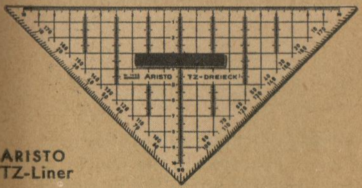
ARISTO



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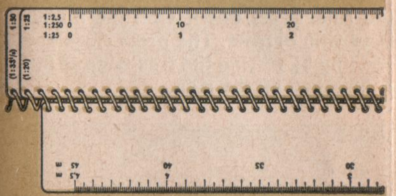
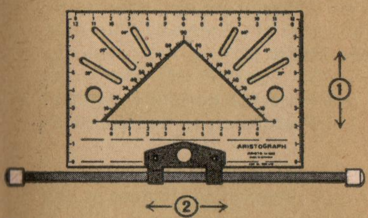


ARISTO Spiral-Scale

ARISTO TRIGON



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DRAFTING EQUIPMENT

ARISTO TZ-LINER

A set square for technical drawing, combining in one instrument scales symmetrical about a centre zero, a parallel ruler and a protractor divided into 360° or 400g.

ARISTO SPIRAL-SCALE

This consists of three 30 cm (12 in.) lengths of white ARISTOPAL, bound together with a plastics spiral. Fifteen scale ratios are displayed by means of multiple figuring on the six divided faces.

ARISTO TRIGON

A full circle protractor, divided to 360° and in radians. For setting out and measurement in either system, or for conversion from one system to the other.

ARISTOGRAPH

A drafting instrument in transparent ARISTOPAL, for quick and neat sketching, embodying a protractor divided to 180° and millimeter scales on the edges. The set square, of sides 85 × 130 mm., can be moved (1) as a parallel ruler on a roller of 200 mm length and (2) be shifted simultaneously, laterally, along the roller parallel to a given line.

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Slide rules · Circular computers
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INSTRUCTIONS
FOR USE

ARISTO

SURVEYOR

0958

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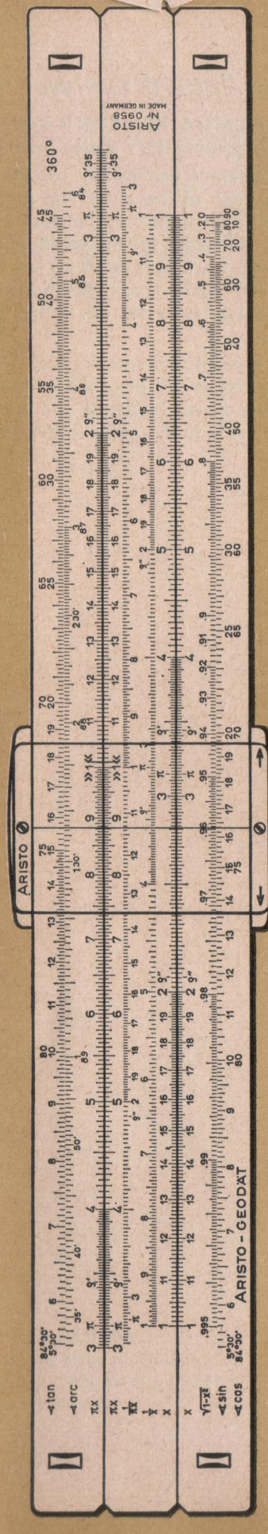

The ARISTO Surveyor Slide Rule 0958

The ARISTO SURVEYOR is a double faced slide rule of modern design, incorporating supplementary scales for the special problems of surveying practice.

For normal day-to-day calculations, all necessary scales are grouped on the front face of the rule. The folded scales permit multiplication, tabulation and error distribution to be performed without traversing the slide. The trigonometrical scales for the solution of commonly occurring problems involving right triangles are arranged on the body of the rule and are available graduated in the 360° system, sexagesimally divided, or the 400^g system, decimally divided, whichever is preferred. In this instruction book, all examples are solved in both systems.

The special tachymetric scales and several auxiliary scales of the well known Rietz system are disposed on the back of the rule. The scales on both faces are in perfect correspondence and the cursor is adjusted so that transfer from one face of the rule to the other in the course of a calculation is freely possible.

Printed in Germany by R. Borek KG 16975

CONTENTS

1. The Scales

Standard Side:

Scale of Tangents figured in black 5° 30′ to 45° or 6^g to 50^g; figured in red, counter-clockwise, 45° to 84° 30′ or 5^g to 94^g. Also available for Cotangents	∢ tan	Upper panel of body
Scale of Small Angles, in radians, 33′ to 6° or 0.6^g to 6.5^g	∢ arc	
Folded scale	πx	
Folded scale	πx	On slide
Reciprocal scale of πx	$1/\pi x$	
Reciprocal scale of x	$1/x$	
Fundamental scale	x	
Fundamental scale	x	Lower panel of body
Pythagoras scale	$\sqrt{1-x^2}$	
Scale of Sines figured in black, 5° 30′ to 90° or 6^g to 100^g, figured in red, counter-clockwise for Cosines 0° to 84° 30′ or 0^g to 94^g	∢ sin ∢ cos	

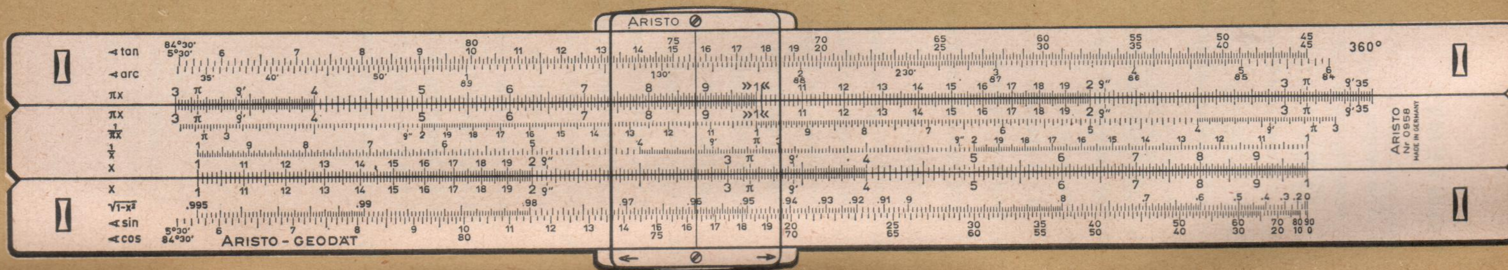

Fig. 1 Standard Side

Tachymetric Side:

Scale, in two sections, for Reduction of Distances measured by horizontal rod and angles of inclination between 15′ and 28° or 25^g and 30^g, referred to scale of x^2	1 — cos	Upper panel of body
Scale of Squares	x^2	
Scale of Squares	x^2	On slide
Scale for Offset Checks	$1/\tan\frac{\alpha}{2}$	
Scale, in two sections, for Differences in Elevation, between 30′ and 45° or 0.6^g and 50^g	sin cos	
Scale for Reduction of Distances measured by vertical staff and with angles between 0° and 45° or 0^g and 50^g	$\cos^2$	
Fundamental Scale	x	Lower panel of body
Mantissa Scale	lg x	
Scale of Cubes	x^3	

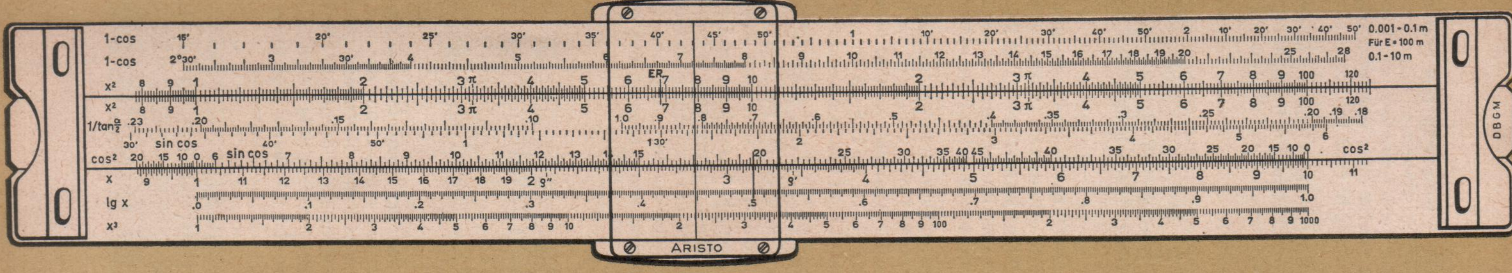

Fig. 2 Tachymetric Side

2. Reading the Scales

To use the slide rule efficiently for rapid calculations is essentially a matter of learning to read the scales easily and correctly.

For guidance in learning to read the scales refer to the figs. 3—6. They show the general pattern of the scales and give examples of several specific settings on the most frequently used fundamental scales, x, on body and slide.

Fig. 3 The Main Intervals

You will notice that there are ten so-called primary intervals, marked by long dividing lines and labeled with the numbers 1—10 in large print (fig. 3). The "10" can be marked "1" on the slide rule, for the reason that the end line of a scale can also be regarded as the first line of an identical scale imaginable as the continuation to the right of the first scale.

Each primary interval is again divided into ten secondary intervals. Between the primary 1 and 2 each of the ten secondary intervals is labeled with somewhat smaller numerals so that here the second digit in a number can be actually read (1—1, 1—2, 1—3 etc.), whereas in the following ranges from 2 to 10 the second digit has to be counted off.

The scale intervals diminish progressively and thus three systems of subdivision must be used for the smallest, i. e., the tertiary intervals, to avoid crowding of the lines in the region toward the end. Therefore all tertiary division lines will only be found between 1 and 2. In this first section of the scale the reading is, therefore, comparable to the reading of a rule with metric graduation, so that all numbers can here be actually read to the third digit. The fourth digits can be easily estimated as the sample settings in fig. 4 demonstrate. Do not overlook the zero when reading the intervals immediately following the labeled marks (see 1007, 1095 in fig. 4).

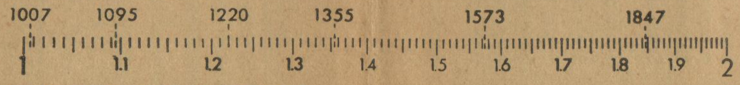

Fig. 4 Reading in the Range 1 to 2

In the next sector, between figured primary intervals 2 and 4, secondary intervals are marked but are not fully figured. Tertiary intervals are marked in units of 2 (fig. 5). Hence, the third digit of even numbers can be read directly from the scale divisions. The third digit of odd numbers must be visually estimated. After a little practical experience you will even be able to estimate the place for the fourth digit fairly accurately.

Fig. 5 Reading in the Range 2 to 4

Between 4 and 10 the scale is kept open by marking tertiary intervals in units of 5. Only numbers, the third digit of which is 0 or 5 can be directly referred to tertiary marks. Third digits other than 0 or 5 must be located by inspection.

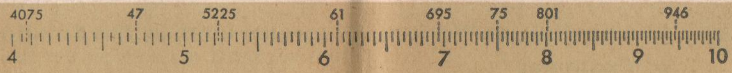

Fig. 6 Reading in the Range 4 to 10

The three systems of subdivision explained above are employed in all other scales and their interpretation will be no problem if you apply to them your knowledge of the fundamental scales. To avoid reading errors it is good policy to pronounce all numbers mentally digit for digit, as for instance, one-two-eight; not one hundred and twenty-eight. The reading gives no information about the decimal point and may signify any decimal variation, such as .128, 1.28, 12.8, 128 etc.

Slide rule results only supply digits in consecutive order. The decimal point is therefore at first entirely ignored and determined by a rough calculation with strongly rounded-off numbers when the computation is completed. This is a check on the order of magnitude of the result as well as an independent check on the correctness of the slide rule manipulation in a broader sense.

2.1 The Slide rule Principle

The calculating procedure and the required manipulations are easy to learn by thinking through and observing how a simple addition can be performed by sliding one ordinary graduated metric rule alongside a second similar rule.

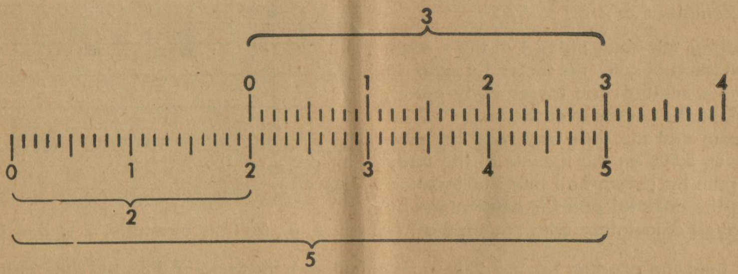

Fig. 7 Graphic Addition by Use of two Ordinary Scales

Fig. 7 shows how a mechanical addition is made with two such scales. When, for instance, the tip 0 of the uppermost scale is moved so as to coincide with the value 2 of the lower scale, we shall find the sum 5 under the value 3 of the upper scale.

Subtraction is the same process in reversed order, i. e. the length 3 of the upper scale is deducted from the total length 5 of the lower scale. It follows that, by simply setting the value 3 over 5, we can read the answer 2 under 0 of the upper scale.

Multiplication and division by use of the slide rule is exactly the same process as that described above, except that we are now dealing with logarithmic lengths, so that by adding or subtracting two segments of line we actually accomplish either a multiplication or a division. In more refined form the above discussed principle of two separate scales is embodied in the slide, movably tongued and grooved to the body of the rule. A cursor is provided to facilitate setting and reading of values to hairbreadth accuracy.

3. Explanation of Working Diagrams used in the Solution of Examples

In the following text an easily memorized method of explanation will be employed, so as to show the step-by-step operations in the respective computation with greater clarity than in the customary form of a facsimile slide rule. Parallel lines bearing their corresponding marginal labels represent the scales and the following symbols will make the diagrams very easy to interpret:

Initial setting
Each subsequent setting
Final result
Setting or reading
of an intermediate result
Reverse the rule
Direction and sequence of movements
Hairline of the cursor

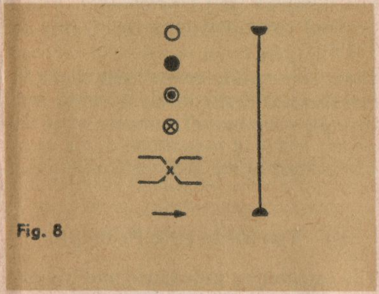

Fig. 8

4. Multiplication

(two logarithmic scale lengths are added).

Example: $1.6 \times 3.7 = 5.92$

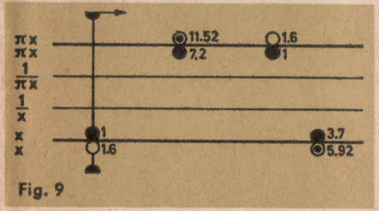

Fig. 9

Set the left hand index 1 of the x scale on the slide over the factor 1.6 read on the x scale of the lower body panel. Move the cursor over the factor 3.7 on scale x of the slide and read the product, 5.92, on the x scale of the body under the cursor hair line. The location of the decimal point is determined by rough calculation with rounded-off figures, e. g., for this example, $2 \times 3 = 6$.

If next the product 1.6×7.2, is to be calculated, the factor 7.2 on the slide scale x will be outside the range of the body scale x. In this event the slide must be "reset", i. e., the right hand index 1 of the slide scale x must be brought over the factor 1.6. Beneath 7.2 on slide scale x will then be the product 11.52 on body scale x.

4.1 Multiplication with the πx scales

The folded scales are identical with the fundamental scales but are laterally displaced by the distance corresponding to $\pi = 3.142$. This results in the value π of the folded scale lying over the begining or end of the fundamental scale, with the value "1" of the folded scale near the middle of the rule. The folded scales may be regarded as extensions of the fundamental scales, providing a means of reducing slide resetting.

The product 1.6×7.2 can be found by the same setting of the slide, but by use of the πx scales, if factor 7.2 is found on slide scale πx and the result 11.52 read over it on the πx scale of the body. Multiplication can with advantage be begun on the folded scales, since no question then arises about the index of the fundamental scale to by used.

Slide scales x and πx are tinted yellow because especially when tabulating, factors on slide scale x are to be set over bady scale x whereas factors on slide scale πx must be set under body scale πx.

5. Division

(two logarithmic scale lengths are subtracted, the reverse of multiplication).

Example: $\frac{47.5}{22.2} = 2.24$

Approximation: $\frac{50}{25} = 2$

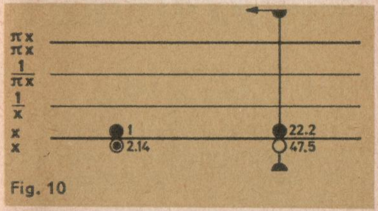

Fig. 10

Usual procedure: Find the numerator 47.5 on the x scale of the body and draw 22.2 on the x scale of the slide into coincidence. The quotient, 2.14 is found on body scale x, under the index (1) of the slide. With other examples, the result is found under the terminal index of the slide (e. g., $47.5 \div 6.05 = 7.85$). The result is, naturally, readable over index 1 of the slide scale πx, in any case.

The same slide setting achieves the multiplication $2.14 \times 22.2 = 47.5$. The difference between multiplication and division lies merely in the order in which the factors are taken. In division, the result is found on the body scale under the initial or terminal index of the slide scale — slide traversing is of no significance. The advantage of this will be exploited again in the following sections.

An advantage of performing division on the πx scales is that the factors are set as in a normal fraction, numerator over denominator.

6. Multiplication and Division Combined

In problems of the type $\frac{a \times b}{c}$ division is usually taken first, followed by multiplication. With several factors in numerator and denominator, divide and multiply alternately.

Example: $\frac{32}{12} \times 27 = 72$

Approximation: $\frac{30}{10} \times 30 = 90$

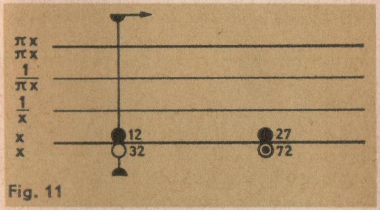

Fig. 11

Following the division of 32 by 12, (fig. 11), the intermediate result 2.66 need not be read, since the rule is set for the final multiplication. The cursor is moved over 27 on the slide scale of x and the result, 72, found on the body scale of x under the cursor hairline.

7. The Reciprocal Scales 1/x and 1/πx

The reciprocal scales are the exact duplicates of the respective fundamental scales, except that their graduations run from right to left and embody red numerals. This means that all values on the fundamental scales x and πx on the slide are permanently coordinated with their reciprocals 1/x and 1/πx. The reciprocity of this scale arrangement has the advantage that a multiplication may be substituted at will by a division, and vice versa, as, for instance:

$$\frac{4}{5} = 4 \times \frac{1}{5} \text{ and } 4 \times 5 = \frac{4}{1/5}$$

The reciprocal scales are mostly used in solving for $a \times b \times c$ or $\frac{a}{b \times c \times d}$

Example: $2.18 \times 42.5 \times 0.85 = 78.7$ Roughly: $2 \times 40 \times 1 = 80$

Procedure: $\frac{2.18}{1/42.5} \times 0.85$

Set 42.5 on the reciprocal scale 1/x over 2.18 on fundamental scale x and then do the multiplication by 0.85 by moving the cursor.

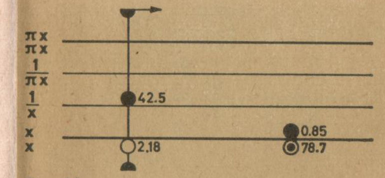

Fig. 12

In this class of computation, too, it is an advantage to be able to cross over to the scales πx and 1/πx. This will often save a resetting of the slide.

8. Proportions

The slide rule is particularly convenient for computations involving proportions of the shape $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ for the reason that with one setting of the given ratio all the required terms can be obtained by simply passing the cursor along the scales. The joint between the scales of the slide and the body can be regarded as the dividing line in a common fraction.

Whenever it is possible to express a problem in the form of a proportion this type of computation should be preferred. So, for instance, the problem in fig. 11 can be easily rearranged to read: $\frac{12}{32} = \frac{27}{72}$. A little practice in this direction will make the slide rule user more independent of the orthodox methods of computation. It really makes no difference whether the given ratio is set $\frac{a}{b}$ or $\frac{b}{a}$ as long as the other ratios are read in conformity with the order of the first setting.

Example:

Conversion of inches to millimeters: How many millimeters are in 3, 5, 7 inches? The initial ratio is known to be 1 in = 25.4 mm. Hence, expressing our problem in proportional form, we write.

$$\frac{1}{25.4} = \frac{3}{x} = \frac{5}{y} = \frac{7}{z}$$

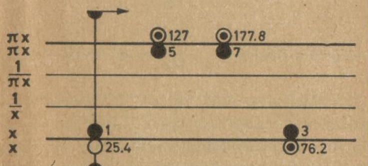

Fig. 13 Ratios and Proportions

Once the given ratio 1 : 25.4 is set on the rule the other terms can be found by merely moving the cursor as required: x = 76.2 mm, y = 127.0 mm and z = 177.8 mm.

8.1 Tabulation

Only one setting of the slide is necessary when many values are required to be multiplied with the same factor. No "resetting" of the slide will be necessary.

Computations of this class are quite frequent in surveying practice, as in the distribution of errors, accounting for paper shrinkage of plans, adjustment of triangulation etc.

Illustrative example of Error Distribution.

It is required to distribute an error of 11.2 cm in a traverse of 243.5 m, proportionately over the segments of the line, the segments being: 34.24, 51.4, 87.25, 125.4, 173.5 m respectively.

After setting 243.5 over 11.2, multiplication is done by successive cursor movements.

$$x = \frac{11.2}{243.5} \times 34.24 = 1.57 \text{ cm.}$$

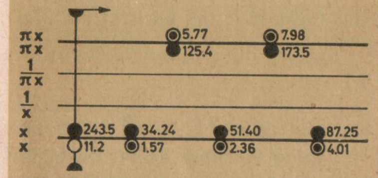

Fig. 14

It is also possible to establish the ratio 243.5/11.2, i. e., to exchange the function of the scales, provided only that any subsequent ratios are read on the appropriate scale. In the calculation of proportions, it is significant to note that the solution of the previous examples is not confined to the method described above. In the course of these instructions the principle of ratio calculation will often be used. The numerator will be set on the slide and the denominator on the body scale. The fraction will then be in the form in which it is usually written, the parting line between slide and body scales corresponding to the division line in a common fraction.

9. Scales of Squares

The problems of multiplication and division so far discussed can also be solved with the scales of squares, exactly as described. The readings obtained, however, will be less accurate.

9.1 Powers and Roots a^2, $\sqrt{a}$, a^3, $\sqrt[3]{a}$, $a^{2/3}$, $a^{3/2}$

By setting the hairline over any value on scale x, its square can be read on scale x^2 and its cube on scale x^3. In the reverse operation the respective square and cube roots are obtained.

Analogously we can find the powers and roots of the angle functions by turning the rule over.

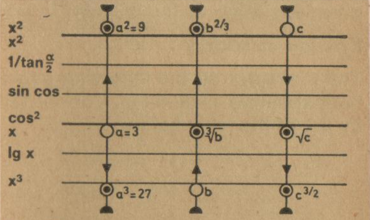

Fig. 15

If a value c is set on scale x^2 we can read $c^{3/2}$ from x^3, the scale of cubes. Conversely, for a value b found on x^3 we can read $b^{2/3}$ from the scale of squares, x^2.

10. Logarithms

Logarithms to base 10 are found by use of scale log x. This scale supplies the mantissas corresponding to the numbers (antilogs) set on scale x. The characteristic is then added as usual. The slide rule is therefore another form of Log Table reading to 3—4 figures and can also be used for powers higher than the cube.

11. The Pythagoras Scale

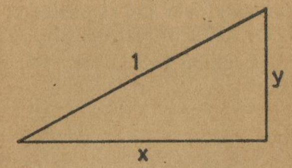

Fig. 16

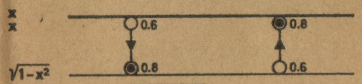

Fig. 17 $y = \sqrt{1-0{,}6^2} = 0{,}8$

The notation for a right-angle triangle with the hypotenuse 1 reads

$y = \sqrt{1-x^2}$ and $x = \sqrt{1-y^2}$

This formula is the basis of the reciprocity between the scales x and $\sqrt{1-x^2}$

Example: $y = \sqrt{1-0.6^2} = 0.8$

The value 0.6 may be set on either scale; the solution will always appear under the cursor hairline on the associated scale. Which scale to prefer for the setting is a matter of deciding where the more accurate reading can be expected. In the example $\sqrt{1-0.15^2} = 0{,}9887$ for instance, it is obvious that 0.15 should be set on x, as this gives us a four-digit result; the advantage over the other form of setting is quite apparent.

Scale $\sqrt{1-x^2}$ runs contrary to the usual order and has red numbers. Its particular usefulness is in the reciprocity of the sine and cosine functions and in trigonometric solutions of right-angled triangles.

$$\cos\alpha = \sqrt{1-\sin^2\alpha} \qquad \sin\alpha = \sqrt{1-\cos^2\alpha}$$

The sine as well as the cosine of an angle can be obtained with one cursor setting.

12. Trigonometrical Functions

The scales of all trigonometrical functions are referred to the fundamental scale x on the body and are divided either in the 360° or the 400^g system. Because all function scales are disposed on the body of the rule, the function values for any given angle can be found under the cursor line on scale x, on scale 1/x or on the Pythagoras scale $\sqrt{1-x^2}$.

The examples on the following pages have been worked out for the 360° and the 400^g circle graduation. Conversions of one system to the other can be effected by use of the table below:

$1^\circ = 1.111^g$	$1^g = 54'$
$1' = 1.852^c$	$1^c = 32.4'$
$1'' = 3.086^{cc}$	$1^{cc} = 0.324''$

The following schedule is a table of comparable trigonometric functions for use in the reduction of angles to the first quadrant.

	$\pm\alpha$	$\left.\begin{matrix}90^\circ\\100^g\end{matrix}\right\}\pm\alpha$	$\left.\begin{matrix}180^\circ\\200^g\end{matrix}\right\}\pm\alpha$	$\left.\begin{matrix}270^\circ\\300^g\end{matrix}\right\}\pm\alpha$
sin	$\pm\sin\alpha$	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$
cos	$+\cos\alpha$	$\mp\sin\alpha$	$-\cos\alpha$	$\pm\sin\alpha$
tan	$\pm\tan\alpha$	$\mp\cot\alpha$	$\pm\tan\alpha$	$\mp\cot\alpha$
cot	$\pm\cot\alpha$	$\mp\tan\alpha$	$\pm\cot\alpha$	$\mp\tan\alpha$

12.1 Sines and Cosines

If an angle is set by cursor line on the scale of sines (∢ sin) then the corresponding sine function is read on scale x. Because $\cos\alpha = \sin(90^\circ-\alpha)$, the scale of sines also provides values of cosines and is thus figured in red, from right to left. All values read on the fundamental scale x, as sine or cosine functions, are less than unity (are prefixed by 0).

The sines of larger angles and the cosines of smaller angles can only be read or set approximately, by the method described above. It is better to find the angle as its co-function and to read the function value, to four or five places of decimals, from the Pythagoras scale, $\sqrt{1-x^2}$.

The colour rule will be helpful:

Sines . . . . . . All settings and readings in like colours.

Cosines . . . Settings and readings in unlike colours.

$\sin 26^\circ 30' = 0.446$

$\sin 73^\circ 15 = \sqrt{1-\cos^2 73^\circ 15'} = 0.9576$

$\sin 133^\circ 24' = \cos 43^\circ 24' = 0.726$

$\text{arc}\sin 0.543 = 32^\circ 53', 147^\circ 07'$, etc.

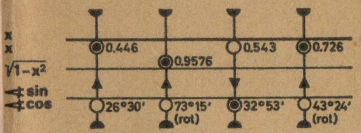

Fig. 18a (rot) = (red)

$\sin 26.50^g = 0.404$

$\sin 80.25^g = \sqrt{1-\cos^2 80.25^g} = 0.9522$

$\sin 139.48^g = \cos 39.78^g = 0.811$

$\text{arc}\sin 0.543 = 36.54^g, 163.46^g$, etc.

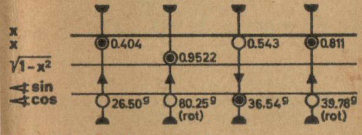

Fig. 18b (rot) = (red)

$\cos 75^\circ = 0.2588$

$\cos 23^\circ 50' = \sqrt{1-\sin^2 23^\circ 50'} = 0.9147$

$\cos 245^\circ 35' = -\cos 65^\circ 35' = -0.413$

$\text{arc}\cos 0.1372 = 82^\circ 07'$

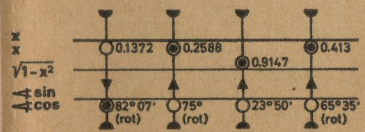

Fig. 19a (rot) = (red)

$\cos 84^g$	$=$	0.2487
$\cos 26.28^g$	$=$	$\sqrt{1 - \sin^2 26.28^g}$
	$=$	0.9160
$\cos 271.50^g$	$=$	$-\cos 71.50^g$
	$=$	-0.433
$\text{arc}\cos 0.1372$	$=$	91.24^g

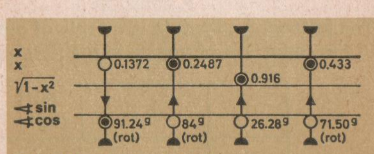

Fig. 19b (rot) = (red)

12.2 Tangents and Cotangents

To find these function values the scale ∢ tan is used in association with scale x and its reciprocal scale, 1/x. This necessitates, as a first step, bringing the initial and final index marks of the x scales on slide and body into correspondence.

The tangent values 0.1 to 1.0, for angles $\alpha < 45°$ will be found in scale x, those between 1.0 and 10, for angles $\alpha > 45°$ in scale 1/x, in agreement with the relationship

$$\tan \alpha = \frac{1}{\tan (90° - \alpha)} .$$

With scales graduated in the 400^g system, this formula becomes:

$$\tan \alpha = \frac{1}{\tan (100^g - \alpha)}$$

Cotangents, using the formula $\cot \alpha = 1/\tan \alpha$, are read as reciprocals of the tangents.

$\tan 14° 20' = 0.2555$

$\tan 67° = \frac{1}{\cot 67°} = 2.356$

$\tan 230° 25' = \tan 50° 25'$

$= \frac{1}{\cot 50° 25'} = 1.210$

arc tan 0.555 = 29° 02′, 209° 02′, etc.

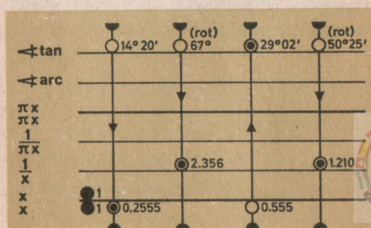

Fig. 20 a (rot) = (red)

$\tan 16.30^g = 0.2618$

$\tan 70^g = \frac{1}{\cot 70^g} = 1.963$

$\tan 255.41^g = \tan 55.41^g$

$= \frac{1}{\cot 55.41^g} = 1.186$

arc tan 0.555 = 32.25^g; 232.25^g, etc.

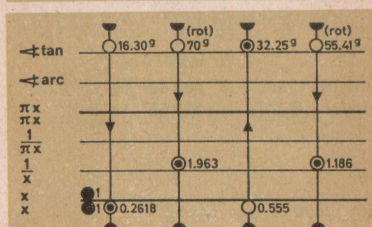

Fig. 20 b (rot) = (red)

For tangent values, set and read in **like** colours.

$\cot 77° = 0.2309$

$\cot (-9°) = -\cot 9°$

$= -\frac{1}{\tan 9°} = -6.31$

$\cot 14° 49' = \frac{1}{\tan 14° 49'} = 3.78$

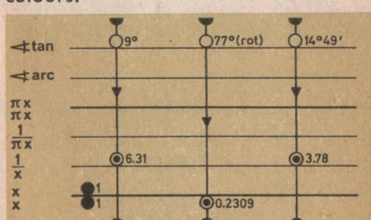

Fig. 21a (rot) = (red)

$\cot 83^g = 0.2736$

$\cot (-11^g) = -\cot 11^g$

$= -\frac{1}{\tan 11^g} = -5.73$

$\cot 37.73^g = \frac{1}{\tan 37.73^g} = 1.485$

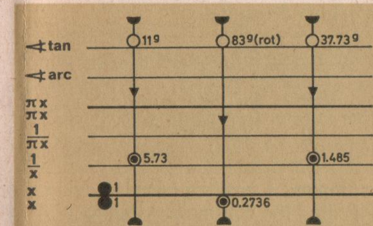

Fig. 21b (rot) = (red)

For values of cotangents, set and read in **unlike** colours.

12.3 Small angles

If $\sin \alpha$ and $\tan \alpha$ for α less than 6.5^g or 5.5°, or for $\cos \alpha$ and $\cot \alpha$ for α greater than 83.5^g or 84° are required, the approximation:

$$\sin \alpha \approx \tan \alpha \approx \cos (90° - \alpha) \approx \cot (90° - \alpha) \approx \text{arc}\, \alpha° = \frac{\pi}{180} \alpha = 0.01745 \alpha$$

$$\text{or } \sin \alpha \approx \tan \alpha \approx \cos (100^g - \alpha) \approx \cot (100^g - \alpha) \approx \text{arc}\, \alpha^g = \frac{\pi}{200} \alpha = 0.01571 \alpha$$

should be used.

The angular scale ∢ arc provides exact values for radian measure and makes possible the simultaneous reading of sine, tangent, and arc functions on the fundamental scale.

The red figures, running from right to left on the ∢ arc scale, provide the corresponding cosine and cotangent values.

There is correspondence once again between angular values and function values found on scale x. It is to be noted, however, that the decimal point is moved one place to the left. The function value for small angles begins 0.0 . . .

$\sin 3° 15' = 0.0566$	$\sin 3.15^g = 0.0495$
$\tan 52' = 0.01513$	$\tan 0.72^g = 0.01131$
$\cot 88° 40' = \tan 1° 20' = 0.0233$	$\cot 98.40^g = \tan 1.60^g = 0.0251$
$\cos 86° 20' = \sin 3° 40' = 0.0640$	$\cos 96.23^g = \sin 3.77^g = 0.0592$

The agreement between the function values for the sine, tangent and arc is very good up to 4°. For larger angles, the precision can at times be increased by applying the formula:

$$\sin \alpha = \frac{\sin 6°}{6} \alpha \qquad \tan \alpha = \frac{\tan 6°}{6} \alpha$$

12.4 The Mark ϱ

Computations involving small angles are often made with the constant ϱ.

$\varrho° = \frac{180}{\pi} = 57.3$ $\qquad \varrho^g = \frac{200}{\pi} = 63.66$

$\varrho' = \frac{180 \times 60}{\pi} = 3{,}438$ $\qquad \varrho^c = \frac{200 \times 100}{\pi} = 6{,}366$

$\varrho'' = \frac{180 \times 60 \times 60}{\pi} = 206{,}265$ $\qquad \varrho^{cc} = \frac{200 \times 100 \times 100}{\pi} = 636{,}600$

The slide rule contains the marks ϱ' and ϱ'' on the scales x, πx, 1/x and 1/πx; in the 400^g graduation the single mark ϱ serves equally for ϱ^g, ϱ^c and ϱ^{cc}.

Working Formulas:

$\alpha = \frac{b}{r} \times \varrho$ for determining the angle

$b = \frac{\alpha \times r}{\varrho}$ for determining the length of arc

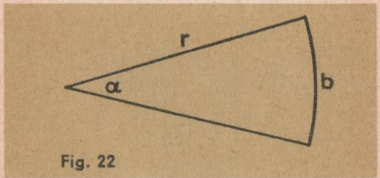

Fig. 22

Examples: A leveling bubble having a sensitivity of 30″ of arc per 2 mm is used in adjusting the collimation line to ± 3″ of precision. Determine the possible error in a rod reading at 50 m distance attributable to 3″ of questionable reliability.

$$\Delta\, b = \frac{3 \times 50{,}000}{\varrho''} = \frac{150{,}000}{\varrho''} = 0.73 \text{ mm}$$

Assuming that, in the course of a traverse, the ranging pole is displaced laterally by 5 cm from the true station point, determine the angular error thereby caused in a line of length 185 m.

(Calculation for the 360° circle graduation).

$$\Delta\, \alpha = \frac{5}{185{,}000} \times \varrho'' = 5.57''$$

$$\text{Roughly: } \frac{5 \times 200{,}000}{200{,}000} = 5$$

(Calculation for the 400^g circle graduation).

$$\Delta\, \alpha = \frac{5}{185{,}000} \times \varrho^{cc} = 17.2^{cc}$$

$$\text{Roughly: } \frac{5 \times 600{,}000}{200{,}000} = 15$$

13. Trigonometrical and Tachymetrical calculations

By committing the following explanatory diagrams to memory you will find much more frequent use for your slide rule in the daily problems of surveying practice.

13.1 Calculations using the Sine Rule

The law of sines is usually stated in the form of the proportion:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2\, r$$

After setting one of the ratios by placing the value for the length of the side on the slide scale x over the sine of the opposite angle on the body scale, the side corresponding to any angle of the triangle or the angle for any side can be read by moving the cursor as required.

This method of solution is of particular importance in the evaluation of right-angled triangles.

13.2 Trigonometrical calculations with right triangles

For right-angled triangles the law of sines can be rearranged to read

$$\frac{c}{1} = \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{a}{\cos \beta} = \frac{b}{\cos \alpha}$$

Furthermore $\tan \alpha = \frac{a}{b}$

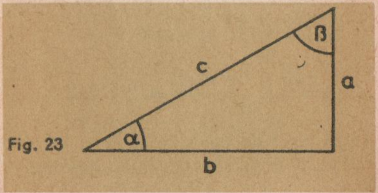

Fig. 23

Fig. 24 shows how the various ratios appear on the slide rule.

Depending on which parts are given, there are two distinctive methods of calculation viz.

1) Given any two parts (except the case 2)
2) Given the two small sides

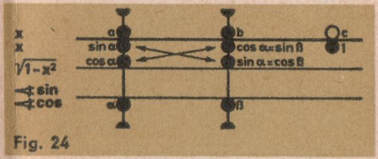

Fig. 24

13.2.1 Example for method A:

Given c = 5, a = 3.

Required: α, β, b

Procedure: Set the hypotenuse c on slide scale x, over either the left or the right index of body scale x. Then use the cursor to set the remaining ratios. Read the value of α on scale ∢ sin under side a and transfer the angle so obtained, with the cursor, to the red figured scale ∢ cos. The value of b is then found by cursor on scale x on the slide. (See fig. 25 a and 25 b.)

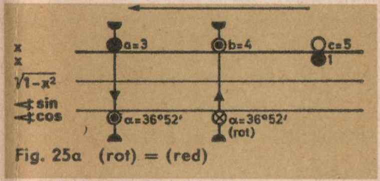

Fig. 25a (rot) = (red)

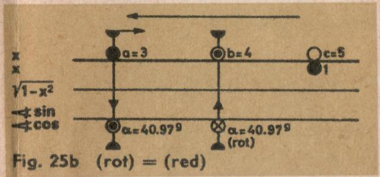

Fig. 25b (rot) = (red)

Fig. 25 c shows another form of solution by use of the scale $\sqrt{1 - x^2}$ whose decimal subdivision sometimes affords closer readings.

When the hypotenuse is unknown computation begins with one ratio of the sine law and c is then found on the slide scale x over the value 1 of the body scale x.

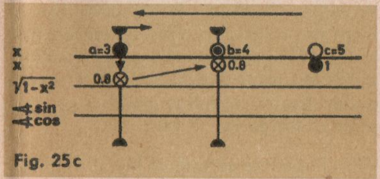

Fig. 25c

13.2.2 Latitude and Departure by Coordinate Differences

The Coordinate Differences

$$\Delta\, x = s \cos \alpha$$
$$\Delta\, y = s \sin \alpha$$

are easily found by the process shown in fig. 26. Following the setting of the hypotenuse, angle α is found successively on the ∢ sin and ∢ cos scales and thence, with the cursor, Δ x and Δ y respectively, read on slide scale x. Traversing of the slide can often be avoided, if the hypotenuse is set on the folded scale πx instead of on scale x and the folded scale used to read the coordinate differences.

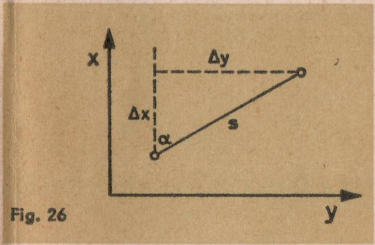

Fig. 26

Example:

Given: s = 150.20 m
α = 30°25′ or 32.24^g

Required: Δ x, Δ y

Result (see fig. 27 a): Δ x = 76.0 m
Δ y = 129.6 m

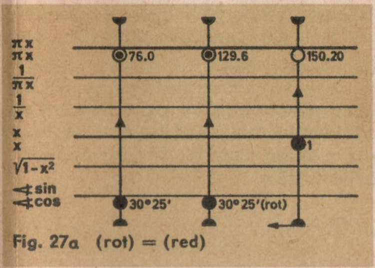

Fig. 27a (rot) = (red)

Result (see fig. 27 b) $\Delta x = 72.9$ m
$\Delta y = 131.4$ m

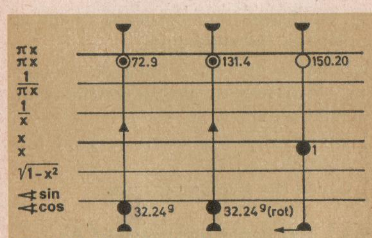

Fig. 27 b (rot) = (red)

13.2.3 Finding the hypotenuse, given the other two sides

Given: $a = 3$, $c = 4$

Required: c, α, β

$$\tan \alpha = \frac{3}{4} = 3 \times \frac{1}{4}$$

$$\sin \alpha = \frac{a}{c} = 3 \times \frac{1}{c}$$

The symmetry of the expressions for $\tan \alpha$ and $\sin \alpha$ permits solution with one setting of the slide.

Working method: First solve for α from $\tan \alpha = 3 \times 1/4$, by setting the index of the slide over 3 on body scale x. Move cursor over 4 on scale 1/x and under the cursor line read α on the scale of tangents, ∢ tan.

With the same setting of the slide, find the angle $\alpha = 36° 52'$ or 40.97^g on the scale of sines and under the cursor line read the hypotenuse, $c = 5$, on the scale of reciprocals 1/x.

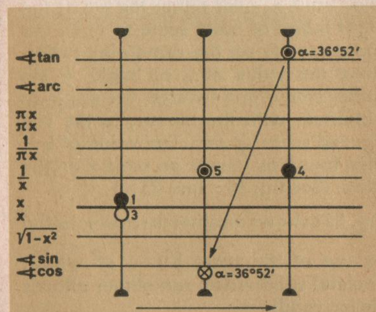

Fig. 28 a

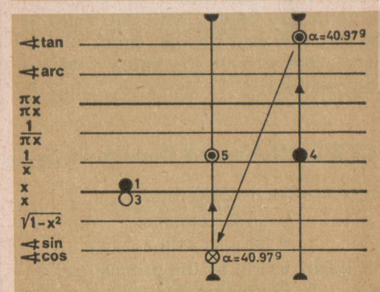

Fig. 28 b

13.2.4 Direction angle and distance by Coordinate Differences

Given: $\Delta x = 85.7$
$\Delta y = 80.3$

Required: α and s.

In this example, slide traversing can be avoided by the use of the $1/\pi x$ scale in place of the scale for 1/x, for the reciprocals of x.

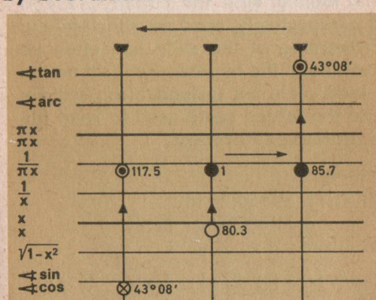

Fig. 29 a $\alpha = 43° 08'$ $s = 117.5$

Results (see fig. 29 a and 29 b):

$\alpha = 43° 08'$ or 47.93^g

$s = 117.5$

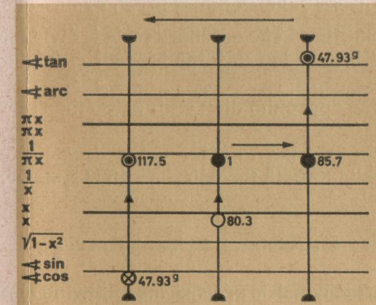

Fig. 29 b $\alpha = 47.93^g$ $s = 117.5$

13.2.5 Offset Checks

The method shown in figs. 28 and 29 suffices for offset checks when the hypotenuse exceeds 20 m in length. If the setting fails to yield the required accuracy, the difference c—b can easily be calculated:

$$\tan \alpha = \frac{a}{b}$$

$$c - b = x = a \tan \alpha/2$$

$$c = b + x$$

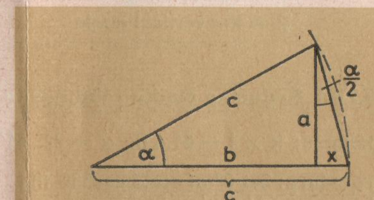

Fig. 30

Slide scale 1/tan $\alpha/2$, on the Tachymetric face of the rule, facilitates the solution of such problems.

Example: $a = 6$, $b = 8$

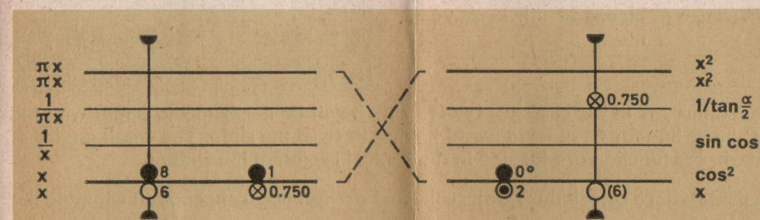

Fig. 31

Procedure: First perform normally the division 6 ÷ 8 reading the quotient 0.750 under the right slide index. Next, without moving the cursor (because side $a = 6$ will be required for the final step), turn the rule over and bring the value 0.750 of scale 1/tan $\alpha/2$ under the hairline, reading the answer $c - b = 2.0$ on scale x under either the 0° line of the $\cos^2$ scale or under the index 1 of the x scale on the slide. The 0° line coincides with the index on the side carrying the standard scales. The length of the hypotenuse is $8 + 2 = 10$.

This method of calculation can also be used for checking traverses, the length s being recalculated from the differences of coordinates.

For $\Delta y = 51.24$ and $\Delta x = 129.28$, the slide rule provides the difference 9.78 m, whence $s = 129.28 + 9.78 = 139.06$ m to the nearest centimeter.

The example of fig. 29 gives, by this method, the more refined value s = 117.44.

Caution! The auxiliary scale is divided as tan $\alpha/2$ but is figured with the function values of tan α. If the tangent < 0.1, the scale tan $\alpha/2$ can no longer be used. The approximation $c - b \approx \frac{a^2}{2\,b}$ then serves.

Because the angular value does not enter into the calculation, this scale is valid in both the 360° and 400^g systems.

13.2.6 Problems in Tachymetry

In tachymetric practice the following class of computations is constantly met with.

$E = k\,l\cos^2\alpha = k\,l - k\,l\sin^2\alpha$ for vertical rod readings
$E = k\,l\cos\alpha = k\,l - k\,l\,(1-\cos\alpha)$ for horizontal rod readings
$\Delta h = k\,l\sin\alpha\cos\alpha$

where in k = multiplication factor (usually 100)
l = rod reading
α = the vertical angle

13.2.6.1 Reducing stadia readings to horizontal

Example: k × l = 81.4 ft., α = 22° 41′ or 25.20^g

Procedure: Set 0° of scale $\cos^2$ to the distance read k × l = 81.4 on scale x. Move cursor to the value of α on scale $\cos^2$ and read below on scale x the horizontal distance E = 69.3 ft. Now shift the cursor to the value of the angle α on the sin cos scale and read the difference in elevation Δh = 28.96 ft. on x.

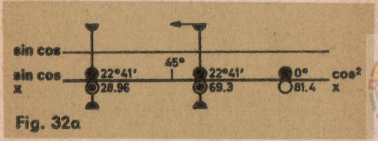

Fig. 32a

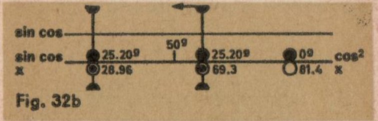

Fig. 32b

Sometimes, as in the case k × l = 115.7 ft., it will be necessary to begin with the left-hand line for 0° so as to avoid having to reset the slide. The small angles of the sin cos function are located in the central region of the slide.

In cases where calculations with scale $\cos^2$ are not accurate enough compute the required correction $k\,l\sin^2\alpha$ in the formula $E = k\,l - k\,l\sin^2\alpha$.

Example: k × l = 215.7 ft. α = 4° 50′ or 5.37^g

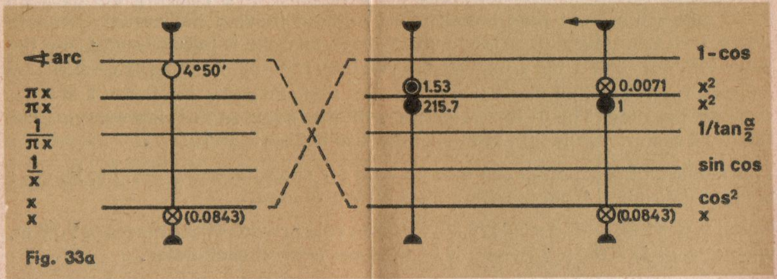

Fig. 33a

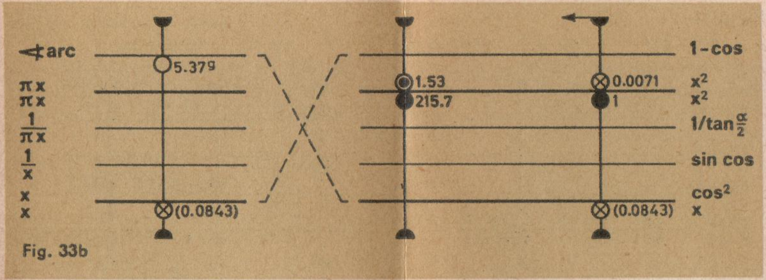

Fig. 33b

Procedure: Set cursor to 4°50′ on scale ∢ arc. Reverse the rule, without moving the cursor and draw the index of slide scale x^2 under the cursor line. The values for sin α (see fig. 33 in parentheses) and $\sin^2\alpha$ need not be read. The multiplication is at once performed with the scale of squares and the result, 1.53, read on the body scale of x^2.
E = 215.7 — 1.5 = 214.2 ft.

13.2.6.2 Reducing distances measured with horizontal rods

For measurements of slant distances made with a double image glass wedge, sighted on a horizontal rod, reductions are made with the two part scale 1 —cos, which is referred to the scale of squares.

Example: k × l = 145.8 ft
α = 8° 25′ or 9.35^g
$E = k\,l - k\,l\,(1-\cos)$

Procedure: Set the cursor to 8° 25′ on scale 1 — cos. Bring the centre index of scale x^2 of the slide under the hairline. Then multiply on the x^2 scales by reading the product, $k\,l\,(1-\cos\alpha)$ on body scale x^2 over 145.8 on slide scale x^2

E = 145.8 — 1.6 = 144.2 ft

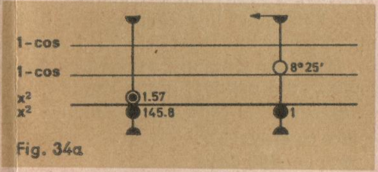

Fig. 34a

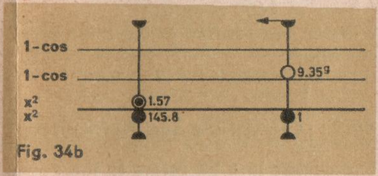

Fig. 34b

13.2.7 Computing the Coefficient of Direction in the adjustment of triangulation

In the adjustment of rectangular coordinates we will constantly need the coefficient.

$$a = -\frac{\varrho''}{10}\,\frac{\sin\alpha\cos\alpha}{\Delta x} \qquad b = \frac{\varrho''}{10}\,\frac{\sin\alpha\cos\alpha}{\Delta y}$$

In these computations scale, sin cos, will be found very helpful provided that the equations are suitable rearranged for the use of the reciprocal, as shown below:

$$a = -\frac{\sin\alpha\cos\alpha}{1/\varrho''}\times\frac{1}{\Delta x}\times 0.1 \qquad b = \frac{\sin\alpha\cos\alpha}{1/\varrho''}\times\frac{1}{\Delta y}\times 0.1$$

Example:

$\alpha = 202° 17'$ $\Delta x = -2498$ (Lat.) $\Delta y = -1024$ (Dep.)

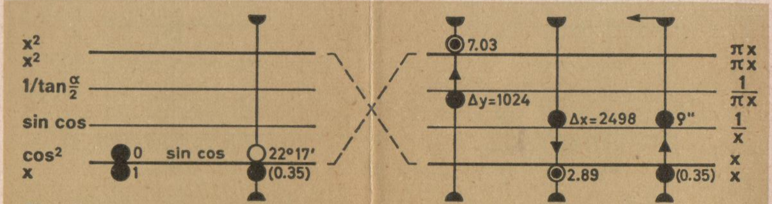

Fig. 35

Procedure: With the slide in "neutral" set 22° 17′ on scale sin cos. Turn the slide rule over, leave the cursor in place and bring ϱ'' of scale 1/x under the hairline. With the slide in this position set first Δx and then Δy on the reciprocal scales 1/x and $1/\pi x$ by use of the cursor. The solutions appear on the body scales x and πx, respectively: $a = + 2.89$, $b = - 7.03$.

The sin cos scale is applicable for all angles because the cofunctions will give again the expression sin cos. These values are positive in the first and third quadrants and negative in the second and fourth quadrants.

14. The Detachable Cursor and its Lines

14.1 The Mark 36

The cursor bears, on its front face, a short line to the right of the main reference line. This short line corresponds to the scale value 36 on scale πx, when the main cursor line is set to index 1 of scale x. With this auxiliary line, multiplication by 36 is conveniently performed by reading, for any scale value selected on scale x, the product of the reading by the factor 36 on scale πx. This is particularly useful in the conversions:

Years to Days:
1 year = 360 days
Hours to Seconds:
1 hour = 3600 seconds
1 meter per second = 3.6 kilometers per hour
Degrees to Seconds: 1° = 3600″

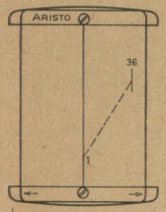

Fig. 36

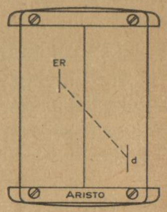

Fig. 37

14.2 Curvature and Refraction

The cursor line ER refers to the scale of squares and serves to determine the correction for earth's curvature and refraction. The distance between the lower right cursor line and the ER line is equivalent to the constant

$$\frac{1-k}{2r^2} = 0.0683.$$

When the distance of the object in km on scale x is located under the right hairline the correction term in metres can be read under the ER line on scale x^2.

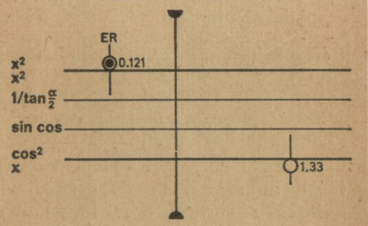

Fig. 38

Example: For E = 1.33 km read 0.121 m under the ER mark.

14.3 Circular Areas

The distance between the lower right and the centre cursor lines is so arranged that circle areas can be determined in the usual way with the scale of squares. Each setting of the diameter under the right cursor line on scale x gives the area under the centre hairline on the scale of squares in accordance with the formula $A = d^2 \frac{\pi}{4}$.

14.4 Detaching the Cursor

The hairlines of the two cursor glasses are precisely matched so that the user can pass from one face of the rule to the other when required in the course of a problem. The accuracy of its adjustment is not disturbed when the cursor is taken off for cleaning.

The cursor glasses are secured, on one face with four screws and on the other with two screws, of special construction. These two screws act similarly to press studs. To remove the cursor from the slide rule, press down with the thumb nails on the ends of the cursor bridge marked with arrows. The press studs are thereby released. The upper press stud is opening by raising the free edge of the cursor glass and the cursor can then easily be taken off the rule.

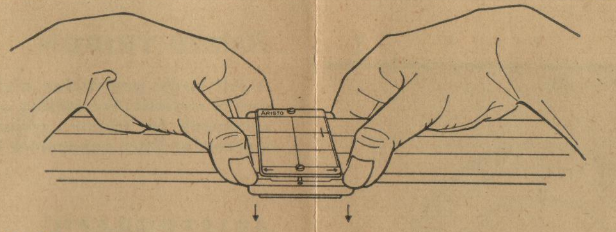

Fig. 39

14.5 Adjustment of the Cursor

Even though the cursor hairs are reliably adjusted, violent jarring of the rule may throw them out of alignment. In such a case loosen the four screws on the cursor face with the ER mark. Turn the slide rule over and shift the other glass until the hairline is accurately aligned to the index lines of the scales. Holding the adjusted glass firmly in position, turn the slide rule over, and adjust the first glass in a similar manner. Tighten all screws carefully to prevent renewed dislocation of the hairlines.

15. Treatment of the ARISTO Slide Rule

The instrument is a valuable calculating aid and deserves careful treatment. Scales and cursor should be protected from dirt and scratches, so that the reading accuracy may not suffer.

It is advisable to give the rule an occasional treatment with the special cleanser fluid DEPAROL followed by a dry polishing. Avoid chemical substances of any description as they are almost certain to spoil the scales.

The slide rule must be protected from plastic erasers and their abrasive dusts, which can damage the surface of the material ARISTOPAL. Avoid also exposure to hot surfaces or bright sunshine, because at temperatures of about 60° C (140° F), distortion occurs. Rules so damaged will not be exchanged free of charge.