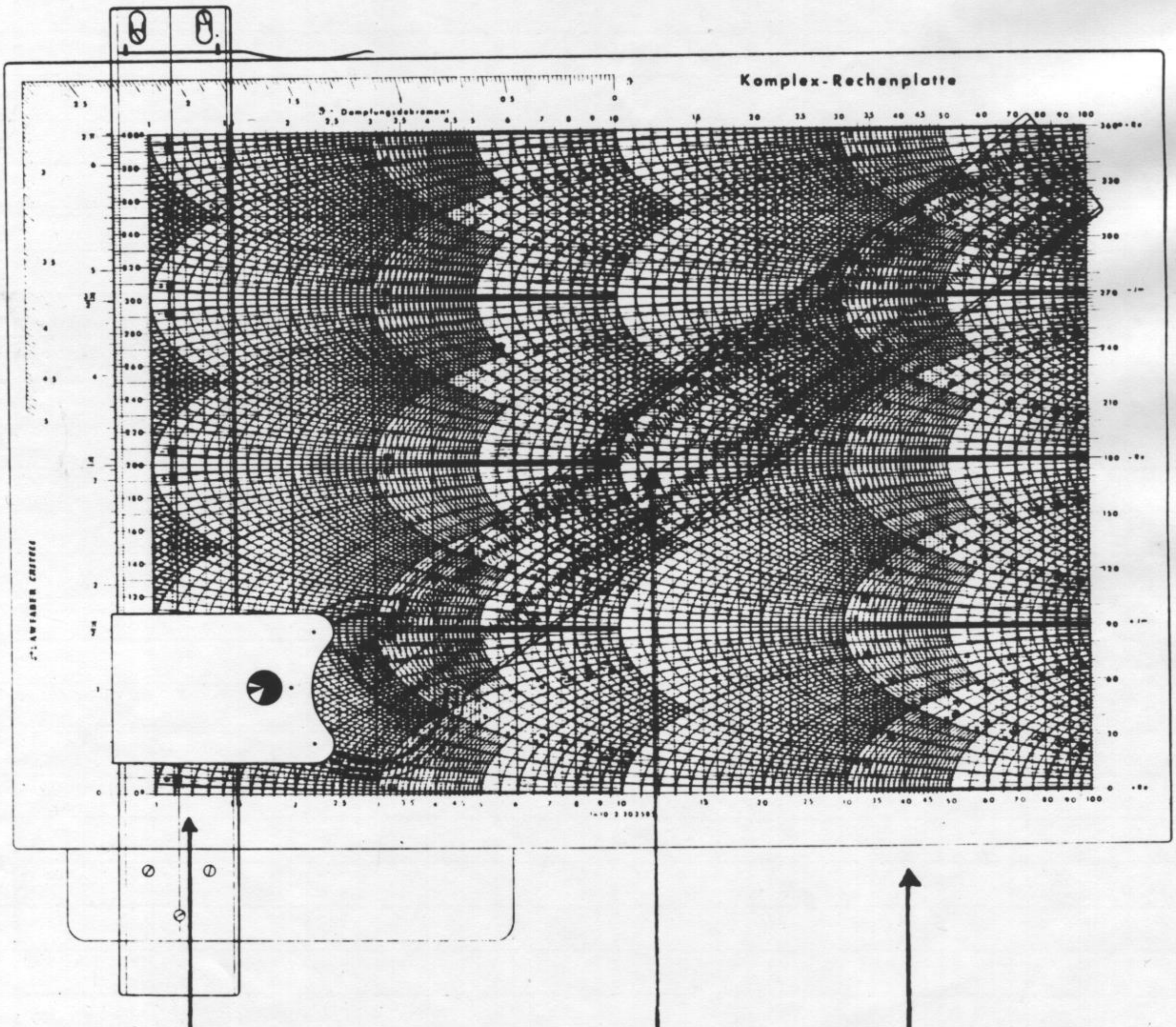
FABER-CASTELL

INSTRUCTIONS

for use of

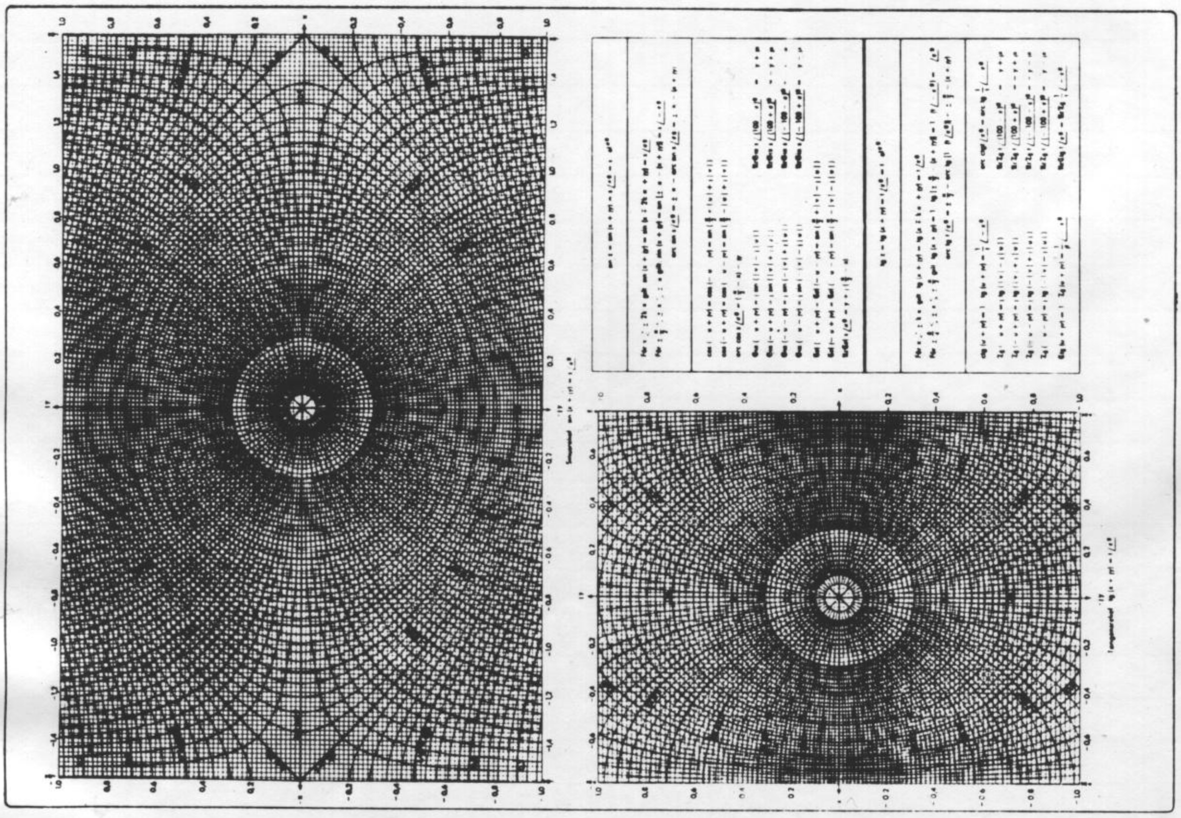
CASTELL COMPLEX CALCULATOR

No. 989



A. W. FABER-CASTELL, STEIN BEI NURNBERG

(GERMANY)



IMPORTANT NOTES

1. Although the single relief diagrams are based upon new degrees (400g), the angles may also be read off in 360 degrees (360°) from the narrow edge to the right.

2. The "special type symbols" in chapters 15 e - g and 15m - p are to be interpreted as follows:

𝔖𝔦𝔫 =	sinh	(hyperbolic sine)
ℭ𝔬𝔰 =	cosh	(hyperbolic cosine)
𝔗𝔤 =	tanh	(hyperbolic tangent)
ℭ𝔱𝔤 =	cotanh	(hyperbolic cotangent)

𝔄𝔯 𝔖𝔦𝔫 = read = " area sinus hyperbolicus "

etc. for 𝔄𝔯 𝔗𝔤 etc.

COMPLEX CALCULATOR.

Castell No. 989.

INSTRUCTIONS FOR USE.

A. W. FABER - CASTELL · STEIN BEI NÜRNBERG

THE COMPLEX CALCULATOR.

The system in which complex numbers are represented by points in the "Gauss Plane" is now widely used both in pure mathematics and in the technical sciences. In electrical engineering, in particular, the method of calculation based on complex numbers has become absolutely indispensable in drawing the vectors for alternating current values. The complex plane of numbers, however, is nothing more than a medium of representation and is hardly suitable for carrying out practical calculations. The use of the formula

$$\ln (a_1 + ja_2) = \ln (|a|\ e^{j\varphi}) = \ln |a| + j\varphi$$

enables complex numbers to be suitably indicated in a semi-logarithmic system of coordinates. In converting the complex plane of numbers into the semi-logarithmic system we obtain the basis for a practical medium of calculation, which at the same time has the merit of remaining "synoptical".

On this apparatus, known briefly as the COMPLEX CALCULATOR, the aforementioned semi-logarithmic system of co-ordinates is shown in red on a base plate. In accordance with $\ln |a| + j\varphi$ the angle (versor) appears in linear form as the ordinate, while the absolute (vector) value is marked logarithmically as the abscissa. The converted lines from the Gauss plane of numbers are marked in as groupings of curves on this basic grid, the real components being shown in black and the imaginary components in blue. The total "calculation surface" contains all four quadrants (with the first in the lower and the fourth in the upper zone). A special pivoted arm or scale, of which the angle can be adjusted, enables multiplication and division to be carried out and powers or roots to be extracted on the Complex Calculator, in the same way as with the ordinary Slide Rule. As the values can be analysed into their normal components in a simple manner, addition and subtraction can also be effected without difficulty.

Since the Complex Calculator contains all four quadrants, thus providing for the whole angular frequency, both harmonic and exponentially damped oscillations can be represented by straight lines and adequately dealt with in the calculations.

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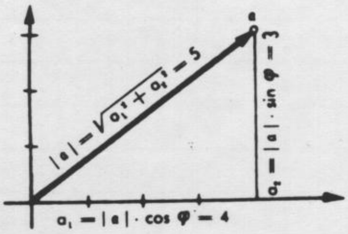
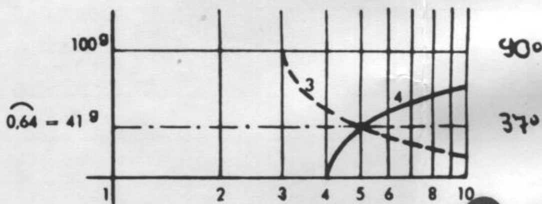
1. Representation of Complex Numbers.

Gauss Plane of Numbers



Complex Calculator

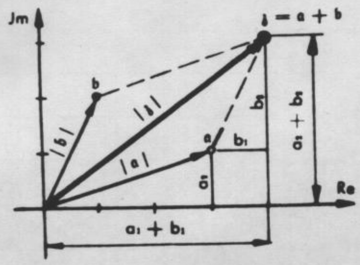


$$= (a_1 + j\,a_2) = (4 + j\,3) = 5\,e^{j0,64} = 5\underline{/41^g} = 5\underline{/37^o}$$

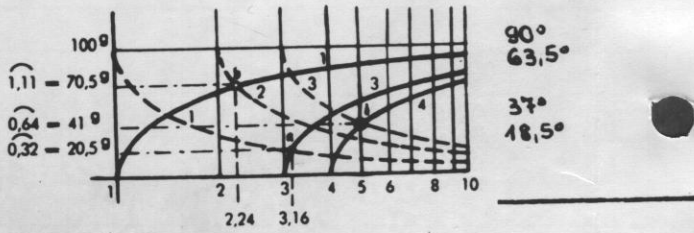
The Complex Calculator indicates - for complex numbers - the relationship between absolute (vector) value and angle (versor) and also between real component and imaginary component.

2. Addition of Complex Numbers.

Gauss Plane of Numbers.



Complex Calculator.



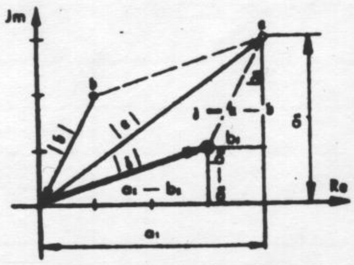
$$a + b = |a|\,e^{j\varphi} + |b|\,e^{j\psi} = a\underline{/\varphi} + b\underline{/\psi} = (a_1 + ja_2) + (b_1 + jb_2) = (a_1 + b_1) + j(a_2 + b_2)$$

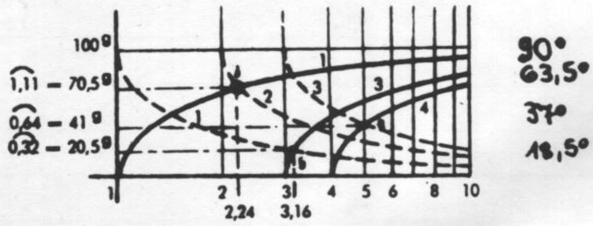
$$3{,}16\,e^{j0,32} + 2{,}24\,e^{j1,11} = 3{,}16\underline{/20{,}5^g} + 2{,}24\underline{/70{,}5^g} = 3{,}16\underline{/18{,}5^o} + 2{,}24\underline{/63{,}5^o} = (3+j1) + (1+j2) = 4+j3) = 5\,e^{j0,64} = 5\underline{/41^g} = 5\underline{/37^o}$$

As the Calculator provides readings of the real and imaginary components of each of these complex numbers, the said components need only be combined and the new point traced; vector value and versor can also be read off at the same time.

3. Subtraction of Complex Numbers

Gauss Plane of Numbers   Complex Calculator





$$a - b = |a|\, e^{j\varphi} - |b|\, e^{j\psi} = a\underline{/\varphi} - b\underline{/\psi} = (a_1 + ja_2) - (b_1 + jb_2) = (a_1 - b_1) + j\,(a_2 - b_2)$$

$5\ e^{j0,64} - 3,16\ e^{j0,32} = 5\underline{/41^g} - 3,16\underline{/20,5^g} = 5\underline{/37^o} - 3,16\underline{/18,5^o}$
$= (4 + j3) - (3 + j1) = (1 + j2) = 2,24\ e^{j1,11} = 2,24\underline{/70,5^g}$
$= 2,24\underline{/63,5^o}$

This is an analogous process to that involved in the addition, except that here the components are subtracted.

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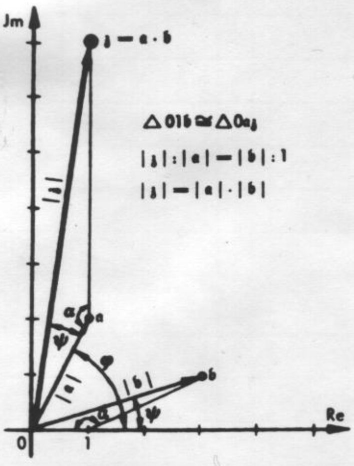
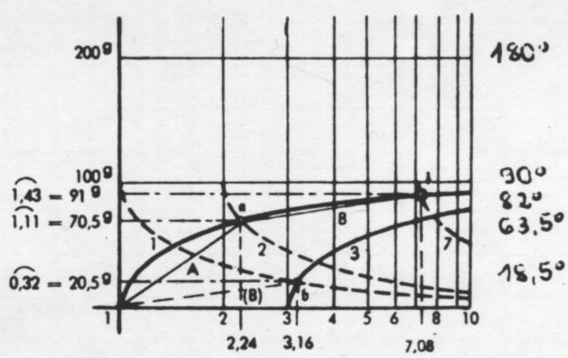
4. Multiplication of Complex Numbers

Gauss Plane of Numbers   Complex Calculator.





$$a \cdot b = |a|\, e^{j\varphi} \cdot |b|\, e^{j\psi} = a\underline{/\varphi} \cdot b\underline{/\psi} = (a_1 + ja_2) \cdot (b_1 + jb_2) = a_1 b_1 - a_2 b_2 + j(a_1 b_2 + a_2 b_1) = |a| \cdot |b|\, e^{j(\varphi+\psi)} = ab\underline{/\varphi+\psi}$$

$2,24\ e^{j1,11} \cdot 3,16\ e^{j0,32} = 2,24\underline{/70,5^g} \cdot 3,16\underline{/20,5^g} = 2,24\underline{/63,5^o} \cdot$
$3,16\underline{/18,5} = (1 + j2) \cdot (3 + j1) = (1 + j7) = 7,08\underline{/91^g} = 7,08\ e^{j1,43}$
$= 7,08/82^o$

When The two series A and B are geometrically added, with the aid of the adjustable pivoting scale, the point   is found immediately, the result being $7,08\underline{/91^g} = 7,08\underline{/82^o} = (1 + j7)$ $= 7,08\ e^{j1,43}$.

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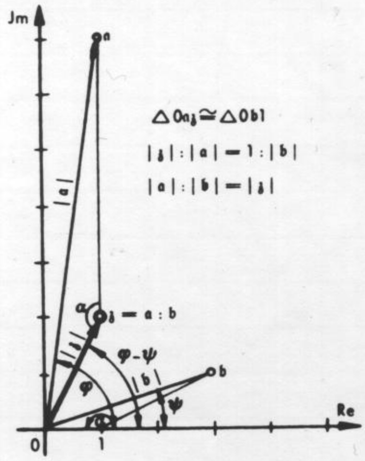
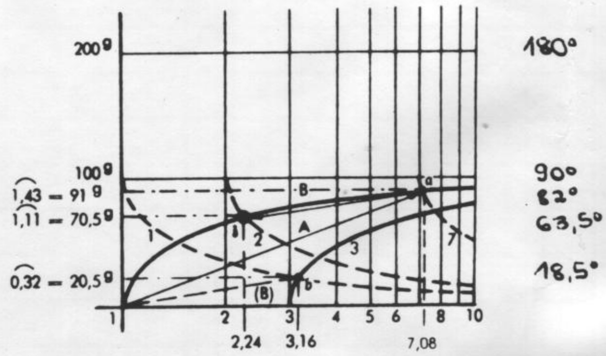
5. Division of Complex Numbers

Gauss Plane of Numbers



Complex Calculator



$$a : b = |a|\, e^{j\varphi} : |b|\, e^{j\psi} = a\underline{/\varphi} : b\underline{/\psi} = (a_1 + ja_2) : (b_1 + jb_2)$$

$7{,}08\ e^{j1{,}43} : 3{,}16\ e^{j0{,}32} = 7{,}08\underline{/91^g} : 3{,}16\underline{/2015^g} = 7{,}08\underline{/82^o} : 3{,}16\underline{/18{,}5^o} = (1 + j7) : (3 + j1) = (1 + j2) = 2{,}24\underline{/70{,}5^g} = 2{,}24\ e^{j1{,}11} = 2{,}24\underline{/63{,}5^o}$

When series B is subtracted geometrically from series A, with the aid of the adjustable pivoting scale, the point is found, with the values $2{,}24\underline{/70{,}5^g} = 2{,}24\underline{/63{,}5^o} = (1 + j2) = 2{,}24\ e^{j1{,}11}$ as the result.

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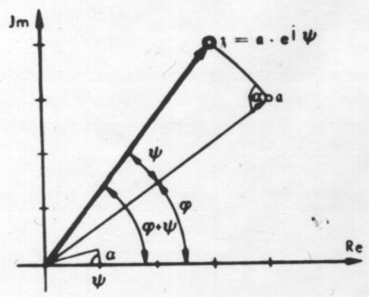
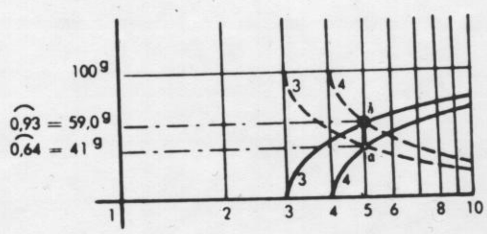
6. Multiplication or Division by $e^{j\psi}$

Gauss Plane of Numbers.



Complex Calculator.



$$a \cdot e^{j\psi} = |a|\, e^{j\varphi} \cdot e^{j\psi} = a\underline{/\varphi} \cdot 1\underline{/\psi} = |a|\, e^{j(\varphi+\psi)} = a\underline{/\varphi+\psi}$$

$5\ e^{j0{,}64} \cdot e^{j0{,}29} = 5\underline{/41^g} \cdot 1\underline{/18^g} = 5\underline{/37^o} \cdot 1\underline{/16^o} = 5\ e^{j0{,}93} = 5\underline{/59{,}0^g} = 5\underline{/53^o}$

$(4 + j3) \cdot (0{,}97 + j0{,}29) = (3{,}88 - 9{,}88) + j(1{,}1 + 2{,}9) = 3 + j4$

The anti-clockwise representation of the rotation of the vector a about the angle ψ in the Gauss plane of numbers -- corresponding to multiplication by the unit vector $1/\psi$ -- is shown on the Calculator as a mere upward vertical displacement of point a, reaching the ordinate $\varphi + \psi$ at point z.

The operations corresponding to the division are then found to be, in the Gauss plane, a clockwise rotation of the vector , and on the Calculator, a downward vertical displacement.

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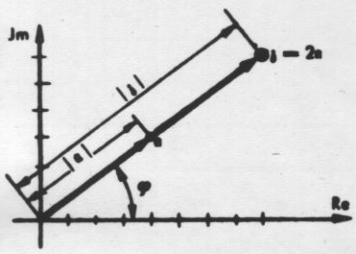
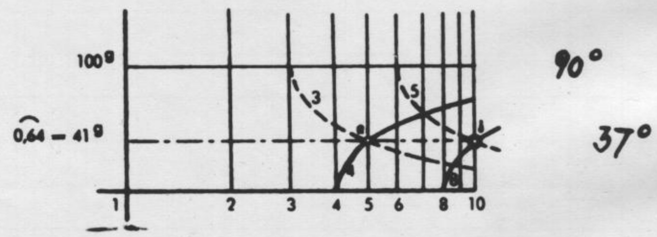
7. Multiplication or Division of a Complex Number by a Real Number.

Gauss Plane of Numbers.



Complex Calculator.



$$a \cdot p = p \cdot |a|\, e^{j\varphi} = p \cdot a/\underline{\varphi} = (a_1 + ja_2) \cdot p = (pa_1 + jpa_2)$$

$$2 \cdot 5\, e^{j0,64} = 2 \cdot 5/\underline{41^g} = (4 + j3) \cdot 2 = 10\, e^{j0,64} = 10/\underline{41^g} = (8 + j6)$$

The multiplication of a complex number a by a real number p, involves an increase of the vector value to p times its original value, the angle φ being retained, and on the Complex Calculator it takes the form of a horizontal displacement of the point to the right, by the distance d, to the abscissa value at point z.

Correspondingly, the division involves a reduction of the vector value to the "p-th" part of its original value, the angle φ being retained, and on the Complex Calculator it takes the form of a horizontal displacement of the point z towards the left, by the distance , to the abscissa value $|z|/p$ at point a.

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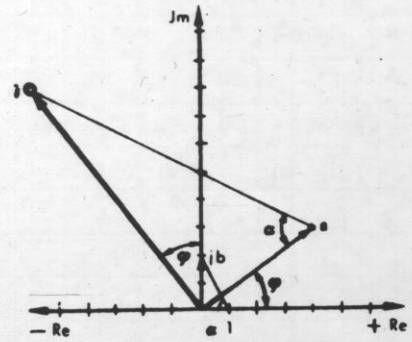
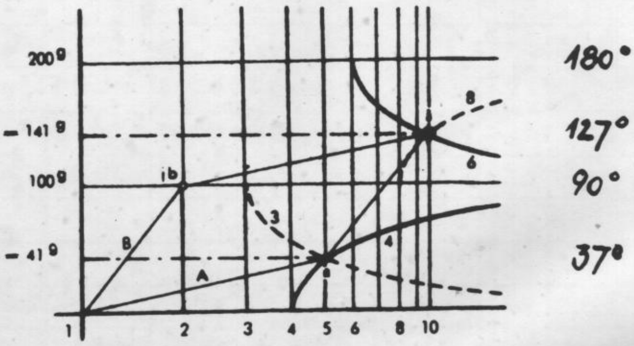
8. Multiplication or Division of a Complex Number by an Imaginary Number

Gauss Plane of Numbers



Complex Calculator.



$e^{j\pi/2} = j = 1\underline{/100^g} = 1\underline{/90^o}$; $e^{j\pi/2} = -1 = 1\underline{/200^g} = 1\underline{/180^o}$, $e^{-j\pi/2} =$
$-j = 1\underline{/300^g} = 1\underline{/270^o}$ etc.

$$a \cdot jb = |a| e^{j\varphi} \cdot b\, e^{j\pi/2} = a\underline{/\varphi} \cdot b/100^g = (a_1 + ja_2) \cdot jb = (-a_2 b + ja_1 b) = |a| \cdot b\, e^{j(\varphi + \pi/2)} = ab\underline{/\varphi + 100^g}$$

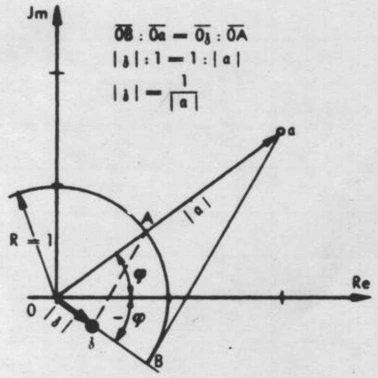
$$5\, e^{j0,64} \cdot 2\, e^{j\pi/2} = 5/41^g \cdot 2/100^g = (4 + j3) \cdot j2 = 10\, e^{j2,21} = (-6 + j8) = \underline{10/141^g}$$

The operation carried out on the Complex Calculator corresponds exactly to that involved in the multiplication of complex numbers, i. e. to the geometrical addition of the distances A and B.

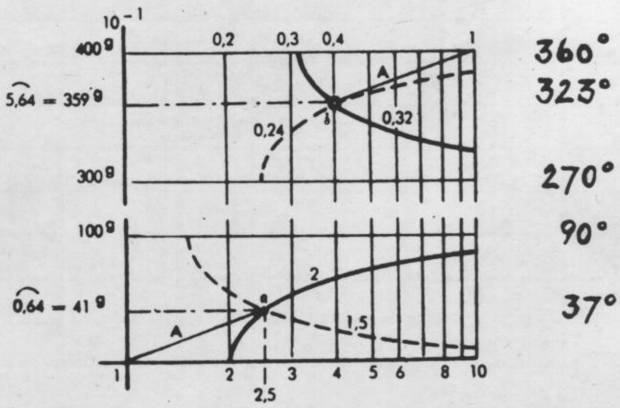
The division takes the form of a geometrical subtraction (as in the case of the division of complex numbers).

9. The Reciprocal of a Complex Number.

Gauss Plane of Numbers.



Complex Calculator.



$$\frac{1}{a} = \frac{1}{|a|\, e^{j\varphi}} = \frac{1}{a\underline{/\varphi}} = \frac{1}{a_1 + ja_2} = \frac{a_1 - ja_2}{a_1^2 + a_2^2} = \frac{a_1}{a_1^2 + a_2^2} - j\frac{a_2}{a_1^2 + a_2^2}$$

$$\frac{1}{2,5\, e^{j0,64}} = \frac{1}{2,5\underline{/41^g}} = \frac{1}{2 + j1,5} = \frac{2}{6,25} - j\frac{1,5}{6,25} = 0,32 - j0,24 = 0,4\underline{/-41^g} = 0,4\underline{/359^g} = 0,4\, e^{j5,64}$$

The operation fo forming the reciprocal of a complex number corresponds to the division of 1 by the value in question, and is carried out on the Complex Calculator by a geometrical subtraction from point 1. It must be borne in mind, however, that the 4th quadrant of the preceding period is not shown on the plate and must be replaced by the 4th quadrant to be conceived of as displaced towards the left by one decimal place (or by two where necessary).

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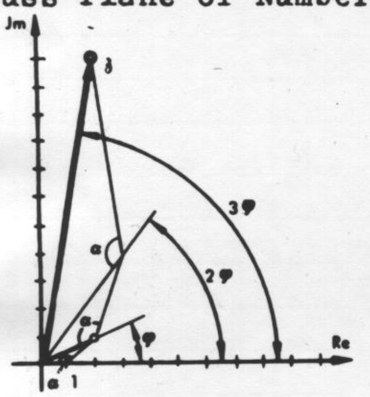
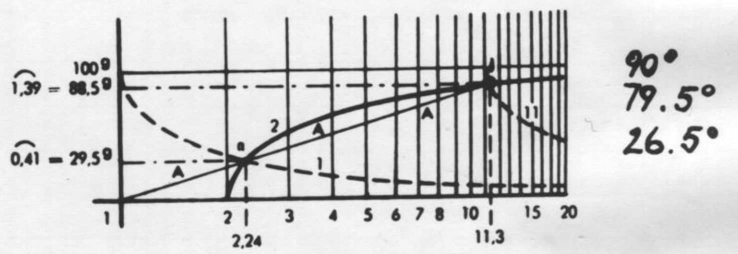
10. Raising a Complex Number to a Given Power.

$$a^n = (|a|\, e^{j\varphi})^n = (a/\underline{\varphi})^n = (a_1+ja_2)^n = |a|^n \cdot e^{jn\varphi} = a^n\,\underline{/n\varphi} = z\underline{/\psi}$$

$$(2{,}24\, e^{j0{,}46})^3 = (2{,}24/\underline{29{,}5^g})^3 = (2+j1)^3 = 2+j11 = 11{,}3 e^{j1{,}39} = 11{,}3\,\underline{/88{,}5^g}$$

Gauss Plane of Numbers. Complex Calculator.





Powers are found on the Complex Calculator by placing distances A by side the number of times corresponding to the exponent. For this purpose it is of advantage to employ a linear graduation (e. g. the In scale) on the pivoting scale.

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11. To Find a given Root of a Complex Number.

The method for the extraction of roots emerges from the preceding diagram, as a reversal of that employed for raising a number to a given power The division of the distance A -- measured with one of the scales of the pivoting scale -- should preferably be carried out, as a subsideary calculation, with an ordinary slide rule.

$$\sqrt[n]{\mathfrak{z}} = \sqrt[n]{|\mathfrak{z}| \cdot e^{j\psi}} = \sqrt[n]{z\underline{/\psi}} = \sqrt[n]{z_1+jz_2} = \sqrt[n]{|\mathfrak{z}|} \cdot e^{j\frac{\psi}{n}} = \sqrt[n]{z}\,\underline{/\psi/n} = a/\varphi = |a|\, e^{j\varphi} = a$$

12. Finding the Logarithms of Complex Numbers.

$\ln \underline{a} = \ln(|a|\, e^{j\varphi}) = \ln|a| + j\varphi + j2k\pi$ wherein $k = 0; \pm 1; \pm 2 \ldots$

We have: $\ln j = j\,\pi/2;\ \ln -1 = j\pi;\ \ln(-j) = j\,\frac{3\pi}{2}$.

$\mathrm{In}(4+j3) = \mathrm{In}(e^{j0,64}) = 1,61 + j\,0,64 = 1,74\underline{/24,18} = 1,74\underline{/27,7^\circ}$
$= 1,74\ e^{j0,377}$

The natural logarithm of the absolute value a can immediately be read off from the top of the pivoting scale, by the aid of the In scale (linear graduation up to 4.6).

The Complex Calculator thus also enables powers and roots to be found with fractional exponents.

$(4+j3)^{1,2} = (5e^{j0,64})^{1,2} = \mathfrak{z};\ \ln\mathfrak{z} = 1,2 \cdot \ln(5e^{j0,64}) = 1,2\,(1,61+j0,64)$
$\ln\mathfrak{z} = 1,93+j0,77;\ \mathfrak{z} = 6,9\ e^{j0.77} = 6,9/\underline{49,0^\circ} = 4,95+j4,81$

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13. The Harmonic Oscillation.

A line parallel to the axis of the ordinates represents a rotating and constant vector -- and thus a cosine wave when the intersecting points of the black curves in the system are referred to, or a sine wave where the readings are based on the points of intersection on the blue lines. This method of representation, at the same time, provides a clear picture of the interrelationship of the two plane oscillations of the same applitude, which are perpendicular to each other and which are completed to form a circular oscillation.

The two lower scales on the adjustable pivoted scale -- which correspond to the graduations of the ordinates -- also provide a helpful means of representing leading and lagging oscillations. In this case the pivoting scale is placed parallel to the axis of the ordinates and then moved in the appropriate direction (according ro whether we are dealing with a lead or a lag).

By adding together the individual values we can determine both added and multiplied superimposed oscillations.

---

14. The Exponentially Damped Oscillation.

A line which is inclined in respect of the axis of the ordinates represents -- on the Complex Calculator -- an exponentially damped oscillation with a damping decrement of ϑ governed by the angle of inclination. To set the Complex Calculator to a damped oscillation of this kind, the adjustable pivoting scale or arm, together with its guide, is detached from the vertical rule; it is then turned over and replaced in such a manner that the hinged scale is then to be found to the left of the vertical slide. The adjustable pivot-point is then in the vicinity of the lower edge of the Calculator. The pivoted arm should now be so adjusted that its centre line coincides with the "10" of the lower basic scale for real values and with the required damping decrement of the scale on the upper edge of the Calculator. The centre line should be displaced so that it comes to rest over the initial vector value R; readings can then be taken of all indermediate values from 0 to 2π -- both as vector value and versor and as real and imaginary components. If further readings of the damped oscillation are required beyond the first period, all that is necessary is for the vector value $r_{2\pi}$ obtained at 2π to be transferred to the lower basic scale, the centre line of the adjustable pivoted scale then being set to this new value. The continued application of this process enables the oscillations to be ascertained to any extent desired -- of necessary until the damping is completely terminated.

For the exponenteally damped oscillation we have the formula:

$$Y = e^{-\frac{\varphi}{2\pi}} \cdot R\,(\cos\varphi + j\sin\varphi) \quad \text{or} \quad Y = e^{-\frac{\omega t}{2\pi}} \cdot R\left(\cos(\omega t) + j\sin(\omega t)\right)$$

We also have:

$$r = e^{-\frac{\varphi}{2\pi}} \cdot R \text{ and } x = r \cdot \cos\varphi \text{ or } y = r \cdot j\sin\varphi$$

A further illustration is now given in connection with an exponentially damped oscillation with an initial vector value R of 10 and a damping decrement ϑ of 1:

In this case we have the following equations:

For $\varphi = 2\pi$

$$Y = e^{-1} \cdot 10\,(\cos 2\pi + j\sin 2\pi)$$
$$Y = 10/e \cdot (1 + j0) = 10/e = 3{,}68$$

for $\varphi = 4\pi$

$$Y = e^{-2} \cdot 10\,(\cos 4\pi + j\sin 4\pi)$$
$$Y = 10/e^2 = 10/7{,}39 = 1{,}35$$

The damping decrement is as follows:

$$\vartheta = \ln\frac{R}{r_{2\pi}} = \ln\frac{r_{2\pi}}{r_{4\pi}} = \ldots = \ln\frac{10}{3{,}68} = \ln\frac{3{,}68}{1{,}35} = 1$$

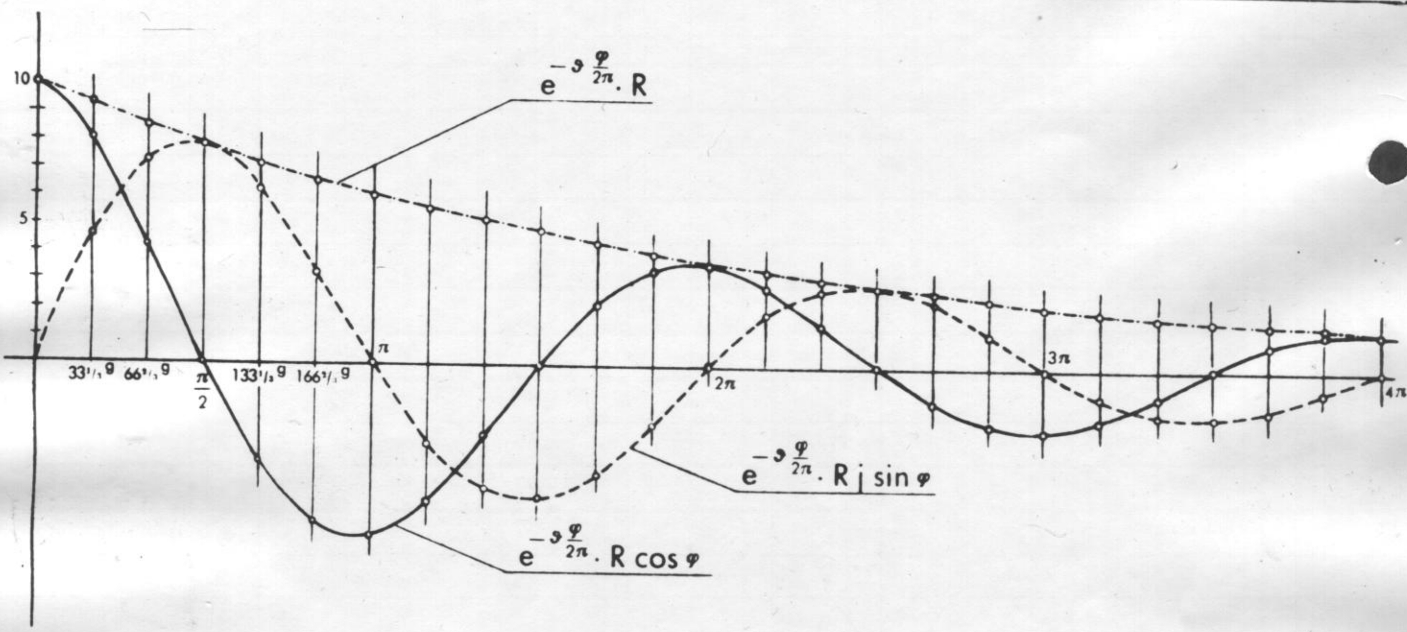
The function takes the course shown in the accompanying table and diagram.

Exponentially Damped Oscillation (where ϑ = 1 and R = 10).

$$Y = e^{-\vartheta \frac{\varphi}{2\pi}} R(\cos\varphi + j\sin\varphi) = e^{-1\frac{\varphi}{2\pi}} 10(\cos\varphi + j\sin\varphi);\ r = e^{-\vartheta\frac{\varphi}{2\pi}} \cdot R;\ x = r\cdot\cos\varphi;\ jy = r\cdot j\sin\varphi$$

Damping decrement $\vartheta = \ln\frac{R}{r_{2\pi}} = \ln\frac{r_{2\pi}}{r_{4\pi}} = \ln\frac{10}{3,68} = \ln\frac{3,68}{1,35} = \ln e = 1$

Versor/ϑ	0	33,3^g	66,6^g	$\frac{\pi}{2}$	133,3^g	166,6^g	π	233,3^g	266,6^g	$\frac{3\pi}{2}$	333,3^g	366,6^g	2π	433,3^g	466,6^g	$\frac{5\pi}{2}$	533,3^g	566,6^g	3π	633,3^g	666,6^g	$\frac{7\pi}{2}$	733,3^g	766,6^g	4π
Vector: r	10	9,25	8,50	7,80	7,20	6,65	6,10	5,60	5.15	4,75	4,40	4,00	3,68	3,40	3,12	2,85	2,63	2,43	2,23	2,05	1,90	1,73	1,60	1,47	1,35
Real Component: x	10	8,00	4,25	0	-3,60	-5,75	-6,10	-4,80	-2,60	0	2,20	3,50	3,68	2,95	1,55	0	-1,32	-2,10	-2,23	-1,78	-0,95	0	0,80	1,28	1,35
Imaginary Component: jy	0	4.60	7,30	7,80	6,25	3,35	0	-2,80	-4,45	-4,75	-3,80	-2,00	0	1,70	2,70	2,85	2,30	1,20	0	-1,03	-1,65	-1,73	-1,40	-0,74	0



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15. Circular and Hyperbolic Functions of Complex Arguments.

The two diagrams (sine relief and tangent relief) provided on the back of the Complex Calculator are used for ascertaining the circular and hyperbolic functions of complex arguments. In this diagram the Gauss Plane of Numbers is subdivided by a rectangular network, shown in red, so that the real part and the imaginary part of the argument can be read off at any point. Superimposed on it is a network of orthogonal curves, shown in black, enabling the function in accordance with the vector value and the versor to be found.

The tangent relief diagram is accompanied by a detailed system of formulae, to enable the hyperbolic functions to be found from the circular functions.

a. Sine of a Complex Number.

If the argument is given with the real and imaginary part, i. e. in the form $x + jy$, then the value s and the versor σ^g of the functional value can be read off direct from the sine relief.

$$\sin z = \sin (x + jy) = s\underline{/\sigma^g} = s \cdot e^{j\sigma^g}$$

Example:

$$\sin\ (0{,}9 + j\ 0{,}46) = 0{,}92\underline{/21^g} = 0{,}92\underline{/19^o}$$
$$\sin\ (-0{,}38 - j\ 0{,}53) = 0{,}67\underline{/144^g} = 0{,}67\underline{/129{,}5^o}$$
$$\sin\ (-0{,}20 - j\ 0{,}90) = 0{,}94\underline{/-118^g} = 0{,}94\underline{/106^o}$$
$$\sin\ (0{,}8 - j\ 0{,}48) = 0{,}88\underline{/-26^g} = 0{,}88\underline{/24^o}$$

For we have:

$$\sin\ (6{,}08 - j\ 0{,}9) = \sin(6{,}08 - 6{,}28 - j\ 0{,}9)$$
$$= \sin(-0{,}2 - j\ 0{,}9) = 0{,}94\underline{/-118^g} = 0{,}94\underline{/-106^o}$$
$$\sin\ (-6{,}66 + j\ 0{,}53) = \sin\ (-6{,}66 + 6{,}28 + j\ 0{,}53)$$
$$= \sin\ (-0{,}38 + j0{,}53) = 0{,}67\underline{/144^g} = 0{,}67\underline{/129{,}5^o}$$

For we have:

$$\sin\ (2{,}24 + j\ 0{,}46) = \sin(3{,}14 - (2{,}24 + j0{,}46))$$
$$= \sin(0{,}9 - j0{,}46) = 0{,}92\underline{/-21^g} = 0{,}92\underline{/-19^o}$$
$$\sin\ (-2{,}34 - j0{,}48) = \sin\ (-3{,}14 - (-2{,}34 - j0{,}48))$$
$$= \sin(-0{,}8 - j0{,}48) = 0{,}88\underline{/174^g} = 0{,}88\underline{/156{,}5^o}$$

b. Sine Value Known -- Argument Required.

If the sine value is given, in the form $a + jb$, it is converted into the form $r\underline{/\varphi^g} - s\underline{/\sigma^g}$, using the front side of the Complex Calculator. With s and , the values x and y, including the relevant sign, can be read off from the sine relief diagram.

Example:

arc sin $0{,}92\underline{/-21^g} = 0{,}92\underline{/-19^o} = 0{,}9 - j\ 046$ or, alternatively, in accordance with $\pi - (x + jy) = 2{,}24 + j\ 0{,}46$ x)

arc sin $0{,}88\underline{/174^g} = 0{,}88\underline{/156{,}5^o} = -0{,}8 + j\ 0{,}48$ or, alternatively, in accordance with $-\pi - (x + jy) = -2{,}34 - j\ 0{,}48$ x)

x) For solutions capable of different interpretations the extended sine relief should be used, for the sake of greater clarity.

c. The Cosine of a Complex Number.

The cosine is converted into the sine in accordance with the following formulae:

$$\cos(u + jv) = \cos(-u - jv) = \sin\left(\frac{\pi}{2} + |u| + j|v|\right)$$

$$\cos(-u + jv) = \cos(u - jv) = \sin\left(\frac{\pi}{2} - |u| + j|v|\right)$$

Example:

$\cos(0{,}67 + j0{,}46) = \sin(1{,}57 + 0{,}67 + j0{,}46) = \sin(2{,}24 + j0{,}46)$
$= 0{,}92\angle\underline{-21^g} = 0{,}92\angle\underline{-19^o}$

$\cos(1{,}83 - j\,0{,}3) = \sin\,1{,}57 - 1{,}83 + j0{,}3) = \sin(-0{,}26 + j\,0{,}3)$
$= 0{,}4\angle\underline{148^g} = 0{,}4\angle\underline{133^o}$

d. Cosine Value Known -- Argument Required.

Proceed as under (b) and insert the resulting values x and y, with their sign, in the following formula:

$$\text{arc}\cos s\angle\underline{\sigma^g} = \left(\frac{\pi}{2} - x\right) - jy$$

Example:

$\text{arc}\cos 0{,}4\angle\underline{148^g}$; $= \text{arc}\cos 0{,}4\angle\underline{133^o}$ with $= 148^g$ and $s =$ 0.4 we obtain a value of -0.26 for x and 0.3 for y from the sine relief.

$\text{arc}\cos 0{,}4\angle\underline{148^g} = (1{,}57 + 0{,}26) - j\,0{,}3 = 1{,}83 - j\,0{,}3$

$\text{arc}\cos 0{,}67\angle\underline{-144^g}$; $(x = -0{,}38$ and $y = -0{,}53)$

$\text{arc}\cos 0{,}67\angle\underline{144^g} = \text{arc}\cos 0{,}67\angle\underline{-129{,}5^o} = (1{,}57 + 0{,}38) + j\,0{,}53$
$= 1{,}95 + j\,0{,}53$

e. The Hyperbolic Sine of a Complex Number.

The hyperbolic sine is transferred to the circular sine in accordance with the following formulae:

$$\mathfrak{Sin}(u + jv) = j\sin(|v| - j|u|); \quad \mathfrak{Sin}(-u - jv) = j\sin(-|v| + j|u|)$$

$$\mathfrak{Sin}(-u + jv) = j\sin(|v| + j|u|); \quad \mathfrak{Sin}(u - jv) = j\sin(-|v| - j|u|)$$

Example:

$\mathfrak{Sin}(0{,}8 - j0{,}22) = j\sin(-0{,}22 - j0{,}8) = j\,0{,}91\angle\underline{-121^g} =$
$j0{,}91\angle\underline{109^o}$; multiplication by j is equivalent to a rotation through an angle of -100^g (or -90^o)

$\mathfrak{Sin}(0{,}8 - j\,0{,}22) = 0{,}91\angle\underline{-21^g} = 0{,}91\angle\underline{-19^o}$

$\mathfrak{Sin}(0{,}35 + j6{,}49) = j\sin(6{,}49 - j0{,}35)$; $6{,}49 - 2\pi = 6{,}49 - 6{,}28 = 0{,}21$;
$= j\sin(0{,}21 - j\,0{,}35) = j\,0{,}42\angle\underline{-62^g} = j0{,}42\angle\underline{-56^o}$
$= 0{,}42\angle\underline{38^g} = 0{,}42\angle\underline{34^o}$

f. Hyperbolic Sine Known -- Argument Required.

According to whether φ' lies in the 1st, 2nd, 3rd or 4th quadrant, it is equated with $(100-\sigma)^g$, $(100-\sigma)^g$ or $(-100-\sigma^g)$ as the case may be, x and y being read off from the sine relief diagram and the argument obtained according to the following formulae:

$$\mathfrak{Ar\,Sin}\; s\,\underline{/(100-\sigma)^g} = y + jx; \quad \mathfrak{Ar\,Sin}\; s\,\underline{/(-100-\sigma)^g} = -y - jx$$
$$\mathfrak{Ar\,Sin}\; s\,\underline{/(100+\sigma)^g} = -y + jx; \quad \mathfrak{Ar\,Sin}\; s\,\underline{/(-100+\sigma)^g} = y - jx$$

Example:

$$\mathfrak{Ar\,Sin}\; 0{,}91\,\underline{/-21^g}; \quad s\,\underline{/(-100+\sigma)^g} = 0{,}91\,\underline{/79^g}; \; x=0{,}22,\; y=0{,}8$$
$$\mathfrak{Ar\,Sin}\; 0{,}91\,\underline{/-21^g} = 0{,}8 - j\,0{,}22$$
$$\mathfrak{Ar\,Sin}\; 0{,}42\,\underline{/38^g}; \quad s\,\underline{/(100-\sigma)^g} = 0{,}42\,\underline{/62^g}; \; x=0{,}22,\; y=0{,}35$$
$$\mathfrak{Ar\,Sin}\; 0{,}42\,\underline{/38^g} = 0{,}35 + j\,0{,}22$$

g. Hyperbolic Cosine of a Complex Number.

The hyperbolic cosine is transferred to the circular sine in accordance with the following formulae:

$$\mathfrak{Cof}(u + jv) = \mathfrak{Cof}(-u - jv) = \sin\left(\frac{\pi}{2} + |v| - j\,|u|\right)$$
$$\mathfrak{Cof}(-u + jv) = \mathfrak{Cof}(u - jv) = \sin\left(\frac{\pi}{2} - |v| - j\,|u|\right)$$

Example:

$$\mathfrak{Cof}(-0{,}42 + j\,0{,}45) = \sin(1{,}57 - 0{,}45 - j\,0{,}42) = \sin(1{,}12 - j\,0{,}42) = 1{,}0\,/-12^g$$
$$\mathfrak{Cof}(-0{,}46 - j\,0{,}67) = \sin(1{,}57 + 0{,}67 - j\,0{,}46) = \sin(2{,}24 - j\,0{,}46) = \sin(\pi - (x + jy)) = \sin(0{,}9 + j\,0{,}46) = 0{,}92\,\underline{/21^g}$$

h. Hyperbolic Cosine Known -- Argument Required.

Proceed as under (b), the resulting values x and y, with their sign, being inserted in the following formula:

$$s\,\underline{/\sigma^g} = y + j\left(\frac{\pi}{2} - x\right)$$

Example:

$1{,}0\,\underline{/-10{,}8^o} = 1{,}0\,\underline{/-12^g}$; 4. Quadrant; x = 1,12, y = -0,42

$1{,}0\,\underline{/-10{,}8^o} = 1{,}0\,\underline{/-12^g} = -0{,}42 + j(1{,}57 - 1{,}12) = -0{,}42 + j\,0{,}45$

or alternatively: x = 2,02; y=0,42 with $\pi - (x + jy)$ and thus:

$1{,}0\,\underline{/-10{,}8^o} = 1{,}0\,\underline{/-12^g} = 0{,}42 + j(1{,}57 - 2{,}02) = 0{,}42 - j\,0{,}45$

$0{,}92\,\underline{/19^o} = 0{,}92\,\underline{/21^g}$; 1. Quadrant; x=0,9; y=0,46 or x = 2,24; y=-0,46

$0{,}92\,\underline{/19^o} = 0{,}92\,\underline{/21^g} = 0{,}46 + j\,0{,}67$ or $= -0{,}46 - j\,0{,}67$

i. Tangent of a Complex Number:

If the argument is presented with the real and imaginary part, i. e. in the form x-jy, then the vector value t and the versor τ' of the functional value can be read off direct from the tangent relief diagram.

$$tg\,z = tg(x + jy) = t\,\underline{/\tau^g} = t \cdot e^{j\tau^g}$$

Example:

$$\mathrm{tg}\,(0{,}36+j0{,}6) = 0{,}64\angle\underline{74^g} = 0{,}64\angle\underline{66{,}5^\circ}$$
$$\mathrm{tg}\,(-0{,}44-j0{,}38) = 0{,}58/-147^g = 0{,}58\angle\underline{-132^\circ}$$

For $x \leq \pm k\pi$: $\mathrm{tg}(x+jy) = \mathrm{tg}(x \mp k\pi + jy) = t\angle\underline{\tau^g}$

$\mathrm{tg}(-9{,}69-j0{,}34) = (-x+k\pi-jy) = (-9{,}69+3\pi-0{,}34j) = (-0{,}26-j0{,}34) = 0{,}42\angle\underline{-138^g}$

For $\pm\frac{\pi}{4} \leq \pm x \leq \pm\frac{\pi}{2}$: $\mathrm{tg}(x-jy) = 1:\mathrm{tg}(\pm\frac{\pi}{2}-(x+jy)) = 1:(t\angle\underline{-\tau^g}) = \frac{1}{t}\angle\underline{\tau^g}$

$\mathrm{tg}(1{,}22-j0{,}86) = 1:\mathrm{tg}(1{,}57-1{,}22+j0{,}86) = 1:\mathrm{tg}(0{,}35+j0{,}86) = 1:(0{,}76\angle\underline{85^g}) = 1{,}32\angle\underline{-8}$

j. Tangent Value Known -- Argument Required.

If the tangent value is given in the form $a+jb$, it is converted into the form $r\angle\underline{\varphi^g} = t\angle\underline{\tau^g}$. With t and τ, the values x and y are ascertained from the tangent relief (including their sign).

Example:

arc tg $0{,}58\angle\underline{-132^\circ}$ = arc tg $0{,}58\angle\underline{-147^g}$ = $-0{,}44 - j0{,}38$ and also
$= -\frac{\pi}{2}$ - arc tg $1{,}72\angle\underline{147^g}$ = $-1{,}13-j0{,}38$ x)
arc tg $1{,}32\angle\underline{-76{,}5^\circ}$ = arc tg $1{,}32\angle\underline{-85^g}$ = $\frac{\pi}{2}$ - arc tg $0{,}76\angle\underline{85^g}$ =
$1{,}22 - j0{,}86$ and also = $0{,}35 - j\,0{,}86$ x)

k. Cotangent of a Complex Number.

The cotangent is converted into the tangent in accordance with the following formula:

$$\mathrm{ctg}\,(u + jv) = 1:\mathrm{tg}(u + jv) = \frac{1}{t}\angle\underline{-\tau^g}$$

Example:

ctg $(0{,}5+j0{,}67)$ = 1 : tg$(0{,}5-j0{,}67)$ = 1:$(0{,}76\angle\underline{72^g})$ = $1{,}32\angle\underline{-72^g}$
= $1{,}32\angle\underline{-65^\circ}$
ctg $(-0{,}38-j0{,}74)$ = 1:tg$(-0{,}38-j0{,}74)$ = 1:$(0{,}72\angle\underline{-120^g})$ =
$1{,}39\angle\underline{120^g}$ = $1{,}39\angle\underline{108^\circ}$

l. Cotangent Value Known -- Argument Required.

The equation arc ctg $\angle\underline{\tau^g}$ = arc tg $\frac{1}{t}\angle\underline{-\tau^g}$ is employed.

Example:

arc ctg $2{,}08\angle\underline{129{,}5^\circ}$ = arc ctg $2{,}08\angle\underline{-144^g}$ = arc tg $\angle\underline{144^g}$
= arc tg $0{,}48\angle\underline{144^g}$ = $-0{,}34 - j\,0{,}35$

arc tg $0{,}76\angle\underline{76{,}5^\circ}$ arc ctg $0{,}76\angle\underline{85^g}$ = arc tg $1{,}32\angle\underline{-85^g}$
= $1{,}22 - j\,0{,}86$

x) For solutions capable of different interpretations the extended tangent relief should be used, for the sake of greater clarity.

m. Hyperbolic Tangent of a Complex Number.

The hyperbolic tangent is converted into the circular tangent in accordance with the following formulae:

$$\mathfrak{Tg}(u+jv) = j\,tg(|v| - j|u|);\quad \mathfrak{Tg}(-u-jv) = j\,tg(-|v| + j|u|)$$
$$\mathfrak{Tg}(-u+jv) = j\,tg(|v| + j|u|);\quad \mathfrak{Tg}(u-jv) = j\,tg(-|v| - j|u|)$$

Example:

$\mathfrak{Tg}(-0{,}64+j0{,}22) = j\ tg(+0{,}22+j0{,}64) = j\ 0{,}60\underline{/84^g}$; multiplication by j amounts to a rotation through an angle of $+100^g$ (or 90^o)

$\mathfrak{Tg}(-0{,}64+j0{,}22) = 0{,}6\underline{/184^g} = 0{,}6\underline{/165{,}5^o}$

$\mathfrak{Tg}(-0{,}54-j0{,}44) = j\ tg\ (-0{,}44 + j\ 0{,}54) = j0{,}66\underline{/134^g} = 0{,}66\underline{/-166^g}$
$\doteq 0{,}66\underline{/-149^o}$

n. Hyperbolic Tangent Known -- Argument Required.

According to whether φ^g lies in the 1st, 2nd, 3rd or 4th quadrant it is equated to $(100-\tau)^g$, $(100+\tau)^g$, $(-100-\tau)^g$ or $(-100+\tau)$ as the case may be, x and y being read off from the tangent relief and the argument being found in accordance with the following formulae:

$$\mathfrak{Ar}\,\mathfrak{Tg}\ t\ \underline{/(100-\tau)^g} = y+jx;\quad \mathfrak{Ar}\,\mathfrak{Tg}\ t\ \underline{/(-100-\tau)^g} = -y-jx$$
$$\mathfrak{Ar}\,\mathfrak{Tg}\ t\ \underline{/(100+\tau)^g} = -y+jx;\quad \mathfrak{Ar}\,\mathfrak{Tg}\ t\ \underline{/(-100+\tau)^g} = y-jx$$

Example:

Ar tanh $0{,}6\underline{/165{,}5^o}$ = Ar tanh $0{,}6\underline{/184^g}$; s $\underline{/100+\tau)^g} = 0{,}6\underline{/84^g}$;
x = 0,22; y = 0,64
Ar tanh $0{,}6\underline{/184^g} = -0{,}64 - j0{,}22$
Ar tanh $0{,}66\underline{/-149^o}$ = Ar tanh $0{,}66\underline{/-166^g}$; s$\underline{/(-100-\tau)^g} = 0{,}66\underline{/-66^g}$;
x = 0,44; y = 0,54
Ar tanh $0{,}66\underline{/-166^g} = -0{,}54 - j0{,}44$

o. Hyperbolic Cotangent of a Complex Number.

The Hyperbolic cotangent is converted into the hyperbolic tangent by means of the following formula:

$$\coth(u+jv) = 1 : \tanh(u+jv) = \frac{1}{jt}\underline{/-\tau^g}$$

Example.

coth (0,76-j0,18) = 1 : tanh(0,76-j0,18) = 1 : jtg(-0,18-j0,76)
= 1 : $(j0{,}66\underline{/-110^g}) = 1{,}52\underline{/110^g}) : j$

Division by j amounts to a rotation through an angle of -100^g (or -90^o)

$\mathfrak{Ctg}\ (0{,}76-j0{,}18) = 1{,}52\ \underline{/10^g}$
$\mathfrak{Ctg}\ (-0{,}18-j0{,}76) = 1 : \mathfrak{Tg}\ (-0{,}18-j0{,}76) = 1 : j tg(-0{,}76+j0{,}18)$
$= 1 : (j0{,}94\underline{/178^g}) = (1{,}06\underline{/-178^g}) : j$
$\mathfrak{Ctg}\ (-0{,}18-j0{,}76) = 1{,}06\ \underline{/122^g}$

p. Hyperbolic Cotangent Known -- Argument Required.

The equation Ar Coth $t\underline{/\tau^g}$ = Ar Tanh $\frac{1}{t}\underline{/-\tau^g}$

is employed.

Example:

Ar Coth $1{,}52\underline{/10^g}$ = Ar Tanh $0{,}66/-10^g$; $t\underline{/(-100+90)^g}$;
x = 0,18; y = 0,76 = 0,76 - j 0,18

Ar Coth $1{,}06\underline{/122^g}$ = Ar Tanh $0{,}94\ \underline{/-122^g}$; $t\underline{/(-100-22)^g}$;
x = 0,76; y = 0,18 = -0,18 - j 0,76

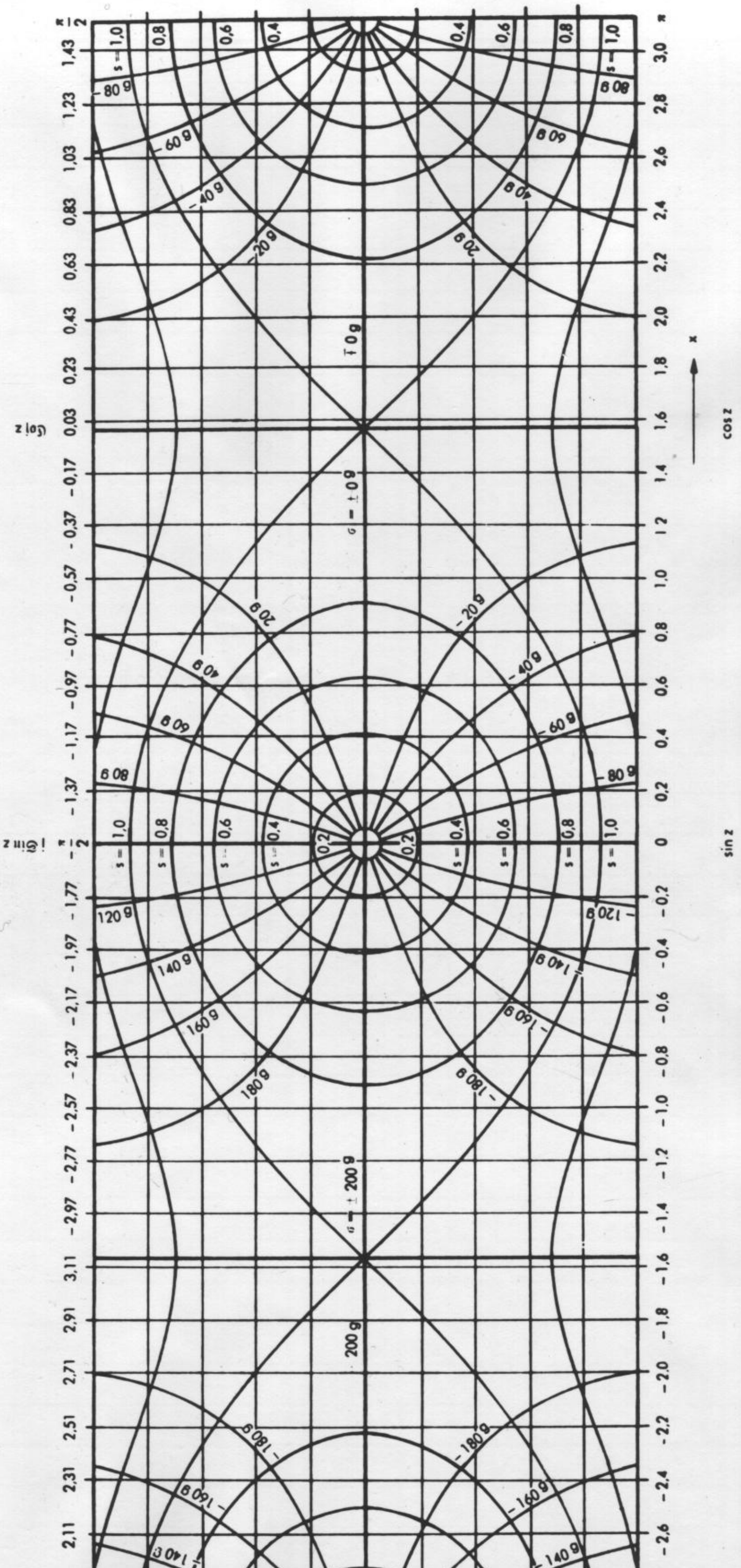
Note:

Pages 22 and 23 contain an extended relief diagram of the sine function and tangent function respectively, to enable errors to be avoided and to provide a view of the possible solutions in cases where there is more than one interpretation.

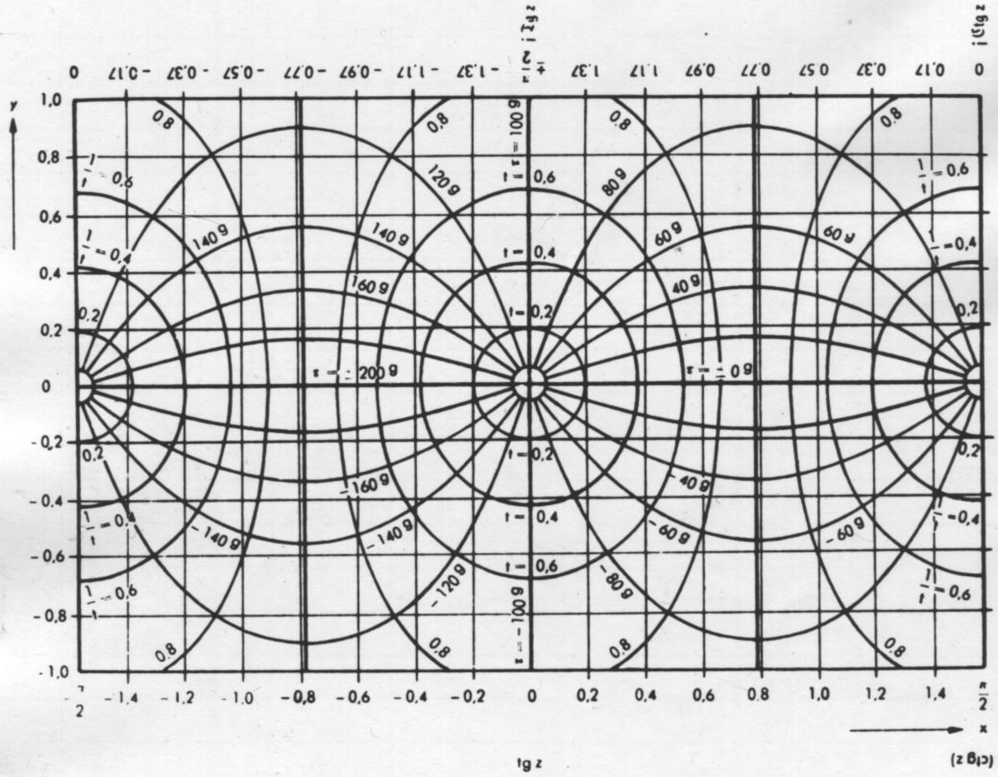
By examining these extended relief charts from different sides, we obtain, according to the zero point selected, the following functions: sin z, cos z, cosh z, j sinh z or tan z, j tanh z, j coth z, + ctg z.

The exact values, however, must be obtained from the relief diagrams on the back of the Complex Calculator.

If users desire to insert or superimpose additional single values or curves on the Complex Calculator, we recommend the use of our special CRISTALLOGRAPH Pencil No. 2241 - obtainable from dealers, in 5 colours.



Sinusrelief: $\sin z = \sin(x + iy) = s\,\underline{/\sigma}$



Tangensrelief: $tg z = tg(x + jy) = t \angle \tau$