Approximate cancellation of the numbers will fix the position of decimal point in the answer. 22.5 divides 86.2 approximately 4, and the 4 thus obtained divides 11.45 nearly 3, which leaves 6 in the numerator when divided into 18; .8 into 1.7 gives roughly 2, and the result is approximately  $6 \times 2 \times .05 = 12 \times .05 = .6$ . The answer, therefore, is .627.

Frequently the position of decimal point may be determined without resorting to the approximation indicated above, e.g., suppose the fraction  $\frac{51.9}{69.7}$  is desired as a percentage. Using the slide rule to divide 519 by 697, the result, 745, obtained, is obviously 74.5 per cent.

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sum of the number of digits preceding the decimal points of the original numbers. When dividing, if the answer appears opposite 1 in C the number of digits preceding the decimal point of the answer is one greater than the difference obtained by subtracting the number of digits lying before the decimal point of the divisor from the number of digits appearing before the decimal point of the dividend, but if the answer is found opposite 10 in C the number of digits preceding its decimal point is the same as the difference between the numbers of digits appearing before the decimal points of dividend and divisor respectively. When the numbers to be multiplied together or divided are of values less than unity, the number of ciphers immediately following the decimal points must be taken into account and reckoned as negative in the application of the rules for fixing the position of decimal point in the answer.

**SQUARES.** Numbers may be squared by multiplication direct, but results are more readily obtained by reading in scale A the square of numbers directly opposite in scale D, the cursor, or preferably the slide, being used to project from one scale to the other.

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**CUBE ROOTS.** Find the number whose cube root is required in scale A and place X over it. Move the slide until the number in scale B, directly under X, is exactly the same as that in D opposite 1 or 10 in C. There will be three positions of the slide satisfying these conditions, and care must be taken to select, by inspection, the one giving the correct value.

Squares, square roots, cubes and cube roots may also be evaluated with the aid of the log-log scale, sometimes with a higher degree of accuracy than is possible with scales A, B, C and D.

**LOGARITHMS.** The common log of any number is obtained by finding the number in the LU or LL scale, placing X over it and moving the slide so that the number 2.303 in C lies under X. The log will then be found in D opposite either 1 or 10 of C. Suppose the common log of 18.75 is required: place X over 18.75 in LL; and move the slide until 2.303 in C lies under X. The log 1.273 appears in D opposite 1 in C. Alternatively, the slide may be adjusted so that the 1 or 10 of scale C lies immediately above or below the number in LL or LU, which is base of the system of logs employed (in the case of common logs

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To find the value of  $\frac{82}{3.6 \times .78}$  find 82 in D and bring 36 in C into coincidence. Then opposite 78 in R find the result 29.2 in D, the decimal point being inserted by inspection.

### LOG-LOG COMPUTATIONS

The tenth powers of all numbers in LU lie immediately below in LL, and the tenth roots of numbers in LL lie directly above in LU.

- Examples (a)  $1.8^{10} = 357$   
 (b)  $12^{10} = (1.2 \times 10)^{10} = (1.2^{10}) \times (10^{10}) = 6.2 \times 10^{10}$   
 (c)  $10\sqrt{50} = 1.48$   
 (d)  $10\sqrt[10]{2} = 10\sqrt[10]{\frac{20}{10}} = \frac{1.35}{1.259} = 1.072$

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THE arithmetical operations of multiplication and division occur frequently in practical calculations. Very often the work involved would be tedious if effected by the ordinary rules of arithmetic, and a great saving in time would result from the use of some mechanical means of computing.

The slide rule has been designed with this end in view, and with its aid results sufficiently accurate for most practical purposes may be readily obtained. Compared with ordinary or contracted methods of multiplication and division, or with the use of logarithms, computation by slide rule is less laborious, less liable to error, and very much more expeditious.

An illustration of the 10 in. 'UNIQUE' Log-Log Slide Rule is given on pp. 12 and 13 and an inspection of the rule itself shows that the essential parts consist of four scales, denoted for reference in the illustration by the letters A, B, C and D, and a log-log scale running along the top and bottom edges, denoted by LU and LL. A transparent cursor with a fine line drawn across it is supplied to assist in certain operations. The cursor (indicator) index is referred to as X in the instructions which follow.

The scales are in most cases divided in decimals, and practice in reading them may be necessary. It is obviously quite impossible to number every line, and in reading a value in any scale the nearest number

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in the example above the answer is 1125, but the manipulation of the slide rule would be exactly the same in the multiplication of any two numbers in which two five and four five are the only significant figures; for example,  $25 \times 45 = 1125$ ;  $2.5 \times 45 = 112.5$ ;  $.25 \times 4.5 = 1.125$ ;  $.025 \times .45 = .01125$ . The position of the decimal point in the answer is easily determined by inspection. When three or more numbers are to be multiplied together the computation is effected by a series of operations, X being used to mark the intermediate answers until the final result is reached.

**DIVISION.** Set the slide so that the divisor on scale C is coincident with the dividend on scale D. The result will be found in D opposite 1 or 10 in C.

For example, suppose it is desired to divide 13.9 by 5.65. Adjust the slide so that 565 in scale C is coincident with 139 in scale D. Opposite 10 in C will be found the result in D, viz., 246. Inserting the decimal point, the result 2.46 is obtained.

In computing the value of an expression such as the following,  $\frac{86.2 \times .049 \times 18 \times 1.7}{22.5 \times 11.45 \times .8}$ , it is evident

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Those using the slide rule for the first time are advised to master the operations of multiplication and division, as explained above, before reading any further. Practice with simple numbers giving results easily checked is recommended, e.g., using scales C and D evaluate  $\frac{2 \times 12 \times 6}{4 \times 9}$  and see if the answer is 4. Now repeat, taking the numbers in a different order, and see if the result is the same. Take note of the time saved by dividing and multiplying alternately, as described in the example given earlier. Half an hour spent on similar simple examples will suffice to teach the use of the rule for the fundamental operations of multiplication and division.

The following rules, based upon the manipulation of the slide rule, are sometimes used to fix the decimal point, but their use is not recommended. In multiplication, when the 1 of scale C is used in the setting of the slide, the number of digits occurring before the decimal point of the answer is one less than the sum of numbers of digits appearing before the decimal points of the original numbers. When the 10 of scale C is used in setting, the number of digits before the decimal point of the answer is the same as the

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The calculation of the area of a circle from the diameter is a computation often desired. Find the number representing diameter on D and bring the 1 or 10 of scale C into coincidence with it. The answer appears in A opposite the value of 785 in B.

**SQUARE ROOTS.** The square roots of all numbers in scale A appear directly below in scale D. Since, however, any number appears twice in scale A, care is necessary in selecting the one to be used. The rule is: If the original number has an odd number of digits preceding its decimal point, or, when less than unity, has an odd number of ciphers immediately following its decimal point, the left-hand half of scale A must be used. When the number of digits preceding, or the ciphers immediately following the decimal point in the original number is even, the right-hand half of scale A must be used.

**CUBES** of numbers may be found by repeated multiplication, or more quickly by moving the 1 or 10 of scale C into coincidence with the number to be cubed in D, and reading the answer in A directly opposite the original number in B.

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the number 10). The logs of all numbers in LU and LL will then be found by using the cursor to project into scale C. By these methods the complete logarithm, characteristic and mantissa, is obtained. In certain models there is a gauge mark, denoted by U, at 2.303 in C to assist in finding common logs.

### RECIPROCAL SCALE

Certain types of Log-Log Rules are equipped with a reversed C scale (subsequently referred to as the R scale) placed along the middle of the slide. The uses of this scale are indicated in the following examples: Reciprocals are obtained by projecting, from C to R or vice versa, e.g., 4 in C projects into .25, i.e.  $\frac{1}{4}$ , in R.

**Multiplication and Division.** To compute the value of an expression such as  $2.8 \times 3.2 \times 6.5$ , find 2.8 in scale D, then with the aid of the cursor bring 3.2 in R into coincidence and read the result, 58.2 in D, opposite 6.5 in C, with one setting of the slide. The factors may be selected in any order and the operations repeated, if necessary, to cover any number of factors.

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**NATURAL LOGARITHMS** may be obtained by reading opposite the number whose logarithm is desired in LU or LL, the logarithm in D.

Examples (e)  $\log_e 9 = 2.2$  (f)  $\log_e 1.5 = .405$

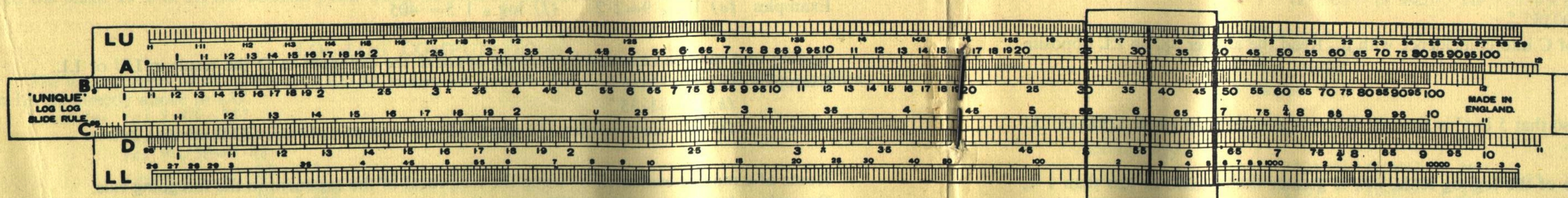
Powers of e may be obtained by reading opposite the exponent in D the result in LU or LL.

- Examples (g)  $e^4 = 54.6$   
 (h)  $e^{-3} = 1.35$   
 (i)  $e^{12} = (e^4)^3 =$  from (g) above  $(54.6)^3 = (5.46 \times 10)^3 = 162000$  see (k) below.

Roots of e may be evaluated by using the reciprocal of the exponent in the foregoing rule.

Example (j)  ${}^3\sqrt{e} = e^{.333} = 1.395$

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All 'UNIQUE' slide rules have the basic A, B, C and D scales.

Some models also have scales for reciprocals and trigonometrical ratios.

The most useful purpose which the log-log scale serves is computing powers and roots when exponents are fractional.

Example (k) To evaluate  $6 \cdot 4^3 \cdot 2^1$

Set X over 6.4 in LL and bring 1 of C into coincidence with it. Read the answer 387 in LL opposite 3.21 in C, again using X.

Example (l) To evaluate  $5\sqrt[3]{30}$

Place X over 30 in LL, move slide so that 5 in C is brought under X and read the result 1.973 in LU opposite 10 in C, again using X.

If the answer lies outside the range of the log-log scale, i.e., is greater than 40,000 or less than 1.1, it may be found as indicated below.

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If the exponent is negative, proceed as with a positive exponent and then find the reciprocal of the result.

Find the value of \$860 after  $6\frac{1}{2}$  years, compound interest at 5% per annum being allowed.

First calculate for \$1 capital.

\$1 at the end of one year becomes  $(1 + .05) = \$1.05$ . At the end of two years \$1 becomes  $\$1.05(1 + .05) = \$1.1025$ , and so on.

At the end of  $6\frac{1}{2}$  years, \$1 at 5% compound interest becomes  $\$(1.05)^{6.5}$   
Proceeding as at (k) above.

$$\$(1.05)^{6.5} = \left(\frac{2.1}{2}\right)^{6.5} = \frac{\$124}{90.5} = \$1.372$$

Now multiply by 860.  $860 \times 1.372 = \$1180$ , which is very near the correct amount.

The low reading log-log scale on the reverse of the slide of the 5 in. Universal rule (Code U1/1) extends from .00001 to .99. It reads in increasing values from right to left. With the slide inverted, calculations

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$$\begin{aligned} \text{E.g.} - \sin 3^\circ 10' &= .0552 \\ \sin 20^\circ 40' &= .353 \end{aligned}$$

Cosines of angles are obtained by finding the sines of the complementary angles.

$$\text{E.g.} - \cos 36^\circ = \sin 54^\circ = .809$$

Tangents of angles are obtained by projecting from the T scale into the D scale, using the cursor. The tangent scale starts just below  $6^\circ$  and finishes at  $45^\circ$  and all values lie between .1 and 1.

$$\text{E.g.} - \tan 27^\circ 30' = .521$$

Tangents of angles between  $45^\circ$  and  $90^\circ$  are obtained easily by finding the reciprocal of the tangent of the complementary angles.

$$\text{E.g.} - \tan 72^\circ = \frac{1}{\tan 18^\circ} = 3.078$$

In this case,  $18^\circ$  in the T scale is projected into the R scale, with the slide in its central position.

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### NAVIGATIONAL RULE

This rule has S and T scales on both the slide and the body. When much trigonometrical work has to be included, such as in navigational calculations, this rule is probably the best obtainable. It is literally unique, and can be obtained only from the manufacturers of these rules.

Detailed instructions are not given here for the Universal II and the Navigational Rules, since the variety of work is extensive and it is assumed that users of these rules will be familiar with their operation. (A special technical instruction is available for the Navigational Rule.)

### 5/10 and 10/20 PRECISION RULES

It is assumed that users of these Rules are familiar with the slide rule in its ordinary form.

Slight modifications of the instructions given above are necessary. The log-log instructions do not apply to the precision type of rule, which has no log-log scale.

The C and D scales, which are twice the usual length, are divided into two parts and occupy the positions of the A, B, C and D scales in the standard rule.

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and the lower figures in the central log scale refer to logs of numbers in the lower C scale. The vernier may be used to read the fourth figure of the mantissa if desired. To use the vernier, move the slide into its central position, i.e., with the ends of the C and D scale coincident. Place X over the number whose logarithm is desired, and read the first three figures of the mantissa directly from the log scale. Move the slide to the right, so that the line in the log scale immediately to the left is brought exactly under X, and now read the vernier to obtain the fourth figure of the mantissa.

Example.—To find the log of 485. Adjust slide to mid-position. Place X over 485, which lies in the lower parts of the C and D scales, and read the log scale, viz., .6840. Move slide so that the line immediately to the left of cursor is brought under X and note that the vernier reading is 17. Add .6840 to .0017 and so obtain the correct mantissa .6857. Now add the characteristic 2 and complete the logarithm, 2.6857.

N.B.—The log scale is omitted from the 10/20 Rule, which is designed for great accuracy in ordinary calculations.

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### NOTES

Example (m) Evaluate  $2 \cdot 1^{20}$

$$\begin{aligned} (2 \cdot 1)^{20} &= (2 \cdot 14)^5 = (19 \cdot 45)^5 \text{ see (k)} \\ (19 \cdot 45)^5 &= (1 \cdot 945)^5 \times 10^5 = 27 \cdot 9 \times 10^5 \text{ see (k)} \\ &= 2,790,000 \end{aligned}$$

$$\begin{aligned} (2 \cdot 1)^{20} &= (2 \cdot 110)^2 = (1670)^2 \text{ see (a)} \\ (1670)^2 &= (1 \cdot 670)^2 \times (1000)^2 \\ &= 2 \cdot 79 \times (1000)^2 \\ &= 2,790,000 \end{aligned}$$

Example (n) Evaluate  $1 \cdot 2^{13}$

Alternatively

$$1 \cdot 2^{13} = \left(\frac{12}{10}\right)^{13} = \frac{(12)^{13}}{(10)^{13}} = \frac{1 \cdot 38}{1 \cdot 35} = 1 \cdot 023$$

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similar to those above may be effected, and a little practice will ensure familiarity with this useful scale, seldom seen even in expensive slide rules.

SINES, COSINES, TANGENTS. The table on the back of the Log-Log and 5/10 Rules gives values of Sines, Cosines, Tangents and Cotangents, of all angles. Values should be taken from the table and used in computations when necessary.

### UNIVERSAL RULES

In addition to the scales of the Log-Log Rule, universal rules are equipped with Sine and Tangent scales, denoted by S and T respectively, for trigonometrical calculations.

Sines of angles are found by using the cursor to project from the S scale to the A scale. If the result lies between 1 and 10 of A, the decimal point and a cipher precede the number found in scale A. If the result lies between 10 and 100 in A, the decimal point only should be prefixed.

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Cosecants, Secants and Cotangents are found as the reciprocals of sines, cosines and tangents respectively. When sin or tan terms appear as factors, the cursor is used in conjunction with the appropriate angles in the S or T scales.

Example.—The sides of a triangle are 3.5 and 7.2 feet long. The included angle is  $25^\circ$ . The area is required. Place X over 25 in S. Move slide to bring 20 of B under X. Now move X to 3.5 in B. Bring 10 of B under X and read the result: 5.33 sq. feet in A, opposite 7.2 in B.

### UNIVERSAL II RULE

This rule has the S and T scales on the slide instead of on the body. For certain calculations this arrangement of scales is more convenient than that of the Universal I Rule, since it allows of multiplication or division of any series of numbers and functions of angles.

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Multiplication and Division are effected by the C and D scales, but since numbers on either edge of the slide cannot be brought into direct coincidence with numbers on the opposite side of the body the cursor must be used in setting the slide in such cases. Otherwise the manipulation of the rule is similar to that of the standard type and any uncertainty in reading the result may be avoided if it is remembered that, if in setting the rule it is necessary to use the cursor to cross the slide, it will be necessary to cross the slide again when reading the result.

A little practice with simple examples will overcome any initial difficulty.

Squares and Square Roots are obtained easily by projecting from the C and D scales to the scales lying on the edges of the stock.

Logarithms (5/10 Rule only). The mantissa of the logarithm of any number is found by projecting from the C scale to the evenly divided scale lying in the centre of the slide. The figures along the top of this scale are used when the number whose logarithm is required appears in the upper part of the C scale,

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### NOTES